SOME PROPERTIES OF LIQUID HELIUM

AT VERY LOW TEMPERATURES

ABSTRACT

of a Thesis

Submitted for the Degree of Doctor of Philosophy

in the University of Oxford

by

L.J. CHALLIS B.A.

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ABSTRACT

Measurements have been made of the thermal conductivity of gaseous ^3He and ^4He and of liquid ^3He and ^4He by using a modified parallel plate arrangement of small volume (0.12 ccs.) which is capable of measuring conductivities of the order of 5.10^{-6} watt units. Before this work could be carried out, the tritium with which the ^3He gas was contaminated was removed by passing the gas through a copper trap immersed in liquid helium. The thesis includes a description of the apparatus used in cleaning this ^3He and the techniques employed in transporting the gas from the storage container to the measuring apparatus. Comparatively small quantities of gas were available so that all apparatus had to be of small volume, and of a robust construction to reduce the possibility of loss.

The transport properties of gases at low temperatures are of interest because of the appreciable quantum effects, and, of all gases, helium is the most suitable since it is monatomic, light and remains in the gas phase to very low temperatures. There are also the advantages that it exists in two isotopic forms so that the effect of statistics may be studied, and that the interaction potential between two helium atoms which is used in the calculation of the transport properties should be a simpler function of distance than the potential between more complicated atoms.

No measurements have been reported for ^3He gas, and in view of the considerable difference in the conductivity of the two isotopes suggested by the theoretical calculations (see de Boer 1955), it was decided to make these measurements. For comparison purposes and in order to check that the apparatus was behaving correctly, measurements were also made on gaseous ^4He. Values were obtained for both gases in the temperature range 1.5 - 4.0°K. The conductivity of ^4He gas at these temperatures has been measured by Ubbink and de Haas (1943) and the results of the present work are in good agreement with their values. The results for ^3He gas are significantly different from those of ^4He confirming the predictions, and the values for both gases are compared with the theoretical values of Keller (1957) when it is found that the agreement obtained is good for ^4He and moderate for ^3He. A comparison is also made with the conductivities derived from the viscosity data using the expression $K = \frac{5}{2} \eta C_v$.

The conductivity of liquid ^3He is found to be of the same order of magnitude as that of gaseous ^3He and, as it also rises with temperature like a gas, some sort of gas-model would seem appropriate. The results we obtain are slightly below those of Lee, Donnelly and Fairbank (1957) and we attribute this to a boundary resistance which is present in our cell though not in theirs. This conclusion has recently been confirmed by some experiments of H.A. Fairbank and Lee (1957) and our values for the conductivity of

^{4}HeI indicate that a boundary resistance is also present in this liquid.

A resistance of the same order of magnitude is known to exist at surfaces in contact with ^{4}HeII when it is referred to as the Kapitza resistance (Kapitza 1941) and the present demonstration of its existence in ^{4}HeI would appear to rule out explanations based on the peculiar properties of helium below the λ-point. A theory independent of these properties is by Khalatnikov (1952) who considers the vibrations of the surface of a solid in contact with liquid helium and attributes the resistance to the acoustic mismatch of the phonons, or quantised vibrational motions, in the solid at the interface. We suggest that this theory should be modified in some way to account for the dense adsorbed layer of helium at this interface; the thickness of the layer is of the same order of magnitude as that of the dominant wavelength of the sound radiated into the helium and it would appear that this invalidates the assumption that the helium is a continuous medium of constant density right up to the interface. The wavelengths just above the dominant wavelength will be of the order of the interatomic distances so that the concept of wave motion for the energy radiated at these wavelengths is becoming a less useful one.

The conductivities of liquid ^{3}He and liquid ^{4}HeI together with their viscosities have been discussed briefly in the light of the model proposed to describe the behaviour of these liquids

and they have been compared with other simple liquids such as liquid argon and nitrogen.

Measurements have also been made on the velocity of sound in liquid ^3He. A method has been used in which the time taken for a pulse of ultrasonics to travel a known distance (to a reflector and back again) is measured. Before the completion of this work, two independent sets of values were reported (Laquer, Sydoriak and Roberts 1957, and Flicker and Atkins 1957) and in view of this, only a few measurements were made; these lie between the values obtained by the two groups of workers mentined above. From this velocity, we calculate the ratio of the specific heats C_p/C_v from thermodynamic formulae and hence show that C_{sat}, which has hitherto been used for C_v in calculations of the entropy, differs appreciably from C_v above 1.5°K.

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" 'tis precious of itself "

(Troilus and Cressida

Act 2, Sc. 2, Shakespeare

1601)

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PART I

The Thermal Conductivity of Gaseous and Liquid Helium

CHAPTER 1

INTRODUCTION

Thermal conductivity of liquid helium

There is still no general theory which describes the behaviour of both the quantum liquids, ^{3}He and ^{4}He. The properties of liquid ^{4}He have been examined in detail particularly below the so-called lambda-point where the liquid is very unusual, but liquid ^{3}He has not yet been investigated so thoroughly. The behaviour of liquid ^{3}He is not only intrinsically interesting but also provides material for a comparative study of the two isotopes.

Quantum liquids such as ^{3}He and ^{4}He consist of interacting particles which can be considered in terms of their cooperative motions or excitations and a flow of these excitations in the presence of a temperature gradient results in thermal conduction. Thermal conductivity measurements provide information about the dominant excitations at any temperature or alternatively may indicate whether the concept of cooperative motions or of free-particle motion as in a gas is the more accurate. It was with this in mind that it was decided to measure the thermal conductivity of liquid ^{3}He.

^{3}He was first liquified in 1949 by Sydoriak, Grilly and Hammel and since then a considerable amount of work has been carried out on the liquid. It differs from ^{4}He in that no λ-anomaly in the specific heat has been observed down to 0.23°K and that none of the unusual properties associated with superfluidity, such as internal convertion resulting in very efficient heat transfer, have been

detected in ^{3}He. The ^{3}He nucleus has a spin 1/2 $\hbar$ so that there are phenomena associated with this which do not occur in ^{4}He. Reviews exist of the properties of the liquid (e.g. Hammel 1955) and we propose to discuss very briefly some of the models which have been suggested to explain the properties of the liquid, since we shall consider later the measurements made on the thermal conductivity in the light of certain of these models.

Like ^{4}He liquid ^{3}He possesses a very large molar volume so that as in ^{4}He it was thought that to a first approximation the effect of molecular interactions might be ignored and the liquid treated theoretically as a gas of particles obeying the appropriate statistics (Singwi 1952; Goldstein 1948). For ^{4}He these are Bose-Einstein and the moderate success of the idal gas theory is well known (see London 1954) but for ^{3}He, with an odd number of particles in the nucleus, the statistics are Fermi-Dirac.

The thermodynamic quantities can be calculated for a Fermi-Dirac gas in terms of the mass of the atoms and density of the gas. It is convenient to discuss the results in terms of a characteristic temperature, the so-called degeneracy temperature which is a function of these two properties , $kT_o = \frac{h^2}{8m}\left(\frac{3\rho}{\pi m}\right)^{2/3}$, and if the values of the density of the liquid and the atomic mass are substituted in this formula a value for T_o of about 5°K is obtained. At temperatures well below this, the liquid is said to be degenerate and the thermodynamic functions become simple functions of temperature - the specific heat and entropy are both linearly dependent on temperature - but at temperatures of the same order of magnitude as or greater than T_o, further

terms must be considered in the expressions for these functions. No measurements have been made for C_v but as we shall discuss later, the difference between the specific heat at constant volume and at the saturation vapour pressure (C_{sat}) is small at the lower temperatures. The entropy has been calculated from values of C_{sat} and such properties as the heat of vaporization and the vapour pressure (Osborne, Abraham and Weinstock 1953, 1954, 1955; Roberts and Sydoriak 1954, 1955; de Vries and Daunt 1953, 1954) and is similar in form to that of a Fermi-Dirac gas with a value of T_o of 5.0°K. The entropy is further complicated by the existence of a spin system : when the spins are disorientated, there is an additional contribution of $R\ln_e 2$ to the entropy, but as the temperature of the ^3He is reduced, the spins become aligned and this contribution becomes less.

Measurements of the nuclear magnetic susceptibility have been made down to .23°K (Fairbank, Ard and Walters 1954) and show that if a Fermi-Dirac gas model is to be used, good agreement is obtained for $T_o = 0.45$°K whereas the agreement for $T_o = 5$°K is very poor.

It seems then that the molecular interactions modify the properties of the FermiDirac system in such a way that a pure gas model is inappropriate.

This free particle model can be adapted slightly by considering no longer a gas of particles of mass m obeying Fermi-Dirac statistics, but to assume cooperative motion with a resulting gas of excitations of effective mass μ, but obeying the same statistics. Pomeranchuk (1950) and Landau (1957) have calculated the properties of such a Fermi liquid and Landau concludes that at low temperatures Bose-

Einstein excitations (phonons) could also exist.

Other models include the cell model which has been applied to helium in various forms by de Boer (1953), Prigogine and Philpott (1952) , Temperley (1955), Price (1955) and Rice (1955). In these models atoms are imagined as being maintained within a 'cell' bordered by the average positions of their neighbours. In the simplest model the motion of a single atom is investigated and the motion of the complete system described in terms of this, but in other models cell-clusters are imagined and the possibility of atoms being closely linked to neighbours considered as in the so-called pair rotator models. These models predict the existence of Debye waves and are capable of describing a number of features of the liquids.

Measurements have also been made on liquid ^4He. The properties of this liquid have been reviewed extensively (see for example Wilks, 1957) and the conductivity is known to be due to three different mechanisms. Above the λ-point (2.2°K) the conductivity is similar to that of a gas, from 2.2°K to 0.6°K, heat transfer is by a convection process between the normal and the superfluid or, in terms of excitations, by a flow of excitations under the fountain pressure produced by a temperature gradient. This flow is limited by the viscosity of the normal fluid which is the sum of the viscosities of the rotons and the phonons. Below 0.6°K, the number of rotons present is negligible and the mean free path of the phonons is limited only by the dimensions of the apparatus so that the conduction is similar to that of a dielectric solid at low temperatures (Casimir, 1938).

The present work is concerned with the region above 2.2°K where the conductivity is very small.

Thermal conductivity of gaseous helium

The transport properties of gaseous helium have been examined in some detail particularly at low temperatures where quantum effects predominate (see de Boer, 1955). In a simple treatment based on classical kinetic theory, the conductivity of a gas is given by the expression $K = \frac{1}{3}\rho\bar{c}\lambda C_v$ where the mean free path is related to the molecular diameter by $\lambda = \frac{1}{\sqrt{2}\, n\pi\sigma^2}$. This formula agrees well with the experimental results at high temperatures. If quantum methods are used to calculate the conductivity, departures from this classical expression are observed and an indication of the extent of these departures can be obtained by making the assumption that molecules behave roughly as in classical theory if we suppose them to be surrounded by a 'wave' field whose linear extension is given by the de Broglie wavelength $\lambda = \frac{h}{p}$. The momentum p is equal to the mass m of a molecule multiplied by the relative velocity g and the mean value of g is of the order of $\sqrt{\frac{RT}{m}}$ so that $p \sim \sqrt{mkT}$ and $\lambda \sim \frac{h}{\sqrt{mkT}}$.

Deflections of passing molecules occur if this wave field is penetrated and so the departure from classical theory becomes appreciable when λ is no longer negligible compared with the molecular diameter σ . An examination of the expression for λ reveals that this is the case when m, σ and T are small and it is found that the gas which is most suitable for a study of these quantum corrections is

helium in which m and σ are small and the molecule is monatomic. Experiments can be carried out on gaseous helium down to about 1.5°K before the vapour pressure becomes prohibitively small. For ^4He at 1.5°K, $\lambda \sim 1.8\sigma$. There are also the advantages that helium exists in two stable isotopic forms which obey different statistics, so that the effect of these statistics can be studied. The interaction potential which, being electronic, is the same for both isotopes, can in principle, be calculated directly.

An outline of the theoretical treatment of these transport properties is included in the discussion of the present work. It will be seen that a knowledge of the interaction potential is required before the calculations can be carried out. Unfortunately this potential cannot be calculated to the desired accuracy and it is usual therefore to assume a potential curve of the type indicated by the theoretical calculations but containing constants which are derived from such data as the second virial coefficient. Once the potential is known the expression for the transport coefficients can be calculated so, conversely, by comparison with the experimental results, the most accurate form of potential can be determined. It is found that the properties of ^3He are more dependent on the potential than those of ^4He so that measurements on ^3He should provide a more sensitive test to the accuracy of the potential function chosen.

We have made measurements on the thermal conductivity of ^3He and ^4He. No previous values for ^3He have been reported. Ubbink and de Haas (1943) measured the conductivity of ^4He down to 1.6°K and

our measurements were made to confirm these values and to test the apparatus with the more expendable isotope. A number of workers (Schwarze 1903, Weber 1917, Curie and Lepape 1931, Dickins 1934, Kannaluik and Martin 1934) have made measurements on ^4He at 0°C and above and Eucken (1913) obtained values down to 20°K.

HELIUM 3 TRANSPORT SYSTEM. 2

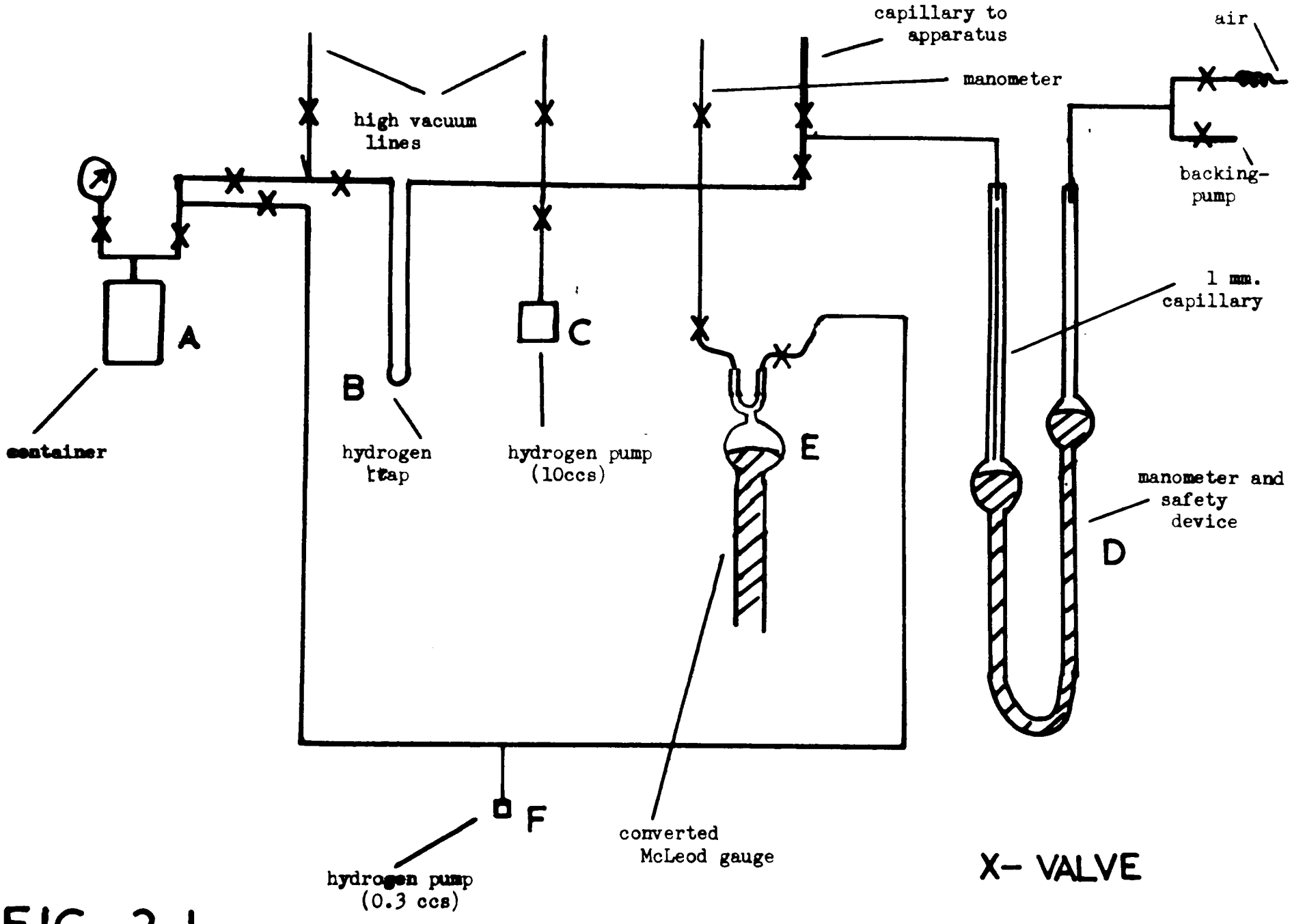


FIG. 2.1

CHAPTER 2

THE ^3He CLEANING AND TRANSPORT APPARATUS

Introduction

The ^3He used in these experiments was supplied by A.E.R.E., Harwell.

^3He is obtained by neutron irradiation of lithium in the reactions

$$^6Li + n \longrightarrow {}^3H + {}^4He$$

$$^3H \xrightarrow{12.5 \text{ years}} {}^3He + \beta^-$$

The tritium is separated from the ^4He by allowing it to diffuse through heated palladium, and this tritium decays to ^3He which is then removed using the same technique. In the sample supplied, ^4He and tritium were present as impurities to the extent of about 5% and 10^{-6}% respectively. With a concentration of ^4He of this order, the accepted method of purification is by fractional distillation, but it was decided that with the amount of ^3He initially available the yield of pure ^3He obtained might be too small to be of use. The tritium had been detected in the ^3He used in neutron counters and spectrometers (Batchelor, Aves and Skyrme, 1955) where the background radiation due to its decay had set the limit to resolution. In this chapter we describe the apparatus used in the cleaning and transport systems. Owing to the expensive nature of the helium, great care has been taken throughout this work to select reliable methods of handling.

Valves

The valves used in the ^3He systems were designed at Harwell for use with ^3He and possess very little dead volume. They are described by Batchelor et al. (1955).

Pumps

The action of a transfer pump depends on the principle that if two volumes containing gas are connected, flow takes place from one to the other until the pressures are the same in both. In order to pass gas from a vessel A to a vessel B, the pump P is connected to A and its volume made as large as possible. A is then isolated, the volume of P reduced and opened to B. If there is initially no gas in the pump or B, the amount transferred in one stroke is

$$a\left(\frac{V_{pmax.}}{V_A + V_{pmax}}\right)\times\left(\frac{V_B}{V_B + V_{pmin.}}\right)$$

where V_A is the volume of container A

V_B is the volume of container B

V_{Pmax} is the maximum volume of the pump

V_{Pmin} is the minimum volume of the pump

a is the mass of the gas

From this expression it is seen that for the pump to be efficient, V_{Pmax} should be made large and V_{Pmin} small; the compression ratio V_{pmax}/V_{pmin} should be large. It is important too that V_{Pmax} should be at least of the same order of magnitude as V_A. Three ways

of achieving a compression ratio were considered : (a) a rigid piston moving inside a cylinder, (b) a mercury piston moving inside a cylinder, (c) a thermal pump in which the effective volume of the pump is increased by reducing its temperature. All three types of pump have been used.

A rigid piston pump was used in the purification process carried out at A.E.R.E., Harwell but this belonged to the Establishment, and, owing to the difficulties in constructing such a pump so that it should not leak when the piston is being moved, we did not consider it practicable at that time to build our own. A description of the Harwell pump is given by Batchelor et al.(1955). Mercury piston and thermal pumps have been constructed and used with success. The pumps will now be considered in more detail.

Mercury pumps

The pump used consists of a 100 cc. glass bulb fitted at the top with two capillary taps. The bulb is waxed securely into a steel cylinder and replaces the glass part of a McLeod gauge. Mercury may be raised or lowered in the bulb by rotating a stout wheel which raises or lowers a reservoir. This arrangement, although modified to provide a larger traverse cannot be used at pressures above 1/3 atmosphere but has the advantage that a high degree of manual control is possible. Another pump in which the mercury is raised or lowered by changing the pressure on the outside level will be described under the heading 'Manometer and safety device'. This can be used at larger pressures.

Thermal pumps

A number of thermal pumps were built; they are very similar in design, each consisting of a copper vessel supported by a german-

silver tube which connects to the rest of the system through a valve. The vessel is arrangd so that it may be immersed in liquid hydrogen. At the temperature of liquid hydrogen, the effective volume of the vessel is nearly 15V, where V is the volume at room temperature, and so the compression ratio is nearly 15. When a larger ratio is needed, liquid helium pumped to just above the λ-point is used as a refrigerant so that a ratio of 130 is obtained. The vessel is precooled in liquid oxygen before being immersed in liquid hydrogen or helium and may be warmed rapidly by immersion in hot water.

The action is identical to that of other pumps mentioned and the pump is easy to construct, convenient and robust as it possesses no moving parts. It is also moderately efficient; if a pump of volume 10 ccs. is used to transfer gas from a container of volume 150 ccs. to another of the same volume, 80% of the gas is transferred in two strokes (this takes about 5 minutes) and 87% in four strokes.

Mercury manometer and safety device

Whenever possible, the helium in the system was maintained at a pressure of less than one atmosphere. Under these conditions a leak results in contamination of the gas with air and this can easily be remedied by passing the gas through a U-tube immersed in liquid hydrogen. If the helium were at an overpressure the helium leaking out could not be reclaimed. This of course applies to small leaks; if a breakage occurred in any glassware then the helium would very probably be lost whatever the pressure happened to be and for this reason we reduced the amount of glassware to a minimum.

For some of the experiments on the liquid, it was found

desirable that the pressure on the liquid should have a value just above the critical pressure (1.15 ats). A device was required which would make it possible to increase the pressure in the system to about 1.15 ats., and would maintain this pressure approximately constant and would also protect the apparatus against a sudden increase in pressure due to , say, the complete evaporation of the liquid helium by a sudden large rise in temperature.

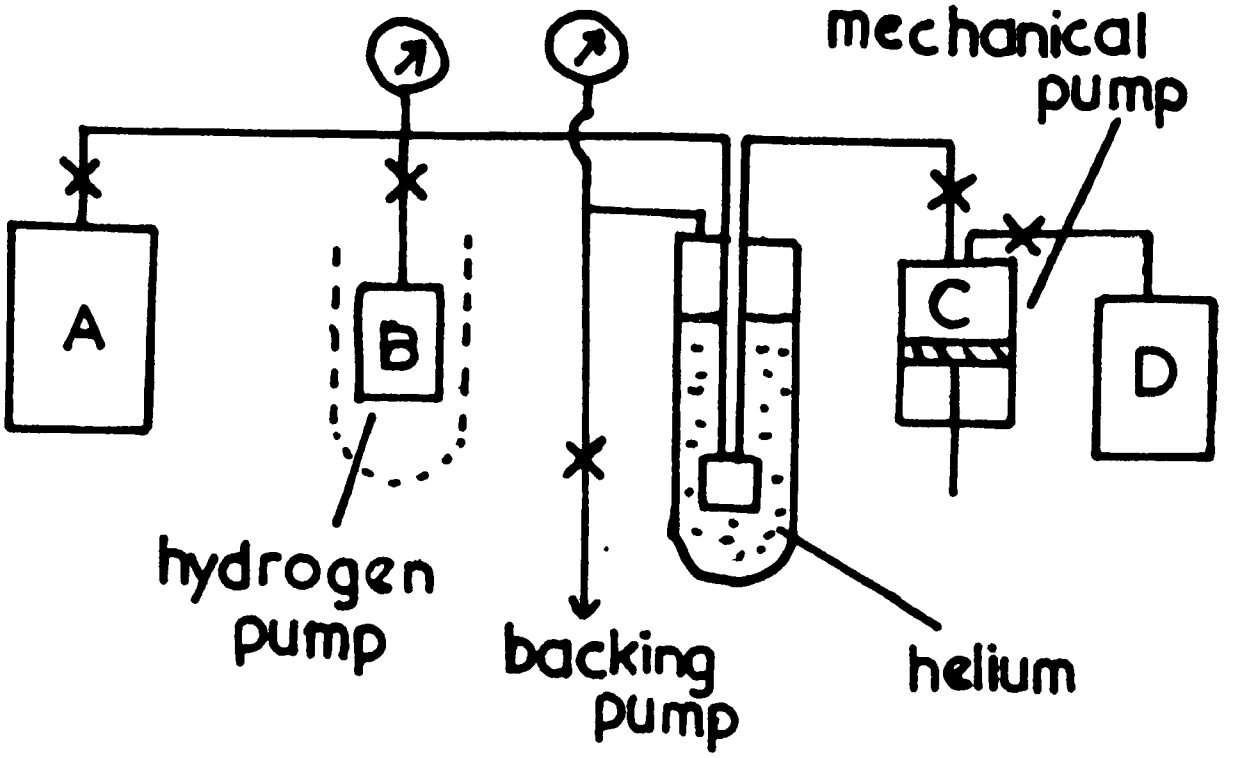
The final arrangement is shown on the left of figure 23. It comprises a glass U-tube in which the limb connecting the capillary with the thermal conductivity cell is of 1 mm. diameter and opens out to a 200 cc. bulb. The other limb is of 1 cm. diameter and also opens out to a similar bulb. The U-tube is filled with mercury as indicated in the diagram. The pressure of air above the mercury in the wider limb is controlled by the system of taps shown ; one of these is connected to a backing pump and the other to the atmosphere through a short length of capillary which prevents violent motion of the mercury when the tap is opened. The device is normally operated with this outer limb connected to the atmosphere.

The arrangement may be used as a Toepler pump; the mercury piston is raised or lowered by changing the pressure on the outer limb. In this way a pressure of up to about 1.4 ats. may be produced in the system. The capillary limb serves as a useful manometer particularly for pressures below one atmosphere when the level in the wider tube remains almost stationary. Besides measuring the pressure it alsohelps to stabilise it. A typical situation will be considered where the thermal conductivity cell is full of liquid at 1.2°K and the filling-

capillary full to a height where the temperature is 3.35°K tthe critical temperature). The rest of the capillary and the U-tube is filled with gas at a pressure of 1.2 ats. (just above the critical pressure). The cryostat is then warmed up to 3.2°K. The total volume of the liquid increases as the density falls from .081 to .059 gms/cc. but the helium can still only exist in the liquid state up to about the same point in the tube. Consequently a fraction $\frac{.081-.059}{.081}$ of the helium which originally formed the liquid phase (say 100 ccs.) evaporates, raising the pressure. This depresses the mercury in the bulk of the U-tube. Since the pressure change is small it may be calculated approximately in the following way. The bore of the bulb when 1/4 filled with helium is about 10 sq. cms. so that, to incorporate the increased volume of gas (27 ccs.) at about 1.2 ats., the level of mercury will drop by $\frac{27}{1.2 \times 10}$ cms. which is approximately 2.2 cm. The increase in pressure then is 4.4 cm. or 5%. This calculation demonstrates that the pressure remains constant to 5% for measurements made over the temperature range 1.2 to 3.2°K. It was verified approximately by experiment. We have found experimentally that a change in pressure of 1/2 atmosphere produces no measurable change in the thermal conductivity so that the measure of pressure stability achieved is adequate.

In the example considered above, the effect of a temperature increase to a value still below the critical temperature was determined. We shall now consider the more drastic effect of a sudden increase of temperature to room temperature. This could be produced deliberately

REMOVAL OF TRITIUM



YIELD AND CONCENTRATION AS A FUNCTION OF n (number of strokes)

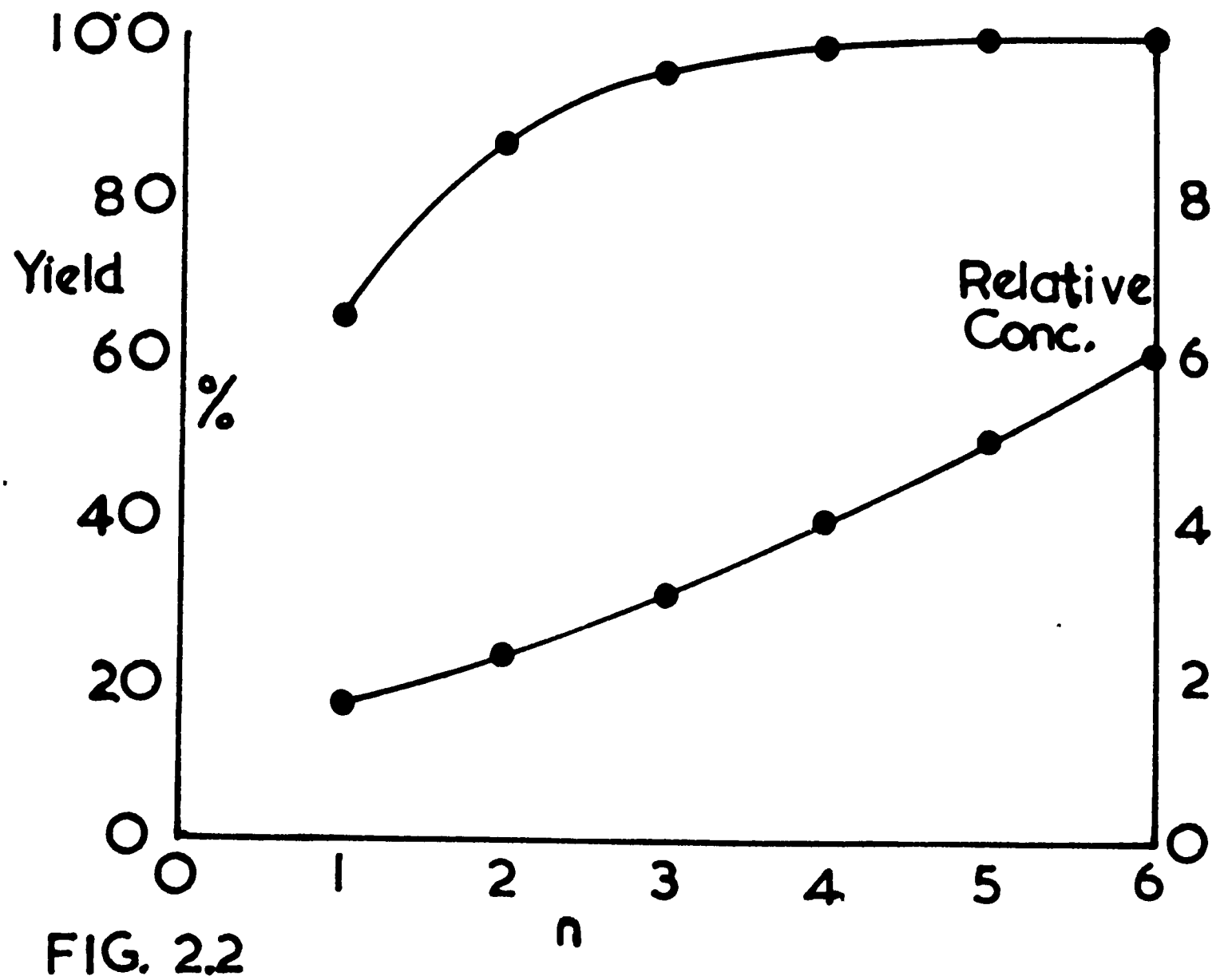


FIG. 2.2

on warming up the apparatus or accidentally if, say, the dewar vessel containing liquid hydrogen broke in the middle of an experiment and started a leak into the outer vacuum jacket (a full description of the low temperature part of the apparatus will be given in another chapter). The volume of cell and filling-tube is about 1 cc. and the volume of evaporated gas 150 ccs. at N.T.P. so that the mercury level is depressed until the relation P.V. = 150 is fulfilled. It will be seen from the dimensions that this will occur for a volume of about 110 ccs. and a pressure of about 1.4 ats. This shows that the maximum pressure which can occur in the system is about 1.4 ats.

The removal of tritium

The original tritium concentration in the ^3He was one part in 10^8. This was estimated from the background radiation in the ^3He proportional counter used by Batchelor et al. It was desirable to reduce this concentration to a value as small as possible in both the helium used in the present work and the helium used in the proportional counter.

The gas was passed through a trap which consisted of a small copper vessel immersed in liquid helium. The vessel which has a capacity of 4 ccs. is filled with copper swarf to provide a larger area of contact, and is connected to the rest of the apparatus by two german-silver tubes. The system also includes a hydrogen pump. A diagram of the apparatus is shown in figure 2.2. The unit was transported to Harwell where it was connected into the transport system already there.

Two preliminary experiments were carried out. In the first of these, a sample of ^4He contaminated with tritium was mixed

with a small quantity of hydrogen (1 cc.) as discussed below, and passed through the apparatus into a proportional counter. The reduction in tritium content was found to be satisfactory. In the second, the helium emerging from the trap was passed into the counter via a rigid piston transfer pump. The result was again satisfactory.

Several runs were then carried out with ^3He, 1 cc. of hydrogen being added as before. The procedure which was similar in each run will be outlined below.

Helium is transferred from a container A of (say) volume 1000 ccs. to a container C, which was usually the transfer pump, of volume 2000 ccs. by means of a hydrogen pump of volume 300 ccs. The effective volume of this pump at hydrogen temperature is $300 \times 15 = 4{,}500$ ccs. In the first cycle, a fraction

$$\left(\frac{4{,}500}{4{,}500+1{,}000}\right)\times\left(\frac{2{,}000}{2{,}000+300}\right) = 71\%$$

is passed through the trap into C. The trap was immersed in liquid helium contained in a transport vessel of the type used in the Clarendon Laboratory with slight modifications and the helium is pumped until the temperature of the bath is just above the λ-point. Below the λ-point, in an arrangement of this nature, the heat influx due to film flow becomes prohibitively large. At the end of this cycle, if the effect of the addition of hydrogen is ignored, C contains this fraction .71 of the helium together with an amount of tritium given by

$$Y = 2000 \times \text{Vapour Pressure of tritium at } T_1 \times K$$

where T_1 is the temperature of the copper can,

The factor K is included because at these pressures, some thermal transpiration may take place. The magnitude of this factor is then between 1 and $\sqrt{\frac{T_2}{T_1}} = 11$, where T_2 is the temperature of C. However, the molecules in this gas are colliding with the walls of the vessel C and a considerable fraction of them may be adsorbed so that more gas flows from the tritium in the trap. In this way the concentration of tritium measured by a proportional counter could differ from the calculated concentration by many orders of magnitude.

Another factor complicates the calculation of the final concentration of tritium expected. It was thought likely that traces of hydrogen might also be present in the ^3He in greater quantity than tritium. If this were so, the outside monolayer of molecules in a crystal of solid hydrogen would contain tritium as an impurity, and if this layer were removed by sublimation, the concentration of tritium in the gas phase would be equal to that in the solid phase. From this it is seen that the pressure exerted by the tritium is $\chi_{T_2} \times P_{H_2}$ where χ_{T_2} is the concentration of tritium in the solid and P_{H_2} is the vapour pressure of hydrogen. This being the case, it was decided to reduce the pressure even more by deliberate dilution of the tritium by hydrogen. It is assumed that the amount of tritium transferred in each stroke is constant whereas the amount of helium transferred decreases very rapidly in each successive stroke. Hence the concentration of tritium in the transferred gas rises with the number of strokes until eventually a linear dependence is approached. This implies that as the yield of purified helium increases so also does the concentration of tritium and a compromise between maximum yield

HELIUM 3 TRANSPORT SYSTEM I

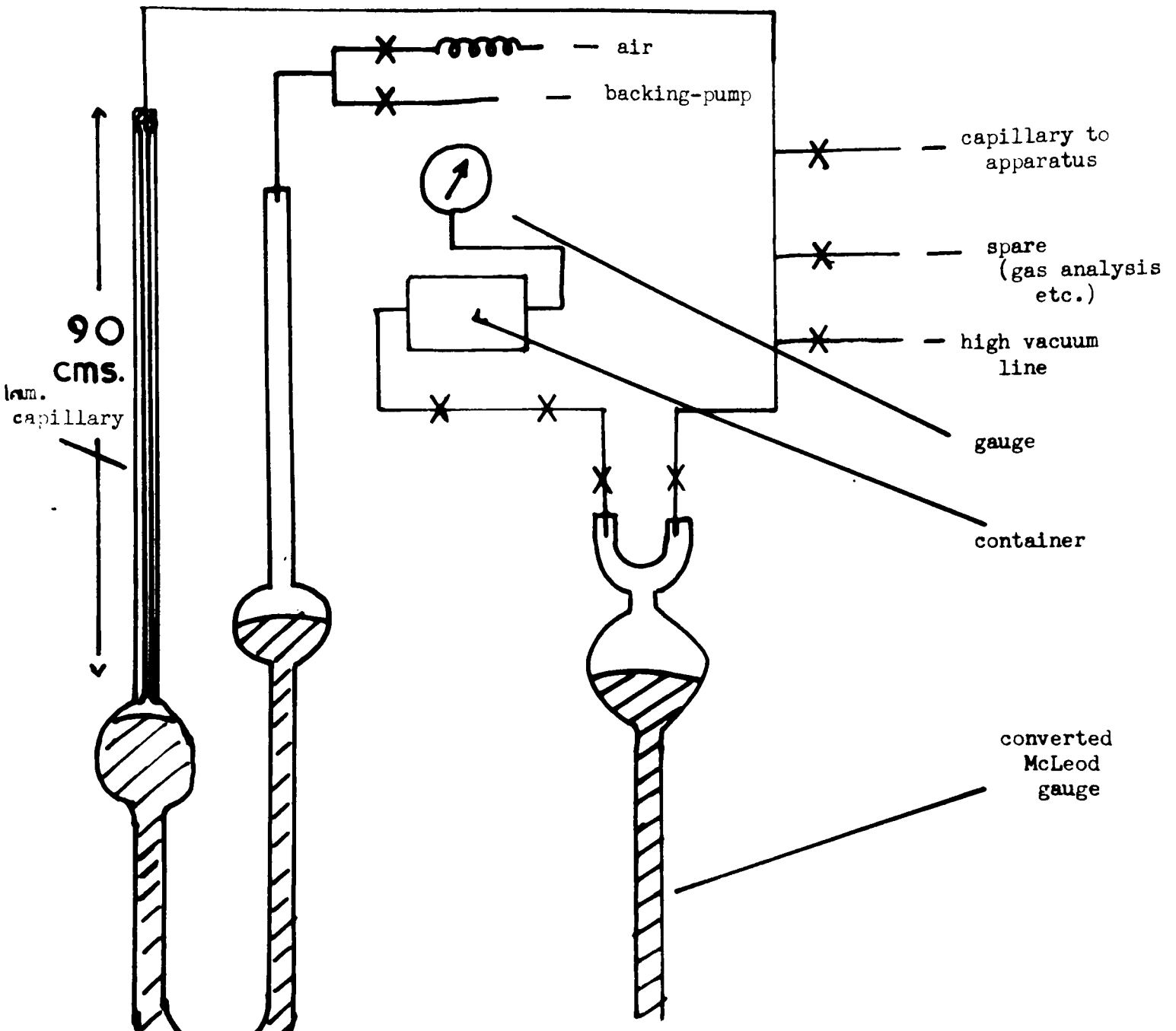


FIG. 2.3

and maximum purity must be effected. The results of a calculation of yield and purity as a function of the number of strokes of the pump for a typical operation are shown in the graph (figure 2.2). In this case the operation was stopped after 5 strokes.

Transport apparatus

The ^3He is kept in a storage container which is connected through a pump to the capillary leading to the conductivity cell. The original arrangement is depicted in figure 2.3 and comprised a mercury pump, a U-tube safety device and a high vacuum apparatus. The storage vessel (for 20 ccs. ^3He) was a brass cylinder fitted with a Bourdon gauge, the whole being mounted on a board and protected by a perspex shield. All tubing in this apparatus was of capillary of inside diameter either 0.3 mm. or 0.8 mm. so that the amount of helium left outside the storage vessel could be made very small by pumping. The preliminary experiments were carried out with this arrangement.

Fig2.1

The later apparatus, which is the one being used at present, is designed to transport up to 250 ccs. of helium. Most of the unit is mounted on a board attached to a Dexion trolley of dimensions 3 x 2 x 6 ft. The trolley contains a high vacuum line incorporating a two stage mercury pump, and is on wheels so that the apparatus can be used for experiments in different parts of the building. In the description which follows reference will be made to the lettering on the diagram.

The storage vessel A which is similar, but larger, to the one described above is fitted with two entrance valves. The first of

these leads to a U-tube of 1 mm. german-silver, B, which is immersed in liquid hydrogen during experiments so that if the helium should contain any air, it will solidify in this trap. The U-tube is surrounded by a 16 mm. german-silver tube for mechanical protection. After passing through the trap, the helium reaches a hydrogen pump, C, of volume 10 ccs. and then the manometer and safety device, D, which is kept permanently connected to the capillary leading to the conductivity cell. There are also two leads to the high vacuum line, one spare, and one lead to the mercury pump, E.

To condense gas from the storage container into the conductivity cell, it is first pulled into the hydrogen pumped through the trap. The hydrogen is removed and the gas is forced into the capillary (what actually happens is that the gas depresses the mercury in the U-tube and this provides the pressure under which the gas condenses). The hydrogen pump is emptied by using the U-tube as a Toepler pump. For the experiments on the liquid, a pressure of about 1.2 ats. was used so that this process was repeated until,in equilibrium, some helium remained in the bulb of the U-tube. The temperature of the cell was kept at about 2.4 - 2.5°K.

To remove the helium, the low temperature apparatus is warmed up to room temperature. The helium evaporates and fills the bulb of the U-tube. The storage vessel is opened, and the helium flows from the bulb to the vessel. The storage vessel is isolated. The rest of the transport apparatus of volume 1 - 2 ccs. contains helium at the pressure of the vessel (about 1/2 ats.) and there may be a similar

amount of gas in the hydrogen pump. These traces are pulled into the mercury pump and then transferred into another hydrogen pump, F, of volume 0.3 ccs. (volume at hydrogen temperature is 4.5 ccs), whence it is returned to the storage vessel. (This hydrogen pump, F, is needed because the mercury pump will only operate up to 1/3 ats. and gas is to be transferred to the storage container at 1/2 ats) The operation of condensing in takes about 1 - 2 hours, and of returning the hdium, about 1 hour.

THE CRYOGENIC APPARATUS (SCHEMATIC)

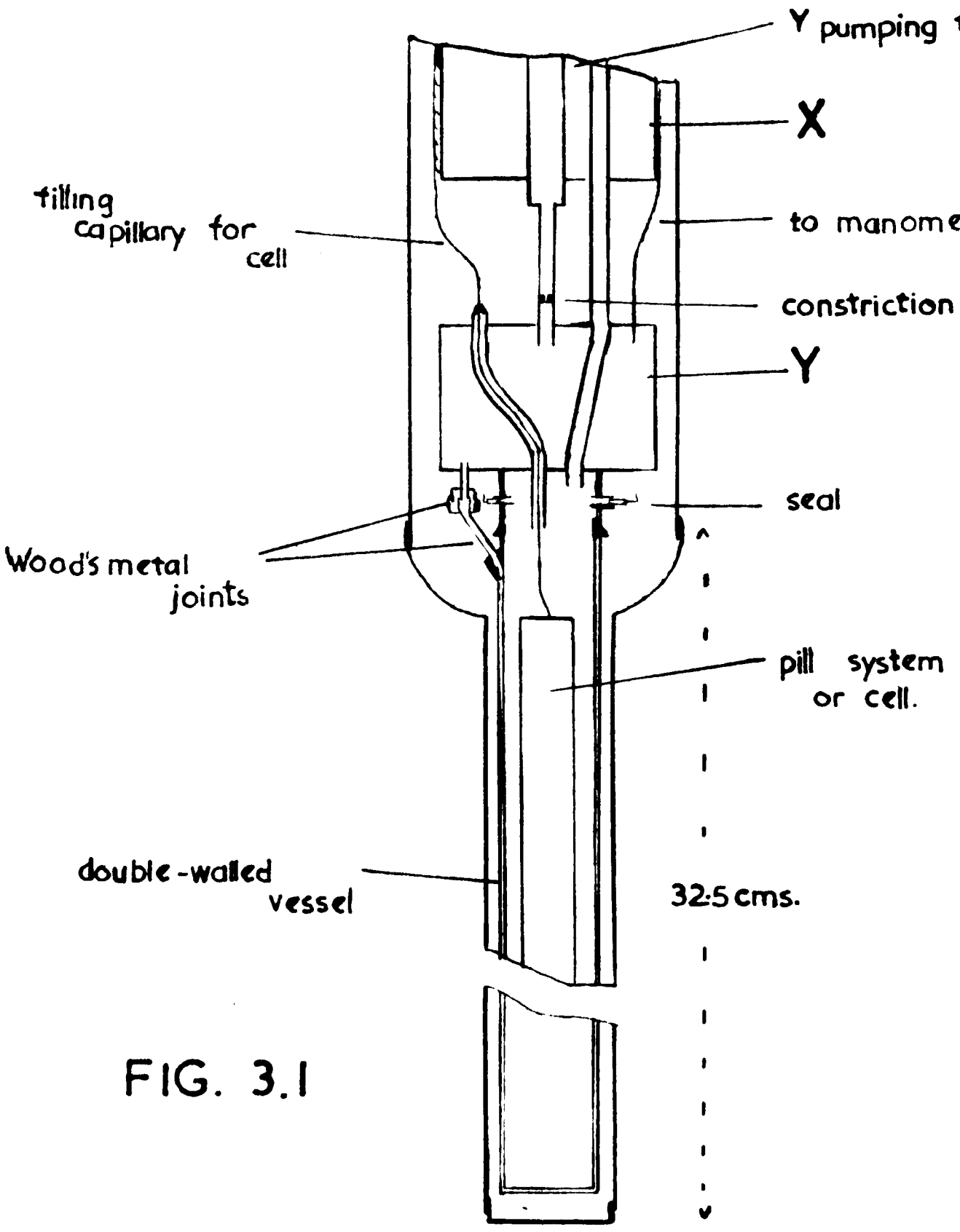


FIG. 3.1

scale approx. half full-size

CHAPTER 3

THE APPARATUS AND EXPERIMENTAL PROCEDURE

1. The Apparatus

The cryogenic apparatus

An apparatus was used of the "free helium " type which was built and described by Dr. G.R. Hercus (1953). Liquid helium produced in a separate liquefier is siphoned into a vessel X contained inside the apparatus and helium gas at a slight overpressure is then condensed into the cryostat chamber Y. This evaporates most of the liquid in X which is then refilled by siphoning in further helium. The helium in the cryostat can be cooled by pumping with a rotary pump to less than 1.2°K, and lasts for up to 16 hours. The part of the apparatus peculiar to these experiments is shown in Figure 3.1.

The capillary leading to the conductivity cell passes through a tube in the cryostat. The electrical leads reach the outside of the cryostat through a glass udder waxed into a tube connecting to the jacket and then pass through platinum-glass seals which were originally soft-soldered into position. These seals provided vacuum problems which will be discussed in the section describing the conductivity cell.

The tube through which the capillary was to pass was found to be partially blocked in the original cryostat, and the next two cryostats made included porous metal, both brass and copper. Finally, a cryostat was turned from a solid block of copper and found to be non-porous.

Oscillations

During the process of siphoning in the liquid helium, oscillations in the helium gas of considerable amplitude were detected, which were often so vident that it was impossible to transfer the helium. Theoretical treatments of these vibrations exist (Kramers 1949; ter Haar 1955) but their elimination is still largely a matter of trial and error. It is known that large heat leaks may induce these oscillations, which themselves cause considerable energy dissipation, and it is also known that the amplitude of the oscillations is linked very closely with such variables as the length of the vacuum jacket on the siphon (Hill 1952). Oscillations in temperature of about 1 c/s have been observed in pumping tubes above liquid helium (Kapitza 1941; Hercus 1953) and can be removed by including a large smoothing volume (8 litres is the volume used in the present apparatus) in series with the backing pump line. Attempts have been made to reduce the oscillations in the transfer tube in a similar way. This technique has been used with effect by some workers but was not successful in our experiments. Hercus suggested that the oscillations he obtained during transfer were due to helium gas leaking into the vacuum jacket at helium temperatures and we investigated this possibility, but found that there was no evidence of any leak in the present case. The siphon was carefully reassembled with the spacers (small metal foils to prevent the inside tube from touching theexternal jacket) removed but the oscillations reappeared with renewed violence. It was found that the amplitude could occasionally be reduced by switching off

backing pumps operating in the room and that this applied to pumps directly linked to the helium by a pumping line as well as to those completely independent of the system and presumably only cupled to it acoustically. The problem was finally ameliorated by lengthening the vacuum jacketing around the siphon tube from 7-1/2" to 9-1/2" at the same time fitting a baffle in the X-pumping tube designed to damp the oscillations. This consisted of 5 copper washers, 8 mms. apart, each with a quadrant missing, arranged so that each was rotated at an angle of 90° to the next. It is unfortunate that both these changes were introduced simultaneously but it was felt that a more complete study of oscillations would have to be made befre a method of elimination could be said to be applicable to any system and the problem of siphoning helium into the apparatus was a more immediate one.

In these experiments the end of the siphon was sealed after transfer with a small length of thin rubber tubing closed at the lower end and oscillations of fairly small amplitude could be detected by pressing the rubber between forefinger and thumb of one hand. It was found that in all the later experiments, oscillations on this column of about 5 - 10 c/s were present during the early part of condensation and if an attempt were made to accelerate the process by increasing the overpressure used, the oscillations would increase in amplitude and condensation would cease. These oscillations usually decayed after about fifteen minutes and permitted further condensation.

The thermal conductivity cells

The original purpose of the experiments was to measure the thermal conductivity of liquid ^{3}He. In a later section, we shall describe the several suggestions that have been made of the possible magnitude and temperature variation of the thermal conductivity. No evidence of superfluidity, which in liquid ^{4}He is associated with a very large heat conductivity had been observed in liquid ^{3}He at temperatures down to 1.04°K, and it was hoped to extend the temperature range by making thermal conductivity measurements down to about 0.3°K.

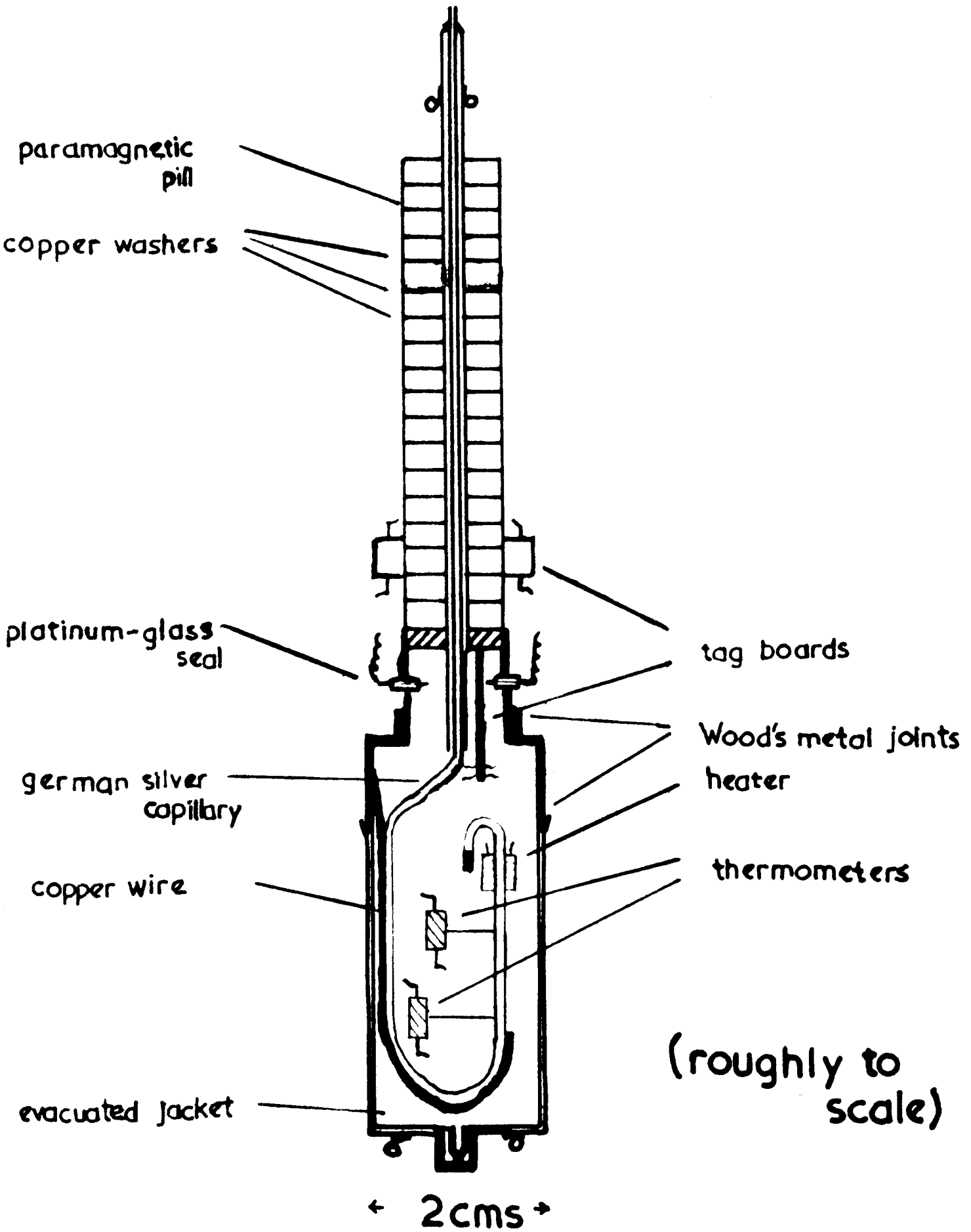
Preliminary cells

An apparatus was constructed similar to that used by H.A. Fairbank and Wilks (1955) to measure the thermal conductivity of liquid ^{4}He below 1°K. In their arrangement, the liquid was contained in a vertical german-silver capillary to which were soldered a heater and two carbon resistance thermometers. The heater was placed vertically above the two thermometers to avoid convection currents. They found that for ^{4}He the conduction taking place down the walls of the capillary was a negligible fraction of the total heat flow. German-silver was chosen for the capillary because it has a very small conductivity at low temperatures ($\sim 5.10^{-3}$ watt units) and also may be drawn into capillaries of very small wall thickness 0.01 cm.

The amount of ^{3}He available for the experiments was just under 20 ccs. of gas at N.T.P. or about .03 ccs. of liquid at 1.0°K but the ^{3}He is never completely in the liquid phase. The capillary leading to the conductivity cell has a temperature which varies from

THERMAL CONDUCTIVITY CELL 1α

FIG. 3.2



about 300°K to below 3.2°K and a rough calculation can be made of the volume of gas at N.T.P. necessary to fill it to any pressure. It was assumed at the start of these experiments that measurements would be carried out at the vapour pressure of the helium in the cell, but in fact a pressure greater than this was needed to prevent the formation of a gas phase at the hottest part of the cell.

German-silver capillaries of external diameter 0.5 and 1.0 mm. are readily available and have internal diameters of 0.3 mm. and 0.8 mm. and internal cross-sections of $7 \cdot 1 . 10^{-4}$ sq. cms. and 5.10^{-3} sq. cms. respectively. If we assume all the ^3He to be in the liquid state, at 1.0°K, .03 ccs. of liquid would fill 40 cms. of the smaller capillary but only 6 cm. of the larger, and as there is in fact a certain amount in the gas phase these figures are appreciably reduced. We estimated that the length of capillary in a suitable arrangement would be greater than 6 cms. so that 1 mm. capillary could not be used. The cell was therefore constructed of 0.5 mm. capillary.

Cell 1a

The arrangement is shown schematically in Figure 3.2. The german-silver capillary passes through the centre of a copper tube which opens out into a copper vessel of diameter 1.9 cm. which is the maximum permitted by the dimensions of the cryostat. A heater and the thermal leads of the carbon resistance thermometers are soft-soldered to the capillary in the positions shown, it being assumed that the expansion coefficient of liquid ^3He remains positive below 1°K. The heater is placed 1 cm. above the thermometer to ensure that the flow

lines are coaxial. A copper wire of diameter 1 mm. which is attached to the copper vessel is sweated to the capillary as far as the lower thermometab to provide good thermal contact between the conductivity cell and the copper vessel. Four platinum-glass seals of 2 mm. diameter are soft-soldered into the side of the vessel and the leads passing through them are connected to the lower copper wires of the tag-board A, which screws into the base of the vessel. This tag-board was constructed by winding four cotton-covered and shellaced copper wires of 22 SWG around a copper rod. The wires were then covered in bakelite varnish and baked to ensure good thermal contact with the rod.

The heater is of cotton-covered 47 SWG manganin wire wound on a cylindrical copper former, has a resistance of about 1,000 ohms, and is earthed at one end to the former. The connections between the tag-board and the heater and thermometers are alternative paths for heat flow and to reduce the heat loss to negligible proportions they are made of 46 SWG manganin wire.

Around the copper tube through which the capillary passes is pressed a paramagnetic pill of manganous ammonium sulphate interspersed with discs of copper foil which provide thermal equilibrium. This pill is coated with bakelite varnish and condenser paper to retard dehydration.

The lower end of the vessel is completely enclosed by a copper jacket which prevents the adsorption of residual exchange gas on to the capillary. The jacket,which is Wood's metalled to the vessel, is fitted with a copper tube which can be connected to a high vacuum line. After ensuring that no leaks existed in the jacket or

through the platinum seals, the copper tube was pinched in a vice, sawn off as close to the evacuated jacket as possible and fitted with a cap which was Wood's metalled into position. As an example of the importance of the vacuum round the capillary, it should be mentioned that in one experiment in which a seal was later found to have been leaking, conduction down the german-silver was completely shorted by convection processes associated with helium film on the capillary.

The outer ends of the platinum-glass seals are connected to the tag-board B, which is a short perspex cylinder into which are fixed four copper wires. The upper ends of the tag-board are connected to cotton-covered 42SWG manganin wires which lead to the further tag-board attached to the cryostat.

The pill system is supported by nylon threads in a brass cage which is screwed into the top of the cryostat.

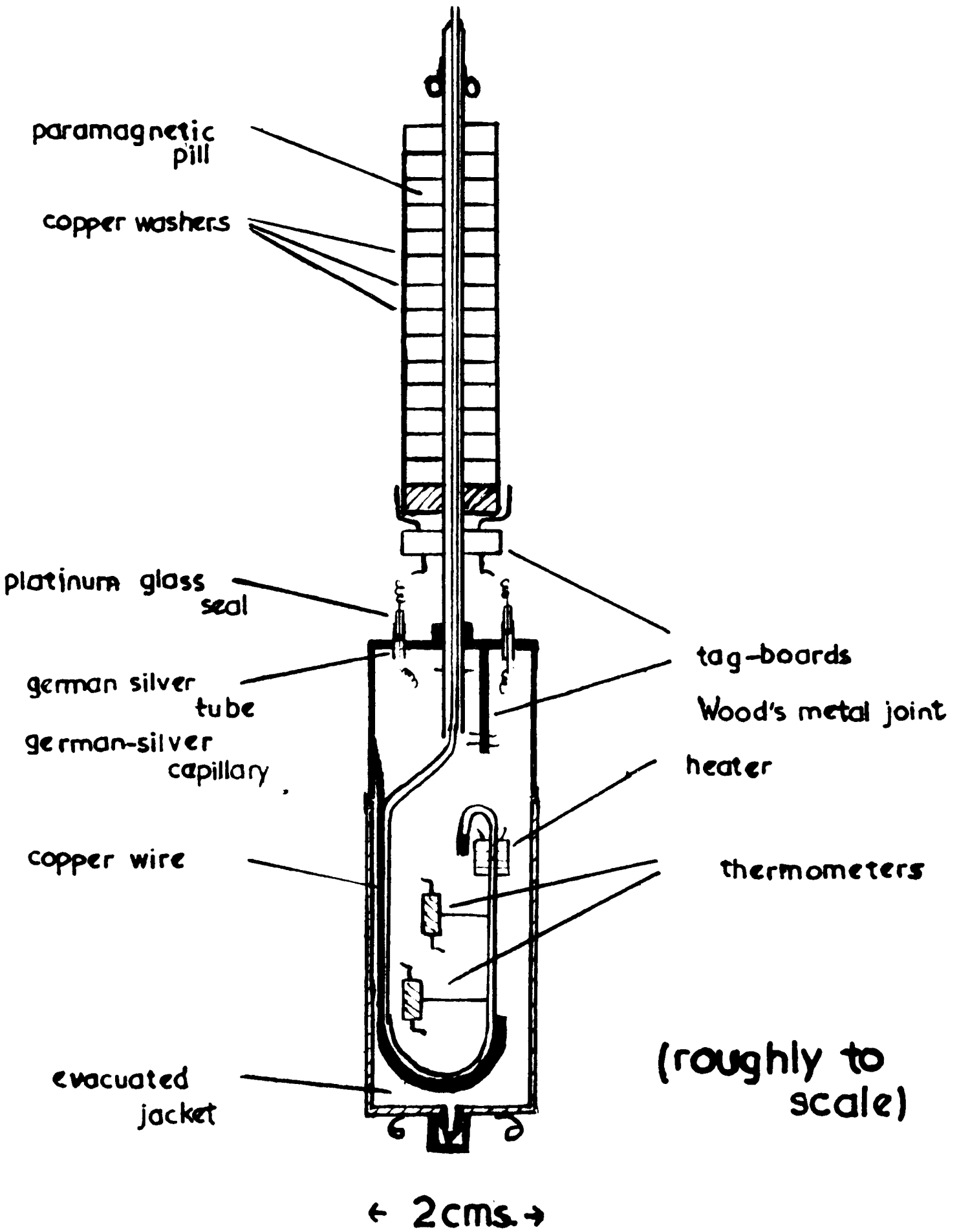
The cell was originally constructed with a jacket which was Wood's metalled on to the narrower diameter CD but this was found to be too small and so the shoulder CDBA was Wood's metalled on to CD and the jacket attached to XY. This arrangement was very unsatisfactory. Considerable difficulties were provided by the platinum-glass seals and in any arrangement of this nature, a replacement involves the removal of the jacket which must then be fitted with another copper tube. In this particular arrangement there are further inconveniences; in order to remove a seal from the side of the copper vessel, an area around the seal must be heated to the melting temperature of soft-solder and the seals are so close to the pill and the joint CD that both suffer damage

even when protected by asbestos string soaked in water. It was not possible to soft-solder CD into position as this would introduce a closed superconducting ring near the pill at low temperatures.

Before describing the second cell in which most of these difficulties were overcome, we shall describe the problems associated with platinum-glass seals. These seals consist of a cylinder of platinum inside which is fused a small globule of glass through which passes a platinum wire. The seals are non-magnetic and so are suitable for use near to paramagnetic pills; they are constructed in the glass-blowing workshops of the laboratory and have been used with success at the lowest temperatures (see, for example, Fairbank and Wilks, 1955). A few of the seals were found to leak at room temperature after they had been soft-soldered into position. This was surprising as they had been annealed in the workshop and great care was taken in soft-soldering them into position; two others leaked only after a period of time of up to two or three weeks. Others leaked at low temperatures although they were perfectly satisfactory at room temperature. The magnitude of these leaks was very small and it was only just possible to detect the leaks using a fine jet of hydrogen in conjunctinn with a 5 μ-amp meter attached to a Pirani. An arrangement was constructed in which seals could be soldered and tested and the whole cooled to liquid oxygen or hydrogen temperatures. Seals were constructed with platinum taken from a different batch in case the trouble was due to a flaw on the inside of the tube. It was found that if a seal was Wood's metalled into position most of these difficulties disappeared

THERMAL CONDUCTIVITY CELL Ib

FIG. 3.3



and in the second cell, this procedure was adopted.

Cell 1b

The arrangement is shown schematically in Figure 3.3 . The capillary arrangement is the same as in the first cell and the two differ mainly in that the seals are Wood's metalled into german-silver sleeves soft-soldered into the copper. The seals are moved to the base of the vessel where they have greater protection and in order that the diameter of the vessel can be larger. The pill is in thermal contact with the vessel through the copper tube and a new pill may be constructed and reconnected to the system without removing the seals or capillary system. The pill itself was constructed as in the first arrangement.

For the experimentss above 1°K it was more convenient to connect the copper tube thermally to the cryostat with a copper heat link rather than by using exchange gas. This avoided the necessity of closing off the jacket. Hydrogen exchange gas was used to speed the cooling of the arrangement down to hydrogen temperatures.

This apparatus was used to determine an upper limit of the thermal conductivity down to 1.2°K. To produce a pressure greater than the vapour pressure of the liquid in the cell, the capillary is filled with liquid to a point where the temperature is such that the vapour pressure is equal to the pressure reuired. A pressure just about the critical pressure (1.15 ats) was used so that the capillary was filled to the point where the temperature was equal to the critical temperature (3.34°K). It is seen in Figure 3.1 that this point

on the capillary occurs somewhere between the cryostat (temperature between 1.2° and 3.2°K) and the vessel X which is normally at 4.2°K. To reduce the amount of the liquid required, the critical point on the capillary was brought lower than usual by sweating the upper part of the capillary between the cryostat and X to a copper tube connected to X. In this way a saving of perhaps 3 cms. length of liquid was achieved but even then there was not sufficient ^{3}He to fill the capillary to the required height. We decided therefore to concentrate on the region above 1°K and so removed the pill and most of the copper tube and raised the copper vessel by about 8 cms. (a saving of $5.6.10^{-3}$ ccs. of liquid or 3.4 ccs. of gas at N.T.P.) It was then possible to fill the capillary to the required height.

The conductivity of liquid ^{3}He was found to be so small that at 1.2°K the additional conduction down the helium was less than 4% (the estimated limit of detection) of the conduction down the capillary. It was not considered profitable to continue the investigations below 1°K with this apparatus particularly as more ^{3}He had become available.

Carbon thermometers

Carbon resistors were chosen as thermometers because of their sensitivity and because empirical formulae can usually be found to describe their behaviour over limited temperature ranges. The calibrations of the thermometers used in this work were found to vary only slightly between runs. The thermometers were required to have their maximum useful sensitivity below 1°K and the resistors chosen

were I.R.C. nominally 220 ohms. A roughly matched pair was selected by measuring the resistance of 3 thermometers at temperatures down to 4.2°K. The results are shown below together with the value at 1.4°K obtained later:

		300°K	90°K	20°K	4.2°K	(1.4°K)
	A	250	280	319	380	
Resistances (Ω)	B	223	252	290	350	435
	C	218	243	282	340	

and thermometers B and C were used as a pair. The value of $\frac{dR}{dT}$ at 1.5°K was about 40 ohms/°K and the more significant parameter $\frac{1}{R}\frac{dR}{dT}$ where R now refers to the total resistance including the leads, had a value of $5 \cdot 7.10^{-2}$ degrees^{-1}.

The thermometers are constructed by winding cotton-covered shellaced 22 SWG copper wire round the resistor. The two ends are twisted together to provide a thermal lead and the thermometer is painted with bakelite varnish and then baked carefully to provide good thermal contact. In many arrangements, it is possible to use the electrical leads as thermal connections but in these experiments an A.C. bridge was used to measure the resistances of the thermometers, and to avoid an earth loop it was important that the electrical leads should not be earthed.

Final Arrangement

The amount of ^{3}He available was now 120 ccs. of gas at N.T.P. or about .19 ccs. of liquid at 1°K.

Factors influencing the design

The cell was made of volume .12 ccs. calculated so that there would again be sufficient ^{3}He to fill the capillary with liquid to the critical point and the rest of the capillary with gas at 1.2 atmospheres pressures. The upper limit obtained with the preliminary apparatus showed that the conductivity of liquid ^{3}He was of the order of or less than that for ^{4}He liquid above the lambda point. We decided to build an apparatus which could be used to make measurements of conductivity of gaseous ^{4}He, gaseous ^{3}He and liquid ^{4}HeI and liquid ^{3}He.

An apparatus similar to the preliminary apparatus but of volume .12 ccs. and length 1 cm. would have a diameter of 4 mms. The problem of coaxial flow could be eliminated by winding the heater on the conducting plug closing the cell. If such an arrangement were used to measure the conductivity of ^{4}HeI, nearly 30% of the heat flow would flow through the helium. This fraction could be increased slightly be reducing the length of the cell but only at the expense of the accuracy to which the form factor would be known. Our preliminary experiments had however shown that a conductivity of the order of that possessed by liquid HeI was the upper limit to the conductivity of liquid ^{3}He and if the actual value were say 4 or 5 times smaller than that of ^{4}HeI, the contribution to heat flow through the ^{3}He in an apparatus of the type described above would be only 6 or 7%. It was decided therefore to build a cell in which conductivities ten times smaller than that of ^{4}HeI could be measured.

A brief description of some of the methods which have been used to measure conductivities of liquids and gases will be given.

It was mentioned earlier that if a knowledge of the form factor is required to say 4%, in an apparatus of the Fairbank and Wilks type, the thermometers must be spaced by about 1 cm. - the uncertainty in position of each thermometer is rarely less than .2 mm. In the Parallel Plate or Film method of measuring conductivities, in which the heat flows through a thin film of the fluid enclosed between two flat circular plates of a good conductor (Lees, 1898) the thermometers are mounted on the plates. The distance between the plates can usually be measured with some accuracy so that the errors in the form factor are largely eliminated. The thickness of the film aan be reduced very considerably so that, if the apparatus is to be of a fixed volume, the diameter can be increased.

In this simple arrangement the conduction down the surrounding walls can be reduced very considerably. It can be eliminated completely by using a guard-ring method (Todd, 1909; Hercus and Laby, 1918). There are the additional advantages in these thin film methods in that convection is strongly inhibited and that equilibrium times are short. With thick films or columns of liquid, both of these problems are of importance. In the measurements of Bowers (1952) on the conductivity of liquid ^{4}HeI in which heat flowed down a wide column of liquid, there was a large scatter of points attributed to convection currents, and the equilibrium times were of the order of two hours.

The principal disadvantage in the parallel plate methods described above is that any temperature jump at the walls is included

in the measurements. In gases, this jump or sudden departure from the constant temperature gradient existing in the bulk of the gas, is attributed to the restriction of movement of molecules by the boundary . The molecules striking the wall are partly specularly reflected and partly reflected after inelastic collisions with the molecules of the wall. In simple kinetic theory, a treatment of this problem exists (Kennard 1938). On the average, molecules striking the wall suffered their last molecular collision a distance λ away and possessed a velocity characteristic of that distance from the wall. These molecules suffer no further molecular collisions until they have been reflected from the wall and travelled a further distance λ. In this region λ from the wall then, Knudsen flow exists and the transport of energy depends on the efficiency of the wall-collisions which is described by the accommodation coefficient f. The temperature 'jump' across the two walls due to this process is calculated as

$$\Delta T_w = \lambda \cdot \frac{2(2-f)}{f} \cdot \frac{1}{K} \cdot \dot{Q}$$

and the temperature difference across the rest of the gas is

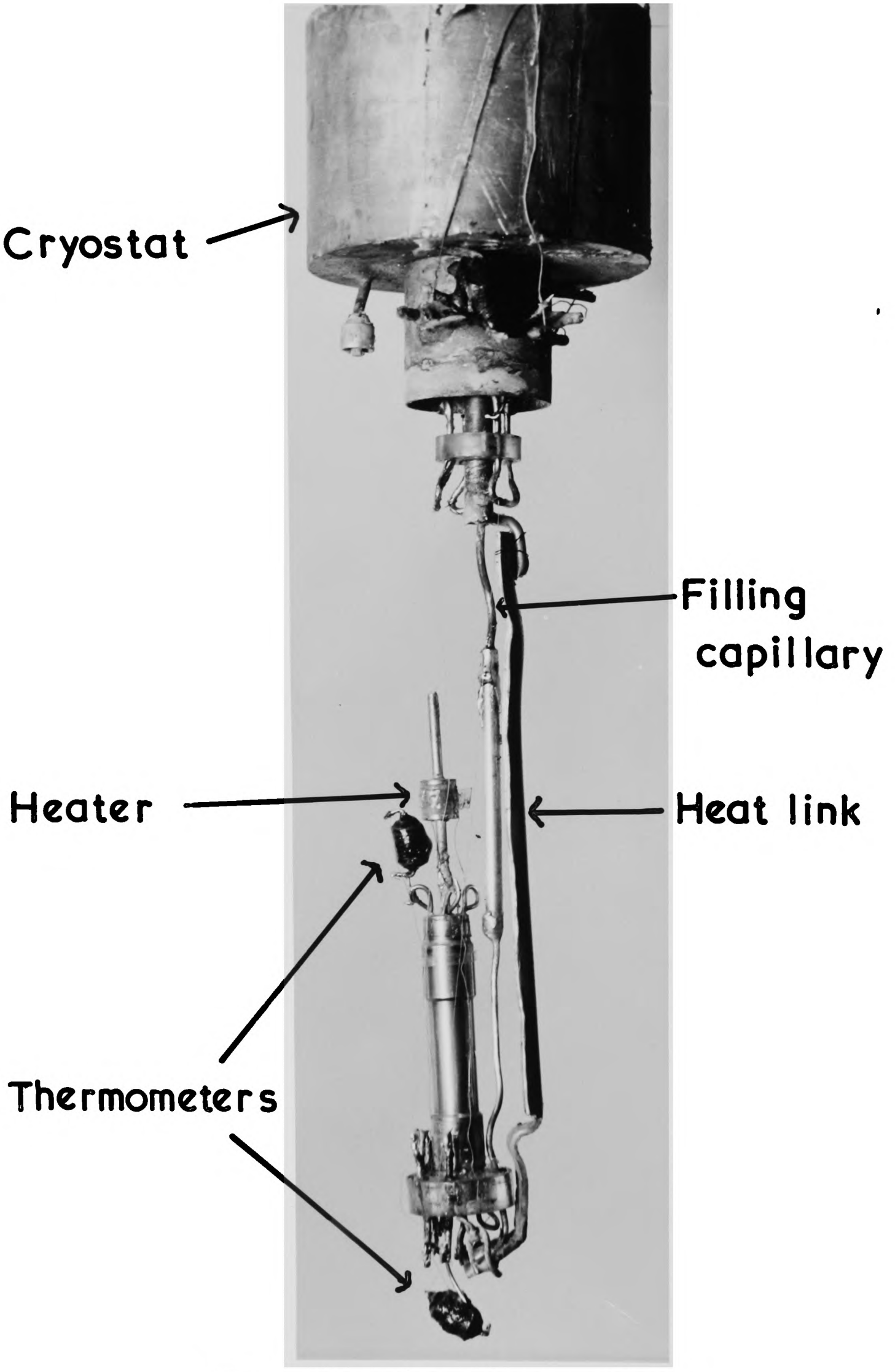
$$\Delta T = (d-2\lambda) \cdot \frac{1}{K} \cdot \dot{Q}$$

$\frac{2(2-f)}{f}$ is of the order unity so that the 'jump' is negligible if $\lambda \ll d$. A more sophisticated treatment is due to Welander (1954) but leads essentially to the same condition. These calculations are in agreement with experiment although it is not usually possible to measure the accommodation coefficient directly.

This temperature jump is eliminated in a method used by Ubbink and de Haas (1943) and they also claim to eliminate the effects of radiation and conduction to the walls. Their apparatus comprises two parallel plates and the distance between the plates can be varied. With this apparatus the conductivity of gaseous ^{4}He and the hydrogen isotopes have been measured. It should be noted that the temperature jump is proportional to λ and therefore inversely to thepressure. In the later measurements of Ubbink (1947) a fixed plate arrangement was used in which the thickness of the helium film was .1 mm.

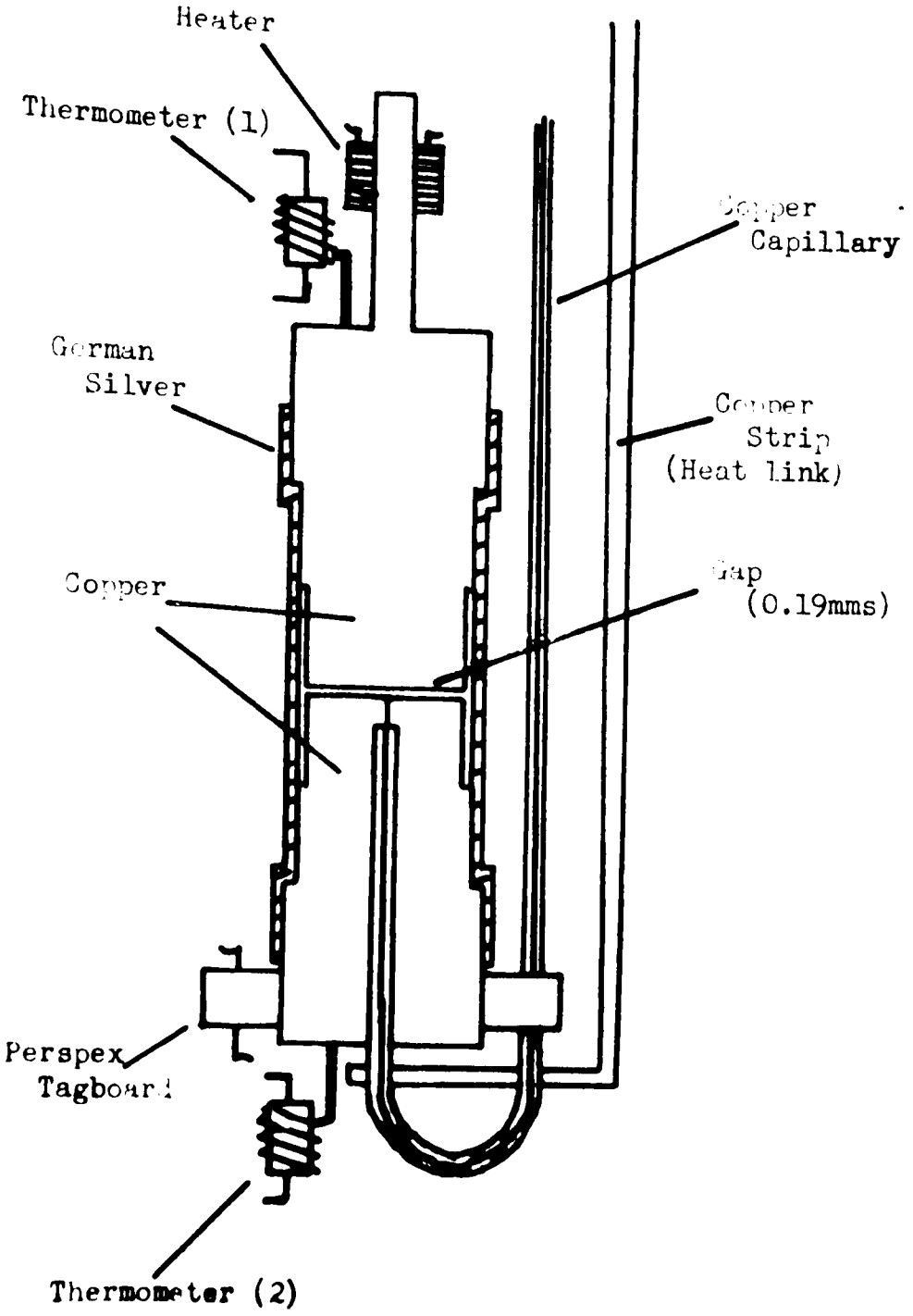
The variation of apparent conductivity $\frac{\dot{Q}}{A\left(\frac{T-T_0}{d}\right)}$ in these experiments is illuminating : a sharp curving rise occurs at low pressures ($p < 5$ cms) which disappears and is replaced by a steady linear increase with a slope of 4% per atmosphere. The first rise is interpreted as being due to temperature jump and the second as a genuine increase in conductivity with pressure; a result anticipated in the theory of dense gases (see, for example, Chapman and Cowling 1952).

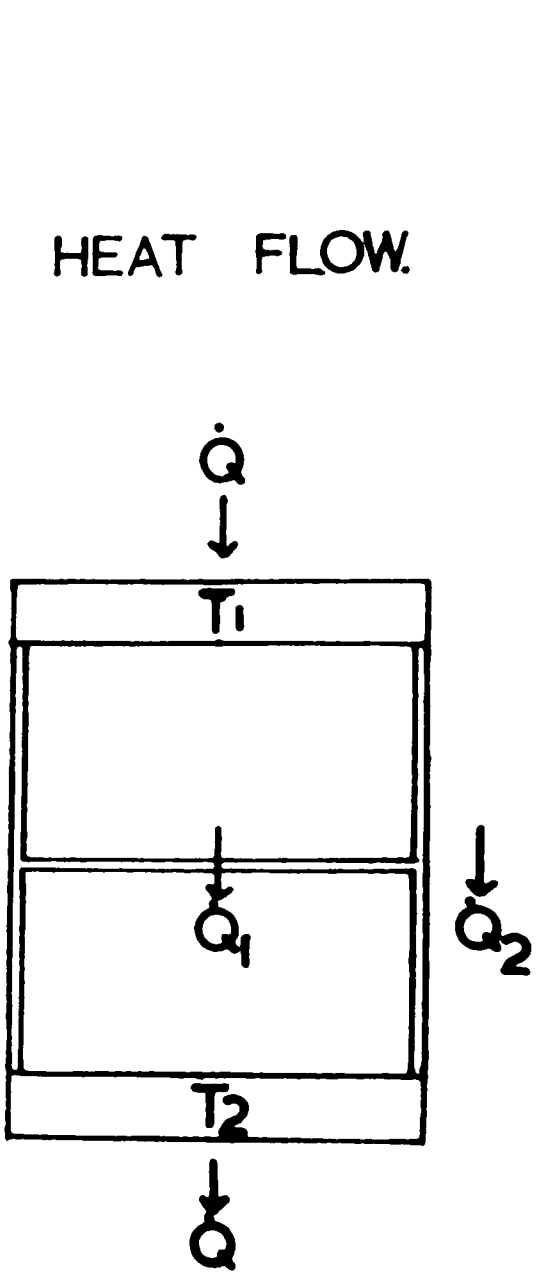
It is seen then that there is very strong theoretical and experimental evidence that for 'normal' pressures the effect of temperature jump in gaseous helium is negligible. There was no evidence at the time of the construction of the apparatus of a temperature jump in either liquid ^{4}HeI or liquid ^{3}He. The conductivity of liquid ^{4}He is said to be gas-like (for example, London, 1954) and as the conductivity of a gas increases with pressure, ^{4}HeI might be thought of as a compressed gas or a gas in which λ is very small, perhaps of the order



The Conductivity Cell

THERMAL CONDUCTIVITY APPARATUS 2





$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2$$

FIG. 3.4

1cm.

of the interatomic distance. If this were the case the temperature jump would be negligible and this assumption was made for both liquid ^{3}He and liquid ^{4}HeI.

This assumption was almost a necessity. A method similar to that used by Ubbink and de Haas would not be suitable for liquid ^{3}He as the moving parts in the apparatus would not be compatible with the precautionary measures necessary to avoid loss of the ^{3}He, although it has been used for liquid ^{4}HeI (Grenier 1951). A guard-ring is excluded because of volume considerations. The simple Lees method would be inadequate to measure very small conductivities since the diameter of the cell is restricted by the diameter of the cryostat.

The cell

The apparatus is shown schematically in figure 3.4 The helium fills a container in the shape of an H rotated vertically about an axis through the centre of the horizontal limb. The two copper blocks are surrounded by a german-silver cylinder of thickness 0.1 mm. to leave a disc of diameter 8 mm. and thickness 0.19 mm. and a cylinder of length 1.0 cm. and thickness 0.40 mm. A carbon resistance thermometer is mounted on each block and a heater on the upper block. The lower block is linked thermally with the cryostat by a copper strip. Helium is introduced into the cell by a copper capillary of internal diameter 0.3 mm.

When the apparatus is evacuated the flow taking place from the upper to the lower block is entirely by conduction down the german-silver tube , $\dot{Q}_G \sim K_G \cdot \Delta T \cdot \frac{\pi \times \cdot 8 \times \cdot 01}{1}$ watt units (K_g is the thermal conductivity of the german-silver). When the apparatus is filled with helium there is additional heat flow across the vertical gap

$\dot{Q}_H \sim K_H \cdot \Delta T \cdot \frac{\pi \cdot .4^2}{.02}$ watt units. (K_H is the thermal conductivity of the helium). There is also a radial component of flow across the cylinder of helium to the german-silver. The magnitude of this is calculated in Appendix 2 and depends on the ratios of the conductivities of german-silver and helium. Even without this component the increase in heat flow when the system is filled with helium is $\frac{8\pi K_H}{8\pi K_G \cdot 10^{-3}}$ or $\frac{K_H}{K_G} \cdot 10^3$ and the increase is 50% for a helium conductivity of $K_G \cdot 10^{-3}$ or 5.10^{-6} watt units which is approximately 40 times smaller than the conductivity of liquid ^{4}HeI.

The thickness of the vertical gap and the various other geometrical distances were chosen with the restriction that the volume should be not more than 0.12 ccs. Various values were tried and the ratios of heat flow computed. Other considerations were, that the form of the function determining the radial component of the heat flow should be simple to avoid solving transcandental equations, and that all distances should be sufficiently large to be measured with accuracy.

The optimum value of the gap was found to be 0.2 mm. and this was achieved by making the copper blocks and german-silver tube with shoulders at measured distances. A rough check could be made by measuring the distances between the ends of the copper blocks with a micrometer but the magnitude of the gap appears as the difference between two large distances and an alternative method was desirable. The apparatus was radiographed by placing it on a strip of film beneath the exit port of an X-ray machine. The greatest contrast was obtained with V = 130 KV, i = 1·5 m.a., an exposure time of 0.5 secs. and an object distance of 21 cms. A simple pendulum was employed for alignment

FIG. 3.5



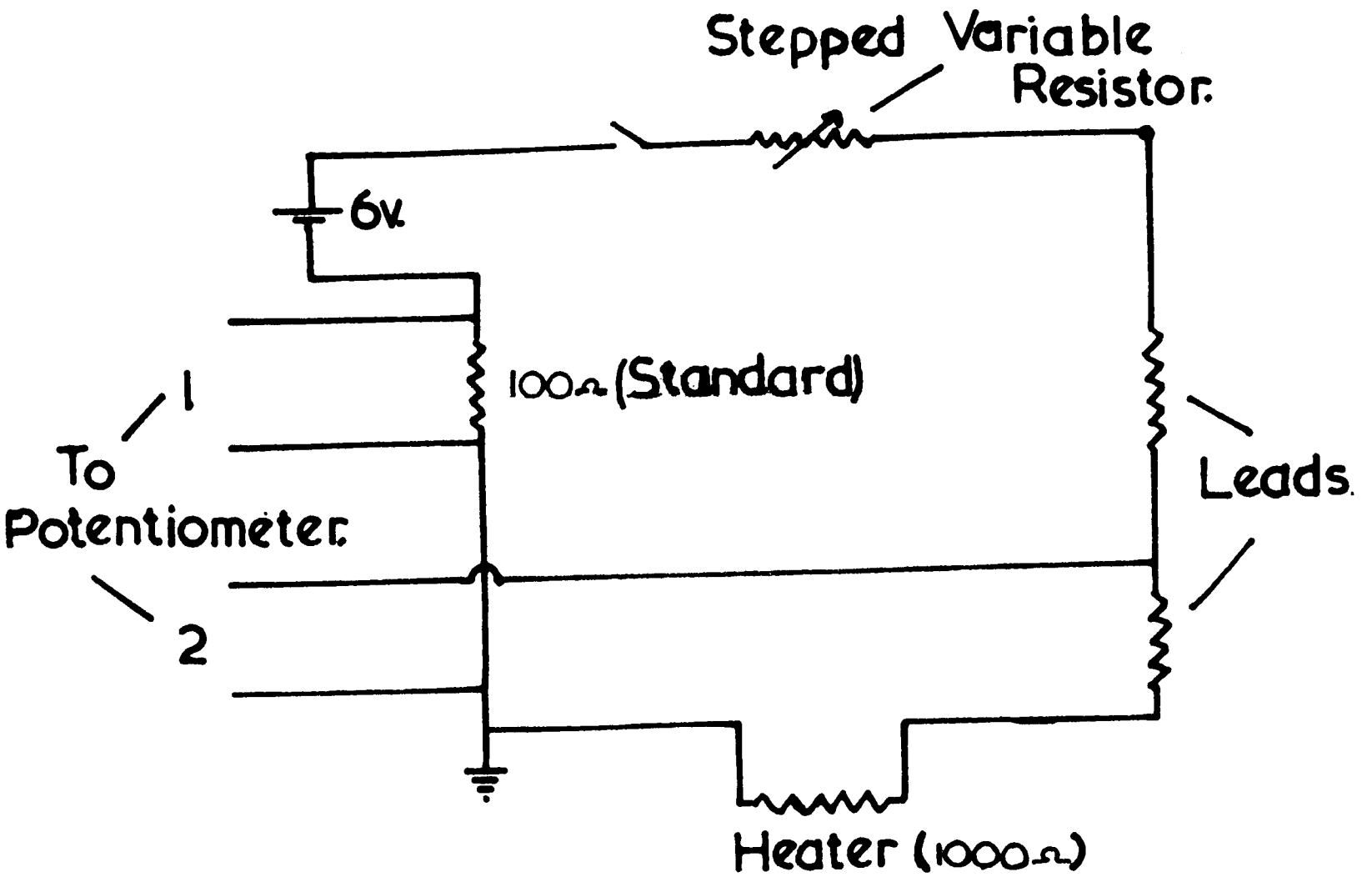
Radiograph of the Conductivity Cell
(X3)

purposes. The film was examined with a travelling microscope and the distance of the gap estimated as 0.19 mm. A reproduction of the radiograph is shown in Figure 3.5; the contrast has been heightened for convenience of display. Experiments had been carried out prior to X-raying the cell, with two copper plugs separated by a circular spacer made from .008" polythene sheet. The thickness of this spacer could be measured accurately with a micrometer screw gauge, and the experiment showed that the distance measured on the radiograph was equal to this thickness. The accuracy to which the distance on the radiograph could be measured was only about 5% as there was a diffuse edge to the shadows caused by scattering in the film.

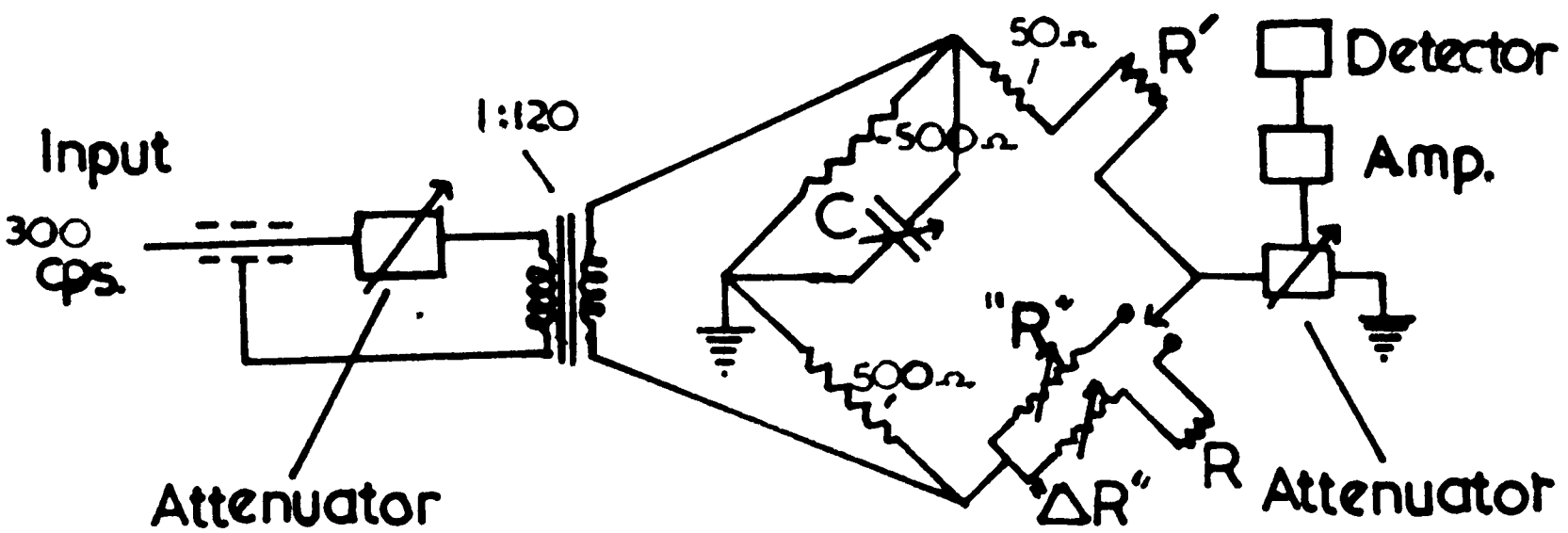
The thickness was measured at the end of the experiments by milling two slits in the german-silver tube and observing the gap with a travelling microscope. The observatims were also made with the cell immersed in liquid oxygen. The gap contracted by about 5% - the german-silver has a larger expansion coefficient than the copper. The further contraction of 0.5% to helium temperatures was estimated by assuming the two metals obey Grüneisen's law to some degree of approximation. The form of the specific heat curves is known roughly so that the extra contraction can be computed.

The measurement of the other distances was carried out by sawing the german-silver away from the copper and then using a travelling microscope. The thickness of the german-silver, which was distorted after removal from the cell, was calculated from its volume and area. The volume was measured by weighing in air and various liquids but, due to the magnitude of the surface tension effects, the error may be appreciable.

HEATER CIRCUIT FIG. 3.6



BRIDGE CIRCUIT



The results of the measurements are given below.

$d = 20{\cdot}5.10^{-4}$ cms. (vertical gap between the blocks)

$h = 1{\cdot}004$ cms. (length of cell)

$a = 1{\cdot}21.10^{-2}$ cms. (thickness of german-silver)

$l = 3{\cdot}77.10^{-2}$ cms. (width of annular gap)

$r = 4{\cdot}04.10^{-1}$ cms. (mean radius of german-silver)

$r = 3{\cdot}79.10^{-1}$ cms. (mean radius of gap)

$r_c = 3{\cdot}60.10^{-1}$ cms. (radius of copper block)

Carbon thermometers

These were constructed as in the preliminary experiments, the difference being that 33 ohm 1/4 watt LAB resistors were used which have their maximum convenient sensitivity in the temperature range 4.2°K - 1°K. The resistance of these thermometers varies from about 160 ohms at 4.2°K to 1,400 at 1.2°K and the value of $\frac{dR}{dT}$ at 1.5°K is about 1000 and $\frac{1}{R} \cdot \frac{dR}{dT}$, 0.9.

The thermometers R and R are arranged in the bridge circuit shown in Figure 3.6. With the switch S in the upper position the resistance of R is measured by balancing the bridge with the decade box "R" (0 - 10,000 Ω); and when the switch is thrown over the difference in resistance ($\Delta R \mp R - R'$) is measured by obtaining the new balance position with the decade box "ΔR" (0 - 110 Ω). This method has the advantage that "ΔR" is a sensitive indication of the temperature difference while at the same time being rather insensitive to the absolute values of temperatures. Included in the measurements of the

resistance are the lead resistances, but these are matched and almost temperature independent so that a negligible error is introduced.

A 300 c/s oscillator is connected across the bridge and the out of balance voltage amplified and fed into a phase-sensitive detector.

Oscillator and bridge

The oscillator has an output of 2.4 volts R.M.S. at 300 c/s and this signal passes through a stepped attenuator to the input transformer (1:120) of the bridge. The bridge is essentially a Wheatstone's bridge with a variable condenser connected across one limb to correct for the permanent unbalance due to one side of the bridge being more 'earthy' than the other, and the effect of any stray capacities or inductive parts of the impedances. The fixed limbs of the bridge are matched 500 Ω wirewound resistors.

Amplifier and detector

The output from the bridge passes through another attenuator to the input resistance of the amplifying system. This is a Dynatron 1,008 pulse amplifier with a built-in attenuator and is preceded by a low noise head amplifier which was modified by removing the feedback loop and by replacing the 1 MΩ input resistor by a 1,000 Ω resistor to provide a better match with the bridge. The arrangement is capable of a maximum voltage gain of $1{\cdot}5.10^5$.

The phase-sensitive detector differs only slightly from that used and described by Templeton (1953). A brief outline of the mode of operation will be given; the circuit and description of the

method of adjustment is given in full in the reference quoted above. The amplified output signal at 300 c/s is passed into the first stage of the detector which is a tuned cascade amplifier. This removes any unbalanced harmonics and 50 c/s main pick-up which would otherwise saturate the detector. It also reduced the spectrum of noise generated in the resistances and the first valve to a narrow bandwidth.

The output signal from the bridge is in phase or 180° out of phase with the oscillator according to whether the balancing resistor is too large or too small. A reference signal in phase with the oscillator is incorporated in the detector in such a way that the detector output is not only proportional to the magnitude of the out of balance voltage from the bridge but is positive or negative according to its phase. In the arrangement used by Templeton to measure noise, the output was either observed on a 5 m. amp. meter or recorded on magnetic tape. This output was from a bridge fed with an A.C. signal and balanced very carefully so that the average current passing through the meter was zero but oscillations about the zero position were observed due to unbalanced current noise in the bridge components. In the experiments of Fairbank and Wilks, when a similar system was used, the output signal obtained for an unbalance in resistance of say 1 part in 10^4 was sufficiently large compared with this noise that the balance position could be found with accuracy. The output signal is proportional to the input signal to the bridge and so to the square root of the power dissipated in each thermometer. This power must be kept negligible compared with the power supplied to the

heater and in the measurements of Fairbank and Wilks this was very small so necessitating thermometric powers of less than 10^{-8} watts. In our preliminary experiments even smaller heat inputs had to be used even above 1°K and the power dissipated was limited to 10^{-9} watts.

The amplitude of the noise was found to be about 2.10^{-2} μ volts R.M.S. and was independent of the current through the bridge. Any noise generated in the input to the bridge would be balanced out and it could be also shown that the noise was not generated solely in the amplifier. It is thought that the noise occurred partly in the bridge so that an improvement due to this could not be achieved by using an output transformer.

In our arrangement the smallest signal which can be detected in the absence of noise is $2{\cdot}7.10^{-4}$ volts R.M.S. which corresponds to an input voltage to the amplifier of 4.10^{-3} μ volts. This could have been further reduced by using a battery pre-amplifier, but any increase in amplification which resulted in an amplification of the 50 c/s pick-up would cause saturation of the detector. In the presence of noise, the smallest detectable signal in this arrangement is about 3 or 4.10^{-2} μ volts. This corresponds to a balance to about 1 part in 10^4 and to improve slightly on this we added a further stage to the detector which could be used to increase the time constant of the output stage to 5, 10 or 25 seconds. It consisted of a balanced D.C. amplifier arrangement which included a series of circuits of large

RC. By this means the noise was effectively removed although small long-term movements could be detected. These integrating circuits were only switched in very near balance and the output from the bridge was disconnected very frequently to ensure that the zero position had not drifted - a common fault in D.C. amplifiers. With this arrangement the bridge could be balanced to $2/10^5$: the last figure in the resistance was in fact inferred from the meter reading.

The A.C. bridge compares favourably with D.C. methods for measuring resistances with small power dissipation. For sensitivities of this order using D.C., a galvanometer amplifier is generally used but these have the disadvantage that drift can be significant and that all thermoelectric E.M.F.s have to be carefully balanced and the temperature of all external connections maintained constant.

In the later experiments the heats supplied to the heater were very much larger than in the first experiments and the sensitivity was improved by increasing the voltage of the input to the bridge.

Heater circuit

Heat is supplied to the specimen by means of the circuit shown in Figure 3.6. The stepped variable resistance provides a large range of currents which are measured with a Tinsley potentiometer and a standard 100 ohm resistance. The potential over the heater is measured in the same way. To save introducing a further platinum-glass seal, one lead from the heater is earthed inside the apparatus and one side of the battery to the outside of the apparatus. This introduces a thermoelectric E.M.F. which, in comparison with the heater voltage, is always very small and often negligible.

2. The Experimental Procedure

The procedure discussed in this section was that employed in the experiments using the final apparatus but in general it also applied to the preliminary apparatus.

The carbon thermometers were calibrated against the helium vapour pressure thermometers throughout the range 4.2°K - 1.2°K. Below 1.4°, the vapour pressure thermometer becomes unreliable because of the long equilibrium time associated with it together with the onset of the region where the mean free path is of the same order of magnitude as the relevant apparatus dimensions. A position of thermal equilibrium was determined using the oil manometer differentially for temperatures above 2.2°K and a carbon thermometer mounted on the cryostat for temperatures below this. It was found that for experiments in which the cell was evacuated or filled with gas, the relaxation time before the carbon thermometer mounted on the cell came into equilibrium with the cryostat was less than a minute and that this was a very much more sensitive indication of equilibrium than the resistance of the thermometer on the cryostat measured with a simple bridge.

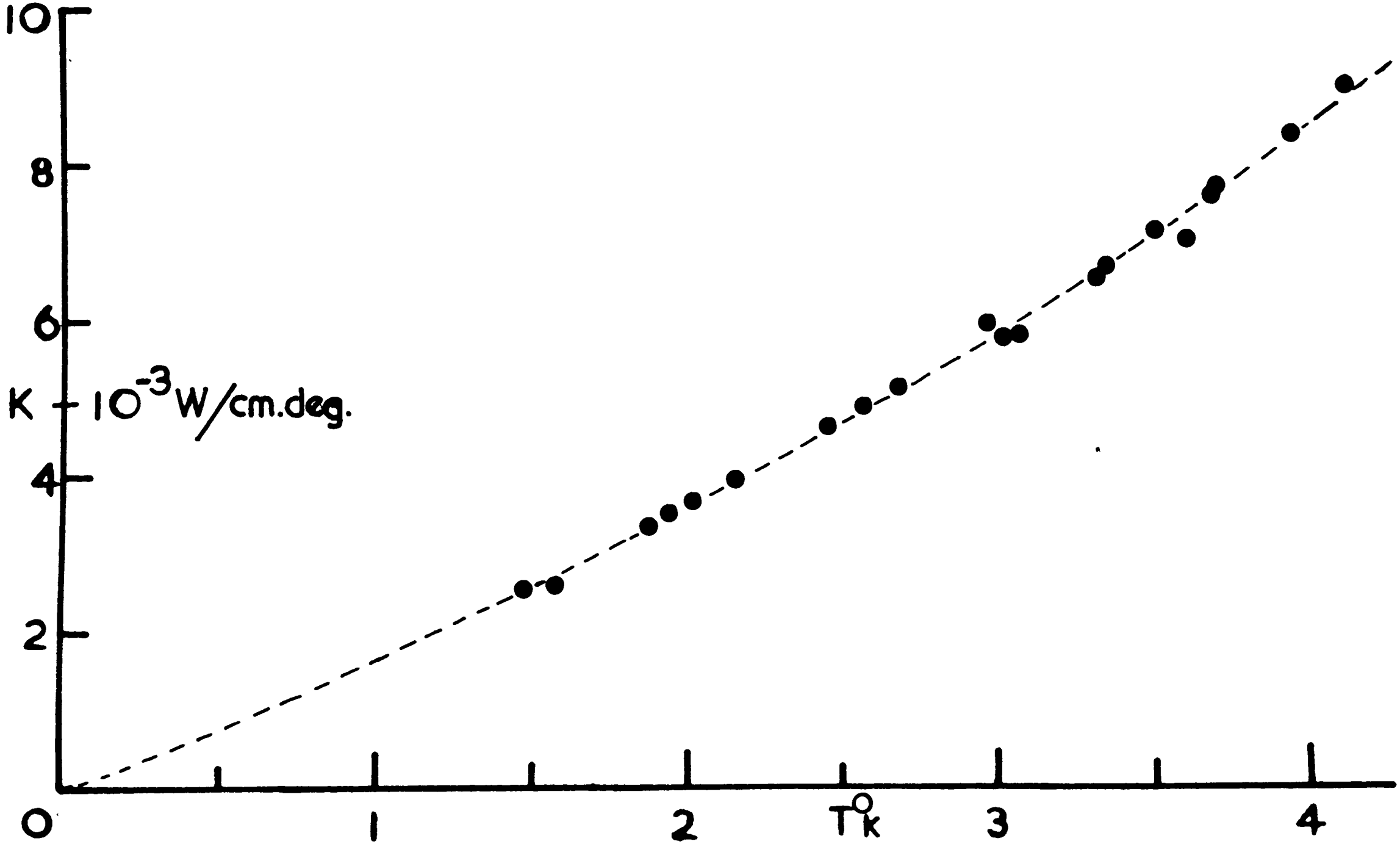
Calibration points were taken at intervals of approximately 0.4°K and at each position of equilibrium several thermal conductivity measurements were made. When heat was supplied to the cell, a temperature difference was produced between the cell and the cryostat as well as between the copper blocks in the cell. Consequently each value of heat input corresponded to a different cell temperature as well as a different temperature gradient across the cell. In this way

values of conductivity over a range of temperatures could be produced at each calibration point. A useful check that the apparatus was behaving correctly was obtained by making measurements on the conduction across the empty cell before filling it with liquid or gaseous helium. The conduction divided by the temperature difference should be the same in each experiment.

The possible variation in gas conductivity with pressure was investigated. This was carried out at the higher temperatures where the pressure range possible is much greater. A variation of 25 cms. was used for ^3He at 3.0°K and for ^4He at 4.0°K. At temperatures below these, the maximum pressure possible is determined by the saturation vapour pressure. As no appreciable effect was noted, it was usual to operate at constant gas density rather than constant pressure. Care had to be taken at the lowest temperatures to ensure that the pressure was in fact less than the saturation value and this limited the range over which measurements could be made.

When the cell was full of liquid helium, its thermal capacity was greatly increased so that the cell took considerably longer to reach equilibrium with the cryostat. It was desirable therefore to fill the cell at as low a temperature as possible so that the calibration could be carried out with the cell empty or full of gas only. Again the lower the temperature, the more rapid the condensation of helium became, since this was controlled to a large extent by the difference between vapour pressure of the liquid in the cell and the pressure at the upper end of the capillary, and this difference increases with decreasing temperature.

THERMAL CONDUCTIVITY OF GERMAN SILVER



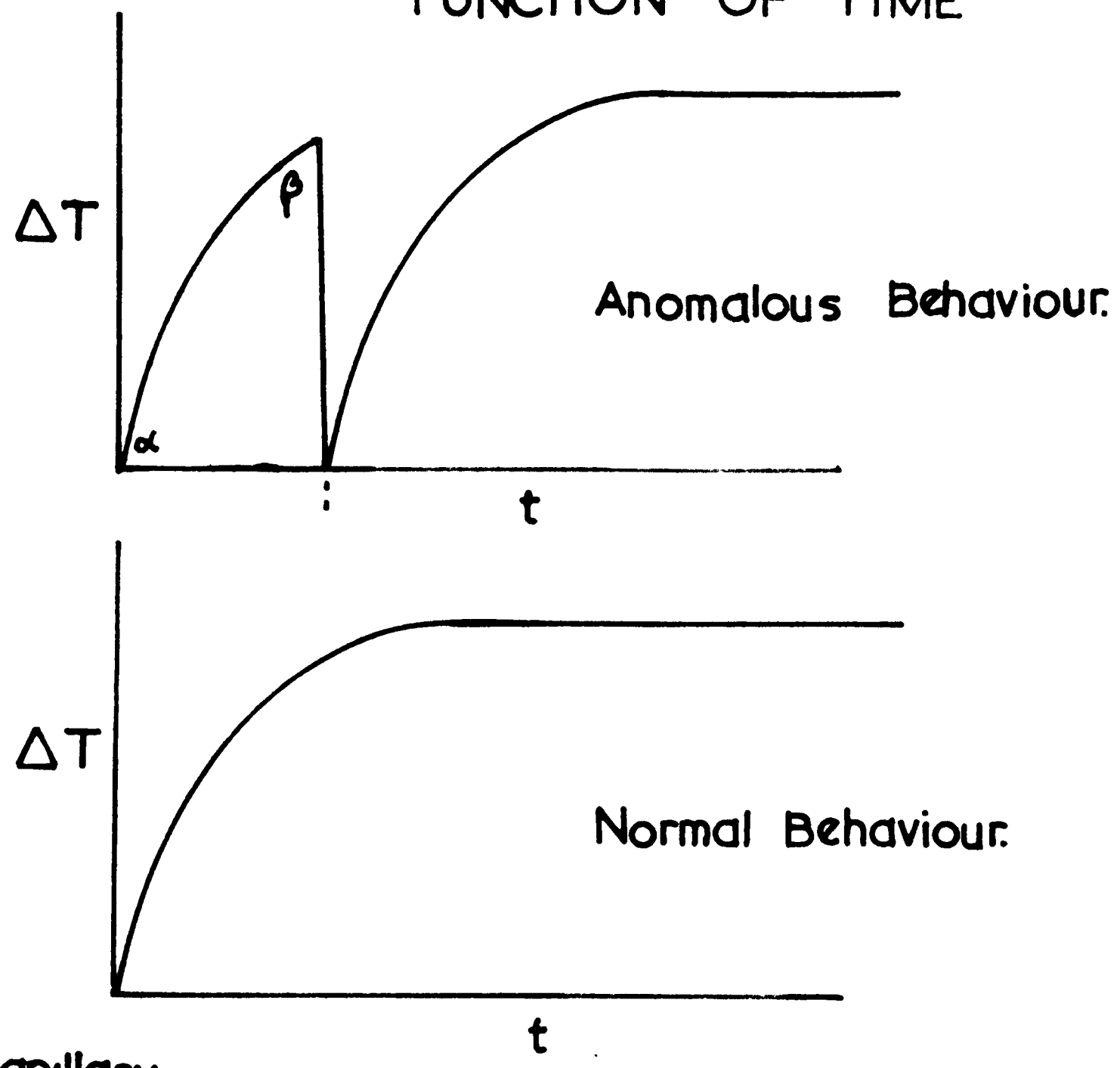
The ^{3}He was condensed in at 1.2°K and since the density of the liquid decreases with temperature, it was known that if the cell could be filled at 1.2°K (at a pressure of one atmosphere) then it would remain full as the temperature was increased. Measurements were made using different heat inputs so that the cell temperature was varied between 1.2°K and 3.2°K. By condensing helium into the cryostat to increase its temperature, new equilibrium positions were obtained and further measurements made.

The ^{4}He was condensed in at 2.6°K. The range of measurements was to be from about 2.2°K to 4.2°K but in this range the cryostat temperature cannot be increased to a new equilibrium position by condensing in helium, due to the poor conductivity of ^{4}HeI, so that in order to make measurements with different temperatures of the cryostat, the temperature had to be reduced and not increased as in the case of ^{3}He.

Measurements were made on the evacuated conductivity cell over the complete range and again, with the thermometers soldered on to the germansilver, so that the conductivity of the german-silver tube could be measured. A further run was made, when the cell had been dismantled, to measure the conductivity of the copper which formed part of the cell.

The description of procedure also applies in the main to the experiments on liquid ^{3}He carried out in the preliminary apparatus. The necessity for using pressures on the liquid greater than the saturation vapour pressure - a state we describe by the word overpressure -

TEMPERATURE DIFFERENCE AS A FUNCTION OF TIME



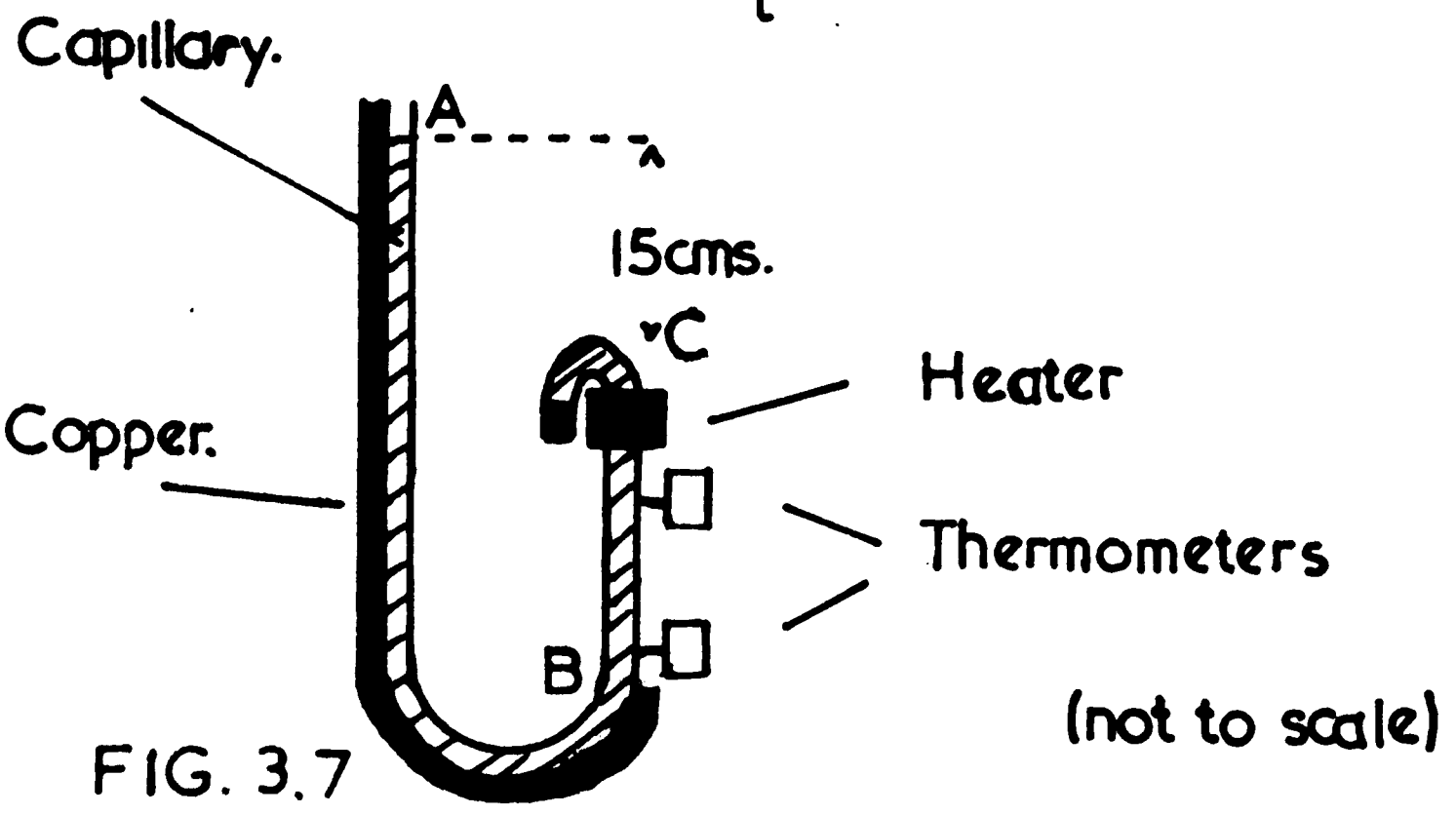


FIG. 3.7

was illustrated by one of the preliminary experiments. The capillary had been filled with liquid ^{3}He to a length of 20 cms. at 1.3°K so that the whole of the region between the heater and the lower thermometer was filled with liquid. Measurements of conductivity had been made in the normal way. Rough calculations carried out during the experiment indicated that the additional heat conducted by the helium was a small and perhaps negligible fraction of that carried by the german-silver. The cryostat temperature was eventually reduced to 1.2°K. A measurement of conductivity was attempted : current was passed through the heater but it was noticed that the meter indicating ΔR behaved in an unusual fashion. A sketch of the temperature difference recorded by this meter is shown in figure 3.7 together with a sketch of the behaviour usually observed. It is seen that the temperature difference increased from zero in the normal manner but that after perhaps half or three quarters of the equilibrium time, it returned very rapidly to zero; sometimes the meter even indicated a negative temperature difference. The time taken for this drop was usually about a second or less. The temperature difference then slowly increased again and reached the final equilibrium value. For some heat inputs and cryostat temperatures, the drop occurred a second time before equilibrium was finally established. The final temperature difference obtained with a particular heat input at any of these low temperatures was, to within experimental accuracy, equal to that obtained when the capillary was evacuated.

The explanation of this curious phenomenon appears to be the following.

In the absence of a temperature gradient, the pressure on the ^3He in the region of the heater is equal to the vapour pressure of ^3He at 1.2°K plus the weight of a column of liquid ^3He 15 cms. long (see figure 3.7). When heat flows down the cell, the copper attached to the capillary ensures that the helium at the surface A is at the same temperature as at B, which is nearly that of the bottom thermometer. Consequently the temperature of the heater would become greater than that of the surface C by an amount of the order of $2\Delta T$ where ΔT is the difference in temperature of the two thermometers, if the helium surface remained stationary. The increase in temperature of the heater is associated with a pressure increase $2\Delta T\left(\frac{dP}{dT}\right)$ which at 1.2°K is 120 ΔT mms. and if ΔT = 0.1°K, ΔP = 12 mms. of Hg = 200 cms. of ^3He. This is greater than the pressure difference due to the height of the surface of the ^3He so that evaporation takes place by the heater and the new surface is depressed to a point where the temperature is less and $\delta T\left(\frac{dP}{dT}\right)$ = 15 cms. of ^3He.

The thermometers are affected by this movement. When the heat is switched on, two processes are occurring : a temperature gradient is slowly established along the german-silver corresponding to the part $\alpha\beta$ of the sketch, and evaporation is taking place corresponding to a depression of the levelof the liquid. The level which is at the temperature of the heater moves down the capillary and passes the upper thermometer but produces only a small change since the temperature at this point is nearly that of the heater. It then reaches the lower thermometer and since the conduction through the german-silver and helium is very small, the change in temperature of the thermometer is almost

instantaneous. The temperature increases to that of the heater while the upper thermometer has cooled slightly to just below this temperature so that negative values of ΔT are possible. The level continues to fall and the temperature gradient along the gas filled capillary re-establishes itself.

When the level reaches this cooler part of the capillary, the gas condenses and the process may be repeated.

For larger heat inputs all this takes place so rapidly that it is not observed and the equilibrium position corresponding to a gas filled capillary is reached. It is possible to derive an upper limit of the liquid conductivity from the occurrence of this phenomenon - if the conductivity of the helium were large, the ΔT produced by these small heat inputs would be such that $2\Delta T\left(\frac{dP}{dT}\right) < 15$ cms. and no evaporation would occur. So for this to occur :

$$2\Delta T \cdot \left(\frac{dP}{dT}\right) > \frac{15 \times \cdot 08}{13.6} \text{ cm Hg}$$

$$\therefore \quad 12\Delta T > \cdot 09 \ ^\circ K$$

$$\Delta T > 7 \cdot 10^{-3} \ ^\circ K$$

These effects were observed when a final temperature difference of .04°K was obtained and this corresponds to a difference with helium liquid present of

$$\Delta T = \frac{K_G A_G}{K_G A_G + K_H A_H} \times \cdot 04$$

Thus $$\frac{K_G A_G}{K_G A_G + K_H A_H} \cdot 4 \cdot 10^{-2} > 7 \cdot 10^{-3}$$

which gives $K_H < 9 K_G <$ $2.3 \cdot 10^{-2}$ watt units

(A_G, A_H are the cross-sectional areas of german-silver and helium respectively).

All this demonstrated very clearly the importance of the use of an overpressure whilst the measurement of thermal conductivities of liquids is being carried out.

We consider the most sensitive measurement of the thermal conductivity of liquid ^{3}He made with the preliminary apparatus, to be one in which liquid ^{3}He at an overpressure was introduced to the capillary during a measurement of the conductivity of the evacuated capillary. With the cryostat at 1.2°K, heat was applied so that a certain temperature difference was produced between the two thermometers. When ^{3}He was introduced, the capillary was naturally warmed but when equilibrium was re-established, it was observed that the temperature difference was the same to within 4% - the estimated maximum error. From this it was deduced that the conductivity of liquid ^{3}He at 1.2°K is less than 3.10^{-4} watt units.

CHAPTER 4

THE RESULTS

1. Calculation of results

Introduction

In the previous chapter a description was given of the experimental procedure and the measurements made during a run. From these measurements, the conductivity may be calculated. The expression for the heat flow in the final conductivity cell is

$$\frac{\dot{Q}}{\Delta T} = F_1 K_H + f(K_H, K_G)$$

The first term represents the heat flowing across the parallel plate part of the apparatus and the second that flowing down the enclosing case at the middle section. The first term is about four times greater than the second term. The derivation of this expression and the calculation of K from it will follow; at the beginning of this section we shall deal with the calculation of the heat input, the temperature and the temperature difference from these measurements.

Heat input

In a previous section we have described how the current through the heater and the voltage across both the heater and a small section of the leads were measured with a potentiometer. This voltage also included a small thermoelectric potential which was measured two or three times during the run and the mean value subtracted from all voltage measurements. This potential was never more than 3% of the total potential. The power dissipated $\dot{Q} = VI$ was multiplied by the factor 0.98 which makes allowance for the voltage drop over the leads and the fact that half of the heat produced in part of these leads

must be added to that produced in the heater.

Temperature difference and temperature of the specimen

An expression of the form

$$Y = \frac{1}{R-R_0} = AT - B + Ce^{-DT}$$

is found to agree with the calibration points to within about 4 millidegrees down to 1.4°K. Above 2.6°K the last term is negligible. The magnitude of R_0 was found by plotting Y against T for the values above 2.6°K until a straight line was obtained. This value was used throughout all of the subsequent runs and the values of A and B were determined for each run by using the method of least squares on the points above 2.6°K.

The formula was then checked by direct substitution and found to agree with the calibration points to an accuracy which was usually of the order of 1 or 2 millidegrees although an occasional discrepancy, particularly above 4°K, of 3 or 4 millidegrees was found; however, this is still within the accuracy of calibration. Below 2.6°K a departure from linearity appeared. This could be accounted for by an additional term $\Delta Y = Ce^{-DT}$. Below 1.4°K a further departure was observed but at these temperatures the calibration errors are considerable so that even if a satisfactory formula could be obtained, the errors inherent in it would be large. The constants A, B, C, D, changed slightly from run to run. The expression for $\frac{dR}{dT}$ is

$$\frac{dR}{dT} = \frac{A - CDe^{-DT}}{(AT - B + Ce^{-DT})^2}$$

In Appendix 1, the expression $\delta R' = \Delta T \left(\frac{dR}{dT}\right)_{T''}$ is derived. T'' is the average temperature of the specimen and $\delta R'$ the

difference between the value of $\Delta R'$ measured when heat is flowing through the specimen, and the value ΔR when no heat is flowing and the temperature of the specimen is T''. At the calibration points, values of ΔR corresponding to zero heat input were obtained and these are plotted against temperature. It is seen then that if T'' is known accurately $\left(\frac{dR}{dT}\right)_{T''}$ and $\delta R'$ may be calculated and so ΔT.

The temperature at which the conductivity is being measured can be taken as the mean temperature of the specimen provided the temperature difference is small. For conductivities of the form $K = \alpha T$, this restriction is unnecessary but when for example $K = \alpha T^2$, the temperature difference should be less than about 0.1 T.

From the formula quoted above, the temperature of the upper thermometer may be calculated from the measured value of R. The value of ΔR corresponding to this temperature is obtained from the graph of ΔR against T. To obtain the temperature of the lower thermometer and so the mean temperature an approximation must be made. This is, that the value of ΔR obtained from the graph at the temperature of the upper thermometer is equal to its value at the mean temperature. With this approximation the temperature of the lower thermometer is calculated from R'

$$R' \approx R + \Delta R' - \Delta R_u$$

This introduces an error in T'' of less than 2 millidegrees. The method of calculation also provides a value for the temperature difference ΔT between the two thermometers which supplies a useful check on the more accurate value.

At this temperature T'' , ΔR is taken from the graph and substituted in the formula $\delta R' = \Delta R' - \Delta R$ and $\left(\frac{dR}{dT}\right)_{T''}$ is calculated.

Finally $\Delta T = \dfrac{\delta R'}{\left(\frac{dR}{dT}\right)_{T''}}$

Calculation of the conductivities

The expression for the heat conducted from one copper block to the other is

$$\dot{Q} = \Delta T\left\{ K_H F_{disc} + K_G \cdot \frac{2\pi \bar{r} a}{h} \cdot \frac{\frac{\alpha h}{2}}{\tanh \frac{\alpha h}{2}} \right\}$$

where $\alpha^2 = \frac{K_H}{K_G} \cdot \frac{r'}{\bar{r}} \cdot \frac{1}{\ell a}$ and $F_{disc} = \frac{\pi r_0^2}{d}$

A small correction must be applied to this to account for a temperature drop across the copper blocks. This amounted to between 2 and 3% depending on the value of K_H.

In the section dealing with the design and construction of the apparatus, it was explained that there was considerable freedom in choosing the values of l, h, r and a. By making l and a small, and h large, the ratio $\frac{\alpha h}{2}$ was made greater than 2.5 for the smallest conductivity, which was assumed to be gaseous ^{4}He, so that the expression simplified to

$$\dot{Q} = \Delta T\left\{ K_H F_{disc} + \sqrt{K_H} \cdot \sqrt{K_G} \sqrt{\frac{\pi^2 a r' \bar{r}}{\ell}} \right\}$$

where ΔT is the value corrected for the 2 - 3% drop in temperature over the copper.

Since the geometry of the arrangement is known and K_G has been measured over the whole temperature range, the expression is of the form

$$\alpha x^2 + \beta x - \frac{\dot{Q}}{\Delta T} = 0$$

where $\alpha = \frac{\pi r_0^2}{d}$

$$\beta = \sqrt{\frac{\pi^2 a^4 \bar{r}^4}{l}} \cdot \sqrt{K_G}.$$

and $$X^2 = K_H$$

The quadratic is solved in the usual way and so the value of K_H calculated.

2. Experimental errors

We shall consider the final cell only.

Errors in 'Form Factor'

These arise from three sources : the possible error in the measurements of the apparatus dimensions, the error in the thermal conductivity of the german-silver used in the apparatus, and the error in the form of the expression used for $\dot{Q}$.

(a) Error in dimensions

Apart from the gap between the two copper blocks, all distances were measured at room temperature only but the error introduced in this way is very small - for copper $\int_0^{300} \alpha dT \approx \alpha_{300} \times 190$ $\approx 3.10^{-3}$. The contraction is appreciable for the gap (5) because although it arises from differential contraction between copper and germansilver only, this contraction in 1 cm. is observed in a gap of only 0.02 mms. The gap was measured at 290°K - $21{\cdot}6 \pm 0{\cdot}1.10^{-4}$ cms. and at 90°K $20.6 \pm 0{\cdot}1.10^{-4}$ cms. and the value at 3°K is then

$$20{\cdot}6\left(1 - \int_{3{\cdot}0}^{90} (\alpha_G - \alpha_{Cu})\, dT\right)$$

If Grüneisen's law is assumed, this further contraction is only

$0{\cdot}1.10^{-4}$ cms. so that a considerable departure from this law will introduce only small error.

A travelling microscope was used to measure distances (except for the thickness of the german-silver) and the estimated maximum errors are listed below.

gap (d)	= 1%
radii (r_o, $\bar{r}$, r')	< ½ %
annular gap (l)	< ½%
thickness of german-silver (a)	= 5%

The value of l was not the same for the upper and lower blocks but differed by 10% so the error in this was taken as 5%.

The total systematic error in K_H due to these individual errors may be found by examining the expression

$$\dot{Q} = \Delta T \left\{ K_H \left(\frac{\pi r_o^2}{d}\right) + \sqrt{K_H} \cdot \sqrt{K_G} \left(\frac{\pi^2 a \bar{r} r'}{\ell}\right)^{\frac{1}{2}} \right\}$$

or $$\dot{Q} = \Delta T \{ \quad A \quad + \quad B \quad \}$$

and for the middle of the temperature range it is found that for

^{3}He liquid	A	~	84%	B	~ 16%
^{4}He liquid	A	~	82%	B	~ 18%
^{3}He gas	A	~	81%	B	~ 19%
^{4}He gas	A	~	74%	B	~ 26%

Due to the individual errors, the maximum error in A is (less than) 2%, in B is 5% and so the error in K_H is, for ^{3}He liquid

(.84 x 2 + .16 x 5)% = 2.48%. For all of the fluids, the maximum systematic error due to this cause is between 2.5 and 3.0%.

(b) Thermal conductivity of german-silver

This contains errors in Form Factor A/ℓ and similar errors in $\dot{Q}$ and ΔT to those present in the measurements of helium. These will be discussed in a later section; the estimated maximum error in $\dot{Q}/\Delta T$ is 4%. The area of the specimen is $2\pi r'$ x a, and the errors in r' and a are less than $\frac{1}{2}$ % and 5% respectively; the error in l is 1% making a total of $6\frac{1}{2}$% in form factor. Adding these errors we arrive at the expression for the total maximum estimated error in K_g of $10\frac{1}{2}$%. In B this represents an error of 5% and in K_H of about 1%.

(c) Error in expression for $\dot{Q}$

The conduction between the two blocks directly across the disc of helium will probably contain only small errors associated with the slight departures from normal flow lines at the edges of the blocks. The second term in the expression includes the effect of flow down the german-silver but not the flow across the meridian plane down the annular gap of helium. This is discussed in Appendix 2b; the area of this plane is 10% of the area of the blocks but the temperature gradient decreases rapidly as the radius increases from r_o, the radius of the blocks. If the gradient were constant as far as the germansilver, this flow would represent 8% of the total heat flow and it is thought that an error of 2% in the total flow is possible by neglecting it. The error in the expression for the flow down the german-silver is thought to be less than this. A rough estimate

of 3% is made which represents the systematic error inherent in the experiments.

Errors in heat input $\dot{Q}$

The errors in the voltage and current are negligible. The factor 0.98 allowing for the heat dissipated in the leads is also accurately computed. Negligible loss is thought to occur by radiation or heat conduction through any remaining hydrogen exchange gas.

Errors in Temperature difference

ΔT is calculated from the expression $\frac{\delta R'}{\left(\frac{dR}{dT}\right)_{T''}}$ where

$\delta R' = \Delta R' - \Delta R$. $\Delta R'$ is measured directly to an accuracy of 0.1 ohm or better above about 1.6°K but to about 0.3 ohms below this, and ΔR is taken from a graph once the value of T'' is known. The error in reading ΔR is partly random and partly systematic in each drawing of $\Delta R = f(T)$. The curve of $\Delta R = f(T)$ includes the errors of 0.1 ohm - 0.3 ohms although these at high temperatures are probably reduced by smoothing as ΔR is nearly constant. All these errors increase sharply with decreasing temperature and are considerable below 1.4°K.

Consequently the error in $\delta R'$ is random error of about 0.15 ohm and systematic error (for each experiment) of 0.15 ohm at temperatures above 1.8°K which increase to about 0.3 ohm and 0.7 ohm respectively below this value. These results expressed as percentages depend on the values of $\delta R'$ used. For ^{4}HeI these were from 13.5 ohm to 7.5 ohm and so the maximum errors in ^{4}HeI due to this cause are

random error	= 2%	(1.5%)	Figures in brackets are
systematic error (per run)	= 2%	(1.5%)	for average points.

For the other fluids we have used only values of $\delta R >$ 10 Ω .

	Above 1.8°K		Below 1.8°K	
random error	1.5%	(1%)	3%	(1%)
systematic error	1.5%	(1%)	7%	(2 %)

$\left(\frac{dR}{dT}\right)_{T''}$ is calculated from the empirical expression derived for R. The temperature scale T_{55E} was used which is thought to be accurate to better than 2 millidegrees and this introduces a negligible error in $\left(\frac{dR}{dT}\right)_{T''}$. A possible source of error is due to inaccuracies in T'' : a relative error of δT produces an error of $\delta T \left(\frac{d^2R}{dT^2}\right)_{T''}$ but this can be shown to be less than $\frac{1}{2}$ % for the estimated errors in T'' . The error in $\left(\frac{dR}{dT}\right)_{T''}$ due to inaccuracies in the form of $R = f(T)$ is estimated as 1% making a total systematic error in $\left(\frac{dR}{dT}\right)_{T''}$ of $1\frac{1}{2}$%.

The errors in ΔT are determined by adding the errors in $\delta R'$ and $\left(\frac{dR}{dT}\right)_{T''}$.

Temperature drop over the copper

The correction for this was never more than 2 or 3%. The thermal conductivity of the copper was measured to an accuracy of about 5% so that the error introduced in this correction term is negligible.

The absolute error in temperature

This is small and is equivalent to less than 0.1% in thermal conductivity.

Total Maximum Error	^{4}HeI	^{4}He gas, ^{3}He gas, ^{3}He liquid	
Systematic error	6%	6%	
Variable systematic error	2%	1.5%	these are values above 1.8°K; below the values are slightly higher
Random error	2%	1.5%	

The variable systematic error may be different in each run and so this figure represents the maximum variation in K_H expected from run to run. The random error represents the maximum scatter expected - the occasional point outside this limit is attributed to careless error. A number of points were taken and in some cases in more than one run. The smoothed curve obtained from two runs probably eliminates most of the random and variable systematic errors. Points on this curve have a maximum absolute error of 6% and a probable error of considerably less than this.

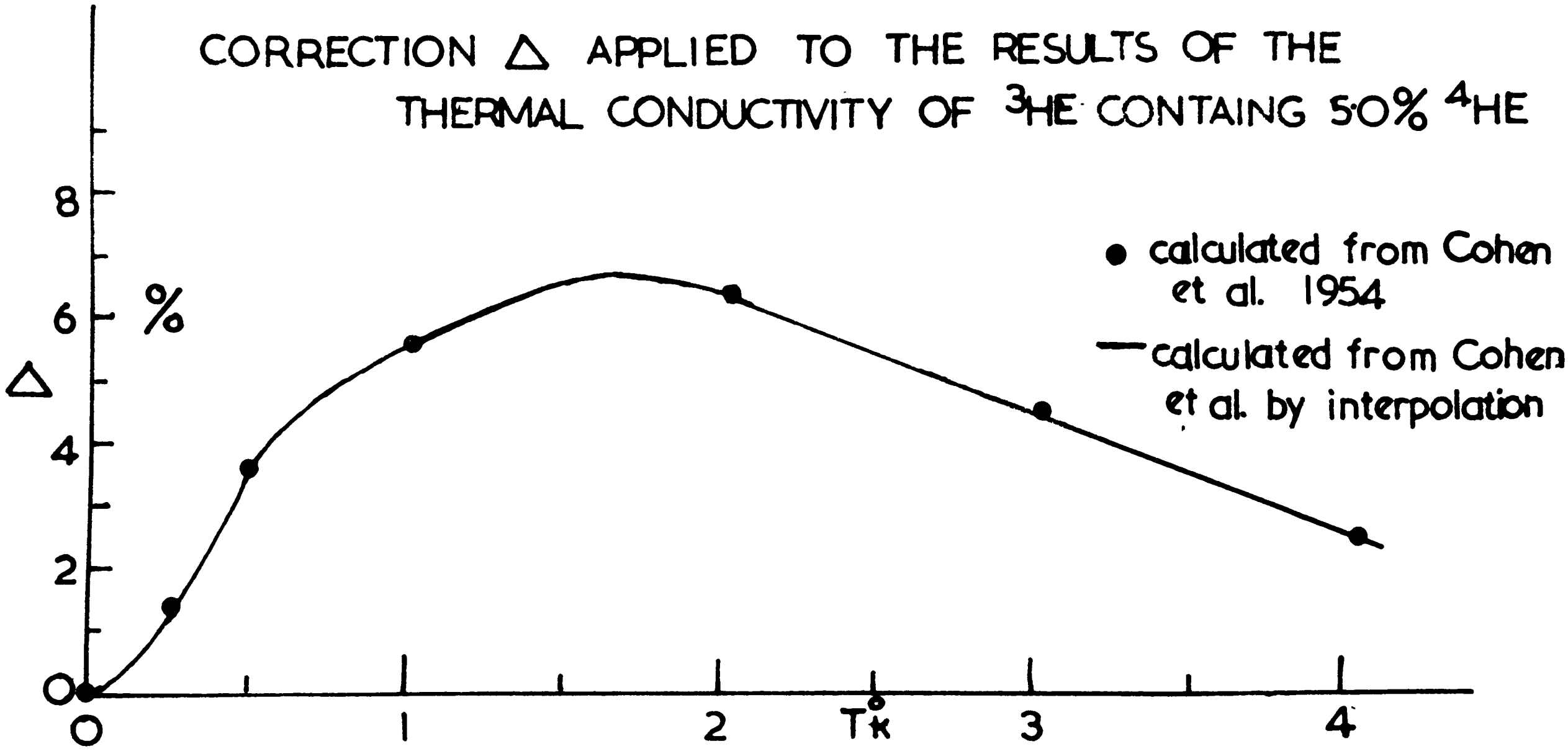
Corrections applied to results for ^{3}He gas containing ^{4}He

The ^{3}He used in the experiments on the thermal conductivity of gaseous ^{3}He contained 5% ^{4}He. A correction has been applied to the results to obtain the conductivity of pure ^{3}He. The viscosity measurements of Becker, Misenta and Schmeissner (1954) have been corrected by us in a similar fashion.

Cohen, Offerhaus and de Boer (1954) calculated the transport properties of a mixture of ^{3}He and ^{4}He to the first approximation. Their formulae are quoted below.

^{3}HE GAS

FIG. 4.1



$$\frac{1}{(\eta_{mix})_1} = \frac{\dfrac{H_3 x_3^2}{(\eta_3)_1} + \dfrac{2H_{34}x_3x_4}{(\eta_{34})_1} + \dfrac{H_4 x_4^2}{(\eta_4)_1}}{H_3 x_3^2 + 2K_{34} x_3 x_4 + H_4 x_4^2}$$

$$\frac{1}{(K_{mix})_1} = \frac{\dfrac{L_3 x_3^2}{(K_3)_1} + \dfrac{2L_{34}x_3x_4}{(K_{34})_1} + \dfrac{L_4 x_4^2}{(K_4)_1}}{L_3 x_3^2 + 2M_{34}x_3x_4 + L_4 x_4^2}$$

x_3 and x_4 are the concentrations of ^{3}He and ^{4}He respectively. The coefficients H_3, H_4, H_{34}, K_{34} and L_3, L_4, L_{34}, M_{34} are complicated functions of the integrals Ω. The values given for these coefficients are shown in Tables 4.1 and 4.2 together with the values of viscosity and thermal conductivity calculated by de Boer et al. (see de Boer 1955)

We have used these values to calculate the thermal conductivity of a 95% ^{3}He, 5% ^{4}He mixture and the viscosity of a 94% ^{3}He, 6% ^{4}He mixture. The ratios

$$\frac{\Delta K}{K} = \frac{(K_3)_1 - (K_{mix})_1}{(K_{mix})_1} \quad \text{and} \quad \frac{\Delta \eta}{\eta} = \frac{(\eta_3)_1 - (\eta_{mix})_1}{(\eta_{mix})_1}$$

have been calculated as percentages and the first is shown in Figure 4.1 as a function of temperature and is seen to reach a maximum at about 2°K which corresponds to the temperature at which the conductivities of pure ^{3}He and ^{4}He differ most. These fractions were also calculated at intermediate temperatures from the coefficients H_3, H_4 etc. obtained from the smooth curve drawn through the known values of these coefficients. The calculations of the transport properties in the first approximation by de Boer et al. agree well with the experimental values, and it appears that these fractions, which are in any case used only as correction terms, are probably quite accurate.

Finally a graph of $\frac{\Delta K}{K}$ against temperature was made so that, at any temperature, our measured values could be increased by this fraction to give the values for pure ^{3}He. A similar method was used to estimate the viscosity of pure ^{3}He from the values obtained by Becker et al.

No correction has been made to the results for liquid ^{3}He. The magnitudes of the conductivity of liquid ^{4}HeI and ^{3}He are very similar and the correction factor is probably small.

TABLE 4.1

Data for viscosity of ^{3}He, ^{4}He and mixtures in first approximation

T °K	$(\eta_3)_1$ μP	$(\eta_4)_1$ μP	$(\eta_{34})_1$ μP	H_3	H_4	H_{34}	K_{34}
0	0	0	0	1.45	1.80		
0.256	2.35	0.54	0.98	1.40	1.71	1.41	1.69
0.511	3.87	2.50	2.80	1.41	1.72	1.45	1.48
1.02	7.20	3.44	5.14	1.43	1.77	1.63	1.72
2.04	11.3	5.01	7.49	1.52	1.93	1.69	1.82
3.07	12.4	7.45	9.68	1.53	1.95	1.74	1.80
4.09	13.1	10.9	11.9	1.52	1.92	1.70	1.72

TABLE 4.2

Data for heat conductivity of ^{3}He, ^{4}He and mixtures in first approximation

T °K	$(K_3)_1$ 10^2 erg s^{-1} cm^{-1} deg^{-1}	$(K_4)_1$	$(K_{34})_1$	L_3	L_4	L_{34}	M_{34}
0	0	0	0	1.10	0.93		
0.256	2.44	0.418	0.888	1.05	0.88	0.92	1.06
0.511	4.02	1.95	2.55	1.08	0.92	0.97	0.97
1.02	7.49	2.69	4.68	1.08	0.91	1.03	1.07
2.04	11.8	3.91	6.75	1.13	0.95	1.04	1.12
3.07	12.9	5.81	8.81	1.13	0.96	1.06	1.09
4.09	13.6	8.51	10.8	1.12	0.95	1.04	1.04

Results

The Thermal Conductivity of Gaseous ^{4}He (Jan 7, 1957)

T°K	ΔT (2 sig.figs)	K.10^{-5}w.u.	T°K	ΔT (2 sig. figs.)	K.10^{-5}w.u.
1.678	.016	3.10	2.818	.37	5.11
1.735	.036	3.24	2.844	.16	4.94
1.768	.036	3.33	2.979	.33	5.70
1.772	.036	3.2[illegible]	3.095	.23	5.88
1.827	.068	3.33	3.105	.42	5.83
1.880	.084	3.51	3.250	.39	6.34
2.014	.032	3.71	3.296	.29	6.38
2.035	.13	3.81	3.543	.35	6.97
2.090	.059	3.87	3.563	.20	7.04
2.262	.20	3.86	3.735	.25	7.48
2.269	.12	4.20	3.860	.56	7.72
2.575	.28	4.83	3.948	.31	7.90
2.681	.10	5.18	3.976	.53	8.15
2.766	.27	5.21			

The Thermal Conductivity of Gaseous ^3He (Jan 10, 1957)

T°K	ΔT (2 sig. figs.)	K. 10^{-5} w.u. (uncorrected)	K. 10^{-5} w.u. (corrected for ^4He)
1.388	.038	6.59	7.01
1.434	.046	7.18	7.64
1.532	.011	8.07	8.59
1.572	.016	8.23	8.76
1.664	.028	8.63	9.20
1.717	.036	8.87	.46
1.860	.060	9.30	9.91
1.866	.10	9.21	9.82
1.914	.026	9.43	10.04
1.993	.033	9.77	10.40
2.089	.093	10.19	10.83
2.119	.055	10.16	10.79
2.211	.15	10.53	11.17
2.311	.084	10.69	11.32
2.388	.14	10.92	11.54
2.583	.14	11.15	11.74
2.615	.032	11.22	11.80

The Thermal Conductivity of Gaseous ^3He (Jan 10, 1957)
(continued)

T°K	ΔT (2 sig. figs.)	K. 10^{-5} w.u. (uncorrected)	K. 10^{-5} w.u. (corrected for ^4He)
2.625	.18	11.21	11.79
2.778	.24	11.53	12.09
2.804	.18	11.66	12.23
2.865	.13	11.71	12.26
2.906	.23	11.78	12.33
3.067	.23	11.91	12.43
3.069	.17	11.95	12.48
3.170	.12	12.14	12.65
3.308	.21	12.24	12.72
3.353	.16	12.57	13.05
3.579	.37	13.07	13.50
3.581	.20	12.93	13.36
3.665	.36	12.85	13.26
3.779	.36	13.23	13.61
3.880	.18	13.57	13.94
3.984	.34	13.43	13.77
4.210	.34	13.81	14.10

The Thermal Conductivity of Liquid ^{4}HeI

January 8, 1957

T°K	ΔT(2 sig.figs.)	K.10^{-5}w.u.
2.695	.058	15.5
2.993	.09	17.1
3.132	.12	17.3
3.394	.15	18.5
4.002	.15	21.3

March 18, 1957

T°K	ΔT(2 sig.figs.)	K.10^{-5}w.u.
2.411	.037	14.5
2.576	.060	15.3
2.791	.093	16.0
3.117	.14	17.1
3.376	.18	18.1
3.601	.13	18.8
3.705	.22	18.9
3.831	.16	19.7

The Thermal Conductivity of Liquid ^3He

T°K	ΔT(2 sig.figs.)	K.10^{-5}w.u.	T°K	ΔT(2 sig.figs.)	K.10^{-5}w.u.
March 22, 1957					
2.184	.039	13.28	1.612	.043	11.14
2.348	.062	14.34	1.829	.071	11.57
2.400	.10	14.61	1.938	.031	11.82
2.541	.057	15.54	1.984	.039	12.21
2.596	.094	15.78	2.105	.104	13.37
2.764	.090	16.46	2.154	.062	13.12
2.799	.15	16.43	2.305	.094	14.70
2.952	.14	17.24	2.506	.053	15.35
April 8, 1957			2.523	.15	15.38
1.303	.0059	10.36	2.717	.084	16.44
1.323	.0085	10.12	2.750	.14	16.42
1.363	.013	10.37	2.821	.18	16.79
1.417	.018	10.48	3.030	.17	17.50
1.536	.034	10.87	3.031	.13	17.46

CHAPTER 5

DISCUSSION OF RESULTS

Thermal conductivity of gaseous helium

It is convenient to discuss the thermal conductivity together with the viscosity because in all kinetic theories there exists the relationship $K = f\eta C_v$ where f depends on the model used, and, in the classical treatments of a monatomic gas, usually has a value between 2.49 and 2.52. In the quantum mechanical treatment f is equal to 2.50 to the first approximation and to within 1% of the second. A useful criterion to check the consistency of both experiments and theory can be made therefore by examining the experimental values of f. Since no values existed for the conductivity of ^3He the following discussion applies only to ^4He.

C_v is taken to be $\frac{3}{2}\frac{R}{M}$ and η has been measured by a large number of workers at room temperature (see Keesom 1942) and by Kamerlingh Onnes and Weber (1913), Vogel (1914), van Itterbeek and Keesom (1938) and Becker and Misenta (1955) at 20°K. These values of the viscosity are in good agreement with each other. The values of the conductivity obtained at room temperature are also in agreement (see Chapter 1) and using the values for K and η at room temperature in the expression for f, an answer of about 2.44 is obtained. However at 20°K there is a discrepancy of 20% between the values of Eucken and those of Ubbink and de Haas and the corresponding values of f are 1.92 and 2.35. Thus in view of the improved techniques now available, it would appear likely that the values of Ubbink et al. are

the more accurate and f remains at least approximately constant down to 20°K.

At helium temperatures, there are three conflicting sets of measurements for the viscosity, (van Itterbeek and Keesom, 1938 - 2 points only, van Itterbeek, Schapink, van den Berg and van Beek, 1953; and Becker, Misenta and Schmeissner, 1954) and the value of f depends of course on the viscosity used. The conductivity had been measured only by Ubbink et al. and using their values and the viscosity data of Becker et al., f = 2.58, of van Itterbeek, Schapink et al., f = 2.32 and of van Itterbeek and Keesom, f = 2.10.

It is seen then that although the measurements at low temperatures are often in disagreement, there is no definite evidence of a fall in f for ^{4}He at low temperatures, and if the conductivity values of Ubbink and de Haas are used at 20°K in preference to those used of Eucken, and the viscosity values of Becker et al. are used at 4°K, then f is seen to remain constant and about 2.50 to within the possible combined experimental errors.

Quantum-mechanical calculations at low temperatures can also be compared directly with the experimental wlues of η and K (below 5°K - ^{4}He : Buckingham, Hamilton and Massey 1941; de Boer 1943; Buckingham and Scriven 1952; Keller 1957; - ^{3}He : de Boer and Cohen 1951; Buckingham and Scriven 1952; Keller 1957). In a review article de Boer (1955) makes this comparison with theoretical curves calculated using different potential functions and this is repeated for two potentials by Keller (1957). Keller recalculates K and η for the potentials

used by de Boer et al. with the aid of an electronic computer. Small discrepancies appear and in view of this, we have preferred to use the data of Keller.

Keller calculates η for two potentials which he designates as LJ1 and MR5. LJ1 is a Lennard-Jones 12-6 potential containing two constants and is the potential calculated by de Boer from virial coefficient measurements and used by him to derive the transport coefficients. MR5 is an exponential - 6 potential containing three constants : two from low temperature values of the second virial coefficient, and a third from the high temperature ^{4}He viscosity data. The form of these potentials is explained at the end of this section. Keller points out that the virial coefficients calculated from MR5 agree better with experiment than those calculated from LJ1 and so one should perhaps expect MR5 to be the better potential of the two when used in calculating the transport properties. This does not appear to be the case, however, and the inference from this is that the true potential is neither of these. It would obviously be of interest to examine the values obtained from other potentials in comparison with experimental values of both the virial and transport coefficients.

Theoretical treatment of the transport properties at low temperatures

A complete description of the quantum theory of transport phenomena in gases would be inappropriate here; the theory is given in full by Chapman and Cowling (1952) and in part in a review article on the theoretical treatment applied to gaseous helium at low temperatures by de Boer (1955). The treatment is sketched briefly below in order

that certain qualitative comments may be made.

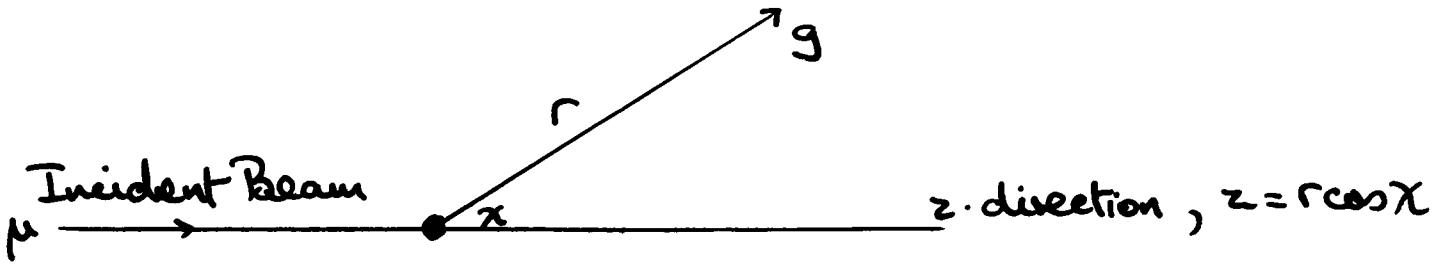
The transport properties are calculated from the collision cross-sections for two helium atoms which appear in the Boltzmann integral equation. To calculate these cross-sections, a knowledge of the interaction potential is required and this being electronic is the same for both helium isotopes. The curve of the potential was calculated by Slater and Kirkwood (1931) in the form $\phi(r) = b\exp(-ar) - cr^{-6}$ (an 'exponential-six potential') but there are technical problems associated with the convergence of the perturbation calculations and the potential curve has not yet been calculated to the precision required. Accordingly the method adopted is to assume an expression for this interaction potential which includes undetermined constants but which is of the form indicated by the theoretical calculations. These constants are then chosen to give the best fit with such experimental datum as the second virial coefficient of helium gas. Two forms of potential have been used in this work : an 'exponential-six potential' proposed by Massey and Buckingham (1938) and a Lennard-Jones potential of the form $\phi(r) = ar^{-n} + br^{-6}$ proposed by de Boer and Michels (1939).

The collision cross-section is calculated in terms of this potential by considering a stream of molecules of mass m and velocity c_1 (range dc_1) which are moving in another stream of molecules of mass m and velocity c_2 (range dc_2). The number of molecules leaving in a particular direction at angle χ with the incident direction, within a solid angle $d\omega$, is proportional to the number densities of the two streams n_1 and n_2 and also depends on χ, and g, the relative velocity.

It is written

$$n_1 n_2 \alpha_{12}(g,\chi)\, d\underline{n}$$

The collisions can be considered as being between a molecule of mass μ (the reduced mass $\mu = \frac{m \cdot m}{m+m} = \frac{m}{2}$) and velocity g moving towards a stationary force centre.



The Schrödinger wave equation is then

$$\frac{\hbar^2}{2\mu}\nabla^2\psi + \left(\frac{\mu g^2}{2} - V\right)\psi = 0$$

The solution required must be symmetrical about the incident direction of g and finite at the origin. The equation is rewritten as

$$\nabla^2\psi + \left(j^2 - \frac{2\mu V}{\hbar^2}\right)\psi = 0$$

where j is the wave number $\frac{\mu g}{\hbar}$ and one solution is $\psi = u_n(r)\frac{P_n(\cos\chi)}{r}$ where $P_n(\cos\chi)$ is the Legendre polynomial of order n and $u_n(r)$ satisfies the equation

$$\frac{d^2u_n}{dr^2} + \left(j^2 - \frac{2\mu V}{\hbar^2} - \frac{n(n+1)}{r^2}\right)u_n = 0$$

The solution appropriate to the problem is a linear combination of solutions $\psi = \frac{1}{r}\sum_{n=0}^{\infty} A_n u_n(r) P_n(\cos\chi)$ where A_n is some constant. This may be split up into two wave functions : the first ψ' corresponds to the incident beam and the second ψ'' to the scattered molecules. The first, for unit number density and like molecules, is written as $\psi' = \frac{1}{\sqrt{2}}\left(e^{ijz} \pm e^{-ijz}\right)$ (the sign depends on the particular statistics obeyed by the molecules) and this can be

written in a similar form to the solution for ψ . By subtraction, the wave function of the scattered molecules is obtained.

At a large distance r from the stationary force centre, these scattered molecules are travelling radially outwards with a velocity g. The intensity of the molecules as a function of χ and r is given by $|\psi''|^2$ so that the number of molecules crossing a small area dA on the surface of a sphere of mius r is $|\psi''|^2 dA = |\psi''|^2 r^2 d\underline{n}$ ($dn = \frac{dA}{r^2}$) and the number crossing per second is $g|\psi''|^2 r^2 d\underline{n}$

We defined this earlier for a beam of unit number density as $\alpha_{12}(g,\chi)\, d\underline{n}$

Therefore $\alpha_{12}(g,\chi)\, d\underline{n} = |\psi''|^2 g r^2 d\underline{n}$

or $\alpha_{12}(g,\chi) = |\psi''|^2 g r^2$

so from the calculated expression for ψ'' the crossection is derived.

To obtain the total probability of collision of a molecule of velocity c_1 colliding with a molecule of velocity c_2 we must integrate α_{12}

Now $dn = \frac{2\pi r \sin\chi \cdot r\, d\chi}{r^2} = 2\pi \sin\chi\, d\chi$ so the total probability is

$$2\pi \int_0^{\pi} \alpha_{12}(g,\chi) \sin\chi\, d\chi$$

However, the solution of the Boltzmann integral equation shows that the so-called total crossection includes 'weight-functions' which are either $2\sin^2\frac{1}{2}\chi$ or $\sin^2\chi$

$$Q^{(r)}(g) = 2\pi \int_0^{\pi} (1 - \cos^r\chi)\, \alpha_{12}(g,\chi) \sin\chi\, d\chi \qquad r = 1 \text{ or } 2$$

We quote now the final expressions obtained from the Boltzmann integral equation (e.g. Hirschfelder, Curtiss and Byron Bird, 1954).

$$[\eta]_1 = \frac{5kT}{8\Omega_1^{(2,2)}} \quad \text{and} \quad [K]_1 = \tfrac{5}{2} C_v \cdot \frac{5kT}{8\Omega_1^{(2,2)}}$$

where $\Omega_1^{(2,2)} = \frac{1}{2}\pi^{-\frac{1}{2}} \int e^{-g^2} g^6 Q_1^{(2)}(g)\, dg$ $\qquad C_v = \frac{3k}{2m}$

The coefficients of viscosity and thermal conductivity obtained in this way are values to the first approximation. The second approximations may be written as

$$[\eta]_2 = [\eta]_1(1 + \varepsilon_\eta) \quad \text{and} \quad [K]_2 = [K]_1(1 + \varepsilon_K)$$

and the correction terms are functions of $\Omega_1^{(2,5)}$ (see Chapman and Cowling 1952). These correction terms are small; for hard spheres the first approximation is too small by about 2% and the second by .2%.

It may be noted that

(1) The ratio $\frac{[K]_1}{[\eta]_1}$ is, in the first approximation, exactly $\frac{5}{2} C_v$ and, because the further approximations are small, the ratio $\frac{[K]_2}{[\eta]_2}$ is also equal to this to within about 1%.

(2) These results apply to gases of 'normal' density; that is to gases at sufficiently low densities that the chance of triple collisions is negligible, but at sufficiently high densities that the mean free path is small compared with the apparatus dimensions.

(3) From the equation

$$\frac{\hbar^2}{2\mu} \nabla^2\psi + \left(\frac{\mu g^2}{2} - V\right)\psi = 0$$

it is seen that small changes in V would be expected to produce larger changes in ψ and so in K and η for $\mu = 3/2$ (m=3; ^{3}He) than for $\mu = 2$ (m=4; ^{4}He). Conversely, it would be expected that a comparison between the experimental and theoretical values of K and η would

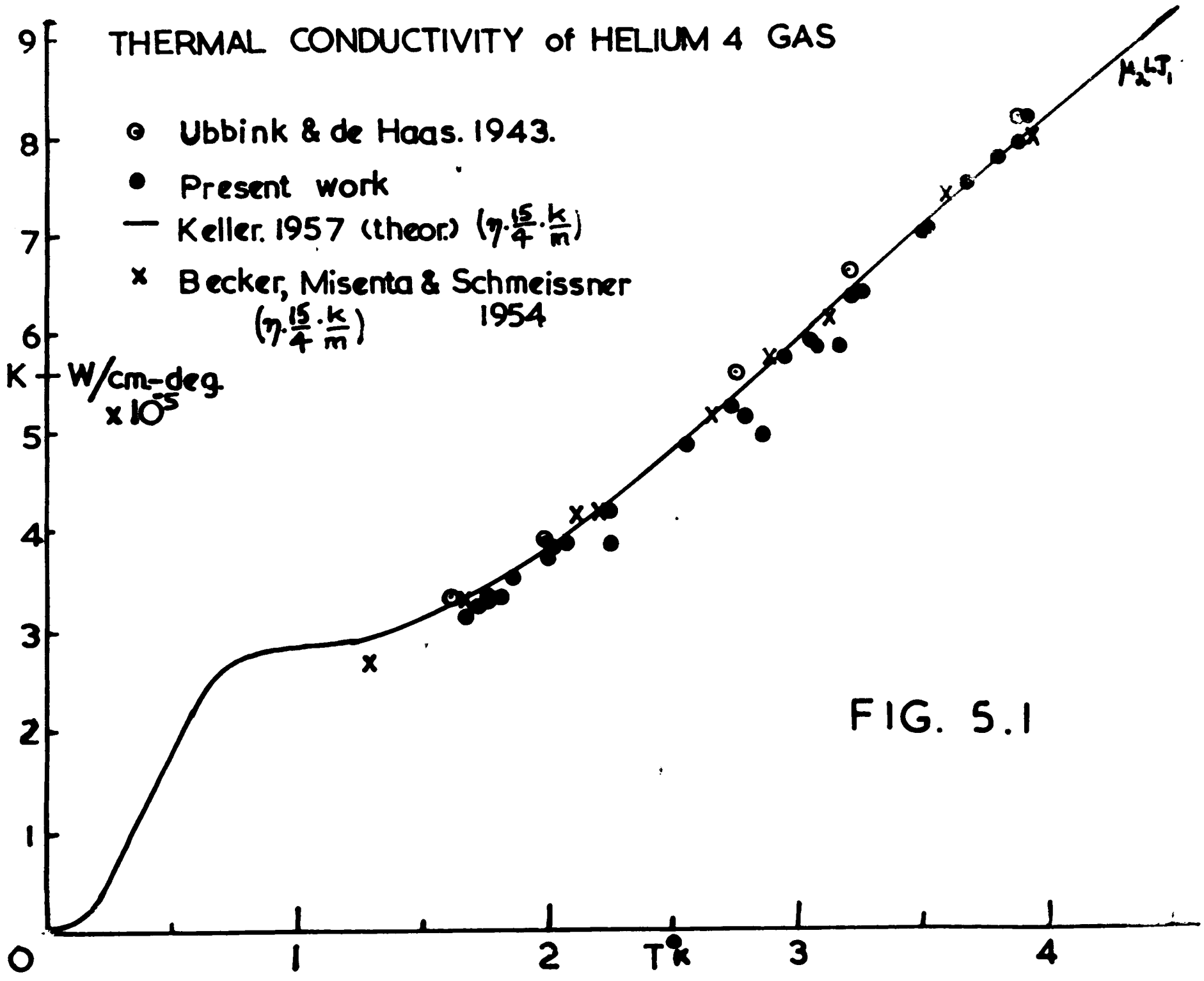

THERMAL CONDUCTIVITY of HELIUM 4 GAS
Ubbink & de Haas. 1943.
Present work
Keller. 1957 (theor.) $(\eta \cdot \frac{15}{4} \cdot \frac{k}{m})$
Becker, Misenta & Schmeissner 1954
$(\eta \cdot \frac{15}{4} \cdot \frac{k}{m})$
K W/cm-deg. x 10^5
$\mu_2 L T_1$
9
8
7
6
5
4
3
2
1
0
1
2
T°K
3
4

FIG. 5.1

provide a more sensitive test of the form of potential used in the case of ^{3}He than in the case of ^{4}He.

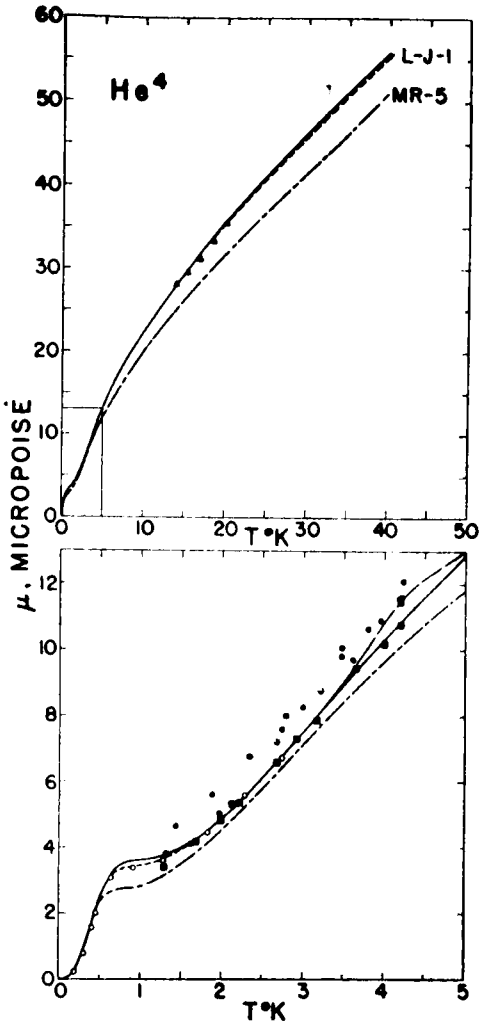
(4) In the calculations of the 'total cross-sections' the influence of the statistics is considerable. The cross-sections, when integrated, are in the form of a summation from $j = 0$ to $j = \infty$ $\left(j = \frac{\mu g}{\hbar}\right)$. For ^{4}He, which obeys Bose-Einstein statistics, the summation is restricted to even values of j, while for ^{3}He, which obeys Fermi-Dirac statistics and which has a spin of $\frac{1}{2}\hbar$, the cross-section is obtained by taking $\frac{1}{4}$ of the summation restricted to even values of j (the symmetrical cross-section) and $\frac{3}{4}$ of the summation restricted to even values of j (the asymetrical cross-section). The factors $\frac{1}{4}$ and $\frac{3}{4}$ arise from the ratio $\frac{1}{2}\rho(\rho+1) \Big/ \frac{1}{2}\rho(\rho-1)$ where $\rho = 2s+1$ and $s = \frac{1}{2}$

This together with the difference in mass leads to completely different values for the viscosity and thermal conductivity of gaseous ^{3}He and ^{4}He.

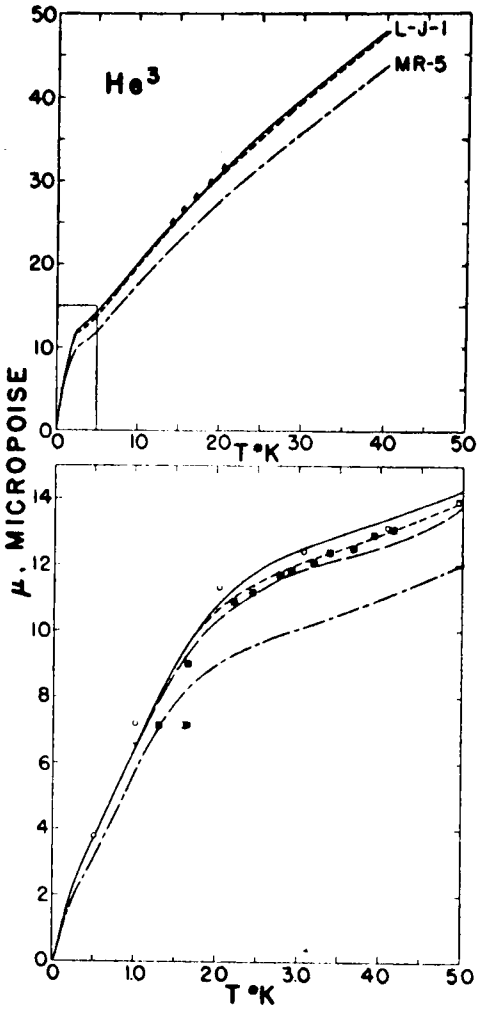
<u>^{4}He gas results</u>

The results for ^{4}He gas are in good agreement with those of Ubbink and de Haas (1943) (figure 5.1). Included in this figure are values of the conductivity derived from the experimental viscosity data of Becker, Misenta and Schmeissner (1954) and the theoretical viscosity values (η_2 LJ1) of Keller using the conversion factor $\frac{K}{\eta} = \frac{5}{2}C_v = \frac{15}{4}\frac{R}{M}$

The agreement with these values is good and it appears likely that the measurements of Becker et al. provide the most accurate values of the viscosity at these temperatures. The values of van Itterbeek, Schapink et al. are compared with those of Becker et al. and the



Comparison of calculated and measured viscosity data of gaseous He^4 as a function of temperature. Lower section shows details below 5°K. Calculated results: ——μ_2—LJ1, present work; - - -μ_1—LJ1, present work; —— - ——μ_1—MR5, present work; ○ μ_1—LJ1, de Boer[7]; —— ——μ_1, Buckingham and Scriven.[7] Measured results: ▲ Becker and Misenta[11]; ■ Becker, Misenta, and Schmeissner[12]; ● van Itterbeek *et al.*[14]



Comparison of calculated and measured viscosity data of gaseous He^3 as a function of temperature. Lower section shows details below 5°K. Calculated results: ——μ_2—LJ1, present work; - - -μ_1—LJ1, present work; —— - ——μ_1—MR5, present work; ○ μ_1—LJ1, de Boer and Cohen[8]; —— ——μ_1, Buckingham and Scriven.[7] Measured results: ▲ Becker and Misenta[11]; ■ Becker, Misenta, and Schmeissner.[12]

FIG. 5.2

theoretical values (η LJ1 and η MR5) of Keller in figure 5.2, the subscripts referring to the first and second approximation respectively. The two values of van Itterbeek and Keesom are not shown.

It appears then that the theoretical values calculated by Keller using the Lennard-Jones potential proposed by de Boer agree very well with the experimental measurements of the viscosity (in both the helium and hydrogen ranges) - see figure 5.2 and also with the conductivity values obtained in the present work and in the work of Ubbink et al.

The results are slightly dependent on pressure; this has been investigated at about 4.0°K where it is possible to vary the pressure over an appreciable range. The increase in conductivity is about 1.5% for an increase in pressure of 25 cms. or approximately 4.5% per atmosphere which is just outside the experimental error. An increase is indicated by the Enskog theory of dense gases and has been reported by Ubbink (1947) who observed that as the pressure was increased from zero, the heat conducted across this cell increased sharply, indicating that the thermal boundary resistance was decreasing rapidly with pressure and then showed a gradual linear increase attributed to a genuine increase in conductivity. The percentage increase found by Ubbink was 4% per atmosphere and since no larger increase was found in our experiments, it appears that in accordance with the theory the effects of resistance in the arrangement used are negligible. The results were also found to be independent of temperature difference which together with the absence of any unexplained pressure dependence,

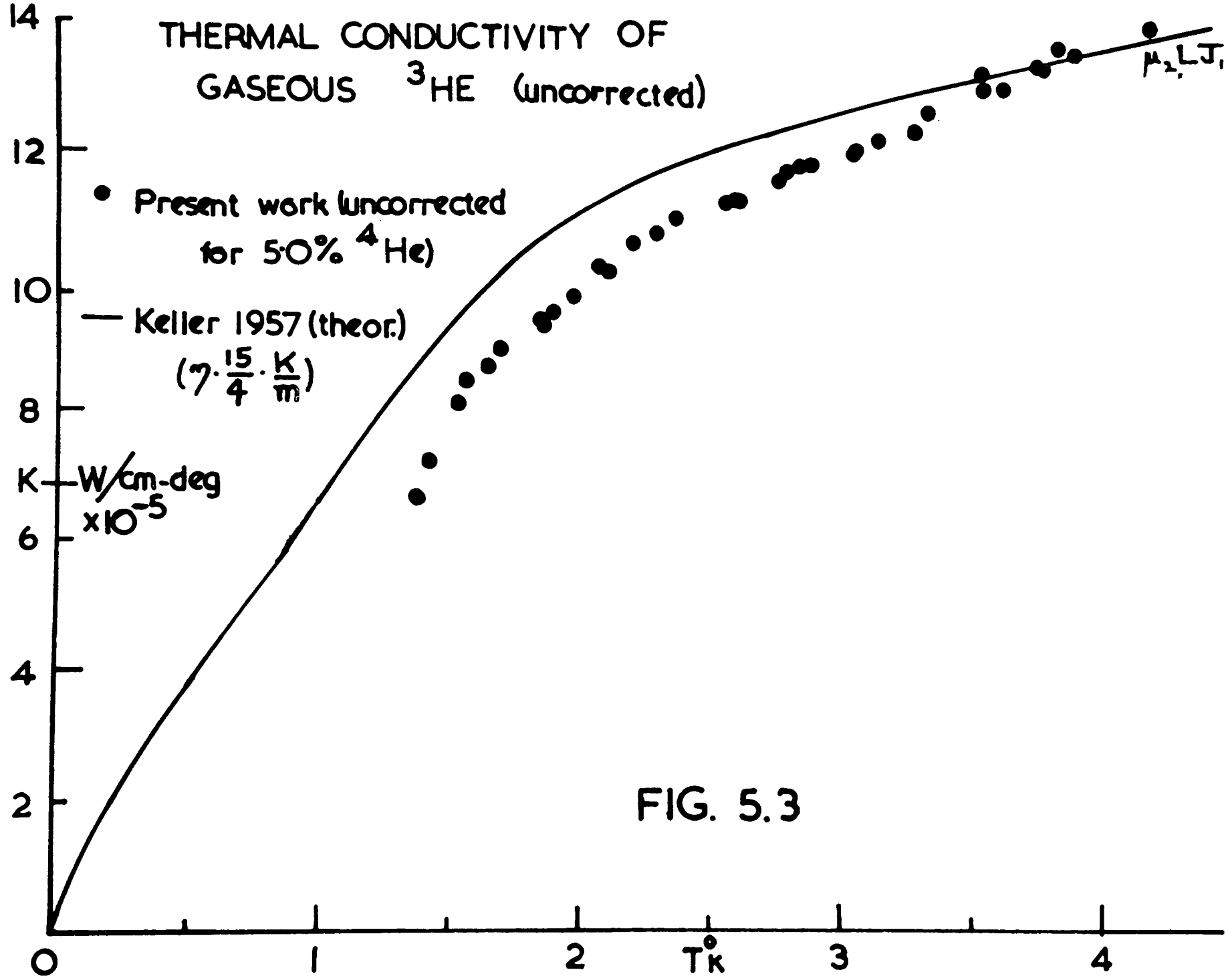

THERMAL CONDUCTIVITY OF
GASEOUS ^{3}HE (uncorrected)
Present work (uncorrected
for 5·0% ^{4}He)
Keller 1957 (theor.)
$(7 \cdot \frac{15}{4} \cdot \frac{K}{m})$
K W/cm-deg
$\times 10^{-5}$
14
12
10
8
6
4
2
0
1
2
3
4
T°K
$\mu_2 LJ_1$

FIG. 5.3

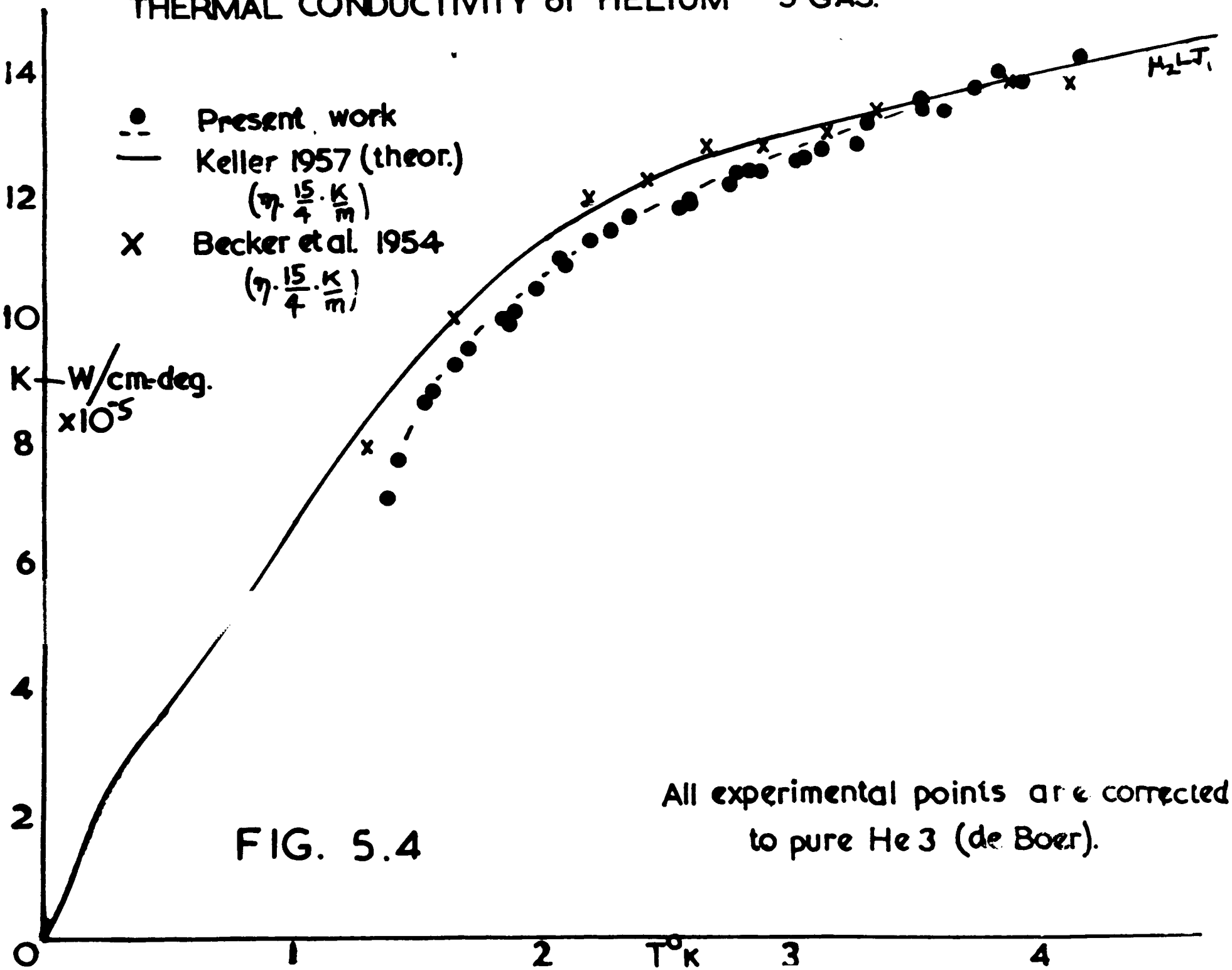


FIG. 5.4

All experimental points are corrected to pure He 3 (de Boer).

indicates that the effects of convection are negligible.

^{3}He gas results

The results for ^{3}He gas are shown in figure 5.3 together with the theoretical curve of $\frac{15}{4}\cdot\frac{a}{m}\cdot\eta_2 L_{J1}$ calculated by Keller. The ^{3}He contained 5.0% of ^{4}He and since the ^{4}He conductivity in this temperature range is considerably smaller than the ^{3}He conductivity, it is to be expected that the results for pure ^{3}He would be a few percent larger. Cohen et al. (1954) have calculated the viscosities and thermal conductivities of mixtures of ^{3}He and ^{4}He in terms of the values of the pure components and their findings have been confirmed, although for larger concentrations than we have used, by Becket et al. (1954) who measured the viscosity as a function of concentration at several temperatures. We have used the results of these calculations to determine the correction which should be applied to our values to obtain the conductivity of pure ^{3}He, and have also calculated the viscosity of pure ^{3}He from the results of Becker et al. for a mixture containing 6.0% ^{4}He (see Chapter 4).

The corrected values of the thermal conductivity are shown in figure 5.4 together with the values of the corrected viscosity measurements of Becker et al. and the theoretical curve of Keller ($\eta_2 L_{J1}$) multiplied by $\frac{15}{4}\cdot\frac{R}{m}$. It is seen that the agreement between the thermal conductivity and the viscosity is very good at the highest temperatures but becomes steadily worse as the temperature decreases although the good agreement between the theoretical curve of Keller and the corrected viscosity values still exists at the lower temperatures.

Our results confirm that there is a large difference between the behaviour of ^3He and ^4He and the general correctness of the calculation of the heat conductivity of ^3He. The reason for the disagreement in detail is not known with certainty but we consider various possibilities below.

If we assume our results at 4.0°K to be correct there seems no reason why the values at lower temperatures, should contain any but random error resulting only in scatter, because systematic errors due to the geometry of the cell are practically constant over the range 1.8° - 4.0°K. However, in our correction for the ^4He present we assumed the concentration to be that of the bulk fluid whereas, due to the effects of thermal diffusion, the temperature gradient between the cell and the small volume at room temperature is accompanied by a concentration gradient and so there is a larger concentration of ^4He in the cell than in the total volume. The cell being designed for use with liquid ^3He is very small and even when its volume at say 3.0°K is multiplied by the temperature ratio $\frac{300}{3} = 100$, the effective volume of the cell is only about 85 - 90% of the total volume. The coefficient of thermal diffusion has been calculated by Cohen, Offerhaus and de Boer (1954), and Buckingham and Scriven (1952) and is very sensitive to the form of potential used, but no confirmatory measurements have been made. Using the values of Cohen et al. we may calculate the difference in ^4He concentration between the helium in the cell and that in the volume at room temperature. At 3.0°K this is 3.5%. The concentration of ^4He in the cell is then $\left(5.0 + \frac{3.5r}{1+r}\right)\%$ where r is the ratio

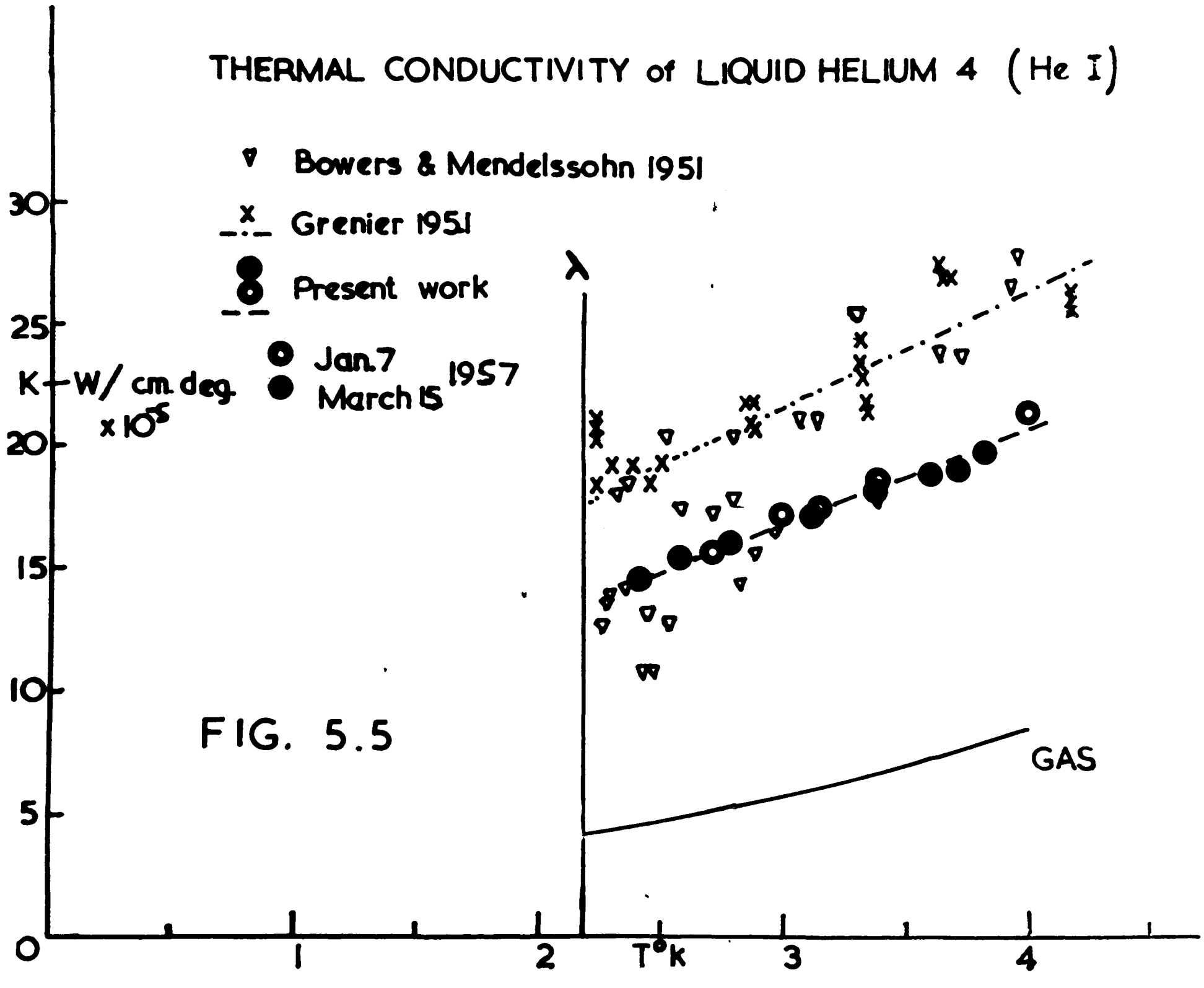

THERMAL CONDUCTIVITY of LIQUID HELIUM 4 (He I)
Bowers & Mendelssohn 1951
Grenier 1951
Present work
Jan.7
March 15
1957
K — W/ cm. deg. x 10^-5
λ
GAS
T°K
0
5
10
15
20
25
30
1
2
3
4

FIG. 5.5

of the mass of gas at room temperature to the mass of gas in the cell which we estimate to be about 0.13 at 3.0°K. Hence the concentration in the cell at 3.0°K is about 5.4% and a similar calculation shows that at 2.0°K it is about 5.5%. The corrections due to these increases are not sufficient to account for the difference between the experimental and theoretical values of the conductivity.

Possibly the first approximation factor $\frac{15}{4}\frac{R}{M}$ is no longer sufficiently accurate at low temperatures but as corrections have had to be applied to allow for the ^{4}He present, and for thermal diffusion using unconfirmed values of the diffusion coefficient, the evidence is not yet conclusive.

It was also found that the results were slightly dependent on pressure and independent of temperature difference, indicating that no convection is occurring. The dependence on pressure was again measured at the higher temperatures only. For ^{3}He the increase was about 2% for 25 cms. which is approximately 6% per atmosphere. This is similar to the increase in the conductivity of ^{4}He reported by Ubbink (1947) and also observed in the present work, and so it is probable that the pressure dependent thermal resistance between the copper and the gas is negligible at the values of pressures used.

Liquid ^{4}HeI

In figure 5.5 our values of the conductivity are shown together with those of previous workers (Bowers 1952; Grenier 1951; the point obtained by Keesom (1936) is not shown). The scatter of the points obtained by Bowers is too large to permit a detailed comparison but it appears that our values at the higher temperatures are

are lower than his. The values obtained by Grenier are less scattered and it is found that both sets of results have roughly the same temperature dependence but that our values are about 20% lower than his. Grenier estimates that his absolute values of the conductivity are accurate to within 10% and so it seems that the discrepancy is in our measurements. This is confirmed by some recent results by Lee, Donnelly and Fairbank (unpublished) which agree fairly closely with the values obtained by Grenier.

We estimate that our absolute values of conductivity are accurate to better than 7%. The potential error is made up of possible systematic errors in form factor and errors in the measurements and calculation of the heat input and temperature difference, but it precludes the possibility of a temperature jump at the copper-helium interface. It appears therefore that this jump, or boundary resistance, which does not occur in the methods of Bowers, and Lee et al., is eliminated by the method of Grenier, but is of necessity present in our apparatus, is responsible for the 20% difference in the results. A rough calculation of the boundary resistance at each copper-helium interface using the values of the earlier work together with our results shows that it probably remains constant over the temperature range from 2.5° to 4°K to within a factor of 2 or 3 and has a value of about 100 $cm^2 deg/watts$. or $Q = 10^{-2}$ $watts/cm^2$ deg. It is hoped to investigate this boundary resistance by using an arrangement similar to that used in the present experiments, as the scatter is less than in other methods, but to vary the distance between the copper blocks.

The results will be discussed in more detail under two separate headings : thermal conductivity and the boundary resistance which will include the results obtained for ^{3}He.

Thermal conductivity

In figure 5.5 which shows the results for ^{4}HeI, we have included values obtained for ^{4}He gas in the same temperature region. The curves for liquid and gas vary in a similar manner with temperature in direct contrast with all other substances with the exception of ^{3}He. Normally the liquid conductivity under the saturated vapour pressure always decreases rapidly with temperature while the gas conductivity rises, but because of the large molar volume of liquid helium (due to its zero point energy, Simon 1934), it is to be expected that helium I would behave more like a gas than most liquids do and the question as to whether all the properties of liquid helium can be interpreted on a pure gas-like model is worthy of consideration.

The earlier measurements on the viscosity showed that it rose steadily with temperature as in a gas and this, together with the similar rise in the conductivity, was taken to be strong evidence of gas-like behaviour .

Other evidence which has been cited in favour of the gas-like model is the small specific heat, C_v, which is even less than that of a monatomic gas, and the fact that the thermal conductivity, viscosity, and specific heat at constant volume obey (very roughly) a relationship of the form $K = \text{const.}\ \eta\, C_v$ (Grenier 1951, Taylor and Dash 1957). An exact relationship of this form would indicate that the mechanisms for

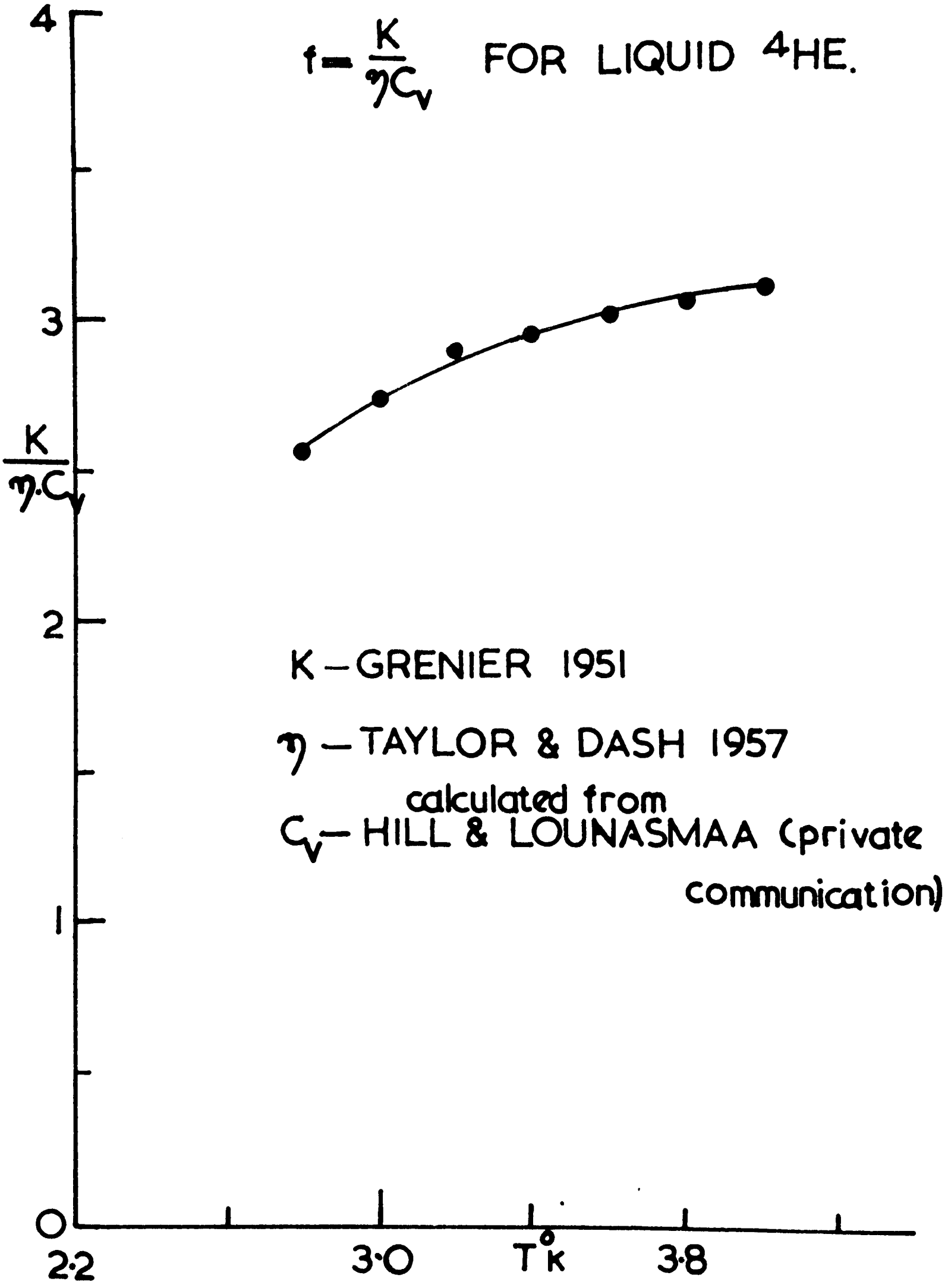

$f = \frac{K}{\eta C_v}$ FOR LIQUID ^{4}HE.
4
3
2
1
0
$\frac{K}{\eta . C_v}$
2·2
3·0
T°K
3·8
K – GRENIER 1951
η – TAYLOR & DASH 1957
calculated from
C_v – HILL & LOUNASMAA (private communication)

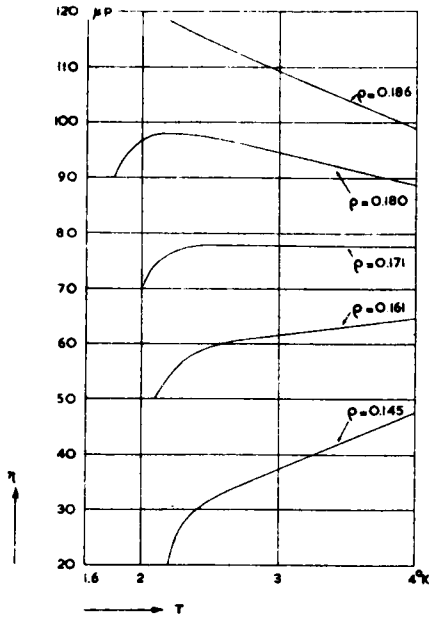
FIG. 5.6

heat transport and momentum transport are the same. In a monatomic gas, the relationship is accurately obeyed and the constant is very nearly 2.5. In figure 5.6, the ratio $K/\eta C_v$ is plotted as a function of temperature for liquid ^4He using the data of Grenier for K, Taylor and Dash for η (who find the temperature dependence of η which is now generally accepted) and some extrapolated values of C_v taken from the data of Hill and Lounasmaa (private communication).

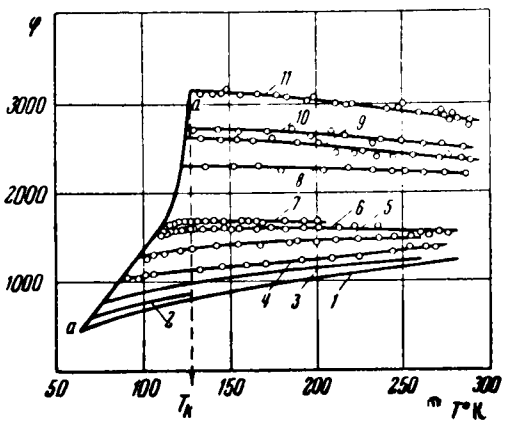
The viscosity and thermal conductivity values used are those at the saturation vapour pressure so that the specific heat C_v must also be the value at this pressure. It can be derived by slight extrapolation of the curves of C_v against density at that temperature. Such curves have been obtained recently by Hill and Lounasmaa and this method of deriving C_v was used in our analysis.

It is very important that C_v should be used in this calculation; Taylor and Dash (1957) have used C_{sat} in place of C_v and as this is 80% larger than C_v at 4°K, a considerable error is present in their values for f. Grenier used the best values of C_v available which were a few points at densities up to 20% greater than the vapour pressure (Keesom and Clusius 1932) but these differ by less than 4% from the required values over most of the temperature range. He found even larger values for f than ours as he used the data of Bowers and Mendelssohn (1949) in which η rises steadily with temperature. The viscosity above 3°K is now thought to fall slightly with increasing temperature unlike a gas (de Troyer, van Itterbeek and van den Berg 1951, Taylor and Dash 1957, Zinovieva - reported by Mendelssohn 1957).

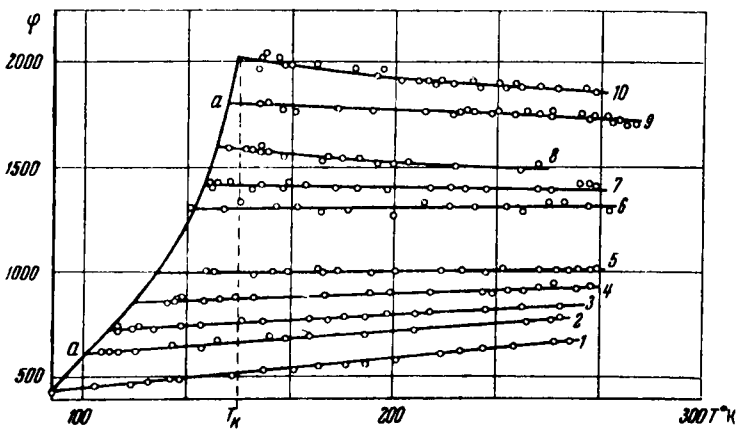
FIG. 5.7



The viscosity of liquid helium I as a function of the temperature at constant density.



Temperature dependence of viscosity in liquid nitrogen: *a* – under equilibrium vapor pressure; ρ equal to: *1* – 0.861, *2* – 0.832, *3* – 0.798, *4* – 0.746, *5* – 0.69, *6* – 0.63, *7* – 0.60, *8* – 0.50, *9* – 0.47, *10* – 0.43, *11* – 0.38 gm/cm^3.



Temperature dependence of fluidity of liquid argon in equilibrium with its vapor pressure; *1* – ρ = 1.37 g/cm^3; *2* – 1.31 g/cm^3; *3* – 1.22 g/cm^3; *4* – 1.16 g/cm^3; *5* – 1/10 g/cm^3; *6* – 1.02 g/cm^3; *7* – 0.95 g/cm^3; *8* – 0.88 g/cm^3, *9* – 0.78 g/cm^3; *10* – 0.70 g/cm^3

In view of these later viscosity measurements and the fact that our results indicate that a large thermal boundary resistance exists in ^{4}HeI unlike a gas (see next section) it appears that a short analysis of the strength of the evidence for a gas-like picture would not be inappropriate.

The temperature variation of viscosity and thermal conductivity at constant density is much more significant than the variation at the saturation vapour pressure or at constant pressure. The variation with temperature at constant pressure is due more to the expansion of the liquid than a change in the mechanism of transport and this expansion can obscure differences in mechanisms. Measurements have been made of the temperature variation of viscosity of ^{4}HeI by Tjerkstra (1952) and have been plotted both as a function of pressure and of density, and the temperature variations of viscosity at constant density of liquid argon and nitrogen have also been obtained (Zhdanova 1957). The values at constant density are shown in figure 5.7; the argon and nitrogen results are for fluidity $\phi = \frac{1}{\eta}$. The curves for argon, nitrogen and helium are very similar : each is 'gas-like', that is to say $\left(\frac{\partial \eta}{\partial T}\right)_\rho > 0$, at low densities but $\left(\frac{\partial \eta}{\partial T}\right)_\rho$ decreases to zero and at high densities becomes negative.

Zhdanova suggests that this behaviour is common to a number of simple liquids : it is tempting to suppose it to be a completely general result and as Andrade points out (1934), J.D. van der Waals Jr. postulated in 1918 that $\left(\frac{\partial \eta}{\partial T}\right)_\rho$ should be positive for all liquids at low densities. However, Bridgman (1931) has shown that for a number of organic liquids and mercury $\left(\frac{\partial \eta}{\partial T}\right)_\rho < 0$ at room tempera-

tures at least. The properties of all but the simple liquids are obscured by association, polar bonding and other complications and no attempt is made in this thesis to do more than to quote a few general results for these liquids. From the evidence of the viscosities it appears then that the mechanism of momentum transport in liquid ^{4}HeI is the same as in other simple liquids with the comment that the density is small due to zero point energy.

No measurements have been made on the thermal conductivity at constant density of nitrogen, argon or helium. Calculations of $\left(\frac{\partial K}{\partial T}\right)_\rho$ have been made for liquid argon from measurements of $\left(\frac{\partial K}{\partial T}\right)_p$ for a large range of pressure and $\left(\frac{\partial P}{\partial T}\right)_\rho$ (Uhlir 1952). They show that the conductivity is nearly constant with temperature though has a slight negative slope, but it is not certain whether the accuracy of the calculation is such to exclude the possibility of a slight positive slope. The isobaric values for liquid helium (Grenier etc) are very different from those of argon or nitrogen in which K drops very rapidly with temperature (Uhlir 1952, Burton and Ziebland 1957) but the significance of this may again be connected with the expansion of the liquid rather than the mechanism of heat transport. It seems very likely though that $\left(\frac{\partial K}{\partial P}\right)_T$ for helium would be greater than zero so that, as in the viscosity, the thermal conductivity at constant density would still rise with temperature as in a gas.

Finally we come to the parameter $f = K/\eta C_v$. It should be noted that for a large number of liquids, except when conduction is electronic as in liquid metals, f has a value between 3.3 and 0.34

(Enskog - see Chapman and Cowling 1952) and it is apparent for most of these liquids that transport of momentum and of heat cannot be by the same process; for example, the addition of a few drops of glycerine to water does not affect the thermal conductivity but can considerably increase the viscosity, and again, as the melting point is approached $\eta \to \infty$ while K remains finite. Consequently, the significance of the magnitude of f should not be over-estimated. For comparison with our values for liquid ^{4}HeI we have calculated f for liquid argon but there is a shortage of information regarding C_v so that some extrapolation was necessary. f rises at constant pressure from 1.5 at 120°K to 1.97 at 140°K.

The evidence indicates then that at low densities the transport properties of liquid helium I, argon and nitrogen are not dissimilar from those of a gas. Liquids are usually assumed to be in a state between that of a gas and that of a solid; the viscosity of a gas rises with temperature and , in a quasi-solid is calculated to fall with temperature as $e^{-A/T}$ (see for example Andrade 1934, Frenkel 1946) so that if a gradual transition from gas to solid were to take place we should expect the temperature variation of viscosity to be very similar to that actually observed in these simple liquids as the density is increased.

The change in the properties of a gas as the density is increased has been considered by Enskog (see Chapman and Cowling 1952). Enskog's theory however applied to hard spheres and since the solid-like behaviour of the liquids at high densities is almost certainly due to the intermolecular binding, it is not surprising that the theory fails

show this change in behaviour. The theory is in moderate agreement with experiment for compressed gases (Michels et al. 1931, 1953a, 1953b and 1954) and has also been applied to liquid argon and nitrogen by Uhlir with success. The theory appears to describe the thermal conductivity better than the viscosity but as Uhlir points out this may be because the measurements of the viscosity available to him were mainly at higher densities than those used for the thermal conductivity. It would be interesting to see a comparison between the theory with the recent results of Zhdanova.

Enskog derived the expressions :

$$\eta = \eta_0 \left\{ \frac{1}{\chi} + 0{\cdot}8000 b\rho + 0{\cdot}7614 (b\rho)^2 \chi \right\}.$$

$$K = K_0 \left\{ \frac{1}{\chi} + 1{\cdot}2000 b\rho + 0{\cdot}7574 (b\rho)^2 \chi \right\}$$

η_0 is the viscosity and K_0 the conductivity of the gas at low density.

Classically $\eta_0 = \frac{5}{16\sigma^2} \cdot \left(\frac{mkT}{\pi} \right)^{1/2}$

$b = \frac{2}{3}\pi N \sigma^3$ the excluded volume of the molecules

ρ is the density of the gas

χ is a collision parameter which has a value of 1 at low densities and increases with density.

Tjerkstra has applied this expression for the viscosity to helium I. The expression is for a hard spheres gas and to be consistent he has assumed the classical hard sphere values for η_0 - the gas viscosity. Since this is 35% higher at 2.5°K than the measured values of Becker et al. the good agreement he obtains with the experimental values by varying one parameter only must be regarded as largely

fortuitous. With the considerable departures from classical theory existing at these temperatures, a further quantitiative analysis using the classical Enskog theory would not be very profitable; qualitatively the classical theory indicates that $\left(\frac{\partial \eta}{\partial T}\right)_\rho = \left(\frac{\partial \eta_0}{\partial T}\right)_\rho$ so that as we have mentioned the theory is incapable of showing a change in sign of $\left(\frac{\partial \eta}{\partial T}\right)_\rho$ such as is found in the simple liquids.

Tjerkstra also shows that the Andrade expression for the viscosity near the melting point $\eta = \frac{4}{3}\frac{\lambda m}{a}$ where λ is taken as $\frac{k}{h}\Theta$ - the Debye Θ, m the molecular mass and a the intermolecular distance, is obeyed well by helium as in a large number of other liquids (see Andrade 1952).

The Andrade expression $\eta \propto e^{-4/T}$ is also in agreement with experiment for a number of liquids including the simple liquids (e.g. argon and nitrogen, Zhdanova 1957).

Hence it appears that the transport properties of compressed gases and simple liquids are qualitatively understood and at high and low densities semi-quantitative agreement with the two limiting theories of gas-like and solid-like models is obtained.

A recent theory of Taylor and Dash to account for the viscosity of ^{3}He and ^{4}He assumes the kinetic energy of the helium atoms to be $E = E_o + 3/2\ kT$ where E_o is the zero point energy, and may be calculated from the known density by the London formula (1936). η is then calculated from the kinetic relationship

$$\eta = \frac{m\bar{v}}{2\sqrt{2}\pi\sigma^2} = \frac{(\bar{E}m)^{1/2}}{2\pi\sigma^2}$$

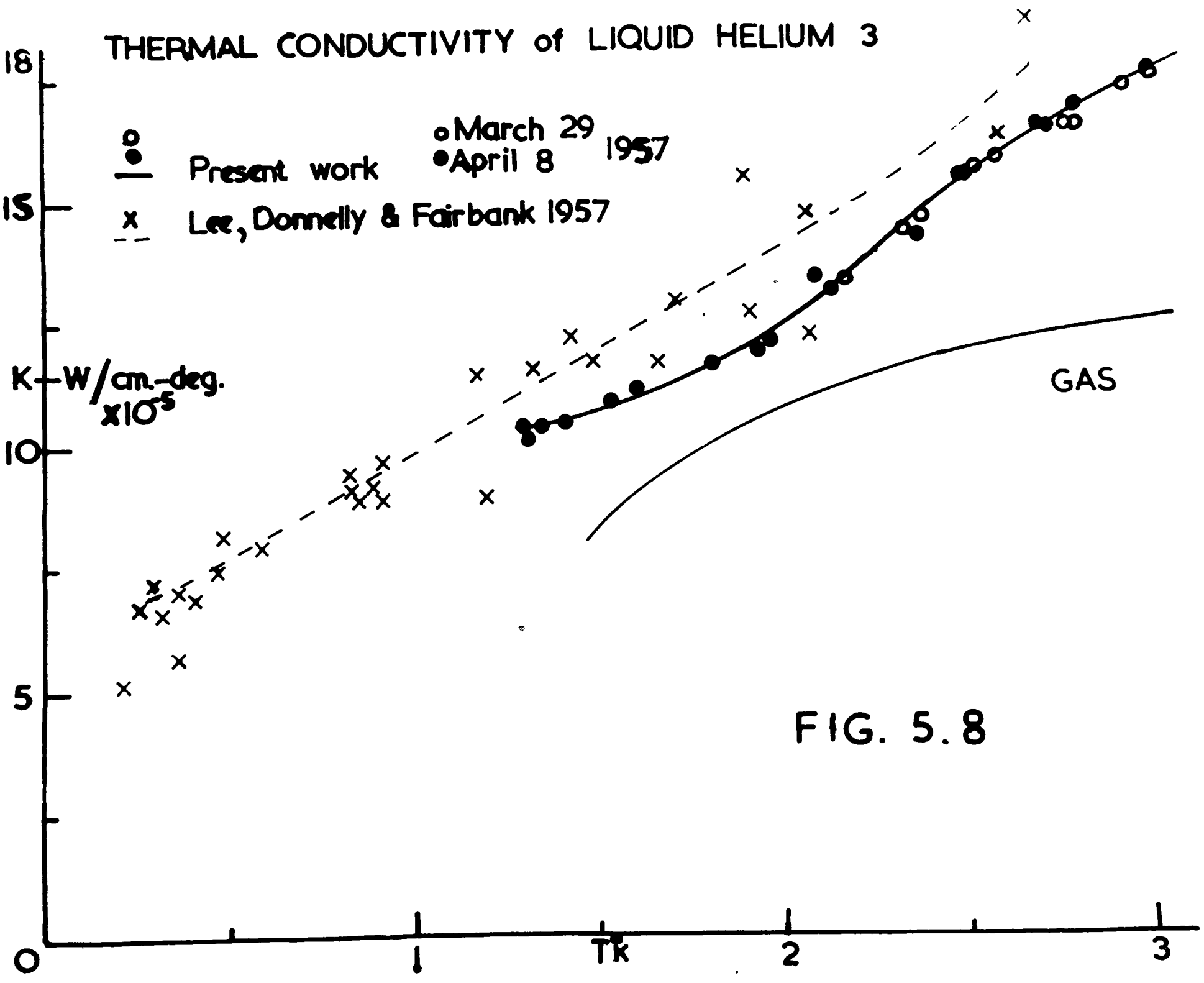

THERMAL CONDUCTIVITY of LIQUID HELIUM 3
o March 29
• April 8 1957
Present work
x Lee, Donnelly & Fairbank 1957
K - W/cm.-deg. x10⁻⁵
GAS
18
15
10
5
0
1
Tk
2
3

FIG. 5.8

$\pi\sigma^2$ is the collision cross-section. By normalising this expression for ^{4}HeI at one temperature, they obtain good agreement with experiment above 2.5°K over the rest of the temperature range, and also a value for $\pi\sigma^2$ of $1.77.10^{-15}$ cm^2. This value is very reasonable and is in almost exact agreement with the value of the effective atomic diameter used by London to explain best the latent heat and density data. The model breaks down at higher densities where no change in sign of the calculated $\left(\frac{\partial m}{\partial T}\right)_\rho$ is observed. Unfortunately, the agreement with the ^{3}He results is not so good.

Finally it should be mentioned that our results show no evidence of an increase in conductivity near the λ-point. Below the λ-point heat is transported through the helium with great efficiency and it is of interest to discover whether any increase in thermal conductivity occurs just above the λ-point. The results of Grenier show no large increase occurring down to 2.24°K and our results indicate that there is no increase greater than about 3% down to 2.41°K. This is in contrast with the viscosity which starts to fall more rapidly at 3.0°K, and the specific heat which starts to rise at 2.5°K.

Liquid ^{3}He

In figure 5.8 our values of the conductivity are shown together with those of Lee, Donnelly and Fairbank (1957). We note that our values are significantly lower, and in this case by about 15%. We attribute this to the boundary-resistance at the copper-helium interface occurring in our method and this conclusion has recently been confirmed by H.A. Fairbank and Lee (1957), who have measured the resis-

tance down to 0.25°K. The value of the resistance calculated from the difference in measured conductivities is about 200 cm^2/deg watts at 2.0°K and 100 cm^2/deg watts at 2.5°K but this difference could be due to scatter. The value given by Fairbank et al. is $130/T^2$ cm^2/deg watts. The values of the conductivity given by Lee et al. do not include a boundary resistance since a method similar to that used by H.A. Fairbank and Wilks to measure the conductivity of liquid ^{4}He was employed.

We shall discuss the conductivity separately and the temperature jump together with the results obtained for ^{4}He.

Thermal conductivity

In the section on the thermal conductivity of liquid ^{4}HeI we compared the values obtained with those of the gas at the same temperature and the figure 5.8 showing the measurements made on liquid ^{3}He also shows the curve obtained for the gas. It is noticeable that the curves for ^{3}He liquid and gas lie very much closer together than those for ^{4}HeI and this is in accord with the supposition that ^{3}He and ^{4}He behave like compressed gases : ^{3}He has a larger molar volume than ^{4}He due to the effect of zero point energy. It is informative to consider the behaviour of ^{3}He in terms of the theoretical predictions for the transport properties.

Calculations have been made of these properties on the assumption that the liquid behaves like a non-degenerate Fermi-Dirac gas (Buckingham and Temperley 1950) and like a degenerate gas (ter Haar and Wergeland 1949; Singwi and Kothari 1949; Pomeranchuk 1950). The conduct-

ivity of a simple or non-degenerate gas of the density of liquid ^3He would be larger than the conductivity of the same gas at lower densities. On the Enskog model for hard spheres, an increase in viscosity or conductivity due to the increased density occurs because transport of either momentum or heat takes place not only by diffusion but also because the transfer at collisions is of importance. Allowance is made for the increase in the probability of collisions due to the finite size of the molecules being no longer negligible compared with the volume occupied by the gas, and the decrease due to the shielding of one molecule by another. These are essentially non-quantum effects : at low temperatures when the de Broglie wavelength associated with an atom becomes larger than the atomic diameter, a new form of congestion is possible. This is called degeneracy and is possible for molecules of either finite or point size. A brief discussion of the properties of liquid ^3He on the assumption that it is a degenerate Fermi-Dirac gas has been given in the Introduction.

ter Haar and Wegeland calculate η on this assumption from the kinetic expression

$$\eta = \alpha_\eta \cdot \tfrac{1}{3}\rho \bar{c} \lambda$$

where α_η is a numerical factor.

They assume $$\lambda = \frac{m}{\sqrt{2} \cdot \pi d^2}$$

This expression is modified by Singwi and Kothari, and Pomeranchuk who account for the increase in the mean free path at low temperature because collisions are limited by the exclusion principle. The expressions obtained by Singwi and Kothari are :-

S is the collision X-section

$$K = \alpha_K \cdot \frac{\pi^2}{12} \cdot \frac{k^3}{m^2} \cdot \frac{3\rho}{8\pi m} \cdot \frac{1}{ST}.$$

$$\eta = \alpha_\eta \cdot \frac{1}{3} \cdot \frac{k^5}{2^{5/2} \cdot m^2} \cdot \left(\frac{3\rho}{8\pi m}\right)^{5/3} \cdot \frac{1}{S} \cdot \frac{1}{(kT)^2}$$

Our values of thetmal conductivity (which include boundary resistance) decrease with decreasing temperature but the slope is becoming less at the lower temperatures. Lee et al. draw a straight line through their points which extend to lower temperatures but there is sufficient scatter to permit slight departures from this linearity; there is however a steady decrease in thermal conductivity down to 0.3°K. It would appear then that the ideal degenerate Fermi-Dirac model is agin inadequate. The viscosity has also been measured by Weinstock, Osborne and Abraham (1949), Taylor and Dash (1957), Zinovieva - (reported by Mendelssohn 1957). Above 1.0°K it decreases slowly with increasing temperature vaying as about $T^{-0.4}$ but at lower temperatures it decreases more rapidly (Zinovieva). The decrease with temperature above 1°K cannot be taken as evidence for degeneracy. As has been explained earlier, the variation of the viscosity under the vapour pressure can be rather misleading and the viscosity of liquid ^{3}He at constant density may well rise with temperature as in liquid ^{4}He, argon and nitrogen. Below about 1°K where the expansion coefficient is smaller and the viscosity decreases more rapidly with temperature, the behaviour would probably be similar at constant density so that there may be a minimum in the viscosity-temperature curve at constant density. It appears then that the behaviour below 1°K is probably evidence of degeneracy but it is

$f = \frac{K}{\gamma C_v}$ FOR LIQUID ^{3}HE

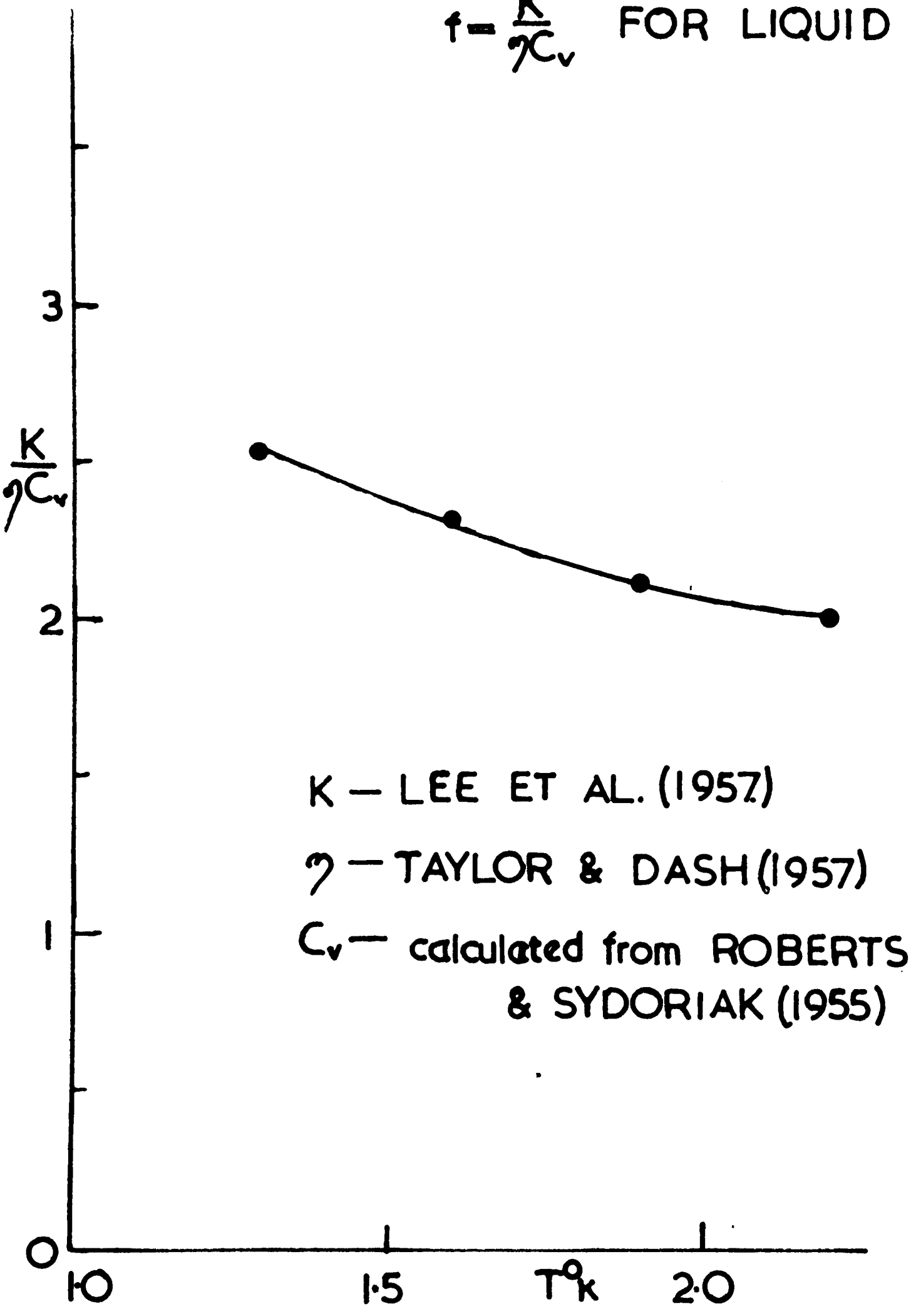


FIG. 5.9

perhaps surprising that this is not reflected in the thermal conductivity. The parameter $f = \frac{K}{\eta C_v}$ is plotted in figure 5.9 where K has been taken from the data of Lee et al. and η from the data of Taylor and Dash. C_v has not been measured and must be calculated from C_{sat}.

In comparisons between the specific heats obtained experimentally and those derived from the consideration of a theoretical model, C_{sat} has hitherto been used throughout the literature because although of course it was realised that a correction should be applied insufficient data was available for this to be possible. Abraham, Osborne and Weinstock (1955) pointed out that as $C_{sat} > C_v$, any correction is in a direction that makes the disagreement with the ideal gas model even larger and Weinstock (unpublished) has calculated $(C_p - C_v)$ at 1.2°K from

$$C_p - C_v = \frac{TV\alpha^2}{\beta}$$

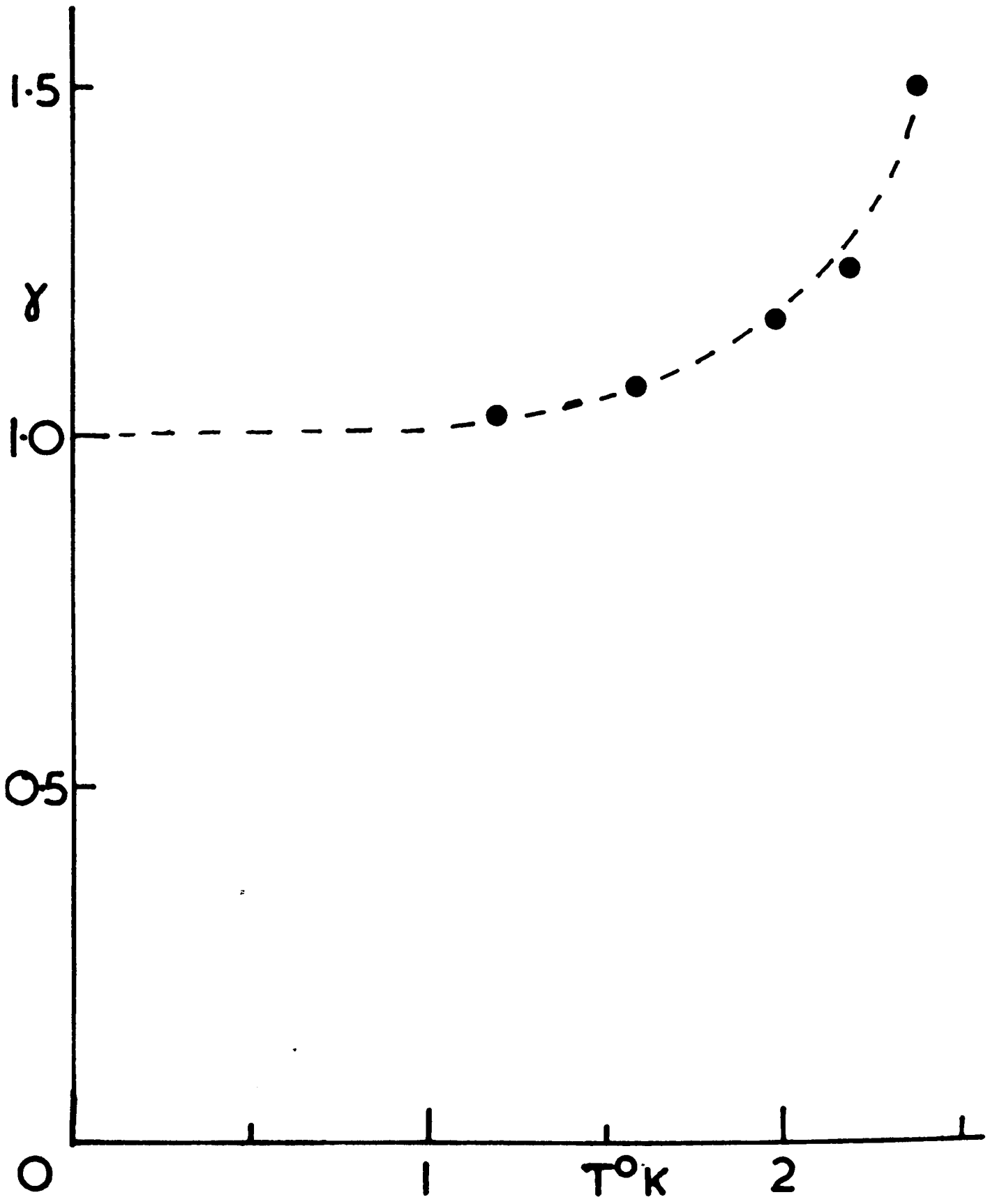
where $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$, $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$

and α can be calculated from the density values of Kerr (1954) and β has been measured (Walters and Fairbank 1956) at 1.2°K. C_{sat} is equal to C_p to a fair degree of approximation. The value of $\frac{C_p - C_v}{C_v}$ at 1.2°K is about 2%.

Recently fresh data has become available. The adiabatic compressibility may be calculated from the velocity of sound (see for example, Laquer, Sydoriak and Roberts 1957) and as $\frac{C_p}{C_v} = \frac{\beta_T}{\beta_S}$

γ — ^{3}HE LIQUID

FIG. 5.10



$$C_p - C_v = \frac{TV\alpha^2}{\beta_S\left(\frac{C_p}{C_v}\right)} \quad \text{or} \quad \gamma - 1 = \frac{TV\alpha^2}{C_p \cdot \beta_S}.$$

and as $c^2 = \frac{1}{\rho \beta_S}$ we get

$$\gamma - 1 = \frac{TV\alpha^2 c^2 \rho}{C_p} = \frac{TM\alpha^2 c^2}{C_p}$$

Unfortunately $(C_p - C_v)/C_v$ becomes larger at higher temperatures being 16% at 2°K and 49% at 2.4°K. Our calculated values are shown in figure 5.10. It seems then that the entropy curves should be recalculated in view of these differences although fortunately this does not affect the entropy at low temperatures where the entropy is of the order of Rln2.

With these values of C_v, f is seen to vary from 2.6 to 2.0. The significance of this parameter has already been discussed for ^{4}He and it appears that the same conclusion, that this is not in contradiction with a compressed gas-like model, is obtained.

It is interesting to note that it has been suggested that phonons should be present in liquid ^{3}He (see for example Landau 1957). At sufficiently low temperatures, the mean free path of these phonons could be limited by the apparatus dimensions to give an approximate T^3 size dependent conductivity as in liquid ^{4}He below 0.6°K (Fairbank and Wilks, 1954, 1955; Whitworth, 1955). The phonon conductivity is given by

$$K = \tfrac{1}{3} C_v \bar{c} \lambda$$

$$\text{and } C_v = \frac{16\pi^5 k^4}{15 h^3 c^3} \cdot T^3$$

$$\therefore K_{ph} = \frac{16\pi^5 k^4}{15 h^3 c^3} \cdot \frac{1}{3} \cdot c\lambda T^3 = \frac{16\pi^5 k^4}{45 h^3} \cdot \lambda c^{-2} T^3$$

$$\text{and } \therefore \frac{K_{ph}\,{}^3He}{K_{ph}\,{}^4He} = \frac{c_4^2}{c_3^2} = \frac{237^2}{183^2} \sim 1.7$$

and in a tube of 0.3 mms. diameter, $K_{ph}\,{}^4He = 0.03$ watts/cm deg at 0.3°K so that $K_{ph}\,{}^3He = 1.9\ T^3$ watts/cm deg.

At 0.1°K $K_{ph} = 1{\cdot}9.10^{-3}$ watts

0.037°K $K_{ph} = 10.10^{-5}$ watts/cm deg which is equal to the total conductivity measured at 1.3°K. The conductivity certainly decreases to 0.3°K (Lee et al) and if it were certain that it decreased steadily to absolute zero, it could be said that a pure phonon region would not be observed in 0.3 mm. tubing above a temperature considerably less than .037°K.

Boundary resistance in ^3He and ^4He liquids

A boundary resistance is now known to exist in ^4HeII (Kapitza 1941; White, Gonzales and Johnston 1953; Beenakker, Taconis, Lynton, Dokoupil and van Souest 1952; H.A. Fairbank and Wilks 1955; Dransfeld and Wilks 1957; and Andronikashvilli and Mirskaya 1955) and in ^3He (Fairbank and Lee 1957); our results indicate that a similar resistance exists in ^4HeI. The magnitudes of the resistances in ^4HeII, ^3He and ^4HeI are all of the same order. The temperature and density dependence for ^4HeI and ^3He is not known - the measurements of Lee and

Fairbank for ^{3}He are scattered - but a T^2 dependence has been fitted to the points. The existence of this temperature jump in ^{4}HeI and ^{3}He would appear to rule out theoretical treatments based on the two-fluid model in which the temperature jump is assumed to be due to the conversion of the normal to the superfluid taking place at the interface (Gorter, Taconis and Beenakker 1951, Kronig and Thellung 1950). A second approach is due to Khalatnikov (1952) and independently by Masu and Onsager (unpublished).

In his treatment, Khalatnikov shows that heat transfer can take place from a solid body to liquid HeII in contact with it, by the radiation of phonons from the wall and by the loss or gain of energy of phonons or rotons in inelastic collision with the wall (processes analogous to the accommodation effects occurring in a gas). He shows that at temperatures below the λ-point, the radiation effect predominates over the accommodation processes and so the resistance on this theory is probably a particular case of the contact resistance occurring between two solids in which the excitations are phonons (see for example Mendoza 1948, Robinson 1954). The resistance arises essentially because of acoustic mismatch at the interface, the phonons radiated into the helium above 0.6°K being mainly converted into rotons.

The calculation consists of determining the velocity of vibration of the solid material normal to the surface and to do this, the assumption is made that the interface is between solid and vacuum and is well justified because of the small density of helium. The energy carried away in unit time is

$$\dot{Q} = \rho_H c_H \int |\dot{u}_n|^2 dS$$

P_H = helium density

C_H = velocity of sound in helium

which for a plane boundary becomes $\dot{q}(\omega) = \rho_H c_H |\dot{u}_n|^2$ for the rate of flow of energy of phonons of energy $\hbar\omega$, and to the total energy flowing from the body to the helium is found by integrating $\dot{q}(\omega)$ multiplied by the appropriate distribution function. The amount flowing the opposite way is also found and subtracted.

$$\Delta W = \frac{\rho_H c_H}{\rho_T} \cdot \frac{16\pi^5}{15} \cdot \frac{(kT)^3}{(2\pi\hbar c_t)^3} \cdot F(\eta) \cdot k\Delta T$$

is a parameter calculated from the elastic constants of the body and F is a given function. For glass $F(\eta) \approx 2.5$ and for platinum $F(\eta) \approx 2.0$. The magnitude of this expression is roughly correct but variation of the heat flow is T^n where n = 3, whereas experiment shows that below 1°K n = 2 (H.A. Fairbank and Wilks 1955) and above 1°K in most of the later experiments (White et al., Dransfeld and Wilks) n was usually between 2 and 3. In the experiments of Dransfeld et al. interfaces between HeII and a number of metals and a dielectric were investigated at different pressures, and with copper-helium boundaries, measurements were made using both polished and etched copper surfaces. The variation in the resistance between solids of widely differing elastic properties is considerably smaller than that predicted by the acoustical model of Khalatnikov and indeed is no larger than that produced by

changing from polished copper to etched copper. Accurate comparisons are probably not possible because of the uncertainty in the effective areas of each surface. Another discrepancy with the theory is the variation of the resistance with the helium density at any temperature; the experiments of Dransfeld and Wilks show that between 1.3°K and the λ-point the resistance is practically density independent in complete disagreement with the theory although changes of about 25% were possible.

We suggest that the Khalatnikov treatment, which considers a continuous medium of density ρ_H, should be modified to allow for the presence of the dense adsorbed layer of helium found at the interface. The dense layer is due to van der Waals' forces between the solid and the helium atoms. The net force on any atom due to a plane surface of infinite extent is obtained by integrating the forces over the surface and varies as r^{-4} where r is the perpendicular distance from the plane, so that the density and pressure fall off as the distance from the surface increases. The exact thickness of the layer is not easy to define since the density falls continuously from that of a solid to that of the bulk liquid, but is usually taken to be about 4 atomic layers ($\sim$10 - 15 Å).

The possible frequencies of vibrations of the surface of the metal range from 0 - ν_m where ν_m for copper is $6{\cdot}6.10^{12}$ cps, but at low temperatures the energy radiated at the highest frequencies is negligible. The energy spectrum at temperature T has a fairly high peak at the frequency

$$\nu \sim \frac{3kT}{h} \qquad \text{(Ziman 1956)}$$

so that vibrations are induced in the helium mainly at frequencies of this order. Now the Debye temperature Θ is defined by $k\Theta = h\nu_m$ so that

$$\nu \sim 3 \cdot \left(\frac{T}{\Theta}\right) \cdot \nu_m$$

$$\sim 3 \cdot \left(\frac{T}{\Theta}\right) \cdot 6 \cdot 8 \cdot 10^{12}$$

and for copper $\Theta \sim 300^\circ K$ so that at $3.0^\circ K$

$$\nu \sim 3 \cdot \left(\frac{3}{300}\right) \cdot 6 \cdot 8 \cdot 10^{12} \sim 2 \cdot 10^{11} \text{ c.p.s.}$$

This corresponds to a wavelength in the helium of $\frac{2 \cdot 4 \cdot 10^4}{2 \cdot 10^{11}}$ cms. or 12 Å which is the order of the three or four atomic distances.

It would seem then that since the energy is radiated into the dense layer the effect of this layer of thickness about a wavelength cannot be ignored. If a formula of the form $\dot{q}(\omega) = \rho_H c_H |\dot{u}_n|^2$ is still applicable, the values for ρ_H, c_H should be for the dense layer rather than the bulk liquid. The problem is complicated however as the density and velocity are varying rapidly in a distance comparable with the wavelength so that the normal wave equation may not be applicable.

A modification of the equation for $\dot{q}(\omega)$ of the type indicated above would explain qualitatively certain features of the experiments which are in disagreement with the theory.

For example, the density ρ_H to be used in the formula is large - corresponding to solid helium or liquid helium under pressure - so that a considerable change in pressure on the bulk liquid would produce a much smaller fractional change in the density of this solid or

nearly solid helium than in the bulk liquid and this would explain why the boundary resistance is less dependent on pressure than indicated by Khalatnikov.

The explanation for the T^2 or T^n dependence of the energy flow, where n lies between 2 and 3, would seem to be due to the longer thermal waves being preferentially transmitted (Fairbank and Wilks 1955). In view of the magnitude of the frequencies involved, this is not surprising. The frequencies higher than the frequency $\nu = 2.10^{11}$ cps result in wavelengths of the order of, or less than, interatomic distances which would perhaps result in their being emitted less efficiently than the smaller frequencies.

Appendix 1.

In this appendix, we derive the expression $\delta R' = \Delta T \left(\frac{dR}{dT}\right)_{T''}$ used in calculating the results (see Chapter 4). We consider the specimen of uniform cross-section shown in figure A.1. Two thermometers of resistance R and R′ are attached to the specimen as shown and heat Q may be supplied at the upper end of the specimen. The temperature of each thermometer is denoted by a subscript e.g. R_T'. Two situations are considered (a) when no heat is supplied or $\dot{Q} = 0$, which corresponds to conditions during calibration and (b) when a heat input is supplied or $\dot{Q} \neq 0$ and the temperatures of the two thermometers are no longer equal; this corresponds to the conditions during a measurement.

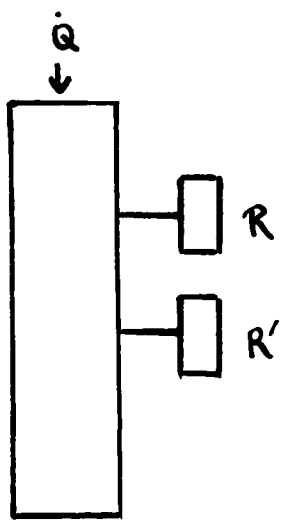


Fig A.1

(a) $\dot{Q} = 0$

R and R − R′ are measured as f(T).

(b) $\dot{Q} \neq 0$

R and R − R′ are measured. If the temperatures of R and R′ are T and T′ then the mean temperature of the specimen is

$$T'' = \tfrac{1}{2}(T + T')$$

Now $$R_T = R_{T''} + (T-T'')\left(\frac{dR}{dT}\right)_{T''} + \tfrac{1}{2}(T-T'')^2\left(\frac{d^2R}{dT^2}\right)_{T''} + \cdots$$

and $$R'_{T'} = R'_{T''} + (T'-T'')\left(\frac{dR'}{dT}\right)_{T''} + \tfrac{1}{2}(T'-T'')^2\left(\frac{d^2R'}{dT^2}\right)_{T''} + \cdots$$

$$\therefore \left(R_T - R'_{T'}\right) - \left(R_{T''} - R'_{T''}\right) = (T-T')\left(\frac{dR}{dT}\right)_{T''} + \frac{1}{3}\left(\frac{T-T'}{2}\right)^3\left(\frac{d^3R}{dT^3}\right)_{T''} + \cdots (1)$$

we have assumed that, for these matched resistors,

$$\left(\frac{dR}{dT}\right)_{T''} = \left(\frac{dR'}{dT}\right)_{T''}, \left(\frac{d^2R}{dT^2}\right)_{T''} = \left(\frac{d^2R'}{dT^2}\right)_{T''}, \left(\frac{d^3R}{dT^3}\right)_{T''} = \left(\frac{d^3R'}{dT^3}\right)_{T''}$$

$(R_T - R'_{T'})$ is the quantity measured during a measurement which we call $\Delta R'$. $(R_{T''} - R'_{T''})$ is the quantity $(R - R')$ which is taken from the calibration plot of $(R - R')$ against temperature at temperature T''. We call this ΔR.

$R_T - R'_{T'} = \Delta R'$

$R_{T''} - R'_{T''} = \Delta R$

and we let $\delta R' = \Delta R' - \Delta R$

$\Delta T = T - T'$

So from (1)

$$\delta R' = \Delta T\left(\frac{dR}{dT}\right)_{T''} + \left(\frac{d^3R}{dT^3}\right)_{T''}\frac{\Delta T^3}{24} + \cdots$$

and with the thermometers used in this work, the second term is negligible for $\Delta T < 0.1\ T$. With this restriction which was maintained in the present work

$$\delta R' = \Delta T\left(\frac{dR}{dT}\right)_{T''}$$

Appendix 2

(a) Heat conduction between two concentric cylinders

The result of this section will be used in the second part of this Appendix which deals with the calculation of the heat flow in the apparatus used in the experiments.

We shall consider the radial strip of length δy and radius a shown in figure A.2.

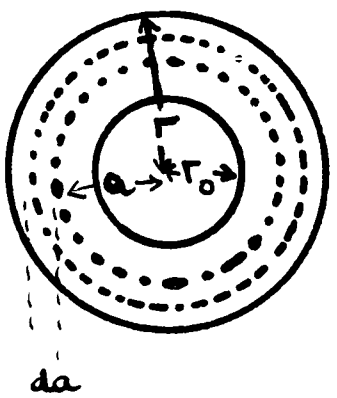


Fig A.2

$$\dot{Q} = K.2\pi a.\delta y.\left(\frac{dT}{da}\right)$$

$$\therefore \frac{\dot{Q}}{K.2\pi.\delta y}\int_{r_0}^{r}\frac{da}{a} = \int_{T_0}^{T} dT$$

$$\therefore \frac{\dot{Q}}{K.2\pi.\delta y}.\log_e r/r_0 = T-T_0 \quad \text{or} \quad \dot{Q} = \frac{K.2\pi\delta y(T-T_0)}{\ln r/r_0}$$

We shall show that the following approximation is justified.

$$\dot{Q} = K.2\pi r'.\delta y.\left(\frac{T-T_0}{l}\right) \qquad -(1)$$

where $r' = \frac{r+r_0}{2}$ mean radius

$l = r-r_0$ the gap

Now $\log_e r/r_0 = \log_e\left[1+\left(\frac{r-r_0}{r_0}\right)\right]$

$$= \left(\frac{r-r_0}{r_0}\right) - \frac{1}{2}\left(\frac{r-r_0}{r_0}\right)^2 + \frac{1}{3}\left(\frac{r-r_0}{r_0}\right)^3 + \cdots$$

$$= \frac{r-r_0}{r_0}\left\{1 - \frac{1}{2}\left(\frac{r-r_0}{r_0}\right) + \frac{1}{3}\left(\frac{r-r_0}{r_0}\right)^2 \cdots\right\}.$$

and $l/r' = \frac{2(r-r_0)}{r+r_0} = \frac{2(r-r_0)}{2(r_0+\frac{r-r_0}{2})} = \left(\frac{r-r_0}{r_0}\right)\left(1+\frac{r-r_0}{2r_0}\right)^{-1}$

$$= \frac{r-r_0}{r_0}\left\{1 - \frac{1}{2}\left(\frac{r-r_0}{r_0}\right) + \frac{1}{4}\left(\frac{r-r_0}{r_0}\right)^2 + \cdots\right\}.$$

In the apparatus used

$$r - r_o = 3{\cdot}77.10^{-2} \text{ cms.}$$

$$r_o = 3{\cdot}60.10^{-1} \text{ cms.}$$

so that $\frac{r-r_o}{r_o} \sim 0{\cdot}1$ and the approximation is valid to about 0.3%.

(b) Heat conduction in the cell

The conductivity cell has a cross-section of the form of an H. In figure A.3, the whole cross-section and details of the cross-section are shown.

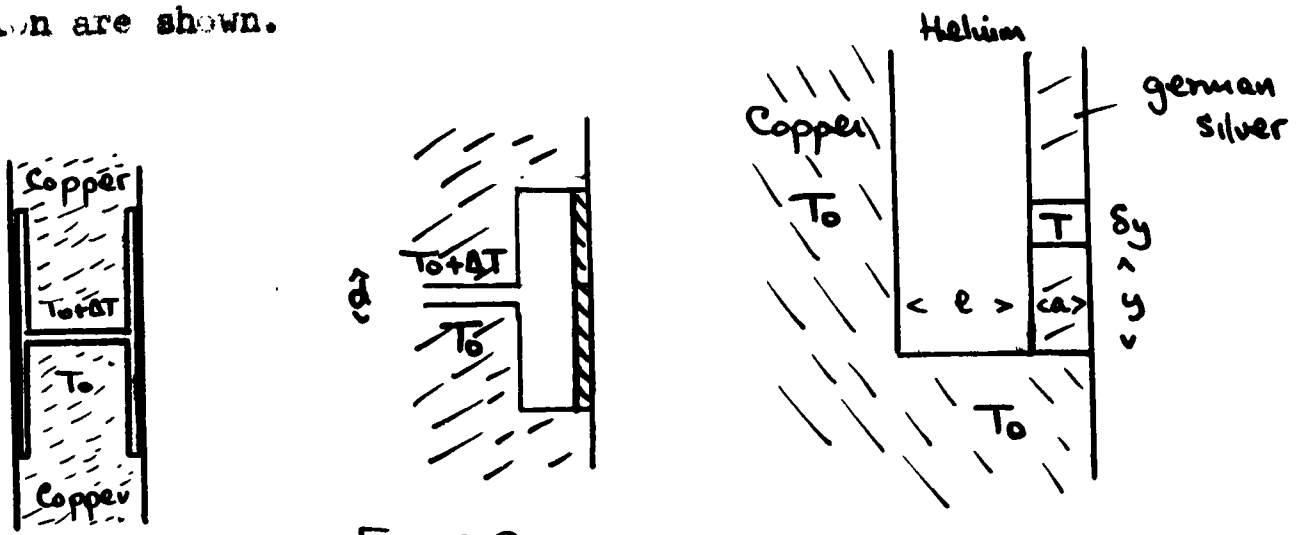


Fig A.3.

For convenience, the symbols listed in Chapter 3 are described again here.

d = vertical gap between the blocks ($2{\cdot}5.10^{-4}$cms)

h = length of cell (1.004 cms)

a = thickness of german-silver ($1{\cdot}21.10^{-2}$cms)

l = width of annular gap ($3{\cdot}77.10^{-2}$cms)

$\bar{r}$ = mean radius of german-silver ($4{\cdot}04.10^{-1}$cms)

r' = mean radius of gap ($3.79.10^{-1}$cms)

r_o = radius of copper block ($3.60.10^{-1}$cms)

r = inner radius of german-silver = $(r_o + l)$

K_H = conductivity of the fluid

K_G = conductivity of the german-silver

The heat flow in the steady state will be considered. It is assumed that the lines of thermal flux are radial except in the gap between the two copper blocks ($r < r_o$) when the lines are assumed to be vertical. The validity of this assumption will be discussed briefly at the end of this section.

An elemental volume of german-silver of length δy is considered. From equation 1 of Appendix 2(a) the heat flow through the helium to the copper (from this elemental volume) is

$$\frac{2\pi r'}{l} \cdot \delta y \cdot K_H (T-T_o)$$

This is balanced by the net flow into the german-silver

so that

$$K_G \cdot 2\pi \bar{r} a \left(\frac{\partial T}{\partial y}\right)_{T+\delta T} + K_H \cdot 2\pi r' \cdot \frac{\delta y}{l} (T-T_o) = K_G \cdot 2\pi \bar{r} a \left(\frac{\partial T}{\partial y}\right)_T$$

$$\therefore K_G \cdot 2\pi \bar{r} a \left(\frac{\partial^2 T}{\partial y^2}\right) = K_H \cdot 2\pi r' \frac{\delta y}{l} (T-T_o)$$

$$\therefore \frac{K_G}{K_H} \cdot \frac{\bar{r}}{r'} \cdot la \left(\frac{\partial^2 T}{\partial y^2}\right) = T-T_o \qquad -(2).$$

The solution of this equation is

$$T-T_o = Ae^{\alpha y} + Be^{-\alpha y} \qquad \alpha^2 = \frac{K_H}{K_G} \cdot \frac{r'}{\bar{r}} \cdot \frac{1}{la}$$

Boundary conditions

At the bottom of the german-silver the temperature is equal to that of the copper

or when $y = 0$, $T = T_o$ $\quad \therefore A+B = 0$

$y = h/2$, $T = T_o + \frac{\Delta T}{2}$ (symmetry) $\quad \therefore Ae^{\alpha h/2} + Be^{-\alpha h/2} = \Delta T/2$

$$\therefore \frac{\Delta T}{2} = A\left(e^{\alpha h/2} - e^{-\alpha h/2}\right) = 2A \sinh \frac{\alpha h}{2}.$$

or $$A = \frac{\Delta T}{4} \operatorname{cosec} \frac{\alpha h}{2}$$

Finally $$(T - T_0) = A\left(e^{\alpha y} - e^{-\alpha y}\right) = 2A \sinh \alpha y$$

$$= \frac{\Delta T}{2} \operatorname{cosec} \frac{h}{2}\sqrt{\frac{K_H}{K_G} \cdot \frac{r'}{r} \cdot \frac{1}{la}} \sinh y\sqrt{\frac{K_H}{K_G} \cdot \frac{r'}{r} \cdot \frac{1}{la}} \quad (3)$$

This result will be discussed very briefly. If $K_H \gg K_G$, the german-silver temperature is very nearly the same as that of the neighbouring copper block, and most of the fall in temperature ΔT along the german-silver takes place over a distance equal to that of the gap. As the conductivity ratio is reduced, this abrupt change in temperature becomes replaced by a more even temperature distribution until for a zero fluid conductivity, the distribution is linear.

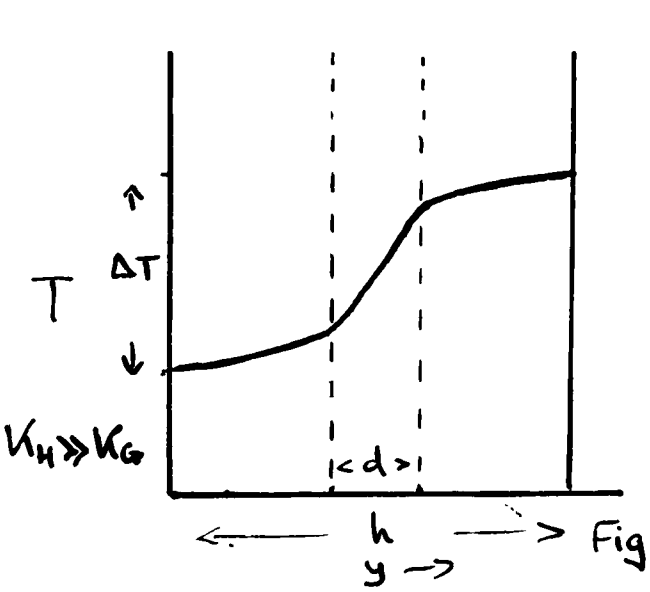
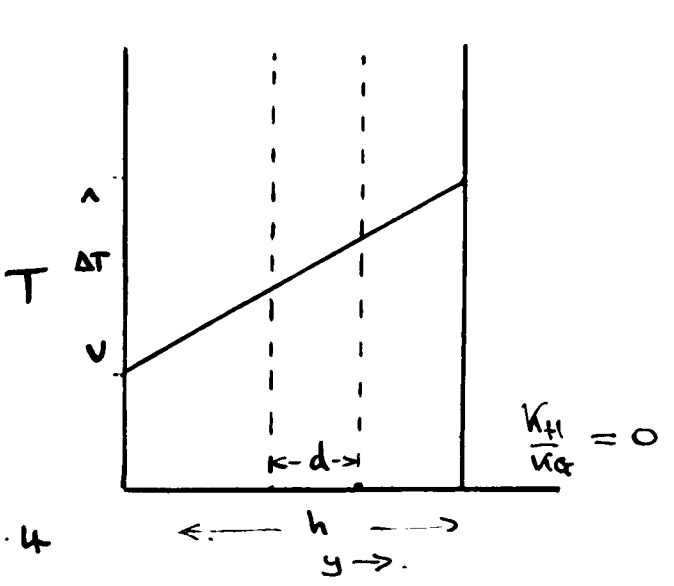


Fig. A·4



Accordingly, heat conduction takes place from the upper copper block across the fluid to the german-silver, down the german-silver and across to the lower block. The total heat flowing from the upper to the lower block is equal to the heat crossing the meridian plane XY and is given by

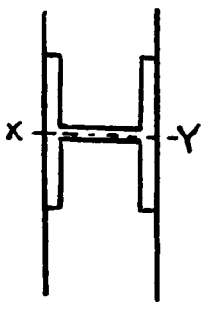


Fig A.5

$$F_{disc} \cdot K_H \cdot \Delta T + K_G \cdot 2\pi \bar{r} a \left(\frac{\partial T}{\partial y}\right)_{h/2} \qquad F_{disc} = \frac{\pi r_0^2}{d}$$

But from (3) $$(T-T_0) = \frac{\Delta T}{2} \cdot \frac{\cosh \alpha h}{\cosh \frac{\alpha h}{2}} \cdot \sinh \alpha y$$

so $$\left(\frac{\partial T}{\partial y}\right) = \frac{\Delta T}{2} \frac{\cosh \alpha h}{\cosh \frac{\alpha h}{2}} \cdot \cosh \alpha y \cdot \alpha$$

$$\therefore \left(\frac{\partial T}{\partial y}\right)_{h/2} = \frac{\Delta T}{h} \cdot \frac{\frac{\alpha h}{2}}{\tanh \frac{\alpha h}{2}}$$

When $\frac{\alpha h}{2} > 2.5$ $\quad \frac{\frac{\alpha h}{2}}{\tanh \frac{\alpha h}{2}} = \frac{\alpha h}{2}$

$\frac{\alpha h}{2} < 0.2$ $\quad \frac{\frac{\alpha h}{2}}{\tanh \frac{\alpha h}{2}} = 1$ } to within 1%

In general

$$\dot{Q} = \Delta T \left\{ K_H \cdot F_{disc} + K_G \cdot \frac{2\pi \bar{r} a}{h} \cdot \frac{\frac{\alpha h}{2}}{\tanh \frac{\alpha h}{2}} \right\} \quad -(4)$$

It is interesting to consider once more the limiting cases. When $K_H \gg K_G$ $\frac{\alpha h}{2} \gg 2.5$ and the expression for the radial flow becomes

$$\dot{Q}_r = K_G \cdot \pi \bar{r} a \sqrt{\frac{K_H}{K_G} \cdot \frac{r'}{\bar{r}} \cdot \frac{1}{\ell_a}} \cdot \Delta T = \sqrt{K_G} \cdot \sqrt{K_H} \cdot \left(\sqrt{\pi^2 \bar{r} r' a}\right) \Delta T$$

and when $K_H \ll K_G$ it becomes $K_G \cdot \frac{2\pi \bar{r} a}{h} \cdot \Delta T$ which is the result obtained in the usual way in the case when the apparatus is evacuated.

The assumption that the thermal flux lines are perpendicular to the walls is never entirely justified. In Figure A.6 a sketch is drawn of the probable lines of constant temperature and the lines of thermal flux.

FLOW LINES IN CELL

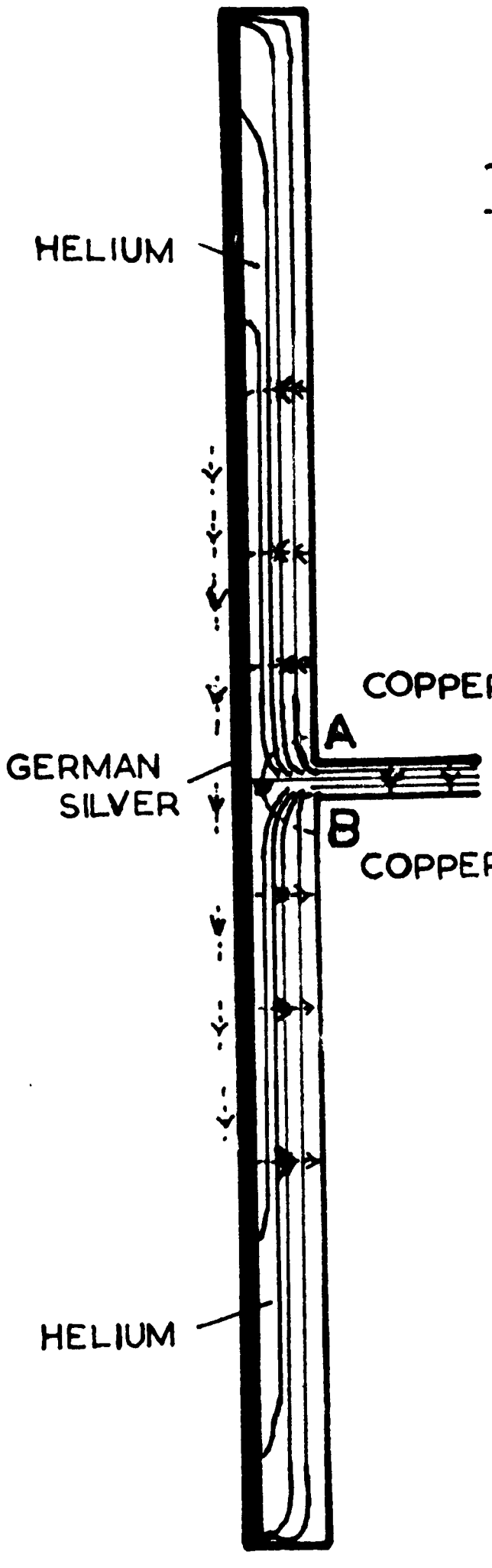


FIG. A.6

The heat flow at the meridian plane is

$$A \cdot K_G \cdot \left(\frac{\partial T_G}{\partial y}\right)_{y/2} + \int_0^r K_H \left(\frac{\partial T_H}{\partial y}\right)_{y/2,r} \cdot 2\pi r\,dr$$

$$= 2\pi \bar{r}\, a\, K_G \cdot \left(\frac{\partial T_G}{\partial y}\right)_{y/2} + \int_0^{r_0} K_H \left(\frac{\partial T_H}{\partial y}\right)_{y/2} \cdot 2\pi r\,dr + \int_{r_0}^{r} K_H \left(\frac{\partial T}{\partial y}\right)_{y/2} \cdot 2\pi r\,dr$$

With the assumption that the flow lines are normally radial, but vertical when $r < r_0$ only, the third term becomes zero. However this excludes paths such as AB which will certainly exist. To estimate the effect of this, two methods have been used.

(a)

Now $$K_H \int_{r_0}^{r} \left(\frac{\partial T_H}{\partial y}\right)_{y/2} \cdot 2\pi r\,dr \quad < K_H \cdot \left(\frac{\partial T_H}{\partial y}\right)_{y/2\,max} \cdot \int_{r_0}^{r} 2\pi r\,dr$$

and $\left(\frac{\partial T_H}{\partial y}\right)_{y/2}$ has its maximum value in the disc of helium between the blocks and is equal to $\frac{\Delta T}{d}$. When this value is substituted in, we find

$$I = K_H \cdot \int_{r_0}^{r} \left(\frac{\partial T_H}{\partial y}\right)_{y/2} \cdot 2\pi r\,dr < 20\% \int_0^r K_H \cdot \left(\frac{\partial T_H}{\partial y}\right)_{y/2} \cdot 2\pi r\,dr$$

or the total heat flow along paths such as AB is certainly less than 20% of the flow across the disc. The actual value depends on the rate at which the temperature gradient decreases (as the radius increases from r to r_0) to its value in the german-silver which for conductivities of the orders used in this work, is from 5 - 8% of the maximum value $\frac{\Delta T}{d}$.

This rate of decrease would be difficult to calculate - any reasonable estimate shows that it is fairly rapid so that over most of the area the gradient is perhaps 10% of gradient between the disc. In other words, the integral I_2 amounts to less than 5% of the heat flowing across the disc. This heat flow has been partially accounted for by radial flow and then vertical flow down the german-silver so that the inaccuracy is less than 5% - say 3%. The flow across the disc is about 80% of the total so the error introduced is about 2.5%.

(b) The heat flow along paths such as AB occurrs because the thermal resistance is less than in the path we have postulated. In other words, the distance along AB is less than twice the mdial distance between the copper and the german-silver : the distance travelled in the german-silver may be ignored since the conductivity is about 100 times greater than that of the helium. The length of the curve AB is less than πAB and of the order of 2AB so that

$$2AB < 0.8 \text{ mms.}$$

$$\text{or} \quad AB < 0.4 \text{ mms.}$$

but AB = 0.2 + 2x where x is a vertical length on the circumference of the copper block.

The area then from which lines AB emanate is $2\pi r \times 0.1$ or 5% of the area of the disc. Since the length of these paths is about 2AB, the average gradient along the path is $\frac{\Delta T}{2AB}$ and the heat flow therefore about 3% of that across the disc. Consequently, the error in calculating the flow, with the assumption of the existence of radial and vertical flow lines only, is again small.

The agreement with the experimental results of Ubbink et al. for gaseous ^{4}He indicates that these approximations are probably fairly accurate. If a greater degree of accuracy were required it would be possible to construct an electrical analogue of the system in 2 dimensions - with copper blocks and either a graphite block or a thin alloy sheet replacing the german-silver. The whole could be immersed in distilled water. The equi-potentials could be plotted and the total resistance measured as a function of the conductivity of the water as its acidity was increased. A complete calculation would involve the solution of Laplace's equation $\nabla^2 T = 0$ using the appropriate boundary conditions.

PART II

The Velocity of Sound in Liquid Helium

CHAPTER 6

THE VELOCITY OF SOUND IN LIQUID ^{3}He

Introduction

Measurements on the velocity of sound in fluids can give valuable information about thermodynamic quantities as the velocity of sound is directly related to the adiabatic compressibility. In quantum liquids the velocity (extrapolated to absolute zero) has the additional significance of being the velocity at which phonon excitations travel through the liquid and so for example the specific heat of these phonons and the thermal conductivity, when the phonon mean free path is limited by the walls, are also calculated in terms of this velocity. These calculations have been made for a solid at low temperatures (Debye 1912; Casimir 1938) and may be applied immediately to liquid ^{4}He, when it is found that, below 0.6°K, the measured specific heat is in good agreement with the phonon specific heat (Wiebes, Niels-Hakkenberg and Kramers 1957 and Markham, Pearce, Netzel and Dillinger 1957) and the measured thermal conductivity is explained by phonon conduction if, as seem. likely, a degree of specular reflection occurs (H.A. Fairbank and Wilks 1954, 1955; Whitworth 1955).

We decided to measure the velocity of sound in liquid ^{3}He so that similar calculations could be carried out. The only estimate of the velocity which had been published at the start of this work was that of Walters and W.M. Fairbank (1956) who measured the isothermal compressibility of liquid ^{3}He by investigating the increase in the nuclear magnetic resonance absorption signal as the pressure was

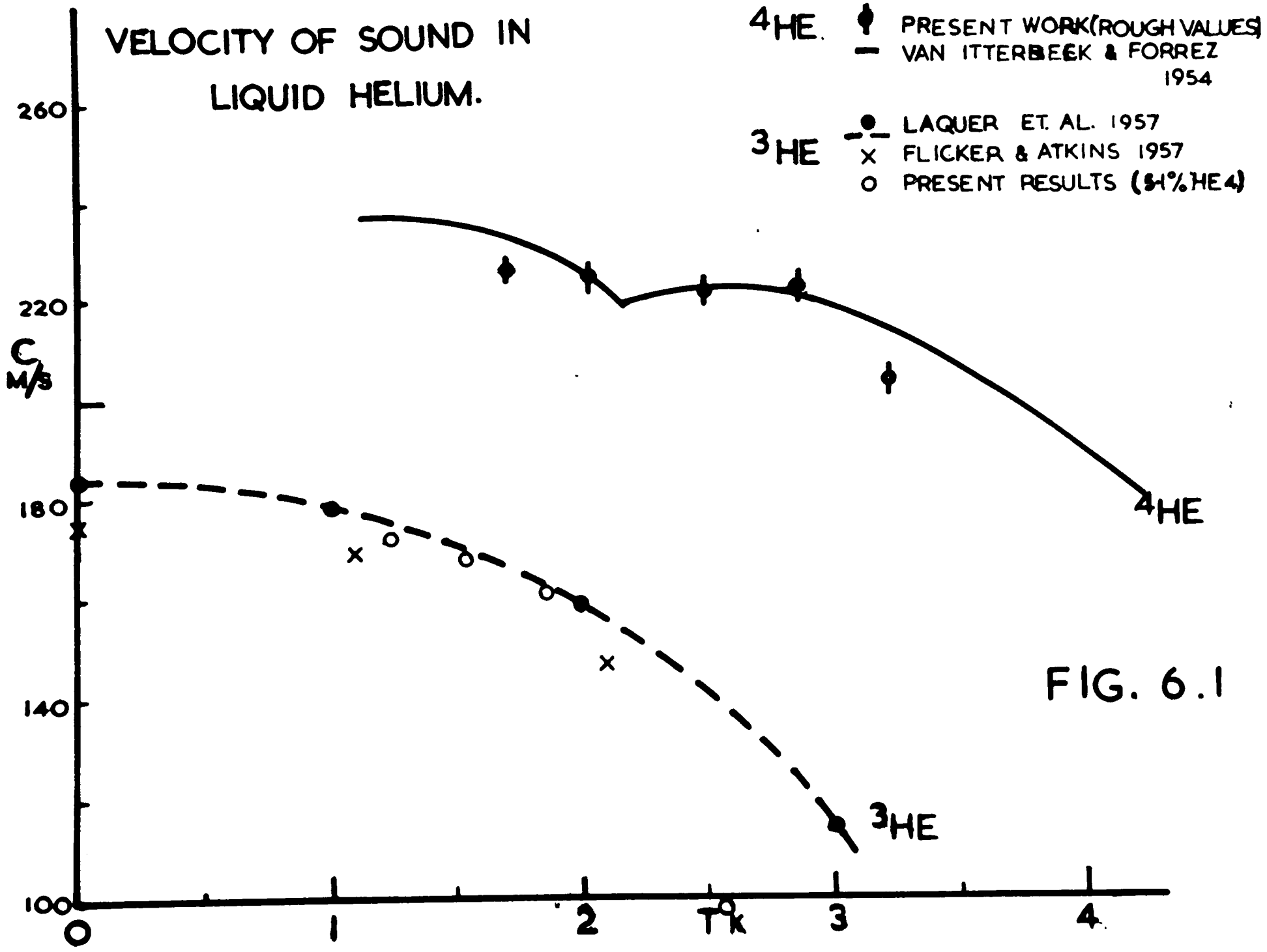

VELOCITY OF SOUND IN
LIQUID HELIUM.
4HE. PRESENT WORK(ROUGH VALUES)
VAN ITTERBEEK & FORREZ
1954
3HE LAQUER ET. AL. 1957
FLICKER & ATKINS 1957
PRESENT RESULTS (5H% HE4)
260
220
180
140
100
C
M/S
4HE
3HE
0
1
2
3
4
Tk

FIG. 6.1

increased. They assumed this signal to be proportional to the density and so obtained a value of the compressibility at 1.2°K. From this, making the assumption that the isothermal and adiabatic compressibilities were equal, they calculated the velocity of sound in liquid ^{3}He at 1.2°K to be 195 metres/second or about 0.8 of the velocity in liquid ^{4}He.

Before this present work was completed, results were reported by two independent groups of workers (Flicker and Atkins 1957, Laquer, Sydoriak and Roberts 1957). Flicker et al. made measurements in the range 1.1 - 2.1°K using a frequency of 13 mc/s and quote their results as being accurate to within 3 metres/second, while Laquer et al. made measurements in the range 0.4 - 3.1°K at a frequency of 5 mc/s and quote their results as being accurate to 0.3 metres/second at 2.0°K. The values are shown in Figure 6.1. A broken dotted line is drawn through the published points of Laquer et al. to give an indication of the variation of velocity with temperature but is not intended to imply that these, rather than points of Flicker et al., are the true values. The values at absolute zero obtained by extrapolation are also shown in the figure.

In view of the existence of this work, it was not considered worthwhile carrying out an extended series of measurements of the velocity of sound.

The apparatus

(a) The sonic chamber

A diagram of the apparatus is shown in Figure 6.2 . An etalon of fixed length 2.482 cms. was used and the velocity calculated

VELOCITY OF SOUND APPARATUS.

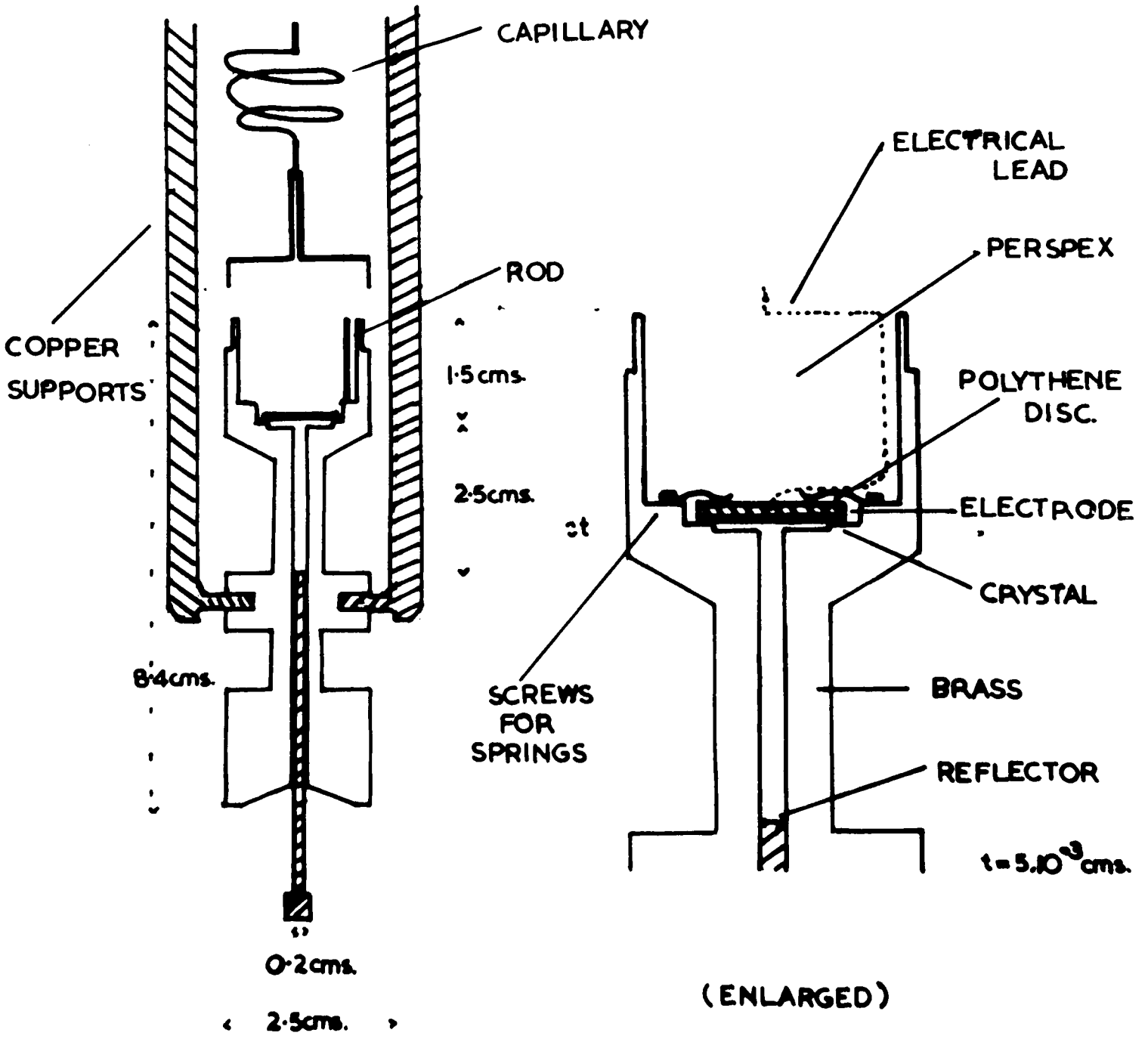


FIG. 6.2

from the time for a pulse of ultrasonics to travel from the radiating quartz crystal through the helium to the end of the etalon and back again.

The sonic chamber was designed to have a small volume consists of a cylindrical brass block turned down to the shape shown in the figure. The lower reflecting end of the etalon is a polished brass piston soldered into position. The liquid forms a cylinder of diameter 0.2 cms. and length 2.48 cms. which opens out to a thin disc of thickness 5.10^{-3} cms. (.002"). A quartz crystal is supported round its rim on a shoulder of brass between two electrodes. The lower electrode comprises a thin film of silver which had been evaporated on to the crystal and, being in contact with the brass chamber, is at earth potential. A brass disc resting on the top surface of the crystal acts as the upper electrode. The crystal is held in position by three springs of copper sheet which are insulated from the upper electrode by a thin polythene ring.

A perspex cylinder fills the space above the crystal and is slotted at the lower end to accommodate the springs, and a rod screwed into the brass acts as a locating pin. A brass lid is Wood's metalled to the top of the chamber. The ^{3}He is introduced to the chamber by a 0.5 mm. capillary soldered into this lid and connected at the upper end to the ^{3}He transport apparatus described in Part I of this thesis. The electrical lead to the upper electrode of the crystal passes down the capillary. The chamber is supported by two copper rods in a jacket also connected to the ^{3}He transport apparatus.

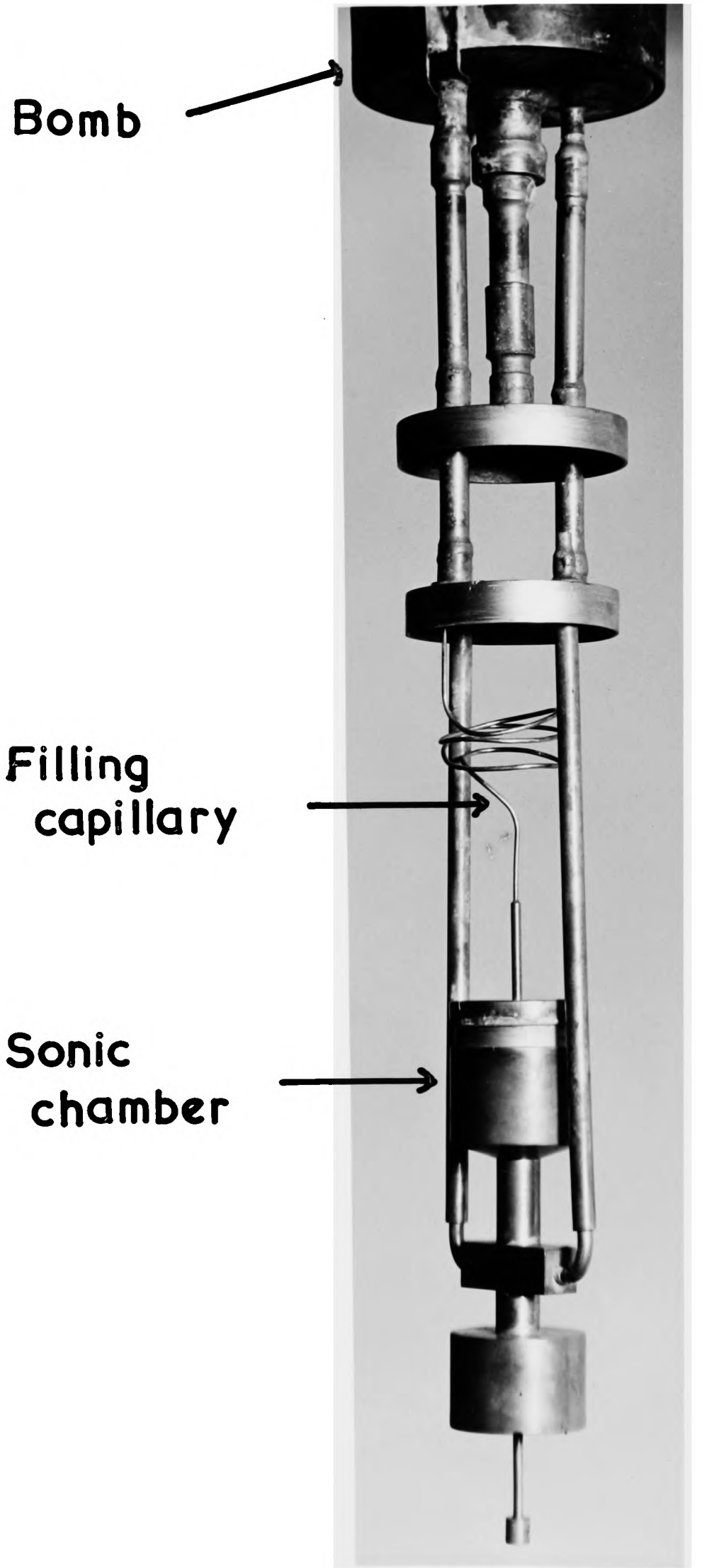


FIG. 6.3
The Velocity of Sound Apparatus

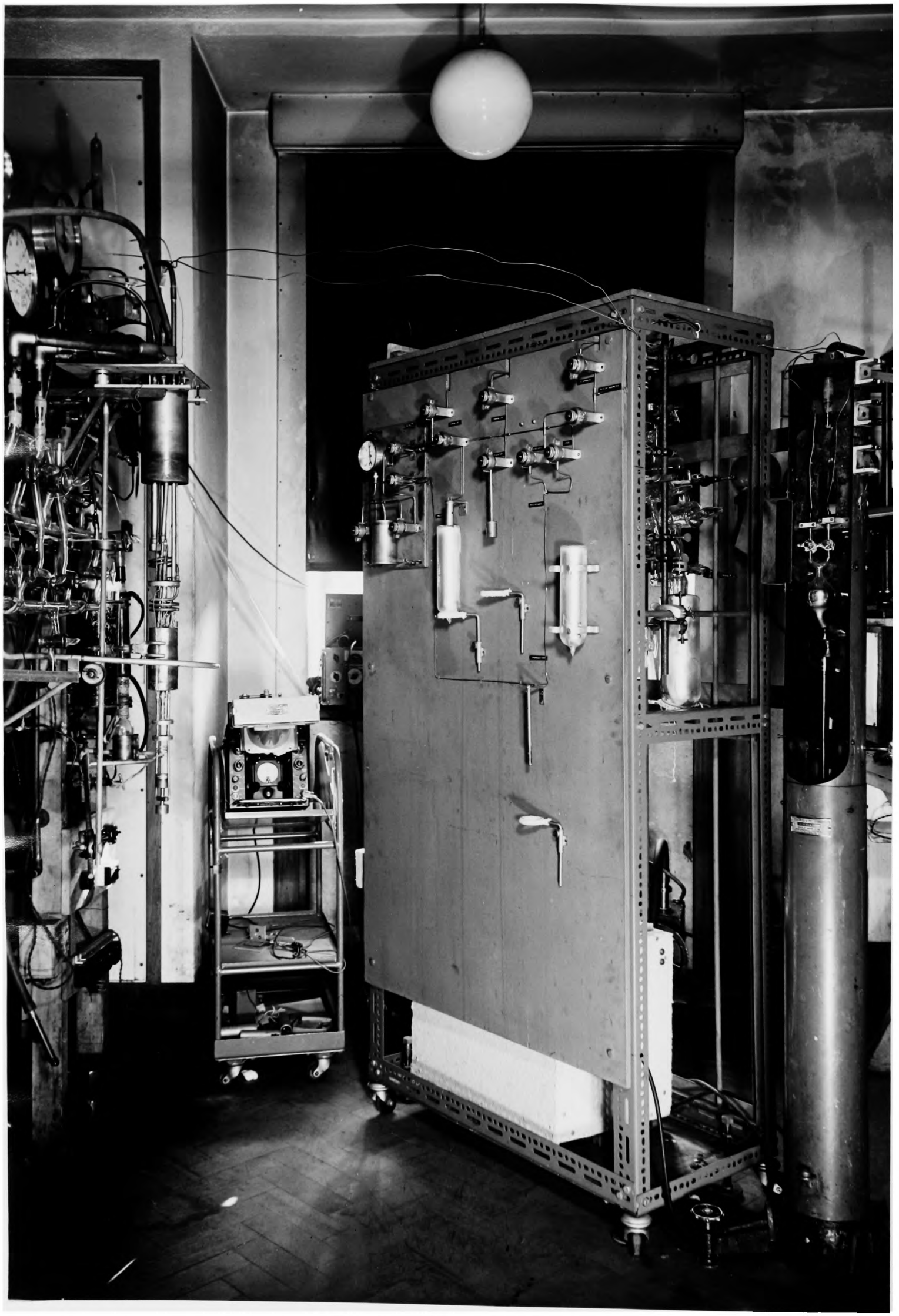
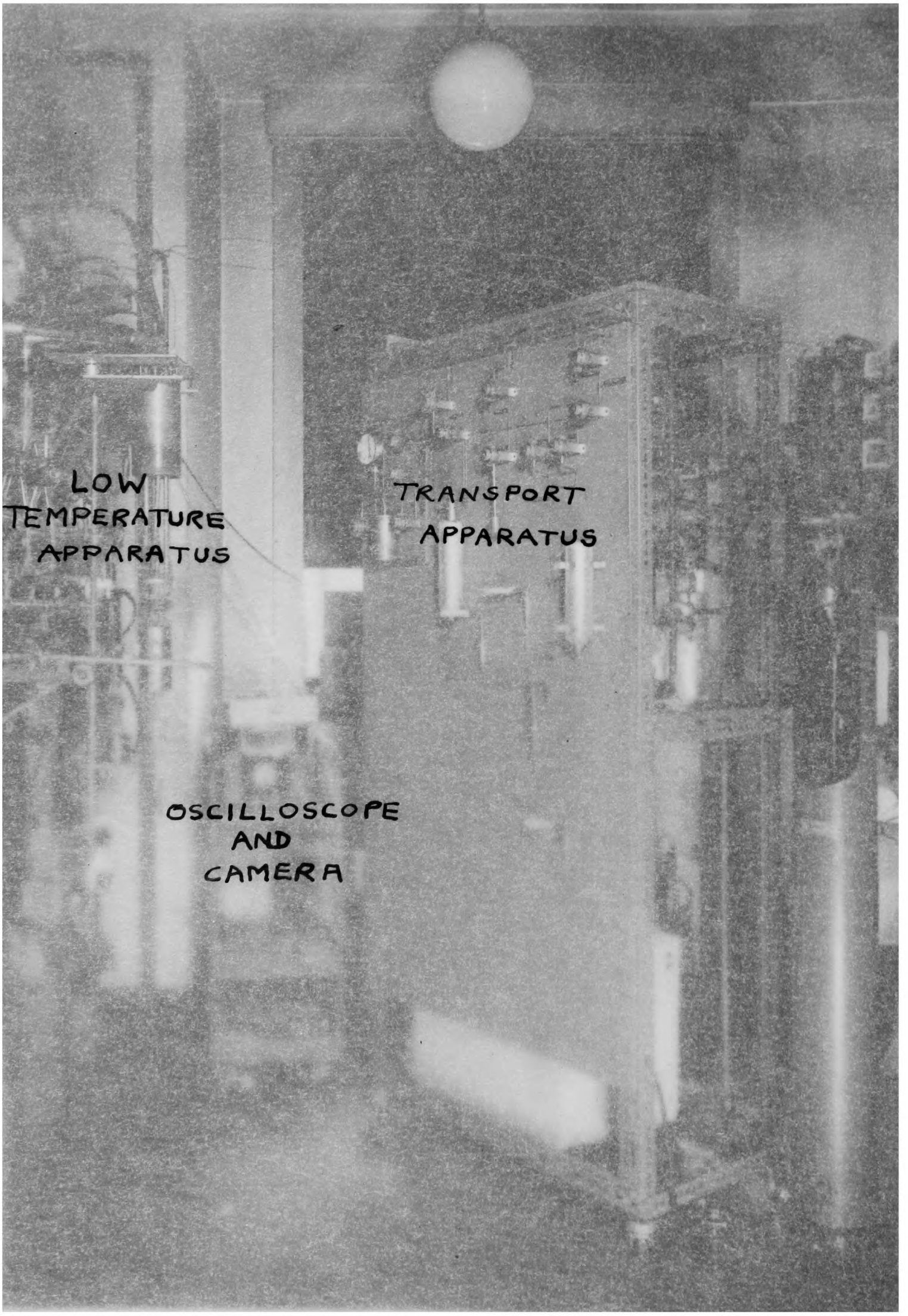
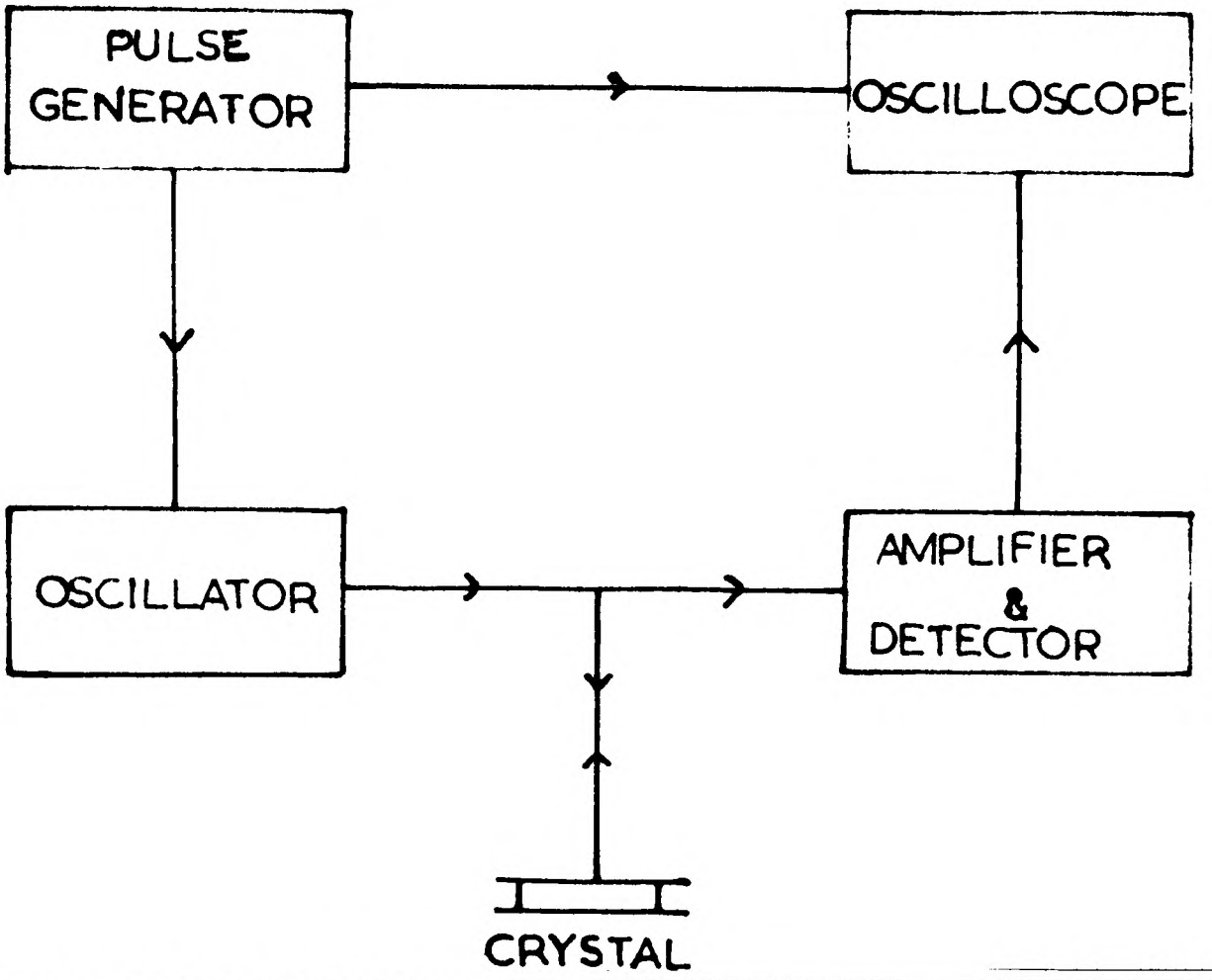


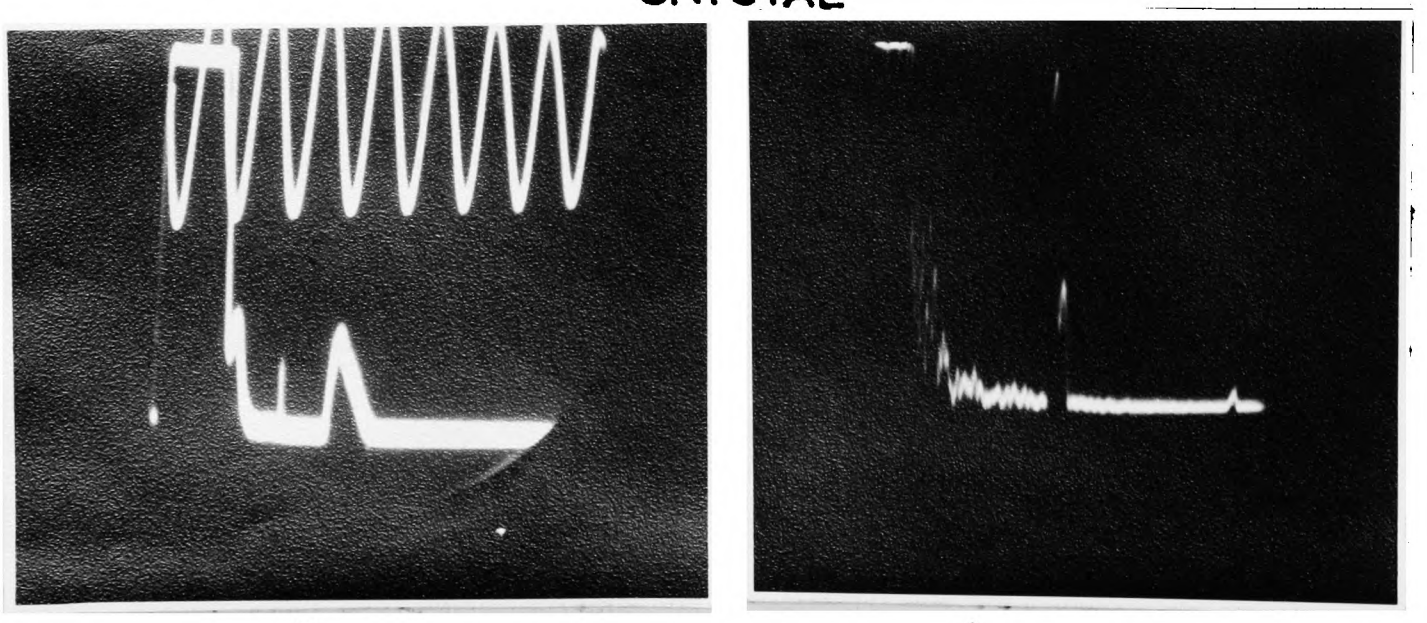
FIG. 6.4


LOW
TEMPERATURE
APPARATUS
TRANSPORT
APPARATUS
OSCILLOSCOPE
AND
CAMERA

BLOCK DIAGRAM OF THE ELECTRONIC APPARATUS

FIG. 6.5





^3He ^4He

Oscilloscope traces

This jacket acts as a safety space so that in the event of a leak on the chamber, the ^3He could be returned to the container.

Oscillations of the crystal were induced when an A.C. voltage with the natural frequency of the crystal was applied to the upper electrode. This voltage is modulated so that a pulse of ultrasonics is radiated into the liquid. Each reflected pise causes the crystal to vibrate and so an electrical signal to be generated. In this way, the crystal is used as both generator and detector.

(b) The cryogenic apparatus

A Simon liquifier was used to produce liquid helium in a bomb and the cryostat which consisted of a jacket fitting closely to the safety jacket was filled by condensing in helium gas cooled by exchange with the evaporating helium in the bomb. The cryostat could be pumped to a temperature of about 1.1°K. A photograph of the sonic chamber in position below the bomb is shown in Figure 6.3 and again together with the ^3He transport apparatus in Figure 6.4.

(c) The electronic apparatus

The electronic apparatus was built and described by Dr. J.A. Newell (1955) and was used by him together with Dr. K. Dransfeld and Dr. J. Wilks to measure the absorption of sound in liquid ^4He (see for example Dransfeld, Newell and Wilks 1957). The part of the apparatus used in these experiments is shown in a block diagram in Figure 6.5. An R.F. oscillator tuned to 14 mc/s (the natural frequency of the crystal) is triggered by a free running pulse generator. The 10 μs pulses produced are applied directly to the upper electrode of the crystal and

the signals produced by the reflected pulses are amplified, detected and then applied to an oscilloscope previously triggered by the pulse generator. The pulses produced on the screen of the oscilloscope are photographed together with a calibration signal from the oscilloscope which establishes the time scale.

Experimental procedure

(a) Aligning the crystal

Newell (1955) has discussed the necessity for the crystal and reflector to be accurately aligned in measurements on the absorption of sound. This applies to a large extent in the present work since the amplitude of the signal depends very critically on the angle of alignment and the accuracy of the measurements is influenced by the signal to noise ratio. By the method of construction, the reflector and crystal support are made accurately parallel so that the alignment consisted of adjusting the spring pressure until the crystal was held rigidly and of changing the position of the crystal with relation to the axis of the chamber. The preliminary alignment was carried out with the chamber filled with methyl alcohol which was found to be suitable as it is volatile and does not attack perspex. It was found that the optimum position was with the centre of the crystal displaced from the axis as far as the geometry permitted, and that this position was reproducible so that the crystal could be aligned in the absence of a liquid.

The experiments

A preliminary experiment was made with liquid ^{4}He to check

the behaviour of the apparatus. The ^{4}He was condensed in at 4.2°K and the temperature was reduced by pumping on the cryostat. Photographs of the oscilloscope trace of the pulses together with the calibration signal were taken at various temperatures.

With ^{3}He liquid, the chamber was filled at the lowest cryostat temperature and higher temperatures (up to the λ-point) were obtained by condensing in helium into the cryostat.

In the earlier experiments, the reflected pulses were obscured by the "ringing" of the crystal. At the end of an electrical pulse, the amplitude of the applied R.F. signal is zero but the crystal still possesses mechanical energy of vibration. In dense liquids of large viscosity this energy is quickly dissipated but in gases and very light mobile liquids such as helium, the time during which the amplitude of the decaying vibrations is appreciable compared with the amplitude of the reflected signal is much longer. These decaying vibrations result in decaying R.F. signals so that on the oscilloscope trace the input pulse is seen to decrease slowly in amplitude as a function of time. This is known as crystal ringing. The strength of the reflected signal is smaller in the present apparatus then in apparatuses of larger volume and so the effect of the ringing was enhanced. In the earlier experiments on ^{3}He, the ringing, which is larger than in ^{4}He because of the smaller density, completely obscured the reflected pulses. It was reduced by placing "Nonaq." grease between the brass disc and the crystal and, although this also had the effect of reducing the amplitude of the reflected signals, it was found that an overall improvement was obtained so that the reflected pulses in liquid ^{3}He were no longer obscured.

Examples of the pulses obtained are shown in Figure 6.5 : the very narrow pulse on the trace shown for ^3He is a comparison pulse which may be used for measuring pulse heights.

The measurements

(a) Etalon length

The distance between the plane of the crystal and the reflector was measured by mounting the chamber horizontally in a chuck on the head stock of a lathe. A thin steel rod, curved at one end to avoid scratching the reflector, was held loosely in the tool post and the rod was moved along its length until one end touched the reflector. The position of the other end was noted with a travelling microscope clamped on the bed of the lathe. The rod was then pulled from the cylindrical hole, the tool post was moved in a direction at right angles to the axis of the chamber and the rod was moved along its length until it touched the brass shoulder on which the crystal rests when mounted. The position of the end of the rod was again noted with the microscope and the difference between the readings is the length of the etalon at room temperature. A small correction of 0.3% was applied to allow for the contraction of the brass which takes place on cooling to helium temperatures.

(b) Time

The distances between the leading edge of the input pulse and the leading edges of the reflected pulses on the oscilloscope trace are proportional to the time taken for the pulse of ultrasonics to travel twice the length of the etalon. The wavelength of the trace of the calibration signal establishes the time scale and so the time for a pulse

to travel twice the length of the etalon is calculated. The frequency of the calibration signal obtained from the oscilloscope was measured by photographing its trace together with that of a standard signal from a crystal oscillator, and it was found that the calibration frequency differed from its nominal value by about 1%.

In the experiment carried out on ^{4}He, the photographs of the pulses were mainly underexposed and the measurement of time could not be made so accurately as in the experiments made on ^{3}He. However, it was not considered worthwhile to repeat this experiment in view of the values already in existence for ^{4}He.

Results

The results are tabulated below and shown graphically in figure 6.1 The ^{3}He contained 5.1% ^{4}He and so a correction must be applied to the results to obtain values for pure ^{3}He.

Velocity of sound in liquid ^{4}He (approximate values)

Temperature (°K)	1.72	2.04	2.52	2.94	3.24
Velocity of sound (m/s)	226	224	221	222	203

Velocity of sound in liquid ^{3}He (containing 5.1% ^{4}He)

Temperature (°K)	1.23	1.53	1.86
Velocity of sound (m/s)	172	168	161

Experimental errors

In ^{4}He, the errors due to the difficulty in measuring the distance between two pulses on the underexposed films is larger than

any other error and results in scatter. The errors in the measurements of the velocity in ^{3}He will be considered individually.

The probable error in the length of the etalon is 0.1% and the correction due to contraction should introduce negligible errors so that the error in measuring the length is 0.1%. The error in the calibration frequency is due to errors in measuring lengths on film and is about 0.5%. The systematic error is then about 0.6%.

There are also random errors in the measurements of the wavelengths of the calibration signal and the distance between the pulse on each trace. These total about 0.5%. The total error in the values obtained is then about 1.1%.

Discussion of results

(a) ^{4}He liquid

To within the probable accuracy of the experiments, agreement is obtained with the results of other workers (Findlay, Pitt, Grayson Smith and Wilhelm 1938, 1939; Pellam and Squire 1947; Atkins and Chase 1951; Atkins and Stasior 1953; van Itterbeek and Forrez 1954). In figure 6.1 the values are shown together with those of van Itterbeek et al. (1954).

(b) ^{3}He liquid

The values obtained in the present work lie between those of Laquer et al. and Flicker et al. There is a difference of about 6% between the values obtained by these two groups of workers which is well outside their quoted errors. The reason for this is not known - it is perhaps possible that it is a genuine difference due to dispersion.

The ^3He used in the present work contains 5.1% ^4He. To a first approximation, it might be expected that the velocity of sound of this mixture would be a linear combination of the velocities of the two components and if this were so, the values for pure ^3He would be about 1.7% less than the values obtained in the present work. These values, corrected for this ^4He, would lie about 2% below those of Laquer et al., which is just outside the combined experimental errors, and about 4% above those of Flicker et al.

It is interesting to note that a phonon specific heat has been calculated from their extrapolated value of the velocity by Flicker et al. They estimate that a phonon specific heat could account for 6% of the measured value for liquid ^3He at 1°K and perhaps 30% at 2°K. We have determined the value of a phonon conductivity for liquid ^3He in the Casimir region using this extrapolated value of the velocity and have also used the adiabatic compressibility calculated from the velocity to calculate $C_p - C_v$ (see Chapter 5 of this thesis).

In view of the difference in the results of Flicker et al. and Laquer et al., new measurements may be made when pure ^3He becomes available. It also appears that it may be possible to adapt the present apparatus to make measurements on the absorption of sound in liquid ^3He.

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