



The Importance of Additive Reasoning in Children's Mathematical Achievement:  
A Longitudinal Study

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## **Abstract**

The aim of this thesis is to examine the relative importance of working memory, counting ability, and additive reasoning in children's mathematics learning. One hundred and fifteen 6-year-old Chinese children in Hong Kong participated in two waves of assessments. At the first time point (T1 – first grade), they were assessed using non-verbal intelligence, working memory (central executive, phonological loop, and visuospatial sketchpad), counting ability (procedural counting and conceptual knowledge of counting), additive reasoning (knowledge of the commutativity and complement principles), and mathematical achievement (calculation and story problem solving). Approximately 10 months later (T2 – second grade), children's mathematical achievement in calculation and story problem solving were evaluated once again. The extent to which various cognitive factors longitudinally predicted children's mathematical achievement was evaluated in this study. Several key findings were identified through two sets of analyses – multiple regression models and latent profile analysis.

The multiple regression analyses showed that counting ability accounted for a significant amount of variance in T1 and T2 calculation beyond the effects of age, IQ, and working memory, in which conceptual knowledge of counting, but not procedural counting, was a unique predictor. However, counting ability did not contribute significantly to story problem solving at both time points. When additive reasoning was also included in the regression model, counting ability made a unique contribution to T1 calculation only, but not T2 calculation.

By contrast, additive reasoning and working memory appeared to be more stable and stronger predictors of children's performance in calculation and story problem solving at both time points than counting ability. Additive reasoning explained a substantial and significant amount of variance in calculation and story problem solving at both time points after the effects of age, IQ, working memory, and counting ability were controlled for – Both knowledge of the commutativity and complement principles were unique predictors. Similarly, working memory also accounted for a significant amount of variance in calculation and story problem solving at both time points beyond the influence of age, IQ, counting ability, and additive reasoning. Among the three components of working memory, only the central executive was a unique predictor for all measures of mathematical achievement.

Autoregressive analyses provided strong evidence for the longitudinal predictive powers of additive reasoning and working memory. The analyses showed that both additive reasoning and working memory remained significant predictors of T2 mathematical achievement (calculation and story problem solving) even after the effects of children's previous performance were taken

into account (i.e. T1 mathematical achievement). Overall, additive reasoning accounted for the greatest amount of variance in mathematical achievement both concurrently and longitudinally among all the other factors. This finding underscores the importance of additive reasoning in the teaching and learning of mathematics in young children.

Because additive reasoning (as indicated by the knowledge of the commutativity and complement principles) is a critical variable in this thesis and relatively scarce research has examined this construct, particular concern was paid to the measurement of additive reasoning. It was measured in two ways in the present study: with the support of concrete materials (the concrete condition) and without the support of concrete materials (the abstract condition). Latent profile analysis showed that all children who performed well in the abstract conditions also did well in the concrete conditions, whereas it did not reveal a group of children who performed well in the abstract conditions, but not in the concrete conditions as well. Another interesting finding was that all children who obtained high scores on tasks that assessed their knowledge of the complement principle also obtained high scores in tasks that assessed their understanding of the commutativity principle. The overall pattern of profiles provides initial evidence suggesting that additive reasoning may develop from thinking in the context of specific quantities to thinking about more abstract symbols, and children acquire the knowledge of the commutativity principle in abstract tasks before they start to acquire the knowledge of the complement principle.

This finding demonstrated that patterns of individual differences are present in the development of different aspects of additive reasoning. If teachers possess some knowledge about the particular strengths and weaknesses of each child, it would be easier for them to devise teaching strategies that are tailored to the needs of different children, which may relate to the developmental order of the commutativity and complement principles, and the role of concrete materials in this development. Thus, this study contributes to the literature by showing that assessing additive reasoning in different ways and identifying profiles with classification analyses may be useful for educators to understand more about the developmental stage where each child is placed. It appears that a more fine-grained assessment of additive reasoning can be achieved by incorporating both concrete materials and relatively abstract symbols in the assessment.

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## **Chapter One    Introduction**

The purpose of this thesis is to investigate the relative importance of working memory, counting ability, and additive reasoning in children's mathematical achievement. Mathematical achievement has an influence on individuals' performance in college and choice of careers (National Mathematics Advisory Panel, 2008). The mathematical skills and knowledge at an early age have been shown to predict mathematical achievement test scores in both primary and high schools (e.g., Jordan, Kaplan, Ramineni, & Locuniak, 2009; Locuniak & Jordan, 2008; Mazzocco & Thompson, 2005). Thus, providing children with a strong foundation of mathematical competence is important for success in school and beyond.

In my thesis, I begin with this chapter by asking what it means to be mathematically competent. The definition of 'mathematical competence' is important because it affects what kinds of mathematical skills will be examined in this study. The first section of this chapter describes and analyses two theoretical perspectives on children's mathematics learning and lays the foundation from which I identify factors that may form the basis of my definition of 'mathematical competence'. The second section discusses various potential pillars of mathematical competence that lead to the hypotheses of this study. The third section contains the definitions of key terms in this thesis. The fourth section turns to the general research design that I used to address the hypotheses. The final section illustrates the proposed contents of each chapter, explaining how they link together and develop the thesis.

### **1.1    Defining Mathematical Competence**

What does it mean to be competent in mathematics? 'Competence' indicates sufficiency of knowledge and skills that enable a person to act in a wide variety of situations. To illustrate, if a person is said to have competence in a particular language, she or he should be able to understand and interpret oral narratives and written texts in that language. She or he should also be able to express her- or himself in speech and in writing. Also, a person who is competent in a language can read, write, listen, and speak about different things and in different ways in that language. By contrast, a person who can only listen and speak in a language about certain topics is not competent enough. This analogy with linguistic competence can be an inspiration to answering the following question: What are the characteristics of a person who can deal with a wide range of situations that involves mathematical thinking successfully? 'Mathematical competence' is the term that I have chosen to denote this collective and complex entity. In this chapter, I start from reviewing two different theoretical perspectives on children's mathematics



learning. Proponents of different approaches have a different focus about the characteristics of a young child who is mathematically competent. The task of the following section is to reflect on and theoretically analyse different perspectives as the point of departure of my work, from which I identify the essential pillars that form the basis of mathematical competence to be examined in this study.

### **1.1.1 Number Sense Perspective**

‘Number sense’ has been considered as an inborn characteristic of children that forms the foundation for mathematics learning (e.g., Gelman & Butterworth, 2005; Gelman & Gallistel, 1978; Dehaene, 1997; Dehaene & Cohen, 1995; Siegler & Booth, 2005). A review of the literature (Siegler & Booth, 2005) suggests one definition of number sense – ‘a process of translating between alternative quantitative representations.’ The translations can be between the representations of spatial and numerical information (e.g., ‘About how many feet wide is this classroom?’), the representations of temporal and tactile information (e.g., ‘tap your finger once every 5 seconds’), and so on. It has been argued that an accurate estimation of numerical magnitudes is the basis for children to learn mathematics. There are different theories about number sense, but space precludes a full discussion of each theory here. In this chapter, I have chosen one prominent theory of number sense proposed by Stanislas Dehaene to make a case in point (Dehaene, 1997; Dehaene & Cohen, 1995; see also Dehaene, Molko, Cohen, & Wilson, 2004; Dehaene, Piazza, Pinel, & Cohen, 2005).

#### **1.1.1.1 Dehaene’s triple-code theory**

Dehaene (1997) argues that the foundations of arithmetic rest upon our ability to represent and manipulate quantities on a mental ‘number line’ and he believes that this representation has a long evolutionary history and a specific brain substrate. His ‘triple-code model’ postulates three fundamental hypotheses. First, numerical information can be manipulated in three different formats in our mind: (1) an analogical representation of quantities – numbers are represented as activation patterns on a mental number line; (2) a verbal representation – numbers are represented as strings of words (e.g., fifty-three); (3) a visual Arabic representation – numbers are represented as a string of digits (e.g., 53). Second, there are some ‘transcoding procedures’ that help us translate one code directly to the other. Third, we rely on a fixed set of ‘input codes’ and ‘output codes’ to calculate. For example, Dehaene suggests that we compare different numbers by coding them as quantities on the number line. He also argues that we

memorise multiplication tables as verbal associations between numbers that are represented as string of words, and we perform multi-digit operations with the visual Arabic code.

#### **1.1.1.2 Characteristics of the mental number line**

The core of number sense seems to be the presence of a mental number line, which is based on the hypothesis that numbers are arranged spatially on a continuum (from left to right for cultures using left-to-right orthographies) (Siegler & Booth, 2005). One crucial characteristics of the number line in young children is that it is a fuzzy representation of quantities and it takes time for children to develop the mental number line perfectly. The form of mental number line representations has been measured with a number line estimation task. In this task, participants are presented with some lines with a number at each end (e.g., 0 and 10, 0 and 100, 0 and 1000). They are asked to estimate the location of a third number (e.g., 42) on the line. This task is thought to be measuring the mental number line because it parallels the ratio characteristics of the number system – 80 is four times greater than 20, so the distance of the estimated position of 80 from 0 should be four times greater than the distance of the estimated position of 20 from 0. Likewise, the distance between 0 and 20 should be the same as the distance between 20 and 40, 80 and 100, 140 and 160, and so on, which gives a perfect linear function of estimates when they are plotted against the correct place on the number line.

Although this kind of numerical estimation is not difficult for most adults, it takes some time for young children to grasp. It has been suggested (e.g., Siegler & Booth, 2004; Siegler & Opfer, 2003) that young children have difficulties in representing the magnitudes accurately but this improves with age. Young children are bad at estimating the position of small and large numbers – They overestimate small numbers and underestimate larger numbers, which gives a logarithmic description of their estimates when they are plotted against the correct place on the number line. For example, kindergartners at the age of 5 and 6 were found to exhibit a clear logarithmic pattern of number representation (Siegler & Booth, 2004). It was also indicated that half of the first graders at the age of 6 and 7 showed logarithmic patterns, whereas the other half fit a linear pattern. Second graders at the age of 7 and 8 revealed numerical representations that were best fit by a linear function. Similar developmental patterns have been demonstrated with other types of numerical magnitude estimation, such as numerosity estimation (e.g., ‘Here is a cup with one dot and here is a cup with 1000 dots. Put about N dots in this empty cup on the screen’) and measurement estimation (e.g., ‘Here is a line 1 zip long and here is a line 1000 zips long. Draw a line N zips long.’).

### **1.1.1.3 Numerical magnitude estimation and calculation**

Why might numerical magnitude estimation be related to computational proficiency? Some researchers suggest that when a person solves an arithmetic problem, the rote verbal representation of the answer to the problem and an approximate representation of the answer's magnitude will be activated (Ansari, 2008; Hanich, Jordan, Kaplan, & Dick, 2001). If an approximate representation has more activation strength concentrated on the right answer and the numbers around it, the person is more likely to retrieve the correct answer. An accurate magnitude representation also allows for the rejection of implausible answers and recalculation in cases when a person has retrieved implausible answers. In contrast, approximate representations in which activation strength is more widely distributed among different numbers are likely to lead to the retrieval of wrong answers.

An accurate representation of numerical magnitudes may be related to the development of a variety of computational estimation strategies (e.g., Dowker, Flood, Griffiths, Harriss, & Hook, 1996). Examples of these strategies are (1) 'rounding' – converting one or both operands to the nearest number than ends in a zero (e.g., on  $197 \times 196$ , both multiplicands are converted to 200), (2) 'translation' – simplifying an equation (e.g., on  $44 + 53 + 51 + 47$ , multiplying  $50 \times 4$ ) and (3) 'prior compensation' – rounding the second operand in the opposite direction of the first before calculation (e.g., on  $197 \times 196$ , 196 can be rounded to 190 rather than 200 to compensate the effect of rounding 197 to 200). Thus, according to this view, the ability to represent magnitude accurately may help children retrieve correct answers to novel addition problems (Booth & Siegler, 2008; Siegler & Ramani, 2009). It was found that children whose number line estimates better fit a linear function had better performance on a range of other numerical tasks, such as magnitude comparison, memory for numbers, calculation, and standardised mathematical achievement tests (e.g., Booth & Siegler, 2006; Geary, 2011; Geary, Hoard, Byrd-Craven, Nugent & Numtee, 2007).

### **1.1.1.4 Educational implications of the number sense perspective**

Given the importance of numerical estimation in calculation, proponents of the number sense perspective suggest that one strategy to help children learn mathematics is to improve their number sense through helping them generate a mental number line that relates numbers to nonverbal numerical representations more precisely and accurately. One such activity is playing linear number board games (Ramani & Siegler, 2008), which provide multiple cues to numerical magnitudes. It is suggested that a child who plays these games would gradually

recognise that the greater the number in a square, the greater (1) the distance she or he has moved a token, (2) the number of moves that she or he has made, (3) the number of words that she or he has said and heard, and (4) the amount of time passed. These cues are said to be effective in helping children represent and discriminate numerical magnitudes. According to the National Mathematics Advisory Panel (2008) of the United States, a strong sense of number, which is defined as “the ability to estimate the results of computations and thereby to estimate orders of magnitude” (p.17-18), is crucial in children’s mathematics learning. The panels have suggested in the report that teachers should “broaden instruction in computational estimation beyond rounding” and that textbooks should “explicitly explain that the purpose of estimation is to produce an appropriate approximation.” (p.27)

#### **1.1.1.5 Evaluation of the number sense perspective**

In summary, number sense is one approach that I may reference for the definition of mathematical competence in my thesis. It is one essential domain of mathematical competence in young children because it may foster the growth of computational facility. However, the definition of mathematical competence within this theoretical framework is limited in several ways. First, according to this perspective, children are born with a fuzzy representation of quantities and the representations become more accurate with age. Perceived numerosity is hypothesised to provide children with the foundation for understanding number words. However, every number has its exact meaning that is not simply an estimation (Sarnecka & Gelman, 2004). For instance, even very young children understand that if they add one object to eight objects, they will no longer have eight objects. It is obvious to them that ‘8’ is not the same as ‘approximately 8’. This theory does not explain how young children, from the starting point of an imprecise analogue representation, suddenly come to understand the precise meanings of number.

The number sense perspective does not define what a number is, but it seems uncontroversial that every number has its own precise meanings. My view is that how we conceptualise ‘number’ is important in mathematics education because it affects the approach that we use to teach mathematics to children. I will come back to what may constitute the ‘meanings of number’ from another perspective shortly in this chapter. At the moment, I consider that the number sense perspective does not offer a good account of the concept of ‘number’, which should be precise and fundamental to mathematics learning.

Second, although the number sense perspective may explain variation in computational proficiency (e.g., Booth & Siegler, 2006; Geary, 2011; Geary, Hoard, Byrd-Craven, Nugent & Numtee, 2007), it is difficult to understand from this perspective how magnitude estimation allows us to solve problems in a variety of mathematical situations. Consider additive reasoning, three different kinds of situations that involve part-whole relations have been identified (Carpenter, Hiebert, & Moser, 1981; De Corte, & Verschaffel, 1985; 1987; Ginsburg, 1982; Hudson, 1982; Nesher, 1982; Stern, 1993; Svenson, & Broquist, 1975; Vergnaud, 1979; 1982). The first kind of situations involves transformations. For example, 'William had 13 cookies, he gave 3 away; how many does he have now?' The transformation can be additive or subtractive and the unknown in the problem can be the initial quantity, the transformation, or the final quantity. Thus, six different types of problems can result from situations that involve transformations, by combining the position for the unknown and each type of transformation.

The second kind of situations involves composition of two quantities. For instance, Becky has 3 blue and 5 black marbles. How many marbles does she have?' There are only two possible types of problem in this type of situation, either the total is missing or one of the parts is missing. The third kind of situation involves comparing relations. For example, 'Matt has 4 pencils and Andrew has 6 pencils. How many more pencils does Matt have than Andrew?' The unknown in comparison situations can be the comparative relation (2 in this case), the 'unknown reference set' ('Susie 6 flowers. She has 2 flowers more than Alex. How many flowers does Alex have?') or the 'unknown compare set' ('Terry has 6 books, he has 2 more books than Winnie; how many books does Winnie have?').

Research that has examined children's performance on these types of problems (e.g., (Carpenter, Hiebert, & Moser, 1981; Verschaffel, 1994) has shown findings that challenge the number sense perspective. First, children's accuracy rates for problems that require the same calculation are different. For example, in transformation situations, problems with the final quantity unknown are significantly easier than those in which the initial quantity is unknown. Second, when the situation involves the composition of two quantities, finding the whole is significantly easier than finding a part. Third, reference set problems are significantly more difficult than other types of problems that involve comparisons even when the quantities in the problems are the same. In these studies, the demands for arithmetic computation (e.g., quantities and types of calculation) are controlled for while the quantitative reasoning demand vary. Thus, the differences in the rates of correct responses are not likely due to individual

differences in estimating numerical magnitudes, but in their ability to reason about the relations of quantities in the problems.

In order to reason about the relations of quantities, it is not necessary to process the numerosity – a person can reason about differences between quantities without knowing the actual quantities. For example, if I know that David is 5 cm taller than Peter, and Peter is 2 cm taller than Sam, there is no doubt that David is 7 cm taller than Sam, although we do not know how tall each person is. Therefore, perceiving differences between quantities is distinct from reasoning about relations between quantities. The number sense perspective is limited in a sense that it touches upon quantities only, but it does not entertain the idea that children need to understand relations between quantities in order to choose relevant arithmetic operations to solve a variety of problems. Again, I will come back to talk about reasoning shortly in this chapter from another perspective. In brief, the number sense perspective does not provide a basis for us to understand how people solve mathematical problems in different situations.

In summary, the number sense perspective may be useful for explaining children's growth in computational proficiency. The ability to estimate numerical magnitudes may contribute to arithmetic competence through the development of a variety of computational estimation strategies. However, this theoretical framework suffers several limitations. First, it is not clear how the link between an imprecise analogue system and a precise system of numbers can be forged. It does not give a precise conceptualisation of the meanings of number. Second, it cannot explain how we can base on numerical estimation to solve mathematical problems in a variety of situations. Therefore, it appears that we need to turn to another theoretical approach to search for a better definition of mathematical competence.

### **1.1.2 Mathematical Thinking Perspective**

The second view of children's mathematics learning, which I term as 'mathematical thinking perspective', focuses on how children think about mathematics logically (e.g., Bryant, 1995; Carpenter & Moser, 1982; Ginsburg, Klein, & Starkey, 1998; Nunes & Bryant, 1996, 2015; Nunes, Bryant, Barros, & Sylva, 2012; Piaget, 1952; Piaget & Inhelder, 1975; Thompson, 1993, 1994; Vergnaud, 1997, 2009). Mathematical thinking involves the understanding of the meanings of number. The development of mathematical thinking is to some extent similar to language learning. In order to progress in mathematical thinking, children need to learn mathematical symbols and their meanings and to connect them sensibly, just as one has to combine words

sensibly in sentences. Quantitative reasoning involves using numbers to represent quantities and relations between quantities as well as operating on the numbers to reach conclusions about the quantities (Thompson, 1993). Thus, one core intellectual demand to understand the meanings of number is the need to understand relations between quantities, rather than merely understanding things in isolation. The subsequent discussions explain what constitutes a true understanding of number from the mathematical thinking perspective and illustrate with some examples how this understanding contributes to the success in calculation and solving mathematical problems in various situations.

#### **1.1.2.1 The nature of understanding number**

In the context of mathematical thinking, in order to say that a child has an understanding of number, we would expect some demonstration that the child understands the relational meanings of number. For example, Jean Piaget (1952) pioneered this view when he argued that we need to examine whether children understand the equivalence between sets in order to credit children with an understanding of cardinality.

Suppose Mary has 5 sweets and she exchanges with Annie one sweet that she has for 1 sticker. If Mary understands cardinality, she should know that, by the end of this exchange, she would have 5 stickers without having to count. If Mary is able to count the sweets and say there are 5, but she does not know that how many stickers she has after sharing on a one-to-one basis, according to the mathematical thinking perspective, we can only say that Mary can count, but we cannot say that she understands cardinality. In short, Piaget considers cardinality as the number that relates one set of objects to other sets. If there are 5 objects in this set, then it has the same quantity as any other set with 5 objects.

Piaget based his claim on his observations that some young children did not have an understanding of one-to-one correspondence even though they count well. He found that the children often made mistakes when they were shown one set of items (e.g., eggs) and were asked to make another set (e.g., eggcup) of the same number. Children of 4 and 5 years of age often match the new set with the old one on the basis of irrelevant criteria, such as lengths of the rows, and did not make any effort to put the items into one-to-one correspondence. Although their ability to establish one-to-one correspondence between sets emerges over time, it cannot be assumed in every child who counts well.

Another crucial aspect of the nature of understanding number, which plays an important part in Piaget's theory, is logical inferences. All quantities (e.g., number, height, temperature) can be arranged in a particular order from smaller to larger. In order to grasp the nature of this order, we have to master a fundamental logical rule called transitivity. If quantity A is greater than quantity B, and B is greater than quantity C, then it follows that A must also be greater than C. Some children may only know that 3 is more than 2 and 2 is more than 1, but they cannot work out the relation between 3 and 1 which they cannot directly compare. According to the mathematical thinking perspective, these children are demonstrating an incomplete understanding of the relations between different numbers. This aspect of number knowledge is known as the ordinal concept of number.

The cardinal and ordinal concepts of number are requirements for the most basic mathematical activity of all – counting. However, Piaget's list of logical requirements goes further than this. He contends that all mathematical procedures have their own logical demands. For example, soon after children have learned to count, they start to learn addition and subtraction, and then multiplication and division later at school. Proponents of the mathematical thinking perspective argue that it is important for children to learn about the connections between these operations.

One obvious connection is inversion. This is the idea that each operation has its converse. For example, the inverse relation to addition is subtraction, and vice versa; the inverse of multiplication is division, and vice versa. The understanding of the inversion principle is a fundamental aspect of learning about number. Piaget (1952) argues that it is not possible to grasp the 'additive composition of number' without understanding the inversion principle. Additive composition of number refers to the fact that numbers are made up of other numbers. For example, 9 consists of 4 and 5 or 6 and 3, and it follows that if you subtract 6 from 9, you will be left with 3. Piaget argues that it is not sufficient for children to know or be able to calculate that  $5 + 3 = 8$  and that  $8 - 3 = 5$ , they must also realise why each of these relations automatically follows from the other. According to this view, numbers are not simply a series of words in a constant order, but they also reflect the part-whole logic of the number system – each number word encompasses the previous ones additively (8 means  $7 + 1$ ,  $6 + 2$ ,  $5 + 3$  etc.). Nunes and Bryant (2015) call this idea the 'analytical meanings of number' because the meaning is given by definitions within a number system.



These particular examples make the point that, according to the mathematical thinking perspective, children need to grasp certain logical principles in order to do well in mathematics. Examples of the relational meaning of number involve the cardinal and ordinal concept of numbers and the inversion principles. It is also reasonable to suggest that understanding the additive composition of number may contribute to the development of a more accurate estimation of numerical magnitudes on a number line. My view is that once children have understood the additive composition of number, it would be easier for them to represent the relative magnitudes of quantities and numbers. In other words, understanding relations may actually support the development of numerical magnitude representation. Therefore, compared with the number sense account, the mathematical thinking perspective appears to give a more comprehensive, precise, and parsimonious model that captures the fundamental concept of number.

### **1.1.2.2 Mathematical thinking and computational proficiency**

Why is grasping the nature of number important in learning mathematics? One possible reason is that an understanding of the analytical meaning of numbers contributes to the success in calculation. According to the mathematical thinking perspective, arithmetic is the study and use of relations between numbers to come to conclusions and this is always carried out using a number system, which has specific characteristics. One characteristic is the inverse relation between addition and subtraction.

The inversion principle may underlie the understanding of the exchanges in addition and subtraction of multi-digit numbers. Some researchers (e.g., Fuson, 1990; Nunes & Bryant, 1996) have suggested that understanding carrying and borrowing demands the knowledge of the inverse relation between addition and subtraction. For example, Fuson argued that, when we are adding 8 tens and 7 tens, in order to understand the ‘ten-for-one to the left exchange’, we have to recognise that we are taking 100 away from the tens place and adding 100 to the hundreds place, so that the total value is not changed. We also need similar reasoning to subtract 63 from 1657 – in order to understand the conservation of the minuend, we need to understand that taking away 100 from the hundreds place and adding 100 in the form of 10 tens to the tens place does not alter the quantity.

An understanding of the inversion principle may also contribute to the use of a computational strategy called ‘indirect addition’ in which children can use additions to solve subtraction problems effectively if the numbers are close to each other. For example, to solve

'21 – 18', it is less likely to make mistakes if they count up from 18 to 21. The use of the inverse relation between addition and subtraction to calculate has been observed in oral arithmetic. For example, Nunes, Schliemann and Carraher (1993) reported two different ways street vendors in Brazil used the inverse relation to solve computations. Problems about change were commonly solved with indirect addition. For instance, when someone bought something valued 80 Cruzeiros and paid with a 500 note, a child vendor said 'Eighty, ninety, one hundred. Four twenty' (Nunes et al., 1993, p.25). In this case, the child calculated the change through indirect addition. In another example, a child used the inverse relation effectively in a different way. He solved the problem  $243 - 75$  by simplifying the problem – At first he subtracted 143 from 243, which becomes  $100 - 75$ , and then he added 143 back to reach the answer. In short, if children understand the inverse relation between addition and subtraction, they are able to think of various ways to simplify an arithmetic problem with their logical understanding of number in order to enhance their computational proficiency (Canobi, 2004; Canobi, Reeve, & Pattison, 2003).

According to the mathematical thinking perspective, learning mathematics should be based on understanding the relations between quantities and operating on the numbers to reach conclusions about the quantities. However, it seems common to observe that children learn computational algorithms in a meaningless fashion. For instance, previous studies showed that children often encountered difficulties in solving multi-digit addition and subtraction (e.g., Brown & Burton, 1978; Brown, & VanLehn, 1982; Carpenter, Franke, Jacobs, Fennema, & Empson, 1997; Carraher & Schlieman, 1985; Carraher, Carraher, & Schliemann, 1985; Fuson, 1990; Hennessy, 1994; Hiebert, & Wearne, 1996; Resnick, 1982, 1989, 1992, 1994; Selter, 2001; Young & O'Shea, 1981). Their difficulties can be understood in terms of the implementation of faulty procedures (Brown, & VanLehn, 1982). For example, when calculating  $237 - 49$ , the children obtain the answer 212 by taking 7 away from 9 and 3 away from 4 presumably because they assume that one cannot take a larger number from a smaller number. Another example of faulty procedures is that when facing a subtraction such as  $607 - 8$ , the children obtain 699, by subtracting 8 from 17. These children have correctly borrowed and added to the ones column, making the 0 into a 9 because 1 had been borrowed from the tens column. However, they forgot that something had been borrowed from the hundreds column. These are typical faulty procedures and well known to primary school teachers. Brown and VanLehn (1982) suggest that they are not merely a result of lack of attention. Instead, the mistakes follow from a systematic

application of erroneous algorithms across different kinds of problems by the same children. The mathematical thinking approach suggests that if children do not have a clear understanding of analytical meaning of number i.e. the relation between numbers, they are more likely to make calculation mistakes because of the use of faulty procedures.

Compared with the number sense perspective, the mathematical thinking perspective also addresses how children solve arithmetic calculation. While the number sense perspective postulates that magnitude estimation supports the retrieval of a correct answer, the mathematical thinking perspective suggests that reasoning about the relations between numbers can be a basis of effective calculation. From this perspective, arithmetic is not just about memorizing number facts. Instead, the crux of a successful problem solver of arithmetic calculation refers to the ability to understand the relational or analytical meaning of number.

### **1.1.2.3 Mathematical thinking and solving problems in different situations**

Up to now I have highlighted the importance of understanding the analytical meaning of number in arithmetic calculation. Now I turn to what Nunes and Bryant (2015) have called the ‘representational meaning of number’, which is about working out relations between quantities. Thompson (1994) highlights the importance of a logical “comprehension of a situation” (Thompson, 1994, p.187-188) in solving mathematical problems in different situations. He argues that it is important to analyse the underlying quantitative structures of mathematical problems – “a prominent characteristic of reasoning quantitatively is that numbers and numeric relationships are of secondary importance, and do not enter into the primary analysis of a situation. What is important is relationships among quantities” (Thompson, 1993, p.165).

The solution to many story problems rests upon the knowledge of the underlying relations between the quantities in the problem. Occasionally, this set of relations is not obvious to problem solvers. This applies to some story problems whose solutions rely on the understanding of the inverse relation between addition and subtraction. For example, a Change problem is easy when the missing information is the result of the change (e.g., ‘David had 8 books. Then Peter gave him 3 more books. How many books does David have now?’) because the action in the story and the arithmetic operation required to solve the problem are directly related. In other words, a problem that involves a change that increases the quantity can be solved by addition, while one that decreases the quantity can be solved by subtraction.

In contrast, when the starting situation is not known (e.g., 'Alex had some cookies. He gave 3 cookies to his mother and had 8 cookies left. How many cookies did he have before?'), one must decide which arithmetic operation to use for calculation on the basis of the information about the change and its end result. This type of start-unknown problems is more difficult (e.g., Carpenter, Hiebert, & Moser, 1981; De Corte, & Verschaffel, 1982, 1987; Ginsburg, 1982) because the relation between the action described in the story and the operation is inverse, i.e., A problem that involves a change that decreases the quantity has to be solved by addition. Thus, students must understand that the operation 'addition' can be conceived as the inverse of 'subtraction' and analyse the quantitative relations underlying the problem situation.

Verschaffel (1994) examined the difficulty of comparison problems which also demand inverse reasoning, but applied to relations rather than operations. In one type of comparison problem, the relation between quantities can be described as 'more than' but the problem solver has to think of its inverse to solve the problem e.g., when the reference set is the missing quantity (e.g. 'Pete has 29 nuts. He has 14 more nuts than Rita. How many nuts does Rita have?'). Verschaffel asked 5th graders in Belgium (aged about 11 years) to solve comparison problems in which the relation was consistent with the operation (i.e. the relation was described as 'more than' and the operation to be used to solve the problem was an addition e.g., 'Timothy has 29 cups. Jenny has 14 more nuts than Timothy. How many cups does Jenny has?') or was inconsistent (i.e. the relation was described as 'more than' and the operation to be used to solve the problem was a subtraction, as in the problem presented above). When the relation and the operation were consistent, the correct rate was 92.5%; when the relation and the operation were inconsistent, the correct rate was 72.5%. Because the numbers involved in these problems are the same, the ability to reason about the relation between quantities is likely to be the reason that explains the difference in accuracy. Therefore, students does not only have to learn that addition is the inverse of subtraction and vice versa, but also that the relation 'more than' can be seen as the inverse of 'less than' and vice versa.

In summary, according to the mathematical thinking perspective, quantitative reasoning that is based on relations between quantities is crucial in solving mathematical problems in a variety of situations, whereas the numbers used to represent the quantities are of secondary importance. Some additive reasoning situations involve just quantities whereas others involve quantities and relations. If a problem requires reasoning about relations, it is significantly more difficult than a similar one that involves just quantities. Children have to reason in a

sophisticated manner about the underlying structure of the quantitative relations in the story, in order to choose whether to add or subtract and solve the problem successfully. Thus, compared with the number sense perspective, the mathematical thinking perspective definitely has an edge by providing a good account of how children solve mathematical problems in different situations.

### **1.1.3 Definition of Mathematical Competence in the Present Thesis**

The mathematical thinking perspective focuses on the understanding of the meanings of number – analytical and representational. The analytical meaning of number is defined by a number system, whereas the representational meaning refers to the use of numbers to represent quantities (Nunes & Bryant, 2015). Comparing the two approaches to children's mathematics learning, the mathematical thinking perspective appears to provide a better theoretical framework to understand mathematical competence in children. As I have argued earlier in this chapter, competence indicates sufficiency of knowledge and skills that enable a person to act in a wide variety of situations. Translated it into mathematics learning, mathematical competence should refer to the ability to understand and use mathematics in a variety of situations in which mathematics is useful. As the National Mathematics Advisory Panel (2008) has recommended in their report, "the curriculum must simultaneously develop conceptual understanding, computational fluency, and problem-solving skills" (p.xix). Clearly, the number sense perspective touches upon computational fluency only. By contrast, the mathematical thinking approach addresses all three aspects of mathematical achievement.

In summary, my view is that it is not sufficient to say that a child who possesses a good sense of number is competent in mathematics. What is more important is the ability to think mathematically – that is, the ability to think about the relations between quantities and numbers. A child who is competent in mathematical thinking means that she or he has a good understanding of the meanings of numbers and quantities. This understanding appears to support his or her ability to excel in a variety of mathematical tasks. For example, as I have discussed in the previous sections, mathematical thinking allows children (1) to represent, formulate, and solve mathematical problems in different situations, and (2) to carry out calculation procedures accurately and flexibly. Therefore, I consider that mathematical thinking is a better way to conceive of mathematical competence and the rest of the present thesis would be largely based on this theoretical approach.

## 1.2 Cognitive Foundations of Mathematical Thinking in Children

After defining mathematical competence as mathematical thinking in the previous section, I explore its cognitive foundations in children. What are the pillars of mathematical thinking? What kinds of skills do children need to possess in order to have mathematical competence? I am going to analyse some aspects in the subsequent sections. The skills required for mathematical competence may not be the same for children of different ages. In this thesis, I am interested in studying children at the age of around 6, who have acquired some skills in counting and begin to learn addition and subtraction. Thus, I would focus on these aspects in the following discussion.

### 1.2.1 Working Memory

It seems uncontroversial that learning and using mathematics must draw on some general cognitive resources. For example, in order to solve the following problem, ‘David had 8 books. Then Peter gave him 3 more books. How many books does David have now?’, we need to (1) pay attention to the information, (2) select, remember, and reason about the relevant parts of this information, and (3) execute arithmetic operations that help us answer the problem. Likewise, when children have to solve a calculation or an applied problem, they have to keep in mind the information in the problem and the steps to execute the solution, while monitoring what they have done and what remains to be done. ‘Working memory’ is the term researchers use to denote the ability to keep track of information and operate on it simultaneously. It is expected that working memory should influence how well individuals retain information in mind and think mathematically.

There are different theoretical models of working memory, such as, Baddeley-Hitch model of working memory (Baddeley & Hitch, 1974), Engle’s model of controlled attention (Engle, Kane, & Tuholski, 1999), and Miyake’s executive model (Miyake, Friedman, Emerson, Witzki, Howerter, & Wager, 2000). Different theoretical models lead to different definitions and corresponding measures for working memory in different studies. However, most previous studies testing the connection between working memory and mathematics learning in children have used the model proposed by Baddeley and Hitch (e.g., Alloway & Alloway, 2010; Bull, Espy, & Wiebe, 2008; Geary, 1993; Huttenlocher, Jordan, & Levine, 1994; Rasmussen & Bisanz, 2005; Swanson, 2011; Welsh, Nix, Blair, Bierman, & Nelson, 2010). Working memory has been defined as ‘a brain system that provides temporary storage and manipulation of the information necessary for ...complex cognitive tasks’ (Baddeley, 1992, p.556). Baddeley’s working memory model provides

a unified and parsimonious theoretical framework that comprises three key components – the phonological loop, the visuospatial sketchpad, and the central executive. The phonological loop serves to hold speech-based information temporarily, whereas the visuospatial sketchpad holds visual and spatial information for a short period of time. The central executive component is responsible for focusing, dividing, and switching attention, which provides an overall monitoring and regulation of the entire working memory system and coordination of the activities among different components in the system. In the present study, I would base on Baddeley's working memory model (Baddeley & Hitch, 1974, Baddeley, 1992) to test the relation between working memory and children's mathematical achievements.

The evidence regarding the close connection between working memory and children's mathematical achievements will be presented in the next chapter. However, it is likely that working memory is important for learning and performance across all academic domains. Thus, the relation between working memory and mathematical achievements may not be specific. The non-specificity of working memory suggests that, in order to understand what factors predict children's success in mathematics learning, we need to look at other abilities that are more specifically related to mathematics. It is reasonable to speculate that these domain-specific abilities would explain variation in children's mathematical achievements beyond general cognitive resources, such as working memory.

### **1.2.2 Counting Ability**

Counting is one type of domain-specific abilities central to children's mathematical thinking because learning to count provides children with words to represent quantities. This activity helps children reflect upon and develop the logical concept of one-to-one correspondence, ordinality, and cardinality. The definitions of conceptual knowledge of counting vary in the literature. In the following, different ways of conceptualising counting are explored because they influence how we interpret the findings from research that examines the connection between counting and children's mathematical achievement.

One popular theory about counting was proposed by Gelman and Gallistel (1978). The researchers suggest three 'how-to-count' principles that are necessary for correct counting, including the 'one-to-one correspondence', 'stable order', and 'cardinality' principles. The one-to-one correspondence principle refers to the understanding that one must only tag an object in an array with one and only one label for each individual object. In other words, children who are proficient in counting should be able to separate the items that have already been counted and

those have not, and to use only one tag for each item. Some children may fail to recognise this principle and thus skip an item or count an item more than once, or use the same tag more than once.

The stable order principle requires the person who counts to choose tags that correspond to items in an array in a stable order, which should stay the same regardless of the number of items. The cardinality principle, according to Gelman and Gallistel (1978), is defined as the understanding that the number tag assigned to the final item in an array represents the total quantity of the set. Understanding the cardinality principle may underlie the use of more efficient counting strategies to solve problems. For example, children who understand cardinality can use the 'first' procedure to solve an arithmetic problem e.g.  $8 + 4$ . If they know the cardinal value of the first number, they can use this number as the shortcut to count more efficiently: they would start from '8' and count '8, 9, 10, 11, 12' to solve  $8 + 4$ , rather than start all the way from '1' and count '2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12' to reach the answer.

It has been suggested that these three counting principles govern the counting behavior of young children (Gelman & Meck, 1983). The proponents of this view suggest that young children's conceptual understanding of these essential features of counting precedes their acquisition of counting procedures. However, there is an alternative view that children do not start with an adequate understanding of the counting principles when they count. Instead, they start from imitating other people's counting behavior and induce some common features of counting from the observation (Briars & Siegler, 1984; Fuson, 1988). These common features are called 'unessential' features of counting because they may are not necessary for correct counting. Briars and Sielger (1984) identified four such unessential characteristics, including (1) standard direction (items must be counted from left to right), (2) adjacency (items must be counted contiguously), (3) pointing (items have to pointed at during counting), and (4) start-at-the-end (items must be counted from one end of an array of objects). Although we do not need to follow these rules if we want to do a correct counting, some children believe that these four features of counting are necessary for it. This suggests that the conceptual understanding of counting of some young children is still rigid and not yet fully developed.

In short, these researchers suggest that conceptual knowledge of counting refers to the understanding of what is necessary for correct counting. Children who have a thorough conceptual knowledge of counting should be able to abide by the essential principles and not to mistake the unessential characteristics of counting as the criteria for correct counting. It is necessary to respect each of these principles because they are part of the analytical meaning of



number. Counting activity is important for children to learn mathematic because it helps children think about the meanings of number.

However, it is not enough for children to know individual counting principles separately, but they also need to coordinate their knowledge of the principles in order to understand the analytical meaning of number. For example, the last number word of a counting sequence denotes the cardinal value of a set (cardinality principle) should only hold when the counting follows the one-to-one correspondence principle. If one skips an object in the middle of the counting sequence, she or he should not say that the last number word is the cardinal value of the set. There is evidence that some children failed to coordinate their knowledge of the counting principles even though they demonstrated competence in reciting the number sequence and applied it to objects and events (e.g., Bermejo, Morales, & deOsuna, 2004; Freeman, Antonnucchia, & Lewis, 2000; Sarnecka & Gelman, 2004; Sophian, 1988). These studies suggest that knowing how to count does not necessarily imply a full understanding of number. However, much of the research on counting analysed children's knowledge of these principles separately (e.g., Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Barrouillet, Fayol, & Lathulière, 1997; Koponen, Aunola, Ahonen, & Nurmi, 2007; Passolunghi, Vercelloni, & Schadee, 2007). I will review the studies regarding counting more thoroughly in the next chapter.

In summary, counting is a useful starting point from which children learn to develop mathematical thinking. It is an activity that young children can use to learn the ordinal and cardinal meanings of number, but it takes some time for them to achieve a full understanding of counting. Because counting is more specific than working memory to mathematics learning, it is expected that individual differences in counting ability would explain variation in children's mathematical achievements beyond general cognitive capacities, such as working memory and general intelligence. It appears that the measures of counting have to be chosen with care, which should capture children's true understanding of counting. Therefore, to measure counting ability in this study, I would use various tasks, including (1) procedural counting (the ability to correctly say a number-word sequence) and (2) conceptual knowledge of counting, which refers to the awareness of Gelman and Gallistel's five counting principles as well as the ability to coordinate different principles to determine the cardinal number of a set.

### 1.2.3. Additive Reasoning

Another domain-specific ability that is important for young children to learn mathematics is additive reasoning. Additive reasoning is based on quantities connected by part-whole relations. Two central properties of part-whole relations involve (1) commutativity and (2) the inverse relation between addition and subtraction (some researchers called it the 'complement principle' e.g., Canobi, Reeve, & Pattison, 2003). Commutativity refers to the irrelevance of addend order to the sum, i.e. ' $a + b = c$ ' implies ' $b + a = c$ ', whereas the complement principle refers to the inverse relation between addition and subtraction, i.e. ' $a + b = c$ ' implies ' $c - a = b$ '. These two principles are considered important in children's mathematics learning because they contribute to the understanding of the relational meanings of numbers and quantities. It is clear that the mastery of additive reasoning requires an integration of the principles – one should understand that three quantities e.g.,  $3 + 4 = 7$  can be expressed in four mathematical relations, e.g.,  $7 - 3 = 4$ ,  $4 + 3 = 7$ ,  $7 - 4 = 3$ , and  $3 + 4 = 7$ , and that these four expressions can be deduced from each other. A thorough understanding of the part-whole relations of quantities involves the recognition that these expressions are essentially describing the same relation.

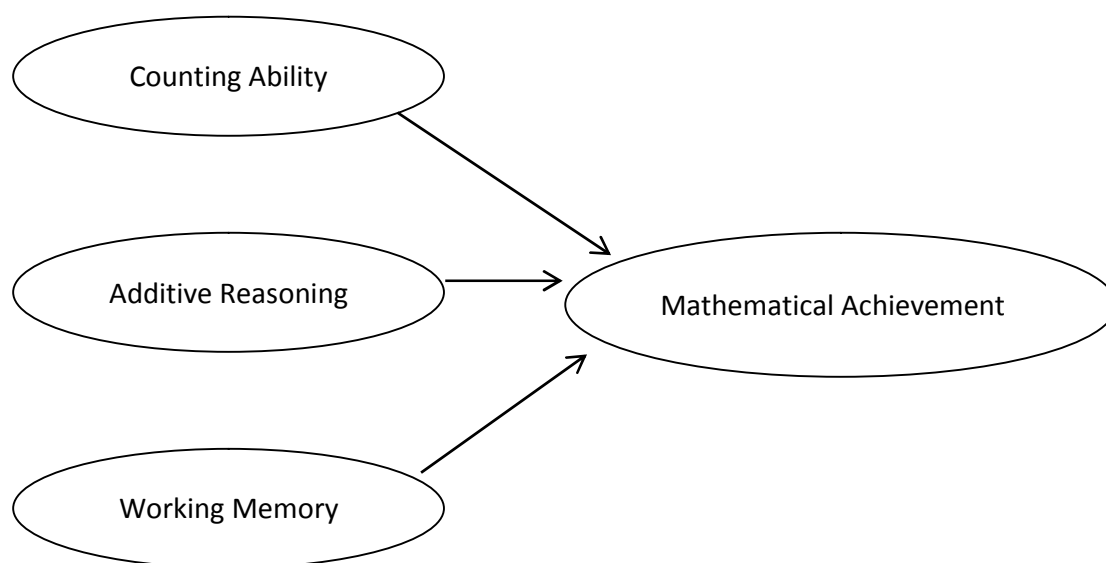
It has been suggested that additive reasoning is not easy to measure empirically (Bisanz & LeFevre, 1992) because children can do things with one kind of representations that they cannot do with another. There is very little disagreement that young children sometimes can solve problems that they cannot talk about. For instance, they can reason about certain things with concrete materials before they can use symbols to express them (e.g., Bruner, 1966; Piaget, 1952; Kozulin, 1986). Thus, it is possible that in additive reasoning, children at first succeed in tasks that assess their knowledge of each of these principles when the problems are set in the presence of concrete materials, and succeed only later when the problems refer to more abstract symbols, in the absence of objects. It remains to be verified whether there is a progress in children's understanding of mathematical principles from concrete to abstract situations.

There could be an order of acquisition for the commutativity principle and the complement principle. Commutativity refers to the simplest logical aspect of part-whole, which is that the order in which you add the parts does not affect the whole ( $a + b = b + a$ ). Knowledge of the complement principle ( $a + b = c$  implies  $c - a = b$ ) has a logical relation to commutativity, but it requires more than that. In order to understand that if  $a + b = c$ , then  $c - b = a$ , and  $c - a = b$ , children need to think of (1)  $a$  and  $b$  as interchangeable parts (i.e. they need to understand commutativity) and that (2) if you take the first part away from the whole, you are left with the

second, and if you take the second part away from the whole, you are left with the first. Thus, the knowledge of the commutativity principle and that of the complement principle may develop over time rather than emerge in an all-or-nothing fashion. More evidence is needed to evaluate whether there is an order of understanding of the principles (from the commutativity to the complement principle).

In summary, additive reasoning is crucial for children to learn mathematics. Understanding commutativity and the inverse relation between addition and subtraction are part of the construct of additive reasoning. This knowledge seems to be distinct from and developmentally more advanced than the understanding of ordinality and cardinality. Thus, it is expected that individual differences in additive reasoning would explain variation in children's mathematical achievements beyond counting ability and general cognitive capacities, such as working memory and general intelligence.

#### 1.2.4 Hypotheses



*Figure 1.1*

*The hypothesised contributions of three foundational abilities to mathematical achievement*

Mathematical competence refers to the cognitive foundations for learning mathematics. I argue that working memory, counting ability, and additive reasoning contributes to this foundation because they affect how well children think mathematically about the relations between quantities and numbers. It is hypothesised that each of these factors relates to

children's performance in mathematical problems that are typically assessed in primary school, such as calculation and solving story problems, because children's success in these mathematical tasks depends on their ability to think mathematically. Throughout the present thesis, I use 'mathematical achievement' to refer to children's performance on these tasks.

Overall, this study aims to test whether working memory, counting ability, and additive reasoning contributes to mathematical achievement in children of around 6 years of age. In summary, the above analyses lead to the following hypotheses:

1. Counting ability is important in children's mathematics learning and its influence is independent from that of general cognitive capacities, such as working memory.
2. Additive reasoning (as assessed by knowledge of commutativity and the complement principle) is independent from and more important than counting ability and general cognitive capacities, such as working memory in children's mathematics learning.
3. Working memory, as a domain-general factor, makes a contribution to mathematical achievement, even when one accounted for children's specific mathematical knowledge such as their knowledge of counting and additive reasoning.
4. Knowledge of the commutativity and of the complement principles develops from thinking in the context of specific quantities to thinking about more abstract symbols.
5. There is an order of understanding of the principles – from the commutativity to the complement principle.

### **1.3 Definitions of Key Terms**

#### **1.3.1. Working Memory**

Working memory is defined according to Baddeley and Hitch's (1974) model of working memory. It comprises three components – the central executive, phonological loop, and visuospatial sketchpad. The central executive component is responsible for focusing, dividing, and switching attention, which provides an overall monitoring and regulation of the entire working memory system and coordination of the activities among different components in the system. The phonological loop holds speech-based information temporarily, whereas the visuospatial sketchpad holds visual and spatial information for a short period of time.

#### **1.3.2 Counting Ability**

Counting ability is defined as children's procedural counting skills and conceptual understanding of counting. Procedural counting skill refers to the ability to correctly say a

number-word sequence. Simply observing children's procedural counting skills, however, is not sufficient to determine whether they have conceptual understanding. A child who demonstrates success in counting a set of objects is credited with procedural skills, but if the same child cannot detect errors in a puppet's counting or coordinate different counting principles to recognise the cardinality of a set, then s/he may not have a full understanding of counting. Thus, in this study, conceptual knowledge of counting refers to children's understanding of counting procedures, which includes an awareness of Gelman and Gallistel's five counting principles and the ability to coordinate the principles to determine the cardinal number of a set.

### **1.3.3 Additive Reasoning**

Additive reasoning is defined as the understanding of two crucial principles for additive problem solving i.e. the commutativity and complement principles. The commutativity principle refers to the fact that the order of addends does not lead to different results. If  $a + b = c$ , then  $b + a$  is also equal to  $c$ , and  $a + b = b + a$ . The complement principle refers to the fact that addition and subtraction are different sides of the same coin. If  $a + b = c$ , then  $c - b = a$ , and  $c - a = b$ . Additive reasoning basically refers to the understanding of a part-whole schema, which involves an integration of knowledge of the commutativity and complement principles.

### **1.3.4 Mathematical Achievement**

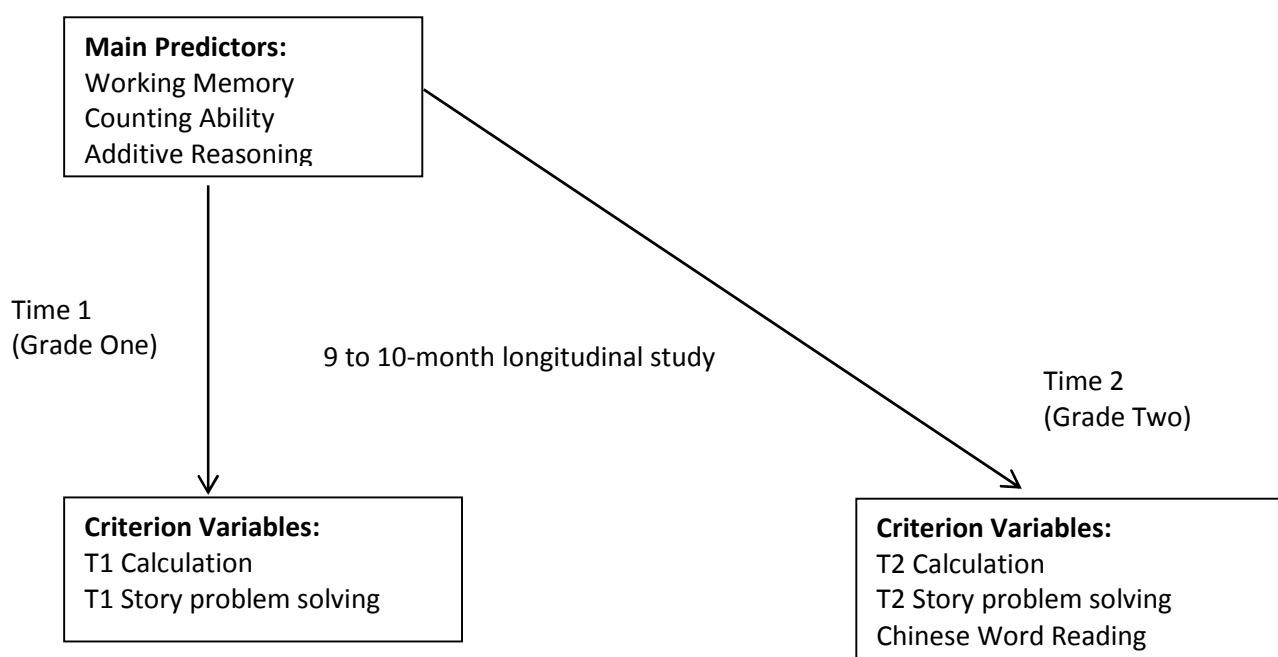
Mathematical achievement is defined as children's performance on arithmetic calculation and story problem solving. Calculation tasks measure children's accuracy of carrying out addition and subtraction operations, whereas story problem solving tasks measure children's abilities to solve mathematical problems in different situations, such as Combine, Change, Compare, Equalize, and de-combining transformation problems.

## **1.4 Overview of Research Design**

According to Bradley and Bryant (1983), both longitudinal and intervention studies are important for establishing a causal relation between variables. Intervention studies can be used to discern the causal relation between certain skills and mathematical achievement. Through this type of design, we can find out, for example, whether training additive reasoning leads to an improvement of these skills and mathematical achievement. But before we implement an intervention study, we need to identify factors that are important for children's mathematics learning. Longitudinal studies give us a good opportunity to examine the temporal order of

events. It is important for us to know whether a predictor precedes mathematical achievement because it is a necessary condition for determining causal relation between variables. Through statistical techniques, such as multiple regression analysis, we can identify the direction and strengths of associations between a predictor and mathematical achievement and compare the unique contributions of each predictor to variation in the outcome. Thus, longitudinal study is considered as an important first step for developing an intervention.

In this study, I use a longitudinal design, which spans around 10 months, to examine whether the main predictors (working memory, counting ability, and additive reasoning) uniquely predict children's mathematical achievement (calculation and story problem solving) (Figure 1.2).



*Figure 1.2 An overview of research design*

The main predictors in this study are working memory, counting ability, and additive reasoning. Working memory is defined as children's performance on three tasks including digit span forward (phonological loop), Corsi span (visuospatial sketchpad), counting recall and digit span backward (central executive). Counting ability is operationalised as children's procedural counting skills and conceptual knowledge of counting; whereas additive reasoning is operationalised as children's understanding of the commutativity and complement principles which are assessed in two contexts (with the support of physical objects and without). All of these main predictors are assessed at the first wave of data collection (Time 1).

A number of control variables, such as, general intelligence and demographic characteristics, are also measured at Time 1 to ensure that any observed associations between predictors and outcome measures are not due to an extraneous factor that may affect the relations. Mathematical achievement is operationalised as children's performance on calculation and a variety of word problem solving tasks. It is assessed at both Time 1 and Time 2. Assessing mathematical achievement at both time points provides an opportunity to test whether the main predictors would predict mathematical achievement concurrently (T1) and longitudinally (T2). It also enables me to assess whether these predictors remain significant predictors of T2 mathematical achievement after the effects of T1 mathematical achievement is statistically controlled for.

A literacy measure is also used as one of the Time 2 outcome measures for testing the specificity of predictors for mathematical achievement (Bradley & Bryant, 1983). If certain aspects of competence are crucial specifically for mathematics learning, they should predict children's performance on mathematical tasks significantly much better than their performance on non-mathematical tasks.

### **1.5 Content of Each Chapter**

This thesis is divided into five chapters. Chapter 2 examines the plausibility and novelty of the hypotheses by considering previous research. It starts with a literature review on the contribution of counting ability to mathematical achievement. It is followed by a review of studies that suggest the importance of additive reasoning in mathematics learning in young children. The chapter then goes on to present evidence that highlights the importance of working memory. This is followed by a brief review of research on the mathematics learning of East Asian children (related to the sample of this study). The literature review then turns to studies that examined the roles of concrete materials in children's understanding of mathematical concepts. The chapter continues with an overview of the present research and ends with a recapitulation of its aims and hypotheses.

Chapter 3 gives an overview of the study and described the method in detail, including the sample characteristics, the rationale and operational definitions of variables, and procedures. The findings are presented in Chapter 4 and 5. In the beginning of Chapter 4, I present the preliminary analyses, including the descriptive statistics, demographic differences, and bivariate correlations. The next part of this chapter comprises the main analyses that address the hypotheses, including a series of multiple regression models that assess the relative

contributions of working memory, counting ability, and additive reasoning to explaining variance in calculation and story problem solving at both time points of data collection.

Chapter 5 contains the latent profile analysis in which I explore children's profiles in additive reasoning and their relations to mathematical achievement. With latent profile analysis, I explore the individual differences in children's development of the knowledge of the commutativity and complement principles and the role of concrete materials in this development. Chapter 6 consists of a discussion of the findings with regard to previous research. This chapter involves an evaluation of the theoretical and practical contributions as well as the limitations of this study. Suggestions for future research are also presented, followed by the conclusion of this thesis.



## **Chapter Two    Literature Review**

In Chapter One, I have analysed two theoretical perspectives to search for a definition of ‘mathematical competence’ in this thesis. Mathematical thinking – understanding the meanings of number – appears to be a better way than number sense to conceptualise mathematical competence. Three essential abilities are proposed to be the foundations of mathematical thinking – working memory, counting ability, and additive reasoning. In order to make the argument that these three factors are important for children’s mathematics learning, I conduct a critical review of the literature in the current chapter. This review is based on three types of evidence: cross-sectional, longitudinal, and experimental studies. From cross-sectional studies, we can obtain evidence about the association between a particular factor and children’s performance in mathematics. Statistical techniques, such as multiple regression analysis, allow researchers to assess the independent contribution of a particular variable to mathematical achievement. Longitudinal studies provide stronger evidence for the connection between a predictor and a mathematical outcome. On the basis of longitudinal evidence, we can know whether a predictor precedes mathematical achievement, which establish a necessary condition for a possible causal relation between the variables. Finally, experimental studies allow researchers to make the claim of causality, and whether it is possible to enhance a particular mathematical ability that leads to mathematical progress.

The first section of this chapter reviews research that investigated the contributions of counting to mathematical achievement in children. The second section continues to review evidence that suggests the importance of additive reasoning in children’s mathematics learning. The third section turns to studies that examined the role of working memory in mathematical performance. The fourth section reviews research that investigated the mathematics learning in East Asian children (Chinese children in particular), which is relevant to the sample of the present study. This is followed by the fifth section in which I briefly review studies on the roles of concrete materials in children’s understanding of mathematical concepts. This section is related to how I assess children’s additive reasoning ability. The chapter goes on to present an overview of the present research and concludes with a recapitulation of its aims and hypotheses.

## **2.1 Counting Ability and Mathematics Learning**

On the basis of the mathematical thinking perspective, mathematics learning is based on understanding the relations of quantities and numbers. Counting is regarded as a foundational skill for the development of mathematical thinking in children because learning to count provides children with words to represent quantities. This activity helps children reflect on some essential aspects of the concept of number, such as ordinality and cardinality. Thus, It is hypothesised that counting ability is important in children's mathematics learning and its influence is independent from that of general cognitive capacities, such as working memory.

In this section, I review three types of evidence regarding the role of counting ability in children's mathematics learning: cross-sectional, longitudinal, and experimental studies. Limitations on the research design of particular studies are also discussed. It should be noted that past studies used different ways to operationalise counting ability, which can be broadly categorised into two types of measures: (1) procedural counting (the ability to correctly say a number-word sequence) and (2) conceptual knowledge of counting, which refers to the awareness of the logic of counting. Because I consider mathematical competence as the ability to understand the meanings behind mathematical activities, the use of procedural counting as the sole indicator of counting ability does not adequately capture 'counting ability' from this perspective. I will elaborate more on this argument with research evidence in the following review.

### **2.1.1 Cross-Sectional Studies**

Cross-sectional studies regarding the contribution of counting ability to mathematical achievement have focused on comparing the counting performance between children with mathematical difficulties and their typically developing peers. This type of research addresses the connection between counting ability and mathematical achievement by examining whether children at the lower ends of the distribution in mathematics demonstrate co-occurring deficits in counting.

For example, Geary (1990) conducted a study with first-grade children with mathematical difficulties and compared their accuracy, speed, and strategies used to solve simple addition problems with typically developing children. The children with mathematical difficulties were chosen on the basis of a remediation program in mathematics at the end of kindergarten. Based on the children's performance at the end of the first grade, Geary (1990) classified two groups of children with mathematical difficulties, namely the improved group and the no-change group.

He found that these two groups of children and children without mathematical difficulties in the same age group used similar types of strategies when solving addition problems. For example, all of them used their fingers to count, counted verbally and retrieved answers directly from memory. However, compared with children without mathematical difficulties and the MD-improved group, the MD-no-change group committed significantly more counting and retrieval errors; they also used count-all procedures predominantly to solve the problems. Although the MD-no-change group did not differ from the other two groups in their average counting speed, their counting speed varied much more considerably within the group.

Russell and Ginsburg (1984) examined the contributions of counting to various arithmetic tasks in a group of older children. They compared the performance of fourth-grade children with mathematical difficulties with that of typically developing children in the third and fourth grade. In this study, children were asked to solve simple e.g.,  $4+7$  and complex arithmetic problems e.g.,  $16+35$ . They found that the children with mathematical difficulties performed significantly less well than the typically developing children in third and fourth grade on simple problems. As for more complex problems, the children with mathematical difficulties performed similarly with the typically developing children in the third grade, but less well than the typically developing children in the fourth grade. Similar to Geary (1990), Russell and Ginsburg (1984) observed that all groups of children used similar types of strategies, whereas the children with mathematical difficulties committed significantly more counting errors when solving the problems.

More recently, Geary and colleagues (Geary, Hamson, & Hoard, 2000; Geary, Hoard, & Hamson, 1999) compared the computational strategies and arithmetic performance among typically developing children, children with mathematical difficulties, and children with reading disability. Similar to previous studies (e.g., Geary, 1990; Russell & Ginsburg, 1984), they found that children with mathematical difficulties committed significantly more errors in counting and used more count-all procedures to solve problems, compared with the other two groups of children of the same age. They also observed a change of strategy selection from first to second grade in both typically developing children and children with reading disability. The strategies of these children shifted from finger counting to verbal counting and direct retrieval, and they also improved in problem solving accuracy. In contrast, children with mathematical difficulties did not demonstrate the same shift – they still predominately relied on finger counting in both grades and made significantly more errors. Similar findings have been replicated in different countries, including the United States (Geary & Brown, 1991; Jordan & Montani, 1997; Jordan,

Hanich, & Kaplan, 2003), Europe (Barrouillet, Fayol, & Lathulière, 1997; Ostad, 1997, Ostad, 1999, 2000; Svenson & Broquist, 1975), and Israel (Gross-Tsur et al., 1996).

Differences in counting accuracy and types of predominant strategies used may reflect differences in the underlying conceptual knowledge of counting. For example, Ohlsson and Rees (1991) have argued that counting knowledge and skill at identifying counting errors allows children to correct miscounts. Thus, Geary, Bow-Thomas, and Yao (1992) examined whether the tendency of children with mathematical difficulties to use less sophisticated counting procedures and commit more computational errors stems from their delay in their conceptual understanding of counting. They argue that conceptual knowledge of counting acts as a standard against which computational procedure is evaluated. So children will modify their procedures if they are aware that the execution of a counting procedure does not conform to the conceptual rules of counting.

In their study, children with mathematical difficulties and typically achieving children were given various counting tasks, which were created to test children's understanding of the three 'how-to-count' principles described by Gelman and Gallistel (1978). For example, the children had to judge whether the counting procedures done by a puppet were correctly executed. The researchers also looked into whether children believed that some unessential features of counting (e.g., adjacency and start counting from one end; Briars & Siegler, 1984) were important to determine if a counting procedure was correct or not. For example, the two groups of children were compared on two types of counting trials, namely pseudoerror and error. In the pseudoerror trials, an array of objects was counted from, for instance, the second, fourth, sixth, eighth items and then from the first, third, fifth items. The counting is correct but does not conform to the adjacency rule, which is not an essential feature of counting. This trial aimed at testing whether children understand the order-irrelevance principle. In the error trials, some of the items in a set were double-counted. To examine the connection between conceptual knowledge of counting and arithmetic competence, the children were asked to solve some addition problems and their problem solving strategies were recorded.

It was found that children with mathematical difficulties were less accurate in solving the addition problems and committed more counting errors, compared with their typically achieving peers in the same age group. Children with mathematical difficulties also used fewer developmentally advanced computational procedures, such as, counting-up, to solve problems. Findings from the counting tasks suggest that the poorer performance of children with mathematical difficulties may stem from their immature conceptual understanding of counting.

This study showed that children with mathematical difficulties were poorer at identifying counting errors and they were also more likely to believe that adjacency was an essential feature of correct counting. This incomplete knowledge of counting is related to their choice of counting strategies and computational accuracy. A composite of conceptual knowledge was significantly and positively correlated ( $r = .47$ ) with the frequency of count-up procedures.

A significantly negative correlation ( $r = -.44$ ) was also demonstrated between the index of conceptual knowledge and the number of counting errors. After the influence of conceptual knowledge was controlled for, the differences in the use of count-up procedure and counting errors between the two groups were eliminated. Subsequent studies that controlled for the effects of intelligence and reading achievement levels reported converging findings (Geary, Hoard, & Hamson, 1999; Geary, Hamson, & Hoard, 2000). These studies suggest that (1) children with mathematical difficulties had a poorer conceptual understanding of counting relative to their typically achieving peers, and (2) counting knowledge contributes to children's development of different types of computational skills in problem solving as well as their arithmetic competence.

In summary, studies that compare children with and without mathematical difficulties provide one type of evidence that is useful for exploring the association between counting and children's mathematics learning. However, most studies (e.g., Barrouillet, Fayol, & Lathulière, 1997; Geary, Hamson, & Hoard, 2000; Geary, Hoard, & Hamson, 1999; Svenson & Broquist, 1975) reveal one key limitation in the research design – They simply involved a group of typically developing children of the same age as the control group (CA control), but not also a group of children that are matched by mathematical ability. The disadvantage of the CA match-only design is that it cannot distinguish cause from effect (Backman, Mamen, & Ferguson, 1984; Bryant & Goswami, 1986). In this type of design, the two groups (MD group versus no MD-CA control) have different mathematical abilities (e.g., measured by scores on standardised mathematical tests), so any discrepancies between them (e.g., counting ability) could just be a product of their different mathematical abilities as a cause of these differences. In other words, whatever form the positive results in a CA match take, one cannot rule out the possibility that they are partly or wholly the consequence of differences in mathematical skills. A better design would be studies that include two types of control groups: (1) children that are matched on chronological age (i.e. the no MD-CA control) and (1) children that are matched on mathematical ability. If the children are at the same level of mathematical ability, any difference between

groups cannot be attributed to one group being more successful than the other at mathematics. If the children with mathematical difficulties are worse than their controls on a measure (e.g., counting) in this type of experiment, it is more credible to conclude that the measure has something to do with the cause of their mathematical difficulties.

### **2.1.2 Longitudinal Studies**

However, as has been argued by some researchers (e.g., Bradley & Bryant, 1983), it is better to combine longitudinal and intervention studies in order to infer the causal relation between variables. Cross-sectional studies do not allow us to make convincing inferences about how variables are related to one another. Thus, it is necessary to review evidence to find out whether counting ability at an earlier age would predict children's later success in mathematics, i.e., longitudinal studies. This type of research observes participants of a given age over a period of time, which enables us to test whether a consequence is contingent upon the influence of certain antecedents across time in a controlled manner (Baltes & Nesselroade, 1979). The time ordering nature of longitudinal study allows us to make some reasonable speculations about causality. This section reviews longitudinal studies that examine the extent to which children's counting ability predicts their mathematical achievements.

For example, Passolunghi, Vercelloni, and Schadee (2007) conducted a one-year longitudinal study to examine the relation between basic numerical abilities, including counting, and children's later success in learning mathematics. The participants were young children who were attending the first year of primary school in Italy. In this study, counting skills were assessed with 3 tasks. The first task was called the 'counting knowledge task' in which children were asked to judge whether a puppet was counting in the right way. The second task was 'verbal counting' in which children were asked to count from 1 to 10 as fast as possible. The third task was called the 'counting speed task' in which children were given balls of different colours and asked to count the number of balls of a particular colour. It was found that overall counting ability was a unique predictor of children's performance on a standardised mathematical achievement test measured one year later beyond the effects of general cognitive variables, such as working memory.

In Finland, Aunio and Niemivirta (2010) investigated whether children's early numerical skills evaluated in kindergarten would predict their mathematical attainment in the first grade. After partialing out the effects of age, gender, and parents' education, they found that early counting knowledge (the use and understanding of number-word sequence) before formal education independently predicted children's acquisition of arithmetical skills and overall mathematical

achievements in grade one. In the United Kingdom, Muldoon, Towse, Simms, Perra, and Menzies (2013) examined the connections between procedural counting, number line estimation, and general mathematical achievement. In this study, 5-year-old children were assessed on 4 occasions at 3 monthly intervals. Counting ability was measured by children's ability (1) to recite the number string as high as they could and (2) to count different sets of dots. It was shown that counting ability was a unique predictor of children's overall mathematical achievement. Cowan, Donlan, Shepherd, Cole-Fletcher, Saxton, and Hurry (2011) examined the development of basic calculation fluency and its associations with mathematical achievement and other factors by following a group of children from second to third grade. Procedural counting, knowledge of number sequence, tasks that involved identification of numerical magnitudes were included as a composite measure of number system knowledge. This study showed that a Grade 2 number composite made independent contributions to explaining variation in Grade 3 calculation proficiency, conceptual knowledge of calculation principles, and scores on a standardised mathematical achievement test, beyond the effects of age, working memory, processing speed, and oral language. However, it is not certain whether procedural counting was a unique predictor of the outcome measures in this study because it was not analysed separately from other number knowledge measures.

Spanning over a longer period of time, Aunola, Leskinen, Lerkkanen, and Nurmi (2004) examined the development of mathematical achievement during children's transition from preschool to Grade 2 and the cognitive antecedents of this development. They followed 194 children 6 times, twice for each year across a 3-year period. Several cognitive abilities, that is, procedural counting ability, visual attention, metacognitive knowledge, and listening comprehension, were assessed at the first time point. They found that both the initial level and the growth of mathematics performance were best predicted by procedural counting ability. Studying an older age group of children, Koponen, Aunola, Ahonen, and Nurmi (2007) investigated the degree to which cognitive abilities in kindergarten predicted children's performance in single-digit and procedural calculation in Grade 4. They measured 178 children's visual attention, linguistic skills and basic number skills including procedural counting ability at kindergarten. In Grade 4, they tested the children's competence in calculation and reading. With path analyses, the researchers found that children's procedural counting abilities at the age of 5 made independent contributions to their performance in single-digit and procedural calculation.

Zhang, Koponen, Räsänen, Aunola, Lerkkanen, and Nurmi (2014) conducted two studies to investigate how early spatial and linguistic skills predicted children's development of arithmetic

and whether procedural counting ability mediated these associations. In their first study, they measured letter knowledge and spatial visualisation in a large sample ( $N = 1,880$ ) of Finnish children in kindergarten. They found that these skills predicted a significant amount of variance in arithmetic in the first grade and the growth of arithmetic competence through third grade. In their second study with 378 children, they found that procedural counting skills mediated the association between letter knowledge and spatial visualisation.

Koponen, Salmi, Eklund, and Aro (2013) conducted a longitudinal study that lasted for a longer period of time (5 years) to investigate whether counting was a significant predictor of both reading fluency and arithmetic calculation. Children were followed from age 5 to 10 years and their counting competence was measured by 2 tasks, (1) counting forward and (2) counting backward. After controlling for the effects of phonological awareness and verbal short-term memory, they found that children's overall counting ability remained a strong predictor of both calculation and reading fluency. In another 5-year longitudinal study conducted in the United States, Geary (2011) examined the predictive values of a variety of quantitative competencies on children's mathematical achievements from the beginning of 1st grade through 5th grade. These quantitative competencies included measures of basic numerical knowledge, counting, and arithmetic skills. Working memory, speed of processing and general intelligence was also measured. Multilevel models showed that intelligence, speed of processing, and the central executive dimension of working memory were significant predictors of achievement growth in mathematics and word reading. After controlling for the effects of these variables, Geary (2011) found that the accuracy of using sophisticated counting procedures to solve addition problems was independently predictive of mathematical achievement.

In summary, these longitudinal studies have shown that children's earlier counting ability predicts their mathematical achievement in older age. Although general cognitive capacities may affect counting performance, the significant predictions of counting ability to mathematical performance seem to be independent of general cognitive factors, such as working memory. However, most of these predictive studies have suffered one limitation – They measured procedural counting only. This is a limitation because children can count a numerical sequence without understanding the logic behind the activity. Thus, procedural counting may not be a sufficient indicator of children's counting ability. In the following, I review evidence that suggests that a complete understanding of numbers cannot be taken for granted in young children.



Knowing how to count and understanding numbers are not the same thing. According to Piaget, cardinality is not just saying how many objects are in sets, but is about understanding that sets are equivalent in number if they are in one-to-one correspondence. Piaget contended that children could only be said to understand numbers if they established a connection between numbers and the relations between quantities that are implied by numbers. Piaget based his claim on his observations that some young children did not have an understanding of one-to-one correspondence even though they count well. He found that the children often made mistakes when they were shown one set of items (e.g., eggs) and were asked to make another set (e.g., eggcup) of the same number. Children of 4 and 5 years of age often match the new set with the old one on the basis of irrelevant criteria, such as lengths of the rows, and did not make any effort to put the items into one-to-one correspondence. Although their ability to establish one-to-one correspondence between sets emerges over time, it cannot be assumed in every child who counts well.

Other research that provides more quantitative information has also demonstrated that children who count well do not necessarily mean that they understand numbers. For example, Gréco (1962) tested the understanding of numbers in 4 to 8 year-old children with three different versions of conservation task. The first task was the traditional conservation problem (Piaget, 1952), in which children were shown two identical-looking sets and asked to judge that the two sets were equal in number. The experimenter then altered the appearance of one of the sets (spread it out) and asked once again the children to compare the quantity of the two sets. The second task was largely identical with the first one, except that the experimenter asked the children to count one of the sets after the transformation, then required the children to determine the number of the second set. In the third task, the children were asked to count both sets after the transformation and then asked to judge whether the sets were equal in quantity. Gréco found that most children younger than 6 years old failed all three tasks. They were happy to say that the set that was more spread-out contained more objects than the other set, even though in the third task they had counted both sets and arrived at the same number for both sets. The fact that these children who judged a set that had 'six' objects was more numerous than another set that also had 'six' objects suggests that they have an imperfect understanding of what the word 'six' means.

More recently, Sarnecka and Gelman (2004) showed that some children did not recognise that the same quantity implied the same number. They asked the children to compare the number of snacks that a frog and a lion would be given. Then, the children had to state whether

the animals had the same amount or different amounts of snacks. After that, the snacks were put away into a closed box and in their absence, the children were told that the frog had a particular number of strawberries (e.g., five). Then, they were asked whether the lion had five or six strawberries. If we know the number of snacks in a box, we should be able to predict that the number will stay the same if nothing is added or taken away. So if the children had previously known that the animals had the same number of snacks, they should answer 'five' in this case. By contrast, if they said that the animals had different amounts of snacks, they should answer a different number. Sarnecka and Gelman showed that not all children who had previously said that the quantities were identical inferred that the number should also be the same. Even for those children who had demonstrated in another task that they could recognise five and six objects correctly, not all of them answered this question correctly: They were on average correct in 80% of the trials. As for those children who could not recognise five and six objects correctly, but could recognise two and three, had only a chance-level performance when both animals had five or six snacks: These children did no better than somebody who was just guessing. Thus, it appears that, when there were two boxes, these children could not make a parallel inference about number from one box to the other. This study suggests that even if a child can use a number to label a quantity, it does not follow that she or he can think about relations between quantities.

Some children may be able to establish a one-to-one correspondence between two sets, but they do not necessarily know that counting the items in one set gives them information on how many items there are in the other set. Frydman and Bryant (1988) showed that many 4-year-old children could not infer that equivalent sets have the same number of items even if they understood one-to-one correspondence well enough to share fairly. The researchers asked the children to share a set of 'chocolates' (i.e. blocks that represent chocolates) to two recipients on a one-for-A, one-for-B basis. After the child had finished sharing the chocolates, the researchers counted out the number of chocolates that had been given to one recipient (e.g., six) and asked the child how many chocolates the other recipient had received. Frydman and Bryant found that none of the children made the correct inference immediately: Instead, all children began to count the second set. In each case, the researchers then interrupted the child's counting and asked her or him whether there were alternative ways of knowing the number of chocolates in the second recipient's share. They found that only 40% of the 4-year-old inferred correctly that the second recipient had received six chocolates. More than half of the children failed this task. Most of the children knew that the two recipients had an equal share and one of them had

received a particular number of chocolates. However, they did not connect what they knew about the relation between the quantities to the numerical symbols. It was argued that making this connection is a significant step in understanding cardinality.

The research of the coordination of counting principles also highlights the difference between knowing how to count and understanding number. Fuson (1988), for example, examined whether children recognised that the failure to follow one-to-one correspondence in counting or the order of numerical labels would inevitably result in an erroneous number label for the set. She asked a group of 3-year-olds, who appeared to know that the last number word used in counting gave the number label for the set, to observe a puppet counting. Then, they were required to indicate how many items were in the set. The puppet did not start counting from one, but from two. Counting in this unusual manner should prompt the children to reject the last word as the cardinal label for the set. However, some children used the last count word as the cardinal label for the set.

In another study, Freeman, Antonucci, and Lewis (2000) evaluated 3- and 5-year-old children's rejection of the last word after counting if the puppet had made mistake in counting. In one of the tasks, a puppet miscounted a set of 3 or 5 items either by skipping an item or counting an item twice. The researchers then asked the children whether the puppet had counted correctly. If the children said that the puppet had not, they were asked (1) how many objects the puppet thought there were and (2) how many there were in reality. All children could count 5 items accurately, but their ability to count does not give us assurance that they recognised that the puppet's answer was incorrect after miscounting: Only about one third of the children said that the answer was wrong. The children's performance increased with age: over 80% of the 5-year-old children rejected the puppet's answer in all trials in which a mistake was made by the puppet. However, the majority of the children were not able to identify the cardinal for the set was without recounting. They did not say immediately the previous number when the puppet counted an item twice nor used the next number when the puppet had skipped one item. Such findings suggest that whereas children may be able to implement the counting procedures, they may not be able to understanding the logic inherent within the number system. Most children who had rejected the last numerical label as the cardinal did not deduce what the right cardinal should be. The ability to infer the cardinal number from the knowledge of the puppet's error would have shown a relatively good mastery of the logic of the number system.

Similar findings were obtained by Bermejo, Morales and deOsuna (2004), who used a different method: If a person counts a set of items by saying 'three, two, one' and s/he reaches the last item when s/he says 'one', s/he should know that 'one' is not the number of items in the set. Bermejo and colleagues observed that the 4- and 6-year-old children, who could say there are three items in a set when a person counts forward, could not necessarily understand that if a person count backward from four and the last numerical label is 'two', this does not mean that the set contains two objects in total. In this study, some children were not aware of the contradiction between the two answers – they could tell that the set contains three objects if you count forward, whereas the same set contains two objects if you count backward. This finding shows a lack of understanding of cardinality of numbers, because it is fundamental to the concept of cardinality that two sets have the same cardinal if the items are in one-to-one correspondence.

More recently, Rodríguez, Lago, Enesco, and Guerrero (2013) asked Spanish children attending kindergarten, first or second grade to evaluate whether had counted a set of items in a correct manner, and then to make a numerical inference. The researchers argued that if children had a true understanding of the logic of counting, they should be able to distinguish real errors, which violate the counting principles, from pseudo-errors, which are unusual routines but do not violate the logic of counting. One example of pseudo-errors was indicated by the performance of a character who sometimes counted an object with pointing while sometimes without pointing. Rodríguez and colleagues found that older children were more likely to distinguish real errors from pseudo-errors. However, even children in the second grade did not uniformly accept counting routines with pseudo-errors as correct counting. This finding suggests that some children may recognise at first that some rules must be respected when counting and that the failure to follow any of these rules implies that there is a mistake in counting. However, knowing these counting principles or rules does not guarantee an understanding of the logic of numbers. One initial step in understanding the meaning of numbers appears to rely on the coordinated use of the counting principles, which children who can cite a counting sequence may not demonstrate.

In summary, procedural counting (being able to say numbers in a particular order) does not guarantee children's understanding of numbers. Some studies show that children may count two sets and know that the numerical label for them is the same, but they did not know that the sets have the same quantity. Other studies demonstrate that children may know that two sets have the same quantity of items, but they fail to infer that the cardinal number for the two sets

should be identical. Research also shows that children may know how to count proficiently, but they fail to coordinate different counting principles in order to determine the cardinal value of a set.

On the basis of the mathematical thinking perspective, learning numbers by themselves may not be adequate for learning mathematics. In the case of counting, it is important for children to think about the connection between numerical symbols and quantities. Learning to count should contribute to children's mathematics learning because the counting activity helps them to think mathematically. However, evidence shows that even if a child can use a number to label a quantity or count verbally in a proficient manner, it does not follow that she or he can think about relations between quantities. It appears that there are individual differences in how far children use counting to think about the logic of numbers: Some children may seem to count proficiently, but they do not understand what they are doing. As such, procedural counting alone may not be a good indicator of counting ability from the mathematical thinking perspective. Thus, it is necessary for researchers who examine the contributions of counting ability to mathematics learning in children to include a variety of counting tasks in a study. These tasks should also assess children's understanding of the counting logic, such as the awareness and coordination of various counting principles.

### **2.1.3 Experimental Studies**

To address the question of causality, we need to turn to experimental/intervention studies that provide stronger evidence for whether an improvement of counting ability would lead to an enhancement in children's performance in mathematics. Tournaki's (2003) examined the differential effects of teaching addition through instructions of strategic counting versus drill and practice in children with and without learning disabilities. In this study, second-grade children received instructions on simple, one-digit addition problem solving (e.g.,  $4 + 3 = ?$ ). Some of these children were identified as having learning disabilities, whereas some did not have any disabilities. Children from each of these two groups were randomly assigned to one of the three conditions: (1) receiving instruction on count-up strategy, (2) drill and practice, and (3) control. In the experimental groups, children received eight 15-minute individually administered sessions supplemental to regular math instruction in classrooms, whereas in the control group, children received pre-test and post-test only without receiving any instruction. The relative effectiveness of different instructional methods was measured through children's performance on a simple addition post-test and a more difficult transfer task (e.g.,  $5 - 3 + 7 = ?$ ). Tournaki found that the

improvements of children without learning disabilities in both the strategic counting and the drill and practice group were significantly greater than that of the children in the control group. The researcher also observed that children with learning disabilities in the strategic counting group improved significantly more than those who had received drill-and-practice instruction. Considering their performance on the transfer tasks, all children, regardless of having disabilities or not, did significantly better in the strategic counting group compared with their peers in the other groups.

In a more recent study, Fuchs and her colleagues (2010) investigated the influence of an intervention of strategic counting on arithmetic skills in children with mathematical difficulties. Third-grade children were stratified on the status of mathematical difficulties (whether they had mathematical difficulties only or had both mathematical and reading difficulties) and location (near versus distant from the intervention developer). The children were randomly assigned to one of the three conditions: (1) strategic counting instruction with deliberate practice, (2) strategic counting instruction without deliberate practice, (3) control in which children did not receive any strategic counting instruction. In both experimental conditions, children received instructions on strategic counting for addition and subtraction. The counting strategy taught for addition was the min strategy, i.e. start counting with the larger number and count up the smaller number. The counting strategy taught for subtraction was addend strategy – Children were firstly introduced two new vocabularies, i.e. the number just after the minus sign was a ‘minus number’ and the first number in an equation was ‘the number you start with’. To solve a subtraction problem, children were taught to begin with the ‘minus number’ and count up to the ‘number you start with’.

In the first strategic counting condition, children received instruction on strategic counting and participated in 4- to 6-minute deliberate strategic counting practice per session, whereas in the second strategic counting condition, the children only received instruction on strategic counting. The interventions lasted for 16 weeks, 3 sessions per week with each session spanning 20 to 30 minutes. The researchers found that the children in both strategic counting groups produced significantly greater improvements in arithmetic performance than the children in the control group. Compared to the group without practice, children who received strategic instruction with deliberate practice showed significantly better performance. These intervention studies provide some evidence that suggests that counting strategies may causally lead to the development of arithmetic competence. However, it remains unclear whether training on counting leads to improvements in solving story problems in different situations.

In conclusion, this review has presented three types of empirical evidence that suggests that children's counting ability is important for mathematics learning. First, performance on counting tasks could differentiate children with mathematical difficulties from typically achieving children at the same age level. However, without control groups that are matched on mathematical abilities, it remains contentious whether the difference in counting abilities are a cause of mathematical difficulty or simply the consequence of being worse at mathematics. Second, longitudinal studies show that individual differences in counting ability predict children's mathematical achievements over time. Although general cognitive capacities may affect how well children count, evidence shows that the connection between counting ability and mathematical performance appears not to be explained solely by general cognitive capacities, such as working memory. One crucial limitation of most predictive studies, however, is that they use procedural counting as the sole measure of counting ability, whereas some evidence suggests that children's proficiency in counting number sequences does not guarantee an understanding of the logic of counting. There can be a disconnection between learning numbers and understanding its relations to quantities. From the mathematical thinking perspective, learning numbers alone is thus not sufficient to be competent in mathematics. In order to capture counting ability sufficiently, research that assesses counting should not only involve procedural counting tasks, but also incorporate tasks that measure children's awareness and coordinated use of various counting principles. Finally, a few intervention studies have suggested that a causal relationship may exist between counting ability and arithmetical performance in children. More research is needed on whether training counting would enhance children's ability to solve mathematical problems in different situations.

## **2.2 The Importance of Additive Reasoning in Mathematics Learning**

In the previous chapter, I have argued that mathematical thinking, i.e. understanding the relational meanings of number and quantities, is the crux of mathematics learning. Children who are competent in mathematics should be able to understand the logic inherent in the number system (as in calculation) and how they can use numerical symbols to represent the relations between quantities (as in solving different kinds of problems). It seems that additive reasoning is involved in the understanding of both types of meanings of number, which may contribute to mathematical achievement in children. The subsequent section reviews two lines of evidence that examined this idea – (a) the importance of analysing the underlying quantitative structure

of a mathematical problem in different situations, and (b) studies that directly examined the connection between quantitative reasoning and mathematical achievement.

### **2.2.1 The Importance of Understanding Relations in Mathematical Problem Solving**

In the following review, I focus on studies that examined children's performance on Combine, Change, and Compare problems in order to illustrate how the type of problem situation has a strong impact on the difficulty of addition and subtraction story problems. We shall see why reasoning about a problem and carrying out calculation are two different things and why it is important to consider the significance of children's making a connection between quantitative relations and numbers in mathematical development.

#### **2.2.1.1 The cases of Combine and Change problems**

The easiest additive reasoning situations are questions in which elements are added to or taken away from sets. Even though most pre-school children do not appear to have mastered addition and subtraction facts, they are still able to solve the simplest Combine and Change problems by joining or separating two sets (using objects or fingers) and counting to find the answers. Carpenter and Moser (1982) showed that for Combine problems, at the beginning and end of pre-school, respectively, 75% and 82% of the answers were correct for small numbers (<10) and 50% and 71% for larger numbers. For Change problems, the correct rates were 42% and 61% for larger numbers at each of the testing sessions (the researchers did not report the results for small numbers).

However, the simple distinction between Combine and Change problems is not adequate to characterise its level of difficulty. Which information is unknown is also important. For example, a Change problem is easy when the missing information is the result of the change (i.e. change-unknown problems) because the action in the story and the arithmetic operation required to solve the problem are directly related. In other words, a change that increases the quantity can be solved by addition, while one that decreases the quantity can be solved by subtraction.

In contrast, when the starting situation is not known, one must decide, on the basis of the information about the change and its end result, which arithmetic operation to use for calculation. This type of start-unknown problems is more difficult because the relation between the action described in the story and the operation is inverse. Consider this example, 'Alex had some cookies. He gave 3 cookies to his mother and had 8 cookies left. How many cookies did he have before?' a change that decreases the quantity has to be solved by addition. Vergnaud



(1982) showed that 5-year-old children found start-unknown Change problems (26% correct) significantly more difficult than result-unknown Change problems (nearly 50% correct). Nunes and Bryant (1996) demonstrated similar results by giving children calculators to help them in problem solving – they found that most children chose the wrong operation on the calculator if they failed to reason logically to obtain the correct solution for the problems.

Vergnaud interpreted the varying levels of difficulty of different problem types by proposing two types of calculations in mathematical problem solving – numerical and relational calculation. Numerical calculation refers to the arithmetic operations that are required for children to solve a problem. In the context of additive reasoning, addition and subtraction are the relevant operations. Relational calculation refers to logical move that children must make in order to understand the relations between quantities in the problem. In the case of aforementioned start-unknown Change problems, for example, the relational calculation is the awareness that the answer requires using the inverse of subtraction to reach the initial state from the end state, while the numerical calculation would be  $8 + 3$ . Vergnaud suggests that children carry these relational calculations implicitly. In his terminology, they depend on theorems in action (Vergnaud, 2009) – the children may not be able to articulate the fact that addition is the inverse of subtraction, but they may just know that they have to add in order to solve the problem.

#### **2.2.1.2 The case of Compare problems**

Vergnaud also contends that the important elements in additive problem solving are quantities, transformations and relations. It becomes even more difficult than Combine and Change problems for children when they are asked to quantify relations. For example: David has 8 cookies. Chris has 3 cookies. How many more cookies does David have than Chris? The question in this problem is not about quantity (i.e. David or Chris's cookies) or about a transformation (because neither David nor Chris got more or less cookies); instead it is about the relation between two quantities. Most young children understand that David has more cookies, but the majority of them are not able to quantify the relation between the two. Carpenter and his colleagues showed that 53% of the first-grade children who were asked 'how many more does A have than B' answered the number that A has. If the children were asked, in the same problem, who has fewer cookies and how many fewer, most of their answers were three. They did not recognise that the relation between quantities is the same regardless of how the question is framed: 'a more than b' and 'b more than a' are expressions about the same relation. The mistakes cannot be explained by children's lack of knowledge of addition and subtraction

because approximately 85% of them chose the correct operations to solve the simplest Combine and Change problems. Thus, it takes time for children to realize that different expressions mean the same thing (Stern, 1993).

Compare problems can take another form by stating how many items A has, then the numerical value of the relation between A's and B's quantities, and then asking how much B has. For example, in the problem 'Susan has 8 cookies. Betty has 3 more cookies than Susan. How many cookies does Betty have?' the relation is expressed as '3 more cookies' and the answer is solved by addition. The framing of the problem involves consistent language that 'more' implies addition intuitively. In another problem 'Peter has 5 books, Peter has 3 more books than Gary. How many books does Gary have?' the relation in the problem is expressed as '3 more books', but the answer is solved by subtraction. This type of problem involves inconsistent language because 'more' does not imply addition in this case. Verschaffel (1994) found that children in sixth grade gave 82% correct answers to problems with consistent language, and 71% correct answers to problems with inconsistent language.

In summary, the type of problem situation has a strong impact on the difficulty of addition and subtraction word problems. The exact phrasing of the problem influences the relative difficulty of a problem. The different levels of difficulty of problems that demand exactly the same arithmetic solution (the same additions or subtractions) suggest that reasoning about a problem and carrying out computations are two different things. Evidence suggests that there is more to mathematical development than computational proficiency. These studies point to the significance of children's making a connection between quantitative relations and numbers in mathematical development. Children may become better able to learn mathematics as they establish these connections.

### **2.2.2 Cross-Sectional Studies**

Is there any evidence regarding the connection between understanding the relational meanings of number and quantities and children's mathematical achievement? There are some cross-sectional studies that have addressed this question and the findings are inconsistent. For example, Bryant, Christie, and Rendu (1999) examined children's understanding of the inverse relation between addition and subtraction. They compared the performance on three-term inverse problems (e.g.,  $14 + 7 - 7$ ) and matched control problems (e.g.,  $9 + 9 - 4$ ) in a group of 5- to 8-year-old children. On the basis of factor analysis, they found that children's understanding of the inversion principle was not related to their accuracy on calculation (addition and

subtraction problems). In contrast, Canobi (2004) investigated the associations between conceptual knowledge and problem solving in 90 6- to 8-year-old children. Conceptual knowledge was tested by a judgment task in which children made and justified judgments of a puppet's solving problems that involved part-whole relations. She identified patterns of conceptual and problem solving profiles with cluster analysis and found that advanced conceptual profiles were associated with skilled problem solving. Approximately all children who recognised part-whole relations used more efficient strategies, such as retrieval and decomposition, to solve problems. Children with a more advanced conceptual understanding of part-whole relations also demonstrated higher accuracy and lower solution time than those with less advanced conceptual profiles.

Rasmussen, Ho, and Bisanz (2003) examined the use of the inversion principle in 24 preschool children and 24 children in Grade 1. They found that both preschool and Grade 1 children indicated evidence of understanding the inversion principle in a fully quantitative manner. The researchers also demonstrated that the relation between inversion understanding and calculation varied with age. They found that the preschool children did not show evidence of an association between their performance on inversion problems and arithmetic calculation. However, they identified a significant correlation between inversion understanding and accuracy of arithmetic calculation in Grade 1 children. Gilmore and Bryant (2006) used cluster analyses to analyse different patterns of inversion understanding and calculation skills among 6- to 9-year-old children. They identified three distinct subgroups, including one group showing good inversion understanding and good calculation skills, a second group demonstrating poor performance in both inversion understanding and calculation, and the final group having good inversion understanding but poor calculation performance. This finding suggests that individual differences in conceptual knowledge do not correspond with arithmetic competence directly.

In summary, one common characteristic of these studies is that they are cross-sectional in nature without taking into account other factors that may contribute to the relation between additive reasoning and children's mathematical achievement. These studies also focused exclusively on only one dimension of mathematical achievement, i.e., arithmetic calculation. Thus, it is not clear how understanding the relational meaning of number relates to other related but different aspects, such as story problem solving. More research is needed to elucidate the connection with studies that involve a wider range of question types and studies that have a more rigorous research design, such as, longitudinal studies.

### 2.2.3 Longitudinal Studies

There are a few longitudinal studies that demonstrated that quantitative reasoning predicted children's later success in mathematical achievement. Nunes and colleagues (2007) investigated whether children's quantitative reasoning measured at school entry was a significant predictor of mathematical achievement 16 months later, which was assessed by Standardised Achievement Tasks, Mathematics Section (SATs-Maths). There were four measures for quantitative reasoning: inversion between addition and subtraction, additive composition, one-to-one and one-to-many correspondence and seriation. At school entry, the children were also tested with the British Ability Scale-II as an assessment of their general cognitive ability and a counting recall task as an assessment of their working memory. Through multiple regression analyses, Nunes and colleagues (2007) found that quantitative reasoning was a significant and specific predictor of children's mathematical achievement. The relation was specific because quantitative reasoning remained a significant predictor after the effects of general intelligence and working memory were statistically controlled for.

In this article, Nunes and colleagues (2007) did not report the analyses regarding the connection of each of the four logical relation measures with mathematical achievement. However, Nunes, Bryant, and Watson (2007) reported this analysis about two of these relations: inverse relation between addition and subtraction and one-to-many correspondence. On the basis of fixed-order regression analyses, they demonstrated that both types of quantitative reasoning made significant predictions on children's mathematical achievement above and beyond the effects of age, general intelligence, and working memory. Children's performance in inversion explained 12% additional variance of mathematical achievement, whereas one-to-many correspondence made further 6% of the variance after all other factors were taken into account. These findings suggest that at the start of primary school, the ability to reason about the relations between quantities plays a significant role in children's learning mathematics.

Nunes and colleagues (2012) conducted another longitudinal study to evaluate whether quantitative reasoning and arithmetic skills are independent predictors of children's mathematical achievement in an older age (Key Stage 2 at 11 years of age and Key Stage 3 at 14 years of age). They found that quantitative reasoning made a unique contribution to the prediction of children's mathematical achievement at 11 and 14 years beyond and above the effects of age, general intelligence, working memory, and arithmetic skills. The amount of variance that was uniquely explained by quantitative reasoning was 8%. They also found that arithmetic ability also made an independent contribution to children's mathematical

achievement at 11 and 14 years after the effects of other factors were statistically controlled for. It explained an additional 3% of the variance of children's mathematical attainment. This study suggests that quantitative reasoning is a strong predictor of children's mathematical achievement. Nunes and colleagues (2012) also reported that the contributions of quantitative reasoning and arithmetic skills to mathematical achievement was specific because these measures were more strongly correlated with mathematics than to English or science achievement. In contrast, intelligence and working memory were not specific predictors of mathematics because intelligence predicted more variance in science than mathematics while working memory contributed to English and mathematics equally well. A study by Stern (2005) also provided some support for the importance of reasoning about quantities and relations for later mathematics learning. She found that a measure of children's understanding of the inverse relation between addition and subtraction when they were eight years old predicted their performance in algebra later in university. The significant prediction remained even after individual differences in intelligence were statistically controlled for.

#### **2.2.4 Experimental Studies**

Two recent intervention studies (Nunes et al., 2007; Nunes, Bryant, Evans, Bell, & Barros, 2012) have shown that quantitative reasoning may have a causal relation to children's mathematical problem solving. In one study, Nunes and colleagues (2007) identified children who underperformed in a logical assessment at the start of their first year in school and divided them into a control and an intervention group. The intervention group received instruction on a variety of reasoning principles for an hour per week for 12 weeks. In the control group, the children did not receive specialised instruction on quantitative reasoning, but participated in normal mathematics lessons. Thus, in the intervention group, the children did not receive extra time on mathematics, but they only received specialised teaching on the reasoning principles. Nunes and colleagues (2007) found that the intervention group performed significantly better than the control group on a state-designed mathematical achievement test. The mean for the control group in this test was at the 28<sup>th</sup> percentile, whereas the mean for the intervention group was just above the 50<sup>th</sup> percentile. Thus, through this intervention, a group of children who appeared to be at risk for mathematical difficulties caught up with their peers in the norm. However, three reasoning principles were taught together in this intervention, so it is not possible to tease apart the independent effects of each of these principles on mathematical performance.

In a more recent study, Nunes and colleagues (2012) focused on the inversion principle. They evaluated two methods that aimed to help children solve problems that involve relational calculus. This study used a pre-test, 2-session intervention, post-test design that included a control group and two intervention groups. Children who were 7 to 8 years old were randomly assigned to each of the groups. The control group received teaching that encouraged them to use counting, memory, and number line to solve arithmetic problems. In the intervention groups, children solved two types of problems, including (1) result-unknown, direct problems and (2) start-unknown, inverse problems. Children in both intervention groups learned how to use inversion to solve problems that require them to reason logically on quantities. The first intervention group differed from the second group in terms of how the problems were ordered – the ‘block group’ were taught to solve all direct problems in one session and all inverse problems in another session; whereas the ‘mixed group’ were taught to solve both types of problems in each session. All participants participated in a pre-test, an immediate post-test, and a delayed post-test that was conducted 8 weeks after the intervention. The pre-test, immediate post-test, and delayed post-test consisted of three types of story problems, including (1) direct problems that did not require relational calculus, (2) start-unknown problems that required relational calculus, and (3) change-unknown problems that was not included in the teaching in neither the intervention nor control groups.

The analyses of the immediate post-test scores showed that there were no significant differences between the control group and intervention groups in children’s performance on result-unknown problems, but the analyses of the delayed post-test showed that the mixed group, but not the block group, made significantly more progress than the control group for the result-unknown problems, but the effect size was small. As for the start-unknown problems, both intervention groups did not make significantly greater progress than the control group for start-unknown problems in the immediate post-test; whereas the mixed intervention group, but not the block group, showed significantly greater improvement than the control group for the start-unknown problems in the delayed post-test. As for the change-unknown problems, children in the block group, but not the mixed group, made significantly more progress than the control group in the immediate post-test, whereas both intervention groups showed significantly more progress than the control group in the delayed post-test.

These findings suggest that enhancing children’s inversion understanding may not increase children’s ability to solve start-unknown and change-unknown problems immediately, but it takes some time for these children to gain the benefits from the training in the long run. This

study has also suggested that incorporating different types of problems in the same training session (the mixed group) may be more effective than teaching different types of problems in separate sessions (the block group).

In conclusion, this section has reviewed empirical evidence that suggests that mathematical thinking is central to mathematics learning in children. First, the analysis of children's solving story problems suggests that understanding the relations between number and quantities is important in solving problems in different situations. Children need to know the logic and apply it into appropriate situations. Second, cross-sectional correlational studies show inconsistent findings between understanding the relational meanings of number/quantities and arithmetic calculation. However, with proper controls, some longitudinal studies demonstrate that quantitative reasoning has a strong and independent contribution to children's mathematical achievement. Finally, a few intervention studies have shown that an improvement in quantitative reasoning facilitates children's learning mathematics. As I have argued in the previous chapter, additive reasoning is an important dimension of mathematical competence. Understanding commutativity and the inverse relation between addition and subtraction are part of the construct of additive reasoning. This knowledge seems to be distinct from and developmentally more advanced than the understanding of ordinality and cardinality. Thus, it is expected that individual differences in additive reasoning would explain variation in children's mathematical achievements beyond counting ability and general cognitive capacities, such as working memory and general intelligence.

### **2.3 Working Memory and Mathematics Learning**

When evaluating the contributions of domain-specific abilities, such as counting ability and additive reasoning to mathematical achievement, it is important not to overlook the importance of general cognitive resources. One such factor that has received considerable attention is working memory (e.g., Geary, 1993, 2001; Geary et al., 2007). Working memory refers to the ability to keep track of information and operate on it simultaneously. It is considered important for mathematical thinking because it influences how well people retain mathematical information in mind. Therefore, it is expected that working memory makes an independent contribution to children's mathematical achievement. The following section reviews empirical research on the basis of Baddeley's model of working memory (Baddeley & Hitch, 1974), which

has been the most thoroughly explored theory of working memory in relation to children's mathematical development.

### **2.3.1 Cross-Sectional Studies**

Some cross-sectional studies have shown that working memory is independently associated with children's mathematical achievement. For example, Andersson (2007) examined the contribution of working memory to mathematical word problem solving in 69 children from Grades 2 to 4. These children were given tasks of fluid IQ, working memory, reading, arithmetical calculation, and mathematical problem solving. Multiple regression analyses demonstrated that the central executive components of working memory explained unique variance to mathematical problem solving after the effects of age, IQ, reading were statistically controlled for. Performance on the digit span task continued to explain a significant portion of variance to mathematical problem solving when the influence of age, IQ, arithmetical calculation, and reading was also taken into account. Berg (2008) investigated the role of verbal and visual working memory in a slightly older group of children's (3rd to 6th grade) performance in a general mathematical test that involved a variety of skills, such as, arithmetic, fractions, and algebra. Both types of working memory explained a significant portion of variance in mathematical performance, in the presence of the effects of all other variables including short-term memory, literacy, and speed of processing.

However, the connection between working memory and children's mathematical achievement is not as simple as one might have expected because some research shows that the relation is moderated by other factors. For example, the relation between working memory and children's performance in mathematics is complicated by the age of children and specific mathematical outcomes. For example, Huttenlocher, Jordan, and Levine (1994) have proposed that children at a very young age start to use visual information to process mathematical information, and they become increasingly reliant on verbal codes to solve mathematical problems as they acquire a higher level of language proficiency. Consistent with this idea, some studies showed that visual-spatial working memory was the best predictor of preschool children's performance on nonverbal addition and subtraction tasks, but it was not significantly related to their performance on similar tasks at Grade 1 (Klein & Bisanz, 2000; Rasmussen & Bisanz, 2005). At Grade 1, verbal working memory was significantly associated with their performance on verbal addition and subtraction problems.



The age of children may also determine the ways in which different aspects of working memory and mathematical performance are related. For example, Holmes and Adams (2006) investigated whether different components of working memory contributed to different mathematical skills in children aged from 8 to 10 years. In this study, working memory was measured by 3 tasks: a non-word list recall task (phonological), a listening span task (central executive), and maze memory task (visuospatial). A variety of mathematical attainment indicators included number and algebra, knowledge in geometry, measurement, data handling, and mental arithmetic. A cluster analysis showed two separate categories for the mathematical items and the items in these categories varied according to children's age. In the younger age group, items were categorised into 'pure mathematics' and 'applied mathematics'. Pure mathematics refers to number, algebra, and mental arithmetic items; whereas applied mathematics refers to items that measured geometry, measurement, and data handling. As for older children, the two clusters were 'easy' and 'difficult' items.

The researchers revealed some interesting patterns of associations between different components of working memory and different mathematical tasks (clusters) across younger and older children. In the younger children, the central executive and visuospatial working memory were significantly associated with their performance in both the pure and applied domains of mathematics. As for older children, phonological working memory was associated with performance on the easy items, whereas visuospatial working memory was related to the difficult items. The central executive component of working memory significantly contributed to performance on both easy and difficult items in older children.

A meta-analysis of 28 studies that compared the cognitive profiles of children with and without mathematical difficulties demonstrated that verbal working memory significantly differentiated children with mathematical difficulties from their typically achieving peers even after the effects of other variables, such as, age, intelligence, naming speed, and short-term memory were statistically controlled for (Swanson & Jerman, 2006). For example, Swanson and Beebe-Frankenberger (2004) examined the cognitive processes that underpin individual variation in working memory and word problem solving in a group of young children (1st to 3rd grade) at risk and not at risk for serious mathematical difficulties. They found that children who were at risk for mathematical difficulties had poorer performance on working memory, arithmetic, and word problem solving, than children who were not at risk for mathematical difficulties. After controlling for the influence of other cognitive variables, such as, intelligence, phonological processing, short-term memory, arithmetic competence etc., the researchers

found that a composite of working memory (verbal and visual-spatial) remained a significant predictor for children's accuracy in solving word problems.

However, most research that compared children with and without mathematical difficulties on working memory involved control groups that were matched on chronological age only. As I have argued earlier in the review of studies on counting, a better experimental design should also include a group of children who are matched on mathematical difficulties to rule out the possibility that the differences in working memory are simply due to differences in mathematical ability. A second limitation of this type of research is that very different diagnostic criteria of 'mathematical difficulties' were used in different studies that examined working memory, which led to a broad range of prevalence estimates (from 1.3% to 10.3%) (Devine, Soltesz, Nobes, Goswami, & Szucs, 2013). For example, some studies defined mathematical difficulty by performance cutoffs on standardised tests (Badian, 1983, 1999; Barahmand, 2008; Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005; Lewis, Hitch, & Walker, 1994). These cutoffs varied from performance below the 3<sup>rd</sup> percentile to performance below the 25<sup>th</sup> percentile. Some studies categorised children who had mathematical difficulty if their mathematical performance was equal to or below the average level of children two years younger (e.g., Gross-Tsur, Manor, & Shalev, 1996; Ramaa & Gowramma, 2002). Some researchers (e.g., Desoete, Roeyers, & De Clerq, 2004) defined mathematical difficulty as children exhibiting resistance to mathematics intervention. Thus, it is important to interpret the findings from these studies with caution.

In summary, these studies suggest that there is an independent association between working memory and children's mathematical achievement, whereas the relation is complicated by many factors, such as the age of children and the type of tasks used for assessing working memory and mathematical outcomes. However, these studies are correlational in nature that does not allow us to make inferences about how variables are connected with one another.

### **2.3.2 Longitudinal Studies**

The independent relation between working memory and children's mathematical achievement has also been found in some longitudinal studies. For example, Monette, Bigras, and Guay (2011) investigated the independent contributions of executive functions in children's early school achievement. In their study, eighty-five kindergartners were administered measures of executive functioning, including inhibition, flexibility, and working memory, and school readiness. Academic achievements at school were then evaluated at the end of Grade 1. The researchers found that children's mathematics and reading/writing skills at the end of Grade 1

were associated with the levels of executive functions at kindergarten. However, only working memory made a unique contribution to the variance in mathematical achievement after a number of covariates (pre-academic abilities, affective factors, and family characteristics) was statistically controlled for.

Spanning over a longer period of time in their longitudinal study, Alloway and Alloway (2010) examined whether working memory was merely a proxy for general intelligence or whether there was an independent contribution of working memory to learning outcomes in literacy and numeracy. They showed that children's working memory abilities at 5 years of age were the best predictor of literacy and numeracy 6 years later. General intelligence, in contrast, explained a small amount of unique variance in these learning outcomes. This study suggests that working memory is not a proxy for general intelligence, but rather represents a cognitive skill that has unique connections with academic achievements.

However, only measuring mathematical performance at a later time point does not necessarily reflect the learning of new materials. A better design of studies may be the one that predicts the growth in mathematical performance between two time points. Welsh, Nix, Blair, Bierman, and Nelson (2010) studied the developmental connections between growth in domain-general cognitive skills (working memory and attention control) and growth in domain-specific skills (emergent numeracy and literacy) across the pre-kindergarten period and their relative contributions to kindergarten mathematics and reading achievement. One hundred and sixty-four children were followed longitudinally. Using path analyses, the researchers found that working memory and attention control predicted growth in emergent numeracy and literacy abilities during the pre-kindergarten period. They also showed that the growth in the general cognitive abilities made independent contributions to the prediction of mathematics and reading achievements at kindergarten, after controlling for the growth in domain-specific skills. These findings suggest that prior to school entry, working memory and attention control play an important role in learning mathematics.

LeFevre, Berrigana, Vendettia, Kamawara, Bisanz, Skwarchukd, and Smith-Chante (2013) investigated the role of executive attention, which included the central executive component of working memory, in children's learning of two dimensions of mathematical skills – (a) knowledge of the number system, such as place value, and of arithmetic procedures, such as multi-digit addition and (b) arithmetic fluency (i.e. the speed of solutions to questions, such as  $3 + 4$  and  $8 - 5$ ). The sample consisted of 157 children in Grades 2 and 3 who completed tasks that measured executive attention and mathematical achievement. The mathematical tasks were administered

again 1 year later. Structural equation modeling showed that executive attention was concurrently associated with both knowledge of the number system and arithmetic fluency. They also found that executive attention did not predict growth in knowledge, but it predicted growth in fluency.

Swanson (2011) examined the growth in children's competence in mathematical problem solving in a longitudinal study of 127 children. This study showed that children's working memory at Grade 1 was one of the predictors that explained unique variance in their performance in word problem solving at Grade 3. The researcher also found that the growth in the executive component of working memory was significantly associated with the growth in the accuracy of word problem solving. The connection between the executive component of working memory and word problem solving performance remained significant beyond the influence of other cognitive measures, such as attention, inhibition, and processing speed. Hecht, Torgesen, Wagner, and Rashotte (2001) conducted a longitudinal study that lasted for a longer period of time with children from second to fifth grades to examine the unique contribution of working memory on growth in mathematical computation skills. After controlling for early mathematical skills, literacy, phonological processing abilities, and vocabulary, they found that working memory remained a significant predictor of the growth of mathematical performance from second to third grade.

Bull, Espy, and Wiebe (2008) examined the role of short-term memory, working memory, and executive functioning of preschool children in predicting their later performance in academic achievement at age 7. Children were tested on their cognitive abilities as well as literacy and mathematics competence at three time points: at the beginning and the end of the first year, and at the end of the third year of elementary school. Growth curve analyses demonstrated that visuospatial working memory was a specific predictor of children's performance in mathematics, whereas verbal working memory and inhibitory control predicted learning in both literacy and mathematics.

In summary, these longitudinal studies have shown that children's earlier working memory skills predict their mathematical achievement in an older age. The predictions seem to be independent of other number skills and general cognitive factors. However, the time-ordering nature of longitudinal studies represents only one necessary, but not sufficient, condition for inferring causal relation (Baltes & Nesselroade, 1979; Bradley & Bryant, 1983; Campbell, 1988; Farrington, 1991; Pellegrini, 1996; Wohlwill, 1973). To address the question of causality, we

need stronger evidence from experimental studies to examine whether a change in working memory leads to a change in children's performance in mathematics.

### **2.3.3 Experimental Studies**

The contributions of working memory to particular cognitive tasks can be investigated by examining individuals' performance on 'dual task experiments'. Dual task experiments require participants to solve a primary task while performing a secondary task at the same time. For example, participants are asked to solve simple addition problems while simultaneously articulating syllables. The secondary tasks are designed to represent various dimensions of a working memory system, such as, verbal tasks taxing the phonological loop (e.g., Hecht, 2002) and finger tapping (e.g., Seitz & Schumann-Hengsteler, 2000) taxing the visuospatial sketchpad. If the cognitive resources that are required to solve the tasks are overlapped, the success rate of the primary task will decline when the secondary task becomes more difficult. It has been suggested that dual task experiments offer evidence for understanding what cognitive processes are involved in mathematical processing. On the basis of this type of experiment, some research has suggested that working memory is one of the crucial elements in children's learning in mathematics.

For example, McKenzie, Bull, and Gray (2003) investigated the influence of phonological and visuospatial interference on children's performance in solving arithmetic problems. This study aimed to examine the cognitive processes and strategies being used in solving arithmetic problems in children of different age. Children were divided into two age groups (6-7 and 8-9 years) and asked to solve arithmetic problems under 3 conditions: baseline control, phonological interference, and visuospatial interference. In the phonological interference condition, children listened to a story while solving the arithmetic problems; whereas in the visuospatial interference condition, they were shown a matrix of black and white squares of which the colours alternate between black and white randomly during problem solving.

It was found that younger children's performance was negatively influenced by visuospatial disruption, whereas older children were affected to a significantly smaller extent. In contrast, younger children were not affected by phonological interference when solving arithmetic problems, but older children's performance markedly declined in the presence of phonological disruption. These findings suggest that children may resort to different strategies to solve arithmetic problems at different age – Younger children may use more visuospatial working

memory to support their mathematical processing, whereas older children may employ relatively more verbal working memory resources in mathematical problem solving.

Imbo and Vandierendonck (2007) investigated the role of executive resources in solving simple arithmetic problems among an older group of children from 4th to 6th grades. Children were asked to solve single-digit addition problems with and without a dual task requirement under 5 conditions. These conditions were (1) naming (i.e. reading an answer provided on the computer screen), (2) choice (i.e. children could choose their own strategies to solve the problem and had to report what strategies they had used for each problem), (3) no-choice – retrieval (children had no choice in strategy use, instead they were required to use retrieval to solve the problem), (4) no-choice – decomposition, and (5) no choice – transformation. The dual task was a judgment task (a choice reaction time task) involving presentation of tones that vary in pitch. Children were asked to press a computer key for low tones and another for high tones.

It was found that the dual task effects were significant for all strategies that children used. They also showed that the influence of working memory load decreased across grades if children were asked to solve problems using retrieval and counting strategies, but this effect was not observed in children who had to use decomposition strategies to solve problems. Imbo and Vandierendonck argued that older children might become more efficient in using retrieval and counting strategies to solve addition problems, which reduced the need for working memory resources to support the processes.

These dual-task experiments suggest that working memory is involved in children's mathematical processing and different aspects of working memory may interact with age that contributes to the process. In other words, it is likely that the roles of different types of working memory in mathematical processing differ for children who are at different stages of learning mathematics. However, most of these dual-task experiments focused on children's mental arithmetic only, and they rarely controlled for other factors that may also be involved in children's mathematical processing.

Experimental evidence can also come from intervention studies. If working memory is important for children's development of mathematical ability, training working memory should improve their performance in solving mathematical problems. Compared with cross-sectional and longitudinal studies, there are much fewer intervention studies in the literature. One of these studies was conducted by Holmes, Gathercole, and Dunning (2009). They used a computerised working memory task to train a group of 9-10 year-old children with poor working

memory skills. They found that the working memory training led to an improvement in mathematical achievement at a delayed post-test administered 6 months after the training. However, it is difficult to connect this improvement specifically to the working memory training because there was no comparison group at the delayed post-test.

St Clair-Thompson, Stevens, Hunt, and Bolder (2010) conducted similar computerized working memory intervention in a group of 5- to 8-year-old children. Compared with a control group, the intervention resulted in a significant improvement in working memory skills and mental arithmetic. However, they did not observe any impact of the intervention on a standardised test of mathematical achievement either at an immediate post-test or five months later. Using a different intervention strategy, Kroesbergen, van't Noordende and Kolkman (2014) conducted small group teaching sessions in 5-year-old children. The working memory training program involved non-computerised games with either numerical or non-numerical content. It was demonstrated that the visuospatial working memory in both groups significantly improved compared to controls, but counting skills improved only in the numerical training group.

Taken together, these studies have shown that their intervention programs improve performance on working memory tests. However, it is not clear whether they have succeeded in training the processes that enable the transfer of this improvement to other mathematical areas. Further research on the mechanisms and pathways by which working memory supports mathematics learning will help elucidate the processes that are necessary to be trained in order for real-world benefits to be observed.

In summary, this section has reviewed three types of research evidence that suggest that working memory may play an important role in children's mathematical learning. Cross-sectional studies have demonstrated that working memory is significantly and independently associated with mathematical achievement in children. However, some studies have also shown that the relations between different aspects of working memory and mathematical performance on particular tasks are complicated by the age of children. The wide range of tasks that measure working memory and mathematical outcomes has also rendered definitive conclusions difficult. Regarding the studies that compared children with and without mathematical difficulties, it remains unclear whether the difference in working memory are a cause of mathematical difficulty or simply the result of being worse at mathematics because most studies did not include control groups that are matched on children's mathematical abilities. An additional problem of comparing children with and without mathematical difficulties on working memory is

that there is no consensus with regard to the definition of ‘mathematical difficulties’.

Longitudinal studies have provided some evidence showing that working memory is predictive of children’s performance in mathematics over time. Dual task experiments have shown that working memory is involved in mathematical processing in children. However, a review of the intervention studies suggest that it remains to be established whether working memory has a causal relation with children’s mathematical attainment.

## **2.4 Mathematics Learning in East Asian Children**

It is interesting to examine mathematics learning in different cultures. One reason is that language differences may impact how the number system is structured in individuals’ mind. In other words, it influences how people in different cultures reason about mathematical information. For example, cross-cultural comparisons of young children’s mathematical skills have demonstrated that the mathematical performance of East Asian children is on average better than that of their non-Asian counterparts. Chinese children outperformed their Western peers in object and abstract counting as well as in concrete and mental addition and subtraction (Ginsburg, Choi, Lopez, Netly, & Chi, 1997; Huntsinger, Jose, Liaw, & Ching, 1997; Miller, Smith, Zhu, & Zhang, 1995). These differences can be attributable to a variety of factors, including the influence of language and the broader cultural contexts.

### **2.4.1 The Influence of Language on Numerical Processing**

The structure of the numerical sequence in different languages may influence the emergence of counting abilities and the cognitive representation of numbers. The East Asian number naming system (e.g., Chinese) may be beneficial for children’s learning mathematics. In Chinese, for example, the relation between the words for numbers in different denominations is transparent. The word for the number ‘14’ is the equivalent of ‘ten-four’. So once a person knows the Chinese words for 4, 2, and 10, he/she can figure out the words for 14, 40, 42, and 24. This is not always true for English-speaking people. When the Chinese say the equivalent of ‘ten-two’ for 12, English-speaking people say ‘twelve’. The Chinese words for 20 and 30 are the equivalent of ‘two-ten’ and ‘three-ten’, the English words for these numbers are ‘twenty’ and ‘thirty’. If we understand the logic of a numeration system, we can generate numbers that we have not heard before. It has been suggested that presumably because of the different ways to name numbers, Chinese-speaking children made fewer mistakes in saying the numbers to 19 and learned the numerical sequence between 109 and 2000 earlier than English-speaking children in



the United States (e.g., Fuson & Kwon, 1992). In Finland, sixty-three percent of the 7-year-old children could say number words accurately from 1 to 50 (Kinnunen, Lehtinen, & Vauras, 1994), whereas almost 100% of the 6-year-old Chinese-speaking children could say from 1 to 100 (Yang et al., 1985).

Miller and Stigler (1987) compared the way in which 4- to 6-year-old Taiwanese and American children counted and demonstrated striking differences. They found that the Taiwanese children performed much better than the American children in abstract counting (i.e. producing numerical sequences in correct orders). The American children were especially weak at counting the teens. When the children counted objects, there was no difference between the two groups of children in terms of their success in counting each object once (i.e. Gelman and Gallistel's one-to-one principle). However, the Taiwanese children again performed significantly better than their American peers in producing the right number words in the correct order. Miller, Smith, and Zhang (1995) conducted a longitudinal study to examine the counting skills between 2- to 4-year-old children in China and the United States. They found that the 2-year-old children in both countries had difficulty in reciting a list of 10 items. However, their performance started to diverge around 3 and 4 years. By the age of 4, the Chinese children could recite number names up to 100, while only a few children from the United States could do so.

The differences in the number naming system may go far beyond East Asian children's success in counting. It is possible that children who start to learn mathematics would benefit from a transparently regular system, such as Chinese. This kind of system helps children organise a notation system. When they use place value to write numbers, the digit on the right represents units, whereas the digit just to the left of it represents tens. Because the relation between the words for numbers in different denominations is transparent in Chinese, the same structure used in counting becomes a source of organization for the writing of numbers. Thus, computations become both economic and efficient.

The number words in a regular system may also make additive composition rather explicit. Additive composition of number refers to a property of numbers that any number 'n' can be broken down into two others that are smaller than it and that these two numbers add up exactly to 'n'. Understanding a base system involves the awareness that 14 can be decomposed into one ten plus four ones. In a regular numeration system, the words used with 'ten' and 'four' highlight this particular way of decomposing this number. This may contribute to the relatively early mastery of the concept of additive composition in children who use a more regular numeration system.

Miller (1987) investigated the potential influence of East Asian languages on numerical representations among Japanese- and English-speaking first graders who resided in the United States. The children were first introduced some base-ten blocks. There were two types of blocks – small cubes that represented a unit of one ('one' blocks), and three-dimensional rectangles that represented a unit of ten ('ten' blocks). The children were asked to show a set of five two-digit numbers with the blocks. Miura found that Japanese-speaking children tended to represent the two-digit numbers as a combination of the ten and one blocks (e.g., they used one 'ten' block and three 'one' blocks to represent '13'). In contrast, English-speaking children seldom chose to use 'ten' blocks to represent the two-digit numbers. Instead, they used 'one' blocks only (e.g., they used thirteen 'one' blocks for '13'). Miller found that around 75% of the Japanese-speaking children used correct combinations of ten and one blocks for all five two-digit numbers, whereas only around 50% English-speaking children did so. This study provided some early evidence that the number naming system may affect how children represent numbers. However, Miller's (1987) study was not without limitations. For example, all of the Japanese-speaking children lived in the United States, so it is likely that they were not immersed in a Japanese oral language environment to the same extent as those children who resided in Japan.

Follow-up studies by Miura and colleagues involved children who lived in the target country and spoke the official language of that country only. These studies were done within the first grade, so the children had not yet learned two-digit numbers in schools. The researchers have demonstrated evidence remarkably similar to Miller's (1987) initial findings. For example, first graders who speak East Asian languages (Chinese, Korean, and Japanese) tended to use combinations of 'ten' blocks and 'one' blocks to represent two-digit numbers (Miura, Kim, Chang, & Okamoto, 1988), whereas those who speak European languages (English, French, and Swedish) tended to use 'one' blocks only (Miura, Okamoto, Kim, Chang, Steere, & Fayol, 1994). All East Asian children in their studies showed at least one of the five numbers with the correct combinations of tens and ones. In contrast, around 50% of the English-speaking children did not use this kind of number representation at all (Miura, Kim, Chang, & Okamoto, 1988). These studies suggest that East Asian children may differ significantly from non-East Asian children in the types of representation they used for two-digit numbers. More recently, Miura and Okamoto (2003) investigated children with no experience with base-ten blocks and systematic learning about place value concepts. They replicated findings from previous studies by showing that the majority of first graders from France, Sweden, and the United States used one-to-one correspondence to represent two-digit numbers, whereas the first graders from Korea and Japan

tended to represent the numerals with a canonical base-ten representation. This finding suggests that East Asian children may have developed the concept of place values as an integral part of their numerical representations at a young age, while at the same time the English-speaking children may represent numbers as a group of counted objects.

Counting with the Chinese number words may also relate to the type of strategies that children used to solve mathematical problems. For example, Geary, Bow-Thomas, Fan, and Siegler (1993) found that Chinese children abandoned finger counting and used verbal counting during preschool years, while children in the United States shifted from finger counting to verbal counting until the end of first grade. The researchers also found that the Chinese children solved 3 times more addition problems than the children from the United States at the same age. Geary and colleagues (1993) argued that the relatively shorter Chinese number words facilitate children to keep track of the numbers, so they acquire the ability to count verbally earlier than English-speaking children. The spare working memory resources are important for mathematical development because they could be used for carrying out more efficient problem solving procedures.

#### **2.4.2 The Role of Culture in Mathematics Learning**

Language itself cannot be the only factor that explains the superior performance in mathematics in East Asian children. We need to consider the broader cultural context in order to obtain a more thorough understanding of why East Asian children had higher performance on the whole. This evidence is reviewed briefly.

For example, at a very young age, Chinese children have already received a high level of family support in learning mathematics (Cai, Lin, & Fan, 2004). Parents in major Chinese cities taught basic arithmetic calculation to their children at home before formal schooling (Zhang & Zhou, 2003). Chinese parents placed higher expectations on their children to perform well in mathematics than do Caucasian parents (Chen & Stevenson, 1995). Cai (2003) also found that Chinese parents were more involved than their American counterparts in their children's mathematics learning and this involvement was a significant predictor of children's mathematical attainment.

Teachers' and classroom practice may also explain the differences in mathematics performance between Chinese and Caucasian children. Stigler, Lee, Lucker, and Stevenson (1982) found that Chinese and Japanese teachers spent more time in teaching mathematical knowledge in class, compared with teachers from the United States. Leung (2005) found that the

mathematical curriculum of East Asian children were generally more difficult and thematically more coherent than the curriculum from English-speaking countries. Murata (2008) compared the textbook contents from Grades 1 to 6 between the Japanese curriculum and the American curriculum. She found that the contents of the Japanese textbooks were more inclined to stimulate students' thinking of mathematical situations. Correa, Perry, Sims, Miller and Fang (2008) found that Chinese teachers focused on students' interest in mathematics and how the knowledge could be applied to daily life. In contrast, teachers from the United States emphasized individual differences in learning styles and rote learning of mathematical problems. However, it should be noted that all of these studies did not directly address the association between teachers' practice and children's mathematical competence.

In summary, the review has shown that language is an important factor that affects children's understanding of the number structure, counting, and arithmetic calculation. Chinese-speaking children on average performed better than English-speaking children in place value understanding, verbal counting, and simple calculation. The regular structure of the numerical sequence in the Chinese language may be beneficial for the acquisition of counting skills and the cognitive representation of numbers. Cultural factors, such as, parental expectations, teachers' practice, and curriculum may also impact children's development of mathematical knowledge. However, studies that examined the mathematical development of Chinese children have focused on number skills and the factors that affect children's numerical competence. No study has been done on aspects of mathematical thinking in Chinese children. Given that there are a number of potential differences between Chinese and Caucasian children's learning mathematics, the strengths and relations among various cognitive factors may also be different. Thus, it would be interesting to examine whether the findings regarding the contributions of various cognitive factors could be replicated in this understudied cultural context.

## **2.5 Assessment of Additive Reasoning – The Role of Concrete Materials**

When conducting a study that involves quantitative reasoning, it is important to consider the ways in which we assess this construct. Some researchers argue that children can reason about certain things with concrete objects before they can use symbols to express them (Bruner, 1966; Piaget & Inhelder, 1971; Resnick, 1992). Thus, whether the children are given concrete materials to act on some quantities in an assessment setting may make a difference in their performance in additive reasoning.

In classic developmental theories, the acquisition of symbolic competence is believed to proceed through a concrete-to-abstract shift: the progression from thinking that is based on concrete reality to thinking that is less constrained by context. For example, Piaget (1952) postulates that the development of the ability to reason with abstract hypothetical propositions without the help of more concrete information was the final stage of cognitive development. Piaget observed that children at the concrete operational stage had difficulty in reasoning about false propositions that included relations that could not happen in the real world. For instance, if these children are shown the statements, 'If squirrels are bigger than tigers and tigers are bigger than elephants', they normally cannot deduce 'then squirrels are bigger than elephants.' These kinds of problems require the children to reason about the relations as given abstractly, rather than about the actual relations in the world. According to Piaget, these children do not succeed in these tasks because there is no concrete reference from which they can reason about and solve the problem.

Other popular theories have also seen development in terms of a transition from concrete to abstract. For example, in research of early categorization, Bruner (1966) argues that conceptual development is a perceptual-to-conceptual shift. At first, children can only think of objects in terms of the features that are directly available to their senses. After that, they start to consider abstract properties of objects. For instance, children may regard that bats and birds belong to the same category because they can fly. With development, children become realise that objects and living things can be categorised based on abstract and non-observable information. So eventually children understand that bats and birds should be in different categories because bats are mammals, and that even an animal that does not fly, such as penguins, can be in the bird category. Thus, the developmental shift is from a reliance on concrete properties to more abstract ones. Bruner argues "what is most important for teaching basic concepts is that the child be helped to pass progressively from concrete thinking to the utilization of more conceptually adequate modes of thought" (Bruner, 1960, p. 38).

Some of the Vygotsky's writings (Kozulin, 1986) also mention about the concrete-to-abstract transition. For example, Vygotsky suggests that young children classify things in terms of themes that are highly concrete and salient properties. He also argues for the important role of concreteness in symbolic play. When young children play, they often use concrete objects to substitute things in the real world (e.g., a stick for a horse). He thinks that the use of concrete objects in this manner is an initial form of symbolisation because the children are less constrained to the properties of the objects in the game. Thus, pretend play is important for

children's cognitive development in a way that it helps children to recognise that physical objects can be considered as a representation of something else.

On the basis of these theories, it is also reasonable to speculate that in additive reasoning, children at first succeed in problems that are set in the presence of concrete materials, and succeed only later when the problems refer to more abstract symbols, in the absence of objects. In one study, Hughes (1981) gave three kinds of tasks to children at the age of 3, 4, and 5. One task was called the 'closed box task' in which the children were shown a box containing a certain number of objects, and then the experimenter added or took away a certain number of objects. The second task was called a 'hypothetical' task in which the children were told about additions and subtractions in a hypothetical situation (either a box or a shop). In the first and second tasks, the children were asked to solve a problem that refers to quantities, such as, 'I put 2 bricks in this box and then I put 1 brick in this box; how many bricks are in the box now?' The third kind of task was called the 'formal code' task in which the children were given numerical problems without reference to any concrete materials or situations. For example, the children were asked, 'What is 2 plus 1?' Hughes found that the children performed well in the first two types of task that consisted of real or imagined concrete materials. Their performance was particularly good when small numbers were being added or subtracted. In contrast, the children performed poorly in the formal code task in which no concrete referents were used or mentioned. This study suggests that the kind of task that children are given make a difference. Thus, some children may need concrete materials in order to make sense of the problem that they are going to solve.

More recently, Bryant, Christie, and Rendu (1999) showed that the accuracy for children to solve three-term inverse problems ( $a + b - b = ?$ ) was significantly higher when the problems were presented with concrete objects. Studying an older age group (6-9 years old), Gilmore and Bryant (2006) also found that children were more accurate at solving inversion problems when they were presented in pictures. More recently, Sherman and Bisanz (2009) found that children (7-9 year-old) who solved equivalence problems (e.g.,  $5 + 4 = 7 + ?$ ) in a concrete condition had significantly higher accuracy rates than children who solved the same problems in a numerical context. Cowan and Renton (1996), however, did not find any difference between children's performance on recognising commutativity ( $a + b = b + a$ ) in concrete and abstract contexts. These studies suggest that the context in which conceptual knowledge is assessed may affect children's performance.

However, the findings have to be interpreted with caution because different researchers examined different conceptual principles with children in a diverse range of age groups. The inconsistent findings may also have resulted from the possibility that certain principles in additive relations are more difficult to understand than others, even though they belong to the same conceptual category 'additive reasoning'. For example, Canobi (2004) found that almost all 6- to 8-year-old children grasped the commutativity principle, but only 11 out of 90 children demonstrated partial understanding of the inversion and complement principles (e.g., if  $a + b = c$ , then  $b = c - a$ ). Recently, Torbeyns, Peters, de Smedt, Ghesquière, and Verschaffel (2016) showed a wide range of individual differences in children's knowledge of the complement principle even at the ages of 9 to 10 years. Most children at this age group should have mastered the commutativity principle, but the researchers found that the children did not understand the complement principle even though it is also based on part-whole relations in additive reasoning. Canobi, Reeve and Pattison (2003) assessed additive reasoning in different contexts (e.g., concrete and numerical) and revealed some interesting interactions between the use of concrete materials and the understanding of the logical relations in additive reasoning. They found that concrete materials only aided children in the understanding of the associativity principle but not the commutativity principle. However, some limitations in Canobi et al. (2003) assessments of conceptual knowledge merit attention in further research.

First, in the task that the researchers assessed the commutativity knowledge, children were asked to judge if ' $a + b$ ' was helpful for a puppet to solve ' $b + a$ '. A child may truly understand the commutativity principle in order to answer the question correctly. However, it is also likely that some children produce a correct response without paying attention to the operation. That is, they may say ' $a + b$ ' is helpful for a puppet to solve ' $b + a$ ' simply because both numbers ' $a$ ' and ' $b$ ' are present. To design a better measure, we may need to involve a control item, i.e. ' $b - a$ ' to ensure that children have a true understanding of the commutativity principle. Because the commutativity principle is applicable to addition only but not subtraction, children should recognise that ' $a + b$ ' is helpful for a puppet to solve ' $b + a$ ' but not ' $b - a$ ' if they truly understand the concept. Thus, children's both performance on the ' $a + b$ ' and ' $a - b$ ' items should be considered together in order to calculate an index for an understanding of the principle.

Second, the problems in Canobi et al. (2003) study were presented in numerical format and the numbers and operations were written down and shown to the children during the assessment (e.g., whether  $3 + 4 = 7$  is helpful to solve  $7 - 4 = ?$ ). One potential problem of this

presentation format is that the children may formulate a rule to solve the problems easily, for example, by shifting a number from one side to another and change the sign. An alternative format of presentation is to embed the numbers in story problems. To reduce the working memory demands of the task, we may need to present the problems orally to the children and at the same time present the written form of the story problems to the children until they gave an answer. In this way, children are easier to keep track of the contents and to make relevant judgments accordingly.

In conclusion, there is evidence that suggest that concrete materials may make a difference in children's performance on mathematical reasoning tasks. Concrete materials may help some children to think about the part-whole relation in additive reasoning. However, little research has examined the role of concrete materials in children's development of mathematical principles and the findings are mixed. Thus, it remains to be verified whether there is a progress in children's understanding of mathematical principle from concrete to more abstract problems. With regard to additive reasoning, more evidence is needed to evaluate whether there is an order of understanding of the principles (from the commutativity to the complement principle). Assessing additive reasoning in various ways (with and without concrete materials) would also allow us to examine the developmental sequences of the two principles and the role of concrete materials in this development. This way of assessment may also capture more variability in children's knowledge of the principles.

## **2.6 The Present Study**

In summary, this chapter has reviewed evidence regarding the roles of working memory, counting ability, and additive reasoning in children's mathematics learning. On the basis of the literature review, several research gaps are identified. First, from the mathematical thinking perspective, it is important to assess the conceptual aspects of counting. Learning to count matters for children to learn mathematics because it helps children reflect on the analytical and representational meanings of number. Thus, if a child counts without understanding what she or he is doing, she or he should not be considered as mathematically competent from the mathematical thinking perspective. Related to this argument is that procedural tasks alone, such as counting number sequences, are not good indicators of counting ability because they do not necessarily reflect children's understanding of the logic of counting. Therefore, measures that capture children's knowledge of the relations of counting to quantities such as, the ability to



identify the cardinality of a set, should also be included in research studies. However, to the best of my knowledge, most predictive studies used procedural counting as the sole indicator of counting ability, which is a limitation that needs to be addressed in future studies.

Second, the relation between additive reasoning (as measured by the knowledge of commutativity and the inverse relation between addition and subtraction) and mathematical achievement remains unclear. There are mixed findings regarding its connection with children's calculation ability, however, there seems to be no study that examines its relation to children's ability to solve different types of story problems. More research that incorporates both calculation and story problem solving as the outcome measures are needed because cognitive correlates of mathematical ability may vary across mathematical tasks (Chong & Siegel, 2008; Cowan & Powell, 2014; Hughes, 1981; Geary, Hoard, Nugent, & Byrd-Craven, 2008).

Third, the contribution of additive reasoning to mathematical achievement has never been investigated in a non-Caucasian cultural context. Given that the dominant language and other cultural factors may differ from one country to another, it is important to evaluate the predictive power of various cognitive factors on children's mathematical achievement in a different culture. Finally, the measures of additive reasoning also merit empirical attention because it remains controversial whether concrete materials make a difference in children's performance on mathematical reasoning tasks. It is interesting to explore whether these materials help children think about the part-whole relation in additive reasoning and whether there is an order of the understanding of the principles (from the commutativity to the complement principle).

To recapitulate, the aim of the present thesis is to examine the extent to which working memory, counting ability, and additive reasoning uniquely contributes to individual differences in mathematical achievement in a sample of Chinese-speaking children. I use a longitudinal design, which spans around 10 months, to examine whether the main predictors would independently predict children's mathematical achievement (calculation and story problem solving). This study represents the first longitudinal study that investigates the relative contributions of these three factors to mathematical achievement in Chinese children. Using different ways to assess additive reasoning may also reveal interesting findings regarding the developmental nature of the understanding of the commutativity and complement principles in the domain of addition and subtraction.

On the basis of theories and literature review, the present study addresses the following hypotheses:

1. Counting ability is important in children's mathematics learning and its influence is independent from that of general cognitive capacities, such as working memory.
2. Additive reasoning (as assessed by knowledge of commutativity and the complement principle) is independent from and more important than counting ability and general cognitive capacities, such as working memory in children's mathematics learning.
3. Working memory, as a domain-general factor, makes a contribution to mathematical achievement, even when one accounted for children's specific mathematical knowledge such as their knowledge of counting and additive reasoning.
4. Knowledge of the commutativity and of the complement principles develops from thinking in the context of specific quantities to thinking about more abstract symbols.
5. There is an order of understanding of the principles – from the commutativity to the complement principle.

## **Chapter Three    Method**

In Chapter Two, I have reviewed evidence regarding the contributions of working memory, counting ability, and additive reasoning to variations in children's mathematical performance. The first section of the current chapter is a discussion of the research design of the present study. It is followed by a description of the characteristics of the participating children. After that, I turn to the operational definitions of the main predictors, control variables, and mathematical achievement, which involve a description of the tasks that were used to assess various constructs. This chapter concludes with the procedure of the study.

### **3.1    Research Design**

On the basis of the mathematical thinking approach, it is important for children to understand the relational meanings of numbers and quantities in order to succeed in mathematics. Three cognitive foundations were hypothesised to support children to think mathematically, which contributed to their performance in tasks of mathematical achievement, such as calculation and story problem solving. The first hypothesis concerns the contribution of counting ability. It was hypothesised that counting ability is important in children's mathematics learning and its influence is independent from that of general cognitive capacities, such as working memory. The second hypothesis states that additive reasoning is independent from and more important than counting ability and general cognitive capacities, such working memory for children to learn mathematics. The third hypothesis considers the role of working memory. It was hypothesised that working memory makes a contribution to mathematical achievement, even when one considered the influence of children's specific mathematical knowledge, such as, counting ability and additive reasoning.

To address these hypotheses, a 10-month longitudinal design was employed in the present study. Longitudinal studies allow researchers to examine the temporal order of events. If children's performance in the main predictors was assessed at an earlier time point than their performance in mathematical achievement, it could be more credible that the predictors are antecedents of mathematical achievement. The evidence regarding the temporal order of events is important because it relates to what we should focus on teaching mathematics. For example, if additive reasoning is seen to be a positive antecedent to mathematical achievement, intervention programs may offer a way to teach children that make them directly experience more about reasoning in their learning. In order to determine the direction and strengths of associations between a predictor and mathematical achievement, statistical techniques, such as

multiple regression analysis, can be used to evaluate the unique contributions of each predictor to variations in mathematical achievement.

Testing took place on two occasions in this study. On the first testing occasion, Time 1 (during the first grade of the participating children), measure of demographic characteristics, non-verbal intelligence [Raven's Standard Progressive Matrices (Raven, Raven, & Court, 2003)], four measures of working memory from the Working Memory Test Battery for Children (WMTB-C) (Pickering & Gathercole, 2001), two measures of counting ability (procedural and conceptual knowledge of counting), the additive reasoning tasks (testing the commutativity and complement principles), and two measures of mathematical achievement (calculation and story problem solving) were administered. The main predictors were working memory, counting ability, and additive reasoning, whereas outcome variable at Time 1 was children's performance in calculation and story problem solving.

To assess mathematical achievement, two measures including calculation and story problem solving in the domain of addition and subtraction were used in this study for three reasons. First, both tasks are commonly assessed in research and school. Second, children at this age are expected to learn addition and subtraction in school. Third, some researchers (Chong & Siegel, 2008; Cowan & Powell, 2014; Hughes, 1981; Geary, Hoard, Nugent, & Byrd-Craven, 2008) have shown that cognitive correlates of mathematical ability may vary across mathematical tasks, which supports the rationale to examine multiple indicators of mathematical ability in this study.

In order to evaluate whether any association between the main predictors and mathematical achievement was also explained by other factors, several control variables were selected for the present study. The first was non-verbal intelligence. Cattell and Horn (Cattell, 1971; Horn, 1968) classify intelligence into two main types: fluid intelligence and crystallised intelligence. Fluid intelligence comprises the ability of abstract reasoning, whereas crystallised intelligence refers to knowledge that is accumulated with age and education. Non-verbal intelligence has been considered a kind of fluid intelligence and it represents the ability to recognise the relationships of visually presented materials, such as similarities and differences between shapes and patterns. Because tests on non-verbal reasoning do not require reading, they give insight into an individual's competence to reason with materials abstractly without the constraints by language skills. Some studies showed that general intelligence was an independent predictor of a range of academic achievements, including mathematics (e.g., Deary, Strand, Smith, & Fernandes, 2007; Jensen, 1998; Stevenson, Parker, Wilkinson, Hegion, & Fish, 1976; Taub, Floyd, Keith, & McGrew, 2008; Walberg, 1984). For example, in a large-scale 5-year longitudinal study involving 70,000

students, Deary et al. demonstrated that intelligence contributed to 60% of the variance of students' performance on a national mathematics test. Thus, non-verbal intelligence was included as one of the control variables in the present study.

Other control variables include demographic characteristics, such as sex of children and their mothers' highest educational levels. Some studies showed that girls' performance was better than boys' on the basic arithmetic tasks in the British National Curriculum Key Stage 1 measurements (Demie, 2001, Gorard, Rees, & Salisbury, 2001; Strand, 1997; Strand, 1999). Strand (1999) found that girls performed better than boys at the age of four and seven and showed that boys made significantly more improvement than girls over three years. In both Australia and Finland, girls' competence in early numeracy was also better than boys' (e.g., Aunio, Aubrey, Godfrey, Yuejuan, & Liu, 2008; Aunio, Hautamäki, Heiskari, & Van Luit, 2006; Boardman, 2006). However, in the United States, Jordan, Kaplan, Oláh, and Locuniak (2006) found that boys in kindergarten performed better than girls in numeracy skills. Some studies showed no significant differences between boys and girls in mathematical performance (e.g., Aubrey & Godfrey, 2003, Aunola, Leskinen, Lerkkanen, & Nurmi, 2004, Carr & Jessup, 1997, Fennema, Carpenter, Jacobs, Franke, & Levi, 1998; Nunes, Bryant, Sylva, & Barros, 2009). Thus, research findings regarding the sex differences in mathematics are still mixed, but the possible influence of the sex of children on mathematical achievement would be analysed in this study.

Mothers' highest educational level has been regarded as a proxy variable for socioeconomic status. Children may have more opportunities to participate in a wider range of mathematical activities if their parents are more educated (Clement & Sarama, 2007). The socioeconomic gap in children's mathematical development in kindergarten and primary schools has been well documented (e.g., Aunola et al., 2004; Jordan, Huttenlocher, & Levine, 1992; Jordan et al., 2006, Sammons & Smeeds, 1998, Saxe, Guberman, & Gearhart, 1987, Strand, 1999; Tzouriadou, Barbas, & Bonti, 2002). Among various indicators of socioeconomic status, Nunes, Bryant, Sylva, and Barros (2009) showed that mothers' highest educational level was the best predictor of mathematical achievement. Therefore, it was selected as the indicator of socioeconomic status in the present study.

The second testing occasion, Time 2 (during the second grade of the participating children), comprised two measures of mathematical achievement (calculation and story problem solving) and a measure of Chinese word reading. Chinese word reading was included as the outcome control measure in order to test the specificity of certain variables on mathematical performance. If additive reasoning (i.e. complement and commutativity principles) is specifically

relevant to mathematics learning, children's performance on this task should predict much better for their success in mathematical tasks than non-mathematical tasks, such as, Chinese word reading. By contrast, general cognitive ability should correlate with both mathematical and non-mathematical tasks because all of these tasks demand cognitive resources, such as, working memory and general intelligence. This kind of design has been adopted in some longitudinal research of children's reading (e.g., Bradley & Bryant, 1983) but it is rare in studies that address children's mathematics learning (e.g., Nunes, Bryant, Barros, & Sylva, 2012).

The independent contributions of each predictor measured at T1 to mathematical achievement at the second testing occasion was assessed by taking into account the effects of age, non-verbal intelligence and demographic factors. For each child, the interval between the first and second wave of assessments was between 9 and 11 months, with 10 months being the commonest interval (83%). Table 3.1 summarises the different measures used on the two testing occasions.

*Table 3.1 Measures used in the longitudinal study*

Time 1	Time 2
Non-verbal intelligence	Mathematical achievement
<ul style="list-style-type: none"> <li>• Raven's Standard Progressive Matrices</li> </ul>	<ul style="list-style-type: none"> <li>• Calculation</li> </ul>
Demographic variables	<ul style="list-style-type: none"> <li>• Story problem solving</li> </ul>
<ul style="list-style-type: none"> <li>• Sex of children</li> <li>• Mother's educational level (socioeconomic status)</li> <li>• School</li> </ul>	Chinese word reading
Working memory	
<ul style="list-style-type: none"> <li>• Counting recall (central executive)</li> <li>• Digit span backward (central executive)</li> <li>• Digit span forward (phonological loop)</li> <li>• Corsi blocks (visuospatial)</li> </ul>	
Counting ability	
<ul style="list-style-type: none"> <li>• Procedural counting</li> <li>• Conceptual understanding of counting</li> </ul>	
Additive reasoning	
<ul style="list-style-type: none"> <li>• Commutativity principle</li> <li>• Complement principle</li> </ul>	
Mathematical achievement	
<ul style="list-style-type: none"> <li>• Calculation</li> <li>• Story problem solving</li> </ul>	

## 3.2 Participants

One hundred and fifteen children (61 boys, 54 girls) studying in three primary schools in Hong Kong participated in both waves of assessments in this longitudinal study. All of these children spoke Cantonese and attended the first year of primary school, with a mean age of 76.32 months ( $SD = 2.81$  months, ranging from 67.8 to 82.1 months), during the first wave of assessment. The mean age of the children during the second wave of assessment was 86.34 months ( $SD = 2.81$  months, ranging from 77.8 to 92.1 months). All of the children were reported to have intelligence within the range accepted as normal for their ages, and did not have learning difficulties or emotional/behavioral problems, such as, dyslexia, specific language impairments, attention deficits and hyperactivity disorders, or any neurological disorders.

On the basis of previous studies (e.g., Canobi et al., 2003; Gilmore & Bryant, 2006; Nunes et al., 2007, 2012), an *a priori* power analysis (Cohen, 1988; GPower 3.1; Faul, Erdfelder, Lang, & Buchner, 2007) indicated that a minimum sample size of 91 was needed to detect a medium effect size [using Cohen's (1988) criteria] with an alpha of .05 and power of .80 using multiple regression analyses, therefore the current sample size was considered sufficient.

The highest educational levels attained by the mothers of the children in the sample were as follows: No schooling/pre-primary school level – 5.2%, primary school graduates – 20.8%, secondary school graduates – 57.4%, and university graduates – 16.5%. According to the Hong Kong Population Census (2011), the distribution of educational attainment (highest level attained) was: No schooling/pre-primary school level – 10%, primary school graduates – 19.2%, secondary school graduates – 46.6%, and university graduates – 24.1%. Thus, the relative distribution of educational levels was similar to that of the overall Hong Kong population, in which the majority of the population was secondary school graduates whereas a small proportion received no schooling or had pre-primary educational level.

## 3.3 Measures

### 3.3.1 Main Predictors

#### 3.3.1.1 Working memory

Working memory was assessed at Time 1 as one of the main predictors. Working memory plays an important role in mathematical thinking because it determines how well individuals maintain and process relevant information. Thus, it was hypothesised that working memory would make a contribution to mathematical achievement, even when one accounted for

children's specific mathematical knowledge, such as their knowledge of counting and additive reasoning.

This construct was operationalised on the basis of Baddeley's working memory model. In this model, there are three essential components of working memory: phonological loop, visuospatial memory, and central executive. In this study, the relations of all three components to mathematical achievement were tested. Working memory was assessed with four tasks, including (1) digit span forward (phonological loop), (2) digit span backward (central executive), (3) counting recall (central executive), and (4) Corsi blocks (visuospatial). There were two tasks for central executive memory because previous research showed that measures of the central executive were stronger predictors of children's mathematical performance than other working memory measures (e.g., Cowan & Powell, 2014; Gathercole & Pickering, 2000; Holmes & Adams, 2006; Keeler & Swanson, 2001; Lee, Ng, Ng, & Lim, 2004; Lehto, 1995; Noel, Seron, & Trovarelli, 2004; Swanson & Beebe-Frankenberger, 2004; Wilson & Swanson, 2001).

The working memory task refers to the Working Memory Test Battery for Children (WMTB-C) (Pickering & Gathercole, 2001). In the digit span forward task, children listened to a series of single-digit numbers and were asked to repeat the numbers in the correct order. All digits were presented at a rate of one per second. The series of numbers initially consisted of two numbers, and increased by one number after every other presentation, to a maximum of nine. Children were given one point for each sequence correctly recalled. The maximum possible score for this task was 16. The internal consistency of this task was satisfactory (Cronbach's  $\alpha = 0.81$ ).

The digit span backward was similar to the digit span forward, except that the children were asked to recite the numbers backward. In counting recall, children were asked to count the triangles in a series of shape arrays and then to recall the total number of triangles in each series. The number of arrays started from two and increased by one array after every other presentation to a maximum of nine. The total number of correct trials was used as an indicator of participants' performance on these tasks. The maximum possible score for this task was 16. The internal consistency of this task was satisfactory (Cronbach's  $\alpha = 0.89$ ). The listening recall task was not used in this study because it cannot be simply translated into Cantonese without a proper investigation of how it works in this language.

The Corsi block task involved nine blocks and the experimenter tapped a sequence of blocks at a rate of one per second. Then, children were asked to replicate the sequence. The sequence involved two blocks initially and increased by one block every other presentation, to a maximum



of nine. The maximum possible score for this task was 16. The internal consistency of this task was satisfactory (Cronbach's  $\alpha = 0.83$ ).

For each of the above tasks, there were two trials for each span length and testing was terminated when a child failed two trials of the same length. In each task, two practice items were given to the children and no feedback was given to the children in any of the testing trials.

### **3.3.1.2 Counting ability**

Counting activity may help children think about the meanings of number. Thus, counting ability was included at Time 1 as one of the main predictors of mathematical achievement. It was hypothesised that counting ability is important in children's mathematics learning and its influence is independent from that of general cognitive capacities, such as working memory. Counting ability was operationalised as (1) children's ability to count with accuracy (procedural counting) and (2) their ability to recognise the counting principles and the coordinated use of various counting principles (conceptual knowledge of counting).

Procedural counting was assessed with two tasks: oral rote counting and object counting. In oral rote counting, children counted some numerical sequences verbally in ascending and descending orders. They were first asked to count from 5 to 16 as a practice trial. There were then eight testing trials in which children were asked to count a set of numbers in ascending orders (e.g., 25 to 32; 56 to 63; 76 to 81; 118 to 123) and in descending orders (e.g., 46 to 38; 73 to 65; 34 to 27; 121 to 115). Testing within a set was discontinued when a child had committed errors on two sequences in a set. Children received one point for each sequence completed correctly.

Another task, object counting, was also included as one of the measures of procedural counting to test whether the children could count correctly using one-to-one correspondence between words and objects. In object counting, they were required to count two trials of geometric shapes (e.g., circles, squares) and two trials of recognizable objects (e.g., pens, rubber). The numbers of objects were 6, 9, 13, and 15 for rubber, pens, squares, and circles, respectively. On any given trials, the objects were identical in appearance. The order of task presentations was counterbalanced across participants. Children received one point for each correct counting. The total scores for procedural knowledge of counting for each child was the sum of his/her performance on the oral rote and object counting tasks. The maximum possible score was 12. The internal consistency of this procedural counting task was satisfactory

(Cronbach's  $\alpha = 0.71$ ). Table 3.2 summarises the types of trials in the measures of procedural counting.

*Table 3.2 Types of trials of procedural counting*

Types of trial	Description
<i>Oral rote counting</i>	
In ascending order	25-32; 56-63; 76-81; 118-123
In descending order	46-38; 73-65; 34-27; 121-115
<i>Object counting</i>	
Geometric shapes	13 squares, 15 circles
Recognisable objects	6 rubbers; 9 pens

Conceptual knowledge of counting was assessed with a counting judgment task adapted from previous work (e.g., Briars & Siegler, 1984; Freeman, Antonucci, & Lewis, 2000; LeFevre et al., 2006). As shown in Table 3.3, three types of trials were used – (1) correct counts (four trials), (2) incorrect counts (six trials), and (3) correct but unusual counts (six trials). Thus, children evaluated a total of sixteen counts. On each trial, a set of objects ranging in number from 6 to 12 was shown to children. A puppet 'Pika' was introduced, and the researcher (the author) explained to the children that Pika was just learning to count.

*Table 3.3 Counting judgment task (adapted from LeFevre et al., 2006)*

Type of trial	Trial	N	Description
<i>Correct counts</i>	1, 4, 13, 15	6, 8, 9, 12	Conventional left-to-right count
<i>Incorrect counts</i>			Violations of word-object correspondence
Repeated words	2, 11	7, 10	Pika used an incorrect (repeated) number word that did not correspond to an item (i.e., one, two, two)
Skipped object	7, 16	6, 11	Pika missed counting an item in the regular sequence and never returned to it
Double count	8, 14	12, 8	Pika counted one item twice
<i>Unusual counts</i>			Violations of conventional features
Reverse direction	3, 12	7, 12	Pika counted from the right to the left
Start in the middle	5, 9	11, 9	Pika started counting in the middle of the set, counted to the right end, and then went back to the beginning to finish
Double point	6, 10	9, 10	Pika hopped twice on an item but repeated the correct number word twice (e.g., eight eight)

*Note.* Trial refers to position in the order of presentation. N refers to the number of items for each trial.

The same protocol designed by previous researchers (e.g., Freeman, Antonucci, & Lewis, 2000) was used: 'This is Pika and he would like you to play a counting game with him. He is going to count the things on the table. But he is just learning to count, and sometimes he makes mistakes. Sometimes he counts in ways that are okay, but sometimes he counts in ways that are not okay and he was wrong. Watch carefully while he counts. When he has finished counting, you tell me if he counted okay or not okay.' Items were put in a row at 1 cm intervals. Pika faced each child and always counted one item per second from the child's left to right except on the reverse direction trials. After each trial, children were asked whether the count was ok or not okay ('error detection'). Then, they were given 10 seconds to answer the question that tested their understanding of cardinality – 'How many things are there in total?' The maximum possible scores for 'error detection' and 'cardinality' were 16, respectively. The internal consistency of the entire conceptual knowledge of counting measure was satisfactory (Cronbach's  $\alpha = 0.85$ ).

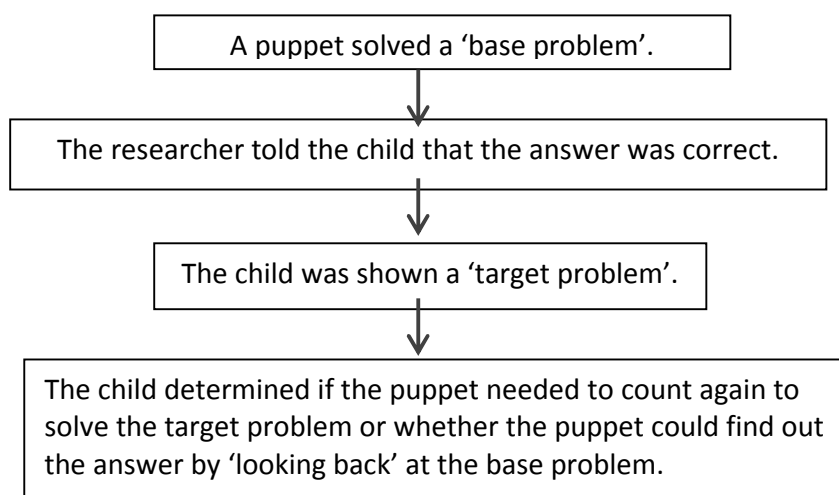
### **3.3.1.3 Additive reasoning (the commutativity and complement principles)**

Additive reasoning was included at Time 1 as one of the main predictors of mathematical achievement. The mastery of additive reasoning involves learning about the properties of addition and subtraction. In this study, it was operationalised as children's understanding of the commutativity and complement principles. The commutativity principle refers to the irrelevance of addend order to the sum, i.e. ' $a + b = c$ ' implies ' $b + a = c$ ', whereas the complement principle refers to the inverse relation between addition and subtraction, i.e. ' $a + b = c$ ' implies ' $c - a = b$ '. It was hypothesised that additive reasoning is independent from and more important than counting ability and general cognitive capacities, such working memory for children to learn mathematics.

This study adapted a similar conceptual task used by Canobi, Reeve, and Pattison (2003) in which children were tested whether they could recognise conceptual relations between pairs of addition/subtraction problems. The general design of this task is summarised in Figure 3.1. Children were shown a puppet that was going to solve two problems, namely base and target problems. The puppet 'solved' the base problem by counting very quickly and told the answer to the researcher (the author), who then told the children that the answer was correct. After that, the children were shown a target problem and were asked to determine whether the puppet needed to count again to solve the problem or whether the puppet could find out the answer by 'looking back' at the base problem.

All of the problems were presented as story problems that involved a change in quantity (e.g., Mary has 3 fish and her mother gave her 5 more). Change problems were used instead of Combine problems because Change problems appeared to be a more rigorous test of understanding of the commutativity principle of addition (De Corte, & Verschaffel, 1987; Wilkins, Baroody, & Tiilikainen, 2001). De Corte and Verschaffel (1987) found that young children tended to apply commutativity in the context of a Change problem than in the context of a Combine problem.

The number of words in each problem did not vary considerably. The experimenter presented each child with a written version of the problem as it was read and kept it in front of the child until the problem was solved. For example, after showing the base and target problems that were printed on two separate cards, the researcher asked, 'Now look at these two problems. If we gave Pika (the puppet) this problem next (pointing to the target problem), do you think Pika would need to count to work out the answer or could Pika look back at the problem he has already done (pointing to the base problem)?'



*Figure 3.1. An overview of the general design of the conceptual task*

The conceptual judgment task involved two parts: a 'testing session' immediately after a 'warm-up session'. In the 'warm-up session', the children were given six practice problems to familiarise with the procedure. Half of the practice problems were identical (e.g., base:  $4 + 4$  and target:  $4 + 4$ ) and half of them were different (e.g., base:  $4 + 3$  and target:  $6 + 7$ ). Children were given feedback on whether they were correct in judging the same/different relation between

the target and base problems. The answers were 100% correct for all participants in this session, indicating that they understood the task instructions.

In the 'testing session', the researcher showed six target problems in random order after asking the puppet to solve the base problem (e.g., Mary has 3 fish and her mother gave her 5 more). The target problems included (1) an identity problem, which was identical with the target problem (e.g., Mary has 3 fish and her mother gave her 5 more); (2) a different problem, which was completely unrelated to the target problem (e.g., Mary has 7 fish and her mother gave her 2 more). The identity and different problems were designed to detect possible responses biases, which may involve inattention, difficulty in understanding the procedure, and random responses. The accuracy rates for all the identity and different problems were 100%.

To assess children's knowledge in each of the additive reasoning principles, two types of items were used: test items and control items. Examples of these items are presented in Table 3.4. The test items were designed to assess children's understanding of a particular principle. They included (1) commutativity test items, which were related to the corresponding base problems on the basis of the commutativity principle (e.g.,  $5 + 3$ ); (2) complement test items, which were related to the corresponding base problems according to the complement principle (e.g.,  $8 - 5$ ).

Control items were included to detect whether children answer the question correctly because of biases. For example, children may answer that ' $3 + 5 = 8$ ' is helpful for solving ' $5 + 3$ ' correctly just because they realise that two numbers in the base problem (i.e. 3 and 5) are present in the target problem ( $5 + 3$ ). These children may not understand the commutativity principle but simply have a response bias to say 'yes' when the numbers are the same. Children with such a response bias would also answer that ' $3 + 5 = 8$ ' helps to solve the question ' $5 - 3$ '. These control items included (1) commutativity controls: subtraction items that evaluate whether the children did not simply ignore the operation to make a judgment (e.g.,  $5 - 3$ ); and (2) complement controls, which involved addition problems that comprised the sum and one term of the base problem added together (e.g.,  $8 + 5$ ).

Table 3.4 Types of target problems, examples, and their purpose

Base Problem	Types of its Corresponding Target Problems	Purpose of the Target Problems
Mary has 3 fish and her mother gave her 5 more. How many fish does Mary have now? (Answer: $3 + 5 = 8$ )	<i>Commutativity test item:</i> 'Mary has 5 fish and her mother gave her 3 more. How many fish does Mary have now?'	To test children's understanding of the commutativity principle. The base problem should be helpful to solve this item because the answer of ' $5 + 3$ ' can be deduced by ' $3 + 5 = 8$ ' according to the commutativity principle.
	<i>Commutativity control item:</i> 'Mary has 5 fish and her mother took away 3 from her. How many fish does Mary have now?'	To allow for a correction for response biases. The base problem should not be helpful to solve this control item because the answer of ' $5 - 3$ ' cannot be deduced by ' $3 + 5 = 8$ ' according to the commutativity principle.
	<i>Complement test item:</i> 'Mary has 8 fish and her mother took away 5 from her. How many fish does Mary have now?'	To test children's understanding of the complement principle. The base problem should be helpful to solve this item because the answer of ' $8 - 5$ ' can be deduced by ' $3 + 5 = 8$ ' according to the complement principle.
	<i>Complement control item:</i> 'Mary has 8 fish and her mother gave 5 more to her. How many fish does Mary have now?'	To allow for a correction for response biases. The base problem should not be helpful to solve this control item because the answer of ' $8 + 5$ ' cannot be deduced by ' $3 + 5 = 8$ ' according to the complement principle.

Thus, the control items did not serve to measure the constructs, but they were there to allow for a correction for response biases. A child was only credited one point if they answered both the test and the control items correctly (see Table 3.3). There were 6 commutativity items and 6 control items for commutativity; if the child passed one commutativity item and its control, the child was awarded one point; otherwise, no points were awarded. Similarly, there were 6 complement items and 6 control items for the complement principle; if the child passed one complement item and its control, the child was awarded one point; otherwise, no points were awarded.

Table 3.5 Scoring method for children's commutativity and complement knowledge

Commutativity Test items	Commutativity Control items	Commutativity Score	Complement Test Items	Complement Control Items	Complement Score
Correct	Correct	1	Correct	Correct	1
Correct	Incorrect	0	Correct	Incorrect	0
Incorrect	Incorrect	0	Incorrect	Incorrect	0

The problems were presented to children in two conditions: one with bricks (concrete condition) versus another without bricks (abstract condition). In the concrete condition, children were presented with the same problems as in the abstract condition but with bricks that represented the addends of the problems. In the warm-up session of the concrete condition, the experimenter modeled the problems by attaching or pulling apart groups of bricks. The bricks were attached to each other so that they could not be counted easily. Prior to the judgment task, the experimenter allowed the children to play with the bricks to ensure that the children felt comfortable with the bricks and recognised that the bricks were attached to each other. All of the bricks were of the same size and colour.

In the concrete condition, the experimenter moved the bricks from a base problem card to a target problem card and at the same time attached and separated bricks in front of the children in order to help children identify the part-whole relations of the quantities. For example, the commutativity problems involved swapping the order of addends by moving the bricks representing these addends so that they were arranged in a different order. The complement problems involved adding or taking away the bricks representing the addends or subtrahends, respectively. The concrete and abstract conditions were conducted on two separate days. The possible range of scores for each principle (commutativity and complement) in each condition (concrete versus abstract) was 0 to 6. In all trials, feedback was not given. The internal consistencies of the additive reasoning measures were satisfactory (Commutativity: Cronbach's  $\alpha = 0.81$ ; Complement principle: Cronbach's  $\alpha = 0.85$ ).

In all conditions, half of the problems had sums less than 10 (small number) and half of them had sums between 15 and 25 (large number). If the children solve the problems with their understanding of the commutativity and complement principles, they should be able to provide correct answers regardless of the size of the numbers involved in the problems. If the children solve the problems by calculating, they should make significantly more mistakes for large-number problems than small-number problems. Another indicator of the use calculation to solve problems is how long it takes for the children to respond. If they rely on calculation, exceptionally long response latency can be observed. In this study, it is likely that the children solved the additive reasoning tasks on the basis of conceptual knowledge rather than calculation because all of them responded within a short period of time (within 12 seconds) and the number size did not make a significant difference in accuracy rates for all conditions ( $ps > .05$ ). However, the response latencies were not used for testing the hypotheses because it is not clear what

they represent. A child who has long response latency may indicate that she or he is not good at the concept or simply slow to respond in general.

### **3.3.2 Control Variables**

#### **3.3.2.1 General intelligence**

Children's general intelligence was measured with Raven's Standard Progressive Matrices (Raven, Raven, & Court, 2003) at Time 1. This test was considered because it has been a robust measure of non-verbal aspect of intelligence and has been used widely in previous research. It is a standardised test including five sets of twelve items each. Each item involves a target matrix with a missing piece. Children were asked to choose, from six or eight alternatives, the best figure to complete the target matrix. One mark was given for the correct answer for each item.

#### **3.3.2.2 Demographic characteristics**

Other control variables included demographic information reported by parents in a questionnaire, namely children's sex and mothers' highest education level at Time 1.

### **3.3.3 Outcome Measures**

#### **3.3.3.1 Mathematical achievement – Calculation**

A calculation task was adapted on the basis of a recent study conducted with children studying primary school in Hong Kong (Wong, Ho, & Tang, 2014). In this task, children were asked to solve simple addition and subtraction problems (e.g.,  $2 + 7 = ?$ ,  $11 + 6 = ?$ ) which was designed with reference to the local mathematics curriculum. This study showed that the task had good internal reliability and children's performance on this task correlated with that on a standardised test in mathematical achievement in Hong Kong. All items of the arithmetic task were available in the published article, and were used as a reference to develop a similar task for the present study.

Fifty items were tested in a pilot study with item analyses, which evaluated whether the task had an appropriate level of difficulty and discriminating power (Crocker & Algina, 1986; Gronlund & Linn, 1990; Pedhazur & Schemlkin, 1991; Thorndike, Cunningham, Thorndike, & Hagen, 1991). In order to select age appropriate items for testing, thirty-one children at Grade 1 and twenty-three children at Grade 2 participated in the pilot study.

In the pilot, the  $p$  and  $d$  indexes were calculated for each outcome measures (calculation and story problem solving). The  $p$  index was determined by the proportion of children who answered



individual items correctly. An easier item would have a higher  $p$  index (Wood, 1960). The  $d$  index measured the discriminating power of items. It compared the number of people with high test scores who correctly answered an item with the number of people with low scores who got a correct answer for the same item. If an item has a high discriminating power between high and low scorers, there should be more people in the top-scoring group who answer the item correctly. According to Wiersma and Jurs (1990), the  $d$  index is computed as follows. The children were divided into three groups on the basis of their performances on the task as a whole – an upper group with 27% who had the highest scores, a lower group with 27% who had the lowest scores, and a middle group consisting of the rest 46%. The  $d$  index was calculated by the number of people in the upper group who got the right answer minus the number of people in the lower group who got the right answer, divided by the number of people in the largest of the two groups. According to Ebel and Frisbie (1986), items with a  $d$  index 0.3-0.39 are considered as good items, 0.4 or above as very good.

*Table 3.6 Calculation tasks at Time 1 and Time 2*

T1 Calculation (First Grade)	<p>Eight addition of numbers up to 25:  <math>6 + 7</math>; <math>3 + 8</math>; <math>2 + 6</math>; <math>9 + 16</math>; <math>7 + 4</math>; <math>2 + 16</math>; <math>14 + 4</math>; <math>11 + 7</math></p> <p>Eight subtraction from numbers less than 25:  <math>7 - 5</math>; <math>9 - 6</math>; <math>6 - 4</math>; <math>12 - 3</math>; <math>21 - 16</math>; <math>22 - 18</math>; <math>25 - 6</math>; <math>18 - 5</math></p>
T2 Calculation (Second Grade)	<p>Five addition of sum larger than 25:  <math>24 + 4</math>; <math>8 + 19</math>; <math>7 + 23</math>; <math>21 + 5</math>; <math>9 + 19</math></p> <p>Five subtraction from numbers more than 25:  <math>28 - 9</math>; <math>31 - 8</math>; <math>27 - 5</math>; <math>28 - 19</math>; <math>26 - 8</math></p> <p>Three 3-addend single digit problems:  <math>3 + 9 + 2</math>, <math>7 + 2 + 4</math>; <math>8 + 5 + 2</math></p> <p>Three 3-subtrahend single digit problems:  <math>8 - 4 - 3</math>; <math>13 - 3 - 8</math>; <math>15 - 7 - 5</math></p>

Fifty items (25 addition and 25 subtraction) were constructed and tested in the pilot. On the basis of the  $p$  and  $d$  indexes, sixteen items (8 addition and 8 subtraction) were selected for each wave of data collection. Table 3.6 shows the items used in the main study. At Time 1, children were orally presented with addition and subtraction combinations that involved eight addition of numbers up to 25 and eight subtractions from numbers less than 25 (see Table 3.5). At time 2, children were orally presented with ten addition and subtraction problems with large

numbers, three 3-addend single digit problems, three 3-subtrahend single digit problems. A printed version of each calculation problem was presented as each problem was read and kept in full view of the child during problem solving. Feedback was not provided and no time limit was set. The maximum possible score for calculation was 16. The measures appeared to have good internal consistency (T1:  $\alpha = .87$ ; T2:  $\alpha = .92$ ).

### **3.3.3.2 Mathematical achievement – Story problem solving**

Similarly, eight types of word problems were tested from the same pilot study as in calculation. Thirty-two problems were chosen in the main study on the basis of the  $p$  and  $d$  indexes. Table 3.7 shows the examples of the story problem problems. On the basis of Riley, Greeno, and Heller's (1983) classification of story problems, Time 1 assessment included four result unknown Change problems, four start unknown Change problems, four change unknown Change problems, four unknown difference set Compare problems, four unknown compare set Compare problems, four unknown reference set Compare problems, four different unknown Combine problems, and four Equalize problems.

At Time 2, the number of each type of problems was the same, except that two Combine problems were replaced by two more difficult 'de-combine transformations problems' (e.g., John played two games of marbles. In the second game he lost seven marbles. His final result, with the two games together, was that he had won three marbles. What happened in the first game?). For each type of problems, half of them (i.e. two for each type) involved small numbers (sum < 10), whereas half of them involved larger numbers (10 < sum < 20).

To reduce the working memory demands of the task, the experimenter presented the each child with a written version of the story problem as it was read and kept it in front of the child until the problem was solved. In this way, children were easier to keep track of the contents and to make relevant judgments accordingly. The maximum possible score for story problem solving was 32. The measures appeared to have good internal consistency (T1:  $\alpha = .92$ ; T2:  $\alpha = .91$ ).

Table 3.7 Examples of Story Problems

<p><b>Change problems (Time 1 and Time 2)</b></p> <p><u>Result unknown</u></p> <ul style="list-style-type: none"> <li>David had 7 books. Then Peter gave him 5 more books. How many books does David have now?</li> <li>John had 16 candies. Then he gave 4 candies to Harry. How many candies does John have now?</li> </ul> <p><u>Change unknown</u></p> <ul style="list-style-type: none"> <li>Alan had 4 books. Then Kenny gave him some more books. Now Alan has 5 books. How many books did Kenny give him?</li> <li>Susan had 17 flowers. Then she gave some flowers to Mary. Now Susan has 6 flowers. How many flowers did she give to Mary?</li> </ul> <p><u>Start unknown</u></p> <ul style="list-style-type: none"> <li>Emily had some candies. Then Annie gave her 3 candies. Now Emily has 5 candies. How many candies did Emily have in the beginning?</li> <li>Jason had some flowers. Then he gave 6 flowers to Terry. Now Jason has 18 flowers. How many flowers did Jason have in the beginning?</li> </ul> <p><b>Equalize problems (Time 1 and Time 2)</b></p> <ul style="list-style-type: none"> <li>Rose has 7 books. Doris has 10 books. How many books must Rose get from Doris in order to have as many as Doris?</li> <li>Sue has 6 flowers. Eva has 3 flowers. How many flowers does Sue need to give away to Eva in order to have as many as Eva?</li> </ul>	<p><b>Combine problems (Time 1 and Time 2)</b></p> <p><u>Difference unknown</u></p> <ul style="list-style-type: none"> <li>Edison and Ray have 16 candles altogether. Edison has 5 candles. How many candles does Ray have?</li> </ul> <p><b>Compare problems (Time 1 and Time 2)</b></p> <p><u>Unknown difference set</u></p> <ul style="list-style-type: none"> <li>Janet has 8 flowers. May has 3 flowers. How many flowers does Janet have more than May?</li> <li>Ben has 18 candies. Lois has 6 candies. How many candies does Lois have fewer than Ben?</li> </ul> <p><u>Unknown compare set</u></p> <ul style="list-style-type: none"> <li>Billy has 9 books. Jamie has 7 more books than Billy. How many books does Jamie have?</li> <li>Joe has 15 books. Steven has 4 books less than Joe. How many books does Steven have?</li> </ul> <p><u>Unknown reference set</u></p> <ul style="list-style-type: none"> <li>Tom has 10 pencils. He has 4 more pencils than Jack. How many pencils does Jack have?</li> <li>Ivan has 19 flowers. He has 7 flowers less than Samuel. How many flowers does Samuel have?</li> </ul> <p><b>De-combine transformations problems (Time 2 only)</b></p> <ul style="list-style-type: none"> <li>John played two games of marbles. In the second game he lost 7 marbles. His final result, with the two games together, was that he had won 3 marbles. What happened in the first game?</li> </ul>
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### 3.3.3.3 Chinese word reading

On the basis of the Hong Kong Lexical Lists for Primary Learning (Hong Kong Education Bureau, 2013), a word recognition task was constructed. According to the corpus, there are 4,914 words in Key Stage One (grade one to three). Fifty words from this corpus were chosen for the pilot test. On the basis of the  $p$  and  $d$  indexes, thirty words were included in the word recognition task in the main study. Of these 30 items, eight were easy items (average correct rate: 70-100%), twelve had moderate difficulty (average correct rate: 40-70%), and ten were difficult (average correct rate: 0-40%). All items, except the easy items, had a  $d$  index over 0.3. The items were arranged from the easiest words at the beginning to the most difficult ones towards the end of the test. In this task, children were shown written two-character Chinese words and asked to read aloud. One point was given for each correct response. No feedback was given. The maximum possible score for this task was 30. The internal consistency of this measure was satisfactory (Cronbach's  $\alpha = 0.93$ ).

## 3.4 Procedure

This study was approved by the Central University Research Ethics Committee (CUREC) that reviews research studies on their ethical aspects. Participating children were recruited through local schools and non-profit child-related community centres in Hong Kong. Parents were informed of the study via letters sent home by teachers or/and administrators. Upon receipt of parental consent, the children were asked for verbal assent and participated individually with the author in a quiet location, which was separate from other children in the primary school or centre. At Time 1 (first grade), the children were tested in two 30-40 min sessions separated by approximately 1 week. For all children, order of task presentation was the same. The first session included Raven's Standard Progressive Matrices, the central executive, phonological loop, and visuospatial sketchpad tasks, as well as the tasks that assessed children's knowledge of the commutativity and complement principles in the abstract condition additive reasoning. The second session involved tasks that assessed procedural and conceptual knowledge of counting, additive reasoning (concrete condition), calculation and story problem solving. At Time 2 (second grade), the children were tested in one session that lasted for approximately 20 to 30 minutes in which the Chinese word reading, calculation, and story problem solving tasks were administered. For each child, the interval between the first and second wave of assessments was between 9 and 11 months, with 10 months being the commonest interval (83%). For all children, testing was conducted by the author in Cantonese during the day.

## **Chapter Four     Multiple Regression Analyses**

The aim of the present study was to investigate the relative importance of working memory, counting ability, and additive reasoning in children's mathematical achievement. Multiple regression analysis is used to address the hypotheses of this study because it allows us to quantify the relation between a particular factor and mathematical achievement while holding the effects of other factors constant (Cohen & Cohen, 1983; Field, 2013; Pedhazur, 1994; Stevens, 2002; Tabachnick & Fidell, 2001). An overview of the analysis plan is described as follows.

The first section of this chapter presents preliminary analyses. First, the descriptive statistics are examined. In particular, the distribution of the scores of calculation and story problem solving at both waves of assessment are evaluated, because one assumption of regression analysis is that the outcome measure has to be normally distributed. If this assumption is violated, regression analysis may not be an appropriate tool for analysing the data.

Second, the effects of demographic variables on mathematical achievement are investigated. Demographic variables (e.g., the sex of children, the highest educational levels of mothers, and different schools that the children attend) may explain both children's performance in working memory, counting ability, and additive reasoning, on one hand, and their mathematical achievement, on the other hand. Thus, it could be a third factor that explains the relation between a main predictor and mathematical achievement (Bradley & Bryant, 1983). There is a need to include these factors in subsequent regression models if children's performance on the main predictors and mathematical achievement differs across any of the demographic variables.

Third, the associations between variables are examined. The correlation patterns of variables can be used to examine how different measures relate to a particular construct. For example, two measures (e.g., backward digit span and counting recall) can be used to represent one single construct (e.g., the central executive component of working memory). The scores should be significantly correlated with each other if the measures indicate the same construct. This can serve as one of the justifications, in addition to theoretical considerations, for combining the scores to form a composite that represents the construct. Examining the correlations also allows us to have an initial idea of how each variable relates to mathematical achievement.

In the second section, different sets of fixed order multiple regression analyses are presented. In this study, the main predictors are classified into three sets of factors, namely working memory, counting ability, and additive reasoning. Each factor (e.g., working memory) is

entered as a block after age and IQ to examine the amount of variance in mathematical achievement that a particular factor can explain when the effects of age and IQ are controlled for. Each factor is also entered as the final block of each of the models, which provides information about how much variance each factor accounts for mathematical achievement after the effects of all the other factors have been controlled for. In each of the regression models, we can also examine whether individual variables within a factor (e.g., the central executive in working memory) are unique predictors of mathematical achievement. Thus, fixed order multiple regression analyses offer a good test of the hypotheses of this study because they tell us about the relative contributions of each variable to mathematical achievement (Cohen & Cohen, 1983; Field, 2013; Pedhazur, 1994; Stevens, 2002; Tabachnick & Fidell, 2001).

In the final part of this section, findings of autoregressive analyses are presented to assess whether the unique predictors identified in the previous regression analyses remain significant predictors for T2 mathematical achievement when the effects of T1 mathematical achievement and all the other factors are controlled for. What these factors had in common with mathematical achievement at T1 may explain their longitudinal predictive power. If they remain significant longitudinal predictors of variance after all these controls, the case for their predictive value can be considered very strong.

#### **4.1 Revisiting the Hypotheses and Predictions**

The first hypothesis is that counting ability is important in children's mathematics learning and its influence is independent from that of general cognitive capacities, such as working memory. According to this hypothesis, counting ability should explain an additional amount of variance in children's performance in calculation and story problem solving concurrently and longitudinally beyond the effects of age, IQ, and working memory.

The second hypothesis states that additive reasoning (as assessed by knowledge of commutativity and the complement principle) is independent from and more important than counting ability and working memory in children's mathematics learning. If the results show that additive reasoning accounts for an additional amount of variance in children's performance in calculation and story problem solving concurrently and longitudinally after the effects of counting ability and working memory were statistically controlled for and it explains a larger amount of the variance than counting ability and working memory, that would provide support for this hypothesis.

The third hypothesis states that working memory makes a contribution to mathematical achievement, even when one accounted for children's specific mathematical knowledge, such as their knowledge of counting and additive reasoning. If working memory explains a significant amount of variance in children's performance in calculation and story problem solving concurrently and longitudinally, even when the effects of other factors, such as counting ability and additive reasoning were taken into account, this hypothesis is supported.

## **4.2 Preliminary Analyses**

### **4.2.1 Descriptive Statistics**

Table 4.1 shows the descriptive statistics for each variable. Of particular concern is whether the scores of the outcome measures of mathematical achievement are normally distributed. It is necessary to examine whether the normality assumption of regression analysis is violated in order to evaluate whether regression is an appropriate statistical tool to address the hypotheses of this study (Cohen & Cohen, 1983; Field, 2013; Pedhazur, 1994; Stevens, 2002; Tabachnick & Fidell, 2001).

Children's performance in calculation and story problem solving at Time 1 and Time 2 were used as the indicators of mathematical achievement. The distributions of children's scores on calculation and story problem solving were analysed with regard to the z-values of Skewness and Kurtosis of each outcome variable. The z-value of Skewness was calculated by dividing the Skewness value by its standard errors, whereas z-value of Kurtosis was calculated by dividing the Kurtosis value by its standard error. Table 4.1 shows that none of the z-values are higher than 1.96, suggesting that the scores do not violate the normality assumption.

Table 4.1 Descriptive statistics for domain-general factors, counting ability, additive reasoning, and mathematical achievement (N = 115)

	Reliability ( $\alpha$ coefficients)	Possible range	Mean	Standard deviations	Minimum	Maximum	Skewness z-value	Kurtosis z-value
<i>Domain-general factors</i>								
Age in months	N.A.	N.A.	76.32	2.81	67.8	82.1	-1.87	-0.34
Non-verbal intelligence: Raven's raw scores	0.81	0-60	19.45	2.94	15	26	1.52	-1.81
Working memory: Central executive	0.89							
Digit span backward		0-16	7.51	1.88	4	10	-0.01	-1.12
Counting recall		0-16	7.93	1.61	6	10	0.06	-1.45
Working memory: Phonological loop	0.81							
Digit span forward		0-16	11.37	2.3	8	16	0.5	-2.2
Working memory: Visuospatial sketchpad								
Corsi span	0.83	0-16	10.47	2.01	6	16	0.65	-1.1
<i>Counting ability</i>								
Procedural counting	0.71	0-12	11.02	1.08	9	12	-3.39	-1.6
Conceptual knowledge of counting	0.85							
Error detection		0-16	13.37	1.91	10	16	-0.4	-0.66
Cardinality		0-16	14.23	1.14	12	16	-0.13	-0.74
<i>Additive reasoning</i>								
Commutativity principle	0.81							
Concrete condition		0-6	4.21	1.17	1	6	-3.79	1.1
Abstract condition		0-6	4.14	1.39	1	6	-3.57	0.32
Complement principle	0.85							
Concrete condition		0-6	2.75	1.43	0	5	-0.6	-1.79
Abstract condition		0-6	2.05	1.54	0	5	1.8	-2.11
<i>Mathematical achievement</i>								
Time 1 calculation	0.87	0-16	11.03	2.85	5	16	-0.09	-1.73
Time 2 calculation	0.92	0-16	10.95	3.04	5	16	0.39	-1.85
Time 1 story problem solving	0.92	0-32	22.38	4.43	14	32	1.21	-1.13
Time 2 story problem solving	0.91	0-32	23.63	3.57	16	30	-1.04	-1.17



#### **4.2.2 Examining the Influence of Demographic Variables**

Demographic variables may explain differences in children's scores on the main predictors (working memory, counting ability, and additive reasoning) as well as the scores in mathematical achievement. In other words, they may be extraneous variables that accounts for the relation between a main predictor and mathematical achievement (Bradley & Bryant, 1983). If children's performance on the main predictors and mathematical achievement differs across any of the demographic variables, it is necessary to include these factors in subsequent regression models.

To assess the effects of demographic variables on mathematical achievement, I conducted independent t-tests with children's sex and one-way analyses of variance (ANOVA) with mothers' educational level and school separately as a fixed factor for each predictor and each measure of mathematical achievement. The results are reported in Table 4.2, 4.3, and 4.4. All variables showed no evidence of significant influence of children's sex, mothers' educational level, and school. Intraclass correlations (ICC) were also calculated according to Cohen, Cohen, West, and Aiken (2003) to examine whether there was any evidence of clustering. All ICCs ranged from 0.01 to 0.07, which were close to 0. The very low within-cluster correlations suggest that there is no clustering in the present data. Thus, these variables were not included in the regression analyses.

Table 4.2 Sex differences in all predictors and mathematical achievement (N = 115)

	Sex	N	Mean	Std. Deviation	Levene's <i>F</i> -tests	<i>p</i> -values	T-tests ( <i>t</i> -values)	<i>p</i> -values
Age in months	Boys	61	76.24	2.85	0.01	0.94	-0.29	0.77
	Girls	54	76.40	2.80				
Raven's raw scores	Boys	61	19.89	2.73	1.14	0.29	1.69	0.09
	Girls	54	18.96	3.12				
Procedural counting	Boys	61	11.03	1.08	0.36	0.55	0.16	0.87
	Girls	54	11.00	1.08				
Counting error detection	Boys	61	13.67	1.88	0.56	0.46	1.80	0.08
	Girls	54	13.04	1.90				
Cardinality understanding	Boys	61	14.31	1.19	1.10	0.30	0.85	0.40
	Girls	54	14.13	1.08				
Digit span backward	Boys	61	7.54	2.01	2.00	0.16	0.17	0.87
	Girls	54	7.48	1.75				
Counting recall	Boys	61	7.77	1.64	0.66	0.42	-1.13	0.26
	Girls	54	8.11	1.57				
Digit span forward	Boys	61	11.21	2.17	1.65	0.20	-0.80	0.43
	Girls	54	11.56	2.45				
Corsi span	Boys	61	10.39	2.12	1.24	0.27	-0.43	0.67
	Girls	54	10.56	1.88				
Commutativity-concrete	Boys	61	4.11	1.14	0.73	0.39	-0.92	0.36
	Girls	54	4.31	1.19				
Commutativity-abstract	Boys	61	4.11	1.42	0.00	0.96	-0.20	0.84
	Girls	54	4.17	1.38				
Complement-concrete	Boys	61	2.72	1.53	3.28	0.07	-0.21	0.83
	Girls	54	2.78	1.31				
Complement-abstract	Boys	61	2.00	1.54	0.46	0.50	-0.38	0.70
	Girls	54	2.11	1.56				
T1 Calculation	Boys	61	11.20	2.79	0.05	0.83	0.65	0.52
	Girls	54	10.85	2.93				
T2 Calculation	Boys	61	10.95	3.10	0.80	0.37	0.01	0.99
	Girls	54	10.94	3.01				
T1 Story problem solving	Boys	61	22.53	4.38	0.09	0.77	0.36	0.72
	Girls	54	22.22	4.53				
T2 Story problem solving	Boys	61	23.69	3.68	0.60	0.44	0.17	0.87
	Girls	54	23.57	3.47				

*Table 4.3 Socioeconomic status (indicated by mothers' highest education levels) differences in all predictors and mathematical achievement (N = 115)*

	SES	N	Mean	Std. Deviation	ANOVA F-values	p-values
Raven's raw scores	1	6	17.17	1.83	2.297	0.082
	2	24	19.58	2.57		
	3	66	19.29	3.01		
	4	19	20.58	3.10		
Procedural counting	1	6	10.83	1.17	1.52	0.213
	2	24	10.71	1.27		
	3	66	11.20	0.96		
	4	19	10.84	1.12		
Counting error detection	1	6	12.67	1.63	0.307	0.821
	2	24	13.50	2.21		
	3	66	13.39	1.82		
	4	19	13.37	1.98		
Cardinality understanding	1	6	14.00	1.10	0.104	0.958
	2	24	14.29	1.20		
	3	66	14.23	1.16		
	4	19	14.21	1.08		
Digit span backward	1	6	7.67	1.51	0.374	0.772
	2	24	7.17	1.95		
	3	66	7.64	1.89		
	4	19	7.47	1.98		
Counting recall	1	6	8.33	1.51	0.409	0.747
	2	24	7.67	1.63		
	3	66	7.94	1.64		
	4	19	8.11	1.56		
Digit span forward	1	6	10.33	1.97	0.739	0.531
	2	24	11.33	2.48		
	3	66	11.33	2.41		
	4	19	11.89	1.70		
Corsi span	1	6	10.67	1.03	0.499	0.684
	2	24	10.25	2.31		
	3	66	10.39	1.93		
	4	19	10.95	2.15		

1 = No schooling/pre-primary school level, 2 = primary school graduates, 3 = secondary school graduates, 4 = university degree holders

*Table 4.3 Socioeconomic status (indicated by mothers' highest education levels) differences in all predictors and mathematical achievement – continued (N = 115)*

	SES	N	Mean	Std. Deviation	ANOVA F-values	p-values
Commutativity-concrete	1	6	4.33	1.37	0.465	0.707
	2	24	3.96	1.40		
	3	66	4.27	1.09		
	4	19	4.26	1.10		
Commutativity-abstract	1	6	4.00	1.26	0.023	0.995
	2	24	4.13	1.65		
	3	66	4.15	1.28		
	4	19	4.16	1.57		
Complement-concrete	1	6	3.00	1.26	0.293	0.831
	2	24	2.54	1.28		
	3	66	2.76	1.48		
	4	19	2.89	1.52		
Complement-abstract	1	6	1.83	1.72	0.46	0.711
	2	24	1.79	1.35		
	3	66	2.09	1.56		
	4	19	2.32	1.73		
T1 Calculation	1	6	11.33	2.50	0.743	0.528
	2	24	10.83	2.28		
	3	66	10.83	3.01		
	4	19	11.90	3.07		
T2 Calculation	1	6	11.00	2.00	0.116	0.951
	2	24	10.63	3.19		
	3	66	11.02	3.10		
	4	19	11.11	3.11		
T1 Story problem solving	1	6	23.33	2.42	0.553	0.647
	2	24	21.52	4.16		
	3	66	22.84	4.50		
	4	19	24.21	5.08		
T2 Story problem solving	1	6	25.00	2.10	0.706	0.551
	2	24	22.88	3.88		
	3	66	23.70	3.55		
	4	19	23.64	3.57		

1 = No schooling / pre-primary school level, 2 = primary school graduates, 3 = secondary school graduates, 4 = university degree holders

*Table 4.4 School differences in all predictors and mathematical achievement (N = 115)*

	School	N	Mean	Std. Deviation	ANOVA F-values	p-values
Raven's raw scores	1	34	19.68	3.23	0.431	0.651
	2	45	19.13	2.82		
	3	36	19.64	2.86		
Procedural counting	1	34	10.88	1.10	0.616	0.542
	2	45	11.00	1.15		
	3	36	11.17	0.97		
Counting error detection	1	34	13.18	1.59	0.257	0.774
	2	45	13.44	1.80		
	3	36	13.47	2.32		
Cardinality understanding	1	34	14.00	1.05	1.08	0.344
	2	45	14.38	1.07		
	3	36	14.25	1.30		
Digit span backward	1	34	7.59	1.83	0.347	0.707
	2	45	7.33	2.00		
	3	36	7.67	1.82		
Counting recall	1	34	7.94	1.74	0.018	0.982
	2	45	7.96	1.45		
	3	36	7.89	1.72		
Digit span forward	1	34	11.59	2.35	0.7	0.499
	2	45	11.51	2.18		
	3	36	11.00	2.41		
Corsi span	1	34	10.18	1.99	0.522	0.595
	2	45	10.62	1.95		
	3	36	10.56	2.12		
Commutativity-concrete	1	34	4.50	0.90	1.98	0.143
	2	45	3.98	1.23		
	3	36	4.22	1.27		
Commutativity-abstract	1	34	4.44	1.13	2.11	0.126
	2	45	3.82	1.45		
	3	36	4.25	1.50		
Complement-concrete	1	34	2.91	1.42	0.47	0.626
	2	45	2.60	1.54		
	3	36	2.78	1.29		
Complement-abstract	1	34	2.09	1.29	0.798	0.453
	2	45	1.84	1.55		
	3	36	2.28	1.75		
T1 Calculation	1	34	11.12	2.36	0.129	0.879
	2	45	10.87	3.06		
	3	36	11.17	3.07		
T2 Calculation	1	34	11.09	2.76	0.074	0.928
	2	45	10.96	3.02		
	3	36	10.81	3.39		
T1 Story problem solving	1	34	23.00	3.93	0.537	0.586
	2	45	21.96	4.45		
	3	36	22.33	4.89		
T2 Story problem solving	1	34	24.18	3.04	0.563	0.571
	2	45	23.36	3.59		
	3	36	23.47	4.01		

#### 4.2.3 Associations between Variables

In this study, composite scores of some variables were created on the basis of theoretical and empirical reasons (previous research and the correlations between variables in the present data). According to Baddeley (Baddeley & Hitch, 1974, Baddeley, 1992), working memory consists of three components: the central executive, phonological loop, and visuospatial sketchpad. The central executive has been commonly measured by two tasks in previous studies: digit span backward and counting recall. The correlation between the scores in these tasks in the present sample was also significant ( $r = .43$ ). According to Cohen (1988), this correlation value indicates a moderate effect ( $.30 < r < .50$ ). Thus, a composite score was formed for central executive by averaging the standardised scores of the constituent measures. In subsequent analyses, the three components of working memory were considered separately because of two reasons: First, in theory (Baddeley & Hitch, 1974, Baddeley, 1992), the central executive, phonological loop, and visuospatial sketchpad are three related but separate components in working memory. Second, most researchers have treated them as three distinct factors in previous studies (e.g., Gathercole & Pickering, 2000; Henry & MacLean, 2003; Holmes & Adams, 2006; Keeler & Swanson, 2001; Lee, Ng, Ng, & Lim, 2004; Lehto, 1995; Noel, Seron, & Trovarelli, 2004; Swanson, 1994; Swanson & Beebe-Frankenberger, 2004; Wilson & Swanson, 2001).

Three tasks were used to measure children's counting ability: procedural counting, counting error detection, and cardinality understanding. Theoretically the latter two tasks explicitly measure children's understanding of the counting principles, and in the present study, the correlation between the scores in these two tasks was moderate and significant ( $r = .39$ ). Therefore, a composite score that represented 'conceptual knowledge of counting' was formed by averaging the standardised scores of these measures. Although the scores of procedural counting strongly correlated with the composite score of conceptual knowledge of counting ( $r = .62$ ) (Cohen, 1988;  $r > .50$  indicates a strong correlation), they were considered separately in subsequent analyses because there is evidence that some children failed to coordinate their knowledge of the three counting principles in these tasks even though they demonstrated competence in reciting the number sequence and applied it to objects and events (e.g., Bermejo, Morales, & deOsuna, 2004; Freeman, Antonucci, & Lewis, 2000; Sarnecka & Gelman, 2004; Sophian, 1988).

Additive reasoning was measured by children's performance on two tasks that assessed their understanding of the commutativity and complement principles in two different testing conditions (concrete versus abstract conditions). The scores for the commutativity knowledge in the two testing conditions were strongly correlated with each other ( $r = .73$ ), and the correlation between the scores of the complement knowledge in the two testing conditions was also high ( $r = .51$ ) (Cohen, 1988). Thus, the standardised scores of children's performance in the concrete and abstract conditions were averaged to form a composite score. Similarly, another composite score that represented the complement knowledge was formed by averaging the standardised scores of children's performance in the two testing conditions.

The next set of analyses explores the correlations between the main predictors (i.e. working memory, counting ability, and additive reasoning) and each measure of mathematical achievement at the two waves of assessment (Time 1 and Time 2). Table 4.5 shows the bivariate correlations between the variables. Several key findings are identified.

First, The scores of central executive, digit span forward, and Corsi span significantly correlated with each other. This result is consistent with the theoretical model of working memory (Baddeley & Hitch, 1974; Baddeley, 1992) that these three components are related to each other. However, the correlations of the scores on these measures with mathematical achievement varied. Children's performance in the central executive correlated moderately (all coefficients  $> .30$ ) with the scores in calculation and story problem solving at both Time 1 and Time 2. By contrast, the scores of visuospatial sketchpad had no significant correlation with any measure of mathematical achievement at both time points. The scores of phonological loop had significant correlations with calculation at Time 1 and Time 2, but did not correlate with story problem solving at both time points.

Second, both indicators of the construct 'counting ability', procedural and conceptual knowledge of counting, had significant correlations with children's performance in calculation concurrently and longitudinally. However, only conceptual knowledge of counting correlated with children's performance in story problem solving. Finally, All measures of additive reasoning had strong (all coefficients close to and larger than 0.50) and significant correlations with both calculation and story problem solving at both time points.

Table 4.5 Bivariate correlations among standardised variables (N = 115)

	Counting ability				Working memory			Additive reasoning			Mathematical achievement				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1. Age in months	1														
2. IQ (Raven's raw scores)	0.33**	1													
Counting ability															
3. Procedural counting	0.08	0.01	1												
4. Counting knowledge	0.05	0.05	0.62**	1											
Working memory															
5. Central executive	0.15	0.16	0.15	0.13	1										
6. Digit span forward	0.12	0.14	0.18	0.20*	0.25**	1									
7. Corsi span	0.02	0.15	0.12	0.13	0.24**	0.29**	1								
Additive reasoning															
8. Commutativity-concrete	0.16	0.17	0.16	0.14	0.02	0.14	0.05	1							
9. Commutativity-abstract	0.11	0.15	0.16	0.17	0.10	0.14	0.04	0.73**	1						
10. Complement-concrete	0.01	0.13	0.17	0.12	0.13	0.09	0.08	0.41**	0.44**	1					
11. Complement-abstract	0.09	0.15	0.15	0.16	0.14	0.02	0.03	0.36**	0.37**	0.51**	1				
Mathematical achievement															
12. T1 Calculation	0.07	0.13	0.22*	0.31**	0.35**	0.19*	0.06	0.47**	0.50**	0.48**	0.54**	1			
13. T2 Calculation	0.08	0.17	0.24**	0.29**	0.42**	0.25**	0.14	0.51**	0.51**	0.51**	0.54**	0.81**	1		
14. T1 Story problem solving	0.15	0.18	0.16	0.20*	0.33**	0.04	0.04	0.55**	0.56**	0.49**	0.56**	0.55**	0.64**	1	
15. T2 Story problem solving	0.09	0.20*	0.13	0.21*	0.35**	0.10	0.05	0.54**	0.52**	0.52**	0.62**	0.57**	0.68**	0.75**	1

\*\* Correlation is significant at the 0.01 level (2-tailed)

\* Correlation is significant at the 0.05 level (2-tailed)



In summary, the strengths of the associations between working memory and mathematical achievement varied according to the types of working memory measures and the nature of mathematical achievement tasks. Among all working memory measures, it appeared that the central executive was the strongest and most stable variable that contributed to mathematical achievement. The correlations for counting ability suggest that both conceptual knowledge of counting and procedural counting were important for calculation. However, because these two variables were strongly correlated, there may share a substantial amount of variance in common that explained children's performance in calculation. As for story problem solving, it seems that conceptual knowledge of counting was the only relevant variable within the construct of counting ability. Finally, additive reasoning appeared to be a stronger predictor for mathematical achievement than working memory and counting ability. Both knowledge of both commutativity and complement principles had high correlations with children's performance in calculation and story problem solving concurrently and longitudinally.

#### **4.3 Main Analyses – Multiple Regression Analyses**

Having obtained significant correlations between a predictor and mathematical achievement is not sufficient to conclude that the contributions of that predictor is unique, because the different predictors may share variance that relates to the measure of mathematical achievement. Thus, multiple regression analyses were used that allow the examination of the independent contributions of individual predictors to an outcome variable. When we include multiple variables in a regression model, we can estimate the linear association between each predictor and mathematical achievement while controlling for the effects of other variables (Cohen & Cohen, 1983; Field, 2013; Pedhazur, 1994; Stevens, 2002; Tabachnick & Fidell, 2001). It provides a beta regression coefficient for each explanatory variable included in the regression equation, allowing us to separate out the independent effect of each predictor on the mathematical achievement. The beta regression coefficients also allow us to evaluate the strength of the relation between each predictor to mathematical achievement. The larger the beta value, the greater is the impact of the predictor. The analysis also provides information about the amount of variance in mathematical achievement that is explained by a regression model (i.e.  $R^2$ ). The larger the value of the  $R^2$ , the better the model fits the data.

In the subsequent sections, sets of fixed-order regression analyses are reported to assess the contributions of working memory, counting ability, and additive reasoning to explaining individual differences in calculation and story problem solving at Time 1 and Time 2. Prior to

each of the following analyses, assumptions of regression analyses (Cohen & Cohen, 1983; Field, 2013; Pedhazur, 1994; Stevens, 2002; Tabachnick & Fidell, 2001) were checked and showed no breaches to normality, linearity, homoscedasticity, multicollinearity, and auto-correlation. The plots regarding the assumptions for each regression analysis are presented in Appendix B.

Skewness as well as kurtosis z-values, and visual inspections of the normal probability plots indicated that the normality assumption was not violated. No missing data and outliers were noted. Examinations of residual plots (plots of the standardised residuals as a function of standardised predicted values) indicated that (1) the relations between the criterion variables and all predictors were linear and (2) the residuals were randomly scattered around the horizontal line (homoscedasticity). None of the variance inflation factors of all predictors in the regression models were substantially greater than 1, suggesting that the assumption of multicollinearity was met (Hair, Anderson, Tatham, & Black, 1995; Kennedy, 1992; Marquardt, 1970; Neter, Wasserman, & Kutner, 1989). All values of Durbin-Watson's d (Durbin & Watson, 1951) were close to 2, suggesting that the residuals were not linearly auto-correlated. All predictors and criterion variables were standardised in subsequent regression analyses.

### 4.3.1 Concurrent Predictions – Outcome Variable: Calculation

#### 4.3.1.1 Independent contribution of counting ability to T1 calculation

The first set of regression analyses regarding the concurrent predictions of children's performance in calculation concerns the unique contributions of counting ability. As shown in Table 4.6, counting ability accounted for an additional 9.4% of variance in calculation beyond the effects of age and IQ. Table 4.7 indicates that conceptual knowledge of counting was a unique predictor of children's performance in T1 calculation ( $\beta = 0.235$ ,  $t = 2.449$ ,  $p = .016$ ). However, another measure of counting ability, procedural counting, did not make a significant contribution to T1 calculation in the regression model ( $p > 0.05$ ).

*Table 4.6 The additional amount of variance of T1 calculation explained by counting ability beyond age and IQ (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.004	0.004	0.475	0.492	(1, 113)
2	Age in months Non-verbal intelligence	0.016	0.012	1.368	0.245	(1, 112)
3	Age in months Non-verbal intelligence Procedural counting Counting knowledge	0.111	0.094	5.834**	0.004	(2, 110)

\*\*significant at the 0.01 level

*Table 4.7*

*Independent contribution of counting ability to T1 calculation beyond age and IQ (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.065	0.094	0.065	0.689	0.492
2	Age in months	0.027	0.099	0.027	0.271	0.787
	Non-verbal intelligence	0.116	0.099	0.116	1.17	0.245
3	Age in months	0.015	0.095	0.015	0.16	0.873
	Non-verbal intelligence	0.104	0.095	0.104	1.087	0.279
	Procedural counting	0.133	0.096	0.133	1.388	0.168
	Counting knowledge	0.289	0.118	0.235	2.449*	0.016

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level

The first hypothesis of this study states that counting ability is important in children's mathematics learning and its influence is independent from that of general cognitive capacities, such as working memory. To test this hypothesis, variables of counting ability were entered in the last block of a regression model after age, IQ, and working memory (the central executive, phonological loop, and visuospatial sketchpad). Table 4.8 shows that counting ability explained an additional 5.4% of variance in T1 calculation beyond the effects of age, IQ, and working memory. This finding supported the first hypothesis. Table 4.9 indicates that conceptual knowledge of counting was an independent predictor of T1 calculation in the final block ( $\beta = 0.191$ ,  $t = 2.104$ ,  $p < .05$ ). By contrast, procedural counting was not a significant predictor ( $p > 0.05$ ).

*Table 4.8 The additional amount of variance of T1 calculation explained by counting ability beyond age, IQ, and working memory (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.004	0.004	0.475	0.492	(1, 113)
2	Age in months Non-verbal intelligence	0.016	0.012	1.368	0.245	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad	0.188	0.172	7.694***	<0.001	(3, 109)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge	0.243	0.054	3.843*	0.024	(2, 107)

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level

Table 4.9

*Independent contribution of counting ability to T1 calculation beyond age, IQ, and working memory (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.065	0.094	0.065	0.689	0.492
2	Age in months	0.027	0.099	0.027	0.271	0.787
	Non-verbal intelligence	0.116	0.099	0.116	1.17	0.245
3	Age in months	0.032	0.092	0.032	0.351	0.726
	Non-verbal intelligence	0.074	0.092	0.074	0.804	0.423
	Central executive	0.498	0.114	0.407	4.371***	<0.001
	Phonological loop	0.086	0.093	0.086	0.921	0.359
	Visuospatial sketchpad	0.076	0.092	0.076	0.823	0.413
4	Age in months	0.033	0.09	0.033	0.365	0.716
	Non-verbal intelligence	0.07	0.09	0.07	0.785	0.434
	Central executive	0.46	0.112	0.375	4.104***	<0.001
	Phonological loop	0.041	0.093	0.041	0.442	0.66
	Visuospatial sketchpad	0.06	0.09	0.06	0.668	0.506
	Procedural counting	0.095	0.091	0.095	1.043	0.299
	Counting knowledge	0.234	0.111	0.191	2.103*	0.038

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level

Does counting ability make a significant contribution to T1 calculation when all the other factors are taken into account? Variables of counting ability were entered in the final step of a regression model in which all the other factors, including age, IQ, working memory, and additive reasoning, were present. Table 4.10 indicates that counting ability explained an additional 2.5% of variance in T1 calculation when the effects of all the other factors were controlled for. The increase in variance explained was marginally significant ( $p = 0.063$ ). Table 4.11 demonstrates that conceptual knowledge of counting remained a significant predictor of T1 calculation in the final block ( $\beta = 0.16$ ,  $t = 2.212$ ,  $p < .05$ ), whereas procedural counting was not a significant predictor ( $p > 0.05$ ).

*Table 4.10 The additional amount of variance of T1 calculation explained by counting ability beyond all the other factors (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.004	0.004	0.475	0.492	(1, 113)
2	Age in months Non-verbal intelligence	0.016	0.012	1.368	0.245	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Commutativity knowledge Complement knowledge	0.505	0.489	21.117***	<0.001	(5, 107)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Commutativity knowledge Complement knowledge Procedural counting Counting knowledge	0.530	0.025	2.838	0.063	(2, 105)

\*\*\*significant at the 0.001 level

Table 4.11

Independent contribution of counting ability to T1 calculation beyond all the other factors (N = 115)

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.065	0.094	0.065	0.689	0.492
2	Age in months	0.027	0.099	0.027	0.271	0.787
	Non-verbal intelligence	0.116	0.099	0.116	1.17	0.245
3	Age in months	0.057	0.073	0.057	0.781	0.437
	Non-verbal intelligence	0.001	0.073	0.001	0.01	0.992
	Central executive	0.385	0.091	0.315	4.218***	<0.001
	Phonological loop	0.042	0.074	0.042	0.567	0.572
	Visuospatial sketchpad	0.037	0.073	0.037	0.511	0.61
	Commutativity knowledge	0.378	0.086	0.352	4.38***	<0.001
	Complement knowledge	0.371	0.092	0.323	4.05***	<0.001
4	Age in months	0.053	0.072	0.053	0.731	0.466
	Non-verbal intelligence	0.004	0.072	0.004	0.051	0.959
	Central executive	0.365	0.09	0.298	4.044***	<0.001
	Phonological loop	0.017	0.074	0.017	0.223	0.824
	Visuospatial sketchpad	0.029	0.072	0.029	0.405	0.687
	Commutativity knowledge	0.361	0.091	0.313	3.978***	<0.001
	Complement knowledge	0.365	0.085	0.34	4.286***	<0.001
	Procedural counting	0.012	0.073	0.012	0.162	0.871
	Counting knowledge	0.196	0.089	0.16	2.212*	0.029

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level

#### 4.3.1.2 Independent contribution of additive reasoning to T1 calculation

The second set of regression analyses regarding the concurrent predictions of children's performance in calculation concerns the unique contributions of additive reasoning. Table 4.12 shows the extent to which the variables of additive reasoning (commutativity knowledge and complement knowledge) explained variance in T1 calculation in addition to age and IQ. It was found that the contribution of additive reasoning is substantial and significant (39.3%). As shown in Table 4.13, both commutativity knowledge ( $\beta = 0.36$ ,  $t = 4.231$ ,  $p < .001$ ) and complement knowledge ( $\beta = 0.378$ ,  $t = 4.499$ ,  $p < .001$ ) were unique predictors of T1 calculation.

*Table 4.12 The additional amount of variance of T1 calculation explained by additive reasoning beyond age and IQ (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.004	0.004	0.475	0.492	(1, 113)
2	Age in months Non-verbal intelligence	0.016	0.012	1.368	0.245	(1, 112)
3	Age in months Non-verbal intelligence Commutativity knowledge Complement knowledge	0.409	0.393	36.615***	<0.001	(2, 110)

\*\*\*significant at the 0.001 level

*Table 4.13*

*Independent contribution of additive reasoning to T1 calculation beyond age and IQ (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.065	0.094	0.065	0.689	0.492
2	Age in months	0.027	0.099	0.027	0.271	0.787
	Non-verbal intelligence	0.116	0.099	0.116	1.17	0.245
3	Age in months	0.015	0.078	0.015	0.189	0.851
	Non-verbal intelligence	0.023	0.078	0.023	0.288	0.774
	Commutativity knowledge	0.387	0.091	0.36	4.231***	<0.001
	Complement knowledge	0.436	0.097	0.378	4.499***	<0.001

\*\*\*significant at the 0.001 level



The second hypothesis of this study is that additive reasoning (as assessed by knowledge of commutativity and the complement principle) is independent from and more important than counting ability and general cognitive capacities, such as working memory in children's mathematics learning. To address this hypothesis, variables of additive reasoning were entered in the final step of a regression model after all the other factors, including age, IQ, working memory, and counting ability were controlled for. Table 4.14 shows that additive reasoning accounted for an additional 28.8% of variance in T1 calculation beyond the influence of all the other factors. Both commutativity knowledge ( $\beta = 0.313$ ,  $t = 39.78$ ,  $p < .001$ ) and complement knowledge ( $\beta = 0.34$ ,  $t = 4.286$ ,  $p < .001$ ) remained significant and independent predictors of T1 calculation in the final model (Table 4.15). Thus, the second hypothesis of the present study was strongly supported.

*Table 4.14 The additional amount of variance of T1 calculation explained by additive reasoning beyond all the other factors (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.004	0.004	0.475	0.492	(1, 113)
2	Age in months Non-verbal intelligence	0.016	0.012	1.368	0.245	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge	0.243	0.226	6.394***	<0.001	(5, 107)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.530	0.288	32.152***	<0.001	(2, 105)

\*\*\*significant at the 0.001 level

Table 4.15

*Independent contribution of additive reasoning to T1 calculation beyond all the other factors (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	<i>t</i> values	<i>p</i> values
		B	Std. Error	Beta		
1	Age in months	0.065	0.094	0.065	0.689	0.492
2	Age in months	0.027	0.099	0.027	0.271	0.787
	Non-verbal intelligence	0.116	0.099	0.116	1.17	0.245
3	Age in months	0.033	0.09	0.033	0.365	0.716
	Non-verbal intelligence	0.07	0.09	0.07	0.785	0.434
	Central executive	0.46	0.112	0.375	4.104***	<0.001
	Phonological loop	0.041	0.093	0.041	0.442	0.66
	Visuospatial sketchpad	0.06	0.09	0.06	0.668	0.506
	Procedural counting	0.095	0.091	0.095	1.043	0.299
	Counting knowledge	0.234	0.111	0.191	2.103*	0.038
4	Age in months	0.053	0.072	0.053	0.731	0.466
	Non-verbal intelligence	0.004	0.072	0.004	0.051	0.959
	Central executive	0.365	0.09	0.298	4.044***	<0.001
	Phonological loop	0.017	0.074	0.017	0.223	0.824
	Visuospatial sketchpad	0.029	0.072	0.029	0.405	0.687
	Procedural counting	0.012	0.073	0.012	0.162	0.871
	Counting knowledge	0.196	0.089	0.16	2.212*	0.029
	Commutativity knowledge	0.361	0.091	0.313	3.978***	<0.001
	Complement knowledge	0.365	0.085	0.34	4.286***	<0.001

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level

### 4.3.1.3 Independent contribution of working memory to T1 calculation

The third set of regression analyses regarding the concurrent predictions of children's performance in calculation concerns the unique contributions of working memory. Table 4.16 shows that working memory made a significant contribution to T1 calculation beyond the effect of age and IQ. An additional 17.2% of variance in T1 calculation was explained by working memory. As indicated in Table 4.17, not all working memory measures contributed significantly to explaining the variance. Only the central executive ( $\beta = 0.407$ ,  $t = 4.371$ ,  $p < .001$ ) was a unique predictor of T1 calculation, other variables of working memory (phonological loop and visuospatial sketchpad) did not contribute to it significantly (both  $p$  values  $> 0.05$ ).

*Table 4.16 The additional amount of variance of T1 calculation explained by working memory beyond age and IQ (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.004	0.004	0.475	0.492	(1, 113)
2	Age in months Non-verbal intelligence	0.016	0.012	1.368	0.245	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad	0.188	0.172	7.694***	<0.001	(3, 109)

\*\*\*significant at the 0.001 level

*Table 4.17*

*Independent contribution of working memory to T1 calculation beyond age and IQ (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.065	0.094	0.065	0.689	0.492
2	Age in months	0.027	0.099	0.027	0.271	0.787
	Non-verbal intelligence	0.116	0.099	0.116	1.17	0.245
3	Age in months	0.032	0.092	0.032	0.351	0.726
	Non-verbal intelligence	0.074	0.092	0.074	0.804	0.423
	Central executive	0.498	0.114	0.407	4.371***	<0.001
	Phonological loop	0.086	0.093	0.086	0.921	0.359
	Visuospatial sketchpad	0.076	0.092	0.076	0.823	0.413

\*\*\*significant at the 0.001 level

The third hypothesis of this study states that working memory makes a contribution to mathematical achievement, even when one accounted for children's specific mathematical knowledge such as their knowledge of counting and additive reasoning. To examine this hypothesis, variables of working memory were entered as the final step of a regression model after all the other factors, such as age, IQ, counting ability, and additive reasoning. Table 4.18 shows that working memory accounted for an additional 8% of variance in T1 calculation after the effects of all the other factors were controlled for. This finding supports the third hypothesis of the present study.

Table 4.19 indicates that, among the variables in working memory, the central executive was the only significant predictor of children's performance in T1 calculation ( $\beta = 0.298$ ,  $t = 4.044$ ,  $p < .001$ ). It shows that the central executive makes a unique contribution to T1 calculation even when the influence of counting ability and additive reasoning was controlled for. By contrast, the impacts of phonological loop and visuospatial sketchpad on children's performance on T1 calculation were weak.

*Table 4.18 The additional amount of variance of T1 calculation explained by working memory beyond all the other factors (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.004	0.004	0.475	0.492	(1, 113)
2	Age in months Non-verbal intelligence	0.016	0.012	1.368	0.245	(1, 112)
3	Age in months Non-verbal intelligence Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.450	0.434	21.308***	<0.001	(4, 108)
4	Age in months Non-verbal intelligence Procedural counting Counting knowledge Commutativity knowledge Complement knowledge Central executive Phonological loop Visuospatial sketchpad	0.530	0.080	5.966***	=0.001	(3, 105)

\*\*\*significant at the 0.001 level

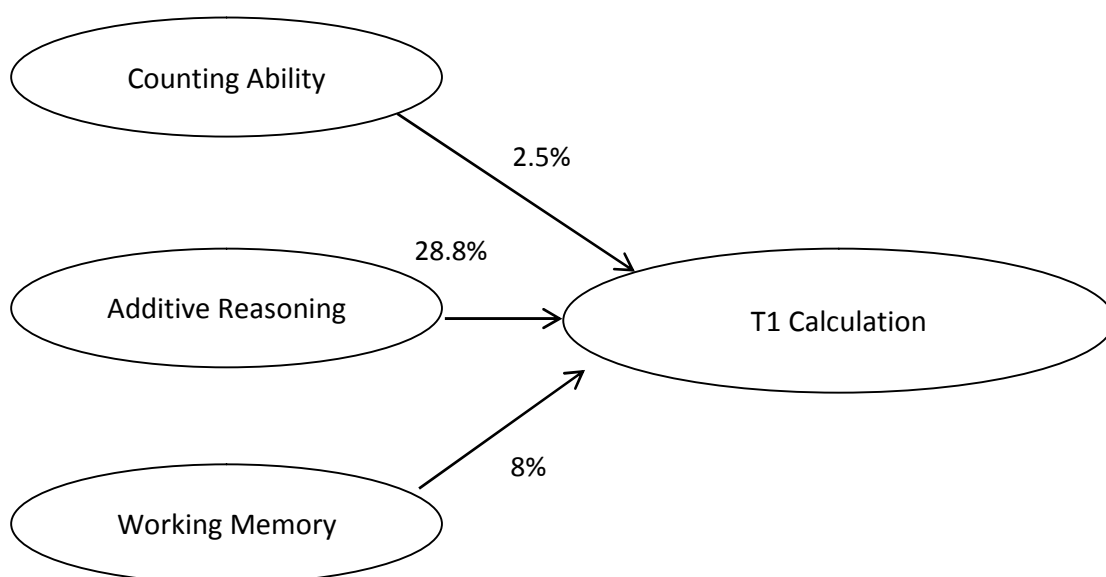
Table 4.19

*Independent contribution of working memory to T1 calculation beyond all the other factors (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.065	0.094	0.065	0.689	0.492
2	Age in months	0.027	0.099	0.027	0.271	0.787
	Non-verbal intelligence	0.116	0.099	0.116	1.17	0.245
3	Age in months	0.015	0.076	0.015	0.202	0.84
	Non-verbal intelligence	0.016	0.076	0.016	0.213	0.831
	Procedural counting	0.034	0.077	0.034	0.436	0.664
	Counting knowledge	0.234	0.094	0.191	2.496**	0.014
	Commutativity knowledge	0.364	0.09	0.338	4.059***	<0.001
	Complement knowledge	0.418	0.095	0.363	4.403***	<0.001
4	Age in months	0.053	0.072	0.053	0.731	0.466
	Non-verbal intelligence	0.004	0.072	0.004	0.051	0.959
	Procedural counting	0.012	0.073	0.012	0.162	0.871
	Counting knowledge	0.196	0.089	0.16	2.212*	0.029
	Commutativity knowledge	0.361	0.091	0.313	3.978***	<0.001
	Complement knowledge	0.365	0.085	0.34	4.286***	<0.001
	Central executive	0.365	0.09	0.298	4.044***	<0.001
	Phonological loop	0.017	0.074	0.017	0.223	0.824
	Visuospatial sketchpad	0.029	0.072	0.029	0.405	0.687

\*significant at the 0.05 level, \*\*significant at the 0.01 level, \*\*\*significant at the 0.001 level

Figure 4.1 summarises the amount of variance in T1 calculation that was explained by each of the main predictors (counting ability, additive reasoning, and working memory) when the effects of all the other factors were controlled for. It shows that additive reasoning and working memory made significant contributions to the variance and the contribution of counting ability was marginally significant ( $p = .063$ ). In particular, conceptual knowledge of counting, the central executive component of working memory, as well as the knowledge of the commutativity and complement principles were the variables that uniquely and significantly accounted for variance in T1 calculation.



*Figure 4.1*

*The amount of variance in T1 calculation that is explained by each of the main predictors (counting ability, additive reasoning, and working memory) when the effects of all the other factors were controlled for*

### 4.3.2 Concurrent Predictions – Outcome Variable: Story Problem Solving

#### 4.3.2.1 Independent contribution of counting ability to T1 story problem solving

Is the concurrent contribution of counting ability to T1 story problem solving similar to that to T1 calculation? Table 4.20 shows that counting ability did not make a significant contribution (only 3.2%, which was not significant) to T1 story problem solving after the influence of age and IQ was controlled for. Both conceptual knowledge of counting and procedural counting did not uniquely account for variance in T1 story problem solving (all  $p$  values  $> 0.05$ ) (Table 4.21). This finding is in contrast to that for T1 calculation in which it was found that conceptual knowledge of counting was a significant predictor after the effects of age and IQ were controlled for.

*Table 4.20 The additional amount of variance of T1 story problem solving explained by counting ability beyond age and IQ (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.024	0.024	2.738	0.101	(1, 113)
2	Age in months Non-verbal intelligence	0.04	0.016	1.907	0.17	(1, 112)
3	Age in months Non-verbal intelligence Procedural counting Counting knowledge	0.072	0.032	1.888	0.156	(2, 110)

*Table 4.21*

*Independent contribution of counting ability to T1 story problem solving beyond age and IQ (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.154	0.093	0.154	1.655	0.101
2	Age in months	0.11	0.098	0.11	1.121	0.265
	Non-verbal intelligence	0.135	0.098	0.135	1.381	0.17
3	Age in months	0.099	0.098	0.099	1.019	0.31
	Non-verbal intelligence	0.131	0.097	0.131	1.346	0.181
	Procedural counting	0.121	0.098	0.121	1.233	0.22
	Counting knowledge	0.118	0.12	0.096	0.983	0.328

Because variables of counting ability were not unique predictors of T1 story problem solving beyond age and IQ, it was expected that it would not contribute significantly to T1 story problem solving when the effects of working memory were also taken into account. If this is the case, the first hypothesis of this study was not supported. Table 4.22 shows that counting ability did not explain a significant amount of variance in T1 story problem solving (2% only). As shown in Table 4.23, both variables of counting ability did not make significant contributions to T1 story problem solving (all  $p$  values > 0.05). Therefore, the first hypothesis was not supported in the analyses for T1 story problem solving.

*Table 4.22 The additional amount of variance of T1 story problem solving explained by counting ability beyond age, IQ, and working memory (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.024	0.024	2.738	0.101	(1, 113)
2	Age in months Non-verbal intelligence	0.04	0.016	1.907	0.17	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad	0.142	0.102	4.328**	0.006	(3, 109)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge	0.162	0.020	1.292	0.279	(2, 107)

\*\*significant at the 0.01 level



Table 4.23

*Independent contribution of counting ability to T1 story problem solving beyond age, IQ, and working memory (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.154	0.093	0.154	1.655	0.101
2	Age in months	0.11	0.098	0.11	1.121	0.265
	Non-verbal intelligence	0.135	0.098	0.135	1.381	0.17
3	Age in months	0.071	0.095	0.071	0.753	0.453
	Non-verbal intelligence	0.113	0.094	0.113	1.195	0.235
	Central executive	0.42	0.117	0.343	3.585***	0.001
	Phonological loop	0.067	0.096	0.067	0.702	0.484
	Visuospatial sketchpad	0.037	0.095	0.037	0.394	0.695
4	Age in months	0.067	0.095	0.067	0.71	0.479
	Non-verbal intelligence	0.114	0.094	0.114	1.207	0.23
	Central executive	0.397	0.118	0.324	3.367***	0.001
	Phonological loop	0.096	0.097	0.096	0.982	0.329
	Visuospatial sketchpad	0.028	0.095	0.028	0.292	0.771
	Procedural counting	0.103	0.096	0.103	1.083	0.281
	Counting knowledge	0.092	0.117	0.075	0.781	0.437

\*\*\*significant at the 0.001 level

Similarly, Table 4.24 shows that when counting ability was entered as the final step after all the other factors, it only explained a very small additional amount of variance (0.2%, which was not significant) in T1 story problem solving. Table 4.25 indicates that both conceptual knowledge of counting and procedural counting were not unique predictors of T2 story problem solving when the effects of all the other factors were controlled for (all  $p$  values  $> 0.05$ ). This result is again in contrast to that for T1 calculation in which it was found that conceptual knowledge of counting was a significant predictor after the effects of all the other factors were controlled for. Thus, counting ability appeared to be more important in children's performance in T1 calculation, but not in T1 story problem solving.

*Table 4.24 The additional amount of variance of T1 story problem solving explained by counting ability beyond all the other factors (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.024	0.024	2.738	0.101	(1, 113)
2	Age in months Non-verbal intelligence	0.04	0.016	1.907	0.17	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Commutativity knowledge Complement knowledge	0.548	0.508	24.098***	<0.001	(5, 107)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Commutativity knowledge Complement knowledge Procedural counting Counting knowledge	0.550	0.002	0.195	0.823	(2, 105)

\*\*\*significant at the 0.001 level

Table 4.25

*Independent contribution of counting ability to T1 story problem solving beyond all the other factors (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.154	0.093	0.154	1.655	0.101
2	Age in months	0.11	0.098	0.11	1.121	0.265
	Non-verbal intelligence	0.135	0.098	0.135	1.381	0.17
3	Age in months	0.049	0.07	0.049	0.709	0.48
	Non-verbal intelligence	0.03	0.07	0.03	0.433	0.666
	Central executive	0.283	0.087	0.231	3.249**	0.002
	Phonological loop	0.107	0.071	0.107	1.505	0.135
	Visuospatial sketchpad	0.003	0.07	0.003	0.036	0.971
	Commutativity knowledge	0.354	0.082	0.329	4.293***	<0.001
	Complement knowledge	0.497	0.088	0.432	5.674***	<0.001
4	Age in months	0.05	0.07	0.05	0.715	0.476
	Non-verbal intelligence	0.03	0.07	0.03	0.421	0.675
	Central executive	0.278	0.088	0.227	3.148**	0.002
	Phonological loop	0.114	0.073	0.114	1.566	0.12
	Visuospatial sketchpad	<0.001	0.071	<0.001	0.004	0.997
	Commutativity knowledge	0.35	0.083	0.326	4.203***	<0.001
	Complement knowledge	0.494	0.089	0.429	5.568***	<0.001
	Procedural counting	0.006	0.071	0.006	0.087	0.931
	Counting knowledge	0.049	0.087	0.04	0.562	0.575

\*\*significant at the 0.01 level, \*\*\*significant at the 0.001 level

#### 4.3.2.2 Independent contribution of additive reasoning to T1 story problem solving

The second set of regression analyses regarding the concurrent predictions of children's performance in story problem solving concerns the unique contributions of additive reasoning. Table 4.26 shows that additive reasoning explained a significant amount of variance in T1 story problem solving (46%) beyond the effects of age and IQ. Table 4.27 indicates that both commutativity knowledge ( $\beta = 0.314$ ,  $t = 4.012$ ,  $p < .001$ ) and complement knowledge ( $\beta = 0.478$ ,  $t = 6.18$ ,  $p < .001$ ) were unique and strong predictors of T1 story problem solving in this model.

*Table 4.26 The additional amount of variance of T1 story problem solving explained by additive reasoning beyond age and IQ (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.024	0.024	2.738	0.101	(1, 113)
2	Age in months Non-verbal intelligence	0.04	0.016	1.907	0.17	(1, 112)
3	Age in months Non-verbal intelligence Commutativity knowledge Complement knowledge	0.500	0.460	50.684***	<0.001	(2, 110)

\*\*\*significant at the 0.001 level

*Table 4.27*

*Independent contribution of additive reasoning to T1 story problem solving beyond age and IQ (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.154	0.093	0.154	1.655	0.101
2	Age in months	0.11	0.098	0.11	1.121	0.265
	Non-verbal intelligence	0.135	0.098	0.135	1.381	0.17
3	Age in months	0.071	0.072	0.071	0.992	0.323
	Non-verbal intelligence	0.037	0.072	0.037	0.508	0.612
	Commutativity knowledge	0.338	0.084	0.314	4.012***	<0.001
	Complement knowledge	0.551	0.089	0.478	6.18***	<0.001

\*\*\*significant at the 0.001 level

The regression analyses for T1 calculation supported the second hypothesis by demonstrating that additive reasoning was the strongest predictor even after the influence of age, IQ, counting ability, and working memory was controlled for. Can this finding be replicated in T1 story problem solving? Table 4.28 shows that additive reasoning explained a substantial and significant amount of variance in T1 story problem solving beyond the effects of all the other factors (38.8%). Both variables of additive reasoning made unique contributions to accounting for the variance: commutativity knowledge ( $\beta = 0.326$ ,  $t = 4.203$ ,  $p < .001$ ) and complement knowledge ( $\beta = 0.429$ ,  $t = 5.568$ ,  $p < .001$ ). Therefore, this finding concurs with that for T1 calculation and the second hypothesis was strongly supported.

*Table 4.28 The additional amount of variance of T1 story problem solving explained by additive reasoning beyond all the other factors (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.024	0.024	2.738	0.101	(1, 113)
2	Age in months Non-verbal intelligence	0.04	0.016	1.907	0.17	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge	0.162	0.122	3.128**	0.011	(5, 107)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.550	0.388	45.246***	<0.001	(2, 105)

\*\*significant at the 0.01 level, \*\*\*significant at the 0.001 level

Table 4.29

*Independent contribution of additive reasoning to T1 story problem solving beyond all the other factors (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.154	0.093	0.154	1.655	0.101
2	Age in months	0.11	0.098	0.11	1.121	0.265
	Non-verbal intelligence	0.135	0.098	0.135	1.381	0.17
3	Age in months	0.067	0.095	0.067	0.71	0.479
	Non-verbal intelligence	0.114	0.094	0.114	1.207	0.23
	Central executive	0.397	0.118	0.324	3.367***	0.001
	Phonological loop	0.096	0.097	0.096	0.982	0.329
	Visuospatial sketchpad	0.028	0.095	0.028	0.292	0.771
	Procedural counting	0.103	0.096	0.103	1.083	0.281
	Counting knowledge	0.092	0.117	0.075	0.781	0.437
4	Age in months	0.05	0.07	0.05	0.715	0.476
	Non-verbal intelligence	0.03	0.07	0.03	0.421	0.675
	Central executive	0.278	0.088	0.227	3.148**	0.002
	Phonological loop	0.114	0.073	0.114	1.566	0.12
	Visuospatial sketchpad	<0.001	0.071	<0.001	0.004	0.997
	Procedural counting	0.006	0.071	0.006	0.087	0.931
	Counting knowledge	0.049	0.087	0.04	0.562	0.575
	Commutativity knowledge	0.35	0.083	0.326	4.203***	<0.001
	Complement knowledge	0.494	0.089	0.429	5.568***	<0.001

\*\*significant at the 0.01 level, \*\*\*significant at the 0.001 level

#### 4.3.2.3 Independent contribution of working memory to T1 story problem solving

The third set of regression analyses regarding the concurrent predictions of children's performance in story problem solving concerns the unique contributions of working memory. In line with the findings for T1 calculation, Table 4.30 shows that working memory accounted for a significant amount of variance (10.2%) in T1 story problem solving beyond the effects of age and IQ. The central executive was the only variable in working memory that contributed to variance in T1 story problem solving ( $\beta = 0.343$ ,  $t = 3.585$ ,  $p = .001$ ).

*Table 4.30 The additional amount of variance of T1 story problem solving explained by working memory beyond age and IQ (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.024	0.024	2.738	0.101	(1, 113)
2	Age in months Non-verbal intelligence	0.04	0.016	1.907	0.17	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad	0.142	0.102	4.328**	0.006	(3, 109)

\*\*significant at the 0.01 level

*Table 4.31 Independent contribution of working memory to T1 story problem solving beyond age and IQ (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.154	0.093	0.154	1.655	0.101
2	Age in months	0.11	0.098	0.11	1.121	0.265
	Non-verbal intelligence	0.135	0.098	0.135	1.381	0.17
3	Age in months	0.071	0.095	0.071	0.753	0.453
	Non-verbal intelligence	0.113	0.094	0.113	1.195	0.235
	Central executive	0.42	0.117	0.343	3.585***	0.001
	Phonological loop	0.067	0.096	0.067	0.702	0.484
	Visuospatial sketchpad	0.037	0.095	0.037	0.394	0.695

\*\*\*significant at the 0.001 level

Is the third hypothesis supported by the regression analyses for T1 story problem solving? Table 4.32 shows that when variables of working memory were entered in the last block after all the other factors were controlled for, they continued to explain a significant amount of variance (4.7%) in T1 story problem solving. Therefore, the third hypothesis of the present study was supported by the findings for both calculation and story problem solving at T1. Similar to T1 calculation, the only significant variable in working memory uniquely accounting for variance in T1 story problem solving was the central executive ( $\beta = 0.227$ ,  $t = 3.148$ ,  $p = .002$ ).

*Table 4.32 The additional amount of variance of T1 story problem solving explained by working memory beyond all the other factors (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.024	0.024	2.738	0.101	(1, 113)
2	Age in months Non-verbal intelligence	0.04	0.016	1.907	0.17	(1, 112)
3	Age in months Non-verbal intelligence Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.503	0.463	25.155***	<0.001	(4, 108)
4	Age in months Non-verbal intelligence Procedural counting Counting knowledge Commutativity knowledge Complement knowledge Central executive Phonological loop Visuospatial sketchpad	0.550	0.047	3.666*	=0.015	(3, 105)

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level



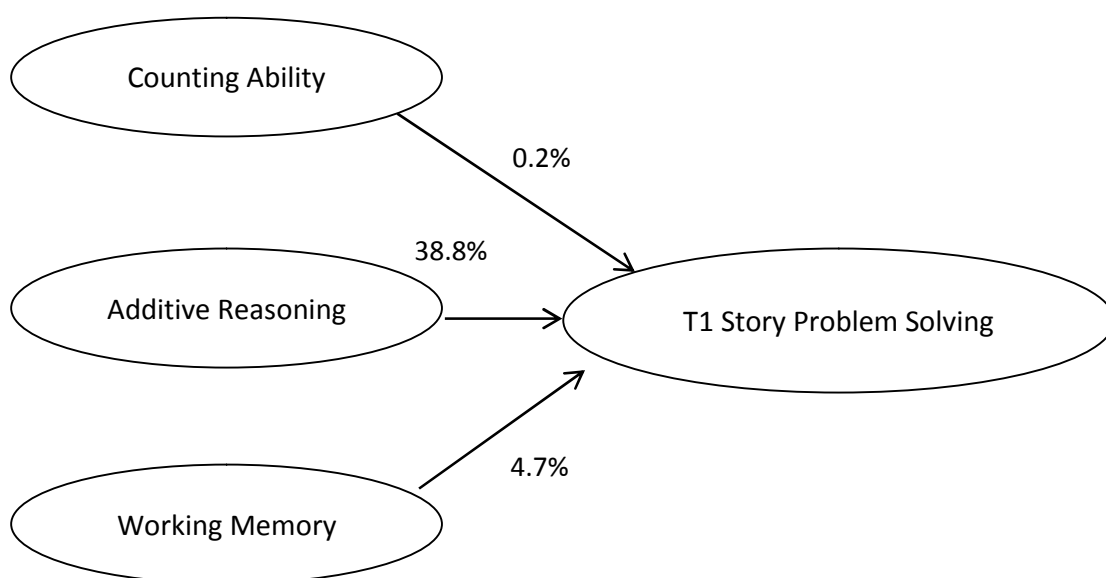
Table 4.33

*Independent contribution of working memory to T1 story problem solving beyond all the other factors (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.154	0.093	0.154	1.655	0.101
2	Age in months	0.11	0.098	0.11	1.121	0.265
	Non-verbal intelligence	0.135	0.098	0.135	1.381	0.17
3	Age in months	0.071	0.072	0.071	0.98	0.329
	Non-verbal intelligence	0.035	0.073	0.035	0.482	0.631
	Procedural counting	0.009	0.073	0.009	0.117	0.907
	Counting knowledge	0.059	0.089	0.048	0.665	0.508
	Commutativity knowledge	0.332	0.085	0.309	3.893***	<0.001
	Complement knowledge	0.546	0.09	0.474	6.054***	<0.001
4	Age in months	0.05	0.07	0.05	0.715	0.476
	Non-verbal intelligence	0.03	0.07	0.03	0.421	0.675
	Procedural counting	0.006	0.071	0.006	0.087	0.931
	Counting knowledge	0.049	0.087	0.04	0.562	0.575
	Commutativity knowledge	0.35	0.083	0.326	4.203***	<0.001
	Complement knowledge	0.494	0.089	0.429	5.568***	<0.001
	Central executive	0.278	0.088	0.227	3.148**	0.002
	Phonological loop	0.114	0.073	0.114	1.566	0.12
	Visuospatial sketchpad	<0.001	0.071	<0.001	0.004	0.997

\*\*significant at the 0.01 level, \*\*\*significant at the 0.001 level

Figure 4.2 summarises the amount of variance in T1 story problem solving that is explained by each of the main predictors (counting ability, additive reasoning, and working memory) when the effects of all the other factors were controlled for. The contributions of additive reasoning and working memory were significant, however counting ability did not contribute significantly to explaining variance in T1 story problem solving. Specifically, the central executive component of working memory, as well as the knowledge of the commutativity and complement principles were the variables that uniquely and significantly accounted for variance in T1 story problem solving.



*Figure 4.2*

*The amount of variance in T1 story problem solving that is explained by each of the main predictors (counting ability, additive reasoning, and working memory) when the effects of all the other factors were controlled for*

### 4.3.3 Longitudinal Predictions – Outcome Variable: Calculation

#### 4.3.3.1 Independent contribution of counting ability to T2 calculation

The first set of regression analyses regarding the longitudinal predictions of children's performance in calculation concerns the unique contributions of counting ability. Similar to T1 calculation, counting ability accounted for a significant amount of variance in T2 calculation (8.4%) when entered after age and IQ (Table 4.34). However, conceptual knowledge of counting was only marginally significant in predicting T2 calculation ( $\beta = 0.181$ ,  $t = 1.894$ ,  $p = .061$ ). It is also noted that procedural counting was also marginally significant in the model ( $\beta = 0.173$ ,  $t = 1.815$ ,  $p = .072$ ). It is possible that both conceptual knowledge of counting and procedural counting shared much variance of predicting T2 calculation beyond the effects of age and IQ. This may be the reason for the results that counting ability as a whole made a significant contribution to explaining variance in T2 calculation, while individual variables within this construct were not unique predictors.

*Table 4.34 The additional amount of variance of T2 calculation explained by counting ability beyond age and IQ (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.006	0.006	0.722	0.397	(1, 113)
2	Age in months Non-verbal intelligence	0.034	0.028	3.197	0.076	(1, 112)
3	Age in months Non-verbal intelligence Procedural counting Counting knowledge	0.118	0.084	5.257**	0.007	(2, 110)

\*\*significant at the 0.01 level

*Table 4.35*

*Independent contribution of counting ability to T2 calculation beyond age and IQ (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.08	0.094	0.08	0.85	0.397
2	Age in months	0.022	0.098	0.022	0.228	0.82
	Non-verbal intelligence	0.176	0.098	0.176	1.788	0.076
3	Age in months	0.007	0.095	0.007	0.072	0.943
	Non-verbal intelligence	0.168	0.095	0.168	1.767	0.08
	Procedural counting	0.213	0.117	0.173	1.815	0.072
	Counting knowledge	0.181	0.096	0.181	1.894	0.061

Similar to T1 calculation, when entered after age, IQ, and working memory (Table 4.36), counting ability accounted for a significant amount of variance in T2 calculation (4.1%). However, presumably because of the shared variance of T2 calculation explained by conceptual knowledge of counting and procedural counting, both variables were not unique predictors of T2 calculation ( $p$  values  $> 0.05$ ). Because counting ability as a whole explained a significant amount of variance in T2 calculation beyond the effects of age, IQ, and working memory, this finding was considered as supporting evidence for the first hypothesis of the present study.

*Table 4.36 The additional amount of variance of T2 calculation explained by counting ability beyond age, IQ, and working memory (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.006	0.006	0.722	0.397	(1, 113)
2	Age in months Non-verbal intelligence	0.034	0.028	3.197	0.076	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad	0.247	0.212	10.269***	<0.001	(3, 109)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge	0.288	0.041	3.843*	0.048	(2, 107)

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level

Table 4.37

*Independent contribution of counting ability to T2 calculation beyond age, IQ, and working memory (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.08	0.094	0.08	0.85	0.397
2	Age in months	0.022	0.098	0.022	0.228	0.82
	Non-verbal intelligence	0.176	0.098	0.176	1.788	0.076
3	Age in months	0.043	0.089	0.043	0.479	0.633
	Non-verbal intelligence	0.127	0.088	0.127	1.43	0.156
	Central executive	0.526	0.11	0.429	4.788***	<0.001
	Phonological loop	0.114	0.09	0.114	1.265	0.209
	Visuospatial sketchpad	0.014	0.089	0.014	0.156	0.877
4	Age in months	0.047	0.087	0.047	0.541	0.59
	Non-verbal intelligence	0.127	0.087	0.127	1.462	0.147
	Central executive	0.492	0.109	0.401	4.529***	<0.001
	Phonological loop	0.073	0.09	0.073	0.816	0.417
	Visuospatial sketchpad	<0.001	0.087	<0.001	0.001	0.999
	Procedural counting	0.135	0.088	0.135	1.537	0.127
	Counting knowledge	0.149	0.108	0.122	1.383	0.17

\*\*\*significant at the 0.001 level

Unlike T1 calculation, when entered in the last block (Table 4.38), counting ability did not explain a significant amount of variance in T2 calculation after all the other factors were taken into account (1.3% only). Table 4.39 shows that both conceptual knowledge of counting and procedural counting were not independent predictors of T2 calculation ( $p$  values  $> 0.05$ ). Thus, this finding was in contrast to that for T1 calculation in which conceptual knowledge of counting had a marginally significant contribution to explaining individual differences in T1 calculation beyond the effects of age, IQ, working memory, and additive reasoning. It is possible that counting ability may be more important for affecting children's performance in calculation when they are at a younger age or when the task is less difficult.

*Table 4.38 The additional amount of variance of T2 calculation explained by counting ability beyond all the other factors (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.006	0.006	0.722	0.397	(1, 113)
2	Age in months Non-verbal intelligence	0.034	0.028	3.197	0.076	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Commutativity knowledge Complement knowledge	0.576	0.542	27.362***	<0.001	(5, 107)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Commutativity knowledge Complement knowledge Procedural counting Counting knowledge	0.589	0.013	1.616	0.204	(2, 105)

\*\*\*significant at the 0.001 level

Table 4.39

*Independent contribution of counting ability to T2 calculation beyond all the other factors (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	<i>t</i> values	<i>p</i> values
		B	Std. Error	Beta		
1	Age in months	0.08	0.094	0.08	0.85	0.397
2	Age in months	0.022	0.098	0.022	0.228	0.82
	Non-verbal intelligence	0.176	0.098	0.176	1.788	0.076
3	Age in months	0.066	0.067	0.066	0.982	0.328
	Non-verbal intelligence	0.051	0.068	0.051	0.753	0.453
	Central executive	0.409	0.085	0.334	4.833***	<0.001
	Phonological loop	0.071	0.069	0.071	1.034	0.303
	Visuospatial sketchpad	0.023	0.068	0.023	0.345	0.731
	Commutativity knowledge	0.369	0.08	0.344	4.622***	<0.001
	Complement knowledge	0.396	0.085	0.344	4.668***	<0.001
4	Age in months	0.066	0.067	0.066	0.983	0.328
	Non-verbal intelligence	0.052	0.067	0.052	0.768	0.444
	Central executive	0.393	0.084	0.321	4.653***	<0.001
	Phonological loop	0.05	0.07	0.05	0.724	0.471
	Visuospatial sketchpad	0.03	0.067	0.03	0.443	0.658
	Commutativity knowledge	0.358	0.08	0.333	4.491***	<0.001
	Complement knowledge	0.384	0.085	0.334	4.533***	<0.001
	Procedural counting	0.05	0.068	0.05	0.737	0.463
	Counting knowledge	0.111	0.083	0.09	1.334	0.185

\*\*\*significant at the 0.001 level

#### 4.3.3.2 Independent contribution of additive reasoning to T2 calculation

The second set of regression analyses regarding the longitudinal predictions of children's performance in calculation concerns the unique contributions of additive reasoning. Table 4.40 shows that additive reasoning accounted for a substantial amount of variance in T2 calculation (45%) after the effects of age and IQ were controlled for. Table 4.41 indicates that both variables of additive reasoning were unique predictors of T2 calculation beyond age and IQ: commutativity knowledge ( $\beta = 0.349$ ,  $t = 4.247$ ,  $p < .001$ ) and complement knowledge ( $\beta = 0.41$ ,  $t = 5.047$ ,  $p < .001$ ).

*Table 4.40 The additional amount of variance of T2 calculation explained by additive reasoning beyond age and IQ (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.006	0.006	0.722	0.397	(1, 113)
2	Age in months Non-verbal intelligence	0.034	0.028	3.197	0.076	(1, 112)
3	Age in months Non-verbal intelligence Commutativity knowledge Complement knowledge	0.450	0.416	41.589***	<0.001	(2, 110)

\*\*\*significant at the 0.001 level

*Table 4.41*

*Independent contribution of additive reasoning to T2 calculation beyond age and IQ (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.08	0.094	0.08	0.85	0.397
2	Age in months	0.022	0.098	0.022	0.228	0.82
	Non-verbal intelligence	0.176	0.098	0.176	1.788	0.076
3	Age in months	0.019	0.075	0.019	0.247	0.806
	Non-verbal intelligence	0.08	0.076	0.08	1.061	0.291
	Commutativity knowledge	0.375	0.088	0.349	4.247***	<0.001
	Complement knowledge	0.472	0.093	0.41	5.047***	<0.001

\*\*\*significant at the 0.001 level



The results of T1 calculation and story problem solving strongly supported the second hypothesis that additive reasoning is independent of and more important than working memory and counting ability in children's mathematics learning. Consistent with the hypothesis, Table 4.42 shows that when variables of additive reasoning were entered in the last step, they continued to account for a substantial and significant amount of variance in T2 calculation (30%). Table 4.43 demonstrates that the independent contributions of commutativity ( $\beta = 0.333$ ,  $t = 4.491$ ,  $p < .001$ ) and complement knowledge ( $\beta = 0.334$ ,  $t = 4.533$ ,  $p < .001$ ) remained significant after all the other factors were controlled for. Therefore, consistent with the findings for T1 calculation and story problem solving, this evidence strongly supported the second hypothesis.

*Table 4.42 The additional amount of variance of T2 calculation explained by additive reasoning beyond all the other factors (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.006	0.006	0.722	0.397	(1, 113)
2	Age in months Non-verbal intelligence	0.034	0.028	3.197	0.076	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge	0.288	0.242	7.655***	<0.001	(5, 107)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.589	0.300	38.32***	<0.001	(2, 105)

\*\*\*significant at the 0.001 level

Table 4.43

*Independent contribution of additive reasoning to T2 calculation beyond all the other factors (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	<i>t</i> values	<i>p</i> values
		B	Std. Error	Beta		
1	Age in months	0.08	0.094	0.08	0.85	0.397
2	Age in months	0.022	0.098	0.022	0.228	0.82
	Non-verbal intelligence	0.176	0.098	0.176	1.788	0.076
3	Age in months	0.047	0.087	0.047	0.541	0.59
	Non-verbal intelligence	0.127	0.087	0.127	1.462	0.147
	Central executive	0.492	0.109	0.401	4.529***	<0.001
	Phonological loop	0.073	0.09	0.073	0.816	0.417
	Visuospatial sketchpad	<0.001	0.087	<0.001	0.001	0.999
	Procedural counting	0.135	0.088	0.135	1.537	0.127
	Counting knowledge	0.149	0.108	0.122	1.383	0.17
4	Age in months	0.066	0.067	0.066	0.983	0.328
	Non-verbal intelligence	0.052	0.067	0.052	0.768	0.444
	Central executive	0.393	0.084	0.321	4.653***	<0.001
	Phonological loop	0.05	0.07	0.05	0.724	0.471
	Visuospatial sketchpad	0.03	0.067	0.03	0.443	0.658
	Procedural counting	0.05	0.068	0.05	0.737	0.463
	Counting knowledge	0.111	0.083	0.09	1.334	0.185
	Commutativity knowledge	0.358	0.08	0.333	4.491***	<0.001
	Complement knowledge	0.384	0.085	0.334	4.533***	<0.001

\*\*\*significant at the 0.001 level

### 4.3.3.3 Independent contribution of working memory to T2 calculation

The third set of regression analyses regarding the concurrent predictions of children's performance in calculation concerning the unique contributions of working memory. Table 4.44 shows that working memory explained a significant amount of variance in T2 calculation (21.3%) beyond the effects of age and IQ. The only significant variable in working memory that made this contribution was the central executive variable ( $\beta = 0.429$ ,  $t = 4.788$ ,  $p < .001$ ).

*Table 4.44 The additional amount of variance of T2 calculation explained by working memory beyond age and IQ (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.006	0.006	0.722	0.397	(1, 113)
2	Age in months Non-verbal intelligence	0.034	0.028	3.197	0.076	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad	0.247	0.213	10.269***	<0.001	(3, 109)

\*\*\*significant at the 0.001 level

*Table 4.45*

*Independent contribution of working memory to T2 calculation beyond age and IQ (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.08	0.094	0.08	0.85	0.397
2	Age in months	0.022	0.098	0.022	0.228	0.82
	Non-verbal intelligence	0.176	0.098	0.176	1.788	0.076
3	Age in months	0.043	0.089	0.043	0.479	0.633
	Non-verbal intelligence	0.127	0.088	0.127	1.43	0.156
	Central executive	0.526	0.11	0.429	4.788***	<0.001
	Phonological loop	0.114	0.09	0.114	1.265	0.209
	Visuospatial sketchpad	0.014	0.089	0.014	0.156	0.877

\*\*\*significant at the 0.001 level

Similar to the analyses on T1 calculation and story problem solving, Table 4.46 demonstrates that working memory explained a significant amount of variance in T2 calculation (11%) when the effects of all other factors were taken into account. This finding was consistent with the third hypothesis that working memory makes a unique contribution to children's mathematics learning beyond the specific mathematical knowledge, such as counting ability and additive reasoning. Among the variables in working memory, Table 4.47 shows that only the central executive was a unique predictor of T2 calculation ( $\beta = 0.321$ ,  $t = 4.653$ ,  $p < .001$ ) when the effects of all other variables were controlled for.

*Table 4.46 The additional amount of variance of T2 calculation explained by working memory beyond all the other factors (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.006	0.006	0.722	0.397	(1, 113)
2	Age in months Non-verbal intelligence	0.034	0.028	3.197	0.076	(1, 112)
3	Age in months Non-verbal intelligence Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.478	0.444	22.996***	<0.001	(4, 108)
4	Age in months Non-verbal intelligence Procedural counting Counting knowledge Commutativity knowledge Complement knowledge Central executive Phonological loop Visuospatial sketchpad	0.589	0.110	9.394***	<0.001	(3, 105)

\*\*\*significant at the 0.001 level

Table 4.47

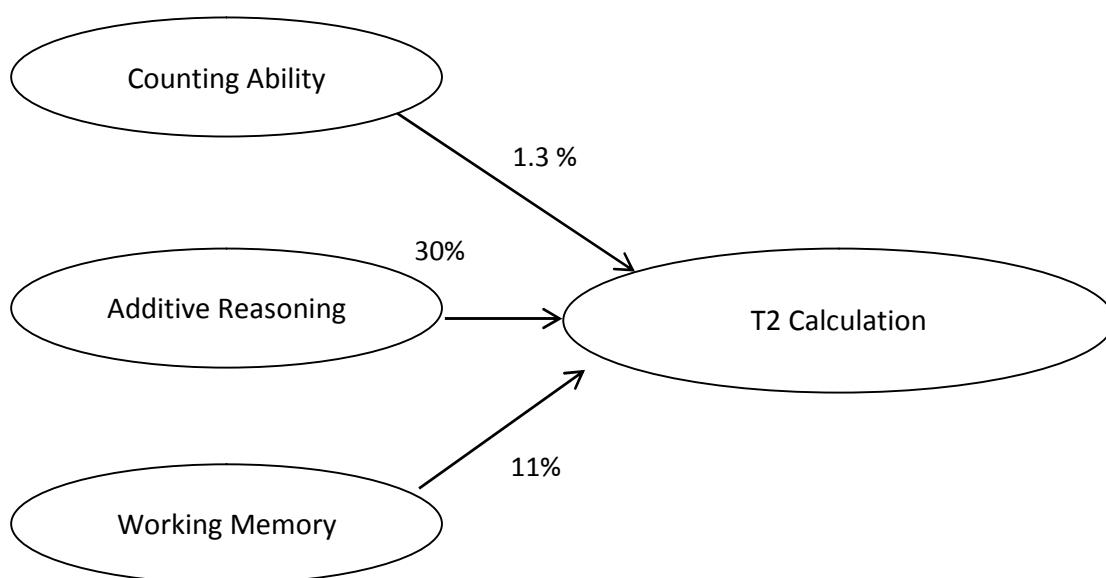
*Independent contribution of working memory to T2 calculation beyond all the other factors (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.08	0.094	0.08	0.85	0.397
2	Age in months	0.022	0.098	0.022	0.228	0.82
	Non-verbal intelligence	0.176	0.098	0.176	1.788	0.076
3	Age in months	0.023	0.074	0.023	0.311	0.756
	Non-verbal intelligence	0.078	0.074	0.078	1.052	0.295
	Procedural counting	0.078	0.075	0.078	1.042	0.3
	Counting knowledge	0.157	0.091	0.128	1.723	0.088
	Commutativity knowledge	0.353	0.087	0.328	4.042***	<0.001
	Complement knowledge	0.452	0.092	0.393	4.892***	<0.001
4	Age in months	0.066	0.067	0.066	0.983	0.328
	Non-verbal intelligence	0.052	0.067	0.052	0.768	0.444
	Procedural counting	0.05	0.068	0.05	0.737	0.463
	Counting knowledge	0.111	0.083	0.09	1.334	0.185
	Commutativity knowledge	0.358	0.08	0.333	4.491***	<0.001
	Complement knowledge	0.384	0.085	0.334	4.533***	<0.001
	Central executive	0.393	0.084	0.321	4.653***	<0.001
	Phonological loop	0.05	0.07	0.05	0.724	0.471
	Visuospatial sketchpad	0.03	0.067	0.03	0.443	0.658

\*\*\*significant at the 0.001 level

Figure 4.3 summarises the amount of variance in T2 calculation that was explained by each of the main predictors (counting ability, additive reasoning, and working memory) when the effects of all the other factors were controlled for. When the effects of age, IQ, and working memory were taken into account, counting ability made significant contributions to both T1 and T2 calculation. However, when all the other factors including additive reasoning were considered, counting ability did not contribute significantly to T2 calculation. This result differed from that for T1 calculation in which counting ability accounted for a marginally significant amount of variance beyond the effects of all the other factors.

Both additive reasoning and working memory accounted for a significant amount of variance of T2 calculation when the influence of all the other factors was controlled for. In particular, the central executive component of working memory, as well as the knowledge of the commutativity and complement principles were the variables that uniquely and significantly accounted for variance in T2 calculation.



*Figure 4.3*

*The amount of variance in T2 calculation that is explained by each of the main predictors (counting ability, additive reasoning, and working memory) when the effects of all the other factors were controlled for*

#### 4.3.4 Longitudinal Predictions – Outcome Variable: Story Problem Solving

##### 4.3.4.1 Independent contribution of counting ability to T2 story problem solving

The first set of regression analyses regarding the longitudinal predictions of children's performance in story problem solving concerns the unique contributions of counting ability. Table 4.48 shows that counting ability did not make a contribution to T2 story problem solving beyond the effects of age and IQ. Similar to the finding for T1 story problem solving, both conceptual knowledge of counting and procedural counting were not unique predictors of T2 story problem solving when the influence of age and IQ was controlled for ( $p$  values  $> 0.05$ ).

*Table 4.48 The additional amount of variance of T2 story problem solving explained by counting ability beyond age and IQ (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.008	0.008	0.865	0.354	(1, 113)
2	Age in months Non-verbal intelligence	0.042	0.034	4.032*	0.047	(1, 112)
3	Age in months Non-verbal intelligence Procedural counting Counting knowledge	0.079	0.037	2.231	0.112	(2, 110)

\*significant at the 0.05 level

*Table 4.49*

*Independent contribution of counting ability to T2 story problem solving beyond age and IQ (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.087	0.094	0.087	0.93	0.354
2	Age in months	0.023	0.098	0.023	0.237	0.813
	Non-verbal intelligence	0.196	0.098	0.196	2.008*	0.047
3	Age in months	0.016	0.097	0.016	0.169	0.866
	Non-verbal intelligence	0.188	0.097	0.188	1.942	0.055
	Procedural counting	0.078	0.098	0.078	0.794	0.429
	Counting knowledge	0.187	0.12	0.153	1.563	0.121

\*significant at 0.05 level

Because counting ability did not contribute significantly to T2 story problem solving beyond age and IQ, it is unlikely that it would make a unique contribution to it when the effect of working memory is also controlled for. Consistent with this prediction, Table 4.50 shows that counting ability only accounted for 2% of variance in T2 story problem solving beyond the influence of age, IQ, and working memory. Table 4.51 indicates that both conceptual knowledge of counting and procedural counting were not independent predictors ( $p$  values  $> 0.05$ ). This finding is consistent with that of T1 story problem solving in which counting ability was also not a good predictor. Therefore, the first hypothesis was not supported by the result for children's performance in story problem solving at both T1 and T2.

*Table 4.50 The additional amount of variance of T2 story problem solving explained by counting ability beyond age, IQ, and working memory (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.008	0.008	0.865	0.354	(1, 113)
2	Age in months Non-verbal intelligence	0.042	0.034	4.032*	0.047	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad	0.175	0.133	5.836***	0.001	(3, 109)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge	0.195	0.020	1.322	0.271	(2, 107)

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level



Table 4.51

*Independent contribution of counting ability to T2 story problem solving beyond age, IQ, and working memory (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.087	0.094	0.087	0.93	0.354
2	Age in months	0.023	0.098	0.023	0.237	0.813
	Non-verbal intelligence	0.196	0.098	0.196	2.008*	0.047
3	Age in months	0.025	0.093	0.025	0.265	0.792
	Non-verbal intelligence	0.166	0.093	0.166	1.792	0.076
	Central executive	0.471	0.115	0.385	4.101***	<0.001
	Phonological loop	0.019	0.094	0.019	0.203	0.839
	Visuospatial sketchpad	0.053	0.093	0.053	0.567	0.572
4	Age in months	0.024	0.093	0.024	0.262	0.794
	Non-verbal intelligence	0.163	0.093	0.163	1.767	0.08
	Central executive	0.448	0.115	0.366	3.881***	<0.001
	Phonological loop	0.046	0.095	0.046	0.483	0.63
	Visuospatial sketchpad	0.043	0.093	0.043	0.467	0.642
	Procedural counting	0.05	0.094	0.05	0.537	0.593
	Counting knowledge	0.148	0.115	0.12	1.284	0.202

\*significant at 0.05 level, \*\*\*significant at the 0.001 level

Table 4.52 shows that counting ability was a weak predictor of T2 story problem solving when all the other factors were controlled for. It only explained an additional 0.6% of the variance. Both conceptual knowledge of counting and procedural counting were not significant predictors of T2 story problem solving beyond the effects of all the other factors (Table 4.53).

*Table 4.52 The additional amount of variance of T2 story problem solving explained by counting ability beyond all the other factors (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.008	0.008	0.865	0.354	(1, 113)
2	Age in months Non-verbal intelligence	0.042	0.034	4.032*	0.047	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Commutativity knowledge Complement knowledge	0.575	0.532	26.779***	<0.001	(5, 107)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Commutativity knowledge Complement knowledge Procedural counting Counting knowledge	0.581	0.006	0.803	0.451	(2, 105)

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level

Table 4.53

*Independent contribution of counting ability to T2 story problem solving beyond all the other factors (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.087	0.094	0.087	0.93	0.354
2	Age in months	0.023	0.098	0.023	0.237	0.813
	Non-verbal intelligence	0.196	0.098	0.196	2.008*	0.047
3	Age in months	0.053	0.068	0.053	0.786	0.433
	Non-verbal intelligence	0.082	0.068	0.082	1.211	0.228
	Central executive	0.346	0.085	0.283	4.087***	<0.001
	Phonological loop	0.07	0.069	0.07	1.014	0.313
	Visuospatial sketchpad	0.008	0.068	0.008	0.122	0.903
	Commutativity knowledge	0.405	0.085	0.351	4.758***	<0.001
	Complement knowledge	0.437	0.08	0.406	5.455***	<0.001
4	Age in months	0.048	0.068	0.048	0.709	0.48
	Non-verbal intelligence	0.077	0.068	0.077	1.139	0.257
	Central executive	0.34	0.085	0.278	3.989***	<0.001
	Phonological loop	0.076	0.07	0.076	1.08	0.282
	Visuospatial sketchpad	0.006	0.068	0.006	0.091	0.928
	Commutativity knowledge	0.405	0.085	0.351	4.758***	<0.001
	Complement knowledge	0.437	0.08	0.406	5.455***	<0.001
	Procedural counting	0.046	0.069	0.046	0.661	0.51
	Counting knowledge	0.103	0.084	0.084	1.232	0.221

\*significant at 0.05 level, \*\*\*significant at the 0.001 level

#### 4.3.4.2 Independent contribution of additive reasoning to T2 story problem solving

The second set of regression analyses regarding the concurrent predictions of children's performance in story problem solving concerning the unique contributions of additive reasoning. Table 4.54 shows that additive reasoning contributed substantially to variance in T2 story problem solving (46.4%) beyond the effects of age and IQ. Both commutativity knowledge ( $\beta = 0.396$ ,  $t = 5.093$ ,  $p < .001$ ) and complement knowledge ( $\beta = 0.406$ ,  $t = 5.284$ ,  $p < .001$ ) were unique predictors of T2 story problem solving in the model.

*Table 4.54 The additional amount of variance of T2 story problem solving explained by additive reasoning beyond age and IQ (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.008	0.008	0.865	0.354	(1, 113)
2	Age in months Non-verbal intelligence	0.042	0.034	4.032*	0.047	(1, 112)
3	Age in months Non-verbal intelligence Commutativity knowledge Complement knowledge	0.506	0.464	51.716***	<0.001	(2, 110)

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level

*Table 4.55*

*Independent contribution of additive reasoning to T2 story problem solving beyond age and IQ (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.087	0.094	0.087	0.93	0.354
2	Age in months	0.023	0.098	0.023	0.237	0.813
	Non-verbal intelligence	0.196	0.098	0.196	2.008*	0.047
3	Age in months	0.022	0.071	0.022	0.315	0.753
	Non-verbal intelligence	0.095	0.072	0.095	1.324	0.188
	Commutativity knowledge	0.426	0.084	0.396	5.093***	<0.001
	Complement knowledge	0.468	0.089	0.406	5.284***	<0.001

\*significant at 0.05 level, \*\*\*significant at the 0.001 level

When additive reasoning was entered in the last step after all the other factors were controlled for, Table 4.56 shows that it continued to explain a large amount of variance in T2 story problem solving (38.6%). This finding is consistent with that of T1 and T2 calculation as well as T1 story problem solving that strongly support the second hypothesis regarding the importance of additive reasoning in children's mathematics learning. Table 4.57 indicates that both commutativity knowledge ( $\beta = 0.351$ ,  $t = 4.758$ ,  $p < .001$ ) and complement knowledge ( $\beta = 0.406$ ,  $t = 5.455$ ,  $p < .001$ ) made significant contributions, independently of all the other factors, to T2 story problem solving.

*Table 4.56 The additional amount of variance of T2 story problem solving explained by additive reasoning beyond all the other factors (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.008	0.008	0.865	0.354	(1, 113)
2	Age in months Non-verbal intelligence	0.042	0.034	4.032*	0.047	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge	0.195	0.152	4.051**	0.002	(5, 107)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.581	0.386	48.403***	<0.001	(2, 105)

\*significant at the 0.05, \*\*significant at the 0.01 level, \*\*\*significant at the 0.001 level

Table 4.57

*Independent contribution of additive reasoning to T2 story problem solving beyond all the other factors (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.087	0.094	0.087	0.93	0.354
2	Age in months	0.023	0.098	0.023	0.237	0.813
	Non-verbal intelligence	0.196	0.098	0.196	2.008*	0.047
3	Age in months	0.024	0.093	0.024	0.262	0.794
	Non-verbal intelligence	0.163	0.093	0.163	1.767	0.08
	Central executive	0.448	0.115	0.366	3.881***	<0.001
	Phonological loop	0.046	0.095	0.046	0.483	0.63
	Visuospatial sketchpad	0.043	0.093	0.043	0.467	0.642
	Procedural counting	0.05	0.094	0.05	0.537	0.593
	Counting knowledge	0.148	0.115	0.12	1.284	0.202
4	Age in months	0.048	0.068	0.048	0.709	0.48
	Non-verbal intelligence	0.077	0.068	0.077	1.139	0.257
	Central executive	0.34	0.085	0.278	3.989***	<0.001
	Phonological loop	0.076	0.07	0.076	1.08	0.282
	Visuospatial sketchpad	0.006	0.068	0.006	0.091	0.928
	Procedural counting	0.046	0.069	0.046	0.661	0.51
	Counting knowledge	0.103	0.084	0.084	1.232	0.221
	Commutativity knowledge	0.405	0.085	0.351	4.758***	<0.001
	Complement knowledge	0.437	0.08	0.406	5.455***	<0.001

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level

#### 4.3.4.3 Independent contribution of working memory to T2 story problem solving

The third set of regression analyses regarding the concurrent predictions of children's performance in story problem solving concerning the unique contributions of working memory. Table 4.58 indicates that working memory explained a significant amount of variance in T2 story problem solving (13.3%) beyond the effects of age and IQ. Only the central executive made a unique contribution to T2 story problem solving in the model ( $\beta = 0.385$ ,  $t = 4.101$ ,  $p < .001$ ).

*Table 4.58 The additional amount of variance of T2 story problem solving explained by working memory beyond age and IQ (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.008	0.008	0.865	0.354	(1, 113)
2	Age in months Non-verbal intelligence	0.042	0.034	4.032*	0.047	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad	0.175	0.133	5.836***	0.001	(3, 109)

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level

*Table 4.59 Independent contribution of working memory to T2 story problem solving beyond age and IQ (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.087	0.094	0.087	0.93	0.354
2	Age in months	0.023	0.098	0.023	0.237	0.813
	Non-verbal intelligence	0.196	0.098	0.196	2.008*	0.047
3	Age in months	0.025	0.093	0.025	0.265	0.792
	Non-verbal intelligence	0.166	0.093	0.166	1.792	0.076
	Central executive	0.471	0.115	0.385	4.101***	<0.001
	Phonological loop	0.019	0.094	0.019	0.203	0.839
	Visuospatial sketchpad	0.053	0.093	0.053	0.567	0.572

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level

When the influence of all the other factors were controlled for, working memory continued to explain a significant amount of variance in T2 story problem solving (6.6%). The central executive was the only variable in working memory that made a unique contribution to T2 story problem solving beyond the effects of age, IQ, counting ability, and additive reasoning ( $\beta = 0.278$ ,  $t = 3.989$ ,  $p < .001$ ). These results were consistent with that in T1 and T2 calculation as well as T2 story problem solving. All these findings consistently support the third hypothesis that working memory is a factor that contributes to mathematics learning independently of children's ability to count and reason additively.

*Table 4.60 The additional amount of variance of T2 story problem solving explained by working memory beyond all the other factors (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.008	0.008	0.865	0.354	(1, 113)
2	Age in months Non-verbal intelligence	0.042	0.034	4.032*	0.047	(1, 112)
3	Age in months Non-verbal intelligence Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.515	0.473	26.372***	<0.001	(4, 108)
4	Age in months Non-verbal intelligence Procedural counting Counting knowledge Commutativity knowledge Complement knowledge Central executive Phonological loop Visuospatial sketchpad	0.581	0.066	5.472**	=0.002	(3, 105)

\*significant at the 0.05 level, \*\*significant at the 0.01 level, \*\*\*significant at the 0.001 level



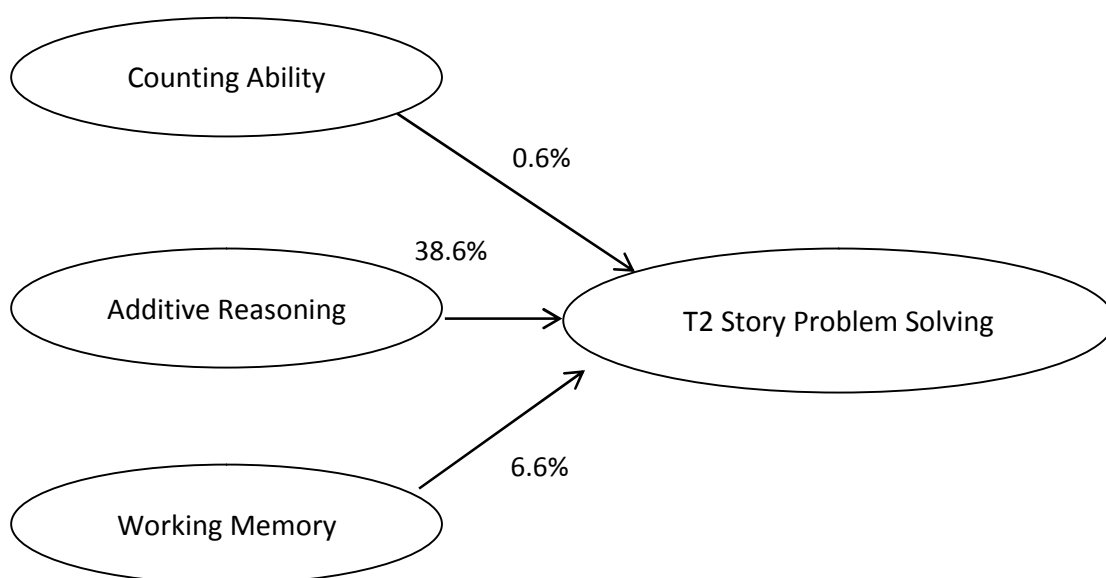
Table 4.61

*Independent contribution of working memory to T2 story problem solving beyond all the other factors (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.087	0.094	0.087	0.93	0.354
2	Age in months	0.023	0.098	0.023	0.237	0.813
	Non-verbal intelligence	0.196	0.098	0.196	2.008*	0.047
3	Age in months	0.019	0.071	0.019	0.266	0.791
	Non-verbal intelligence	0.089	0.072	0.089	1.243	0.216
	Procedural counting	0.035	0.073	0.035	0.488	0.627
	Counting knowledge	0.125	0.088	0.102	1.424	0.157
	Commutativity knowledge	0.42	0.084	0.39	4.988***	<0.001
	Complement knowledge	0.466	0.089	0.404	5.227***	<0.001
4	Age in months	0.048	0.068	0.048	0.709	0.48
	Non-verbal intelligence	0.077	0.068	0.077	1.139	0.257
	Procedural counting	0.046	0.069	0.046	0.661	0.51
	Counting knowledge	0.103	0.084	0.084	1.232	0.221
	Commutativity knowledge	0.405	0.086	0.352	4.736***	<0.001
	Complement knowledge	0.434	0.08	0.404	5.398***	<0.001
	Central executive	0.34	0.085	0.278	3.989***	<0.001
	Phonological loop	0.076	0.07	0.076	1.08	0.282
	Visuospatial sketchpad	0.006	0.068	0.006	0.091	0.928

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level

Figure 4.4 summarises the amount of variance in T2 story problem solving that was explained by each of the main predictors (counting ability, additive reasoning, and working memory) when the effects of all the other factors were controlled for. The contributions of additive reasoning and working memory were significant. However, counting ability did not have a unique contribution to explaining variance in T2 story problem solving. Specifically, the central executive component of working memory, as well as the knowledge of the commutativity and complement principles were the variables that uniquely and significantly accounted for variance in T2 story problem solving.



*Figure 4.4*

*The amount of variance in T2 story problem solving that is explained by each of the main predictors (counting ability, additive reasoning, and working memory) when the effects of all the other factors were controlled for*

### 4.3.5 Overall Summary – Regression Analyses of Calculation and Story Problem Solving

Table 4.62 A summary of the findings of the regression analyses

Hypotheses	T1 calculation	T1 story problem solving	T2 calculation	T2 story problem solving
<i>Hypothesis 1:</i> Counting ability is important in children's mathematics learning and its influence is independent from that of general cognitive capacities, such as working memory.	✓	×	✓	×
<i>Hypothesis 2:</i> Additive reasoning (as assessed by knowledge of commutativity and the complement principle) is independent from and more important than counting ability and general cognitive capacities, such as working memory in children's mathematics learning.	✓	✓	✓	✓
<i>Hypothesis 3:</i> Working memory, as a domain-general factor, makes a contribution to mathematical achievement, even when one accounted for children's specific mathematical knowledge such as their knowledge of counting ability and additive reasoning.	✓	✓	✓	✓

*Note.* ✓ indicates that the hypothesis was supported, x indicates that the hypothesis was not supported

Several key findings from the regression analyses with respect to the hypotheses are summarised as follows. First, it was hypothesised that counting ability is important in children's mathematics learning and its influence is independent from that of general cognitive capacities, such as working memory. This hypothesis is only partially supported. Counting ability contributes significantly to explaining the variance in calculation beyond the influence of age, IQ, and working memory. However, it does not make a unique contribution to story problem solving. Procedural counting appears not to be a good predictor of children's performance in both calculation and story problem solving in the present study. However, conceptual knowledge of

counting is a unique predictor of calculation, independent of age, IQ, and working memory, at both time points.

Second, it was hypothesised that additive reasoning (as assessed by knowledge of commutativity and the complement principle) is independent from and more important than counting ability and general cognitive capacities, such as working memory in children's mathematics learning. The findings show that commutativity and complement knowledge are independently and strongly related to of children's performance in calculation and story problem solving concurrently and longitudinally. The amount of variance explained by additive reasoning is the largest among all the other factors, such as counting ability and working memory. Therefore, the second hypothesis is strongly supported.

Third, it was hypothesised that working memory is important in its own right in explaining variations in mathematical achievement. In support of this hypothesis, working memory explains a significant amount of variance in calculation and story problem solving concurrently and longitudinally beyond the effects of all the other factors. The central executive component of working memory is a unique predictor of children's performance in calculation and story problem solving concurrently and longitudinally, even after the effects of other factors, such as counting ability and additive reasoning are controlled for. However, the phonological loop and visuospatial sketchpad appear not to make significant contributions to children's performance in both calculation and story problem solving.

Taken together, among the three main predictors of interest in the present study, working memory and additive reasoning seem to be more important than counting ability for children's mathematics learning. The present study suggests that the central executive component of working memory as well as the knowledge of the commutativity and complement principles are particularly crucial.

#### **4.3.6 Autoregressive Models of Calculation and Story Problem Solving**

##### **4.3.6.1 Autoregressive analysis for T2 calculation**

Previous multiple regression models show that working memory and additive reasoning (both commutativity and complement knowledge) are significant longitudinal predictors of children's performance in calculation and story problem solving. One question remains: Are these variables strong enough to be unique predictors of mathematical achievement (Time 2) even when children's previous performance in mathematical achievement (Time 1) is taken into account? This question is important because it is possible that what these predictors had in

common with mathematical achievement at T1 actually explains their longitudinal power. If they remain significant longitudinal predictors of variance after children's performance in mathematical achievement at T1 is controlled for, the case for their predictive value is very strong. Thus, two sets of autoregressive analyses were conducted to examine this question.

When conducting autoregressive analyses, there is a risk for non-independence of errors because of the use of a variable (e.g., T1 calculation) to predict another variable of a similar nature (e.g., T2 calculation) over time. There may also be a risk of multicollinearity because the variables are significantly correlated. Thus, before conducting each of the following regression analyses, attention is paid to the Durbin-Watson's *d* and variance inflation factor (VIF) to check whether these two assumptions of regression are violated. Typically, a value close to 2 for the Durbin-Watson's *d* suggests that the residuals are not linearly auto-correlated (Durbin & Watson, 1951; Field, 2013). The general rule of thumb for VIF is that a value exceeding 4 warrants further examination, whereas a value higher than 10 is a sign of serious multicollinearity requiring correction (Hair, Anderson, Tatham, & Black, 1995; Kennedy, 1992; Marquardt, 1970; Neter, Wasserman, & Kutner, 1989).

The Durbin-Watson's *d* for the autoregressive model of T2 calculation was 2.077 and the highest VIF value was 2.129, thus these two assumptions of regression analyses appeared not to be violated. Examinations of residual plots indicated that the relations between T2 calculation and all predictors were linear and the residuals were randomly scattered around the horizontal line (homoscedasticity). In the following autoregressive models, age, IQ, and counting ability were considered as control variables because the previous regression analyses indicated that they were not unique predictors of T2 calculation and story problem solving when the effects of working memory and additive reasoning were taken into account.

Table 4.63 shows that when the variables of additive reasoning were entered in the final step, the amount of variance in T2 calculation that they accounted for additionally was significant (3.9%). Table 4.64 shows that both commutativity and complement knowledge remained unique predictors of T2 calculation even when children's previous performance in calculation at T1 and all the other factors were taken into account; commutativity knowledge ( $\beta = 0.144$ ,  $t = 2.217$ ,  $p < .05$ ) and complement knowledge ( $\beta = 0.159$ ,  $t = 2.50$ ,  $p = .01$ ).

Table 4.65 shows that when the variables of working memory were entered in the final step after the effects of T1 calculation and all the other factors are controlled for. Working memory explained a significant amount of variance in T2 calculation (2.9%). Table 4.66 shows that the central executive was a unique predictor of T2 calculation even when children's previous performance in calculation at T1 and all the other factors were considered ( $\beta = 0.155$ ,  $t = 2.588$ ,  $p = .011$ ).

*Table 4.63 The additional amount of variance of T2 calculation explained by additive reasoning beyond T1 calculation and all the other factors (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.006	0.006	0.722	0.397	(1, 113)
2	Age in months Non-verbal intelligence	0.034	0.028	3.197	0.076	(1, 112)
3	Age in months Non-verbal intelligence T1 calculation	0.668	0.635	212.426***	<0.001	(1, 111)
4	Age in months Non-verbal intelligence T1 calculation Procedural counting Counting knowledge	0.674	0.005	0.846	0.432	(2, 109)
5	Age in months Non-verbal intelligence T1 calculation Procedural counting Counting knowledge Central executive Phonological loop Visuospatial sketchpad	0.696	0.022	2.781*	0.047	(3, 106)
6	Age in months Non-verbal intelligence T1 calculation Procedural counting Counting knowledge Central executive Phonological loop Visuospatial sketchpad Commutativity knowledge Complement knowledge	0.734	0.039	7.567***	0.001	(2, 104)

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level

Table 4.64

*Autoregressive effects of additive reasoning on T2 calculation: Additive reasoning entered in the final step (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.08	0.094	0.08	0.85	0.397
2	Age in months	0.022	0.098	0.022	0.228	0.82
	Non-verbal intelligence	0.176	0.098	0.176	1.788	0.076
3	Age in months	0.001	0.058	0.001	0.014	0.989
	Non-verbal intelligence	0.083	0.058	0.083	1.419	0.159
	T1 calculation	0.803	0.055	0.803	14.575***	<0.001
4	Age in months	0.005	0.058	0.005	0.09	0.928
	Non-verbal intelligence	0.086	0.058	0.086	1.471	0.144
	T1 calculation	0.79	0.058	0.79	13.616***	<0.001
	Procedural counting	0.076	0.059	0.076	1.286	0.201
	Counting knowledge	0.015	0.074	0.012	0.205	0.838
5	Age in months	0.023	0.057	0.023	0.403	0.688
	Non-verbal intelligence	0.075	0.057	0.075	1.317	0.191
	T1 calculation	0.733	0.062	0.733	11.912***	<0.001
	Procedural counting	0.066	0.058	0.066	1.133	0.26
	Counting knowledge	0.022	0.072	0.018	0.311	0.757
	Central executive	0.155	0.077	0.126	2.015*	0.046
	Phonological loop	0.043	0.059	0.043	0.732	0.466
	Visuospatial sketchpad	0.044	0.057	0.044	0.769	0.443
6	Age in months	0.037	0.054	0.037	0.677	0.5
	Non-verbal intelligence	0.054	0.054	0.054	0.989	0.325
	T1 calculation	0.557	0.074	0.557	7.553***	<0.001
	Procedural counting	0.044	0.055	0.044	0.793	0.429
	Counting knowledge	0.001	0.069	0.001	0.021	0.984
	Central executive	0.19	0.073	0.155	2.588**	0.011
	Phonological loop	0.041	0.056	0.041	0.732	0.466
	Visuospatial sketchpad	0.046	0.055	0.046	0.847	0.399
	Commutativity knowledge	0.155	0.07	0.144	2.217*	0.029
	Complement knowledge	0.184	0.073	0.159	2.5**	0.014

\*significant at the 0.05 level, \*\*significant at the 0.01 level, \*\*\*significant at the 0.001 level



*Table 4.65 The additional amount of variance of T2 calculation explained by working memory beyond T1 calculation and all the other factors (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.006	0.006	0.722	0.397	(1, 113)
2	Age in months Non-verbal intelligence	0.034	0.028	3.197	0.076	(1, 112)
3	Age in months Non-verbal intelligence T1 calculation	0.668	0.635	212.426***	<0.001	(1, 111)
4	Age in months Non-verbal intelligence T1 calculation Procedural counting Counting knowledge	0.674	0.005	0.846	0.432	(2, 109)
5	Age in months Non-verbal intelligence T1 calculation Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.705	0.032	5.767**	0.004	(2, 107)
6	Age in months Non-verbal intelligence T1 calculation Procedural counting Counting knowledge Commutativity knowledge Complement knowledge Central executive Phonological loop Visuospatial sketchpad	0.734	0.029	3.8**	0.012	(3, 104)

\*\*significant at the 0.01 level, \*\*\*significant at the 0.001 level

Table 4.66

*Autoregressive effects of working memoryr on T2 calculation: Working memory entered in the final step (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.08	0.094	0.08	0.85	0.397
2	Age in months	0.022	0.098	0.022	0.228	0.82
	Non-verbal intelligence	0.176	0.098	0.176	1.788	0.076
3	Age in months	0.001	0.058	0.001	0.014	0.989
	Non-verbal intelligence	0.083	0.058	0.083	1.419	0.159
	T1 calculation	0.803	0.055	0.803	14.575***	<0.001
4	Age in months	0.005	0.058	0.005	0.09	0.928
	Non-verbal intelligence	0.086	0.058	0.086	1.471	0.144
	T1 calculation	0.79	0.058	0.79	13.616***	<0.001
	Procedural counting	0.076	0.059	0.076	1.286	0.201
	Counting knowledge	0.015	0.074	0.012	0.205	0.838
5	Age in months	0.013	0.056	0.013	0.235	0.815
	Non-verbal intelligence	0.068	0.056	0.068	1.207	0.23
	T1 calculation	0.643	0.071	0.643	9.078***	<0.001
	Procedural counting	0.057	0.057	0.057	0.999	0.32
	Counting knowledge	0.007	0.071	0.006	0.099	0.921
	Commutativity knowledge	0.119	0.071	0.111	1.683	0.095
	Complement knowledge	0.184	0.076	0.159	2.423*	0.017
6	Age in months	0.037	0.054	0.037	0.677	0.5
	Non-verbal intelligence	0.054	0.054	0.054	0.989	0.325
	T1 calculation	0.557	0.074	0.557	7.553***	<0.001
	Procedural counting	0.044	0.055	0.044	0.793	0.429
	Counting knowledge	0.001	0.069	0.001	0.021	0.984
	Commutativity knowledge	0.155	0.07	0.144	2.217*	0.029
	Complement knowledge	0.184	0.073	0.159	2.5**	0.014
	Central executive	0.19	0.073	0.155	2.588**	0.011
	Phonological loop	0.041	0.056	0.041	0.732	0.466
	Visuospatial sketchpad	0.046	0.055	0.046	0.847	0.399

\*significant at the 0.05 level, \*\*significant at the 0.01 level, \*\*\*significant at the 0.001 level

#### 4.3.6.2 Autoregressive analysis for T2 story problem solving

Table 4.67 shows that when the variables of additive reasoning were entered in the final step, they accounted for a significant amount of variance in T2 story problem solving (6.9%) when T1 story problem solving and all the other factors were controlled for. Table 4.68 demonstrates that both commutativity and complement knowledge remained unique predictors of T2 story problem solving even when children's previous performance in story problem solving at T1 and all the other factors were taken into account; commutativity knowledge ( $\beta = 0.164$ ,  $t = 2.168$ ,  $p < .05$ ) and complement knowledge ( $\beta = 0.261$ ,  $t = 3.607$ ,  $p < .001$ ).

Table 4.69 shows that when the variables of working memory were entered in the final step after the effects of T1 story problem solving and all the other factors are controlled for. Working memory explained a significant amount of variance in T2 story problem solving (2.8%). Table 4.70 indicates that the central executive was a unique predictor of T2 story problem solving even when children's previous performance in story problem solving at T1 and all the other factors were taken into account ( $\beta = 0.178$ ,  $t = 2.734$ ,  $p < .01$ ).

In summary, the central executive component of working memory as well as the knowledge of the commutativity and complement principles are strong predictors for children's mathematical achievement. These variables continued to account for significant amounts of variance in calculation and story problem solving longitudinally even when children's previous performance in the mathematical achievement tasks was taken into account. Thus, the autoregressive analyses confirm the power and importance of additive reasoning and working memory in children's mathematics learning.

*Table 4.67 The additional amount of variance of T2 story problem solving explained by additive reasoning beyond T1 story problem solving and all the other factors (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.008	0.008	0.865	0.354	(1, 113)
2	Age in months Non-verbal intelligence	0.042	0.034	4.032*	0.047	(1, 112)
3	Age in months Non-verbal intelligence T1 story problem solving	0.574	0.532	138.703***	<0.001	(1, 111)
4	Age in months Non-verbal intelligence T1 story problem solving Procedural counting Counting knowledge	0.58	0.006	0.787	0.458	(2, 109)
5	Age in months Non-verbal intelligence T1 story problem solving Procedural counting Counting knowledge Central executive Phonological loop Visuospatial sketchpad	0.601	0.021	1.739	0.109	(3, 106)
6	Age in months Non-verbal intelligence T1 story problem solving Procedural counting Counting knowledge Central executive Phonological loop Visuospatial sketchpad Commutativity knowledge Complement knowledge	0.67	0.069	10.852***	<0.001	(2, 104)

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level

Table 4.68

*Autoregressive effects of additive reasoning on T2 story problem solving: Additive reasoning entered in the final step (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	t values	p values
		B	Std. Error	Beta		
1	Age in months	0.087	0.094	0.087	0.93	0.354
2	Age in months	0.023	0.098	0.023	0.237	0.813
	Non-verbal intelligence	0.196	0.098	0.196	2.008*	0.047
3	Age in months	0.059	0.066	0.059	0.889	0.376
	Non-verbal intelligence	0.096	0.066	0.096	1.449	0.15
	T1 story problem solving	0.744	0.063	0.744	11.777***	<0.001
4	Age in months	0.057	0.066	0.057	0.855	0.394
	Non-verbal intelligence	0.092	0.066	0.092	1.387	0.168
	T1 story problem solving	0.735	0.064	0.735	11.404***	<0.001
	Procedural counting	0.011	0.067	0.011	0.169	0.866
	Counting knowledge	0.1	0.082	0.082	1.229	0.222
5	Age in months	0.071	0.066	0.071	1.075	0.285
	Non-verbal intelligence	0.084	0.066	0.084	1.277	0.204
	T1 story problem solving	0.694	0.067	0.694	10.308***	<0.001
	Procedural counting	0.022	0.067	0.022	0.322	0.748
	Counting knowledge	0.084	0.082	0.069	1.027	0.307
	Central executive	0.173	0.086	0.141	2.007*	0.047
	Phonological loop	0.02	0.068	0.02	0.297	0.767
	Visuospatial sketchpad	0.024	0.066	0.024	0.366	0.715
6	Age in months	0.07	0.061	0.07	1.151	0.252
	Non-verbal intelligence	0.064	0.061	0.064	1.059	0.292
	T1 story problem solving	0.438	0.084	0.438	5.193***	<0.001
	Procedural counting	0.048	0.062	0.048	0.782	0.436
	Counting knowledge	0.082	0.075	0.067	1.09	0.278
	Central executive	0.218	0.08	0.178	2.734**	0.007
	Phonological loop	0.026	0.064	0.026	0.408	0.684
	Visuospatial sketchpad	0.006	0.061	0.006	0.099	0.921
	Commutativity knowledge	0.189	0.087	0.164	2.168*	0.032
	Complement knowledge	0.281	0.078	0.261	3.607***	<0.001

\*significant at the 0.05 level, \*\*significant at the 0.01 level, \*\*\*significant at the 0.001 level

*Table 4.69 The additional amount of variance of T2 story problem solving explained by working memory beyond T1 story problem solving and all the other factors (N = 115)*

Model	Variables entered into model	R <sup>2</sup>	R <sup>2</sup> change	F change	Sig. F change	(df)
1	Age in months	0.008	0.008	0.865	0.354	(1, 113)
2	Age in months Non-verbal intelligence	0.042	0.034	4.032*	0.047	(1, 112)
3	Age in months Non-verbal intelligence T1 story problem solving	0.574	0.532	138.703***	<0.001	(1, 111)
4	Age in months Non-verbal intelligence T1 story problem solving Procedural counting Counting knowledge	0.58	0.006	0.787	0.458	(2, 109)
5	Age in months Non-verbal intelligence T1 story problem solving Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.642	0.062	9.29***	<0.001	(2, 107)
6	Age in months Non-verbal intelligence T1 story problem solving Procedural counting Counting knowledge Commutativity knowledge Complement knowledge Central executive Phonological loop Visuospatial sketchpad	0.67	0.028	3.791*	0.047	(3, 104)

\*significant at the 0.05 level, \*\*\*significant at the 0.001 level

Table 4.70

*Autoregressive effects of working memory on T2 story problem solving: Working memory entered in the final step (N = 115)*

Model		Unstandardised Coefficients		Standardised Coefficients	<i>t</i> values	<i>p</i> values
		B	Std. Error	Beta		
1	Age in months	0.087	0.094	0.087	0.93	0.354
2	Age in months	0.023	0.098	0.023	0.237	0.813
	Non-verbal intelligence	0.196	0.098	0.196	2.008*	0.047
3	Age in months	0.059	0.066	0.059	0.889	0.376
	Non-verbal intelligence	0.096	0.066	0.096	1.449	0.15
	T1 story problem solving	0.744	0.063	0.744	11.777***	<0.001
4	Age in months	0.057	0.066	0.057	0.855	0.394
	Non-verbal intelligence	0.092	0.066	0.092	1.387	0.168
	T1 story problem solving	0.735	0.064	0.735	11.404***	<0.001
	Procedural counting	0.011	0.067	0.011	0.169	0.866
	Counting knowledge	0.1	0.082	0.082	1.229	0.222
5	Age in months	0.055	0.062	0.055	0.885	0.378
	Non-verbal intelligence	0.071	0.062	0.071	1.153	0.251
	T1 story problem solving	0.505	0.082	0.505	6.162***	<0.001
	Procedural counting	0.04	0.063	0.04	0.635	0.527
	Counting knowledge	0.095	0.076	0.078	1.253	0.213
	Commutativity knowledge	0.19	0.089	0.165	2.131*	0.035
	Complement knowledge	0.252	0.078	0.234	3.25**	0.002
6	Age in months	0.07	0.061	0.07	1.151	0.252
	Non-verbal intelligence	0.064	0.061	0.064	1.059	0.292
	T1 story problem solving	0.438	0.084	0.438	5.193***	<0.001
	Procedural counting	0.048	0.062	0.048	0.782	0.436
	Counting knowledge	0.082	0.075	0.067	1.09	0.278
	Commutativity knowledge	0.189	0.087	0.164	2.168*	0.032
	Complement knowledge	0.281	0.078	0.261	3.607***	<0.001
	Central executive	0.218	0.08	0.178	2.734**	0.007
	Phonological loop	0.026	0.064	0.026	0.408	0.684
	Visuospatial sketchpad	0.006	0.061	0.006	0.099	0.921

\*significant at the 0.05 level, \*\*significant at the 0.01 level, \*\*\*significant at the 0.001 level

### 4.3.7 Specificity of Predictions Made by Additive Reasoning Tasks

Previous analyses have established a strong relation between children's ability to reason mathematically and their achievement in mathematics, but it is possible that their performance in the additive reasoning tasks may predict their attainment in other academic subjects as well. Examining this possibility is important because it will let us know more about the reason why additive reasoning predicts mathematical achievement so strongly. This may be because the associations that children have to reason about in these tasks are specific to mathematics learning, in which case it would not be likely that children's performance in additive reasoning would predict children's performance in a subject that does not involve mathematical reasoning, such as word reading. If additive reasoning tasks predict mathematics learning just because they measure reasoning in general, they should also predict children's performance in other non-mathematical academic subjects.

Chinese word reading was also used as one of the Time 2 outcome measures for testing the specificity of predictors for mathematical performance. All of the stimuli in the reading tasks involve two characters. Sometimes Chinese children can reason from one character as a cue to guess the pronunciation of another character and the meaning of the word. Thus, this task may also demand some aspects of general reasoning. Bivariate correlation shows that the central executive significantly correlated with children's performance in reading ( $r = 0.271, p < .01$ ). By contrast, there were no significant correlations between both measures of additive reasoning and children's scores in reading ( $r = 0.124, p > .05$ ). Therefore, the fact that the additive reasoning tasks predicted children's mathematical achievement much better than in Chinese word reading confirms the specificity and importance of additive reasoning in supporting children to learn mathematics.

## 4.4 Concluding Remarks

The mathematical thinking perspective postulates that what is important in mathematics learning is the ability to think about the relations between quantities and numbers. A child who is competent in mathematical thinking should have a good understanding of the meanings of numbers and quantities. Working memory, counting ability, and additive reasoning seem to form the cognitive foundations for children to think mathematically. Therefore, these factors should affect their success in mathematical achievement, such as calculation and solving story problems.



Taken together, consistent with the mathematical thinking perspective, the results from the regression analyses may contribute to the development of a model for young children's mathematics learning that highlights the importance of logical understanding. It appears that the contents of the model may vary slightly by the age of children and the nature of mathematical tasks. For example, all three hypothesised factors (working memory, counting ability, and additive reasoning) make independent contributions to explaining individual differences in calculation beyond the effects of all the other factors in first graders. However, only working memory and additive reasoning are unique longitudinal predictors for children's performance in calculation beyond all the other factors one year later.

One possible reason is that the role of additive reasoning in children's mathematics learning may increase with age because the mathematical tasks children are exposed to usually become increasingly challenging, so that the relative importance of counting ability decreases over time. The mathematical tasks that children need to solve at the age of 7 are likely to be more difficult than those they are exposed to at the age of 6. For example, larger numbers and more two-digit numbers are involved in calculation and the calculation problems may include more than two numbers. Because there was a need to develop an age-appropriate measure in the present study, the calculation tasks used in the second wave of assessment were more challenging than those in the first wave of assessment. Those children who use more advanced calculation strategies (e.g., indirect addition, decomposing ' $16 + 9$ ' into ' $16 + 10 - 1$ ', transforming ' $3 + 6 + 7$ ' into ' $(3 + 7) + 6$ ') may be more successful in solving the more difficult calculation problems. Understanding the logical relations between numbers, according to the mathematical thinking perspective, may be conducive to children's development of more efficient strategies. Working memory is also required for the cognitive processes that monitor and coordinate the operations of various calculation strategies. Therefore, both additive reasoning and working memory are important for children to deal with the increasing mathematical demands when they grow older.

Effective story problem solving may demand a different set of cognitive skills. Counting ability may be a necessary factor that affects children's ability to solve story problems because it is a skill that young children begins to learn calculate and solving story problems also involves calculation. However, it appears not to be a sufficient factor that contributes to the success in solving story problems because it must involve conceptual analysis of the underlying quantitative relations in the problems. Thus, compared to additive reasoning and working memory, the present study suggests that counting ability is less important. Having a good

understanding of the relations between quantities in a story problem helps children solve problems that are framed in different ways. For example, start-unknown change problems require problem solvers to conceive addition as the inverse of subtraction, and vice versa. A great deal of general cognitive resources, such as working memory, is also required for problem solvers to keep track of the relations between quantities in a story problem. Therefore, on the basis of the mathematical thinking perspective, both additive reasoning and working memory should be important for children's success in solving story problems. Consistent with this view, this study suggests that both additive reasoning and working memory are consistent concurrent and longitudinal predictors for children's performance of story problem solving.

The findings from this chapter entail some educational implications. The strong relation between additive reasoning and mathematical achievement is evidence that it is important for teachers to provide more explicit instruction on mathematical relations. Children should be encouraged to understand what mathematical relations are and how to reason about them logically. Educational programmes that combine working memory training and the learning of additive reasoning may be promising for enhancing the mathematical development in young children.

## Chapter Five Latent Profile Analysis

Regression analyses in the previous chapter have shown that additive reasoning is a strong predictor of children's mathematical achievement. One implication of this finding is that it is important for educational professionals to spend more time on teaching children how to reason about relations between quantities. However, on the basis of a continuous score alone, it is difficult for teachers to know where to start. For example, a total score of '6' on additive reasoning may represent 4 correct answers on the commutativity tasks and 2 correct answers on the complement tasks. It may also consist of 2 correct answers on the commutativity tasks and 4 correct answers on the complement tasks. It would not be easy for teachers to devise strategies that are tailored to the needs of different children if they do not have knowledge about the particular strengths and weaknesses of each child.

The needs of different children may relate to the developmental order of the understanding of the commutativity and complement principles, and the role of concrete materials in this development. Are there distinct groups of children who share similar patterns of strengths and weaknesses in learning these mathematical principles because they are at different developmental stages? I address this question in the current chapter with latent profile analysis. This statistical tool allows one to identify distinct groups of children on the basis of their performance in additive reasoning in a parsimonious way.

### 5.1 Revisiting the Hypotheses

It was hypothesised that knowledge of the commutativity and the complement principles develops over time rather than emerges in an all-or-nothing fashion. At first, children succeed in tasks that assess their knowledge of each of these principles when the problems are set in the presence of concrete materials, and succeed only later when the problems refer to more abstract materials, in the absence of objects. It was also hypothesised that there is an order of acquisition for these two principles. Both principles rest on children's understanding of part-whole relations, but commutativity refers to the simplest logical aspect of part-whole, which is that the order in which you add the parts does not affect the whole ( $a + b = b + a$ ). Knowledge of the complement principle appears to be logically related to, but to require more than commutativity. In order to understand that if  $a + b = c$ , then  $c - b = a$ , and  $c - a = b$ , children need to think of (1)  $a$  and  $b$  as interchangeable parts (i.e. they need to understand commutativity) and that (2) if you take the first part away from the whole, you are left with the second, and if you take the second part away from the whole, you are left with the first.

Thus, two basic hypotheses are being tested in this study: one refers to children's progress within the understanding of one principle (from concrete to symbolic problems) and the other refers to the order of understanding of the principles (from the commutativity to the complement principle). These hypotheses lead to the following predictions.

First, if children initially understand a principle in the context of concrete referents, all children who perform well for the principle in the abstract condition should also do well in the concrete condition. It is not expected to find a group of children who perform well in the abstract condition, but not in the concrete condition as well. Identification of the latter group of children would lead to falsifying the first hypothesis.

Second, if children develop the understanding of the commutativity principle before the complement principle, then all children who obtain high scores in the complement tasks should also perform well in the commutativity tasks, and one should not find children who perform well in the complement tasks but not in the commutativity tasks. If a sizeable group of children with the latter profile of performance were to be identified, this could not be attributed simply to error of measurement, but would be evidence against the hypothesis of an order of acquisition of these two principles.

One aspect in this developmental sequence remains unclear: Do children need to master the commutativity principle in the more abstract condition before they start to succeed in the complement principle tasks? Or is it sufficient to succeed in the commutativity tasks with concrete materials to make progress in the complement tasks with concrete materials?

If the first alternative is true, one would expect to find four groups of children who display some knowledge of the principles: the first would succeed in commutativity tasks with concrete materials but in no other tasks; the second could succeed in commutativity tasks in both the concrete and abstract conditions, but not in complement tasks; the third would succeed in all commutativity tasks and in complement tasks with concrete materials; the final group would succeed in all the tasks.

However, if the second alternative is true, and it is sufficient to succeed in commutativity tasks with concrete materials in order to make some progress in the complement principle, one should identify a fifth group, that succeeds in commutativity and complement tasks with concrete materials but in neither task in the abstract conditions only.

It is, of course, possible to identify yet a sixth group, which has no success at all in any of the tasks. However, its identification is not relevant to the test of these alternatives.

## 5.2 Latent Profile Analysis and Predictions

A useful statistical approach to test these hypotheses is the identification of children's profiles of success and failure in the tasks. The aim of this analysis is to uncover groups of children who show similarities and differences in their performance on the reasoning tasks in the different testing conditions (with or without concrete materials). Latent profile analysis provides an effective way to identify homogeneous subgroups of individuals, with each subgroup possessing a unique set of characteristics that distinguishes it from other subgroups.

The profiles are not directly observed themselves: they are inferred from the patterns of observed responses. For this reason, the statistical technique used to identify these groups is termed latent profile analysis. In this analysis, people are classified into latent classes on the basis of similar patterns of observed data. In this study, the observed distribution of 'additive reasoning' may be a combination of two or more subgroups of children. Latent profile analysis can be used to classify children probabilistically into subgroups by inferring each child's membership to latent classes from the data. Then, every child has his/her own probabilities computed for his/her membership in all of the latent classes estimated (when added up they equal to 1). The assignment of each child to each latent class is based on these probabilities. Thus, this analytical tool is a useful tool that can be used in the present study to test the hypotheses about the groupings of children according to their performance on reasoning tasks.

On the basis of the hypotheses, I expected to find five groups of children with latent profile analysis (Table 5.1):

*Table 5.1 Predictions of children's profiles on their understanding of mathematical principles*

	Commutativity- concrete	Commutativity- abstract/numerical symbols	Complement- concrete	Complement- abstract/numerical symbols
Class 1	High score	High score	High score	High score
Class 2	High score	High score	High score	Low score
Class 3	High score	High score	Low score	Low score
Class 4	High score	Low score	Low score	Low score
Class 5	Low score	Low score	Low score	Low score

The first group of children would obtain high scores in all the tasks. The second group would have high scores in all commutativity tasks and in complement tasks with concrete materials only. The third group would get high scores in commutativity tasks in both concrete and abstract

conditions, but not in complement tasks. The fourth group would have high scores in commutativity tasks with concrete materials but not in other tasks. The final group would get low scores in all of the tasks. This group is not relevant to the test of the hypothesis, but it is reasonable to expect that some children will not succeed in any of the tasks.

There are classification techniques other than latent profile analysis that serve similar analytical purposes, such as cluster analysis, but latent profile analysis is a better technique to address the hypotheses of this study. First, cluster analysis is a non-inferential procedure, which means that we cannot assume the identified clusters are applicable to the population. The groups are arbitrarily based on the similarities among individuals in a particular sample. In contrast, it is assumed in latent profile analysis that there is an unobserved latent variable in the population that explains responses on the indicator variables. Thus, it takes a confirmatory approach, which is more appropriate than cluster analysis for the present study. It is because the hypotheses and predictions were set up according to developmental theories and previous research.

Second, no statistical tests are available to evaluate the clustering solution in cluster analysis. Thus, the decisions for the groupings are rather subjective (Hair, Anderson, Tatham, & Black, 1998; Magidson & Vermunt, 2002; Whiteman & Loken, 2006). In contrast, latent profile analysis employs rigorous statistical measures of fit to facilitate group identifications in a given population (Pastor, Barron, Miller, & Davis, 2007). The application of statistical tests of model-data fit confers a major advantage of mixture modeling over cluster analysis by allowing researchers to identify groups in a more objective manner. Comparative and simulation research has demonstrated that latent profile analysis offers more accurate grouping solutions than cluster analysis (DiStefano & Kamphaus, 2006; Magidson & Vermunt, 2002; Whiteman & Loken 2006).

In practice, researchers usually start by hypothesising about the number of classes and assess the fit between the sampled data and the model. Then they specify another model, estimate the fit of the data to that model, and compare to the first model. This procedure is repeated with various specified models until the best-fitting solution is identified (Magidson & Vermunt, 2002; Pastor & Gagné, 2013).

For this study, latent profile analysis was conducted using MPlus7 (Muthén & Muthén, 2007). In order to determine the optimal number of classes, I evaluated various models against a number of criteria (Table 5.2). First, models with different numbers of classes were compared

using information criteria (IC)-based fit statistics. These include the Bayesian Information Criteria (BIC; Schwartz, 1978), Akaike Information Criteria (AIC; Akaike, 1987), and Adjusted BIC (Sclove, 1987). Lower values on these fit statistics represent better model fit. Second, the accuracy with which models group children into their most likely class was examined. Entropy is a statistic that evaluates this accuracy, and can range from 0 to 1, with higher scores indicating higher classification accuracy. On the basis of the hypotheses, I expected to obtain the lowest values on the IC statistics and the highest value on Entropy in the five-class model.

Third, a statistical model comparison likelihood ratio test was used – the Lo–Mendell–Rubin test (LMR; Lo, Mendell, & Rubin, 2001). This test compares the fit of a target model (e.g., four-class model) to a comparison model that specifies one less class (e.g., three-class model). The  $p$ -value generated for the LMR test indicates whether the solution with more classes ( $p < .05$ ) or less classes ( $p > .05$ ) fits better. In the subsequent analysis, I expected that the five-class model would fit better than the four-class model, whereas the inclusion of more classes would not show improvement, suggesting that the five-class model is the most parsimonious model.

An additional consideration is the size of each class because a small class could lead to unstable parameter estimates if the sample size is not large. The final and the most important consideration of model selection is whether the classes make theoretical sense. Thus, I would examine the patterns of variables within each class to see whether a particular classification provides any meaningful information (McLachlan & Peel, 2000; Pastor & Gagné, 2013). In summary, deciding on the number of classes involve the consideration of theories, statistical indices, parsimony, and the substantive meaning of each solution (Bauer & Curran, 2003).

### 5.3 Results

#### 5.3.1 Profile Identification

*Table 5.2 Model fit indices for each class model (N = 115)*

	3 classes	4 classes	5 classes	6 classes
AIC	1356.996	1341.798	1344.278	1340.597
BIC	1406.405	1407.412	1418.656	1431.179
SSABIC	1349.51	1330.154	1334.713	1326.873
Entropy	0.962	0.898	0.862	0.870
LMR test	2 vs 3	3 vs 4	4 vs 5	5 vs 6
	value = 69.70	value = 21.80	value = 11.98	value = 10.75
	$p < 0.01$	$p = 0.02$	$p = 0.16$	$p = 0.33$
Number of children	C1=21	C1=21	C1=21	C1=9
for each class (C)	C2=59	C2=21	C2=38	C2=12
	C3=35	C3=38	C3=21	C3=38
		C4=35	C4=17	C4=18
			C5=18	C5=21
				C6=17

Latent profile models containing 3, 4, 5, and 6 classes were fit to the data. The model fit indices for each LPA are available in Table 5.2. To determine the optimal number of classes, I began by reviewing the IC indices (AIC, BIC, and SSABIC) and the entropy values. The indices of all profile models were close to each other. Thus, it is difficult to make a clear-cut decision merely based on the IC indices and the entropy values. However, the three-class model was not selected as the optimal model because Class 2 contains all children except the high-performing and low-performing children. This model is not meaningful in theoretical sense because this classification made no distinctions in the middle group. The LMR test also showed that the inclusion of one more class (i.e. the four-class model) fit better than the three-class model ( $p < 0.05$ ).

The six-class model was also not chosen as the optimal model because it yielded a class size that was too small to be of substantive value (9 children only). The LMR test also indicated that it did not fit better than the five-class model ( $p > 0.05$ ).

Comparing the four- and five-class models, the LMR test showed that the five-class model was not better than the four-class model ( $p > 0.05$ ). The four-class model also revealed lower AIC, BIC, and SSABIC values than the five-class model.

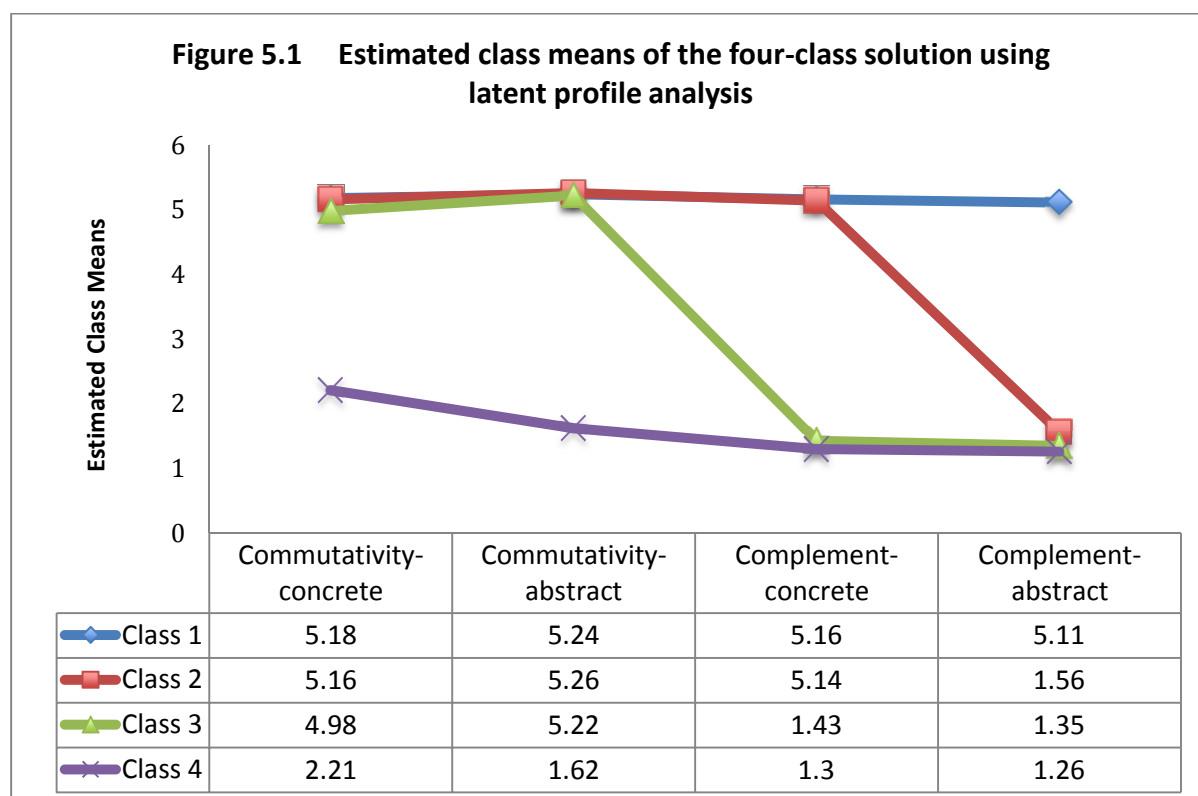
In summary, the four-class model gave a solution that fits the model better than the three-class model and the other comparisons did not show an improvement. Thus, the latent profile



analysis led me to choose the four-class model as the best fit to the data in this study. This finding did not support my hypothesis because I expected to find five distinct groups of children. Having decided on the four-class model, the next step is to examine whether these classes show a relation to the groups identified in my hypothesis. It is thus necessary to examine the characteristics of the children within each class to identify which hypothetical group of children is missing.

### 5.3.2 Description of Classes

The estimated class means for each class for the four-class model are depicted in Figure 5.1.



Children in Class 1 had good performance of the commutativity and complement principles in both the concrete and abstract conditions. Within-group comparisons showed that the children in this Class did not differ significantly in their performance across the four conditions (all comparisons showed  $p > .05$ ).

Children in Class 2 demonstrated good performance on the commutativity tasks in both conditions, but their performance on the complement tasks was high only in the presence of concrete referents. It appears that they could not think about the inverse relation between addition and subtraction without the support of concrete materials. Within-group comparisons showed that the only significant differences happened between the 'complement-abstract' condition and the other three conditions (all comparisons showed  $p < .05$ ).

In Class 3, children demonstrated good performance on the commutativity tasks, but they did not perform well on the complement tasks. Within group comparisons indicated that the children in this Class performed significantly better in the two commutativity conditions than the two complement conditions (all comparisons showed  $p < .05$ ).

Finally, children in Class 4 showed good performance on the commutativity tasks in the presence of concrete materials only, but they did not do well on all of the complement tasks. Although it may appear that the children in this Class performed poorly across all four conditions, a closer look at the performance difference between the ‘commutativity-concrete’ and the ‘commutativity-abstract’ conditions showed that they were significantly better at understanding the commutativity principle with objects than in the abstract condition ( $p < .05$ ).

A comparison of the present results and my hypothetical predictions of children’s profiles is summarised in Table 5.3. It shows that while the hypothetical Class 5 (children who demonstrate low competence in all tasks) was absent in the data, the existence of the first four Classes were supported by the latent profile analysis. Although Class 4 did not have a truly ‘high’ score (only 2.21) on the commutativity task in the concrete condition, the score was significantly higher than that on the same task in the abstract condition (1.62), which supports my prediction.

*Table 5.3 A comparison of the results with the hypothetical predictions of children’s profiles*

	Commutativity- concrete	Commutativity- abstract/numerical symbols	Complement- concrete	Complement- abstract	Results
Class 1	High score	High score	High score	High score	<b>Present</b>
Class 2	High score	High score	High score	Low score	<b>Present</b>
Class 3	High score	High score	Low score	Low score	<b>Present</b>
Class 4	High score	Low score	Low score	Low score	<b>Present</b>
Class 5	Low score	Low score	Low score	Low score	<b>Absent</b>

This set of analyses has a clear theoretical implication for the understanding of the development of mathematical principles in children. The findings suggest that mathematical development is not akin to turning on a light. Children do not lack a mathematical concept one day, and then that becomes full-blown in the next day. Instead, they begin with some mathematical insights, enjoying moments of proficiency in one context while losing it in others, until in the end their performance becomes steady across a variety of situations. In other words, they may master various pieces of the concepts at different times, such that incomplete knowledge interact for some time before children reliably demonstrate what might be regarded as mature understanding. Consistent with my hypothesis, this study suggests that additive reasoning develops over time rather than emerges in an all-or-nothing manner. There seems to

be an order of acquisition for the two principles essential to additive reasoning (from commutativity to complement principle) and children initially understand each of these concepts in the context of concrete referents.

Do children need to master the commutativity principle in the more abstract tasks before they start to succeed in the complement principle tasks? Or is it sufficient to succeed in the commutativity tasks with concrete materials to make progress in the complement tasks with concrete materials? The findings showed that the second alternative is not tenable because I did not identify a group of children who succeeded commutativity and complement tasks with concrete materials but in neither task in the abstract conditions only. The existence of the four classes suggests that there are individual differences in the development of additive reasoning. It is likely that children acquire the knowledge of the commutativity principle in the more abstract tasks before they start to acquire the knowledge of the complement principle.

### **5.3.3 Validation Analyses – Mean differences in Time 1 Mathematical Achievement**

In order to provide additional evidence for the meaningfulness of the classes, I examined whether the profiles predict children's mathematical achievement. In this study, it is hypothesised that children with a higher additive reasoning ability perform better in mathematics. Thus, if I find that a group of children that is characterised by high levels of quantitative reasoning shows significantly higher scores on calculation and story problem solving than a group of children that exhibits a weaker profile, both concurrently and longitudinally, then I can demonstrate some validity for the four-class model.

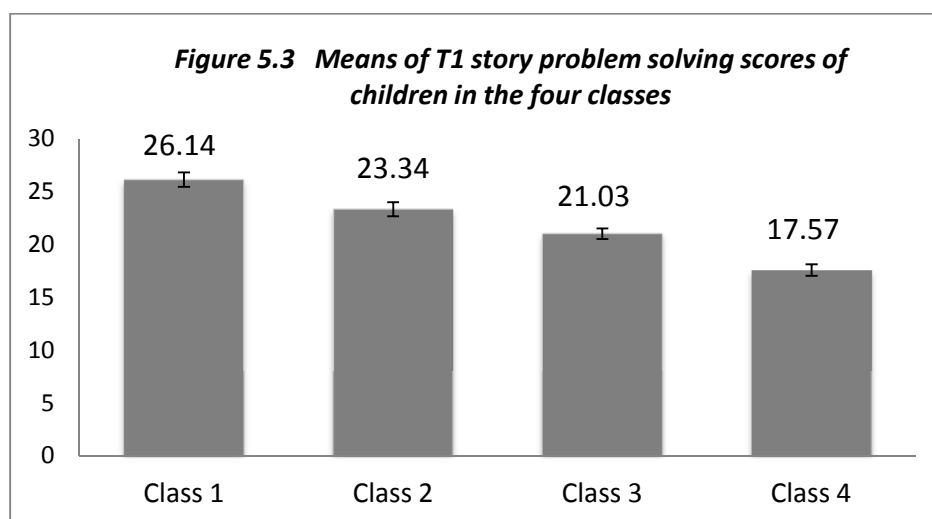
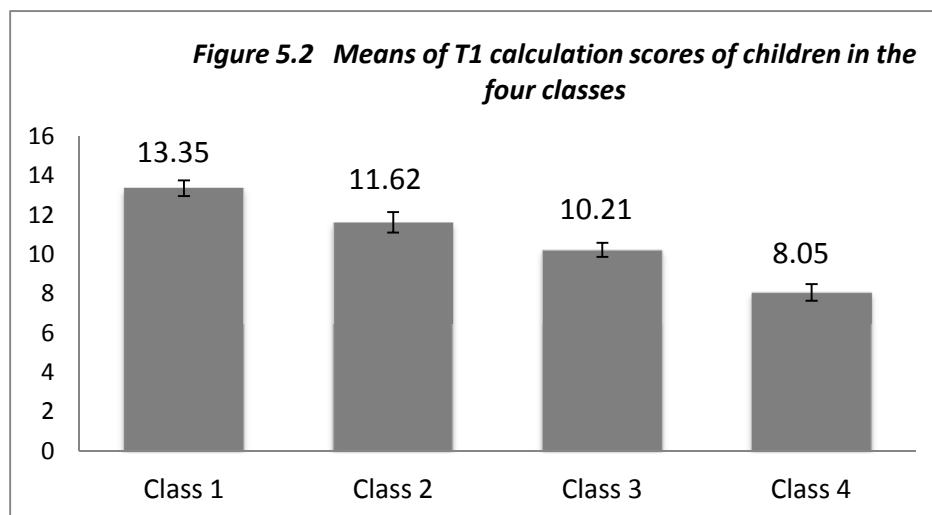
I entered the two variables – calculation scores and scores on solving story problem at Time 1 – in the latent profile analysis as 'auxiliary variables' using MPlus7 to assess the equality of means. Comparisons of the means of validity variables across classes are summarised in Table 5.4 and the differences are graphically depicted in Figures 5.2 and 5.3. The results showed that all four classes were statistically differed from each other on calculation scores and story problem solving scores at Time 1 (Class 1 > Class 2 > Class 3 > Class 4). This finding provides some evidence for the validity of the four-class model.

**Table 5.4** Means and standard errors of each class on calculation and story problem solving at T1

	Means (standard errors)			
	Class 1	Class 2	Class 3	Class 4
Time 1 Calculation	13.35 (0.40)	11.62 (0.52)	10.21 (0.36)	8.05 (0.43)
Time 1 Story Problem Solving	26.14 (0.69)	23.34 (0.68)	21.03 (0.50)	17.57 (0.55)

*Note.* The maximum scores of T1 and T2 calculation are 16.

The maximum scores of T1 and T2 story problem solving are 32.



### 5.3.4 Validation Analyses: Using Profiles to Predict Time 2 Mathematical Achievement

The test of validity can be obtained from analyses that assess whether the profiles that were obtained at an earlier time predict children's performance in calculation and story problem solving at a later time beyond the effects of age, IQ, working memory, and counting ability. Because the latent classes are categorical, they were dummy-coded and entered into a series of multiple regression analyses.

Class 4 served as the comparison group (i.e. the group coded 0) in the first set of regression analyses on T2 calculation and story problem solving as criterion variables. Table 5.5 shows that the means of Classes 1 to 3 were significantly higher than the mean of Class 4 [all standardised beta-values were significant and positive ( $ps < .001$ )]. Using Class 3 as the comparison group, the model showed that the means of Classes 1 and 2 were significantly higher than the mean of Class 3 ( $ps < .001$ ). When the comparison group was Class 2, the standardised beta-value indicated that the mean of Class 1 was significantly higher than that of Class 2 ( $ps < .001$ ). Table 5.6 summarises the significant mean differences among the four classes, showing that the differences between every pair of classes were statistically significant.

*Table 5.5 Standardised beta-values of dummy-coded cluster variables in multiple regression analyses on children's performance on calculation and story problem solving at Time 2*

	T2 Calculation		T2 Story problem solving	
	<i>standardised betas</i>	<i>t-values</i>	<i>standardised betas</i>	<i>t-values</i>
Class 1 vs Class 4	0.78***	9.03	0.85***	9.50
Class 2 vs Class 4	0.55***	6.05	0.66***	7.02
Class 3 vs Class 4	0.27***	3.28	0.36***	4.16
Class 1 vs Class 3	0.49***	6.17	0.47***	5.72
Class 2 vs Class 3	0.25**	2.98	0.26**	3.04
Class 4 vs Class 3	-0.25***	-3.28	-0.33***	-4.16
Class 1 vs Class 2	0.26***	3.55	0.22**	3.00
Class 3 vs Class 2	-0.23**	-2.98	-0.23**	-3.04
Class 4 vs Class 2	-0.45***	-9.06	-0.54***	-7.02

\*\*  $p$ -values significant at the 0.01 level, \*\*\*  $p$ -values significant at the 0.001 level

*Table 5.6 Significant mean differences among the four classes*

	Class 1	Class 2	Class 3	Class 4
Class 1	--	✓	✓	✓
Class 2	✓	--	✓	✓
Class 3	✓	✓	--	✓
Class 4	✓	✓	✓	--

Note. ✓ denotes significant mean differences

## 5.4 Concluding Remarks

In summary, the latent profile analysis provides evidence for the developmental order of the acquisition of the commutativity and complement principles. It suggests that children acquire the commutativity knowledge prior to the complement knowledge. Concrete materials may also play a role in this development because children seem to grasp each of these principles in the context of concrete referents before being able to reason with more abstract symbols. A

continuous score that represents additive reasoning does not capture this developmental sequence. Thus, the use of classification-typed analytical approach contributes to the identification of specific strengths and weaknesses of children's learning of mathematical concepts. This information is important in teaching because it helps educators develop pedagogical practice that tailors to the needs of children at different stages of development.

To illustrate with an analogy, one could think that the development of additive reasoning in children might be analogous to motor development: Children crawl before they are able to stand up and walk. However, the experience of crawling may not be developmentally irrelevant. It tells young children about the feelings of balance, the characteristics of different types of surface, navigation through space, and so on. These are the things that they need to learn before they can walk. Thus, motor skills in children do not develop in an all-or-none fashion. For some children, there is also a temporary bridge that facilitates them to transit from crawling to walking. This is cruising, which refers to using furniture to support moving upright. Cruising is not a 'proper' way to walk, but it connects the previous experience of crawling with the additional skills that are required in walking, such as standing upright.

What if additive reasoning works in the same way? Children who only understand the commutativity principle (like crawling) are far from achieving success in understanding additive reasoning, but knowledge of commutativity allows the child to understand the simplest logical aspect of part-whole relations, which forms the basis for the understanding of the more complicated complement principle. In the end, the integration of the knowledge of the two principles makes the child come closer to a better understanding of additive reasoning.

Some children may need some external support for reasoning (just like cruising) before they can internalise and achieve the understanding of a mathematical concept with symbols. In this regard, physical objects may help these children to learn the concept by providing concrete representations of the quantities and the relations between quantities. These materials serve similar functions as the furniture in cruising, which supports the children to think about the relations between quantities and hence develop more advanced reasoning abilities gradually.

In conclusion, there are patterns of individual differences in the development of additive reasoning. Thus, when teaching additive reasoning to children, educators may need to consider more about the developmental stage where each child is placed in order to provide more appropriate support. Assessing reasoning in different ways and identifying profiles with classification analyses may be helpful in this respect. It appears that complementing commonly used symbolic assessments (e.g., the use of pure numbers) with concrete materials would place children better in the continuum of mastery of a mathematical concept.

## Chapter Six Discussion

The purpose of this thesis is to evaluate the relative importance of working memory, counting ability, and additive reasoning in children's mathematics learning. The key findings of this study have contributed to the literature in several ways. First, Nunes and colleagues (2007, 2012) found that quantitative reasoning was a significant and specific predictor of children's mathematical achievement beyond IQ and working memory. The present study replicated the finding regarding the close connection between quantitative reasoning and mathematical achievement in a non-Caucasian cultural context. Second, whereas previous studies demonstrated a strong link between a global measure of quantitative reasoning and test scores on general mathematical achievement, the present study showed that mathematical reasoning in the domain of addition and subtraction in particular related significantly to both calculation and story problem solving concurrently and longitudinally. Third, the autoregressive analyses indicated that variables in additive reasoning and the central executive component of working memory remained independent predictors of T2 mathematical achievement beyond the influence of children's performance on T1 mathematical achievement. This is strong evidence for the predictive powers of these variables. Fourth, this study incorporated procedural and conceptual counting as the indicators of counting ability and showed that only conceptual counting was uniquely predictive of calculation, but not of story problem solving. Finally, the latent profile analysis suggests that patterns of individual differences in the development of additive reasoning might represent developmental changes. It is likely that additive reasoning develops from thinking in the context of specific quantities to thinking about more abstract symbols, and that children acquire knowledge of the commutativity principle in the symbolic tasks before they start to acquire knowledge of the complement principle.

In the following sections, I discuss the key findings with respect to the hypotheses of this study and the extent to which the findings are consistent with previously published knowledge on the topic. Then, the theoretical and educational implications of the results on children's mathematics learning and education are discussed. Finally, limitations of this study are identified and suggestions for future research are made towards the end of this chapter.

### 6.1 Contributions of Counting Ability

On the basis of the mathematical thinking perspective, counting ability was hypothesised to be one of the important cognitive foundations for children's mathematics learning. The mathematical thinking perspective emphasises that children need to understand the meanings

of number in order to perform well in mathematics. The knowledge of the meanings of number refers to the understanding of the relations between numbers and quantities. Learning to count is relevant in this respect because it provides children with words to represent quantities. It also helps children reflect upon and develop the concept of one-to-one correspondence, ordinality, and cardinality as well as the coordinated use of these counting principles. According to the mathematical thinking perspective, grasping the conceptual knowledge of counting is more important than reciting a counting sequence in children's mathematics learning.

The regression analyses of this study show that counting ability accounted for a significant amount of variance in calculation (both concurrently and longitudinally) after the effects of age, IQ, and working memory were controlled for. Thus, the first hypothesis was supported for calculation in this study. Among the counting measures, conceptual knowledge of counting was a unique predictor of children's performance in calculation beyond the influence of age, IQ, and working memory. When children learn to calculate, they usually start from counting all of the numbers presented (i.e. the count-all procedure) and later shift to counting on from the cardinal value of the first or larger number presented. (Fuson, 1982). The more efficient counting-on procedures may rely on conceptual knowledge of counting, such as the understanding of cardinality. Thus, conceptual knowledge of counting contributes to children's success in calculation.

However, another measure of counting ability, procedural counting, did not make independent contributions to any measure of mathematical achievement. This finding is at odds with those from previous longitudinal studies (e.g., Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Koponen, Aunola, Ahonen, & Nurmi, 2007; Koponen, Salmi, Eklund, & Aro, 2013; Passolunghi, Vercelloni, & Schadee, 2007; Zhang, Koponen, Rasanen, Aunola, Lerkkanen, & Nurmi, 2014). For example, one study (Zhang et al., 2014) showed that the impact of procedural counting was so strong that it fully mediated the longitudinal association between spatial visualisation and letter knowledge with a group of Finnish children's performance in arithmetic. The discrepancy of the findings could be attributable to the ceiling performance of children in the present study, which may relate to the languages that the participating children speak in different research.

In counting, there are units of different sizes that can be counted within different classes. For example, we have the class of ones, the class of tens, the class of hundreds and so on. Because most of us use a base-ten system, when we have ten units of any size, we regroup these into units of the next size. For instance, ten 'ones' make up one 'ten', and ten 'tens' make up one 'hundred'. In Chinese number words, the base structure of the number system is transparent.



Counting with Chinese number words makes children recognise easily that they are counting different units and they can repeat the same reasoning indefinitely to generate number words that they have not been taught formally before. Thus, the systematic relation between the number words in Chinese language and the underlying base-10 values may contribute to Chinese-speaking children's early mastery of procedural counting (Miller & Stigler, 1987; Miller, Smith, & Zhang, 2004; Miura, Kim, Chang, & Okamoto, 1988). This may be one of the reasons that the children in the present study had exceptional performance in procedural counting. Thus, the variation of children's performance on this task was small in this study, which might have influenced the strength of its relation to mathematical achievement.

By contrast, most of the children who participated in previous longitudinal research spoke European languages (e.g., English and Finnish). In these languages, the base structure of the number system is only partially reflected in the language, for example, 'tens' are counted with different names like ten, twenty, and thirty etc. This may render it more difficult for some young children to grasp the underlying structure of the counting system, thereby contributing to greater variation in procedural counting in these children. Thus, the impact of languages on structuring the number system in different cultures may explain the divergent findings regarding procedural counting across studies.

It was demonstrated in the present study that conceptual knowledge of counting was a stronger predictor than procedural counting of all measures of mathematical achievement. The ceiling effect may be one of the explanations of the results. Another possible interpretation is that conceptual knowledge of counting is more important than procedural counting in children's mathematics learning. In the present study, conceptual counting was measured by a task that required children to identify incorrect ways of counting and to coordinate their knowledge of various counting principles to determine the cardinal value of a set. It has been argued that it is necessary for children to coordinate different counting principles in order to understand the logic of numbers (Nunes & Bryant, 2015). On the basis of the mathematical thinking perspective, counting involves not only the memorisation of the number words in a fixed order but also the understanding of how number labels are generated in order to surpass simple memorisation of labels. It has been suggested the reason that counting ability makes contributions to explaining variation in mathematical achievement is that it helps children reflect on the relations between quantities and numbers (e.g., Piaget, 1952; Piaget & Inhelder, 1975; Nunes & Bryant, 2015). Thus, if a child can only generate number words proficiently but fails to understand the logic of

counting, she or he is not likely to do well in mathematics according to the mathematical thinking perspective. Consistent with this view, the findings of this study show that conceptual knowledge of counting had a stronger connection than procedural counting with both calculation and story problem solving concurrently and longitudinally.

Thus, the present study adds to the literature that individual differences in conceptual knowledge of counting may matter more than procedural counting in mathematical achievement. This finding has several implications. First, from a methodological perspective, this evidence suggests that conceptual knowledge of counting may be a better measure of counting ability than procedural counting, especially for children who speak a language in which the organisation of number words fits well with the base-ten system. Second, from an educational viewpoint, the finding suggests that learning the numerical symbols of counting by themselves is not sufficient for children to succeed in mathematics. Past evidence showed that there could be a disconnection between using numbers and understanding the logic of counting (e.g., Bermejo, Morales, & deOsuna, 2004; Freeman, Antonucci, & Lewis, 2000; Sarnecka & Gelman, 2004; Sophian, 1988). Thus, teachers and parents need to ensure that children learn not only to count fluently, but also learn to think about the logical connections of the numbers they use for counting with quantities.

It is noted that the contribution of conceptual knowledge of counting continued to be marginally significant even after the effects of all other factors, including additive reasoning were controlled for in T1 calculation. However, it ceased to contribute to T2 calculation significantly when the influence of all the other factors was taken into account. It is worth noting that the mathematical achievement tasks in this study differed between first and second waves of assessment. The differences in task complexity may explain the relative contributions of various factors to mathematical achievement. For example, children can solve problems  $4 + 3$  easily using counting, but when they are exposed to addition problems with larger addends (e.g.,  $16 + 8$ ), other cognitive skills, such as additive reasoning and working memory, may have larger impact. Similarly, story problems may demand a different set of cognitive skills. Counting ability may be a necessary factor in story problem solving because it also involves calculation. However, relative to other cognitive skills, the present study shows that counting ability is a less important one. For example, counting ability did not make a significant contribution to story problem solving concurrently and longitudinally when the effects of age, IQ, and working memory were controlled for. Thus, it is possible that the relative importance of counting might decrease when

more complicated tasks were used. There seems to be other kinds of cognitive competence that play a more important role than counting ability in children's mathematics learning. On the basis of the mathematical thinking perspective, one such competence variable may be additive reasoning.

## 6.2 Contributions of Additive Reasoning

The second hypothesis of this study is that additive reasoning is independent from and more important than working memory and counting ability in children's mathematics learning. This hypothesis is strongly supported by the findings. Consistent with this hypothesis, additive reasoning was shown to make independent contributions to explaining variance in calculation and story problem solving beyond and above the effects of age, IQ, working memory, and counting ability at both waves of assessments. The regression analyses showed that the additional amount of variance explained by additive reasoning beyond all the other factors was substantial (close to 30% for both calculation and story problem solving).

On the basis of Piaget's logical operations framework, some researchers (e.g., Nunes & Bryant, 1996, 2015; Thompson, 1993, 1994; Vergnaud, 1997, 2009) have argued that children's competencies to reason about quantities logically are of primary importance for mathematical development. In the domain of additive reasoning, it is important to understand that quantities are connected by part-whole relations. Two central properties of part-whole relations involve (1) commutativity and (2) the inverse relation between addition and subtraction. Commutativity refers to the irrelevance of addend order to the sum, i.e. ' $a + b = c$ ' implies ' $b + a = c$ ', whereas the complement principle refers to the inverse relation between addition and subtraction, i.e. ' $a + b = c$ ' implies ' $c - a = b$ '. A few studies have shown that global measures of quantitative reasoning are main predictors of children's later mathematical achievements (Nunes et al., 2007, 2012; Stern, 2005). The findings of the present study extend these results to a sample of non-Caucasian children and establish that reasoning about part-whole relations in particular is critical for success in both calculation and story problem solving.

Another important and novel finding of this research is that when the effects of T1 mathematical achievement were controlled for, the influence of additive reasoning on T2 mathematical achievement remained significant. This evidence is important because it shows that what additive reasoning had in common with mathematical achievement at T1 did not explain their longitudinal predictive power. The fact that additive reasoning remained a significant longitudinal predictor of variance after children's performance in mathematical

achievement at T1 was controlled for suggests that the predictive value of additive reasoning is very strong.

This study also demonstrated a strong specificity of the additive reasoning tasks. The tasks were intended to measure the mathematical reasoning ability of children, but it is also possible that children need to rely heavily on other skills, such as general cognitive resources in order to complete the tasks. In other words, the tasks may not just measure mathematical reasoning, but reasoning in general. These two possibilities were ruled out by two findings. First, the scores of all tasks of additive reasoning did not correlate significantly with IQ, working memory, and counting ability. Thus, it appears that the additive reasoning tasks did not load on these other cognitive competence. Second, if they measure general reasoning ability, rather than mathematical reasoning ability, they should correlate significantly with the scores on Chinese word reading. The result showed that there was no significant correlation between additive reasoning and word reading. These results suggest that (1) the tasks were tapping a specific aspect of reasoning i.e. additive reasoning and that (2) additive reasoning predicts mathematics because it is a measure of competence specific to mathematics learning.

Why is there a strong link between additive reasoning and mathematical achievement? One possibility derives from considering the contributions of additive reasoning to the understanding of the nature of number and the use of more efficient problem solving strategies. According to the mathematical thinking perspective, arithmetic is the study and use of relations between numbers to solve problems and this is always carried out using a number system, which has specific characteristics. From this perspective, arithmetic is not just about memorising number facts. Instead, the process of calculation requires a deep understanding of number and of relations between operations. This understanding may form the basis for developing more advanced computational strategies that help children modify complex problems to make them easier to solve (e.g., Canobi, 2004; Canobi, Reeve, Pattison, 2003; Carraher, Schliemann & Carraher, 1993; Fuson, 1990; Nunes & Bryant, 1996, 2015).

For example, some efficient computational strategies (Gaschler, Vaterrodt, Frensch, Eichler, & Haider, 2013; Shrager & Siegler, 1998), such as counting-all starting with the larger addend (CAL) and counting-on from the larger addend (COL), require the understanding that numerical order does not affect the outcome in addition (i.e. the commutativity principle). The commutativity knowledge may also relate to the development of other strategies, such as the 'ten-strategy' and 'addends-compare strategy'. The ten-strategy refers to individuals' reordering

different addends within a problem in an attempt to exploit the circumstance that non-adjacent numbers add up to ten. For instance, children who understand the commutativity principle can transform the problem ' $3 + 6 + 7$ ' into ' $(3 + 7) + 6$ ' that is easier for them to solve. For some arithmetic problems, computation can become unnecessary if one recognises that the identical addends had been shown (though in different order e.g., ' $2 + 7 + 8$ ') in a previous problem that had already been solved e.g., ' $8 + 7 + 2$ '. This addends-compare strategy also demands the application of the commutativity knowledge between problems (Gaschler et al., 2013)

The complement principle may contribute to the use of a strategy called 'indirect addition' in which children can use additions to solve subtraction problems effectively if the numbers are close to each other. For example, to solve ' $21 - 18$ ', it is less likely to make mistakes if they count up from 18 to 21. Thus, the use of more advanced computational strategies may be one of the reasons that children with better understanding of the commutativity and complement principles performed better on the calculation tasks.

Knowledge of these principles may also help the children analyse the mathematical situations presented in story problems more effectively. According to the mathematical thinking perspective, learning mathematics should be based on understanding the relations between quantities and operating on the numbers to reach conclusions about the quantities. Story problems are texts that involve information about quantities, which typically 'describe(s) a situation assumed familiar to the reader and pose(s) a quantitative question, an answer to which can be derived by mathematical operations performed on the data provided in the text, or otherwise inferred' (Greer, Verschaffel, & De Corte, 2002, p 271). Solving an additive story problem has been viewed as selecting and activating appropriate cognitive schema and filling the empty 'slots' of the activated schema with information provided in the story text. Some of these problem types (e.g., result-unknown Change problems and total set-unknown Combine problems) are suggested to link easily to counting or calculation schemes readily available in individuals' cognitive repertoire (Carpenter, Hiebert, & Moser, 1981; De Corte, & Verschaffel, 1985; 1987; Ginsburg, 1982). Other more difficult problems (e.g., start-unknown Change problems) require additional re-representational steps that involve the application of the part-whole schema before a connection with a proper counting or operation scheme could be formed. The understanding of the inverse relation between addition and subtraction (the complement principle) and the commutativity nature of quantities may help children reason about the underlying structure of the quantitative relations in the story.

For example, Vergnaud (1979) hypothesised that children have an idea of addition on the basis of the schema of action of joining sets. This schema is useful for solving problems that involve transformation of a quantity by addition (e.g., 'David had 8 books. Then Peter gave him 3 more books. How many books does David have now?') or problems that involve a combination of quantities into a single whole (e.g., 'Grace has 3 flowers. Henry has 5 flowers. How many flowers do they have altogether?'). However, this schema is not sufficient for solving problems in a transformation situation in which the quantity decreases while the initial quantity is unknown (e.g., 'Alex had some cookies. He gave 3 cookies to his mother and had 8 cookies left. How many cookies did he have before?'). To solve this kind of problem, addition has to be seen as the inverse of subtraction. Thus, an understanding of the complement principle, which is based on the inverse relation between addition and subtraction, may be helpful for transforming a story that on the surface is about addition into an subtraction problem and vice versa.

In some problems that involve comparisons, children must also think of the relation 'more than' as the inverse of 'less than' (e.g., Verschaffel, 1994). For example, if the reference set is the missing information (e.g., 'Tom has 9 cups. He has 5 more cups than Ivy. How many cups does Ivy have?'), the relation (described as 'more than') in this problem is inconsistent with the operation (i.e. subtraction) to be used to solve the problem. If children are not able to conceive the relation 'more than' as the inverse of 'less than' and vice versa, this kind of reference-set comparison problems is likely to be difficult for them.

Knowledge of the commutativity principle may also be related to children's solving some missing addend problems. Consider this example 'Jane had 3 cookies, got some more and now has 7. How many more cookies did she get?' Children can easily solve this problem by representing the first addend with 3 fingers, counting up to the final state i.e. 7 fingers, and evaluated how many fingers they had to add in the process. However, if the problem has the first rather than the second addend missing e.g., 'Jane had some cookies; her mother gave her 4 more and now she has 7; how many did she have to start with?' the children have to understand that the order does not affect the total. Those who understand the commutativity principle can start from the second addend i.e. 4, add up to 7, and count how many were added. Children who do not understand commutativity may find this problem difficult to solve because they do not know how many cookies Jane to start with (Nunes & Bryant, 2015).

### 6.3 Contributions of Working Memory

Learning and using mathematics, including thinking mathematically, must draw on some general cognitive resources, such as working memory. Thus, it was hypothesised that working memory makes a contribution to mathematical achievement, even when one has accounted for children's specific mathematical knowledge such as their knowledge of counting ability and additive reasoning. Consistent with this hypothesis, working memory continued to account for a significant amount of variance in all measures of mathematical achievement after all the other factors were controlled for. This suggests that working memory is a stable factor that contributes to children's mathematical achievement from the first to second grade. In contrast to additive reasoning, working memory was not a specific predictor for mathematical achievement because there was also a significant correlation between working memory and Chinese word reading.

Among the three components of working memory, the central executive appeared to be a stronger predictor than phonological loop and visuospatial sketchpad. The central executive was found to be a unique and significant predictor of variations in calculation and story problem solving at both time points. By contrast, visuospatial sketchpad did not correlate with mathematical achievement at all, whereas phonological loop did not make a unique contribution to mathematical achievement when the effect of central executive is taken into account. The finding is consistent with previous research that shows that measures of the central executive are especially strong predictors of children's performance in mathematics (e.g., Cowan & Powell, 2014; Gathercole & Pickering, 2000; Holmes & Adams, 2006; Keeler & Swanson, 2001; Lee, Ng, Ng, & Lim, 2004; Lehto, 1995; Noel, Seron, & Trovarelli, 2004; Swanson & Beebe-Frankenberger, 2004; Wilson & Swanson, 2001). Most of these studies have measured central executive by memory span tasks that demand simultaneous monitoring and storage of information. The evidence suggests that the particular central executive function of monitoring and coordinating concurrent processing and storage of information is crucial for children's performance on mathematical tasks. From the mathematical thinking perspective, the central executive may support children to think about the relations between numbers, to make a decision about appropriate strategy use to calculate, and then allocate attentional resources to implement the selected strategy. One study showed that the central executive component, rather than the phonological loop and visuospatial sketchpad, was associated with strategy use in calculation (Wu, Meyer, Maeda, Salimpoor, Tomiyama, Geary, & Menon, 2008). When solving story problems, the central executive may also support children to reason about the underlying

quantitative structure of story problems, to identify the operations required to solve the problem, while working out the solution.

The finding that visuospatial sketchpad did not correlate with mathematical achievement in this study may be explained by the age of the participants. Some studies suggest that preschool children tend to have better performance on nonverbal compared with verbal arithmetic tasks and that individual differences in visuospatial sketchpad are the best predictor of mathematical achievement in this age group (Levine, Jordan & Huttenlocher, 1992; McKenzie, Bull & Gray, 2003; Rasmussen & Bisanz, 2005; Simmons, Chris & Horne, 2008). However, it has been argued that from primary school onwards, children become increasingly reliant on verbal rehearsal to retain materials in memory (Hitch, Halliday, Schaafstal & Schraagen, 1988). Consistent with this idea, Rasmussen and Bisanz showed that by the first grade, children performed equally well on verbal and nonverbal mathematical tasks, and that phonological loop became the best predictor of children's performance on verbal problems (Rasmussen & Bisanz, 2005). The non-significant correlation between visuospatial sketchpad and mathematical achievement in the present study also suggests that this component of working memory may not be important for the children in this study at the age of around 6 to 7 to perform calculation and to solve story problems.

The present study also showed that the phonological loop significantly correlated with calculation at both time points, suggesting that it may be important for mathematics learning in this age group. However, it was not a unique predictor when the effect of central executive is controlled for. One straightforward interpretation is that central executive is more important than phonological loop in children's mathematics learning. However, it has to be noted that the central executive component of working memory was assessed by 'counting span' and 'backward digit span' tasks. Thus, both the central executive and phonological loop tasks may draw on verbal processing of materials (Savage, Lavers, & Pillay, 2007). Thus, this may reduce the likelihood that a unique association between phonological loop and mathematical achievement is observed in a regression model in which central executive measures are also included. Future research may explore ways to measure the central executive component of working memory non-verbally and examine its association, relative to the phonological loop, with mathematical achievement.

#### **6.4 The Unexpected Result on Non-Verbal Intelligence**

Children's performance on the Raven's Progressive Matrices was considered as a measure of non-verbal intelligence. Cattell and Horn (Cattell, 1971; Horn, 1968) propose a two-factor model



of intelligence that include fluid intelligence and crystallised intelligence. Fluid intelligence refers to the ability of abstract reasoning, whereas crystallised intelligence represents a command of language and knowledge that are accumulated with age and education. Non-verbal intelligence has been regarded as a kind of fluid intelligence and it comprises the ability to identify relationships of materials presented in visual format, such as similarities and differences between shapes and patterns. This aspect of general cognitive functioning was included in the present study because it seems strongly related to mathematical achievement (e.g., deJong and van der Leji, 1999; Kytala & Lehto, 2008; Rourke & Conway, 1997; Rourke, 1993). Rourke (1993) suggests that specific poor mathematical achievement can be associated with so-called non-verbal learning difficulties that are characterised by poor visuospatial organisation ability. Although one study (Wu et al., 2008) found that the IQ scores (measured by the Weschler Abbreviated Scales of Intelligence, Weschler, 1999) of typically achieving (in mathematics) children did not significantly differ from that of low-achieving children, most of the published studies show that non-verbal intelligence is a significant predictor of children's mathematics learning.

In contrast to these findings, the present study did not show significant correlations between non-verbal intelligence and three measures of mathematical achievement. Only story problem solving at T2 had significant correlation with non-verbal intelligence ( $r = 0.20$ ). I can only speculate why non-verbal intelligence turned out to be a non-significant predictor. The present sample consisted of 115 children and the value of a correlation coefficient in order to be significant is 0.19. The correlation between IQ and T2 calculation was 0.18 and that between IQ and T1 story problem solving was 0.17. There is a possibility that if a larger sample is collected, significant results may emerge for these pairs. However, this does not seem to be a good explanation because it is equally possible that recruiting a larger sample may reduce the correlation and the correlation between IQ and T1 calculation ( $r = 0.13$ ) is likely to remain non-significant. The different strengths of the associations may be explained by the relative importance of non-verbal abstract reasoning in performing the tasks. Because the calculation task at T1 is relatively simple, it may demand less of abstract reasoning ability. By contrast, non-verbal reasoning may matter more in more difficult tasks, such as involving double-digit numbers in the calculation task at T2 and story problems. That may explain why the correlation between IQ and T1 calculation was smaller than that between IQ and other mathematical achievement tasks in this study.

An additional speculation about the non-significant correlation between IQ and mathematical achievement may relate to the sample of children. Recently, two studies that examined children's reading performance in Hong Kong did not demonstrate a close connection between IQ and the outcome measures. For example, Ching and Nunes (2015) found that scores on the Raven's Standard Progressive Matrices did not make a unique prediction of Chinese word reading in both hearing children and deaf and hard-of-hearing children. Chan (2015) showed that scores on the Raven's Standard Progressive Matrices were not associated with children's performance in a reading comprehension test ( $r = -0.081$ ,  $p > .05$ ). There may be some unusual characteristics in the children in Hong Kong that make the Raven's test scores not valid measures, a possibility that is worth exploring in the future. We may need to have a combination of IQ tests to form a stronger test of non-verbal intelligence for this group of children.

## 6.5 Possible Developmental Order of Additive Reasoning

Overall, the regression analyses show that additive reasoning is a strong predictor of children's mathematical achievement. As I have mentioned earlier in this chapter, one implication of this finding is that children should be given more opportunities to explore the additive relations between quantities and numbers. It would be easier for educators to devise strategies that are tailored to needs of different children, which may relate to the individual differences in the developmental order of the two principles in additive reasoning. The fourth hypothesis states that knowledge of the commutativity and of the complement principles develops from thinking in the context of specific quantities to thinking about more abstract symbols. It was hypothesised that at first, children succeed in tasks that assess their additive reasoning when the problems are set in the presence of concrete materials, and succeed only later when the problems refer to symbols, in the absence of objects. This hypothesis is supported by the findings that all children who performed well for the principles in the abstract condition also did well in the concrete condition. There was not a group of children who performed well in the abstract condition but not in the concrete condition.

On the basis of theories and research that suggest that children's thinking becomes increasingly more abstract as they grow older (e.g., Bruner, 1966; Bryant, Christie, & Rendu, 1999; Piaget & Inhelder, 1971; Sherman & Bisanz, 2009), a concrete-to-abstract ordering in children's responses to both commutativity and complement problems is expected. The conceptual profiles suggest that some children may initially understand additive reasoning in the context of concrete referents. Helping the children to represent reasoning problems with

concrete materials may lead children to pay more attention to the relations between quantities. From an educational perspective, some children may need support from concrete materials to reason about how quantities can be physically combined and separated in order to learn about addition and subtraction concepts.

In contrast to the findings by Canobi, Reeve, and Pattison (2003) that children performed equally well on commutativity tasks in concrete and abstract conditions, the present study showed that children were more accurate at recognising the commutativity principle when they were given some concrete materials to think about quantities. One factor that may explain the divergent results is that a more stringent test for commutativity was adopted in this study. In both studies, children were asked to judge whether a puppet could use a solved problem to figure out the answer to another. In Canobi et al.'s study, children were asked to judge if ' $a + b$ ' was helpful for a puppet to solve ' $b + a$ '. For these tasks, there are two possibilities for why children can give a correct answer: they may truly understand the commutativity principle, but it is also likely that they produce a correct response without paying attention to the order of the addends. That is, they may say ' $a + b$ ' is helpful for a puppet to solve ' $b + a$ ' simply because both numbers ' $a$ ' and ' $b$ ' are present. If the second possibility is true, then the commutativity task used in Canobi et al.'s study may overestimate children's understanding of the principle, which may also reduce the variations observed in their study.

By contrast, the present study used a more stringent measure that required children to solve two types of items: test items and control items. The test items were designed to assess children's understanding of a particular principle (e.g., whether the puppet can solve ' $5 + 3$ ' without counting if it knows that ' $3 + 5 = 8$ '), whereas the control items (e.g., whether the puppet can solve ' $5 - 3$ ' without counting if it knows that ' $3 + 5 = 8$ ') were used to allow for a correction for response biases. Children were considered to understand the principles only if they answered both test and control items correctly. This more stringent test may represent a more valid measure of commutativity understanding and may capture more variations in children's responses.

It was also hypothesised that there is an order of understanding of the principles – from the commutativity to the complement principle. The finding that all children who obtained high scores in the complement tasks also performed well in the commutativity tasks supported this hypothesis. There was not a group of children who performed well in the complement tasks but not in the commutativity tasks. There has been a discussion in the literature concerning the

sequences in which children learn about the commutativity and complement principles. For example, Resnick (1986) has argued that the principles are not distinct and arise from a single part-whole schema, whereas other researchers (e.g., Canobi, Reeve, & Pattison, 2003; Canobi, 2004) have suggested that some principles are developed earlier. Recently, Torbeyns, Peters, de Smedt, Ghesquière, and Verschaffel (2016) showed substantial individual differences in children's understanding of the complement principle even at the ages of 9 to 10 years. They found that a larger number of children at this age group did not show evidence for understanding the complement principle. It was commented that such difficulties are in contrast with children's acquisition of other principles that are also based on part-whole relations. For example, children at this age group are likely to have mastered the commutativity principle. Consistent with this view, the present findings suggest that some children develop commutativity knowledge prior to their complement knowledge. One interpretation of this finding is that for these children, the knowledge of the commutativity principle may represent part of the conceptual basis for understanding the complement principles that involve the appreciation of the consequences of subtracting a part from the whole. They may find it easier to master how two parts are added together to form the whole before understanding concepts that involve subtraction.

Do children need to master the commutativity principle in the more abstract tasks before they start to succeed in the complement principle tasks? Or is it sufficient to succeed in the commutativity tasks with concrete materials to make progress in the complement tasks with concrete materials? The present findings showed that the first alternative is more likely to be true. On the basis of additive reasoning performance, four groups of children were identified: The first group succeeded in commutativity tasks with concrete materials but in no other tasks; the second succeeded in commutativity tasks in both concrete and abstract conditions, but not in complement tasks; the third group succeeded in all commutativity tasks and in complement tasks with concrete materials, and the final group succeeded in all the tasks. The existence of the four classes suggests that the individual differences in the development of additive reasoning may reflect a developmental trend. It is likely that children acquire the knowledge of the commutativity principle in the more abstract tasks before they start to acquire the knowledge of the complement principle. If the second alternative were true, that it is sufficient to succeed in commutativity tasks with concrete materials in order to make some progress in the complement principle, one would have identified a fifth group, that succeeded in commutativity and

complement tasks with concrete materials but in neither task in the abstract conditions only. In contrast to this prediction, this group of children was not found in the present study.

However, a solid conclusion on the development of additive reasoning cannot be made on the basis of cross-sectional data because there can be different interpretations of the findings regarding the developmental trajectory of the two types of knowledge. The first possibility is that the children in different subgroups develop along the same developmental path but in different rates. That is, all children go through the same developmental process (from commutativity to complement, from concrete to abstract for each principle) in which some children are faster in completing their understanding of the part-whole relations in addition and subtraction. The second possibility is that the profiles may indicate different paths of development. Whereas some children develop the concepts in a piecemeal fashion, for other children both types of understanding develop together. For example, those children who succeeded in all tasks may have acquired the understanding of commutativity and complement principles at the same time. Thus, longitudinal studies that keep track of the conceptual development of the same group of children are needed to examine whether the distinct profiles found in the present study indicate a single path of development or differential paths of development.

## **6.6 Theoretical Implications**

In the introductory chapter, two theoretical perspectives on mathematics learning were highlighted and compared, namely the number sense perspective and the mathematical thinking perspective. The present thesis is designed on the basis of the latter perspective that focuses on how children think about mathematics logically and meaningfully. According to this view, one core intellectual demand to learn mathematics is the need to understand relations between quantities, rather than merely understanding things in isolation. For example, Nunes and Bryant (2015) propose that there are two meanings of number – analytical and representational. The analytical meaning of number is defined by a number system, whereas the representational meaning refers to the use of numbers to represent quantities. A child who is competent in mathematical thinking means that she or he has a good understanding of the relational meanings of numbers and quantities. This understanding appears to support his or her ability to excel in a variety of mathematical tasks.

The findings of this study strongly suggest that the mathematical thinking perspective is an excellent theoretical framework for understanding mathematics learning and education. The

final regression models that combine all factors hypothesised to relate to mathematical achievement explained over 50% of the variance in calculation and story problem solving, both concurrently and longitudinally. In particular, conceptual knowledge of counting is more important than procedural counting in predicting children's mathematical achievement in calculation. Additive reasoning, as measured by the knowledge of the commutativity and complement principles, explained variance in T2 mathematical achievement that was not accounted for by T1 mathematical achievement and all the other factors.

Other results that are theoretically important concern the development of additive reasoning. This study showed that children might initially understand additive reasoning in the context of concrete materials before more abstract symbols. Mathematics models various aspects of the world effectively by creating abstract structures that have properties shared with its real-world counterpart. We can manipulate and use the mathematical model to predict and make conclusions about events if the model acts in ways that truly corresponds to it. Some researchers have proposed that concrete materials can be used as an intermediary between the symbolic-mathematical world and the real world (Bruner, 1960; 1966; Piaget, 1952; Resnick, 1992). Thus, they may act as a vehicle through which children model the quantitative aspects of the real world. The finding in the present study suggests that some children's additive reasoning develops from thinking in the context of specific quantities to thinking about more abstract symbols, but none of the participating children showed evidence that suggests the reverse order. In other words, learning about numbers and arithmetic may start from situations in which children are invited to connect it meaningfully to the reality, rather than the other way round. One implication is that learning about how quantities are logically connected to each other may be assigned a higher priority than learning formal operations of addition and subtraction in early mathematics education. Educational implications for mathematics education of the findings are discussed further in the subsequent section.

## **6.7 Educational Implications**

One educational implication of the present thesis is that quantitative reasoning should be a central aspect addressed in mathematics education curricula. Children need to learn to reason about relations between quantities in order to solve problems, not only about arithmetic. A traditional assumption in early mathematics education is that knowledge of arithmetic comes first. Quantitative reasoning is usually introduced only after children have learned arithmetic.

The assumption behind this practice is that children will learn to apply the acquired formal arithmetic operations to deal with various kinds of problem situations.

The present study showed that some young children, who failed to reason about addition and subtraction in more abstract forms, could solve the same kind of problems using concrete materials that model closely the quantitative relations in the problems. Concrete materials are important because they enhance children's ability to represent quantities, changes in quantities, and relations between quantities. Thus, in order to search for meaningful mathematics teaching, educators should find ways to keep teaching connected to quantities in the world. One of the ways to achieve this is to avoid a predominant focus on learning procedures without any connection to understanding or applications that require these procedures. Additive reasoning should be considered as a domain of teaching and learning on its own right and numbers should not be taught in isolation from quantities from the start. Children may also be given simple representational tools, such as blocks and diagrams that represent information about relations to solve problems, before they are taught about formalisations.

Research has identified a number of ways to promote quantitative reasoning. One simple way to do so is to engage children to reflect and discuss about the problem. For example, Bermejo, Morales, and deOsuna (2004) showed that children who were asked to discuss what was the number of objects in a set when the counting was carried out backwards made significant improvement in tasks where counting was done in a non-conventional way, such as counting from two. This study suggests that reflection and discussion could be one of the strategies that educators can use to promote the coordination of counting principles.

Muldoon, Lewis, and Francis (2007) demonstrated evidence for the effectiveness of using self-explanation in enhancing children's understanding of counting. Four- and five-year-old who were at first used length to compare quantities were trained to use counting instead. The children asked to watch a puppet using counting to compare quantities. Sometimes the puppet counted correctly and sometimes not. One group of children was asked to explain whether they could use the puppet's counting to compare the quantities between sets, whereas other groups of children only received feedback on their judgment about the use of the numbers by the puppet at the end of counting. It was shown that the training was only successful when children were asked to explain why they could or could not use the puppet's counting. By contrast, those children who were given feedback only did not learn to use counting to compare quantities spontaneously. These studies suggest that activities that explicitly require children to pay attention to the logic of mathematical activity could promote quantitative reasoning.

On the basis of Piaget's (1952) theory, mathematics education researchers (e.g. Cobb, & Von Glasersfeld, 1983; Nunes & Bryant, 1996; Steffe, 1994; Steffe, & Thompson, 2000; Vergnaud, 2009; Von Glasersfeld, 1981) suggest that educators should focus on helping children form schemes of action to understand different types of situations. Schema based instruction in problem solving may represent another promising way to promote quantitative reasoning (e.g. Chen, 1999; Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004; Jitendra, & Hoff, 1996; Marshall, 1995). The main idea behind this way of teaching is that children can learn to classify problems into problem types and design a path to solution on the basis of what they know is similar to a particular problem. For example, teachers may first present some prototypical problems in lessons and exemplify the paths to solution. The students are then asked to model the solutions. This teaching strategy encourages students to identify analogous problems and thus resort to similar pathways to solution. Because the classification of problems requires teachers to have knowledge about what kinds of reasoning is required to solve problems in a particular situation, it is important for teacher education programmes to ensure teachers to become knowledgeable about the ways through which different problem situations are classified, such as based on different schemes of action for different situations.

Another educational implication of the present thesis concerns the assessment of additive reasoning in young children. The latent profile analysis suggests that individual differences are present in the development of different aspects of additive reasoning. It would not be easy for teachers to devise strategies that are tailored to the needs of different children if they do not have knowledge about the particular strengths and weaknesses of each child. The needs of different children may relate to the developmental order of the commutativity and complement principles, and the role of concrete materials in this development. Thus, in order to provide more appropriate teaching support, teachers may need to understand more about the developmental stage where each child is placed. This study shows that assessing additive reasoning in different ways and identifying profiles with classification analyses may be useful in this respect. It seems that a more fine-grained assessment of additive reasoning can be achieved by incorporating both concrete materials and numerical symbols in the assessment. Intervention efforts can then target the specific areas of deficit associated with a given profile.

## **6.8 Limitations and Future Directions**

With regard to the limitations of this study, some suggestions for future directions are made in this section. First, the present study has employed a longitudinal design that does not allow us



to determine the causal relation between variables. In order to establish whether additive reasoning and mathematical achievement are in a causal relation, both longitudinal and intervention studies have to be used (Bradley and Bryant, 1983). The present study shows that additive reasoning made an independent contribution to explaining individual differences in mathematical achievement beyond and above working memory and counting ability. This finding addresses the first step of the paradigm to determine whether additive reasoning is a potential cause of children's mathematical achievement. A possible research project in the future may focus on implementing intervention programmes aimed to enhance children's additive reasoning and examining whether an improvement in additive reasoning would result in significant progress in mathematics learning.

Second, additive reasoning was only measured once at Time 1 in this study. The present study has shown that there are individual differences in developing different aspects of additive reasoning. However, without measuring additive reasoning at two time points, the findings from the latent profile analysis based on the cross-sectional data do not allow us to make credible conclusions about the developmental trajectory of additive reasoning. Longitudinal studies that keep track of the conceptual development of the same group of children are needed to examine whether the distinct profiles found in the present study indicate a single path of development or differential paths of development. It would be interesting to explore whether the number of classes remains the same and whether each child's class membership is stable or changes over time.

Third, the introductory chapter compares the number sense and mathematical thinking perspectives. The present study shows that the mathematical thinking perspective is a useful theoretical framework for understanding mathematics learning and education. However, one cannot draw any conclusion regarding whether it is a better perspective than the number sense approach. To test this research question, future studies may incorporate measures of number sense (e.g., numerical magnitude comparisons, number facts, number line estimation) to evaluate the relative importance of number sense and quantitative reasoning.

Finally, future research may also include other types of tasks to assess additive reasoning, such as justification tasks (Prather & Alibali, 2009). The use of multi-faceted assessments may contribute to our knowledge about the variety in developmental trajectories of additive reasoning. However, justifying the use of a logical principle is not easy for young and mathematically weaker children. For example, a couple of studies have shown only minor successes for children to adequately justify the application of the complement principle

(Baroody, 1999; Dowker, 2014; Torbeyns, Peters, de Smedt, Ghesquière, & Verschaffel, 2016). Thus, it appears that developing age-appropriate versions of this type of measures for young children remains a challenge.

## 6.9 Conclusion

In conclusion, the present thesis has provided some evidence regarding the contributions of working memory, counting ability, and additive reasoning to children's mathematical achievement. This study is guided by the mathematical thinking perspective that emphasises the importance of understanding the relations between quantities in mathematics learning. Conceptual knowledge of counting, but not procedural counting, was a unique predictor of calculation ability beyond age, IQ, and working memory, but it did not contribute significantly to story problem solving. It was found that the central executive component of working memory made independent contributions to explaining variations in calculation and story problem solving beyond the effects of all the other factors. Additive reasoning (as assessed by knowledge of commutativity and the complement principle) was shown to be more important than counting ability and working memory for children's mathematics learning. It appears to be an independent and the strongest predictor of children's mathematical achievement. Consistent profiles of individual differences in the development of the knowledge of the commutativity and complement principles were also observed using latent profile analysis. It seems that additive reasoning may develop from thinking in the context of specific quantities to thinking about more abstract symbols, and children acquire the knowledge of the commutativity principle in more abstract tasks before they start to acquire the knowledge of the complement principle. These findings suggest that it is important for educators to help children recognise the power of reasoning and to engage them to represent their reasoning process. Despite several limitations, this study offers some novel and exciting directions for more empirical investigations in mathematics learning and education with a focus on quantitative reasoning in the future.

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## Appendix A1    Research Ethics Approval

**Oxford 18th June, 2014**

**Dear Bobby Ho-Hong Ching**

### **Application Approval**

**Title: Contributions of counting ability, working memory, and additive logical principles to different types of additive problem solving in Chinese children**

The above application has been considered on behalf of the Departmental Research Ethics Committee (DREC) in accordance with the procedures laid down by the University for ethical approval of all research involving human participants.

I am pleased to inform you that, on the basis of the information provided to DREC, the proposed research has been judged as meeting appropriate ethical standards, and accordingly, approval has been granted.

If your research involves participants whose ability to give free and informed consent is in question (this includes those under 18 and vulnerable adults), then it is advisable to read the following NSPCC professional reporting requirements for cases of suspected abuse [http://www.nspcc.org.uk/Inform/research/questions/reporting\\_child\\_abuse\\_wda74908.html](http://www.nspcc.org.uk/Inform/research/questions/reporting_child_abuse_wda74908.html) Should there be any subsequent changes to the project which raise ethical issues not covered in the original application you should submit details to [research.office@education.ox.ac.uk](mailto:research.office@education.ox.ac.uk) for consideration.

Good luck with your research study.

Best wishes

Lars-Erik Malmberg  
Member of DREC

Lars-Erik Malmberg, Docent, Dr.  
Associate Professor,  
Department of Education, University of Oxford,  
15 Norham Gardens, OX2 6PY, UK (+44) 01865-274047  
Fellow at St Cross College

## Appendix A2 CUREC Application Form

### University of Oxford CENTRAL UNIVERSITY RESEARCH ETHICS COMMITTEE (CUREC) CUREC/1A Checklist for the Social Sciences and Humanities

The University of Oxford places a high value on the knowledge, expertise, and integrity of its members and their ability to conduct research to high standards of scholarship and ethics. The research ethics clearance procedures have been established to ensure that the University is meeting its obligations as a responsible institution. They start from the presumption that all members of the University will take their responsibilities and obligations seriously and will ensure that their research on human subjects is conducted according to the established principles and good practice in their fields and in accordance, where appropriate, with legal requirements. Since the requirements of research ethics review will vary from field to field and from project to project, the University accepts that different guidelines and procedures will be appropriate. Please check the CUREC website to ensure that you have the correct form for your project.

This form does not cover research governance, satisfactory methodology, compliance with the requirements of publishers when administering their tests or questionnaires, or the health and safety of employees and students. As principal investigator, it is your responsibility to ensure that requirements in these areas are met. Please carry out a risk assessment of the project, in consultation with all researchers involved, using the checklist and CUREC's other documentation.

The use of an asterisk in this form indicates a phrase defined in the glossary. The glossary and further information on the University's research ethics procedures are available from the CUREC website:

<http://www.admin.ox.ac.uk/curec>

This form is designed largely for research that falls within the Divisions of Social Sciences and Humanities and which does not involve a high level of risk to the subjects. Elite interviews, field work and oral history are included in the CUREC process. Please take a moment to read through it and if you have any questions or doubts as to whether it is the appropriate form, please review Section A or consult the CUREC website.

**Note on anonymised data and audit:** you do not need to obtain ethical approval for your study if:

- you are using previously collected **anonymised data** about people which neither you nor anyone else involved in your study can trace back to the individuals who provided them (e.g. census data, administrative data, secondary analysis). Please refer to the definition of \*personal data in the glossary and FAQ A4 for further guidance; or
- you are conducting research on behalf of or at the request of a service provider that matches the definition of \*audit in the glossary.

If your research is audit or uses prior-anonymised data, please check this box: ☐

You do not need to seek ethical approval from CUREC, and you do not need to complete any more of this form. However, please check with your department's requirements, as some departments require you to lodge this form with them.

Office use only: IDREC Ref. No. \_\_\_\_\_

Date of confirmation that checklist accepted on behalf of IDREC:    //    //

SECTION A	Yes	No
1) Are you using research methodologies commonly used in biomedical or behavioural laboratory sciences?		✓
2) Is there a significant risk that the research will induce anxiety, stress or other harmful psychological states in participants that might persist beyond the duration of any test or interview in which they are participating?		✓
3) Will the research involve human participants recruited by means of their status as present or past NHS patients or their relatives or carers?		✓
4) Does the research involve *human participants aged 16 and over who do not have *capacity to consent for themselves? See the Mental Capacity Act 2005		✓
5) Is the study to be funded by the US National Institutes of Health or another US federal funding agency?		✓

If you answered 'yes', please **stop** work on this checklist and

- **for questions 1 and 2**, complete CUREC/1 instead (available from [www.admin.ox.ac.uk/curec](http://www.admin.ox.ac.uk/curec));
- **for questions 3 and 4**, submit your proposal to the appropriate NHS ethics committee (see [www.nres.nhs.uk](http://www.nres.nhs.uk) and <http://www.admin.ox.ac.uk/researchsupport/ctrq/> for further information);
- **for question 5**, or if you answered 'yes' to questions 1, 2 or 4 and your research will take place outside the EU and is a biomedical study, submit your proposal to OXTREC, which uses separate documentation. **Applications to OXTREC using this form will not be accepted.** If your research is not a biomedical study and does not have US funding, but will take place outside the EU, you may use this form to submit your application for approval to the Social Sciences and Humanities IDREC.

If you have answered 'no' to all questions in Section A, please complete Sections B-E. This form and any supporting materials should be typewritten.

## SECTION B

<b>*Principal investigator/ supervisor/student researcher (title and name):</b>	Boby Ho-Hong CHING
<b>Name of supervisor (STUDENT RESEARCH PROJECTS ONLY):</b>	1 <sup>st</sup> supervisor: Professor Terezinha Nunes 2 <sup>nd</sup> supervisor: Dr. Maria Evangelou
<b>Degree programme, e.g. DPhil, MPhil, MSc (STUDENT RESEARCH PROJECTS ONLY):</b>	DPhil Education
<b>Department or institute:</b>	Department of Education
<b>Address for correspondence (if different):</b>	
<b>Email and phone contact:</b>	boby.ching@education.ox.ac.uk
<b>Title of research project:</b>	Contributions of the understanding of the part-whole relationships in additive reasoning to children's success in story problem solving: A longitudinal study

**Title and brief lay description of \*research (about 150 words), plus description (about 200 words) of the nature of participants (including the criteria for inclusion/exclusion & method of recruitment, how professional guidelines are being applied (if applicable) and use to which the results/data will be put.**

**Please describe how you will obtain informed consent, citing and attaching, where applicable, documentation produced in support of your application such as generic recruitment and advertisement material, participant information sheet(s), consent form(s) and debriefing document(s).**

The purpose of this study is to investigate the extent to which mathematical reasoning predict the performance on additive word problem solving in Chinese children. This research project adopts a longitudinal design that includes tracking the same group of children over 10 months. Whereas there is a lack of longitudinal studies with proper control measures, the findings of this study may contribute to better curriculum designs and pedagogical practices for mathematics learning for young children.

This study will include approximately 70-90 children in Hong Kong, whose age range falls between 6-7 years old at Time 1 of the study and between 7-8 years old at Time 2. All children will be confirmed to have intelligence within the range accepted as normal for their ages, and not to have specific learning disability, ADHD, or any neurological disorders. Participants will be recruited by the researchers through contacting primary schools and child-related not-for-profit organizations, and individual parents directly in Hong Kong. The children will be asked to participate in paper and pencil tasks measuring their cognitive skills related to mathematics learning, such as, counting ability, working memory, basic number knowledge, non-verbal reasoning, conceptual knowledge of additive reasoning, spontaneous attention to numerosity, and additive problem solving.

Invitation letters and information sheets of this study will be given to school principals/teachers/day-care workers/parents. Parents or guardians will give consent for the children. Informed consent will also be gained from school principals and teachers when children are recruited through schools if data collection is conducted in classrooms. If the children are recruited through child-related non-profit-organizations, informed consent will be gained from day-care workers of particular centres and data collection will be conducted in the centres. Both the children and parents will be reminded that their participation in this research project is completely voluntary – they have the right to decline and choose not to answer specific questions or to withdraw at any time without any consequence(s) on the children's grades or evaluations. The data for this project will be kept confidential and stored in a safe place only accessible by the researcher.

The data from this study will be analyzed and used as part of my doctoral thesis at the Department of Education, University of Oxford. The findings may be published or presented at professional meetings, but the identities of all research participants will remain anonymous.

**List actual or probable location(s) where project will be conducted, if known:**

Probable locations

PLK Fung Ching Primary School, HK  
T.W.G.Hs Yiu Dak Chi Memorial Primary School, HK

**If your research involves overseas travel or fieldwork, have you completed and returned a travel risk assessment form? (Bear in mind that this may be necessary to ensure that the travel or fieldwork is covered by the University's travel insurance – see <http://www.admin.ox.ac.uk/finance/insurance/travel/>.)**

YES

**Anticipated duration of project:**

12 months

**Anticipated start date:**

24<sup>th</sup> November 2014

**Anticipated end date:**

31<sup>st</sup> October 2015

**Name and status (e.g. 3<sup>rd</sup> year undergraduate; post-doctoral research assistant) of others taking part in the project:**

N/A

<b>Please indicate what training on research ethics the researchers involved with this study have received, e.g. the title of the online or in-person course, and date completed (online training available at <a href="http://www.admin.ox.ac.uk/researchsupport/integrity/human/">www.admin.ox.ac.uk/researchsupport/integrity/human/</a>):</b>	Attended the following lectures on research ethics: Lecture title: Research Ethics and Accessing the Field Course: Foundations of Educational Research 1 Date completed: 7 <sup>th</sup> November 2013 Location: Department of Education, University of Oxford
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### SECTION C

Methods to be used in the study (**tick** as many as apply: this information will help the committee understand the nature of your research and may be used for audit).

	Please tick
<b>Interview</b>	
<b>Questionnaire</b>	
<b>Analysis of existing records</b>	
<b>Participant performs verbal/paper and pencil/computer based task</b>	√
<b>Measurement/recording of motor behaviour</b>	
<b>Audio recording of participant</b>	
<b>Video recording or photography of participant</b>	
<b>Physiological recording from participant</b>	
<b>Participant observation</b>	
<b>Covert observation</b>	
<b>Systematic observation</b>	
<b>Observation of specific organisational practices</b>	
<b>Other (please specify)</b>	

### SECTION D

Have you read one or more of the following professional guidelines and do you undertake to use the principles listed there as a guide for your own work? Please note that this is not intended to be an exhaustive list. Links to the guidelines listed below are included on the CUREC website.

	Please tick
<b>British Society of Criminology: Code of Ethics for Researchers in the Field of Criminology</b> [ <a href="http://britsocrim.org/new/?q=node/22">http://britsocrim.org/new/?q=node/22</a> ]	
<b>British Educational Research Association Ethical Guidelines for Educational Research</b> [ <a href="http://www.bera.ac.uk/guidelines">www.bera.ac.uk/guidelines</a> ]	√
<b>Academy of Management's Code of Ethics</b> [ <a href="http://www.aomonline.org/aom.asp?ID=&amp;page_ID=242">www.aomonline.org/aom.asp?ID=&amp;page_ID=242</a> ]	
<b>Association of American Geographers Statement on Professional Ethics</b> [ <a href="http://www.aag.org/cs/resolutions/ethics">www.aag.org/cs/resolutions/ethics</a> ]	
<b>Oral History Society of the UK Ethical Guidelines</b> <a href="http://www.ohs.org.uk/ethics.php">http://www.ohs.org.uk/ethics.php</a>	
<b>American Political Science Association (APSA) Guide to Professional Ethics in Political Science (Section H)</b> [ <a href="http://www.apsanet.org/content_9350.cfm">www.apsanet.org/content_9350.cfm</a> ]	
<b>Political Studies Association Guide to Good Professional Conduct</b> [ <a href="http://www.psa.ac.uk/psa">www.psa.ac.uk/psa</a> ]	
<b>British Psychological Society Code of Ethics and Conduct</b> [ <a href="http://www.bps.org.uk/what-we-do/ethics-standards/ethics-standards">www.bps.org.uk/what-we-do/ethics-standards/ethics-standards</a> ]	√

<b>Ethics Guidelines of the Association of Social Anthropologists of the UK and Commonwealth</b> [ <a href="http://www.theasa.org/ethics/guidelines.shtml">www.theasa.org/ethics/guidelines.shtml</a> ]	
<b>Social Research Association: Ethical Guidelines</b> [ <a href="http://the-sra.org.uk/sra_resources/research-ethics/ethics-guidelines/">http://the-sra.org.uk/sra_resources/research-ethics/ethics-guidelines/</a> ]	
<b>Statement of Principles of Ethical Research Practice from the Socio-Legal Studies Association</b> [ <a href="http://www.slsa.ac.uk/content/view/247/270/">www.slsa.ac.uk/content/view/247/270/</a> ]	
<b>Statement of Ethical Practice for the British Sociological Association</b> [ <a href="http://www.britisoc.co.uk/about/equality/statement-of-ethical-practice.aspx">www.britisoc.co.uk/about/equality/statement-of-ethical-practice.aspx</a> ]	
<b>Other professional guidelines (please specify):</b>	

## SECTION E

Please put a tick in the yes/no column as appropriate to indicate your response.

<b>1) Will you obtain informed consent according to good practice in your discipline before participation?</b>	Yes	No
	√	
<b>2) Will you ensure that *personal data collected directly from participants or via a *third party is held and processed in accordance with the provisions of the Data Protection Act?</b>	Yes	No
	√	
<b>3) Does the research involve as participants *people whose ability to give free and informed consent is in question?</b>	Yes	No
<i>Note: participants aged under 16 are generally considered to require consent of a parent or guardian (only answer 'no' to this question if you can cite one of the protocols listed under 'children'). For participants aged 16-17, consult FAQ C13</i>	√	
<b>4) As a consequence of taking part in the research, will participants be at serious risk of rendering themselves liable to criminal prosecution (e.g. by providing information on drug abuse or child abuse)?</b>	Yes	No
		√
<b>5) Does the research involve the *deception of participants, as part of the investigation/experiment?</b>	Yes	No
		√
If any of your answers above are in a shaded box, please indicate whether those aspects of your project are fully covered by the following.		
<b>6) Research protocol(s) which has/ve received IDREC/CUREC approval?</b>	Yes	No
<i>If yes, please give protocol number(s):</i> MSD/IDREC/2005/P2.1.2	√	
<b>7) Professional guidelines that you will be following, as noted under Section D?</b>	Yes	No
	√	

If any of your answers in Section E are in a shaded box and are not covered by a protocol or by professional guidelines, please complete CUREC/2, available to download from the CUREC website. Then submit both this form (you need not complete section F) and the CUREC/2 to the Social Sciences and Humanities IDREC.

If all your answers in Section E are in the unshaded boxes or your answers in shaded boxes are covered by a protocol or professional guidelines, complete Section F and submit this form and any accompanying documents to the Social Sciences and Humanities IDREC or to the relevant officer/committee at departmental level (see notes and address below).

## FINAL CHECK

Please check each of the following before submitting the checklist. **If the appropriate supporting documentation is not included with your application, you will then be asked to provide this separately. This may well delay the ethical review process, and thus the start of your research.**

Have you completed Sections A-E? ☒

Have you defined all technical terms and abbreviations used? ☒

If you have produced any documentation in support of your application (which might include questionnaires, participant information, consent forms/form or note of procedure for recording oral consent, advertisements and surveys), have you attached a copy of these? ☒

Are all pages (including appendices and attachments) numbered? ☒

## SECTION F

**You can submit this checklist by email and/or as a signed hard copy;** if it is being sent by email only, the checklist, and any email from the head of department or nominee separately endorsing its submission, must be sent from a University of Oxford email address (i.e. as a minimum, the checklist and supporting documents must be submitted by the head of department or nominee indicating his/her approval from a University of Oxford email account).

**Complete this section only if you do not need to submit form CUREC/2.**

I understand my responsibilities as principal researcher/supervisor/student researcher as outlined in the CUREC glossary and guidance on the CUREC website.

I declare that the answers above accurately describe my research as presently designed and that I will submit a new checklist should the design of my research change in a way which would alter any of the above responses so as to require completion of CUREC/2 (involving full scrutiny by an IDREC). I will inform the relevant IDREC if I cease to be the principal researcher on this project and supply the name and contact details of my successor if appropriate.

**Signed by principal researcher/supervisor/student researcher:** .....

**Date:**...12<sup>th</sup> June 2014.....

**Print name** (block capitals).....BOBY HO-HONG CHING.....

**Signed by supervisor:**... ..(for student projects)

**Date:**.....12<sup>th</sup> June 2014.....

**Print name** (block capitals) Dr Maria Evangelou.....

I understand the questions and answers that have been entered above describing the research, and I will ensure that my practice in this research complies with these answers, subject to any modifications made by the principal researcher properly authorised by the CUREC system.

**Signed by associate/other researcher:** .....

**Print name** (block capitals).....

**Date** .....

I have read the research project application named above. On the basis of the information available to me, I:

- (i) consider the principal researcher/supervisor/student researcher to be aware of her/his ethical responsibilities in regard to this research;
- (ii) consider that any ethical issues raised have been satisfactorily resolved or are covered by relevant professional guidelines and/or CUREC approved protocols, and that it is appropriate for the research to proceed without further formal ethical scrutiny at this stage (noting the principal researcher's obligation to report should the design of the research change in a way which would alter any of the above responses so as to require completion of a CUREC/2 full application);
- (iii) am satisfied that the proposed project has been/will be subject to appropriate \*peer review and is likely to contribute something useful to existing knowledge and/or to the education and training of the researcher(s) and that it is in the \*public interest.
- (iv) [FOR DEPARTMENTS/FACULTIES WITH A DEPARTMENTAL RESEARCH ETHICS COMMITTEE (DREC) OR EQUIVALENT BODY - PLEASE DELETE IF NOT APPLICABLE] confirm that this checklist (and associated research outline) has been reviewed by the Department's Research Ethics Committee (DREC)/equivalent body, and attach the associated report from that body.

**Signed:**.....

**(Head of department or nominee e.g Chair of DREC, Director of Graduate Studies for postgraduate student projects)**

**Print name (block capitals)**.....

**Date:**.....

**If your research involves participants recruited by means of their status as current or former NHS staff, or the research will, in whole or in part, be carried out on NHS premises, use NHS facilities or assess NHS facilities or services, please see FAQ B3 ([www.admin.ox.ac.uk/curec/faqs/](http://www.admin.ox.ac.uk/curec/faqs/)).**

**Please check with your department about its procedures for the approval of CUREC forms.** If your department indicates that the checklist should be submitted directly to the IDREC, please send it, together with any supporting documentation, to the following address(es), keeping a copy for yourself:

Secretary of the Medical Sciences IDREC Email: [ethics@medsci.ox.ac.uk](mailto:ethics@medsci.ox.ac.uk)  
 Research Services  
 University of Oxford  
 Wellington Square  
 Oxford, OX1 2JD

Secretary of the Social Sciences and Humanities IDREC Email: [ethics@socsci.ox.ac.uk](mailto:ethics@socsci.ox.ac.uk)  
 Research Services  
 University of Oxford  
 Wellington Square  
 Oxford, OX1 2JD

IDRECs and/or CUREC will review a sample of completed checklists and may ask for further details of any project.



**Appendix B Risk Assessment Fieldwork Safety and Insurance Form****Declaration**

*Please note: electronic signatures are not acceptable*

**Student Declaration**

- ☐ I have completed the risk assessment relevant to my fieldwork.
- ☐ The information given on this form is correct to the best of my knowledge and I will ensure that it is updated as necessary (referring to the FCO website if relevant).
- ☐ I have familiarised myself with current university Health & Safety Policies (and university statement of safety organisation)
- ☐ I have also signed the travel insurance form!

Signed \_\_\_\_\_

Date: \_\_\_\_\_

**Supervisor Declaration**

- ☐ I have read this risk assessment form and discussed it with the student
- ☐ I have completed the supervisor check list with comments as appropriate
- ☐ I am aware of my responsibilities to act as a point of contact for the student during travel

Signed (supervisor) \_\_\_\_\_

Date: \_\_\_\_\_

**Departmental Administrator**

I have signed the insurance form! (please tick to confirm) ☐

**DGS / HoD Approval**

(HoD Approval required when region/area to be visited is deemed 'inadvisable' by the FCO)

Signed \_\_\_\_\_

Date: \_\_\_\_\_

## **Fieldwork Safety, Risk Assessment and Insurance**

This **entire** form **must** be completed by **all** students intending to carry out fieldwork. The Head of Department or their representative is required to approve all overseas fieldwork. It is mandatory to submit this form to the Higher Degrees Office **at least five weeks** in advance of carrying out your research. No travel should be booked until approval has been obtained by your supervisor and the Head of Department (or their representative). Please note that even if you're travelling to your home country on fieldwork, it still falls under university business and **must** be treated with equal consideration. **All signatures must be original.**

### **1. General information**

<b>First name</b>	<b>Surname</b>
<b>Nationality</b>	<b>Supervisor</b>
<b>Contact Address prior to fieldwork:</b>	
<b>Contact email prior to fieldwork</b>	<b>Contact phone number prior to fieldwork</b>

### **2. Itinerary**

Where is the fieldwork taking place? <i>Please be specific if you are visiting particular areas within a city, or region. (For example, Cowley, not just Oxford).</i>	
According to the FCO Website, the area I am visiting has been identified as:	<b>Low Risk</b>
Date(s) of visit <i>Please note, this form can only cover you for a single period. If you plan to return at a later date, you will have to submit another form.</i>	
Contact details during fieldwork: <b>Accommodation</b> <i>Please give as much as information as possible (address, contact name, phone, mobile phone, fax, email)</i>	
Contact details during fieldwork: <b>Placements / Local Contact</b> <i>Please give as much as information as possible (address, contact name, phone, mobile phone, fax, email)</i>	
If you are not a British national, does your own government advise against travel to the area you propose to visit	No
What activities will you be undertaking?	
What institutions will you be visiting?	
Will you be travelling alone at night, and what precautions will you take?	

### **3. Health/medical and emergency procedures**

Have you taken advice from the University Occupational Health Service on the health risks associated with travel to your field site ( <a href="http://www.admin.ox.ac.uk/uohs/at-work/travel/">http://www.admin.ox.ac.uk/uohs/at-work/travel/</a> )?	<b>Yes</b>
If no please state why:	
All accidents, incidents or near misses must be reported to Erica Oakes ( <a href="mailto:erica.oakes@education.ox.ac.uk">erica.oakes@education.ox.ac.uk</a> ) at the Department of Education and your supervisor. Please check the box to confirm that you will ensure this takes place while you are on fieldwork: <input checked="" type="checkbox"/>	
Next of Kin: <i>Please state full name, address, telephone number, email and their relationship to you</i>	
Passport / visa numbers (overseas travel only)	
What first aid training have you received or what first aid is available to you?	
Will you need to take any first aid provisions?	
If you're visiting an area that is fairly isolated, what access do you have to emergency services and medical help? If relevant, please also provide details of emergency plans in place (e.g. for evacuation from the research area, or if there is an injury to a worker in a small team)?	

#### 4. Training and or experience

Have you received any training for your overseas travel? Please give details.	Have you experience in similar work? Please give details.
Training Available from: <a href="https://www.admin.ox.ac.uk/safety/safetytraining/safetytraining/course/?crsID=104">https://www.admin.ox.ac.uk/safety/safetytraining/safetytraining/course/?crsID=104</a>	

#### 5. Supervision

A fieldworker will still be under supervision with regards to health and safety, even though the supervisor may not be present. Please provide details of your supervisor, or other nominated person, who you will contact on a regular basis to ensure your safety. The nominated person should be provided with the contact details of a person in the Department, in case of emergency etc. Specify contact period times (e.g. daily/weekly).

<b>Name</b>	<b>Contact Period (when and how often you will contact your supervisor)</b>
<b>Address</b>	<b>Telephone</b>
<b>Email</b>	
How will contact be made? You must ensure that this is possible (e.g. if reliance is put on a mobile phone, you must ensure there is adequate signal):	

#### 6. Risk Assessment

A risk assessment must be carried out in order to comply with University Safety Policy. You will need to read UPS S3/07 Overseas Travel and UPS S5/07 Safety in Fieldwork.

Please remember to consider not only possible risks to your own health and safety, but also the effect that your work may have on other people or the environment.

**The competency of external agencies hosting a fieldworker must be considered. Arrangements and responsibilities for the fieldwork must be agreed and documented as part of the risk assessment.**

Hazard identified (please specify) (see checklists in appendix to UPS S5/07)	Risk(s) involved and control measures to be put in place to minimise risk	Estimated level of risk
Personal (e.g. lone work, first aid, violence, crime, travel, handling cash, transport)		Low
Physical (e.g. extreme weather, fitness and medical risks)		Low
Equipment (will you be taking a laptop, mobile, cash, internet access etc? These should be specified in your insurance form)		Low
Environmental and surroundings (e.g. pollution, waste, language barriers, local culture and awareness of this)		Low
Other		Not Applicable

## 7. Supervisor Risk Assessment check list

*Supervisor to complete, then sign the front page of the form.*

		Additional comments if relevant
I have discussed with the student the general risks associated with the planned fieldwork.	<input checked="" type="checkbox"/>	
I have discussed with the student the potential additional specific safety issues and risks associated with this fieldwork, and appropriate measures to reduce them.	<input checked="" type="checkbox"/>	
I believe that travel arrangements discussed are satisfactory and safe.	<input checked="" type="checkbox"/>	
I have discussed with the student any specific health issues associated with the area they are travelling to and any issues regarding their research.	<input checked="" type="checkbox"/>	
We have identified specific requirements for additional guidance and advice (indicate as necessary) and this is being sought by the student/researcher	<input checked="" type="checkbox"/>	
I am satisfied that the student has adequately assessed the risks associated with the planned work, and agrees to carry out the work in a manner that reduces risks to health and safety to a satisfactory level.	<input checked="" type="checkbox"/>	

## Travel Insurance Application

Your travel insurance document provides cover for both you and your personal belongings while travelling on University business. Details of the limits of cover are set out on the Confirmation & Summary of Cover, available from [www.admin.ox.ac.uk/finance/insurance/travel](http://www.admin.ox.ac.uk/finance/insurance/travel). Please note:

1. All travel insurance claims are settled net of a standard excess of £50.
2. Please list all personal items over £500 in value below. We strongly advise you not to take valuable items with you while travelling.
3. All **theft/lost property** claims must be supported by a local police/security report.
4. All **property damage** claims must be supported by an estimate for repair, detailing the extent of the damage and the cost of repair. If the item is beyond economical repair a quotation for replacement must be supplied with the claim. You may also be asked to supply a receipt for the original item.

Please complete the following information:

<b>Full name of traveller</b>	<b>Department</b>
<b>Inclusive dates of travel</b> From <b>24<sup>th</sup> November 2014</b> To <b>31<sup>st</sup> October 2015</b>	<b>Destination(s) (City &amp; Country)</b>
<b>Attach Completed Risk Assessment</b> <i>(See University Policy Statement S3/07 point 3 for further information)</i>	<input checked="" type="checkbox"/>
<b>Name of Supervisor</b>	<b>Name of Head of Department</b>
<b>Home address &amp; emergency contact no.</b>	<b>Next of kin name, address &amp; contact no.</b>
<b>Accommodation addresses and contact numbers</b>	
<b>Reason for travel</b>	
<b>Outbound flight number</b> N/A at the moment	<b>Airport of departure</b>  <b>Destination airport</b>
<b>Inbound flight number</b> N/A at the moment	<b>Airport of departure</b>  <b>Destination airport</b>

<b>List of personal items &gt;£500 value</b>	
<b>Item</b>	<b>Estimated value</b>

Signature of Applicant:

Date:

*Cover will operate from this date in the event of a cancellation claim*

Signature of Departmental Administrator:

Date:

**NOTES:**

There is a maximum of £3000 cover for personal property (and in the event of a claim, there is a £50 excess). As a consequence, if you are taking a personal laptop (or any other electrical item) with a value greater than £1000, you should arrange all-risks insurance cover for the item personally (and declare this on your University travel insurance application form). Please check website at: <http://www.admin.ox.ac.uk/finance/insurance/travel.shtml> for any changes to this information. If you hire a car overseas for local travel you must arrange car insurance (comprehensive) locally. **Cover is NOT provided under the University travel insurance scheme.**

Please note, repatriation for medical reasons provides for emergency return to Britain and not your home country. The cover does not provide for medical treatment in Britain as it expects you to be covered under the NHS. If you are not covered under the NHS you should make your own insurance arrangements for medical treatment in this country. You should take details of the Emergency Medical Assistance contact numbers. Further copies can be obtained from the Higher Degrees Administrator. The details are also available on the University's Travel Insurance web page (see:

<http://www.admin.ox.ac.uk/finance/insurance/travel.shtml>). **Please ensure you have this information before you travel.** If you do need medical treatment overseas please ensure you obtain receipts. They will be essential for the claim which will be made to the insurers. (Please note the excess charges.)

**Appendix A3 School Head Information Letter**

Dear [Principal/Head Teacher/Social Worker's name],

I am writing to enquire if it would be possible to conduct research with the children in your school/organization during October/November 2014 and September/October 2015. As part of my studies in the Department of Education at the University of Oxford, I am investigating children's cognitive skills and their associations with mathematical achievement. From this study, the researchers hope to learn more about what kinds of cognitive skills are good for children to learn mathematics. The findings of this study may contribute to better curriculum designs and pedagogical practices for mathematics learning for children in Hong Kong.

The project will require approximately 30 to 40 of your children from Primary 1. There will be two sessions: one during October/November 2014 and another during September/October 2015. During the sessions, each child will complete a few simple word tasks. I can be very flexible with the timings of implementation and will plan the programme around the needs of the classroom teacher/workers. I will provide information to parents beforehand. All children will have the right to withdraw at any stage, and anonymity will be maintained in all reports to ensure your school/organization's and the children's confidentiality. All data collected for this research will be used solely for research purposes.

The research project will be carried out by Mr. Bobby Ching of Oxford University, who is a graduate student under the supervision of Professor Terezinha Nunes. The University of Oxford Central University Ethics Committee has approved this study and the researcher has obtained the required clearance to work with children in schools.

Very little research has been conducted in relevant areas, so I do hope that your school/organization will consider participating in this project. I would be delighted to talk to you about this in more detail. If you feel that your school/organization would like to take part, or you have any queries, please contact me by email at [boby.ching@education.ox.ac.uk](mailto:boby.ching@education.ox.ac.uk) or by phone at 92774457. Thank you very much for your time and wish you and your school/organization a very successful term.

Yours sincerely,  
Bobby Ching

## **Appendix A4    Parent Consent Letter**

### **An Investigation of Mathematics Learning in Young Children**

Dear Parents/Guardians,

I am writing to ask for your permission to allow your child to take part in a study about young children's mathematics learning. Your child's teacher has kindly agreed to cooperate with the project and now I am asking for your permission for your child to be included.

I am investigating children's cognitive skills and their associations with mathematical achievement. From this study, the researchers hope to learn more about what kinds of cognitive skills are good for children to learn mathematics. The findings of this study may contribute to better curriculum designs and pedagogical practices for mathematics learning for children in Hong Kong. Each child will be asked to play some fun paper-and-pencil and computer-based games. In similar studies the children have enjoyed participating. The study will take place in a classroom at the school that your child is attending under the supervision of their teachers.

All data collected for this research will be used solely for research purposes and is not intended as a specific evaluation of the child. Your child may choose to stop participating at any time if they wish. All names will be removed from the files. Access to the information collected from your child will be restricted to the research team.

The research project will be carried out by Mr. Bobby Ching of Oxford University, who is a graduate student under the supervision of Professor Terezinha Nunes. The University of Oxford Central University Ethics Committee has approved this study and the researcher has obtained the required clearance to work with children in schools. If you have any questions you would like to ask before replying, please do not hesitate to contact me by email at [boby.ching@education.ox.ac.uk](mailto:boby.ching@education.ox.ac.uk).

Very little research has been conducted in relevant areas, so I do hope that you will agree your children to participate in this project. In order for your child to take part in the project, please complete and return the attached form to your child's class teacher. Unfortunately, children who do not return the form cannot participate.

Thank you very much for your time.

Yours sincerely,  
Bobby Ching



### **An Investigation of Mathematics Learning in Young Children**

- ✓ Your child's school has agreed to take part in a study run by Oxford University looking at how children learn to read.
  - ✓ If your child takes part, a researcher would do some activities and play some fun games with them.
  - ✓ Your child's participation will contribute to better curriculum designs and pedagogical practices for mathematics learning for children in Hong Kong
  - ✓ If you are happy for your child to take part, please fill in the form below and return it to your child's class teacher.
- 

#### **To be returned to your child's teacher**

I have read and understood the details of the above study. I understand that the project has received ethics clearance through the University of Oxford's ethical approval process for research involving human participants, and understand that the information will be kept confidential and only be used for research purpose. I understand that participation is voluntary and that I and my child are free to withdraw at any time without consequences.

I agree to my child's participation in this study.

Child's name: .....

Parent signature: .....

Date: .....

## Appendix B Plots – Assumptions for Multiple Regression Analyses

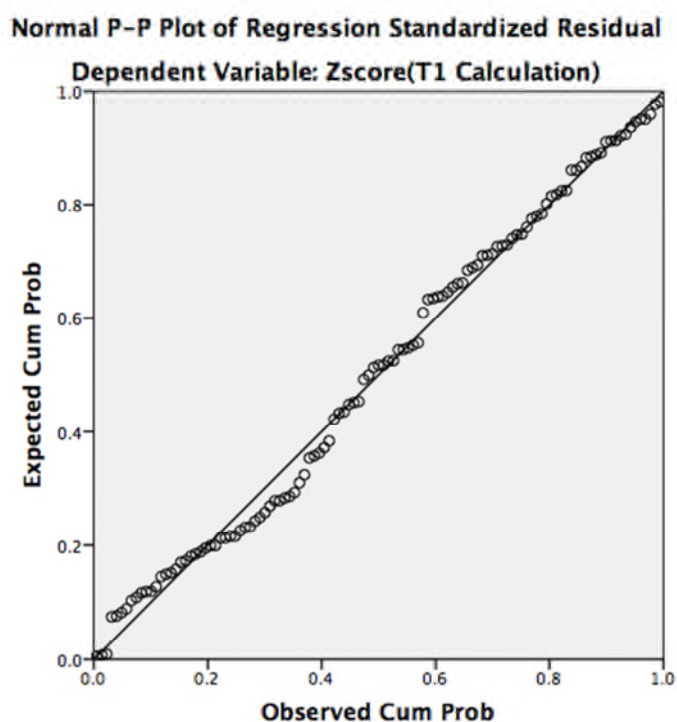


Figure B.1 Normality probability plot of regression standardised residual for T1 calculation as the outcome measure (age, IQ, counting ability, working memory, additive reasoning as predictors)

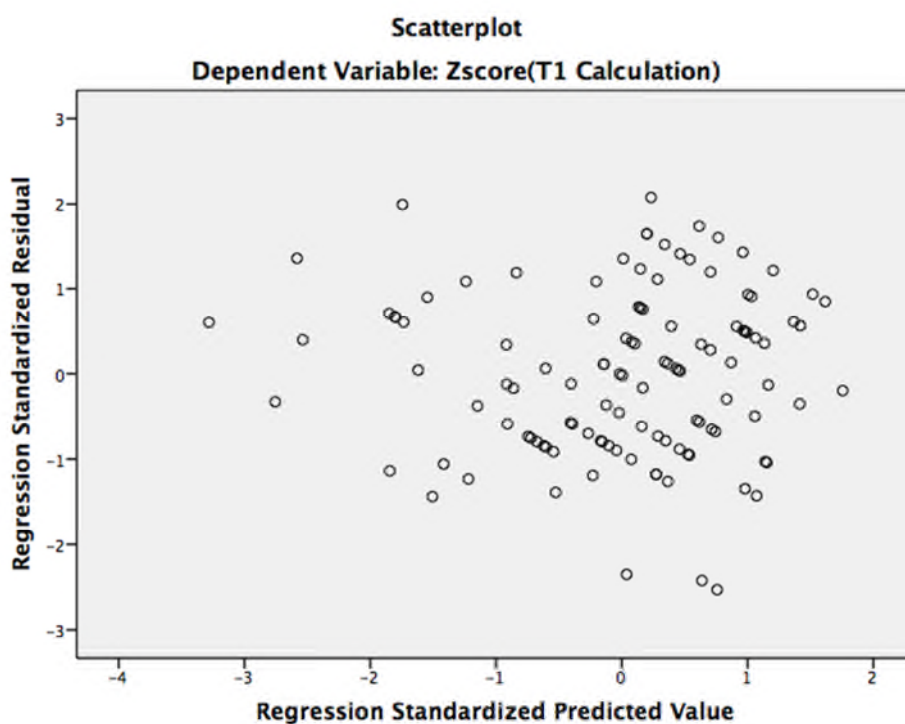


Figure B.2 Scatterplot of the regression standardised predicted values compared with the regression standardised residuals for T1 calculation as the outcome measure (age, IQ, counting ability, working memory, additive reasoning as predictors)

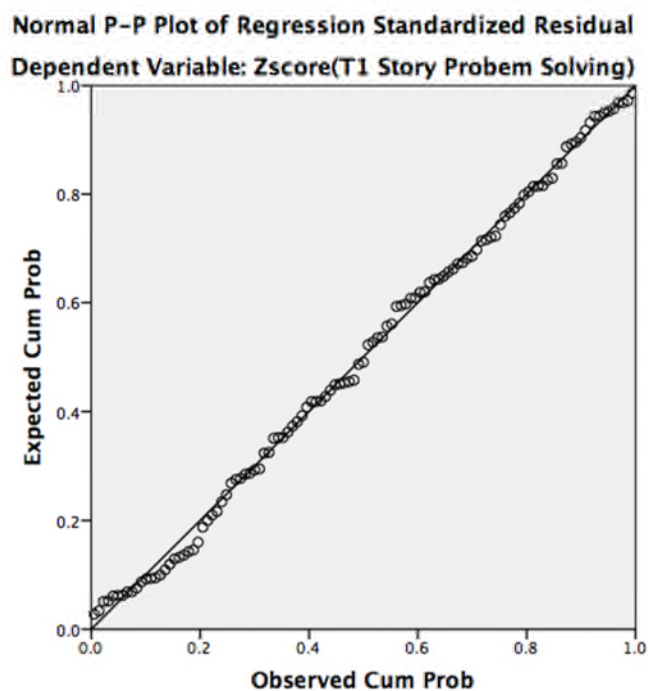


Figure B.3 Normality probability plot of regression standardised residual for T1 story problem solving as the outcome measure (age, IQ, counting ability, working memory, additive reasoning as predictors)

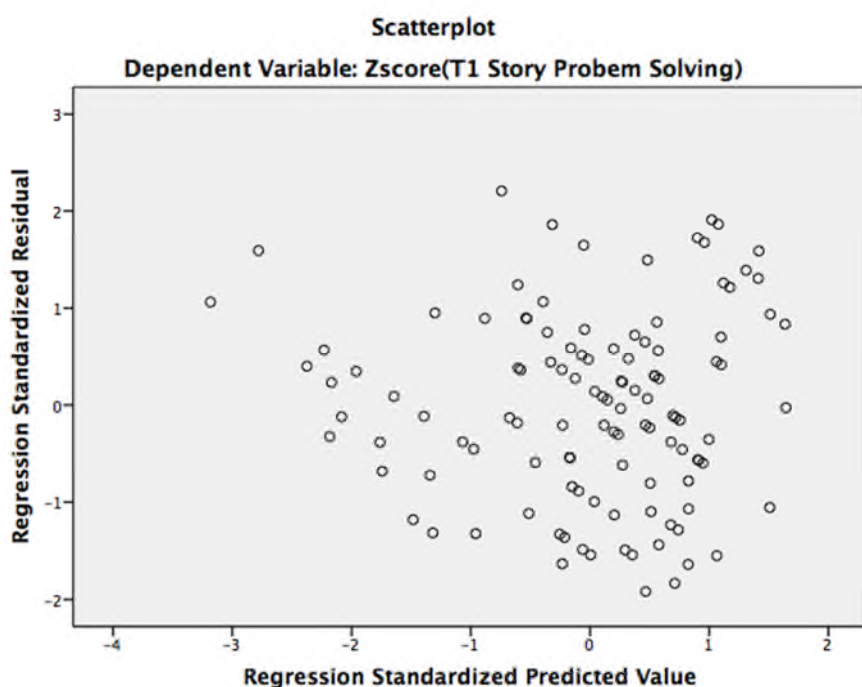


Figure B.4 Scatterplot of the regression standardised predicted values compared with the regression standardised residuals for T1 story problem solving as the outcome measure (age, IQ, counting ability, working memory, additive reasoning as predictors)

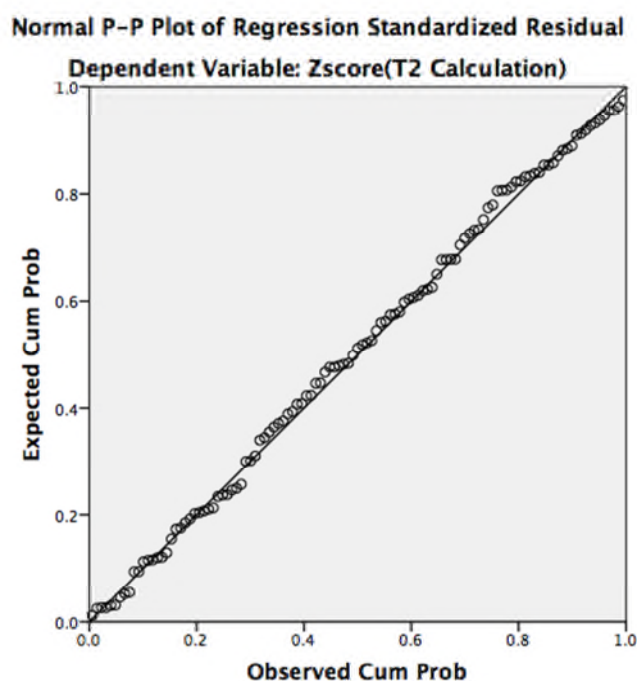


Figure B.5 Normality probability plot of regression standardised residual for T2 calculation as the outcome measure (age, IQ, counting ability, working memory, additive reasoning as predictors)

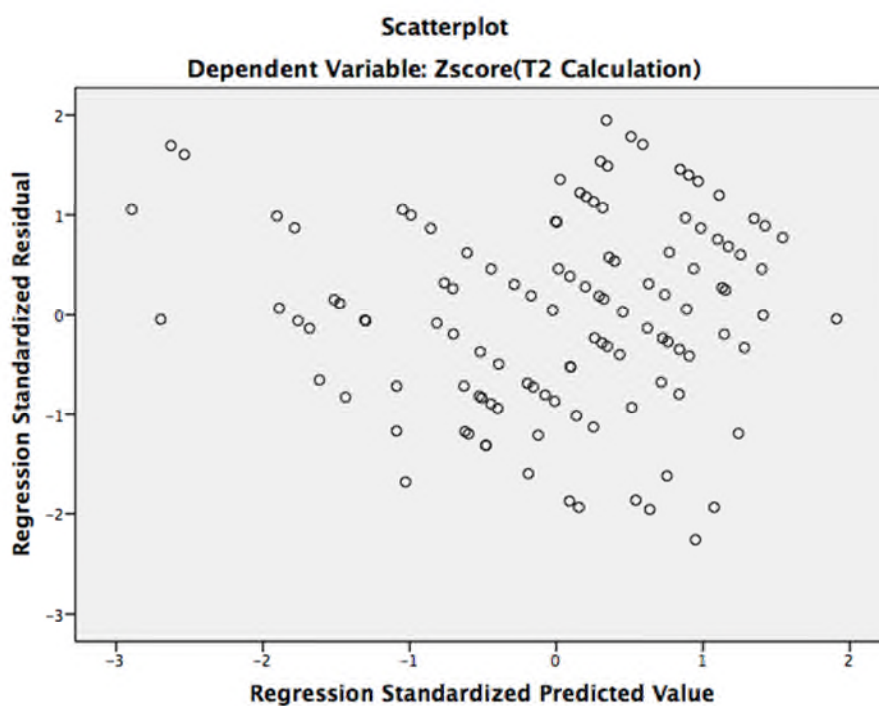


Figure B.6 Scatterplot of the regression standardised predicted values compared with the regression standardised residuals for T2 calculation as the outcome measure (age, IQ, counting ability, working memory, additive reasoning as predictors)

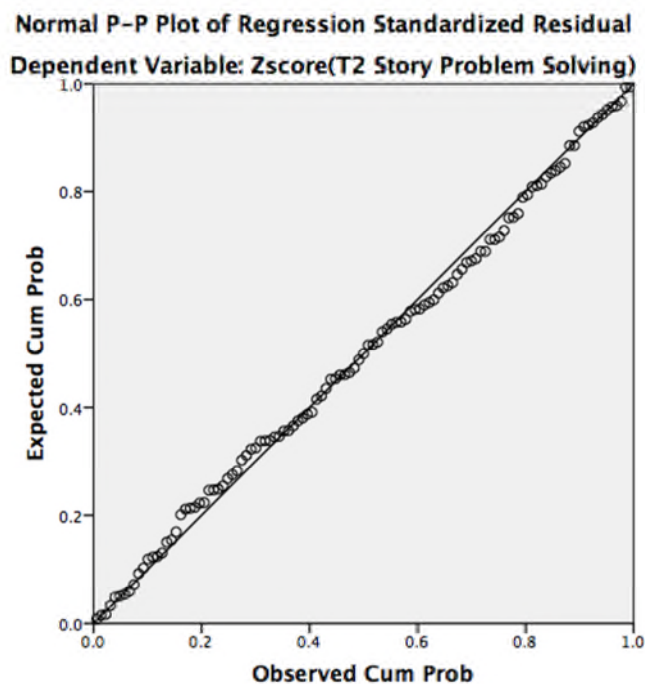


Figure B.7 Normality probability plot of regression standardised residual for T2 story problem solving as the outcome measure (age, IQ, counting ability, working memory, additive reasoning as predictors)

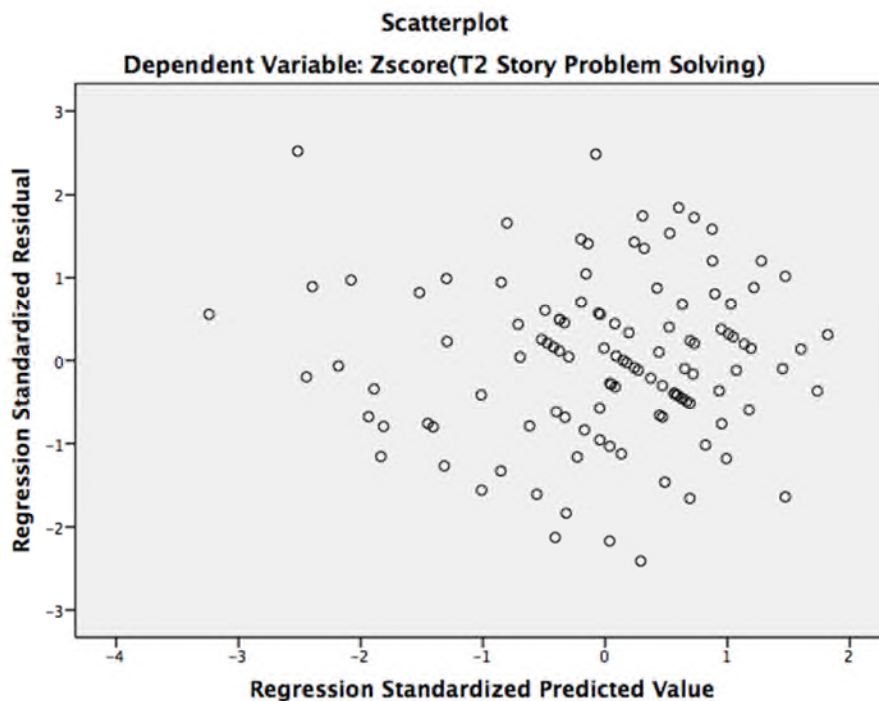


Figure B.8 Scatterplot of the regression standardised predicted values compared with the regression standardised residuals for T2 story problem solving as the outcome measure (age, IQ, counting ability, working memory, additive reasoning as predictors)