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**COMPETITIVE NONLINEAR PRICING AND BUNDLING**

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# Competitive Nonlinear Pricing and Bundling\*

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## Abstract

We examine the impact of multiproduct nonlinear pricing on profit, consumer surplus and welfare in a duopoly. When consumers buy all their products from one firm (the one-stop shopping model), nonlinear pricing leads to higher profit and welfare, but often lower consumer surplus, than linear pricing. By contrast, in a unit-demand model where consumers may buy one product from one firm and another product from another firm, bundling generally acts to reduce profit and welfare and to boost consumer surplus. In a more general model where consumers may buy from more than one firm and where consumers have elastic demands for each product, nonlinear pricing has ambiguous effects. Compared with linear pricing, nonlinear pricing tends to raise profit but harm consumer surplus when: (i) demand is elastic, (ii) there is substantial product differentiation, (iii) there is substantial heterogeneity in consumer demand, (iv) consumers face substantial shopping costs when visiting more than one firm, and (v) a consumer's brand preference for one product is strongly correlated with her brand preference for another product. Nonlinear pricing is more likely to lead to welfare gains when (i), (ii), (iv) and (v) hold, but (iii) does not.

## 1 Introduction

Most economic analysis of imperfect competition is based on the assumption of linear pricing, where the price of a combination of purchases from a firm, whether of one or more products, is equal to the sum of the prices of the component parts. While many markets operate on that basis, an increasing number feature nonlinear pricing—for example, discounts for purchases of larger volumes or of more products. In the absence of easy arbitrage, which

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would undermine it, such pricing can be observed in more or less competitive markets as well as in some with market power.

Examples include energy markets, which traditionally were monopolies but are now open to varying degrees of competition. Consumers are often able to buy their gas and electricity from a single supplier or from two suppliers. In each case, consumers typically face nonlinear tariffs, and they will often enjoy an additional discount if they purchase both services from a single supplier. Similarly, consumers nowadays can often source telecommunications, cable television and internet services from a single supplier or from several, and nonlinear pricing and bundling are commonplace. Nonlinear pricing is also to be seen in areas such as air travel (both individual and corporate) and supermarkets, where loyalty schemes have become more prevalent with the advent of electronic point-of-sale information. The economic importance of nonlinear pricing goes wider still. Thus labour contracts may include extra pay for longer tenure, so that a two-period worker gets a better deal than two one-period workers.<sup>1</sup> While the provision of incentives is doubtless a prime reason for such nonlinear wages, the nature of multi-period competition by firms for labour may also be of relevance, and there may be parallels with multi-product competition among firms for consumers.

The aim of this paper is to compare linear and nonlinear pricing in various settings with imperfect (duopoly) competition. We explain why equilibrium nonlinear pricing is better for profit and welfare and worse for consumers than equilibrium linear pricing in some settings, and why the opposite is true in other settings.

There is an extensive literature on nonlinear pricing and bundling.<sup>2</sup> In the context of monopoly supply, analyses include Adams and Yellen (1976), McAfee, McMillan, and Whinston (1989), Armstrong (1996), and Rochet and Choné (1998). In particular, the latter two papers demonstrate the complexity of multiproduct nonlinear pricing for a monopolist, and the profit-maximizing tariff can be derived only in a few isolated examples. Whinston (1990) and Nalebuff (2004) have explored aspects of the relationship between (pure) bundling and entry deterrence by an incumbent monopolist. There is also a growing literature on (more or less) competitive nonlinear pricing and bundling by symmetrically-placed firms. For example Spulber (1979), Stole (1995), Armstrong and Vickers (2001) and Rochet and Stole (2002) examine competitive nonlinear pricing in situations in which consumers purchase all products from a single supplier. The latter two papers suggest that marginal-cost pricing often emerges as a nonlinear pricing equilibrium, in which case welfare is boosted when firms offer such tariffs compared with linear pricing.<sup>3</sup> More generally, a theme in Armstrong and Vickers (2001) is that when consumers are one-stop shoppers, firms choose socially desirable tariffs if given freedom to do so.

But how reasonable is the assumption that consumers buy all relevant products from

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<sup>1</sup>See, for example, Stevens (2004).

<sup>2</sup>Armstrong (2006) and Stole (2006) are recent surveys.

<sup>3</sup>There is no systematic comparison between nonlinear and linear pricing in terms of profit and consumer surplus in those papers, although Armstrong and Vickers (2001) show when consumers have homogeneous demands that profit rises and consumer surplus falls when two-part tariffs are employed.

a single supplier? The competitive bundling literature, including the contributions by Matutes and Regibeau (1992), Anderson and Leruth (1993), Reisinger (2006) and Thanassoulis (2006), investigates nonlinear pricing when some consumers wish to buy products from more than one supplier. This line of analysis suggests that bundling tends to harm profit and welfare but to be pro-consumer. It tends to proceed by way of worked examples and assumes that consumers wish to buy only one unit of a product. Moreover, the models make the polar opposite assumption from the one-stop shopping models that consumers face no intrinsic extra “shopping cost” if they buy from more than one firm. Thus, the literature on competitive nonlinear pricing and bundling has yielded starkly conflicting results about the pros and cons of nonlinear pricing, which our analysis seeks to reconcile.

The rest of the paper, and the main results, can be summarized as follows. Our point of departure in section 2 is a Hotelling model of product differentiation with one-stop shopping and consumers with heterogeneous demands.<sup>4</sup> We show that with nonlinear pricing and all consumers served, the unique symmetric equilibrium of the model has, in effect, two-part tariffs with marginal prices equal to marginal costs. Welfare is maximized with such nonlinear pricing, whereas with linear pricing there is the problem of *excessive marginal prices*. Profit is also higher than with linear pricing for two reinforcing reasons that relate to elasticity of demand and the extent of consumer heterogeneity. Consumer surplus is however typically lower than with linear pricing.

A limitation of this model is the assumption that each consumer patronizes just one firm. Section 3 examines the possibility of two-stop shopping—where consumers choose on the basis of prices and brand preferences whether to buy from both firms or just one—in a two-dimensional model of product differentiation with inelastic demand.<sup>5</sup> Discounts for joint purchase—mixed bundling—are a general feature of the nonlinear pricing equilibrium, and have a simple characterization. These discounts, however, give rise to the problem of *excessive loyalty*, in that there is too much one-stop shopping. Therefore, in contrast to the one-stop shopping model, welfare is lower when nonlinear pricing is used. Remarkably, the profit and consumer surplus comparisons between linear and nonlinear pricing are also exactly the opposite of those from the one-stop shopping model.

A unifying model, which allows for consumers to have elastic and heterogeneous demands, is analyzed in section 4. With nonlinear pricing permitted and all consumers buying some of both products, there is an equilibrium with efficient two-part tariffs—i.e., with marginal prices equal to marginal costs. Moreover, the fixed elements of the tariffs are precisely the same equilibrium prices, with the same discount for one-stop shopping, as in the model of section 3 with inelastic demand. Therefore, the nonlinear pricing equilibrium, unlike that with linear pricing, is free from the excessive marginal price problem, but it does suffer from the excessive loyalty problem. The models in sections 2 and 3 each had just one of these effects; hence their contrasting results. Perhaps surprisingly, the analysis of competitive

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<sup>4</sup>This develops the models of Armstrong and Vickers (2001, section 4) and Rochet and Stole (2002).

<sup>5</sup>This develops the model of Matutes and Regibeau (1992).

linear pricing sometimes turns out to be simpler than either (a) monopoly nonlinear pricing or (b) competitive linear pricing.

Section 5 pursues the comparison between linear and nonlinear pricing in terms of five underlying economic influences: (i) demand elasticity, (ii) product differentiation, (iii) consumer heterogeneity, (iv) costs of going to more than one supplier, and (v) correlation in brand preferences. Table 1 indicates whether an increase in each of these influences tends to add to or subtract from the merits of nonlinear pricing relative to linear pricing for welfare, profit and consumer surplus.

	Welfare	Profit	Consumers
(i) demand elasticity	+	+	–
(ii) product differentiation	+	+	–
(iii) consumer heterogeneity	–	+	–
(iv) shopping costs	+	+	–
(v) brand preference correlation	+	+	–

Table 1: Effects on the relative merits of nonlinear pricing

The excessive marginal prices effect becomes more important relative to the excessive loyalty effect—which favours nonlinear pricing in welfare terms—as demand elasticity, product differentiation, consumer homogeneity, shopping costs and brand preference correlation increase. For example, the excessive marginal prices effect and the excessive loyalty effect respectively increase in importance with more elastic demand (because deadweight welfare loss ‘triangles’ open up) and as shopping costs fall (because more consumers two-stop shop).

The direction of effects on profit tends to be the opposite of that on consumer surplus. In respect of (i), (ii), (iv) and (v), the profit effect has the same sign as the welfare effect. But with greater consumer heterogeneity, the relative merits of nonlinear pricing for welfare and consumers tend to be lower. That is because heterogeneity sharpens linear price competition, which is good for consumers and welfare (as the excessive marginal price effect diminishes) but is bad for profit.

Section 6 concludes by suggesting directions for future research on this topic.

## 2 Nonlinear Pricing and One-Stop Shopping

Suppose there are  $n \geq 1$  products, each supplied by two firms, denoted  $A$  and  $B$ . For exogenous reasons, suppose consumers buy all their suppliers from one firm or the other, i.e., consumers are one-stop shoppers. Consumers differ in their brand preference for the two firms,  $x$ , and their preference for the products,  $\theta$ . The parameter  $x$  is uniformly distributed between 0 (where firm  $A$  is located) and 1 (where  $B$  is located), while  $\theta$  is independently

distributed according to some distribution function  $F(\theta)$ .<sup>6</sup> Gross utility (excluding transport costs) is  $u(\theta, q)$  if a type- $\theta$  consumer buys quantities  $q$ . Lump-sum transport cost is  $t$  per unit of distance. In sum, a type- $(x, \theta)$  consumer's net utility if she buys quantities  $q^A$  from firm  $A$  in return for payment  $T^A$  is

$$u(\theta, q^A) - tx - T^A ,$$

and if she buys quantities  $q^B$  from  $B$  with payment  $T^B$  her utility is

$$u(\theta, q^B) - t(1 - x) - T^B .$$

In the following analysis, we make the important assumption, which is satisfied subject to conditions on willingness-to-pay relative to costs, that all consumers choose to participate in the market with the relevant range of tariffs. Finally, suppose each firm has constant unit cost  $c_i$  for supplying product  $i$ .

Our first result derives the equilibrium nonlinear tariff, which we will use to compare with linear pricing.<sup>7,8</sup>

**Proposition 1** *Suppose that over the relevant range of nonlinear tariffs all consumers are served. Then the unique symmetric equilibrium outcome with nonlinear pricing involves efficient consumption, and a consumer who buys quantities  $q$  makes payment*

$$T(q) = t + \sum_i c_i q_i . \quad (1)$$

*Equilibrium industry profit is  $\pi_{NL} = t$ .*

**Proof.** That the cost-based two-part tariff in (1) is an equilibrium was established in Armstrong and Vickers (2001, Proposition 5). That this tariff induces the *unique* symmetric equilibrium outcome can be argued by contradiction. Suppose that in an alternative symmetric equilibrium, industry profit is  $\Pi$ . Suppose further that the equilibrium tariff  $T(\cdot)$  does not satisfy  $\frac{\partial}{\partial q_i} T(\cdot) \equiv c_i$ . Suppose firm  $A$  deviates from the candidate equilibrium, and instead sets the tariff  $\hat{T}(q) = \Pi + \sum_i c_i q_i$ . A consumer located at  $x$  will buy from firm  $A$  if  $v^*(\theta) - \Pi - tx \geq v(\theta) - t(1 - x)$ , where

$$v^*(\theta) = \max_q : u(\theta, q) - \sum_i c_i q_i$$

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<sup>6</sup>The assumption that  $x$  is uniform is made purely for expositional convenience, and none of the following results depend on this assumption. If  $x$  were instead distributed on  $[0, 1]$  with density  $f(x)$  which is symmetric about  $x = \frac{1}{2}$ , then subject to regularity conditions on  $f(\cdot)$  the following results remain valid if ‘ $t$ ’ is replaced by ‘ $t/f(\frac{1}{2})$ ’.

<sup>7</sup>Note that this result need not hold if (i)  $\theta$  and  $x$  are correlated, (ii) firms have different costs, or (iii) if some consumers do not buy from either firm. See Rochet and Stole (2002) for further details.

<sup>8</sup>We do not claim that the tariff in (1) is the unique symmetric tariff. For instance, if all consumers purchased quantities above some positive lower bound, how the tariff is defined for quantities below this lower bound cannot be uniquely determined.

and

$$v(\theta) = \max_q : u(\theta, q) - T(q) .$$

Let

$$x(\theta) = \frac{1}{2} + \frac{v^*(\theta) - \Pi - v(\theta)}{2t}$$

be the marginal  $x$  consumer in the  $\theta$ -segment. Then  $A$ 's deviation profit is  $E_\theta[x(\theta)] \times \Pi$ , and so the deviation is profitable if  $E_\theta[x(\theta)] > \frac{1}{2}$ , i.e., if

$$E_\theta[v^*(\theta)] > \Pi + E_\theta[v(\theta)] .$$

(Here and elsewhere,  $E_\theta[\cdot]$  refers to taking expectations with respect to  $\theta$  using  $F(\theta)$ .) However, the left-hand side of the above is total surplus with marginal cost pricing, and the right-hand side is total surplus with candidate tariff (which does not always involve marginal cost pricing). Therefore, this inequality is indeed satisfied, and the unique symmetric equilibrium outcome involves marginal-cost pricing. ■

Consider next the outcome when linear prices are used. Suppose that the type- $\theta$  consumer has demand functions  $q_i(\theta, p)$ , where  $p$  is the vector of linear prices  $p = (p_1, \dots, p_n)$ . (These demands are the quantities  $q$  which maximize  $u(\theta, q) - \sum_{i=1}^n p_i q_i$ .) Let

$$v(\theta, p) = \max_q : u(\theta, q) - pq$$

be type- $\theta$  consumer surplus with prices  $p$ . Let  $\pi(\theta, p)$  be type- $\theta$  profit, i.e.,  $\pi(\theta, p) \equiv \sum_{i=1}^n q_i(\theta, p)(p_i - c_i)$ . It is convenient to introduce one further piece of notation: for a candidate symmetric equilibrium price vector  $p$ , define

$$s(\theta, r) \equiv v(\theta, c + r(p - c)) ,$$

where  $r$  is a scalar. This allows us to conduct the analysis in terms of the scalar  $r$  rather than the vector  $p$ . Then  $s(\theta, 0) = v(\theta, c)$  and  $s(\theta, 1) = v(\theta, p)$ . Since  $v$  is convex in  $p$ ,  $s$  is convex in  $r$  and so  $s_{rr}(\theta, r) \geq 0$ . Note that

$$\pi(\theta, c + r(p - c)) = -rs_r(\theta, r) .$$

In particular  $\pi(\theta, p) = -s_r(\theta, 1)$ . For  $p$  to be a symmetric equilibrium price vector, it is necessary that  $r = 1$  maximizes a firm's profit

$$E_\theta \left[ \left( \frac{1}{2} + \frac{s(\theta, r) - s(\theta, 1)}{2t} \right) (-rs_r(\theta, r)) \right] .$$

This has the first-order condition<sup>9</sup>

$$\pi_L = E_\theta[-s_r(\theta, 1)] = E_\theta \left[ s_{rr}(\theta, 1) + \frac{(s_r(\theta, 1))^2}{t} \right] , \quad (2)$$

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<sup>9</sup>The second-order condition on  $r$  is satisfied provided  $E_\theta[s_{rrr}(\theta, 1)]$  is not strongly negative, which we assume henceforth.

where  $\pi_L$  is the equilibrium industry profit with linear prices.

Expression (2) implies that

$$\pi_L = E_\theta \left[ s_{rr}(\theta, 1) + \frac{(s_r(\theta, 1))^2}{t} \right] \geq \frac{1}{t} E_\theta [(s_r(\theta, 1))^2] \geq \frac{1}{t} (E_\theta [s_r(\theta, 1)])^2 = \frac{1}{t} \pi_L^2 . \quad (3)$$

The first inequality follows since  $s_{rr} \geq 0$ , and the inequality is strict if consumer demand is elastic. We can refer to this as the “elasticity” effect. The second inequality follows from Jensen’s inequality. It is strict if there is some consumer heterogeneity (i.e., if the variance of  $s_r(\theta, 1)$  is positive); this is the “heterogeneity” effect. Expression (3) shows that  $\pi_L \leq t = \pi_{NL}$ . Hence profits are lower when linear pricing is employed than with nonlinear pricing.<sup>10</sup> The reason for this is a combination of the elasticity and heterogeneity effects, both of which act in the same direction. (These two effects are discussed in more detail in sections 5.1 and 5.3.)

Clearly, the marginal-cost pricing that is implemented by nonlinear pricing is unambiguously beneficial for welfare (as well as for profits, as we have seen). A more subtle issue is the impact of nonlinear pricing on consumer surplus. For a type- $\theta$  consumer, the welfare loss of using linear prices relative to nonlinear prices, which we know involves prices equal to marginal costs, is

$$v(\theta, c) - [v(\theta, p) + \pi(\theta, p)] = s(\theta, 0) - s(\theta, 1) + s_r(\theta, 1) .$$

If consumers have linear demands (or generally when  $t$  is small) this welfare loss is  $\frac{1}{2} s_{rr}(\theta, 1)$ . (The reason for this is that with linear demands  $s(\theta, r)$  is quadratic in  $r$ , and so a second-order Taylor expansion about  $r = 1$  is exactly accurate.) In this case, the aggregate welfare gain when firms employ nonlinear pricing is  $\Delta W = \frac{1}{2} E_\theta [s_{rr}(\theta, 1)]$ . From (3) we see

$$2\Delta W = \pi_L - \frac{1}{t} E_\theta [(s_r(\theta, 1))^2] \leq \pi_L - \frac{1}{t} (E_\theta [s_r(\theta, 1)])^2 = \pi_L - \frac{1}{t} \pi_L^2 ,$$

i.e.,

$$\Delta W \leq \frac{1}{2} \frac{\pi_L}{t} (t - \pi_L) \leq \frac{1}{2} (t - \pi_L) ,$$

where the second inequality follows from our previous result that  $\pi_L \leq t$ . Therefore, with linear demands the welfare gain is less than half the gain in industry profit, and so consumers in aggregate are worse off when nonlinear tariffs are used.<sup>11</sup>

We can summarise our analysis of the one-stop shopping model as:

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<sup>10</sup>Note that this profit comparison seems to be valid in an alternative framework in which consumers incur transport costs on a per-unit rather than a lump-sum basis (at least when transport costs are small), although the analysis is considerably more complicated. For instance, see Proposition 5 in Yin (2004) for analysis with linear demand.

<sup>11</sup>The result that consumers are worse off with nonlinear pricing can be established under weaker assumptions than linear demand. For example, let  $\zeta(\theta)$  be the elasticity for a type- $\theta$  consumer of welfare loss relative to the first best with respect to  $r$ , evaluated at  $r = 1$ . If  $\zeta(\theta) > 1$  for all  $\theta$ , then the result holds. (With linear demands  $\zeta = 2$ .)



**Proposition 2** *Suppose that over the relevant range of tariffs all consumers are served. Then compared to the outcome with linear pricing, industry profit and total welfare are higher with nonlinear pricing. In addition, if each consumer’s demand is linear in prices (or if  $t$  is small), consumer surplus is lower with nonlinear pricing.*

In the unique symmetric equilibrium with nonlinear pricing, firms use cost-reflective (so efficient) two-part tariffs. The optimal fixed fee ( $t$ ) in such a tariff balances (i) the firm’s loss of profit on existing consumers against (ii) its gain in profitable consumers from the other firm. Demand per consumer does not change: it is as if consumers had identical inelastic demands. By contrast, if firms are restricted to linear prices, a firm choosing its price(s) will balance (i) against not only (ii) but also (iii) its gain in profit per consumer. Profit per consumer can change for two reasons. First, with elastic demands, lower prices expand demand from each type of consumer. Second, and less obviously, if consumers are heterogeneous, lowering prices can alter the *mix* of consumer types coming to a firm—in particular by winning proportionately more high demand (hence high profit) consumers than low demand consumers from the other firm. This second reason why average profit-per-consumer may rise as linear prices are reduced occurs even if consumers have inelastic demands. These two reasons are reflected in the two inequalities in equation (3). They are both reasons why there is more incentive to lower prices—hence why consumers do better—with linear pricing. Yet welfare is higher with nonlinear pricing because there is no marginal price inefficiency. It follows that profits must be higher then too.

### 3 Bundling and Two-Stop Shopping

In this section we analyze a model of multi-product competition where consumers can “mix-and-match” products from two firms. We assume here that consumers have unit demands. This case is of interest in its own right; it will also provide the key to the more general elastic demand analysis in sections 4 and 5.

#### 3.1 The Framework

Consider a two-dimensional Hotelling model. Two firms,  $A$  and  $B$ , each offer their own brand of two products, 1 and 2. The location (or brand preference) of a consumer is denoted  $(x_1, x_2) \in [0, 1]^2$ . Let  $x_1$  represent a consumer’s distance from firm  $A$ ’s brand of product 1 and  $x_2$  represent a consumer’s distance from the same firm’s brand of product 2. The density of  $(x_1, x_2)$  is  $f(x_1, x_2)$  and firms are symmetrically placed in terms of consumers:

$$f(x_1, x_2) \equiv f(1 - x_1, 1 - x_2) . \quad (4)$$

The transport cost (or brand preference) parameter is  $t_1$  for product 1 and  $t_2$  for product 2. In addition to these transport costs, consumers face an additional “shopping cost”  $z \geq 0$

when they source supplies from two firms. This shopping cost might represent the time or cost involved in visiting two shops rather than one, or it might measure a consumer's perceived cost of dealing with two firms (paying two bills rather than one, for instance). In order to have some two-stop shoppers in equilibrium, it is necessary to constrain the shopping cost not to be too large, and we assume that

$$z < \min\{t_1, t_2\} .$$

If this inequality does not hold, all consumers will choose to buy both items from one firm or the other, and the distinction between bundling and linear pricing vanishes.

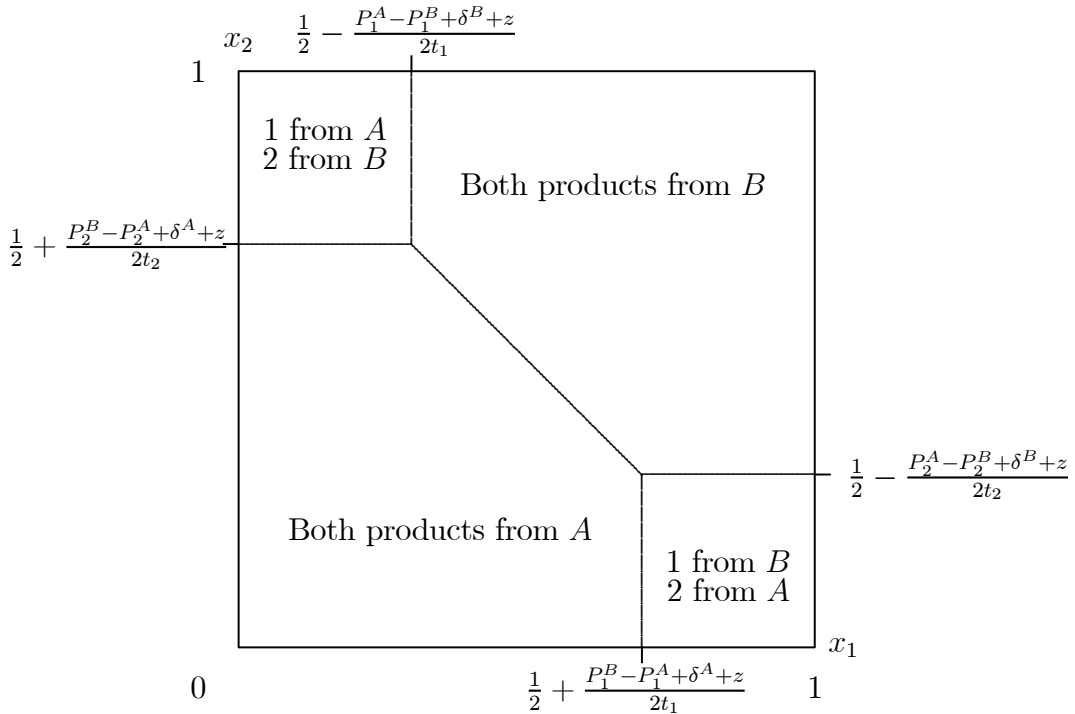


Figure 1: The Pattern of Consumer Demand

Since consumers have unit demand for each product, a firm's tariff consists of three prices. Let  $P_1^i$  denote firm  $i$ 's stand-alone price for its product 1, let  $P_2^i$  be its stand-alone price for its product 2, and let  $\delta^i$  be its discount if a consumer buys both products, so the total charge for buying both products from firm  $i$  is  $P_1^i + P_2^i - \delta^i$ . For simplicity, suppose that conditions in the market are such that all consumers buy both products. The type- $(x_1, x_2)$  consumer's total outlay if she buys both products from firm  $A$  is  $P_1^A + P_2^A - \delta^A + t_1x_1 + t_2x_2$ , her total outlay if she buys both products from  $B$  is  $P_1^B + P_2^B - \delta^B + t_1(1-x_1) + t_2(1-x_2)$ , and her total outlay if she buys product  $i$  from  $A$  and product  $j \neq i$  from  $B$  is  $P_i^A + P_j^B + t_ix_i + t_j(1-x_j) + z$ .

The consumer located at  $(x_1, x_2)$  will choose the option from among the four possibilities which involves the smallest outlay. The pattern of demand is as shown in Figure 1.

For simplicity of notation, suppose that production is costless. (This is without loss of generality if marginal costs are constant, and the prices  $P_i$  derived below can be considered to be prices net of marginal costs.) In this case, when firms offer the same tariff with stand-alone prices  $P_1$  and  $P_2$  and discount (if applicable)  $\delta$ , industry profit is

$$(P_1 + P_2) - \delta \times \{\text{proportion of one-stop shoppers}\} . \quad (5)$$

In the following analysis, it is useful to introduce some further notation.

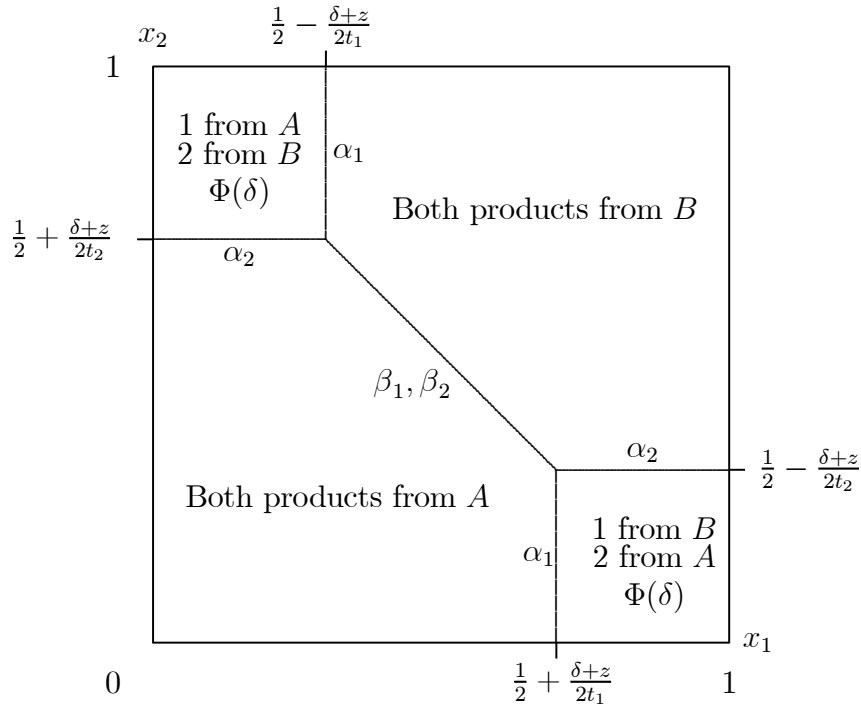


Figure 2: Notation Used in the Analysis

Define the (decreasing) function

$$\Phi(\delta) \equiv 2 \int_0^{\frac{1}{2} - \frac{\delta+z}{2t_1}} \int_{\frac{1}{2} + \frac{\delta+z}{2t_2}}^1 f(x_1, x_2) dx_2 dx_1$$

to be the proportion of consumers who are two-stop shoppers when the firms set the same tariff which involves the discount  $\delta$ . The function  $\Phi$  summarizes many of the economically-relevant features of this market, and it is depicted on Figure 2. (The symmetry assumption (4) implies that the two rectangles of two-stop shoppers each contain the same proportion of consumers.)

It will be useful to know the derivative of  $\Phi$ . Write

$$\alpha_1(\delta) = \int_{\frac{1}{2} + \frac{\delta+z}{2t_2}}^1 f\left(\frac{1}{2} - \frac{\delta+z}{2t_1}, x_2\right) dx_2 ; \quad \alpha_2(\delta) = \int_0^{\frac{1}{2} - \frac{\delta+z}{2t_1}} f\left(x_1, \frac{1}{2} + \frac{\delta+z}{2t_2}\right) dx_1 \quad (6)$$

for the line integrals depicted on Figure 2. (From (4), the two integrals marked  $\alpha_1$  have the same value, as do those marked  $\alpha_2$ .) Then

$$\Phi'(\delta) = - \left( \frac{\alpha_1(\delta)}{t_1} + \frac{\alpha_2(\delta)}{t_2} \right) . \quad (7)$$

Finally, write

$$\beta_1(\delta) = \int_{\frac{1}{2} - \frac{\delta}{2t_2}}^{\frac{1}{2} + \frac{\delta}{2t_2}} f\left(\frac{1}{2} + \frac{t_2}{t_1}\left(\frac{1}{2} - x_2\right), x_2\right) dx_2 ; \quad \beta_2(\delta) = \int_{\frac{1}{2} - \frac{\delta}{2t_1}}^{\frac{1}{2} + \frac{\delta}{2t_1}} f\left(x_1, \frac{1}{2} + \frac{t_1}{t_2}\left(\frac{1}{2} - x_1\right)\right) dx_1 \quad (8)$$

for the line integrals along the diagonal segment depicted on Figure 2. Here,  $\beta_1$  is the integral in the vertical direction, and  $\beta_2$  is the integral in the horizontal direction, and they are related by  $t_2\beta_1 = t_1\beta_2$ .

### 3.2 Linear Pricing

Consider first the case where firms compete with linear prices—that is to say with no bundling discounts in the present context—so  $\delta^A = \delta^B = 0$ . For simplicity, write  $\alpha_i^0 = \alpha_i(0)$  and  $\beta_i^0 = \beta_i(0)$ . Suppose the two firms initially set the same pair of linear prices  $P_1$  and  $P_2$ . Consider firm  $A$ 's incentive to reduce its price  $P_1$  by  $\varepsilon$ , keeping its price for product 2 unchanged at  $P_2$ . At a symmetric equilibrium, half the consumers buy product 1 from firm  $A$ , and the firm loses revenue  $\varepsilon$  from each of these infra-marginal consumers. The price reduction shifts the boundary of the set of consumers who buy product 1 from the firm uniformly to the right by  $\varepsilon/(2t_1)$ , as depicted on Figure 3.

The profit from these marginal consumers is not constant along this boundary. Those consumers on the two vertical boundaries  $\alpha_1^0$  on the figure generate profit  $P_1$  to the firm, while those on the diagonal boundary  $\beta_1^0$  generate “double” profit ( $P_1 + P_2$ ). The total profit of these marginal consumers is therefore

$$\frac{\varepsilon}{2t_1} \times \{2\alpha_1^0 P_1 + \beta_1^0 (P_1 + P_2)\} .$$

Since the profit gained from the marginal consumers must equal the profit lost from the infra-marginal consumers, it follows that in equilibrium

$$2\alpha_1^0 P_1 + \beta_1^0 (P_1 + P_2) = t_1 . \quad (9)$$

Similarly, the first-order condition for the stand-alone price  $P_2$  is

$$2\alpha_2^0 P_2 + \beta_2^0 (P_1 + P_2) = t_2 . \quad (10)$$

Solving these linear simultaneous equations in  $(P_1, P_2)$  yields explicit formulae for the (unique) equilibrium prices:

$$P_1 = \frac{\alpha_2^0 t_1}{\alpha_1^0 \beta_2^0 + \beta_1^0 \alpha_2^0 + 2\alpha_1^0 \alpha_2^0} ; P_2 = \frac{\alpha_1^0 t_2}{\alpha_1^0 \beta_2^0 + \beta_1^0 \alpha_2^0 + 2\alpha_1^0 \alpha_2^0} . \quad (11)$$

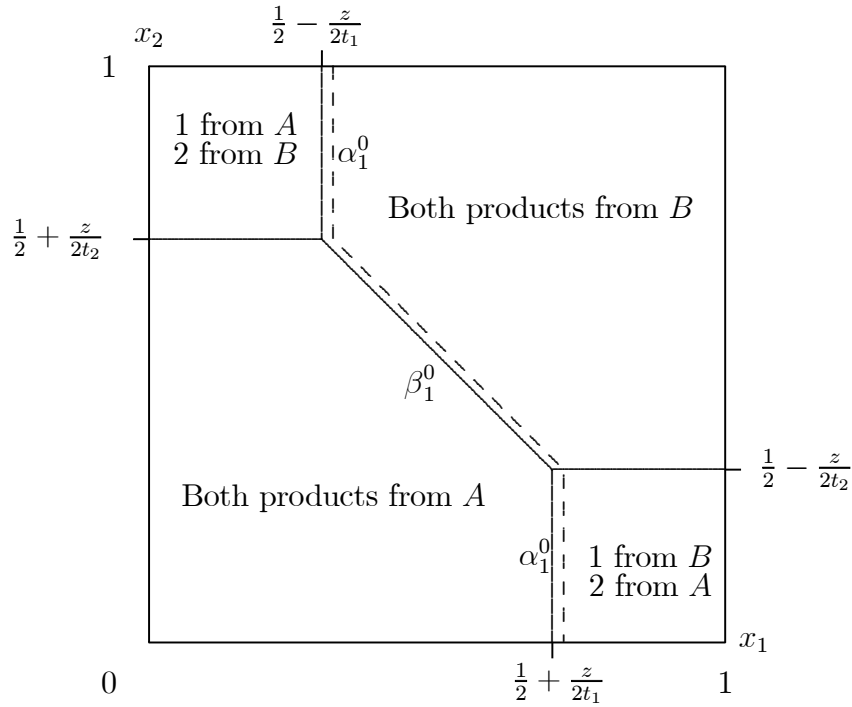


Figure 3: Incentive to Reduce Linear Price for Product 1

In most cases, prices and profit falls as the shopping cost  $z$  becomes more significant.<sup>12</sup> The reason is that, when  $z$  is large, the number of consumers (measured by  $\beta_i^0$ ) who are indifferent between buying both products from  $A$  and buying both products from  $B$  increases. These marginal consumers are “doubly profitable” with their two-unit demands. So firms compete hard to attract these consumers, with the result that prices decrease with  $z$ .

<sup>12</sup>For instance, when product preferences  $x_1$  and  $x_2$  are independently distributed, the method of proof for Proposition 6 can show that the sum of the linear prices decreases with  $z$  (subject to mild regularity conditions).

It is useful to illustrate these, and subsequent, results by means of a simple example.<sup>13</sup>

**Uniform Example:**  $t_1 = t_2 = t$  and  $f(x_1, x_2) \equiv 1$ .

Here,  $\alpha_1^0 = \alpha_2^0 = \frac{1}{2} - \frac{z}{2t}$  and  $\beta_1^0 = \beta_2^0 = \frac{z}{t}$ . From (11) the equilibrium linear price for each product is

$$P_1 = P_2 = \frac{t^2}{t + z} , \quad (12)$$

which is decreasing in  $z$ . Therefore, with linear pricing the shopping cost makes the market more competitive, in much the same way as firms selling their products with a bundling discount does so.

### 3.3 Bundling

Suppose next that firms can offer discounts for joint consumption. It turns out that a firm always has a unilateral incentive to do so.

**Proposition 3** *Suppose the two firms initially offer the equilibrium linear prices (11). Then a firm's profit increases if it unilaterally introduces a small discount  $\delta > 0$  for joint purchase.*

**Proof.** Suppose both firms initially offer the linear prices in (11), and consider the effect on firm  $A$ 's profit when it introduces a small joint-purchase discount  $\delta > 0$ . Let  $N < \frac{1}{2}$  denote the proportion of consumers who choose to buy both items from firm  $A$  when symmetric linear prices are offered. Here, the firm loses revenue  $\delta$  from the  $N$  consumers who previously purchased both products from it in any case, but the discount induces some two-stop shoppers to buy both products from  $A$ . See Figure 4.

Specifically, from Figure 4 one sees that  $\delta\alpha_1^0/(2t_1)$  consumers switch from buying product 1 from  $B$  and product 2 from  $A$  to buying both from  $A$ , and each of these consumers brings in additional profit  $P_1 - \delta$ . Similarly,  $\delta\alpha_2^0/(2t_2)$  consumers switch from buying product 1 from  $A$  and product 2 from  $B$  to buying both from  $A$ , and each of these consumers brings additional profit  $P_2 - \delta$ . Finally,  $\delta\beta_1^0/(2t_1)$  consumers switch from buying both items from  $B$  to buying both from  $A$ , and these “doubly profitable” consumers bring profit  $P_1 + P_2 - \delta$ . In sum, ignoring terms in  $\delta^2$ , the effect on firm  $A$ 's profit of introducing the small discount  $\delta$  is approximately

$$\frac{\delta\alpha_1^0}{2t_1}P_1 + \frac{\delta\alpha_2^0}{2t_2}P_2 + \frac{\delta\beta_1^0}{2t_1}(P_1 + P_2) - \delta N = \delta(\frac{1}{2} - N) > 0 .$$

---

<sup>13</sup>This example (with  $z = 0$ ) was first analyzed by Matutes and Regibeau (1992), and extended in Armstrong (2006, section 4.2) to situations where the products were not symmetric in the sense that  $t_1 \neq t_2$ . These earlier analyses took a “brute force” approach by simply calculating the areas of the various regions in Figure 1. This method is only practical when the distribution of  $(x_1, x_2)$  is uniform, and it cannot be used to derive more general results, such as Propositions 3, 4 and 6 in this paper.

Here, the equality follows from expressions (9)–(10), which proves the result. ■

This argument is in the same spirit as that used in the monopoly context by McAfee, McMillan, and Whinston (1989).<sup>14</sup> However, in our oligopoly framework a firm *always* has an incentive to introduce a positive discount, whereas in the monopoly context it was not always the case that a discount was profitable.

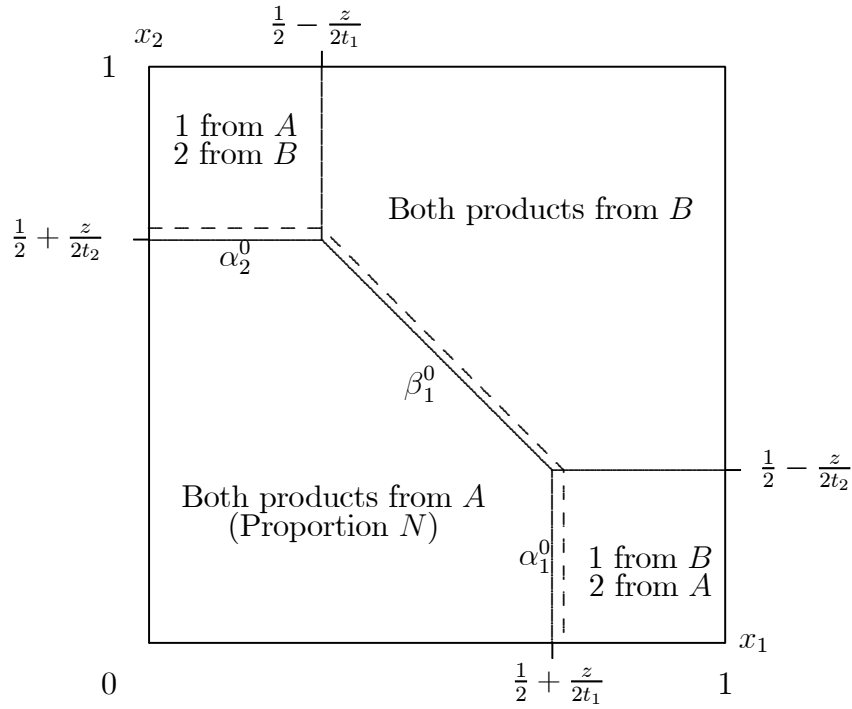


Figure 4: Incentive to Introduce Bundling Discount

An intuition for this result goes as follows. Starting from symmetric equilibrium with linear pricing, there would be no first-order effect on the profits of a firm that reduced each of its standalone prices by  $\frac{1}{2}\delta$  for small  $\delta$  because those prices are optimal for the firm (i.e., the envelope theorem applies). Suppose instead that the firm does not change its stand-alone prices but introduces a discount  $\delta$  for joint purchase. From (4), this brings the same gain in custom for each of the two products as the stand-alone price cuts. But it involves strictly less foregone profit on intra-marginal custom because the discount is restricted to one-stop shoppers rather than all the firm's consumers. The firm, having been roughly indifferent about the stand-alone price cuts, will therefore strictly gain from the bundling discount. In sum, offer a bundling discount is a more cost-effective way to boost market share than cuts in linear prices.

<sup>14</sup>See Thanassoulis (2006) for a related result.

Notice, however, that if *both* firms offer the same (small) joint purchase discount  $\delta$ , each firm's profit falls (by approximately  $N\delta$ ) compared to the profit generated with linear pricing. This illustrates the prisoner's dilemma nature of competitive bundling.

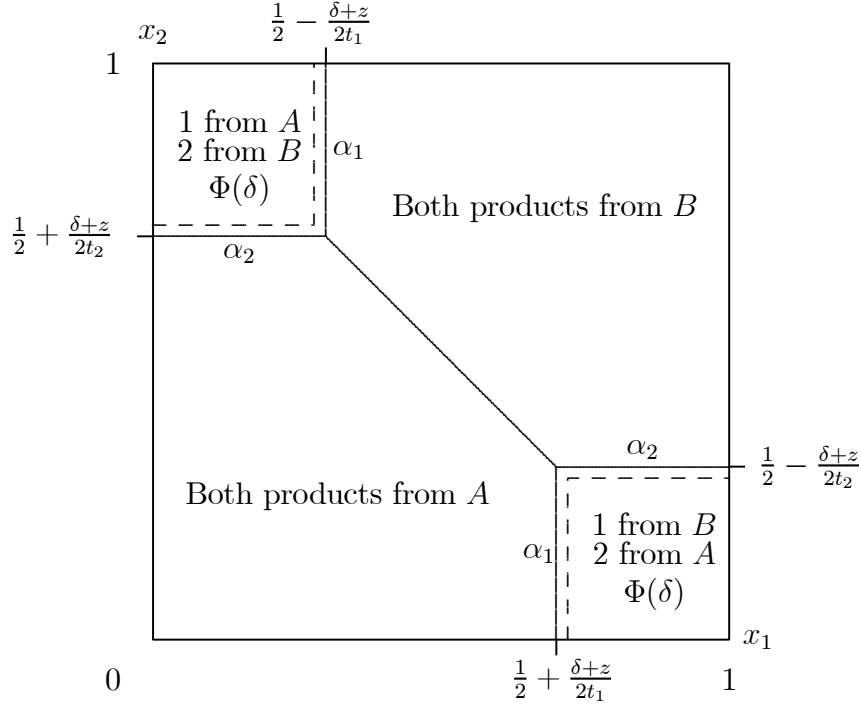


Figure 5: Effect of Changing the Bundle Discount

Having established that linear pricing cannot be an equilibrium when firms can offer bundling discounts, we turn next to analysis of the equilibrium nonlinear tariffs. It turns out that there is a simple and general formula for the equilibrium discount  $\delta$ :

**Proposition 4** *In a symmetric equilibrium the discount  $\delta$  satisfies*

$$\Phi(\delta) + \frac{1}{2}\Phi'(\delta)\delta = 0. \quad (13)$$

**Proof.** Suppose that the symmetric equilibrium nonlinear tariff involves the stand-alone price  $P_1$  for product 1, the stand-alone price  $P_2$  for product 2, and the bundling discount  $\delta$ . Consider the effect on firm A's profit if it increases its discount by a small amount  $2\varepsilon$  and simultaneously increases each of its stand-alone prices by  $\varepsilon$ . The result is that its total charge to one-stop shoppers is unchanged, but the total charge for each two-stop shopping option rises by  $\varepsilon$ . The net effect on the firm's profit is depicted in Figure 5, where the two-stop shopping regions shrink to the dashed regions: the deviation moves the upper boundary marked  $\alpha_1$  to the left by  $\varepsilon/(2t_1)$  and lower boundary  $\alpha_1$  to the right by the same amount;



the upper boundary  $\alpha_2$  is moved up by  $\varepsilon/(2t_2)$  and the lower boundary  $\alpha_2$  down by the same amount. The net effect on firm  $A$ 's profit is then approximately

$$(\varepsilon\Phi(\delta)) + \frac{\varepsilon}{2t_1}\alpha_1(P_1 - \delta) - \frac{\varepsilon}{2t_1}\alpha_1P_1 + \frac{\varepsilon}{2t_2}\alpha_2(P_2 - \delta) - \frac{\varepsilon}{2t_2}\alpha_2P_2 = \varepsilon \left( \Phi(\delta) + \frac{1}{2}\Phi'(\delta)\delta \right) ,$$

where the equality follows from (7). For this deviation to be unprofitable, expression (13) must hold in equilibrium. ■

A simple corollary of this result is that  $\delta$  tends to zero as the shopping cost  $z$  approaches  $\min\{t_1, t_2\}$ . That is to say, as almost all consumers are anyway one-stop shoppers, there is little benefit to firms in inducing still more consumers to become one-stop shoppers by means of a bundle discount. In this sense, the shopping cost reduces the incentive to bundle. For instance, in the *Uniform Example* we have  $\Phi(\delta) = 2(\frac{1}{2} - \frac{\delta+z}{2t})^2$ , and so expression (13) implies that  $\delta = \frac{1}{2}(t - z)$ .

A crucial point is that welfare is reduced when firms offer discounts for joint purchase: there is *excessive loyalty*, as more consumers than is efficient buy both products from the same firm. The efficient pattern of consumption requires there to be no bundling discount, so that the pattern of demand is as depicted in Figure 3.

By examining Figure 5, one sees that when the discount  $\delta$  is increased by  $\varepsilon$ , the extra consumers who buy product 1 from the less preferred firm is equal to  $\frac{\alpha_1}{t_1}\varepsilon$ , and each of these consumers incurs the extra travel cost  $(\delta+z)$  compared to buying the product from the closer firm, although these consumers also save the shopping cost  $z$ . Thus, their net disutility is  $\delta$ . Similarly,  $\frac{\alpha_2}{t_2}\varepsilon$  extra consumers buy product 2 from the less preferred firm, and these each incur a net disutility  $\delta$ .

In sum, the extra welfare loss caused by increasing  $\delta$  by  $\varepsilon$  is

$$\varepsilon\delta \times \left( \frac{\alpha_1}{t_1} + \frac{\alpha_2}{t_2} \right) = \varepsilon\delta \times (-\Phi'(\delta)) .$$

Denote by  $w(\delta)$  the level of welfare corresponding to the bundling discount  $\delta$  relative to the first-best welfare level (i.e., the welfare level corresponding to the case of linear pricing when  $\delta = 0$ ). Thus  $-w(\delta)$  is welfare loss relative to the first best. It follows that

$$w'(\delta) = \delta\Phi'(\delta) . \tag{14}$$

This condition and  $w(0) = 0$  yield  $w(\delta)$ . For instance, in the *Uniform Example* we have  $\Phi(\delta) = 2(\frac{1}{2} - \frac{\delta+z}{2t})^2$  and  $\delta = \frac{1}{2}(t - z)$ , in which case (14) implies that the equilibrium welfare loss relative to the case of linear pricing is

$$\frac{(t - z)^3}{12t^2} . \tag{15}$$

As expected, as the shopping cost becomes large, this welfare loss falls to zero since the bundling discount also falls to zero.

Having derived the equilibrium discount  $\delta$  as in (13), we can now derive the equilibrium stand-alone prices  $P_1$  and  $P_2$  in the same way in which the linear prices were derived in section 3.2. For simplicity, given the equilibrium  $\delta$ , write  $\alpha_i = \alpha_i(\delta)$  and  $\beta_i = \beta_i(\delta)$ . The pattern of demand is as shown on Figure 6.

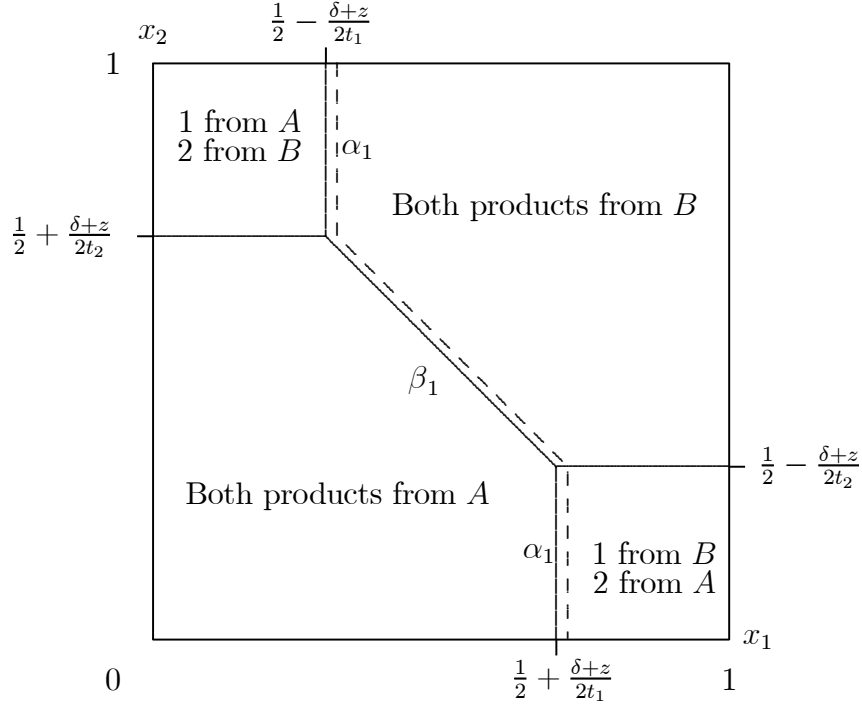


Figure 6: Incentive to Reduce Stand-Alone Price for Product 1

Consider firm  $A$ 's incentive to reduce its stand-alone price  $P_1$  by  $\varepsilon$ , keeping its discount unchanged at  $\delta$  and its stand-alone price for product 2 unchanged at  $P_2$ . At a symmetric equilibrium, half the consumers buy product 1 from firm  $A$ , and the firm loses revenue  $\varepsilon$  from each of these infra-marginal consumers. The price reduction shifts the boundary of the set of consumers who buy product 1 from the firm uniformly to the right by  $\varepsilon/(2t_1)$ . Those consumers on the upper boundary  $\alpha_1$  on the figure generate profit  $P_1$  to the firm, those on the diagonal boundary  $\beta_1$  generate profit  $(P_1 + P_2 - \delta)$ , and those on the lower boundary  $\alpha_1$  generate incremental profit  $(P_1 - \delta)$ .

Since the profit gained from the marginal consumers must equal the profit lost from the infra-marginal consumers, it follows that in equilibrium

$$\alpha_1 P_1 + \beta_1 (P_1 + P_2 - \delta) + \alpha_1 (P_1 - \delta) = t_1 .$$

Similarly, the first-order condition for the stand-alone price  $P_2$  is

$$\alpha_2 P_2 + \beta_2 (P_1 + P_2 - \delta) + \alpha_2 (P_2 - \delta) = t_2 .$$

Solving these linear simultaneous equations in  $(P_1, P_2)$  yields explicit formulae for the stand-alone prices, as reported in the next result.

**Proposition 5** *At a symmetric equilibrium, the discount  $\delta$  satisfies (13) and the stand-alone prices are*

$$P_1 = \frac{\delta}{2} + \frac{\alpha_2 t_1}{\alpha_1 \beta_2 + \beta_1 \alpha_2 + 2\alpha_1 \alpha_2} ; P_2 = \frac{\delta}{2} + \frac{\alpha_1 t_2}{\alpha_1 \beta_2 + \beta_1 \alpha_2 + 2\alpha_1 \alpha_2} . \quad (16)$$

Notice that the stand-alone prices in (16) are uniquely determined given the discount  $\delta$ . Then, provided there is a unique solution to the first-order condition for the discount in (13), which can be ensured by assuming regularity conditions on the shape of  $\Phi$ , there is then only one possible nonlinear pricing equilibrium with some two-stop shoppers.<sup>15,16</sup>

In the *Uniform Example*, with the equilibrium discount  $\delta = \frac{1}{2}(t - z)$  it follows that  $\alpha_1 = \alpha_2 = \frac{1}{4} - \frac{z}{4t}$  and  $\beta_1 = \beta_2 = \frac{1}{2} + \frac{z}{2t}$ . From (16) the equilibrium tariff is then<sup>17</sup>

$$P_1 = P_2 = \frac{1}{4}(t - z) + \frac{2t^2}{3t + z} ; \delta = \frac{1}{2}(t - z) . \quad (17)$$

Therefore, as in the linear pricing regime, the shopping cost  $z$  acts to lower all prices. The reason is the same: the shopping cost expands the margin of “doubly profitable” consumers, which intensifies competition.

### 3.4 Comparing the Regimes

The regimes of linear pricing and bundling are easily compared in the *Uniform Example*. First, notice that as  $z$  tends to  $t$ , so that the proportion of two-stop shoppers vanishes, the tariffs associated with linear pricing (12) and with bundling (17) converge. That implies that profit, consumer surplus and welfare also converge for large  $z$ .

Figure 7 depicts relative profit, welfare and consumer surplus for all  $z \leq t$ . The analytic expression for industry profit with bundling is not illuminating, but can be calculated using

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<sup>15</sup>We have not investigated second-order conditions in general for the tariff in Proposition 5, so cannot be sure that any (pure strategy) equilibria exists. However, we have examined the case of the uniform distribution for  $(x_1, x_2)$ , and verified that this tariff is a global best response for one firm if the rival offers the same tariff.

<sup>16</sup>There is always another, less interesting, *pure bundling* equilibrium. Suppose one firm offers a tariff involving extremely high stand-alone prices. Then no consumer will ever be a two-stop shopper, and so the rival might as well offer a similar tariff. In the case where  $t_1 = t_2 = t$  and  $(x_1, x_2)$  uniformly distributed, one can show that this pure bundling equilibrium involves setting a price for the bundle equal to  $t$  (and setting the stand-alone prices prohibitively high). However, there are good reasons to believe that this second equilibrium is non-robust. For instance, it involves firms playing weakly dominated strategies. Moreover, Thanassoulis (2006) shows that when there are some consumers who wish to buy just one item, this ceases to be an equilibrium.

<sup>17</sup>The model of Matutes and Regibeau (1992) corresponds to the case  $z = 0$ , when  $P_i = \frac{11}{12}t$  and  $\delta = \frac{1}{2}t$ .

expression (5). We depict the bundling profit relative to that with linear pricing (divided by  $t$ ) as the thin line in the figure. Here,  $z$  ranges from zero, where the profit loss with bundling relative to linear pricing is about 30%, to  $t$ , where profit is the same in the two regimes. In particular, bundling acts to destroy profit relative to linear pricing. The thick line depicts the difference in aggregate consumer surplus (divided by  $t$ ) in the two regimes, which is positive but decreases to zero as  $z$  approaches  $t$ .<sup>18</sup> In particular, consumers in aggregate are always better off in the bundling regime. Finally, the dotted line depicts welfare with bundling (15) relative to linear pricing (divided by  $t$ ). Of course, welfare falls when bundling is used. For instance, when  $z = 0$  only an eighth of consumers are two-stop shoppers with bundling, whereas maximal efficiency requires that half the consumers should use two suppliers (as occurs with linear pricing).

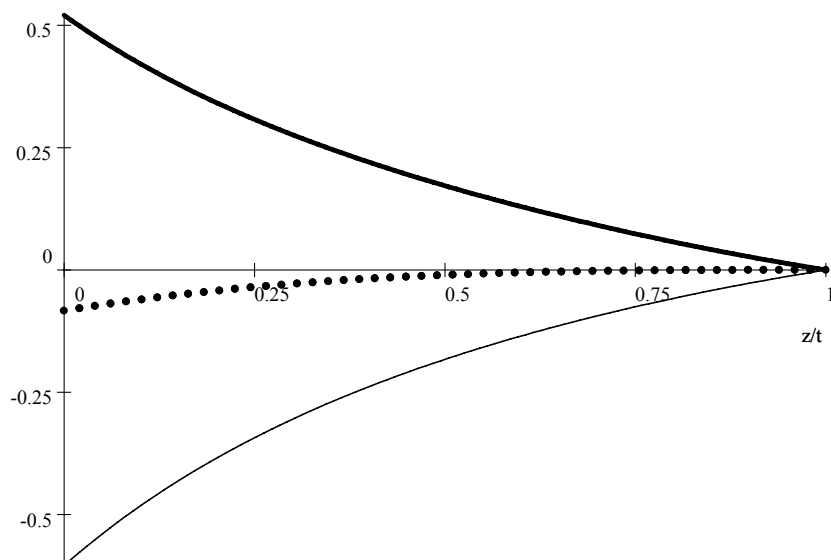


Figure 7: The Effect of  $z$  on Profit, Consumer Surplus and Welfare

In this example, the stand-alone price with bundling in (17) is lower than the corresponding linear price in expression (12) whenever  $z$  is sufficiently small.<sup>19</sup> In such cases, all prices fall when bundling is used, and so all consumers benefit. For larger  $z$ , the two-stop shoppers are worse off in the bundling regime, although (as shown on the figure) overall consumer surplus is always higher with bundling.

<sup>18</sup>Thanassoulis (2006) examines effects of mixed bundling on consumer welfare in a version of this framework where there are some “small” consumers who just want one product. He too finds that with no extra costs of two-stop shopping mixed bundling is better for consumers and worse for firms than stand-alone pricing, but with (prohibitive) shopping costs the opposite can occur as competition for two-product consumers protects one-product consumers if mixed bundling is disallowed.

<sup>19</sup>The stand-alone price with bundling is lower than the equilibrium linear price when  $z < (\sqrt{5} - 2)t$ .

Moving beyond this example, it appears to be widely true that aggregate consumer surplus rises, and profit (and of course welfare) falls, when bundling is used. The next result establishes this in the context of independently distributed product preferences, while a simple form of correlated brand preferences is analyzed subsequently.

**Proposition 6** *Suppose that  $x_1$  and  $x_2$  are independently distributed, with respective density functions  $f_1(x_1)$  and  $f_2(x_2)$  and respective distribution functions  $F_1(x_1)$  and  $F_2(x_2)$ . Suppose the distributions satisfy the hazard rate conditions*

$$\frac{d}{dx} \frac{F_1(x)}{f_1(x)} \geq \frac{1}{4} ; \quad \frac{d}{dx} \frac{F_2(x)}{f_2(x)} \geq \frac{1}{4} \quad (18)$$

(for  $x \leq \frac{1}{2}$ ). Then compared to the outcome with linear pricing, industry profit and welfare fall and aggregate consumer surplus rises when bundling is used.

**Proof.** See appendix. ■

Before discussing this result in more detail, it is worth checking its robustness to the presence of correlation in brand preferences. In many situations, it is plausible that if a consumer prefers firm  $A$ 's product 1 then she is more likely to prefer the same firm's product 2, so that  $x_1$  and  $x_2$  are correlated. That is to say, firm-level brand preference may be an important factor in a consumer's brand preferences over individual products. As is the case with substantial shopping costs, this situation implies that the proportion of two-stop shoppers is smaller than in the uncorrelated case. How does this impact on the incentive to engage in bundling?

In general, the impact of correlation on the bundling discount is not obvious. In rough terms, an increase in correlation will cause there to be fewer consumers in the “north-west” and “south-east” portions of the square, i.e., for the function  $\Phi(\cdot)$  to be shifted downwards. However, without further information expression (13) does not indicate what happens to equilibrium  $\delta$  when  $\Phi$  is shifted downwards.

For instance, consider the following variant of the *Uniform Example*.<sup>20</sup> For simplicity, suppose products are symmetric ( $t_1 = t_2 = t$ ). Suppose that a fraction  $\rho$  of consumers have perfectly correlated preferences, so that  $x_1 = x_2$ , and this common brand preference is uniform on  $[0, 1]$ . The remaining  $1 - \rho$  consumers have locations  $(x_1, x_2)$  uniformly distributed on the unit square.

In this example,  $\alpha_1^0 = \alpha_2^0 = (1 - \rho)(\frac{1}{2} - \frac{z}{2t})$  and  $\beta_1^0 = \beta_2^0 = \frac{1}{2} - (1 - \rho)(\frac{1}{2} - \frac{z}{t})$ .<sup>21</sup> Therefore,

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<sup>20</sup>A similar framework is used in Nalebuff (2004, section III.C). See Reisinger (2006) for further analysis of the effect of correlation in a related framework.

<sup>21</sup>Strictly speaking,  $\beta_i(\delta)$  in (8) is not defined in this example, since there is no (two-dimensional) “density” on the line  $x_2 = x_1$ . However, by examining small changes in  $P_i$  as depicted on Figure 3, one can verify that this value for  $\beta_i^0$  is the appropriate value to use in expression (11).

from (11), the equilibrium linear price for each product is

$$P_1 = P_2 = \frac{t^2}{t + (1 - \rho)z} .$$

Thus, positive correlation acts to relax competition (unless there are no shopping costs, in which case correlation has no impact on linear prices).

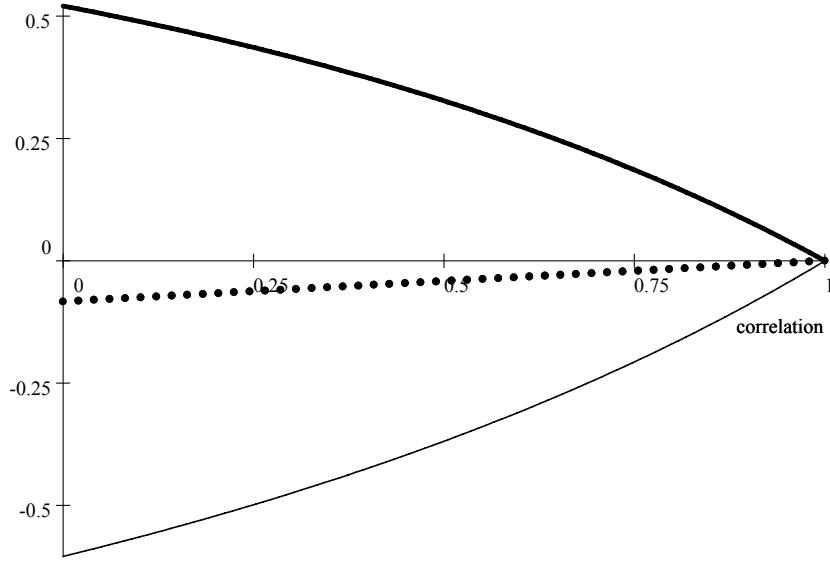


Figure 8: The Effect of  $\rho$  on Profit, Consumer Surplus and Welfare

Turning to the case where bundling is employed,  $\Phi(\delta)$  is here equal to  $2(1 - \rho)(\frac{1}{2} - \frac{\delta+z}{2t})^2$ , and so expression (13) shows that the equilibrium discount does not depend on  $\rho$  and equals  $\delta = \frac{1}{2}(t - z)$ . From (14), the welfare cost of bundling with this discount is just scaled down by the factor  $(1 - \rho)$ , and so from (15) this welfare cost is  $(1 - \rho)\frac{(t-z)^3}{12t^2}$ . With the discount  $\delta = \frac{1}{2}(t - z)$ , we have  $\alpha_1 = \alpha_2 = \frac{1}{4}(1 - \rho)(1 - \frac{z}{t})$  and  $\beta_1 = \beta_2 = \frac{1}{2} + \frac{1}{2}(1 - \rho)\frac{z}{t}$ . Expression (16) then shows that the equilibrium bundling tariff is

$$P_1 = P_2 = \frac{1}{4}(t - z) + \frac{2t^2}{2t + (1 - \rho)(t + z)} ; \delta = \frac{1}{2}(t - z) . \quad (19)$$

Thus, like the linear prices, the equilibrium bundling prices are increasing with correlation. The stand-alone prices with bundling are higher than the equilibrium linear prices whenever correlation is sufficiently strong (e.g., when  $z = 0$ , stand-alone prices rise with bundling whenever  $\rho > \frac{1}{3}$ ). When there is perfect correlation (or if  $z = t$ ), there are no two-stop shoppers in equilibrium, and the outcomes with and without bundling coincide.

Figure 8 represents the relative profit (thin line), welfare (dotted line) and consumer surplus (thick line) associated with bundling versus linear pricing (all divided by  $t$ ), as correlation  $\rho$  varies. (The shopping cost  $z$  is set equal to zero in the figure.) Thus, the profit-destroying effect of bundling is mitigated, though not overturned, when there is positive correlation. By comparing Figures 7 and 8 we see that effects of increased correlation in product brand preferences is qualitatively similar to the effects of increasing the shopping cost.

In this model why do firms do worse, and consumers do better, with nonlinear pricing than with linear pricing? With nonlinear pricing, Propositions 3 and 4 establish that firms will choose to offer discounts to one-stop shoppers. Such discounts can intensify price competition generally so that stand-alone purchases might become cheaper too (or at least they do not rise by enough to overturn the consumer benefits of the discount). When there are discounts for joint purchase, a wider margin of competition for one-stop shoppers opens up—i.e., consumers for whom the operative choice is to buy both products from firm  $A$  or both from  $B$ . Such consumers are “doubly profitable”; they bring two profit margins (less the discount for joint purchase) so their existence often intensifies price competition generally. Thus larger bundling discounts, by creating more doubly profitable consumers, may strengthen incentives to reduce stand-alone prices as well. (The effect is similar to the impact of the shopping cost  $z$ .) Bundling discounts do not necessarily do so, because the discount itself reduces the profit obtained from the doubly profitable consumers. Nonetheless, even when bundling raises the stand-alone prices, the price rise is not (under the conditions of Proposition 6) sufficient to outweigh the consumer benefits of the discount.

Therefore consumers usually do better with nonlinear pricing than with linear pricing, when these effects are lessened. Yet discounts induce the excessive loyalty inefficiency, so depress welfare. It follows that profit is higher with linear pricing.

It is striking that these are exactly the opposite comparative statics to those obtained in the one-stop shopping model presented in Proposition 2. In that model, where demand was elastic, giving greater pricing freedom to firms—in particular the freedom to engage in nonlinear pricing—enhanced equilibrium profit and welfare but was detrimental to consumers. But with consumers able to choose between one- and two-stop shopping, and with inelastic demand, giving firms the freedom to offer discounts for joint purchase lowers profit and welfare but is good for consumers. We will now analyze a model general enough to encompass and reconcile these contrasting findings.

## 4 Bundling and Elastic Demand

In this section we extend the unit-demand bundling model to allow consumers to have elastic and heterogeneous demands. As well as being of interest for its own sake, the more general model will enable us to explain the sharp contrast between the results from the one-stop shopping model (section 2) and the unit-demand bundling model (section 3).

We make the innocuous assumption that consumers buy all their supplies of a given product from one firm or the other. In general, firm  $i$ 's tariff consists of three options:  $T_1^i(q_1)$  is the charge for  $q_1$  units of product 1 if the consumer does not buy any product 2 from the firm;  $T_2^i(q_2)$  is the corresponding tariff if the consumer only buys product 2 from the firm, and  $T_{12}^i(q_1, q_2)$  is the tariff if the consumer buys all her supplies from the firm.

Suppose a type- $\theta$  consumer has gross utility  $u(\theta, q_1, q_2)$  if quantity  $q_i$  of product  $i$  is consumed. If the type- $(\theta, x_1, x_2)$  consumer buys quantities  $(q_1, q_2)$  from firm  $A$ , her net utility is  $u(\theta, q_1, q_2) - T_{12}^A(q_1, q_2) - t_1x_1 - t_2x_2$ . If she buys quantities  $(q_1, q_2)$  from firm  $B$ , her net utility is  $u(\theta, q_1, q_2) - T_{12}^B(q_1, q_2) - t_1(1 - x_1) - t_2(1 - x_2)$ . And if she buys quantity  $q_i$  of product  $i$  from  $A$  and quantity  $q_j$  of product  $j$  from  $B$ , her utility is  $u(\theta, q_1, q_2) - T_i^A(q_i) - T_j^B(q_j) - t_ix_i - t_j(1 - x_j) - z$ . The consumer will choose quantities and suppliers to maximize this utility. As in section 2, suppose that  $\theta$  is distributed independently from the brand preference parameters  $(x_1, x_2)$ . Suppose that each firm incurs a marginal cost  $c_i$  for serving a consumer with a unit of product  $i$ . Finally, suppose that parameters are such that all consumers wish to buy some of each product.

In this model two kinds of inefficiency can arise. First, marginal prices may diverge from marginal costs, and as a result there will be a sub-optimal amount of each product being consumed. This welfare effect was seen with linear pricing in the one-stop shopping analysis of section 2, but not in the basic bundling model of section 3 where the unit demand framework meant that high prices had no welfare impact. This is the excessive marginal price effect. Second, tariffs may encourage excessive one-stop shopping, and consumers may be induced to one-stop shop more often than is socially efficient. This inefficiency was seen in the basic bundling model, but did not arise in the one-stop shopping framework. This is the excessive loyalty effect. In the unified model presented in this section, both inefficiencies can be present. We will see that with linear pricing, inefficiencies arise from excessive marginal prices but not from excessive loyalty, whereas with nonlinear pricing the reverse is true.

## 4.1 Linear Pricing

Write  $v(\theta, p_1, p_2) = \max_{q_1, q_2} u(\theta, q_1, q_2) - p_1q_1 - p_2q_2$  for the consumer surplus of the type- $\theta$  consumer when linear prices are  $p_1$  and  $p_2$ . If both firms set the same linear prices, the pattern of consumer demand is exactly as depicted on Figure 3. (The  $\theta$  parameter has no impact on a consumer's choice of firm when firms offer the same tariff.) In particular, in a symmetric equilibrium there will be no inefficiency due to excessive loyalty, although there will be inefficiency due to excessive marginal prices.

If firm  $A$  undercuts  $B$ 's product 1 price by  $\varepsilon$ , this shifts the boundary of the set of type- $\theta$  consumers who buy the product from  $A$  uniformly to the right by  $\frac{\varepsilon}{2t_1}q_1(\theta, p_1, p_2)$ . Each of the marginal consumers on the vertical boundaries  $\alpha_1^0$  brings extra profit  $(p_1 - c_1)q_1(\theta, p_1, p_2)$ , while each "doubly profitable" consumer on the diagonal boundary  $\beta_1^0$  brings profit  $(p_1 - c_1)q_1(\theta, p_1, p_2) + (p_2 - c_2)q_2(\theta, p_1, p_2)$ . Set against this is the impact of the price cut on the profit from infra-marginal consumers. Each of  $A$ 's existing consumers of product



1 contribute product 1 profit which is changed by  $\varepsilon[q_1(\theta, p_1, p_2) + (p_1 - c_1)\frac{\partial}{\partial p_1}q_1(\theta, p_1, p_2)]$ , and in a symmetric equilibrium the proportion of such consumers equals a half. Finally, there is the effect of changing  $p_1$  on demand for the firm's product 2. This is only relevant for the firm's one-stop shoppers, who are  $N$  in number in equilibrium. Each of these one-stop shoppers generates product 2 profit which is changed by  $\varepsilon(p_2 - c_2)\frac{\partial}{\partial p_1}q_2(\theta, p_1, p_2)$ .

Putting all this together and taking expectations over  $\theta$  implies that the first-order condition for  $p_1$  to be the equilibrium price for product 1 is this generalization of (9):

$$\begin{aligned} E_\theta \left[ (2\alpha_1^0(p_1 - c_1)q_1 + \beta_1^0((p_1 - c_1)q_1 + (p_2 - c_2)q_2)) q_1 \right] \\ = t_1 \times E_\theta \left[ q_1 + (p_1 - c_1)\frac{\partial q_1}{\partial p_1} + 2N(p_2 - c_2)\frac{\partial q_2}{\partial p_1} \right]. \end{aligned} \quad (20)$$

(Here, the dependence of demands  $q_i$  on  $p_1, p_2$  and  $\theta$  has been suppressed.) A similar expression holds for product 2.

Formula (20) is complex, and reflects the effects of own and cross-price elasticities, consumer heterogeneity (via the quadratic terms  $q_1^2$  and  $q_1q_2$ ), the extent of product differentiation, the shopping cost, and correlation in product brand preferences (via the size of  $N$ ). An extension to the *Uniform Example* illustrates some of the main effects.

**Linear Uniform Example:**  $t_i = t$ ;  $f(x_1, x_2) \equiv 1$ ;  $q_i(\theta_1, \theta_2, p_1, p_2) = \theta_i(1 - bp_i)$ ;  $c_i = 0$ .

Thus, demand functions are linear and exhibit no cross-price effects, and consumer heterogeneity is represented by an idiosyncratic multiplicative term  $\theta_i$ . Here, suppose that each  $\theta_i$  has mean 1 and variance  $\sigma^2$ , and let the covariance of  $\theta_1$  and  $\theta_2$  be  $\kappa\sigma^2$  for  $-1 \leq \kappa \leq 1$ . It is quite natural to suppose that there is positive correlation in the scale of demands for the two products across consumers (i.e.,  $\kappa > 0$ ), since it is likely that a consumer's income will be positively correlated with her demand for each product. The parameter  $b$  represents the sensitivity of demand to marginal price. With a linear price  $p$  for each product, industry profit is  $2p(1 - bp)$  and welfare relative to the first best is  $-bp^2$ .

As in section 3.2,  $\alpha_i^0 = \frac{1}{2} - \frac{z}{2t}$  and  $\beta_i^0 = \frac{z}{t}$ , and so (20) implies that the equilibrium linear price for a unit of either product,  $p$ , satisfies

$$\frac{p(1 - bp)^2}{1 - 2bp} = \frac{t^2}{t + z + \sigma^2[t + \kappa z]}, \quad (21)$$

which generalizes (12). This equilibrium price increases with  $t$  and falls with  $z, b, \sigma^2$  and  $\kappa$ .<sup>22</sup> The impact of these comparative statics, together with their intuition, will be explored in section 5.

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<sup>22</sup>The left-hand side of (21) is increasing in  $p$  and  $b$  over the relevant range  $0 \leq p \leq 1/(2b)$ . The right-hand side is increasing in  $t$  and decreasing in  $z, \sigma^2$  and  $\kappa$ .

## 4.2 Nonlinear Pricing

In this section we establish that it is an equilibrium for firms to offer tariffs with marginal prices equal to marginal costs, and with fixed charges corresponding to the bundling prices derived in section 3.<sup>23</sup>

**Proposition 7** *Suppose that over the relevant range of tariffs all consumers wish to purchase both products. Then it is an equilibrium for each firm to offer the following tariff:*

$$T_1(q_1) = P_1 + c_1 q_1 ; T_2(q_2) = P_2 + c_2 q_2 ; T_{12}(q_1, q_2) = T_1(q_1) + T_2(q_2) - \delta , \quad (22)$$

where  $P_1, P_2$  and  $\delta$  comprise the mixed bundling tariff described in Proposition 5.

**Proof.** See appendix. ■

Despite the generality of the demand structure and consumer heterogeneity, this equilibrium is remarkably simple. As with Proposition 1 the shape and heterogeneity of consumer demand functions have no impact on either industry profit or on welfare relative to the first best, so long as all consumers participate. (Welfare continues to be determined by expression (14).) In particular, and perhaps surprisingly, whether or not the two products are complements or substitutes in consumer demand makes no difference to the equilibrium incentive to offer bundling discounts. In this equilibrium there is efficient marginal-cost pricing, but there is inefficiency due to excessive loyalty. In this more general model, the impact of nonlinear pricing on profit, consumer surplus and welfare is ambiguous. In the next section, we discuss how the various aspects of consumer preferences determine the net impact of nonlinear pricing.

## 5 Comparing the Regimes

In this section we compare linear pricing and fully flexible nonlinear tariffs.<sup>24</sup> At the end of section 3 we noted the contrast between the model of one-stop shopping (with elastic demand) and the model with consumer choice between one- and two-stop shopping (with inelastic demand) in respect of the consequences for profit, welfare and consumer surplus of allowing

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<sup>23</sup>Notice that, unlike Proposition 1, we have not been able to show that this is the *unique* symmetric equilibrium. One reason is the existence of at least one other, pure bundling equilibrium (see footnote 16).

<sup>24</sup>One could also consider an intermediate regime in which firms offer quantity discounts for a particular product, but cannot offer bundling discounts across products. In welfare (and profit) terms, this pricing regime delivers the best of both worlds, since the excessive loyalty effect and the excessive marginal price effect are both avoided. However, it would be hard for public policy to enforce this intermediate regime, since in practice it is not always clear what constitutes a distinct product. (For instance, should a season ticket to a concert series count as an inter-product or intra-product discount?) For this reason, we focus on the transparent distinction between linear pricing and fully flexible nonlinear tariffs.

firms freedom to depart from linear pricing. The analysis of the unified model of section 4 now allows us to reconcile these findings and to undertake a more general comparison of linear and nonlinear pricing, and to show the importance of five kinds of economic effect: (i) demand elasticity, (ii) product differentiation, (iii) consumer heterogeneity, (iv) shopping costs, and (v) correlation in brand preferences. The impact of these effects was summarised in Table 1 in the introduction.

## 5.1 The Effect of Demand Elasticity

When nonlinear pricing is used, the shape of the demand functions has no effect on profit or welfare (see Proposition 7). With linear pricing, on the other hand, it is plausible that profit and welfare are lower when demand is elastic. For instance, in section 2 we suggested that the “elasticity effect” was one reason why linear prices yielded lower profits than nonlinear prices. As far as the welfare comparison is concerned, linear pricing has the advantage that there is no excessive loyalty inefficiency, but there is the inefficiency due to excessive marginal prices. This may be expected to become more prominent as demand becomes more elastic, because for a given price the welfare loss is larger if demand is more elastic. On the other hand, greater demand elasticity lowers the equilibrium price. However, when consumer demand is sufficiently inelastic, the model is approximated by the unit demand model of section 3, in which case there is no excessive pricing inefficiency and nonlinear pricing harms profit and welfare, but benefits consumers, relative to the case of linear pricing.

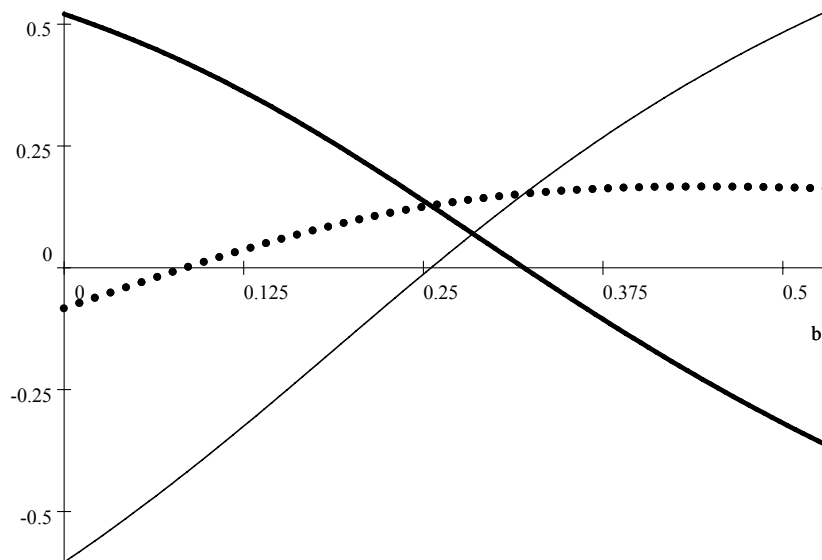


Figure 9: The Effect of Elasticity on Profit, Consumer Surplus and Welfare

To explore these elasticity effects further consider the *Linear Uniform Example*, where the linear price is given by expression (21). With nonlinear pricing, Proposition 7 shows that profit does not depend on the elasticity parameter  $b$ . However, in this example at least, profit with linear pricing is decreasing in  $b$ . When demand is sufficiently sensitive to price, profit with nonlinear pricing exceeds that with linear pricing. Thus, in the unified model of section 4, the impact of nonlinear pricing on profit is ambiguous.

Figure 9 plots (as the thin solid line) the difference between the profit with nonlinear pricing and with linear pricing as a function of the elasticity term  $b$  (here  $t = 1$ ,  $z = 0$  and  $\sigma^2 = 0$ ). The welfare difference between nonlinear and linear pricing is plotted as the dotted line on the figure, while the difference in consumer surplus is the thick solid line. When  $b = 0$  we return to the unit demand setting, where profit and welfare are reduced with bundling, while consumers benefit. With sufficiently elastic demand, the reverse holds. In this example, welfare is improved by nonlinear pricing even for relatively inelastic demand. Finally, it is worth noting that for intermediate elasticities, both profit and consumer surplus rise with nonlinear pricing. Thus, firms and consumers do not inevitably have opposing interests when it comes to price discrimination.

## 5.2 The Effect of Product Differentiation

Generally, when  $(t_1, t_2)$  is small, all prices converge to marginal costs, whether linear or nonlinear pricing is employed, and so the two regimes yield approximately the same profit, welfare and consumer surplus.

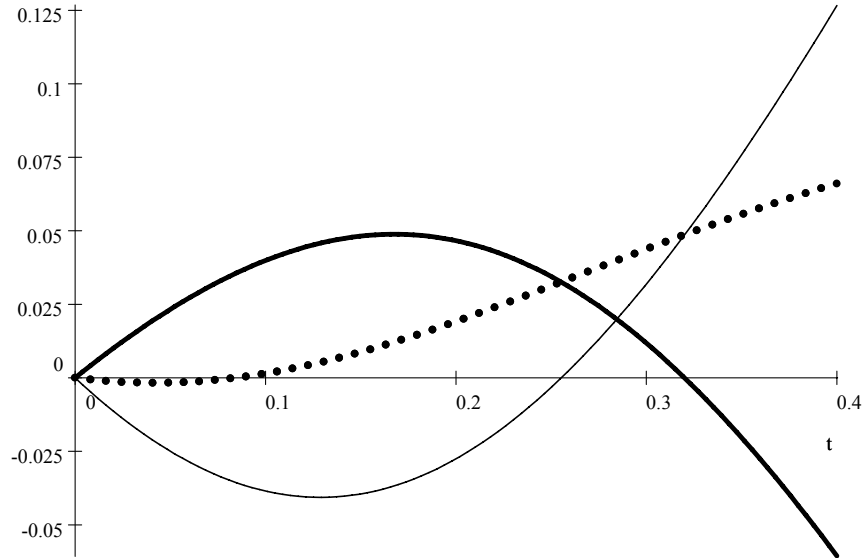


Figure 10: The Effect of Product Differentiation on Profit, Consumer Surplus and Welfare

For instance, consider the *Linear Uniform Example*. Figure 10 shows relative profit (the thin line), consumer surplus (the thick line) and welfare (the dotted line) associated with nonlinear versus linear pricing as  $t$  varies. (Here,  $b = 1$ ,  $z = 0$  and  $\sigma^2 = 0$ .) In this example welfare is—very slightly—reduced by nonlinear pricing when the products offered by the two firms are close substitutes. This is true quite generally. The reason is that, when  $z = 0$ , the excessive loyalty welfare loss is proportional to  $t$ ,<sup>25</sup> while the welfare loss due to excessive marginal prices is of order  $(t^2)$ .<sup>26</sup> Thus, for small  $t$ , the welfare losses due to excessive loyalty will dominate the losses due to excessive pricing, and so nonlinear pricing lowers welfare.<sup>27</sup> (The analysis in the next section can show that the separate impact of nonlinear pricing on consumers and profit is ambiguous, even when  $t$  is small.) However, Figure 10 indicates that the welfare loss may be rather small. Moreover, this discussion has assumed that exogenous shopping costs are zero. Once  $z > 0$ , whenever  $t$  becomes sufficiently small, consumers will become one-stop shoppers regardless of whether nonlinear or linear pricing is used. In this case there is no *extra* welfare cost involved in bundling, and the excessive marginal pricing problem dominates. So nonlinear pricing is then sure to yield higher welfare (as was demonstrated in section 2).

### 5.3 The Effect of Consumer Heterogeneity

The discussion in section 2 mentioned the heterogeneity effect, where the presence of consumers with different demands acted to intensify linear price competition and to reduce the profit obtained with linear pricing. In this section we explore this in more detail. The reason why consumer heterogeneity acts to depress the equilibrium linear prices can be seen from expression (20). Speaking loosely, a “mean-preserving spread” of the demand functions has no impact on the right-hand side of (20), but it raises the left-hand side via the quadratic terms. The impact of this is similar to a reduction in the product differentiation parameter  $t_1$ , which will typically causes prices to fall. The economic intuition is that with heterogeneity a price cut attracts proportionally more high demand (hence high profit) consumers from the rival firm—so improves, at the margin, the *mix* of consumers (an effect absent with homogeneity). Since prices are then closer to marginal costs, this acts to boost the welfare

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<sup>25</sup>When  $z = 0$ , the discount  $\delta$  in (13) and the prices  $P_i$  in (16) are homogeneous degree one in  $(t_1, t_2)$ . The pattern of demand illustrated in Figure 5 is unaffected if  $(t_1, t_2)$  are scaled up proportionally, so long as all consumers continue to buy both products, and so the equilibrium fraction of one-stop shoppers is homogeneous degree zero in  $(t_1, t_2)$ . The equilibrium welfare difference  $w$  is also homogeneous degree one in  $(t_1, t_2)$ . In particular, when  $t_1 = t_2 = t$ , the welfare loss is proportional to  $t$ .

<sup>26</sup>With linear pricing price-cost margins are roughly proportional to  $t$  for small  $t$ , which implies that the welfare loss from excessive marginal prices is of order  $t^2$ .

<sup>27</sup>The result contrasts with Armstrong and Vickers (2001, Proposition 3), where we argued that price discrimination led to welfare gains in markets where firms offered closely substitutable products. In the earlier paper, we assumed a one-stop shopping framework, so the first-order effect of excessive loyalty caused by price discrimination was absent. As a result, the comparison focussed on the second-order effects associated with excessive pricing (which indeed favour price discrimination).

and consumer surplus associated with linear pricing, but to depress profits. We deduce that welfare and consumer surplus is more likely to be higher with linear pricing when there is substantial consumer heterogeneity, while profit is then more likely to be higher with nonlinear pricing. However, the reason is not that heterogeneity boosts the profits from engaging in price discrimination (as one might perhaps have expected), but rather that it harms profit when linear prices are used.

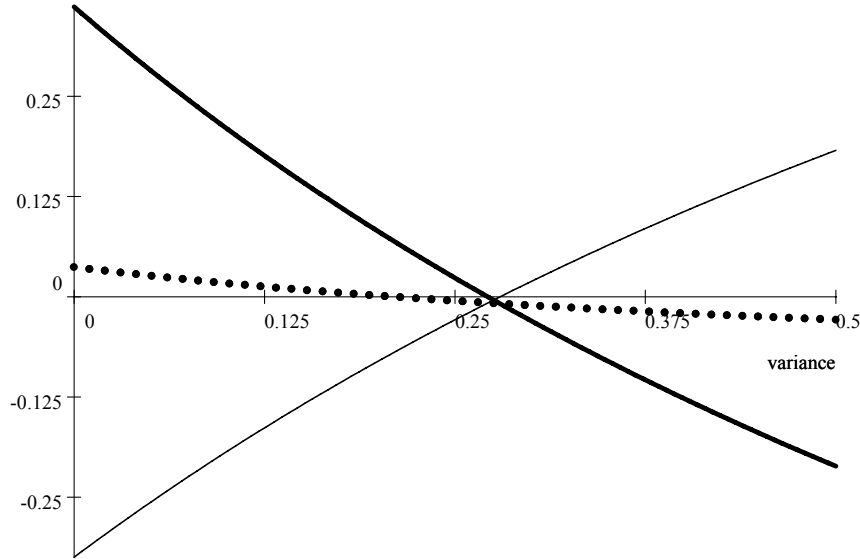


Figure 11: The Effect of Heterogeneity on Profit, Consumer Surplus and Welfare

The effect is illustrated by the *Linear Uniform Example*. Expression (21) shows that the equilibrium linear price for each product is decreasing in  $\sigma^2$ , which confirms the intuition that heterogeneity in consumer demand pushes down linear prices. In addition, (21) shows that the price is decreasing in the correlation in demands for the two products,  $\kappa$ , at least when there is a positive shopping cost  $z$ . The intuition for this is as follows. When  $z > 0$ , there is a set of consumers for whom the relevant margin is whether to buy both products from either firm  $A$  or firm  $B$ . When there is correlation in the scale of demand for the two products, the variance of the total profit from both products rises, and this intensifies competition for these one-stop shoppers yet further.

By contrast, when firms use nonlinear tariffs, Proposition 7 shows that profit, welfare and consumer surplus do not depend on the variance or correlation in the scale of consumer demand. Thus, the relative profitability of using nonlinear pricing increases with  $\sigma^2$  and  $\kappa$ . The reductions in linear prices caused by increased heterogeneity result in relative welfare with nonlinear pricing decreasing with  $\sigma^2$  and  $\kappa$ . Consumers are better off with nonlinear pricing when  $\sigma^2$  is relatively small, but when their demands are more varied consumers (in aggregate) prefer linear pricing. This is illustrated in Figure 11, which shows relative profit

(the thin line), consumer surplus (the thick line) and welfare (the dotted line) associated with nonlinear versus linear pricing as  $\sigma^2$  varies. (Here,  $b = \frac{1}{8}$ ,  $t = 1$ ,  $z = 0$  and  $\kappa = 0$ .)

## 5.4 The Effect of the Shopping Cost

When the shopping cost  $z$  becomes large, the equilibrium bundling discount  $\delta$  becomes small and the excessive loyalty welfare loss becomes small—see Figure 7 above—and the only relevant factor for welfare is the excessive marginal price problem associated with linear pricing. We can deduce that for large  $z$ , the welfare effect of nonlinear pricing is always positive. (Of course, the welfare effect of nonlinear pricing may be positive for small  $z$  too, depending on demand elasticities and the other factors we have discussed.) Similarly, for large  $z$  the impact of nonlinear pricing on profit is surely positive, while the impact on consumer surplus is ultimately negative. In sum, as  $z$  becomes large, the one-stop shopping analysis in section 2 applies.

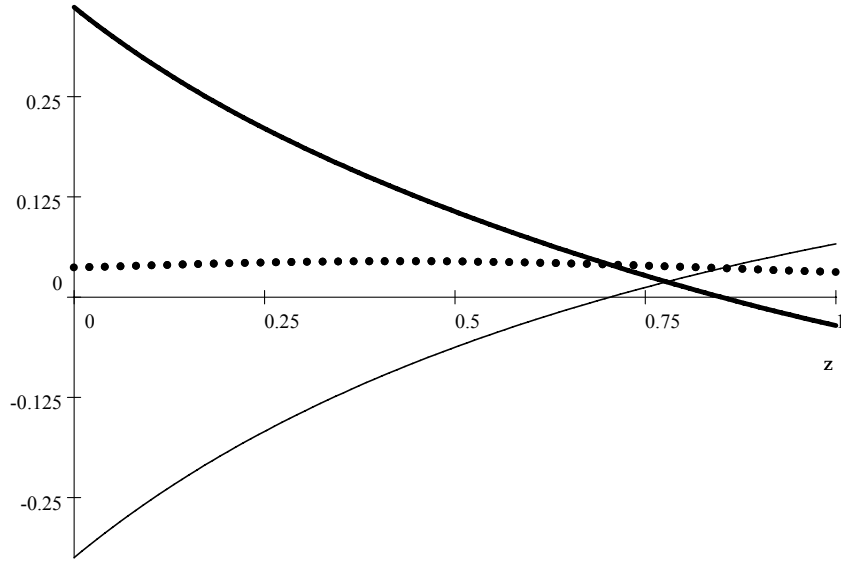


Figure 12: The Effect of Shopping Costs on Profit, Consumer Surplus and Welfare

This discussion can be illustrated with the *Linear Uniform Example*. Figure 12 shows relative profit (the thin line), consumer surplus (the thick line) and welfare (the almost flat dotted line) associated with nonlinear versus linear pricing as  $z$  varies. (Here,  $b = \frac{1}{8}$ ,  $t = 1$  and  $\sigma^2 = 0$ .)

## 5.5 The Effect of Correlation in Brand Preferences

Section 5.3 argued that correlation in the scale of demands for the two products,  $\kappa$ , tends to intensify linear pricing competition (while leaving the outcome with nonlinear pricing unchanged). The impact of this form of correlation is to increase effective consumer heterogeneity, and this decreases welfare and consumer surplus associated with nonlinear pricing relative to linear pricing. In this section we consider an alternative form of correlation—that involving the brand preferences for the two products—which has distinct economic effects.

The impact of high correlation in consumer brand preferences is qualitatively similar to that of large shopping costs. When this form of correlation is strong, the fraction of consumers who might be two-stop shoppers is small, and the effect of nonlinear pricing mirrors the analysis of one-stop shopping in section 2: profits and welfare rise, while consumers are harmed. This can be illustrated using the *Linear Uniform Example* modified to allow for a fraction  $\rho$  of consumers to have perfectly correlated brand preferences (as suggested in section 3.4). Figure 13 shows relative profit (the thin line), consumer surplus (the thick line) and welfare (the dotted line) associated with nonlinear versus linear pricing as  $\rho$  varies. (Here,  $b = \frac{1}{8}$ ,  $t = 1$ ,  $z = 0$  and  $\sigma^2 = 0$ .)

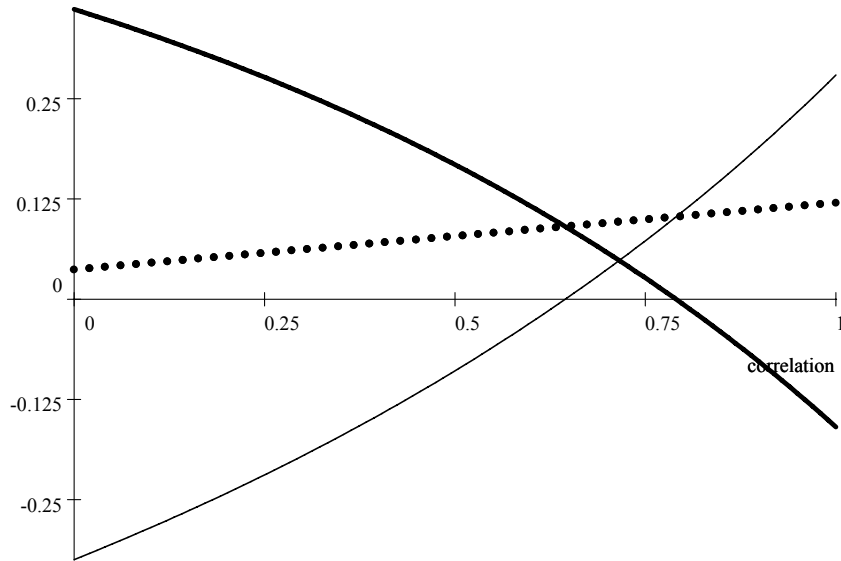


Figure 13: The Effect of Brand Preference Correlation

## 6 Conclusions

Our analysis of competitive nonlinear pricing and bundling—and its effects on consumer surplus, profit and welfare—has proceeded from the special models of sections 2 and 3 to



the considerably broader framework of section 4, which allowed for consumer heterogeneity, elastic demands, consumer choice between one- and two-stop shopping, and shopping costs. Yet the broader model was shown to have simple and economically intuitive equilibria when firms offered nonlinear tariffs. The model illuminated the importance of keen competition for one-stop shoppers when bundling is feasible, and the inefficiencies arising from excessive marginal prices (with linear pricing) and excessive loyalty (with nonlinear pricing). We identified five economic influences on these sources of inefficiency—and hence on the pros and cons of nonlinear pricing relative to linear pricing—which were summarised in the introduction.

The economic effects discussed in this paper may operate more widely, along with other influences no doubt. However, though general in some respects, our framework has been confined to static competition between symmetric two-product duopolists in a setting where all consumers buy some of each product. Natural next steps would be to relax these restrictions.

For example, allowing for free entry instead of duopoly would open up the issue of the effect of nonlinear pricing on the equilibrium number of firms.<sup>28</sup> In the unit demand bundling model, nonlinear pricing acts to depress profit and welfare. With free entry, this implies that the equilibrium number of firms will fall, and this could act to mitigate possible “excess entry”. Bundling might then have a positive impact on welfare, despite the excessive loyalty problem.

Another extension would be to make the model dynamic, and to allow for “customer poaching”. Existing models of customer poaching, where a firm sets its current price on the basis of whether a buyer is a previous or a new customer of the firm, assume unit demands in each period.<sup>29</sup> In the basic version of these models, firms do not commit to their future prices, and so price low to their rival’s existing customer base. Like the (static) bundling analysis in this paper, customer poaching tends to benefit consumers and to harm firms and welfare relative to linear pricing. However, the welfare problem is quite different: since firms offer low prices to existing customers of their rival (rather than discounts to their own loyal customers as in the bundling framework), customer poaching models involve socially *insufficient* rather than excessive loyalty. It would be interesting to see whether the extension of those models to elastic demand means that nonlinear pricing can benefit firms and welfare, just as we have shown it can benefit firms and welfare in static bundling models.

These are but two lines of possible further analysis. There is much more to be understood about the economics of competitive nonlinear pricing and bundling.

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<sup>28</sup>See Stole (1995) for early work in this direction in the one-stop shopping context.

<sup>29</sup>See Chen (1997) and Fudenberg and Tirole (2000).

## APPENDIX

**Proof of Proposition 6:** From (16), for any  $\delta$  the sum of the stand-alone prices is

$$P_1 + P_2 = \delta + \frac{t_1\alpha_2 + t_2\alpha_1}{\alpha_1\beta_2 + \alpha_2\beta_1 + 2\alpha_1\alpha_2} ,$$

or

$$P_1 + P_2 = \delta + \frac{t_1}{\hat{\alpha} + \beta_1}$$

where

$$\hat{\alpha} = \frac{2\alpha_1\alpha_2}{\frac{t_2}{t_1}\alpha_1 + \alpha_2} .$$

Therefore, for a particular  $\delta$  industry profit is

$$\begin{aligned} \pi(\delta) &= \delta + \frac{t_1}{\hat{\alpha} + \beta_1} - \delta \times \{\text{proportion of consumers who one-stop shop}\} \\ &= \delta + \frac{t_1}{\hat{\alpha} + \beta_1} - \delta(1 - \Phi(\delta)) , \end{aligned}$$

where  $\Phi$  is defined in (13). Therefore,

$$\pi(\delta) = \delta\Phi(\delta) + \frac{t_1}{\hat{\alpha} + \beta_1} . \quad (23)$$

From (23), aggregate consumer surplus relative to the case of linear pricing, denoted  $v(\delta)$ , satisfies

$$v(\delta) = w(\delta) - \left[ \delta\Phi(\delta) + \frac{t_1}{\hat{\alpha} + \beta_1} \right]$$

and so

$$v'(\delta) = -\Phi(\delta) - \frac{d}{d\delta} \left[ \frac{t_1}{\hat{\alpha} + \beta_1} \right] . \quad (24)$$

(Here,  $w(\delta)$  is total welfare with discount  $\delta$  which satisfies expression (14).) Therefore, a sufficient condition for consumer surplus to rise with bundling is that expression (24) be positive. (Consumer surplus with linear pricing corresponds to  $\delta = 0$ .)

To make progress, specialize to the case of independence, so that  $f(x_1, x_2) \equiv f_1(x_1)f_2(x_2)$ . Write  $F_i(\cdot)$  for the distribution function corresponding to  $f_i(\cdot)$  and write

$$\eta_i(\delta) \equiv \frac{f_i(\frac{1}{2} - \frac{\delta+z}{2t_i})}{F_i(\frac{1}{2} - \frac{\delta+z}{2t_i})} .$$

Then one can show

$$\frac{d}{d\delta}(\hat{\alpha} + \beta_1) = 2t_1F_1F_2 \frac{t_1\eta_2^2\eta_1' + t_2\eta_1^2\eta_2'}{(t_1\eta_2 + t_2\eta_1)^2} = 2t_1F_1F_2\phi' , \quad (25)$$

where

$$\phi(\delta) \equiv \frac{\eta_1(\delta)\eta_2(\delta)}{t_1\eta_2(\delta) + t_2\eta_1(\delta)}$$

and  $F_i = F_i(\frac{1}{2} - \frac{\delta+z}{2t_i})$ . Then (24) implies that

$$v'(\delta) = -2F_1F_2 + 2t_1^2 \frac{F_1F_2\phi'}{(\hat{\alpha} + \beta_1)^2} . \quad (26)$$

Assumption (18) implies that  $\eta'_i(\delta) \geq 0$ , which in turn implies  $\phi'(\delta) \geq 0$ . Notice that (25) implies

$$\frac{d}{d\delta}(\hat{\alpha} + \beta_1) \leq \frac{1}{2}t_1\phi' .$$

And since  $\hat{\alpha} + \beta_1 = \frac{1}{2}t_1\phi$  when  $\delta = -z$ , we deduce that

$$\hat{\alpha} + \beta_1 \leq \frac{1}{2}t_1\phi .$$

From (26) it follows that  $v(\delta)$  is increasing if

$$\frac{\phi'}{\phi^2} = \frac{t_1\eta'_1}{\eta_1^2} + \frac{t_2\eta'_2}{\eta_2^2} \geq \frac{1}{4} .$$

A sufficient condition for this inequality to hold is the (relatively mild) hazard rate condition (18). In sum, if condition (18) holds, consumer surplus necessarily rises when bundling is used.

Since welfare  $w(\delta)$  is decreasing in  $\delta$ , it follows immediately that industry profit (which equals  $w(\delta) - v(\delta)$ ) is decreasing in  $\delta$  under the same conditions. This proves Proposition 6.

**Proof of Proposition 7:** Suppose that firm  $B$  offers the menu of two-part tariffs  $T_1(\cdot)$ ,  $T_2(\cdot)$  and  $T_{12}(\cdot, \cdot)$  described in (22). We establish that firm  $A$ 's best response is to use the same tariff by means of the following argument.<sup>30</sup>

Suppose firm  $A$  can directly observe a consumer's parameter  $\theta$  (but not  $x_1$  or  $x_2$ ). We will calculate firm  $A$ 's best response to  $B$ 's tariff, given  $\theta$ . Suppose firm  $A$ 's tariff is  $\{T_1^A(q_1), T_2^A(q_2), T_{12}^A(q_1, q_2)\}$ . Then let

$$\begin{aligned} U_1 &= \max_{q_1, q_2} : u(\theta, q_1, q_2) - T_1^A(q_1) - T_2(q_2) \\ U_2 &= \max_{q_1, q_2} : u(\theta, q_1, q_2) - T_1(q_1) - T_2^A(q_2) \\ U_{12} &= \max_{q_1, q_2} : u(\theta, q_1, q_2) - T_{12}^A(q_1, q_2) \end{aligned}$$

be the type- $\theta$  consumer's gross utility (i.e., the utility excluding travel and shopping costs) when she buys only product 1 from  $A$ , only product 2 from  $A$  or buys all supplies from  $A$ ,

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<sup>30</sup> A similar argument was used to prove Armstrong and Vickers (2001, Proposition 5).

respectively. Next, consider firm  $A$ 's most profitable way to generate the utilities  $U_1$ ,  $U_2$  and  $U_{12}$ . The most profitable way to generate utility  $U_1$  is the solution to the problem

$$\max_{T_1, q_1} : T_1 - c_1 q_1 \text{ subject to } U_1 = \max_{q_2} u(\theta, q_1, q_2) - T_1 - T_2(q_2)$$

which is the same problem as

$$\max_{q_1, q_2} : u(\theta, q_1, q_2) - c_1 q_1 - T_2(q_2) - U_1 .$$

Clearly, the solution to this problem involves marginal-cost pricing for product 1. That is to say, given  $\theta$  it is a dominant strategy for firm  $A$  to choose a tariff of the form  $T_1^A(q_1) = P_1^A(\theta) + c_1 q_1$  for some fixed charge  $P_1^A(\theta)$ . (We write the fixed charge as a function of  $\theta$ , since in general it will depend on  $\theta$ .) A similar argument serves to show that  $T_2^A(q_2) = P_2^A(\theta) + c_2 q_2$  and  $T_{12}^A(q_1, q_2) = P_1^A(\theta) + P_2^A(\theta) - \delta^A(\theta) + c_1 q_1 + c_2 q_2$  for some choice of  $P_2^A(\theta)$  and  $\delta^A(\theta)$ .

Since both firms are setting marginal prices equal to marginal costs, the net utility of the consumer is

$$v^*(\theta) - [P_1 + P_2 - \delta + t_1(1 - x_1) + t_2(1 - x_2)]$$

if both products are purchased from  $B$ ,

$$v^*(\theta) - [P_1^A + P_2^A - \delta^A + t_1 x_1 + t_2 x_2]$$

if both products are purchased from  $A$ , and

$$v^*(\theta) - [P_i^A + P_j + t_i x_i + t_j(1 - x_j) + z]$$

if product  $i$  is purchased from  $A$  and product  $j$  is purchased from  $B$ . Here,  $v^*(\theta) = \max_{q_1, q_2} u(\theta, q_1, q_2) - c_1 q_1 - c_2 q_2$  is consumer surplus with marginal-cost pricing (excluding transport and shopping costs). In particular, the consumer's decision over where to buy depends only on her total outlay (the terms in square brackets above), and the benefit from consumption,  $v^*(\theta)$ , does not depend on her choice of suppliers. Therefore, given  $\theta$ , firm  $A$  will choose the fixed charges  $P_1^A$ ,  $P_2^A$  and the bundling discount  $\delta^A$  in order to maximize its profit, given  $B$ 's tariff (22). But this is precisely the problem analyzed in section 3, where Proposition 5 established that the optimal response to the bundling tariff  $(P_1, P_2, \delta)$  in (16) is to use the same bundling tariff.

In sum, we have shown that (i) if firm  $B$  sets the tariff (22) and (ii) if firm  $A$  can observe a consumer's type  $\theta$ , then the tariff (22) maximizes  $A$ 's profit. However, since the tariff (22) does not depend on  $\theta$ , this tariff must also be the firm's best response in the more constrained problem in which  $A$  cannot observe  $\theta$ , which proves the result.

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