

Age velocity dispersion relations and heating histories in disc galaxies

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ABSTRACT

We analyse the heating of stellar discs by non axisymmetric structures and giant molecular clouds (GMCs) in N -body simulations of growing disc galaxies. The analysis resolves long-standing discrepancies between models and data by demonstrating the importance of distinguishing between measured age-velocity dispersion relations (AVRs) and the heating histories of the stars that make up the AVR. We fit both AVRs and heating histories with formulae $\propto t^\beta$ and determine the exponents β_R and β_z derived from in-plane and vertical AVRs and $\tilde{\beta}_R$ and $\tilde{\beta}_z$ from heating histories. Values of β_z are in almost all simulations larger than values of $\tilde{\beta}_z$, whereas values of β_R are similar to or mildly larger than values of $\tilde{\beta}_R$. Moreover, values of β_z ($\tilde{\beta}_z$) are generally larger than values of β_R ($\tilde{\beta}_R$). The dominant cause of these relations is the decline over the life of the disc in importance of GMCs as heating agents relative to spiral structure and the bar. We examine how age errors and biases in solar neighbourhood surveys influence the measured AVR: they tend to decrease β values by smearing out ages and thus measured dispersions. We compare AVRs and velocity ellipsoid shapes σ_z/σ_R from simulations to Solar-neighbourhood data. We conclude that for the expected disc mass and dark halo structure, combined GMC and spiral/bar heating can explain the AVR of the Galactic thin disc. Strong departures of the disc mass or the dark halo structure from expectation spoil fits to the data.

Key words: methods: numerical - galaxies:evolution - galaxies:spiral - Galaxy: disc - Galaxy: kinematics and dynamics;

1 INTRODUCTION

When the stars in the Solar neighbourhood (Snhd) are binned by age, the velocity dispersion of each bin increases with its age. This age-velocity dispersion relation (AVR) has been known and studied for decades (e.g. [Strömberg 1946](#); [Parenago 1950](#); [Wielen 1977](#)) and similar relations have now also been inferred for external galactic discs ([Beasley et al. 2015](#) for M33, [Dorman et al. 2015](#) for M31). It is generally agreed that understanding the physics that establishes the AVR would be a significant step towards understanding how galaxies form and evolve.

Despite many efforts, the shape of the AVR is still not adequately constrained. The major constraints on the AVR come from observations of stars in the Snhd. The measured ages of stars suffer from substantial errors, and samples of stars with measured ages typically have a selection function that favours young stars over old ([Nordström et al. 2004](#)). Alternatively modelling the velocity dispersions as functions

of stellar colour has been used to determine the shape of the AVR ([Aumer & Binney 2009](#), hereafter AB09).

Whereas the vertical density profile of the Milky Way (MW) is well fitted by a double-exponential in $|z|$, the AVR is typically described as a simple power-law in age $\sigma(\tau) \propto \tau^\beta$ with exponent β . [Quillen & Garnett \(2001\)](#) claimed to detect a jump in the vertical AVR for ages $\tau > 10$ Gyr and connected this jump to the double-exponential nature of the density profile. However, more recent studies find that the AVR in the Snhd can be reasonably described by a single power-law ([Holmberg et al. 2009](#), AB09). Moreover, [Schönrich & Binney \(2009\)](#) demonstrated that the observed vertical density profile is fitted well by a model in which the histories of star formation and disc heating are continuous, and [Bovy et al. \(2012\)](#) showed that subsets of stars selected at different points in the $([\alpha/\text{Fe}], [\text{Fe}/\text{H}])$ abundance plane have scale heights that vary continuously, with no evidence for a dichotomy.

To constrain the heating processes responsible for the Snhd AVR, three diagnostics have been used in addition to the value of β : (i) the axis ratios σ_z/σ_R of the velocity ellipsoids of old and young populations; (ii) the magnitude

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σ_{old} of σ_z for the oldest populations; (iii) the vertical density profile of the MW's disc.

Several physical processes work together to establish the AVR. The goals of this paper are to deepen our understanding of the collaboration of secular heating processes and to show how the AVR responds to one or the other process playing a more prominent role. External heating due to interaction with dark substructure is not considered in this paper.

Spitzer & Schwarzschild (1953) showed that scattering of stars off small-scale irregularities in the potential of the Galactic disc would transform the nearly circular orbits of young stars, which reflect their origin from the gas disc, into orbits with higher radial and vertical energy. The discovery of Giant molecular clouds (GMCs) with masses $M_{\text{GMC}} \sim 10^{5-7} M_{\odot}$ provided a suitable candidate for the heating agent. Using analytic arguments, Lacey (1984) showed that GMC heating could have contributed significantly to the observed AVR, but that the dispersion σ_{old} of the oldest stars could be reproduced only if the masses and / or number density of GMCs was significantly higher in the past than they are now. Lacey further derived values $\sigma_z/\sigma_R \sim 0.8$ and $\beta \sim 0.25$ that are, respectively, larger and smaller than the data indicate. Ida et al. (1993) used analytical calculations more sophisticated than that of Lacey to show that scattering by GMCs in discs actually yields values $\sigma_z/\sigma_R \sim 0.5 - 0.6$ that are consistent with observations. This was confirmed with particle simulations by Shiidsuka & Ida (1999) and Hänninen & Flynn (2002): to obtain correct values it is essential to take into account that in a thin disc impact parameters are concentrated towards the galactic plane, whereas Lacey had assumed an isotropic distribution of impact parameters (Sellwood 2008). However, the conflict between observations and Lacey's values for β and σ_{old} remained (Hänninen & Flynn 2002).

GMCs are not the only sources of a non-axisymmetric component to the gravitational field experienced by disc stars, and any such component will heat the disc. Barbanis & Woltjer (1967) showed that spiral structure could significantly heat the disc, when the spiral pattern has either a very high density contrast or is of a transient and recurring nature. Two-dimensional simulations of discs continuously fed with young, cold particles showed that transient and recurrent spiral structure is always present in star forming discs and provides sufficient heating to explain the in-plane AVR (Sellwood & Carlberg 1984; Carlberg & Sellwood 1985). From such two-dimensional simulations and analytical arguments for vertical cloud heating, Carlberg (1987) concluded that a combination of GMC and spiral heating could explain the observations. However, spirals do not directly increase σ_z significantly (e.g. Sellwood 2013; Martinez-Medina et al. 2015), and the question remained open whether deflections by GMCs can convert in-plane heat to vertical heat in an appropriate manner. Jenkins & Binney (1990) used analytic arguments to examine this question for growing discs. They concluded that the observed value of σ_z/σ_R could be explained, but the observed value of σ_{old} was problematic.

Another continuous secular heating process that has been discussed in the literature is heating by the bar (Saha et al. 2010; Grand et al. 2016). The interaction of galactic discs with satellite galaxies and the corresponding dark matter substructure can also cause disc heating (Velazquez

& White 1999). However, in the case of the MW this process has likely only made a minor contribution to the observed AVRs (Moetazedian & Just 2016).

Another contributor to the AVR could be a decline over cosmic time in the velocity dispersion of stars at their time of birth as discs have become less gas-rich and and less turbulent (Bournaud et al. 2009; Forbes et al. 2012). These models are motivated by observations of gas kinematics in disc galaxies at various redshifts, mostly based on H α emission. These observations have revealed significantly ($\sim 3 - 10$ times) higher velocity dispersions σ at redshifts $z \sim 2 - 4$ than in corresponding observations of the local universe (e.g. Förster Schreiber et al. 2009), and a decline of σ with decreasing redshift (Wisnioski et al. 2015). It is, however, unclear how the kinematics of young stars which form from cold gas, relate to these observations.

Fully cosmological hydrodynamical simulations of galaxy formation have recently reached reasonable levels of success in reproducing MW like disc galaxies and the AVRs in some of these simulations have been studied (House et al. 2011; Bird et al. 2013; Martig et al. 2014; Grand et al. 2016). At $z = 0$ the stellar populations in the majority of these simulations are significantly hotter than the stars in the MW at all ages (but see model *g92* of Martig et al.). Especially young stars have overly high σ_z which has been linked to numerical effects and insufficient resolution and shown to depend on the specifics of the numerical sub-grid star-formation models (House et al. 2011; Martig et al. 2014). Better agreement with the Snhd AVR is generally found for galaxies unaffected by mergers. Martig et al. find that the thin disc stars in their more successful models are born cold and heated with time, which they attribute to heating by spirals and overdensities and possibly the coupling between transient spirals and weak bending waves (Masset & Tagger 1997). Note that GMCs are not resolved in these simulations.

For many years it was assumed that the chemodynamical evolution of any annulus of the Galactic disc could be modelled in isolation of other annuli. Now there is clear evidence that radial migration of stars within discs is an important process (Sellwood & Binney 2002; Roškar et al. 2008; Schönrich & Binney 2009; Kordopatis et al. 2015), with the consequence that the production of a hot population in one annulus, for example through the action of a bar, can subsequently endow a distant annulus with a hot population that could not have been locally heated. On account of radial migration, it is essential to understand the origin of the AVR *globally*, that is by tracking the evolution of the disc at all radii. In general we expect the mean birth radius of a coeval cohort of Snhd stars will decrease with increasing age, and on account of the radial gradient in velocity dispersion the decrease in birth radius will be reflected in the AVR (Schönrich & Binney 2009).

In this paper we use the simulations presented in Aumer et al. (2016, hereafter ABS16) to study the formation of the AVR. Unlike the previously cited studies, these simulations include simultaneously all the following important aspects: growing discs with multiple coeval populations, GMCs, recurring spiral structure with evolving properties, a bar, an evolving GMC-to-stellar mass fraction, radial migration and sufficiently cold young stars. ABS16 showed that although the vertical profiles of their models do not show a thick disc

like that of the MW, some models do provide quite good fits to the AVR of the Sphd. Hence they concluded that the thick disc requires additional sources of heat, but the thin disc can be explained by combined GMC and spiral heating. They showed that the efficiency of GMC heating declines over time because the fraction of the disc's mass contained in GMCs falls steadily as a consequence of a declining star-formation rate (SFR) and a growing disc mass. Their simulations are thus a promising tool to study what shapes the AVR in thin galactic discs.

Two major conclusions will emerge from our study: (i) biased ages and age uncertainties cause measured AVRs to deviate significantly from the true AVRs; (ii) it is vital to distinguish between an AVR $\sigma(\tau)$, which gives velocity dispersion as a function of age for stars that are now co-located, and a *heating history* $\sigma(t - t_b)$, which gives the velocity dispersion as a function of time for a cohort of currently co-located stars that were born at a given time t_b . Whereas the AVR $\sigma(\tau)$ for $\tau \approx 4.5$ Gyr quantifies the current kinematics of stars born contemporaneously with the Sun, the heating history $\sigma(t - t_b)$ for $t - t_b \approx 4.5$ Gyr quantifies the kinematics of stars 4.5 Gyr after they were born, which would be 5.5 Gyr ago in the case of 10 Gyr old stars in the disc. If stars were born into a statistically stationary environment providing heating processes which are constant in time, the cohort born 10 Gyr ago would 5.5 Gyr ago have been in the same dynamical state that the Sun's cohort is in now. That is, given a stationary environment the AVR would be the same function of τ that the heating history is of $t - t_b$. If a galaxy undergoes a major merger, stars born before and after the merger will undergo different heating histories $\sigma(t - t_b)$. Here we argue, that even in the absence of mergers or declining birth dispersions, the thin discs of galaxies change beyond recognition over cosmological timescales, so the environment is very far from stationary, and the heating experienced by stars born 10 Gyr ago during the first Gyr of their lives was very different from the environment experienced by recently born stars during the first Gyr of their lives. Consequently heating histories are described by entirely different functions from the AVR.

Nevertheless we will find that both AVRs and heating histories can be well approximated by the modified power law

$$\sigma(x) = \sigma_{10} \left(\frac{x + x_1}{10 \text{ Gyr} + x_1} \right)^\beta \quad (1)$$

used by AB09, with $x = \tau$ or t . To differentiate between parameters derived from AVRs and heating histories, we will mark the latter with a tilde, i.e. $\tilde{\beta}$, $\tilde{\sigma}_{10}$ etc. We will find that the indices β and $\tilde{\beta}$ of these power laws are often dissimilar. Moreover, we find that in the case of a heating history the value of $\tilde{\sigma}_{10}$ can evolve strongly with the time t_b of the cohort's birth, whereas in most models $\tilde{\beta}$ evolves only mildly.

Our paper is organised as follows: In Section 2 we briefly describe the simulations. In Section 3 we examine the effects of observational age errors and biases on the AVR. In Section 4 we describe the model AVRs and compare them to local data. Topics discussed include the uncertainties of the comparisons arising from azimuthal variations in the model AVRs (Sections 4.1 and 4.2), and the diagnostic content of the axis ratios of velocity ellipsoids (Section 4.3), and power-law fits to AVRs (Section 4.5). In Section 5 we consider the

heating histories for different populations of coeval model stars and show how these relate to AVRs. In Section 6 we relate our findings to the physics of star scattering, and we conclude in Section 7.

2 SIMULATIONS

The simulations analysed in this paper are a subset of the models presented in ABS16. These are simulations of growing disc galaxies within non-growing live dark matter haloes made using the Tree Smoothed Particle Hydrodynamics (TreeSPH) code GADGET-3, last described in Springel (2005). We focus on standard-resolution models, which contain $N = 5 \times 10^6$ particles in the final disc and the same number of halo particles. Most of the simulations are collisionless, but a subset contains an isothermal gas component with pressure $P = \rho c_s^2$ and sound speed $c_s = 10 \text{ km s}^{-1}$. The global gas fraction in these discs is kept roughly constant over time at $f_g = 10$ per cent. In addition, most simulations contain a population of short-lived, massive particles representing GMCs. The force softening lengths are $\epsilon_{\text{bar}} = 30 \text{ pc}$ for baryonic particles (including GMCs) and $\epsilon_{\text{DM}} = 134 \text{ pc}$ for DM particles.

Table 1 gives an overview of the models cited here. We give only a brief account of the meaning of the model parameters – a full description can be found in ABS16. Standard models contain GMCs, but no gas and all models have evolved over a simulation time of $t_f = 10 \text{ Gyr}$. The presence of gas in a model is marked by a 'G' in its name, while the absence of GMCs is marked by an 'N'.

The initial conditions (ICs) were created using the GALIC code (Yurin & Springel 2014). The details of the ICs can be found in Table 1 of ABS16. The models discussed here all start with a spherical dark matter halo with a Hernquist (1990) profile and a mass of $10^{12} M_\odot$. The F model differs from the others in that the scale length of its halo is $a_{\text{halo}} = 51.7 \text{ kpc}$ rather than 30.2 kpc .

All models analysed here contain an IC disc with a mass profile

$$\rho_{\text{disc},i}(R, z) = \frac{M_{b,i}}{4\pi z_{0,\text{disc}} h_{R,\text{disc}}^2} \text{sech}^2 \left(\frac{z}{z_{0,\text{disc}}} \right) \exp \left(\frac{-R}{h_{R,\text{disc}}} \right). \quad (2)$$

Here $h_{R,\text{disc}}$ is the IC disc exponential scalelength and a radially constant isothermal vertical profile with scaleheight $z_{0,\text{disc}}$ is assumed. The Y and F models start with a baryonic disc of mass $M_{b,i} = 5 \times 10^9 M_\odot$, which is compact ($h_{R,\text{disc}} = 1.5 \text{ kpc}$) and thin ($z_{0,\text{disc}} = 0.1 \text{ kpc}$). The A models contain a thicker and more massive IC disc ($z_{0,\text{disc}} = 0.8 \text{ kpc}$, $M_{b,i} = 10 \times 10^9 M_\odot$) and the IC disc in the E models is even more massive, thicker and more extended ($h_{R,\text{disc}} = 2.5 \text{ kpc}$, $z_{0,\text{disc}} = 1.2 \text{ kpc}$, $M_{b,i} = 15 \times 10^9 M_\odot$).

Stellar particles are continuously added to the disc on near-circular orbits. The young stellar populations are assigned birth velocity dispersions σ_0 in all three directions R , ϕ and z . The standard choice is $\sigma_0 = 6 \text{ km s}^{-1}$, but we also consider a simulation with $\sigma_0 = 10 \text{ km s}^{-1}$ (Y1f σ) and one (A2 τ) in which the birth velocity dispersion declines exponentially with time

$$\sigma_0(t) = (6 + 30e^{-t/1.5 \text{ Gyr}}) \text{ km s}^{-1}. \quad (3)$$

Table 1. List of models analysed in this paper. *1st Column:* Model Name; *2nd Column:* Initial Conditions; *3rd Column:* Total baryonic IC mass $M_{b,i}$; *4th Column:* IC DM halo scalelength a_{halo} ; *5th Column:* IC radial disc scalelength $h_{R,\text{disc}}$; *6th Column:* IC vertical disc scaleheight $z_{0,\text{disc}}$; *7th Column:* GMCs Yes/No; *8th Column:* Cutoff: Adaptive (no new particles in bar region) or fixed (pre-defined evolving inner cutoff for new particles); *9th Column:* Total inserted baryonic model mass M_f (including initial baryonic mass); *10th Column:* Initial disc scalelength $h_{R,i}$; *11th Column:* Final disc scalelength $h_{R,f}$; *12th Column:* Scalelength growth parameter ξ ; *13th Column:* Exponential decay timescale t_{SFR} for the star formation rate; *14th Column:* Initial velocity dispersion for inserted stellar particles, σ_0 ; *15th Column:* GMC star formation efficiency ζ ;

1st Name	2nd ICs	3rd $M_{b,i}$ [$10^9 M_\odot$]	4th a_{halo} kpc	5th $h_{R,\text{disc}}$ kpc	6th $z_{0,\text{disc}}$ kpc	7th GMCs	8th Cutoff	9th M_f / M_\odot [10^{10}]	10th $h_{R,i}$ kpc	11th $h_{R,f}$ kpc	12th ξ	13th t_{SFR} [Gyr]	14th σ_0 [km s^{-1}]	15th ζ
Y1	Y	5	30.2	1.5	0.1	Yes	Adap	5	1.5	4.3	0.5	8.0	6	0.08
Y1s2	Y	5	30.2	1.5	0.1	Yes	Adap	5	1.5	4.3	0.5	16.0	6	0.08
Y1 ζ -	Y	5	30.2	1.5	0.1	Yes	Adap	5	1.5	4.3	0.5	8.0	6	0.04
Y1f σ	Y	5	30.2	1.5	0.1	Yes	Fix	5	1.5	4.3	0.5	8.0	10	0.08
Y2	Y	5	30.2	1.5	0.1	Yes	Adap	5	2.5	2.5	0.0	8.0	6	0.08
Y2Mb-	Y	5	30.2	1.5	0.1	Yes	Adap	3	2.5	2.5	0.0	8.0	6	0.08
Y2Mb+	Y	5	30.2	1.5	0.1	Yes	Adap	7.5	2.5	2.5	0.0	8.0	6	0.08
Y4f ζ -	Y	5	30.2	1.5	0.1	Yes	Fix	5	1.5	2.2	0.5	8.0	6	0.04
YG1	YG	5	30.2	1.5	0.1	Yes	Adap	5	1.5	4.3	0.5	8.0	6	0.08
YN1	Y	5	30.2	1.5	0.1	No	Adap	5	1.5	4.3	0.5	8.0	6	–
A2 τ	A	10	30.2	1.5	0.8	Yes	Adap	5	2.5	2.5	0.0	8.0	$6 + 30e^{-t/1.5\text{Gyr}}$	0.08
E2	E	15	30.2	2.5	1.2	Yes	Adap	5	2.5	2.5	0.0	8.0	6	0.08
F2	F	5	51.7	1.5	0.1	Yes	Adap	5	2.5	2.5	0.0	8.0	6	0.08

The star-formation rate is

$$\text{SFR}(t) = \text{SFR}_0 \times \exp(-t/t_{\text{SFR}}), \quad (4)$$

with $t_{\text{SFR}} = 8$ or 16 Gyr. The constant SFR_0 is adjusted to produce at $t = t_f$ a target final baryonic mass M_f in the range $3 - 7.5 \times 10^{10} M_\odot$. Mass growth is smooth in time and sufficiently slow for the process to be effectively adiabatic.

Particles are added randomly distributed in azimuth every five Myr with an exponential radial density profile $\Sigma_{\text{SF}}(R) \propto \exp(-R/h_R(t))$. The scalelength $h_R(t)$ of the newly added particles grows in time as

$$h_R(t) = h_{R,i} + (h_{R,f} - h_{R,i})(t/t_f)^\xi. \quad (5)$$

To avoid inserting particles in the bar region, where near-circular orbits do not exist, particles are not added inside the cutoff radius R_{cut} , which is either determined by the current bar length ('adaptive cutoff'), or given by a pre-determined formula $R_{\text{cut}}(t) = \left(0.67 + \frac{0.33t}{1\text{Gyr}}\right)$ kpc ('fixed cutoff').

GMCs are modelled as a population of massive particles drawn from a mass function of the form $dN/dM \propto M^\gamma$ with lower and upper mass limits $M_{\text{low}} = 10^5 M_\odot$ and $M_{\text{up}} = 10^7 M_\odot$ and a power law exponent $\gamma = -1.6$. Their radial density is proportional to Σ_{SF} , and their azimuthal density is given by

$$\rho_{\text{GMC}}(\phi) \propto [\rho_{\text{ys}}(\phi)]^\alpha, \quad (6)$$

where ρ_{ys} is the density of young stars and $\alpha = 1$. The mass in GMCs is determined by the SFR efficiency ζ . Specifically, for each Δm_{stars} of stars formed, a total GMC mass $\Delta m_{\text{GMC}} = \Delta m_{\text{stars}}/\zeta$ is created. GMC particles live for only 50 Myr: for 25 Myr their masses grow with time, and for the final 25 Myr of their lives their masses are constant.

In ABS16 we presented an overview of the properties of the simulated galaxies. Important findings of ABS16 that are relevant to the results of this paper, are:

(i) In the absence of GMCs, the models are too cold vertically to explain the vertical profile of the MW thin disc. Heating by GMCs creates remarkably exponential vertical profiles, the scaleheights of which agree roughly with that inferred for the MW thin disc for our standard GMC mass function and $\zeta = 0.04$ (Y1 ζ -). These discs have radially constant vertical profiles.

(ii) GMC heating is particularly efficient early on, when SFRs are high and stellar disc masses are small.

(iii) No thick discs similar to the one observed in the MW form in the models with thin IC discs. Thicker discs can form in models with high baryon fractions (Y4f ζ -, F2), but they are too hot radially.

(iv) Spurious two-body heating due to halo particles is negligible when the halo is resolved with at least 5×10^6 particles.

(v) The output scalelength h_R of a stellar population can differ significantly from the input scalelength, as bars and spirals drive radial redistribution and lead to an increase of h_R in the outer disc.

(vi) Bars are stronger for more compact models and weaker for models with an isothermal gas component.

(vii) Isothermal gas components mildly increase the efficiency of GMC heating, presumably as GMC particles attract wakes of gas which increase their effective mass.

Unless otherwise noted, we will analyse our models at a solar-like radius $R_0 = 8$ kpc. The local exponential scalelength h_R at R_0 can differ significantly between models (2–5 kpc), but the extreme values are caused by deviations from simple exponential profiles. The differences in final surface density at R_0 are small (Fig. 12 in ABS16) and for our standard disc mass the models agree reasonably with estimates for the Snhd (Holmberg & Flynn 2004).

3 THE IMPACT ON AVRS OF BIASES AND ERRORS IN AGES

Before we can compare the AVRs from the *Geneva-Copenhagen Survey* (GCS) (Nordström et al. 2004; Casagrande et al. 2011) to data from simulations, we need to model the impact on the observations of the survey’s selection function and errors in the estimated ages of stars (see also Holmberg et al. 2007; Martig et al. 2014).

We start by selecting GCS stars with $[\text{Fe}/\text{H}] > -0.8$ and heliocentric azimuthal velocity $V > -150 \text{ km s}^{-1}$ and a ‘good’ age determination: from Casagrande et al., we use the maximum likelihood ages τ and the ages τ_{16} and τ_{84} of the 16 and 84% quantiles of the probability distribution in age. If either $\tau_{84} - \tau_{16} < 2 \text{ Gyr}$ or $2(\tau_{84} - \tau_{16}) / (\tau_{84} + \tau_{16}) < 0.5$ the age τ is deemed good and the star enters our sample. These ~ 7500 stars are ordered by age and placed in bins with 200 stars each to calculate $\sigma(\tau)$. Every 10 stars a new point is plotted and thus every 20th point is statistically independent of its predecessors.

The blue curve in Fig. 1 shows the strongly biased age distribution of this sample. In addition to the bias of this mid-plane survey towards kinematically colder and hence younger stars, the GCS is largely magnitude limited, favouring more luminous, younger stars. The exclusion of hot/blue stars with $T_{\text{eff}} \gtrsim 7000 \text{ K}$ biases the sample against very young stars, and in particular excludes massive stars that could have safe age-determinations with $\tau \lesssim 1.5 \text{ Gyr}$. The small upturn in the number densities at $\tau < 0.5 \text{ Gyr}$ can be mostly ascribed to a pile-up of maximum likelihood values from main-sequence stars with temperature and/or metallicity over-estimates, and hence underestimated maximum likelihood ages. Consistently, there is little evolution in GCS velocity dispersions for ages below $\tau \sim 2 \text{ Gyr}$, stalling at values typical for $\sim 1.5 \text{ Gyr}$ old stars and far above the values derived by AB09 for their bluest stars.

Blue *Hipparcos* stars can be used to determine velocity dispersions of very young stars. The lowest dispersions found by AB09 for their bluest bins in $B-V$ are $\sigma_z = 5.5 \text{ km s}^{-1}$ and $\sigma_R = 8 \text{ km s}^{-1}$. These stars will, however, be kinematically biased, as a majority of them belong to a small number of moving groups of young stars. It is interesting that σ_R increases very strongly between $B-V = -0.2$ and $B-V = 0$, so the smallest value of $\sigma_z/\sigma_R \sim 1/3$ occurs at $B-V \sim 0$.

We impose the GCS age bias and errors of ~ 20 per cent in ages on data from a model galaxy as follows: first we select star particles at $R = 8 \pm 0.5 \text{ kpc}$ and $|z| \leq 0.1 \text{ kpc}$, which approximates the GCS volume. Stars already present in the ICs are randomly assigned ages $\tau \in [t_f, t_f + 1 \text{ Gyr}]$. To each star i we assign a weight $w_i \geq 1$ so that the distribution of true ages of the weighted sample agrees with the one of the GCS sample (blue curve in Fig. 1). We then determine w_i ‘observed’ ages for each star by assuming a Gaussian error distribution with standard deviation 20 per cent of the true age. The resulting distribution of all assigned ages is shown as the red curve in Fig. 1. For the models all bins have width $\Delta\tau = 0.25 \text{ Gyr}$. Note that the GCS selection function also depends on metallicity, which is not taken into account here.

The black curve in the left panel of Fig. 1 shows for Model Y1 at $t_f = 10 \text{ Gyr}$ the actual age distribution of ‘solar neighbourhood’ stars. This distribution peaks at the

youngest ages because young stars are confined to the plane, where we select particles. At intermediate ages the distribution is rather flat because heating and inside-out growth are balanced by the declining SFR. The oldest stars are under-represented as they are hot and centrally concentrated. The green curve is obtained on folding the black curve with our assumed errors in age. Now stars with ages above 5 Gyr are smeared into a tail that extends to 14 Gyr. The striking difference between this green curve and the blue curve for the age distribution of our GCS sample shows how strongly the survey’s selection function biases the data.

In the right panel of Fig. 1 we show the derived AVRs $\sigma_z(\tau)$ for Models Y1 (green) and E2 (red). The dashed lines show the AVRs for true age distributions, and the solid lines the AVRs for the GCS-like samples just described. The jump at 10 Gyr in the red dashed curve reflects the hot proto-thick disc included in the ICs of Model E2. The main difference between the dashed and full curves is extension of the end point from 10 to 14 Gyr and elimination of the step in the dashed curve for E2 (see also Martig et al. 2014). At low ages the solid curves lie slightly above the dashed curves as a consequence of stars with ages $\sim 2 \text{ Gyr}$ being scattered to lower ‘observed’ ages.

4 AVRS AND VELOCITY ELLIPSOID SHAPES

In the last section we explained how we extract comparable AVRs from the models and the GCS. Here we compare $\sigma_z(\tau)$, $\sigma_R(\tau)$ and the ratio $\sigma_z/\sigma_R(\tau)$ between models and the GCS sample.

4.1 Azimuthal variation of the AVR

Whereas in our models we can select star particles from every azimuth, all GCS stars are drawn from azimuths near that of the Sun. Just as the density of stars in disc galaxies varies with azimuth, we expect the age and velocity distributions of stars to vary as well. Moreover, a bar or spiral arm drives non-axisymmetric streaming (e.g. Dehnen 2000), and streaming velocities will boost the recovered velocity dispersion when star particles are binned regardless of azimuth.

Here we attempt to quantify the azimuthal variation of the AVR in our models. We make no attempt to take into account the actual inter-arm location of the Sun because this task requires (i) reliable knowledge of the Galaxy’s non-axisymmetries, (ii) precise control of the non-axisymmetries within the models, (iii) sufficient star particles within $\sim 0.1 \text{ kpc}$ of the Sun to make Poisson noise unimportant, and (iv) detailed modelling of the selection function. None of these three conditions being satisfied, we confine ourselves to the estimation of the uncertainties arising from uncharted non-axisymmetric structures.

For each model we divided the annulus $R = 7.5 - 8.5 \text{ kpc}$ at $|z| < 0.3 \text{ kpc}$ into 36 sectors of 10 deg width and determined mean motions and dispersions for each sector. These 36 separate AVRs are displayed as grey lines in Fig. 2. The area occupied by these lines should be regarded as the region within which the AVR of a mock Sphd could fall. On average we find that higher velocity dispersions are found in regions of higher star density, but the scatter is significant. For most models a typical azimuthal spread in $\sigma_z(\tau)$ is $\sim \pm 1 \text{ km s}^{-1}$ at

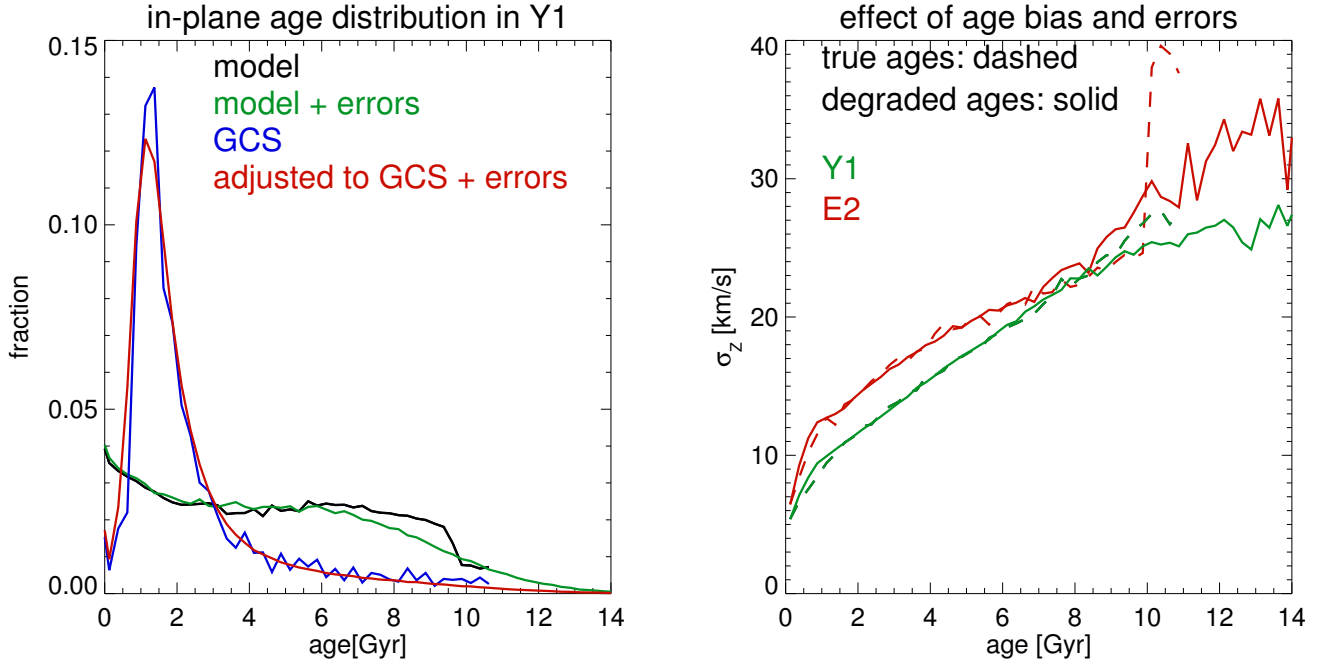


Figure 1. Correcting for age bias and errors to compare models to the GCS data. Left: The black curve shows the age distribution of stars in Model Y1 at 10 Gyr and at $R = 8 \pm 0.5$ kpc and $z = 0 \pm 0.1$ kpc. If we assume observational 1σ errors of 20 per cent we predict the green line. The blue line shows the age distribution in the GCS sample. The red line is what we get when we weight stars in the model so that they match the blue line and then assume 20 per cent age errors. Right: Vertical AVRs $\sigma_z(\tau)$ for Models Y1 (green) and E2 (red) at 10 Gyr and at $R = 8 \pm 0.5$ kpc and $|z| \leq 0.1$ kpc. Dashed lines are for the true age distribution (black line in left panel) and solid lines are for the degraded age distribution (red line in left panel).

young ages and $\sim 2 - 3 \text{ km s}^{-1}$ at older ages. Models such as Y4f ζ - and E2 with bars that nearly reach $R = 8$ kpc, show larger spreads, even at young ages, because bars have strong effects on stellar orbits and significant differences in orbit populations occur in regions positioned differently with respect to the bar. Bars and spiral structure excite streaming motions more parallel to the plane than vertically, so the fractional azimuthal spreads are larger for $\sigma_R(\tau)$ than for $\sigma_z(\tau)$.

In Fig. 2 we also plot in red the azimuthal average for $|z| < 0.3$ kpc and in blue for $|z| < 0.1$ kpc, which is the most relevant region to compare to the GCS data. Unfortunately, when we subdivide the data by azimuth, Poisson noise is unacceptably large in the data for $|z| < 0.1$ kpc. The biggest discrepancies between red and blue lines are found for $\sigma_z(\tau)$ in Models E2 and F2, which have thicker discs, and in Model YN1, which has no GMCs. The discrepancy indicates vertical dispersions which increase with altitude at a given age, as the red lines probe higher altitudes. For most models dispersions hardly change with altitude and red and blue lines are very similar indicating that our use of bins extending to $|z| = 0.3$ kpc will not mislead.

In Fig. 2 the Snhd data from the GCS are shown in pink. Green horizontal lines at low ages indicate the lowest values found for each of the three quantities for blue *Hipparcos* stars by AB09 to give an indication of the uncertainties at low ages.

4.2 Specific models

We begin by analysing models, which ABS16 considered to have problematic AVRs to reconsider this judgement in light of the azimuthal spreads shown in Fig. 2.

Models Y2Mb+ and Y2Mb- have an abnormally high and low final disc mass, respectively. In these models the azimuthal variations are too small to account for the conflict with observation. Similarly, the vertical dispersions in Model YN1, which lacks GMCs, remain too small and the dispersions in the thickest Y model, Y4f ζ -, remain too high. Finally, the characteristic feature in the radial dispersion at $\tau \sim 4$ Gyr for Y4f ζ - shows up clearly at all azimuthal positions.

ABS16 showed that models that start from IC F, which has a low-density dark halo, produce thicker discs, which are, however, too hot. In particular Model F2 has a vertical profile that is significantly thicker than those of our standard models, run from initial condition Y, and is slightly double-exponential, but still significantly thinner than that of the MW. When the azimuthal variation in its AVRs is taken into account, Model F2 becomes marginally acceptable because at the right end of the lower row of panels in Fig. 2, the lowest grey curve for $\sigma_R(\tau)$ is consistent with the observations.

We now consider models run from the E and Y ICs. These models have standard dark halos and total baryonic masses $M_f = 5 \times 10^{10} M_\odot$. Model Y1 is slightly too cold radially for old stars, and slightly too cold vertically for stars of intermediate age. The more compact Model Y2 has a radial AVR that fits observations better because it has higher

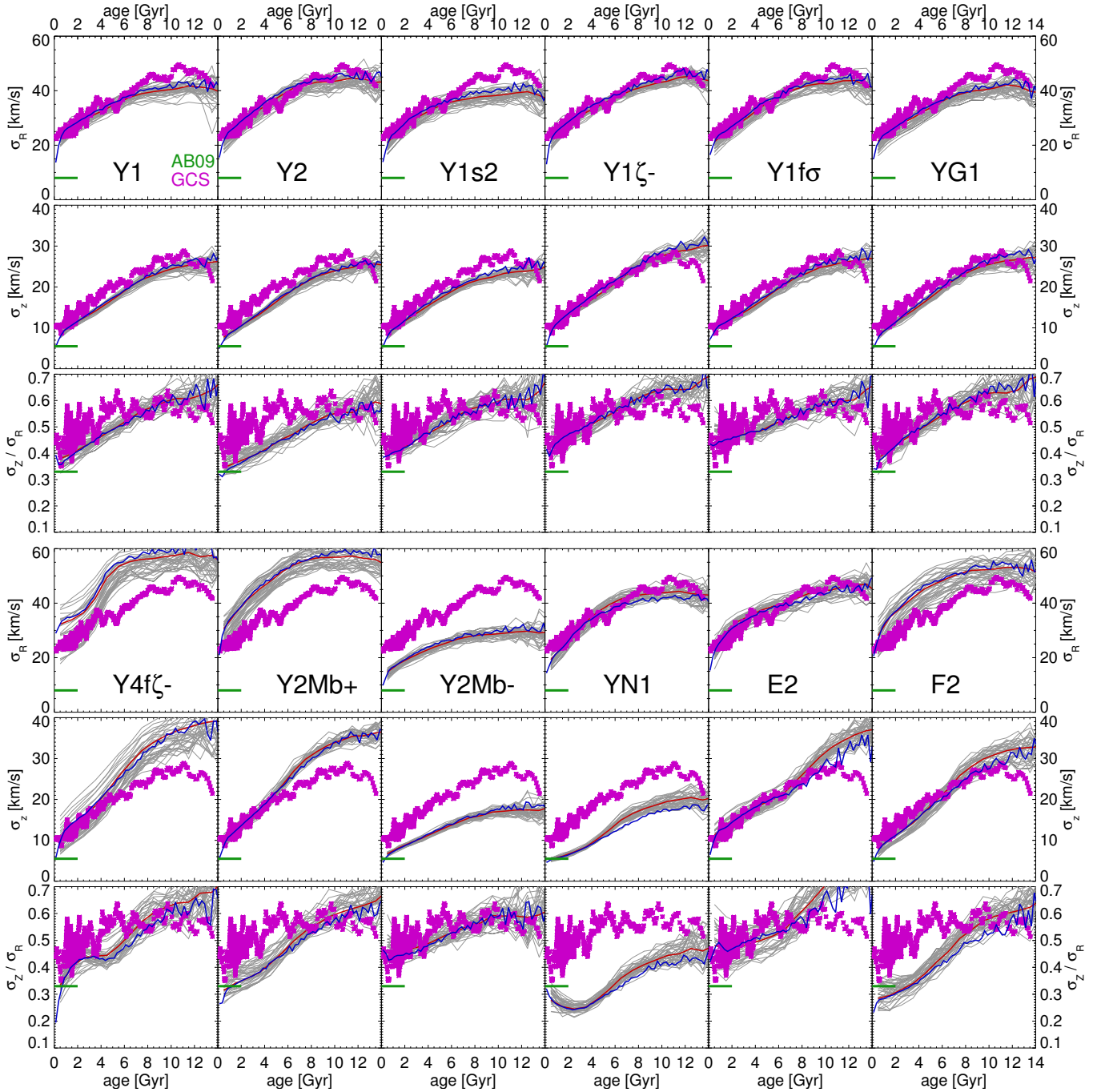


Figure 2. The AVRs of a variety of models. For each model we show $\sigma_R(\tau)$ (top), $\sigma_z(\tau)$ (middle) and $\sigma_z/\sigma_R(\tau)$ (bottom). The pink asterisks are extracted from the GCS Sph data of Casagrande et al. (2011). The blue lines are the azimuthally averaged AVRs from the models for $|z| < 0.1$ kpc and the red lines are for $|z| < 0.3$ kpc. The set of grey lines are for 36 equally spaced azimuthal bins at $|z| < 0.3$ kpc. All model data are for $R = 8 \pm 0.5$ kpc. The green lines at low ages mark the lowest values found by AB09 for each quantity with blue *Hipparcos* stars. The GCS stars with good ages are placed in bins with 200 stars each and every 10 stars a new point is plotted, so that every 20th point is statistically independent. At $\tau \sim 2$ Gyr, there are 200 stars per ~ 100 Myr, whereas at $\tau > 6$ Gyr there are only ~ 1000 stars in total.

surface densities and thus stronger non-axisymmetric structures. In Model Y1s2 the SF timescale is longer so the disc grows more slowly. The consequence is old populations that are too cold radially and vertically because the total mass and the GMC mass fraction at early times are both low. Model Y1fσ, which uses a fixed cutoff and has a higher value

for the input velocity dispersion σ_0 , shows a mildly better fit to the data than Y1, as does Model YG1, which has an isothermal gas component. The best fits by a Y model are provided by Model Y1ζ-, which has a lower SF efficiency and thus a higher total GMC mass at all times.

Whereas the Y models start from a small, compact and

thin disc, Model E2 starts from a more extended and more massive thick disc. If we consider that we should not trust any points at $\tau > 12$ Gyr and that the blue lines in Fig. 2 are more relevant than the red lines, E2 also provides acceptable fits to both $\sigma_R(\tau)$ and $\sigma_z(\tau)$. At old ages its AVR $\sigma_z(\tau)$ differs from that of Model Y2 on account of the thick IC, and at young ages it differs on account of its extended and buckling bar.

4.3 Shape of the velocity ellipsoid

We now consider $\sigma_z/\sigma_R(\tau)$, which is plotted for each model in the third and sixth rows of Fig. 2. The (pink) observational values show substantial scatter around $\sigma_z/\sigma_R \sim 0.5$, with lower values at young ages and higher values at old ages. Although the Casagrande et al. (2011) values lie in (0.4, 0.6), AB09 found values as low as 0.33 for the bluest stars in the *Hipparcos* catalogue, which, as we noted above, are excluded from the GCS sample, but may be biased by moving groups. At old ages, $\tau \gtrsim 7$ Gyr, $\sigma_z/\sigma_R \approx 0.55$ –0.6 appears rather constant. Values of this order are predicted by simulations of heating by GMCs (Hänninen & Flynn 2002; Sellwood 2008).

In all models stars are, by construction, added with $\sigma_z = \sigma_R = \sigma_0$. Non-axisymmetries almost instantaneously increase in σ_R to a value $\gg \sigma_0$, while σ_z increases much more gradually. Consequently, quite soon $\sigma_z/\sigma_R < 0.5$, as observed for the youngest stars. Moreover, σ_z/σ_R increases with age, again as observed.

Although Model Y2Mb- with a low-mass disc provides a very good fit to the observed $\sigma_z/\sigma_R(\tau)$, its velocity dispersions are too low to be consistent with observations. Model F2 with a low-density halo and a marginally acceptable AVR, can now be excluded because in it $\sigma_z/\sigma_R(\tau)$ is too low at young ages.

The E and Y models have standard dark haloes. In Model Y1 $\sigma_z/\sigma_R(\tau)$ is lower than in the observations at $\tau \sim 5$ Gyr, and higher for $\tau \gtrsim 10$ Gyr. Model Y2 with a more compact disc fits σ_z/σ_R less well. Model Y1f σ , which has an abnormally high value of the birth dispersion parameter $\sigma_0 = 10 \text{ km s}^{-1}$, fits the observations better. Model Y1 ζ -, which has an abnormally low star-formation efficiency, provides an excellent fit except at ages $\tau \gtrsim 8$ Gyr. Model Y1s2, which has a more slowly declining SFR, provides a similar quality of fit to that of Model Y1.

Model E2, which has a massive and extended primordial thick disc, provides a good fit to the observed $\sigma_z/\sigma_R(\tau)$ at $\tau < 9$ Gyr, but provides unacceptably high values at older ages. However, this conclusion must be moderated by two caveats: (i) the grey lines should be moved downwards by the separation between the blue line for $|z| < 0.1$ kpc and the red line for $|z| < 0.3$ kpc; (ii) no model has stars with $\tau > 11$ Gyr (ICs stars are assigned ages at 10–11 Gyr), so we must be careful when drawing conclusions regarding this age range. In light of these caveats, we consider that Model E2 also provides an acceptable fit to $\sigma_z/\sigma_R(\tau)$.

As expected for a model without GMCs, $\sigma_z/\sigma_R(\tau)$ is too low at all ages τ in YN1. We note that the increase in $\sigma_z/\sigma_R(\tau)$ for older stars may in part be caused by spurious collisional relaxation (Sellwood 2013).

4.4 Which models can reproduce the Snhd AVR?

Combining the results of all three quantities plotted in Fig. 2, we conclude that only models with a standard halo (Y and E models) and standard baryonic mass ($M_f = 5 \times 10^{10} M_\odot$) are compatible with the Snhd data. Lower or higher disc masses, or lower density haloes, all fail to reproduce the data. As far as models with thicker disc components are concerned, inclusion of a thick disc in the ICs (Model E2) is clearly favoured over a thick disc that emerges during the simulations (Models F2 and Y4f ζ -).

Since our models include significant idealizations, we cannot expect any model to provide a perfect fit to the data and we do not seek to reproduce the MW disc precisely. Not only do the models still lack heating processes, such as interactions with satellite galaxies (e.g. Moetazedian & Just 2016), that will modify the predictions, but even after our effort to take age biases and errors into account, the comparison of data from models and observations must be imperfect. Moreover, the resolution of our models is still too low and the history of the MW is too rich in events to allow the detailed modelling of velocity distributions of stars in the ~ 100 pc sphere covered by the GCS.

Considering all the shortcomings listed above and despite their failure to model properly the thick disc component of the MW, models such as Y1 ζ - or E2 provide very good fits to the data. We thus emphasise the conclusions already drawn in ABS16: (i) combined disc heating by GMCs and non-axisymmetries in the disc is very likely responsible for the overall shape of the AVR in the Snhd; (ii) the models clearly favour a baryonic disc mass $5 \times 10^{10} M_\odot$ and the cosmologically inferred dark-halo mass parameters.

4.5 Power-law indices of AVRs

We now examine the values of β that are obtained by fitting AVRs to equation (1) with $x = \tau$. Small values indicate larger differences in the velocity dispersions of young stars and stars of intermediate ages than between the stars of intermediate and old ages. Over the last half century values of β between 0.25 and 0.6 have been found from various samples of the Snhd stars (see e.g. Table 1 in Hänninen & Flynn 2002). Data from the *Hipparcos* and *GCS* surveys indicate that $\beta_z \sim 0.45$ –0.53 is larger than $\beta_R \sim 0.31$ –0.39 (Holmberg et al. 2009, AB09). In view of the influence of age biases and errors on empirical AVRs, the spread of values is not surprising.

Fig. 3 displays for the endpoints of several models the AVRs for the stars in the annulus $R = 8 \pm 0.5$ kpc and at $|z| < 0.1$ kpc. There are two panels for each model because the AVRs using true ages are plotted in the left panel, while the right panel shows the AVRs yielded by ages degraded as described in Section 3. The model data are displayed as black (σ_R), green (σ_ϕ) and grey points (σ_z). Fits of equation (1) to these data are over-plotted in red (σ_R), pink (σ_ϕ) and blue (σ_z). The corresponding heating exponents β_i are displayed in the lower right corner of each panel.

In this section we analyse also the azimuthal velocity dispersion σ_ϕ . For the most part we exclude σ_ϕ from our analysis because (i) the skewness of the v_ϕ distribution (see e.g. Binney & Tremaine 2008, Section 4.4.3) renders σ_ϕ hard to interpret, and (ii) it is tightly coupled by dynamics to

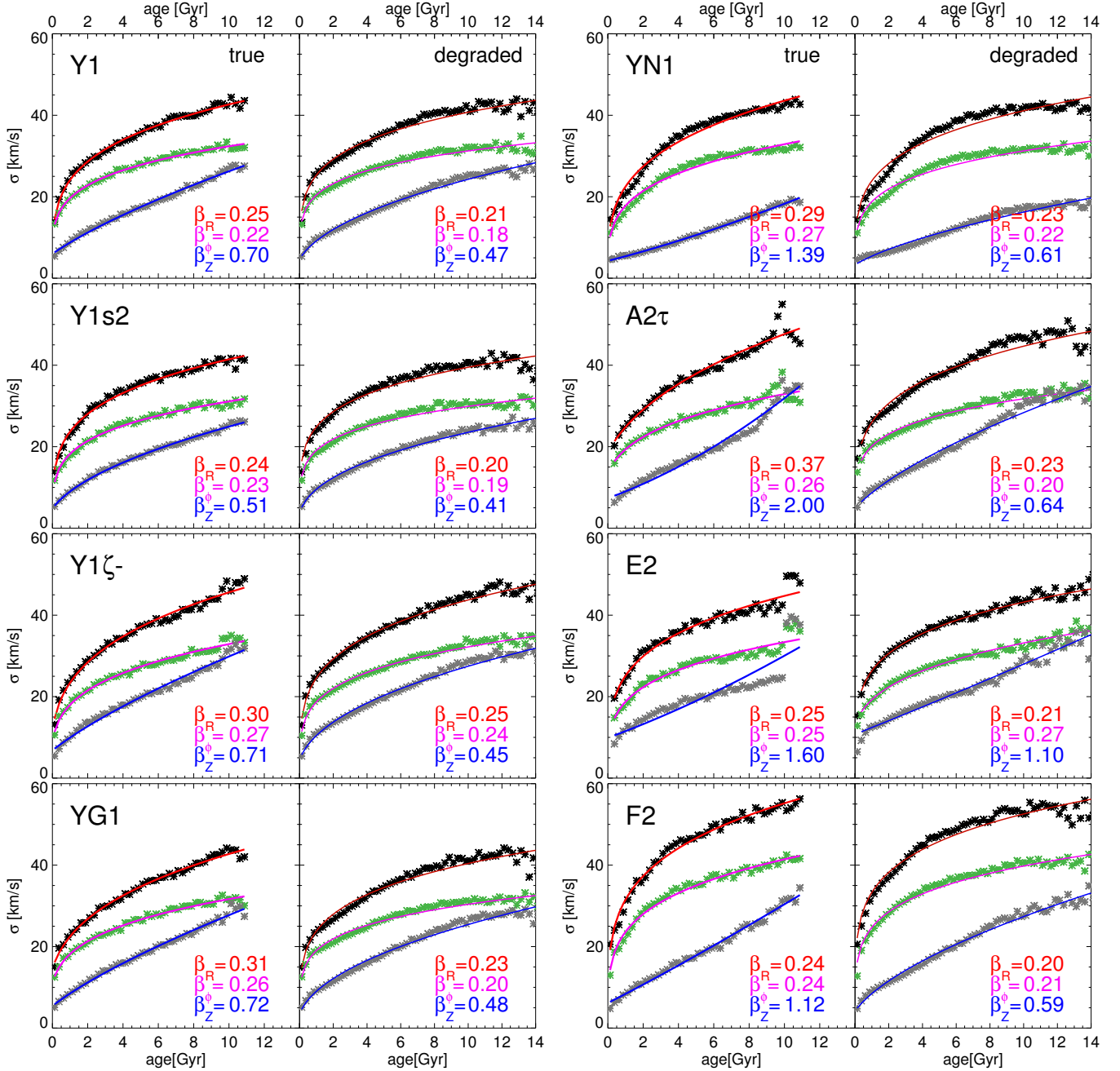


Figure 3. Asterisks display AVRs for all three directions R (black), ϕ (green) and z (grey) at $R = 8 \pm 0.5$ kpc, $z = 0 \pm 0.1$ kpc and $t = 10$ Gyr for several models. Overplotted are fits of the form $\sigma_{10} [(\tau + \tau_1) / (10 \text{ Gyr} + \tau_1)]^\beta$ in red (R), pink (ϕ) and blue (z). For each model we show AVRs for both true (left) and degraded ages (right) according to Section 3. The fitted values for β are displayed in corresponding colours in the bottom right of each panel.

σ_R , so it does not provide independent diagnostic information. Models yield $\sigma_\phi/\sigma_R \sim 0.75 - 0.8$ at $R = 8$ kpc and $|z| < 0.1$ kpc, whereas observations of the Sphd yield smaller values, $\sigma_\phi/\sigma_R \sim 0.65$. Azimuthal variation can lower the observed value by $0.05 - 0.1$ for some azimuths, which only brings some models to marginal agreement with the Sphd data. We interpret this discrepancy as a result of selection biases. Any selection bias exerting a preference in metallicity has a tendency to lower σ_ϕ/σ_R . This kind of bias has been

discussed in Schönrich et al. (2010). We leave the detailed investigation of this phenomenon to a future paper.

Fig. 3 shows that the data can usually be nicely fitted by equation (1). Notable exceptions are, as expected, the AVRs from true ages for Models A2 τ , which features a declining input dispersion (eq. 3), and E2 which has a thick disc in its IC. Consequently, in both models all three dispersions, but especially σ_z , have sharp upturns at the oldest ages.

The true AVRs of the other models have heating expo-

nents in the range $\beta_R = 0.24 - 0.31$, $\beta_\phi = 0.22 - 0.27$ and $\beta_z = 0.51 - 1.39$. The in-plane coefficients show hardly any scatter as heating is dominated by non-axisymmetries, which are similar in all models. $\beta_\phi < \beta_R$ holds in all models. AB09 favoured $\beta_\phi > \beta_R$ for the Snhd data but could not exclude $\beta_\phi < \beta_R$.

The scatter in β_z is significant. The lowest value is found for Model Y1s2. In Section 5, we will show that this arises from this model's flatter SF history, which implies a slower decline with time in the total mass of GMCs. The heating exponent of Model F2 with a low-density dark halo, $\beta_z = 1.12$, is unusually high. In this model an extended bar forms very early on, which leads to a high inner cutoff radius R_{cut} for the insertion of star and GMC particles. As a consequence, the GMC total mass at $R = 8$ kpc is at early times higher than in other models and the decline towards late time is thus stronger. Moreover in this model vertical heating by the extended $m = 2$ non-axisymmetries is non-negligible. Model YN1, which lacks GMCs, has the highest value of β_z because it is heated vertically by large-scale non-axisymmetries rather than GMCs. We find that this heating mechanism generally produces higher values of β_z .

Degradation of the ages flattens AVR's because the observations are dominated by stars with true ages $\tau \sim 2$ Gyr, which are scattered into all age bins by observational errors. In our models this flattening is greatest at the oldest ages, but this is to some degree artificial: we only have stars with true ages up to 11 Gyr (IC stars are assigned ages 10–11 Gyr) and ages can be scattered up 14 Gyr, so we may be overestimating the total flattening. However, detailed aspects of the selection function, such as the exclusion of young blue stars, and of how ages were determined, for example the restriction $\tau \leq 14$ Gyr on the ages of GCS stars, and of the error distribution, which is significantly non-Gaussian, were not modelled here, and could significantly change the parameters of observed AVR's.

Flattening of the AVR leads to a noticeable reduction in β , most prominently for β_z . For models with the Y IC that have GMCs (left column in Fig. 3), the reduced ranges are $\beta_R = 0.20 - 0.25$, $\beta_\phi = 0.18 - 0.24$ and $\beta_z = 0.41 - 0.48$. The extremely high values of β_z for Models YN1 and F2 are reduced to 0.61 and 0.59, respectively. For Models A2 τ and E2 that have hot old components in their ICs, degradation of the ages smears out the AVR's at old ages and leads to better fits by equation (1). Nonetheless, even after flattening $\beta_z = 0.64$ for Model A2 τ is high, and for Model E2 $\sigma_z(\tau)$ is fitted best by an almost straight line: $\beta_z = 1.1$.

Despite the agreement between degraded AVR's of Model E2 and Snhd data noted in Section 4.4, the model's values for β conflict with the data. Most prominently $\beta_z = 1.1$ is significantly higher than $\beta_z \sim 0.45 - 0.53$ found in the Snhd (Holmberg et al. 2009, AB09). By contrast, the Y models with GMCs yield values of β that are consistent with the data, even though these models lack a thick disc. As far as in-plane heating is concerned, β_R is lower in all models than inferred for the Snhd, where $\beta_R \sim 0.31 - 0.39$.

4.5.1 Radial variation of AVR's

Fig. 4 explores how AVR's are predicted to vary with radius in four models selected because they show different behaviours. Each panel in the first three rows shows eight sets

of dots, with each such set showing the AVR given by the true ages of stars found at a radius in the range (3,10) kpc: the larger the value of R , the lower the curve lies in the panel. Fits of equation (1) to each curve are over-plotted in colours that move through the spectrum from blue to orange as R increases. The values of β for these curves are plotted in the bottom row of panels, with black stars from $\sigma_R(\tau)$, green crosses from $\sigma_\phi(\tau)$ and red diamonds from $\sigma_z(\tau)$. For $R = 8$ kpc, we also show how using degraded rather than true ages changes β . The in-plane values for β are small but increase outwards. The vertical value of β is much larger and its radial variation is rather various.

Model Y1s2, shown in the second column of Fig. 4, is a typical model with GMCs and a weak bar at $R \lesssim 3$ kpc. Its AVR's are well fitted by equation (1) at all radii. The in-plane velocity dispersions for the youngest age bins increase mildly with decreasing radius, whereas young stars at all radii are equally cold vertically. Outside $R = 3$ kpc the in-plane values of β are almost independent of R , while β_z declines gently with increasing R . The shape of the AVR thus depends only mildly on radius.

As far as vertical heating in the outer disc is concerned, we note that stars which have migrated outwards have higher velocity dispersions than stars which have not migrated, as qualitatively predicted by Schönrich & Binney (2009, 2012). Non-migrating stars still show a clear increase of σ_z with age due to local GMC heating.

In contrast to Model Y1s2, Model Y1, shown in the leftmost column of Fig. 4, has a strong buckled bar of length $L_{\text{bar}} \sim 4$ kpc. Its vertical AVR's behave like those of Model Y1s2, but the bar significantly changes the in-plane AVR's. In particular, at the youngest ages the velocity dispersion increases rapidly with decreasing R because the bar's gravitational field deforms orbits from circular ones. On account of our use of an adaptive cutoff, no young stars were inserted into the bar region after $t \sim 7$ Gyr, and the youngest stars at $R = 3 - 4$ kpc are there because they have been captured onto bar orbits. The influence of the bar moves the disc region, in which the shape of the AVR is almost independent of R , outwards to $R \gtrsim 6$ kpc.

Model Y4fz-, shown in the third column of Fig. 4, has a thicker disc due to the formation of extended ($R \sim 10$ kpc) $m = 3$ and $m = 2$ non-axisymmetric structures at $t \sim 6$ Gyr. At young ages the in-plane AVR's of this model are affected even more strongly by global non-axisymmetries. A feature in the AVR caused by the event at $t \sim 6$ Gyr is visible at $\tau \sim 4$ Gyr at all displayed radii for the in-plane dispersions. On account of these features, equation (1) provides an unusually poor fit to the data.

Model E2 in the rightmost column of Fig. 4 features a thick disc in its IC and has a long bar ($L_{\text{bar}} \sim 6$ kpc) at $t = 10$ Gyr. In the bottom right panel we see the impact on the vertical AVR of the buckling of this bar, which created an X-shaped region out to $R \sim 4$ kpc. In contrast to the other models, at the end of the third row of Fig. 4 we see that at the youngest ages σ_z increases significantly with decreasing radius. The thick disc from the IC shows up in the AVR as an abrupt increase in $\sigma(\tau)$ at $\tau = 10$ Gyr: this increase is particularly evident for $\sigma_z(\tau)$ at the largest radii. On account of this step in σ , equation (1) provides an unusually poor fit to the data.

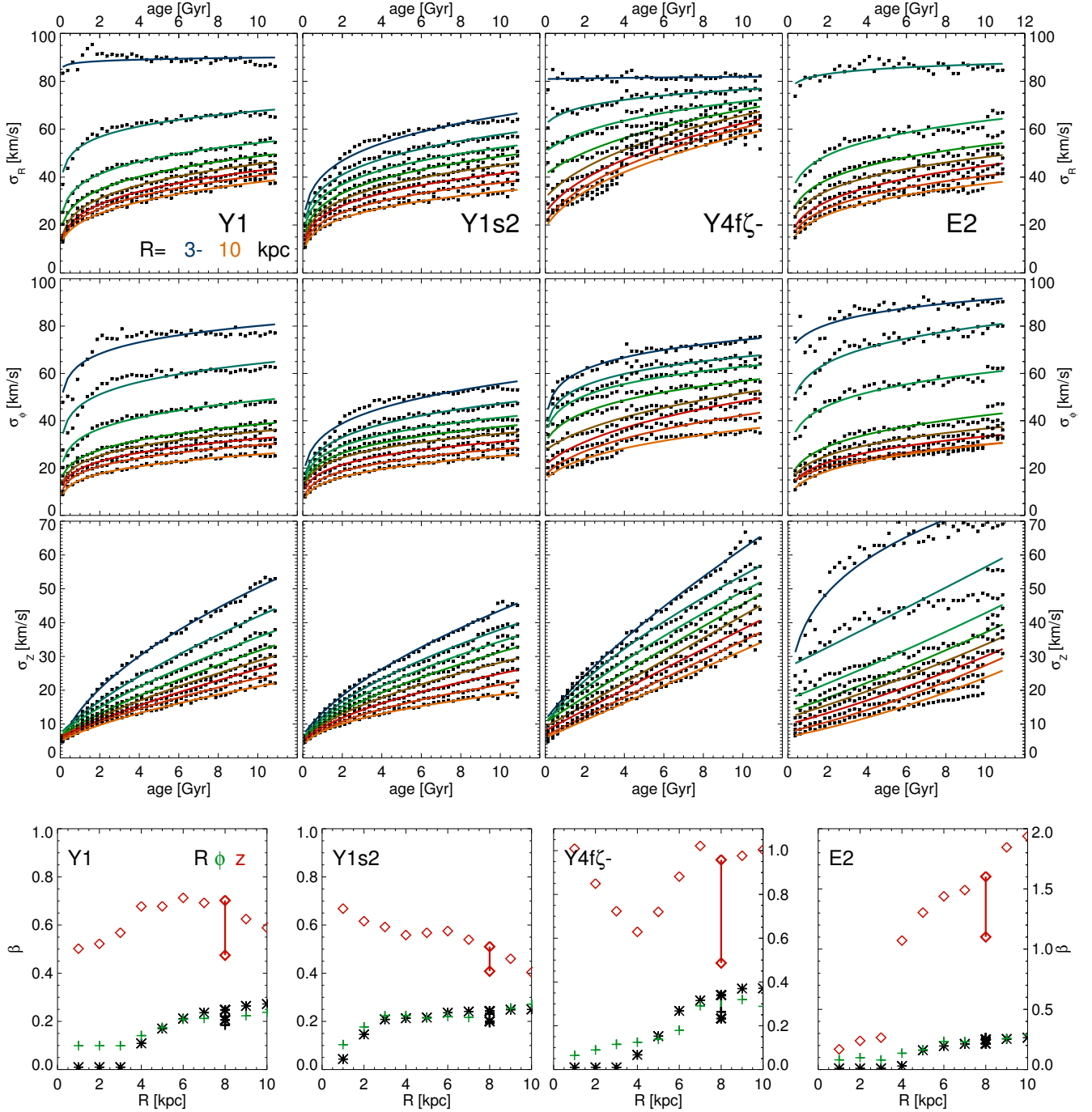


Figure 4. The AVRs $\sigma_i(\tau)$ at $t = 10$ Gyr for true ages and at radii $R = 3, 4, \dots, 10$ kpc and $z = 0 \pm 0.1$ kpc in directions R (top row), ϕ (second row) and z (third row). Overplotted are fits of the form $\sigma_{10} [(\tau + \tau_1) / (10 \text{ Gyr} + \tau_1)]^\beta$. The bottom row plots the heating parameters β obtained from fits in all three directions as a function of R . Vertical lines at $R = 8$ kpc show how β changes when ages are degraded by errors and observational bias.

5 HEATING HISTORIES OF COEVAL POPULATIONS

We now extract the intrinsic heating histories of coeval populations that make up the AVR at a certain time and place. To relate this to the Snhd AVR we select stars at $t = 10$ Gyr and $R = 8 \pm 1$ kpc, then divide them into age

groups $\Delta\tau = 0.05$ Gyr wide and track them from their time of birth t_b until $t_f = 10$ Gyr. In this way, we calculate velocity dispersions of each group at every output and thus assemble a heating history for each coeval cohort.

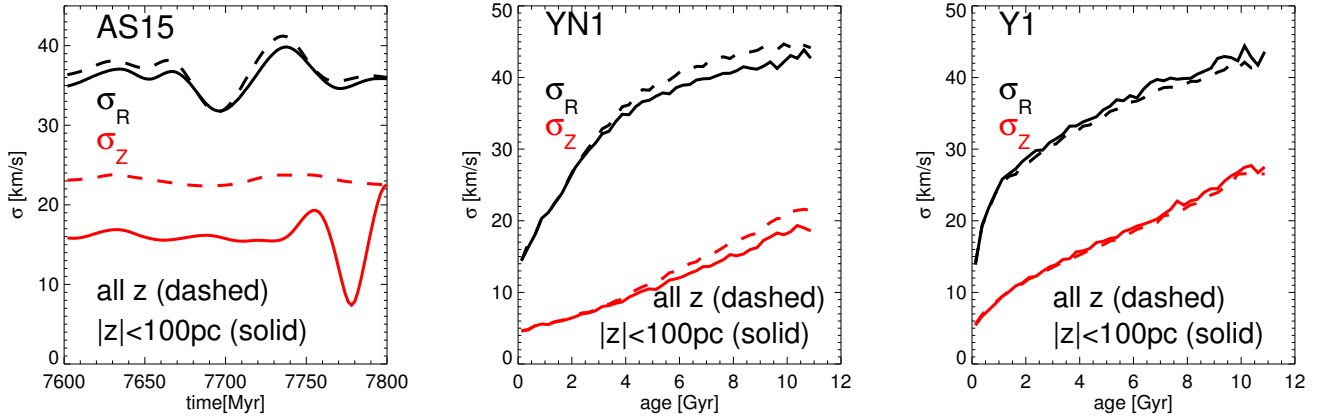


Figure 5. Effect of spatial selection on velocity dispersions. *Left:* IC stars selected from the model of Aumer & Schönrich (2015) at $R = 8 \pm 1$ kpc and either $|z| < 0.1$ kpc (solid) or no restriction in z (dashed). We track these stars back in time and at each output calculate their velocity dispersions in radial (black) and vertical (red) directions. *Middle and right:* AVRs for true ages in models YN1 and Y1 in radial (black) and vertical (red) directions at $R = 8 \pm 0.5$ kpc. Solid lines are for stars at $|z| < 100$ pc and dashed lines are for all stars irrespective of z position.

5.1 Selection effects

Selecting stars from a limited spatial volume introduces phase correlations between stars which influence the values of velocity dispersions at the time of selection and before. For example, when stars are selected close to $z = 0$, they are all close to their maxima in $|v_z|$. Tracking them back in time, their vertical velocity dispersions thus have to be lower than at the time of selection. Unfortunately, the output frequency (snapshots at 50 Myr intervals) of the simulations discussed here is too low to demonstrate this effect clearly. So we illustrate the effect with a simulation from Aumer & Schönrich (2015) that is similar to YN1, but has a lower number of particles in the dark halo and a gas component. For this model we have snapshots at 1 Myr intervals.

From the Aumer & Schönrich (2015) model, we select IC (i.e. old) stars at $t = 7.8$ Gyr, $R = 8 \pm 1$ kpc and $|z| \leq 0.1$ kpc and track them back for 200 Myr. From the full red curve in the left panel of Fig. 5 we see that the vertical dispersion σ_z of these stars undergoes an oscillation with a period of ~ 50 Myr, which is only distinctly visible in the last period before selection. This period is the average vertical oscillation period of these stars and phase mixing diminishes the correlation with increasing time before selection near the midplane. Repeating the experiment with only radial and no vertical selection bounds imposed (dashed lines), the effect disappears and σ_z is almost constant.

Selecting stars in radius also imposes phase correlations. E.g., if a disc galaxy has an old, centrally concentrated disc population, old stars at $R = 8 \pm 1$ kpc will be dominated by stars on eccentric orbits, which have their guiding centre radii further in and will therefore be selected close to apocentre. This effect alone however cannot explain the tracked-back radial velocity dispersions (black lines) in the left panel of Fig. 5. As predicted, they show oscillations of ~ 10 per cent, but these oscillations appear to have more than one underlying period of ~ 100 Myr, irrespective of the z selection of the stars.

On account of these selection effects, we decided to track

heating histories in the following way: we select stars at $t = 10$ Gyr and $R = 8 \pm 1$ kpc, but do not select in z . To prevent oscillations impacting our results, we exclude the last 500 Myr from our analysis – that is, we fit $\sigma(t - t_b)$ between $t = t_b$ and $t = 9.5$ Gyr. We fit the curves $\sigma(t - t_b)$ to equation (1) with $x = t - t_b$. To avoid confusion, we mark parameters derived from heating histories with a tilde, i.e. we write $\tilde{\beta}$, $\tilde{\sigma}_{10}$ etc.

We compare these parameters to the parameters from AVRs determined at $t = 10$ Gyr and $R = 8 \pm 0.5$ kpc, without a z selection. Fig. 2 has already shown that AVRs depend little on vertical selection range. We reiterate this point in the middle and right panels of Fig. 5, where we show for models YN1 and Y1 radial and vertical AVRs for $|z| < 100$ pc (solid lines) and for all stars irrespective of z position (dashed lines). Standard models with a single-exponential vertical profile, such as Y1, show no significant difference between solid and dashed lines. Other models, like YN1, show only mildly higher dispersions for old stars when considering stars at all z . The AVR parameters thus differ little between the two ways of selection of stars for all models. The most notable difference is that β tends to be mildly higher when the selection of stars is unrestricted in z .

5.2 How heating histories shape AVRs

In Fig. 6 we plot the heating histories of several coeval populations from Model Y1 that end up at $R = 8$ kpc. The extracted data are shown as black (σ_R) and blue asterisks (σ_z), whereas the fits are overplotted as red (σ_R) and green lines (σ_z). Equation (1) again provides very good fits to the data, regardless of the population's time of birth. In each panel, we give the fitted values of $\tilde{\beta}$. We note that $\tilde{\beta}_R$ fluctuates between 0.18 and 0.26, whereas $\tilde{\beta}_z$ slowly increases with t_b from 0.21 to 0.41. Comparing these values to the ones found for Y1 in the AVRs of Fig. 3 we note that $\tilde{\beta}_R$ shows similar values as β_R , whereas $\tilde{\beta}_z$ is always smaller than β_z , irrespective of the choice of true or degraded ages.

In Fig. 7 we plot the heating curves shown in Fig. 6 in

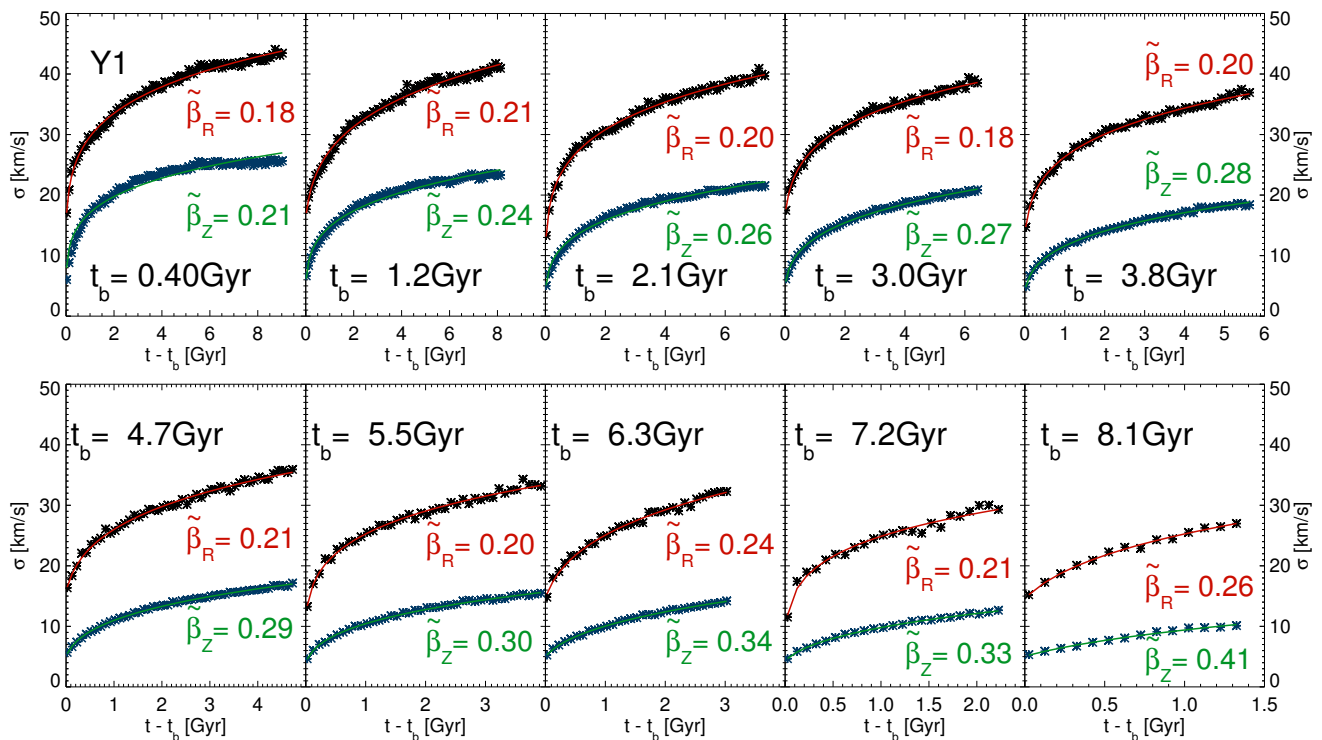


Figure 6. The evolution with time $t - t_b$ of the vertical and radial velocity dispersions in Model Y1 for stars with different birth times t_b . The stars are selected to be at $R = 8 \pm 0.5$ kpc at $t = 10$ Gyr. Displayed are curves for ten different age cohorts. Overplotted are fits of $\sigma(t) = \bar{\sigma}_{10} [(t + \bar{\tau}_1) / (10 \text{ Gyr} + \bar{\tau}_1)]^{\bar{\beta}}$. The fitted values of $\bar{\beta}$ are displayed in red ($\bar{\beta}_R$) and green ($\bar{\beta}_z$).

a different way by showing several curves corresponding to populations of different birth times (encoded in colour) on top of each other. The left panel shows σ_z , whereas the right panel shows σ_R . In dotted lines of corresponding colours we overplot fits of equation (1) to the curves and extend these fitted curves to 10 Gyr to show how the populations would evolve if they continued to heat according to equation (1).

The curves for $\sigma_z(t - t_b)$ clearly demonstrate that the heating histories vary with time of birth. This has already been indicated by the increase in $\bar{\beta}_z$ with t_b shown in Fig. 6. But this alone would not cause the large differences. The value $\sigma_z(t - t_b)$ would reach after 10 Gyr, as encoded by the parameter $\bar{\sigma}_{10}$ from equation (1), also decreases strongly with increasing t_b . The reason is easily understood. As ABS16 showed, vertical heating in these models is dominated by GMCs, and for standard parameters (e.g., Model Y1), the fraction of disc mass residing in GMCs is ~ 30 per cent at early times and steadily decreasing. At $t = 10$ Gyr, < 5 per cent of the mass remains in GMCs, consistent with observations of the MW. Consequently, the global efficiency of GMC heating decreases and the intrinsic heating history evolves towards smaller $\bar{\sigma}_{10}$ with mildly varying $\bar{\beta}_z$.

The AVR for any model is given by the lower envelope of the solid heating curves in its analogue of Fig. 7. A glance at the left panel of Fig. 7 shows that the exponent β_z required to fit this envelope must be higher than the values of $\bar{\beta}_z$ associated with any individual heating history. Consequently the AVR $\sigma_z(\tau)$ is shaped by the *evolution* of the heating history and there is no universal heating history valid for

all stellar populations as was assumed by AB09 and many other authors.

The situation regarding the heating histories $\sigma_R(t - t_b)$ in the right panel of Figure 7 is markedly different. The tendency to stronger heating of populations born at the earliest times remains visible, but curves for $t_b \gtrsim 1$ Gyr lie almost on top of one another, indicating that after ~ 1 Gyr the heating history hardly changes. From this fact it follows that the difference between the in-plane AVRs and heating histories is mild. In the models of ABS16, after the first Gyr, when the high mass fraction of GMCs can lead to powerful radial heating by clumps of GMC particles, in-plane heating is dominated by spiral structure and the bar. These non-axisymmetries are constantly excited by the addition of cold stellar populations: in the region of the disc, in which its rotation curve is not dominated by the contribution from the dark halo, and which lies outside of the bar, Toomre's Q (Toomre 1964) parameter settles to a characteristic value $Q \sim 1.5$. This requires the velocity dispersion σ_R of the entire disc to increase (see Fig. 9 in ABS16). In consequence, young stars which are born cold need to be heated efficiently at all times, which results in almost constant in-plane heating histories.

5.3 Heating histories in standard models

In Fig. 8 we plot for the heating histories of several models the parameters $\bar{\beta}$ (first and fourth rows), $\bar{\sigma}_{10}$ (second and fifth rows) and $\bar{\sigma}(t = 0)$ (third and sixth rows) as functions of t_b for both vertical (red asterisks) and radial (black as-

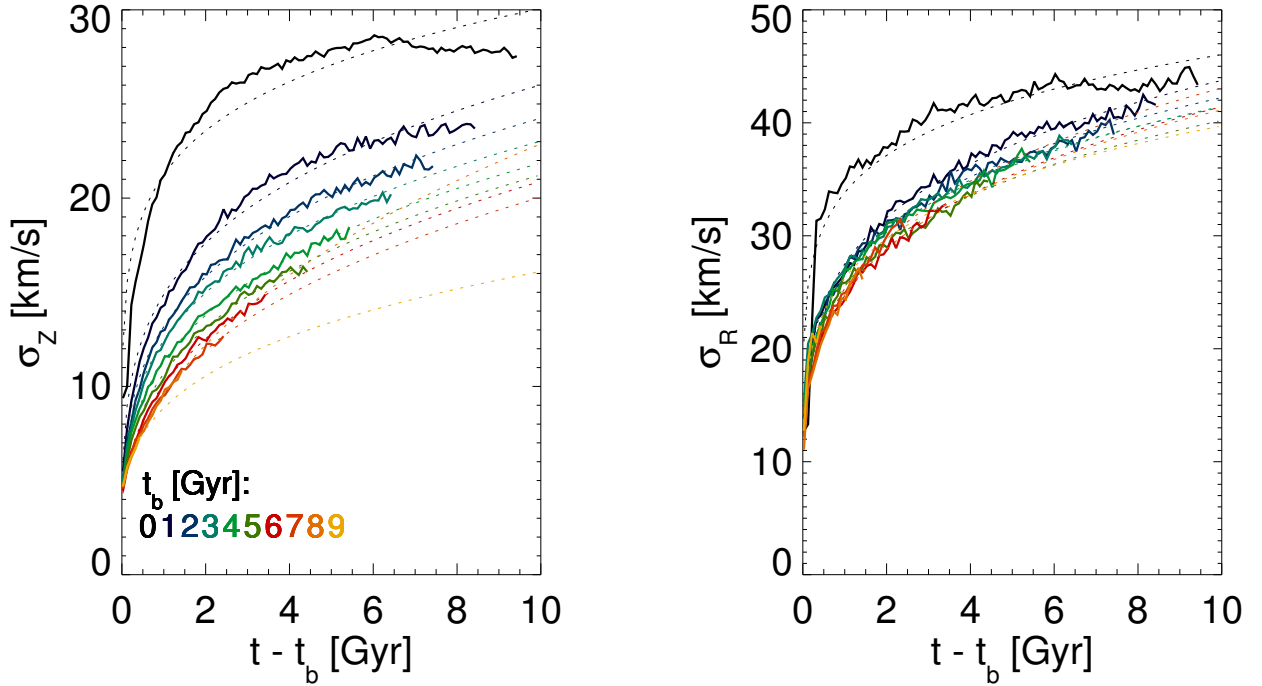


Figure 7. The evolution with time $t - t_b$ of the vertical (left panel) and radial (right panel) velocity dispersions in Model Y1 at $R = 8$ kpc for stars with different birth times ranging from IC stars (black) to stars born after $t = 8$ Gyr (orange). Overplotted as dotted lines of the same colour are fits of equation (1) to the curves, which are extended to 10 Gyr.

terisks) heating histories extracted from analogues of Fig. 6. $\bar{\sigma}(t=0)$ is determined by the parameter $\bar{\tau}_1$ of equation (1), which is required to be $\bar{\tau}_1 \geq 0$, so that $\bar{\sigma}(t=0) \geq 0$. To display the differences between heating histories and AVRs at $R = 8$ kpc we show in each panel by horizontal lines the values of the vertical (in gold) and radial heating parameters (blue) extracted from the corresponding AVRs at $R = 8 \pm 0.5$ kpc without z restriction: full lines for the values obtained using true ages and dashed lines for values obtained using degraded ages. When $\beta > 1$, so the line lies beyond the top of the panel, the value is printed in the panel. Note that due to the difference in z selection the AVR parameters differ mildly from those found in Section 4.5.

The models analysed in the first three rows of Fig. 8 are variations on the standard Model Y1. They all have single-exponential vertical profiles and bars of varying extent and strength. We note that the β values change little with the time of birth of the populations. The analytic treatment of scattering by GMCs in Lacey (1984) yielded $\beta = 0.25$, and Hämmen & Flynn (2002) found $\beta_R = 0.21$ and $\beta_z = 0.26$ from numerical simulations of heating by GMCs. De Simone et al. (2004) found that β_R caused by transient spirals can lead to a wide range of $\beta_R \sim 0.2$ – 0.7 depending on properties of the spirals. These models did not strictly distinguish between AVRs and heating histories. As their heating indices were derived from the evolution of velocity dispersions, we assign tildes. The values for the heating index $\tilde{\beta}$ from our simulations agree well with these results, as $0.15 < \tilde{\beta}_R < 0.25$ and $0.20 < \tilde{\beta}_z < 0.33$.

For the oldest populations in all models, we find $\tilde{\beta}_z \approx$

$\tilde{\beta}_R$, whereas for the younger populations $\tilde{\beta}_z > \tilde{\beta}_R$. Lower $\tilde{\beta}$ indicate a stronger initial increase in velocity dispersions and an earlier saturation of heating. For the younger populations the explanation is likely that after in-plane heating, driven by spiral structure, has essentially saturated, GMCs continue to increase σ_z by deflecting stars from eccentric near-planar orbits to less eccentric and more highly inclined orbits. For the populations born early on, GMCs have a high mass fraction in the disc and can efficiently heat the disc both vertically and radially. Consequently, the high GMC mass fraction in Y1 ζ - leads to $\tilde{\beta}_z \approx \tilde{\beta}_R$ for a wider range of t_b , whereas in Y2Mb- the lower disc mass leads to less efficient spiral heating and thus a longer period of GMCs heating the disc both radially and vertically.

For Models Y1, Y1s2 and Y1 ζ -, which grow inside-out, there is a mild increase in $\tilde{\beta}_z$ visible with increasing time of birth whereas for Models Y2 and Y2Mb-, which have a constant input scalelength $h_R = 2.5$ kpc, there is a very mild decrease in $\tilde{\beta}_R$. It is striking that $\tilde{\beta}_z$ for all these five models and for the vast majority of times of birth are lower than the values of β_z inferred from the AVR. In contrast, the values of $\tilde{\beta}_R$ scatter around the values β_R from AVRs for all models with the exception of Model Y1 ζ -, which has a higher fraction of its mass in GMCs and for which the value of β_R is consistently higher than $\tilde{\beta}_R$ by ~ 0.05 – 0.15 depending on which ages are used.

For all models, the values of $\bar{\sigma}_{10}$ scatter around the values σ_{10} found from AVRs. Naively one might think the heating history of the oldest cohort to agree with the AVR at the oldest ages. However, in several panels of Fig. 8 $\bar{\sigma}_{10,R}$

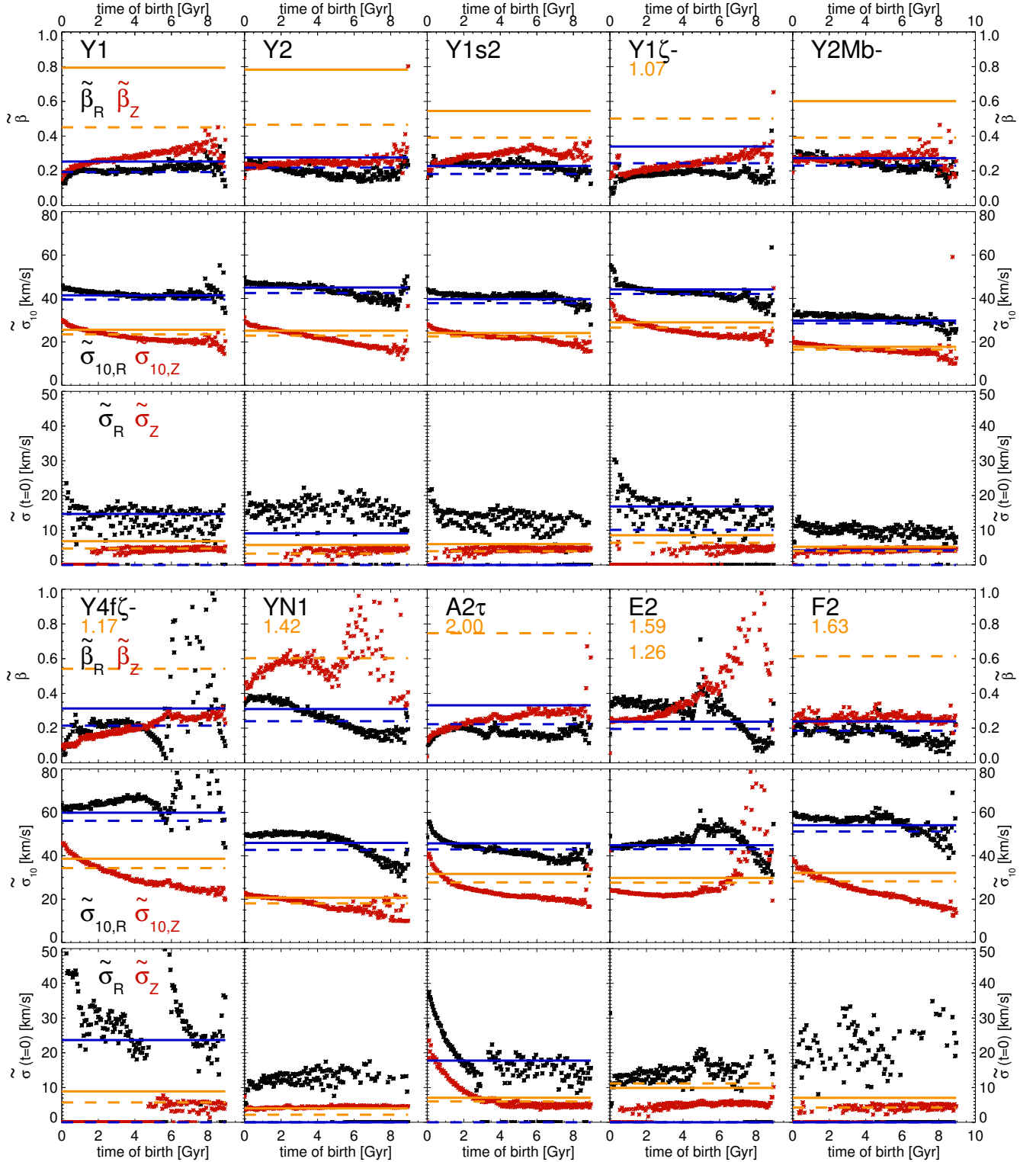


Figure 8. Parameters from fits of equation (1) to heating histories $\sigma(t - t_b)$ as functions of time of birth t_b of stars found at $R = 8$ kpc after $t = 10$ Gyr. Three panels are shown for each model: β (upper); σ_{10} (middle); $\sigma(t = 0)$ (lower) (the latter is determined by the parameter $\tau_1 > 0$). Parameter values β , σ_{10} and $\sigma(t = 0)$ found by fitting equation (1) to the AVRs are indicated by horizontal lines: solid lines when true ages are used and dashed lines when degraded ages are used. When the AVR yields $\beta > 1$, the value is printed on the panel's left side.

for $t_b = 0$ is larger than expected by this reckoning. This discordance arises, as Fig. 7 illustrates, because equation (1) often provides a poor fit to the heating history of the very oldest stars, and tends to over-estimate their σ at large values of $t - t_b$: in reality for these stars σ saturates earlier than equation (1) predicts.

Fits to $\sigma_z(t - t_b)$ yield values of $\tilde{\sigma}_{10}$ that decrease with t_b , just as Fig. 7 indicates for Model Y1. This decline is weaker in Y1s2 as its SFR and thus its total GMC mass decline on a longer timescale. Consequently, the AVR of this model yields a smaller value of β_z than the AVRs of other models. As far as radial heating histories are concerned, Y1 ζ - and to a lesser degree Y2 show a more significant decline with increasing t_b in $\tilde{\sigma}_{R,10}$ than Y1, which explains why AVR values of β_R are mildly larger than the $\tilde{\beta}_R$ values in these models.

$\tilde{\sigma}_R(t = 0)$ for all models mostly scatters between 10 and 15 km s^{-1} , whereas $\tilde{\sigma}_z(t = 0)$ mostly lies between 0 and 6 km s^{-1} . We note that the fitted $\tilde{\sigma}_i(t = 0)$ does not necessarily describe the actual σ_i of very young stars well, as a best fit can deviate from the fitted data. Still, this finding reiterates that the radial dispersions of stars increase almost instantaneously after insertion in reaction to the local non-axisymmetries, whereas the vertical dispersions increase more slowly. For some fits to radial AVRs, $\sigma_R(t = 0) = 0$ is favoured and thus differs from the typical $\tilde{\sigma}_R(t = 0)$ values, whereas for others the values from AVRs and heating histories are similar. We note that for low t_b in Model Y1 ζ -, we find $\tilde{\sigma}_R(t = 0) > 20 \text{ km s}^{-1}$ to be higher than average, indicating that a high mass fraction of GMCs can significantly influence the radial heating.

5.4 Heating histories in non-standard models

In the fourth to sixth rows of Figure 8, we show models that do not finish with single-exponential vertical profiles. Model F2 has a lower density halo than Model Y2, but shows similar evolution of the heating exponents. It develops $m = 2$ non-axisymmetries that extend to $R \sim 10 \text{ kpc}$. These lead to smaller values $\tilde{\beta}_R \sim 0.1$ for stars born at late times. The long bar causes high values of $\tilde{\sigma}_{R,10} \sim 60 \text{ km s}^{-1}$. The values of $\tilde{\beta}$ and $\tilde{\sigma}_{10}$ scatter much less than do the values of $\tilde{\sigma}_R(t = 0)$, which cover the range $(0, 35) \text{ km s}^{-1}$, presumably because this parameter has the smallest influence on the fits. Although we have seen that the AVRs of Models F2 and Y2 yield significantly different values of β_z , the heating histories of these two models yield almost identical values of $\tilde{\beta}_z$. As was discussed in Section 4.5, Model F2 has a large bar early on and thus at early times the cutoff radius and the early surface density of GMCs at $R \sim 8 \text{ kpc}$ are both larger than in Y2. Consequently, the efficiency of GMC heating and thus $\tilde{\sigma}_{z,10}$ decline more strongly in F2, causing the fit to the AVR $\sigma_z(\tau)$ to yield a larger value of β_z . Direct vertical heating by the bar likely plays a role for this model as well, but disentangling the effects is beyond the scope of this paper.

Model A2 τ has a thicker disc in its ICs than Model Y2 and an input velocity dispersion σ_0 that decreases with time (eq. 3), so stars at early times are born significantly hotter than stars at late times, which are born as cold as those in Model Y2. The declining input dispersions are nicely visible in $\tilde{\sigma}(t = 0)$. As noted in ABS16, vertical dispersions for younger stars are lower than input dispersions σ_0 as all

stars are added at $z = 0$ and thus lose kinetic energy when moving away from the midplane. Interestingly, this decline in $\tilde{\sigma}(t = 0)$ is also reflected in a milder decline in $\tilde{\sigma}_{10}$ and smaller values of $\tilde{\beta}$ in both vertical and radial directions for stars born at early times. Small values of $\tilde{\beta}$ are likely connected to an earlier saturation of heating due to higher initial dispersions. At late times we find only mild differences between the heating indices of Model A2 τ and Model Y2.

Model E2 has an even thicker and more extended disc in its IC than Model A2 τ , and the same small, constant input dispersion σ_0 as Model Y2. At early times the thick, extended disc suppresses spiral and bar formation below the level seen in the thin, compact disc in the IC of Model Y2. Consequently, early on radial heating is less powerful in Model E2, with the consequence that the oldest populations heat more slowly in Model E2 and thus have large $\tilde{\beta}_R > 0.3$ as the saturation phase happens later (fourth panel in second row). A long ($L_{\text{bar}} \sim 6 \text{ kpc}$), strong and buckled bar forms in E2 after $t \sim 6 \text{ Gyr}$. This bar significantly influences the heating of the populations which end up at $R = 8 \text{ kpc}$. For populations born after bar formation, $\tilde{\beta}_R$ decreases strongly as a strong bar leads to a very fast increase in σ_R for young stars and thus a quicker saturation and a low value of $\tilde{\beta}_R$. At the same time $\tilde{\sigma}_{R,10}$ decreases, implying that populations born at late times are expected to attain lower velocity dispersions after 10 Gyr. This is likely driven by the lower $\tilde{\beta}_R$, as for these stars there is no information available at $t \gtrsim 4 \text{ Gyr}$ and thus the fits likely over-predict the saturation effect and thus under-predict the continuous in-plane heating. By adding an additional vertical heating mechanism for young stars, bar buckling increases $\tilde{\beta}_z$ and $\tilde{\sigma}_{z,10}$. Then the fits very likely over-predict the vertical dispersion these stars would attain after 10 Gyr.

A different situation is found in the leftmost panel of the second row for Model Y4f ζ -. In this model, which has a very compact feeding history, a fixed cutoff and a high GMC mass fraction, strong and extended $m = 3$ and $m = 2$ modes form around $t \sim 6 \text{ Gyr}$. These events lead to deviations from simple t^β AVRs and heating histories as was shown in ABS16 and in Fig. 4. Consequently the quality of fits to equation (1) to the heating histories is worse than for other models and there are strong variations in the radial heating parameters. $\tilde{\beta}_R$ varies between 0 and 1 for stars born after 5 Gyr, $\tilde{\sigma}_{R,10}$ is generally high at $\sim 60 \text{ km s}^{-1}$ due to the strong non-axisymmetries and also scatters strongly for stars born after 5 Gyr, and $\tilde{\sigma}_R(t = 0)$ varies systematically between 0 and 50 km s^{-1} . The decrease in $\tilde{\sigma}_{z,10}$ is strong, which explains the high AVR value of β_z .

Finally, we also show one model without GMCs, Model YN1 (second panel second row). Both the radial and vertical heating histories have $\tilde{\beta}$ values that differ sharply from those of the standard Y models as the heating curves are shaped only by non-axisymmetric structure. $\tilde{\beta}_R$ decreases from 0.4 to 0.15 with increasing time of birth, whereas $\tilde{\beta}_z$ is rather high, with typical values $\tilde{\beta}_z \sim 0.5 - 0.6$ and a tendency to increase with t_b . As in model E2, the decrease in $\tilde{\beta}_R$ and the decrease in $\tilde{\sigma}_{R,10}$ for stars born at late times, is likely caused by a strong bar, which in YN1 forms early and extends to $R \sim 5 \text{ kpc}$ by the end of the simulation. On account of the change in the main source of vertical heat, the value of $\tilde{\beta}_z$ for Model YN1 differs sharply from the values yielded by models with GMCs.

6 DISCUSSION

In a galactic disc the random velocities of stars are increased by the fluctuating non-axisymmetric component of the gravitational field. GMCs and spiral arms are both major contributors to this component.

In a naive picture of the heating of the solar neighbourhood, stars diffuse through velocity space from the velocities of circular orbits under the influence of fluctuations that constitute a stationary random process (Wielen 1977). If the diffusion coefficient that governed this process were independent of \mathbf{v} , the velocity dispersion of a coeval cohort of stars would grow as $(t - t_b)^{1/2}$. In reality the diffusion coefficient must be a declining function of $|\mathbf{v}|$ because a given potential fluctuation deflects fast stars through smaller angles than slow stars (typically $\delta\theta \propto v^{-2}$). Hence if the fluctuating component of the gravitational potential is statistically stationary, the exponent in the heating law $\sigma \propto (t - t_b)^\beta$ has to be less than 0.5: Lacey (1984) found $\beta = 0.25$ for analytical models of GMC heating. In the case of spiral structure the concept of a deflection angle is problematic, but stars with significant random velocities and therefore eccentric orbits, encounter a given spiral wave at a variety of orbital phases, so the time-averaged impact of the wave on a fast star is small. That is, very general physical principles leave no doubt that in the presence of statistically stationary fluctuations, the exponent in any heating law is $\lesssim 0.25$ (Binney 2013).

The black and red asterisks in the first and forth rows of Fig. 8 plot the exponents of heating histories rather than heating laws: they relate to the variation with $t - t_b$ of σ_R and σ_z for a group of stars that are now at $R = 8$ kpc rather than the variation with $t - t_b$ of a group of stars that formed at $R = 8$ kpc. Nonetheless, these two groups are similar, so it is interesting to see that in the standard Y models β_R tends to be constant at a value $\lesssim 0.25$, consistent with fairly stationary fluctuations. In some Y models β_z is constant at a similar or slightly larger value than β_R , and in other models it increases mildly with t_b , remaining at $\beta_z < 0.33$ for most coeval cohorts.

Spiral structure in discs with $Q > 1$ is maintained by swing amplification of noise (Toomre 1981; Fouvy et al. 2015). Since the efficacy of swing amplification rises very sharply as Toomre's

$$Q \equiv \frac{\sigma_R \kappa}{3.36 G \Sigma} \quad (7)$$

drops towards unity, a cold ($Q \lesssim 1$) stellar disc very rapidly heats until $Q \approx 1.5$. Hence from an early time, a galactic disc will have $Q \approx 1.5$. As the disc grows in mass, its surface density Σ rises and σ_R is driven upwards by spiral structure to keep Q approximately constant (see Fig. 9 in ABS16). Hence, the time dependence of σ_R for stars of all ages at a given radius is essentially prescribed by the history of gas accretion. As a consequence, young stars which are born cold are heated efficiently by constantly excited spiral structure at all times.

Spiral structure heats within the plane but does not directly increase σ_z . GMCs, by contrast heat the disc vertically as well as horizontally. Consequently, the ratio σ_z/σ_R is a measure of the relative importance of GMCs and spirals as heating agents.

Fig. 8 reveals that for almost all heating histories,

$\beta_z > \beta_R$. Consequently, the velocity ellipsoids of groups of coeval stars tend to become rounder over time. The natural explanation is that after in-plane heating, driven by spiral structure, has essentially saturated, GMCs continue to increase σ_z by deflecting stars from eccentric near-planar orbits to less eccentric and more highly inclined orbits.

One may plausibly argue, as ABS16 did, that the mass of gas in all GMCs is proportional to the SFR, since over the lifetime of a GMC a fraction ζ of the GMC's mass is converted to stars. Hence, the rate at which GMCs heat the disc declines with the SFR. Moreover, early on the disc has a low mass, so each GMC represents a larger fraction of the total disc mass. Since the rate at which an individual GMC heats scales with its mass relative to the mass of the Galaxy interior to its orbit, each GMC is individually a more effective heating agent early in the life of the disc. Hence over time the heating power of the ensemble of GMCs declines faster than the SFR. Large-scale spiral structure, by contrast, is associated with the self-gravity of the stellar disc, which grows steadily over time from a small initial value. That is, over the life of the disc, the impact of spiral structure has increased relative to that of GMCs.

If the fluctuations that heat the disc were a stationary random process and the disc were homogeneous, heating histories would be independent of t_b and have the same functional form as the AVR. We have seen that in the models heating histories do depend on t_b and are very different from the AVR. While the dependence of heating histories on t_b and their deviation from the AVR could be entirely attributed to the non-stationary nature of the fluctuations, a contributing factor is undoubtedly radial migration (Sellwood & Binney 2002; Schönrich & Binney 2009), which adds significant complexity to the problem by mixing stars that were born at small and large radii. Motivated by the desire to understand data for the Snhd, we have studied samples of stars that are currently at $R \approx R_0$. In a future paper we will consider groups of stars with a common birth radius.

7 CONCLUSIONS

In this paper, we have used a series of N -body simulations of growing disc galaxies (ABS16) to study (i) age-velocity-dispersion relations (AVRs) and (ii) the heating histories of the coeval cohorts of stars which make up the AVRs. As these models feature heavy GMC particles, secular heating is dominated by a combination of scattering of stars off GMCs and non-axisymmetric disc structures.

To be able to compare these simulations to observational data from the Snhd, we analysed the impact on the AVR of biases and errors in measured stellar ages. Stars with ages $\tau \sim 2$ Gyr are very much over-represented in the GCS data (Fig. 1). Scattering of such stars to young ages artificially boosts $\sigma(\tau)$ at the youngest ages, and depresses $\sigma(\tau)$ at the oldest ages. When a power law in τ is fitted to the measured AVR, lower values of the exponent β are recovered than would be in the absence of errors (see also Martig et al. 2014). The reduction in β is particularly marked in the case of σ_z (Fig. 3).

On account of spiral structure and bars, AVRs vary with azimuth as well as radius. Fig. 2 quantifies the extent of this azimuthal variation, which must be borne in mind

when considering whether a given model is consistent with data for the Snhd, which are measured at one particular azimuth. After taking azimuthal variation into account, we concluded that the GCS data are consistent with some models in ABS16 that have the expected disc mass ($5 \times 10^{10} M_{\odot}$) and the cosmologically motivated dark halo ($M = 10^{12} M_{\odot}$, $a = 30.2 \text{ kpc}$). Models with a significantly different disc mass or a less concentrated dark halo are inconsistent with data for the Snhd. The data also favour the model that starts with a massive, extended thick disc over models in which (a rather inadequate) thick disc forms as a consequence of powerful non-axisymmetries developing in the thin disc. As we do not self-consistently form appropriate thick discs and as we lack heating by dark matter substructure, which may contribute a minor part of the observed disc heating, we are not able to put tight constraints on our model parameters.

AVRs vary with radius. At locations currently inside the bar, the AVR's index β_R is generally very small, $\beta_R < 0.1$, as there are no circular orbits and young stars thus acquire high σ_R rapidly. At the end of the bar, β_R rises abruptly and is thereafter constant or slowly rising with R (Fig. 4). By contrast, β_z sometimes increases and sometimes decreases at the end of the bar. A buckling bar can lead to exceptionally high σ_z for young stars in the bar regions.

The heating history, $\sigma(t - t_b)$, of stars now in the Snhd that were born at time t_b can also be fitted by the power-law (1). We mark the corresponding parameters with a tilde. For standard models, the heating history depends on t_b more strongly in the case of σ_z than σ_R . Smaller values of the exponent $\tilde{\beta}$ are required to fit heating histories than AVRs. In fact, values of $\tilde{\beta}_R$ are consistent with the predictions of dynamics in the case that the fluctuating gravitational potential is a stationary random process. The values of $\tilde{\beta}_z$ are generally somewhat larger than is consistent with a stationary random process, but in agreement with numerical simulations of stationary GMC heating (Hänninen & Flynn 2002).

The AVR reflects the history of star and bar formation. The past SFR strongly affects the AVR for two reasons: the time integral of the SFR determines the mass of the disc, and thus the fraction of the gravitational force on a star that derives from the disc rather than the dark halo. At early times this fraction is small, so spiral arm formation is already suppressed by a low value of σ_R . As the mass of the disc increases, spiral structure increases σ_R to keep Toomre's Q nearly constant. If the SFR is rapidly declining, the rate at which σ_R increases will decline rapidly, and a relatively small value of $\tilde{\beta}_R$ will be required to fit the heating histories of the oldest stellar groups.

In contrast, the vertical heating is dominated by GMCs. By analysing histories of stars that end up at $R = 8 \text{ kpc}$, we showed that the heating histories of older stars reach higher σ_z after 10 Gyr ($\tilde{\sigma}_{10}$) than those of younger stars. The corresponding $\tilde{\beta}_z$ values only vary mildly with t_b . This decline in heating efficiency is connected to the declining influence of GMCs, the total mass of which declines due to a declining SFR and the mass fraction of which declines due to a growing stellar disc. When coeval cohorts, whose $\tilde{\sigma}_{10}$ values decline with t_b , are superposed to form an AVR, a value of β_z in excess of 0.5 is needed to fit the curve.

By combining all these results we have been able to clarify long standing discrepancies between the observed AVR

and theoretical predictions for combined spiral and GMC heating. Some of our models correctly reproduce the general shape of both $\sigma_R(\tau)$ and $\sigma_z(\tau)$ as observed in the Snhd and thus also the ratio of the two components. The key ingredient is that each coeval cohort of stars that contributes to the AVR has undergone a different heating history and the AVR is not produced by a single stationary heating law. We conclude that combined GMC and spiral/bar heating has likely shaped the MW thin disc AVR.

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