

Atmospheric Circulation on Slowly Rotating Planets in the Solar System, and Beyond



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Non est ad astra mollis e terris via

Abstract

Atmospheric circulation on Venus and Titan is characterised by prograde superrotating jets at the equator, and meridional overturning circulations that extend a significant distance towards the poles. Circulation of this type is characteristic of planets that are slowly rotating. By contrast, the atmospheres of the Earth and Mars, which are more rapidly rotating, feature zonal jets in mid-latitudes, and overturning circulation that is confined to the tropics. However, while planetary rotation is likely the main determinant of a given planet's atmospheric circulation, other factors also have an important influence. This has been illustrated by numerical experiments using models of slowly rotating Earth-like planets, which do not automatically generate Venus or Titan-like circulations.

The aim of this thesis is to improve our understanding of the different circulation regimes that may be obtained on slowly rotating planets. This is achieved by developing a theoretical framework within which the circulation of different slowly rotating planets can be easily compared. Additionally, for the case of Titan, this thesis presents an in-depth investigation of the specific atmospheric phenomena that differentiate it from other slow rotators.

Recently, the array of possible circulation regimes has expanded significantly, due to the discovery of tidally locked planets orbiting stars other than the Sun. If we restrict our attention to temperate planets, then theoretical considerations suggest most will be slowly rotating. Tidally locked planets have a permanent day side and night side, which strongly influences their atmospheric circulation. Consequently, these planets occupy circulation regimes that are different from those exhibited by the slowly rotating planets in the Solar System.

To characterise the atmospheres of slowly rotating tidally locked planets, this thesis proposes a new analysis technique that divides the total circulation into physically meaningful constituent components, in much the same way as the circulation of the Solar System planets is divided into zonal jets and meridional overturning. It is also shown that the proposed decomposition of the circulation can be used to aid the observational characterisation of these planets, by showing how different parts of the decomposed circulation contribute distinct features to a planet's thermal emission.

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Introduction

1.1 Atmospheric circulation on rocky planets in the Solar System

The Solar System contains four *rocky* planetary bodies that possess substantial atmospheres. Three – Venus, the Earth and Mars – are planets. The fourth is Titan, which is the largest moon of Saturn. Each of these bodies' atmospheres are in motion, termed *atmospheric circulation*, driven by differential solar heating in latitude and planetary rotation. However, the atmospheric circulation on each planet (henceforth taken to include Titan) differs significantly in form and strength.

Figure 1.1 shows the zonally-averaged circulation for the Earth, Mars, Venus and Titan in the latitude-pressure plane. The Earth's atmospheric circulation is characterised by prograde zonal *jets* in the sub-tropics and mid-latitudes with a maximum velocity of roughly 40 m s^{-1} ; clockwise, thermally-direct overturning *Hadley cells* (after Hadley, 1735), confined to the tropics; and anti-clockwise *Ferrel cells* (after Ferrel, 1856), in the mid-latitudes. The Earth's jets are sometimes separated into two distinct components: the *sub-tropical jets*, which are located at the outflow of the Hadley circulation, and the *mid-latitude eddy driven jets*, which are located poleward of the Hadley circulation and extend more strongly down towards the surface. This distinction is made as the two types of jet have different generation mechanisms (Vallis, 2017); the sub-tropical jet is associated with conservation of angular momentum in the poleward moving branch of the Hadley circulation, while the mid-latitude jet is associated with momentum convergence induced by atmospheric waves.

Mars' atmospheric circulation is similar in many ways to the Earth's. Distinct clockwise and anti-clockwise cells can be identified in each hemisphere, as can mid-latitude zonal jets. However, the circulation on Mars is 'expanded' in the meridional direction when compared with the Earth's. The Hadley cells have a greater poleward extent, the Ferrel cells are compacted close to the poles, and it is much harder to make a distinction between the sub-tropical and mid-latitude jets, which have essentially merged into one. The maximum zonal velocity is around 90 m s^{-1} , which is faster than that reached in the Earth's atmosphere.

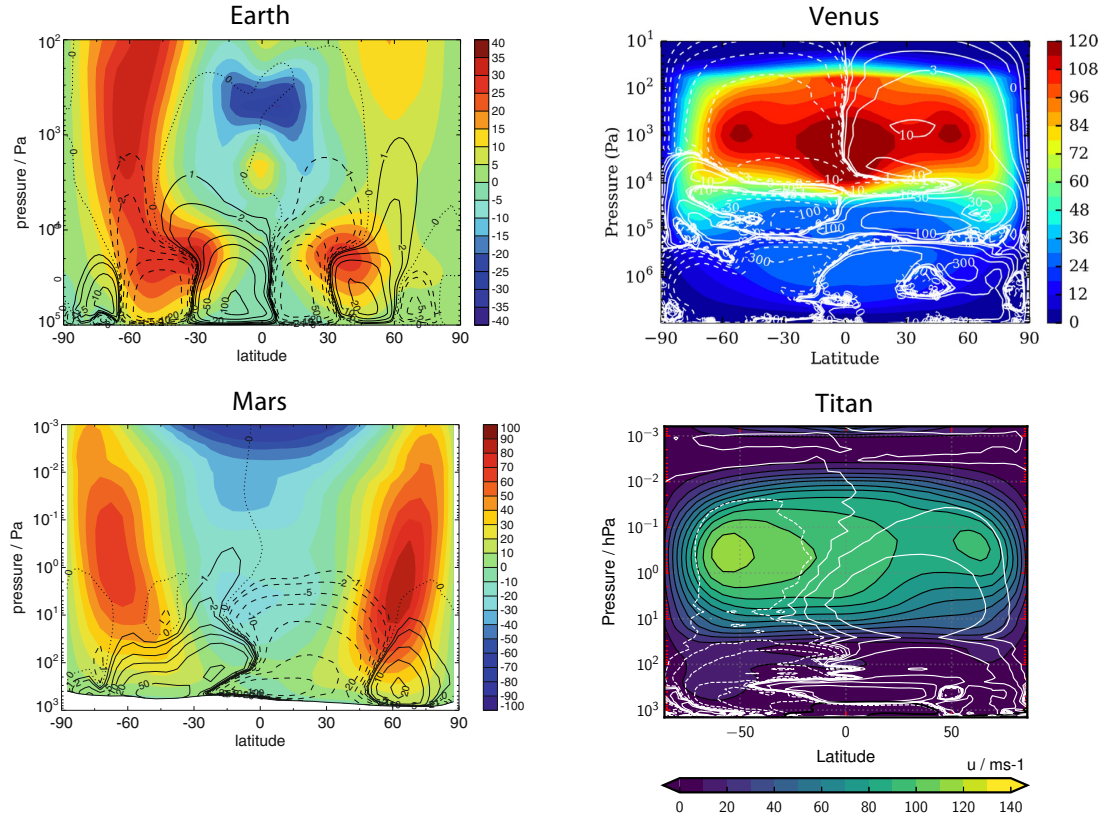


Figure 1.1: Atmospheric circulation in the Solar System. In each panel, colour contours show the zonal-mean zonal wind $\bar{u}$, and solid contours show the Eulerian mean meridional mass streamfunction Ψ , which measures the strength of overturning circulation in the ϑ - p plane (positive contours indicate clockwise motion). Panels for Earth and Mars show annually averaged circulation computed from the ERA-40 reanalysis dataset (Uppala et al., 2005), and the Mars Climate Database (Lewis et al., 1999; Millour et al., 2018), respectively. The data shown for Venus and Titan is output from General Circulation Model simulations described in Lebonnois et al. (2016) and Lora et al. (2015). The ‘Earth’ and ‘Mars’ panels are obtained from Read et al. (2018). The ‘Venus’ panel is obtained from Lebonnois et al. (2016), and the ‘Titan’ panel is from Lewis et al. (2022, *in review*; also Chapter 4 of this thesis).

The atmospheric circulation on Venus and Titan has striking differences to that on the Earth and Mars. Pronounced prograde zonal jets are present at the equator, and so Venus’ and Titan’s atmospheres are *superrotating*, which describes a phenomenon where the specific axial angular momentum of the zonal wind exceeds that of the solid planet at the equator (Read, 1986b). The maximum zonal wind velocity obtained on Venus and Titan is greater than on the Earth and Mars. Equatorial wind velocities in excess of 100 m s^{-1} have been measured in both atmospheres *in situ*, by the Pioneer Venus (at Venus; Counselman et al. 1980), and Huygens (at Titan; Bird et al. 2005) descent probes. In addition, the Hadley circulation on each planet extends the whole way to the poles. On Venus, it also takes on a more complex ‘stacked’ structure in the vertical with multiple distinct cells layered on top of one another.

Planet	a (km)	Ω (s ⁻¹)	U (m s ⁻¹)	$\mathcal{R}$
Earth	6371	7.29×10^{-5}	40	0.09
Mars	3396	7.09×10^{-5}	70	0.29
Titan	2575	4.56×10^{-6}	120	10
Venus	6051	2.99×10^{-7}	120	66

Table 1.1: Estimates of the Rossby number $\mathcal{R}$ for the Earth, Mars, Venus and Titan.

1.2 Factors affecting atmospheric circulation

Given the differences in atmospheric circulation on each planet, and in particular between the circulation of Earth and Mars compared with Venus and Titan, it is natural to ask what their origin is?

The most obvious difference between the two pairs of planets – the Earth and Mars, and Venus and Titan – comes in through the importance of planetary rotation in determining their atmospheric dynamics. This can be quantified by the Rossby number $\mathcal{R}$, defined as the ratio between advection and Coriolis forces in the horizontal momentum equations

$$\mathcal{R} \equiv \frac{U}{fL} \quad (1.1)$$

Above, U is a characteristic horizontal velocity scale and L a characteristic horizontal length scale, and $f = 2\Omega \sin \vartheta$ is the Coriolis parameter. Ω is the planetary rotation rate and ϑ is latitude. Estimates of $\mathcal{R}$ computed for each of the Earth, Mars, Venus and Titan, assuming $L = a$ where a is the planetary radius, are tabulated in [Table 1.1](#). When $\mathcal{R} \ll 1$ the dynamics are dominated by planetary rotation.

On the Earth and Mars, $\mathcal{R} < 1$, whereas on Venus and Titan $\mathcal{R} > 1$. Venus and Titan's large $\mathcal{R}$ comes from their slow rotation (relative to the Earth), and additionally in the case of Titan, its small size. The circulations shown in [Figure 1.1](#) suggest that planets with small $\mathcal{R}$ may generally feature pronounced mid-latitude jets and overturning circulation confined to the tropics, while large $\mathcal{R}$ planets may feature equatorial superrotation and planetary-scale overturning circulations. This notion is supported by theoretical models of atmospheric circulation, which suggest that the Hadley circulation should expand polewards as $\mathcal{R}$ is increased (Held and Hou, 1980), and numerical modeling that suggests superrotating circulation is to be expected for large $\mathcal{R}$ whereas mid-latitude jets prevail for small $\mathcal{R}$ (Mitchell and Vallis, 2010; Kaspi and Showman, 2015; Wang et al., 2018).

Planetary rotation is also important for sustaining various waves in the atmosphere, and has an influence on their size. In mid-latitudes, waves take a horizontal scale defined by the *Rossby radius of deformation* l_d . Closer to the equator, this scale is given by the

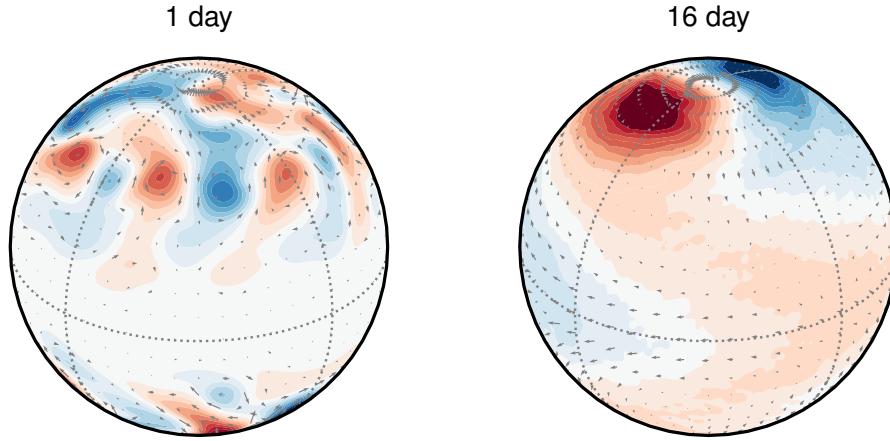


Figure 1.2: Mid-troposphere cross-sections of an ‘Earth-like’ atmosphere with the Earth’s rotation rate (left) and 1/16 times the Earth’s rotation rate (right). Fields plotted are eddy geopotential height (colour) and eddy horizontal wind (quivers). The colour contour interval in the left-hand panel is four times that in the right-hand panel. Data was produced with numerical simulations following Held and Suarez (1994) using the Isca modeling framework (Vallis et al., 2018).

equatorial radius of deformation l_e . These are defined as (Vallis, 2017, Chapters 6 and 8)

$$l_d = \frac{N}{f m}, \quad (1.2)$$

$$l_e = \left(\frac{N}{\beta m} \right)^{\frac{1}{2}} \quad (1.3)$$

where N is the buoyancy frequency, which is a measure of stratification, m is the vertical wavenumber of the wave, related to the vertical wavelength λ_v via $m = 2\pi/\lambda_v$, and $\beta = df/dy = 2\Omega \cos \vartheta/a$. Both deformation radii depend inversely on rotation rate, with $l_d \propto \Omega^{-1}$ and $l_e \propto \Omega^{-\frac{1}{2}}$, which means that slower planetary rotation leads to a larger horizontal scale for atmospheric waves. This phenomenology emerges naturally in simulations of the Earth’s atmosphere when planetary rotation rate is varied (Figure 1.2). The existence of large, planetary-scale waves has been inferred from observations of cloud patterns in the atmospheres of Venus and Titan (del Genio and Rossow, 1990; Mitchell et al., 2011).

However, while the importance of planetary rotation is likely the most important factor determining the ‘flavour’ of a planet’s atmospheric circulation, its value cannot be used single-handedly to explain the differences between the circulations of the Earth and Mars, and Venus and Titan. Simulations of atmospheric circulation using numerical models of the Earth’s atmosphere, but with a rotation rate similar to Venus’, do obtain a superrotating circulation, but the zonal wind velocity is far slower (by roughly a factor of 10) than that obtained in the real Venus atmosphere (e.g., Colyer and Vallis, 2019).

This suggests that other features distinguishing Venus and Titan from the Earth are potentially important in determining their atmospheric circulation. Notable amongst these are the mass of Venus' and Titan's atmospheres, which are approximately $100\times$ and $10\times$ more massive than the Earth's, respectively (Griffith et al., 2013; Bullock and Grinspoon, 2013), and the presence of ultraviolet absorbing substances which absorb a significant fraction of (net) incident solar radiation before it reaches the surface (approximately 90% for both planets; Tomasko et al., 1980, 2008). Properties of a planet's orbit can also affect its atmospheric circulation. Venus' atmosphere features large diurnal and semi-diurnal thermal tides due to Venus' long solar day (116 Earth days), and Titan experiences a seasonal variation in insolation similar to the Earth's due to Saturn's non-zero obliquity (26.7° inclination). Venus' thermal tides have been shown to be important for accelerating its superrotating jet (Lebonnois et al., 2010; Horinouchi et al., 2020).

1.3 Atmospheric circulation on tidally locked exoplanets

Recently, the number of known rocky planets has increased significantly. This is due to the discovery of numerous planets orbiting stars other than the sun, called extrasolar planets (or *exoplanets* for short). At the time of writing, roughly 100 exoplanets with a size and mass similar to the Earth's have been discovered. These planets are presented in Figure 1.3 as a function of their orbital distance and mass. The radius of each planet and the spectral class of its host star is also shown.

Due to the limitations of current observational techniques, most observationally accessible exoplanets are on close-in, short-period orbits around stars cooler, and thus dimmer, than the Sun (predominantly stars with spectral class K or M; Showman et al., 2013). Many of these planets are sufficiently close to their host star for gravitational interaction between star and planet to 'spin down' the planet's rotation so that their orbital and rotational periods are synchronised (Dole, 1964). A planet in this state is referred to as *tidally locked*. If we restrict our interest to temperate planets that might support surface temperatures similar to the Earth's, then we implicitly require planets orbiting M and K stars to have orbital periods (and thus rotation periods) on the order of approximately 5–50 days¹ (Leconte et al., 2015). A good example of a temperate, tidally locked rocky exoplanet is Trappist-1e, which is part of a seven planet (Trappist 1b–h) chain of planets orbiting an ultracool M star (Gillon et al., 2017). It has a rotation period of 6 days and a radius of $0.91a_E$ where a_E is the Earth's radius (Table 1.1), and numerical

¹Inferred by applying Kepler's Third Law to the boundaries of the shaded habitable zone in Figure 3 of Leconte et al. (2015).

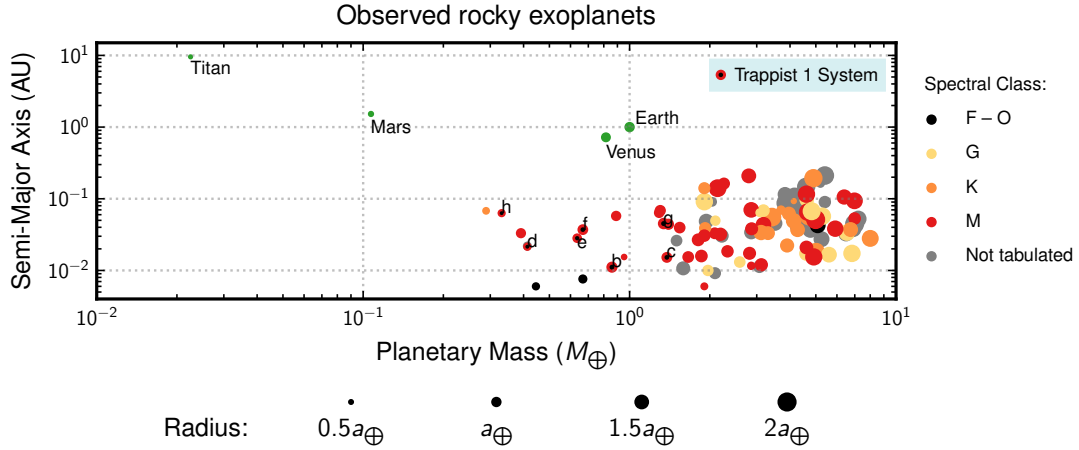


Figure 1.3: 104 observed rocky exoplanets, with Solar System counterparts shown for context. Data is obtained from The Extrasolar Planets Encyclopaedia (<https://exoplanet.eu>). Planets are plotted as a function of their orbital distance (semi-major axis, in AU) and planetary mass (in Earth masses). The radius of each planet is indicated by its marker size, and the spectral class of a planet's host star is indicated by marker colour. 131 planets are returned from the catalogue if the following filters are applied: $0.5 < \frac{a}{a_{\oplus}} < 2$ and $0.5^3 < \frac{M}{M_{\oplus}} < 2^3$, where a is radius and M is planetary mass, and $a_{\oplus}$ and $M_{\oplus}$ are the radius and mass of the Earth, respectively. Only those with a known orbital distance (semi-major axis) are shown, leaving 104 planets. The planets in the Trappist 1 system are marked with central black dots and letter labels (b–h).

climate simulations suggest that its atmosphere may possibly support temperatures such that liquid water can be sustained at the surface (Turbet et al., 2018).

Tidal locking means that these planets always present the same face to their host star, and thus have a permanent *day side* and *night side*. This forcing has a more pronounced zonal asymmetry than that experienced by the Solar System planets, which leads to differences in their atmospheric circulation. This is illustrated in Figure 1.4, which shows snapshots of mid-troposphere horizontal wind and temperature for two simulations of atmospheric circulation on tidally locked rocky exoplanets, with rotation periods of 4 days and 10 days, respectively. Some similarities with trends emergent in the ‘Earth-like’ simulations shown in Figure 1.2 can be identified. For example, the circulation for the 10 day rotator has a broader latitudinal extent, and is less wavy, than that of the 4 day rotator. However, the imprint of the day-night forcing imposed in the two simulations can clearly be identified in the circulation. Temperatures are hottest on the day side, near the sub-stellar point (located at the origin), and vortices associated with stationary waves are prominent on the night side. Superrotation is present in both experiments, generated by momentum convergence associated with the stationary waves (Showman and Polvani, 2011).

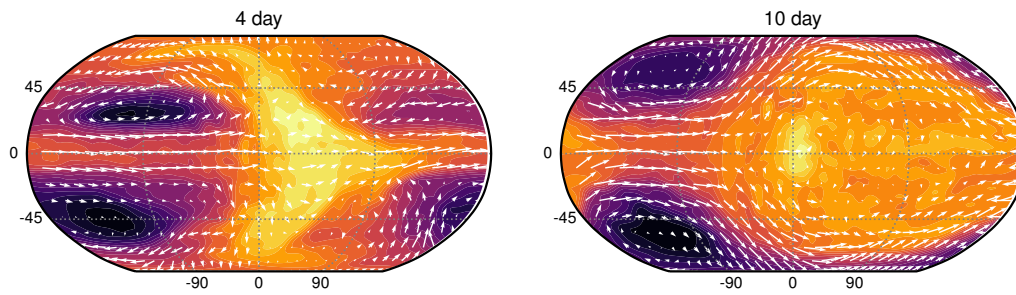


Figure 1.4: Mid-troposphere cross-sections of atmospheric circulation on a tidally locked planet. Simulations with $1/4$ times the Earth’s rotation rate (left) and $1/16$ times the Earth’s rotation rate (right) are shown. Fields plotted are temperature (colour) and horizontal wind (quivers). Data was produced with numerical simulations following Hammond and Pierrehumbert (2018) using the Isca modeling framework (Vallis et al., 2018).

1.4 Thesis overview

In this thesis, my aim is to improve our understanding of the atmospheric circulation regimes that may be obtained on slowly rotating rocky planets, both in the Solar System and elsewhere.

The work in this thesis is conducted by designing, running and analysing *numerical* experiments using General Circulation Model (GCM) simulations of atmospheric circulation. GCMs solve the equations of fluid motion in a rotating frame, subject to forcing from solar heating and dissipation through radiative cooling, parametrised diffusion and surface friction. A variety of GCMs are used for the work in this thesis, including idealised models of slowly rotating Earth-like planets and tidally locked planets, as well as a sophisticated model of Titan’s atmosphere. These models are described in the chapters where they are introduced. The remainder of this thesis is structured as follows.

[Chapter 2](#) begins with a survey of previous work which has investigated the factors that determine the large-scale circulation of planetary atmospheres. This review begins by discussing the basic dependence of planetary atmospheric circulation on key parameters such as planetary rotation rate, radius, and atmospheric mass, before focussing on processes and phenomena of particular relevance to slowly rotating planets, such as equatorial waves and superrotation. The atmospheric circulation of tidally locked terrestrial planets is also reviewed in detail.

In [Chapter 3](#), I examine how the different regimes of atmospheric circulation identified in [Chapter 2](#) may be characterised and compared using a diagnostic called the ‘global superrotation index’. In [Chapter 4](#) we turn our attention towards atmospheric circulation on Titan specifically, and I present an investigation of the wave processes that maintain superrotation in Titan’s atmosphere.

[Chapter 5](#) and [Chapter 6](#) focus on atmospheric circulation on tidally locked planets. [Chapter 5](#) introduces a technique that allows the circulation to be partitioned into its constituent phenomena (i.e., similar to partitioning the Earth’s atmospheric circulation into zonal jets and a meridional overturning circulation). Then, in [Chapter 6](#) I use this decomposition to interpret the dynamical balances that determine atmospheric temperature structure on tidally locked planets.

Finally, [Chapter 7](#) gives a summary of the outcomes of each of these investigations, and outlines some opportunities for future work.

Theory and Models of Planetary Atmospheric Circulation

The aim of this chapter is to provide an overview of previous work to understand the factors governing planetary atmospheric circulation, focusing on rocky planets similar to the Earth.

Fluid motion in an atmosphere or ocean is governed by equations for conservation of momentum, mass, and internal energy, applied in a rotating spherical coordinate system, combined with an equation of state. Collectively these equations are often referred to as the *equations of motion*. The equations of motion are a nonlinear set of partial differential equations that cannot be solved directly. Instead, they must either be integrated numerically, or reduced to a simpler form for which approximate analytic solutions can be obtained. Motivated by a need to understand weather and climate on Earth, both of these approaches have been explored extensively (Schneider, 2006; Held, 2018). With increasing frequency, the theory and models of the Earth’s atmospheric circulation have been adapted and applied to understanding atmospheric circulation on planets other than the Earth (Read, 2011; Showman et al., 2013; Mitchell et al., 2019). This effort began with the Solar System planets, motivated by the availability of high quality observations made by orbiting spacecraft, and has more recently expanded to speculate on the possible atmospheric circulations of newly discovered exoplanets.

Our discussion of this work begins by introducing the equations of motion (Section 2.1). This is followed with a non-dimensionalisation of the equations, so as identify the key dimensionless numbers that determine the circulation regime occupied by a particular planetary atmosphere (Section 2.2). Studies that investigate the parameter dependence of atmospheric circulation are then reviewed, drawing on results from idealised general circulation model experiments (Section 2.3). The next three sections focus on phenomena of particular relevance to slowly rotating terrestrial planets, namely axisymmetric overturning circulations (Section 2.4), equatorial waves (Section 2.5), and equatorial superrotation (Section 2.6). Finally, we consider the special case of tidally locked planets, for which the dynamics are strongly influenced by a stationary heating contrast between their day side and night side (Section 2.7). The chapter concludes with a list of open questions motivated by the work discussed in this review (Section 2.8).

2.1 Equations of motion

For atmospheric circulation at large scales (made precise below), the dynamics are well-represented by an equation set called the *hydrostatic primitive equations*, which may be written (Andrews et al., 1987, Chapter 3; Vallis, 2017, Chapter 2)

$$\frac{D\mathbf{u}}{Dt} + \mathbf{f} \times \mathbf{u} = -\nabla_p \phi - D\mathbf{u}, \quad (2.1)$$

$$\frac{\partial \phi}{\partial \ln p} = -RT, \quad (2.2)$$

$$\nabla_p \cdot \mathbf{u} + \frac{\partial \omega}{\partial p} = 0, \quad (2.3)$$

$$\frac{DT}{Dt} - \frac{\kappa T}{p} \omega = \dot{Q} - D_T, \quad (2.4)$$

$$p = \rho RT. \quad (2.5)$$

This form of the primitive equations uses pressure p as the vertical coordinate. $\mathbf{u} = (u, v)$ is the horizontal velocity in the longitude λ (east-west) and latitude ϑ (north-south) directions, respectively, $\omega = Dp/Dt$ is the pressure vertical velocity, $\phi = gz$ is the geopotential, where g is the acceleration due to gravity and z is height, T is temperature and ρ is density. $\kappa = R/c_p$ is the ratio of the specific gas constant and specific heat capacity at constant pressure. $\dot{Q}$ describes the net radiative heating or cooling rate, and D describes dissipation due to viscous effects and friction near the surface.

Above, D/Dt is the material derivative, defined in pressure coordinates as

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla_p + \omega \frac{\partial}{\partial p}. \quad (2.6)$$

When applied to the vector horizontal wind $\mathbf{u} = u\mathbf{i} + v\mathbf{j}$, care must be taken to account for the rate of change of the unit vectors $\mathbf{i}$ and $\mathbf{j}$. One way of achieving this is to expand the material derivative as (Vallis, 2017, Chapter 2)

$$\begin{aligned} \frac{D\mathbf{u}}{Dt} &= \frac{Du}{Dt}\mathbf{i} + \frac{Dv}{Dt}\mathbf{j} + u\frac{D\mathbf{i}}{Dt} + v\frac{D\mathbf{j}}{Dt}, \\ &= \frac{Du}{Dt}\mathbf{i} + \frac{Dv}{Dt}\mathbf{j} + \frac{u^2 \tan \vartheta}{a}\mathbf{j} - \frac{uv \tan \vartheta}{a}\mathbf{i}. \end{aligned} \quad (2.7)$$

∇_p is the horizontal gradient operator acting along surfaces of constant pressure. When applied to a scalar φ it has the form

$$\nabla_p \varphi = \left(\frac{1}{a \cos \theta} \frac{\partial \varphi}{\partial \lambda} \right) \mathbf{i} + \left(\frac{1}{a} \frac{\partial \varphi}{\partial \vartheta} \right) \mathbf{j}. \quad (2.8)$$

The horizontal divergence is expanded as

$$\nabla_p \cdot \mathbf{u} = \frac{1}{a \cos \vartheta} \frac{\partial u}{\partial \lambda} + \frac{1}{a \cos \vartheta} \frac{\partial v \cos \vartheta}{\partial \vartheta}. \quad (2.9)$$

It is worth noting that the thermodynamic equation (2.4) can be rewritten in terms of the potential temperature $\theta = \Lambda T$, where $\Lambda = (p_0/p)^{R/c_p}$, so that

$$\frac{D\theta}{Dt} = \dot{Q}_\theta - D_\theta. \quad (2.10)$$

The potential temperature is a marker for specific entropy, and is materially conserved in the absence of diabatic heating and cooling.

The primitive equations are a simplified version of the full Navier–Stokes equations, derived from the full equations by i) assuming that the vertical momentum equation can be approximated by a statement of *hydrostatic balance* (2.2), ii) simplifying derivatives in the radial direction $r = a + z$ by replacing r with a except where r is the differentiating argument (Vallis, 2017, Chapter 2), e.g.,

$$\frac{1}{r^2} \frac{\partial(r^2 u)}{\partial r} \rightarrow \frac{\partial u}{\partial z}, \quad (2.11)$$

and iii) neglecting Coriolis and metric terms involving the vertical velocity w . Approximations ii) and iii) are often called the *shallow-fluid approximation* and the *traditional approximation* respectively. They are valid when $\gamma = H/a \ll 1$, where $H = RT_0/g$ is a pressure scale height. The hydrostatic approximation is valid when the aspect ratio of the motion is small, namely $\alpha = H/L \ll 1$, where L is the horizontal scale of motion. For large-scale phenomena $L \sim a$, and the approximations are coincident.

The simple form of the primitive equations compared with the full equations facilitates their use for diagnostic analysis of meteorological phenomena, and the derivation of some approximate analytic solutions. Additionally, the approximations made to obtain the primitive equations exclude fast vertically propagating sound waves, which enables the use of longer timesteps when integrating the equations numerically. For these reasons the work in this thesis will use the primitive equations, and further simplifications to them, almost exclusively.

2.2 Non-dimensionalisation

Generally speaking, the different dynamical regimes that can be obtained by a planetary atmosphere will correspond to different balances between the various terms in the

equations of motion. In order to elucidate the relative sizes of each of the terms, we can non-dimensionalise the equations.

To begin, we set $\mathbf{u} = U\hat{\mathbf{u}}$ and assume that the Lagrangian rate of change scales with the advective terms so that $D/Dt = \tau_{\text{adv}}^{-1}D/\hat{D}t$ where $\tau_{\text{adv}} = aU^{-1}$. From the continuity equation, we set $\omega = a^{-1}Up_s\hat{\omega}$. Following Hoskins and James (2014), we make use of a reference atmosphere for the thermodynamic variables, so that $\phi = \phi_R(p) + \phi'$, and $\theta = \theta_R(p) + \theta'$. The reference atmosphere is chosen so that it is in hydrostatic balance. The potential temperature perturbation is non-dimensionalised as $\theta' = \delta\theta\hat{\theta}'$, and from hydrostatic balance $\phi' = R\delta\theta\hat{\phi}'$. The heating and dissipation terms are non-dimensionalised as $D_u = \tau_d^{-1}U\hat{D}_u$, $D_\theta = \tau_d^{-1}\delta\theta\hat{D}_\theta$ and $\dot{Q}_\theta = \tau_r^{-1}\delta\theta\hat{\dot{Q}}_\theta$, where τ_d is a drag timescale and τ_r is the radiative timescale.

With these substitutions, the horizontal momentum equation may be written

$$\mathcal{R}\frac{D\hat{\mathbf{u}}}{\hat{D}t} + \hat{\mathbf{f}} \times \hat{\mathbf{u}} = -\frac{R\delta\theta}{U\Omega a}\hat{\nabla}_p\hat{\phi}' - \mathcal{E}\hat{D}_u, \quad (2.12)$$

where

$$\mathcal{R} \equiv \frac{U}{\Omega a}, \quad (2.13) \quad \text{and} \quad \mathcal{E} \equiv \frac{1}{\Omega\tau_d}. \quad (2.14)$$

$\mathcal{R}$ is a global Rossby number, and $\mathcal{E}$ is an Ekman number. The value of the Ekman number indicates the relative importance of surface dissipative effects to rotation, and is generally small away from the surface (where friction is important). Assuming small $\mathcal{E}$, then if the Rossby number is small, a balance called *geostrophic balance* between Coriolis forces and horizontal pressure gradients dominates the dynamics. If the Rossby number is order one or greater, then the dominant balance is between nonlinear terms contained within the material derivative and horizontal pressure contrasts, termed *cyclostrophic balance*.

For the thermodynamic equation, it is simplest to work with the potential temperature version given by (2.10). To non-dimensionalise the $\partial\theta_R/\partial p$ term, we make use of the definition of the Buoyancy frequency N , $N^2 = g/\theta(\partial\theta/\partial z)$ to set $\partial\theta_R/\partial p = H^2N^2/(Rp)(\partial\hat{\theta}_R/\partial\hat{p})$, where N is a characteristic value for the buoyancy frequency, and $H = R\theta_0/g$ is a characteristic value for the pressure scale height. Then, the thermodynamic equation may be written

$$\mathcal{R}\frac{D\hat{\theta}'}{\hat{D}t} + \frac{U\Omega a}{R\delta\theta}\mathcal{B}\hat{\omega}\frac{\partial\hat{\theta}_R}{\partial\hat{p}} = \mathcal{I}\hat{\dot{Q}}_\theta - \mathcal{E}\hat{D}_\theta, \quad (2.15)$$

where

$$\mathcal{B} \equiv \frac{N^2 H^2}{\Omega^2 a^2} \quad (2.16)$$

is a global Burger number, which is the square of the ratio of the deformation scale l_d , defined by (1.2), to the planetary radius. The Burger number indicates the importance of the stratification in determining the dynamics. $\mathcal{E}$ is the Ekman number, as before, and $\mathcal{I}$ is a radiative damping number, defined

$$\mathcal{I} \equiv \frac{1}{\Omega \tau_r} \quad (2.17)$$

that determines the importance of radiative heating and cooling for the dynamics.

In both non-dimensionalised equations, there is a factor $R\delta\theta/(U\Omega a)$ present in one of the terms. In order to simplify these equations, it is common (e.g., Read, 2011; Vallis, 2019) to set $U = R\delta\theta/(\Omega a)$ based on a ‘thermal wind’ scale. With this substitution, the two equations read

$$\mathcal{R}_T \frac{D\hat{\mathbf{u}}}{D\hat{t}} + \hat{\mathbf{f}} \times \hat{\mathbf{u}} = -\hat{\nabla}_p \hat{\phi}' - \mathcal{E} \hat{D}_u, \quad (2.18)$$

and

$$\mathcal{R}_T \frac{D\hat{\theta}'}{D\hat{t}} + \mathcal{B} \hat{\omega} \frac{\partial \hat{\theta}_R}{\partial \hat{p}} = \mathcal{I} \hat{Q}_\theta - \mathcal{E} \hat{D}_\theta, \quad (2.19)$$

where

$$\mathcal{R}_T = \frac{R\delta\theta}{\Omega^2 a^2} \quad (2.20)$$

is the called the thermal Rossby number.

2.3 Circulation driven by zonally symmetric, time-independent forcing

To investigate the influence of various factors on planetary atmospheric circulation, a number of authors have leveraged idealised general circulation models to survey the parameter space discussed in the previous section (as reviewed by Showman et al., 2010; Read, 2011; Read et al., 2018; Mitchell et al., 2019).

Perhaps the simplest model that can be constructed for this task is one where the external forcing and dissipation are defined to be zonally symmetric and time-independent. This involves excluding zonal asymmetries from the surface (i.e., excluding topography), as well as diurnal and seasonal variations from the radiative heating. One way to achieve this is to parametrise the forcing term on the right-hand-side of the thermodynamic equation with a linear Newtonian cooling scheme, so that

$$\dot{Q} = -\frac{T - T_r}{\tau_r}, \quad (2.21)$$

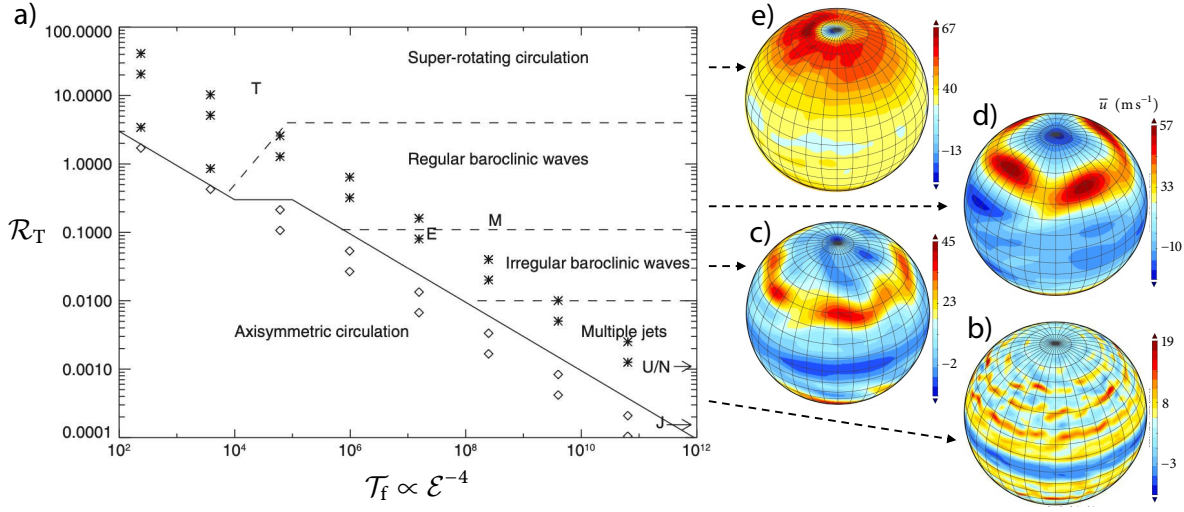


Figure 2.1: Regime diagram adapted from Wang et al. (2018), showing the circulation regimes obtained in an idealised general circulation model as a function of thermal Rossby number $\mathcal{R}_T$ and frictional Taylor number $\mathcal{T}_f$. The approximate locations of some of the solar system planets are indicated by their initial letters. Snapshots of zonal wind at 200 hPa for each regime are shown on the right.

where T_r is a relaxation temperature field that is zonally symmetric and time-independent. T_r is commonly defined following Held and Suarez (1994) to describe the Earth’s annually-averaged radiative-convective equilibrium temperature (Potter et al., 2014; Wang et al., 2018), although some authors have also constructed T_r to yield a temperature structure characteristic of Venus (Yamamoto and Takahashi, 2016; Tsunoda et al., 2021). A further option is to represent $\dot{Q}$ with a radiative transfer scheme where the solar forcing is defined to be zonally and time-independent, although often still defined to yield a temperature structure similar to the Earth’s (Kaspi and Showman, 2015).

2.3.1 Importance of the thermal Rossby number

Experiments conducted with simplified models of this type have revealed a dominant role for the thermal Rossby number $\mathcal{R}_T$ (which measures the importance of planetary rotation) in determining the atmospheric circulation regime obtained (Potter et al., 2014; Kaspi and Showman, 2015; Wang et al., 2018). These regimes are schematised in the *regime diagram* shown in Figure 2.1a, obtained from Wang et al. (2018), by analogy with regime diagrams constructed from the results of rotating annulus experiments (see, e.g., Read et al., 2014, for a review). Experiments with small $\mathcal{R}_T \ll 1$ (i.e., rapid rotation, large radius) occupy a regime characterised by multiple zonal jets in each hemisphere (Figure 2.1b), consistent with observations of multiple jets on Jupiter (which is large and rapidly rotating). As $\mathcal{R}_T$ is increased (reducing rotation rate, radius), regimes characterised by baroclinic planetary waves and a single jet in mid-latitudes are obtained

(Figure 2.1c). For $\mathcal{R}_T = 0.1$, characteristic of Earth, these waves are highly irregular with mixed wavenumbers and aperiodic time dependence. When $\mathcal{R}_T$ is made slightly larger, so as to be characteristic of Mars, the planetary waves assume a structure that is far more regular in spatial and temporal structure (Figure 2.1d). Finally, when $\mathcal{R}_T$ is made large (slow rotation, small planetary size) a regime characterised by equatorial superrotation is obtained (Figure 2.1e). This is consistent with the observation of superrotation on Venus and Titan, although it should be noted that the wind speed obtained in the idealised GCM ($\bar{u} \sim 40 \text{ m s}^{-1}$) is much slower than on either Venus or Titan ($\bar{u} \gtrsim 100 \text{ m s}^{-1}$).

Zonal-mean cross-sections of the atmospheric circulation obtained in similar experiments performed by Kaspi and Showman (2015), but with semi-grey radiative transfer in place of the Newtonian cooling used by Wang et al. (2018), are shown in Figure 2.2. A similar dependence of the atmospheric circulation on rotation rate (or $\mathcal{R}_T$) can be identified: the simulation with $\Omega = 4\Omega_E$ features multiple jets in mid-latitudes, and when the rotation rate is reduced to $\Omega = \Omega_E/4$, a circulation featuring a superrotating equatorial jet is obtained. Accompanying these different circulation regimes are marked differences in the eddy momentum fluxes ($\overline{u'v'}$, shown in the right-hand column of Figure 2.2 as solid contours). The experiments with $\Omega = 4\Omega_E$ and Ω_E feature horizontal eddy momentum flux convergence ($\partial\overline{u'v'}/\partial y < 0$) in mid-latitudes, whereas the superrotating $\Omega = \Omega_E/4$ simulation features eddy momentum flux convergence at the equator.

Some further trends in the atmospheric circulation can be identified in Figure 2.2. In this figure, the meridional overturning (Hadley) circulation is shown using the Eulerian mean meridional mass streamfunction, defined as

$$\Psi = \frac{2\pi a \cos \vartheta}{g} \int_0^{p_s} \bar{v} dp \quad (2.22)$$

(left-hand column, colour contours). Positive values correspond to clockwise overturning, and negative values correspond to anti-clockwise overturning. The Hadley circulation can be identified as pairs of cells that feature air rising at the equator, before moving polewards in the upper troposphere (around $p = 400 \text{ hPa}$). A clear dependence of the Hadley cell width on rotation rate can be identified in Figure 2.2, with the meridional extent of the cells extending from $\pm 5^\circ$ latitude when $\Omega = 4\Omega_E$ (small $\mathcal{R}_T$) to $\pm 30^\circ$ latitude when $\Omega = \Omega_E/4$ (large $\mathcal{R}_T$). This is accompanied by an increase in the strength of the overturning circulation, as well as an expansion of the region of ‘flat’ temperature contours (right column, colour contours) associated with meridional heat transport by the overturning circulation. Similar phenomenology is also identified in the Newtonian cooling experiments of Wang et al. (2018) that are shown in Figure 2.1. Figure 2.3 shows

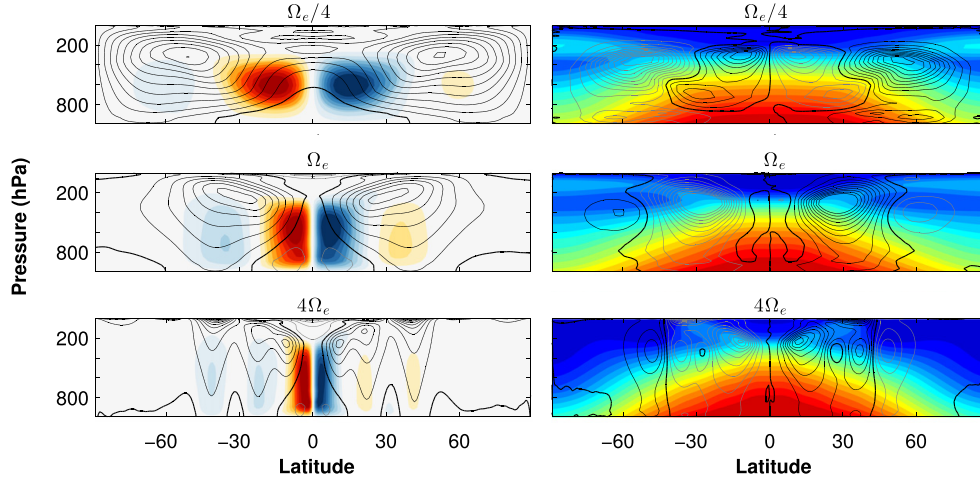


Figure 2.2: Zonal-mean circulation obtained from idealised GCM experiments varying the planetary rotation rate, adapted from Kaspi and Showman (2015). The left-hand column shows zonal-mean zonal wind (contours, interval 5 m s^{-1} with zero contour bold) and the Eulerian mean meridional mass streamfunction (colours, red indicates clockwise circulation, maximum values are: 4.2 , 1.9 , and $0.6 \times 10^{11} \text{ kg s}^{-1}$, from top to bottom). The right-hand column shows zonal-mean temperature (colour, from 210 to 290 K) and eddy momentum flux $u'v'$ (contours, with positive values black and negative values grey, indicating northward and southward transport of momentum, respectively).

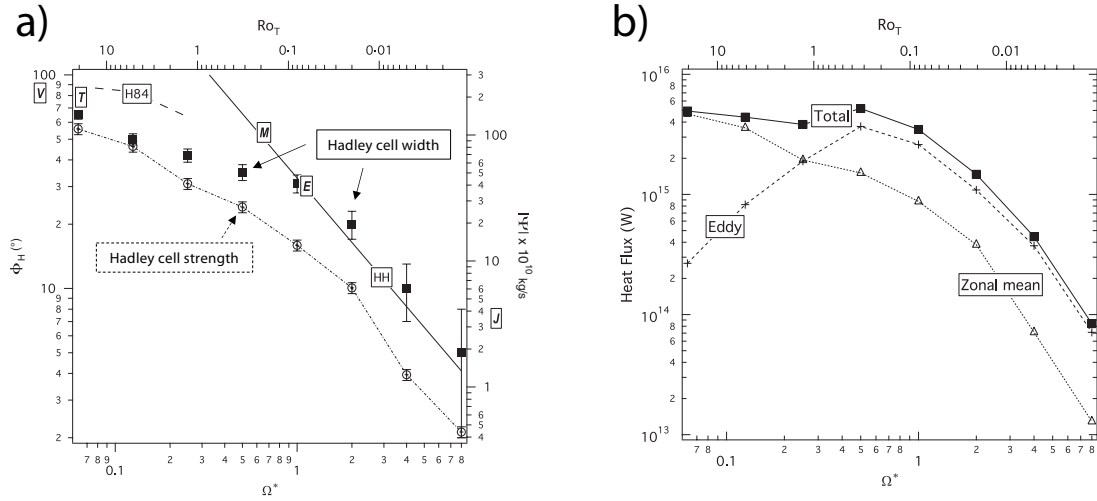


Figure 2.3: Dependence of a) Hadley cell properties and b) Maximum meridional heat transport on planetary rotation rate in idealised simulations, adapted from Wang et al. (2018). Panel a) shows the Hadley cell width (left y-axis, solid markers) and meridional mass streamfunction strength (right y-axis, open markers), as well as theoretical scaling relations for the Hadley cell width from Held and Hou (1980) and Hou (1984) (marked HH and H84, respectively). Panel b) shows the mass-weighted vertical integral of the meridional heat flux $\int c_p \bar{v} T' dp/g$ (solid line) separated into contributions from the zonal mean circulation ($\bar{v} T'$, dotted line) and eddies ($v' T'$, dashed line).

the thermal Rossby number dependence of Hadley cell strength and width (panel a), as well as the meridional heat flux due to the mean flow and eddies (panel b), obtained by Wang et al. (2018). The dependence of the Hadley cell width on the thermal Rossby number is in approximate agreement with scaling relations derived from axisymmetric theory (also shown in the figure; due to Held and Hou, 1980, for the rapidly rotating limit, and Hou, 1984, for the slowly rotating limit). Figure 2.3b shows that as rotation rate is decreased, the maximum meridional heat flux, $\overline{vT}$, increases. This is partially due to an increase in zonal-mean heat transport, $\overline{vT}$, by the Hadley circulation as it expands and strengthens when rotation rate is reduced. For more rapidly rotating simulations, however, the dependency of meridional heat transport is dominated by the contribution from eddies, $\overline{v'T'}$, which increases as rotation rate is decreased towards $\Omega_E/2$ due to an increase in the mid-latitude deformation scale (Equation 1.2; Kaspi and Showman, 2015; Wang et al., 2018). For rotation rates less than $\Omega_E/2$, the eddy heat transport term is reduced as baroclinic instability is suppressed in favour of barotropic instability at low rotation rates.

2.3.2 Other input parameters

Thus far, we have only discussed the dependence of planetary atmospheric circulation on the rotation rate or radius, through their effect on the thermal Rossby number. In Section 2.2 three further parameters that influence atmospheric dynamics were identified, namely the Ekman number $\mathcal{E}$, radiative damping number $\mathcal{I}$, and Burger number $\mathcal{B}$. Of these parameters, $\mathcal{E}$ and $\mathcal{I}$ have the effect of driving the atmospheric dynamics towards axisymmetry if they are made large (i.e., strong damping). This is highlighted in the lower left-hand corner of the regime diagram shown in Figure 2.1, and illustrated in Figure 2.4 with a series of idealised GCM experiments configured following Held and Suarez (1994), for a planet with $a = 2a_E$, and different friction and radiative timescales. Eddies in mid-latitudes are clearly damped in the center panel (increased friction) when compared with the left-hand panel (control experiment), and the circulation in the right-hand panel (increased friction and radiative cooling) is almost entirely axisymmetric. The zonal-mean zonal wind in these simulations (not shown) transitions from a regime with multiple eddy driven jets in the control experiment, to a regime with a single sub-tropical jet associated with angular momentum conservation in the overturning circulation. These results are consistent with those presented in Kaspi and Showman (2015) and Chemke and Kaspi (2017), who investigate the effect of varying the atmospheric mass on atmospheric circulation. Both studies found that the energy containing wavenumber decreased as surface pressure was reduced (increased radiative

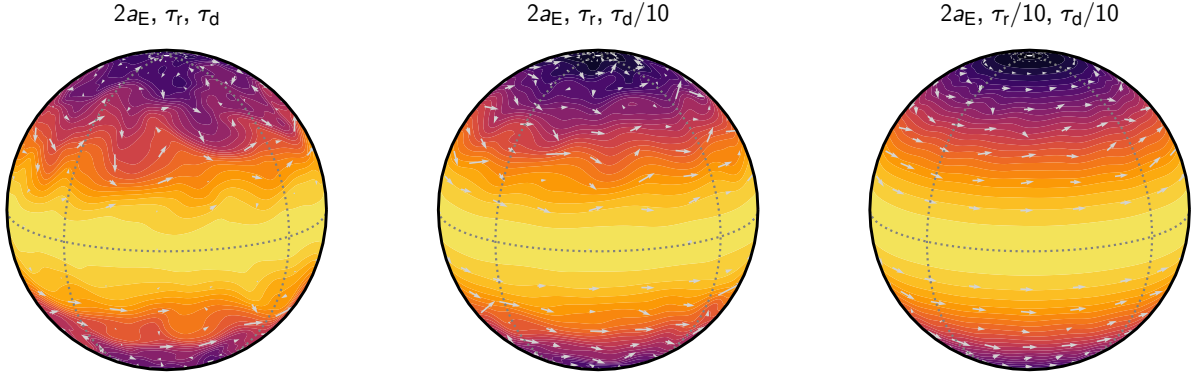


Figure 2.4: Geopotential height (colour) and velocity (quivers) at 600 hPa for three simulations showing the effect of friction and damping on atmospheric circulation. The left-hand panel shows a simulation with radius $a = 2a_E$, radiative timescale $\tau_r = 40$ days and friction timescale $\tau_d = 1$ day. The centre panel shows a simulation where the drag timescale has been reduced by a factor of 10, and the right-hand panel shows a simulation where the drag timescale *and* the radiative timescale have been reduced by a factor of 10. Data was produced with numerical simulations following Held and Suarez (1994) using the Isca modeling framework (Vallis et al., 2018).

cooling), and Chemke and Kaspi (2017) also show that the circulation transitions from a multiple eddy-driven jet circulation to a single sub-tropical jet circulation when the surface pressure is reduced. For planets in the large $\mathcal{R}_T$, slowly rotating regime, Dias Pinto and Mitchell (2014) show that large $\mathcal{I}$ can act to inhibit superrotation. This is because stronger radiative damping induces a stronger overturning circulation that can more effectively communicate the effect of surface friction to the jet region aloft, decelerating the superrotation.

Finally, we briefly discuss the role of the Burger number $\mathcal{B}$. The Burger number differs from $\mathcal{R}_T$ only by a factor of $\Delta\theta_v/\delta\theta$ (where $\Delta\theta_v$ is a measure of the mean vertical potential temperature contrast in the atmosphere), and so generally there will not be a significant distinction between ‘high (or low) Rossby number’ circulations and ‘high (or low) Burger number’ circulations. However, in principle, the two parameters can be varied independently via the stratification $\Delta\theta_v$. Studies that vary the stratification independently in GCM simulations have shown that increasing the stratification for a planet with an Earth-like thermal Rossby number leads to a poleward expansion of the Hadley circulation (Korty and Schneider, 2008; Levine and Schneider, 2015). This is associated with the suppression of baroclinic instability when the stratification is strong (Pierrehumbert and Swanson, 1995)¹. Kidston et al. (2011) showed that the eddy length scale increases as the stratification is increased, as might be expected from the

¹Baroclinic instability has been proposed to set the Hadley circulation extent on rapidly rotating planets (Walker and Schneider, 2006).

dependence of the Rossby deformation radius l_d (1.2) on the buoyancy frequency N . The effect of the stratification on the circulation of slowly rotating planets has not yet received significant attention in the literature, although we note that enhanced stratification can lead to the onset of superrotation (again, associated with the suppression of mid-latitude baroclinic instability; Williams, 2003; Laraia and Schneider, 2015), which could be of relevance to the processes that generate and maintain superrotation on slowly rotating planets (Zurita-Gotor et al., 2022).

To facilitate the comparison of circulation regimes obtained in idealised models, we have discussed them in terms of non-dimensional *input* parameters (such as the thermal Rossby number, $\mathcal{R}_T$). However, the utility of input parameters for this task is limited by the sensitivity of atmospheric circulation to multiple input parameters. This topic is examined further in Chapter 3, which investigates the extent to which different circulation regimes can be characterised in terms of a bulk *diagnostic* called the *global superrotation index*, S . The global superrotation index compares the mass-weighted integral of the specific zonal angular momentum m with a state of solid-body corotation with the underlying planet. It is defined as (Read, 1986b; Read and Lebonnois, 2018)

$$S \equiv \frac{\int \rho m dV}{\int \rho \Omega a^2 \cos^2 \vartheta dV} - 1, \quad (2.23)$$

where

$$m = a \cos \vartheta (\Omega a \cos \vartheta + u), \quad (2.24)$$

and dV is an element of volume.

2.4 Axisymmetric circulation

In the previous two sections, we have discussed how atmospheric circulation depends on planetary parameters, with a focus on the thermal Rossby number $\mathcal{R}_T$ (or, in terms of dimensional parameters, the planetary rotation rate and radius). In the next three sections, we will focus our attention onto phenomena of particular importance in the large $\mathcal{R}_T$ (small radius or rotation rate) regime. This section begins by discussing the dynamics of an axisymmetric (two-dimensional) atmosphere, which yields insight into the mechanisms that determine the width of the meridional overturning (Hadley) circulation, as well as the importance of eddies in a fully three-dimensional atmosphere for accelerating superrotation. Then, in Section 2.5 and Section 2.6 we will discuss three-dimensional phenomena: first, the theory of linear waves on the equatorial beta plane, and second, the proposed mechanisms that give rise to superrotation on planets with large $\mathcal{R}_T$.

2.4.1 Held and Hou model for the Hadley circulation

The idealised GCM experiments discussed in [Section 2.3](#) show that both the strength and width of the Hadley circulation depend strongly on the thermal Rossby number (Kaspi and Showman, 2015; Wang et al., 2018).

There are two mechanisms known to constrain the width of the Hadley circulation on Earth to the sub-tropics. The first is that it terminates at the latitude where an angular momentum conserving overturning circulation would become baroclinically unstable. This idea was discussed by Lorenz (1969), and numerical studies have since shown that baroclinic eddies do have an important influence on the Hadley circulation (e.g., Walker and Schneider, 2006). However, this mechanism does not explain why the Hadley circulation extent is limited in GCM simulations with slower rotation rates within which baroclinic instability is suppressed, such as those shown in [Figure 2.2](#) and [Figure 2.3](#), where, even for the most slowly rotating simulations, the Hadley circulation terminates before reaching the poles.

The mechanism by which the Hadley circulation is restricted in the absence of non-axisymmetric effects was demonstrated by Held and Hou (1980; hereafter [HH](#)), building on earlier work by Schneider (1977). These authors showed that an idealised axisymmetric circulation can be constrained by the combined requirements for it to conserve energy and angular momentum.

The [HH](#) model is derived from the Boussinesq hydrostatic primitive equations on a sphere. It consists of essentially two layers; a lower layer with $u \approx 0$, and an upper layer (the free atmosphere) which is in approximate gradient wind balance with the model temperature field:

$$\frac{\partial}{\partial z} \left(f u + \frac{u^2 \tan \vartheta}{a} \right) = -\frac{g}{a \theta_0} \frac{\partial \theta}{\partial \vartheta}, \quad (2.25)$$

where the height is being used as the vertical coordinate. The circulation is forced by a linear relaxation to a radiative-convective equilibrium potential temperature field, given by

$$\frac{\theta_{\text{eq}}}{\theta_0} = 1 - \frac{2}{3} \Delta_h P_2(\sin \vartheta) + \Delta_v \left(\frac{z}{H} - \frac{1}{2} \right), \quad (2.26)$$

where $\theta_0 \Delta_h$ is the equator-to-pole surface temperature difference, and $\theta_0 \Delta_v$ is the vertical potential temperature difference. $P_2(y) = (3y^2 - 1)/2$ is the second Legendre polynomial. These equations may be vertically integrated to yield

$$f u + \frac{u^2 \tan \vartheta}{a} = -\frac{g H}{a \theta_0} \frac{\partial \hat{\theta}}{\partial \vartheta} \quad \text{at} \quad z = H, \quad (2.27)$$

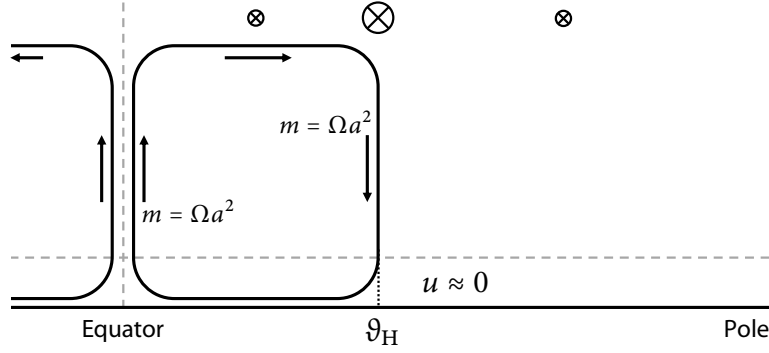


Figure 2.5: A schematic for the atmospheric circulation in an axisymmetric atmosphere. The atmosphere is divided into a quiescent boundary layer where $u \approx 0$, and the free atmosphere where $u \neq 0$. An angular momentum conserving overturning cell extends to some latitude ϑ_H , beyond which there is no meridional overturning. The crossed circles indicate the magnitude of prograde flow aloft.

and

$$\hat{\theta}_{\text{eq}} = \theta_0 \left[1 - \frac{2}{3} \Delta_h P_2(\sin \vartheta) \right], \quad (2.28)$$

where a hat denotes a vertical average $H^{-1} \int_0^H dz$, and H is the domain height.

At $z = H$, the circulation is divided into two regions: a tropical region, and an extra-tropical region. In the extra-tropical region, $v = 0$, and so u is determined through gradient wind balance with the radiative equilibrium temperature field (i.e., u_{ET} is given by Equation 2.27 with $\hat{\theta} = \hat{\theta}_{\text{eq}}$). In the Hadley cell region, u is determined by conservation of specific angular momentum m , which, if $u = 0$ at the equator, implies $m = \Omega a^2$ and $u_{\text{HC}} = \Omega a \sin^2 \vartheta / \cos \vartheta$. The vertically averaged potential temperature field in the Hadley cell region is then obtained from gradient wind balance, which yields

$$\frac{\hat{\theta}(0) - \hat{\theta}}{\theta_0} = \frac{u_{\text{HC}}^2}{2gH}. \quad (2.29)$$

The circulation in the HH model is schematised in Figure 2.5.

To obtain expressions for the boundary latitude ϑ_H , and the vertically-averaged potential temperature at the equator $\hat{\theta}(0)$, HH enforce two matching conditions:

$$\hat{\theta}_{\text{eq}}(\vartheta_H) = \hat{\theta}(\vartheta_H), \quad (2.30)$$

and

$$\int_0^{\vartheta_H} \hat{\theta} \cos \vartheta d\vartheta = \int_0^{\vartheta_H} \hat{\theta}_{\text{eq}} \cos \vartheta d\vartheta. \quad (2.31)$$

The first requires that potential temperature be continuous across the Hadley cell boundary, and the second requires that the Hadley cell is energetically closed. If the expressions

for temperature in the tropical and extra-tropical regions, (2.28) and (2.29), are substituted into (2.30) and (2.31) then the following expression can be obtained for the Hadley cell latitude:

$$\mathcal{R}_T = \frac{3}{4} \left[\frac{1}{3} + \frac{1}{y_H^2} + \frac{y_H^2}{1 - y_H^2} - \frac{1}{2y_H^3} \ln \left(\frac{1 + y_H}{1 - y_H} \right) \right], \quad (2.32)$$

where $y_H = \sin \vartheta_H$, and $\mathcal{R}_T = gH\Delta_h/(\Omega^2 a^2)$. For limiting values of $\mathcal{R}_T$, approximate analytic solutions to (2.32) can be obtained:

$$\sin \vartheta_H = \begin{cases} \left(\frac{5}{3} \mathcal{R}_T \right)^{\frac{1}{2}}, & \mathcal{R}_T \ll 1, \\ 1 - \frac{3}{8\mathcal{R}_T}, & \mathcal{R}_T \gg 1. \end{cases} \quad (2.33)$$

Noting that $\sin \vartheta_H \approx \vartheta_H$ when ϑ_H is small, the result in the $\mathcal{R}_T \ll 1$ limit is Held and Hou's celebrated scaling for the Hadley cell width. The second limit is that obtained by Hou (1984), which is appropriate for slowly rotating or small sized planets with $\mathcal{R}_T \gg 1$.

For finite values of $\mathcal{R}_T$, the theory predicts that $\vartheta_H < \pi/2$, that is, that the Hadley circulation is constrained to terminate before it reaches the pole. This is the key result of HH, namely that a thermally-driven overturning circulation is restricted from extending all of the way towards the pole, even in the absence of baroclinic instabilities (or other non-axisymmetric effects). Furthermore, the theory finds that the meridional extent of the Hadley circulation is determined entirely by the thermal Rossby number, and expands as $\mathcal{R}_T$ is increased. This underscores the importance of this parameter for determining the dynamical regime occupied by a planetary atmosphere.

The scalings given by (2.33) generally compare well against those obtained from fully 3D GCM simulations, suggesting that the axisymmetric theory is useful for interpreting the phenomenology of real atmospheres (each scaling is shown in Figure 2.3, alongside results for ϑ_H obtained from the idealised GCM simulations of Wang et al., 2018). For example, substitution of $\mathcal{R}_T = 0.1$ (in the right ballpark for Earth) into (2.33) yields $\vartheta_H = 24.3^\circ$ latitude (in the small $\mathcal{R}_T$ limit), which is roughly where the Earth's Hadley cell terminates. Similarly, setting $\mathcal{R}_T = 20$, which is appropriate for Titan (Read and Lebonnois, 2018), yields $\vartheta_H = 79^\circ$, which is in rough agreement with that obtained in Titan general circulation models (Lora et al., 2019).

The HH model has subsequently been generalised by other authors. One deficiency of the model is that the zonal wind profile is discontinuous at the Hadley cell edge, particularly for large $\mathcal{R}_T$ (a phenomenon not observed in real atmospheres). This arises due to the 'continuity of potential temperature' matching condition at the Hadley cell

boundary (Equation 2.30). Colyer and Vallis (2019) consider the effects of a different matching condition, namely that the zonal velocity is continuous at the Hadley cell boundary (which introduces a discontinuity in temperature). They find that the original HH model gives a better prediction of the Hadley cell extent obtained in numerical simulations, but that their variant gives a better prediction of the sub-tropical jet speed than either u_{HC} or u_{ET} evaluated at ϑ_{H} for the original model. A further limitation of the HH model is that its forcing is highly idealised, using a linear Newtonian cooling formulation. Caballero et al. (2008) extend the model by replacing the linear forcing with a simplified treatment of non-grey radiative transfer and convective adjustment. They recover the same scaling as HH in the small angle approximation, multiplied by a pre-factor $\mathcal{F} < 1$ that depends on the radiative properties of the atmosphere. They note that the difference between their prediction for ϑ_{H} and that of HH will generally only be large for atmospheres with an optically thick stratosphere. Caballero et al. (2008) also relaxed the Boussinesq approximation when deriving their expression, and thus their results suggest that neither the Boussinesq approximation, nor the representation of radiative transfer as a linear relaxation, severely restrict the applicability of the HH model to real atmospheres. In Chapter 3 I will use the Held–Hou model to develop an axisymmetric scaling theory for the global superrotation index S defined by (2.23).

2.4.2 Hide’s theorem

Atmospheres are inherently three-dimensional, and the biggest limitation of the HH model is that it is axisymmetric. On slowly rotating planets, eddies are critical for accelerating superrotation at the equator, and consequently the HH model does not admit superrotation (Hou, 1984).

This can be appreciated by considering the zonally-averaged zonal momentum equation, which may be written (Vallis, 2017, Chapter 14)

$$\frac{\partial \bar{u}}{\partial t} + \left(\frac{1}{a \cos \vartheta} \frac{\partial \bar{u} \cos \vartheta}{\partial \vartheta} - f \right) \bar{v} + \bar{\omega} \frac{\partial \bar{u}}{\partial p} = - \frac{1}{a \cos^2 \vartheta} \frac{\partial}{\partial \vartheta} (\overline{u'v'} \cos^2 \vartheta) - \frac{\partial \overline{u'\omega'}}{\partial p} + \mathcal{V}, \quad (2.34)$$

where the $\mathcal{V}$ represents the possible effects of viscous friction, an overbar indicates a zonal-average, and a prime indicates a deviation from the zonal average. Equation (2.34) may be re-written in terms of the axial component of specific angular momentum m (defined by Equation 2.24) which yields

$$\frac{D\bar{m}}{Dt} = - \frac{1}{a \cos \vartheta} \frac{\partial}{\partial \vartheta} (\overline{m'v'} \cos \vartheta) - \frac{\partial \overline{m'\omega'}}{\partial p} + \mathcal{V}_m. \quad (2.35)$$

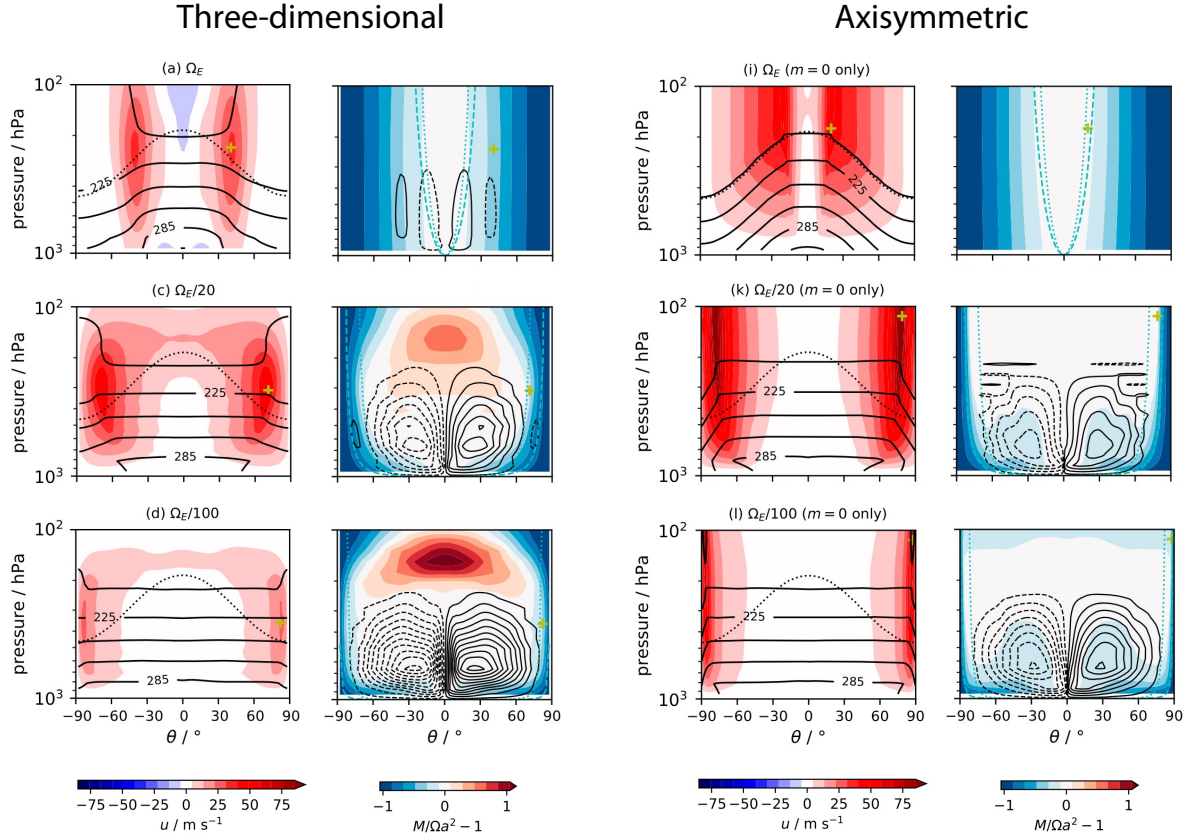


Figure 2.6: Zonally-averaged circulation obtained from idealised GCM experiments with rotation rates Ω_E , $\Omega_E/20$, and $\Omega_E/100$. The left-hand set of panels shows output from three-dimensional simulations and the right-hand set shows output from axisymmetric simulations. Within each set the left-hand column shows the zonal-mean zonal wind (colours) and temperature structure (black contours), and the right-hand column shows the local superrotation index (colours, defined by Equation 2.37) and the Eulerian mean meridional overturning streamfunction (black contours, negative dashed, defined by Equation 2.22). Figure adapted from Colyer and Vallis (2019).

If the atmosphere is axisymmetric ($X = \bar{X}$), and additionally inviscid, (2.35) reduces to

$$\frac{Dm}{Dt} = 0, \quad (2.36)$$

which states that the specific zonal angular momentum of any given air mass is materially conserved. It is this result that forms the basis of the HH model.

In such an atmosphere, there is therefore no mechanism by which m can be transported up-gradient. Given that air rising at the equator will be equilibrated with the underlying surface, so that $u = 0$ and $m = \Omega a^2$ (cf. Figure 2.5), this implies that $m \leq \Omega a^2$ everywhere in the atmosphere, and $u \leq 0$ at the equator. As such, an axisymmetric atmosphere may not superrotate unless it is initialised as such. This result was first presented by Hide (1969), and is often referred to as *Hide's theorem*. These considerations

also motivate the definition of a *local superrotation index* (Read, 1986a,b)

$$s \equiv \frac{m}{\Omega a^2} - 1, \quad (2.37)$$

defined so that Hide's theorem requires $s \leq 0$ in an axisymmetric atmosphere, and $s > 0$ indicates superrotation.

Generalising to an axisymmetric, but viscous atmosphere (i.e. $\mathcal{V} \neq 0$), Read (1986b) notes that *molecular viscosity*, which acts towards an end-point of uniform *angular velocity*, can transport angular momentum up-gradient, and so in principle could generate local superrotation. In practice, typical molecular viscosity coefficients for gases are far too small to balance the down-gradient transport of m by advection. A corollary that may be drawn here is that in the absence of an unphysically large molecular viscosity, non-axisymmetric disturbances that act to transport m up-gradient are *required* for the existence of a region that is superrotating. This is illustrated in Figure 2.6, which shows output from three-dimensional and axisymmetric GCM simulations, obtained from Colyer and Vallis (2019). At low rotation rate, local superrotation ($s > 0$) is present in the three-dimensional simulations, but not the axisymmetric simulations, where instead the Hadley cell fills the atmosphere with air that has $s = 0$ (or $m = \Omega a^2$, as in Figure 2.5; see also Mitchell and Vallis, 2010).

2.5 Equatorial waves

Atmospheric waves are an important feature of the atmospheric circulation of slowly rotating or small sized planets with large $\mathcal{R}_T$. In particular, their capacity to induce eddy momentum fluxes provides a mechanism through which equatorial superrotation can be maintained against poleward transport of angular momentum by meridional overturning circulation (Gierasch, 1975; Rossow and Williams, 1979).

On a slowly rotating planet, the atmospheric dynamics may be thought to occupy an 'all tropics' regime (Showman et al., 2013). The first reason for this is that the Hadley circulation expands towards the poles (as discussed in the previous section). In addition, the atmospheric wave dynamics become decidedly equatorial as their characteristic meridional scale (given by the equatorial deformation radius; Equation 1.3) becomes comparable to the planetary radius. In this section, we review the linear theory of equatorial waves on the beta plane. Then, in the next section, we discuss the mechanisms through which equatorial waves have been shown to accelerate superrotation.

Following Matsuno (1966) and Wheeler and Nguyen (2015), we begin with the linearised shallow water equations on the equatorial beta plane (Vallis, 2017, Chapter 3)

$$\frac{\partial u_l}{\partial t} - \beta y v_l = -\frac{\partial \phi_l}{\partial x}, \quad (2.38)$$

$$\frac{\partial v_l}{\partial t} + \beta y u_l = -\frac{\partial \phi_l}{\partial y}, \quad (2.39)$$

$$\frac{\partial \phi_l}{\partial t} + g h_l \left(\frac{\partial u_l}{\partial x} + \frac{\partial v_l}{\partial y} \right) = 0, \quad (2.40)$$

which correspond to the equations that govern the vertical normal modes of the continuously stratified equations (indicated by the subscript l). Working on the equatorial beta plane introduces the approximation $f = \beta y$. x is the east–west direction, y is the north–south direction, and h_l is the equivalent depth of a fluid layer.

To obtain a dispersion relation for the equatorial wave modes, we seek wave-like solutions of the form (Matsuno, 1966; Wheeler and Nguyen, 2015)

$$(u_l, v_l, \phi_l) = \text{Re} \left\{ [\hat{u}(y), \hat{v}(y), \hat{\phi}(y)] \exp[i(kx - \omega t)] \right\}, \quad (2.41)$$

where $\hat{u}$, $\hat{v}$, and $\hat{\phi}$ are functions that describe the meridional structure of the solutions, k is the zonal wavenumber, and ω is the frequency. If it is required that the solutions described by (2.41) remain bounded far from the equator, then substitution into the shallow water equations (2.38)–(2.40) yields the general dispersion relation

$$\frac{\sqrt{g h_l}}{\beta} \left(\frac{\omega^2}{g h_l} - k^2 - \frac{k}{\omega} \beta \right) = 2n + 1; \quad n = 0, 1, 2, \dots, \quad (2.42)$$

where n is a positive integer called the meridional mode number. Approximate solutions to (2.42) may be obtained by considering the limits where $\omega \gg 1$ and $\omega \ll 1$. If $n \geq 1$, then the high frequency approximation to (2.42) is

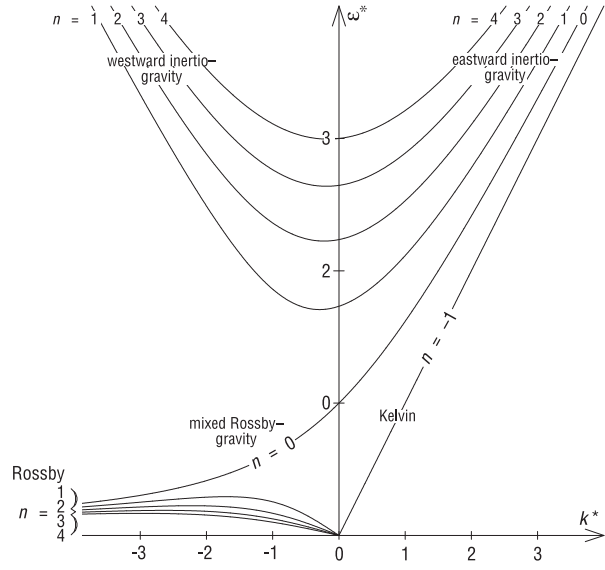
$$\omega_{\text{IG}} \approx \pm \left[(2n + 1) \beta \sqrt{g h_l} + k^2 g h_l \right]^{\frac{1}{2}}, \quad (2.43)$$

and the low frequency approximation is

$$\omega_{\text{ER}} \approx -\frac{\beta k}{k^2 + (2n + 1) \beta / \sqrt{g h_l}}. \quad (2.44)$$

Equation (2.43) has two roots corresponding to eastward (positive root) and westward propagating (negative root) inertio–gravity waves, respectively, and (2.44) has one root corresponding to westward propagating equatorial Rossby waves.

Figure 2.7: Dispersion curves for waves on the equatorial beta plane, shown as a function of frequency ω and zonal wavenumber k . Eastward propagating waves are shown with a positive wavenumber, and westward propagating waves are shown with a negative wavenumber. Obtained from Wheeler and Nguyen (2015).



For $n = 0$, an additional, exact solution to (2.42) exists,

$$\omega_{n=0} = k\sqrt{gh_l} \left[\frac{1}{2} \pm \frac{1}{2} \left(1 + \frac{4\beta}{k^2\sqrt{gh_l}} \right)^{1/2} \right]. \quad (2.45)$$

The two roots of (2.45) correspond to an eastward propagating inertio–gravity wave (positive root) and a mixed Rossby–gravity wave (negative root). A further exact wave-like solution exists to (2.38)–(2.40) if $\hat{v} = 0$ is set, namely

$$\omega_K = \sqrt{gh_l}k, \quad (2.46)$$

which corresponds to an eastward propagating Kelvin wave. The dispersion relations for each of these waves are shown in Figure 2.7 (obtained from Wheeler and Nguyen, 2015). The solutions shown are obtained without approximation from (2.42) for the inertio–gravity waves, mixed Rossby–gravity wave, and the equatorial Rossby wave, and from (2.46) for the Kelvin wave (also without approximation). Figure 2.8 shows the spatial structure of the zonal wavenumber-1 Kelvin, mixed Rossby–gravity, and $n = 1$ equatorial Rossby waves.

Each of the solutions to (2.38)–(2.40) have meridional structure functions that decay exponentially away from the equator according to $\hat{X}(y) \propto \exp[-y^2/(2l_e^2)]$, where l_e is the equatorial deformation radius, and defined by

$$l_e^2 = \frac{\sqrt{gh_l}}{\beta}. \quad (2.47)$$

These waves are therefore *equatorially trapped*, and can only propagate in the zonal and vertical directions (Andrews et al., 1987, Chapter 4). The shallow water equivalent depth

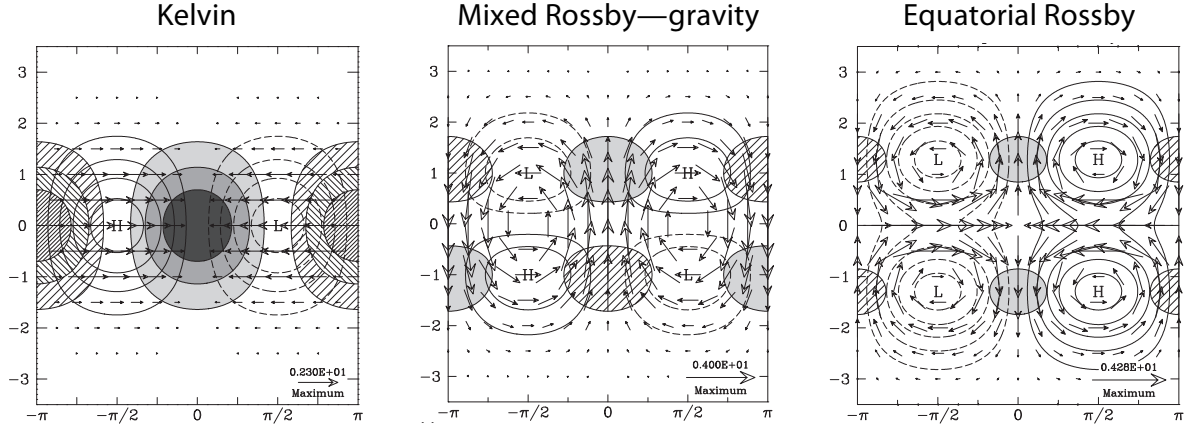


Figure 2.8: Horizontal structures of waves on the equatorial beta plane. Unshaded contours show the geopotential (negative dashed) and quivers show the velocity. Filled and hatched shading corresponds to regions of convergence and divergence, respectively. Adapted from Wheeler and Nguyen (2015).

is related to the vertical wavenumber m via

$$\sqrt{gh_l} = \frac{N}{[m^2 + 1/(4H^2)]^{1/2}}, \quad (2.48)$$

and so the expression for l_e given by (2.47) reduces to that quoted in the introduction (1.3) if the $1/(4H^2)$ term is neglected, which corresponds to making the Boussinesq approximation (Andrews et al., 1987, Chapter 4; Vallis, 2017, Chapter 7). For a slowly rotating or small sized planet, β is small, which means that l_e (and thus the waves) may be large, and comparable in meridional extent to the planetary radius. In this scenario, the beta plane approximation is no longer formally valid (although still useful), and the spherical geometry of the planet will affect the structure of the waves. An important example of this concerns the Kelvin wave, which has a non-zero meridional velocity on the sphere, in contrast with its counterpart on the beta plane (Yamamoto, 2019).

In Chapter 4 I will use space-time cross-spectral analysis to isolate the equatorial waves present in a GCM simulation of Titan’s atmosphere performed with the Titan Atmospheric Model (Lora et al., 2015). These waves are then discussed and identified in terms of those discussed in this section, obtained from linear theory on the equatorial beta plane.

2.6 Generation and maintenance of equatorial superrotation

2.6.1 Superrotation in idealised models

Equatorial superrotation appears to be a ubiquitous phenomenon in the atmospheres of planets that are slowly rotating or have a small radius (large $\mathcal{R}_T$) (Read and Lebonnois, 2018; Imamura et al., 2020). Both Venus (very slowly rotating) and Titan (small compared

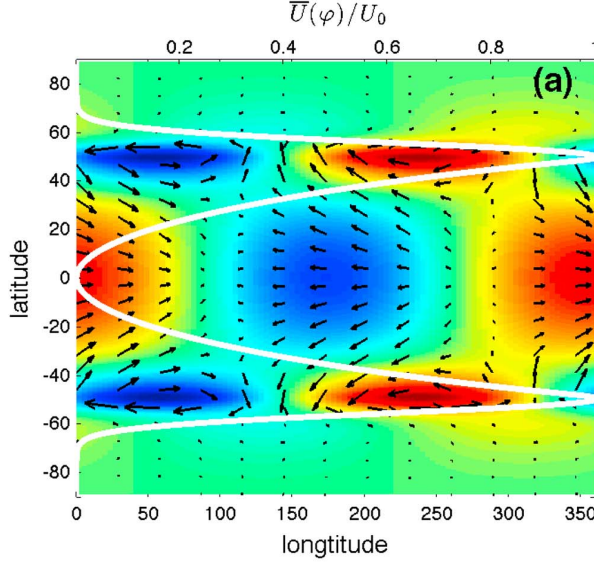


Figure 2.9: Horizontal structure of an Rossby–Kelvin mode obtained from a stability analysis of the primitive equations linearised about a background zonal wind profile. Horizontal velocities are shown as quivers and the pressure perturbation associated with the wave is shown as colour contours. The background zonal-mean zonal wind profile is shown as a white line. Obtained from Wang and Mitchell (2014).

to Earth, and slowly rotating) feature superrotation in their atmospheres — Read and Lebonnois (2018) estimate local superrotation indices (using Equation 2.37) of $s_V \approx 65$ and $s_T \approx 15$, respectively for Venus and Titan. Superrotation is also present in the slowly rotating Earth-like GCM simulations of Kaspi and Showman (2015) and Wang et al. (2018) shown in Section 2.3. Given the requirement of Hide’s theorem that equatorial superrotation is accelerated by up-gradient angular momentum transport due to non-axisymmetric disturbances, these observations motivate investigation into the phenomena that induce the required angular momentum transport, and why they are most pronounced in the large $\mathcal{R}_T$ regime.

To address this issue, a number of authors have investigated the transition to superrotation on large $\mathcal{R}_T$ planets using idealised Earth-like GCMs (del Genio et al., 1993; Williams, 2003; Mitchell and Vallis, 2010; Potter et al., 2014; Laraia and Schneider, 2015; Lu and Yamamoto, 2020). These studies have shown that equatorial superrotation emerges naturally in this context when either the planetary radius of rotation rate is reduced from Earth’s. Stronger superrotation is obtained when the planetary radius is reduced when compared with that due to an equivalent reduction in the planetary rotation rate (Dias Pinto and Mitchell, 2014). This occurs as a reduction of the rotation rate reduces the ratio of the radiative damping timescale to the advective timescale (i.e., the radiative damping number $\mathcal{I}$ in Section 2.2 is large), whereas a reduction in the radius does not.

In the idealised Earth-like setting, superrotation is commonly found to be generated by a planetary-scale, zonal wavenumber-1 equatorial wave mode that comprises a Kelvin component phase locked with a Rossby-like component in mid-latitudes (Mitchell and Vallis, 2010). An example of the horizontal structure of this type of wave is shown in

Figure 2.9, obtained from Wang and Mitchell (2014). Iga and Matsuda (2005) and Wang and Mitchell (2014) have shown that this sort of mode emerges due to a barotropic, ageostrophic instability, called *Rossby–Kelvin instability* (Sakai, 1989). This instability occurs when the deformation radius is large enough that the two component waves have some spatial overlap, and the meridional shear in the background wind (the white line in Figure 2.9) is substantial enough that the Rossby component (moving west relative to the mean flow) is sufficiently Doppler-shifted by the mid-latitude jet that it can phase-lock with the Kelvin component (moving east relative to the mean flow). Both of these phenomena are more likely to occur on planets with large $\mathcal{R}_T$, due to the dependence of the deformation radius and the sub-tropical jet speed on the rotation rate or radius.

An additional factor that contributes to the generation of superrotation at large $\mathcal{R}_T$ is that baroclinic instability in the extra-tropics is suppressed (Williams, 2003; Laraia and Schneider, 2015; Zurita-Gotor et al., 2022). This is due to the action of baroclinic eddies to converge momentum into their source region, which means that they decelerate the sub-tropical jet when they are active in the extra-tropics. Zurita-Gotor et al. (2022) show that this results in less favorable conditions for Rossby–Kelvin instability, as the Rossby component experiences a weaker Doppler-shift by the zonal-mean zonal wind. This explains the results of other studies that have found equatorial superrotation to emerge in idealised models with the Earth’s planetary parameters when the equator-to-pole radiative-convective equilibrium temperature contrast in a model with Newtonian cooling is made weaker (Williams, 2003; Polichtchouk and Cho, 2016).

In some circumstances, a Rossby–Kelvin mode generates superrotation in an idealised model’s spin-up phase (initiated from rest), but then disappears when the circulation equilibrates (Mitchell and Vallis, 2010; Dias Pinto and Mitchell, 2016). This occurs when the accelerated equatorial superrotation is strong enough to remove the meridional shear required for the Kelvin and Rossby components to phase lock. In this scenario, after spin-up, Dias Pinto and Mitchell (2016) show that equatorial Rossby waves, generated by a mixed barotropic–baroclinic instability, can induce the acceleration required to maintain superrotation.

2.6.2 *Superrotation in the Solar System*

Idealised models of Earth-like atmospheres show that superrotation emerges naturally when the thermal Rossby number is made large. However, the strength of superrotation in these models is generally far weaker ($\approx 20\text{--}50\text{ m s}^{-1}$) than that observed on Venus or Titan ($> 100\text{ m s}^{-1}$). This suggests that other properties of Venus and Titan

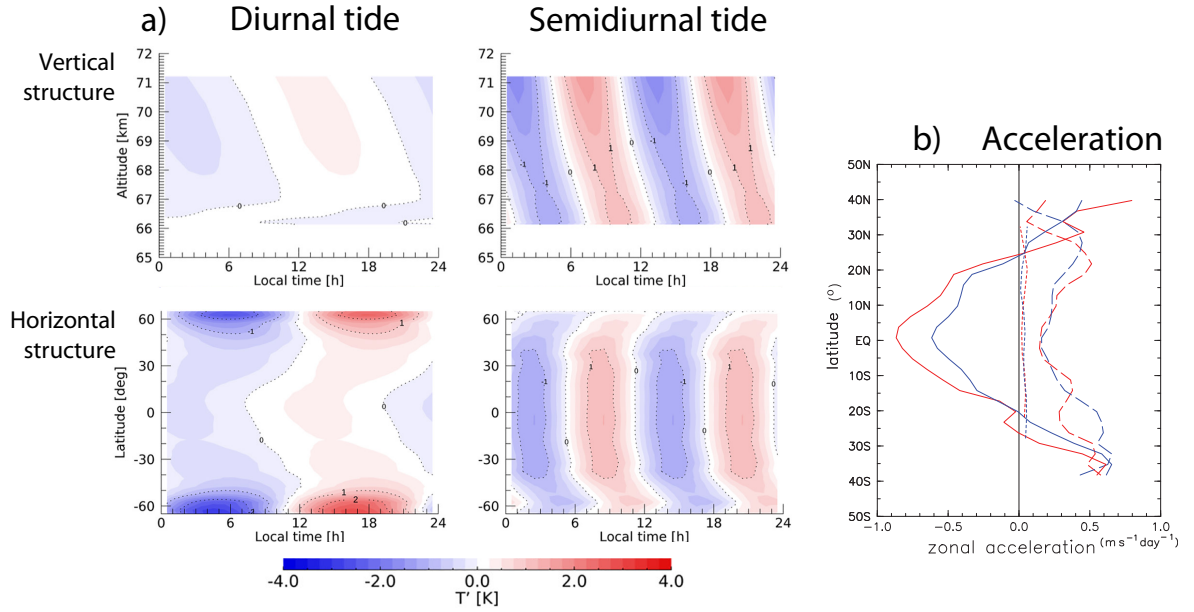


Figure 2.10: a) Spatial structure of Venus' thermal tides derived from measurements of thermal radiation made by the Longwave Infrared Camera instrument on board Akatsuki. The left column shows the diurnal tide and the right column shows the semidiurnal tide. The vertical structure displayed is representative of the region between 0° and 30° latitude. The horizontal structure is shown for at an altitude of 69 km. b) Zonal acceleration due to tides (solid lines), low frequency transients (dotted lines) and high frequency transients (dashed lines) obtained from cloud tracked winds (65–70 km altitude) derived from measurements made with the Ultraviolet Imager on board Akatsuki at 283 nm (red) and 365 nm (blue). The 283 nm results capture winds at a slightly higher altitude than the 365 nm results. Note that a negative acceleration in panel (b) indicates acceleration, not deceleration, of the superrotating jet, which is retrograde on Venus. The panels labelled (a) are adapted from Akiba et al. (2021), and panel (b) is obtained from Horinouchi et al. (2020).

play an important role in determining their zonal-mean atmospheric dynamics. Both Venus and Titan have atmospheres that are relatively massive compared to the Earth's (Griffith et al., 2013; Bullock and Grinspoon, 2013). Additionally, their atmospheres contain substances that strongly absorb incoming solar radiation (Tomasko et al., 2008; Bullock and Grinspoon, 2013). On Venus, the combination of direct solar heating in the atmosphere and Venus' long solar day leads to the generation of large amplitude thermal tides (forced gravity waves; Schofield and Taylor, 1983; Zasova et al., 2007; Akiba et al., 2021). The spatial structure of Venus' thermal tides is shown in Figure 2.10a, derived from measurements of thermal emission taken by the Longwave Infrared Camera on board the JAXA *Akatsuki* orbiter (adapted from Akiba et al., 2021). On Titan, seasonal variation in insolation (due to Saturn's obliquity of 27°) leads to pronounced seasonal variation in Titan's stratospheric winds (Achterberg et al., 2008; Sharkey et al., 2021).

A number of general circulation models have been developed to study Venus' atmospheric dynamics. Early models of Venus' atmosphere (Yamamoto and Takahashi,

2003; Lee et al., 2007), similar to the idealised Earth-like GCMs discussed in Section 2.3 but forced with a Venus-like relaxation temperature profile (derived from observations), obtained superrotation of a comparable magnitude to that observed in the real Venusian atmosphere. In this setting, the superrotation was shown to be accelerated by transient, planetary scale equatorial waves, similar to those discussed in the previous subsection. Comprehensive models of Venus’ atmosphere have since been developed that include a realistic representation of radiative transfer that includes Venus’ diurnal cycle (Lebonnois et al., 2010; Mendonça and Read, 2016). These models identify a dominant role for Venus’ thermal tides (particularly the semidiurnal tide) in accelerating superrotation. Both studies show that thermal tides act to transport angular momentum vertically downward in the equatorial region, which prevents it from being transported poleward by the Hadley circulation (which extends from equator to pole). They also identify a role for transient equatorial waves in Venus’ upper cloud layer in converging angular momentum horizontally from higher latitudes towards the equator, as in the idealised models. Horinouchi et al. (2020) derived estimates for horizontal and vertical angular momentum transport in Venus’ atmosphere due to tides and waves from cloud tracked wind velocities obtained using the Ultraviolet Imager (UVI) on board Akatsuki (sensing approximately 65 km altitude). They found that the tides act to converge momentum meridionally (shown in Figure 2.10b) and vertically into the cloud region, accelerating the superrotation, while transients or turbulence (i.e., mostly high-frequency variability) act to *decelerate* the cloud region. This apparent inconsistency regarding the role of transients remains to be resolved, although it is possible that the UVI instrument is sensing a different altitude to that where the transients contribute to accelerating superrotation in the GCMs (Lebonnois, 2020).

The nature of the eddies that accelerate superrotation on Titan is less clear. The idealised modeling studies described in the previous subsection do not consider the effects of a seasonal cycle, thus omitting an important feature of Titan. In particular, it is not obvious *a priori* what effect the hemispheric asymmetry present in Titan’s background zonal-mean zonal wind will have on the emergence of the Rossby-Kelvin modes found to accelerate superrotation in idealised models. Using an idealised model *with a seasonal cycle*, Mitchell et al. (2014) show that seasonal forcing can suppress superrotation, by inducing cross-equatorial momentum transport that decelerates the tropics in favour of the winter-hemisphere jet, and overwhelms equatorward momentum transport due to transients. They further show that if radiative damping time is set to be very long (100 times that in the standard, Earth-like set-up), then this effect can be

overcome as seasonal overturning is suppressed. The radiative timescale on Titan is long compared with Earth's due to Titan's larger atmospheric mass and lower temperatures, and Mitchell et al. (2014) suggest this may be critical for the development of strong superrotation there. However, observations (Achterberg et al., 2008; Sharkey et al., 2021) and more comprehensive GCMs with realistic radiative transfer (Newman et al., 2011; Lebonnois et al., 2012a; Lora et al., 2015) do feature substantial seasonal variation in the upper-stratospheric zonal-mean zonal wind where equatorial superrotation is strongest. Newman et al. (2011) and Lebonnois et al. (2012a) find that eddy acceleration of the superrotating jet is strongest around the solstices, when hemispheric asymmetries in the zonal-mean zonal wind are strongest. Both studies identify that the acceleration is due to waves with low zonal wavenumbers, which they suggest are generated by barotropic instability in sub-tropical latitudes. However, it is yet to be determined whether these waves are similar or different to those obtained in idealised models, as neither study characterised their spatial structure. This will be investigated in Chapter 4.

2.7 Tidally locked exoplanets

Finally, we conclude our review of the circulation regimes that may be obtained on slowly rotating planets by considering the case of tidally locked planets. These planets always present the same face to their host star. This means that they have a permanent day side, where they are heated by stellar radiation, and night side, where they cool to space. The day–night heating contrast on tidally locked planets gives rise to atmospheric circulations that have distinct differences from those obtained in idealised Earth-like GCMs, or observed in the Solar System.

Most observationally accessible extrasolar planets are likely to be tidally locked (Dole, 1964; Guillot et al., 1996), which has motivated a large body of work that seeks to understand the processes that influence observable features of these planets in thermal emission and reflected visible light, as well as their climates and potential habitability. Understanding atmospheric circulation is central to each of these objectives. Circulation transports heat, chemical species, and clouds around a planet, which affect the observable properties of exoplanets in both the visible (reflected light) and infrared (thermal emission) (Burrows, 2014; Crossfield, 2015; Parmentier and Crossfield, 2018). In addition, the circulation may be crucial to supporting a habitable climate on a rocky planet, with sufficient heat transport from the day side to the night side required to prevent volatile species from condensing on the cold night side, causing atmospheric collapse (Joshi et al., 1997; Heng and Kopparla, 2012; Wordsworth, 2015).

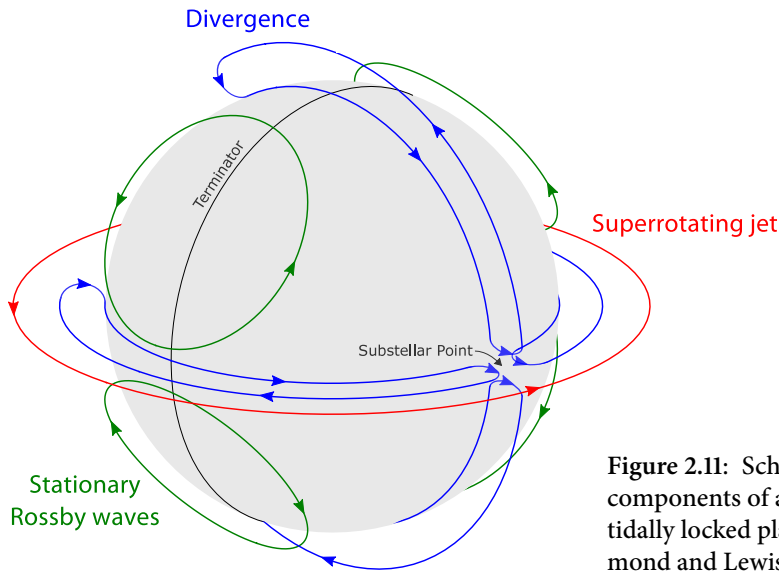


Figure 2.11: Schematic showing the main components of atmospheric circulation on tidally locked planets. Obtained from Hammond and Lewis (2021).

Theoretical work and modeling studies using GCMs have shown that atmospheric circulation on tidally locked planets is strongly influenced by the day-night pattern of heating and cooling (as reviewed by Showman et al., 2013; Pierrehumbert and Hammond, 2019; Showman et al., 2020). A schematic displaying the main components of atmospheric circulation on tidally locked planets is shown in Figure 2.11. Heating on the day side drives vertical motion at the substellar point and divergence aloft (Showman et al., 2013), which in turn generates stationary Rossby waves (Showman and Polvani, 2010; see also Sardeshmukh and Hoskins, 1988). Interaction between these two circulation components can lead to momentum convergence at the equator and the acceleration of a westerly superrotating jet (Showman and Polvani, 2011). In Chapter 5 I will show how these components can be separated from one another using a *Helmholtz decomposition*, which partitions the horizontal velocity into components associated with rotational (divergence-free) and divergent (vorticity-free) components. Then, in Chapter 6, I will use the Helmholtz decomposition to show how each component of the circulation contributes to atmospheric temperature structure by analysing balance relations in the divergence equation (obtained by taking the divergence of the momentum equations).

2.7.1 Stationary wave response to day-night heating

One of the prominent features of atmospheric circulation on tidally locked planets is a pronounced stationary wave response to the day-night pattern of heating and cooling. This is exemplified in Figure 2.12, which shows output from an idealised, dry GCM simulation of a tidally locked terrestrial planet with a 10 day orbital period ($\Omega = \Omega_E/10$), obtained from Hammond and Pierrehumbert (2018). The substellar point is located

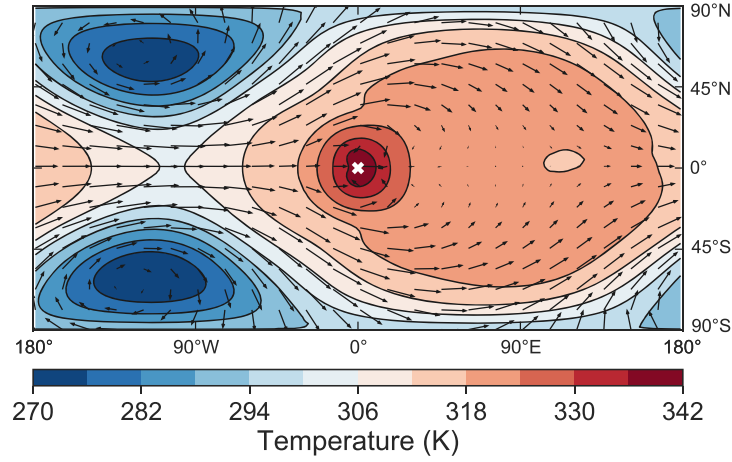


Figure 2.12: Temperature (colour contours) and horizontal velocity (quivers) output from an idealised GCM simulation of an Earth-like, tidally locked terrestrial planet, with an orbital period of $P = 10$ days. Obtained from (Hammond and Pierrehumbert, 2018).

at $(\lambda, \vartheta) = (0^\circ, 0^\circ)$. A wavenumber-1 modulation is present in the temperature field (colour) at the equator, phase shifted eastwards by around 70° longitude from the heating, and equatorial Rossby wave-like gyres are also present in higher latitudes (around $\pm 60^\circ$).

Showman and Polvani (2011) showed that the stationary wave response to day-night heating on tidally locked planets is analogous to that identified and analysed by Matsuno (1966) and Gill (1980) for the response to localised heating in the Earth’s tropics. These studies sought wave-like solutions to the linearised shallow water equations for stationary circulation in the presence of linear friction, which may be written

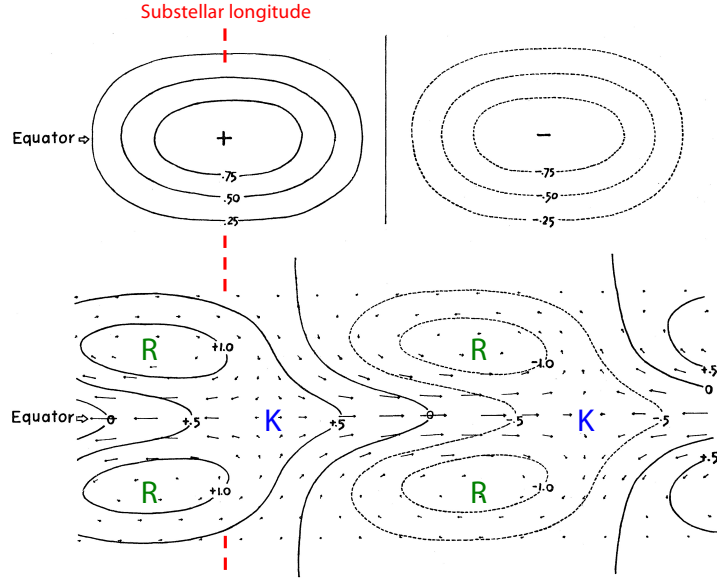
$$-\beta y v_l = -\frac{\partial \phi_l}{\partial x} - \frac{u_l}{\tau_{\text{drag}}}, \quad (2.49)$$

$$\beta y u_l = -\frac{\partial \phi_l}{\partial x} - \frac{v_l}{\tau_{\text{drag}}}, \quad (2.50)$$

$$gh_l \left(\frac{\partial u_l}{\partial x} + \frac{\partial v_l}{\partial y} \right) = Q_l(x, y) - \frac{\phi_l}{\tau_{\text{rad}}}, \quad (2.51)$$

where the notation is the same as in Section 2.5 for the unsteady shallow water equations (Equations 2.38–2.40), and the boundary conditions once again require solutions that remain bounded far from the equator. In the equations above, the linear drag terms essentially replace the time derivatives in (2.38)–(2.40). Additionally, a mass source or sink term Q_l has been introduced that represents the tropical heating (or, for a tidally locked planet, the day-night forcing). Matsuno (1966) shows that it is possible to express both the forcing (Q_l) and solutions (u_l, v_l, ϕ_l) of the equations for forced,

Figure 2.13: Stationary wave response (bottom) to tropical heating (top) that is sinusoidal in the longitude direction and has a Gaussian decay in the latitude direction. The wave response is comprised of a Kelvin component and an equatorial Rossby component (labelled K and R, respectively). Adapted from Matsuno (1966).



stationary circulation (2.49)–(2.51) as a linear combination of the free wave solutions to (2.38)–(2.40) discussed in Section 2.5.

For the special case where $\tau_{\text{drag}} = \tau_{\text{rad}}$ and the forcing Q_l takes the idealised form $Q_l = Q_0 \exp(ix/a) \exp[-y^2/(2l_e)^2]$, Matsuno (1966) shows that the forced wave response is solely comprised of the zonal wavenumber-1 Kelvin and $n = 1$ equatorial Rossby wave solutions. The resultant wave pattern is shown in Figure 2.13 along with the spatial structure of the forcing function Q_l . This pattern has a similar structure to that of the GCM simulation shown in Figure 2.12, but there are also some notable differences. For example, only the Rossby gyres with a negative anomaly are identifiable in the GCM simulation, and the wave response in the GCM simulation is phase-shifted eastwards by about 45° compared with the Matsuno–Gill solution.

These differences are due to the effect of the zonal-mean flow on the waves obtained in the GCM simulation (Tsai et al., 2014; Hammond and Pierrehumbert, 2018), which features an equatorial superrotating jet (whereas the shallow water system given by Equations 2.49–2.51 is linearised about a state of rest). This is illustrated in Figure 2.14, which shows numerical solutions to the shallow water equations on the sphere, linearised about a background zonal-mean zonal jet (adapted from Hammond and Pierrehumbert, 2018). The center panel shows the effect of the background jet on the stationary waves, which phase-shifts the Rossby waves eastwards relative to their position in the Matsuno–Gill solution with no background jet. Additionally, it appears to suppress the Kelvin wave response, which is difficult to identify in Figure 2.14. In the right-hand panel, the circulation associated with the background jet, which has a Gaussian structure

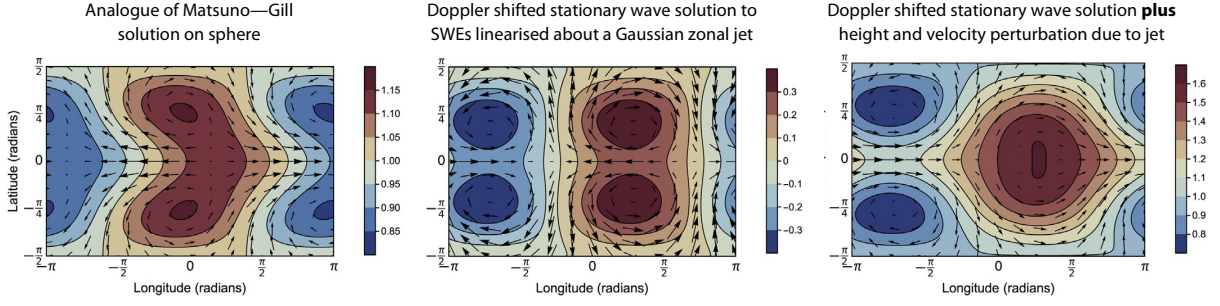


Figure 2.14: Left panel: Equivalent of Matsuno–Gill solution shown in Figure 2.13 but in a spherical geometry. The substellar point is at $(\lambda, \vartheta) = (0, 0)$. Center panel: Stationary wave solution for a shallow water system linearised about a Gaussian zonal jet of the form $\bar{u} = \cos \vartheta \exp[-(\vartheta/\vartheta_0)^2]$. Right panel: The stationary wave solution shown in the centre panel with the height and velocity perturbation due to the Gaussian jet added on. Notably, the circulation in this panel is very similar to that shown in Figure 2.12 obtained from a GCM simulation. Adapted from Hammond and Pierrehumbert (2018).

that is maximal on the equator, has been added to the stationary wave solution. The perturbation fields associated with the zonal jet obscure the Rossby gyres with a positive height anomaly, and the total circulation now resembles that in the three-dimensional GCM. One notable remaining difference between the GCM simulation and the shallow water solutions shown in Figure 2.14 is the sharp temperature maximum at the substellar point in the GCM simulation, which is not present in the shallow water solutions. In Chapter 6 I will show that this is associated with thermally-driven overturning circulation, which is not resolved in the shallow water system.

2.7.2 Equatorial superrotation

General Circulation Model simulations indicate that equatorial superrotation is likely to be a ubiquitous feature of atmospheric circulation on tidally locked planets (Heng and Showman, 2015; Turbet et al., 2021). These model results are supported by observations of exoplanets which exhibit eastward offsets in maximum thermal emission relative to the substellar point (also known as *hotspot shifts*; Knutson et al., 2007; Majeau et al., 2012; Demory et al., 2016), which have been suggested to be associated with equatorial superrotation, both directly due to advection of heat by the equatorial jet (Zhang and Showman, 2017), and indirectly by phase-shifting the stationary waves eastwards (and thus the temperature structure associated with them, cf. Figure 2.12 and Figure 2.14; Tsai et al., 2014; Hammond and Pierrehumbert, 2018).

As for non-tidally locked planets, Hide’s theorem (Section 2.4; Hide, 1969) requires that equatorial superrotation is accelerated by up-gradient momentum transport due to non-axisymmetric disturbances. For the case of tidally locked planets, equatorial

momentum convergence is induced by the stationary wave response to the day-night heating (Showman and Polvani, 2010; 2011).

The phase shift between the Kelvin and Rossby components of the wave response in the Matsuno–Gill model gives rise to velocity vectors that are tilted northwest to southeast, which implies $\partial \bar{u} / \partial t = \dots - \partial \bar{u}' v' / \partial y > 0$. In the shallow water system (Equations 2.49–2.51), this momentum convergence does not actually lead to an acceleration of the Eulerian-mean zonal velocity, as mass is also converged towards the equator, and the eddy momentum flux balances this exactly so as to keep the zonal momentum per unit mass constant (Showman and Polvani, 2010; see also Zurita-Gotor and Held, 2018). However, Showman and Polvani (2010) show that if an additional term is included in (2.49) and (2.50) that represents the effects of momentum advection from a quiescent lower layer, then equatorial superrotation does emerge in the Matsuno–Gill model as a result of equatorward momentum convergence by the stationary waves. In three-dimensional models, equatorial superrotation emerges without the need for an additional term, as vertical motion is resolved in this case, and consequently so is the required vertical momentum advection (Showman and Polvani, 2011).

2.7.3 *Circulation in three-dimensional models, and its dependence on rotation rate*

The three-dimensional atmospheric circulation on tidally locked planets has been investigated extensively using general circulation models. This subsection discusses some of the circulation features that emerge in these simulations, with a focus on studies that model *terrestrial planets*. Terrestrial planets are rocky planets of a similar size and mass to Earth (typically defined to be those with radii less than $2a_E$; see introductory discussion in Dressing and Charbonneau, 2015). The study of atmospheric circulation on terrestrial tidally locked planets has been focussed on temperate planets, which might host climates that are suitable for supporting life (Showman et al., 2013; Forget and Leconte, 2014). The atmospheric dynamics of hotter terrestrial *lava* planets such as 55 Cancri e has also received some attention (e.g., Hammond and Pierrehumbert, 2017), motivated by the availability of observations of thermal emission (Demory et al., 2016).

Figure 2.15 shows horizontal wind and temperature in the mid-troposphere obtained from idealised GCM simulations performed by Koll and Abbot (2016). Their model is configured with semi-grey radiative transfer, and a diffusive boundary layer code that exchanges sensible heat with the surface and parametrises surface friction. The substellar point in each panel is located at 270° longitude. Three simulations are presented that vary $\mathcal{R}_T$ through the planetary rotation period, from $P = 60$ days (‘slow rotation’ – large

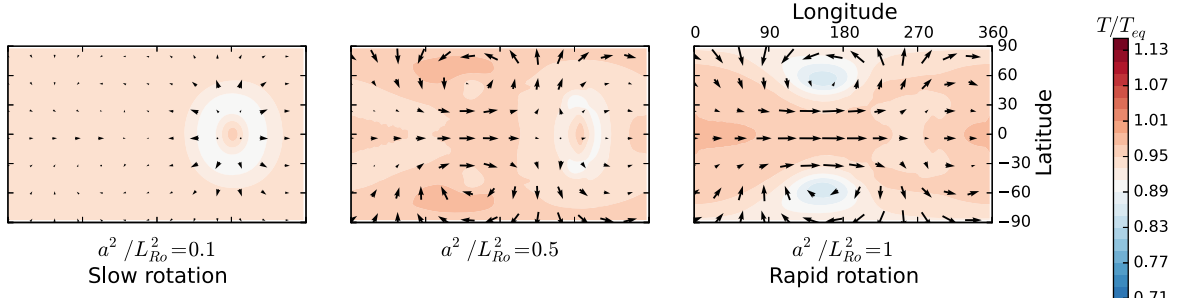


Figure 2.15: Horizontal velocity (quivers) and temperature (colour contours) obtained from idealised terrestrial, tidally locked GCM simulations with varying rotation rates, corresponding to orbital periods of approximately 60 days, 12 days, and 6 days respectively, from left to right, and $T_{\text{eq}} \approx 255$ K. The substellar point is located at 270° longitude. Adapted from Koll and Abbot (2016).

$\mathcal{R}_T \approx 50$) to $P = 6$ days (‘rapid rotation’ – although still slowly rotating compared with the Earth – smaller $\mathcal{R}_T \approx 5$). The $P = 6$ days simulation exhibits a pronounced stationary wave response to the tropical heating, and a superrotating jet at the equator. By contrast, both the stationary waves and equatorial superrotation are essentially absent in the $P = 60$ days simulation, which instead features roughly isotropic divergent flow away from the substellar point, which in the vertical takes the form of an overturning circulation (Koll and Abbot, 2016). The $P = 12$ days simulation appears to represent an intermediate case, with less pronounced stationary waves and weaker superrotation. It has been proposed that the weakening of the stationary wave response, evident $P = 12$ days and $P = 60$ days cases, is related to the deformation radius becoming larger than the planetary radius (Noda et al., 2017; Haqq-Misra et al., 2018), although the extent to which the deformation radius (which only has formal importance in the limit where the beta plane approximation is valid) is a useful indicator of the behaviour of waves that are global in scale is unclear. Similar rotation rate dependence has been identified by a number of other studies (Merlis and Schneider, 2010; Edson et al., 2011; Carone et al., 2015, 2016; Noda et al., 2017; Haqq-Misra et al., 2018; Pierrehumbert and Hammond, 2019).

The structure of the overturning circulation on tidally locked planets is more complicated than that found in idealised GCM simulations with axisymmetric forcing, or in the atmospheres of the Solar System planets which experience a moving, diurnal forcing. In these latter two cases, overturning is predominantly meridional (i.e., in the north–south direction) and axisymmetric in nature (Section 2.3 and Section 2.4). By contrast, atmospheric circulation on tidally locked planets displays distinct zonal asymmetry due to the stationary forcing associated with their orbital configuration. This means that overturning circulation on tidally locked planets has a component in the zonal direction, which makes it difficult to analyse using traditional zonally-averaged diagnostics such

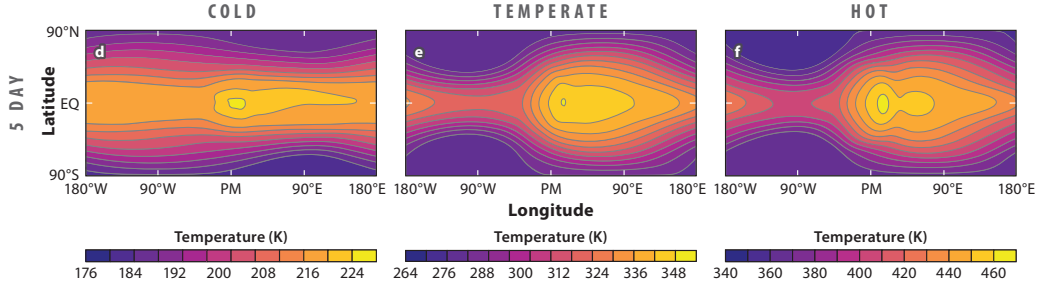


Figure 2.16: Mid-tropospheric temperature obtained from idealised, dry GCM simulations configured with an orbital period of $P = 5$ days and three different values of instellation: 271.8 (left), 1376 (center, the same as Earth), and 4320 W m^{-2} (right). Adapted from Pierrehumbert and Hammond (2019).

as the Eulerian mean meridional mass streamfunction (defined by Equation 2.22). At present, overturning circulation on terrestrial tidally locked planets is understood in terms of a superposition of various different zonal and meridional circulations. Ascent on the day side paired with meridional horizontal motion and descent near the poles has been described as a Hadley circulation (Merlis and Schneider, 2010; Edson et al., 2011; Heng et al., 2011; Carone et al., 2014; Haqq-Misra et al., 2018), and overturning circulation in the zonal direction has been described as a Walker circulation (Carone et al., 2014; Haqq-Misra et al., 2018; by analogy with zonal overturning in the Earth’s tropical pacific. On the night side, meridional motion has been described as an ‘anti-Hadley circulation’ (Heng et al., 2011; Charnay et al., 2015; Haqq-Misra et al., 2018). The partitioning of meridional overturning into separate day side and night side components in previous work has been informed by computing the Eulerian mean mass streamfunction Ψ over a limited longitude range (e.g., over just the night side). However, this approach is difficult to motivate as Ψ is derived by integrating the continuity equation over all longitudes. In Chapter 5 I will show that, for slowly rotating planets, the overturning circulation, which is purely divergent, can be separated from the jet and stationary waves, which are mostly rotational, using the Helmholtz decomposition.

As with circulation forced by zonally symmetric heating (as in Section 2.3), the circulation on tidally locked planets is also sensitive to parameters beyond the thermal Rossby number $\mathcal{R}_T$. Pierrehumbert and Hammond (2019) present results from idealised GCM simulations similar to those of Koll and Abbot (2016) where they vary the strength of incoming stellar radiation, which in non-dimensional terms corresponds primarily to varying the thermal damping number $\mathcal{I}$. Figure 2.16 shows a subset of their simulations, run with a rotation period of $P = 5$ days and three values for the instellation: 271.8, 1376 (the same as Earth), and 4320 W m^{-2} . These simulations show that hotter planets with stronger instellation feature a more pronounced stationary wave response compared

with cooler planets that have a more zonally symmetric circulation. Finally, Heng and Vogt (2011) and Carone et al. (2016) present results from idealised GCM simulations where they vary the effect of surface friction (i.e., the Ekman number $\mathcal{E}$). They find that as the strength of surface friction is increased, the circulation moves away from a regime characterised by pronounced stationary waves, towards one dominated by isotropic day side to night side overturning (similar to the left-hand panel in Figure 2.15).

2.8 Open questions

This chapter has discussed the different circulation regimes that can arise in planetary atmospheres, with an emphasis on slowly rotating rocky planets with large thermal Rossby number $\mathcal{R}_T$. The dynamics in the ‘slowly rotating regime’ are generally characterised by equatorial superrotation and planetary-scale overturning circulations, and the general reasons for the emergence of each of these features when $\mathcal{R}_T$ is large are understood in terms of idealised theoretical models and results from numerical experiments.

However, we have identified that the details of atmospheric circulation on slowly rotating planets, for example: the superrotating jet speed and the mechanisms that accelerate it, the nature of atmospheric waves, and the structure of the overturning circulation, are highly sensitive to the specific properties of an individual planet or class of planets (e.g., tidally locked planets), such as the radiative properties of its atmosphere and its orbital configuration. The relative importance of these factors in determining circulation in the slowly rotating regime is not yet fully understood. A non-exhaustive list of open questions raised by the discussion in this chapter is given below:

1. Can quantitative metrics be used to characterise the different circulation regimes identified in idealised GCMs (e.g., in the regime diagram shown in Figure 2.1), as well as the Solar System atmospheres?
2. How does the strength of stratification affect the dynamics of planets in the slowly rotating (large $\mathcal{R}_T$) regime?
3. Are the mechanisms that accelerate superrotation in idealised models relevant to Venus and Titan?
4. What role do free transient waves play in Venus’ zonal momentum budget?
5. Which properties of Venus and Titan (e.g., their atmospheric mass, radiative heating structure, and orbital configuration) are most important in determining their atmospheric dynamics?

6. What does the regime diagram for atmospheric circulation on tidally locked planets look like, and what are the key nondimensional control parameters?
7. How do different components of circulation on tidally locked planets contribute to processes such as heat and volatile species transport, and observable properties such as thermal emission?
8. How do the characteristics of equatorial waves on tidally locked and non-tidally locked planets depend on planetary parameters, when the equatorial beta plane approximation is no longer formally valid?

I aim to seek answers to some of these questions during the remainder of this thesis. Specifically, In [Chapter 3](#), I investigate whether circulation regimes can be quantitatively characterised using a metric called the global superrotation index, addressing Question 1. In [Chapter 4](#), I study the waves that accelerate superrotation in a simulation of Titan's atmosphere, and discuss the instabilities that give rise to them, addressing Question 3. [Chapter 5](#) and [Chapter 6](#) form a pair of chapters that seek to tackle Question 7. In [Chapter 5](#), I introduce a method for decomposing circulation on tidally locked planets into its constituent parts, and assess how each component contributes to heat transport. In [Chapter 6](#), I build on this work and assesses how each different components of the circulation contribute to temperature structure via diagnostic balance relationships.

Characterising Regimes of Atmospheric Circulation in Terms of Their Global Superrotation

The contents of this chapter are a modified version of N. T. Lewis, G. J. Colyer, and P. L. Read (2021): Characterising Regimes of Atmospheric Circulation in Terms of Their Global Superrotation, in: *Journal of the Atmospheric Sciences*, 78 (4), pp. 1245-1258.

Author contributions: NTL and PLR identified the study topic. NTL designed, ran and analysed the numerical experiments, and lead the writing of the paper. GJC and PLR proof read the paper, and advised on the interpretation and discussion of results.

3.1 Introduction

Planetary atmospheres can occupy a variety of different circulation regimes. This is borne out by the dynamics of the Solar System atmospheres (Chapter 1). Earth’s atmospheric circulation is characterised by overturning (Hadley) cells which feature air rising near the equator and sinking in the subtropics. In the midlatitudes, prograde jets with zonal velocities of approximately $30\text{--}40\text{ m s}^{-1}$ are present, while in the tropics weak retrograde flow is found (Schneider, 2006). By contrast, Venus and Titan (which are slowly rotating with respect to Earth, and in the case of Titan also smaller) host prograde equatorial jets with zonal wind velocities in excess of 100 m s^{-1} (Counselman et al., 1980; Bird et al., 2005). Numerical modeling suggests these atmospheres also feature planetary-scale overturning circulations formed of Hadley-like cells with a nearly meridional extent (Lebonnois et al., 2013; Lora et al., 2019). The atmospheres of the gas giant planets Jupiter and Saturn (much larger and more rapidly rotating with respect to the Earth) feature zonal velocity profiles with strongly prograde jets alternating with weakly retrograde jets in a banded pattern (Porco et al., 2003; Porco et al., 2005).

There is now a large body of work that seeks to understand how atmospheric circulation is sensitive to key parameters and processes (as reviewed in Chapter 2). Idealised simulations using Earth-like General Circulation Models have found a dominant role for the thermal Rossby number $\mathcal{R}_T$ (2.20) in determining the dynamical regime occupied by a planetary atmosphere. Atmospheric circulation has been found to occupy a regime characterised by geostrophic balance and multiple zonal jets when $\mathcal{R}_T \ll 1$, and a regime characterised by cyclostrophic balance and equatorial superrotation when $\mathcal{R}_T \gg 1$ (Mitchell and Vallis, 2010; Kaspi and Showman, 2015; Wang et al., 2018).

A planet's atmospheric circulation is not solely determined by $\mathcal{R}_T$, however, as it is sensitive to other nondimensional control parameters (such as the Ekman number, and the radiative damping number; see [Chapter 2](#), Sections 2.2 and 2.3). For example, Dias Pinto and Mitchell (2014) show that when $\mathcal{R}_T$ is large, a strongly superrotating state — analogous to that of Venus' and Titan's atmospheres — is not obtained if the radiative damping number $\mathcal{I}$ is large (as may be typical for a slowly rotating planet).

To facilitate the comparison of circulation regimes obtained in different planetary atmospheres, it is useful to develop bulk quantities that indicate the specific regime they occupy. However, the utility of $\mathcal{R}_T$ and other input parameters for this task is limited by the sensitivity of atmospheric circulation to multiple input parameters and atmospheric properties. Instead, it is necessary to use diagnostic quantities that can be computed from the equilibrium flow structure in a planetary atmosphere (or a set of numerical simulations).

In this chapter, I will investigate the utility of the global superrotation index S for characterising and comparing the circulation regimes occupied by different planetary atmospheres. S is defined (Read, 1986a,b; Read and Lebonnois, 2018)

$$S \equiv \frac{\int \rho m \, dV}{\int \rho \Omega a^2 \cos^2 \vartheta \, dV} - 1, \quad (2.23, \text{revisited})$$

and compares the mass-weighted global integral of the specific zonal angular momentum m with a state of solid-body co-rotation with the underlying planet (when $S > 0$ the atmosphere has more zonal angular momentum than if it were co-rotating with the planetary surface). Above, $dV = a^2 \cos \vartheta \, d\lambda \, d\vartheta \, dz$ is an element of volume.

The global superrotation index S can be regarded as the quotient of a mean zonal velocity U , and Ωa , and thus shares some characteristics of a zonal Rossby number (Read, 1986a). This suggests that S may be a useful quantity to compute when comparing planetary atmospheres, as its value is *indicative* of the dynamical regime occupied by an atmosphere. $S \ll 1$ is suggestive of a regime where the dynamics are dominated by geostrophic balance, and $S \gg 1$ suggests a regime where the dynamics are dominated by cyclostrophic balance. S is given further value by the fact that it is constrained to be $S \leq 1/2$ in an axisymmetric, inviscid atmosphere. The constraint is due to Hide's theorem (Hide, 1969; [Chapter 2](#), Section 2.4), which states that in an axisymmetric, inviscid setting, $Dm/Dt = 0$. If we assume that in a thin boundary layer, surface friction acts to restore m to a state of co-rotation with the underlying planet, then the maximum value of m that may be obtained in the boundary layer is $m = \Omega a^2$ (the value of m for a parcel of air co-rotating with the planetary surface at the equator). Parcels of air that are

then transported into the free atmosphere (e.g., by an overturning circulation) will then also have $m \leq \Omega a^2$, as m is materially conserved. This ideal circulation is schematised in [Figure 2.5](#). The $S = 1/2$ limit corresponds to the scenario where $m = \Omega a^2$ everywhere (and the boundary layer has infinitesimal depth), and if $S > 1/2$ then non-axisymmetric disturbances which transport m up its local gradient must be present in the atmosphere.

The global superrotation index has been used previously to compare the dynamics of the Solar System atmospheres (Read and Lebonnois, 2018), and to compare different Venus and Titan GCMs (Lebonnois et al., 2012b, 2013). In addition, the value of S estimated from zonal velocity measurements in Venus' atmosphere has been compared with S computed from the output of Venus GCMs to assess how well they reproduce the observed circulation (Mendonça and Read, 2016; Read and Lebonnois, 2018). The atmospheres of Venus and Titan, for which $S_V \approx 7.7$ and $S_T \approx 2$ (Read and Lebonnois, 2018), exhibit the greatest degree of global superrotation within the Solar System, and $S \gg 1$ is indicative of the dominance of cyclostrophic balance in the local momentum budget throughout the atmosphere. For Mars and the Earth, $S_M \approx 0.04$ and $S_E \approx 0.0135$ (Read and Lebonnois, 2018), which is consistent with the dominance of geostrophic balance in their atmospheres. The fact that $S > 1/2$ for Venus and Titan means that eddy processes, which transport m up its local gradient, *must* be important in determining the dominant (cyclostrophic) dynamical balance that characterises their atmospheric circulations.

3.1.1 Aim of this chapter

The work in this chapter aims to investigate the utility of S for characterising regimes of atmospheric circulation.

To date, this has only been investigated using axisymmetric models with an eddy-viscosity parametrisation that diffuses angular velocity, which allows for up-gradient transport of m (Read, 1986a; Yamamoto et al., 2009; Yamamoto and Yoden, 2013). These studies have shown that S scales with Ω^{-2} ($S \propto \mathcal{R}_T$) at high rotation rate (small $\mathcal{R}_T$), and that at low rotation rate (large $\mathcal{R}_T$) S either saturates ($S = \text{constant}$) when parametrised up-gradient transport of m is small, or scales with Ω^{-1} or $\Omega^{-2/3}$ ($S \propto \sqrt{\mathcal{R}_T}$ or $S \propto \mathcal{R}_T^{1/3}$) when the eddy diffusion coefficient is made large and up-gradient transport of m is significant. Yamamoto and Yoden (2013) develop a theory that relates S to $\mathcal{R}_T$, valid for slowly rotating planets with extreme up-gradient transport of m . By considering the limiting behaviour of their theory, they are able to explicitly demonstrate that $S \propto \sqrt{\mathcal{R}_T}$ or $S \propto \mathcal{R}_T^{1/3}$ is associated with the dominance of cyclostrophic balance in the zonal

momentum budget. I aim to extend the analysis of Read (1986a), Yamamoto et al. (2009) and Yamamoto and Yoden (2013) by investigating the dependence of S on Ω (or $\mathcal{R}_T$) in idealised three-dimensional Earth-like GCM simulations. Experiments of this type are an obvious extension to the work of Read (1986a), Yamamoto et al. (2009), and Yamamoto and Yoden (2013). Unlike in these previous studies, up-gradient transport of m in the model used here is not parametrised, and the existence, direction and strength of materially non-conservative transport of m is associated with the existence and characteristics of non-axisymmetric disturbances that can induce stresses on the zonal flow. To elucidate the role of eddies in determining the dependence of S on Ω , a set of axisymmetric (2D) experiments are also run for comparison with the 3D experiments.

For both the 3D and 2D experiments, it will be shown that S scales with Ω^{-2} ($S \propto \mathcal{R}_T$) for rapid rotation rates, while for low rotation rates (large $\mathcal{R}_T$) $S = \text{constant}$. To interpret these scalings for S , a theory is derived for S in terms of $\mathcal{R}_T$ from the axisymmetric model of Held and Hou (1980), that captures the scaling behaviour of S in the numerical experiments. As in Yamamoto and Yoden (2013), the limiting behaviour of the theory presented here is considered, to show how different dependencies of S on $\mathcal{R}_T$ correspond to different dominant dynamical balances.

The rest of this chapter is structured as follows. The model configuration and experiment design is described in Section 3.2. To provide context for the rest of the chapter, the zonally-averaged circulation obtained in the GCM experiments is described in Section 3.3. In Section 3.4, S is computed for each experiment, the dependence of S on Ω is discussed, and differences in S between the 3D and axisymmetric experiments are investigated. This section also includes the derivation of a theory for S for the case of an axisymmetric atmosphere in the inviscid limit. Discussion focussed on illustrating the relationship between S and circulation regime is provided in Section 3.5. Section 3.6 summarises the findings of this work.

3.2 Description of experiments

3.2.1 Numerical model

The GCM used in this study is build using Isca, a framework for building idealised general circulation models of varying complexity. Isca is built on top of the GFDL¹ primitive equation spectral dynamical core. In the present work, Isca is configured as a dry dynamical core forced by Newtonian cooling to a statically stable axisymmetric

¹Geophysical Fluid Dynamics Laboratory, Princeton.

<https://www.gfdl.noaa.gov/idealized-spectral-models-quickstart/>.

radiative–convective equilibrium temperature profile.

Dynamical core

The dynamical core integrates the primitive equations forwards in time, using a semi-implicit leapfrog scheme with a Robert–Asselin time filter. The equations are solved on a thin spherical shell, using a pseudospectral method in the horizontal (prognostic fields are represented by a triangular truncation of spherical harmonics), and a finite difference method in the vertical. For numerical efficiency, and to avoid the formulation of vector fields on the spectral grid, the primitive equations are solved in terms of the scalar vorticity and divergence. The vertical coordinate is a terrain-following coordinate, defined as $\sigma = p/p_s$. There are $N = 80$ levels, distributed according to $\sigma = \exp[-6(0.5\tilde{z} + 0.5\tilde{z}^{7.5})]$ where $\tilde{z}$ is equally spaced on the unit interval. This configuration yields vertical layers that are roughly evenly spaced in the troposphere, with enhanced resolution in the stratosphere.

Thermal forcing

The thermal relaxation temperature profile used for the Newtonian cooling is that described in Wang et al. (2018), similar to that of Held and Suarez (1994), and is written

$$T_r = T_{r,z} + T_{r,\vartheta}, \quad (3.1)$$

where

$$\begin{aligned} T_{r,z} &= T_{\text{tp}} + \sqrt{\left[\frac{L}{2}(z_{\text{tp}} - z)\right]^2 + K^2} + \frac{L}{2}(z_{\text{tp}} - z), \quad \text{and} \\ T_{r,\vartheta} &= h(\sigma) \Delta T_h \left(\frac{1}{3} - \sin^2 \vartheta\right), \end{aligned} \quad (3.2)$$

with

$$h = \begin{cases} \sin\left[\frac{\pi}{2}\left(\frac{\sigma - \sigma_{\text{tp}}}{1 - \sigma_{\text{tp}}}\right)\right], & \text{if } \sigma \geq \sigma_{\text{tp}}, \\ 0, & \text{otherwise.} \end{cases} \quad (3.3)$$

Above, $z_{\text{tp}} = 12$ km is the tropopause height (σ_{tp} is the corresponding σ level), $L = 6.5 \text{ K km}^{-1}$ is the vertical lapse rate, and $T_{\text{tp}} = T_0 - Lz_{\text{tp}}$ is the temperature at the tropopause, with $T_0 = 288 \text{ K}$ the globally-averaged surface temperature. $\Delta T_h = 60 \text{ K}$ is the equator-to-pole temperature difference at the surface. $K = 2 \text{ K}$ is a smoothing parameter that ensures a continuous temperature change across the tropopause. The radiative relaxation timescale is 2.5 days in the boundary layer ($\sigma > 0.8$), and 30 days in the free atmosphere.

Drag and dissipation

A linear drag is applied in the boundary layer, operating on a timescale of 0.6 days, and in the upper-atmosphere ($\sigma < 0.01$, applied to the eddy components of the flow only), operating on a timescale of 0.5 days. The drag in the upper atmosphere is included as a sponge layer, to avoid the reflection of waves at the model top.

A hyperviscosity term is applied to the momentum and thermodynamic equations:

$$\frac{\partial \zeta}{\partial t} = \dots + \nu \nabla^{2n} \zeta, \quad (3.4)$$

$$\frac{\partial \delta}{\partial t} = \dots + \nu \nabla^{2n} \delta, \quad (3.5)$$

$$\frac{\partial T}{\partial t} = \dots + \kappa \nabla^{2n} T, \quad (3.6)$$

operating on a timescale of 0.25 days at the grid scale. $n = 4$ is the damping order. ζ , δ , and T are the vorticity, divergence, and temperature. This term is included to parametrise the dissipation of energy by sub-grid scale dynamics.

3.2.2 Description of model runs

GCM simulations are run over a wide range of rotation rates, from $8\Omega_E$ to $\Omega_E/512$, where $\Omega_E = 7.29 \times 10^{-5} \text{ s}^{-1}$ is the Earth's rotation rate. All other planetary parameters are taken to be Earth-like: $a = 6.4 \times 10^6 \text{ m}$, $g = 9.81 \text{ m s}^{-2}$, and surface pressure $p_0 = 1 \text{ bar}$.

Three-dimensional experiments are run at both T127 and T42 spectral resolution (corresponding to approximately 1° and 2.8° latitude–longitude resolution, respectively, at the equator). Axisymmetric experiments are run with the same meridional resolution as the T127 three-dimensional experiments. The axisymmetric runs are constructed by retaining only zonal wavenumber $m = 0$ coefficients in the spectral dynamical core at each time step. The rotation rates run in each configuration are shown in [Table 3.1](#).

Each of the experiments is run to equilibrium, by which it is meant that S is no longer evolving with time. To achieve this, three-dimensional experiments with $\Omega \geq \Omega_E/4$ are run for 3 (Earth) years, then $\Omega = \Omega_E/8$ experiment is runs for 6 years, and experiments with $\Omega \leq \Omega_E/16$ are run for 8 years. The axisymmetric experiments are all run for 3 years.

3.2.3 Data analysis

The 3D model data analysed in this chapter is output in daily-averaged format, and the axisymmetric model data is output in monthly-averaged format. Isca outputs data on the σ coordinate vertical grid, which is subsequently interpolated onto pure pressure

Config.	8×	4×	2×	Ω_E	1/2	1/4	1/8	1/16	1/32	1/64	1/128	1/256	1/512
3D, T127	✓	✓	✓	✓		✓		✓		✓			
3D, T42				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
2D		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		

Table 3.1: Rotation rates for each model configuration. $\Omega_E = 7.29 \times 10^{-5} \text{ s}^{-1}$ is the Earth’s rotation rate. ‘2D’ refers to axisymmetric experiments.

levels $p_e = \sigma_e p_0$. Occasionally $p_e > p_s(t, \lambda, \vartheta)$, in which case the data is set to NaN (the grid-point being interpolated onto is below the surface). In order to correctly take account of these subterranean grid points when computing integrals and averages, we make use of the techniques described by Boer (1982).

3.3 Zonally-averaged circulations

To provide context for the rest of the chapter, I begin the presentation of the experiment results by discussing some aspects of their zonally-averaged circulations. Each of the experiments discussed in this section was run at T127 resolution. The structure of the circulation obtained in the numerical experiments run for this study is consistent with that presented in previous work (e.g., Dias Pinto and Mitchell, 2014; Kaspi and Showman, 2015; Wang et al., 2018; and Colyer and Vallis, 2019).

The zonal-mean zonal wind $\bar{u}$ and meridional mass streamfunction Ψ (2.22) are shown in Figure 3.1, and the local superrotation index s (2.37) and Ψ are shown in Figure 3.2. In each figure, output from the 3D experiments is shown in the top row, and from the axisymmetric experiments in the bottom row.

In all of the experiments, thermally direct Hadley cells which feature air rising at the equator and sinking at higher latitudes are present. The action of the Hadley cell to fill the domain with air that has $\bar{m} \approx \Omega a^2$ (equilibrated with the equatorial surface) is clearly visible in the s field (Figure 3.2), where the structure of the subtropical jets in the $\bar{u}$ (Figure 3.1) is replaced by a region within the Hadley cell where $s \approx 0$. The Hadley cells are flanked by a subtropical jet associated with angular momentum conservation in the poleward moving branch of each cell. In the axisymmetric experiments, the maximum subtropical jet velocity increases from as Ω is decreased from $4\Omega_E$ to $\Omega_E/16$, before decreasing when Ω is decreased further. This result is consistent with the predictions of axisymmetric theory (Held and Hou, 1980; Hou, 1984; Colyer and Vallis, 2019). A similar trend can be identified for the subtropical jets in the 3D experiments.

Three-dimensional experiments with $\Omega \geq \Omega_E$ feature additional mid-latitude, eddy-

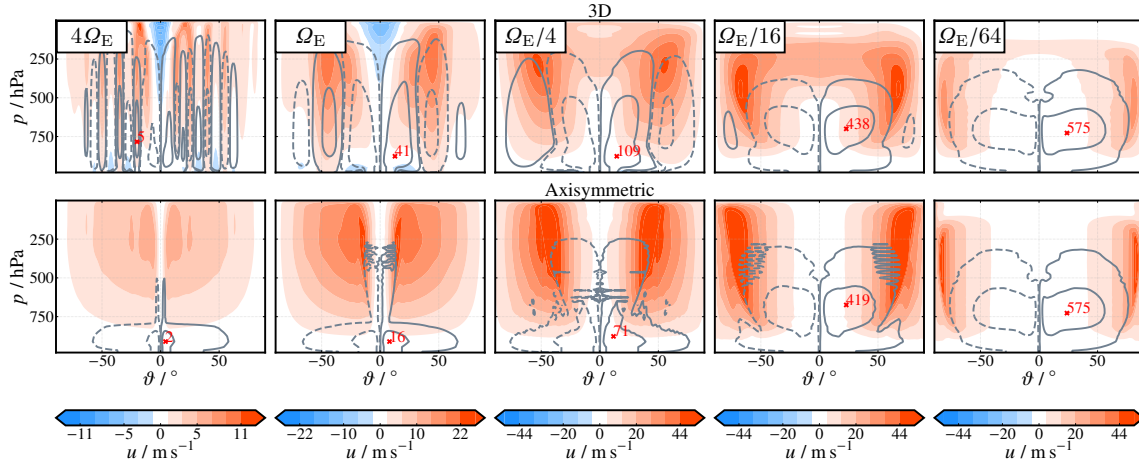


Figure 3.1: Zonal-mean zonal wind u (colour), and meridional mass streamfunction Ψ (contours). All experiments shown were run with T127 horizontal resolution. The rotation rate for each experiment is indicated in the top-left corner of each panel. Note that the colour scale for u varies between panels. The red cross (and number) indicates the location (and value) where Ψ is maximum. The maximum value for Ψ has units $\times 10^9 \text{ kg s}^{-1}$. The solid contours delimit 5% and 50% of the maximum, and the dashed contours show $-1 \times$ the solid contours. As the contour levels are percentages of the maximum value for Ψ , they vary between panels.

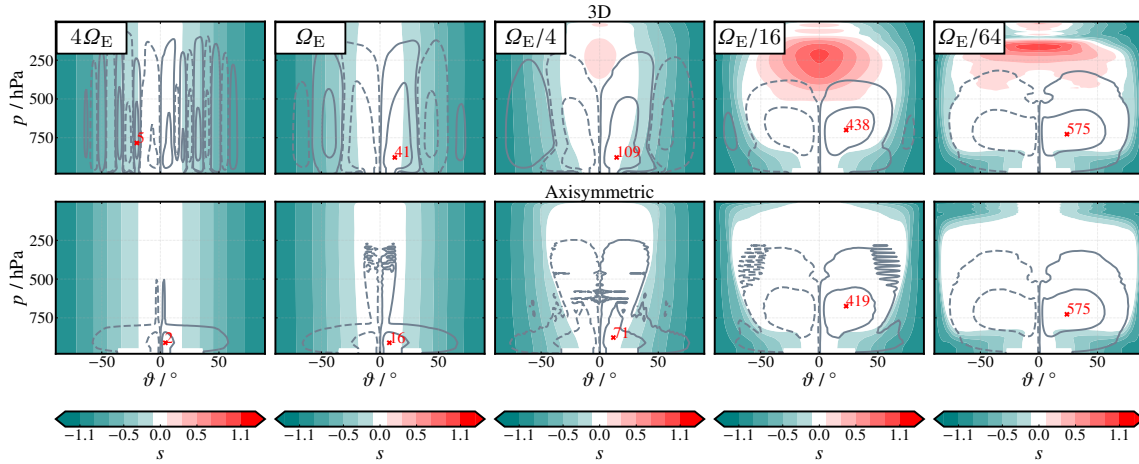


Figure 3.2: Local superrotation index, s . As in Figure 3.1, the red label and contours show the meridional mass streamfunction Ψ (see Figure 3.1 caption for details). All experiments shown were run with T127 horizontal resolution.

driven jets poleward of each sub-tropical jet (Figure 3.1). As the rotation rate is increased from Ω_E to $4\Omega_E$, the number of jets increases as the characteristic length scale of baroclinic instability decreases (Williams, 1978; Lee, 2005). As the mid-latitude jets present in the 3D experiments are eddy driven, they are absent in the axisymmetric experiments at the same rotation rate, which leads to a very different $\bar{u}$ field. However, when the zonal flow is displayed in terms of s (Figure 3.2), the 3D experiments appear very similar to the axisymmetric experiments, and the complex structure of the eddy driven jets is

no longer present. This is explained by the fact that, for rapidly rotating planets, the *relative* component of m ($m_r = u \times a \cos \vartheta$) is much smaller than the contribution to m from the background rotation of the planet ($m_p = \Omega a^2 \cos^2 \vartheta$). Eddies are important for the angular momentum *budget* (as $\partial m / \partial t = \partial m_r / \partial t$), but the absolute value of m is essentially set by m_p .

At rotation rates $\leq \Omega_E/4$, a prograde (locally superrotating) jet emerges at the equator in the 3D experiments (Figure 3.1). This is associated with planetary-scale tropical disturbances which converge momentum towards the equator (Mitchell and Vallis, 2010), which are generated when $\mathcal{R}_T$ is increased to an $\mathcal{O}(1)$ value (Wang and Mitchell, 2014). Regions of local superrotation ($s > 0$, Figure 3.2) can clearly be identified for the 3D experiments, while in the axisymmetric experiments, local superrotation is no longer present, in accordance with Hide's theorem. In the superrotating 3D experiments, s is only significantly positive in the region above the Hadley circulation. This is because within the Hadley cell, air ascending at the equator acts to communicate the effects of friction at the surface to the atmosphere aloft. The meridional extent of the superrotating region is confined within the bounds of the Hadley circulation, indicating that momentum is only converged towards the equator from within the region where it is deposited by meridional overturning (Gierasch, 1975; Rossow and Williams, 1979).

In the 3D experiments, the maximum (superrotating) zonal wind velocity at the equator is found in the $\Omega_E/16$ experiment (28 m s^{-1}). It is of interest to note that while equatorial velocities in the $\Omega_E/4$ and $\Omega_E/16$ experiments are similar (Figure 3.1), the maximum local superrotation *relative* to the underlying planet is much weaker in the $\Omega_E/4$ experiment when compared with the $\Omega_E/16$ experiment (Figure 3.2). When the rotation rate is further decreased to $\Omega_E/64$ the maximum velocity at the equator is reduced (8 m s^{-1}) (Figure 3.1). The trend for the maximum equatorial velocity to decrease when $\Omega \ll \Omega_E$ is reduced is also present in the T42 experiments: the maximum equatorial velocity is 1.2 m s^{-1} at $\Omega_E/256$, and just 0.4 m s^{-1} at $\Omega_E/512$ (not shown).

3.4 Dependence of global superrotation on rotation rate

3.4.1 Results from numerical experiments

We now turn our discussion towards the dependence of the global superrotation index S on planetary rotation rate Ω . Figure 3.3 shows S vs. $\mathcal{R}_T$ for each of the experiments run for this work. S is shown as a function of $\mathcal{R}_T$ instead of Ω in order to make a cleaner comparison between the numerical experiments, and estimates of S for the real planets (Venus, Titan, Mars, the Earth).

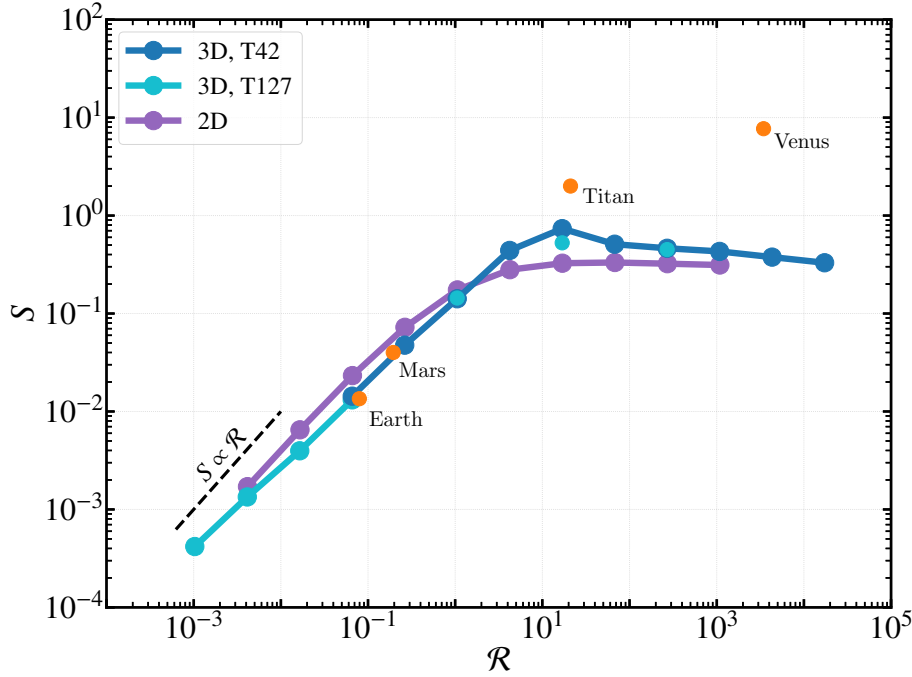


Figure 3.3: Global superrotation index S vs. thermal Rossby number $\mathcal{R}_T$. 3D experiments are shown in blue, and axisymmetric experiments in purple. Experiments run at T42 are plotted in dark blue, and those run at T127 are plotted in light blue. The orange dots show estimates for S for the Earth, Mars, Titan and Venus, and are taken from Read and Lebonnois (2018). The black dashed line indicates a Ω^{-2} scaling.

In both the 3D and the axisymmetric experiments, S (S_{3D} and S_{ax} , respectively) increases with decreasing Ω (increasing $\mathcal{R}_T$) when $\mathcal{R}_T$ is small (rapid rotation rate), and $S \ll 1$, which indicates that the dynamics are in a regime where the local momentum budget is dominated by geostrophic balance throughout the domain. When $\mathcal{R}_T$ is large (slow rotation rate), S saturates so that it becomes virtually independent of Ω (and $\mathcal{R}_T$). S saturates at a value $S < 1$, indicating that a regime where the atmospheric dynamics are dominated by cyclostrophic balance is not obtained – even for the 3D experiments. The sole exception to this is the 3D $\Omega = \Omega_E/16$ experiment (T42 resolution), for which S is close to unity.

For the axisymmetric experiments, S_{ax} appears to scale inversely with Ω^2 when $\mathcal{R}_T$ is small; this may be inferred by comparing the black dashed line (proportional to Ω^{-2}) with the solid purple line. At high $\mathcal{R}_T$, S_{ax} appears to saturate at a value $S_{ax} \approx 0.3$. This is less than the limiting value of $S_{ax} = 1/2$ for an axisymmetric atmosphere.

The normalised difference $\Delta S = (S_{3D} - S_{ax})/S_{ax}$ (shown in Figure 3.4) between S_{3D} and S_{ax} is generally $|\Delta S| < 1$, with the exception of the $\Omega_E/16$ T42 experiment, for which $\Delta S = 1.25$. For the T127 experiments, ΔS is maximal when $\Omega = \Omega_E/16$, for which $\Delta S = 0.61$, and minimal (most negative) when $\Omega = \Omega_E$, for which $\Delta S = -0.43$. The

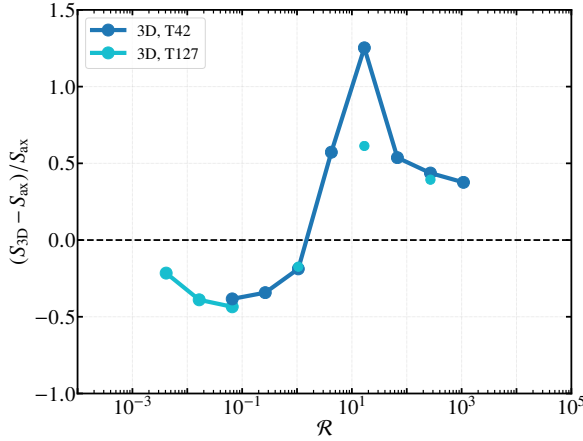


Figure 3.4: Difference between S_{3D} and S_{ax} , normalised by S_{ax} . The shade of blue indicates the resolution of the 3D experiment. All axisymmetric experiments were run with meridional resolution equivalent to the T127 3D experiments.

values taken by S_{3D} appear to be perturbations around S_{ax} (i.e., S in each case is of the same order of magnitude). In the idealised GCM simulations considered here, eddies are unable to substantially modify the globally integrated atmospheric angular momentum away from that realised in axisymmetric setting.

When $\mathcal{R}_T$ is small, S_{3D} is less than S_{ax} , and when $\mathcal{R}_T$ is large, S_{3D} is greater than S_{ax} . Each of the numerical experiments is initiated with $u = 0$ everywhere, corresponding to $S = 0$, and so the equilibrium value of S is a measure of the total zonal angular momentum injected into the atmosphere during spin-up; $dS/dt = 0$ when the globally integrated surface stress $\mathcal{T} = 0$. Figure 3.5 shows S as function of time over the first 120 days of the $\Omega = \Omega_E$ experiment, and the first 360 days of the $\Omega = \Omega_E/16$ experiment (both T127 resolution). For both experiments shown, S_{3D} and S_{ax} are initially nearly identical. During this initial period, angular momentum is lifted into the atmosphere by the mean flow, and there is little vertical momentum transport due to eddy activity. After day 30 for the $\Omega = \Omega_E$ experiment, and day 180 for the $\Omega = \Omega_E/16$ experiment, non-axisymmetric disturbances cause S_{3D} and S_{ax} to diverge. For the $\Omega_E/16$ case, eddies which induce an up-gradient transport of m allow the mean flow to lift more angular momentum into the atmosphere by removing angular momentum from the descending branch of the Hadley circulation and depositing it at the equator (Gierasch, 1975; Rossow and Williams, 1979). This leads to $S_{3D} > S_{ax}$. For the Ω_E case, Rossby waves generated by baroclinic instability induce a down-gradient transfer of m into the boundary layer (Andrews et al., 1983), which counteracts upward transport of m by the mean flow. This causes the Ω_E experiment to equilibrate with $S_{3D} < S_{ax}$.

The difference between the 3D and axisymmetric experiments $|\Delta S|$ is largest for intermediate values of $\mathcal{R}_T \approx 0.1 - 10$. When $\mathcal{R}_T \ll 1$ and $\mathcal{R} \gg 1$ (i.e., at very high and very low rotation rate) S_{3D} converges towards S_{ax} . This is consistent with a decrease in the

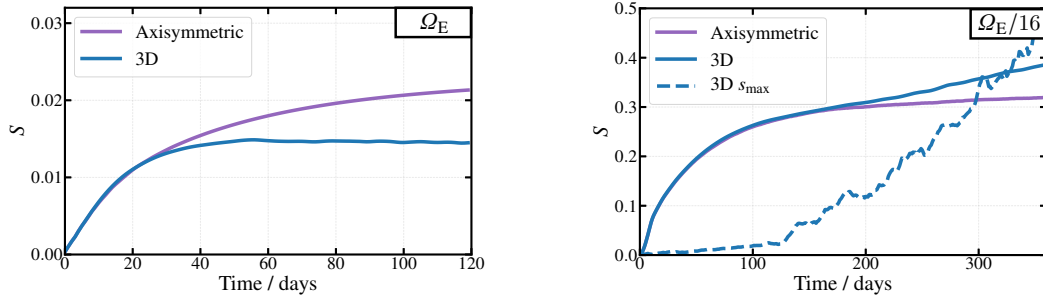


Figure 3.5: S vs. t for axisymmetric and 3D experiments. Left panel: S vs. t for the first 120 days of the Ω_E experiments. Right panel: S vs. t for the first 360 days of the $\Omega_E/16$ experiments. Blue curves show 3D experiments and purple curves show axisymmetric experiments. All output shown is from experiments with T127 resolution.

characteristic scale of baroclinic instability as the rotation rate increased in the $\mathcal{R}_T \ll 1$ regime (Lee, 2005), and the drop-off in the magnitude of equatorial superrotation as the rotation rate is decreased in the $\mathcal{R}_T \gg 1$ regime.

Estimates of S for the Earth, Mars, Venus, and Titan (taken from Read and Lebonnois, 2018) are shown as orange dots in Figure 3.3. The 3D experiments with $\mathcal{R}_T$ closest to the Earth (Ω_E) and Mars ($\Omega_E/2$) obtain values for S similar to those obtained by the two real planets. At low rotation rate, however, experiments with $\mathcal{R}_T$ characteristic of Venus ($\Omega_E/128$) and Titan ($\Omega_E/16$) do not generate global superrotation of a similar order of magnitude to that estimated for the real planets (particularly for Venus), underscoring the fact that the processes responsible for storing excess m in the atmospheres of Venus and Titan are not (fully) represented in an Earth-like numerical model.

3.4.2 Theory for S in axisymmetric case

The objective of this section is to understand which processes contribute to the $\mathcal{R} \ll 1$ and $\mathcal{R} \gg 1$ scalings for S obtained in the numerical experiments (Figure 3.3). To do so, a theory is developed for S in terms of $\mathcal{R}_T$, for the case of an axisymmetric, inviscid atmosphere, based on the model of Held and Hou (1980; the HH model).

To relate S to $\mathcal{R}_T$, we make use of the zonal velocity profiles found for the tropical and extra-tropical regions of the HH model. We will assume

$$u = \begin{cases} u_{\text{HC}}, & \vartheta \in [0, \vartheta_H], \\ u_{\text{ET}}, & \vartheta \in (\vartheta_H, \pi/2], \end{cases} \quad (3.7)$$

where u_{HC} is given by

$$u_{\text{HC}} = \frac{\Omega a \sin^2 \vartheta}{\cos \vartheta} \quad (3.8)$$

for all $p/p_s < 0.8$ (i.e., in the free atmosphere), and $u_{\text{HC}} = 0$ in the boundary layer. u_{ET} is given by (Held and Hou, 1980)

$$u_{\text{ET}} = \Omega a \cos \vartheta \left(\sqrt{2\mathcal{R}_T \frac{z}{H} + 1} - 1 \right), \quad (3.9)$$

through the depth of the atmosphere (this expression for u_{ET} can be obtained by substituting $\theta = \theta_{\text{eq}}$ into the expression for gradient wind balance given by Equation 2.27). By assuming that u_{HC} is independent of z in the free atmosphere, we are assuming that, above the boundary layer, $\partial\theta/\partial\vartheta \rightarrow 0$ within the Hadley cell (Fang and Tung, 1999).

It is worth noting that in the numerical simulations, the Hadley cell boundary contracts equatorwards with decreasing altitude, and consequently so does the region where $u = u_{\text{HC}}$. Therefore, the choice for u_{HC} may lead to an overestimate of S , although this overestimate will likely be small, as the Hadley cell width exhibits a weak altitude dependence above 800 hPa in the majority of the numerical experiments (see the boundary of the white contour in Figure 3.2).

The definition of S may be re-written

$$S = \frac{\int \rho a \cos \vartheta u \, dV}{\int \rho \Omega a^2 \cos^2 \vartheta \, dV}, \quad (3.10)$$

as the -1 in (2.23) cancels with the planetary ($\Omega a^2 \cos^2 \vartheta$) contribution to m in the numerator. S may then be split into contributions from the Hadley cell and extra-tropical regions

$$S = S_{\text{HC}} + S_{\text{ET}} \quad (3.11)$$

where

$$S_{\text{HC}} = \frac{\int_0^{\frac{4}{5}p_s} \int_0^{\vartheta_{\text{HC}}} u_{\text{HC}} \cos^2 \vartheta \, d\vartheta \, dp/g}{\int_0^{p_s} \int_0^{\frac{\pi}{2}} \Omega a \cos^3 \vartheta \, d\vartheta \, dp/g}, \quad (3.12)$$

and,

$$S_{\text{ET}} = \frac{\int_0^H \int_{\vartheta_{\text{HC}}}^{\frac{\pi}{2}} u_{\text{ET}} \cos^2 \vartheta \, d\vartheta \, \rho_0 dz}{\int_0^H \int_0^{\frac{\pi}{2}} \Omega a \cos^3 \vartheta \, d\vartheta \, \rho_0 dz}. \quad (3.13)$$

Note that in the Boussinesq framework, $\rho = \rho_0$ is a constant reference density, and the hydrostatic relation $dp = -g\rho_0 dz$ has been used to write (3.12) as an integration over pressure. The integration $\int d\lambda$ in each expression cancels between the numerator and denominator due to axisymmetry, and is omitted.

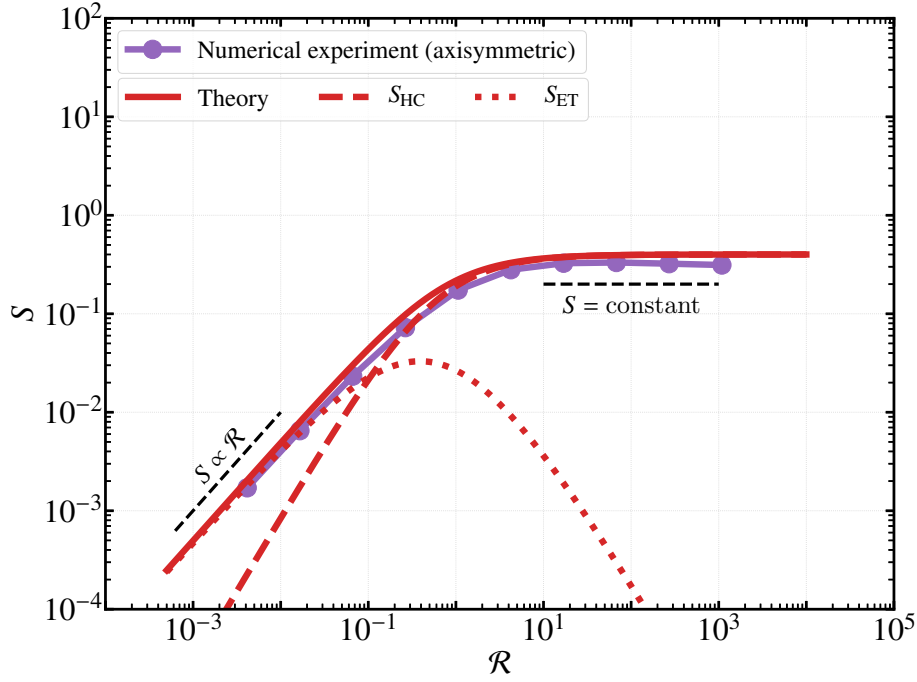


Figure 3.6: Theoretical prediction for S from Equation 3.14 vs. $\mathcal{R}_T$ for the case of an axisymmetric atmosphere (solid red curve). Also shown are the contributions to the theoretical S from the Hadley cell and extra-tropical regions, S_{HC} and S_{ET} . The purple line shows S for the axisymmetric numerical experiments.

Equations (3.12) and (3.13) may then be evaluated analytically using (3.8) and (3.9) to obtain

$$\begin{aligned} S &= S_{HC} + S_{ET}, \\ &= \frac{2}{5} \sin^3 \vartheta_H + \frac{1}{12} \left[\frac{(2\mathcal{R}_T + 1)^{\frac{3}{2}} - 1}{2\mathcal{R}_T} - \frac{3}{2} \right] (8 - 9 \sin \vartheta_H - \sin 3\vartheta_H). \end{aligned} \quad (3.14)$$

As ϑ_H depends only on $\mathcal{R}_T$ (obtained by solving Equation 2.32 in Chapter 2, Section 2.4), S is now determined solely by $\mathcal{R}_T$.

3.4.3 Comparison between theoretical S and simulation results

Figure 3.6 shows S given by (3.14) plotted against $\mathcal{R}_T$. The values for S calculated for the axisymmetric numerical experiments are also shown, and there is good agreement between the theoretical prediction for S and the numerical experiments in both the high and low rotation rate limits. For intermediate $\mathcal{R}_T$, the theoretical prediction for S is a slight overestimate of that obtained in the numerical experiments; this difference arises from the simplifying assumption that ϑ_H is independent of height, which is most strongly violated by experiments with intermediate $\mathcal{R}_T$ (see, for example, the $\Omega_E/4$ experiment in Figure 3.2).

Equation (3.14) is comprised of two terms, one that measures the contribution to S from the Hadley cell region (S_{HC}), and one that measures the contribution from the extra-tropical region (S_{ET}). These are shown in Figure 3.6 as a dashed and dotted curve, respectively. At high rotation rates, the extra-tropical term dominates the total value of S , as in this regime the Hadley cell is small and the ‘extra-tropics’ essentially comprise the whole atmosphere. At low rotation rates, the contribution to S from the Hadley cell dominates, as the Hadley cell expands to fill the domain. The transition between the extra-tropical and Hadley-cell dominated regimes occurs when the first and second terms in (3.14) are equal. This occurs when $\mathcal{R}_T = 0.11$, corresponding to a Hadley cell latitude $\vartheta_H = 22.8^\circ$.

It is instructive to consider the behaviour of the axisymmetric theory in the limits $\mathcal{R}_T \ll 1$ and $\mathcal{R}_T \gg 1$. When $\mathcal{R}_T \ll 1$ (rapid rotation)

$$\frac{(2\mathcal{R}_T + 1)^{\frac{3}{2}} - 1}{2\mathcal{R}_T} - \frac{3}{2} \approx \frac{3\mathcal{R}_T}{4} \quad (3.15)$$

(via Taylor expansion). Additionally $\vartheta_H \rightarrow 0$, meaning that $\sin \vartheta_H$, $\sin 3\vartheta_H$, and $\sin^3 \vartheta_H \rightarrow 0$. From (3.14), we obtain

$$S \approx \frac{1}{2}\mathcal{R}_T. \quad (3.16)$$

In the rapidly rotating regime, S is dominated by the extra-tropical region (as the Hadley Cell is small), and (3.16) predicts that S scales linearly with $\mathcal{R}_T$, which, in terms of the rotation rate, corresponds to a Ω^{-2} scaling.

The $\mathcal{R}_T$ dependence in (3.14) comes from the expression for u_{ET} (3.9), which originally derives from enforcing gradient wind balance in the extra-tropical region with the radiative-convective equilibrium forcing profile. This tells us that the $S \propto \Omega^{-2}$ scaling in the rapidly rotating regime is simply that implied by geostrophic thermal wind balance, which in this scenario is the dominant dynamical balance throughout the domain.

When $\mathcal{R}_T \gg 1$ (slow rotation) then $\vartheta_H \rightarrow \pi/2$, so that $\sin \vartheta_H$ and $\sin^3 \vartheta_H \rightarrow 1$, while $\sin 3\vartheta_H \rightarrow -1$. In this scenario, the axisymmetric theory approaches

$$S \approx \frac{2}{5} = \text{constant}. \quad (3.17)$$

The $\mathcal{R}_T \gg 1$ limit is the scenario where the Hadley cell has filled the entire domain. This is qualitatively the same as the $S = 1/2$ limit suggested in the introduction to this chapter, and the additional factor $4/5$ comes from assuming $u = 0$ in the boundary layer ($p/p_s > 0.8$). In this limit, the entire free atmosphere has specific zonal angular momentum $m = \Omega a^2$ due to the equilibration of air with the equatorial surface, and

conservation of m in the free atmosphere. In this scenario, the dominant term in the local momentum budget varies through the domain; at low latitudes, geostrophic balance is dominant, and at high latitudes, cyclostrophic balance dominates. This may be appreciated by considering the ratio (cyclostrophic term)/(geostrophic term) in the gradient wind equation (2.27) when $u = u_{\text{HC}}$, i.e., $u_{\text{HC}} \tan \vartheta / (2\Omega a \sin \vartheta) = \tan^2 \vartheta / 2$, which is greater than one when ϑ is large, and less than one when ϑ is small.

In summary, the results from the theoretical model for S suggest that in the axisymmetric experiments, the dependence of S on $\mathcal{R}_T$ at high rotation rate is determined by geostrophic thermal wind balance, while at low rotation rate, it is determined by approximate conservation of angular momentum. The presence of eddies does not alter the dependence of S on $\mathcal{R}_T$ obtained in the 3D experiments from that in the axisymmetric experiments, at least to first order (see Section 3.4), and so it is likely that the same processes also determine the $\mathcal{R}_T \ll 1$ and $\mathcal{R}_T \gg 1$ behaviour of S in the 3D model.

3.5 Relationship between S and circulation regime

Part of the motivation for studying S is due to its similarity with a zonal Rossby number, which means that the value of S is indicative of the dynamical regime occupied by a planetary atmosphere (Read, 1986a).

By considering the limits of the axisymmetric theory for S , it was shown that the relationship between S and circulation regime is manifest in the local dependence of S on $\mathcal{R}_T$. In the rapid rotation rate limit, $S \propto \mathcal{R}_T \ll 1$ defines a regime where geostrophic balance is dominant in the local momentum budget, throughout the atmosphere. At low rotation rate, $S = \text{constant}$ defines a regime where the dynamics are determined by approximate conservation of angular momentum within the Hadley circulation, which fills the domain at low rotation rate, and the local momentum budget is characterised by a mixed gradient wind balance.

As discussed in the introduction, $S \gg 1$ indicates a further regime, characteristic of the atmospheres of Venus and Titan, where the local momentum budget is dominated by cyclostrophic balance throughout the domain. It is not possible to comment on the cyclostrophic regime within the framework provided by the axisymmetric theory; $S > 1/2$ requires some representation of up-gradient angular momentum transport, and is not obtainable in an axisymmetric, inviscid model.

However, we can draw on the results of Yamamoto and Yoden (2013; henceforth the ‘YY theory’), who derive an expression for a non-dimensional measure of superrotation

strength $S' = 4S/3$ in terms of $\mathcal{R}_T$,

$$\left[S'^2 + 2S' + BS' \left(\frac{2 + S'}{1 + S'} \right) \right] \left[\frac{AS'}{2} \left(\frac{2 + S'}{1 + S'} \right) + 1 \right] = 2\mathcal{R}_T, \quad (3.18)$$

derived from the Boussinesq primitive equations on a sphere forced by Newtonian relaxation (e.g., as in the HH model), *in the presence of diffusion of angular velocity*. S depends on the external parameters A , B , and $\mathcal{R}_T$. $\mathcal{R}_T$ is the same as for the axisymmetric theory, and $A = \pi^2 \tau / \tau_V$ and $B = 20\pi^2 (\Omega^{-1} / \sqrt{\tau_H \tau_V})^2$, where τ is the radiative relaxation timescale, τ_V is the timescale for vertical eddy momentum diffusion, and τ_H is the timescale for horizontal eddy momentum diffusion. In the YY theory, diffusion terms are formulated in analogy with molecular viscosity and so can transport angular momentum up-gradient (Read, 1986b). Yamamoto and Yoden (2013) assume that the relaxation time for horizontal diffusion is much shorter than the turnover time of the meridional circulation, and the relaxation time for vertical diffusion, which allows the YY theory to access the cyclostrophic regime where $S \gg 1$, occupied by Titan and Venus.

The dependence of S on $\mathcal{R}_T$ given by the YY theory is shown in Figure 3.7 alongside values for S computed from the axisymmetric, inviscid theory developed in Section 3.4.2 and from the numerical experiments (Section 3.4.1). S from the YY theory is only shown for $\mathcal{R}_T > 0.1$, as when $\mathcal{R}_T \ll 1$ it is generally no longer appropriate to assume up-gradient eddy angular momentum transport. Multiple curves are shown, with different curves corresponding to different values for A . The values for A are chosen to be in the ‘correct ballpark’ for Venus and Titan (see Section 5.5 of YY), and multiple values are shown to demonstrate the dependence of S on A . For all YY curves, $B = 0.3$ is set. The parameter B is only important in determining the behaviour of S at low and intermediate $\mathcal{R}_T$ (high and intermediate rotation rate), and has no effect on the $\mathcal{R}_T \gg 1$ asymptote. $B < 1$ puts the curves in the geostrophic regime occupied by the idealised GCM simulations, the Earth, and Mars, and the $B = 0.3$ is chosen so that the YY theory curves join up with the axisymmetric theory developed in this work at approximately $\mathcal{R}_T = 0.1$.

YY show that for intermediate values for $\mathcal{R}_T$, $1 < \mathcal{R}_T < 10$, S scales with $\sqrt{2\mathcal{R}_T} \propto \Omega^{-1}$, which corresponds to either (i) a scaling associated with the dominance of cyclostrophic balance in the local momentum budget, or (ii) a scaling associated with the dominance of geostrophic balance, modified from $S \propto \mathcal{R}_T$ (i.e., the scaling given by Equation 3.16) to $S \propto \sqrt{\mathcal{R}_T}$. This modified-geostrophic scaling arises due to a significant reduction in the equator-to-pole temperature difference ΔT with respect to its radiative equilibrium value ΔT_{eq} . Regime (i) is realised when the timescale for vertical eddy momentum diffusion τ_V is much longer than the radiative relaxation time τ , $\tau_V \gg \tau$, while regime

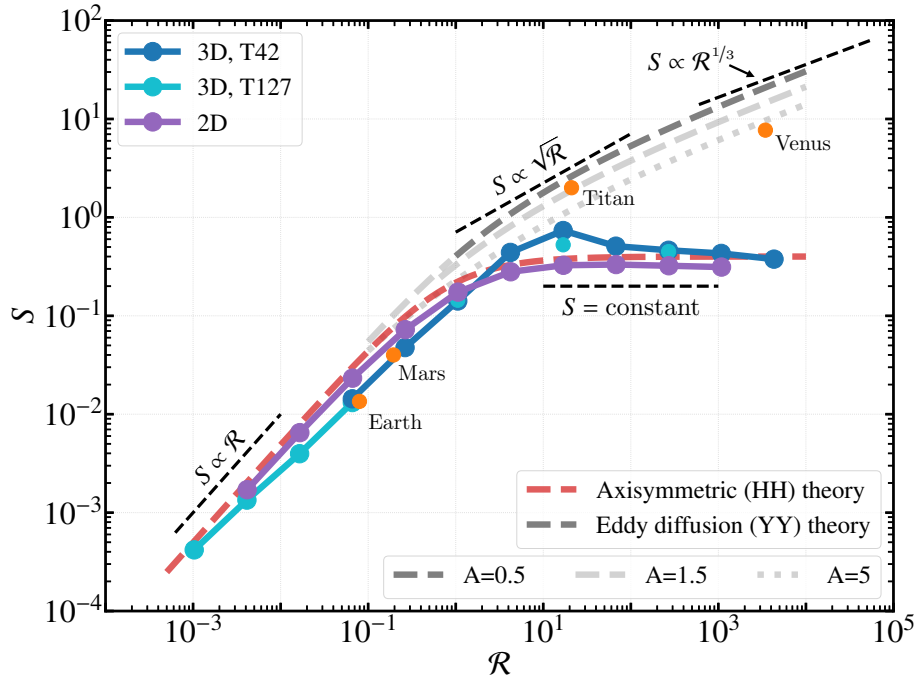


Figure 3.7: S vs. $\mathcal{R}_T$ calculated from numerical experiments (solid curves) and theoretical predictions (dashed curves). 3D experiments are shown in blue, and axisymmetric experiments in purple (see Figure 3.3 caption for details). The orange dots show estimates for S for the Earth, Mars, Titan and Venus taken from Read and Lebonnois (2018). The red dashed curve shows S given by the axisymmetric theory (3.14). The grey dashed curves show S given by the quasi-axisymmetric YY theory (3.18). In the YY theory, S depends on parameters A and B in addition to $\mathcal{R}_T$. Parameter values for A are shown in the legend, and B is set to $B = 0.3$ (see text for a definition of A and B , and justification of the values chosen for each). S from the YY theory is only shown for $\mathcal{R}_T > 0.1$. This choice is made both to enhance the clarity of the figure (for $B = 0.3$ the YY theory curves are essentially identical to that given by the axisymmetric theory when $\mathcal{R}_T \ll 1$), and because, when $\mathcal{R}_T \ll 1$, it is difficult to place a physical interpretation on the circulation implied by the YY theory due to the assumption of strong horizontal diffusion (see footnote 2 in the text, and Section 5.5 of Yamamoto and Yoden, 2013).

(ii) corresponds to the opposite scenario where $\tau_V \ll \tau$. For large $\mathcal{R}_T > 10$ ($S \gg 1$), S scales with $\mathcal{R}_T^{1/3} \propto \Omega^{-2/3}$, indicating a modified-cyclostrophic regime (again, due to a reduction in ΔT with respect to ΔT_{eq}).

Strong up-gradient angular momentum transport allows the YY theory to access the cyclostrophic regime, but it precludes their theory from describing the angular momentum conserving regime occupied by the numerical experiments presented in this work. Additionally, the strong-diffusion assumption makes it difficult to place a physical interpretation on the behaviour of the YY theory when $\mathcal{R}_T \ll 1$ (not shown in Figure 3.7)². However, when the theory for the axisymmetric, inviscid case, developed

²The YY theory does have a geostrophic regime for $\mathcal{R}_T \ll 1$ where $S \propto \mathcal{R}_T$ (if additionally $B \ll 1$), but the implied meridional circulation structure is quite different from that assumed for the theory developed in this chapter, due to the assumed importance of horizontal diffusion in Yamamoto and

in this chapter, is taken in tandem with the YY theory, the breadth of dynamical regimes accessed by the idealised numerical experiments shown in Figure 3.1 and Figure 3.2, and the real planets is captured.

Figure 3.7 makes clear the utility of the global superrotation index S for characterising and comparing the dynamical regimes occupied by different atmospheres. The Earth and Mars clearly occupy the geostrophic regime defined by $S \ll 1$ and $S \propto \mathcal{R}_T$, while Titan and Venus clearly occupy the modified-geostrophic or cyclostrophic regimes for which $S \gg 1$ and $S \propto \sqrt{\mathcal{R}_T}$ or $S \propto \mathcal{R}_T^{\frac{1}{3}}$. It is also apparent that the regime towards which the Earth-like atmospheric model asymptotes when $\mathcal{R}_T \gg 1$, defined by $S = \text{constant} \approx 1/2$, is fundamentally different from that occupied by Venus and Titan. Figure 3.7 shows that the circulation regime associated with a given value of $\mathcal{R}_T$ is degenerate when $\mathcal{R}_T$ is made large. This is due to the high sensitivity of the atmospheric circulation to other input parameters in the slowly rotating regime (as discussed in the introduction to this chapter, cf. Dias Pinto and Mitchell, 2014). Unlike the thermal Rossby number $\mathcal{R}_T$, the combination of the value of S and the local dependence of S on $\mathcal{R}_T$ uniquely define the circulation regime occupied by a planetary atmosphere.

3.6 Summary

The work presented in this chapter has investigated the dependence of global superrotation on planetary rotation rate using a combination of numerical and theoretical modelling. The aim of this work was to investigate the utility of the global superrotation index S for characterising and comparing the circulation regimes occupied by different planetary atmospheres. The results from this study are summarised graphically in Figure 3.7.

The dependence of S on Ω was studied using an idealised Earth-like general circulation model in both 3D and axisymmetric configurations (Figure 3.3). In both configurations, $\mathcal{R}_T \ll 1$ (rapid rotation rate) $S \ll 1$, and S increases as Ω is decreased. When Ω is decreased sufficiently so that $\mathcal{R}_T \gg 1$, S stops increasing with decreasing Ω , and

Yoden (2013). Even at high rotation rate, a single Hadley cell spans each hemisphere in their study (see, e.g., the left-most columns Figure 4 in Yamamoto and Yoden, 2013). This feature of the circulation in the YY theory, combined with the sensitivity of the YY theory to the parameter B , makes it difficult to give a physical interpretation on its behaviour when $\mathcal{R}_T \ll 1$. In particular, in the axisymmetric theory developed in this chapter, the transition from the geostrophic to angular momentum conserving scaling for S is associated with the expansion of the Hadley cell region (and contraction of the extra-tropical region). In the YY theory, the physical interpretation of the transition away from the geostrophic regime is less clear. The limitations of the YY theory when $\mathcal{R}_T \ll 1$ are discussed in greater depth in Section 5.5 of Yamamoto and Yoden (2013).

instead saturates at a constant value $S \approx 1/2$. Over the entire parameter space considered, S in the 3D and axisymmetric experiments (S_{3D} and S_{ax}) are always of the same order of magnitude (Figure 3.4). Eddies are unable to substantially modify S_{3D} away from S_{ax} , and instead S_{3D} takes the form of perturbations about S_{ax} . At high rotation rate, eddies induce down-gradient transport of m which causes $S_{3D} < S_{ax}$, while at low rotation rate, eddies induce up-gradient transport of m which leads to $S_{3D} > S_{ax}$. At both *very high* and *very low* rotation rates S_{3D} and S_{ax} converge.

The dependence of S on Ω (or more generally $\mathcal{R}_T$) obtained in the numerical experiments can be captured with an axisymmetric theoretical model for S (Figure 3.6), derived from the model of Held and Hou (1980). In the rapid rotation rate limit ($\mathcal{R}_T \ll 1$), the theoretical model predicts $S \approx \mathcal{R}_T/2$, and in the low rotation rate limit ($\mathcal{R}_T \gg 1$), the theoretical model predicts that $S \approx 2/5$, both of which are consistent with the results of the numerical experiments shown in Section 3.3. By considering the limiting behaviour of the theory, it is shown that the value of S and its local dependence on $\mathcal{R}_T$ is related to the dominance of different dynamical balances in the atmosphere. $S \ll 1$ and $S \propto \mathcal{R}_T$ defines a regime where geostrophic thermal wind balance is dominant throughout the domain, and $S \approx 1/2 = \text{constant}$ defines a regime dominated by conservation of angular momentum within the overturning circulation. In this scenario, the dominant momentum balance is geostrophic at low latitudes, and cyclostrophic at middle and high latitudes.

By consulting the axisymmetric theory in tandem with an alternative theory for S proposed by Yamamoto and Yoden (2013), which defines a cyclostrophic regime when $S \gg 1$ and $S \propto \sqrt{\mathcal{R}_T}$ or $S \propto \mathcal{R}_T^{1/3}$, it is possible to characterise each of the circulation regimes accessed by the idealised numerical experiments presented in this chapter, and the Solar System planets (Figure 3.7). This analysis suggests that the atmospheres of Venus and Titan, characterised by cyclostrophic balance and strong equatorial superrotation, are in a fundamentally different dynamical regime to the idealised Earth-like simulations in their low rotation rate limit. With this in mind, it is argued that slowly rotating planets do not enter a cyclostrophic, strongly superrotating regime by ‘default’, as is sometimes assumed.

The work presented in this chapter demonstrates that the global superrotation index S is a useful quantity to study when comparing the dynamics of different planetary atmospheres. It is hoped that this work, in combination with that of Yamamoto and Yoden (2013), will provide a framework for future work to make more use of S .

Equatorial Waves and Superrotation in the Stratosphere of a Titan General Circulation Model

The contents of this chapter are a modified version of N. T. Lewis, N. A. Lombardo, P. L. Read, and J. M. Lora (2022): Equatorial Waves and Superrotation in the Stratosphere of a Titan General Circulation Model, submitted to *The Planetary Science Journal* (pending first referee report).

Author contributions: NTL proposed the research. JML and NAL designed and ran the GCM simulation. NTL conducted the analysis and wrote the paper. NAL, JML and PLR proof read the paper, and advised on the interpretation and discussion of results.

4.1 Introduction

Titan's stratospheric circulation is dominated by a broad prograde zonal jet, which extends from pole to pole and reaches speeds in excess of 200 m s^{-1} (Achterberg et al., 2008; Sharkey et al., 2021). The structure of this jet varies with Titan's seasonal cycle, with maximum zonal velocities obtained in mid-autumn (Sharkey et al., 2021). A property of Titan's zonal jet is that it is superrotating, which means that it has more axial angular momentum, m , than the solid planet at the equator (Read, 1986b; Read and Lebonnois, 2018). As discussed in Chapter 2, the existence of superrotation requires that non-axisymmetric eddies exist that act to mix angular momentum up-gradient (Hide, 1969; Gierasch, 1975; Rossow and Williams, 1979; Read, 1986b). In this chapter, my aim is to characterise the waves present in a simulation of Titan's atmospheric circulation, and determine which are responsible for accelerating equatorial superrotation. This work is partially motivated by the discussion of the global superrotation index presented in Chapter 3, and may be viewed as an attempt to determine why Titan's atmospheric circulation occupies the 'cyclotrophic regime' as opposed to the 'angular momentum conserving regime' (cf. Figure 3.7).

In an effort to understand and interpret Titan's observed atmospheric circulation, a number of general circulation models (GCMs) of Titan's atmosphere have been developed. They include: the Köln Titan model (Tokano et al., 1999), TitanCAM (Friedson et al., 2009), TitanWRF (Newman et al., 2011), the IPSL Titan model (Lebonnois et al., 2012a), and the Titan Atmospheric Model (TAM; Lora et al., 2015). Each of these models integrates the hydrostatic primitive equations forwards in time to simulate Titan's

atmospheric circulation, forced by radiative heating designed to imitate that in Titan's real atmosphere. A major success of TitanWRF, the IPSL model, and TAM has been reproducing Titan's stratospheric superrotation with a strength similar to that inferred from Cassini observations (Newman et al., 2011; Lebonnois et al., 2012a; Lora et al., 2015). Strong superrotation is not generated in TitanCAM or the Köln model, possibly due to poor conservation of angular momentum in their dynamical cores (Lebonnois et al., 2012b).

TitanWRF, the IPSL model, and TAM present an opportunity to investigate the properties of wave-like disturbances that exist under Titan-like atmospheric conditions, and lead to the acceleration of superrotation. To this end, Newman et al. (2011) and Lebonnois et al. (2012a) analyse the angular momentum budget and waves obtained in their respective models, TitanWRF and the IPSL model. In both cases, eddy acceleration of the superrotating jet is strongest around the solstices, and acts to balance deceleration at the equator due to mean-flow advection of angular momentum into the winter hemisphere. In the TitanWRF model, eddy accelerations appear to be concentrated into short, intense 'transfer events'. Both Newman et al. (2011) and Lebonnois et al. (2012a) find that the acceleration of superrotation is predominantly due to waves with low zonal wavenumbers, which in their models likely originate from barotropic instabilities in sub-tropical latitudes. An equivalent analysis has not been performed for TAM.

More recently, a number of studies have proposed that coupling between Rossby waves and equatorial Kelvin waves can lead to the acceleration of superrotation on small and/or slowly rotating planets (Mitchell and Vallis, 2010; Wang and Mitchell, 2014). However, this is based on modeling results from idealised Earth-like GCMs where the planetary radius or rotation rate is reduced from Earth's, and it has not yet been investigated whether or not coupled Rossby–Kelvin waves play a role in accelerating Titan's superrotation.

4.1.1 *Chapter outline*

In this chapter, I present an analysis of the equatorial waves present in a simulation of Titan's atmosphere run using the TAM general circulation model (Lora et al., 2015), focusing on the stratosphere. This analysis has three main objectives. The first is to characterise the different equatorial waves that are supported in the TAM simulation, with an emphasis on investigating their three-dimensional structure, as exploration of this topic in previous work has been limited in scope. The second objective is to determine which of the waves contribute most to accelerating Titan's equatorial superrotation, and

understand whether or not these waves are affected by Titan's seasonal cycle. Finally, the third objective is to investigate the mechanisms that generate the waves identified in the model atmosphere.

The rest of this chapter is organised as follows. The TAM general circulation model and the diagnostics used to analyse the waves in the simulation are described in [Section 4.2](#). The basic properties of the atmospheric circulation obtained in the TAM simulation and its seasonal variation are described in [Section 4.3](#). The main results of this study are then presented in [Section 4.4](#) to [Section 4.6](#). [Section 4.4](#) and [Section 4.5](#) analyse the waves present in the model atmosphere in northern hemisphere autumn and winter, respectively. [Section 4.6](#) discusses the likely sources of instability that generate the waves in the simulation. Finally, discussion is presented in [Section 4.7](#), and the chapter's main conclusions are summarised in [Section 4.8](#).

4.2 Methodology

4.2.1 General Circulation Model

The work in this chapter uses results from a simulation run using the Titan Atmospheric Model GCM. TAM is introduced and described in detail in Lora et al. (2015), and updated in Lora and Ádámkovics (2017) and Battalio et al. (2022). The version of TAM used for this study is based on the configuration described in Lombardo and Lora (2022, submitted to Icarus), which features an updated radiative transfer code. There are two differences between the configuration used in this work and that of Lombardo and Lora (2022). First, the model top has been extended from 2×10^{-3} hPa (14 scale heights) to 6×10^{-4} hPa (15 scale heights) and the number of layers in the vertical has been increased from 30 to 50. Second, the radiative transfer computation makes use of static instead of seasonally varying profiles for the abundances of radiative species. The main features of TAM are summarised below.

The model is built on the GFDL primitive equation spectral dynamical core (the same as the dynamical core used by the Isca framework, introduced in [Chapter 3](#)). To recap, the equations are solved on a spherical domain using a pseudospectral method in the horizontal, and a finite difference method in the vertical. The vertical coordinate is a hybrid 'sigma' coordinate, $\sigma = p/p_s$, that transitions to a pure pressure, p , coordinate around 100 hPa.

The model is run with a T21 triangular truncation, which corresponds to a horizontal latitude–longitude resolution of roughly 5.6° at the equator. This resolution is

relatively low, but necessary to avoid the computational expense of the model becoming prohibitive. There are 50 layers in the vertical direction up to a top pressure of 6×10^{-4} hPa (approximately 465 km altitude). They are distributed in σ according to $\sigma = \exp[-15(0.1\tilde{z} + 0.9\tilde{z}^{2.1})]$, where $\tilde{z}$ is equally spaced in the unit interval. To stabilise the model, an eighth-order horizontal hyperdiffusion acting on a timescale of 3.7 days is applied throughout the atmosphere. A Laplacian ‘sponge’ is included in the uppermost model layer, and a Rayleigh drag is applied above 2.5×10^{-3} hPa, to prevent wave reflection at the model top. The sponge damps eddy zonal and meridional winds on a timescale of 0.16 days at the grid scale. The Rayleigh drag is applied to the full zonal and meridional wind, and acts on a timescale of 1 day.

TAM is adapted to Titan through the inclusion of a multi-band radiative transfer code with opacities based on the composition of Titan’s atmosphere (described in Lombardo and Lora, 2022), a representation of Titan’s methane hydrological cycle (Lora and Ádámkovics, 2017; Battalio et al., 2022), and an astronomy calculation adapted to give Titan’s seasonal cycle of insolation. A Mellor–Yamada scheme is used to parametrise boundary layer turbulence (Mellor and Yamada, 1982) and the ‘Simple Betts–Miller’ scheme of Frierson (2007) is used to parametrise convection.

To spin-up the model, the zonal wind is nudged towards a prescribed, hemispherically symmetric superrotating state. Once the total angular momentum of the model’s zonal winds reaches that of the prescribed state, the nudging is switched off, and the model allowed to run freely until the circulation reaches a statistically steady state. A detailed description of this procedure is given in Lombardo and Lora (2022). The simulation is then run for a further half Titan year (14.7 Earth years; solar longitudes $L_s = 180^\circ - 360^\circ$) producing output every 6 hours, which is interpolated onto a pure pressure coordinate prior to analysis.

4.2.2 *Transformed Eulerian Mean momentum budget*

In this chapter, the *Transformed Eulerian Mean* (TEM) formulation of the zonally-averaged zonal momentum equation is used to analyse the acceleration of the mean flow due to eddies. In a log-pressure coordinate, the TEM momentum budget may be written (Andrews et al., 1987, Chapter 3)

$$\frac{D^*\bar{u}}{Dt} - \frac{\bar{u}\bar{v}^* \tan \vartheta}{a} - f\bar{v}^* = \frac{1}{\rho_0 a \cos \vartheta} \nabla \cdot \mathbf{F} - \bar{D}, \quad (4.1)$$

where

$$\frac{D^*}{Dt} = \frac{\partial}{\partial t} + \frac{\bar{v}^*}{a} \frac{\partial}{\partial \vartheta} + \bar{w}^* \frac{\partial}{\partial z^*} \quad (4.2)$$

is the material derivative following the residual mean meridional circulation $(\bar{v}^*, \bar{w}^*)$, which is defined by

$$\bar{v}^* = \bar{v} - \frac{1}{\rho_0} \frac{\partial \rho_0 \psi}{\partial z^*}, \quad (4.3)$$

$$\bar{w}^* = \bar{w} + \frac{1}{a \cos \vartheta} \frac{\partial \psi \cos \vartheta}{\partial \vartheta}, \quad (4.4)$$

and $\psi = \overline{v'\theta'}/(\partial\bar{\theta}/\partial z^*)$. The vector $\mathbf{F} = (F^{(\vartheta)}, F^{(z^*)})$ is the Eliassen–Palm flux (EP flux), defined

$$F^{(\vartheta)} = \rho_0 a \cos \vartheta \left(\frac{\partial \bar{u}}{\partial z^*} \psi - \overline{v'u'} \right), \quad (4.5)$$

$$F^{(z^*)} = \rho_0 a \cos \vartheta \left[\left(f - \frac{1}{a \cos \vartheta} \frac{\partial \bar{u} \cos \vartheta}{\partial \vartheta} \right) \psi - \overline{w'u'} \right], \quad (4.6)$$

with

$$\nabla \cdot \mathbf{F} = \frac{1}{a \cos \vartheta} \frac{\partial F^{(\vartheta)} \cos \vartheta}{\partial \vartheta} + \frac{\partial F^{(z^*)}}{\partial z^*}. \quad (4.7)$$

Above, ϑ is latitude and $z^* = -H \log(p/p_0)$ is log-pressure ‘pseudoheight’, with $H = RT_s/g$ a pressure scale-height and p_0 a reference pressure (e.g. the mean surface pressure). $\rho_0 = \rho_s e^{-z^*/H}$ is a basic state density profile, where $\rho_s = p_0/(RT_s)$. $\theta = T(p_0/p)^{R/c_p}$ is the potential temperature. R is the specific gas constant, c_p is the specific heat capacity at constant pressure, T_s the globally-averaged surface temperature, and g the acceleration due to gravity. $w = Dz^*/Dt$ is the vertical velocity in the log pressure coordinate system. $\bar{D}$ symbolises the effects of friction and dissipation. A number of variables are decomposed into their zonal-average and a deviation from the zonal average, using the notation $X = \bar{X} + X'$.

It is worth noting that (4.1) can be simplified considerably by re-writing it in terms of the axial component of angular momentum m , yielding

$$\rho_0 \frac{D^* \bar{m}}{Dt} = \nabla \cdot \mathbf{F} - \rho_0 \bar{D}_m, \quad (4.8)$$

with $D_m = a \cos \vartheta D$. From (4.8) it is clear that if D_m transports angular momentum down gradient, then up gradient transport can only be achieved by $\nabla \cdot \mathbf{F}$ (i.e., non-axisymmetric disturbances; cf. Hide’s theorem; Hide, 1969; also Chapter 2, Section 2.4).

The TEM formulation of the zonally-averaged momentum budget has two main advantages over the regular Eulerian mean momentum budget (Andrews et al., 1987). First, the residual mean circulation approximates that driven by diabatic heating, and

thus more closely resembles the Lagrangian mean meridional mass flow. Residual mean meridional overturning can be quantified via definition of a streamfunction

$$\Psi^* = \frac{2\pi a \cos \vartheta}{g} \int_0^p \bar{v}^* dp. \quad (4.9)$$

Second, in the quasigeostrophic limit, $\nabla \cdot \mathbf{F}$ describes the total wave-induced forcing of the TEM circulation. Additionally, $\nabla \cdot \mathbf{F}$ depends on the basic physical properties of wave or eddy disturbances in such a way that it is zero if the disturbances are steady, linear, frictionless and adiabatic.

4.2.3 Spectral diagnostics

In order to characterise the disturbances present in the model atmosphere, spectral diagnostics are used to transform time series data from real space (t, λ) to spectral space (ω, k) . Here, t is time, λ is longitude, ω is frequency and k is the zonal wavenumber, related to the number of wavelengths k_n that fit around a latitude circle via $k = k_n / (a \cos \vartheta)$.

For some quantity $g(t, \lambda)$ defined in real space, this is achieved via the complex Fourier transform

$$\tilde{G}(\omega, k) = \frac{1}{2\pi T} \int_0^T \int_0^{2\pi} g(t, \lambda) e^{-i(k_n \lambda - \omega t)} d\lambda dt. \quad (4.10)$$

Here, k (or k_n) is positive, and $\omega > 0$ and $\omega < 0$ correspond to eastward and westward propagating modes, respectively. It is worth noting that the absolute frequency ω of a transient wave can be decomposed into a contribution from the intrinsic frequency of the wave ω^+ , and a Doppler-shift due to the background zonal-mean zonal wind, so that (Andrews et al., 1987)

$$\omega^+ = \omega - k\bar{u}. \quad (4.11)$$

ω^+ is then positive for waves that propagate eastward relative to the background zonal wind (prograde waves), and negative for waves that propagate westward relative to the background wind (retrograde waves).

Given time series f and/or g , cross-spectral power density can be computed via

$$C_{\omega,k} = 2 \langle \text{Re} \{ \tilde{F} \tilde{G}^* \} \rangle, \quad (4.12)$$

where the asterisk denotes the complex conjugate, and angle brackets represent averaging over a frequency bandwidth. The method of Hayashi (1971) is used for the computation of (4.12), and averaging in frequency is achieved by application of a Gaussian window following Randel and Held (1991). Spectra are computed from subsets of the data $10^\circ L_s$

in duration, corresponding to roughly 300 (Earth) days (1196 data points). This yields a raw spectral resolution of $2\pi \times \Delta\omega = 3.34 \times 10^{-3} \text{ days}^{-1}$, smoothed to an effective resolution of $2\pi \times \Delta\omega_{\text{eff}} = 1.67 \times 10^{-2} \text{ days}^{-1}$ by the Gaussian window.

The propagation of waves relative to the mean flow can also be assessed by transforming spectra in frequency-wavenumber (ω, k) space to phase speed-wavenumber (c_p, k) space, where $c_p = \omega/k$. This is achieved via the relation

$$C_{c_p, k} = k C_{\omega, k}, \quad (4.13)$$

again following Randel and Held (1991). The lower bounds on raw and effective resolution in phase-speed space are $\Delta c_p = 0.626 \text{ m s}^{-1}$ and $\Delta c_{p, \text{eff}} = 3.13 \text{ m s}^{-1}$, respectively (corresponding to $\Delta\omega$ and $\Delta\omega_{\text{eff}}$ above for $k_n = 1$ at the equator). Phase speed spectra will be shown as a function of c_p and some other spatial dimension x (latitude or pressure). This is achieved by computing $C_{c_p, k}$ for each x and then summing over wavenumber.

The spatial structure of selected waves (in either latitude–longitude or longitude–pressure) is obtained by computing $\tilde{G}$ for perturbation quantities (u' , v' , and z' where z is the geopotential height) and filtering for frequencies and wavenumbers of interest (by zeroing the spectra outside of a desired frequency and wavenumber range). Each wave shown is filtered for a single zonal wavenumber k_{wave} , and a narrow range of frequencies (a spectral window) centered on the frequency ω_{wave} of a peak in the z' ω – k spectra associated with a particular wave. As the fields are being filtered for a single wavenumber, the spectral window corresponds to a range of phase speeds $c_{p, \text{wave}} \pm c_{p, \text{diff}}$, centred on $c_{p, \text{wave}} = \omega_{\text{wave}}/k_{\text{wave}}$. The width of the spectral window used to obtain the structure of each wave is indicated by quoting $c_{p, \text{diff}}$ in the relevant figure caption, and range from $\pm 1 \text{ m s}^{-1}$ to $\pm 5 \text{ m s}^{-1}$ (corresponding to 3 – 15 frequency bins in the discretised FFT). After filtering, the perturbation quantities are transformed back into real space, and shown as snapshots at the time when the filtered wave's z' amplitude is largest.

4.3 Basic circulation and seasonal cycle

Before proceeding to analyse the waves present in the TAM simulation, it is useful to discuss some aspects of the simulation's zonally-averaged circulation (on which the waves themselves depend).

Figure 4.1 shows the circulation averaged over two periods close to the northern hemisphere autumn equinox ($L_s = 180^\circ - 190^\circ$) and winter solstice ($L_s = 260^\circ - 280^\circ$), respectively. Colour contours show the zonal-mean zonal wind $\bar{u}$, and white contours show the TEM streamfunction, Ψ^* . The circulation displays a clear dependence on the

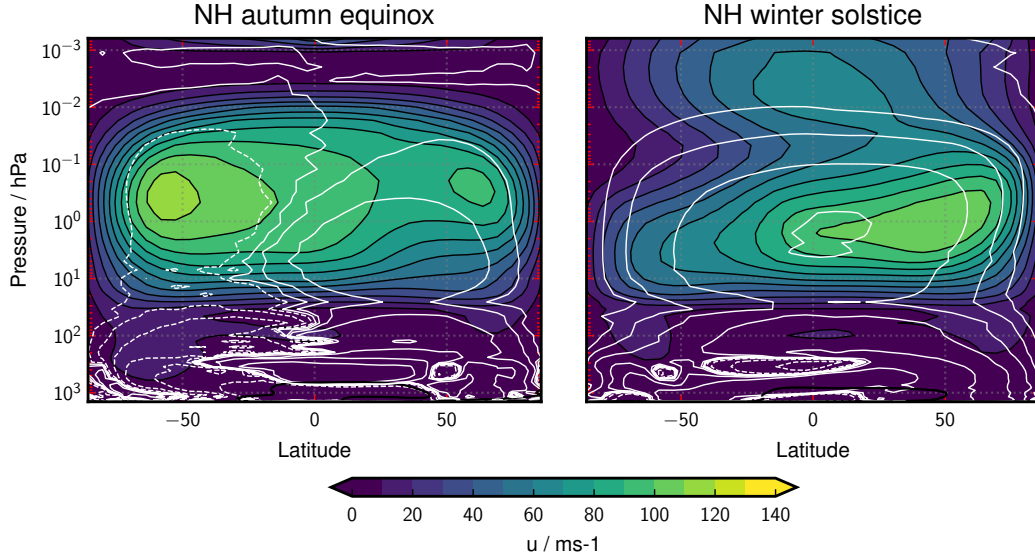


Figure 4.1: Zonal-mean zonal wind $\bar{u}$ and TEM streamfunction Ψ^* for northern hemisphere autumn equinox and winter solstice. Six contour levels are shown for Ψ^* at $\pm 10^7$, $\pm 10^8$ and $\pm 10^9$ kg s⁻¹ (negative contours dashed). Positive Ψ^* indicates clockwise overturning.

seasonal cycle, most pronounced in the stratosphere (roughly $p > 40$ hPa). At equinox, the circulation has some hemispheric symmetry, whereas around the solstice it is very asymmetric, with $\bar{u}$ strongest in the winter hemisphere, and Ψ^* taking the form of a single cell rising near the summer pole and sinking near the winter pole. The structure of Ψ^* is similar to that of the regular Eulerian mean streamfunction computed in previous studies (e.g., Lora et al., 2019); this is due to the meridional eddy heat flux being small (so ψ in Section 4.2 is small, and $\bar{v}^* \approx \bar{v}$). Also noteworthy is a pronounced shear in the zonal wind, in both the vertical and meridional directions. The vertical shear is strongest in the range $p = 10 - 100$ hPa and does not vary that substantially with the seasonal cycle. Meridional shear is most pronounced around $p = 1$ hPa and is strongest around the solstice.

The temperature structure of an atmosphere also has important effects on the characteristics of atmospheric waves, for example through the dependence of the deformation radius on the buoyancy frequency N (see Chapter 2). N is given by

$$N = \left[\frac{R}{H} \left(\frac{\partial T}{\partial z^*} + \frac{\kappa T}{H} \right) \right]^{\frac{1}{2}}, \quad (4.14)$$

where $\kappa = R/c_p$. Figure 4.2 shows T and N vs. p , at the equator and in mid-latitudes, for northern hemisphere autumn equinox and winter solstice. Below roughly $p = 60$ hPa, temperature decreases with height in a region that could be termed the troposphere. In the troposphere, there is little seasonal variation in T (and so likewise N), and additionally

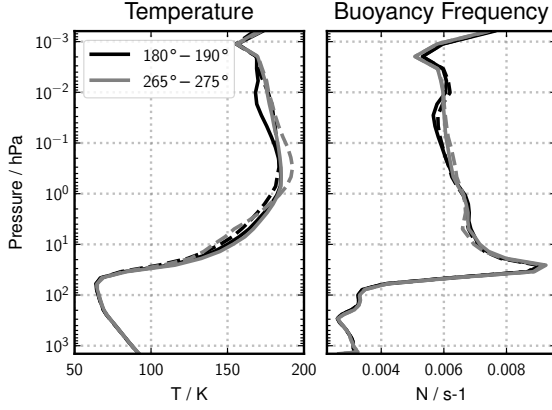


Figure 4.2: Vertical profile of temperature T and buoyancy frequency N at the equator (solid) and 60° N (dashed), around northern hemisphere autumn equinox (black) and winter solstice (grey).

meridional temperature contrasts are very weak (Mitchell and Lora, 2016). Above $p = 40$ hPa, temperature increases sharply with height until $p \approx 1$ hPa, above which the atmosphere is roughly isothermal. In this chapter, the region between $p = 40$ hPa and 0.04 hPa will be referred to as the stratosphere. Within the stratosphere, meridional temperature contrasts are somewhat more pronounced, and there is also some evidence for seasonal variation of the temperature structure, as in observations (Achterberg et al., 2008; Schinder et al., 2011). The sharp increase in temperature in the lower stratosphere notably gives rise to a pronounced increase in N between $p = 60$ hPa and $p = 20$ hPa. There is a ‘kink’ in the temperature structure in the top few model layers ($p \gtrsim 3 \times 10^{-3}$ hPa) which is due to the sponge layer.

The aim of this chapter is to investigate eddy momentum fluxes and wave structures in the stratosphere of the TAM simulation. With this in mind, Figure 4.3 presents a more granular view of seasonal variation in the zonally-averaged circulation over the pressure range $p = 0.04 - 40$ hPa. The left-hand column shows $\bar{u}$ and Ψ^* , as in Figure 4.1, and the right-hand column shows the EP flux (quivers) and its divergence $\nabla \cdot \mathbf{F}$ (colours).

Figure 4.3 shows that through much of the year, the stratospheric zonal-mean circulation is strongly asymmetric about the equator. Over the whole period $L_s = 215^\circ - 325^\circ$, the TEM streamfunction indicates that overturning takes the form of a single cell rising in the summer hemisphere and sinking in the winter hemisphere, and only for a brief period around equinox is a two cell structure apparent. Likewise, throughout most of the year zonal wind velocities are clearly much stronger in the winter hemisphere, with the strongest meridional shear occurring in mid-autumn (around $L_s = 225^\circ$).

Across a range of pressures, the EP flux divergence is positive at or near the equator, consistent with the requirement for equatorial superrotation to be maintained by eddy accelerations. Seasonal variation is also evident in the EP flux and its divergence.

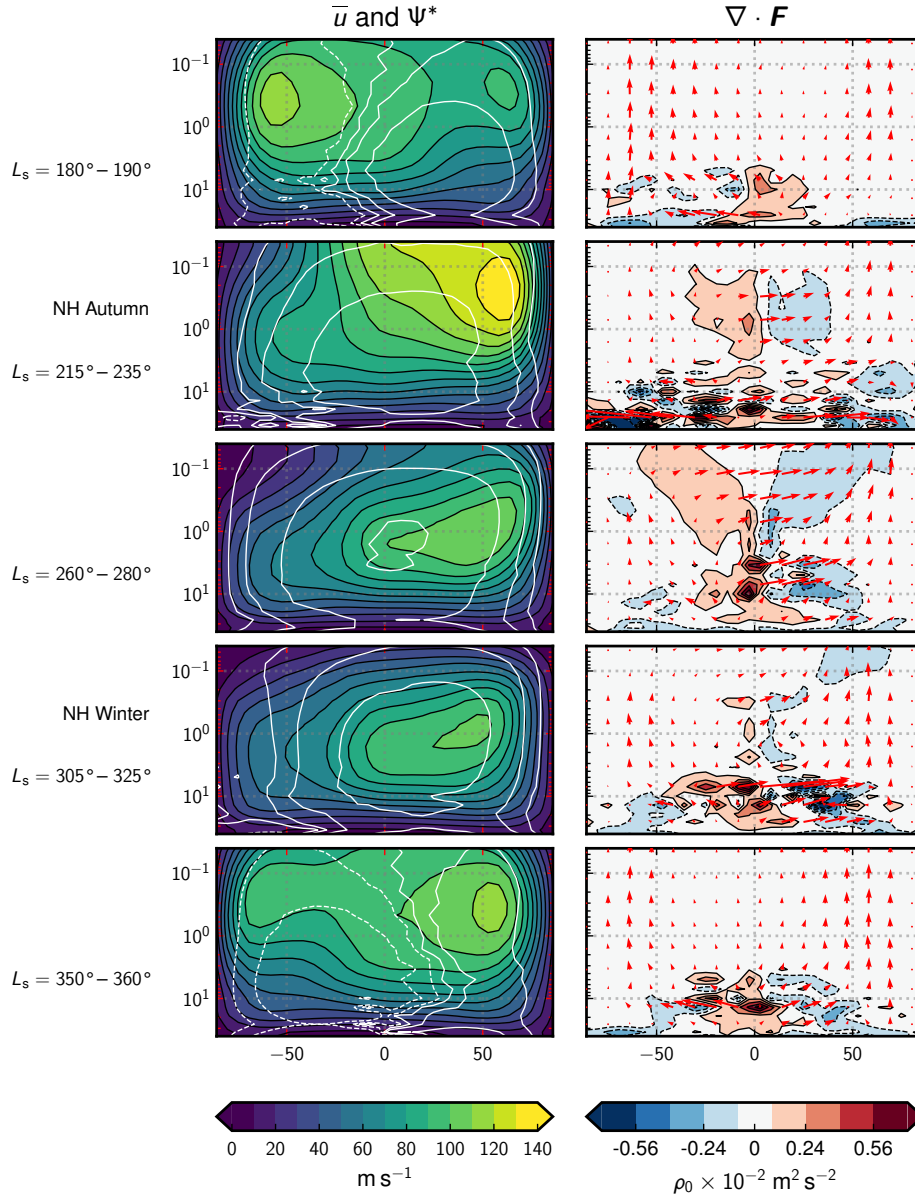


Figure 4.3: Seasonal cycle of $\bar{u}$, Ψ^* , F and $\nabla \cdot F$ across the pressure range $p = 0.04 - 40$ hPa. Left column: Zonal-mean zonal wind $\bar{u}$ and TEM streamfunction Ψ^* , as in Figure 4.1. Right column: the EP flux F (quivers) and its divergence $\nabla \cdot F$ (colour contour). The horizontal and vertical components of F are rescaled as $(\pi a)^{-1} F^{(\theta)}$ and $(\Delta z^*)^{-1} F^{(z^*)}$, respectively, where $\Delta z^* = -H \log(p_t/p_b)$ with $p_t = 4 \times 10^{-2}$ hPa and $p_b = 40$ hPa. The EP flux divergence has units of density $\times$ specific angular momentum per unit time. Red colour indicates acceleration.

This is particularly true in the upper stratosphere ($p \lesssim 4$ hPa), where, during northern hemisphere mid-autumn to mid-winter, eddies induce a southward flux of angular momentum that balances northward momentum transport by the mean flow, whereas around the equinoxes eddy acceleration is greatly reduced. Lower down in the stratosphere ($p \gtrsim 4$ hPa), eddy accelerations display much weaker seasonal variation.

The direction of the EP flux vectors is indicative of the direction of wave propagation, although whether $\mathbf{F}$ is roughly parallel or anti-parallel to the wave propagation depends on whether a wave has a prograde or retrograde intrinsic frequency (Andrews et al., 1987). Nonetheless, it is clear from Figure 4.3 that waves propagate both vertically and horizontally. Note that for waves that have a horizontal size close to the planetary scale, ‘horizontal propagation’ should be taken to mean that the waves are preferentially generated at one latitude and dissipated at another (without necessarily physically propagating in the meridional direction). In regions where the EP flux divergence is strongest, $\mathbf{F}$ is very close to horizontal, and a decomposition of $\nabla \cdot \mathbf{F}$ into contributions from $F^{(\theta)}$ and $F^{(z^*)}$ reveals that eddy acceleration is almost entirely due to horizontal divergence (not shown).

In the next two sections, I will characterise the waves present in the TAM simulation and identify those that contribute most to the acceleration of equatorial superrotation. In Section 4.4, the analysis focuses on waves in northern hemisphere autumn, where there is significant eddy momentum flux divergence in both the upper and lower stratosphere. The main differences between those waves and waves in winter are then described in Section 4.5, focusing on the persistence of eddy momentum flux divergence in the lower stratosphere and its absence in the upper stratosphere.

4.4 Waves in autumn

This section characterises the waves present during northern hemisphere mid-autumn. All of the analysis for this season will focus on the period $L_s = 220^\circ - 230^\circ$. Analysing a short period makes for simpler spectral analysis, as it reduces the effect of a time-varying Doppler-shift of the waves due to seasonal variation in $\bar{u}$. The period $L_s = 220^\circ - 230^\circ$ was chosen because it yielded the cleanest results as far as visual presentation is concerned. However, it is stressed that the results described in this section are characteristic of the entire period $L_s = 215^\circ - 280^\circ$.

We begin by returning to the EP flux and its divergence, shown again in Figure 4.4, but now separated into contributions from prograde and retrograde disturbances. Both prograde and retrograde disturbances contribute to $\nabla \cdot \mathbf{F}$, but in different locations. Prograde waves mostly accelerate the mean flow in the upper stratosphere, whereas retrograde waves have a greater contribution in the lower stratosphere. The purple contours indicate the contribution of $\partial F^{(z^*)}/\partial z^*$ to $\nabla \cdot \mathbf{F}$, which is generally very weak, and acceleration of the mean flow by eddies is identified to be mostly due to horizontal convergence of eddy angular momentum fluxes.

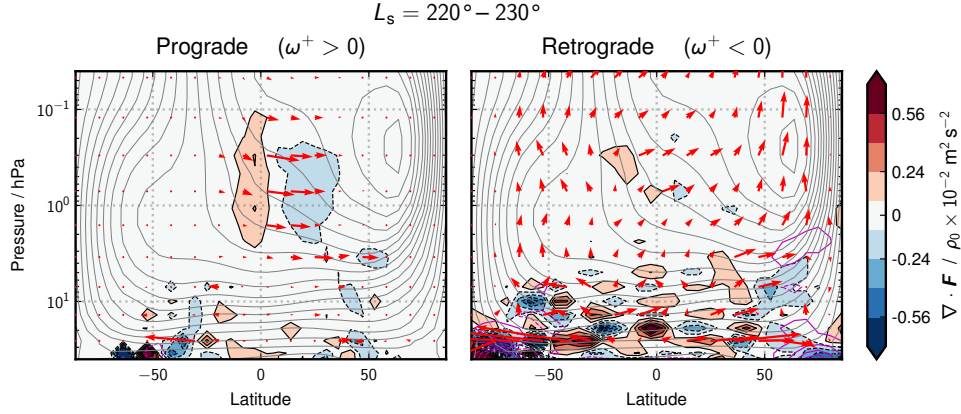


Figure 4.4: EP flux F (quivers) and its divergence $\nabla \cdot F$ (colour contour) for prograde (left) and retrograde (right) waves during the period $L_s = 220^\circ - 230^\circ$. F is scaled as in Figure 4.3, but the quiver length has been doubled. Grey contours show the zonal-mean zonal wind $\bar{u}$ with the same contour levels as in Figure 4.1 and Figure 4.3. Purple contours show the contribution to $\nabla \cdot F$ from $\partial F^{(z^*)}/\partial z^*$ (using the same contour levels).

The direction of F indicates the predominant direction of wave propagation for each case. Noting that F is aligned with the direction of propagation for retrograde waves, and opposite to the direction of propagation for prograde waves (Andrews et al., 1987; Imamura, 2006), it can be determined from Figure 4.4 that prograde waves appear to propagate equatorwards and, to a lesser extent, upwards from source regions in the middle stratosphere around $p = 4$ hPa in northern hemisphere sub-tropical latitudes. Meanwhile, retrograde waves propagate polewards and upwards from a source lower in the stratosphere, around $p = 20$ hPa and near to the equator. A spectral analysis of the waves that contribute to the general behaviour shown in Figure 4.4 is presented in the next two subsections.

4.4.1 Upper stratosphere

This subsection presents an analysis of the waves present in the upper stratosphere, focusing on the pressure level $p = 0.7$ hPa. Figure 4.5 shows latitude–phase speed and frequency–wavenumber (averaged between $\pm 6^\circ$ latitude) spectra for the eddy geopotential height anomaly and eddy angular momentum flux convergence. Dispersion curves are overlaid on the frequency–wavenumber spectra, computed from the general dispersion relation given by (2.42) in Chapter 2, Section 2.5¹.

¹Dispersion curves are shown for vertical wavenumbers $m_n = 1$ and $m_n = 4$ for all waves, and additionally $m_n = 8$ for equatorial Rossby waves and Kelvin waves. Here, m_n is the number of wavelengths λ_v that fits into the total altitude range Δz^* occupied by a wave, i.e., $m_n = \Delta z^* / \lambda_v$ where $\Delta z^* = -H \log(p_t/p_b)$. As we are considering waves in the stratosphere, $p_t = 4 \times 10^{-2}$ hPa and $p_b = 40$ hPa are set. m_n is related to the vertical wavenumber m that appears in the dispersion relationships quoted in Chapter 2, Section 2.5

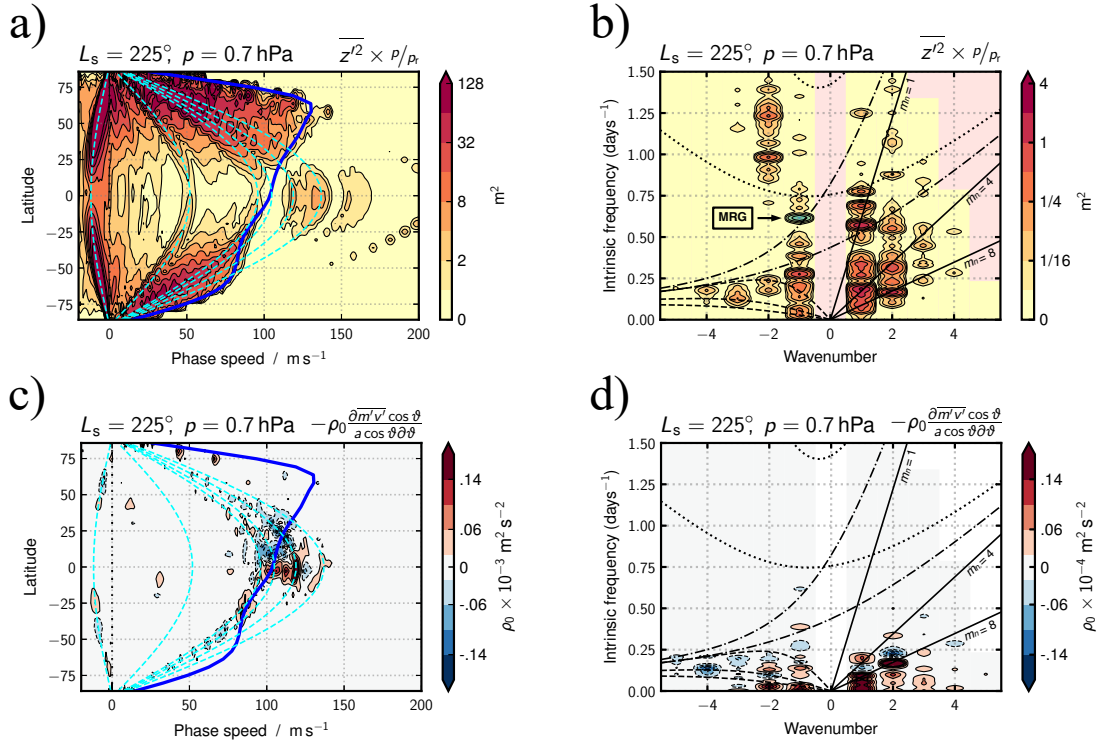


Figure 4.5: Spectra of eddy geopotential height and horizontal angular momentum flux convergence presented as a function of latitude and phase speed (panels a and c), and wavenumber and frequency (panels b and d), averaged between $\pm 6^\circ$ latitude. Analysis is shown for $p = 0.7$ hPa, $L_s = 220^\circ - 230^\circ$. The dashed cyan lines in panels a) and c) are curves of constant angular velocity ($c_p \times \cos \theta$), and solid blue lines show the zonal-mean zonal wind $\bar{u}$. In panels b) and d), prograde modes are plotted with a positive wavenumber and retrograde modes are plotted with a negative wavenumber. Blue colour in panel b) indicates modes anti-symmetric across the equator (only one is present, a mixed Rossby–gravity mode at $\omega^+ = 0.625$ days and $k_n = 1$; labeled in panel). The curves in panels b) and d) are dispersion curves for Kelvin waves (solid), mixed Rossby–gravity / $n = 0$ eastward inertio–gravity waves (dash-dot), $n = 1$ inertio–gravity waves (dotted), and $n = 1$ equatorial Rossby waves (dashed), where n is the meridional mode number. Curves are shown for vertical wavenumbers $m_n = 1$ and 4 for all waves, and additionally $m_n = 8$ for equatorial Rossby waves and Kelvin waves, corresponding to approximate vertical wavelengths $\lambda_{v1} \approx 136$ km, $\lambda_{v4} \approx 34$ km, and $\lambda_{v8} \approx 16$ km, respectively (see footnote 1 for details). The dispersion curves are calculated assuming a rotation rate $\Omega_{\text{eff}} = \bar{u}/a$, which is the effective rotation rate moving with the superrotating jet. For details of the computation, see footnote 1.

via $m = 2\pi/\lambda_v = 2\pi m_n/\Delta z^*$. The vertical wavenumbers selected for plotting in Figure 4.5 are based on those of the waves identified later in the chapter (Figure 4.10). The dispersion relation also has a dependency on the buoyancy frequency N and pressure scale height H . Recall that $H = RT_s/g$, and when computing the dispersion curves, $T_s = 92$ K is set. For the buoyancy frequency, $N = 0.006$ s $^{-1}$ is set, based on Figure 4.2. For a strongly superrotating atmosphere, it is appropriate to set the planetary rotation rate Ω to that rotating with the equatorial jet (del Genio and Rossow, 1990; Imamura, 2006), so $\beta = 2\Omega_{\text{eff}}/a = 2\bar{u}/a^2$ is used when computing dispersion curves, where $\bar{u} = 100$ m s $^{-1}$ at $p = 0.7$ hPa (for Figure 4.5), and $\bar{u} = 50$ m s $^{-1}$ at $p = 13.8$ hPa (for Figure 4.13).

From Figure 4.5a, it can be identified that both prograde ($c_p > \bar{u}$) and retrograde ($c_p < \bar{u}$) disturbances are present at the equator. A number of structures have coherence along curves of constant angular velocity. This is particularly clear for a series of retrograde modes with broad meridional extent (examples with equatorial phase speeds $c_p = -12 \text{ m s}^{-1}$, 52 m s^{-1} , and 96 m s^{-1} are indicated by dashed cyan curves of constant angular velocity). Figure 4.5b shows that power in the equatorial disturbances is generally concentrated into low wavenumbers, but across a broad range of frequencies (although it is important to note that the large mean flow contribution to ω means that disturbances with moderate eastward intrinsic frequencies ω^+ and wavenumbers, i.e. the upper-right hand corner of Figure 4.5b and d, are not resolved in the output data).

In Figure 4.5b, retrograde waves are plotted with a negative wavenumber, while prograde waves are plotted with a positive wavenumber. The $\omega^+ - k$ distribution of $\overline{z'^2}$ suggests that a number of equatorial waves are supported in the model atmosphere, including low frequency equatorial Rossby waves (retrograde) and Kelvin waves (prograde), as well as higher frequency inertio-gravity modes (retrograde and prograde). Colour in Figure 4.5b is used to differentiate between modes that are symmetric (red) and anti-symmetric (blue) across the equator (see Wheeler and Kiladis, 1999, for method). Almost all disturbances are symmetric, aside from one prominent example with $\omega^+ = -0.625 \text{ days}^{-1}$ and $k_n = 1$. This corresponds to the structure at $c_p = -12 \text{ m s}^{-1}$ shown in Figure 4.5a, which is later identified as a mixed Rossby-gravity wave (Figure 4.6f).

Figure 4.5c and d present a spectral decomposition of the eddy angular momentum flux divergence associated with each of these disturbances. The high-frequency inertio-gravity waves and the mixed Rossby-gravity wave do not play a role in the acceleration of superrotation. Instead, the largest eddy accelerations are due to modes with phase speeds similar to the mean flow speed (i.e., small ω^+) and very low wavenumbers $k_n = 1, 2$. Generally these disturbances have $\omega^+ > 0$ at latitudes where $\overline{m'v'}$ is convergent, and $\omega^+ < 0$ where $\overline{m'v'}$ is divergent.

Figure 4.6 shows snapshots of the horizontal structure of selected waves, highlighted in Figure 4.5a,c with dashed cyan lines. Three prograde modes and three retrograde modes are presented. For each panel, filtered u' and v' are shown as quivers, and filtered z' as colour contours. The filtered fields are obtained following the method described in Section 4.2. To simplify the analysis, attention is restricted to modes with $k_n = 1$. As described in Section 4.2, frequencies are filtered using a spectra window centred on a peak in the z' power spectrum (Figure 4.5a) associated with a particular wave of interest, which, given k_n , corresponds to a range of phase speeds $c_{p,\text{wave}} \pm c_{p,\text{diff}}$. In

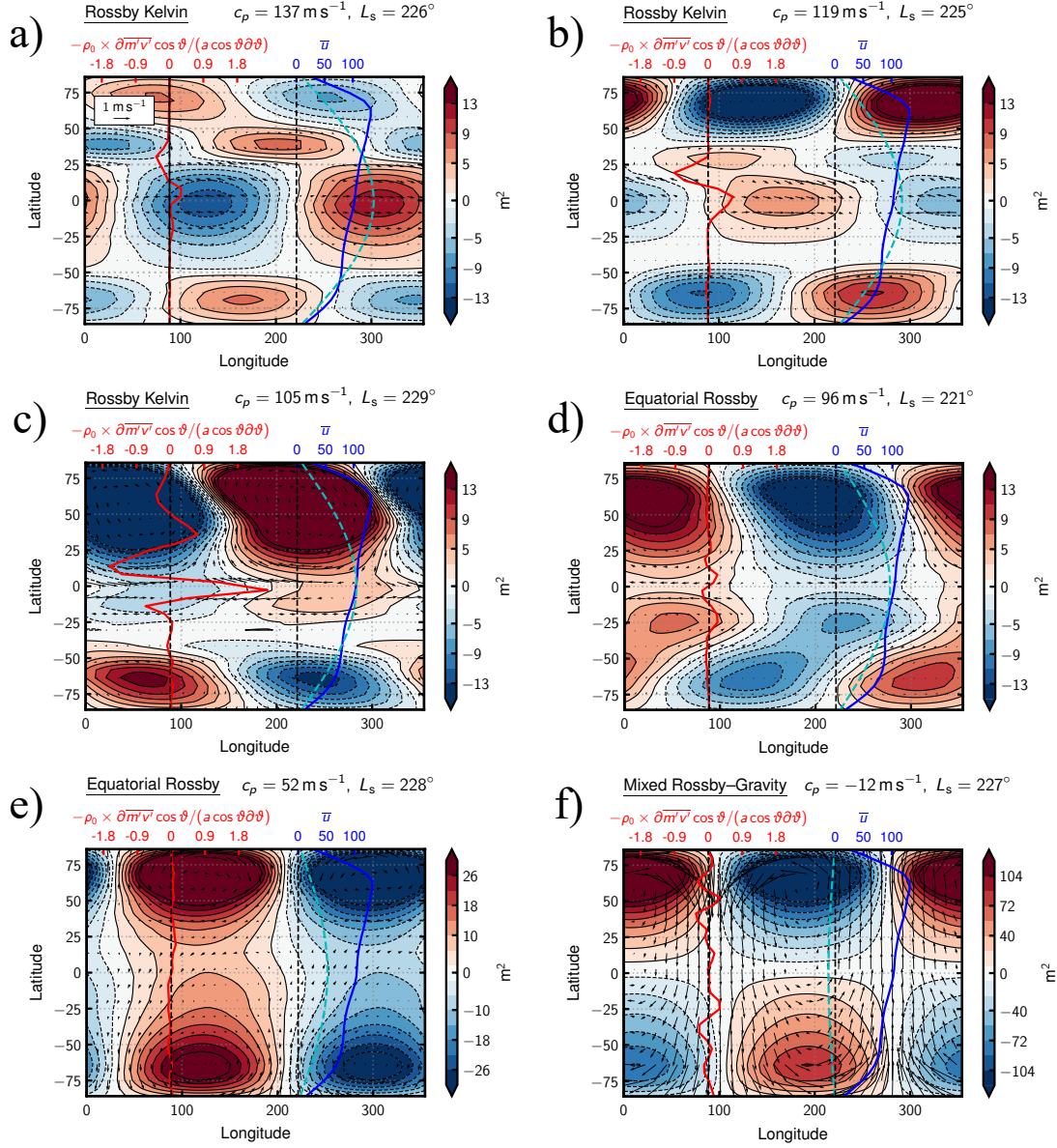


Figure 4.6: Snapshots showing the horizontal structure for various waves identified at $p = 0.7$ hPa during the period $L_s = 220^\circ - 230^\circ$. Eddy velocity components (u', v') are shown as quivers, and eddy geopotential height is shown as colour contours. The quiver scale is constant between panels. Each panel shows fields filtered for $k_n = 1$ and frequencies corresponding to $c_p = c_{p,\text{wave}} \pm c_{p,\text{diff}}$. $c_{p,\text{wave}}$ for each wave is indicated above each panel, and corresponds to one of the cyan lines in Figure 4.5a,c. $c_{p,\text{diff}} = 2, 2, 5, 1, 3,$ and 1 m s^{-1} , for panels a)–f) respectively. The red line in each panel shows the horizontal angular momentum flux convergence computed directly from each wave's filtered eddy velocity field. It has units of $\rho_0 \times 10^{-3} \text{ m}^2 \text{ s}^{-2}$. Blue lines show the zonal-mean zonal wind $\bar{u}$, and the dashed cyan line shows $c_p \times \cos \vartheta$ (both with units m s^{-1}). The solar longitude at which each snapshot was obtained is indicated at the top-right of each panel.

Figure 4.6, $c_{p,\text{wave}}$ is indicated at the top-right of each panel, and corresponds to a cyan line in Figure 4.5a,c, while $c_{p,\text{diff}}$ is stated in the figure caption for each wave. Each panel shows a snapshot of the filtered fields at the time where the wave geopotential height anomaly is maximum (over the period $L_s = 220^\circ - 230^\circ$).

Figure 4.6a and b present filtered fields for the phase speeds $c_p = 137 \text{ m s}^{-1}$ and $c_p = 119 \text{ m s}^{-1}$. The waves in these panels have a similar structure, composed of a prograde equatorial Kelvin wave-like component on the equator, phase-locked with a retrograde, Rossby wave-like component in northern mid-latitudes (cf. Figure 2.7 in Chapter 2; Matsuno, 1966; Kiladis et al., 2009). This sort of wave mode has been identified in previous studies of superrotating atmospheres, and is often referred to as a ‘Rossby–Kelvin’ mode (cf. Figure 2.9 in Chapter 2; Iga and Matsuda, 2005; Mitchell and Vallis, 2010; Wang and Mitchell, 2014).

The $c_p = 137 \text{ m s}^{-1}$ mode has larger amplitude ($\sim 13 \text{ m}^2$) and meridional extent ($\pm 47^\circ$) than the $c_p = 119 \text{ m s}^{-1}$ mode ($\sim 7 \text{ m}^2$ and $\pm 35^\circ$) which is consistent with the prediction of linear theory when the intrinsic frequency ω^+ is reduced (Andrews et al., 1987). These modes correspond to the Kelvin modes in Figure 4.5b with wavenumber 1 that are intersected by the $m_n = 4$ and $m_n = 8$ dispersion curves, where m_n is the number of wavelengths λ_v that fits into the altitude range Δz^* spanned by the stratosphere (see footnote 1 for more information). The equatorial deformation radii² l_e expected for these modes from linear theory are $l_e/a = 0.79$, corresponding to $\pm 45^\circ$ latitude, for the $m_n = 4$ mode ($c_p = 137 \text{ m s}^{-1}$), and $l_e/a = 0.56$, corresponding to $\pm 32^\circ$ latitude, for the $m_n = 8$ mode ($c_p = 119 \text{ m s}^{-1}$), which agrees well with the meridional extent of each mode shown in Figure 4.6. For the phase speeds shown in Figure 4.6a and b there are also Rossby-like waves in northern and southern high latitudes ($|\vartheta| > 50^\circ$) which are not directly coupled to the Rossby–Kelvin mode (cf. Figure 4.5a, where there is no clear evidence of phase locking in the $\overline{z'^2}$ field between sub-tropical and extra-tropical latitudes). These high-latitude waves appear in the analysis as they are present across a broad range of phase speeds, including those filtered for here.

Phase locking between the prograde and retrograde components is made possible by the meridional shear in the background wind (see the blue line in each panel), and due to the asymmetry of the background shear, the combined Rossby–Kelvin mode is asymmetric about the equator, with no Rossby-like component in the southern hemisphere.

²Equation (2.47) in Chapter 2 is used to compute the theoretical deformation radii quoted in the text. N , H and Ω are set as described for the dispersion curves in footnote 1. When l_e is quoted as a ‘meridional extent’ (degrees latitude) in the text, this quantity is computed as $\text{meridional extent} = (180^\circ/\pi) \times (l_e/a)$.

Spatial overlap between the Rossby wave-like component and the Kelvin wave-like component creates a velocity field structure that converges momentum towards the equator. This overlap is greater for the $c_p = 119 \text{ m s}^{-1}$ wave, and thus the associated equatorward momentum flux is stronger. Note that there are additional, faster Kelvin waves with $c_p \gtrsim 175 \text{ m s}^{-1}$ that do not have a Rossby-like component in mid-latitudes (e.g., the prograde disturbance with equatorial phase speed $c_p > 200 \text{ m s}^{-1}$ in Figure 4.5a; spatial structure not shown), likely because their high phase speed prevents phase locking with a retrograde component in sub-tropical or middle latitudes.

Figure 4.6c presents an additional Rossby–Kelvin wave, travelling with a phase speed $c_p = 105 \text{ m s}^{-1}$. Consistent with the trend above, it appears to have a smaller amplitude ($\sim 5 \text{ m}^2$) and meridional extent ($\pm 25^\circ$) than either of the other two Rossby–Kelvin modes shown. It is also more symmetric about the equator, and partial phase-locking between a Rossby-like component in the southern hemisphere, and the Kelvin component, is apparent. This is made possible by the proximity of the wave’s phase speed to the mean flow speed at the equator, which allows for $(c_{p,\text{wave}} - c_{p,\text{diff}}) \cos \vartheta|_{\vartheta=-25^\circ} < \bar{u}|_{\vartheta=-25^\circ}$. Consequently, momentum is converged to the equator from both the northern and southern hemispheres. At the time of the snapshot shown in Figure 4.6c, momentum transport from the northern hemisphere into the equatorial region appears to be enhanced by interaction with a Rossby wave in northern high latitudes.

Three retrograde modes are presented in Figure 4.6d, e and f. Two (Figure 4.6d,e; $c_p = 96 \text{ m s}^{-1}$ and $c_p = 52 \text{ m s}^{-1}$) have a horizontal structure that is characteristic of an equatorial Rossby wave (cf. Figure 2.7; Matsuno, 1966; Kiladis et al., 2009). As with the prograde modes, the amplitude and meridional extent of these equatorial Rossby waves increases with increasing intrinsic frequency ω^+ . The $c_p = 96 \text{ m s}^{-1}$ mode corresponds to the wavenumber 1 mode in Figure 4.5b that is intersected by the $m_n = 8$ equatorial Rossby wave dispersion curve, which has an associated theoretical deformation radius corresponding to a meridional extent of $\pm 32^\circ$ latitude. The $c_p = 52 \text{ m s}^{-1}$ mode is situated just above the $m_n = 1$ dispersion curve. Setting $m_n = 1/3$ for this mode yields a theoretical deformation radius of $l_e/a = 1.98$, which corresponds to a meridional extent of $\pm 113^\circ$ latitude (i.e., the simulated mode has grown to fill the domain). Similarly to Figure 4.6a–c, there are also Rossby-like waves in northern and southern high latitudes ($|\vartheta| > 50^\circ$) for the panel showing $c_p = 96 \text{ m s}^{-1}$. While there is a clear overlap between these tropical and extra-tropical modes at the time shown, they are not strongly coupled in general. Regarding the $c_p = 52 \text{ m s}^{-1}$ equatorial Rossby wave, it is difficult to make any distinction between high-latitude Rossby waves and the equatorial wave as the meridional extent of

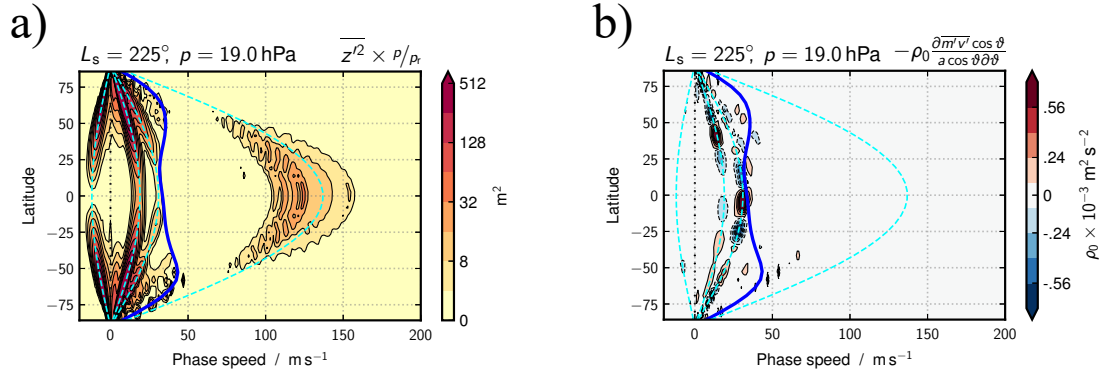


Figure 4.7: Spectra of eddy geopotential height and horizontal angular momentum flux convergence presented as a function of latitude and phase speed, as in Figure 4.5a,c but for $p = 19.0$ hPa. The dashed cyan lines are curves of constant angular velocity ($c_p \times \cos \vartheta$), and solid blue lines show the zonal-mean zonal wind $\overline{u}$.

the equatorial wave is global. This wave has quite a ‘messy’ structure when compared with that expected from linear theory in the absence of a background wind shear, and notably the meridional component of the velocity is not zero at the equator (i.e., there is cross-equatorial flow). This is likely due to the effects of the background shear on the wave, and possibly also due to overlap between the equatorial Rossby wave and high-latitude Rossby waves that are not symmetric about the equator.

The final mode shown in Figure 4.6f has the structure of a mixed Rossby–gravity wave (Kiladis et al., 2009), which travels with a much higher intrinsic frequency than do the equatorial Rossby waves. The effect of each of these retrograde waves on the angular momentum budget in the equatorial area is minimal compared with that of the Rossby–Kelvin waves shown in Figure 4.6b and c, which is consistent with the separation of $\nabla \cdot \mathbf{F}$ shown in Figure 4.4, where prograde modes are responsible for acceleration near the equator in the upper stratosphere.

4.4.2 Lower stratosphere

We now turn our attention towards the lower stratosphere, focusing on the pressure level $p = 19$ hPa, in the lower stratosphere, where the equatorial eddy angular momentum flux convergence is strongest (see Figure 4.4). Figure 4.7 shows $\overline{z'^2}$ and the horizontal eddy angular momentum flux convergence as a function of latitude and phase speed, as in Figure 4.5a,c.

The picture is quite different to that in the upper stratosphere. First, zonal-mean zonal wind is more symmetric about the equator, and this appears to be reflected in the spectra shown in Figure 4.7. Prograde and retrograde disturbances are both present (Figure 4.7a), as before, but eddy angular momentum flux convergence is now primarily

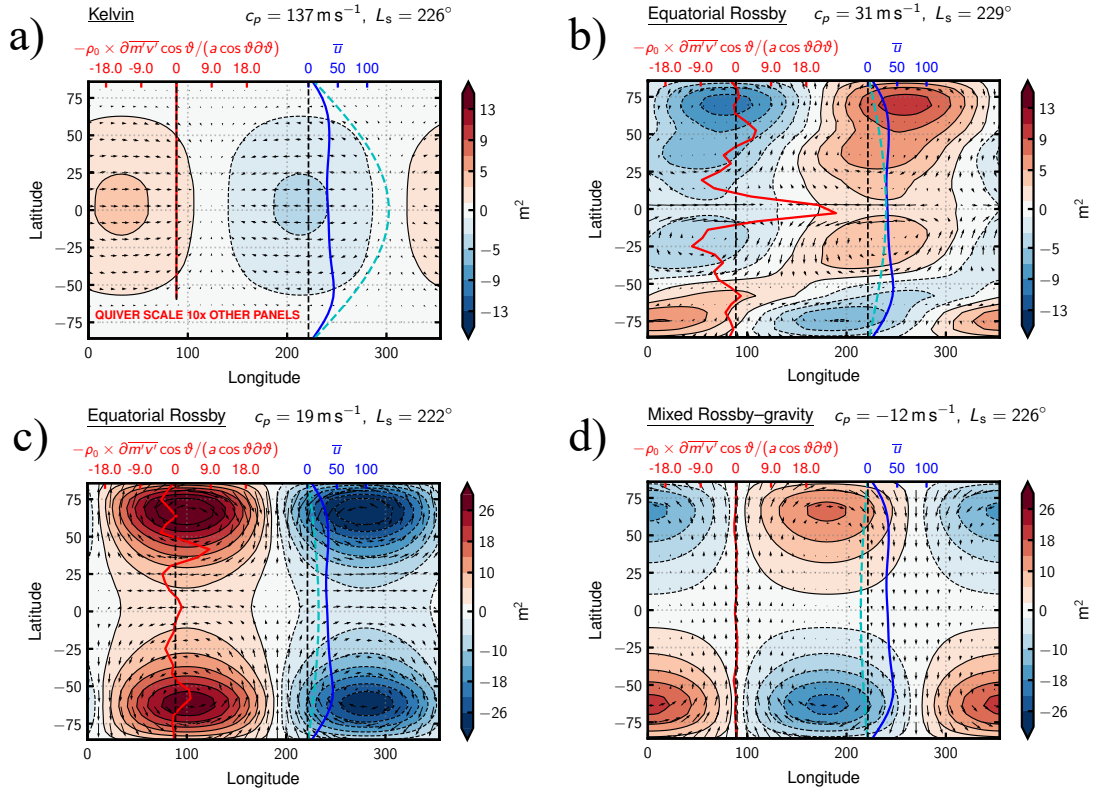


Figure 4.8: Snapshots showing the horizontal structure for various waves as in Figure 4.6 but for $p = 19.0$ hPa. For panels b)–d) the quiver scale is constant, and the same as in Figure 4.6. For panel a) the quiver length has been multiplied a factor of 10. As before, frequencies correspond to $c_p = c_{p,\text{wave}} \pm c_{p,\text{diff}}$. $c_{p,\text{wave}}$ is indicated above each panel, and $c_{p,\text{diff}} = 2, 2, 1,$ and 1 m s^{-1} , for panels a)–d) respectively. The solar longitude at which each snapshot was obtained is indicated at the top-right of each panel.

due to retrograde waves. It is notable that prograde modes appear to exist at phase speeds similar to those in the upper atmosphere, implying that the Kelvin waves may have a somewhat coherent vertical structure. A similar observation can also be made at the phase speed of the mixed Rossby–gravity wave identified previously ($c_p = -12 \text{ m s}^{-1}$). The vertical structure of the waves present in the atmosphere for this time period ($L_s = 220^\circ - 230^\circ$) will be analysed in detail in the next subsection.

The horizontal structures of selected disturbances (indicated by cyan lines in Figure 4.7) are shown in Figure 4.8. The prograde disturbance with $c_p = 137 \text{ m s}^{-1}$ has a structure that is very reminiscent of the solution for a linear Kelvin wave on the equatorial beta plane (cf. Figure 2.7; Matsuno, 1966; Kiladis et al., 2009), aside from the fact that it has non-zero meridional velocity; likely a product of the spherical geometry (Yamamoto, 2019). Notably, this mode is not accompanied by Rossby wave-like component in mid-latitudes, unlike the disturbances with similar c_p identified at $p = 0.7$ hPa. This underscores the importance of horizontal shear flow in the upper stratosphere for

generating these features; without meridional shear that is comparable in magnitude to the phase speed of the prograde wave, prograde and retrograde components are unable to phase-lock (Wang and Mitchell, 2014). In the absence of an accompanying Rossby-like component, this mode is unable to converge momentum towards the equator as might be expected for predominantly divergent motion (Iga and Matsuda, 2005).

Figure 4.8b shows the retrograde wave which is responsible for horizontal eddy angular momentum convergence at the equator. This wave has a structure characteristic of an equatorial Rossby wave, which at the time shown appears to be interacting with a further Rossby wave in high-latitudes (although at other times, this interaction is not as clear or is not present at all). The vertical shear in the background zonal-mean wind appears to induce an equatorward meridional phase tilt in the wave (Imamura, 2006), which allows it to converge momentum towards the equator. Dias Pinto and Mitchell (2016) have shown that similar disturbances are responsible for accelerating equatorial superrotation in idealised GCM simulations of an Earth-like planet with a smaller (Titan-like) planetary radius.

The final two waves shown in Figure 4.8c and d are additional retrograde modes. The first (Figure 4.8c) resembles an equatorial Rossby wave, while the second (Figure 4.8d) appears to be the same mixed Rossby–gravity wave identified in the upper stratosphere. Neither of these waves induce a significant momentum flux near the equator. As in the upper stratosphere, it appears that this is less likely to occur for with a phase speed that is significantly different from the mean flow speed.

4.4.3 Vertical structure

Some of the waves identified in the analysis of the $p = 0.7$ hPa and $p = 19$ hPa levels appear to have a coherent vertical structure. Notably, a mixed Rossby–gravity wave was identified in both the upper and lower stratosphere with $c_p = -12$ m s⁻¹. Additionally, the eddy geopotential height anomaly analysed at $p = 19$ hPa has a Kelvin wave-like feature that appears across a broad range of phase speeds, $c_p = 100 - 150$ m s⁻¹, similar to those of the Rossby–Kelvin mode identified at $p = 0.7$ hPa. Using this as a motivation, this section presents an analysis of the vertical structure of the waves present during the period $L_s = 220^\circ - 230^\circ$.

Figure 4.9 shows $\overline{z'^2}$ as a function of pressure and phase speed, averaged between $\pm 10^\circ$ latitude. The background zonal-mean zonal wind is also plotted as a solid blue line. This analysis is presented over a broad range of pressures, from 4×10^2 hPa to 4×10^{-3} hPa.

A number of disturbances with a coherent vertical structure can be identified. This is

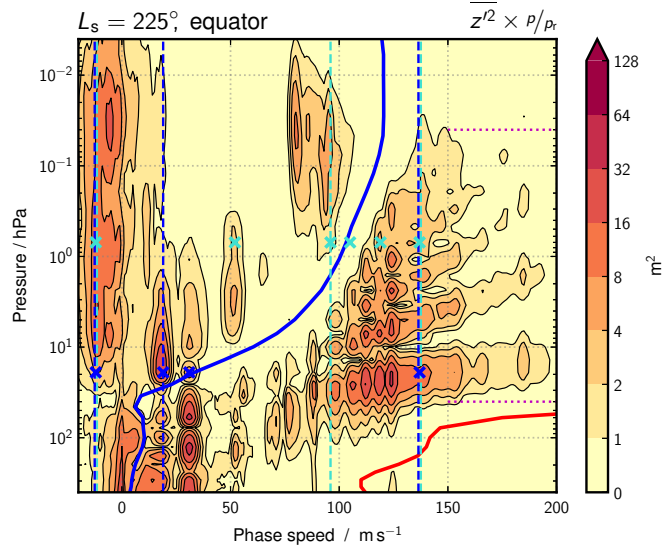


Figure 4.9: Eddy geopotential height spectrum presented as a function of pressure and phase speed, averaged within $\pm 10^\circ$ latitude for $L_s = 220^\circ - 230^\circ$. Vertical dashed lines correspond to c_p associated with waves whose spatial structure in the λ - p plane is shown in Figure 4.10. Points marked by cyan and blue crosses correspond to the locations of waves whose horizontal structures are shown in Figure 4.6 and Figure 4.8, respectively. The solid blue curve shows the zonal-mean zonal wind $\bar{u}$. The red curve indicates the pressure below which waves should be evanescent for a given phase speed (see text). The dotted purple lines indicate the pressure range spanned in Figure 4.3 and Figure 4.4.

particularly evident for retrograde disturbances (with $c_p < \bar{u}$); the signature of the mixed Rossby-gravity wave is present at -12 m s^{-1} and appears to extend from roughly 40 hPa to 1×10^{-2} hPa, and each of the equatorial Rossby waves identified in Figure 4.6 and Figure 4.8, with phase speeds $c_p = 19, 31, 50$, and 96 m s^{-1} , appear over broad pressure ranges.

Prograde waves also exist across a broad range of phase speeds and pressures, but with a complicated vertical structure in $\bar{z}^{1/2}$. In the lower stratosphere, $\bar{z}^{1/2}$ appears to be amplified at certain pressures for certain phase speeds. This distribution for $\bar{z}^{1/2}$ is difficult to interpret, although the region where it is most prominent notably coincides with sharp vertical contrasts in the background wind $\bar{u}$ and the buoyancy frequency N (see Figure 4.2). This effect is not realised for the retrograde waves, and less so for prograde waves in the upper stratosphere, which both have a more evenly spread $\bar{z}^{1/2}$ distribution in the vertical. Finally, there is evidence of a critical region in Figure 4.9 close to the mean flow speed (i.e., where $|c_p - \bar{u}|$ is small), through which disturbances cannot propagate vertically and the wave amplitudes become small.

Figure 4.10 presents the vertical structure of selected waves in the longitude–pressure plane, averaged between $\pm 20^\circ$ latitude. The filtered fields shown are obtained in the same way as those presented in Figure 4.6 and Figure 4.8. Each wave is indicated by a vertical

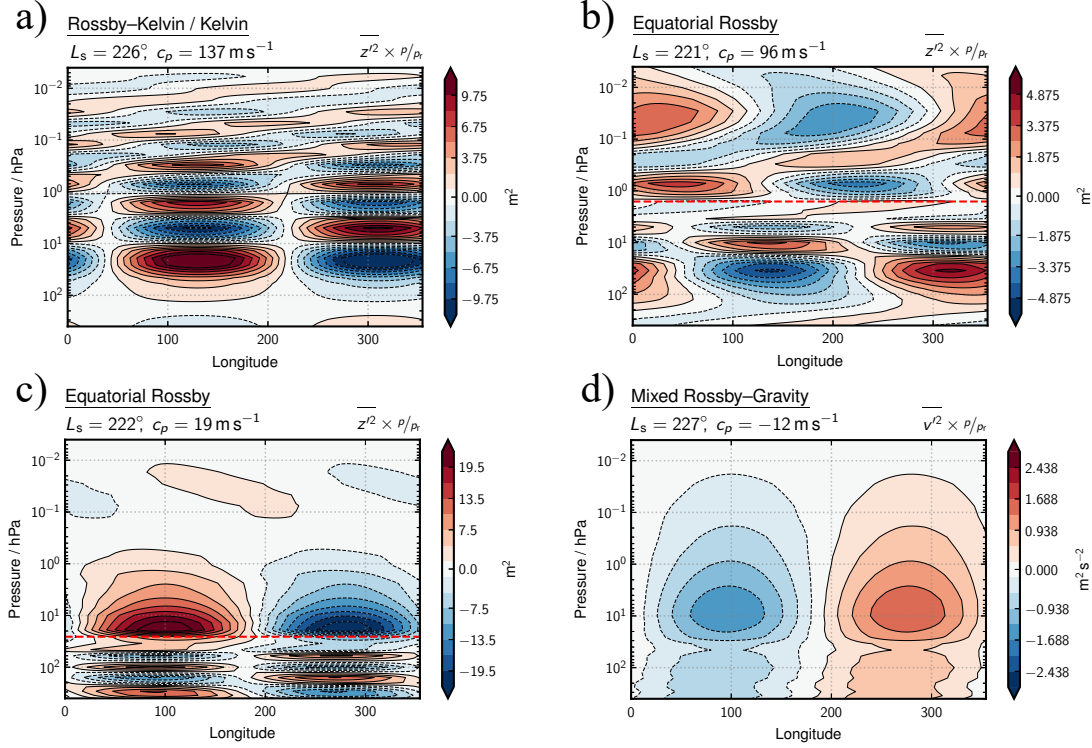


Figure 4.10: Snapshots of filtered geopotential height anomaly (a–c) and meridional velocity (d) averaged between $\pm 20^\circ$ latitude for selected waves. Red dashed lines in panels b) and c) indicate the pressure where $c_p = \bar{u}$ for the selected phase speed. Above these lines, waves are retrograde, and below they are prograde. As in Figure 4.6 and Figure 4.8, fields are filtered for frequencies corresponding to a range of phase speeds $c_p = c_{p,\text{wave}} \pm c_{p,\text{diff}}$, with $c_{p,\text{diff}} = 2, 1, 1$ and 1 m s^{-1} for panels a)–d) respectively. Meridional velocity is shown in place of geopotential height for the mixed Rossby–gravity wave as it has a vanishing amplitude in z' at the equator (see Figure 4.6f; Figure 4.8d). The solar longitude at which each snapshot was obtained is indicated at the top-left of each panel.

dashed line in Figure 4.9, and corresponds to one shown in Figure 4.6 or Figure 4.8 (or both). The vertical snapshots are shown at the same time as the horizontal ones (for the same wave). In panels b and c, red dashed lines indicate the pressure where $c_p = \bar{u}$. In these panels, disturbances are retrograde above the red dashed line, and prograde below.

Figure 4.10a shows the vertical structure of $\overline{z'^2}$ for $c_p = 137 \text{ m s}^{-1}$. This corresponds to the phase speed of the Rossby–Kelvin wave shown in Figure 4.6a at $p = 0.7 \text{ hPa}$, that appeared more similar to a pure Kelvin wave in Figure 4.8a, in the lower stratosphere. This mode has a high vertical wavenumber, with a wavelength that decreases with altitude. The vertical wavelength is consistent with that selected to produce the $m_n = 4$ Kelvin wave dispersion curve shown in Figure 4.5b and d.

For linear Kelvin waves on the equatorial beta plane, the intrinsic frequency obeys the dispersion relationship (Andrews et al., 1987)

$$\omega_{\text{kelvin}}^+ = -\frac{Nk}{[m^2 + 1/(4H^2)]^{1/2}}, \quad (4.15)$$

where m is the vertical wavenumber, related to the vertical wavelength λ_v via $m = 2\pi/\lambda_v = 2\pi m_n/\Delta z^*$ (see footnote 1 for more detail). For waves with smaller ω^+ , travelling at a speed close to the mean flow speed, we expect larger m . Furthermore, for prograde waves propagating vertically in a positive (increasing with altitude) shear, it is necessary for the vertical wavelength to shorten with altitude so as to preserve a coherent c_p in the vertical (Andrews et al., 1987; Imamura, 2006). Above $p \approx 2$ hPa, the wave shown in Figure 4.10a displays a clear phase tilt with altitude which, for a prograde Kelvin wave, is consistent with upward vertical propagation (see, e.g., Figure 4.19 of Andrews et al., 1987), and suggestive of a source region for the wave around $p = 2$ hPa. Sources of instability that lead to the generation of this wave, and others discussed later, will be investigated in Section 4.6.

Below $p \approx 2$ hPa, the wave appears to take the form of a standing wave that does not propagate vertically, where instead we might expect it to propagate downward away from the apparent source region. This standing structure is coincident with the vertical range in Figure 4.9 where $\overline{z'^2}$ is concentrated at particular pressures. This behaviour is difficult to interpret using the linear theory, within which equatorial Kelvin waves only become vertically trapped when $N^2 k^2 / \omega^{+2} < 1/(4H^2)$, so that m in (4.15) is imaginary (del Genio and Rossow, 1990). This is not satisfied in the simulation around $p \approx 2$ hPa, but instead much deeper in the atmosphere (below the red curve in Figure 4.9, which, notably, is roughly where $\overline{z'^2}$ goes to zero). Gossard and Hooke (1975) show that the curvature of the background wind can also inhibit vertical propagation, if $N^2 k / |\omega^+| < |d^2 \bar{u} / dz^2|$, but this condition is not satisfied anywhere in the model atmosphere. Possible explanations for apparent disagreement between the linear theory and the standing structure of the Kelvin wave are discussed further in Section 4.7.3.

Figure 4.10b presents the vertical structure of the equatorial Rossby wave identified at $p = 0.7$ hPa with $c_p = 96 \text{ m s}^{-1}$ (Figure 4.6d). This mode also displays a clear phase tilt in the vertical, consistent with upward propagation for a retrograde wave above roughly 3×10^{-2} hPa, and downward propagation below (compare with, e.g., Figure 13 of Yang et al., 2011, and accompanying text), suggestive of a source region in the upper stratosphere. This wave has a longer vertical wavelength than does the Rossby–Kelvin/Kelvin wave, consistent with larger separation between c_p and $\bar{u}$ (at the altitude it is generated). There

is less evidence for variation of the vertical wavelength with height, consistent with the reduced vertical shear of the background zonal wind in the upper stratosphere. Note that a prograde wave can also be identified in Figure 4.10b at lower altitude, below the pressure where $c_p - \bar{u}$ changes sign. As in Figure 4.10a, this wave appears to propagate vertically away from a source region, this time located around $p = 4$ hPa.

Figure 4.10c and d show the vertical structure of the equatorial Rossby wave presented in Figure 4.8c, and the mixed Rossby–gravity wave presented in Figure 4.6f and Figure 4.8d, respectively. Each of these waves has a barotropic structure, with a slight phase tilt indicative of upward propagation away from a source around $p = 40$ hPa. The increased vertical wavelength of these modes, with respect to the others shown previously, is once again consistent with an increase in $|c_p - \bar{u}|$. In Figure 4.10c, a prograde wave exhibiting similar properties to those in Figure 4.10a and b is also present.

Unlike the modes presented in Figure 4.10, the Rossby–Kelvin wave with $c_p = 105 \text{ m s}^{-1}$, at $p = 0.7$ hPa, and the equatorial Rossby wave with $c_p = 31 \text{ m s}^{-1}$, at $p = 19$ hPa, which contribute the greatest acceleration of the mean flow on their respective pressure levels, are confined to one model level. It is interesting to note from Figure 4.9 that disturbances with the same phase speed as the equatorial Rossby wave exist at pressures above and below $p = 16$ hPa. Below, there is a prograde mode with a vertical structure similar to those in the lower part of Figure 4.10b and c, and a horizontal structure similar to that in Figure 4.8a. Above, there is an additional retrograde disturbance that takes the form of a further equatorial Rossby wave with a much broader meridional extent (similar to that in Figure 4.8c) and a longer vertical wavelength. Regarding the Rossby–Kelvin wave at $c_p = 105 \text{ m s}^{-1}$, $p = 0.7$ hPa, note that its amplitude is sufficiently small that it cannot be identified in the $\overline{z'^2}$ co-spectrum shown in Figure 4.9.

4.5 Waves in winter

So far, this chapter has only presented an analysis of waves characteristic of northern hemisphere autumn. However, the waves present in the atmosphere vary with the seasonal cycle, due to seasonal variation in the background zonal wind and temperature structure. This is manifested in the seasonal variation exhibited by the EP flux vectors shown in Figure 4.3. This section presents an analysis of the waves present during the period $L_s = 350^\circ - 360^\circ$ (Figure 4.3, bottom row), corresponding to northern hemisphere late-winter, during which the EP flux and its divergence have a very different structure to that in autumn. In particular, significant eddy acceleration of the mean flow is now only present in the lower stratosphere. For brevity, the analysis in this section will focus

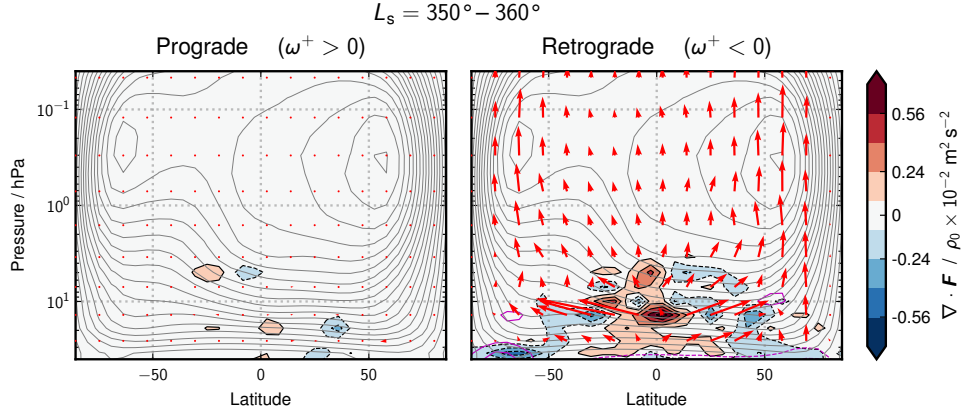


Figure 4.11: EP flux F (quivers) and its divergence $\nabla \cdot F$ (colour contour) for prograde (left) and retrograde (right) waves, as in Figure 4.4 but for $L_s = 350^\circ - 360^\circ$. See Figure 4.4 caption for more details.

only on notable differences between the autumn and winter periods, and on waves that contribute the most to equatorward transport of angular momentum.

Figure 4.11 shows $\nabla \cdot F$ and EP flux vectors for the period $L_s = 350^\circ - 360^\circ$, decomposed into contributions from prograde and retrograde waves. In contrast with the analysis presented for northern hemisphere autumn, acceleration of the equatorial zonal flow is almost entirely due to retrograde waves and is confined to the lower stratosphere ($p \gtrsim 4$ hPa). This indicates that the generation mechanism for the retrograde waves that accelerate superrotation in the lower stratosphere persists throughout the Titan year, whereas the mechanism generating prograde waves in the upper stratosphere is only active when the background circulation has a more pronounced hemispheric asymmetry.

Latitude–phase speed spectra of the eddy geopotential height anomaly and eddy angular momentum flux convergence during this time period are shown in Figure 4.12, for two pressure levels, $p = 0.7$ hPa and $p = 13.8$ hPa. $p = 0.7$ hPa corresponds to the pressure level in the upper stratosphere that was analysed in the previous section, and $p = 13.8$ hPa corresponds to the pressure where the equatorial acceleration due to retrograde disturbances is greatest in Figure 4.11.

For both pressures shown, Figure 4.12a,c shows that essentially all disturbances with significant amplitude in z' are retrograde, with only one prograde disturbance, at $c_p = 153$ m s⁻¹, clearly identifiable. At the equator, retrograde disturbances generally have $c_p \lesssim 50$ m s⁻¹, and a few structures have a broad meridional extent that is suggestive of phase locking between equatorial and mid-latitude disturbances. In the lower stratosphere ($p = 13.8$ hPa), Figure 4.12d shows that eddy angular momentum flux convergence at the equator is entirely due to one mode with a phase speed $c_p = 50$ m s⁻¹ that is very close to

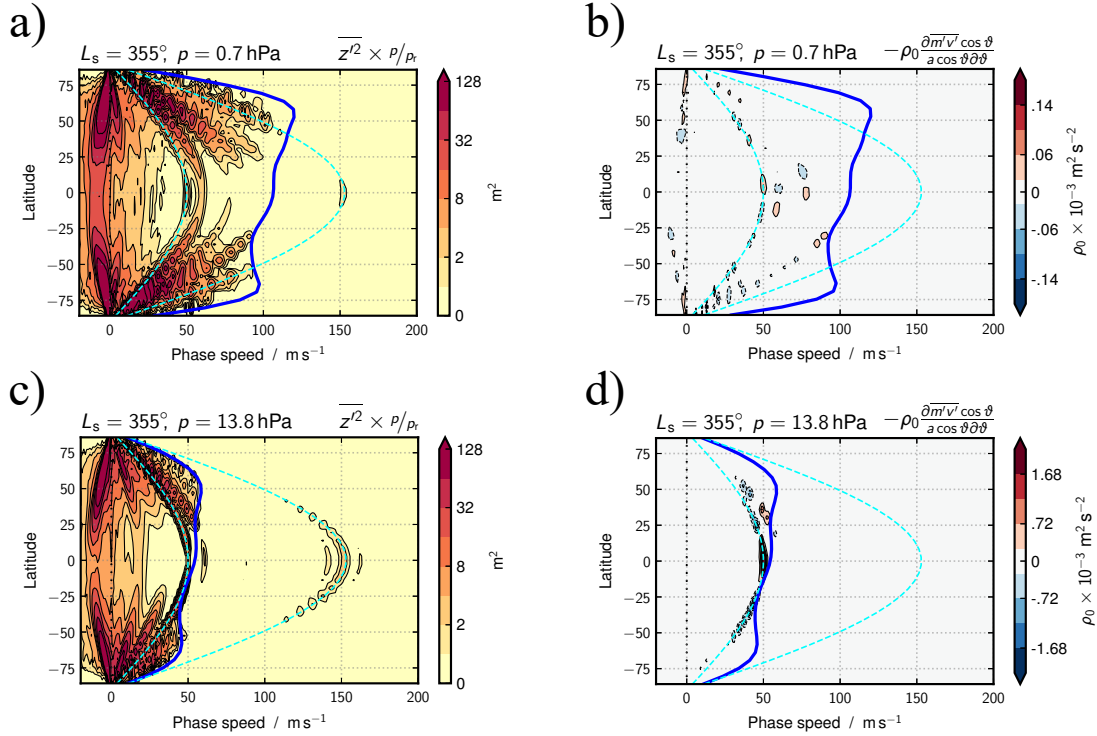


Figure 4.12: Eddy geopotential height and horizontal angular momentum flux convergence presented as a function of latitude and phase speed, as in Figure 4.5a,c, but for $L_s = 350^\circ - 360^\circ$, $p = 0.7$ hPa (panels a and b) and $p = 13.8$ hPa (panels c and d). The dashed cyan lines are curves of constant angular velocity ($c_p \times \cos \vartheta$), and solid blue lines show the zonal-mean zonal wind $\bar{u}$. Note that the contour spacing changes between panels b) and d), and that the acceleration is much larger in panel d) compared with b).

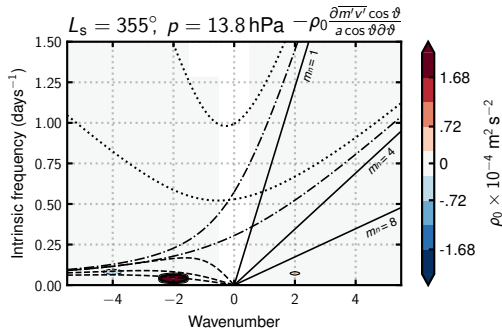


Figure 4.13: Spectra of eddy momentum flux convergence co-spectra averaged between $\pm 6^\circ$ latitude, as in Figure 4.5d but for $L_s = 350^\circ - 360^\circ$, $p = 13.8$ hPa. Black lines show dispersion curves for various equatorial waves (see Figure 4.5 caption and footnote 1 for details).

the mean flow speed. Figure 4.13 presents the wavenumber–frequency co-spectrum for the eddy momentum flux at this altitude, which reveals that the disturbance contributing this acceleration has zonal wavenumber $k_n = 2$. In Figure 4.12b, there is no disturbance that is able to induce a significant acceleration at the $p = 0.7$ hPa level, likely because none have c_p that is close enough to $\bar{u}$.

Figure 4.14 presents the horizontal structure for two of the waves present in Figure 4.12. Figure 4.14a and b show the structure of the prograde disturbance with

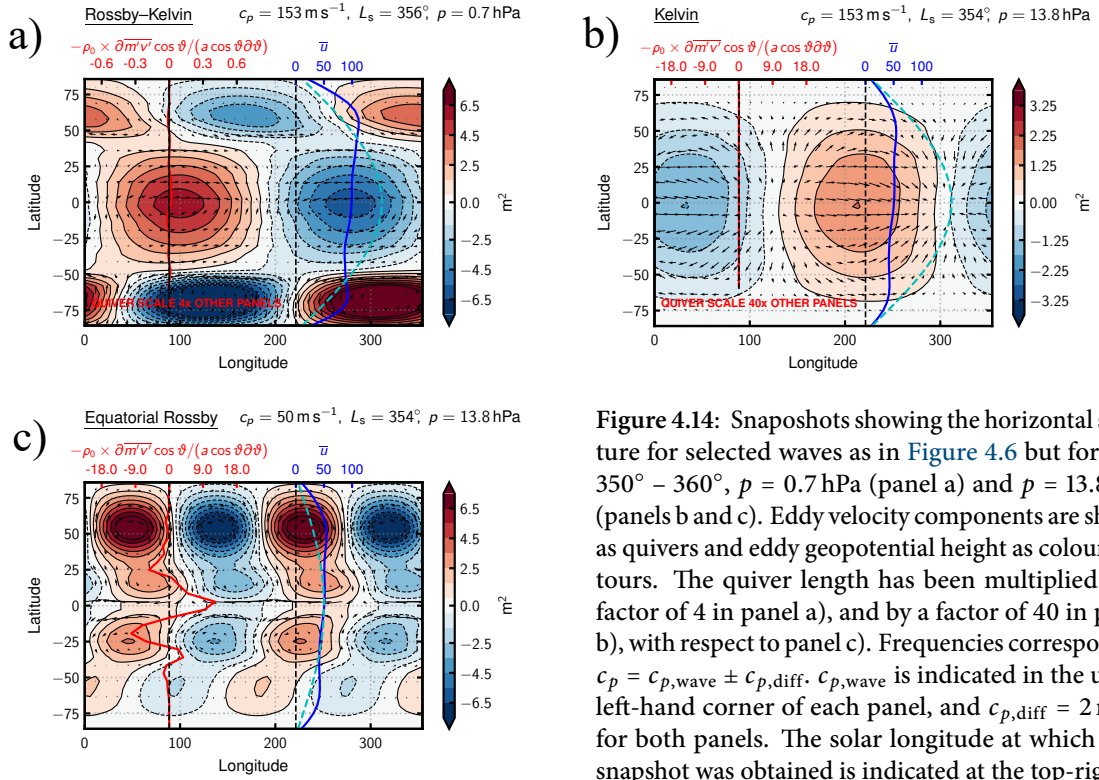


Figure 4.14: Snapshots showing the horizontal structure for selected waves as in Figure 4.6 but for $L_s = 350^\circ - 360^\circ$, $p = 0.7$ hPa (panel a) and $p = 13.8$ hPa (panels b and c). Eddy velocity components are shown as quivers and eddy geopotential height as colour contours. The quiver length has been multiplied by a factor of 4 in panel a), and by a factor of 40 in panel b), with respect to panel c). Frequencies correspond to $c_p = c_{p,\text{wave}} \pm c_{p,\text{diff}}$. $c_{p,\text{wave}}$ is indicated in the upper left-hand corner of each panel, and $c_{p,\text{diff}} = 2 \text{ m s}^{-1}$ for both panels. The solar longitude at which each snapshot was obtained is indicated at the top-right of each panel.

$c_p = 153 \text{ m s}^{-1}$, which is associated with $k_n = 1$, at $p = 0.7$ hPa and $p = 13.8$ hPa. At both pressures, this mode has a Kelvin wave-like component at the equator. In the upper stratosphere it also features a Rossby-like component in higher latitudes. In this sense, the mode is similar to that shown in the previous section for $L_s = 220^\circ - 230^\circ$ with $c_p = 137 \text{ m s}^{-1}$ (see Figure 4.6a and Figure 4.8a). Unlike the disturbances shown in the previous section for the upper stratosphere, this mode is approximately symmetric about the equator, reflecting the enhanced hemispheric symmetry of the background zonal wind. Phase-locking between the prograde and retrograde components of the wave at $p = 0.7$ hPa appears to be made possible by modest meridional shear in the background wind at high latitudes. However, the amplitude of the waves' velocity field is too small to induce a strong equatorward flux of angular momentum (although note that eddy angular momentum flux convergence due to this mode is positive at the equator).

Figure 4.14c shows the horizontal structure of the $k_n = 2$ retrograde mode associated with equatorial eddy angular momentum flux convergence at $p = 13.8$ hPa. This wave resembles an equatorial Rossby wave with an extra-tropical projection in higher latitudes (similar to those identified by Dias Pinto and Mitchell, 2016). The meridional extent of this mode is consistent with the theoretical deformation radius for modes with $m_n =$

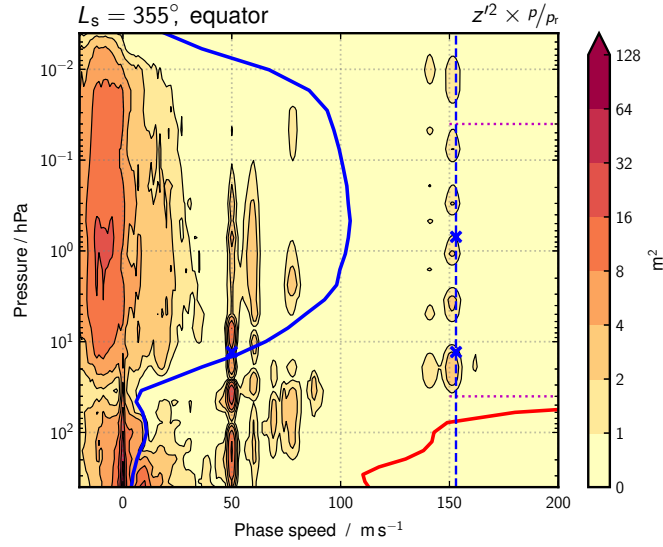


Figure 4.15: Eddy geopotential height presented as a function of pressure and phase speed, averaged within $\pm 10^\circ$, as in Figure 4.9 but for $L_s = 350^\circ - 360^\circ$. The vertical dashed line indicates c_p for the Kelvin wave whose vertical structure is displayed in Figure 4.16. Points marked by blue crosses correspond to the locations of waves whose horizontal structure is shown in Figure 4.14. Other curves have the same meaning as in Figure 4.9.

8 ($\pm 32^\circ$ latitude; see footnote 2 for details). As with the equatorial Rossby wave at $p = 19.0$ hPa identified during $L_s = 220^\circ - 230^\circ$ (Figure 4.8b), convergence of angular momentum at the equator is facilitated by a meridional phase tilt in the equatorial Rossby wave.

The distribution of $\overline{z'^2}$ in pressure and phase speed is shown in Figure 4.15 averaged between $\pm 10^\circ$ latitude. This figure reveals that prograde disturbances are almost completely absent across the full range of pressures that demarcate the stratosphere (40 hPa to 4×10^{-2} hPa, indicated by the horizontal purple lines), although it should be noted that there is still some significant prograde wave activity outside of this range in the upper troposphere ($p > 40$ hPa). By contrast, retrograde disturbances are identifiable across a range of pressures in the stratosphere. The mixed-Rossby gravity wave identified during northern hemisphere autumn at $c_p = -12$ m s $^{-1}$ is still present, as are other retrograde disturbances with phase speeds closer to the mean flow speed. The $k_n = 2$ equatorial Rossby wave that accelerates superrotation at $p = 13.8$ hPa is difficult to identify in Figure 4.15. This is because it is confined to one pressure level, similar to the $c_p = 105$ m s $^{-1}$ Rossby-Kelvin wave identified during $L_s = 220^\circ - 230^\circ$ at $p = 0.7$ hPa. However, it is clear from Figure 4.15 that other waves are also generated with this phase speed. At lower pressures, the $c_p = 50$ m s $^{-1}$ disturbance takes the form of a $k_n = 2$ mixed Rossby-gravity wave, and at higher pressures (where $c_p > \bar{u}$) it takes the form of a $k_n = 2$ Kelvin wave.

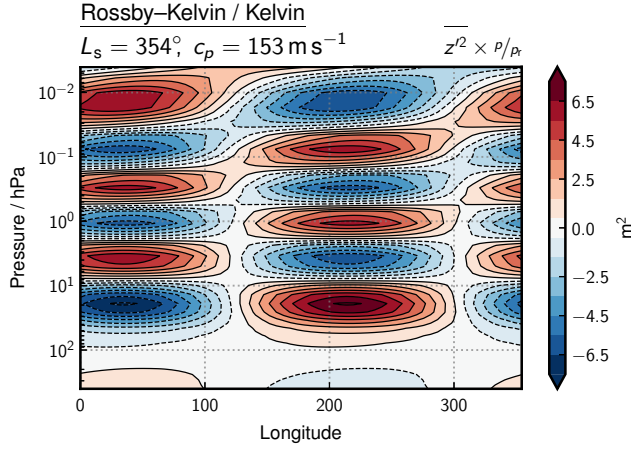


Figure 4.16: Snapshot of filtered geopotential height anomaly for the Rossby-Kelvin/Kelvin wave identified in Figure 4.14a and b, averaged between $\pm 20^\circ$ latitude at $L_s = 354^\circ$. As in Figure 4.14, fields are filtered for a range of phase speeds $c_p = c_{p,\text{wave}} \pm c_{p,\text{diff}}$, with $c_{p,\text{wave}} = 153 \text{ m s}^{-1}$ and $c_{p,\text{diff}} = 2 \text{ m s}^{-1}$.

There is a weak signature in $\overline{z'^2}$ at $c_p = 153 \text{ m s}^{-1}$, corresponding to the phase speed of the Rossby-Kelvin / Kelvin wave shown in Figure 4.14a,b, across a range of pressures. It appears that, as in the previous section, this prograde mode has a somewhat coherent vertical structure, shown in the longitude-pressure plane in Figure 4.16. However, there is very little phase tilt with altitude in the upper stratosphere, above $p \approx 2 \text{ hPa}$, indicative of only modest or no vertical propagation. There is also no longer evidence of the vertical wavelength shortening with increasing altitude, which is consistent with the lack of vertical shear in $\bar{u}$ in the upper stratosphere during this period.

4.6 Wave generation mechanisms

In this section, the possible mechanisms for the generation of the waves identified in the previous two sections are investigated. Both i) ageostrophic, barotropic instabilities, similar to that discussed by Iga and Matsuda (2005) and Wang and Mitchell (2014) (Section 4.6.1), as well as ii) conventional quasigeostrophic barotropic instability (see, e.g., Read et al., 2020) (Section 4.6.2) are considered.

4.6.1 Seasonal generation of prograde modes by ageostrophic, barotropic instabilities

The analysis in the previous sections has shown that the acceleration of superrotation in the upper stratosphere is due to waves that consist of an equatorial Kelvin-like component, coupled with a Rossby-like component in mid- and high latitudes. This mode grows when there is strong meridional shear in the horizontal wind, which is most pronounced around solstice. It is generally asymmetric with respect to the equator, with a Rossby-like component located in the winter hemisphere but not the summer hemisphere, due to asymmetry in the background wind.

Previous work has shown that coupled Rossby-Kelvin waves similar to those identi-

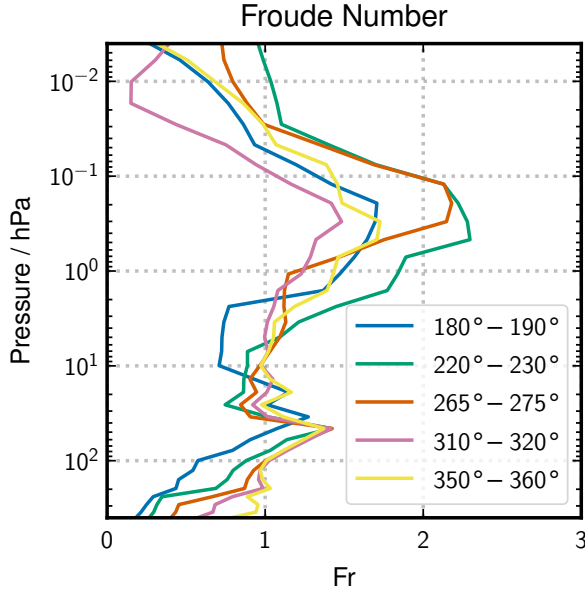


Figure 4.17: Froude number as in Wang and Mitchell (2014), indicating the ratio of the Kelvin wave frequency to the Rossby wave frequency (see Equation 4.16 and accompanying text for details). Lines are shown for different ranges of L_s , indicated in the legend.

fied in this chapter can grow due to an ageostrophic, barotropic instability that exists on slowly rotating and/or small-sized planets if there is meridional shear in the background wind (such that the velocity in mid-latitudes is greater than that at the equator; Iga and Matsuda, 2005; Wang and Mitchell, 2014). The instability requires the deformation radius to be large enough that the Rossby and Kelvin components have some spatial overlap, and that the two component waves have similar frequencies.

The first condition is generally true in the slowly rotating/small planet regime as both the equatorial and mid-latitude deformation radii depend inversely on the planetary rotation rate and radius. The second ‘frequency matching’ condition can be quantified via the definition of a Froude number that measures the ratio of the Doppler-shifted frequency of mid- or high-latitude Rossby waves to that of the equatorial Kelvin wave (Wang and Mitchell, 2014):

$$\text{Fr} \equiv \frac{\omega_{\text{rossby}}}{\omega_{\text{kelvin}}} \approx \frac{\bar{u}_{\text{ml}} / \cos \vartheta_{\text{ml}}}{\bar{u}_{\text{eq}} + \omega_{\text{kelvin}}^+ / k}, \quad (4.16)$$

where $\omega_{\text{kelvin}}^+ / k$ is given by (4.15), $\bar{u}_{\text{eq}}$ is the zonal velocity at the equator, $\bar{u}_{\text{ml}}$ is the mid-latitude speed at the latitude, ϑ_{ml} , where it is maximal, and the intrinsic frequency of the Rossby component has been assumed small and neglected. In principle, $\text{Fr} = 1$ for a Rossby–Kelvin mode. In practise, however, Wang and Mitchell (2014) find that neglecting the intrinsic component of the Rossby wave frequency means that Rossby–Kelvin instability occurs for a range of $\text{Fr} > 1$. Specifically, they find the growth rate (and equatorward eddy momentum flux due to the wave) is maximised when $\text{Fr} \approx 2$, and drops off as Fr is varied away from 2 (but within the range $\text{Fr} = 1 - 3$, outside of which

the instability is inactive).

Figure 4.17 shows Fr as a function of pressure at different times during the Titan year (corresponding to those shown in Figure 4.3). Each line assumes $m_n = 4$, corresponding to an approximate vertical wavelength $\lambda_{v4} \approx 34$ km (see footnote 1). For each period shown, $1 \lesssim Fr < 2.5$ throughout the depth of the stratosphere, indicating conditions favorable for Rossby–Kelvin instability. Notably, $Fr \approx 2$ during the period when coupled Rossby–Kelvin modes induce the greatest acceleration in the upper stratosphere, as in Wang and Mitchell (2014). Furthermore, $Fr \approx 1.5$ in winter is consistent with the existence of weaker Rossby–Kelvin modes (e.g., Figure 4.14a) that do not induce a significant acceleration.

This analysis is not fully conclusive regarding the origin of the waves responsible for accelerating superrotation in the upper stratosphere, but it is at least *consistent* with the waves being generated by an ageostrophic shear instability that is most active during autumn, when the meridional shear in the background wind is maximised and $Fr \approx 2$.

4.6.2 Year-round generation of retrograde modes by conventional barotropic instability

In the lower stratosphere, superrotation is maintained by equatorial Rossby-like waves, and the equatorial eddy angular momentum flux convergence due to these waves displays far less seasonal variation. More generally, retrograde modes exist throughout the year in the stratosphere, whereas prograde modes do not (compare Figure 4.9 and Figure 4.15). This suggests a generation mechanism for retrograde waves that does not vary as strongly with the seasonal cycle as the ageostrophic, barotropic instability described in the previous subsection. A possible mechanism is conventional quasigeostrophic barotropic instability, which has been shown to occur close to the equator in other Titan GCMs (Newman et al., 2011; Lebonnois et al., 2012a).

A necessary criterion for quasigeostrophic barotropic instability is that the meridional gradient of zonally-averaged potential vorticity, taken on isentropic surfaces, change sign somewhere within the domain (Charney and Stern, 1962).

The criterion for barotropic instability can be expressed mathematically as

$$\xi \equiv \partial \bar{Q} / \partial \vartheta|_{\theta} = 0, \quad \text{somewhere,} \quad (4.17)$$

where the potential vorticity Q is defined

$$Q = -g(f + \mathbf{k} \cdot \nabla_{\theta} \times \mathbf{v}) / (\partial p / \partial \theta). \quad (4.18)$$

Above, ∇_{θ} is the horizontal gradient acting along surfaces of constant potential temperature, θ (Hoskins et al., 1985).

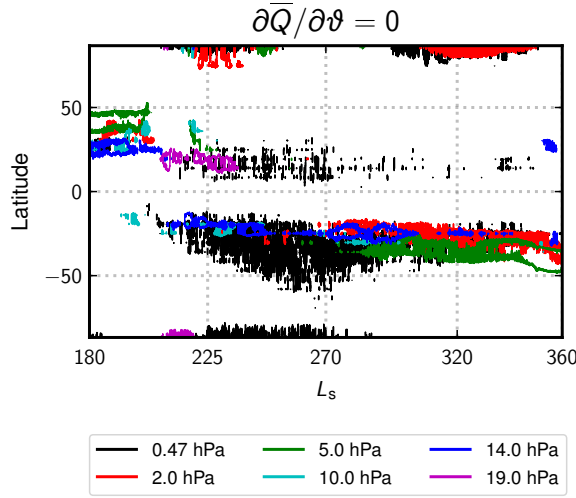


Figure 4.18: Criterion for barotropic instability plotted as zero contours for $\partial\bar{Q}/\partial\vartheta|_\theta$ (see text). Contours are shown for a range of pressures indicated in the legend.

Figure 4.18 shows zero-contours for ξ at a range of pressures in the stratosphere, over the period $L_s = 180^\circ - 360^\circ$. The criterion for barotropic instability is satisfied year-round at pressures from which the retrograde waves shown in Figure 4.10 and Figure 4.16 appear to originate. $\xi = 0$ generally occurs at latitudes coincident with the equatorward flank of the summer hemisphere mid-latitude jet. It is worth noting that $\xi = 0$ never occurs directly on the equator. However, the occurrence of barotropic instability within a deformation radius of the equator is sufficient to generate equatorial wave modes (as is the case here).

The fact that $\xi = 0$ for most of the year in regions where retrograde equatorial waves appear to be generated points to barotropic instability as a likely source of excitation. However, as before, it cannot be definitively asserted that this is the case based solely on the analysis presented here. Instead, this suggestion, and the suggestion that ageostrophic barotropic instability leads to the generation of Rossby–Kelvin type waves in the upper stratosphere, should be investigated further by conducting stability analyses of mean circulation states that are characteristic of Titan.

4.7 Discussion

4.7.1 Relationship to results from idealised modeling studies

As discussed in Chapter 2, Section 2.6 and summarised in the introduction to this chapter, the generation and maintenance of superrotation in the atmospheres of small and/or slowly rotating planets has been investigated extensively with idealised Earth-like GCMs forced by time-independent, hemispherically symmetric heating and cooling. However, it is not obvious *a priori* that mechanisms leading to the generation and

maintainance of superrotation in a simplified modeling setting are of direct relevance to Titan's atmosphere, which is complicated by seasonally-varying, hemispherically asymmetric meridional shear.

Studies with idealised GCMs have shown that a global mode that couples equatorial Kelvin waves with Rossby-like waves in midlatitudes leads to the generation of superrotation during the model spin-up period (Mitchell and Vallis, 2010). Phase locking of the Rossby and Kelvin components is possible as the Rossby waves are Doppler-shifted eastwards by a midlatitude jet that emerges very early in the spin-up phase (and is associated with conservation of angular momentum within the Hadley cell). However, once the Rossby–Kelvin wave has accelerated the zonal wind at the equator and the simulation equilibrates, the Kelvin wave also experiences a Doppler-shift (due to the equatorial jet) and phase locking between the Rossby and Kelvin components is no longer possible. Dias Pinto and Mitchell (2014, 2016) have shown that in equilibrium superrotation is maintained by equatorial Rossby waves, which develop a meridional phase tilt in vertical shear that facilitates eddy momentum flux convergence (Imamura, 2006).

The results presented in this chapter suggest that direct analogies between the generation scenario *and* the maintenance scenario may be drawn with different aspects of Titan's stratospheric circulation. The seasonal *maintenance* of superrotation by a Rossby–Kelvin like wave in the upper stratosphere is analogous to the *generation* of superrotation by Rossby–Kelvin waves in idealised Earth-like GCMs during their spin-up phase. In Titan's upper stratosphere, the circulation in each hemisphere never really equilibrates, and is instead constantly 'spinning up' and 'spinning down' with the seasonal cycle. Meanwhile, the *maintenance* mechanism for superrotation in Titan's lower stratosphere, where there is less seasonal variation, is very comparable to the *maintenance* mechanism in idealised GCMs, with equatorial Rossby waves playing a central role in both cases.

Mitchell et al. (2014) have shown that when a seasonal cycle is included in an idealised Earth-like model, the generation of equatorial superrotation is inhibited. They suggest this is due to two effects: i) cross-equatorial advection of momentum by the overturning circulation, and ii) suppression of the Rossby–Kelvin instability in hemispherically asymmetric meridional shear. To achieve a Titan-like superrotating circulation, they increase the radiative cooling time in their model by a factor of 100 (appropriate for Titan), the effect of which is to reduce the strength of overturning which, in turn, damps each of the superrotation-inhibiting seasonal effects described above.

The circulation in the TAM simulation used for this study *does feature* strong meridional shear, but this actually appears to give rise to the asymmetric Rossby–Kelvin type

wave that accelerates superrotation in the upper stratosphere (notably, this disturbance is very similar to one shown in Mitchell et al., 2014, for a Titan-like case that does not develop strong superrotation; compare Figure 4.6b with their Figure 8). This indicates that Rossby–Kelvin modes that are not symmetric about the equator can provide sufficient momentum transport to maintain superrotation, counter to the suggestion of Mitchell et al. (2014). However, it is likely that the superrotation in TAM does benefit from effect ii) above, namely ‘reduced mean flow momentum transport due to weaker overturning’, as a result of Titan’s long radiative timescale, although this will be reduced in the stratosphere where the pressure is low. Additionally, note that in the TAM simulation used for this work, the overturning in the stratosphere is partially detached from that in the troposphere, which will limit the extent to which the mean flow can communicate the effects of friction near the surface to the superrotating jet (see, e.g., Figure 4.1, where it could be argued that overturning in the stratosphere takes the form of an isolated cell sitting above another overturning cell in the troposphere).

4.7.2 *Comparison with other Titan GCMs*

This subsection compares the results presented in this chapter with those of Newman et al. (2011) and Lebonnois et al. (2012a), who analysed the waves responsible for accelerating superrotation in TitanWRF and the IPSL Titan GCM, respectively.

Each of these studies found that eddy angular momentum transport is strongest in the autumn/winter hemisphere, and induces acceleration close to the equator paired with deceleration in mid-latitudes, which is consistent with the seasonal cycle of EP flux divergence shown in Figure 4.3.

Spectral analysis of the waves in the IPSL model also appears to be consistent with some of the results presented here (Lebonnois et al., 2012a). In particular, the acceleration of superrotation above $p = 10$ hPa in their model seems to be due to modes that are prograde at the equator, and, notably, have a similar spatial structure in latitude and longitude to the hemispherically asymmetric Rossby–Kelvin wave identified in this study (compare the filtered u' in Lebonnois et al., 2012a’s Figure 16b with the zonal component of the quivers in Figure 4.6b). A few modes have a coherent structure in the vertical direction in their Figure 14, which shows u' and v' as a function of frequency and pressure (similar to Figure 4.10 and Figure 4.16 in this work), although not as obviously as in the analysis presented here. There appear to be some retrograde modes in the lower stratosphere of the IPSL simulation, but Lebonnois et al. (2012a) do not show whether or not they contribute to the angular momentum budget at the equator.

Newman et al. (2011) compute latitude–phase speed eddy momentum flux co-spectra for TitanWRF, which show that acceleration in TitanWRF’s upper stratosphere is due to a mode with strong coherence along a line of $c_p \times \cos \vartheta$, which has $c_p > \bar{u}$ at the equator and $c_p < \bar{u}$ in mid-latitudes. They interpret this in terms of a wave propagating meridionally from the equator and breaking in mid-latitudes after crossing a critical line where $c_p = \bar{u}$, whereas the analysis in this chapter indicates that this sort of signature is due to a coupled Rossby–Kelvin mode that does not propagate meridionally. However, Newman et al. (2011) do not show the horizontal latitude–longitude structure of the wave that accelerates superrotation in the upper stratosphere of TitanWRF, so it is difficult to comment on whether or not it is similar to those identified in this study.

It is worth noting that Newman et al. (2011) present evidence that angular momentum transport from the winter hemisphere to the equator is concentrated into short-lived ‘transfer events’ (duration corresponding to $< 5^\circ L_s$). While there is eddy angular momentum flux variability in the TAM simulation used here that occurs on a timescale corresponding to a few degrees L_s , no events of comparable magnitude to those in TitanWRF have been identified. Newman et al. (2011) show that the timing of these transfer events is closely related to instances when barotropic instability might operate (i.e., times when $\xi = 0$), which also occurs in short-lived events (note that TAM does not exhibit this sort of behaviour, cf. Figure 4.18). This might indicate that the waves that accelerate superrotation in the upper stratosphere of TitanWRF are generated by a different mechanism to those here.

4.7.3 Vertical structure of equatorial Kelvin waves

In this study, a number of waves which resemble an equatorial Kelvin wave at the equator have been identified (see, e.g., Figure 4.6a,b, and c, Figure 4.8a, and Figure 4.14a and b). These waves generally have short vertical wavelengths, consistent with their phase speeds being close to the mean flow speed (cf. Equation 4.15). Furthermore, there is evidence that their wavelength varies due to vertical propagation in a vertically-sheared background zonal wind profile (e.g., Figure 4.10a), which, again, is consistent with our expectation from linear theory.

However, these Kelvin waves also exhibit some unusual characteristics. Specifically, a number appear to be vertically trapped (i.e., unable to propagate) in the region where the background vertical wind shear is strongest (Figure 4.10a and Figure 4.16). This behaviour is not predicted by the linear theory for waves on the equatorial beta plane. However, the linear theory is only valid for the case of a slowly varying background $\bar{u}$

and N (see, e.g., Appendix 4A in Andrews et al., 1987), which is clearly not satisfied in Titan's stratosphere. It is therefore *possible* that unusual vertically trapped behaviour exhibited by the Kelvin waves is a physical response to the strong vertical variation in N and $\bar{u}$ in the simulation.

Notwithstanding the possible influence of strong vertical variation in $\bar{u}$ and N , it is likely that the short vertical wavelength of the equatorial waves identified in this work means that they are not properly resolved with the model's current vertical resolution. At least four grid points per wavelength are required for a wave to be represented accurately, i.e., $\lambda_v/(4\delta z^*) > 1$, where δz^* is the vertical grid spacing. This is not always satisfied in TAM; for example, $\lambda_v/(4\delta z^*) \approx 0.875$ for the Rossby–Kelvin / Kelvin wave shown in Figure 4.10a. Boville and Randel (1992) investigated the effect of vertical resolution on equatorial waves in the stratosphere of a GCM designed for Earth. They found that for $\lambda_v/(4\delta z^*) \lesssim 1$, the amplitude of equatorial waves depended strongly on the vertical resolution (increasing amplitude for increasing resolution). This, in turn, affects eddy acceleration of the mean flow, as the EP flux is formed from covariances of wave properties, and so depends on the wave amplitude squared. Boville and Randel (1992) also note that insufficient resolution in regions with strong vertical shear in the background wind can lead to poor representation of vertically propagating equatorial waves; notably, it is in such regions that the equatorial Kelvin waves identified in this study exhibit unusual behaviour.

It is therefore possible that the EP flux and its divergence in the TAM simulation used here only partially capture the way in which waves interact with the mean flow in Titan's real atmosphere. The vertical resolution used for this study is similar to that of other Titan GCMs (e.g., TitanWRF and the IPSL Titan Model; Newman et al., 2011; Lebonnois et al., 2012a), and thus it is reasonable to expect that the issues regarding vertical resolution discussed here for TAM extend to those models as well. Poor representation of the vertical propagation and structure of equatorial waves may explain why current Titan GCMs struggle to produce the 'zonal wind minimum' present in the Huygens descent profile (at $p = 20$ hPa in Figure 2 of Bird et al., 2005), which could be formed by a vertical eddy momentum flux divergence (e.g., due to prograde waves propagating vertically away from this region). Taken together, the arguments discussed in this subsection motivate the development of Titan GCMs with higher vertical resolution in the stratosphere.

4.8 Conclusions

The aim of this chapter was to characterise the different waves that are supported in a simulation of Titan's stratosphere using the Titan Atmospheric Model (Lora et al., 2015; Lombardo and Lora, 2022), with a focus on those that contribute most to the acceleration of equatorial superrotation.

The key conclusions of this study are as follows:

1. A variety of equatorial waves are identified in TAM's stratosphere, including equatorial Kelvin waves, equatorial Rossby waves, and mixed Rossby–Gravity waves.
2. In the upper stratosphere, equatorial waves are strongly affected by the seasonal cycle and feature pronounced hemispheric asymmetries when meridional shear in the background wind is strongest. This leads to interactions between equatorial Kelvin waves and Rossby-like waves in the winter hemisphere, which converge momentum from winter mid-latitudes towards the equator, opposing momentum transport away from the equator by the mean flow.
3. In the lower stratosphere, equatorial waves have more hemispheric symmetry, and equatorial superrotation is accelerated throughout the year. The strongest eddy momentum fluxes in the lower stratosphere are due to equatorial Rossby waves.
4. Seasonal variation in the Rossby–Kelvin type waves that accelerate superrotation in the upper stratosphere is consistent with generation by ageostrophic, barotropic ‘Rossby–Kelvin instability’ (cf. Wang and Mitchell, 2014). Meanwhile, conventional quasigeostrophic barotropic instability is a possible source of waves that accelerate superrotation in the lower stratosphere.
5. Equatorial waves that are most important for accelerating superrotation generally have phase speeds that are close to the zonal velocity of the mean flow (i.e., small intrinsic frequency ω^+). Small ω^+ requires these modes to have short vertical wavelengths, and, therefore, it is likely that they are underresolved in the vertical. In light of this analysis, it is suggested that Titan modelers should work to increase the vertical resolution of Titan GCMs. This should be complemented by theoretical investigation of the expected properties of equatorial waves in Titan-like background conditions with strong vertical variation in the background wind and buoyancy frequency, and hemispherically asymmetric meridional shear.

Rotational and Divergent Components of Atmospheric Circulation on Tidally Locked Planets

The contents of this chapter are a modified version of M. Hammond and N. T. Lewis (2021): The Rotational and Divergent Components of Atmospheric Circulation on Tidally Locked Planets, in: *Proceedings of the National Academy of Sciences*, 118 (13), e2022705118.

Author contributions: NTL proposed the research. NTL and MH analysed the numerical experiments and wrote the paper. The analysis in the paper is applied to two simulations: one of a terrestrial planet, and the other a hot Jupiter. NTL lead the analysis and writing for the terrestrial planet, and MH lead the analysis and writing for the hot Jupiter. This chapter only presents results from the terrestrial planet simulation. NTL and MH are listed alphabetically on the publication.

5.1 Introduction

In this chapter and the next, we turn our attention away from the Solar System and towards planets orbiting stars other than the sun, called exoplanets. The most observationally accessible exoplanets are likely to be tidally locked, which means that they always present the same face to their host star, and thus have a permanent day side and a permanent night side (Dole, 1964; Guillot et al., 1996).

As discussed in Chapter 2, the day-night heating contrast on tidally locked planets strongly influences their atmospheric circulation. Notably, it introduces pronounced zonal asymmetries in the form of stationary equatorial waves (Showman and Polvani, 2010; Showman and Polvani, 2011; Hammond and Pierrehumbert, 2018) and divergent overturning motion which features air rising on the day side and sinking on the night side (Showman et al., 2013). Interaction between these circulation components can lead to the acceleration of a superrotating equatorial jet (Showman and Polvani, 2011). The day-night asymmetry of atmospheric circulation on tidally locked planets is in contrast to that on non-tidally locked planets, the statistics of which are approximately zonally symmetric (see, e.g., Figure 2.1).

The superposition of different stationary circulation features on tidally locked planets makes them difficult to analyse. In particular, it is no longer obviously meaningful to diagnose overturning circulation using the Eulerian mean meridional mass streamfunction (as in Chapter 2, Section 2.3), which is a zonally averaged diagnostic, and the stationary waves cannot be separated from the rest of the circulation using spectral diagnostics,

as was done for Titan in [Chapter 4](#). Consequently, the relative contribution of each of the circulation components on tidally locked planets — namely, the superrotating jet, stationary waves, and divergent circulation — to the total circulation, remains poorly understood.

In this chapter, I will show how this issue can be addressed by partitioning the circulation on tidally locked planets using a Helmholtz decomposition, which uniquely divides the total circulation $\mathbf{u} = (u, v)$ into divergent (vorticity free) and rotational (divergence free) components (Holton, 2004, Chapter 11):

$$\begin{aligned}\mathbf{u} &= \mathbf{u}_d + \mathbf{u}_r \\ &= \nabla_p \chi + \mathbf{k} \times \nabla_p \psi.\end{aligned}\tag{5.1}$$

Above, χ is the velocity potential, and ψ is a streamfunction. χ and ψ are obtained from

$$\nabla_p^2 \chi = \delta,\tag{5.2}$$

$$\nabla_p^2 \psi = \zeta,\tag{5.3}$$

where $\delta = \nabla_p \cdot \mathbf{u}_d$ is the divergence and $\zeta = \mathbf{k} \cdot \nabla_p \times \mathbf{u}$ is the vorticity.

The utility of the Helmholtz decomposition for analysing circulation on tidally locked planets is investigated by applying it to a well-studied simulation of a terrestrial planet, run using the GCM Exo-FMS (Hammond and Pierrehumbert, 2018; Hammond et al., 2020). Hammond and Lewis (2021) also present results from a simulation of a giant gaseous planet (a hot Jupiter) run using the GCM THOR (Mendonça et al., 2016; Deitrick et al., 2020), and these results are discussed in the text where they are relevant. [Section 5.2](#) describes the configuration of the Exo-FMS GCM, and also presents the model's basic atmospheric circulation. [Section 5.3](#) presents the results of this study. Finally, in [Section 5.4](#) some opportunities for future work using the Helmholtz decomposition are outlined, and the main conclusions of this chapter are summarised.

5.2 Description of numerical simulation

5.2.1 Model configuration

The GCM simulation was run using Exo-FMS (Hammond and Pierrehumbert, 2018), which solves the primitive equations on a cubed-sphere grid with a vertical sigma-pressure coordinate (Lin, 2004). Exo-FMS is configured as a dry model with no representation of water vapour or cloud condensate, with the planetary and atmospheric parameters listed in [Table 5.1](#). The simulation has an orbital period of $P = 10$ days, which

Parameter	
Radius a (m)	6.371×10^6
Rotation rate Ω (s^{-1})	7.29×10^{-6}
Gravitational acceleration g (m s^{-2})	9.81
Lower boundary pressure p_0 (Pa)	10^5
Gas constant R_d ($\text{J kg}^{-1} \text{K}^{-1}$)	287
Specific heat capacity c_p ($\text{J kg}^{-1} \text{K}^{-1}$)	1005
Instellation F_s (W m^{-2})	1000

Table 5.1: Input parameters of the Exo-FMS GCM simulation.

is an approximate representation of a tidally locked Earth-like planet orbiting an M dwarf, similar to the Trappist-1 planets (Figure 1.3 in Chapter 1; Gillon et al., 2017).

The simulation used for this work has the same configuration as the “1000 W m^{-2} ” instellation experiment presented in Hammond and Pierrehumbert (2018). It uses semi-grey two-stream radiative transfer scheme and dry convective adjustment (see Section 6.2 in Chapter 6 for a description of the semi-grey radiative transfer formulation). The atmosphere is assumed to be transparent to shortwave radiation (incoming stellar radiation), and grey in the longwave (thermal emission), with grey optical depth $\tau_{\text{lw}} = 1$ at the surface, which yields Earth-like surface temperatures. The surface albedo is set to zero (the effect of albedo is folded into the instellation, as these parameters are degenerate when the atmosphere is transparent to shortwave radiation).

Each of the six faces of the cubed-sphere grid has 48 by 48 cells, which corresponds to an approximate latitude–longitude resolution of 1.9° , and the model has 48 levels in the vertical. The simulation is stabilised with a fourth-order hyperdiffusion, with a coefficient that is the same as that in the default distribution of the dynamical core (Lin, 2004). A linear drag is applied to the horizontal velocity at the surface on a timescale of 1 Earth days, which decreases linearly (in pressure) to a value of zero at 700 hPa.

The simulation was spun up for 1000 Earth days, until the total angular momentum and total outgoing longwave radiation were equilibrated. The output data were then taken as an average over the next 1000 days, and interpolated onto a pure pressure coordinate prior to analysis.

5.2.2 Simulated circulation

Figure 5.1 shows the global circulation of the terrestrial planet GCM simulation used for this study. The left-hand panel of Figure 5.1 shows the geopotential height and horizontal velocity fields at 400 hPa, and the right-hand panel shows the zonal-mean

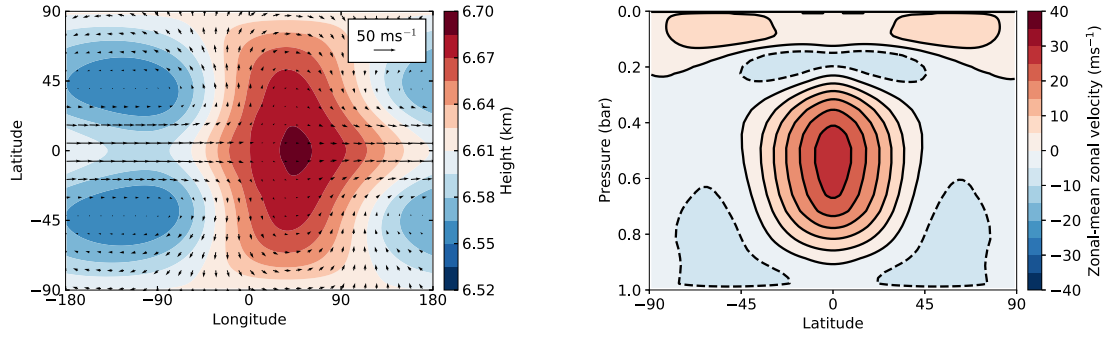


Figure 5.1: The global circulation of the terrestrial tidally locked planet simulated using Exo-FMS. Left panel: Geopotential height (colour contours) and horizontal velocity field (quivers) at 400 hPa. The substellar point is located at (0°, 0°). Right panel: zonal-mean zonal velocity. The simulation features an eastward equatorial jet and eastward hot-spot shift typical to tidally locked planets.

zonal wind. These fields show the key features of the circulation in the simulation: a hot spot shifted eastward of the substellar point, stationary planetary-scale waves, and an eastward equatorial jet.

5.3 Decomposition of circulation

In this section, the Helmholtz decomposition given by (5.1) is applied to the GCM simulation shown in Figure 5.1. The horizontal wind is decomposed into its rotational and divergent components in Section 5.3.1. In Section 5.3.2, the divergent circulation is discussed in terms of a ‘tidally locked streamfunction’ which describes an overturning cell featuring air rising on the day side and sinking on the night side. Finally, the contributions of the rotational and divergent circulations to day–night heat transport are evaluated in Section 5.3.3.

5.3.1 Application of the Helmholtz decomposition to horizontal velocities

Figure 5.2 shows $\mathbf{u}_r$ and $\mathbf{u}_d$ for the terrestrial GCM simulation shown in Figure 5.1. The rotational circulation is characterised primarily by zonal flow around the planet’s axis of rotation, superimposed over a stationary wave pattern, and the divergent circulation is characterised by roughly isotropic flow away from the substellar point. The analysis of the hot Jupiter presented in Hammond and Lewis (2021) yields a similar decomposition.

The rotational circulation has two phenomenological components, which are manifest in the zonal mean of $\mathbf{u}_r$ and the eddy component of $\mathbf{u}_r$ (Figure 5.3). The eddy component corresponds to planetary waves, primarily stationary Rossby waves with zonal wavenumber-1 (Showman and Polvani, 2011; Tsai et al., 2014; Hammond and Pierrehumbert, 2018), which are excited by the divergent circulation (Sardeshmukh and

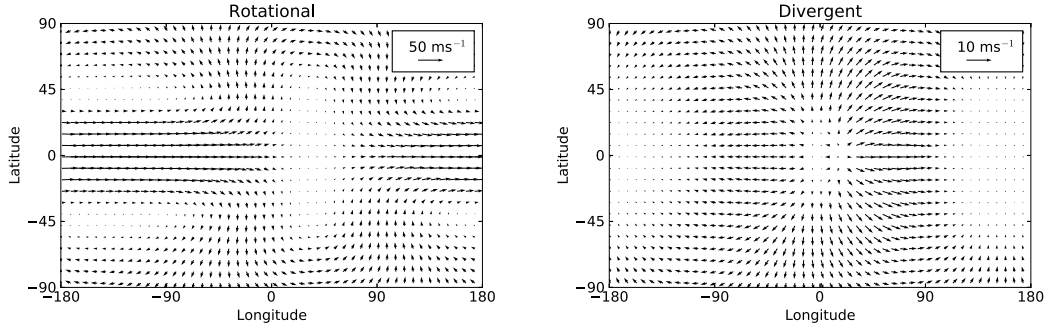


Figure 5.2: Helmholtz decomposition of horizontal velocity $\mathbf{u}$ at 0.4 bar. Left panel: Rotational component of $\mathbf{u}$, $\mathbf{u}_r$. Figure 5.3 shows that this component is composed of the zonal-mean eastward equatorial jet and a stationary wave with zonal wavenumber-1. Right panel: Divergent component of $\mathbf{u}$, $\mathbf{u}_d$. At the pressure level shown here, the divergent component is dominated by an isotropic flow away from the substellar point (located at 0° longitude).

Hoskins, 1988). The zonal-mean component of $\mathbf{u}_r$ corresponds to the zonal-mean super-rotating jet, which is produced by momentum flux convergence at the equator associated with the stationary waves (Showman and Polvani, 2010; Showman and Polvani, 2011). The zonally-averaged meridional rotational velocity is zero, as $v_r = \partial\psi/\partial x$ vanishes when integrated over longitude (assuming there is no surface topography to produce a discontinuity in ψ).

Existing theory of the atmospheric circulation on tidally locked planets is built largely on the results of shallow water models, which represent the first baroclinic mode of an atmosphere as a single fluid layer (Chapter 2, Section 2.7). Showman and Polvani (2011) show that forcing the shallow water system with a day-night heating contrast produces stationary waves which converge westerly momentum towards the equator that accelerate a superrotating jet. Tsai et al. (2014) and Hammond and Pierrehumbert (2018) used shallow water modelling to show how a superrotating jet phase-shifts the

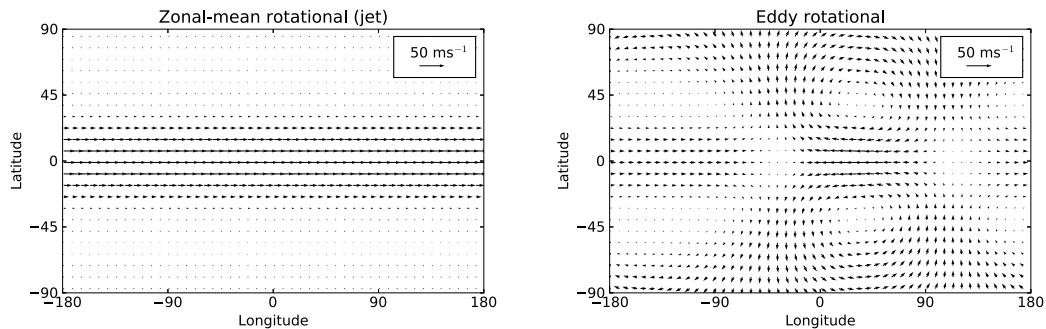


Figure 5.3: The two physical components of the rotational circulation in Figure 5.2. Left panel: Zonal-mean part of $\mathbf{u}_r$, $\bar{\mathbf{u}}_r$, which is the eastward equatorial jet. Right panel: Eddy (total minus zonal mean) part of $\mathbf{u}_r$, $\mathbf{u}'_r$, which is dominated by a wave with zonal wavenumber-1.

stationary waves to produce an eastward hot spot shift (as in [Figure 5.1](#)). Each of these shallow water models *assumes* that the atmosphere is dominated by a stationary wave response with zonal wavenumber-1 and a zonal-mean zonal flow, and that these two features have separable effects. They also *exclude* all aspects of the overturning circulation. [Figure 5.3](#) is therefore an important confirmation of the assumptions made in shallow water models, as it shows that their stationary wave response corresponds to a specific part of the rotational circulation, which is indeed dominated by a component with zonal wavenumber-1. It also suggests that the overturning circulation is a physically distinct feature from both the jet and stationary waves, so that it is reasonable to model the latter components in isolation in a shallow water system.

Divergent horizontal motion occurs due to convergent vertical motion (ignoring non-hydrostatic effects), and, phenomenologically speaking, the divergent circulation in the Exo-FMS simulation corresponds to an overturning circulation driven by heating at the substellar point. The divergent horizontal wind is shown in [Figure 5.4](#) at three different pressure levels: 250 hPa, 400 hPa, and 886 hPa, with the pressure vertical velocity ω underlaid as color contours (the quivers at 400 hPa are the same as those in [Figure 5.2](#)). In the free atmosphere (250 hPa and 400 hPa), divergent horizontal motion is primarily a flow away from the substellar point, where radiative heating is strongest and air rises (negative ω) and diverges. This flow extends roughly isotropically away from the substellar point in all directions, before starting to subside on the night side (positive ω). In the boundary layer (886 hPa), air returns from regions of subsidence on the night side towards the substellar point. The convergent return flow in the boundary layer is roughly isotropic, like the divergent flow aloft. There is some anisotropy in the boundary layer ascent (i.e., ω) near the substellar point, which takes the form of a wide, horseshoe-shaped feature. This feature appears similar in structure to a weather front in the Earth's atmosphere ([Hoskins, 1982](#)), and forms at the boundary where cold air, advected from the night side by the rotational circulation (which is predominantly westward in the boundary layer), meets the warm air near the substellar point.

At 400 hPa, it is notable that there are two main regions of convergence just beyond each terminator, rather than a single region at the antistellar point, suggesting that the overturning circulation is not sufficiently strong to reach all the way to the antistellar point in the mid-troposphere for the specific terrestrial planet considered here. Higher up in the atmosphere at 250 hPa, divergent flow extends further and convergence primarily occurs near the antistellar point. The divergent circulation is slightly stronger going east from the substellar point, compared to its westward part, likely due to the presence of the

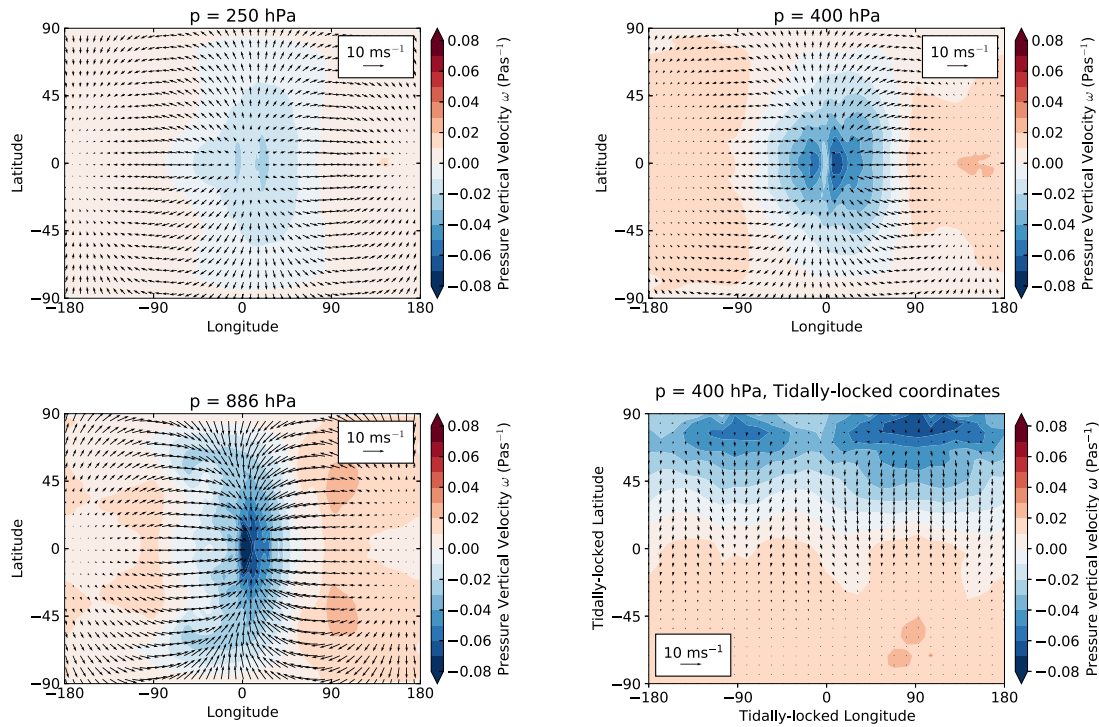


Figure 5.4: The spatial structure of the divergent circulation. Each panel shows vertical velocity (shaded) and horizontal velocity (quivers). First to third panels: Latitude-longitude slices at 250 hPa (top left), 400 hPa (top right), and 886 hPa (bottom left), showing the day-night overturning cell. Fourth panel (bottom right): the 400 hPa slice shown in tidally locked coordinates (see text; Koll and Abbot, 2015).

zonal-mean eastward equatorial jet; notably, this effect is more pronounced at 400 hPa where the jet is stronger.

5.3.2 Overturning circulation in tidally locked coordinates

Past interpretation of the overturning circulation on tidally locked planets has described the circulation as a superposition of various *different* overturning circulations. Ascent on the day side paired with meridional horizontal motion and descent near the poles has been described as a sort of Hadley circulation (Merlis and Schneider, 2010; Edson et al., 2011; Heng et al., 2011; Carone et al., 2014; Haqq-Misra et al., 2018), and overturning circulation in the zonal direction has been described as a Walker circulation (Carone et al., 2014; Haqq-Misra et al., 2018), by analogy with their namesakes in the Earth's tropical atmosphere. On the night side, meridional motion has been described as an 'anti-Hadley circulation' (Heng et al., 2011; Haqq-Misra et al., 2018). The analysis of the divergent circulation presented here suggests that this interpretation of overturning in a tidally locked atmosphere is overly complicated, and misleading; the overturning circulation is not three different circulations, but instead a single thermally-direct and (roughly) isotropic circulation from the day side to the night side.

Given the roughly isotropic nature of the overturning circulation about the substellar point, it is instructive to visualise it within a coordinate system that exploits symmetry about the axis connecting the substellar and antistellar points. To do this, the GCM output is transformed from latitude–longitude coordinates to the ‘tidally locked coordinates’ of Koll and Abbot (2015). In tidally locked coordinates, latitude ϑ' is measured as the angle from the terminator, and longitude λ' is the angle about the axis connecting the substellar and antistellar points. Mathematically, they are defined

$$\vartheta' = \arcsin(\cos \vartheta \cos \lambda), \quad (5.4)$$

$$\lambda' = \arctan\left(\frac{\sin \lambda}{\tan \vartheta}\right). \quad (5.5)$$

Lines at constant tidally locked longitude (i.e., tidally locked meridians) are arcs that connect the substellar and antistellar points. Tidally locked velocities u' and v' are then defined by

$$u' \equiv a \cos \vartheta' \frac{D\lambda'}{Dt} = \cos \vartheta' \left(\frac{\partial \lambda'}{\partial \lambda} \frac{u}{\cos \vartheta} + \frac{\partial \lambda'}{\partial \vartheta} v \right), \quad (5.6)$$

and

$$v' \equiv a \frac{D\vartheta'}{Dt} = \frac{\partial \vartheta'}{\partial \lambda} \frac{u}{\cos \vartheta} + \frac{\partial \vartheta'}{\partial \vartheta} v, \quad (5.7)$$

in the λ' and ϑ' directions respectively. The point $(\vartheta', \lambda') = (0, 0)$ is chosen to coincide with the north pole. Note that the Helmholtz decomposition is invariant under the coordinate system rotation from latitude–longitude to tidally locked coordinates. This follows from the invariance of the ∇_p^2 operator under rotation, and the fact that as ζ and δ are scalar fields, they are also invariant under rotation by definition.

The bottom panel of Figure 5.4 shows the divergent velocity $\mathbf{u}_d$ in the tidally locked coordinate system at 400 hPa. Comparison with the upper right-hand panel in Figure 5.4, which shows $\mathbf{u}_d$ in regular latitude–longitude coordinates, illustrates how moving to tidally locked coordinates transforms the data. The substellar point, which was a point on the equator in the latitude–longitude coordinate, is now a ‘pole’ in the new coordinate system. Consequently, all substellar ascending vertical motion is now at $\vartheta' = 90^\circ$. The ‘equator’ in the tidally locked coordinate system is the terminator. ‘North’ of the terminator (i.e., on the day side), there is ascending vertical motion, and ‘south’ of the terminator (i.e., on the night side), there is descending vertical motion. The dominance of substellar to antistellar flow is clear in this coordinate system, taking the form of ‘southward’ flow along lines of constant λ' . The isotropic nature of the divergent circulation appears in the form of approximate zonal symmetry in λ' .

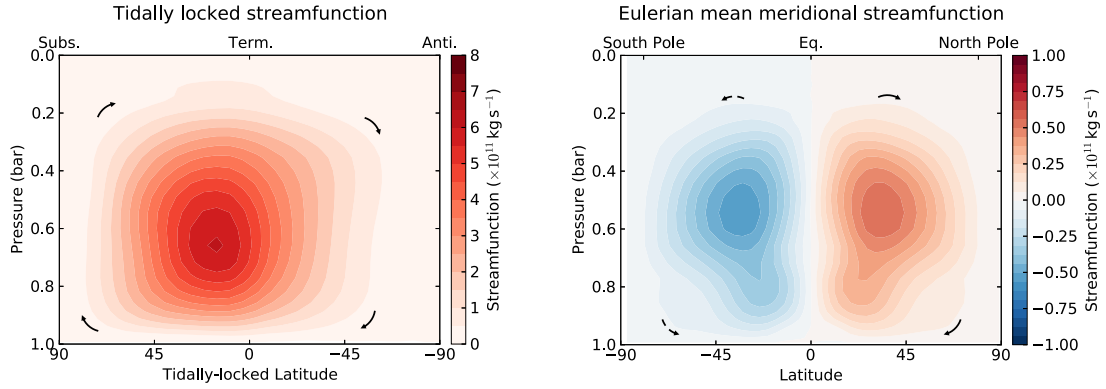


Figure 5.5: Mass streamfunctions showing the structure of overturning circulation on a tidally locked planet. Left panel: Tidally locked streamfunction Ψ' defined by (5.9). Right panel: Regular Eulerian mean meridional streamfunction Ψ . Mass flux follows lines of constant Ψ' or Ψ . The direction of overturning is indicated by arrows and the sign of the streamfunction. Positive values indicate clockwise motion, negative values indicate anti-clockwise motion. Note that the colour scale differs in magnitude between the two panels.

By analogy with the continuity equation in regular λ – ϑ coordinates, the continuity equation in tidally locked coordinates is

$$\frac{1}{a \cos \vartheta'} \frac{\partial u'}{\partial \lambda'} + \frac{1}{a \cos \vartheta'} \frac{\partial}{\partial \vartheta'} (v' \cos \vartheta') + \frac{\partial \omega}{\partial p} = 0. \quad (5.8)$$

Symmetry in the λ' direction motivates integrating (5.8) over λ' , which enables the definition of a tidally locked meridional mass streamfunction

$$\Psi' = \frac{2\pi a \cos \vartheta'}{g} \int_0^p [v']_{\lambda'} dp, \quad (5.9)$$

where $[\cdot]_{\lambda'}$ indicates an average over tidally locked longitude. Ψ' is the total northward (in the ϑ' direction, i.e. towards the antistellar point) mass flux above a particular pressure p , and mass flux in the ϑ' – p plane follows lines of constant Ψ' .

The tidally locked streamfunction Ψ' is shown Figure 5.5. The overturning circulation viewed in the ϑ' – p plane takes the form of a single overturning cell from the day side to the night side, and reveals each of the features of the circulation inferred from the λ – ϑ slices of $\mathbf{u}_d$ and ω which were shown in Figure 5.4. Air rises near the substellar point ($\vartheta' = 90^\circ$), before moving towards the night side aloft. Descent begins as air passes across the terminator ($\vartheta' = 0^\circ$), and eventually air enters a near-surface return flow back towards the substellar point.

Visualising the overturning circulation in terms of Ψ' clearly captures the nature of the circulation better than the regular meridional mass streamfunction (Equation 2.22; also shown in Figure 5.5). The circulation strength indicated by Ψ is far weaker than

that indicated by Ψ' due to cancellation between day side and night side meridional mass fluxes. The regular streamfunction Ψ also captures none of the overturning in the zonal direction, or cross-polar meridional motion. Day–night differences in the overturning circulation have been described in previous works where Ψ is averaged over a limited longitude range (i.e., over *just* the day side or night side; e.g., Heng et al., 2011; Haqq-Misra et al., 2018). However, this approach is difficult to motivate, as Ψ is derived by integrating the continuity equation over *all* longitudes.

5.3.3 Day–night heat transport

The instellation gradient between the day and night sides of tidally locked planets naturally produces a day–night heat transport by the atmospheric circulation. This heat transport is a key feature of these atmospheres, due to its effect on climate stability (preventing night side temperatures from becoming so cold that the atmosphere condenses; Heng and Kopparla, 2012; Wordsworth, 2015; Turbet et al., 2018) and observations of thermal phase curves (Demory et al., 2016; Hammond and Pierrehumbert, 2017).

In the absence of a condensable substance, atmospheric circulation transports heat by transporting dry static energy (DSE), defined as

$$s = c_p T + \phi, \quad (5.10)$$

where T is temperature and ϕ is the geopotential. Local heating is due to the divergence of the dry static energy flux, and, column integrated, the local energy budget is

$$\langle \nabla_p \cdot (s\mathbf{u}) \rangle + F_S - F_{\text{OLR}} = 0, \quad (5.11)$$

where $\langle \cdot \rangle$ denotes a vertical (mass-weighted) integration, F_S is the net radiative flux entering the atmosphere due to instellation, and F_{OLR} is the outgoing longwave radiation. The meridional average of (5.11) is shown in Figure 5.6, decomposed into contributions from the rotational and divergent circulations. This budget does not exactly sum to zero, as there is a small residual introduced by re-gridding the model output data on the cubed-sphere to a latitude–longitude grid.

The mean day side instellation is 315 W m^{-2} , of which 70% is reradiated from the day side and 26% is transported to the night side. Of the transport to the night side, 96% is due to the divergent circulation, 6% due to the zonal-mean rotational circulation, and –2% is by the eddy rotational circulation (i.e., heat is returned from the night side to the day side). This is a surprising result, as Figure 5.2 shows that the divergent circulation is significantly weaker than the rotational circulation in terms of absolute velocity. The

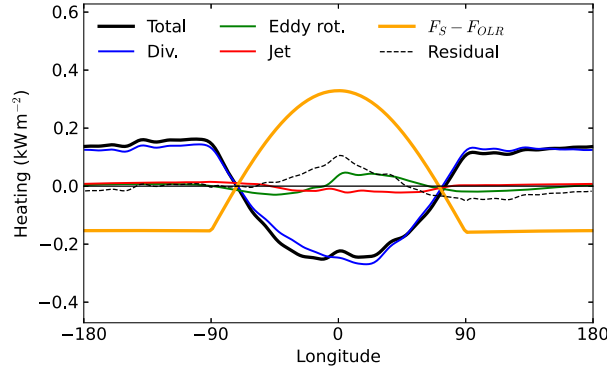


Figure 5.6: The dry static energy budget for the Exo-FMS GCM simulation. The DSE is flux divergence is decomposed into contributions from the zonal-mean rotational $\bar{\mathbf{u}}_r$ (red), eddy rotational $\mathbf{u}'_r$ (green), and divergent $\mathbf{u}_d$ (blue), components of the circulation. The day–night heat transport is dominated by the divergent circulation.

divergent heat transport is roughly symmetric in the eastward and westward directions, reflecting the isotropic nature of the divergent circulation.

The dominance of the divergent circulation can be explained by the fact that the atmosphere is in a *weak temperature gradient* (WTG) regime (Mills and Abbot, 2013; Pierrehumbert and Hammond, 2019). In this regime, which is typical of planets that are slowly rotating, the zonal momentum equation is dominated by a balance between the local temperature gradient and a term that depends on the square of the local velocity (rather than the Coriolis force; Hammond et al., 2020). This means that horizontal temperature gradients are relatively small. Expanding the local heating due to the divergence of the DSE flux as

$$\nabla_p \cdot (s\mathbf{u}) = s\nabla_p \cdot \mathbf{u} + \mathbf{u} \cdot \nabla_p s \quad (5.12)$$

shows that the contribution of the rotational velocity $\mathbf{u}_r$ will only be through the second term on the right-hand side, as $\nabla_p \cdot \mathbf{u}_r = 0$ by definition. However, the horizontal gradient of the DSE $\nabla_p s$ is small in the WTG regime, so the heating due to the $\mathbf{u} \cdot \nabla_p s$ terms is small, meaning that the total heating due to the rotational circulation is small. On the other hand, the divergent velocity $\mathbf{u}_d$ has non-zero divergence so contributes to the local heating via both terms on the right-hand side of (5.12). This means that the divergent circulation plays a dominant role in the energy budget, as $s\nabla_p \cdot \mathbf{u}_d$ is much greater than $\mathbf{u}_r \cdot \nabla_p s$ (or $\mathbf{u}_d \cdot \nabla_p s$). For the hot Jupiter simulation presented in Hammond and Lewis (2021), most of the day–night heat transport (78%) is also due to the divergent circulation, although not to the same extent as in the terrestrial simulation shown here; this is because the hot Jupiter is less firmly in the WTG regime.

5.4 Discussion

5.4.1 Summary

In this chapter, the atmospheric circulation on tidally locked planets has been analysed by applying the Helmholtz decomposition (5.1) to output from a GCM simulation of a terrestrial, tidally locked planet.

This work has three main conclusions. First, the Helmholtz decomposition isolated the three main components that make up the atmospheric circulation in the GCM simulation – the superrotating jet, stationary waves, and overturning circulation (see Figure 2.11 in Chapter 2). The rotational velocities contain the jet (zonal-mean rotational) and wavenumber-1 stationary wave response (eddy rotational), and the divergent velocities contains the overturning circulation. The stationary wave response corresponds to the shallow water Rossby waves analysed by previous studies (Showman and Polvani, 2011; Tsai et al., 2014; Hammond and Pierrehumbert, 2018). The divergent circulation is weaker than the rotational circulation, but not negligible, with velocities only a single order of magnitude less than those associated with the rotational circulation. The overturning circulation is similar in some respects to a combination of the Hadley and Walker circulations on Earth. However, its isotropy means that there is little distinction between overturning in the zonal or meridional directions. With this in mind, it is suggested that it is not useful, and potentially misleading, to define distinct Hadley and Walker circulations for tidally locked planets.

Second, the overturning circulation is represented naturally by a mass streamfunction in tidally locked coordinates (Koll and Abbot, 2015), as it is dominated by isotropic motion away from the substellar point. This reveals that the overturning circulation is dominated by a single cell which features air rising near the substellar point and descending on the night side. By contrast, the traditional zonal mean meridional mass streamfunction defined in latitude–longitude coordinates fails to capture most of the overturning and contains no information about cross-polar flow or overturning in the zonal direction.

Third, for the terrestrial simulation analysed in this chapter (and also the hot Jupiter in Hammond and Lewis, 2021), the divergent circulation dominates day–night heat transport even when its absolute velocity is weaker than the other components. This is due to the circulation occupying a weak temperature gradient regime, typical of tidally locked planets (Pierrehumbert and Hammond, 2019), which inhibits advection of heat by the superrotating jet (or stationary waves). The importance of the divergent circulation

in the day–night heat budget for both the terrestrial case and the hot Jupiter illustrates that each of the circulation components are important to the whole circulation, even in regimes where one component has much higher velocities than the others.

5.4.2 Future work

A few opportunities for future research motivated by this study are now outlined. First, in this chapter we have considered a single *exemplar* benchmark simulation of a terrestrial tidally locked planet. While the terrestrial results presented here are given some generality by their similarity with those obtained for the hot Jupiter presented in Hammond and Lewis (2021), future work should explore this further by applying the Helmholtz decomposition to simulations of different types of exoplanet spanning a wide parameter space. In addition, simulations of the circulation on *both* terrestrial planets and hot Jupiters have been shown to be sensitive to choices made in model configuration. For example, the circulation of a hot Jupiter has been shown to be sensitive to imposed drag and radiative timescales (Komacek and Showman, 2016) and the lower boundary pressure (Carone et al., 2020). On terrestrial tidally locked planets, the presence of absence of a land surface (as opposed to a homogeneous ocean surface) has been shown to affect the atmospheric circulation (Lewis et al., 2018). Additionally, on habitable planets, water vapour and clouds will likely influence the atmospheric dynamics and heat transport, through both their radiative effects and the effect of latent heating (Merlis and Schneider, 2010). The simulation analysed in this chapter is dry (i.e., there is no water vapour or cloud condensate in the model atmosphere), and examining the effect of moisture on the Helmholtz decomposition of the circulation and heat transport is an obvious follow-up to this work.

The fact that the majority of day–night heat transport is due to the divergent circulation, for both the terrestrial planet and the hot Jupiter, is an important result, as some previous work studying day–night heat transport on tidally locked planets has focused entirely on the role of the jet and stationary waves (Perez-Becker and Showman, 2013; Hammond and Pierrehumbert, 2017; Mendonça, 2020). The red line in Figure 5.6 (minus the known instellation) is related to the broadband thermal phase curve that could be observed for the planet (Cowan and Agol, 2008; Koll and Abbot, 2015), and consequently the energy budget can be constrained by observations. Phase curves observed from hot Jupiters and high-temperature rocky planets (Demory et al., 2016; Kreidberg et al., 2019) are already available and will increase in quality in the coming years (Bean et al., 2018). Similar observations should become available for terrestrial tidally locked planets using

next-generation observatories such as the *James Webb Space Telescope* (Deming et al., 2009). Separating the contributions of the rotational and divergent circulations to heat transport on tidally locked planets over a broad range of parameters could be crucial to interpreting phase curve observations. Temperature and atmospheric circulation are also related *diagnostically* through balances of terms in the momentum equations. In Chapter 6, I will show that it is possible to partition the geopotential, and thus temperature, into components associated with the rotational and divergent circulations using diagnostic balance relations (Trenberth and Chen, 1988; Holton, 2004, Chapter 11). These physically-based temperature components could be used by future work as basis functions for fitting thermal phase curves, which have previously been fitted to a sum of spherical harmonics (Rauscher et al., 2018) that do not correspond to physical components of the circulation.

In this chapter I have shown how the Helmholtz decomposition can be used to divide the atmospheric circulation of tidally locked planets into distinct, physically meaningful components, just as is done by dividing the circulation of non-tidally locked planets into contributions from zonal jets, travelling waves, and meridional overturning. On a tidally locked planet, the components isolated by the Helmholtz decomposition contain the superrotating jet, stationary Rossby waves, and divergent (overturning) circulation. Understanding each of these components is important, even in circumstances where one or two components dominate the total circulation. It is hoped that the method and results presented in this chapter will form a basis for future work on the behaviour of each of these components, and their role in the global circulation.

Temperature Structure Associated with Circulation Components on Tidally Locked Planets

The contents of this chapter are a modified version of N. T. Lewis and M. Hammond (2022): Temperature Structures Associated with Different Components of Atmospheric Circulation on Tidally Locked Exoplanets, submitted to *The Astrophysical Journal* (pending first referee report).

Author contributions: NTL proposed the research. NTL designed and ran the numerical experiments, conducted the analysis, and wrote the paper. MH proof read the paper, and advised on the interpretation and discussion of results.

6.1 Introduction

Tidally locked exoplanets can be characterised by measuring their thermal emission (Burrows, 2014; Crossfield, 2015). Time-resolved observations of thermal emission are known as thermal phase curves, and show the disk-integrated infrared flux emitted from a planet as it orbits its host star (Cowan and Agol, 2008; Demory et al., 2016), revealing the longitudinal structure of its brightness temperature. Eclipse maps show the two-dimensional structure of brightness temperature on a planet’s day side (Majeau et al., 2012).

Atmospheric temperatures and atmospheric circulation are closely related. This is not only due to the ability of an atmosphere to transport heat (Koll and Abbot, 2016), and thus maintain a temperature structure against differential heating and cooling (Showman et al., 2013), but also through the pressure gradient forces in the horizontal momentum equations. These pressure gradient forces mean that any distribution of winds must be balanced by a particular temperature distribution. The most familiar example of such a balance is the geostrophic thermal wind relation (Holton, 2004, Chapter 3)

$$f \frac{\partial u}{\partial \ln p} = \frac{R}{a} \frac{\partial T}{\partial \vartheta}; \quad f \frac{\partial v}{\partial \ln p} = -\frac{R}{a \cos \vartheta} \frac{\partial T}{\partial \lambda}; \quad (6.1)$$

which can be derived by assuming the flow is in hydrostatic and geostrophic balance. The thermal wind relation states that vertical variation in the horizontal wind $\mathbf{u} = (u, v)$ must be accompanied by horizontal variation in the temperature T . The utility of geostrophic balance itself is limited to scenarios where the Coriolis term is dominant in the momentum equations, but the requirement that accelerations in the horizontal momentum

equations are balanced by horizontal pressure contrasts is a generic one. In this chapter, I will show how an understanding of the balances between different components of the horizontal velocity field and atmospheric temperature aids interpretation of observations of thermal emission from extrasolar planets.

Chapter 5 showed that each of the circulation features on tidally locked planets – the superrotating jet, stationary Rossby waves, and divergent circulation – can be separated out from one another using a Helmholtz decomposition, which divides the horizontal velocity into a rotational (divergence free) and divergent (vorticity free) component. The zonal-mean zonal jet and stationary Rossby waves are contained within the zonal-mean and eddy components of $\mathbf{u}_r = \bar{\mathbf{u}}_r + \mathbf{u}'_r$ respectively, and the divergent circulation (comprising, e.g., thermally-direct overturning; cf. Chapter 5, and Kelvin waves; see Appendix 6.A) is contained within $\mathbf{u}_d$.

The aim of this chapter is to use the Helmholtz decomposition to understand how different components of atmospheric circulation contribute to the distribution of atmospheric temperature, and by extension thermal emission, on tidally locked planets. This is achieved by decomposing the geopotential ϕ into a component that is balanced by purely rotational winds (‘the rotational geopotential’), and a component that is due to divergent winds and rotational–divergent interactions (‘the divergent geopotential’) (Section 6.3; see also Trenberth and Chen, 1988). The hydrostatic relation

$$\frac{\partial \phi}{\partial \ln p} = -RT \quad (2.2, \text{revisited})$$

is then used to split the global temperature field into rotational and divergent components (Section 6.4). Finally, the contribution of each temperature component to simulated thermal phase curves is evaluated by linearly expanding $F = \sigma T^4$ around the horizontally averaged temperature T_0 (Section 6.5). The chapter concludes with a summary of the key results of this work (Section 6.6).

In each section, the diagnostics are applied to idealised general circulation model (GCM) simulations of atmospheric circulation on dry, temperate terrestrial tidally locked planets. The model is constructed using the Isca modeling framework (Vallis et al., 2018), and is described in detail in Section 6.2. Results are presented from four simulations with different rotation periods, $P = 4, 8, 16$, and ∞ days (i.e., zero rotation). The zero-rotator is included to illustrate the effect of rotation in the other simulations.

In this chapter, results are only presented from simulations of terrestrial planets. However, there is no reason why the techniques described herein cannot be applied to

gaseous planets (e.g., hot Jupiters or mini Neptunes) and this is something I aim to do in the near future.

6.2 Model description

The GCM simulations analysed in this chapter were configured and run using *Isca*, which is a framework for building idealised general circulation models of varying complexity (Vallis et al., 2018; also used in Chapter 3). In the present study a dry GCM configuration is used, i.e., water vapour and moist processes are not included in the model atmosphere. The model is configured to simulate the atmospheric circulation of terrestrial planets with a lower boundary modeled as a ‘slab ocean’. All planetary parameters aside from the rotation rate and properties of the stellar irradiation (e.g., radius, surface pressure, gravitational acceleration) are set to be those of the Earth. Four rotation periods are used, $P = 4, 8, 16$, and ∞ days, and the distribution of stellar irradiation is set to be that of a tidally locked planet (defined below). The main components of the model are described below.

6.2.1 Dynamical core

Isca uses a pseudospectral dynamical core to integrate the primitive equations forwards in time (described in more detail in Chapter 3). The vertical coordinate used is a terrain-following ‘sigma’ coordinate, defined $\sigma = p/p_s$, where p is pressure and p_s is surface pressure. The simulations presented here use a T42 triangular truncation, which corresponds to approximately 2.8° latitude–longitude resolution, and 25 unevenly spaced levels in the vertical, distributed according to $\sigma = \exp[-5(0.05\tilde{z} + 0.95\tilde{z}^3)]$ where $\tilde{z}$ is equally spaced in the unit interval. A ∇^8 hyperviscosity term is applied to the momentum and thermodynamic equations operating on a timescale of 0.1 days at the grid scale. Prior to analysis, the model output is interpolated from the model’s sigma coordinate to a pure pressure coordinate.

6.2.2 Radiative transfer

The model uses a semi-grey radiative transfer code to model radiative heating and cooling. Radiative fluxes are split into two wavelength bands: one covering longwave (thermal emission) wavelengths, and the other covering shortwave (stellar heating) wavelengths.

In the shortwave band, the atmosphere is assumed to be transparent, and all incoming radiation is absorbed at the surface (i.e., the surface albedo has been folded into the

solar constant). The instellation at the top-of-atmosphere is imposed with the following distribution, appropriate for a tidally locked planet:

$$S = \begin{cases} S_0 \cos \vartheta \cos (\lambda - \lambda_0) & \text{on the day side,} \\ 0 & \text{on the night side,} \end{cases} \quad (6.2)$$

where $\lambda_0 = 0^\circ$ is the substellar longitude, and $S_0 = 1000 \text{ W m}^{-2}$.

In the longwave band, upward and downward radiative fluxes are computed according to

$$\frac{dF^\uparrow}{d\tau} = F^\uparrow - \sigma T^4 \quad (6.3)$$

$$\frac{dF^\downarrow}{d\tau} = \sigma T^4 - F^\downarrow \quad (6.4)$$

where $\tau = \tau_0(p/p_0)$ is the longwave optical depth and $\tau_0 = 1$ is the optical depth at $p_0 = p_s$. The longwave radiative heating in the model is then

$$\frac{\partial T}{\partial t} = \dots + \frac{g}{c_p} \frac{\partial (F^\uparrow - F^\downarrow)}{\partial p}. \quad (6.5)$$

The boundary conditions for the longwave fluxes are $F^\downarrow(p=0) = 0$ and $F^\uparrow(p=p_s) = \sigma T_s^4$ where T_s is the surface temperature.

6.2.3 Surface energy budget, boundary layer sensible heat transport, and convection

The surface is modeled as a static slab ocean that can exchange energy with the atmosphere in the vertical but cannot transport heat horizontally. The surface temperature evolves according to

$$C \frac{\partial T_s}{\partial t} = S + F_{\text{sfc}}^\downarrow - \sigma T_s^4 - \mathcal{H} \quad (6.6)$$

where C is a specified heat capacity that corresponds to an ocean mixed-layer depth of 2.5 m, $F_{\text{sfc}}^\downarrow$ is the downward flux of longwave radiation incident on the surface, and $\mathcal{H}$ is the surface sensible heat flux. $\mathcal{H}$ is computed according to

$$\mathcal{H} = \rho_a c_p \mathcal{C} |\mathbf{u}_a| (T_s - T_a), \quad (6.7)$$

where ρ is the density, the subscript ‘a’ indicates quantities that are evaluated at the lowest model level, and $\mathcal{C} = 0.001$ is a dimensionless bulk transfer coefficient.

Turbulent vertical transport of heat within the boundary layer is parametrised following Thatcher and Jablonowski (2016) as

$$\mathcal{D}_T = \overline{w'\theta'} = -\mathcal{K} \frac{\partial \theta}{\partial z}, \quad (6.8)$$

where z is height, $w = Dz/Dt$ is the vertical velocity, and θ is the potential temperature. The diffusion coefficient $\mathcal{K}$ is calculated according to

$$\mathcal{K} = \begin{cases} \mathcal{C} |\mathbf{u}_a| z_a & \text{for } p > p_{\text{pbl}}, \\ \mathcal{C} |\mathbf{u}_a| z_a \exp \left[- \left(\frac{p_{\text{pbl}} - p}{p_{\text{strat}}} \right)^2 \right] & \text{for } p \leq p_{\text{pbl}}. \end{cases} \quad (6.9)$$

$p_{\text{pbl}} = 850$ hPa is the top of the boundary layer, and $p_{\text{strat}} = 100$ hPa controls the rate of decrease of boundary layer diffusion with height. The contribution to the model temperature tendency from the boundary layer diffusion is then

$$\frac{\partial T}{\partial t} = \dots + \frac{g}{c_p} \frac{\partial \mathcal{D}_T}{\partial p}. \quad (6.10)$$

Finally, the model includes a dry convection scheme, which instantaneously restores the atmospheric temperature structure to the dry adiabat whenever it is convectively unstable.

6.2.4 Surface friction and sponge layer

Surface friction is parametrised following Held and Suarez (1994) as a linear Rayleigh drag

$$\frac{\partial \mathbf{u}}{\partial t} = \dots - k_f \mathbf{u} \quad (6.11)$$

where k_f is defined as

$$k_f = k_{f,s} \max \left(0, \frac{\sigma - \sigma_b}{1 - \sigma_b} \right). \quad (6.12)$$

$k_{f,s} = 1 \text{ day}^{-1}$ is the drag timescale at the surface, and the top of the frictional boundary layer is located at $\sigma_b = 0.7$. A linear drag is also applied in the upper atmosphere above $p_{\text{sponge}} = 50$ hPa following Polvani and Kushner (2002), which damps the horizontal winds on a timescale of 0.5 days^{-1} . This acts as a sponge layer, and is included to suppress wave reflection at the top of the model atmosphere.

6.3 Rotational and divergent geopotential components

6.3.1 Definition of rotational geopotential

The horizontal momentum equations (Equation 2.1 in Chapter 2) can be written in the form (Holton, 2004, Chapter 4)

$$\frac{\partial \mathbf{u}}{\partial t} + \omega \frac{\partial \mathbf{u}}{\partial p} + (f + \zeta) \mathbf{k} \times \mathbf{u} = -\nabla_p \left(\phi + \frac{1}{2} |\mathbf{u}|^2 \right) - k_f \mathbf{u}, \quad (6.13)$$

where $|\mathbf{u}|^2 = u^2 + v^2$, and k_f is a linear drag coefficient that is non-zero close to the surface. Equations for the vorticity $\zeta = \mathbf{k} \cdot \nabla_p \times \mathbf{u}$ and divergence $\delta = \nabla_p \cdot \mathbf{u}$ can then be obtained by taking $\mathbf{k} \cdot \nabla_p \times$ (6.13) and $\nabla_p \cdot$ (6.13), respectively,

$$\frac{D(f + \zeta)}{Dt} + (f + \zeta) \delta + \mathbf{k} \cdot \left(\frac{\partial \mathbf{u}}{\partial p} \times \nabla_p \omega \right) = -k_f \zeta, \quad (6.14)$$

$$\frac{\partial \delta}{\partial t} + \nabla_p \cdot \left(\omega \frac{\partial \mathbf{u}}{\partial p} \right) + \nabla_p \cdot [(f + \zeta) \mathbf{k} \times \mathbf{u}] = -\nabla_p^2 \left(\phi + \frac{|\mathbf{u}|^2}{2} \right) - k_f \delta. \quad (6.15)$$

The geopotential ϕ does not appear in the vorticity equation (6.14), and thus all of the dynamics that determines ϕ is described by the divergence equation (6.15).

In order to separate the geopotential into components associated with the rotational and divergent winds, $\mathbf{u} = \mathbf{u}_r + \mathbf{u}_d$ is substituted into Equation (6.15), which yields

$$\begin{aligned} \frac{\partial \delta}{\partial t} + \nabla_p \cdot \left[\omega \frac{\partial (\mathbf{u}_r + \mathbf{u}_d)}{\partial p} \right] + u_d \beta + J(\chi, \nabla_p^2 \psi) - \nabla_p \cdot [(f + \nabla_p^2 \psi) \nabla_p \psi] = \\ -\nabla_p^2 \left[\phi + \frac{|\nabla_p \psi|^2}{2} + \frac{|\nabla_p \chi|^2}{2} + J(\psi, \chi) \right] - k_f \delta \end{aligned} \quad (6.16)$$

where ψ is a streamfunction and χ is the velocity potential function, as in Chapter 5. $J(\psi, \chi) = \mathbf{k} \cdot (\nabla_p \psi \times \nabla_p \chi)$ is the Jacobian, and $\beta = a^{-1} \partial f / \partial \vartheta$. A derivation of (6.16) is given in Appendix 6.B.

To decompose the geopotential into components associated with the rotational and divergent circulation, it can be expanded as $\phi = \phi_0 + \phi_r + \phi_d$, where $\phi_0(t, p)$ is the horizontal-mean geopotential, which does not contribute to (6.16) as $\nabla_p^2 \phi_0 = 0$. Following Trenberth and Chen (1988), the ‘rotational geopotential’ is *defined* to be the component of ϕ which is balanced by purely rotational circulation (with no vertical motion). When applied to (6.16), this definition yields the following equation for ϕ_r

$$\nabla_p^2 \phi_r \equiv \nabla_p \cdot [(f + \nabla_p^2 \psi) \nabla_p \psi] - \nabla_p^2 \left(\frac{|\nabla_p \psi|^2}{2} \right). \quad (6.17)$$

This equation is called the *non-linear balance equation* (Lorenz, 1960) and takes the place of the divergence equation for purely non-divergent horizontal circulation (see, e.g., Holton, 2004, Chapter 11). For stationary circularly-symmetric flow, it is equivalent to the gradient wind approximation. If the streamfunction ψ (i.e., the rotational wind distribution) is known, then (6.17) can be inverted to obtain ϕ_r .

6.3.2 Divergent geopotential

Given the rotational geopotential as defined by (6.17), the ‘divergent geopotential’ is then defined following Trenberth and Chen (1988) to be that obtained as a residual from

$$\phi_d \equiv \phi - (\phi_0 + \phi_r). \quad (6.18)$$

The remaining terms in (6.16) which determine ϕ_d involve terms that are purely divergent, and terms due to interaction between the rotational and divergent winds.

6.3.3 A note on the meaning of balances in the divergence equation

In the next subsection, the definitions (6.17) and (6.18) are used to partition the geopotential in the GCM simulations into rotational and divergent components, and the terms in (6.16) and (6.17) that contribute the most to the decomposed height fields are discussed. In anticipation of this, it is important to note that a large term in, e.g., Equation (6.17), which defines ϕ_r , does not *necessarily* correspond to a dominant balance in the momentum equation for $\mathbf{u}_r$. This is because an additional term is introduced if (6.17) is integrated to obtain an equation for $\mathbf{u}_r$; the same applies if (6.16) (minus the solely rotational terms) is integrated to obtain an equation for $\mathbf{u}_d$. This is illustrated for the linearised shallow water equations on the sphere in Appendix 6.A. In this appendix I also show that the momentum equations for $\mathbf{u}_r$ and $\mathbf{u}_d$ can be used to understand the dynamical balance that supports the existence a linear Kelvin-like wave on the sphere, in the absence of drag.

6.3.4 Application to simulations

Figure 6.1 shows the horizontal wind $\mathbf{u} = (u, v)$ and geopotential height z in the mid-troposphere ($p = 480$ hPa) for each GCM simulation. The full circulation is shown in the top row, and subsequent rows show the rotational circulation – split into contributions from the zonal-mean (jet) and eddies (stationary waves) – and the divergent circulation. Figure 6.2 shows the vertical structure of the full circulation, eddy rotational circulation, and divergent circulation averaged between $\pm 40^\circ$ latitude. In the $P = 4$ days simulation, the circulation is mostly contained within the rotational component, whereas in the $P = 8$ and $P = 16$ days simulations it is more evenly divided into rotational and divergent contributions. Finally, the circulation in the $P = \infty$ days simulation is purely divergent. In this subsection, the important features of the circulation components shown in Figure 6.1 and Figure 6.2 are described. Appendix 6.C is referred to throughout; it contains additional analysis of the contributions of individual terms in the expanded divergence equation (6.16) to each of the rotational and divergent height fields.

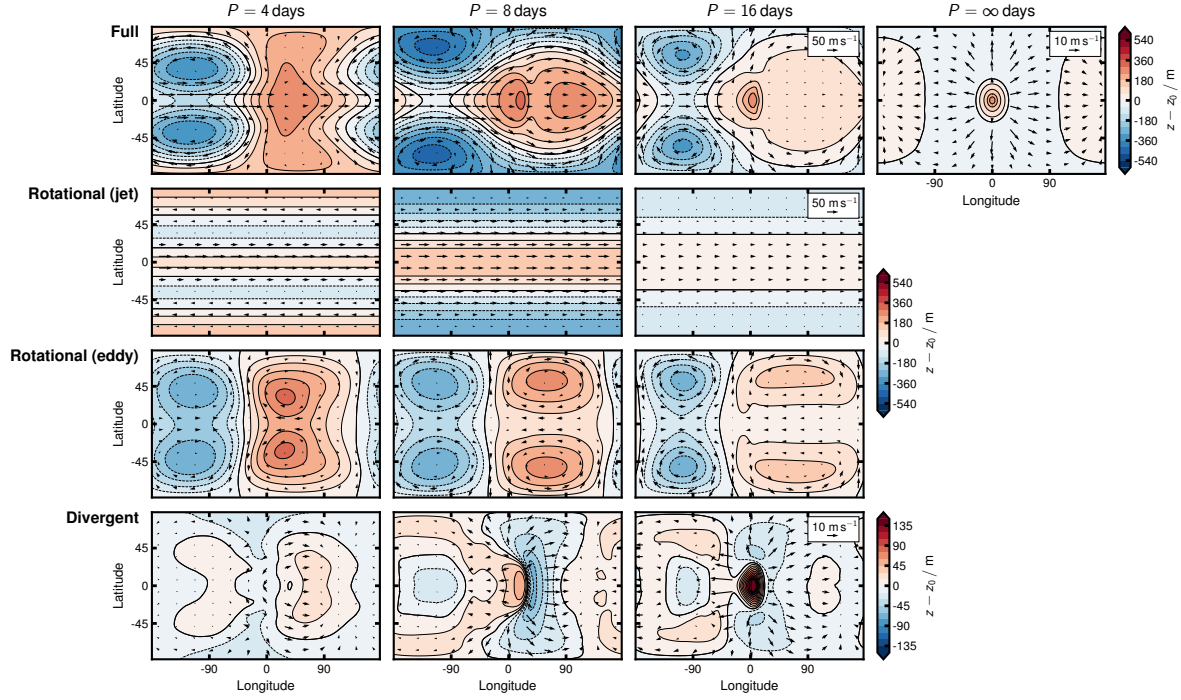


Figure 6.1: Geopotential height (contours) and horizontal velocity (quivers) at $p = 480$ hPa for each GCM simulation. For each simulation, full fields are shown in the top row. The next three rows show the zonal-mean rotational circulation, eddy rotational circulation, and divergent circulation. Note that the circulation in the $P = \infty$ days simulation has no rotational component.

The zonal-mean rotational height field $\bar{\phi}_r$ in each of the $P = 4, 8$, and 16 days simulations is in approximate geostrophic balance with the zonal-mean rotational wind $\bar{u}_r$:

$$f\bar{u}_r \approx -\frac{1}{a} \frac{\partial \bar{\phi}_r}{\partial \vartheta}, \quad (6.19)$$

which gives rise to a local height maximum centred on the equator, associated with the superrotating equatorial jet (Hammond and Pierrehumbert, 2018). In the $P = 4$ days simulation there are additional retrograde jets poleward of $\pm 50^\circ$ in each hemisphere which lead to a secondary maximum in the zonal-mean rotational height field at each pole.

The eddy-rotational circulation in each simulation has a horizontal wind and geopotential structure characteristic of a stationary equatorial Rossby wave. The longitude of the eddy rotational height maximum is located eastwards of the substellar point due to a phase-shift by the zonal-mean zonal wind (Tsai et al., 2014). For the rapidly rotating $P = 4$ days case, the structure and phase offset of the Rossby waves is very similar to that obtained in a linearised shallow water model with a prescribed background jet (e.g., Figure 10 in Hammond and Pierrehumbert, 2018). As the rotation period is reduced, the

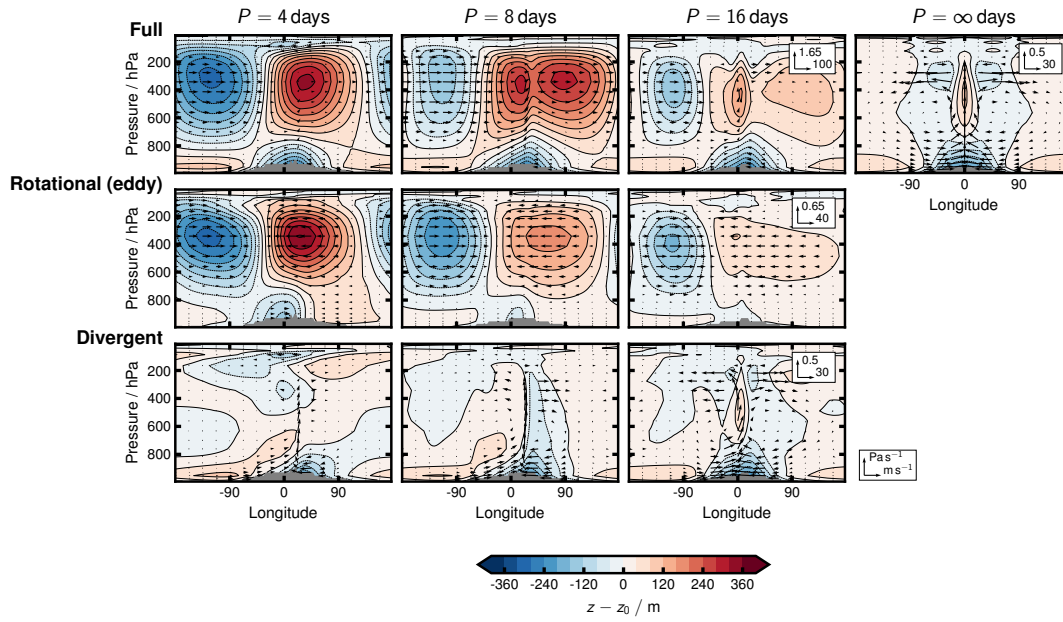


Figure 6.2: Geopotential height (contours) and velocity (u , ω ; quivers) in the longitude–pressure plane, averaged between $\pm 40^\circ$ latitude, shown for each GCM simulation. The top row shows the full circulation. Contributions to the full circulation from the eddy-rotational and divergent components are shown in the next two rows. Data is averaged over the tropics only in order to show the baroclinic structure of the eddy rotational wind.

Rossby lobes with a positive geopotential anomaly become elongated with respect to those with a negative geopotential anomaly. Similar phenomenology is also exhibited in the GCM simulations presented in Edson et al. (2011, see their Figure 5). In [Appendix 6.C](#) it is shown that this is due to non-linear effects by partitioning the rotational height into contributions from the linear and non-linear terms in the non-linear balance equation (6.17). In the vertical, the stationary Rossby waves have a wavenumber-1 modulation (Tsai et al., 2014), as can be identified in the second row of [Figure 6.2](#) (most clearly evident in the $P = 4$ and 8 days simulations).

The divergent circulation generally displays the most complex structure of all of the circulation components in [Figure 6.1](#) and [Figure 6.2](#). The exception to this is the $P = \infty$ days simulation, for which the circulation is purely divergent and takes the form of a single isotropic, thermally-direct, overturning cell that features air rising on the day side, near the substellar point, and sinking on the night side (cf. [Chapter 5](#)). The geopotential structure associated with the overturning circulation is predominantly associated with the $\nabla_p \cdot (\omega \partial \mathbf{u}_d / \partial p)$ term in (6.16), which is dominant in regions of ascent and descent. In the boundary layer the surface friction term is important, and in the outflow regions aloft the horizontal non-linear $-\nabla_p^2 [|\nabla_p \chi|^2 / 2]$ term is important (see [Appendix 6.C](#)).

The balance of terms in the expanded divergence equation (6.16) for the $P = 16$ days is similar to that in the zero-rotation simulation. In particular, the $\nabla_p \cdot (\omega \partial \mathbf{u}_d / \partial p)$ term is still important in these simulations in the region of strong ascent near the substellar point, and the general characteristics of the divergent circulation are the same as those described as overturning circulation in Chapter 5. However, additional terms in (6.16) that describe interactions between the rotational (mostly $\bar{u}_r$) and divergent circulations are also important (Appendix 6.C), which suggests that the overturning circulation in the $P = 16$ days is ‘tipped over’ by the eastward superrotating jet.

Finally, in the $P = 4$ days simulation, the structure of the divergent height field is very different from that in the $P = 16$ or $P = \infty$ days simulations, and is harder to interpret. Both its horizontal and vertical structure are somewhat wave-like, although very different to the structure of a linear Kelvin wave on the equatorial beta plane (e.g., as in Chapter 2, Figure 2.7). In Appendix 6.C it is shown that the total divergent height structure shown in Figure 6.1 and Figure 6.2 for the $P = 4$ days simulation is mostly due to the linear $u_d \beta$ term, which takes the form of a Kelvin wave in a linearised shallow water model (Appendix 6.A), and terms describing interaction between rotational and divergent horizontal winds. Finally, in the $P = 8$ days simulation, the divergent height field has features that are characteristic of both the slowly rotating simulations, with thermally-direct overturning, and the $P = 4$ days simulation, with non-linear and wave-like contributions to the height, suggesting that it resides in an intermediate regime.

6.4 Relationship between geopotential components and temperature

In this section, the hydrostatic relation is used to obtain components of the atmospheric temperature that are associated with the rotational and divergent geopotential components. This is achieved by requiring that each component of the geopotential (ϕ_0 , ϕ_r and ϕ_d) is in hydrostatic balance with the corresponding component of the temperature (T_0 , T_r and T_d), i.e.,

$$\frac{\partial \phi_0}{\partial \ln p} \equiv -RT_0; \quad \frac{\partial \phi_r}{\partial \ln p} \equiv -RT_r; \quad \frac{\partial \phi_d}{\partial \ln p} \equiv -RT_d. \quad (6.20)$$

Understanding how each component of the circulation contributes to atmospheric temperature structure is important for interpreting observations of thermal emission from tidally locked planets, such as eclipse maps (Majeau et al., 2012) and phase curves (Cowan and Agol, 2008; Demory et al., 2016; Morris et al., 2022).

Figure 6.3 shows the temperature structure in the lower troposphere ($p = 750$ hPa) for each GCM simulation. As with the geopotential height shown in Figure 6.1, the

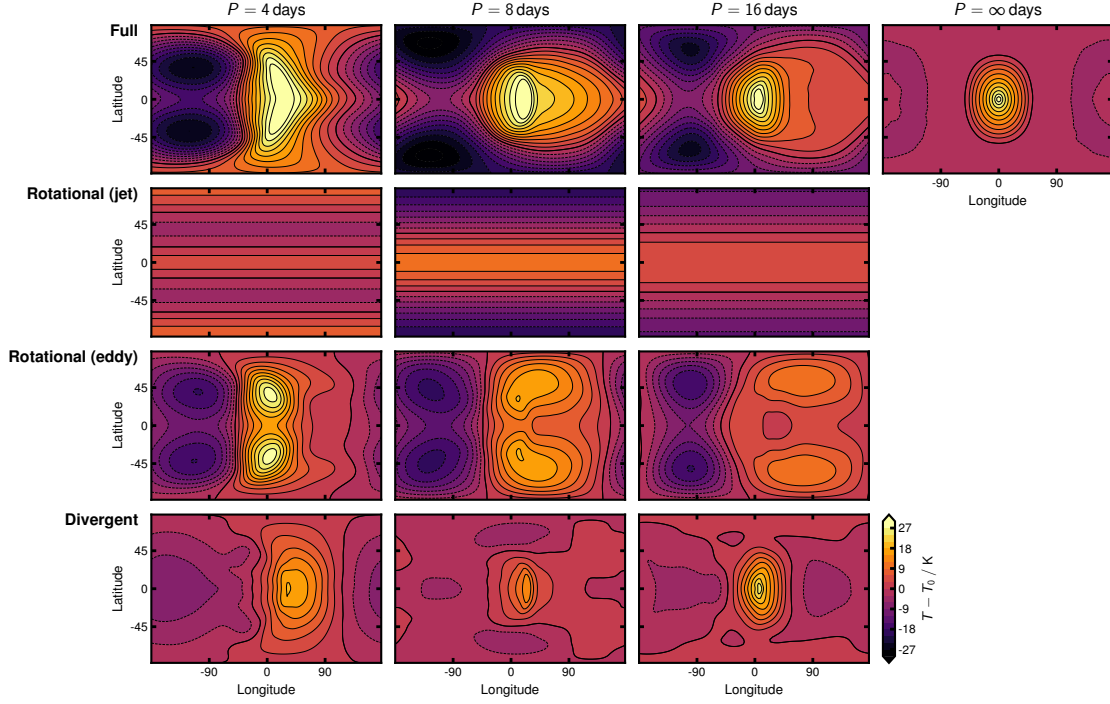


Figure 6.3: Temperature at $p = 750$ hPa for each GCM simulation. For each simulation, full fields are shown in the top row. The next three rows show the zonal-mean rotational circulation, eddy rotational circulation, and divergent circulation, respectively.

temperature is decomposed into contributions from the zonal-mean and eddy rotational circulations, and the divergent circulation. Figure 6.4 shows the vertical structure of the full circulation, eddy rotational circulation, and divergent circulation, averaged meridionally over all latitudes for consistency with the disk-integrated phase curves shown later.

Generally, the horizontal and vertical structures of the temperature components shown in Figure 6.3 and Figure 6.4 are similar to the structure of the height components shown in Figure 6.1 and Figure 6.2. The main difference between them is a phase offset in their vertical structures, which appears as the temperature is obtained from the *vertical derivative* of the geopotential (this is why the temperature in Figure 6.3 is plotted deeper in the atmosphere than the height field in Figure 6.1).

In each of the simulations with rotation, there is an eastward hot spot shift from the substellar point. In the $P = 8$ and 16 days simulations, this is almost exclusively due to the rotational standing Rossby waves which are phase-shifted eastwards from the substellar point by the zonal-mean jet (Tsai et al., 2014; Hammond and Pierrehumbert, 2018). The zonal-mean rotational temperature that balances the zonal-mean rotational wind cannot contribute a longitudinal shift to the temperature structure by definition, although it does

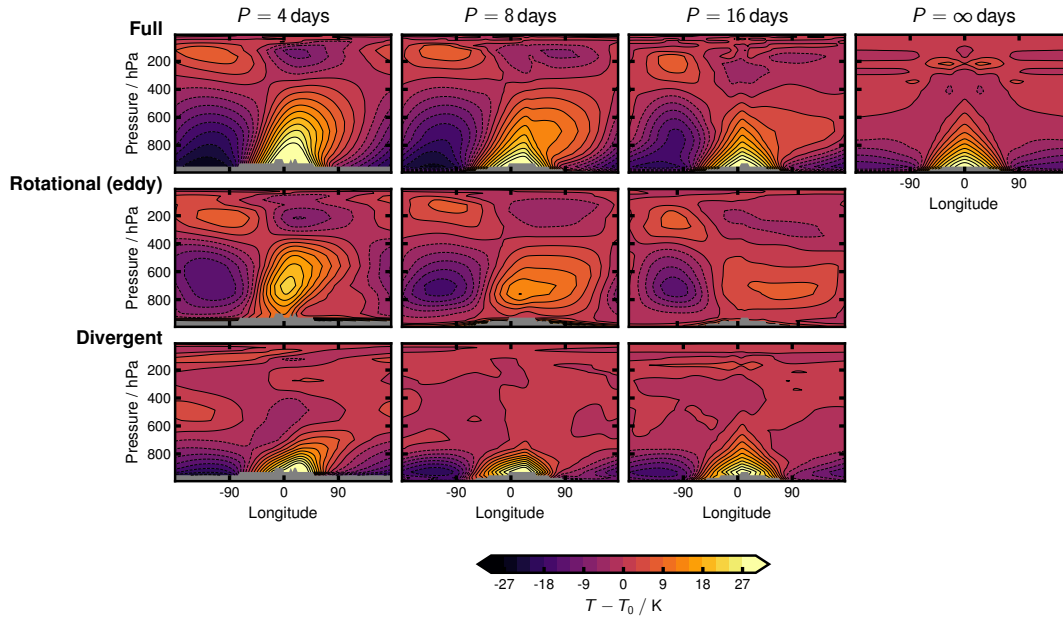


Figure 6.4: Temperature (contours) in the longitude–pressure plane, averaged meridionally from pole to pole ($\pm 90^\circ$ latitude), shown for each GCM simulation. The top row shows the full circulation. Contributions to the full circulation from the eddy-rotational and divergent components are shown in the next two rows.

contribute towards ‘brightening’ (i.e., warming) the equator with respect to the poles in the zonal-mean. The divergent temperature structure in the $P = 16$ days simulation is similar to that of the $P = \infty$ days simulation which only features overturning circulation. The overturning circulation is a direct response to heating at the substellar point, and it appears that this strong coupling to the surface prevents it from being phase-shifted much by the jet.

By contrast, in the $P = 4$ days simulation, the divergent temperature structure has a *greater* eastward shift than the eddy rotational circulation (which is now only shifted by a few degrees longitude from the substellar point), although its amplitude is comparatively lower. The eastward shift of the divergent temperature in this simulation is consistent with the wave-like nature of its divergent circulation and height field, discussed in the previous section. The temperature structure associated with the zonal mean rotational circulation is also different to that in the $P = 8$ and 16 days simulations, due to the retrograde jets that exist in this simulation in mid-latitudes, and now contributes a brightening to both the equator and the poles, relative to the mid-latitudes.

6.5 Thermal phase curves

Thermal phase curves show the infrared flux emitted from a planet as it orbits its star. As a planet progresses through its orbit we see emission from different angles, so these observations characterise the longitudinal distribution of temperature on extrasolar planets (Cowan and Agol, 2008; Koll and Abbot, 2015).

In this section, thermal phase curves are computed from the decomposed temperature structures under the approximation that the broadband thermal emission $I^\uparrow$ originates from a specific radiating level p_{rad} :

$$I^\uparrow(p = p_{\text{rad}}) = \sigma T^4(p = p_{\text{rad}}), \quad (6.21)$$

to show how each component contributes to their observable features. The thermal phase curves are computed using a radiating level to simulate observing at a wavelength at which the atmosphere is optically thick, instead of using the model's actual semi-grey outgoing longwave radiation (OLR). This is because the OLR in the simulations used for this work is dominated by the surface emission, and so is not an instructive demonstration of the potential contribution to the phase curve from the atmosphere.

As (6.21) is non-linear in T , the contributions to $I^\uparrow$ from each of T_r and T_d cannot be easily separated. To make progress, (6.21) is approximated by the linear relation

$$I^\uparrow \approx \sigma T_0^4 + 4\sigma T_0^3 (T - T_0) = \sigma T_0^4 + 4\sigma T_0^3 (T_r + T_d), \quad (6.22)$$

where T_0 is the horizontal mean temperature. Normalised thermal phase curves are then computed according to

$$I(\xi) = \frac{1}{\pi \sigma T_0^4} \int_{-\xi-\pi/2}^{-\xi+\pi/2} \int_{-\pi/2}^{\pi/2} I^\uparrow \cos(\lambda + \xi) \cos^2 \vartheta \, d\vartheta \, d\lambda. \quad (6.23)$$

where ξ is the phase angle of the planet in its orbit (Cowan and Agol, 2008).

Figure 6.5 shows phase curves computed for each GCM simulation using (6.23), for a radiating level located at $p = 750$ hPa, which corresponds to the level for which the horizontal temperature structure was shown in Figure 6.3. Solid black curves show the phase curve computed using the non-linear expression given by (6.21), and dashed grey curves show the phase curve computed using the linear approximation given by (6.22). There is generally good agreement between the phase curves computed using the non-linear and linear expressions, and the difference between the two is reduced with increasing rotation period as deviations from the horizontal-mean temperature become smaller. This shows that the decomposition of the phase curve into rotational

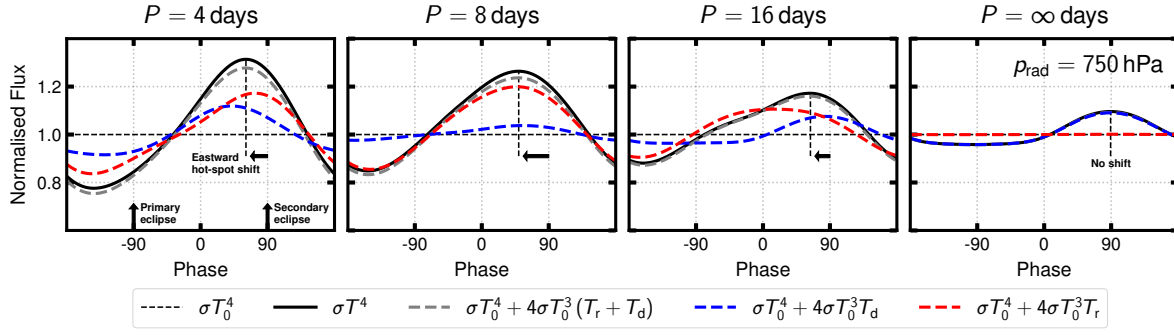


Figure 6.5: Phase curves calculated using the broadband thermal emission from a radiating level at $p_{\text{rad}} = 750$ hPa, for each GCM simulation. The secondary and primary eclipses would be at phase 90° and -90° . The solid black curve shows emission calculated using the full temperature field, and the dashed grey curve shows emission using the linear approximation given by (6.22). The dashed red and blue curves show the contributions to the linear approximation from the rotational and divergent temperature fields, respectively.

and divergent components by the linear approximation is useful for the simulations considered here. For hotter planets, it may be necessary to include higher order terms in the approximation given by (6.22).

The contributions from the rotational and divergent components of the temperature to the linear approximation are shown in Figure 6.5 as dashed red and blue lines, respectively. In each simulation with rotation, the amplitude of the rotational contribution to the phase curve is larger than that of the divergent contribution, and leads to a negative offset in the phase curve peak from $\xi = 90^\circ$ (corresponding to an eastward hot-spot shift in the temperature maps shown in Figure 6.3). As discussed in Section 6.4, this is entirely due to the phase-shifted Rossby waves, as the temperature structure that balances the jet cannot itself contribute a direct hot-spot shift by definition (although it is the velocity of the jet that phase-shifts the waves).

The negative phase offset associated with the stationary Rossby waves increases monotonically with increasing period. However, this trend is not reflected in the full phase curves, due to contributions from the divergent circulation. In the $P = 4$ days simulation, the divergent circulation, which in this simulation resembles a non-linear wave that is phase-shifted eastwards, also contributes an eastward phase shift, that is in fact larger than that associated with the rotational circulation. This means that the peak of the full phase curve has a greater negative phase offset from $\xi = 90^\circ$ than it would if only the rotational circulation (stationary Rossby waves) contributed. By contrast, in the $P = 16$ days simulation, the divergent circulation now takes the form of overturning circulation, and is restricted to be close to the substellar point, due to strong coupling between the overturning circulation and the instellation. This means that the divergent

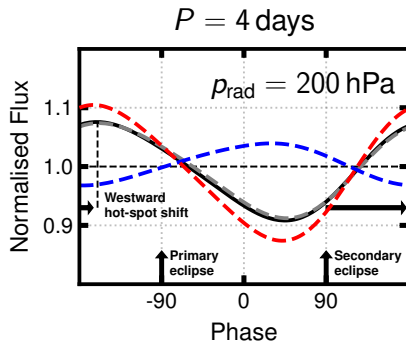


Figure 6.6: Phase curve calculated using broadband thermal emission from a radiating level at $p = 200$ hPa for the $P = 4$ days simulation. See Figure 6.5 caption for the meaning of each curve.

contribution to the phase curve is not offset from $\xi = 90^\circ$, and the full phase curve has a lesser negative phase offset than it would if due to the stationary Rossby waves alone.

The vertical temperature structure shown for each simulation in Figure 6.4 indicates that the phase curves in Figure 6.5 will depend strongly on the choice of radiating level. To illustrate this an additional phase curve is shown in Figure 6.6 for the $P = 4$ days simulation, this time for $p_{\text{rad}} = 200$ hPa. In this figure, the sign of the normalised flux associated with the rotational circulation is essentially flipped with respect to that shown in Figure 6.5, and the peak of the full phase curve has a positive offset from $\xi = 0^\circ$, which would correspond to a westward hot-spot shift in temperature. This is due to the wavenumber-1 structure of the stationary Rossby waves in the vertical (see Figure 6.4), which means that they can contribute either a negative or a positive shift to the phase curve peak depending on the exact pressure level from which they contribute to thermal emission. This is a key result of understanding thermal phase shifts as due to stationary wave shifts rather than heat advection by a jet, and is a potential explanation for unexpected observations of westward phase shifts (Dang et al., 2018).

6.6 Summary

The work presented in Chapter 5 showed that the Helmholtz decomposition $\mathbf{u} = \mathbf{u}_r + \mathbf{u}_d$ is a useful tool for separating out different components of the atmospheric circulation on tidally locked planets. The jet and stationary Rossby waves are contained within $\mathbf{u}_r$ and divergent circulation is contained within $\mathbf{u}_d$, which takes the form of thermally-direct overturning for slowly rotating planets (e.g., our $P = 8$ and $P = 16$ days simulations; see also Chapter 5), but has a complex linear Kelvin / non-linear wave structure for more rapidly rotating planets (cf. our $P = 4$ days GCM simulation, and the shallow water results in Appendix 6.A).

The objective of this chapter was to build upon Chapter 5 by relating the atmospheric temperature structure to each of the circulation components contained within $\mathbf{u}_r$ and $\mathbf{u}_d$.

This was achieved by defining rotational and divergent components of the geopotential using balance relations in the divergence equation, which were then used to define a temperature decomposition via the hydrostatic relation.

Decomposed temperature maps (Figure 6.3) computed from idealised GCM simulations of atmospheric circulation on terrestrial tidally locked planets show that both the rotational and divergent circulations make non-negligible contributions to the horizontal temperature structure. Understanding what determines the relative strength of these circulations, and how they depend on planetary parameters, will be crucial for correctly interpreting observations of thermal emission such as phase curves (Cowan and Agol, 2008) and eclipse maps (Majeau et al., 2012).

The analysis presented in this chapter shows that both circulation components can contribute to the hot spot shifts that have been inferred from observations of thermal emission (e.g., Demory et al., 2016) (Figure 6.3 and Figure 6.5). In the GCM simulations, both the rotational circulation and the divergent circulation contribute an eastward hot spot shift if thermal emission is assumed to come from the lower troposphere ($p = 750$ hPa). For the $P = 8$ and $P = 16$ days cases, the hot spot shift is almost entirely due to the rotational stationary Rossby waves (which are phase-shifted eastwards by the superrotating jet). In the $P = 8$ days case, this is because the amplitude of the rotational temperature component is much larger than the divergent temperature component. In the $P = 16$ days case, both components have a similar amplitude, but the divergent component does not contribute a hot spot shift as it is dominated by thermally-driven overturning, which means it is strongly coupled to the pattern of instellation which is centered on the substellar point. By contrast, in the $P = 4$ days simulation, both the rotational component and the divergent component (which is now more wave-like) contribute an eastward hot spot shift if emission is from the lower troposphere. However, it is shown that the rotational component can also contribute a westward hot spot shift if emission comes from higher up in the atmosphere (Figure 6.6), which is due to its wavenumber-1 structure in the vertical (Figure 6.4).

Based on the results presented in this chapter, it is suggested that the temperature structure of the atmosphere of a tidally locked planet can be divided into four physically meaningful components:

1. The overturning divergent component, providing a temperature peak at the substellar point and uniform temperature elsewhere.
2. A wave-like divergent component, providing a zonal wavenumber-1 sinusoidal

temperature modulation, maximal on the equator which can be phase-shifted by a zonal jet.

3. The Rossby-wave-like eddy rotational component, providing a zonal wavenumber-1 sinusoidal temperature modulation with maxima off the equator, which can be phase-shifted by a zonal jet.
4. The zonal-mean rotational component, i.e. the zonal jet, providing a zonally uniform temperature field with a meridional gradient determined by the gradient of the jet itself.

These components could be used to fit thermal phase curves or eclipse maps with physically meaningful functions. It is hoped that the techniques and analysis presented herein will be of use for interpreting observations of thermal emission on tidally locked planets.

Appendices for Chapter Six

6.A Rotational and divergent circulations in linear shallow water

In this appendix, the linearised shallow water equations on the sphere are used as an example to show how partitioning the height field into rotational and divergent components can be used to obtain momentum equations for the rotational and divergent wind. These equations are then used to interpret the existence of a steady linear Kelvin wave in the spherical geometry, in the absence of drag, which does not exist on the equatorial beta plane.

In the absence of friction, the linearised shallow water equations may be written (Vallis, 2017, Chapter 3)

$$\frac{\partial \mathbf{u}}{\partial t} + f \mathbf{k} \times \mathbf{u} = -g \nabla \eta, \quad (6.A.1)$$

$$\frac{\partial \eta}{\partial t} + H \delta = Q. \quad (6.A.2)$$

Above, η is the free surface displacement (or height), H is the mean fluid depth, and Q is a mass source or sink term.

As with the primitive equations, equations for the vorticity and divergence can be obtained by taking $\mathbf{k} \cdot \nabla \times$ (6.A.1) and $\nabla \cdot$ (6.A.1), respectively,

$$\frac{\partial \zeta}{\partial t} + f\delta + (v_r + v_d)\beta = 0, \quad (6.A.3)$$

$$\frac{\partial \delta}{\partial t} - \nabla \cdot [f\nabla\psi] + u_d\beta = -g\nabla^2\eta. \quad (6.A.4)$$

Assuming a steady state, the divergence equation can be partitioned into two equations that define the rotational and divergent height, η_r and η_d :

$$\nabla \cdot [f\nabla\psi] \equiv g\nabla^2\eta_r; \quad -u_d\beta \equiv g\nabla^2\eta_d, \quad (6.A.5)$$

which can then be integrated to yield momentum equations for the rotational and divergent circulations in a steady state (Trenberth and Chen, 1988)

$$f\mathbf{k} \times \mathbf{u}_r = -g(\nabla\eta_r + \mathbf{k} \times \nabla\gamma), \quad (6.A.6)$$

$$f\mathbf{k} \times \mathbf{u}_d = -g(\nabla\eta_d - \mathbf{k} \times \nabla\gamma), \quad (6.A.7)$$

where γ is defined by

$$g\nabla^2\gamma \equiv \beta v_r = -(f\delta + \beta v_d). \quad (6.A.8)$$

Equation (6.A.8) can be obtained by taking the curl of either (6.A.6) or (6.A.7), and the second equality comes from the vorticity equation in steady state.

Equations (6.A.6) and (6.A.7) each contain an additional term involving the potential function γ , when compared with the momentum equation for the full horizontal velocity $\mathbf{u}$. These terms arise as an integration constant and are due to the variability of f with latitude, and their proper representation is contingent on working in a spherical geometry (Trenberth and Chen, 1988).

Figure 6.A.1 shows horizontal velocity and height fields from a linear shallow water simulation using the GFDL spectral shallow water model. The model was run with T85 resolution for 500 days, and the output shown is averaged over the final 100 days of the run. The mass source term is configured for a tidally locked planet as

$$Q = \frac{\eta_{\text{eq}} - \eta}{\tau_{\text{rad}}}, \quad (6.A.9)$$

following Perez-Becker and Showman (2013), where η_{eq} is given by

$$\eta_{\text{eq}} = \begin{cases} H + \Delta\eta_{\text{eq}} \cos \lambda \cos \vartheta & \text{on the day side,} \\ H & \text{on the night side.} \end{cases} \quad (6.A.10)$$

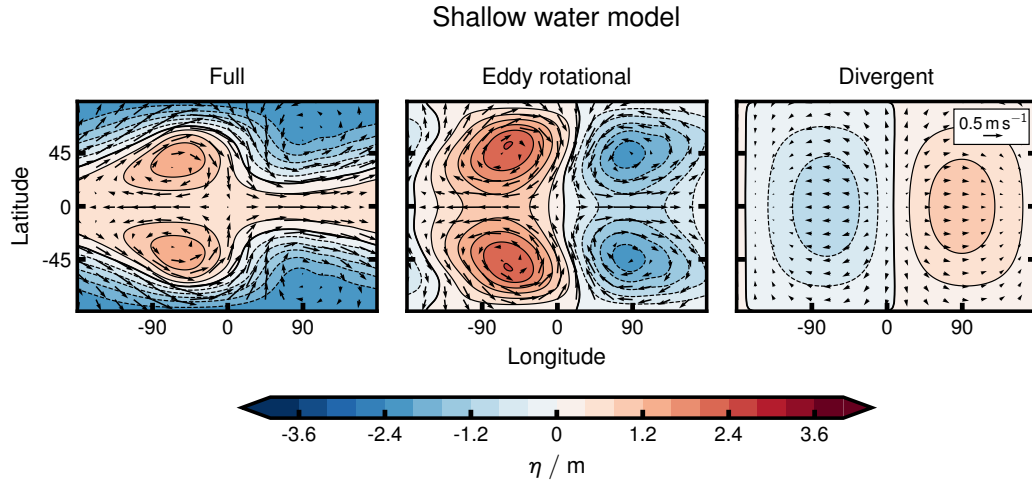
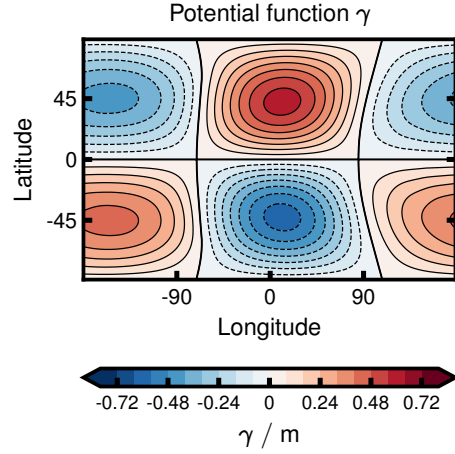


Figure 6.A.1: Free surface displacement (height; colour contours) and horizontal velocity (quivers) obtained in a linear shallow water simulation without drag ($P = 8$ days, Earth’s radius). The height and velocity fields are decomposed into eddy rotational (center panel) and divergent (right panel) components. The zonal-mean rotational component is small and so is not shown.

$\Delta\eta_{\text{eq}}$ is the difference in the radiative equilibrium height between the substellar point and the terminator. In (6.A.9), τ_{rad} is the radiative relaxation timescale. The GFDL model solves the full non-linear shallow water equations, and the solution was kept in a linear regime by using a small forcing amplitude, $\eta_{\text{eq}}/H \ll 1$, following Perez-Becker and Showman (2013). An important feature of the model is that it does not include a linear drag (i.e., $\tau_{\text{drag}} = \infty$ in Perez-Becker and Showman, 2013). The planetary parameters used for this simulation were: $a = 6.371 \times 10^6 \text{ m}$, $P = 8$ days, and $g = 9.81 \text{ m s}^{-2}$. The equilibrium height was $H = 10000 \text{ m}$, the magnitude of the forcing was $\Delta\eta_{\text{eq}} = 10 \text{ m}$, and the radiative timescale was $\tau_{\text{rad}} = 0.1$ days. These parameters were chosen for demonstrative purposes as they give rotational and divergent components of similar magnitude, but the conclusions of this appendix are not sensitive to the chosen parameters as long as the forcing is linear. In Figure 6.A.1 the velocity and height is split into eddy rotational and divergent components. The zonal mean rotational component is not shown as it is small.

The decomposed rotational and divergent circulations shown in Figure 6.A.1 have structures that resemble the linear solutions for equatorial Rossby and Kelvin waves, respectively, on the equatorial beta plane (cf. Figure 2.7, Chapter 2), although they are modified by the spherical geometry. This partitioning of the Rossby waves into the rotational component and the Kelvin waves into the divergent component is consistent with Vanneste and Vial (1994), who show that in the limit of small Lamb parameter, $\Lambda \equiv 4\Omega^2 a^2 / (gH) \ll 1$ (in the shallow water simulation $\Lambda \approx 0.1$), Rossby waves are purely

Figure 6.A.2: Structure of the potential function γ that appears in the momentum equations for the rotational and divergent circulations, obtained from the linear shallow water simulation.



rotational and Kelvin waves are purely divergent. The Kelvin wave obtained here has non-zero meridional velocity, which derives from the spherical geometry (Yamamoto, 2019). The Rossby component is very similar to that obtained by Showman and Polvani (2011) on the equatorial beta plane in the absence of friction (as is the case here; cf. their Figure 14). Notably, Showman and Polvani (2011, their Appendix C) show that in the absence of drag, an equatorial Kelvin wave *cannot exist* on the equatorial beta plane.

In a steady state, the zonal momentum equation is

$$fv = \frac{g}{a \cos \vartheta} \frac{\partial \eta}{\partial \lambda}. \quad (6.A.11)$$

As $f = 0$ at the equator (and also $v = 0$ due to the hemispheric symmetry of the problem), (6.A.11) requires that $\partial \eta / \partial \lambda = 0$ at the equator. This is satisfied in the shallow water simulation (see the left-hand panel in Figure 6.A.1). However, Showman and Polvani (2011) further show that on the equatorial beta plane, equatorial Rossby wave solutions have no longitudinal η variation at the equator, which by (6.A.11), means that an equatorial Kelvin component cannot exist (as it would introduce a non-zero $\partial \eta / \partial \lambda$).

By contrast, both the equatorial Rossby and Kelvin components in Figure 6.A.1 have a non-zero height modulations that cancel one another so that the total $\partial \eta / \partial \lambda$ at the equator is still zero. This is due to the effect of the spherical geometry on the waves, which now are governed by a zonal momentum equation of the form (e.g., for u_r , from Equation 6.A.6)

$$fv_r = g \left(\frac{1}{a \cos \vartheta} \frac{\partial \eta_r}{\partial \lambda} + \frac{1}{a} \frac{\partial \gamma}{\partial \theta} \right) \quad (6.A.12)$$

If $\partial \gamma / \partial \theta$ is non-zero at the equator (which will be the case if v is symmetric about the equator; γ obtained in the shallow water model is shown in Figure 6.A.2 for reference), then neither will $\partial \eta_r / \partial \lambda|_{\vartheta=0} = -\partial \eta_d / \partial \lambda|_{\vartheta=0}$. As a consequence, on the sphere and in

the limit of small Lamb parameter, the existence of a rotational stationary Rossby wave requires the existence of a divergent stationary Kelvin wave, and vice versa. Showman and Polvani (2011) introduced a linear drag to their beta plane model to generate the stationary Kelvin wave necessary for the acceleration of a zonal jet; here it is shown that this linear drag is not necessary for the formation of a Kelvin (-like) wave on a sphere rather than on a beta plane.

6.B Derivation of expanded divergence equation

This appendix describes how the divergence equation (Equation 6.15 in the main text)

$$\frac{\partial \delta}{\partial t} + \nabla_p \cdot \left(\omega \frac{\partial \mathbf{u}}{\partial p} \right) + \nabla_p \cdot [(f + \zeta) \mathbf{k} \times \mathbf{u}] = -\nabla_p^2 \left(\phi + \frac{|\mathbf{u}|^2}{2} \right) - k_f \delta. \quad (6.B.1)$$

can be expanded into a form that separates contributions from the rotational and divergent winds (i.e., Equation 6.16 in the main text).

First, applying the Helmholtz decomposition to $\nabla_p \cdot [(f + \zeta) \mathbf{k} \times \mathbf{u}]$, four terms are obtained:

1. $\nabla_p \cdot [f \mathbf{k} \times \mathbf{u}_r] = \nabla_p \cdot [f \mathbf{k} \times \mathbf{k} \times \nabla_p \psi] = -\nabla_p \cdot [f \nabla_p \psi],$
2. $\nabla_p \cdot [\zeta \mathbf{k} \times \mathbf{u}_r] = -\nabla_p \cdot [\nabla_p^2 \psi \nabla_p \psi],$
3. $\nabla_p \cdot [f \mathbf{k} \times \mathbf{u}_d] = \mathbf{k} \times \mathbf{u}_d \cdot \nabla_p f = u_d \beta,$
4. $\nabla_p \cdot [\zeta \mathbf{k} \times \mathbf{u}_d] = \mathbf{k} \times \nabla_p \chi \cdot \nabla_p \zeta = \mathbf{k} \cdot (\nabla_p \chi \times \nabla_p \zeta) = J(\chi, \zeta) = J(\chi, \nabla_p^2 \psi),$

where $\beta = \partial f / \partial \theta$, for the third and fourth terms we have used $(f + \zeta) \nabla_p \cdot (\mathbf{k} \times \mathbf{u}_d) = -(f + \zeta) (\nabla_p \times \mathbf{u}_d) = 0$, and in the fourth term the Jacobian

$$J(\chi, \zeta) \equiv \mathbf{k} \cdot (\nabla_p \chi \times \nabla_p \zeta) \quad (6.B.2)$$

has been introduced. The $|\mathbf{u}|^2$ term can also be expanded into contributions from $\mathbf{u}_r$ and $\mathbf{u}_d$ as follows:

$$\frac{|\mathbf{u}|^2}{2} = \frac{|\mathbf{u}_r|^2}{2} + \frac{|\mathbf{u}_d|^2}{2} + \mathbf{u}_r \cdot \mathbf{u}_d = \frac{|\nabla_p \psi|^2}{2} + \frac{|\nabla_p \chi|^2}{2} + J(\psi, \chi). \quad (6.B.3)$$

With these substitutions, the divergence Equation (6.15) becomes

$$\begin{aligned} \frac{\partial \delta}{\partial t} + \nabla_p \cdot \left[\omega \frac{\partial (\mathbf{u}_r + \mathbf{u}_d)}{\partial p} \right] + u_d \beta + J(\chi, \nabla_p^2 \psi) - \nabla_p \cdot \left[(f + \nabla_p^2 \psi) \nabla_p \psi \right] = \\ - \nabla_p^2 \left[\phi + \frac{|\nabla_p \psi|^2}{2} + \frac{|\nabla_p \chi|^2}{2} + J(\psi, \chi) \right] - k_f \delta \end{aligned} \quad (6.B.4)$$

which is Equation (6.16) in the main text.

6.C Subdivision of height field

This appendix includes some additional analysis of the terms in the expanded divergence equation (Equation 6.16), to identify those that contribute most strongly to the rotational and divergent height components in the GCM simulations. In particular, the characterisation of the divergent circulation is refined by highlighting the term that corresponds to overturning circulation, and comparing it to the terms that correspond to wave-like divergent circulation and non-linear rotational-divergent interactions.

In the $P = 4$ and $P = 8$ days simulations, the eddy rotational height (Figure 6.1) is dominated by the linear $\nabla_p \cdot [f \nabla_p \psi]$ term (not shown). It has the structure of a stationary equatorial Rossby wave, that is phase-shifted eastwards by the zonal-mean jet (Tsai et al., 2014; Hammond and Pierrehumbert, 2018). In the $P = 16$ days simulation, the eddy rotational height also resembles a stationary Rossby wave, but the lobes with a positive geopotential anomaly are noticeably elongated with respect to those with a negative geopotential anomaly (Figure 6.1). Figure 6.C.1 shows the eddy rotational height field for the $P = 16$ days simulation, divided into contributions from the linear term, and the non-linear $\nabla_p \cdot [\nabla_p^2 \nabla_p \psi] - \nabla_p^2 [|\nabla_p \psi|^2/2]$ terms. This subdivision of the height field shows that the elongation of the stationary Rossby is mostly due to non-linear rotational-rotational interactions.

The divergent height field has a complex structure in each of the simulations presented in Figure 6.1 and Figure 6.2. In order to interpret it, Figure 6.C.2 and Figure 6.C.3 show the divergent height field, decomposed into contributions from various terms in (6.16).

The $P = \infty$ days simulation represents a limiting case where the divergent circulation takes the form of an isotropic, overturning cell (see Figure 6.1 and Figure 6.2). In this case, Figure 6.C.3 shows that the divergent height structure is due to the $\nabla_p \cdot (\omega \partial \mathbf{u}_d \partial p)$ term in regions of ascent near the substellar point, and descent on the night side. Meanwhile, the $k_f \delta$ (surface friction) contribution is important in the boundary layer, and non-linear horizontal divergent-divergent interactions are important in the outflow area

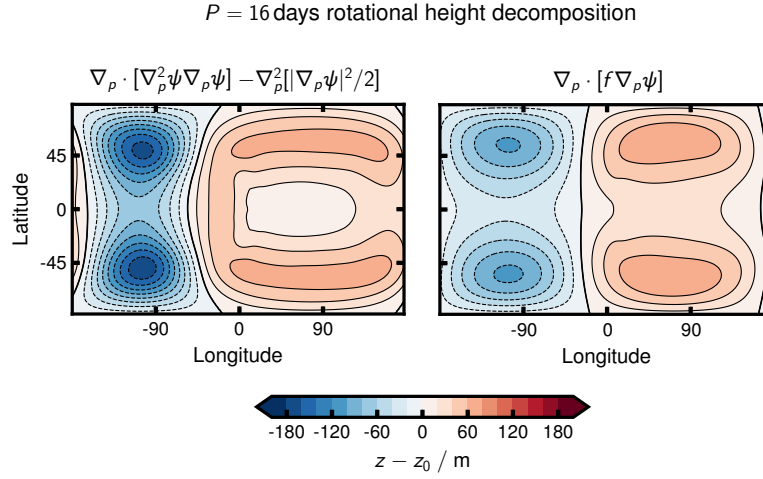


Figure 6.C.1: Subdivision of rotational geopotential height in the $P = 16$ days simulation into contributions from non-linear (left-hand panel) and linear (right-hand panel) terms.

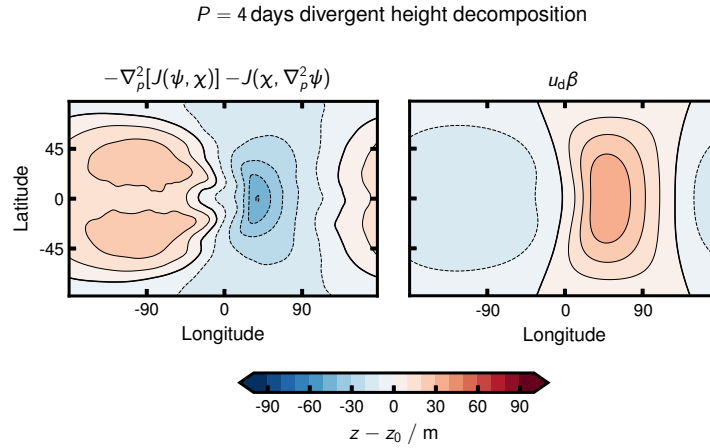


Figure 6.C.2: Subdivision of divergent geopotential height in the $P = 4$ days simulation into contributions from the non-linear rotational-divergent terms (left-hand panel) and linear $u_d \beta$ term (right-hand panel).

surrounding the region of ascent near the substellar point. The balance of terms in the $P = 16$ days simulation is similar to that in the $P = \infty$ days case, although there is a contribution to the divergent height field from the horizontal non-linear rotational-divergent interaction terms, $-\nabla_p^2 [J(\psi, \chi)] - J(\chi, \nabla_p^2 \psi)$ (mostly through $\bar{u}_r$). This suggests the interpretation that the divergent height structure in the $P = 16$ days simulation is consistent with a thermally-direct overturning circulation that is ‘tipped over’ by the superrotating jet.

By contrast, the most important contributions to the divergent height in the $P = 4$ days simulation are from the linear $u_d \beta$ term, and non-linear horizontal rotational-divergent interactions (note that surface friction is still important in the boundary layer).

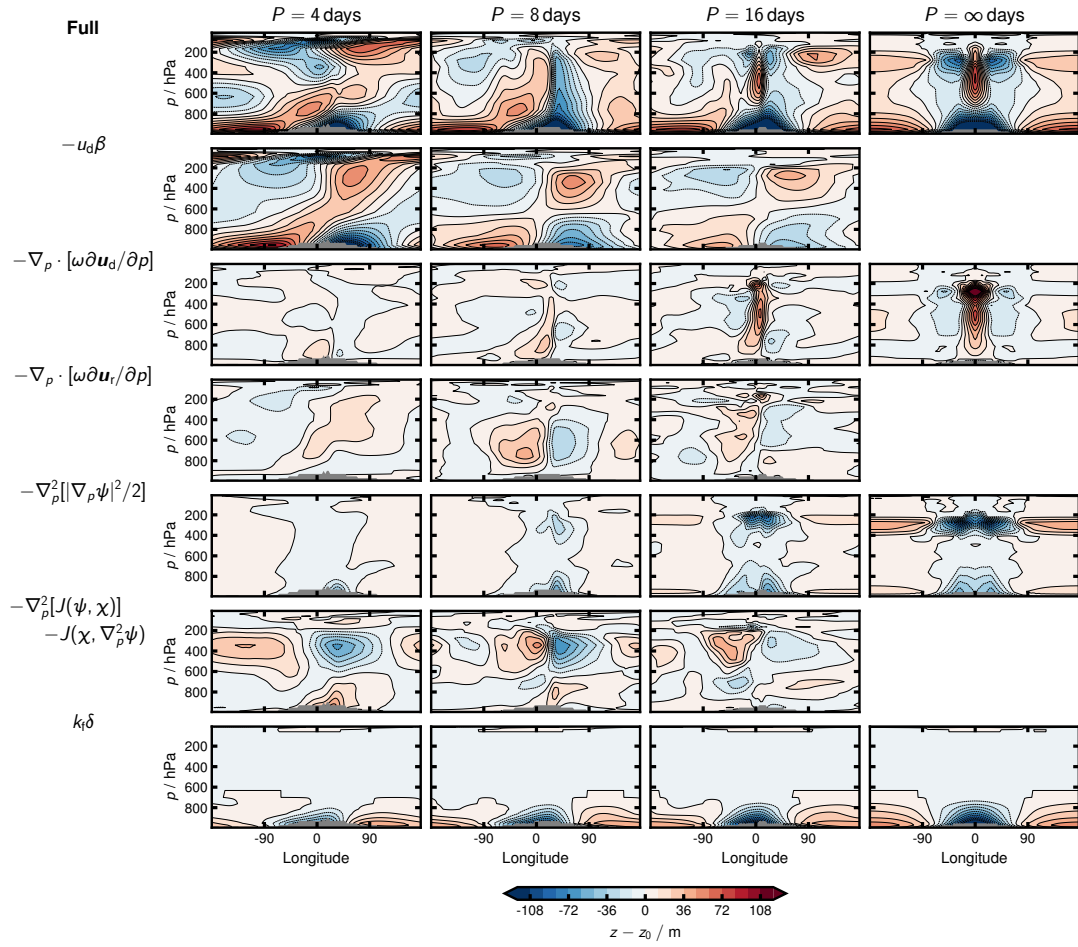


Figure 6.C.3: Subdivision of divergent geopotential height into contributions from different terms in the divergence equation (Equation 6.16 in the main text). The top row shows the total divergent height field for each simulation, and subsequent rows show contributions to the divergent height from different terms (labelled to the left of each row). Contributions from terms in the full divergence equation that are due to solely rotational circulation are not shown, as they do not contribute to the divergent geopotential by definition.

Figure 6.C.2 shows the horizontal structure of the linear $u_d \beta$ term and horizontal non-linear rotational-divergent interaction terms for the $P = 4$ days simulation. The height field associated with the $u_d \beta$ term is qualitatively similar to that of the linear Kelvin wave obtained in the linear shallow water simulation presented in Appendix 6.A. In the vertical, the height associated with this term has a wavenumber-1 structure (Figure 6.C.3). However, unlike in the linear shallow water simulation, the divergent height field is modified by the contribution from the non-linear rotational-divergent interaction term (largest in the upper troposphere), which yields the complex structure shown in Figure 6.1. The divergent height in the $P = 8$ days simulation has contributions from all of the terms discussed for both the $P = 4$ days and $P = 16$ days simulations, and it appears that this

simulation is an intermediate case between the Kelvin/non-linear wave regime of the $P = 4$ days simulation, and the thermally-direct overturning regime of the $P = 16$ and $P = \infty$ days simulations.

Conclusion

The aim of this thesis was “to improve our understanding of the atmospheric circulation regimes that may be obtained on slowly rotating rocky planets, both in the Solar System and elsewhere”.

The crucial role of planetary rotation in determining atmospheric circulation is exemplified by the Solar System terrestrial planets. Atmospheric circulation on the Earth and Mars is characterised by zonal jets in mid-latitudes and meridional overturning circulations that terminate some distance before the poles, so that these planets have distinct regions where ‘tropical dynamics’ and ‘mid-latitude dynamics’ prevail. By contrast, circulation on Venus and Titan, which are both slowly rotating when compared with the Earth and Mars, is characterised by planetary-scale overturning circulation and equatorial superrotation, and these slowly rotating planets could be described as residing in an ‘all tropics’ regime.

However, the atmospheres on Venus and Titan also illustrate that circulation on slowly rotating planets is highly sensitive to other planetary parameters and atmospheric properties. The maximum velocity of the superrotating jet on each of these planets is far greater than that obtained in models of the Earth’s atmosphere where the rotation rate is reduced, and so other features of Venus and Titan that differentiate them from the Earth must be important in determining their circulation (for example their large atmospheric masses or the presence of shortwave absorbing substances in their atmospheres). This sensitivity is also demonstrated by the atmospheric circulation of tidally locked planets, typically slowly rotating, which exhibits significant differences from the circulation on non-tidally locked planets in the form of stationary zonal asymmetries driven by day side–night side forcing contrasts.

In this thesis, my objective has been to try to characterise and understand the different regimes of atmospheric circulation that may be obtained on slowly rotating planets dominated by tropical dynamics. This work has considered both the Solar System planets and tidally locked exoplanets.

In [Chapter 3](#), I investigated the extent to which different ‘flavours’ of circulation on slowly rotating planets can be quantitatively differentiated using a metric called the global superrotation index S , which compares the total axial angular momentum of a planetary atmosphere with that due to solid body co-rotation with the underlying planet. By considering the limiting behaviour of different scaling theories for S , this work revealed that it can be used effectively to distinguish between the strongly superrotating circulation regime occupied by Venus and Titan, which is dominated by cyclostrophic balance, and the weakly or non-superrotating regime obtained in idealised models of a slowly rotating Earth like atmosphere, which is characterised (in a bulk sense) by conservation of angular momentum. I hope that the results presented in [Chapter 3](#) will be of use to others when comparing the atmospheric circulations of different planets (or sets of numerical general circulation model simulations). In the future, it would be interesting to investigate how the global superrotation index depends on rotation rate for the case of tidally locked planets. Both of the scaling theories for S discussed in [Chapter 3](#) placed a central emphasis on the importance of meridional overturning circulation. However, as was shown in [Chapter 5](#), the overturning circulation on tidally locked planets takes a different form — namely isotropic divergence away from the substellar point, in response to the day-night heating contrast. It would be interesting to know whether the different form of overturning circulation on tidally locked planets has a significant impact on the rotation rate dependence of their atmospheric circulation.

In [Chapter 4](#), I focused specifically on Titan’s atmospheric circulation. My aim in undertaking this project was to characterise the waves that accelerate superrotation under Titan-like atmospheric conditions, and to understand whether or not they are similar to those that generate and maintain superrotation in idealised Earth like models. Using spectral diagnostics, this chapter identified a range of equatorial waves — including Kelvin, equatorial Rossby, inertio-gravity, and mixed Rossby-gravity waves — to be present in a realistic simulation of Titan’s atmospheric circulation. Of these waves, superrotation was found to be accelerated by coupled Rossby–Kelvin modes in the upper stratosphere, and equatorial Rossby waves in the lower stratosphere. Both of these classes of wave have been shown to be important for accelerating superrotation in idealised Earth like models, and thus the results of this chapter serve as an important bridge between the ‘idealised’ and ‘realistic’ modeling approaches applied to understand superrotation. However, this chapter also showed that Titan’s seasonal cycle does not act to inhibit the formation of superrotation, in contrast with results from idealised modeling. It is suggested that this is due to the fact that the circulation in Titan’s stratosphere is ‘decoupled’ from the

meridional overturning in the troposphere. I hope to explore this suggestion further in the future in the idealised modeling context by introducing a stratosphere to these models, which to date have been used in essentially ‘troposphere only’ configurations. Finally, [Chapter 4](#) also highlights a potential issue with current Titan GCMs – namely that equatorial waves are not properly resolved in the vertical, which may affect the way in which they interact with the mean flow. I hope by raising this issue, the results from this work will be a useful guide to future development of Titan models.

[Chapter 5](#) and [Chapter 6](#) formed a pair of chapters within which I investigated the utility of the Helmholtz decomposition for characterising atmospheric circulation on tidally locked planets. [Chapter 5](#) showed that dividing the circulation of a slowly rotating terrestrial planet into rotational (divergence free) and divergent (vorticity free) components usefully separates out different phenomenological features of the atmospheric circulation on tidally locked planets: the rotational component is shown to ‘contain’ the superrotating jet (in the zonal mean) and stationary waves (deviation from the zonal mean), and the divergent component contains the overturning circulation. It has been exciting for me to see that, in the year following the publication of the results presented in [Chapter 5](#), a number of authors have used the Helmholtz decomposition to analyse their own GCM simulations of circulation on tidally locked planets (Turbet et al., 2021; Sergeev et al., 2021; Macdonald et al., 2022; Innes and Pierrehumbert, 2022; Paradise et al., 2022). It would be interesting in the future to explore the utility of the Helmholtz decomposition for analysing circulation on slowly rotating planets that are not tidally locked (e.g., Venus, or slowly rotating Earth-like, ‘nearly’ tidally-locked planets, as discussed by Leconte et al., 2015 and Penn and Vallis, 2018). An example of such a study would be an analysis of Venus’ atmospheric circulation using the Helmholtz decomposition, where I expect it would isolate divergent motion driven by Venus’ (long) diurnal cycle of insolation from the total circulation. Such an analysis could form the basis for a comparative study of Venus’ atmospheric circulation and the circulation on *Venus-like* tidally locked planets (cf. Kane, 2022).

[Chapter 6](#) built on the results of [Chapter 5](#), and showed that it is possible to define components of the full atmospheric temperature field that balance the rotational and divergent components of the circulation. The contributions of each of these components to thermal emission were evaluated, and it was demonstrated that ‘hot spot shifts’, observable in thermal phase curves, are due to Doppler-shifted stationary Rossby waves, and, in some cases, complex non-linear interactions between the rotational and divergent circulation. I hope that the results obtained in [Chapter 6](#) will be of use in the near future

for interpreting observations made by next generation telescopes, including the recently launched James Webb Space Telescope. My ambition is to extend the ideas discussed in [Chapter 6](#) to develop a method by which the two-dimensional velocity distribution can be retrieved from spatially resolved observations of thermal emission.

In the near term, I look forward to seeing how the results of this thesis are used to aid study of atmospheric circulation in the Solar System, and on exoplanets. In the longer term, I look forward to seeing how they are built upon, and ultimately made redundant, by theories yet to be developed, and analyses yet to be conducted!

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