

In memoriam: Marco Avellaneda (1955–2022)

Rama Cont 

Mathematical Institute, University of
Oxford, Oxford, UK

Correspondence

Rama Cont, Mathematical Institute,
University of Oxford, Oxford, UK.
Email: Rama.Cont@maths.ox.ac.uk

Abstract

Marco Avellaneda (1955–2022) was a leading figure in the development of mathematical modeling in finance and its dissemination among market practitioners. We provide a sketch of his trajectory and outline some of his main research contributions to mathematical finance.

1 | EARLY YEARS

Marco Marcelo Avellaneda was born on February 16, 1955 in Miramar, Argentina, to an illustrious Argentinian family. His great-grandfather Nicolas Avellaneda was Argentina's youngest president, at the end of the 19th century. 1955 was also the year Perón was ousted in a military coup, and the decade that followed was a turbulent one in Argentinian politics. Shortly afterwards, Marco's family left the chaos of Argentina for Brazil, where he spent a peaceful childhood with his family in the seaside village of Buzios (Tudball, 2013).

A turning point in Marco's life occurred when his parents moved to Paris in 1967 to serve as diplomats at UNESCO. Then a teenager, Marco attended the *Lycée International de Sèvres* from 1968 to 1973. There he discovered a flair for mathematics, a subject which was emphasized in the French secondary school curriculum. Moving back to Buzios in 1973, he maintained an interest in mathematics, and finally ended up enrolling at the University of Buenos Aires in 1977 to study mathematics.

Spotting Marco's mathematical talent, one of his professors at the University of Buenos Aires encouraged him to apply for a Ph.D. in mathematics at the University of Minnesota, where he had some connections. Marco arrived in Minneapolis in 1981 as a Ph.D. student, and graduated in 1985 with a thesis on Brownian motion on manifolds (Avellaneda, 1985), his first foray into the nascent field of stochastic analysis, which would lead him to NYU's Courant Institute.

This is an open access article under the terms of the [Creative Commons Attribution-NonCommercial-NoDerivs](https://creativecommons.org/licenses/by-nc-nd/4.0/) License, which permits use and distribution in any medium, provided the original work is properly cited, the use is non-commercial and no modifications or adaptations are made.

© 2023 The Authors. *Mathematical Finance* published by Wiley Periodicals LLC.

2 | IMMIGRANT MATHEMATICIAN

Following his Ph.D., Marco was hired in 1985 as an Instructor at the Courant Institute of Mathematical Sciences. At Courant, Marco found a group of top probability theorists such as George Papanicolaou and Raghu Varadhan, who had done pioneering work on the theory of *homogenization* in random media (Papanicolaou & Varadhan, 1979; Guo et al., 1988), whereby one derives effective equations for the meso-scale behavior of systems with random microscopic structures by averaging over the latter. Given his background in partial differential equations and stochastic analysis, he embraced the topic and embarked on a decade of work on the mathematics of random media and multiscale physical systems. Together with Fanghua Lin, a fellow Minnesota graduate who had joined Courant the same year, Marco derived several new results on the *quantitative* theory of homogenization and lifted the regularity theory of the homogenized limit to heterogeneous situations (Avellaneda and Lin, 1987a, 1989a, 1989b, 1991, 1989c, 1987b).

One of the motivations of homogenization theory was to model composite materials, which possess heterogeneous polycrystalline or randomly structured arrangements of atoms at different scales. Building on earlier work by Papanicolaou, Marco became interested in the problem of deriving effective equations describing such media through *iterated homogenization* (Avellaneda, 1987a, 1996), a technique which he successfully applied to polycrystals (Avellaneda, 1989) and composite materials (Avellaneda, 1987b, 1991a; Avellaneda et al., 2000).

Marco's work on the use of iterated homogenization to study problems with multiple scales led to extensions and applications to other multiscale problems, such as the analysis of turbulent flows, a topic on which he made important contributions in collaboration with his Courant colleague Andrew Majda (Avellaneda and Majda, 1991a, 1994, 1992c, 1993, 1992a), Weinan E (Avellaneda & E, 1995; Avellaneda et al., 1995), and Reade Ryan (Ryan & Avellaneda, 1999), in particular during his period at the Institute for Advanced Studies in Princeton (1994–95). Mathematical fluid dynamics remained one of his research interests throughout the 1990s, and he worked on this theme with Peter Laurence (Laurence & Avellaneda, 1993a, 1993b, 1991), as well as a certain Alex Lipton (Lipton & Avellaneda, 1990).

It was during this period that Marco met and married his life companion Cassandra Richmond, who would always remain at his side. He settled in New York, became a U.S. citizen, and was proud to present himself as an “immigrant mathematician.”

3 | A MATHEMATICIAN ON WALL STREET

Marco's arrival at Courant coincided with the advent of mathematical models for option pricing: the work of Black, Scholes, and Merton, published in 1973, had gradually made its way into Wall Street and mathematicians were being called to collaborate with industry to further develop these models and implement them. The 1987 stock market crash, partly triggered by “portfolio insurance strategies,” made market participants realize that risk management is not a luxury and further accelerated the development and adoption of mathematical models and computational software in finance. Robert C. Merton published in 1992 his magistral exposition of “continuous-time finance” (Merton, 1992), in which Ito calculus appeared alongside with partial differential equations.

As a mathematician in New York, Marco was curious about these developments and keen to explore the mathematics underlying them. He started teaching a course at Courant using a graduate textbook (Duffie, 1988) that Darrell Duffie had recently published. Graduate programs in

mathematical finance were popping up in many universities and Marco helped found a masters program at Courant, which remains one of the most well-known ones.

Marco approached mathematical modeling in finance with his characteristic style. Unlike some mathematicians who took the mathematical models developed by finance experts as given and focused on studying or “solving” them, Marco did not take these mathematical models for granted and deployed his mathematical modeling skills to develop several original ideas, which would become influential in the burgeoning field of computational finance: the Uncertain Volatility Model (UVM) (Avellaneda et al., 1995; Avellaneda & Paras, 1996), the Weighted Monte Carlo method (Avellaneda et al., 2001), and the constrained stochastic control approach to model calibration (Avellaneda et al., 1997).

This work was done in collaboration with a group of bright graduate students, including Robert Buff, Craig Friedman, Peter Friz, Nicolas Grandchamps, Antonio Paras, Dominick Samperi, and many others, as well as his friend Mike Lipkin.

Marco decided to take a “deep dive” in the world of finance during his sabbatical year 1996–97: he became Vice-President of the Fixed-Income research and Derivative Products Group at Morgan Stanley, where he implemented many of the ideas he had developed on derivatives pricing and hedging and model calibration. This experience also solidly grounded his research in applications, and gave him a lot of new research ideas, which he subsequently developed back at Courant.

The mathematical community took notice of Marco’s original and influential work: in 1998, Marco was invited to speak at the International Congress of Mathematicians in Berlin, where he presented his work on the use of entropy methods for the construction and calibration of pricing models (Avellaneda, 1998b). In the same year, the Courant Institute decided to create a “Division of Financial Mathematics” with Marco as the Director.

In 1998, the *Société Mathématique de France* decided to transgress the Bourbaki tradition and recognize that applied mathematics were worthwhile after all, and invited three distinguished speakers for its annual gathering in Paris to speak about “three applications of mathematics”: Vladimir Arnold, Jean-Michel Morel, and Marco Avellaneda (Arnold et al., 1998). The lectures were held in a packed auditorium at the *Institut Henri Poincaré* and Marco was the last lecturer. Then a young postdoc, I attended these lectures and was impressed by Marco’s presentation on his use of entropy methods for constructing market-consistent pricing models. We had a follow-up conversation, which was the beginning of a long friendship.

I visited Marco in New York in 1999, and he proposed that we jointly organize a session on “Mathematical Methods in Financial Modeling” at the first AMS-SMF Joint Congress in Lyon, France in 2001, the first irruption of mathematical finance in an AMS (or SMF) congress. I was honored by this proposal and we put together an excellent line-up of speakers. It was a memorable event, which he attended with Cassandra, and led to a special journal issue we edited together¹. It was around this time that Marco published his textbook on derivatives pricing with Peter Laurence (Avellaneda & Laurence, 2000).

My contacts with Marco intensified when I moved to New York in 2007 to join Columbia University. We met regularly and launched a joint “New York Quantitative Finance Seminar” together: a monthly event, held at a midtown location halfway between Columbia and NYU, which became a lively forum for exchange of ideas between quants, risk managers, academics, and students interested in new developments in quantitative finance.

Then came the subprime crisis, and the demise of Lehman Brothers and AIG in September 2008: financial derivatives were on the front page of news media and risk management became intermingled with policy issues. A consortium of dealer banks set up a proposal for a new central counterparty (CCP) for credit default swaps, and called on Marco and I to help with the design and stress testing of the risk management system. This led to several follow-up projects with other

major CCPs, in the United States, Asia, and eventually... Brazil, taking us back to the country of Marco's childhood, where we spent memorable days working on the *CloseOut Risk Evaluation* (CORE) approach (Avellaneda & Cont, 2013) with the Brazilian CCP.

In parallel, there was intense regulatory activity surrounding the Dodd-Frank Act and related reforms, to which Marco and I were called to contribute through various assignments, including a series of reports on OTC market transparency for a joint industry-regulator working group (Avellaneda & Cont, 2010a, 2010c, 2010b).

During this period, there were two countries Marco kept returning to regularly: Brazil, where he organized the annual Research In Options (RIO) conference every year with his friends Bruno Dupire and Jorge Zubelli, and France, where he had many friends and collaborators—Claude Bardos and Raphael Douady deserve a special mention—and where he spent his sabbatical in 2003–2004.

These numerous activities and projects with industry and regulators did not prevent Marco from simultaneously engaging in multiple research projects, which he pursued to the very end. I attempt to summarize below some of his main research contributions in mathematical finance.

4 | 1990–2000: DERIVATIVES PRICING AND COMPUTATIONAL FINANCE

Marco's early work in mathematical finance in the 1990s was focused on computational methods for pricing and hedging of derivatives. Rather than focusing on the derivation of analytical results for option prices, which were the focus of much of the literature back then, he developed several novel ideas for designing models for pricing and hedging, which remain relevant today. Marco realized early on that, unlike mathematical models in physics, which are derived from first principles and physical laws, mathematical models in finance are subject to *model uncertainty* and need to be grounded in market data. This led him to develop the *UVM* (Avellaneda et al., 1995) as well as several ideas on the *calibration* of stochastic models to market data (Avellaneda & Paras, 1996; Avellaneda, 1998c; Avellaneda et al., 2001, 1997).

4.1 | The Uncertain Volatility Model

Marco's best known contribution to mathematical finance is perhaps the Uncertain Volatility Model (Avellaneda et al., 1995). Rather than assuming some stochastic asset dynamics as known, Marco incorporated *volatility uncertainty* explicitly into the modeling framework: what can be said about option values and hedging strategies if we only have an upper and lower bound for volatility?

Consider a price process driven by a Brownian motion W :

$$dS_t = S_t \sigma_t dW_t \quad (1)$$

with unknown volatility σ_t satisfying some bounds: $\sigma_{\min} \leq \sigma_t \leq \sigma_{\max}$. Denote by \mathcal{P} the set of probability measures on the space of continuous paths corresponding to such price processes. The value V_t at time t of a derivative contract with maturity T and final payoff $H(S_T)$ then satisfies

$$W^-(t, S_t) \leq V_t \leq W^+(t, S_t), \quad (2)$$

where the superhedging bounds

$$\begin{aligned} W^-(t, S) &= \inf_{P \in \mathcal{P}} \mathbb{E}^P [e^{-r(T-t)} H(S_T) | S_t = S] \\ W^+(t, S) &= \sup_{P \in \mathcal{P}} \mathbb{E}_t^P [e^{-r(T-t)} H(S_T) | S_t = S] \end{aligned} \quad (3)$$

satisfy the following nonlinear PDE, baptized the “Black–Scholes–Barenblatt” equation, with a wink to Marco’s previous work in mathematical fluid dynamics:

$$\frac{\partial W^\pm}{\partial t} + r \left(\frac{\partial W^\pm}{\partial S} S - W^\pm \right) + \frac{1}{2} \Sigma \left(\frac{\partial^2 W^\pm}{\partial S^2} \right) S^2 \frac{\partial^2 W^\pm}{\partial S^2} = 0 \quad (4)$$

with terminal condition

$$W^\pm(S, T) = H(S_T), \quad (5)$$

where

$$\Sigma \left(\frac{\partial^2 W^\pm}{\partial S^2} \right) = \begin{cases} \sigma_{\max}^2 & \text{if } \frac{\partial^2 W^\pm}{\partial S^2} \geq 0 \\ \sigma_{\min}^2 & \text{if } \frac{\partial^2 W^\pm}{\partial S^2} < 0 \end{cases} \quad \text{and} \quad \Sigma \left(\frac{\partial^2 W^\pm}{\partial S^2} \right) = \begin{cases} \sigma_{\max}^2 & \text{if } \frac{\partial^2 W^\pm}{\partial S^2} \leq 0 \\ \sigma_{\min}^2 & \text{if } \frac{\partial^2 W^\pm}{\partial S^2} > 0. \end{cases} \quad (6)$$

This 17-page paper (Avellaneda et al., 1995), packed with theoretical ideas but also algorithms and implementation on market data, contained many ideas in germ and was influential well beyond mathematical finance. The nonlinear semigroup associated with the UVM is in fact a fundamental object, which arises in other contexts and inspired Shige Peng’s theory of “nonlinear expectations,” (Peng, 2019) which explores various extensions and probabilistic ramifications of these ideas.

Similar ideas, though formulated in a different way, were developed at the same time by Terry Lyons, in a paper (Lyons, 1995) which, remarkably, appeared in the same journal issue as the Avellaneda–Levy–Paras paper!

4.2 | Stochastic control approach to model calibration

At the time of publication of the UVM paper, market practice was to fit volatility models to market prices of options, by numerically solving gruesome nonlinear, and often ill-posed, optimization problems. This was done as a preliminary step, following which volatility uncertainty was not taken into account.

In an elegant sequel to the UVM paper, Avellaneda and Paras (1996) showed that one can instead think of market prices of options as *constraints* in the hedging problem, leading to a formulation of the hedging problem under volatility uncertainty as a stochastic control problem under constraints expressed as expectations. They further showed that the dual problem corresponds to the problem of dynamic hedging with the underlying asset together with static hedging in options, a practice which was already being used by market participants, with the Lagrange multipliers for the calibration constraints being identified as a static hedging portfolio. The introduction of

market prices of options greatly reduces the uncertainty and leads to a realistic cost of superhedging. This "Lagrangian UVM (Avellaneda & Paras, 1996) contains many of the ideas which were subsequently developed under the name of "Martingale Optimal transport" in the mathematical finance and probability literature.

This stochastic control approach to model calibration proved a fruitful idea, which Marco pursued in an interesting paper with Friedman, Holmes, and Samperi (Avellaneda et al., 1997). To calibrate a diffusion model with unknown volatility, for example,

$$\mathbb{P}^\sigma : \quad dS_t = S_t \sigma(t, S_t) dW_t, \quad (7)$$

they proposed to minimize a convex criterion under calibration constraints:

$$\inf_{\sigma} J(\sigma) = E^{\mathbb{P}^\sigma} \left[\int_0^T dt \, \eta(\sigma^2(t, S_t)) \right] \quad \text{under} \quad E^{\mathbb{P}^\sigma} [H_i] = C_i, \quad i = 1..n. \quad (8)$$

where η is a strictly convex function. The dual problem may be expressed as a sequence of stochastic control problems, which may be solved iteratively through a system of Hamilton–Jacobi–Bellman equations (Avellaneda et al., 1997). Again, Lagrange multipliers for the calibration constraints play the role of static hedge ratios for options.

The duality results underlying these approaches are far from obvious, as one is dealing with nonconvex, infinite-dimensional optimization problems. Some of these issues were studied by Samperi (2002) and Denis and Martini (2006), and related questions continue to be studied in the literature on martingale optimal transport and model calibration.

4.3 | Weighted Monte Carlo

The year spent in the fixed income team at Morgan Stanley brought Marco into contact with the large scale deployment of Monte Carlo simulation methods in finance, especially in fixed-income markets where models tend to be high dimensional. This led him to the idea of the Weighted Monte Carlo method (Avellaneda et al., 2001). Given N scenarios $\Omega_N = \{\omega_1, \dots, \omega_N\}$ simulated from a prior model, the idea is to assign weights $(Q_N(\omega_i), i = 1..N)$ to the paths in order to satisfy the calibration constraints. The weights are constructed by minimizing relative entropy with respect to the prior, under calibration constraints

$$\inf_{Q_N \in \mathcal{P}(\Omega_N)} \sum_{i=1}^N Q_N(\omega_i) \ln \frac{Q_N(\omega_i)}{P_N(\omega_i)} \quad \text{under} \quad \sum_{i=1}^N Q_N(\omega_i) H_j(\omega_i) = C_j. \quad (9)$$

This constrained optimization problem is solved by duality (Avellaneda et al., 2001): the dual has an explicit solution (Avellaneda, 1998c). A (discounted) payoff H is then priced using the same set of simulated paths via

$$E^{Q_N} [H] = \sum_{i=1}^N Q_N(\omega_i) H(\omega_i) = \frac{1}{N} \sum_{i=1}^N \frac{Q_N(\omega_i)}{P_N(\omega_i)} H(\omega_i). \quad (10)$$

The benchmark payoffs H_j play the role of *biased* control variates, leading to variance reduction (Glasserman & Yu, 2005):

$$E^{\mathbb{Q}_N}[X] = E^{\mathbb{Q}_N}\left[H - \sum_{i=1}^I \alpha_i H_i\right] + \sum_{i=1}^I \alpha_i C_i. \quad (11)$$

This method yields as a by-product a static hedge portfolio α_i^* , which minimizes the variance in Equation (11). This approach is generic in nature and has been applied to a variety of settings for model calibration.

5 | 2000–2010: MATHEMATICAL MODELING IN FINANCE

A hallmark of Marco's work in mathematical finance was his practical knowledge of financial markets and his ability to use mathematics to model real market phenomena ignored in idealized models. In the words of Alex Langnau²:

“In a way that is different from many other academics, Avellaneda looks at real market situations and then forms a model in the simplest mathematical way that describes the effect. It goes straight to the point without being dragged down by formalism, which is what makes his work so valuable.”

We give here a few examples of Marco's work in this area.

5.1 | Stock pinning

A nice example of Marco's modeling skills is his work on stock-pinning (Avellaneda and Lipkin, 2003; Avellaneda et al., 2012) with his longtime friend Mike Lipkin, a constant presence at the Courant seminars. “Pinning the strike” designates the situation where a stock price ends at or near the strike price of heavily traded options. It was known that stock prices tend to pin the strike when there is significant open interest in an option that is close to being in the money, and some papers by economists had suggested that this may be due to market manipulation. Avellaneda and Lipkin (Avellaneda and Lipkin, 2003; Avellaneda et al., 2012) showed that this results in fact from ... delta hedging by options market makers! By taking into account the market impact of delta-hedging, they obtained a *singular drift* term in the stock dynamics, which may drive the price to the strike of the most liquid option, resulting in pinning.

5.2 | Hard-to-borrow stocks

Another collaboration with Lipkin focused on the price dynamics of “hard-to-borrow” stocks (Avellaneda and Lipkin, 2009): these are stocks either subject to short-selling restrictions or which have insufficient liquidity for lending. Market participants with short positions risk being “squeezed” and put-call parity may fail for such stocks. These phenomena were well known to market participants but absent from theoretical models. Avellaneda and Lipkin modeled this phenomenon with a system of coupled stochastic differential equations describing the stock price

and the “buy-in rate,” an additional factor absent in standard models, and showed that short-sale restrictions result in increased prices and volatilities. This paper led Marco to be named “2010 Quant of the Year” by RISK Magazine³.

5.3 | High-frequency trading in a limit order book

Aside from our mathematical interests, Marco and I also had in common a collaborator: Sasha Stoikov. Sasha arrived in New York shortly after his Ph.D. in Mathematical Finance with Thaleia Zariphopoulou at the University of Texas, and started working with Marco at Courant on a well-known problem in market microstructure: optimal market making. This problem had been studied in game theoretic settings in one- or two-period toy models, too simple to be applicable in a real setting, and also in continuous-time models using stochastic control methods, leading to Hamilton–Jacobi–Bellman equations, which were intractable, which made them impossible to use in real-time applications.

Marco and Sasha found an elegant way out of this dilemma (Avellaneda & Stoikov, 2008): by focusing on a special parameterization where the problem becomes analytically tractable, they obtained simple and intuitive analytical results, which combine the main insights of the Ho and Stoll (1981) with information contained in the limit order book. Their paper (Avellaneda & Stoikov, 2008) is now a classic on the topic of optimal market making. The Avellaneda–Stoikov market-making algorithm became an industry reference, and variants of it have been implemented in many electronic markets worldwide.

5.4 | Leveraged exchange-traded funds (LETFs)

Marco’s 2010 paper with Stanley Zhang on leveraged exchange-traded funds (ETFs) (Avellaneda & Zhang, 2010) deserves a special mention. A “leveraged ETF” (LETf) is a contract which promises to pay out a multiple of the return on an index or ETF. By the mid-2000s, such contracts had become popular and were being marketed to a wide range of retail clients. Investors had noted that these (LETfs) did *not* reproduce the corresponding multiple of index returns over extended (e.g., annual) investment horizons: in 2008–2009, most LETfs had underperformed the corresponding static strategies.

Using an intuitive argument, Avellaneda and Zhang (2010) showed that, in addition to the multiple of the benchmark ETF, leveraged ETFs exposed investors to volatility risk. Using the Ito formula, they gave an exact formula linking the return of a leveraged fund with the corresponding multiple of the return of the unleveraged fund and its realized variance.

I handled this paper as an Associate Editor of a journal, whose editor was reluctant to publish it, arguing that “it was just an application of the Ito formula.” I recall having to insist to get it published. It subsequently became the most cited paper in this journal ...

6 | 2010–2022: STATISTICAL MODELING AND “BIG DATA” IN FINANCE

Another theme in Marco’s research in quantitative finance was the statistical analysis and modeling of financial data. Marco was quick to realize that the advent of big data would change quantitative finance and focused his interest on statistical modeling and data-driven approaches.

Marco's paper with Jeong-Hyun Lee on statistical arbitrage (Avellaneda & Lee, 2010) is a classic on the topic: it demystified many concepts related to statistical arbitrage and connected them with well-known statistical and mathematical modeling techniques, laying the ground for many studies on the topic.

Marco liked to explore the wealth of information in financial data and uncover their structure before jumping into analytical modeling. Examples of his work on the statistical analysis and modeling of financial data are his work on principal component analysis (PCA) for implied volatility surfaces (Avellaneda et al., 2020), hierarchical PCA of stock returns (Avellaneda, 2020), the analysis of eigenportfolios for US equities (Avellaneda et al., 2022), and the statistical analysis and modeling of VIX (Avellaneda and Papanicolaou, 2019). Interestingly, some of this work was done in collaboration with George Papanicolaou, who had guided Marco as a rookie at Courant, and his son Andrew Papanicolaou.

7 | ADIOS, AMIGO

In 2021, I invited Marco to present his recent work on hierarchical PCA of stock returns (Avellaneda, 2020) in an online seminar. He was already weakened by his illness but was keen to give a presentation. When introducing him to the audience, I said: "There are three Argentinians you should know: Diego Maradona, Luis Caffarelli, and... Marco Avellaneda!" Marco laughed, and went on to give a lively and interesting presentation, as usual. I did not know it then, but it would be his last seminar.

Marco passed away in June 2022, leaving behind not only a scientific legacy, which will continue to inspire many researchers in his field and beyond, but also an indelible trace among his many friends and colleagues who will remember for his optimism and enthusiasm.

ACKNOWLEDGMENTS

The author sincerely thanks Cassandra Avellaneda-Richmond for providing the cover portrait of Marco Avellaneda and for her support and encouragement.

ORCID

Rama Cont  <https://orcid.org/0000-0003-1164-6053>

ENDNOTES

¹<https://www.tandfonline.com/toc/rquf20/2/1?nav=tocList>

²<https://www.risk.net/awards/1567801/quant-of-the-year-marco-avellaneda>

³<https://www.risk.net/awards/1567801/quant-of-the-year-marco-avellaneda>

REFERENCES

- Achdou, Y., & Avellaneda, M. (1992). Influence of pore roughness and pore-size dispersion in estimating the permeability of a porous medium from electrical measurements. *Physics of fluids A*, 4(12), 2651–2673.
- Alama, S., Avellaneda, M., Deift, P. A., & Hempel, R. (1994). On the existence of eigenvalues of a divergence-form operator $A + \lambda B$ in a gap of $\sigma(A)$. *Asymptotic Anal.*, 8(4), 311–344.
- Alanko, S., & Avellaneda, M. (2013). Reducing variance in the numerical solution of BSDEs. *C. R. Math. Acad. Sci. Paris*, 351(3–4), 135–138.
- Apelian, C., Holmes, R. L., & Avellaneda, M. (1997). A turbulent transport model: streamline results for a class of random velocity fields in the plane. *Comm. Pure Applied Mathematics*, 50(11), 1053–1088.

- Arnold, V., Avellaneda, M., & Morel, J.-M. (1998). *Trois applications des mathématiques*. Société Mathématique de France.
- Avellaneda, M. (1987a). Iterated homogenization, differential effective medium theory and applications. *Comm. Pure Applied Mathematics*, 40(5), 527–554.
- Avellaneda, M. (1987b). Optimal bounds and microgeometries for elastic two-phase composites. *SIAM J. Applied Mathematics*, 47(6), 1216–1228.
- Avellaneda, M. (1989). Iterated homogenization and the effective properties of polycrystals. In *Control of boundaries and stabilization (Clermont-Ferrand, 1988). Lecture notes in control and information sciences* (Vol. 125, pp. 66–74). Springer.
- Avellaneda, M. (1991a). Bounds on the effective elastic constants of two-phase composite materials. In *Nonlinear partial differential equations and their applications. Collège de France Seminar, Vol. X (Paris, 1987–1988). Pitman research notes in mathematics series* (Vol. 220, pp. 1–34). Longman Scientific and Technical.
- Avellaneda, M. (1991b). Enhanced diffusivity and intercell transition layers in 2-D models of passive advection. *J. Math. Phys.*, 32(11), 3209–3212.
- Avellaneda, M. (1995). Statistical properties of shocks in Burgers turbulence. II. Tail probabilities for velocities, shock-strengths and rarefaction intervals. *Comm. Math. Phys.*, 169(1), 45–59.
- Avellaneda, M. (1996). Homogenization and renormalization: The mathematics of multi-scale random media and turbulent diffusion. In *Dynamical systems and probabilistic methods in partial differential equations (Berkeley, CA, 1994). Lectures in applied mathematics* (Vol. 31, pp. 251–268). American Mathematical Society.
- Avellaneda, M. (1998a). The minimum-entropy algorithm and related methods for calibrating asset-pricing model. In *Trois applications des mathématiques. SMF journ. annu.* (Vol. 1998, pp. 51–86). Societe Mathematique de France.
- Avellaneda, M. (1998b). The minimum-entropy algorithm and related methods for calibrating asset-pricing models. In *Proceedings of the International Congress of Mathematicians, Vol. III (Berlin, 1998)*, (Vol. III, pp. 545–563).
- Avellaneda, M. (1998c). Minimum-relative-entropy calibration of asset-pricing models. *International Journal of Theoretical and Applied Finance*, 01(04), 447–472.
- Avellaneda, M. (2004). A look ahead at options pricing and volatility. *Quant. Finance*, 4(5), C51–C54.
- Avellaneda, M. (2020). Hierarchical PCA and applications to portfolio management. *Revista Mexicana de economía y finanzas*, 15(7), 1–16.
- Avellaneda, M., Bardos, C., & Rauch, J. (1992). Contrôlabilité exacte, homogénéisation et localisation d'ondes dans un milieu non-homogène. *Asymptotic Anal.*, 5(6), 481–494.
- Avellaneda, M., Berlyand, L., & Clouet, J.-F. (2000). Frequency-dependent acoustics of composites with interfaces. *SIAM J. Applied Mathematics*, 60(6), 2143–2181.
- Avellaneda, M., Boyer-Olson, D., Busca, J., & Friz, P. (2003). Application of large deviation methods to the pricing of index options in finance. *C. R. Math. Acad. Sci. Paris*, 336(3), 263–266.
- Avellaneda, M., & Bruno, O. (1990). Effective conductivity and average polarizability of random polycrystals. *J. Math. Phys.*, 31(8), 2047–2056.
- Avellaneda, M., Buff, R., Friedman, C., Grandchamp, N., Kruk, L., & Newman, J. (2001). Weighted Monte Carlo: A new technique for calibrating asset-pricing models. *Int. J. Theor. Appl. Finance*, 4(1), 91–119.
- Avellaneda, M., & Cont, R. (2002). Introduction to the Special issue on volatility modelling. *Quantitative Finance*, 2(1), 6–7.
- Avellaneda, M., & Cont, R. (2010a). *Transparency in credit default swap markets. ISDA-Fed-SEC working group on OTC market transparency*. ISDA.
- Avellaneda, M., & Cont, R. (2010b). *Transparency in OTC equity markets. ISDA-Fed-SEC working group on OTC market transparency*. ISDA.
- Avellaneda, M., & Cont, R. (2010c). *Transparency in over-the-counter interest rate derivatives. ISDA-Fed-SEC working group on OTC market transparency*. ISDA.
- Avellaneda, M., & Cont, R. (2013). *Close-out risk evaluation (CORE): A new risk management approach for central counterparties* (Technical Report). Finance Concepts.
- Avellaneda, M., & E, W. (1995). Statistical properties of shocks in Burgers turbulence. *Comm. Math. Phys.*, 172(1), 13–38.
- Avellaneda, M., Elliott, Jr, F., & Apelian, C. (1993). Trapping, percolation, and anomalous diffusion of particles in a two-dimensional random field. *J. Statist. Phys.*, 72(5–6), 1227–1304.

- Avellaneda, M., Friedman, C., Holmes, R., & Samperi, D. (1997). Calibrating volatility surfaces via relative-entropy minimization. *Applied Mathematical Finance*, 4(1), 37–64.
- Avellaneda, M., & Gamba, R. (2002). Conquering the Greeks in Monte Carlo: efficient calculation of the market sensitivities and hedge-ratios of financial assets by direct numerical simulation. In *Mathematical finance—Bachelier Congress, 2000 (Paris)*, Springer finance (pp. 93–109). Springer.
- Avellaneda, M., Healy, B., Papanicolaou, A., & Papanicolaou, G. (2020). PCA for implied volatility surfaces. *The Journal of Financial Data Science*, 2(2), 85–109.
- Avellaneda, M., Healy, B., Papanicolaou, A., & Papanicolaou, G. (2022). Principal eigenportfolios for U.S. equities. *SIAM J. Financial Math.*, 13(3), 702–744.
- Avellaneda, M., Hou, T. Y., & Papanicolaou, G. C. (1991). Finite difference approximations for partial differential equations with rapidly oscillating coefficients. *RAIRO Modél. Math. Anal. Numér.*, 25(6), 693–710.
- Avellaneda, M., Kasyan, G., & Lipkin, M. D. (2012). Mathematical models for stock pinning near option expiration dates. *Comm. Pure Applied Mathematics*, 65(7), 949–974.
- Avellaneda, M., & Laurence, P. (2000). *Quantitative modeling of derivative securities*. Chapman & Hall/CRC.
- Avellaneda, M., & Lee, J.-H. (2010). Statistical arbitrage in the US equities market. *Quant. Finance*, 10(7), 761–782.
- Avellaneda, M., Levy, A., & Parás, A. (1995). Pricing and hedging derivative securities in markets with uncertain volatilities. *Applied Mathematical Finance*, 2(2), 73–88.
- Avellaneda, M., Li, T. N., Papanicolaou, A., & Wang, G. (2021). Trading signals in VIX futures. *Applied Mathematics Finance*, 28(3), 275–298.
- Avellaneda, M., & Lin, F.-H. (1987a). Compactness methods in the theory of homogenization. *Comm. Pure Applied Mathematics*, 40(6), 803–847.
- Avellaneda, M., & Lin, F.-H. (1987b). Counterexamples related to high-frequency oscillation of Poisson's kernel. *Applied Mathematics Optim.*, 15(2), 109–119.
- Avellaneda, M., & Lin, F.-H. (1987c). Homogenization of elliptic problems with L^p boundary data. *Applied Mathematics Optim.*, 15(2), 93–107.
- Avellaneda, M., & Lin, F. H. (1988). Fonctions quasi affines et minimisation de $R|\nabla u|^p$. *C. R. Acad Sci Paris Sér I Math*, 306, 355–358.
- Avellaneda, M., & Lin, F.-H. (1989a). Compactness methods in the theory of homogenization. II. Equations in nondivergence form. *Comm. Pure Applied Mathematics*, 42(2), 139–172.
- Avellaneda, M., & Lin, F.-H. (1989b). Homogenization of Poisson's kernel and applications to boundary control. *J. Math. Pures Appl.* (9), 68(1), 1–29.
- Avellaneda, M., & Lin, F.-H. (1989c). Un théorème de Liouville pour des équations elliptiques à coefficients périodiques. *C. R. Acad. Sci. Paris Sér. I Math.*, 309(5), 245–250.
- Avellaneda, M., & Lin, F.-H. (1991). L^p bounds on singular integrals in homogenization. *Comm. Pure Applied Mathematics*, 44(8–9), 897–910.
- Avellaneda, M., & Lipkin, M. (2009). A dynamic model for hard-to-borrow stocks. *Risk*, 22(6), 92–97.
- Avellaneda, M., & Lipkin, M. D. (2003). A market-induced mechanism for stock pinning. *Quant. Finance*, 3(6), 417–425.
- Avellaneda, M., & Majda, A. (1992a). Mathematical models with exact renormalization for turbulent transport. II. Fractal interfaces, non-Gaussian statistics and the sweeping effect. *Comm. Math. Phys.*, 146(1), 139–204.
- Avellaneda, M., & Majda, A. J. (1990). Mathematical models with exact renormalization for turbulent transport. *Comm. Math. Phys.*, 131(2), 381–429.
- Avellaneda, M., & Majda, A. J. (1991a). Homogenization and renormalization of multiple-scattering expansions for Green functions in turbulent transport. In *Composite media and homogenization theory (Trieste, 1990)*. Progress in nonlinear differential equations and their applications (Vol. 5, pp. 13–35). Birkhäuser.
- Avellaneda, M., & Majda, A. J. (1991b). An integral representation and bounds on the effective diffusivity in passive advection by laminar and turbulent flows. *Comm. Math. Phys.*, 138(2), 339–391.
- Avellaneda, M., & Majda, A. J. (1992b). Approximate and exact renormalization theories for a model for turbulent transport. *Physics of fluids A*, 4(1), 41–57.
- Avellaneda, M., & Majda, A. J. (1992c). Superdiffusion in nearly stratified flows. *J. Statist. Phys.*, 69(3–4), 689–729.
- Avellaneda, M., & Majda, A. J. (1993). Application of an approximate R-N-G theory, to a model for turbulent transport, with exact renormalization. In *Turbulence in fluid flows. The IMA volumes in mathematics and its applications* (Vol. 55, pp. 1–31). Springer.

- Avellaneda, M., & Majda, A. J. (1994). Simple examples with features of renormalization for turbulent transport. *Philos. Trans. Roy. Soc. London Ser. A*, 346(1679), 205–233.
- Avellaneda, M., & Milton, G. W. (1989). Optimal bounds on the effective bulk modulus of polycrystals. *SIAM J. Applied Mathematics*, 49(3), 824–837.
- Avellaneda, M., & Papanicolaou, A. (2019). Statistics of VIX futures and applications to trading volatility exchange-traded products. *Int. J. Theor. Appl. Finance*, 22(1), 1850061, 30.
- Avellaneda, M., & Paras, A. (1994). Dynamical hedging strategies for derivative securities in the presence of large transaction costs. *Applied Mathematical Finance*, 1, 165–93.
- Avellaneda, M., & Paras, A. (1996). Managing the volatility risk of portfolios of derivative securities: The Lagrangian uncertain volatility model. *Applied Mathematical Finance*, 3(1), 21–52.
- Avellaneda, M., Reed, J., & Stoikov, S. (2011). Forecasting prices from level-I quotes in the presence of hidden liquidity. *Algorithmic Finance*, 1(1), 35–43.
- Avellaneda, M., Ryan, R., & E. W. (1995). PDFs for velocity and velocity gradients in Burgers' turbulence. *Phys. Fluids*, 7(12), 3067–3071.
- Avellaneda, M., & Stoikov, S. (2008). High-frequency trading in a limit order book. *Quant. Finance*, 8(3), 217–224.
- Avellaneda, M., & Torquato, S. (1991). Rigorous link between fluid permeability, electrical conductivity, and relaxation times for transport in porous media. *Physics of fluids A*, 3(11), 2529–2540.
- Avellaneda, M., & Vergassola, M. (1995). Stieltjes integral representation of effective diffusivities in time-dependent flows. *Phys. Rev. E* (3), 52(3), 3249–3251.
- Avellaneda, M., & Wu, L. (2001). Credit contagion: Pricing cross-country risk in Brady debt markets. *Int. J. Theor. Appl. Finance*, 4(6), 921–938.
- Avellaneda, M., & Zhang, S. (2010). Path-dependence of leveraged ETF returns. *SIAM J. Financial Math.*, 1(1), 586–603.
- Avellaneda, M. M. (1985). *Large deviation estimate and the homological behavior of Brownian motion on manifolds* [Ph.D. thesis, University of Minnesota]. ProQuest.
- Cont, R., & Avellaneda, M. (2002). Introduction to the special issue on volatility modelling. *Quantitative Finance*, 2, 6–7.
- Davis, G., Mallat, S., & Avellaneda, M. (1997). Adaptive greedy approximations. *Constr. Approx.*, 13(1), 57–98.
- De Genaro, A., & Avellaneda, M. (2018). Pricing interest rate derivatives under monetary changes. *Int. J. Theor. Appl. Finance*, 21(6), 1850037.
- Denis, L., & Martini, C. (2006). A theoretical framework for the pricing of contingent claims in the presence of model uncertainty. *Annals of Applied Probability*, 16(2), 827–852.
- Duffie, D. (1988). *Security markets: Stochastic models*. Academic Press.
- Glasserman, P., & Yu, Y. (2005). Large sample properties of weighted Monte Carlo estimators. *Operations Research*, 53(2), 298–312.
- Guo, M., Papanicolaou, G., & Varadhan, S. (1988). Nonlinear diffusion limit for a system with nearest neighbor interactions. *Communications in Mathematical Physics*, 118(1), 31–59.
- Harrison, J., & Pliska, S. R. (1981). Martingales and stochastic integrals in the theory of continuous trading. *Stochastic Processes and their Applications*, 11, 215–260.
- Ho, T., & Stoll, H. R. (1981). Optimal dealer pricing under transactions and return uncertainty. *Journal of Financial economics*, 9(1), 47–73.
- Laurence, P., & Avellaneda, M. (1991). On Woltjer's variational principle for force-free fields. *J. Math. Phys.*, 32(5), 1240–1253.
- Laurence, P., & Avellaneda, M. (1993a). A Moffatt-Arnold formula for the mutual helicity of linked flux tubes. *Geophys. Astrophys. Fluid Dynam.*, 69(1–4), 243–256.
- Laurence, P., & Avellaneda, M. (1993b). Woltjer's variational principle. II. The case of unbounded domains. *Geophys. Astrophys. Fluid Dynam.*, 69(1–4), 201–241.
- Lipton, R., & Avellaneda, M. (1990). Darcy's law for slow viscous flow past a stationary array of bubbles. *Proc. Roy. Soc. Edinburgh Sect. A*, 114(1–2), 71–79.
- Lyons, T. J. (1995). Uncertain volatility and the risk-free synthesis of derivatives. *Applied Mathematical Finance*, 2(2), 117–133.
- Merton, R. C. (1992). *Continuous-time finance*. Blackwell.

- Papanicolaou, G. C., & Varadhan, S. (1979). Boundary value problems with rapidly oscillating random coefficients. In *Colloquia Mathematica Societatis, Janos Bolyai*, (Vol. 27, pp. 853–873).
- Peng, S. (2019). *Nonlinear expectations and stochastic calculus under uncertainty: With robust CLT and G-Brownian motion*. Springer.
- Ryan, R., & Avellaneda, M. (1999). The one-point statistics of viscous Burgers turbulence initialized with Gaussian data. *Comm. Math. Phys.*, 200(1), 1–23.
- Samperi, D. (2002). Calibration a diffusion pricing model with uncertain volatility: Regularization and stability. *Mathematical Finance*, 12(1), 71–87.
- Tudball, D. (2013). That crazy boy from Búzios! *Wilmott*, 2013(67), 28–43.
- Vergassola, M., & Avellaneda, M. (1997). Scalar transport in compressible flow. *Phys. D*, 106(1–2), 148–166.