Rationing and Peasant Household Behaviour

WPS/94.9

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1994

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We gratefully acknowledge the helpful comments of participants at seminars at the Centre for the Study of African Economies, University of Oxford, and the University of Nottingham, Reading and Warwick, at which earlier drafts f this papers were presented, and to others who provided comments in informal discussion. We would especially like to thank, without in any way implicating, the following: David Bevan, Paul Collier, Norman Gemmell, Pramila Krishnan, Mark McGillivray, Steve O'Connell, Andrew Powell, Neil Rankin and Francis Teal

1 Introduction

In many developing countries, a large proportion of agricultural production is undertaken by small-scale peasant households. Such peasant households are typically complex economic units, given that they are engaged, *inter alia*, in the economic activities of production, consumption and factor supply. Notwithstanding such complexity, an accurate characterisation of their behaviour is important from the point of view of development policy. This is especially the case with regard to their responsiveness to price and other market signals[1].

This issue is of particular importance for stabilisation and structural adjustment policies, many of which place emphasis on raising the prices of key cash crops in the hope of generating a strong supply response. Other components of such programmes, such as exchange rate devaluation or the removal of subsidies, may affect, intentionally or incidentally, prices of other agricultural inputs and outputs, and consumer goods. The production and welfare consequences of such changes is an issue of considerable policy importance; these consequences will depend on how, and to what extent, peasant households respond to changing prices.

Empirical evidence suggests that supply response in agriculture is weak, at least at the aggregate level (Binswanger *et al*, 1987) and perhaps also at the level of individual crops (Bond, 1983). Historically, evidence of this type was used to counteract concern about the effects of the anti-agricultural terms of trade bias resulting from development strategies based on the extraction of agricultural surplus. In the present context, such evidence suggests that attempts to reverse this terms of trade bias, which are often implicit or explicit elements of structural adjustment and stabilisation policies, may only have limited effects on increasing agricultural production and reducing rural poverty.

In the light of this and other issues, the question of the behaviour of peasant households has attracted significant attention in the economic literature, from both theoretical and empirical points of view. In this regard, the models associated with the names of Barnum and Squire (1979a) and Singh, Squire and Strauss (1986a, 1986b) represent seminal contributions. These models capture the various household decisions in a framework incorporating the conventional microeconomic theory of consumer and producer behaviour. Implicit in such models is the assumption that peasants are potentially responsive to price. However, a number of studies have recognised that the markets in which peasants operate do not necessarily function perfectly; peasants may face rationing in some markets (Bevan, Bigsten, Collier and Gunning, 1987; Berthelemy and Morrisson, 1987),

[1] In this regard, de Janvry, Fafchamps and Sadoulet (1991) distinguish two schools of thought in characterising peasant behaviour: a "substantivist" school, in which peasants are regarded as not engaging in optimising behaviour and as being unresponsive to market signals, and a "formalist" school, in which peasants are characterised by optimising behaviour, and so will respond to market signals.

whereas other markets may be absent (Strauss, 1986; de Janvry, Fafchamps and Sadoulet, 1991). This absence or imperfection of markets will not only constrain household choice in the markets directly affected, but also in other markets in which they transact. Such market failures might be expected to reduce the responsiveness of peasant households to price and other signals. They might also be expected to have adverse effects on welfare and agricultural output.

Based on the assumption that peasant households are potentially responsive to market signals, but are frequently constrained in their response by market failures, this paper seeks to set out a general model of peasant behaviour in an environment of rationed or missing markets, and to trace out the implications for production and consumption behaviour. Starting from the major contribution of Strauss (1986), to whose formulation this paper owes much, we seek to set out a general model which encompasses existing models of peasant households in the literature, but which generalises and develops them in some directions.

The paper is organised as follows. Building on a behavioural model of a peasant household in the absence of rationing (set out in section 2), a general model of a rationed peasant household is set out in section 3, based on a generalisation of Strauss's (1986) model. The comparative static properties of the model are then examined in two sections, with section 4 deriving general expressions applying to changes in any exogenous variable and section 5 discussing specific instances, in particular the effects of changes in prices. Section 6 concludes.

2 The Basic Model

As noted above, the purpose of this section is to state a general model of an agricultural household developed by Strauss (1986) such that:[2]

(a) the similarities of other relevant models can be highlighted;

(b) the conditions of recursiveness are spelt out; and

(c) the basic framework for (later) analysing rationing is defined.

Consider a farm household which:

(a) maximises utility subject to a production function, a cash income (or explicit budget) constraint, and a time constraint;

(b) produces a food crop (Q_a), a part of which it consumes, and a cash crop (Q_c), all of which it supplies to the market;[3]

[2] Here we just summarise the Strauss (1986) model with the same notation, though making some relations and results more explicit or concise.

[3] As noted by Strauss (1986), we can have more than one food crop, cash crop, variable input, fixed input and consumption good. In this case vectors rather than scalars have to be used.

(c) applies its own labour (F) in farm production; and

(d) participates in the labour market.

2.1 Assumptions

In this section the following assumptions are made:

(a) All relevant markets exist and clear, and the household is a price-taker in all of them.

(b) There exists a single household utility function (U), with household consumption of the food crop (X_a), a market purchased consumer good (X_m) [for greater relevance we call this a manufactured consumer good], and leisure (X_l) as its arguments, which is twice continuously differentiable, monotonically increasing and quasi-concave.

(c) The farm production technology is expressed by an implicit production function $G(Q_a, Q_c, L, V, A, K) = 0$, where L, V, A, and K are total labour input, a variable input, land area, and a fixed input respectively. This function is twice continuously differentiable, increasing in outputs and decreasing in inputs, and quasi-convex.[4]

(d) Farm production is risk-free.

2.2 The Model

Under these assumptions, the short-run (i.e. single agricultural cycle) optimisation problem of the farm household becomes:

Maximise:

$$U(X_a, X_l, X_m) \tag{2.1}$$

subject to:
cash income constraint:

$$p_m X_m \leq p_a(Q_a - X_a) + p_c Q_c - p_l(L - F) - p_v V + E \tag{2.2}$$

time constraint:

$$X_l + F \leq T \tag{2.3}$$

production function:

$$G(Q_a, Q_c, L, V, A, K) = 0 \tag{2.4}$$

[4]This general formulation in terms of an implicit production function is a more general formulation than having separate explicit production functions for the cash crop and the food crop. Specifically, the former representation encompasses the latter.

where: p_i = market price of commodity i, $i = a, l, m, c, v$; $(Q_a - X_a)$ = marketed surplus of the food crop; $(L - F)$ = hired labour [hired-in if positive, hired-out if negative]; E = non-wage, non-farm, net other income; T = household's time endowment.

Assuming that constraints 2.2 - 2.4 bind, and substituting for F in 2.2 from 2.3, we can write the Lagrangean as:

$$\begin{aligned}\phi = \; & U(X_a, X_l, X_m) + \lambda_1[p_l T + (p_a Q_a + p_c Q_c - p_l L - p_v V) + \\ & E - p_a X_a - p_l X_l - p_m X_m] + \lambda_2[G(Q_a, Q_c, L, V, A, K)]\end{aligned} \quad (2.5)$$

With interior solutions, first order necessary conditions are:

$$U_a = \lambda_1 p_a \quad (2.6)$$

$$U_l = \lambda_1 p_l \quad (2.7)$$

$$U_m = \lambda_1 p_m \quad (2.8)$$

$$p_l T + (p_a Q_a + p_c Q_c - p_l L - p_v V) + E = p_a X_a + p_l X_l + p_m X_m \quad (2.9)$$

$$p_l = -\frac{\lambda_2}{\lambda_1} G_l \quad (2.10)$$

$$p_v = -\frac{\lambda_2}{\lambda_1} G_v \quad (2.11)$$

$$p_a = \frac{\lambda_2}{\lambda_1} G_a \quad (2.12)$$

$$p_c = \frac{\lambda_2}{\lambda_1} G_c \quad (2.13)$$

$$G(Q_a, Q_c, L, V, A, K) = 0 \quad (2.14)$$

where: $U_i = \frac{\partial U}{\partial X_i}$, partial derivative of U with respect to its i^{th} argument ($i = a, l, m$); G_j = partial derivative of G with respect to its j^{th} argument ($j = a, c, l, v$).

Strauss (1986) shows that totally differentiating these first order conditions results in a block-diagonal system of equations. In other words, the model is recursive. It means that production and consumption decisions, though temporally simultaneous, are logically separable such that the household makes the former independently and incorporates them in reaching the latter. This separability requires the following assumptions relating to goods both produced (or used in production) and consumed by the farm household (Strauss, 1986):

(a) the markets for all such goods exist and clear;

(b) the household is a price-taker in all these markets; and

(c) such goods are homogeneous.[5]

Given recursiveness, the logical sequence of optimisation then is:

(a) Maximise full income by maximising farm profit subject to the production function and given the levels of fixed inputs and input/output prices [2.10 - 2.14 in the above set of conditions]. This solves for output supplies and input demands, a solution which can be summarised by a profit function π:[6]

$$\pi = p_a Q_a + p_c Q_c - p_l L - p_v V = \pi(p_a, p_c, p_l, p_v, A, K) \qquad (2.15)$$

Thus we have maximised full income, Y_f, as:

$$\begin{aligned} Y_f &= p_l T + [p_a Q_a + p_c Q_c - p_l L - p_v V] + E \\ &= p_l T + \pi(p_a, p_c, p_l, p_v, A, K) + E \qquad (2.16) \end{aligned}$$

(b) Maximise utility subject to the full income constraint and given consumption goods' prices [2.6 - 2.9 in the above set of conditions]. This gives Marshallian (or uncompensated) commodity demands:

$$X_i = X_i(p_a, p_l, p_m, p_l T + \pi(p_a, p_c, p_l, p_v, A, K) + E) \qquad (2.17)$$

where: $i = a, l, m$.

Moreover, once leisure demand is known, family farm labour supply is determined by the constraint $X_l + F = T$.

The above utility maximisation can be formulated as an expenditure (or cost of utility) minimisation problem using duality. Indeed, this reformulation plays a fundamental role in the analysis of rationing in later sections. In this regard, given the producer-consumer nature of the agricultural household, there is a need for defining an expenditure function equivalent to the pure consumer case. Two modifications are evident. First, the arguments of that function have to be net demands, with supplies defined as negative demands. Second, the exogenous income (expenditure) analogue, i.e. E, has to be used. In brief, the expenditure that the household minimises is:

$$E = p_m X_m + p_l(L - F) + p_v V - p_a(Q_a - X_a) - p_c Q_c \qquad (2.18)$$

[5]This assumption can be relaxed if we make, as we do, the common assumption of interior solutions (Strauss, 1986).

[6]Note that by the properties of the production function, the profit function is convex in all prices, increasing in output prices and decreasing in input prices (McFadden, 1978).

The right hand side of 2.18 is the sum of the values of the net demands for all the commodities in the model. Once we substitute for F from the time endowment constraint and re- arrange, the household can be thought of as minimising E subject to the production function and to obtaining a given level of utility $\bar{U}$, i.e.:

Minimise:

$$p_a X_a + p_l X_l + p_m X_m - (p_a Q_a + p_c Q_c - p_l L - p_v V) - p_l T \tag{2.19}$$

subject to:

$$U(X_a, X_l, X_m) \geq \bar{U} \tag{2.20}$$

$$G(Q_a, Q_c, L, V, A, K) = 0 \tag{2.21}$$

Duality ensures that the first-order necessary conditions for this problem are the same as those of the utility maximisation problem. The resultant expenditure function is:

$$\begin{aligned} e'(p_a, p_c, p_l, p_m, p_v, T, A, K, \bar{U}) &= p_a X_a + p_l X_l + p_m X_m \\ &- (p_a Q_a + p_c Q_c - p_l L - p_v V) - p_l T \end{aligned} \tag{2.22}$$

e' is the minimum exogenous income (expenditure) required to achieve $\bar{U}$, and using 2.15 may be written:

$$e'(p_a, p_c, p_l, p_m, p_v, T, A, K, \bar{U}) = e(p_a, p_l, p_m, \bar{U}) - \pi(p_a, p_c, p_l, p_v, A, K) - p_l T \tag{2.23}$$

where $e = p_a X_a + p_l X_l + p_m X_m$.[7] We can have a more compact relation if we define a new full income $Y_f^* = \pi + p_l T$. Then

$$e'(.) = e(.) - Y_f^* \tag{2.24}$$

Obviously, given the properties of $\bar{U}$, e' is twice continuously differentiable and concave. Moreover, by Shepherd's Lemma, its first order partial derivatives with respect to prices are Hicksian net demands, while those of e are Hicksian total demands. Accordingly, in order to equate Marshallian and Hicksian demands, we have to define the former at e' or, more precisely, in equilibrium $E = e'$, so that:

[7] Strauss (1986) suggests that $e(p_a, p_l, p_m, \bar{U})$ is the expenditure function which is required in order to relate compensated and uncompensated virtual prices. In fact, formally the dual of the utility maximisation problem of the agricultural household in general is the minimisation of E. In the special case of a recursive model, however, the distinction between e' and e is not crucial, since Y_f is exogenous to consumption decisions.

$$X_i(p_a, p_l, p_m, e'(p_a, p_c, p_l, p_m, p_v, T, A, K, \bar{U})) = X_i^c(p_a, p_l, p_m, \bar{U}) \qquad (2.25)$$

with ^{c} denoting compensated demand. This equality holds even under rationing, except that in that case virtual prices have to be used.

Before we close this section, we briefly note the generality of the basic model considered above. Its generality follows from its ability to encompass most agricultural household models formulated in the absence of rationing, including the models of Barnum and Squire (1979a, 1979b); Singh, Squire and Strauss (1986a, 1986b); Lundahl and Ndulu (1987); and the recursive or sequential versions of the models of de Janvry *et al* (1991), Benjamin (1992), Lambert and Magnac (1992). And as will be shown in the next section, the rationed version of this model is also able to encompass many of the models of agricultural household under rationing.

3 A general peasant household model with rationing

A number of strong assumptions underlie the peasant household model set out in the previous section. In this paper we wish to focus on the consequences of the relaxation of one of these assumptions, the assumption of the existence of, and absence of rationing in, all markets in which the household trades, whether as producer or consumer and whether as supplier or demander. The interest in relaxing this assumption stems from the fact that in many developing country contexts it is not an appropriate characterisation of the situation in which many peasant households find themselves. For a variety of policy, institutional, infrastructural and other reasons, many markets in rural areas may be subject to rationing (quantity constraints) or be missing; others may exist in general, although particular households may choose not to undertake transactions in those markets (de Janvry *et al*, 1991). The rationing or absence of markets, whether it affects all or just some households, represents an additional constraint under which households operate which therefore must be taken into account in modelling their behaviour. Indeed, the failure to allow for rationing in a situation in which rationing is present will lead to misleading inferences.

In this section, the model of the previous section is modified to provide a general model of peasant household behaviour under rationing. In an analogous way to the case of consumer behaviour under rationing (Neary and Roberts, 1980; Deaton and Muellbauer, 1980) the rations may be incorporated by the inclusion of additional constraints in the model, taking the form of quantity constraints on the rationed commodity.

Of the five markets in which the household trades (those for the cash crop, the food crop, labour, the purchased input and the consumer good) it will be assumed in the general model that all are potentially subject to rationing except the market for the cash crop, in which it is assumed that the household can sell as much as it wishes. This latter assumption is likely to be reasonable in the case of an export crop for a small country, or in other cases in which the output of the peasant producing sector in the country is small relative to the size of the market demand. At a theoretical level for the model to be non-redundant it is necessary that at least one market is unrationed.

Cases in which a smaller number of markets are subject to rationing may be derived as special cases of the general model; some such instances which may be of particular interest are discussed later. Also, cases in which certain markets are missing may be derived as special cases of rationed markets in which the ration level is zero. In the presence of rationing in the markets for the food crop, labour, the purchased input and the consumer good, the model of the previous section is modified by the inclusion of the following additional constraints:

$$X_a - Q_a \leq \bar{Q}_a \tag{3.1}$$

$$L - F \leq \bar{L} \tag{3.2}$$

$$V \leq \bar{V} \tag{3.3}$$

$$X_m \leq \bar{X}_m \tag{3.4}$$

Constraints 3.1 and 3.2 relate to commodities which are both supplied and demanded by the household; in such cases the rations are assumed to apply to net demands. In the case of 3.1 it is assumed that the household's net demand for the food crop is subject to an upper bound $\bar{Q}_a$; note, however, that there is no reason why $\bar{Q}_a$ should not be negative in which case it is the household's net *supply* of the food crop which is subject to an upper bound. In either case, however, the analysis is the same. Constraint 3.2 implies an upper bound constraint of $\bar{L}$ on off-farm labour demand by the household; again, however, $\bar{L}$ can be negative which translates into an upper bound on off-farm labour supply by the household (Benjamin, 1992). Note that $\bar{Q}_a$ in 3.1 and $\bar{L}$ in 3.2 may be zero; this would correspond to the market in question being missing, at least for the household in question, so that it is required to be self- sufficient in that commodity.

Constraints 3.3 and 3.4 relate to commodities which are demanded but not supplied by the household, and operate like conventional quantity constraints; for these constraints the requirements $V \geq 0$ and $X_m \geq 0$ apply.[8] The model with rationing may be solved by incorporating the constraints 3.1 - 3.4 in the Lagrangean, and proceeding in the usual manner to obtain the consumer demand functions, input demand functions and output supply functions under rationing. Assuming that each of the constraints 3.1 - 3.4 binds and so holds with equality, then each of these functions will depend on the ration levels. In the case of V

[8]Note that for interior solutions, strict inequality will apply.

and X_m the rationed demand functions are trivially defined by 3.3 and 3.4; the other functions will depend in general on all the ration levels, subject to the fact that 3.1 and 3.2 must hold.

However, a more elegant and analytically useful, although ultimately equivalent, approach to the model under rationing is in terms of virtual prices (Neary and Roberts, 1980; Deaton and Muellbauer, 1980; Strauss, 1986). Here the virtual prices are those prices for the rationed commodities (X_a, X_l, X_m, V) which would lead to the ration levels being freely chosen by an unrationed household. These virtual prices may be defined on either a compensated or uncompensated basis, the difference being that in the former case the prices are calculated at constant utility, whereas in the latter case the virtual prices are calculated at a constant level of exogenous income E.[9]

Assuming that all four rations bind, the compensated virtual prices ($\bar{p_l^*}$, $\bar{p_a^*}$, $\bar{p_m^*}$, $\bar{p_v^*}$) must be such as to simultaneouly solve the following system of equations:

$$X_a^c(\bar{p_l^*}, \bar{p_a^*}, \bar{p_m^*}, \bar{U}) - Q_a(p_c, \bar{p_a^*}, \bar{p_l^*}, \bar{p_v^*}, A, K) = \bar{Q}_a \qquad (3.5)$$

$$L(p_c, \bar{p_a^*}, \bar{p_l^*}, \bar{p_v^*}, A, K) + X_l^c(\bar{p_l^*}, \bar{p_a^*}, \bar{p_m^*}, \bar{U}) - T = \bar{L} \qquad (3.6)$$

$$X_m^c(\bar{p_l^*}, \bar{p_a^*}, \bar{p_m^*}, \bar{U}) = \bar{X}_m \qquad (3.7)$$

$$V(p_c, \bar{p_a^*}, \bar{p_l^*}, \bar{p_v^*}, A, K) = \bar{V} \qquad (3.8)$$

The compensated virtual prices are such that the exogenously given ration levels (the right hand side of equations 3.5 - 3.8) are the outcomes of free choice by an unrationed household facing these prices (the left hand side of equations 3.5 - 3.8) and compensated for the adverse income/welfare effects of the rations. The virtual prices may be obtained by simultaneously solving the equation system 3.5 - 3.8:

$$\bar{p_j^*} = \bar{p_j^*}(p_c, \bar{X}_m, \bar{Q}_a, \bar{L}, \bar{V}, T, A, K, \bar{U}) \qquad (3.9)$$

where: $j = a, l, m, v$.

In general each of the virtual prices is dependent on all the ration levels, and on all the other exogenous variables in the system. It follows therefore that a change in any exogenous variable, including any of the ration levels, will change all of the virtual prices, and these changes will need to be taken into account in any comparative static analysis. In other words, the virtual prices are now endogenous

[9]The distinction between compensated and uncompensated virtual prices is important, for the same reason the distinction between compensated and uncompensated demand functions is important in conventional (unrationed) consumer theory. They can be set equal to each other in equilibrium, but will not generally respond in the same way to a change in an exogenous variable. Neary and Roberts (1980) do not make this distinction, and appear to use the two concepts interchangeably.

variables. Having defined the compensated virtual prices, the behaviour of the rationed peasant household may now be represented as equivalent to that of an unrationed peasant household facing virtual prices. As the virtual prices are defined on a compensated basis the dual representation is appropriate:

Minimise:

$$\bar{p_l^*} X_l + \bar{p_a^*} X_a + \bar{p_m^*} X_m - \bar{p_l^*} T - p_c Q_c - \bar{p_a^*} Q_a + \bar{p_l^*} L + \bar{p_v^*} V \tag{3.10}$$

subject to:

$$U(X_m, X_a, X_l) \geq \bar{U} \tag{3.11}$$

$$G(Q_c, Q_a, L, V, A, K) = 0 \tag{3.12}$$

This model may be solved in the same way as in section 2 for the unrationed household, to give the same solution except that for the rationed commodities compensated virtual prices replace the actual market prices. The solution can then be represented by the profit function:

$$\tilde{\pi} = \pi(p_c, \bar{p_a^*}, \bar{p_l^*}, \bar{p_v^*}, A, K) \tag{3.13}$$

and the set of (Hicksian) demand functions:

$$\tilde{X}_i^c = X_i^c(\bar{p_m^*}, \bar{p_a^*}, \bar{p_l^*}, \bar{U}) \tag{3.14}$$

where: $i = a, l, m$, and the $\tilde{}$ superscript denotes profits or demands under rationing. The resultant expenditure function is then the following:

$$e'(p_c, \bar{p_a^*}, \bar{p_l^*}, \bar{p_m^*}, \bar{p_v^*}, T, A, K, \bar{U}) \tag{3.15}$$

The rations are represented in this model by the use of virtual prices ($\bar{p_l^*}$, $\bar{p_a^*}$, $\bar{p_m^*}$, $\bar{p_v^*}$), defined according to equation 3.9, in place of the market prices (p_l, p_a, p_m, p_v) in the unrationed model.

One important implication of the endogeneity of the virtual prices in this general model is that the model loses its property of recursiveness, with the consequence that production and consumption decisions are logically simultaneous rather than logically sequential (as in the unrationed case). The reason for this is that two of the virtual prices, $\bar{p_l^*}$ and $\bar{p_a^*}$, depend both on consumption and production decisions of the household. In the case of the food crop for example the virtual price is obtained according to equation 3.5 by relating an unrationed supply function, $Q_a(.)$, which depends on production parameters, to an unrationed demand function, $X_a(.)$, which depends on consumption parameters. The virtual price will depend on the position of both of these functions (as well as on the ration level, $\bar{Q}_a$), and so is affected by both production and consumption parameters. A change in a consumption parameter will therefore affect a key variable, $\bar{p_a^*}$, in the production decision, and so recursiveness no longer holds. An analogous argument applies in the case of $\bar{p}_l^*$. Thus if there is rationing in either the labour market or the market for the food crop then recursiveness no longer holds and the model must be solved simultaneously.

However, if these two markets are both unrationed then recursiveness will still hold, even if the markets for X_m and V are still rationed. Rationing of V will of course affect production, thereby affecting income and so consumption decisions, but the consumption decision will not affect the production decision. Unlike in the case of labour and the food crop, the variable input V is a pure production commodity. As the ration level $\bar{V}$ is exogenous, the virtual price depends only on production parameters and the exogenous ration level $\bar{V}$. Consumption parameters do not therefore affect the production decision and so the model remains recursive. In the case of rationing of X_m only, this is a pure consumption effect; its virtual price neither affects nor depends on the production decision, and so the model remains recursive. Rationing of X_m by itself does not affect the production decision, and so the situation is the same as the standard case of consumer theory under rationing (Neary and Roberts, 1980).

Duality theory continues to apply to the rationed peasant household as it applies to the unrationed peasant household. Thus the primal representation of household behaviour may be used in place of the dual representation used above. However, in this case, it is necessary to use uncompensated virtual prices, evaluated at constant exogenous income, in place of the compensated virtual prices used in the dual formulation. Additionally, full income needs to be evaluated at uncompensated virtual prices. The uncompensated virtual prices, are defined as the set of prices $(p_a^*, p_l^*, p_m^*, p_v^*)$ which solve the following set of four simultaneous equations:

$$X_a(p_l^*, p_a^*, p_m^*, p_l^*T + \pi(p_c, p_a^*, p_l^*, p_v^*, A, K) + E) - Q_a(p_c, p_a^*, p_l^*, p_v^*, A, K) = \bar{Q}_a \quad (3.16)$$

$$L(p_c, p_a^*, p_l^*, p_v^*, A, K) + X_l(p_l^*, p_a^*, p_m^*, p_l^*T + \pi(p_c, p_a^*, p_l^*, p_v^*, A, K) + E) - T = \bar{L} \quad (3.17)$$

$$X_m(p_l^*, p_a^*, p_m^*, p_l^*T + \pi(p_c, p_a^*, p_l^*, p_v^*, A, K) + E) = \bar{X}_m \quad (3.18)$$

$$V(p_c, p_a^*, p_l^*, p_v^*, A, K) = \bar{V} \quad (3.19)$$

Solving these equations, the uncompensated virtual prices will depend on all the exogenous variables in the model:

$$p_j^* = p_j^*(p_c, \bar{X}_m, \bar{Q}_a, \bar{L}, \bar{V}, T, A, K, E) \quad (3.20)$$

where: $j = a, l, m, v$.

It is then possible to define the primal representation of the behaviour of the rationed peasant household, which will be the same as in the unrationed case but with virtual prices for the rationed commodities in place of their market prices. Thus the resulting Marshallian demand functions and profit function will also have virtual prices in place of market prices for the rationed commodities.

The uncompensated virtual prices, defined for a level of exogenous income E, can be equated to the compensated virtual prices, defined for a constant level of utility $\bar{U}$, when E is equal to the minimum level of income (expenditure) required to attain the utility level $\bar{U}$, in other words, when $E = e'(p_c, \bar{p}_a^*, \bar{p}_l^*, \bar{p}_m^*, \bar{p}_v^*, T, A, K, \bar{U})$. This result will be exploited frequently in the following sections examining the comparative statics of this model.

Finally, it is appropriate to note that the model set out in this section is a general model, in the sense that it encompasses a number of other models which have examined the behaviour of peasant households under rationing. It is based on a generalisation of the model set out by Strauss (1986). However, as was the case for the unrationed model, the essential features of a number of other models of peasant behaviour under rationing can be incorporated within this framework, including the models of Bevan, Bigsten, Collier and Gunning (1987); Besley (1988); Bevan, Collier and Gunning (1991); de Janvry, Fafchamps and Sadoulet (1991); Benjamin (1992); Lambert and Magnac (1992); and Eriksson (1993).[10] It can therefore be used to evaluate the theoretical results obtained by some of these authors in a more general case.

4 Consumption and Production Comparative Statics in the General Model

Having set out a general model of the rationed peasant household in the previous section, it is now possible to investigate its comparative static properties. This will be considered in two parts, firstly, in this section, deriving general expression for production and consumption responses of unrationed variables to changes in any exogenous variables, and secondly, in section 5, discussing some of the more interesting specific cases in more detail. At all points we attempt to make clear the relationship between the results derived from our model and those derived from other similar models in the literature.

The expressions derived in this section are general in the sense that they apply to all relevant exogenous variables and to all variants of our model of an agricultural household. The procedure is to drop those terms which are inapplicable to an exogenous variable and/or a particular variant of the model, and insert

[10]Note that in the case of Eriksson's model, both the neoclassical and non-Walrasian versions of the model are in fact characterised by the absence of the labour market at the household level (Eriksson, 1993, p. 29).

the appropriately specified impact of exogenous variables on the virtual prices. Equipped with these general expressions we can then explore - in the next section - production and consumption effects in specific models.

4.1 Changes in the virtual prices

Relative to the unrationed model, the comparative statics in the rationed model are complicated by the fact that a change in any exogenous variable will in general change all the virtual prices, which will in turn affect production and consumption decisions. The magnitude of the change in the virtual prices resulting from a change in an exogenous variable, denoted by α, may be derived, following Strauss (1986), by first equating the uncompensated and compensated virtual prices, the former calculated for a level of exogenous income equal to $e'(p_c, \bar{p_a^*}, \bar{p_l^*}, \bar{p_m^*}, \bar{p_v^*}, T, A, K, U)$:

$$\bar{p_j^*}(p_c, \bar{X}_m, \bar{Q}_a, \bar{L}, \bar{V}, T, A, K, \bar{U}) = \\ p_j^*(p_c, \bar{X}_m, \bar{Q}_a, \bar{L}, \bar{V}, A, K, e'(p_c, \bar{p_a^*}, \bar{p_l^*}, \bar{p_m^*}, \bar{p_v^*}, T, A, K, \bar{U})) \tag{4.1}$$

Differentiating 4.1 with respect to the general exogenous variable α, the change in the uncompensated virtual price may be represented as follows:

$$\frac{\partial p_j^*}{\partial \alpha} = \frac{\partial \bar{p_j^*}}{\partial \alpha} - \frac{\partial p_j^*}{\partial E}\frac{\partial e'}{\partial \alpha} \tag{4.2}$$

The first term is the change in the compensated virtual price, or the virtual price at constant utility, whereas the second term gives the effect on the virtual price of the income changes associated with the change in the exogenous variable.

An expression for the first term of 4.2 can be derived by noting the fact that the rationing constraints 3.1 - 3.4 can be written in terms of the expenditure function e' as follows:

$$e'_a = \bar{Q}_a \tag{4.3}$$

$$e'_l = \bar{L} \tag{4.4}$$

$$e'_m = \bar{X}_m \tag{4.5}$$

$$e'_v = \bar{V} \tag{4.6}$$

where e'_j denotes the partial derivative of e' with respect to $\bar{p_j^*}$. A change in an exogenous variable α may affect $e'(.)$ directly, and indirectly through changing all the virtual prices. Differentiating 4.3 - 4.6 with respect to α, and taking this fact into account, gives rise to the following:

$$\begin{bmatrix} e'_{a,a} & e'_{a,l} & e'_{a,m} & e'_{a,v} \\ e'_{l,a} & e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,a} & e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,a} & e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix} \begin{bmatrix} \frac{\partial \bar{p}_a^*}{\partial \alpha} \\ \frac{\partial \bar{p}_l^*}{\partial \alpha} \\ \frac{\partial \bar{p}_m^*}{\partial \alpha} \\ \frac{\partial \bar{p}_v^*}{\partial \alpha} \end{bmatrix} = \begin{bmatrix} \frac{\partial \bar{Q}_a}{\partial \alpha} - e'_{a,\alpha} \\ \frac{\partial \bar{L}}{\partial \alpha} - e'_{l,\alpha} \\ \frac{\partial \bar{X}_m}{\partial \alpha} - e'_{m,\alpha} \\ \frac{\partial \bar{V}}{\partial \alpha} - e'_{v,\alpha} \end{bmatrix} \tag{4.7}$$

where $e'_{i,j}$ denotes the second partial derivative of $e'(.)$ with respect to $\bar{p}_i^*$ and $\bar{p}_j^*$ and $e'_{i,\alpha}$ its partial derivative with respect to $\bar{p}_i^*$ and α. Solving equation 4.7 using Cramer's Rule gives the following expressions for the changes in the compensated virtual prices resulting from a change in α:

$$\frac{\partial \bar{p}_a^*}{\partial \alpha} = \frac{\det \begin{bmatrix} \frac{\partial \bar{Q}_a}{\partial \alpha} - e'_{a,\alpha} & e'_{a,l} & e'_{a,m} & e'_{a,v} \\ \frac{\partial \bar{L}}{\partial \alpha} - e'_{l,\alpha} & e'_{l,l} & e'_{l,m} & e'_{l,v} \\ \frac{\partial \bar{X}_m}{\partial \alpha} - e'_{m,\alpha} & e'_{m,l} & e'_{m,m} & e'_{m,v} \\ \frac{\partial \bar{V}}{\partial \alpha} - e'_{v,\alpha} & e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}}{\det \begin{bmatrix} e'_{a,a} & e'_{a,l} & e'_{a,m} & e'_{a,v} \\ e'_{l,a} & e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,a} & e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,a} & e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}} \tag{4.8}$$

$$\frac{\partial \bar{p}_l^*}{\partial \alpha} = \frac{-\det \begin{bmatrix} \frac{\partial \bar{Q}_a}{\partial \alpha} - e'_{a,\alpha} & e'_{a,a} & e'_{a,m} & e'_{a,v} \\ \frac{\partial \bar{L}}{\partial \alpha} - e'_{l,\alpha} & e'_{l,a} & e'_{l,m} & e'_{l,v} \\ \frac{\partial \bar{X}_m}{\partial \alpha} - e'_{m,\alpha} & e'_{m,a} & e'_{m,m} & e'_{m,v} \\ \frac{\partial \bar{V}}{\partial \alpha} - e'_{v,\alpha} & e'_{v,a} & e'_{v,m} & e'_{v,v} \end{bmatrix}}{\det \begin{bmatrix} e'_{a,a} & e'_{a,l} & e'_{a,m} & e'_{a,v} \\ e'_{l,a} & e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,a} & e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,a} & e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}} \tag{4.9}$$

$$\frac{\partial \bar{p}_m^*}{\partial \alpha} = \frac{\det \begin{bmatrix} \frac{\partial \bar{Q}_a}{\partial \alpha} - e'_{a,\alpha} & e'_{a,a} & e'_{a,l} & e'_{a,v} \\ \frac{\partial \bar{L}}{\partial \alpha} - e'_{l,\alpha} & e'_{l,a} & e'_{l,l} & e'_{l,v} \\ \frac{\partial \bar{X}_m}{\partial \alpha} - e'_{m,\alpha} & e'_{m,a} & e'_{m,l} & e'_{m,v} \\ \frac{\partial \bar{V}}{\partial \alpha} - e'_{v,\alpha} & e'_{v,a} & e'_{v,l} & e'_{v,v} \end{bmatrix}}{\det \begin{bmatrix} e'_{a,a} & e'_{a,l} & e'_{a,m} & e'_{a,v} \\ e'_{l,a} & e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,a} & e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,a} & e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}} \tag{4.10}$$

$$\frac{\partial \bar{p}_v^*}{\partial \alpha} = \frac{-\det \begin{bmatrix} \frac{\partial \bar{Q}_a}{\partial \alpha} - e'_{a,\alpha} & e'_{a,a} & e'_{a,l} & e'_{a,m} \\ \frac{\partial \bar{L}}{\partial \alpha} - e'_{l,\alpha} & e'_{l,a} & e'_{l,l} & e'_{l,m} \\ \frac{\partial \bar{X}_m}{\partial \alpha} - e'_{m,\alpha} & e'_{m,a} & e'_{m,l} & e'_{m,m} \\ \frac{\partial \bar{V}}{\partial \alpha} - e'_{v,\alpha} & e'_{v,a} & e'_{v,l} & e'_{v,m} \end{bmatrix}}{\det \begin{bmatrix} e'_{a,a} & e'_{a,l} & e'_{a,m} & e'_{a,v} \\ e'_{l,a} & e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,a} & e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,a} & e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}} \quad (4.11)$$

Substituting the appropriate one of equations 4.8 - 4.11 in 4.2 gives the total change in the uncompensated virtual price. This result may then be used to investigate the comparative statics of a change in an exogenous variable.

4.2 Comparative Statics: General Expressions

The presence of rationing in some markets will potentially also affect the production and/or consumption of unrationed commodities. Here we examine how the household's production and consumption behaviour in a rationed situation is affected by changes in exogenous variables. We emphasise production, consumption and net demand/supply surplus responses in cases in which the commodity in question is unrationed. For this reason, some cases will be considered in which rationing is less pervasive than in the general model.

Production, consumption and net demand responses will be considered in turn.

4.2.1 Production responses

While in the general model set out in section 3 the only production commodity not subject to any rationing is the output of the cash crop, in less rationed versions of the model either of the two outputs (Q_a, Q_c) or two variable inputs (L, V) might be affected by changes in exogenous parameters. Thus, in the discussion which follows, changes in each of these variables will be considered, although in many rationed regimes they will not all be free to respond, or will only be able to respond in a restricted way (subject to the net demand rations, equations 3.1 - 3.4).

Of particular interest is the behaviour of the output variables, the supply of which is given (by Hotelling's Lemma) by the partial derivatives of the profit function (equation 3.13) with respect to the corresponding prices:

$$\tilde{Q}_k = Q_k(p_c, p_a^*, p_l^*, p_m^*, p_v^*, A, K) \quad (4.12)$$

where: $k = a, c$.[11] [12] Consider the effect of a change in an exogenous variable, α, on the output Q_k of crop k by differentiating the supply function 4.12 with respect to α:

$$\frac{\partial \tilde{Q}_k}{\partial \alpha} = \frac{\partial Q_k}{\partial \alpha} + \sum_j \frac{\partial Q_k}{\partial p_j^*} \frac{\partial p_j^*}{\partial \alpha} \tag{4.13}$$

where: $k = a, c$; and the summation occurs over all commodities subject to rationing, j. In the absence of rationing this equation would comprise simply the first term, which is equal to $\pi_{k,\alpha}$, the second derivative of the profit function $\pi(.)$ with respect to p_k^* and α. However, in the presence of rationing the supply response of the crop in question to a change in an exogenous variable is complicated by the virtual price effects. Decomposing the virtual price effects according to 4.2, and using the fact that $\frac{\partial Q_k}{\partial p_j^*} = \pi_{k,j}$ gives the following equation:

$$\frac{\partial \tilde{Q}_k}{\partial \alpha} = \pi_{k,\alpha} + \sum_j \pi_{k,j} \frac{\partial \bar{p}_j^*}{\partial \alpha} - \sum_j \pi_{k,j} \frac{\partial p_j^*}{\partial E} \frac{\partial e'}{\partial \alpha} \tag{4.14}$$

The first two terms of equation 4.14 constitute the effect of a change in the exogenous variable α at constant utility, comprising both the direct effect ($\pi_{k,\alpha}$) and the effects of the virtual price changes at constant utility. The last term represents the effects of the income changes resulting from the change of α on the virtual prices.

This is a general expression for the response of output supplies to a change in an exogenous variable. It can be simplified by substituting for the effect of a change in α on the compensated virtual prices using equations 4.8 - 4.11 above. The specific elements of the summations will depend on how many and which markets are subject to rationing. In the most general rationed model, the only output supply which is not directly rationed is that of the cash crop. In this case, substituting for $\frac{\partial \bar{p}_j^*}{\partial \alpha}, j = (a, l, m, v)$ from equations 4.8 - 4.11, equation 4.14 may be written as follows:

[11] Note that, in fact, output supply will be independent of the virtual price of the market purchased good X_m. The virtual price p_m^* is included in equation 4.12 for later analytic convenience.

[12] Note that if the food crop is unrationed, p_a^* in equation 4.12 should be replaced by p_a.

$$\frac{\partial \tilde{Q}_c}{\partial \alpha} = \frac{-det\begin{bmatrix} e'_{c,\alpha} & e'_{c,a} & e'_{c,l} & e'_{c,m} & e'_{c,v} \\ e'_{a,\alpha} - \frac{\partial \bar{Q}_a}{\partial \alpha} & e'_{a,a} & e'_{a,l} & e'_{a,m} & e'_{a,v} \\ e'_{l,\alpha} - \frac{\partial \bar{L}}{\partial \alpha} & e'_{l,a} & e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,\alpha} - \frac{\partial \bar{X}_m}{\partial \alpha} & e'_{m,a} & e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,\alpha} - \frac{\partial \bar{V}}{\partial \alpha} & e'_{v,a} & e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}}{\det\begin{bmatrix} e'_{a,a} & e'_{a,l} & e'_{a,m} & e'_{a,v} \\ e'_{l,a} & e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,a} & e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,a} & e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}} - \sum_j \pi_{c,j} \frac{\partial p_j^*}{\partial E} \frac{\partial e'}{\partial \alpha} \quad (4.15)$$

This equation will apply for all exogenous variables affecting the supply of the cash crop, in other words its own price, the ration levels and the levels of the fixed inputs, although in some of these cases its form will simplify. Equation 4.15 relates to the general model in which all markets are subject to rationing except for that for the cash crop; however, the analysis is easily applicable to cases in which fewer markets are affected by rationing.

In the case in which the food crop market is now unrationed, it is possible to derive an analogous expression for its supply response to a change in the general exogenous variable α. However, as the food crop appears in both the consumption and production decisions, the response of total output at constant utility cannot be expressed just in terms of the expenditure function $e'(.)$ but rather in terms of both e' and the profit function $\pi(.)$. Assuming that the other rations continue to apply and bind, the supply response of the food crop is given by the following:

$$\frac{\partial \tilde{Q}_a}{\partial \alpha} = \frac{-det\begin{bmatrix} -\pi_{a,\alpha} & -\pi_{a,l} & -\pi_{a,m} & -\pi_{a,v} \\ e'_{l,\alpha} - \frac{\partial \bar{L}}{\partial \alpha} & e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,\alpha} - \frac{\partial \bar{X}_m}{\partial \alpha} & e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,\alpha} - \frac{\partial \bar{V}}{\partial \alpha} & e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}}{\det\begin{bmatrix} e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}} - \sum_j \pi_{a,j} \frac{\partial p_j^*}{\partial E} \frac{\partial e'}{\partial \alpha} \quad (4.16)$$

where: $j = l, m, v$. As in the case of equation 4.15, the first term gives the effect at constant utility, whereas the second captures the effect of the income changes on the virtual prices.

Finally, if the relevant input market is unrationed, it is possible to derive analogous expressions for the response of input demands to changes in exogenous variables. In the case of the variable input V, which is only a production-side variable, an expression analogous to 4.15 applies to demand, whereas in the case of L, a variable which appears on both the production and consumption sides, an expression analogous to 4.16 will apply. Of course, the sign will be reversed in each case, as the expressions relate to inputs rather than outputs.

4.2.2 Consumption responses

We now turn to the general comparative static results pertaining to consumption, developing a general expression for the response of commodity demands to changes in exogenous variables. To begin, recall that the following relations from the unrationed model in section 2:

$$e' = e - \pi - p_l T \tag{4.17}$$

$$Y_f^* = \pi + p_l T \tag{4.18}$$

These will hold in all versions of the model, though in the rationed situation virtual prices must be used in place of market prices.

The obvious implication is that in all cases a change in an exogenous variable may impact on commodity demands directly and through full income. Rationing, however, introduces virtual prices (i.e., endogenizes the prices of those goods to which it applies), consequently giving rise to two modifications:

(a) e', π, Y_f^* and X_i, $i = a, l, m, c, v$ should now be evaluated at the relevant virtual prices; and

(b) in addition to its direct and full income effects, a change in an exogenous variable now affects endogenous variables via its impact on virtual prices.

Our model has five goods and thus rationing in four markets (Q_a, X_m, V, L) is the most rationed case. We use that version to derive the aforementioned general expression for the comparative static response of rationed demands.

To begin with, we note that the rationed Marshallian demand for a commodity X_i may be equated to its Marshallian demand in the absence of rationing if the latter is evaluated at virtual prices, and if the consumer is compensated for the income effects of the rations. The income (or expenditure) effect of the rations is equal to $\sum_j (\bar{p}_j^* - p_j) R_j$, where: $j = a, l, m, v$, and $\bar{p}_j^*$, p_j and R_j represent respectively the compensated virtual price, the market price and the ration level associated with the j^{th} rationed good. This amount should be added to E as compensation for the rationing and thus the unrationed Marshallian demands should be evaluated at $E + \sum_j (\bar{p}_j^* - p_j) R_j$. Therefore we can write:

$$\begin{aligned} \tilde{X}_i(p_a, p_l, p_m, p_c, p_v, \bar{Q}_a, \bar{X}_m, \bar{L}, \bar{V}, T, A, K, E) &= \\ X_i(p_a^*, p_l^*, p_m^*, p_v^*, p_c, T, A, K, E + \sum_j (\bar{p}_j^* - p_j) R_j) \end{aligned} \tag{4.19}$$

Then, at constant exogenous income E, a change in an exogenous variable α impacts on $\tilde{X}_i$ 'directly', through the virtual prices, and through changes in the compensation required, i.e.,

$$\frac{\partial \tilde{X}_i}{\partial \alpha} = \frac{\partial X_i}{\partial \alpha} + \sum_j \frac{\partial X_i}{\partial p_j^*}\frac{\partial p_j^*}{\partial \alpha} + \sum_j \frac{\partial X_i}{\partial E}(R_j \frac{\partial \bar{p}_j^*}{\partial \alpha} + (\bar{p}_j^* - p_j)\frac{\partial R_j}{\partial \alpha}) \tag{4.20}$$

where: $i = a, l, m; j = a, l, m, v; \alpha = p_c, \bar{Q}_a, \bar{X}_m., \bar{L}, \bar{V}, A, K.$

An analogue to the Slutsky equation can be derived for the comparative static responses of rationed demands, which can be used to derive a more informative expression for $\frac{\partial \tilde{X}_i}{\partial \alpha}$. This expression may be obtained by noting that, in the rationed situation, the unrationed Marshallian and Hicksian demands may be equated at a utility level $\bar{U}$ if the latter is evaluated at compensated virtual prices and the former is evaluated at uncompensated virtual prices and an exogenous income level e':

$$X_i(p_a^*, p_l^*, p_m^*, p_v^*, p_c, T, A, K, e'(.)) = X_i^c(\bar{p}_a^*, \bar{p}_l^*, \bar{p}_m^*, \bar{p}_v^*, p_c, T, A, K, \bar{U}) \tag{4.21}$$

where e' is evaluated at the compensated virtual prices, and where compensated and uncompensated virtual prices are related by equation 4.1. Differentiating 4.21 with respect to the general exogenous variable α, and taking these facts into account gives the following relationship:

$$\frac{\partial \tilde{X}_i}{\partial \alpha} + \sum_j \frac{\partial X_i}{\partial p_j^*}(\frac{\partial p_j^*}{\partial \alpha} + \frac{\partial p_j^*}{\partial E}\frac{\partial e'}{\partial \alpha}) + \frac{\partial X_i}{\partial E}(\frac{\partial e'}{\partial \alpha} + \sum_j \frac{\partial e'}{\partial \bar{p}_j^*}\frac{\partial \bar{p}_j^*}{\partial \alpha}) =$$
$$\frac{\partial X_i^c}{\partial \alpha} + \sum_j \frac{\partial X_i^c}{\partial \bar{p}_j^*}\frac{\partial \bar{p}_j^*}{\partial \alpha} \tag{4.22}$$

The first term of the left hand side gives the effect on the change in α on demand at constant money income and constant virtual prices. The second term is the virtual price effect, which is comprised of two components, a direct effect, evaluated at constant money income, and an indirect effect, which is the compensation needed to preserve the equality between compensated and uuncompensated virtual prices needed so that 4.21 continues to hold. The final term on the left hand side is the income or compensation effect resulting from the change in α; again it is composed of a direct effect, at constant compensated virtual prices, and an indirect effect operating through virtual prices. The right hand side of 4.22 shows the response at constant utility; again this comprises a direct effect, at which virtual prices remain constant, and an indirect effect due to the changing virtual prices as α changes.

By re-arranging equation 4.22, and also making use of the relationships: $e' = e - Y_f^*$ (from equations 4.17 and 4.18), and $\frac{\partial e'}{\partial \bar{p}_j^*} = R_j$ (from Shepherd's Lemma), it is now possible to substitute for the terms on the right hand side of equation 4.20, to give the desired general expression for the comparative static response of rationed demand to a change in any exogenous variable, represented by α:

$$\begin{aligned}\frac{\partial \tilde{X}_i}{\partial \alpha} &= \frac{\partial X_i^c}{\partial \alpha} - \frac{\partial X_i}{\partial E}\frac{\partial e}{\partial \alpha} + \frac{\partial X_i}{\partial E}\frac{\partial Y_f^*}{\partial \alpha} \\ &+ \sum_j \frac{\partial X_i^c}{\partial \bar{p}_j^*}\frac{\partial \bar{p}_j^*}{\partial \alpha} - \sum_j \frac{\partial X_i}{\partial p_j^*}\frac{\partial p_j^*}{\partial E}\frac{\partial e'}{\partial \alpha} + \sum_j \frac{\partial X_i}{\partial E}(\bar{p}_j^* - p_j)\frac{\partial R_j}{\partial \alpha} \quad (4.23)\end{aligned}$$

Consistent with earlier characterisation of the possible impact of an exogenous variable, equation 4.23 has four component parts. The first two terms constitute the direct effect decomposed into substitution and income effects respectively. The third term is the profit effect. The next two terms are effects operating through virtual price effects, which are introduced by the rationing which the household faces. In this regard, the fourth term gives the effect of changes in virtual prices at constant utility (the substitution effect of virtual prices), while the fifth term gives their "income" effect. More precisely, the fifth term represents the change in compensation induced by a change in virtual prices. The sixth term is a compensation effect which will only apply in the case of a change in the ration level. Thus, we have the two substitution effect terms (the first and the fourth) and four "income/compensation" effect terms. Moreover, 4.23 is the consumption analogue of equation 4.14 in section 4.2.1, with the exception that the equivalent of the second, third and sixth terms of the former do not appear in the latter because analogous "income" effects do not operate in production.

Clearly, all these effects may or may not occur together depending on which of the exogenous variables is changing. The different specific instances will be discussed in more detail in section 5.

4.2.3 Net demands and supplies

Two of the commodities in the model, the food crop and labour, arise in both the production and consumption decisions of the household. In the production decision the food crop is supplied and labour is demanded, whereas in the consumption decision the food crop is demanded and labour supplied. For these commodities the production and consumption responses to changes in exogenous variables may be combined to derive a general expression for the response of net demand for the commodity in question. As will be seen, this will enable additional insights to be derived. Of course, net demand response will be zero if the market in question is rationed. Assume therefore, for example, that the market for the food crop is unrationed. In this case the response of its total supply to a change in a general exogenous variable α will be given by equation 4.14 for the case in which $k = a$:

$$\frac{\partial \tilde{Q}_a}{\partial \alpha} = \pi_{a,\alpha} + \sum_j \pi_{a,j} \frac{\partial \bar{p}_j^*}{\partial \alpha} - \sum_j \pi_{a,j} \frac{\partial p_j^*}{\partial E} \frac{\partial e'}{\partial \alpha} \tag{4.24}$$

and the response of food consumption to the change in the same exogenous variable α will be given by 4.23 with $i = a$:

$$\begin{aligned} \frac{\partial \tilde{X}_a}{\partial \alpha} &= \frac{\partial X_a^c}{\partial \alpha} - \frac{\partial X_a}{\partial E} \frac{\partial e}{\partial \alpha} + \frac{\partial X_a}{\partial E} \frac{\partial Y_f^*}{\partial \alpha} \\ &\quad + \sum_j \frac{\partial X_a^c}{\partial p_j^*} \frac{\partial \bar{p}_j^*}{\partial \alpha} - \sum_j \frac{\partial X_a}{\partial p_j^*} \frac{\partial p_j^*}{\partial E} \frac{\partial e'}{\partial \alpha} + \sum_j \frac{\partial X_i}{\partial E} (\bar{p}_j^* - p_j) \frac{\partial R_j}{\partial \alpha} \end{aligned} \tag{4.25}$$

The response of net demand is therefore given by the difference between 4.25 and 4.24, which, using the facts that $\frac{\partial X_a^c}{\partial \alpha} = e_{a,\alpha}$; and $e' = e - \pi - p_l T$, may be written as follows:

$$\begin{aligned} \frac{\partial (\tilde{X}_a - \tilde{Q}_a)}{\partial \alpha} &= (e_{a,\alpha} - \pi_{a,\alpha}) + (e_{a,l} - \pi_{a,l}) \frac{\partial \bar{p}_l^*}{\partial \alpha} + (e_{a,m} - \pi_{a,m}) \frac{\partial \bar{p}_m^*}{\partial \alpha} \\ &\quad + (e_{a,v} - \pi_{v,l}) \frac{\partial \bar{p}_v^*}{\partial \alpha} - \frac{\partial X_a}{\partial E} \frac{\partial e}{\partial \alpha} + \frac{\partial X_a}{\partial E} \frac{\partial Y_f^*}{\partial \alpha} \\ &\quad + \sum_j \left(\frac{\partial \pi_a}{\partial p_j^*} - \frac{\partial X_a}{\partial p_j^*} \right) \frac{\partial p_j^*}{\partial E} \frac{\partial e'}{\partial \alpha} + \sum_j \frac{\partial X_i}{\partial E} (\bar{p}_j^* - p_j) \frac{\partial R_j}{\partial \alpha} \end{aligned} \tag{4.26}$$

This is a general expression for the response of the net demand for the food crop to a change in the exogenous variable. As before, the first four terms capture the response at constant utility, the remaining terms representing "income" and "virtual price" type effects.

An equivalent expression will apply for the net demand for labour. More generally, it is possible to represent the responses of all commodities to changes in exogenous variables in an equation of the form of 4.26, even if the commodities in question are only demanded or only supplied by the household. For commodities which are only demanded by the household, total demand and net demand are equal, so that total demand response is given by the appropriate special case of 4.26, in which some terms will be zero and so drop out. For commodities which are only supplied by the household, total supply is equal to the negative of net demand; in this case supply response will be given by the negative of the appropriate special case of 4.26. Again, many of the terms may be zero; in the case of the cash crop all terms relating to consumption effects are zero, and so 4.26 reduces to 4.15.

Thus, equation 4.26 nests as special cases the pure production and consumption responses given by 4.14 and 4.23 respectively. The attraction of this point is that it enables some general results to be derived.

5 Comparative Statics: Specific Results

The expressions for production, consumption and net demand responses are general in the sense that they apply to all exogenous variables in the model, and (suitably modified where necessary) irrespective of the number of markets assumed to be rationed or missing. Because they are general, it is not possible for the most part to discuss the direction and precise nature of the effects they capture. However, once the exogenous variable in question is specified, then in most cases the general expressions will simplify in form and/or interpretation.

The exogenous variables can be considered in three groups: exogenous prices, ration levels and fixed inputs. In this section these three groups of variables will be considered in turn. Most emphasis will, however, be placed on price changes, as these are of greatest interest from a policy point of view and have attracted most attention in the literature.

5.1 Responses to price changes

While in principle each of the five market prices in the model is exogenous, the net demand for rationed commodities cannot respond to a change in their own price, assuming that the rations continue to bind after the price change. Changes in such prices will, however, give rise to income effects on unrationed commodities. And of course, unrationed commodities will continue to respond to changes in their own prices, and in the prices of other unrationed commodities, though the magnitude of these responses are likely to be affected by the presence of rationing affecting other commodities.

For this reason we focus mostly on the behaviour of unrationed commodities to changes in their own price, considering different numbers of markets as subject to rationing depending on the case in question. Following the structure of section 4, production, consumption and net demand responses will be considered in turn, while noting again that the production and consumption responses are special cases of the more general net demand responses.

5.1.1 Production responses

The question of the supply response in the rationed situation of the cash crop to changes in its price is of particular interest. Considering the most general rationed model, this may be obtained by replacing α by p_c in equation 4.15 which after simplifying gives the following:

$$\frac{\partial \tilde{Q}_c}{\partial p_c} = \frac{-det \begin{bmatrix} e'_{c,c} & e'_{c,a} & e'_{c,l} & e'_{c,m} & e'_{c,v} \\ e'_{a,c} & e'_{a,a} & e'_{a,l} & e'_{a,m} & e'_{a,v} \\ e'_{l,c} & e'_{l,a} & e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,c} & e'_{m,a} & e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,c} & e'_{v,a} & e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}}{\det \begin{bmatrix} e'_{a,a} & e'_{a,l} & e'_{a,m} & e'_{a,v} \\ e'_{l,a} & e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,a} & e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,a} & e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}}$$
$$+ \pi_{c,a} Q_c \tfrac{\partial p_a^*}{\partial E} + \pi_{c,l} Q_c \tfrac{\partial p_l^*}{\partial E} + \pi_{c,m} Q_c \tfrac{\partial p_m^*}{\partial E} + \pi_{c,v} Q_c \tfrac{\partial p_v^*}{\partial E} \qquad (5.1)$$

The first term in equation 5.1 gives the supply response at constant utility. Given the concavity of $e'(.)$, this expression is unambiguously positive, and so the supply response at constant utility is positive. However, the overall supply response will depend also on the income effect terms, the sign of which is ambiguous. This introduces the possibility of negative supply response in the presence of rationing, if the income effects in aggregate are negative and of a magnitude greater than the first term in equation 5.1. From this, it is clear that the possibility of a negative supply response in the presence of rationing is entirely due to the income effects of the virtual price changes, a point which has previously been made by Besley (1988).

The combined income effects in equation 5.1 in fact reduce to the following:

$$\pi_{c,a} Q_c \frac{\partial p_a^*}{\partial E} + \pi_{c,l} Q_c \frac{\partial p_l^*}{\partial E} \qquad (5.2)$$

as a result of the facts that: (i) $\pi_{c,m} = 0$, because the profit function is independent of p_m which is purely a consumption parameter; and (ii) $\frac{\partial p_v^*}{\partial E} = 0$, because the virtual price of the variable input will depend only on the household's demand for V which will depend only on production parameters (remembering that the ration level $\bar{V}$ is exogenous). The other two income effect terms, as given by equation 5.2, are non-zero in general; further, the direction of these two effects, individually and in combination, cannot be specified in general and will depend on the particular case. However, if $\pi_{c,a} < 0$ and $\pi_{c,l} < 0$, both of which are likely to hold, and both leisure and the food crop are normal goods in consumption, so that $\frac{\partial p_a^*}{\partial E} > 0$ and $\frac{\partial p_l^*}{\partial E} > 0$, then the combined income effect will be negative. If it is sufficiently negative to outweigh the positive effect at constant utility then the household's supply response to an increase in the price of the cash crop will be negative.

While the above assumptions giving rise to a negative combined income effect appear plausible, even if they do apply, the direction of the overall supply response is not necessarily negative. The rations present in this general model introduce the possibility of a negative supply response, but do not guarantee it.

This analysis of supply response of the cash crop to its own price applies irrespective of the number of markets which are rationed. In particular, the supply response at constant utility will always be positive. For example in a case in which the market for the variable input is unrationed, but in which the markets for labour, the food crop and the market purchased good are rationed as before, the supply response is given by

$$\frac{\partial \tilde{Q}_c}{\partial p_c} = \frac{-det \begin{bmatrix} e'_{c,c} & e'_{c,a} & e'_{c,l} & e'_{c,m} \\ e'_{a,c} & e'_{a,a} & e'_{a,l} & e'_{a,m} \\ e'_{l,c} & e'_{l,a} & e'_{l,l} & e'_{l,m} \\ e'_{m,c} & e'_{m,a} & e'_{m,l} & e'_{m,m} \end{bmatrix}}{\det \begin{bmatrix} e'_{a,a} & e'_{a,l} & e'_{a,m} \\ e'_{l,a} & e'_{l,l} & e'_{l,m} \\ e'_{m,a} & e'_{m,l} & e'_{m,m} \end{bmatrix}} + \pi_{c,a} Q_c \frac{\partial p_a^*}{\partial E} + \pi_{c,l} Q_c \frac{\partial p_l^*}{\partial E} \qquad (5.3)$$

This is essentially the characterisation used by Bevan, Bigsten, Collier and Gunning (1987; hereafter referred to as BBCG) in discussing the effects of rationing on supply response, although they are not explicit about their assumptions regarding the labour market. They argue that in such a situation peasant supply response will be perverse. However, as this analysis shows, and as previously pointed out by Besley (1988), this will not be the case in general. At constant utility supply response will be unambiguously positive (the first term in equation 5.3)); overall supply response will only be negative if the combined income effects are sufficiently negative to outweigh the effect at constant utility. The fact that Bevan *et al.* obtain an unambiguous result is a consequence of the restrictive assumptions of their model, specifically the fact that they do not allow for substitution possibilities in production or consumption; in a more general context, but with the same rations (or missing markets) applying, their unambiguous result will no longer apply[13].

Based on this analysis of supply response, an analysis which encompasses and generalises the models of BBCG and Besley, it is clear that a necessary condition for the *possibility* of the cash crop displaying a negative supply response to own price changes is that either the labour market or the market for the food crop (or both) be subject to rationing or missing. Only in these cases do the income effect terms in equation 5.1 come into play, and without these income effects

[13]Besley (1988) makes this point using a peasant household model similar to the one presented here, with three rations affecting the markets for labour (which is essentially absent), for the market purchased consumer good and for the variable input. However, the market for the food crop is assumed to exist. As pointed out by Bevan, Collier and Gunning (1991) this is an important point of difference with the original BBCG model; however, this observation does not undermine the validity of Besley's basic point, which equally applies in the more generally rationed model here in which the market for the food crop is also subject to rationing

there is no possibility of a negative supply response in this model. However, as previously noted, this is *not* a sufficient condition for a negative supply response.

A corollary of this point is that a negative supply response cannot occur in the framework of this model if only the markets for V and/or X_m are rationed. In the case in which only V is subject to rationing, equation 5.1 reduces to the following:

$$\frac{\partial Q_c}{\partial p_c} = \frac{-det \begin{bmatrix} e'_{c,c} & e'_{c,v} \\ e'_{v,c} & e'_{v,v} \end{bmatrix}}{e'_{v,v}} = -e'_{c,c} + \frac{(e'_{c,v})^2}{e'_{v,v}} = \pi_{c,c} - \frac{(\pi_{c,v})^2}{\pi_{v,v}} \qquad (5.4)$$

which is necessarily (i) less than $\pi_{c,c}$, the supply response in the unrationed situation, given that $\pi_{v,v} > 0$; and (ii) positive, given the convexity of the profit function. Thus the supply response is unambiguously reduced by rationing affecting the variable input, which is the standard Le Chatelier result (Samuelson, 1947), but remains strictly positive because of the absence of income effects. In the case in which only X_m is rationed, equation 5.1 reduces to simply:

$$\frac{\partial Q_c}{\partial p_c} = -e'_{c,c} = \pi_{c,c} \qquad (5.5)$$

in other words, the same as in the unrationed situation. In other words, *by itself* rationing of X_m only affects consumption behaviour, and has no effect on production behaviour. This is in contrast to the assertion of Bevan et al (1987, p. 431) that this is the cause of negative supply response. As long as one allows for some substitution possibilities in consumption, rationing of the market purchased good will not by itself give rise to negative supply response; this can only arise if rationing affects markets in which the household acts both as consumer and producer, in other words the labour and food crop markets in this model. However, it is true, as is demonstrated elsewehere (McKay and Taffesse, 1994), that if either the labour market or the food crop market is affected by rationing, then additional rationing in the market for X_m cannot increase, and will in general reduce, the price elasticity of supply of the cash crop at constant utility. In fact, in this particular case, although not in general, it is possible to make a stronger statement; because the introduction of rationing of X_m does not introduce any additional income effects, it will not increase, and will in general reduce, supply response *overall*. Thus, in this situation, rationing of X_m will increase the probability that the supply response is negative, but is not the fundamental cause of this negative supply response.

5.1.2 Consumption responses

Consumption responses to price changes may be derived based on equation 4.23, by replacing α with the relevant price under consideration. In any particular instance, however, some of the terms in 4.23 may be zero. In this respect, the following general observations can be made:

(a) The market or virtual price of a pure consumer good (in our case X_m) cannot directly produce a profit effect. However, if such a good is unrationed its market price may affect full income via relevant virtual prices.

(b) The market or virtual price of a good which is either exclusively produced for sale or solely used in production (in our model Q_c and V, respectively) cannot have the direct effect. Again, if unrationed, it can indirectly induce such an effect through pertinent virtual prices.

(c) The sixth term of 4.23 will not apply in the case of a price change.

We begin with the most general rationed model, in which the only unrationed commodity is the cash crop. Changes in its price, however, have no direct consumption effect. On the other hand, it is possible for food crop and leisure consumption to vary within the limits imposed by $\bar{Q}_a$, $\bar{L}$, production technology, and household preferences. Consequently, the impact of p_c on consumption takes the form of virtual price and profit effects on X_a and X_l. These effects can be summarised by 4.23 with $\alpha = p_c$, so that the first two terms (the direct effect) on the right hand side drop out.

$$\frac{\partial \tilde{X}_i}{\partial p_c} = \frac{\partial X_i}{\partial E}\frac{\partial Y_f^*}{\partial p_c} + \sum_j \frac{\partial X_i^c}{\partial p_j^*}\frac{\partial \bar{p}_j^*}{\partial p_c} - \sum_j \frac{\partial X_i}{\partial p_j^*}\frac{\partial p_j^*}{\partial E}\frac{\partial e'}{\partial p_c} \tag{5.6}$$

where: $i = a, l$; $j = a, l, m, v$.

The virtual price of the purchased variable input V can only have a profit effect, i.e. $\frac{\partial X_i^c}{\partial p_v^*} = \frac{\partial p_v^*}{\partial E} = 0$. Obviously the direction of impact of p_c on the total demand for X_i is indeterminate *a priori*. The net demand for it, however, remains the same due to rationing. Of course, Q_a and L can, and very likely will, respond to p_c, but again with the limits noted above. It is not possible to make more substantive general statements about the ultimate effect of p_c.

Changes in other exogenous prices can be considered in the same way. However, in the most fully rationed model net demand will remain unchanged, though total demands and supplies may change as a result of income effects.

Let us now remove the rationing in the food crop market and consider the model with rationing in X_m, L and V. Accordingly, we have two exogenous prices, p_a and p_c. As to the possibilities of consumption responses, now the household can "freely" vary its consumption (and production for that matter) of the food crop. It can also change its leisure consumption so long as the binding ration, $\bar{L}$, is satisfied.

The effect of p_c on consumption is essentially the same as in the fully rationed case to the extent that it will still be composed of virtual price and profit effects. For that reason, and because it leads to results which mirror those relating to the supply response of Q_c, the effect of a change in p_a is more interesting. Because the food crop is a good which is both produced and consumed by the household, changes in its price generate all three of the effects captured by 4.23, namely the direct, virtual price and profit effects. Thus, focusing on X_a, since it is unrationed, we have

$$\begin{aligned}\frac{\partial \tilde{X}_a}{\partial p_a} &= \frac{\partial X_a^c}{\partial p_a} + \sum_j \frac{\partial X_i^c}{\partial p_j^*}\frac{\partial \bar{p}_j^*}{\partial p_a} - \frac{\partial X_a}{\partial E}\frac{\partial e}{\partial p_a} + \frac{\partial X_a}{\partial E}\frac{\partial Y_f^*}{\partial p_a} \\ &\quad - \sum_j \frac{\partial X_a}{\partial p_j^*}\frac{\partial p_j^*}{\partial E}\frac{\partial e'}{\partial p_a} \qquad (5.7)\end{aligned}$$

where: $j = l, m, v$. Little can be said about this expression except that:

(a) the direct subsititution effect is negative by concavity of e;

(b) $\frac{\partial X_i^c}{\partial p_v^*} = \frac{\partial p_v^*}{\partial E} = 0$;

(c) the ultimate result reflects substitution possibilities in consumption and the sign of "income" effects.

As stated in considering the general case above, and as will be discussed for the specific case of a price change shortly, additional insights can be obtained if we consider the net demand for the food crop.

An expression equivalent to 5.7 applies for leisure demand in the case in which the labour market is unrationed. Similar remarks apply to its interpretation.

Finally, we consider the case where the household faces rationing in the labour and food crop markets, but is unrationed in that of X_m. Equation 4.23 still applies with the qualification that p_m has no profit effect and the relevant virtual prices are those of labour and the food crop. Hence the impact of a change in p_m on the demand for X_m is:

$$\begin{aligned}\frac{\partial \tilde{X}_m}{\partial p_m} &= \frac{\partial X_m^c}{\partial p_m} + \sum_j \frac{\partial X_m^c}{\partial p_j^*}\frac{\partial \bar{p}_j^*}{\partial p_m} - \frac{\partial X_m}{\partial E}\frac{\partial e}{\partial p_m} \\ &\quad - \sum_j \frac{\partial X_a}{\partial p_j^*}\frac{\partial p_j^*}{\partial E}\frac{\partial e'}{\partial p_a} \qquad (5.8)\end{aligned}$$

where $j = a, l$.

5.1.3 Net demand responses

As noted in discussing the general comparative static expression, the production and consumption responses to price changes are special cases of a more general net demand response. As in section 4, this will be discussed in terms of the food crop, assuming that its market is unrationed.

The response of net demand for the food crop to a change in its own price is given by replacing α by p_a in 4.26:

$$\begin{aligned}\frac{\partial(\tilde{X}_a - \tilde{Q}_a)}{\partial p_a} &= (e_{a,a} - \pi_{a,a}) + (e_{a,l} - \pi_{a,l})\frac{\partial \bar{p}_l^*}{\partial p_a} + (e_{a,m} - \pi_{a,m})\frac{\partial \bar{p}_m^*}{\partial p_a} \\ &+ (e_{a,v} - \pi_{v,l})\frac{\partial p_v^*}{\partial p_a} - \frac{\partial X_a}{\partial E}\frac{\partial e}{\partial p_a} + \frac{\partial X_a}{\partial E}\frac{\partial Y_f^*}{\partial p_a} \\ &+ \sum_j\left(\frac{\partial \pi_a}{\partial p_j^*} - \frac{\partial X_a}{\partial p_j^*}\right)\frac{\partial p_j^*}{\partial E}\frac{\partial e'}{\partial p_a} \end{aligned} \qquad (5.9)$$

In this case, the changes in the virtual prices resulting from the change in p_a are again given by solving an equation of the form 4.7, though of one dimension less to allow for the fact that the food market is now unrationed:

$$\begin{bmatrix} e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix} \begin{bmatrix} \frac{\partial \bar{p}_l^*}{\partial \alpha} \\ \frac{\partial \bar{p}_m^*}{\partial \alpha} \\ \frac{\partial \bar{p}_v^*}{\partial \alpha} \end{bmatrix} = \begin{bmatrix} \frac{\partial \bar{L}}{\partial \alpha} - e'_{L\alpha} \\ \frac{\partial \bar{X}_m}{\partial \alpha} - e'_{m\alpha} \\ \frac{\partial \bar{V}}{\partial \alpha} - e'_{v\alpha} \end{bmatrix} \qquad (5.10)$$

Replacing α by p_a, and noting that exogenous rations are unaffected by prices, we use Cramer's Rule to solve for each $\frac{\partial \bar{p}_j^*}{\partial p_a}$. Substituting into 5.9 are re-arranging then gives the following expression for the own price response of net demand for the food crop:

$$\begin{aligned}\frac{\partial(\tilde{X}_a - \tilde{Q}_a)}{\partial p_a} &= \frac{det\begin{bmatrix} e'_{a,a} & e'_{a,l} & e'_{a,m} & e'_{a,v} \\ e'_{l,a} & e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,a} & e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,a} & e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}}{\det\begin{bmatrix} e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}} - \frac{\partial X_a}{\partial E}\frac{\partial e}{\partial p_a} + \frac{\partial X_a}{\partial E}\frac{\partial Y_f^*}{\partial p_a} \\ &+ \sum_j\left(\frac{\partial \pi_a}{\partial p_j^*} - \frac{\partial X_a}{\partial p_j^*}\right)\frac{\partial p_j^*}{\partial E}\frac{\partial e'}{\partial p_a} \end{aligned} \qquad (5.11)$$

The first term on the right hand side of 5.11 gives the response of net demand for the food crop at constant utility; this term gives the total substitution effect, comprising the direct substitution effect of the change in p_a and the indirect substitution effects resulting from the changes in the compensated virtual prices. By the concavity of e', this effect is unambiguously negative. The remaining terms are essentially income/profit effects, operating either directly (according to the magnitude and direction of the net demand *ex ante* or indirectly through virtual prices. These terms make the direction of the overall response of net demand for the food crop to a change in its own price ambiguous, as the sign and magnitude of the direct and indirect income effect terms is indeterminate. Note, for example, that net demand for food *may* respond positively to an increase in its own price, without it necessarily being a Giffen or an inferior good. This, however, is not fundamentally a consequence of the rationing; this possibility also arises in the unrationed model, where it is a consequence of the profit effect (the third term in 5.11; Singh, Squire and Strauss, 1986a).

This result implies that, at constant utility, the response of the marketed surplus of the food crop (the negative of its net demand) to a change in its own price will be unambiguously positive. This is the analogue of the result derived for the cash crop in equation 5.1 above.

As previously observed in discussing the general expression above, an equivalent analysis may be conducted for the response of the net demands for all unrationed commodities to changes in their own prices, including those which are only supplied or only demanded by the household. The general form would be the same as 5.11, although its specific form would differ according to how many and which markets are rationed. For example, the analysis of the suppply response of the cash crop may be set out in terms of an expression equivalent to 5.11, dropping the consumption terms, and reversing the sign to obtain supply response.

From this analysis, we note that at constant utility, the response of net demand for a commodity to a change in its own price is unambiguouly negative. The standard consumer theory result, that the substitution effect is always negative, applies in this framework for commodities which are consumed but not produced by the household. For commodities both supplied and demanded by the household, such as the food crop and labour in this case, an equivalent result holds in terms of net demands. On the production side, at constant utility net supplies will always respond positively to changes in the price of the commodity in question, so that is the specific case of a crop produced but not consumed by the household, supply response at constant utility will always be positive.

Finally, we note that results relating to the unrationed household may be derived as special cases of 5.11 in which all virtual price effects disappear.

5.2 Changes in the ration levels

Changes in exogenously determined ration levels can affect unrationed commodity demands and supplies only through virtual prices, and through - on the consumption side - the income effect of the relaxation of the ration. This is so because unrationed demands, unrationed expenditure and full income, all defined at virtual prices, cannot be directly affected by such ration levels. The result is intuitive in that it is through changes in virtual prices that the household adjusts to new ration levels.

In particular, a change in the ration level has no direct effect on the function

$e'(.)$, it only has indirect effects on $e'(.)$ through the resultant changes in the virtual prices, all of which will change in general when one ration level changes. This absence of a direct effect on $e'(.)$ means that changes in compensated and uncompensated virtual prices are equal to each other. The general production and consumption responses simplify somewhat relative to the general case, given by 4.14 and 4.23 respectively.

5.2.1 Production responses

On the production side, the fact that the ration level does not affect e' means that that the third term in equation 4.14 drops out. The overall response is thus equal to the response at constant utility.

For example, in the case of a change in the ration level $\bar{Q}_a$, the response of the cash crop will be given by equation 4.15, replacing α by $\bar{Q}_a$. After simplifying, this gives the following:

$$\frac{\partial \tilde{Q}_c}{\partial \bar{Q}^*_a} = \frac{-det \begin{bmatrix} e'_{c,a} & e'_{c,l} & e'_{c,m} & e'_{c,v} \\ e'_{l,a} & e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,a} & e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,a} & e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}}{\det \begin{bmatrix} e'_{a,a} & e'_{a,l} & e'_{a,m} & e'_{a,v} \\ e'_{l,a} & e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,a} & e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,a} & e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}} \qquad (5.12)$$

with equivalent expressions holding for the other ration levels. Note that there are no income effects in this case. The change in the ration level for one good changes the compensated and uncompensated virtual prices for all rationed goods, but these remain equal to each other. This change in the virtual prices leads to a change in the supply of the cash crop according to equation 5.12; however, in general the direction of this response is indeterminate.

Again, this result will generalise in the same way as discussed above to cases with fewer rations and to the other production commodities.

5.2.2 Consumption responses

As previously noted, a change in the k^{th} ration level, denoted by R_k, will lead to a change in compensated and uncompensated virtual prices, but it will not affect $e'(.)$ directly, nor will it have a direct effect on consumption. Taking these facts into account, equation 4.23 simplifies to the following:

$$\frac{\partial \tilde{X}_i}{\partial R_k} = \sum_j \frac{\partial X^c_i}{\partial \bar{p}^*_j} \frac{\partial \bar{p}^*_j}{\partial R_k} + (\bar{p}^*_k - p_k) \frac{\partial X_i}{\partial E} \qquad (5.13)$$

As in the case of the pure consumer under rationing (Cornes, 1992), the first term on the right hand side of 5.13 gives the pure substitution response, whereas the second term gives the real income response, which arises from the compensation necessary as a result of the income effect of the ration. As in the case of the production response, it is possible to substitute for $\frac{\partial \bar{p}_j^*}{\partial R_k}$ using equations of the form of 4.8 - 4.11 of the appropriate dimension for the case in question. However, the direction of the pure substitution response will be ambiguous in general. The sign of the real income effect will depend on whether or not X_i is a normal good, and on whether the net demand ration R_k is positive or negative (which determines the sign of $(\bar{p}_k^* - p_k)$).

The situation simplifies if only one market is rationed, as in this case $\frac{\partial \bar{p}_j^*}{\partial R_k}$ will always be negative. The overall effect will then depend on the balance of the two terms on the right hand side of 5.13.

5.3 Changes in fixed inputs

The general expressions 4.14 and 4.23 for production and consumption responses respectively can also be used to examine the effect of changes in the level of fixed inputs. Changes in fixed inputs will operate through their direct effects on e', and their indirect effects on virtual prices. However, it is not possible to generalise about the directions of these effects without making further assumptions.

5.3.1 Production responses

In the general rationed case, the response of the supply of the cash crop to changes in the levels of the fixed inputs (A, K) may be derived in an analogous way, by substituting the appropriate variable for α in equation 4.14. For example, in the case of a change in land area A, the change in the cash crop will be given by:

$$\frac{\partial \tilde{Q}_c}{\partial A} = \frac{-det \begin{bmatrix} e'_{c,A} & e'_{c,a} & e'_{c,l} & e'_{c,m} & e'_{c,v} \\ e'_{a,A} & e'_{a,a} & e'_{a,l} & e'_{a,m} & e'_{a,v} \\ e'_{l,A} & e'_{l,a} & e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,A} & e'_{m,a} & e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,A} & e'_{v,a} & e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}}{\det \begin{bmatrix} e'_{a,a} & e'_{a,l} & e'_{a,m} & e'_{a,v} \\ e'_{l,a} & e'_{l,l} & e'_{l,m} & e'_{l,v} \\ e'_{m,a} & e'_{m,l} & e'_{m,m} & e'_{m,v} \\ e'_{v,a} & e'_{v,l} & e'_{v,m} & e'_{v,v} \end{bmatrix}} - \pi_{c,a} \frac{\partial p_a^*}{\partial E} \frac{\partial e'}{\partial A} - \pi_{c,l} \frac{\partial p_l^*}{\partial E} \frac{\partial e'}{\partial A} \quad (5.14)$$

with an exactly equivalent expression applying for the case of the fixed input K.

As before, the first term gives the effect at constant utility, the direction of which is indeterminate in general. As usual, the sign of the "income effect" terms (the second and third terms of equation 5.14) is also indeterminate and dependent on the particular case in question. For example, if X_a and X_l are normal goods in consumption, and the profit function has the following properties: $\pi_{c,a} < 0$, $\pi_{c,l} < 0$, then the combined income effect of the increase in cultivated area will be to reduce supply of the cash crop. Similar analogues to those discussed under price responses above will also apply to cases with fewer rations, and to the responses of the other production commodities when they are unrationed.

5.3.2 Consumption responses

Changes in the level of fixed inputs, in general, produce both virtual price and profit effects. Again these effects can be expressed by 4.23 by setting α to A or K, and dropping the first two and sixth terms (the direct effects and the ration relaxation term, respectively). For example, in the case of a change in A:

$$\frac{\partial \tilde{X}_i}{\partial A} = \frac{\partial X_i}{\partial E}\frac{\partial Y^*}{\partial A} + \sum_j \frac{\partial X_i^c}{\partial \bar{p}_j^*}\frac{\partial \bar{p}_j^*}{\partial A} - \sum_j \frac{\partial X_i}{\partial p_j^*}\frac{\partial p_j^*}{\partial E}\frac{\partial e'}{\partial A} \tag{5.15}$$

As in the case of the production effects, it is not possible to unambiguously identify the direction of consumption changes.

6 Conclusions

This paper has attempted to set out a general model of the behaviour of a peasant household faced by rationing, and to set out the implications of rationing for its production and consumption behaviour. Rationing imposes additional constraints on household choice, and so will adversely affect the flexibility of households in responding to changes in exogenous variables such as prices. At constant utility, the supply response in production and the substitution effect in consumption are reduced in absolute magnitude by rationing, but their signs do not change. However, the overall (uncompensated) effect is indeterminate because of the presence of "income" type effects, the magnitude and direction of which cannot be specified *a priori*. In the case of production, these income effects can, but will not necessarily, give rise to perverse supply response to price changes.

The model used in this paper has enabled us to derive general expressions for the response of the peasant household's production and consumption behaviour to changes in exogenous variables, which can be applied to any commodity and any exogenous variable in the model, and irrespective of the extent of rationing. Furthermore, the model can be easily generalised to an N-good case (McKay and Taffesse, 1994).[14]

[14] The N good model can also be used to derive some general conclusions about behaviour of producers and consumers under rationing (McKay and Taffesse, 1994).

However, some significant features of the environment in which the peasant household operates are not capured by the present version of the model. Two important directions in which the model could usefully be developed are the following:

(a) the development of a dynamic equivalent to the present static model;

(b) the incorporation of risk aversion on the part of peasant households.

The incorporation of these features can be expected to further modify the responsiveness of the household to changes in exogenous variables.

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