The following have been added to the original version of the thesis upon the suggestion of the examiners:

(i) Page S1 immediately before page 80

(ii) the word "interior" in line 25 of page 85

(iii) to the statement of Proposition 1 on page 92,

"Assume that the maximisation problems have well-defined interior solutions..."

Information and Contracts: A Study of Principal-Agent Relationships*

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Abstract

This thesis is concentrated broadly in the field of mathematical industrial economics and more specifically upon what is known in the literature as principal-agent relationships. It focuses on investigating the nature of optimal contracts between, say, owners of the firm and the manager appointed by them to run the affairs of the firm or yet again between the owners and the workers employed in the firm.

Chapter 1 introduces by first establishing the background of the analysis and then summarising the results of the thesis. The background consists mainly of implicit contract models, both of the symmetric and asymmetric information kind, and models of moral-hazard. The results of the thesis are contained in four chapters following the introduction.

Chapters 2 and 3 are concerned primarily with the use made of principal-agent models in the asymmetric-information implicit contract literature. This literature attempts to explain involuntary unemployment by showing that the inefficiency generated by the asymmetry in information between the principal (firm) and the workers (agent) manifests itself in employment lower than the efficient level. We show instead that results are altered in quite striking ways depending not only on the eventual asymmetry of information but also the asymmetry prevailing, say, when the agent takes his action, but before production occurs.

Chapter 4 makes the case in favour of using the first-order approach in solving principal-agent models by proposing a weakening of the sufficient conditions which make this approach valid. Such weakening extends the range of cases - given by particular configurations of utility and density functions - for which the analytical convenience of the first-order approach may be utilised.

Chapter 5 uses moral-hazard models and the first-order approach to answer the specific question " Should owner-managed firms with limited liability be taxed a higher rate than similar firms with unlimited liability?". The answer is "Yes, but only under certain conditions".

Chapter 6 summarises and draws together the various strands of the arguments presented in the thesis.

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A thesis submitted to Board of the Faculty of Social Studies in fulfilment of the requirements for the degree of Doctor of Philosophy, University of Oxford.

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CONTENTS

Chapter 5: Moral Hazard, Limited Liability and the Corporation Tax.

CHAPTER 1: INTRODUCTION

1. Introduction

This thesis is a collection of five (including this introduction) extended chapters on principal-agent relationships. Recently, the 'principal-agent model' has provided a paradigm for the study of the superior-subordinate relationship. The models focus on the differences in information available to the principal and the agent, and on the difficulty the principal may have in monitoring the agent's actions and information. This leads to a potential conflict between (1) the need to share risk optimally between the principal and the agent and (2) the need to provide the agent with effective incentives. Analysis of such models shows how this conflict can lead to a loss of efficiency compared to what is potentially available with the given distribution of information. Principal-agent models have been put to influential use by Grossman and Hart, among others, in attempting to explain involuntary unemployment by showing that the inefficiency generated by the asymmetry in information between the principal and the agent manifests itself in employment lower than the efficient level. The dominant message of Chapters 2 and 3, following this introduction, is that it is necessary to look not only at the eventual asymmetry of information between the principal and the agent but also at the asymmetry prevailing when the agent takes his action in determining whether the inefficiency is expressed in underemployment or overemployment or indeed whether there exists any scope for the efficient contract to be implemented.

In order to make the arguments somewhat more concrete we shall illustrate the paradigmal principal-agent model here in terms of a relationship between an owner and a manager.

The owner of an enterprise wants to put it in the hands of a manager. In each of a number of successive periods (month, quarter, year) the profit of the enterprise will depend both on the actions of the manager and on the environment in which he is operating. The owner cannot directly monitor the agent's actions, nor can he costlessly observe all the relevant aspects of the environment. The owner and the manager will have to agree on how the manager is to be compensated, and the owner wants to pick a compensation mechanism that will motivate the manager to provide a good return on his investment net of the payments to the manager. This is the well known 'principal-agent' problem.

Principal-agent relationships are apparent in a large number of economic institutional contexts. Some other examples are: client-lawyer, customer-supplier, regulator-public utility, and insurer-insured. It may further be reasonably argued that the firm is essentially a series of interlinked principal-agent relationships - for example between the owner of the firm or shareholders and the manager appointed by them to run the affairs of the firm, between the shareholders and the workers employed in the firm, or yet again between the manager of the firm and its workers.

The term 'principal' perhaps justly evokes a notion of power. He determines the structure of contracts signed and uses the incentive schemes to induce actions which will maximise his (principal's) expected utility. The agent on the other hand has the monopoly on certain information which he can use to his advantage. The important notion in all such contracts is that of the symmetry or asymmetry of information.

When all parties to the contract have the same information set both at the time of the signing of the contract and when the outcome is realised, information is said to be symmetric. Any deviation from this bench-mark information state is termed variously asymmetric or imperfect information. It is important to draw terminological and indeed practical distinctions regarding the information sets facing the various parties to the contract both ex-ante when (1) the contract is signed and (2) the action is taken by the agent and ex-post, that is when production occurs and payments must be made. Such informational assumptions crucially determine the nature of the contracts signed and the inefficiency generated.

We shall illustrate the contention of the above paragraph with a simple example, this time relating to the insurance market. In this setting, the insuring company is the principal and the insured (or those seeking insurance) the agents. It is clear that in order to discuss the problem with reference to a single agent representative of all other agents, we require that:

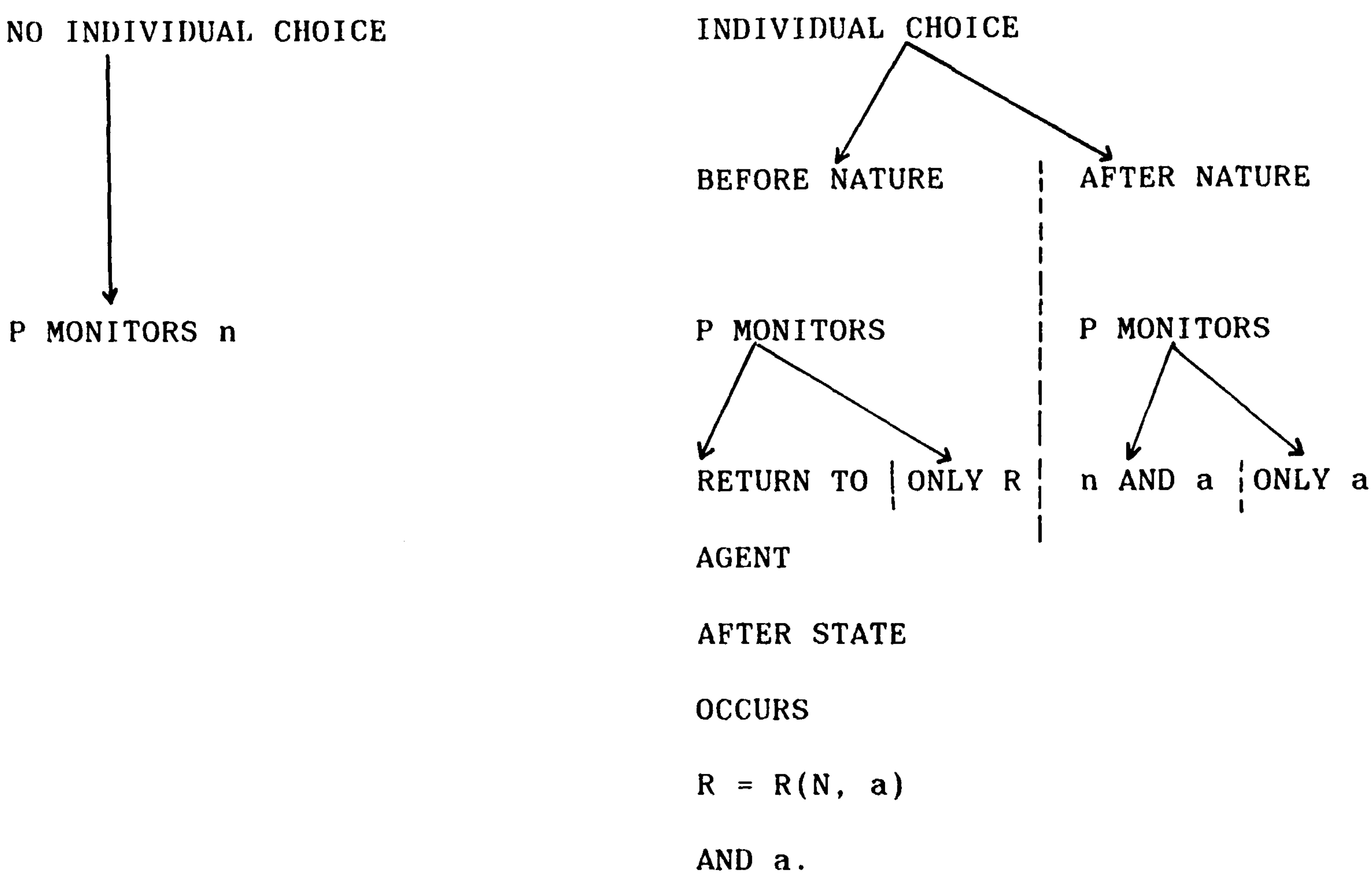
(1) there is substantial independence in the action nature takes (which is a determinant of payoffs to the agents) with respect to the different insured individuals and

(2) all individuals have identical prospects, resources and utility functions.

The arguments of the representative individual's utility function U are w, n, and a, where 'n' is an act of nature, 'a' is the individual's action and 'w' is his overall wealth level. We have therefore,

(1.1) $U = U(w, n, a)$

The presence or absence of individual choice determines two different classes of contracts. Further, where there is room for individual action, the sequencing of moves between the individual and nature and the information set S available to the principal become important. The following categorisation scheme is possible:



With no individual choice and the principal monitoring 'n', the information sets are clearly symmetric both ex-ante and ex-post. If there is individual choice and he chooses before nature and ex-post the insurer observed R and 'a', this is clearly equivalent to monitoring n as well since the payoff,

(1.2) $R = R(n, a)$ and n may therefore be expressed as,

(1.3) $n = S(R, a)$.

Where 'a' is unobservable, the insurer hypothesises that the insured maximises his expected utility given the payment scheme,

(1.4) $g = g(R)$ say.

Taking this into account, the insurer must set the payment scheme accordingly in order to maximise his own returns. The information sets are the same when the contract is signed since no action has been taken and neither the principal nor the agent has observed n since the state of nature has not yet been realised. Ex-post matters are quite different since action 'a' has not been observed by the principal.

The discussion in the previous paragraph is intended to highlight the role of information in the formation of contracts. The categorisation described above is by no means complete. A fuller categorisation appears in Spence and Zeckhauser (1971). The information sets of course critically determine upon what variables the payment scheme can be conditioned. The obvious distrust that exists among parties means that contracts can be conditioned only upon unobservables. Take, as an example, the model in Grossman and Hart (1983a) of contracts under asymmetric information between a firm and a worker, with the sequence of moves set forth by them as follows:

Date 0: The firm decides whether to sign a contract with a worker. No actual production or employment takes place at this date.

Date 1: Assuming that a contract has been signed, production and employment can occur. If production occurs at date 1, that is the worker is employed, the firm's output is 's'. If production does not occur, the worker is unemployed and the firm's ouput is zero.

At date 0, when the contract is signed, although the distribution function of the state of nature is known to both parties, the realisation of 's' is known to neither. Thus at date 0, there is symmetric information. At date 1 however, before any employment decisions have to be made, the firm learns the realisation of s. Thus there is asymmetric information at date 1 and since payments decisions have to be made at this date, payments cannot be made conditional upon the realisation of the state of nature because of the asymmetry of information. This leads to inefficient levels of labour employment but we shall come to that in Section 5 of this chapter. We simply note here that in the Grossman-Hart example, the asymmetry of information ex-post concerns the ignorance, of one of the parties to the contract, of the state of nature. In the insurance example, the ignorance of one of the parties related to the action taken by the other party. If the insurer had been able to observe the action ex-post, he would have conditioned his payment,

(1.5) $g = g(R,a)$ rather than as in (1.4).

In the Grossman-Hart model, payments could have been conditioned simply on the state of nature, if only it were observable to all.

The complexities of both the 'asymmetric information model' and the 'moral-hazard model' can be incorporated within a slightly extended model with (1) a principal (2) a manager who takes an action 'a' unobservable by the principal and (3) a worker. The sequencing of moves between the observation of the state of nature and the action of the manager determines to which one of the simpler models this extended model gets reduced. If the manager chooses his action after the state of nature becomes known to him, as in Hart (1983), we are in the realm of an asymmetric information model with the advantage of information held by the manager. However, if the manager chooses 'a' before the state of nature is realised, as in Chapter 2, we are in the second - or moral-hazard - kind of model. In both cases, the state of nature is ultimately revealed only to the manager, As far as the principal and the worker are concerned, their total ignorance of 'a' implies their total ignorance of the state of nature and therefore their information sets remain the same under both sequences of moves by nature. The 'worker' is included in the extended model simply to retain conceptual continuity with the previous asymmetric information framework by having a party in a subordinate position with an informational disadvantage and also to focus sharply and importantly on employment decisions. The principal-manager relationship, when modelled explicitly, also helps us to move away from the prevailing use of a monolithic principal in the literature. As might be expected, the results under the two alternative sequences differ significantly and this is precisely the thrust of the analysis in Chapter 2. We turn now to a more detailed discussion of the results proved in the chapters.

2. Summary of Results

Chapter 2 focuses attention on a critical assumption, made by much of the asymmetric information literature, that the principal (by which is meant the employer) is risk-averse. In Section 4 of the present chapter, we demonstrate that, given the specification of the utility function for the risk-averse agent, it is primarily the assumption of a concave utility function for the principal that drives the inefficiency results derived by Hart (1983).

The concept of risk-aversion can be described, for completeness, in the following terms. Consider a decision maker with assets x and utility function U. The risk-premium 'r' is the amount such that he would be indifferent between receiving the risk z andd receiving a non-random amount E(z) - r.

In other words,

(2.1a) $U(z - r) = E(U(z))$

implies,

(2.1b) $U(E(z)) > E(U(z)) \; ; \quad r > 0$

The assumption appears surprising especially in the context of large firms with many shareholders and easily diversifiable risk. We can do no better than quote Arrow and Lind (1970) on this point

> "...One might raise the question as to whether risk-spreading is not associated with large corporations...(T)his can be made more precise by assuming there were n shareholders who were identical in the sense that their utility functions were identical, their incomes were represented by identically distributed random variables, and they had the same share in the company. When the corporation undertakes an investment with a return in a given year represented by B, each stockholder's income is represented by A + (1/n)B... Clearly, ... if n is large, the total cost of risk-bearing to the stockholders will be negligible..."

It would not however appear intuitively unreasonable for the case of a small owner managed firm with limited resources. Grossman and Hart's rationalisation for a risk-averse large firm is that an agent is appointed to run the firm and it is he who behaves in a risk-averse manner. Even more importantly, this manager takes an unobservable action 'a'. This unobservability serves the additional crucial role of justifying the existence of asymmetry of information in large firms where both profits and employment are observed. In terms of Grossman and Hart's own formulation,

(2.2) $P = P(\ell, \theta)$

where P is a symbol for gross (net) profits while ℓ is employment. If P and ℓ are both observed, θ is observed too and the asymmetry of information disappears. The introduction of unobservable 'a' as a determinant of profits restores unobservability of θ.

In Chapter 2 it is demonstrated (on the lines of Hart (1983)) that inefficiency results can be generated with a risk-neutral principal if a risk-averse manager taking an unobservable action is introduced into the model. Such an arrangment therefore restores the results of a model with a monolithic risk-averse principal without in any sense justifying such an assumption. It is further demonstrated that the above inefficient underemployment results follow from a particular specification of moves made by the manager and the degree and nature of informational advantage he has over the principal (refer to discussion in Section 1, page 11). A familiar alternative and, in our view, more reasonable specification of moves is proposed and it is shown how this may yield fully efficient results. It is shown that Hart's specification of moves removes all notion of 'risk-averse behaviour' by the manager as too much is known by him with certainty. The alternative specification is more realistic for capturing risk-averse behaviour but may lead to quite contrary results in the shape of over-employment. On the basis of the analysis, an index of risk-averse behaviour is proposed in terms of the concavity of the manager's utility function with respect to the profits of the firm. The argument therefore in favour of introducing the alternative moral-hazard specification of moves is two-fold; first to model the rationale for a risk-averse principal and second to demonstrate the sensitivity of the results to the altered sequence of moves. A third purpose is also served, with a useful method of approximating incentive schemes in the neighbourhood of the first-best being derived as an essential part of the analysis.

The analysis of Chapter 2 is extended in the next chapter by allowing for a risk-averse principal. The emphasis is on the implementability of the first-best contract given that all the parties to the contract are risk-averse. An important restriction on the interpretation of the action by the manager needs to be imposed. In this chapter we work without a disutility of effort variable. The analysis is conditioned on the assumption that the absence of such disutility does not mean that the manager automatically chooses the action most preferred by the principal. Divergence is possible because of the varying preferences to risk of the principal and the manager. We find that the first-best is incentive compatible if and only if the principal, the manager and the worker possess affine risk-tolerance utility functions. The analysis is conducted in two stages. The first stage concentrates on the principal-manager relationship and proves several propositions on first-best implementability. The analysis is close, conceptually, to Ross (1973,74) but differs from the earlier analyses in concentrating on proving the necessity as opposed to the sufficiency of linear risk-tolerance functions for first-best implementability of contract. The idea of necessity is implicit in the Ross analyses but the proofs are rather obscure and unintuitive. The second stage of the analysis incorporates a risk-averse worker in addition to the manager and shows that linear risk-tolerance utility again guarantee first-best implementability. This is called the 'extended similarity principle'. The inclusion of the worker links Chapters 2 and 3 and deduces the existence of another class of cases where the moral-hazard framework leads to fully efficient results. The absence of the disutility of effort variable allows us to operate with any general specification of distribution of profits. Therefore, the loss in generality (vis-a-vis Part 1) involved in restricting utility functions is made up somewhat by the gain in generality obtained by allowing arbitrary distribution functions. The restrictions in each of the two parts are there to fulfil their special roles.

Chapter 4 is the first in a series of two chapters justifying the use of the techniques developed in the first two chapters and applying the same to specific problems. The emphasis in Chapter 4 is on justifying the use of the first-order approach to principal-agent problems. The first-order approach involves relaxing the constraint that the agent chooses an action which is utility maximising to require instead only that the agent choose an action at which his utility is at a stationary point. The problems inherent in using this approach are by now well known. Essentially, they arise from the inability to guarantee the concavity of the agent's expected utility in action and hence the attainment of a global maximum as opposed to local maxima, minima or saddle-points. Papers rehabilitating the use of the first-order approach, notably Rogerson (1985) have not proceeded further beyond identifying sufficient conditions for the validity of the approach. Chapter 4 argues that the sufficient conditions proposed are too strong and impose unnecessary structure on the behaviour of the agent's expected utility. We argue instead that by imposing a positive minimum expected utility condition on the agent, the sufficient conditions can be weakened to a 'quasi-concavity' or 'concave when positive' condition. An alternative set of sufficient conditions is proposed. Further by justifying the use of the first-order approach on the basis of the new set of conditions, they are shown to be less demanding than the Rogerson conditions. This follows from the simple additional observation that, in the example presented, the Rogerson conditions are not satisfied. The use of the first-order approach is also justified for distributions from perfect observability by small variance shifts. In sum, the aim of Chapter 4 is to argue that the analytical convenience of the first-order approach may be retained for a much larger of economically relevant cases than those allowed by the Rogerson conditions.

Finally, Chapter 5 uses a model of manager's behaviour under uncertainty to analyse the effect of limited liability on optimal profits taxes. In particular, it addresses the question of whether the benefits of limited liability should be taxed. Two interpretations of manager's behaviour are examined. First, a manager chooses his effort level which affects the distribution of profits according to first-order stochastic dominance. Second, the manager chooses an investment portfolio which alters the distribution of outcomes according to mean preserving spread of utility. The analysis is conducted under four alternative configurations determined by limited liability arrangements and interpretation of managers' behaviour. We show that limited liability affects the tax and, under certain conditions, according to the formula that limited liability implies higher taxes.

A brief concluding chapter completes the thesis.

The analysis in all these chapters derives its ultimate motivation from the so-called implicit contract models of employment. The aim is to discover when efficient results are obtainable and when this is not possible. The models in both Chapters 2 and 3 can be essentially thought of as extended implicit contract models of employment. There are several bench-line or paradigmal models and concepts that will occur repeatedly during the thesis. The rest of this chapter is therefore devoted to describing fully three of these concepts (models):

(1) A symmetric information implicit contract model

(2) The notion of incentive compatibility and

(3) Following from (2), an asymmetric-information implicit contract model.

The remaining important model, that is the 'moral-hazard model' is described fully at the beginning of Chapter 2 and will not be repeated here. Taken together, these models are useful reference points on which more complicated analyses will be based. It is only natural to gather them together for ease of reference and also to serve as a brief survey of the existing literature.

3. Symmetric Information Implicit Contract Models.

Prominent among the early implicit contract theorists were Azariadis, Baily and Gordon. They used their models to try and explain the phenomenon of wage rigidity in the face of large amounts of cyclical unemployment. They also tried to explain involuntary unemployment. Although wage-rigidity followed quite easily from the manipulations, the layoffs they induced could hardly be termed involuntary. Workers were assumed to be attached to firms owing to high mobility and training costs. In the long run, all the workers would share equally in the layoffs. The workers would evaluate the mean utility from a job offer, not just over the current state. Normally, layoffs would be followed by re-hires and although the laid-off worker is ex-post worse off, his unemployment cannot be said to be involuntary. In their market clearing and symmetric information assumptions, they were sticking too closely to the Walrasian paradigm.

Implicit contract models had risk-neutral principals maximising expected profits. There were n possible states of nature, each with probability of occurrence q_i. Principals thus maximise, subject to their choices of w_i and L_i, the wage rate and labour employed in state i respectively,

(3.1) $\sum_i q_i P_i$ where P_i = profits in state i = $p_i f(L_i) - w_i L_i$

Firms face a labour pool constraint,

(3.2) $L_i \leqslant L^*$

Workers are assumed risk-averse and need to be guaranteed a minimum utility level of U*, i.e

(3.3) $\sum_i \{q_i (L_i/L) U(w_i) + q_i (1-L_i/L) U(z)\} \geqslant V^*$,

where $q_i L_i/L$ is the probability that the worker is employed in state i and z, unemployment compensation, is externally financed.

Setting up the maximisation problem and differentiating with respect to w_i easily shows the condition that (real) wages would be state invariant. This follows directly from the assumption of the risk-aversion of workers. The concavity of the utility function means that they would prefer a wage with certainty less than the expected value of a fluctuating wage, the difference between the two being the so-called risk-premium.

With unemployment compensation externally financed there were two strands in the optimal symmetric information contract working in opposite directions. The risk-aversion of the workers meant that they preferred contracts which ensured certainty of wages and employment over all possible states of nature while the second element of the theory 'unemployment compensation' implied that the firm stood to gain from unemployment fluctuations since they were able to make use of the subsidy offered by the government to the unemployed. The net effect according to the theory would be constancy of w but either constancy or fluctuation of L_i depending upon the relative strengths of unemployment compensation and risk-aversion involved. Thus, there entered an element of circularity in the argument: if the unemployment compensation was the cause of unemployment, then why was it introduced in the first place? The policy prescriptions were neo-Classical rather than Keynesian (which was, after all, we suspect, the school of thought from which these writers drew their initial inspiration). Abolish unemployment compensation or let it be financed by the firm and all will be well. This theme was taken up by Rosen (1983). He showed that in a model where workers are financed by the firm, if they are laid off, then in equilibrium, for all unemployed workers,

(3.4) total of lump-sum payment from government

+ compensation received from firm = employment wages.

Therefore the worker ends up being indifferent between working and being laid-off and their unemployment cannot be said to be voluntary.

Another criticism of symmetric-information implicit contract models came in a paper by Akerlof and Miyazaki (1980). They showed how simple consideration of the concavity of the utility functions of the workers would lead to a full-employment contract with guaranteed fixed wages dominating all other contracts. The firm would be indifferent between
(a) a fluctuating employment-fixed wage contract and
(b) a fixed employment - fixed wage contract as long its wage bill from employing all the workers remained unchanged. This could be achieved by lowering the wage slightly below that which would obtain under (a) and by exploiting the certainty equivalence it would also be possible to leave the workers strictly preferring (a) to (b) or being indifferent between the two. In sum, it was shown that in the absence of externally financed unemployment compensation and ex-post mobility of labour across labour pools, an implicit contract model with symmetric information could not result in any unemployment and there was no obvious reason for looking to such theories for explanations of involuntary unemployment.

A cleaner break from the Walrasian informational assumptions was required and was provided in recent papers on asymmetric-information implicit contracts. Once the assumption that the workers can observe the state of nature is dropped, the optimal labour contract involves variability of employment.

4. ASYMMETRIC INFORMATION AND INCENTIVE COMPATIBILITY

If all individuals are fully informed about the state of nature prevailing in an uncertain environment, by sharing risks either indirectly, by trading in contingent claims markets or directly by writing detailed contingent contracts, they will achieve Pareto efficient production and risk sharing. These properties may fail to hold when information is private, since allocative mechanisms will not provide individuals with sufficient incentives to reveal truthfully to others information in their exclusive possession. Consider the following example:

A simple exchange economy with one physical commodity, two agents A and B and two equiprobable states of nature 1 and 2. Suppose further that A is risk neutral and B is risk averse. The endowment matrix is shown below:

		Agent A	Agent B	
(4.1)	State 1	1	$1-a$	
	State 2	1	$1+a$	$0 < a < 1$

If information is symmetric, then one Pareto optimal allocation of risks is for two individuals to exchange endowments; i.e for B to pay A, $s-1$ units where s is B's actual endowment. This mechanism will break down if A knows the probability distribution of s but not its realised value, for it will be in B's interest to claim that $s=1-a$ even and especially when $s=1+a$. A natural way to counter incentive problems of this type is to write contracts consistent with truth telling; such contracts however by restricting the choice set available to individuals under symmetric information typically result in second best allocations.

Optimum contracts under symmetric information provide optimum insurance and provide for productive efficiency. One can find productively efficient contracts that satisfy incentive compatibility but are not optimal given the incentive structure because they cannot simultaneously distribute risks well and satisfy incentive compatibility. In a range of environments the optimum second best contract violates productive efficiency as well.

We proceed to discuss the notion of incentive compatibility in more formal terms. Suppose in a firm, the agents(workers) know the set S of possible states of nature and G the probability distribution but unlike the Principal (entrepreneuer) they cannot observe directly the actual state or anything correlated with it, it is sensible to design mechanisms that elicit truthful information from entrepreneurs.
Let $s \varepsilon S$ be the actual state of nature and let $\theta \varepsilon S$ be the state announced by the entrepreneur. A contract δ say, consists of two functions of
$w: S \to R_+$ and $\ell: S \to R_+$ mapping state announced by the firm into total compensation and employment.

(4.2) $\pi(s,\theta/\delta)$=Consumption by entrepreneur under contract if s is the actual state and θ the announced state.

(4.3) $c(s,\theta/\delta)$=Consumption by worker

δ is feasible if $\ell(\theta) \leqslant 1$(where labour supply has been normalised to 1).

The contract is incentive compatible if lying is never in the entrepreneur's own interesti.e for all $(s,\theta)\varepsilon S$

(4.4) $\pi(s,\theta) \leqslant \pi(s,s/\delta)$ (*)

The inequalities starred above are the self-selection or incentive compatibility constraints.

Asymmetric-information implicit contract literature focuses exclusively on incentive compatible contracts. There is no loss of generality involved in doing this as the following lemma due to Myerson(1979) shows.

The Revelation Principle : Associated with any contract is an equivalent incentive compatible contract.

Proof: First assume a production function $f(\ell,s) = sg(\ell)$. This simplifies the exposition of the proof which in any case can be proved more generally. There is one worker paid wage $w(\theta)$.
Suppose that $\theta^*:S\to S$ maximises $\pi(s,\theta/\delta)$given s and δ. Consider the contract $\hat{\delta}=\{\hat{w}(s),\hat{\ell}(s)\}$ where $\hat{w}(s)=w(\theta^*(s))$. Clearly δ yields the same allocation as δ; in addition it is incentive compatiblé since for θ

$$
\begin{aligned}
(4.5)\quad \pi(s,\theta/\hat{\delta}) &= sg(\hat{\ell}(\theta))-\hat{w}(\theta)\\
&= sg[\ell(\theta^*(\theta))]- w[\theta^*(\theta)]\\
&\leqslant sg[\ell(\theta^*(s))]- w[\theta^*(s)] \text{ by definition of } \theta^*(s)\\
&= sg[\hat{\ell}(s)]-\hat{w}[s] \text{ by definition of } \ell(.) \text{ and } w(.)\\
&= \pi(s,s/\hat{\delta}) \quad //
\end{aligned}
$$

A couple of points must be noted at this stage:

(1) Since the incentive compatibility constraints must hold for all s,θ there are usually an unwieldy number of incentive compatibility constraints. The usual practice in the literature has been to try and reduce the number of incentive compatibility constraints. This is done by ordering the states in decreasing or increasing order of productivity and then adopting suitable restrictive assumptions such that only,say, the downward or adjacent incentive compatibility constraints need to be considered since the system (or at least the solution to the system) with these reduced number of constraints is equivalent to the larger system. Let us illustrate what we mean:

(i) In Hart(1983) the states are ordered $s_1, \ldots\ldots, s_n$ and under the reduced model, incentive compatibility constraints are given by (in our notation):

(4.6) $\pi(s_i, s_i/\delta) \geqslant \pi(s_i, s_{i-1}/\delta)$

(ii) In Moore(1984c) a large part of this paper is devoted to proving that if the the contract satisfies downward incentive compatibility (DIC) then it automatically satisfies upward incentive compatibility (UIC) and consequently half of the incentive compatibility constraints can be dropped. The DIC constraints are given by:

(4.7) $\pi(s_i, s_i/\delta) \geqslant \pi(s_i, s_j/\delta) \quad \forall\ i \geqslant j$

(2) The second point considers the role of signals r tó the ill informed agent which may help him to gather more information. Grossman,Hart and Maskin(1983) show that if the contracts are conditioned on r, the inefficiencies induced by asymmetric information are not eliminated.

The discussion in this section enables us to proceed to a description of the third paradigmal model, an asymmetric-information implicit contract model.

5. THE GROSSMAN AND HART MODEL

This model is presented in Grossman and Hart(1983a).The assumptions are as follows:

(i) There exists a continuum of states labelled s with positive support on an interval (s_1, s_2). The density function is labelled g(.), G(s) is the distribution function of s with $G(s_1)=0$; $G(s_2)=1$.

(ii) The occurrence of the state of nature,which affects the productivity of the worker is known only to the manager and not to the worker.

(iii) The firm is able to employ at most one worker with output denoted by a continuous random variable s.

(iv) All potential workers are assumed identical with VN-M utility functions U. If the worker is employed,his utility is U(w-R) and if he is unemployed, his utility is U(w') where w (respectively w') is the worker's wealth when employed (respectively unemployed). The particular form of the utility function of the worker is important and we shall discuss it further in Chapter 2.

Under symmetric information, i.e when s is common knowledge at date 1, the wage function must be chosen.

This involves specifying

$$w(s)=w_u \mid L(s) = 0$$

$$\text{(5.1)} \qquad \text{and} \quad [V'(s-w(s))/V'(-w_u)] = [U'(w(s)-R)/U'(w_u)] \mid L(s) = 1$$

Note that the contract is characterised by firm's maximising expected utility E(V) subject to the constraint that the worker's expected utility is at least U*.

Under asymmetric information,concentrating only on incentive compatible contracts we have the following considerations:

Let E = {s| L(s)=1} be the set of employment states denoted by the contract and U = {s| L(s)=0} be the set of unemployment states denoted by the contract.

Then the condition that truth telling is optimal implies that $w(s)=w_e$ on E and that w(s) equals a (possibly) different constant w_u on U. For if $w(s_A)>w(s_B)$ and both s_A and $s_B \varepsilon E$ the firm will pretend that $s=s_B$ when $s=s_A$ since by so doing this it can reduce wages without sacrificing employment. Similarly if $w(s_A)>w(s_B)$ and s_A and s_B εU.

Thus under asymmetric information, a contract specifies two wages w_e and w_u when the worker is employed and unemployed respectively.

Let $k = w_e - w_u$. Then the condition that truth telling is an optimal strategy tells us that:

(5.2) $E \equiv \{s \mid L(s)=1\} = \{s \mid s>k\}$

(5.3) $U \equiv \{s \mid L(s)=0\} = \{s \mid s<k\}$

since when $s \in E$ the firm gains more from employing the worker than it has to pay for this employment and vice versa when $s \in U$.

Thus where only the firm observes the state at date 1, a contract is characterised by two numbers w_u and k. Also $w_e = w_u + k$.

If the Principal is risk neutral, from (5.1), under symmetric information,

$w(s)=w_u+R$ when V' is constant for $s \mid L(s) = 1$.

However this wage path can be realised under asymmetric information by setting k=R and this yields efficient employment.

The dissipative effect of asymmetric information is eliminated if the principal is risk-neutral and consequently the first-best contract is implementable. Therefore, the credibility of the asymmetric-information implicit contract literature hinges on the assumption of a risk-averse firm. It also of course hinges on how far unobservability of profits can be justified in large firms with widely publicised profit statements. Grossman and Hart (1983) and Hart (1983) are aware of these criticisms. Hart (1983) spends some time extending his model to the case of large firms with a risk-neutral principal. As discussed in Section 1, we find that his results depend on a particular sequencing of moves and if we are really interested in modelling 'risk-averse behaviour' an alternative sequence, leading to contrary results, should be considered. For the moment, we shall concentrate on concluding this chapter with a brief description of the results of the Grossman-Hart model with a risk-averse principal.

An optimal contract under asymmetric information in their model,is a pair (w_u,k) , $s_1 \leq k \leq s_2$ which maximises

$$(5.5) \int_k^{s_2} V(s - w_u - k)g(s)ds + \int_{s_1}^{k} V(-w_u)g(s)ds$$

subject to

$$U(w_u+k-R)(1-G(k)) + U(w_u)G(k) \geq U^*$$

and appropriate regularity conditions

Solving the above program,Grossman and Hart show that $k > R$.Thus

$$U(w_u+k -R) = U(w_e - R) \geq U(w_u).$$

Hence the worker regrets the fact that he is unemployed and is therefore in a sense involuntarily unemployed.

The intuition behind all this may be summarised as follows. If the worker cannot observe wage s, such as employment, wages cannot be conditioned on s directly, but only on observable variables determined by s, such as employment. As a result in order to obtain the right degree of risk sharing, employment varies more than is necessary from the efficiency standpoint. In particular, unemployment may occur in situations where $k>s>R$ i.e where the marginal product of labour exceeds the reservation wage. This is because the only way risk averse firms can reduce their wage bill in times when s is low is by sacrificing employment. Firms would like to reduce wages without sacrificing employment but this is not possible under asymmetric information.

6. Conclusions

It is worrying to see a theory on something as important as involuntary unemployment depending so crucially on the assumption of the risk aversion of firms. The earlier symmetric information literature avoided making this assumption, but ended up with the indifference result. Furthermore, given that the models operated within the parameters of workers being attached to firms owing to high mobility and training costs, in the long run, all the workers would share equally in the layoffs. The workers would evaluate the mean utility from a job offer,not just over the current state. Normally, layoffs would be followed by re-hires and although the laid off worker is ex-post worse off, his unemployment cannot be said to be involuntary.

To get away from these results, the asymmetric information literature assumed, obviously, asymmetry of information but also risk aversion of the principal. What really drove the results in their model was not asymmetric information per se but the assumption of risk aversion. Thus while the symmetric information literature suffered from implausible results, the asymmetric information literature suffers from potentially implausible assumptions unless ways can be found of generating similar results using principal-manager combinations. This way of justifying the assumption turns out to be quite difficult and is the concern of Chapters 2 and 3. The important point that we stress here is that while risk aversion is crucial to many of the results obtained in the recent asymmetric information literature, any attempt to justify such risk-aversion leads to over-employment or under-employment results depending on the particular structure of the model. The mechanisms generating over-employment are of independent interest - emphasising as they do the need to look closely at the assumptions of particular models because subtle changes alter results - as are the techniques developed in Chapter 2 in approximating incentive schemes. We also argue the need to look closely at the assumptions of particular models as even subtle changes alter results significantly. Chapters 4 and 5 deal primarily with the mechanics of principal-agent models and investigate their properties and ways of solving for effort-choice, incentive schemes and a variety of other issues. Thus, while proceeding by using fairly simple models, the implications of the issues addressed in this thesis are more wide-ranging.

A Refutation of Inefficient Underemployment Results in Imperfect Information Implicit Contract Models - Part 1*

Abstract:

The chapter takes as its starting point the seemingly obvious proposition that the special results of the implicit contract literature with asymmetric information stem from the assumption of a risk-averse principal. This assumption appears surprising especially in the context of large firms with many shareholders and easily diversifiable risk. It is demonstrated, on the lines of Hart (1983), that inefficiency results can be generated with a risk-neutral principal if a risk-averse manager, taking an unobservable action, is introduced into the model. Such an arrangement therefore simply restores the results of a model with a monolithic risk-averse principal without in any sense justifying such an assumption. It is further demonstrated that the above inefficiency results follow from a particular specification of moves made by the manager and the degree and nature of informational advantage he has over the principal. A familiar alternative, and in our view, more reasonable specification is proposed and it is shown how this may yield fully efficient results. A distinction is made between 'risk-aversion' and 'risk-averse behaviour'. It is shown that Hart's specification of moves removes all notion of 'risk-averse behaviour' by the manager as too much is known by him with certainty. The alternative specification is more realistic for capturing 'risk-averse' behaviour but may lead to contrary results by generating over-employment. On the basis of the analysis, an index of 'risk-averse behaviour' is proposed in terms of the concavity of the manager's utility function with respect to the profits of the firm. The argument in favour of introducing the alternative moral-hazard specification is two-fold; first to model the rationale for a risk-averse principal in asymmetric information models and second to demonstrate the sensitivity of the results to the altered sequence of moves. A third purpose is also served with a useful method of approximating incentive schemes in the neighbourhood of the first-best being derived as an essential part of the analysis.

1. Introduction.

This chapter is an investigation of some important strands of the principal-agent literature. It starts from the observation that assumptions about the risk-preferences of various parties to the contract affect the efficiency of the optimal contract. Section 2 of the chapter is therefore particularly concerned with the assumption made by Grossman and Hart, in several of their papers, of a risk-averse principal. Although presented as an apparently minor and uncontroversial assumption, the elaborate and influential theory of asymmetric information labour contracts developed in recent years hinges rather crucially on it.

There are two major reasons for concentrating attention on the fine details of implicit contract models. Firstly it constitutes an example - of a striking kind - of how assumptions on risk preferences, even of a simple and seemingly innocuous kind, alter the efficiency of contracts. This was demonstrated in Section 5 of Chapter 1 and we shall extend the discussion in the next section of this chapter. Secondly, it should be noted that the literature on implicit contracts under asymmetric information developed as a response to the symmetric information literature. The conclusions of the latter were rather disheartening, as an explanation of involuntary unemployment, in so far as they predicted that workers would end up being indifferent between employment and unemployment [1]. This conclusion followed from the analysis regardless of whether the principal was risk-averse or risk neutral. The symmetry of information generated the result. The more reasonable conclusions of the asymmetric information literature, notably its description of involuntary unemployment and the inefficiency of the contracts signed, are based on the assumption of a risk-averse principal, given the specification of the utility function for the risk-averse workers. Risk-aversion as much as asymmetry drives these results. Asymmetry of information is easy to justify in general terms and has indeed been the focus of a great deal of recent economic research. Risk-aversion of the principal is rather more difficult to support. The emphasis is on Grossman-Hart's (1983a,b) own concession that risk-aversion, as a postulate of their model, is less realistic in large firms with many shareholders each with diversified portfolios. Their rationalisation for a risk-averse large firm is that an agent is appointed to run the firm and it is he who behaves in a risk-averse manner. They state

> "...the risk-aversion of the principal simply reflects the risk-aversion of the manager.." (Hart (1983) p.19)

Even more importantly, this manager takes an unobservable action 'a'. This unobservability serves the additional crucial role of justifying the existence of asymmetry of information in large firms where both profits and employment are observed. In terms of Grossman and Hart's own formulation,

(1.1.1) $P = P(\ell, \theta)$

where P is a symbol for gross (net) profits while ℓ is employment. If P and ℓ are both observed, θ is observed too and the asymmetry of information disappears. The introduction of unobservable 'a' as a determinant of profits restores unobservability of θ.

In this chapter it is demonstrated (on the lines of Hart (1983)) that inefficiency results can be generated with a risk-neutral principal if a risk-averse manager taking an unobservable action is introduced into the model. Such an arrangement therefore restores the results of a model with a monolithic risk-averse principal without, in any sense, justifying such an assumption. However, it is further demonstrated that the above inefficent underemployment results follow from a particular specification of moves made by the manager and the degree and nature of informational advantage he has over the principal. A familiar alternative and, in our view, more reasonable specification of moves is proposed and it is shown how this may yield fully efficient results. A distinction is made between 'risk-aversion' and 'risk-averse behaviour'. It is shown that Hart's specification of moves removes all notion of 'risk-averse' behaviour by the manager as too much is known by him with certainty. The alternative specification is more realistic for capturing risk-averse behaviour but may lead to contrary results in the shape of over-employment. On the basis if the analysis, an index of risk-averse behaviour is proposed in terms of the concavity of the manager's utility function with respect to the profits of the firm. The argument therefore in favour of introducing the alternative moral-hazard specification of moves is two-fold; first to model the rationale for a risk-averse principal and second to illustrate the consequences of the altered sequence of moves for the employment results and while these form the two most important themes of this chapter, a third purpose is also served, with a useful method of approximating incentive schemes in the neighbourhood of the first-best being derived as an essential part of the analysis. This overcomes an often heard criticism of principal-agent models - namely that incentive schemes are often extremely difficult to solve for explicitly without recourse to numerical techniques. The techniques developed in this chapter therefore have general relevance as aids to working out the effects of moral hazard on effort choice, of the agent, in quite general models. Much of the succeeding analysis is undertaken within the framework of a principal-agent model. This is quite natural as principal-agent models have been extensively used in the macro-economics literature. Furthermore, the firm may be viewed primarily as a series of principal-agent relationships, for example between the owner of the firm or shareholders and the manager appointed by them to run the affairs of the firm, between the shareholders and the workers employed in the firm or yet again between the manager of the firm and its workers.

The structure of the chapter is as follows. Section 2 returns to the arguments presented in Section 5 of the first chapter but concentrates on the specification of the utility function of the agent. It also discusses in detail the rationalisation for a risk-averse principal suggested by Grossman and Hart and presents an alternative specification. The stage is then set for the chapter to pursue its two major themes. The moral-hazard model is presented in detail in Section 3 and we then proceed to a definition of risk-averse behaviour and demonstrate some potential ambiguities. Section 4 contains an illustrative example, showing how there may be a concurrence of the risk-neutral principal's interests with those of the risk-averse agent's (taking an action 'a') casting some doubt on the Grossman-Hart rationalisation. Section 5 extends the arguments of Section 3 by developing approximation techniques based on small amounts of uncertainty in the distribution of profits. This helps clarify some previous ambiguities but leads to others with important consequences for employment results. The analysis is presented using a special case in Section 5 while the general case is developed in Section 6. The method of approximating second-best incentive schemes, in the neighbourhood of the efficient scheme, even for analytically intractable cases, is an important component of the analysis in this section - which also concludes the investigation of the first theme of the chapter. Section 7 uses the preceding analysis to show that in several cases inefficient employment results will prevail but in a direction opposite to that commonly supposed. The conclusions are mainly independent of any special specification of utility or density functions. .

2. The Grossman-Hart Model: Why Assumptions Matter.

It was shown in Section 5 of Chapter 1 that the assumptions of the Grossman and Hart model - per se - led to fully efficient results in the presence of a risk-neutral principal. Farmer (1984) succeeded in obtaining inefficient employment results with a risk-neutral principal by introducing external institutional arrangements into the model such as bankruptcy and limited liability.[2] Various arguments have been presented by Grossman and Hart themselves in justifying the risk-aversion of the principal, arguments which form the basis for discussion in a substantial part of this chapter.

Before proceeding to this discussion it is worth noting that the specification of the utility function of the worker is also important for the implementability of the first-best contract with a risk-neutral principal. This can be seen from the following example. Suppose the utility function of the worker were given by,

$U(w, \ell) = U(w) - \ell$, where U is strictly concave.

If the principal is risk-neutral, the first-best contract implies that the wage paid to the worker is independent of the employment state. This means that in the first-best the worker's utility is lower the higher ℓ is, that is in good states, where ℓ is high, the worker is badly off. We no longer get coinsurance. This scheme cannot be implemented under asymmetric information. For, if the wage is state independent, then it is in the firm's interest always to report that a good state has occurred since, by doing this, it obtains more labour and pays no higher wages. In such cases, there is a tendency for the firm to overemploy the worker. It turns out, however, that this inefficiency or distortion can be overcome if a third party is included in the contractual negotiations, in particular, if the firm and worker had access to a risk-neutral insurance company at date 0. If the firm is risk-neutral, the first-best contract can now be implemented. However, if the firm is risk-averse, the first-best and the second-best are no longer the same, even with a risk-neutral third party. This explains why we do not focus much attention on the specification of the utility function of the worker because it is clear that implementability of the first-best contract rests essentially on the specification of the risk-preferences of the principal.

Grossman and Hart concede that while in a small owner managed firm it seems intuitively reasonable that the owner is risk-averse, this rationalisation breaks down in large publicly owned firms with n shareholders because n may be very large and with each shareholder holding a very small stake in the firm the usual risk-spreading arguments apply . For the case of large firms, the justification takes the form of the introduction of a manager.

> "... We now argue that the firm is run by a manager and that the firm's profits depend upon how hard this manager works..."
>
> Hart (1983 p.18)

and further,

> "... the introduction of the manager who takes an action 'a' also justifies the assumption made throughout that the firm is risk-averse. The risk-aversion of the firm simply reflects the risk-aversion of the manager. Furthermore, the manager is risk-averse because the owners, in order to make him work hard gives him an incentive scheme...which makes

him bear a lot of risk.."
Hart (1983 p.19)

We do not take exception to the first rationalisation. Rather we pick up the second and model the relationship between the principal who is risk-neutral (representing the shareholders) and the agent (manager) who is risk - averse. The major part of the analysis in Sections 3, 4, 5 and 6 is devoted to making precise the sense in which the manager may (or may not) be said to be behaving in a risk - averse manner.

There are at least two points that should be noted following on from the second paragraph quoted above. First, the risk-aversion of the manager claimed above cannot refer to the commonly stated notion of risk-aversion mainly because the utility function of the manager is defined in a rather complicated fashion over income and action. Let us therefore adopt the convention that henceforth, unless otherwise stated, whenever we refer to 'risk-aversion' we are actually using the term as short-hand for 'is seen to behave in a risk-averse fashion'. The term is meant to apply as a description of the net behaviour of the principal-manager combination and **not** as a property of the utility function. Secondly, in presenting our arguments we differ from Hart (1983) on a substantial point. It concerns the timing of the choice of action by the manager. The differences are depicted schematically below:

<u>HART SEQUENCE</u>

Stage 0: Contract signed between shareholders and manager and manager giving incentive scheme I(.) as a function of observables

$$\downarrow$$

Stage 1: State occurs. Not observed by firm but observed by manager

↓

Stage 2: 'a' chosen by manager. Profits are realised and the contract is implemented.

We argue instead that in the above sequence of events, it is hard to justify the behaviour of the manager as reflecting anything about his preferences towards risk because when he chooses 'a' he knows everything with certainty. We shall therefore specify and frame our definition of risk-averse behaviour of the manager in terms of an alternative sequence of moves from that postulated by Hart. The alternative sequence we propose simply reverses the order of stages 1 and 2. Thus the agent has to choose 'a' before the random state occurs. The following sequence is postulated,

ALTERNATIVE SEQUENCE

STAGE 0 : Contract signed between shareholders and manager I(.) as a function of observables

↓

STAGE 1: Manager chooses 'a' before state of nature occurs.

↓

State of nature occurs

↓

STAGE 2: Contract implemented, profits accrue

Thus, our model differs from Hart's in one crucial respect - the timing of the agent's actions. We discuss, in detail, the consequences of this re-specification for employment results in Section 7 - as this is the focus of Hart's analysis - but we shall first concentrate on isolating possible risk-averse behaviour by the manager or principal - manager combinations.

We define risk-averse behaviour of the manager with respect to the profits of the firm in terms of the concavity of

$$W(P) = V(I(P))$$

where V is the strictly concave utility function of the manager and P is output or profits suitably defined. We ask whether the structure of the optimal incentive scheme I(P) is such that

$$W''(P) < 0$$

where we know that $V'' < 0$.

Note therefore that

$$W''(P) = V''(I(P))(I'(P))^2 + V'(I(P))I''(P)$$

and may therefore be ambiguous if I(P) is a convex function of P. The concavity of I is of course sufficient to guarantee the concavity of W. Since we conduct the analysis under the alternative sequence of moves we shall use a moral-hazard model, where the manager (agent) faces uncertainty when making his effort choice. The moral-hazard model on which the the analysis is based is presented in the next section.

3. The Moral-Hazard Model.

We shall introduce the basic moral-hazard model and then proceed to the main discussion in this section. This discussion has general relevance for all the chapters following and we shall refer back to it wherever appropriate. The model is adapted from that appearing in

Holmstrom (1979) and the notation established here is used throughout the thesis. The assumptions are as follow:

(1) The principal's utility function is defined over wealth alone and is given by $U(w)$.

(2) The agent's utility function is $V(w, a)$ and is defined over wealth and action. In our formulation,

$V(w, a) = V(w) - C(a)$ where $C''(a) \geq 0$

(3) $I(P)$ denotes the share of P that goes to the agent and $H(P)=P-I(P)$ denotes the share that goes to the principal.

(4) It is assumed that both parties agree on the probability distribution of θ, the random variable that along with 'a' determines P, since $P = P(a, \theta)$. The agent privately chooses 'a' before θ is known.

The constrained Pareto-optimal sharing rules I(P) and H(P) are generated by the program:

(3.1) $\max\limits_{I(P),a} \quad E[U(H(P))]$

subject to,

(3.2) $E[V(I(P)), a] \geq V_0$

(3.3) $a \in \operatorname*{argmax}\limits_{a' \in A} E[V(I(P)), a']$

Following Holmstrom (1979) we distinguish between a first-best and a second-best solution. (3.2) guarantees the agent a minimum expected utility. (3.3) reflects the restriction that the principal can observe P but not 'a'. If he could also observe 'a', a forcing contract could be used to guarantee that the agent selects a proper action even when I(P) is chosen to solve (3.1) - (3.2) ignoring (3.3). This is the first-best solution that entails optimal risk-sharing. It differs from what is termed the second-best solution of (3.1) subject to (3.2) and (3.3).

Equivalently :

(3.4) $\underset{I(.),a}{\mathrm{Max}} \int U(H(P))f(P, a)dP$

subject to

(3.5) $\int V(I(P))f(P,a)dP - C(a) \geq V_0$

(3.6) $\int V(I(P))f_a(P,a)dP - C'(a) = 0$

The assertion that the programs (3.1) - (3.3) and (3.4) - (3.6) are equivalent is valid only under certain circumstances. Solving (3.4) - (3.6) to determine the optimal incentive scheme and optimal action choice is termed the first-order approach to solving moral-hazard models. As might be expected such an approach is not always valid, in the obvious sense that we cannot be sure of obtaining a maximum unless certain conditions on the density function and distribution function of profits are satisfied. Grossman and Hart (1983b) and Mirrlees (1979) describe how the use of this approach can often lead to misleading results. For the moment note that the simplest and the strongest possible sufficient conditions for the validity of the first-order approach are the Mirrlees/Rogerson conditions. These are conditions on the density and distribution functions of P and ensure the concavity of the agent's expected utility function in action.

Condition 1: The Monotonic Likelihood Ratio Condition(MLRC)

The function f(P, a) is said to satisfy MLRC if $f_a(P, a)/f(P, a)$ is non-decreasing in P for every 'a'. MLRC also implies $F_a(P, a) \leq 0$ which is the so-called stochastic dominance condition (SDC).

Condition 2: The Convexity of the Distribution Function Condition (CDFC)

This is simply that $F_{aa}(P,a) > 0$ for all $a \in A$.

Researchers in the optimal incentives literature are well aware that in general, for most reasonable specifications of density functions and their corresponding distribution functions, the <u>sufficient</u> conditions will <u>not</u> be satisfied. In particular the class of distribution functions satisfying Condition 2 is restrictive. However, since these are precisely <u>sufficient</u> and not <u>necessary</u> conditions, we shall make the assumption that the first-order approach still gives us the correct result in the examples where we choose to use it.[3]

Under the standard interpretation of the effort variable 'a' given a distribution of θ, if F(P, a) is the distribution of P imposed on P via the relationship via P = P(a, θ), $F_a(P, a) \leqslant 0$ for all P, with strict inequality holding for some P values. This means that a change in 'a' has a non-trivial effect on the distribution of P. In particular, it will shift the distribution to the right in the sense of first-order stochastic dominance. Under this assumption and the additional Mirrlees/Rogerson conditions, it may be proved that I is an increasing function of P. A similar conclusion holds if the first-order approach is valid despite the Mirrlees/Rogerson conditions not being satisfied. This is proved quite easily via a lemma which appears in Holmstrom (1979) and Rogerson (1985).

<u>Lemma</u> : Assume V' > 0 , $F_a \leqslant 0$, with strict inequality for some P values, MLRC holds and the first-order approach is valid. Then μ, which is the Lagrange multiplier associated with the constraint (3.6), is greater than zero.

Note that λ is the multiplier associated with the constraint (3.5).
We are now in a position to work out the expressions for I'(P) and I"(P) which are needed to evaluate W"(P). The importance of the above Lemma

is simply that it enables us to establish the monotonicity of I in P ; that is we can be sure that $I'(P) > 0$. This is evident from the following expression for $I'(P)$ which may be worked out from the model:

$$(3.7)\quad I'(P) = \frac{(V'(I(P))U''(H(P)) - \mu\{\partial(f_a/f)/\partial P\}}{(V'U'' + U'V'')}$$

The monotonicity result claimed now follows easily from the assumptions made about the signs of the various component parts of the above expression. In particular, MLRC is very important for signing $I'(P)$ unambiguously.

We conclude this section by listing for future reference the expressions for $I''(P)$. A little tedious algebra yields,

$$(3.8)\quad I''(P) = \{\psi(P) - I'(P)\alpha'(P)\}/\{V'U'' + U'V''\}$$

where

$$(3.9)\quad \psi(P) = V''(I(P))I'(P)U''(H(P)) + V'(I(P))U'''(H(P))(1-I'(P)) - \mu[\partial^2(f_a/f)/\partial P^2](V'(I(P)))^2 - 2\mu\{\partial(f_a/f)/\partial P\}V'(I(P))V''(I(P))I'(P)$$

$$(3.10)\quad \alpha'(P) = V''(I(P))I'(P)U''(H(P)) + V'(I(P))U'''(H(P))(1-I'(P)) + U'(H(P))V'''(I(P))I'(P) + V''(I(P))U''(H(P))(1-I'(P))$$

$I''(P)$ is therefore ambiguous in sign. Some simplification is possible in the case of a risk-neutral principal. Even so the ambiguity remains. The significance of (3.7) and (3.8) will become clear in due course as we proceed to the main propositions of this chapter.

There is at least one example where we need not worry about the obvious ambiguities in the sign of $W''(P)$. In this example, adapted from Mirrlees (1979) and presented in the next section, it seems unncessary to be worried about risk-averse behaviour of the agent. There exists a clear possibility for the first-best contract to be implemented - which is desired by the risk-neutral principal - and therefore issues such as risk-averse behaviour of the agent are redundant. This is also an illustration of an important consequence of working with the alternative specification of moves because under the Hart sequence the probability of the first-best contract being implemented is infinitesmal. In general however, the need to tackle the ambiguity of $W''(P)$ over the entire range of Ps forces us to evaluate $W''(P)$ in the neighbourhood of the first-best incentive scheme. The argument is therefore an approximation one with unobservability of 'a' induced by small variances σ in the distribution of profits and is discussed in detail in Sections 5 and 6.

4. An Interesting Example.

We use this example, adapted from Mirrlees (1979), to demonstrate the claim that for certain specifications of density functions of profits, the agent may be made indifferent across a wide-range of effort choices. He will in particular be indifferent between a high variance choice and a low variance choice (for values of variances in a particular range) and thus it is difficult to agree with statements like 'the risk-aversion of the firm simply reflects the risk-aversion of the manager'. The agent is clearly behaving in a 'risk-neutral' fashion by displaying indifference across a wide range of variance choices.

The assumptions of this example are as follow:

(i) Risk-neutral principal

(ii) Profits P distributed uniformly, that is,

(4.1) $P \sim \text{Uniform } [0, h(a)]$ where 'a' is the effort variable

$h'(a) > 0$

$h''(a) < 0$

The agent will choose 'a' to maximise,

(4.2) $h^{-1}.\{\int_0^h V(P-H(P))dP\} - a$

Let us denote the optimum value of utility by V and the optimum a by a*.

Write,

(4.3) $\int_0^h V(P-H(P))dP = (V+a)h - w(h)$

When $h = h^* = h(a^*)$, $w = 0$

For other h, $w \geq 0$

Differentiating (4.3) with respect to 'a' we obtain,

(4.4) $V(h - H(h)) = (V+a) + h/h'(a) - w'(h)$

We now pose the question,

When $h''(a) < 0$, is the optimum of the form

(4.5) $w^*(h) = 0$ for all $h \in [0, h^*]$?

Suppose not. That is, let the contract be such that

(4.6) $w \geq 0$ for all $h \in [0, h^*)$

$w(h^*) = 0 \quad h = h^*$

We compare this with another policy such that the following two conditions hold for the policy:

(4.7a) $\int_0^h H^*(P)dP = \int_0^h H(P)dP$

(4.7b) $w^*(h) = 0$ for all $h \in [0, h^*]$

Now for the old contract,

(4.8) $V(h - H(h)) = (V+a) + h/h'(a) - w'(h)$

With the new contract,

(4.9) $V(h-H^*(h)) = V^* + a + h/h'(a)$

Thus,

(4.10) $V(h-H(h)) - V(h-H^*(h)) = V - V^* - w'(h)$

By the inequality for concave functions, this implies,

(4.11) $[H^*(h) - H(h)]V'(h-H^*(h)) \geq V - V^* - w'(h)$

Thus,

(4.12) $\int_0^{h^*}[H^*(h) - H(h)]dh \geq (V-V^*).\int_0^{h^*}V'(h-H^*(h))^{-1}dh$
$- \int_0^{h^*}w'(h)\{V'(h-H^*(h))\}^{-1}dh$

By (4.7a), the left hand side of (4.12) is zero.

Integrating by parts,

(4.13) $(V-V^*).\int_0^{h^*}[V'(h-H^*(h))]^{-1}dh \leq -w(0)[V'(-H^*(0))]^{-1}$
$+ \int_0^{h^*}V''w[d\{(h-H^*(h))\}/dh](V')^{-2}dh$

Now from (4.9) above,

(4.14) $V'[d\{(h-H^*(h))\}/dh] = 2/h'(a) - h.h''/(h')^2 > 0$, given that $h''(a)<0$.

Therefore,

(4.15) $(V-V^*).\int_0^{h^*}[V'(h-H^*(h))]^{-1}dh < 0$ since $w \geq 0$ also.

This implies $V < V^*$.

Thus the new contract is Pareto superior to the old contract and to any contract that has $w(h) \geq 0$ for $0 \leq h \leq h^*$.

The optimal contract is the one that makes the agent indifferent across a range of choices of 'a' and there is no possibility of 'apparent risk-averse' behaviour. Furthermore, suppose the first-best choice of 'a' is given by solving,

(4.16) $\mathrm{Max}_a\ h(a)^{-1}\int_0^h\{U(P-I(P) + \lambda V(I(P))\}dP$ leading to an optimal choice of $h = h' \in [0, h(a^*)]$ then the first-best contract is implementable and the production results are entirely efficient.

This example is a fairly important case to consider and is not as lacking in generality ,as it might initially seem, for we have allowed the agent to have any concave utility function. It has also illustrated the dangers of taking Grossman and Hart's particular rationalisation too literally. This theme is developed in the remaining sections.

5. The Grossman and Hart Rationalisation - A Special Case

We shall work with the second specification of the sequence of events and therefore within the moral-hazard framework. Risk-averse behaviour of the manager with respect to the profits of the firm will be defined in terms of the concavity of $W(P)=V(I(P)$. We ask whether the structure of the optimal incentive scheme $I(P)$ meant that $W''(P) < 0$ where we know that $V'' < 0$. Note therefore that, $W''(P) = V''.(I'(P))^2 + V'.(I''(P))$ and therefore may be ambiguous if $I(P)$ is a convex function of P. The concavity of I is of course sufficient to guarantee the concavity of W. The concavity (respectively convexity) of $W(P)$ is a useful guide to determining potential risk-averse (respectively risk-preferring behaviour) of the firm as do Grossman and Hart(1983a p.149). In the example discussed in this section there is no ambiguity in the sign of $W''(P)$ despite the convexity of the optimal incentive scheme in P. In general however, we choose to evaluate $W''(P)$ in the neighbourhood of the first best incentive scheme and use its sign as an index of risk-averse (preferring) behaviour. The aim is to discover how the manager is likely to behave when exposed to small amounts of risk as represented by a value of the variance (or of any suitable measure of the spread of the distribution of profits) in the neighbourhood of zero. Suppose, on the basis of the index, there exists potential for risk-preferring behaviour by the manager. We conjecture that if there exists an additional dimension of choice, relating purely to the choice of riskiness of the project, he would choose the riskier project. We formalise this argument, briefly, a little later.

The value of using such approximation techniques is that they also enable us to derive explicit expressions for the second-best choice of effort and consequently the deviation from the first-best - or symmetric information - choice of 'a' without resorting to numerical optimisation by computers. The expressions for the second-best choice of effort are of course valid only in the neighbourhood of the first-best. We shall pay particular attention, in the remaining sections of this chapter to

(a) the sign of $W''(P)$ and

(b) the deviation of the second-best effort choice from the first best.

While the relevance of (a) has already been discussed and will be discussed again later, the importance of (b) will become evident in Section 7 where we derive the consequences for employment results of our alternative specification.

To make the argument more formal, we are interested in comparing two situations. The first where profits are known with certainty, that is,

(5.1) $P = a$ where 'a' is the effort choice of the agent .

This is the situation of no moral-hazard - perfect observability of effort - and the outcome is the first-best outcome. Let it lead to a choice of $a = a_0$, the first-best effort choice. Now, consider a second situation where

(5.2) $P = a + \sigma q$ where $q \sim (0, 1)$.

This is the situation of imperfect observability of effort. The purpose of this analysis is to work out the optimal choice of effort in the second situation for small σ (that is $\sigma \varepsilon N(0)$)[4] and thereby deduce the sign of $\partial a/\partial \sigma$ in the neighbourhood of the first-best choice of effort. The comparative static results, notably the change in 'a' with a change in 'σ' are fairly easy to solve for explicitly out in special cases. In the more general case where the utility and density functions are not specified, approximation methods, such as those developed in Section 6, must be used.

Strictly speaking, the analysis is not yet complete. In comparing just the two situations above we will still have some difficulty in unscrambling the effects of what we shall call 'pure moral-hazard or pure unobservability' and 'pure risk-aversion' although such a comparison helps to provide a strongly suggestive view of the way a particular agent is behaving. Therefore, to tie up the analysis completely we would like to consider a third situation,

(5.3) $P = \sigma q\beta + a$ and the agent chooses 'a' and 'β' the first with a disutility of C(a) and the second costlessly. He thus has an influence on the choice of the variance of the project. This choice, by allowing a second dimension of choice to the agent, does seem important in describing properly notions of risk. We are interested in evaluating the changes in the choice of 'a' at each stage locally at $\sigma = 0$. The prospect of solving a 5 X 5 matrix system, with all the associated ambiguities, deters us, at least for the present, from considering the third situation.

This section is now developed in two stages. First, the first-best solution is characterised and second, comparative static results are worked out for a special case. The general case is described in the next section.

Stage 1 : Perfect Observability

$\sigma = 0$

The problem reduces simply to

(5.4) Max $\{a - I(a)\}$

subject to

(5.5) $\{V(I(a)) - C(a)\} \geqslant V_0$

We have therefore,

(5.6) $L = a - I(a) + \lambda\ [V(I(a)) - C(a) - V_0]$

and the solution to the system is given by

(5.7a) $C'(a) = \lambda^{-1}$

(5.7b) $\lambda V'(I^*) = 1$

implies,

(5.8a) $V'(I^*)/C'(a^*) = 1$

(5.8b) $V(I^*) - C(a^*) = V_0$ because from (5.7b), $\lambda > 0$.

Note that these equations are identical to those we would have obtained if we had solved for the symmetric information (observability of 'a') outcome but with uncertainty attached.

Stage 2: Imperfect Observability - Special Case.

The special case is constructed as follows.

(1) We have a risk-neutral principal

(2) The utility function of the agent is given by $V(I) = 2I^{1/2}$

(3) The cost of effort function is given by $2C(a) = 2.\exp(a)$

(4) The reservation expected utility of the agent is $2V_0$.

(5) Profits are distributed normally with mean equal to 'a', the effort choice by the agent.

$f(P, a) = (2\pi\sigma^2)^{-1/2}.\exp\{-(P-a)^2/2\sigma^2\}$

$f_a(P,a)/f(P,a) = \sigma^{-2}.(P-a)$

Next consider the system comprised of (3.4) - (3.6). Forming the appropriate Lagrangean and differentiating with respect to I(.), we have $\lambda + \nu f_a/f = (V')^{-1}$, where ν [5] is the Lagrangean multiplier associated with (3.6), and in the special case,

$$\left.\begin{array}{ll} (5.9a) & \lambda + \mu.(P - a^*) \leqslant I^{1/2} \\ (5.9b) & I \geqslant 0 \end{array}\right\} \quad \text{Complementarily slack}$$

a^* is the second best optimal choice of effort, $\mu = \nu/\sigma^2$ which is assumed to have a finite positive limit [6] and the complementary slackness conditions must hold because of the structure of the utility function.

We also have the condition (3.5),

$\int V(I(P))f(P,a)dP - C(a) = V_0$

From (5.9a) and (b) we have

(5.10a) $I = 0$ for all $P < -\lambda/\mu + a^*$ which also implies

(5.10b) $V(I) = 0$ for all $P < -\lambda/\mu + a^*$

Also

(5.10c) $2(\lambda + \mu.(P - a^*)) = 2.I^{1/2} = V(I)$ for all $P \geqslant -\lambda/\mu + a^*$

Substituting in (3.5) and noticing the relevant range of P, we have

$$[\int_{a^* -\lambda/\mu}^{\infty} (2\pi\sigma^2)^{-1/2}(\lambda + \mu(P-a^*)).\exp\{-(P-a)^2/2\sigma^2\}dP - C(a) - V_0] = A$$

Making the change of variable $q = (P-a)/\sigma$ and noticing that $(P-a^*)$ can be written as $(P-a) + (a-a^*)$ we can write

$$\hat{A} = 2[\{\lambda + \mu(a-a^*)\}.\int_{s}^{\infty} (2\pi)^{-1/2}.\exp(-q^2/2)dq$$
$$+ \mu\sigma(2\pi)^{-1/2}.\exp(-(a^*-a-\lambda/\mu)^2/2\sigma^2)$$
$$- C(a) - V_0]$$

where $s = (a^* - a - \lambda/\mu)\sigma^{-1}$

Symbolically,

$$\hat{A} = 2[U(a) - C(a) - V_0]$$

$\Rightarrow A(a,\lambda,\mu,\sigma) = \hat{A}(a,\lambda,\mu.\sigma)/2 = 0$

in rather obvious notation.

We also have (3.6), the first order condition with respect to choice of effort,

$B = A_a(a,\lambda,\mu,\sigma) = B(a,\lambda,\mu,\sigma) - 0$

We shall derive B explicitly later and ignore multiplicative constants. First let us note that we have a third equation available to us for the analysis. This is the derivative of the Lagrangean with respect to a.

$L = \int (P-I(P))f(P, a)dP + \lambda\Lambda + 2\sigma^2\mu.B$

Therefore, substituting for the optimal incentive scheme and differentiating L with respect to 'a' we have,

$$H = \partial L/\partial a = 1 - 2\mu.\int_s^\infty (\lambda + \mu\sigma q + \mu(a-a^*))f(q)dq + 2\sigma^2\mu.B$$

Thus $H = 1 - 2\mu.U(a) + 2\sigma^2\mu\ [\ U''(a) - C''(a)] = 0$

Summarising, we have the three equations,

(5.11a) $A(a,\lambda,\mu,\sigma) = 0$

(5.11b) $B(a,\lambda,\mu,\sigma) = 0$

(5.11c) $H(a,\lambda,\mu,\sigma) = 0$

Linearising the system by differentiating each of (5.11a) - (5.11c) with respect to σ, we have,

(5.12a) $A_1.\partial a/\partial\sigma + A_2.\partial\lambda/\partial\sigma + A_3.\partial\mu/\partial\sigma + A_4 = 0$

(5.12b) $B_1.\partial a/\partial\sigma + B_2.\partial\lambda/\partial\sigma + B_3.\partial\mu/\partial\sigma + B_4 = 0$

(5.12c) $H_1.\partial a/\partial\sigma + H_2.\partial\lambda/\partial\sigma + H_3.\partial\mu/\partial\sigma + H_4 = 0$

The task is to evaluate (at least the signs) A_1, A_2, A_3, A_4, etc. so that by using Cramer's rule, we can deduce the signs and possibly the precise magnitudes of $\partial a/\partial\sigma$, $\partial\lambda/\partial\sigma$ and $\partial\mu/\partial\sigma$. We are especially interested in the sign of $\partial a/\partial\sigma$ for reasons stated before.

The following expressions may be derived by some tedious algebra.

$$A_1 = \mu \int_s^\infty (2\pi)^{-1/2}\exp(-q^2/2)dq - C'(a) \qquad (5.13)$$

$$A_2 = \int_s^\infty (2\pi)^{-1/2}\exp(-q^2/2)dq \qquad (5.14)$$

$$A_3 = (a-a^*)\int_s^\infty (2\pi)^{-1/2}\exp(-q^2/2)dq$$

$$+ \sigma(2\pi)^{-1}\exp(-(a^*-a-\lambda/\mu)^2/2\sigma^2) \qquad (5.15)$$

$$A_4 = \mu(2\pi)^{-1/2}\exp(-(a^*-a-\lambda/\mu)^2/2\sigma^2) \qquad (5.16)$$

$$B_1 = \mu(2\pi\sigma^2)^{-1/2}\exp(-(a^*-a-\lambda/\mu)^2/2\sigma^2) - C''(a)$$

$$(5.17)$$

$$B_2 = (2\pi\sigma^2)^{-1/2}\exp(-(a^*-a-\lambda/\mu)^2/2\sigma^2) \qquad (5.18)$$

$$B_3 = -\lambda\mu^{-1}(2\pi\sigma^2)^{-1/2}\exp(-(a^*-a-\lambda/\mu)^2/2\sigma^2)$$

$$+ \int_s^\infty (2\pi)^{-1/2}\exp(-q^2/2)dq \qquad (5.19)$$

$$B_4 = (a^*-a-\lambda/\mu)\mu.\exp(-(a^*-a-\lambda/\mu)^2/2\sigma^2).\sigma^{\ 2} \qquad (5.20)$$

$$H_1 = -2\mu^2\int_s^\infty (2\pi)^{-1/2}\exp(-q^2/2)dq - 2\sigma^2\mu.C'''(a)$$

$$+ 2\mu^2(a^*-a-\lambda/\mu)(2\pi\sigma^2)^{-1/2}\exp(-(a^*-a-\lambda/\mu)^2/2\sigma^2)$$
$$(5.21)$$

$$H_2 = -2\mu.\int_s^\infty (2\pi)^{-1/2}\exp(-1/2q^2)dq$$

$$+ 2\mu(a^*-a-\lambda/\mu).(2\pi)^{-1/2}\exp(-(a^*-a-\lambda/\mu)^2/2\sigma^2)$$
$$(5.22)$$

$$H_3 = -2(\lambda+2\mu(a-a^*)).\int_s^\infty (2\pi)^{-1/2}\exp(-q^2/2)dq$$

$$-[2\lambda\sigma^{-2}(a^*-a-\lambda/\mu)]\sigma(2\pi)^{-1/2}(\exp(-(a^*-a-\lambda/\mu)^2/2\sigma^2)$$

$$(5.23)$$

$$H_4 = [-4\mu^2+2\sigma^{-2}(a^*-a-\lambda/\mu)^2](2\pi)^{-1/2}\ \exp(-(a^*-a-\lambda/\mu)^2/2\sigma^2)$$

$$+4\sigma\mu B_a \qquad (5.24)$$

We evaluate (5.13) - (5.24) at a=a* and subject them to two alternative processes. Note first that $\int f(q)dq \rightarrow 1$ as $\sigma \rightarrow 0$ because $s \rightarrow -\infty$ as $\sigma \rightarrow 0$. Process 1 is a complete limiting process i.e terms like $l_1 = \sigma^{-1}\exp(-\sigma^{-2}\eta(a))$ <u>and</u> $l_2 = \sigma^{-2}\exp(-\sigma^{-2}\eta(a))$ both vanish as $\sigma \rightarrow 0$, where $\eta(a)$ is any arbitrary function. In general it will make far more sense to evaluate the derivatives at a value of σ auch that terms like l_1 vanish but terms like l_2 still have finite <u>non-zero</u> values. This is denoted Process 2. We list below the signs of the derivatives under both processes 1 and 2 and also in general (i.e for any arbitrary value of σ not in neighbourhood of 0). The reason for looking at values of derivatives for small values of σ is obvious from the previous disçussion. We are primarily interested in considering small deviations from the first-best.

<u>Table 1</u>.

Sign of	In general	Under process 1	Under process 2
A_1	0 by ass.[7]	0	0
A_2	>0	>0	>0
A_3	>0	0	0
A_4	>0	0	0
B_1	<0 by ass.[8]	<0	<0
B_2	>0	0	0
B_3	AMBIGUOUS	>0	>0
B_4	<0	0	<0
H_1	<0	<0	<0

H_2	<0	<0	<0
H_3	AMBIGUOUS	<0	<0
H_4	AMBIGUOUS	0	>0

We now use Cramer's rule to work out the signs of $\partial a/\partial\sigma$, $\partial\lambda/\partial\sigma$ and $\partial\mu/\partial\sigma$. It may be confirmed that in general, the signs are ambiguous. This follows quite obviously from the ambiguities in the signs of the partial derivatives B_3, H_3 and H_4. This is not at all worrying because we are interested primarily in evaluating all the derivatives for $\sigma \in N(0)$. Under process 1 they all equal zero. This is natural, for if the limiting process is completed the solution approximates the first best very closely and 'a' and 'λ' have their first-best values while $\mu \rightarrow 0$ as the incentive compatibility constraint on the agent is redundant. However, most interestingly, still in the neighbourhood of zero but under process 2, the following results can easily be checked:

$\partial\lambda/\partial\sigma = 0$

$\partial\mu/\partial\sigma > 0$ unambiguously

$\partial a/\partial\sigma < 0$ if and only if $B_4H_3 > B_3H_4$ and less than zero otherwise. In this case $\partial a/\partial\sigma < 0$. The ambiguity in the sign of $\partial a/\partial\sigma$ is illuminating and we shall have much more to say about it in the next section. The positive sign on $\partial\mu/\partial\sigma$ is not surprising as there are several indications, as expected ,that in the second-best solution $\mu > 0$. It is possible to calculate explicitly the precise magnitudes of $\partial a/\partial\sigma$ and $\partial\mu/\partial\sigma$ and also the second-best choice of effort a* when $\sigma \in N(0)$. In order to solve the model explicitly for any $\sigma \in N(0)$, numerical optimisation techniques must be used.[9] We shall reserve judgment on this section until later. We have demonstrated how comparative static results may be worked out quite easily by using the equations generated by the principal-agent model. In the next section we extend the argument to cover any specification of well-behaved density or utility functions (but will have to resort to approximation techniques) and conclude the chapter with a definition of 'risk-averse behaviour of the firm'.

6. Grossman and Hart Rationalisation - The General Case.

Refer back to the system of equations (3.3) - (3.6). Let g be the density function of the random variable q satisfying Property P below:

(P) $\int (g'/g)^{i} g dq = 0$ for all $i \in \{1, 3, 5, \dots\}$

that is we shall work with symmetric density functions.

By making the change of variable $P = a + \sigma q$, the following equations may be derived:

(6.1) $\lambda - \mu\sigma^{-1}(g'/g) = (V')^{-1}$

(6.2) $\int V(I(q))g(q)dq - C(a) = W_0$

(6.3) $\int V(I(q))g'(q)dq + \sigma C'(a) = 0$

(6.4) $1 + \sigma^{-1}\int Ig'dq + \mu\sigma^{-2}\int V(I(q))g''(q)dq = \mu C''(a)$

where,

g(.) is the density function of the random variable q

(6.2) is the minimum utility condition,

(6.3) is simply the first-order condition for choice of effort,

while (6.4) is obtained by differentiating the Lagrangean formed from the system (3.3) - (3.6) with respect to 'a'.

μ is the multiplier associated with the constraint (6.3) [10].

These equations appeared in Section 6 in the slightly modified guises of $A(a,\lambda,\mu,\sigma)$, $B(a,\lambda,\mu,\sigma)$ and $H(a,\lambda,\mu,\sigma)$.

(6.1) is obtained by differentiating the Lagrangean with respect to I(.).

We shall focus on the sign of I"(P) and the first-order change, with respect to the first-best, in action choice, denoted a_1. We need the first expression for determining the concavity (convexity) of W(P) and the second for the employment results deduced in the next section.

In order to work with almost any specification of utility function and symmetric density function we shall need to use approximations of incentives schemes, utility levels and action choices around their first-best levels. Denoting , as before, the first-best levels of a, λ and μ by a_0, λ_0 and μ_0, we use (P) above to deduce most importantly the following approximations in powers of σ^2:

(6.5a) $a = a_0 + a_1\sigma^2 + a_2\sigma^4 + \ldots\ldots$

(6.5b) $\lambda = \lambda_0 + \lambda_1\sigma^2 + \lambda_2\sigma^4 + \ldots\ldots$

(6.5c) $\mu = \mu_0 + \mu_1\sigma^2 + \mu_2\sigma^4 + \ldots\ldots$

$= \mu_1\sigma^2 + \mu_2\sigma^4$

In order to obtain expressions for a_1, λ_1, μ_1 a_2, λ_2 and μ_2, we shall expand, using Taylor series, the expressions for the incentive scheme and exepcted utilities of the principal and the agent in powers of σ^2 and then gather terms of the same order to obtain a system of equations which may be solved for a_1, λ_1, μ_1 etc.

To a first-order approximation,

(6.6) $(V')^{-1} = (V'_0)^{-1} - (V''_0/V'^2_0)(I - I_0)$

$= \lambda_0 + \alpha.(I-I_0)$

where all expressions subscripted by 0 refer to first-best levels and

$\alpha = -(V''_0)/(V'^2_0) > 0$

Substituting (6.6) into (6.1) gives, to first-order,

(6.7) $(\lambda - \lambda_0).\alpha^{-1} - \mu.(\alpha\sigma)^{-1}(g'/g) = (I - I_0)$

We shall however derive a second-order approximation by using (6.7).

To second-order,

(6.8) $(V')^{-1} = \lambda_0 + \alpha.(I-I_0\ ) + \beta.(I-I_0)^2$ where $\beta = 1/2.(\partial\alpha/\partial I)|_{I=I_o}$

Substituting for $(I-I_0)$ from first stage, and using (6.1) again,

(6.9) $I-I_0 = \alpha^{-1}(\lambda-\lambda_0)-\mu(\alpha\sigma)^{-1}(g'/g)-\beta\alpha^{-1}\{\alpha^{-1}.(\lambda-\lambda_0)-\mu(\alpha\sigma)^{-1}(g'/g)\}^2$

Then substituting for $(\lambda-\lambda_0)$ and μ from (6.5a) - (6.5c) into (6.9), expanding and gathering terms, we have to $O(\sigma^4)$.

$$(6.10)\quad I - I_0 = -\mu_1\alpha^{-1}(g'/g)\sigma + \{\lambda_1\alpha^{-1} - \beta\alpha^{-3}\mu_1^{2}(g'/g)^2\}\sigma^2$$
$$-(\mu_2\alpha^{-1} - 2\beta\lambda_1\mu_1\alpha^{-3})(g'/g)\sigma^3$$
$$+\{\lambda_2\alpha^{-1} - \beta\lambda_1^{2}\alpha^{-3} - 2\beta\mu_1\mu_2\alpha^{-3}(g'/g)^2\}\sigma^4 + \ldots$$

Similarly, to $O(\sigma^4)$

$$(6.11)\quad (I - I_0)^2 = \mu_1^{2}\alpha^{-2}(g'/g)^2\sigma^2 + \{2\beta\mu_1^{3}\alpha^{-4}(g'/g)^3-2\mu_1\lambda_1\alpha^{-2}(g'/g)\}\sigma^3$$
$$+\{\lambda_1^{2}\alpha^{-2}-6\beta\lambda_1\mu_1^{2}\alpha^{-4}(g'/g)^2+2\mu_1\mu_2\alpha^{-2}(g'/g)^2$$
$$+\beta^2\mu_1^{4}\alpha^{-6}(g'/g)^4\}\sigma^4$$

Also, to second order,

$$(6.12)\quad \int Vgdq = \cancel{V_0} + V'_0.\int (I-I_0)gdq + (V''_0/2)\int (I-I_0)^2 gdq$$
$$= C(a) + W_0$$
$$= \cancel{C(a_0)} + C'(a_0)(a-a_0) + (C''(a_0)/2)(a-a_0)^2 + \cancel{W_0} \text{ by the}$$

minimum utility condition and expanding $C(a)$ to second order around a_0.

Substituting (6.12) into (6.2) and noting also from the first-best solution that, (6.13) $C'(a_0) = V'_0$ we have the following re-written version of (6.2)- specially labelled below as Equation 1:

Equation 1

$$(6.14)\quad \int (I-I_0)gdq + (V''_0/2V'_0)\int(I-I_0)^2gdq = a_1\sigma^2 + \{a_2+(C''/2C')a_1^{2}\}\sigma^4$$

again to $O(\sigma^4)$.

We have also used the approximation for $(a - a_0)$ given by (6.5a).

By (6.10) and Property (P),

$$(6.15)\quad \int (I-I_0)gdq = (\lambda_1\alpha^{-1} - \beta\alpha^{-3}\mu_1^{2}\gamma_2)\sigma^2$$
$$+(\lambda_2\alpha^{-1} - \beta\lambda_1^{2}\alpha^{-3} - 2\beta\mu_1\mu_2\gamma_2\alpha^{-3})\sigma^4 + \ldots$$

where $\gamma_i \equiv \int (g'/g)^i gdq$

$$(6.16)\quad \int (I-I_0)^2gdq = \mu_1^{2}\alpha^{-2}\gamma_2\sigma^2 + \{\lambda_1^{2}\alpha^{-2}-(6\beta\lambda_1\mu_1^{2}\alpha^{-2}-2\mu_1\mu_2)\gamma_2\alpha^{-2}$$

$$+ \beta^2\mu_1{}^4\alpha^{-6}\gamma_4\}\sigma^4 + \ldots$$

Now we substitute (6.16) and (6.15) into l.h.s of (6.14). Next, gathering terms of $O(\sigma^2)$ and $O(\sigma^4)$ on r.h.s and l.h.s of (6.14), we have,

(6.17a) $\lambda_1\alpha^{-1} - \mu_1{}^2\gamma_2\delta^{-2} = a_1$ where $\delta \equiv \beta\alpha^{-1} - (V''_0/2V'_0)$

Also,

(6.17b) $\lambda_2\alpha^{-1} - \delta(\lambda_1{}^2\alpha^{-2} + 2\mu_1\mu_2\gamma_2\alpha^{-2})$

$$- 6\beta\lambda_1\alpha^{-4}\gamma_2\mu_1{}^2(V''_0/2V'_0) + \beta^2\alpha^{-6}\mu_1{}^4(V''_0/2V'_0)$$

$$= (C''/2C')a_1{}^2 + a_2$$

Note now that,

(6.18) $\int Vg'dq = V'_0.\int(I-I_0)g'dq + (V''_0/2).\int (I-I_0)^2g'dq$

where,

(6.19) $\int (I-I_0)g'dq = -\mu_1\alpha^{-1}\gamma_2\sigma - (\mu_2-2\beta\lambda_1\mu_1\alpha^{-3})\gamma_2\sigma^3$ using (6.10) and Property (P)

and,

(6.20) $\int (I-I_0)^2g'dq=(2\beta\mu_1{}^3\gamma_4\alpha^{-4} - 2\mu_1\lambda_1\alpha^{-2}\gamma_2)\sigma^3$ using (6.11) and (P) again

Also, to second order,

(6.21) $\sigma C' = \sigma\{C'_0 + C''_0(a-a_0) + (C'''/2)(a-a_0)^2\}$

$$= \sigma C'_0 + \sigma^3 a_1 C''_0 \text{ to } O(\sigma^4).$$

We have from (6.3),

Equation 2

$\int Vg'dq = -\sigma C'$

Next, substitute (6.19) and (6.20) into (6.18) and insert in l.h.s of Equation 2. (6.21) is inserted into r.h.s of Equation 2. Now gathering terms of $O(\sigma)$ and $O(\sigma^3)$ on l.h.s and r.h.s of Equation 2, we have,

(6.22a) $\mu_1 = \alpha\gamma_2{}^{-1}$

$$(6.22b)\quad 2\lambda_1\mu_1\gamma_2\alpha^{-2}\delta - \mu_2\gamma_2\alpha^{-1} + \beta\mu_1{}^3\gamma_4\alpha^{-4}(V''_0/V'_0) = -a_1(C''_0/C'_0)$$

Equation 3

Finally, from (6.4),

$$(6.23)\quad 1 + \underset{(i)}{\sigma^{-1}\int Ig'dq} + \underset{(ii)}{(\mu_1 + \mu_2\sigma^2)\int Vg''dq} = \mu_1\sigma^2C''_0 \quad \text{to } O(\sigma^2).$$

In this equation, we shall equate terms up to order σ^2 because in order to legitimately equate terms of $O(\sigma^4)$ we need to expand $\int Ig'dq$ in (i) to $O(\sigma^5)$ which we have not done. In any case, such an expansion is unnecessary for the purposes of our analysis.

$$(6.24)\quad \sigma^{-1}\int Ig'dq = -\mu_1\gamma_2\alpha^{-1} - (\mu_2\alpha^{-1} - 2\beta\lambda_1\mu_1\alpha^{-3})\gamma_2\sigma^2 + \ldots$$
$$= -1 - (\mu_2\alpha^{-1} - 2\alpha\lambda_1\mu_1\alpha^{-3})\gamma_2\sigma^2 + \ldots$$

$$(6.25)\quad \int Vg''dq = V'_0\int(I-I_0)g''dq + (V''_0/2)\int(I-I_0)^2g''dq$$

$$(6.26)\quad V'_0(\mu_1+\mu_2\sigma^2)\int(I-I_0)g''dq = -(2/3)\beta\alpha^{-3}\mu_1{}^3(V'_0)\gamma_4\sigma^2 + \ldots\text{to } O(\sigma^2)$$

where we have used calculations like

$$\int(g'/g)^2g''dq = \int (1/g)^2d\{(1/3)(g')^3\}$$
$$=(2/3)\int ((g')^4/g^3)dq$$
$$=(2/3)\int(g'/g)^4gdq$$
$$=(2/3)\gamma_4 \text{ and then used Property (P).}$$

Similarly it may be shown that,

$$(6.27)\quad (V''_0/2)(\mu_1+\mu_2\sigma^2)\int(I-I_0)^2g''dq = (2/3)\mu_1{}^3\alpha^{-2}\gamma_4(V''_0/2)\sigma^2+\ldots$$

Substituting (6.26) and (6.27) into (6.25), then substituting (6.25) and (6.24) into (6.23) and gathering terms of $O(\sigma^2)$ on l.h.s and r.h.s of (6.23) (while noting that $\mu_1 = \alpha\gamma_2{}^{-1}$), we have

$$(6.28)\quad 2\beta\lambda_1\mu_1\alpha^{-3}\gamma_2 - \mu_2\alpha^{-1}\gamma_2 - (2/3)V'_0\mu_1{}^3\alpha^{-2}\gamma_4\delta = \mu_1C''_0 .$$

From (6.22a), we have the value for μ_1 in terms of exogenously determined parameters.

$$\mu = \alpha\gamma_2^{-1}$$

Consider the system of equations formed by (6.17a), (6.22b) and (6.28) having substituted for μ_1.

$$\begin{array}{llllll}
(6.17a) & \alpha a_1 & -\ 1.\lambda_1 & +\ 0.\mu_2 & = & -\alpha\delta\gamma_2^{-1} \\
(6.22b) & -(C''_0/C'_0)a_1 & -2\delta\alpha^{-1}\lambda_1 & +\ \gamma_2\alpha^{-1}\mu_2 & = & \beta\alpha^{-1}\gamma_4(\gamma_2)^{-3}(V''_0/V'_0) \\
(6.28) & 0.a_1 & +2\beta\alpha^{-2}\lambda_1 & -\ \gamma_2\alpha^{-1}\mu_2 & = & \alpha\gamma_2^{-1}C''_0 + (2/3)V'_0\alpha\gamma_4\delta(\gamma_2)^{-3}
\end{array}$$

The system can be solved for a_1, μ_2 and λ_1. a_2 and λ_2 remain linked by equation (6.17b) and cannot be identified without considering higher order expansions in σ^2, that is terms of order σ^6. The principle is quite clear - the system can be worked out recursively to as high an order as necessary but as we are primarily interested in variations of 'a' in the neighbourhood of the first-best we have gone as far as we need to for our present purpose.

We are now in a position to state the two important propositions of this section.

Proposition 1: $\beta > (<)\ 0$ [11] implies that the incentive scheme is concave (convex) in P.

Proof: This is seen by working out the leading terms of the expression for I(P) in (6.10). For the normal density,

(6.29) $$I(P) = I_0 + \mu_1\alpha^{-1}(P-a) - \beta\alpha^{-3}\mu_1^{2}(P-a)^2 + \ldots \quad //$$

The index of risk-aversion is simply β which determines the convexity/concavity of the incentive schedule and therefore the convexity or concavity of $W(P) = V(I(P))$. Refer to the definition of β on page 57. It may be seen that,

$\beta > 0$ if and only if $\partial(A^{-1})/\partial I < 1$ where A is the index of absolute risk-aversion evaluated at I_0. To provide some intuition for this condition note that for the case of utility functions with affine risk-tolerance functions,

$A^{-1} = k_1 I + k_2 \; ; \; k_1 > 0$

It therefore follows that for this class at least,

$\beta > 0$ if and only if $0 < k_1 < 1$

Definition : If $W''(P) > 0$ then for small amounts of risk facing the manager (agent), which is after all what we have dealt with here, he will prefer the risky outcome to a certain payment and will be said to display risk-preferring behaviour.

Risk-averse behaviour is symmetrically defined. We conjecture that if there exists a second dimension of choice, involved explicitly with choosing the riskiness of a project, an agent displaying potential risk-preference, in terms of our definition, would be likely to choose the more risky project.

Proposition 2: a_1 is less than zero for the case of the normal distribution. It is ambiguous in general. Hence to first order, **sgn** $(a-a_0)$ is ambiguous.

Proof: Applying Cramer's rule to the system above, and simplifying to the normal density function,

$$a_1 = \frac{\delta(-V''_0/V'_0) + [\gamma_4\{\beta\alpha^{-1}(V''_0/V'_0) + (2/3)V'_0.\alpha\delta\} + \alpha C''_0]}{[-C''_0/C'_0 + V''_0/V'_0]}$$

In the special case of the normal density function with $\gamma_2 = 1$ and $\gamma_4 = 3$, the above expression simplifies to,

$$a_1 = [\ (3/2)A^2 + \alpha C''_0]/[-(C''_0/C'_0) + (V''_0/V'_0] < 0\ . \quad //$$

In general, the point to be noted about the expressions for a_1 is that the denominator is unambiguously negative for strictly concave utility functions. Thus the sign of the numerator determines the sign of a_1 and whether or not the first-best choice of effort is greater or less than the moral-hazard or second-best choice. In the special example we discussed in Section 5 we had $\beta < 0$ and $\partial a/\partial \sigma \leqslant 0$. Note too that in the example $W''(P) = 0$ and by our definition the manager displays risk-neutrality with respect to the profits of the firm. This case is consistent with the expression for a_1 above. The normal density forms a convenient reference point for all our discussions, especially because we allow for any specification of utility and cost functions. The arguments are however quite general although we suspect we will not find such simple and intuitive conditions for signing a_1 in the general case.

7. Consequences for Employment Results of Moral Hazard Specification.

The analysis of Section 6 will now be used to obtain propositions on employment in a model with

(a) a risk-neutral principal

(b) a risk-averse manager taking an unobservable action 'a' and

(c) a risk-averse worker.

The structure is derived from Hart (1983) who uses such a model to derive inefficient underemployment results with a risk-neutral principal (refer Section 2 for an extended discussion of this issue). Since our model is a modification of Hart's, it will be useful to first present his model and describe the main results derived by him. We then propose our alternative specification and derive results based on it.

The Hart Model

The firm is run by a manager and the firm's profits depend on how hard the manager works. In particular, let gross revenue P be given by,

(4.1) $P = F(\ell, a, \theta)$ where

'ℓ' is the labour force employed by the firm,

'θ' is a state of nature variable, and

'a' is managerial effort. It is assumed further that the workers and owners can observe 'ℓ' and P but not 'θ' or 'a'. Thus the manager is rewarded according to the observed performance of the firm, that is his salary function is $I(\ell, P)$. The manager is assumed to observe θ before choosing 'a' and 'ℓ'. It is further assumed that owners are risk-neutral. The manager, ceteris paribus, dislikes working hard. The manager's utility is given by,

(7.2) $V(I(\ell, P) - C(a))$ where $C(a)$, which is increasing and convex, represents the cost of effort. The worker's utility function is $U(w-R\ell)$ as before.

Since the worker is risk-averse, it is clearly optimal for the risk-neutral owners to offer him complete assurance that is to set $w(\theta_i) - R\ell(\theta_i) = w^*$ where $U(w^*)=U_0$, the reservation utility level of workers, R is the disutility of one unit of labour and $\theta_i \in (\theta_1, \theta_2 \ldots \theta_n)$, the n possible states. The probability of occurrence of each of these n states is known a priori and not affected by the actions of the agent. The owner's expected profits are then given by

(7.3) $E(P - I(\ell,P) - R\ell(\theta_i) - w^*)$

Thinking of the manager's choice variables as being ℓ and P rather than ℓ and 'a', the manager's expected utility can be written as,

(7.4) $EV(I(\ell,P) - C(a(\ell,P,\theta)))$ where $a=a(\ell,P,\theta)$ is the inverse of (7.1), i.e

(7.5) $P = F(\ell,\theta,a(\ell,P,\theta))$

Incentive compatibility tells us that in state s, the manager will choose $\ell(\theta)$, $P(\theta)$ such that

(7.6) $\ell(\theta)$, $P(\theta)$ maximise $I(\ell,P) - C(a(\ell,P,\theta))$

An optimal contract consists of a salary schedule $I(\ell, P)$ and employment and profit functions $\ell(\theta)$, $P(\theta)$ which maximise (7.4) subject to (7.6) and the condition that $(7.3)\geq R_0$. The last constraint reflects the fact that the owners require a minimum expected profit to invest in the firm. Of course this problem is formally equivalent to maximising the profits of the owners subject to a minimum utility constraint for the agent and the appropriate incentive compatibility constraint. The inefficiency result now follows.

<u>RESULT</u> : ℓ and P are below their first-best levels. Hence the contract is inefficient and the revised model restores the behaviour of the model with a monolithic risk-averse principal.

PROOF: This result follows as an application of Result 4 in Hart (1983) p.17 - 18 [12], and will not be repeated here. The intuition underlying it is quite clear. When times are bad, the manager would like the owners to accept a lower payment P - I. If θ is not publicly observable, however, the only way to do this is to gear owners' and workers' payments to ℓ and P. ℓ and P are useful signals about the state

θ but both are imperfect in view of the extra input, managerial effort. Therefore, in order to "prove" that things are really bad, the manager will have to set both ℓ and P too low from an efficiency point of view.

The introduction of a risk-averse manager, in a model with a risk-neutral principal, who acts after the state of nature has occurred, leads to inefficient underemployment results. We have however already doubted the value of this particular sequence of moves in modelling the risk-averse behaviour of a manager. Under the moral-hazard specification and in terms of our definition in Section 6 we saw that it would not always be true to say that

> '... the introduction of the manager who takes an action 'a' also justifies the assumption made throughout that the firm is risk-averse..'

especially if the manager happen to be risk-preferring with respect to the profits of the firm, that is W"(P) were greater than zero. Further, the difference in the two specifications carries over into differing employment results. Before demonstrating this claim we shall re-specify in detail the two sequences of moves.

HART SEQUENCE

Stage 0: Contract signed between shareholders and manager and shareholder and worker giving I(ℓ, P) and between principal and worker w(.)

$\downarrow$

Stage 1: θ occurs. Not observed by firm or workers but observed by agent

|

|

↓

Stage 2: a and ℓ chosen at inefficient levels

ALTERNATIVE SEQUENCE

Stage 0: Contract signed between shareholders and manager I(.) as a function of observables and between principal and worker w(.)

|

|

↓

Stage 1: Manager chooses 'a' before state of nature occurs

|

|

↓

State of nature occurs

|

|

↓

Stage 2: Manager chooses 'ℓ', the proportion of labour force employed. Gross profits $P = F(a, \ell, \theta)$ accrue and I(.) and w(.) are paid out to manager and worker respectively.

Our model differs from Hart's in one crucial respect. While in Hart's model 'a', alongwith 'ℓ', is chosen by the manager after the state of nature occurs, in our model 'a' is chosen before the state of nature occurs. Thus while in Hart's model both 'a' and 'ℓ' fulfil pretty much equivalent roles with 'a' and 'ℓ' being chosen below their efficient levels to signal to the principal and the worker that times are bad [note that unobservability of 'a' implies unobservability of θ by the principal and the worker], in our model 'a' serves a conventional 'moral hazard' role while 'ℓ' fulfils an 'asymmetric information' role. It is still possible for 'ℓ' to be chosen below its efficient level to signal to the principal and the worker but since 'a' is chosen before the state of nature occurs it has limited value in signalling the state of nature. The heart of our analysis will be the comparison of first-best and second-best choices of 'a' since in the example that we discuss the manager's incentive scheme makes him indifferent to ex-post choices of 'ℓ'. Conditional on the ex-ante choice of 'a' therefore 'ℓ' is chosen efficiently and the problem boils down entirely to a comparison of the second-best and first-best choices of 'a', and in particular the direction of the inefficiency, in determining whether ex-post we have employment higher or lower than efficient levels. Thus in our model, not only do we have ex-post efficient choice of 'ℓ' (conditional on 'a') the final inefficiency in choice of 'a', and hence 'ℓ', is no longer uni-directional as in the Hart model. All that is required for this drastic difference in outcomes of the two models is a simple re-sequencing of moves between nature and the manager.

We shall consider a simple and rather specific technology, chosen simply in the interests of analytical tractability and the following analysis should be regarded as illustrative of the role of re-sequencing of moves. The implications of the analysis are more general. Let,

(7.7) $F(a, \ell, \theta) = 2a\theta\pi(\ell)$ and for simplicity, we shall take $\pi(\ell) = \ell^{1/2}$.

Let maximum labour supply be artificially normalised at one unit. Thus when we speak of 'ℓ' we are effectively talking of fractions of that single unit of labour endowment supplied by the single worker. Let disutility accruing to the worker be given by R. Since we have a risk-neutral principal and a risk-averse worker with no moral-hazard (at least in this simple case) between the two, the first-best risk-sharing contract between them is implementable and is given by,

(7.8) $[U'(w-R\ell)]^{-1} = \text{constant}$

implies

$w(\ell) = R\ell + c$ and let $c = w^*$

implies

$w(\ell) = R\ell + w^*$

that is marginal utility of worker (as a function of wage income net of disutility) is equated across states. The principal faces a wage bill of $R\ell + w^*$ when a fraction ℓ of the labour force is employed. R may therefore be regarded as shadow wage per unit of labour. Suppose now that the manager is made indifferent across all choices of 'ℓ'. This will be the case if, say,

(7.9) $I(F, \ell) = I(F/\pi(\ell)) = I(a\theta)$

The manager is completely indifferent across all choices of ℓ made ex-post for all θ, conditional on 'a' chosen ex-ante of course, and he might as well choose the ex-post efficient level of 'ℓ' given by,

(7.10) $2s\pi'(\ell) = R$ where $s = a\theta$

(7.10) follows from realising that the net profits of the principal are given by,

$$F(a, \ell, \theta) - I(a\theta) - w^* - R\ell.$$

Note incidentally, that this way of formulating the problem explains why unemployment persists.

Suppose $\ell = 0$ for some $\theta = \theta'$. Then the worker may offer to work for some wage $w' < w^*$ but then,

$U(w'-R\ell') < U(w^* - R\ell') < U(w^*)$ and he ends up worse off not only because he incurs disutility by working but he also has to settle for a lower wage, both effects making him worse off. Besides, conditional on 'a', the principal always sets employment level efficiently with,

$2s\pi'(\ell) = R$, independently of w, hence the principal would not change his employment decision either. This also explains the persistence of over-employment (should it be predicted) in this model. Everything in the analysis is conditional upon the ex-ante choice of 'a'. We turn now to a comparison of the second-best choice a* with the first-best choice a_0 in this simple model.

$$F(a, \ell, \theta) = 2s\pi(\ell) = 2sl^{1/2}$$

$$I(F, \ell) = I(F/\pi(\ell)) = I(a\theta) = I(s)$$

The problem now simplifies, to a standard moral-hazard problem.

$$(7.11a) \quad \max_{I(.),\, \ell,\, a} \int [2s\ell^{1/2} - I(s) - w^* - R\ell]g(s/a)a^{-1}ds$$

subject to,

$$(7.11b) \quad \max_{a} \int V(I(s))g(s/a)a^{-1}ds - a$$

$$(7.11c) \quad \int V(I(s))g(s/a)a^{-1}ds - a \geq V_0.$$

At the second stage, conditional on 'a', l is chosen such that

$$2s\pi'(\ell) = R$$

This is the efficient choice of 'ℓ' conditional on 'a'. In our special case,

$$(7.12) \quad \ell = (s/R)^2$$

Therefore we now have the problem reduced to,

$$\max_{I(.),a} \int [s^2/R - I(s)]g(s/a)a^{-1}ds - a$$

subject to

$$\max_{a} \int (V(I(s))g(s/a)a^{-1}\, ds - a$$

$$\int (V(I(s))g(s/a)a^{-1}\, ds - a \geq V_0$$

It is now clear that the second sequence leads in fact to the model discussed in Section 3. Here the actions of the manager affect the probability distribution of the states of nature and by choosing high effort levels he will reduce the probability of low outcomes with certainty and therefore in a certain sense, may reflect his aversion to riskiness. The important points to note however that under the second sequence of events, there is a considerable element of uncertainty when the agent chooses 'a' although the informational state vis-a-vis the shareholder or the worker remains quite unaltered from the first sequence. The agent can ex-post deduce the state of nature from the functional relationship $P = F(a, \ell, \theta)$ because of his knowledge of 'a'. The shareholder and the worker, given their ignorance of 'a' remain unaware of both 'a' and θ and profits P are an imperfect indicator of managerial performance as before.

This simple change in the sequence of events and the information available to the manager when making his choice of effort has important and striking effects on the efficiency results. An example presented in shows how the outcomes, in moral hazard models might lead to over-employment results. The argument is extended to more general models, using the analysis of Section 6, following the example.

Proceeding with the analysis in the previous sections, it is clear that the first-best contract is given by,

$$(7.13)\quad \max_{I(.),\, a} \int [s^2/R - I(s)]g(s/a)a^{-1}ds + \lambda[\int V(I(s))g(s/a)a^{-1} - a]$$

subject to,

$$\int V(I(s))g(s/a)a^{-1}ds - a \geq V_0$$

The Lagrangean Y is given by,

$$(7.14)\quad Y = \int [s^2/R-I(s)]g(s/a)a^{-1}ds + \lambda[\int V(I(s))g(s/a)a^{-1}ds-a]$$

Following standard practice, the following is true, (7.15a) $I(s) = c_1$

Thus,

(7.15b) $\partial Y/\partial a = R^{-1}.\partial[E(s^2)]/\partial a - \lambda$

Let us specialise further to get a quantitative handle on the problem.

Take $g(s/a) = \gamma^{-1}\exp(-s/a\gamma)$ and take $\gamma \equiv 1$

Then,

(7.16) $E(s^2/R) = R^{-1}.E(s^2) = 2a^2/R$

Thus,

(7.17) $4a/R = \lambda$ at $a = a_0$

implies

$$a_0 = \lambda R/4$$

Take $R = 8$

(7.18) $a_0 = 2\lambda$; the first-best choice of 'a'.

The second-best choice of 'a' is considerably more difficult to solve and we shall do so only partially, enough to derive some quantitative predictions and to indicate how the model might potentially be solved completely.

In the second-best solution, (where we also have the agent's first-order condition for effort as a constraint), differentiating the Lagrangean with respect to I(.), we have

$$\left.\begin{array}{ll} \text{(7.19a)} & \lambda + \mu.a^{*-2}(s-a^*) \leqslant I^{1/2} \\ \text{(7.19b)} & I \geqslant 0 \end{array}\right\} \quad \text{complementarily slack}$$

The agent's expected utility (assuming $V(I) = 2I^{1/2}$) is given by,

$$\int V(I(s)g(s/a)a^{-1}ds - a = 2\int_{a^*-\lambda a^{*2}/\mu}^{\infty} \{\lambda+\mu.a^{*-2}(s-a^*)\}a^{-1}e^{-s/a}ds - a$$

$$= \{2\lambda-2\mu.a^{*-1}\}\{e^{-k/a}\}+2\mu.(a+k).a^{*-2}\{e^{-k/a}\} - a \quad (7.20)$$

where $k \equiv k(a^*, \lambda, \mu) = a^* - \lambda a^{*2}/\mu$

We know,

$\int V(I(s)g(s/a)a^{-1}ds - a = V_0$ at $a = a^*$, the second-best solution.

Then substituting from (7.20) above and putting a = a*, we have,

(7.21) $2\lambda - V_0 e^{k/a^*} + 2\mu(ke^{-k/a^*})/a^{*2} = a^* e^{k/a^*}$

Also, differentiating (7.20) once with respect to 'a' and setting the derivative equal to zero at a=a*, we have,

(7.22) $2\lambda(ke^{-k/a^*})/a^{*2} + 2\mu.(e^{-k/a^*})(k^2 + 1)/a^{*2} = 1$

implies,

(7.23) $\lambda k + \mu = (a^{*2} e^{k/a^*})/2(k^2+1)$

We need to solve for λ, μ and a* and so far we have two equations (7.20) and (7.23). The third equation is obtainable from differentiating the Lagrangean associated with the second-best programme with respect to 'a'. For this we need the second derivative (with respect to 'a') of the agent's expected utility and also an expression for $\int I(s)g(s/a)a^{-1}ds$. The analytic complications multiply rapidly if we try to solve the model explicitly and in any case some powerful predictions follow just by looking at (7.20).

Suppose, without loss of generality, k>0. Then,

(7.24) $a^* < a^*.e^{k/a^*} = 2\lambda - V_0 e^{k/a^*} + 2\mu k/a^{*2}$

$\Rightarrow a^* < 2\mu/a^* - V_0 e^{k/a^*} < 2\mu/a^*$ assuming $V_0 > 0$

$\Rightarrow a^* < (2\mu)^{1/2}$

Say, $a^* = \xi(2\mu)^{1/2}$ $0 < \xi < 1$

Then,

$a^* \geqslant a_0 \iff \xi\sqrt{2\mu}^{1/2} > 2\lambda$

It is therefore perfectly possible to construct examples where $a^* > a_0$. This is especially true for R sufficiently small.

Denoting $\ell(\theta| a_0, R)$ as the first-best choice of 'ℓ' for all θ and $\ell(\theta| a^*, R)$ as the second-best choice,

(7.25) $a^* \geqslant (<) a_0$ <u>iff</u> $\ell(\theta|a^*, R) \geqslant (<) \ell(\theta|a_0, R)$

Thus on our model the inefficient employment results are definitely not uni-directional, in the sense of always having under-employment. An interesting question remains. How does the argument presented above carry over more generally? For this we need to resort to the approximation techniques which are presented in Section 6.

The General Case

Return now to (7.11a) - (7.11c). Some judicious changes of variables is required here to cast this problem in the familiar framework of the earlier part of this section.

Look at the managerial expected utility expression first. This is given by,

(7.26) $\int V(I(s))g(s/a)a^{-1}ds - a$

Take $s = e^z$ and $a = e^\zeta$

Then (7.26) may be re-written as,

(7.27) $\int V(I(z))g(z-\zeta)e^{z-\zeta}dz - e^\zeta$

$= \int V(I(z))n(z-\zeta)dz - e^\zeta$ where $n(z-\zeta) \equiv g(z-\zeta)e^{z-\zeta}$

Let $z - \zeta \sim (0, \nu^2)$ and

(7.28) $z = \zeta + \nu.q$ where $q \sim (0, 1)$

(7.29) $N(z-\zeta) = P(z-\zeta \leqslant Z-\zeta)$

$= P(q \leqslant Z-\zeta/\nu)$

$= F(Z-\zeta/\nu)$

F(.) is the distribution function associated with the random variable 'q'.

Thus

(7.30) $n(z-\zeta) = \nu^{-1}f(z-\zeta/\nu)$

Managerial utility is given by,

(7.31) $\int V(I(z))f(z-\zeta/\nu)\nu^{-1}dz - e^\zeta$

The most convenient thing about this formulation is that we have proved [13] the validity of the first-order approach for precisely such a specification of the cost function and f normal. Approximation approaches work, most evidently, too.

Equations (6.1) - (6.3) are unaltered with ζ playing the role of 'a' and ν the role of σ. Equation (6.4) is altered to the extent that we must deal with the expression

(7.32) $\int (s^2/R)g(s/a)a^{-1}ds \equiv J(\zeta)$, say, after an appropriate change of variable.

The modified version of equation (6.4) is,

(6.4') $J'(\zeta) + \nu^{-1}\int If'dq + \mu\nu^{-2}\int Vf''dq - \mu C''(a) = 0$

Approximating $J(\zeta)$ by a Taylor series around ζ_0, the first-best choice of ζ, we have,

(7.33a) $J(\zeta) = J(\zeta_0) + J_1(\zeta_0)(\zeta - \zeta_0) + J_2(\zeta_0)(\zeta - \zeta_0)^2/2 + \ldots$

(7.33b) $J'(\zeta) = J_1(\zeta_0) + J_2(\zeta_0)(\zeta - \zeta_0) + \ldots$

We now have a modified version of (6.24). Term gathering considerations (of order ν^0) in (6.23) - (6.24) determine the restriction

(7.34) $J_1(\zeta_0) \equiv 1$

while gathering terms of order ν^2 gives,

(6.28') $J_2(\zeta_0)\zeta_1 + 2\beta\lambda_1\mu_1\alpha^{-3}\gamma_2 - \mu_2\alpha^{-1}\gamma_2 - (2/3)V'_0\mu_1^{\,3}\alpha^{-2}\gamma_4\delta$

$$= \mu_1 C''_0$$

The modified system is therefore formed by (6.17a), (6.22b) and (6.28'), with obvious notational changes. The determinant of this system is ambiguous in sign, induced by the additional term $J_2(\zeta_0)$ - leading to ambiguity in the sign of ζ_1 - and even intuitive sufficient conditions for comparing the magnitudes of the first-best effort choice ζ_0 and the second-best ζ^* are hard to come by. This change leads to a further compounding of the problems of ambiguities discussed previously. Hence we conclude, in line with both the special example and the general case,

Proposition 3: Since $\zeta^* \geqslant \zeta_0$ and $\zeta^* < \zeta_0$ are both possible the inefficiency in the employment results may manifest itself either as <u>under-employment</u> or <u>over-employment</u> vis-a-vis the <u>efficient</u> level of employment. Hence in direct contrast with the Hart formulation, the inefficiency results are not uni-directional.

Leaving aside, for the moment, any definition of risk-averse/preferring behaviour which might be in any case considered somewhat arbitrary, we have the stronger result that, for given configurations of utility, density and cost function, for each realisation of the random state, profits (employment) may be higher than the efficient (or equivalently symmetric information) level of profits (employment). In the case of no moral hazard, the agent bears none of the risk associated with the prospect P. If 'a' is unobservable, the incentive scheme (potentially convex) shifts some of the risk on to the agent in such a fashion that the effort level of the agent is increased by him beyond the efficient level. This result therefore run quite contrary to Grossman and Hart's intuition for postulating a risk-averse principal and has force precisely because small amounts of uncertainty are sufficient to generate it. Further, it displays differences in preferences (to a risk σ) between the principal and the agent. For the same degree of riskiness σ attached to a prospect, the agent, under moral-hazard conditions, prefers to choose a higher level of effort (mean) than the principal would choose under symmetric information. We can make these statements without being too specific about the technologies.[14]

The distinction between the two sequences of events is a subtle but important one. Under the Hart sequence, inefficiency is generated by the unobservability of θ by the principal. In the second sequence, over-employment may be generated by the manager increasing his level of effort ex-ante to counter the effects of increased uncertainty. This is of course the sequence in which, as we have argued before, the elements of uncertainty are truly present.

8. Conclusions

The results of this chapter can be summarised rapidly. It was shown how the assumption of risk-aversion of the principal altered the efficiency results in the Grossman-Hart model. This assumption appears very surprising especially in the context of large firms with many shareholders and easily diversifiable risk. It was demonstrated on the lines of Hart (1983) that inefficiency results can be generated with a risk-neutral principal if a risk averse manager, taking an unobservable action is introduced. It was further demonstrated that such efficient results must follow from the particular specification of moves made by the manager and the form of informational advantage available to him and any alternative specification might well lead to contrary results. An alternative and more reasonable sequence of events was proposed which enables us to function in the familiar framework of a moral-hazard model. A definition of apparent risk-averse behaviour was proposed in terms of the concavity (convexity) of the agent's utility function with respect to the profits of the firm. A useful method of approximating incentive schemes was derived as an essential part of the analysis.
The arguments were conducted largely within the context of simple models and more complicated extensions remain as areas for future research.

Notes

1. For example see Azariadis (1975) and Baily (1974).

2. For a more elaborate and ingenious argument, refer to Holmstrom (1983) where the risk-averse choices of managers are explained by reputation effects in a multi-period principal-agent game.

3. This is not a particularly restrictive or unreasonable assumption to make. Indeed, it is shown rigorously that for small σ, the first-order approach is valid for all the examples dealt with in this chapter. The approach is in fact also valid more generally, i.e for any arbitrary positive σ for the special example discussed in Section 5. Proofs of these assertions appear in Chapter 4 of this thesis.

4. This is non-standard notation. N(0) is meant to read 'in the neighbourhood of zero'.

5. We shall work with $\mu = \nu/\sigma^2$ in this example, hence the notational choice of ν as the multiplier for (3.6).

6. This may easily be seen from equation (5.13). As $\sigma \to 0$, $\mu \to (g'(a^*))$. As long as we insist that $g'(a) > 0$ for all 'a', the result follows.

7. From the first-order condition.

8. From the second-order condition.

9. A description of such techniques is available from the author upon request.

10. Not to be confused with μ in Section 5 where it was defined as ν/σ^2.

11. It is easily verified that $\beta > 0$ iff $\partial(A^{-1})/\partial I < 1$.

12 For a version of this proof refer to Ch.4 of Banerjee(1985)

13. See Chapter 4 of this thesis.

14. See Holmstrom (1983) on this point.

Illustration of Proposition 3

Concentrate, as before, on the normal density function. It is possible to see, after the change of variable, that,

(S.1) $J(\zeta) = R^{-1}.E(e^{2z}) = R^{-1}.\exp(2\zeta + 2\nu^2)$

(S.2) $J_2(\zeta) = 2J_1(\zeta)$

Thus,

(S.3) $J_2(\zeta_0) = 2$ given that we require (from (7.34)) that

$J_1(\zeta_0) = 1$

Hence,

(S.4) $\zeta_1 = \{(3/2).A^2 + \mu_1 C''_0\}\{J_2(\zeta_0) + (V''_0/V'_0) - (C''_0/C'_0)\}^{-1}$

Since $C = e^{\zeta}$, $C''/C' = 1$

(S.5) $\zeta_1 = \{(3/2).A^2 + \mu_1 C''_0\}\{1 + (V''_0/V'_0)\}^{-1}$

Hence,

(S.6) $\zeta_1 > (<)\ 0$ according as $(-V''_0/V'_0) < (>)\ 1$

Take the case of the utility function we have used before (and justified its validity with regard to the use of the first-order approach):

(S.7) $V(I) = 2I^{1/2}$

Then,

(S.8) $V''_0/V'_0 = -(2\lambda_0^2)^{-1}$ where λ_0 is the first-best level of λ and is an increasing function of W_0 - the minimum utility level - and by pitching this high enough (respectively low enough), $-V''_0/V'_0$ can be made to be less than (respectively greater than) 1. Similar analyses will follow for different specifications of the utility function.

Chapter 3: A Refutation of Inefficient Underemployment Results in Imperfect Information Implicit Contract Models- Part 2.

Abstract:

The analysis of Part 1 is extended in this chapter by allowing for a risk-averse principal. The emphasis is on the implementability of the first-best contract given that all the parties to the contract are risk-averse. An important restriction on the interpretation of the action by the manager needs to be imposed. In this chapter we shall work without a disutility of effort variable. The analysis is therefore implicitly conditioned on the assumption that the absence of such disutility does not mean that the manager automatically chooses the action most preferred by the principal. Divergence is possible because of the varying preferences to risk of the principal and the manager. We find that the first-best is incentive compatible if and only if the principal, the manager and the worker possess affine risk-tolerance utility functions. The analysis is conducted in two stages. The first stage concentrates on the principal-manager relationship and proves several propositions on first-best implementability. The analysis is close, conceptually, to Ross (1973,74) but differs from the earlier analyses in concentrating on proving the necessity - as opposed to the sufficiency - of affine risk-tolerance functions for first-best implementability of contract. The idea of necessity is implicit in the Ross analyses but the proofs are rather obscure and unintuitive. The second stage of the analysis incorporated a risk-averse worker in addition to the manager and shows that affine risk-tolerance utility again guarantee first-best implementability. This is called the 'extended similarity principle'. The inclusion of the worker links the two chapters (Part 1 and Part 2) and deduces the existence of another class of cases where the moral-hazard framework leads to fully efficient results. The absence of the disutility of effort variable allows us to operate with any general specification of distribution of profits. Therefore, the loss in generality (vis-a-vis Part 1) involved in restricting utility functions is made up somewhat by the gain in generality obtained by allowing arbitrary distribution functions. The restrictions in each of the two parts are there to fulfil their special roles.

1. Introduction

This chapter could have been alternatively entitled 'On the Importance of Affine Incentive Schemes for First-best Implementability of Contracts'. Its purpose is to re-emphasise that Grossman and Hart's inefficiency results follow from their particular specification of moves postulated and also to show that the moral-hazard specification, admittedly with a special interpretation of the effort variable and restricted utility functions, can lead to fully efficient employment results and that the optimal incentive schemes emerging from the analysis have particularly simple forms.

The focus is particularly on the <u>efficiency</u> of results or, to put it slightly differently, the first-best implementability of contracts, as we have already dealt with the possibility of over-employment in Chapter 2. We demonstrate that for a special interpretation of the effort variable and specifications of the utility functions for the principal (shareholders), manager and workers the first-best contract, with efficiency defined in terms of the effort choice of [1]the manager, is fully implementable. In demonstrating this proposition we will in fact use a simple implicit contract model with principal, manager and workers. This is very much in the spirit of the analysis in the previous chapter. We have however gone further by allowing a risk-averse principal and yet deriving efficient results under familiar but restrictive assumptions.

It is the principal-manager relationship which is the core of the analysis. The reader will therefore have to be patient [until the final section] while waiting for the worker to be introduced in the model. Most of the important propositions and the motivation in this chapter relate to the more standard principal-agent relationship, that is a single principal and a single agent taking an unobservable action 'a'. The final sections of this chapter will draw the arguments together in an extended model. We shall concentrate, for the moment, on the standard (but slightly re-interpreted) moral hazard model.

The interpretation of 'a' as the effort variable which unambiguously shifted the probability distribution of the states of nature to the right in the sense of first order stochastic dominance leads to problems on interpretation of risk averse behaviour . This is because under this interpretation, the choice variable relates predominantly to the choice of expected values of projects. In order to properly describe notions of risk, there has to exist, or alternatively we need to be concerned with, a second dimension of choice for the agent - namely the choice of riskiness of a project - and interpret the choice variable as influencing the spread rather the level of the distribution of profits. It was probably this second interpretation that was closer in the minds of the authors on agency models in the late sixties and early seventies who considered solving moral hazard models without imposing a disutility of effort variable on the agent. We make this suggestion because under the conventional interpretation of the choice of effort variable, non-disutility causing effort will lead either to

(a) first-best implementability of the contract if the principal is risk-neutral or

(b) a contradiction in the model.

Taking (b) first - using notation established in Section 3 of Chapter 2 - note that from the first-order condition on the agent's choice of effort without disutility of effort,

(1.1a) $\int V(I(P))f_a dP = 0$

(1.1b) $\Rightarrow - \int V'(I(P))I'(P)F_a dP = 0$

Now, the l.h.s of (1.1b) > 0 because under MLRC (refer p.41 Chapter 2), $I' > 0$ and of course $F_a \leqslant 0$ with strict inequality for some values of P. This is inconsistent with the r.h.s of (b) being equal to zero. This contradiction persists unless the principal is risk-neutral and the first-best contract is implementable with $I' = 0$. The first-best is implementable because with costless effort, the conflict between incentives and risk sharing is resolved. Since effort causes no disutility, a deterministic contract provides sufficient incentive to the agent to exert the first best level of effort. This is discussed at greater length in Section 4 of this chapter.

A way of overcoming the contradiction evident in (b) would be to have F_a varying in sign which must also mean that MLRC does not hold since MLRC => $F_a \leqslant 0$ (proved in Rogerson (1985)). The interpretation of 'a' in this chapter implies that F_a is positive for some values of P. We shall mention the problems involved in dropping MLRC, thereby violating one of Rogerson's conditions, a little later.

First, to elaborate on the alternative interpretation, it is not entirely clear why the agent in the absence of disutility of effort would not exert as much effort as possible to reduce the probability of bad profit outcomes without any disutlity to himself. In other words, for us to work without a cost of effort, there must be an element in the choice of action that dissuades the agent from choosing as high a level of effort as possible. We would suggest that choice of action 'a' might affect the riskiness of the project and therefore the agent's optimum choice of 'a' will not necessarily be the maximum possible value of a. This will be due both to elements of risk aversion and moral hazard and at this stage, no attempt is made to unscramble the two effects.

What we must therefore effectively be moving away from is the pure first order stochastic dominance effect of action. To put it more concretely, suppose that an increase in 'a' meant an increase in the variance of the random variable P. Heuristically speaking, this would fatten the tails of the distribution while taking away weight from the central regions. Intuitively then, it is easy to see that F_a is of ambiguous sign. This will mean that the agent's optimal choice of action will not coincide with that of the Principal's even in the absence of disutility of effort. In sum, the two kinds of effort are best distinguished by regarding the interpretation given in Chapter 2 as "effort affecting the levels of the density function of profits" while in this chapter "effort affects the spread of profits". Chapter 5 of this thesis elaborates upon this distinction in the context of a specific problem, namely the effect of limited liability arrangements on effort choice.

Hess(1983) makes the following claim while speaking of action not causing disutility : "...A principal would not be directly interested in a change in portfolio but would care how such a change influences the portfolio's yield. The agent will not shirk but may choose too risky or too conservative a portfolio for the Principal's tastes. The agency must contractually share risks for the mutual benefit of the Principal and the agent.

Levinthal(1984)speaks in terms of omitting the effort variable to examine "...the issue of risk sharing in its purest form.." although he also speaks of the interconnectedness of the many issues involved, notably moral hazard and choices of risk by the agent. We do not agree with Hess when he writes that the agent will not shirk, but the crucial part of the statement relates to the risk preferences of the agent being important in determining his action choice.

The analysis in this chapter is therefore implicitly conditioned on the assumption that the absence of the disutility of effort does not necessarily mean that the agent's choice of action coincides with that of the Principal's desired choice. We are interested in determining circumstances in which such coincidence will occur. We must necessarily be talking of a risk averse principal otherwise the first best contract is implementable. If we were to be extreme, we might say that the choice of action is infact the choice of the variance of the project. We will not specify what effect this has on the mean or any other moments of the distribution.

An important point must be noted here. Once we move away from the conventional moral hazard model, we no longer have the sufficient conditions identified by Rogerson. Section 3 of Chapter 2 is relevant here. MLRC and CDFC will not hold over the entire range of Ps. To be sure that we have obtained a maximum and not a minimum, the second order conditions must be satisfied, namely that the agent's expected utility should be concave in effort and this is difficult to guarantee in the absence of the Rogerson conditions. An extended discussion of the Rogerson conditions appears in the next chapter where a weakening of these conditions is also proposed. We shall therefore not pursue this discussion further here apart from requiring that even in the absence of the sufficient conditions, the problems discussed in this chapter have meaningful <u>interior</u> solutions.

We are interested in finding the answer to the following question: "Are there any necessary and sufficient conditions on the utility functions of the Principal and the agent for the first best to be incentive compatible given that both the Principal and the agent are risk averse?"

The answer turns out to be remarkably simple. The relevant class of utility functions is that of utility functions with affine risk tolerance functions i.e if U is the utility function of the Principal and V that of the agent we require,

(1.2) $-U'/U'' = \alpha_0 x + b$ and $-V'/V'' = \alpha_0 x + e$

where x is the argument of the relevant utility functions and α_0, b and e are constants.

This result has been around in the agency literature for a considerable time although not in exactly this context. It appears in the work of Wilson(1968) and Ross(1973,1974).

Ross (1973), for example, notes:

> ".... We have shown that for an interesting class of utility functions...the need to motivate agents does not conflict with the attainment of Pareto-efficiency..."

He therefore seems to be essentially discussing the sufficiency of affine risk-tolerance utility functions for first-best implementability of the contract. Closer examination does reveal that the idea of necessity - or in other words the uniqueness of the above class for first-best implementability - is also implicit in the analysis. We feel nevertheless that the method which we use and the way we proceed are more intuitive and appealing. The extension of the model to include a third-party, although fairly straightforward, is another novelty of the present analysis.

In the Ross and Wilson papers the principle of similarity is defined.

Given the incentive scheme I(P) if there exist constants such that

(1.3) $U(P-I(P)) = \alpha_1 V(I(P) + \beta_1$ where $\alpha_1 > 0$

then the agent will always choose the act that maximises the Principal's expected utility. In the informational context, if the agent and the Principal have identical preferences over actions under optimal risk sharing then the action 'a' does not have marginal value given P, that is there is no need to observe the action as well. The first best level of effort will be incentive compatible and the moral hazard problem will no longer exist.

This is the notion given rigour in the Proposition later in the chapter. Notions of affine incentive schemes will play an important role in this Proposition so let us preface its statement and proof with some considerations of first best incenive schemes

2. FIRST BEST INCENTIVE SCHEMES UNDER CONSTANT ABSOLUTE RISK AVERSION(CARA) AND CONSTANT RELATIVE RISK AVERSION(CRRA) UTILITY FUNCTIONS

The first best sharing contract is deduced from the first order Pareto condition

(2.1) $U'(P-I(P))/V'(I(P)) = \lambda$

Let us look at the first best contract under two alternative specifications of utility functions of the Principal and the agent.

(i) Let the utility function of the Principal and the agent be identical and characterised by CARA.

(2.2) $U=U(x) = V(x) = k_1 \exp(-c_1 x)$

$$U'(P-I(P)) = \lambda V'(I(P))$$

Thus

(2.3) $\exp\{-c_1(P-I(P)-I(P))\} = \lambda$

$\Rightarrow -c_1P + 2c_1I(P) = \log \lambda$

(2.4) $\Rightarrow I(P) = 0.5P + \log\lambda/2c_1$

(2.5) If $U = k_1\exp(-c_1x)$ and $V(x) = k_2\exp(-c_2x)$ then

(2.6) $[-c_1k_1\exp\{-c_1(P-I(P))\}]/[-c_2k_2\exp\{-c_2(I(P))\}] = \lambda$

$\Rightarrow [\exp\{-c_1(P-I(P))\}]/[\exp\{-c_2(I(P))\}] = \lambda c_2k_2/c_1k_1=\lambda^*$ say.

Thus we have

(2.7) $I(P) = (c_1P + \log \lambda^*).(c_1+c_2)^{-1}$

(ii) The case where both the Principal and the agent have identical utility functions characterised by CRRA.

(2.8) $U(x) = V(x) = \beta x^{1-r}/1-r$ say

Thus

(2.9) $(P-I(P))^{-r}/(I(P))^{-r} = \lambda$

$\Rightarrow P-I(P) = \lambda^{-1/r}I(P)$

(2.10) $\Rightarrow I(P) = [1+ \lambda^{-1/r}]^{-1}P$

(i) and (ii) above are all cases of affine incentive schemes.The utility functions which give rise to these schedules were characterised by constant relative risk aversion and or constant absolute risk aversion.

They are members of the wider class of utility functions with affine risk tolerance functions. Note that to obtain affine schedules from the utility functions we needed the coefficient of relative risk aversion to be the same for the Principal and the agent while the coefficients of absolute risk aversion could be different. This is simply a reflection of the requirement that for first best incentive compatibility we need:

(2.11) $-U'/U'' = \alpha_0 x+b \quad ; \quad -V'/V'' = \alpha_0 x+e$

Affine incentive schemes are sufficient to guarantee incentive compatibility as we shall see since alongwith the Pareto condition they imply

(2.12) $U(P-I(P) = \alpha_1 V(I(P)) + \beta_1$ which in turn implies incentive compatibility.

3. A NECESSARY CONDITION FOR AFFINE INCENTIVE SCHEME

We are aiming towards general results all the time so let us move in the other direction. Given the Pareto condition, suppose the incentive scheme is affine in P. Does this imply anything for the utility functions? This exercise will serve as a useful preface to the proposition stated later.

Suppose we are given the Pareto condition

(3.1) $U'(y) = \lambda V'(x)$ where $y= P-I(P)$

$x=I(P)$

at the appropriate (x,y)

(3.2) Writing $u(y) = \log U'(y)$

$v(x) = \log V'(x)$

$\mu = \log \lambda$

we have

(3.3) $u(y) = \mu + v(x)$

yielding a sharing rule

(3.4) $y=f(x,\mu)$ say.

Suppose we would like x to be affine in $P=(x+y)$ given μ for a range of

μs.

This is equivalent to requiring

(3.5) $y=\alpha(\mu) + \beta(\mu)x$

or equivalently

(3.6) $u(\alpha + \beta x) = v(x) + \mu$ for appropriate (x,μ,α,β).

Differentiating with respect to x we have

(3.7) $\psi(\alpha+\beta x) = \eta(x) - \log(\beta)$

where

(3.8) $\psi(y) = \log u'(y)$; $\eta(x) = \log v'(x)$

There are now two possibilities.

$\beta(\mu)$ independent of μ. In that case α is not. Differentiate (3.7) with respect to α to get

(3.9) $\psi'(y) = 0$

$\psi(y)$ and hence $u'(y)$ are constants and

(3.10) $U'(y) = \exp(u(y)) = \exp(-by + c) = \exp(-by)$ w.l.o.g

(3.10) also implies $U(y) = -\exp(-by)/b$

The corresponding result for the agent is

(3.11) $V(x) = -\exp(-ax)/a$

and we have the case of CARA described above.

The second possibility of course is that $\beta(\mu)$ is variable.

We assume that it is additionally strictly montotonic at least over intervals. Solving for μ in terms of β in such an interval and substituting,we have

(3.12) $\alpha = \alpha^*(\beta)$

Differentiating (3.7) with respect to β we get

(3.13) $\psi'(\alpha + \beta x)(\alpha^{*\prime} + x) = -\beta^{-1}$

(3.14) $y + \{\psi'(y)\}^{-1} = \alpha + \beta x - \beta(\alpha^{*\prime} + x)$

$$= \alpha - \beta d\alpha/d\beta$$

$$= \text{constant}$$

$= -b$ say

The last step follows because the two sides depend on different variables.

Thus

(3.15) $\psi'(y) = -(y+b)^{-1}$; $\psi(y) = B - \log (y+b)$ say

(3.16) $u'(y) = \exp \psi(y) = -c(y+b)^{-1}$ say

$\log U'(y) = u(y) = -c\log(y+b) + B'$ say

(3.17) $U'(y) = (y+b)^{-c}$; $U(y) = (y+b)^{1-c}/1-c$ w.l..o.g.

The corresponding result for the agent is

(3.18) $V(x) = (x+a)^{1-c}/1-c$

The second case is that of constant relative risk aversion also discussed previously.

As is evident from this section,there exists an intimate relationship between constant relative or absolute risk aversion utility functions and (transforms thereof) and affine incentive schemes and correspondingly incentive compatibility. The above discussion has helped to identify links which we usefully exploit in the next section where we prove one of the main results of this chapter. These links, which are indeed well defined, enable one to speak of incentive compatibility if and only if the utility functions have affine risk tolerance functions, to which CARA and CRRA utility functions belong, simply to re-iterate a point made earlier.

Incidentally, this dichotomy between obtaining CARA while ignoring terms of the nature $\beta(\mu)$ and CRRA otherwise is a feature that re-emerges in the next section.

4. A THEOREM ON FIRST BEST IMPLEMENTABILITY

To re-iterate

The first best contract is characterised by the condition:

(4.1) $U'(P-I)/V'(I) = \lambda$

THE FIRST BEST CONTRACT IS INCENTIVE COMPATIBLE IF AND ONLY IF GIVEN I(.) FROM (3.4.1) THE AGENT'S UTILITY MAXIMISATION CHOICE IS PRECISELY THAT DESIRED BY THE PRINCIPAL.

Mathematically we state it as follows and label the statement "The first best is incentive compatible" as A.

A. THE FIRST BEST IS INCENTIVE COMPATIBLE

B. a^* max $\int V(I(P))f(P,a)dP$

$\Rightarrow a^*$ max $\int U(P-I)\, f(P,a)dP$ given I(.) from (4.1).

We add the following statements:

C. Given I(.) from (4.1) $U(P-I(P)) = \alpha_1 V(I(P)) + \beta_1$

D. U and V belong to the following class of utility functions

$-U'/U'' = \alpha_0 x + b \quad ; \quad -V'/V'' = \alpha_0 x + e$

Proposition 1: Assume that the maximisation problems have well-defined interior solutions. Then A,B,C and D are equivalent, that is,

A <==> B <==> C <==> D.

Proof: The proof proceeds via several lemmas.To establish the structure of the proof we proceed as follows:

Lemma 1: A <==> B

Lemma 2: C => B

Lemma 3: C <==> D

Lemma 4: B => C

The final lemma is proved via two sub lemmas.Lemma 4(i) establishes that B and the Pareto condition imply a affine incentive scheme. Lemma 4(ii) establishes that a affine incentive scheme alongwith the Pareto condition imply C. Thus in the presence of the Pareto condition B => C.

We now proceed to the proof of the lemmas. The order in which they are proved simply denote the degree of difficulty of the proof. The more difficult results are left to the end.

Lemma 1: A <==> B

Proof: The first best contract is the solution to the program

$$\max_{I(.);a} \int U(P-I)fdP \quad \text{s.t} \quad \int V(I)fdP \geq V^*$$

The first order condition is given by (4.1) from which we obtain I(.). The Principal under the first best then sets the effort level 'a' to maximise

$$(4.2) \quad \int U(P-I)fdP + \lambda \int V(I)fdP$$

i.e

$$(4.3) \quad \int Uf_a dP + \lambda \int Vf_a dP \,\Big|_{a=\tilde{a}} = 0$$

Under second best conditions if the agent were given the first best incentive scheme, he would choose a* to maximise

(4.4) $\int V(I)fdP$

i.e

(4.5) $\int V(I)f_a dP \,|_{a=a*} = 0$

$\tilde{a}$ and a* would coincide if and only if

(4.6) $\int V(I)\, f_a dP|_{a=a*} = 0$

$\Rightarrow \int U(P-I)f_a dP \,|_{a=a*} = 0$ since then a* would be a solution to (4.3).

Note that if I(P) = k, a constant independent of P, then (4.6) is solved at $a=\tilde{a}$ (among others) which also co-incides with the maximisation derived from (4.3) and the implication

$$\int V(I)f_a dP = 0$$

$$\Rightarrow \int U(P-I(P)f_a dP = 0$$

holds almost trivially for any utility function V. The risk neutral case is in the present context not an interesting one to consider.

Thus the first best contract is incentive compatible

iff a* max $\int V(I)fdP \Rightarrow$ a* max $\int U(P-I)\, f\, dP$ $//_{Q.E.D.}$

Lemma 2: C => B

Proof: We note now that if for certain utility functions

(4.8) $U'(P-I) = \lambda V'(I) \Rightarrow V(I(.)) = \alpha_2 + \beta_2 U(P-I)\ ;\ \beta_2 > 0$

then certainly

(4.9) a* max $\int V(I)f(P,a)dP$

$\Rightarrow$ a* max $\int U(P-I)f(P,a)\, dP$

This is easily seen

(4.10) a* max $\int V(I(P))f(P,a)dP$

(4. $\Rightarrow$ $\int V(I(P))f_a(P,a)dP = 0|_{a=a*}$

but under the optimal program,

(4.12) $V = \alpha_2 + \beta_2 U$

Thus

(4.13) $\int V(I)f_a(P,a)dP = 0 |_{a=a*}$

(4.14) $\Rightarrow \int (\alpha_2 + \beta_2 U)f_a(P,a)dP = 0 |_{a=a*}$

(4.15) $\Rightarrow \beta_2 \int Uf_a dP = 0 |_{a=a*}$

(4.16) $\Rightarrow \int Uf_a dP = 0 |_{a=a*}$. $//_{Q.E.D}$

The converse is more difficult to prove and we shall leave it until later. We have already shown

A <==> B

C => B

The next lemma requires showing

C <==> D

Then we would have D => A. If we could next prove B => C then we would have the required proposition i.e A <==> D. This is done in Lemma 3, 4(i) and 4(ii). Before proving the next lemma let us state and prove a theorem on functional equations. It is one of Pexider's generalisations of Cauchy's equations for functions of several variables.

Theorem P: Let three functions

$$M_1 : R^n_+ \to R \qquad (z_1, z_2 \ldots z_n) = z \to M_1(z) \neq 0$$

(a) $$M_2 : R^n_+ \to R \qquad (x_1, x_2, \ldots x_n) = x \to M_2(x)$$

$$M_3 : R^n_+ \to R \qquad (y_1, y_2, \ldots y_n) = y \to M_3(y)$$

be given where M_2 or M_3 is strictly increasing.

Let $M_1(x_1y_1, x_2y_2 \ldots x_ny_n) = M_2(x)M_3(y)$; $(x,y) \in R^{2n}_+$

Then there exist positive constants (real), $A, B, \alpha_1, \alpha_2 \ldots \alpha_n$ such that

$$M_1(z) = ABz_1^{\alpha_1} . z_2^{\alpha_2} \ldots\ldots z_n^{\alpha_n}$$

(b) $M_2(x) = A\, x_1^{\alpha_1}.\, x_2^{\alpha_2} \ldots\ldots x_n^{\alpha_n}$

$M_3(y) = B\, y_1^{\alpha_1}.\, y_2^{\alpha_2} \ldots\ldots y_n^{\alpha_n}$

Conversely every triple of functions M_1 M_2, M_3 given by (b) above satisfy

$$M_1(x_1y_1, x_2y_2, \ldots x_ny_n) = M_2(x).M_3(y)$$

Proof: Setting first x = (1,1,...1), y = z and then y=(1.1,...1), x = z we get from (a)

(c) $M_1(z) = M_2(1,\ldots.1)M_3\ (z);\ M_1(z) = M_2(z)M_3(1,\ldots)$

If for example A: = $M_2(1,\ldots1) = 0$ then $M_1(z) = 0$ contradicting our assumption of $M_1(z) \neq 0$. In the same way it follows that

$$B: = M_3(1,1,\ldots1) \neq 0.$$

Hence we can write

$$M_3\ (z) = M_1(z)/A;\ M_2(z) = M_1(z)/B$$

The function M: $R^n_+ \to R$ introduced by M(z): = $M_1(z)/AB$ satsfies the functional equation

(d) $M(x_1y_1, x_2y_2\ \ldots x_ny_n) = M_1(x)M_2(y) \quad (x,y) \in R^{2n}_+;\ M:R^n \to R$

since the functions given by

(e) $M_1(z) = ABM(z),\ M_2(z) = AM(z)\ ;\ M_3(z) = BM(z)$ satisfy

equation (a). M is strictly increasing since we assumed the same for M_2 or M_3.

In order to complete the proof we have to show that in view of (e) and (c) the class of strictly increasing solutions of equation(d) is given by

$$M(z) = z_1^{\alpha_1}.z_2^{\alpha_2}\ \ldots\ldots z_n^{\alpha_n} \qquad (\alpha_1, \alpha_2, \ldots, \alpha_n > 0\ )$$

This is indeed the case.

(d) implies

$$M^{\nu}(x_{\nu}y_{\nu}) = M^{\nu}(x_{\nu})M^{\nu}(y_{\nu}) \text{ for } M^{\nu}(z_{\nu}) = M(1,1,\ldots,z_{\nu}\ldots1)$$

$$\nu = 1,2\ldots n$$

According to a theorem of Cauchy's, the strictly increasing solutions of this equation can be written as

$$M^{\nu}(z_{\nu}) = z_{\nu}^{\alpha_{\nu}} \quad (\alpha_{\nu} > 0)$$

Thus from (d)

$M(z) = M^{1}(z_1)M^{2}(z_2)\ldots M^{n}(z_n)$ and we have proved the theorem//

Lemma 3: C<==>D

Proof: Let $U' = \lambda V' \Rightarrow V = \alpha_2 + \beta_2 U$

Write

(4.17) $U=g(U');V=h(V')$

This is possible since $U'' < 0$; $V'' < 0$.

Re-write

(4.18) $V' = pq$

$U' = q$

$\lambda = p^{-1}$

Thus we have

(4.19) $h(pq) = \alpha(p) + \beta(p)g(q)$

<u>Case 1</u>: $\alpha(p) \equiv 0$.Then

(4.20) $h(pq)=\beta(p)g(q)$

Applying Theorem P directly, we have

(4.21) $h(V')=A(V')^{\mu_1}$

i.e

(4.22) $V(\theta) = A[V'(\theta)]^{\mu_1}$and θ is a dummy variable

(4.23) $V'(\theta)=(A^{-1/\mu_1})V(\theta)^{1/\mu_1}$

$=BV(\theta)^{1/\mu_1}$where $B \equiv A^{-1/\mu_1}$

Solving this differential equation gives

(4.24) $V(\theta) = (\alpha_3\theta+\beta_3)^{\gamma}$ where $\alpha_3=B\{(-1+\mu_1)/\mu_1)\}$

β_3 is an arbitrary constant

(4.25) $\gamma=\mu_1/\mu_1-1$

Also from the theorem

(4.26) $U(\theta) = (\alpha_4\theta+\beta_4)^{\gamma}$ where α_4 and β_4 are symmetrically defined.

Putting β_3 and $\beta_4 = 0$ we have the case of constant relative risk aversion. In the limit they are obviously represented by the logarithmic function.

Case 2.

The more general case is identified under the assumption

(4.27) $h(pq) = \alpha(p) + \beta(p)g(q)$

Partially differentiating with respect to q

(4.28) $ph'(pq)=\beta g'(q)$ i.e $h'(pq) = (\beta/p)g'(q)$

Thus using Lemma to derivatives this time,

(4.29) $h'(z) = Az^{\mu}$ where $z \equiv pq$

$h(z) = Az^{\mu+1}/\mu+1 + B$

OR

(4.30) $h(z) = A \log z + B$

Thus

(4.31) $V= A(V')^{\mu+1}/\mu+1 + B$

Therefore

(4.32) $V' = K(V-B)^{1/\mu+1}$ where $K=\{(\mu+1)/A\}^{1/\mu+1}$

OR

(4.33) $V' = \exp\{(V-B)/A\}$

Solving the differential equation gives

(4.34) $V = [K(\mu/\mu+1)\theta + c]^{\mu+1/\mu} + B$ where θ is again a dummy variable.

Also

(4.35) $U = [K_1\theta + c_1]^{\mu+1/\mu} + B_1$

These are simple generalisations of Case 1.

We know that the utility functions derived above are of the class prescribed in D since

(4.36) $-V''/V'=(a\theta+c)^{-1}$

Look at (4.32) above.

$$V''=(K/\mu+1)(V-B)^{-\mu/\mu+1}\ V'$$

$$\Rightarrow V''/V'=(K/\mu+1)(V-B)^{-\mu/\mu+1} = (K/\mu+1)[K(\mu/\mu+1)\theta +c]^{-1}$$

which after re-labelling is exactly what we required since

$$K/\mu+1 = (\mu+1/K)^{-1}$$

Thus

$$V''/V' = [\mu\theta + c(\mu+1)/k]^{-1}$$

Similarly $\quad U''/U' = [\mu\theta + c_2]^{-1}$

We shall briefly discuss the limiting case.

$$V = A\log V' + B$$

(4.37) $\quad V = -A^{-1}\log(e_1\theta + e_2)$ where e_1 and e_2 are suitably defined constants.

The very nature of Pexider's theorem determines the if and only if nature of the proof.

Note that in (4.32) if μ=0,we have

$$V' = K(V-B)$$

Thus

(4.38) $\quad V = H\exp(Kx - B) + B \Rightarrow V-B = H\exp(Kx-B)$ for suitably defined constant H.

$$V' = KH\exp(Kx - B)$$

$$= k_2\exp(Kx) \text{ where } k_2= K.H.\exp(-B)$$

This is evidently the case of CARA and have covered all possible cases.

This completes the proof of the Lemma. //

Lemma 4: B => C

This is proved in two sub lemmas

Lemma 4(i) : Given Pareto efficient sharing condition,B => Affine payment schedule.

Proof: Consider first the case where U,V and λ are data for the problem,that is,say a social planner has fixed λ as the shadow price on the agent's expected utility.

Suppose now that B holds.

i.e given $\quad U'(P-I)/V'(I) = \lambda$

$$\int V(I(P)f_a(P,a)dP = 0$$

$$\Rightarrow \int U(P-I(P)f_a(P,a)dP = 0$$

Integrating by parts and substituting out this is equivalent to saying

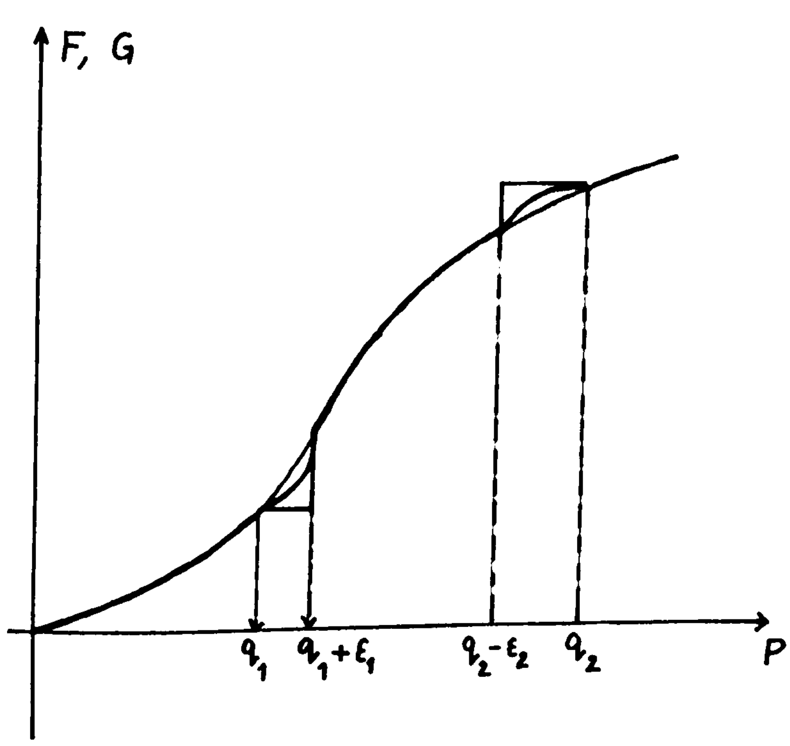
(4.40) $$\int V'(I(P))I'F_a dP = 0$$

$$\Rightarrow \int U'(P-I(P))F_a(1-I'(P))dP = 0$$

(4.41) $$\Rightarrow \int V'(I(P))F_a dP = 0$$

Suppose this holds for all distribution functions F.

In particular therefore it holds for the following distribution function which we construct below.



Let $G(P)$ be an arbitrary distribution function. Construct $F(P,a_2)$ as follows:

$$
\begin{array}{lll}
F(P,a) = G(P) & & |P-q_i| > \varepsilon_i \\
\quad 0< G(P)<1 & & \\
\quad 0< G'(P)<1 & & \\
\quad =(1-a)G(P) + aG(q_1) & & q_1 \le P \le q_1 + \varepsilon_1 \\
\quad =(1-a^2)G(P) + a^2 G(q_2) & & q_2-\varepsilon_2 \le P \le q_2
\end{array}
\tag{4.42}
$$

Differentiating F with respect to a and substituting in (4.40) and (4.41) we have

$$\int_{q_1}^{q_1+\varepsilon_1} V'I'[G(P)-G(q_1)]dP = 2a\int_{q_2-\varepsilon_2}^{q_2} V'I'[G(q_2)-G(P)]dP$$

$$=> \int_{q_1}^{q_1+\varepsilon_1} V'[G(P)-G(q_1)]dP = 2a\int_{q_2-\varepsilon_2}^{q_2} V'[G(q_2)-G(P)]dP \tag{4.43}$$

We have intutively in this variation, two degrees of freedom, first to ensure that $\int fdP = 1$ and second to obtain a choice of 'a' for $0<a<1$. Let ε_1 and ε_2 be chosen such that $a^* = 0.5$

Differentiating under the integral sign with respect to ε_1 we have

$$V'(q_1+\varepsilon_1)I'(q_1+\varepsilon_1)[G(q_1+\varepsilon_1) - G(q_1)]$$

$$=(d\varepsilon_2/d\varepsilon_1)V'(q_2-\varepsilon_2)I'(q_2-\varepsilon_2)[G(q_2)-G(q_2-\varepsilon_2)]$$

$$=>V'(q_1+\varepsilon_1)[G(q_1+\varepsilon_1)-G(q_1)]=(d\varepsilon_2/d\varepsilon_1)V'(q_2-\varepsilon_2)[G(q_2)-G(q_2-\varepsilon_2)]$$

$$=>I'(q_1+\varepsilon_1) = I'(q_2-\varepsilon_2) \tag{4.44}$$

Let $\varepsilon_1, \varepsilon_2 \rightarrow 0$

(4.45) Thus $I'(q_1)=I'(q_2)$

Since q_1 and q_2 were arbitrarily chosen, this implies

$$I'(q) = \text{constant for all } q$$

(4.46) $I(P) = \alpha_3 P + \beta_3$

Thus B => affine incentive schedule when λ is an exogenous parameter of the model. This interpretation is quite important but a simple generalisation extends the result to the case where λ which is the shadow price on the expected utility is not datum for the model. The need simply is to introduce a third degree of freedom into the model to ensure that

(i) The variation in $\varepsilon_1, \varepsilon_2$, and ε_3 leaves $\int f(P,a)dP$ unchanged.

(ii) we are able to enforce a suitable choice of action say a=0.5 and

(iii) the expected utility of the manager is unchanged, so λ and consequently I and I' are unaltered as a result of the variation.

$$
\begin{aligned}
F(P,a) &= G(P) && |P-q_i| \geqslant \varepsilon_i \\
&=(1-a)G(P) + aG(q_1) && q_1 \leqslant P \leqslant q_1 + \varepsilon_1 \\
&=(1-a^2)G(P)+ a^2G(q_2) && q_2-\varepsilon_2 \leqslant P \leqslant q_2 \\
&=(1-a)G(P) + aG(q_3) && q_3 \leqslant P \leqslant q_3 + \varepsilon_3
\end{aligned}
\tag{4.47}
$$

Thus

$$\{-\int_{q_1}^{q_1+\varepsilon_1} V'I'[G(P)-G(q_1)]dP + 2a\int_{q_2-\varepsilon_2}^{q_2} V'I'[G(q_2) - G(P)]dP$$

$$- \int_{q_3}^{q_3+\varepsilon_3} V'I'[G(P)-G(q_3)]dP\} = 0$$

implies

$$\{-\int_{q_1}^{q_1+\varepsilon_1} V'[G(P) - G(q_1)]dP + 2a\int_{q_2-\varepsilon_2}^{q_2} V'[G(q_2) - G(P)]dP$$

$$-\int_{q_3}^{q_3+\varepsilon_3} V'[G(P)-G(q_3)]dP\} = 0 \quad (4.48)$$

Let a* = 0.5

Differentiating with respect to $\varepsilon_1$again gives

$$V'(q_1+\varepsilon_1)I'(q_1 \; +\varepsilon_1) - G(q_1)]$$
$$+[V'(q_3+\varepsilon_3)(d\varepsilon_3/d\varepsilon_1)I'(q_3+\varepsilon_3)][G(q_3+\varepsilon_3) \; -G(q_3)]$$
$$= (d\varepsilon_2/d\varepsilon_1)V'(q_2-\varepsilon_2)I'(q_2-\varepsilon_2)[G(q_2)-G(q_2-\varepsilon_2)]$$

implies

(4.49) $V'(q_1+\varepsilon_1)[G(q_1+\varepsilon_1) - G(q_1)] +V'(q_3+\varepsilon_3)d\varepsilon_3/d\varepsilon_1[G(q_3)-G(q_3+\varepsilon_3)]$
$$=(d\varepsilon_2/d\varepsilon_1)V'(q_2-\varepsilon_2)[G(q_2)-G(q_2-\varepsilon_2)]$$

Since this must hold for arbitrary choices of $q_{1,}$ $q_2$and q_3 and LHS and RHS not identically equal to zero,this implies that

(4.50) $I'(q_1+\varepsilon_1) = I'(q_2-\varepsilon_2) = I'(q_3+\varepsilon_3)$

Let ε_1,ε_2,and ε_3 tend to zero and we again have the result that

$I'(q_1) = I'(q_2) = I'(q_3)$ for arbitrary q_1,$q_2$and q_3.

Therefore even under the more general interpretation,

(4.51) $I(P) = \alpha_3 P + \beta_3$.

Since it is fairly clear that an affine incentive scheme implies incentive compatibility, we have a by-product of the main proposition the rather surprising but appealing result that the first-best is incentive compatible <u>iff</u> affine incentive scheme. This result is intuitively pleasing in several respects. Firstly, it ties in beautifully with the intuition expresses in a recent paper by Holmstrom amd Milgrom (Aggregation and Linearity in the Provision of Incentives: Cowles Foundation D.P. Feb 1985) that incentive schemes tend to take quite complicated non-linear forms in order to use all the information that bears on the agent's actions. In circumstances therefore, as above, where P is a sufficient statistic (the first-best is incentive compatible) the incentive scheme is extremely simple. Secondly of course, such schemes correspond well with real-world incentive schemes which essentially have rather simple structures. Finally, this is another respect in which the current proposition adds to the Ross/Wilson results. The final sections of this chapter will extend this convenient affineness property when, as in Part 1, a worker is introduced into the model. The outcome of the analysis is a proposition on first best implementability of contract <u>iff</u> the utility functions (of the parties to the contract) are characterised by affine risk-tolerance functions <u>iff</u> gross profits are distributed among the parties on a simple proportional shares basis (plus additive constants).

We need a final lemma before the result we are looking for here is finally proved.

Lemma 4(ii): Affine incentive scheme and $U'(P-I) = \lambda V'(I)$ together imply

$$U(P-I(P)) = \alpha_1 V(I(P)) + \beta_1$$

Proof: Substituting $I(P) = \alpha_5 P + \beta_5$ into the Pareto condition gives

(4.52) $\quad U'((1-\alpha_5)P - \beta_5)) = \lambda\, V'(\alpha_5 P + \beta_5)$

Integrating over P we obtain

(4.53) $\quad (1-\alpha_5)^{-1}U[(1-\alpha_5)P - \beta_5] = \lambda(\alpha_5)^{-1}V[\alpha_5 P + \beta_5\ ] + c$

Defining $\alpha_1 = \lambda(1-\alpha_5)/\alpha_5$ and $\beta_1 = c(1-\alpha_5)$ we have the required result.Thus taking 4(i) and 4(ii) together,we have that given the Pareto condition

B => C //Q.E.D.

Thus we have proved the theorem A <==> B <==> C <==> D //

Our present analysis differs from the earlier analyses of Wilson(1968) and Ross(1973,74) especially with respect to Lemma 4. The earlier analyses essentially concentrated on proving the sufficiency for first best implementability of utility functions with affine risk tolerance functions. We have in this case also been able to prove necessity.

5. AN EXTENDED MODEL

We shall now introduce a single representative worker into the model. The aim in this section is to extend the propositions of Sections 2, 3 and 4 to include a worker. The extended model is of the form,

(5.1a) $\max\limits_{I(.),w(.),a} \int W(P - I(P) - w(P))f(P,a)dP$

subject to

(5.1b) $a \max \int V(I(P))f(P,a)dP \qquad \mu$

(5.1c) $\int V(I(P))f(P,a)dP \geq V_0 \qquad \lambda$

(5.1d) $\int U(w(P))f(P,a)dP \geq U_0 \qquad \gamma$

Here W is the utility function of the principal, V that of the manager and U that of the worker. The Lagrange multiplier associated with each constraint is indicated on the right.

Determining w(.) first as,

(5.2) $W'(P - I(P) - w(P))/U'(w(P)) = \gamma$

This simply denotes the fact that there is no moral-hazard for the worker. Note that if the principal is risk-neutral, $w(P)=w^*$. Here we shall concentrate on the case where the principal is not risk-neutral. Next, we can work out I(.) as before.

(5.3a) $W'/V' = \lambda$ First-best contract

(5.3b) $W'/V' = \lambda + \mu f_a/f$ Second-best contract

We also have the condition obtained by differentiation of the extended Lagrangean with respect to 'a' and the manager's maximisation condition but we shall come to these later.

We shall simplify the problem further by insisting that the optimal wage schedule has,

(5.4a) $w(P) = \theta_1.(P-I(P)) + \theta_2$ where $\theta_1 > 0$ and θ_2 are constants. This is weaker than insisting upon

(5.4b) $w(P) = \theta_1 P + \theta_2$

There are four major reasons for insisting upon this specification. First, as we show heuristically in the concluding part of this section, the neat propositions on affineness of I(.) are unlikely to go through in the absence of this specification. Second, using the results of Section 3, it is easy to work out a class of utility functions for which (5.4a) is satisfied, using (5.2). Thirdly, we do not a priori rule out arbitrary behaviour of w as a function of P. The restrictions emerge from the analysis.

The fourth great advantage of working with (5.4a) will now be revealed. From (5.2) and (5.4a), we have,

(5.5) $W'(\theta_3.(P-I(P))-\theta_2) = \gamma U'(\theta_1.(P-I(P))+\theta_2)$

Call $P-I(P) \equiv c$

Then,

(5.6) $W'(\theta_3.c - \theta_2) = \gamma U'(\theta_1.c + \theta_2)$ and integrating with respect to c we have,

(5.7) $\theta_1\theta_3^{-1}\gamma^{-1}.W(\theta_3.c - \theta_2) = U(\theta_1.c + \theta_2) + \theta_4$

Re-writing (5.7) and appropriately defining constants, we have,

(5.8) $U(w(P)) = \theta_5.W(P-I(P)-w(P)) + \theta_6$ thereby imposing similarity of preferences between the principal and the worker. This does not seem at all unreasonable as both are equally ignorant of action 'a' taken by the manager and sufficient flexibility is permitted by similarity in allowing the indices of both absolute and relative risk-aversion, for the utility functions of the principal and the worker, to differ. This throws the current analysis directly into the framework of the previous sections and some re-interpretation of the lemmas, which we shall now perform, extends the propositions on similarity and affineness of incentive schemes.

Lemma 1': The first-best contract between the principal and the worker is in any case always implementable. The first-best contract between the principal and the manager is implementable if and only if given I(.) from (5.3a),

$\int V(I)f_a(P,a)dP \;|_{a=a*} = 0$

implies

$\int W(P-I(P)-w(P))f_a(P,a)dP|_{a=a*} = 0$

This is essentially a re-statement of Lemma 1.

Proof: The first-best contract is the solution to the program,

$$(5.9)\ \max_{I(.),w(.),a} \int W(P-I(P)-w(P))f(P,a)dP \text{ subject to } \int V(I(P))f(P,a)dP \geq V_0$$
$$\int U(w(P))f(P,a)dP \geq U_0$$

The first-best contract between the principal and the manager is given by (5.3a) and between the principal and the worker by (5.2). The principal under the first-best then sets the effort level to maximise (5.9), that is,

$$(5.10)\ \int W(P-I(P)-w(P))f_a dP + \lambda.\int V(I(P))f_a dP + \gamma.\int U(w(P))f_a dP = 0|_{a=\tilde{a}}$$

But we have already shown that $U = \theta_5.W + \theta_6$

Thus we have,

$$(5.11)\ (1+\gamma\theta_5)\int Wf_a dP + \lambda\int Vf_a dP = 0|_{a=\tilde{a}}$$

Under second-best conditions, the manager, if given the first-best incentive scheme, sets a = a* such that

$$(5.12)\ \int V(I))f_a(P,a)dP = 0|_{a=a*}$$

a* and $\tilde{a}$ would coincide if and only if

$\int V(I)f_a dP = 0|_{a=a*}$

implies

$\int W(P-I(P)-w(P))f_a dP = 0|_{a=a^*}$ since then a=a* would be a solution to (5.10) and the lemma is proved.

Lemmas 2 and 3 follow just as before. Lemma 4 is the next focus of concern.

Lemma 4.1': Given the Pareto efficient sharing scheme between the principal and the manager, B implies an affine incentive scheme I(.).

Proof: We sketch as much of the proof as necessary

(5.13) $\int W'.(1-I' - w')F_a dP = 0$

implies

(5.14) $\int \lambda.V'.(1-I'-\theta_1 + \theta_1 I')F_a dP = 0$ using the information on w(.)

implies

(5.15) $\lambda.\int V'(1-\theta_1)F_a dP - \lambda.(1-\theta_1)\int V'I'F_a dP = 0$

Now $\int V'I'F_a dP = 0$

implies

(5.16) $\int V'F_a dP = 0$

implies

I affine in P by Lemma 4.1 which follows as before.

This is the Lemma where the special affineness property of w(.) is crucial and one cannot do without it in proving the extended similarity principle.

Lemma 4(ii) deducing similarity between the principal and the manager now follows routinely. This is called the 'extended similarity principle'. If (and only if) W, V and U have utility functions characterised by affine risk-tolerance functions, the first-best effort contract is incentive compatible and gross profits are shared out in a particularly simple way. This is summarised more intuitively in the following Proposition:

Proposition 2 : Provided (and only if) one has written a contract with the worker which is affine in (P-I(P)) then the first-best effort level is incentive compatible if and only if the contract given to the manager is an affine function of P.

This Proposition emphasises the role of a simple profit sharing schemes, in a firm comprised of a principal, a manager and a worker, for first-best incentive compatibility.

6. Conclusions

A few general remarks are in order by way of concluding this chapter. The proposition proved above is remarkably intuitive as it relates to the aggregation of preferences of the principal, the manager and the worker and the requirements of affineness have their close analogue in the requirement of linear Engel curves for the aggregation of demands.

At one level we have functioned very generally in this chapter having required no specific functional forms either on the utility functions of the Principal or the agents or of the density functions. At another level however we have been extremely specific in working with a class of functions where the first order approach is valid even under the special requirements of costless effort and correpondingly non stochastic dominance effect of the effort variable. Discovering such a class of functions is an interesting matter for research. Admittedly such a class will be extremely restrictive but it must be noted that the possibilities for first best risk sharing become even more remote with the introduction of a cost of effort. It is possible to introduce other complications into the model like non-similarity between the principal and the worker but the insights gained by this are not likely to be much more dramatic than concluding that the outcomes will be inefficient. The purpose of this chapter has simply been to highlight and focus sharply on the restricted class of functions, in an extended model, for which the first best contract is implementable amd moral hazard issues are consequently redundant. The reason for calling the proposition proved in Section 5 'the extended similarity principle ' is therefore clear.

Notes

1. The word 'effort' may be considered inappropriate to describe an activity that causes no disutility to the agent. It is useful to note, to clarify the sense in which the word has been used, that 'effort' and 'action' have been used interchangeably in the text of this chapter.

2. An alternative specification, where $F(P, a)$ varies more continuously with 'a' is given by,

$$F(P, a) = \begin{cases} G(P) & |P - q_i| > \varepsilon_i \\ G(P) - s_1(a, P)\{G(P) - G(q_1)\} & q_1 \leqslant P \leqslant q_1+\varepsilon_1 \\ G(P) - s_2(a, P)\{G(P) - G(q_2)\} & q_2-\varepsilon_2 \leqslant P \leqslant q_2 \end{cases}$$

where,

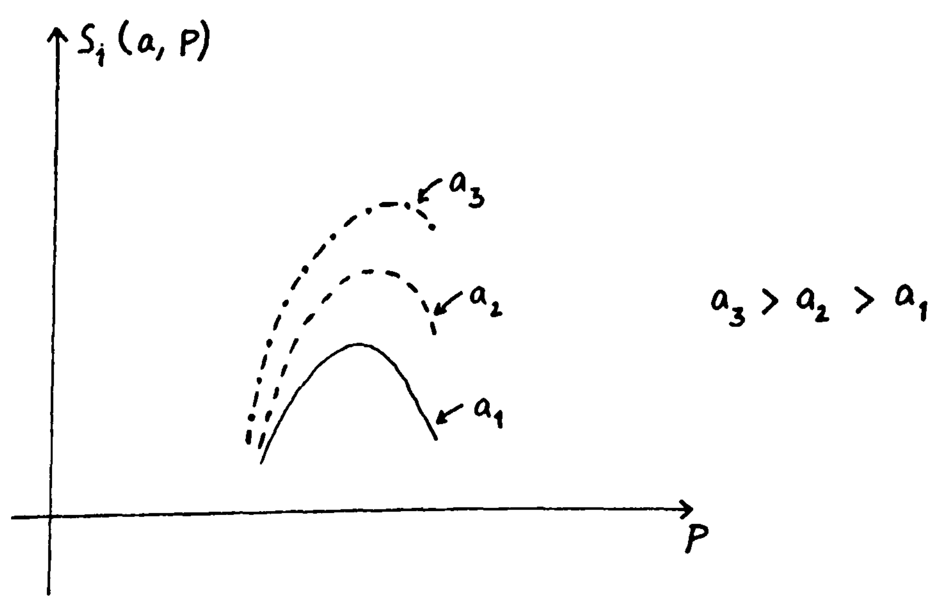
$1 \geqslant s_i(a, P) \geqslant 0$

$s_1(a, q_1+\varepsilon_1) = 0$

$s_2(a, q_2-\varepsilon_2) = 0$

$\partial s_i/\partial a > 0$

$\partial s_i/\partial P$ is depicted diagramatically below.



The analysis follows through, with only minor modifications, as before.

Chapter 4: A Note on the Validity of the First-Order Approach to Principal-Agent Problems.

Abstract:

The first-order approach to principal-agent problems involves relaxing the constraint that the agent chooses an action which is utility maximising to require instead only that the agent choose an action at which his utility is at a stationary point. The problems inherent in using this approach are by now well known. Essentially they arise from the inability to guarantee the concavity of the agent's expected utility in action and hence the attainment of a global maximum as opposed to local maxima, minima or saddle-points. Papers rehabilitating the use of the first-order approach, notably Rogerson (1985) have not proceeded further beyond identifying sufficient conditions for the validity of the approach. This chapter argues that the sufficient conditions proposed are too strong and impose unnecessary structure on the behaviour of the agent's expected utility. We argue that by imposing a positive minimum expected utility condition on the agent, the sufficient conditions can be weakened to a 'quasi-concavity' or 'concave when positive' condition. An alternative set of sufficient conditions is proposed. Further, by justifying the use of the first-order approach on the basis of the new set of conditions in a representative example, they are shown to be much less demanding than the Rogerson conditions. This follows from the simple additional observation that the Rogerson conditions are not satisfied in this example. The use of the first-order approach is also justified for distributions that differ from perfect observability only by small variance shifts. In sum, the aim of this chapter is to argue that the analytical convenience of the first-order approach may be retained for a much larger number of economically relevant cases than those allowed by the Rogerson conditions.

1. Introduction

In Chapter 2 [1], we made use of the first-order approach in establishing propositions on efficiency of outcomes in a principal-agent model. The aim of this chapter is on the one hand to justify the use made of the technique in the earlier chapter and on the other to suggest conditions under which the first-order approach may be more generally valid.

The questions on the validity of the first-order approach are by no means new but the analytical convenience afforded by this approach has led to attempts to identify classes of cases where it is valid. Three papers in particular [Mirrlees (1979), Grossman and Hart (1983) and Rogerson (1985)] have helped to establish sufficient conditions. These have come to be known in the literature as the 'monotone likelihood ratio condition (MLRC)' and the 'convexity of the distribution function condition (CDFC)' - defined precisely in Section 3 of Chapter 2 - which apply to the probability functions determining output as a stochastic function of effort. The immediate problem with using these two sufficient conditions is that the cases satisfying both MLRC and CDFC are extremely limited. While the class of density functions satisfying MLRC is not too restrictive, the same is not true of the class satisfying CDFC. In particular, CDFC is not satisfied by the normal density function which was the case dealt with in Chapter 2 of this thesis. This can be demonstrated straightforwardly:

If, $F(P, a) = \int^{P} (2\pi\sigma^2)^{-1}\exp(-(t-a)^2/2\sigma^2)dt = \int f(t, a)dt$ say,

where the lower limit of integration is $-\infty$,

then, $F_{aa}(P, a) = \int^{P} \{(t-a)^2\sigma^{-4} - \sigma^{-2}\}f(t,a)dt$ which is ambiguous in sign.

We show nevertheless that we were perfectly justified in using the approach within the specific context of the analysis and that the argument can be generalised.

The structure of the chapter is as follows. Section 2 presents in a, by now, standard fashion the argument against the use of the first-order approach. This is presented in some detail both in order to provide background and to add substance to the counter-arguments by making the contrasts between the two sets of sufficient conditions absolutely clear. This section also provides a statement of MLRC and CDFC and indicates, heuristically, how a conjunction of the two conditions guarantees the concavity of the agent's expected utility in action. The reader is referred to Rogerson (1985) for a detailed proof. Our aim is also to obtain such sufficient conditions but sufficient only to guarantee concavity over a restricted but relevant range of action choices of the agent. It is an important contention of this chapter that MLRC and CDFC impose too much structure on the behaviour of the agent's expected utility and that the sufficient conditions can be relaxed if we require only that the agent's expected utility be concave in action when expected utility is positive - in other words, a single-peakedness or quasi-concavity condition. Section 3 clarifies the use of the first-order approach in Chapter 2 and suggests that in case where unobservability of effort [moral-hazard] is induced by only small amounts of variability in profits the first-order approach works for a range of cases. Section 4 starts to generalise the argument and proposes the weakened sufficient conditions. Section 5 extends the example of Section 3 to any arbitrary value of the variance and shows that it satisfies the 'weakened sufficient conditions'. This example also leads to a sharpening of the sufficient conditions presented in Section 4. It needs to be emphasised that the particular importance of the examples in Sections 3 and 4, is that they use a specification of the density function which does not satisfy CDFC and validity of the first-order approach cannot be guaranteed by the Rogerson conditions. It is more than detrimental to the usefulness of the Rogerson conditions for them to exclude the normal density function as it has such a central place in mathematical and statistical analysis. However, the analysis of Section 5, which restores the use of the first-order approach, is encouraging. On the negative side, Section 5 also demonstrates the difficulties in using the 'weakened sufficient conditions'.

2. The Argument Against the Use of the First-Order Approach.

Let $U(I(.), a)$ and $V(I(.), a)$ denote the expected utility to, respectively, the principal and the agent of the contract $I(.)$ when the agent chooses action 'a'. The standard principal agent problem is to solve the following program:

(2.1) $\max_{I(.),a} \quad U(I, a)$

subject to,

(2.2) $\qquad V(I, a) \geq V^*$

(2.3) $a \in \operatorname{argmax}_{\hat{a} \in A} V(I, \hat{a})$

Analysis of the above program is difficult because the incentive compatibility constraint (2.3) is actually a continuum of constraints of the form

(2.4) $\quad V(I, a) \geq V(I, \hat{a})$ for every $\hat{a} \in A$

If (2.3) could be replaced with the requirement that the effort level be a stationary point for the agent,

(2.5) $\quad V_a(I, a) = 0$, then a solution could be calculated using the Kuhn-Tucker theorem. This is called the first-order approach and has been extensively used.

The problem with the use of the first-order approach can be easily illustrated by a graphical example adapted from Mirrlees (1975) and Grossman and Hart (1983).

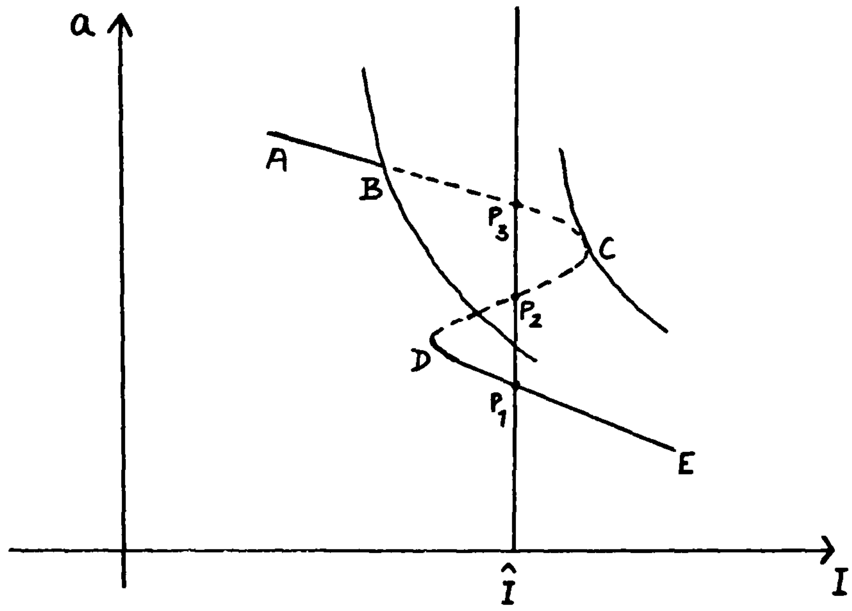


Figure 2.1

The curve ABCDE is the locus of the pairs of actions and incentive schemes which satisfy the agent's first-order conditions - that is, given I, the agent's expected utility is at a stationary point. Of these points, only those lying on the segment AB and DE represent global optima, for example given the incentive scheme $\hat{I}$ the agent's optimal action is at P_1, not at P_2 or P_3. Indifference curves in terms of 'a' and 'I' are drawn for the principal (C is on a higher indifference curve than B). The true feasible set for the principal are segments AB and DE and the optimal outcome for the principal is therefore B. However, B does not satisfy the first-order conditions for the problem: maximise the principal's utility subject to (a, I) lying on ABCDE, that is subject to (a, I) satisfying the agent's first-order conditions (the solution to this problem is at C). In other words, B does not satisfy the necessary conditions for the optimality of the problem. The first-order conditions are in any case not sufficient.

Rogerson (1985) presents and describes the conditions which are then shown to be sufficient for the first-order approach to be valid.[2] The first is the Monotone Likelihood Ratio Condition (MLRC).

<u>Definition</u>: The function $f(P, a)$ is said to satisfy MLRC if $f_a(P, a)/ f(P,a)$ is non-decreasing in P for every 'a'. MLRC also implies the stochastic dominance condition (SDC) which requires that $F_a(P, a) \leq 0 \; \forall \; a \; \varepsilon \; A$

The second property that the probability functions need to satisfy is the Convexity of the Distribution Function Condition (CDFC).

<u>Definition</u>: The function $F(P,a)$ is said to satisfy CDFC if $F_{aa}(P, a) > 0 \quad \forall \; a \; \varepsilon \; A$.

Rogerson's proof of validity of the first-order approach under MLRC, CDFC and certain reasonable regularity assumptions relies on consideration of three maximisation programs.

(i) The unrelaxed Pareto optimisation program given by

$$\max_{I,a} \; U(I, a)$$

subject to,

$$V(I, a) \geq V^*$$

$$a \; \varepsilon \; \arg\max_{a' \varepsilon A} V(I, a')$$

(ii) The relaxed Pareto optimisation program given by,

$$\max_{I,\, a} \; U(I, a)$$

subject to,

$$V(I, a) \geq V^*$$

$$V_a(I, a) = 0$$

and,

(iii) The doubly-relaxed Pareto optimisation program

Max $U(I, a)$

subject to,

$V(I, a) \geqslant V^*$

$V_a(I, a) \geqslant 0$

Under MLRC, CDFC and suitable regularity assumptions and the assumptions that,

(1) A solution to Program (iii) exists

(2) A solution to Program (ii) exists with $a > a_1$ where a_1 is the minimum possible value of 'a',

Rogerson proves the following proposition:

Proposition R: If $\{I, a\}$ is a solution to the doubly-relaxed program, then,

(a) it is also a solution to the relaxed program if $a < a_2$ where a_2 is the maximum possible value of 'a'.

(b) $I'(P) > 0$

(c) $U_a(I, a) \geqslant 0$

(a) and (b) follow from the doubly-relaxed program using MLRC.

(a) implies $V_a(I, a^*) = 0 \quad \forall\, a = a^*$

$V_a(I, a^*) \geqslant 0 \quad$ for $a = a_2$

(b) and CDFC imply that the agent's expected utility is concave in action. This is easily observed by noting that,

$V(I, a) = \int V(I(P))f(P, a)dP - C(a)$ where $C(a)$ is the separable cost of effort.

Thus,

$V_{aa}(I, a) = \int V(I(P))f_{aa}(P, a)dP - C''(a)$

$$= - \int V'(.)I'(P)F_{aa}(P, a)dP - C''(a)$$

$$\leq 0 \qquad \forall\, a \quad \text{using (b) and CDFC.}$$

These conditions imply that the agent's action choice is a global maximum and consequently I*, a* solves the unrelaxed program which in turn implies that the first-order approach is valid.

The reader is referred to Rogerson (1985) for a precise statement and proof of the Proposition. However, the above demonstration has served to identify quite clearly the respective roles of MLRC and CDFC in generating validity of the first-order approach. In a quite obvious sense these are the strongest possible conditions, as the easiest way of guaranteeing $V_{aa}(I, a) \leq 0$ is to insist not only upon $I'(P) > 0$ but also to require that $F_{aa}(P, a) > 0$ everywhere. While $I'(P) > 0$ may be considered a minimal requirement for incentive schemes[3] and is not too difficult to guarantee, as MLRC, at least, is satisfied by a large number of density functions, the same cannot be said for the second sufficient condition. The second condition severely restricts the cases to which the first-order approach is immediately applicable by virtue of their satisfying the sufficient conditions. Rogerson suggests only one example of a family of densities satisfying the MLRC and CDFC and that too does not appear at all attractive for the purposes of simple analysis.

The extended discussion so far simply leads to the observation that the MLRC and CDFC are too obvious, in the sense of being the strongest possible, and weaker, although still sufficient conditions, ought to be demonstrated to exist. Such demonstration is the purpose of the remaining sections of this chapter. The approach by and large is to proceed by inspection of the behaviour of the agent's expected utility as a function of his action. The aim is to impose a minimum expected utility condition on the agent and to allow a wide range of behaviour of expected utility over the region where it is non-positive rather than insisting upon strict concavity throughout. The achievements of this chapter are as significant as the reader chooses to regard them. At a minimum, it justifies the use of the first-order approach in several special contexts, especially in Chapter 2, while at the other extreme it suggests a significant weakening of the Rogerson sufficient conditions. The simplicity of the weakening, it is hoped, will help to make the present analysis even more attractive.

3. An Illustrative Example

This example is taken from Chapter 2. It demonstrates the validity of the first-order approach in cases where profits are determined stochastically as,

(3.1) $P = a + \sigma.q$ where $q \sim (0, 1)$

and $\sigma \varepsilon N(0)$ [4] .

Effort of the agent is imperfectly observable but this imperfection is induced only by small variance σ. We now fill in the details of the example.

<u>Assumptions</u>

A1. We have a risk-neutral principal.

A2. The utility function of the agent is given by $V(I) = 2.I^{1/2}$

A3. The cost of effort function is given by 2.C(a) with C(a), C'(a) and $C''(a) > 0 \ \forall \ a > -\infty$

A4. The reservation expected utility of the agent is $2.U_0$ where $U_0 > 0$.

A5. Profits are distributed normally with mean equal to 'a', the effort choice of the agent, that is,

$f(P, a) = (2\pi\sigma^2)^{-1/2}.\exp\{-(P-a)^2/2\sigma^2\}$

$f_a(P, a)/f(P, a) = \sigma^{-2}(P-a)$

A6. An interior solution to the maximisation programme exists.

Next, consider the system comprised of (2.1), (2.2) and (2.5), the so-called relaxed Pareto optimisation program. Forming the appropriate Lagrangean and differentiating with respect to I(.), we have,

(3.2) $\lambda + \nu.f_a/f = (V')^{-1}$ where ν is the Lagrange multiplier associated with (2.5)

Thus,

$$\left.\begin{array}{ll}(3.3a) & \lambda + \mu.(P-a^*) \leqslant I^{1/2} \\ (3.3b) & I \geqslant 0\end{array}\right\}\text{ complementarily slack}$$

a^* is the second-best optimal choice of effort, $\mu = \nu/\sigma^2$ [5] which has a finite positive limit and the complementary slackness conditions must hold because of the structure of the utility function.

From the complementary slackness condition,

(3.4a) $I = 0 \quad \forall\, P < a^* - \lambda/\mu$

(3.4b) $V(I) = 0 \quad \forall\, P < a^* - \lambda/\mu$

Also,

(3.4c) $2(\lambda + \mu.(P-a^*)) = 2.I^{1/2} = V(I) \quad \forall\, P \geqslant a^* - \lambda/\mu$

Concentrating entirely on the agent's expected utility and remuneration, we have,

$$(3.5)\quad A = 2\{\int V(I)f(P,\ a)dP - C(a) - U_0\}$$

$$= 2\{\int_{a^* - \lambda/\mu}^{\infty} [\lambda + \mu.(P-a^*)].(2\pi\sigma^2)^{-1/2}.\exp[-(P-a)^2/2\sigma^2]dP - C(a) - U_0\}$$

Making the change of variable $P = a + \sigma.q$ and noting that $(P-a^*) = (P-a) + (a-a^*)$, we can re-write (3.5) as,

$$A = 2.\{\int_{s}^{\infty} [\lambda + \mu\sigma q + \mu(a-a^*)](2\pi)^{-1}\exp(-q^2/2]dq - C(a) - U_0\} \qquad (3.6)$$

$$= 2.\{V(a) - C(a) - U_0\}$$

where $s = (a^* - a - \lambda/\mu)/\sigma$

It is as well to highlight a notational convention here which is followed for the rest of the chapter. The expression for the expected utility of the agent is expressed as the sum of two component parts $V(a)$ and $- C(a)$ where,

$V(a) = \int V(I^*)f(P,a)\ dP$

and I^* is the optimal incentive scheme, conditional on a^*.

C(a) is the disutility of action to the agent.

The object of the analysis in this section is to show that $\forall\ \sigma \in N(0)$, $\partial^2 A/\partial a^2 < 0$, that is expected utility of the agent is concave in action and hence the first-order approach is valid. Section 5 will show that A is concave in the relevant range [that is where $V(a) - C(a) > 0$] for any σ and a reasonably wide specification of cost of effort function. Section 3 however provides all the important clues for weakening the Mirrlees/ Rogerson conditions.

We shall look separately at the behaviour of V(a) and C(a). U_0 is not at all relevant once the component parts of A are differentiated. It plays a crucial role in a different setting as we shall see.

Ignoring multiplicative constants,

(3.7) $V'(a) = \mu.\Pr[q \geqslant s] \geqslant 0$

(3.8) $V''(a) = \mu.(2\pi\sigma^2)^{-1/2}\exp(-s^2/2) \geqslant 0$

Therefore, in general V(a) looks like,

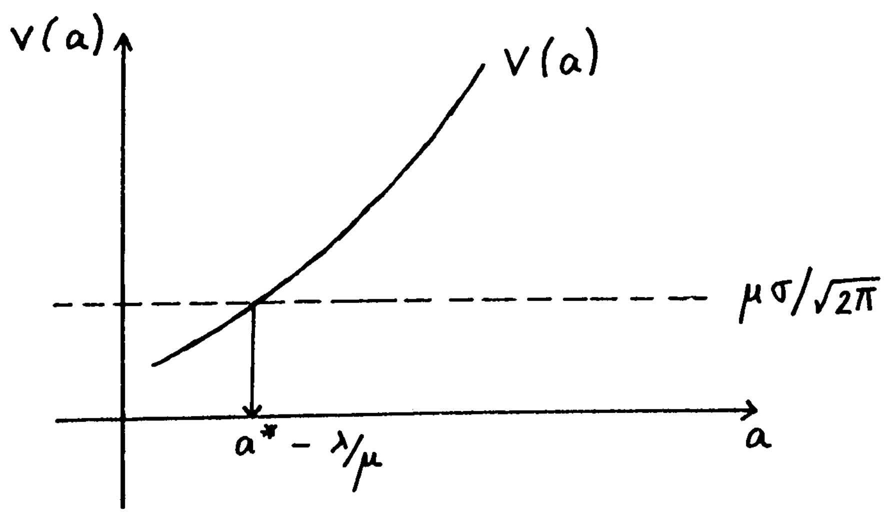


Figure 3.1

Thus, in general, for the first-order approach to be valid, we need C(a) to be sufficiently convex in order to make,

(3.9) $V''(a) - C''(a) < 0$

Look at the limiting behaviour of V(a) as $\sigma \to 0$.

$$(3.10)\quad V(a) = \underbrace{[\lambda + \mu.(a-a^*)].\Pr[q \geq s]}_{(i)} + \underbrace{\mu\sigma(2\pi)^{-1}\exp(-s^2/2)}_{(ii)}$$

(ii) in (3.10) tends strongly to zero as $\sigma \to 0$. Thus we must concentrate on the behaviour of (i) as $\sigma \to 0$.

$$(3.11)\quad \mathrm{Lim}_{\sigma\to 0} \Pr[q \geq s] = 1 \text{ if } a > a^* - \lambda/\mu$$
$$= 1/2 \text{ if } a = a^* - \lambda/\mu$$
$$= 0 \text{ if } a < a^* - \lambda/\mu$$

It follows from (3.9) and (3.10) that,

$$(3.12)\quad \mathrm{Lim}_{\sigma\to 0} V(a) = \lambda + \mu.(a-a^*) \qquad a > a^* - \lambda/\mu$$
$$= 0 \qquad a \leq a^* - \lambda/\mu$$

This is depicted diagramatically below.

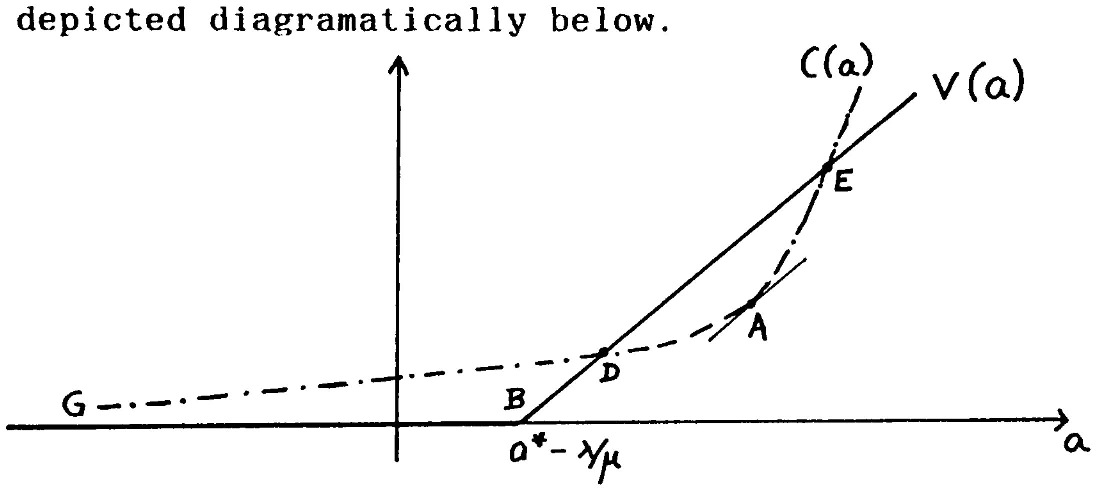


Figure 3.2

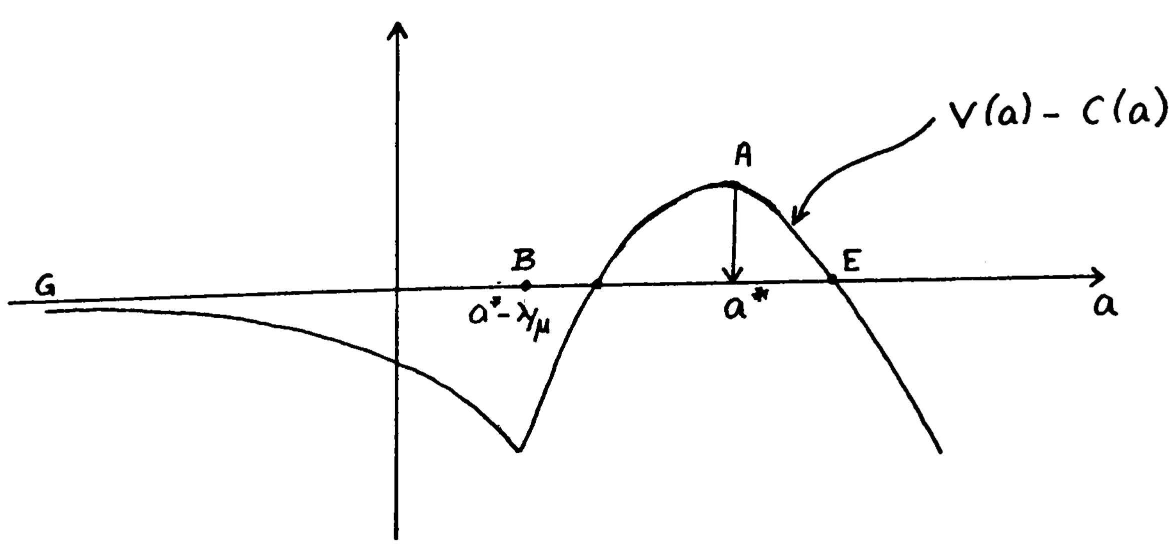


Figure 3.3

Figures 3.1 and 3.2 are alternative representations of the same story.

There is no problem with identifying A as the solution to the agent's problem. Figure 3.2 in particular highlights the importance of the 'concave when positive' condition. The positive minimum utility restriction means that we need concentrate on the region DE over which V - C is concave. The possibility of a kink at B is clarified in Section 5 when we allow for convex regions in the curve V - C. The story is completed by computing V' and V" and drawing one more appropriate diagram.

From (3.7),

$$V'(a) = \mu . \int_{s}^{\infty} (2\pi)^{-1} \exp(-q^2/2)\,dq \qquad (3.13)$$

As $\sigma \to 0$,

(3.14) $V'(a) \to \mu$ if $a > a^* - \lambda/\mu$

$\to 0$ if $a \leqslant a^* - \lambda/\mu$

Indeed, $a^* > a^* - \lambda/\mu$ as λ, $\mu > 0$

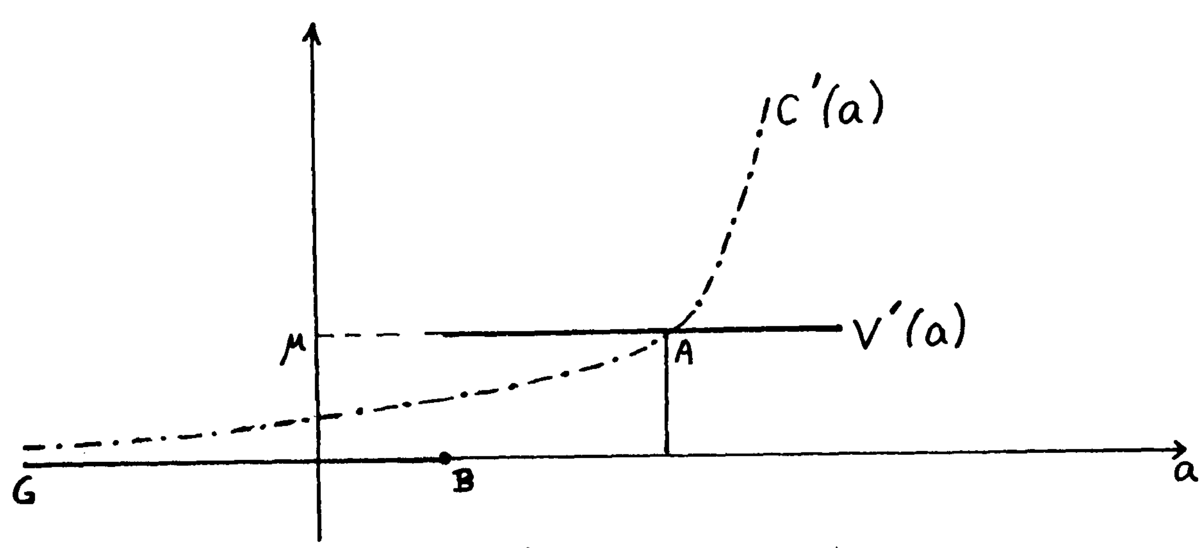


Figure 3.4

From (3.8),

$$V''(a) = \mu . (2\pi\sigma^2)^{-1/2} \exp(-(s^2))$$

$\to 0$ as $\sigma \to 0$ because evidently,

$\exp(-\sigma^{-2}/2) \to 0$ faster than $\sigma^{-1} \to \infty$ as $\sigma \to 0$

Thus in the limit $V''(a) = 0$ everywhere as all the diagrams confirm. We have however to be careful on one remaining point. We need

$V'(a) = C'(a)$ to occur at a point like A in figure 3.1 and not at a point like G. This is effectively to rule out maximisation in the convex region of the V(a) schedule. By setting $U_0 > 0$, (assumption A4) we rule out region GD of V(a).

$V''(a)-C''(a) < 0$ in any case.

It was useful to insist upon a property (assumption A3) of the cost function:

$C(a),\ C'(a) > 0 \ \forall\ a > -\infty$

$a^* = -\infty$ is not an interesting solution. Thus for $a^* > -\infty$, the point $V'(a) = C'(a)$ can only occur at A. Finally, what guarantee is there that

(1) $V'(a) = C'(a)$ anywhere and

(2) $V(a) - C(a) > 0$ for some interval $[a_1, a_2]$?

We arrange it so (by assumption A6) for suitable values of λ and μ. (1) and (2) are at any rate necessary for the maximisation program to have an interior solution. The conclusions of this section can be summarised rapidly in the following proposition.

Proposition 1:

(1) Firstly, under minimal assumptions like $U_0 > 0$;

$C(a),\ C'(a) > 0 \ \forall\ a > -\infty$, we find that the first-order approach is valid for all $\sigma \in N(0)$.

(2) Secondly, $\mu \to C'(a^*)$ which is a very intuitive result as it states simply that in the limit, the marginal cost of effort equals the marginal gain. It also justifies the claim that μ has a finite positive limit [recall assumption on C(a)] and also shows, $\nu = \sigma^2\mu \to 0$ as $\sigma \to 0$ i.e the solution approximates the first-best outcome.

(3) Thirdly, it suggests a weakening of the sufficiency conditions for

the first-order approach to work. This is the theme of the next section while Section 5 generalises the analysis of this section to $\sigma \notin N(0)$ using the arguments of Section 4.

(4) Finally and most importantly, it justifies the approximation analysis in Chapter 2 as the arguments for this section are easily extended to more general specifications of utility and density functions and $\sigma \in N(0)$. This is seen by writing V(a) as a power series in σ (or σ^2) and looking at its behaviour as $\sigma \to 0$.[6]

4. Sufficient Conditions for the Validity of the First-Order Approach: A Fresh Look

We are interested in particular in the behaviour of V - C. What sort of behaviour of the function $\psi(a) = V(a) - C(a)$ will ensure straightforwardly the validity of the first-order approach in general? We have attacked the question in several stages in this chapter with Section 2 above suggesting the most obvious answer.

Answer 1: The first-order approach works if V(a) is concave in 'a' everywhere. This may be guaranteed by insisting upon the fulfilment of the sufficient conditions MLRC and CDFC.

This answer is too restrictive because the class of density functions satisfying CDFC is restrictive. Section 3 therefore, albeit with the primary aim of justifying the use of the first-order approach in a special example, suggests a partial weakening of the sufficient conditions.

Answer 2: For small variances and fairly general specifications of the cost of effort function and density function, provided $C'(a) > 0$ and $C''(a) > 0$, the first-order approach is valid. The usefulness of the 'concave when positive' condition also emerged in Section 3 and constitutes an important clue in extending the analysis.

The remaining sections of this chapter therefore have the following three-fold task:

(1) To provide a more general answer than either Answer 1 or Answer 2 above on when the first-order approach will work.

(2) To use the analysis in (1) to extend the special example in Section 3 to $\sigma \notin N(0)$ and

(3) To discover why the first-order approach may not work $\forall\ \sigma > 0$ and for all specifications of C(a) using the special example.

We proceed from the intuition given us from dealing with the local case.

We require, that at $a = a^*$, if

$$\left.\begin{array}{ll} \text{(i)} & V(a^*) - C(a^*) > 0 \\ & \text{This is equivalent to insisting upon} \\ & \text{the minimum utility constraint } U_0 > 0 \\ \text{(ii)} & V'(a^*) = C'(a^*) \\ \text{(iii)} & V''(a^*) < C''(a^*) \end{array}\right\}, \quad V-C|_{a=a^*} > V-C|_{a\neq a^*}$$

that is 'a*' is the optimal choice of effort, in the sense of a global maximum.

Essentially for (i), (ii) and (iii) above to imply a global maximum, we require concavity of $\psi(a)$ when $\psi \geq 0$ and $\psi \geq 0$ only in one interval of values for 'a'. That at least one such interval exists follows from the assumption of the existence of a solution to the program with the minimum positive utility constraint imposed.

A curve like the following for ψ will do it,

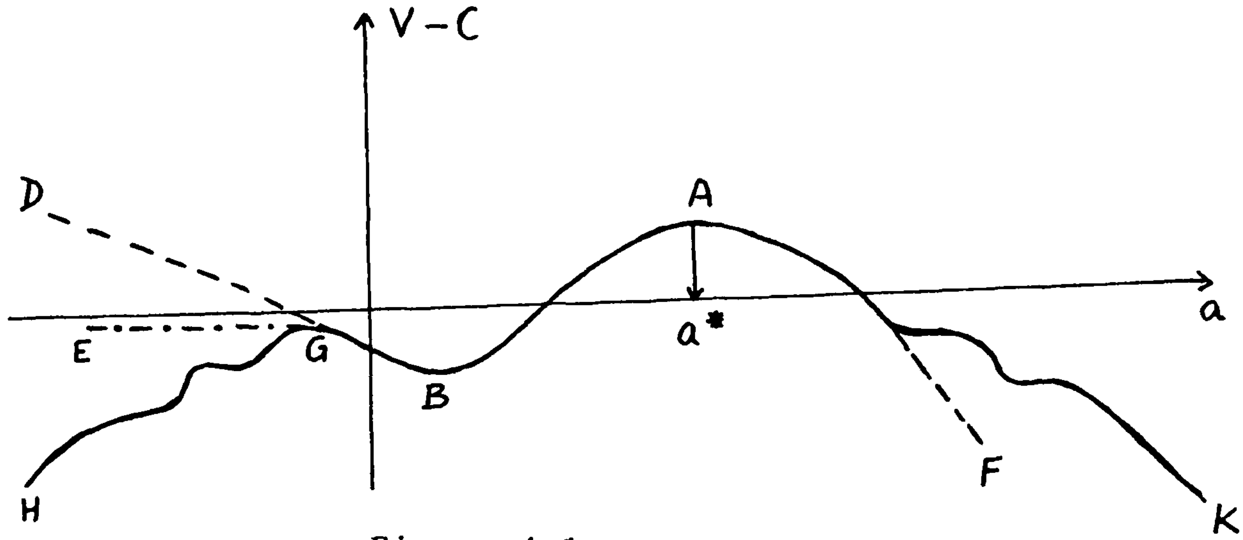


Figure 4.1

What we are doing effectively, by drawing a curve like the bold curve above, is to restrict all but one of the stationary points to the region such that $V - C < 0$. The stationary point such that $V - C > 0$ also has the right sign for $V'' - C''$. In a real sense we are bludgeoning the utilities into behaving properly

A turning point like G is required, otherwise the curve would follow the dotted line GD and lead to a corner solution or one where we would have two local maxima. Of course, the turning point or intersection of the bold curve with the x-axis could occur at $-\infty$ in which case we allow for asymptotic behaviour at the lower end indicated by the path GE.

What we need to exclude, if we are to derive unambiguous results, are curves like Figure 4.2 below which might well occur for many specifications of utility, density and cost functions. The only alternative in this case would be to compare the local maxima directly.

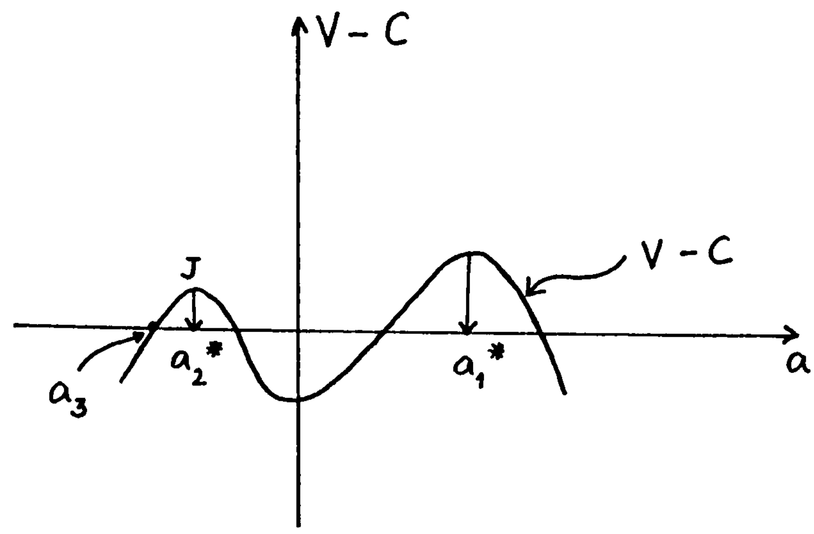


Figure 4.2

These ambiguities arise when we have more than one region over which V–C is positive, leading to the possibility of local maxima and minima. We are now in a position to state the revised sufficient conditions for the validity of the first - order approach.

The Revised Sufficient Conditions

Assume a solution to the relaxed programme exists. Then the first-order approach is valid if,

S1. $V'' - C'' < 0 \quad \forall\ a \in [-\infty, \alpha_1]$

S2. $V'' - C'' > 0 \quad \forall\ a \in [\alpha_1, \alpha_2]$

S3. $V'' - C'' < 0 \quad \forall\ a \in [\alpha_2, \infty]$

S4. $V - C \to 0$ as $a \to -\infty$ if " not $[\alpha_1 = \alpha_2 = -\infty]$"

α_1 and α_2 are the measure zero points where $V'' - C'' = 0$

That is, we require the V - g curve to be concave first, then convex [this region may collapse to a single point] and then concave again, with asymptotic behaviour at the lower end-point. The Mirrlees/Rogerson conditions imply $\alpha_1 = \alpha_2 = -\infty$, obviating the need for S4. The example discussed in Section 3 has $\alpha_1 = \alpha_2$. The asymptotic behaviour of V - C is crucial to the analysis as it restricts V - C to behaving like EGBAF in Figure 4.1 while ruling out potentially damaging exotic behaviour like HGBAK. Turning points like J in Figure 4.2 with $a_2^* > -\infty$ are ruled out because with positive slope, the curve would intersect the x-axis at, say, $a_3 > -\infty$, giving $V - C < 0$ for $a < a_3$, violating S4 [7].

We have, on the one hand imposed rather strong conditions by allowing one path for V - C. However, on the other we have allowed for a more flexible behaviour of V - C when V - C is negative, using the $U_0 > 0$ to exclude all possible choices of 'a' in this region. This certainly assures greater flexibility in the behaviour of V - C than the Mirrlees/Rogerson conditions which indeed imply special cases of our sufficient conditions.

Our approach has also been fairly heuristic and has proceeded mainly by inspection. The lesson to be learnt is to concentrate on the behaviour of the functions V - C themselves rather than relying too much on conditions like CDFC and MLRC. The examples presented in the final section of this chapter show how the sufficient conditions might be used and suggest classes of cases which will satisfy them.

5. Using the Sufficient Conditions: A Few Examples.

In order to motivate the discussion further let us work with the exponential cost function. As before,

$V'' = k.\exp(-(\bar{a} - a)^2/2\sigma^2))$ where $k \equiv \mu/(2\pi\sigma^2)^{1/2}$

$\bar{a} \equiv a^* - \lambda/\mu$

$C(a) = e^a$ and $\sigma > 0$ and $\sigma \notin N(0)$

We are interested, in particular, in the behaviour of,

$h(a)/k$ where $h(a) = V''/C''$.

$h'(a)/k = (C(a)^{-1}).\ \{(\bar{a} - a).\sigma^{-2} - 1\}.\exp\{-(\bar{a} - a)^2/2\sigma^2\}$

Thus,

$h'(a) \geq 0 \iff a \leq \bar{a} - \sigma^2$

$h'(a) < 0 \iff a > \bar{a} - \sigma^2$

Also,

$h''(a)/k = (C(a)^{-1}).[\{(\bar{a} - a).\sigma^{-2} -1\}^2 - \sigma^{-2}].\exp\{-(\bar{a} - a)^2/2\sigma^2\}$

Thus,

$h''(a) \geq 0 \iff a < \bar{a} - \sigma^2 - \sigma$ OR $a > \bar{a} - \sigma^2 + \sigma$

$< 0 \iff \bar{a} - \sigma^2 - \sigma < a < \bar{a} - \sigma^2 + \sigma$

Thus h(a) looks like,

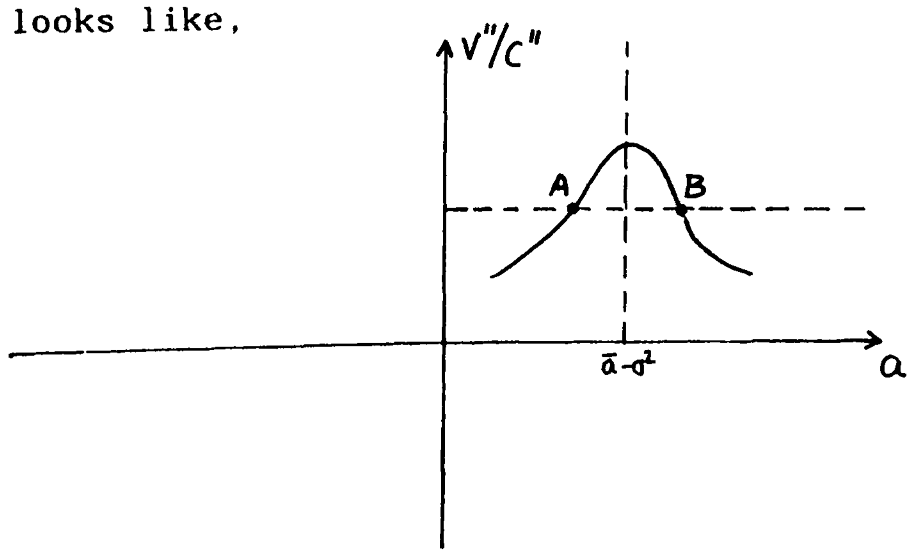


Figure 5.1

The next task is to work out the co-ordinates of A and B, that is where $h(a) = 1$.

$h(a) = 1$, provided,

$k.\exp(-(\bar{a} - a)^2/2\sigma^2) = e^a$

that is,

$a^2/2\sigma^2 + a.[1 - \bar{a}/\sigma^2] + \bar{a}^2/2\sigma^2 - \log k = 0$

The roots are given by,

$a = \bar{a} - \sigma^2 \pm \sigma^2(1 - 2\bar{a}/\sigma^2 + 4\log k/2\sigma^2)^{1/2}$

There is no particular significance to the precise numbers attached to the co-ordinates. We must assume however, at least, that real roots exist. This is the simplest case to deal with. If no real roots exist, then either

$V'' > C''$ $\forall$ a, in which case the problem becomes meaningless or,

$V'' < C''$ $\forall$ a, and the first-order approach is always valid. The case, for particular configuration of a, λ, μ, that we discuss here, is therefore the most general.

Given that V''/C'' behaves as in Figure 5.1, V - C looks like the curve in Figure 5.2. However, we can use some other information to fix the precise position of the curve.

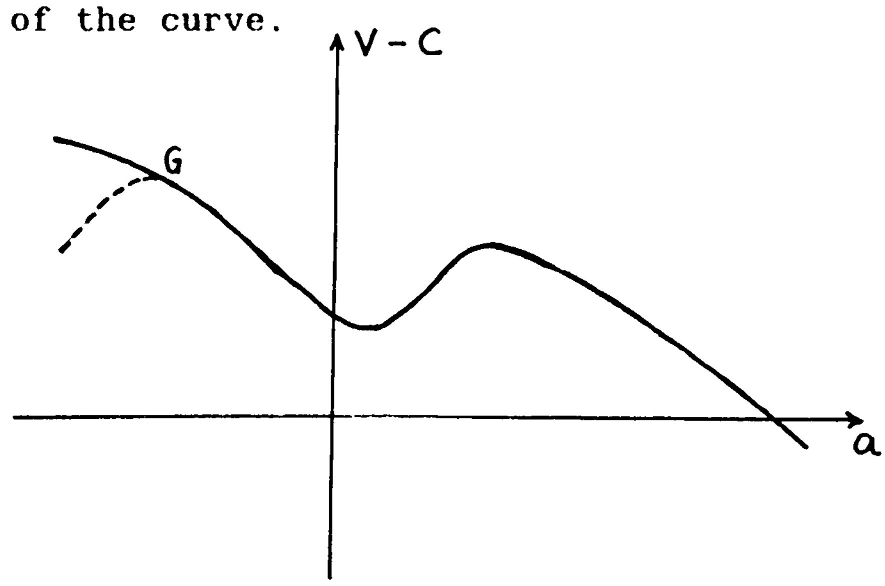


Figure 5.2

First, it cannot have a turning point at G of the sort indicated by the dotted line because,

$$V' - C' = \mu.\int_s^\infty f(q)dq - e^a \to 0 \text{ as } a \to -\infty$$

Hence, there exists another turning point for a large negative value of 'a'. If there is a turning point at G additionally, then there would exist a second convex region for V - C which is ruled out by Fig 5.1. Thus, if anything, V - C would follow the bold curve. Note now that $V - C \rightarrow 0$ as $a \rightarrow -\infty$

This is because,

$$V = \underbrace{[\lambda + \mu(a-a^*)].\int_s^\infty f(q)dq}_{(i)} + \underbrace{\mu\sigma(2\pi)^{-1/2}.[\exp\{-s^2/2\}]}_{(ii)}$$

(ii) $\rightarrow 0$ as $a \rightarrow -\infty$, without any doubt.

Looking at (i), by a standard result in statistical theory, as $a \rightarrow -\infty$ (and therefore for large s)

$$(i) \sim [\underbrace{-\sigma\mu + \sigma^2\mu.(a^*-a-\lambda/\mu)^{-2} - \ldots}_{(iii)}].\underbrace{f(s)}_{(iv)}$$

The more terms we expand to, the better the approximation.

(iii) $\rightarrow -\sigma\mu$ as $a \rightarrow -\infty$

(iv) $\rightarrow 0$ as $a \rightarrow -\infty$

Thus (i) $\rightarrow 0$ as $a \rightarrow -\infty$

$C(a) = e^a \rightarrow 0$ as $a \rightarrow -\infty$

Hence, we have proved $V - C \rightarrow 0$ as $a \rightarrow -\infty$.

This provides another reason why V - C cannot have a turning point at G because none of the consequent paths sketched below are permissible. There will have to exist a convex region either by the argument that $V' - C' \rightarrow 0$ as $a \rightarrow -\infty$ which applies to Path 1 or by the need to have $V - C \rightarrow 0$ as $a \rightarrow -\infty$ in the case of Path 2. Taken in total, V - C must look like Path 3 in Figure 5.4.

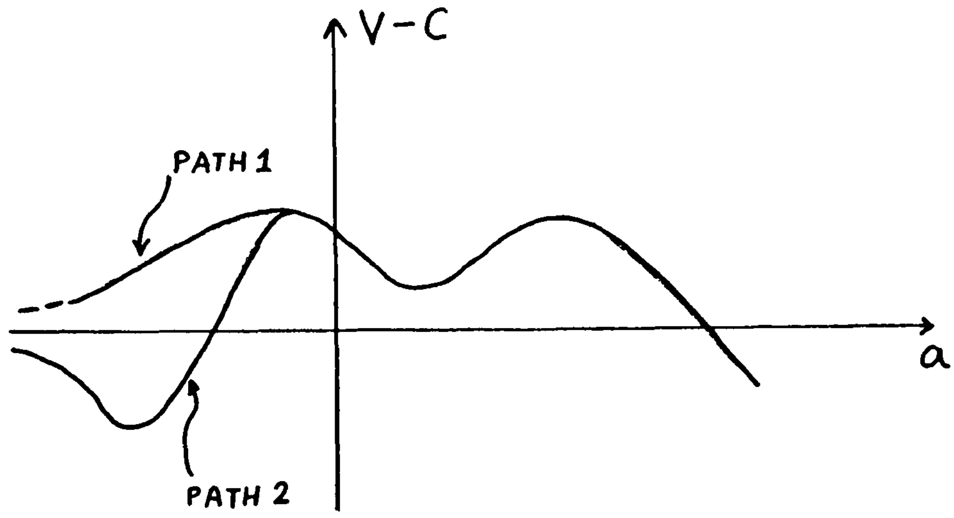


Figure 5.3

The bold curve indicated in Figure 5.2 is not possible as $V - C \to 0$ (as $a \to -\infty$) there. The need only is to shift the curve downwards so as to have Figure 5.4 as the only curve that is consistent with all the restrictions, which are,

(i) The curve V - C is CONCAVE $\overset{\text{then}}{-}$ CONVEX $\overset{\text{then}}{-}$ CONCAVE

(ii) $\lim_{a \to -\infty} (V - C) = 0$

(iii) $\lim_{a \to -\infty} (V' - C') = 0$

(iv) $\lim_{a \to -\infty} (V'' - C'') = 0$

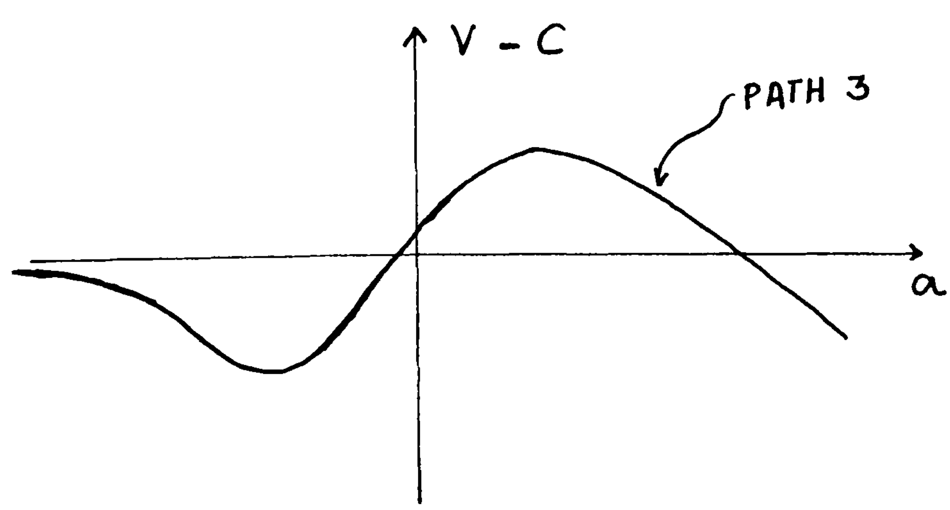


Figure 5.4

and we are back to the case of Figure 3.3. That V - C has a positive part is an assumption necessary for the existence of a solution with $U_0 > 0$. The same story is expressed alternatively in Figure 5.5 which is very much the ideal picture.

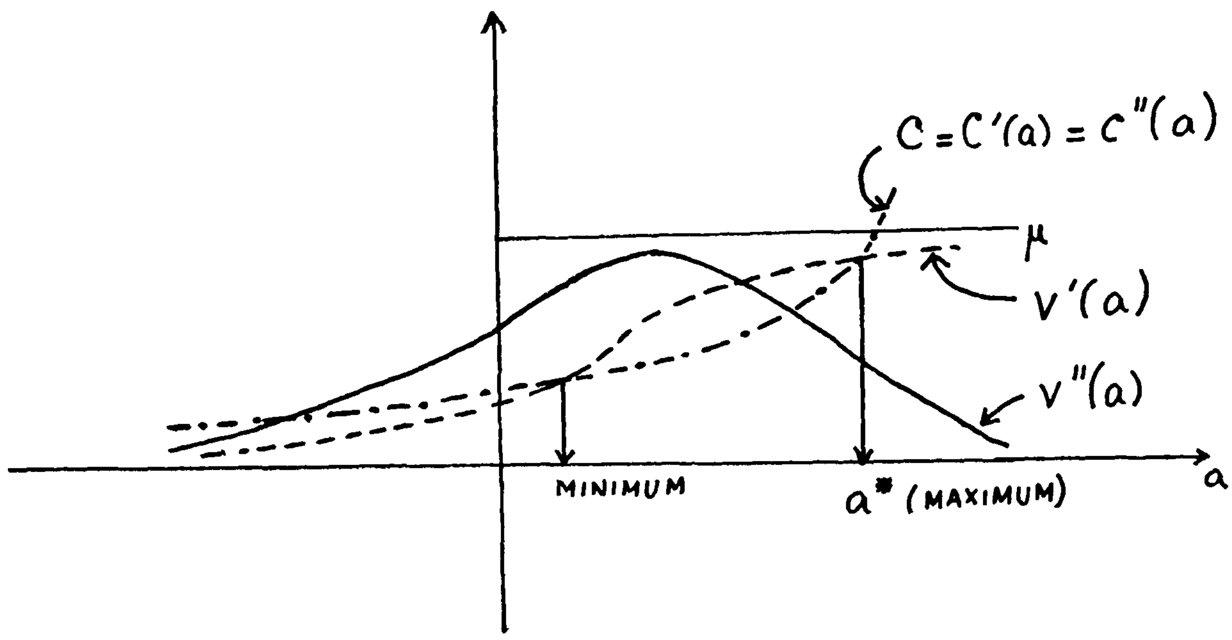


Figure 5.5

To summarise,

Proposition 2

(1) The first-order approach works here for the class of exponential cost functions by virtue of our having shown that the revised sufficient conditions are satisfied.

(2) It should also be noted, that where C" = constant and V" has the shape indicated in Figure 5.5, V"/C" will have the shape indicated in Figure 5.1 and the first-order approach will work.

The moral of the analysis in this section is quite clear. We propose the following scheme for establishing whether the first-order approach will work under the weakened set of conditions.

Step 1: Work out V"

Step 2: It is next easy to work out conditions C" must satisfy for V"/C" to have the shape indicated in Figure 5.1.

Step 3: If additionally, $V - C \to 0$ as $a \to -\infty$, the sufficient conditions S1 - S4 are satisfied and the first-order approach will work.

This method of approaching the issue of validity of the first-order approach will help to extend the class of cases and we shall be able to move away from the rigid structure on expected utility imposed by the Mirrlees/Rogerson conditions.

We shall end this section on a note of caution by asking the following question :
'Does the first-order approach work $\forall\ \sigma$ and for every specification of the cost function in this special example?'
The answer, unsurprisingly, appears to be: 'No, not generally' as the following calculations show.

$V(a) - C(a) = \int \{\eta(q, a) - C(a)\}f(q)dq$ where
$\eta(q, a) = [\hat{\lambda} + \mu\sigma q + \mu a]_+$ where $\hat{\lambda} = \dot{\lambda} - \mu a^*$
$\eta(q, a) - C(a)$ is obviously concave when positive [for given q and λ] as a function of 'a', as long as $\mu < C'(\infty)$; $0 < C'(-\infty)$, and is positive over only one region.

We need to show that summing over all q, this concavity property is retained. This turns out to be the difficulty.
To show the difficulty, let q adopt two values q_1 and q_2 with, say, equal probability.
Denote by $\omega_i(a) \equiv \eta(q_i, a) - C(a)$; $i = 1,2$
Now,
$\omega_i(a)$ is concave $\forall$ a such that $\omega_i(a) \geqslant 0$; $i = 1, 2$
Look at $\Omega = \omega_1(a) + \omega_2(a)$.
There are four possible cases to consider.
<u>Case 1</u>: $\omega_i(a) \geqslant 0$; $i=1, 2$ with at least one of the inequalities strict.
Here there is no problem in proving the concavity of Ω when $\Omega \geqslant 0$.
<u>Case 2</u>: $\omega_i(a) \leqslant 0$; $i=1, 2$. This is not relevant.

Case 3: $\omega_1(a) \geq 0$; $\omega_2(a) \leq 0$ such that $\Omega \geq 0$

Case 4: Symmetric to Case 3 above.

As we have made no assumption about convexity or concavity when $\omega_i(a)$ is negative (that is it could be either concave or convex as the previous diagrams have demonstrated) we must allow for either possibility. If $\omega_2(a)$ happens to concave, say in Case 3 above then there is no problem. However, should it be convex, then there seems to be no way of proving any 'concave when positive' condition for Ω. A similar analysis applies to Case 4 and highlights the negative answer to the question posed originally. It also leads us to expect that for some specifications of cost functions even the weakened sufficient conditions will not be satisfied for this example. More generally, it is not too difficult to think of examples where the first-order approach will not work.[8] Nevertheless, this is only to be expected and in at least being able to extend the cases for which the first-order approach does work we regard the task of this chapter accomplished.

6. Conclusions

This chapter took as its starting point the observation detailed in Mirrlees (1979), Rogerson (1985) and Grossman and Hart (1983) that the first-order approach to solving principal-agent problems is not valid in general. The sufficient conditions, identified in the above papers, which guarantee validity, impose unnecessary structure on the behaviour of expected utilities. We argued that by looking directly at the behaviour of the expected utility of the agent and not concentrating on the sufficient conditions on density and cumulative density functions weaker sufficient conditions could be found, enabling us to benefit from the analytic convenience of the first-order approach than those allowed by the Mirrlees/Rogerson conditions. These weaker sufficient conditions required a particular pattern of behaviour for V"/C". These conditions were used to justify the use of the first-order approach for a class of cases not satisfying the Mirrlees/Rogerson conditions. It was also demonstrated that for distributions which differ from perfect observability by small additive variance shifts, the first-order approach is perfectly valid. This demonstration also justifies the techniques used in Chapter 2. It is hoped therefore that the analysis in this chapter will help to rehabilitate faith in the use of the first-order approach - and its accompanying convenience - in principal-agent models.

Notes

1. Chapter 2 compared two situations:
(1) P = a (Perfect observability) and (2) $P = a + \sigma.q$
where $q \sim (0, 1)$ and deduced results on signs and magnitudes of $\partial a/\partial\sigma$ etc. See Section 3 of this chapter for further details.

2. These conditions were first proposed by Mirrlees (1979) and therefore ought to be called properly the Mirrlees/Rogerson conditions. Rogerson (1985) was the first to rigorously prove sufficiency simply.

3. Not so reasonable if effort is interpreted as choice of riskiness of project in the sense of Ross (1973/74). See Chapter 3 for further details and also Grossman and Hart (1983a) p28 - 29. The optimal action for the principal might require the agent not being too cautious in choosing projects and it may pay the principal to pay high amounts for bad outcomes (to discourage excessive caution).

4. This is non-standard notation. N(0) is meant to read 'in the neighbourhood of zero.

5. This may be esily seen from (3.13). As $\sigma \to 0$, $\mu \to g'(a^*)$. As long as we insist upon $C'(a) > 0 \ \forall\ a > -\infty$, the result follows.

6. See Chapter 2 for further details

7. We reason as follows. Unless, at points $a \in N(a_2^*)$, V - C is parallel to the x-axis, the intersection of V - C with the x-axis would

occur at $a_3 > -\infty$. By strict concavity of V - C in the region under consideration, even if V - C were parallel in the neighbourhood of a_2*, it would have positive slope for sufficiently negative 'a'. Hence at $a_3 > -\infty$, V -C = 0 and for all $a < a_3$, V - C < 0, violating S4.

8. Section 4 of Chapter 2 presents one such example.

Chapter 5: Moral Hazard, Limited Liability and the Corporation Tax.

Abstract:

This chapter uses a model of managers' behaviour under uncertainty to analyse the effects of limited liability on optimal profits taxes. In particular, it addresses the question of whether the benefits of limited liability should be taxed. Two interpretations of managers' behaviour are examined. First, a manager chooses his effort level which affects the distribution of profits according to first-order stochastic dominance. Second, the manager chooses an investment portfolio which alters the distribution of outcomes according to a mean preserving spread of utility. The analysis is conducted under four alternative configurations determined by limited liability arrangements and interpretation of managers' behaviour. We show that limited liability does affect the tax and, under certain conditions, according to the formula that limited liability implies higher positive taxes.

1. Introduction

This chapter uses a model of managers' behaviour under uncertainty to cast light on the validity of an argument which is often used in Public Finance theory. The argument is well put in the Meade report:

> "... it may be thought that the privileges conferred by incorporation, and in particular the benefits of limited liability, justify the levy of an additional tax on such concerns."
>
> Meade (p.27, 1978)

The argument given above is by no means universally accepted. Kay and King (1982), for example, suggest that:

> "...[A]t first sight this argument has some appeal, but on closer inspection it becomes less attractive. There is no reason to believe that the benefits of incorporation are proportional to profits (indeed the reverse might be the case) and one might as well argue for a license fee for companies."
>
> Kay and King (p.172, 1982)

The purpose of this chapter is to obtain, at least, a partial resolution of the controversy. Our aim is to consider whether the benefits conveyed by limited liability imply that taxes should be levied, that is there should be differential rates of tax between incorporated and unincorporated sectors and, more specifically, whether the tax rate should be higher in the corporate sector. We will look specifically at the effects of limited liability on optimal profits taxes using a simple model with (a) creditors who loan a sum of money to the firm (b) the owner-manager of a small firm and (c) the government.

The government and the owner-manager interact as principal and agent while the creditors loan money on a passive basis. The sort of situation we have in mind is where say customers of mail-order firms send off money in advance of receiving the goods. Institutional limited liability arrangements determine how much of the money they are repaid should the firm decide to cease trading.

The analysis is conducted primarily under two regimes.

(1) A regime where the owner-manager has unlimited liablity for the losses incurred by the firm; in other words, the amount loaned to the firm must be repaid under all circumstances and .

(2) an alternative regime with limited liability indexed by a (potentially small in magnitude) parameter ε.

Profits of the firm, from which the owner-manager derives utility, are defined as the value of output (or revenue) net of any loan repayments, while the creditor derives utility from the repayment net of loan. Repayments never exceed the loan and revenue is assumed to be positive always. The limited liability arrangements we consider may be illustrated by the following example.

Suppose the owner manager is loaned £ 20 for which he is fully liable. He will therefore have to repay the money irrespective of his income. Thus the argument of his utility function takes values ranging from - 20 to N = Q - 20 where Q is the maximum possible value of revenue. In such a situation, the creditors always get their loan repaid.

In the presence of limited liability, however, the owner-manager is not liable for any losses (revenue less repayment) in excess of £15; that is should he earn revenue of only £4 he will declare himself bankrupt and pay out £4 + £15 = £19 to the creditor. Here the argument of his utility function, when his loan is repaid, takes values ranging from - 15 to Q - 20, while losses in excess of £15 are met by the creditor. In this example, the creditor bears the loss of £1. (Any payment out of the capital stock owned by the firm can easily be incorporated into the liquidation arrangements). Since any loan repayment in excess of earnings is met presumably by drawing from the owner-manager's own assets, or by carrying forward his liability, under limited liability arrangements, the creditors have only a finite claim on the manager's assets. Should the manager not be liable for any losses in excess of £0, the creditors have no claim at all on the manager's assets. The two extreme cases are therefore unlimited liability and no liability for losses.

Two versions of a principal-agent model [1] are considered under each regime. In the first, the manager of a firm chooses his effort level which affects the probability distribution of profits in the sense of first-order stochastic dominance. Under the second interpretation, the action of the agent represents the choice of riskiness of the investment portfolio. The results that we obtain in this chapter therefore relate to four possible configurations of action choice and limited liability arrangements.

A. Unlimited liability - Interpretation 1 of action by manager

B. Limited liability - Interpretation 1 of action by manager

C. Unlimited liability - Interpretation 2 of action by manager

D. Limited liability - Interpretation 2 of action by manager.

The comparisons are made between A and B and between C and D respectively. The emphasis is on deriving propositions concerning the effects on taxation and action (effort) of limited liability. We show, quite naturally, that in Regime A, limited liability raises the probability of bankruptcy by lowering effort. We derive the necessary conditions under A for the optimal tax on profits when the budget is balanced [2]. We derive the necessary conditions under which the sign of the optimal tax may be determined. [3] Having done this we move to B and show that for certain cases introducing more limited liability increases taxes on profits. Hence profits will be more heavily taxed in the sector which has limited liability. This result is, therefore, evidence in favour of Meade's proposition. [4]

The role of the creditors is a passive one, both in the presence and absence of unlimited liability. In the first case, since he always encounters full repayment, his utility is maintained constant. With the introduction of limited liability, and the accompanying possibility of default should revenue fall below a certain level, the government introduces a lump-sum transfer to the creditors, in addition to the lump-sum payment to the owner-manager. This is in order to maintain the expected utility of the creditors at a pre-determined participation constraint utility level, especially since they bear the brunt of the worst outcomes. The introduction of the lump-sum transfer to the creditors also brings them into the tax system as it is otherwise assumed that profits taxes are levied only on the positive profits of the owner-manager. Thus if a manager is granted some limited liability, he is prepared to bear more taxes. This is because taxes lower his expected utility, ceteris paribus, while limited liability increases it. On the other hand, a creditor can have his expected utility maintained when limited liability is introduced by being given a lump-sum transfer. This leads us to the result that limited liability implies higher taxes.

Under C, we again establish a proposition linking the tax rate to the effect of taxation on the margin of risk-taking. We show that under certain conditions, limited liability increases risk-taking. The comparisons between C and D are more difficult given that the action does not induce stochastic dominance. Further, as we emphasise in the conclusion, enthusiasm for any assertions that one can make must be tempered by considering the simplicity of the framework that we are using. The analysis simply serves as a crutch for our intuitions.

In the chapter we show that it is both important and useful to distinguish between balanced budget changes in effort or risk-taking and changes when tax proceeds, which are re-distributed back to the manager in lump-sum form, are held fixed. This is because taxation not only distorts behaviour at the margin but may also have an effect if it offers some element of insurance in the way that it operates.

The chapter is structured as follows. In Section 2 we lay out describe in detail the basic model and derive comparisons between configurations A and B. In Section 3 we take the alternative interpretation, namely that of portfolio choice and compare configurations C and D. Section 4 uses the results derived in these two sections to suggest circumstances in which the argument concerning limited liability and the corporation tax can be supported by our findings. These suggestions form the conclusions from our work.

2.1 The Model

This sub-section introduces the basic model that we shall use throughout the chapter. It will serve as a useful reference point for the entire analysis. More specific details will be introduced in later sections whenever necessary. This is done in the interests of clearer exposition.

The model has three component parts - the owner-manager, the creditors and the government. We shall deal with each of these in turn under (a) unlimited liability and (b) limited liability. We shall start with a description of the owner-manager's choices.

The Owner-Manager

We use what is now familiarly referred to in the literature as a 'principal-agent' model. Furthermore, the 'first-order' approach is used to solve these models. This approach involves maximising the utility of, say, the principal subject to (i) a minimum utility constraint for the agent and (ii) the agent's utility being at a stationary point. The large volume of literature that has developed in the recent past [5] criticising the use of the first-order approach we consider largely irrelevant for the purposes of this chapter for two distinct reasons. First, the convenience afforded by the first-order method far outweighs any considerations of mathematical elegance. This is especially true in light of the aim of this chapter which is simply, and above all, to get a handle on the intuitive arguments advanced in the introduction rather than to construct a totally water-tight theory. Second, as we have shown elsewhere [6], the first-order approach can be generalised and applied to a larger number of cases than those allowed by Rogerson's (1985) sufficient conditions.

The assumptions of this part of the model are as follow:

1. The agent privately takes an action 'a ' ε A, A being the set of all possible actions.
2. A random state of nature $\theta \varepsilon \Theta$ together with 'a ' determines the monetary outcome or payoff:

   $\pi = \pi(a, \theta) = R(a, \theta) - M$ where M is the amount borrowed by the firm.

   $R(a, \theta) \geqslant 0$
3. The agent's utility function is V(w, a) defined over wealth 'w ' and action 'a '. In our formulation,

   $V(w, a) = V(w) - C(a)$ with $C'(a) > 0$, $C''(a) > 0$.

In this simplified set up, with no government intervention and consequently no moral hazard, the optimal programme is simply to maximise,

$\int V(\pi)f(\pi, a)d\pi - C(a)$

4. Given a distribution of θ, $F(\pi, a)$ is simply the distribution imposed on π via the relationship $\pi = \pi(a, \theta)$.

5. For all 'a ', assume (1) $f_a(\pi, a)/f(\pi, a)$ is an increasing function of π and (2) $F_{aa}(\pi, a) \geq 0$ with strict inequality for a range of π s.

   These are the Mirrlees/Rogerson sufficient conditions, referred to respectively as the Monotonic Likelihood Ratio Condition (MLRC) and the Convexity of the Distribution Function Condition (CDFC).

6. $V'(.) > 0$; $V''(.) < 0$; $V'''(.) > 0$

We now discuss how limited liability arrangements affect the owner-manager. Under unlimited liability, if the owner-manager loans M from the creditors and then it must be re-paid for all realisations of π. Thus the expected utility of the owner-manager is given by,

$$\int_{-M}^{N} V(\pi)f(\pi, a)d\pi - C(a) \qquad (2.1.1)$$

Now suppose that liability is limited. If π is above $- M + \varepsilon < 0$ then the manager must pay out the full amount of the loan. If however π falls below $-M+\varepsilon$ then the owner-manager declares himself bankrupt - with bankruptcy costs artificially normalised at zero - and pays out $R + M - \varepsilon$. Thus, in this case, the expected utility of the manager is given by,

$$F(-M+\varepsilon, a)V(-M+\varepsilon) + \int_{-M+\varepsilon}^{N} V(\pi)f(\pi, a)d\pi - C(a). \qquad (2.1.2)$$

Any lump-sum transfer from the government to the manager alongwith any taxes levied, may be incorporated in the standard way into the expressions for the expected utility of the manager.

The Creditors

With unlimited liability, the creditors are always repaid the full amount. Thus his expected utility is given by a constant $\Omega(0)$. In the presence of the limited liability arrangement described above, his expected utility is given by,

$$\underbrace{\Omega(0)[1 - F(-M + \varepsilon, a)]}_{(i)} + \underbrace{\int_{-M}^{-M+\varepsilon} \Omega(\pi + M - \varepsilon)f(\pi, a)d\pi}_{(ii)} \qquad (2.1.3)$$

(i) represents the expected utility of the creditors should the manager not be able to declare himself bankrupt (if $\pi \geqslant -M + \varepsilon$) while (ii) is his expected utility if the manager goes bankrupt. Any lump-sum transfer from the government is also easily incorporated into (2.1.3) while the creditors are never taxed because they never actually get more than their loan repaid.

The Government

The government is assumed to introduce balanced budget taxes. Under unlimited liability, the budget constraint of the government is given by,

$$t.\int_{0}^{N} \pi f(\pi, a)d\pi - \beta_1 = 0. \qquad (2.1.4)$$

Taxes are levied only on positive profits and no lump-sum transfer is made to the creditors. β_1 represents the lump-sum transfer to the manager. Under limited liability, the budget constraint is given by,

$$t.\int_0^N \pi f(\pi, a)d\pi - \beta_1 - \beta_2 = 0$$

where β_2 represents the lump-sum transfer to the creditors to compensate for the loss in utility faced by them should the manager default.

2.2 Some Institutional Details.

In order to examine further the effects of limited liability, imagine that the manager is liable, as before, up to an amount $-M + \varepsilon$, that is, his expected utility is,

$$U(\varepsilon)= F((-M+\varepsilon),a\ )V(-M+\varepsilon) + \int_{-M+\varepsilon}^{N} V(\pi)f(\pi, a)d\pi - C(a) \quad (2.2.2)$$

Note first that an increase in limited liability increases the utility level of the agent. This can be seen by writing out the expression for $\partial U/\partial\varepsilon$.

$$\partial U/\partial\varepsilon = F((-M+\varepsilon), a).V'(-M+\varepsilon)$$

$$+ \{F_a.V + \int_{-M+\varepsilon}^{N} V(\pi)f_a(\pi, a)d\pi - C'(a)\}\partial a/\partial\varepsilon \qquad (2.2.3)$$

$$= F((-M+\varepsilon), a).\ V'(-M+\varepsilon) \qquad (2.2.4)$$

The term in curly brackets disappears because it is the first-order condition for the agent's choice of effort and at the optimum is set equal to zero. The result follows by inspection.

We now prove a further effect of limited liability.

Result 1: An increase in limited liability reduces the level of effort chosen by the agent, that is, $\partial a/\partial\varepsilon < 0$

Proof: The first-order condition for effort choice is given by,

$$F_a((-M+\varepsilon, a)V(-M+\varepsilon) + \int_{-M+\varepsilon}^{N} V(\pi)f_a(\pi, a)d\pi - C'(a) = 0 \qquad (2.2.5)$$

Differentiating (2.2.5) with respect to ε and re-arranging terms, gives,

$$\partial a/\partial\varepsilon = -[\Delta]^{-1}[F_a(-M+\varepsilon, a).V'(-M+\varepsilon)] \qquad (2.2.6)$$

where

$$\Delta \equiv [F_{aa}.V(-M+\varepsilon) + \int_{-M+\varepsilon}^{N} V(\pi)f_{aa}(\pi, a)d\pi - C''(a)] \qquad (2.2.7)$$

$$= \underbrace{-\int_{-M+\varepsilon}^{N} V'(\pi)F_{aa}(\pi, a)d\pi}_{(i)} \quad \underbrace{- C''(a)}_{(ii)}$$

Δ is the second-order condition for the agent's choice of effort and is less than zero. By CDFC, (i) is less than zero, while (ii) is always less than zero (by assumption of strictly convex cost of effort function).

The result now follows, under the assumption that the second-order condition is satisfied, by using the stochastic dominance condition that $F_a < 0$.

2.3 Taxation and Effort Choice under Configuration A: The Case of Unlimited Liability

We have already observed that the introduction of limited liability reduces the level of effort by the agent. We list below the main results of Sections 2.3 - 2.5.

(1) In Sections 2.3 - 2.4 we are concerned with configuration A discussed in the introduction. We have a government levying a balanced budget tax t on positive profits and giving a corresponding lump-sum transfer of β to the owner-manager. The creditors need not be included in the discussion here because they always obtain full repayment of their loans in this regime of unlimited liability. It is shown that where taxation reduces effort, the optimal tax is in fact positive that is,

$\partial a/\partial t \leqslant 0 \Rightarrow t > 0$.

(2) An intuition for this result is presented in Section 2.4. Interestingly it turns out that the implication cannot be reversed, that is,

$t > 0$ does not imply $\partial a/\partial t \leqslant 0$.

An explanation for this finding is also proposed.

(3) We are also able to deduce sufficient conditions to sign the optimal tax for a special case. We show that provided the third derivative of the agent's utility function is non-negative, the optimal tax is strictly positive.

(4) In Section 2.5 we move to configuration B and show that the introduction of limited liability raises taxes on profits. Hence profits will be more heavily taxed in the sector which has limited liability. This result is therefore evidence in favour of Meade's proposition. An extended discussion in this section presents the intuition for this result.

The Analysis under Configuration A

The government's budget constraint is given by,

$$t.\int_0^N \pi f d\pi - \beta = 0 \qquad (2.3.1)$$

where t is the rate of tax on profits and β is a lump-sum tax. [7]

We have assumed implicitly here that the government is risk-neutral.

Expected utility of the owner-manager is,

$$\int_{-M}^{0} V(\pi+\beta)fd\pi + \int_0^N V(\pi(1-t) + \beta)fd\pi - C(a) \qquad (2.3.2)$$

as we assume that the government levies taxes only on positive profits. Note also from the agent's first-order condition (2.2.5), using integration by parts,

$$\partial a/\partial t = (\Delta)^{-1}[\int_0^N V'\pi f_a d\pi] = -[\Delta]^{-1}[\int_0^N V'(1-R)F_a d\pi \qquad (2.3.3)$$

where $R \equiv -(V''/V').\pi(1-t)$ and is the index of partial risk-aversion, [8] and,

$$\partial a/\partial\beta=(\Delta)^{-1}.[\int_{-M}^{0} V''(\pi+ \beta)F_a d\pi+(1-t)\int_0^N V''(\pi(1-t)+\beta)F_a d\pi] \qquad (2.3.4)$$

Therefore,

$\partial a/\partial t \geqslant (<) 0$ according as $R \geqslant (<) 1$ and

$\partial a/\partial\beta < 0$ always.

The government's problem is to maximise (2.3.2) subject to the government budget constraint (2.3.1), by choosing t and β.

Look at the Lagrangean, for choosing optimal t and β,

$$Y = \int_{-M}^{0} V(\pi+\beta)f d\pi + \int_{0}^{N} V[\pi(1-t) + \beta]f d\pi - C(a) + \lambda[t\int_{0}^{N} \pi f d\pi - \beta] \qquad (2.3.5)$$

Note that we have implicitly already solved out for 'a' in terms of t and β from the agent's first-order conditions, that is, $a = a(t, \beta)$ and therefore t and β are left to be determined.

$$\partial Y/\partial\beta = \int_{-M}^{0} V'(\pi+\beta)f d\pi + \int_{0}^{N} V' f d\pi - \lambda + \lambda t.\int_{0}^{N} \pi f_a d\pi.\partial a/\partial\beta = 0 \qquad (2.3.6)$$

$$\partial Y/\partial t = \lambda\int_{0}^{N} \pi f d\pi + \lambda t\int_{0}^{N} \pi f_a d\pi.\partial a/\partial t - \int_{0}^{N} V'\pi f d\pi = 0 \qquad (2.3.7)$$

The derivation of $\partial Y/\partial\beta$ and $\partial Y/\partial t$ make use of the agent's first-order condition.

It is useful to note the following re-arrangement of (2.3.6) and (2.3.7) above,

$$t = \frac{\lambda - \int_{0}^{N} V' f d\pi - \int_{-M}^{0} V'(\pi+\beta)f d\pi}{\lambda. \int_{0}^{N} \pi f_a d\pi.\partial a/\partial\beta} \qquad (2.3.8)$$

re-arranging (2.3.6) and,

$$t = \frac{\int_{0}^{N} V'\pi f d\pi - \lambda.\int_{0}^{N} \pi f d\pi}{\lambda. \int_{0}^{N} \pi f_a d\pi.\partial a/\partial t} \qquad (2.3.9)$$

re-arranging (2.3.7).

We may now proceed to the determination of the sign of the optimal tax.

Proposition 1: At an optimum under limited liability $\partial a/\partial t < 0$ implies $t > 0$.

Proof: The proof is by contradiction. Suppose the proposition is false; then $t < 0$ and $\partial a/\partial t < 0$ are not mutually contradictory. From (2.3.7),

$$\lambda = \frac{\int_0^N V'\pi f d\pi}{[\int_0^N \pi f d\pi + (t\int_0^N \pi f_a d\pi)\partial a/\partial t]} > 0 \qquad (2.3.10)$$

if $\partial a/\partial t < 0$ and $t < 0$.

Hence from (2.3.8),

$$t < 0 \text{ implies } \lambda - \int_0^N V'fd\pi - \int_{-M}^0 V'fd\pi > 0$$

$$\text{so that } \lambda > \int_0^N V'fd\pi \qquad (2.3.11)$$

Using (2.3.11), the numerator of (2.3.9) satisfied,

$$\int_0^N V'\pi fd\pi - \lambda \int_0^N \pi fd\pi < \int_0^N V'\pi fd\pi - \int_0^N V'fd\pi . \int_0^N \pi fd\pi < 0 \quad (2.3.12)$$

where the last inequality follows from the assumption of $V''' > 0$ and using the fact that $V'(.)$ is decreasing (risk-aversion) and π is increasing (trivially) in π.

Using (2.3.9) we now see,

$$t < 0 \text{ implies } \partial a/\partial t \geqslant 0. \qquad (2.3.13)$$

The term $\partial a/\partial t$ cannot equal zero at the optimum since then $t = -\infty$.

The above result states (i) that if, ceteris paribus, optimal effort choice is reduced by higher tax rates (reading the implication in its equivalent alternative form, $\partial a/\partial t < 0$ implies $t>0$) profits should be taxed. Also, (ii) the implication cannot be reversed. The next sub-section is devoted to providing some intuition for (i) and (ii) above, in the hope of making the findings entirely believable.

2.4 Intuition for Proposition 1.

As given, our result is only partial; it does not rule out $t > 0$ with $\partial a/\partial t > 0$. We will now argue that there is a good reason for this being so once a distinction between balanced budget changes (denoted BBC) in 'a' and ceteris paribus changes (denoted CPC) in 'a', where β, the lump-sum transfer, is held constant, is made.

Under BBC, 'a' must satisfy the government budget constraint as well as the agent's first-order condition. Denote expected utility of the agent as a function of effort, the tax rate, and the lump-sum transfer as ,

$$W(a, \beta, t) \qquad (2.4.1)$$

The first-order condition is,

$$W_a(a, \beta, t) = 0 \qquad (2.4.2)$$

This is just an alternative representation of (2.2.5). Denote the government's budget constraint (2.3.1) in similar fashion; that is, as a function of the same three variables,

$$G(a, \beta, t) = 0 \equiv \beta - t\int \pi f d\pi \qquad (2.4.3)$$

Any BBC in t and 'a' must satisfy (2.4.2) and (2.4.3). Totally differentiating these two equations and regarding these differentials as simultaneous equations yields,

$$\begin{bmatrix} W_{aa} & W_{a\beta} \\ G_a & G_\beta \end{bmatrix} \begin{bmatrix} da \\ d\beta \end{bmatrix} = - \begin{bmatrix} W_{at} \\ G_t \end{bmatrix} dt \qquad (2.4.4)$$

where obvious notation for derivatives has been used. We may solve (2.4.4) to get the BBC in 'a ' and 'β '.

$$da/dt = (W_{aa}G_\beta - W_{a\beta}G_a)^{-1}.(-W_{at}G_\beta + G_tW_{a\beta}) \qquad (2.4.5)$$

Using (2.3.3) and (2.3.4) it should be clear that,

$$\text{sign } \partial a/\partial t = \text{sign } W_{at} \qquad (2.4.6)$$

and that $W_{a\beta}$ is negative. Routine operations will establish that $G_t<0$, $G_a<0$ if $t > 0$, $G_\beta>0$. Using these, we see that da/dt and $\partial a/\partial t$ may well differ in sign. The following relationship may be seen using (2.4.5), and using Proposition 1,

$$da/dt < 0 \text{ if } \partial a/\partial t < 0 \qquad (2.4.7)$$

No obvious relationship seems to exist between the signs of da/dt and $\partial a/\partial t$ when $da/dt > 0$.

The reason for the difference in the signs of the BBC and CPC in 'a' with respect to changes in t lies in the fact that when considering a BBC in the rate of tax t, we should allow for the effect on β of a change in taxes. In effect, higher tax rates raise β and lower effort because they act like insurance. We shall see shortly that it is this feature of the tax system which means that only a partial relation between the signs of t and $\partial a/\partial t$ can be established. The tax system not only distorts at the margin (represented by CPC), as we have in the conventional analysis of the incidence of taxation, but offers insurance [9] and alters behaviour through this channel also.

To see the importance of this, consider the question of whether a balanced budget change in taxation yields a welfare improvement beginning from an arbitrary initial position. From (2.4.3),

$$d\beta/dt = t\int \pi f_a d\pi . da/dt + \int \pi f d\pi \quad (2.4.8)$$

when the balanced budget is maintained [10]. Whether welfare rises or falls when such a change is implemented is determined from,

$$dW/dt = \partial W/\partial t + \partial W/\partial \beta . d\beta/dt \quad (2.4.9)$$

From (2.3.2),

$$\partial W/\partial t = - \int_0^N V'\pi f d\pi \quad (2.4.10)$$

$$\partial W/\partial \beta = \int_{-M}^{0} V'(\pi+\beta) f d\pi + \int_0^N V'(\pi(1-t) + \beta) f d\pi$$

Using these in (2.4.9) yields (after a little manipulation),

$$dW/dt = [\int_0^N V' f d\pi . \int_0^N \pi f d\pi - \int_0^N V'\pi f d\pi] + \int_{-M}^{0} V'(\pi+\beta) f d\pi . \int_0^N \pi f d\pi$$

$$+ t . \int_0^N \pi f_a d\pi . da/dt . [\int_{-M}^{0} V' f d\pi + \int_0^N V' f d\pi] \quad (2.4.11)$$

The first term in (2.4.11) is minus the covariance between profits and the marginal utility of income and is positive. The second term is also positive as is the part of the final term enclosed in square brackets. Equation (2.4.11) helps to establish a further proposition which assists our characterisation of the optimal tax regime.

Proposition 2: Under BBC, a necessary condition for an optimum is

(a) $t \neq 0$

(b) $t . da/dt < 0$

Proof:

Part (a): This follows by inspection of (2.4.11) (provided $da/dt \neq 0$). Note that $dW/dt > 0$ if $t=0$ and thus an increase in the tax rate, starting from $t=0$, raises welfare.

Part(b): Suppose not. Then $t . da/dt \geqslant 0$ may be an optimum. But from

(2.4.11), if this were true then a balanced budget change in the tax will raise welfare but this contradicts the assumption that we are at an optimum.

Note that this characterisation applies only with a BBC in 'a' and not with a CPC. This is an important observation to make. The link between the two propositions that we have established, the first referring to CPC and the second to BBC, lies in equations (2.4.7).

The lesson to be learnt from this analysis is that it is important, when analysing the incidence of taxation under uncertainty to distinguish between the effects of taxes on the margin of effort and possible insurance effects when the proceeds of taxation are re-distributed back to the manager in lump-sum form. When the analysis of taxation is undertaken under certainty, the latter effect manifests itself in income effects. Our effects are somewhat different since they pertain to real changes in the payoff structure experienced by agents due to taxation offering insurance. Since our analysis of limited liability is emphasising the role of limited liability as inducing moral-hazard, it is not surprising that we will find further importance for this distinction when we characterise the optimal response of taxation to limited liability.

Before proceeding to discuss the case of limited liability, it is interesting to discover whether the case $t < 0$ can in fact occur.

Proposition 3:[11] For the case where $\pi = a + \varepsilon$ where ε is a random variable and $\pi - a$ has density function f with $f(-M)=f(N) = 0$, $V''' > 0 \Rightarrow t > 0$

Proof: We have already shown in Proposition 2 that $t = 0$ cannot be an optimum, in general. We proceed now to rule out $t < 0$ for this special case.

$$W = \int_{-M}^{0} V(\pi + \beta)f(\pi-a)d\pi + \int_{0}^{N} V(\pi(1-t) + \beta)f(\pi-a)d\pi - C(a)$$

The government budget constraint is given simply by,

$$\beta = t.\int_{0}^{N} \pi f(\pi-a)d\pi$$
$$= t.z(a)$$

$$z(a) > 0;\ z'(a) > 0$$

Differentiating W with respect to t, using the government budget constraint and also taking advantage of the agent's first-order condition, we have,

$$dW/dt = \int_{-M}^{0} V'.(z(a)+t.z'(a)da/dt)f(\pi-a)d\pi$$
$$+ \int_{0}^{N} V'(z(a)-\pi + t.z'(a)da/dt)f(\pi-a)d\pi \qquad (2.4.12)$$
$$> \int_{-M}^{0} V'.fd\pi.tz'(a)da/dt + \int_{0}^{N} V'.fd\pi.tz'(a)da/dt \quad (2.4.13)$$

because,

(i) $\int_{-M}^{0} V'.z(a)f(\pi-a)d\pi > 0$

and,

(ii) V' is positively correlated with $(z(a) - \pi)$ over the sub-interval [0, N]. Thus,

$E[V'(z(a)-\pi)] > E(V')E(z(a)-\pi) = E(V')(z(a) - E(\pi)) = 0.$

We want to show that (2.4.13) > 0 when $t < 0$ and that is implied if 'a' is non-increasing in t.

We know that 'a' as a function of t satisfies,

$$\int_{-M}^{0} V(\pi+\beta)f'd\pi + \int_{0}^{N} V(\pi(1-t) + \beta)f'd\pi + C'(a) = 0 \quad (2.4.14)$$

$$\Rightarrow \int_{-M}^{0} V'(\pi+\beta) f d\pi + (1-t).\int_{0}^{N} V'((1-t)\pi + tz(a)) f d\pi = C' \quad (2.4.15)$$

after integrating (2.4.14) by parts.

The sign of da/dt is therefore the sign of W_{at} (that is, (2.4.15) differentiated partially with respect to t).

This is given by,

$$-\int_{0}^{N} V' f d\pi \;-\; (1-t).\int_{0}^{N} (\pi - z(a)) V'' f d\pi \;+\; \int_{-M}^{0} V'' z(a) f d\pi$$

$$(1) \qquad\qquad\qquad (2) \qquad\qquad\qquad (3)$$

It is clear that (1) and (3) are negative. (2) < 0 if $V''' > 0$ (using an argument similar to that used above).

Thus da/dt < 0 and (2.4.13) > 0 if t < 0.

Hence, t > 0 as required. //

Proposition 3 strengthens Proposition 2 derived earlier. It shows that for a special case, defined by a plausible sufficient condition, da/dt < 0 is the only possibility and hence t > 0 at an optimum.

2.5 The Analysis under Configuration B: The Case of Limited Liability

We again ask the question that we posed in the introduction. Is it always optimal for taxes to increase [or subsidies to decrease] with the introduction of limited liability? However, before proceeding to a resolution of that question it is worthwhile to dwell a little longer in extending Propositions 1 and 2 to the case of limited liability.

The Lagrangean - equation (2.3.6)- must now take account of the creditors' expected utility function. This is given by,

$$C = \int_{-M}^{-M+\varepsilon} \Omega(\pi + M-\varepsilon + \beta_2)f(\pi,a)d\pi + (1 - F(-M+\varepsilon,a))\Omega(\beta_2)$$

Equations (2.3.6) and (2.3.7) must now contain terms $C_a.\partial a/\partial\beta$ and $C_a.\partial a/\partial t$ respectively which by rendering terms on the r.h.s of (2.3.8) and (2.3.9) ambiguous obstruct the proof of Proposition 1, at least for finite non-zero ε. The Proposition is however restored as $\varepsilon \to 0$ as in that case $C_a \to 0$. A similar comment applies in the case of Proposition 2. Hence for sufficiently small ε, the results in Propositions 1 and 2 still hold.

Now consider a contract $D = (t, \beta)$ with $t, \beta > 0$ and let this be optimal in the absence of limited liability. We will characterise the conditions under which an alternative contract,

$$D' = (t + \gamma\varepsilon, \beta_1 + \gamma\delta_1, \beta_2 + \gamma\delta_2) \qquad (2.5.1)$$

yields a Pareto improving change, that is taking limited liability into account is beneficial, remembering that in configuration B the government provides a lump-sum transfer β_2 to the creditors. Expected utility now depends upon ε, δ_1, δ_2 and γ.

We shall look at the changes in the expected utilities of

(1) the manager

(2) the creditors

(3) the government

for a given change in γ.

Change in Manager's expected utility

$$W = F(-M+\varepsilon,a)V(-M+\varepsilon+\beta_1) + \int_{-M+\varepsilon}^{0} V(\pi+\beta_1)f(\pi,a)d\pi$$

$$+ \int_{0}^{N} V(\pi(1-t)+\beta_1)f(\pi,a)d\pi$$

$$- C(a) \qquad (2.5.2)$$

$$\partial W(a,\beta,\gamma,\delta,\varepsilon,t)/\partial\gamma|_{\gamma=0} = \underbrace{\delta_1[F(.)V'(-M+\varepsilon+\beta_1) + \int_{-M+\varepsilon}^{0} V'(\pi + \beta_1)dF + \int_{0}^{N} V'(\pi(1-t)+\beta_1)dF]}_{(i)}$$

$$\underbrace{-\varepsilon.\int_{0}^{N} V'\pi f d\pi}_{(ii)} \qquad (2.5.3)$$

Change in Creditors' Expected Utility

$$\partial C/\partial\gamma = \underbrace{\delta_2\{\int_{-M}^{-M+\varepsilon} \Omega'(.)f(\pi,a)d\pi + (1 - F(.))\Omega'(\beta_2)\}}_{(iii)}$$

$$\underbrace{+ \{\int_{-M}^{-M+\varepsilon} \Omega f_a d\pi - F_a(-M+\varepsilon).\Omega(\beta_2)\}\partial a/\partial\gamma}_{(iv)} \qquad (2.5.4)$$

Change in Government Revenue

$$\partial G/\partial\gamma = \varepsilon\int_{0}^{N} \pi f d\pi + t.\{\int_{0}^{N} \pi f_a d\pi\}\partial a/\partial\gamma - (\delta_1 + \delta_2) \qquad (2.5.5)$$

It is next necessary to work out $\partial a/\partial\gamma$

$$\partial a/\partial\gamma = -W_{a\gamma}/W_{aa} \text{ [12]} \qquad (2.5.6)$$

From (2.2.5),

$$W_{a\gamma}=\underbrace{\delta_1 . [F_a(.)V'(-M+\varepsilon+\beta_1)+\int_{-M+\varepsilon}^{0} V'(\pi+\beta_1)f_a d\pi+\int_0^N V'(\pi(1-t)+\beta_1)f_a d\pi]}_{(v)} \underbrace{- \varepsilon.\int_0^N \pi V' f_a d\pi}_{(vi)} \qquad (2.5.7)$$

To work out the possibilities of a Pareto improving change, note that,

(i) $> 0 \ \forall \ \delta_1 > 0$

(ii) < 0

(iii) $> 0 \ \forall \ \delta_2 > 0$

(iv) $\geq (<) \ 0$ according as $\partial a/\partial\gamma \geq (<) \ 0$

(v) $< 0 \ \forall \ \delta_1 > 0$

(vi) $\geq (<) \ 0$ according as $\int \pi V' f_a d\pi \leq (>) \ 0$

We list these signs in order to illustrate, as dramatically as possible, that the arguments in favour of introducing higher taxes under limited liability are not as clear-cut as some may like them to be. This is for the simple reason that there are several competing effects at work. For the manager higher taxes lower expected utility but the correspondingly higher lump-sum subsidy and limited liability increase it. In the case of the creditors, while higher limited liability lowers utility, the higher lump-sum subsidy increases it. Similar conflicting effects are seen acting on the government's budget constraint.

But we have enough degrees of freedom to bring about a Pareto improving change in certain circumstances. The case in which the results are most clear-cut is where $\partial a/\partial\gamma > 0$. For this to be true, we require, $W_{a\gamma} > 0$ for positive δ_1 and ε. A necessary condition for this to hold is,

$$\int \pi V'(\pi(1-t)+\beta_1)f_a(\pi,a)d\pi < 0 \qquad (*)$$

which in turn requires the coefficient of partial relative risk-aversion to be greater than one in some range.

Suppose (*) holds and $t > 0$. The sufficient condition for $t > 0$ is given in Proposition 3 (which also holds here, for small amounts of limited liability ε.

For given ε, choose γ and δ_1 small, such that $W_{a\gamma} \geqslant 0$; $W_\gamma \geqslant 0$

Then, $\partial a/\partial\gamma \geqslant 0$.

In particular, choose γ and δ_1 such that $W_{a\gamma} = 0$ and $W_\gamma = 0$. Then for all $\delta_2 > 0$, $\partial C/\partial\gamma > 0$.

Choose $\delta_2 > 0$ such that $\partial G/\partial\gamma \geqslant 0$ and we have a Pareto improvement.

This is summarised as a Proposition below.

Proposition 4: There exists scope for an increase in taxes, in order to take account of the introduction of limited liability, to bring about a Pareto improvement.

The existence, for given ε, of such γ, δ_1 and δ_2 are not guaranteed. Furthermore suppose (*) does not hold. Then the arguments for raising taxes in the presence of limited liability are even less clear-cut. Raising taxes means a corresponding positive δ_1 and δ_2 which implies $\partial a/\partial\gamma < 0$. This in turn implies ambiguity in the sign of $\partial C/\partial\gamma$ in addition to the already prevailing ambiguities in the signs of W_γ and $\partial G/\partial\gamma$.

Proposition 4, when interpreted in a slightly different light helps us to establish some idea of when limited liability is itself a worthwhile institution given a world in which taxes are already in use. The argument is essentially of a second-best variety. Once some margin of distortion has been created by taxes then it may be optimal to introduce a further distortion viz limited liability. There is, therefore, a closer relationship between limited liability and taxation than may at first be thought, when they are both construed as instruments of the government. The second-best nature of the argument is observed by noting that it is never a Pareto improving change to introduce limited liability into a world in which there is no corporate taxation. The reason for this can be demonstrated straightforwardly by differentiating C with respect to ε.

$$\partial C/\partial \varepsilon = - \int_{-M}^{-M+\varepsilon} \Omega'(\pi + M - \varepsilon) f(\pi, a) d\pi + \int_{-M}^{-M+\varepsilon} \Omega f_a d\pi . \, \partial a/\partial \varepsilon - F_a . \partial a/\partial \varepsilon . \Omega(\beta_2) < 0$$

This ceases to be valid in world in which (even optimal) taxes are in operation. Furthermore, by referring to 2.5.2 - 2.5.7, as we have shown, if t is positive it is always worthwhile to introduce some limited liability. This is because taxes and limited liability are complementary instruments in a second-best world. Specifically, if a manager is granted some limited liability, he may be prepared to bear more taxes. This is because taxes lower his expected utility *ceteris paribus* while limited liability increases it. On the other hand, a creditor can have his expected utility maintained in the presence of limited liability is introduced by being given a lump-sum transfer. The government is prepared to bear this additional expense in return for higher taxes. In Proposition 4, we have shown that such compensation can be made such that at least one of the parties in this miniature economy is strictly better off with the remaining two no worse off, for some level of limited liability ε. This suggests that it may be worthwhile introducing some limited liability in a world of positive taxes, in conjunction with an increase in taxes.

3. An Alternative Interpretation of 'a '

In this section, we make a specialisation of the model that we used in the previous section in order to focus attention on taxation and risk-taking. We are therefore in configuration C. As might be expected, many of the results carry over qualitatively from the last section, although there may be significant quantitative differences. Let us regard 'a ' as the choice of an investment portfolio. As 'a' increases, we suppose that it generates a riskier distribution of π. When 'a 'is a choice of riskiness, no sense can be attached to a cost of effort so we assume this to be zero in this case. The first-order condition for the choice of 'a 'is [13],

$$\int_{-M}^{N} Vf_a d\pi = 0 \tag{3.1}$$

The second-order condition is,

$$\Delta = \int_{-M}^{N} Vf_{aa} d\pi < 0 \tag{3.2}$$

A natural notion of increased riskiness is that of Rothschild and Stiglitz (1970) who suggested the mean-preserving spread. In our case this is not appropriate since the risk-neutral government would then be indifferent among different levels of 'a' being chosen as it is only concerned with its average revenue, which would be unaffected by changes in 'a' if it induced a mean-preserving spread. A necessary condition for one distribution to be called riskier than another would seem to be that it has more weight in the tails whether or not its mean is the same. In other words, we would have $f_a(.)$ positive at either end of the distribution.

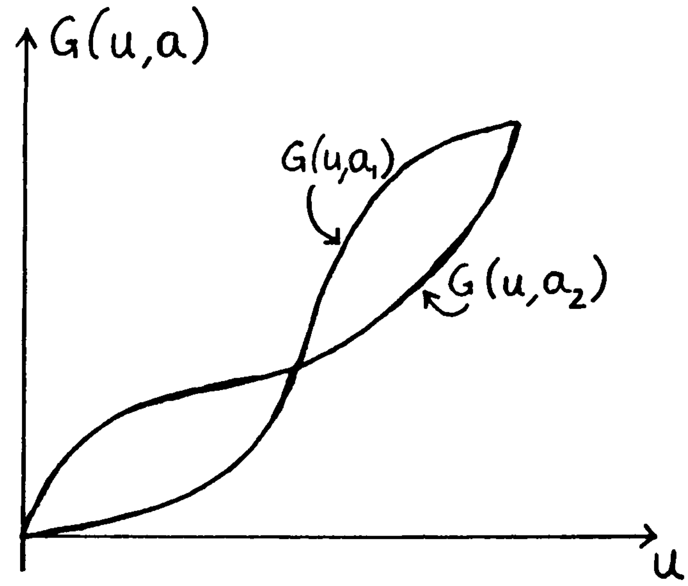


Figure 3.1

The probability density functions illustrated in the figure above satisfies this property without leaving the mean unchanged. Such a configuration may be shown to be implied by a change in 'a ' which induces a mean utility preserving spread according to a single crossing property of the distribution of utility. [14] To see this, define the distribution of utility induced by variations in π as,

$$G(u, a) = F(V^{-1}(u), a) \qquad (3.3)$$

and note that for 'a' to induce a mean utility preserving spread, we need,

$$\int_{u(-M)}^{u(N)} G_a(u, a)du = 0 \qquad (3.4)$$

$$\int_{u(-M)}^{x} G_a(u, a)du \leq 0 \qquad \text{for } x < u(N)$$

That f_a (.) is positive at either end of the distribution is necessary in conjunction with (3.1) for the conditions (3.4) to hold at the optimum.

Given our discussion therefore, we make the following assumptions:

1. $f_a(.)$ is positive within a neighbourhood of either end of the

distribution of profits,

2. The effect of 'a' in the optimum is to induce a mean utility preserving increase in spread.

Note that with the single crossing property the second implies the first. We will now look at how this different specification of 'a' affects the results stated in the previous section.

It is clear that the introduction of limited liability <u>increases</u> risk-taking, that is $\partial a/\partial \varepsilon > 0$. This can be checked by totally differentiating (3.1) (where (3.1) must take account of limited liability), which gives us,

$$\partial a/\partial \varepsilon = -(\Lambda)^{-1} F_a(-M + \varepsilon).V'(-M+\varepsilon) \qquad (3.5)$$

$F_a > 0$ in the lower regions of π (see Figure 2) and the result follows and is intuitively clear. It says that a rise in limited liability increases risk-taking. Since agents are less responsible for their losses in this case, the result is not surprising.

We are now in a position to analyse optimal taxation in the risk-taking model. The first-order conditions will be as in the previous section. Before examining their implications, it will be useful to make the following natural assumption,

$$\int_{-M}^{N} \pi f_a d\pi = - \int_{-M}^{N} F_a > 0 \qquad (3.6)$$

Otherwise if an increase in 'a ' both increases the variance and reduces the mean, the agent will have no incentive to increase 'a '. From this assumption follows the surprisingly simple Lemma,

Lemma: If a rise in 'a ' has the single crossing property then,

$$\int_{0}^{N} \pi f_a d\pi = - \int_{0}^{N} F_a d\pi > 0 \qquad (3.7)$$

Proof:

To prove this result, consider the implications of a mean utility preserving increase in risk for the distribution of profits. The diagram below summarises the likely possibility.[15] It is now clear that irrespective of the position of the point $\pi = 0$ (either Z_1 or Z_2) relative to the crossing over point c_1 the property of the integral, as stated in the lemma, must hold because at worst we are omitting, from the interval of integration, the region where $- F_a$ is negative and hence, if the integral over the region [-M, N] is positive, deleting regions with $- F_a$ negative must increase the value of the new integral and maintain the positive sign.

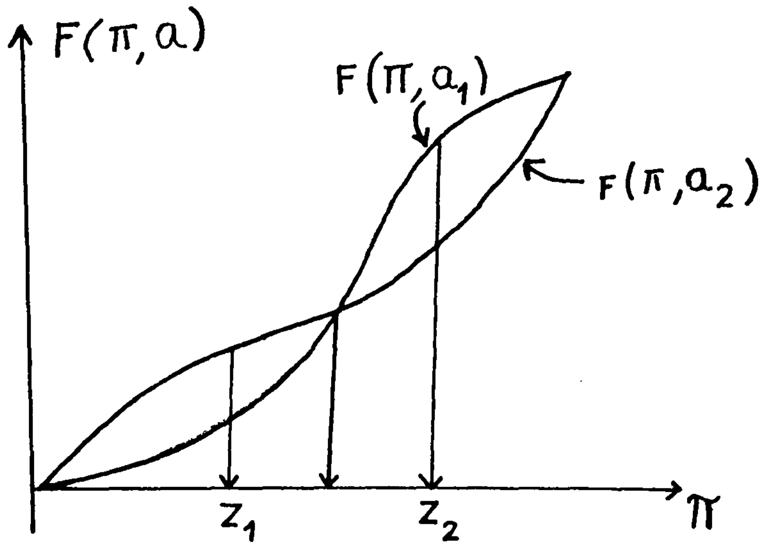


Figure 3.2

The Lemma stated above enables us to establish the following result for the risk-taking case.

Proposition 5: At an optimum, $\partial a/\partial t \geqslant 0$ implies $t < 0$ if $\partial a/\partial \beta > 0$

$t < 0$ implies $\partial a/\partial t \geqslant 0$ if $\partial a/\partial \beta < 0$

Proof: The proof follows exactly the same route as that followed in Proposition 1. The important feature to note is the Lemma above. The proof now follows as in Proposition 1.

The intuition for the first part of the proposition can be explained as follows. A tax acts like insurance, since tax proceeds are re-distributed back to the firm in lump-sum form. A subsidy has the opposite effect of insurance and hence discourages risk-taking. A subsidy is optimal in just those cases where insurance leads to 'riskier behaviour'.

Finally, moving quickly on to configuration D, it is possible to demonstrate circumstances in which an increase in taxes, to take account of limited liability Pareto improving. The method of proof is very similar to that adopted in Proposition 2 and will not be repeated here.

This completes our analysis of the risk-taking model and of the four configurations A, B, C and D. We draw together implications of the analysis of the last two sections for the argument discussed in the introduction, in the next section.

4. Summary and Conclusions

The simple model analysed in this chapter has been used to shed some light on the argument that firms should be taxed because they receive benefits from limited liability. The proposition that has been supported by the analysis is somewhat weaker than this. The results of our model suggest that we should always take limited liability into account when setting tax rates for profits under uncertainty. This can be thought of, at one level, as simply a corollary of a general result on optimal incentive schemes due to Holmstrom (1979). This result states if a signal is informative then it is optimal to condition the incentive scheme for an agent upon it. Limited liability is just such an informative signal. Its existence was shown to allow us to make qualitative predictions about the effort level (or risk-taking behaviour) of the manager. As such, its existence is a signal as to how managers will behave. Our result is then perfectly consonant with the existing literature.

A disappointing feature of the analysis undertaken is perhaps the inability to derive necessary and sufficient conditions for the sign of the effect of taxes on managers' actions. We discussed in the text of the chapter why only partial results can be derived. The model is already special and we do not wish to simplify further in order to get a characterisation in terms of necessary and sufficient conditions. The results do however throw light on the argument in the Public Finance literature which we sought to illuminate.

Firstly, the argument that we should have positive profit taxes at all, does not seem straightforward under uncertainty. There are conditions under which subsidies on profits are optimal. Taxes which are used to finance lump-sum transfers offer insurance and hence may lead to moral-hazard. This in turn leads to subsidies on profits financed by lump-sum levies on corporations being optimal. We could envisage a scheme operating by charging an annual fee, say, to corporations which is paid back according to profits performance. The favourable incentives created by such a scheme are manifest, as are the difficulties of operating such a scheme in practice, and in no sense could it be advocated based on the simple model that we have analysed. Nevertheless, it does lead us to question whether current arrangements are necessarily optimal and even simple models which lead to question conventional wisdom may be valuable.

Secondly, we showed that the optimal response to limited liability involves an aim to counteract its effect upon managers' actions. Hence in the risk-taking case taxes or subsidies should be adjusted so as to reduce risk-taking, once limited liability is introduced. In the

'effort taking' case taxes or subsidies should be adjusted so as to encourage more effort. Proposition 4 demonstrates a case where the argument in favour of a higher tax for owner - managed companies with limited liability is apparently valid. Nevertheless we cannot support the argument in general. It does seem, however, that that a tax system should allow for the existence of limited liability when it is being designed. Nevertheless, even our simple model cannot give clear qualitative guidelines to assist the deliberations of policy makers except for the special case discussed in Proposition 3.
Let us suppose the following. We are in a world where, at the optimum, higher taxes encourage riskier choices because proceeds find their way back to managers via the tax system and taxation therefore acts like insurance. In addition let effort be reduced by taxes, again because of the insurance effect. Whether taxes are optimally positive or negative depends upon which of these influences is dominant.

We do not claim that our model captures all that is relevant in the minds of policy makers when they design a tax system. The model we have presented here does suggest a perspective from which the issue of differential tax rates between incorporated and unincorporated enterprises might be assessed . The observation that limited liability may be an important difference between firms is suggestive of differential tax rates. It is the qualitative propositions that emerge that seem open to question.

Notes

1. For a general analysis of the Principal-agent problem, see Holmstrom (1979).

2. We choose a balanced budget so as to abstract from the 'revenue raising' argument for corporate taxation.

3. See below for conditions for this to hold.

4. It explains why we might have differential tax rates between the incorporated and unincorporated sectors. In a later section we suggest reasons why taxes should be introduced because of limited liability.

5. See for example Mirrlees (1979), Grossman and Hart (1983) and Rogerson (1985).

6. See Chapter 4.

7. We will assume that the the lump-sum tax β is paid by the government to all firms and that the tax on profits is levied only when profits are positive.

8. It is 'partial ' rather than 'relative ' because the argument of the utility function is $\pi(1-t) + \beta$. Also note that although the expressions from $\partial a/\partial\beta$ and $\partial a/\partial t$ have been obtained from the agent's first order conditions by total differentiation they are

properly denoted by partial rather than full derivatives because changes in t are ignored while calculating the expression for $\partial a/\partial\beta$ and changes in β ignored while calculating $\partial a/\partial t$.

9. Equivalently makes an agent bear more risk.

10. Note that it is the balanced budget change in effort which is important.

11. We are grateful to Jim Mirrlees for suggesting this Proposition.

12. The reason for using partial as opposed to total derivative signs is identical to that in note 8 above.

13. At present we are assuming no limited liability and no taxes.

14. For an exposition of this concept see Diamond and Stiglitz (1974).

15. See also Figure 2 in Rothschild and Stiglitz (1974)

CHAPTER 6: CONCLUSIONS

The aims of this thesis can be summarised under three broad headings. First, to investigate in close detail the assumptions on which the implicit contract employment literature is based. These assumptions include not only the more obvious ones, for example, on the specification of utility functions for the various parties to the contract, but also those of a more subtle kind relating to the sequencing of moves between nature and the participants in the models. Particular attention was paid here to implicit contract models with imperfect information. The discussion, of necessity, therefore included both models of asymmetric information as appearing in the work of Grossman-Hart (1983a, b), Hart (1983) and moral-hazard models structured on Holmstrom (1979).

The second and closely related aim was to look at the consequences, for employment results, of alternative specifications of imperfect information.

Finally, the models of moral-hazard and implicit contracts were used to derive results predominantly within the framework of a firm, where the owner of the firm, the manager and workers interact in principal-agent relationships of varying degrees of complexity.

The division of attention in this thesis - among the three aims - is quite clear. Chapters 3 and 5 concentrated purely on the application of techniques for solving models of imperfect information while Chapters 2 and 4 undertook a scrutiny of the foundations of the models and the techniques.

Chapter 2 started from the observation, that the special results of the implicit contract literature with asymmetric information proceed from the particular specifications of the utility functions of the principal and the agent. In particular, the assumption of a risk-averse principal is crucial in deriving the inefficient underemployment results.

There appear to be three levels at which such an argument may be justified. At the first level, it may be claimed, that even in the case of large firms with many shareholders, the shareholder-portfolios are not diversified enough or the components of the portfolios are not uncorrelated and thus the assumption of a risk-averse principal is not an unlikely one.

At the second and more important level however, it is often argued that, in large firms, the shareholders do not interact directly with the workers of the firm, that a manager is delegated to run the affairs of the firm and it is he who behaves in a risk-averse manner. The risk-aversion of the principal simply reflects the risk-aversion of the manager.

At the third and final level, one might look at the results of the model and see whether they are, in Friedman's terminology, " as if the firm were run by a risk-averse principal".

The second and third levels of argument formed the concerns of Chapter 2. The first level of argument represents, of course, an a- priori belief in risk-averse principals.

The introduction of a manager into the model, taking an unobservable action 'a', restored the inefficiency results in the Hart framework, with a risk-neutral principal. More crucially the unobservable action 'a' played the vital role of retaining the asymmetry of information between two parties - the firm and the worker - in large firms where profits and employment were both observable and thus under the original framework of asymmetric information models, the first-best contract would be implementable. It was shown, in Chapter 2 that such inefficiency results follow from the particular specification of moves made by the manager and the degree and nature of informational advantage he has over the principal when making his effort choice. It was further shown that under a familiar alternative, and in our view, more reasonable specification may lead to fully efficient results or inefficiency in the sense not commonly supposed, that is over-employment. The crucial distinction between the two specification of moves lies in the choice by the manager of 'a' after the state of nature has occurred in the Hart sequence and before the state of nature has occurred in the moral-hazard sequence of moves. It was shown that Hart's specification of moves removed all notion of risk-averse behaviour by the manager as too much was known by him with certainty when making his choice. On the basis of the analysis, an index of risk-averse behaviour was proposed and it was demonstrated that, at least in terms of the definition, it would not always be true to claim that the firm could be seen to be behaving in a risk-averse manner. The argument in favour of introducing the alternative moral-hazard specification may therefore seen to be two-fold; first to model the rationale for a risk-averse principal in asymmetric information models and second to demonstrate the sensitivity of the results to the altered sequence of moves. A third purpose was also served with a useful method of approximating incentive schemes in the neighbourhood of the first-best being derived as an essential part of the analysis.

Chapter 3 focused on the first-best implementability of contracts in a model with a risk-averse principal, a risk-averse manager taking an unobservable action 'a', where the action here relates to the choice of riskiness of a portfolio, and a worker. We found that the first-best contract was incentive compatible if and only if the principal, the manager and the worker possessed affine risk-tolerance utility functions with identical slope co-efficients.

Chapters 2 and 3 both worked with models with a potentially hierarchic structure with,

$$\begin{array}{c} \text{principal (shareholders)} \\ \Big\downarrow \\ \text{manager} \\ \Big\downarrow \\ \text{worker} \end{array}$$

This hierarchic structure was necessary to demonstrate the main propositions of this thesis. Although an exciting extension of the traditional bilateral principal-agent framework, inadequate attention has been paid in the literature to such multilateral contracts. This thesis follows much the same trend, by not attacking the framework in its full complexity, with the multilateral structure being reduced to a more tractable bilateral one, usually under suitable specifications of the utility functions of the three parties. Some work is still required before the theory of multilateral contracts, with asymmetries at each stage, can be fully developed.

The essential propositions of this thesis are based on the use of the moral-hazard framework and, in particular, the use of the first-order approach to solving moral-hazard models. The first-order approach has been the subject of much recent critical attention, almost all of it unfavourable, towards its use as an analytically convenient technique. Chapter 4 undertook an investigation of the approach not only to justify its use in earlier and subsequent chapters but also to restore, more generally, faith in its use. It is well known in the literature that the problems arise from the inability to guarantee the concavity of the agent's expected utility in action and hence the attainment of a global maximum as opposed to local maxima, minima or saddle-points. Papers, notably Rogerson (1985) rehabilitating the use of the first-order approach have not proceeded further beyond identifying sufficient conditions for the validity of the approach. Chapter 4 argued that the sufficient conditions proposed are too strong and impose unnecessary structure on the behaviour of the agent's expected utility. We argued that by imposing a positve minimum expected utility condition on the agent, the sufficient conditions can be weakened to a 'quasi-concave' or 'concave when positive' condition. An alternative set of sufficient conditions was proposed and their use demonstrated by justifying the use of the first-order approach in an example where the Rogerson conditions were not satisfied. The use of the first-order approach was also justified for distributions that differ from perfect observability only by small variance shifts. In sum, the aim of this chapter was to argue that the analytical convenience of the first-order approach could be retained for a larger number of cases than those allowed by the Rogerson conditions.

Chapter 5 used a moral-hazard model of managers' behaviour under uncertainty to analyse the effects of limited liability on optimal profits taxes. In particular, it addressed the question of whether the benefits of limited liability should be taxed. Two interpretations of managers' behaviour were examined. First, a manager chose his effort level according to first-order stochastic dominance. Second, the manager chose an investment portfolio which altered the distribution of outcomes according to a mean-preserving spread of utility. The analysis was conducted under four alternative configurations determined by limited liability arrangements and interpretation of managers' behaviour. We showed that limited liability affected the tax and, under certain conditions, according to the formula that limited liability implied higher taxes. The concerns of this chapter were, in a sense, distinct from the issues dealt with in the earlier part of the thesis, where the focus was on the efficiency and implementability of contracts. However, the models of choice under uncertainty used here had been discussed in detail in Chapters 2 and 3 and in applying them to answer a special question we demonstrated the versatility of the moral-hazard framework.

In sum, the thesis investigated - what might essentially be thought of as - the mechanics of principal-agent models. The issues dealt with include, for example, the sensitivity of results to the assumptions of the model, the sequencing of moves and the methods used to solve them. The models were also used to answer specific questions. In developing approximation techniques and extending the range of utility and density functions for which the first-order approach is valid, the thesis also helped to make the moral-hazard framework more analytically tractable.

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