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Optomechanical to mechanical entanglement transformation

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Abstract. We present a scheme for generating entanglement between two mechanical oscillators that have never interacted with each other by using an entanglement-swapping protocol. The system under study consists of a Michelson–Morley interferometer comprising mechanical systems embodied by two cantilevers. Each of them is coupled to a field mode via the radiation pressure mechanism. Entanglement between the two mechanical systems is set by measuring the output modes of the interferometer. We also propose a control mechanism for the amount of entanglement based on path-length difference between the two arms.

Contents

Our understanding of the mechanisms responsible for optomechanical dynamics has received an enormous boost in the last few years, due to the first significant demonstrations of experimental controllability of small mechanical devices by means of electrical or optical probes [1]. In particular, the coherent coupling between electromagnetic fields and massive micromechanical structures integrated in optical cavities opens up intriguing possibilities for the study of the behaviour of mesoscopic devices that are closer to the boundary between classical and quantum domains. The fabrication of microstructures having large mechanical quality factor and optical reflectivity [2], together with the improvement of cooling abilities by means of passive and active techniques, will soon allow the achievement of a fully quantum regime in such systems. Theoretically, investigations of the conditions for the establishment of entanglement involving optical and mechanical degrees of freedom have revealed a surprising resilience of quantum correlations to the thermal character of the mechanical system and to its intrinsic damping rate [3, 4], while unexpected genuinely multipartite entangled states can be created by means of optomechanical couplings involving a single micromechanical oscillator at larger temperature and two optical fields [5]. These expectations are awaiting an experimental demonstration that would represent a major step forward in both quantum control and technology.

Here, we present a scheme for entangling two spatially remote mechanical devices by means of a mechanism that is closely related to the well-known concept of entanglement swapping [6]. Differently from the proposal put forward by Zhang *et al* [7], where purely optical entanglement is transferred to mechanical systems by means of bilocal interactions, our protocol does not require any pre-built entangled resource. We simply transform bilocal optomechanical entanglement, established through radiation–pressure coupling, into purely mechanical entanglement by means of postselection operated over two ancillary optical modes. Our scheme is also distinguished with respect to previous proposals for mechanical entanglement established by means of the joint interaction of two mechanical systems with common optical buses [4]. On the other hand, it shares some features with the scheme put forward in [8], where proper joint postselection of the sidebands of the light reflected by two identical micromirrors is able to create pure mechanical entanglement. Such a sideband-filtering process, however, may imply some practical difficulties when the mechanical systems are embodied by vibrating mirrors of optical cavities. Our proposal does not rely on similar processes and is based on a standard interferometric configuration closely related to proposals that are currently under active experimental investigation [9]. Moreover, we highlight a mechanism that allows the control of the amount of mechanical entanglement based on the management of the width of optical pulses overlapping at a beam splitter (BS), thus providing a handy knob for tunable entanglement.

The remainder of the paper is organized as follows: in section 1, we describe in detail the protocol we propose. First, we assess the dynamics within a single optomechanical subsystem consisting of a cavity with a vibrating mirror fed by a pulse and then we address the role played by the considered interferometric setting. Section 2 describes the swapping mechanism that transforms hybrid entanglement of optomechanical nature into purely mechanical entanglement. We quantify the entanglement between the vibrating mirrors finding that even Geiger-like detectors can be used in such a process, although the ability to resolve the population of the photonic part of the system is key to achieving large amounts of entanglement. We then briefly address the mechanism for controlling entanglement by means of optical pulses. In section 3, we summarize our findings.

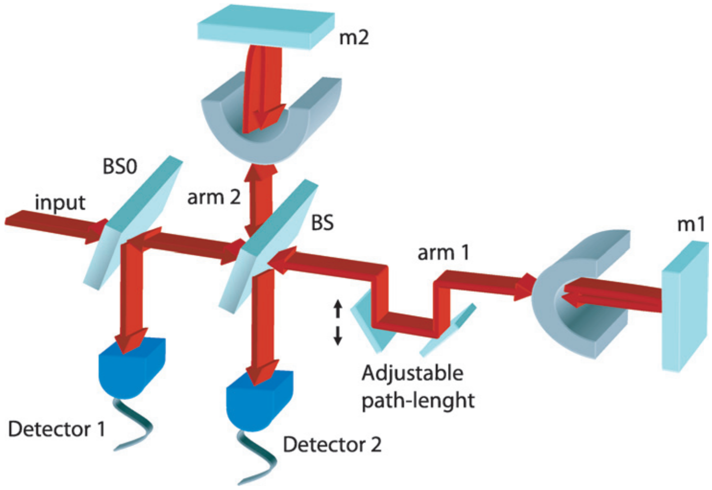

Figure 1. Sketch of the interferometrical setting for optomechanical to mechanical entanglement transformation. Each arm of a Michelson–Morley interferometer includes an optical cavity with a light-vibrating mirror (labelled $m1$ and $m2$, respectively). An adjustable pathlength difference can be set between the two arms. An input coherent pulse feeds the interferometer. We also show two BSs: BS0 is a polarizing-BS needed to redirect the reflected light to detector 1, while BS is polarization insensitive and superimposes the fields propagating along arms 1 and 2.

1. Description of the dynamics

We now give an outline of the dynamics within the setup we propose, which consists of the Michelson–Morley interferometer shown in figure 1. The interferometer comprises two identical Fabry–Perot cavities with fixed input mirrors and light vibrating rear ones. Each vibrating mirror is modelled as a harmonic oscillator with fundamental frequency ω_{m}. We also include a tunable path-length difference δx between the two arms of the interferometer, which will be used later in order to control the mechanical entanglement. The input to the 50 : 50 BS shown in figure 1 is taken to be a coherent pulse of time-length Δt. Given the classical nature of the input, the BS is unable to entangle the output modes, which end up in two coherent-state pulses. After propagation along the arms of the interferometer, the two pulses enter the cavities and interact, via radiation–pressure coupling, with the vibrating mirrors. In general, this creates a tensorial product of entangled optomechanical states of a vibrating mirror and the corresponding field: while each optomechanical subsystem is in general entangled, no quantum correlation is shared by them. However, the reflected pulses might carry enough 'non-classicality' to give rise, after the BS, to entangled output modes. This possibility is at the basis of our entanglement swapping mechanism. In particular, if the path difference δx is arranged in a way so as to set a time-mismatch $\delta t = \delta x/c$ larger than the pulse-length Δt, the two reflected pulses will not meet at the BS. Differently, if $\delta t < \Delta t$, the BS superimposes the reflected pulses, thus determining a correlation between the two optomechanical subsystems. In general, this leaves us with a four-body entangled state comprising both the vibrating mirrors and the optical fields. A postselective measurement operated over these latter will project the mirrors onto an entangled state.

Now that we have given the *rationale* of the proposed physical mechanism, we can address the details of our scheme. Let us consider a coherent pulse $|\{\alpha(\omega)\}\rangle$ entering the interferometer with $\alpha(\omega)$ a function centred at the carrier frequency ω_0 and with width $\Delta\omega$. The pulse is prepared in a coherent state described via a multi-mode displacement operator as

$$|\{\alpha(\omega)\}\rangle = \exp\left(\int \mathrm{d}\omega[\alpha(\omega)\hat{a}^\dagger(\omega) - \mathrm{h.c.}]\right)|\{0\}\rangle, \tag{1}$$

where $a(\omega)$ $[a^\dagger(\omega)]$ is the annihilation (creation) operator of a photon at frequency ω and $|\{0\}\rangle = |0\rangle \otimes |0\rangle \otimes ..|0\rangle$ is a collection of vacuum modes. Free evolution of each component of the pulse leads to the state at position x and time t given by $|\Psi(t,x)\rangle = |\{\alpha(\omega)\mathrm{e}^{\mathrm{i}\omega(t-x/c)}\}\rangle = \exp(\int \mathrm{d}\omega[\alpha(\omega)\mathrm{e}^{\mathrm{i}\omega(t-x/c)}\hat{a}^\dagger(\omega) - \mathrm{h.c.}])|\{0\}\rangle$. We now consider the long-pulse approximation within which only a narrow range of frequencies around the carrier one ω_0 are taken into account and such that $\hat{a}(\omega) \simeq \hat{a}(\omega_0)$. Within this approximation, the field operator can be taken out of the integration, leaving us with

$$|\Psi(t,x)\rangle = \exp\left(F(t-x/c)\hat{a}^\dagger(\omega_0) - F^*(t-x/c)\hat{a}(\omega_0)\right)|0\rangle, \tag{2}$$

where $F(t-x/c) = \int \mathrm{d}\omega\, \alpha(\omega)\mathrm{e}^{\mathrm{i}\omega(t-x/c)}$ is the Fourier transform of $\alpha(\omega)$. Equation (2) accounts for a travelling signal described as a coherent state of the carrier mode ω_0 whose amplitude is modulated in time and space.

We now address the coupling between one field and the corresponding vibrating mirror, which occurs via radiation–pressure effects. Clearly, in the picture above only one component of the original pulse is considered, so that the Hamiltonian describing the radiation–pressure interaction between light and the intra-cavity mechanical system is [10] (we set $\hbar = 1$ throughout the paper)

$$\hat{H} = \omega_{\mathrm{m}}\hat{b}^\dagger\hat{b} + \omega_0\hat{a}^\dagger\hat{a} + G\hat{a}^\dagger\hat{a}(\hat{b}^\dagger + \hat{b}), \tag{3}$$

where $\hat{b}$ $(\hat{b}^\dagger)$ annihilates (creates) a phonon of the mechanical system, G is the field–mirror coupling strength (which depends on ω_0 and the length of the cavity) and ω_{m} is the vibrating frequency of the mechanical system. Within our unitary description, the time-propagator associated with $\hat{H}$ is given by a generalization of the expression that has been explicitly derived in [11]

$$\hat{U}(t) = \mathrm{e}^{-\mathrm{i}\omega_0 t\hat{a}^\dagger\hat{a}}\mathrm{e}^{\mathrm{i}g^2(\hat{a}^\dagger\hat{a})^2\Lambda(t)}\hat{D}_{\mathrm{m}}[g\hat{a}^\dagger\hat{a}\eta(t)], \tag{4}$$

where $\Lambda(t) = \omega_{\mathrm{m}}t - \sin(\omega_{\mathrm{m}}t)$, $\eta(t) = 1 - \mathrm{e}^{-\mathrm{i}\omega_{\mathrm{m}}t}$, $g = G/\omega_{\mathrm{m}}$ and $\hat{D}_{\mathrm{m}}[\beta]$ is the mirror's displacement operator of amplitude β. The term containing $(\hat{a}^\dagger\hat{a})^2$ in equation (4), which is obtained using the Campbell–Baker–Hausdorff theorem, arises by virtue of the dependence of the dynamics of $\hat{b}$ and $\hat{b}^\dagger$ on the amplitude of the cavity field. Note that $\hat{U}(t)$ describes the time evolution only when the pulse is inside the cavity. When the pulse is travelling in free space, the time evolution is implicitly taken into the account in equation (2). Such monochromatic approximation not only is mathematically convenient but is also physically important. In fact, we now show that, in order to have field–mirror entanglement, the pulse has to be considered to be for all practical purposes, almost monochromatic. From equation (4), a necessary condition for optomechanical entanglement is to have $g\eta(t)$ large [9], so that the mirror-displacement is actually effective (rigorously, the amplitude of the displacement has to be larger than the spread of the initial wavepacket describing the initial state of the mechanical system). A rough timescale for the interaction within the cavity is given by $\mathcal{N}\Delta t$, where $\mathcal{N}$ is the number of round-trips of a photon inside the cavity. This means that $\Delta t \approx \pi/(\mathcal{N}\omega_{\rm m})$. On the other hand, the Hamiltonian in equation (4) holds for the cavity's free spectral range Ω much larger than the mirror oscillation frequency, i.e. $\Omega \gg \omega_{\rm m}$. These two conditions imply that $\Omega \gg \Delta\omega/(2\mathcal{N})$ with $\Delta\omega$ the pulse width in the frequency domain. The above condition is satisfied when the pulse-width is narrower than the free spectral range, i.e. when the field is treated as quasi-monochromatic. Clearly, the bandwidth of the pulse should also fall within the linewidth of the cavity it enters. This guarantees that only a negligible fraction of the incident light is reflected at the cavity input mirror, thus optimizing the coupling with the mechanical system. The quasi-monochromatic nature of the pulse guarantees this condition to be matched and we will assume this from now on.

We now consider in detail the role of the interferometer. We take $\alpha(\omega)$ to be a Gaussian function of carrier amplitude α_0. In this way, we can set $F(t-x/c)=\alpha_0 f(t-x/c)$ with $f(t-x/c)$ a Gaussian function of t, centred at x/c. A coherent pulse $|\sqrt{2}\alpha_0 f(t-x/c)\rangle$ at the input of the interferometer is split at the BS into two equal coherent pulses of modulated amplitude $\alpha_0 f(t-x/c)$, each propagating in one of the arms of the interferometer. In order to illustrate the basic features of our proposal, we now address the case of mechanical systems being initially prepared in their ground state. Although not yet achieved, this case is not far fetched: current progress in optomechanical cooling, either assisted by feedback or entirely passive [1], together with advancements in the fabrication techniques [2], hold promise for the achievement of mechanical free energies comparable with the ground state energy of a harmonic oscillator. This will put the vibrating mirror fully within the quantum regime, as predicted by recent theoretical studies [12]. The case of mechanical systems being prepared in thermal states of small average phonon number is the subject of our ongoing work and presents some technical difficulties related to the quantification of entanglement in high-dimensional bipartite mixed states [13]. We just mention that one could use an approach close to the one presented in [14, 15]. Under these conditions, the state of the optomechanical system after the BS reads

$$|\psi(t,x_1,x_2)\rangle = \left|\alpha_0 f\left(t-\frac{x_1}{c}\right),\ \alpha_0 f\left(t-\frac{x_2}{c}\right)\right\rangle_{f1,f2} |0,0\rangle_{m1,m2}, \tag{5}$$

where the first ket refers to the state of the field modes in the two arms (labelled $f1$ and $f2$) and the second to the state of the mirrors (labelled $m1$ and $m2$, respectively). In our notation, x_1 (x_2) is the position of the pulse in the first (second) arm of the interferometer.

Each mirror–field interaction begins when one of the pulses enters the respective cavity, which happens at $x_1 = x_0$ and $x_2 = x_0 + \delta x$ with x_0 the length of the shorter arm. Formally, these are the positions at which the radiation pressure-driven evolution has to be 'switched on'. This is taken into account by writing

$$\hat{U}(x_1,x_2) = [\bar{\delta}(x_1-x_0)\hat{\mathbb{1}} + \delta(x_1-x_0)\hat{U}_1(t)] \otimes [\bar{\delta}(x_2-x_0-\delta x)\hat{\mathbb{1}} + \delta(x_2-x_0-\delta x)\hat{U}_2(t)], \tag{6}$$

where $\hat{\mathbb{1}}$ is the identity operator, $\hat{U}_1(x)$ and $\hat{U}_2(x)$ have the same form as in equation (4) and $\bar{\delta}(x) = 1-\delta(x)$. We stress that the formal expression in (6) accounts for the influences of spatial degrees of freedom on the interaction mechanism. On the temporal scale, on the other hand, the interaction depends on the time each pulse spends inside a cavity and the corresponding pulse's time-width, as detailed later. The full time-evolution of the system is given by the application of the operator in equation (6) to the state given in equation (5)

$$|\psi(t,x_1,x_2)\rangle = \hat{U}(x_1,x_2)|\psi_{\rm bi}(t,x_1,x_2)\rangle. \tag{7}$$

When the two pulses have fully entered the respective cavity, i.e. after a time Δt, the evolved optomechanical state is

$$|\psi(t+\Delta t, x_1, x_2)\rangle = \hat{U}_1(\Delta t)\otimes\hat{U}_2(\Delta t)|\psi(t, x_1, x_2)\rangle. \tag{8}$$

The explicit form of this state is easily gathered by expanding each coherent state in terms of Fock states and applying the time evolution operator. We find that

$$|\psi'(t, x_1, x_2)\rangle = \sum_{n_1,n_2} c_{n_1}(t, x_1)c_{n_2}(t, x_2)|n_1, n_2\rangle_{f1,f2}\otimes|n_1\zeta(\Delta t), n_2\zeta(\Delta t)\rangle_{m1,m2}, \tag{9}$$

where $\zeta(\Delta t) = g\eta(\Delta t)$ and

$$c_p(t, x) = \frac{[\alpha_0 f(t, x)]^p}{\sqrt{p!}}\mathrm{e}^{-\mathrm{i}p\omega_0\Delta t}\,\mathrm{e}^{\mathrm{i}g^2p^2\Lambda(\Delta t)}. \tag{10}$$

The two pulses getting out of the cavities are then superimposed by the BS when fields $f1$ and $f2$ have covered a distance $2x_0$, $2x_0+2\delta x$ along arms 1 and 2, respectively. This transforms the state into

$$\begin{aligned}|\psi_{\mathrm{out}}\rangle &= \hat{B}|\psi'(t, 2x_0, 2x_0+2\delta x)\rangle\\ &= \sum_{n_1,n_2} c_{n_1}(t, 2x_0)c_{n_2}(t, 2x_0+2\delta x)\hat{B}|n_1n_2\rangle_{f1,f2}\otimes|n_1\zeta(\Delta t), n_2\zeta(\Delta t)\rangle_{m1,m2}\end{aligned} \tag{11}$$

with $\hat{B} = \exp\frac{\pi}{4}(a_1^\dagger a_2 - a_1 a_2^\dagger)$ the operator of a 50:50 BS and $\hat{a}_{1,2}$ ($\hat{a}_{1,2}^\dagger$) the annihilation (creation) operator of the fields in the two arms of the interferometer. The action of the BS over a Fock state is accounted for by introducing the coefficients $B_{n_1n_2}^{N_1N_2} ={}_{f1,f2}\langle N_1, N_2|\hat{B}\,|n_1, n_2\rangle_{f1,f2}$ [16], where N_j ($j=1,2$) is the number of photons in the jth output mode. The output state can then be written as

$$|\psi_{\mathrm{out}}\rangle = \sum_{N_1,N_2}|N_1N_2\rangle_{f1,f2}\otimes|M_{N_1N_2}\rangle_{m1,m2}, \tag{12}$$

where

$$|M_{N_1N_2}\rangle_{m1,m2}=\sum_{n_1,n_2}\Gamma_{n_1n_2}^{N_1N_2}(t, x_0)|n_1\zeta(\Delta t), n_2\zeta(\Delta t)\rangle_{m1,m2} \tag{13}$$

with $\Gamma_{n_1n_2}^{N_1N_2}(t, x_0) = c_{n_1}(t, 2x_0)c_{n_2}(t, 2x_0+2\delta x)B_{n_1n_2}^{N_1N_2}$. Equation (11) or, equivalently, equation (12) is in general an entangled state involving four parties. This result is central to our discussion as it is the basis for our protocol for the transformation of entanglement from optomechanical to purely mechanical.

2. Entanglement swapping

We now pass to the description of the entanglement swapping protocol and the quantification of the purely mechanical entanglement that is achievable by means of this procedure. As we stated, equations (12) and (13) describe the state of the mirrors and the output modes. We can now perform a measure of the number of photons in the output field modes at time $t = 2x_0/c$, i.e. the instant of time at which the first pulse arrives at the BS. If $\delta x > c\Delta t$, the first pulse has passed the BS before the second has reached it and the coefficients $c_{n_2>0}$ in equation (13) are identically zero. Therefore, the state of the mirrors after a projective measurement detecting N_1 photons in the mode travelling along the first arm of the interferometer and N_2 photons in the mode travelling along the second one is given by the unnormalized state

$$|\psi_{\text{out}}^{N_1 N_2}\rangle_{m1,m2} = \sum_{n_1} \Gamma_{n_1 0}^{N_1 N_2}(t, 2x_0)|n_1\zeta(\Delta t), 0\rangle_{m1,m2}, \tag{14}$$

which is a separable state of the two mirrors. Clearly, this should be expected because, if the two pulses do not arrive simultaneously at the BS, this is unable to entangle them and a projective measurement of the output modes simply achieves a local effect.

On the other hand, it is rather interesting to investigate how much entanglement is set between the vibrating mirrors, after the postselective measurement at the output field, when these have a considerable mutual overlap at the BS. We will show that such entanglement is strongly related to the length-difference δx between the interferometric arms. Within the limits of validity of our assumptions over the initial state of the mirrors and the unitarity of the dynamics, the Von Neumann entropy S of the reduced density matrix can be used as a faithful entanglement measure. It is straightforward to calculate the amount of entanglement between the mechanical systems after a measurement revealing $\tilde{N}_1$ and $\tilde{N}_2$ photons in modes $f1$ and $f2$, respectively, as

$$E(N_1, N_2) = S\left[\text{Tr}_{m_2}\left(\mathcal{M}|M_{\tilde{N}_1,\tilde{N}_2}\rangle\langle M_{\tilde{N}_1,\tilde{N}_2}|\right)\right], \tag{15}$$

where Tr_{m_2} denotes the partial trace over one of the second mechanical oscillator and $\mathcal{M}$ is the normalization factor. From equation (14), we know that $|M_{\tilde{N}_1,\tilde{N}_2}\rangle$ is a superposition of coherent states of the two mechanical systems. For values of $g \gtrsim 1$ and arranging for conditions such that $\eta(\Delta t)$ is maximized (which means taking $\Delta t = \pi/\omega_{\text{m}}$), the mechanical coherent states that enter such a superposition are sufficiently far in phase-space to be considered as quasi-orthogonal and be used in order to perform the partial trace. In what follows, we concentrate on the case of fields that simultaneously impinge on the BS, i.e we consider $\delta x = 0$. Moreover, still within a picture where the number of photons arriving at one of the detectors can be discriminated, we consider the probability of counting N_1 photons in the first output field and N_2 in the second one. This is clearly given by $\mathcal{M}^{-1} \equiv P(N_1, N_2) = |\langle M_{N_1,N_2}|M_{N_1,N_2}\rangle|^2$. In figure 2, we show such counting probabilities for $\alpha = 1$ and $t = 2x_0/c$. Evidently, for such a small amplitude of the input field, the Hilbert space of the overall system is strongly reduced: only a few photons are likely to be counted in the fields. This allows the performance of analytic calculations without requiring heavy numerical efforts. Clearly, enlarging α makes the cut-off in the dimensionality of the Hilbert space larger.

Figure 3 presents a plot of $E(N_1, N_2)$ versus N_1 and N_2, for $\alpha_0 = 1$. In figure 3(b), brighter regions correspond to larger mechanical entanglement. Purely mechanical entanglement is set for any configuration of outcomes that is associated with a nonzero probability. Clearly, the two-side no-click case of $N_1 = N_2 = 0$ leads to a fully separable mechanical system. Moreover, we find that for $N_1 = N_2 = 2$ no entanglement is established, as is evident in figure 3(a). We interpret this result as a clear manifestation of the bosonic bunching effect: if prior to the BS, fields $f1$ and $f2$ each contain only two photons, a state proportional to $(|4, 0\rangle + |0, 4\rangle - \sqrt{2/3}\,|2, 2\rangle)_{f1,f2}$ arrives at the detectors. A balanced detection event revealing

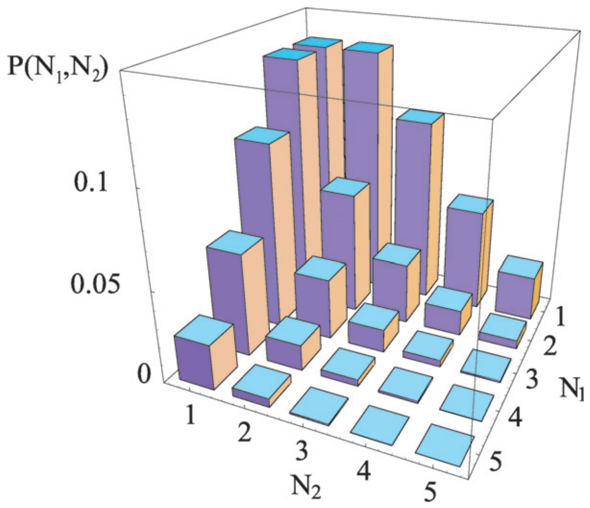

Figure 2. Probability $P(N_1, N_2)$ that N_1 photons are measured in mode $f1$, whereas N_2 are in $f2$ for $\alpha_0 = 1$, $\delta x = 0$ and for $\Delta t = \pi/\omega_{\mathrm{m}}$ and $g \gtrsim 1$.

two photons per field thus projects the mechanical systems onto a separable state. While the two-side no-click event can be discarded with data postselection, the $N_1 = N_2 = 2$ event cannot be identified unless we are provided with perfectly efficient photo-discriminating detectors. As this is a rather demanding requirement, we now address the realistic case, where the swapping experiment is performed by means of Geiger-mode operating detectors (ON/OFF detectors). This situation can be described by replacing the projective measurements used so far with the single-mode positive-operator-valued-measurement (POVM)

$$\hat{\Pi}^{\mathrm{OFF}}_{fj} = |0\rangle_{fj}\langle 0|\,, \quad \hat{\Pi}^{\mathrm{ON}}_{fj} = \hat{\mathbb{1}} - \hat{\Pi}^{\mathrm{OFF}}_{fj}, \quad (j = 1, 2). \tag{16}$$

The state of the mechanical system after an ON/OFF detection corresponding to clicks at both the detectors is found to be

$$\begin{aligned}\rho_{m1,m2} &= \mathrm{Tr}_{f1,f2}\left[\hat{\Pi}^{\mathrm{ON}}_{f1} \otimes \hat{\Pi}^{\mathrm{ON}}_{f2}\,|\psi_{\mathrm{out}}\rangle\,\langle\psi_{\mathrm{out}}|\right] \\ &= \sum_{N_1 N_2} \left|M_{N_1 N_2}\right\rangle_{m1,m2}\!\left\langle M_{N_1 N_2}\right| - |M_{00}\rangle_{m1,m2}\langle M_{00}| \\ &\quad - \sum_{N_2} \left|M_{0N_2}\right\rangle_{m1,m2}\!\left\langle M_{0N_2}\right| - \sum_{N_1} \left|M_{N_1 0}\right\rangle_{m1,m2}\!\left\langle M_{N_1 0}\right|.\end{aligned} \tag{17}$$

State (17) is a mixed state for which the entropy of entanglement is no longer a good measure of quantum correlations. On the other hand, we can use the entanglement measure based on the positivity of partial transposition (PPT) criterion [17]. The partially transposed density matrix $\tilde{\varrho}$ is obtained from any given bipartite quantum state ϱ by transposing the variables of only one of the two subsystems. The PPT criterion then simply reads $\tilde{\varrho} \geqslant 0$. Such a criterion is necessary for the separability of any quantum state regardless of the dimension of the system's Hilbert space but is also sufficient for $2 \otimes 2$, $3 \otimes 2$ and $\infty \otimes \infty$ Hilbert spaces. The 'negativity' $\mathcal{E}_N$ (first envisaged in [18] and later discussed in [19]), which can be easily determined from the knowledge of the density matrix (upon diagonalization of $\tilde{\varrho}$), is simply defined as the absolute

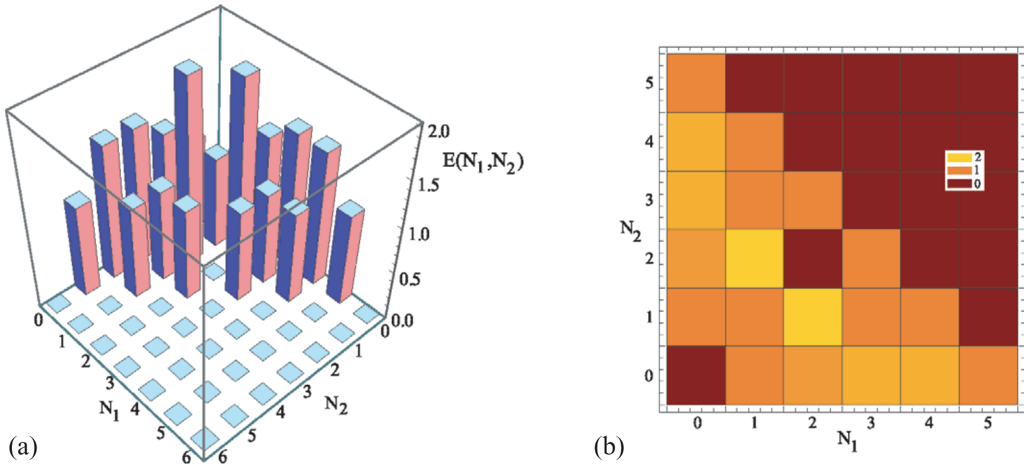

Figure 3. (a) Entropy of entanglement of the mechanical state after the projective measurement of the fields' state versus N_1 and N_2 for $\alpha_0 = 1$, $\delta x = 0$, $\Delta t = \pi/\omega_m$ and $g \gtrsim 1$. (b) top view of the plot in (a). Lighter regions correspond to larger entanglement, as indicated in the legend.

value of the sum of the negative eigenvalues of $\tilde{\varrho}$ and directly quantifies the violation of the PPT criterion. For the realistic choices of $g \gtrsim 1$, $\alpha_0 = 1$, $\omega_{\rm m} = 2\pi 10^5$ Hz and $\omega_0 \sim 2\pi 10^{14}$ Hz, and the optimized interaction time, we get $\mathcal{E}_N \simeq 0.35$ for equation (17), showing that the mechanical systems remain entangled (although the degree of quantum correlations is small) even if our resolution of the number of photons impinging at the detectors is limited.

Finally, we briefly assess the effect of a mismatch in the pulses' overlap at the BS. For the sake of clarity of our argument, we address the case of a pure mechanical state arising from photon-resolving detections. The entanglement entropy for a given value of N_1 and N_2 as a function of the length difference between the two arms δx is evaluated as a straightforward generalization of the calculations that have been previously performed. A global figure of merit of the performance of the protocol is given by the average entanglement over all the possible nonzero detection outcomes

$$\bar{E}(\delta t) = \sum_{N_1, N_2} P(N_1, N_2) E(N_1, N_2, \delta t), \tag{18}$$

where $E(N_1, N_2, \delta t)$ generalizes equation (15) to the case of a nonzero time-delay δt between the pulses. In figure 4, we see the behaviour of the average entanglement $\bar{E}$ between the two mirrors as a function of $\delta x = c\delta t$. As expected, the amount of entanglement becomes smaller when $\delta x/c$ is close to Δt, and goes to zero when $\delta x/c > \Delta t$. This demonstrates the controllability of purely mechanical entanglement achieved through the swapping mechanisms that we have addressed by means of time-mismatch control.

3. Conclusion and remarks

We have proposed an interferometric setup that is able to transform hybrid entanglement of optomechanical nature into quantum correlations shared by two mechanical systems that can

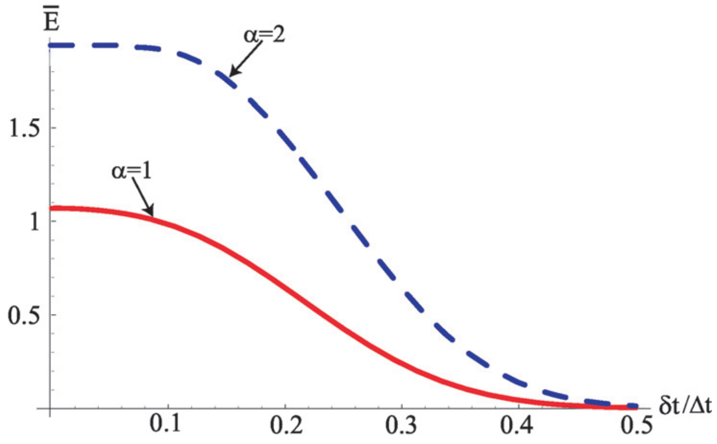

Figure 4. Average entanglement $\bar{E}$ versus the dimensionless time-mismatch $\delta t/\Delta t$ for $\alpha_0 = 1$ and $\alpha_0 = 2$.

have, in principle, mesoscopic (up to macroscopic) physical characteristics (such as its mass). Our scheme requires faint pulses with a narrow Gaussian spectrum and the (experimentally feasible (see for instance [20])[5], although demanding) discrimination of up to two photons in each field mode used in order to set optomechanical entangled states. Our protocol provides an example of control and manipulation of hybrid states involving photonic and mechanical degrees of freedom. It contributes to the grounding of optomechanical devices as intriguing systems for fundamental investigations and quantum information processing.

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Appendix. Quasi-monochromatic approximation

Here we address in some detail the quasi-monochromatic approximation. In equation (1), we have written a coherent pulse as

$$|\{\alpha(\omega)\}\rangle = \exp\left[\int d\omega[\alpha(\omega)a^\dagger(\omega) - \text{h.c.}]\right]|0\rangle. \tag{A.1}$$

Using the approach developed in [21], based on the introduction of a complete set of discrete orthogonal functions $\xi_i(\omega)$ such that $\gamma_i = \int d\omega\alpha(\omega)\xi_i^*(\omega)$, we can change picture from

[5] Recently, Avenhaus *et al* [20] have used photo-resolving detectors for the full characterization of the number statistics in multimode parametric down-conversion.

continuous to discrete, where $\hat{c}_i^\dagger = \int d\omega \xi_i(\omega)\hat{a}^\dagger(\omega)$. The continuous-picture radiation pressure Hamiltonian [10] reads

$$\hat{H}_{\rm rp} = \omega_{\rm m}\hat{b}^\dagger\hat{b} + \int d\omega[\omega\hat{a}^\dagger(\omega)\hat{a}(\omega) + K\omega\hat{a}^\dagger(\omega)\hat{a}(\omega)(\hat{b}^\dagger + \hat{b})], \tag{A.2}$$

where K is the coupling strength. By replacing the expression for $\hat{a}(\omega)$ given in terms of $\hat{c}_i$ in $\hat{H}_{\rm rp}$ we find

$$\hat{H}_{\rm rp} = \omega_{\rm m}\hat{b}^\dagger\hat{b} + \sum_{ij}\lambda_{ij}\hat{c}_i^\dagger\hat{c}_j + K\lambda_{ij}\hat{c}_i^\dagger\hat{c}_j(\hat{b}^\dagger + \hat{b}), \tag{A.3}$$

where $\lambda_{ij} = \int d\omega\omega\xi_i^*(\omega)\xi_j(\omega)$. As we have discussed, we consider $\alpha(\omega)$ to be a Gaussian function. As the set of $\xi_i(\omega)$'s is arbitrary, we can take the Hermite polynomials, in such a way that it becomes straightforward to show that $\lambda_{i,j} \neq 0$ only if $j = i, i \pm 1$. In addition, we find that $\lambda_{ii} \propto \omega_0$, and $\lambda_{i,i\pm 1} \propto \Delta\omega$. So, if the pulse's width $\Delta\omega$ is small compared with the carrier frequency of the pulse ω_0, we are allowed to consider only the coefficients λ_{ii}. By taking $\xi_0 = \alpha(\omega)$ and exploiting the orthogonality of γ_is, we finally get

$$\hat{H}_{\rm rp} = \omega_{\rm m}\hat{b}^\dagger\hat{b} + \lambda\hat{c}^\dagger\hat{c} + K\lambda\hat{c}^\dagger\hat{c}(\hat{b}^\dagger + \hat{b}), \tag{A.4}$$

where $\lambda = \lambda_0$ and $c^\dagger = c_0^\dagger$. This expression is formally identical to the Hamiltonian valid for the purely monochromatic case.

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