

# Contract Bargaining with Symmetric Information\*

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## Abstract

This paper reviews a recent literature that extends the Rubinstein/Stahl bargaining model to the case of contract bargaining. Theoretical issues such as the appropriate game form, existence and uniqueness of equilibria are discussed. The paper finishes with a brief overview of some applications of the framework.

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# 1 Introduction

Contracts are ubiquitous. They are important for economic transactions that have component parts not all carried out simultaneously. Renting an apartment, repaying a mortgage or loan, employment, long term supply of goods and services all start with an agreement with provisions carried out over time. Many contracts involve bargaining. Even where initial contract negotiation takes place in a perfectly competitive market, renegotiation and renewal often do not. At renewal, the parties may incur search or switching costs from finding another trading partner and may have made specific investments that will not be recouped if they switch. Legally, moreover, renegotiation is always possible as long as it is by mutual consent. It is also common — annual wage increases, rent reviews, and so on. For all these reasons bargaining is an important element in contract negotiation. This paper reviews some of the implications.

A standard approach to bargaining in economic models is the Rubinstein/Stahl strategic approach in which the buyer and seller exchange offers until a trading price is agreed, at which point trade takes place and the game ends. This model has the property, convenient for applications, that the unique subgame perfect equilibrium payoffs can be expressed in terms of the Nash bargaining solution, as shown by Binmore, Rubinstein and Wolinsky (1986). But the assumption that the transaction is completed (and the game ends) as soon as agreement is reached neglects the important element of contract bargaining that time will elapse before the transaction is completed. There is thus time for parties to renege and, if they wish, renegotiate the contract. This possibility has an important impact on equilibrium outcomes in the Rubinstein (1982) model. With infinite horizon bargaining, subgame perfect equilibria are no longer necessarily unique. See Muthoo (1990).

A characteristic of many contracts is that trade and payment are more frequent than contract renegotiation. Renting an apartment involves an agreement to pay for services on a monthly basis but the agreement is typically renegotiated only infrequently. Mortgages are similar. Wages and salaries are typically paid monthly, bi-weekly or weekly for services performed daily, yet are renegotiated much less frequently, maybe yearly or after even longer periods of time. We capture this characteristic in the present paper by focusing on contract negotiation over the future division of a *flow* of goods

and services.<sup>1</sup> With contracting over a flow, agents not only decide on a trading price in the contract but subsequently decide whether to continue trading at that price, to breach, or to renegotiate. This serves as a paradigmatic representation of many long term relationships, including employment relationships, long term supply of inputs to a manufacturing firm, financial contracts, etc. Models of this type can throw light on the conditions under which renegotiation will occur. This is important for understanding actual contracts. If a new contract is negotiated each time trade occurs, the current contract will reflect only current market conditions, which is not consistent with the evidence in, for example, Beaudry and DiNardo (1991) that earlier labour market conditions significantly affect the current wage.

In section 2, we present the model of contract bargaining over flows. Instead of using the alternating offers model of Rubinstein (1982), we use the random proposer model first studied by Binmore (1987) in which each bargaining round nature chooses with constant probability which player may offer a new contract. The probability of being chosen provides a simple way to represent the bargaining power of a player. In section 3, we study the equilibria for a finite bargaining horizon. The finite horizon ensures (generically) unique subgame perfect equilibrium payoffs. Moreover, whenever renegotiation occurs, these payoffs converge as bargaining becomes frictionless (in the sense that the number of bargaining rounds per period becomes large) to the payoffs given by the Nash bargaining solution with the weight attached to a party's utility gain equal to the probability of being chosen to make an offer. Thus, this convenient property of infinite horizon models established in Binmore, Rubinstein and Wolinsky (1986) holds despite the finite horizon.

In section 4, the effect of an infinite horizon is studied. The results of Haller and Holden (1990) and Fernandez and Glazer (1991) demonstrate that in this case subgame perfect equilibrium is not unique. In fact equilibrium is not even necessarily efficient. Fernandez and Glazer (1991) interpret the existence of inefficient equilibria in terms of strikes — one side refuses to trade for several periods, after which a new agreement is reached. If, however, the equilibria in the infinite horizon game are required to be *renegotiation proof* as well as subgame perfect, the set of equilibrium payoffs as bargaining becomes frictionless is the same as with a finite bargaining horizon.

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<sup>1</sup>In the incomplete contracts models of Hart and Moore (1988) and Aghion, Dewatripont and Rey (1994), trade is a once off event and so does not capture this characteristic.

In section 5, we discuss Shaked's (1987) results on outside options in the infinite horizon model and apply them to the finite horizon model. An outside option is an alternative market opportunity that, if taken up, stops the game. Shaked (1987) has shown that the consequences of adding an outside option to the model depend on the particular extensive form representation. If *both* parties can take up their outside options immediately after an offer has been rejected, there are multiple equilibria in the infinite horizon game and there is no well defined limit for frictionless bargaining in the finite horizon game. The consequences are different if the party refusing an offer can always make another offer before the other party can take up an outside option. Shaked (1987) calls such a situation a *bazaar* because in a bazaar "no self respecting seller would allow a customer to leave without making the last offer." In such a market the outside options act not as threat points in the sense of the Nash Bargaining Solution but as constraints on the outcome. If bargaining in the absence of outside options would result in an outcome that is better for both parties than the outside options, the existence of outside options has no effect on the equilibria. If, however, one party would be better off choosing an outside option, that party receives exactly his or her outside option value.

In section 6, some applications are discussed. An important application is to relationships that involve specific investments. The model can be used to show how indexing contracts can sometimes overcome the hold-up problem discussed at length in Williamson (1985). It can also be used to show how a relationship can be structured in such a way that, even in the absence of a formal contract, outside options can result in efficient investments. Another application is to the endogenous determination of contract length. Section 7 contains concluding remarks.

## 2 A Model of Contract Bargaining

Consider a buyer  $B$  and seller  $S$  bargaining for the supply of a flow of a good or service over the time interval  $[0, T]$ , where  $T$  may be infinite. The value of the supply per unit of time to the buyer is normalized to  $1 + c$ , where  $c$  is the cost per unit of time to the seller of supplying the good, so there is a net benefit of 1 per unit of time from exchange. Time zero denotes the start of current negotiations. To cover renegotiation of an existing contract as well as initial contract negotiation, we let  $p_0$  denote the trade price in the existing

contract. If there is no such contract, then  $p_0 = 0$ . Consistent with contract law, agents can, if they wish, renegotiate an existing contract but, in the absence of a mutually agreed new contract, the existing contract remains in force.

The time interval  $T$  is divided into discrete periods of length  $\Delta = T/N$ , where  $N$  is the number of bargaining rounds in the game. Bargaining rounds are indexed by  $n$  (starting with  $n = 0$ ), the start of round  $n$  corresponding to time  $n\Delta$ . We follow Binmore (1987) in supposing that, at the start of round  $n$ , nature ( $\mathcal{N}$ ) chooses which agent can make an offer, the buyer being chosen with probability  $\pi$ . In the original Rubinstein (1982) model, agents take turns to make offers, which makes the equilibrium payoffs at the beginning of the game depend on who makes the first offer. Using a randomly chosen proposer avoids this. Moreover, the choice of  $\pi$  provides a convenient way to parameterize the relative bargaining powers of the two agents, a higher  $\pi$  endowing the buyer with greater bargaining power. This framework gives the same limiting results as the alternating offers model when  $\pi = 0.5$ .

In each bargaining round  $n$ , the agent selected to make an offer suggests a new price,  $\tilde{p}_n$ , for the good. The responding agent then decides whether to accept (A) or reject (R) this new price. If the new price is accepted,  $\tilde{p}_n$  becomes the price for trade in round  $n$ ,  $p_n$ . If it is rejected, the price for trade in round  $n$  is that specified by the most recently agreed contract, which is just the price for the previous period if  $n > 0$  ( $p_n = p_{n-1}$ ) and the price agreed in the initial contract if  $n = 0$ . If no contract is in force, the current price is zero. These moves are illustrated by the choices at stages n.1 and n.2 in figure 1.

Since exchange is voluntary, each agent can decide whether or not to trade at the contract price in force for round  $n$ . This is illustrated by stages n.3 and n.4 in figure 1. For convenience we assume that the buyer decides first whether he wants to trade, with the seller then deciding whether she wants to do so. The order of moves at this point can be reversed without affecting the results that follow. If both parties want to trade, trade occurs for the  $\Delta$  units of time of round  $n$ .

Agents maximize their discounted flow payoffs from the game. Let  $r$  denote the discount rate and  $\delta = e^{-r}$  the discount factor for one unit of time.

The seller's payoff is then

$$U = \frac{1 - \delta^\Delta}{r} \left( \sum_{n=0}^{N-1} \delta^{n\Delta} u_n \right), \quad (1)$$

where  $u_n$  is the flow payoff to the seller in round  $n$ , normalized to 0 if no trade occurs and to  $p_n - c$  if there is trade at price  $p_n$ . The factor  $(1 - \delta^\Delta)/r = \int_0^\Delta e^{-rt} dt$  is the discounted payoff for a flow of one unit over a period of length  $\Delta$ . The buyer's payoff is

$$V = \frac{1 - \delta^\Delta}{r} \left( \sum_{n=0}^{N-1} \delta^{n\Delta} v_n \right), \quad (2)$$

where  $v_n$  is the flow payoff to the buyer in round  $n$ , normalized to 0 if there is no trade and to  $1 + c - p_n$  if there is trade at price  $p_n$ .<sup>2</sup> Let  $s_n = p_n - c$ . Since  $c$  is exogenous, we can then think of a contract for round  $n$  as specifying  $s_n$  rather than  $p_n$  and we shall do this where convenient in what follows. When there is no contract in force the sale price is set to zero as a matter of convention. In this case  $s_n = -c$ , that is the sale price is zero and the seller is making a loss of  $-c$  on each unit sold. The flow payoffs when there is trade are given by  $u_n = s_n$  and  $v_n = 1 - s_n$ . Thus  $s_n$  is the seller's share of the gains from trade in bargaining round  $n$ . Note that the buyer gains from trade as long as  $s_n \leq 1$ , the seller as long as  $s_n \geq 0$ . In what follows, it is often convenient to express payoffs in flow terms since these are independent of the bargaining horizon  $T$ .

### 3 Finite Horizon Bargaining

In this section we analyze finite horizon bargaining ( $T < \infty$ ). The first issue we consider is the appropriate equilibrium concept. One widely used concept is Nash equilibrium, which requires only that each party plays a strategy that yields a payoff as high as any other strategy given the strategy of the other party. With this equilibrium concept, however, contracts have little role — the fact that the parties already have a contract has no effect on the

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<sup>2</sup>The normalizations must take account of any breach penalties the courts would impose for nonfulfilment of an existing contract so that, if trade occurs,  $u_n$  measures the seller's, and  $v_n$  the buyer's, gain from trading relative to breaching.

set of equilibrium shares. To see this, suppose the parties have an existing contract with a price giving the seller a share  $s_0 \in (0, 1)$ , so both parties strictly prefer trading on the terms of the contract to not trading. Suppose also the seller thinks the price should be renegotiated upwards to give her a share  $s \in (s_0, 1]$  and adopts the following *tough strategy* in order to achieve this.

*Tough strategy for the seller*

1. at n.1, offer the contract  $s$  whenever given the opportunity to make an offer;
2. at n.2, reject all offers that give a share less than  $s$ ;
3. at n.4, refuse to trade until a new deal has been struck.

With the seller adopting this strategy, the buyer has a choice between accepting the price increase (which, since  $s \leq 1$ , is never less profitable than not trading) and receiving a payoff of zero while the seller holds up trade. Clearly the best thing for the buyer is to accept the seller's offer of  $s$  if the seller is chosen to make the first offer and to offer  $s$  himself if he is chosen to make the first offer. Moreover, against this strategy of the buyer, the tough strategy is the best the seller can do, so  $s$  is a Nash equilibrium share. A symmetric argument can be used to show that, with the buyer adopting a tough strategy,  $s \in [0, s_0)$  is also a Nash equilibrium share. Note that the argument applies *a fortiori* if  $s \notin (0, 1)$  since then one party would lose nothing by refusing to trade in the absence of renegotiation. We thus have the following result.

**Proposition 1** *Suppose the bargaining horizon is finite. Then, regardless of the shares agreed in an existing contract, for every  $s \in [0, 1]$  there is a Nash equilibrium in which the contract giving seller's share  $s$  is offered and accepted in the first bargaining round, and trade takes place in every bargaining round without further renegotiation.*

Having an existing contract thus makes no difference to the Nash equilibrium shares. This is not very satisfactory, either practically or theoretically. The practical objection is that, since contracts are widely used, the parties presumably think they make a difference to the outcome. The theoretical

objection is that, when  $s_0 \in (0, 1)$  and  $T < \infty$ , the result depends on the use of incredible threats. An incredible threat is one which, if an agent were actually faced with the decision to carry it out, would not maximize that agent's payoff given the strategy of the other agent. The concept of *subgame perfect equilibrium* rules out such threats by requiring that strategies form a Nash equilibrium *at every stage of the game, regardless of what has happened previously*.

To see that proposition 1 depends on the use of incredible threats when  $s_0 \in (0, 1)$ , suppose that we wish to support an equilibrium price with the property  $s \neq s_0$ . Consider the last bargaining round when all previous offers have been rejected, so no renegotiation has occurred. At stages n.3 and n.4, both agents prefer trade to no trade given  $s_0$  and, since there is no further play, trading in this round will not affect payoffs in future rounds. Therefore the seller would always wish to trade at stage n.4. Anticipating this, the buyer will choose to trade at stage n.3. However, if both agents anticipate trade at  $s_0$ , any offer of renegotiation at stage n.1 that one party would be prepared to offer would be rejected by the other at stage n.2 — since trade will occur even without renegotiation, one of the two parties must lose out if the price is changed. Consequently no renegotiation will occur in the last round and trade will take place with the seller receiving share  $s_0$ .

The outcome in the last bargaining round is thus uniquely determined. Since trading in the previous round makes no difference to what happens in the last round, both agents gain from trading at  $s_0$  then too. Anticipating this, one or the other will block any proposed renegotiation in the penultimate round. This argument can be continued backwards in time to conclude that  $s_0$  will never be renegotiated and trade will take place in every round. Thus, in any subgame perfect equilibrium, the parties will stick to any existing contract that gives the seller share  $s_0 \in (0, 1)$ . We then have the following result.

**Proposition 2** *If the bargaining horizon is finite and both parties would gain from trade under an existing contract (that is,  $s_0 \in (0, 1)$ ), this contract is not renegotiated in any subgame perfect equilibrium. Furthermore, trade takes place in every bargaining round.*

The essential ingredient in this result is that, when there are gains to both parties from trading under an existing contract, it is never credible to hold up



exchange. One way to see this result is to think in terms of the employment contract in which there are no separation payments. Suppose that a wage contract for the coming year has been agreed upon at  $w$  per hour. Further suppose that the worker has no alternative employment, and that the worker is carrying out valuable work for the firm that is worth more than  $w$  per hour. The worker could try to get a higher salary by threatening not to work. This result states that in a finite horizon model such a threat will never work since the firm will always refuse to renegotiate. The worker then chooses to work at  $w$  per hour rather than stay home and earn nothing. In contrast, if the value of leisure is great than  $w$ , or the value of the worker to the firm is less than  $w$  there is an incentive for one party to refuse to trade. This corresponds to having the existing contract,  $s_0$ , lying outside the interval  $[0, 1]$ . Trade will not then occur until the contract price is renegotiated. In this case too, there is a unique subgame perfect equilibrium.

**Proposition 3** *If the bargaining horizon is finite and the seller's share  $s_0$  of the gains from trade agreed in the existing contract satisfies  $s_0 \in (-\infty, 0) \cup (1, \infty)$ , the subgame perfect equilibrium payoffs are unique, independent of  $s_0$ , and involve trade in every bargaining round. The limits of these payoffs as the time interval between successive offers goes to zero ( $\Delta \rightarrow 0$ ) are also unique and correspond to the seller receiving the flow payoff  $u^* = (1 - \pi)$  [that is, the limiting payoffs to the seller and buyer are  $U^* = (1 - \pi)(1 - e^{-rT})/r$  and  $V^* = \pi(1 - e^{-rT})/r$ , respectively].*

The proof of this proposition follows from the proof of a more general result with outside options contained in the appendix. It follows immediately from lemma 11 in the appendix with the outside options set to yield payoffs of zero.

In this case the renegotiated contract is *independent* of the previous contract, rather the new contract is a function of the relative bargaining power of agents, as indexed by  $\pi$ , and of the payoffs in the absence of exchange (in our case these have been normalized to zero). For the Rubinstein model with an infinite horizon, Binmore, Rubinstein and Wolinsky (1986) demonstrate the relationship between the limiting equilibrium shares as the time between offers goes to zero and the payoffs derived from the Nash bargaining solution. The asymmetric Nash bargaining solution expressed in terms of flow payoffs

is given by (see Osborne and Rubinstein, 1990, page 13)

$$\arg \max_{v,u} (v - v^o)^\alpha \cdot (u - u^o)^{1-\alpha} \text{ subject to } v + u \leq 1, \quad (3)$$

where  $(v^o, u^o)$  is the threat point and  $\alpha$  the bargaining weight of the buyer. With  $v^o = u^o = 0$  (the payoffs in the absence of trade) and  $\alpha = \pi$ , this gives exactly the limiting payoffs in proposition 3. Note that these payoffs necessarily apply if there is no existing contract since then  $s_0 = -c$ . This convenient practical relationship between the limiting equilibrium payoffs and the Nash bargaining solution continues to hold despite the finite horizon.

There are two values of  $s_0$  not covered by either proposition 2 or proposition 3, namely  $s_0 = 0$  and  $s_0 = 1$ . For these values, one of the parties is indifferent between trading and not trading under the existing contract. It turns out that there is then a continuum of subgame perfect equilibrium payoffs. For completeness, we state the formal result, which is a special case of proposition 8 that is proved in the appendix.

**Proposition 4** *If the bargaining horizon is finite and there is an existing contract with either  $s_0 = 0$  or  $s_0 = 1$ , there exists a continuum of subgame perfect equilibrium payoffs, all involving trade in every bargaining round. As the time interval between offers goes to zero ( $\Delta \rightarrow 0$ ), the seller's flow payoff  $u^*$  satisfies*

$$u^* \in \begin{cases} [0, 1 - \pi], & \text{if } s_0 = 0; \\ [1 - \pi, 1], & \text{if } s_0 = 1. \end{cases}$$

*In each case, the buyer's flow payoff  $v^*$  satisfies  $v^* = 1 - u^*$ .*

In summary, for a finite horizon bargaining model the subgame perfect equilibrium payoffs are unique except when  $s_0 = 0$  and  $s_0 = 1$ . If both parties would gain from trade under an existing contract, that contract is not renegotiated. If at least one party prefers not trading to trading under the existing contract, the price specified in that contract has no effect on the renegotiated contract terms, which simply reflect the relative bargaining powers of the two parties. Figure 2 illustrates the equilibrium payoff for the buyer as a function of the normalized contract price  $s_0$ .

## 4 Infinite Horizon Bargaining

In this section we discuss the infinite horizon version of the bargaining game of section 2. In practice relationships are not infinitely lived but, as Osborne and Rubinstein (1990) have suggested, the infinite horizon game can be viewed as a model of a situation in which both agents behave each period as if the relationship is going to last at least one more period. In the original Rubinstein (1982) game with a single trade and an infinite horizon, the subgame perfect equilibrium payoffs are unique. This is no longer the case if players have an opportunity to decide not to trade after reaching an agreement, as Muthoo (1990) has shown. With bargaining over a flow and an infinite horizon, Haller and Holden (1990) and Fernandez and Glazer (1991) show that the addition of a trade decision also results in a multiplicity of equilibrium payoffs. Among these are inefficient equilibria in which trade does not commence in the first bargaining round. In the context of union bargaining, Fernandez and Glazer (1991) interpret such equilibria as involving a strike. Two features of the model are important for this. One is trade decisions that are separate from the agreement on a contract. The other is the infinite horizon.

It is helpful to start by noting that continuing to trade under a contract which is mutually beneficial is always an equilibrium (though not, as we shall see, the only equilibrium).

**Proposition 5** *With an infinite bargaining horizon, if neither party loses from trade under an existing contract (that is,  $s_0 \in [0, 1]$ ), continuing to trade in every bargaining round under that contract without renegotiation is a subgame perfect equilibrium.*

**Proof.** Consider the following *no renegotiation strategy* for both buyer and seller.

*No renegotiation strategy*

1. at stage n.1, the buyer offers a new contract with  $s < s_0$ , the seller a new contract with  $s > s_0$ , whenever chosen to make an offer;
2. at stage n.2, the buyer rejects any new contract offered unless  $s < s_0$ , the seller any new contract offered unless  $s > s_0$ ;

3. at stages n.3 and n.4, both buyer and seller offer to trade on the terms of the existing contract.

Given 1 and 2, it is clear that 3 is always a best response since, with  $s_0 \in [0, 1]$ , neither party loses in the current round from trading and doing so has no effect on future payoffs. It is then clear that neither buyer nor seller can improve on 1 and 2 given that the other is following the no renegotiation strategy. With both following this strategy, trade continues under the existing contract with no renegotiation. ■

We next characterize the set of efficient equilibria, those in which a trading price is agreed, and trade commences, in the first bargaining round. Suppose the existing contract price is between  $c$  and  $1 + c$  (that is,  $0 \leq s_0 \leq 1$ ), so the seller has a maximum potential gain of  $1 - s_0$  from renegotiation and the buyer a maximum potential gain of  $s_0$ . We solve for the efficient equilibrium payoffs using the method developed by Shaked and Sutton (1984) and explored further in Sutton (1986). Consider first the best renegotiation for the seller. Let  $\bar{U}$  denote the best efficient equilibrium payoff the seller could hope to get at the start of the game (before stage 0.0). Since an efficient outcome involves immediate agreement, the corresponding equilibrium payoff to the buyer is  $\frac{1}{r} - \bar{U}$ . Let  $\bar{U}^b$  be the corresponding payoff to the seller at stage 0.1 if the buyer is chosen to make the offer,  $\bar{U}^s$  the corresponding payoff to the seller at stage 0.1 if she is chosen to make the offer at that stage. If the buyer makes the offer, the best the seller could hope for is that the buyer would make an offer that leaves her just indifferent between accepting the offer and delaying agreement for one bargaining round, while trading at the existing contract price in the meantime.<sup>3</sup> Because of the stationarity that results from the infinite horizon, the best payoff the seller could hope for after a delay of one period is just  $\bar{U}$ . This gives us the following formula for  $\bar{U}^b$ :

$$\bar{U}^b = \frac{1 - \delta^\Delta}{r} s_0 + \delta^\Delta \bar{U}. \quad (4)$$

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<sup>3</sup>Note that it cannot be an equilibrium for the buyer to offer a contract that makes the seller strictly better off than she would be by turning it down. For such a contract, the buyer could always offer a slightly worse deal that is accepted by the seller with probability one. Therefore, the only possible equilibrium offer is one that makes the seller indifferent between accepting and refusing. This result obviously depends on the assumption that offers are infinitely divisible. See van Damme, Selten and Winter (1990) for a study of bargaining when this assumption is dropped.

The second term on the right hand side of this is the best equilibrium payoff the seller could hope to get from reaching an agreement one bargaining round of length  $\Delta$  later. The first term is the payoff from trading at the existing contract price in the meantime. Note that by trading during this period the seller lowers her cost of waiting and thereby tilts the buyer's offer in her favour.

If the seller is chosen to make the offer at stage 0.1, she makes an offer that leaves the buyer indifferent between accepting and delaying agreement by one round. In this case the seller increases the cost of delay to the buyer by refusing to trade if the buyer rejects her offer. Thus her offer gives the buyer a payoff  $(\frac{1}{r} - \bar{U}^s)$  that satisfies

$$\frac{1}{r} - \bar{U}^s = 0 + \delta^\Delta (\frac{1}{r} - \bar{U}). \quad (5)$$

The second term on the right hand side of this is the buyer's payoff from agreement one bargaining round of length  $\Delta$  later. The first is the zero payoff from having no trade occur in the meantime.

Since the buyer gets to make the offer at stage 0.1 with probability  $\pi$  and the seller with probability  $(1 - \pi)$ , it follows that  $\bar{U} = \pi \bar{U}^b + (1 - \pi) \bar{U}^s$ . Thus equations (4) and (5) imply that the best equilibrium payoff (in flow terms) the seller could hope for is  $\bar{s}$  defined by

$$\bar{s} \equiv r\bar{U} = (1 - \pi) + \pi s_0 = s_0 + (1 - \pi)(1 - s_0). \quad (6)$$

This corresponds to the share from the existing contract  $s_0$  plus a proportion  $(1 - \pi)$  of the seller's maximum potential gain from renegotiation,  $(1 - s_0)$ .

Now consider the best renegotiation the buyer could hope to get out of the maximum potential gain of  $s_0$ . Denote the resulting payoff by  $\bar{V}$ . Symmetrically to the case just discussed, the worst threat by the buyer is to trade whenever he rejects a bad offer by the seller and to refuse to trade whenever the seller refuses his offer. When the buyer gets to make the offer of a contract at stage 0.1, he will offer a contract that makes the seller indifferent between accepting and delaying agreement by one round with no trade occurring in the meantime. Thus he will offer  $\bar{V}^b$  satisfying

$$\frac{1}{r} - \bar{V}^b = 0 + \delta^\Delta (\frac{1}{r} - \bar{V}). \quad (7)$$

When the seller gets to make the offer at stage 0.1, she makes an offer that leaves the buyer indifferent between accepting and delaying agreement for one round while trading at the existing contract price in the meantime. Thus she offers  $\bar{V}^s$  that satisfies

$$\bar{V}^s = \frac{1 - \delta^\Delta}{r}(1 - s_0) + \delta^\Delta \bar{V}. \quad (8)$$

Since  $\bar{V} = \pi \bar{V}^b + (1 - \pi) \bar{V}^s$ , equations (7) and (8) imply that the best equilibrium payoff the buyer could hope for is

$$\bar{V} = \frac{(1 - s_0) + \pi s_0}{r}, \quad (9)$$

with corresponding payoff  $\underline{s}$  (in flow terms) to the seller of

$$\underline{s} \equiv (1 - r\bar{V}) = (1 - \pi)s_0. \quad (10)$$

This defines the lowest equilibrium (flow) share the buyer can hope the seller will get.

The payoffs in (6) and (10) define the best that the seller and the buyer could hope for from renegotiation. To show that they are indeed equilibrium payoffs, we must specify subgame perfect strategies that support them. Consider first the case in which the seller tries to renegotiate the price upwards to give her the share  $\bar{s}$ . Note that, if she achieves this, it follows from proposition 5 that, since  $\bar{s} \in [0, 1]$ , it is a subgame perfect equilibrium to continue trading under that contract with no further renegotiation. Thus, to show  $\bar{s}$  is an equilibrium share, it is enough to show that there exist equilibrium strategies that result in  $\bar{s}$  being offered in the first period and immediately accepted.

The derivation of  $\bar{s}$  depended on (a) both seller and buyer offering to trade at the existing contract price if the seller rejects an offer by the buyer, and (b) the seller offering the contract  $\bar{s}$  and refusing to trade if the buyer rejects this. The first of these is always part of an equilibrium strategy given that  $s_0 \in [0, 1]$  provided future decisions are not conditioned on whether or not trade has taken place in the past — neither party loses in the current period so, if future decisions are unaffected, neither loses overall. The second can always be made part of an equilibrium strategy if  $\Delta$  is sufficiently small. We know from proposition 5 that it would always be a continuation equilibrium

not to renegotiate away from  $s_0 \in [0, 1]$ . Thus strategies that call for both parties to move to the no renegotiation strategy (defined in the proof of proposition 5) if the seller ever trades after the buyer rejects an offer of  $\bar{s}$  are certainly subgame perfect from that round on. With such strategies, demanding  $\bar{s}$  now and, if that demand is rejected, refusing to trade in order to get  $\bar{s}$  agreed in the next bargaining round is optimal for the seller as long as

$$\delta^\Delta \bar{s} \geq s_0. \quad (11)$$

Since  $\bar{s} > s_0$ , this inequality is satisfied whenever  $\Delta$  is sufficiently small or, equivalently, the costs of delay are sufficiently low. Strategies that support immediate renegotiation to  $\bar{s}$  followed by continuing trade under that contract can thus be summarized as follows.

*Seller's  $\bar{s}$  strategy*

1. At the beginning of the game and after any history in which the seller has refused to trade whenever the buyer has rejected the seller's offer:
  - (a) at stage n.1, demand  $\bar{s}$  whenever given the opportunity to make an offer;
  - (b) at stage n.2, reject all offers with  $s < \bar{s}$ ;
  - (c) at stage n.4, offer to trade whenever the buyer made the offer at n.1; refuse to trade whenever the buyer has rejected the seller's offer at n.1 until a new deal has been struck.
2. After any history in which the seller has traded after having an offer rejected by the buyer, follow the *no renegotiation strategy* described in the proof of proposition 5.

*Buyer's  $\bar{s}$  strategy*

1. At the beginning of the game and after any history in which the seller has refused to trade whenever the buyer has rejected the seller's offer:
  - (a) at stage n.1, offer  $\bar{s}$  whenever given the opportunity to make an offer;
  - (b) at stage n.2, reject all offers with  $s > \bar{s}$ ;

(c) at stage n.3, offer to trade.

2. After any history in which the seller has traded after having an offer rejected by the buyer, follow the *no renegotiation strategy* described in the proof of proposition 5.

Corresponding strategies can be used to show that immediately agreeing on  $\underline{s}$  is a subgame perfect equilibrium. This establishes that immediate renegotiation to  $\bar{s}$  and to  $\underline{s}$ , followed in both cases by continuing trade without further renegotiation, are both equilibrium outcomes. Now consider immediate renegotiation to  $s \in (\underline{s}, \bar{s})$ , followed by continuing trade without further renegotiation. We can construct equilibrium strategies supporting such an outcome by supposing that, in round 0, both parties offer  $s$  if given the chance, accept  $s$  if it is offered, and then follow the *no renegotiation strategy* given the contract  $s$ . We know from proposition 5 that, once  $s$  has been accepted, it is an equilibrium to continue trading under this contract without further renegotiation. Moreover, we can ensure that it is optimal for  $s$  to be offered and accepted in round 0 by the following punishment strategies: if the buyer deviates, the parties continue by playing the equilibrium with seller's share  $\bar{s}$  from round 1 on. If the seller deviates, they continue with the equilibrium with seller's share  $\underline{s}$ . We then have the following result.

**Proposition 6** *Suppose the bargaining horizon is infinite and the existing contract satisfies  $s_0 \in [0, 1]$ . For any  $s \in [\underline{s}, \bar{s}]$  with  $\underline{s}$  and  $\bar{s}$  defined by (10) and (6) respectively, there is a bargaining round of length  $\Delta$  sufficiently short that there exists a subgame perfect equilibrium with immediate renegotiation to the contract with seller's share  $s$  of the gains from trade, followed by continuing trade in every bargaining round without further renegotiation. Furthermore, there exist no efficient equilibria for  $s \notin [\underline{s}, \bar{s}]$ .*

Proposition 6 is concerned with  $s_0 \in [0, 1]$ . When  $s_0 \notin [0, 1]$ , at least one of the parties will refuse to trade until a new contract is agreed. By following the argument leading to proposition 6 but amending the payoffs to reflect this refusal to trade, it is straightforward to show that the efficient equilibrium payoffs are unique with, as  $\Delta \rightarrow 0$ , the seller receiving the limiting share



$s = (1 - \pi)$ .<sup>4</sup> The set of possible efficient equilibria as a function of the existing contract price is shown in figure 3. The shaded area represents the possible payoffs for the seller given an initial contract price  $s_0$ .

Fernandez and Glazer (1991) argue that the multiple equilibrium result of proposition 6 can be used as the basis for a theory of strikes. For any  $s_0 \in [0, 1]$ , a strike can occur in a subgame perfect equilibrium with the following strike strategies.

*Strike strategies*

1. The seller refuses to trade at all times  $t \in [0, T']$ . In the first bargaining round after time  $T'$ , both parties play a subgame perfect equilibrium that results in continuing trade with the seller's share of the gains  $\bar{s}$ .
2. Should the seller agree to trade at any time before  $T'$ , both parties move immediately to the subgame perfect equilibrium with the seller's share of the gains from trade  $\underline{s}$ .

For  $\Delta$  sufficiently small, these strategies can support any strike of length  $T'$  that satisfies

$$\ln \bar{s} - \ln \underline{s} > rT'. \quad (12)$$

Substitution of the values of  $\bar{s}$  and  $\underline{s}$  from (6) and (10) results in the expression

$$\ln \left[ \frac{1}{s_0} + \frac{\pi}{1 - \pi} \right] > rT'. \quad (13)$$

Note that, if the existing contract price is low so that the seller gains little from trade ( $s_0$  close to zero) or if the bargaining power of the seller is very low ( $\pi$  close to one), we can get very long strikes. Corresponding expressions can be obtained for a lock-out (or exchange hold up) by the buyer. In that case, long lock-outs can be obtained when  $s_0$  is close to one or the bargaining power of the seller is very high.

As a theory of strikes, this model has features that are unsatisfactory for both practical and theoretical reasons. On the practical side, it provides

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<sup>4</sup>This is different from the result in Muthoo (1990) because Muthoo uses the original Rubinstein (1982) assumption that trade occurs at most once, instead of being a flow as here. That gives the parties a greater incentive to hold out for a good offer because, when trade occurs at most once, trading on a bad offer now necessarily reduces potential future payoffs.

little guidance as to when strikes will occur and how long they will last. With a low existing contract price, we know that long strikes are possible equilibria. However, for the same set of observable characteristics, immediate agreement without a strike is also an equilibrium outcome. The model also provides little guidance as to when, and in which direction, renegotiation will occur. Along an equilibrium path, the trading price may be renegotiated at any time and, provided the contract currently in force has  $s \in (\underline{s}, \bar{s})$ , in either direction.

On the theoretical side, the theory runs into a problem noted by Hicks (1963). A strike is inefficient since neither party receives the potential gains from trade. Thus, if the parties know they will settle on a particular  $s$  at time  $T$ , it is hard to believe that they would not find some way of agreeing on  $s$  now and so make both better off. Every equilibrium in this model requires that both parties understand the rules and have consistent expectations about future behaviour. The essence of a strike or refusal to trade is that the two parties cannot agree on a new contract. Given this, it is difficult to understand why they should implicitly agree upon a single set of strategies with the appropriate punishments to support a particular equilibrium allocation. Indeed, one common view of play in a repeated game is that agents know and understand each other. In such circumstances, Hicks' argument seems very reasonable and one would expect rational agents to negotiate outcomes that cannot be improved upon by renegotiation — that is, outcomes that are *renegotiation proof*.

There are several concepts of renegotiation proofness, not all of which are appropriate for all games. See Bergin and MacLeod (1993) for a full discussion. Where it exists, however, there is some preference for the concept of *strong renegotiation proofness* of Farrell and Maskin (1989). This requires that at every node of the game agents agree to play a subgame perfect equilibrium that is Pareto efficient in the set of subgame perfect equilibrium payoffs at that node. Suppose at date  $n.3$  and  $n.4$  the contract in force specifies  $s_n \in (0, 1)$  and the subgame perfect equilibrium requires that no trade occur. Both agents can be made better off by agreeing to trade under the current contract and subsequently continuing with the equilibrium they would have played in the absence of trade. Thus a renegotiation proof equilibrium requires trade to occur whenever  $s_n \in (0, 1)$ . As with a finite horizon, no existing contract with  $s_0 \in (0, 1)$  is then renegotiated — since trade will take place in any case, renegotiation of the price will make one party worse off and

that party will thus block it. We have already shown that, for  $s_0 \notin (0, 1)$ , efficient equilibrium payoffs are unique and give the same shares of the gains from trade as in the finite horizon case as  $\Delta \rightarrow 0$ . Consequently, as  $\Delta \rightarrow 0$ , the set of strong renegotiation proof equilibrium flow payoffs in the infinite horizon bargaining game is the same as the set of subgame perfect equilibrium flow payoffs for the finite horizon game. (Note that, imposing strong renegotiation proofness in the finite horizon case makes no difference to the equilibrium payoffs because these are in any case efficient.)

**Proposition 7** *With an infinite bargaining horizon, all strong renegotiation proof equilibria involve trade in every bargaining round. Moreover, the limiting set of strong renegotiation proof equilibrium flow payoffs as the time interval between offers goes to zero ( $\Delta \rightarrow 0$ ), expressed as a function of the seller's share  $s_0$  of the gains from trade under the existing contract, are  $u^*$  for the seller and  $v^*$  for the buyer given by*

$$(u^*, v^*) = \begin{cases} [(1 - \pi), \pi], & \text{if } s_0 \in (-\infty, 0) \cup (1, \infty); \\ [(u, v) | u + v = 1 \text{ and } (u, v) \geq (0, \pi)], & \text{if } s_0 = 0; \\ (s_0, 1 - s_0), & \text{if } s_0 \in (0, 1); \\ [(u, v) | u + v = 1 \text{ and } (u, v) \geq (1 - \pi, 0)], & \text{if } s_0 = 1. \end{cases}$$

These payoffs are illustrated in figure 3. In each case, agreement is reached in the first bargaining round and trade continues at every date without further renegotiation. Therefore, if we interpret full rationality between two traders in a long term relationship with no asymmetry of information as requiring efficient play from the set of subgame perfect equilibrium payoffs, rational play never leads to delay in reaching agreement. This suggests that models of the type analysed here are not appropriate for a satisfactory theory of delay in reaching agreement.<sup>5</sup>

## 5 Markets and the Outside Option Principle

Consider now an extension to the bargaining model in which agents are not restricted to trading with just one partner but also have opportunities to

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<sup>5</sup>Kennan and Wilson (1993) provide an overview of bargaining with private information. One of the main issues they discuss is when the existence of private information results in costly delay in reaching agreement.

trade with other agents in the market. The form such market opportunities take is important for the outcome of bargaining. Formal modelling helps us to understand how.

One form of market opportunity is the possibility of trading with third parties during the negotiation process. In this case, the only difference from the models of the previous sections is that the payoffs from such trade affect the default payoffs of the bargaining partners in the absence of agreement. The default payoffs in the previous sections were normalized to zero and the gains from trade to one. Thus, for this form of market opportunity, the equilibrium payoffs of the previous sections continue to apply provided they are interpreted as the proportions of the gains from reaching agreement that are received by the parties over and above the payoffs from trading with third parties during the negotiation process.

An alternative form of market opportunity is one that, once taken up, effectively terminates the current relationship. This is what is known in the literature as an *outside option*. This form of market opportunity is important when starting a new trading relationship that involves sunk costs, such as switching costs or relationship specific investments. Changing one's job, for example, involves a series of investments such as moving, learning new skills and forming new relationships. If these sunk costs are sufficiently high, they will be incurred only to establish a new long term relationship in which case, once an outside option is selected, the current bargaining game ends.

Shaked (1987) has shown that the set of equilibria is sensitive to the time at which agents are able to exercise their outside option. He considers two possibilities, "*hi-tech*" markets and *bazaars*. Shaked defines a "hi-tech" market as one in which an agent who makes an offer that is refused is able to take up an outside option before there is any possibility of the other agent making a counter offer. An example that illustrates why such a market is called "hi-tech" is securities trading over the telephone. The trader makes a verbal offer to a trader on one line and, if it is refused, can immediately switch to bargaining with another trader on another line without waiting for a counter offer. In contrast, in a bazaar the seller always has time to make a counter offer before the buyer can reach the door.

In the "hi-tech" case, there may be multiple equilibria with an infinite bargaining horizon. With a finite bargaining horizon, equilibrium is unique but sensitive to the number of bargaining rounds. To see why, consider the renegotiation proof equilibrium in the infinite horizon bargaining game with

no existing contract ( $s_0 = -c$ ). Then, by proposition 7, the equilibrium payoff to the buyer in the absence of outside options is  $\frac{\pi}{r}$ . Now suppose that, in the first bargaining round only, the buyer has an outside option with a value of  $\frac{\pi}{r} - \epsilon$  (for some small positive  $\epsilon$ ) that he may take up after the offer of a new contract has been accepted or rejected but before the trade stage. Even though this outside option is worth less than the buyer's equilibrium payoff in the absence of the outside option, it can have a large impact on the game. Consider  $\epsilon$  sufficiently small (or  $\Delta$  sufficiently large) that

$$\frac{\pi}{r} - \epsilon \geq \delta^\Delta \frac{\pi}{r}. \quad (14)$$

Since the outside option is available only in the first bargaining round, the game from the second round on is exactly the same as the game in the previous section, so the buyer will expect payoff of  $\delta^\Delta \frac{\pi}{r}$  if there is no trade in the first round. (The factor  $\delta^\Delta$  appears because of the one period delay in reaching agreement.) Thus, if the seller refuses an offer by the buyer in the first round, the buyer is better off taking up the outside option worth  $\frac{\pi}{r} - \epsilon$  (thus giving the seller a payoff of zero) than waiting one round to receive  $\delta^\Delta \frac{\pi}{r}$ . Consequently, the seller will get a payoff of zero if she refuses the buyer's offer because the buyer will choose the outside option instead of delaying. The existence of this outside option thus gives the buyer *all* the bargaining power. In the infinite horizon model, allowing agents to choose an outside option in any period after having an offer refused may result in multiple equilibria. In the finite horizon model with no existing contract, equilibrium is still unique but the equilibrium payoffs may cycle as the length of the bargaining round  $\Delta$  is decreased, which implies that the limit payoff to each party as  $\Delta \rightarrow 0$  is a set of points.

This effect is somewhat unnatural. For many markets, however, the bazaar model seems more appropriate. In labour markets, for example, an employer typically has a chance to respond to outside offers an employee receives before the employee actually moves. A bazaar is modelled formally by allowing an outside option to be taken up only by an agent who has just received an offer. The agent making the offer must wait to take up an outside option until he or she has received a counter offer. The extensive form for a bazaar is shown in figure 4 with O used to represent the choice of an outside option. (For simplicity, the trade decisions are not illustrated — they are the same as in figure 1.)

In the case of a bazaar, the bargaining game with a finite horizon has well defined limiting payoffs (as  $N \rightarrow \infty$ ) with a simple structure. If the equilibrium payoffs in the absence of outside options are preferred by both parties to their respective outside options, adding these outside options to the game leaves the equilibrium payoffs unchanged. If the equilibrium payoffs in the absence of outside options are worse than the outside option for one agent, the effect of adding the outside options to the game is that this agent receives a payoff exactly equal to his or her outside option. These results are summarized in the following proposition, in which the outside option payoffs for the seller and the buyer are denoted by  $\bar{u}$  and  $\bar{v}$  respectively.

**Proposition 8** *Consider a finite horizon bazaar in which it is efficient for the parties to trade with each other (that is,  $\bar{u} + \bar{v} < 1$ ) and in which both can do better by choosing an outside option than by not trading at all (that is,  $\bar{u}, \bar{v} > 0$ ). In every subgame perfect equilibrium, trade takes place in every bargaining round. Moreover, the limits of the subgame perfect equilibrium flow payoffs as the time interval between offers goes to zero ( $\Delta \rightarrow 0$ ) are  $u^*$  for the seller and  $v^*$  for the buyer given by*

$$(u^*, v^*) = \begin{cases} (1 - \pi, \pi), & \text{if } s_0 \in (-\infty, 0) \cup (1, \infty) \text{ and } \pi \in [\bar{v}, 1 - \bar{u}]; & \text{(i)} \\ [(u, v) \mid u + v = 1 \text{ and } (u, v) \geq (\bar{u}, \pi)], & \text{if } s_0 = 0; & \text{(ii)} \\ (\bar{u}, 1 - \bar{u}), & \text{if } s_0 \in (0, \bar{u}), \text{ or if } s_0 \in (-\infty, 0) \cup (1, \infty) \text{ and } 1 - \pi < \bar{u}; & \text{(iii)} \\ (s_0, 1 - s_0), & \text{if } s_0 \in [\bar{u}, 1 - \bar{v}]; & \text{(iv)} \\ (1 - \bar{v}, \bar{v}), & \text{if } s_0 \in (1 - \bar{v}, 1), \text{ or if } s_0 \in (-\infty, 0) \cup (1, \infty) \text{ and } \pi < \bar{v}; & \text{(v)} \\ [(u, v) \mid u + v = 1 \text{ and } (u, v) \geq (1 - \pi, \bar{v})], & \text{if } s_0 = 1. & \text{(vi)} \end{cases}$$

The proof of this proposition is in the appendix. The effect of an existing contract on the buyer's equilibrium payoff is illustrated in figure 5. If the existing contract price  $s_0$  is below 0, the seller refuses to trade unless the price is renegotiated, in which case the seller gets the share  $1 - \pi$  of the gains from trade. If the existing contract price  $s_0$  is above 1, the buyer refuses to trade unless the price is renegotiated and again the seller gets share  $1 - \pi$  of the gains from trade. If the existing contract price  $s_0$  lies within the range  $(0, 1)$  but trade at this price would make one party worse off than an outside option, the price is renegotiated to give that party a payoff exactly equal to that from the outside option. In contrast, if the existing contract price is better for both parties than their outside options (that is, it results in a payoff to the seller in the region  $[\bar{u}, 1 - \bar{v}]$ ), it is not renegotiated. This last

property plays an important role in the applications described in the next section.

A final point about proposition 8. If the existing contract has  $s_0 \in (-\infty, 0) \cup (1, \infty)$ , which includes the case of having no contract ( $s_0 = -c$ ), the limiting equilibrium flow payoffs are the same as those derived from the Nash bargaining solution defined in (3) with  $v^0 = u^0 = 0$  and  $\alpha = \pi$ , subject to the constraints that each player receives at least the value of her/his outside option. Thus, as in the case without outside options, this convenient property is preserved in finite horizon bargaining over flows.

## 6 Applications

In this section, we illustrate how the results of previous sections can be used to derive insights about economic issues. We consider two applications, the first to contracting when there are specific investments, the second to repeated shocks to the environment and the endogenous determination of contract length.

### 6.1 Specific Investments

Although allowing for the bargainers to have an existing contract at the start of bargaining, the models of the previous sections did not make explicit the reasons why they should want to do so. Even without a contract, they trade whenever it is efficient. All that an existing contract does is affect how the gains from trade are divided.

An existing contract has an additional role if it is efficient for the parties to make specific investments before trade begins. Without a contract, as Grout (1984) and Williamson (1985) have emphasized, investment will typically be inefficient. The reason is that the *ex post* gains from trade include the returns on the investments. If the parties share these gains as a result of bargaining, each will get a share of the return on the other's investment. But an investor who does not receive all the return on an investment will not choose the efficient level of that investment. Williamson calls this the *hold-up* problem.

The results in the previous sections can be used to derive a form of initial contract that overcomes the hold-up problem. Suppose the seller and the buyer make specific investments of  $i_s$  and  $i_b$  respectively, the seller's in-

vestment reducing the cost of supplying the good, the buyer's increasing the utility from acquiring it. Suppose also these investments are made while there is still some uncertainty about what the seller's cost and the buyer's utility will be. We can then represent the actual flow payoffs to the seller and the buyer respectively by

$$u = p - c(i_s, \theta) - i_s \quad (15)$$

$$v = y(i_b, \theta) - p - i_b, \quad (16)$$

where  $p$  is the actual trading price,  $c(i_s, \theta)$  the seller's cost,  $y(i_b, \theta)$  the buyer's utility from the good, and  $\theta$  a random variable whose realization is not known at the time the investments are made but becomes known before renegotiation/trade takes place. (Since it is convenient to work with flow payoffs, the costs of the investments  $i_s$  and  $i_b$  are expressed in flow terms too but the total costs of the investments are assumed to be incurred before the renegotiation/trade stage.)

With these payoffs, the efficient levels of investment are  $i_s^*$  and  $i_b^*$  given by the first order conditions

$$E\{c'(i_s^*, \theta)\} = -1 \text{ and } E\{y'(i_b^*, \theta)\} = 1. \quad (17)$$

If there is no initial contract, these efficient levels will not be attained. The *ex post* gains from trade (once the investments have been made and  $\theta$  become known) are  $y(i_b, \theta) - c(i_s, \theta)$ . By proposition 3 (for the finite horizon case) or proposition 7 (for the infinite horizon case), the buyer will receive a share  $\pi$  of these gains when there is no existing contract ( $s_0 = -c$ ), so the actual trading price, denoted  $p^*(i_s, i_b, \theta)$ , must satisfy

$$y(i_b, \theta) - p^*(i_s, i_b, \theta) = \pi [y(i_b, \theta) - c(i_s, \theta)] \quad (18)$$

so that

$$p^*(i_s, i_b, \theta) = (1 - \pi)y(i_b, \theta) + \pi c(i_s, \theta). \quad (19)$$

With this substituted for  $p$  in (15) and (16), one gets the first order conditions for the actual choices of investments

$$E\{\pi c'(i_s, \theta)\} = -1 \text{ and } E\{(1 - \pi)y'(i_b, \theta)\} = 1. \quad (20)$$

Because of the factors  $\pi$  and  $(1 - \pi)$  that enter, (20) results in levels of investment different from the efficient levels defined in (17). When net benefit



is a concave function of the investments these conditions imply that both agents will under invest in the relationship. This is a formal statement of the hold-up problem.

The hold-up problem could be overcome if the parties agreed to an enforceable initial contract that specified the trading price as a function of the investment levels and  $\theta$ . But, as the literature has emphasized, the nature of specific investments often makes it difficult to specify them fully in a contract. Suppose, however, it is possible to make the price specified in the contract contingent on  $\theta$ . (It is common for long term contracts for coal to have built in provisions for changing the price in response to changing circumstances, see Joskow (1988).) Then, if there exists a contract price  $p(\theta)$  such that

$$y(i_b, \theta) > p(\theta) > c(i_s, \theta), \forall i_b, i_s \geq 0 \text{ and } \forall \theta, \quad (21)$$

and if the parties agree to an initial contract that specifies this price, it follows from proposition 5 (for the finite horizon case) and proposition 7 (for the infinite horizon case) that the initial contract will not be renegotiated regardless of the levels of investment by the two agents. Thus the investments are chosen *ex ante* to satisfy

$$\hat{u} = \max_{i_s} E\{p(\theta) - c(i_s, \theta)\} - i_s \quad (22)$$

$$\hat{v} = \max_{i_b} E\{y(i_b, \theta) - p(\theta)\} - i_b, \quad (23)$$

which results in the efficient levels of investment specified in (17). Note that for this result the contract price is not required to be conditioned on the levels of investments, only on the random variable  $\theta$ .

This model can throw light on the long term coal contracts studied by Joskow (1988). The puzzle in Joskow's data set is the observation that public utilities often sign long term contracts (20 years or more) with coal suppliers that contain a large number of escalator clauses. If the motive for writing a contract were insurance, we should expect the contract price and market price to deviate over time. In fact what we observe is that the escalator clauses do a very good job of following the market price for coal. A natural question is why firms would expend resources on signing a complex contract instead of depending on the market for their supply. Joskow's interpretation is that this is to protect specific investments, such as the construction of a train line from the mine mouth to the utility. In the context of the

present model, what is required to protect specific investments is avoidance of renegotiation, since renegotiation would lead to sharing of the returns on those specific investments. The model would predict, in sharp contrast to the insurance model, that firms would include escalator clauses to make the trading price follow the market price and thus preclude any incentive to renegotiate. Joskow (1990) certainly emphasizes how few significant price renegotiations occurred even with substantial changes in market conditions, which suggests that actual contracts did a good job of achieving what theory says they ought to achieve. For further discussion of these issues, see MacLeod and Malcomson (1993b).

In other cases, it may not be possible to index the contract price to the random variable  $\theta$  or there may be no indexed price that satisfies the condition (21) necessary to avoid *ex post* renegotiation. An alternative way to induce efficient investment is to restructure the relationship so that one party makes all the specific investments and make use of the outside option principle in the way suggested by Hart and Moore (1988).<sup>6</sup> Consider, for example, an employment relationship with specific investments in both training and in relocation of the employee. If the firm (the buyer) pays for the training costs  $i_b$  and the employee (the seller) meets her or his own moving expenses  $i_s$ , specific investments are made by both parties. A common practice, however, is to have the firm pay both training costs and moving expenses. Then the flow payoffs to the employee and the firm corresponding to (15) and (16) are

$$u = p - c(\theta) \quad (24)$$

$$v = y(i_b, i_s, \theta) - p - i_b - i_s. \quad (25)$$

(Note that  $c(\theta)$  now represents the disutility of work.) Let  $\bar{u}(\theta)$  denote the *ex post* flow payoff the employee could get by going back to the market and looking for another job and suppose that the bargaining power of the buyer  $\pi$  is sufficiently high that

$$(1 - \pi) [y(i_b, i_s, \theta) - c(\theta)] \leq \bar{u}(\theta), \forall i_b, i_s \geq 0 \text{ and } \forall \theta. \quad (26)$$

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<sup>6</sup>In Hart and Moore, trade between the investing parties must occur at a single fixed date for the investment to yield any return, so delay ends the game in exactly the same way as choosing an outside option. As a result, the only outcomes other than trading at the existing contract price involve renegotiation so that one party or the other receives the outside option value and there is no sharing of the gains from trade as in the outcome in case (i) of proposition 8.

The expression  $[y(i_b, i_s, \theta) - c(\theta)]$  is the gain from trade, of which the employee would receive the share  $(1 - \pi)$  if the contract were renegotiated in the absence of outside options. When condition (26) holds, this would give the employee a payoff lower than the outside option. Thus proposition 8 implies that renegotiation results in the employee receiving a payoff just equal to the value of the outside option  $\bar{u}(\theta)$ . By (24), this requires the actual trading price to be  $p = \bar{u}(\theta) + c(\theta)$ , so the firm's payoff is  $v = y(i_b, i_s, \theta) - \bar{u}(\theta) - c(\theta) - i_b - i_s$ . The firm therefore chooses the investments by maximizing the expectation over  $\theta$  of this expression, which will result in it selecting the efficient investment levels. Intuitively, since the employee's payoff is independent of the investment, the employee receives none of the return on the firm's investments, so the firm invests efficiently.

This result is in contrast to Becker's (1975) conclusion that the parties should share the return to specific training in order to reduce inefficient turnover, with the result that specific investment by the firm generates a tenure effect on wages. When condition (26) is satisfied, sharing the returns is detrimental to the efficiency of investment and the efficient arrangement analysed here generates no such tenure effect. The difference arises because Becker assumes no *ex post* renegotiation, whereas in the present analysis *ex post* renegotiation is important because it ensures that the employee never quits when it is efficient for her/him to stay. Some degree of sharing would arise even with *ex post* renegotiation if the employee's outside option value  $\bar{u}(\theta)$  was known with certainty only to the employee and not to the firm. Becker's analysis can be interpreted as the case in which *ex post* renegotiation is possible but never occurs because the firm has no more information about  $\bar{u}(\theta)$  after  $\theta$  is revealed than before. The conclusion is that specific investment does not necessarily result in a tenure effect on wages, as commonly assumed. It need not if the firm knows  $\bar{u}(\theta)$  *ex post* and this may well differ from one case to another. For further discussion of how tenure effects may arise as a result of specific investments by both parties even when *ex post* renegotiation is possible and the outside option values are known to both parties, see MacLeod and Malcomson (1993a).

The same framework can also be applied to the analysis of horizontal and vertical integration. Suppose the investing buyer in the previous example is a downstream firm using an input from an upstream seller. As long as the buyer and seller are separate firms, with the seller able to supply to third parties if she wishes, the outside option argument used in that example continues to

apply and investment will be efficient as long as (26) holds. But if the buyer takes over the seller and, as a result, constrains the manager of the upstream plant from selling to third parties, the outside option is no longer available and it may no longer be possible to induce efficient investment. Bolton and Whinston (1993) use a similar bargaining framework to analyse the effects of different ownership structures when two investing downstream firms compete for inputs from an upstream firm.

## 6.2 Repeated Shocks and Endogenous Contract Length

So far, the only random events we have considered are those that occur after investments have been made but before trade takes place. The finite horizon model is, however, readily adapted to the situation in which random events affect the gains from trade even after trade has started. Suppose we divide the total time horizon into the time periods between successive shocks. We know from proposition 8 that the flow payoffs are independent of the length of the time horizon as the time interval between successive offers goes to zero. Thus, in the final period, the payoffs are given by proposition 8 conditional on the contract in force and on the realization of the random variable at the start of that period. In the penultimate period, the only effect of having a subsequent period is thus to add a known expected payoff to each of the possible renegotiated outcomes, so it is in principle straightforward to compute the equilibrium payoffs conditional on the contract in force and the realization of the random variable at the start of the penultimate period.<sup>7</sup> The whole game can be solved by working backwards in this way.

Contract length will then be endogenous. How long it is before the initial contract is renegotiated depends on the shocks that occur. That renegotiation may not occur every period despite the shocks implies that the price in one period will be correlated with the price in the next. See MacLeod and Malcomson (1993b) for further discussion of this issue.

The solution in the face of repeated shocks is particularly simple when the contract in force is of fixed duration (as, for example, with union contracts in the US) and this duration corresponds to the periods between shocks. With

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<sup>7</sup>The expected payoffs from the final period are unique despite the multiplicity of equilibria in proposition 8 for  $q_0 = 0$  and  $q_0 = 1$  provided that the random variable has a distribution with no mass points so that these non-unique outcomes occur with probability zero.

contracts of a fixed duration, payoffs for subsequent periods are independent of the contract negotiated in the current period. Thus the same expected payoff in future periods attaches to each outcome of the current negotiations and the choice between actions in the current period is unaffected by the existence of the future periods. The shares of the gains from trade in any given period conditional on the contract  $s_0$  in force at the start of that period are therefore just those given by proposition 8.

In its current form, the model does not capture all the characteristics observed in actual contracts. It provides no explanation for contract lengths other than *either* the length of the period between shocks *or* indefinite duration with renegotiation when required. Nor does it have anything to say about how often payments are made during the period the contract is in force though, since in the model that is arbitrary, it can be determined by whatever factors external to the model make convenient. Moreover, the model predicts that the terms of any renegotiated contract should be independent of those previously agreed when, as assumed above, the default payoffs from not trading are unaffected by the previous agreement. Yet Abowd and Lemieux (1993) find that even after including controls for firm profitability and union alternatives, the previous contract wage has a large significant impact on the new contract. One possible explanation is that contract renegotiation is in effect an implicit way of incorporating the escalator clauses that, as discussed above, are needed to achieve efficient specific investments. Another is that, as in Moene (1988) and Holden (1994), employees can use some form of “go slow” or “work to rule”, during which they are still paid something, as an alternative to an all out strike and their default payoff in the absence of agreement is thus affected by their existing contract. Further developments of the basic framework are required to formalize these and other possibilities and further empirical work to discriminate between them.

## 7 Conclusions

We have argued in this paper that the standard Rubinstein (1982) bargaining model is not, in its original form, appropriate for analyzing contract bargaining. Contracts are used where the components of transactions are not all completed simultaneously and this inevitably provides scope for the parties to renegotiate before the transaction is completed. To analyze contract bar-

gaining, it is thus important to use a model that allows for renegotiation. One way to allow for this is to treat trade as a flow that takes place under the terms of any contract already agreed. Then either party can refuse to trade unless the terms are renegotiated. This is a natural way to represent many long term relationships such as employment, the long term supply of inputs, and financial contracts.

We have shown in this paper that, when trade is a flow and the bargaining horizon finite, the equilibrium properties are nevertheless similar to those of the original Rubinstein model and its extensions by Shaked and Sutton (1984) to include outside options. When there is no contract in force at the start of the game, the subgame perfect equilibrium payoffs are unique and, no matter how short the bargaining horizon, have the convenient representation in terms of the Nash bargaining solution as the time interval between successive offers goes to zero that was noted by Binmore, Rubinstein and Wolinsky (1986). The model also gives rise to intuitive conditions for when an existing contract will not be renegotiated.

With an infinite bargaining horizon, the results are more complicated even when trade is a flow. In many cases, subgame perfect equilibrium payoffs are then not unique, as Haller and Holden (1990) and Fernandez and Glazer (1991) have shown. There may also be delay in reaching agreement despite perfect information, a delay that Fernandez and Glazer interpret as a strike in the context of union wage bargaining. However, if one imposes a not unreasonable condition of renegotiation proofness on equilibrium, no delay occurs and the equilibrium payoffs are exactly the same as for the finite horizon model. This at least throws doubt on whether this model can be used to provide a satisfactory theory of strikes.

It can, however, be used to throw light on other economic issues. The present paper considers two applications. One application is to the Williamson (1985) hold-up problem that arises when the parties make investment specific relationships. The model can be used to derive forms of contract that overcome this problem even when the levels of investment cannot themselves be specified enforceably in the contract. The other application is to bargaining when there are repeated shocks that affect the gains from trade. The model can then be used to generate a theory of endogenous contract length.

## A Appendix

This appendix develops the basic results for the finite horizon bargaining game, which are then applied to proposition 8. For simplicity we set  $T = 1$  (the extension to  $T \neq 1$  is trivial) and do not deal explicitly with the trade/no trade decision at stages n.3 and n.4. The game form is illustrated in figure 4. As discussed in the text, with a finite horizon agents trade if and only if they are made better off in the current bargaining round by doing so. The best responses are not unique only when one of the players is indifferent between trade and no trade. In this case the equilibrium is indeterminate, as illustrated by the vertical sections for the set of renegotiation proof payoffs in figure 4.

Let  $U_N(n)$  and  $V_N(n)$  be the subgame perfect equilibrium payoffs to the seller and buyer respectively for the subgame starting with period  $n$  in the  $N$  period bargaining game under the assumption that no agreement has been reached in periods 0 to  $n - 1$ . Since the game stops in period  $n = N$ , then  $U_N(N) = V_N(N) = 0$ . The following result is an immediate consequence of applying backward dynamic programming to this problem.

**Lemma 9** *The subgame perfect equilibrium payoffs for the  $N$  period finite horizon bargaining game are unique. The payoffs for the subgame starting in period  $n$ , under the assumption that an agreement has not previously been reached, are given by*

$$\begin{aligned} V_N(n) &= \beta(n\Delta) - U_N(n) \\ U_N(n) &= F_N[U_N(n+1), n], \end{aligned} \tag{27}$$

where  $\beta(n\Delta) = (1 - \delta^{1-n\Delta})/r$  is the total surplus from the relationship from time  $n\Delta$  until the end of the game at time  $T = 1$  and

$$\begin{aligned} F_N(U, n) = & \pi \cdot \max \left\{ \beta(n\Delta)\bar{u}, \delta^\Delta U \right\} \\ & + (1 - \pi) \left\{ \beta(n\Delta) - \max \left\{ \beta(n\Delta)\bar{v}, \delta^\Delta [\beta((n+1)\Delta) - U] \right\} \right\}. \end{aligned} \tag{28}$$

**Proof.** Suppose the recursive equations (27) hold in period  $n + 1$  (except at  $n = N$  when  $U_N(0) = V_N(0) = 0$ ) and consider the decision faced by the seller at node  $n.2$  after receiving an offer  $p_n$  from the buyer. The seller will accept such an offer only if it provides a payoff at least as large as

$U' \equiv \max \{ \beta(n\Delta)\bar{u}, \delta^\Delta U_N(n+1) \}$ , the better of either taking the outside option or delaying for one period. If the offer has greater value it will be accepted with probability one. This means that the buyer never offers the seller a price resulting in a payoff that is higher than  $U'$ .

In equilibrium the buyer makes an offer of  $U'$  that is accepted with probability one. Since there is a loss of surplus from waiting one period or taking the outside option if the seller is expected to reject the offer, the buyer can offer slightly more than  $U'$  to ensure that the seller accepts with probability one. Rejection by the seller cannot therefore be part of equilibrium play.

Together these imply that in equilibrium plays the buyer makes an offer yielding payoff  $U'$  to the seller and the seller accepts with probability one. By a similar argument, the seller receives the payoff  $\beta(n\Delta) - \max \{ \beta(n\Delta)\bar{v}, \delta^\Delta [\beta((n+1)\Delta) - U_N(n+1)] \}$  if she is selected by nature to make the offer at node  $n.1$ . Thus the agents reach an efficient agreement in period  $n$ . The form of the recursion in (28) follows from taking expectations at node  $n.0$ . ■

It is convenient to express payoffs as flow values  $u_N(n)$  and  $v_N(n)$ , as follows

$$\begin{aligned} u_N(n) &= U_N(n)/\beta(n\Delta), \text{ for } n < N \\ v_N(n) &= V_N(n)/\beta(n\Delta), \text{ for } n < N. \end{aligned}$$

With this normalization,  $u_N(n) + v_N(n) = 1$  for  $n < N$ .

Consider first the case with one period of play ( $N = 1$ ). In this case the agent chosen at time 0 makes a “take it or leave it” offer for one period of trade, with no potential for renegotiation. From equation (27) we then have the following corollary.

**Corollary 10** *If  $N = 1$ , the unique subgame perfect equilibrium payoffs are given by  $u_1(0) = (1 - \pi)(1 - \bar{u} - \bar{v}) + \bar{u}$  and  $v_1(0) = \pi(1 - \bar{u} - \bar{v}) + \bar{v}$ .*

This implies that, expressed in terms of the Nash bargaining solution, the values of the outside options  $(\bar{v}, \bar{u})$  are the threat point and  $\pi$  is the bargaining weight. To see this note that setting  $v^o = \bar{v}$ ,  $u^o = \bar{u}$  and  $\alpha = \pi$  in the expression for the Nash bargaining solution (3) gives exactly the payoffs in corollary 10.

Therefore in the absence of renegotiation outside options can act as threat points in the sense of the Nash bargaining solution. Increasing the number



of bargaining rounds introduces the possibility of waiting as a strategy, significantly changing the structure of the equilibrium. We characterize the equilibrium payoffs for frictionless bargaining given by the limit of the equilibrium payoffs as  $\Delta \rightarrow 0$ . To express these in flow terms, let  $[x]$  denote the integer part of  $x$ , and let  $\tau$  be the amount of time already elapsed in bargaining. Define the normalized flow payoffs for the frictionless game by:

$$u^*(\tau) = \lim_{\Delta \rightarrow 0} U_{[1/\Delta]}([\tau/\Delta])/\beta(\tau) \quad (29)$$

$$v^*(\tau) = \lim_{\Delta \rightarrow 0} V_{[1/\Delta]}([\tau/\Delta])/\beta(\tau), \quad (30)$$

where, consistent with the notation in lemma 9,  $\beta(\tau) = (1 - \delta^{1-\tau})/r$ . Thus  $u^*(\tau)$  [ $v^*(\tau)$ ] is the flow payoff to the seller [buyer] in frictionless equilibrium when  $\tau$  is the time elapsed. The payoffs are characterized in the following proposition. It demonstrates that, as bargaining becomes frictionless, the equilibrium payoffs approach the Nash bargaining solution when the defaults are zero (no trade), with the qualification that the outside options act as bounds on the equilibrium payoffs.

**Lemma 11** *For  $\tau \in [0, 1)$  the limiting flow payoffs to bargaining in a finite horizon bazaar are given by*

$$u^*(\tau) = \begin{cases} (1 - \pi), & \text{if } (1 - \pi) \in (\bar{u}, 1 - \bar{v}); \\ \bar{u}, & \text{if } (1 - \pi) \leq \bar{u}; \\ (1 - \bar{v}), & \text{if } \pi \geq \bar{v}; \end{cases}$$

$$v^*(\tau) = 1 - u^*(\tau).$$

**Proof.** Because the payoff to the seller is bounded, there is a function  $U^*(\tau)$  that is a limit of some sequence  $U_{[1/\Delta]}([\tau/\Delta])$  as  $\Delta \rightarrow 0$ . An increase in the number of periods corresponds to a finer partition of the interval  $[0, 1]$ . Eq. (27) for  $U$  specifies the change in the seller's payoff over a period of length  $\Delta$ . Dividing the change by  $\Delta$  defines the rate of change for  $U_{[1/\Delta]}([\tau/\Delta])$  and thus, in the limit,  $dU^*(\tau)/d\tau$  if it exists. Combined with the boundary condition  $U^*(1) = 0$ , this defines a unique  $U^*(\tau)$ . The rate of change of the seller's payoff when  $\tau$  is the time elapsed and  $U$  the equilibrium payoff in period  $\tau + \Delta$  is defined by

$$G(U, \tau) = \lim_{\Delta \rightarrow 0} \{U - F_{[1/\Delta]}(U, [\tau/\Delta])\} / \Delta,$$

where  $F_N(U, n)$  is defined in lemma 9. A straightforward computation yields the following limits:

$$G(U, \tau) = \begin{cases} -\infty, & \text{if } U > \beta(\tau)(1 - \bar{v}); \\ rU - (1 - \pi), & \text{if } \beta(\tau)(1 - \bar{v}) > U > \beta(\tau)\bar{u}; \\ +\infty, & \text{if } U < \beta(\tau)\bar{u}. \end{cases} \quad (31)$$

When  $U = \beta(\tau)(1 - \bar{v})$  or  $\beta(\tau)\bar{u}$ , a well defined limit does not exist, rather a range of possible values exists depending on how the limit is taken. Given the monotonicity of the function  $F_N(U, n)$  in  $U$ , the range of possible values is given by  $(-\infty, rU - (1 - \pi)]$  if  $U = \beta(\tau)(1 - \bar{v})$  and  $[rU - (1 - \pi), +\infty)$  if  $U = \beta(\tau)\bar{u}$ . In these cases set

$$G(U, \tau) = \begin{cases} (-\infty, rU - (1 - \pi)], & \text{if } U = \beta(\tau)(1 - \bar{v}); \\ [rU - (1 - \pi), +\infty), & \text{if } U = \beta(\tau)\bar{u}. \end{cases} \quad (32)$$

Together (31) and (32) define a differential inclusion that places necessary restrictions on the limit points of the bargaining game. In particular, it is impossible to have  $U^*(\tau) < \beta(\tau)\bar{u}$  or  $U^*(\tau) > \beta(\tau)(1 - \bar{v})$ . Moreover, if  $U^*(\tau) \in (\beta(\tau)\bar{u}, \beta(\tau)(1 - \bar{v}))$ , it must be differentiable and satisfy  $dU^*(\tau)/d\tau = G[U^*(\tau), \tau]$ . The limit payoffs at all points of differentiability must therefore satisfy

$$dU^*(\tau)/d\tau \in G[U^*(\tau), \tau], \text{ for all } \tau \in [0, 1], U^*(1) = 0. \quad (33)$$

This inclusion has a unique solution. To see this, first suppose that  $(1 - \pi) \in (\bar{u}, 1 - \bar{v})$  and note that  $dU/d\tau = rU - (1 - \pi)$  has the unique solution  $U(\tau) = \beta(\tau)(1 - \pi)$ . If the limit follows the seller's outside option, that is  $U(\tau) = \beta(\tau)\bar{u}$ , then  $dU/d\tau = -\delta^{(1-\tau)}\bar{u}$ . But if  $(1 - \pi) > \bar{u}$ , (31) implies that  $dU/d\tau = rU - (1 - \pi) = \beta(\tau)\bar{u} - (1 - \pi) < -\delta^{(1-\tau)}\bar{u}$ , and hence (33) cannot be satisfied. A similar argument shows that  $U^*(\tau) < \beta(\tau)(1 - \bar{v})$ . Therefore the unique solution to (33) when  $(1 - \pi) \in (\bar{u}, 1 - \bar{v})$  is  $U^*(\tau) = \beta(\tau)(1 - \pi)$ .

If  $(1 - \pi) \leq \bar{u}$ , any  $U(\tau)$  larger than  $\beta(\tau)\bar{u}$  grows in value no faster than  $\beta(\tau)\bar{u}$ , so the only solution to (33) in this case is  $U^*(\tau) = \beta(\tau)\bar{u}$  with  $dU^*(\tau)/d\tau = -\delta^{(1-\tau)}\bar{u} \in [rU - (1 - \pi), +\infty)$ . Similarly when  $(1 - \pi) \geq (1 - \bar{v})$ , then  $U^*(\tau) = \beta(\tau)(1 - \bar{v})$ .

The flow payoff  $u^*(\tau)$  in the proposition is obtained by dividing these expressions for  $U^*(\tau)$  by  $\beta(\tau)$ . The flow payoff for the buyer is given by  $v^*(\tau) = 1 - u^*(\tau)$ . ■

These results can now be used to prove proposition 8.

**Proof of proposition 8.** In case (i) either the buyer or seller prefers not to trade at the price  $s_0$  and therefore no trade occurs until renegotiation occurs. The result then follows from an application of lemma 2. In case (iv) both parties prefer to trade rather than not to trade or take up the outside option. The result follows from the backward dynamic argument in lemma 1 combined with the fact that trade occurs every period at the price  $s_0$ . Cases (iii) and (v) follow from the outside option principle given in lemma 2. When  $s_0 \in (0, \bar{u})$  or  $s_0 \in (1 - \bar{v}, 1)$  simply replace  $(1 - \pi)$  by  $s_0$  in lemma 2.

In case (ii) the seller is indifferent between trading and refusing to trade at the price  $s_0$ . At one extreme the seller could refuse to trade every period until a new agreement is reached. In that event the equilibrium payoff is given by case (i). At the other extreme the seller might agree to trade at a zero price, then the outside option is binding with the equilibrium payoff given by case (iii). From lemma 1, we know that agents reach an efficient agreement immediately. Thus all the remaining payoffs in this case are generated by the seller randomizing the trade decision when  $s_0 = 0$ . This gives the remaining payoffs in (ii). The demonstration of case (vi) is similar. ■

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