

Read this expression out loud:

$$6-2$$

You might have said 'six minus two', 'six subtract two', or 'six take away two'. You might have said 'two less than six', or 'the difference between six and two'. If you were thinking about different ways to help students understand the expression, you might have said something like 'start with six and remove two', 'start with six and *go back* two', or 'start at six and *go down* two'. These last two descriptions lean on an understanding of numbers as an ordered sequence. Children might be able to move forwards and backwards in the sequence by recalling the names of the numbers in order: 'six, five, four', or they might use a number line representation, either drawn or imagined. Most number lines are drawn horizontally (rather unintuitively), so 'go back' or 'go down' might mean 'move to the left'.

Now read this expression out loud:

$$-5--3$$

What words or phrases did you use for each of the dashes?

Did you use the same for all of them, or did your names or descriptions change depending on where in the expression the symbol is? Do the symbols mean the same thing?

The dash in mathematics can be interpreted as an *operation*: a prompt to 'take-away' or remove. In $6-2$, we are being told to 'subtract' or 'minus' two from six. With the number line representation, we can picture moving two places down (or to the left). 'Subtract' is a *verb*, an instruction to do something. Using words and phrases such as 'take-away' and 'subtract' can communicate this idea of an operation or something we have to do.

What word did you use to describe the dash in -3 ? Here, the dash could be interpreted as assigning the number a *property*, telling us it is less than zero. In this case you might have used a word such as 'negative', which differentiates it from an instruction to perform a calculation. 'Negative', in this case, is an *adjective*.

These choices of words to use when reading the dash symbol can influence how we think about calculations such as $8\div-4$; I might use words like 'eight divided by minus four', 'eight divided by subtract four' or

'eight divided by negative four'. If these words prompt me to think of $- \div$ as an operator, then how might I make sense of $\div - \div$?

We might, then, read $-5--3$ as 'negative five subtract negative three', interpreting the expression as a number (-5), an operator $\div$, and another number (-3). This might suggest to us a 'holistic' view of number, where the dash and the digit are both integral parts of the number. Here, the $- \div$ is part of the noun phrase -3 . This can be reinforced with the use of double-sided counters, where -5 can be represented by five red counters, and $--3$ can be interpreted as taking away three of those red counters.

A different way of interpreting the calculation $-5--3$, might be to think about a starting point (-5), a direction $\div$, and a distance to move (3). In this conception of the expression, we treat -5 holistically, with the dash being part of the number (behaving as an adjective). But we treat -3 differently, disconnecting the dash from the 3 and combining it with the dash in the middle to create an instruction to move (changing it from behaving as an adjective, to a verb). How might you read the expression aloud to convey this?

You might be thinking in terms of numbers as having both magnitude and direction; not as static *positions* on a number line, but *movements*. In this situation you might be thinking about $-5--3$ as two movements: starting from zero, move 'down 5', then subtract a movement of 'down 3'. Here, the dash is consistently a verb: an instruction to *move* (if imagining a number line), or more generally, *change the magnitude* of the number. In this case, numbers could be modelled as arrows rather than points on a number line. What words could be used when reading out loud that convey this interpretation?

What language do you use when naming the dash in mathematics? Does this change depending on the context?

If you would like to take part in an informal survey and see how other people have answered some of the questions above, please use this link (or use the QR code):

https://oxfordeducation.eu.qualtrics.com/jfe/form/SV_3OhsTVF43860FEy



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