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**COMPETITION IN POSTED PRICES WITH STOCHASTIC
DISCOUNTS**

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Competition in Posted Prices With Stochastic Discounts

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Abstract

We study price competition between firms over public list or posted prices when a fraction of consumers (termed ‘bargainers’) can subsequently receive discounts with some probability. Such stochastic discounts are a feature of markets in which some consumers bargain explicitly; of markets in which sellers use the marketing practice of couponing; and of markets in which sellers offer both simple-to-understand tariffs (the posted prices) alongside complex or opaque tariffs that might offer a discount. Even though bargainers receive reductions off the posted prices, the potential to discount dampens competitive pressure in the market by reducing the incentive to undercut a rival’s posted price, thus raising all prices and increasing profits. Welfare falls because of the stochastic nature of the discounts, which generates some misallocation of products to consumers. We also find that stochastic discounts facilitate collusion by reducing the market share that can be gained from a deviation.

Keywords: Posted prices; list prices; collusion; bargaining; negotiation; haggling; discounting; coupons; obfuscation; flat rate bias; price takers.

JEL Classification: C78; D43; L13.

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1 Introduction

Firms often offer discounts off public list or posted prices. For example, private automobile buyer discounts have been documented as being large for some consumers, while others purchase with no discount off list price.¹ A similar pattern holds for discounts off realtor fees in the United Kingdom,² while public commentators have referred to evidence of bargaining off the list price in retail stores such as jewellers, shoe shops, travel agents, furniture stores and electrical retailers.³

In these examples, bargainers secure a reduction with some probability dependent upon the specifics of the interaction between the consumer and the given sales representative. Such stochastic discounting is also a feature of the marketing strategy of offering discount coupons, and the perhaps more sinister strategy of offering both simple and complex tariffs. In the case of coupons, retail firms target coupons that offer a given reduction off the list price at specific consumer segments. Such vouchers can be delivered by direct mail, or by being placed in certain media outlets. These reductions are, however, only received by the target consumers with some probability. In the case of obfuscation, not all consumers are willing to study complicated tariff structures to determine whether they offer a discount off simple-to-understand tariffs, and not all who try will succeed in understanding the total cost of the package to them. Consumers unwilling to decipher complex tariffs are labeled in the marketing literature as having a “flat rate bias.” The existence of such consumers has been documented in, for example, the market for internet access, gym membership and telephony (see Lambrecht and Skiera, 2006). The United Kingdom retail electricity market is good example of a market with simple headline prices and complicated tariffs that often work out cheaper for particular customers.

In this paper, we study the effect of discounting on price competition between firms when a fraction of consumers (termed ‘bargainers’) can be strategically offered discounts off list prices that they receive with some probability. We find that even though bargainers often receive reductions off list prices, stochastic discounts raise all prices and cause a misallocation of goods to consumers that lowers total welfare. Furthermore, when the firms interact repeatedly, discounts facilitate collusion by the firms.

We develop a tractable model of differentiated product price competition followed by strategically chosen stochastic discounts. First, the two firms simultaneously set list prices that become

¹The Competition Commission (2000) offer evidence that private automobile buyer discounts in the United Kingdom in 1997/98 averaged 11% off Ford’s list prices, averaged 12% off Vauxhall’s list prices, and averaged 10% off Fiat’s list prices. Yet many consumers received no reduction at all (e.g., 13% in the case of Ford). According to Goldberg (1996, p. 641), data from the United States “reveal substantial variation in dealer discounts, a large fraction of which cannot be explained just by financing, or model- and time-specific variables.”

²The Office of Fair Trading (2004) report on the United Kingdom estate agency (realtor) market found that almost 50% of house sellers using an estate agent had tried to negotiate fees, with 80% of those receiving a reduction (Section 4.48).

³See, for instance, Sunday Times (2008) and Daily Telegraph (2009).

common knowledge. ‘Price takers’ buy at list prices. After the list prices become known, the firms can also choose to offer a discount price that ‘bargainers’ secure with some probability less than one. Both categories of consumer are uniformly distributed along a Hotelling line, and so share a common view of product differentiation.

We offer three interpretations of this model of stochastic discounts (see Section 3.1 for a more detailed discussion):

- Our leading interpretation is that the model captures explicit bargaining in a simple and tractable way. ‘Price takers’ do not attempt to bargain, while ‘bargainers’ approach both firms to ask for a better price than the one posted. A bargainer receives a particular firm’s reduced price offer with probability less than one. This assumption captures in a simple way the fact that bargaining is uncertain: the psychological costs and tension of bargaining and the danger of frayed emotions lead to the possibility that negotiations between the sales representative and the consumer break down.⁴
- Our second interpretation is that the model captures the use of discount coupons. ‘Bargainers’ regularly visit websites or newspapers in which firms offer discount coupons, while ‘price takers’ do not. The assumption that discount price offers are received with probability less than one captures the fact that the bargainers do not always find a firm’s coupon (e.g., due to inattention, not visiting the website at the right time, not getting the right issue of the newspaper).
- Our third interpretation is that the model captures settings in which firms obfuscate pricing by simultaneously using both simple and complicated tariffs. ‘Price takers’ buy at the simple-to-understand tariffs (the posted prices), while other consumers (the ‘bargainers’) explore the opaque tariffs to determine whether they offer a discount relative to the simple tariffs. These consumers only explore and then understand a particular firm’s opaque tariff with probability less than one.

Stochastic discounts affect the optimal pricing strategy in important ways. Firms offer bargainers discounts off the list prices since each firm is worried that a bargainer will be tempted by a more attractive discount price from the rival firm. However, discounts also affect the firms’ strategic incentives when they set their list prices because the firms anticipate how their chosen list prices will affect the competition for bargainers. We demonstrate that list prices and the stochastic discounts are strategic complements. If a firm undercuts its rival’s list price, it will

⁴Bargaining models with an exogenous probability of breakdown have been studied by, for instance, Binmore et al. (1986), Stole and Zwiebel (1996) and de Fontenay and Gans (2005).

win some price takers; however, the lower list price will encourage an aggressive response to bargainers from the rival seller. This is because the rival firm will find it harder to win bargainers who do not receive the initial firm's discounted price, but can purchase at the reduced list price: the rival will respond to this increased competitiveness by making its discounts more attractive. Anticipating this response, the list-price-reducing firm will also lower its offer to bargainers, and so the result will be lower equilibrium discount prices for bargainers. The presence of the bargainers therefore dampens the competitive pressure that exists in the market for price takers, since the strategic response from the rival means that any list price reduction lowers equilibrium profits from the bargainers. This effect allows list prices to rise above the standard Hotelling equilibrium level. The higher list prices in turn allow the discount prices to rise above the Hotelling level.

The moderating influence on competition brought about by bargainers and stochastic discounts not only increases prices, but also raises profits and lowers both consumer surplus and welfare. Prices are a transfer from consumers to firms: hence higher prices are welfare neutral. However, stochastic discounts also lower welfare: a bargainer who happens to receive a discount price offer from only one firm might be left with the choice between paying a high list price for the product she prefers and a lower discount price for the less attractive product. The consumer can, in effect, be bribed to accept a less-ideal product. This generates some misallocation of products to consumers, which lowers the efficiency of the market. Furthermore, as the proportion of bargainers in the market increases, all prices rise at an increasing rate (both list prices and the prices net of the stochastic discount) and welfare falls at an increasing rate.

We also study the effect of stochastic discounts on the ability of the industry to sustain collusive outcomes, thus extending our work to a dynamic setting. To simplify the analysis, we consider the case where the products are perfect substitutes. However, we also extend our analysis by allowing any number of firms $N \geq 2$ to compete in the market. We find that discounts facilitate collusion. The mechanism is that discounts lower the profits available from deviating on a collusive agreement. If a firm deviates only in the discounts it offers, it foregoes any increase in the market share of price takers. If, instead, a firm deviates by lowering its publicly posted list price, then the rival firms can respond by discounting more aggressively. In either case, the increase in market share available to firms from a deviation is reduced, thus allowing collusion to be sustained at lower discount factors than would be possible without discounting.

The paper proceeds as follows: Section 2 relates our results to the existing literature; Section 3 sets out the model; Section 4 characterizes the equilibria; Section 5 conducts comparative statics on prices, profits and welfare; Section 6 considers collusion; and Section 7 concludes. All proofs are relegated to the Appendix.

2 Relation to the literature

Our model allows us to extend the literature on differentiated product price competition by characterizing the effect of strategically chosen stochastic discounts off previously posted list prices on those list prices, discount price offers, profits and welfare.

A small and recent literature considers the consequences of discounts and bargaining when some consumers take list or posted prices as given. Korn (2007) and Zeng et al. (2007) consider monopolists, and so are silent about the implications of discounts on competitive outcomes, which is the focus of this study.⁵ Desai and Purohit (2004) and Zeng et al. (2007) focus on the marketing decision of whether to permit bargaining or not. In Desai and Purohit (2004), when both firms permit bargaining, the list prices are irrelevant to the bargainers since they are never effective outside options; thus the strategic interaction between list prices and discount prices is severed. Raskovich (2007) finds that a big enough proportion of bargainers causes list prices to jump from marginal cost to the monopoly price. The mechanism is different than ours: Raskovich (2007) assumes the firms that post higher list prices are weaker bargainers and so are more attractive to bargaining consumers. Finally, Gill and Thanassoulis (2009) present a homogeneous product quantity-setting model in which a Cournot auctioneer sets a binding public list price. There, bargaining was modeled as an application of Burdett and Judd (1983) search, resulting in mixed-strategy equilibria instead of the pure-strategy equilibria that we find here, welfare results were not available and collusion was not studied.

Our work also contributes to the study of couponing and of obfuscatory pricing. As noted above, Lambrecht and Skiera (2006) empirically document a preference among some consumers for simple tariffs. Spiegel (2006) studies the incentives of firms to obfuscate when competing for a single consumer. Much of the work on obfuscation has focused on firms increasing the costs of search for consumers, rather than the preference some consumers have for simple tariffs (Ellison, 2005; Wilson, 2010; Ellison and Wolitzky, 2012). A notable exception is Carlin (2009) who studies banks that compete by adding complexity to their account terms so that only a proportion of the consumers become informed. In Carlin (2009), uninformed consumers do not see a price and so must choose randomly, while we allow for list prices that have the effect of committing the firms to simple tariffs that uninformed consumers can always choose.

On competition with couponing we add to the marketing literature that has explored how couponing may alter competitive outcomes. Our contribution is to offer a model that allows for the strategic effect of previously set list prices on price reductions. This more involved strategic

⁵Kuo et al. (2011) and Kuo et al. (2012) also consider monopolists, with a focus on, respectively, inventory management and supply chain relationships. As well as setting a posted price, in these papers the monopolist is allowed to commit to a lower bound below which it will never sell.

analysis has, to our knowledge, not yet been explored. This strategic interaction is key to understanding why stochastic discounts raise prices. Shaffer and Zhang (1995) assume that coupon reductions are set simultaneously with list prices, so severing the strategic interaction between list and reduced prices. Narasimhan (1984) notes that coupons can be used to target different prices at different consumer segments, but explores this in a monopoly context. Narasimhan (1988) considers competition, but in a setting where firms can only set one price and so again there is no strategic interaction between list and reduced prices.

To the best of our knowledge, we are the first to study dynamic repeated competition in markets with discounts and list prices, and so the mechanism by which discounts facilitate collusion in markets with public list prices is new. Our result complements the existing literature that shows how collusion can be facilitated when firms operate in more than one market. Bernheim and Whinston (1990) find that competing in multiple markets can make collusion easier since firms are able to transfer excess punishment capacity from one market to another. Spector (2007) shows that if a firm is a monopolist in one market but competes in another, then bundling can help collusion by shrinking the demand available in the competitive market.⁶ In both cases, linkages across markets make collusion more sustainable, while our complementary results show that linkages across segments within a single market can also make collusion easier to sustain.

Our analysis also complements a broader literature in which firms sell to two different types of consumer. In Stahl (1989)'s model of search, consumers have high or low search costs. In Rosenthal (1980), Varian (1980) and Narasimhan (1988), some consumers only consider buying from one firm while others buy at the lowest price. In these papers, the two categories of consumer give rise to mixed-strategy pricing equilibria because of the inability of the firms to price discriminate.⁷ In this paper, the competing firms are able to discriminate between the consumer segments (bargainers and price takers) by setting different list and then discount prices. Compared to these earlier papers, this allows dynamic strategic interaction between the price choices for the different segments since the list prices form the competitive price offer for some of the bargainers. The product differentiation in our model yields pure-strategy pricing equilibria. When we study collusion with perfect substitutes in Section 6, we do find that immediately following a deviation in list prices, and so off the equilibrium path, the firms set discount prices according to a mixed strategy. The mechanism giving rise to mixing for this segment is related to that which gives rise to mixing for the whole market in these earlier papers.

Much of the rest of the literature exploring bargaining in consumer markets examines the

⁶In contrast, Montero and Johnson (2012) identify a setting with multiple markets in which collusion is inhibited by bundling.

⁷In Burdett and Judd (1983)'s model of non-sequential search, consumers vary in how many firms they search, again giving rise to mixed-strategy equilibria.

choice between committing to a fixed price and allowing consumers to bargain in the absence of a posted price (e.g., Bester, 1993, Wang, 1995, Arnold and Lippman, 1998, Camera and Delacroix, 2004, and Myatt and Rasmusen, 2009.) There is also a small literature on bargaining below a posted price when all consumers bargain (e.g., Chen and Rosenthal, 1996a, 1996b, and Camera and Selcuk, 2009).

3 The model of stochastic discounts

Two competing firms sell a differentiated product and compete in prices. The firms have the same constant marginal cost of production $c \geq 0$, have no fixed costs, and seek to maximize their expected profits. To capture product differentiation, we adopt the standard Hotelling framework: the two firms are located at the opposite ends of a Hotelling line of length 1 with a uniform density of consumers along it, and the consumers have a linear Hotelling ‘transport cost’ $t > 0$. As in Hotelling (1929), every consumer purchases exactly one unit and the market is always covered. There are two types of consumer. A proportion $\mu \in (0, 1)$ are ‘price takers’, and the remaining proportion $1 - \mu$ are ‘bargainers’. A consumer’s type is independent of her location on the Hotelling line, and firms cannot observe a consumer’s location. We capture stochastic discounts through the following two-stage game:

1. List-price-setting stage: The firms simultaneously choose publicly posted list prices $l_i \geq 0$ and $l_j \geq 0$. Each price taker purchases at the list price that gives her the highest surplus net of transport costs.
2. Discount stage: The firms simultaneously choose discount prices $p_i \in [0, l_i]$ and $p_j \in [0, l_j]$. Each bargainer receives a particular firm’s discount price offer with probability $\beta \in (0, 1)$; if the bargainer does not receive the price offer, she is still able to purchase at the public list price.⁸ Each bargainer buys at the available price that gives her the highest surplus net of transport costs.⁹

3.1 Three interpretations

We offer three interpretations of this model of stochastic discounts.

3.1.1 Explicit bargaining

Our leading interpretation is that the model captures explicit bargaining in a simple and tractable way. Reflecting real-world bargaining, bargainers actively approach both firms to

⁸The random draws that determine whether discount price offers are received are independent across firms and bargainers.

⁹All consumers randomize in the event of a tie.

ask for a better price than the one posted. The firms simultaneously choose the discount prices to offer to bargainers, which act as final take-it-or-leave-it offers. The strategic choice over the price to offer bargainers captures that firms can adjust their bargaining policy in response to their rival's choice of list price. The assumption that discount price offers are received with probability β captures in a simple way the fact that bargaining is uncertain: the psychological costs and tension of bargaining and the danger of frayed emotions lead to the possibility that negotiations between the sales representative and the consumer break down. However, the list prices are binding and thus are always available.

Under this interpretation, the difference between price takers and bargainers can be motivated by consumers having either high or low personal costs of bargaining.¹⁰ We can think of price takers as consumers who suffer significant bargaining costs: the costs could be real, e.g., time costs, or psychological, e.g., the embarrassment of starting a negotiation; alternatively, the price takers are not aware that discounts might be available. Bargainers, on the other hand, have low costs of bargaining and are aware that firms are willing to negotiate. Given their low costs of bargaining, it is natural to assume that the bargainers approach both firms for a better price offer.

3.1.2 Discount coupons

Our second interpretation is that the model captures settings in which firms offer discount coupons through media sources such as particular websites or newspapers. Discount coupons allow consumers to buy at a fixed discounted price. A proportion of consumers ('bargainers') regularly visit the media source (e.g., visit a particular website or regularly read a particular newspaper), while others ('price takers') do not and so must buy at the list prices as they are unaware of the discount opportunity. The assumption that discount price offers are received with probability β captures the fact that consumers who visit the media source do not always find a firm's coupon (e.g., due to inattention, not visiting the website at the right time, not getting the right issue of the newspaper). Instead of being passive recipients of coupon discounts, we could also think of the bargainers as consumers with a low cost of time who find it worthwhile actively to seek out discount opportunities, while price takers have a high cost of time and so do not pay attention to, or search for, discount coupons.

¹⁰This heterogeneity in bargaining costs parallels the heterogeneity in personal search costs adopted in the search literature. For example, in Stahl (1989) search costs are low or high. Note, however, that under this interpretation bargainers are doing more than searching: they are actively inviting sellers to beat their publicly posted list prices.

3.1.3 Obfuscation and opaque tariffs

Our third interpretation is that the model captures settings in which firms obfuscate pricing using a combination of both simple and complicated tariffs. ‘Price takers’ buy at the simple-to-understand tariffs (the posted prices). These consumers can, for example, be interpreted as the consumers who exhibit a “flat rate bias” as identified in the marketing literature (Lambrecht and Skiera, 2006), perhaps because they have a high cost of time and so are unwilling to attempt to decipher opaque tariffs. Other consumers (the ‘bargainers’) explore the firms’ opaque tariffs to determine whether they offer a discount relative to the simple tariffs. These consumers only engage with and understand (or equivalently, are able to decipher) a particular firm’s opaque tariff with probability β , and choose the best tariff that they understand.¹¹ We can interpret the bargainers as astute consumers who are better able to determine the likely overall cost to them of an opaque tariff (see Financial Times, 2012), or as consumers who have a low cost of time making it worthwhile to attempt to decipher the implications of the complex tariff for the final price.

4 Equilibrium analysis

For any given pair of list prices, we first characterize pure-strategy Nash equilibria of the strategic discounting stage. As discussed above, the prices offered at this stage can be interpreted as the outcome of bargaining, or of strategic price reductions more broadly. We focus on candidate discount prices that yield an interior solution, that is discount prices $\{p_1, p_2\}$ at which the firms sell to a strictly positive share of bargainers who receive both firms’ discount price offers and of bargainers who receive only one discount price offer. We then characterize symmetric pure-strategy Nash equilibria of the list-price-setting stage, under the maintained assumption that interior pure-strategy Nash equilibria are played at the discount stage. The solution concept is subgame-perfect Nash equilibrium.¹² We restrict attention to the case where $t > 0$, i.e., there is some product differentiation. In Section 6, we will consider the case where, instead, the products are undifferentiated.

Consider first the price discounting subgame that ensues following list prices $\{l_1, l_2\}$. Formally, the discount prices yield an interior solution if they satisfy the following condition:

¹¹Our model does not allow firms to set opaque tariffs that are more expensive than their simple tariff. Under our assumption that consumers choose the best tariff that they understand, the firms would never have an incentive to set more expensive opaque tariffs. Consumers might be cautious in the face of obfuscation if they have encountered firms that attempt to fool naive consumers into buying expensive opaque tariffs.

¹²The two list prices are common knowledge; furthermore, in the equilibria we study they are equal. This holds throughout the manuscript. Nonetheless, we need to solve the discount stage given unequal list prices to allow us to consider the effect of deviations at the list-price-setting stage.

Definition 1 *Discount prices yield an interior solution of the discount stage if $p_i \in (p_j - t, p_j + t)$ and $p_i \in (l_j - t, l_j + t)$ for $i \in \{1, 2\}$.*

Given transport costs of t , definition 1 ensures that each firm's list price remains competitive for some bargainers against the rival firm's discount price. Proposition 1 characterizes the equilibrium discount price offers that yield an interior solution.

Proposition 1 *Given any list prices $l_i \geq l_j$, an interior pure-strategy Nash equilibrium of the discount stage must be given by:*

1. $\left\{ p_i^* = \frac{1}{2-\beta} (t + c) + \left(1 - \frac{1}{2-\beta}\right) \left(\frac{2l_j + \beta l_i}{2+\beta}\right), p_j^* = \frac{1}{2-\beta} (t + c) + \left(1 - \frac{1}{2-\beta}\right) \left(\frac{2l_i + \beta l_j}{2+\beta}\right) \right\}$
when $\frac{1}{2-\beta} (t + c) + \left(1 - \frac{1}{2-\beta}\right) \left(\frac{2l_i + \beta l_j}{2+\beta}\right) \leq l_j$; and
2. $\left\{ p_i^* = \min \left\{ \frac{t+c+l_j}{2}, l_i \right\}, p_j^* = l_j \right\}$ when $\frac{1}{2-\beta} (t + c) + \left(1 - \frac{1}{2-\beta}\right) \left(\frac{2l_i + \beta l_j}{2+\beta}\right) > l_j$.

We can see that the discount price offers are increasing in the list prices. That is, list prices and subsequently strategically chosen discount prices are strategic complements. If firm 1, say, raises its list price then firm 1's list price is less of a competitive threat to firm 2 at the discount stage. Specifically, firm 2 will be competing against firm 1's higher list price for all bargainers who fail to receive firm 1's discounted price offer, but receive firm 2's discount price. This softening of the competitive constraint imposed on firm 2 by firm 1 encourages firm 2 to raise its discount price: the proof of Proposition 1 gives the linear reaction function $p_2 = [t + c + (1 - \beta) l_1 + \beta p_1] / 2$ with slope $dp_2/dl_1 = (1 - \beta)/2 > 0$.

This effect applies to more than just bargaining. In the couponing interpretation of our model, a high rival list price implies that customers who fail to pick up a rival's discount coupon can be attracted with a less generous coupon, thus pushing down the reductions that coupons offer. Likewise, in the obfuscatory tariffs interpretation, an expensive simple tariff set by the rival implies that astute customers who explore the non-simple tariffs, but who fail to understand the rival's complex tariff, can be won with a less generous offer.

Given the effect of firm 1's list price on firm 2's discount price, firm 1 also finds it optimal to raise its discount price offer in the equilibrium of the discount stage after an increase in its own list price. The increase in firm 2's discount price softens competition for bargainers who receive both firms' discount price offers, leading firm 1 to also raise its discount price. That is, the discount prices themselves are strategic complements at the discount stage: we can see from the reaction function that $dp_1/dp_2 = \beta/2 > 0$.

This strategic dynamic link between list prices and the subsequent strategic discounting is key to the results we will generate in our analysis. The Corollary to Proposition 1 below describes the equilibrium discount price offers for symmetric list prices.

Corollary 1 (to Proposition 1) *Given symmetric list prices $l_i = l_j = l$, any interior pure-strategy Nash equilibrium of the discount stage must be given by:*

1. $p_i^* = p_j^* = \frac{1}{2-\beta} (t + c) + \left(1 - \frac{1}{2-\beta}\right) l < l$ when $l > t + c$; and
2. $p_i^* = p_j^* = l$ when $l \leq t + c$.

We can see that when the list prices are above the standard Hotelling level of $t + c$, the discount price offers are a weighted average of the standard Hotelling price and the list prices. Thus, when responding to the bargainers, the firms push their discount price offers away from the standard Hotelling level and up towards the prevailing list prices. How far the discount price offers are from the list prices depends upon the probability β that the consumers receive the firms' discount price offers. The higher is this probability, the less relevant the list prices become since there will be fewer bargainers who receive only one of the discount price offers and so compare that discount price to the rival's list price when deciding which product to purchase.

We now turn to the firms' decision about what list prices to set. If the firms considered only the price takers, prices would be set just as in the standard Hotelling model, and so the equilibrium list prices would be $l^* = t + c$. However, when thinking about how to set list prices, the firms also anticipate how the chosen list prices will affect competition at the discount stage. We saw above that list prices and discounted prices are strategic complements: the higher the list prices, the weaker the competitive challenge for the custom of those bargainers who do not receive the stochastic discount from the rival, and so the higher the discount price offers that can be supported in equilibrium. This effect implies that the presence of the bargainers moderates competitive forces in the market by reducing the incentive to undercut any given rival list price. Proposition 2 describes equilibria in which this moderating force allows list prices to rise above the standard Hotelling level.

Proposition 2 *Any symmetric pure-strategy Nash equilibrium of the list-price-setting stage in which $l^* > t + c$ must be given by:*¹³

$$l^* = c + t + t \left[\frac{2(2-\beta)(1-\beta)\beta}{\left(\frac{\mu}{1-\mu}\right)(4-\beta^2)(2-\beta) + 2(1-\beta)(4-2\beta+\beta^2)} \right]. \quad (1)$$

Furthermore, Proposition 3 below shows that no symmetric equilibrium can exist in which list prices are below the standard Hotelling level. At list (and thus discount prices) below the

¹³The list price in (1) is strictly larger than the Hotelling level for any probability of receiving a discount $\beta \in (0, 1)$. Formally, we do not consider the case where $\beta = 1$, that is when discounts are always received. In that case, the strategic complementarity between list prices and discount prices is broken, since there is no subset of bargainers who fail to receive the rival's discount price offer and yet receive the firm's own discount price offer. Thus list prices (and consequently discount prices) remain at the Hotelling level. In the limit as β tends to 1, prices tend to the Hotelling level.

standard Hotelling level, an upward list price deviation towards the Hotelling level would raise profits from both bargainers and price takers.

Proposition 3 *There can be no symmetric pure-strategy Nash equilibrium of the list-price-setting stage in which $l^* < t + c$.*

We have now characterized the unique symmetric equilibrium with list prices in excess of the standard Hotelling level, that is with $l > t + c$, and we have also shown that no equilibrium can exist with list prices below the standard Hotelling level, that is with $l < t + c$. We cannot rule out an equilibrium in which the firms set list prices at the standard Hotelling level, that is with $l = t + c$. In such an equilibrium, by Corollary 1 the discount prices are also at the Hotelling level. However, this equilibrium is not a compelling one to study or to expect for at least two reasons.

First, from the firms' perspective an equilibrium at Hotelling prices is Pareto dominated by the equilibrium with $l > t + c$ (profits in this equilibrium are higher since all prices are higher and the market is covered). Second, the equilibrium is not stable in the following sense: if firm 2, say, were to set a list price of $l_2 = t + c + \varepsilon$ for some arbitrarily small $\varepsilon > 0$, then at this slightly higher list price firm 1 would have an incentive to increase its list price beyond $l_2 = t + c + \varepsilon$, and thus move its list price even further from the Hotelling level. By contrast, the equilibrium offered in Proposition 2 is stable in this sense.¹⁴ This formalizes the idea that there is something knife-edge about prices at the Hotelling level. At any symmetric list prices between the Hotelling level and the equilibrium with $l > t + c$, there is upward pressure on the list prices: an increase in a firm's list price raises the rival's discount price, thus softening competition for the bargainers and increasing that firm's total profits. However, exactly at the Hotelling list prices, the discount prices hit their upper bound, and hence a small upward list price deviation cannot induce a corresponding upward shift in the rival's discount price offer. For both of these reasons it appears to us more defensible to focus on the price-increasing equilibrium. Thus, in the next section, we focus on the more interesting and Pareto-dominant equilibrium with $l > t + c$ in which the bargainers do in fact affect competition between the firms.

This section has characterized the properties of the symmetric pure-strategy Nash equilibrium of our model of list price competition followed by stochastic discounting. Appendix B demonstrates existence of the equilibrium with $l > t + c$. This is not a simple endeavor as our work allows for the fact that firms can respond strategically in the price discounting phase to

¹⁴By construction of the symmetric equilibrium $l^* > t + c$, $[d\pi_i/dl_i]_{l_i=l_j=l^*} = 0$. From (5) in the proof of Proposition 2, setting $l_i = l_j = l > t + c$, $[d\pi_i/dl_i]_{l_i=l_j=l}$ is a linear function of l . An algebraic exercise confirms that the coefficient of l is negative. This implies that for $l \in (t + c, l^*)$ we must have $[d\pi_i/dl_i]_{l_i=l_j=l} > 0$ and for $l > l^*$ we must have $[d\pi_i/dl_i]_{l_i=l_j=l} < 0$.

the rival's list price. Showing existence thus requires us to know what the global Nash equilibrium of the discount stage will be for any list price deviation, in order to determine whether any such list price deviation is profitable. Appendix B proceeds in two steps. First, we show analytically that list price deviations within a certain distance from the equilibrium result in interior global Nash equilibria of the discount stage and are not profitable. In particular, we show that deviations to $l_i \in (\underline{l}, l^*)$ and to $l_i \in (l^*, \bar{l})$ are never profitable, where the bounds are given analytically in (19) and (20). The proof of this step is already rather involved. List price deviations beyond these bounds lead to corner solutions. An analytic proof would require us to find the global corner Nash equilibrium in discount prices for every possible list price deviation, and then we would have to stitch together the different functional forms to calculate profits in the initial list-price-setting stage: this is not practical to do algebraically. Instead, in the second step we do this numerically and show that list price deviations beyond the bounds are never profitable. Appendix B provides the details: for every list price deviation on a fine grid, and for each of nearly 10,000 parameter combinations, we calculate all of the discount stage equilibria on a fine grid of discount prices, and then use this to confirm that there are no profitable list price deviations.

5 Prices, profits and welfare

In this section, we analyze the properties of the equilibrium with list prices above the standard Hotelling level, that is with $l > t + c$, outlined in Proposition 2.¹⁵ Using Part 1 of the Corollary to Proposition 1, the equilibrium discount price offers are given by

$$p = \frac{1}{2 - \beta} (t + c) + \left(1 - \frac{1}{2 - \beta}\right) l, \quad (2)$$

a weighted average of the list prices and the standard Hotelling price.

Proposition 4 *Compared to the benchmark with only price takers, the presence of bargainers: (i) raises the list prices and discount price offers; (ii) raises the firms' profits; (iii) lowers consumer surplus; and (iv) lowers total welfare.*

Proposition 4 is a key result of our study of list price competition with stochastic discounts. Once some of the consumers become 'bargainers', that is they are willing to bargain explicitly, collect coupons or decipher complex tariffs, the list prices rise. Section 4 described in detail how the presence of bargainers moderates competition in the market. We saw there that the list prices form the competitive constraint for the bargainers who receive only one discount price

¹⁵Throughout this section and the related proofs, for notational clarity we omit the stars when referring to equilibrium prices.

offer at the discount stage. Thus, the presence of the bargainers results in higher equilibrium list prices: the incentive to undercut the rival's list price is reduced because the firms anticipate that profits from the bargainers would be pulled down by the strategic retaliation of the rival firm. Furthermore, we explained in Section 4 that the higher list prices allow discount prices to rise since the competition for bargainers is relaxed: that is, the list prices and discount prices act as strategic complements. Since in the presence of bargainers both the discount price offers and the list prices rise above the standard Hotelling level, profits must rise given the market is covered. This delivers results (i), (ii) and (iii) of Proposition 4. The higher prices are a transfer from consumers to firms, and so are welfare neutral using a total surplus criterion.¹⁶ Nonetheless, welfare falls because of the stochastic nature of the discounts. A bargainer who happens to receive a discount price offer from only one firm might be left with the choice between paying a high list price for the product she prefers and a lower discount price for the less-attractive product. This generates some misallocation of products to consumers, which lowers the efficiency of the market, yielding result (iv) of Proposition 4.

So far we have compared a market with a positive fraction of bargainers to the standard case where all consumers buy at the list prices. Next, we turn to a comparative statics analysis of how prices, profits and welfare vary in the proportion of bargainers.

Proposition 5 *As the proportion of bargainers increases: (i) the list prices rise; (ii) the discount price offers rise; and (iii) the difference between the list prices and the discount price offers also rises. Furthermore, all three rise at an increasing rate.*

Proposition 5 considers the effect of a change in the proportion of bargainers on prices. The moderating influence of bargainers on competition means that both list prices and discount price offers go up as we increase the proportion of bargainers in the market. The list prices become increasingly set to allow profits to be made from the bargainers at the discount stage. The proposition further tells us that the list prices go up faster than the discount prices, so the difference between the list prices and the discount prices also increases in the proportion of bargainers: the gap becomes larger since discount prices are a weighted average of the list prices and the standard Hotelling price (see (2) above) where the weights do not depend on the proportion of bargainers (given the list prices, discount prices only target the bargainers). Finally, we can see that the prices go up at an increasing rate: as bargainers become more prevalent in the population of consumers, the reduction in the competitive pressure on list prices becomes increasingly powerful, resulting in a convex increase in the list prices. Since the discount prices are a weighted average of the list prices and the standard Hotelling price, the discount prices also inherit this convexity. The convexity means that as we increase the

¹⁶Many competition authorities focus mainly on consumer surplus instead of on the sum of consumer and producer surplus. This is the case in the UK and the EU for example. For such market regulators an increase in prices as compared to the Hotelling benchmark would be unwelcome *per se*.

number of bargainers, the marginal effect on prices of adding yet more bargainers to the market is compounded.

Proposition 6 *As the proportion of bargainers increases, total welfare falls. Furthermore, total welfare falls at an increasing rate.*

Proposition 6 tells us that welfare is always decreasing in the proportion of bargainers. Since the market is covered, and so prices are just a transfer from consumers to firms, the effect of bargainers on welfare depends only on how the bargainers affect the (mis)allocation of goods to consumers. We just noted (Proposition 5, result (iii)) that the greater the proportion of bargainers, the greater the difference between the list prices and the discount prices. This increasing disparity makes bargainers who receive a discount price offer from only one of the firms more likely to settle for a lower-priced but less-attractive product, thus worsening the misallocation of goods and hence lowering welfare. The proposition also tells us that welfare falls at an increasing rate: this concavity of welfare in the proportion of bargainers follows from the convexity of the difference between list prices and discount prices reported in Proposition 5. Summarizing, increasing the number of bargainers raises prices, lowers welfare, and exacerbates the marginal effect on prices and welfare of adding yet more bargainers.

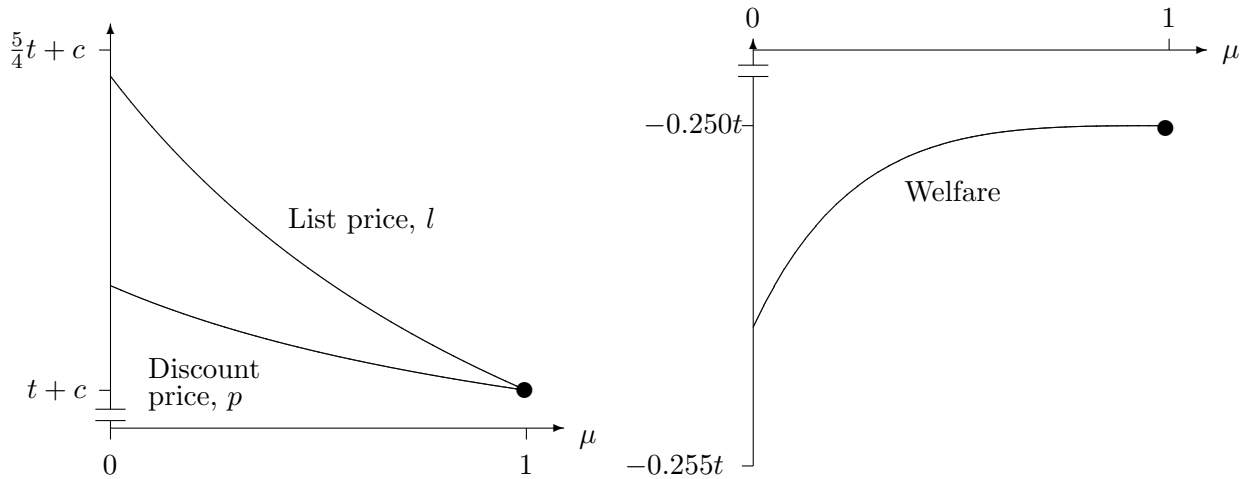


Figure 1: Prices and Welfare as Proportion of Bargainers Changes

Notes: The proportion of bargainers increases from 0 to 1 as the proportion of price takers μ falls from 1 to 0. The dot in each graph gives the benchmark case of Hotelling competition with only price takers. Both graphs are drawn to scale, with $\beta = 1/2$. The welfare graph normalizes surplus from a product at a consumer's exact location on the Hotelling line to zero.

Figure 1 shows graphically how prices and welfare change in the proportion $1 - \mu$ of bargainers. These results are immediately interpretable in the three different settings we described in the model development (Section 3). If the proportion of consumers who bargain explicitly with

sellers should rise, then the list prices and bargained prices will rise. Likewise, if the proportion of consumers who search for and use coupons rises, then the competitive pressure on prices is relaxed. Some consumers use the coupons to buy products that are not, *ceteris paribus*, their preferred varieties, leading to welfare losses that increase in the proportion of consumers that search for coupons. Finally, if the proportion of consumers who seek to understand and engage with complex tariffs rises, then the simple-to-understand tariffs purchased by the remaining less astute consumers will become more expensive, as will the complex tariffs.

Finally, we consider how profits and consumer surplus change in the proportion of bargainers. We have just seen that total welfare always decreases as the number of bargainers goes up. The effect on profits and consumer surplus is not so clear cut. Proposition 5 shows that increasing the number of bargainers raises both discount prices and list prices. Clearly, this hurts consumers who remain price takers or were already bargainers. On the other hand, the consumers who become bargainers switch from a high list price to a lower discount price and so pay less. It is possible to construct examples in which profits and consumer surplus are not monotonic in the proportion of bargainers. However, Proposition 7 shows that when the probability β that the consumers receive the firms' discount price offers is not too large, profits always go up and consumer surplus always declines in the proportion of bargainers.¹⁷ In that case, the price reduction secured by new bargainers is always outweighed by the increase in prices for the other consumers.

Proposition 7 *When the stochastic discount technology is such that $\beta \in (0, 0.62]$, the firms' profits rise and consumer surplus falls as the proportion of bargainers increases.*

6 Collusion

Thus far we have analyzed one-shot competition and demonstrated that the presence of bargainers reduces the competitive pressure on firms when setting their list prices, thus allowing list prices, and subsequently discount prices, to rise. In this section we study dynamic competition in markets with both bargainers and price takers, and we will conclude that the presence of bargainers can facilitate collusion by lowering the critical discount factor above which collusion can be sustained.

6.1 Dynamic model of stochastic discounts

We approach the question of collusion in the standard way of seeking a symmetric subgame-perfect Nash equilibrium in which the firms collude on a common price $z > c$. As our analysis

¹⁷The proof shows only that the range of β is sufficient for profits to increase and consumer surplus to fall in the proportion of bargainers. Numerical analysis suggests that the result extends to a wider range of β .

allows for both list price setting and discounts, we investigate collusion in which the firms collude on the same price z at both the list-price-setting stage and the discount stage.¹⁸ We extend our analysis by allowing any number of firms $N \geq 2$ to compete in the market. At the same time, we simplify the analysis by only considering the case $t = 0$, perfect substitutes. As is common in collusion analyses, we focus on collusive equilibria supported by the threat of reversion to the lowest-payoff non-collusive symmetric equilibrium. Formally, the two-stage one-shot game of Section 3 is assumed to be infinitely repeated, with all firms sharing a discount factor δ .

6.2 One-shot game analysis

Our first task is to establish equilibrium behavior when the two-stage game is played only once. Just as in Section 4, we first consider equilibrium behavior in the discount stage for any list prices, and then use this to work out the equilibrium at the list-price-setting stage.

Lemma 1 demonstrates that the equilibrium of the discount stage involves the firms setting discount prices according to a mixed strategy.¹⁹

Lemma 1 *Suppose that the list prices are given by $\{l_j : j \in \{1, 2, \dots, N\}\}$ and suppose that $n \geq 1$ firms, including firm i , set the lowest list price $\underline{l} \equiv \min \{l_j\} > c$. Then there is a unique symmetric Nash equilibrium of the discount stage, in which all firms offer prices*

$$p \in ((1 - \beta)^{N-1}(\underline{l} - c) + c, \underline{l})$$

drawn from the distribution

$$F(p) = \frac{1}{\beta} - \left(\frac{1 - \beta}{\beta} \right) \left(\frac{\underline{l} - c}{p - c} \right)^{\frac{1}{N-1}}. \quad (3)$$

Firm i makes expected profits from the bargainers of:

$$\pi_i = (l_i - c) (1 - \beta)^{N-1} \left(\frac{1 - \beta}{n} + \beta \right).$$

This recourse to mixed strategies is a consequence of the assumption of perfect substitutes: if a firm sets a discount price of $p_i > c$ for sure, then a rival could gain the business of all the bargainers who receive both discount price offers (and no lower offers from other firms) at essentially zero cost by just undercutting p_i . In the mixed-strategy equilibrium, a firm trades off the incentive to price high to profit from bargainers who receive few discount price offers against the incentive to price low to increase the probability of selling to bargainers who receive

¹⁸If we introduce a common maximum willingness to pay $v > c$, then our analysis applies for any collusive price $z \in (c, v]$. When $z = v$, the firms collude on the monopoly price.

¹⁹It makes no difference to the analysis whether (i) for every bargainer a firm draws a price from the pricing distribution or (ii) a firm draws a single price from the pricing distribution, which it then offers to all bargainers.

many price offers.

Lemma 2 studies the symmetric equilibrium of the two-stage one-shot game.

Lemma 2 *The unique symmetric pure-strategy Nash Equilibrium of the one-shot game has all prices at marginal cost and profits are zero.*

Since the products are perfect substitutes, in the standard way a firm which undercuts its rivals' list price by an arbitrarily small amount secures the business of all the price takers. In addition, Lemma 1 shows that undercutting the rivals' list price also increases profits from the bargainers: the reason is that some bargainers will fail to receive discount price offers from any firm, and these consumers will all buy from the firm with the lower list price. This remorseless Bertrand logic pushes list prices down to marginal cost, and so the discount prices are forced down to cost also.

6.3 Dynamic game analysis

We are now in a position to consider collusion in the dynamic game.²⁰ Proposition 8 shows that bargainers facilitate collusion.

Proposition 8 *Consider a market with $N \geq 2$ firms. When goods are perfect substitutes, the presence of bargainers facilitates collusion compared to the benchmark with only price takers: the critical discount factor that allows collusion to be subgame perfect is strictly lower with bargainers.*

The presence of the bargainers means that there are two ways for firms to deviate on a collusive equilibrium. Despite this, the presence of the bargainers reduces the profits available from deviation, thus making collusion easier to sustain. First, a colluding firm could deviate at the list-price-setting stage. In the period in which the deviation occurred, rival firms, observing this deviation, would be in a position to respond by setting discount prices more aggressively (Lemma 1 gives the rivals' discount price distribution immediately after the deviation). Alternatively, a colluding firm could maintain the collusive list prices, but deviate at the discount stage. In that case the deviant firm would be able to sell to only $1/N$ of the price takers. With or without bargainers, the continuation payoff in the periods after the deviation is zero (Lemma 2 proves this for the case with bargainers; the case without is standard). Which of the two possible deviations is optimal depends upon the parameters; in both cases, however, the deviant firm will be unable to secure the entirety of the market.

²⁰If the game were only finitely repeated, then since the stage game has a unique symmetric pure-strategy equilibrium (Lemma 2), standard backward induction would guarantee that no symmetric pure-strategy collusive equilibrium could be sustained.

7 Conclusion

Discounts off public list prices are commonplace. In this paper we have developed and analyzed a model of dynamic price competition between firms when some consumers buy at discount prices while others buy at list prices. We extend the literature by studying strategically chosen stochastic discounts in markets with prior list-price-setting competition. We document the effect of competition in posted prices with stochastic discounts on discount and list prices, profits and welfare.

We demonstrate that the effect of having a greater proportion of consumers in the population who receive discounts with some probability is to raise list prices. The main driver behind our results is the dynamic link between the list prices and subsequent discount prices. Discounts respond to list price deviations – and the list price will be the competing offer for those consumers who do not receive the rival’s discount price offer, due to failed bargaining, inattention to coupons, or failure to engage with and understand the rival’s complex tariff. The dynamic nature of the interaction causes the discount prices to become strategic complements to the list prices.

The insights of this model apply in a wide variety of settings. In markets, such as the automobile market, where some consumers buy at list prices and others may receive bargained discounts, the predictions are immediate: the greater the proportion of the population who seek to bargain, the higher list prices will be. Similarly, in the case of couponing: if a large proportion of consumers are targeted by coupons then the competitive pressure on list prices will be reduced and these list prices will rise. This results in a welfare loss since misallocation of consumers to firms results. In the case of complex tariffs offered alongside simple tariffs, the larger the proportion of consumers who engage with complex tariffs and seek to understand them, the more expensive the alternative simple tariff (chosen by the less astute or those with a ‘flat rate bias’) becomes.

Appendix

Appendix A: Proofs

Proof of Proposition 1. At the discount stage, p_i only affects profits in the case when both discount price offers are received and the case where only the firm's offer is received. Thus, relevant profits in the interior are given by

$$(\text{constant}) + \beta^2 (p_i - c) \left(\frac{t + p_j - p_i}{2t} \right) + \beta (1 - \beta) (p_i - c) \left(\frac{t + l_j - p_i}{2t} \right),$$

and are strictly concave in p_i . The first-order condition,

$$\beta^2 [(p_i - c)(-1) + (t + p_j - p_i)] + \beta (1 - \beta) [(p_i - c)(-1) + (t + l_j - p_i)] = 0,$$

gives the linear reaction function $p_i = [t + c + (1 - \beta) l_j + \beta p_j] / 2$. Solving simultaneously for p_i and p_j gives

$$\hat{p}_i = \frac{1}{2 - \beta} (t + c) + \left(1 - \frac{1}{2 - \beta} \right) \left(\frac{2l_j + \beta l_i}{2 + \beta} \right) \text{ and } \hat{p}_j = \frac{1}{2 - \beta} (t + c) + \left(1 - \frac{1}{2 - \beta} \right) \left(\frac{2l_i + \beta l_j}{2 + \beta} \right).$$

Note that $\hat{p}_i \leq \hat{p}_j$, given $l_i \geq l_j$ and $\beta < 1$. Thus, when $\hat{p}_j \leq l_j$, $p_i^* = \hat{p}_i$ and $p_j^* = \hat{p}_j$. When instead $\hat{p}_j > l_j$, the fact the reaction functions intersect only once implies that at least one firm must be constrained by its list price. We cannot have an interior equilibrium with $p_i^* = l_i$ and $p_j^* < l_j$ since then $p_i^* > p_j^* = [t + c + (1 - \beta) l_i + \beta p_i^*] / 2 > [t + c + (1 - \beta) l_j + \beta p_j^*] / 2$, so firm i would want to deviate downward given concavity of profits in the interior. Instead, any interior equilibrium must be given by $p_j^* = l_j$ and

$$p_i^* = \min \left\{ \frac{t + c + (1 - \beta) l_j + \beta l_j}{2}, l_i \right\} = \min \left\{ \frac{t + c + l_j}{2}, l_i \right\}.$$

Note that firm i will hit its constraint when $(t + c + l_j) / 2 > l_i$ given concavity of profits in the interior. ■

Proof of Proposition 2. We use the maintained assumption that interior pure-strategy Nash equilibria are played at the discount stage. If the interior pure-strategy Nash Equilibrium of the discount stage given in Proposition 1 exists at list prices $\{l_i, l_j\}$, then firm i 's profits are given by

$$\begin{aligned}
2t \cdot \pi_i(l_i, p_i^*(l_i), p_j^*(l_i)) &= \left[\mu + (1 - \mu)(1 - \beta)^2 \right] (l_i - c)(t + l_j - l_i) \\
&+ (1 - \mu)\beta(1 - \beta)(p_i^* - c)(t + l_j - p_i^*) \\
&+ (1 - \mu)(1 - \beta)\beta(l_i - c)(t + p_j^* - l_i) \\
&+ (1 - \mu)\beta^2(p_i^* - c)(t + p_j^* - p_i^*). \tag{4}
\end{aligned}$$

The first line gives firm i 's profits from the proportion μ of price takers, and from the bargainers who receive neither discount price offer; firm i 's market share is given by $(t + l_j - l_i)/2t$. The second (third) line gives profit from the bargainers who receive only firm i 's (firm j 's) offer. The final line gives profits from the bargainers who receive both offers. The total derivative of profits with respect to the list price l_i is thus given by

$$\begin{aligned}
2t \cdot \frac{d\pi_i}{dl_i} &= \left[\mu + (1 - \mu)(1 - \beta)^2 \right] (t + l_j - 2l_i + c) \\
&+ (1 - \mu)\beta(1 - \beta)(t + l_j - 2p_i^* + c) \frac{dp_i^*}{dl_i} \\
&+ (1 - \mu)(1 - \beta)\beta \left[(t + p_j^* - 2l_i + c) + (l_i - c) \frac{dp_j^*}{dl_i} \right] \\
&+ (1 - \mu)\beta^2 \left[(t + p_j^* - 2p_i^* + c) \frac{dp_i^*}{dl_i} + (p_i^* - c) \frac{dp_j^*}{dl_i} \right]. \tag{5}
\end{aligned}$$

Using Proposition 1 and its Corollary, and the assumption that $l > t + c$, at a symmetric list price equilibrium: (i) $l_i = l_j = l$; (ii) $p_i^* = p_j^* = \frac{1}{2-\beta}(t + c) + \left(1 - \frac{1}{2-\beta}\right)l$; and (iii)

$$\frac{dp_i^*}{dl_i} = \frac{\beta(1 - \beta)}{4 - \beta^2} \text{ and } \frac{dp_j^*}{dl_i} = \frac{2(1 - \beta)}{4 - \beta^2}. \tag{6}$$

Note also that at a symmetric list price equilibrium with $l > t + c$, the profit function is strictly concave in l_i .²¹ The result follows by substituting (6) into the first-order condition given by setting (5) = 0, and then solving for l . ■

Proof of Proposition 3. We use the maintained assumption that interior pure-strategy equilibria are played at the discount stage. Suppose a symmetric equilibrium exists with $l < t + c$. If the interior pure-strategy Nash Equilibrium of the discount stage given in Proposition 1 exists, we know from Proposition 1 and its Corollary that $p_i^* = p_j^* = l$ and that, if firm i raises its list price to $l_i = l + \varepsilon$ for some small $\varepsilon > 0$, then p_j^* remains unchanged at l while p_i^* rises to

²¹Some tedious algebra reduces the second derivative to $2t(4 - \beta^2)^2 \frac{d^2\pi_i}{d(l_i)^2} = -2\mu(4 - \beta^2)^2 - 2(1 - \mu)(1 - \beta)[(4 - \beta^2)(4 - 2\beta + \beta^2) - \beta^3(1 - \beta)]$. Thus $\frac{d^2\pi_i}{d(l_i)^2} < 0$ since $4 - \beta^2 > \beta^3$ and $4 - 2\beta + \beta^2 > 1 - \beta$ given $\beta \in (0, 1)$.

$l_i = l + \varepsilon$. Since $l < t + c$, that is below the standard Hotelling equilibrium level, a deviation by firm i to $l + \varepsilon$ therefore strictly increases expected profits, yielding the desired contradiction. ■

Proof of Proposition 4. From (1) and (2) and given $\beta \in (0, 1)$, when $\mu < 1$ the list prices and discount price offers rise above the standard Hotelling price in the absence of bargainers of $t + c$. Since the market is covered, every consumer buys at a price above $t + c$, and the equilibrium is symmetric, each firm's profits must exceed the standard Hotelling level of $t/2$. Since the market is covered, total welfare falls linearly in transport costs. Transport costs are higher than the standard Hotelling level, since the indifferent bargainer who receives only one discount price offer lies away from $1/2$ on the Hotelling line. Finally, since total welfare is lower and profits are higher, consumer surplus must be lower. ■

Proof of Proposition 5. The proportion of bargainers rises as the proportion μ of price takers falls. Let

$$g \equiv \mu (4 - \beta^2) (2 - \beta) + 2 (1 - \mu) (1 - \beta) (4 - 2\beta + \beta^2).$$

Given $\mu \in (0, 1)$ and $\beta \in (0, 1)$, $g > 0$. Using (1),

$$\frac{dl}{d\mu} = -2\beta (1 - \beta) (2 - \beta)^2 (4 - \beta^2) (tg^{-2}) < 0; \quad (7)$$

$$\frac{d^2l}{d\mu^2} = 4\beta^2 (1 - \beta) (2 - \beta)^2 (4 - \beta^2) [3\beta^2 + 8(1 - \beta)] (tg^{-3}) > 0. \quad (8)$$

(7) gives (i), while the convexity of list prices shown in (8) implies that the list prices rise at an increasing rate. Using (2), (7) and (8),

$$\frac{dp}{d\mu} = \left(\frac{1 - \beta}{2 - \beta} \right) \frac{dl}{d\mu} < 0 \quad \text{and} \quad \frac{d^2p}{d\mu^2} = \left(\frac{1 - \beta}{2 - \beta} \right) \frac{d^2l}{d\mu^2} > 0; \quad (9)$$

hence (ii) holds and the discount price offers rise at an increasing rate. Finally, using (1), (2), (7) and (8),

$$\frac{d(l - p)}{d\mu} = \frac{d}{d\mu} \left(\frac{1}{2 - \beta} [l - (t + c)] \right) = \left(\frac{1}{2 - \beta} \right) \frac{dl}{d\mu} < 0; \quad (10)$$

$$\frac{d^2(l - p)}{d\mu^2} = \left(\frac{1}{2 - \beta} \right) \frac{d^2l}{d\mu^2} > 0. \quad (11)$$

Therefore (iii) holds and the difference rises at an increasing rate. ■

Proof of Proposition 6. Since the market is covered, welfare falls linearly in transport costs.

Denoting transport costs by T ,

$$T = \left\{ \mu + (1 - \mu) \left[(1 - \beta)^2 + \beta^2 \right] \right\} \left(\frac{t}{4} \right) + (1 - \mu) \beta (1 - \beta) 2 \left(\int_0^{\frac{1}{2} + \frac{l-p}{2t}} t x dx + \int_{\frac{1}{2} + \frac{l-p}{2t}}^1 t (1 - x) dx \right).$$

The first line captures the average transport cost of those who choose between two equal prices.

The second line captures the transport cost of the bargainers who receive only one discount price offer. Differentiating and simplifying yields:

$$\begin{aligned} \frac{dT}{d\mu} &= \frac{\beta(1-\beta)}{t} \left[-\frac{(l-p)^2}{2} + (1-\mu)(l-p) \frac{d(l-p)}{d\mu} \right]; \\ \frac{d^2T}{d\mu^2} &= \frac{\beta(1-\beta)}{t} \left\{ \left[-2(l-p) + (1-\mu) \frac{d(l-p)}{d\mu} \right] \frac{d(l-p)}{d\mu} + (1-\mu)(l-p) \frac{d^2(l-p)}{d\mu^2} \right\}. \end{aligned}$$

Using (10), $dT/d\mu < 0$, and so total welfare falls in the proportion of bargainers. Using (10) and (11), $d^2T/d\mu^2 > 0$, and so total welfare falls at an increasing rate. ■

Proof of Proposition 7. Using (4) to give a firm's profit π and differentiating:

$$\begin{aligned} \pi &= \frac{1}{2} [\beta\mu + (1-\beta)] (l-c) + \frac{1}{2} (1-\mu) \beta (p-c) - \frac{1}{2} (1-\mu) (1-\beta) \beta t^{-1} (l-p)^2; \text{ and} \\ \frac{d\pi}{d\mu} &= \frac{1}{2} \beta (l-c) + \frac{1}{2} [\beta\mu + (1-\beta)] \frac{dl}{d\mu} - \frac{1}{2} \beta (p-c) + \frac{1}{2} (1-\mu) \beta \frac{dp}{d\mu} \\ &\quad + \frac{1}{2} (1-\beta) \beta t^{-1} (l-p)^2 - (1-\mu) (1-\beta) \beta t^{-1} (l-p) \frac{d(l-p)}{d\mu}. \end{aligned}$$

Using (1), (2), (7), (9) and (10), and after some manipulation,

$$\frac{\frac{d\pi}{d\mu}}{\frac{dl}{d\mu}} = \frac{1}{2} + \left(\frac{1-\mu}{2-\beta} \right) \frac{\beta}{2} \left\{ 1 - \underbrace{\left[\mu + (1-\mu) \frac{(2-2\beta)(4-2\beta+\beta^2)}{(2-\beta)(4-\beta^2)} + 2 \right]}_{(i)} \left[1 + (1-\beta) \left(\frac{l-p}{t} \right) \right] \right\}.$$

Since $2-\beta > 2-2\beta$ and $4-\beta^2 > 4-2\beta+\beta^2$, (i) < 3 . Furthermore, part 1 of Corollary 1 implies that $l-p = (l-t-c)/(2-\beta)$, hence Proposition 2 yields that $(l-p)/t < \beta(4-2\beta+\beta^2)^{-1}$.

Hence we can determine a bound:

$$\begin{aligned} \frac{\frac{d\pi}{d\mu}}{\frac{dl}{d\mu}} &> \frac{1}{2} + \left(\frac{1-\mu}{2-\beta} \right) \frac{\beta}{2} \left[-2 - 3 \frac{(1-\beta)\beta}{(4-2\beta+\beta^2)} \right] \\ &= \frac{8-16\beta+5\beta^2+\mu\beta(8-\beta-\beta^2)}{2(2-\beta)(4-2\beta+\beta^2)}. \end{aligned} \tag{12}$$

Now (12) > 0 for any $\mu > 0$ if $8 - 16\beta + 5\beta^2 > 0$, which in turn requires $\beta < (8 - 2\sqrt{6})/5 \approx 0.6202$. From (7) $dl/d\mu < 0$, and so for $\beta \in (0, (8 - 2\sqrt{6})/5]$ we have $d\pi/d\mu < 0$, and hence profits increase in the proportion of bargainers. From Proposition 6, total welfare always falls in the proportion of bargainers, and hence consumer surplus falls when profits increase. ■

Proof of Lemma 1. We start by showing that any symmetric equilibrium must be mixed with: (i) $F(\underline{l}) = 1$; (ii) no mass points in the density function; and (iii) $F(p) < 1$ for $p < \underline{l}$.

(i) Recall from Section 3 that $p_i \leq l_i$, and by assumption firm i is one of the $n \geq 1$ firms setting the lowest list price \underline{l} , so $p_i \leq \underline{l}$. Thus, in a symmetric equilibrium $F(\underline{l}) = 1$.

(ii) If there were a mass point at price $p > c$, a firm could deviate profitably by lowering its discount price to $p - \varepsilon$ just below the mass point whenever it would have offered p . This increases sales by a discrete amount (when the bargainer also receives an offer of p from a rival firm and receives no lower offers) in return for a vanishingly small loss and so is a profitable deviation. If there were a mass point at $p = c$, a firm would deviate upward to sell at a strictly positive profit to bargainers who receive its offer and not any of the rivals’.

(iii) Suppose that the support of F is bounded above at $p < \underline{l}$. From (ii), the probability that any of the rival firms offers this highest price to the bargainers is zero. Thus any firm offering the highest discount price could deviate profitably by raising price towards \underline{l} since the firm will continue to sell at the offered price if and only if the bargainer receives its offer and not any of the rivals’.²²

Now suppose that firm i offers a discount price $p < \underline{l}$, and the rival firms draw their discount prices from the same distribution F . If a bargainer does not receive firm i ’s offer, then the firm sells at l_i with probability $1/n$ when the bargainer also fails to receive any of the other firms’ offers. If, instead, a bargainer does receive the offer, then the firm sells at p when p is below any other offers received by the bargainer. The probability that a bargainer receives k of the $N - 1$ rival firms’ offers is

$$\beta^k (1 - \beta)^{N-1-k} \binom{N-1}{k}$$

where the binomial coefficient counts the number of (unordered) combinations of k rivals that can be constructed out of a set of $N - 1$. Combining, we can write firm i ’s expected profit from

²²Even if the density is zero at the highest price in the support of the distribution, by continuity profit at this price must be the same as for prices in the interior of the mixing distribution.

the bargainers at any offered price $p < l_i$ as

$$\pi_i(p) = (l_i - c)(1 - \beta)^N \frac{1}{n} + (p - c)\beta \sum_{k=0}^{N-1} \beta^k (1 - \beta)^{N-1-k} \binom{N-1}{k} (1 - F(p))^k.$$

Using the Binomial Theorem (e.g., Kreyszig, 1993, p. 1165), we have

$$\begin{aligned} \pi_i(p) &= (l_i - c)(1 - \beta)^N \frac{1}{n} + (p - c)\beta [1 - \beta + \beta(1 - F(p))]^{N-1} \\ &= (l_i - c)(1 - \beta)^N \frac{1}{n} + (p - c)\beta (1 - \beta F(p))^{N-1}. \end{aligned} \quad (13)$$

For firm i to be willing to randomize, its profit must be constant at all points in the support of F . To find π_i , we consider firm i 's profit from setting a price which tends to the upper bound of the support of F , that is $l_i = \underline{l}$:

$$\begin{aligned} \pi_i &= \lim_{p \rightarrow \underline{l}} \pi_i(p) = (l_i - c)(1 - \beta)^N \frac{1}{n} + (l_i - c)\beta (1 - \beta)^{N-1} \\ &= (l_i - c)(1 - \beta)^{N-1} \left(\frac{1 - \beta}{n} + \beta \right). \end{aligned} \quad (14)$$

Equating (13) and (14) yields

$$\left(\frac{l_i - c}{p - c} \right) (1 - \beta)^{N-1} = (1 - \beta F(p))^{N-1}.$$

This can be solved to yield (3). The lower bound of the support \underline{p} can then be determined by setting $F(\underline{p}) = 0$.

Clearly, firm i has no incentive to deviate downward from \underline{p} : from (ii) there can't be a mass point at \underline{p} , so the firm would continue to sell to the same proportion of bargainers. It is readily confirmed that any firm j whose list price is above \underline{l} has the same pricing distribution since the profit for such a firm j is the same as for firm i , except that firm j makes no profit when its offer is not received. We also have to check that such a firm j has no incentive to deviate to $p_j \in [\underline{l}, l_j]$: at $p_j = \underline{l}$ the firm would sell to only $1/(n+1)$ of the bargainers who receive only its offer; at $p_j > \underline{l}$ the firm would fail to sell to any of the bargainers when its offer is received. ■

Proof of Lemma 2. Suppose first that there is a symmetric equilibrium with list prices $l^* > c$. A given firm i could deviate profitably by lowering its list price to $l^* - \varepsilon$. The firm would then sell to all the price takers. From Lemma 1, profits from bargainers would also rise, since there would then be $n = 1$ firms with the lowest list price as opposed to $n = N$. Symmetric list prices $l^* = c$ form an equilibrium, since profits from price takers and bargainers are zero at $l_i \geq \{l_j^* : j \neq i\} = c$. ■

Proof of Proposition 8. Recall that we are looking for subgame-perfect Nash equilibria in which the firms collude on a price z at the list-price-setting stage and the discount stage, supported by the threat of reversion to the lowest-payoff non-collusive symmetric equilibrium. In the benchmark case with only price takers, it is well-known that such collusion can be sustained by the threat of reversion to the zero-profit one-shot equilibrium when the discount factor $\delta \geq 1 - 1/N$.

Lemma 1 with $n = 1$ gives profits in the unique non-collusive symmetric equilibrium in the discount stage during a period in which a deviation from $z > c$ occurred at the list-price-setting stage. By Lemma 2, the lowest-payoff non-collusive symmetric equilibrium of the one-shot game must give profits of zero in the periods after a deviation occurred. Deviating at the list-price-setting stage to $l_i = z - \varepsilon$ wins all the price takers. Using Lemmas 1 and 2, total profit from this deviation is given by

$$\pi^{\text{dev1}} \simeq \mu(z - c) + (1 - \mu)(z - c)(1 - \beta)^{N-1} = (z - c) \left[\mu + (1 - \mu)(1 - \beta)^{N-1} \right].$$

An alternative deviation would be to deviate at the discount stage to $p_i = z - \varepsilon$, instead of deviating at the list-price-setting stage. The deviant firm would capture all the bargainers whenever its price offer was received and $1/N$ of the bargainers otherwise. Thus, total profit from this deviation (including profit at the list price-setting-stage preceding the deviation) is given by

$$\pi^{\text{dev2}} \simeq \mu \left(\frac{z - c}{N} \right) + (1 - \mu) \left[\beta(z - c) + (1 - \beta) \left(\frac{z - c}{N} \right) \right] = \left(\frac{z - c}{N} \right) [1 + (1 - \mu)\beta(N - 1)].$$

Hence the collusion can be sustained at all discount factors $\delta \geq \delta^\dagger$, where

$$\begin{aligned} \frac{z - c}{N(1 - \delta^\dagger)} &= \left(\frac{z - c}{N} \right) \max \left\{ N \left[\mu + (1 - \mu)(1 - \beta)^{N-1} \right], 1 + (1 - \mu)\beta(N - 1) \right\}, \text{ i.e.,} \\ \delta^\dagger &= 1 - \frac{1}{\max \left\{ N \left[\mu + (1 - \mu)(1 - \beta)^{N-1} \right], 1 + (1 - \mu)\beta(N - 1) \right\}}. \end{aligned}$$

Clearly, $\delta^\dagger < 1 - 1/N$, since

$$\max \left\{ N \left[\mu + (1 - \mu)(1 - \beta)^{N-1} \right], 1 + (1 - \mu)\beta(N - 1) \right\} < N$$

given $\mu \in (0, 1)$ and $\beta \in (0, 1)$. ■

Appendix B: Existence

In this appendix, we demonstrate existence of the candidate symmetric list price equilibrium l^* given in Proposition 2. We do this in two steps. In Step 1, we show analytically that deviations to $l_i \in (\underline{l}, l^*)$ and to $l_i \in (l^*, \bar{l})$ are never profitable, where the bounds are given in (19) and (20). In Step 2, we show numerically that list price deviations beyond these bounds are never profitable. The final paragraph of Section 4 discusses why we use numerical analysis for large list price deviations.

Step 1: Analytical

We proceed in two sub-steps. First, Lemma 3 shows that if, after one firm deviates from the list prices l^* given in Proposition 2, the discount prices given in Proposition 1 do indeed yield an interior solution, then these discount prices form a global Nash equilibrium at the discount stage. Second, Proposition 9 below shows that such a global Nash equilibrium at the discount stage exists for list price deviations within the bounds given in (19) and (20), and that (assuming this global Nash is played at the discount stage) there is no profitable list price deviation within these bounds.

Proposition 2 identifies a candidate symmetric list price equilibrium l^* given explicitly in (1). By inspection we have that for all $\mu \in (0, 1)$:

$$c + t < l^* < c + t \left[1 + \frac{(2 - \beta)\beta}{(4 - 2\beta + \beta^2)} \right] < c + 2t \text{ for all } \beta \in (0, 1). \quad (15)$$

We explore deviations from this candidate symmetric equilibrium. Consider a price discounting subgame in which firm 1 has deviated in its list price and set a list price of l_1 , while firm 2 has not deviated and so has list price l^* . The firms set discounted prices $\{p_1, p_2\}$. Proposition 1 yields the functional form of any interior pure-strategy Nash equilibrium prices such that $p_i < l_i$ as:

$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \frac{1}{2 - \beta} (t + c) + \frac{(1 - \beta)}{(2 - \beta)(2 + \beta)} \begin{pmatrix} 2l^* + \beta l_1 \\ \beta l^* + 2l_1 \end{pmatrix}. \quad (16)$$

Subtracting one equation from the other implies:

$$p_1 - p_2 = \frac{1 - \beta}{2 + \beta} (l^* - l_1). \quad (17)$$

Lemma 3 *Consider the discount stage after firm 2 has set list price $l_2 = l^*$. Whatever the list price l_1 of firm 1, if discount prices (16) yield an interior solution (see Definition 1), and if $p_i < l_i$ for $i \in \{1, 2\}$, then these discount prices form a global Nash equilibrium of the discount stage.*

Proof. Profits for firm 1 (those for firm 2 are analogous) from the price discounting subgame are given by

$$\begin{aligned} & \pi_1^{\text{barg}}(p_1, p_2) \\ = & (\text{constant}) \\ & + \begin{cases} 0 & p_1 > l^* + t \\ \beta(1-\beta) \frac{1}{2t} (p_1 - c)(t + l^* - p_1) & p_2 + t < p_1 \leq l^* + t \quad (i) \\ \beta^2 \frac{1}{2t} (p_1 - c)(t + p_2 - p_1) + \beta(1-\beta) \frac{1}{2t} (p_1 - c)(t + l^* - p_1) & l^* - t < p_1 \leq p_2 + t \quad (ii) \\ \beta^2 \frac{1}{2t} (p_1 - c)(t + p_2 - p_1) + \beta(1-\beta)(p_1 - c) & p_2 - t \leq p_1 \leq l^* - t \quad (iii) \\ \beta(p_1 - c) & p_1 < p_2 - t \quad (iv) \end{cases} \end{aligned} \quad (18)$$

The (constant) term is independent of the discounted prices $\{p_1, p_2\}$. Prices are interior if they lie strictly inside region (ii). Simple algebra confirms that if $l_1 = l^*$ then the prices (16) are interior, and Corollary 1 yields that $p_i < l^*$.

Consider discounted prices (16) that are interior and such that $p_i < l_i$. By the proof of Proposition 1 these prices are local maxima of the profit function in row (ii) of (18). Given p_2 on region (ii), π_1^{barg} is a negative quadratic in p_1 and so concave. Thus there is no profitable deviation for firm 1 within region (ii). Next consider region (iii) and note that the profit function on region (iii) is smaller than the continuation of the profit function from region (ii) onto the range $p_2 - t \leq p_1 \leq l^* - t$. But the profit function of region (ii) is concave and maximized within region (ii) so there can exist no profitable deviation to region (iii). Next note that the profit available by deviating to the bottom edge of region (iii), that is to $p_1 = p_2 - t$, dominates the profit available from deviating to region (iv). Hence there is no profitable deviation to region (iv) either.

Now we show there is no profitable deviation for firm 1 to region (i). The profit function in region (i) is concave with maximum at a deviation of $p_1 = (t + l^*)/2$. If this maximum point lies below the allowable range, that is if $(t + l^*)/2 < p_2 + t$, then remaining in region (ii) dominates a deviation to region (i). By the concavity of the profit function in (ii) and the fact that we have a strict interior local optimum, the candidate equilibrium p_1 must then yield strictly higher profit for firm 1. From (16), using that $l_1 \geq 0$ we have $p_2 \geq \frac{1}{2-\beta} \left[t + c + \frac{(1-\beta)}{(2+\beta)} \beta l^* \right]$. Therefore we have $p_2 + t > (t + l^*)/2$ as required if

$$\frac{1}{2-\beta} \left[t + c + \frac{(1-\beta)}{(2+\beta)} \beta l^* \right] > \frac{1}{2} (l^* - t).$$

As $1/(2-\beta) > 1/2$ this follows if $2t + c > l^* - \frac{(1-\beta)}{(2+\beta)} \beta l^*$ and this inequality always holds given that $l^* < 2t + c$ (equation 15) and that $\beta^2 < \beta$. Therefore there is no profitable deviation for

firm 1 into region (i), even ignoring the constraint $p_1 \leq l_1$.

Finally note that firm 1 cannot prefer a mixed strategy. Any mixed strategy would require at least two discounted prices at which firm 1's profits from bargainers would be equal in response to firm 2's p_2 , and this profit level must be greater than or equal to that achieved at the interior prices $\{p_1, p_2\}$. We have shown that if prices (16) are interior then this is not possible, and so mixing cannot be an optimal deviation.

The proof that firm 2 will not seek to deviate into regions (iii) or (iv) is analogous. Simple modifications are needed to the proof to ensure firm 2 would not seek to deviate into region (i).²³ The profit function in region (i) is concave with maximum at a deviation of $p_2 = \min\{(t + l_1)/2, l^*\}$ as firm 2 cannot raise its discounted price above its list price of l^* . If this maximum point lies to the left of the allowable range of (i), that is if $p_1 + t > \min\{(t + l_1)/2, l^*\}$, then remaining in region (ii) dominates a deviation to region (i). This is true if $p_1 + t > l^*$. From (16), using that $l_1 \geq c$ we have $p_1 \geq \frac{1}{2-\beta}(t + c) + \frac{(1-\beta)}{(2-\beta)(2+\beta)}(2l^* + \beta c)$. Therefore we have $p_1 + t > l^*$ as required if

$$\begin{aligned} \left[\frac{1}{2-\beta} + 1 \right] t + \frac{c}{2-\beta} \left[1 + \frac{\beta(1-\beta)}{2+\beta} \right] + \frac{(1-\beta)}{(2-\beta)(2+\beta)} 2l^* &> l^* \\ (2+\beta)(3-\beta)t + c(2+2\beta-\beta^2) &> l^*(2+2\beta-\beta^2). \end{aligned}$$

As $l^* < 2t + c$ the result follows if

$$6 + \beta - \beta^2 \geq 2(2 + 2\beta - \beta^2) \Leftrightarrow (2 - \beta)(1 - \beta) \geq 0.$$

Which is true for $\beta \in (0, 1)$. Therefore there is no profitable deviation for firm 2 into region (i).

That firm 2 also has no profitable deviation to a mixed strategy equilibrium follows as above.

Therefore, since there is no profitable global deviation for either firm 1 or 2, the result follows.

■

Proposition 9 *There is no profitable list price deviation from the candidate symmetric list price equilibrium l^* given in Proposition 2 to any list price $l_1 \in (\underline{l}, \bar{l})$ where:*

$$\underline{l} = \frac{(2+\beta)(t+c) + 2(1-\beta)l^*}{4-\beta}; \text{ and} \quad (19)$$

$$\bar{l} = \min \left\{ \frac{(2+\beta)(3-\beta)t + (2+\beta)c + \beta(1-\beta)l^*}{(2+2\beta-\beta^2)}, \frac{(4-\beta)l^* - (2+\beta)(t+c)}{2(1-\beta)} \right\}. \quad (20)$$

Before proving the proposition, we note that the candidate symmetric equilibrium price,

²³The complication arises as we allow deviations $l_1 > 2t + c$.

$l_1 = l^*$, is strictly contained within the bounds. This follows as $\underline{l} < l^*$ as $l^* > t + c$, and $\bar{l} > l^*$ as $l^* < (3 - \beta)t + c$ in the first case, and $l^* > t + c$ in the second case.

Proof. Consider a deviation by firm 1 to list price $l_1 \in (\underline{l}, \bar{l})$. Assume that $(\underline{l}, \bar{l}) \subset [l^* - t, l^* + t]$. We will confirm this at the end of the proof. Assume that at list prices $\{l_1, l^*\}$ prices (16) are in the interior and satisfy $p_i < l_i$. Lemma 3 then implies that prices (16) are a Nash equilibrium of the price discounting subgame. The profit from bargainers is given, expanding out the constant term in row (ii) of (18), by

$$\begin{aligned} \pi_1^{\text{barg}} &= \beta^2 \frac{1}{2t} (p_1 - c) (t + p_2 - p_1) + \beta (1 - \beta) \frac{1}{2t} (p_1 - c) (t + l^* - p_1) \\ &\quad + (1 - \beta) \beta (l_1 - c) \frac{1}{2t} (t + p_2 - l_1) + (1 - \beta)^2 (l_1 - c) \frac{1}{2t} (t + l^* - l_1). \end{aligned} \quad (21)$$

From (16) the discounted prices are positive linear functions of the deviation list price, l_1 , such that $0 < \partial p_1 / \partial l_1, \partial p_2 / \partial l_1 < 1$. By inspection therefore profit (21) is concave in l_1 . The profit from the price takers is also concave. As the sum of two concave functions is concave, it follows that the profit of firm 1 is concave on the set $l_1 \in (\underline{l}, \bar{l})$. Firm 1's profit has a local maximum at $l_1 = l^*$ by construction of l^* (Proposition 2). Hence there can be no incentive to deviate to any list price $l_1 \in (\underline{l}, \bar{l})$.

Now we show that for any $l_1 \in (\underline{l}, \bar{l})$ prices (16) are in the interior, and $p_i < l_i$. Consider first lowering l_1 down from $l_1 = l^*$. As $0 < \partial p_1 / \partial l_1, \partial p_2 / \partial l_1 < 1$ both discounted prices fall. The constraint $p_2 < l^*$ remains satisfied. The constraint $p_1 < l_1$ is broken when

$$p_1 \geq l_1 \Leftrightarrow \frac{1}{2 - \beta} (t + c) + \frac{(1 - \beta)}{(2 - \beta)(2 + \beta)} (2l^* + \beta l_1) \geq l_1,$$

which simplifies to yield \underline{l} in (19). Confining attention to $l_1 \in (\underline{l}, \bar{l})$, firm 2 moves out of the interior if we break $l_1 - t < p_2 < p_1 + t$. However as l_1 falls $p_2 - p_1$ shrinks from (17), and p_2 does not fall as fast as l_1 . Hence if a firm leaves the interior it will be firm 1. Firm 1 moves out of the interior if we break $l^* - t < p_1 < p_2 + t$. But using that $l_1 \geq c$, and that $l^* < 2t + c$ in (17), gives that $p_1 - p_2 < 2t \left(\frac{1 - \beta}{2 + \beta} \right) < t$. Hence if firm 1 moves out of the interior, region (ii) of (18), it moves into region (iii) where $p_1 \leq l^* - t$. Using (16), $p_1 \leq l^* - t$ implies that

$$\begin{aligned} \frac{1}{2 - \beta} (t + c) + \frac{(1 - \beta)}{(2 - \beta)(2 + \beta)} (2l^* + \beta l_1) &\leq l^* - t \\ \frac{c}{2 - \beta} + \frac{(1 - \beta)}{(2 - \beta)(2 + \beta)} \beta l_1 &\leq l^* \left[1 - \frac{2(1 - \beta)}{(2 - \beta)(2 + \beta)} \right] - t \frac{3 - \beta}{2 - \beta} \\ \frac{c}{2 - \beta} - c \left[1 - \frac{2(1 - \beta)}{(2 - \beta)(2 + \beta)} \right] + \frac{(1 - \beta)}{(2 - \beta)(2 + \beta)} \beta l_1 &< 2t \left[1 - \frac{2(1 - \beta)}{(2 - \beta)(2 + \beta)} \right] - t \frac{3 - \beta}{2 - \beta}, \end{aligned}$$

where the final line uses the fact that $l^* < 2t + c$. Hence we would require:

$$(1 - \beta) \beta (l_1 - c) < 2t [2 + 2\beta - \beta^2] - t(3 - \beta)(2 + \beta).$$

But the right-hand side is negative, and so as $l_1 \geq c$ we have a contradiction. Hence if firm 1 lowers its list price to any point above \underline{l} then discounted prices (16) remain in the interior with $p_i < l_i$, and so the deviation is not profitable.

Now consider increasing l_1 from l^* . From (16), both discounted prices rise. The constraint $p_1 < l_1$ remains satisfied as $\partial p_1 / \partial l_1 < 1$. The constraint $p_2 < l^*$ is broken when we have

$$\begin{aligned} p_2 \geq l^* &\Rightarrow \frac{1}{2 - \beta} (t + c) + \frac{(1 - \beta)}{(2 - \beta)(2 + \beta)} (\beta l^* + 2l_1) \geq l^* \\ &\Rightarrow \frac{2(1 - \beta)}{(2 - \beta)(2 + \beta)} l_1 \geq \left[1 - \frac{\beta(1 - \beta)}{(2 - \beta)(2 + \beta)} \right] l^* - \frac{1}{2 - \beta} (t + c), \end{aligned}$$

which simplifies to yield the second term in (20).

Firm 1 moves out of the interior if we break $l^* - t < p_1 < p_2 + t$. However as we are increasing l_1 , p_1 rises. Further, from (17), as l_1 rises, $p_1 - p_2$ shrinks. Hence firm 1 does not move out of the interior. If a firm moves out of the interior as l_1 rises, it will be firm 2. Firm 2 moves out of the interior if we break $l_1 - t < p_2 < p_1 + t$. Note that we cannot have $p_2 \geq p_1 + t$ as in the range $l_1 \leq l^* + t$, from (17), $p_1 - p_2 \geq -\frac{1 - \beta}{2 + \beta} t > -t$. However, it is possible that the other constraint can be broken: $p_2 \leq l_1 - t$. This constraint yields, using (16),

$$\begin{aligned} p_2 \leq l_1 - t &\Rightarrow \frac{1}{2 - \beta} (t + c) + \frac{(1 - \beta)}{(2 - \beta)(2 + \beta)} (\beta l^* + 2l_1) \leq l_1 - t \\ &\Rightarrow l_1 \left[1 - \frac{2(1 - \beta)}{(2 - \beta)(2 + \beta)} \right] \geq \frac{3 - \beta}{2 - \beta} t + \frac{c}{2 - \beta} + \frac{\beta(1 - \beta)}{(2 - \beta)(2 + \beta)} l^*. \end{aligned}$$

Simplifying yields the first term in (20).

Finally we now show that $(\underline{l}, \bar{l}) \subset [l^* - t, l^* + t]$. We have

$$\begin{aligned} \underline{l} \geq l^* - t &\Leftrightarrow (2 + \beta)(t + c) + 2(1 - \beta)l^* \geq (4 - \beta)(l^* - t) \\ &\Leftrightarrow l^*(2 + \beta) \leq (2 + \beta)c + 6t. \end{aligned}$$

And this follows as $l^* < 2t + c$. For the upper bound, if either of the terms in (20) is weakly less than $l^* + t$ then we have $\bar{l} \leq l^* + t$ as required. For the first term in (20) this requires

$$\begin{aligned} \bar{l} \leq l^* + t &\Leftarrow (3 - \beta)(2 + \beta)t + (2 + \beta)c + \beta(1 - \beta)l^* \leq (2 + 2\beta - \beta^2)(l^* + t) \\ &\Leftrightarrow l^*(2 + \beta) \geq (2 + \beta)c + t(4 - \beta). \end{aligned} \tag{22}$$

For the second term in (20) this requires

$$\begin{aligned}\bar{l} \leq l^* + t &\Leftrightarrow (4 - \beta)l^* - (2 + \beta)(t + c) \leq 2(1 - \beta)(l^* + t) \\ &\Leftrightarrow l^*(2 + \beta) \leq (2 + \beta)c + t(4 - \beta).\end{aligned}\tag{23}$$

As one of (22) or (23) must hold we have the required result. ■

Step 2: Numerical

Proposition 9 shows analytically that deviations from the candidate symmetric list price equilibrium l^* given in Proposition 2 to $l_i \in (\underline{l}, l^*)$ and to $l_i \in (l^*, \bar{l})$ are never profitable. We now show numerically that list price deviations beyond these bounds are never profitable. We set marginal cost $c = 0$ and normalize the transport cost t to 1, which leaves the candidate symmetric list price equilibrium $l^* \in (1, \frac{4}{3})$ for any $\mu \in (0, 1)$ and $\beta \in (0, 1)$. Our algorithm proceeds as follows:

- Step A: Fixing μ and β , for every list price deviation on the grid $\{0, 0.01, \dots, 2.00\}$ that is not in (\underline{l}, \bar{l}) : (i) calculate all the pure-strategy Nash equilibria of the discount stage on the discount prices grid $\{0, 0.01, \dots, 2.00\} \times \{0, 0.01, \dots, 2.00\}$; (ii) check that there is always at least one such equilibrium; and (iii) for each discount stage equilibrium, compare total profits from the deviation to total profits at l^* to check that the deviation is not profitable.²⁴
- Step B: Iterate Step A over all μ and β combinations on the grid $\{0.01, 0.02, \dots, 0.99\} \times \{0.01, 0.02, \dots, 0.99\}$, thus iterating over $99 \times 99 = 9,801$ parameter combinations.

²⁴Sometimes the algorithm finds marginally profitable deviations because the grid of discount prices is not fine enough. When this happens, the procedure doubles the number of discount prices on the grid for each firm until the grid is fine enough that we confirm that there is no longer a profitable deviation.

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