

DIRECT WELFARE ANALYSIS OF RELATIVE PRICE REGULATION*

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Abstract

The paper synthesizes and develops the welfare analysis of regulating relative prices, for example price differences, of which banning price discrimination is a special case. Welfare results are derived directly by convexity arguments using functions of welfare levels. The method is also used to obtain results about effects on consumer surplus.

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1 INTRODUCTION

How do constraints on the *relative* prices charged by a profit-maximizing monopolist affect social welfare and consumer surplus? The literature on price discrimination addresses this question by comparing laissez-faire with the case where no price differences are allowed – see, for example, Varian [1985], Aguirre, Cowan and Vickers [2010] (henceforth ‘ACV’), and the subsequent contributions by Cowan [2012, 2016]. The present paper introduces a method for comparing outcomes by defining market variables directly as functions of welfare or consumer surplus. The method not only yields results from the literature on monopoly price discrimination; it generalises them and adds some new findings. Moreover, the method can be applied to forms of relative price regulation other than banning price differences altogether.

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Whether it is good or bad to allow a monopolist to charge different prices to different groups of consumers for products with the same unit cost is an old question. Discrimination is bad for welfare unless total output increases because, unlike with uniform pricing, output is suboptimally allocated across consumer groups.¹ Pigou [1920] showed a century ago that with linear demands and all markets served, total output was the same with and without discrimination. A number of contributions to the literature have pursued Robinson’s [1933] approach of relating the output effect to demand curvature, and ACV obtained a series of welfare results in terms of demand curvature.

The present paper offers a new perspective on those results, based on two observations. The first is that, in the standard market setting with a single product, choosing the price level is equivalent to choosing the level of consumer surplus. Moreover, with downward-sloping (as distinct from inelastic) demand, choosing price is equivalent to choosing the level of welfare (i.e. consumer surplus plus profit) because welfare is strictly decreasing in price. We can therefore think of a monopolist as directly choosing how much welfare, or consumer surplus, to supply in each market that it serves. This approach is in the spirit of the ‘competition in utility space’ analysis of Armstrong and Vickers [2001], who model firms as supplying surplus directly to consumers. But here the setting is monopoly rather than competition, and welfare levels are the primary focus.

The second observation is that, whether discrimination is allowed or not, the monopolist will not wish to change all its prices by the same absolute amount at its optimum. Indeed that is also true under any form of price *difference* regulation that places no constraint on parallel price shifts. The firm’s marginal incentives to raise prices in parallel therefore sum to zero across markets even when price difference constraints bind, as for example when discrimination is banned.

The analytical strategy of the paper, which the next section outlines, is to combine these two observations by considering the marginal price-raising incentives, which sum to zero, as functions of *welfare* in each market. If they are convex functions of welfare, we then obtain (in section 3) direct welfare comparisons – not only between uniform pricing and laissez-faire, but also between those outcomes and other situations when price difference constraints apply. The method of analysis works just as easily for n markets as for two. In some leading cases, welfare comparisons depend only on the convexity properties of price

¹Varian [1985] proved this quite generally and stated bounds for welfare change with discrimination.

incentives as functions of welfare. More generally, covariance conditions are shown to be important. Section 4 obtains further results on the comparison between uniform pricing and laissez-faire by way of auxiliary functions defined in terms of welfare that again exploit the fact that the firm's marginal incentives to change prices sum to zero with uniform pricing. Section 5 adapts the method to yield results on how relative price regulation affects consumer surplus. The method yields new results as well as a new perspective on established results. For example, with constant elasticities of demand, discrimination is bad for consumer surplus if elasticities differ by no more than a factor of two. Section 6 concludes.

2 THE GENERAL METHOD

A product with constant unit cost $c > 0$ is supplied in n markets by a profit-maximizing monopolist. Demand in market i is $x_i(p_i)$, which is assumed to be a smooth, strictly decreasing function of price p_i , and inverse demand is $p_i(x_i)$. Profit from that market is $\pi_i(x_i) = (p_i(x_i) - c)x_i$. The firm's total profit is $\Pi = \sum_i \pi_i$. The firm chooses the vector of quantities $x = (x_1, \dots, x_n)$ to maximize Π subject to price difference constraints except in the case of laissez-faire. The prime question is how the constraints affect total welfare $W = \sum_i w_i$, where w_i is welfare in market i .

Consider the welfare comparison between laissez-faire, which yields welfare level w_i^* in market i , and a requirement of uniform pricing, which yields welfare w_i^0 . (Superscripts $*$ and 0 generally denote outcomes with laissez-faire and uniform pricing respectively, which are assumed to differ.) To see where the analysis is headed, imagine that there is a strictly convex function $\gamma_i(w_i)$ for each market such that

$$\sum_i \gamma_i(w_i^0) = \sum_i \gamma_i(w_i^*) = 0 . \quad (1)$$

With $w_i^0 \neq w_i^*$ strict convexity implies that

$$(w_i^0 - w_i^*)\gamma'_i(w_i^*) < \gamma_i(w_i^0) - \gamma_i(w_i^*) < (w_i^0 - w_i^*)\gamma'_i(w_i^0)$$

and therefore by summing

$$\sum_i (w_i^0 - w_i^*)\gamma'_i(w_i^*) < \sum_i [\gamma_i(w_i^0) - \gamma_i(w_i^*)] < \sum_i (w_i^0 - w_i^*)\gamma'_i(w_i^0) .$$

From (1) the middle term in this chain is zero, and so

$$\sum_i (w_i^0 - w_i^*)\gamma'_i(w_i^*) < 0 < \sum_i (w_i^0 - w_i^*)\gamma'_i(w_i^0) . \quad (2)$$

If, moreover, all $\gamma'_i(w_i^0) > 0$ and were the same, we would immediately have from (2) that total welfare W^0 with uniform pricing was greater than welfare W^* with laissez-faire. If, on the other hand, all $\gamma'_i(w_i^*) > 0$ and were the same, the opposite would be true. More generally, let $E[\cdot]$ denote the average of a variable across the n markets, and write

$$\begin{aligned}\sum_i (w_i^0 - w_i^*) \gamma'_i(w_i^0) &= (W^0 - W^*) E[\gamma'_i(w_i^0)] + \sum_i (w_i^0 - w_i^*) (\gamma'_i(w_i^0) - E[\gamma'_i(w_i^0)]) \\ &= (W^0 - W^*) E[\gamma'_i(w_i^0)] + n \text{Cov}[(w_i^0 - w_i^*), \gamma'_i(w_i^0)] .\end{aligned}\quad (3)$$

If the covariance term is negative, (2) and (3) again imply that $W^0 > W^*$ so long as $E[\gamma'_i(w_i^0)] > 0$. But the opposite holds if $\text{Cov}[(w_i^0 - w_i^*), \gamma'_i(w_i^*)] > 0$ and $E[\gamma'_i(w_i^*)] > 0$.

The property in (1) of summing to zero under both uniform pricing and laissez faire applies to the price-raising incentive functions described in the Introduction, but their convexity, for example, is another matter. We now explore which properties of market demand allow the general method outlined above to yield welfare results.

3 WELFARE ANALYSIS WITH PRICE DIFFERENCE REGULATION

Consider any regime of price difference regulation – i.e. constraints of the form

$$p_i(x_i) - p_j(x_j) \leq r_{ij} \quad (4)$$

for $i \neq j$, where all constants $r_{ij} \geq 0$. At most half of the constraints (4) will bind. Price discrimination is banned when all $r_{ij} = 0$. If price vector (p_1, \dots, p_n) meets constraints (4), then so does the vector $(p_1 + t, \dots, p_n + t)$ for a scalar t . That is to say, *parallel* price shifts are unconstrained. So if we define $\gamma_i(w_i)$ as the *marginal profit incentive to increase p_i* , expressed as a function of w_i , then at the firm's optimum

$$\sum_i \gamma_i(w_i) = 0 \quad (5)$$

as in (1). Welfare vectors that satisfy (5) are central to the analysis that follows.

Definition 1 *Any welfare vector that satisfies (5) is said to be difference-compatible.*

To see how $\gamma_i(w_i)$ relates to market parameters, define $\eta_i(x_i) \equiv -\frac{p_i(x_i)}{x_i p'_i(x_i)} > 0$ as demand elasticity in market i and $\mu_i(x_i) \equiv \frac{p_i(x_i) - c}{p_i(x_i)}$ as the mark-up. A useful term is $\xi_i(x_i) \equiv$

$\eta_i(x_i)\mu_i(x_i)$, and let $\sigma_i(x_i) \equiv -\frac{x_i p_i''(x_i)}{p_i'(x_i)}$ be the elasticity of the slope of inverse demand, a familiar measure of (inverse) demand curvature, which is related to ξ_i by

$$x_i \xi_i' = -[1 + (1 - \sigma_i)\xi_i] . \quad (6)$$

It is assumed that $\sigma_i(x_i) < 2$ for all i , so profit in each market is single-peaked in price. The mild further assumption is made that $E[\sigma_i^0 \xi_i^0] < 2$ at the profit-maximizing uniform price p^0 .²

Define

$$\theta_i(x_i) \equiv x_i + (p_i(x_i) - c)x_i'(p_i(x_i)) = x_i[1 - \xi_i(x_i)] \quad (7)$$

as the marginal effect on π_i of a small increase in the price of product i . Thus $\theta_i(x_i^*) = 0$ and from $\xi_i(x_i^*) = 1$ we have that $p_i^* - c = \frac{c}{\eta_i^* - 1}$. If relative price constraints bind, the vector x that solves the firm's maximization problem also satisfies $\sum_i \theta_i(x_i) = 0$ because each p_i could be increased by the same dp while still meeting the constraints. Relative to uniform pricing the unconstrained monopolist will want to increase prices in some markets and reduce them in others.

Definition 2 $H \equiv \{i : p_i^* > p_i^0\}$ is the set of markets in which the firm would like to charge more than the profit-maximizing uniform price, and L is the set of other markets.

For all constraints r_{ij} the firm will choose $x_i \in [x_i^*, x_i^0]$ in H -markets and $x_i \in [x_i^0, x_i^*]$ in L -markets. Thus we restrict attention to the set of quantity vectors $\mathfrak{X} \equiv \{x : x_i \in [\min(x_i^*, x_i^0), \max(x_i^*, x_i^0)]\}$ and the associated prices.

Since for all $x \in \mathfrak{X}$ we have that $p_i(x_i) \geq c$, welfare $w_i(x_i)$ in market i is a monotonically increasing function with $w_i' = p_i - c$. The inverse function $X_i(w)$ can be defined by $w_i(X_i(w)) \equiv w$. Thus define

$$\gamma_i(w_i) \equiv \theta_i(X_i(w_i)) \quad (8)$$

and since

$$\theta_i'(x_i) = 2 - \sigma_i(x_i)\xi_i(x_i)$$

we have that

$$\gamma_i'(w_i) = \theta_i'(X_i(w_i))X_i'(w_i) = \frac{2 - \sigma_i \xi_i}{p_i - c} . \quad (9)$$

²This last condition is closely related to the second-order condition for p^0 that $\sum_i [2 - \sigma_i^0 \xi_i^0] x_i'(p^0) < 0$. The assumption that π_i is single-peaked in p_i is weaker than ACV's assumption of concavity, which corresponds to each $\sigma_i \xi_i < 2$.

From (9) we see that with $\sigma_i < 2$ all $\gamma_i(w_i^*) > 0$, and the assumption that $E[\sigma_i^0 \xi_i^0] < 2$ implies that $E[\gamma_i'(w_i^0)] > 0$. The following definition concerns the second derivative of $\gamma_i(w_i)$.

Definition 3 *Market i is regular if $\gamma_i(w_i)$ is convex.*

This regularity property is equivalent to ACV's increasing ratio condition that the RHS term in (9), which equals $[\frac{2}{p_i - c} + \frac{x_i''(p_i)}{x_i'(p_i)}]$, is decreasing in p_i . ACV show that condition to be met for a wide range of demand functions.

We can now illustrate the method of section 2. First, suppose that demands take the form

$$x_i(p_i) = a_i - k_i f(p_i) \quad (10)$$

with f the same for all i and $f' > 0$. Linear demands are an instance of (10). Then

$$\gamma_i'(w_i^0) = \frac{2}{p^0 - c} + \frac{f''(p^0)}{f'(p^0)}$$

independently of i .³ So $W^0 > W^*$ from the second inequality in (2), and by the same reasoning W^0 is the maximum total welfare attainable subject to constraints of the form (4).

For a second illustration, let demands have the logistic form

$$x_i(p_i) = \frac{1}{a_i e^{b_i p_i} + k_i} \quad (11)$$

from Cowan [2016], with $a_i, b_i > 0$. Then

$$\gamma_i'(w_i^*) = \frac{2 - \sigma_i^*}{p_i^* - c} = (2 - \sigma_i^*) \frac{\eta_i^*}{p_i^*} = b_i. \quad (12)$$

If the b_i are the same across markets, then from the first inequality in (2) we have that laissez-faire achieves higher welfare than any constrained optimum subject to (4). In particular $W^* > W^0$.

Using (3) these special cases can be generalized as follows.

Proposition 1 *If all markets are regular, among difference-compatible outcomes total welfare (i) is maximized by uniform pricing if $\sigma_i^0 \eta_i^0$ is higher in H -markets in the sense that $\text{Cov}[\sigma_i^0 \eta_i^0, (w_i^0 - w_i)] \geq 0$, but (ii) is maximized by laissez-faire if $(2 - \sigma_i^*)(\eta_i^* - 1)$ is higher in H -markets in the sense that $\text{Cov}[(2 - \sigma_i^*)(\eta_i^* - 1), (w_i - w_i^*)] \geq 0$.*

³Regularity is the condition that this last expression is decreasing in p^0 .

Proof. (i) Under the stated condition $Cov[\gamma'_i(w_i^0), (w_i^0 - w_i)] \leq 0$ because $\gamma'_i(w_i^0) = \frac{2}{p^0 - c} - \frac{\sigma_i^0 \eta_i^0}{p^0}$. So from (5)

$$0 = \sum_i [\gamma_i(w_i^0) - \gamma_i(w_i)] \leq \sum_i (w_i^0 - w_i) \gamma'_i(w_i^0) \leq (W^0 - W) E[\gamma'_i(w_i^0)]$$

and so $W^0 \geq W$.

(ii) In this case $Cov[\gamma'_i(w_i^*), (w_i - w_i^*)] \geq 0$ because $\gamma'_i(w_i^*) = (2 - \sigma_i^*)(\eta_i^* - 1)/c$. So

$$0 = \sum_i [\gamma_i(w_i) - \gamma_i(w_i^*)] \geq \sum_i (w_i - w_i^*) \gamma'_i(w_i^*) \geq (W - W^*) E[\gamma'_i(w_i^*)]$$

and so $W^* \geq W$. ■

Parts (i) and (ii) of Proposition 1 respectively generalize Propositions 1 and 2 of ACV to the multi-market case, and by comparing w^0 and w^* with all difference-compatible w . It is of course quite possible that neither condition in Proposition 1 holds in a given market situation. But when one of them does, the Proposition and its proof show how directly welfare implications follow.

4 WELFARE COMPARISON BETWEEN LAISSEZ FAIRE AND UNIFORM PRICING

Now we focus on the comparison between laissez-faire and uniform pricing, rather than all difference-compatible outcomes. This widens the scope for $\gamma_i(w_i)$ functions to which the method of section 2 can be applied. The key is to note that for any smooth auxiliary functions $g_i(x_i)$ such that $g_i(x_i^0) = x_i^0$ we can define

$$\hat{\gamma}_i(w_i(x_i)) \equiv \frac{g_i(x_i)}{x_i} \gamma(w_i(x_i)) = g_i(x_i) [1 - \xi_i(x_i)] \quad (13)$$

and (1) remains true with $\hat{\gamma}_i(w_i)$ in place of $\gamma_i(w_i)$ because $\hat{\gamma}_i(w_i^*) = 0$ and $\hat{\gamma}_i(w_i^0) = \gamma_i(w_i^0)$ by the construction of $g_i(x_i)$. The economic interpretation of $\hat{\gamma}_i(w_i)$ is however less clear than that of $\gamma_i(w_i)$, though an economic interpretation is offered for Example 4 below. Rather, the justification for the method is essentially pragmatic. For suitable choices of $g_i(x_i)$ we can use further convexity arguments to sign $(W^* - W^0)$, as the following two examples illustrate. They are chosen to yield results in ACV and in Cowan [2012, 2016].

Example 1: Let $g_i(x_i) \equiv x_i^0$. Then from (6)

$$\hat{\gamma}'_i(w_i) = -\frac{x_i^0}{p_i - c} \xi'_i(x_i) = \frac{x_i^0}{\pi_i} [1 + (1 - \sigma_i) \xi_i] . \quad (14)$$

This example is well-suited to the case of constant elasticities because $\sigma_i = 1 + \frac{1}{\eta_i}$ and so (14) implies that

$$\hat{\gamma}'_i(w_i) = \frac{x_i^0}{\pi_i}(1 - \mu_i) = \frac{cx_i^0}{p_i\pi_i} \quad (15)$$

and in particular $\hat{\gamma}'_i(w_i^0) = \frac{c}{p^0(p^0-c)}$ independently of i .⁴ Then we have from (2) that $W^0 > W^*$ if the $\hat{\gamma}_i(w_i)$ are convex. From (15) that is so if all $p_i\pi_i$ are increasing in p_i , that is if

$$0 < \pi_i + p_i\theta_i = \pi_i[1 + \frac{1}{\mu_i} - \eta_i] .$$

That always holds if the largest $\eta_i - 1 \leq \frac{1}{\mu^0} = \eta^0 \equiv \frac{E[x_i^0\eta_i(x_i^0)]}{E[x_i^0]}$, the average elasticity weighted by quantities at uniform pricing. Therefore $W^0 \geq W^*$ with constant elasticities if elasticities do not differ too much, confirming Proposition 6(ii) of ACV. For this it clearly suffices that elasticities differ by no more than 1, but Aguirre and Cowan [2015] show that discrimination can be good for welfare with constant elasticities of 1.5 and 3 in the two-market case – a difference of 1.5.

Example 2: If $g_i(x_i) = \frac{x_i p'_i(x_i)}{p'_i(x_i^0)}$, then

$$\hat{\gamma}'_i(w_i^0) = \frac{2 - \sigma_i^0}{p^0 - c} .$$

So from (3), if the $\hat{\gamma}_i(w_i)$ are convex and σ_i^0 is greater in H -markets than L -markets in the sense that $Cov[\sigma_i^0, (w_i^0 - w_i^*)] > 0$, then $W^0 > W^*$. But with concave $\hat{\gamma}_i(w_i)$, if σ_i^0 is smaller in H -markets than L -markets, then $W^* > W^0$. Convex and concave $\hat{\gamma}_i(w_i)$ are both possible. For example, with constant σ_i in this example $\hat{\gamma}_i''(w_i)$ has the sign of $1 - \sigma_i\xi_i$. That is positive, so the $\hat{\gamma}_i(w_i)$ are convex, if $\max(\sigma_i, \sigma_i\xi_i^0) \leq 1$ for all i , which holds if $\sigma_h \leq 1$ for $h \in H$ and $\sigma_l \frac{\eta_l^0}{\eta^0} \leq 1$ for $l \in L$. But the $\hat{\gamma}_i(w_i)$ are concave if $\min(\sigma_i, \sigma_i\xi_i^0) \geq 1$ for all i . These considerations yield ACV's Proposition 5.

Relating Propositions 5 and 6(ii) of ACV to the multi-market case, these implications of Examples 1 and 2 can be summarised as follows.

Proposition 2 (i) *With constant elasticities of demand η_i , uniform pricing is better for welfare than laissez-faire if all $\eta_i - 1 \leq \eta^0$.* (ii) *With constant curvatures of inverse demand σ_i , uniform pricing is (a) better for welfare than laissez-faire if σ_i is higher in*

⁴More generally demands of the form $x_i(p_i) = a_i e^{-h_i f(p_i)}$ imply that $\hat{\gamma}'_i(w_i^0) = \frac{1}{p^0 - c} + \frac{f''(p^0)}{f'(p^0)}$ for all i . Constant elasticities has $f(p) = \ln p$ and $h_i = \eta_i$.

H -markets in the sense that $\text{Cov}[\sigma_i, (w_i^0 - w_i^*)] \geq 0$, all $\sigma_i \leq 1$ and $\sigma_l \frac{\eta_l^0}{\eta^0} \leq 1$ for all $l \in L$, but (b) worse for welfare than laissez-faire if σ_i is lower in H -markets in the sense that $\text{Cov}[\sigma_i, (w_i^0 - w_i^*)] \leq 0$, all $\sigma_i \geq 1$ and $\sigma_h \frac{\eta_h^0}{\eta^0} \geq 1$ for all $h \in H$.

5 CONSUMER SURPLUS ANALYSIS

Similar methods to those used above yield results on the effect of price discrimination on consumer surplus. Clearly price restrictions benefit consumers whenever they increase welfare because they reduce profit. But discrimination might be bad for consumers when positive (or ambiguous) for welfare, or good for consumers as well as for welfare. To explore these possibilities, define $s_i(x_i) \equiv w_i(x_i) - \pi_i(x_i)$ as consumer surplus in market i , and note that $s'_i(x_i) = -x_i p'_i(x_i) > 0$, so there exists an inverse function $\tilde{X}_i(s)$ defined by $s_i(\tilde{X}_i(s)) \equiv s$. Parallel to (8) let

$$\psi_i(s_i) \equiv \theta_i(\tilde{X}_i(s_i)) . \quad (16)$$

be the marginal incentive to increase p_i expressed as a function of consumer surplus. Then

$$\psi'_i(s_i(x_i)) = \frac{\theta'_i(x_i)}{s'_i(x_i)} = (2 - \sigma_i \xi_i) \frac{\eta_i}{p_i}$$

and $\psi''_i(s_i)$ is ambiguous in sign – for example, negative with linear demand, when $\sigma_i = 0$, but positive with exponential demand, when $\sigma_i = 1$. We can however obtain curvature conditions, by way of auxiliary functions $g_i(x_i)$ as in section 4, by defining

$$\hat{\psi}_i(s_i(x_i)) \equiv \frac{g_i(x_i)}{x_i} \psi_i(s(x_i)) = g_i(x_i) [1 - \xi_i(x_i)]$$

for $g_i(x_i)$ functions such that $g_i(x_i^0) = x_i^0$.

For instance, as in Example 2 above, if

$$\hat{\psi}_i(s_i(x_i)) = \frac{p'_i(x_i)}{p'_i(x_i^0)} x_i (1 - \xi_i(x_i))$$

then

$$\hat{\psi}'_i(s_i(x_i)) = \frac{2 - \sigma_i}{x_i} , \quad (17)$$

which – equivalently to Cowan's [2012] 'passthrough assumption' and Chen and Schwartz's [2015] demand-curvature condition – is decreasing in x_i for a wide range of demand specifications. Chen and Schwartz [2015, page 448] show that this condition is equivalent to

consumer surplus with monopoly pricing being a convex function of unit cost c . Then $\hat{\psi}_i(s_i)$ is concave, and so

$$\begin{aligned} 0 &= \sum_i [\hat{\psi}_i(s_i^0) - \hat{\psi}_i(s_i^*)] \geq \sum_i (s_i^0 - s_i^*) \hat{\psi}_i'(s_i^0) \\ &= \frac{1}{p^0} \sum_i (s_i^0 - s_i^*) (2 - \sigma_i^0) \eta_i^0 . \end{aligned}$$

If $(2 - \sigma_i^0) \eta_i^0$ is higher in H -markets in the sense that $Cov[(2 - \sigma_i^0) \eta_i^0, (s_i^0 - s_i^*)] \geq 0$, then consumer surplus is greater with laissez-faire than uniform pricing, as in Proposition 1(i) of Cowan [2012]. With logistic demands (11) and all $b_i = b$, from (12) and (17) we have $-p_i'(x_i^0) \hat{\psi}_i' = -b p_i'(x_i)$, so $\hat{\psi}_i'(s_i^0) = b$. Thus $\hat{\psi}_i''$ has the sign of $-p_i''(x_i)$, which in turn has the sign of $(2k_i x_i - 1)$. If that is negative for the relevant range of x_i for all i then consumer surplus higher under laissez-faire than uniform pricing.⁵ But the opposite holds if $x_i > \frac{1}{2k_i} > 0$ for all x_i in range.

Now consider two further examples, which yield new results.

Example 3: If $g_i(x_i) = \frac{\xi_i^0}{\xi_i(x_i)} x_i^0$, then $\hat{\psi}_i(s_i(x_i)) = x_i^0 \xi_i^0 [\frac{1}{\xi_i(x_i)} - 1]$ and

$$\hat{\psi}_i'(s_i) = [1 + (1 - \sigma_i) \xi_i] \frac{x_i^0 \xi_i^0}{\pi_i \xi_i} .$$

In particular

$$\hat{\psi}_i'(s_i^0) = \frac{1 + (1 - \sigma_i^0) \xi_i^0}{p^0 - c} ,$$

which equals $\frac{c}{p^0(p^0 - c)}$ for all i with constant elasticities. Then total consumer surplus is greater with uniform pricing than with laissez-faire if the $\hat{\psi}_i(s_i)$ functions are convex, for which the condition is that $(p_i - c) \pi_i$ is increasing in p_i , that is if

$$0 < \pi_i + (p_i - c) \theta_i = \pi_i (2 - \mu_i \eta_i) ,$$

which always holds if all $\eta_i \leq 2\eta^0$.⁶

Example 4: Finally, if $g_i(x_i) = \frac{\pi_i(x_i)}{p^0 - c}$, then

$$(p^0 - c) \hat{\psi}_i' = 3\xi_i - \sigma_i \xi_i^2 - 1 ,$$

⁵This yields and extends Cowan's [2016] Proposition 2, which has logistic demands and $k_i = k < 0$.

⁶However, if constant elasticities differ enough, consumer surplus may be greater with discrimination than with uniform pricing. Thus, for example, Aguirre and Cowan [2015] show that with two markets and elasticities of 4 and 11 – a ratio of 2.75 – consumer surplus is higher with discrimination if the more elastic market is relatively small.

and if the σ_i are constant, $\hat{\psi}_i''$ has the sign of $(2\sigma_i\xi_i - 3)$ if $\xi_i' < 0$, which holds if $(\sigma_l - 1)\xi_l^0 < 1$ for all $l \in L$. Then the $\hat{\psi}_i(s_i)$ are concave if $\min(\sigma_i, \sigma_i\xi_i^0) \leq \frac{3}{2}$ for all i , so

$$0 \leq \sum_i (s_i^0 - s_i^*) \hat{\psi}_i'(s_i^*) = \frac{1}{p^0 - c} \sum_i (s_i^0 - s_i^*) (2 - \sigma_i) ,$$

and consumer surplus is greater with uniform pricing than laissez-faire if σ_i is greater in H -markets than L -markets. But if the $\hat{\psi}_i(s_i)$ are convex, as with constant σ_i such that $\min(\sigma_i, \sigma_i\xi_i^0) \geq \frac{3}{2}$ for all i , then consumer surplus is higher with laissez-faire if σ_i is smaller in H -markets than L -markets. Exponential demand $x_i(p_i) = a_i e^{-b_i p_i}$, an instance of (11), has all $\sigma_i = 1$ and the $\hat{\psi}_i(s_i)$ are concave – so uniform pricing is better for consumers in aggregate – unless markets differ substantially.⁷ It may be noted that Example 4 for consumer surplus mirrors Example 1 for welfare, but with the critical level for σ_i being around $\frac{3}{2}$ rather than 1.

Parallel with Proposition 2 these implications of Examples 3 and 4 can be summarised as follows.

Proposition 3 (i) *With constant σ_i uniform pricing is (a) better for consumer surplus than laissez-faire if σ_i is higher in H -markets in the sense that $\text{Cov}[\sigma_i, (s_i^0 - s_i^*)] \geq 0$, all $\sigma_i \leq 1$ and $\sigma_l \frac{\eta_l^0}{\eta^0} \leq \frac{3}{2}$ for all $l \in L$, but (b) worse for consumers surplus than laissez-faire if σ_i is lower in H -markets in the sense that $\text{Cov}[\sigma_i, (s_i^0 - s_i^*)] \leq 0$, all $\sigma_i \geq 1$, $\frac{\eta_l^0}{\eta^0} < \frac{1}{\sigma_l - 1}$ for all $l \in L$ and $\sigma_h \frac{\eta_h^0}{\eta^0} \geq \frac{3}{2}$ for all $h \in H$. (ii) *With constant η_i uniform pricing is better for consumer surplus than laissez-faire if all $\eta_i \leq 2\eta^0$.**

The following economic interpretation can be given to Example 4. Consider a scheme of relative price regulation that, by contrast to (4), limits divergence of price-cost *ratios*:

$$\frac{p_i - c}{p_j - c} \leq r_{ij} . \quad (18)$$

At any constrained (or unconstrained) optimum, the firm is free to increase all $(p_i - c)$ proportionately. So at such an optimum

$$0 = \sum_i (p_i - c) \frac{d\pi_i}{dp_i} = (p_0 - c) \sum_i g_i(x_i) (1 - \xi_i(x_i))$$

with $g_i(x_i)$ as in Example 4. The example therefore allows comparison not only between laissez-faire and uniform pricing, but also among outcomes satisfying the relative price regulation scheme (18) more broadly.

⁷But as Simon Cowan has noted (by email), consumer surplus can be higher with price discrimination than uniform pricing with exponential demands if markets differ enough. An example of this is with demands $x_1(p) = e^{-p}$ and $x_2(p) = e^{-p/4}$ and $c = 0$. The concavity condition fails in such cases.

6 CONCLUSION

The analytical method used in this paper has been based on the observations that (i) in standard single-product monopoly settings there equivalence between choosing price and choosing the level of consumer surplus (or welfare), and (ii) at its optimum the incentives of a multi-market monopolist to raise prices in parallel sum to zero across markets whether or not there are binding price difference constraints, for example a requirement of uniform pricing. Using convexity properties, the method directly delivers known welfare results on third-degree monopoly price discrimination, such as those in ACV, and somewhat generalises them by relaxing the concavity of $\pi_i(p_i)$, by extending beyond the 2-market case, and by applying to price difference constraints more broadly than a ban on price discrimination. The method also yields up some new results, for example that monopolistic price discrimination is bad for consumers with constant elasticities that differ by no more than a factor of two. Whether the approach can be applied to other contexts involving welfare comparison between constrained and unconstrained optima, and whether its economic interpretation can be strengthened, remain to be seen.

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