

Credit Ratings and Structured Finance*

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Abstract

The poor performance of credit ratings of structured finance products in the financial crisis has prompted investigation into the role of credit rating agencies (CRAs) in designing and marketing these products. We analyze a two-period reputation model in which a CRA both designs and rates securities that are sold both to investors who are constrained to purchase highly rated securities and investors who are unconstrained. Assets are pooled and senior and junior tranches are issued with a waterfall structure. When the rating constraint is lax, the CRA will include only risky assets in the securitization pool, serving both types of investors without any rating inflation. Rating inflation is decreasing in the tightness of the rating constraint locally. But rating inflation may be non-monotonic in the rating constraint globally, with no rating inflation when the constraint is lax or tight.

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1 Introduction

The recent financial crisis has prompted much investigation into the role of credit-rating agencies (CRAs). With the dramatic increase in the use of structured finance products, the agencies quickly expanded their business and earned outsized profits (Moody's, for example, tripled its profits between 2002 and 2006). Ratings quality seems to have suffered, as structured finance products were increasingly given top ratings shortly before the financial markets collapsed. In this paper, we ask how CRAs influence the structure of such products, and how the products' structure changes with market incentives.

The structuring process is marked by close collaboration between issuers and rating agencies. Issuers depend on rating agencies to certify quality and to be able to sell to regulated investors. Beyond directly paying CRAs for ratings (the "issuer pays" system), Griffin and Tang (2012) write that "the CRA and underwriter may engage in discussion and iteration over assumptions made in the valuation process." Agencies also provide their models to issuers even before the negotiations take place (Benmelech and Dlugosz, 2009). These products are characterized by careful selection of the underlying asset pool and private information about asset quality.

We present a reputation-based two-period model of rating structured products. The model incorporates the endogenous structuring and rating of securities. Each period, an issuer has a set of safe and risky assets that it can put into a pool and issue securities against. A monopoly CRA assists in the structuring of these securities and rates them. The prospect of earning future profits can give the CRA reputational incentives to provide accurate ratings.

We model reputation by positing that the CRA is long-lived and can be one of two types: truthful or opportunistic. Securities are sold to rational investors who cannot observe the type of the CRA or the quality of the securities, but who make inferences from the payoff realizations, ratings and the size of the asset pool. The type of the CRA is revealed between periods with an endogenous probability that depends directly on the amount of rating inflation.

The issuer and CRA design securities that have a waterfall/priority structure, i.e. with junior securities that face the first losses and senior securities that pay out their promised amount until the value of the junior securities has gone to zero. This is a typical structure for structured finance products (Coval, Jurek, and Stafford, 2009).

A principal motivation for securitization is to appeal to investor groups with heterogeneous preferences. The *senior* securities are designed to appeal

to constrained investors, who can only purchase investment grade securities, i.e. securities with a rating above a certain level. Constrained investors may be regulated institutions, such as banks, pension funds, and insurance companies. The securities are designed using a rating constraint, a probability that the senior securities will not be paid in full. This probability is exogenous in the model, but may be determined by the requirements of the CRA, the regulator, and/or the investors. The *junior* securities are designed to appeal to unconstrained investors, such as hedge funds, who can purchase securities with any type of risk profile.

The CRA may include safe assets in the pool to be able to sell to constrained investors. This creates scope for rating inflation, which relies on passing off risky assets as safe ones. The trade-off for the opportunistic CRA is that this allows it to extract more rents from the issuer, who gets to retain the safe assets, but makes it more likely that the CRA will be identified as opportunistic and thereby decreases its expected future profits.

Our first result is that, for some parameters, the equilibrium has both types of CRA only including risky assets in the asset pool. This implies there is no rating inflation, as there is no scope for substituting risky for safe assets. Nevertheless, through the structuring process, there will still be senior securities to sell. And this achieves the first best allocation, as there are larger gains from trade for risky assets. This equilibrium will be more likely to occur when there is less demand for safe securities, more demand for risky ones, and the rating constraint is lax.

When the CRA includes safe assets in the asset pool, rating inflation is possible. Rating inflation is important to examine as it is directly related to surplus. Rating inflation (a) measures the amount of adverse selection/misallocation there is, and (b) leads to excess risk being taken on by investors (which, outside of the model, may be surplus reducing due to systemic effects).

We examine how rating inflation depends on the rating constraint. For parameters such that rating inflation is possible, rating inflation is decreasing in the tightness of the rating constraint. First, a tighter rating constraint increases the probability that the opportunistic CRA is discovered if it inflates in the first period. Second, a tighter rating constraint increases the difference between the second-period profits when the inflating opportunistic CRA is not discovered and when it is discovered, making it more desirable to avoid discovery. These two effects dominate a possible increase in the opportunity cost of being truthful in the first period.

Nevertheless, given that for some parameters, rating inflation is not possible, the relationship between rating inflation and the tightness of the rating constraint may be non-monotonic. When the rating constraint is lax, only

risky assets are included, which, as discussed above, leads to no rating inflation. When the rating constraint is tight, there may not be any rating inflation due to a higher likelihood of being caught. Intermediate tightness of the rating constraint involves positive amounts of rating inflation. Interestingly, this indicates that new markets that don't have many constrained investors may not need to be heavily scrutinized by regulators. More advanced markets, consequently, deserve stricter scrutiny.

We show that rating inflation is increasing in the wealth of unconstrained investors, as it allows more risky assets into the pool, decreasing the likelihood that inflation is discovered. Rating inflation is also increasing in the value of retaining safe assets, as this makes it more desirable to substitute risky for safe assets. Temporary increases in wealth or the demand for securities increase the incentive to inflate ratings to capture higher profits.

Finally, we provide two new motivations for the pooling of assets: (a) a mechanical reason of tailoring products for constrained investors; and (b) a novel explanation of the CRA balancing the informational advantage over investors with the need to maintain its reputation by choosing the right mix of safe and risky assets to include.

The paper is organized as follows. In the next subsection, we review the theoretical literature. In Section 2, we set up the model. In Section 3, we analyze the equilibrium of the second period and in Section 4 the equilibrium of the first period. In Section 5, we look at the determinants of rating inflation in the model. Section 6 reviews the empirical implications of the model. Section 7 concludes. The Appendix contains all mathematical proofs not in the text.

1.1 Theory Literature

In the theory literature, Mathis, McAndrews, and Rochet (2009), Fulghieri, Strobl, and Xia (2014), Bar-Isaac and Shapiro (2013), and Strausz (2005) examine dynamic models of certification agencies with reputation concerns.¹ Our model of reputation is similar, but we allow for multiple risky assets, which permits the CRA to tranche securities as well as rate them.

Daley, Green, and Vanasco (2017a, 2017b) examine the interaction between retention, security design, ratings, and origination. Ratings in their main model are similar to public information. We focus on a CRA's strategic incentives to undertake security design and ratings.

Opp, Opp, and Harris (2013) examine how ratings-contingent regulation

¹There is a large recent theoretical literature on CRAs, including Sangiorgi and Spatt (2017), Cohn, Rajan, and Strobl (2016), and Bolton, Freixas, and Shapiro (2012).

affects the informativeness of ratings in a setting where a CRA rates risky projects with binary outcomes. The constrained investors in our model rely on a rating that states it respects a rating constraint. The rating constraint may depend on regulation. However, the constraint may be violated in our model by the CRA. Moreover, we allow for security design.

2 Model

There are three types of agents in the model: issuers, investors, and a CRA. All agents are risk-neutral. We begin by focusing on issuers.

2.1 Issuers

The issuer has two types of assets that it would like to sell to a set of investors: risky assets and safe assets. Risky assets pay off \tilde{X} per unit, a random variable distributed uniformly over the unit interval. For simplicity, the payoffs of different risky assets are assumed to be perfectly correlated with each other. Risky assets are worth $r \in (0, 1/2)$ to the issuer. Safe assets pay off 1 per unit with probability one. They are worth $s \in (2r, 1)$ to the issuer.

The issuer's valuations of the assets are lower than the investors' values for the assets. This can occur for several reasons: the issuer may have valuable alternative investment opportunities, regulatory capital requirements for holding the assets, and/or the need to transfer risk off of its balance sheet. The assumption that $s > 2r$ implies that under full information the net profits to the issuer from selling a dollar of risky assets is greater than the net profits from selling a dollar of safe assets.

The issuer's supply of safe assets is $\sigma > 0$ and its supply of risky assets is large (for simplicity, we assume it is infinite).² We denote the measure of safe assets the issuer includes in the securitization pool by σ^I and the measure of risky assets by ρ^I .

2.2 Investors

There are two types of investors: unconstrained, U , with aggregate wealth $w_U > 0$ and constrained, C , with aggregate wealth $w_C > 0$.³ Constrained

²Note that assuming that the issuer has a large supply of risky assets guarantees that an issuer can create a pool of any size that contains only risky assets. This will be important for the lemons problem we analyze.

³We assume that investors are credit-constrained, which might arise from borrowing frictions (see, for example, Boot and Thakor, 1993).

investors can only purchase securities that receive an investment grade rating. We define investment grade ratings below. Constrained investors may be constrained by regulations (for example, banks, pension funds, and insurance companies are often restricted in the types of assets they may hold), internal by-law restrictions / investment mandates (e.g., Baghai, Becker, and Pitschner, 2018), or their portfolio hedging requirements. The unconstrained investors are willing to purchase any security. These investors may be hedge funds or other institutional investors. We assume that both types of investors are rational, in the sense that they update given available information and maximize their expected payoffs.⁴ Investors' reservation utility is normalized to zero.

2.3 Securitization

With the help of a credit rating agency (CRA), the issuer can issue securities backed by a portfolio of safe and risky assets for investors through *securitization*. We define securitization as creating a senior and junior tranche with a waterfall/priority structure. This means that if the payoff of the underlying pool of assets is sufficiently large, both tranches receive a payoff; the senior tranche receives its promised repayment and the junior tranche receives the residual. When the payoff of the pool is sufficiently low, the junior tranche receives nothing, and the senior tranche receives the whole payoff (which may be below the payoff promised to them). This model is a stylized version of how securitization works (see Coval, Jurek, and Stafford (2009) for a detailed description of the process).⁵

Formally, suppose that the CRA in conjunction with the issuer decides on the assets to include in the pool and how to structure it. To simplify this process for the model, we assume the CRA's fee gives it a fraction f of the surplus from the issuance.⁶ The issuer and the CRA decide jointly on the number of safe and risky assets to include in the securitization and the specification of the structure of the senior and junior tranches. For simplicity of exposition, we will refer to all decisions as being made by the CRA.

More precisely, suppose the CRA can market the securities of a junior

⁴There has been much discussion about the naïveté of investors in the RMBS market; e.g. Bolton, Freixas, and Shapiro (2012). However, not all structured finance markets are necessarily characterized in such a way, as Stanton and Wallace (2018) point out: “All agents in the CMBS market can reasonably be viewed as sophisticated, informed investors.”

⁵In a previous version of this paper, we modelled pass-through securities, where investors get pro-rata shares of cash flows from the underlying assets.

⁶We discuss the fee in further detail in subsection 2.4.

tranche and a senior tranche. The junior tranche securities are intended for unconstrained investors, while the senior tranche securities are intended for constrained investors. Let $z^I \in [0, \sigma^I + \rho^I]$ be the face value of the senior tranche – i.e., the total payoff of the senior securities if they are paid in full. The senior tranche securities are paid in full if and only if the payoff of all assets in the pool is sufficient to make this payment:

$$\sigma^I + \rho^I \tilde{X} \geq z^I.$$

Define x^I as the realization of \tilde{X} such that the inequality binds:

$$\sigma^I + \rho^I x^I = z^I.$$

For realizations x of \tilde{X} below x^I , the senior tranche receives the total payoff of the pool and the junior tranche receives nothing. Notice that having the CRA choose the face value of the senior tranche, z^I , is equivalent to having the CRA choose the cutoff $x^I \in [0, 1]$. As it will be notationally simpler to use the cutoff x^I , we do so in most of the paper.

The realized payoff of the senior tranche is thus:

$$\sigma^I + \rho^I \min \{x, x^I\},$$

and its expected payoff:

$$\begin{aligned} & \sigma^I + \rho^I (1 - x^I) x^I + \rho^I x^I (x^I/2) \\ = & \sigma^I + \rho^I x^I (1 - x^I/2). \end{aligned} \tag{1}$$

The realized payoff of the junior tranche is the residual value of the pool:

$$\max \{ \rho^I (x - x^I), 0 \},$$

and its expected value:

$$\begin{aligned} & \rho^I (1 - x^I) ((1 - x^I) + 0) / 2 \\ = & \rho^I (1 - x^I)^2 / 2. \end{aligned} \tag{2}$$

From the above, it follows that the realized payoff of the entire pool equals:

$$\sigma^I + \rho^I x$$

and its expected payoff:

$$\sigma^I + \rho^I / 2.$$

Figure 1 illustrates the realized payoffs of the two tranches as a function of x .

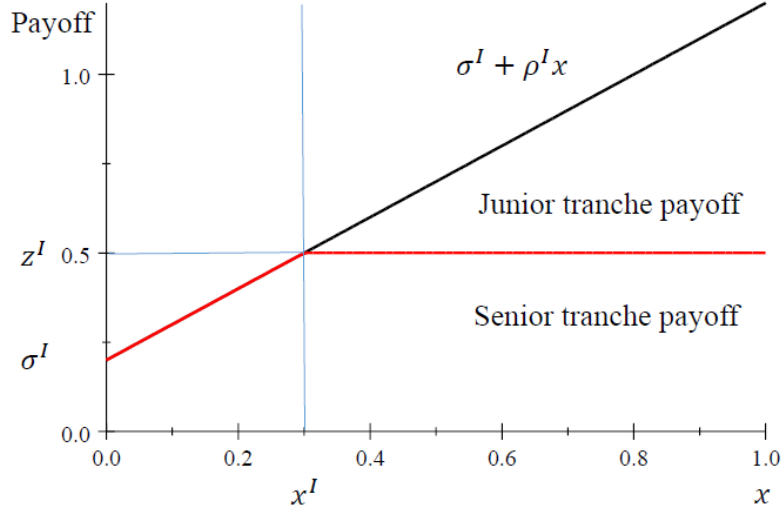


Figure 1: Pool payoff as a function of the realization of \tilde{X} for $\sigma^I = 0.2$ and $\rho^I = 1$. The payoff of the senior tranche is given by the red curve and the payoff of the junior tranche by the difference between the black and the red curves.

2.4 The CRA

We assumed that constrained investors are constrained in the sense that they may only purchase investment grade securities. We thus need to define the criteria a CRA uses for ratings.

2.4.1 Ratings

The simplest approach to define the rating criteria is to quantify the probability of default. We define an investment grade rating as signifying that the probability the senior tranche is not paid in full is less than or equal to an exogenous probability $P \in [0, 1]$.⁷ This gives us the following ratings constraint:

$$\Pr \left(\sigma^I + \rho^I \tilde{X} \leq \sigma^I + \rho^I x^I \right) \leq P,$$

which, given the uniform distribution assumption, is equivalent to stating that:

$$x^I \leq P.$$

⁷We could make this definition more lax by stating that the probability that the realized loss is larger than a pre-specified amount is smaller than the exogenous probability P . The results are qualitatively similar in this case; the proofs are available upon request.

It is natural to think of the probability of default of the senior tranche, P , as exogenous to the given securitization problem. This probability may represent the historical default rate for highly rated securities that the CRA wishes to maintain, a more lax constraint that the CRA applies to securitizations, or a more conservative constraint that the CRA applies due to pressure from regulators or investors. We will examine how changes to this constraint affect ratings.

To simplify the problem, we will use a transformed version of the rating constraint:

$$(1 - x^I)^2 - R \geq 0,$$

where $R \equiv (1 - P)^2$. A higher R thus means a more demanding (tighter) rating constraint.

Lastly, we assume that an issuer can't sell rated securities on its own. It may sell securities, but without ratings it will not be able to access constrained investors. This is the first role of ratings in our model; a regulatory (or institutionalized) license to access certain clientele. Given this, the issuer may still sell to unconstrained investors. As the issuer is short lived, it faces a lemons problem, and can only include risky assets - if investors believed it included safe assets it would switch them for risky assets. This is the second role of ratings in our model; reputation allows the CRA to overcome the lemons problem and sell safe assets. If the issuer decides not to deal with the CRA, it then receives an outside option where it sells as many risky assets/securities as possible to unconstrained investors, $\rho^I = 2w_U$ for a payoff of:

$$w_U(1 - 2r). \tag{3}$$

This outside option partially determines the fee paid to the CRA (which is a share of the surplus).

2.4.2 Reputation

We focus on a monopoly rating agency. The CRA reduces the lemons problem through the reputation it acquires over time. There are two types of rating agencies: truthful (T) and opportunistic (O).⁸ The opportunistic CRA's announcement and its choice of tranche structure depend on its incentives. The truthful CRA is behavioral in the sense that it (a) is restricted to truthful announcements and (b) structures the securities assuming that all investors

⁸This follows the approach of Fulghieri, Strobl, and Xia (2014) and Mathis, McAndrews, and Rochet (2009) (who, in turn, follow the classic approach of modeling reputation of Kreps and Wilson, 1984, and Milgrom and Roberts, 1984).

believe it is truthful.⁹ The literature generally uses the behavioral player as a device to create reputational incentives for the opportunistic player. In our model, this limits the amount of rating inflation that the opportunistic CRA chooses in the first period.

Our model has two periods. The CRA is the same for both periods. The probability that investors place on facing a truthful CRA at the beginning of the period is given by θ_t , where $t \in \{1, 2\}$. In period 1, the probability is a prior given by nature, and in period 2, the probability is a posterior. We assume the prior, the structure of the game, and payoffs, are common knowledge to investors, issuers, and the opportunistic CRA.

The CRA perfectly observes the quality of the issuer's assets. As part of its services, the CRA structures and rates the securities offered by the issuer for a fee equal to a fraction $f > 0$ of the surplus generated. We assume the fraction f is exogenous and it is the same in both periods and for both types of CRAs.¹⁰ The actual fee is paid when the payoffs are realized.

There is a different issuer in each period. The issuer knows the type of the CRA. Due to the surplus sharing rule, the interests of the CRA and the issuer are perfectly aligned, and the CRA can easily signal its type (if needed) to the issuer through the asset composition.

While in practice the issuer will initially structure the securities and get feedback from the rating agencies about modifications necessary to achieve certain ratings,¹¹ we incorporate this back and forth into one step for simplicity. We defined the issuer's outside option in equation (3). The CRA's outside option is zero.

Denote a message that is sent to investors by a CRA of type $d \in \{T, O\}$ by $\bar{m}^d = (\bar{\sigma}^d, \bar{\rho}^d)$, where $\bar{\sigma}^d$ ($\bar{\rho}^d$) is the reported measure of safe (risky) assets in the pool. Denote the true measures of assets by σ^d and ρ^d . We assume that the total quantity of assets $\sigma^d + \rho^d$ is observable. The term z^d represents the face value of the senior tranche, which is also observable.

In each period, an action by a CRA of type d is defined by $s^d = (\bar{m}^d, \sigma^d, \rho^d, z^d)$.

⁹We follow the reputation literature to assume a behavioral player. Nevertheless, point (b) rarely arises in the literature. Hartman-Glaser (2017) is the only example we know in which the behavioral player can make a strategic decision against another player/type (a truthful issuer decides how much to retain of a security). In another version of this paper, we allow the truthful CRA to be strategic in structuring the securities.

¹⁰Fulghieri, Strobl, and Xia (2014) also use a surplus splitting rule.

¹¹See details in Griffin and Tang (2012). Rating agencies also provide their basic model to issuers to communicate further. For example, Benmelech and Dlugosz (2009) write, "The CDO Evaluator software [from S&P, publicly available] enabled issuers to structure their CDOs to achieve the highest possible credit rating at the lowest possible cost... the model provided a sensitivity analysis feature that made it easy for issuers to target the highest possible credit rating at the lowest cost."

Since we assume that the true total quantities of assets are observable to investors, any message \bar{m}^d must fulfill $\bar{\sigma}^d + \bar{\rho}^d = \sigma^d + \rho^d$. If the CRA is truthful, then the strategy space is further restricted such that $(\bar{\sigma}^T, \bar{\rho}^T) \equiv (\sigma^T, \rho^T)$.

In the model, the announcement of \bar{m}^d and the observation of $\sigma^d + \rho^d$ and z^d supply investors with enough information to directly calculate the expected values and probabilities of realizations of the senior and junior tranches. The opportunistic CRA may substitute risky assets for safe assets and thus worsen the actual payoffs and inflate ratings. We say that *rating inflation* occurs when the opportunistic CRA includes a larger amount of risky assets than reported.

To summarize, the timing of the first period game with is as follows:

0. Nature draws the type of the CRA. With probability θ_1 it's truthful and with probability $1 - \theta_1$ it's opportunistic.
1. The CRA specifies the measures of safe and risky assets to be included, the face value of the senior tranche z^d , and that ratings will be produced.
2. The CRA reports measures of safe and risky assets (ratings) to investors.¹²
3. Investors observe the size of the pool of assets, the face value of the senior tranche, and the announcement of the CRA, and buy securities at their conditional expected value.
4. Payoffs are realized and the CRA receives its fee.

We suppose that steps 1 to 4 are repeated in a second period. Based on the message reported and the payoff realization in the first period, the second-period investors update their prior about the type of the CRA accordingly. If the messages of the two types of CRAs are different, investors can deduce the type of the CRA with probability one. If the messages are identical, investors update their priors by observing the realization of security payoffs at the end of the first period. We describe this updating process in detail in our analysis of the first period.

In what follows, we work our way backward in solving the model, beginning with the second period. We use Perfect Bayesian Equilibrium as our equilibrium concept.

¹²The actual ratings to be reported could also be included in the contract, without altering the conclusions in the paper. However, such an inclusion could expose the opportunistic CRA (and the issuer) to lawsuits should the ratings not match the proposed asset quality. As the type of the CRA is known to the issuer (see earlier discussion), the ratings will be known by the issuer without contracting on them.

3 The Second Period

In this section, we will analyze the second period of the game. Since this is the last period, the opportunistic CRA has no reputation concerns. We begin by describing the behavior of the truthful CRA, as it does not act strategically (i.e., it doesn't take into account the fact that investors perceive it to be opportunistic with positive probability). Note that the behavior of the truthful CRA will be the same in both periods, as it doesn't recognize the need for reputation management and the issuers in each period have the same characteristics. Thereafter we derive the equilibrium behavior of the opportunistic CRA.

3.1 The truthful CRA

The truthful CRA maximizes its revenues disregarding the behavior of the opportunistic CRA. It solves the following program:

$$\begin{aligned}
& \max_{\sigma^T, \check{\rho}^T, \check{x}^T \geq 0} \left\{ \check{\sigma}^T (1 - s) + \check{\rho}^T (1/2 - r) - w_U (1 - 2r) \right\} \\
& \quad s.t. \quad \sigma - \check{\sigma}^T \geq 0 \\
& \quad \quad (1 - \check{x}^T)^2 - R \geq 0 \\
& \quad \quad w_C - \check{\sigma}^T - \check{\rho}^T \check{x}^T (1 - \check{x}^T/2) \geq 0 \\
& \quad \quad w_U - \check{\rho}^T (1 - \check{x}^T)^2 / 2 \geq 0
\end{aligned}$$

The objective function is the expected surplus generated by the CRA from selling securities to investors. The expected fee is a fraction $f > 0$ of this expression. The first constraint is the resource constraint for safe assets. The second constraint is the rating constraint, which implies that the probability that the senior tranche will pay out less than its full amount is P or lower. The third constraint is the budget constraint of constrained investors, which makes use of the expected payoff of the senior tranche in equation (1). The fourth constraint is the budget constraint of unconstrained investors, which makes use of the expected payoff of the junior tranche in equation (2).

In the Appendix, we prove that the budget constraint of the unconstrained investors binds in any solution (σ^T, ρ^T, x^T) to the truthful CRA's optimization program, and moreover that any solution has $\rho^T > 0$. This allows us to simplify the program considerably and to prove the following proposition, where we make use of the definition $W \equiv w_C + w_U - w_U/R$, which is discussed further below.

Proposition 1 *The solution to the truthful CRA's problem has the following properties:*

- i. *If $W \leq 0$, the budget constraint of the constrained investors and the non-negativity constraint for safe assets bind, implying:*

$$\begin{aligned}\sigma^T &= 0, \\ \rho^T &= 2(w_C + w_U), \\ x^T &= 1 - \sqrt{w_U/(w_C + w_U)}.\end{aligned}$$

- ii. *If $0 < W \leq \sigma$, the rating constraint and the budget constraint of the constrained investors bind, implying:*

$$\begin{aligned}\sigma^T &= w_C + w_U(1 - 1/R), \\ \rho^T &= 2w_U/R, \\ x^T &= P.\end{aligned}$$

- iii. *If $W > \sigma$, the rating constraint and the resource constraint bind, implying:*

$$\begin{aligned}\sigma^T &= \sigma, \\ \rho^T &= 2w_U/R, \\ x^T &= P.\end{aligned}$$

The variable W represents the value of safe assets that the truthful CRA prefers to allocate to the pool when all it has to worry about is the rating constraint and the budgets of investors. When constrained investors have more wealth, the truthful CRA finds it more profitable to include more safe assets and appeal to them. The opposite effect holds for unconstrained investors. When the rating constraint is tighter (higher R), the truthful CRA includes more safe assets to satisfy it. Since the measures of included assets can't be negative, the truthful CRA will not include any safe assets when W is negative. Finally, if W is larger than the supply of safe assets, σ , the truthful CRA will include a measure of safe assets equal to this supply.

We will start by solving for an equilibrium where a positive amount of safe assets are included in the securitization pool ($W > 0$), and revisit the case where only risky assets are included in the pool ($W \leq 0$) in the next section. Note that for $W > 0$, $\rho^T = 2w_U/R$, whereas $\sigma^T = \min\{w_C + w_U(1 - 1/R), \sigma\}$.

3.2 The opportunistic CRA

In this subsection, we examine the strategic choices of the opportunistic CRA.

Lemma 1 *In equilibrium, any securitization rated by the opportunistic CRA in the second period will only include risky assets in the pool of assets.*

The lemons problem here has two elements. First, risky assets have a higher margin than safe ones for the CRA. Second, investors do not observe the actual composition of assets. Therefore if the opportunistic CRA includes some safe assets and investor beliefs are fixed such that they anticipate these safe assets will be included, the opportunistic CRA has the incentive to replace the safe assets with risky assets. This problem arises directly from the existence of constrained investors - it is profitable to include safe assets in the pool (when $W > 0$) to be able to sell to constrained investors, which then allows for rating inflation (replacing safe assets with risky ones) to take place.¹³

We say that the two types of CRAs *pool* at an information set where they are both called upon to act, if they report the same messages, include the same quantity of assets in their asset pools, and choose the same face value of their senior tranches in equilibrium. If the two types of CRAs do not pool at such an information set, we say that they *separate*.

Lemma 2 *For $W > 0$, if the type of the opportunistic CRA is not fully revealed in the first period, the opportunistic CRA pools with the truthful CRA in the second period.*

If there were an equilibrium where the two types separated in the second period in spite of a positive posterior, the opportunistic CRA would be recognized, and would therefore only be able to sell securities backed by risky assets. For $W > 0$, this gives strictly lower revenues than the truthful CRA's pool that included safe assets, and therefore there is a profitable deviation. It is easy to see that a pooling equilibrium exists for off-the-equilibrium path beliefs such that any deviation is believed to be the opportunistic CRA for sure.

¹³Note that this differs from rating inflation in much of the theoretical literature on CRAs. There, the issuer usually has one good (NPV positive) or bad (NPV negative) asset to sell/issue a bond against. Investors don't want to invest in the bad asset, and rely on the CRA to screen the asset. However, the CRA may earn more by rating a bad asset good. For examples of this type of model, see Fulghieri et al (2014) and Piccolo and Shapiro (2018). Here, there are no bad assets, as risky assets are also NPV positive and investors are risk neutral.

For a given posterior $\theta_2 \in (0, 1)$, the corresponding second-period surplus created by the opportunistic CRA is given by:

$$\begin{aligned}\pi_2(\theta_2) &= \theta_2 (\sigma^T + \rho^T/2) + (1 - \theta_2) (\sigma^T + \rho^T) / 2 \\ &\quad - (\sigma^T + \rho^T) r - w_U (1 - 2r) .\end{aligned}$$

The revenues received depend on investors' beliefs about the type of the CRA. With probability θ_2 , the CRA is truthful, and includes σ^T safe assets worth 1 per unit and ρ^T risky assets worth on average $1/2$ per unit. With the complementary probability, the CRA is opportunistic and includes only risky assets, but ensures that the number of assets is equal to the total number of assets the truthful CRA includes ($\sigma^T + \rho^T$). As these assets are all risky, they are worth on average $1/2$. The opportunity cost of selling off those assets for the issuer is the quantity of assets multiplied by their payoff if retained r . Lastly, the issuer can earn $w_U (1 - 2r)$ by securitizing without the CRA's help.

We can simplify this expression:

$$\pi_2(\theta_2) = \theta_2 \sigma^T / 2 + (\sigma^T + \rho^T - 2w_U) (1 - 2r) / 2.$$

If the type of the opportunistic CRA is revealed to investors in period one ($\theta_2 = 0$), then it will include only risky assets. We derive the optimal tranching by the opportunistic CRA for this case in the following lemma (see Appendix B for a proof):

Lemma 3 *If investors know the type of the opportunistic CRA ($\theta_2 = 0$) in period 2, it will include a measure $\rho_2^O = \rho^T = 2w_U/R$ of risky assets and no safe assets in the pool. The corresponding surplus is given by:*

$$\begin{aligned}\pi_2(0) &= (1 - 2r) (\rho^T - 2w_U) / 2 \\ &= (1 - 2r) w_U (1/R - 1) .\end{aligned}$$

Notice that even though only risky assets are included in the result of the lemma, the opportunistic CRA can still sell securities to constrained investors, as they are made safer through tranching.

Lastly, we derive the difference between the second period surplus when the CRA is pooling and when it is separating:

$$\pi_2(\theta_2) - \pi_2(0) = \sigma^T (\theta_2 + 1 - 2r) / 2 > 0. \quad (4)$$

4 The First Period

In this section, we analyze the strategic choice of the opportunistic CRA in the first period. The following lemma demonstrates that there are no separating equilibria in the first period.

Lemma 4 *There is no equilibrium where the opportunistic CRA separates in the first period.*

If the opportunistic CRA separated in the first period, it would have a strictly lower payoff in the second period than the payoff from pooling (see equation (4)). The truthful CRA makes the same choices in the first period as in the second period. Therefore, the opportunistic CRA can also guarantee a higher payoff by pooling with the truthful CRA in the first period, using the logic of Lemma 2 and the knowledge that it will get a higher payoff than separating in the second period. We can thus restrict ourselves to looking only at pooling equilibria in the first period.

In any pooling equilibrium, the opportunistic CRA chooses the same size of the pool of assets as the truthful CRA ($\sigma^T + \rho^T$) and the same face value of the senior tranche (z^T), but may include a larger measure of risky assets, ρ_1^O .

A property of the uniform distribution is used here to simplify the problem. The likelihood ratio between the density function of the aggregate payoff of the assets pooled by the truthful CRA and the aggregate payoff of the assets pooled by the opportunistic CRA is constant for aggregate payoff realizations above σ^T . Hence, if investors know the aggregate payoff realization is above this level, then no additional information can be learned about the type of the CRA from knowing the exact aggregate payoff realization.

Moreover, if the aggregate payoff could have come from either type of CRA, then no inference can be made about the type of the CRA by observing the different payoff realizations of the two tranches. The reason is that, identical aggregate payoff realizations are split in the same fashion between the junior and senior tranches created by truthful and opportunistic CRAs.

In conclusion, no inference can be made about the type of the CRA, unless the aggregate payoff realization is below σ^T , the minimum payoff of the truthful CRA's pool, as this could not have been generated by a truthful CRA. Given the distribution of the risky asset's payoffs, the opportunistic type is therefore discovered ($\theta_2 = 0$) with probability:

$$\Pr((\sigma^T + \rho^T - \rho_1^O) + \rho_1^O \tilde{X} < \sigma^T) = 1 - \rho^T / \rho_1^O.$$

The probability depends on the amount of risky assets the opportunistic CRA's includes in the pool relative to the amount that it reports/the truthful

CRA includes. As more risky assets are put in relative to the reported amount, the probability of being discovered increases.

With the complementary probability, ρ^T/ρ_1^O , the type of the opportunistic CRA will not be revealed by the payoff realization and instead investors will increase their posterior probability that the CRA is truthful in the second period to:

$$\theta_2 = \theta_1 / (\theta_1 + (1 - \theta_1)(\rho^T/\rho_1^O)).$$

In any pooling equilibrium, the opportunistic CRA's choice of the measure of risky assets to include in the pool, ρ_1^O , must be optimal given the first-period choice of the truthful CRA, (σ^T, ρ^T, z^T) . Furthermore, the beliefs of investors are held fixed when the opportunistic CRA chooses the amount of risky assets to include, meaning that the choice does not affect the revenues received. We denote the amount of risky assets that investors expect to be included in the pool by an opportunistic CRA by ρ_1^{Oe} . More specifically, ρ_1^O must be a solution to the following maximization problem:

$$\max_{\check{\rho}_1^O \in [\rho^T, \sigma^T + \rho^T]} \left\{ \begin{array}{l} \theta_1 (\sigma^T + \rho^T/2) + (1 - \theta_1) (\sigma^T + \rho^T - \rho_1^{Oe} + \rho_1^{Oe}/2) \\ - (\sigma^T + \rho^T - \check{\rho}_1^O) s - \check{\rho}_1^O r - w_U(1 - 2r) + \\ \delta \pi_2 (\theta_1 / (\theta_1 + (1 - \theta_1)(\rho^T/\rho_1^{Oe}))) \rho^T / \check{\rho}_1^O + \\ + \delta \pi_2(0) (1 - \rho^T/\check{\rho}_1^O) \end{array} \right\}$$

The first line represents the revenues in the first period. As the price depends on the equilibrium beliefs of investors, and the quantity is observable and identical for both types of CRAs, revenues are held fixed in the decision problem for the opportunistic CRA. The second line represents the issuer's opportunity cost of not holding on to the assets and the payoff it could obtain without the CRA. The third and fourth lines represent the expected second-period surplus, which depends on the probability that the CRA is discovered or not. Note that the probability depends on the opportunistic CRA's choice, as more distortion away from the reported value make it more likely to be discovered. The equilibrium second-period surplus does not depend on this choice, as the beliefs of investors about the updated type of the CRA are held fixed. Thus, the trade-off for the opportunistic CRA is to increase its payoff by retaining more safe assets and placing more risky assets in the pool versus having a higher probability of enjoying future rents. Finally, as the CRA receives a constant fraction of the surplus f in both periods, we leave it out of the maximization problem.

Due to the assumption of a uniformly distributed \tilde{X} , the above program is convex in $\check{\rho}_1^O$. To see this, note that the second derivative equals:

$$2\delta (\pi_2(\theta_2) - \pi_2(0)) \rho^T / (\check{\rho}_1^O)^3 > 0.$$

Hence, we will have a corner solution. The two possible solutions are (a) maximal rating inflation, in which the opportunistic CRA includes only risky assets ($\rho_1^O = \sigma^T + \rho^T$) or (b) zero rating inflation, where the opportunistic CRA includes the same measure of risky assets as the truthful CRA ($\rho_1^O = \rho^T$). Under maximal rating inflation, the posterior belief about the probability that the CRA is truthful is:

$$\theta'_2 = \frac{\theta_1}{\theta_1 + (1 - \theta_1)\rho^T/(\sigma^T + \rho^T)}.$$

An equilibrium with maximal inflation can be sustained if the expected surplus with the truthful CRA's amount of risky assets $\rho_1^O = \rho^T$ is smaller than the expected surplus with the maximal amount of risky assets $\rho_1^O = \sigma^T + \rho^T$ under the beliefs $\theta_2 = \theta'_2$:

$$\delta\pi_2(\theta'_2) - \sigma^T s - \rho^T r \leq \frac{\rho^T}{\sigma^T + \rho^T} \delta\pi_2(\theta'_2) + \frac{\sigma^T}{\sigma^T + \rho^T} \delta\pi_2(0) - (\sigma^T + \rho^T)r.$$

Notice that here, the first period revenues are the same in both scenarios and are not included, as this looks at a deviation where beliefs are held fixed. The first period opportunity cost of including assets differ, as the assets included are different.

For zero rating inflation, the posterior belief is equal to the prior, θ_1 . Therefore, an equilibrium with zero rating inflation can be sustained if the expected surplus with the same amount of risky assets as the truthful CRA, $\rho_1^O = \rho^T$, is larger than the expected surplus with the maximal amount of risky assets $\rho_1^O = \sigma^T + \rho^T$ under the beliefs $\theta_2 = \theta_1$:

$$\delta\pi_2(\theta_1) - \sigma^T s - \rho^T r \geq \frac{\rho^T}{\sigma^T + \rho^T} \delta\pi_2(\theta_1) + \frac{\sigma^T}{\sigma^T + \rho^T} \delta\pi_2(0) - (\sigma^T + \rho^T)r.$$

The above implies the following equilibrium actions:

$$\rho_1^O = \begin{cases} \sigma^T + \rho^T & \text{for } \delta(\pi_2(\theta'_2) - \pi_2(0)) \leq (\sigma^T + \rho^T)(s - r) \\ \rho^T & \text{for } \delta(\pi_2(\theta_1) - \pi_2(0)) \geq (\sigma^T + \rho^T)(s - r) \end{cases} \quad (5)$$

The intuition behind these expressions is simple. If the current gain from inflating ratings (substituting risky assets for safe assets) is higher than the present value of the future surplus from being more truthful, then the opportunistic CRA prefers to inflate ratings as much as possible. If the present value of the future surplus from truthful ratings is higher than the current gain from inflating ratings, then the opportunistic CRA prefers not to inflate ratings at all.

These expressions define discount factors for which maximal rating inflation and truth-telling are equilibria. Maximal rating inflation is an equilibrium when the discount factor is below a cutoff, which we denote $\underline{\delta}$. Plugging in second-period payoffs, this cutoff is defined as:

$$\underline{\delta} \equiv \frac{2(s-r)}{\theta'_2 + 1 - 2r} \frac{\sigma^T + \rho^T}{\sigma^T} \quad (6)$$

No rating inflation is an equilibrium when the discount factor is above a cutoff, which we denote $\bar{\delta}$. Plugging in second-period payoffs, this cutoff is defined as:

$$\bar{\delta} \equiv \frac{2(s-r)}{\theta_1 + 1 - 2r} \frac{\sigma^T + \rho^T}{\sigma^T} \quad (7)$$

Note also that since second-period surplus is increasing in the posterior beliefs, $\theta_1 < \theta'_2$ and thus $\bar{\delta} > \underline{\delta}$, for $W > 0$. This means that there is a range of δ such that neither maximal nor minimal inflation is part of an equilibrium. For such δ , we conjecture that there are equilibria in mixed strategies where the opportunistic CRA chooses maximal rating inflation with probability $p \in (0, 1)$ and zero rating inflation with probability $1 - p$ in period one.¹⁴ The posterior in period two if the opportunistic CRA is not discovered will be:

$$\theta_2^{mix}(p) \equiv \frac{\theta_1}{\theta_1 + (1 - \theta_1)(1 - p\sigma^T/(\sigma^T + \rho^T))}$$

Note that $\theta_2^{mix}(p)$ is an increasing function of p such that $\theta_2^{mix}(0) = \theta_1$ and $\theta_2^{mix}(1) = \theta'_2$.

The opportunistic CRA will be indifferent between the two extremes whenever:

$$\delta(\pi_2(\theta_2^{mix}(p)) - \pi_2(0)) = (s - r)(\sigma^T + \rho^T). \quad (8)$$

Hence, for every δ such that

$$\frac{(s-r)(\sigma^T + \rho^T)}{\pi_2(\theta'_2) - \pi_2(0)} < \delta < \frac{(s-r)(\sigma^T + \rho^T)}{\pi_2(\theta_1) - \pi_2(0)},$$

there is a unique mixed equilibrium p^* . We can solve for this equilibrium using the indifference condition (equation (8)) and plugging in the profits from equation (4):

$$\delta\sigma^T(\theta_2^{mix}(p^*) + 1 - 2r) = 2(s-r)(\sigma^T + \rho^T),$$

¹⁴Due to the convexity of the objective function, we can rule out mixed equilibria where asset allocations with intermediate inflation are played with positive probability.

giving:

$$p^* = \frac{1}{1 - \theta_1} \left(1 + \rho^T / \sigma^T - \frac{\theta_1}{2(s - r) / \delta - (1 - 2r)\sigma^T / (\sigma^T + \rho^T)} \right). \quad (9)$$

We have thus established the following results.

Proposition 2 *If $W > 0$, in the first period, the opportunistic CRA's equilibrium choice is:*

- (a) *inflating maximally if $\delta \leq \underline{\delta}$,*
- (b) *reporting truthfully if $\delta \geq \bar{\delta}$, and*
- (c) *inflating maximally with probability $p^* \in (0, 1)$ and reporting truthfully with probability $1 - p^*$ if $\delta \in (\underline{\delta}, \bar{\delta})$.*

Recall that for any $W > 0$, the measure of risky assets included by the truthful CRA is given by the formula $\rho^T = 2w_U/R$ and the measure of safe assets $\sigma^T = \min\{w_C + w_U(1 - 1/R), \sigma\}$. We make these substitutions in the expressions for $\underline{\delta}$, $\bar{\delta}$ and p^* in Appendix C.

4.1 Only risky assets

If $W \leq 0$, the truthful CRA includes only risky assets. This implies that the opportunistic CRA will not substitute safe assets for risky ones in equilibrium and will include only risky assets as well. The opportunistic CRA will issue the same type of securities as the truthful CRA in both periods and there will be *no rating inflation*. Even though there are only risky assets, due to tranching, there will still be a safer senior tranche (with $x^T = 1 - \sqrt{w_U/(w_C + w_U)}$) and a riskier junior tranche.

Rewriting the inequality $W < 0$ as

$$w_C < w_U(1/R - 1)$$

reveals when it is preferable to include only risky assets and not inflate ratings. First, when the wealth/demand of constrained investors is sufficiently low there is little need to use safe assets. Second, when the wealth/demand of unconstrained investors is sufficiently high, it is easier to sell risky assets. Lastly, when the rating constraint is sufficiently lax (low R), it is easier to create safe securities out of risky assets.

This equilibrium therefore will be more likely to occur when there is less demand for safe securities, more demand for risky ones, and the rating constraint is lax.

In addition to the absence of rating inflation, this equilibrium configuration has the benefit of maximizing the expected surplus given this set of parameters, since the budget of the constrained and unconstrained investors is exhausted, the profit margin is higher for risky than for safe assets, and the opportunistic CRA will be hired in period two with probability one.

5 Rating Inflation

Rating inflation is a direct measure of surplus in our model. It measures:

- The size of the lemons problem: The first best allocation is given by the solution of the truthful CRA; when $W > 0$, it is optimal to sell safe assets as well as risky assets. Distortions away from this allocation reduce surplus. The opportunistic CRA can't resist substituting risky assets for safe ones, but it is to its own detriment, as investors take into account this behavior. The extreme form of this behavior is when it fully inflates ratings, and includes only risky assets. Reputation allows the opportunistic CRA to partially circumvent this problem and include some safe assets.
- How much risk investors are taking on: Regulators often outsource the assessment of how much risk certain financial institutions (banks, insurance companies, pension funds) take on to the ratings industry. We don't model directly why the investment grade threshold is important, but its use in monitoring for regulatory purposes demonstrates that there are negative consequences from circumventing the threshold. Becker and Ivashina (2015) and Eling (2018) document institutions' (insurance companies and banks, respectively) efforts to arbitrage ratings by reaching for yield. Rating inflation in our model is a measure of by how much the opportunistic CRA is violating the investment grade threshold - causing a buildup of risk and facilitating the reach for yield.

In this section, we will first analyze the effect of small changes of some of the parameters on rating inflation and, thereafter, large changes.

5.1 Local comparative statics

We use three metrics to measure increases in rating inflation for $W > 0$:

1. There is no rating inflation when $\delta \geq \bar{\delta}$. Therefore if $\bar{\delta}$ increases, the range of discount factors for which there is no rating inflation shrinks.

2. There is maximal rating inflation when $\delta \leq \underline{\delta}$. Therefore if $\underline{\delta}$ increases, the range of discount factors for which there is maximal rating inflation increases.
3. In the mixed equilibrium ($\underline{\delta} < \delta < \bar{\delta}$), p^* is the probability with which maximal rating inflation is chosen. Thus, we interpret an increase in p^* as an increase in rating inflation.

We examine how the parameters of the model affect rating inflation in the following proposition:

Proposition 3 *For $W > 0$, rating inflation is:*

1. *Decreasing in the tightness of the rating constraint and the prior that the CRA is truthful. Formally, p^* , $\bar{\delta}$ and $\underline{\delta}$ are decreasing in R and θ_1 .*
2. *Increasing in the wealth of the unconstrained investors and the retention value of the safe assets. Formally, p^* , $\bar{\delta}$ and $\underline{\delta}$ are increasing in w_U and s .*
3. *Decreasing in the wealth of constrained investors if $W \leq \sigma$. Formally, p^* , $\bar{\delta}$ and $\underline{\delta}$ are decreasing in w_C .*
4. *Decreasing in the supply of safe assets if $W > \sigma$. Formally, p^* , $\bar{\delta}$ and $\underline{\delta}$ are decreasing in σ .*

The above results follow immediately from the expressions for p^* , $\bar{\delta}$ and $\underline{\delta}$ (see Appendix C), but we include a proof of the comparative statics with respect to the prior θ_1 in Appendix D, since it is slightly more involved.

We highlight three intriguing results from the proposition and summarize the rest. First, rating inflation is decreasing in the tightness of the rating constraint (an increase in R). There are two main effects that reduce the incentives to inflate ratings. The first is that a tighter rating constraint increases the probability that the opportunistic CRA is discovered if it inflates ratings in the first period. The second is that a tighter rating constraint increases the difference between the second-period profits when the inflating opportunistic CRA is not discovered and when it is discovered, making it more desirable to not be discovered (and therefore not inflate). This difference in profits increases because (i) a tighter rating constraint induces the truthful CRA to include more safe assets (increasing the revenues for an opportunistic CRA who pools with the truthful CRA) and (ii) the opportunistic CRA has a better reputation conditional on not being caught. These two effects dominate a third effect: a possible increase in the opportunity cost of

being truthful in the first period.¹⁵ The increase in the tightness of the rating constraint may come from investor scrutiny or regulation, and is effective.

Second, rating inflation is increasing in the wealth of unconstrained investors. Increasing demand from such investors makes the truthful CRA include more risky assets in the pool, decreasing the likelihood that an inflating opportunistic CRA is discovered. In addition, it reduces the difference between the second-period surplus when the opportunistic CRA is discovered and when it is not. Thus an inflow of money/investment (perhaps due to easy lending) by unconstrained investors such as hedge funds can foster an environment of rating inflation.

Third, rating inflation increases in the value of retaining safe assets, as the opportunistic CRA will have a higher desire to substitute risky for safe assets. The value of retaining safe assets may depend on the demand for safe assets, which have extra value due to their money-like features and use for collateral (Diamond, 2017).

We briefly summarize the other effects. Rating inflation decreases in the prior that the CRA is truthful, due to the increase in the period 2 payoff of not getting caught. In the intermediate case ($0 < W \leq \sigma$), when the constrained investors' budget constraint binds, rating inflation decreases in the wealth of constrained investors. The reason is that their wealth improves the second-period surplus, while at the same time increasing the probability that the opportunistic CRA is discovered after the first period. A similar result holds for the amount of safe assets when the resource constraint for safe assets binds (which occurs when $W > \sigma$).

It is instructive to also examine the effect on rating inflation of period one (only) wealth shocks. The shocks may be temporary fluctuations rather than permanent changes as examined above. This is also of interest because wealth here may proxy for demand.

¹⁵A tighter ratings constraint makes the truthful CRA include more safe assets in the first period, increasing the opportunistic CRA's payoff from substituting safe assets for risky ones. This effect is overwhelmed by the effect on the probability of being discovered. To see this, note that the lower discount (upper) cutoff can be written:

$$\frac{\sigma^T (s - r)}{\sigma^T / (\sigma^T + \rho^T)} \frac{1}{\pi_2(\theta_2) - \pi_2(0)},$$

where $\theta_2 = \theta'_2$ ($\theta_2 = \theta_1$). The numerator of the first term is the value of substituting risky for safe assets. When $W \in (0, \sigma)$, this is increasing in R and there is a larger incentive to inflate ratings. The denominator of the first term is the probability that a maximally inflating CRA gets caught. It is straightforward to see that this eliminates the effect of the numerator. This, of course, depends on the technology we have assumed, particularly on the uniform distribution of the payoff of the risky asset.

We denote wealth for the two types of investors in period $t \in \{1, 2\}$ by w_C^t and w_U^t . Analogously, we define $W^t \equiv w_C^t + w_U^t - w_U^t/R$.

Proposition 4 *For $W^1, W^2 > 0$, rating inflation is:*

1. *Increasing in the first-period wealth of unconstrained investors, w_U^1 .*
2. *Increasing in the first-period wealth of constrained investors, w_C^1 , for $W^1 < \sigma$ and independent of w_C^1 otherwise.*

Increasing w_U^1 strictly increases the quantity of risky assets and weakly decreases the quantity of safe assets included in the pool by the truthful CRA (see Appendix E). This decreases the probability of discovery when the opportunistic CRA inflates ratings. It also reduces the posterior that the CRA is truthful in the second period, which decreases second-period profits in the event that the inflating opportunistic CRA is not discovered.

If $W^1 \geq \sigma$, the resource constraint binds and increasing w_C^1 has no effect on the quantity of safe assets included by the truthful CRA. If, on the other hand, $W^1 < \sigma$, then increasing w_C^1 makes the truthful CRA include more safe assets in the first period. This improves the opportunistic CRA's payoff from substituting safe assets for risky ones.

5.2 Global comparative statics

In this subsection, we analyze the effect of large changes in R on rating inflation. Define $\underline{\delta}(R)$ ($\bar{\delta}(R)$) as the lower (upper) discount threshold evaluated at R . Assume $\delta > \bar{\delta}(R = 1)$, and let \underline{R} be defined implicitly by $\delta = \underline{\delta}(R = \underline{R})$ and \bar{R} by $\delta = \bar{\delta}(R = \bar{R})$ (in Appendix F we show that both exist and are unique under this assumption). Figure 2 depicts the discount factor thresholds in (δ, R) space.

Notice that these functions aren't defined for all R . This is because when $W \leq 0$, the truthful CRA does not include any safe assets and hence there is no possibility of rating inflation. Define $\hat{R} \equiv w_U / (w_U + w_C)$, the value of R such that $W = 0$. It follows immediately that $\hat{R} \in (0, 1)$. In Figure 2, $\hat{R} = 0.25$, the point where the discount factor curves asymptotically approach infinity from the right.

Using Proposition 3, it is straightforward to derive the following proposition, where we write $p^*(R)$ to denote the mixed strategy evaluated at $R \in (\underline{R}, \bar{R})$.

Proposition 5 *For any finite $\delta > \bar{\delta}(1)$, in the first period the opportunistic CRA:*

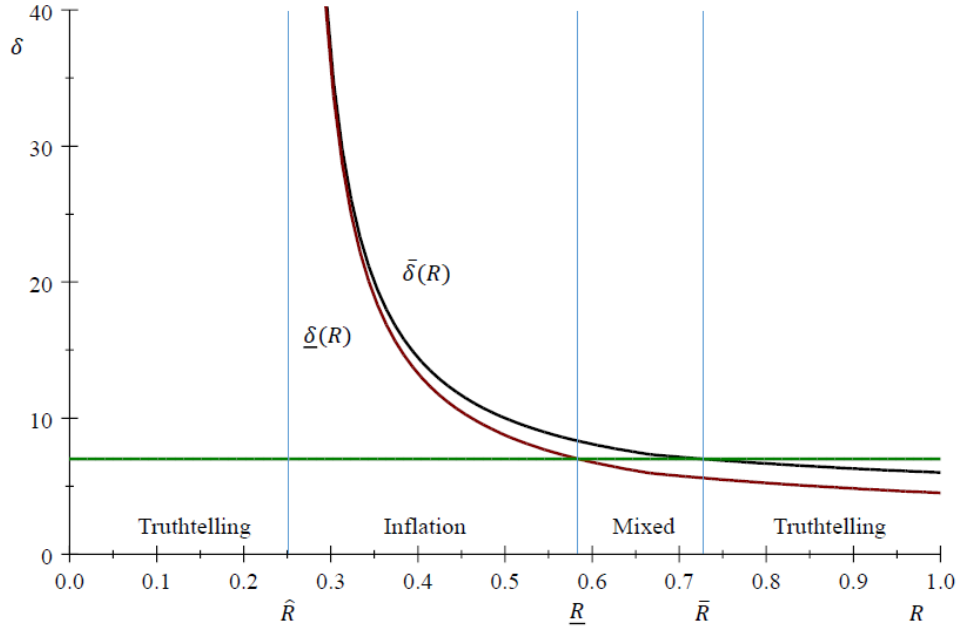


Figure 2: The cutoffs for the discount factor are depicted for the parameter values $w_C = 3$, $w_U = 1$, $\sigma = 2.5$, $\theta = 0.1$, $r = 0.4$, and $s = 0.9$. Assuming a discount factor of $\delta = 7$, we obtain $\hat{R} = 0.25$, $\underline{R} \approx 0.584$, and $\bar{R} \approx 0.727$.

- (a) reports truthfully for $R \leq \hat{R}$
- (b) inflates maximally for $R \in (\hat{R}, \underline{R}]$,
- (c) inflates maximally with probability $p^*(R)$ and reports truthfully with probability $1 - p^*(R)$ for $R \in (\underline{R}, \bar{R})$,
- (d) reports truthfully for $R \in [\bar{R}, 1]$.

This proposition demonstrates that rating inflation may be non-monotonic in the rating constraint R .¹⁶ For low R , it is more profitable for the truthful CRA to include only risky assets. Hence, there is no room for the opportunistic CRA to inflate. For intermediate R , it is more profitable for the truthful CRA to include safe assets. Hence, the opportunistic CRA will inflate ratings, substituting risky assets for safe assets. For high R , part (d) of the Proposition shows that ratings may return to being truthful. Here, the truthful CRA will include more safe assets which makes it easier to discover a cheating opportunistic CRA.

One might imagine that lax rating constraints prevail when financial products are new and have little track record or established models to estimate their risk. In this case the CRA will only include risky assets, but is still able to create some safe securities from them. As these products become more well understood, rating constraints are tightened, but that opens up the possibility of rating inflation. Finally, if rating inflation becomes too pervasive and disruptive, regulatory and investor pressure may increase the tightness of the constraint further, which could reduce rating inflation.

Nevertheless, within the model (i.e. abstracting away from the externalities of rating inflation and risk buildup), the rating constraint is an additional constraint imposed on the solution. Therefore, the solution with lax rating constraint and no inflation has higher surplus than the solution with tight rating constraint and no inflation.

6 Empirical implications

In this section we provide two sets of empirical implications. First, we examine evidence that supports the model. Second, we develop testable implications of the model that have not been analyzed in the literature.

¹⁶Note that if we change the assumption in the proposition to $\delta \leq \bar{\delta}(1)$, the opportunistic CRA will still inflate maximally for intermediate values of R , but it will no longer report truthfully with probability one for high enough R . More precisely, if $\delta \leq \underline{\delta}(1)$, it will inflate maximally for all $R \in (\hat{R}, 1]$; and if $\bar{\delta}(1) < \delta \leq \underline{\delta}(1)$, it will inflate maximally for $R \in (\hat{R}, \underline{R}]$ and play the mixed equilibrium for $R \in (\underline{R}, 1]$.

There is substantial evidence of asymmetric information and strategic asset pool selection for structured finance products. Downing et al (2009) compare the performance of pools of mortgages that are pass-through MBS with no tranching with securitized REMICs (Real Estate Mortgage Investment Conduits) with tranching. The extra layer of securitization and anonymity in sales allows for a selection of worse performing pools due to private information. This is shown to be true with ex-post performance data. Moreover, there is a “lemons spread” due to rational discounting of these securities. An et al (2011) show that portfolio lenders use private information to pass off lower quality loans to commercial mortgage backed securities (CMBS). Conduit lenders, who originate loans for direct sale into securitization markets do not select loans and hence have higher quality loans conditioning on the observables. The analysis shows a lemons discount for portfolio loans. This lemons discount is lower for multifamily loans, which have lower levels of uncertainty and lender private information than retail, office, and industrial loans. Elul (2011) demonstrates that securitized mortgages perform worse than portfolio loans, with the largest differences in prime mortgages in private (non-GSE) securitizations, consistent with the presence of adverse selection. Ashcraft et al (2011) find that the MBS deals that were most likely to underperform were the ones with more interest-only loans (because of limited performance history) and lower documentation, that is, loans that were more opaque or difficult to evaluate.

We find that ratings inflation is an important element of structured finance. Cornaggia et al (2017) find that structured products are overrated compared to all other asset classes.¹⁷ Ashcraft et al (2011) find that as MBS issuance volume shot up between 2005 and mid-2007, ratings quality declined. Specifically, subordination levels¹⁸ for subprime and Alt-A MBS deals decreased over this period when conditioning on the overall risk of the deal. Subsequent ratings downgrades for the 2005 to mid-2007 cohorts were dramatically larger than for previous cohorts. Vickery (2012) shows that ratings inflation occurred for subprime mortgage backed securities at all investment grade rating levels, not just AAA. Griffin and Tang (2012) show that CRA adjustments to their models’ predictions of credit risk in the CDO market were positively related to future downgrades. These adjustments were overwhelmingly positive and the amount adjusted (the width of

¹⁷Gorton and Metrick (2012) also show that AAA-rated asset backed securities have significantly higher cumulative default rates compared to AAA-rated corporate bonds. This is also true for lower rating categories, but the differences lessen as ratings worsen.

¹⁸The subordination level they use is the fraction of the deal that is junior to the AAA tranche. A smaller fraction means that the AAA tranche is less “protected” from defaults, and therefore less costly from the issuer’s point of view.

the AAA tranche) increased sharply from 2003 to 2007 (from 6% to 18.2%). He et al (2012) find that top rated MBS tranches sold by larger issuers¹⁹ performed significantly worse (prices drop more) and had higher initial yields than those sold by small issuers during the boom period of 2004 to 2006.

The model gives us several additional predictions that we believe deserve empirical scrutiny. First, we predict an absence of rating inflation for a sufficiently lax rating constraint. When structured finance products were first issued, they received little regulatory scrutiny and rating agencies were still refining their ratings models for these products. Cornaggia et al (2017) offer suggestive evidence that there was little rating inflation before 2002.

Second, when the rating constraint is tighter, we predict positive but decreasing rating inflation as the rating constraint tightens. This is difficult to test, as there are few exogenous shocks to ratings constraints that we are aware of. Nevertheless, Dimitrov et al (2015) examine the effect of Dodd-Frank on corporate bonds. They point to Dodd-Frank significantly increasing CRAs' liability for inaccurate ratings and making it easier for the SEC to impose sanctions on CRAs and to bring claims against CRAs for material misstatements and fraud. This might be examined in the structured finance context as well, though it would be faced with the confounding effect that the structured market dried up significantly right after the financial crisis.

Third, the model predicts that rating inflation is decreasing in the prior that the CRA is truthful. Baghai and Becker (2018) demonstrate that subsequent to a drop in its business volume caused by a negative shock to its reputation in commercial mortgage backed securities, S&P assigned higher ratings than other raters, particularly for large deals and for deals from important issuers.

Fourth, our model predicts rating inflation is increasing in the retention value of safe assets. At the macro-level, the value of safe assets depends on U.S. treasury bond issuance in terms of amount and maturity, the demand for safe assets, and the supply of other money-like claims by the financial sector (see Sundaram, 2015, and Gorton, 2017). Fluctuations in treasury bond issuance might be used to identify changes in the value of safe assets. At the micro-level, the value of safe assets at financial institutions may depend on regulatory constraints (e.g., Becker and Ivashina, 2015; Eling, 2018; and Merrill et al, forthcoming). Stanton and Wallace (2018) study a change in the risk weighting of CMBS. Demir et al (2017) look at Turkey's adoption of Basel II. Nevertheless, any test will have difficulty isolating the prediction, as the retention value of safe assets to the issuer will likely be correlated with

¹⁹They define larger by market share in terms of deals. As a robustness check, they also look at market share in terms of dollars and find similar results.

the value of safe assets to the end investors, the supply of safe assets, and the wealth of investors.

Lastly, the model states that temporary increases in unconstrained or constrained wealth (i.e., an increase in the first period), generally increase rating inflation. As we state in the text, wealth in the model may also proxy for demand. Consistent with this prediction, many papers find that rating inflation was concentrated in the boom years right before the latest financial crisis (e.g. He et al, 2012; Dilly and Mählmann, 2015; and Auh, 2015). In order to test this, one would need an exogenous wealth or demand shock. One direction is provided by Benmelech and Dlugosz (2009), who detail that modifications to the Employee Retirement Income Security Act of 1974 (ERISA) over the past 20 years have gradually expanded the range of structured finance securities that pension funds can hold. This could possibly be used as a shock to constrained wealth/demand.

7 Conclusion

In this paper, we derive the equilibrium of a simple model of security design where a CRA with reputation concerns both designs and rates securities. We show that the observed equilibrium outcome and rating inflation depend on rating quality constraints as well as the relative demand by constrained and unconstrained investors. Intriguingly, for some parameters, the most efficient outcome (which has no rating inflation), is observed for lax rating constraint. The non-monotonicity of rating inflation with respect to the rating constraint may be an important concern to regulators. It would also be of interest to study further possible systemic effects of rating inflation.

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Appendix

A Proof of Proposition 1 (The truthful CRA's solution)

In order to simplify the optimization program, the following lemma is useful.

Lemma 5 *The budget constraint for the unconstrained investors must bind in any solution.*

Proof: Suppose this is not the case. There are two cases to consider: $x^T = 0$ and $x^T > 0$. In the first case, by continuity, ρ^T can be increased without violating the unconstrained investors' budget constraint. Neither will any of the other constraints be violated since none of them depends on $\check{\rho}^T$ for $x^T = 0$. However, this implies that the suggested solution can be improved upon since the objective function is strictly increasing in $\check{\rho}^T$ – a contradiction.

In the second case, by continuity, it is possible to reduce \check{x}^T slightly without violating the unconstrained investors' budget constraint. This relaxes all of the other constraints except the first, which remains unaffected (if $\check{\sigma}^T = w_C$ and $\check{\rho}^T = 0$, the constrained investors' budget constraint cannot be relaxed, but this cannot be a solution since a strictly higher payoff can be achieved by setting $\check{x}^T = 0$, $\check{\sigma}^T = w_C$, and $\check{\rho}^T = 2w_U$). Hence, after the reduction in \check{x}^T it is possible to increase $\check{\rho}^T$ slightly without violating any constraint. Once again, the suggested solution can be improved upon since the objective function is strictly increasing in $\check{\rho}^T$ – a contradiction. ■

Corollary 1 $\rho^T > 0$ in any solution.

Proof: This follows immediately from Lemma 5. ■

The above allows us to solve for x^T from the binding budget constraint:

$$x^T = 1 - \sqrt{2w_U/\check{\rho}^T}.$$

We can now simplify the optimization program by substituting for x^T and rewriting the rating constraint:

$$\begin{aligned} \max_{\check{\sigma}^T, \check{\rho}^T \geq 0} \quad & \{\check{\sigma}^T (1 - s) + \check{\rho}^T (1/2 - r) - w_U (1 - 2r)\} \\ \text{s.t.} \quad & \sigma - \check{\sigma}^T \geq 0 \\ & w_U - R\check{\rho}^T/2 \geq 0 \\ & w_C + w_U - \check{\sigma}^T - \check{\rho}^T/2 \geq 0 \end{aligned}$$

By the Karush–Kuhn–Tucker theorem, in any solution there are multipliers $\lambda_1, \lambda_2, \lambda_3 \geq 0$, one for each constraint, such that the following conditions holds, where the first condition holds with equality if $\check{\sigma}^T > 0$ and the second condition holds with equality due to Corollary 1.

$$\begin{aligned}\frac{\partial L}{\partial \sigma^T} &= 1 - s - \lambda_1 - \lambda_3 \leq 0, \\ \frac{\partial L}{\partial \rho^T} &= 1/2 - r - \lambda_2 R/2 - \lambda_3/2 = 0.\end{aligned}$$

We can immediately rule out certain solutions by observing the following:

1. If $\lambda_2 = 0$, then $\lambda_3 = 1 - 2r$, implying that the budget constraint of constrained investors binds. Substituting in the first first-order condition reveals that this must be negative and thus $\sigma^T = 0$.
2. If $\lambda_1 = 0$ and $\sigma^T > 0$, then $\lambda_3 = 1 - s$ and $\lambda_2 R = 1 - 2r - (1 - s) > 0$. Hence, the rating and the budget constraints of constrained investors bind.
3. If $\lambda_3 = 0$, then $\lambda_1 > 0$ and $\lambda_2 > 0$. Hence, the rating and the resource constraints bind.

This means that we have the following possible solutions:

1. The budget constraints of the constrained investors binds and $\sigma^T = 0$.
2. The budget constraints of the constrained investors and the rating constraint bind.
3. The resource and rating constraints bind.
4. $x^T = 0$.

A.1 Only risky assets

If the budget constraints of the constrained binds and there are no safe assets, we have $\sigma^T = 0$,

$$\begin{aligned}\rho^T &= 2(w_C + w_U), \\ x^T &= 1 - \sqrt{w_U / (w_C + w_U)},\end{aligned}$$

and the expected surplus is

$$(1 - 2r)w_C.$$

A necessary condition for this solution is that the rating condition is not violated. This holds if and only if:

$$w_C + w_U (1 - 1/R) \leq 0.$$

A.2 Budget and rating constraints binding

If the budget constraints of the constrained and the rating constraint bind, we have:

$$\begin{aligned}\sigma^T &= w_C + w_U (1 - 1/R), \\ \rho^T &= 2w_U/R, \\ x &= P.\end{aligned}$$

The associated surplus is given by:

$$(1 - s)(w_C + w_U (1 - 1/R)) + (1 - 2r)w_U (1/R - 1).$$

A necessary condition for this solution is that the resource constraint is not violated, which is true if and only if:

$$w_C + w_U (1 - 1/R) \leq \sigma.$$

In addition, the non-negativity constraint of σ^T must not be violated:

$$w_C + w_U (1 - 1/R) \geq 0.$$

Note that if the last inequality holds with equality, this solution is identical to the one with only risky assets.

A.3 Resource and rating constraints binding

If the resource and rating constraints bind, we have:

$$\begin{aligned}\sigma^T &= \sigma, \\ \rho^T &= 2w_U/R, \\ x^T &= P,\end{aligned}$$

and a surplus of

$$(1 - s)\sigma + (1 - 2r)w_U (1/R - 1).$$

A necessary condition is that the budget constraint of the constrained investors is not violated. This holds if and only if:

$$w_C + w_U (1 - 1/R) \geq \sigma.$$

Note that if the last inequality holds with equality, this solution is identical to the one where the budget and rating constraints bind.

A.4 Safe senior tranche

If the non-negativity constraint for x binds we obtain the following solution: $x^T = 0$, $\sigma^T = \min\{\sigma, w_C\}$, $\rho^T = 2w_U$, with corresponding surplus:

$$(1 - s) \min\{\sigma, w_C\}.$$

A necessary condition for this solution is that the rating constraint is not violated, but this is always true for $x^T = 0$. It is also easy to see that the budget constraint of the constrained investors is not violated. However, it is straightforward to demonstrate that the payoff in this case is strictly less than the payoff in all of the previous cases. Hence, this cannot be a solution to the optimization program.

A.5 Comparing the cases

1. The budget constraints of the constrained investors binds and $\sigma^T = 0$:
 $w_C + w_U (1 - 1/R) \leq 0$.
2. The rating constraint and the budget constraint of the constrained investors bind:
 $0 < w_C + w_U (1 - 1/R) \leq \sigma$.
3. The resource and rating constraints bind:
 $w_C + w_U (1 - 1/R) > \sigma$.

These are the cases listed in Proposition 1. ■

B Proof of Lemma 3 (Opportunistic CRA revealed in period two)

Suppose that the opportunistic CRA's type is revealed in period two. What securities would it sell if it were hired? From Lemma 1, it follows that the opportunistic CRA would not include any safe assets. From analogous arguments to those in Lemma 5, it follows that the budget constraint of the unconstrained investors will bind, meaning that $x_2^O = 1 - \sqrt{2w_U/\check{\rho}_2^O}$ and giving the following optimization program.

$$\begin{aligned}
& \max_{\check{\rho}_2 \geq 0} \{ \check{\rho}_2^O (1/2 - r) - w_U (1 - 2r) \} \\
& \quad s.t. \quad w_U - R\check{\rho}_2^O/2 \geq 0 \\
& \quad w_C + w_U - \check{\rho}_2^O/2 \geq 0.
\end{aligned}$$

It is easy to see that this program has two solutions:

1. $\rho_2^O = 2(w_C + w_U)$ if $W \leq 0$.
2. $\rho_2^O = 2w_U/R$ if $W > 0$. ■

C Discount-factor cutoffs and mixed equilibrium

C.1 Intermediate W

If $W \in (0, \sigma]$, $\rho^T = 2w_U/R$ and $\sigma^T = w_C + w_U - w_U/R$. Substituting in the formulas for the discount-factor cutoffs and the mixed-strategy equilibrium gives:

$$\begin{aligned}
\underline{\delta} &= \frac{2(s-r)}{\frac{\theta_1}{\theta_1 + 2w_U/(R(w_C + w_U) - w_U)} + (1-2r) \frac{w_C + w_U - w_U/R}{w_C + w_U + w_U/R}}, \\
\bar{\delta} &= \frac{2(s-r)}{\frac{\theta_1}{1 + 2w_U/(R(w_C + w_U) - w_U)} + (1-2r) \frac{w_C + w_U - w_U/R}{w_C + w_U + w_U/R}}, \\
p^* &= \frac{1}{1 - \theta_1} \left(\frac{1 + 2w_U/(R(w_C + w_U) - w_U)}{-\frac{\theta_1}{2(s-r)/\bar{\delta} - (1-2r)(w_C + w_U - w_U/R)/(w_C + w_U + w_U/R)}} \right).
\end{aligned}$$

Note that p^* equals zero for $\delta = \bar{\delta}$ and one for $\delta = \underline{\delta}$.

C.2 High W

If $W > \sigma$, $\rho^T = 2w_U/R$ and $\sigma^T = \sigma$. Substituting in the formulas for the discount-factor cutoffs and the mixed-strategy equilibrium gives:

$$\begin{aligned}
\underline{\delta} &= \frac{2(s-r)}{\theta_1/(\theta_1 + 2w_U/(\sigma R)) + (1-2r) \sigma/(\sigma + 2w_U/R)}, \\
\bar{\delta} &= \frac{2(s-r)}{\theta_1/(1 + 2w_U/(\sigma R)) + (1-2r) \sigma/(\sigma + 2w_U/R)}, \\
p^* &= \frac{1}{1 - \theta_1} \left(\frac{1 + 2w_U/(\sigma R)}{-\frac{\theta_1}{2(s-r)/\bar{\delta} - (1-2r)\sigma/(\sigma + 2w_U/R)}} \right).
\end{aligned}$$

Analogously to the previous case, p^* equals zero for $\delta = \bar{\delta}$ and one for $\delta = \underline{\delta}$.

D Proof of Proposition 3 (Local comparative statics with respect to θ_1)

The only comparative static which is non-trivial to demonstrate is the change in p^* with respect to θ_1 . We prove that p^* is decreasing in θ_1 . Define:

$$\begin{aligned} A &\equiv 1 + \rho^T / \sigma^T, \\ C(\delta) &\equiv \frac{1}{2(s-r)/\delta - (1-2r)\sigma^T / (\sigma^T + \rho^T)}, \end{aligned}$$

and note that $A > 1$ since $\rho^T > 0$. Using this notation, the mixed strategy can be written:

$$p^* = \frac{A - \theta_1 C(\delta)}{1 - \theta_1}.$$

This expression equals one for $\delta = \underline{\delta}$. Hence, we can solve for $C(\underline{\delta})$.

$$C(\underline{\delta}) = \frac{A - 1 + \theta_1}{\theta_1}.$$

Differentiating p^* with respect to θ_1 gives:

$$\frac{\partial p^*}{\partial \theta_1} = \frac{A - C(\delta)}{(1 - \theta_1)^2}.$$

We will show that this expression is negative. The mixed strategy is defined for $\delta \in [\underline{\delta}, \bar{\delta}]$ (at the boundaries it is degenerate), but since $C(\delta)$ is increasing in δ , it is enough to show the sign for $\delta = \underline{\delta}$.

$$\begin{aligned} \frac{\partial p^*}{\partial \theta_1}(\delta = \underline{\delta}) &= \frac{A - C(\underline{\delta})}{(1 - \theta_1)^2} \\ &= \frac{1 - A}{\theta_1(1 - \theta_1)} < 0. \end{aligned}$$

■

E Proof of Proposition 4 (First-period wealth shocks)

Suppose that wealth of investors in period $t \in \{1, 2\}$ is (w_U^t, w_C^t) . We will consider the case where rating inflation is possible, $W^1, W^2 > 0$. With investor wealth varying over time, the truthful CRA's choice of asset pool composition (σ_t^T, ρ_t^T) will be different in the two periods. Plugging into the upper discount threshold for $W^1 > 0$ gives:

$$\bar{\delta} \equiv \frac{2(s-r)}{\theta_1 + 1 - 2r} \frac{\sigma_1^T + \rho_1^T}{\sigma_2^T}.$$

Since $\rho_1^T = 2w_U^1/R$ and $\sigma_1^T = \min\{w_C^1 + w_U^1(1 - 1/R), \sigma\}$, this implies that the discount threshold is increasing in w_C^1 for $W^1 < \sigma$ and independent of w_C^1 otherwise. The upper discount threshold is strictly increasing in w_U^1 .

The lower discount threshold can be written as:

$$\underline{\delta} \equiv \frac{2(s-r)}{\theta'_2 + 1 - 2r} \frac{\sigma_1^T + \rho_1^T}{\sigma_2^T},$$

where

$$\theta'_2 = \frac{\theta_1}{\theta_1 + (1 - \theta_1)\rho_1^T/(\sigma_1^T + \rho_1^T)}.$$

It is straightforward to demonstrate that $\bar{\delta}$ is increasing in σ_1^T and ρ_1^T . This implies that $\bar{\delta}$ is increasing in w_C^1 for $W^1 < \sigma$ and independent of w_C^1 otherwise. The discount threshold is strictly increasing in w_U^1 .

Lastly, in the equilibrium where the CRA randomizes, the probability of maximal inflation can be written:

$$p^* = \frac{1}{1 - \theta_1} \frac{1}{\sigma_1^T} \left(\sigma_1^T + \rho_1^T - \frac{\theta_1}{2 \frac{s-r}{\delta \sigma_2^T} - \frac{1-2r}{\rho_1^T + \sigma_1^T}} \right).$$

If $W^1 \geq \sigma$:

$$p^* = \frac{1}{1 - \theta_1} \frac{1}{\sigma} \left(\sigma + 2w_U^1/R - \frac{\theta_1}{2 \frac{s-r}{\delta \sigma_2^T} - \frac{1-2r}{\sigma + 2w_U^1/R}} \right).$$

From the last expression, it follows immediately that p^* is independent of w_C^1 and increasing in w_U^1 .

If $0 < W^1 < \sigma$:

$$p^* = \frac{1}{1 - \theta_1} \frac{1}{w_C^1 + w_U^1 (1 - 1/R)} \left(w_C^1 + w_U^1 (1 + 1/R) - \frac{\theta_1}{2 \frac{s-r}{\delta \sigma_2^T} - \frac{1-2r}{w_C^1 + w_U^1 (1+1/R)}} \right).$$

Calculating the partial with respect to w_C^1 gives:

$$\begin{aligned} \partial p^* / \partial w_C^1 &= -\frac{1}{1 - \theta_1} \frac{1}{(w_C^1 + w_U^1 (1 - 1/R))^2} \left(w_C^1 + w_U^1 (1 + 1/R) - \frac{\theta_1}{2 \frac{s-r}{\delta \sigma_2^T} - \frac{1-2r}{w_C^1 + w_U^1 (1+1/R)}} \right) \\ &\quad + \frac{1}{1 - \theta_1} \frac{1}{w_C^1 + w_U^1 (1 - 1/R)} \left(1 + \frac{\theta_1 \frac{1-2r}{(w_C^1 + w_U^1 (1+1/R))^2}}{\left(2 \frac{s-r}{\delta \sigma_2^T} - \frac{1-2r}{w_C^1 + w_U^1 (1+1/R)} \right)^2} \right) \\ &\geq \frac{-1}{w_C^1 + w_U^1 (1 - 1/R)} + \frac{1}{1 - \theta_1} \frac{1}{w_C^1 + w_U^1 (1 - 1/R)} \left(1 + \frac{\theta_1 \frac{1-2r}{(w_C^1 + w_U^1 (1+1/R))^2}}{\left(2 \frac{s-r}{\delta \sigma_2^T} - \frac{1-2r}{w_C^1 + w_U^1 (1+1/R)} \right)^2} \right) \end{aligned}$$

where the last inequality follows since $p^* \leq 1$. The last expression is clearly greater than zero.

Finally, the partial with respect to w_U^1 is:

$$\begin{aligned} \partial p^* / \partial w_U^1 &= \frac{1}{1 - \theta_1} \frac{-(1 - 1/R)}{(w_C^1 + w_U^1 (1 - 1/R))^2} \left(w_C^1 + w_U^1 (1 + 1/R) - \frac{\theta_1}{2 \frac{s-r}{\delta \sigma_2^T} - \frac{1-2r}{w_C^1 + w_U^1 (1+1/R)}} \right) \\ &\quad + \frac{1}{1 - \theta_1} \frac{1 + 1/R}{w_C^1 + w_U^1 (1 - 1/R)} \left(1 + \frac{\theta_1 \frac{1-2r}{(w_C^1 + w_U^1 (1+1/R))^2}}{\left(2 \frac{s-r}{\delta \sigma_2^T} - \frac{1-2r}{w_C^1 + w_U^1 (1+1/R)} \right)^2} \right). \end{aligned}$$

The first term is positive since it equals p^* multiplied with the positive factor $-(1 - 1/R) / (w_C^1 + w_U^1 (1 - 1/R))$. It is easy to see that the second term is also positive. ■

F Proof of Proposition 5 (Global comparative statics with respect to R)

Lemma 6 *For any finite $\delta > \bar{\delta}(R = 1)$, there are unique values \underline{R} and \bar{R} such that $\underline{\delta}(\underline{R}) = \delta$ and $\bar{\delta}(\bar{R}) = \delta$.*

Proof: For $R \in (\hat{R}, 1]$, the functions $\underline{\delta}(R)$ and $\bar{\delta}(R)$ are continuous in R since they are both continuous in σ^T for $\sigma^T > 0$ and since $\sigma^T = \min\{w_U + w_C - w_U/R, \sigma\}$ is positive and continuous in R over the same interval. It is easy to show that $\underline{\delta}(R) < \bar{\delta}(R)$ for $R > \hat{R}$ and moreover that:

$$\lim_{R \downarrow \hat{R}} \underline{\delta}(R) = \lim_{R \downarrow \hat{R}} \bar{\delta}(R) = \infty.$$

Hence, there exists $a \in (\hat{R}, 1]$ such that $\delta < \underline{\delta}(a) < \bar{\delta}(a)$. By assumption $\delta > \bar{\delta}(R = 1)$. The intermediate value theorem can thus be applied to the functions over the closed and bounded interval $[a, 1]$, proving existence of \underline{R} and \bar{R} . Uniqueness follows since the functions $\underline{\delta}(R)$ and $\bar{\delta}(R)$ are decreasing in R for $R \in (\hat{R}, 1]$ by Proposition 3. ■