Status, Signalling and Positional Competition



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Abstract

The first chapter analyses a game between the desired and undesired agents and the Decision Maker (DM), who only wants to admit the desired agents. To influence the DM's decision, both types of agents can purchase an ostentatious good at the same price, yet the undesired agent sometimes fails to make the right impression. I investigate how the imposition of a capacity constraint can improve the outcomes for the DM. I show that committing to ignore the message received from the agent is an even better policy for the DM. I then add a second, counterfeit good to the model, which is cheaper than the original and only guarantees success with the same, low probability for both types. I show that while full separation might not be obtainable where it used to be without counterfeits, the DM's expected payoff may increase compared to the baseline model.

The second chapter analyses markets in which several *ex-ante* identical brands are offered and then purchased by status-conscious consumers. In equilibrium, the brands are acquired by agents with different incomes and their quality, or the status they confer, is thus *ex-post* differentiated. I analyse different market structures and provide comparisons *vis-à-vis* benchmark models of product differentiation. When status matters even to the poorest agent, the monopolist over-produces each brand, which can be rectified by a flat per-unit tax. Moreover, while an equilibrium does not always exist in more competitive market structures, I find that sequential entry and large enough differences in cost structures are sufficient to guarantee one. In such equilibria, however, both consumer surplus and total welfare are lower than they would be under a monopoly.

In the third chapter, I build a model of cardinal positional competition, in which agents compare their own positional consumption to the average positional consumption. In equilibrium, agents over-consume compared to the social optimum and lower average positional consumption increases the utilities of all agents. An increase in income inequality raises the average positional consumption if the demand for the positional good is convex in income. I then apply the model in a scenario where agents first have to join one of potentially heterogeneous communities, and only then engage in the positional competition against their fellow community members. I find that moderate differences in qualities of the communities are necessary to support a stratified equilibrium. If assignment to the communities is externally determined, any relocation of agents from the richer to the poorer community reduces average positional consumption in both communities.

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Chapter 1

Selection and Incentives with Signalling Agents

1.1 Introduction

It is a well-known sociological phenomenon that individuals strive to obtain access to various groups, clubs, associations, organisations or social circles (thereafter collectively called 'clubs') whose membership is to some extent exclusive. This poses two kinds of questions: those regarding what the admission policies are and should be, and those on how the applicants adjust their behaviour to influence the admission decision. In the simplest context, the club's decision boils down to separating the *desired* agents from the *undesired* agents, where the meaning of the labels depends on the particular application. For example, a golf club may only want to admit applicants whose handicap is low enough,[1] while certain liberal arts colleges may only want to admit applicants committed to charitable work.[2] In the latter example, the desired types really care about the charitable causes, while the undesired types

[1]The Old Course in St Andrews, for instance, requires players to have at most a handicap of 24 for men and 36 for women, see https://www.standrews.com/play/courses/old-course/playing-the-old-course

[2]In the US, 58 per cent of admission officers surveyed by x2VOL actively look for participation in community or voluntary services in applicants, see http://trackservicehours.x2vol.com/encouraging-community-service-whitepaper

would like to get in for some other, non-altruistic, reason.

The incentives of the club are not fully aligned with those of all prospective members, because all applicants would like to get in, but the club would only like to admit the desired ones. As the desirability of an applicant from the club's perspective is unobservable, the club may demand that the applicant provides a certain signal which helps the club to categorise him. The personal essay, in which the proof of prior charity work may be included, serves this role in the college admissions example. The cost-benefit balance of sending such a signal may not be the same between both types of applicants. For one, it may be the case that it costs the undesired type more to send the signal; the inherently mediocre golf player would find it considerably more frustrating to obtain the required handicap. Another reason, and the focus of this chapter, is that, despite equal costs, the undesired type sometimes fails at producing the right signal. In the college admissions example, the undesired type is not really interested in charitable causes. The admissions officer may with a positive probability infer this fact from the rest of his application, despite the seemingly right signal, by for example discovering an incoherence in his personal statement.

The problem becomes more complex when in addition to the main signal, there also exists a fake one, which can be acquired at a lower cost, but only sometimes works. In the context of university applications, examples include a bogus high-school diploma, fake letters of recommendation or a made-up volunteering experience. If the applicant is not found out, these signals work just as well as the originals would, but if he is, it sends a different signal which would probably be interpreted very negatively by the college.

An excellent example that encompasses all these themes in the broader positional context of this thesis is provided by prestigious nightclubs and their door policies. Rivera (2010), who analysed this topic through interviews with doormen and ethnographic research in one of the "plushest" clubs in Chicago, finds that the club's profit

motive pushes it to select patrons who are big spenders and who make the other patrons feel comfortable. Door policies are therefore designed to adequately sort the prospective patrons into those admitted and those refused entry. Rivera (2010) finds that coming dressed in fancy clothes and with an expensive watch sends the right signal about the patron being of the "right" kind and significantly impacts the chances of being let in. Moreover, it may so happen that the "uncool" patrons will be recognised as such despite their best efforts to indicate otherwise. This can take the form of wearing expensive, but mismatched clothes, or otherwise misjudging the type of signals that are desirable. Purchasing an expensive Armani tracksuit may cost as much as an Armani shirt, but it does not correlate well with the image of the club's target customer and thus is unlikely to secure entry. Likewise, while a Rolex may help to secure entry, the discovery that it is a counterfeit most certainly ruins the chances of obtaining admission.

In this chapter, I develop a model of club admissions, where the agent's type is his private knowledge and the club must rely on imperfect indicators of the candidate's suitability, chiefly the signals sent by the agents themselves, to make an admission decision. There are two kinds of players: a club, called the Decision Maker (DM thereafter), and the agents, who may be either desired (called Good) or undesired (called Bad). The DM has to decide whether to admit the candidates or not. It would however only like to admit the Good agents and reject all Bad agents. The agents of both types, however, always prefer to be admitted. Each agent privately observes his own type, and then decides whether to purchase an ostentatious good, such as a Rolex watch. If the sender is Good, the signal is transmitted without interruption, but if he is Bad, the signal may get corrupted with a certain, fixed probability. This is the only source of heterogeneity between agents in this model, and it captures the above-mentioned observation that despite trying, the Bad agents may be unable to send the right kind of signal.

In Section 1.2, I formally set up the model described above. The kind of equi-

librium which arises depends on the parameters of the model. When the cost of the ostentatious good is high relative to the Bad type's probability of success, the DM may obtain full separation, which is the DM's preferred outcome. Otherwise, when the proportion of the Good types in the population is high compared to the Bad type's chances of success, all agents purchase the good and all successful purchasers are admitted. When the proportion is low, the Good types purchase, the Bad types mix between purchasing and not purchasing, and the DM mixes between admitting and rejecting upon the observation of a success.

I subsequently examine three different measures which may help the DM to achieve a better outcome. First, I look at a capacity constraint, which limits the total number of agents which can be admitted. Here, the nightclub could announce that it only let in a certain number of people on any particular night. Second, I consider the DM's commitment to ignoring the received message with a fixed probability. Here, the nightclub could make an informed decision on a randomly chosen, fixed-size subset of people in the queue, while always admitting or always rejecting the rest. Third, the DM can obtain additional, objective information about the agents who seek to gain entry and condition its decision on the results of such an exercise. To achieve this, the nightclub could ask for frequent guests' opinion about prospective members and only let in those who receive a recommendation. In all of these analyses, I am interested not only in the impact on the DM's expected payoff, but also in the overall spending on the ostentatious good, because it may be considered to be wasteful and thus socially undesirable.

In Section 1.3, when a capacity constraint binds, the agents may reduce their purchasing of the ostentatious good, because their chances of getting in are diminished. As the constraint becomes tighter, the Bad type reduces its purchasing before the Good type does. In fact, the Bad type may at some point be completely discouraged from purchasing. There are two main consequences of a capacity constraint. First, under mild conditions, some undesirable equilibria are eliminated, such as

those in which no agents purchase and everyone is admitted. Second, the DM may actually *want to* impose the capacity constraint upon itself, as it improves the DM's expected payoff, unless the proportion of the Good types in the population is high.

In Section 1.4, I examine how the ability of the DM to limit its attention, so that it only observes the message sent by the agents with a certain, fixed probability, impacts its expected payoff. There are two effects to consider. On the one hand, the DM ignores potentially useful information and has to rely solely on its prior. On the other hand, the incentives may be diminished sufficiently for the Bad agent not to purchase the good anymore. Therefore, the optimal level of ignorance induces the Bad agent to *just* not purchase, unless it remains preferable to continue to pay full attention, which may be the case when the cost of the good is low and the proportion of the Good types in the population is high. While their mechanisms are similar, commitment to ignorance turns out to be a superior measure to a capacity constraint, as not only can the DM obtain a weakly higher expected payoff, but also the overall spending is reduced.

Finally, Section 1.5 introduces counterfeits to the model. Counterfeits cost less than originals and give some probability of success, which is the same for both types and lower than the chances of the Bad type succeeding when he purchases the original. The Bad types may therefore substitute purchasing of the counterfeit for purchasing of the original or not purchasing at all. As long as the cost of the counterfeit is lower than its probability of success, the DM cannot obtain full separation anymore despite having control over the price of the original. However, the DM's expected payoff may go up when counterfeits are bought in place of the originals whenever the originals are moderately priced, because such a switch lowers the Bad type's chances of success, and thus improves separation between agents. When commitment to ignorance is added, there are two additional possibilities compared to the baseline model, that the DM has to consider. In particular, an optimal setting for the attention level either just discourages the Bad type from purchasing

the counterfeit or makes the Bad type switch from purchasing the original to the counterfeit. To the best of my knowledge, this is the first attempt to extend the signalling literature in the direction of including counterfeits.

1.1.1 Related Work

While the topic of this chapter may point the reader towards the classical theory of clubs (Buchanan 1965), my focus in not on the provision of the club good, or the resulting congestion and optimal membership size. Indeed, the value of the membership in the clubs that I examine does not as such decrease with the number of members. Rather, it goes up when the desired agents join, and down when undesired agents do. Therefore, the key consideration becomes the selection problem: how to ensure that only the desired agents are admitted to the club?

There is a natural parallel between my signalling model in this chapter and the seminal Spence (1973) paper, where workers signal their ability to prospective employers. When the more able workers can take observable, costly and unproductive actions that the less able workers cannot take, they often choose to do so. Wages are then paid differentially conditional on the signal sent. My model, however, deviates from Spence (1973) in two aspects. First, both types of agents can choose to purchase at the same price, but the probability with which they succeed is different. Second, and more importantly, the club's decision is binary (to admit or to reject) rather than spanning the full interval of possible wage offers. Hence, the signals received by the club form the basis of its screening to distinguish the undesired from the desired types. This leads to problems with equilibrium selection because the number of possible out-of-equilibrium strategies is smaller, and thus the usual refinements have less bite.

While the focus of this chapter is on signalling through the purchase of an ostentatious good, my approach can be also interpreted in the context of communication models. In particular, it resembles the cheap talk models akin to Crawford and Sobel

(1982), where the sender can lie costlessly without fear of detection. In my model "lying" is re-interpreted as purchasing of the ostentatious good by the undesired type, while "telling the truth" is re-interpreted as purchasing of the ostentatious good by the desired agent. My model differs from that of Crawford and Sobel (1982) in a number of important respects. First, purchasing of the ostentatious good is always costly. Second, the preferences take an extreme form: the desired agent's preferences are fully aligned with the club's, while the undesired agent's are fully misaligned, which is a difference compared to the upward bias of Crawford and Sobel (1982). Finally, the club is able to tell apart the undesired types from the desired types with an exogenous probability, despite both of them purchasing the ostentatious good. This brings my game closer to the "bluffing game" described by Holm (2010). Overall, these factors make the game much less favourable to the undesired types, as they incur the cost of signalling and face the possibility of failure.

Somewhat closer to this chapter is the model of Kartik (2009), because he assumes that lying is costly. However, the receiver remains unable to detect lying and the sender's preferences differ from the receiver's preferences by an upward bias. The state space in Kartik (2009) is bounded above, as in my model, which leads to the separation becoming incomplete and the high types pooling together. Hertel and Smith (2013) build upon these results to demonstrate that when the sender and the receiver have aligned preferences, the *no intention to separate (NITS)* condition always results in the largest possible number of actions in the equilibrium. However, when preferences are not aligned - as is the case in my model - *NITS* does not guarantee uniqueness of the equilibrium. An increase in the communication costs can nonetheless improve communication. My model supports this conclusion in a number of contexts.

Due to its payoff structure, even closer to my model is a paper by Dziuda and Salas (2017), where in addition to costly lying, the receiver wants to take an action corresponding to the state of the world, while the sender wants the receiver to take

the highest possible action. Also similar to my model, with a positive probability the receiver learns that an untrue claim is a lie. Dziuda and Salas (2017) show that in such a setting partially informative equilibria exist, in which if the state of the world is moderate or high, the sender tells the truth, while if the state of the world is low, the sender lies by pretending that it is high. This corresponds to my model, with a suitable relabelling of the desired type as the "high state of the world" and the undesired type as the "low state of the world".

Strategic communication literature has explored different measures which may improve the receiver's expected payoff. Blume et al. (2007) add a communication error, or noise, to the standard cheap talk model with costless signals. In such a set-up, observed messages are drawn from a fixed error distribution with some probability, independent of the message sent, and otherwise go through as sent. They find that such a mechanism can improve welfare thorough inducing the sender to reveal more information than he otherwise would, despite the clear loss of information for the receiver. Their findings are confirmed by Goltsman et al. (2009), who show that in the optimal mediation scheme, the mediators filter the unmediated communication and introduce noise to it. My approach is somewhat similar when I allow the club to commit to ignorance with an *ex-ante* determined probability. The difference, however, is that this probability is chosen optimally by the club and not determined exogenously, which further improves the expected payoff.

Finally, in the last section of this chapter, I consider the introduction of counterfeits. Following Grossman and Shapiro (1998), counterfeits can be classified as either deceptive, which claim to be genuine goods and are hard to distinguish for the consumer, or as non-deceptive, which are deliberately chosen by the consumers for they offer an improvement on the quality-status trade-off compared to the genuine good. My counterfeits are non-deceptive, but there is no distinction between quality and status, as only their role as a signal matters. Thus, the chances of success with the counterfeit are lower than with the original, but the incurred cost is also lower.

My model of counterfeits resembles that of Van Kempen (2003), where consumers care about their social status. Consumers first consume the counterfeit, which is observed by the receiver, who assesses the signal and determines the purchased good's status quality. The receiver then derives a perceived level of income, which is subsequently endowed to the sender in the form of status. The better the counterfeit quality, the higher the status, which without the counterfeit deception would be too expensive for the consumer to acquire. Unlike Van Kempen (2003), in my model there is no difference in the incomes of the agents and they do not care about status *per se*. Rather, agents care about the membership of an exclusive club, which is only granted to a chosen subset of them. Purchasing the counterfeit is therefore an alternative means to convince the club that the agent is desired whenever purchasing the original is uneconomical from the agent's perspective.

1.2 Baseline Model

The game is played between the DM and the population of agents who apply to the DM. The type of each agent, θ, can be Good ($\theta = G$) or Bad ($\theta = B$), and the type constitutes the agent's private information. The proportion of Good agents in the population is ρ, where $\rho \in (0, 1)$. The DM must take one of two possible actions: it either admits the applying agent ($a = A$) or rejects him ($a = R$). The DM would like only to admit the Good agents, as they give it a positive payoff and the Bad agents give it a negative payoff. Rejecting either type does not impact the DM's payoff. Assuming that the magnitude of the positive and negative payoffs is identical and normalised, the DM obtains $v(a, \theta) = 1$ if $a = A$ and $\theta = G$, $v(a, \theta) = -1$ if $a = A$ and $\theta = B$, and $v(a, \theta) = 0$ if $a = R$ and $\theta \in \{G, B\}$. Regardless of the type, all agents obtain the same payoff if admitted, and obtain another, identical payoff if rejected. Normalisation of payoffs yields $w(a) = 1$ if $a = A$ and $w(a) = 0$ if $a = R$,

so both types always prefer to be admitted than rejected.[3] The DM as well as the agents are risk-neutral expected utility maximisers.

The agents may try to influence the DM's decision by purchasing an ostentatious good at a cost $C \in (0,1)$. In other words, the agent makes a decision of whether to purchase ($d = P$) or not to purchase ($d = N$). The agent's decision is then translated into a message $m \in \{S, F, N\}$ that is observed by the DM, where S corresponds to a "success", F to a "failure", and N to a lack of purchase, or "nothing". The only source of asymmetry between the agents lies in the fact that while the Good type's purchase is always successful, $\Pr(m = S \mid \theta = G, d = P) = 1$, the Bad type sometimes fails, $\Pr(m = S \mid \theta = B, d = P) = y$ and $\Pr(m = F \mid \theta = B, d = P) = 1 - y$. $y \in (0,1)$ is the probability that the Bad agent successfully obtains and displays the ostentatious good, and can reflect the skill of the Bad agent or constitute a characteristic of the particular ostentatious good. If the agent does not purchase the good, the DM always observes the message $m = N$.

The game proceeds as follows: (i) the agents privately observe their type and decide whether to purchase the ostentatious good, (ii) the DM receives a message based on whether the agent purchased the good and whether he was successful in doing so, (iii) the DM forms an inference about the agent's type based on the message received and decides whether to admit the agent, and (iv) the payoffs are realised.

Upon observing a message m, the DM forms an inference about the probability that the agent is Good. This posterior belief is given by $\beta_m = \Pr(\theta = G \mid m)$. The DM's expected payoff is $\mathbb{E}[v(\theta, a = A)] = \beta_m - (1 - \beta_m)$ if the agent is admitted, and $\mathbb{E}[v(\theta, a = R)] = 0$ if he is rejected, so the DM's decision rule is as follows: admit if $\beta_m > \frac{1}{2}$, be indifferent between admitting and rejecting if $\beta_m = \frac{1}{2}$, and

[3] We might imagine that the value of being admitted to the club depends on the composition of its membership. Thus, an alternative formulation, which better captures such an externality, might assume that the agents' payoffs depend on the proportion of Good types which ends up admitted by the DM. However, this complicates the analysis and does not lead to additional insights in the baseline model. Therefore, I opt for the simpler specification.

reject if $\beta_m < \frac{1}{2}$. Thus, let the DM's strategy be given by $\delta = (\delta_S, \delta_F, \delta_N)$, where $\delta_m = \Pr(a = A \mid m)$ denotes the probability with which the DM admits upon observing message m.

The agent's expected payoff when he does not purchase is $u(\theta = G, d = N, \delta) = u(\theta = B, d = N, \delta) = \delta_N w(A) + (1 - \delta_N) w(R) = \delta_N$. The expected payoff from purchasing is $u(\theta = G, d = P, \delta) = \delta_S w(A) + (1 - \delta_S) w(R) - C = \delta_S - C$ if the agent is Good, and $u(\theta = B, d = P, \delta) = y(\delta_S w(A) + (1 - \delta_S) w(R)) + (1 - y)(\delta_F w(A) + (1 - \delta_F) w(R)) - C = y\delta_S + (1 - y)\delta_F - C$ if the agent is Bad. Each agent purchases whenever $u(\theta, d = P) > u(\theta, d = N)$, is indifferent between purchasing and not purchasing if $u(\theta, d = P) = u(\theta, d = N)$, and decides not to purchase if $u(\theta, d = P) < u(\theta, d = N)$. Thus, the agents' strategies are given by $\sigma = (\sigma_G, \sigma_B)$, where $\sigma_\theta = \Pr(d = P \mid \theta)$ denotes the probability with which the agent of type θ purchases the ostentatious good. Denote the agent's conditional expected payoff when he makes a decision d by $v_\theta^d = u(\theta, d, \delta^*)$ and the unconditional expected payoff by $v_\theta^* = u(\theta, \sigma^*, \delta^*)$. Note that I assume that all agents apply to the DM and there is no outside option available, which, if it were to guarantee a payoff of zero, is innocuous as the agent can always obtain at least zero by not buying, $u(\theta, d = N) = \delta_N \geq 0$.

The relevant solution concept is a Perfect Bayesian Equilibrium (thereafter equilibrium): a triple (σ^*, δ^*, β^*), where (i) $\sigma^* = (\sigma_G^*, \sigma_B^*)$ maximises the agents' expected payoffs, $u(\theta, \sigma_\theta, \delta^*)$ for $\theta \in \{G, B\}$, given the DM's strategy δ^* and the belief β^*; (ii) $\delta^* = (\delta_S^*, \delta_F^*, \delta_N^*)$ maximises the DM's expected payoff, $\mathbb{E}[v(\delta_m, \theta)]$, given the agents' strategies σ^* and the belief β^*; and (iii) $\beta^* = (\beta_S^*, \beta_F^*, \beta_N^*)$ is derived by Bayes rule from the prior distribution over $\{G, B\}$ and the agents' strategies σ^*, whenever possible. Whenever multiple equilibria exist, I employ the D1 criterion to bring their number down. In this context, the D1 criterion requires that when message m is received by the DM, zero weight be put on type θ if there is another type $\theta' \neq \theta$ such that type θ' always strictly benefits from the deviation whenever

type θ benefits from the deviation.

Note that because the Good agent can never fail, we have $\beta_F = 0$ whenever the Bad agent purchases with a positive probability. The DM never admits an agent upon observing $m = F$, so that $\delta_F^* = 0$ holds in every equilibrium. While such a conclusion is commonsensical, whenever the Bad agents do not purchase in the equilibrium, the belief function β_F is technically undefined. But then any suitable refinement, such as the trembling hand perfection, would force the beliefs to be $\beta_F = 0$, as the Good player has no way of ending up with message $m = F$. Thus I will maintain that $\beta_F = 0$ and $\delta_F^* = 0$ for the rest of this chapter.[4]

1.2.1 Equilibrium

The following proposition characterises all possible equilibria of the game:[5]

Proposition 1.1. *When $\rho \leq \frac{1}{2}$, the game has a unique equilibrium in which the agents' and the DM's strategies are:*

(i) if $C < y$ and $\frac{y}{1+y} \leq \rho < 1$, then $(\sigma_G^*, \sigma_B^*) = (1, 1)$ and $(\delta_S^*, \delta_N^*) = (1, 0)$ (equilibrium A),

(ii) if $C < y$ and $0 < \rho < \frac{y}{1+y}$, then $(\sigma_G^*, \sigma_B^*) = (1, \frac{\rho}{(1-\rho)y})$ and $(\delta_S^*, \delta_N^*) = (\frac{C}{y}, 0)$ (equilibrium B),

(iii) if $C \geq y$, then $(\sigma_G^*, \sigma_B^*) = (1, 0)$ and $(\delta_S^*, \delta_N^*) = (1, 0)$ (equilibrium C).

[4] The assumption that the Good agent always succeeds in sending the right signal is a simplification. A more realistic model might assume that $\Pr(m = S \mid d = P, \theta) = y_\theta$, where $0 < y_B < y_G \leq 1$. While the increased number of parameters complicates the analysis, the set of equilibria is similar to the simpler model, and in the limit where $y_G \to 1$ the models converge. Therefore, I will not pursue this analysis further in this chapter.

[5] Throughout this chapter, in the boundary cases where there exists an equality relationship between the parameters of the model, there may arise multiple equilibria. In such cases, I ignore the multiplicity and choose the equilibrium preferred by the agents. For example, when both agents purchase the good and $\rho = \frac{y}{1+y}$, there exists an equilibrium where $\delta_N^* = 0$ for any $\delta_S^* \in (\frac{C}{y}, 1)$, while when $\rho > \frac{y}{1+y}$, there exists a unique equilibrium where $\delta_N^* = 0$ for $\delta_S^* = 1$. As the agents-preferred former equilibrium converges to the latter equilibrium, I assume that the latter equilibrium exists for $\rho \geq \frac{y}{1+y}$.

When $\rho > \frac{1}{2}$, equilibria A and C still exist, but they are not unique anymore as there also exist two other equilibria in which the agents' and the DM's strategies are:

(iv) $(\sigma_G^*, \sigma_B^*) = (0, 0)$ *and* $(\delta_S^*, \delta_N^*) = (1, 1)$ *(equilibrium D),*

(v) $(\sigma_G^*, \sigma_B^*) = (\frac{2\rho-1}{\rho}, 0)$ *and* $(\delta_S^*, \delta_N^*) = (1, 1-C)$ *(equilibrium E).*

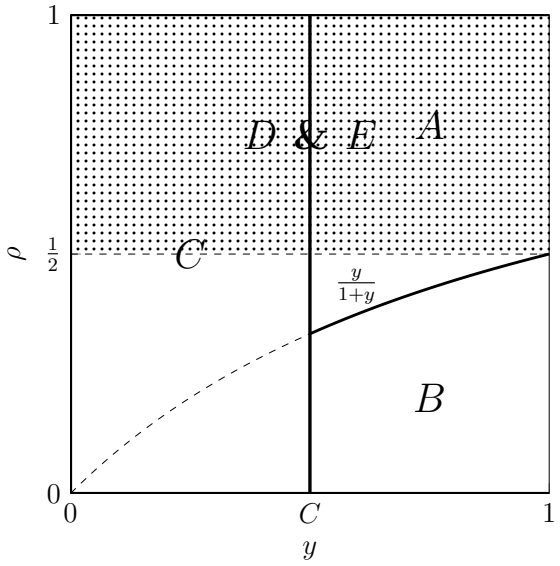


Figure 1.1: All possible equilibria of the baseline model.

The equilibria are illustrated in Figure 1.1. The no-purchase equilibrium for $\rho \leq \frac{1}{2}$ has been eliminated by the D1 criterion. The troubling feature appears to be the multiplicity of equilibria whenever $\rho > \frac{1}{2}$. Standard equilibrium refinements, such as the D1 criterion, do not help in this case as either there are no out-of-equilibrium beliefs, or the agents receive the maximum possible payoff from the game, so neither type would ever want to deviate regardless of the out-of-equilibrium beliefs. In the latter case (equilibrium D), this is akin to the "babbling" equilibrium of the Crawford and Sobel (1982) cheap-talk model: when nobody purchases, the message received by the DM is completely uninformative.

For a number of reasons, however, this multiplicity of equilibria is less troubling than it seems. First, it would be natural to assume that the Good types are "rarer" than the Bad types, or that $\rho \leq \frac{1}{2}$. Such an assumption, by itself, would eliminate

equilibria D and E, which rely on the DM's expected payoff from admitting a randomly chosen agent being positive. Second, there may be an additional stigma for the DM when it admits an agent upon observing no purchase. An alternative interpretation of this scenario might be that the DM actually cares not only about admitting the Good agents, but it also likes to admit owners of the ostentatious good. This could in turn force $\delta_N = 0$, thus eliminating equilibria D and E. Finally, note that both of these equilibria admit a large number of agents, in particular equilibrium D, where the whole population is admitted. Therefore, in the presence of a suitable capacity constraint, these equilibria would not be possible.[6] In light of the arguments above, I define an additional condition, which will be employed throughout this chapter, except where indicated otherwise:

Assumption 1.1. An equilibrium satisfies *Condition N*0 if $\delta_N^* = 0$.

Some further comments pertaining to the remaining equilibria A, B and C are warranted. Consider what happens when the ostentatious good is cheap, $C < y$, so that the Bad type may find it worthwhile to purchase. To what extent this happens depends on the DM's strategy, which in turn depends on the relationship between the proportion of Good types in the population and a function of the success rate for the Bad types. When $\rho \geq \frac{y}{1+y}$, which guarantees that $\beta_S \geq \frac{1}{2}$ when both types purchase, the payoff from admitting agents at random is high, so the DM is willing to admit all the successful purchasers. Thus, all agents purchase the ostentatious good and the only ones rejected are the Bad types who fail. For lower values of $\rho < \frac{y}{1+y}$, which corresponds to $\beta_S < \frac{1}{2}$ when both types purchase, it is no longer optimal for the DM to admit all successful agents. At the same time, $\beta_S = 1$ when only the Good types purchase and the Bad types do not, so there is no equilibrium in pure strategies. Thus, there must exist a mixed-strategy equilibrium, where the Bad agent is indifferent between purchasing and not purchasing, while the DM is indifferent between admitting and rejecting upon success. Finally consider what

[6] This point is discussed in detail in the subsequent sections.

happens when $y \leq C$. The cost of acquiring the ostentatious good outweighs the maximum expected payoff from being admitted for the Bad type. Thus, regardless of ρ, there is always a fully separating equilibrium in which the Good agents purchase, the Bad agents do not, and the DM obtains exactly its desired membership structure.

Throughout this chapter, I look at two welfare objectives. First, I consider the DM's *ex-ante* expected payoff, which matters either because we care about the DM's well-being as such (e.g. a nightclub), or because we have reasons to believe that successful selection is socially beneficial (e.g. university admissions). Second, I consider the minimisation of the overall spending on ostentatious goods, which is justified as long as we assume that it constitutes completely wasteful spending. To simplify notation, let $\Pi_R^* = \mathbb{E}[v(\theta, \delta^*, \sigma^*)]$ denote the DM's expected payoff in equilibrium R, and $\Sigma_R^* = \Pr(d = P) = (\rho \Pr(d = P \mid \theta = G) + (1 - \rho) \Pr(d = P \mid \theta = B))$ denote the proportion of agents who purchase in the equilibrium R, thereafter referred to as the overall spending. The following proposition summarises the welfare results:

Proposition 1.2. *The DM's expected payoff and the overall spending weakly increase with C. The DM's expected payoff weakly decreases with y, but the overall spending does not change monotonously with y. The DM's expected payoff is maximised and the overall spending is minimised in equilibrium C, so if the DM can choose C, it optimally sets $C \in (y, 1)$.*

In equilibrium C, the DM's expected payoff is the highest obtainable, $\Pi_C^* = \rho$, as all Good agents, who are the only ones that purchase, are admitted. It is unsurprising that the DM would always like to find itself in this equilibrium, and thus when it can choose the cost of the ostentatious good, it would ensure that it is high enough to force full separation. In equilibria A and B, the DM obtains a payoff of one on all Good agents, of whom there is ρ, and a payoff of minus one on the successful Bad agents, of whom there is $y(1 - \rho)$ and ρ, respectively. Expected payoffs are therefore $\Pi_A^* = \rho - y(1 - \rho)$ and $\Pi_B^* = 0$. In equilibrium A, the expected

payoff falls in y, because as the Bad types become more successful, they end up being admitted more often, and rises in ρ, because this increases the DM's prior. In equilibrium B, the DM is indifferent between admitting and rejecting upon success, so its expected payoff is the same if it rejected regardless of the message observed, i.e. equals zero. As for the overall spending, in equilibrium C, $\Sigma_C^* = \rho$ as only the Good agents purchase. In equilibrium A, $\Sigma_A^* = 1$ because the chances of getting in for successful purchasers are so high that everyone purchases. In equilibrium B, $\Sigma_B^* = \rho\frac{1+y}{y}$ as Good agents always purchase, while Bad agents, of whom there is $1-\rho$, purchase with probability $\frac{\rho}{y(1-\rho)}$. As the chances of getting in fall in y, the Bad types reduce their purchasing accordingly. As $\Sigma_C^* < \Sigma_B^* < \Sigma_A^*$, equilibrium C is preferred not only by the DM, but also from the spending perspective. In fact, the equilibrium in which the cost of the ostentatious good is set just so that it discourages the Bad type from purchasing, $C^* = y + \epsilon$, where ϵ is small, minimises the overall spending while guaranteeing the maximum obtainable expected payoff for the DM.[7]

1.3 Capacity Constraint

In this section, I analyse how the equilibrium changes when the DM faces a capacity constraint, and whether the DM may deliberately want to set a capacity constraint. I begin by considering all equilibria, including those that violate Condition $N0$, to demonstrate why we might want to discard them.

The framework in which I will examine this problem is as follows: let π denote the proportion of the whole population that is admitted by the DM. I assume that the maximum proportion of the total population that can be admitted by the DM,

[7] If the DM could charge either an application fee or an admission fee, which lower the agents' relative gains from being admitted, it would also set it so that it just discourages the Bad type from purchasing. The type of fee does not matter, because the DM can fully extract the surplus from the Good agent through any combination of them which does not discourage the Good type from purchasing. Note that the application fee requires adding the possibility of not applying to the set of the agents' decisions, and that it is essentially equivalent to an outside option for the agents.

either due to an external factors such as the venue size, or due to the DM's own decision such as in case of a private members' club, is given by $\gamma \in (0, 1]$. Thus, the capacity constraint (CC thereafter) can be expressed as an inequality $\pi(\sigma, \delta) \leq \gamma$. The conditions under which the baseline equilibria continue to exist when the CC does not bind are described below:[8]

Lemma 1.1. *The equilibria from Proposition 1.1 remain the equilibria with a CC that does not bind when:* $\gamma \geq \rho + y(1-\rho)$ *for equilibrium A,* $\gamma \geq \frac{2\rho C}{y}$ *for equilibrium B,* $\gamma \geq \rho$ *for equilibrium C,* $\gamma = 1$ *for equilibrium D, and* $\gamma \geq 1 - 2C(1-\rho)$ *for equilibrium E.*

The following proposition defines a new set of equilibria in which the CC does bind:

Proposition 1.3. *When the CC binds, the following equilibria which satisfy Condition N*0 *exist:*

(i) if $y > C$ *and* $\rho > \frac{y}{1+y}$ *and* $\rho + y(1-\rho) > \gamma \geq \frac{C}{y}(\rho + y(1-\rho))$*, then* $(\sigma_G^*, \sigma_N^*) = (1, 1)$ *and* $(\delta_S^*, \delta_N^*) = (\frac{\gamma}{\rho + y(1-\rho)}, 0)$ *(equilibrium A′),*

(ii) if $y > C$*, and either* $\rho < \frac{y}{1+y}$ *and* $\frac{2\rho C}{y} > \gamma \geq \frac{\rho C}{y}$*, or* $\rho > \frac{y}{1+y}$ *and* $\frac{C}{y}(\rho + y(1-\rho)) > \gamma \geq \frac{\rho C}{y}$*, then* $(\sigma_G^*, \sigma_N^*) = (1, \frac{y\gamma - \rho C}{y(1-\rho)C})$ *and* $(\delta_S^*, \delta_N^*) = (\frac{C}{y}, 0)$ *(equilibrium B′),*

(iii) if $y > C$ *and* $\frac{\rho C}{y} > \gamma \geq \rho C$*, or* $y < C$ *and* $\rho > \gamma \geq \rho C$*, then* $(\sigma_G^*, \sigma_N^*) = (1, 0)$ *and* $(\delta_S^*, \delta_N^*) = (\frac{\gamma}{\rho}, 0)$ *(equilibrium C′),*

(iv) if $\rho C > \gamma > 0$*, then* $(\sigma_G^*, \sigma_N^*) = (\frac{\gamma}{\rho C}, 0)$ *and* $(\delta_S^*, \delta_N^*) = (C, 0)$ *(equilibrium C″).*

In addition, when the CC binds, for $\rho > \frac{1}{2}$ *the following equilibria which do not satisfy Condition N*0 *also exist:*

[8]Note that the DM generally cannot commit to strategy in this model, and thus it might seem problematic to introduce a capacity constraint on the *ex-ante* probability of admission. However, this is less of a problem if we focus on the equilibria which satisfy Condition $N0$, since the ex-ante formulation of the capacity constraint translates immediately into a constraint on the probability of admission conditional on $m = S$.

(v) if $1 > \gamma > 1 - C$*, then* $(\sigma_G^*, \sigma_N^*) = (0, 0)$ *and* $(\delta_S^*, \delta_N^*) = (0, \gamma)$ *(equilibrium* D'*),*

(vi) if $1 - 2C(1-\rho) > \gamma > 1 - C$*, then* $(\sigma_G^*, \sigma_N^*) = (\frac{\gamma + C - 1}{\rho C}, 0)$ *and* $(\delta_S^*, \delta_N^*) = (1, 1 - C)$ *(equilibrium* E'*).*

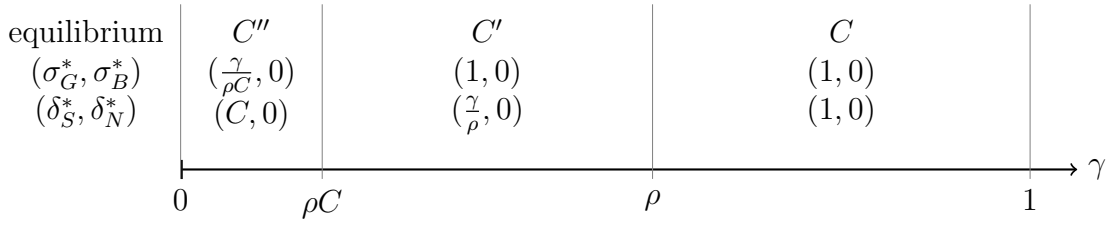
The lemma below provides further justification for Condition $N0$, i.e. the earlier assumption to disregard equilibria where some agents are admitted when they do not purchase:

Lemma 1.2. *Equilibria in regions A,* A'*,* B*,* B'*,* C*,* C' *and* C'' *are unique when* $\rho \leq \frac{1}{2}$*, or when* $\rho > \frac{1}{2}$ *and* $\gamma \leq 1 - C$*.*

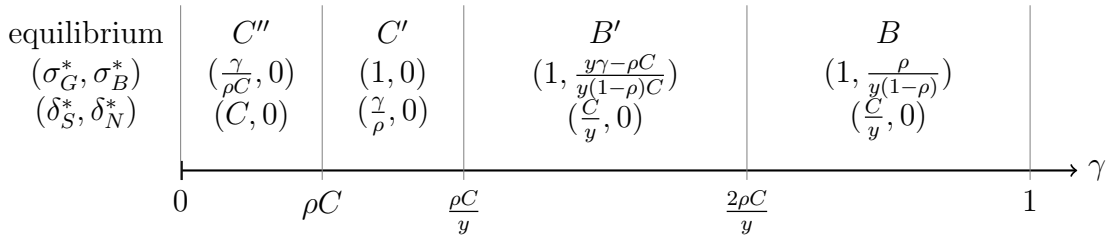
In equilibrium D', both types of agents prefer not to purchase when the probability of being admitted is lowered to satisfy the CC and exactly γ of all agents are admitted upon no purchase. Note however that the D1 criterion postulates that probability one be put on the type of agent who deviates to purchasing being Good, for he always benefits at least to the extent that the Bad agent does and sometimes more. Thus, we must have $\delta_S^* = 1$, and $\gamma > 1 - C$ is required for not purchasing to be optimal for the Good agent. For lower values of γ, the Good agents prefer to purchase. The no-purchase equilibrium then breaks down and agents move to the relevant equilibrium which satisfies Condition $N0$. The situation is similar in equilibrium E, only here for $1 \geq \gamma \geq 1 - 2(1-\rho)C$ the equilibrium is unchanged compared to the baseline model as the constraint does not bind, while the rationing happens for $1 - 2(1-\rho)C > \gamma > 1 - C$ (equilibrium E').[9] For γ below $1 - C$ this kind of equilibrium does not exist too. Taking these facts into account, whenever $\gamma \leq 1 - C$, neither type of equilibrium that violates Condition $N0$ exists. Consequently, if $1 - C \geq \rho + y(1-\rho)$, or $C \leq 1 - (\rho + y(1-\rho))$, any value of γ such that $\gamma \leq 1 - C$ eliminates the possibility of multiple equilibria and guarantees uniqueness, while maintaining the possibility of obtaining any other equilibrium described

[9] Note that in this equilibrium the Good agent purchases *less* often when the CC tightens, even though purchasing is the only means of ensuring that he is admitted. This is a counter-intuitive feature of the mixed-strategy equilibrium as this behaviour is driven by the necessity to make the DM indifferent between admitting and rejecting.

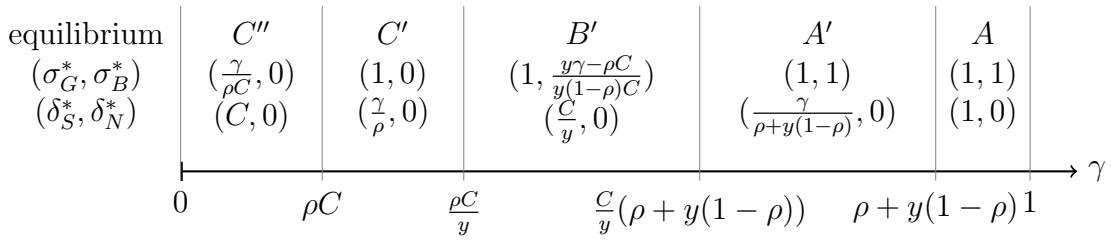
by Lemma 1.1 and Proposition 1.3. Hence, from now on, I only focus on equilibria that satisfy Condition $N0$.



(a) when $y \leq C$



(b) when $y > C$ and $\rho < \frac{y}{1+y}$

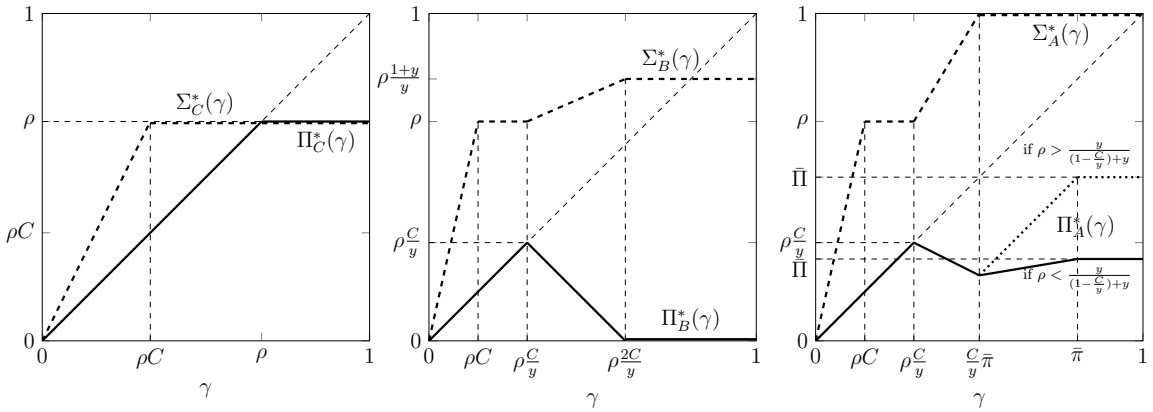


(c) when $y > C$ and $\rho \geq \frac{y}{1+y}$

Figure 1.2: Evolution of the equilibrium as well as the agents' and the DM's equilibrium strategies as γ is varied.

Assume initially that the CC is exogenously imposed, so that the DM has no influence over it. The evolution of the equilibrium as γ is varied is illustrated in Figure 1.2. For high values of γ, the CC does not bind, so the equilibria correspond to those of Proposition 1.1. When the constraint is lowered below a certain threshold, despite the DM's willingness to admit all successful agents, it is not able to, and hence must resort to rationing admission. At a certain point, the chances of getting in become so low that the expected payoff from purchasing falls to the level or below the expected payoff from not purchasing. When $y \leq C$, as the Bad type never purchases anyway, this happens to the Good type, who becomes indifferent between purchasing and not purchasing. As γ falls further, the Good type simply

reduces the probability of purchasing accordingly. When $y > C$, if both agents used to purchase ($\rho \geq \frac{y}{1+y}$ case), as the chances of getting in keep falling, first the Bad agents begin to mix with a decreasing probability of purchasing, then they stop purchasing altogether, and then even the Good agents begin to mix with a decreasing probability. If only the Good agents always purchased and the Bad agents mixed ($\rho < \frac{y}{1+y}$ case), first the Bad agents reduce their probability of purchasing, then they stop purchasing altogether, and finally, again, even the Good agents begin to mix.



(a) $C \geq y$ (b) $C < y$ and $\rho < \frac{y}{1+y}$ (c) $C < y$ and $\rho \geq \frac{y}{1+y}$

Figure 1.3: The DM's expected payoff, $\Pi^*(\gamma)$, and overall spending, $\Sigma^*(\gamma)$, as functions of the capacity constraint γ ($\bar{\pi} = \rho + y(1-\rho)$ and $\bar{\Pi} = \rho + y(1-\rho)$).

The DM's expected payoff and the overall spending on the positional good are illustrated in Figure 1.3. For low values of $0 < \gamma < \rho \min\{1, \frac{C}{y}\}$, it is only the Good agent who may ever purchase, so the DM admits upon success with the highest probability it can, obtaining an expected payoff of $\Pi^*_{C''} = \Pi^*_{C'} = \gamma$. What happens for higher values of γ depends on the values of other parameters. If $y \leq C$, for any $\gamma \geq \rho$, the DM continues to only admit the successful agents, who are all of the Good type. As the CC does not bind anymore, the DM simply obtains a payoff of ρ. For $y > C$ the situation is more complex, for at least some Bad agents purchase in the equilibrium. As $\frac{C}{y}\rho < \rho$ is the highest level of the CC which guarantees that

they are discouraged from purchasing, it can never be the case that all Good agents are admitted in the equilibrium.

If $\rho < \frac{y}{1+y}$, for $\frac{C}{y}\rho \leq \gamma < \frac{2C}{y}\rho$, the DM's expected payoff falls when γ rises. This may appear counter-intuitive, but it results from the fact that the Bad agent's payoff becomes high enough for him to sometimes purchase. In fact, his payoff increases as γ goes up, making him purchase more often. Consequently, the composition of successful agents deteriorates from the DM's perspective and its expected payoff falls. This is despite the fact that it may be the case that the CC becomes so lax that all Good agents could be admitted. When $\gamma \geq \frac{2\rho}{y}C$, the CC does not bind anymore and the DM becomes indifferent between admitting and rejecting, so that its payoff is as if it always rejected. If $\rho \geq \frac{y}{1+y}$, for $\frac{C}{y}\rho \leq \gamma < \frac{C}{y}(\rho + y(1-\rho))$, the DM's payoff is the same as above and falls with γ for the same reason as above. However, the situation is different when $\frac{C}{y}(\rho + y(1-\rho)) \leq \gamma < \rho + y(1-\rho)$, as all agents purchase and the DM admits as many successful purchasers as it can, because the expected payoff from admitting any such agent is positive. The relaxation of the CC has no effect on the incentives to purchase, but it allows the DM to admit more agents, each of whom bring a positive expected payoff, so $\Pi^*_{A'} = \frac{\gamma}{\rho+y(1-\rho)}(\rho - y(1-\rho))$. Thus, the DM's payoff increases until $\gamma = \rho + y(1-\rho)$, at which point the CC does not bind and the DM's expected payoff is not affected by γ anymore.

The following proposition drops the assumption that the CC is exogenously determined in favour of the DM setting it to maximise its expected payoff:[10]

Proposition 1.4. *When the DM has the ability to choose the CC to maximise its expected payoff, if $y \leq C$, or $y > C$ and $\rho \geq \frac{y}{(1-\frac{C}{y})+y}$, it chooses not to do so, while if $y > C$ and $\rho < \frac{y}{(1-\frac{C}{y})+y}$, it sets $\gamma^* = \frac{C}{y}\rho \equiv \bar{\gamma}$.*

Trivially, no CC is needed when there is full separation in the equilibrium for $y \leq C$. When $y > C$ and $\rho < \frac{y}{(1-\frac{C}{y})+y}$, the DM's payoff is maximised when γ is set to just discourage all Bad agent from purchasing, and thus obtain perfect

[10] I assume that the DM chooses not to set the CC whenever the optimal CC would be slack.

separation. While this results in a payoff of $\gamma^* = \frac{\rho}{y}C < \rho$, so it would seem as though a higher payoff were possible, it is unattainable as any increase in γ would convince some Bad types to purchase. Conversely, any further reduction in γ hurts the DM's expected payoff, because it does not impact the incentives to purchase, while it reduces the DM's ability to admit the only agents who purchase, i.e. the Good types. When $y > C$ and $\rho \geq \frac{y}{(1-\frac{C}{y})+y}$, the DM maximises its expected payoff by admitting all successful purchasers without any CC. The reason for this is that the Bad types always purchase, because they understand that there are numerous Good types in the population, which makes the DM inclined to admit often. As a result, the capacity would have to be severely constrained to make the Bad agents choose not to purchase. This, however, would cause the DM to be forced to often reject the Good types, of whom there are plenty. Hence, the DM prefers to take the chance of sometimes admitting the Bad agent, while retaining the ability to admit all of the Good agents.

We may obtain further intuition about the threshold value of ρ by re-writing the condition for no CC to be optimal as $\rho(1 - \frac{C}{y}) > (1 - \rho)y$. The LHS reflects the cost of setting a CC: in the fully separating equilibrium with $\gamma = \bar{\gamma}$, the successful agent is rejected with probability $1 - \frac{C}{y}$, and he must be of the Good type, so with probability $\rho(1 - \frac{C}{y})$ the DM does not obtain a payoff of one. The RHS reflects the benefit from discouraging the Bad types: were it not for the CC, all Bad types would purchase in the equilibrium and the DM would make the mistake of admitting them and obtaining a payoff of minus one when they are successful, i.e. with probability $(1 - \rho)y$. When the cost outweighs the benefit, it is optimal for the DM not to set the CC.

Note that the ability of the DM to choose the CC is somewhat similar to its ability to choose the cost of the ostentatious good. Both measures ensure that the Bad types are discouraged from purchasing. The difference however is that the minimum sufficient CC requires setting $\gamma = \bar{\gamma} < \rho$, so that it is impossible to admit

all Good agents.

Unsurprisingly, overall spending weakly increases in γ because all agents purchase weakly more. Moreover, whenever it is optimal to set a binding CC, this limits the overall spending compared to the baseline model, as $\Sigma^*_{\gamma=\bar{\gamma}} = \rho$ is smaller than Σ^*_A and Σ^*_B.

1.4 Commitment to Ignorance

Sometimes, the DM may be able to commit to ignore the message sent by the agent with a positive probability. This section analyses when such strategy is optimal for the DM and shows this problem's near equivalence to the optimal capacity constraint problem discussed before.

The way I model ignorance is by assuming that the DM only receives the message as in the baseline model with probability $\xi \in (0, 1)$, while with probability $1 - \xi$ it makes the decision based solely on the prior probability of the agent being Good. This is equivalent to augmenting the message set to $m \in \{S, F, N, I\}$, where $m = I$, rather than the actual message, is randomly observed by the DM with probability $1 - \xi$. As $\beta_I = \rho$, the DM admits the agent only if the prior is sufficiently high, $\rho \geq \frac{1}{2}$, and rejects the agent if $\rho < \frac{1}{2}$, so that $\delta^*_I = \mathbb{1}_{\rho \geq \frac{1}{2}}$. Trivially, the baseline model of Section 1.2 corresponds to setting $\xi = 1$.

When $\xi < 1$, so that the DM sometimes ignores the message sent by the agent, there are two opposing effects on the DM's expected payoff. On the one hand, it lessens the incentives of the Bad agents to purchase, because the message is less likely to be observed, and thus to influence the DM's decision. As a result, separation in the equilibrium is improved. On the other hand, it means that potentially valuable information is consciously thrown away and the DM has to rely solely on the prior. The following lemma describes this trade-off:

Lemma 1.3. *Whenever it is optimal for the DM to commit to ignorance rather than*

to set $\xi = 1$, it ignores the message just often enough to discourage the Bad types from purchasing by setting $\xi = \bar{\xi} \equiv \frac{C}{y}$.

When $y \leq C$, the Bad type never purchases, so perfect separation is obtained without the need to throw away potentially useful information. When $y > C$, this is not the case, but the introduction of ignorance aims to achieve the same fully separating equilibrium artificially. The goal of the DM is to completely disincentivise the Bad agent from purchasing by ignoring the message sufficiently often. The Bad agent's expected utilities are $\upsilon_N^P = \xi y \delta_S + (1-\xi)\delta_I - C$ and $\upsilon_B^N = (1-\xi)\delta_I$, and in the desired separating equilibrium we have $\delta_S^* = 1$, so that $\upsilon_B^N < \upsilon_B^P$ whenever $\xi < \frac{C}{y}$. The DM, however, also wants to set ξ as high as possible, because it is useful to know whether the agent purchases and whether he is successful in doing so. In sum, the DM would like to set ξ as high as possible, subject to it being low enough to discourage purchasing by the Bad agents. Thus, when positive level of ignorance is optimal, the DM would like to set $\xi = \bar{\xi} - \epsilon$, where ϵ is small. With this in mind, I will assume that when $\xi = \bar{\xi}$, the Bad type does not purchase.

1.4.1 Optimal Level of Ignorance

The following proposition compares the DM's expected payoff with commitment to ignorance, when $\xi = \bar{\xi}$, and with full attention, when $\xi = 1$:

Proposition 1.5. *When $C < y$, the DM can improve its expected payoff with commitment to ignorance by setting $\xi^* = \hat{\xi}$, unless $C < \min\{\frac{y}{\rho}(\rho - y(1-\rho)), y(1-y)\}$, when it is optimal for the DM to set $\xi^* = 1$.*

The summary of results are presented in Table 1.1 (alongside comparison with a capacity constraint, which is discussed below). The DM never wants to commit to ignorance when the cost is high enough to discourage the Bad types from purchasing on its own, $C > y$. Proposition 1.5 demonstrates, however, that full attention may also be optimal when the cost is low, while committing to ignorance by setting $\xi^* = \bar{\xi}$

is only optimal when the cost is moderate. An alternative interpretation of these results is that when $y > C > y(1-y)$, it is always optimal to commit to ignorance, while when $C < y(1-y)$, it is only optimal to commit to ignorance if the proportion of the Good types in the population is not too high, $\rho < \frac{y}{(1-\frac{C}{y})+y}$. With regards to the overall spending, it is always lower in the fully separating equilibrium obtained by setting $\xi = \bar{\xi}$, where $\Sigma^*_{\xi=\bar{\xi}} = \rho$, than in either of the full attention equilibria when $C < y$. From this perspective, commitment to ignorance is beneficial.

$\rho \in$	ξ^*	Π^*_ξ	Σ^*_ξ	γ^*	Π^*_γ	Σ^*_γ	Comparison
$(0, \frac{y}{1+y})$	$\bar{\xi}$	$\frac{C}{y}\rho$	ρ	$\bar{\gamma}$	$\frac{C}{y}\rho$	ρ	$\Pi^*_\xi = \Pi^*_\gamma$
$[\frac{y}{1+y}, \frac{y}{(1-\frac{C}{y})+y})$	$\bar{\xi}$	$\frac{C}{y}\rho$	ρ	$\bar{\gamma}$	$\frac{C}{y}\rho$	ρ	and
$[\frac{y}{(1-\frac{C}{y})+y}, \frac{1}{2})$	1	$\breve{\rho}$	1	–	$\breve{\rho}$	1	$\Sigma^*_\xi = \Sigma^*_\gamma$
$[\frac{1}{2}, 1)$	1	$\breve{\rho}$	1	–	$\breve{\rho}$	1	

(a) when $0 < C < y(1-y)$

$\rho \in$	ξ^*	Π^*_ξ	Σ^*_ξ	γ^*	Π^*_γ	Σ^*_γ	Comparison
$(0, \frac{y}{1+y})$	$\bar{\xi}$	$\frac{C}{y}\rho$	ρ	$\bar{\gamma}$	$\frac{C}{y}\rho$	ρ	$\Pi^*_\xi = \Pi^*_\gamma$
$[\frac{y}{1+y}, \frac{1}{2})$	$\bar{\xi}$	$\frac{C}{y}\rho$	ρ	$\bar{\gamma}$	$\frac{C}{y}\rho$	ρ	and $\Sigma^*_\xi = \Sigma^*_\gamma$
$[\frac{1}{2}, \frac{y}{(1-\frac{C}{y})+y})$	$\bar{\xi}$	$\rho\dagger$	ρ	$\bar{\gamma}$	$\frac{C}{y}\rho$	ρ	$\Pi^*_\xi > \Pi^*_\gamma$ and $\Sigma^*_\xi = \Sigma^*_\gamma$
$[\frac{y}{(1-\frac{C}{y})+y}, 1)$	$\bar{\xi}$	$\rho\dagger$	ρ	–	$\breve{\rho}$	1	$\Pi^*_\xi > \Pi^*_\gamma$ and $\Sigma^*_\xi < \Sigma^*_\gamma$

(b) when $y(1-y) \leq C < y$

Table 1.1: The optimal strategy, the DM's maximised expected payoff, Π^*, and the overall spending, Σ^*, under commitment to ignorance and under capacity constraint ($\breve{\rho} = \rho - y(1-\rho)$ and $\rho\dagger = \rho - (1-\frac{C}{y})(1-\rho)$).

I now examine how changes in C, y and ρ impact the resulting equilibrium. It may be surprising that the DM may choose not to commit to ignorance when the cost is low. But note that a reduction in C makes purchasing of the good more attractive. Therefore, the DM would need to commit to ignore the message with a higher probability to discourage purchasing by the Bad types, which forces the DM to make an uninformed decision more often. At some point, when C is sufficiently low, the DM therefore prefers to always observe all messages and admit all successful buyers, rather than to commit to almost always ignoring all messages, even if it means sometimes letting in the Bad agents.

An increase in y has two effects on the DM's incentives to commit to ignorance. On the one hand, it improves the Bad type's chances of succeeding, and thus the benefit from ignoring the resulting, more misleading, message is higher. On the other hand, the Bad agent is more inclined to purchase, so a higher level of ignorance is required to discourage him from doing so. This increases the cost of ignorance, because potentially valuable information is thrown away more often. The former effect dominates for high values of y, where it is optimal to commit to ignorance.

Finally, an increase in ρ weakly enlarges the set of parameters for which it is optimal for the DM not to ignore the message, with a strict enlargement for $\rho < \frac{1}{2}$. This is because when ρ is higher, the DM's chances of making the right decision at random increase. When $\rho < \frac{1}{2}$, the DM only benefits from this effect when it pays full attention, but not when it sometimes ignores the message, as it never admits the agent upon $m = I$. Hence, higher ρ makes the former choice more appealing. When $\rho \geq \frac{1}{2}$, however, the DM benefits from a higher ρ both when it pays full attention and when it ignores the message. Hence, the set of parameters for which it is optimal for the DM not to ignore the message does not change with ρ.

1.4.2 Relationship to Capacity Constraint

Commitment to ignorance plays in this model a role similar to that of a capacity constraint. Both of them function by reducing the Bad agent's incentives to purchase, thereby improving separation in the equilibrium. But there is always a cost attached: in the case of capacity constraint, it is the inability to admit all Good agents who successfully purchase, while in the case of ignorance, it is the inability to always use the perfectly informative message. The similarity between outcomes of these two measures is characterised in Table 1.1.

When the restrictions are chosen optimally, so that $\xi = \bar{\xi}$ and $\gamma = \bar{\gamma}$, the benefit from purchasing to the Bad type is identical in both cases, so he behaves in the same manner regardless of the measure used. While the measures have the same impact

on the incentives of the agents, this is not necessarily true about their impact on the DM's expected payoff. The reason for this is the additional flexibility allowed by the commitment to ignorance. Specifically, whenever the DM chooses a capacity constraint, it may never be exceeded, even when $\rho \geq \frac{1}{2}$, in which case admitting an agent at random contributes positively to the overall payoff. In contrast, when the DM commits to ignorance, it may still admit the agent whenever it ignores the message. This allows it to retain the desirable influence on the agents' incentives without giving up the ability to admit more agents whenever the circumstances are right:

Proposition 1.6. *Optimal commitment to ignorance results is the DM's expected payoff being weakly higher, and overall spending being weakly lower than the optimal capacity constraint.*

When $y(1-y) < C < y$ and $\rho \geq \frac{1}{2}$, the maximised DM's expected payoff obtained under commitment to ignorance is weakly higher than under the capacity constraint, where the expected payoff is *as if* ρ were below $\frac{1}{2}$ under commitment to ignorance. Initially, for values of ρ not much above $\frac{1}{2}$, this does not change the conclusion that committing to ignorance or restricting capacity are the optimal strategies, even though the DM's expected payoff is already higher under commitment to ignorance. For $\rho > \frac{y}{(1-\frac{C}{y})+y}$, however, the choices diverge. Moreover, note that the overall spending is lower under commitment to ignorance than under capacity constraint, because the Bad types end up discouraged from purchasing.

Unfortunately, it is not possible to further improve the DM's payoff by combining the two measures, as it makes sense for the DM to make only one of them binding. When γ is low, there is no need for commitment to ignorance, while when γ is high, $\xi = \bar{\xi}$ is still required to discourage the Bad type from purchasing.

In subsequent sections I will only consider the commitment to ignorance as it is the superior measure. The results for the capacity constraint case will be similar, with differences rooted in the same reasoning as presented above.

1.5 Introduction of Counterfeits

The setting introduced in this chapter compels the DM to address the problem of the existence of counterfeits. This section explores how the equilibrium changes compared to the baseline when counterfeits become available, and how it evolves as the parameters of the model change.

I continue to assume that the genuine good ($d = P$) is available at the price C, which guarantees the Good type, but not the Bad type, a successful sending of the message. But available now is also a counterfeit good ($d = PC$), whose characteristics are identical for both types of agents: it costs c, where $0 < c < C$, and leads to a successful message with probability x, where $0 < x < y$, while with probability $1 - x$ is revealed to be a counterfeit and delivers a message $m = C$. I assume that the DM never deliberately admits anyone upon an observation of a counterfeit, even if it is sure that the applicant is of a Good type, because that would reflect poorly on it among other admitted agents.[11] The counterfeit is indistinguishable from the original upon a success, in that once $m = S$ is observed, the DM cannot tell if the message came from a genuine or a counterfeit good.

The agent's strategy is characterised by a pair $(\sigma_\theta, \mu_\theta)$, where $\sigma_\theta = \Pr(d = P \mid \theta)$, $\mu_\theta = \Pr(d = PC \mid \theta)$ and $\Pr(d = N \mid \theta) = 1 - \sigma_\theta - \mu_\theta$. The complete description of the agents' strategies therefore is given by $\Phi = (\sigma, \mu)$, where $\sigma = (\sigma_G, \sigma_B)$ and $\mu = (\mu_G, \mu_B)$. While there are now four messages, $m \in \{S, F, C, N\}$, Condition $N0$ and the proceeding arguments ensure that the DM can only admit the agents upon a success. The belief functions must be suitably adjusted to account for the agents' strategies. To avoid the equilibria in which only the counterfeits are purchased, I impose an additional condition that the original must always be bought with a positive probability. The following proposition describes the set of equilibria which fulfils these requirements:

[11] In other words, the stigma from admitting a known counterfeit user is higher than the potential gain from admitting a Good agent.

Proposition 1.7. *When counterfeits become available, the agents and the DM's strategies in equilibria which satisfy Condition N0, and in which the original is at least sometimes purchased, are:*

(i) if $1 > \rho \geq \frac{y}{1+y}$, *and either (a)* $C \leq y - x$, *or (b)* $y - x < C < y$ *and* $c > C - (y - x)$, *then* $(\sigma_G^*, \mu_G^*, \sigma_B^*, \mu_B^*) = (1, 0, 1, 0)$ *and* $(\delta_S^*, \delta_N^*) = (1, 0)$ *(equilibrium* A^c*),*

(ii) if $\frac{y}{1+y} > \rho > 0$ *and* $C \leq y$ *and* $c > \frac{x}{y}C$, *then* $(\sigma_G^*, \mu_G^*, \sigma_B^*, \mu_B^*) = (1, 0, \frac{\rho}{(1-\rho)y}, 0)$ *and* $(\delta_S^*, \delta_N^*) = (\frac{C}{y}, 0)$ *(equilibrium* B^c*),*

(iii) if $C > y$ *and* $c > x$, *then* $(\sigma_G^*, \mu_G^*, \sigma_B^*, \mu_B^*) = (1, 0, 0, 0)$ *and* $(\delta_S^*, \delta_N^*) = (1, 0)$ *(equilibrium* C^c*),*

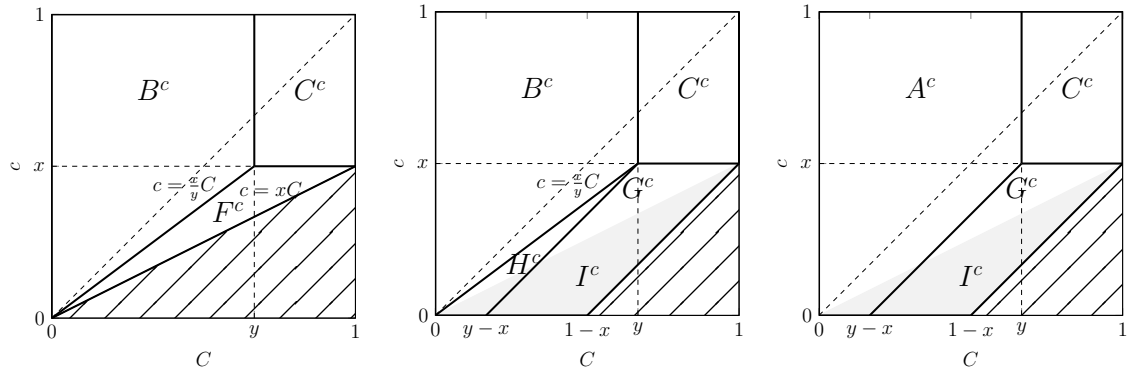
(iv) if $\frac{x}{1+x} > \rho > 0$ *and* $xC < c < \min\{x, \frac{x}{y}C\}$, *then* $(\sigma_G^*, \mu_G^*, \sigma_B^*, \mu_B^*) = (1, 0, 0, \frac{\rho}{(1-\rho)x})$ *and* $(\delta_S^*, \delta_N^*) = (\frac{c}{x}, 0)$ *(equilibrium* F^c*),*

(v) if $1 > \rho \geq \frac{x}{1+x}$ *and* $C-(1-x) < c < \min\{x, C-(y-x)\}$, *then* $(\sigma_G^*, \mu_G^*, \sigma_B^*, \mu_B^*) = (1, 0, 0, 1)$ *and* $(\delta_S^*, \delta_N^*) = (1, 0)$ *(equilibrium* G^c*),*

(vi) if $\frac{y}{1+y} > \rho \geq \frac{x}{1+x}$ *and* $c < \frac{x}{y}C$ *and either (a)* $C \leq y - x$, *or (b)* $y - x < C < y$ *and* $c \geq C - (y - x)$, *then* $(\sigma_G^*, \mu_G^*, \sigma_B^*, \mu_B^*) = (1, 0, \frac{\rho - x(1-\rho)}{(1-\rho)(y-x)}, \frac{\rho - y(1-\rho)}{(1-\rho)(y-x)})$ *and* $(\delta_S^*, \delta_N^*) = (\frac{C-c}{y-x}, 0)$ *(equilibrium* H^c*),*

(vii) if $\frac{1}{2} > \rho \geq \frac{x}{1+x}$ *and* $C - (1 - x) \leq c < xC$, *then* $(\sigma_G^*, \mu_G^*, \sigma_B^*, \mu_B^*) = (\frac{x(1-2\rho)}{\rho(1-x)}, \frac{\rho - x(1-\rho)}{\rho(1-x)}, 0, 1)$ *and* $(\delta_S^*, \delta_N^*) = (\frac{C-c}{1-x}, 0)$ *(equilibrium* I^c*).*

The equilibria are illustrated in Figure 1.4. Note that in the region above the $c = C$ line, nobody ever buys the counterfeit, so the equilibria would be the same as in the baseline model. The excluded sets in the southeast corners in each of the figures, where the cost of the counterfeit is low compared to the original, results from the assumption that the Good types must at least sometimes want to purchase the



(a) when $0 < \rho < \frac{x}{1+x}$ (b) when $\frac{x}{1+x} \leq \rho < \frac{y}{1+y}$ (c) when $\frac{y}{1+y} \leq \rho < 1$

Figure 1.4: All possible equilibria when counterfeits are present. Equilibrium I^c only exists for $\frac{x}{1+x} \leq \rho < \frac{1}{2}$ and overlaps with equilibria A^c, G^c and H^c (as defined by Proposition 1.7).

genuine good whenever success guarantees admission with a positive probability.[12] The relevant condition depends on whether $\rho < \frac{x}{1+x}$, in which case it is $c > xC$, or $\rho \geq \frac{x}{1+x}$, in which case it is $c > C - (1 + x)$. This is because in the former case the proportion of the Good types is so low that the DM does not always admit upon a success, but rather only with probability $\frac{c}{x}$, which is determined by making the Bad type indifferent between purchasing the counterfeit and not purchasing. Hence, the condition for the Good agents to indeed prefer to purchase the original rather than nothing is $v_G^P > v_G^N$, or $c > xC$. In the latter case, when the proportion of the Good agents is higher and the DM admits more happily for the same values of c and C, the Bad type always purchases the counterfeit and the DM always admits. Thus, the relevant condition for a positive purchasing of the original in the equilibrium is $v_G^P > v_G^{PC}$, or $c > C - (1 + x)$.

Note that, given the conditions mentioned, the model essentially has a unique equilibrium for all parameter values, except in the region where equilibrium I^c exists. However, the Pareto-dominance criterion allows us to eliminate the multiplicity of

[12] If we dropped the assumption that the genuine good must be purchased in the equilibrium with a positive probability, in the south-east corner there would be equilibria in which nobody purchases anything, the Good agent mixes between the genuine good and nothing, or both types only purchase counterfeits for $\rho > \frac{1}{2}$, and there will not be an equilibrium that survives the D1 criterion at all for $\rho < \frac{1}{2}$.

equilibria in the subsequent analysis:

Lemma 1.4. *Equilibrium I^c is always weakly Pareto-dominated. If we exclude it from the set of equilibria in Proposition 1.7, the remaining equilibria become unique.*

From now on, I adopt the above-mentioned refinement. Consider the surviving equilibria. In each one of them, the Good agent purchases the genuine good with probability one. Note that equilibria A^c, B^c and C^c are the counterparts of equilibria A, B and C from the baseline model, with the additional requirement that the cost of the counterfeit may not be too low, or it would induce the Bad agent to switch to purchasing the counterfeit. Note that these equilibria always prevail when $c > x$, as then neither type ever wants to purchase the counterfeit.

Unlike the baseline model, there are now also equilibria that involve the Bad agents at least sometimes purchasing the counterfeit. In equilibrium F^c, the Bad agent mixes between purchasing the counterfeit and not purchasing at all, and the DM is indifferent between admitting and rejecting on $m = S$, because the proportion of the Good types is low. When ρ increases, the attractiveness of admitting a successful purchaser increases, so much so that the DM admits with probability one and the Bad type finds it optimal to always purchase the counterfeit as long as the counterfeit is cheap enough (equilibrium G^c). For medium values of ρ, it is possible that the Bad agent prefers to mix between purchasing the original and purchasing the counterfeit, provided that the costs of the original and the counterfeit are low (equilibrium H^c).

I now examine what happens when the parameters of the model are changed for a fixed set of prices C and c. Consider first changes in y. As the ability of the Bad type to successfully purchase the original increases, he purchases more often. This means that full separation happens more sporadically. Moreover, this may lead the Bad agent to substitute originals for counterfeits, leading what used to be an equilibrium in regions F^c, G^c or H^c to become an equilibrium in regions A^c, B^c or H^c.

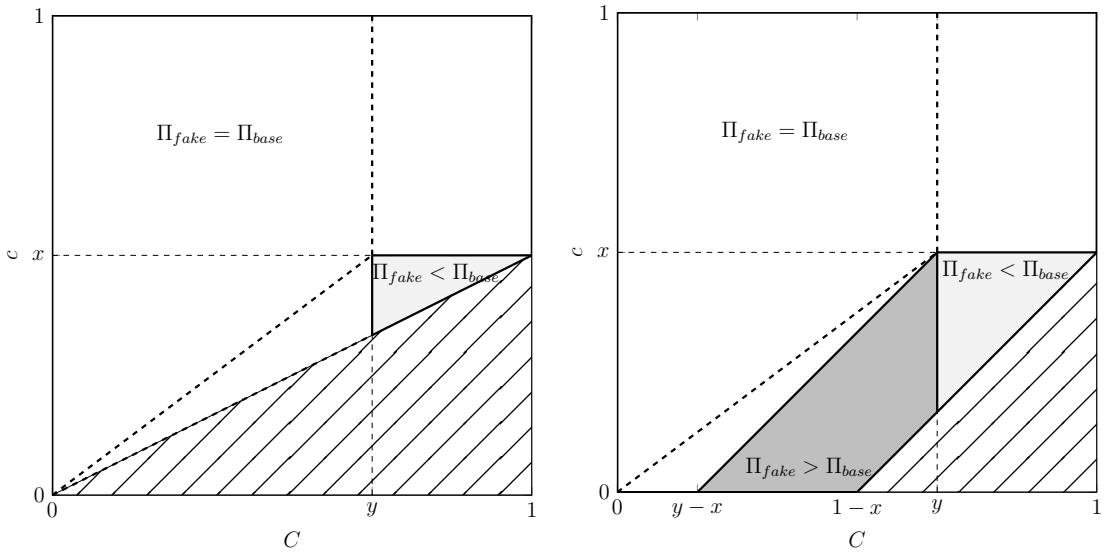
Next, consider potential changes in x. As the quality of the counterfeit increases, the Bad agents purchase it more often. The effect on separation is ambiguous, as it may be positive where the purchase of the counterfeit comes at the expense of the original, such as in a switch from an equilibrium in region B^c to an equilibrium in region G^c. However, it can also be negative when it comes at the expense of not purchasing, such as in a switch from an equilibrium in region C^c to an equilibrium in region G^c.

Finally, consider potential changes in ρ. As the proportion of the Good types increases, the DM becomes keener on admitting the agents, which in turn makes the Bad types more willing to signal successfully, so the probability that they purchase the originals, rather than the counterfeits or nothing, increases. While the strategies in the mixed-strategy equilibria change with ρ, the switch of the kind of equilibrium only happens as ρ moves between the ranges $\rho \in (0, \frac{x}{1+x})$, $\rho \in [\frac{x}{1+x}, \frac{y}{1+y})$ and $\rho \in [\frac{y}{1+y}, 1)$. As a consequence, what used to be equilibrium in region B^c may become an equilibrium in region A^c, similar to the baseline model. An interesting feature is that what used to be an equilibrium in regions F^c or H^c, where the Bad type used to mix between the counterfeit and nothing, or the original and the counterfeit, may become an equilibrium in region B^c or A^c, respectively, where he purchases the original or nothing, or always the original. In these cases, an increase in ρ may lead to a complete elimination of the counterfeit purchasing. Finally, note that it is possible that an equilibrium with a positive purchasing of the originals now exists in south-west part of the diagram where it was previously absent.

1.5.1 Consequences of Introduction of Counterfeits

The main focus of this subsection is to analyse the DM's expected payoff and the overall spending both on the genuine goods as well as on the counterfeits. I begin with the following proposition, which compares the outcomes involving counterfeits to those in the baseline model:

Proposition 1.8. *Introduction of counterfeits decreases the DM's expected payoff when (a)* $\frac{x}{1+x} > \rho > 0$ *and* $C > y$ *and* $xC < c < x$*, or (b)* $1 > \rho \geq \frac{x}{1+x}$ *and* $C > y$ *and* $C - (1-x) < c < x$*. It increases the DM's expected payoff when* $1 > \rho \geq \frac{x}{1+x}$ *and* $C < y$ *and* $C - (1-x) < c < C - (y-x)$*, and does not change the DM's expected payoff otherwise.*



(a) when $0 < \rho < \frac{x}{1+x}$ (b) when $\frac{x}{1+x} \leq \rho < 1$

Figure 1.5: Changes in DM's expected payoff between the baseline model, Π_{base}, and the model with counterfeits, Π_{fake}.

The results are illustrated in Figure 1.5. Where the equilibrium had previously involved full separation, i.e. for $y \leq C$, this may not be the case anymore with counterfeits. Specifically, when the counterfeit is sufficiently cheap, some of the Bad types will now prefer to either mix between not purchasing and purchasing the counterfeit, or even to always purchase the counterfeit when the proportion of the Good types is high. This is bad news for the DM, as perfect separation had ensured the highest obtainable payoff, $\Pi^*_C = \rho$, but in the new equilibria, the payoff are lower, $\Pi^*_{F^c} = 0$ and $\Pi^*_{G^c} = \rho - x(1-\rho)$, as the Bad types sometimes succeed and end up admitted, hurting the DM's expected payoff.

The situation is different for $y > C$. The possible existence of an equilibrium where the Good types purchase the original, while the Bad types purchase the

counterfeit is actually preferred to the baseline equilibrium where the Bad types purchase the original with a positive probability. The reason for this is that the Bad type has a higher chance of succeeding when he purchases the original than when he purchases the counterfeit. Thus, when the cost of the counterfeit is low enough, $c < C-(y-x)$, and the original high enough, $C > y-x$, the DM's expected payoff is somewhat counter-intuitively higher than in the baseline model. This is not just true when the baseline equilibrium used to be B, where the players use mixed strategies, and the DM's payoff is zero, but also when the baseline equilibrium used to be A, where all agents always purchase. Such an improvement is not possible when the proportion of the Good types is very low, $\rho < \frac{x}{1+x}$, because the DM then does not always admit upon success, so its payoff is zero, as if it always rejected. An improvement is also not possible for higher values of ρ when the price of the genuine good is low, $C < y - x$, because in such a case the Bad types prefer to purchase the genuine good with a positive probability. This guarantees them a higher probability of getting in, so the positive effect of counterfeits on separation is not present.

The emergence of the counterfeits weakly decreases the overall spending on the originals, with a strict decrease whenever the Bad type substitutes the originals with counterfeits, i.e. for $C < y$ in equilibrium G^c. However, it may be the case that a higher proportion of the population ends up purchasing some kind of ostentatious good, either original or counterfeit. This happens whenever the Bad type substitutes purchasing of the counterfeits for not purchasing, i.e. for $C > y$ in equilibrium F^c and in equilibrium G^c when $\rho \geq \frac{y}{1+y}$. It may be also the case that these two effects are combined, which happens for $C < y$ in equilibria F^c and H^c, as well as in equilibrium G^c when $\frac{y}{1+y} > \rho \geq \frac{x}{1+x}$.

The key difference when counterfeits are present *versus* the baseline model is that the DM no longer can obtain full separation at will when it has the ability to choose C. This is because c constrains the choice of C in the way described below:

Lemma 1.5. *To maximise its expected payoff, the DM chooses $C^* \in (y, 1)$ when*

$c > x$, $C^* \in (0, \frac{c}{x})$ *when* $c < x$ *and* $\frac{x}{1+x} > \rho > 0$, *and* $C^* \in (c + (y - x), c + (1 - x))$ *when* $c < x$ *and* $1 > \rho \geq \frac{x}{1+x}$.

In the baseline model, the DM had always preferred to achieve full separation by choosing $C^* \in (y, 1)$. As mentioned before, this is no longer always possible. Consider what happens to overall spending on the ostentatious goods when the DM chooses C optimally. I assume that whenever more than one C maximises the DM's expected payoff, it chooses the lowest such value of C. Therefore, in the baseline model, the DM would choose $C^* = y$.[13] With counterfeits, when $c > x$, the DM also chooses $C^* = y$, so the overall spending does not change. When $c < x$ and $\frac{x}{1+x} > \rho > 0$, the new choice is $C^* = 0$, as due to the low price of the counterfeits and low inference about the agent being Good upon the observation of a success, the DM cannot ensure enough separation. While the new equilibrium has poor separation characteristics, it is not wasteful as the overall spending on both the originals and the counterfeits is essentially zero. Finally, when $c < x$ and $1 > \rho \geq \frac{x}{1+x}$, the new choice is $C^* = c + (y - x)$ making the overall spending on originals equal to $\rho(c + (y - x))$, which is lower than ρy in the baseline equilibrium. However, there is now an additional $(1 - \rho)c$ spent on the counterfeits, and the total signalling-related spending, $\rho(c + (y - x)) + (1 - \rho)c$, depending on c, may be larger or smaller than in the baseline equilibrium.[14]

1.5.2 Commitment to Ignorance with Counterfeits

This subsection analyses what happens when the DM can commit to ignorance when counterfeits are available and how the results compare to the model without counterfeits.[15]

[13] More precisely, the DM would choose $C^* = y + \epsilon$, where ϵ is small, but this is dropped here as I assume that the DM can always choose the preferred side of the inequality.

[14] There arises an additional question of whether the spending on counterfeits should be treated differently than the spending on the genuine ostentatious goods from the welfare perspective, which is beyond the scope of this chapter.

[15] Note that the analysis of a capacity constraint would be similar.

Similarly to Section 1.4, sufficiently strong commitment to ignorance, or low ξ, is required to disincentivise purchasing by the Bad type. The difference *versus* the model without counterfeits, however, is that now not only purchasing of the original, but also purchasing of the counterfeit may need to be discouraged. Moreover, whenever it is not feasible to completely disincentivise the Bad type from purchasing, it may be possible to nonetheless improve the DM's expected payoff by inducing the Bad type to switch from purchasing the original to purchasing the counterfeit. In fact, there may exist circumstances when doing so may be preferred to making the Bad agent not purchase at all, because despite poorer separation, the useful information is thrown away more sparingly. The following lemma offers the menu of possible optimal levels of attention:

Lemma 1.6. *The optimal levels of ignorance for the DM are* $\xi \in \{1, \frac{C}{y}, \frac{c}{x}, \frac{C-c}{y-x}\}$. *Whenever it is optimal for the DM to commit to ignorance rather than set* $\xi = 1$, *it ignores the message just often enough to either discourage the Bad type from purchasing the original by setting* $\xi = \frac{C}{y}$ *when* $c > \frac{x}{y}C$, *or discourage the Bad type from purchasing the counterfeit by setting* $\xi = \frac{c}{x}$ *when* $xC < c < \frac{x}{y}C$, *or induce the Bad type to purchase only the counterfeit instead of the original by setting* $\xi = \frac{C-c}{y-x}$ *when* $C - (1-x) < c < \frac{x}{y}C$.

Whenever the DM wants to discourage any purchasing by the Bad agent, the optimal level of ignorance is set to just discourage purchasing of the good which is more attractive from the Bad agent's perspective. Purchasing the original gives the Bad agent at most $v_B^P = y\xi - C$, while the counterfeit gives him at most $v_B^{PC} = x\xi - c$. Thus, for both of these expressions to be lower than $v_B^N = 0$, we require $\xi^* = \bar{\xi} \equiv \min\{\frac{C}{y}, \frac{c}{x}\}$.[16] This results is full separation, or the DM never

[16] As in Section 1.4, in fact $\xi^* = \bar{\xi} - \epsilon$ is required, where ϵ is small, but to simplify notation I assume that the DM can choose its preferred option whenever the agent is indifferent.

ending up with any Bad members, and the expected payoff is:

$$\mathbb{E}[v(\delta^*, \Phi^*; \xi = \bar{\xi})] = \begin{cases} \bar{\xi}\rho & \text{if } 0 < \rho \leq \frac{1}{2} \\ \bar{\xi}\rho + (1 - \bar{\xi})(2\rho - 1) & \text{if } \frac{1}{2} < \rho < 1, \end{cases}$$

with $\bar{\xi} = \frac{C}{y}$ when $c > \frac{x}{y}C$ and $\bar{\xi} = \frac{c}{x}$ when $\frac{x}{y}C > c > xC$. Whenever $c < xC$, the DM cannot discourage the Bad agent from purchasing the counterfeit without simultaneously discouraging the Good type from purchasing the original:

Lemma 1.7. *The DM cannot prevent all Bad agents from purchasing the originals or the counterfeits by committing to ignorance when $c < xC$.*

However, when $c < \frac{x}{y}C$, the DM may also pursue a different strategy whenever $\rho > \frac{x}{1+x}$. It may then be optimal to lower the DM's level of attention only to the extent that just convinces the Bad types to switch from purchasing the originals to purchasing the counterfeits. This is possible for $c < xC$, where full separation proved to be impossible, but may also be optimal as an alternative to fully discouraging the purchase of the counterfeits, because $\frac{C-c}{y-x} > \frac{c}{x}$ in the relevant region, so it only requires a smaller reduction of attention. Such partial discouragement strategy may be beneficial for the DM as the chances of the Bad type succeeding are lower when he purchases the counterfeit instead of the original. The DM's expected payoff in this case is:

$$\mathbb{E}[v(\delta^*, \Phi^*; \xi = \frac{C-c}{y-x})] = \begin{cases} \frac{C-c}{y-x}(\rho - x(1-\rho)) & \text{if } 0 < \rho \leq \frac{1}{2} \\ \frac{C-c}{y-x}(\rho - x(1-\rho)) + (1 - \frac{C-c}{y-x})(2\rho - 1) & \text{if } \frac{1}{2} < \rho < 1. \end{cases}$$

Trivially, when $\xi = 1$ the DM's expected payoffs are the same as in baseline model with counterfeits.

To examine which choice of ξ is optimal, we need to compare the expected payoffs under each level of ignorance in Lemma 1.6. This always involves comparing the expected payoff with ignorance and the expected payoff when $\xi = 1$, but unlike in

Section 1.4, in region where $\frac{x}{y}C > c > xC$ we also need to compare the payoff with $\xi = \frac{c}{x}$ and the payoff with $\xi = \frac{C-c}{y-x}$. This renders the analysis more complex, so instead of providing the full algebraic specification of the solution space, I present the key insights in a set of lemmas. I begin by showing that in some regions we can be sure that the DM prefers to sometimes ignore the message:

Lemma 1.8. *In the baseline regions where the Bad types mix in the equilibrium and the DM's expected payoff is zero, the DM always prefers to ignore the message with a positive probability.*

The main consequence of this lemma is that with optimal ignorance, the DM can always obtain a strictly positive expected payoff regardless of the parameters, similar to the model without counterfeits. From here I proceed in two steps: first, I establish the quasi-optimal level of ignorance when $\xi < 1$, and second, I determine when the expected payoff from the quasi-optimal level of ignorance is larger than from $\xi = 1$:

Lemma 1.9. *If the DM must choose a strictly positive level of ignorance,* $\xi \in \{\frac{C}{y}, \frac{c}{x}, \frac{C-c}{y-x}\}$*:*

(i) $\xi = \frac{C}{y}$ *is quasi-optimal when* $c > \frac{x}{y}C$ *and* $C < y$*,*

(ii) $\xi = \frac{c}{x}$ *is quasi-optimal when* $c < x$*, and either* $\rho < \frac{x}{1+x}$ *and* $\frac{x}{y}C > c > xC$*, or* $\rho > \frac{x}{1+x}$ *and either (a)* $C - (y - x) > c > xC$*, or (b)* $y > 1 - x + x^2$ *and* $\frac{x}{y}C > c > \max\{xC, C - (y - x)\}$*, or (c)* $y < 1 - x + x^2$ *and* $\frac{x}{y}C > c > \max\{C \min\{\frac{x(\rho - x(1-\rho))}{y\rho - x(1-\rho)}, \frac{x(1-x)}{y-x^2}\}, C - (y - x)\}$*,*

(iii) $\xi = \frac{C-c}{y-x}$ *is quasi-optimal when* $\rho > \frac{x}{1+x}$ *and* $xC > c > C - (y - x)$*, or* $\rho > \frac{x(1-x)}{1-y+x(1-x)}$ *and* $y < 1 - x(1 - x)$ *and* $c < C \min\{\frac{x(\rho - x(1-\rho))}{y\rho - x(1-\rho)}, \frac{x(1-x)}{y-x^2}\}$*.*

The region where an equilibrium with $\xi = \frac{C}{y}$ exists does not overlap with regions where equilibria with other ignorance levels exist. Also, regions where an equilibrium with $\xi = \frac{c}{x}$ exists and where an equilibrium with $\xi = \frac{C-c}{y-x}$ exists do not overlap

when $\rho < \frac{x}{1+x}$ or $c < C - (y - x)$ or $c < xC$. The difficulty results from the need to compare the payoff obtained when $\xi = \frac{c}{x}$ and $\xi = \frac{C-c}{y-x}$ in regions A^c or F^c. Whenever $y > 1 - x(1 - x)$, $\xi = \frac{C-c}{y-x}$ is never optimally chosen in the part of these regions where equilibria with both $\xi = \frac{c}{x}$ and $\xi = \frac{C-c}{y-x}$ exist. Whenever $y < 1-x(1-x)$, the part of these regions where equilibria with both $\xi = \frac{c}{x}$ and $\xi = \frac{C-c}{y-x}$ exist is divided: for higher values of c, it may only be optimal to choose $\xi = \frac{c}{x}$, and for low values of c, it may only be optimal to choose $\xi = \frac{C-c}{y-x}$.

As the DM's expected payoffs are zero in the baseline model with counterfeits in regions B^c, F^c and H^c, and thus $\xi = 1$ is never optimal, the above lemma fully describes how the DM optimally chooses ξ in these regions. Whenever the expected payoffs are not zero, as in A^c or G^c, the quasi-optimal expected payoff with ignorance has to be compared with the expected payoff without ignorance:

Proposition 1.9. *In region A^c the optimal level of ignorance is:*

(i) $\xi^* = \frac{C}{y}$ *when* $c > \frac{x}{y}C$ *and* $C > \min\{\frac{y}{\rho}(\rho - y(1-\rho)), y - y^2\}$,

(ii) $\xi^* = \frac{c}{x}$ *when* $\frac{x}{y}C > c > \max\{\max\{xC, C\min\{\frac{x(\rho-x(1-\rho))}{y\rho-x(1-\rho)}, \frac{x(1-x)}{y-x^2}\}\}, \min\{\frac{x}{\rho}(\rho - y(1-\rho)), x - xy\}\}$,

(iii) $\xi^* = \frac{C-c}{y-x}$ *when* $c < \min\{\max\{xC, C\min\{\frac{x(\rho-x(1-\rho))}{y\rho-x(1-\rho)}, \frac{x(1-x)}{y-x^2}\}\}, C - \min\{\frac{\rho-y(1-\rho)}{\rho-x(1-\rho)}(y-x), \frac{1-y}{1-x}(y-x)\}\}$

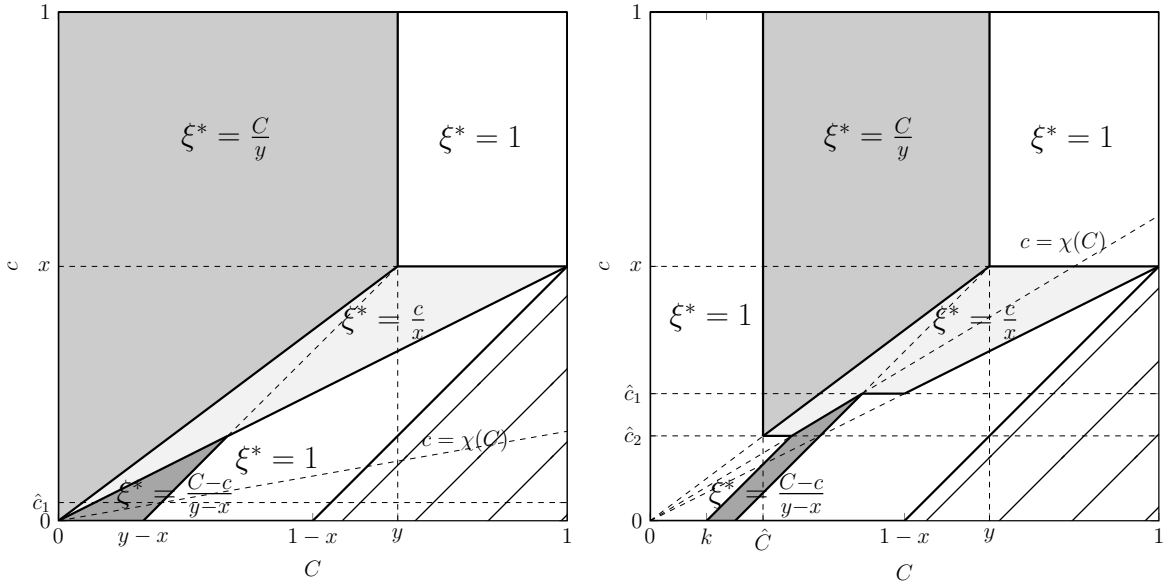
(iv) $\xi = 1$ *otherwise.*

In region G^c the optimal level of ignorance is:

(i) $\xi = \frac{c}{x}$ *when* $c > \max\{xC, \min\{\frac{x}{\rho}(\rho - x(1-\rho)), x - x^2\}\}$,

(ii) $\xi = 1$ *otherwise.*

The proposition above is illustrated in Figure 1.6. There are two key drivers of the results. First, observe that $\xi = \frac{C}{y}$ increases with C and does not change with c, $\xi = \frac{c}{x}$ does not change with C and increases with c, while $\xi = \frac{C-c}{y-x}$ increases with C



(a) when $\frac{x}{1+x} < \rho < \frac{y}{1+y}$ and $y > 1-x$

(b) when $\frac{x(1-x)}{1-y+x(1-x)} < \rho < 1$ and $y < 1-x+x^2$

Figure 1.6: Examples of DM's optimal commitment to ignorance ($\hat{C} = \min\{\frac{y}{\rho}(\rho - y(1-\rho)), y-y^2\}$, $\hat{c}_1 = \min\{\frac{x}{\rho}(\rho - x(1-\rho)), x-x^2\}$, $\hat{c}_2 = \min\{\frac{x}{\rho}(\rho - y(1-\rho)), x-xy\}$, $k = \frac{(y-x)(1-y)}{(1-x)}$ and $\chi(C) = C\min\{\frac{x(\rho-x(1-\rho))}{y\rho-x(1-\rho)}, \frac{x(1-x)}{y-x^2}\}$).

and decreases with c. The comparative statics of the expected payoffs is the same. As a result, when c is increased for low values of C in Figure 1.6a, first $\xi = \frac{C-c}{y-x}$, then $\xi = \frac{c}{x}$, and then $\xi = \frac{C}{y}$ are optimal. Second, observe that the expected payoffs from choosing $\xi = 1$ never depend on C or c. As a result, there exists a horizontal border between the region where $\xi = \frac{c}{x}$ is optimal and the region where $\xi = 1$ is optimal in area G^c of Figure 1.6b.

For any C, there exists a c which makes it optimal for the DM to commit to ignorance, except when $C < \min\{\frac{\rho-y(1-\rho)}{\rho-x(1-\rho)}(y-x), \frac{1-y}{1-x}(y-x)\}$ for $\rho > \frac{y}{1+y}$. That for very low values of C the commitment to ignorance may not help is the equivalent of the similar result in the model without counterfeits: whenever the originals are very cheap, the DM would have to commit to too high a degree of ignorance.

On a final note, observe that while the DM's expected payoff with counterfeits and ignorance can easily be lower than under ignorance only, it is possible for it to be higher when commitment with $\xi = \frac{C}{y}$ used to be optimal, but now it is $\xi = \frac{C-c}{y-x}$. This

happens because $\frac{C-c}{y-x} > \frac{C}{y}$ in the relevant region, so while in the equilibrium without counterfeits the DM had to completely discourage the Bad type from purchasing, which required a significant degree of ignorance, now it suffices to induce him to switch to purchasing counterfeits, which requires a lower degree of ignorance. This development is beneficial for the DM.

1.6 Conclusion

Situations in which a decision maker needs to make an admissions decision are at the heart of how numerous institutions function, from nightclubs to universities. Imperfect information makes the selection process difficult, because while the decision maker knows what kind of people it is seeking, it often does not know whether an individual applicant fits the bill. It is therefore of value to study the measures which the decision maker can employ to improve the outcomes of the selection process.

In this chapter, I have focused on a particular class of selection problems where price discrimination is impossible or implausible. As a result, the decision maker has to rely on the imperfect signals sent by the applicants. Agents engage in signalling through the purchase of an ostentatious good, because they believe that in doing so, they may improve their chances of admission. Unlike the standard Spencian model, I assume that the cost of obtaining the signal is the same for all, but that the undesired types sometimes fail to produce the right signal. To make the subsequent discussion below more tangible, I summarise the main conclusions of this chapter with reference to the nightclub scenario introduced at the onset.

Among the measures I have studied are capacity constraints and commitment to ignorance. Throughout this chapter, I have considered two plausible welfare objectives: the maximisation of the DM's *ex-ante* expected payoff, and the minimisation of the overall wasteful spending on the ostentatious good.

Bringing this back to the nightclub case, the establishment can completely dis-

courage the undesired types from attempting to gain entry by choosing the ostentatious good whose price is high compared to the undesired type's probability of success. Should that not be possible, the nightclub can often improve its expected payoff by restricting its total capacity. Faced with the prospect of not being admitted despite having to purchase the ostentatious good, the undesired agent's incentives to purchase are dampened, which makes distinguishing between the types easier for the nightclub. However, despite the ability to obtain a perfectly informative signal, and thus perfect separation, the constraint is binding. This prevents the nightclub from admitting as many agents, who are known to be desirable, as it would like. For this reason, when the proportion of the desired agents in the population is high, the DM may be better off admitting everyone, even if this means letting in some undesired agents.

An alternative to a capacity constraint is for the nightclub to commit to ignore the message sent by the agent with a given probability. This can be imagined as the nightclub either always rejecting or always admitting a randomly chosen subset of agents of a fixed size. The mechanism is similar to the capacity constraint: it reduces the benefit from purchasing, and with the optimal choice of the ignorance level, the undesired agent can be just discouraged from purchasing. I have demonstrated that because of its greater flexibility, commitment to ignorance is preferable to a capacity constraint from the club's perspective. At the same time, it is also socially preferable as it weakly reduces the overall wasteful spending on the ostentatious good.

As the main extension of the model, I have considered the addition of a counterfeit good, which is cheaper and guarantees success with an equally low probability for both types of club-goers. In this setting, the undesired agents may substitute purchasing of the counterfeit for purchasing of the original or not purchasing at all. Even if the nightclub can choose the ostentatious good and its price, it may not be able to obtain full separation if the counterfeit is sufficiently cheap. However, fake Rolexes are not necessarily bad news for the club, because the undesired agents

are less likely to succeed when they they purchase a counterfeit compared to the original, so when a switch from the original to the counterfeit takes place, the club may actually obtain a better degree of separation. I have finished by analysing how optimal commitment to ignorance changes when counterfeits are available.

Further research could shed more light on the usefulness of external evidence and the extent to which the availability of counterfeits limits it. Moreover, beyond this chapter there are additional extensions which may be interesting to consider in this setting. In particular, making the agent unsure about his true type, or introducing a medium type whose admission is neutral for the club, could further enhance the realism of the model.

Appendix 1.A Omitted Proofs

Proof of Proposition 1.1

I begin by proving a lemma that restricts the set of strategies which may be played in equilibrium:

Lemma 1.10. *In any Nash equilibrium, we must have* $\sigma_G > \sigma_B$, $\sigma_G = \sigma_B = 1$, *or* $\sigma_G = \sigma_B = 0$.

Proof. The proof proceeds by contradiction in two parts. First, it is shown that we must have $\sigma_G \geq \sigma_B$, and second it is shown that both types of agents cannot be using mixing strategies simultaneously.

For the first part, suppose that $\sigma_G < \sigma_B$. Then we must have $\upsilon_B^P \geq \upsilon_B^N$, so that $\upsilon_G^P \leq \upsilon_G^N$. But as $\upsilon_G^N = \upsilon_B^N = \delta_N$, this implies that $\upsilon_G^P \leq \upsilon_B^P$, which is a contradiction as $y \in (0,1)$. If $\upsilon_B^P < \upsilon_B^N$, then $\sigma_B = 0$ and there exists no $\sigma_G \in [0,1]$ that would be smaller than σ_B. Thus, we must have $\sigma_G \geq \sigma_B$.

For the second part, suppose that both agents were mixing. This implies that $\upsilon_G^P = \upsilon_G^N$ and $\upsilon_B^P = \upsilon_B^N$. But $\upsilon_G^N = \upsilon_B^N = \delta_N$, so this implies $\upsilon_G^P = \upsilon_B^P$, which is a contradiction as $y \in (0,1)$.

Thus, $\sigma_G \geq \sigma_B$ with equality only when $\sigma_G = \sigma_B = 0$ or $\sigma_G = \sigma_B = 1$. $\square$

The inference about the types, as a function of the agents' strategies, are $\beta_S = \frac{\rho\sigma_G}{\rho\sigma_G+(1-\rho)y\sigma_B}$ and $\beta_N = \frac{\rho(1-\sigma_G)}{\rho(1-\sigma_G)+(1-\rho)(1-\sigma_B)}$. As per Lemma 1.10, the only strategies which may be played by the agents in an equilibrium are: (i) $\sigma = (1,1)$, (ii) $\sigma = (1,\iota)$, (iii) $\sigma = (1,0)$, (iv) $\sigma = (\iota,0)$, or (v) $\sigma = (0,0)$, where $\iota \in (0,1)$. Whenever one of the agents mixes, the DM must mix as well at one of the messages, $\beta_m = \frac{1}{2}$ for some m, as otherwise we would have $\delta^*_m \in \{0,1\}$ for all m, which leads to a contradiction in the mixing agent's indifference condition.

Consider (i): $\beta_S = \frac{\rho}{\rho+y(1-\rho)}$ and β_N is undefined. When $\beta_S \geq \frac{1}{2}$, or $\rho \geq \frac{y}{1+y}$, we have $\delta^*_S = 1$. Then the Bad agent prefers to purchase if $\upsilon^P_B > \upsilon^N_B$, or $\delta_N < y - C$. As $\delta_N \in [0,1]$, we must have $y > C$ in any such equilibrium. The Good agent prefers to purchase if $\upsilon^P_G > \upsilon^N_G$, or $1 > C + \delta_N$, which is implied by the former inequality. Thus, for $C < y$ and $\rho \geq \frac{y}{1+y}$, there exists a range of equilibria in which $\sigma^* = (1,1)$ and $\delta^* = (1,\delta^*_N)$, where $\delta^*_N \in [0, y-C]$. The D1 criterion does not help to refine the out-of-equilibrium beliefs, as a switch to not purchasing gives the same payoff to both the Good and the Bad type. However, as the outcomes in all such equilibria are the same, I choose the one with $\delta^*_N = 0$ for simplicity. When $\beta_S < \frac{1}{2}$, we have $\delta_S = 0$, but then the Good agent only indeed prefers purchasing, $\upsilon^P_G > \upsilon^N_G$, if $\delta_N + C < 0$, which is impossible for any $\delta_N \in [0,1]$.

Consider (ii): $\beta_S = \frac{\rho}{\rho+y(1-\rho)\sigma_B}$ and $\beta_N = 0$, so $\delta^*_N = 0$. As the Bad agent mixes, we must have $\beta_S = \frac{1}{2}$, so he mixes with probability $\sigma_B = \frac{\rho}{y(1-\rho)}$, and $\sigma_B \in (0,1)$ only if $\rho < \frac{y}{1+y}$. The Bad agent is indeed indifferent, $\upsilon^P_B = \upsilon^N_B$, only if $\delta_S = \frac{C}{y}$, and $\delta_S \in (0,1)$ only if $C < y$. The Good agent prefers to purchase if $\upsilon^P_G > \upsilon^N_G$, or $\frac{C}{y} - C > 0$, which is always satisfied. Thus, for $C < y$ and $\rho < \frac{y}{1+y}$, there exists an equilibrium in which $\sigma^* = (1, \frac{\rho}{y(1-\rho)})$ and $\delta^* = (\frac{C}{y}, 0)$.

Consider (iii): $\beta_S = 1$ and $\beta_N = 0$, so that $\delta^*_S = 1$ and $\delta^*_N = 0$. The Bad agent only prefers not to purchases if $\upsilon^N_B > \upsilon^P_B$, or $y < C$, while the Good agent only prefers to purchase if $\upsilon^P_G > \upsilon^N_G$, or $C < 1$, which is always satisfied. Thus, for

$y < C$, there exists an equilibrium in which $\sigma^* = (1, 0)$ and $\delta^* = (1, 0)$.

Consider (iv): $\beta_S = 1$ and $\beta_N = \frac{\rho(1-\sigma_G)}{\rho(1-\sigma_G)+(1-\rho)}$, so that $\delta_S^* = 1$. As the Good agent mixes, we must have $\beta_N = \frac{1}{2}$, so he must mix with probability $\sigma_G^* = \frac{2\rho-1}{\rho}$, and $\sigma_G \in (0, 1)$ only if $\rho > \frac{1}{2}$. The Good agent is indeed indifferent, $v_G^P = v_G^N$, only if $\delta_N = 1 - C$, and $\delta_N \in (0, 1)$ is always satisfied. The Bad agent prefers not to purchase if $v_B^N > v_B^P$, or $y < 1$, which is always satisfied. Thus, for $\rho > \frac{1}{2}$, there exists an equilibrium in which $\sigma^* = (\frac{2\rho-1}{\rho}, 0)$ and $\delta^* = (1, 1 - C)$.

Consider (v): β_S is undefined and $\beta_N = \rho$, so (a) when $\rho \geq \frac{1}{2}$ the DM sets $\delta_N = 1$, while (b) when $\rho < \frac{1}{2}$ the DM sets $\delta_N = 0$. Consider (a). The Good agent prefers not to purchase only if $v_G^N > v_G^P$, or $\delta_S < 1 + C$, which is always satisfied as $\delta_S \in [0, 1]$. The Bad agent prefers not to purchase only if $v_B^N > v_B^P$, or $\delta_S < \frac{1+C}{y}$, which is also always satisfied as $\delta_S \in [0, 1]$. Thus, when $\rho \geq \frac{1}{2}$, there exists an equilibrium in which $\sigma^* = (0, 0)$ and $\delta^* = (\delta_S^*, 1)$, where $\delta_S^* \in [0, 1]$. The D1 criterion does not help to refine the out-of-equilibrium beliefs, as a switch to purchasing necessarily lowers the payoffs of both the Good and the Bad type. However, as the outcomes in all such equilibria are the same, I choose the equilibrium with $\delta_S^* = 1$ for simplicity. Consider (b). The Good agent prefers not to purchase only if $v_G^N > v_G^P$, or $\delta_S < C$. The Bad agent prefers not to purchase only if $v_B^N > v_B^P$, or $\delta_S < \frac{C}{y}$, which is always satisfied whenever $\delta_S < C$ is satisfied. Thus, when $\rho < \frac{1}{2}$, there exists an equilibrium in which $\sigma^* = (0, 0)$ and $\delta^* = (\delta_S^*, 0)$, where $\delta_S^* \in [0, C)$. In this equilibrium, the Bad type would benefit from a deviation to purchasing whenever $y\delta_S - C > 0$, or $\delta_S \in (\frac{C}{y}, 1]$, while the Good type would benefit from a deviation to purchasing whenever $\delta_S - C > 0$, or $\delta_S \in (C, 1]$. As $(\frac{C}{y}, 1] \subset (C, 1]$, the Good type strictly benefits from a deviation whenever the Bad type benefits from a deviation. Thus, the D1 criterion requires that probability zero be put on the deviating agent being Bad, and hence $\beta_S = 1$, which means that we must have $\delta_S^* = 1$, which is a contradiction as this we must have $\delta_S^* < C$. Thus, this equilibrium is eliminated by the D1 criterion.

Proof of Proposition 1.2

For expected payoff, we have $\Pi_A^* = \rho - y(1-\rho)$, $\Pi_B^* = 0$ and $\Pi_C^* = \rho$. The expected payoff in none of the regions depends directly on C. As C increases, the equilibrium can move from region A or B to C, in which case the expected payoff increases as $\Pi_C^* > \Pi_A^* > \Pi_B^*$. When y increases, the expected payoff in region A falls, and it does not change in regions B or C. Also, the equilibrium can move from region C to A or B, in which case the expected payoff falls.

For overall spending, we have $\Sigma_A^* = 1$, $\Sigma_B^* = \rho\frac{1+y}{y}$ and $\Sigma_C^* = \rho$. The overall spending in none of the regions depends directly on C. As C increases, the equilibrium can move from region A or B to C, in which case the overall spending falls as $\Sigma_C^* < \Sigma_B^* < \Sigma_A^*$. When y increases, the overall spending in region B falls, and it does not change in regions A or C. Also, the equilibrium can move from region C to A or B, in which case the overall spending increases. Therefore, the behaviour of Σ^* is not monotonous in y.

As $\Pi_C^* > \Pi_A^* > \Pi_B^*$, and the expected payoff does not depend on C in equilibrium C, the DM optimally chooses any $C > y$ to find itself in equilibrium C. As $\Sigma_C^* < \Pi_B^* < \Pi_A^*$, the overall spending is minimised in this equilibrium.

Proof of Lemma 1.1

We calculate the proportion of all agents that are admitted by the DM for the equilibria from Proposition 1.1 to obtain $\pi_A = \rho + y(1-\rho)$, $\pi_B = \rho\frac{C}{y} + (1-\rho)\frac{C}{y}y\frac{\rho}{y(1-\rho)} = \frac{2\rho C}{y}$, $\pi_C = \rho$, $\pi_D = \rho + (1-\rho) = 1$ and $\pi_E = \rho(\frac{2\rho-1}{\rho} + (1-\frac{2\rho-1}{\rho})(1-C)) + (1-\rho)(1-C) = 1 - 2C(1-\rho)$. Whenever the capacity constraint is set so that it does not bind or it just binds, this has no impact on the equilibrium.

Proof of Proposition 1.3

Consider the equilibria in which the CC becomes binding, $\pi(\sigma, \delta) = \gamma$. As per Lemma 1.10, the only strategies which may be played by the agents in an equilibrium

are: (i) $\sigma = (1,1)$, (ii) $\sigma = (1,\iota)$, (iii) $\sigma = (1,0)$, (iv) $\sigma = (\iota,0)$, or (v) $\sigma = (0,0)$, where $\iota \in (0,1)$.

Consider (i): $\beta_S = \frac{\rho}{\rho+y(1-\rho)}$ and β_N is undefined. When $\beta_S \geq \frac{1}{2}$, or $\rho \geq \frac{y}{1+y}$, the DM would like to admit all successful agents, which is impossible because the CC binds, so $\pi(\sigma^*,\delta) = \gamma$, or $\rho\delta_S + (1-\rho)y\delta_S = \gamma$. Thus $\delta_S^* = \frac{\gamma}{\rho+y(1-\rho)}$, and $\delta_S^* \in [0,1]$ only if $\gamma < \rho + y(1-\rho)$. Then the Bad agent prefers to purchase only if $\upsilon_B^P > \upsilon_B^N$, or $\delta_N < y\frac{\gamma}{\rho+y(1-\rho)} - C$. But this implies that $\delta_N^* < y\frac{\gamma}{\rho+y(1-\rho)} - C < 1$, so $\beta_N \leq \frac{1}{2} < 1 = \beta_S$, so if the DM is limited in the number of agents that it can admit, it prefers to first admit upon the message that gives a higher chance of selecting the Good agent. Thus, $\delta_N^* = 0$ in the equilibrium. But then the Bad agent only indeed prefers to purchase if $\gamma > \frac{C}{y}(\rho + y(1-\rho))$, so as we must have $\gamma < \rho + y(1-\rho)$, this requires $C < y$. The Good agent prefers to purchase only if $\upsilon_G^P > \upsilon_G^N$, or $\gamma > C(\rho + y(1-\rho))$, which is implied by the former inequality. Thus, for $C < y$ and $\rho \geq \frac{y}{1+y}$ and $\rho + y(1-\rho) > \rho \geq \frac{C}{y}(\rho + y(1-\rho)$, there exists an equilibrium in which $\sigma^* = (1,1)$ and $\delta^* = (\frac{\gamma}{\rho+y(1-\rho)}, 0)$. When $\beta_S < \frac{1}{2}$, we have $\delta_S = 0$, but then the Good agent only indeed prefers purchasing, $\upsilon_G^P > \upsilon_G^N$, if $\delta_N + C < 0$, which is impossible for any $\delta_N \in [0,1]$, regardless of the CC.

Consider (ii): $\beta_S = \frac{\rho}{\rho+y(1-\rho)\sigma_B}$ and $\beta_N = 0$, so $\delta_N^* = 0$. If $\beta_S \geq \frac{1}{2}$, the DM would like to admit more successful agents that it can, so the CC binds, $\pi(\sigma^*,\delta) = \gamma$, or $\rho\delta_S + (1-\rho)y\delta_S\sigma_B^* = \gamma$. The Bad agent is indeed indifferent only if $\upsilon_B^P = \upsilon_B^N$, or $\delta_S^* = \frac{C}{y}$ and $\delta_S \in (0,1)$ only if $C < y$. Then the probability with which the Bad type mixes is calculated from the CC, $\sigma_B^* = \frac{y\gamma-\rho C}{y(1-\rho)C}$, and $\sigma_B \in (0,1)$ only if $\frac{C}{y}(\rho + y(1-\rho)) > \gamma > \frac{\rho C}{y}$. The Good agent prefers to purchase only if $\upsilon_G^P > \upsilon_G^N$, or $\frac{C}{y} - C > 0$, which is always satisfied. It remains to check that $\beta_S \geq \frac{1}{2}$ is indeed satisfied, which is the case only when $\frac{\rho}{\rho+y(1-\rho)\frac{y\gamma-\rho C}{y(1-\rho)C}} \geq \frac{1}{2}$, or $\gamma \leq \frac{2\rho C}{y}$. $\frac{C}{y}(\rho + y(1-\rho)) < \frac{2\rho C}{y}$ whenever $\rho > \frac{y}{1+y}$, so for $C < y$ and (a) $\rho < \frac{y}{1+y}$ and $\frac{2\rho C}{y} > \gamma > \frac{\rho C}{y}$, or (b) $\rho \geq \frac{y}{1+y}$ and $\frac{C}{y}(\rho + y(1-\rho)) > \gamma > \frac{\rho C}{y}$, there exists an equilibrium in which $\sigma^* = (1, \frac{y\gamma-\rho C}{y(1-\rho)C})$ and $\delta^* = (\frac{C}{y}, 0)$. If $\beta_S < \frac{1}{2}$, we must have

$\delta_S = 0$, but then the Bad agent is indeed indifferent only if $\upsilon_B^P = \upsilon_B^N$, or $C = 0$, which is a contradiction.

Consider (iii): $\beta_S = 1$ and $\beta_N = 0$, so that $\delta_N^* = 0$. The CC binds, so $\pi(\sigma^*, \delta) = \gamma$, or $\rho\delta_S = \gamma$. Thus $\delta_S = \frac{\gamma}{\rho}$, and $\delta_S \in (0,1)$ only if $\gamma < \rho$. The Bad agent only prefers not to purchase if $\upsilon_B^N > \upsilon_B^P$, or $\gamma < \frac{\rho C}{y}$, which is always satisfied whenever $\gamma < \rho$ is satisfied, while the Good agent only prefers to purchase if $\upsilon_G^P > \upsilon_G^N$, or $\gamma > \rho C$. Thus, for $y < C$ and $\rho > \gamma > \rho C$, there exists an equilibrium in which $\sigma^* = (1,0)$ and $\delta^* = (\frac{\gamma}{\rho}, 0)$.

Consider (iv): $\beta_S = 1$ and $\beta_N = \frac{\rho(1-\sigma_G)}{\rho(1-\sigma_G)+(1-\rho)}$. As $\beta_S > \beta_N$, the DM would always maximise its expected payoff by admitting all agents upon $m = S$ before admitting any agents upon $m = N$. There are two cases to consider: (a) the CC is lax enough to admit all successful agents and some non-purchasers, or (b) the CC is so strict that not even all successful agents can be admitted. Consider (a). If $\beta_N \geq \frac{1}{2}$, the DM would like to admit all non-purchasers, but the CC binds, so that $\pi(\sigma^*, \delta) = \gamma$, or $\rho(\sigma_G^* + (1-\sigma_G^*)\delta_N) + (1-\rho)\delta_N = \gamma$. The Good agent is indeed indifferent only if $\upsilon_G^P = \upsilon_G^N$, or $\delta_N^* = 1 - C$, and $\delta_N \in (0,1)$ is always satisfied. Then the probability with which the Good type mixes is calculated from the CC, $\sigma_G^* = \frac{\gamma+C-1}{\rho C}$, and $\sigma_G \in (0,1)$ only if $1 - C(1-\rho) > \gamma > 1 - C$. The Bad agent prefers not to purchase only if $\upsilon_B^N > \upsilon_B^P$, or $1 - C > y - C$, which is always satisfied. It remains to check the $\beta_N \geq \frac{1}{2}$ is indeed satisfied, which is the case only when $\frac{\rho(1-\frac{\gamma+C-1}{\rho C})}{\rho(1-\frac{\gamma+C-1}{\rho C})+(1-\rho)} > \frac{1}{2}$, or $\gamma < 1 - 2C(1-\rho)$. Thus, for $\rho > \frac{1}{2}$ and $1 - 2C(1-\rho) > \gamma > 1 - C$, there exists an equilibrium in which $\sigma^* = (\frac{\gamma+C-1}{\rho C}, 0)$ and $\delta^* = (1, 1-C)$. If $\beta_N < \frac{1}{2}$, the DM does not want to admit upon $m = N$, so the CC does not bind, which is a contradiction. Consider (b). We must have $\pi(\sigma^*, \delta) = \gamma$, or $\rho\sigma_G^*\delta_S = \gamma$. The Good agent is indeed indifferent only when $\upsilon_G^P = \upsilon_G^N$, or $\delta_S^* = C$, and δ_S is always in $(0,1)$. Then the probability with which the Good type mixes is calculated from the CC, $\sigma_G^* = \frac{\gamma}{\rho C}$, and $\sigma_G \in (0,1)$ only if $\gamma < \rho C$. Thus, for $\rho C > \gamma > 0$, there exists an equilibrium in which $\sigma^* = (\frac{\gamma}{\rho C}, 0)$ and $\delta^* = (C, 0)$.

Consider (v): β_S is undefined and $\beta_N = \rho$. When $\rho \geq \frac{1}{2}$ the DM would like to admit all agents, but the CC binds, so $\pi(\sigma^*, \delta) = \gamma$, or $\delta_N = \gamma$, and δ_N is always in $[0, 1]$. But now, as opposed to Proposition 1.1, the D1 criterion does help to refine the out-of-equilibrium beliefs β_S. By reasoning from case (v) of Proposition 1.1, we must have $\delta_S^* = 1$. The Good agent thus prefers not to purchase only if $v_G^N > v_G^P$, or $\gamma > 1 - C$. The Bad agent prefers not to purchase only if $v_B^N > v_B^P$, or $\gamma > y - C$, which is also always satisfied whenever $\gamma > 1 - C$ is satisfied. Thus, when $\rho > \frac{1}{2}$, there exists an equilibrium in which $\sigma^* = (0, 0)$ and $\delta^* = (1, \gamma)$. When $\rho < \frac{1}{2}$ and $\gamma > 1 - C$, we must have $\delta_N^* = 0$ in the equilibrium, but any such equilibrium is eliminated by the D1 criterion as in Proposition 1.1.

Proof of Lemma 1.2

The equilibria that do not satisfy Condition $N0$ are D, D', E and E'. They all require $\rho > \frac{1}{2}$ and $\gamma > 1 - C$ for existence. Thus, when these conditions are not met, they do not exist, and the remaining equilibria become unique as they do not overlap for the same parameter values.

Proof of Proposition 1.4

Assume that $C \geq y$. From Figure 1.3a, the DM's expected payoff reaches the maximum for any $\gamma \in [\rho, 1]$. But as $\pi_C = \rho$, for any such γ the CC does not bind, so it is optimal for the DM not to set a CC.

Assume that $C < y$ and $\rho < \frac{y}{1+y}$. From Figure 1.3b, the DM's expected payoff reaches a unique maximum at $\gamma^* = \frac{C}{y}\rho$. As $\pi_B = \frac{2C}{y}\rho > \frac{C}{y}\rho$, it is optimal in this case to set the CC.

Assume that $C < y$ and $\rho \geq \frac{y}{1+y}$. From Figure 1.3c, the candidates for optima are $\gamma = \frac{C}{y}\rho$, where the payoff is $\Pi(\gamma = \frac{C}{y}\rho) = \frac{C}{y}\rho$, or any $\gamma \in [\rho + y(1-\rho), 1]$, where $\Pi_A = \rho - y(1-\rho)$. As $\pi_A = \rho + y(1-\rho)$, the CC does not bind in the latter case, so the DM does not set the CC whenever $\rho - y(1-\rho) > \frac{C}{y}\rho$, or $\rho > \frac{y}{(1-\frac{C}{y})+y}$, where

the threshold is always greater than $\frac{y}{1+y}$. If this condition on ρ is not satisfied, the DM does optimally set the CC at $\gamma^* = \frac{C}{y}\rho$.

Proof of Lemma 1.3

Suppose that $C \geq y$. It follows from Proposition 1.1 that without committing to ignorance, or by setting $\xi^* = 1$, the DM obtains the maximum possible payoff of ρ, so committing to ignorance may only (weakly) lower it.

In what follows, observe that the agents' expected payoffs are $\upsilon_B^P(\xi) = \xi y \delta_S - C + (1-\xi)\delta_I$, $\upsilon_G^P(\xi) = \xi\delta_S - C + (1-\xi)\delta_I$ and $\upsilon_B^N(\xi) = \upsilon_G^N(\xi) = (1-\xi)\delta_I$. Also, $\delta_I^* = 0$ if $\rho \leq \frac{1}{2}$, and $\delta_I^* = 1$ if $\rho > \frac{1}{2}$.

Suppose that $C < y$ and $\xi \leq \bar{\xi} \equiv \frac{C}{y}$. The Bad agent prefers to purchase only if $\upsilon_B^P(\xi) > \upsilon_B^N(\xi)$, or $\xi > \frac{C}{y\delta_S} \geq \frac{C}{y}$, where the second inequality follows from the fact that $\delta_S \in [0,1]$. But this contradicts $\xi \leq \frac{C}{y}$, so the Bad type never purchases, $\sigma_B^* = 0$. Consequently, as we assume that at least some agents purchase, $\beta_S^* = 1$, so that $\delta_S^* = 1$. The Good agent prefers not to purchase only if $\upsilon_G^N(\xi) > \upsilon_G^P(\xi)$, or $\xi < C$. Thus, as we require that at least some agents purchase, we must have $\xi \geq C$. Thus, an equilibrium in which $(\sigma_G^*, \sigma_B^*) = (1,0)$ and $(\delta_S^*, \delta_N^*) = (1,0)$ may only exist when $C \leq \xi \leq \frac{C}{y}$. The DM's expected payoff is $\Pi^*(\xi) = \xi(\delta_S^*(\rho\sigma_G^* - (1-\rho)y\sigma_B^*) + (1-\xi)\delta_I^*(\rho - (1-\rho)) = \xi\rho + (1-\xi)\delta_I^*(2\rho - 1)$, which is strictly increasing in ξ, because the derivative with respect to ξ is $\rho - \delta_I^*(2\rho-1)$, which is positive as $\rho > 2\rho - 1$ and $\delta_I^* \in [0,1]$. Thus the DM would like to set ξ as high as possible, $\xi^* = \bar{\xi}$. Except the boundary cases, the DM's expected payoff from following such a strategy is always strictly positive.

Suppose that $C < y$ and $\bar{\xi} < \xi \leq 1$. As per Lemma 1.10, the only strategies which may be played by the agents in an equilibrium are: (i) $\sigma = (1,1)$, (ii) $\sigma = (1,\iota)$, (iii) $\sigma = (1,0)$, (iv) $\sigma = (\iota,0)$, or (v) $\sigma = (0,0)$, where $\iota \in (0,1)$. But the last candidate can be eliminated as it violates the requirement that at least some agents purchase the good. Whenever one of the agents mixes, the DM must mix as well at one of

the messages, $\beta_m = \frac{1}{2}$ for some m, as otherwise we would have $\delta_m^* \in \{0, 1\}$ for all m, which leads to a contradiction in the mixing agent's indifference condition.

Consider (i): $\beta_S = \frac{\rho}{\rho + y(1-\rho)}$ and β_N is undefined, but by Condition $N0$ we have $\delta_N = 0$. When $\beta_S \geq \frac{1}{2}$, or $\rho \geq \frac{y}{1+y}$, we have $\delta_S = 1$. Then the Bad agent prefers to purchase if $\upsilon_B^P > \upsilon_B^N$, or $\xi > \frac{C}{y}$, while the Good agent prefers to purchase if $\upsilon_G^P > \upsilon_G^N$, or $\xi > C$, which is satisfied whenever $\xi > \frac{C}{y}$ is satisfied. In this equilibrium, where $\sigma = (1, 1)$ and $\delta_S = 1$, the DM's expected payoff is $\Pi_A(\xi) = \xi(\rho - y(1-\rho))$, which increases in ξ, so the DM prefers to set $\xi = 1$. When $\beta_S < \frac{1}{2}$, we have $\delta_S = 0$, but then the Good agent only indeed prefers purchasing, $\upsilon_G^P > \upsilon_G^N$, if $C < 0$, which is a contradiction.

Consider (ii): $\beta_S = \frac{\rho}{\rho + y(1-\rho)\sigma_B}$ and $\beta_N = 0$, so $\delta_N = 0$. As the Bad type mixes, we must have $\beta_S = \frac{1}{2}$, so he mixes with probability $\sigma_B = \frac{\rho}{y(1-\rho)}$, and $\sigma_B \in (0, 1)$ only if $\rho < \frac{y}{1+y}$. Then the Bad agent is indifferent, $\upsilon_B^P = \upsilon_B^N$, only if $\delta_S = \frac{C}{\xi y}$, and $\delta_S \in (0, 1)$ only if $\xi > \frac{C}{y}$. The Good agent prefers to purchase only if $\upsilon_G^P > \upsilon_G^N$, or $\frac{C}{y} - C > 0$, which is always satisfied. In this equilibrium, where $\sigma = (1, \frac{\rho}{y(1-\rho)})$ and $\delta_S = \frac{C}{\xi y}$, the DM's expected payoff is zero regardless of ξ, so the DM can always obtain a higher expected payoff by committing to ignorance with $\xi = \frac{C}{y}$.

Consider (iii): $\beta_S = 1$ and $\beta_N = 0$, so that $\delta_S = 1$ and $\delta_N = 0$. But then the Bad agent only prefers not to purchases if $\upsilon_B^N > \upsilon_B^P$, or $\xi < \frac{C}{y}$, which contradicts the assumption that $\xi > \bar{\xi}$.

Consider (iv): $\beta_S = 1$ and $\beta_N = \frac{\rho\sigma_G}{\rho\sigma_G + (1-\rho)}$, so that $\delta_S = 1$. We cannot have $\beta_N \geq \frac{1}{2}$, as then $\delta_N > 0$, which violates Condition $N0$. When $\beta_N < \frac{1}{2}$, we must have $\delta_N = 0$, but then $\upsilon_G^P = \upsilon_G^N$, or $\xi = C$, but this contradicts the assumption that $\xi > \bar{\xi}$.

Thus, I have established that the DM's expected payoff can only be maximised when $\xi = 1$ or $\xi = \frac{C}{y}$.

Proof of Proposition 1.5

The DM's expected payoff with positive optimal commitment to ignorance is $\Pi_{\xi=\bar{\xi}} = \frac{C}{y}\rho$ for $\rho \leq \frac{1}{2}$ and $\Pi_{\xi=\bar{\xi}} = \frac{C}{y}\rho + (1-\frac{C}{y})(2\rho-1)$ for $\rho > \frac{1}{2}$. As $\frac{y}{1+y} < \frac{1}{2}$, there are three regions in which we need to compare the DM's expected payoffs: (i) $0 < \rho < \frac{y}{1+y}$, (ii) $\frac{y}{1+y} \leq \rho < \frac{1}{2}$, and (iii) $\frac{1}{2} \leq \rho < 1$. In (i), $\Pi_{\xi=\bar{\xi}} > \Pi_B = 0$, so commitment to ignorance is optimal. In (ii), $\Pi_A = \rho - y(1-\rho)$, so commitment to ignorance is better only if $\Pi_A < \Pi_{\xi=\bar{\xi}}$, or $C > \frac{y}{\rho}(\rho - y(1-\rho))$. In (iii), $\Pi_A = \rho - y(1-\rho)$, so commitment to ignorance is better only if $\Pi_A < \Pi_{\xi=\bar{\xi}}$, or $C > y(1-y)$. $\frac{d}{d\rho}\big(\frac{y}{\rho}(\rho - y(1-\rho))\big) = \frac{y^2}{\rho^2} > 0$ and $\lim_{\rho\to\frac{1}{2}} \frac{y}{\rho}(\rho - y(1-\rho)) = y(1-y)$, so the threshold above which commitment to ignorance with $\xi^* = \frac{C}{y}$ is optimal when $\rho > \frac{y}{1+y}$ can be rewritten as $C > \min\{\frac{y}{\rho}(\rho - y(1-\rho)), y(1-y)\}$.

Proof of Proposition 1.6

The proof follows from inspection of Table 1.1.

Proof of Proposition 1.7

I begin by proving a lemma that restricts the set of strategies which may be played in the equilibrium:

Lemma 1.11. *When counterfeits become available, in any equilibrium we must have:*

(i) $\sigma_G \geq \sigma_B$,

(ii) $\mu_G = 0$ *and* $\sigma_G \in (0,1)$ *implies* $\sigma_B = \mu_B = 0$,

(iii) $\sigma_G \in (0,1)$ *and* $\mu_G = 1 - \sigma_G$ *implies* $\sigma_B = 0$ *and* $\mu_B = 1$.

Proof. For all parts note that $\upsilon_B^{PC} = x\delta_S^* - c = \upsilon_G^{PC}$ and $\upsilon_B^N = \delta_N^* = \upsilon_G^N$.

For (i), Suppose $\sigma_G < \sigma_B$, so that the Bad type mixes between $d = P$, $d = PC$ and $d = N$ or always chooses $d = P$, and the Good type mixes between $d = P$, $d = PC$ and $d = N$ or always chooses $d = N$. Then we must have $\upsilon_B^P \geq \upsilon_B^N$ and

$v_G^P \leq v_G^N$. But as $v_B^N = v_G^N$, this implies $v_G^P \leq v_B^P$, which is a contradiction as $y \in (0,1)$. If we had $v_B^P < v_B^N$, then we would have $\sigma_B = 0$ and there would exist no $\sigma_G \in [0,1]$ that would be smaller than σ_B. Thus, we must have $\sigma_G \geq \sigma_B$.

For (ii), $\mu_G = 0$ and $\sigma_G \in (0,1)$ implies that $v_G^P = v_G^N > v_B^{PC}$. By part (i) we cannot have $\sigma_B = 1$. Suppose that $\mu_B = 1$. But then $v_B^{PC} > v_B^N$, which gives a contradiction $v_B^N = v_G^N > v_G^{PC} = v_B^{PC}$. Suppose instead that $\sigma_B \in (0,1)$ and $\mu_B = 1 - \sigma_B$. But then $v_B^P = v_B^{PC} > v_B^N$, which gives a contradiction $v_B^N = v_G^N > v_G^{PC} = v_B^{PC}$. Suppose instead that $\sigma_B \in (0,1)$ and $\mu_B = 0$. But then $v_B^P = v_B^N = v_G^N = v_G^P$, which is a contradiction for $y \in (0,1)$. Suppose instead that $\sigma_B = 0$ and $\mu_B \in (0,1)$. But then $v_B^{PC} = v_B^N$, which gives a contradiction $v_B^N = v_G^N > v_G^{PC} = v_B^{PC}$. Suppose instead that $\sigma_B \in (0,1)$, $\mu_B \in (0,1)$ and $\sigma_B + \mu_B < 1$. But then $v_B^P = v_B^{PC} = v_B^N$, which gives a contradiction $v_B^N = v_G^N > v_G^{PC} = v_B^{PC}$. Suppose instead that $\sigma_B = \mu_B = 0$. Then $v_B^N > v_B^{PC}$ and $v_B^N > v_B^P$, which is always satisfied.

For (iii), $\sigma_G \in (0,1)$ and $\mu_G = 1 - \sigma_G$ implies that $v_G^P = v_G^{PC} > v_G^N$. By (i) we cannot have $\sigma_B = 1$. Suppose that $\sigma_B \in (0,1)$ and $\mu_B = 1 - \sigma_B$. But then $v_B^P = v_B^{PC}$, which gives a contradiction $v_G^P = v_G^{PC} = v_B^{PC} = v_B^P$ for $y \in (0,1)$. Suppose instead that $\sigma_B \in (0,1)$ and $\mu_B = 0$. But then $v_B^P = v_B^N > v_B^{PC} = v_G^P$, which is a contradiction for $y \in (0,1)$. Suppose instead that $\sigma_B = 0$ and $\mu_B \in (0,1)$. But then $v_B^{PC} = v_B^N$, which gives a contradiction $v_B^{PC} = v_G^{PC} > v_G^N = v_B^N$. Suppose instead that $\sigma_B \in (0,1)$, $\mu_B \in (0,1)$ and $\sigma_B + \mu_B < 1$. But then $v_B^P = v_B^{PC} = v_B^N$, which gives a contradiction $v_B^{PC} = v_G^{PC} > v_G^N = v_B^N$. Suppose instead that $\sigma_B = \mu_B = 0$. Then $v_B^N > v_B^{PC}$ and $v_B^N > v_B^P$, which gives a contradiction $v_B^{PC} = v_G^{PC} > v_G^N = v_B^N$. Suppose instead that $\mu_B = 1$. Then $v_B^{PC} > v_B^N$ and $v_B^{PC} > v_B^P$, which is always satisfied for $y \in (0,1)$. □

The assumption that the originals are at least sometimes purchased combined with the first part of Lemma 1.11 gives us $\sigma_G > 0$. Let $\iota \in (0,1)$ be used whenever a mixed strategy is used by the agent. Let $\Phi_R = (\sigma_G, \mu_G, \sigma_B, \mu_B)$ denote the set

of agents' strategies which *may* constitute an equilibrium called R. Thus the Good type may either (i) always purchase the originals, in which case $\sigma_G = 1$, or (ii) mix purchasing the originals with purchasing the counterfeits or not purchasing at all, in which case $\sigma_G = \iota \in (0,1)$. Consider (i). $\sigma_G = 1$ implies that either $\sigma_B = 1$, in which case $\Phi_{A^c} = (1,0,1,0)$, or $\sigma_B = 0$, in which case $\Phi_{G^c} = (1,0,0,1)$, $\Phi_{C^c} = (1,0,0,0)$ or $\Phi_{F^c} = (1,0,0,\iota)$, or $\sigma_B = \iota \in (0,1)$, in which case $\Phi_{B^c} = (1,0,\iota,0)$ or $\Phi_{H^c} = (1,0,\iota,1-\iota)$. Consider (ii). The Good agent can mix between purchasing the original and the counterfeit, in which case $\mu_G = 1 - \iota$, which by the third part of Lemma 1.11 gives $\sigma_B = 0$ and $\mu_B = 1$, so that $\Phi_{I^c} = (\iota, 1-\iota, 0, 1)$, or he can mix between purchasing the original and not purchasing at all, in which case $\mu_G = 0$, which by the second part of Lemma 1.11 gives $\sigma_B = \mu_B = 0$, so that $\Phi_{E^c} = (\iota, 0, 0, 0)$. It is not possible for any agent to mix between all three actions, as that could happen only in boundary cases. Whenever one of the agents mixes, the DM must mix as well at one of the messages, $\beta_m = \frac{1}{2}$ for some m, as otherwise we would have $\delta^*_m \in \{0,1\}$ for all m, which leads to a contradiction in the mixing agent's indifference condition. I now consider each equilibrium candidate to see if it indeed constitutes an equilibrium.

In A^c, $\beta_S = \frac{\rho}{\rho + y(1-\rho)}$ and β_N is undefined, so δ^*_N is also undefined, but as in Proposition 1.1 I assume that $\delta^*_N = 0$. When $\beta_S \geq \frac{1}{2}$, or $\rho \geq \frac{y}{1+y}$, $\delta^*_S = 1$. For the Good agent not to deviate, we need $v^P_G > v^N_G$, which is always satisfied, and $v^P_G > v^{PC}_G$, or $c > C - (1-x)$. For the Bad agent not to deviate, we need $v^P_B > v^N_B$, or $C < y$, and $v^P_B > v^{PC}_B$, or $c > C - (y-x) > C - (1-x)$. Thus, equilibrium A^c can be supported as long as $\rho \geq \frac{y}{1+y}$ and $C < y$ and $c > C - (y-x)$. When $\beta_S < \frac{1}{2}$, it must be that $\delta_S = 0$, but then the Good agent wants to deviate as $v^P_G < v^N_G$, so it cannot be an equilibrium.

In B^c, $\beta_S = \frac{\rho}{\rho + y(1-\rho)\sigma_B}$ and $\beta_N = 0$, so $\delta^*_N = 0$. For the Bad agent to be indifferent, we need $v^P_B = v^N_B$, or $\delta^*_S = \frac{C}{y}$, which requires $C < y$. As the Bad type mixes, we must have $\beta_S = \frac{1}{2}$, or $\sigma^*_B = \frac{\rho}{y(1-\rho)}$. $\sigma^*_B \in (0,1)$ requires that $\rho < \frac{y}{1+y}$. For

the Bad agent not to deviate we need $v_B^P = v_B^N > v_B^{PC}$, or $c > \frac{x}{y}C$. For the Good agent not to deviate we need $v_G^P > v_G^N$, which is always satisfied, and $v_G^P > v_G^{PC}$, or $c > \frac{x-(1-y)}{y}C$, which is always satisfied whenever the stricter condition $c > \frac{x}{y}C$ is satisfied. Thus, equilibrium B^c can be supported as long as $\rho < \frac{y}{1+y}$ and $C < y$ and $c > \frac{x}{y}C$.

In C^c, $\beta_S = 1$ and $\beta_N = 0$, so $\delta_S^* = 1$ and $\delta_N^* = 0$. For the Good agent not to deviate we need $v_G^P > v_G^N$, which is always satisfied, and $v_G^P > v_G^{PC}$, or $c > C-(1-x)$. For the Bad agent not to deviate we need $v_B^N > v_B^P$, or $C > y$, and $v_B^N > v_B^{PC}$, or $c > x$. Thus, equilibrium C^c can be supported as long as $C > y$ and $c > x$.

In E^c, $\beta_S = 1$ and $\beta_N = \frac{\rho(1-\sigma_G)}{\rho(1-\sigma_G)+(1-\rho)}$, so $\delta_S^* = 1$. As the Good agent must be indifferent, we need $v_G^P = v_G^N$, or $\delta_N^* = 1 - C$. But this violates Condition $N0$, so it cannot constitute a relevant equilibrium.

In F^c, $\beta_S = \frac{\rho}{\rho+x(1-\rho)\mu_B}$ and $\beta_N = 0$, so $\delta_N^* = 0$. For the Bad agent to be indifferent, we need $v_B^{PC} = v_B^N$, or $\delta_S^* = \frac{c}{x}$, which requires $c < x$. As the Bad type mixes, we must have $\beta_S = \frac{1}{2}$, or $\mu_B^* = \frac{\rho}{x(1-\rho)}$. $\mu_B^* \in (0,1)$ requires that $\rho < \frac{x}{1+x}$. For the Bad agent not to deviate we need $v_B^{PC} > v_B^P$, or $c < \frac{x}{y}C$. For the Good agent not to deviate we need $v_G^P > v_G^N$, or $c > xC$, and $v_G^P > v_G^{PC}$, or $c > xC$ again. Thus, equilibrium F^c can be supported as long as $\rho < \frac{x}{1+x}$ and $xC < c < \min\{x, \frac{x}{y}C\}$.

In G^c, $\beta_S = \frac{\rho}{\rho+x(1-\rho)}$ and β_N is undefined, so δ_N^* is also undefined, but as in Proposition 1.1 I assume that $\delta_N^* = 0$. When $\beta_S \geq \frac{1}{2}$, or $\rho \geq \frac{x}{1+x}$, the DM admits upon success, $\delta_S^* = 1$. For the Good agent not to deviate we need $v_G^P > v_G^N$, which is always satisfied, and $v_G^P > v_G^{PC}$, or $c > C-(1-x)$. For the Bad agent not to deviate we need $v_B^{PC} > v_B^N$, or $c < x$, and $v_B^{PC} > v_B^P$, or $c < C-(y-x)$. Thus, equilibrium G^c can be supported as long as $\rho \geq \frac{x}{1+x}$ and $c < x$ and $C-(y-x) > c > C-(1-x)$. When $\beta_S < \frac{1}{2}$, it must be that $\delta_S = 0$, but then the Good agent wants to deviate from purchasing the original to not purchasing as $v_G^N > v_G^p$, so it cannot be an equilibrium.

In H^c, $\beta_S = \frac{\rho}{\rho+(1-\rho)(y\sigma_B+x(1-\sigma_B))}$ and β_N is undefined, so δ_N^* is also undefined,

but as in Proposition 1.1 I assume that $\delta_N^* = 0$. For the Bad agent to be indifferent, we need $\upsilon_B^P = \upsilon_B^{PC}$, or $\delta_S^* = \frac{C-c}{y-x}$, which requires $c > C - (y - x)$. As the Bad type mixes, we must have $\beta_S = \frac{1}{2}$, or $\sigma_B^* = \frac{\rho - x(1-\rho)}{(y-x)(1-\rho)}$. $\sigma_B^* \in (0, 1)$ requires that $\frac{x}{1+x} < \rho < \frac{y}{1+y}$. For the Bad agent not to deviate we need $\upsilon_B^P > \upsilon_B^N$, or $c < \frac{x}{y}C$. For the Good agent not to deviate we need $\upsilon_G^P > \upsilon_G^N$, or $c < (1 - (y - x))C$, and $\upsilon_G^P > \upsilon_G^{PC}$, or $(C - c)(1 - y) > 0$, which is always satisfied. $c < (1 - (y - x))C$ is satisfied whenever $c < \frac{x}{y}C$ is satisfied, as we always have $1 - (y - x) > \frac{x}{y}$, or $(y-x)(1-y) > 0$. Thus, equilibrium H^c can be supported as long as $\frac{x}{1+x} \geq \rho < \frac{y}{1+y}$ and $C - (y - x) < c < \frac{x}{y}C$.

In I^c, $\beta_S = \frac{\rho(\sigma_G + x(1-\sigma_G))}{\rho(\sigma_G + x(1-\sigma_G)) + x(1-\rho)}$ and β_N is undefined, so δ_N^* is also undefined, but as in Proposition 1.1 I assume that $\delta_N^* = 0$. For the Good agent to be indifferent, we need $\upsilon_G^P = \upsilon_G^{PC}$, or $\delta_S^* = \frac{C-c}{1-x}$, which requires $c > C - (1 - x)$. As the Good type mixes, we must have $\beta_S = \frac{1}{2}$, or $\sigma_G^* = \frac{x(1-2\rho)}{(1-x)\rho}$. $\sigma_G^* \in (0, 1)$ requires that $\frac{x}{1+x} < \rho < \frac{1}{2}$. For the Good agent not to deviate we need $\upsilon_G^P = \upsilon_G^{PC} > \upsilon_G^N$, or $c < xC$. For the Bad agent not to deviate we need $\upsilon_B^{PC} > \upsilon_B^N$, or $c < xC$ again, and $\upsilon_B^{PC} > \upsilon_B^P$, or $c < xC$ again. Thus, equilibrium I^c can be supported as long as $\frac{x}{1+x} < \rho < \frac{1}{2}$ and $C - (1 - x) < c < xC$.

Proof of Lemma 1.4

As multiple equilibria can only arise where region I^C coincides with other regions, showing the first part of the lemma implies the second part. The region where equilibrium I^c exists is given by $\frac{x}{1+x} < \rho < \frac{1}{2}$ and $C - (1 - x) < c < xC$. The same region is covered by equilibria G^c and H^c when $\frac{x}{1+x} < \rho < \frac{y}{1+y}$ and $y > 1 - x$, and by equilibria A^c and G^c when $\frac{x}{1+x} < \rho < \frac{y}{1+y}$ and $y < 1 - x$, or $\frac{y}{1+y} < \rho < \frac{1}{2}$. I compare the expected payoffs of the DM and both types of agents in these regions to establish weak Pareto-dominance by showing that the expected payoffs in equilibrium I^c are always weakly lower. Calculating the DM's expected payoffs, we obtain $\Pi_{A^c}^* = \rho - y(1 - \rho)$, $\Pi_{G^c}^* = \rho - x(1 - \rho)$ and $\Pi_{H^c}^* = \Pi_{I^c}^* = 0$. Thus $\Pi_{R'}^* \geq \Pi_{I^c}^*$

for $R' \in \{A^c, G^c, H^c\}$. Denote the agent's expected payoff in equilibrium R by $\upsilon^*_\theta(R)$. We have $\upsilon^*_G(A^c) = \upsilon^*_G(G^c) = 1 - C$, $\upsilon^*_G(H^c) = \frac{C-c}{y-x} - C$ and $\upsilon^*_G(I^c) = \frac{xC-y}{1-x}$, as well as $\upsilon^*_B(A^c) = y - C$, $\upsilon^*_B(G^c) = x - c$, $\upsilon^*_B(H^c) = \frac{xC-yc}{y-x}$ and $\upsilon^*_B(I^c) = \frac{xC-c}{1-x}$. Thus, must have $\upsilon^*_G(H^c) > \upsilon^*_G(I^c)$ whenever $(C-c)(1-y) > 0$, which is always true. Moreover, we must have $\upsilon^*_G(A^c) = \upsilon^*_G(G^c) > \upsilon^*_G(I^c)$ whenever $c > C - (1-x)$, which is true by assumption. Thus $\upsilon^*_G(R') > \upsilon^*_G(I^c)$ for $R' \in \{A^c, G^c, H^c\}$. We also must have $\upsilon^*_B(H^c) > \upsilon^*_B(I^c)$ whenever $(C-c)x(1-y) > 0$, which is always true, and $\upsilon^*_B(G^c) > \upsilon^*_B(I^c)$ whenever $c > C - (1-x)$, which is true by assumption. Finally, we must have $\upsilon^*_B(A^c) > \upsilon^*_B(I^c)$, or $c > C - y(1-x)$, because region A^c extends either where $C \leq y - x$, in which case the inequality is satisfied as $c > 0$ is always greater than $-x(1-y) < 0$, or where $y > C > y - x$ and $C < c + (y - x)$, in which the inequality is satisfied as c is always greater than $C - x(1-y)$. Thus $\upsilon^*_B(R') > \upsilon^*_B(I^c)$ for $R' \in \{A^c, G^c, H^c\}$. Consequently, I have shown that every player is at least weakly worse-off in equilibrium I^c, while sometimes some players are strictly worse-off, so equilibrium I^c must be weakly Pareto-dominated.

Proof of Proposition 1.8

For $0 < \rho < \frac{y}{1+y}$, the equilibrium in the baseline model was B for $C < y$, where $\Pi_B = 0$, and C for $C > y$, where $\Pi_C = \rho$. With counterfeits, we need to analyse two disjoint cases: (i) when $0 < \rho < \frac{x}{1+x}$, and (ii) when $\frac{x}{1+x} < \rho < \frac{y}{1+y}$. In (i), the equilibrium for $C < y$ is either B^c or F^c, where $\Pi_{B^c} = \Pi_{F^c} = 0$, so the expected payoff does not change, while for $C > y$, the equilibrium is C^c or F^c, where $\Pi_{C^c} = \rho$ and $\Pi_{F^c} = 0$. So the DM's expected payoff does not change as long as the new equilibrium is C^c, but if falls when the new equilibrium is F^c as $\Pi_{F^c} < \Pi_C$. This happens in region F^c, which requires $xC < c < x$, with the additional condition $C > y$. In (ii), the equilibrium for $C < y$ is B^c, G^c or H^c, where $\Pi_{B^c} = \Pi_{H^c} = 0$ but $\Pi_{G^c} = \rho - x(1-\rho) > \Pi_B$, so the payoff does not change except for the part of region G^c where $C < y$, where it increases. In region G^c,

$C-(1-x) < c < \min\{x, C-(y-x)\}$ is required. For $C > y$, the equilibrium is now C^c or G^c, where $\Pi_{C^c} = \rho = \Pi_C$ and $\Pi_{G^c} = \rho - x(1-\rho) < \Pi_C$, so the DM's expected payoff does not change except in region G^c, which requires $C - (1-x) < c < x$, with the additional condition that $C > y$, where is falls.

For $\frac{y}{1+y} < \rho < 1$, the equilibrium in the baseline model was A for $C < y$, where $\Pi_A = \rho - y(1-\rho)$, and C for $C > y$, where $\Pi_C = \rho$. With counterfeits, the equilibrium for $C < y$ is A^c or G^c, where $\Pi_{A^c} = \rho - y(1-\rho) = \Pi_A$ and $\Pi_{G^c} = \rho - x(1-\rho) > \Pi_A$, so the DM's expected payoff does not change except in region G^c, which requires $C-(1-x) < c < \min\{x, C-(y-x)\}$, with the additional condition that $C < y$, where is rises. For $C > y$, the equilibrium is C^c or G^c, where $\Pi_{C^c} = \rho = \Pi_C$ and $\Pi_{G^c} = \rho - x(1-\rho) < \Pi_C$, so the DM's expected payoff does not change except in region G^c, which requires $C - (1-x) < c < x$, with the additional condition that $C > y$, where is falls.

Proof of Lemma 1.5

Throughout this proof, I use the DM's expected payoffs calculated in the proof of Proposition 1.8. When $c > x$, the possible equilibria are C^c, B^c when $0 < \rho < \frac{y}{1+y}$ and A^c when $\frac{y}{1+y} < \rho < 1$. But $\rho = \Pi_{C^c} > \Pi_{A^c}$ and $\Pi_{C^c} > \Pi_{B^c}$, so the DM always prefers to be in equilibrium C^c. As its payoff in this equilibrium does not depend on C, it may choose any $y < C < 1$. When $c < x$ and $0 < \rho < \frac{x}{1+x}$, the possible equilibria are B^c and F^c. $\Pi_{B^c} = \Pi_{F^c} = 0$, so the choice of C does not matter, but $0 < C < \frac{c}{x}$ for equilibrium existence. When $c < x$ and $\frac{x}{1+x} < \rho < \frac{y}{1+y}$, the possible equilibria are B^c, G^c and H^c. But $\Pi_{B^c} = \Pi_{F^c} < \Pi_{G^c} = \rho - x(1-\rho)$, so the DM always prefers to be in equilibrium G^c. As its payoff in this equilibrium does not depend on C, it may choose any $c + (y-x) < C < c + (1-x)$. Similarly, when $c < x$ and $\frac{y}{1+y} < \rho < 1$, the possible equilibria are A^c and G^c. But $\Pi_{A^c} < \Pi_{G^c} = \rho - x(1-\rho)$, so the DM always prefers to be in equilibrium G^c. As its payoff in this equilibrium does not depend on C, it may choose any $c + (y-x) < C < c + (1-x)$.

Proof of Lemma 1.6

Suppose first that $C \geq y$ and $c \geq x$. It follows from Proposition 1.7 that the DM obtains there a payoff of ρ, which is the maximum possible in this game without committing to ignorance, i.e. by setting $\xi = 1$; commitment to ignorance may only lower this payoff. In what follows, assume that $C < y$ or $c < x$.

Suppose that $c \geq xC$ and $\xi \leq \bar{\xi} = \min\{\frac{C}{y}, \frac{c}{x}\}$. The Bad agent's expected payoffs are $\upsilon_B^P(\xi) = \xi y\delta_S - C + (1-\xi)\delta_I$, $\upsilon_B^{PC}(\xi) = \xi x\delta_S - c + (1-\xi)\delta_I$ and $\upsilon_B^N(\xi) = (1-\xi)\delta_I$. Thus, the Bad agent purchases the original rather than nothing only if $\upsilon_B^P(\xi) > \upsilon_B^N(\xi)$, or $\xi > \frac{C}{y\delta_S} \geq \frac{C}{y}$, where the last inequality follows from the fact that $\delta_S \in [0,1]$. But this is not possible, as we assumed that $\xi \leq \min\{\frac{C}{y}, \frac{c}{x}\} \leq \frac{C}{y}$, so the Bad agent never purchases the original. By similar reasoning, $\upsilon_B^{PC}(\xi) > \upsilon_B^N(\xi)$ only when $\xi > \frac{c}{x\delta_S} \geq \frac{c}{x}$, while we assumed that $\xi \leq \min\{\frac{C}{y}, \frac{c}{x}\} \leq \frac{c}{x}$, so neither does the Bad agent prefer to purchase the counterfeit. Consequently, the Bad type chooses not to purchase at all, $(\sigma_B^*, \mu_B^*) = (0,0)$. As no Bad agents ever purchase anything, the DM's inference upon a success must be $\beta_S^* = 1$ and thus $\delta_S^* = 1$. The Good agent's expected payoffs are $\upsilon_G^P(\xi) = \xi\delta_S - C + (1-\xi)\delta_I$, $\upsilon_G^{PC}(\xi) = \xi x\delta_S - c + (1-\xi)\delta_I$ and $\upsilon_G^N(\xi) = (1-\xi)\delta_I$. The Good agent purchases the counterfeit rather than the original only if $\upsilon_G^{PC}(\xi) > \upsilon_G^P(\xi)$, or $\xi < \frac{C-c}{\delta_S^*(1-x)} = \frac{C-c}{1-x}$. When $c > \frac{x}{y}C$, $\bar{\xi} = \frac{C}{y}$, but then $\frac{C}{y} < \frac{C-c}{1-x}$ is equivalent to $c < (\frac{x}{y} - \frac{1-y}{y})C < \frac{x}{y}C$, which contradicts $c > \frac{x}{y}C$. When $c < \frac{x}{y}C$, $\bar{\xi} = \frac{c}{x}$ but then $\frac{c}{x} < \frac{C-c}{1-x}$ is equivalent to $c < xC$, which contradicts the initial assumption. Thus, the Good agent never prefers to purchase the counterfeit rather than the original. The Good agent prefers to purchase nothing rather than the original only if $\upsilon_G^N(\xi) > \upsilon_G^P(\xi)$, or $\xi < \frac{C}{\delta_S^*} = C$. But this is never true when $\bar{\xi} = \frac{C}{y}$ and would only be true when $\bar{\xi} = \frac{c}{x}$ if $c < xC$, which again contradicts the initial assumption. Thus, the Good agent never prefers not purchasing to purchasing the original. Consequently, the Good agent always prefers to purchase the original to other alternatives, $(\sigma_G^*, \mu_G^*) = (1,0)$. Consequently, the DM's expected payoff is $\mathbb{E}[v(\Phi^*, \delta^*; \xi)] = \xi(\delta_S^*(\rho(\sigma_G^* + x\mu_G^*) - (1-\rho)(y\sigma_B^* + x\mu_B^*))) + (1-\xi)\delta_I^*(\rho - (1-\rho)) =$

$\xi\rho+(1-\xi)\mathbb{1}_{\rho>\frac{1}{2}}(2\rho-1)$, which strictly increases in ξ. For $\rho<\frac{1}{2}$ this is straightforward as the second summand disappears, while for $\rho>\frac{1}{2}$ this follows from the fact that the derivative with respect to ξ is $\rho-(2\rho-1)=1-\rho>0$. Thus the DM would like to set ξ as high as possible, so the optimal choice in this case is $\xi^*=\bar{\xi}$.[17] Except the knife-edge cases, the payoff is always strictly positive.

Suppose that $c<xC$ and $\xi\leq\bar{\xi}$. Then the Good agents prefer to purchase the counterfeit rather than the original, as explained above, and by Lemma 1.11 this precludes the Bad types from purchasing the originals. Consequently, nobody purchases the original, which contradicts the assumption of the model.

Suppose that $\bar{\xi}<\xi\leq\hat{\xi}=\frac{C-c}{y-c}$. Suppose also that $c<\frac{x}{y}C$ which guarantees that such a range exists and that $c\leq C-(y-x)$, so that $\hat{\xi}\leq 1$, and solve for the equilibrium strategies of the agents and the DM. As $c<\frac{x}{y}C$, we have $\bar{\xi}=\frac{c}{x}$. The Bad agent purchases the original rather than the counterfeit only if $\upsilon_B^P(\xi)>\upsilon_B^{PC}(\xi)$, or $\xi>\frac{C-c}{\delta_S(y-x)}\geq\frac{C-c}{y-x}$, where the last inequality follows from the fact that $\delta_S\in[0,1]$. But this is not possible, as we assumed that $\xi\leq\frac{C-c}{y-x}$, so the Bad agent never purchases the original. Thus $\sigma_B=0$, and by Lemma 1.11 and Condition $N0$ the only possible equilibria that are not Pareto-dominated are (i) $\Phi=(1,0,0,1)$, (ii) $\Phi=(1,0,0,\iota)$, (iii) $\Phi=(1,0,0,0)$, where $\iota\in(0,1)$. We must have $\beta_S\geq\frac{1}{2}$ in any equilibrium, as otherwise $\delta_S=0$, so not even the Good types purchase the original, which contradicts the assumption that it is purchased at least sometimes.

Consider (i). Then $\beta_S=\frac{\rho}{\rho+x(1-\rho)}$ and we must have $\beta_S>\frac{1}{2}$, or $\rho>\frac{x}{1+x}$ as we exclude the knife-edge case. Then $\delta_S^*=1$. The Bad agent indeed prefers to purchase the counterfeit rather than nothing only if $\upsilon_B^{PC}(\xi)>\upsilon_B^N(\xi)$, or $\xi>\frac{c}{x}$, which is always true for $\xi>\bar{\xi}$, and rather than the original only if $\upsilon_B^{PC}(\xi)>\upsilon_B^O(\xi)$, or $\xi<\frac{C-c}{y-x}$, which is always true as $\xi<\hat{\xi}$. The Good agent prefers to purchase the original rather than nothing only if $\upsilon_G^P(\xi)>\upsilon_G^N(\xi)$, or $\xi>C$, which is always true for $\xi>\bar{\xi}$, and rather than the counterfeit only if $\upsilon_G^P(\xi)>\upsilon_G^{PC}(\xi)$, or $\xi>\frac{C-c}{1-x}$. As we have $\frac{C-c}{1-x}<\frac{C-c}{y-x}$,

[17] As usual, the more precise expression would be that the DM chooses $\xi^*=\min\{\frac{C}{y},\frac{c}{x}\}-\epsilon$, where ϵ is small.

there exists an equilibrium here for $\xi \in (\frac{C-c}{1-x}, \frac{C-c}{y-x})$. The DM's expected payoff is $\mathbb{E}[v(\Phi^*, \delta^*; \xi)] = \xi(\delta_S^*(\rho(\sigma_G^* + x\mu_G^*) - (1-\rho)(y\sigma_B^* + x\mu_B^*))) + (1-\xi)\delta_I^*(\rho - (1-\rho)) = \xi(\rho - x(1-\rho)) + (1-\xi)\mathbb{1}_{\rho > \frac{1}{2}}(2\rho - 1)$, which strictly increases in ξ as before. Thus the DM would like to set ξ as high as possible, so the optimal choice in this case is $\xi^* = \hat{\xi}$. Except the knife-edge cases, the payoff is always strictly positive.

Consider (ii). Then $\beta_S = \frac{\rho}{\rho + x(1-\rho)\mu_B}$. When $\beta_S > \frac{1}{2}$, we would have $\delta_S = 1$, but then the Bad agent would indeed only mix when $v_B^{PC}(\xi) = v_B^N(\xi)$, or $\xi = \frac{c}{x}$, which is a contradiction. Thus we must have $\beta_S = \frac{1}{2}$, or $\mu_B = \frac{\rho}{(1-\rho)x}$, and $\mu_B \in (0,1)$ only if $\rho < \frac{x}{1+x}$. Then for the Bad type to mix between purchasing the counterfeit and not purchasing we must have $v_B^{PC}(\xi) = v_B^N(\xi)$, or $\delta_S = \frac{c}{\xi x}$, and $\delta_S \in (0,1)$ only if $\xi > \frac{c}{x}$, which is always satisfied. For the Bad agent not to prefer to purchase the original, we must have $v_B^{PC}(\xi) = v_B^N(\xi) > v_B^P(\xi)$, or $c < \frac{x}{y}C$. For the Good agent to prefer choosing the original rather than the counterfeit, we need $v_G^P(\xi) > v_G^{PC}(\xi)$, and rather than nothing, we need $v_G^P(\xi) > v_G^N(\xi)$, which both translate to $c > xC$. Thus there exists an equilibrium here for any $\xi \in (\frac{c}{x}, \frac{C-c}{y-x})$. The DM's expected payoff is $\mathbb{E}[v(\Phi^*, \delta^*; \xi)] = \xi(\delta_S^*(\rho(\sigma_G^* + x\mu_G^*) - (1-\rho)(y\sigma_B^* + x\mu_B^*))) + (1-\xi)\delta_I^*(\rho - (1-\rho)) = (1-\xi)\mathbb{1}_{\rho > \frac{1}{2}}(2\rho - 1)$. But as $\rho < \frac{x}{1+x} < \frac{1}{2}$, the payoff is always zero here, therefore it cannot be ever an improvement on the baseline payoff.

Consider (iii). Then $\beta_S = 1$ and $\beta_N = 0$, so we must have $\delta_S = 1$ and $\delta_N = 0$. But the Bad type indeed prefers to purchase nothing rather than the counterfeit only if $v_B^N(\xi) > v_B^{PC}(\xi)$, or $\xi < \frac{c}{x}$, which is a contradiction as $\xi \geq \bar{\xi}$.

Suppose finally that $\hat{\xi} < \xi \leq 1$ for $c < \frac{x}{y}C$ or $\bar{\xi} < \xi \leq 1$ if $c > \frac{x}{y}C$. As the behaviour of the Bad agent is unchanged compared to the baseline model, all the original equilibria from Proposition 1.7 are possible here, except C^c, which was considered before, and I^c, which is Pareto-dominated again. Similar to Lemma 1.3, in all of these equilibria the DM does not manage to change the agents' behaviour, so they can exist for the same combinations of parameter values as before. The DM's expected payoff is always an increasing function of ξ, and in no equilibrium

is there an upper bound on the value of ξ. Thus, the DM sets the highest possible value, $\xi = 1$, where the payoff is as in the baseline model.

It remains to show, that the DM's expected payoff from committing to ignorance with $\xi = \frac{C}{y}$, $\xi = \frac{c}{x}$ or $\xi = \frac{C-c}{y-x}$ may indeed be the highest obtainable for a given set of parameters. Consider $\rho < \frac{x}{1+x}$. The DM's payoff in baseline equilibrium B^c is zero, while everywhere in this region the DM can obtain a positive expected payoff by committing to $\xi = \frac{C}{y}$, and committing to other values of ξ does not lead to an equilibrium there. Similarly, the DM's payoff in baseline equilibrium F^c is zero, while everywhere in this region the DM can obtain a positive expected payoff by committing to $\xi = \frac{c}{x}$, and committing to other values of ξ does not lead to an equilibrium there. Finally, consider $\frac{x}{1+x} < \rho < \frac{y}{1+y}$. The DM's payoff in the baseline equilibrium H^c is zero, while in the part of this region for which $c < xC$, the DM can obtain a positive expected payoff by committing to $\xi = \frac{C-c}{y-x}$, and committing to other values of ξ does not lead to an equilibrium there. Thus whenever $\xi = 1$ is not optimal, $\xi \in \{\frac{C}{y}, \frac{c}{x}, \frac{C-c}{y-x}\}$ must be optimal.

Proof of Lemma 1.7

The optimal level of attention for the DM, whenever it finds it optimal to discourage the Bad types from purchasing, is $\xi^* = \min\{\frac{C}{y}, \frac{c}{x}\}$. Whenever $c > \frac{x}{y}C$, it is optimal to set $\xi^* = \frac{C}{y}$, and in the resulting equilibrium the expected payoff of the Good agents from following the equilibrium strategy is $\upsilon_G^P(\xi^*) = \delta_S^*\xi^* + (1-\xi^*)\delta_I^* - C = C\frac{1-y}{y} + (1-\frac{C}{y})\delta_I^*$, which is always greater than the payoff from switching to not purchasing, where the expected payoff is $\upsilon_G^N(\xi^*) = \xi^*\delta_N^* + (1-\xi^*)\delta_I^* = (1-\frac{C}{y})\delta_I^*$. It is also greater than the payoff from purchasing the counterfeit, $\upsilon_G^{PC}(\xi^*) = x\delta_S^*\xi^* + (1-\xi^*)\delta_I^* - c = x\frac{C}{y} - c + (1-\frac{C}{y})\delta_I^*$, whenever $c > \frac{x-(1-y)}{y}C$, which is always satisfied when $c > \frac{x}{y}C$ is satisfied. Whenever $c < \frac{x}{y}C$, it is optimal to set $\xi^* = \frac{c}{x}$, and in the resulting equilibrium the expected payoff of the Good agents from following the equilibrium strategy is $\upsilon_G^P(\xi^*) = \delta_S^*\xi^* + (1-\xi^*)\delta_I^* - C = \frac{c}{x} - C + (1-\frac{C}{y})\delta_I^*$,

which is always greater than the payoff from switching to purchasing the counterfeit, $\upsilon_G^{PC}(\xi^*) = x\delta_S^*\xi^* + (1-\xi^*)\delta_I^* - c = (1-\frac{C}{y})\delta_I^*$ It is also greater than the payoff from switching to not purchasing, $\upsilon_G^N(\xi^*) = \xi^*\delta_N^* + (1-\xi^*)\delta_I^* = (1-\frac{C}{y})\delta_I^*$, but only if $\frac{c}{x} - C > 0$, or $c > xC$. If this condition is not met, the Good type prefers not to purchase at all, so no desired separation can be obtained.

Proof of Lemma 1.8

The DM's expected payoff is zero in baseline regions B^c, F^c and H^c as per Proposition 1.8. The expected payoffs that the DM can obtain by committing to the optimal ignorance level, $\xi = \frac{C}{y}$, $\xi = \frac{c}{x}$ or $\xi = \frac{C-c}{y-x}$, are always strictly positive, and they collectively cover the same set of parameters that the baseline set $B^c \cup F^c \cup H^c$ covers. Consequently, in these regions the DM prefers to commit to ignorance rather than to set $\xi = 1$.

Proof of Lemma 1.9

The proof proceeds in two parts. First, I establish in which regions more than one equilibrium exists for $\xi \in \{\frac{C}{y}, \frac{c}{x}, \frac{C-c}{y-x}\}$. Whenever an equilibrium exists for only one such value of ξ, only that value of ξ may be quasi-optimal. Second, I compare the DM's expected payoffs in the co-existing equilibria and derive the conditions under which the DM prefers each value of ξ.

For the equilibrium in which $\xi = \frac{C}{y}$ to exist, we require that $c > \frac{x}{y}C$, for the equilibrium in which $\xi = \frac{c}{x}$ to exist, we require that $\frac{x}{y}C > c > xC$, and for the equilibrium in which $\xi = \frac{C-c}{y-x}$ to exist, we require that $\frac{x}{y}C > c > C-(y-x)$. Thus, for $c > \frac{x}{y}C$, whenever $\xi < 1$ is optimal, it must be the case that $\xi = \frac{C}{y}$. For $\rho < \frac{x}{1+x}$ and $\frac{x}{y}C > c > xC$, or $\rho \geq \frac{x}{1+x}$ and $C-(y-x) > c > xC$ and $c < x$, whenever $\xi < 1$ is optimal, it must be the case that $\xi = \frac{c}{x}$. For $\rho \geq \frac{x}{1+x}$ and $C - xC > c > C-(y-x)$ and $c < x$, whenever $\xi < 1$ is optimal, it must be the case that $\xi = \frac{C-c}{y-x}$.

The only region where more than one equilibrium may exist is $\rho \geq \frac{x}{1+x}$ and $\frac{x}{y}C >$

$c > \max\{xC, C-(y-x)\}$, where we may have either $\xi = \frac{c}{x}$ or $\xi = \frac{C-c}{y-x}$. When $\rho \leq \frac{1}{2}$, the relevant expected payoffs are $\Pi(\xi = \frac{c}{x}) = \frac{c}{x}\rho$ and $\Pi(\xi = \frac{C-c}{y-x}) = \frac{C-c}{y-x}(\rho - x(1-\rho))$, and the latter is greater than the former if and only if $c < C\frac{x(\rho-x(1-\rho))}{y\rho-x^2(1-\rho)} \equiv \hat{c}(C)$. But this is only true when $\rho > \frac{x(1-x)}{1-y+x(1-x)}$, because when $\rho < \frac{x(1-x)}{1-y+x(1-x)}$, we have $\hat{c}(C) < xC$, so for all the points in the region where the two equilibria coexist $\xi = \frac{c}{x}$ is optimal. When $\rho > \frac{1}{2}$, the relevant expected payoffs are $\Pi(\xi = \frac{c}{x}) = \frac{c}{x}\rho + (1-\frac{c}{x})(2\rho-1)$ and $\Pi(\xi = \frac{C-c}{y-x}) = \frac{C-c}{y-x}(\rho - x(1-\rho)) + (1-\frac{C-c}{y-x})(2\rho-1)$, and the latter is greater than the former if and only if $c < C\frac{x(1-x)}{y-x^2}$. But this is only true when $y < 1 - x(1-x)$, because when $y > 1 - x(1-x)$, we have $C\frac{x(1-x)}{y-x^2} < xC$, so for all the points in the region where the two equilibria coexist $\xi = \frac{c}{x}$ is optimal. Whenever $y > 1-x(1-x)$, we must have $\frac{x(1-x)}{1-y+x(1-x)} > \frac{1}{2}$, which implies that it must always be the case that $\xi = \frac{c}{x}$ is preferred. We also have $\frac{d\hat{c}(C)}{d\rho} > 0$ and $\lim_{\rho\to\frac{1}{2}} \hat{c}(c) = C\frac{x(1-x)}{y-x^2}$, so when $y < 1 - x(1-x)$, $\xi = \frac{c}{x}$ is preferred when $c > C\min\{\frac{x(\rho-x(1-\rho))}{y\rho-x(1-\rho)}, \frac{x(1-x)}{y-x^2}\}$, and $\xi = \frac{C-c}{y-x}$ is preferred otherwise.

Proof of Proposition 1.9

Consider region A^c first, which requires that $\rho > \frac{y}{1+y}$ for existence. The payoff in the baseline model, or when $\xi = 1$, is $\Pi^*_{A^c} = \rho - y(1-\rho)$. By Lemma 1.9, in the part where $c > \frac{x}{y}C$, the maximum payoff that can obtained by committing to ignorance by setting $\xi^* = \frac{C}{y}$ is $\Pi^*_{\xi=\frac{C}{y}} = \frac{C}{y}\rho$ for $\frac{y}{1+y} < \rho < \frac{1}{2}$ and $\Pi^*_{\xi=\frac{C}{y}} = \frac{C}{y}\rho + (1-\frac{C}{y})(2\rho-1)$ for $\frac{1}{2} < \rho < 1$. Thus, when $\frac{y}{1+y} < \rho < \frac{1}{2}$, we have $\Pi^*_{\xi=\frac{C}{y}} > \Pi^*_{A^c}$ whenever $C > \frac{y}{\rho}(\rho - y(1-\rho)) \equiv \hat{C}(\rho)$, and when $\frac{1}{2} < \rho < 1$, we have $\Pi^*_{\xi=\frac{C}{y}} > \Pi^*_{A^c}$ whenever $C > y - y^2$. As $\frac{d\hat{C}(\rho)}{d\rho} = (\frac{y}{\rho})^2 > 0$ and $\lim_{\rho\to\frac{1}{2}} \hat{C}(\rho) = y - y^2$, the condition for optimality of committing to ignorance with $\xi^* = \frac{C}{y}$ can be restated as $C > \min\{\frac{y}{\rho}(\rho - y(1-\rho)), y - y^2\}$.

In the part where $xC < c < \frac{x}{y}C$, the maximum payoff that can obtained by committing to ignorance by setting $\xi^* = \frac{c}{x}$ is $\Pi^*_{\xi=\frac{c}{x}} = \frac{c}{x}\rho$ for $\frac{y}{1+y} < \rho < \frac{1}{2}$ and $\Pi^*_{\xi=\frac{c}{x}} = \frac{c}{x}\rho + (1-\frac{c}{x})(2\rho-1)$ for $\frac{1}{2} < \rho < 1$. Thus, when $\frac{y}{1+y} < \rho < \frac{1}{2}$, we

have $\Pi^*_{\xi=\frac{c}{x}} > \Pi^*_{A^c}$ whenever $c > \frac{x}{\rho}(\rho - y(1-\rho))$, and when $\frac{1}{2} < \rho < 1$, we have $\Pi^*_{\xi=\frac{c}{x}} > \Pi^*_{A^c}$ whenever $c > x - xy$. By a similar reasoning as before, the condition for optimality of committing to ignorance with $\xi^* = \frac{c}{x}$ can be restated as $c > \min\{\frac{x}{\rho}(\rho - y(1-\rho)), x - xy\}$. We also need to guarantee that $\Pi^*_{\xi=\frac{c}{x}} \geq \Pi^*_{\xi=\frac{C-c}{y-x}}$, which by Lemma 1.9 happens when $c > \max\{xC, C\min\{\frac{x(\rho - x(1-\rho))}{y\rho - x(1-\rho)}, \frac{x(1-x)}{y-x^2}\}\}$. Combining all these conditions yields the result.

Finally, in the part where $C - (y-x) < c < \frac{x}{y}C$, the maximum payoff that can obtained by committing to ignorance by setting $\xi^* = \frac{C-c}{y-x}$ is $\Pi^*_{\xi=\frac{C-c}{y-x}} = \frac{C-c}{y-x}(\rho - x(1-\rho))$ for $\frac{y}{1+y} < \rho < \frac{1}{2}$ and $\Pi^*_{\xi=\frac{C-c}{y-x}} = \frac{C-c}{y-x}(\rho - x(1-\rho)) + (1 - \frac{C-c}{y-x})(2\rho - 1)$ for $\frac{1}{2} < \rho < 1$. Thus, when $\frac{y}{1+y} < \rho < \frac{1}{2}$, we have $\Pi^*_{\xi=\frac{C-c}{y-x}} > \Pi^*_{A^c}$ whenever $C > \frac{\rho - y(1-\rho)}{\rho - x(1-\rho)}(y-x)$ and $c < C - \frac{\rho - y(1-\rho)}{\rho - x(1-\rho)}(y-x)$, and when $\frac{1}{2} < \rho < 1$, we have $\Pi^*_{\xi=\frac{C}{y}} > \Pi^*_{A^c}$ whenever $C > \frac{1-y}{1-x}(y-x)$ and $c < C - \frac{1-y}{1-x}(y-x)$. By a similar reasoning as before, the conditions for optimality of committing to ignorance with $\xi^* = \frac{C-c}{y-x}$ can be restated as $C > \min\{\frac{\rho - y(1-\rho)}{\rho - x(1-\rho)}(y-x), \frac{1-y}{1-x}(y-x)\}$ and $c < C - \min\{\frac{\rho - y(1-\rho)}{\rho - x(1-\rho)}(y-x), \frac{1-y}{1-x}(y-x)\}$. We also need to guarantee that $\Pi^*_{\xi=\frac{C-c}{y-x}} \geq \Pi^*_{\xi=\frac{c}{x}}$, which by Lemma 1.9 happens when $c < \max\{xC, C\min\{\frac{x(\rho - x(1-\rho))}{y\rho - x(1-\rho)}, \frac{x(1-x)}{y-x^2}\}\}$. Combining all these conditions yields the result.

Consider now region G^c, which requires that $\rho > \frac{x}{1+x}$ for existence. The payoff in the baseline model, or when $\xi = 1$, is $\Pi^*_{G^c} = \rho - x(1-\rho)$. Lemma 1.9 established that if ignorance were to be optimal, the DM would have to set $\xi = \frac{c}{x}$, which requires $c > xC$. Thus, when $\frac{x}{1+x} < \rho < \frac{1}{2}$, we have $\Pi^*_{\xi=\frac{c}{x}} > \Pi_{G^c}$ whenever $c > \frac{x}{\rho}(\rho - x(1-\rho))$, and when $\frac{1}{2} < \rho < 1$, we have $\Pi^*_{\xi=\frac{c}{x}} > \Pi_{G^c}$ whenever $c > x - x^2$. By a similar reasoning as before, the condition for optimality of committing to ignorance with $\xi^* = \frac{c}{x}$ can be restated as $c > \min\{\frac{x}{\rho}(\rho - x(1-\rho)), x - x^2\}$, or incorporating the condition which guaranteed that the Good types purchase, as $c > \max\{xC, \min\{\frac{x}{\rho}(\rho - x(1-\rho)), x - x^2\}\}$.

Chapter 2

Product Differentiation with Status Effects

2.1 Introduction

As societies become more affluent, a rising share of incomes is assigned to fulfilling needs that rank higher in Maslow's hierarchy. Notably, the desire for esteem, or social status, becomes more prominent. Unlike in the past, today social status is very closely linked to the material status of a person. Yet bragging in public about one's wealth is neither seen as credible nor in good taste. This creates room for positional goods: objects, such as designer clothes or sports cars, that are desired not because of their intrinsic value to the user, but because they serve as credible signals about their owner's material, and thus social standing.

In certain cases, such as with the amount of money pledged to charity, it is the quantity of the positional good consumed that matters. In other cases, however, a model reliant on quantity does not seem to provide an accurate description of reality: to wear more than one Rolex may be considered too much. Even then, however, Bagwell and Bernheim (1996) argue that a model of competition in quantities may apply: we can observe e.g. that someone wears a different expensive watch every

day, which indicates that they can afford to spend a lot of money on luxuries. However, their argument becomes less convincing when interactions between agents are rare, such that few conspicuous goods are observed, or when agents cannot afford acquiring a whole stable of conspicuous goods. Under such circumstances, the choice of the type (brand or model) of a watch or a car, rather than the quantity, becomes the key decision with status implications.

To illustrate this point, consider the Armani brand. This well-known maker of fashionable suits is in fact a collection of brands that cater to various target groups. The most expensive and prestigious amongst these is the Giorgio Armani brand, which consists of signature Armani suits and targets consumers aged 35 to 50. Next, there is Armani Collezioni, which aims for a slightly lower market segment of people who want to wear Armani apparel but cannot afford the priciest Giorgio Armani brand. Cheaper still there is Emporio Armani targeting the young professional segment aged 25 to 35. Finally, the lowest range available is Armani Jeans, catering to young adults in the 18 to 30 age group. All garments are made of high-quality fabrics, with arguably imperceptible differences in quality for a significant majority of the population (except perhaps fashion experts). There are however considerable differences in price: the signature Armani suits can be up to 20% more expensive than those from Armani Collezioni.[1] Such market behaviour, i.e. offering a variety of products with little differences in quality but large differences in price, is not confined to Armani. Other famous clothing companies such as Hugo Boss or Tommy Hilfiger also produce several brands, as do companies in other sectors that involve highly conspicuous goods, such as perfume (Calvin Klein, Dior), shoes (Prada, Jimmy Choo), bags (Louis Vuitton), watches (Rolex) or even alcohol (Johnny Walker).

What we observe is a situation where positive demand exists for the pricier of the *ex-ante* identical goods. The literature on vertical differentiation, in which demand for a variety of products may simultaneously be positive, explains such

[1]This description comes from the Marketing91 blog: http://www.marketing91.com/marketing-mix-armani/

a phenomenon by the presence of some intrinsic differences between the products offered. But this cannot be the right explanation here: some consumers deliberately choose the more expensive brand for the sole reason of demonstrating to others that they can afford it, and that by extension their material status is high. Such behaviour fulfils the definition of the Veblen effect: the willingness to pay for the good increases with price.

In order to examine the phenomenon of status goods more closely I build a model where products are *ex-ante* identical in all ways but in price, but turn out to be differentiated *ex post*. A discrete number of brands are offered in the market and agents choose the brand which maximises their utility.[2] However, the utility is not obtained directly from the consumption of the positional good, but rather indirectly from the social status whose determination depends on the chosen brand. Such positional goods thus allow agents who want to be perceived as high-income to signal so indirectly, as their incomes are private information unobservable by others.

The nature of the positional competition in this model is different to what is frequently assumed. The usual status game is a zero-sum game (Frank 1985; Hopkins and Kornienko 2004) because agents care about their rank in the distribution of positional good consumption. Whenever one agent rises by a few spots, some other agent's standing has to fall. In this model, however, status is achieved through imperfect transmission of information similar to the cheap talk models (Crawford and Sobel 1982): as such, smoother revelation of some agent's income may raise his status without lowering the status of other agents. Such a nature of the status rivalry will be the key driver behind the welfare conclusions: the positional externality may be a way of correcting the imperfect information externality and thus constitute a welfare improvement. Moreover, the status aspect makes this model different from the standard vertical differentiation models, e.g. Itoh (1983) or Gabszewicz, Shaked, et al. (1986), where the monopolist can independently choose prices and qualities.

[2] 'Brands' and 'positional goods' will be used interchangeably in this paper.

Here, the vector of prices directly determines the vector of qualities. This aspect needs to be considered by the decision maker when the profit maximisation problem is solved.

In Section 2.2, I develop the model which fulfils the requirements described above. There is a continuum of agents with different incomes who care about status and the consumption of the numéraire good. Status is determined indirectly by the choice of the purchased positional good and the agents' desire for status increases with their wealth. I look at various market structures to examine the equilibrium outcomes and welfare consequences.

In Section 2.3, I begin by considering a monopolist who can produce at most a single brand. While the introduction of a positional good lowers consumer surplus, it weakly improves overall welfare. Whether the monopolist's price choice is optimal from the social point of view depends on whether even the poorest member of society cares about status. Only if he does may a Pigouvian per-unit tax be welfare improving (but not Pareto improving). Moreover, it may be socially beneficial when the marginal cost of production is high. Appendix 2.A assumes a different, triangular distribution of incomes to demonstrate how an increase in inequality may hurt the consumers while improving the overall welfare.

The model is then generalised to a case of N brands in Section 2.4. The monopolist always benefits from the introduction of an additional brand. In fact, if it were allowed to offer a continuum of brands, it would offer the full range so that no bunching of consumers occurs. Whenever a new brand is introduced by the monopolist, there may be agents who benefit, but there are always losers, in particular at the bottom and at the top of the income distribution. However, the marginal profit from an additional brand is diminishing, so in the presence of a semi-fixed cost of brand introduction the number of brands on offer is finite. Moreover, the monopolist always offers too much variety compared to the social optimum and when even the poorest agent cares about status, the monopolist over-supplies each brand. As

above, the same Pigouvian per-unit tax imposed on each brand will improve welfare. Another explanation for why the number of brands observed in the market is usually finite is that the marginal cost of the higher-priced brand is higher than the marginal cost of the lower-priced brand. In such cases, it may be optimal for the monopolist to restrict its product offering to only one brand, provided that the cost difference is sufficiently large.

In Section 2.5, I turn to two duopolists competing in the market by producing one brand each. The key difference between this strategic game and the monopolist's maximisation problem is that it may no longer be assumed that the income thresholds are the choice variables. Consequently, depending on the prices chosen, it may be the case that the agents end up buying: (i) nothing, the cheaper good or the more expensive good; (ii) nothing or the cheaper good; (iii) nothing or the more expensive good; (iv) nothing. I show that a PBE of a simultaneous game does not exist due to an "over-cutting" argument akin to the standard Bertrand game. However, when players enter sequentially, an equilibrium does exist. The first mover chooses to produce the high-priced brand but fails to obtain an advantage over the second mover. Such a strategy is optimal because it prevents the other firm from deviating to "over-cutting". Not only are the prices of both brands higher under oligopoly than under monopoly, but the latter is the preferred outcome from the consumer surplus and total welfare perspective too. Another way to obtain an equilibrium is to introduce asymmetry between the firms, e.g. in the form of different marginal costs. To this end, I relabel the firms as the low-cost producer, with a marginal cost still equal to zero, and the high-cost producer, with a marginal cost higher than zero. I find that if the difference in costs is sufficiently large, a simultaneous-entry equilibrium may exist, where the lower cost firm enjoys a significant advantage. The consumer surplus and total welfare perspectives again favour the monopolist, who would choose to produce only the low-cost brand in identical circumstances.

On a final note I should warn the reader that my model is somewhat stylised and

therefore its conclusions, in particular relating to welfare, should be taken with caution. Most problematically, the most advanced cases developed in this chapter are the multi-brand monopolist and two duopolists producing one brand each. Reality closer resembles the overlap of these two: there are multiple producers who compete in the market by each producing a varying number of brands. In the aforementioned market for clothing, Armani's five brands face competition from four brands of Hugo Boss (BOSS, BOSS Orange, BOSS Green, HUGO), four brands of Versace (Versace, Versace Collection, Versus Versace, Versace Jeans) and so on. However, such a detailed model would be intractable, so I will seek to characterise the simpler baseline models to obtain an intuition of the outcomes in more realistic settings.

2.1.1 Related Work

While no existing paper directly addresses product differentiation in the context of positional goods, there is an extensive body of related literature. Corneo and Jeanne's (1997) setup is perhaps closest to the model of this chapter. In their model, there is a continuum of agents ordered by income, who make a decision about whether or not to purchase the positional good. Only one variety of the positional good is available at a fixed price, and either zero or one unit may be consumed. The semi-separating PBE depends crucially on whether the behaviour of consumers is conformist (utility from the conspicuous good grows when it is more widely consumed) or snobbish (utility from purchasing the good is enhanced by its uncommonness). My model bears greater resemblance to the snob effect. However, Corneo and Jeanne's (1999) specification depends on rank (rather than average income) and thus their game is zero-sum, which is why their welfare conclusions are often different from mine. A similar approach to Corneo and Jeanne (1997) is used by Grilo et al. (2001), who find that when two firms compete for snobbish consumers, it makes sense for them to locate at the two ends of the quality spectrum to suppress price competition.

For my model I follow Bagwell and Bernheim (1996) in taking a more direct approach by explicitly assuming status considerations in the utility function, which results in positional competition in conspicuous consumption. They construct two groups of agents, high and low income, who care about their perceived status. Status is determined indirectly by social contacts - observers, who form inferences based on the type of positional good purchased so as to minimise their loss function which depends on the error of judgement. The best benchmark for my one-brand model is when consumer choice in Bagwell and Bernheim (1996) is restricted by fixing the quality and quantity, and thus leaving to the consumers only the decision of whether or not to buy. In such a situation, the Veblen effect arises in equilibrium, similar to this model. Other models of positional competition, e.g. Ireland (1994) or Frank (1985), provide additional comparisons, but their usefulness is limited by their assumption that agents compete in quantities.

Standard product differentiation literature, in which both price and quality are choice variables, provides valuable benchmarks against which my model can be evaluated. My model diverges from such an approach in that the monopolist or the duopolists cannot choose prices separately from qualities, which are indirectly determined by the prices. In their seminal paper, Mussa and Rosen (1978) consider a monopolist that produces a continuum of qualities. Agents differ by their intensity of taste for quality and only consume at most one unit of the good. The differences in valuations of quality limit the consumer surplus that may be extracted by the monopolist from those who value quality more highly. The monopolist internalises these "upstream" externalities by inducing the less enthusiastic consumers to buy lower quality items. The assumption of an unlimited number of qualities is relaxed by Itoh (1983), who considers a coarser version of product differentiation: there is a finite number of products and the type space is divided into bands of consumers, which buy different qualities (or nothing). He establishes that for an arbitrary distribution of tastes, the emergence of a new product does not impact the prices of

the lower-quality products but changes the prices of the higher-quality products in the same direction. A more accurate benchmark for my model, however, is provided by Gabszewicz, Shaked, et al. (1986), who use a multiplicative (between quality and after-purchase income) utility function and a uniform income distribution. They demonstrate that, under suitable conditions, only one firm can survive in the market as well as that it will produce the maximum number of brands that is allowed. Their conclusions depend heavily on the assumption that the maximum quality is exogenously fixed. I provide a more detailed comparison against their paper in the concluding Section 2.6.

The literature on vertical differentiation in more competitive market structures provides valuable additional insights. The closest benchmark is provided by Gabszewicz and Thisse (1979), who use a framework similar to that of Gabszewicz, Shaked, et al. (1986). They assume that two duopolists compete in the market and each produces a single product of a fixed, exogenous quality. They derive restrictions on parameters, in particular on income dispersion and product qualities, which guarantee certain types of the oligopolistic equilibrium: (i) nothing, low-quality product or high-quality product is consumed, (ii) low-quality or high-quality product is consumed, (iii) only high-quality product is consumed. They find that the resulting demand functions have no discontinuities and unlike in the Hotelling model, the natural forces lead the duopolists to maintain a considerable degree of product differentiation. Moreover, they show that the optimal welfare solution requires a degree of income differentiation which is just sufficient to guarantee that the low-quality product is not sold in the market.

Shaked and Sutton (1982) endogenise the quality choice from Gabszewicz and Thisse (1979), but in a way that it is still chosen independently from the price. The game they present involves the entry decision, the quality choice and the price choice. As qualities in my model depend on prices, the conclusion that at most two duopolists can have positive market shares if income dispersion is not too high does

not carry over. This is because it is unclear whether more competition has to reduce the prices (this constitutes an undesirable reduction in quality at the same time). Shaked and Sutton (1982) assert that with only two firms present, a subgame-perfect equilibrium game always exists in which the firms offer differentiated products and both make positive profits. Economides (1985) modifies the model so that the choice of quality and of price happens simultaneously. He finds that as a result of undercutting, firms have to price at the marginal costs. My model, in contrast, provides a scenario where an equilibrium may fail to exist at all due to discontinuous demand functions, unless the entry is sequential, or the cost structures are sufficiently different.

There have also been numerous other attempts to embed status concerns into monopoly or oligopoly models in which consumers care about the inherent quality of a product. Some notable examples include Deltas and Zacharias (2012), Dritsa and Zacharias (2012) as well as Lambertini and Orsini (2002, 2005). However, due to the presence of the inherent quality as well as the chosen form of the positional concerns akin to the snob case of Corneo and Jeanne (1997), their results are difficult to compare with mine and thus I will not reference them in the body of this chapter.

2.2 Model

There is a unit mass of agent types who vary by income $w \in [0, \theta]$, where $\theta > 0$.[3] Agents' types are their incomes. Income constitutes an agent's private information and is not observable to other players. The distribution of incomes is described by a density function $f(w)$ and a cumulative distribution function $F(w)$. Agents care about their status, which is determined indirectly by their spending choices. An

[3]The normalisation of the lower support of the distribution at zero simplifies the analysis, but it is not innocuous as it guarantees that the market is not fully covered unless the poorest agent does not care about status. This contrasts with Gabszewicz, Shaked, et al. (1986), where a positive lower support of the distribution guarantees the key result that only one firm survives in the market. But such an assumption is less problematic than it may seem as suitable assumptions about the cost structures may also guarantee full coverage, and even the monopoly outcome, whenever the lower support is zero.

agent intends to spend the same fixed proportion of income on positional consumption, yet he may spend more or less than this reference amount and he is always able to cover the positional expenditure within his entire budget (so that the budget constraints may be ignored).[4] The positional consumption does not yield direct utility, but it determines his social status which the agent values; the money that the agent ends up not spending is valued as such. Due to the nature of many positional goods described above I assume that every agent consumes at most one unit of the branded positional good. The model of status is borrowed from Bagwell and Bernheim (1996). They employ a theoretical construct called *social contacts*, who observe the signals sent by the agents, i.e. the type of brand bought, and form an inference about the wealth of the buyer.[5] When it is said that agents care about status, what they in fact care about are these inferences.

Assume that N branded goods are available in the market, from which the consumer chooses brand $k \in \{1, 2, \ldots, N\}$, or no brand at all (which to simplify notation will be described as $k = 0$ with $b_0 = 0$). An agent's utility function is then $u(w, b_k, \rho_k)$, where w denotes income, b_k denotes the price of brand k, and ρ_k is the social status resulting from the inference of the social contact who observes brand k. As will be shown below, under some mild assumptions every agent will have their unique preferred brand, or they will prefer not to consume at all. These choices can be described as a function, $\beta(w) = \text{argmax}_{k \in \{0,1,\ldots,N\}} u(w, b_k, \rho_k)$. Let x_k be the

[4]The *desired* spending on positional consumption, w, is not binding. The agent can spend more or less than the reference amount (the outstanding balance/overdraft provides additional utility/disutility by impacting the overall budget), but he is always able to cover the positional expenditure within his entire budget. This is less problematic than it seems if we assume that everybody intends to spend the same percentage of their overall budget on positional goods. Then the amount that the agent desires to spend is proportional of his overall wealth, while we do not have to worry about the possibility of the agent spending more on the positional good than he could afford. An alternative approach would be to relax this assumption by a more realistic, asymmetric, counterpart: agents still do not have to spend all the money they budgeted for the positional good, but they are not allowed to go above their earmarked amount. Then when the positional concern is strong (α is high), there may be agents who would like to buy the good as indicated by the indifference constraint but are unable to do so as the budget constraint $w \geq b$ binds. In such case the monopolist will simply be proceeding as if it sold a standard non-positional product facing a linear demand. The insights from such analysis are limited and the model becomes intractable for a higher number of brands, so I will not pursue it further.

[5]'Income' and 'wealth' are used interchangeably in this static model.

demand for each brand, which is determined by the proportion of agents who choose it. We shall assume without the loss of generality that the brands are sorted from the least desirable to the most desirable, i.e. if they were given for free, everyone would prefer brand 2 to 1 as it gives a higher social standing: $\rho_N > \rho_{N-1} > \ldots > \rho_1 > \rho_0$. If a monopolist produces all the brands, it may want to exploit this ordering by charging prices $b_N > b_{N-1} > \ldots > b_1$, but the situation is less clear in the duopoly case because of strategic effects.

The social contacts' objective is to minimise the expected error in judging the agent's income. The general loss function depends on the discrepancy between the agent's income and the social contacts' inference about it. As income constitutes agent's private information, the social contacts must make an inference based on the information conveyed in the agent's choice of the brand. To keep the analysis simple a simple quadratic loss function $v(\rho_k, w) = -(\rho_k - w)^2$ is assumed in line with the standard cheap-talk literature.

I begin by assuming that all N brands are produced by a single monopolist active in the market. There may be costs of production for each brand (both variable and semi-fixed), but there are no complementarities in production so the costs are separable between brands and the cost of each brand can be expressed as $C_k(x_k)$. Thus, in the monopoly case, the firm maximises its profit π_N by choosing the set of prices $b_1, \ldots, b_N$, subject to the demand determined by the consumers' choices $x_1, \ldots, x_N$, where $\pi_N = \sum_{k=1}^{N} \big(b_k x_k - C_k(x_k)\big)$.

The timing of this multi-stage game is simple: (i) the monopolist decides the number of brands and chooses the prices; (ii) the agents choose which brand (if any) to buy; (iii) the social contacts form their inferences about expected incomes associated with each brand; (iv) the payoffs are realised. The relevant equilibrium concept here is the Perfect Bayesian Equilibrium (PBE). The equilibrium consists of the monopolist's choice of the vector of prices $(b_1, \ldots, b_N)$, the brand choice (if any) made by an agent of any income $\beta(w)$, the inference about incomes made by

the social contacts ρ_k for $k \in \{0, 1, \ldots, N\}$, and the relevant belief function of the social contacts on which the inference is based, such that actions are optimal given inferences and inferences can be deduced from the actions and the belief function. If there exist multiple PBEs, a further refinement may be necessary and in this essentially sender-receiver game with a continuum of types, the Banks and Sobel's (1987) D1 criterion constitutes the best choice, because when the utility function exhibits increasing differences, it yields a unique equilibrium (Cho and Kreps 1987).

2.2.1 Assumptions

There are three key simplifications which I will use throughout the majority of this chapter to ensure tractability of the model. First, I assume that there are no variable or fixed costs of production, $C_k(x_k) = 0$ for all k, to focus on the implications of the introduction of status effects in the conventional model of a quality-choosing firm. This assumption will be relaxed in the subsequent analysis.

Second, I fix the income distribution to be the uniform distribution on $[0, \theta]$, so that the probability density function is $f(w) = \frac{1}{\theta}$ and the cumulative distribution function is $F(w) = \frac{w}{\theta}$. This way parameter θ measures the degree of income dispersion in the population: the larger it is, the wider a range of incomes is observed. Uniformity simplifies the model at the cost of accurately depicting reality, and the consequences of relaxing it are analysed in Appendix 2.A.

Third, I choose the linear form of agent's utility function, $u(w, b_k, \rho_k) = -b_k + \alpha(w + \epsilon)\rho_k$, where utility depends linearly on status and the income spent on positional goods. Such a form of preferences is common in IO literature for it assumes away income effects.[6] $\alpha > 0$ measures the intensity of the concern for status. $\epsilon \geq 0$ measures the extent to which even the poor care about status: when $\epsilon = 0$, the poorest agent does not care about status at all so he chooses not to buy with certainty,

[6] A natural alternative specification would be a Cobb-Douglas utility function employed by Gabszewicz, Shaked, et al. (1986), but it makes it considerably more difficult to obtain a tractable model.

while if $\epsilon > 0$ he may well want to buy it.[7]

The key assumption about the utility function is that wealth reinforces preferences for status, which may be justified on the grounds of education, habit formation and group interaction (Bourdieu 1984; Veblen 1899).[8] Mathematically, this means that the utility function obeys the single-crossing condition for status with respect to income: $\alpha w_m \rho_k - \alpha w_m \rho_l > \alpha w_n \rho_k - \alpha w_n \rho_l$ for $w_m > w_n$ and $\rho_k > \rho_l$, and therefore whenever a poorer agent prefers good k to good l, so will the richer agent. Such an assumption excludes the possibility that, say, one agent prefers brand l, a richer agent prefers brand k, and a richer still agent prefers brand l again. Thus, there will exist a set of well-defined thresholds $\tilde{w}_k$, such that any person with income $w < \tilde{w}_k$ will prefer the cheaper brand $k-1$ (or no brand at all if $k = 1$), while any person with income $w > \tilde{w}_k$ will prefer the more expensive brand k, and the person with income $w = \tilde{w}_k$ will be just indifferent. The whole distribution of income will be therefore segmented into intervals. To keep the description compact, with some abuse of notation let $\tilde{w}_0 = 0$ and $\tilde{w}_{N+1} = \theta$. Then let I_k, where $k \in \{0, 1, \ldots, N-1, N\}$, be defined as the interval which begins at threshold income level $\tilde{w}_k$ and ends at threshold income level $\tilde{w}_{k+1}$ inside which every agent prefers to choose brand k: $I_k = [\tilde{w}_k, \tilde{w}_{k+1})$.[9] Consequently the brand choice function can be specified as $\beta(w) = k$ if $\tilde{w}_k \leq w < \tilde{w}_{k+1}$. Moreover, given the quadratic form of the social contacts' loss function, its expected value will be minimised at the average income of the interval which buys a certain brand. Thus, upon observing brand

[7]Such a dichotomy is akin to Hopkins and Kornienko (2004) with regards to the minimum guaranteed status.

[8]To see why this matters consider a simple linear specification without increasing differences with respect to status, $u(w, b_k, \rho_k) = -b_k + \alpha\rho_k$. The indifferent agent with wealth $\tilde{w}$ is pinned down by $-0 + \alpha\rho_0 = -\hat{b} + \alpha\rho_1$. Consequently, the only price that guarantees indifference is $\hat{b} = \alpha(\rho_1 - \rho_0)$. This almost completely constrains the monopolist's price choice: if it chooses $b < \hat{b}$, everyone would buy the positional good; if $b > \hat{b}$, nobody would; and if $b = \hat{b}$, there may be multiple equilibria. Clearly, such a model is trivial except for the knife-edge case.

[9]Note that I assumed that the indifferent agent always chooses the more expensive brand. With an atom-less distribution this assumption is innocuous and altering it does not change the analysis or its results. Also, for $k = N$, the interval is closed and contains the upper endpoint, θ

$k \in \{0, 1, \ldots, N\}$, the optimal inference for a uniform distribution is:

$$\rho_k = \frac{\int_{I_k} wf(w)dw}{\int_{I_k} f(w)dw} = \frac{\tilde{w}_k + \tilde{w}_{k+1}}{2}. \tag{2.1}$$

2.3 Results for a Single-Brand Monopolist

To better illustrate some of the key features of the model, I begin by assuming that the monopolist can produce only one brand, $N = 1$. A great if somewhat unbelievable real-life example was the iOS app that could have been found in 2008 in the Apple App Store. The "I Am Rich" app cost $999.99 and offered no functionality at all, other than its buyer's ability to flaunt his material status that allows him to throw away almost a thousand dollars on a whim.[10] The monopolist faces an easy problem: it chooses a single price b that maximises its profit.

Note that two kinds of PBEs are possible in this setting: either a semi-separating equilibrium, where some agents purchase the positional good while others do not, or a pooling equilibrium where all agents either buy or do not buy it. For the latter type the question of out-of-equilibrium path beliefs that the social contacts hold has to be examined. I employ Banks and Sobel's (1987) D1 criterion which requires that zero weight be put on the type w if message b (where $b = 0$ if nothing is bought and $b = b_1$ if the positional good is bought) is sent if there is another type w' such that w' always strictly benefits from the deviation whenever w benefits from the deviation. This results in the social contacts assuming that off-the-equilibrium path the agent who might deviate is the one who stands to benefit most (if all buy, it is the poorest agent, while if no buy, it is the richest agent). Combined with the fact that the monopolist never chooses a price that results in a no-buy equilibrium, only one PBE remains such that all agents buy the positional good. The results are summarised in the following proposition:

[10] http://latimesblogs.latimes.com/technology/2008/08/iphone-i-am-ric.html

Proposition 2.1. *If the monopolist only produces one positional good, then in the unique PBE that survives the D1 criterion:*

(i) if $0 \leq \epsilon < \theta$*, the market will be partially covered as the monopolist sets the price at* $b_1^* = \frac{1}{4}\alpha\theta(\theta+\epsilon)$*, which results in the threshold income of* $\tilde{w}_1^* = \frac{1}{2}(\theta-\epsilon)$ *and the profit of* $\pi^* = \frac{1}{8}\alpha(\theta+\epsilon)^2$*,*

(ii) if $\epsilon \geq \theta$*, the market will be fully covered (*$\tilde{w}_1^* = 0$*) as the monopolist sets the price at* $b_1^* = \frac{1}{2}\alpha\theta\epsilon$*, which results in the profit of* $\pi^* = \frac{1}{2}\alpha\theta\epsilon$*.*

The indifference condition which determines the threshold level of income is standard in the differentiated goods literature: the agents weight having to spend money on a luxury item against the boost in social standing that it will yield at their level of wealth. The role of ϵ is particularly important for this and future results. Partial coverage is guaranteed regardless of θ only if $\epsilon = 0$, so that the poorest agent does not care at all about his status. In such a case, he will never buy any positional good, so the market cannot be fully covered. But if ϵ is positive, it becomes possible that even the poorest agent will care about the status and thus may want to buy the positional good. This explains why the full coverage equilibrium may sometimes arise.

A number of comments pertaining to the comparative statics are warranted. First, observe that a change in α does not impact the threshold value because even though more agents are willing to buy the positional good at the old price, the monopolist responds by raising it and the two effects on demand cancel out. This is due to the linear nature of the utility function: changes in α merely affect the size of the transfer from the consumer to the producer but does not impact the decision whether or not to buy. Similarly, an increase in ϵ increases the desirability of the positional good, but here the thresholds falls even though the price goes up, so the profit must necessarily increase. Finally, an increase in θ makes the positional good more desirable through a higher inference about the buyers, and in this case the

threshold goes down, even though the price goes up as well. This necessarily leads to a higher profit for the monopolist.

While the all-buy equilibrium may appear counter-intuitive, there exist communities where certain positional goods provide the consumers with a sense of belonging, as otherwise they might face ostracism due to their perceived low wealth (assuming that they care enough about status). An expensive pair of Nike basketball shoes among American youngsters or the newest iPhone among white-collar workers are good examples. Agents need to buy an expensive gadget even though it does not boost their status; the only reason for doing so is to avoid losing social belongingness.[11]

2.3.1 Consumer Surplus and Total Welfare

The first conclusion of the welfare analysis is summarised below:

Proposition 2.2. *Introduction of one purely positional brand lowers the aggregate consumer surplus.*

It is unsurprising that banning positional goods increases consumer surplus if the market is fully covered: the inference about agents' incomes does not change (it is the average taken for the population) but surplus is transferred from the consumers to the monopolist. While less obvious, the conclusion stays the same for partial coverage. To see this, observe first that any member of society who ends up not purchasing the positional good must be necessarily worse off. His initial utility is $\alpha(w+\epsilon)\frac{\theta}{2}$, while his new utility, $\alpha(w+\epsilon)\frac{0+\tilde{w}_1}{2}$, is lower for any $w \in [0, \tilde{w}_1]$. Among the buying agents, the one that enjoys the positional good most is the richest agent. But note that $\alpha(\theta+\epsilon)\frac{\theta}{2} > -b_1^* + \alpha(\theta+\epsilon)\frac{\tilde{w}_1^*+\theta}{2}$ is true both for full coverage (trivially)

[11] This is not a universally accepted model and conclusion, and depends on the particular equilibrium refinement chosen. Corneo and Jeanne (1997), for example, use the concept of "passive" beliefs, where social contacts retain their prior after observing out-of-equilibrium actions, and they reach an opposite conclusion: if the price is so low that everyone would buy the good, nobody ends up buying it as it does not serve its only purpose of wealth signalling.

and for partial coverage (where $b_1^* = \frac{1}{4}\alpha\theta(\theta+\epsilon)$ and $\tilde{w}_1^* = \frac{1}{2}(\theta-\epsilon)$), so even his utility must drop. Everyone loses out, although to varying degrees. Such an analysis is more complex and more insightful for the general N-brand case, as discussed in Section 2.4.

Another conclusion of the welfare analysis pertains the effect on overall welfare:

Proposition 2.3. *Introduction of one purely positional brand increases the total welfare in the case that the market is partially covered and does not change the total welfare in the case that the market is fully covered.*

It seems that looking from a welfare perspective and assuming away any distributional issues, there are no grounds to ban the positional goods. This stands in stark contrast to Corneo and Jeanne (1997), where banning positional goods may under some circumstances even constitute a Pareto improvement. This is because their positional competition is a zero-sum game, so resources are wasted without any increase in the aggregate status utility. In my model, the game is not zero-sum as the better revelation of agents' incomes may be socially beneficial: while prohibiting positional goods be Pareto-improving for the agents, the overall welfare would be hurt due to the lower profit enjoyed by the monopolist.

Finally, we can ask what price and income threshold maximise overall welfare:

Proposition 2.4. *With one positional brand:*

(i) when $\epsilon > 0$, the monopolist over-produces the positional good and sells it at too low a price compared to the social optimum,

(ii) when $\epsilon = 0$, the monopolist produces at the social optimum.

That the social planner does not choose full coverage is not surprising: such a choice is simply a transfer with no information revelation gains to be realised. What is interesting however is that contrary to the standard monopoly distortion the positional good is over-supplied (for $\epsilon > 0$) rather than under-supplied and the

price is too low rather than too high. There is a different type of externality at work here: lower income agents flocking to buy the brand reduce the value which the brand has the richer agents. As a consequence, incomes are not revealed as smoothly as the social planner would like.

2.3.2 Introduction of Taxes

In light of the above discussion it is natural to ask whether the social optimum could be achieved through taxation. The most basic instrument at hand is a per-item excise (luxury) tax which prevents some agents from buying.[12] Assume that buyers of the positional good, in addition to paying the price b also incur extra per-unit tax t. This gives us the following:

Proposition 2.5. *When $\epsilon > 0$, first-best level of total welfare can be restored by the introduction of a flat (Pigouvian) per-unit tax, which should increase as the income dispersion, the desirability of the positional good or the strength of the status externality increases. Assuming that tax revenues are redistributed back to those who paid the tax, such a tax does not constitute a Pareto improvement as some agents who used to buy the positional good but do not buy it with the tax are worse-off.*[13]

This result contrasts with that of Ireland (1994) who found that in the presence of a positional good, a Pigouvian tax may be distorting behaviour but it also necessarily brings a Pareto improvement. Here this is not the case, as not every agent over-consumes compared to the social optimum. The result also contrasts with Bagwell and Bernheim (1996), where an excise tax on conspicuous goods amounts to a non-distortionary tax on pure profits. In my model, the behaviour is distorted as the tax pushes the cost of buying the good just above the willingness to pay for some agents,

[12] Due to the linear preferences, *ad valorem* taxes which increase the price of the positional good proportionally do not work in this case: they are fully incorporated by the monopolist, so the quantity of positional goods traded does not change; the only difference is the transfer from the monopolist to the government.

[13] This simplifying assumption is common in literature that abstracts from the use of the tax proceeds and distributional issues (Hopkins and Kornienko 2004; Ireland 1994).

and at the same time it changes the status associated with the good. Note that if we take the welfare-enhancement criterion (as opposed to the Pareto-improvement criterion), whether the tax money is returned to the purchasers of the positional good (so that it could only be spent on non-positional consumption) or just retained by the government for other purposes, is irrelevant in this model because agent's status increases in the income *before* positional spending.

2.3.3 Positive Marginal Cost

Thus far I have abstracted from the issue of costs. Nonetheless, these should be mentioned for they lead to somewhat perverse welfare conclusions. For simplicity I assume that the marginal cost $c > 0$ is constant.[14] The cost plays a role that is similar to that of a per-unit tax in Proposition 2.5 when $\epsilon > 0$: it increases the optimal price and lowers the demand, as would be required for an improvement in welfare. The welfare consequences of the inclusion of a positive marginal cost are summarised in the proposition below:

Proposition 2.6. *Assume that only one positional brand is produced and there is a constant marginal cost that is not too high,* $0 < c \leq \frac{1}{2}\alpha\theta(\theta + \epsilon)$. *The market is fully covered when* $\epsilon \geq \theta$ *and* $c \leq \frac{1}{2}\alpha\theta(\epsilon - \theta)$, *and partially covered otherwise. As c increases, consumer surplus weakly increases, with a strict increase under partial coverage. Total welfare may also go up if c is high enough,* $c > \frac{1}{6}\alpha\theta(2\theta + \epsilon)$.

When c goes up, the monopolist must increase the price to compensate for the rising cost, which is beneficial for the consumers as long as the market is already only partially covered. When $\epsilon = 0$, this is because the presence of the cost limits the ability of the monopolist to extract surplus: the resulting consumers' gain is

[14] If instead the marginal cost were increasing in the quantities produced, the multi-brand analysis becomes similar to Lancaster (1975): the monopolist weights the extra surplus which may be extracted by an introduction of an additional brand against the efficiency loss from not exhausting the economies of scale. This is covered further in the subsequent section in the context of semi-fixed costs.

larger than the loss from poorer revelation of incomes. When $\epsilon > 0$, there is an additional improvement akin to the introduction of a per-item tax. What this means is that from consumers point of view, it is always (weakly) better when the goods that are expensive in production, such as diamond jewellery, serve as status signals. Furthermore, a higher cost may even improve overall welfare but only when the cost is sufficiently elevated. While this conclusion may at first appear counter-intuitive, the rationale behind it is simple: whether the total welfare goes up or down depends on the relative scale and magnitudes of the changes in consumer surplus and profit. When the cost is high, the fall in the latter is much smaller and so not sufficient to outweigh the rise in the former, which is boosted by the increasing differences of the utility function. A paradoxical consequence may therefore be that a spike in the price of gold or diamonds, which increases prices of watches and jewellery, may improve overall welfare.

2.4 Results for a Multiple-Brand Monopolist

Having examined in detail what happens when the monopolist may only produce one brand, I will now expand the analysis to an arbitrary integer number of brands $N \geq 2$. As mentioned, all goods are *ex-ante* identical, but it turns out to be optimal for the monopolist to assign different price tags to them. From now on, for simplicity, I will focus on the case of $\epsilon = 0$ because it makes the analysis neater by "imposing" partial coverage. I will discuss the consequences of this simplification at the end of this section. The lemma and the proposition below summarise the main results of this section:

Lemma 2.1. *The lower end-points of intervals I_k for $2 \leq k \leq N-1$ follow an arithmetic progression, $\tilde{w}_k = \frac{\tilde{w}_{k-1}+\tilde{w}_{k+1}}{2}$, while for $k = 1$ the lower end-point is 0, and for $k = N$ the lower end-point is $\tilde{w}_N = \frac{\tilde{w}_{N-1}+\theta}{2}$.*

Proposition 2.7. *When the monopolists produces $N \geq 2$ brands, it sets the prices to*

be $b_k = \frac{k(k+1)}{2}\frac{\alpha\theta^2}{(N+1)^2}$ *for* $k \in \{1, \ldots, N\}$. *Consumers sort themselves into partitions* $I_0, I_1, \ldots, I_N$, *whose endpoints are given by* $\tilde{w}_k = \frac{k\theta}{N+1}$ *for* $k \in \{0, 1, \ldots, N, N+1\}$. *Social contacts' inference* ρ_k *is given by the mean income in interval* I_k. *The profit made by the monopolist is:*

$$\pi^N = \frac{\alpha\theta^2 N(N+2)}{6(N+1)^2}. \tag{2.2}$$

Several observations about the derived results are interesting. First, the thresholds are evenly spaced, with the distance between them being constant for any given N, $\tilde{w}_{k+1} - \tilde{w}_k = \frac{\theta}{N+1}$ for $1 \leq k < N$. This result is driven by the assumption about the linear demand and the uniform distribution of incomes: the spacing must be as even as possible as this guaranteed the best possible trade-off between the available demand and the feasible price.[15] Such behaviour, however, could not be generated by equally-spaced *prices* as status is more important to the rich than to the poor, so they are willing to pay more for it and the monopolist exploits this. Indeed, the difference in prices between neighbouring goods $k+1$ and k is $\delta_k = b_{k+1} - b_k = \frac{(k+1)\alpha\theta^2}{(N+1)^2} > 0$, i.e. it is positive and depends on k. It is easy to see that the agent would have to spend increasingly more to go one brand up as he acquires the more expensive brands: $\delta_{k+1} - \delta_k = \frac{\alpha\theta^2}{(N+1)^2} > 0$.

It should be obvious from the previous paragraph that when the number of brands is increased, the size of the gap between thresholds must fall. This is a consequence of the "even-spacing" feature of this model. What is more interesting is to examine the price difference between adjacent brands when more brands become available. The difference in δ_k when there are $N+1$ goods and when there are N goods is $-\frac{(k+1)(2N+3)\alpha\theta^2}{(N+1)^2(N+2)^2}$, which is smaller than zero. Thus, the difference between prices of adjacent brands goes down as more brands are offered, which is consistent with the observation about the tightening of the thresholds. This is in contrast

[15] Gabszewicz, Shaked, et al. (1986), who use a multiplicative (Cobb-Douglas) utility function, show that the optimal pattern for product qualities (which are the equivalents of status in my model) is for them to follow a geometric progression.

to Itoh (1983), where an introduction of a new brand does not affect the prices of other brands (including the higher-quality products when the distribution of tastes is uniform). This is because the qualities available are unchanged and still available. In my model, the statuses associated with these lower brands do change, so consumer choices and, in turn, the monopolist's pricing strategy is affected.

2.4.1 Comparative Statics

The comparative statics in this model are straightforward and they are summarised in the following proposition:

Proposition 2.8. *In the multiple brand model:*

(i) a rise in α increases the optimal prices, does not change the thresholds and increases the profit,

(ii) a rise in θ increases the optimal prices, the thresholds and the profit,

(iii) a rise in the number of brands N lowers the optimal prices and the thresholds but increases the profit.

The variations in α do not impact the thresholds but linearly change the respective prices. This is a consequence of the fact that the price enters the utility function linearly and thus the marginal utility of an additional dollar is equal to one. In other words, α acts merely as a scaling factor: it determines the size of the transfer from the consumers to the monopolist, as can be seen from the profit equation. When θ goes up, the monopolist exploits the wealthier society by increasing prices even if this leads to a simultaneous increase in thresholds and to lowering of demand. The increase in the profit is the consequence of the fact that the prices rise faster than the thresholds due to increasing differences.

A larger number of brands always strictly increases the profit. Suppose that the monopolist currently only sells $N-1$ brands and begins to produce one more.

Evidently, the profit does not fall as the monopolist can always keep the old prices unchanged. However, it is easy to see that it can do better than this, e.g. by finding a new price at which only the richest member of the society will buy, so that $\tilde{w}_N = \theta - \tau$, where $\tau \to 0$. This must improve the profit as it leaves the demand almost unchanged while capturing all the extra surplus for the top member of society. The consequence of this result is that given an opportunity to produce at most N brands, the monopolist will always produce the maximum number. This conclusion is equivalent to Itoh (1983), where the profit increases even in the presence of a marginal cost which is convex in quality as well as Gabszewicz, Shaked, et al. (1986), where sufficient income dispersion guarantees that the monopolist produces the maximum allowed number of brands.

The contribution to the profit of an additional brand at the margin is summarised in the following proposition:

Proposition 2.9. *For any $N \geq 3$, the marginal profit is diminishing in the number of brands, i.e. $\pi^{N+1} - \pi^N < \pi^N - \pi^{N-1}$.*

What this means is that the monopolist's incentive to introduce new brands weakens with each additional brand introduced. Moreover, observe that:

$$\lim_{N \to \infty} (\pi^{N+1} - \pi^N) = 0,$$

which means that the marginal contribution of the last brand, as the number of brands increases indefinitely, approaches zero, or in other words that the total profit is bounded. Thus is the presence of any non-decreasing semi-fixed cost of brand introduction that is not too concave there will always be only a finite number of brands in the market.[16] These conditions are likely to be met in reality: any company that owns multiple brands is unlikely to see considerable economies of scale when it introduces new brands as they require their own, separate marketing spend, *et*

[16] Even if the semi-fixed costs rise ever more slowly with introduction of more brands, the number of brands will be bounded as long as the marginal gross profit rises even more slowly.

cetera. This gives us one possible explanation for why we indeed rarely observe extreme brand proliferation in the positional goods spectrum, a topic that will be further analysed at the end of this section.[17]

2.4.2 Welfare Analysis and Taxes

Having established the profit-maximising behaviour of the monopolist we can move to the analysis of its welfare consequences. I begin by looking at the consumer surplus as a function of the number of brands:

Proposition 2.10. *For any $N \geq 2$, consumer surplus decreases as the number of brands goes up.*

In other words, the average consumer is necessarily worse-off when the monopolist increases the range of its offering. Moreover, the loss of consumer surplus grows with the strength of the positional concerns. While the aggregate result is important, it is also interesting to look at the winners and losers in a more granular manner:

[17] We can also consider the continuous version of the model above, which makes the comparison with the benchmark of Mussa and Rosen (1978) easier. Assuming that the agents can spend any amount of money on the positional good that they wish, $b \in [0, \infty)$, they would choose to spend $b^*(w) = \alpha \frac{w^2}{2}$ (which can be found by solving the relevant FOC with an initial condition $b(0) = 0$ and noting that the social contacts' inferences must be confirmed in the equilibrium). Absent costs, the monopolist's profit maximisation problem translates to choosing the menu of prices $B \subseteq [0, \infty)$ that it offers. It can be shown that the profit-maximising choice of the monopolist is the offer the full range of the prices, $B = [0, \alpha \frac{\theta^2}{2}]$, so as to not allow for bunching (which can be deduced by observing that bunching cannot be profitable for the monopolist). Similarly to Mussa and Rosen (1978), the ability of the monopolist to extract the surplus from the wealthier agents is limited by the presence of the lower-price, lower-status brands. As there there are no costs in my model, no agents are excluded from consuming at the bottom of the distribution as the monopolist enjoys a non-negative margin from selling to any agent. The reason why there is no bunching is that given the uniform distribution of incomes and linear preferences, the assignment of statuses (qualities) to incomes (tastes for quality) is strictly increasing, $\rho(b(w)) = w$, and thus the marginal revenue always increases as a function of w. Therefore, there are no regions where "smoothing" would be required and the profit-maximising strategy for the monopolist is to offer the whole range of prices (qualities). The result about offering the full product range is consistent with Gabszewicz, Shaked, et al. (1986), where in the absence of costs and with the lower support of the income distribution equal to zero, this is the behaviour which their model demonstrates as $N \to \infty$. Note also that a direct comparison with Mussa and Rosen's (1978) perfectly competitive market result is unfortunately impossible as it is unclear what happens in my model as the number of firms goes to infinity (see Section 2.5).

Proposition 2.11. *Suppose that the number of brands goes up from N to $N+1$, so that the thresholds change from $\tilde{w}_1^N$, $\tilde{w}_2^N$, ..., $\tilde{w}_N^N$ to $\tilde{w}_1^{N+1}$, $\tilde{w}_2^{N+1}$, ..., $\tilde{w}_N^{N+1}$, $\tilde{w}_{N+1}^{N+1}$. The income distribution will be partitioned into $2(N+1)$ sub-intervals $J_1 = [0, \tilde{w}_1^{N+1}]$, $J_2 = [\tilde{w}_1^{N+1}, \tilde{w}_1^N]$, $J_3 = [\tilde{w}_1^N, \tilde{w}_2^{N+1}]$, ..., $J_{2N+1} = [\tilde{w}_N^N, \tilde{w}_{N+1}^{N+1}]$, $J_{2(N+1)} = [\tilde{w}_{N+1}^{N+1}, \theta]$, where the bounds are determined by the thresholds under N and $N+1$ brands. Then the net change in agent's utility when moving from N brands to $N+1$ brands is negative on J_1 and $J_{2(N+1)}$. Within all other sub-intervals J_j, $j \in \{2, 3, \ldots, 2N+1\}$, for even j, the net change in utility is first negative then positive, while for odd j, the net change in utility is first positive then negative.*

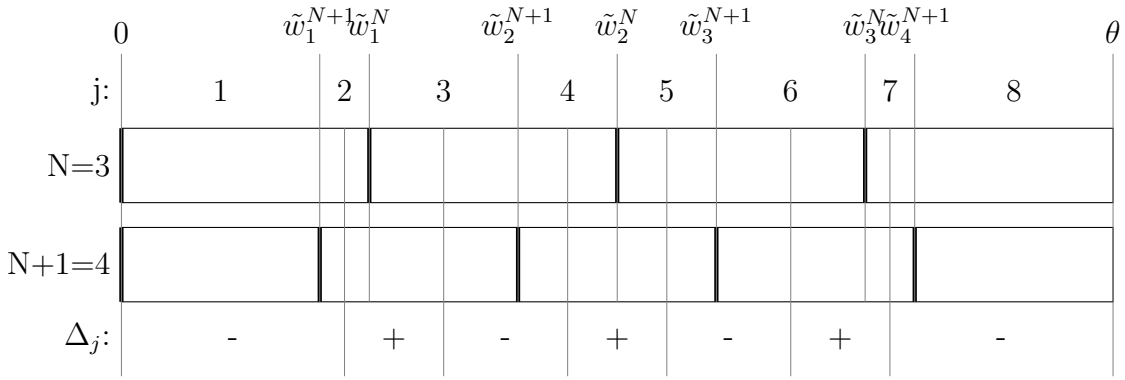


Figure 2.1: Winners and losers from the introduction of a new brand under full coverage when $N = 4$ (thick lines indicate the relevant thresholds).

The results illustrated in Figure 2.1 have two main consequences. First, there is an interlacing pattern on the inner sub-intervals: agents in the vicinity of the old threshold (on either side) gain utility when a new brand is introduced, while agents far away from either of the old thresholds lose out. The reason for this pattern is that for the former kind of agents the original assignment was far from ideal: either they were getting a bad deal (those just below the threshold), i.e. the signalled wealth was significantly lower than their actual wealth, or they forced to pay a high price for signalling while not valuing the status much (those just above the threshold). They would happily accept an offer of a different brand more aligned to their types, and so when the opportunity presents itself, they will do

so. The converse is true for agents who used to be in the middle of the interval. In the new setting it is them who are pushed to the fringe. Second, agents at the extremes of the distribution (either very poor or very rich) always lose out upon the introduction of a new brand. The reasoning here is similar: at the bottom of the income distribution the inferred income falls as the threshold moves down, while at the top of the income distribution the monopolist exploits the opportunity to charge the agents considerably more, which more than counterweights the increase in status obtained.

Similarly to the single-brand monopolist case, it is also of interest to examine the effect of a change in N on the overall welfare:

Proposition 2.12. *The social planner prefers an infinite number of brands.*

This result affirms the intuitive outcome. As the transmission of information about status becomes increasingly less coarse, the overall efficiency improves. This is identical to the move from second-degree to first-degree price discrimination where prefect discrimination is overall beneficial from the efficiency point of view but may have undesirable distributional effects as all the surplus accrues to the monopolist. Given the results from the previous section and the assumption that $\epsilon = 0$, it is unsurprising that for a fixed N, the monopolist's choice of the thresholds coincides with that of the social planner.[18] Thus, the monopolist happens to maximise the overall welfare while maximising its profit. No taxes are required to restore efficiency. However, the above result is no longer true when $\epsilon > 0$. In such case, the social optimum *for a given number of brands* may be attained through flat per-unit taxes:

Proposition 2.13. *When $\epsilon > 0$, the social planner can maximise total welfare by setting the same (Pigouvian) per-unit tax for all positional brands.*

The lack of differentiation in the tax rates can be explained using the combination of the linear utility and the uniform distribution. Parallel to the single-brand case,

[18]The proof is almost identical to Proposition 2.4 and hence will be skipped.

all thresholds go up compared to the monopolist's choice and remain equidistant. Every good is taxed at the same level, so e.g. watches would be taxed identically regardless of whether they are Casios or Rolexes. The tax is essentially regressive as $\frac{t_k^*}{b_k^*}$ falls as k increases. The tax is not Pareto-improving, as there will be agents who used to buy positional good k and care considerably about status, who will be forced to switch to brand $k-1$ as a result of the tax and see a fall in their utilities.

2.4.3 Cost of Brand Introduction

The conclusion that the monopolist would like to produce (and the social planner would like to choose) an infinite number of brands does not however seem to account for certain real-life frictions. A key reason why this may not be the case is that there exist certain costs to the introduction of additional brands. Such costs are different in nature from the variable costs in that they do not depend on the number of units produced. They are, however, not fully fixed either in that they depend on the number of brands produced, hence the label semi-fixed. Their introduction should allow us to re-consider the comparison of the optimal number of brands chosen by the monopolist and the social planner. For simplicity, I employ a specification where the total cost depends directly on the number of brands produced $C(N) = \beta N^{\zeta}$, where $\beta > 0$ and $\zeta > 0$ to ensure that it is increasing in the number of brands. For example, any $\zeta > 1$ makes this function convex in the number of brands, and so is able to capture the empirically most plausible idea that the addition of new brands, presumably targeting ever more affluent agents, is becoming increasingly more costly. Yet, note that convexity is not required: we only have to exclude the possibility of the *total* cost falling in the number of brands to obtain the earlier conclusion that $N \to \infty$.

The model is augmented by a preliminary round in which the monopolist decides on the number of brands that it produces, before choosing the profit-maximising set of prices as before:

Proposition 2.14. *When semi-fixed costs take the form $C(N) = \beta N^{\zeta}$ where $\beta > 0$ and $\zeta > 0$, the monopolist's profit-maximising number of brands will be weakly higher than the socially optimal number of brands. Both weakly go down when the cost parameter β or the convexity of the costs ζ increases.*

A similar conclusion is reached by Mussa and Rosen (1978), who find that when the cost increases in quality, the range of quality under a monopoly is always greater than that under perfect competition, and thus social optimum. This is akin in two respects to the Salop circular model where firms decided whether or not to incur a fixed cost and locate on a circle where they can engage in Bertrand competition for customers. First, the profit-maximising as well as socially optimal numbers of brands, or firms in the Salop model, are inversely related to the size of the semi-fixed cost. Second, the socially optimal number of brands, or firms in the Salop model, is always lower than the number of brands chosen by the monopolist, or the number of firms that decide to enter in the Salop model. While in the Salop model this conclusion is driven by competition and zero-profit condition, the same conclusions are obtained here for a profit-maximising monopolist. This is because the rationale behind the decisions is similar. The gross gain from an additional brand is higher for the monopolist than for the social planner as the former is able to discriminate more efficiently between consumers and extract all of the surplus, while the latter benefits to a lesser extent from the less coarse signalling of incomes. At the same time, the cost of wasted resources is identical for the monopolist and the social planner, hence the latter prefers fewer brands. It should be noted that if $\epsilon > 0$, the excessive proliferation of brands comes on top of the already too high supply of each positional good from the social point of view (see Proposition 2.4).

2.4.4 Different Marginal Costs

In Section 2.3, I discussed how a positive constant marginal cost affects the result in the case of a single-product monopolist. The natural extension is how this result

changes when N goods are produced, and each incurs a different marginal cost. To simplify the analysis, I focus on the case where $N = 2$ and the lower-priced good incurs a marginal cost of zero, while the higher-priced good incurs a marginal cost $c > 0$. It may not always be the case anymore that the best strategy is to produce the maximum possible number of brands:

Proposition 2.15. *Whenever the monopolist can produce up to two brands, of which the lower-priced one incurs a marginal cost of zero, while the higher-priced one incurs a marginal cost of $c > 0$, it chooses to offer both brands only if $c \leq \bar{c} = \frac{1}{8}\alpha\theta^2$. Then consumer surplus always increases with c, while total welfare only increases if c is high enough. If $c > \bar{c}$ the monopolist only offers the lower-priced brand.*

It is intuitive that the monopolist may not want to produce the high-cost brand as it faces a trade-off between the ability to better price-discriminate on the one hand and the necessity to incur the cost on the other. For any value of $c > \bar{c}$, to continue profitably offering the high-cost brand, the monopolist would need to price it so high that it would not obtain any demand. Another interpretation of this result is that in addition to semi-fixed cost, also a sufficient difference in the structure of the variable costs may lead the monopolist to constrain its brand offering. Note that when two brands are offered, an increase in c raises both prices (as well as the resulting thresholds). This is because the second price must increase to counterbalance the higher cost, which results in the increased second threshold. This, in turn, improves the status obtained by the buyers of the lower-priced good, so the monopolist can extract some of this extra surplus from them by setting a higher price for the first good. Similarly to Proposition 2.6, an increase in the cost is beneficial for consumers if two brands are offered. While the surplus of agents who consume the more expensive good decreases, the surplus of the remaining agents increases by enough to make the overall change positive. The impact on total welfare is more complex and depends on how the loss in profit compares to the gain in consumer surplus; as in the single-brand case, a critical value of c exists, above which society

benefits from a higher cost.

2.5 Duopoly and Increased Competition

In reality most positional goods markets are oligopolistic. To better understand their functioning, I now analyse the equilibrium when two duopolists are present and each produces only one brand. As I shall demonstrate the difficulty with this problem relative to that of the monopolist is that it can no longer be assumed that thresholds rather than prices are the choice variables. Moreover, the game becomes "truly" strategic and therefore much closer attention will have to be paid to the beliefs of the social contacts.

Throughout this section I will maintain the assumptions that $\epsilon = 0$ and that the lower support of the income distribution is zero, as these simplify the analysis enormously by ensuring that there will always be agents who do not buy any positional good. I continue to assume that the products are *ex-ante* identical and, without the loss of generality, that good 1 is cheaper than good 2, $b_1 < b_2$. Let us define the utilities of an agent who chooses either to not consume, to consume good 1, or to consume good 2 as $u_0(w) = \alpha w \rho_0$, $u_1(w) = -b_1 + \alpha w \rho_1$ and $u_2(w) = -b_2 + \alpha w \rho_2$, respectively. The agents correctly believe that the rich agents would buy the more expensive brand whenever suitable, as could be predicted from the increasing differences of the utility function. Depending on the prices b_1, b_2 chosen by the oligopolists, the following scenarios, illustrated in Figure 2.2, are possible:

Lemma 2.2. *Let $\hat{b} = \frac{1}{2}\alpha\theta$. Whenever two goods are offered in the market, depending on the prices (b_1, b_2) set by the duopolists, the following behaviour is observed in the unique PBE which satisfies the D1 refinement:*

(i) when $0 \leq b_1 < \hat{b}$ and $\sqrt{\frac{\alpha\theta^2 b_1}{2}} < b_2 < \hat{b}$, agents with $w \in [0, \frac{\theta b_1}{b_2})$ buy nothing, agents with $w \in [\frac{\theta b_1}{b_2}, \frac{2b_2}{\alpha\theta})$ buy good 1 and agents with $w \in [\frac{2b_2}{\alpha\theta}, \theta]$ buy good 2 (region A),

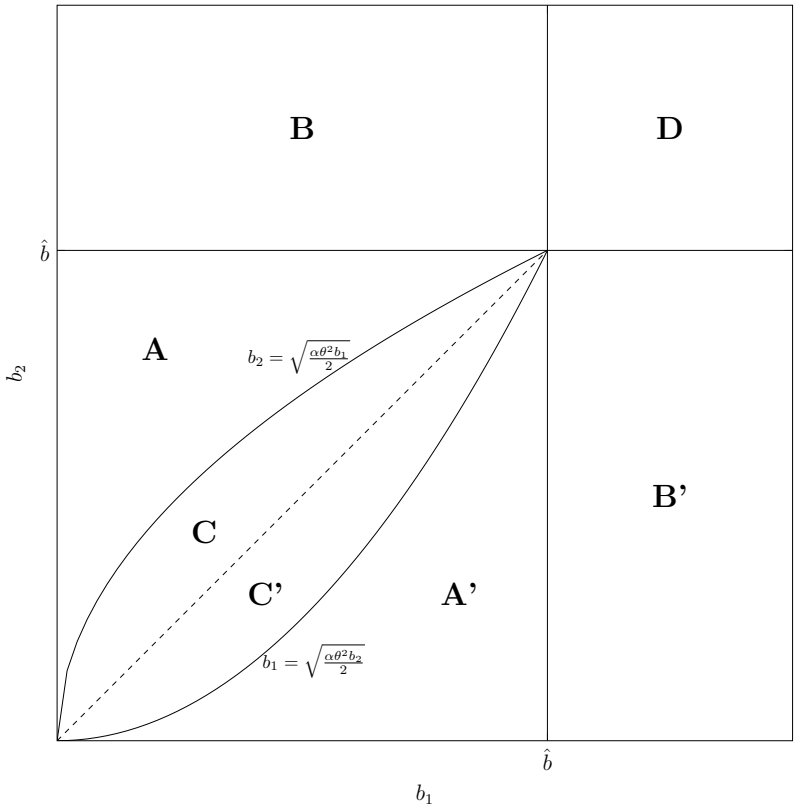


Figure 2.2: Description of agents' behaviour for a set of prices b_1, b_2 (regions as described by Lemma 2.2).

(ii) when $0 \leq b_2 < \hat{b}$ *and* $\sqrt{\frac{\alpha\theta^2 b_2}{2}} < b_1 < \hat{b}$, *agents with* $w \in [0, \frac{\theta b_2}{b_1})$ *buy nothing, agents with* $w \in [\frac{\theta b_2}{b_1}, \frac{2b_1}{\alpha\theta})$ *buy good* 2 *and agents with* $w \in [\frac{2b_1}{\alpha\theta}, \theta]$ *buy good* 1 *(region A'),*

(iii) when $0 \leq b_1 < \hat{b}$ *and* $b_2 \geq \hat{b}$, *agents with* $w \in [0, \frac{2b_1}{\alpha\theta})$ *buy nothing and agents with* $w \in [\frac{2b_1}{\alpha\theta}, \theta]$ *buy good* 1 *(region B),*

(iv) when $0 \leq b_2 < \hat{b}$ *and* $b_1 \geq \hat{b}$, *agents with* $w \in [0, \frac{2b_2}{\alpha\theta})$ *buy nothing and agents with* $w \in [\frac{2b_2}{\alpha\theta}, \theta]$ *buy good* 2 *(region B'),*

(v) when $0 \leq b_1 < \hat{b}$ *and* $b_1 < b_2 \leq \sqrt{\frac{\alpha\theta^2 b_1}{2}}$, *agents with* $w \in [0, \frac{2b_2}{\alpha\theta})$ *buy nothing and agents with* $w \in [\frac{2b_2}{\alpha\theta}, \theta]$ *buy good* 2 *(region C),*

(vi) when $0 \leq b_2 < \hat{b}$ *and* $b_2 < b_1 \leq \sqrt{\frac{\alpha\theta^2 b_2}{2}}$, *agents with* $w \in [0, \frac{2b_1}{\alpha\theta})$ *buy nothing and agents with* $w \in [\frac{2b_1}{\alpha\theta}, \theta]$ *buy good* 1 *(region C'),*

(vii) when $0 \leq b_1 = b_2 < \hat{b}$*, agents with* $w \in [0, \frac{2b_1}{\alpha\theta})$ *buy nothing and agents with* $w \in [\frac{2b_1}{\alpha\theta}, \theta]$ *are indifferent between buying good* 1 *and good* 2 *(border C-C'),*

(viii) when $b_1 \geq \hat{b}$ *and* $b_2 \geq \hat{b}$*, all agents buy nothing (region D).*

The thresholds and profits under each scenario from Lemma 2.2 are summarised in Table 2.1. To see that no other non-trivial beliefs may be confirmed in an equilibrium, observe that if both firms are to make strictly positive profits, it must be the case that status conferred by the pricier good is higher than status conferred by the cheaper good. If this were not the case, the more expensive good would obtain no demand. Moreover, due to the agents' single crossing condition with respect to income and status, if an agent with a certain income chooses a good which confers status ρ, then a richer agent cannot purchase a lower-quality good than ρ. Consequently, there are four types of partitions of the income distribution in relation to the chosen consumption: (i) some agents consume nothing, some agents consume good 1, some agents consume good 2; (ii) some agents consume nothing, some agents consume good 1; (iii) some agents consume nothing, some agents consume good 2; or (iv) no agent consumes anything. But these are exactly the partitions considered by Lemma 2.2, where a unique set of beliefs has been derived for any price pair.

My approach is similar to that of Gabszewicz and Thisse (1979), but the partitioning of the price space is somewhat different because their setup allows for the market to be fully covered, which is impossible here for $\epsilon = 0$ unless the income distribution were to be shifted rightwards. Moreover, their cheaper good never enjoys a positive demand when the demand for the more expensive good is zero, while in my model this needs not necessarily be true. This is because Gabszewicz and Thisse (1979) explicitly assumes that one firm produces a better quality product than the other firm while in my model the qualities are determined implicitly through prices.

Region	$\tilde{w}_1$	$\tilde{w}_2$	π_1	π_2	Consumers' choice
A	$\frac{\theta b_1}{b_2}$	$\frac{2b_2}{\alpha\theta}$	$\frac{2b_1b_2}{\alpha\theta^2} - \frac{b_1^2}{b_2}$	$b_2 - \frac{2b_2^2}{\alpha\theta^2}$	Nothing, cheap or expensive good
A'	$\frac{2b_1}{\alpha\theta}$	$\frac{\theta b_2}{b_1}$	$b_1 - \frac{2b_1^2}{\alpha\theta^2}$	$\frac{2b_1b_2}{\alpha\theta^2} - \frac{b_2^2}{b_1}$	Nothing, cheap or expensive good
B	$\frac{2b_1}{\alpha\theta}$	-	$b_1 - \frac{2b_1^2}{\alpha\theta^2}$	0	Nothing or cheap good
B'	-	$\frac{2b_2}{\alpha\theta}$	0	$b_2 - \frac{2b_2^2}{\alpha\theta^2}$	Nothing or cheap good
C	-	$\frac{2b_2}{\alpha\theta}$	0	$b_2 - \frac{2b_2^2}{\alpha\theta^2}$	Nothing or expensive good
C'	$\frac{2b_1}{\alpha\theta}$	-	$b_1 - \frac{2b_1^2}{\alpha\theta^2}$	0	Nothing or expensive good
C-C'	$\frac{2b}{\alpha\theta}$		$\frac{1}{2}(b - \frac{2b^2}{\alpha\theta^2})$	$\frac{1}{2}(b - \frac{2b^2}{\alpha\theta^2})$	Nothing or either same-priced good
D	-	-	0	0	Nothing

Table 2.1: Thresholds and profits for all regions.

2.5.1 Simultaneous Equilibrium

The first question that I consider concerns the existence of the PBE in this game. To proceed, note that in regions B and C, as well as their mirror images, and in region D, at least one of the duopolists will not be making any profit. This is because at the chosen prices the agents are better off consuming their competitor's good or not consuming at all. Unlike the heterogeneous Bertrand outcome, the lowest price does not guarantee any demand (and thus profit) at all. This paradox results from the price playing both the role of the price as well as the role of quality. In fact, in region C the consumers strictly prefer to consume the more expensive brand than to consume the less expensive brand. If people were unable to alter their choices - so that the threshold stayed in place - the collective switch to consuming good 1 would be a weak Pareto improvement for the consumers. But given that the thresholds are allowed to change, this will not be the case. The lower price will induce more agents to buy good 1 and therefore lower the status signal to the point where this is not an attractive option for anyone.

As a consequence, unlike in the Bertrand competition, no equilibrium in pure strategies exists. The proof proceeds in the following steps: (i) PBE cannot exist in regions B (B'), C (C') or D; (ii) PBE cannot exist on the borders between regions C and C', (iii) PBE cannot exist in region A (A'), (iv) thus no PBE can exist at all. To simplify the exposition, I describe the strategy profile simply as a set of prices

b_1, b_2 chosen by the two oligopolists, as consumer and social contacts' strategies and beliefs are easy to deduce. The result is stated in the proposition below:

Proposition 2.16. *No pure-strategy PBE exists when two identical positional good duopolists simultaneously choose their prices.*

Why does the equilibrium fail to exist? Consider any point where firms set equal prices that are not too high for any demand to exist (i.e. not in region D). This means that the firms divide the total demand in half. But then, either firm has an incentive to set the price marginally higher than its competitor as this allows it to capture the whole market. This allows it to benefit not only from an infinitesimally increased price, but also from a discrete increase in demand. However, following the first deviation, the other firm will have an incentive to deviate by "over-cutting" as well, triggering a process in which firms take turns in "over-cutting" again and again. In all this note that the profit actually increases (for the winner who "wins" the competition) only up until the point where the price is below a certain value, and for higher prices it begins to fall. Consequently, there might be a point where it would actually be beneficial for one of the players to deviate to region A (A') as the lower-priced player, as the profit from doing so will be higher than the profit from further "over-cutting". However, once one of the players switches to charging a much lower price, it becomes optimal for the other player to just "over-cut" the price again, and the whole process continues. Therefore, there could be no equilibrium as firms would constantly change their prices in a cycle. If qualities (or statuses associated with the brands in my model) were exogenously fixed or sufficiently "sticky", the demand functions would be smooth, so such a pattern would not be possible and an equilibrium would exist (Gabszewicz and Thisse 1979). The results become easier to visualise when the best reply functions are derived in the next section and presented in Figure 2.3.

There appear to be three routes to escape this non-existence result. First, the firms could enter the market sequentially which I examine in the next sub-section.

Second, there could be sufficient difference in economies of scale between the two firms to force the high-cost firm to choose a high price to make a positive profit even bearing the cost of low demand. Third, the market might be characterised by a certain stickiness of the brand image and thus the historically "low-priced" firm would not be able to set the price higher than the historically "high-priced" firm as it would face zero demand. I will focus on the first and second of these possibilities in the next sub-sections, while the third case will not be pursued as thorough the right re-parametrisation it becomes similar to the second case.

2.5.2 Sequential Entry

In reality we should rarely expect simultaneous entry in case of positional goods: the usual pattern is for the first-mover to create a fad for a certain type of conspicuous consumption, and then for the imitators to offer similar products at lower prices. In fact, we rarely observe significant price changes that would be predicted by the description of the vicious cycle above; instead, prices of positional goods appear to be "sticky". Therefore, sequential entry might be considered a plausible alternative.

Assume firm 1 moves first, followed by firm 2. I begin by describing fully the best response functions of both players. This is done in two steps: (i) for any b_1 in a certain region k, find the best reply $b_2^k(b_1)$, where $k \in \{A, A', B, B', C, C', D\}$, in that region and calculate the resulting profit for firm 2; (ii) for each possible value of b_1 describe the best reply chosen by firm 2, $b_2^*(b_1)$ (comparing the profits that can be obtained in every region). The result is summarised in the lemma below:

Lemma 2.3. *Player 2's best reply function is:*

$$b_2^*(b_1) = \begin{cases} \frac{1}{4}\alpha\theta^2 & \text{if } b_1 \in [0, \frac{1}{4}\alpha\theta^2) \\ b_1 + \tau & \text{if } b_1 \in [\frac{1}{4}\alpha\theta^2, (\sqrt{2}-1)\alpha\theta^2) \\ \frac{b_1^2}{\alpha\theta^2} & \text{if } b_1 \in [(\sqrt{2}-1)\alpha\theta^2, \frac{1}{2}\alpha\theta^2) \\ \frac{1}{4}\alpha\theta^2 & \text{if } b_1 \in [\frac{1}{2}\alpha\theta^2, \infty). \end{cases}$$

We have derived a full best reply map for firm 2. By symmetry, we can similarly derive the best reply map for firm 1. This allows us to take another look at the lack of simultaneous-entry Nash equilibrium and the subsequent discussion by examining these best reply functions illustrated in Figure 2.3. The sequential-entry equilibrium, which is derived below, is illustrated in Figure 2.3 by a grey circle.

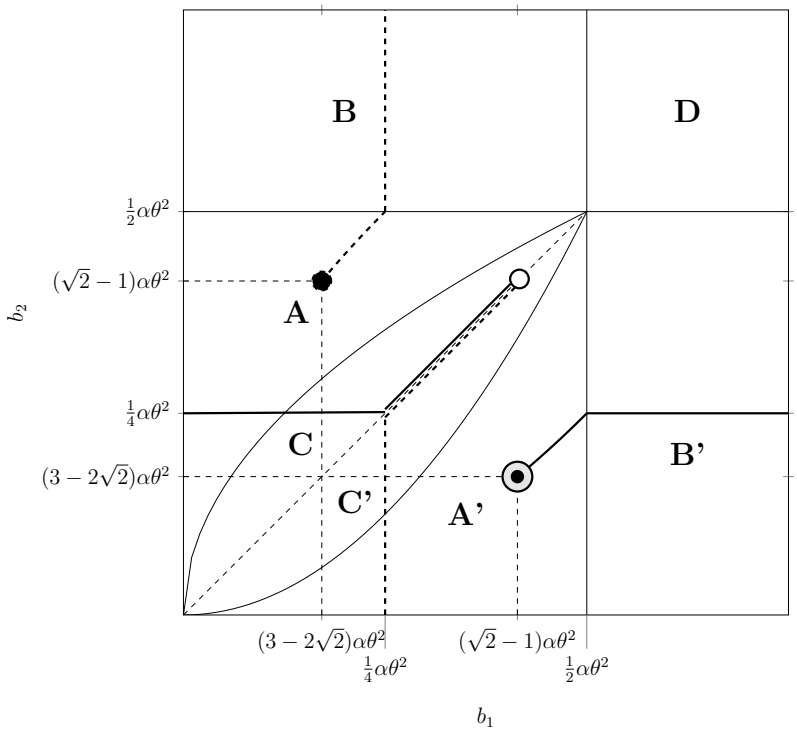


Figure 2.3: Map of the best reply sets for both firms (dashed line for firm 1, solid line for firm 2). Empty and full black circles denote discontinuities. For sequential entry, the grey circle denotes the equilibrium.

We only need two more steps to derive the sequential equilibrium: (i) for each range of b_1 find the value of b_1 that locally maximises firm 1's profit *given* the reply of the other firm; (ii) henceforth choose the profit-maximising b_1 in the range of b_1 which maximises the profit globally. Below is the main result of this section:

Proposition 2.17. *When two identical positional good duopolists choose their prices sequentially, firm* 1 *chooses* $b_1^* = (\sqrt{2}-1)\alpha\theta^2$ *while firm* 2 *chooses* $b_2^* = (3-2\sqrt{2})\alpha\theta^2$. *Their profits are identical, so there is no first-mover advantage.*

The equilibrium consists of the first mover choosing a high price and selling to the

Variable	Monopoly	Duopoly	Comparison
b_1	$\frac{1}{9}\alpha\theta^2$	$(3-2\sqrt{2})\alpha\theta^2$	$b_1^{mon} < b_1^{duo}$
b_2	$\frac{1}{3}\alpha\theta^2$	$(\sqrt{2}-1)\alpha\theta^2$	$b_2^{mon} < b_2^{duo}$
$\tilde{w}_1$	$\frac{1}{3}\theta$	$(\sqrt{2}-1)\theta$	$\tilde{w}_1^{mon} < \tilde{w}_1^{duo}$
$\tilde{w}_2$	$\frac{2}{9}\theta$	$2(\sqrt{2}-1)\theta$	$\tilde{w}_2^{mon} < \tilde{w}_2^{duo}$
π_1	$\frac{1}{27}\alpha\theta^2$	$(5\sqrt{2}-7)\alpha\theta^2$	$\pi_1^{mon} > \pi_1^{duo}$
π_2	$\frac{1}{9}\alpha\theta^2$	$(5\sqrt{2}-7)\alpha\theta^2$	$\pi_2^{mon} > \pi_2^{duo}$
π_{TOTAL}	$\frac{4}{27}\alpha\theta^2$	$2(5\sqrt{2}-7)\alpha\theta^2$	$\pi_{TOTAL}^{mon} > \pi_{TOTAL}^{duo}$
CS	$\frac{19}{108}\alpha\theta^2$	$\frac{1}{4}(53-37\sqrt{2})\alpha\theta^2$	$CS^{mon} > CS^{duo}$
TW	$\frac{35}{108}\alpha\theta^2$	$\frac{3}{4}(\sqrt{2}-1)\alpha\theta^2$	$TW^{mon} > TW^{duo}$

Table 2.2: Comparison of the market outcomes under monopoly and duopoly with sequential entry.

high end of the market as well as the second mover choosing a low price and selling to the low end of the market (yet not to the poorest agents). Key to the existence of the equilibrium is the fact that the second mover does not have an incentive to deviate to "over-cutting" the first mover, as such a deviation lowers its profit (since the deviator would have to increase the price by a small τ). This is the key difference *vis-à-vis* the no-equilibrium result of the simultaneous entry game: the first mover understands that if it lowered the price to what its simultaneous entry best reply function would suggest, the other player would respond by "over-cutting" the price and thus annihilating player 1's profit.

Table 2.2 presents a comparison *versus* a monopolist with $N = 2$.[19] Note that the duopolists set higher prices for both goods than a monopolist would have set: the latter internalises the consequences that its pricing of the low-priced good has on the demand for the high-priced good. It contents itself with a low price at the low end as this keeps the average income in the group of buyers of good 1 low. This renders it unattractive to the high-income agents, who care considerably about status because of the increasing differences. Hence the group feels the need to buy the more expensive good 2, which can be priced higher to extract the extra surplus. In other words, the monopolist makes the bulk of its profit off the wealthier agents:

[19]In the duopoly case, the first mover is re-labelled with subscript 2, while the second mover with subscript 1 for consistency.

its profit from good 2 is three times that from good 1. For the duopolists, such internalisation is impossible due to their strategic competition. If the monopoly prices were to be set, both players would have an incentive to deviate: the low-priced player would like to just "over-cut" the opponent, while the high-priced player would like to lower its prices to increase its market share. Therefore, in equilibrium such behaviours must be prevented by the means of the price of the more expensive good being higher than under monopoly. The consequences are twofold: first, this reduces the incentives to "over-cut" by the lower-priced player as its profit is lower than what could be made as the lower-priced competitor, and second, it allows the other player to charge a price for the less expensive good that is higher than under monopoly, given that the status signal that it conveys is higher (richer agents buy the good compared to monopoly). The first mover understands these facts and thus does not seek to reduce its price.

These effects might help to understand the paradoxical situation when prices of all goods are *higher* under duopoly than under monopoly. As a consequence of the competition, equal profits are made off either good: there is no first-mover advantage. Both consumer surplus and total welfare are higher under the monopoly, which stands in contrast to the standard results. This is hardly surprising given that the monopolist chooses the optimal prices when $\epsilon = 0$, but appears to be highly dependent both on the assumption that the poorest member of society does not care about status and the uniform income distribution. It should be expected, that for small values of ϵ it may still remain the case that the monopoly is preferred from a welfare point of view. This example, therefore, provides an interesting case for the positive role that the market power may play in certain markets. While Corneo and Jeanne (1997) reach the same conclusion, their motivation is different. In their model the monopolist produces less than the competitive supply, while in my model the positive impact of the monopolist results from the improved revelation of incomes.

Unlike other models of imperfect competition, it is not obvious what happens in this model with sequential entry as the number of firms increases and approaches infinity. The standard result about prices approaching marginal costs (zero in this case) would be troubling as identical prices effectively kill any differentiation among the products as they convey no message about their owner's income. Alas no conclusions can be reached without extending the model to an arbitrary number of oligopolists and looking at the behaviour in the limit, but this appears difficult analytically.[20] Likewise, any attempt to extend the model to allow at least one of the duopolists to produce more than one good is also bound to lead to serious issues of tractability. The alternative approach assuming different marginal costs may give us some additional intuition.

2.5.3 Difference in Costs

As was alluded to previously, another way to escape the non-existence result in a simultaneous game is to consider the inherent difference in the cost structures between the duopolists. This can be shown to be isomorphic to a model with inherent differences in the desire to buy a certain brand. For simplicity, I assume that the low-cost duopolist (denoted L) has a constant marginal cost of zero, while the high-cost duopolist (denoted H) has a constant marginal cost of $c > 0$. The proposition below summarises the equilibrium which is illustrated in Figure 2.4:

Proposition 2.18. *When two duopolists labelled L and H have different marginal costs, $c_L = 0$ and $c_H > 0$, and simultaneously enter the market, as long as $\frac{1}{2}(4\sqrt{2} - 5)\alpha\theta^2 \leq c < \frac{1}{2}\alpha\theta^2$ there exists a unique PBE in which firm L makes a higher profit than firm H. Consumer surplus and total welfare are lower than they would be if a monopolist with an identical cost structure were present in the market instead of the duopolists.*

[20] Such an extension has been explored e.g. by Gabszewicz and Thisse (1979) in a simpler setup with fixed, exogenous qualities and has not been too successful.

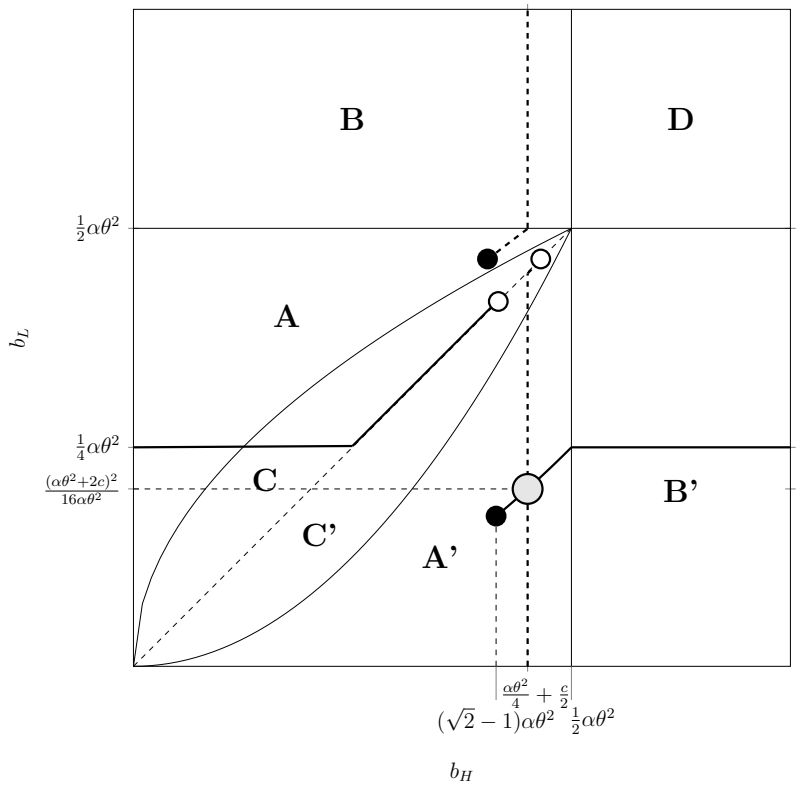


Figure 2.4: Map of the best reply sets for both firms when one of the firms faces a positive marginal cost, $\hat{c} \leq c < \hat{b}$ (dashed line for firm H, solid line for firm L). Empty and full black circles denote discontinuities, while the grey circle denotes the equilibrium.

The high-cost firm must offer the higher-priced good in order to make a positive profit. However, for it to be able to do so, the marginal cost cannot be too high, as otherwise it would face no demand. On the other hand, the marginal cost cannot be too low, as that would lead to the "over-cutting" cycle discussed before, even though firm H always makes a lower profit due to the cost it incurs. This is somewhat similar to Gabszewicz and Thisse (1979), in that the cost must be high enough compared to the measure of income dispersion θ for both firms to produce positive quantities. In the equilibrium, $\frac{(\alpha\theta^2+2c)^3}{64(\alpha\theta^2)^2} = \pi_L^* > \pi_H^* = \frac{(\alpha\theta^2-2c)^2}{8\alpha\theta^2}$, so the low-cost firm always enjoys a higher profit. There are two reasons for this. First, the demand for the lower-priced brand, $\frac{\tilde{w}_H^*-\tilde{w}_L^*}{\theta}$, is greater than the demand for the higher-priced brand, $\frac{\theta-\tilde{w}_H^*}{\theta}$ as due to the strategic effects the spacing of thresholds in no longer equidistant. The high-cost firm is unable to relocate and serve the lower-income agents, because this

would require a reduction in price that it cannot afford. Second, despite its lower price, firm L's margin on each unit sold, b_L^*, is higher than that of firm H, $b_H^* - c$.

As was the case for the sequential entry game, it would seem instructive to compare the above duopoly outcome to that of a monopolist who produces two goods, of which only one requires a positive marginal cost to be incurred. However, as per Proposition 2.15, for the values of c that guarantee existence of the PBE for the duopolists, the monopolist would prefer to offer only one brand whose marginal cost is zero. Then, as per Proposition 2.1, $b_L^* < b^* = \frac{1}{4}\alpha\theta^2 < b_H^*$ and $\tilde{w}_L^* < \tilde{w}^* = \frac{1}{2}\theta < \tilde{w}_H^*$, so the price and the resulting threshold set by the monopolist will be between the prices and thresholds set by the oligopolists. The combined profit enjoyed by the duopolists must by lower than that of the monopolist (who always has to option to choose the same prices as they do). By a reasoning similar to that of Section 2.4, consumer surplus is lower under a duopoly than under a monopoly as less surplus is extracted from the agents. Consequently, a monopoly must be preferred to a duopoly from a total welfare perspective. This conclusion is shared with Gabszewicz and Thisse (1979), who argue that the outcome where only one firm stays in the market is preferred from the welfare perspective, although in their model it would be the high-quality firm.

While it is difficult to obtain an analytically rigorous result, it seems that the reasoning above may be extended to allow for a larger number of brands. The equilibrium may still exist provided that the differences in the cost structures are sufficiently large. Moreover, when the number of firms that produce brands at the same marginal costs increases, it is possible that the prices will collapse towards the marginal costs (which are not all zero anymore) and the consumers will then choose their preferred brands. This appears to be a fruitful direction of further research on this topic.

2.6 Conclusion

This chapter has modelled a particular form of status rivalry where agents want to be perceived as high income and their only means of signalling their wealth is the purchase of branded goods whose quality is *ex-ante* identical. I found that if the monopolist produces several brands, the difference in prices between consecutive brands (ordered by price) increases in the status that the brands confer. This may explain, for example, why the premium commanded by iPhone X over the also newly released iPhone 8 is higher than the premium commanded by iPhone 8 over the older iPhone 7.[21]

My model suggests that we should observe continuous sets of positional goods on offer as they enable the monopolist to extract the maximum amount of surplus and allow for a perfect transmission of information about incomes.[22] In reality, however, we observe a limited number of brands. My model identified two potential reasons for this: the semi-fixed costs of brand introduction and sufficiently large differences in the marginal costs of additional brands. In the former case, the monopolist offers too broad a spectrum of brands from which agents choose (compared to the social optimum). This makes it closer in spirit to Chamberlin (1933) than Dixit and Stiglitz (1977), who demonstrate that the market may give too little variety. In the latter case, if the monopolist faces a positive marginal cost, the overall welfare is augmented when this cost is high. From a practical perspective, this suggests that positional goods should be expensive in production to minimise welfare losses. Such a conclusion has been advocated on several other grounds; see e.g. Mandler (2018) who considers incentives to produce counterfeit positional goods.

My model predicts that the average consumer is usually hurt by the existence of

[21] This obviously disregards the differences in quality between the different models of the iPhone. It may be argued, however, that for a significant proportion of the population the technical improvements in the newest models of mobile phones are less important than the status considerations.

[22] A fictional example would be a sticker whose price would be equal to agent's income and which would state in a non-falsifiable way how much it cost (obviously this abstracts from social customs which render such a device unlikely to ever succeed).

positional goods. Such a result is common in the literature, but unlike e.g. Frank (2005), my model holds that the positional rivalry is not a zero-sum game and the revelation of agents' true incomes can as such be welfare-improving. When even the poorest agent cares about status, a per-unit tax which maximises total welfare can in fact increase the consumer surplus above the baseline level where positional goods are not allowed. Note that the optimal tax is essentially regressive (as the per-unit rate is fixed, while prices of different brands are not), but this result is driven by the assumption of uniform distribution of income, which may be the reason why it is not generally observed.

Identification of the winners and losers when a new brand is introduced constitutes an interesting result, which can be readily applied to real-life examples. Consider the case of iPhone X again, and for simplicity assume that the old iPhone 7 is the only available alternative to it.[23] The people who used to *just* buy iPhone 7 benefit from the introduction of the new model as the monopolist reduces the price of the old one. Thanks to the lower price, the people who used to *just* not buy iPhone 7 benefit as the phone becomes more affordable. However, those at the high end of the income spectrum lose out: while it becomes easier to recognise their high incomes, the monopolist extracts more surplus than before. Those at the low end of the income spectrum also stand to lose, as some are pressured into buying the cheaper iPhone to avoid being perceived as very low income, while those who uphold their decision to not buy see their status deteriorate.

The results of this model may be compared with the aforementioned benchmark of Gabszewicz, Shaked, et al. (1986), who reach two main conclusions. First, they show that when the cost rises only slowly with quality relative to the willingness to pay for quality, there exists an upper bound to the number of firms which enjoy prices above the unit variable costs as well as positive demands in the equilibrium. Thus, if there were a fixed cost of entry, only a single monopolist would survive in the market.

[23] Perhaps to make this example more plausible we should restrict our sample only to people who could in principle afford an iPhone.

My model does not directly address this issue. However, as it assumes that the lower support of the income distribution is zero, which guarantees only partial coverage of the market, this needs not necessarily be true. Second, Gabszewicz, Shaked, et al. (1986) demonstrate that unless the dispersion of incomes is sufficiently large (compared to the difference in production costs), the monopolist will produce the maximum allowed number of brands. However, when the income dispersion falls, the market share of the top-quality good rises continuously, so that at some point it coincides with the entire market. My model concurs that in the absence of costs, the monopolist will produce as many brands as possible. However, due to linear utility and the determination of qualities through prices, falling income dispersion may only lower the number of brands on offer, but not immediately reduce it to one. This is because when one brand is removed from the offering, the income thresholds, and consequently the resulting qualities, would move to restore the equal market shares of all brands.

A more competitive market structure may result in some counter-intuitive welfare results in my model. When the duopolists enter sequentially, they both set the prices higher than the monopolist would. The first mover offers the higher-priced brand but enjoys no first-mover advantage: unlike the standard Stackelberg result, this is the consequence of the market leader having to just discourage the follower from "over-cutting". If the duopolists enter simultaneously, but face different marginal costs, the low-cost firm enjoys a significant profit advantage in the equilibrium. If the non-zero marginal cost is high enough to guarantee the existence of such an equilibrium, but if a monopolist were present in the market instead of the duopolists, it would only offer the low-cost brand in these circumstances. Regardless of the type of the duopolistic equilibrium, both consumer surplus as well as overall welfare are lower under a duopoly than under a monopoly. An interesting real-life consequence of this is the difficulty it poses for the conventional anti-trust policy.

On a final note, this chapter deliberately abstracted from the quality of the brand

to focus on the purely positional aspects of the status goods. Further research into the intrinsic differences in quality might shed some light on the intersection between quality and status (Deltas, Stengos, et al. 2011; Deltas and Zacharias 2012; Dritsa and Zacharias 2012; Lambertini and Orsini 2002, 2005). In addition, a rigorous extension of the duopoly analysis to allow for a larger number of firms, with each producing an arbitrary number of brands, would not only be more realistic, but moreover allow for a better comparison with the non-positional benchmarks of the product differentiation literature.

Appendix 2.A Changes in Inequality

One of the key limitations of the model discussed above is the choice of the uniform income distribution, which is a simplification necessary to enable multiple brand analysis. Such a choice also precludes us from analysing the effects of changes in income inequality. To overcome these limitations, I now employ a symmetric triangular distribution (in a single-brand framework), which also offers the side benefit of accurately representing a society with a sizeable middle class. Let income w be distributed on $[\omega - \tau, \omega + \tau]$, where $0 \leq \tau < \omega$, with a probability density function:

$$g(w) = \begin{cases} 0 & \text{if } w \in [0, \omega - \tau) \cup [\omega + \tau, \infty) \\ \frac{w-(\omega-\tau)}{\tau^2} & \text{if } w \in [\omega - \tau, \omega) \\ \frac{(\omega+\tau)-w}{\tau^2} & \text{if } w \in [\omega, \omega + \tau). \end{cases}$$

The distribution is illustrated in Figure 2.5 for a high and a low value of τ. ω is the mean (mode) of the distribution which is kept fixed, while τ is the spread which may be interpreted as a degree of income inequality ($\tau = 0$ corresponds to a perfectly equal society, while $\tau = \omega$ to a very unequal one). The following lemma re-examines the equilibrium in this new context:

Lemma 2.4. *Under a symmetric triangular distribution described by ω and τ as*

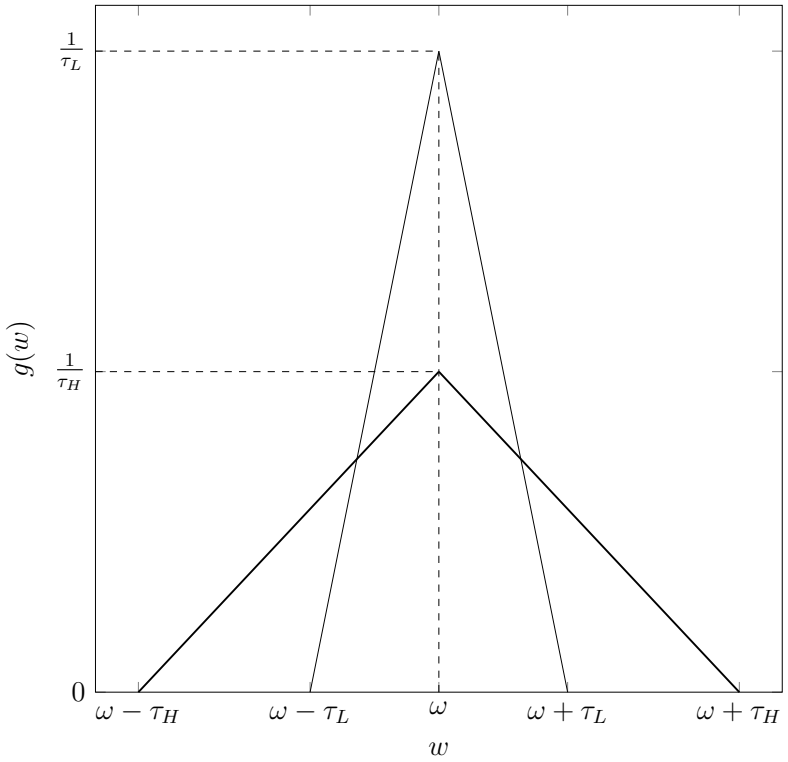


Figure 2.5: Changes in inequality modelled on a triangular distribution ($\tau_L < \tau_H$).

above, in the unique PBE which survives the D1 criterion:

(i) if $\tau > \frac{2}{5}\omega$*, the coverage is partial and the monopolist sets a price that results in the threshold of* $\tilde{w}^* = \frac{1}{4}(2\omega + \tau)$ *and the profit of* $\pi^* = \frac{1}{24}\alpha(2\omega + \tau)^2$*,*

(ii) if $\tau \leq \frac{2}{5}\omega$*, the coverage is full and the monopolist sets the price of* $b^* = \alpha\tau(\omega - \tau)$ *that results in the profit of* $\pi^* = \alpha\tau(\omega - \tau)$*.*

Regardless of the degree of the market coverage, the profit goes up when society becomes more unequal (τ increases). Note that it may be possible that the market may be initially fully covered, but when inequality increases, at a certain point the price will be raised high enough to discourage at least some agents from buying and guarantee partial coverage. When τ goes up, there are two effects on the inference of the social contacts. First, the wealth is transferred from the non-buyers' group to the buyers' group. Second, the composition of both groups changes as fewer agents decide to buy the positional good (the threshold $\tilde{w}$ goes up). As a consequence, with more inequality the social contacts' inference about the non-buyers' incomes is going

down, while the inference about the buyers' incomes is going up. The impact on the profit is driven by the fact that while fewer agents decide to buy the good, they are paying a price that is increasing at a faster pace than the pace of the demand reduction: the increased profit of the monopolist come on the back of the wealthier agents in society.

The following proposition summarises the welfare consequences of an increase in income inequality:

Proposition 2.19. *Assume a symmetric triangular distribution described by ω and τ as above. As τ increases, consumer surplus always strictly falls, while total welfare always weakly increases, with strict increase under partial coverage (or a switch from full to partial coverage).*

The result about the consumer surplus may appear puzzling as the demand is compressed when inequality is high, and fewer purchasers should translate to improved consumer surplus. However, this is not the case because the increase in price outweighs the contraction in demand: the inference about the buyers' average status goes up due to the increased amount of wealth concentrated in the buying region, which results in a larger amount of potential surplus, which in turn results in a considerable rise in the price charged by the monopolist. This is true even if the market is fully covered or when the degree of market coverage changes. In the case of full coverage, the fall in the consumer surplus is in fact exactly equal to the increase in profit, so the total welfare remains unchanged. This is not surprising as under full coverage the total amount of surplus is fixed (and in the given specification this does not depend on τ) and the distribution of surplus between the agents and the monopolist does not matter. However, under partial coverage, this is no longer true: there will be a trade-off between smoother revelation of information about status and over-provision of the positional good. When income inequality increases, the former effect dominates, as there are more extremely rich agents for whom to obtain a higher status is of great importance due to the increasing differences of the utility

function.

Appendix 2.B Omitted Proofs

Proof of Proposition 2.1

There is a single income threshold $\tilde{w}_1$ below which every agent chooses not to consume, and above which every agent chooses to consume. The social contacts' inference is ρ_1 if brand 1 is observed and ρ_0 if no brand is observed. Then the agent $\tilde{w}_1$'s indifference condition is:

$$-b_1 + \alpha(\tilde{w}_1 + \epsilon)\rho_1 = \alpha(\tilde{w}_1 + \epsilon)\rho_0. \tag{2.3}$$

Assume that $\tilde{w}_1 > 0$, so that the market is partially covered, and this indifference condition is meaningful. The only piece of information that the social contacts receive is whether or not the branded good is bought. Subsequently, they infer that only agents with income in $[\tilde{w}_1, \theta]$ buy brand 1. Consequently, as per (2.1) with uniform distribution, their inferences are $\rho_1 = \frac{\tilde{w}_1+\theta}{2}$ and $\rho_0 = \frac{\tilde{w}_1}{2}$. Taking (2.3) and the above facts together allows us to solve for the threshold income, $\tilde{w}_1 = \frac{2b_1}{\alpha\theta} - \epsilon$. The demand for the positional good is $x_1 = \frac{\theta-\tilde{w}_1}{\theta}$. Hence the monopolist's problem becomes choosing b_1 to maximise:

$$\pi_1 = b_1 x_1 - C_1(x_1) = b_1\frac{\theta - \tilde{w}_1}{\theta} - 0 = \frac{b_1\big(-2b_1 + \alpha\theta(\theta+\epsilon)\big)}{\alpha\theta^2}. \tag{2.4}$$

Differentiating with respect to b gives us the FOC, $\frac{-4b_1^*+\alpha\theta(\theta+\epsilon)}{\alpha\theta^2} = 0$. Differentiating again gives us $\frac{\partial\pi_1^2}{\partial b^2} = \frac{-4}{\alpha\theta^2} < 0$, so the profit function is concave and the optimum $b_1^* = \frac{\alpha\theta(\theta+\epsilon)}{4}$ is indeed a maximum. The resulting threshold income is $\tilde{w}_1^* = \frac{2b_1^*}{\alpha\theta} - \epsilon = \frac{\theta-\epsilon}{2}$. The market is indeed partially covered, i.e. $\tilde{w}_1^* > 0$, only when $\epsilon < \theta$. The maximised profit is $\pi_1^* = \frac{\alpha(\theta+\epsilon)^2}{8}$. As this is the unique semi-separating equilibrium, the D1 criterion is clearly not violated.

Suppose now that the market is fully covered and $\epsilon \geq \theta$. The profit under full coverage is $\pi_1 = b_1 x_1 = b_1$. Thus, the monopolist wants to set the price as high as possible subject to no agent having an incentive to deviate from buying. Fix an equilibrium in which all agents buy at price b_1. Assume that upon observing a deviation to not buying the social contacts make an inference $\rho \in [0, \theta]$. An agent with income w only deviates if $\alpha(w+\epsilon)\rho > -b_1 + \alpha(w+\epsilon)\frac{\theta}{2}$, or $\rho > \frac{\theta}{2} - \frac{b}{\alpha(w+\epsilon)} = \hat{\rho}(w)$. Define the following sets: $D_w = \{\rho \in [0, \theta] : \rho > \hat{\rho}(w)\}$, the set of inferences which lead an agent with income w to deviate, and $D_w^0 = \{\rho \in [0, \theta] : \rho = \hat{\rho}(w)\}$, the set of inferences which make the agent with income w indifferent between deviating and not deviating. The D1 criterion requires that we upon observing a deviation we put zero probability on w if there exists a w' such that $D_w \cup D_w^0 \subseteq D_{w'}$. As $\hat{\rho}(w)$ is an increasing function of w, when $w' < w$, we have that $\hat{\rho}(w) > \hat{\rho}(w')$. Consequently, it must be true that $D_w \cup D_w^0 \subseteq D_{w'}$ and through this method the only type of agent on which positive probability is put upon deviation is $w = 0$. This means that the inference of the social contacts upon deviation must be $\rho = 0$. It remains to determine the maximum price that the monopolist can set. If it induces the poorest agent to buy, the other agent will buy as well because of the increasing differences of the utility function, so the condition is $-b_1 + \alpha(0+\epsilon)\frac{\theta}{2} \geq \alpha(0+\epsilon)0$, and thus $b_1^* = \frac{1}{2}\alpha\theta\epsilon$. The maximised profit is $\pi_1^* = b_1^* = \frac{1}{2}\alpha\theta\epsilon$.

Finally, I show that these equilibria are indeed unique by proving that a no-buy equilibrium will not arise. The proof is analogical to the one above, with the agent deviating to buying only if $-b_1 + \alpha(w+\epsilon)\frac{\theta}{2} > \alpha(w+\epsilon)\rho$. Using the D1 criterion, positive probability upon deviation may only be put on the wealthiest agent $w = \theta$, so the inference in this case must be $\rho = \theta$. Then the condition for this to constitute an equilibrium requires that no agent buys, including the wealthiest agent: $\alpha(\theta+\epsilon)\theta > -b_1 + \alpha(\theta+\epsilon)\frac{\theta}{2}$, or $b_1 > \frac{\alpha\theta(\theta+\epsilon)}{2}$. But the optimal prices in both equilibria above do not meet this condition, so they must be unique.

Proof of Proposition 2.2

The consumer surplus (and in absence of a profit also the overall welfare) when positional goods do not exist is $TW_0^* = CS_0^* = \int_0^\theta u(w)f(w)dw = \int_0^\theta \alpha(w+\epsilon)\frac{0+\theta}{2}\frac{1}{\theta}dw = \frac{1}{4}\alpha\theta(\theta+2\epsilon)$. When one positional good appears, consumer surplus becomes:

$$\begin{aligned} CS_1 &= \int_0^{\tilde{w}} \alpha(w+\epsilon)\frac{0+\tilde{w}}{2}\frac{1}{\theta}dw + \int_{\tilde{w}}^{\theta}(-b+\alpha(w+\epsilon)\frac{\tilde{w}+\theta}{2})\frac{1}{\theta}dw \\ &= \frac{\alpha\theta\big(-\tilde{w}^2+\theta\tilde{w}+\theta(\theta+2\epsilon)\big)-4b(\theta-\tilde{w})}{4\theta}, \end{aligned} \tag{2.5}$$

which at the profit-maximising price simplifies to:

$$CS_1^* = \begin{cases} \frac{1}{16}\alpha(3\theta^2+4\theta\epsilon-3\epsilon^2) & \text{if } 0 \le \epsilon < \theta \\ \frac{1}{4}\alpha\theta^2 & \text{if } \epsilon \ge \theta. \end{cases}$$

It is then trivial to verify that $CS_0^* > CS_1^*$.

Proof of Proposition 2.3

Total welfare is the sum of consumer surplus and profit:

$$TW_1^* = CS_1^* + \pi_1^* = \begin{cases} \frac{1}{16}\alpha(5\theta^2+8\theta\epsilon-\epsilon^2) & \text{if } 0 \le \epsilon < \theta \\ \frac{1}{4}\alpha\theta(\theta+2\epsilon) & \text{if } \epsilon \ge \theta. \end{cases}$$

It is trivial to verify that $TW_1^* > TW_0^*$ for partial coverage and $TW_1^* = TW_0^*$ for full coverage.

Proof of Proposition 2.4

Assume partial coverage first. Using (2.4) and (2.5), the total welfare as a function of the price is $TW = CS_1(\tilde{w}, b)+\pi(b) = \frac{\alpha^2\theta^2(\theta^2+\theta\epsilon-\epsilon^2)-4b^2+2\alpha\theta(\theta+2\epsilon)b}{4\alpha\theta^2}$, where the threshold was expressed as a function of price using indifference condition (2.3). The FOC, $\frac{1}{2}+\frac{\epsilon}{\theta}-\frac{2b^{SO}}{\alpha\theta^2} = 0$, gives us the socially optimal price $b^{SO} = \frac{1}{4}\alpha\theta(\theta+2\epsilon)$, and the

resulting socially optimal threshold is $\tilde{w}^{SO} = \frac{\theta}{2}$. The partial coverage outcome must be compared with the total welfare under full coverage, which is strictly increasing in price b. Thus it is maximised at the largest possible value of price, $b = \frac{1}{2}\alpha\theta\epsilon$, and the resulting total welfare is $\frac{1}{4}\alpha\theta(\theta + 2\epsilon)$. But the maximised welfare under partial coverage is $TW^{SO} = \frac{1}{16}\alpha\theta(5\theta + 8\epsilon)$, which is strictly larger. Therefore, the social planner always prefers partial coverage. Clearly if $\epsilon > 0$, $\tilde{w}^{SO} = \frac{\theta}{2} > \frac{\theta-\epsilon}{2} = \tilde{w}^*$ and $\tilde{w}^{SO} > 0$, so regardless of the degree of market coverage under monopoly, the socially optimal threshold is higher than the market outcome. Similarly, if $\epsilon > 0$, $b^{SO} > b^*$ regardless of the degree of coverage. When $\epsilon = 0$, $\tilde{w}^{SO} = \tilde{w}^*$ and $b^{SO} = b^*$.

Proof of Proposition 2.5

The indifference condition (2.3), assuming the market is partially covered, becomes $\alpha(\tilde{w} + \epsilon)\frac{\tilde{w}}{2} = -(b + t) + \alpha(\tilde{w} + \epsilon)\frac{\tilde{w}+\theta}{2}$, while the profit function remains unchanged. Proceeding similarly to Proposition 2.1, we obtain the new profit-maximising price of $b^{**}(t) = \frac{1}{4}\big(\alpha\theta(\theta + \epsilon) - 2t\big)$, which gives the threshold $\tilde{w}^{**}(t) = \frac{1}{2}(\theta - \epsilon) + \frac{t}{\alpha\theta}$. Therefore, to achieve the optimal level of consumption, the government calibrates t so that $\tilde{w}^{**}(t^{**}) = \tilde{w}^{SO} = \frac{\theta}{2}$, i.e. sets $t^{**} = \frac{1}{2}\alpha\theta\epsilon$. Trivially, $\frac{\partial t^{**}}{\partial\theta} > 0$, $\frac{\partial t^{**}}{\partial\epsilon} > 0$ and $\frac{\partial t^{**}}{\partial\alpha} > 0$.

To see whether the tax constitutes a Pareto improvement, note that there will be three groups of agents: agents with incomes $w \in [0, \frac{\theta-\epsilon}{2})$ who used to not buy and still do not buy after the introduction of the tax, agents with incomes $w \in [\frac{\theta-\epsilon}{2}, \frac{\theta}{2})$ who used to buy but do not buy with the tax, and agents with incomes $w \in [\frac{\theta}{2}, \theta]$ who used to buy and still buy with the tax. In the first group, everyone is better off as the inference about their incomes ρ_0 went up. In the third group, agents used to pay $\frac{1}{4}\alpha\theta(\theta + \epsilon)$, but with the tax (which is redistributed back to them) they effectively pay $b^{**}(t^{**}) = \frac{1}{4}\alpha\theta(\theta - 3\epsilon) < \frac{1}{4}\alpha\theta(\theta + \epsilon)$ and obtain a higher status (as the poorest buyer has $w = \frac{\theta}{2}$ rather than $w = \frac{\theta-\epsilon}{2}$), so everybody must be better off. In the second group, however, the old utility is $-\frac{1}{4}\alpha\theta(\theta + \epsilon) + \alpha(w + \epsilon)\frac{\frac{\theta-\epsilon}{2}+\theta}{2}$, while

the new utility is $\alpha(w+\epsilon)\frac{0+\frac{\theta}{2}}{2}$. For the richest member with the strongest positional concern ($w = \frac{\theta}{2}$) this is not an improvement. Therefore, there exist agents who used to buy the positional good, but after the tax is introduced are just forced out of buying, who lose out.

Proof of Proposition 2.6

The profit function with constant marginal cost c becomes $\pi = (b-c)\frac{\theta-\tilde{w}}{\theta}$. We proceed as in Proposition 2.1. It must be the case that $b^* > c$ or otherwise the monopolist makes a loss, so this condition has to be satisfied by the solution. Consider partial coverage first. The FOC of the new profit function with respect to price gives $b^* = \frac{1}{4}\alpha\theta(\theta+\epsilon) + \frac{c}{2}$. To satisfy the condition above, the cost cannot be too high, $c \leq \frac{1}{2}\alpha\theta(\theta+\epsilon) = \bar{c}$. Assume that this is true. The new threshold is then $\tilde{w}^* = \frac{\theta-\epsilon}{2} + \frac{c}{\alpha\theta}$. The resulting profit is $\pi = \frac{(\alpha\theta(\theta+\epsilon)-2c)}{8\alpha\theta^2}$. The partial coverage is guaranteed when $\tilde{w}^* > 0$, which translates to $0 \leq \epsilon < \theta$, or $\epsilon \geq \theta$ while $c > \frac{1}{2}\alpha\theta(\epsilon-\theta)$. If none of the above conditions is not satisfied, the market is fully covered ($\tilde{w}^* = 0$) and the price set at the maximum possible level of $b^* = \frac{\alpha\theta\epsilon}{2}$. Again, the price has to cover the cost for a non-negative profit, so that $c \leq \frac{\alpha\theta\epsilon}{2}$. But note that if $c < \bar{c}$, then it must be true. The profit becomes $\pi = \frac{\alpha\theta\epsilon}{2} - c$.

Under partial coverage, consumer surplus becomes:

$$CS_1 = \frac{4c(c+\alpha\theta\epsilon) + \alpha^2\theta^2(3\theta^2+4\theta\epsilon-3\epsilon^2)}{16\alpha\theta^2},$$

as we plug in the monopolist's choices with positive marginal cost into (2.5). Then $\frac{dCS_1}{dc} = \frac{2c+\alpha\theta\epsilon}{4\alpha\theta^2} > 0$. Under full coverage, as in Proposition 2.2, the consumer surplus is $CS_1 = \frac{1}{4}\alpha\theta^2$, so it does not depend on c. For total welfare, we add the relevant profit to the expression above. Under partial coverage we obtain:

$$TW_1 = CS_1 + \pi_1 = \frac{4c\big(3c-\alpha\theta(\theta+\epsilon)\big) + \alpha^2\theta^2(5\theta^2+8\epsilon\theta-\epsilon^2)}{16\alpha\theta^2}.$$

Then $\frac{dTW_1}{dc} = \frac{6c-\alpha\theta(2\theta+\epsilon)}{4\alpha\theta^2}$, which is less than or equal to zero if $c \leq \frac{1}{6}\alpha\theta(2\theta+\epsilon)$, and greater than zero otherwise. Under full coverage, $TW_1 = \frac{1}{4}\alpha\theta^2 + \frac{\alpha\theta\epsilon}{2} - c$, so $\frac{dTW_1}{dc} < 0$.

Proof of Lemma 2.1

Remembering that we assume $b_0 = 0$, $\tilde{w}_0 = 0$ and $\tilde{w}_{N+1} = \theta$, observe that the indifference conditions $-b_{k-1} + \alpha\tilde{w}_k\frac{\tilde{w}_{k-1}+\tilde{w}_k}{2} = -b_k + \alpha\tilde{w}_k\frac{\tilde{w}_k+\tilde{w}_{k+1}}{2}$ for $k \in \{1,\ldots,N\}$ allow us after sequential substitution to express the prices as:

$$\begin{aligned} b_k &= \frac{1}{2}\alpha\tilde{w}_{k+1}\tilde{w}_k \text{ for } k \in \{1,\ldots,N-1\}, \\ b_k &= \frac{1}{2}\alpha\theta\tilde{w}_k \text{ for } k = N. \end{aligned} \tag{2.6}$$

The profit function is:

$$\begin{aligned} \pi_N &= b_1\frac{\tilde{w}_2 - \tilde{w}_1}{\theta} + \ldots + b_k\frac{\tilde{w}_{k+1} - \tilde{w}_k}{\theta} + b_N\frac{\theta - \tilde{w}_N}{\theta} \\ &= \sum_{k=1}^{N-1} b_k\frac{\tilde{w}_{k+1} - \tilde{w}_k}{\theta} + b_N\frac{\theta - \tilde{w}_N}{\theta}. \end{aligned} \tag{2.7}$$

As prices are fully pinned down by the thresholds, the standard monopolist problem of choosing the prices to maximise the profit is equivalent to choosing the thresholds for each brand to maximise the profit. Thus after substituting in the prices from (2.6) and taking N partial derivatives with respect to the thresholds we obtain the

following set of FOCs:

$$\begin{aligned} 2\tilde{w}_1 - \tilde{w}_2 = 0 \text{ for } k = 1, \\ \alpha(\tilde{w}_3 - \tilde{w}_1)(\tilde{w}_1 - 2\tilde{w}_2 + \tilde{w}_3) = 0 \text{ for } k = 2, \\ \vdots \\ \alpha(\tilde{w}_{k+1} - \tilde{w}_{k-1})(\tilde{w}_{k-1} - 2\tilde{w}_k + \tilde{w}_{k+1}) = 0 \text{ for } k, \\ \vdots \\ \alpha(\theta - \tilde{w}_{N-1})(\tilde{w}_{N-1} - 2\tilde{w}_N + \theta) = 0 \text{ for } k = N. \end{aligned} \tag{2.8}$$

For the solution to make sense, no two thresholds can be identical, so for any $k \geq 2$ the expression in the first pair of brackets cannot be zero, so the second must be zero. Hence the thresholds follow an arithmetic progression, $\tilde{w}_k = \frac{\tilde{w}_{k-1}+\tilde{w}_{k+1}}{2}$.

Proof of Proposition 2.7

As the thresholds $\tilde{w}_k$ for $k \in \{2, \ldots, N\}$ follow an arithmetic progression as in Lemma 2.1, with the first term equal to $\tilde{w}_1$ and the last term equal to θ, the generalised expression for the k^{th} threshold is $\tilde{w}_k = \frac{(N-k+1)\tilde{w}_1+(k-1)\theta}{N}$. Thus $\tilde{w}_2 = \frac{(N-1)\tilde{w}_1+\theta}{N}$. But we also know from the first of the FOCs in (2.8) that $\tilde{w}_2 = 2\tilde{w}_1$, so $\tilde{w}_1 = \frac{\theta}{N+1}$. Plugging this into the generalised expression for the k^{th} threshold and simplifying yields $\tilde{w}_k = \frac{k\theta}{N+1}$. The expression for the price of the k^{th} good is obtained by plugging this expression into (2.6), $b_k = \frac{k(k+1)}{2}\frac{\alpha\theta^2}{(N+1)^2}$. The thresholds are evenly spaced, so that the proportion of consumers who buy any certain good is $\frac{\tilde{w}_{k+1}-\tilde{w}_k}{\theta} = \frac{\frac{\theta}{N+1}}{\theta} = \frac{1}{N+1}$. Thus the expression for profit (2.7) simplifies to:

$$\begin{aligned} \pi_N &= \sum_{k=1}^{N} b_k \frac{1}{N+1} = \frac{1}{N+1}\frac{\alpha\theta^2}{2(N+1)^2}\sum_{k=1}^{N} k^2 + k \\ &= \frac{\alpha\theta^2}{2(N+1)^3}\Big(\frac{\frac{N(N+1)(2N+1)}{6} + N(N+1)}{2}\Big) = \frac{\alpha\theta^2 N(N+2)}{6(N+1)^2}, \end{aligned}$$

where the penultimate equality follows from the facts that $\sum_1^N k = \frac{N(N+1)}{2}$ and $\sum_1^N k^2 = \frac{N(N+1)(2N+1)}{6}$.

Proof of Proposition 2.8

Using the results from Proposition 2.7, for continuous α and θ: $\frac{db_k}{d\alpha} = \frac{k(k+1)\theta^2}{2(N+1)^2} > 0$, $\frac{d\tilde{w}_k}{d\alpha} = 0$, $\frac{d\pi_N}{d\alpha} = \frac{N(N+2)\theta^2}{6(N+1)^2} > 0$, $\frac{db_k}{d\theta} = \frac{k(k+1)\alpha\theta}{(N+1)^2} > 0$, $\frac{d\tilde{w}_k}{d\theta} = \frac{k}{N+1} > 0$, $\frac{d\pi_N}{d\theta} = \frac{N(N+2)\alpha\theta}{3(N+1)^2} > 0$. Similarly for discrete N: $b_k^{N+1} - b_k^N = -\frac{(2N+3)k(k+1)\alpha\theta^2}{2(N+1)^2(N+2)^2} < 0$, $\tilde{w}_k^{N+1} - \tilde{w}_k^N = -\frac{k\theta}{N^2+3N+2} < 0$ and $\pi^{N+1} - \pi^N = \frac{(2N+3)\alpha\theta^2}{6(N+1)^2(N+2)^2} > 0$.

Proof of Proposition 2.9

Using the result from the Proposition 2.8:

$$\begin{aligned}(\pi^{N+1} - \pi^N) - (\pi^N - \pi^{N-1}) &= \frac{(2N+3)\alpha\theta^2}{6(N+1)^2(N+2)^2} - \frac{(2N+1)\alpha\theta^2}{6N^2(N+1)^2} \\ &= -\frac{(3N^2+12N+11)\alpha\theta^2}{3(N+1)^2(N+2)^2(N+3)^2} < 0.\end{aligned}$$

Proof of Proposition 2.10

Consider partitions $I_0, I_1, \ldots, I_N$. For the left-most partition, $0 \leq w < \tilde{w}_1$, the utility is $u_0(w) = \alpha w \frac{\tilde{w}_1}{2}$. For the right-most partition, $\tilde{w}_N \leq w \leq \theta$, the utility is $-b_N + \alpha w \frac{\tilde{w}_N + \theta}{2}$. For any k^{th} interval in the middle the utility is $u_k(w) = -b_k + \alpha w \frac{\tilde{w}_k + \tilde{w}_{k+1}}{2}$. Plugging in the optimal choices of the monopolist from Proposition 2.7, we obtain the following generalised expression:

$$u_k(w) = \frac{(N+1)(2k+1)w - k(k+1)\theta}{2(N+1)^2}\alpha\theta, \tag{2.9}$$

for $k \in \{0, 1, \ldots, N\}$. By integrating this expression within the given intervals, we obtain the generalised expression for the consumer surplus, $U_k = \int_{I_k} u_k(w) f(w) dw =$

$\frac{2k^2+2k+1}{4(N+1)^3}\alpha\theta^2$. Thus, the aggregated consumer surplus is:

$$CS^N = \sum_{k=0}^{N} U_k = \frac{2N^2+4N+3}{12(N+1)^2}\alpha\theta^2, \tag{2.10}$$

and consequently $CS^{N+1} - CS^N = -\frac{(2N+3)}{12(N+1)^2(N+1)^2}\alpha\theta^2 < 0$.

Proof of Proposition 2.11

We use the expressions that describe the utility of agents in every interval that were derived in (2.9). We relabel the utility to u_k^N, where the subscript denotes the good purchased, while the superscript denotes the number of goods available. Consider the first sub-interval J_1, where the agent used to get $u_0^N(w)$, but now obtains $u_0^{N+1}(w)$. This gives a net change of $\Delta_1(w) = u_0^{N+1}(w) - u_0^N(w) = -\frac{w}{2(N^2+3N+2)}\alpha\theta$, which is negative for any value of w. Consider next the sub-interval $J_{2(N+1)}$, where the agent used to get $u_N^N(w)$, but now obtains $u_{N+1}^{N+1}(w)$. This gives a net change of $\Delta_{2(N+1)}(w) = u_{N+1}^{N+1}(w) - u_N^N(w) = -\frac{\theta-w}{2(N+1)(N+2)}\alpha\theta$, which, again, is negative for any value of w.

If $j \neq 1$ and $j \neq 2(N+1)$, for an even j every sub-interval J_j begins at $\tilde{w}_{j/2}^{N+1}$ and ends at $\tilde{w}_{j/2}^N$, while for an odd j it begins at $\tilde{w}_{(j-1)/2}^N$ and ends at $\tilde{w}_{(j+1)/2}^{N+1}$. Consider a sub-interval for which j is even. The utility used to be $u_{j/2-1}^N(w)$, but now is $u_{j/2}^{N+1}(w)$, so that the change is:

$$\begin{aligned}\Delta_j(w) &= u_{j/2}^{N+1}(w) - u_{j/2-1}^N(w) \\ &= \frac{4(N^2+3N+2)(2N-j+3)w - j\big(2(2N^2+6N+5) - j(2N+3)\big)\theta}{8(N+1)^2(N+2)^2}\alpha\theta.\end{aligned}$$

Then $\Delta_j(\tilde{w}_{j/2}^{N+1}) = -\frac{j\left(2(N+2)-j\right)}{8(N+1)^2(N+2)^2}\alpha\theta^2 < 0$ and $\Delta_j(\tilde{w}_{j/2}^N) = \frac{j\left(2(N+1)-j\right)}{8(N+1)^2(N+2)^2}\alpha\theta^2 > 0$, so the difference in utilities is negative at the lower end of the sub-interval J_j and positive at the upper end of the sub-interval J_j. As the equation $\Delta_j(w) = 0$ has only one root, it must sit strictly inside J_j. Therefore, utility first falls and then

increases when j is even. Consider now a sub-interval for which j is odd. The utility used to be $u^N_{(j-1)/2}(w)$, but now is $u^{N+1}_{(j-1)/2}(w)$, so that the change is:

$$\begin{aligned}\Delta_j(w) &= u^{N+1}_{(j-1)/2}(w) - u^N_{(j-1)/2}(w) \\ &= -\frac{4(N^2+3N+2)jw - (2N+3)(j^2-1)\theta}{8(N+1)^2(N+2)^2}\alpha\theta.\end{aligned}$$

Then $\Delta_j(\tilde{w}^N_{(j-1)/2}) = \frac{(2N-j+3)(j-1)}{8(N+1)^2(N+2)^2}\alpha\theta^2 > 0$ and $\Delta_j(\tilde{w}^{N+1}_{(j+1)/2}) = -\frac{(2N-j+3)(j+1)}{8(N+1)^2(N+2)^2}\alpha\theta^2 < 0$, so the difference in utilities is positive at the lower end of the sub-interval J_j and negative at the upper end of the sub-interval J_j. As the equation $\Delta_j(w) = 0$ again has only one root, it must sit strictly inside J_j. Therefore, utility first increases and then falls when j is odd.

Proof of Proposition 2.12

The optimal prices will be chosen by the monopolist for a given N as in Proposition 2.7. Thus the total welfare can be calculated by adding the profit described by (2.2) to the consumer surplus described by (2.10): $TW^N = CS^N + \pi^N = \frac{4N^2+8N+3}{12(N+1)^2}\alpha\theta^2$. The social gain from an additional brand is $TW^{N+1} - TW^N = \frac{2N+3}{12(N+1)^2(N+2)^2}\alpha\theta^2$, which is positive for any value of N. Therefore, the social planner always wants to add an additional brand, i.e. it chooses an infinite number of brands.

Proof of Proposition 2.13

Assume N is fixed and $\epsilon > 0$. The social planner's goal is to maximise total welfare by choosing a set of taxes that makes the after-tax prices socially optimal. But this is equivalent to choosing the thresholds, while the prices will be governed by the agents' indifference conditions. Remembering that $b_0 = 0$, $\tilde{w}_0 = 0$ and $\tilde{w}_{N+1} = \theta$, the utility of the agent is $u_k(w) = -b_k + \alpha(w+\epsilon)\frac{\tilde{w}_k+\tilde{w}_{k+1}}{2}$ for $k \in \{0, 1, \ldots, N\}$, and the resulting surplus in each interval as a function of thresholds and prices is $U_k(w) = \int_{\tilde{w}_k}^{\tilde{w}_{k+1}} u_k(w)f(w)dw = \frac{\alpha}{4\theta}(\tilde{w}_{k+1}-\tilde{w}_k)(\tilde{w}_{k+1}+\tilde{w}_k)(\tilde{w}_{k+1}+\tilde{w}_k+2\epsilon) - b_k\frac{\tilde{w}_{k+1}-\tilde{w}_k}{\theta}$,

while the profits on each interval are $\pi_k = b_k \frac{\tilde{w}_{k+1}-\tilde{w}_k}{\theta}$. In computation of total welfare on each interval the term that involves the price disappears, $TW_k = CS_k + \pi_k = \frac{\alpha}{4\theta}(\tilde{w}_{k+1} - \tilde{w}_k)(\tilde{w}_{k+1} + \tilde{w}_k)(\tilde{w}_{k+1} + \tilde{w}_k + 2\epsilon)$, and in the expression for the total welfare across whole society, $TW = \sum_{k=1}^{N} TW_k$, each term $\tilde{w}_k$ only appears in two summands, so the derivatives with respect to the thresholds are easy to calculate, $\frac{dTW}{d\tilde{w}_k} = \frac{\alpha}{4\theta}(\tilde{w}_{k+1} - \tilde{w}_{k-1})(\tilde{w}_{k+1} - 2\tilde{w}_k + \tilde{w}_{k-1})$. Setting these derivatives for $k \in \{1, \ldots, N\}$ equal to zero gives us the FOCs. For all thresholds the solution will be identical to that of Lemma 2.1 where $\epsilon = 0$: the socially optimal thresholds follow an arithmetic series, so that the general expression becomes $\tilde{w}_k = \frac{k\theta}{N+1}$.

However, these are not the thresholds that the monopolist will indirectly set when $\epsilon > 0$. To correct this, assume that the social planner sets the following set of per-item taxes $t_1, \ldots, t_N$ on goods $1, \ldots, N$ respectively. With these taxes, the new indifference condition becomes $-(b_{k-1} + t_{k-1}) + \alpha(\tilde{w}_k + \epsilon)\frac{\tilde{w}_{k-1}+\tilde{w}_k}{2} = -(b_k + t_k) + \alpha(\tilde{w}_k+\epsilon)\frac{\tilde{w}_k+\tilde{w}_{k+1}}{2}$. Solving for the lowest price first, and then repeating the procedure for the subsequent prices yields $b_k = \frac{\alpha}{2}\big(\tilde{w}_k(\tilde{w}_{k+1}+\epsilon)+\epsilon(\tilde{w}_{k+1}-\tilde{w}_{k-1})\big) - t_k$, and thus the profit on each brand is $\pi_k = \frac{\tilde{w}_{k+1}-\tilde{w}_k}{\theta}\Big(\frac{\alpha}{2}\big(\tilde{w}_k(\tilde{w}_{k+1}+\epsilon)+\epsilon(\tilde{w}_{k+1}-\tilde{w}_{k-1})\big) - t_k\Big)$. In the expression for the total profit across the whole society, $\pi = \sum_{k=1}^{N} \pi_k$, each term $\tilde{w}_k$ only appears in two summands, so the derivatives with respect to the thresholds will be easy to calculate (assume $t_0 = 0$): $\frac{d\pi}{d\tilde{w}_k} = \frac{\alpha}{2\theta}(\tilde{w}_{k+1}-\tilde{w}_{k-1})(\tilde{w}_{k+1}-2\tilde{w}_k+\tilde{w}_{k-1})+ \frac{1}{\theta}(t_k - t_{k-1})$. We obtain the set of FOCs by setting these expressions equal to zero for $k \in \{1, \ldots, N\}$. We plug in the optimal thresholds found above, $\tilde{w}_k = \frac{k\theta}{N+1}$. In FOCs with $k \in \{2, \ldots, N\}$ this is equivalent to $t_k^* = t_{k-1}^*$. From the first FOC we obtain $t_1^* = \frac{\alpha\theta\epsilon}{2}$. So all per-unit tax rates are the same and equal to $\frac{\alpha\theta\epsilon}{2}$.

Proof of Proposition 2.14

We proceed by backward induction. In the last stage of the game, when the number of brands is decided, the game proceeds as in Proposition 2.7. Thus the maximised gross profit is given by (2.2), and in the first stage the net profit becomes $\pi_N =$

$\frac{N(N+2)\alpha\theta^2}{3N(N+1)^2} - \beta N^\zeta$. We take the derivative of profit with respect to N to derive the FOC for the profit-maximising number of brands N^*:

$$\frac{\alpha\theta^2 N^* - 3\beta\zeta(N^*)^\zeta(N^*+1)^3}{3N^*(N^*+1)^3} = 0,$$

which simplifies to $(N^*)^{\zeta-1}(N^*+1)^3 = \frac{1}{3}\frac{\alpha\theta^2}{\beta\zeta}$. Similarly, the maximised gross total welfare is derived in Proposition 2.12, so the net total welfare is $TW_N = \frac{(4N^2+8N+3)\alpha\theta^2}{12(N+1)^2} - \beta N^\zeta$. We take the derivative of total welfare with respect to N to derive the FOC for the welfare-maximising number of brands N^{SC}:

$$\frac{\alpha\theta^2 N^{SC} - 6\beta\zeta(N^{SC})^\zeta(N^{SC}+1)^3}{3N^{SC}(N^{SC}+1)^3} = 0,$$

which simplifies to $(N^{SC})^{\zeta-1}(N^{SC}+1)^3 = \frac{1}{6}\frac{\alpha\theta^2}{\beta\zeta}$. In both cases the optimum must be a maximum, as in the relevant range of $N > 1$, the profit first increases and then decreases in N. Both equations can be re-written as $\Gamma(N) = \phi\frac{\alpha\theta^2}{\beta\zeta}$, where $\phi^* = \frac{1}{3}$ or $\phi^{SC} = \frac{1}{6}$ and $\Gamma(N) = N^{\zeta-1}(N+1)^3$, which increases as N goes up. This is because $\frac{d\Gamma(N)}{dN} = N^{\zeta-2}(N+1)^2\big((\zeta-1)(N+1)+3N\big)$, which is greater than zero whenever $\zeta > \frac{1-2N}{1+N}$. But this is always true for $N > 1$, because $\frac{1-2N}{1+N}$ is falling in N and at $N = 1$ it is equal to $-\frac{1}{2}$ while $\zeta > 0$ by assumption. Then as $\phi^* > \phi^{SC}$, we must have $N^* > N^{SC}$. For both N^* and N^{SC}, when β increases, $\phi\frac{\alpha\theta^2}{\beta\zeta}$ falls, so $\Gamma(N)$ must fall, which can only happen when the relevant N falls too. For $\zeta > 0$, $\Gamma(N)$ shifts outwards when ζ increases. But as the overall value of $\Gamma(N)$ must fall because of a fall in $\phi\frac{\alpha\theta^2}{\beta\zeta}$, this can only be achieved by a fall in N. None of the conclusions can be made strict due to the discreteness of the number of brands because rounding may always cause the number of brands to remain constant.

Proof of Proposition 2.15

Assume first that the monopolist produces two brands, so that is profit is $\pi = b_1\frac{\tilde{w}_2-\tilde{w}_1}{\theta} + (b_2-c)\frac{\theta-\tilde{w}_2}{\theta}$. The thresholds are determined by the indifference conditions

$\alpha\tilde{w}_1\frac{0+\tilde{w}_1}{2} = -b_1 + \alpha\tilde{w}_1\frac{\tilde{w}_1+\tilde{w}_2}{2}$ and $-b_1 + \alpha\tilde{w}_2\frac{\tilde{w}_1+\tilde{w}_2}{2} = -b_2 + \alpha\tilde{w}_2\frac{\tilde{w}_2+\theta}{2}$, so they may be expressed as functions of prices, $\tilde{w}_1 = \frac{\theta b_1}{b_2}$ and $\tilde{w}_2 = \frac{2b_2}{\alpha\theta}$. Consequently the profit expressed as a function of the prices only is $\pi = b_1\frac{\frac{2b_2}{\alpha\theta} - \frac{\theta b_1}{b_2}}{\theta} + (b_2 - c)\frac{\theta - \frac{2b_2}{\alpha\theta}}{\theta}$. The FOC with respect to b_1 is $-\frac{2b_1^*}{b_2^*} + \frac{2b_2^*}{\alpha\theta^2} = 0$ and with respect to b_2 is $\frac{b_1^{*2}}{b_2^{*2}} + \frac{2b_1^* - 4b_2^* + 2c + \alpha\theta^2}{\alpha\theta^2} = 0$. Choosing the root which makes economic sense, the optimal prices are $b_1^* = \frac{1}{9}\big(\theta(5\alpha\theta - 4\sqrt{\alpha(\alpha\theta^2 - 6c)}) - 6c\big)$ and $b_2^* = \frac{1}{3}\theta\big(2\alpha\theta - \sqrt{\alpha(\alpha\theta^2 - 6c)}\big)$. But this results in the higher threshold of $\tilde{w}_2^* = \frac{4\alpha\theta - 2\sqrt{\alpha(\alpha\theta^2 - 6c)}}{3\alpha}$. For this to be a valid threshold, it must be lower than or equal to θ, or otherwise the second brand would not face positive demand demand. But this only happens when $\tilde{w}_2^* \leq \theta$, or $c \leq \frac{1}{8}\alpha\theta^2$. Otherwise the monopolist is better off ignoring the possibility to sell the brand with high marginal cost and proceeds as in Proposition 2.1.

Consumer surplus can be calculated analogously to Proposition 2.13. The surplus of the non-buyers is $U_0^* = \int_0^{\tilde{w}_1^*}(\alpha w\frac{\tilde{w}_1^*}{2})\frac{1}{\theta}dw$, the surplus of the buyers of good 1 is $U_1^* = \int_{\tilde{w}_1^*}^{\tilde{w}_2^*}(-b_1^* + \alpha w\frac{\tilde{w}_1^* + \tilde{w}_2^*}{2})\frac{1}{\theta}dw$ and the surplus of the buyers of good 2 is $U_2^* = \int_{\tilde{w}_2^*}^{\theta}(-b_2^* + \alpha w\frac{\tilde{w}_2^* + \theta}{2})\frac{1}{\theta}dw$. Therefore, the overall consumer surplus is:

$$\begin{aligned} CS^* &= U_0^* + U_1^* + U_2^* \\ &= \frac{\alpha\theta^3(50\alpha\theta - 31\sqrt{\alpha(\alpha\theta^2 - 6c)}) - 12\theta c(\alpha\theta + 2\sqrt{\alpha(\alpha\theta^2 - 6c)}) - 72c^2}{108\theta(2\alpha\theta - \sqrt{\alpha(\alpha\theta^2 - 6c)})}, \end{aligned}$$

where the second equality follows from plugging in the profit-maximising values computed above. Thus, we have $\frac{dCS^*}{dc} = \frac{c}{\theta\sqrt{\alpha(\alpha\theta^2 - 6c)}}$, which is always positive. We analogously calculate total welfare, $TW^* = CS^* + \pi^*$, whose derivative is greater than zero if and only if $c > \frac{1}{75}(9 - \sqrt{6})\alpha\theta^2$.

Proof of Lemma 2.2

Assume initially that $b_2 > b_1$. Consider region A. There are agents who choose not to consume, agents who consume good 1 and agents who consume good 2. Hence, because of increasing differences, there are two thresholds, $\tilde{w}_1 < \tilde{w}_2$ that sit

strictly inside the $(0, \theta)$ interval. Thus the inferences made by the social contacts are $\rho_0 = \frac{0+\tilde{w}_1}{2}$, $\rho_1 = \frac{\tilde{w}_1+\tilde{w}_2}{2}$ and $\rho_2 = \frac{\tilde{w}_2+\theta}{2}$. Consequently, the agents' behaviour is governed by two indifference conditions, $\alpha\tilde{w}_1\frac{0+\tilde{w}_1}{2} = -b_1 + \alpha\tilde{w}_1\frac{\tilde{w}_1+\tilde{w}_2}{2}$ and $-b_1 + \alpha\tilde{w}_2\frac{\tilde{w}_1+\tilde{w}_2}{2} = -b_2 + \alpha\tilde{w}_2\frac{\tilde{w}_2+\theta}{2}$. We solve this set of indifference conditions to find the thresholds as functions of prices and obtain $\tilde{w}_1 = \frac{\theta b_1}{b_2}$ and $\tilde{w}_2 = \frac{2b_2}{\alpha\theta}$. For this to constitute a valid solution, we require that the thresholds are between 0 and θ, and that the order is preserved, $\tilde{w}_2 > \tilde{w}_1$, which happens if and only if $0 < b_1 \leq \frac{1}{2}\alpha\theta^2$ and $\sqrt{\frac{\alpha\theta^2 b_1}{2}} \leq b_2 \leq \frac{1}{2}\alpha\theta^2$.

Consider region B. There are only agents who choose not to consume or consume good 1, but no agents consume good 2. The inferences of the social contacts are $\rho_0 = \frac{0+\tilde{w}}{2}$ and $\rho_1 = \frac{\tilde{w}+\theta}{2}$, where $\tilde{w}$ denotes the agent who is just indifferent between not consuming and consuming good 1. The only out-of-equilibrium belief which survives the D1 refinement is that the agent must be very rich, $w = \theta$, if he were to choose to consume good 2. Hence $\rho_2 = \theta$. The threshold is defined by $\alpha\tilde{w}\frac{0+\tilde{w}}{2} = -b_1 + \alpha\tilde{w}\frac{\tilde{w}+\theta}{2}$, which translates to $\tilde{w} = \frac{2b_1}{\alpha\theta}$. For this to constitute a valid solution we need $0 < \tilde{w} < \theta$ as well as a condition that guarantees that nobody, and in particular the richest agent, deviates to the consumption of good 2, $-b_2 + \alpha\theta\theta < -b_1 + \alpha\theta\frac{\tilde{w}+\theta}{2}$, which happens if and only if $0 \leq b_1 \leq \frac{1}{2}\alpha\theta^2$ and $b_2 > \frac{1}{2}\alpha\theta^2$.

Consider region C. There are only agents who choose not to consume or to consume good 2. This is a perverse behaviour where the agents deliberately choose the more expensive signalling device available. The equilibrium inferences are $\rho_0 = \frac{0+\tilde{w}}{2}$ and $\rho_2 = \frac{\tilde{w}+\theta}{2}$, where $\tilde{w}$ denotes the agent who is just indifferent between not consuming and consuming good 2. The agent that may potentially benefit most from a deviation (whom we consider for the D1 refinement) due to reduction in price if he switches from good 2 or increase in status if he is a first-time buyer, is the marginal person with $w = \tilde{w}$. Thus, the out-of-equilibrium belief is $\rho_1 = \tilde{w}$. The behaviour of the agents will be governed by an indifference condition between not buying and buying good 2, $\alpha\tilde{w}\frac{0+\tilde{w}}{2} = -b_2 + \alpha\tilde{w}\frac{\tilde{w}+\theta}{2}$, which gives the threshold value

of $\tilde{w} = \frac{2b_2}{\alpha\theta}$. For this to constitute a valid solution we need $0 < \tilde{w} < \theta$ as well as a condition that guarantees no deviation, $-b_1 + \alpha\tilde{w}\tilde{w} < -b_2 + \alpha\tilde{w}\frac{\tilde{w}+\theta}{2}$, which happens if and only if $0 \leq b_1 < \frac{1}{2}\alpha\theta^2$ and $b_1 < b_2 < \sqrt{\frac{\alpha\theta^2 b_1}{2}}$.[24]

For $b_1 > b_2$ the description of regions A', B' and C' are just the mirror images of the above. Next, consider the border between region C and C', $0 \leq b_1 = b_2 < \hat{b}$. The agents are indifferent between buying from either producer and the analysis is almost identical to that of region C (C'), except that the demand is split in half between both firms.

Finally, consider region D. Nobody consumes anything, which happens when both goods are too expensive. The inference of the social contacts in the equilibrium is $\rho_0 = \frac{0+\theta}{2}$, while to be consistent with the D1 criterion, any deviation should be treated as if it came from the agent who might benefit most, $w = \theta$, so that $\rho_1 = \rho_2 = \theta$. Clearly, as $b_2 > b_1$, nobody will ever buy good 2, as the same signalling value can be achieved at a lower price out-of-equilibrium. The condition that must be satisfied for this to constitute a solution is therefore $\alpha\theta\frac{0+\theta}{2} < -b_1 + \alpha\theta\theta$, which happens if and only if $b_1 > \frac{1}{2}\alpha\theta^2$ and $b_2 > b_1$.

Proof of Proposition 2.16

The proof proceeds in four parts. First, I show that no PBE in pure strategies can exist in regions B (B'), C (C') or D, excluding the border between C and C'. Consider any price $b_1 \in [0, \hat{b})$. Obviously firm 2 does not choose the price in region B or C' as this guarantees a profit of zero, while a strictly positive profit can be obtained e.g. in region A. If $b_1 \in [\hat{b}, \infty)$, firm 2 never chooses a price in region D as it can obtain a strictly positive profit by playing B' instead. Thus, the best reply to b_1 can never lead to a price in regions B, C' or E. By symmetry, this is also true about B', C and E. As points in these regions are not a part of the best reply sets, they can be never supported in an equilibrium.

[24] We do not need to check whether $u_1(\tilde{w}) < u_0(\tilde{w})$, as $u_0(\tilde{w}) = u_2(\tilde{w})$

Second, I show that no PBE in pure strategies can exist at any point (b_1, b_2) on the border between C and C'. On the line between C and C' where the prices are identical, the assumption that the demand will be split in half when the agent is indifferent means that the standard Bertrand undercutting argument applies: (b, b) cannot be an equilibrium as by increasing the price to $b+\tau$, where τ is small, either firm can double its demand and almost double its profit, so the original strategy could not have been an equilibrium.[25]

Third, I show that no PBE in pure strategies can exist in regions A (A'). Consider region A. If an equilibrium were to exist there it could only be where the duopolists play mutual best responses (confined to region A). The profits in this region are given by $\pi_1 = \frac{2b_1b_2}{\alpha\theta^2} - \frac{b_1^2}{b_2}$ and $\pi_2 = b_2 - \frac{2b_2^2}{\alpha\theta^2}$. Thus, inside A, the best response of firm 1 is $b_1^*(b_2) = \frac{d\pi_1}{db_1} = -\frac{2b_1}{b_2} + \frac{2b_2}{\alpha\theta^2}$ and of firm 2 is $b_2^*(b_1) = 1 - \frac{4b_2}{\alpha\theta^2}$. Solving these two equations simultaneously gives us the solution: $b_1^* = \frac{1}{16}\alpha\theta^2$ and $b_2^* = \frac{1}{4}\alpha\theta^2$, which indeed sits inside region A as $0 < b_1^* < \hat{b} = \frac{1}{2}\alpha\theta^2$ and $\sqrt{\frac{1}{2}\alpha\theta^2 b_1^*} = \frac{\sqrt{2}}{2}b_2^* < b_2^* < \hat{b}$. The resulting profits are $\pi_1^* = \frac{1}{64}\alpha\theta^2$ and $\pi_2^* = \frac{1}{8}\alpha\theta^2$. But this is not a global best reply for firm 1 when firm 2 plays $b_2 = \frac{1}{4}\alpha\theta^2$ as it may deviate to region C' just slightly "over-cutting" firm's 2 price by τ, where τ is small. This will guarantee it a profit of $\lim_{\tau\to 0}(b_2+\tau) - \frac{2(b_2+\tau)^2}{\alpha\theta^2} = \frac{1}{8}\alpha\theta^2 > \pi_1^*$. So this could not have been a global equilibrium as a profitable deviation exists and thus no equilibrium may exist in region A. By symmetry the same argument applies to region A'. Taking all these parts together, there does not exist any point in the (b_1, b_2)-space that could constitute an equilibrium.

Proof of Lemma 2.3

The proof proceeds in two steps. First, I obtain firm 2's best replies in every region. In region A, the relevant FOC, $\frac{\pi_2^A}{db_2} = 0$, gives $b_2^A = \frac{1}{4}\alpha\theta^2$. However, this does only sit inside region A if it is above its southern border, which is given by $b_2 = \sqrt{\frac{1}{2}\alpha\theta^2 b_1}$,

[25]Note that unlike in Bertrand, an "over-cutting" rather than "undercutting" takes place.

i.e. only when $b_1 < \frac{1}{8}\alpha\theta^2$. Otherwise, firm 2 would like to set the price as low as possible, i.e. along the border. In region A' the relevant FOC, $\frac{\pi_2^{A'}}{db_2} = 0$, gives $\frac{2b_1}{\alpha\theta^2} - \frac{2b_2^{A'}}{b_1} = 0$, so that $b_2^{A'} = \frac{b_1^2}{\alpha\theta^2}$, which must indeed sit in A' (and not C') as $b_2^{A'} < \frac{2b_1^2}{\alpha\theta^2}$. In region B' the relevant FOC, $\frac{\pi_2^{B'}}{db_2} = 0$, gives $b_2^{B'} = \frac{1}{4}\alpha\theta^2$, which always sits inside region B'. In region C the relevant FOC, $\frac{\pi_2^{C}}{db_2} = 0$, gives $b_2^{C} = \frac{1}{4}\alpha\theta^2$. However, this does not always sit inside region C: it is below the northern border $b_2 = \sqrt{\frac{1}{2}\alpha\theta^2 b_1}$ only if $b_1 \geq \frac{1}{8}\alpha\theta^2$. Thus, for $b_1 < \frac{1}{8}\alpha\theta^2$, firm 2 sets the price as high as possible, i.e. along the border between C and A. Moreover, $b_2^C = \frac{1}{4}\alpha\theta^2$ sits inside region C only if it is above its southern border $b_2 = b_1$. Thus, firm 2 sets $b_2^C = \frac{1}{4}\alpha\theta^2$ only if $b_1 \in [\frac{1}{8}\alpha\theta^2, \frac{1}{4}\alpha\theta^2)$. When $b_1 \geq \frac{1}{4}\alpha\theta^2$, firm 2 would like to set the price as low as possible, but not at the southern border to avoid splitting the profit in half. Thus, it sets the price marginally higher than its competitor, $b_2^C = b_1 + \tau$, where $\tau \to 0$. In regions B, C' and D firm 2 obtains no demand, so he makes no profit regardless of the price chosen. The profits follow from plugging in the prices above into the respective profit functions. Thus, the per-region best reply function (with a slight abuse of notation) is:

$$
\begin{aligned}
b_2^A &= \begin{cases} \frac{1}{4}\alpha\theta^2 & \text{if } b_1 \in [0, \frac{1}{8}\alpha\theta^2) \\ \sqrt{\frac{1}{2}\alpha\theta^2 b_1} & \text{if } b_1 \in [\frac{1}{8}\alpha\theta^2, \frac{1}{2}\alpha\theta^2), \end{cases} \\
b_2^{A'} &= \frac{b_1^2}{\alpha\theta^2} \text{ if } b_1 \in [0, \frac{1}{2}\alpha\theta^2), \\
b_2^{B'} &= \frac{1}{4}\alpha\theta^2 \text{ if } b_1 \in [0, \frac{1}{2}\alpha\theta^2), \\
b_2^C &= \begin{cases} \sqrt{\frac{1}{2}\alpha\theta^2 b_1} & \text{if } b_1 \in [0, \frac{1}{8}\alpha\theta^2) \\ \frac{1}{4}\alpha\theta^2 & \text{if } b_1 \in [\frac{1}{8}\alpha\theta^2, \frac{1}{4}\alpha\theta^2) \\ b_1 + \tau & \text{if } b_1 \in [\frac{1}{4}\alpha\theta^2, \frac{1}{2}\alpha\theta^2), \end{cases} \\
b_2^{B,C',D} &= \text{ any } b_2 \text{ in region B, C', D,}
\end{aligned}
$$

and the resulting profit function is:

$$\pi_2^A = \begin{cases} \frac{1}{8}\alpha\theta^2 & \text{if } b_1 \in [0, \frac{1}{8}\alpha\theta^2) \\ -b_1 + \sqrt{\frac{1}{2}\alpha\theta^2 b_1} & \text{if } b_1 \in [\frac{1}{8}\alpha\theta^2, \frac{1}{2}\alpha\theta^2), \end{cases}$$

$$\pi_2^{A'} = \frac{b_1^3}{(\alpha\theta^2)^2} \text{ if } b_1 \in [0, \frac{1}{2}\alpha\theta^2),$$

$$\pi_2^{B'} = \frac{1}{8}\alpha\theta^2 \text{ if } b_1 \in [0, \frac{1}{2}\alpha\theta^2),$$

$$\pi_2^C = \begin{cases} -b_1 + \sqrt{\frac{1}{2}\alpha\theta^2 b_1} & \text{if } b_1 \in [0, \frac{1}{8}\alpha\theta^2) \\ \frac{1}{8}\alpha\theta^2 & \text{if } b_1 \in [\frac{1}{8}\alpha\theta^2, \frac{1}{4}\alpha\theta^2) \\ (b_1 + \tau) - \frac{2(b_1+\tau)^2}{\alpha\theta^2} & \text{if } b_1 \in [\frac{1}{4}\alpha\theta^2, \frac{1}{2}\alpha\theta^2), \end{cases}$$

$$\pi_2^{B,C',D} = 0.$$

Second, I derive the global best reply function for every possible value of b_1 by comparing the profits in different regions. Consider the following disjoint sets of b_1 values. If $b_1 \in [0, \frac{1}{4}\alpha\theta^2)$, firm 2 chooses between regions A, A', B, C and C'. The highest possible profit is $\pi_2 = \frac{1}{8}\alpha\theta^2$, which is obtained by choosing the best reply $b_2^*(b_1) = \frac{1}{4}\alpha\theta^2$ in region A when $b_1 \in [0, \frac{1}{8}\alpha\theta^2)$ and in region C when $b_1 \in [\frac{1}{8}\alpha\theta^2, \frac{1}{4}\alpha\theta^2)$. For $b_1 \in [\frac{1}{4}\alpha\theta^2, \frac{1}{2}\alpha\theta^2)$, $\pi_2^C \geq \pi_2^A$, so given that $\pi_2^B = \pi_2^{C'} = 0$, the highest profit is obtained in region C or A'. The inequality $\lim_{\tau\to 0} \pi_2^C > \pi_2^{A'}$, or $b_1 - \frac{2b_1}{\alpha\theta^2} > \frac{b_1^3}{(\alpha\theta^2)^2}$, or $b_1^2 + 2\alpha\theta^2 b_1 - (\alpha\theta^2)^2 < 0$, is solved by $b_1 < (\sqrt{2}-1)\alpha\theta^2$. So for $b_1 \in [\frac{1}{4}\alpha\theta^2, (\sqrt{2}-1)\alpha\theta^2)$ firm 2 chooses the best reply $b_2^*(b_1) = b_1 + \tau$, where $\tau \to 0$, in region C. For $b_1 \in [(\sqrt{2}-1)\alpha\theta^2, \frac{1}{2}\alpha\theta^2]$, however, it chooses the best reply $b_2^*(b_1) = \frac{b_1^2}{\alpha\theta^2}$ in region A'. At $b_1 = (\sqrt{2}-1)\alpha\theta^2$, firm 2 prefers to choose region A' to avoid splitting the profit in half. Finally, when $b_1 \in [\frac{1}{2}\alpha\theta^2, \infty)$, firm 2 chooses between regions B' and D. $\pi_2^{B'} > \pi_2^D = 0$, so over this range $b_2^*(b_1) = \frac{1}{4}\alpha\theta^2$.

Proof of Proposition 2.17

The proof proceeds in two steps. First, I calculate the profit that firm 1 obtains for any choice of b_1 given that firm 2 plays the best reply. Consider the following disjoint

sets of b_1 values. If $b_1 \in [0, \frac{1}{8}\alpha\theta^2)$, firm 2 plays $b_2^* = \frac{1}{4}\alpha\theta^2$ in region A, resulting in profit $\pi_1 = \frac{2b_1 b_2^*}{\alpha\theta^2} - \frac{b_1^2}{b_2^*} = \frac{1}{2}b_1 - \frac{4b_1^2}{\alpha\theta^2}$. If $b_1 \in [\frac{1}{8}\alpha\theta^2, (\sqrt{2}-1)\alpha\theta^2)$, firm 2 plays $b_2^* = b_1 + \tau$, where $\tau \to 0$, in region C, resulting in profit $\pi_1 = 0$. If $b_1 \in [(\sqrt{2}-1)\alpha\theta^2, \frac{1}{2}\alpha\theta^2)$, firm 2 plays $b_2^* = \frac{b_1^2}{\alpha\theta^2}$ in region A', resulting in profit $\pi_1 = b_1 - \frac{2b_1}{\alpha\theta^2}$. If $b_1 \in [\frac{1}{2}\alpha\theta^2, \infty)$, firm 2 plays $b_2^* = \frac{1}{4}\alpha\theta^2$ in region B', resulting in profit $\pi_1 = 0$. Thus, for any chosen price b_1, given the best reply of firm 2, the profit obtained by firm 1 is:

$$\pi_1(b_1) = \begin{cases} \frac{1}{2}b_1 - \frac{4b_1^2}{\alpha\theta^2} & \text{if } b_1 \in [0, \frac{1}{8}\alpha\theta^2) \\ 0 & \text{if } b_1 \in [\frac{1}{8}\alpha\theta^2, (\sqrt{2}-1)\alpha\theta^2) \\ b_1 - \frac{2b_1^2}{\alpha\theta^2} & \text{if } b_1 \in [(\sqrt{2}-1)\alpha\theta^2, \frac{1}{2}\alpha\theta^2) \\ 0 & \text{if } b_1 \in [\frac{1}{2}\alpha\theta^2, \infty). \end{cases}$$

Second, I obtain the global maximum for firm 1. To obtain a positive profit, firm 1 chooses either $b_1 \in [0, \frac{1}{8}\alpha\theta^2)$ or $b_1 \in [(\sqrt{2}-1)\alpha\theta^2, \frac{1}{2}\alpha\theta^2)$ as otherwise the profit is zero. In the former range, firm 1 maximises $\pi_1^\circ(b_1) = \frac{b_1}{2} - \frac{4b_1^2}{\alpha\theta^2}$, so the FOC is $\frac{d\pi_1^\circ}{db_1} = 0$, and the optimum is reached at $b_1^\circ = \frac{1}{16}\alpha\theta^2$ for a maximised profit of $\pi_1^\circ(b_1^\circ) = \frac{1}{64}\alpha\theta^2$. In the latter range, firm 1 maximises $\pi_1^\bullet(b_1) = b_1 - \frac{2b_1^2}{\alpha\theta^2}$, so the FOC is $\frac{d\pi_1^\bullet}{db_1} = 0$, and the optimum is reached at $b_1^\bullet = (\sqrt{2}-1)\alpha\theta^2$ for a maximised profit of $\pi_1^\bullet(b_1^\bullet) = (5\sqrt{2}-7)\alpha\theta^2$. As $\pi_1^\bullet > \pi_1^\circ$, the global maximum is reached when firm 1 plays $b_1^* = b_1^\bullet$. This results in a profit of $\pi_1^* = \pi_1^\bullet(b_1^\bullet)$. Consequently, firm 2 plays the best response in region A', $b_2^* = b_2(b_1^\bullet) = \frac{(b_1^\bullet)^2}{\alpha\theta^2} = (3-2\sqrt{2})\alpha\theta^2$, and obtains a profit of $\pi_2^* = \pi_2(b_1^\bullet) = (5\sqrt{2}-7)\alpha\theta^2$. The profits are indeed the same for both firms and there is no first-mover advantage.

Proof of Proposition 2.18

Firm L's best reply functions and the resulting profit functions remain unchanged compared to Lemma 2.3. For firm H, the cost c cannot be larger than $\hat{b} = \frac{1}{2}\alpha\theta^2$ as that would guarantee it may never profitably participate in the market, for that would involve setting a price in the region where the demand for good H is zero.

Thus, for suitable values of $c < \hat{b}$, the best reply map for firm H shifts outwards, as firm H has to increase the price to recoup some of the variable cost. Due to linear demand and uniform distribution, the maximum profit that may be extracted from the market can be calculated by maximising $\pi_H^{max} = (b_H - c)\frac{\theta - \frac{2b_H}{\alpha\theta}}{\theta}$, which happens at $b_H^* = \frac{\alpha\theta^2}{4} + \frac{c}{2}$. Indeed this is the best reply for firm H in region A' as long as it stays inside this region, which happens if $b_H^* > \sqrt{\frac{\alpha\theta^2 b_L}{2}}$, or $0 \leq b_L < \frac{(\alpha\theta^2+2c)^2}{8\alpha\theta^2}$. Thus it becomes possible that it may cross the best reply correspondence of firm L in region A', which is described by $b_L^*(b_H) = \frac{b_H^2}{\alpha\theta^2}$ on $(\sqrt{2}-1)\alpha\theta^2 \leq b_H < \frac{1}{2}\alpha\theta^2$. This can only happen if the cost is high enough, i.e. $(\sqrt{2}-1)\alpha\theta^2 \leq \frac{\alpha\theta^2}{4} + \frac{c}{2} < \frac{1}{2}\alpha\theta^2$, which happens whenever $\hat{c} = \frac{1}{2}(4\sqrt{2}-5)\alpha\theta^2) \leq c < \hat{b}$. In other words, provided that the difference in the marginal cost is sufficiently large (but not too large to guarantee positive demand for firm H), an equilibrium in simultaneous entry game may exist. In the equilibrium firm H plays b_H^*, while firm L plays $b_L^* = b_L^*(b_H^*) = \frac{(\frac{\alpha\theta^2}{4}+\frac{c}{2})^2}{\alpha\theta^2} = \frac{(\alpha\theta^2+2c)^2}{16\alpha\theta^2}$. As $b_L^*(b_H^*) < \frac{(\alpha\theta^2+2c)^2}{8\alpha\theta^2}$, the solution indeed sits inside region A'. The resulting profits are $\pi_L^* = \frac{2b_H^* b_L^*}{\alpha\theta^2} - \frac{b_H^{*2}}{b_L^*} = \frac{(\alpha\theta^2+2c)^3}{64(\alpha\theta^2)^2}$ and $\pi_H^* = b_L^* - \frac{2b_L^{*2}}{\alpha\theta^2} = \frac{(\alpha\theta^2-2c)^2}{8\alpha\theta^2}$, so we must have $\pi_L^* > \pi_H^*$ for the relevant range of c.

Similarly to the proof of Proposition 2.15, the surplus of the non-buyers is $U_0^* = \int_0^{\tilde{w}_L^*}(\alpha w \frac{\tilde{w}_L^*}{2})\frac{1}{\theta}dw$, the surplus of the buyers of the lower-priced good is $U_L^* = \int_{\tilde{w}_L^*}^{\tilde{w}_H^*}(-b_L^* + \alpha w \frac{\tilde{w}_L^* + \tilde{w}_H^*}{2})\frac{1}{\theta}dw$ and the surplus of the buyers of the higher-priced good is $U_H^* = \int_{\tilde{w}_H^*}^{\theta}(-b_H^* + \alpha w \frac{\tilde{w}_H^* + \theta}{2})\frac{1}{\theta}dw$. Therefore the overall consumer surplus is:

$$CS^{duo} = U_0^* + U_L^* + U_H^* = \frac{23\alpha^3\theta^6 - 6c\alpha^2\theta^4 + 20c^2\alpha\theta^2 - 8c^3}{128\alpha^2\theta^4},$$

where the second equality follows from plugging in the equilibrium values computed above. We analogously calculate total welfare, $TW^{duo} = CS^{duo} + \pi^*$. From Propositions 2.2 and 2.3 we know that $CS^{mon} = \frac{3}{16}\alpha\theta^2$ and $TW^{mon} = \frac{5}{16}\alpha\theta^2$. We can therefore verify that for the relevant values of $c \in [\hat{c}, \hat{b})$, it is always true that $CS^{mon} > CS^{duo}$ and $TW^{mon} > TW^{duo}$.

Proof of Lemma 2.4

Assume partial coverage first. Modifying (2.1), the social contacts' inferences will be given by:

$$\rho_0 = \frac{\int_0^{\tilde{w}} wg(w)dw}{\int_0^{\tilde{w}} g(w)dw} = \begin{cases} \frac{\omega-\tau+2\tilde{w}}{3} & \text{if } \tilde{w} \in [\omega-\tau, \omega) \\ \frac{\omega^3+3\omega^2\tau-3\omega\tau^2+\tau^3-3(\omega+\tau)\tilde{w}^2+2\tilde{w}^3}{3\left(\omega^2+2\omega\tau-\tau^2-2(\omega+\tau)\tilde{w}+\tilde{w}^2\right)} & \text{if } \tilde{w} \in [\omega, \omega+\tau], \end{cases}$$

and:

$$\rho_1 = \frac{\int_{\tilde{w}}^{\theta} wg(w)dw}{\int_{\tilde{w}}^{\theta} g(w)dw} = \begin{cases} \frac{\omega^3-3\omega^2\tau-3\omega\tau^2-\tau^3-3(\omega-\tau)\tilde{w}^2+2\tilde{w}^3}{3\left(\omega^2-2\omega\tau-\tau^2-2(\omega-\tau)\tilde{w}+\tilde{w}^2\right)} & \text{if } \tilde{w} \in [\omega-\tau, \omega) \\ \frac{\omega+\tau+2\tilde{w}}{3} & \text{if } \tilde{w} \in [\omega, \omega+\tau]. \end{cases}$$

Thus the indifference condition (2.3) gives the following values of the price as a function of the threshold:

$$b(\tilde{w}) = \begin{cases} \frac{2\alpha\tau^2(2\tilde{w}-2\omega-\tau)}{3\left(\omega^2-2\omega\tau-\tau^2-2(\omega-\tau)\tilde{w}+\tilde{w}^2\right)} & \text{if } \tilde{w} \in [\omega-\tau, \omega) \\ \frac{2\alpha\tau^2(-2\tilde{w}+2\omega-\tau)}{3\left(\omega^2+2\omega\tau-\tau^2-2(\omega+\tau)\tilde{w}+\tilde{w}^2\right)} & \text{if } \tilde{w} \in [\omega, \omega+\tau]. \end{cases}$$

The demand for the positional good is:

$$D(\tilde{w}) = \int_{\tilde{w}}^{\theta} g(w)dw = \begin{cases} \frac{-\omega^2+2\omega\tau+\tau^2+2(\omega-\tau)\tilde{w}-\tilde{w}^2}{2\tau^2} & \text{if } \tilde{w} \in [\omega-\tau, \omega) \\ \frac{(\omega+\tau-\tilde{w})^2}{2\tau^2} & \text{if } \tilde{w} \in [\omega, \omega+\tau]. \end{cases}$$

Consequently, the monopolist's profit is:

$$\pi(\tilde{w}) = b(\tilde{w})D(\tilde{w}) = \begin{cases} \frac{\alpha\tilde{w}(2\omega+\tau-2\tilde{w})}{3} & \text{if } \tilde{w} \in [\omega-\tau, \omega) \\ \frac{\alpha\tilde{w}(2\omega-\tau-2\tilde{w})(\omega+\tau-\tilde{w})^2}{3\left(\omega^2+2\omega\tau-\tau^2-2(\omega+\tau)\tilde{w}+\tilde{w}^2\right)} & \text{if } \tilde{w} \in [\omega, \omega+\tau]. \end{cases}$$

Suppose first that $\tilde{w}^* \in [\omega, \omega+\tau]$, so that

$$\begin{aligned}\frac{d\pi(\tilde{w})}{d\tilde{w}} = \frac{\alpha}{C}\big(&-4\tilde{w}^5 + 3(6\omega+5\tau)\tilde{w}^4 - 4(8\omega^2+13\omega\tau+\tau^2)\tilde{w}^3 \\ &+ (28\omega^3+66\omega^2\tau+12\omega\tau^2-8\tau^3)\tilde{w}^2 - 12(\omega^3+3\omega^2\tau+\omega\tau^2-\tau^3)\tilde{w} \\ &+ (\omega+\tau)^2(2\omega^3+3\omega^2\tau-4\omega\tau^2+\tau^3)\big)\end{aligned}$$

where $C = 3\big(\omega^2+2\omega\tau-\tau^2-2(\omega+\tau)\tilde{w}+\tilde{w}^2\big)^2$. The FOC has at most 5 roots, one of which is $\tilde{w} = \omega+\tau$, which makes no economic sense as it leads to a profit of zero. Dividing the numerator of the derivative by $(\omega+\tau-\tilde{w})$ we obtain a new polynomial. But it can be shown that this polynomial is negative at ω and $\omega+\tau$, and its derivative is always negative over the relevant range. Therefore, $\Gamma(\tilde{w})$ cannot have any real roots in $[\omega, \omega+\tau]$ and there is no profit-maximising choice of $\tilde{w}$ in this range. Suppose then that $\tilde{w}^* \in [\omega-\tau, \omega)$, so that $\frac{d\pi(\tilde{w})}{d\tilde{w}} = \frac{\alpha(2\omega+\tau-4\tilde{w})}{3}$. The FOC has only one root, $\tilde{w}^* = \frac{2\omega+\tau}{4}$. As $\tau \leq \omega$, $\tilde{w}^*$ is always less than ω. In order for it to be greater than or equal to $\omega-\tau$ we need $\tau > \frac{2}{5}\omega$. Otherwise the market is fully covered. Under partial coverage, the price is $b(\tilde{w}^*) = \frac{4\alpha\tau^2(2\omega+\tau)^2}{3(7\tau^2+20\omega\tau-4\omega^2)}$ and the profit is $\pi(\tilde{w}^*) = \frac{1}{24}\alpha(2\omega+\tau)^2$. Under full coverage, using the D1 refinement, the monopolist sets the highest possible price which guarantees that even the poorest agent prefers buying to not buying, $-b+\alpha(\omega-\tau)\omega \geq \alpha(\omega-\tau)(\omega-\tau)$, so that $b^* = \alpha\tau(\omega-\tau)$. This results in a profit of $\pi^* = b^* = \alpha\tau(\omega-\tau)$.

Proof of Proposition 2.19

For consumer surplus, under partial coverage:

$$\begin{aligned}CS^*_{\tilde{w}^*>\omega-\tau} = CS(b^*, \tilde{w}^*) &= \int_{\omega-\tau}^{\tilde{w}^*} \alpha w \rho_0^* g(w)dw + \int_{\tilde{w}^*}^{\omega+\tau} (-b^* + \alpha w \rho_1^*) g(w)dw \\ &= \frac{\alpha(-208\omega^4+1120\omega^3\tau+240\omega^2\tau^2+16\omega\tau^3+29\tau^4)}{72(-4\omega^2+20\omega\tau+7\tau^2)},\end{aligned}$$

where the second equality is obtained by plugging in the optimal values from Lemma 2.4. Taking the derivative gives:

$$\frac{dCS^*_{\tilde{w}^*>\omega-\tau}}{d\tau} = \frac{\alpha(-160\omega^5 + 496\omega^4\tau - 1616\omega^3\tau^2 + 88\omega^2\tau^3 + 926\omega\tau^4 + 203\tau^5)}{36(-4\omega^2 + 20\omega\tau + 7\tau^2)^2},$$

which is always negative for $\frac{2}{5}\omega < \tau \leq \omega$. Similarly, under full coverage $CS^*_{\tilde{w}^*=\omega-\tau} = CS(b)_{\tilde{w}^*=\omega-\tau} = \int_{\omega-\tau}^{\omega+\tau}(-b + \alpha w\omega)g(w)dw = \alpha(\omega^2 - \omega\tau + \tau^2)$. Taking the derivative gives $\frac{dCS^*_{\tilde{w}^*=\omega-\tau}}{d\tau} = \alpha(-\omega + 2\tau)$, which again is always negative for $0 \leq \tau \leq \frac{2}{5}\omega$. As $\lim_{\tau\to\frac{2}{5}\omega} CS^*_{\tilde{w}^*>\omega-\tau} = \lim_{\tau\to\frac{2}{5}\omega} CS^*_{\tilde{w}^*=\omega-\tau}$ when the degree of coverage changes, consumer surplus strictly falls with τ for all values of τ.

For total welfare, we add up the relevant profit and the consumer surplus. Thus, under partial coverage:

$$\begin{aligned} TW^*_{\tilde{w}^*>\omega-\tau} &= CS^*_{\tilde{w}^*>\omega-\tau} + \pi^*_{\tilde{w}^*>\omega-\tau} \\ &= \frac{\alpha(-128\omega^4 + 656\omega^3\tau + 276\omega^2\tau^2 + 80\omega\tau^3 + 25\tau^4)}{36(-4\omega^2 + 20\omega\tau + 7\tau^2)}, \end{aligned}$$

and the derivative is:

$$\frac{dTW^*_{\tilde{w}^*>\omega-\tau}}{d\tau} = \frac{\alpha(-32\omega^5 - 208\omega^4\tau - 16\omega^3\tau^2 + 1400\omega^2\tau^3 + 1030\omega\tau^4 + 175\tau^5)}{18(-4\omega^2 + 20\omega\tau + 7\tau^2)^2},$$

which is always positive for $\frac{2}{5}\omega < \tau \leq \omega$. Under full coverage we have $TW^*_{\tilde{w}^*=\omega-\tau} = CS^*_{\tilde{w}^*=\omega-\tau} + \pi^*_{\tilde{w}^*=\omega-\tau} = \alpha\omega^2$, which does not depend on τ. Again, $\lim_{\tau\to\frac{2}{5}\omega} TW^*_{\tilde{w}^*>\omega-\tau} = \lim_{\tau\to\frac{2}{5}\omega} TW^*_{\tilde{w}^*=\omega-\tau}$. Hence total welfare weakly increases with τ, with the strict increase under partial coverage (or a switch from full to partial coverage).

Chapter 3

Cardinal Positional Externalities with an Application to Sorting Between Communities

3.1 Introduction

It is a widely observed phenomenon that social status matters to people no less than their physical consumption. In most societies, the most important determinant of a person's status is his income. Incomes, however, cannot be directly observed, which leads to the need for a mechanism through which the social standing can be mutually recognised. Such a mechanism is provided by positional goods, which can be defined as "those things whose value depends relatively strongly on how they compare with things owned by others" (Hirsch 1976). Examples of such goods span from luxuries, such as Rolex watches or luxurious yachts, to charitable giving and education.

Significant amounts of empirical and experimental research have been produced on the importance of positional concerns. I will discuss only a few examples here. Perhaps the most famous one is the Easterlin paradox, which states that in a cross

section happiness varies directly with income, but over time it does not trend upward as income continues to grow (Easterlin 1974). For example, while Japan experienced a six fold increase in real GDP per capita between 1958 and 1991, there had been negligible variation in reported life satisfaction over the same period (Frey and Stutzer 2002). Clark, Frijters, et al. (2008) explain that such an apparent inconsistency is due to the increasing material norms on which judgements of well-being are based: people with higher income tend to be happier, but raising everybody's income does not raise everybody's happiness, since *relative* income does not improve. Using panel data for West Germany, Ferrer-i-Carbonell (2005) finds not only that the average income of the reference group is as important as the individual's own income for the agent's happiness, but also that the agent is happier whenever his income significantly exceeds the average. These empirical findings seem to be confirmed in experiments which aim to quantify how strongly individuals care about status. Alpizar et al. (2005), for example, measure the marginal degree of positionality, defined as the fraction of the total utility change from the last dollar spent, which comes from the relative income, and obtain a number as high as 40-45 *per cent*.

Despite the early efforts by Thorstein Veblen, James S. Dusenberry and Fred Hirsch, standard economic theory still has not adopted positional goods into the mainstream. This is because unlike standard consumption, its relative counterpart is much fuzzier as a concept. Chief amongst the unanswered (or not fully answered) questions about relative concerns is whether positional competition is *ordinal* or *cardinal* in nature. When a person consumes oranges, it is uncontroversial that he cares about the quantity and quality consumed. However, when a person purchases a Rolex to display his wealth, it is unclear if he just wants to have a better watch than his friends, or perhaps if he also cares about how much his watch eclipses theirs. The difference is akin to the difference between an Olympic sprint competition and a poker tournament in which players use their own money. In the former, only the

final placement in the competition matters, but in the latter, it is important how much money was won relative to other players. However, even brushing aside these concerns, to assume that agents compare themselves to everyone else in the population considerably simplifies the problem. In reality, status competition is much more local. On the one hand, the frame of reference for comparisons may be provided by groups that are "social" in nature, such as family, high school friends or work colleagues. Agents almost always prefer to be members of groups of higher quality, even if the only reason for that is the intrinsic pleasure without any economic consequence. On the other hand, there are also groups which are "physical" in nature, such as the residents of a city or members of a club. There, the intrinsic reasons for wanting to join a high-quality group (thereafter referred to as a community) may also be present, albeit more important seem to be the "hard" consequences of such decisions. For example, the cities may differ by the quality of air, while clubs may differ by the level of club good provision. In either case, the choice of the community is only the first step, after which agents begin to compete for status against their follow members in the manner described above.

The topic of the interaction between status and sorting, which can take the form of migration or segregation, amongst others, has been extensively examined in the empirical literature. Stark and Taylor (1991) develop a straightforward econometric model which assumes that migration is a consequence of the desire to increase agent's relative standing in the distribution of incomes. Using evidence from Mexico, they demonstrate a sizeable impact of relative deprivation on the intensity with which workers migrate, both internally and from Mexico to the United States. Cheshire et al. (2003) assert that "not only is the role of positional goods critical in producing segregation but also that the intensity of segregation interacts with the distribution of incomes." (p.85) They give an example of the Reading area in the United Kingdom, where Thames fronting houses are available in limited supply. The increase in inequality between 1984 and 1993, as measured by the Gini coefficient, resulted

in a higher concentration of the most affluent residents among the riverfront house occupiers. As for the "social" type of groups, Clark and Senik (2010) provide an empirical study of the frames of reference commonly used by Europeans. They find that colleagues are the most frequently cited reference group, but friends and family are also found to be significant.

In light of the above discussion, my aim in this chapter is twofold.[1] First, I want to build a model of cardinal positional competition, where agents care about the extent to which they consume more than their peers. Such a model provides an adequate description of the social competition is certain situations. It also allows us to understand the comparison of the mechanisms and outcomes *versus* the ordinal model. Second, I apply the model in a multiple-community scenario to understand the interplay between positional competition and sorting. To achieve these aims, I develop two kinds of models. Initially, I work with the most general version possible and try to elicit a maximal amount of intuition with a minimal number of assumptions. This approach allows the model to cover a wide range of detailed specifications. However, at some point it becomes impossible to draw further conclusions, so I add more structure to the model by choosing two plausible functional forms for the utility and status functions, as well as the uniform distribution of income, and continue the analysis in this framework.

Section 3.2 is devoted to the general version of the model. Agents have conventional utility functions augmented by an additional status component. The only source of heterogeneity in the model is income, which is not observable to other agents. The number of agents is large so that each agent's decision has a negligible effect on the other agents. I initially assume that only one community exists. I make some mild assumptions about the required properties of the utility and status functions, in particular about their concavity with respect to agent's own positional consumption. I also identify two important conditions that a reasonable positional

[1]This chapter is based on my MPhil thesis.

good model should obey, and which are difficult to impose directly through assumptions on the derivatives of the utility and status functions. First, agents must display following-the-crowd behaviour, in that an increase in the positional consumption of others must lead them to increase their positional consumption. Second, the positional good must be a luxury, in that when agent's income increases, a more-than-proportional amount is spent on positional consumption.

Using all these assumptions, I show that agents over-consume the positional good in the equilibrium compared to the social optimum, and that their utilities go up whenever average positional consumption goes down. I show that an equilibrium always exists in this game of status and derive conditions for uniqueness and stability. I also demonstrate that both an increase in incomes (in the sense of first-order stochastic dominance), as well as an increase in income inequality (in the sense of a mean preserving spread) increases average positional consumption. I conclude the analysis of the single-community case by providing a basic characterisation of the optimal Pigouvian tax.

I next relax the assumption that only one community exists. Whenever there are no quality differences between communities, I demonstrate that a stratified equilibrium cannot exist. Moreover, I show that if agents are unable to switch between communities, any external increase in the threshold income which divides the poor community from the rich community necessarily increases average positional consumption in both communities. I conclude by providing a basic characterisation of the case with multiple-communities of varying qualities. I show that the relevant single crossing condition is crucial for existence of a stratified equilibrium, but it is not guaranteed that it is necessarily satisfied. Throughout the general section, I provide a comparison of results *versus* the ordinal model of Hopkins and Kornienko (2004).

Section 3.3 is devoted to the specific models which fulfil the requirements set out by the general discussion. In particular, I propose two tractable models which cap-

ture different aspects of the positional good competition. Throughout the analysis, I use whichever model is more suitable, and try to triangulate the results.

I first repeat some exercises from the general section to elicit additional intuition for the two cardinal models as well as the ordinal model based on Hopkins and Kornienko (2004). I find that because positional competition is zero sum in my model, the cooperative solution corresponds to the socially optimal solution. I thus find that the optimal tax is decreasing in income, because the status distortion is highest at the bottom. As a result of the tax, the choices of the poor agents are distorted to such an extent that the tax does not constitute a Pareto-improvement. I demonstrate that a Pareto-improving tax may nevertheless exist by computing the optimal *linear* consumption tax.

Next, I return to the question of sorting. In the identical qualities' scenario with pre-assigned communities, I show that more "local" competition can be harmful, as whenever the number of communities goes up, total welfare falls. Total welfare also falls when agents compare themselves upwards to the agents in richer communities.

When communities differ in quality, I incorporate the quality component multiplicatively into the utility function, as this captures the complementarity to other types of consumption. I show that such preferences satisfy the single crossing condition and derive conditions under which a stratified equilibrium exists. I then examine the behaviour of the threshold value of income which separates agents into either of the communities and argue that community choice can be understood in terms of the relative strengths of the *within* effect (i.e. status competition) and *between* effect (i.e. desire for community quality). Finally, I endogenise quality determination by assuming that quality is a function of the average income in the community, either for intrinsic reasons (agents just enjoy the prestige) or for more concrete reasons (e.g. the level of public good provision is linked to average income). I show that there exist parameter values for which a stratified equilibrium exists. I conclude by comparing the utilities of agents under the one-community and two-communities'

scenarios.

3.1.1 Related Literature

The literature on positional concerns and status, as well as sorting, is vast. Therefore, below I discuss the papers which I have identified as the most relevant. For broader surveys, see Hopkins (2008) and Kuminoff et al. (2013), respectively.

One of the key categorisations of status is whether its nature is cardinal or ordinal. Given that my model is cardinal, I look to this category first. Congleton (1989) develops a model of status-seeking behaviour in a pure status good context. He assumes that an agent's utility rises in his own status good consumption and falls in the average status consumption, as well as that the marginal utility of status is a decreasing function of the average consumption of the status good. As a consequence, an increase in the average consumption of the status good has two effects - it reduces status produced by a fixed level of agent's own positional consumption, but it also decreases the rate of growth at which additional purchases of the status good generate additional status. Whether the agents follow the behaviour of others depends on whether the former effect dominates the latter effect, which in my model is the case by assumption. Congleton (1989) notes that that the described effects constitute features of the status-assigning mechanism rather than the agent's utility function. This separation is also assumed in my model, because utility and status functions can change independently from each other.

A similar model of cardinal positional externality has been developed by Clark and Oswald (1998). They assume that an agent's total utility depends on a convex combination of his direct private utility and the comparison status utility. Agents compare their choice of the status good consumption to the average either through additive comparisons (agent's own consumption minus the average) or through ratio comparisons (agent's own consumption divided by the average). Their main finding is that the following-the-crowd behaviour requires sufficient degree of concavity of

the comparison utility function. For the additive comparisons, just standard concavity suffices, but for ratio comparisons it is required that the elasticity of the marginal utility of status be higher than one. I obtain the same results, and in the specific model of Section 3.3, I employ ratio-based status with a CRRA transformation which ensures that the condition above is satisfied. Clark and Oswald (1998) also find that when the cost of the status good is convex and the direct utility is linear, agents adjust their status consumption less-than-proportionally and the strength of the following of others depends on the strength of the status concerns.

Turning to ordinal models of status, Frank (1985) first proposed a rigorous model of status based on the rank in the distribution of an observable good. Such a specification makes it a natural zero-sum game, as a gain in rank for one agent must mean a loss of rank for another. Frank (1985) suggests the cooperative bundle - a level of consumption that would be chosen if agents ignored their positional concerns, as a natural benchmark for the competitive equilibrium. In a competitive equilibrium, the ranks in the distribution of positional consumption are the same as the ranks in the distribution of income. Frank's (1985) main conclusion, therefore, is that the positional good is always over-consumed in the equilibrium, which for all agents results in lower utilities than they could have achieved if everyone consumed their preferred cooperative bundle (and nevertheless obtained the same status). His model is extended by Hopkins and Kornienko (2004), who consider a continuous distribution of agents varying by incomes, who compete for rank-based status. Their formulation constitutes an important reference point for this chapter, so a brief summary of their model is provided in Appendix 3.A. As mentioned, the key results, which examine the impact of changes in inequality and the tax policy, are compared and contrasted with my results in the main text. Some other relevant papers examining the implications of ordinal status concerns are Frank (1984), who considers how they can compress the wage schedule compared to the distribution of the workers' marginal products, Hopkins and Kornienko (2009), who consider the

effects of changes in inequality on an agent whose rank (rather than income) remains unchanged, and Hopkins (2018), who considers the relationship between inequality and risk-taking behaviour in the presence of ordinal status effects.

Bilancini and Boncinelli (2008) address the natural question of the circumstances in which the two types of status specifications above lead to different conclusions. They show that if status were cardinal in Frank's (1985) formulation, positional concerns would not affect agents' behaviour for an important class of utility functions. Conversely, if status were ordinal as in Clark and Oswald's (1998) formulation, agents might display following-the-crowd behaviour even if the comparison utility were convex. This chapter supplements this list with further examples in which these specifications differ, particularly in relation to income inequality and sorting.

Positional concerns have also been studied in signalling models, where agents engage in positional consumption in an attempt to obtain a higher level of status than their true types. Ireland (1994) concludes that this is inefficient, and thus a Pareto-improving tax policy must exist. Similar approach has been used by Konrad (1992) with regards to charitable contributions, Kempf et al. (2001) with regards to participation in clubs, and Bagwell and Bernheim (1996) with regards to pure profits for status good manufacturers. Ireland (1998, 2001) examines income tax policy in the signalling framework with quasi-linear preferences and argues that progressive income taxes are not necessarily justified by positional concerns.

Moreover, Cole et al. (1992) propose an "instrumental" approach to status as a means to facilitate trade in non-market goods (in particular, the matching market for marriage). Their status concerns are generated endogenously, and the reduced-form preferences can vary depending on the type of social organisation. In my model, status concerns are *assumed*, as this simpler approach enables the analysis of sorting; nonetheless, critique of such reduced-form approaches applies, as explained e.g. by Postlewaite (1998). Becker, Murphy, and Werning (2005) take a different approach and look at a hedonic market for social positions in a framework where

agents have an innate desire for status. They discover that such an arrangement can be perfectly replicated by a market for a status good together with an appropriate social norm.

Of the literature on sorting in the spirit of Tiebout, the most relevant papers are Westhoff (1977), Epple and Platt (1998), and Fernandez and Rogerson (1996), who analyse a multi-community model in which communities are characterised by different tax rates (determined by the voters in the community) and, consequently, different qualities of schooling. The communities are *ex-ante* identical but differ *ex-post* due to the different characteristics of agents who end up joining them. Indeed, agents must choose their preferred combination from the finite number of available communities. Westhoff (1977) derives conditions for existence of an equilibrium, the most important of which is the single crossing condition of agents' indifference curves in the tax-quality space. Epple and Platt (1998) observe that in equilibrium, three properties characterise sorting by each household type: boundary indifference (agent at the border between communities must be exactly indifferent between them), increasing bundles (if the tax is higher in one community than other, so must be the quality of schooling) and stratification (if an agent of a certain income chooses a community with a certain quality, an agent with a strictly higher income cannot choose a strictly lower-quality community). Fernandez and Rogerson (1996) use a more structured framework to derive some policy recommendations, the most important amongst which is that moving rich agents to poor communities may result in lower taxes and higher quality of schooling for all. I discuss the similarities and differences with these models in the main text of the chapter.

3.2 General Model

In this section, I first formulate the general model which captures the phenomenon described in the introduction. I then derive the restrictions on its structure which

guarantee the empirically observed behaviour and characterise the resulting equilibrium.

There is a unit mass of agents who vary by income $y \in [\underline{y}, \bar{y}]$, where $\bar{y} > \underline{y} > 0$. As income is the only source of heterogeneity, it fully defines the agent's type. The measure of agents whose income is at most y is $F(y)$, and the corresponding probability density function is $f(y)$. The number of agents is large so that each agent has a probability measure zero, and thus his impact on the consumption decisions of other agents is negligible.

There exist N social groups (also referred to as communities) and each agent has to decide which singular one to join. The number of social groups is fixed and determined prior to the agents' decision. Let C_i where $i \in \{1, \ldots, N\}$ denote community i. Each community C_i is characterised by a certain quality q_i, which may be a function of its inherent characteristics as well as the characteristics of the set of agents who join it. This allows the quality of the community to depend on the incomes of its members, as e.g. they are related to the level of public good provision. Let $\beta(y)$ denote the choice of the community made by agent with income y, so that $\beta(y) = i$ if and only if agents with income y choose community C_i.

Agents draw utility from four components: unobservable (*aka* non-positional) consumption x, observable (*aka* positional) consumption z, status s and quality of the social group (*aka* community) q, so that the utility function is $u(x, z, s, q)$. That the utility depends on non-positional consumption is standard; likewise, the direct presence of z in the utility function means that it might not be a *purely* positional good, but it also yields direct utility. The other two components of the utility function - status and community quality - are non-orthodox and constitute the focus of this chapter. The agents face standard budget constraints, $y \geq x + pz$, where p is the relative price of the positional good in terms of the non-positional good, which serves as a *numéraire*. I denote the consumption choices of an agent with income y by $x(y)$ and $z(y)$.

The function which determines status can, in principle, depend on all of the choices of the positional consumption in a given social group. In line with empirical research, however, I assume in this cardinal model that status has only two determinants: $z_i(y)$, the agent's own positional consumption, and Z_i, the *average* positional consumption in his chosen social group C_i, so that $s = s(z, Z)$.[2] This contrasts with *Model HK*, where status is ordinal and depends on the whole *distribution* of the positional consumption rather than its first moment only. Some intuitive specifications of the status function in my cardinal model, which have some grounding in empirical research, are $s(z, Z) = h(z - Z)$ and $s(z, Z) = h(\frac{z}{Z})$, where $h(.)$ is a monotone transformation.[3] The former corresponds to the *difference* (or *deviation*) comparison, while the latter corresponds to the *ratio* comparison.[4]

The game proceeds as follows: (i) the agents simultaneously choose which social group they join; (ii) the agents simultaneously allocate their incomes between non-positional and positional consumption; (iii) quality is determined for each social group, status is determined for every agent based on the average positional consumption in the chosen social group, and payoffs are realised for all agents.

Using backward-induction, once the agents have chosen the community C_i, they maximise their utility $u(x, z, s(z, Z_i), q_i)$ subject to the budget constraint and the non-negativity constraints on x and z. When doing so, they take the quality of the social group q_i and the positional consumption of other agents - and the resulting average positional consumption Z_i - as given. I only focus on equilibria where agents play pure strategies, so because income is the only source of heterogeneity, every agent with income y who resides in social group C_i will choose the same level of positional consumption $z_i(y)$. Consequently, the average positional consumption in

[2]Arguments for such a formulation can also be made on the grounds of prohibitive costs of comparing oneself to everybody else.

[3]The arguments of the functions are suppressed throughout this chapter for conciseness whenever the notation remains unambiguous.

[4]Empirical tests that would allow us make a verdict regarding the choice of the specification are scarce. Some experimental data suggests that the ratio formulation performs better in explaining respondent behaviour, but evidence is weak (Johansson-Stenman et al. 2002).

community C_i upon aggregation becomes $Z_i = \frac{\int_{\underline{y}}^{\bar{y}} z_i(y) \mathbb{1}_{\beta(y)=i} f(y) dy}{\int_{\underline{y}}^{\bar{y}} \mathbb{1}_{\beta(y)=i} f(y) dy}$.

In the first stage, the agents compare the maximised second-stage payoffs that they can obtain by joining each of the communities, $u_i^*(y)$, and choose the community which gives them the highest payoff. If the equilibrium is stratified, as later detailed, the income distribution will be partitioned into communities based on income. The boundaries of the communities will be determined by the agents who are just indifferent between the neighbouring communities. If the threshold incomes for community C_i are $\hat{y}_{i-1}$ and $\hat{y}_i$, where $i \in \{1, \ldots, N\}$, $\hat{y}_0 = \underline{y}$ and $\hat{y}_N = \bar{y}$, the community choice function will be $\beta(y) = i$ if $y \in [y_{i-1}, y_i)$, and the partitioning can be fully described by the set of thresholds $\{\hat{y}_1, \ldots, \hat{y}_{N-1}\}$ instead. If the assignment to the social groups is not decided by the agents themselves, but rather by Nature or the social planner, the agents immediately play the second stage of the game in their allocated community.

For the above-described game, the relevant equilibrium concept is the Sub-game Perfect Equilibrium (thereafter equilibrium), which simplifies to a NE when the agents do not choose in which community to reside. While there is incomplete information in the game, as the reason why agents engage in positional competition is that they have no other way of signalling their incomes, this is not reflected in the choice of the solution concept. The reason is that the model *assumes* that the status function translates agent's own positional consumption and the average positional consumption of his community into status, without explicitly considering beliefs of the agents about the incomes of other community members. In other words, the beliefs of agents do not impact their actions. This is the key difference *versus* the signalling models of status, such as Konrad (1992) or Ireland (1994, 2001). The equilibrium consists of social group choice function $\beta^*(y)$ (or equivalently a set of thresholds in a stratified equilibrium) and the consumption basket $(x_i^*(y), z_i^*(y))$ for every income level present in the group, $y \in C_i$, for all $i \in \{1, \ldots, N\}$, such that no agent has a profitable deviation from his prescribed behaviour.

3.2.1 Assumptions

To provide a good description of the positional goods phenomenon, some additional structure is required for the presented model. First, I assume that $u_r(.) > 0$ for $r \in \{x, s, q\}$ and $u_z(.) \geq 0$.[5], so that the agents display non-satiation in consumption, status and quality of the community, except perhaps for the intrinsic utility from the positional good, which can be zero if the good is purely positional (in which case it only works thorough the status channel). This assumption is standard and means that in the equilibrium the budget constraint will hold with equality, which simplifies the optimisation problem.

Second, I assume that the utility function is weakly concave in all arguments except perhaps q, $u_{rr}(.) \leq 0$ for $r \in \{x, z, s\}$, so that the marginal utilities from either kind of consumption as well as status are diminishing. Such an assumption is required for the maximisation problem to be well defined, in the sense that not all income is spent on only one good. While the diminishing marginal utility of the goods is a standard assumption, its status counterpart requires further justification, as below.

Third, I assume that there is weak complementarity between the intrinsic utility from either of the goods, status and the quality of the community, $u_{rr'}(.) \geq 0$ for $r, r' \in \{x, z, s, q\}$ and $r \neq r'$. This is also uncontroversial with regards to the goods, and there is no reason to believe that status or the quality of the community should behave differently.[6] Note that the cross derivatives are zero if any part of the utility function is separable.

Fourth, status is an increasing function of the agent's own positional consumption and a decreasing function of the average positional consumption. This assumption lies at the heart of the model, as has been explained in the introduction. While the first part is uncontroversial, the second part might not be deemed so. It assumes

[5]I use the subscripts to denote the corresponding partial derivatives.

[6]If the quality of the community signifies the provision of the public good, this assumption becomes standard. For status, see e.g. Becker and Murphy (2000) for a justification.

that, using the terminology of Friedman and Ostrov (2008), the agents display *envy*, as their utility increases as those above them become worse-off, but they also display *pride* (or *downward envy*), as their utility increases as those below them become worse-off. Whereas envy is a well-documented phenomenon, there is less consensus regarding pride. As an alternative assumption to pride, *compassion* is where agents are better off when those below them become better off. While this phenomenon has some empirical grounding it is outside the scope of this chapter.[7]

Fifth, I assume that the status function is (weakly) concave in the agent's own positional consumption, $s_{zz}(.) \leq 0$. The rationale for this is the same as with the utility function, and the need to specify both $u_{ss}(.)$ and $s_{zz}(.)$ results from a very general form of the preferences in the proposed model. In fact, such strong conditions are not required, and it suffices that the reduced-form problem remains concave in the agent's own positional consumption, so that the second-order condition of the maximisation problem is satisfied:

$$\begin{aligned}\frac{d^2u(.)}{dz^2} &= p^2u_{xx}(.) - pu_{xz}(.) - 2pu_{xs}(.)s_z(.) - pu_{xz}(.) + u_{zz}(.) \\ &\quad + 2u_{zs}(.)s_z(.) + u_{ss}(.)s_z^2(.) + u_s(.)s_{zz}(.) < 0.\end{aligned} \tag{3.1}$$

I will refer to this condition as *Condition SOC*. In other words, even if the status function were convex in agent's own positional consumption, this would not be a problem if the utility were sufficiently concave in status to dampen the convexity. Instead, consider what would happen if the reduced-form utility were indeed convex in the agent's own positional consumption. This would result in a *winner-takes-all* economy, where the gains from increasing conspicuous consumption above the mean are ever-increasing. Hence, *Condition SOC* allows us to avoid the death spiral-like pathological cases, where all income might be spent on conspicuous consumption.

Finally, I assume that the appropriate Inada conditions hold for the utility and status functions, in that $\lim_{r\to 0^+} u_r(r, r') = \infty$ and $\lim_{r\to\infty} u_r(r, r') = 0$ for $r' > 0$

[7]For a detailed survey see Hopkins (2008).

when $r, r' \in \{x, z\}$ and $r \neq r'$, as well as $\lim_{z \to 0^+} s_z(.) = \infty$ and $\lim_{z \to \infty} s_z(.) = 0$ for $Z > 0$. These are not necessary, but they simplify the analysis without affecting the conclusions by ensuring that an interior solution always exists. As a result, the non-negativity constraints on x and z can be ignored in the subsequent analysis.

Note that, in principle, the status function is undefined when $z = Z = 0$, i.e. in absence of any positional consumption. This is not important when the positional good in not purely positional, as there always exists some intrinsic demand for the positional good so that the consumption is strictly positive. However, when the positional good becomes purely positional, it might be optimal for the agents not to consume it at all. It is also relevant if the government decided to ban buying and selling of the positional good. Some plausible candidates for $s(0, 0)$ are for the status function to be a constant, or to depend on the comparison of the agent's income and the average income in society.

On top of the direct restrictions on the derivatives of the utility and status functions, there are also two revealed-preference requirements that have been postulated in the introduction. First, I assume that the agents display following-the-crowd behaviour, so that $\frac{dz^*}{dZ} > 0$. As I will show, this is a more general requirement than additional restrictions on the derivatives of the utility and status functions, in particular on the sign of $s_{zZ}(.)$.

Second, I assume that over the range of incomes observed in the population, the positional good is a luxury, so that its income elasticity is greater than one, $\varepsilon_z > 1$. This means that as agents' incomes increase, ever-rising proportions of their budgets are spent on the positional goods. Such an observation is consistent with the Maslow's hierarchy of needs hypothesis and is widely observed in the real world.[8] As I will show, it will be difficult to "obtain" this property by considering the primitives of the utility and status functions, because that would involve making assumptions about their third derivatives, which in light of limited research would

[8]See e.g. Heffetz (2011).

prove impractical.

3.2.2 Single-Community Equilibrium

In this subsection, I solve the model for the simple situation where only one community exists and is equivalent to the whole society. For conciseness, I temporarily drop the quality of the community q from the notation, so that the utility function becomes $u(x, z, s)$. With the budget constraint binding, the unconstrained maximisation problem it for each agent to choose $z^*(y)$ which maximises $u(y-pz, z, s(z, Z))$, taking Z as given. The FOC is satisfied when:

$$-pu_x(.) + u_z(.) + u_s(.)s_z(.) = 0, \tag{3.2}$$

holds at the optimal choice z^*. The SOC for a maximum, $\frac{d^2u(.)}{dz^2} < 0$, is satisfied by assumption because of *Condition SOC*. Thus, equation (3.2) implicitly defines the demand for the positional good at each income level for a given average level of positional consumption, $z = z^*(y, Z)$.

The positional competition described in this chapter involves a negative externality: agents do not incorporate the impact of their actions on other agents into their decisions-making. In particular, whenever an agent chooses to increase his positional consumption, he inadvertently lowers the status of others and forces them to increase their positional consumption. Consequently, the utilitarian welfare is not maximised, and can be improved upon by the social planner by internalising the externality. The social planner's preferred level of positional consumption will be given the following FOC:

$$-pu_x(.) + u_z(.) + u_s(.)s_z(.) + \int_{\underline{y}}^{\bar{y}} u_s(.)s_Z(.)f(y)dy = 0, \tag{3.3}$$

which must be satisfied at the socially optimal level of positional consumption z^{SO}.[9]

[9]The rationale behind this condition can be more easily understood by considering the discrete

The socially optimal FOC differs from the competitive equilibrium FOC by the presence of an additional term in the former, which captures the positional externality mentioned above. The socially optimal demand for the positional good is defined implicitly by the FOC and will be denoted $z = z^{SO}(y, Z)$.

Another candidate for a benchmark, first proposed by Frank (1985) and used also in *Model HK*, is the so-called cooperative level of positional consumption, which is the level of consumption that would be chosen if the positional concerns did not matter (or if status were fixed). The maximisation problem in such a case would be $u(y - pz, z, \hat{s})$, where $\hat{s}$ does not depend on z, and the resulting FOC is:

$$-pu_x(.) + u_z(.) = 0, \tag{3.4}$$

which must be satisfied at the cooperative level of positional consumption, and it implicitly defines the cooperative positional consumption at every income level and for every level of fixed status, $z = z^C(y, \hat{s})$. In an ordinal model, the cooperative solution has a natural appeal because when all agents consume the consumption baskets that are chosen ignoring status, the ranking of status consumption does not change *vis-à-vis* the ranking of the equilibrium consumption or the ranking of income. Thus, any positional consumption above the cooperative level must necessarily be bad: agent's rank remains unchanged, but he ends up further away from his optimal basket (with no status effects) so the direct utility is lower. In my model, this is not the case because the extent to which the agent is ahead of or behind other agents also matters. Therefore, even though treating status as non-existent improves the direct utility, the resulting consumption basket may have an adverse effect on the status component, and thus overall utility.

In fact, for any additive or multiplicative utility function (so that the arbitrary choice of status in the cooperative case does not impact the agents' consumption),

analogue of this continuous problem: to internalise the externality it is necessary to include in the FOC the impact that a given agent has on the utilities of all other agents.

the socially optimal and cooperative level of consumption sometimes coincide. This happens when $u_s(.)s_z(.) + \int_{\underline{y}}^{\bar{y}} u_s(.)s_Z(.)f(y)dy = 0$ for all y at $Z = Z^C = Z^{SO}$. But this condition is essentially equivalent to the game of status being zero-sum, as the above condition is sufficient for $\int_{\underline{y}}^{\bar{y}} u_s(.)(s_z(.) + s_Z(.))f(y)dy = 0$. In light of this, the coincidence is not surprising. In a zero-sum game, all agents can choose their preferred cooperative basket, and thus obtain the same status as they would in a competitive equilibrium. Games of status are often assumed to be zero-sum games, so it is reasonable to expect that the relevant condition may hold in this model. Thus, I will maintain this assumption throughout this chapter, and will use the term socially optimal consumption for $Z^{SO} = Z^C$. In particular, this assumption holds for one of the specific models that I present in Section 3.3.

The following proof establishes existence and uniqueness of the interior solution to the maximisation problem, and compares it to socially optimal solution:

Proposition 3.1. *There exists a unique utility-maximising consumption basket with* $x = x^*(y, Z^*)$ *and* $z = z^*(y, Z^*)$ *for agents of all incomes for a given average positional consumption* Z*, in which all agents consume positive amounts of both goods. The demand for positional good is always strictly greater than what it would be in the cooperative solution, i.e.* $z^*(y, Z^*) > z^{SO}(y, Z^{SO})$.

An agent increases his conspicuous consumption in order to stand out from his peers, but he does not internalise the effect that his actions have on them. Consequently, agents buy too much of the positional good in equilibrium. This is a standard result in the existing literature on status rivalry, independent of the exact specification of status. Frank (1985) explains this phenomenon by noting that the presence of the status component in the utility function acts as an implicit subsidy to the positional good. The similarity to standard (e.g. environmental) externalities suggests that the socially optimal solution can be restored by an adequate Pigouvian tax - a topic which will be discussed later.

Note also that the relative strength of the positional concerns can be captured

by the extent to which $u_s(.)$ increases, for all y, x, z and Z (so that e.g. $u_s'(.) = 2u_s(.)$ means that the strength of the positional concerns doubled everywhere). It is clear from the FOC (3.2), therefore, that an increase in $u_s(.)$ must increase the consumption of the positional good further.

The following lemma shows that all agents benefit from a decrease in the average positional consumption:

Lemma 3.1. *The indirect utility of every agent increases when the average positional consumption is reduced.*

Thus, policies which bring down the equilibrium average positional consumption will have a positive effect on the agents' welfare.

With the stated assumptions it is easy to see that the positional good is a normal good:

Lemma 3.2. *The positional consumption increases in agent's own income.*

However, it is not guaranteed that the assumption about the positional good being a luxury is satisfied out of the box. But it is easy to see geometrically, that $\varepsilon_z > 1$ requires that the demand for the positional good be strictly convex in income. Thus, the question is what restrictions on the utility and status functions generate a demand function with the desired property. However, Lemma 3.2 suggests that such restrictions would involve restrictions regarding the third derivatives of the utility function and status function. In general, these are difficult to assess (particularly for the status function), and so such an analysis does yield much insight. Therefore, I will continue to assume that the relevant conditions are satisfied and we have $\frac{d^2z(y)}{dy^2} > 0$ over the relevant range.[10]

[10]Some additional intuition can be obtained by considering a separable utility function with a purely positional good z. Then, the necessary condition for the convexity of $z^*(y)$ in y is that the ratio of absolute prudence to absolute risk aversion must be smaller for the direct utility function than for the status function. Using the result from Kimball (1990), this may be true e.g. when the utility from private consumption is subject to increasing absolute risk aversion, while status is subject to decreasing absolute risk aversion. In other words, as agents become more affluent, they

The other assumption imposed externally on the model is for the agents to display following-the-crowd-behaviour, which the following lemma explains:

Lemma 3.3. *The agents display following-the-crowd behaviour if and only if* $s_{zZ}(.) > \frac{1}{u_s(.)}\big(pu_{xs}(.)s_Z(.) - u_{ss}s_z(.)s_Z(.) - u_{zs}(.)s_Z(.)\big)$

This means that the marginal status cannot diminish too quickly when the average positional consumption is raised. Otherwise, the incentive to obtain additional status will virtually disappear as it would require an enormous investment in the positional good, so the agent could be better off by essentially maximising his utility as if status did not matter. Note that because the above threshold value for $s_{zZ}(.)$ is negative, a sufficient (but not necessary) condition is for the marginal status to weakly increase in average positional consumption. This conclusion is the same as in Congleton (1989), who considers a similar simple model of status. This also links to the discussion about the model parameters in Gali (1994), where the negative cross derivative makes the consumption of others a substitute for the agent's own consumption (negative consumption externality), while the positive cross derivative results in the "keeping up with the Joneses" phenomenon (positive consumption externality).

Finally, I would like to return to the "natural" specifications of status introduced at the beginning of this section, which I argued could be based on either a deviation from the average or a ratio to the average. The sufficient conditions for the following-the-crowd behaviour are $s_{zz}(.) < 0$ and $s_{zZ}(.) \geq 0$. Any increasing, concave transformation $h(.)$ of the deviation-based status, $z - Z$, meets these criteria, as for $s(z, Z) = h(z-Z)$, we have that $s_{zz}(.) = h''(z-Z) < 0$ and $s_{zZ}(.) = -h''(z-Z) > 0$. In the case of ratio-based status, $s(z, Z) = h(\frac{z}{Z})$, we have that $s_{zz}(.) = \frac{1}{Z^2}h''(\frac{z}{Z}) < 0$ and $s_{zZ}(.) = -\frac{1}{Z^2}\big(h'(\frac{z}{Z}) + \frac{z}{Z}h''(\frac{z}{Z})\big)$. For both of the assumptions above to be satisfied, we need the latter expression to be positive, so we need to impose an additional

have to be willing to take smaller risks with respect to non-positional consumption, while they become increasingly willing to take the gambles with regards to status. The empirical validity of such a conjecture remains to be tested.

assumption on the relative risk aversion, $-\frac{h''(s)s}{h'(s)} \geq 1$ for all $s \geq 0$. In other words, a one per cent increase in status must yield *more than* one per cent increase in marginal status. Clearly, any CRRA function with the coefficient of risk aversion greater than one would meet this criterion. The discussion of these two plausible cases is fully consistent with the earlier work by Clark and Oswald (1996).

Aggregation

So far, the average level of conspicuous consumption Z has been taken as given by the agents. For an equilibrium in the described game of status to exist, I now show that we can find a consistent value of Z:

Lemma 3.4. *When the agents' consumption choices are implicitly defined by* $z = z^*(y, Z)$*, there always exists an equilibrium where the level of average positional consumption is consistent in the sense that* $Z^* = \int_{\underline{y}}^{\bar{y}} z^*(y, Z^*) f(y) dy$.

Next, I consider the uniqueness and stability of the equilibrium. To define stability, I consider a tâtonnement process, where $\dot{Z} = \int_{\underline{y}}^{\bar{y}} z^*(y, Z) f(y) dy - Z$, which could be also understood in terms of the discrete best response dynamics $Z_{t+1} = \lambda \int_{\underline{y}}^{\bar{y}} z^*(y, Z_t) f(y) dy + (1 - \lambda) Z_t$, where $0 < \lambda < 1$. What this means is that if the positional consumption of the small mass of agents were increased or decreased, the original equilibrium would be restored over time. The following lemma gives the sufficient condition for the equilibrium of Lemma 3.4 to be unique and locally stable:

Lemma 3.5. *The equilibrium in which the average level of consumption is* Z^* *is unique and locally stable if* $\int_{\underline{y}}^{\bar{y}} \frac{dz^*(y,Z)}{dZ} f(y) dy < 1$ *for all* Z.

Observe that $\frac{dz^*(y,Z)}{dZ}$ is the degree to which agents follow other agents, and Lemma 3.3 guarantees that it is positive. But for the equilibrium to be stable, the feedback mechanism cannot be too strong, in the sense that the agents cannot increase their consumption by *more* than the average has increased, because that

would lead to an explosive process. In other words, the feedback effect of a change in the average positional consumption cannot be too high.[11]

From now on, I will only consider cases where the equilibrium is unique and stable, and I will impose an even stronger condition that $\frac{dz^*(y,Z)}{dZ} < 1$ for all y and Z, called *Condition S*, which guarantees that the change in Z cannot bring about a more-than-proportional change in individual z for *all* agent regardless of Z (and not only on average). This is a reasonable assumption and simplifies the subsequent analysis.

Changes in Income Distribution

In this subsection, I examine how changes in the income distribution affect the average positional consumption, and thus the agents' utilities. I consider two types of changes: an increase in the mean income, in the sense of first-order stochastic dominance (FOSD), and a reduction in inequality, in the sense of second-order stochastic dominance (SOSD).

Assume that the original distribution $F(y)$ changes to a new distribution $G(y)$, where the supports can, but do not have to be identical. Denote the union of the supports as U. Then, as per Krishna (2010), $G >_{FOSD} F$ if and only if $G(y) \leq F(y)$ for all y in U, while $G >_{SOSD} F$ if and only if $\int_{\underline{y}}^{y} dG(t) \leq \int_{\underline{y}}^{y} dF(t)$, where $\underline{y}$ is the smallest element in U, for all y in U. As we are only interested in changes in inequality, I additionally assume that the average income stays unchanged when the distribution changes.[12]

The following proposition demonstrates the impact of economic growth on the positional consumption:

[11]Note that if the utility function were separable (so that the cross derivatives disappear), a stronger sufficient condition for stability would be $|s_{zz}(.)| > s_{zZ}(.)$ everywhere, which means that the marginal utility of status falls more quickly with additional positional consumption than increases with the average positional consumption (when $s_{zZ}(.) > 0$).

[12]This allows us to obtain the result in Proposition 3.3 for a convex, *non-decreasing* demand function.

Proposition 3.2. *When the income distribution changes from F to G, where $G >_{FOSD} F$, the equilibrium level of average positional consumption, Z^*, goes up.*

The mechanism behind this result is as follows: as the positional good is a normal good, when agents' incomes rise, their positional consumption must also rise (in absolute terms), which causes the average positional consumption to rise as well. But this leads to two other, indirect effects. First, if the agent were to keep his consumption constant, his status would fall because $s_Z(.) < 0$. This effect can be called envy. To keep up with others, all agents therefore increase their positional consumption even further. This effect can be called competition. Both of these indirect effects work against the agents, so *at each income level* their utilities must fall. However, this is not equivalent to saying that the utilities of *all* agents fall. This is because the direct (income) effect of a FOSD change has raised the incomes of agents, making them better off as they can afford more consumption of either kind. The net effect on each agent's utility is ambiguous, and it may positive for some agents and negative for others. Consequently, we are also in no position to assess the impact on the overall welfare. It needs to be mentioned, however, that if the negative effects were to be strong enough, such a model of positional competition would in principle be able to reproduce the Easterlin paradox. The FOSD change leads to a similar conclusion in the ordinal *Model HK*, where the same two indirect effects are at work.

The following proposition demonstrates the impact of a more equal distribution of incomes on the positional consumption:

Proposition 3.3. *When the income distribution changes from F to G, where $G >_{SOSD} F$, the average income does not change and the demand for the positional good is convex in income, $\frac{d^2 z^*(y)}{dy^2} > 0$, the equilibrium level of average positional consumption, Z^*, goes down.*

When income inequality falls, incomes of the poor are raised, so they increase their positional consumption, while incomes of the rich are lowered, so they reduce

theirs. Due to the convexity of demand, the strength of this direct effect is stronger at high income levels, so the average positional consumption falls. Following this change, agents can afford to further reduce their positional consumption as a result of less fierce competition. In other words, the envy and competition effects work in the opposite direction relative to the FOSD case. Hence, utility rises at each income level. Unlike under the FOSD change, however, the income effect may work in either direction, depending on whether the agent's income went up or down. Thus, the overall effect on the utilities is again ambiguous, and so is the effect on the average welfare. The conclusion that positional competition weakens when society becomes more equal is very different from the conclusion of the ordinal *Model HK*. There, utilities of the poorest agents fall in the same circumstances. This is because in a more equal society they have lower rank and therefore status (envy effect). Additionally, if the consumption of the poor goes up when they try to catch up, this would further reduce their utilities by driving the consumption further away from the cooperative level (competition effect). But even if this is not the case (as positional consumption does not have to increase in *Model HK*), the loss of utility from lower status will be larger than the gain in utility from a more favourable consumption bundle, so the net effect on utility is negative. This topic is discussed further in the specific model section.

As both an increase in incomes and an increase in income inequality raise the average positional consumption, their effect must be even more pronounced when the positional concerns are stronger, because each agent reacts more strongly to the change in average positional consumption.

The result that the average positional consumption changes when income inequality changes stands in contrast to Warr's (1983) result that the private provision of a public good is independent of the distribution of income. There are two differences between between his approach and mine which lead to this seeming contradiction. In my model, the positional consumption plays a role of both a pri-

vate good as well as a public good, and each individual agent (who has a measure zero) ignores the effect that his choices have on the average positional consumption. Therefore, when average positional consumption increases, this will create a feedback effect on agent's own positional consumption, a feature that is not present in Warr's (1983) formulation.

Taxes

What follows from the analysis of the previous subsection is that a redistribution of incomes, which essentially causes the resulting income distribution to SOSD the old one, may lead to lower average positional consumption, and thus may reduce the extent of the positional externality. Such a progressive income tax should increase in the strength of the positional externality, which is a result akin the one obtained by Boskin and Sheshinski (1978), who consider linear redistribution plans in a setting where agents care about the consumption of others.

However, more relevant to the topic of this chapter than redistributive taxes is whether a Pigouvian tax exists which brings about the socially optimal consumption levels $z^{SO}(y)$. Assume that the tax rate is $t(y)$, which may or may not depend on income. Thus, the new price of the positional good that the agents face is $p(1 + t(y))$. To avoid introducing a public sector to the model (where the tax revenue could be spent), I follow Hopkins and Kornienko (2004) in focusing on revenue-neutral policies. What this means is that the revenue raised by imposing a levy on positional good is used to effectively subsidise the price of the non-observable good. For example, if the before-tax price of conspicuous consumption were one, and if for a certain income level the calculated $t(y)$ were equal to one, this could be interpreted as introducing a tax rate of a third on z and a subsidy rate of a third on x. This observation prevents us from calling $t(y)$ a progressive tax if it were to be an increasing function of y, because a high tax in this case is coupled with a high subsidy for the non-positional good.

With this in mind, we are looking for a tax rate $t^*(y)$ such that the competitive FOC holds with equality *at* the socially optimal consumption bundle, when the price is suitably adjusted:

$$\begin{aligned} &- p(1+t(y))u_x(y-pz^{SO}, z^{SO}, s(z^{SO}, Z^{SO})) \\ &+ u_z(y-pz^{SO}, z^{SO}, s(z^{SO}, Z^{SO})) \\ &+ u_s(y-pz^{SO}, z^{SO}, s(z^{SO}, Z^{SO}))s_z(z^{SO}, Z^{SO}) = 0. \end{aligned} \tag{3.5}$$

The result below follows:

Proposition 3.4. *The optimal (Pigouvian) consumption tax rate $t(y)$ is unique and positive for all levels of income.*

It is clear from FOC (3.5) that the optimal tax rate increases in $u_s(.)$, which measures the strength of the positional concerns. This is not surprising, as stronger positional concerns increase the distortion, so a higher tax is needed to correct this. Note that it may be the case that the government does not observe the incomes of the agents (which makes particular sense in the model which assumes that incomes are not observable, so agents engage in a positional competition). But then a flat tax rate may eliminate some of the worst effects of the status externality, and thus improve the overall welfare. Unfortunately, it is difficult to reach any further conclusions in this general setting, so I will do so in the more structured framework of the next section.

3.2.3 Multiple-Community Equilibrium

Following the discussion of a single-community society, I now move on to analyse what happens when multiple communities exist. The main goal of this subsection is to analyse the interplay between the positional competition and sorting of heterogeneous agents into (potentially also heterogeneous) communities.

To fix the terminology, whenever the set of agents who choose community C_i is

empty, I will refer to C_i as an *empty community*. Whenever two or more communities follow an assortative order, so that if an agent with income y_k resides in community C_i whose quality is q_i, then an agent with income y_l such that $y_l > y_k$ cannot reside in a community C_j for which $q_j < q_i$, they will be referred to as *stratified communities*.[13] Finally, whenever the distribution of incomes in a community is the same as the distribution of incomes in the whole society, I will refer to it as an *identical community*. Throughout the analysis, for simplicity, I maintain that only two communities exist unless stated otherwise.

As I alluded to in the introduction, there are three distinct possibilities to consider regarding the qualities of the communities. First, there can be communities that are *ex-ante* identical and whose *ex-post* qualities are also identical (even if different types of agents choose to reside in them), e.g. two essentially identical cities. Here, the agents have no reason *per se* to want to switch between communities, other than perhaps the fact that the positional competition may be weaker in some community, which would render it more attractive. I will show that such an arrangement cannot generate a stratified equilibrium, and that all the resulting equilibria are inherently unstable. I will also consider what happens when segmentation into communities nevertheless happens, not as a result of the agents' choices but rather a direct assignment by Nature or the social planner. Second, there can be communities that are *ex-ante* heterogeneous and whose qualities are exogenously determined, e.g. two cities, one of which enjoys a sea view and thus is deemed more desirable. Third, there can be communities that are *ex-ante* identical but whose qualities depend on the characteristics of the agents who comprise them, and so are *ex-post* differentiated, e.g. two cities which are identical except for the level of public good provided, where the level of provision is related the incomes of agents who reside in them. In these two cases, agents may face a trade-off between the

[13] If there are no differences in quality, a stratified equilibrium can be defined as follows: when agents with incomes y_k and y_l reside in community C, where $y_l > y_k$, then agents with any income $y \in (y_k, y_l)$ must also reside in C.

quality of the community and the strength of the status competition inside it. I discuss existence, stability and other properties of these equilibria.

Identical Qualities of the Communities

I begin by assuming that the quality of all communities is identical. Consequently, I continue to work with the simplified version of the utility function which excludes q from the list of arguments. To make the transition from the single-community model as smooth as possible, I begin by considering the assignment to communities made by Nature or the social planner, rather than the agents themselves (the agents only play the second stage game again against other agents who have been assigned to the same community). Such an arrangement may be of interest in its own right, as several social groups, e.g. family, are essentially fixed and immutable.

Consider for simplicity what happens when exactly two communities exist and Nature assigns agents to one or the other based on their income, so that there is a Poor community (denoted P) and a Rich community (denoted R). Such an arrangement fulfils the definition of a stratified "equilibrium" and can be thought of as a model of two social *strata*, whose members only care about the opinions of follow members of their strata. As mentioned, voluntary switching between strata is not allowed. Let $\hat{y}$ be the threshold income above which all agents reside in R, and below which all agents reside in P.[14] Agents only compare themselves to other agents in the same community, so they will look at the average positional consumption in the relevant community:

$$Z_P^* = \frac{\int_{\underline{y}}^{\hat{y}} z_P^*(y, Z_P^*) f(y) dy}{\int_{\underline{y}}^{\hat{y}} f(y) dy} \text{ and } Z_R^* = \frac{\int_{\hat{y}}^{\bar{y}} z_R^*(y, Z_R^*) f(y) dy}{\int_{\hat{y}}^{\bar{y}} f(y) dy}, \tag{3.6}$$

where $z_C^*(y, Z_C^*)$ for $C \in \{P, R\}$ is defined implicitly by the local version of FOC (3.2) from the previous section.

[14]Note that the group of agents whose income is *exactly* $\hat{y}$ has a zero measure, so they can join either P or R without altering the results of the analysis.

The following lemma establishes how the choice of the threshold value affects the positional consumption is both communities:

Lemma 3.6. *When agents with incomes $y \in [\underline{y}, \hat{y}]$ are assigned to P, while agents with incomes $y \in (\hat{y}, \bar{y}]$ are assigned to R, any increase in $\hat{y}$ leads to higher average positional consumption in P and R.*

What this result means is that when the dividing line between the communities in terms of income goes up, this must necessarily increase the positional competition, and hence the positional consumption, in both communities. To see why this is so, consider what happens when a positive mass of agents is relocated from R to P. In P, the new joiners increase the average positional consumption, which activates the envy effect, and in turn induces all the remaining agents to adjust upwards their consumption through the competition effect. This leads to a further increase in the average positional consumption, and so on. In R, the departure of its poorest members also increases the average positional consumption and a similar feedback loop is induced.

The above result has significant welfare consequences which are summarised in the following proposition:

Proposition 3.5. *An increase in $\hat{y}$ reduces the utilities of the agents who have not switched communities, but it is not a Pareto-worsening change.*

The effect on overall welfare cannot be assessed without further assumptions, because it is unclear whether the fall in utilities of the agents who do not switch communities is large enough to balance out the increase in utilities of agents who do switch.

Let us now assume that agents are allowed to relocate freely, and consider whether we could have a stratified equilibrium without quality differences:

Lemma 3.7. *When multiple communities of identical quality exist and agents are allowed to switch freely between them, no division of agents between the commu-*

nities constitutes an equilibrium unless the average positional consumption in all communities is the same.

As the levels of average positional consumption in a stratified "equilibrium" must differ between communities, a stratified arrangement cannot constitute an equilibrium. We can have an equilibrium which consists solely of identical communities, but it would not be stable. Stability in this context means that if a small mass of agents were moved to another community, at least some of the must want to return to their original community.[15] Thus, if we moved a small mass of poor agents from one identical community to another, the average positional consumption in their new community would fall. As it had initially been equal in all communities, the agents who moved would have to be better off. Additionally, the average level of positional consumption in their original community would have to rise. Thus, the agents would have no incentive to return to their original community.

In sum, neither of the arrangements considered can lead to a stable equilibrium. But in reality, we often observe highly heterogeneous communities which are consciously chosen by agents rather than pre-assigned. This points us to quality differences between communities as the factor that may support segmentation in the presence of the positional effects.

Consider how the above results would look like in the ordinal model (assume that the guaranteed level of status is strictly positive). Without quality differences, any stratified arrangement with $\hat{y} \in (\underline{y}, \bar{y})$ is not an equilibrium, because the agent just above the threshold has a rank of zero in community R, but he can achieve a strictly higher rank by relocating to P. Moreover, there cannot exist an equilibrium in which some communities are empty, because any agent who has a rank of zero in one of the other communities will relocate there, increasing his rank without having to change (and possibly even improving) his consumption. Finally, an equilibrium with a number of identical communities may indeed exist because no agent has an

[15] See Fernandez and Rogerson (1996) for a more general discussion of stability.

incentive to deviate, but it may again be unstable.

Differences in Qualities of the Communities

I will now assume that the *ex-post* qualities of the communities are not identical. Therefore, I return to the full version of the utility function $u(x, z, s, q)$. Observe that each community C_i can be characterised by a (Z_i, q_i) pair which describes its quality q_i and average positional consumption Z_i. Agents' utilities increase in the direction of higher q and lower Z. Thus, in any equilibrium which does not involve empty communities, we must have $Z_k > Z_l$ if and only if $q_k > q_l$, for any $k, l \in \{1, \ldots, N\}$. Otherwise, no agent would choose the community which offered higher average positional competition *and* lower quality. Assume, therefore, that two communities called Low (L) and High (H) exist, and can be characterised by $q_L < q_H$ and $Z_L < Z_H$, or more compactly by $(Z_L, q_L) < (Z_H, q_H)$.

As is well-known from the sorting literature, the crucial condition for the existence of the stratified equilibrium is the single crossing condition (Westhoff 1977). Denote by $u^*(y; Z, q)$ the indirect utility function in a community of quality q and average positional consumption Z. In the context of this model, the single crossing condition means that $u(y_i; Z_k, q_k) \geq u(y_i; Z_l, q_l)$ must imply $u(y_j; Z_k, q_k) \geq u(y_j; Z_l, q_l)$ for all $y_j > y_i$ and $(Z_k, q_k) > (Z_l, q_l)$. The intuitive interpretation of this condition is that whenever a poorer agent chooses a community of a certain quality, a richer agent must choose a community of the same or higher quality. Given the definition of the stratified equilibrium stated at the beginning of this subsection, it is clear that a stratified equilibrium may only exist if the single crossing condition is satisfied.

The single crossing condition can be interpreted graphically as a condition for the indifference curves of an agent with income y_k to be steeper than those of an agent with income $y_l < y_k$ (so they can only cross once, hence the name of the condition). Agent's indifference curves can be found by totally differentiating his utility function, $du = (-pu_x(.) + u_z(.) + u_s(.)s_z(.))dz + u_s(.)s_Z(.)dZ + u_q(.)dq$. Observe that the term

in brackets is equal to zero because of the competitive FOC (agents optimise their behaviour in the second stage), so the indifference curve in the $q - Z$ space is given by $\frac{dq}{dZ}\Big|_{du=0} = -\frac{u_s(.)s_Z(.)}{u_q(.)} > 0$. Consequently, the single crossing condition will be satisfied if $\frac{d}{dy}\left(\frac{dq}{dZ}\right) > 0$.

Alas, there is no guarantee that such a condition will be met in this general model and differentiation "by force" gives little insights. Therefore, rather than pursuing this path further in the general framework, in the next section I develop specific models which will not be subject to these limitations.

Before I do so, however, two more comments are warranted. First, observe that the single crossing condition is necessary but not sufficient for a stratified equilibrium. If a community with a sufficiently high quality presents itself, it is possible that *all* agents would relocate to it. Thus, an empty community equilibrium would arise. This suggests that some condition which limits the quality differences between communities is required for a stratified equilibrium to exist. If quality were to be determined endogenously, the relevant question is then whether the quality-generating mechanism is able to satisfy this condition.

Second, observe that with quality differences, it is impossible anymore for an equilibrium with identical communities (in the sense of the income distribution) to be supported. This is for the simple reason that any agent in L would benefit from a switch to H.[16]

In a similar vein, consider how quality differences impact the ordinal model. It is clear that any stratified arrangement with $\hat{y} \in (\underline{y}, \bar{y})$ is not an equilibrium because the agent who obtains the lowest rank in L can always obtain the same rank (without any increase in the positional consumption) and a higher quality by relocating to H. It is possible, however, that for sufficiently large differences in the qualities of the communities, an equilibrium may exist in which all agents choose one community and the other community remains empty.

[16]Note that the mere existence of identical communities under quality differences implies that quality enters the utility function in a way which does not alter second-stage choices.

3.3 Specific Model

With the limited number of conclusions in a general setting, this section adds more structure to the model of cardinal positional externality. I start by proposing two possible models of cardinal status competition, as well as symmetrically further adapt *Model HK* for comparison. Then, I analyse these models to obtain further intuitions, particularly in the multiple-community setting.

As mentioned in the introduction, there are four criteria which can be reasonably expected to be satisfied in a positional good model: (i) the richer agents, who consume more non-positional good, have higher marginal utility of status, $u_{xs}(.) > 0$ (or alternatively $u_{ys}(.) > 0$), (ii) agents display following-the-crowd behaviour, $\frac{dz^*}{dZ} > 0$, (iii) the income elasticity of demand for the positional good is greater than one, so that it is a luxury, $\varepsilon_z > 1$, and (iv) the positional good is not purely positional so that taxes can be analysed in a meaningful manner. Unfortunately, it is difficult to obtain a tractable model which fulfils all these conditions, so I settle for two models which fulfil two different subsets of these conditions.

For a status function, I employ a ratio-based specification and use a suitable transformation to ensure that the condition for the following-the crowd behaviour in Lemma 3.3 is satisfied. As explained earlier, a suitable candidate is a CRRA function with a coefficient of risk aversion greater than one. I choose the coefficient to be two for computational convenience, so that the status function becomes $s(z, Z) = \frac{(\frac{z}{Z})^{1-2}}{1-2} + \alpha = \alpha - \frac{Z}{z}$, where $\alpha > 0$ is the guaranteed level of status that each agent obtains regardless of his positional consumption, akin to *Model HK*. If the utility function were separable between either type of consumption and status, the status function would need to be modified to ensure that the marginal utility of status is increasing in income. To this end I propose another status function, $s(z, Z; y) = \eta y^{\gamma}(\frac{(\frac{z}{Z})^{1-2}}{1-2} + \alpha) = \eta y^{\gamma}(\alpha - \frac{Z}{z})$, where α again is the guaranteed level of status, while η is the strength of the positional concerns.[17] As I will show below, to ensure that the

[17] While the derivations in the previous section would need to be adjusted to account for such a

demand for the positional good is convex in income, which is necessary for $\varepsilon_z > 1$, the complementarity between status and income, here captured by $\gamma > 0$ needs to be sufficiently high. In fact, the demand is convex for any $\gamma > 2$ and I set $\gamma = 4$ for analytic convenience.

I next choose the utility functions of which the first one is multiplicative, while the second one is additive and separable:[18]

Definition 3.1. Let *Model XZS* denote a model where the agents have a utility function $u(x, z, s(z, Z)) = xzs(z, Z) = xz(\alpha - \frac{Z}{z})$.

Definition 3.2. Let *Model XyS* denote a model where the agents have a utility function $u(x, z, s(z, Z)) = x + s(z, Z; y) = x + \eta y^4(\alpha - \frac{Z}{z})$.

Note that the marginal utility of the non-positional good in either model is constant and does not go to infinity as consumption approaches zero, so the optimal consumption bundle may be a corner solution. For expositional simplicity, I will derive conditions for existence of the interior solutions and focus solely on them.

With these assumptions, *Model XZS* fulfils conditions (i), (ii) and (iv), but not (iii) because the resulting demand for the positional good is linear in income. *Model XyS* fulfils conditions (i), (ii) and (iii), but not (iv) because the positional good has to be purely positional to ensure tractability. I will use whichever model is more suitable for the analysis at hand and try to infer the results in an "ideal" model which would fulfil all the conditions (i)-(iv). Thereafter, I refer to these two models collectively as *cardinal* status models.

For comparison, I also further adapt the ordinal *Model HK* by making the positional good purely positional and assuming a concrete simple utility function, which allows us to obtain closed-form solutions.

change in specification, the same results would hold.

[18]Note that the standard Cobb-Douglas model $u(x, z, s(z, Z)) = x^{\delta_1} z^{\delta_2} (s(z, Z))^{\delta_3}$, where $\delta_i \in (0, 1)$ for $i \in \{1, 2, 3\}$ and $\sum_{i=1}^{N} \delta_i = 1$, which is coherent with the standard microeconomic theory, and used e.g. in the ordinal *Model HK*, does not generate the following-the-crowd behaviour in a cardinal model with the ratio-based specification of status, and thus is not useful here. In fact, this can be related to the "keeping up with the Joneses" scenario described by Gali (1994).

Definition 3.3. Let *Model HKS* denote a model where the agents have a utility function $u(x, z, G(.)) = x(G(z) + \alpha)$, where $G(.)$ is the cumulative distribution function, or rank, of the agent's positional consumption and $\alpha > 0$ is the guaranteed level of status.[19]

With regards to the distribution of income, for its considerable analytic simplicity, I employ the uniform with the support on $[\underline{y}, \bar{y}]$, where $\bar{y} > \underline{y} > 0$. Hence, $f(y) = \frac{1}{\bar{y}-\underline{y}}$ and $F(y) = \frac{y-\underline{y}}{\bar{y}-\underline{y}}$.[20] Importantly, the uniform distribution makes possible a simple analysis of changes in income inequality, understood as a symmetric expansion or contraction of the support of the distribution.

Let $u^*(y) = u(y - pz^*(y), z^*(y), s(z^*(y), Z^*))$ be the indirect utility function evaluated at the agent's utility-maximising bundle. Using the above assumptions, the following proposition summarises the equilibrium in each model:

Proposition 3.6. *The equilibrium of the game of status is characterised by the following individual demand for the positional good, average positional consumption and the resulting indirect utility function:*

(i) in Model XZS, $z^*(y) = \frac{y}{2p} + \frac{y+\bar{y}}{4p(2\alpha-1)}$, $Z^* = \frac{\alpha(y+\bar{y})}{2p(2\alpha-1)}$ *and* $u^*(y) = \frac{\alpha(2(2\alpha-1)y-(y+\bar{y}))^2}{16p(2\alpha-1)^2}$ *with* $\alpha > \hat{\alpha}'(\underline{y}, \bar{y}) \equiv \frac{3\underline{y}+\bar{y}}{4\underline{y}}$,

(ii) in Model XyS, $z^*(y) = \frac{\eta y^2(\underline{y}^2+\underline{y}\bar{y}+\bar{y}^2)}{3p}$, $Z^* = \frac{\eta(\underline{y}^2+\underline{y}\bar{y}+\bar{y}^2)^2}{9p}$ *and* $u^*(y) = \frac{1}{3}y(3(1 + \alpha\eta y^3) - 2y(\underline{y}^2 + \underline{y}\bar{y} + \bar{y}^2))$ *with* $\alpha \geq \hat{\alpha}''(\underline{y}, \bar{y}) \equiv \frac{\underline{y}^2+\underline{y}\bar{y}+\bar{y}^2}{3\underline{y}^2}$ *and* $\eta < \hat{\eta}(\underline{y}, \bar{y}) \equiv \frac{3}{\underline{y}^2\bar{y}+\underline{y}\bar{y}^2+\bar{y}^3}$,

(iii) in Model HKS, $z^*(y) = \frac{y^2-\underline{y}^2}{2p(y+\alpha\bar{y}-(1+\alpha)\underline{y})}$, $G(z^*(y)) = F(y) = \frac{y-\underline{y}}{\bar{y}-\underline{y}}$ *and* $u^*(y) = \frac{y^2+\bar{y}^2-2y(\underline{y}-\alpha(\bar{y}-\underline{y}))}{2(\bar{y}-\underline{y})}$.

A number of comments about these models are warranted. In *Model XZS*, the demand for the positional good is $z(y, Z) = \frac{y}{2p} + \frac{Z}{2\alpha}$, so the demand is linear in

[19] Here I only consider the case of $\alpha > 0$ as it makes *Model HK* more comparable to mine. But the result of Proposition 3.6 also holds for $\alpha = 0$ as $\lim_{y\to\underline{y}} z^*(y) = \frac{\underline{y}}{p}$ as predicted by the general *Model HK*.

[20] The results are similar for a unimodal Beta distribution which is significantly more demanding computationally.

income and thus the income elasticity is $\varepsilon_z = \frac{\alpha y}{\alpha y + pZ} < 1$. This is the aforementioned shortcoming of this specification. The condition that α cannot be too low is necessary to ensure that the solution is indeed an interior one; otherwise at least some agents would prefer to only purchase the positional good.

In *Model XyS*, the demand for the positional good is quadratic in income $z(y, Z) = \frac{\eta y^2 \sqrt{Z}}{\sqrt{p}}$, and thus the income elasticity is $\varepsilon_z = 2 > 1$. Note, however, that an additional condition has to be met, as status must not decrease in income, or otherwise the model could have perverse effects. For this to be the case, it is required that $\frac{ds(z,Z;y)}{dy} \geq 0$, or $\alpha \geq \frac{Z}{z^*(y)}$ for all y. But as $z^*(y)$ is an increasing function of y, the required sufficient condition is $\alpha \geq \frac{Z^*}{z^*(\underline{y})}$, which translates into the condition in the proposition. Again, a condition for the interior solution is necessary, which here says that the strength of the positional effect cannot be too strong $\eta < \hat{\eta}(\underline{y}, \bar{y})$, or otherwise at least some agents prefer to only purchase the positional good.

In *Model HKS*, the positional good is purely positional, so the initial condition of the differential equation is $z^*(\underline{y}) = 0$, because the agents consume no positional good in the cooperative bundle. The income elasticity of demand is $\varepsilon_z = \frac{y(y^2 + \underline{y}^2 - 2y(y - \alpha(\bar{y} - \underline{y})))}{(y^2 - \underline{y}^2)(y - \underline{y} + \alpha(\bar{y} - \underline{y}))}$, which is greater than one for sufficiently large values of the guaranteed level of status, or only for the agents with incomes below a certain threshold for lower values of the guaranteed status. Importantly, in *Model HKS* this result is generated endogenously. Agents indeed play a strictly increasing strategy, so that each level of income is associated with a distinct level of positional consumption, and the distribution of the positional consumption ends up the same as the distribution of income, i.e. $G(z^*(y)) = F(y)$.

3.3.1 Comparative Statics and Changes in Inequality

In this subsection, I first consider the changes in the parameters α and η:

Lemma 3.8. *When α increases, $z^*(y)$ decreases in Model XZS as well as Model HKS and stays constant in Model XyS, while $u^*(y)$ increases in each model. When*

η increases in Model XyS, $z^*(y)$ increases, but $u^*(y)$ increases as well.

The guaranteed level of status works as a free substitute of the positional good (and is a perfect substitute in the purely positional good models). Note that the direct effect of an increase in α, i.e. not considering the changes in the agents' behaviour, is necessarily positive. In the additive model *Model XyS*, this is the only effect, as changes in α do not have an impact on the consumption of the positional good. Conversely, due to their multiplicative nature, when α goes up in *Model XZS* or *Model HKS*, the agents reduce their consumption of the positional good as they obtain the free substitute. In *Model HKS*, this indirect effect must have a positive impact on the utilities because the consumption bundles become closer to the optimal cooperative bundles, while the positional consumption ranks remain preserved. In the cardinal *Model XZS*, the indirect effect of the reduction in positional consumption is also positive because envy is reduced and agents can adjust their positional consumption downwards, which is beneficial. Thus, both effects work in the agents' favour in either model.

With an increase in η, the strength of the positional concerns in *Model XyS*, fiercer positional competition causes agents to increase their positional consumption. The effect on the agents' utilities is less clear because the larger positional distortion is balanced out by the fact that the utilities of all agents who obtain positive status go up. As this direct effect is stronger than the indirect effect, all utilities must increase.

In order to analyse the effects of changes in the income distribution, I introduce a simple framework which follows the discussion in the general section:

Definition 3.4. Let an *increase in incomes* denote a change in the support of the income distribution from $[\underline{y}, \bar{y}]$ to $[\underline{y} + \epsilon, \bar{y} + \epsilon]$, where $\epsilon > 0$.

Definition 3.5. Let an *decrease in income inequality* denote a change in the support of the income distribution from $[\underline{y} - \epsilon, \bar{y} + \epsilon]$ to $[\underline{y}, \bar{y}]$, where $\epsilon > 0$.

It follows from the relevant definitions that when incomes increase, the new distribution first-order stochastically dominates the old one. When the incomes become more equal, the new distribution second-order stochastically dominates the old one. The following lemma summarises the effects of changes in the income distribution:

Lemma 3.9. *In Model XyS, when incomes increase, or income inequality increases, Z^* goes up and $u^*(y)$ goes down at each original income level. In Model HKS, when incomes increase, $u^*(y)$ goes down at each original income level, while when income inequality increases, $u^*(y)$ goes up at each original income level.*[21]

I skip the discussion of *Model XZS* because for this particular analysis the income elasticity of the positional good plays a central role. In *Model XyS*, an increase in incomes must necessarily increase the average positional consumption, which, as such, translates into lower utilities for agents at all income levels. This is again due to envy and competition. If all agents were to retain their original endowments, aggregate welfare would fall. However, there is also an income effect, as the incomes of all agents have been increased, so the net change in the overall welfare is ambiguous. With regards to inequality, a more uneven distribution leads to an increase in the average positional consumption, because the rich increase their consumption by more than the poor reduce theirs, which gives a positive net effect. Thus, utilities at every income level fall, with the same effects - envy and competition - driving the result. However, due to the income effect, the overall welfare again does not have to fall. We should expect that the behaviour in a model which fulfils all conditions (i)-(iv) to be similar to the one described above.

In *Model HKS*, an increase in incomes also has a negative effect on the utility at each income level. Similarly to the cardinal models, this is due to envy and competition, although their mechanics is somewhat different. As for envy, agents

[21]The result for *Model HKS* is a strengthening of Proposition 2 from Hopkins and Kornienko (2004) for the chosen utility function and income distribution.

with unchanged incomes see their rank deteriorate because others overtake them: $G(y) < F(y)$ for all y if $G >_{FOSD} F$. As for competition, agents can adjust their positional consumption either upwards or downwards, depending on their income rank. Thus, the effect of competition on the utilities may work in either direction. The impact of envy, however, is always stronger, so the overall effect is negative.

The conclusions between the cardinal and the ordinal models are opposite when inequality increases. In the ordinal *Model HKS*, the envy effect is beneficial for the poor, as their statuses increase, but it hurts the rich. However, because the density of the income distribution goes down, all agents benefit from the reduced competition as they can reduce their positional consumption, and thus bring it closer to the optimal cooperative level. Overall, the competition effect dominates and utilities go up at all fixed income levels.[22]

3.3.2 Welfare and Taxes

In this subsection, I analyse the agents' welfare as well as measures to improve it such as taxes. Note that when the positional good is purely positional, the Pigouvian tax which brings about the cooperative solution is infinite (or, in other words, the optimal policy is to prohibit the buying and selling of the positional good). Therefore, I only focus on *Model XZS* and try to elicit some intuition about the general form of the ordinal *Model HKS*. An additional benefit of *Model XZS* is that the status game is zero-sum, so the socially optimal positional consumption corresponds to the cooperative positional consumption.[23] To avoid confusion, I will refer to this level of consumption as socially optimal in the subsequent analysis.

With this in mind, the socially optimal positional consumption in *Model XZS* is given by:

[22] A more detailed discussion can be found in Hopkins and Kornienko (2004).

[23] To see this, observe that at the cooperative level of positional consumption $u_s(.)s_z(.) + \int_{\underline{y}}^{\bar{y}} u_s(.)s_Z(.)f(y)dy = \frac{yZ^C}{z^C} - \frac{\underline{y}+\bar{y}}{2} = 0$, where the last equality follows from plugging in the cooperative levels of consumption.

Lemma 3.10. *The socially optimal consumption in Model XZS is* $z^{SO}(y) = \frac{y}{2p}$.

The following proposition describes the welfare-maximising Pigouvian tax rate, t^*, which brings about the socially optimal level of consumption:

Proposition 3.7. *In Model XZS, the optimal Pigouvian tax that restores the socially optimal positional consumption is given by* $t^*(y) = \frac{y+\bar{y}}{2y(2\alpha-1)}$ *and decreases in income.*

The way that the Pigouvian taxes work is that the tax rate should be highest where the externality is the strongest. The strength of the externality may be measured by the marginal effect of positional consumption on status. But as $\frac{d^2s(z,Z)}{dz^2} = -2\frac{Z}{z^3} < 0$, the marginal status is diminishing in agent's own positional consumption, so the extent of over-consumption is largest for the poor, who have the strongest incentive to obtain higher status. Consequently, they should be taxed most heavily, so the optimal Pigouvian tax rate is decreasing in income. As a result, the optimal tax is not a Pareto-improvement, as there exist parameter values for which $u^*(y) > u^{SO}(y)$ for low values of y. This contrasts with the optimal choice of the Pigouvian tax rate in *Model HKS*. There, the marginal status is $\frac{ds(z(y),G(.))}{dz} = g(z(y)) = f(y)$, and it is highest where the density of the distribution is the highest. In other words, the competition is strongest where many agents with similar incomes compete for status, and thus it is these agents who should be taxed most heavily. In *Model HKS*, the restoration of the socially optimal (or, equivalently, cooperative) consumption level must always constitute a Pareto-improvement. Returning to the cardinal model, note that if $z^*(y)$ were convex in income, the distortion would increase at the higher incomes, which would be a reason for the Pigouvian tax to increase with income. Thus, this effect would provide some counterbalance to the effect described above, and the overall shape of the tax schedule would be difficult to determine.

The optimal Pigouvian tax falls in α as agents reduce their positional overconsumption. It is also a function of the average income in society, so an increase

in incomes should raise the tax rates across the board, while an increase in income inequality should not affect it. However, the latter conclusion would again be different if the demand for the positional good were convex in income, because then the agents increase their positional consumption, so the rate should go up to counter this.

Having considered the Pigouvian tax, I now look at the more-realistic linear welfare-maximising tax rate, t^{**}, which does not depend on agent's income:

Proposition 3.8. *In Model XZS, the welfare-maximising linear tax is given by* $t^{**} = \frac{6(\alpha-1)(\underline{y}+\bar{y})^2}{(2\alpha-1)(8\alpha(\underline{y}^2+\underline{y}\bar{y}+\bar{y}^2)-(7\underline{y}^2+10\underline{y}\bar{y}+7\bar{y}^2))}$.

Such a tax may constitute a Pareto improvement even if the non-linear tax did not, although overall welfare will be lower. This is because we can think of this tax as of an "averaged" version of the optimal non-linear tax. Thus, t^{**} must be lower for the poor agents than $t^*(y)$. As the poor agents were the losers from the optimal non-linear tax, the linear tax distorts their behaviour less and thus it could constitute a Pareto-improvement.[24] This is illustrated by the resulting indirect utility functions in Figure 3.1.

3.3.3 *Ex-Ante* Homogeneous Communities with Identical *Ex-Post* Qualities

I now move to a multiple-community scenario. Consider first what happens when the boundaries of competition are set externally so that the agents compete for social status only amongst peers from their assigned group, and they cannot change their group assignment. For simplicity, consider two such groups stratified by income, P and R, and let the threshold level of income which separates them be $\hat{y}$.

[24]Indeed, this is the case in this specific model, although there is no reason why it would have to necessarily be the case in general. Also, note that I refrain from addressing the welfare-maximising taxes in *Model HK*. The difficulty of tackling this directly in my specification is obviously that the positional good is purely positional.

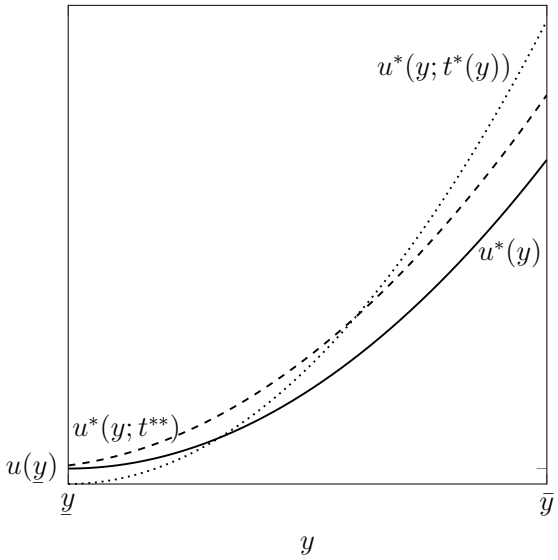


Figure 3.1: The indirect utilities in *Model XZS*: the baseline, $u^*(y)$, with the Pigouvian tax (or in the cooperative solution), $u^*(y;t^*(y))$, and with the welfare-maximising tax, $u^*(y;t^{**})$.

In such a model, the equilibrium will be described as in Proposition 3.6, yet with suitable changes to the boundary incomes: in P, $\bar{y}$ is substituted with $\hat{y}$, while in R $\underline{y}$ is substituted with $\hat{y}$.[25] I continue to assume that Z^* is the average consumption in the single-community equilibrium, while Z_P^*, Z_R^* and Z_{PR}^* denote the average positional consumption in P, R and across the two communities respectively. The following lemma describes the equilibrium average positional consumption:

Lemma 3.11. *When two communities P and R exist, and agents are assigned to the communities so that agents with $y \in [\underline{y}, \hat{y}]$ reside in P and agents with $y \in (\hat{y}, \bar{y}]$ reside in R,*

(i) in Model XZS, $Z_P^* < Z^* = Z_{PR}^* < Z_R^*$,

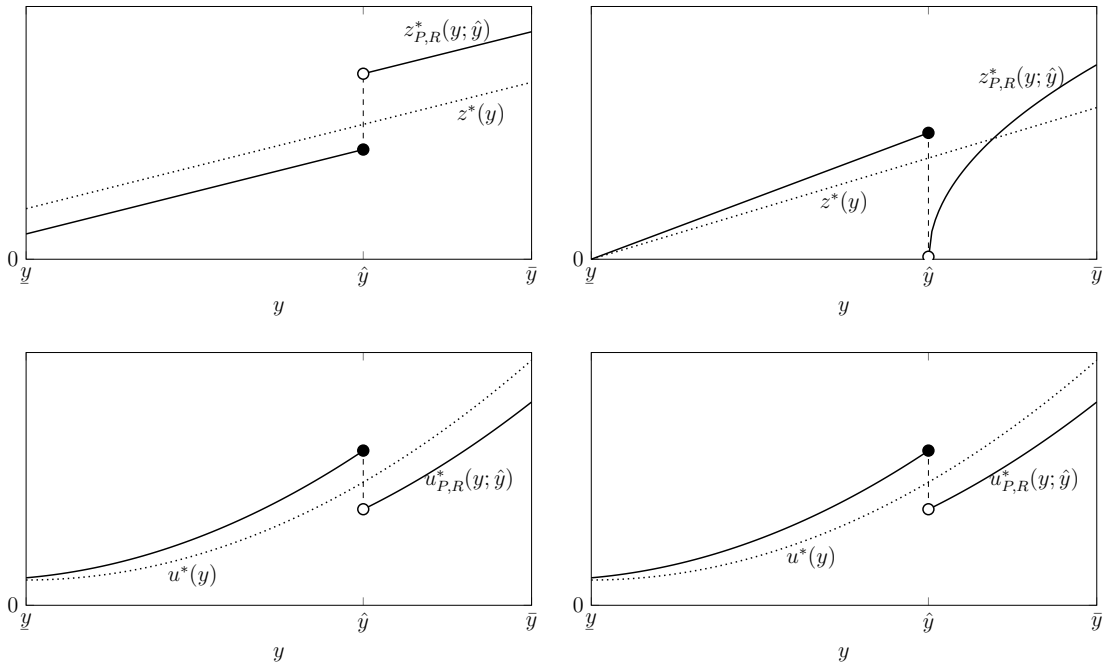
(ii) in Model XyS, $Z_P^* < Z^* < Z_{PR}^* < Z_R^*$.

Due to the linearity of the demand in *Model XZS* in income, the average positional consumption does not change compared to a single-community scenario.

[25] In fact, an additional condition is required to avoid the corner solutions in *either* community, which is given by $\alpha > \frac{2\underline{y}+\hat{y}+\bar{y}}{4\underline{y}}$ in *Model XZS* and $\eta < \frac{1}{\bar{y}^6}$ in *Model XyS*.

But when the demand becomes convex in income, as in *Model XyS*, the increase in the positional consumption in R is larger than the reduction of the positional consumption in P, thus the overall average consumption increases.

As per Lemma 3.1, agents who are assigned to P must be better off, while those assigned to R must be worse off, relative to the single-community scenario. The same is true in *Model HKS*, although for a different reasons. As for envy, the ranks of (almost) all agents in P rise, while the ranks of (almost) all agents in R fall. As for competition, as formation of a new community leads to lower demand for the positional good in P and higher demand in R, agents adjust their consumption accordingly. All these observations are illustrated in Figure 3.2. Due to the higher weight given to the utilities of the richer agents (as a result of the complementarity assumptions), the aggregate welfare must fall in all three models.



(a) in the cardinal *Model XZS* (b) in the ordinal model

Figure 3.2: Comparison between one and two communities, when the threshold income is $\hat{y}$, of the positional consumption $z^*(y)$ and the indirect utility $u^*(y)$.

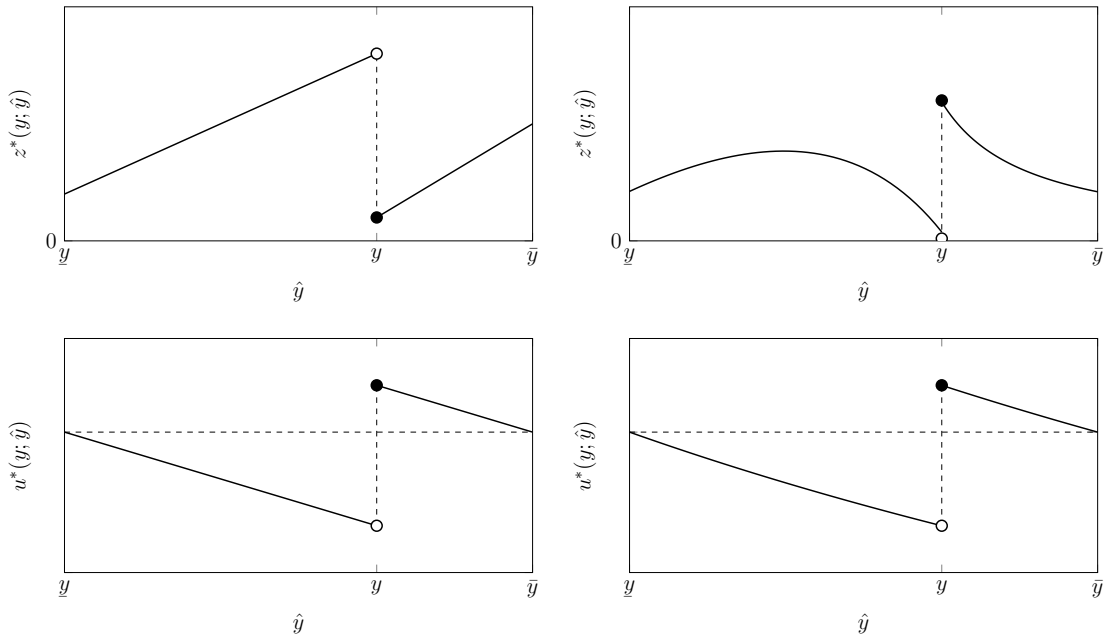
It is clear from Lemma 3.6 that in the cardinal models, the average positional consumption in each community, P and R, increases as the threshold level of in-

come $\hat{y}$ is raised. The reasoning behind these results is similar to the arguments made before. The effect on the average consumption across the whole society is however unclear, because when $\hat{y}$ changes, the composition of the communities also changes. While the utilities of agents whose assignment remained fixed fall, agents who switched from R to P are better off, because the level of positional competition in their new community is lower. Consequently, an increase in $\hat{y}$ is not a Pareto-worsening change. In *Model HKS*, an increase in $\hat{y}$ reduces the utilities of all agents who do not switch communities because the competition in either community becomes fiercer due to the higher average incomes. While the utilities follow the same pattern as $\hat{y}$ is changed, the key difference between the cardinal models and the ordinal model is that when $\hat{y}$ goes up initially, in the latter model the positional consumption does not behave monotonically, while in the cardinal model it always rises. When $\hat{y}$ overtakes y, the positional consumption jumps in both models as agents switch from R to P, and then begins to fall in the ordinal model and rise again in the cardinal models. Figure 3.3 illustrates these observations. Regardless of the model used, overall welfare first falls and then rises with $\hat{y}$, and is maximised at either end of the income distribution. This result, however, seems to be driven by the assumption about the uniform distribution and may not apply more generally.

As predicted in the general section of this chapter, absent any differences between the social groups, any arrangement where $\hat{y} \in (\underline{y}, \bar{y})$ is not an equilibrium. This can be verified by noting that indirect utility of the threshold agent in P is always higher than in R, $u_P(\hat{y}) > u_R(\hat{y})$ (see in Figure 3.2).

To examine what happens when the number of social groups within which the agents compare themselves increases, assume that society consists of N groups of equal size. For simplicity, I only consider *Model XZS*.[26] From the above discussion, it is clear that there may always exist agents who reside near the threshold, whose utilities go up. It turns out, however, that the overall effect on the aggregate welfare

[26] A similar result holds in *Model XyS*.



(a) in the cardinal models (b) in the ordinal model

Figure 3.3: Effect of the changes in $\hat{y}$ on the positional consumption, $z^*(y;\hat{y})$, and indirect utility, $u^*(y;\hat{y})$, of an agent with income y.

is negative:

Proposition 3.9. *In Model XZS, an increase in the number of the equal-sized communities lowers aggregate welfare.*

The effects at work are identical to those when the number of communities is increased from one to two. Moreover, there is an additional question which can be analysed in the cardinal models, which is about the impact of the poor comparing themselves to the rich, rather than their fellow poor.[27]

Proposition 3.10. *In the cardinal models, when agents in P compare themselves to the agents in R, while agents in R also compare themselves to the agents in R, the positional consumption increases and the utility decreases in P compared to the model where comparisons happen within their respective groups.*

This result was anticipated, because in comparing themselves to their richer peers, the agents in P increase their own positional consumption. On top of that,

[27]In *Model HKS*, such a model would be essentially identical to the single-community model.

there is a stronger envy effect, because it is more difficult to compete against the rich than the poor. A possible interpretation of the last two results is that the more localised competition is necessarily socially hurtful, and the welfare effects worsen when agents in any of these groups look up to their richer peers in "higher" groups.

The next question pertains to the effect of the changes in inequality on the outcomes in the model with more than one community. It is summarised by the following proposition:

Proposition 3.11. *In Model XyS, an increase in inequality which does not impact $\hat{y}$ leads to a fall in the average positional consumption in P and a rise in the average positional consumption in R. The utilities of the agents in P rise and in R fall. Overall, the average positional consumption Z^*_{PR} increases. The increase is larger than the increase from an identical change in inequality with only a single community.*

Because the threshold does not change, an increase in income inequality means that agents in P become poorer, while agents in R become richer. Thus, positional consumption falls in P and increases in R, with the overall effect being positive due to the convexity of the demand for the positional good in income. The conclusion regarding the agents' utilities is then clear, but the impact on total welfare is as always unclear because of the direct income effect. The mechanism which causes an increase in inequality to raise the average positional consumption by more with two communities than it would with only one community follows again from convexity. In fact, as the income effect is the same regardless of the number of communities, it is now true that the total welfare is always affected less negatively by inequality (in the sense that $\lim_{\epsilon\to 0} \frac{dW^*_{PR}(\epsilon)}{d\epsilon} < \lim_{\epsilon\to 0} \frac{dW^*(\epsilon)}{d\epsilon}$) when there is only a single community than with multiple communities.[28] In *Model HKS*, the effects on the agent's demand for the positional good are more complex, but it is also true that the utilities at unchanged incomes must rise in P and fall in R, while the increase in the total

[28]The calculations are tedious but straightforward, so they are omitted.

welfare is smaller than it would have been if only one community existed.

As for taxation in the multiple-community framework, the conclusions remain largely unchanged to their single-community counterparts, at least assuming that he social planner can set different tax rates (possibly depending on income) in different communities.[29] As $\frac{dt^*(y)}{d\underline{y}} > 0$ and $\frac{dt^*(y)}{d\bar{y}} > 0$, it must be the case that $t_P^*(y) < t^*(y) < t_R^*(y)$, so that the optimal tax rate falls with y in each community, with a jump at the threshold level of income.

3.3.4 *Ex-Ante* Heterogeneous Communities with Different Exogenous Qualities

Unlike in the general model, the additional structure of the specific model allows us to examine in greater detail what changes in the model with multiple communities when they vary in quality. Even here, though, the problem quickly becomes intractable, so I will constrain my attention to the simplest *Model XZS*.[30]

To proceed, it is necessary to modify the utility function in *Model XZS* to incorporate the quality of the community. To this end, let the new utility function be $u(x, z, s, q) = xzsq^2$. To retain the complementarity with consumption, quality enters the utility function in a multiplicative way. The use of a square of an arbitrary scalar $q \geq 1$ is innocuous and simplifies the resulting solutions. Also for simplicity, I continue to assume that only two communities exist, now labelled L and H, for Low and High quality. I assume that $q_L = 1$ and $q_H = q > 1$. Thus, the utility of an agent in L is $u(y - pz_L^*(y), z_L^*(y), s(z_L^*(y), Z_L^*), 1)$, while for an agent in H it is $u(y - pz_H^*(y), z_H^*(y), s(z_H^*(y), Z_H^*), q)$.

In line with the general discussion, we are looking for a stratified equilibrium,

[29] For the optimal non-linear tax this is a natural assumption, as the social planner already can essentially set a separate tax rate for every agent.

[30] The results in *Model XyS* are similar, but considerably more difficult to obtain. As explained in the general section, there will be no stratified equilibrium in *Model HKS* when the community qualities differ, because the poorest agent would always want to move to the higher-quality community.

in which there exists a threshold income $\hat{y} = \hat{y}^*(q)$, which is now a function of the quality, so that all agents with $y < \hat{y}^*(q)$ reside in L, while all agents with $y \geq \hat{y}^*(q)$ reside in H. The key to existence of a stratified equilibrium is whether the single-crossing condition is satisfied, which is the case for *Model XZS*:

Lemma 3.12. *The utility function is Model XZS satisfies the single-crossing condition.*

The single crossing condition guarantees that a unique stratified equilibrium exists, unless the difference in the qualities of the communities is very small or very large, so that all agents may locate in only one community and leave the other community empty. Assume that a *proper* stratified equilibrium exists, so that the resulting $\hat{y}^*(q)$ sits inside $(\underline{y}, \bar{y})$. Then, once the agents have decided in which community to reside, the second stage game is identical to the one in the previous subsection - the quality does not impact the agents' utilities anymore since it has been fixed. The following proposition identifies the threshold level of income:

Proposition 3.12. *The threshold level of income in the stratified equilibrium is* $\hat{y}^*(q) = \frac{q\bar{y}-\underline{y}}{(4\alpha-3)(q-1)}$. *The proper stratified equilibrium is obtained for* $\underline{q}(\alpha, \underline{y}, \bar{y}) < q < \bar{q}(\alpha, \underline{y}, \bar{y})$, *where* $\underline{q}(\alpha, \underline{y}, \bar{y}) = \frac{4\alpha\bar{y}-3\bar{y}-\underline{y}}{4\alpha\bar{y}-4\bar{y}}$ *and* $\bar{q}(\alpha, \underline{y}, \bar{y}) = \frac{4\alpha\underline{y}-4\underline{y}}{-\bar{y}+4\alpha\underline{y}-3\underline{y}}$.[31]

The result confirms the intuition that when with inherent quality differences, the proper stratified equilibrium exists as long as the difference between qualities is moderate, $\underline{q}(\alpha, \underline{y}, \bar{y}) < q < \bar{q}(\alpha, \underline{y}, \bar{y})$. When the difference between qualities of the communities is small, it might seem that all agents would locate in L. But this clearly cannot be an equilibrium, as the poorest agent benefits from a switch to H. Thus, no equilibrium in pure strategies exists. When the difference in qualities is large, it may be the case that all agents locate in H, but this requires an additional condition that even the poorest agent does not benefit from re-locating to L, where

[31] Again, a high enough $\alpha > \frac{\underline{y}+\bar{y}}{2\underline{y}}$ is required to guarantee interior solutions in both communities.

not only quality but also competition is lower. Should such a condition not be satisfied, again no equilibrium in pure strategies exists.[32]

Figure 3.4 demonstrates why the stratified equilibrium in the *ex-ante* heterogeneous communities model is locally stable under a tâtonnement adjustment process.[33] If a small measure of agents with $y \in [y\dagger, \hat{y}^*(q)]$ were moved to H, that would reduce the value of the threshold, which will now be $y\dagger$. But the figure shows that then the utilities of agents who have moved could be increased if they relocated to their old community, L. Hence, they move back, and equilibrium is restored.

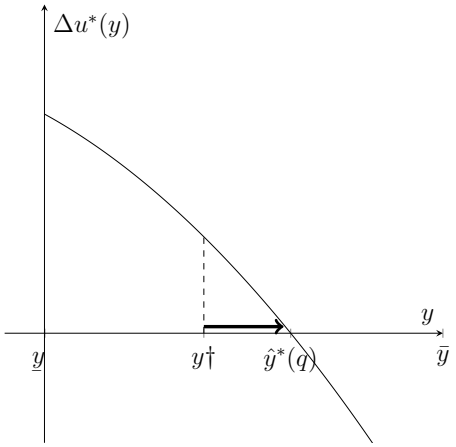


Figure 3.4: Local stability under tâtonnement in *Model XZS* ($\Delta u^*(y) = u_L^*(y; \hat{y}^*(q)) - u_H^*(y; \hat{y}^*(q))$).

The following proposition establishes the comparative statics of the result:

Proposition 3.13. *The equilibrium threshold $\hat{y}^*(q)$ falls when α or q increase. In a modified model where $u(x, z, s, q) = q^\gamma u(x, z, s, q)$, where $\gamma > 0$, the threshold $\hat{y}^*(q)$ also falls when γ increases.*

The impact of changes in q is clear: as the quality of H increases, so does the quality differential *versus* community L, so it becomes more attractive to the agents. As a result, some of the agents in L who are closest to the old threshold switch,

[32]There may, however, exist equilibria in mixed strategies. A way to avoid such a result might be to assume that the community fails to form unless a critical mass of agents join it. Also, note that a similar reasoning may apply to the proper stratified equilibrium when an additional community becomes available. But then such an arrangement indeed cannot be an equilibrium, although there might be an equilibrium where agents stratify across all three communities.

[33]The proof is standard, e.g. in general equilibrium theory, and thus omitted.

despite the prospect of stronger positional competition. The other results can be understood in terms of the strengths of the *within* and *between* effects, where the former denotes the strength of the positional concern inside the (chosen) community, while the latter - the strength of the concern for the quality of the community. When α goes up, the guaranteed level of status increases, which results in a decrease in the importance of the positional consumption; conversely, when γ goes up, the agents care more about the quality of the community in which they reside. Thus, an increase in the strength of the within effect causes the threshold to go up (more people reside in L, where competition is relaxed), while as increase in the strength of the between effect causes the threshold to go down (more agents reside in H to benefit from its high quality).

It is also of interest to examine the effect of the changes in inequality in this new setting:

Proposition 3.14. *An increase in incomes and an increase in inequality lead to an increase in the equilibrium threshold* $\hat{y}^*(q)$.

The result regarding an increase in inequality is interesting, because it arises in *Model XZS* which has so far been invariant to such changes.[34] But here the interpretation of the result is intuitive. Community L becomes poorer, so it reduces its positional consumption, while community R becomes richer, so it increases it. Hence, as a result of increased competition, some agents prefer to switch from H to L, which is equivalent with the threshold going up. If the demand for the positional good were additionally convex in income, this effect would be boosted, and we would expect a sharper increase in the threshold.

Note that the other properties of the equilibrium will be "inherited" from the model with no quality differences, as the only effect that the quality of the community has on the equilibrium is through its impact on the threshold income. The

[34]The same results can be obtained for an arbitrary $\gamma > 0$ in the modified model from Proposition 3.13.

only other difference is that the welfare analysis is not meaningful as the differences in quality are arbitrary. Likewise, it is always true that an increase in q increases the overall welfare because while the indirect effects (adjustments of the threshold as well as envy and competition effects within each community) may work to the detriment of the agents, the stronger direct effect simply increases the utilities of the agents in H, which suffices for the balance to be positive.

3.3.5 *Ex-Ante* Homogeneous Communities with Different Endogenous *Ex-Post* Qualities

This subsection endogenises the community quality determination. To keep specification as simple as possible, I assume that the quality of the community is simply a function of the average income in that community.[35] There are at least two plausible justifications for such a choice. First, participation in a richer community may give the agent inherent utility. In this scenario, quality of the community can be thought of as its prestige; there may also be a more tangible link such as lucrative job opportunities that become more often obtainable when agents reside in a richer community. Second, this may be interpreted in the more orthodox way. Similar to Westhoff (1977) and Fernandez and Rogerson (1996), it may be the case that the level of provision of the public good is different in different communities, but everything is financed with a lump-sum tax imposed on the whole population (so that incomes represent after-tax incomes). Due to their higher lobbying efforts, and despite withholding contributions, richer communities obtain more of the public good.[36]

I retain the assumptions concerning the utility function from the previous section. Thus, the existence and stability of the stratified equilibrium follow (for a certain

[35] Agents must therefore have the ability to observe the average incomes, but not the individual income, as otherwise there would be no need for signalling through positional consumption.

[36] It is possible to relax the assumption that the tax is levied uniformly on the whole population, and let it be decided at the community level through the median voter, but this creates considerable tractability issues.

range of parameters). The quality is given by the average income in the community, $q_i = \frac{\int_{C_i} yf(y)dy}{\int_{C_i} f(y)dy}$. Hence, the threshold level of income which divides agents into the Poor (P) and the Rich (R) community is determined solely by the parameters of the "original" model:

Proposition 3.15. *The threshold level of income in the stratified equilibrium is* $\hat{y}^* = \frac{\underline{y}+\bar{y}}{4(\alpha-1)}$. *The proper stratified equilibrium is obtained for* $\frac{\underline{y}+5\bar{y}}{4\bar{y}} < \alpha < \frac{5\underline{y}+\bar{y}}{4\underline{y}}$. *Otherwise, no stratified equilibrium in pure strategies exists.*

A stratified equilibrium can only exist when α, the guaranteed level of status and an "anti-proxy" for the strength of the positional concerns, takes moderate values. This result can be understood as a requirement that neither the within effect, nor the between effect dominating the other one. Otherwise, a strengthening of the result obtained for the exogenous-quality model applies. Namely, it would seem that all agents locate in only one community, but in fact this is not an equilibrium as the poorest agent always has an incentive to relocate to the empty community.

The threshold level of income falls when α increases because as agents obtain a higher level of guaranteed status, the positional competition is relaxed, and so agents are keener to join the richer community whose quality is higher. An increase in incomes will also predictably increase the threshold, while an increase in income inequality has no effect. If the demand for the positional good were convex in income, we would expect the threshold to increase: as the competition increases by more in R than in P, some agents are induced to switch to the lower-quality community.

Finally, it is of interest to compare the utilities obtained by the agents with two communities to those obtained with only one community (where the quality of the community is simply the average income in the population). Without quality differences, it was the case that all poor agents were better-off with two communities, while all rich agents were worse off. Now the situation is different:

Proposition 3.16. *In the stratified equilibrium with two communities and endogenous quality determination, the utility of the poorest and the richest agent in the*

population is higher than in the equilibrium with only one community, while the utility of the threshold agent is lower.

This is illustrated in Figure 3.5. Endogenous determination of quality essentially "smoothens" the discontinuous indirect utility function from the model with no quality differences, because when an agent switches community, he enjoys a higher community quality in exchange for increased status competition. This allows for the existence of the equilibrium. Compared to the equilibrium with no quality differences, the richest agents are better off, because the complementarity between quality and consumption (or status) increases their utilities. Conversely, the agents in R close to the threshold (i.e. the upper part of the middle class) lose out, because the increased quality is not as important for them as the painful burden of increased positional competition. In P, these effects are reversed. The poorest agents do not care much about quality, but they benefit from reduced status competition, while the agents close to the threshold (i.e. the lower part of the middle class) suffer considerably from the fall in quality. Interestingly, the effects driving the utilities of middle-class agents down are very different depending on whether the agent resides in P or R. These complex effects indicate that it is unfortunately not possible to conduct a meaningful analysis of the impact of endogenous quality determination on total welfare.

3.4 Conclusion

Positional concerns constitute a phenomenon whose importance is set to grow as societies solve their basic economic problem and increasingly concentrate on satisfying their higher-order needs. Positionality, however, poses a challenge to standard economic theory because of the lack of a generally accepted form of status-inclusive preferences. While micro-founded models may provide a solution in certain contexts such as in the matching market for marriage, it is entirely possible that the posi-

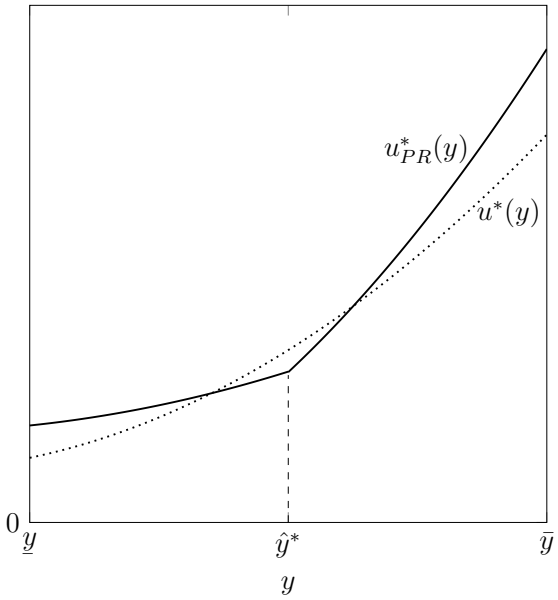


Figure 3.5: Comparison of agents' utilities $u^*(y)$ with endogenous quality determination between a single-community and a two-community model.

tional concerns are innate, and thus we may have to rely on revealed preference to discover their true form.

This chapter has attempted to build a model which captures a class of cardinal status preferences, in which it is not the agent's rank that matters, but rather the degree to which he performs relative to others in positional consumption. Throughout the chapter, I have compared and contrasted my results with those obtained by Hopkins and Kornienko (2004) in an ordinal *Model HK* based on rank. The chapter has moreover sought to examine the interplay between positional competition and community choice, in a setting where agents first choose in which community they wish to reside, and then engage in positional competition only against members of the chosen community.

In line with most of the research on the topic, the general conclusion from my model with a single community is that in the presence of status competition, agents over-consume the positional good compared to the social optimum. Hence, the utilities of all agents can be increased by a reduction in the average level of positional consumption. This result resembles a standard (e.g. environmental) externality,

and thus a Pigouvian tax may be used to improve welfare, but such a tax does not necessarily constitute a Pareto improvement. Economic growth, understood as a rightward shift of the income distribution, increases the average positional consumption, and thus may reduce overall welfare leading to the Easterlin paradox result. An increase in income inequality increases the average positional consumption if the demand for the positional good is convex in income, which I have argued is likely to be the case. Consequently, more inequality reduces the utilities of agents whose incomes have not been impacted and may therefore be harmful.

Compared to the ordinal *Model HK*, my model offers the same predictions with regards to the impact of economic growth on positional competition. The more structured version of my model also gives policy recommendations consistent with *Model HK*. Specifically, both find that the optimal consumption tax rate should be falling in income. This result, however, is dependent on the use of the uniform distribution and the particular form of preferences, so it does not necessarily generalise. Where the models diverge most is with regards to the effects of increasing income inequality. Due to its "locality", *Model HK* predicts that the poor must be worse off in a more equal distribution, while my model predicts that all agents whose incomes have not been changed are better off.

When multiple communities are present, the results in my model depend on whether the qualities of the communities are identical or heterogeneous. In the former case, no stratified equilibrium can exist. However, it may be the case that agents are assigned to a community and cannot freely switch their assignment. Then, an increase in the threshold which divides the population into the poor and the rich increases the average positional consumption in both communities. It is, however, not a Pareto-worsening change, because the agents who are moved to the poorer community are better off. I have found that an increase in the number of communities, as well as the upward comparisons of poorer agents both reduce the overall welfare.

In the latter case, i.e. with heterogeneity in community quality, I have assumed that the agents' preferences are such that they obey the single crossing condition with respect to the average positional consumption and the community quality. This has allowed me to analyse the stratified equilibria. I have considered two quality determination mechanisms. First, quality may be determined exogenously and constitutes the communities' inherent characteristic. A stratified equilibrium exists when neither of the *within-community* and *between-community* effects, which measure the strength of the positional concerns and the desire for high quality, respectively, dominate the other. Otherwise, either an equilibrium exists in which all agents locate in the high-quality community, or no equilibrium exists. Second, quality may be determined endogenously by the agents who end up comprising it. I focus on a specific case where the quality is given by the average income in the community (which can be a result e.g. of different levels of the public good provision proportional to the average income). A stratified equilibrium only exists if the guaranteed level of status takes moderate values. In such an equilibrium, while the poor and the rich are better off compared to the single-community equilibrium, the middle class is worse off. In *Model HK*, it is impossible to support a stratified equilibrium even in the presence of quality differentials, because the poorest agent in the low-quality community always has an incentive to move to the higher-quality community.

My model of multiple communities could be developed further by allowing for greater generality and tractability. The model could also attempt to capture more explicitly some more concrete kinds of community quality generation processes, for example in a matching framework. Other worthwhile extensions include asymmetric preferences, so that agents can display compassion towards those with lower incomes as in Friedman and Ostrov (2008), as well as simultaneous participation is more than one community (e.g. local neighbourhood and family). Finally, the role of prices, and the related question the status production function, may be examined, such as

with differences in housing costs providing another reason for the existence of an equilibrium with multiple communities.

Appendix 3.A Ordinal Model of Positional Competition

As it is an important benchmark for my model, I provide a short characterisation of the ordinal model of positional competition adopted from Hopkins and Kornienko (2004), referred to as *Model HK*. The model being ordinal means that only the agent's rank in the distribution matters, regardless of whether the next competitor is surpassed by a little or by a lot. As a consequence, the competition is more local than in my model - to obtain extra status, the agent only needs to jump over his closest neighbours, because he does not directly compete with the agents further away. Such a formulation of status is clearly a zero-sum game. Formally, status is defined as $s = s(z, G(.))$, where $G(.)$ is the cumulative distribution function of positional consumption. The status function is $s(z, G(.)) = \zeta G(.) + (1-\zeta)G^{-}(.) + \alpha$, where $G^{-}(.) = \lim_{z' \to z^{-}} G(z')$ is the mass of individuals with consumption strictly less than z (essentially a tie-breaking rule) and $\alpha \geq 0$ is the minimum guaranteed level of status. Agent's utility function is multiplicative in the direct utility and status, so that $u = V\big(x, z, s(z, G(.))\big) = v(x, z)s(z, G(.))$.

The crucial aspect of such a formulation is that the behaviour of the poorest agent depends on whether $\alpha = 0$ or $\alpha > 0$. In the former case, the agent with the rank of zero obtains status and thus utility of zero, so he will be desperate in trying to avoid such an outcome. Hence, he ends up spending all of his income on the positional consumption, even though he ends up coming last anyway. In the latter case, the agent realises that he is going to have a rank of zero and a status of $\alpha > 0$, so he ignores the status competition and just consumes his cooperative bundle, a consumption basket which maximises $v(x, z)$, that he would choose in the absence

of positional concerns. The behaviour of the poorest agent defines the boundary condition which influences the behaviour of the other agents.

The main consequence of the ordinal specification of status is that in the equilibrium, the rank in positional consumption corresponds to the original rank in the income distribution, $G(.) = F(.)$. Hence, the positional competition does not allow any agent to improve his status. As a result, agents over-consume in the equilibrium compared to the cooperative solution (except perhaps the poorest agent), even though they could obtain the same status and higher direct utility if all agents collectively switched to purchasing the cooperative bundles.

Appendix 3.B Omitted Proofs

Proof of Proposition 3.1

Existence follows easily form the main text and the assumptions regarding the utility and status functions. Let $q(z) = -pu_x(.) + u_z(.) + u_s(.)s_z(.)$ and $r(z) = -\int_{\underline{y}}^{\bar{y}} u_s(.)s_Z(.)f(y)dy$. Because of the Inada conditions, as $z \to 0^+$, $q(z) \to \infty$, while as $z \to \frac{y}{p}^-$, $q(z) \to -\infty$. Also, because of *Condition SOC*, $q(z)$ must be strictly decreasing. Thus, it can cross the x-axis only once, and the point where it does is the unique level of equilibrium positional consumption $z^*(y)$. Because $u_s(.) > 0$, $s_Z(.) < 0$ and $f(y) > 0$, we must have $r(z) > 0$ everywhere. Also, $r(z)$ does not depend on z, so it is a constant function. Thus, it may only cross $q(z)$ once, and this must happen at $z^{SO}(y) < z^*(y)$.

Proof of Lemma 3.1

This follows from the envelope theorem, as $\frac{du^*(y,Z)}{dZ} = u_s^*(.)s_Z^*(.) < 0$, where the asterisks denote the equilibrium values, because of the assumption that $s_Z(.) < 0$.

Proof of Lemma 3.2

Define $D = -p^2 u_{xx}(.) + 2pu_{xz}(.) - u_{zz}(.) + 2pu_{xs}(.)s_z(.) - 2u_{zs}(.)s_z(.) - u_{ss}(.)s_z^2(.) - u_s(.)s_{zz}(.)$, and note that $D > 0$ when *Condition SOC* is satisfied. By implicitly differentiating the competitive FOC with respect to y, we obtain $\frac{dz^*(y)}{dy} = \frac{-pu_{xx}(.)+u_{xz}(.)+u_{xs}(.)s_z(.)}{D} > 0$, where the inequality follows from the assumptions on the derivatives.

Proof of Lemma 3.3

By implicitly differentiating the competitive FOC with respect to Z, we obtain $\frac{dz^*(y)}{dZ} = \frac{-pu_{xs}(.)s_Z(.)+u_{zs}(.)s_Z(.)+u_{ss}(.)s_z(.)s_Z(.)+u_s(.)s_{zZ}(.)}{D}$, with D defined as in Proposition 3.2. Following-the-crowd behaviour means that $\frac{dz^*}{dZ} > 0$, and as $D > 0$, it is required that $-pu_{xs}(.)s_Z(.) + u_{zs}(.)s_Z(.) + u_{ss}(.)s_z(.)s_Z(.) + u_s(.)s_{zZ}(.) > 0$, or $s_{zZ}(.) > \frac{1}{u_s(.)}\big(pu_{xs}(.)s_Z(.) - u_{ss}s_z(.)s_Z(.) - u_{zs}(.)s_Z(.)\big)$.

Proof of Lemma 3.4

Let $q(Z) = \int_{\underline{y}}^{\bar{y}} z^*(y, Z)f(y)dy$. Due to the continuity of $z^*(y, Z)$ (which follows from the continuity of the utility and status functions), $q(z)$ must be continuous. Consider a closed set $I = [\epsilon, \frac{\bar{y}}{p} - \epsilon]$, where $\epsilon > 0$, which is compact and convex. When a small enough ϵ is chosen, then $z^*(y, \epsilon) > \epsilon$ and $z^*(y, \frac{\bar{y}}{p} - \epsilon) < \frac{\bar{y}}{p} - \epsilon$ due to the Inada conditions and since $q(Z)$ is monotone, it maps I to itself. Thus, $q(Z)$ is a continuous function that maps a compact, convex set I to itself, and so by Brouwer's fixed-point theorem, a fixed point $Z^* = q(Z^*)$ must exist.

Proof of Lemma 3.5

Consider $q(Z)$ defined as in the previous lemma. A sufficient condition for uniqueness is for $q(Z) - Z$ to be decreasing in Z as this guarantees that the 45-degree line is only crossed once. Thus, we need $q'(Z) - 1 < 0$ for any Z, where $q'(Z) = \int_{\underline{y}}^{\bar{y}} \frac{dz^*(y,Z)}{dZ} f(y)dy$,

so this translates to $\int_{\underline{y}}^{\bar{y}} \frac{dz^*(y,Z)}{dZ} f(y)dy < 1$ for all Z. For local stability, the same condition must only hold at the equilibrium $Z = Z^*$, so it must be satisfied as well.

Proof of Proposition 3.2

Assume that $G >_{FOSD} F$ and assume that their supports lie in $[\underline{y}, \bar{y}]$. Then the properties of FOSD guarantee that $\int_{\underline{y}}^{\bar{y}} r(y)g(y)d(y) > \int_{\underline{y}}^{\bar{y}} r(y)f(y)d(y)$ for any increasing function $r(y)$. Consider function $r(y) = z^*(y, Z)$. As per Lemma 3.2 it is increasing in y for any Z, so $q(Z, g(.)) = \int_{\underline{y}}^{\bar{y}} z^*(y, Z)g(y)d(y) > \int_{\underline{y}}^{\bar{y}} z^*(y, Z)f(y)d(y) = q(Z, f(.))$ for any Z. The fixed point of $q(Z, g(.))$ is given by $Z_g^* = q(Z_g^*, g(.))$, while the fixed point of $q(Z, f(.))$ is given by $Z_f^* = q(Z_f^*, f(.))$. Because we assumed that the equilibrium is unique and stable, $q(Z, (.))$ must cross the 45-degree line from above. As $q(Z, g(.)) > q(Z, f(.))$ at any Z, the crossing with the 45-degree line must happen for $q(Z, g(.))$ at a higher value of Z than for $q(Z, f(.))$. Thus, we obtain $Z_g^* > Z_f^*$.

Proof of Proposition 3.3

Assume that $G >_{SOSD} F$ and assume that their supports lie in $[\underline{y}, \bar{y}]$. Then the properties of SOSD with $\int_{\underline{y}}^{\bar{y}} G(y)dy = \int_{\underline{y}}^{\bar{y}} F(y)dy$ guarantee that $\int_{\underline{y}}^{\bar{y}} r(y)g(y)d(y) > \int_{\underline{y}}^{\bar{y}} r(y)f(y)d(y)$ for any *concave* function $r(y)$. Consider function $z^*(y, Z)$. As per the assumption that income elasticity is greater than one, is is convex in y for any Z. Thus, let $r(y) = -z^*(y, Z)$, which is concave in y. Then $q(Z, g(.)) = \int_{\underline{y}}^{\bar{y}} z^*(y, Z)g(y)d(y) < \int_{\underline{y}}^{\bar{y}} z^*(y, Z)f(y)d(y) = q(Z, f(.))$ for any Z. The rest of the proof proceeds as in Proposition 3.2, with Z_g^* and Z_f^* reversed, so we obtain $Z_g^* < Z_f^*$.

Proof of Proposition 3.4

Fix y. Consider the LHS of the FOC (3.5) evaluated at $t(y) = 0$. It equals $-pu_x(.) + u_z(.) + u_s(.)s_z(.)$ evaluated at $z = z^{SO}$ and $Z = Z^{SO}$, which in turn using the socially optimal FOC (3.3) equals $-\int_{\underline{y}}^{\bar{y}} u_s(.)s_Z(.)f(y)d(y)$. But this is always more than zero because $s_Z(.) < 0$. Moreover, the derivative of the LHS of the FOC (3.5) with respect

to $t(y)$ is $-pu_x(.) < 0$. Thus, the LHS of the FOC (3.5) is positive at $t(y) = 0$ and strictly decreasing, so at some point where $t(y) > 0$ it must cross the x-axis, were the FOC (3.5) is satisfied.[37]

Proof of Lemma 3.6

Consider P first. Multiply equation (3.6), which defines Z_P^* by the denominator of the RHS and differentiate with respect to $\hat{y}$.The Leibniz integral rule yields:

$$\frac{dZ_P^*}{d\hat{y}} \int_{\underline{y}}^{\hat{y}} f(y)dy + Z_P^* f(\hat{y}) = z_P^*(\bar{y}, Z_P^*) f(\hat{y}) + \int_{\underline{y}}^{\hat{y}} \frac{dz_P^*(y, Z_P^*)}{dZ} \frac{dZ_P^*}{d\hat{y}} f(y)dy,$$

which can be simplified to:

$$\frac{dZ_P^*}{d\hat{y}} = \frac{f(\hat{y})(z_P^*(\hat{y}, Z_P^*) - Z_P^*)}{\int_{\underline{y}}^{\hat{y}} \left(1 - \frac{dz_P^*(y, Z_P^*)}{dZ}\right) f(y)dy}.$$

Because of *Condition S*, the denominator is positive. As the positional good is normal, the highest-income agent in the community will consume the highest amount of the positional good in that community. Hence $z_P^*(\hat{y}, Z_P^*) > Z_P^*$ for any $\hat{y}$, as the former is the highest positional consumption in P, while the latter is the average consumption. Therefore, the numerator, and thus the whole expression must be positive, which means that average positional consumption in the P group must go up when $\bar{y}$ increases. The proof for Z_R^* is almost identical, and the only difference requires noting that $Z_R^* > z_R^*(\hat{y}, Z_R^*)$.

Proof of Proposition 3.5

Lemma 3.1 shows that the agent's equilibrium utility falls when Z is increased, and Lemma 3.6 shows that when $\hat{y}$ goes up, the average positional consumption increases both in P and R. Thus, the utilities must fall both in P and R, provided that the agent has not changed communities. For the second part of the proposition, consider

[37]Note that no constraint needs to be checked as nothing precludes $t(y) > 1$.

the agent just above the threshold, whose income is $\hat{y}$ and who used to reside in R. As the positional good is a normal good, it must be the case that $Z_R^* > Z_P^*$. Thus, when the threshold is raised, the agent is relocated to P and faces lower average positional consumption. Thus, as per Lemma 3.1, his utility must increase, so an increase in $\hat{y}$ does not make all agents weakly worse off.

Proof of Lemma 3.7

Proof is by contradiction. Suppose that there exists an equilibrium in which the average positional consumption is not equal for all groups. Then, as per Lemma 3.1, any agent from the community with higher average positional consumption has a profitable deviation by switching to the group with lower average positional consumption. Thus, the original arrangement could not have been an equilibrium. If the average positional consumption is the same in all communities, no agent can *strictly* improve his utility.

Proof of Proposition 3.6

The procedure for the cardinal models is as follows: (i) solve FOC (3.2) to obtain $z^*(y, Z)$, the demand for the positional good as a function of income and the average positional consumption, (ii) find the average positional consumption, i.e. the fixed point of the equation $Z^* = \int_{\underline{y}}^{\bar{y}} z^*(y, Z^*) f(y) dy$, (iii) compute $z^*(y) = z^*(y, Z^*)$, the demand for the positional good as a function of income only, and thus compute the indirect utility function, and iv) verify for which parameter values the demand is between zero and $\frac{y}{p}$ for every agent, so that there are no corner solutions.

For *Model XZS*, the FOC is $\alpha y + p(Z - 2\alpha z^*) = 0$, so that $z^*(y, Z) = \frac{\alpha y + pZ}{2\alpha p}$. Using the uniform $f(y)$, $\int_{\underline{y}}^{\bar{y}} z^*(y, Z^*) f(y) dy = \int_{\underline{y}}^{\bar{y}} \frac{\alpha y + pZ}{2\alpha p} \frac{1}{\bar{y} - \underline{y}} dy = \frac{2\alpha(\bar{y} + \underline{y}) + pZ}{4\alpha p}$, so the fixed point is $Z^* = \frac{\alpha(\underline{y} + \bar{y})}{2p(2\alpha - 1)}$. Hence, $z^*(y) = \frac{y}{2p} + \frac{\underline{y} + \bar{y}}{4p(2\alpha - 1)}$ and $u^*(y) = \frac{\alpha(2(2\alpha - 1)y - (\underline{y} + \bar{y}))^2}{16p(2\alpha - 1)^2}$. The set of inequalities $0 < z^*(y) < \frac{y}{p}$ is satisfied if $\alpha > \frac{2y + (\underline{y} + \bar{y})}{4y}$. But the RHS is a decreasing function of y, so for the set of inequalities to be satisfied for any $y \in [\underline{y}, \bar{y}]$,

it must be satisfied for $y = \underline{y}$. Thus we obtain $\alpha > \frac{3\underline{y}+\bar{y}}{4\underline{y}}$.

For *Model XyS*, the FOC is $-p + \frac{\eta y^4 Z}{(z^*)^2} = 0$, so that $z^*(y, Z) = y^2\sqrt{\frac{\eta Z}{p}}$, where the positive root is chosen. Using the uniform $f(y)$, $\int_{\underline{y}}^{\bar{y}} z^*(y, Z^*)f(y)dy$ $= \int_{\underline{y}}^{\bar{y}} y^2\sqrt{\frac{\eta Z}{p}}\frac{1}{\bar{y}-\underline{y}}dy = \sqrt{\frac{\eta Z}{p}}\frac{\underline{y}^2+\underline{y}\bar{y}+\bar{y}^2}{3}$, so the fixed point is $Z^* = \frac{\eta(\underline{y}^2+\underline{y}\bar{y}+\bar{y}^2)^2}{9p}$. Hence, $z^*(y) = \frac{\eta y^2(\underline{y}^2+\underline{y}\bar{y}+\bar{y}^2)}{3p}$ and $u^*(y) = \frac{1}{3}y(3(1+\alpha\eta y^3) - 2y(\underline{y}^2 + \underline{y}\bar{y} + \bar{y}^2))$. The set of inequalities $0 < z^*(y) < \frac{y}{p}$ is satisfied if $\eta^* < \frac{3}{\underline{y}^2\bar{y}+\underline{y}\bar{y}^2+\bar{y}^3}$. Additionally, we require that status never falls in income, $\frac{ds(z,Z;y)}{dy} \geq 0$, which gives $\alpha - \frac{Z}{z} \geq 0$. This is equivalent to the requirement that $\alpha \geq \frac{y^2+y\bar{y}+\bar{y}^2}{3y^2}$, which is always satisfied for all $y \in [\underline{y}, \bar{y}]$ if and only if it is satisfied for $y = \underline{y}$. Thus, we must have $\alpha \geq \frac{\underline{y}^2+\underline{y}\bar{y}+\bar{y}^2}{3\underline{y}^2}$.

For *Model HKS*, the procedure is different. The differential equation which characterises the solution is given by equation 6 of Hopkins and Kornienko (2004). Using the assumptions about the functional form of the utility function and about the uniform distribution, this becomes $z'(y) = \frac{y-pz'(y)}{p(y+\alpha\bar{y}-(1+\alpha)\underline{y})}$. Re-writing, we obtain $z'(y)(y + \alpha\bar{y} - (1+\alpha)\underline{y}) + z(y) = \frac{y}{p}$, which we can solve by using the integrating factor to obtain $z(y)(y+\alpha\bar{y}-(1+\alpha)\underline{y}) = \int \frac{y}{p}dy$, or $z(y) = \frac{y^2+C}{2p(y+\alpha\bar{y}-(1+\alpha)\underline{y})}$, where C is the constant of integration. As per Lemma 1 of Hopkins and Kornienko (2004), if $\alpha > 0$, $z(\underline{y}) = 0$, as no positional good is consumed in the cooperative bundle. Thus the constant of integration must be $\underline{y}^2$, and therefore the optimal choice of positional consumption is $z^*(y) = \frac{y^2-\underline{y}^2}{2p(y+\alpha\bar{y}-(1+\alpha)\underline{y})}$. $z(y)$ is a strictly increasing function, so as per Proposition 1 of Hopkins and Kornienko (2004), rank in the positional good is equal to rank in income, $G(z^*(y)) = F(y)$. The expression for indirect utility follows from plugging $z^*(y)$ into $u(z, G(.))$, which becomes $u^*(y) = z^*(y)F(z^*(y))$.

Proof of Lemma 3.8

We have $\frac{dz^*(y)}{d\alpha} = -\frac{\underline{y}+\bar{y}}{2p(2\alpha-1)^2} < 0$, $\frac{dz^*(y)}{d\alpha} = -\frac{(\bar{y}-\underline{y})(y^2-\underline{y}^2)}{2p(y+\alpha\bar{y}-(1+\alpha)\underline{y})^2} < 0$ and $\frac{dz^*(y)}{d\alpha} = 0$ in *Model XZS*, *Model HKS* and *Model XyS*, respectively. By the envelope theorem, $\frac{\partial u^*(y)}{\partial\alpha} = (y - pz^*(y))z^*(y) > 0$, $\frac{\partial u^*(y)}{\partial\alpha} = z^*(y) > 0$ and $\frac{\partial u^*(y)}{\partial\alpha} = \eta y^4 > 0$ in *Model XZS*, *Model HKS* and *Model XyS*, respectively. Finally, in *Model XyS*, we have

$\frac{dz^*(y)}{d\eta} = \frac{y^2(\underline{y}^2+\underline{y}\bar{y}+\bar{y}^2)}{3p} > 0$ and $\frac{\partial u^*(y)}{\partial \eta} = y^4(\alpha - \frac{Z^*}{z^*(y)}) > 0$.

Proof of Lemma 3.9

In *Model XyS*, from Proposition 3.6 we have $Z^* = \frac{\eta(\underline{y}^2+\underline{y}\bar{y}+\bar{y}^2)^2}{9p}$. For increase in incomes, I substitute $\underline{y}$ with $\underline{y}+\epsilon$ and $\bar{y}$ with $\bar{y}+\epsilon$ to obtain $Z^*(\epsilon)$. Taking the derivative with respect to ϵ and evaluating in the limit $\epsilon \to 0$ gives $\lim_{\epsilon\to 0} \frac{dZ^*(\epsilon)}{d\epsilon} = \frac{2\eta(\bar{y}+\underline{y})(\underline{y}^2+\underline{y}\bar{y}+\bar{y}^2)}{3p} > 0$. For increase in inequality, I substitute $\underline{y}$ with $\underline{y}-\epsilon$ and $\bar{y}$ with $\bar{y}+\epsilon$ to obtain $Z^*(\epsilon)$. Taking the derivative with respect to ϵ and evaluating in the limit $\epsilon \to 0$ gives $\lim_{\epsilon\to 0} \frac{dZ^*(\epsilon)}{d\epsilon} = \frac{2\eta(\bar{y}-\underline{y})(\underline{y}^2+\underline{y}\bar{y}+\bar{y}^2)}{9p} > 0$. As the average positional consumption increases in both cases, the result on utility follows from Lemma 3.1.

In *Model HKS*, the indirect utility function is $u^*(y) = \frac{y^2+\bar{y}^2-2y(y-\alpha(\bar{y}-\underline{y}))}{2(\bar{y}-\underline{y})}$. For increase in incomes, I substitute $\underline{y}$ with $\underline{y}+\epsilon$ and $\bar{y}$ with $\bar{y}+\epsilon$ to obtain $u^*(\epsilon)$. Taking the derivative with respect to ϵ and evaluating in the limit $\epsilon \to 0$ gives $\lim_{\epsilon\to 0} \frac{du^*(\epsilon)}{d\epsilon} = -\frac{y-\underline{y}}{\bar{y}-\underline{y}} < 0$. For increase in inequality, I substitute $\underline{y}$ with $\underline{y}-\epsilon$ and $\bar{y}$ with $\bar{y}+\epsilon$ to obtain $u^*(\epsilon)$. Taking the derivative with respect to ϵ and evaluating in the limit $\epsilon \to 0$ gives $\lim_{\epsilon\to 0} \frac{du^*(\epsilon)}{d\epsilon} = \frac{(y-\underline{y})(\bar{y}-y)}{(\bar{y}-\underline{y})^2} > 0$.

Proof of Lemma 3.10

It follows from the text that the socially optimal consumption is equivalent to the cooperative consumption, so it suffices to find $z^C(y)$. As status enters the utility function multiplicatively in *Model XZS*, the level of status does not impact the agent's maximisation problem. Thus, finding the cooperative consumption reduces to a standard maximisation problem for Cobb-Douglas preferences with the coefficient of $\frac{1}{2}$. The result follows.

Proof of Proposition 3.7

When a tax $t(y)$ is introduced, the proceeds from which are returned to the agents to be spent on non-positional consumption, the FOC (3.2) adjusts to reflect the change

in the relative price from p to $p(1+t(y))$. Thus, the FOC is $-p(1+t(y))u_x(.) + u_z(.) + u_s(.)s_z(.) = 0$, and the equilibrium positional consumption is $z^*(y;t(y)) = \frac{y}{2p(1+t(y))} + \frac{y+\bar{y}}{2p(2\alpha-1)}$. The tax must restore the socially optimal consumption, so that $z^*(y;t(y)) = z^{SO}(y)$, or $\frac{y}{2p(1+t^*(y))} + \frac{y+\bar{y}}{2p(2\alpha-1)} = \frac{y}{2p}$, which gives the optimal tax level $t^*(y) = \frac{y+\bar{y}}{2y(2\alpha-1)}$. $\frac{dt^*(y)}{dy} = -\frac{y+\bar{y}}{2y^2(2\alpha-1)} < 0$, so the optimal tax decreases in income.

Proof of Proposition 3.8

The FOC with an income-independent tax t is derived as in the previous proof, and the indirect utility function becomes $u^t(y;t) = \frac{\alpha(2y(2\alpha-1)-(y+\bar{y}))(2y(1+2t)(1-2t)-(y+\bar{y}))}{16p(2\alpha-1)^2(1+t)^2}$. The resulting social welfare function is:

$$\begin{aligned} W^t(t) &= \int_{\underline{y}}^{\bar{y}} u^t(y;t)f(y)dy \\ &= \frac{1}{D}\alpha(2\underline{y}\bar{y}(11+10t+4\alpha(-5-7t+2\alpha(1+2t))) \\ &+ (\underline{y}^2+\bar{y}^2)(13+14t+4\alpha(-7-11t+4\alpha(1+2t)))), \end{aligned}$$

where $D = 48p(2\alpha-1)^2(1+t)^2$. Taking the derivative with respect to t and solving the resulting FOC for t^{**} gives the desired expression.

Proof of Lemma 3.11

For *Model XZS*, the average positional consumption is $Z_P^* = \frac{\alpha(\underline{y}+\hat{y})}{2p(2\alpha-1)}$ in P and $Z_R^* = \frac{\alpha(\hat{y}+\bar{y})}{2p(2\alpha-1)}$ in R. The average positional consumption across the communities is $Z_{PR}^* = F(\hat{y})Z_P^* + (1-F(\hat{y}))Z_R^* = \frac{\alpha(\underline{y}+\bar{y})}{2p(2\alpha-1)} = Z^*$. The inequality in proposition is trivially obtained.

For *Model XyS*, the average positional consumption is $Z_P^* = \frac{\eta(\underline{y}^2+\underline{y}\hat{y}+\hat{y}^2)^2}{9p}$ in P and $Z_R^* = \frac{\eta(\hat{y}^2+\hat{y}\bar{y}+\bar{y}^2)^2}{9p}$ in R. The average positional consumption across the communities is $Z_{PR}^* = F(\hat{y})Z_P^* + (1-F(\hat{y}))Z_R^* = \frac{\eta((\bar{y}-\hat{y})(\hat{y}^2+\hat{y}\bar{y}+\bar{y}^2)^2+(\hat{y}-\underline{y})(\underline{y}^2+\underline{y}\hat{y}+\hat{y}^2)^2)}{9p(\bar{y}-\underline{y})} > Z^*$. The inequality in proposition is trivially obtained.

Proof of Proposition 3.9

Assume there are N stratified communities and the income thresholds of the k^{th} community ($k \in \{1, \ldots, N\}$) are denoted by $\hat{y}_{k-1}$ and $\hat{y}_k$, where $\hat{y}_{k-1} > \hat{y}_k$, $\hat{y}_0 = \underline{y}$ and $\hat{y}_N = \bar{y}$. The indirect utility in each community can be found by substituting $\underline{y}$ with $\hat{y}_{k-1}$ and $\bar{y}$ with $\hat{y}_k$ in the expression from Proposition 3.6. Thus, we obtain $u_k^*(y) = \frac{\alpha(2(2\alpha-1)y-(\hat{y}_{k-1}+\hat{y}_k))^2}{16p(2\alpha-1)^2}$. As the communities are of equal size, we must have $\hat{y}_k = \underline{y} + \frac{k}{N}(\bar{y} - \underline{y})$, so that $u_k^*(y) = \frac{\alpha((2k-1)(\bar{y}-\underline{y})-2N(y(2\alpha-1)-\underline{y}))^2}{16pN^2(2\alpha-1)^2}$. Thus, the welfare over the range of incomes $[\hat{y}_{k-1}, \hat{y}_k]$ is given by $U_k = \int_{\hat{y}_{k-1}}^{\hat{y}_k} u_k^*(y) f(y) dy = \frac{\alpha(((1-4k(\alpha-1))(\bar{y}-\underline{y})+4N(\alpha-1)\underline{y})^3+((3+4k(\alpha-1)-4\alpha)(\bar{y}-\underline{y})+4N(\alpha-1)\underline{y})^3)}{96pN^3(2\alpha-1)^3(\bar{y}-\underline{y})}$. The overall utility is given by $W_N = \sum_{k=1}^{N} U_k = \frac{\alpha}{48pN^2(2\alpha-1)^2}\big(2\underline{y}\bar{y}(3+8N^2(\alpha-1)^2-4\alpha) - \bar{y}^2(3-16N^2(\alpha-1)^2-4\alpha) - \underline{y}^2(3-16N^2(\alpha-1)^2-4\alpha)\big)$. Thus, the derivative with respect to N is $\frac{dW_N}{dN} = -\frac{\alpha(4\alpha-3)(\bar{y}-\underline{y})^2}{24pN^3(2\alpha-1)^2} < 0$.

Proof of Proposition 3.10

Agents in R contniue to optimise as in Lemma 3.11 without any change. Thus the average positional consumption in R remains Z_R^*. When agents in P compared themselves to other agents in P, the average positional consumption was Z_P^*. When they compare themselves to the rich, they essentially look at Z_R^*. As $Z_R^* > Z_P^*$, the new utilities of the agents in P must be lower as per Lemma 3.1.

Proof of Proposition 3.11

From the proof of Lemma 3.11, $Z_P^* = \frac{\eta(\hat{y}^2+\hat{y}\underline{y}+\underline{y}^2)^2}{9p}$, $Z_R^* = \frac{\eta(\hat{y}^2+\hat{y}\bar{y}+\bar{y}^2)^2}{9p}$ and $Z_{PR}^* = \frac{\eta((\bar{y}-\hat{y})(\hat{y}^2+\hat{y}\bar{y}+\bar{y}^2)^2+(\hat{y}-\underline{y})(\hat{y}^2+\hat{y}\underline{y}+\underline{y}^2)^2)^2}{9p(\bar{y}-\underline{y})}$. Now substitute $\underline{y}$ with $\underline{y} - \epsilon$ and $\bar{y}$ with $\bar{y} + \epsilon$, where $\epsilon > 0$ is small. Taking the derivatives with respect to ϵ and evaluating the limit as ϵ goes to zero gives $\lim_{\epsilon\to 0} \frac{dZ_P^*(\epsilon)}{d\epsilon} = -\frac{2\eta(\hat{y}^3+3\hat{y}^2\underline{y}+3\hat{y}\underline{y}^2+2\underline{y}^3)}{9p} < 0$, $\lim_{\epsilon\to 0} \frac{dZ_R^*(\epsilon)}{d\epsilon} = \frac{2\eta(\hat{y}^3+3\hat{y}^2\bar{y}+3\hat{y}\bar{y}^2+2\bar{y}^3)}{9p} > 0$ and $\lim_{\epsilon\to 0} \frac{dZ_{PR}^*(\epsilon)}{d\epsilon} = \frac{\eta(\bar{y}-\underline{y})(\hat{y}^2+2\hat{y}(\underline{y}+\bar{y})+3\bar{y}^2+4\bar{y}\underline{y}+3\underline{y}^2)}{9p} > 0$. For an agent with a fixed y, the utility must rise (fall) when the average positional consumption falls (rises), because all of the agents over-consume as per Lemma 3.1.

From the proof of Lemma 3.9, $Z^* = \frac{\eta(\bar{y}^2+\bar{y}\underline{y}+\underline{y}^2)^2}{9p}$, so that $\lim_{\epsilon\to 0}\frac{dZ^*(\epsilon)}{d\epsilon} = \frac{2\eta(\bar{y}^3-\underline{y}^3)}{9p}$. Consequently, $\lim_{\epsilon\to 0}\left(\frac{dZ^*_{PR}(\epsilon)}{d\epsilon} - \frac{dZ^*(\epsilon)}{d\epsilon}\right) = \frac{\eta(\bar{y}-\underline{y})(\hat{y}+\bar{y}+\underline{y})^2}{9p} > 0$, so the increase in Z^*_{PR} is greater than in Z^*.

Proof of Lemma 3.12

As explained in the general section, for the single-crossing condition to be satisfied, we must have $\frac{d}{dy}\left(\frac{dZ}{dq}\right) > 0$ because then in the (q, Z) space, the indifference curves of agents of different incomes may cross at most once. In *Model XZS*, by plugging optimal choice of the positional consumption from the FOC into the utility function, we obtain $u^*(y, Z, q) = \frac{q^2(\alpha y - pZ)^2}{4\alpha p}$. In any interior solution, a sufficiently large α is assumed to guarantee $\alpha y - pZ > 0$. Total differentiation gives $du = -\frac{q(\alpha y - pZ)}{2\alpha p}\big(pqdZ - (\alpha y - pZ)dq\big)$. Utility does not change along the indifference curve, $du = 0$, so we obtain that the slope of the indifference curve is $\frac{dZ}{dq} = \frac{\alpha y - pZ}{pq} > 0$. Differentiating with respect to income gives $\frac{d}{dy}\left(\frac{dZ}{dq}\right) = \frac{\alpha}{pq} > 0$, so the slope indeed rises for higher incomes. Thus, the single-crossing condition is satisfied.

Proof of Proposition 3.12

Given that the single crossing condition is satisfied, we only have to identify the threshold agent. Such an agent obtains the same indirect utility in either community, $u^*_L(\hat{y}^*(q)) = u^*_H(\hat{y}^*(q))$. The proof of Lemma 3.11 shows that the indirect utility function with two communities is $u^*_L(y) = \frac{\alpha(2y(2\alpha-1)-(\underline{y}+\hat{y}))^2}{16p(2\alpha-1)^2}$ and $u^*_H(y) = \frac{\alpha q^2(2y(2\alpha-1)-(\bar{y}+\hat{y}))^2}{16p(2\alpha-1)^2}$, where the standard utilities have been multiplied by the respective community qualities. Solving the indifference condition gives two roots, $\hat{y}(q) = \frac{q\bar{y}-\underline{y}}{(4\alpha-3)(q-1)}$ and $\hat{y}(q) = \frac{q\bar{y}+\underline{y}}{(4\alpha-3)(q+1)}$. But considering the necessary condition for the solution of all agents' maximisation problems to be interior, the second root is always smaller than $\underline{y}$, so we choose the first root, $\hat{y}^*(q) = \frac{q\bar{y}-\underline{y}}{(4\alpha-3)(q-1)}$. Finally, we obtain the required range for q by solving the inequality $\underline{y} < \hat{y}^*(q) < \bar{y}$.

Proof of Proposition 3.13

The first part is proved by differentiating the threshold, so that $\frac{d\hat{y}^*(q)}{d\alpha} = -\frac{4(q\bar{y}-\underline{y})}{(4\alpha-3)(q-1)} < 0$ and $\frac{d\hat{y}^*(q)}{dq} = -\frac{\bar{y}-\underline{y}}{(4\alpha-3)(q-1)^2} < 0$. For the result on γ, first the computation of the equilibrium threshold in the proof of Proposition 3.12 needs to be repeated to obtain $\hat{y}^*(q,\gamma)$. Differentiation with respect to γ gives $\frac{d\hat{y}^*(q,\gamma)}{d\gamma} = -\frac{(\bar{y}-\underline{y})(4\alpha-3)(1+\sqrt{q^\gamma})^2 \log q}{2(4\alpha-3)^2(q^\gamma-1)^2} < 0$.

Proof of Proposition 3.14

Using the expression for the optimal threshold $\hat{y}^*(q)$, substitute $\underline{y}$ with $\underline{y} \pm \epsilon$ and $\bar{y}$ with $\bar{y}+\epsilon$, where $\epsilon > 0$ is small and the plus sign is chosen for an increase in incomes while the minus sign for the increase in inequality. Taking the derivative with respect to ϵ and evaluating the limit as ϵ goes to zero gives $\lim_{\epsilon\to 0} \frac{d\hat{y}^*(q;\epsilon)}{d\epsilon} = -\frac{-q\pm 1}{(q-1)(4\alpha-3)} > 0$, where the inequality follows from the assumption on α and $q > 1$. Thus, the equilibrium threshold must increase in both scenarios.

Proof of Proposition 3.15

Similarly to Lemma 3.12, as the single crossing condition is satisfied, we only have to identify the threshold agent. Such an agent obtains the same indirect utility in either community, $u_P^*(\hat{y}^*) = u_R^*(\hat{y}^*)$. The quality of P is $\frac{\underline{y}+\hat{y}}{2}$, while the quality of R is $\frac{\hat{y}+\bar{y}}{2}$. The proof of Lemma 3.11 shows that the indirect utility function with two communities is $u_P^*(y) = (\frac{\underline{y}+\hat{y}}{2})^2 \frac{\alpha(2y(2\alpha-1)-(\underline{y}+\hat{y}))^2}{16p(2\alpha-1)^2}$ and $u_R^*(y) = (\frac{\hat{y}+\bar{y}}{2})^2 \frac{\alpha(2y(2\alpha-1)-(\bar{y}+\hat{y}))^2}{16p(2\alpha-1)^2}$. Solving the indifference condition gives three roots, of which only one, $\hat{y}^* = \frac{\underline{y}+\bar{y}}{4(\alpha-1)}$ makes economic sense. Finally, we obtain the required range for α by solving the inequality $\underline{y} < \hat{y}^*(\alpha) < \bar{y}$.

Proof of Proposition 3.16

The new indirect utility in a single-community equilibrium is derived from Proposition 3.6, by multiplying the utility by $\left(\frac{\underline{y}+\bar{y}}{2}\right)^2$, so that $u^*(y) = (\frac{\underline{y}+\bar{y}}{2})^2 \frac{\alpha(2(2\alpha-1)y-(\underline{y}+\bar{y}))^2}{16p(2\alpha-1)^2}$.

The indirect utilities in P and R are derived by substituting $\bar{y}$ with $\hat{y}^*$ for $u_P^*(y)$ and $\underline{y}$ with $\hat{y}^*$ for $u_R^*(y)$. Then, the direct comparison of utilities gives us that $u_P^*(\underline{y}) > u^*(\underline{y})$ and $u_R^*(\bar{y}) > u^*(\bar{y})$, but $u_P^*(\hat{y}^*) = u_R^*(\hat{y}^*) < u^*(\hat{y}^*)$.

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