

# Nonlinear wave propagation from a discrete annular array: theory

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**Abstract:** Theoretical investigations of finite amplitude wave propagation from axisymmetric, circular piston sources has been considered by several researchers(1-3). Attention has been restricted to analyses of source conditions of either an unfocused piston or a weakly, spherically focused piston. However, little research has been reported on finite amplitude wave propagation from an array of discrete sources. Time domain simulations, based on the Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation, will be discussed for an eight element discrete annular array immersed in fresh and sea water.

## INTRODUCTION

When the peak pressure does not exceed approximately 150 MPa, the KZK nonlinear parabolic wave equation yields numerical solutions in agreement with experiments. An augmented KZK equation may be expressed in the following dimensionless form,

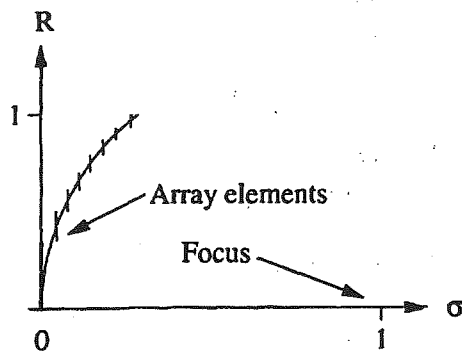
$$\frac{\partial P}{\partial \sigma} = \frac{l}{4z_r} \int_{-\infty}^{\tau} \left( \frac{\partial^2}{\partial R^2} + \frac{1}{R} \frac{\partial}{\partial R} \right) P d\tau' + \alpha_0 l \frac{\partial^2 P}{\partial \tau^2} + \frac{l}{z_s} P \frac{\partial P}{\partial \tau} + \sum_{\nu} \frac{m_{\nu} \omega_0 l}{2c_0} \left( \frac{\tau_{\nu}}{1 + \tau_{\nu} \frac{\partial}{\partial \tau}} \right) \frac{\partial^2 P}{\partial \tau^2}, \quad (1)$$

where diffraction, thermoviscous absorption, and quadratic nonlinearity are described by the first three terms on the right-hand-side (RHS), respectively. The last term on the RHS is a summation over all relaxation processes that may be present in the fluid. The pressure is  $P$ ;  $R = r/a$  and  $\sigma = z/l$  are unitless radial and  $z$  coordinates;  $l$ ,  $z_r = \omega_0 a^2 / 2c_0$ ,  $z_s = \rho_0 c_0^3 / \beta \omega_0 p_0$ , and  $a$  are a reference length, Rayleigh distance, plane-wave shock formation distance, and radius of the piston, respectively. For a focused source,  $l$  is typically taken to be the focal length. The ambient density and small-signal sound speed are  $\rho_0$  and  $c_0$  while  $\omega_0$  is a characteristic angular frequency for the finite pulse. Additionally,  $z_s$  contains the peak positive pressure amplitude at the source,  $p_0$ , and the coefficient of nonlinearity,  $\beta$ . The thermoviscous attenuation coefficient is  $\alpha_0$ , and the  $\nu$ th relaxation process is described by a dimensionless relaxation time  $\tau_{\nu}$  and unitless dispersion parameter  $m_{\nu}$ . Finally,  $\tau$  denotes a dimensionless retarded time.

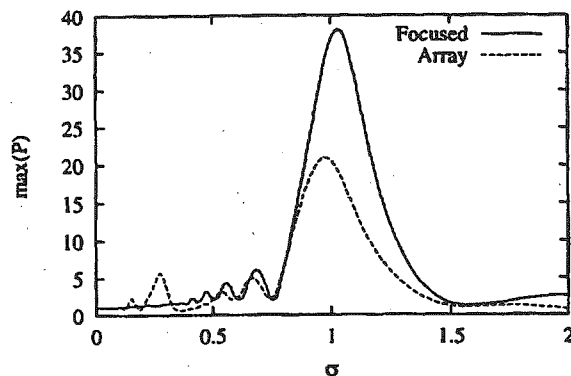
## DISCUSSION

Equation (1) is solved by a variety of numerical techniques including operator splitting, finite difference methods, trapezoidal quadrature, and an analytic solution for the quadratic nonlinearity term. Detailed comments concerning these techniques can be found in (1), but a brief summary is merited. Given an initial finite duration pulse at the source, Eq. (1) permits the forward propagation of the signal in the  $+\sigma$  direction. For a small step-size in  $\sigma$ , each term on the RHS of Eq. (1) can be considered separately, and the order is unimportant. This is the essence of the operator splitting technique. To facilitate the computation of the diffraction term, the integral is first approximated by a trapezoidal quadrature. Both implicit backward and Crank-Nicolson finite difference algorithms are then used for the diffraction, thermoviscous absorption, and relaxation terms. Numerical accuracy is at least first order accurate, and stability depends upon proper choices of the discrete step sizes for  $\sigma$ ,  $R$ , and  $\tau$ . Finally, under the operator splitting, the quadratic nonlinearity term yields an analytic solution. This solution amounts to a finite amplitude distortion of the phase of the pulse at each  $\sigma$  step, and then a resampling of the distorted pulse.

The source condition is such that  $P(R, \sigma = 0, \tau) = g(R) \exp\{-a_t[(2\tau - L)/L]^{2m_t}\} \sin(\tau)$ . Here,  $L$  is the dimensionless temporal length of the pulse and  $a_t$  and  $m_t$  control the temporal envelop.  $g(R)$  provides amplitude shading. When the source is a focused piston, the initial phase of the signal at each discrete radial point  $R_j$  is determined by  $\tau_j = -z_s R_j^2 / l$ . The negative sign indicates points further from the acoustical axis must radiate at earlier times to form a focus at  $l$ . Figure 1 shows the initial data plane  $\sigma = 0$ , and the continuous arc illustrates a wave front of constant phase corresponding to  $\tau_j$ . If the source condition is that of a discrete array, then  $g(R)$  and  $\tau_j$  become  $g(R_n)$  and  $\tau_{jn}$  where  $R_n$  falls within the inner and outer radii of the  $n$ th annular element. The hash marks in Fig. 1 illustrate the relative phase and width for the eight elements of the array. For each element except the center,  $R_n$  is the average radius in the numerical simulation discussed below.



**FIGURE 1.** Unfocused and weakly focused piston source geometry. Relative array element sizes and phases are represented by the hash marks along the arc.



**FIGURE 2.** Peak positive pressure along the acoustical axis of the source.

In a companion paper(4), experiments are described where an eight element annular array is the source and its center frequency is near 500 kHz. A comparison of the predicted peak positive pressure along the acoustical axis of a focused source and the array are shown in Fig. 2 where the focal length is  $l = 0.28$  m. The initial conditions at the source are  $p_0 = 200$  kPa,  $a_t = 2$ ,  $m_t = 3$ , and  $g(R \leq 1) = 1$  (0 otherwise). The pulse duration was 10 cycles, and each cycle contains 200 discrete points. The source was approximated by 200 uniformly distributed discrete points, and the computational domain was sufficiently large to eliminate the possibility of artifacts. The outer radius of the source is 0.07 m, and the surface area of each element is  $0.00189$  m<sup>2</sup>. The fluid medium is fresh water at 17° C and 1 atm of ambient pressure.

### CONCLUSIONS

Figure 2 clearly illustrates that the peak positive pressure at the focus of the array reaches only about 54% of the peak positive pressure predicted for a spherical focused piston. This discrepancy is related solely to the disparity of the phase as depicted in Fig. 1 because of the condition imposed by  $g(R \leq 1) = 1$ .

The acoustic pressure computed for the array and focused source demonstrate shock formation in the vicinity of the focus. The peak pressure for the array occurs prior to the linear geometric focus (i.e.,  $\sigma = 1$ ) while the peak pressure in the focused source simulation occurs beyond the geometric focus.

### ACKNOWLEDGMENTS

Work supported by the Office of Naval Research.

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