

Probabilistic Sensor Network Design*

Jeroen Bergmann, Alison Noble and Mark Thompson

Abstract— Sensor networks are designed to detect events and their applicability is dependent on the likelihood of a correct detection. A network that can't detect events with a high enough probability becomes ineffective. Therefore, it can be very valuable to be able to establish which network design might yield the best detection rate. The endless possibilities in terms of sensor network designs make it difficult to apply a pure experimental method. Computational modelling using statistical techniques can provide a useful tool to explore the sensor network design space. The concept of a probabilistic sensor network (PSN) model is introduced in this paper. A framework is established and examples are given of the PSN model. The PSN model is tested in a hypothetical scenario by computing Root Mean Square Errors (RMSEs) and Absolute Errors between simulation outcomes and the results of the PSN model. The RMSEs between the simulation and the model were approximately 0.02 indicating a close comparison between the simulation and the model. The proposed probabilistic sensor network method provides an intuitive and promising tool to test sensor network designs virtually.

I. INTRODUCTION

Sensor networks have allowed us to develop a more comprehensive view of the world by collecting data across a number of different nodes. These sensor networks can increasingly function without human intervention for extremely long lifetimes [1]. The reliance on these networks combined with the increase of longevity requires careful planning from a design point of view. However, often the location and quantity of these nodes are driven by initial assumptions that are based on empirical knowledge. Network designs are frequently optimised through experimentation, which is a time consuming and expensive process to conduct. Although experimental testing of the sensor network is a key requirement for success it makes it impossible to explore the full range of potential designs that might be available to us. The complexity of the network is further increased when multi-modal approaches are integrated into the system. Design optimization through virtual prototyping provides a concept into which probability can be introduced. Optimisation is distinct from the notion of virtual prototyping, as it goes beyond an initial verification [2]. Established probabilistic methods can be applied to help determine the potential suitability of a given design at the beginning of the design process. Exploring sensor network designs with probabilistic techniques is a new field that can be promising for the assessment of wearable technologies, both in research and industry. The main aim is to use known or estimated probabilities to reduce the number of potential networks that need to be considered for a given case. A straightforward probabilistic framework is applied to explain the concept. In

this paper the idea of probabilistic sensor network (PSN) design is introduced and a preliminary comparison is made with simulation results obtained from a hypothetical scenario. All models and simulations were developed within Matlab (MathWorks, Inc., Natick, MA, USA).

II. PROBABILISTIC SENSOR NETWORK (PSN) MODEL

A conceptual framework is established to provide a starting point for the PSN model. Let's assume a sensor network is being designed to detect a given event. It starts with the notion that there is a genesis of an event that through the environment (A) reaches the sensing element (B). If this event generates a suitable signal for detection then signal conditioning is performed (C). Conditioning of the signal is aimed at conserving the pattern of interest by applying for example filtering or amplification. Successive, signal processing (D) is conducted with the goal to correctly identify the aforementioned event. This whole process (A to D) can be captured in a flow diagram (Fig. 1.i). The steps displayed in the diagram can also be represented as nodes. Each node can be assigned a probability estimate, which represents the chance that an event is correctly passed through that node. These nodes can then be arranged to generate countless design options. The PSN model therefore allows for the exploration of contrasting networks without the need of physically building them. Some initial examples of network designs are given in Fig. 1 (ii to vi).

A variety of designs can be described using this model. This paper will illustrate the application of the PSN model by using a simple initial example. The example in this paper will use a sensor network that contains several similar sensors (e.g. each sensor node has the same probability of detection the event). Let's assume that all the (sensor) nodes are independent and that each sensing element has the same probability estimate for event detection. The detection of a given event is not mutually exclusive between sensors.

The following probabilities are given in this first example; the chance that the environment creates a strong enough environmental signal (A_1) to register the event is set to 0.9; the probability that a sensing element (B_1) detects the signal is 0.8; signal conditioning (C_1) conserves the signal with a probability of 0.8; the signal processing (D_1) correctly identifies an event with a probability of 0.7. If the design consists of a single sensor node (Fig 1.ii) then this can be captured mathematically as

$$P(A_1 \cap B_1 \cap C_1 \cap D_1) = P(A_1) \cdot P(B_1) \cdot P(C_1) \cdot P(D_1) \quad (1)$$

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J.B., A.N. and M.T. are with the Institute of Biomedical Engineering, Department of Engineering Science, University of Oxford, Oxford, OX3

7DQ, UK (phone: 01865 227664; fax: 01865 617701; e-mail: jeroen.bergmann@eng.ox.ac.uk, alison.noble@eng.ox.ac.uk and mark.thompson@eng.ox.ac.uk).

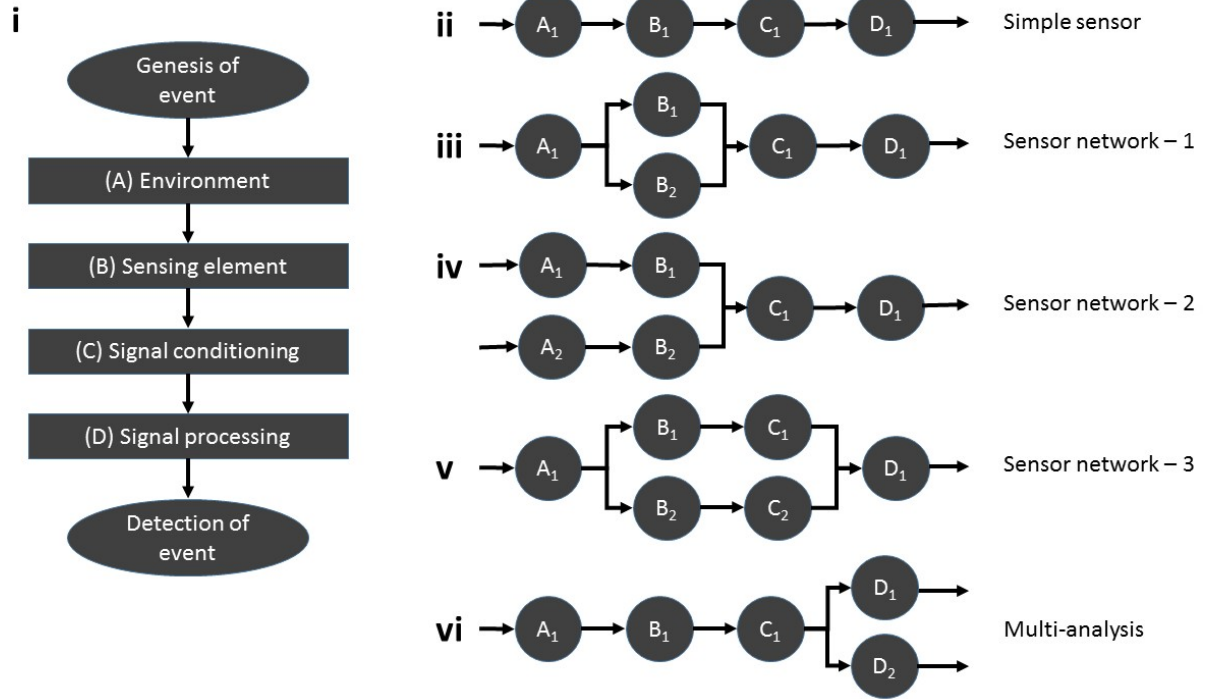


Fig. 1 Conceptual model for event detection. (i) Shows the steps that lead up to the detection of a given event. (ii) shows the most simple design. (iii) displays a network design that consist of two sensors (iv) this is a network that has sensors that sense different "parts" of the environment. (v) shows a network containing two different sensors, which require different signal conditioning (vi) shows a sensing configuration that is processed in two different ways.

Inputting the probabilities gives

$$(0.9) \cdot (0.8) \cdot (0.8) \cdot (0.7) = 0.4032 \quad (2)$$

We can verify this by determining the probability of not detecting an event, which should be 1-0.4032, as

$$P(A'_1 \cup B'_1 \cup C'_1 \cup D'_1) = 1 - P(A_1 \cap B_1 \cap C_1 \cap D_1) \quad (3)$$

This can be rewritten as

$$\begin{aligned} P(A'_1 \cup B'_1 \cup C'_1 \cup D'_1) &= P(A'_1) + P(B'_1) + P(C'_1) + P(D'_1) \\ &- P(A'_1 \cap B'_1) - P(A'_1 \cap C'_1) - P(A'_1 \cap D'_1) \\ &- P(B'_1 \cap C'_1) - P(B'_1 \cap D'_1) - P(C'_1 \cap D'_1) \\ &+ P(A'_1 \cap B'_1 \cap C'_1) + P(A'_1 \cap B'_1 \cap D'_1) \\ &+ P(A'_1 \cap C'_1 \cap D'_1) + P(B'_1 \cap C'_1 \cap D'_1) \\ &- P(A'_1 \cap B'_1 \cap C'_1 \cap D'_1) = 0.5968 \end{aligned} \quad (4)$$

If we add another sensor (Fig 1.iii), which independently detects the same event at another location at a probability of 0.8 we get

$$P(A_1 \cap (B_1 \cup B_2) \cap C_1 \cap D_1) \quad (5)$$

since

$$P(B_1 \cup B_2) = P(B_1) + P(B_2) - P(B_1 \cap B_2) \quad (6)$$

and

$$P(B_1 \cap B_2) = P(B_1) \cdot P(B_2) \quad (7)$$

which we can substitute to

$$P(A_1) \cdot (P(B_1) + P(B_2) - P(B_1) \cdot P(B_2)) \cdot P(C_1) \cdot P(D_1) \quad (8)$$

The sensor network consisting of two sensors yields an event detection probability of

$$(0.9) \cdot (0.96) \cdot (0.8) \cdot (0.7) = 0.48384 \quad (9)$$

Similar, the probability of not detecting an event can also be determined. An output of 0.51616 can be found confirming 1-0.48384. The addition of a second sensor would mean that the probability of detecting the event increases with 0.0806. The PSN model provides an easy method to explore what happens if the number of sensors are further increased. Table 1 contains the outcomes of event detection for this increasing sensor network.

TABLE I. THE PROBABILITY OF EVENT DETECTION BY THE SYSTEM AT A GIVEN NUMBER OF SENSORS. EACH SENSORS HAS A DETECTION PROBABILITY OF 0.8.

Number of sensors (n)	Probability of event detection
1	0.4032
2	0.4838
3	0.5000
4	0.5032
5	0.5038
6	0.5040
7	0.5040

Number of sensors (n)	Probability of event detection
8	0.5040
9	0.5040
10	0.5040

The outcomes show that a network design of 6 sensors produces the maximum possible detection rate at a given accuracy and no noticeable further gain can be observed above this number. However, the detection probability of the sensor was set at a given level and sensors that are better or worse at detecting the event might yield an alternative "optimum". The outcomes for a range of sensors that all have different detection rates can be explored and the results are plotted in Fig. 2. The obtained results demonstrate that "saturation" of detection across the whole system is dependent on the initial detection capabilities of the sensor. Sensors that have low detection rates require a large network in order to reach a desired "saturation" level, while sensors that are very capable of detecting an event require only a small network in order to be successful. An optimum number of sensors can be determined once all probabilities in the system are measured or estimated. It is good to mention that in this example event detection by any sensor is deemed to be correct. In other words, as long as one sensor has detected the event then it is assumed an event has occurred.

III. SIMULATION AND PSN MODEL

The objective is now to simulate a hypothetical scenario to determine if a simulation generates comparable results to those obtained from a PSN model. A comparison is made by calculating the absolute errors and the Root Mean Square Errors (RMSEs) between the simulation data and the PSN model [3]. A network consisting of moving light sensors was simulated, as these photo resistors provide a basic and reproducible scenario. The output of the photo resistors was

used to determine if a nearby (≤ 10 meter) light source was on. The position of the photo resistors were ever changing, while the network tried to detect if the light source is on or not (Fig. 3). The movement of these sensors follow a random uniform distribution generating positions that cover a distance of 0 to 10 meters away from the source. All movements are orthogonal to the light source, as the simulated light intensity was only affected by the nearest distance between source and sensor.

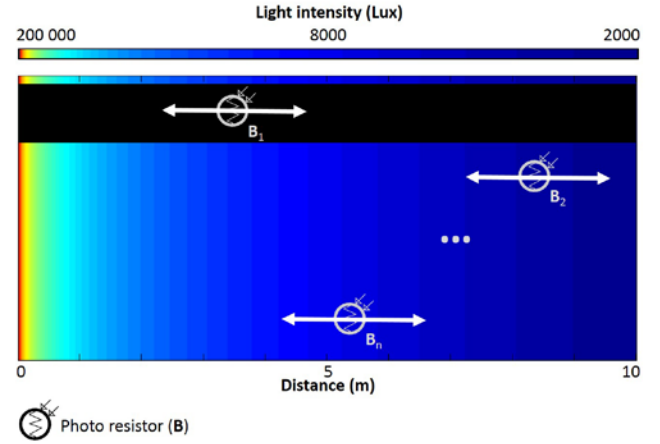


Fig. 3 The figure displays the light intensity across a field in which the photo resistors move. The warmer the colour the greater the light intensity in Lux. Multiple sensors (B_1 to B_n) can be placed within this field to explore different network sizes. The field can be effected by the environment, which can block any light from the source. This is visually represented as a black belt that in this image blocks sensor B_1 .

The sensors are constructed to pick up light intensity with different levels of sensitivity. Sensitive sensors will be able to pick up (and show change) at relative low light intensities, while less sensitive sensors will need to be very close to the source to detect that the light source is on. For this scenario the light source will always be on, thus reflecting that the genesis of the event ("light on") always occurred. However, the

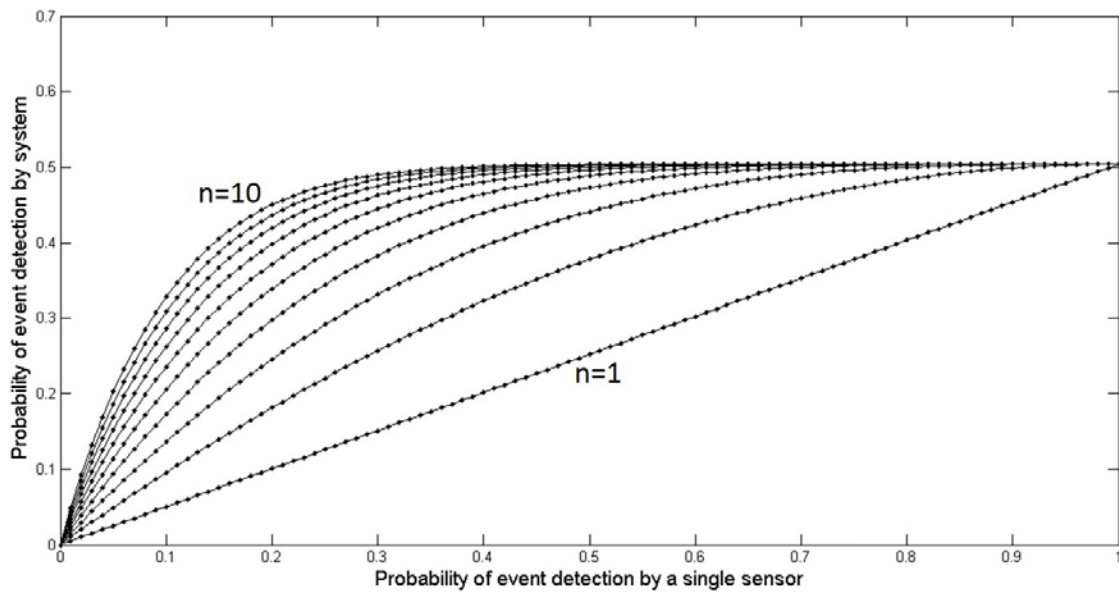


Fig. 2 Probability of event detection by a single sensor set against the probability of event detection of the overall design. The network consist of a number (n) sensors ranging from 1 to 10. The results of the probabilistic model are shown.

environment is simulated to block the light source in 10% of all cases. Blocking was also randomly governed by a uniform distribution. If the source was blocked then the sensors would not pick up any detectable light. In the simulation light intensity is governed by

$$I = \frac{I_0}{x^2} \quad (10)$$

in which illumination (I) is given in Lux and distance (x) from the source is provided in meters (m). The illumination at the source (I_0) was set at 200 000 Lux. The resistance of the sensor decreased with increasing illumination (Fig. 4).

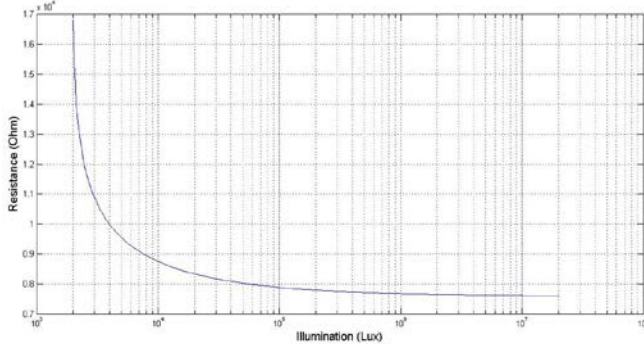


Fig. 4 Resistance (in Ω) of the sensor plotted against illumination (in Lux).

White Gaussian noise was added to the signal from the sensor to generate a slightly more realistic input. This noise level was set to 40 dBW. A threshold was used to determine if the signal indicated that the light source was on or not.

All sensors that were available for a given network were subsequently randomly moved 10 000 times. The sensors could obtain any position between 0 to 10 meters away from the light source. The number of times that the sensor network correctly detected the event was recorded. The detection rate of a sensor network could be any value 1 and 0. A 1 would imply that the network always detected the event and a 0 would be found if it never detected it.

The PSN model was also run for the same scenario (model given in Fig. 5). The scenario translated to an event detection probability within the environment (A) of 0.9. The probability of the sensing element (B) covered the range of 0 to 1 with a step increase of 0.01 for each iteration. Signal conditioning (C) was set to 1, as no conditioning was performed before processing. Processing itself was just a simple threshold, detecting the event with an estimated probability of 0.9. The processing for each sensor was assumed to be independent.

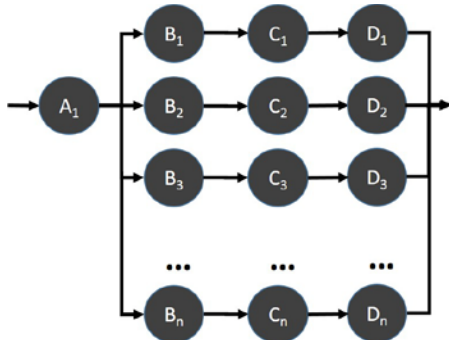


Fig 5. Probabilistic model of the simulated scenario. **A** indicates the probability of event detection based on environmental constraints, **B** shows the probability of the sensing element, **C** represents conditioning and **D** is for signal processing. The number of sensors is given by the subscript (up to n) and each sensor has its own conditioning and processing

The Root Mean Square Error (RMSE) was computed for each of the 10 networks by

$$RMSE = \sqrt{\frac{1}{k} \sum_{j=1}^k (P_{SIM_j} - P_{PSN_j})^2} \quad (11)$$

with P_{sim} as the overall probability obtained from the simulation at iteration j for a given event detection probability of the sensor. The total number of iterations was k. P_{PSN} represented the overall probability obtained from the PSN model. The absolute error was also computed between the simulation and PSN model outcomes.

IV. SIMULATION AND MODEL OUTCOMES

The simulation and the model were run and the results are presented in Fig. 6. The RMSEs and absolute errors for each simulated network are shown in Table 2.

TABLE II. THE ROOT MEAN SQUARE ERROR (RMSE) AND ABSOLUTE ERROR FOR A GIVEN NUMBER OF SENSORS.

Number of sensors (n)	RMSE	Absolute Error		
		Mean	Max	Min
1	0.0127	0.0117	0.0211	0
2	0.02	0.0176	0.0366	0
3	0.0183	0.0154	0.0415	0
4	0.0167	0.0133	0.0538	0
5	0.0157	0.0111	0.0633	0
6	0.0166	0.0101	0.077	0
7	0.016	0.009	0.0845	0
8	0.0184	0.0091	0.0973	0
9	0.0188	0.0086	0.1054	0
10	0.0206	0.0084	0.1188	0

The maximum event detection was reached for a design consisting of two or more sensors. The sensor network could detect the event with a probability 0.9. The RMSE was about 2% in those networks, which reached that level of detection.

V. CONCLUSION

The PSN model and the simulation data show comparable outcomes across the different sensor networks. They both show that a single sensor has distinct drawbacks compared to any network comprised of multiple sensors. This difference is most obvious in the situation where the probability of the individual sensor(s) was set to 1. It can be deduced that this effect is due to the introduction of multiple processing nodes (D), as the probability for sensing (B) and conditioning (C) remained at a value of 1 and thus do not yield any influence on the changing probability.

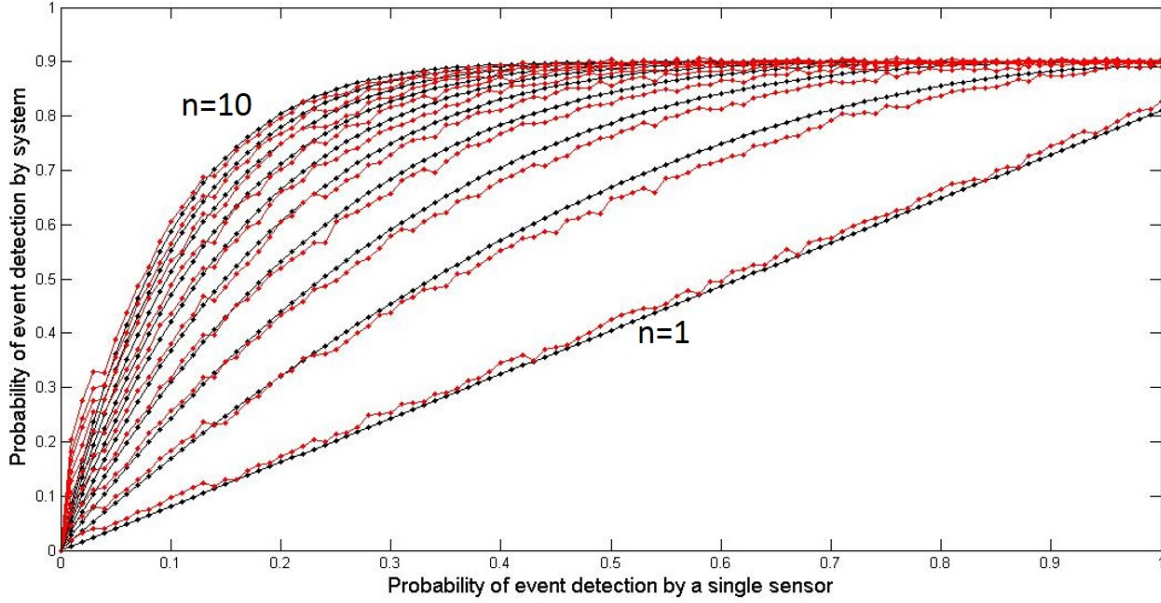


Fig. 2 Probability of event detection by a single sensor set against the probability of event detection of the overall design. The network consist of a number (n) sensors ranging from 1 to 10. The results of the probabilistic model and the simulation are plotted in this figure. Black lines show the model and red the simulation outcomes.

The PSN model makes it easy to assess differences between the networks and allow for clear interpretations to be formulated

The idea of probabilistic sensor network design is to use statistical principles to inform us about the potential suitability of a certain sensor network. The PSN model offers a systematic approach to explore the domain of possible network designs. This paper introduces the PSN topic and specifies a very basic outline of the proposed method. It provided an initial comparison with a simple simulation of a moveable sensor network to show how the model compares to a simulated scenario. The examples were kept simple for introductory purposes and several factors can be modified in future PSN models. One key assumption of the presented model is that the nodes are independent. Yet, two sensors are only truly independent if (7) is true. In practice the event detection between nodes might not be independent and thus a conditional probability needs to be introduced. A measurement taken from one sensor can then influence the possible outcome space of another sensor. This can be captured with a general multiplication rule,

$$P(B_2 \cap B_1) = P(B_1|B_2) \cdot P(B_2)$$

indicating that the probability for sensor 1 (B_1) and sensor 2 (B_2) has a certain combined probability. The inclusion of conditional probability for the PSN model falls outside the scope of this current paper. The presented model also used a uniform distribution and different distributions can be integrated into a PSN model. The scenario in this paper takes into account only false negatives, as the event always occurred (light source was constantly on). The model can be changed to include false positives, as both these errors will determine the probability of a correct detection at a given node. Extra adaptations can be introduced to develop ever more complex and sophisticated PSN models or to adapt the model for a very specific sensor network application. Still, it will be useful to

always reflect on how these more complex models compare to simpler representations.

Further work is still required to determine to what extent the PSN model needs to be modified for the direct practical application. The next phase will be to verify the PSN model with experimental data from sensor networks. The probabilistic modelling presented in this paper can assist in predicting appropriate sensor network designs. It can also help determine what kind of event detection capabilities the sensor need to have to form a network that is acceptable for a given situation. The field of probabilistic sensor network design is a new topic that could bring solutions and allow for new questions to be formulated within the domain of sensor networks.

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