The Importance of Additive Reasoning in Children's Mathematical Achievement: A Longitudinal Study

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Abstract

This longitudinal study examines the relative importance of counting ability, additive reasoning, and working memory in children's mathematical achievement (calculation and story problem solving). One hundred and fifteen 6-year-old Chinese children in Hong Kong participated in two waves of assessments (T1 – first grade and T2 – second grade). Multiple regression analyses showed that counting ability explained a significant amount of variance in T1 and T2 calculation beyond the effects of age, IQ, and working memory, in which conceptual knowledge of counting, but not procedural counting, was a unique predictor. However, counting ability did not contribute significantly to story problem solving at both time points. Additive reasoning explained a substantial and significant amount of variance in calculation and story problem solving at both time points after the effects of age, IQ, working memory, and counting ability were controlled for – Both knowledge of the commutativity and complement principles were unique predictors. Working memory also accounted for a significant amount of variance in calculation and story problem solving at both time points beyond the influence of age, IQ, counting ability, and additive reasoning. Among the three components of working memory, only the central executive was a unique predictor for all measures of mathematical achievement. Autoregressive analyses provided further evidence for the strong predictive powers of additive reasoning and working memory. Overall, additive reasoning accounted for the greatest amount of variance in mathematical achievement both concurrently and longitudinally. This finding underscores the importance of additive reasoning in children's mathematical development.

Keywords: Additive reasoning, counting ability, working memory, mathematical achievement

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The aim of this study was to investigate the relative importance of working memory, counting ability, and additive reasoning in children's mathematical achievement. Mathematical achievement has an influence on individuals' performance in college and choice of careers (National Mathematics Advisory Panel, 2008). The mathematical skills and knowledge at an early age have been shown to predict mathematical achievement test scores in both primary and high schools (e.g., Jordan, Kaplan, Ramineni, & Locuniak, 2009; Locuniak & Jordan, 2008; Mazzocco & Thompson, 2005). Thus, providing children with a strong foundation of mathematical competence is important for success in school and beyond.

Defining Mathematical Competence

What does it mean to be competent in mathematics? The definition of 'mathematical competence' is important because it affects the kinds of mathematical skills examined in this study. In general, 'competence' indicates sufficiency of knowledge and skills that enable a person to act in a wide variety of situations. To illustrate, if a person is said to have competence in a particular language, she or he should be able to understand and interpret oral narratives and written texts in that language. She or he should also be able to express her- or himself in speech and in writing. Also, a person who is competent in a language can read, write, listen, and speak about different things and in different ways in that language. By contrast, a person who can only listen and speak in a language about certain topics is not competent enough. This analogy with linguistic competence can be an inspiration to answering the following question: What are the characteristics of a person who can deal with a wide range of situations that involves mathematical thinking successfully? 'Mathematical competence' is the term that we used to denote this collective and complex entity. In this article, we start from reviewing two different theoretical perspectives on children's mathematics learning. Proponents of different approaches have a different focus about the characteristics of a young child who is mathematically competent. The task of the following section is to reflect on and theoretically analyse different perspectives as the point of departure of our work, from which we hypothesised the essential pillars that might form the basis of mathematical competence.

The number sense perspective.

'Number sense' has been considered as an inborn characteristic of children that forms the foundation for mathematics learning (e.g., Dehaene, 1997; Gelman & Butterworth, 2005; Gelman & Gallistel, 1978; Siegler & Booth, 2005). A review of the literature (Siegler & Booth, 2005) suggests one definition of number sense – 'a process of translating between alternative quantitative representations.' The translations can be between the representations of spatial and numerical information (e.g., 'About how many feet wide is this classroom?'), the representations of temporal and tactile information (e.g., 'tap your finger once every 5 seconds'), and so on. It has been argued that an accurate estimation of numerical magnitudes is the basis for children to learn mathematics.

The core of number sense seems to be the presence of a mental number line, which is based on the hypothesis that numbers are arranged spatially on a continuum (from left to right for cultures using left-to-right orthographies) (Dehaene, 1997; Siegler & Booth, 2005). One crucial characteristics of the number line in young children is that it is a fuzzy representation of quantities and it takes time for children to develop the mental number line perfectly. The form of mental number line representations has been measured with a number line estimation task. In this task, participants are presented with some lines with a number at each end (e.g., 0 and 10, 0 and 100, 0 and 1000). They are asked to estimate the location of a third number (e.g., 42) on the line. This task is thought to be measuring the mental number line because it parallels the ratio characteristics of the number system – 80 is four times greater than 20, so the distance of the estimated position of 80 from 0 should be four times greater than the distance of the estimated position of 20 from 0. Likewise, the distance between 0 and 20 should be the same as the distance between 20 and 40, 80 and 100, 140 and 160, and so on, which gives a perfect linear function of estimates when they are plotted against the correct place on the number line.

Although this kind of numerical estimation is not difficult for most adults, it takes some time for young children to grasp. It has been suggested (e.g., Siegler & Booth, 2004; Siegler & Opfer, 2003) that young children have difficulties in representing the magnitudes accurately but this improves with age. Young children are bad at estimating the position of small and large numbers – They overestimate small numbers and underestimate larger numbers, which gives a logarithmic description of their estimates when they are plotted against the correct place on the number line. For example, kindergartners at the age of 5 and 6 were found to exhibit a clear logarithmic pattern of number representation (Siegler & Booth, 2004). It was also indicated that half of the first graders at the age of 6 and 7 showed logarithmic patterns, whereas the other half fit a linear pattern. Second graders at the age of 7 and 8 revealed numerical representations that were best fit by a linear function.

Why might numerical magnitude estimation be related to computational proficiency? Some researchers suggest that when a person solves an arithmetic problem, the rote verbal representation of the answer to the problem and an approximate representation of the answer's magnitude will be activated (Ansari, 2008; Hanich, Jordan, Kaplan, & Dick, 2001). If an approximate representation has more activation strength concentrated on the right answer and the numbers around it, the person is more likely to retrieve the correct answer. An accurate magnitude representation also allows for the rejection of implausible answers and recalculation in cases when a person has retrieved implausible answers. In contrast, approximate representations in which activation strength is more widely distributed among different numbers are likely to lead to the retrieval of wrong answers. Thus, according to this view, the ability to represent magnitude accurately may help children retrieve correct answers to novel addition problems (Booth & Siegler, 2008; Siegler & Ramani, 2009). An accurate representation of numerical magnitudes may also relate to the development of a variety of computational estimation strategies (e.g., Dowker, Flood, Griffiths, Harriss, & Hook, 1996). It was found that children whose number line estimates better fit a linear function had better perofrmance on a range of other numerical tasks, such as magnitude comparison, memory for numbers, calculation, and standardised mathematical achievement tests (e.g., Booth & Siegler, 2006; Geary, 2011; Geary, Hoard, Byrd-Craven, Nugent & Numtee, 2007).

In summary, number sense is one approach that we may reference for the definition of mathematical competence in this study. It is one essential domain of mathematical competence in young children because it may foster the growth of computational facility. However, the definition of mathematical competence within this theoretical framework is limited in several ways. First, according to this perspective, children are born with a fuzzy representation of quantities and the representations become more accurate with age. Perceived numerosity is hypothesised to provide children with the foundation for understanding number words. However, every number has its exact meaning that is not simply an estimation (Sarnecka & Gelman, 2004). For instance, even very young children understand that if they add one object to eight objects, they will no longer have eight objects. It is obvious to them that '8' is not the same as 'approximately 8'. This theory does not explain how young children, from the starting point of an imprecise analogue representation, suddenly come to understand the precise meanings of number. The number sense perspective does not define what a number is, but our view is that how we conceptualise 'number' is important in mathematics education because it affects the approach that we use to teach mathematics to children. We will come back to what may constitute the 'meanings of number' from another perspective shortly in this article. At the moment, we consider that the number sense perspective does not offer a good account of the concept of 'number', which should be precise and fundamental to mathematics learning.

Second, although the number sense perspective may explain variation in computational proficiency (e.g., Booth & Siegler, 2006; Geary, 2011; Geary, Hoard, Byrd-Craven, Nugent & Numtee, 2007), it is difficult to understand from this perspective how magnitude estimation allows us to solve problems in a variety of mathematical situations. Consider additive reasoning, different kinds of situations that involve part-whole relations have been identified (Carpenter, Hiebert, & Moser, 1981; De Corte, & Verschaffel, 1985; 1987; Ginsburg, 1982; Hudson, 1982; Nesher, 1982; Stern, 1993; Svenson, & Broquist, 1975; Vergnaud, 1979; 1982). Research that has examined children's performance on these types of problems (e.g., Carpenter, Hiebert, & Moser, 1981; Verschaffel, 1994) has shown findings that challenge the number sense perspective. First, children's accuracy rates for problems that require the same calculation are different. For example, in tranformation situations, problems with the final quantity unknown are significantly easier than those in which the initial quantity is unknown. Second, when the situation involves the composition of two quantities, finding the whole is significantly easier than finding a part. Third, reference set problems are significantly more difficult than other types of problems that involve comparisons even when the quantities in the problems are the same. In these studies, the demands for arithmetic computation (e.g., quantities and types of calculation) are controlled for while the quantitative reasoning demand vary. Thus, the differences in the rates of correct responses are not likely due to individual differences in estimating numerical magnitudes, but in their ability to reason about the relations of quantities in the problems. The number sense perspective is limited in a sense that it touches upon quantities only, but it does not entertain the idea that children need to understand relations between quantities in order to choose relevant arithmetic operations to solve a variety of problems. In brief, the number sense perspective does not provide a basis for us to understand how people solve mathematical problems in different situations.

In summary, the number sense perspective may be useful for explaining children's growth in computational proficiency. The ability to estimate numerical magnitudes may contribute to arithmetic competence through the development of a variety of computational estimation strategies. However, this theoretical framework suffers several limitations. First, it is not clear how the link between an imprecise analogue system and a precise system of numbers can be forged. It does not give a precise conceptualisation of the meanings of number. Second, it cannot explain how we can base on numerical estimation to solve mathematical problems in a variety of situations. Therefore, it appears that we need to turn to another theoretical approach to search for a better definition of mathematical competence.

The mathematical thinking perspective.

The second view of children's mathematics learning, which we term as 'mathematical thinking perspective', focuses on how children think about mathematics logically (e.g., Bryant, 1995; Carpenter & Moser, 1983; Ginsburg, Klein, & Starkey, 1998; Nunes & Bryant, 1996, 2015; Nunes, Bryant, Barros, & Sylva, 2012; Piaget, 1952; Piaget & Inhelder, 1975; Thompson, 1993, 1994; Vergnaud, 1997, 2009). Mathematical thinking involves the understanding of the meanings of number. The development of mathematical thinking is to some extent similar to language learning. In order to progress in mathematical thinking, children need to learn mathematical symbols and their meanings and to connect them sensibly, just as one has to combine words sensibly in sentences. Quantitative reasoning involves using numbers to represent quantities and relations between quantities as well as operating on the numbers to reach conclusions about the quantities (Thompson, 1993). Thus, one core intellectual demand to understand the meanings of number is the need to understand relations between quantities, rather than merely understanding things in isolation.

The nature of understanding number.

In the context of mathematical thinking, in order to say that a child has an understanding of number, we would expect some demonstration that the child understands the relational meanings of number. For example, Jean Piaget (1952) pioneered this view when he argued that we need to examine whether children understand the equivalence between sets in order to credit children with an understanding of cardinality. Suppose Mary has 5 sweets and she exchanges with Annie one sweet that she has for 1 sticker. If Mary understands cardinality, she should know that, by the end of this exchange, she would have 5 stickers without having to count. If Mary is able to count the sweets and say there are 5, but she does not know that how many stickers she has after sharing on a one-to-one basis, according to the mathematical thinking perspective, we can only say that Mary can count, but we cannot say that she understands cardinality. In short, Piaget considers cardinality as the number that relates one set of objects to other sets. If there are ‘5’ objects in this set, then it has the same quantity as any other set with ‘5’ objects.

Another crucial aspect of the nature of understanding number, which plays an important part in Piaget’s theory, is logical inferences. All quantities (e.g., number, height, temperature) can be arranged in a particular order from smaller to larger. In order to grasp the nature of this order, we have to master a fundamental logical rule called transitivity. If quantity A is greater than quantity B, and B is greater than quantity C, then it follows that A must also greater than C. Some children may only know that 3 is more than 2 and 2 is more than 1, but they cannot work out the relation between 3 and 1 which they cannot directly compare. According to the mathematical thinking perspective, these children are demonstrating an incomplete understanding of the relations between different numbers. This aspect of number knowledge is known as the ordinal concept of number.

The cardinal and ordinal concepts of number are requirements for the most basic mathematical activity of all – counting. However, Piaget’s list of logical requirement goes further than this. He contends that all mathematical procedures have their own logical demands. For example, soon after children have learned to count, they start to learn addition and subtraction, and then multiplication and division later at school. Proponents of the mathematical thinking perspective argue that it is important for children to learn about the connections between these operations. One obvious connection is inversion. This is the idea that each operation has its converse. For example, the inverse relation to addition is subtraction, and vice versa; the inverse of multiplication is division, and vice versa. The understanding of the inversion principle is a fundamental aspect of learning about number. Piaget (1952) argues that it is not possible to grasp the ‘additive composition of number’ without understanding the inversion principle. Additive composition of number refers to the fact that numbers are made up of other numbers. For example, 9 consist of 4 and 5 or 6 and 3, and it follows that if you subtract 6 from 9, you will be left with 3. Piaget argues that it is not sufficient for children to know or be able to calculate that 5 + 3 = 8 and that 8 – 3 = 5, they must also realise why each of these relations automatically follows from the other. According to this view, numbers are not simply a series of words in a constant order, but they also reflect the part-whole logic of the number system – each number words encompasses the previous ones additively (8 means 7 + 1, 6 + 2, 5 + 3 etc.). Nunes and Bryant (2015) call this idea the 'analytical meanings of number' because the meaning is given by definitions within a number system.

These particular examples make the point that, according to the mathematical thinking perspective, children need to grasp certain logical principles in order to do well in mathematics. Examples of the relational meaning of number involve the cardinal and ordinal concept of numbers and the inversion principles. It is also reasonable to suggest that understanding the additive composition of number may contribute to the development of a more accurate estimation of numerical magnitudes on a number line. It is possible that once children have understood the additive composition of number, it would be easier for them to represent the relative magnitudes of quantities and numbers. In other words, understanding relations may actually support the development of numerical magnitude representation. Therefore, compared with the number sense account, the mathematical thinking perspective appears to give a more comprehensive, precise, and parsimonious model that captures the fundamental concept of number.

Mathematical thinking and computational proficiency.

Why is grasping the nature of number important in learning mathematics? One possible reason is that an understanding of the analytical meaning of numbers contributes to the success in calculation. According to the mathematical thinking perspective, arithmetic is the study and use of relations between numbers to come to conclusions and this is always carried out using a number system, which has specific characteristics. One characteristic is the inverse relation between addition and subtraction.

The inversion principle may underlie the understanding of the exchanges in addition and subtraction of multi-digit numbers. Some researchers (e.g., Fuson, 1990; Nunes & Bryant, 1996) have suggested that understanding carrying and borrowing demands the knowledge of the inverse relation between addition and subtraction. For example, Fuson argued that, when we are adding 8 tens and 7 tens, in order to understand the 'ten-for-one to the left exchange', we have to recognise that we are taking 100 away from the tens place and adding 100 to the hundreds place, so that the total value is not changed. We also need similar reasoning to subtract 63 from 1657 – in order to understand the conservation of the minuend, we need to understand that taking away 100 from the hundreds place and adding 100 in the form of 10 tens to the tens place does not alter the quantity.

An understanding of the inversion principle may also contribute to the use of a computational strategy called 'indirect addition' in which children can use additions to solve subtraction problems effectively if the numbers are close to each other. For example, to solve '21 – 18', it is less likely to make mistakes if they count up from 18 to 21. The use of the inverse relation between addition and subtraction to calculate has been observed in oral arithmetic. For example, Nunes, Schliemann and Carraher (1993) reported two different ways street vendors in Brazil used the inverse relation to solve computations. Problems about change were commonly solved with indirect addition. For instance, when someone bought something valued 80 Cruzeiros and paid with a 500 note, a child vendor said 'Eighty, ninety, one hundred. Four twenty' (Nunes et al., 1993, p.25). In this case, the child calculated the change through indirect addition. In another example, a child used the inverse relation effectively in a different way. He solved the problem 243 – 75 by simplifying the problem – At first he subtracted 143 from 243, which becomes 100 – 75, and then he added 143 back to reach the answer. In short, if children understand the inverse relation between addition and subtraction, they are able to think of various ways to simplify an arithmetic problem with their logical understanding of number in order to enhance their computational proficiency (Canobi, 2004; Canobi, Reeve, & Pattison, 2003).

Compared with the number sense perspective, the mathematical thinking perspective also addresses how children solve arithmetic calculation. It suggests that reasoning about the relations between numbers can be a basis of effective calculation. From this perspective, arithmetic is not just about memorising number facts. Instead, the crux of a successful problem solver of arithmetic calculation refers to the ability to understand the relational or analytical meaning of number.

Mathematical thinking and solving problems in different situations.

Up to now we have highlighted the importance of understanding the analytical meaning of number in arithmetic calculation. Now we turn to what Nunes and Bryant (2015) have called the 'representational meaning of number', which is about working out relations between quantities. Thompson (1994) highlights the importance of a logical "comprehension of a situation" (Thompson, 1994, p.187-188) in solving mathematical problems in different situations. He argues that it is important to analyse the underlying quantitative structures of mathematical problems – "a prominent characteristic of reasoning quantitatively is that numbers and numeric relationships are of secondary importance, and do not enter into the primary analysis of a situation. What is important is relationships among quantities" (Thompson, 1993, p.165).

The solution to many story problems rests upon the knowledge of the underlying relations between the quantities in the problem. Occasionally, this set of relations is not obvious to problem solvers. This applies to some story problems whose solutions rely on the understanding of the inverse relation between addition and subtraction. For example, a Change problem is easy when the missing information is the result of the change (e.g., 'David had 8 books. Then Peter gave him 3 more books. How many books does David have now?') because the action in the story and the arithmetic operation required to solve the problem are directly related. In other words, a problem that involves a change that increases the quantity can be solved by addition, while one that decreases the quantity can be solved by subtraction.

In contrast, when the starting situation is not known (e.g., 'Alex had some cookies. He gave 3 cookies to his mother and had 8 cookies left. How many cookies did he have before?'), one must decide which arithmetic operation to use for calculation on the basis of the information about the change and its end result. This type of start-unknown problems is more difficult (e.g., Carpenter, Hiebert, & Moser, 1981; De Corte, & Verschaffel, 1982, 1987; Ginsburg, 1982) because the relation between the action described in the story and the operation is inverse, i.e., A problem that involves a change that decreases the quantity has to be solved by addition. Thus, students must understand that the operation 'addition' can be conceived as the inverse of 'subtraction' and analyse the quantitative relations underlying the problem situation.

Verschaffel (1994) examined the difficulty of comparison problems which also demand inverse reasoning, but applied to relations rather than operations. In one type of comparison problem, the relation between quantities can be described as 'more than' but the problem solver has to think of its inverse to solve the problem e.g., when the reference set is the missing quantity (e.g. 'Pete has 29 nuts. He has 14 more nuts than Rita. How many nuts does Rita have?'). Verschaffel asked 5th graders in Belgium (aged about 11 years) to solve comparison problems in which the relation was consistent with the operation (i.e. the relation was described as 'more than' and the operation to be used to solve the problem was an addition e.g., 'Timothy has 29 cups. Jenny has 14 more nuts than Timothy. How many cups does Jenny has?') or was inconsistent (i.e. the relation was described as 'more than' and the operation to be used to solve the problem was a subtraction, as in the problem presented above). When the relation and the operation were consistent, the correct rate was 92.5%; when the relation and the operation were inconsistent, the correct rate was 72.5%. Because the numbers involved in these problems are the same, the ability to reason about the relation between quantities is likely to be the reason that explains the difference in accuracy. Therefore, students does not only have to learn that addition is the inverse of subtraction and vice versa, but also that the relation 'more than' can be seen as the inverse of 'less than' and vice versa.

In summary, according to the mathematical thinking perspective, quantitative reasoning that is based on relations between quantities is crucial in solving mathematical problems in a variety of situations, whereas the numbers used to represent the quantities are of secondary importance. Some additive reasoning situations involve just quantities whereas others involve quantities and relations. If a problem requires reasoning about relations, it is significantly more difficult than a similar one that involves just quantities. Children have to reason in a sophisticated manner about the underlying structure of the quantitative relations in the story, in order to choose whether to add or subtract and solve the problem successfully. Thus, compared with the number sense perspective, the mathematical thinking perspective definitely has an edge by providing a good account of how children solve mathematical problems in different situations.

Definition of Mathematical Competence

The mathematical thinking perspective focuses on the understanding of the meanings of number – analytical and representational. The analytical meaning of number is defined by a number system, whereas the representational meaning refers to the use of numbers to represent quantities (Nunes & Bryant, 2015). Comparing the two approaches to children's mathematics learning, the mathematical thinking perspective appears to provide a better theoretical framework to understand mathematical competence in children. As the National Mathematics Advisory Panel (2008) has recommended in their report, "the curriculum must simultaneously develop conceptual understanding, computational fluency, and problem-solving skills" (p.xix). Clearly, the number sense perspective touches upon computational fluency only. By contrast, the mathematical thinking approach addresses all three aspects of mathematical achievement. Our view is that it is not sufficient to say that a child who possesses a good sense of number is competent in mathematics. A child who is competent mathematics should also have a good understanding of the meanings of numbers and quantities. This understanding appears to support his or her ability to excel in a variety of mathematical tasks. Therefore, we consider that mathematical thinking is a better way to conceive of mathematical competence.

Cognitive Foundations of Mathematical Thinking in Children

After defining mathematical competence as mathematical thinking, we explore its cognitive foundations in children. What are the pillars of mathematical thinking? What kinds of skills do children need to possess in order to have mathematical competence? The skills required for mathematical competence may not be the same for children of different ages. In this study, we were interested in studying children at the age of around 6, who have acquired some skills in counting and begin to learn addition and subtraction. Thus, we would focus on these aspects in the following discussion.

Working memory.

It seems uncontroversial that learning and using mathematics must draw on some general cognitive resources. For example, in order to solve the following problem, 'David had 8 books. Then Peter gave him 3 more books. How many books does David have now?', we need to (1) pay attention to the information, (2) select, remember, and reason about the relevant parts of this information, and (3) execute arithmetic operations that help us answer the problem. Likewise, when children have to solve a calculation or an applied problem, they have to keep in mind the information in the problem and the steps to execute the solution, while monitoring what they have done and what remains to be done.

There are different theoretical models of working memory, such as, Baddeley-Hitch model of working memory (Baddeley & Hitch, 1974), Engle's model of controlled attention (Engle, Kane, & Tuholski, 1999), and Miyake's executive model (Miyake, Friedman, Emerson, Witzki, Howerter, & Wager, 2000). Different theoretical models lead to different definitions and corresponding measures for working memory in different studies. Most previous studies that examined the connection between working memory and mathematics learning in children used the model proposed by Baddeley and Hitch (e.g., Cowan & Powell, 2014; Gathercole & Pickering, 2000; Holmes & Adams, 2006; Keeler & Swanson, 2001; Lee, Ng, Ng, & Lim, 2004; Lehto, 1995; Noel, Seron, & Trovarelli, 2004; Swanson & Beebe-Frankenberger, 2004; Wilson & Swanson, 2001). Within this model, working memory has been defined as 'a brain system that provides temporary storage and manipulation of the information necessary for …complex cognitive tasks' (Baddeley, 1992, p.556). It provides a unified and parsimonious theoretical framework that comprises three key components – the phonological loop, the visuospatial sketchpad, and the central executive. The phonological loop serves to hold speech-based information temporarily, whereas the visuospatial sketchpad holds visual and spatial information for a short period of time. The central executive component is responsible for focusing, dividing, and switching attention, which provides an overall monitoring and regulation of the entire working memory system and coordination of the activities among different components in the system. In the present study, we would base on Baddeley's working memory model (Baddeley & Hitch, 1974, Baddeley, 1992) to test the relation between working memory and children's mathematical achievements.

The evidence regarding the close connection between working memory and children's mathematical achievements are well established (e.g., Alloway & Aloway, 2010; Bull, Espy, & Wiebe, 2008; Geary, 1993; Huttenlocher, Jordan, & Levine, 1994; Rasmussen & Bisanz, 2005; Swanson, 2011; Welsh, Nix, Blair, Bierman, & Nelson, 2010). However, it is likely that working memory is important for learning and performance across all academic domains. Thus, the relation between working memory and mathematical achievements may not be specific. The non-specificity of working memory suggests that, in order to understand what factors predict children's success in mathematics learning, we need to look at other abilities that are more specifically related to mathematics. It is reasonable to speculate that these domain-specific abilities would explain variation in children's mathematical achievements beyond general cognitive resources, such as working memory.

Counting ability.

Counting is one type of domain-specific abilities central to children's mathematical thinking because learning to count provides children with words to represent quantities. This activity helps children reflect upon and develop the logical concept of one-to-one correspondence, ordinality, and cardinality. The definitions of conceptual knowledge of counting vary in the literature. In the following, different ways of conceptualising counting are explored because they influence how we interpret the findings from research that examines the connection between counting and children's mathematical achievement.

One popular theory about counting was proposed by Gelman and Gallistel (1978). The researchers suggest three 'how-to-count' principles that are necessary for correct counting, including the 'one-to-one correspondence', 'stable order', and 'cardinality' principles. The one-to-one correspondence principle refers to the understanding that one must only tag an object in an array with one and only one label for each individual object. The stable order principle requires the person who counts to choose tags that correspond to items in an array in a stable order, which should stay the same regardless of the number of items. The cardinality principle, according to Gelman and Gallistel (1978), is defined as the understanding that the number tag assigned to the final item in an array represents the total quantity of the set. Understanding the cardinality principle may underlie the use of more efficient counting strategies to solve problems. For example, children who understand cardinality can use the 'first' procedure to solve an arithmetic problem e.g. '8 + 4'. If they know the cardinal value of the first number, they can use this number as the shortcut to count more efficiently: they would start from '8' and count '8, 9, 10, 11, 12' to solve '8 + 4', rather than start all the way from '1' and count '2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12' to reach the answer.

It has been suggested that these three counting principles govern the counting behavior of young children (Gelman & Meck, 1983). The proponents of this view suggest that young children's conceptual understanding of these essential features of counting precedes their acquisition of counting procedures. However, there is an alternative view that children do not start with an adequate understanding of the counting principles when they count. Instead, they start from imitating other people's counting behavior and induce some common features of counting from the observation (Briars & Siegler, 1984; Fuson, 1988). These common features are called 'unessential' features of counting because they may are not necessary for correct counting. Briars and Sielger (1984) identified four such unessential characteristics, including (1) standard direction (items must be counted from left to right), (2) adjacency (items must be counted contiguously), (3) pointing (items have to pointed at during counting), and (4) start-at-the-end (items must be counted from one end of an array of objects). Although we do not need to follow these rules if we want to do a correct counting, some children believe that these four features of counting are necessary for it. This suggests that the conceptual understanding of counting of some young children is still rigid and not yet fully developed.

In short, these researchers suggest that conceptual knowledge of counting refers to the understanding of what is necessary for correct counting. Children who have a thorough conceptual knowledge of counting should be able to abide by the essential principles and not to mistake the unessential characteristics of counting as the criteria for correct counting. It is necessary to respect each of these principles because they are part of the analytical meaning of number. Counting activity is important for children to learn mathematic because it helps children think about the meanings of number.

However, it is not enough for children to know individual counting principles separately, but they also need to coordinate their knowledge of the principles in order to understand the analytical meaning of number. For example, the last number word of a counting sequence denotes the cardinal value of a set (cardinality principle) should only hold when the counting follows the one-to-one correspondence principle. If one skips an object in the middle of the counting sequence, she or he should not say that the last number word is the cardinal value of the set. There is evidence that some children failed to coordinate their knowledge of the counting principles even though they demonstrated competence in reciting the number sequence and applied it to objects and events (e.g., Bermejo, Morales, & deOsuna, 2004; Freeman, Antonnuccia, & Lewis, 2000; Sarneka & Gelman, 2004; Sophian, 1988). These studies suggest that knowing how to count does not necessarily imply a full understanding of number. For example, Bermejo and colleagues observed that the 4- and 6-year-old children, who could say there are three items in a set when a person counts forward, could not necessarily understand that if a person count backward from four and the last numerical label is ‘two’, this does not mean that the set contains two objects in total. In this study, some children were not aware of the contradiction between the two answers – they could tell that the set contains three objects if you count forward, whereas the same set contains two objects of you count backward. This finding shows a lack of understanding of cardinality of numbers, because it is fundamental to the concept of cardinality that two sets have the same cardinal if the items are in one-to-one correspondence. Despite the findings from these studies, much of the research on counting analysed children's knowledge of these principles separately (e.g., Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Barrouillet, Fayol, & Lathulière, 1997; Koponen, Aunola, Ahonen, & Nurmi, 2007; Passolunghi, Vercelloni, & Schadee, 2007).

In summary, counting is a useful starting point from which children learn to develop mathematical thinking. It is an activity that young children can use to learn the ordinal and cardinal meanings of number, but it takes some time for them to achieve a full understanding of counting. Because counting is more specific than working memory to mathematics learning, it is expected that individual differences in counting ability would explain variation in children’s mathematical achievements beyond general cognitive capacities, such as working memory and general intelligence. It appears that the measures of counting have to be chosen with care, which should capture children’s true understanding of counting. Therefore, to measure counting ability in this study, we would use various tasks, including (1) procedural counting (the ability to correctly say a number-word sequence) and (2) conceptual knowledge of counting, which refers to the awareness of Gelman and Gallistel's five counting principles as well as the ability to coordinate different principles to determine the cardinal number of a set.

Additive reasoning.

Another domain-specific ability that is important for young children to learn mathematics is additive reasoning. Additive reasoning is based on quantities connected by part-whole relations. Two central properties of part-whole relations involve (1) commutativity and (2) the inverse relation between addition and subtraction (some researchers called it the 'complement principle' e.g., Canobi, Reeve, & Pattison, 2003). Commutativity refers to the irrelevance of addend order to the sum, i.e. 'a + b = c' implies 'b + a = c', whereas the complement principle refers to the inverse relation between addition and subtraction, i.e. 'a + b = c' implies 'c - a = b'. These two principles are considered important in children's mathematics learning because they contribute to the understanding of the relational meanings of numbers and quantities. It is clear that the mastery of additive reasoning requires an integration of the principles – one should understand that three quantities e.g., 3 + 4 = 7 can be expressed in four mathematical relations, e.g., 7 – 3 = 4, 4 + 3 = 7, 7 – 4 = 3, and 3 + 4 = 7, and that these four expressions can be deduced from each other. A thorough understanding of the part-whole relations of quantities involves the recognition that these expressions are essentially describing the same relation.

Is there any evidence regarding the connection between understanding the relational meanings of number and quantities and children's mathematical achievement? There are some cross-sectional studies that have addressed this question and the findings are inconsistent. For example, Bryant, Christie, and Rendu (1999) examined children's understanding of the inverse relation between addition and subtraction. They compared the performance on three-term inverse problems (e.g., 14 + 7 – 7) and matched control problems (e.g., 9 + 9 – 4) in a group of 5- to 8-year-old children. On the basis of factor analysis, they found that children's understanding of the inversion principle was not related to their accuracy on calculation (addition and subtraction problems). In contrast, Canobi (2004) investigated the associations between conceptual knowledge and problem solving in 90 6- to 8-year-old children. Conceptual knowledge was tested by a judgment task in which children made and justified judgments of a puppet's solving problems that involved part-whole relations. She identified patterns of conceptual and problem solving profiles with cluster analysis and found that advanced conceptual profiles were associated with skilled problem solving. Approximately all children who recognised part-whole relations used more efficient strategies, such as retrieval and decomposition, to solve problems. Children with a more advanced conceptual understanding of part-whole relations also demonstrated higher accuracy and lower solution time than those with less advanced conceptual profiles.

Rasmussen, Ho, and Bisanz (2003) examined the use of the inversion principle in 24 preschool children and 24 children in Grade 1. They found that both preschool and Grade 1 children indicated evidence of understanding the inversion principle in a fully quantitative manner. The researchers also demonstrated that the relation between inversion understanding and calculation varied with age. They found that the preschool children did not show evidence of an association between their performance on inversion problems and arithmetic calculation. However, they identified a significant correlation between inversion understanding and accuracy of arithmetic calculation in Grade 1 children. Gilmore and Bryant (2006) used cluster analyses to analyse different patterns of inversion understanding and calculation skills among 6- to 9-year-old children. They identified three distinct subgroups, including one group showing good inversion understanding and good calculation skills, a second group demonstrating poor performance in both inversion understanding and calculation, and the final group having good inversion understanding but poor calculation performance. This finding suggests that individual differences in conceptual knowledge do not correspond with arithmetic competence directly.

There are a few longitudinal studies that demonstrated that quantitative reasoning predicted children's later success in mathematical achievement. Nunes and colleagues (2007) investigated whether children's quantitative reasoning measured at school entry was a significant predictor of mathematical achievement 16 months later, which was assessed by Standardised Achievement Tasks, Mathematics Section (SATs-Maths). They found that quantitative reasoning was a significant and specific predictor of children's mathematical achievement. The relation was specific because quantitative reasoning remained a significant predictor after the effects of general intelligence and working memory were statistically controlled for. Nunes and colleagues (2012) conducted another longitudinal study to evaluate whether quantitative reasoning and arithmetic skills are independent predictors of children's mathematical achievement in an older age (Key Stage 2 at 11 years of age and Key Stage 3 at 14 years of age). They found that quantitative reasoning made a unique contribution to the prediction of children's mathematical achievement at 11 and 14 years beyond and above the effects of age, general intelligence, working memory, and arithmetic skills.

In summary, additive reasoning appears to be important for children to learn mathematics. Understanding commutativity and the inverse relation between addition and subtraction are part of the construct of additive reasoning. This knowledge seems to be distinct from and developmentally more advanced than the understanding of ordinality and cardinality. Thus, it is expected that individual differences in additive reasoning would explain variation in children's mathematical achievements beyond counting ability and general cognitive capacities, such as working memory and general intelligence.

The Present Study

On the basis of the literature review, several research gaps are identified. First, from the mathematical thinking perspective, it is important to assess the conceptual aspects of counting. Learning to count matters for children to learn mathematics because it helps children reflect on the analytical and representational meanings of number. Thus, if a child counts without understanding what she or he is doing, she or he should not be considered as mathematically competent from the mathematical thinking perspective. Related to this argument is that procedural tasks alone, such as counting number sequences, are not good indicators of counting ability because they do not necessarily reflect children's understanding of the logic of counting. Therefore, measures that capture children's knowledge of the relations of counting to quantities such as, the ability to identify the cardinality of a set, should also be included in research studies. However, to the best of our knowledge, most predictive studies used procedural counting as the sole indicator of counting ability, which is a limitation that needs to be addressed in future studies.

Second, the relation between additive reasoning (as measured by the knowledge of commutativity and the inverse relation between addition and subtraction) and mathematical achievement remains unclear. There are mixed findings regarding its connection with children's calculation ability, however, there seems to be no study that examines its relation to children's ability to solve different types of story problems. More research that incorporates both calculation and story problem solving as the outcome measures are needed because cogntiive correlates of mathematical ability may vary across mathematical tasks (Chong & Siegel, 2008; Cowan & Powell, 2014; Hughes, 1981; Geary, Hoard, Nugent, & Byrd-Craven, 2008).

Third, the contribution of additive reasoning to mathematical achievement has never been investigated in a non-Caucasian cultural context. Given that the dominant language and other cultural factors may differ from one country to another (Miller & Stigler, 1987; Miller, Smith, & Zhang, 2004; Miura, Kim, Chang, & Okamoto, 1988), it is important to evaluate the predictive power of various cognitive factors on children’s mathematical achievement in a different culture.

Overall, this study aims to test whether working memory, counting ability, and additive reasoning contributes to mathematical achievement in children of around 6 years of age. In summary, the above analyses lead to the following hypotheses:

- Counting ability is important in children’s mathematics learning and its influence is independent from that of general cognitive capacities, such as working memory.
- Additive reasoning (as assessed by knowledge of commutativity and the complement principle) is independent from and more important than counting ability and general cognitive capacities, such as working memory in children’s mathematics learning.
- Working memory, as a domain-general factor, makes a contribution to mathematical achievement, even when one accounted for children's specific mathematical knowledge such as their knowledge of counting and additive reasoning.

Method

Overview of Research Design

To address the hypotheses, we employed a longitudinal design in this study. According to Bradley and Bryant (1983), both longitudinal and intervention studies are important for establishing a causal relation between variables. Intervention studies can be used to discern the causal relation between certain skills and mathematical achievement. Through this type of design, we can find out, for example, whether training additive reasoning leads to an improvement of these skills and mathematical achievement. But before we implement an intervention study, we need to identify factors that are important for children’s mathematics learning. Longitudinal studies give us a good opportunity to examine the temporal order of events. It is important to know whether a predictor precedes mathematical achievement because it is a necessary condition for determining causal relation between variables. Through statistical techniques, such as multiple regression analysis, we can identify the direction and strengths of associations between a predictor and mathematical achievement and compare the unique contributions of each predictor to variation in the outcome. Thus, longitudinal study is considered as an important first step for developing an intervention.

In this study, we used a longitudinal design, which spanned around 10 months, to examine whether the main predictors (working memory, counting ability, and additive reasoning) uniquely predicted children’s mathematical achievement (calculation and story problem solving).

The main predictors in this study were working memory, counting ability, and additive reasoning. Working memory was defined as children's performance on three tasks including digit span forward (the phonological loop), Corsi span (the visuospatial sketchpad), counting recall and digit span backward (the central executive). Counting ability was operationalised as children's procedural counting skills and conceptual knowledge of counting; whereas additive reasoning iwas operationalised as children's understanding of the commutativity and complement principles. All of these main predictors were assessed at the first wave of data collection (Time 1 – during the first grade of the participating children). A number of control variables, such as, general intelligence and demographic characteristics, were also measured at Time 1 to ensure that any observed associations between predictors and outcome measures are not due to an extraneous factor that may affect the relations.

The second testing occasion, Time 2 (during the second grade of the participating children), comprised two measures of mathematical achievement and a measure of Chinese word reading. To assess mathematical achievement, two measures including calculation and story problem solving in the domain of addition and subtraction were used in this study for three reasons. First, both tasks are commonly assessed in research and school. Second, children at this age are expected to learn addition and subtraction in school. Third, some reseachers (Chong & Siegel, 2008; Cowan & Powell, 2014; Hughes, 1981; Geary, Hoard, Nugent, & Byrd-Craven, 2008) have shown that cogntive correlates of mathematical ability may vary across mathematical tasks, which supports the rationale to examine multiple indicators of mathematical ability in this study. Mathematical achievement was assessed at both Time 1 and Time 2. Assessing mathematical achievement at both time points provides an opportunity to test whether the main predictors would predict mathematical achievement concurrently (T1) and longitudinally (T2). It also enables us to assess whether these predictors remain significant predictors of T2 mathematical achievement after the effects of T1 mathematical achievement was statistically controlled for.

Chinese word reading was included as the outcome control measure in order to test the specificity of certain variables on mathematical performance. If additive reasoning is specifically relevant to mathematics learning, children's performance on this task should predict much better for their success in mathematical tasks than non-mathematical tasks, such as, Chinese word reading. By contrast, general cognitive ability should correlate with both mathematical and non-mathematical tasks because all of these tasks demand cognitive resources, such as, working memory and general intelligence. This kind of design has been adopted in some longitudinal research of children's reading (e.g., Bradley & Bryant, 1983) but it is rare in studies that address children's mathematics learning (e.g., Nunes, Bryant, Barros, & Sylva, 2012).

The independent contributions of each predictor measured at T1 to mathematical achievement at the second testing occasion was assessed by taking into account the effects of age, non-verbal intelligence and demographic factors. For each child, the interval between the first and second wave of assessments was between 9 and 11 months, with 10 months being the commonest interval (83%).

Participants

One hundred and fifteen children (61 boys, 54 girls) studying in three primary schools in Hong Kong participated in both waves of assessments in this longitudinal study. All of these children spoke Cantonese and attended the first year of primary school, with a mean age of 76.32 months (*SD* = 2.81 months, ranging from 67.8 to 82.1 months), during the first wave of assessment. The mean age of the children during the second wave of assessment was 86.34 months (*SD* = 2.81 months, ranging from 77.8 to 92.1 months). All of the children were reported to have intelligence within the range accepted as normal for their ages, and did not have learning difficulties or emotional/behavioral problems, such as, dyslexia, specific language impairments, attention deficits and hyperactivity disorders, or any neurological disorders.

On the basis of previous studies (e.g., Canobi et al., 2003; Gilmore & Bryant, 2006; Nunes et al., 2007, 2012), an *a priori* power analysis (Cohen, 1988; GPower 3.1; Faul, Erdfelder, Lang, & Buchner, 2007) indicated that a minimum sample size of 91 was needed to detect a medium effect size [using Cohen's (1988) criteria] with an alpha of .05 and power of .80 using multiple regression analyses, therefore the current sample size was considered sufficient.

The highest educational levels attained by the mothers of the children in the sample were as follows: No schooling/pre-primary school level – 5.2%, primary school graduates – 20.8%, secondary school graduates – 57.4%, and university graduates – 16.5%. According to the Hong Kong Population Census (2011), the distribution of educational attainment (highest level attained) was: No schooling/pre-primary school level – 10%, primary school graduates – 19.2%, secondary school graduates – 46.6%, and university graduates – 24.1%. Thus, the relative distribution of educational levels was similar to that of the overall Hong Kong population, in which the majority of the population was secondary school graduates whereas a small proportion received no schooling or had pre-primary educational level.

Measures

Working memory.

Working memory was assessed with four tasks, including (1) digit span forward (the phonological loop), (2) digit span backward (the central executive), (3) counting recall (the central executive), and (4) Corsi blocks (the visuospatial sketchpad). There were two tasks for the central executive because previous research showed that measures of the central executive were stronger predictors of children's mathematical performance than other working memory measures (e.g., Cowan & Powell, 2014; Gathercole & Pickering, 2000; Holmes & Adams, 2006; Keeler & Swanson, 2001; Lee, Ng, Ng, & Lim, 2004; Lehto, 1995; Noel, Seron, & Trovarelli, 2004; Swanson & Beebe-Frankenberger, 2004; Wilson & Swanson, 2001).

The working memory task refers to the Working Memory Test Battery for Children (WMTB-C) (Pickering & Gathercole, 2001). In the digit span forward task, children listened to a series of single-digit numbers and were asked to repeat the numbers in the correct order. All digits were presented at a rate of one per second. The series of numbers initially consisted of two numbers, and increased by one number after every other presentation, to a maximum of nine. Children were given one point for each sequence correctly recalled. The maximum possible score for this task was 16. The internal consistency of this task was satisfactory (Cronbach's $\alpha = 0.81$).

The digit span backward was similar to the digit span forward, except that the children were asked to recite the numbers backward. In counting recall, children were asked to count the triangles in a series of shape arrays and then to recall the total number of triangles in each series. The number of arrays started from two and increased by one array after every other presentation to a maximum of nine. The total number of correct trials was used as an indicator of participants' performance on these tasks. The maximum possible score for this task was 16. The internal consistency of this task was satisfactory (Cronbach's $\alpha = 0.89$). The listening recall task was not used in this study because it cannot be simply translated into Cantonese without a proper investigation of how it works in this language.

The Corsi block task involved nine blocks and the experimenter tapped a sequence of blocks at a rate of one per second. Then, children were asked to replicate the sequence. The sequence involved two blocks initially and increased by one block every other presentation, to a maximum of nine. The maximum possible score for this task was 16. The internal consistency of this task was satisfactory (Cronbach's $\alpha = 0.83$). For each of the above tasks, there were two trials for each span length and testing was terminated when a child failed two trials of the same length. In each task, two practice items were given to the children and no feedback was given to the children in any of the testing trials.

Counting ability.

Counting ability was operationalised as (1) children's ability to count with accuracy (procedural counting) and (2) their ability to recognise the counting principles and the coordinated use of various counting principles (conceptual knowledge of counting).

Procedural counting was assessed with two tasks: oral rote counting and object counting. In oral rote counting, children counted some numerical sequences verbally in ascending and descending orders. They were first asked to count from 5 to 16 as a practice trial. There were then eight testing trials in which children were asked to count a set of numbers in ascending orders (e.g., 25 to 32; 56 to 63; 76 to 81; 118 to 123) and in descending orders (e.g., 46 to 38; 73 to 65; 34 to 27; 121 to 115). Testing within a set was discontinued when a child had committed errors on two sequences in a set. Children received one point for each sequence completed correctly.

Another task, object counting, was also included as one of the measures of procedural counting to test whether the children could count correctly using one-to-one correspondence between words and objects. In object counting, they were required to count two trials of geometric shapes (e.g., circles, squares) and two trials of recognizable objects (e.g., pens, rubber). The numbers of objects were 6, 9, 13, and 15 for rubber, pens, squares, and circles, respectively. On any given trials, the objects were identical in appearance. The order of task presentations was counterbalanced across participants. Children received one point for each correct counting. The total scores for procedural knowledge of counting for each child was the sum of his/her performance on the oral rote and object counting tasks. The maximum possible score was 12. The internal consistency of this procedural counting task was satisfactory (Cronbach's $\alpha = 0.71$).

Conceptual knowledge of counting was assessed with a counting judgment task adapted from previous work (e.g., Briars & Siegler, 1984; Freeman, Antonucci, & Lewis, 2000; LeFevre et al., 2006). As shown in Table 1, three types of trials were used – (1) correct counts (four trials), (2) incorrect counts (six trials), and (3) correct but unusual counts (six trials). Thus, children evaluated a total of sixteen counts. On each trial, a set of objects ranging in number from 6 to 12 was shown to children. In this task, a puppet 'Pika' was introduced, and the researcher (the author) explained to the children that Pika was just learning to count.

Insert Table 1 about here

The same protocol designed by previous researchers (e.g., Freeman, Antonucci, & Lewis, 2000) was used: ‘This is Pika and he would like you to play a counting game with him. He is going to count the things on the table. But he is just learning to count, and sometimes he makes mistakes. Sometimes he counts in ways that are okay, but sometimes he counts in ways that are not okay and he was wrong. Watch carefully while he counts. When he has finished counting, you tell me if he counted okay or not okay.’ Items were put in a row at 1 cm intervals. Pika faced each child and always counted one item per second from the child’s left to right except on the reverse direction trials. After each trial, children were asked whether the count was ok or not okay (‘error detection’). Then, they were given 10 seconds to answer the question that tested their understanding of cardinality – ‘How many things are there in total?’ The maximum possible scores for ‘error detection’ and ‘cardinality’ were 16, respectively. The internal consistency of the entire conceptual knowledge of counting measure was satisfactory (Cronbach’s α = 0.85).

Additive reasoning (the commutativity and complement principles).

Additive reasoning was operationalised as children’s understanding of the commutativity and complement principles. The commutativity principle refers to the irrelevance of addend order to the sum, i.e. ‘a + b = c’ implies ‘b + a = c’, whereas the complement principle refers to the inverse relation between addition and subtraction, i.e. ‘a + b = c’ implies ‘c - a = b’.

This study adapted a similar conceptual task used by Canobi, Reeve, and Pattison (2003) in which children were tested whether they could recognise conceptual relations between pairs of addition/subtraction problems. In general, children were shown a puppet that was going to solve two problems, namely base and target problems. The puppet ‘solved’ the base problem by counting very quickly and told the answer to the researcher, who then told the children that the answer was correct. After that, the children were shown a target problem and were asked to determine whether the puppet needed to count again to solve the problem or whether the puppet could find out the answer by ‘looking back’ at the base problem.

All of the problems were presented as story problems that involved a change in quantity (e.g., Mary has 3 fish and her mother gave her 5 more). Change problems were used instead of Combine problems because children’s performance on the Combine problems reached ceiling in the pilot test. The number of words in each problem did not vary considerably. The researcher presented each child with a written version of the problem as it was read and kept it in front of the child until the problem was solved. For example, after showing the base and target problems that were printed on two separate cards, the researcher asked, ‘Now look at these two problems. If we gave Pika (the puppet) this problem next (pointing to the target problem), do you think Pika would need to count to work out the answer or could Pika look back at the problem he has already done (pointing to the base problem)?’

The conceptual judgment task involved two parts: a ‘testing session’ immediately after a ‘warm-up session’. In the ‘warm-up session’, the children were given six practice problems to familiarise with the procedure. Half of the practice problems were identical (e.g., base: 4 + 4 and target: 4 + 4) and half of them were different (e.g., base: 4 + 3 and target: 6 + 7). Children were given feedback on whether they were correct in judging the same/different relation between the target and base problems. The answers were 100% correct for all participants in this session, indicating that they understood the task instructions.

In the ‘testing session’, the researcher showed six target problems in random order after asking the puppet to solve the base problem (e.g., Mary has 3 fish and her mother gave her 5 more). The target problems included (1) an identity problem, which was identical with the target problem (e.g., Mary has 3 fish and her mother gave her 5 more); (2) a different problem, which was completely unrelated to the target problem (e.g., Mary has 7 fish and her mother gave her 2 more). The identity and different problems were designed to detect possible responses biases, which may involve inattention, difficulty in understanding the procedure, and random responses. The accuracy rates for all the identity and different problems were 100%.

To assess children’s knowledge in each of the additive reasoning principles, two types of items were used: test items and control items. Examples of these items are presented in Table 2.

Insert Table 2 about here

The test items were designed to assess children’s understanding of a particular principle. They included (1) commutativity test items, which were related to the corresponding base problems on the basis of the commutativity principle (e.g., 5 + 3); (2) complement test items, which were related to the corresponding base problems according to the complement principle (e.g., 8 – 5). Half of the problems had sums less than 10 (small number) and half of them had sums between 15 and 25 (large number).

Control items were included to detect whether children answer the question correctly because of biases. For example, children may answer that '3 + 5 = 8' is helpful for solving '5 + 3' correctly just because they realise that two numbers in the base problem (i.e. 3 and 5) are present in the target problem (5 + 3). These children may not understand the commutativity principle but simply have a response bias to say 'yes' when the numbers are the same. Children with such a response bias would also answer that '3 + 5 = 8' helps to solve the question '5 – 3'. Thus, we included two types of control item in this study – (1) commutativity controls: subtraction items that evaluated whether the children did not simply ignore the operation to make a judgment (e.g., 5 – 3); and (2) complement controls, which involved addition problems that comprised the sum and one term of the base problem added together (e.g., 8 + 5).

Thus, the control items did not serve to measure the constructs, but they were there to allow for a correction for response biases. A child was only credited one point if they answered both the test and the control items correctly. There were 6 commutativity items and 6 control items for commutativity; if the child passed one commutativity item and its control, the child was awarded one point; otherwise, no points were awarded. Similarly, there were 6 complement items and 6 control items for the complement principle; if the child passed one complement item and its control, the child was awarded one point; otherwise, no points were awarded. The internal consistencies of the additive reasoning measures were statisfactory (Commutativity: Cronbach's α = 0.81; Complement prinicple: Cronbach's α = 0.85).

General intelligence.

Children's general intelligence was measured with Raven's Standard Progressive Matrices (Raven, Raven, & Court, 2003) at Time 1. This test was considered because it has been a robust measure of non-verbal aspect of intelligence and has been used widely in previous research. It is a standardised test including five sets of twelve items each. Each item involves a target matrix with a missing piece. Children were asked to choose, from six or eight alternatives, the best figure to complete the target matrix. One mark was given for the correct answer for each item.

Demographic characteristics.

Other control variables included demographic information reported by parents in a questionnaire, namely children's sex and mothers' highest education level at Time 1.

Mathematical achievement – calculation.

Mathematics achievement was measured by children's performance on sixteen simple calculation tasks and thirty-two story problems, all of which were designed with reference to the curriculum guide developed by the Hong Kong Education Bureau. Thirty items (15 addition and 15 subtraction) were constructed and tested in the pilot. On the basis of pilot findings, sixteen items (8 addition and 8 subtraction) were selected for each wave of data collection in the main study. Of these sixteen items, four are considered as "easy" (average correct rate: 70-100%), six are "moderate" in difficulty (average correct rate: 40-70%), and six are "difficult" items (average correct rate: 0-40%). At Time 1, children were orally presented with addition and subtraction combinations that involved eight addition of numbers up to 25 and eight subtractions from numbers less than 25 (i.e. 6 + 7; 3 + 8; 2 + 6; 9 + 16; 7 + 4; 2 + 16; 14 + 4; 11 + 7; 7 – 5; 9 – 6; 6 – 4; 12 – 3; 21 – 16; 22 – 18; 25 – 6; 18 – 5). At time 2, children were orally presented with ten addition and subtraction problems with large numbers (24 + 4; 8 + 19; 7 + 23; 21 + 5; 9 + 19; 28 – 9; 31 – 8; 27 – 5; 28 – 19; 26 – 8); three 3-addend single digit problems (3 + 9 + 2, 7 + 2 + 4; 8 + 5 + 2), three 3-subtrahend single digit problems (8 – 4 – 3; 13 – 3 – 8; 15 – 7 – 5). A printed version of each calculation problem was presented as each problem was read and kept in full view of the child during problem solving. Feedback was not provided and no time limit was set. The maximum possible score for calculation was 16. The measures appeared to have good internal consistency (T1: α = .87; T2: α = .92).

Mathematical achievement – story problem solving.

Similarly, eight types of word problems were tested from the same pilot study as in calculation. Thirty-two problems were chosen in the main study – On the basis of Riley, Greeno, and Heller's (1983) classification of story problems, Time 1 assessment included four result unknown Change problems, four start unknown Change problems, four change unknown Change problems, four unknown difference set Compare problems, four unknown compare set Compare problems, four unknown reference set Compare problems, four different unknown Combine problems, and four Equalize problems. At Time 2, the number of each type of problems was the same, except that two Combine problems were replaced by two more difficult 'de-combine transformations problems' (e.g., John played two games of marbles. In the second game he lost seven marbles. His final result, with the two games together, was that he had won three marbles. What happened in the first game?). For each type of problems, half of them (i.e. two for each type) involved small numbers (sum < 10), whereas half of them involved larger numbers

(10 < sum > 20). To reduce the working memory demands of the task, the experimenter presented the each child with a written version of the story problem as it was read and kept it in front of the child until the problem was solved. In this way, children were easier to keep track of the contents and to make relevant judgments accordingly. The maximum possible score for story problem solving was 32. The measures appeared to have good internal consistency (T1: α = .92; T2: α = .91).

Chinese word reading.

On the basis of the Hong Kong Lexical Lists for Primary Learning (Hong Kong Education Bureau, 2013), a word recognition task was constructed. According to the corpus, there are 4,914 words in Key Stage One (grade one to three). Fifty words from this corpus were chosen for the pilot test. On the basis of the pilot findings, thirty words were included in the word recognition task in the main study. Of these 30 items, eight were easy items (average correct rate: 70-100%), twelve had moderate difficulty (average correct rate: 40-70%), and ten were difficult (average correct rate: 0-40%). The items were arranged from the easiest words at the beginning to the most difficult ones towards the end of the test. In this task, children were shown written two-character Chinese words and asked to read aloud. One point was given for each correct response. No feedback was given. The maximum possible score for this task was 30. The internal consistency of this measure was satisfactory (Cronbach's α = 0.93).

Procedure

This study was approved by a research ethics committee of the university. Participating children were recruited through local schools and non-profit child-related community centres in Hong Kong. Parents were informed of the study via letters sent home by teachers or/and administrators. Upon receipt of parental consent, the children were asked for verbal assent and participated individually with the author in a quiet location, which was separate from other children in the primary school or centre. At Time 1 (first grade), the children were tested in two 30-40 min sessions separated by approximately 1 week. For all children, order of task presentation was the same. The first session included Raven's Standard Progressive Matrices, the central executive, phonological loop, and visuospatial sketchpad tasks, as well as the tasks that assessed children's knowledge of the commutativity and complement principles. The second session involved tasks that assessed procedural and conceptual knowledge of counting, calculation and story problem solving. At Time 2 (second grade), the children were tested in one session that lasted for approximately 20 to 30 minutes in which the Chinese word reading, calculation, and story problem solving tasks were administered. For each child, the interval between the first and second wave of assessments was between 9 and 11 months, with 10 months being the commonest interval (83%). For all children, testing was conducted by a researcher in Cantonese during the day.

Results

Preliminary Analyses

Descriptive statistics.

Table 3 shows the descriptive statistics for each variable. Of particular concern is whether the scores of the outcome measures of mathematical achievement are normally distributed. It is necessary to examine whether the normality assumption of regression analysis is violated in order to evaluate whether regression is an appropriate statistical tool to address the hypotheses of this study (Cohen & Cohen, 1983). Thus, the distributions of children's scores on calculation and story problem solving were analysed with regard to the z-values of Skewness and Kurtosis of each outcome variable. The z-value of Skewness was calculated by dividing the Skewness value by its standard errors, whereas z-value of Kurtosis was calculated by dividing the Kurtosis value by its standard error. Table 3 shows that none of the z-values are higher than 1.96, suggesting that the scores do not violate the normality assumption.

Insert Table 3 about here

Examining the influence of demographic variables.

Demographic variables may explain differences in children's scores on the main predictors (working memory, counting ability, and additive reasoning) as well as the scores in mathematical achievement. To assess the effects of demographic variables on mathematical achievement, we conducted independent t-tests with children's sex and one-way analyses of variance (ANOVA) with mothers' educational level and school separately as a fixed factor for each predictor and each measure of mathematical achievement. All variables showed no evidence of significant influence of children's sex, mothers' educational level, and school. Intraclass correlations (ICC) were also calculated according to Cohen, Cohen, West, and Aiken (2003) to examine whether there was any evidence of clustering. All ICCs ranged from 0.01 to 0.07, which were close to 0.

The very low within-cluster correlations suggest that there is no clustering in the present data. Thus, these variables were not included in the regression analyses.

Associations between variables.

Insert Table 4 about here

In this study, composite scores of some variables were created on the basis of theoretical and empirical reasons (previous research and the correlations between variables in the present data). According to Baddeley (Baddeley & Hitch, 1974, Baddeley, 1992), working memory consists of three components: the central executive, phonological loop, and visuospatial sketchpad. The central executive has been commonly measured by two tasks in previous studies: digit span backward and counting recall. The correlation between the scores in these tasks in the present sample was also significant ($r = .43$). According to Cohen (1988), this correlation value indicates a moderate effect ($.30 < r < .50$). Thus, a composite score was formed for central executive by averaging the standardised scores of the constituent measures. In subsequent analyses, the three components of working memory were considered separately because of two reasons: First, in theory (Baddeley & Hitch, 1974, Baddeley, 1992), the central executive, phonological loop, and visuospatial sketchpad are three related but separate components in working memory. Second, most researchers have treated them as three distinct factors in previous studies (e.g., Gathercole & Pickering, 2000; Holmes & Adams, 2006; Keeler & Swanson, 2001; Lee, Ng, Ng, & Lim, 2004; Lehto, 1995; Noel, Seron, & Trovarelli, 2004; Swanson, 1994; Swanson & Beebe-Frankenberger, 2004; Wilson & Swanson, 2001).

Three tasks were used to measure children’s counting ability: procedural counting, counting error detection, and cardinality understanding. Theoretically the latter two tasks explicitly measure children’s understanding of the counting principles, and in the present study, the correlation between the scores in these two tasks was moderate and significant ($r = .39$). Therefore, a composite score that represented ‘conceptual knowledge of counting’ was formed by averaging the standardised scores of these measures. Although the scores of procedural counting strongly correlated with the composite score of conceptual knowledge of counting ($r = .62$) (Cohen, 1988; $r > .50$ indicates a strong correlation), they were considered separately in subsequent analyses because there is evidence that some children failed to coordinate their knowledge of the three counting principles in these tasks even though they demonstrated competence in reciting the number sequence and applied it to objects and events (e.g., Bermejo, Morales, & deOsuna, 2004; Freeman, Antonucci, & Lewis, 2000; Sarneka & Gelman, 2004; Sophian, 1988).

Additive reasoning was measured by children's performance on two tasks that assessed their understanding of the commutativity and complement principles. The scores for the commutativity knowledge were moderately correlated with the scores for the complement knowledge (r = .37). Although they are related constructs, they were considered separately in subsequent analyses because some studies suggested that some children might master the commutativity priniplce before they could do so for the complement principle (e.g., Canobi, Reeve, & Pattison, 2003; Torbeyns, Peters, de Smedt, Ghesquière, & Vershaffel, 2016).

The next set of analyses explores the correlations between the main predictors (i.e. working memory, counting ability, and additive reasoning) and each measure of mathematical achievement at the two waves of assessment (Time 1 and Time 2). Table 4 shows the bivariate correlations between the variables. Several key findings are identified. First, The scores of central executive, digit span forward, and Corsi span significantly correlated with each other. This result is consistent with the theoretical model of working memory (Baddeley & Hitch, 1974; Baddeley, 1992) that these three components are related to each other. However, the correlations of the scores on these measures with mathematical achievement varied. Children's performance in the central executive correlated moderately (all coefficients > .30) with the scores in calculation and story problem solving at both Time 1 and Time 2. By contrast, the scores of visuospatial sketchpad had no significant correlation with any measure of mathematical achievement at both time points. The scores of phonological loop had significant correlations with calculation at Time 1 and Time 2, but did not correlate with story problem solving at both time points. Second, both indicators of the construct 'counting ability', procedural and conceptual knowledge of counting, had significant correlations with children's performance in calculation concurrently and longitudinally. However, only conceptual knowledge of counting correlated with children's performance in story problem solving. Finally, All measures of additive reasoning had strong (all coefficients close to and larger than 0.50) and significant correlations with both calculation and story problem solving at both time points.

Main Analyses – Multiple Regression Analyses

Having obtained significant correlations between a predictor and mathematical achievement is not sufficient to conclude that the contributions of that predictor is unique, because the different predictors may share variance that relates to the measure of mathematical achievement. Thus, we used multiple regression analyses to examine the independent contributions of individual predictors to an outcome variable. In the subsequent sections, sets of fixed-order regression analyses are reported to assess the contributions of working memory, counting ability, and additive reasoning to explaining individual differences in calculation and story problem solving at Time 1 and Time 2. Prior to each of the following analyses, assumptions of regression analyses were checked and showed no breaches to normality, linearity, homoscedasticity, multicollinearity, and auto-correction.

Concurrent predictions – outcome variable: calculation.

The first hypothesis of this study states that counting ability is important in children's mathematics learning and its influence is independent from that of general cognitive capacities, such as working memory. To test this hypothesis, variables of counting ability were entered in the last block of a regression model after age, IQ, and working memory (the central executive, phonological loop, and visuospatial sketchpad). Table 5 shows that counting ability explained an additional 5.4% of variance in T1 calculation beyond the effects of age, IQ, and working memory. This finding supported the first hypothesis. Conceptual knowledge of counting was an independent predictor of T1 calculation in the final block ($\beta = 0.191$, $t = 2.104$, $p < .05$). By contrast, procedural counting was not a significant predictor ($p > 0.05$).

Insert Table 5 about here

The second hypothesis of this study is that additive reasoning (as assessed by knowledge of commutativity and the complement principle) is independent from and more important than counting ability and general cognitive capacities, such as working memory in children's mathematics learning. To address this hypothesis, variables of additive reasoning were entered in the final step of a regression model after all the other factors, including age, IQ, working memory, and counting ability were controlled for. Table 6 shows that additive reasoning accounted for an additional 28.8% of variance in T1 calculation beyond the influence of all the other factors. Both commutativity knowledge ($\beta = 0.313$, $t = 39.78$, $p < .001$) and complement knowledge ($\beta = 0.34$, $t = 4.286$, $p < .001$) remained significant and independent predictors of T1 calculation in the final model. Thus, the second hypothesis of the present study was strongly supported.

Insert Table 6 about here

The third hypothesis of this study states that working memory makes a contribution to mathematical achievement, even when one accounted for children's specific mathematical knowledge such as their knowledge of counting and additive reasoning. To examine this hypothesis, variables of working memory were entered as the final step of a regression model after all the other factors, such as age, IQ, counting ability, and additive reasoning. Table 7 shows that working memory accounted for an additional 8% of variance in T1 calculation after the effects of all the other factors were controlled for. This finding supports the third hypothesis of the present study. Among the variables in working memory, the central executive was the only significant predictor of children’s performance in T1 calculation ($\beta = 0.298$, $t = 4.044$, $p < .001$). It shows that the central executive makes a unique contribution to T1 calculation even when the influence of counting ability and additive reasoning was controlled for. By contrast, the contributions of phonological loop and visuospatial sketchpad on children’s performance on T1 calculation were not statistically significant.

Insert Table 7 about here

Concurrent predictions – outcome variable: story problem solving.

Is the concurrent contribution of counting ability to T1 story problem solving similar to that to T1 calculation?. Table 8 shows that counting ability did not explain a significant amount of variance in T1 story problem solving (2% only). Both variables of counting ability did not make significant contributions to T1 story problem solving (all p values > 0.05). Therefore, the first hypothesis was not supported in the analyses for T1 story problem solving.

Insert Table 8 about here

The regression analyses for T1 calculation supported the second hypothesis by demonstrating that additive reasoning was the strongest predictor even after the influence of age, IQ, counting ability, and working memory was controlled for. Can this finding be replicated in T1 story problem solving? Table 9 shows that additive reasoning explained a substantial and significant amount of variance in T1 story problem solving beyond the effects of all the other factors (38.8%). Both variables of additive reasoning made unique contributions to accounting for the variance: commutativity knowledge ($\beta = 0.326$, $t = 4.203$, $p < .001$) and complement knowledge (β = 0.429, t = 5.568, p < .001). Therefore, this finding concurs with that for T1 calculation and the second hypothesis was strongly supported.

Insert Table 9 about here

Table 10 shows that when variables of working memory were entered in the last block after all the other factors were controlled for, they continued to explain a significant amount of variance (4.7%) in T1 story problem solving. Therefore, the third hypothesis of the present study was supported by the findings for both calculation and story problem solving at T1. Similar to T1 calculation, the only significant variable in working memory uniquely accounting for variance in T1 story problem solving was the central executive (β = 0.227, t = 3.148, p = .002).

Insert Table 10 about here

Longitudinal predictions – outcome variable: calculation.

The first set of regression analyses regarding the longitudinal predictions of children's performance in calculation concerns the unique contributions of counting ability. Similar to T1 calculation, when entered after age, IQ, and working memory (Table 11), counting ability accounted for a significant amount of variance in T2 calculation (4.1%). However, presumably because of the shared variance of T2 calculation explained by conceptual knowledge of counting and procedural counting, both variables were not unique predictors of T2 calculation (p values > 0.05). Because counting ability as a whole explained a significant amount of variance in T2 calculation beyond the effects of age, IQ, and working memory, this finding was considered as supporting evidence for the first hypothesis of the present study.

Insert Table 11 about here

The second set of regression analyses regarding the longitudinal predictions of children's performance in calculation concerns the unique contributions of additive reasoning. Consistent with the hypothesis, Table 12 shows that when variables of additive reasoning were entered in the last step, they continued to account for a substantial and significant amount of variance in T2 calculation (30%). The independent contributions of commutativity (β = 0.333, t = 4.491, p < .001) and complement knowledge (β = 0.334, t = 4.533, p < .001) remained significant after all the other factors were controlled for. Therefore, consistent with the findings for T1 calculation and story problem solving, this evidence strongly supported the second hypothesis.

Insert Table 12 about here

Similar to the analyses on T1 calculation and story problem solving, Table 13 demonstrates that working memory explained a significant amount of variance in T2 calculation (11%) when the effects of all other factors were taken into account. This finding was consistent with the third hypothesis that working memory makes a unique contribution to children's mathematics learning beyond the specific mathematical knowledge, such as counting ability and additive reasoning. Among the variables in working memory, only the central executive was a unique predictor of T2 calculation ($\beta = 0.321$, $t = 4.653$, $p < .001$) when the effects of all other variables were controlled for.

Insert Table 13 about here

Longitudinal predictions – outcome variable: story problem solving.

Because counting ability did not correate significantly with T2 story problem solving, it is unlikely that it would make a unique contribution to it when the effect of working memory is also controlled for. Consistent with this prediction, Table 14 shows that counting ability only accounted for 2% of variance in T2 story problem solving beyond the influence of age, IQ, and working memory. Both conceptual knowledge of counting and procedural counting were not independent predictors (*p* values > 0.05). This finding is consistent with that of T1 story problem solving in which counting ability was also not a good predictor. Therefore, the first hypothesis was not supported by the result for children's performance in story problem solving at both T1 and T2.

Insert Table 14 about here

When additive reasoning was entered in the last step after all the other factors were controlled for, Table 15 shows that it continued to explain a large amount of variance in T2 story problem solving (38.6%). This finding is consistent with that of T1 and T2 calculation as well as T1 story problem solving that strongly support the second hypothesis regarding the importance of additive reasoning in children's mathematics learning. Both commutativity knowledge ($\beta = 0.351$, $t = 4.758$, $p < .001$) and complement knowledge ($\beta = 0.406$, $t = 5.455$, $p < .001$) made significant contributions, independently of all the other factors, to T2 story problem solving.

Insert Table 15 about here

When the influence of all the other factors were controlled for (Table 16), working memory continued to explain a significant amount of variance in T2 story problem solving (6.6%). The central executive was the only variable in working memory that made a unique contribution to T2 story problem solving beyond the effects of age, IQ, counting ability, and additive reasoning ($\beta = 0.278$, $t = 3.989$, $p < .001$). These results were consistent with that in T1 and T2 calculation as well as T2 story problem solving. All these findings consistently support the third hypothesis that working memory is a factor that contributes to mathematics learning independently of children's ability to count and reason additively.

Insert Table 16 about here

Overall Summary of the Regression Analyses

Several key findings from the regression analyses with respect to the hypotheses are summarised as follows. First, it was hypothesised that counting ability is important in children's mathematics learning and its influence is independent from that of general cognitive capacities, such as working memory. This hypothesis is only partially supported. Counting ability contributes significantly to explaining the variance in calculation beyond the influence of age, IQ, and working memory. However, it does not make a unique contribution to story problem solving. Procedural counting appears not to be a good predictor of children's performance in both calculation and story problem solving in the present study. However, conceptual knowledge of counting is a unique predictor of calculation, independent of age, IQ, and working memory, at both time points.

Second, it was hypothesised that additive reasoning (as assessed by knowledge of commutativity and the complement principle) is independent from and more important than counting ability and general cognitive capacities, such as working memory in children's mathematics learning. The findings show that commutativity and complement knowledge are independently and strongly related to of children's performance in calculation and story problem solving concurrently and longitudinally. The amount of variance explained by additive reasoning is the largest among all the other factors, such as counting ability and working memory. Therefore, the second hypothesis is strongly supported.

Third, it was hypothesised that working memory is important in its own right in explaining variations in mathematical achievement. In support of this hypothesis, working memory explains a significant amount of variance in calculation and story problem solving concurrently and longitudinally beyond the effects of all the other factors. The central executive component of working memory is a unique predictor of children's performance in calculation and story problem solving concurrently and longitudinally, even after the effects of other factors, such as counting ability and additive reasoning are controlled for. However, the phonological loop and visuospatial sketchpad appear not to make significant contributions to children's performance in both calculation and story problem solving.

Taken together, among the three main predictors of interest in the present study, working memory and additive reasoning seem to be more important than counting ability for children's mathematics learning. The present study suggests that the central executive component of working memory as well as the knowledge of the commutativity and complement principles are particularly crucial.

Autoregressive Models of Calculation and Story Problem Solving

Autoregressive analysis for T2 calculation.

Previous multiple regression models show that working memory and additive reasoning are significant longitudinal predictors of children's performance in calculation and story problem solving. One question remains: Are these variables strong enough to be unique predictors of mathematical achievement (Time 2) even when children's previous performance in mathematical achievement (Time 1) is taken into account? This question is important because it is possible that what these predictors had in common with mathematical achievement at T1 actually explains their longitudinal power. If they remain significant longitudinal predictors of variance after children's performance in mathematical achievement at T1 is controlled for, the case for their predictive value is very strong. Thus, two sets of autoregressive analyses were conducted to examine this question.

Table 17 shows that when the variables of additive reasoning were entered in the final step, the amount of variance in T2 calculation that they accounted for additionally was significant (3.9%). Both commutativity and complement knowledge remained unique predictors of T2 calculation even when children's previous performance in calculation at T1 and all the other factors were taken into account; commutativity knowledge ($\beta = 0.144$, $t = 2.217$, $p < .05$) and complement knowledge ($\beta = 0.159$, $t = 2.50$, $p = .01$).

Insert Table 17 about here

When the variables of working memory were entered in the final step after the effects of T1 calculation and all the other factors are controlled for (Table 18), working memory explained a significant amount of variance in T2 calculation (2.9%). The central executive was a unique predictor of T2 calculation even when children's previous performance in calculation at T1 and all the other factors were considered (β = 0.155, t = 2.588, p = .011).

Insert Table 18 about here

Autoregressive analysis for T2 story problem solving.

Table 19 shows that when the variables of additive reasoning were entered in the final step, they accounted for a significant amount of variance in T2 story problem solving (6.9%) when T1 story problem solving and all the other factors were controlled for. Both commutativity and complement knowledge remained unique predictors of T2 story problem solving even when children's previous performance in story problem solving at T1 and all the other factors were taken into account; commutativity knowledge (β = 0.164, t = 2.168, $p < .05$) and complement knowledge (β = 0.261, t = 3.607, $p < .001$).

Insert Table 19 about here

Table 20 shows that when the variables of working memory were entered in the final step after the effects of T1 story problem solving and all the other factors are controlled for. Working memory explained a significant amount of variance in T2 story problem solving (2.8%). The central executive was a unique predictor of T2 story problem solving even when children's previous performance in story problem solving at T1 and all the other factors were taken into account (β = 0.178, t = 2.734, $p < .01$).

Insert Table 20 about here

In summary, the central executive component of working memory as well as the knowledge of the commutativity and complement principles are strong predictors for children's mathematical achievement. These variables continued to account for significant amounts of variance in calculation and story problem solving longitudinally even when children's previous performance in the mathematical achievement tasks was taken into account. Thus, the autoregressive analyses confirm the power and importance of additive reasoning and working memory in children's mathematics learning.

Specificity of Predictions Made by Additive Reasoning Tasks

Previous analyses have established a strong relation between children's ability to reason mathematically and their achievement in mathematics, but it is possible that their performance in the additive reasoning tasks may predict their attainment in other academic subjects as well. Examining this possibility is important because it will let us know more about the reason why additive reasoning predicts mathematical achievement so strongly. This may be because the associations that children have to reason about in these tasks are specific to mathematics learning, in which case it would not be likely that children's performance in additive reasoning would predict children's performance in a subject that does not involve mathematical reasoning, such as word reading. If additive reasoning tasks predict mathematics learning just because they measure reasoning in general, they should also predict children's performance in other non-mathematical academic subjects.

Chinese word reading was also used as one of the Time 2 outcome measures for testing the specificity of predictors for mathematical performance. All of the stimuli in the reading tasks involve two characters. Sometimes Chinese children can reason from one character as a cue to guess the pronunciation of another character and the meaning of the word. Thus, this task may also demand some aspects of general reasoning. Bivariate correlation shows that the central executive significantly correlated with children's performance in reading ($r = 0.271$, $p < .01$). By contrast, there were no significant correlations between both measures of additive reasoning and children's scores in reading ($r = 0.124$, $p > .05$). Therefore, the fact that the additive reasoning tasks predicted children's mathematical achievement much better than in Chinese word reading confirms the specificity and importance of additive reasoning in supporting children to learn mathematics.

Discussion

The purpose of this study was to evaluate the relative importance of working memory, counting ability, and additive reasoning in children's mathematics learning. The key findings of this study have contributed to the literature in several ways. First, Nunes and colleagues (2007,

2012) found that quantitative reasoning was a significant and specific predictor of children's mathematical achievement beyond IQ and working memory. The present study replicated the finding regarding the close connection between quantitative reasoning and mathematical achievement in a non-Caucasian cultural context. Second, whereas previous studies demonstrated a strong link between a global measure of quantitative reasoning and test scores on general mathematical achievement, the present study showed that mathematical reasoning in the domain of addition and subtraction in particular related significantly to both calculation and story problem solving concurrently and longitudinally. Third, the autogressive analyses indicated that variables in additive reasoning and the central executive component of working memory remained independent predictors of T2 mathematical achievement beyond the influence of children's performance on T1 mathematical achievement. This is strong evidence for the predictive powers of these variables. Fourth, this study incorporated procedural and conceptual counting as the indicators of counting ability and showed that only conceptual counting was uniquely predictive of calculation, but not of story problem solving.

In the following sections, we discuss the key findings with respect to the hypotheses of this study and the extent to which the findings are consistent with previously published knowledge on the topic. Then, the theoretical and educational implications of the results on children's mathematics learning and education are discussed. Finally, limitations of this study are identified and suggestions for future research are made towards the end of this article.

Contributions of Counting Ability

On the basis of the mathematical thinking perspective, counting ability was hypothesised to be one of the important cognitive foundations for children's mathematics learning. The mathematical thinking perspective emphasises that children need to understand the meanings of number in order to perform well in mathematics. The knowledge of the meanings of number refers to the understanding of the relations between numbers and quantities. Learning to count is relevant in this respect because it provides children with words to represent quantities. It also helps children reflect upon and develop the concept of one-to-one correspondence, ordinality, and cardinality as well as the coordinated use of these counting principles. According to the mathematical thinking perspective, grasping the conceptual knowledge of counting is more important that reciting a counting sequence in children's mathematics learning.

The regression analyses of this study show that counting ability accounted for a significant amount of variance in calculation (both concurrently and longitudinally) after the effects of age, IQ, and working memory were controlled for. Thus, the first hypothesis was supported for calculation in this study. Among the counting measures, conceptual knowledge of counting was a unique predictor of children's performance in calculation beyond the influence of age, IQ, and working memory. When children learn to calculate, they usually start from counting all of the numbers presented (i.e. the count-all procedure) and later shift to counting on from the cardinal value of the first or larger number presented. (Fuson, 1982). The more efficient counting-on procedures may rely on conceptual knowledge of counting, such as the understanding of cardinality. Thus, conceptual knowledge of counting contributes to children's success in calculation.

However, another measure of counting ability, procedural counting, did not make independent contributions to any measure of mathematical achievement. This finding is at odds with those from previous longitudinal studies (e.g., Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Koponen, Aunola, Ahonen, & Nurmi, 2007; Koponen, Salmi, Eklund, & Aro, 2013; Passolunghi, Vercelloni, & Schadee, 2007; Zhang, Koponen, Rasanen, Aunola, Lerkkanen, & Nurmi, 2014). For example, one study (Zhang et al., 2014) showed that the impact of procedural counting was so strong that it fully mediated the longitudinal association between spatial visualisation and letter knowledge with a group of Finnish children's performance in arithmetic. The discrepancy of the findings could be attributable to the ceiling performance of children in the present study, which may relate to the languages that the participating children speak in different research.

In counting, there are units of different sizes that can be counted within different classes. For example, we have the class of ones, the class of tens, the class of hundreds and so on. Because most of us use a base-ten system, when we have ten units of any size, we regroup these into units of the next size. For instance, ten 'ones' make up one 'ten', and ten 'tens' make up one 'hundred'. In Chinese number words, the base structure of the number system is transparent. Counting with Chinese number words makes children recognise easily that they are counting different units and they can repeat the same reasoning indefinitely to generate number words that they have not been taught formally before. Thus, the systematic relation between the number words in Chinese language and the underlying base-10 values may contribute to Chinese-speaking children's early mastery of procedural counting (Miller & Stigler, 1987; Miller, Smith, & Zhang, 2004; Miura, Kim, Chang, & Okamoto, 1988). This may be one of the reasons that the children in the present study had exceptional performance in procedural counting. Thus, the variation of children's performance on this task was small in this study, which might have influenced the strength of its relation to mathematical achievement.

By contrast, most of the children who participated in previous longitudinal research spoke European languages (e.g., English and Finnish). In these languages, the base structure of the number system is only partially reflected in the language, for example, 'tens' are counted with different names like ten, twenty, and thirty etc. This may render it more difficult for some young children to grasp the underlying structure of the counting system, thereby contributing to greater variation in procedural counting in these children. Thus, the impact of languages on structuring the number system in different cultures may explain the divergent findings regarding procedural counting across studies.

It was demonstrated in the present study that conceptual knowledge of counting was a stronger predictor than procedural counting of all measures of mathematical achievement. The ceiling effect may be one of the explanations of the results. Another possible interpretation is that conceptual knowledge of counting is more important than procedural counting in children's mathematics learning. In the present study, conceptual counting was measured by a task that required children to identify incorrect ways of counting and to coordinate their knowledge of various counting principles to determine the cardinal value of a set. It has been argued that it is necessary for children to coordinate different counting principles in order to understand the logic of numbers (Nunes & Bryant, 2015). On the basis of the mathematical thinking perspective, counting involves not only the memorisation of the number words in a fixed order but also the understanding of how number labels are generated in order to surpass simple memorisation of labels. It has been suggested the reason that counting ability makes contributions to explaining variation in mathematical achievement is that it helps children reflect on the relations between quantities and numbers (e.g., Piaget, 1952; Piaget & Inhelder, 1975; Nunes & Bryant, 2015). Thus, if a child can only generate number words proficiently but fails to understand the logic of counting, she or he is not likely to do well in mathematics according to the mathematical thinking perspective. Consistent with this view, the findings of this study show that conceptual knowledge of counting had a stronger connection than procedural counting with both calculation and story problem solving concurrently and longitudinally.

Thus, the present study adds to the literature that individual differences in conceptual knowledge of counting may matter more than procedural counting in mathematical achievement. This finding has several implications. First, from a methodological perspective, this evidence suggests that conceptual knowledge of counting may be a better measure of counting ability than procedural counting, especially for children who speak a language in which the organisation of number words fits well with the base-ten system. Second, from an educational viewpoint, the finding suggests that learning the numerical symbols of counting by themselves is not sufficient for children to succeed in mathematics. Past evidence showed that there could be a disconnection between using numbers and understanding the logic of counting (e.g., Bermejo, Morales, & deOsuna, 2004; Freeman, Antonnucci, & Lewis, 2000; Sarneka & Gelman, 2004; Sophian, 1988). Thus, teachers and parents need to ensure that children learn not only to count fluently, but also learn to think about the logical connections of the numbers they use for counting with quantities.

Contributions of Additive Reasoning

The second hypothesis of this study is that additive reasoning is independent from and more important than working memory and counting ability in children's mathematics learning. This hypothesis is strongly supported by the findings. Consistent with this hypothesis, additive reasoning was shown to make independent contributions to explaining variance in calculation and story problem solving beyond and above the effects of age, IQ, working memory, and counting ability at both waves of assessments. The regression analyses showed that the additional amount of variance explained by additive reasoning beyond all the other factors was substantial (close to 30% for both calculation and story problem solving).

On the basis of Piaget's logical operations framework, some researchers (e.g., Nunes & Bryant, 1996, 2015; Thompson, 1993, 1994; Vergnaud, 1997, 2009) have argued that children's competencies to reason about quantities logically are of primary importance for mathematical development. In the domain of additive reasoning, it is important to understand that quantities are connected by part-whole relations. Two central properties of part-whole relations involve (1) commutativity and (2) the inverse relation between addition and subtraction. Commutativity refers to the irrelevance of addend order to the sum, i.e. 'a + b = c' implies 'b + a = c', whereas the complement principle refers to the inverse relation between addition and subtraction, i.e. 'a + b = c' implies 'c - a = b'. A few studies have shown that global measures of quantitative reasoning are main predictors of children's later mathematical achievements (Nunes et al., 2007, 2012; Stern, 2005). The findings of the present study extend these results to a sample of non-Caucasian children and establish that reasoning about part-whole relations in particular is critical for success in both calculation and story problem solving.

Another important and novel finding of this research is that when the effects of T1 mathematical achievement were controlled for, the influence of additive reasoning on T2 mathematical achievement remained significant. This evidence is important because it shows that what additive reasoning had in common with mathematical achievement at T1 did not explain their longitudinal predictive power. The fact that additive reasoning remained a significant longitudinal predictor of variance after children's performance in mathematical achievement at T1 was controlled for suggests that the predictive value of additive reasoning is very strong.

This study also demonstrated a strong specificity of the additive reasoning tasks. The tasks were intended to measure the mathematical reasoning ability of children, but it is also possible that children need to rely heavily on other skills, such as general cognitive resources in order to complete the tasks. In other words, the tasks may not just measure mathematical reasoning, but reasoning in general. These two possibilities were ruled out by two findings. First, the scores of all tasks of additive reasoning did not correlate significantly with IQ, working memory, and counting ability. Thus, it appears that the additive reasoning tasks did not load on these other cognitive competence. Second, if they measure general reasoning ability, rather than mathematical reasoning ability, they should correlate significantly with the scores on Chinese word reading. The result showed that there was no significant correlation between additive reasoning and word reading. These results suggest that (1) the tasks were tapping a specific aspect of reasoning i.e. additive reasoning and that (2) additive reasoning predicts mathematics because it is a measure of competence specific to mathematics learning.

Why is there a strong link between additive reasoning and mathematical achievement? One possibility derives from considering the contributions of additive reasoning to the understanding of the nature of number and the use of more efficient problem solving strategies. According to the mathematical thinking perspective, arithmetic is the study and use of relations between numbers to solve problems and this is always carried out using a number system, which has specific characteristics. From this perspective, arithmetic is not just about memorising number facts. Instead, the process of calculation requires a deep understanding of number and of relations between operations. This understanding may form the basis for developing more advanced computational strategies that help children modify complex problems to make them easier to solve (e.g., Bryant & Nunes, 2009; Canobi, 2004; Canobi, Reeve, Pattison, 2003; Fuson, 1990; Gilmore, 2006, Nunes & Bryant, 1996, 2015).

For example, some efficient computational strategies (Gaschler, Vaterrodt, Frensch, Eichler, & Haider, 2013; Shrager & Siegler, 1998), such as counting-all starting with the larger addend (CAL) and counting-on from the larger addend (COL), require the understanding that numerical order does not affect the outcome in addition (i.e. the commutativity principle). The commutativity knowledge may also relate to the development of other strategies, such as the 'ten-strategy' and 'addends-compare strategy'. The ten-strategy refers to individuals' reordering different addends within a problem in an attempt to exploit the circumstance that non-adjacent numbers add up to ten. For instance, children who understand the commutativity principle can transform the problem '3 + 6 + 7' into '(3 + 7) + 6' that is easier for them to solve. For some arithmetic problems, computation can become unnecessary if one recognises that the identical addends had been shown (though in different order e.g., '2 + 7 + 8') in a previous problem that had already been solved e.g., '8 + 7 + 2'. This addends-compare strategy also demands the application of the commutativity knowledge between problems (Gaschler et al., 2013)

The complement principle may contribute to the use of a strategy called 'indirect addition' in which children can use additions to solve subtraction problems effectively if the numbers are close to each other. For example, to solve '21 – 18', it is less likely to make mistakes if they count up from 18 to 21. Thus, the use of more advanced computational strategies may be one of the reasons that children with better understanding of the commutativity and complement principles performed better on the calculation tasks.

Knowledge of these principles may also help the children analyse the mathematical situations presented in story problems more effectively. According to the mathematical thinking perspective, learning mathematics should be based on understanding the relations between quantities and operating on the numbers to reach conclusions about the quantities. Story problems are texts that involve information about quantities, which typically 'describe(s) a situation assumed familiar to the reader and pose(s) a quantitative question, an answer to which can be derived by mathematical operations performed on the data provided in the text, or otherwise inferred' (Greer, Verschaffel, & De Corte, 2002, p 271). Solving an additive story problem has been viewed as selecting and activating appropriate cognitive schema and filling the empty 'slots' of the activated schema with information provided in the story text. Some of these problem types (e.g., result-unknown Change problems and total set-unknown Combine problems) are suggested to link easily to counting or calculation schemes readily available in individuals' cognitive repertoire (Carpenter, Hiebert, & Moser, 1981; De Corte, & Verschaffel, 1985; 1987; Ginsburg, 1982). Other more difficult problems (e.g., start-unknown Change problems) require additional re-representational steps that involve the application of the part-whole schema before a connection with a proper counting or operation scheme could be formed. The understanding of the inverse relation between addition and subtraction (the complement principle) and the commutativity nature of quantities may help children reason about the underlying structure of the quantitative relations in the story.

Knowledge of the commutativity principle may also be related to children's solving some missing addend problems (Nunes & Bryant, 2015). Consider this example 'Jane had 3 cookies, got some more and now has 7. How many more cookies did she get?' Children can easily solve this problem by representing the first addend with 3 fingers, counting up to the final state i.e. 7 fingers, and evaluated how many fingers they had to add in the process. However, if the problem has the first rather than the second addend missing e.g., 'Jane had some cookies; her mother gave her 4 more and now she has 7; how many did she have to start with?' the children have to understand that the order does not affect the total. Those who understand the commutativity principle can start from the second addend i.e. 4, add up to 7, and count how many were added. Children who do not understand commutativity may find this problem difficult to solve because they do not know how many cookies Jane to start with.

Contributions of Working Memory

Learning and using mathematics, including thinking mathematically, must draw on some general cognitive resources, such as working memory. Thus, it was hypothesised that working memory makes a contribution to mathematical achievement, even when one has accounted for children's specific mathematical knowledge such as their knowledge of counting ability and additive reasoning. Consistent with this hypothesis, working memory continued to account for a significant amount of variance in all measures of mathematical achievement after all the other factors were controlled for. This suggests that working memory is a stable factor that contributes to children's mathematical achievement from the first to second grade. In contrast to additive reasoning, working memory was not a specific predictor for mathematical achievement because there was also a significant correlation between working memory and Chinese word reading.

Among the three components of working memory, the central executive appeared to be a stronger predictor than phonological loop and visuospatial sketchpad. The central executive was found to be a unique and significant predictor of variations in calculation and story problem solving at both time points. By contrast, visuospatial sketchpad did not correlate with mathematical achievement at all, whereas phonological loop did not make a unique contribution to mathematical achievement when the effect of central executive is taken into account. The finding is consistent with previous research that shows that measures of the central executive are especially strong predictors of children's performance in mathematics (e.g., Cowan & Powell, 2014; Gathercole & Pickering, 2000; Holmes & Adams, 2006; Keeler & Swanson, 2001; Lee, Ng, Ng, & Lim, 2004; Lehto, 1995; Noel, Seron, & Trovarelli, 2004; Swanson & Beebe-Frankenberger, 2004; Wilson & Swanson, 2001). Most of these studies have measured central executive by memory span tasks that demand simultaneous monitoring and storage of information. The evidence suggests that the particular central executive function of monitoring and coordinating concurrent processing and storage of information is crucial for children's performance on mathematical tasks. From the mathematical thinking perspective, the central executive may support children to think about the relations between numbers, to make a decision about appropriate strategy use to calculate, and then allocate attentional resources to implement the selected strategy. One study showed that the central executive component, rather than the phonological loop and visuospatial sketchpad, was associated with strategy use in calculation (Wu, Meyer, Maeda, Salimpoor, Tomiyama, Geary, & Menon, 2008). When solving story problems, the central executive may also support children to reason about the underlying quantitative structure of story problems, to identify the operations required to solve the problem, while working out the solution.

The finding that visuospatial sketchpad did not correlate with mathematical achievement in this study may be explained by the age of the participants. Some studies suggest that preschool children tend to have better performance on nonverbal compared with verbal arithmetic tasks and that individual differences in visuospatial sketchpad are the best predictor of mathematical achievement in this age group (McKenzie, Bull & Gray, 2003; Rasmussen & Bisanz, 2005; Simmons, Chris & Horne, 2008). However, it has been argued that from primary school onwards, children become increasingly reliant on verbal rehearsal to retain materials in memory (Hitch, Halliday, Schaafstal & Schraagen, 1988). Consistent with this idea, Rasmussen and Bisanz showed that by the first grade, children performed equally well on verbal and nonverbal mathematical tasks, and that phonological loop became the best predictor of children's performance on verbal problems (Rasmussen & Bisanz, 2005). The non-significant correlation between visuospatial sketchpad and mathematical achievement in the present study also suggests that this component of working memory may not be important for the children in this study at the age of around 6 to 7 to perform calculation and to solve story problems.

The present study also showed that the phonological loop significantly correlated with calculation at both time points, suggesting that it may be important for mathematics learning in this age group. However, it was not a unique predictor when the effect of central executive is controlled for. One straightforward interpretation is that central executive is more important than phonological loop in children's mathematics learning. However, it has to be noted that the central executive component of working memory was assessed by 'counting span' and 'backward digit span' tasks. Thus, both the central executive and phonological loop tasks may draw on verbal processing of materials (Savage, Lavers, & Pillay, 2007). Thus, this may reduce the likelihood that a unique association between phonological loop and mathematical achievement is observed in a regression model in which central executive measures are also included. Future research may explore ways to measure the central executive component of working memory non-verbally and examine its association, relative to the phonological loop, with mathematical achievement.

Theoretical Implications

In the introduction, two theoretical perspectives on mathematics learning were highlighted and compared, namely the number sense perspective and the mathematical thinking perspective. The present study was designed on the basis of the latter perspective that focuses on how children think about mathematics logically and meaningfully. According to this view, one core intellectual demand to learn mathematics is the need to understand relations between quantities, rather than merely understanding things in isolation. For example, Nunes and Bryant (2015) propose that there are two meanings of number – analytical and representational. The analytical meaning of number is defined by a number system, whereas the representational meaning refers to the use of numbers to represent quantities. A child who is competent in mathematical thinking means that she or he has a good understanding of the relational meanings of numbers and quantities. This understanding appears to support his or her ability to excel in a variety of mathematical tasks.

The findings of this study strongly suggest that the mathematical thinking perspective is an excellent theoretical framework for understanding mathematics learning and education. The final regression models that combine all factors hypothesised to relate to mathematical achievement explained over 50% of the variance in calculation and story problem solving, both concurrently and longitudinally. In particular, conceptual knowledge of counting is more important than procedural counting in predicting children's mathematical achievement in calculation. Additive reasoning, as measured by the knowledge of the commutativity and complement principles, explained variance in T2 mathematical achievement that was not accounted for by T1 mathematical achievement and all the other factors.

Educational Implications

One educational implication of this study is that quantitative reasoning should be a central aspect addressed in mathematics education curricula. Children need to learn to reason about relations between quantities in order to solve problems, not only about arithmetic.
A traditional assumption in early mathematics education is that knowledge of arithmetic comes first. Quantitative reasoning is usually introduced only after children have learned arithmetic. The assumption behind this practice is that children will learn to apply the acquired formal arithmetic operations to deal with various kinds of problem situations.

However, it seems common to observe that children learn computational algorithms in a meaningless fashion. For instance, previous studies showed that children often encountered difficulties in solving multi-digit addition and subtraction (e.g., Brown & Burton, 1978; Brown, & VanLehn, 1982; Carpenter, Franke, Jacobs, Fennema, & Empson, 1997; Carraher & Schlieman, 1985; Carraher, Carraher, & Schliemann, 1985; Fuson, 1990; Hennessy, 1994; Hiebert, & Wearne, 1996; Resnick, 1982; 1992; 1994; Selter, 2001; Young & O'Shea, 1981). Their difficulties can be understood in terms of the implementation of faulty procedures (Brown, & VanLehn, 1982). For example, when calculating 237 – 49, the children obtain the answer 212 by taking 7 away from 9 and 3 away from 4 presumably because they assume that one cannot take a larger number from a smaller number. Another example of faulty procedures is that when facing a subtraction such as 607 – 8, the children obtain 699, by subtracting 8 from 17. These children have correctly borrowed and added to the ones column, making the 0 into a 9 because 1 had been borrowed from the tens column. However, they forgot that something had been borrowed from the hundreds column. These are typical faulty procedures and well known to primary school teachers. Brown and VanLehn (1982) suggest that they are not merely a result of lack of attention. Instead, the mistakes follow from a systematic application of erroneous algorithms across different kinds of problems by the same children. The mathematical thinking approach suggests that if children do not have a clear understanding of analytical meaning of number i.e. the relation between numbers, they are more likely to make calculation mistakes because of the use of faulty procedures.

Thus, in order to search for meaningful mathematics teaching, educators should find ways to keep teaching connected to quantities in the world. One of the ways to achieve this is to avoid a predominant focus on learning procedures without any connection to understanding or applications that require these procedures. Additive reasoning should be considered as a domain of teaching and learning on its own right and numbers should not be taught in isolation from quantities and relations from the start. Children may also be given simple representational tools, such as blocks and diagrams that represent information about relations to solve problems, before they are taught about formalisations. Research has identified a number of ways to promote quantitative reasoning. One simple way to do so is to engage children to reflect and discuss about the problem. For example, Bermejo, Morales, and deOsuna (2004) showed that children who were asked to discuss what was the number of objects in a set when the counting was carried out backwards made significant improvement in tasks where counting was done in a non-conventional way, such as counting from two. This study suggests that reflection and discussion could be one of the strategies that educators can use to promote the coordination of counting principles.

On the basis of Piaget's (1952) theory, some mathematics education researchers (e.g. Nunes & Bryant, 1996; Steffe, 1994; Steffe, & Thompson, 2000; Vergnaud, 2009) suggest that educators should focus on helping children form schemes of action to understand different types of situations. Schema based instruction in problem solving may represent another promising way to promote quantitative reasoning (e.g. Chen, 1999; Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004; Jitendra, & Hoff, 1996; Marshall, 1995). The main idea behind this way of teaching is that children can learn to classify problems into problem types and design a path to solution on the basis of what they know is similar to a particular problem. For example, teachers may first present some prototypical problems in lessons and exemplify the paths to solution. The students are then asked to model the solutions. This teaching strategy encourages students to identify analogous problems and thus resort to similar pathways to solution. Because the classification of problems requires teachers to have knowledge about what kinds of reasoning is required to solve problems in a particular situation, it is important for teacher education programmes to ensure teachers to become knowledgeable about the ways through which different problem situations are classified, such as based on different schemes of action for different situations.

Limitations and Future Directions

With regard to the limitations of this study, some suggestions for future directions are made in this section. First, the present study has employed a longitudinal design that does not allow us to determine the causal relation between variables. In order to establish whether additive reasoning and mathematical achievement are in a causal relation, both longitudinal and intervention studies have to be used (Bradley and Bryant, 1983). The present study shows that additive reasoning made an independent contribution to explaining individual differences in mathematical achievement beyond and above working memory and counting ability. This finding addresses the first step of the paradigm to determine whether additive reasoning is a potential cause of children's mathematical achievement. A possible research project in the future may focus on implementing intervention programmes aimed to enhance children's additive reasoning and examining whether an improvement in additive reasoning would result in significant progress in mathematics learning. Second, although the present study shows that the mathematical thinking perspective is a useful theoretical framework for understanding mathematics learning and education, one cannot draw any conclusion regarding whether it is a better perspective than the number sense approach. To test this research question, future studies may incorporate measures of number sense (e.g., numerical magnitude comparisons, number facts, number line estimation) to evaluate the relative importance of number sense and quantitative reasoning.

Conclusion

In conclusion, the present study has provided some evidence regarding the contributions of working memory, counting ability, and additive reasoning to children's mathematical achievement. This study is guided by the mathematical thinking perspective that emphasises the importance of understanding the relations between quantities in mathematics learning Conceptual knowledge of counting, but not procedural counting, was a unique predictor of calculation ability beyond age, IQ, and working memory, but it did not contribute significantly to story problem solving. It was found that the central executive component of working memory made independent contributions to explaining variations in calculation and story problem solving beyond the effects of all the other factors. Additive reasoning (as assessed by knowledge of commutativity and the complement principle) was shown to be more important than counting ability and working memory for children's mathematics learning. It appears to be an independent and the strongest predictor of children's mathematical achievement. Despite several limitations, this study offers some novel and exciting directions for more empirical investigations in mathematics learning and education with a focus on quantitative reasoning in the future.

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Table 1. Counting judgment task (adapted from LeFevre et al., 2006)

Type of trial	Trial	N	Description
Correct counts	1, 4, 13, 15	6, 8, 9, 12	Conventional left-to-right count
Incorrect counts			Violations of word–object correspondence
Repeated words	2, 11	7, 10	Pika used an incorrect (repeated) number word that did not correspond to an item (i.e., one, two, two)
Skipped object	7, 16	6, 11	Pika missed counting an item in the regular sequence and never returned to it
Double count	8, 14	12, 8	Pika counted one item twice
Unusual counts			Violations of conventional features
Reverse direction	3, 12	7, 12	Pika counted from the right to the left
Start in the middle	5, 9	11, 9	Pika started counting in the middle of the set, counted to the right end, and then went back to the beginning to finish
Double point	6, 10	9, 10	Pika hopped twice on an item but repeated the correct number word twice (e.g., eight eight)

Note. Trial refers to position in the order of presentation. N refers to the number of items for each trial.

Table 2. Types of target problems, examples, and their purpose

Base Problem	Types of its Corresponding Target Problems	Purpose of the Target Problems
Mary has 3 fish and her mother gave her 5 more. How many fish does Mary have now? (Answer: 3 + 5 = 8)	*Commutativity test item:* 'Mary has 5 fish and her mother gave her 3 more. How many fish does Mary have now?'	To test children's understanding of the commutativity principle. The base problem should be helpful to solve this item because the answer of '5 + 3' can be deduced by '3 + 5 = 8' according to the commutativity principle.
	Commutativity control item: 'Mary has 5 fish and her mother took away 3 from her. How many fish does Mary have now?'	To allow for a correction for response biases. The base problem should not be helpful to solve this control item because the answer of '5 - 3' cannot be deduced by '3 + 5 = 8' according to the commutativity principle.
	Complement test item: 'Mary has 8 fish and her mother took away 5 from her. How many fish does Mary have now?'	To test children's understanding of the complement principle. The base problem should be helpful to solve this item because the answer of '8 – 5' can be deduced by '3 + 5 = 8' according to the complement principle.
	Complement control item: 'Mary has 8 fish and her mother gave 5 more to her. How many fish does Mary have now?'	To allow for a correction for response biases. The base problem should not be helpful to solve this control item because the answer of '8 + 5' cannot be deduced by '3 + 5 = 8' according to the complement principle.

Table 3. Descriptive statistics for domain-general factors, counting ability, additive reasoning, and mathematical achievement (N = 115)

	Reliability (α coefficients)	Possible range	Mean	Standard deviations	Minimum	Maximum	Skewness z-value	Kurtosis z-value
Domain-general factors								
Age in months	N.A.	N.A.	76.32	2.81	67.8	82.1	-1.87	-0.34
Non-verbal intelligence: Raven's raw scores	0.81	0-60	19.45	2.94	15	26	1.52	-1.81
Working memory: Central executive	0.89							
Digit span backward		0-16	7.51	1.88	4	10	-0.01	-1.12
Counting recall		0-16	7.93	1.61	6	10	0.06	-1.45
Working memory: Phonological loop	0.81							
Digit span forward		0-16	11.37	2.3	8	16	0.5	-2.2
Working memory: Visuospatial sketchpad	0.83							
Corsi span		0-16	10.47	2.01	6	16	0.65	-1.1
Counting ability								
Procedural counting	0.71	0-12	11.02	1.08	9	12	-3.39	-1.6
Conceptual knowledge of counting	0.85							
Error detection		0-16	13.37	1.91	10	16	-0.4	-0.66
Cardinality		0-16	14.23	1.14	12	16	-0.13	-0.74
Additive reasoning								
Commutativity principle	0.81	0-6	4.14	1.39	1	6	-3.57	0.32
Complement principle	0.85	0-6	2.05	1.54	0	5	1.8	-2.11
Mathematical achievement								
Time 1 calculation	0.87	0-16	11.03	2.85	5	16	-0.09	-1.73
Time 2 calculation	0.92	0-16	10.95	3.04	5	16	0.39	-1.85
Time 1 story problem solving	0.92	0-32	22.38	4.43	14	32	1.21	-1.13
Time 2 story problem solving	0.91	0-32	23.63	3.57	16	30	-1.04	-1.17

Table 4. Bivariate correlations among standardised variables (N = 115)

			Counting ability		*Working memory*			*Additive reasoning*		*Mathematical achievement*			
	1	2	3	4	5	6	7	8	9	10	11	12	13
1. Age in months	1												
2. IQ (Raven's scores)	0.33**	1											
Counting ability													
3. Procedural counting	0.08	0.01	1										
4. Counting knowledge	0.05	0.05	0.62**	1									
Working memory													
5. Central executive	0.15	0.16	0.15	0.13	1								
6. Digit span forward	0.12	0.14	0.18	0.20*	0.25**	1							
7. Corsi span	0.02	0.15	0.12	0.13	0.24**	0.29**	1						
Additive reasoning													
8. Commutativity knowledge	0.11	0.15	0.16	0.17	0.10	0.14	0.04	1					
9. Complement knowledge	0.09	0.15	0.15	0.16	0.14	0.02	0.03	0.37**	1				
Mathematical achievement													
10. T1 Calculation	0.07	0.13	0.22*	0.31**	0.35**	0.19*	0.06	0.50**	0.54**	1			
11. T2 Calculation	0.08	0.17	0.24**	0.29**	0.42**	0.25**	0.14	0.51**	0.54**	0.81**	1		
12. T1 Story problem solving	0.15	0.18	0.16	0.20*	0.33**	0.04	0.04	0.56**	0.56**	0.55**	0.64**	1	
13. T2 Story problem solving	0.09	0.20*	0.13	0.21*	0.35**	0.10	0.05	0.52**	0.62**	0.57**	0.68**	0.75**	1

** Correlation is significant at the 0.01 level (2-tailed)

* Correlation is significant at the 0.05 level (2-tailed)

Table 5. The additional amount of variance of T1 calculation explained by counting ability beyond age, IQ, and working memory (N = 115)

Model	Variables entered into model	R^2	R^2 change	*F* change	*Sig. F change*	(df)
1	Age in months	0.004	0.004	0.475	0.492	(1, 113)
2	Age in months Non-verbal intelligence	0.016	0.012	1.368	0.245	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad	0.188	0.172	7.694***	<0.001	(3, 109)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge	0.243	0.054	3.843*	0.024	(2, 107)

*significant at the 0.05 level, ***significant at the 0.001 level

Table 6. The additional amount of variance of T1 calculation explained by additive reasoning beyond all the other factors (N = 115)

Model	Variables entered into model	R^2	R^2 change	*F* change	*Sig. F change*	(df)
1	Age in months	0.004	0.004	0.475	0.492	(1, 113)
2	Age in months Non-verbal intelligence	0.016	0.012	1.368	0.245	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge	0.243	0.226	6.394***	<0.001	(5, 107)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.530	0.288	32.152***	<0.001	(2, 105)

***significant at the 0.001 level

Table 7. The additional amount of variance of T1 calculation explained by working memory beyond all the other factors (N = 115)

Model	Variables entered into model	R^2	R^2 change	*F* change	*Sig. F change*	(df)
1	Age in months	0.004	0.004	0.475	0.492	(1, 113)
2	Age in months Non-verbal intelligence	0.016	0.012	1.368	0.245	(1, 112)
3	Age in months Non-verbal intelligence Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.450	0.434	21.308***	<0.001	(4, 108)
4	Age in months Non-verbal intelligence Procedural counting Counting knowledge Commutativity knowledge Complement knowledge Central executive Phonological loop Visuospatial sketchpad	0.530	0.080	5.966***	=0.001	(3, 105)

***significant at the 0.001 level

Table 8. The additional amount of variance of T1 story problem solving explained by counting ability beyond age, IQ, and working memory (N = 115)

Model	Variables entered into model	R^2	R^2 change	*F* change	*Sig. F change*	(df)
1	Age in months	0.024	0.024	2.738	0.101	(1, 113)
2	Age in months Non-verbal intelligence	0.04	0.016	1.907	0.17	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad	0.142	0.102	4.328**	0.006	(3, 109)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge	0.162	0.020	1.292	0.279	(2, 107)

**significant at the 0.01 level

Table 9. The additional amount of variance of T1 story problem solving explained by additive reasoning beyond all the other factors (N = 115)

Model	Variables entered into model	R^2	R^2 change	*F* change	*Sig. F change*	(df)
1	Age in months	0.024	0.024	2.738	0.101	(1, 113)
2	Age in months Non-verbal intelligence	0.04	0.016	1.907	0.17	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge	0.162	0.122	3.128**	0.011	(5, 107)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.550	0.388	45.246***	<0.001	(2, 105)

**significant at the 0.01 level, ***significant at the 0.001 level

Table 10. The additional amount of variance of T1 story problem solving explained by working memory beyond all the other factors (N = 115)

Model	Variables entered into model	R^2	R^2 change	*F* change	*Sig. F change*	(df)
1	Age in months	0.024	0.024	2.738	0.101	(1, 113)
2	Age in months Non-verbal intelligence	0.04	0.016	1.907	0.17	(1, 112)
3	Age in months Non-verbal intelligence Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.503	0.463	25.155***	<0.001	(4, 108)
4	Age in months Non-verbal intelligence Procedural counting Counting knowledge Commutativity knowledge Complement knowledge Central executive Phonological loop Visuospatial sketchpad	0.550	0.047	3.666*	=0.015	(3, 105)

*significant at the 0.05 level, ***significant at the 0.001 level

Table 11. The additional amount of variance of T2 calculation explained by counting ability beyond age, IQ, and working memory (N = 115)

Model	Variables entered into model	R^2	R^2 change	*F* change	*Sig. F change*	(df)
1	Age in months	0.006	0.006	0.722	0.397	(1, 113)
2	Age in months Non-verbal intelligence	0.034	0.028	3.197	0.076	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad	0.247	0.212	10.269***	<0.001	(3, 109)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge	0.288	0.041	3.843*	0.048	(2, 107)

*significant at the 0.05 level, ***significant at the 0.001 level

Table 12. The additional amount of variance of T2 calculation explained by additive reasoning beyond all the other factors (N = 115)

Model	Variables entered into model	R^2	R^2 change	*F* change	*Sig. F change*	(df)
1	Age in months	0.006	0.006	0.722	0.397	(1, 113)
2	Age in months Non-verbal intelligence	0.034	0.028	3.197	0.076	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge	0.288	0.242	7.655***	<0.001	(5, 107)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.589	0.300	38.32***	<0.001	(2, 105)

***significant at the 0.001 level

Table 13. The additional amount of variance of T2 calculation explained by working memory beyond all the other factors (N = 115)

Model	Variables entered into model	R^2	R^2 change	*F* change	*Sig. F change*	(df)
1	Age in months	0.006	0.006	0.722	0.397	(1, 113)
2	Age in months Non-verbal intelligence	0.034	0.028	3.197	0.076	(1, 112)
3	Age in months Non-verbal intelligence Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.478	0.444	22.996***	<0.001	(4, 108)
4	Age in months Non-verbal intelligence Procedural counting Counting knowledge Commutativity knowledge Complement knowledge Central executive Phonological loop Visuospatial sketchpad	0.589	0.110	9.394***	<0.001	(3, 105)

***significant at the 0.001 level

Table 14. The additional amount of variance of T2 story problem solving explained by counting ability beyond age, IQ, and working memory (N = 115)

Model	Variables entered into model	R^2	R^2 change	*F* change	*Sig. F change*	(df)
1	Age in months	0.008	0.008	0.865	0.354	(1, 113)
2	Age in months Non-verbal intelligence	0.042	0.034	4.032*	0.047	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad	0.175	0.133	5.836***	0.001	(3, 109)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge	0.195	0.020	1.322	0.271	(2, 107)

*significant at the 0.05 level, ***significant at the 0.001 level

Table 15. The additional amount of variance of T2 story problem solving explained by additive reasoning beyond all the other factors (N = 115)

Model	Variables entered into model	R^2	R^2 change	*F* change	*Sig. F change*	(df)
1	Age in months	0.008	0.008	0.865	0.354	(1, 113)
2	Age in months Non-verbal intelligence	0.042	0.034	4.032*	0.047	(1, 112)
3	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge	0.195	0.152	4.051**	0.002	(5, 107)
4	Age in months Non-verbal intelligence Central executive Phonological loop Visuospatial sketchpad Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.581	0.386	48.403***	<0.001	(2, 105)

*significant at the 0.05, **significant at the 0.01 level, ***significant at the 0.001 level

Table 16. The additional amount of variance of T2 story problem solving explained by working memory beyond all the other factors (N = 115)

Model	Variables entered into model	R^2	R^2 change	*F* change	*Sig. F change*	(df)
1	Age in months	0.008	0.008	0.865	0.354	(1, 113)
2	Age in months Non-verbal intelligence	0.042	0.034	4.032*	0.047	(1, 112)
3	Age in months Non-verbal intelligence Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.515	0.473	26.372***	<0.001	(4, 108)
4	Age in months Non-verbal intelligence Procedural counting Counting knowledge Commutativity knowledge Complement knowledge Central executive Phonological loop Visuospatial sketchpad	0.581	0.066	5.472**	=0.002	(3, 105)

*significant at the 0.05 level, **significant at the 0.01 level, ***significant at the 0.001 level

Table 17. The additional amount of variance of T2 calculation explained by additive reasoning beyond T1 calculation and all the other factors (N = 115)

Model	Variables entered into model	R^2	R^2 change	*F* change	*Sig. F change*	(df)
1	Age in months	0.006	0.006	0.722	0.397	(1, 113)
2	Age in months Non-verbal intelligence	0.034	0.028	3.197	0.076	(1, 112)
3	Age in months Non-verbal intelligence T1 calculation	0.668	0.635	212.426***	<0.001	(1, 111)
4	Age in months Non-verbal intelligence T1 calculation Procedural counting Counting knowledge	0.674	0.005	0.846	0.432	(2, 109)
5	Age in months Non-verbal intelligence T1 calculation Procedural counting Counting knowledge Central executive Phonological loop Visuospatial sketchpad	0.696	0.022	2.781*	0.047	(3, 106)
6	Age in months Non-verbal intelligence T1 calculation Procedural counting Counting knowledge Central executive Phonological loop Visuospatial sketchpad Commutativity knowledge Complement knowledge	0.734	0.039	7.567***	0.001	(2, 104)

*significant at the 0.05 level, ***significant at the 0.001 level

Table 18. The additional amount of variance of T2 calculation explained by working memory beyond T1 calculation and all the other factors (N = 115)

Model	Variables entered into model	R^2	R^2 change	*F* change	*Sig. F change*	(df)
1	Age in months	0.006	0.006	0.722	0.397	(1, 113)
2	Age in months Non-verbal intelligence	0.034	0.028	3.197	0.076	(1, 112)
3	Age in months Non-verbal intelligence T1 calculation	0.668	0.635	212.426***	<0.001	(1, 111)
4	Age in months Non-verbal intelligence T1 calculation Procedural counting Counting knowledge	0.674	0.005	0.846	0.432	(2, 109)
5	Age in months Non-verbal intelligence T1 calculation Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.705	0.032	5.767**	0.004	(2, 107)
6	Age in months Non-verbal intelligence T1 calculation Procedural counting Counting knowledge Commutativity knowledge Complement knowledge Central executive Phonological loop Visuospatial sketchpad	0.734	0.029	3.8**	0.012	(3, 104)

**significant at the 0.01 level, ***significant at the 0.001 level

Table 19. The additional amount of variance of T2 story problem solving explained by additive reasoning beyond T1 story problem solving and all the other factors (N = 115)

Model	Variables entered into model	R^2	R^2 change	*F* change	*Sig. F change*	(df)
1	Age in months	0.008	0.008	0.865	0.354	(1, 113)
2	Age in months Non-verbal intelligence	0.042	0.034	4.032*	0.047	(1, 112)
3	Age in months Non-verbal intelligence T1 story problem solving	0.574	0.532	138.703***	<0.001	(1, 111)
4	Age in months Non-verbal intelligence T1 story problem solving Procedural counting Counting knowledge	0.58	0.006	0.787	0.458	(2, 109)
5	Age in months Non-verbal intelligence T1 story problem solving Procedural counting Counting knowledge Central executive Phonological loop Visuospatial sketchpad	0.601	0.021	1.739	0.109	(3, 106)
6	Age in months Non-verbal intelligence T1 story problem solving Procedural counting Counting knowledge Central executive Phonological loop Visuospatial sketchpad Commutativity knowledge Complement knowledge	0.67	0.069	10.852***	<0.001	(2, 104)

*significant at the 0.05 level, ***significant at the 0.001 level

Table 20. The additional amount of variance of T2 story problem solving explained by working memory beyond T1 story problem solving and all the other factors (N = 115)

Model	Variables entered into model	R^2	R^2 change	*F* change	*Sig. F change*	(df)
1	Age in months	0.008	0.008	0.865	0.354	(1, 113)
2	Age in months Non-verbal intelligence	0.042	0.034	4.032*	0.047	(1, 112)
3	Age in months Non-verbal intelligence T1 story problem solving	0.574	0.532	138.703***	<0.001	(1, 111)
4	Age in months Non-verbal intelligence T1 story problem solving Procedural counting Counting knowledge	0.58	0.006	0.787	0.458	(2, 109)
5	Age in months Non-verbal intelligence T1 story problem solving Procedural counting Counting knowledge Commutativity knowledge Complement knowledge	0.642	0.062	9.29***	<0.001	(2, 107)
6	Age in months Non-verbal intelligence T1 story problem solving Procedural counting Counting knowledge Commutativity knowledge Complement knowledge Central executive Phonological loop Visuospatial sketchpad	0.67	0.028	3.791*	0.047	(3, 104)

*significant at the 0.05 level, ***significant at the 0.001 level