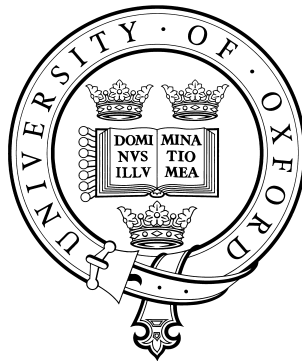

Stabilization and Control In a Linear Ion Trap

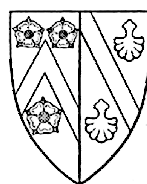
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Abstract

This thesis describes experimental work towards developing a trapped ion quantum information processor. An existing ion trap apparatus was capable of trapping and laser-cooling single ions or small ion strings of $^{40}\text{Ca}^+$, and had been used for studies of quantum jumps and natural lifetime measurements in Ca. This thesis describes improvements in this apparatus, which have allowed the stability and the flexibility of experimental control of the ions to be greatly increased. This enabled experiments to read out the spin state of a single trapped ion, and to load ions with isotope selectivity through photoionization.

The optical systems were improved by installation of new lasers, optical reference cavities, and a system of acousto-optic modulators for laser intensity switching and frequency control. The photon counting for fluorescence detection was improved, and a new photon time-of-arrival correlation circuit developed. This has permitted rapid and more sensitive detection of micromotion, and hence cancellation of stray fields in the trap.

A study of resonant circuits in the low RF, high voltage (10 MHz, 1 kV) regime was carried out with a view to developing a new RF supply for the Paul trap with reduced noise and increased power. A new supply based on a helical resonator was built and used to trap ions. This technique has reduced noise and will permit higher secular frequencies to be attained in the future.

A magnetic field B in the ion trap is used to define a quantization axis, and in one series of experiments was required to be of order 100 G to provide a substantial Zeeman splitting. A set of magnetic field coils to control the size and direction of B is described. The design of these posed some problems owing to an unforeseen issue with the vacuum chamber. In short, it is magnetizable and acts to first approximation like a magnetic shield. The field coils had to be sufficiently substantial to produce the desired field at the ion even in the presence of this shielding effect, and dark resonance (and other) spectra with Zeeman splitting were obtained to calibrate the field using the ion as a probe.

Finally, the thesis describes the successful loading of the ion trap by laser photoionization from a weak atomic beam. This involved two new lasers at 423 nm and 389 nm. Saturated absorption spectroscopy of neutral calcium is first described, then transverse excitation of an atomic beam in our vacuum chamber is used to identify all the main isotopes of calcium and confirm their abundances in our source (a heated sample of natural calcium). Finally, photoionization is used to load the trap. This has three advantages over electron-impact ionization. By avoiding an electron gun, we avoid charging of insulating patches and subsequent electric field drift as they discharge; the flux in the atomic beam and hence calcium (and other) deposits on the electrodes can be greatly reduced; and most importantly, the photoionization is isotope selective. Evidence is presented which suggests that even with a non-enriched source, the rare isotope ^{43}Ca can be loaded with reasonable efficiency. This isotope is advantageous for quantum information experiments for several reasons, but chiefly because its ground state hyperfine structure can act as a stable qubit.

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Chapter 1

Introduction

1.1 A brief history of spectroscopy

Historically, atomic spectroscopy has consisted of the investigation of fundamental properties of atomic physics using detailed analysis of radiation emitted by atomic samples [1]. Its initial formulation was in terms of models, which yielded the *ad hoc* early quantum mechanics as a result of phenomena inexplicable using classical mechanics. More recently spectroscopy has tested many aspects of atomic and nuclear quantum physics, and resulted in frequency standards, and measurements of fundamental physical constants, of very high precision.

The advances in atom/ion trapping and laser cooling of atomic samples [2, 3] have revolutionized spectroscopy [4] almost as much as the invention of the laser. The past half-century has yielded new and striking directions in atomic physics [5]. Experiments previously considered *gedanken*, e.g. the observation of a single ion, or of quantum jumps in a three-level atom [6], were realised in the laboratory [7, 8].

In this thesis we only consider ion traps, although atom trapping is a well-developed field in its own right [9, 10, 11]. The precursor to the three-dimensional ion trap was the Kingdon trap, initially proposed in 1923 [12], consisting of coaxial electrodes which utilized the conservation of angular momentum to prevent ion adsorption to the axial wire electrode. The invention of Paul [13] and Penning traps [14] improved ion trapping lifetimes enormously. These trapped in all three dimensions, the former by time-varying electric fields and the latter by a combination of static electric and magnetic fields. The traps were aided by the application of the method of laser cooling to bound absorbers [15, 16].

The theoretical Doppler limit of standard laser cooling models was improved using subtle techniques. Exploitation of the AC Stark effect in the polarization gradient of a laser standing wave yielded sub-Doppler cooling using the Sisyphus effect [17]. The motional sidebands of a sufficiently narrow transition could be addressed to cool the ion below the Doppler limit [18]. More recently, the coherent phenomenon of electromagnetically induced transparency (EIT) has been employed to provide fast sideband cooling [19].

Trapped atoms and ions provide the experimentalist with an atomic system that suffers from very few unwanted perturbations. As such it is possible to conduct accurate spectroscopic experiments [20]. Many of these are geared towards the time and measurement industry e.g. references [21, 22, 23]. Confined atoms and ions provide us with the most accurate frequency standards in physics; currently these are the caesium atomic clocks in LPTF, Paris [24] and NIST, Boulder CO [25].

1.2 Computation theory and practice

1.2.1 Classical computation and information theory

The principles of classical information theory, extrapolated into the quantum regime, provide a language for quantum mechanics. This language permits philosophical interpretations of quantum theory and provides a paradigm for experimental work. For a history of classical computation and information, see reference [26]. We summarize as follows. The ‘prehistory’ of computation theory arguably begins with the Difference and Analytical Engines of Charles Babbage and Ada Lovelace (ca. 1833 and 1842 respectively). These attempts at automating algorithms prefigured the work of Alan Turing (1936), who used conceptual machines to illustrate one possible model for the implementation of logic; this mirrored the work of Alonzo Church (1936). In principle any classical computing machine could be imitated by the Turing machine, leading to conclusions on what could and could not be deduced from logic alone.

An important abstraction from the Church–Turing principle is evident in Claude Shannon’s information theory (1948). Here we consider the storage and transmission of arbitrary data, rather than computer programs. The data need not be interpreted and thus, whereas Turing’s results pertained to computation, Shannon’s put limits on the flow of information. It is clear in hindsight that all information requires a physical representation, and that Shannon’s statements about information flow can be interpreted as delineations of the underlying physical processes.

1.2.2 Quantum computational theory

The physical representation underpinning a computing machine is fundamentally quantum mechanical, not classical. The idea of a quantum computer results from bringing together a number of different disciplines [27]. Classical information theory provides us with a model with which to describe a classical physical system in terms of information flow; quantum mechanics requires an adaptation of the classical theory to take into account the computational possibilities of a large Hilbert space; engineering and practical considerations place limits on the utility of the resulting machines.

Early work examined some of the ways in which computation can be framed in quantum-mechanical terms [28, 29]. An important paper by David Deutsch [30] brought together most of the concepts which now form the theoretical model for the quantum computer. This laid one of the foundations of quantum information theory. Initially this theory was apparently just a more complete version of classical information theory, permitting all classical algorithms to be recast in a quantum analogue which, computationally speaking, did not speed up the processing time for a given algorithm. However, ground-breaking algorithms were discovered which provided an exponential increase of speed. An important example is the factorization of large numbers [31], a necessary procedure in cryptography and code-breaking. A further important discovery was the ‘needle in a haystack’ algorithm [32] which greatly increased the speed of finding an article in an unordered list.

These procedures are useful solutions to engineering and communication problems, but they arguably do not constitute sufficient physical motivation for the construction of a quantum computer. Instead, one must turn to a third, and physically more interesting, role for the quantum computer. A quantum device can ‘mimic’ quantum behaviour in a way entirely different from simulations on a classical computer: using a quantum system improves exponentially the speed of many calculations concerning quantum behaviour [33]. A quantum information processor can also be used to

make measurements of fundamental constants and physical properties of the system used to build the computer. A quantum information processor is a subtle experimental tool. If built from single atoms, then it would permit complex and revealing experiments on those atoms; if built in solid state, it could be used to investigate the properties of semiconductors. Attempts to implement quantum algorithms such as factorization in current experiments are a convenient method to explore quantum-mechanical behaviour and the limits of current technology.

The major drawback of truly quantum computers follows from the inherently delicate nature of coherence in any physical system. This severely limits the length of the data strings that can be successfully handled by the computer. A breakthrough in quantum information theory was the invention of quantum error correction [34, 35], a method for correcting decoherence errors mid-computation without incoherent observation of the state of the computer. This typically involved constructing parity checks on encrypted data, and then observing the parity bits instead.

Many of the operations required during error correction have been demonstrated using a bulk NMR-based quantum computer. As we shall discuss later these do not give an unambiguous demonstration of true quantum computation. In addition, predictions of the efficacy of quantum error correction in a practical situation vary [36]. It is therefore of the utmost importance to the future of the quantum computer that this important building block of quantum information processing be tested in a quantum-computational environment. This is one of the central goals of current experimental work.

1.2.3 Physical implementations of a quantum computer

There are many possible ways of implementing quantum computation. Experimental work is still in its infancy, however, and it is not clear which implementation will ultimately provide us with a large quantum computer [37]. We provide here a brief discussion of the different techniques available, and our motivation for using cold trapped ions. This section is not intended as a comprehensive review of the entire field. As we shall discuss, almost every area of laboratory physics has associated with it a proposal for quantum computation. In addition, work on quantum computation is moving at a rapidly increasing pace [38], and to produce a review of the entire field¹ is beyond the scope of this thesis. We therefore restrict ourselves to a brief discussion of the overall advantages and disadvantages of the physical systems most frequently proposed.

A physical system must satisfy several requirements, if it is to be capable of acting as the basis of a quantum computer. DiVincenzo has usefully summarized these requirements [40]. The main criteria, not including data transmission between computers, are:

1. **Qubits** The pairs of states which constitute qubits must be well-characterized, and the system must be scalable in number of qubits without serious loss of performance [41].
2. **Initialization** It must be possible to prepare cold initial states e.g. the thermal ground state of the system. Where hot states are used, there must be available a decoherence-free subspace [42] that is not affected by thermal noise.
3. **Decoherence** Preferably our states should decohere slowly enough that the effect has no impact on quantum computation. In practice we would like decoherence to be at a level where we can introduce quantum error correction.

¹For a more detailed review the reader is directed to e.g. reference [33], or the special issue of *Fortschritte der Physik* [39].

4. **Logic gates** We must be able to implement one- and two-qubit gates within the system. Arbitrary single-qubit rotations and any conditional two-qubit flips are sufficient [43].
5. **Measurement** After computation we must be able to reliably read out the (projected) state of each qubit. Depending on the algorithm we compute we may require many such computation and readout stages in order to amass statistics on the quantum processor's final state.

We discuss the most likely possibilities for scaleable quantum computation below in the light of this list. Although the list is not definitive, and every viable scheme has proposals for compensating for its individual limitations and difficulties, DiVincenzo's list remains a useful benchmark for the feasibility of implementing quantum computation in any given medium.

1.2.3.1 Ion traps

In principle the ion trap forms the basis for a modestly scalable quantum computer [44] as we shall discuss more fully in chapter 2. In the Paul trap, the ions are trapped by radio-frequency and DC electric fields. The arrangement of the ions in the trap, relative to each other, is influenced by the Coulomb repulsion between ions. At certain trap voltage parameters linear crystals of ions will form along the centre of the trap. In this situation the ions can be individually addressed by laser beams. Each qubit is represented by two internal long-lived states of a single ion. Initialization and rotation of each qubit is performed by the lasers, usually involving a third atomic level directly or virtually in each case.

The axial motion of the entire string of ions is analysed as a set of orthogonal normal modes, each of which is quantized. By tuning a laser to a motional sideband we are able to effect transitions which move the internal superposition state of a given ion to the total motional state, and back. Thus we have an interaction that is present always; but the laser interaction provides a means whereby the influence of this interaction on the qubits can be 'turned on and off.' A logic gate between any two ions may for example consist of: transferring the information content of one qubit into the ground and first excited states of the motional state of the string; performing logic on a second qubit conditional on the motional state; transferring the information back from the motional states to the first qubit's states.

The ion trap is a good candidate for quantum computation because its qubits are easily distinguished, and the slowest coherent process—transitions involving the motional state—are much faster than the decoherence rates of the qubits. Measurement of the state of the ions typically involves the presence or absence of fluorescence on a strong optical transition and is thus very reliable. Ultimately, the scalability of a single ion trap is questionable, because large ion crystals eventually succumb to trap instabilities and heating effects. Multiplexing schemes involving moving ions in arrays of traps have been proposed, and it may eventually be necessary to have networks of traps connected by 'flying qubit' photons carrying quantum information between them.

1.2.3.2 Cavity QED

A weak atomic beam can deliver qubits—atoms typically in a high Rydberg state—into an interaction region with reasonable probability of only one atom being in the region at once. The atoms can be reliably polarized, and then rotated to a particular qubit state superposition. When an atom passes through a high-finesse microwave cavity then it interacts in a state-dependent manner with the cavity. This interaction is described by the theory of cavity QED (CQED). If the cavity photon number begins in e.g. a Fock state, then after the first interaction it holds the superposition state

of the first atom, and then changes the state of the next atom passing through it, dependent on the state of the first: this provides a two-qubit gate. When the outermost electron is close to the continuum, atomic state measurement can proceed by state-dependent ionization, which has high state discrimination efficiency.

The advantages of CQED methods are that single qubit rotations, and one-off two- and three-qubit gates, are relatively easy to implement and have already been demonstrated [45]. However, the fact that the qubits are not re-used after the single gate complicates the implementation of long algorithms. Until cavity QED and atom or ion traps are combined reliably in order to keep the qubits within the interaction region then this difficulty remains.

1.2.3.3 Optical lattices

Another atomic method consists of atoms trapped at the sites of an optical lattice [46]. Counter-propagating laser beams create a standing wave potential in which atoms reside. By careful preparation of the potential it is theoretically possible to have a single atom at each lattice site. As in all other systems discussed thus far, the qubit states are two internal states of each atom.

The structure of a 2D or 3D atomic lattice appears to be close to the ideal quantum information processor: single qubits, easy to prepare, separated in space from other qubits, capable of controlled interactions with each of their six neighbours. The main difficulties with this system are: atoms are more difficult to trap and keep separate than ions; controlled collisions must be carried out to implement gates, and these are technically demanding; and the state measurement must not destabilize the traps.

1.2.3.4 NMR and other nuclear systems

Bulk NMR quantum information processing utilizes nuclear spins in complex (typically organic) molecules as individual qubits [47]. Given that the sample is at room temperature, NMR relies on pseudo-pure states for the quantum-computational manifold. This ‘pseudo-pure’ state is a way of using the fact that the observable signal in NMR depends only on a small part of the (mixed) thermal state of the sample, and with suitable averaging this small part gives the same signal as a pure state would. Two-qubit gates arise from the nuclear dipole–dipole interaction: this interaction is present always, and much of the work involved in an NMR experiment consists of nulling the two-qubit interaction whenever it is not computationally required. The measurement process consists of measuring the mean magnetization of the entire sample: the precession of all the qubits in the liquid is picked up by a tuned coil around the sample, and the strength of this signal translated into a state population.

Bulk NMR has already provided a demonstration of many of the necessary processes required for quantum computation [48]. However, it is not clear whether the ‘massively parallel’ nature of bulk NMR computation arises from true entangled behaviour, or from each of the very many atoms present in the sample all providing parallel, but classical, computation. For such reasons, bulk NMR remains unconvincing as a demonstration of true entangled behaviour. Re-initialization of qubits—an essential ingredient of e.g. error correction—is also difficult and remains a serious computational barrier in NMR proposals generally. The scalability of liquid bulk NMR is also ruled out, since the addition of each new qubit decreases the observed signal by a constant fraction. This exponential loss of signal with number of qubits spoils any exponential speed-up we might hope to obtain.

It would be useful to combine NMR techniques with the solid-state technology of classical computers, but such systems exhibit fragile, short-lived entanglement; experiments are underway to confront these issues by spatial localization of NMR spins in a solid-state lattice [49] (as opposed to localizing the spins in frequency space), but the system is not yet proven to be scalable. Trapped-atom and photon schemes also exist, yet these too have their disadvantages.

1.2.3.5 Solid-state quantum dots

The majority of solid-state QC proposals can be described collectively as the exploitation of imperfections in semiconductors, deliberately introduced by methods such as microfabrication. These imperfections localize electron wavefunctions and yield a qubit system for each ‘quantum dot.’ Each electron’s spin state is a qubit and, in a many-electron system, the Pauli exclusion principle requires that the symmetries of the internal and external superpositions of many electrons must correlate. Thus a symmetrical two-electron spin state has an antisymmetric spatial state, leading to lower likelihood of finding the electrons close together and hence a lower Coulomb repulsion. The combination of exclusion and repulsion can be represented by an effective Heisenberg spin Hamiltonian $\mathcal{J}\mathbf{S}_1 \cdot \mathbf{S}_2$ i.e. a state-dependent interaction. This interaction can be switched off by varying the qualities of the pure solid-state medium between the dots: if the potential barrier between the dots is increased then the tunnelling between dots is negligible and the electrons cease to experience the exchange interaction. Thus we have a two-qubit gate, and the building blocks of a quantum information processor.

In the far future it seems likely that solid-state systems will ultimately replace cumbersome single-atom, isolated systems (much as valves have been superseded by transistors, except in certain hybridized applications). However, solid-state systems have very short decoherence times. Decoherence in solid-state quantum computers has several sources, and is severe. The primary decoherence route is via interactions with nuclear spins [50], although the electrons also interact with the lattice, and the complex, many-body interactions of all dots is difficult to track. The decay times of coherence in solid-state systems can vary from milliseconds to picoseconds, depending on the degree of freedom in question. Although one- and two-bit gates also have short timescales, it is not yet clear whether decoherence will remain a fundamental limit of the solid state systems. In addition, although in principle the microfabrication techniques exist, in practice constructing single dots reliably and scalably (alongside their single-electron transistors for measurement) is difficult. Despite these limitations, a two-qubit gate has recently been demonstrated using superconducting charge qubits [51].

1.2.3.6 Miscellaneous other proposals

Where there is a rich enough quantized system, there is a possibility of implementing quantum computation: so long as DiVincenzo’s list applies. Magnetic flux is quantized in a superconducting system, and thus Josephson junctions [52] provide one possibility for qubit states. These qubits interact via a communal electrical inductor, and measurement can be carried out using single-electron transistors. Liquid helium provides a substrate for electrons, and the electron spin then becomes a qubit [53]. These and other systems provide at the very least a validation of the universality of quantum-information techniques, that so many different systems can be cast clearly in a QIP representation.

1.2.4 Milestones in the implementation of ion trap quantum computation

Progress has been achieved in various groups, in the implementation of the ion trap quantum computer. We do not consider the implementations involving cavity QED, as these do not directly relate to our intended method of quantum computation. Since the original Cirac–Zoller proposal, cooling of a single ion to the ground state in one, two and three dimensions [18, 54] has been achieved. Two trapped ions have been cooled to the motional ground state [55].

Single ions in a crystal have been individually addressed [56], and entanglement has been generated deterministically between two [57] and four ions [58], although the latter has not been implemented by addressing only one qubit at a time and therefore does not result in a set of unitary operations sufficient for quantum computation. The entanglement of several ions has resulted in a quasi-Schrödinger Cat experiment [59] and a Bell experiment [60]. Ions have been passed between two traps in order to demonstrate one scheme intended to make ion trap quantum computers scalable [61]. The state of one ion has been encoded into a decoherence-free subspace of two ions [62], and the *in situ* transferral of quantum information has been demonstrated [63, 64]. In the first reference the internal state of an ion was mapped onto its external state with high fidelity; in the latter, the two-qubit CNOT gate—a spin-flip of one qubit dependent on the state of the other—was demonstrated with approximately 80% fidelity. In addition the Deutsch algorithm [65] has been demonstrated [66], a computation which only requires a single ion, whose internal and motional states provide two qubits.

1.3 Quantum computation in the Oxford Ion Trap

A central aim of the Oxford Ion Trap group is to demonstrate quantum error correction in an ion trap. Such a demonstration requires at least three qubits. It is clear in theory how one might progress towards such a goal. Beginning with a single qubit one improves methods of cooling, readout and qubit rotation. Then a two-qubit gate is implemented. Finally the quantum network for three-qubit error correction is built up from single laser pulses, and each stage of the calculation verified. Once three ions can be manipulated, simple algorithms can also be demonstrated.

We trapped ions in early 1999, shortly before the work in this thesis was begun. We performed an accurate spectroscopic measurement on the lifetime of a metastable level in the Ca II manifold [67] and used this transition to search for unexplained correlated events found in other ion traps [68].

In the course of the work described in this thesis, the shelving method initially proposed for our experiment [69] was successfully demonstrated, although difficulties with transient magnetic fields required that we rethink the shelving process. The supplies for trap voltages have been overhauled and imperfections in the trapping potential reduced. We were able to demonstrate photoionization as a way to prepare ions rather than the indiscriminate method of electron bombardment. We implemented AOM control of all but one of the lasers in the experiment, replacing shutters and reducing noise in the laboratory which contributed to laser destabilization. We have demonstrated optical pumping into either qubit state, and achieved control over the static magnetic field at the ion. Static-field experiments yielded new possible candidates for the shelving process. Eventually we moved to a two-photon shelving scheme and this has just recently yielded successful results.

Our current short-term goal is to perform single-qubit rotations using either a Raman transition between the qubit states, or direct stimulation of the radio-frequency transition between the states [70]. We plan to observe Rabi oscillations of the qubit states' populations, and then move to manipulation of the motional state of several ions. This will lead to sideband cooling which will

initialize the ‘data bus’ of the quantum register. After this we will implement optics to address single ions in a string, and perform arbitrary two-qubit transformations. Once we are able to control at least three qubits in a coherent manner then we can implement an error correction scheme and demonstrate its practicability in an ion trap environment.

1.4 Structure of the thesis

The contents of the rest of this thesis are structured as follows. In chapter 2 we give a brief description of the layout of the Oxford Ion Trap, both at the beginning of the work described in this thesis, and including any major changes to the apparatus not discussed elsewhere. Chapter 3 is concerned with three construction projects by the author which have been integrated with the rest of the Ion Trap experiment. The theory of Paul traps and its relation to the temperature and heating rates of our ion are discussed in chapter 4. Here we combine established theory with a rate equation approach that infers thermal properties of our ion from simple experimental observations.

In Chapters 5, 6 and 7 we discuss the experimental work to improve the stability and control of trapped ions. Chapter 5 discusses the construction of a new, low-noise supply for the radio-frequency potentials in the trap. We describe experiments with resonant RF circuits and helical resonators to determine the approach that both satisfies our requirements and integrates best with the existing apparatus. Chapter 6 describes magnetic field considerations—including the desired field at the ion and the construction of field coils in order to obtain it. The chapter discusses the design considerations, and gives details of atomic-physics experiments carried out on a single ion to study the field at trap centre. In chapter 7 we describe spectroscopy of the 423 nm transition in neutral calcium. We then use the knowledge acquired to implement photoionization in the ion trap as a method of generating ions. We discuss the possibility of trapping other isotopes than ^{40}Ca , especially ^{43}Ca which, with its hyperfine structure, is an interesting candidate for quantum computation. Finally, chapter 8 concludes the thesis.

Chapter 2

Overview of the Oxford Ion Trap experiment

2.1 Introduction

In this chapter we give a description of the most recent experimental arrangements in the Oxford Ion Trap laboratory. This includes both the state of the experiment at the beginning of the work reported in this thesis, and important changes to the experiment during that time e.g. new laser systems and experimental results. The experimental methods and restrictions provide a context for the rest of the thesis. Here the apparatus and the necessary physics are discussed as background for the other chapters. More in-depth descriptions are given elsewhere as indicated; most importantly in chapter 3 we discuss the main construction projects carried out by the author.

We begin with a broad overview of the entire experiment, discussing the fundamentals of ion trapping and the lasers used, and briefly discussing the problems and solutions presented in this thesis. We then move methodically through the experiment, beginning by considering the energy levels of both neutral and singly-ionized calcium that are of interest, specifically those of the isotope ^{40}Ca . Methods of ionization are discussed. Next we consider storage of ions: the ion trap structure and the surrounding vacuum components. We discuss the lasers used to interrogate the ions; the frequency references used to stabilize the lasers; and the imaging and acquisition methods used to measure fluorescence from the ion. We discuss to what extent our experiment could eventually be used to implement quantum algorithms. Finally we give details of methods used to prepare the trap, capture ions, move to a single-ion system and then experiment on this system.

2.2 Brief discussion of the experiment

In this section we provide a general introduction to the Oxford ion trap. We discuss the principles of trap operation, and the Cirac–Zoller scheme for quantum computation in the ion trap with respect to our system. We then discuss the lasers and other apparatus that we use in the trapping and manipulation of ions. Finally we consider some of the problems with the existing apparatus, and how these are addressed in this thesis. Later sections of this chapter will introduce these aspects of the experiment in more detail.

2.2.1 Principle of ion trapping

In order to bring about the physical situation of cold ions trapped by external fields, an experiment must proceed as follows: ions must be produced; a confining potential must be applied around the ions; the ion temperature must be decreased to avoid loss of the ions owing to collisions or heating.

Until recently, ions were created in the Oxford ion trap using electron bombardment. Counterpropagating beams of hot (of order 700 K) atoms and fast electrons intersected in the trapping region, and random collisions led to ionization of the calcium gas. In chapter 7 an alternative photoionization method is described, in which the outermost electron of each atom is excited by lasers tuned to atomic transitions, until it is no longer bound by the atom.

When considering methods of trapping the ions, we must take into account the fact that Maxwell's equations forbid a trapping potential in all three directions using only external DC electric fields. We therefore cannot use static electric fields alone to confine the ions. However, a rapidly-varying field can yield an overall confining potential. As the particle is driven into oscillatory motion by a radio-frequency electric field, then the mean field strength around the orbit is not equal to the mean field strength at the centre of the orbit (which, for a sinusoidal field amplitude, is zero). The oscillating field is averaged over one RF cycle to give a mean trapping field.

We proceed by first confining along one axis with a DC field produced by two 'endcap' electrodes. This defines the axis of a linear ion trap. Four electrodes parallel to this axis are then connected to the RF supply: each diagonal pair is connected together, and the two pairs are in antiphase with respect to each other. A schematic of this arrangement is shown in figure 2.1. By making the axial trapping weaker than the radial trapping the ions are made to lie along the trap axis; the ion-ion Coulomb interaction leads to a quasi-crystal of ions, separated by tens of micrometres.

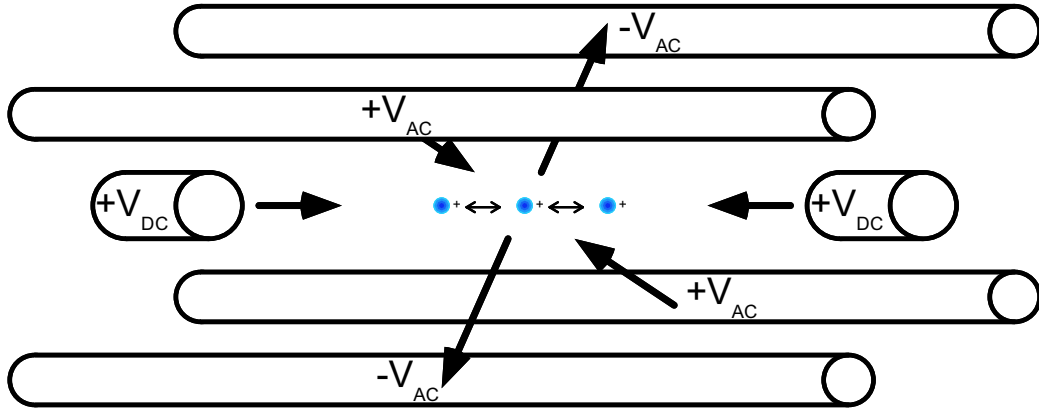


Figure 2.1: Schematic of three ions in a linear ion trap, not to scale. The four long electrodes carry an RF oscillating voltage V_{AC} , with the voltage on one diagonal pair in antiphase with the voltage on the other diagonal pair. The two short endcap electrodes carry a DC voltage V_{DC} . The small arrows represent ion-ion repulsion owing to their like (positive) charges, and the large arrows show the direction of the forces applied by the electrodes: the DC electrodes repel the ions always; over one half of the RF cycle two AC electrodes attract the ion and two repel it; over the other half the roles of the two pairs are reversed. Over the entire cycle a charged particle sees a mean field which is confining in all directions, thus trapping the particle.

Once the ions are within the trapping region, they must be cooled. In the Oxford ion trap we use laser cooling to reduce the ion temperature. When an ion scatters a counterpropagating photon then its momentum is reduced; if the photon is copropagating then the ion momentum is increased. The lasers used to address ionic transitions are therefore detuned to the red side of the transition: the Doppler shift experienced by a moving ion then makes it more probable that an

atom absorb counterpropagating photons. When the photon is scattered in a random direction, this second momentum exchange leads to an increase in temperature owing to its randomness. The final temperature of the ion therefore depends on the ratio of the number of photons scattered, to the differential in photon scattering with respect to velocity: we will show in section 4.3 that it is at a minimum when a low-intensity laser is red-detuned to the half-width of the transition.

2.2.2 The Cirac–Zoller scheme in the Oxford ion trap

We present here the general principle of how our system can be described in terms of the Cirac–Zoller proposal for quantum computation in ion traps.

Each qubit consists of the two spin sublevels of the lowest level in singly-ionized calcium. Two laser beams far-detuned from the cooling transition effect Raman transitions between the two qubit states. The frequency of one beam is detuned from the other to match the Zeeman splitting between the qubit states.

One way to implement a two-qubit gate in our trap is as follows. An intense, tightly-focussed laser beam is far detuned from a transition. Such an electromagnetic field gradient induces a dipole in the ion and, depending on the sign of the frequency detuning relative to the transition, the laser causes the ion to move towards or away from the most intense part of the beam. As the ion moves relative to the other ions in the trap, the wavefunction phase changes owing to the change in the ion–ion repulsion. Thus if we make the dipole force state-selective—only acting on the ion if it is in one qubit state—then we implement a two-qubit phase gate in the trap. A two-qubit phase gate applies a rotation (about the z -axis of the Bloch sphere) to one qubit, dependent on the quantum state of the other. Single-qubit rotations plus a phase gate are sufficient for general quantum computation [41, 71].

2.2.3 Manipulating the state of the ions

Below are some of the methods for manipulating both the external state (temperature) of the ions and their internal (electronic) states. The primary tools are lasers, and each laser is introduced here in the context of the functions it typically performs in the experiment. More details on laser construction are given in section 2.5, and in section 2.9 experimental methods are discussed further.

2.2.3.1 Laser cooling and fluorescence

During trapping, the **397 nm cooling laser** and **866 nm cooling repumper** are used to cool ions and keep them close to the bottom of the trap. At the same time, fluorescence on the strong near-UV cooling transition is observed by camera and photomultiplier tube. The UV cooling laser replaces an old master/slave system at **794 nm**. This system was frequency-doubled to reach the cooling transition, but was abandoned after irreparable damage to the system.

By varying the tuning of either the cooling laser or the repumper, we are able to manipulate the temperature of an ion in the trap and the population of the excited state. The cooling manifold in calcium consists of a three-level Λ -system—a ground level and a metastable level connected by transitions to the same excited level (see figure 2.2)—with degeneracy and slow decay into the lowest level. In section 4.3 we develop a detailed model of the behaviour and temperature of such a system, and apply our results to a single calcium ion.

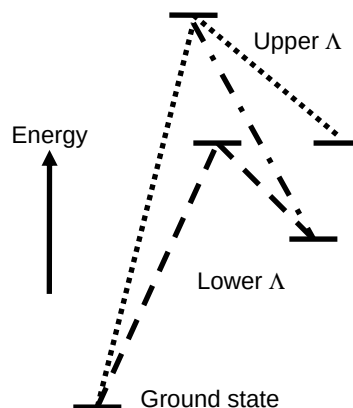


Figure 2.2: Schematic of the double Λ -system in singly-ionized calcium (not to scale). The manifold decomposes as follows: (a) cooling is effected using lasers tuned to the transitions in the lower three levels (dashed-line Λ); (b) during sublevel discrimination, a fraction of ground-level population is excited into the topmost and rightmost levels (dotted-line Λ); (c) a final transition connects these two manifolds further (dash-dotted line). See text for details.

2.2.3.2 State preparation

By choosing the polarization of either cooling laser, ions are selectively excited on different components of the corresponding transition. Certain internal states of the ion decouple from the polarized fields: such states are not excited and eventually excitation of other states leads to accumulation of population in these decoupled states. In section 6.4.4 we discuss this example of ‘optical pumping’ in more detail, with a view to using it as a diagnostic of magnetic field direction in the trap.

2.2.3.3 Shelving and deshelling

Above the cooling manifold in $^{40}\text{Ca}^+$ there is a second Λ -system, sharing the same lowest ground state. This is also shown in figure 2.2. This system can in principle be used for cooling: we have demonstrated continuous fluorescence from this manifold. However, we use it for ‘population shelving’: the excitation of a fraction of the population to a metastable level in order to make this fraction optically dark to the cooling lasers. If shelving is state-selective then it provides us with a method of state discrimination by fluorescence observation.

Addressing the transition from the ground state in this second system is the **393 nm UV shelving laser**. When an ion is excited on this transition, it can either decay back to the ground state, or into the two metastable levels of the shelving and cooling manifolds. The **850 nm IR shelving laser** connects the two manifolds (from cooling metastable state to shelving excited state) and can therefore repump population back into the shelving manifold to eventually decay to the shelving metastable state.

After detecting whether or not the ion is shelved (e.g. by observing fluorescence on the cooling cycle), we must return this population to the cooling cycle to recommence cooling and state preparation. We excite population out of the shelf—the metastable state in the shelving manifold—using the **854 nm shelving repumper**.

2.2.3.4 Two-photon transitions: EIT and dark resonances

A significant feature of Λ -systems is that, when the detunings of the two lasers from their respective transitions are similar, then they drive two-photon Raman transitions. This leads to a dressed system

whose eigenstates are either atypically bright or dark—i.e. the atom scatters more or less photons than simple rate-equation theory would suggest—depending on the laser detunings.

During the work carried out in this thesis, population has been moved incoherently between levels. The coherent methods necessary are not yet in place for unitary state evolution and hence quantum computation. Two-photon resonances manifest themselves in incoherent measurements as modifications to the fluorescence spectrum. The ion is optically pumped into either the bright or dark superposition of states at certain detunings as one laser is scanned across the broad atomic resonance. The dark features are an example of electromagnetically induced transparency (EIT), where one light field renders an atom transparent to a second. We thus observe losses of fluorescence, in features whose linewidth is much smaller than the natural linewidth of the transition. These features are termed ‘dark resonances.’ We use them to diagnose the magnetic field direction and strength in chapter 6.

More importantly for the future of the experiment, EIT provides us with a shelving method that strongly discriminates between the two ground sublevels.

2.2.3.5 Photoionization

Photoionization of calcium can proceed by a two-step process, and we have chosen this method. Firstly the electron is excited by the **423 nm neutral fluorescence laser** and then the **389 nm ionizing laser** is used to excite this electron further, to the continuum of ionized states. Laser spectroscopy of the 423 nm transition is performed using a crossed-beam method, where the neutral fluorescence laser is directed perpendicular to the diffuse atomic beam emerging from the oven.

2.2.4 Improvements required in the Oxford ion trap

The Oxford ion trap is a developing experiment, it has been necessary to make several improvements to the system. It has undergone gradual improvement in order to transform it from a working ion trap, in which single ions were trapped and observed, to a prototype quantum information processor, where ions might be manipulated coherently as carriers of quantum information.

Initially we were without a laser at 850 nm. We were unable to implement the necessary shelving methods without it, and therefore the IR shelving laser was built to provide us with a laser at this wavelength (see section 3.3). Our laser control was also based around a system of shutters. This was slow and noisy—the noise caused jitter in laser frequencies, which translated to poor state preparation and discrimination, and broadening of all transitions—and we therefore replaced it with an AOM-based system (see section 3.4).

In order to distinguish between Zeeman sublevels in the ion, two of which we wish to use as qubit states, we required good knowledge of and control over the magnitude and direction of the magnetic field in the trap. We built magnetic field coils and positioned these around the ion trap, as described in chapter 6. A series of diagnostic experiments was performed to deduce the field at the ion.

The ion trap RF voltages were both noisy and insufficiently strong. In chapter 5 we discuss the design, testing and construction of a new power supply for these RF voltages, and explain how it constitutes an improvement over the old supply.

Finally, we addressed our method of ion creation. The old method of ionizing calcium was electron bombardment, stripping the outermost electron from each atom during a collision. This is inefficient and does not select the isotope of calcium that we require. Often we would load isotopically impure ion crystals which were unstable and the loading procedure had to be repeated.

In addition this method affected the long-term stability of electric fields in the ion trap. After loading we found that charge had accumulated in the trap, leading to slowly-decaying fields which had to be compensated for, the compensation changing every few hours. With all of these problems in mind we implemented a new method of photoionization, using the two lasers discussed above and described later. Photoionization as a method of single-ion production has reduced preparation times from days to minutes, vastly improved the reliability of loading a particular isotope, and led to stable electric fields.

2.3 Physical properties of singly-ionized Calcium-40

2.3.1 Properties of the element and preparation of the ion

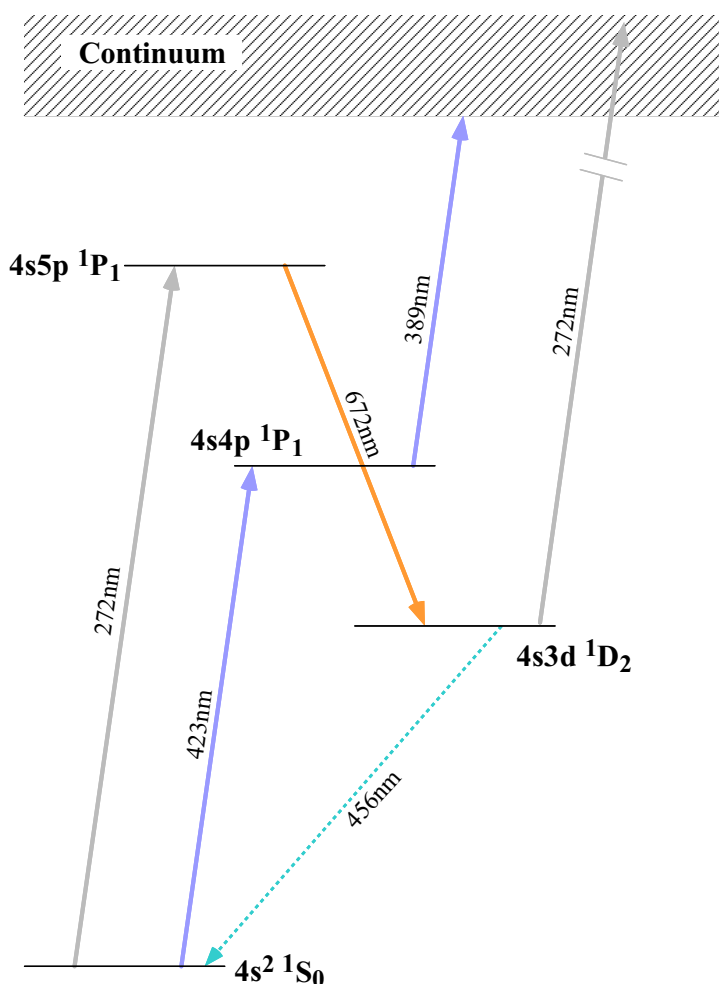


Figure 2.3: Structure and transition wavelengths of the energy levels in Ca I. The levels shown are used for two different photoionization schemes: we use the 423 nm/389 nm method, whereas some groups use intense 272 nm light in a three-stage process including decay on 672 nm. The dotted arrows indicate metastable transitions. The diagram is not to scale.

Calcium is a group II alkaline earth element, atomic number 20. The singly-ionized species is isoelectronic with potassium and therefore has a alkali-like energy-level structure [72]. The isotope

with atomic mass 40 has an even number of both protons and neutrons in the nucleus. Hence it has no nuclear spin and no hyperfine structure. It is by far the most abundant isotope in natural, unenriched calcium which consists of 96.9% ^{40}Ca [73].

Calcium may be ionized by many physical processes. The Oxford ion trap has, until recently, used collisional excitation/ionization. An oven effuses calcium gas, and this crosses the beam of an electron gun. Electrons collide with the output from the oven—all of the naturally-occurring calcium isotopes plus any impurities—and strip species of their valence electrons. The method is not selective. An alternative method is to photoionize neutral calcium. A schematic of transitions and levels important to this process is shown in figure 2.3, and we discuss the various processes more fully in section 7.2.

2.3.2 Electronic structure of $^{40}\text{Ca}^+$

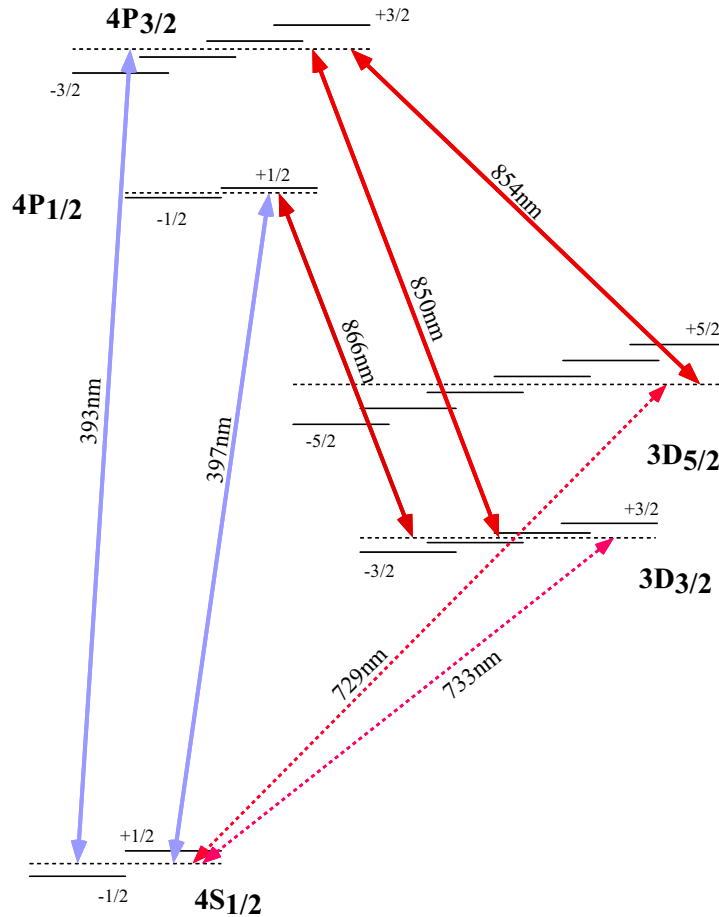


Figure 2.4: Structure and transition wavelengths of the five lowest-lying energy levels in $^{40}\text{Ca}^+$. The dotted arrows indicate metastable transitions that we do not address with lasers in our experiment. All Zeeman splittings are to scale relative to one another, but not relative to the level-level energies. The g_J factors for the splittings are given in table 2.1.

The lowest five energy levels of $^{40}\text{Ca}^+$ are shown in figure 2.4. The ground configuration is $4s^2S_{1/2}$. Above this are the $3d^2D_{3/2}$ and $3d^2D_{5/2}$ levels, and above those the $4p^2P_{1/2}$ and $4p^2P_{3/2}$ levels. The separations between these levels arises predominantly from the electrostatic interaction between the outermost electron and the ‘central field’ of the nucleus and remaining electrons

$S_{1/2}$	$P_{1/2}$	$P_{3/2}$	$D_{3/2}$	$D_{5/2}$
2	2/3	4/3	4/5	6/5

Table 2.1: Landé g -factor for the five lowest levels in the $^{40}\text{Ca}^+$ manifold.

(Russell–Saunders or LS -coupling [74]). The system is then perturbed by a relativistic interaction between the electron’s magnetic dipole moment and the central field, which has a magnetic component in the electron’s rest frame (spin-orbit coupling).

In an external magnetic field $\mathbf{B}$ the degeneracy of a level is lifted. The interaction energy between an atomic state of angular momentum $\mathbf{J}$ and the field is $g_J e \mathbf{J} \cdot \mathbf{B} / 2m_e$, where m_e is the electron’s mass and $g_J(L, S, J)$ is the Landé g -factor. For the levels of interest we list g_J in table 2.1. In first-order perturbation theory the energy shift of the sublevel $|M_J \in \{-J \dots J\}\rangle$ is $\mathcal{H}'_B = g_J \mu_B M_J B$, where μ_B is the Bohr magneton.

In the figure we see the sublevels detuned from the centre frequency of a transition (marked by a horizontal dotted line). This sublevel splitting is $\Delta E = g_J \mu_B B$ and the splittings in the diagram show the relative magnitudes of g_J .

2.3.3 Transition wavelengths and rates

The manifold under consideration consists of five levels and eighteen sublevels [75]. It has two dipole transitions from the ground level: to the $P_{3/2}$ and $P_{1/2}$ levels. Both of these excited levels are unstable with cyclical linewidths $\Gamma_{3/2}=22.9\text{ MHz}$ and $\Gamma_{1/2}=22.4\text{ MHz}$, and their transition to the ground state results in a photon of wavelength 393.5 nm or 397.0 nm respectively. The decay of $|P_{1/2}\rangle$ has a branching fraction of 6% to the metastable state $|D_{3/2}\rangle$ i.e. the A coefficient of this transition is $0.06 \times 2\pi \times 22.4\text{ MHz} = 8.4\text{ Ms}^{-1}$. This transition involves photons of wavelength 866.5 nm. Similarly $|P_{3/2}\rangle$ will decay to $|D_{5/2}\rangle$ or $|D_{3/2}\rangle$, at wavelengths 854.4 nm and 850.0 nm, with branching fractions 5.3% and 0.63% respectively. These three transitions to metastable states are all in the near-IR region of the spectrum. Finally there are two transitions from the metastable D levels to the $S_{1/2}$ level. These are one-photon electric-quadrupole transitions: the $|D_{5/2}\rangle \rightarrow |S_{1/2}\rangle$ transition is at 729.3 nm; the $|D_{3/2}\rangle \rightarrow |S_{1/2}\rangle$ transition is at 732.6 nm. These levels have lifetimes of 1.168 s and 1.20 s respectively.

2.4 The ion trap and vacuum system

2.4.1 Electrode structure

Our ion trap has square, axial symmetry. It is shown in figure 2.5. The dimensions are given in reference [37]¹. Two support blocks of Macor® are positioned on four threaded M5 bolts. These bolts are themselves connected to a feedthrough conflat in the vacuum system. Holes are drilled through these supports for the following electrodes:

4 off. DC electrodes 1.6 mm dia. The distance from trap axis to the nearest surface of each is $r_0 = 5.18\text{ mm}$.

4 off. AC electrodes 1.2 mm dia. The distance from trap axis to the nearest surface of each is $\rho_0 = 1.22\text{ mm}$.

¹Reference [76] gives different values. However, our recent experiments agree with reference [37] and we therefore believe these to be correct.

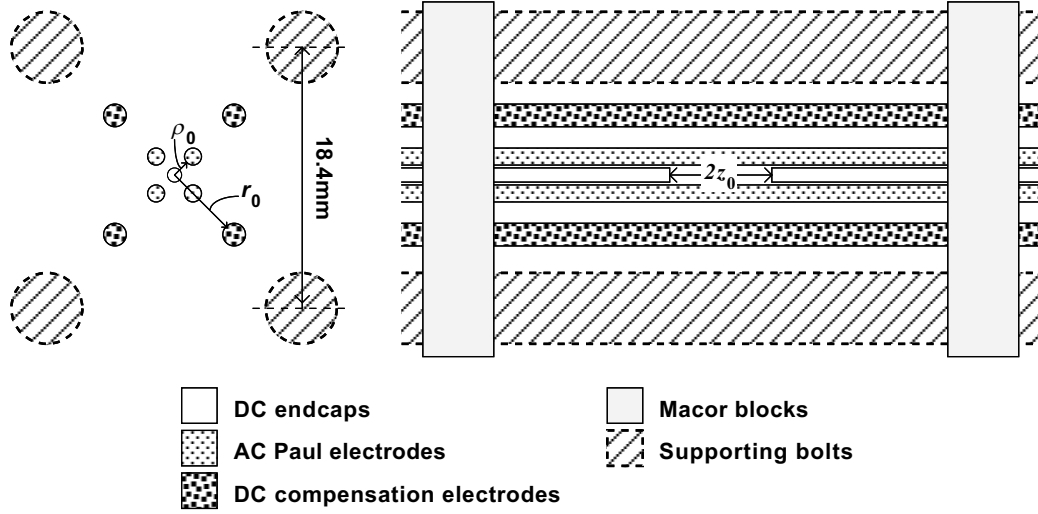


Figure 2.5: Schematic of the ion trap. Dimensions are given in the text. AC Paul electrodes carry the radio-frequency Paul trapping potential; potentials on the DC compensation electrodes are used to push the ions into the central node along the axis of electrode symmetry. The schematic has been scaled up by 1.8:1.

2 off. DC endcaps These are co-propagating with the trap axis, and have diameter 1.0 mm. The two endcaps are separated by a distance of $2z_0 = 7.2$ mm.

2.4.2 The form of the Paul trapping potential

The AC electrodes carry radio-frequency voltages. With the ‘old’ RF supply the voltages on each diagonal pair are in phase; the two pairs are in anti-phase with respect to each other. The new supplies ground one of the diagonal pairs: see chapter 5 for more details. The ideal Paul trap has hyperbolic electrodes [12]. The theory is approximately applicable to other geometries, however, and relies on a quadrupole expansion of the potential close to the centre of the trap. The results are approximately the same as in the hyperbolic case, with the potential scaled by ‘geometrical factors’ α_{AC} radially and α_z axially. In section 4.2 we discuss the nature of the potential in the trap.

Further out from the centre of the trap lie four more rod electrodes. These are compensation electrodes. DC voltages on these electrodes move the centre of the potential in the trap until it coincides with the node in the RF along the trap axis. An ion that does not lie along this nodal line has (undesirable) driven motion at the same frequency as the AC voltage: this is discussed in section 4.2.

2.4.3 Surrounding components and vacuum can

The closeness of electrodes to the trapping region limits the solid angle of light that can be collected to approximately 0.20 sterad. To reduce the scattered light entering the imaging system shielding has been placed over the top of the electrode arrangement with a hole of sufficient size to admit the required solid angle.

The vacuum can has hexagonal symmetry. The upper vertical conflat has a window, whereas the lower has a feedthrough and is otherwise blanked. This feedthrough carries current for the calcium oven and electron gun used for ion preparation. The trap axis lies along the long axis of the bench: one of the horizontal conflat along this axis is a feedthrough for the RF and DC voltages; the other

conflat leads via piping to a Varian VacIon ion pump model no. 911-5036.

The other four horizontal ports are windowed in order to permit laser access to the trapping region. On one axis the windows are separated from the can only by the conflat (the ‘close conflat’ axis). On the remaining axis (the ‘far’ axis) there are sections of piping between the central can and the windows. This piping provides vertical vacuum connections for an ion gauge (Varian model no. L8350301) and getter pump (SAES Getters GP50 2F series).

The precise location of the vacuum equipment and conflats dictates not only the possible axes along which lasers may be introduced to the trap but also the locations of magnetic coils. Coil construction will be described in detail in section 6.3. Circular coils are positioned over the top and bottom conflats, fitting tightly. Horizontally it is unclear whether one ought to follow the hexagonal symmetry in order to position the coils conveniently (and necessarily suffer coupled applied fields) or to uncouple the fields by placing coil pairs on orthogonal axes (and require the coils to be further away from the can). As we shall see both possibilities were investigated, and there were issues with each configuration.

2.5 Lasers used to address $^{40}\text{Ca}^+$ transitions

2.5.1 Grating-stabilized semiconductor lasers at IR and UV wavelengths

An advantage of using the low-lying manifold of singly-ionized calcium is that all the transitions are accessible by semiconductor laser technology [77]. It is possible to obtain more powerful light for such transitions using e.g. Ti:sapphire lasers [78]. However, this would still require a frequency-doubling cavity for the near-UV transitions and we therefore prefer to use blue laser diodes [79]. Diode technology also requires comparatively little maintenance.

The action of the semiconductor laser is based on electron-hole recombination [80]. The simple homojunction device is a forward-biased p - n junction. Electrons pass from the n -type conductor into a diffusion region (typically $1\text{ }\mu\text{m}$) where they recombine with holes and emit radiation. The laser mode diffuses out of the homojunction, mostly in the desired emission direction, but also into the surrounding material, where the gain coefficient is negative and the light is absorbed. This means that a very high current density is required for a net round-trip gain. Homojunction devices are run at cryogenic temperatures for this reason. The heterojunction principle sandwiches the active region in between a p - n junction with different band structure and refractive index. This simultaneously causes a high density of electron-hole pairs in the active medium, and refracts the laser back into the high-gain region; thus the threshold current density is reduced and the devices can be used at room temperature. Nonetheless, temperature control is necessary in these devices in order to sink the heat produced when the medium is active and laser output is observed.

The solitary heterojunction laser diode has a wide gain profile in frequency space and can therefore produce laser light at several different frequencies and be susceptible to undesirable optical feedback. In order to suppress the solitary resonances in favour of tunable resonances, the internal cavity is spoiled by an antireflection coating on the exit facet of the medium. Then an external cavity is set up. Such a cavity must combine partial retroreflection of the laser light with a frequency-dependent filter.

Typically we use a Littrow-configuration grating, retroreflecting the first order back along the path of incident light while permitting the zeroth order to emerge from the laser case [76]. Alignment of the retroreflection determines the frequency at which the external cavity is most likely to oscillate, and this mode quickly utilizes the majority of the available population in the gain medium.

Such external cavity feedback permits phase-continuous tuning of the laser², where the grating is translated and laser current varied synchronously so that both remaining internal modes and dominant external modes follow each other in frequency.

For economic reasons the availability of laser diodes changes over time. We had always considered moving from the frequency-doubled IR system for 397 nm light to a near-UV system. However, once the 794S laser diode had been destroyed, it was a major factor in our immediate transition to the blue laser diode that it was no longer possible to obtain IR diodes at 794 nm. Similarly the blue laser diode technology has moved to very specific wavelengths, and it is unlikely that more diodes will be constructed at wavelengths we require.

2.5.2 The 794 nm master/slave system

Until early 2001 light at 397 nm was provided by frequency-doubling the output from two IR laser diodes. These were arranged in a master/slave arrangement: the lower power diode was more stable in frequency and injected its output mode into the higher power diode, which had no other stabilization. The master laser 794M (commercially-built EOS2010, 15 mW output power) was tuned to 794 nm using the grating method. The output was coupled into the third port of an optical isolator and fed into the slave laser 794S thus. The 794S laser was constructed by David Stevens³ from an SDL 5422-H1 diode head, and had a maximum output of 150 mW.

The output of the two lasers was coupled into the TEM₀₀ mode of a bow-tie cavity containing a Brewster-cut lithium triborate (LBO) crystal. The cavity was aligned so that this mode optimized the efficiency of frequency doubling in the crystal by phase matching [80]. The cavity was locked to the laser frequency by the Hänsch–Couillaud method [81]. The output power of the cavity was initially 350 μ W. Overheating of the 794S laser reduced this to approximately 200 μ W. Over several years the quality of the cavity decreased, and at the point where it was finally dismantled the cavity was only producing around 80 μ W of 397 nm light.

The master/slave system was eventually replaced with a single blue diode laser. The frequency-doubling cavity was difficult to maintain. Its lock was unstable (although it relocked reliably) and its alignment drifted over the course of a few days. The cavity was kept in a non-airtight plastic enclosure to protect it from dust and sudden draughts. However, replacement was eventually made necessary by the loss of the 794S laser diode. This was connected up to a new power supply, which we believe gave a sudden voltage spike that damaged the diode junction irreparably. Lasers at such wavelengths and powers were no longer on the market and we dismantled the 794 nm, frequency-doubling system.

2.5.3 397 nm light and the 397 cooling laser

The frequency-doubling cavity typically gave out tens of microWatts of power. This was split using a $\lambda/2$ -waveplate and PBS cube, to give two beams: the 397D beam is generally used for Doppler cooling of the ion; and the 397 σ beam for optical pumping using circular ($\sigma^\pm$) polarization. The second beam was sometimes aligned (with linear polarization) through the top conflat of the vacuum system as shown in figure 2.7.

²By ‘phase-continuous’ we mean tuning without mode hops. This process is distinct from e.g. tuning using an intracavity filter to favour one of the comb of fixed external cavity modes. The latter may be termed ‘pseudo-continuous’ or ‘stepwise’ tuning [77].

³More details on this arrangement are available in Dr Stevens’ thesis [76].

Light at 397 nm is now provided by one laser, the 397 cooling laser, typically split into two beams. The laser and power supply were built by Toptica⁴. It consists of a 3 mW UV laser diode on a temperature stabilized mount, with collimating optic, grating and a steering optic. The diode is fabricated by Nichia [79] and is based on a GaN junction. This delivered several saturation intensities to the trap, in a similar size spot to the frequency-doubling cavity's output. With the AOM scheme in place (see section 3.4) it once more delivers around 30 μ W of power.

2.5.4 The 866 nm laser

The 866 cooling repumper laser was constructed within the group, using an SDL diode head model 5412-H1. It is grating-stabilized, with the grating mounted on a micrometer tilt stage. This stage has a very slow (several weeks) relaxation which gradually reduces the feedback on the desired mode. However it is also very easy to realign when its mode stability weakens.

We first used an absorption feature in iodine bromide gas to determine frequency relative to the $^{40}\text{Ca}^+$ 866 nm resonance (see later). Eventually high-finesse cavities from NPL replaced the IBr frequency standard. We currently lock the 866 laser to the side of a TEM_{00} transmission peak. When locked its linewidth is ~ 1 MHz. This laser is aligned along the 397 nm light into the trap. The 866 laser delivers approximately 2 mW to the trap.

2.5.5 The 854 nm/850 nm lasers

A third SDL diode package (model 5412-H1, 100 mW) was incorporated into a home-made system. The laser was assembled on a grating-stabilized mount. Its schematic is shown in figure 3.5. Its typical output power is $\lesssim 50$ mW. Because of the relatively broad gain profile of the IR diode lasers, it was possible to retune this laser between the two ionic resonances at 854 nm and 850 nm, and we did so several times depending on the experiment we wished to perform. Eventually the 850 IR shelving laser was built (see section 3.3) and the older laser was then permanently the 854 shelving repumper. The 850 laser delivers approximately 7 mW out of a fibre to the trap; the 854 laser gives less than 2 mW because of the low efficiency of an AOM in its beam path.

The 854 laser was for some time on a second optical table. It was sent down the same monomode polarization-preserving fibre that is discussed in subsection 3.3.8.1. This fibre was then aligned into the conflat windows at sixty degrees to the cooling lasers; when it was later used for the 850 laser the light entered the vacuum can antiparallel to this direction (see subsection 2.5.7). Because the 854 laser was well-behaved—it was often turned off and on, and always readily current-tuned back to the same wavelength—it is typically aligned only into a wavemeter. This made tweaking the grating alignment difficult: it was hard to see what, qualitatively, the laser mode was doing during such tweaks. However, such realignment is rarely necessary.

2.5.6 The 389 nm, 393 nm and 423 nm lasers

Like the 397 cooling laser, the 393 UV shelving laser and 423 neutral fluorescence laser were built by Toptica using TUI Optics components. The diode powers are 3 mW, 5 mW and 30 mW respectively, although the gratings in the 423 and 393 systems approximately halve their powers.

The 389 ionizing laser was built from a Nichia NLHU500X diode package, temperature-stabilized and mounted in a metal enclosure. The output mode was collimated [82] but not reflected off a grating. The laser provides sufficient power at a wavelength connecting the $4p\ ^1P_1$ state in Ca

⁴Then TUI Optics.

I to the continuum that it does not need to be grating-stabilized or kept resonant with the lowest edge of the continuum. The 423 laser mount was constructed so that the beam was aligned into an Oz Optics fibre coupler⁵. The 389 laser was also aligned into this fibre, with the two beams overlapping at a polarizing beam splitter. This permitted control of both beams simultaneously at the trap.

The 423 and 389 lasers are discussed further in section 7.3.

2.5.7 Directions of laser beams in the vacuum can

Depending on the experiment and our own convenience we realigned the lasers through different ports of the vacuum can at different times. A schematic of the can is shown in figure 2.6. We see six directions marked on the schematic. A and C, and B and D, make two antiparallel pairs. These pairs are at sixty degrees to each other, and to the trap axis (horizontal in this diagram). Directions B' and B'' are at a small angle to B (approx $1/20$ rad). E is a beam that first travels parallel to the bench at a height of 230 mm before reflecting off a mirror, 74 mm from the axis of the imaging system, to an angle of 26° from the vertical. This geometry is shown in figure 2.7. Later we aligned the 850 and 393 lasers almost co-propagating—the 850 laser is directed along B''—in order to reduce the Doppler width of the dressed state in our readout scheme, which is based on electromagnetically induced transparency (EIT).

Some configurations of lasers used in different experiments are given in table 2.2. These configurations correspond to the following experiments. We began by aligning the cooling lasers—397D and 866—along A. In order to observe Doppler effects in fluorescence along both radial trapping directions we sent the 397σ beam along E. Precision measurements of the $|D_{5/2}\rangle \rightarrow |S_{1/2}\rangle$ transition required the 854 laser, tuned to 850 nm, to shelve population into $|D_{5/2}\rangle$, and this was aligned along B.

Once we had added the 393 laser to the table we began observing the corresponding transition in Ca^+ . So that we would not have to wait for the ~ 1 s decay time from the $D_{5/2}$ level we retuned the 854 laser to 854 nm. We observed fluorescence and shelving on this transition, and used the 397σ laser for population preparation. We required independent polarization control over the 397σ (B) and 393 (B') lasers and so had to separate their beam paths by a few degrees. This necessarily gave a systematic polarization impurity during EIT experiments: the magnetic field was aligned along B (later B'') and the 850 laser. However, the power in the 393 laser during the experiment is much less than a saturation intensity, minimizing the effect of wrong polarizations.

2.6 Frequency references and measurement

2.6.1 Absolute frequency measurement

We used two wavemeters for frequency measurements. The wavemeter measures wavelengths by the interferometry of two beam paths of varying lengths [83]. A moving retroreflector inside the meter permits a photodiode to count interference fringes on the input beam and a He-Ne beam. The ratio of the number of fringes is equal to the ratio of wavelengths in air. The He-Ne beam also acts as a reference beam along which to align the incoming laser light.

The Burleigh WA-200 was replaced by the Burleigh WA-1500. The newer model works with much lower input powers ($20 \mu\text{W}$ as opposed to 0.1 mW), is more accurate ($\pm 0.2 \text{ ppm}$ rather than

⁵Coupler part number HPUCO-23A-442-P-3.9AS, serial number 58723-1.

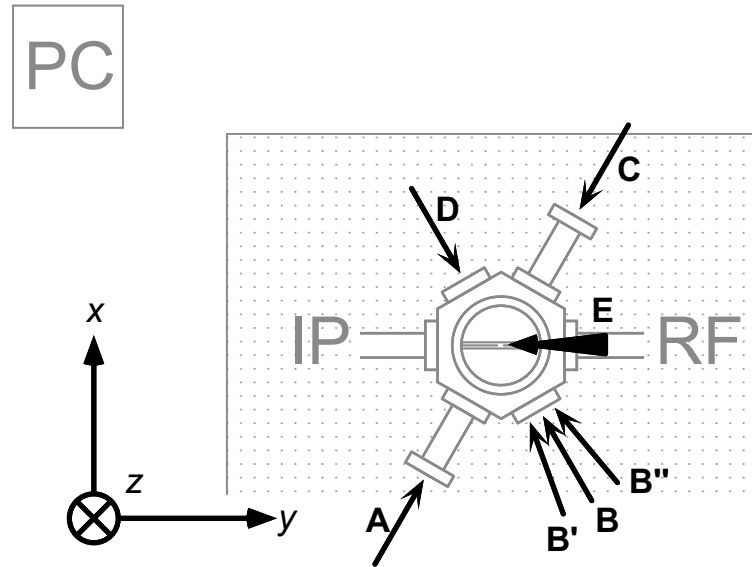


Figure 2.6: Paths of the laser beams into the vacuum can. This is a schematic of one corner of the main optical table seen from above. The ion pump IP and the RF feedthrough are connected to the vacuum chamber in the positions shown, and the acquisition PC is on a separate table. Depending on the particular experiment, different beams were sent along different paths. The vertical path E is both towards the ion pump IP and into the diagram; a side view is in figure 2.7. See table 2.2 for more details.

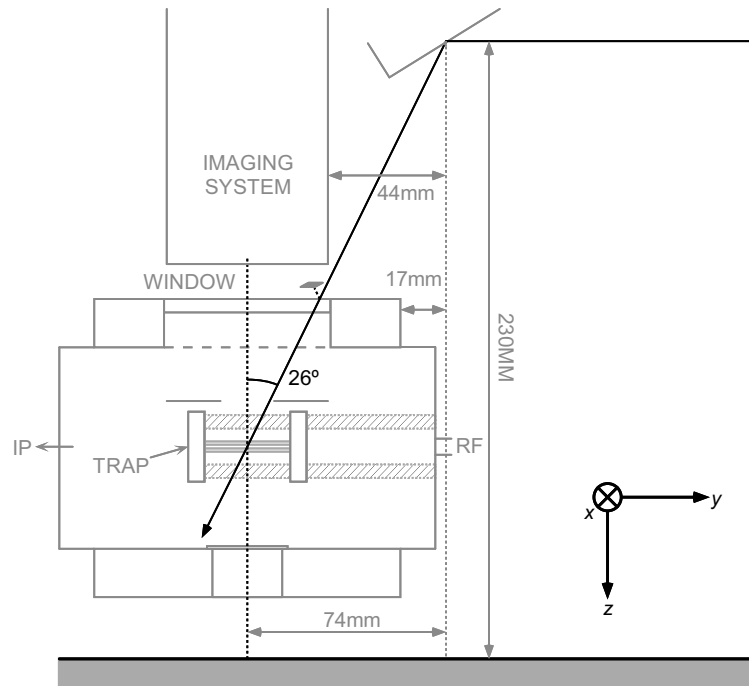


Figure 2.7: Alignment of the vertical 397 beam into the vacuum can.

Directions on figures 2.6 and 2.7						Notes
A	B	B'	B''	C	D	
397D,866						397σ
	'850'					Cooling, micromot ⁿ comp.
						Observe $ D_{5/2}\rangle \rightarrow S_{1/2}\rangle$
						Dark resonances
	397σ	393		854		One-photon readout; shelving
	397σ	393			850	EIT readout, counter-prop.
	397σ	393	850			EIT readout, co-prop.
	397σ	393	850			389,423
						Addition of photoionization

Table 2.2: Different configurations of laser beam alignment. With reference to figure 2.6 beams were aligned along the seven directions tabulated. Beams were rearranged over time, for reasons given above in note form. The 850 beam in quotation marks was actually the 854 laser, tuned to 850 nm. See text for more details

$\pm 3\text{ppm}$), and has separate modes for infra-red and ultra-violet measurement. Switching between these modes requires the wavemeter to be turned off and on: this was a disadvantage as the switch would occasionally spark, disturbing laser locking electronics and causing long-term damage to the wavemeter's internal fuse. The WA-1500 is able to correct for the refractive index of air.

2.6.2 Signals from gas lamps

It is relatively easy to find absorptions in a calcium hollow-cathode cell which correspond to neutral calcium transitions. In section 7.4 we show how the 423 neutral fluorescence laser was used to perform saturated absorption spectroscopy in such a cell. Far more difficult is to obtain information on the sparsely-populated singly-ionized calcium in the cell. This was, however, implemented as a frequency reference using optogalvanic methods [76]. Optogalvanism is the process whereby a laser's redistribution of atomic populations within a gas cell change the impedance of the cell. The hollow-cathode lamp we used was the Hamamatsu L2783-20NE-Ca. It consisted of a T-shaped, UV glass bulb with Brewster-angle windows. This glass bulb contained a buffer gas of neon at 6 torr. The maximum recommended discharge current was 20 mA at continuous operation. The lamp was powered by the constant-current Juniper 3–30 mA, model 127 power supply, model 1196-1. These voltage signals are small for the ionized species, even when the lamp was overdriven at 30 mA. The signal was observed by phase-sensitive detection.

For the 866 cooling repumper a better frequency reference was found in iodine bromide vapour (IBr). A T-shaped cell containing IBr gas was heated to 75 °C, while the windows were heated to 100 °C to avoid bromide deposition. The resonance line centre was found to be displaced from the $^{40}\text{Ca}^+$ line to the red by 650 MHz [75]. To avoid this frequency shift the laser was triple-passed through an AOM that operated preferentially at ~ 200 MHz. Because the IBr absorption feature was broad, the lock on the 866 laser was very slack. Although this was not important for initial trapping of atoms, later experiments needed a tighter lock and an NPL cavity (see below) was used instead.

2.6.3 Etalons for laser locking

Locks to low-drift etalons provide sufficiently accurate frequency references for our current experiments. In table 2.3 we list the specifications of the etalons used by the Oxford ion trap. The finesse gives the width $\Delta\nu$ of the cavity transmission fringe thus:

$$\mathcal{F} \equiv \frac{\pi\sqrt{R}}{1-R} = \frac{c/2l}{\Delta\nu}, \quad (2.1)$$

where R is the intensity reflectivity of one cavity mirror, and $c/2l$ is the free spectral range (fsr), dependent on the cavity length l .

Cavity	Mirror R / %	$\sqrt{R}$ / %	$\mathcal{F}$	$\Delta\nu$ / MHz	l / cm	$\frac{c}{2l}$ / GHz	Confocal?
Short, 794	57	75.5	5.5	359	7.578	1.9780	✓
Short, 850	95	97.5	61	32			×
Long	60	77.5	6.1	98	25.0	0.600	✓
NPL 1&2	UV/IR: 97	98.5	103	15	10	1.5	×
NPL 3&4	UV/IR: 99.4 UV-423 nm : > 95	99.70 97.5	522 61	2.9 25			×

Table 2.3: Parameters for the etalons used in laser locking. See text for more details.

Initially we worked with two home-made cavities. We inherited a cavity with a short spacer from a previous experiment [84] and used it to lock the 794M laser. This had mirrors with reflectivity optimized at a different wavelength from 794 nm, giving a low finesse for our purposes. The cavity had a slow drift rate of <2 MHz hr $^{-1}$, and its medium-term drift ± 200 kHz. Short-term noise of ~ 1 MHz was believed to originate in the locking electronics.

The second cavity was longer, and was assembled with mirrors of reflectivity 60% both in the UV (393–398 nm) and IR (840–880 nm) wavelength ranges [85]. This was initially used to lock the 850 IR shelving laser, and then moved to the 397 cooling laser. The mirrors in the short cavity were replaced with ones coated specifically for high reflectivity ($>95\%$) in both UV and IR ranges, and this cavity was used for the 850 laser lock until the arrival of new commercial cavities.

The home-made cavities were not piezo-tunable: the mirrors were mounted directly onto Zerodur spacers and did not have a piezoelectric crystal to vary the mirror spacing. Only the longer cavity was pressure-tunable; the shorter was continuously pumped down. They could both be slowly temperature-tuned. Four piezo-tunable cavities were obtained from National Physical Laboratories. The first two NPL cavities had 97% reflectivity at UV and IR wavelengths. The second two had higher reflectivity in order to permit tighter locks.

In addition a Melles–Griot spectrum analyser (model 13SAE906) was used as an etalon. This etalon has finesse ~ 100 at 400 nm, and cavity length $l = 250$ mm gives a fsr of 300 MHz. We found that the drift of this etalon was slow, of order 10 MHz hr $^{-1}$ [86], when its piezo was not scanned.

2.7 Cameras and imaging

Our optics for imaging the trapping region are designed to observe near-UV photons (wavelength ~ 400 nm). A wide aperture Nikon compound lens images the trap onto a small moveable aperture with a magnification of 4.1. The aperture blocks stray light. It is re-imaged onto a PMT with unity magnification. A moveable beam splitter directs a fraction of the light from the small aperture onto a camera, with magnification factor 2. More details are available in reference [37].

Initially we used the EEV SuperPhoton Low Light ICCD camera (ICAM2-07-06E) to capture real-time images of the trapped ions. This camera had an integral image intensifier. It was very sensitive to scattered light, and an automatic shutter was required to protect it from high light

levels, especially the black-body radiation of the hot electron gun. The EEV was not triggerable, but transmitted frames at a rate of 50 Hz.

The EEV was replaced by an Andor DV437-BU2 CCD camera (serial no. CCD-2667-LM-C). The integral Marconi CCD57-10 device has a slower readout rate, which reduces readout noise; it also has lower dark current. To increase its speed, additional on-chip integrated circuits in the Andor CCD system permit caching of horizontal CCD tracks, so that several narrow images can be stored on the device, before the contents of the whole device are acquired. The camera can be gated at any point and has a variable shutter speed. Furthermore, a Peltier unit in the camera permits the CCD to be cooled under vacuum to -75°C , reducing the dark current still further.

Imaging of laser beams is carried out using the Pulnix and Hitachi cameras, and several Maplin camera modules mounted in homemade boxes. The Pulnix PE2015 (serial no. 01700) has a minimum signal-to-noise of 50 dB; the Hitachi KP-120E/K has a slightly higher signal-to-noise ratio and minimum illumination. However, in practice both cameras were used interchangeably. The Maplin MS39N module has a low signal-to-noise ratio, but also has an automatic gain modulation. This means that the output data are not linearly dependent on the light falling on each pixel. Thus the Maplin is predominantly used for alignment of e.g. beams through cavities, rather than intensity-dependent measurements e.g. profiles of the laser beams at the trap position.

Acquisition of data from all cameras except the Andor CCD is achieved via the National Instruments PCI-1408 IMAQ card. A set of MATLAB routines were designed by Dr David Stevens to use the IMAQ drivers. Images could then be manipulated in MATLAB as 2D arrays. The Andor CCD system includes a PCI controller card, controlled by a proprietary graphical user interface. This interface was able to save acquired images in several different formats for later manipulation.

2.8 Quantum computation in the ion trap

2.8.1 Introduction

The Cirac–Zoller scheme [44] revealed the potential of a physical realization of quantum computation in an ion trap. An ion trap is an appealing system for quantum computation. The ions can be well isolated from the environment and the internal states of one ion will in general have a negligible effect on those of another. Yet interactions between ions, and coherent manipulation of the state of one ion, can be switched on instantaneously using either optical or radio-frequency transitions.

In this section we discuss the feasibility of our ion trap as a quantum information processor. As quantum computation is a long-term goal of the project, it is useful to see how different aspects of the trapped ions correspond to logical components in a quantum computer. The general mapping of any ion trap’s physical components onto DiVincenzo’s list (given in section 1.2 has been carried out elsewhere [87]. Here we specifically discuss the Oxford trap and how well it fulfils the five criteria.

2.8.2 Qubit states and scalability

In the $^{40}\text{Ca}^+$ manifold we use the Zeeman sublevels of the ground state as qubit states. Whether these states are well-defined or not depends on the stability of the magnetic field within the vacuum can. We shall show in section 6.5 that the ambient laboratory field uncertainty is sufficiently low to permit reliable qubit rotation. We have observed no evidence for correlated decay of $|D_{5/2}\rangle$ in our ion trap [67]. This rules out one possible method of cross-talk between qubits which has been observed in other laboratories.

2.8.3 Initialization and temperature

We believe we can prepare each qubit in either of its states with close to 100% reliability. As our readout method is still in its infancy [88] this cannot yet be confirmed. However, optical pumping on the σ^+ 397 nm transition (see section 6.4 for details) yields fluorescence levels low enough to imply such preparation.

We are currently operating at the Doppler cooling limit in our ion trap. A temperature of $\hbar\Gamma_{1/2}/2k_B = 0.5$ mK implies mean phonon numbers of 6 and 62 in the radial and axial motional states respectively (see section 5.2). Sub-Doppler cooling has been demonstrated in ion traps elsewhere e.g. reference [55]. Our intention is to initially perfect the readout method. Then we will be able to diagnose Raman transitions between the $S_{1/2}$ sublevels, stimulated by the 397 cooling laser far-red detuned from the 397 nm $|S_{1/2}\rangle \leftrightarrow |P_{1/2}\rangle$ transition. We will implement sub-Doppler cooling by e.g. detuning to a sideband of the carrier transition. These sidebands correspond to changing the phonon number in the quantized motion of the ion crystal, and sufficient pumping on the red sideband will reduce the temperature of the ions to close to zero.

2.8.4 Decoherence rates

In principle, the ion trap provides an environment with decoherence low enough to implement error correction (qubit manipulation errors less than 1 part in 10^{-4}). The heating effects in our ion trap are discussed in section 4.3. We observe that we have been able to keep an ion overnight in the trap, and this restricts one type of heating mechanism to a level where it does not limit the decoherence of states in the trap. Other sources of decoherence are not yet characterized. It is expected that the Raman transitions will not be limited by laser bandwidth: if both Raman beams are generated by the same laser, and have a well-defined frequency difference between them, then fluctuations in the absolute frequency of the laser will have only a second-order effect in qubit rotations. Magnetic field noise is likely to be the limiting process.

2.8.5 Logic gates within the $^{40}\text{Ca}^+$ manifold

So far we have not demonstrated coherent behaviour in the ion trap. Several methods to implement two-qubit logic gates in ion traps have been proposed, and some have been implemented e.g. reference [89]. We believe the most promising for our experiment are those based on a state-dependent pushing force applied to the ions [41].

This gate is implemented as follows. A laser is far-red detuned from a transition exciting only one of the two qubit states. This beam is then focussed to a tight spot, close to the ions. A semi-classical dipole force arises owing to the electric field gradient of an intense beam. The AC Stark shift of the energy of the lower level in the transition is downwards i.e. each ion will tend to move towards the more intense part of the beam. When the beam is switched off the ions return to their original positions. However, if the ions' states are such that they have been temporarily moved relative to each other, their wavefunctions will accumulate a phase shift from the ion-ion electrostatic interaction. Thus the total wavefunction will have a phase shift dependent on the states of more than one ion, and we have implemented a phase gate.

In the Oxford ion trap we will detune the 397 laser from its corresponding transition. By ensuring the laser is circularly polarized then one of the two qubit states will decouple from this light field and hence an ion in this state will not experience the dipole force.

2.8.6 State measurement

Our choice of qubit limits our ability to read out the state of the ion.

Other schemes, which use hyperfine sublevels [90] or one of the states in the $D_{5/2}$ level [91], make the readout easier. However, a hyperfine-split level hierarchy requires more lasers to address all transitions, and addressing the dipole-forbidden $|D_{3/2}\rangle \leftrightarrow |S_{1/2}\rangle$ transition requires a complex laser system with high power and/or narrow linewidth.

We hope to carry out cooling, state preparation and qubit rotations and interactions in a low magnetic field (~ 1 G). As the ambient field is increased, all of these operations suffer. Cooling is less efficient as the cooling transition is broadened. State preparation and ion addressing is made more difficult as the lasers must be detuned between components of each transition. Eventually the splitting of the levels is comparable to hyperfine splitting and we encounter precisely the difficulties inherent in those hierarchies. However, the electric-dipole allowed transitions from the ground state have linewidths much greater than the Zeeman splitting at such low fields: they cannot distinguish well enough between the $|0\rangle$ and $|1\rangle$ states. Therefore some aspects of the experiment favour application of much higher fields. This trade-off between the use of small and large fields implies that we should switch between two different field regimes during quantum information processing and readout.⁶

To overcome the difficulty of distinguishing between the two Zeeman states in low field, our readout method is split into two parts. Firstly we move the population of one qubit state—not necessarily coherently—into another metastable state, a process known as ‘shelving.’ This state must be outside the cooling manifold, and stable enough that we can observe fluorescence from the remaining ground state population before the metastable state decays. The obvious choice from figure 2.4 is $|D_{5/2}\rangle$.

I. SHELVING ONE QUBIT STATE Our difficulty in perfecting an efficient readout method has been the shelving process. Initially we hoped to quickly ramp on a high magnetic field, and excite the $S_{1/2} \leftrightarrow P_{3/2}$ transition to pump the ion into the $D_{5/2}$ state [69]. However, as we shall see in chapter 6, the field switching created problems in state preparation. Thereafter we moved to two- or multi-photon methods. Our current readout scheme involves driving the $|D_{3/2}\rangle \leftrightarrow |P_{3/2}\rangle$ transition with very intense 850 nm light. When the much weaker 393 UV shelving laser is scanned across the $|S_{1/2}\rangle \leftrightarrow |P_{3/2}\rangle$ transition then, depending on the polarizations of the two lasers, population from only one ground-state sublevel is excited to $|P_{3/2}\rangle$ and thence shelved. Recently we have had our first successes with this shelving method. We observe⁷

$$P_+ \sim 0.6 \quad , \quad P_- \sim 0.2. \quad (2.2)$$

For readout experiments we interpret this as follows. Suppose the populations of the two qubit states undergo Rabi oscillations. Because of imperfect state discrimination the oscillations observed over a large number of experiments will have visibility $\Upsilon = 0.5$.

II. FLUORESCENCE DISCRIMINATION WITH THE EEV We studied the statistics of images of fluorescing ions acquired with the EEV camera. The frame acquisition time was 10 ms with negligible dead time. 1000 frames were analysed, and we show 25 taken at random in figure 2.8(b).

⁶Difficulties in field switching led us eventually to an intermediate field regime, and a subtle method for state discrimination. See chapter 6 for more details.

⁷This notation is discussed in appendix A.2

In each picture we analysed the fluorescence in a number of pixels over an ion, and a number of pixels over the vacant position at the end of the crystal. This gives us a good estimate of the systematic effect of both noise and the edge of a nearest-neighbour ion's fluorescence profile on the possibility of discrimination. A histogram of the results is shown in figure 2.8(a).

If, in our analysis, we optimize the threshold level below which we assume the ion is in a dark state, then we obtain 98.4% discrimination efficiency from a single frame i.e. at 10 ms acquisition time. If we were to observe for e.g. twice as long, then we improve our efficiency to 100% within the accuracy of our statistics. However, the decay fraction of the shelved population increases from 0.9% to 1.7%. This decay occurs to either qubit state, spoiling our discrimination. There is no noticeable difference if we average over 7×7 pixels (as we did in the histogram shown) or e.g. 5×5 pixels. This implies that the signal-to-noise could be improved by focussing the ions onto fewer pixels using the telescopic section of the imaging optics. However, this improvement is not currently necessary, as shelving is the predominant limitation on the accuracy of our readout method.

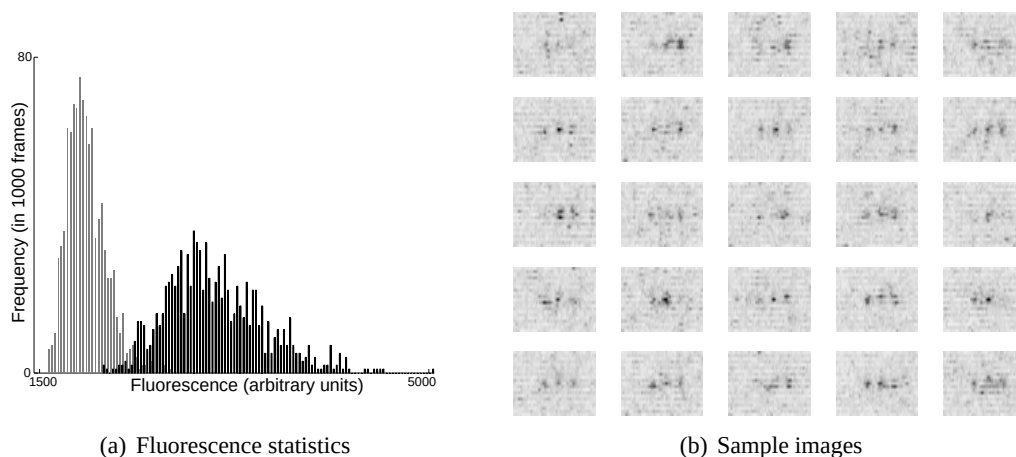


Figure 2.8: Discriminating between fluorescing and non-fluorescing ions. 1000 camera images of a three-ion crystal were analysed (25 sample images are shown on the right). The histogram on the left is obtained by summing the signal from pixels over an ion, and the noise from other pixels, for each camera image. The grey data is the background noise; the black data is the signal from a fluorescing ion. By varying the threshold on the x -axis, below which we assume an ion to not be fluorescing, we were able to discriminate between a fluorescing ion and the noise with 98.4% accuracy.

2.9 Experimental procedure

2.9.1 Principle

Currently we mostly perform experiments on a single ion. This avoids complications associated with ion crystals, such as phase transitions. It can take anywhere between half an hour and a whole day to move from switched-off apparatus to a single ion. Much of this time is dependent on how much apparatus must be prepared. An entirely ‘cold’ experiment can take much more than a day to switch on, and to ensure the lasers are at the right frequencies and alignments. It is rare for our cavities’ pumps and ion gauges to be switched off, but if they are then the larger homemade cavity can take over a day to pump down to a steady state.

A less predictable source of delay is the removal of many ions from a crystal. This is in part a stochastic procedure, and often we must reload a cloud of ions several times, having removed

both ions from a two-ion string instead of only one. Since the introduction of the photoionization loading procedure (see chapter 7) this delay has decreased. It is now possible to load either a few ions or a single ion in a matter of minutes.

2.9.2 Preparation

The experiment is prepared as follows. All cavity pumps and rack power supplies are switched on, if they are not already on. The blue diode lasers, and any immediately necessary IR lasers (typically the 866 cooling repumper) are put at a lasing current. The blue diode lasers especially need to warm up for an hour before they can reach the required frequency at their highest current modes. AOMs and shutter supplies to control the lasers are turned on. The imaging optics is switched on, including any cameras on laser beams. Both experimental computers are switched on, and the PC begins acquisition of PMT counts.

The lasers' alignment is checked. Initial checks are performed on the cameras. We verify that all beams are entering and (where appropriate) leaving the vacuum can. We acquire images using MATLAB routines on the second PC, and check the position of the centres of the beams. Using the Pulnix and Hitachi cameras we verify that the beams are close to the positions used in the previous day's experiments. Later we will align most lasers using the ion fluorescence. However, we note that the ion's fluorescence is not a monotonic function of the 866 nm laser intensity. As we shall see there are a number of reasons for this. Two possibilities are optical pumping effects that we shall discuss in chapter 6: the laser detunings may be close to resonance on a particular two-photon transition which pumps the ion into a dark state, and this may be intensity-broadened as the 866 laser is peaked up on the ion; optical pumping on the 866 nm transition at low magnetic field puts the ion into the extended M_J states. At any rate we align the 397 cooling laser on the ions, then mark this position. The next day we align both the 397 and 866 lasers on this same position.

Electrode voltages are set. We typically trap with endcap DC fields of $V_{DC}=95$ V and oscillating RF voltages of $V_{AC}=80$ V at $\Omega_{RF} = 2\pi 6.29$ MHz. After trapping these are reduced to 30 V and 40–50 V respectively. The voltages on the top-left (TL) and top-right (TR) compensation electrodes are set dependent on the most recent behaviour of the experiment.

2.9.3 Trapping and removing ions

The 397 laser must be locked during ion loading. This is to stop its frequency wandering either to the blue of the transition or too far to the red to provide efficient cooling. The 866 laser needs only to repump, not to cool, and can therefore be unlocked.

The oven is switched on at 4–6 A and allowed to warm up. The photoionization lasers can be shone on the atomic beam shortly afterwards. If the electron gun is used then empirically we find that it must be switched on a couple of minutes after the oven. Depending on the intensity of the light scattered into the imaging optics we close the shutter leading to the camera/PMT. When the ionizing agent is switched off we reopen the shutter. Scattering is observed from the 397 laser. We therefore test for the presence of ions by checking for a difference in the signal. If there are sufficient ions this difference is clear, compared to the PMT counts before loading. If we have only loaded one or two poorly-compensated ions, then we observe PMT counts both with and without the 866 repumper (see section 3.2). The difference between the counts is ion fluorescence.

If more than the desired number of ions has been loaded by photoionization, then the trap is emptied by destabilizing it, and the loading is recommenced. However, when we use the electron gun to ionize calcium atoms it is preferable to remove ions selectively from a load rather than restart

the loading procedure. We can heat the ions by applying a ‘tickle’ voltage to the endcaps. We apply a low radio-frequency voltage that sweeps through the secular frequency of the trap. As it does so we see a loss of fluorescence, and if the tickle voltage is high enough then the signal does not fully recover: ions have been lost. Another method of removing ions is to destabilize the trap. As the radio-frequency Paul voltage’s amplitude is changed the trap becomes unstable for certain charge-to-mass ratios. The stability boundary is ‘fuzzy’ and we can therefore destabilize the trap sufficiently that only a fraction of $^{40}\text{Ca}^+$ is lost. In principle we can also purify the isotopic content of the trap this way, by destabilizing the trap for other calcium isotopes and non-calcium species. With collisional ionization we often trap large clouds of ions. When only a few ions remain they undergo a phase transition and form a linear crystal. Species other than $^{40}\text{Ca}^+$ do not fluoresce at the same laser detunings—they are ‘dark ions’—and merely act to heat up ion crystals and prevent their formation.

We compensate for stray DC fields in the trap by applying voltages to the compensation electrodes. We find that changing the compensation voltages is necessary if either the electron gun has been used recently (which generate patch potentials, decaying exponentially with a time constant of 6.3(7) hr [37]) or if the RF supply has been swapped. This may be because the impedance of the electrodes is different at different frequencies, leading to a change in the position of the potential. We can compensate a large cloud or an ion string, but it is easier to compensate a single ion. In addition the single ion often needs slightly different compensation voltages from the many-ion crystal or cloud.

With our setup it is sufficient to apply voltages to the top-left (TL) and top-right (TR) electrodes and ground the other two. If we change the difference between the voltages, this moves the ions horizontally; if we change the sum of the voltages they move vertically. The micromotion of an ion in a 2D Paul potential is orthogonal to its displacement from the centre of the trap. We are sensitive to the Doppler effect of driven motion of the ions along a laser beam axis; hence to compensate fully we require both horizontal and vertical 397 beams. We note in passing that, when the new RF systems are connected (with two diagonally opposite electrodes earthed), the resulting trap is a 3D Paul potential, and there is micromotion along all three orthogonal axes.

2.9.4 Example experiments

Once we have determined that the required number of ions remain, and that there are no dark ions, we remove the beam splitter to the camera and image all fluorescence on the photomultiplier tube. This maximizes the signal at the expense of knowledge of the localization of the fluorescence. When we wish to perform many-ion readout we will require either the camera or a well-focussed laser beam that only addresses one ion at a time.

We perform most experiments with one or two ions. The photon counting program works in real-time and we can therefore use it to observe transient signals and check the alignment of laser, field and imaging optics. Other experiments require the program to be used in a passive data-acquisition mode. These typically involving varying one or several parameters and repeatedly observing fluorescence in order to either construct a spectrum or build up statistics. It is simplest to change a laser frequency, trap voltage or coil current in small increments. Laser polarizations and orientations are not currently controlled by the computer; some laser intensities are through variable attenuation on their AOMs.

For spectra we scan e.g. either laser in the cooling transition and observe the single-photon transition plus dark resonances, as shown in section 6.5. This scan can be performed for any other lasers e.g. with the 393/854/850 lasers addressing the ion and providing an alternative cooling

manifold.

Pulsed experiments are often more complicated. A sequence of laser pulses cools and manipulates the internal state of the ion in some way, and then fluorescence is detected for a short period. This is repeated many times; then a parameter is adjusted and the sequence recommences. Our readout method is an example of the pulsed experimental method. For readout we collect statistics on the number of times fluorescence is observed. Thus we are able to measure the relative populations of our qubit states and, through variation of parameters, determine the precise orientation of the qubit's spin before readout.

Chapter 3

Construction, design and improvement of apparatus

3.1 Introduction

This chapter is concerned with the construction work behind this thesis. The Oxford ion trap is an experiment under ongoing development. At the beginning of the work discussed in this thesis, it was possible to trap clouds and strings of ions, and single ions. In order to proceed with the ‘grand plan’ of the Oxford group—to manipulate the internal and external states of all ions within a string coherently and demonstrate simple quantum-computation algorithms—it was necessary both to improve existing apparatus and to build new items. Where construction has been part of a larger project discussed elsewhere in this thesis, then it is described along with the project rather than in this chapter.

There are three construction projects in this chapter. The first is a redesign of the photon-acquisition system. As described in section 2.7, we collect a fraction of the ion’s fluorescence and image it on both a PMT and a camera. A system was built, including the experimental PC, to both count the photon arrivals in a specific time interval, and to compare the statistics of the delay between photon arrival times and the trapping RF potential. Next we discuss the design and construction of a new diode laser at 850 nm. Finally we describe the conversion of our shutter-based laser intensity switching into a system based entirely around AOMs. The new system had the advantage of speed and minimum vibrational disturbance to the lasers, whilst having disadvantages of lower laser intensity and alignment difficulties.

3.2 Observation of photons

3.2.1 Introduction

All of our deductions of the internal state of an ion in our trap begin with either an observation of ion fluorescence or of its absence. As described in the experimental overview in chapter 2, an imaging system above the trap collects a fraction of the ion’s fluorescence and focuses it on either a CCD camera or a PMT. Signals from the PMT may then be collected and processed.

Until recently this processing was effected by the Stanford SR400 gated photon counter. This was separate from the experimental PC. As a black box solution it was an extra experimental complication and communicated slowly with the PC. It has since been replaced, therefore, with a

circuit constructed from simple modular components and acquisition cards already present in the PC. The new system is faster and more versatile.

This section touches briefly on the old system, and then goes on to describe the new circuit in detail. The circuit separates neatly into two parts. The first observes photons and records them as a fluorescence rate. It is complicated by the need to gate the photon counters synchronously with the 866 cooling repumper, and this is discussed. The second part of the circuit deals with time-amplitude conversion (TAC) of the delay between photon arrivals and a particular phase of the RF trapping voltages. Correlations are to be observed in a histogram of fluorescence against this delay.

3.2.2 The Stanford SR400 system

This system is described in Charles Donald's thesis [37] and we summarize it here. The Stanford SR400 photon counter was used both for fluorescence observation and for correlation experiments. For the latter it was triggered by the positive zero-crossing of the RF voltage. It then waited a variable delay Δ before acquiring photons in a gate of width 10 ns. The photon counts of $2 \cdot 10^6$ such gates were summed into a counting bin which was communicated to the PC. If each cycle of the RF triggered the SR400 then approximately 0.32 s was required to acquire one bin; however, the counters required a delay of $2 \mu\text{s}$ between acquisitions, and one data point was thus acquired every 4 s. Δ was then incremented by 10 ns and acquisition continued for the next bin. Nearly 16 bins were required to span the whole of one period of the RF.

As a tool for manoeuvring the ion into the central node of the RF potential the SR400 was capable, and provided useful information on the ion's correlation spectrum and the harmonics of the RF therein. The acquisition time—over a minute for one period of the RF—limited its usefulness. In addition communication between the acquisition PC and the SR400 was via a serial cable. This was a severe limiting factor on acquisition speed for both fluorescence rate and correlation measurements.

3.2.3 Fluorescence rate measurements using the 8254 counter

The circuit shown in figure 3.1 now replaces the SR400/serial interface during fluorescence observation. The acquisitional component of this circuit is the Keithley DAS8A0 ISA card within the PC. This card is a counter/DAC and has an Intel 8254 programmable timer with three counters (C0, C1 and C2). The configuration of the counters during typical fluorescence acquisition is shown diagrammatically in figure 3.2, and described below.

In general we count with a timebase period of $\tau_{\text{PC}} = 8333 \mu\text{s}$. This reference frequency is provided by C2. It is programmed to count $8333 \times 10/2$ cycles of the underlying 10 MHz clock frequency before inverting the TTL logic state of its output C2 OUT. C2 OUT is connected to the GATES of C0 and (via an inverter) C1. When a GATE is high the corresponding counter is active. Thus C0 counts incoming photon pulses for half of the timebase period, and C1 for the other half. All counters are sensitive to TTL logic pulses of duration greater than 50 ns, and so the 9 ns TTL pulses from the PMT are stretched by a pulse stretcher designed by Johan Fopma. The module reference number is CEG j01/079 and its timing is shown in figure 3.4. The 9 ns pulse is stretched to $t_1 = 25\text{--}100$ ns, with a dead time of t_1 immediately afterwards. When $t_1 = 50$ ns this ensures a maximum pulse rate of 10 MHz, the maximum specification of the counters.

We call the C0 counts 'A' and the C1 counts 'B'. In software we sum a number (typically 20) of A, B pairs of 'triggers' into a counting bin and record this count. The PC can display both sets of counts separately, at the same time as either their sum or difference. If we have a good signal from

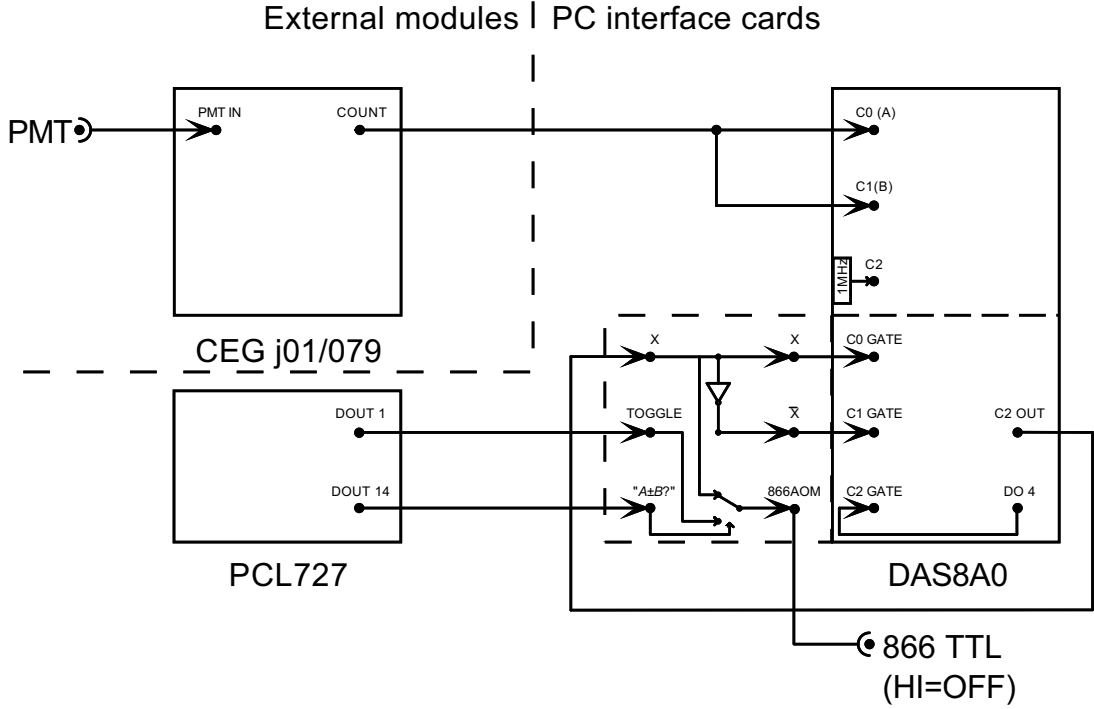


Figure 3.1: Circuitry for observing ion fluorescence. The PMT signals are stretched so that the DAS8A0 counters can reliably register their arrival. The rest of the circuit is used to synchronise fluorescence observation with the 866 laser's on/off state and is described in the text.

an ion or a cloud we display $A + B$. If, on the other hand, we are searching for small fluorescence signals, we connect the 866 AOM TTL input line to C2. The 866 laser is alternately deflected into the trap or blocked, at the double-pass AOM. During the A half-cycle the laser repumps the ion from the $D_{3/2}$ level and we observe fluorescence at 397 nm; during the B half of the cycle the ion is quickly pumped into $|D_{3/2}\rangle$ and remains dark. Hence the $A - B$ counting mode improves the visibility of small, noisy signals. Logic by Simon Webster gives automatic control over the 866 AOM. A digital output DOUT 14 on the Advantech PCL727 board operates a solid-state relay depending on the counting mode ($A \pm B$) and permits control either by a user command on the PC or C2 OUT.

3.2.4 Photon correlation measurements

We discuss in this section an extension of the circuit used to observe fluorescence rates. The extra connections and modules are shown in figure 3.3. This new circuit compares the arrival times of the photons with the phase of the RF. The time delay between the two arrivals is converted into an analogue voltage which the PC's analogue-to-digital converter (ADC) samples. Software on the PC then acquires a histogram of very many TAC voltages and displays it appropriately.

The Ortec 566 TAC module requires NIM-compatible pulses rather than TTL. Therefore the pulse stretcher module described above also produces a NIM pulse at the leading edge of the 9 ns PMT pulse, also 9 ns long. The pulse is fed into the TAC. This is the heart of the photon correlation measurement. The PMT pulse starts the TAC circuit which begins to time the delay between its arrival and the arrival of the stop pulse.

The RF voltage passes through 0 V in a positive-going direction once each cycle. The Ortec 583

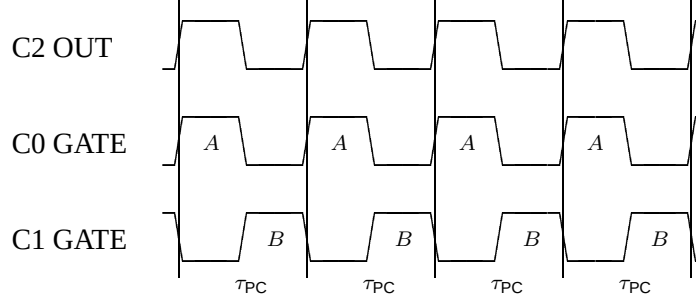


Figure 3.2: Timing diagram for fluorescence acquisition. C2 OUT is a square wave which gates the two other counters on the 8254 chip. When gated, these counters count incoming TTL pulses from the PMT.

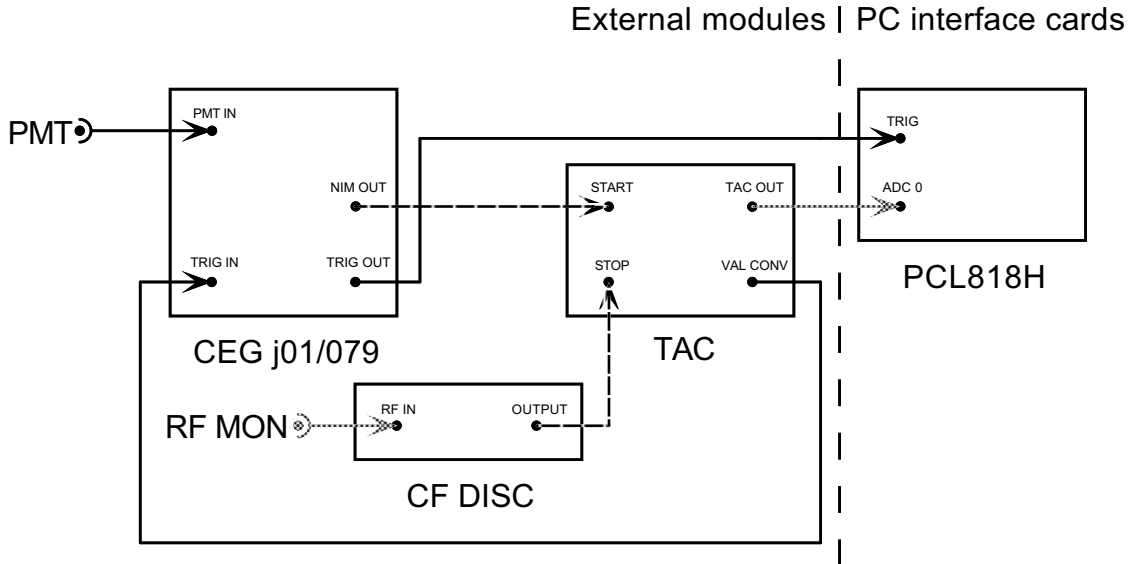


Figure 3.3: Circuitry for observing the correlation of photon arrival time with the phase of the trap RF. The circuit behaviour is described in the text. All signals are TTL-compatible except grey arrows (analogue voltages) and hashed arrows (NIM-compatible).

constant-fraction discriminator (CF DISC) produces a NIM pulse when the monitor voltage V_{MON} (see section 5.2) passes through a preset voltage level in a preset direction. This pulse stops the TAC counter. Manual controls on the CF DISC permit configuration of the pulse length. During the stop pulse the TAC does not register incoming start pulses from the pulse stretcher module; long stop pulses therefore contribute dead time in the photon correlation system.

Once the TAC has received a start and stop pulse it raises TAC OUT to a voltage proportional to the time delay between the two pulses. This voltage has been calibrated at:

$$V_{\text{TAC}} = 0.0426t, \quad (3.1)$$

where V_{TAC} is in volts and the time delay t in nanoseconds. It maintains this pulse for a duration specified by the WIDTH control. From the TAC this output is fed into the Advantech PCL818H 100 kHz ADC.

The TAC's settings are as follows. The RANGE is set to 200 ns, as we require at least 160 ns

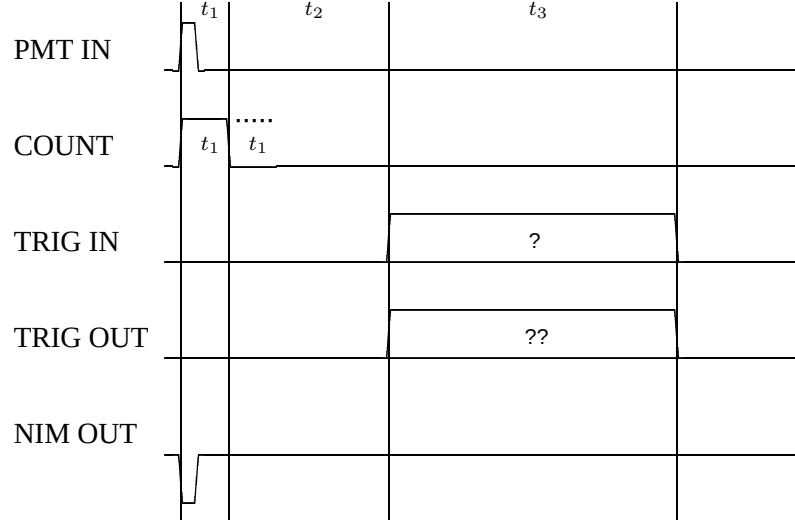


Figure 3.4: Timing diagram for the pulse stretcher/NIM converter. A short input pulse (PMT IN, 9 ns) is stretched for the Intel 8254 counters (COUNT) to a time t_1 . Also a trigger, delayed by t_2 and of duration t_3 , for the ADC may be produced, and a NIM-compatible pulse (NIM OUT) is begun at the leading edge of the PMT IN. Production of this trigger depends on the stretcher's settings. See text for details.

to observe one RF period. STROBE is set to INTERNAL. The WIDTH of the output pulse must be at least $3\ \mu\text{s}$ to allow both the TAC output and the ADC sample-and-hold circuitry to stabilize. Although the ADC is specified at 100 kHz its acquisition bandwidth is sufficiently high to measure this 300 kHz TAC pulse.

The pulse stretcher's TTL output must arrive at the ADC TRIGGER input just before the falling edge of the TAC OUT. This TRIG OUT can either be generated INTERNALLY each time a photon signal is processed by the pulse stretcher, or EXTERNALLY each time the TAC converts a photon/RF signal delay into an analogue voltage. Successful TAC operation causes the TAC's VAL CONV output to produce a TTL pulse. The advantage of EXTERNAL triggering is that the TAC saturates at high photon counts and does not convert every pair of NIM pulses into a voltage. Thus if the ADC is triggered from the arrival of a signal at the pulse stretcher:

$$\begin{array}{ccc} \text{TRIG OUT} & \rightarrow & \text{TRIG} \\ \text{at stretcher} & & \text{at ADC} \end{array}$$

then it observes many spurious 'conversions,' giving a large zero-delay count; whereas if it is triggered via the pulse stretcher, but from the TAC:

$$\begin{array}{ccccccc} \text{VAL CONV} & \rightarrow & \text{TRIG IN} & \rightarrow & \text{TRIG OUT} & \rightarrow & \text{TRIG} \\ \text{at TAC} & & \text{at stretcher} & & \text{at stretcher} & & \text{at ADC} \end{array}$$

then it will only trigger when the TAC makes a VALID CONVERSION.

3.3 The 850 IR shelving laser

3.3.1 Initial considerations

In this section we detail the design, construction and diagnosis of the 850 IR shelving laser [92]. A two-photon process is to be used to shelve atomic population, moving it from one of the two

ground-state $S_{1/2}$ sublevels—our qubit states—into the $D_{5/2}$ level. Section 2.8 briefly discusses this method and the motivations for using it; it is more fully discussed, with results, in reference [88]. In order to accomplish this shelving, and to then repump the population to the qubit states after measurement, we require lasers on both the 850 nm and 854 nm transitions.

The shelving process requires high-intensity light exciting the $|D_{3/2}\rangle \leftrightarrow |P_{3/2}\rangle$ transition and low intensity addressing $|S_{1/2}\rangle \leftrightarrow |P_{3/2}\rangle$. Both the 393 UV shelving laser and the 850 laser must be detuned from the transition by the same frequency (~ 1 GHz) and have linewidths $\lesssim 1$ MHz. This places limitations on the 850 nm laser, therefore. It must be frequency stabilized with a narrow linewidth. When not locked to a reference cavity it must be dynamically tunable over several GHz. In addition, it must deliver as much power as possible to the trap.

3.3.2 Laser medium

The laser medium is the semiconducting junction of a diode. The package is from SDL lasers, model no. SDL-5402-H1 and serial no. AS814. The H1 series consists of a small (2.5 mm) window on the active medium, set in a relatively large (19 mm) case. Without any feedback the laser is specified to operate at 50 mW, with a CW driving current of 66 mA; the threshold current for laser operation is 17 mA. Its unstabilized emission wavelength is 852 nm. The laser head is accompanied in the case by a monitor photodiode (mon. PD) and a thermoelectric cooler (head TEC). Wiring of the laser's inputs and controls is achieved via eight pins arranged in a circle in the back of the metal case. Viewed clockwise from a gap in the circle these pins correspond to inputs: positive head TEC; thermistor $\times 2$; negative laser; positive laser (and case ground); positive and negative mon. PD; negative head TEC. The mon. PD can be used in a constant-intensity lock, but was not wired up in this case. Instead of soldering these wires directly to bulkhead outputs on the enclosure they were passed via a terminal block. This permits easy unwiring and removal of the laser mount from the delicate optics.

3.3.3 Power supply

The power supply is built by the Central Electronics Group. It is a standard design, model EW1206. Similar supplies have powered the 389 ionizing laser and the 866 cooling repumper. It is capable of delivering 200 mA/5 V. The laser is specified at a maximum safe current of 66 mA, but we expect grating feedback to reduce this to approximately 60 mA.

3.3.4 Mounting and temperature stabilization

The laser enclosure is a RFI-shielded, diecast aluminium box from Farnell (279-122), with a chloroprene seal to improve draught-tightness (if not airtightness). The laser egress is also sealed as we shall describe later. The mount is made of brass and consists of three separate pieces. The base of the mount is shown in figure 3.5. The laser diode is mounted on a short upright, with a recess for the laser can, and a milled hole straight through for mounting a collimation optic. The third piece of the mount is intended for the feedback grating (see figure 3.6), and consists of a triangular prism of brass mounted flush to the base, with a lip on the face of the prism closest to the laser diode. A screw pushing against this lip enables fine tuning of the grating orientation about a vertical axis. Horizontal tilting of the grating is performed using a second screw on a lip in the base of the mount. Other groups have reported [93] loss of elasticity with brass over long timescales, but we will rarely wish to tune the mount drastically and this is not expected to be a problem.

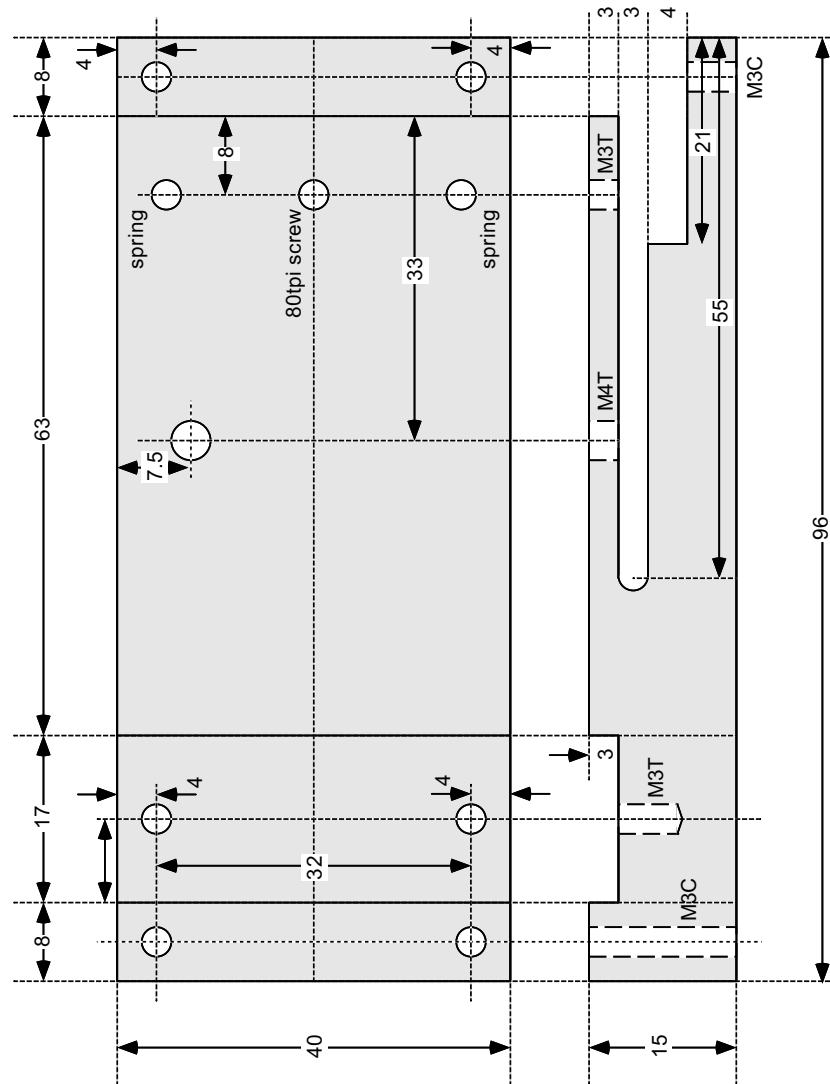


Figure 3.5: Schematic of the laser mount. This mount design is also used for the 389 laser and the 854 shelving repumper.

The exit hole for the laser beam was covered with a 12.7 mm dia, 3 mm thick glass blank, AR-coated on both surfaces for 780–870 nm by Chris Goodwin. This was pressed between two ‘O’-rings and mounted in brass, glued to the side of the laser enclosure. Again, draught-tightness was intended.

The base of the brass laser mount is separated from a Dural heatsink by a flat Peltier element (also referred to as the ‘mount TEC’). This element is from Melcor, serial no. CP 1.0-127-06L. It has dimensions $30 \times 30 \times 3.6$ mm, and with a maximum current (voltage) of 3.9 A (15.4 V) it can sustain a temperature difference of 67°C . Thermally conductive compound was spread lightly on either side of the mount TEC. The mount is held in place with nylon screws. This reduces the possibility of recirculation of energy sunk into the Dural, and hence strain on the mount TEC. The heatsink is then fastened tightly to the base of the diecast box. It is important that the heatsink is well coupled to the enclosure, and thence to the environment: if the heatsink block, mount and TEC are a closed system, their net effect is to generate, not dissipate, heat.

Temperature stabilization is performed at two locations within the mounted laser. Inside the laser can there is a thermistor/TEC pair, very close to the active medium. In addition the mount TEC is accompanied by a glass-bead thermistor: a PC5H multilayer actuator from RS (256-045). This thermistor is sunk into the mount and packed with conductive compound. The two TECs, and the two thermistor monitors, are connected to a Laser Diode Temperature Controller designed by David T. Smith in the departmental CEG. The simplified overview of the controller consists of two independent feedback circuits, driving a current $|I| < 10$ A through the mount TEC and $|I| < 3$ A through the head TEC. This is 2.5 and 1.5 times the specified maximum currents respectively. However, the thermal output of the laser at maximum operation is so small that it was not considered necessary to strengthen the controller's constraints.

The Controller's gain was tuned coarsely to avoid both oscillation of the circuit and slow response. The feedback of the dual op-amps IC2-2A and IC2-2B was in part performed by a resistor R2-13, 22 M Ω . Other resistors were soldered in parallel with this resistor, and it was found that a 1 M Ω resistor (giving a total resistance of 0.96 M Ω) gave roughly critical damping. The mount stabilization circuit was tested without the diode case *in situ*. The data in table 3.1 is in agreement with the standard formula for the temperature dependence of a thermistor's impedance [94].

Pot. reading	Temperature / $^{\circ}\text{C}$	Resistance / k Ω
4.50	25	8.73
5.00	24	9.70
7.50	17	13.58
8.50	14	16.48

Table 3.1: Resistance of the mount thermistor for different values of the controller potentiometer, with corresponding thermocouple temperatures.

3.3.5 Grating feedback and tuning

Grating feedback was required to stabilize the laser at a single mode. The grating is positioned in the Littrow [80] arrangement. Alignment of the grating is such that the -1st order diffracted beam passes exactly antiparallel to the input beam, and the zero-order specular reflection is used as the output of the laser. The grating used is a Edmund Optics gold grating NT43347: 1200 lines per mm; 12.7 $\times$ 12.7mm active area; design wavelength 750 nm. Good feedback from this grating reduced the laser's threshold current from 16 mA to 12 mA. In addition the grating is blazed at an angle of 26 $^{\circ}$. This shifts diffracted power from the zero-order and 1st order (the latter of which is at any rate entirely lost) into the -1st order.

The construction of the part of the brass mount holding the grating is shown in figure 3.6. The brass was sandblasted to avoid specular reflections. The laser light from the diode head strikes the grating mount at around 15mm above its base, and the grating was glued using Permabond so that the beam reflected off the centre of the grating surface. There is a small piezoelectric crystal between the tuning screw and the lip against which it presses. The crystal is a Thorlabs AE0203D04, and can be driven short-term at 0–150V. The recommended long-term maximum is 100V, to prevent premature ageing. This crystal is powered by the Thorlabs 3-axis piezo controller MDT693. Gross alignment of the grating is given by the formula

$$d(\sin \theta' - \sin \theta) = m\lambda, \quad (3.2)$$

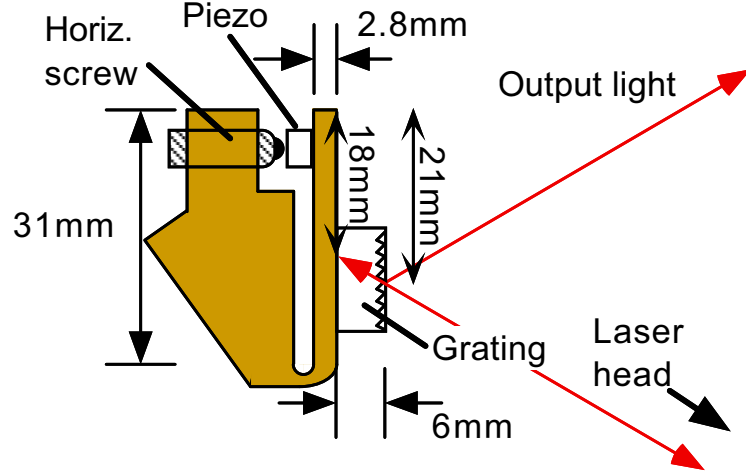


Figure 3.6: Approximate dimensions of the grating and its tunable mount. Depth of this piece of brass is 25mm.

with $\theta = -\theta'$, $m = -1$. For $d = (1/1240)$ mm, this gives $\theta = 31.8^\circ$, consistent with the coarse alignment of both the 850 and 854 lasers. In addition, we would potentially wish to tune the laser to 854 nm, or possibly further. Differentiating equation (3.2) we obtain:

$$\delta\lambda \approx 2d\delta\theta / \cos\theta. \quad (3.3)$$

For a detuning of order 10 nm we would only need to move the grating 0.3° , equivalent to bending the lip 0.2 mm away from the rest of the mount. Obviously for such a large change this would be done slowly and by hand; there would be no hope of the laser tuning dynamically over such a range. Interestingly, we found that the laser would operate at a single mode, with controlled feedback, from approximately 842.5 nm to 866.5 nm. While this horizontal tuning was performed, it was unnecessary to peak up the vertical orientation of the grating. This is over 20 nm of operation, and permits us to even reach the $|D_{3/2}\rangle \leftrightarrow |P_{1/2}\rangle$ transition if the need arises.

The laser can be tuned smoothly over almost 5.5 GHz by increasing the piezo voltage over its full range. Simultaneously a feed-forward signal is applied to the power supply's current modulation input, to reduce the current and maintain monomode behaviour. Without the feed-forward the laser will scan over $\lesssim 2$ GHz before jumping to a different mode.

With grating feedback the laser was seen to jump between modes as the current was increased on its own. This is because of the competition between feedback on the internal and external cavities set up by the facet of the laser medium and the grating respectively. We expect the external cavity of length $l \sim 4$ cm to cause the current modes to be separated by $\Delta\bar{\nu} = 1/2l \sim 0.13 \text{ cm}^{-1}$, and find the separation in practice to be $0.13(1) \text{ cm}^{-1}$.

3.3.6 Beam quality and steering

The light from the laser is specified with FWHM divergence angles from the diode medium of 9° vertically (30° horizontally). It is vertically polarized and has a reasonable transverse mode.

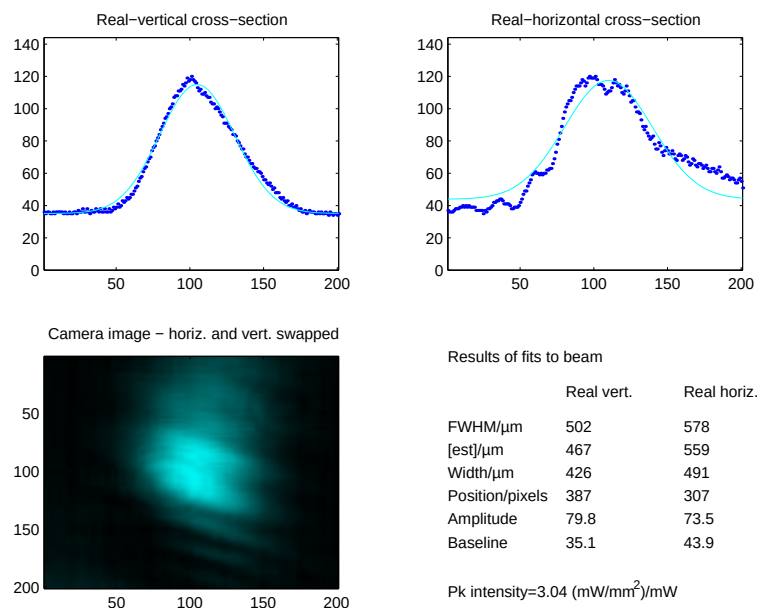


Figure 3.7: Cross-section of the 850 nm laser beam, approximately 320 mm from the diecast enclosure. Model Gaussians have been fitted to two pixel rows through the peak intensity position. The camera image is rotated through 90° . Diffraction fringes can be seen in the horizontal direction (vertical in the camera image) and this skews the horizontal fit.

3.3.6.1 Collimating lens

In order to collimate the beam, we mounted a lens in front of the laser. The laser case was positioned in its mount from one side; from the other side, a screw thread permitted a collimating optic to be positioned at a variable distance from the active laser medium. The Thorlabs mounted optic C330TM-B had a focal length $f = 3.10$ mm. Unfortunately, the brass mount was slightly too thick, and the optic's mount was lathed down by 1mm to permit it to be screwed closer to the laser medium. The laser light was shone along an optical table, and we attempted to get a well-focussed beam as far away as possible. The resultant beam was collimated but the cross-section was not of excellent quality. The difficulty of translating the lens in directions orthogonal to the beam, compounded by slack in the screw thread, meant that the wings of the beam were clipped by the lens mount. In the medium-field this resulted in Fresnel diffraction and hence fringes or lobes on one side of the cross-section. These lobes are shown in figure 3.7. The lens was glued in place with Permabond.

For reference, the screw-thread method was not found to be optimal for ensuring a good quality, collimated beam. In other lasers we have used a different method. The optic is glued to a pin which holds it from above. Translation of this pin optimizes the optic's position, and at this point two wedges are placed gently under the lens and glued to both it and the mount. Once this glue is dry, the connection to the pin is either dissolved or broken, and the lens is well-aligned.

3.3.6.2 Compensation of beam translation with grating tilt

A thin brass mount was added to the rest of the grating mount, holding a mirror nearly parallel to the grating. We see in figure 3.8 a schematic of this arrangement. To tune the laser we rotate the grating. The zero-order beam from the grating suffers a small angular displacement which, far from the laser, would lead to a detrimental translation of the beam. However, the beam is then reflected

off a mirror. The grating and mirror both pivot around the same point (the hinge of the mount in figure 3.6) and remain at the same, nearly parallel, angle to each other. Reflection off first the grating and then the mirror thus leads to translation of the beam rather than angular displacement, and the beam emerging from these two reflectors is parallel to the beam from the laser.

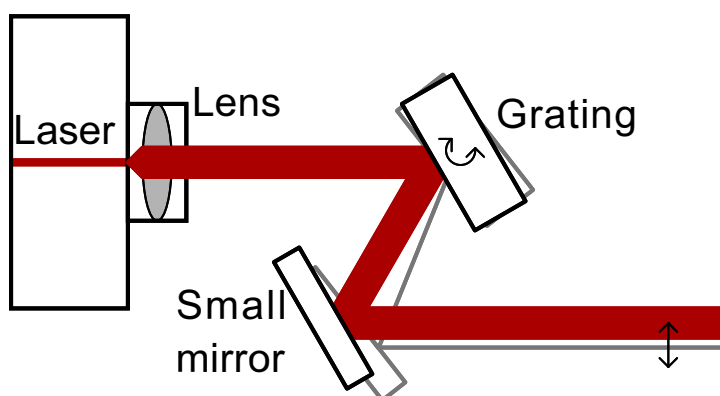


Figure 3.8: Schematic of the steering mirror used to compensate for grating tilt. As the grating is turned to tune the laser frequency, the small optic remains at a constant (almost parallel) angle to it. This optical arrangement converts angular displacement off the grating into translation of the beam.

The mount was milled out of brass to a thickness of 2mm. Attempts to produce a mount through bending a 2 mm strip proved fruitless, because the extension was springy even before the mirror optic was positioned on it, and therefore a mechanical liability. We used a 12.7 mm dia, 3 mm thick mirror, coated by Melles Griot for 794 nm at 13° , $> 99.7\%$ reflection. Its precise angular orientation was not important: misalignment merely introduces an angular offset. Theoretical predictions of the coating behaviour at 850 nm/ 30° by Chris Goodwin in the department suggested intensity losses of around 5%, although empirical measurements with the 854nm laser fixed this at 3%.

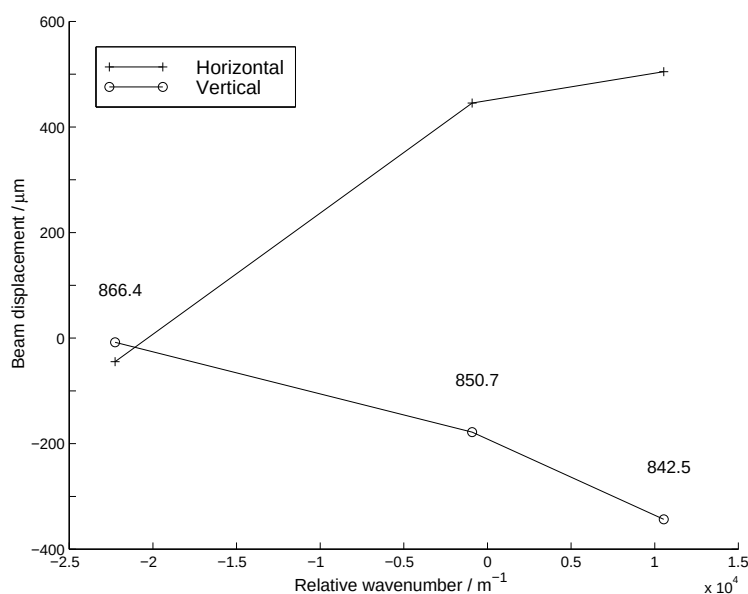


Figure 3.9: Residual beam wander during coarse tuning. The wavenumber is quoted relative to $1/850$ nm. The points are labelled with the wavelength in nm.

With this optic in place the laser was tuned with grating feedback over its full range. The beam was shone onto a Maplin camera approximately 1.5 m from the grating. Images were captured from this camera and fitted with Gaussians to determine the beam centre. The camera has been calibrated at $4.95 \mu\text{m}/\text{pixel}$ horizontally and $4.86 \mu\text{m}/\text{pixel}$ vertically, and this calibration gives us a beam displacement of $550 \mu\text{m}$ in the horizontal and $340 \mu\text{m}$ in the vertical. Without the correcting optic we expect the vertical displacement during this coarse tuning to be zero and the horizontal to be approximately 19 mm from equation (3.3). Moreover, as the uncorrected horizontal displacement is the result of an angular change, then it will increase linearly with distance from the grating. The displacement after correction did not appear to change with distance from the laser.

3.3.7 Beam manipulation

Before the beam is sent down an optical fibre, it must pass through a number of stages. We require a pick-off beam—a weak diagnostic reflected off an uncoated mirror blank—to align into a potentially high-finesse cavity. Unless the cavity is both well-aligned and on resonance there will be a substantial reflection back along the direction of the original beam. Hence the laser must be isolated from any returning light. The transverse intensity distribution of the laser must be massaged in order to match well into a fibre. The lasing medium is taller than it is wide, leading to astigmatism of the emergent beam. Also as we have mentioned there was misalignment of the collimating optic. These two factors together necessitated a tilted-lens arrangement to correct them. The laser polarization must match the fibre’s preferred polarization direction, and there must be sufficient degrees of freedom to match into the fibre tip. Finally weak diagnostic beams must be picked off by a glass blank to inject into: an optical cavity for laser locking; a spectrum analyser; the Burleigh WA1500 wavemeter.

3.3.7.1 Isolation

The Faraday isolator used initially was not optimized for 850 nm light, but for 794 nm (owing to availability). The model was an Isowave I-80-U2, serial no. 26082. This meant that neither the transmission nor the isolation was very good. However, in later work an isolator designed for 852 nm (model I-80-2, serial no. 22701) was used. The transmission is now 73%.

3.3.7.2 Beam shape and propagation

From the quoted divergence angles, the collimated beam from the laser will have an aspect ratio of $\tan 30^\circ / \tan 9^\circ = 3.65$. In order to reduce this astigmatism, a Thorlabs mounted prism pair (PS881-B) was purchased. It corrects for an aspect ratio of exactly 3.5, leaving a residual ratio of 1.04. It was intended to mount this prism pair within the laser enclosure, a ‘black box’ approach that would see the aligned laser-plus-prisms as a portable, single unit.

Unfortunately the minor misalignment of the collimation optics meant that the beam was not propagating in a collimated manner in both horizontal and vertical directions. This effect was mostly seen in the horizontal direction. The beam shape cross-section is shown in figure 3.7. This image was taken with the Pulnix PE-2015 camera (serial no. 01700); a glass pick-off and neutral-density (ND) filters were used in order not to saturate the camera’s CCD. The camera as mounted was rotated through 90° , making its horizontal the real-vertical and vice versa. The fitting program found the peak intensity pixel, and then fitted two Gaussians to the row and column through that pixel.

In this image the beam has little astigmatism. As the camera was moved along the beam's extent through around 100mm it was clear that an asymmetry was developing in the cross-section. The beam appeared to be approximately collimated in the real-vertical direction. By simple geometrical optics the real-horizontal beam width contracts by approximately $70\text{ }\mu\text{m}$ in 40 mm, giving a focal angle of 0.0018 and a focus some 240 mm away. The behaviour of both beam widths is plotted in figure 3.10. Errors arise from repeated measurements with different orientations of the ND filters in front of the camera, and were at the level of $\pm 5\%$.

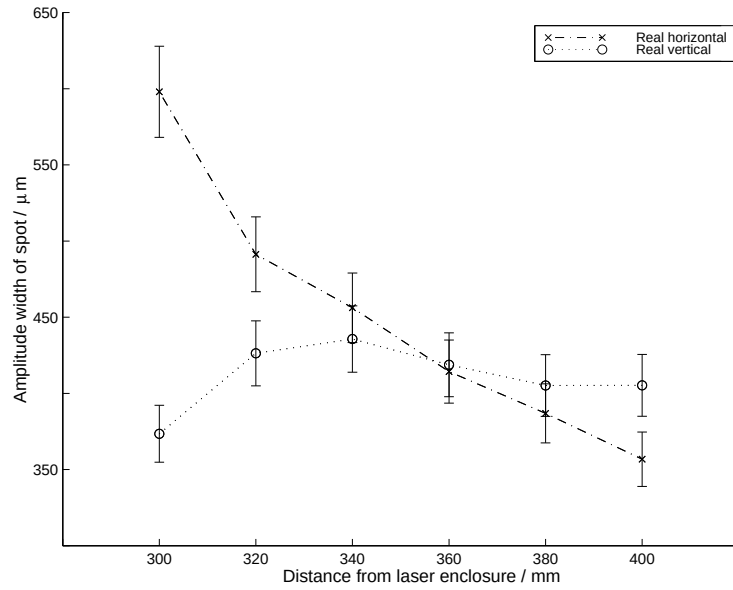


Figure 3.10: Cross-section of the 850 nm laser beam, approximately 320mm from the diecast enclosure. Model Gaussians have been fitted to two pixel rows through the peak intensity position. The camera image is rotated through 90° .

Although the prisms had reasonable results with the (collimated but non-Gaussian) 854 nm beam—they reduced its astigmatism from 2.92 to 1.20, in the same direction—they were eventually deemed unsuitable for the 850 nm beam. In their place was put a tilted-lens telescope. The theory of a tilted-lens system is as follows.

A lens of focal length f is tilted about an axis by an angle θ . In our case this axis is vertical. The lens acts with a different focal length in the horizontal and vertical planes:

$$\begin{aligned} f_v &= f \cos \theta \leq f && \text{(tangential)} \\ f_h &= f / \cos \theta \geq f && \text{(sagittal)} \end{aligned}$$

We would like to obtain a collimated beam without astigmatism. One method of doing this is to place the tilted lens where the spot sizes are equal (i.e. the beam is momentarily circular) and adjust the tilt so that both directions are brought to a focus at the same point. We then image this point at infinity with a second lens. As we can see the horizontal waist is already gently focussing. The lens is therefore tilted about the vertical axis to reduce its focussing power in the horizontal plane.

This is discussed in reference [95] with consideration of geometrical optics. The MATLAB function `tilt lens.m` solves the equations therein. Using the data in figure 3.10 it calculated a tilt angle of around 32° . A lens of normal-incidence $f = 100\text{ mm}$ has focal lengths of 85 mm and 118 mm when it is tilted thus.

A telescope was constructed either side of the optical isolator. This consisted of the 100 mm, AR-BB coated lens at a variable tilt, and a 48 mm, uncoated lens on a translation stage in order to optimize matching into the fibre. The beam widths w_t, w_s at the 100 mm lens were approximately $420\text{ }\mu\text{m}$, and at the required tilt this arrangement will provide a magnification of approximately 48/85 i.e. a collimated, circular beam of width $w_r \approx 240\text{ }\mu\text{m}$.

3.3.8 Alignment into low-finesse cavity and lock

Initially we locked the 850 laser to one of the first two high-finesse NPL cavities listed in table 2.3. However, these were both required for other laser locks, and hence this laser was put into the long home-made cavity.

This confocal cavity was constructed by Dr Matthew McDonnell, and consists of a 250 mm Zerodur spacer with the lenses at either end coated for 60% reflectivity both at infra-red and ultra-violet wavelengths. The finesse of the cavity was approximately 6. Electronic feedback was provided by taking a signal from a photodiode behind the cavity and locking to the half-maximum of a cavity peak using the ‘Clairon I’ circuit [96]. The resulting locked linewidth was estimated at $\sim 1\text{ MHz}$ by considering the remaining noise on the lock signal.

Later, the laser was moved back to a high-finesse NPL etalon. Irrespective of the fringe width, the remaining frequency jitter of the laser remained $\sim 1\text{ MHz}$. This implies that the slackness of the lock originates in either the laser control electronics or the lock circuit. The Clairon I has a low-pass input bandwidth of 60 kHz, and this limits the ability of the circuit to compensate for high-frequency noise. However, attempts with the ‘Clairon II’ circuit (which provides proportional and integrated-signal current control [97]) to tighten this lock failed. The second circuit is more suited to implementation in a modulated locking scheme rather than simple side-of-fringe locking.

3.3.8.1 Fibre alignment and transmission

The 850 nm light was required to pass down a monomode polarization-preserving fibre. Matching into this fibre is discussed elsewhere [98] and we recap briefly. Laser light at 876 nm was shone down the fibre from the opposite direction, and fits made to the beam propagation after the fibre. The fibre tip has an integral collimating lens, which leads to a much wider emergent beam than the monomode nature of the fibre would otherwise imply (of order of the wavelength of the light). It was found that the waist was at the fibre tip, with a beam width¹ of $w_0 = 311\text{ }\mu\text{m}$. This is somewhat larger than our 850 nm beam after the tilted-lens telescope. The second lens was therefore translated over several millimetres, and at each point the alignment into the fibre was peaked up using two mirrors between the telescope and the fibre.

It was found that coupling into the fibre was a reasonable and flat function of this distance. Peak transmission was 6.65 mW from an input 10.8 mW (62%), and transmission fell to 4.8 mW after a few millimetres of lens translation. With the 850 nm laser running at maximum safe current (60 mA) the above alignment gave 8.0 mW at the trap. More careful matching has in the past yielded a transmission of approximately 75%. However, this had been obtained using a good Gaussian input mode, whereas our laser has been clipped in the horizontal direction, reducing its overlap with a Gaussian function.

¹See appendix A.1.

3.3.8.2 Beam parameters at ion

The fibre preserves the polarization vector. The accuracy to which the polarization is maintained is dependent on the thermal and mechanical stability of the fibre. The ambient impurity fraction is approximately 1.1%, measured with a $\lambda/2$ waveplate and a polarizing beam splitter. This takes the form of a fast jitter. Superimposed on this will be a slow wander from thermal changes and the natural oscillation of the optical tables relative to each other. ‘Severe’ pressure on the fibre (bending it and the plastic tubing used to protect it) increased the impurity to $\sim 5\%$. We therefore expect the slow wander to be much less than this.

After the fibre the light passes through several optics before reaching the trap: a waveplate, a mirror and a lens. There is also an iris to block scattering of the counter-propagating 393 laser from the lens, which is close to the trap and hence scatters eventually into the PMT, swamping a typical single-ion signal.

The intensity at the trap has been measured both with a camera at the same optical distance from the last lens, and with an ion itself [88]. From this data we see that the current setup is capable of delivering approximately 1400–1800 saturation intensities (as defined in equation (4.19)) to the trap. All measurements are consistent to within a factor of 2.

3.4 An all-AOM system for laser intensity switching

3.4.1 Introduction

It was decided that the experiment should move from primarily shutter-controlled beam extinction to a system consisting entirely of acousto-optic modulators. Although this was a long-term goal, it was also motivated by preliminary results from our EIT readout method experiments. These are discussed in reference [88], and briefly described in chapter 2. These preliminary results were frequency-broadened which spoiled the efficiency of the qubit readout. It was believed that a major source of this broadening was the effect of acoustic shutter noise on the 393 UV shelving laser lock; hence the shutters were replaced.

The 866 laser was previously triple-passed through its AOM [76]. This was because the laser was locked to an absorption feature in iodine bromide vapour that was approximately 650 MHz away from the $^{40}\text{Ca}^+ |D_{3/2}\rangle \leftrightarrow |P_{1/2}\rangle$ transition, yet the AOM/VCO had a low conversion efficiency at $\gtrsim 300$ MHz; this prohibited only single- or double-passing. Once the laser was locked to a tunable, high-finesse cavity instead, we no longer needed the high frequency shift and this was therefore changed to a double-pass arrangement.

The 397 cooling laser was aligned in the double-pass configuration, with a prism to provide a displacement between the incident and double-passed beams. This produced a master beam, 397M, which was then split and single-passed through two other AOMs to give the Doppler 397D and circularly-polarized 397 σ beams. The 393 laser was double-passed using polarization selectivity to separate the incident and emergent beams. Finally the 854 shelving repumper was aligned in a single-pass through its AOM. The 850 IR shelving laser was not put into an AOM. This is because it was on a separate optical table where it had a low impact on the 393 laser lock; also we required as much 850 nm intensity as possible for the EIT experiments and did not wish to lose power during diffraction.

3.4.2 Product specifications

We already owned four Model 3000-121 AOMs from Crystal Technology for use at visible and near-IR wavelengths. The two used in the experiment on the 866 and 854 beams had serial numbers 14393 and 16165 respectively. Diffraction efficiency of this model was specified at $\gtrsim 80\%$ with a $\sim 100 \mu\text{m}$ waist at the AOM and at the tuned centre frequency of 200 MHz. The AOMs had SMB connectors to which we attached homemade voltage-controlled oscillators (VCOs) and amplifiers. This is described in section 2.5.

In addition to these AOMs we acquired six modulators for use at near-UV wavelengths from IntraAction, model ASM-2001.5B8². These had UV-grade fused silica crystals. Sufficiently intense UV light can damage TeO_2 crystals used in the other AOMs. The quoted diffraction efficiency of these six AOMs was 70(1)% at 200 MHz drive frequency. These AOMs did not require focussing of the input beams, and had large input facets. RF signals were applied using SMA connectors. Four of these AOMs were connected up to IntraAction DE-2002EM26 driver boxes³. It is intended in future experiments to use the remaining two AOMs to generate a precise difference frequency between two Raman beams, using customized driving electronics.

3.4.3 Alignment methods

Owing to different requirements for the AOMs, different beams were aligned through their respective AOMs in different ways. The methods used are shown in figure 3.11. Beams into AOMs were aligned to satisfy the Bragg condition [99]; therefore the angles the incident and zero/first-order beams make with the axis of the AOM are all the same. The schematic shows zero-order beams travelling along the axis instead, for simplicity.

For beams which do not have critical attenuation (or where the majority of attenuation is handled by another component), and which do not have their frequency tuned dynamically by the AOM, the single-pass method at the top of the figure was used. In our setup the 397D, 397 σ and 854 beams were aligned in this manner. The 854 laser was necessarily focussed through the AOM's small aperture, and the lens after the AOM is positioned so that the first-order beam passes through its centre to minimize aberration. The IR AOM is optimized for a beam waist in the crystal.

The standard double-pass arrangement is shown in the middle subfigure. This is used for the 866 and 393 beams. Vertically polarized light is reflected off a polarizing beam splitter (PBS) cube into the AOM. The light is focussed onto the AOM crystal. After the AOM the zero-order beam is stopped, and the first-order beam is deflected by a lens whose focal point is the centre of the AOM. This beam travels, collimated and parallel to the system's optical axis, through a $\lambda/4$ -waveplate at 45° to the polarization vector. It undergoes two passes through the waveplate (effectively one pass through a $\lambda/2$ -waveplate at the same angle) while being reflected off the mirror M in between the passes. The return beam is again aligned to satisfy the Bragg condition, and with its polarization vector rotated through 90° it is transmitted by the PBS cube and continues towards the rest of the experiment.

The IntraAction AOMs have a preferred polarization direction: the diffraction efficiency of horizontally-polarized light is approximately 26% that of vertically-polarized light. This preference in efficiencies is sufficiently strong that we decided to set up the 397M beam/AOM in a double-pass configuration that did not have an unwanted polarization in the AOM on either pass. This necessitated beam displacement by a prism as shown in the lowest subfigure of figure 3.11. The

²Serial numbers 413916–413921 inclusive.

³Serial numbers 813833–813836 inclusive.

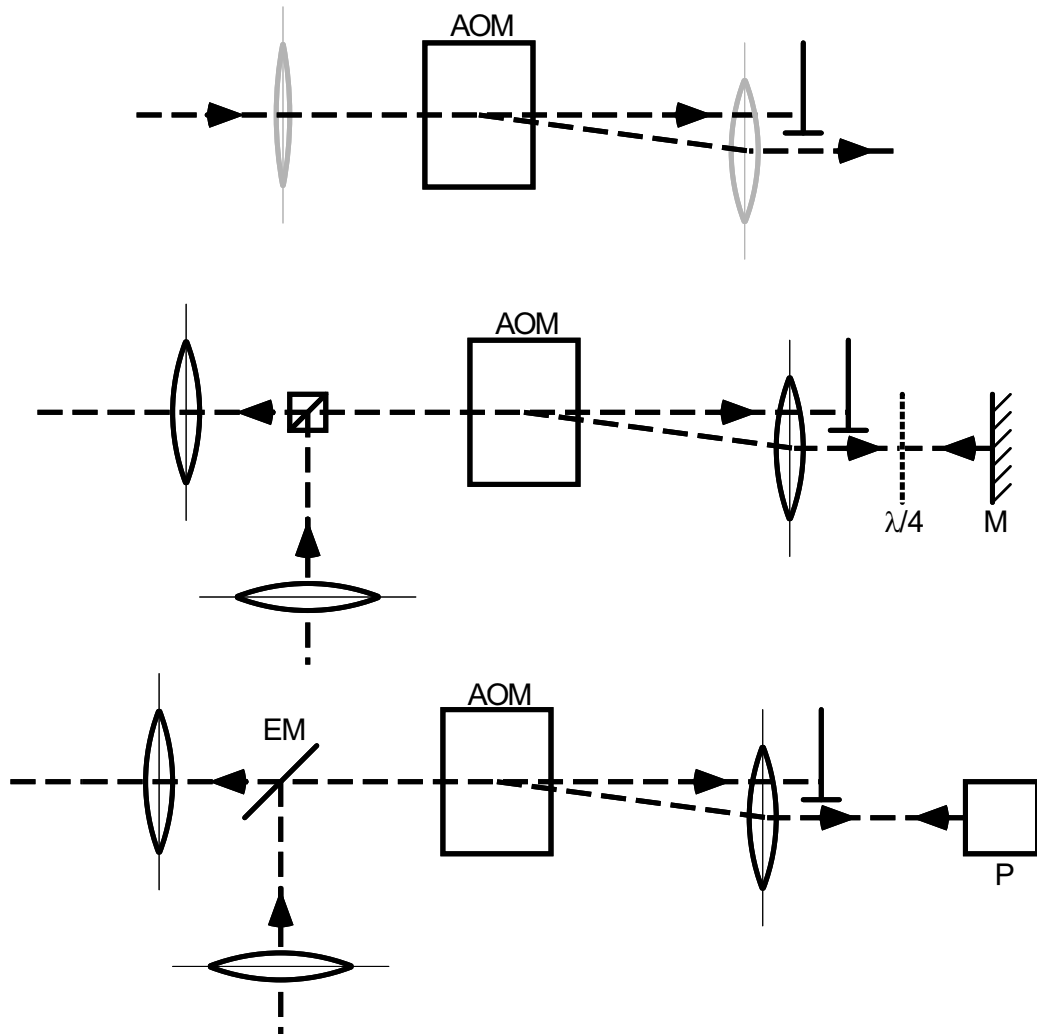


Figure 3.11: Schematics of the AOM alignments constructed. From the top these are single-pass, double-pass with polarization alteration, and double-pass without polarization alteration. M is a mirror that reflects the beam back along its incident path; P is a right-angled prism that translates the beam vertically, resulting in an angular displacement after the lens; EM is an edge mirror that reflects the incoming beam but not the beam emerging from the AOM. The two lenses shown in grey are only present in the 854 single-pass arrangement.

edge mirror EM is positioned so as to reflect the incident beam into the AOM. The AOM and prism P are both at focal points of the lens. The incoming beam is collimated and has an angular displacement from the deflection at the AOM. The lens focuses this beam at P, while compensating for the angular displacement as a translation of the beam between the lens and P. The vertical translation caused by the prism is converted into a vertical angular displacement by this lens. The displaced beam, after its second pass through the AOM, misses the edge mirror and proceeds to the experiment. Because we only require low 393 nm intensities we used a $\lambda/4$ -waveplate on the 393 beam path instead of a prism.

3.4.4 Results

The intensity of all beams in and out of each AOM (see figure 3.12 for nomenclature) were measured with a power meter. For some beams this was difficult owing to e.g. the position of other apparatus. In addition the closeness of some beams meant that power measurements were a realistic upper limit; scattered light was inevitable. The measurements are recorded in table 3.2, and the associated efficiencies given in table 3.3.

The 397M AOM has good diffraction efficiencies on both passes, whereas the other two 397 AOMs do not convert well. It is believed this is a result of the size of the beam. In the first AOM the beam is collimated and wide; in the other two it is focussing (albeit gently) and much narrower. Initially the two single-pass 397 beams were narrower still, and the conversion efficiencies were then yet lower. The 393 AOM has low efficiencies; this is a combination of the preferred direction of light polarization in the AOM and the narrow beam.

The 397M AOM was intended to provide frequency tuning without beam displacement owing to its double-pass configuration, but did not do this well. Without changing the alignment of the beams its 3 dB bandwidth was approximately ± 12 MHz. With constant realigning the AOM was tuned over a large range and approximately matched its specification of ± 50 MHz.

beam/AOM	$P_i/\mu\text{W}$	$P_0/\mu\text{W}$	$P'_0/\mu\text{W}$	$P_1/\mu\text{W}$	$P'_1/\mu\text{W}$	$P_{1\text{AM}}/\mu\text{W}$
397M 1st	1010	928	306	2.05	594	1.6
2nd	'527' ⁴	— ⁵	— ⁶	<0.01	331	<0.01
397 σ	96	95	62	<0.05	38	<0.05
397D	111	103	59.7	<0.036	41	<0.036
854	16900	16000	'11800'	<'15'	'3860'	— ⁷
866 1st	14200	13700	'3750'	<88	'10730'	<76.3
(+ cube) 2nd	7960	—	2360	<0.22	'5470'	<0.19
393 1st	380	378	318	<0.209	31.8	<0.181
(+ cube) 2nd	'30.9'	—	15.7	<0.004	5.6	<0.002

Table 3.2: Observed powers of beams through the AOMs. The conventions in figure 3.12 are used here. Some double-pass arrangements are such that second-pass beams can only be measured after a PBS cube, as shown.

⁴All powers enclosed in quotes are inferred; that is, they are the result of a measurement of the beam after e.g. a reflection from a surface of known reflectivity.

⁵With double-pass AOMs, the power in the zero-order beam when the TTL is off has no meaning in terms of transmission efficiency.

⁶Because of the shallowness of the beam angle introduced by the prism, it is not possible to measure this beam before it is blocked entirely by the edge of a mirror.

⁷The 854 AOM has no external AM control.

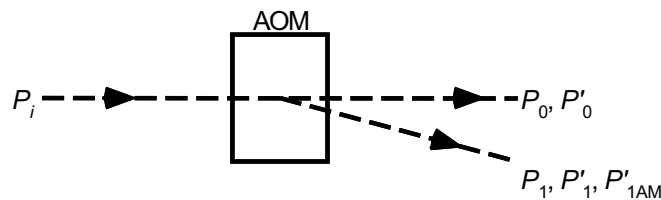


Figure 3.12: Conventions used in power measurements. P_i and P_0 are the input and output powers, and P_1 is the power into the first order; a prime after the power signifies that the AOM is switched ON (TTL input high). P_{1AM} is the power of the AOM, not merely turned off, but with the amplitude modulation turned as low as possible.

beam/AOM	zero-RF transmission P_0/P_i	efficiency, 1st order = P'_1/P_i	extinction, 1st order = P'_1/P_1
397M 1st pass	91.9%	58.8%	290
2nd pass	—	61.8%	$>3 \cdot 10^4$
397 σ	99.0%	39.6%	>760
397D	92.8%	36.9%	$>1.14 \cdot 10^3$
854	94.7%	22.8%	>260
866 1st pass	96.5%	75.6%	>122
2nd pass	—	68.7%	$>1.1 \cdot 10^4$
393 1st pass	99.5%	8.37%	>152
2nd pass	—	18.1%	>1400

Table 3.3:

Single-pass transmission fractions, efficiencies and extinction ratios for the AOMs discussed here. The ‘zero-RF’ transmission is non-unity owing just to reflection losses at the inert crystal surface. In the case of double-pass arrangements, the efficiency for each pass separately is shown i.e. the total efficiency of the double-pass 397M AOM is $58.8\% \times 61.8\% = 36.3\%$. This enables comparison of e.g. polarization-specific losses in certain AOMs.

In table 3.4 we see the results of measurements of the power output from the driver boxes used to control the AOMs. These were made with the Agilent ESA-E spectrum analyser, model E4405B, and inline attenuators to protect the analyser. The IntraAction boxes and the 866 VCO behaved approximately according to their specifications. However, the 854 AOM was below specification at ‘maximum power’ (by approximately 10 dB) and did not provide sufficient extinction. This is believed [100] to be a problem with the earth connection on the box: the perfect DC earth is not a good AC earth at radio-frequencies.

The control TTL was switched on and off to determine the extinction ratio, and similarly with the AM control. The extinction from the AM control was optimized by eye. The dial on the front of the box was at maximum, whilst an analogue (negative) voltage was applied to the AM input of the box. As the control voltage was decreased the signal reached a minimum, then began to increase again. This is probably because the input voltage is added to the voltage from the potentiometer dial, and then this DC voltage mixed with the RF voltage. Hence when the input AM voltage is sufficiently low then the RF is being mixed with a negative DC voltage: it begins to increase in power again, but with a π change of phase.

3.4.5 Summary

All laser beams on the main optical table have been aligned through AOMs. With certain beam qualities the UV AOMs have been seen to give conversion efficiencies similar to the ideal test con-

Beam	Driver box	TTL on/ dBW	TTL off/ dBm	with AM/ dBm
397 σ	813833	3.07	2.21	-29
393	813834	3.22	-3.56	-25
397M	813835	3.72	2.41	-20
397D	813836	3.13	0.48	-17

Table 3.4: Behaviour of the four IntraAction driver boxes. The output power of each driver box is tabulated for different control settings. A TTL attenuator and an analogue AM attenuation stage are both employed.

ditions under which they were specified. Conversion efficiency is strongly dependent on alignment, and the size, polarization and collimation of the beam. There are problems with the homemade VCO circuits in principle, although in practice these only manifest themselves in the 854 circuit. The poor extinction on this AOM means that certain experiments necessitated replacement with a shutter.

The IntraAction driver boxes have TTL extinction ratios of 30.9–32.7 dB. This compares favourably with the specified 30 dB. The extra extinction of the analogue control was less impressive: around 20 dB when the specified extinction was 40 dB. This is because the output power is linear in the control voltage. Hence the control voltage must be set with a further accuracy of 20 dBV if we wish the full 40 dB extinction. A possible solution is to optimize the input control voltage using electronic feedback to minimize the RF power.

Whilst the reduced intensity in the 397 beams has not restricted the experiments we have been able to conduct, it would be preferable to have more intensity in this beam. We wished therefore to remove the grating we currently use to filter out amplified spontaneous emission (ASE [77]) spikes in the spectrum of the blue laser diodes from Nichia. Although the spikes in the 397 laser are small as far away as 393 nm, nonetheless we and other groups [101] have observed shelving into $|D_{5/2}\rangle$ when a diffraction grating has not been used. Shelving events have been seen at a rate of $\sim 1 \text{ s}^{-1}$ per ion.

This grating approximately halves the intensity of light at the correct wavelength in the beam. We therefore considered the frequency selectivity of AOMs, to determine whether an AOM could act as a frequency filter. We align light of wavevector $\mathbf{k}$ into an AOM of wavevector shift $\mathbf{K}$ at an angle θ . In the approximation that the emergent light wavevector is also $\mathbf{k}$ maximum conversion efficiency occurs when this angle satisfies the Bragg condition

$$K \approx 2k \sin \theta \approx 2k\theta \quad (3.4)$$

$$\sin \theta \approx \frac{1}{2} \frac{\Omega}{\omega} \frac{c}{v}, \quad (3.5)$$

where Ω, ω are the frequencies of the AOM and the light, and v is the speed of sound in the AOM crystal. From this expression it is clear that, for two laser frequencies separated by $\delta\omega$ and two AOM frequencies separated by $\delta\Omega$,

$$\frac{\delta\theta}{\theta} \approx -\frac{\delta\omega}{\omega} \approx \frac{\delta\lambda}{\lambda} \quad , \quad \frac{\delta\theta}{\theta} \approx +\frac{\delta\Omega}{\Omega}. \quad (3.6)$$

When we tuned the 397M AOM without peaking up its alignment there was a 3 dB rolloff in conversion efficiency at $\delta\Omega_{1/2} \sim 12 \text{ MHz} \sim 0.06\Omega$. For ASE at 393 nm the fractional wavelength change is only 0.01. Even after three passes through IntraAction AOMs this filtering is insufficient to reduce the ASE to a level where we will see negligible shelving.

3.5 Conclusions

We assembled a new photon counting/correlation measurement system, constructed a laser to function at 850 nm, and introduced AOMs into the experiment.

The new photon acquisition system has functioned as specified, and has provided more accurate measurements of the correlation between photon emission and the phase of the RF trapping voltage than were previously available. The system saturates at high photon counts ($\sim 10^6 \text{ s}^{-1}$) but this is well understood and easily accounted for.

The 850 IR shelving laser has proven to be one of our most stable lasers, and performs well at a wide range of frequencies. We require reasonable power from the system for current experiments, and the output from the laser system—diode head, feedback, pick-offs and fibre—is sufficient for our requirements.

We do not obtain the efficiencies and extinction from our AOMs that the manufacturers claim: some conversion efficiencies are reduced by a factor of two. In addition the scan range, at least for the blue beams, is not as good. Jonathan Home is currently performing experiments which we hope will improve the tuning range of the AOMs. Nonetheless, the AOM system works well apart from the poor conversion efficiencies.

Chapter 4

The motion and temperature of a trapped, charged particle

4.1 Introduction

This chapter describes the motion of a charged particle in a Paul trap, and discusses the effect of laser cooling and extraneous heating on this motion. We need: to understand how to increase the tightness of our trap (this is required for chapter 5); and to use our experimental observations to learn about heating rates in the trap.

This chapter begins with a brief discussion of ion motion. We begin by discussing the arrangement of electric fields in a Paul trap. We then summarize the motion of a classical particle, separating it into its thermal motion and driven motion. This is extended to the quantum-mechanical regime, and we discuss the coupling of light to the internal and external degrees of freedom of the ion. In the second section we develop laser cooling to determine the temperature and heating of a single ion in our trap. We use a 3-level rate equation approach combined with thermal occupancy of quantized motional states.

4.2 The motion of trapped charged particles

4.2.1 Introduction

In this section we outline the motion of a charged particle in an electric potential which has periodic time-varying components. Within the field of ion-trap spectroscopy this theory has already been discussed at length [102]. It is not our intention to repeat in full such analyses here. Work has also been done with closer attention to our ion trap [76]. We intend the following to be complementary to this work: it will repeat certain early results but then concentrate on behaviour relevant to the rest of this thesis.

One singly-ionized Ca^+ ion moves in a Paul trap's electric potential. The potential satisfies Maxwell's equations and is thus only ever trapping in two dimensions. However, the radio-frequency oscillations of an inhomogeneous potential average out over time to give a pseudopotential that traps in all three directions. This gives rise to a harmonic secular motion of the ion, with the radio-frequency oscillations—micromotion—superimposed on this orbit. The quality of the signals to the Paul trap electrodes is reflected in the temperature and driven micromotion of a Doppler-cooled ion, and we can observe these properties indirectly. If the centre or spring constant

of the pseudopotential fluctuates then this has been shown [103] to contribute to heating effects, and we discuss this in section 4.3.

4.2.2 Quadrupole potentials in a linear ion trap

Our trap is linear: the electrode arrangement has a square, axial symmetry. The full details of the arrangement are discussed in section 2.4.1. We quote here that the characteristic dimensions of the trap are $2z_0 = 7.2$ mm and $\rho_0 = 1.9$ mm. Let us define our Cartesian axes such that the z -axis lies along the trap. The nearest surfaces of the AC electrodes lie at x - y coordinates $\rho_0\{\pm 1, 0\}$ and $\rho_0\{0, \pm 1\}$. A DC voltage V_{DC} is applied to the two endcaps, and a voltage oscillating at Ω_{RF} with amplitude V_{AC} is applied to the AC electrodes. Here the AC voltage is defined as in appendix A.3. The precise relationship between V_{AC} and the amplitude of the signal on any one electrode is discussed more fully there.

As we discussed in section 2.4, if the resultant potential V_{Ω} were applied by hyperbolic surfaces instead of our circular electrodes then it would be perfectly quadrupolar. As it is the approximation is very good close to the centre of the trap, with a geometrical correction factor α for each quadrupolar component:

$$Q_{\text{AC}} = \frac{\alpha_{\text{AC}} V_{\text{AC}}}{\rho_0^2} \quad , \quad Q_z = \frac{\alpha_z V_{\text{DC}}}{z_0^2} ,$$

giving

$$V_{\Omega}(x, y, z, t) = Q_{\text{AC}}(x^2 - y^2) \cos \Omega_{\text{RF}} t + Q_z \left(z^2 - \frac{1}{2}(x^2 + y^2) \right) . \quad (4.1)$$

where the x - and y -axes are at 45° to their usual definitions. We quote from earlier work [37]

$$\alpha_{\text{AC}} = 1.1(2) \quad , \quad \alpha_z = 0.0237(9). \quad (4.2)$$

4.2.3 Ion motion and the Mathieu equation

An ion's motion in the trap is governed by the classical equation of motion $M\ddot{\mathbf{r}} = -e\nabla V_{\Omega}$, where M is the ion's mass. Along the z -axis of the potential in equation (4.1) the resultant motion is strictly simple harmonic—no micromotion—with frequency $\omega_z = \sqrt{2eQ_z/M}$. Let us treat each radial direction as a one-dimensional Paul trap. We will discuss later the motion in the two-dimensional potential. The motion along the x -axis can then be parametrized so that it is equivalent to the Mathieu equation [104]

$$\frac{d^2 x}{d\theta^2} + (a - 2q \cos 2\theta)x = 0, \quad (4.3)$$

where

$$a = -\frac{4eQ_z}{\Omega_{\text{RF}}^2 M} \quad , \quad q = \frac{4eQ_{\text{AC}}}{\Omega_{\text{RF}}^2 M} \quad , \quad \theta = \Omega_{\text{RF}} t/2, \quad (4.4)$$

and the equivalent equation holds true along the y -axis. This equation has standard analytical solutions which we will not discuss here. However, it should be noted that the orbits are only stable for certain ranges of the parameters $\{a, q\}$. The first stability region is the overlap of the areas

$a < 0$, $|a| < q^2/2$ close to the a -axis and we typically operate the trap in this region. Outside these stability regions either the radial or axial motion becomes unstable and the ion eventually leaves the trap.

4.2.4 Secular motion and micromotion

We parametrize the time-dependent ion motion $x = \bar{x} + \xi(\theta)$ and similarly for y . The first term varies slowly, and is assumed constant over one cycle of the RF. The second term is a displacement that oscillates at Ω_{RF} . Respectively these components are the secular motion and micromotion. Assuming also that the micromotion is small, and that $a \ll q$, we obtain

$$\xi \approx \frac{1}{2}q\bar{x} \cos \Omega_{\text{RF}} t \quad (4.5)$$

$$\frac{d^2 \bar{x}}{dt^2} \approx -\omega_r^2 \bar{x} \quad (4.6)$$

$$\Rightarrow x \approx x_0 \cos \omega_r t (1 + \frac{1}{2}q \cos \Omega_{\text{RF}} t). \quad (4.7)$$

We now have both axial and radial frequencies for our secular motion:

$$\omega_z = \sqrt{\frac{2eQ_z}{M}} \quad , \quad \omega_r = \frac{\Omega_{\text{RF}}}{2} \sqrt{\frac{q^2}{2} - \frac{2\omega_z^2}{\Omega_{\text{RF}}^2}}. \quad (4.8)$$

The correspondence principle now implies we can describe the secular motion in terms of the occupancy of a superposition/mixture of simple harmonic oscillator states. The radial and axial states are arranged in energy space as a ladder, each state $|n\rangle$ separated from its neighbours $|n \pm 1\rangle$ by $\hbar\omega_r$ and $\hbar\omega_z$ respectively.

4.2.5 Effect of micromotion on ion fluorescence

The effect of the driven velocity component on an ionic resonance is as follows. Let us consider the cooling transitions: the ion is addressed by the 397 cooling laser and 866 cooling repumper as it makes a classical secular orbit. The excitation of the ion into the scattering $P_{1/2}$ level when the 397 laser is at a frequency ν is given by a Lorentzian¹ $\mathcal{L}(\nu)$ parametrized by the centre frequency of the ionic resonance ν_0 , the power-broadened width $\Gamma_I \geq \Gamma_0$ and the broad-band scattering rate A . If both lasers are kept at a constant detuning, therefore, we see over several cycles of the RF an average fluorescence from the ion. As the micromotion of the ion tends to zero this fluorescence is $A_{21}\mathcal{L}(\nu)$; increasing the micromotion broadens this time-averaged response and makes it non-Lorentzian as we shall explain shortly.

Let us now consider timescales much shorter than an RF period. We use a photon correlation method (see section 3.2) to observe the relationship between photon arrival time and the phase of the RF. Over such timescales the ion makes a negligible secular excursion $\Delta\{\bar{x}\}$ but its micromotion ξ completes an orbit every RF period. The instantaneous excitation of the ion into the scattering $P_{1/2}$ level is modulated with a fixed phase relationship to the RF signal. This effect is explained if we consider the Doppler effect of the motion on the excitation of the ion. The sinusoidally varying

¹For hot ions, this is approximately a Voigt profile, a convolution of Lorentzian and Gaussian owing to the thermal Doppler width [80]; for colder ions the quantized simple harmonic motion causes the Lorentzian to be convolved with several delta functions instead, separated by $\nu_r = \omega_r/2\pi$ [102].

velocity causes phase modulation (PM) of the light field in the reference frame of the ion. This field has frequency $\nu = \omega/2\pi$ and wavevector $\mathbf{k}$, and an electric field vector given by:

$$\mathbf{E}(t) = \mathbf{E}_0 \exp(i\mathbf{k} \cdot \mathbf{x} - 2i\omega t) \quad (4.9)$$

The phase modulation is calculated in equivalent ways as either the integral of the frequency Doppler shift $2\pi \int \Delta\nu_0 dt = 2\pi\nu_0\xi \cos 30^\circ/c$ or as the variation in the spatial phase $\mathbf{k}_0 \cdot \boldsymbol{\xi} = k_0\xi \cos 30^\circ$ [105]. The cosine is the result of the 30° angle between the radial motion and the laser wavevector. The resultant field is a sum over harmonics [106]:

$$\mathbf{E}'(t) = \mathbf{E} \exp\left(i\frac{\sqrt{3}}{2}qk\bar{x}_0 \sin(\Omega_{\text{RF}}t)\right) \quad (4.10)$$

$$= \mathbf{E} \sum_{p=-\infty}^{\infty} i^p J_p\left(\frac{\sqrt{3}}{2}qk\bar{x}_0\right) \exp i(\omega_0 + p\Omega_{\text{RF}})t. \quad (4.11)$$

$J_p(a)$ is the p th-order Bessel function of the first kind evaluated at a , the modulation index. The bandwidth of this summation is $\sim \pm s\Omega/2\pi$ in cyclical units. For our ion trap in section 5.2 we calculate $k\bar{x}_0 = 0.506$, $q = 0.365 \rightarrow s = 0.16$; hence the bandwidth is $\sim \pm 1$ MHz when the ion is in thermal motion about the centre of the trap. This is much smaller than $\Omega_{\text{RF}}/2\pi = 6.29$ MHz, and therefore all harmonics are suppressed.

It is clear therefore what effect the ion's micromotion has on a spectral line. The incoming light is converted into a sum of infinitely many frequencies by PM, owing to the ion's motion. Each frequency excites atomic population independent of the others in the steady state. Thus the resultant atomic resonance is a convolution of the underlying, free-particle resonance curve with delta-functions corresponding to the frequency spectrum of the PM light. A harmonic frequency excursion is mapped via the underlying Lorentzian onto an anharmonic 'fluorescence excursion.' The resultant higher harmonics are seen in the fluorescence [37].

4.2.6 Atom-laser interactions and η

The interaction between a trapped ion and a laser field is different from that between a free particle and the same field [102]. When a free particle is excited by a photon it absorbs this photon and its momentum is changed. A particle confined in a trap does not have this freedom: its motional transitions must be made with respect to the ladder of eigenstates of the trapping potential, of energy $E_m = h\nu_z(m + \frac{1}{2})$. Such energy eigenstates are also therefore eigenstates in 'vibrational phonon number' m , and this phonon number can be raised (lowered) by the quantum-mechanical operator $a_z^\dagger$ (a_z).

Let us concentrate on the effect that motional quantization has on the atom-laser interaction Hamiltonian [107]. This Hamiltonian has a term of order $e^{i\mathbf{k} \cdot \mathbf{r}}$, where $\mathbf{k}$ is the wavevector of the light and $\mathbf{r}$ the position vector of the atom. Specifically we consider the exponential in $k_z z$. We define a Lamb-Dicke parameter η_z such that

$$k_z z \equiv \eta_z (a_z^\dagger + a_z). \quad (4.12)$$

Consider the transition element between two states of phonon number m, m' . For small η_z we can expand the exponential in the Hamiltonian in powers of η_z :

$$\langle m | e^{ik_z z} | m' \rangle \equiv \langle m | \left[1 - i\eta_z(a_z^\dagger + a_z) - (\eta_z^2/2!)(a_z^\dagger + a_z)^2 + \dots \right] | m' \rangle. \quad (4.13)$$

In the limit $\eta_z \ll 1$ this approximates to

$$\langle m | e^{ik_z z} | m' \rangle \approx \delta_{mm'} \left[1 - \frac{1}{2}\eta_z^2(2m+1) \right] - i\eta_z(\delta_{m',m+1}\sqrt{m'} + \delta_{m',m-1}\sqrt{m}). \quad (4.14)$$

Thus we have an expression for the transition matrix element between two adjacent vibrational states when coupled by the light field [15]:

$$\langle m | e^{ik_z z} | m-1 \rangle \propto \sqrt{m}\eta + \mathcal{O}(\eta^3), \quad (4.15)$$

The Lamb-Dicke parameter can be interpreted in a number of ways. It is proportional to the ratio of the spatial extent a_0 of the ground state of the harmonic oscillator to the wavelength λ of the light field addressing the ion; it is the square-root of the ratio of the single-photon recoil energy E_R to the vibrational energy-level spacing:

$$\eta \equiv \frac{\pi a_0}{\lambda} \equiv \sqrt{\frac{E_R}{h\nu_z}}. \quad (4.16)$$

The limit of $\eta \ll 1$ implies the ion is tightly bound to the trap, and the recoil momentum is passed predominantly to the macroscopic trap instead of changing the motional state of the ion. Equivalently the wavepacket of an ion in the ground state of such a trap has a width small compared to the wavelength of light, and this means that the phase difference across the wavepacket is small. This phase difference causes changes in the motional state of the ion in equation (4.14), and therefore these transitions are ‘frozen out’ by a tight trap.

4.3 Laser cooling theory for a three-level atom in a confining potential

4.3.1 Introduction

When a single ion is stored for long time periods in the trap it is typically addressed by the ‘cooling’ lasers: the 866 cooling repumper (unlocked, but tuned close to the resonant frequency) and the 397nm laser (several GHz detuned from resonance). These two transitions cycle population among the three levels $4S_{1/2}$, $4P_{1/2}$ and $3D_{3/2}$. Each level has angular momentum J_i and a sublevel degeneracy $2J_i + 1$, making in total a manifold of eight eigenstates in the ion.

We wish to discuss the implications of these storage conditions. Strictly speaking we require the eight-level Bloch equations to do so. However, there are certain conditions under which we can neglect the coherences that necessitate a full quantum-mechanical treatment. If the magnetic field direction and lasers’ polarizations are such that the ion is not polarized, we may average over Zeeman sublevels to reduce the problem to a three-level case with degeneracy. Furthermore, if the difference between the detunings of the lasers from their respective transitions is large, then any coherence between levels that results from a two-photon interaction will precess rapidly and can therefore be neglected.

In this section we use rate-equations to study the three-level Λ -system [108]. We consider an artificial case where the $D_{3/2}$ level does not decay, as well as the case where its decay is included. We also examine the impact of a mode hop in the 866 laser which either moves its centre frequency far-detuned or makes the laser multimode; in either case it no longer excites the ion. From these

calculations we obtain information about the ion's temperature, and the maximum heating rates in the trap.

4.3.2 The Einstein A coefficient and saturation intensity

We wish to determine the probability of excitation between two levels, owing to an interaction between an electromagnetic field and an electric dipole. Much of the theory in this case deals with broadband radiation for historical reasons [109]: an integral over the atomic response is implicit in the transition probabilities while the spectral density of radiation is explicit and not integrated.

Given that, in our experiments, the laser bandwidths are much less than the linewidths of the 4 P states then we do not wish to assume the laser spectral density to be constant over the atomic response. We note, however, that it is possible to determine Einstein A and B coefficients from a consideration of thermodynamic equilibrium [1]. In a similar manner the steady-state populations of two energy levels derived from both the rate equation and the Bloch equations may be compared [110]. Once Rabi oscillations have been compelled by spontaneous decay to settle to a steady state, then the two approaches should yield identical results.

With this in mind we can take as our starting point the standard expression for excitation probabilities [109, eq. (9.40)] and consider narrow-band radiation at a cyclical frequency ν . The atom starts off in a sublevel $|J_i, M_{J_i}\rangle$ and makes its transition to $|J_k, M_{J_k}\rangle$, where the labels correspond to angular momentum quantum number J and its projection M_J on a particular axis. Light polarized along the x -axis illuminates the ion. The magnetic field at the ion is not along this same axis, and its magnitude is sufficient to induce precession within the $D_{3/2}$ level and hence prevent optical pumping into the $M_J = \pm J_i$ states. The probability for a stimulated transition from the level $|k\rangle$ to $|i\rangle$ (averaging over sublevels) is then

$$P(J_k \rightarrow J_i) = P_{ki} = \frac{I}{\pi\epsilon_0\hbar^2c\Gamma_0} \frac{1}{2J_k + 1} \sum_{M_{J_i}, M_{J_k}} |\langle J_i, M_{J_i} | -e\hat{x} | J_k, M_{J_k} \rangle|^2, \quad (4.17)$$

where the light has local intensity I and all other symbols have their usual meaning. The line strength S is defined as [111]

$$S_{ki} = S_{ik} = \sum_{M_{J_i}, M_{J_k}} |\langle J_i, M_{J_i} | -e\hat{\mathbf{r}} | J_k, M_{J_k} \rangle|^2.$$

In this expression the overlap integral has x , y and z terms and thus the overlap in equation (4.17) is equal to $S/3$, given no particular orientation of the quantization axis. The relationship between S and A_{ki} [111] yields

$$P_{ki} = A_{ki} \frac{I\lambda^3}{4\pi^2\hbar c\Gamma_0}. \quad (4.18)$$

We may define the saturation intensity I_s on a particular transition as the intensity for which $P_{ki} = A_{ki}$ i.e. the stimulated emission rate from state i to state k is equal to the spontaneous decay on the same transition:

$$I_s = \frac{4\pi^2\hbar c\Gamma_0}{\lambda^3}. \quad (4.19)$$

The fluorescent response of this transition as the laser is scanned is power broadened (see appendix B.1.1).

4.3.3 The three-level system

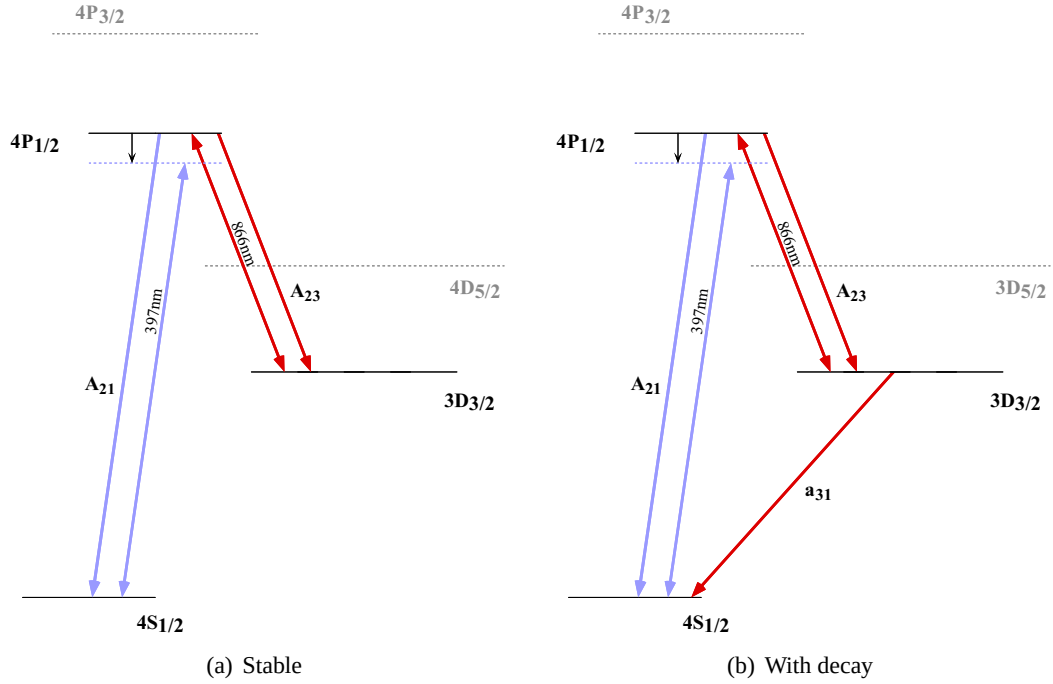


Figure 4.1: Three-level system involved in laser cooling. In (a) the $D_{3/2}$ level is considered stable (the limit of metastable); in (b) it decays to the $S_{1/2}$ level.

When the ion is unpolarized then all Zeeman components of each transition are addressed and we can ignore optical pumping into any particular sublevel. As in the introduction we average over all Zeeman states and treat the eight-sublevel system as a three-level system with degeneracy. Figure 4.1 shows two special cases of this three-level system. In the first case $|D_{3/2}\rangle$ is considered stable and does not decay to the ground state $|S_{1/2}\rangle$. In the second case a small A coefficient a_{31} governs slow, metastable decay on this transition.

When the 866 laser is intense and tuned to the ionic resonance, the primary channel for population leaving $|D_{3/2}\rangle$ is by absorption to $|P_{1/2}\rangle$ and not by spontaneous emission to $|S_{1/2}\rangle$. The rates of the two channels differ by many orders of magnitude, and in this situation we can therefore consider the level to be not merely metastable but (to a good approximation) stable. When, on the other hand, we wish to see the effect on the system of removing the 866 laser, we should consider the metastable case instead.

4.3.3.1 Three-level system with a stable excited state

Consider the level system in figure 4.1(a). The $S_{1/2}$, $P_{1/2}$ and $D_{3/2}$ manifold can be studied as a three-level system of $|1\rangle$, $|2\rangle$ and $|3\rangle$ respectively. $|2\rangle$ decays to $|1\rangle$ and $|3\rangle$ with rates A_{21} and A_{23} , and frequencies ν_{01} and ν_{03} , respectively. Its linewidth is $2\pi\Gamma_2 = (A_{21} + A_{23})$.

The i th state has a degeneracy in angular momentum of $2J_i + 1$, and we define a degeneracy ratio $r_i = (2J_2 + 1)/(2J_i + 1)$. The $|1\rangle \leftrightarrow |2\rangle$ transition is addressed by a laser of intensity I_1 (measured in units of the saturation intensity of the transition I_{s1}). This laser is detuned by δ_1 to the red i.e. cooling side of the transition, where the detuning is measured in units of $\Gamma_2/2$ [112]:

$$\delta_1 = \frac{\nu_{01} - \nu_1}{\Gamma_2/2}. \quad (4.20)$$

Hence $\delta_1 > 0$ for cooling on the 397 nm transition. The laser has linewidth $\Gamma_{\text{las}2} \ll \Gamma_2$. Similarly the transition $|3\rangle \leftrightarrow |2\rangle$ is excited by intensity I_3 at a detuning δ_3 .

We will consider the steady-state solution. In addition we will assume that the relative detuning of the lasers $|\delta_1 - \delta_3| \gg 0$. This avoids the possibility of two-photon coherences between levels e.g. dark resonances (see later). With these requirements we may be satisfied that rate equations based on the Einstein A and B coefficients will be equivalent to the results that would be obtained from the optical Bloch equations. The rate equations for the two stable levels are

$$\frac{dn_1}{dt} = n_2 A_{21} + n_2 A_{21} \mathcal{L}(\nu_1) I_1 - r_1 n_1 A_{23} \mathcal{L}(\nu_1) I_1 \quad (4.21)$$

$$\frac{dn_3}{dt} = n_2 A_{23} + n_2 A_{23} \mathcal{L}(\nu_3) I_3 - r_3 n_3 A_{23} \mathcal{L}(\nu_3) I_3. \quad (4.22)$$

In these rate equations the Lorentzian $\mathcal{L}$ has baseline $B = 0$ and scaling factor $A = \frac{1}{2}\pi\Gamma_2$. This notation is outlined in appendix A.4.1, and should not to be confused with the Einstein coefficients. This normalization of the Lorentzian sets its line-centre value $\mathcal{L}(\nu_0) = 1$. If we include the condition that the total population $n_1 + n_2 + n_3 \equiv 1$ is conserved then the steady state solution for the excited population is [67]

$$n_2 = \left\{ 1 + \frac{1}{r_1} + \frac{1}{r_3} + \frac{\delta_1^2 + 1}{r_1 I_1} + \frac{\delta_3^2 + 1}{r_3 I_3} \right\}^{-1}. \quad (4.23)$$

and the population in $|3\rangle$ is

$$n_3 = n_2 \left[\frac{\delta_3^2 + 1 + I_3}{r_3 I_3} \right]. \quad (4.24)$$

4.3.3.2 Three-level system with decay to the ground state

$|D_{3/2}\rangle$ has a lifetime of $a_{31} = 1.20(1)$ s, as shown in figure 4.1(b). The spontaneous decay channel acts to recycle population from the metastable level to the other levels, and the calculation from rate equations is considered separately in appendix B.1.2 for clarity. Equations (4.23) and (4.24) become:

$$n_2 = \left\{ \frac{1}{r_1} + \frac{\delta^2 + 1}{r_1 I_1} \left(1 + \frac{a_{31}}{A_{21}} \left[\frac{\delta_3^2 + 1 + I_3}{r_3 I_3 + \frac{a_{31}}{A_{23}}(\delta_3^2 + 1)} \right] \right) + 1 + \left[\frac{\delta_3^2 + 1 + I_3}{r_3 I_3 + \frac{a_{31}}{A_{23}}(\delta_3^2 + 1)} \right] \right\}^{-1} \quad (4.25)$$

$$n_3 = n_2 \left[\frac{\delta_3^2 + 1 + I_3}{r_3 I_3 + \frac{a_{31}}{A_{23}}(\delta_3^2 + 1)} \right]. \quad (4.26)$$

i.e. equations (B.7) and (B.5).

4.3.4 Results

Table 4.1 gives the numerical values of various parameters used in this section. Typical laser intensities are:

$$I_{397} \equiv I_1 \sim I_{s1} \quad I_{s1} = 2757 \mu\text{W mm}^{-2} \quad (4.27)$$

$$I_{866} \equiv I_3 \simeq I_{s3} \times 80 \quad I_{s3} = 266 \mu\text{W mm}^{-2}. \quad (4.28)$$

The variations in the 866 nm intensity from one day to the next were much less than those of the 397 nm light, which varied owing to alignment of the 794 lasers into the frequency-doubling cavity.

Description	Symbol	Freq. / Hz	Energy/eV	Temperature	Phonon no.	Other
Trap spacing	ν_z	$207 \cdot 10^3$	$8.56 \cdot 10^{-10}$	$9.93 \mu\text{K}$	1	
Natural width	Γ_2	$22 \cdot 10^6$	$9.10 \cdot 10^{-8}$	1.06 mK	107	
Trap depth	E_{trap} etc.	$2.4 \cdot 10^{15}$	10	0.12 MK	$1.17 \cdot 10^{10}$	
Lamb-Dicke	η					0.391

Table 4.1: Numerical values of parameters. If the parameter is an energy, frequency or phonon occupation then it is listed in all its forms. The column marked ‘other’ contains dimensionless parameter values.

4.3.4.1 Fluorescence and temperature

We wish to relate the photon scattering rate $n_2 A_{21}$ to the ion temperature, and to cooling and heating rates within the trap. Initially we only consider heating from photon scattering, although we will discuss the possibility of other heating mechanisms later.

In order to relate the scattering rates to a static temperature, we need to understand how the scattered photons contribute to both cooling and heating of the ion. To a good approximation the ion is trapped in a simple-harmonic potential of characteristic frequency ν_z .

We begin by considering the ladder of states of the quantum harmonic oscillator $|m\rangle$, having energies $E_m = h\nu_z(m + \frac{1}{2})$, $m \in \{0 \dots\}$. Figure 4.2(a) details this system, with the transitions $|m\rangle \leftrightarrow |m-1\rangle$. The excitation rate $|m-1\rangle \rightarrow |m\rangle$ is $m A_-$; the de-excitation rate $|m-1\rangle \leftarrow |m\rangle$ is $m A_+$.

In figure 4.2(b) we consider these two motional states coupled to the internal states of the ion. The lower two states correspond to the ground electronic state $|1\rangle$; the upper two correspond to the excited state $|2\rangle$. Two routes² for single-phonon creation in figure 4.2(a) are shown explicitly: equivalent routes govern phonon annihilation. In the notation of section 4.2 we assume the Lamb-Dicke parameter $\eta \ll 1$ and ignore $\mathcal{O}(\eta^2)$ terms in the atom-laser coupling Hamiltonian. Therefore the rates of excitation to, and de-excitation from, $|m\rangle$ are [113]

$$m A_+ = m \eta^2 A_{21} (n_2(\delta_1)\alpha + n_2(\delta_1 + 2\nu_z/\Gamma_2)) \quad (4.29a)$$

$$m A_- = m \eta^2 A_{21} (n_2(\delta_1)\alpha + n_2(\delta_1 - 2\nu_z/\Gamma_2)) \quad (4.29b)$$

The right-hand side of each equation has two terms. The first is absorption on the carrier frequency, followed by a spontaneous emission with $\delta m \neq 0$; the second is absorption on a sideband, followed

²Transitions are possible with $\Delta m > 1$ but we only consider detailed balance between two adjacent harmonic energy levels.

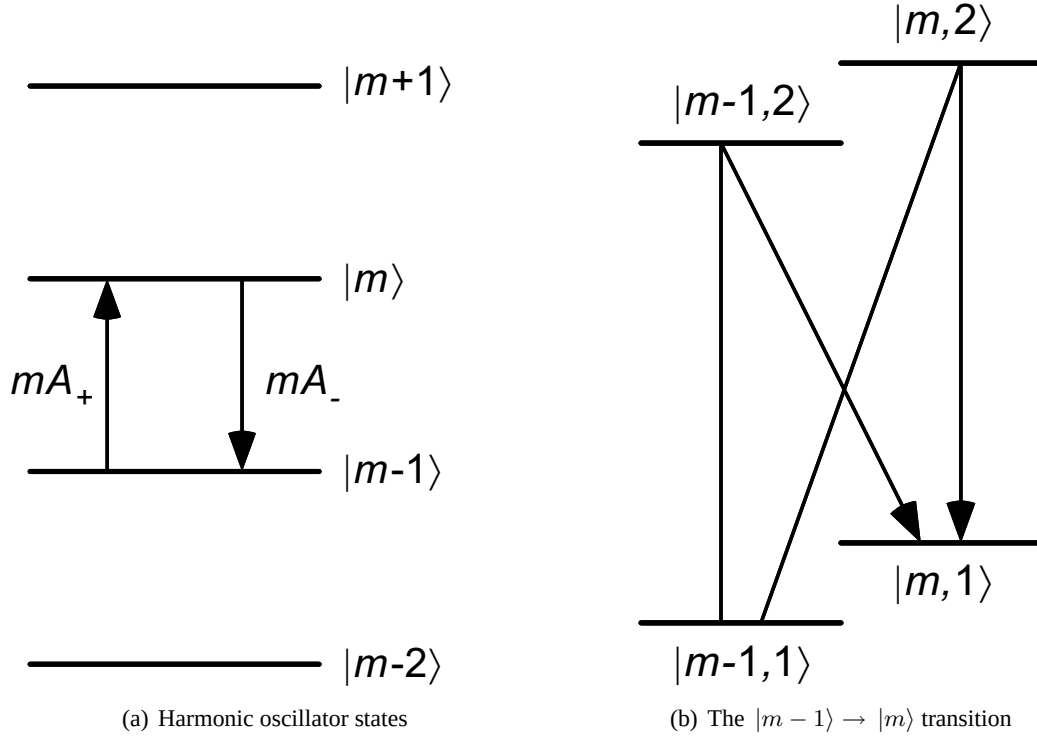


Figure 4.2: Excitation rates in the harmonic oscillator. (a) We consider detailed balance between two adjacent motional states. (b) Here we show the motional states $|m\rangle, |m-1\rangle$ (small splitting) coupled to the electronic state of the ion $|1\rangle, |2\rangle$ (large splitting). In the limit $\eta \ll 1$ only carrier and sideband transitions are considered.

by spontaneous emission on the carrier $\Delta m = 0$. The factor of $\alpha\eta$, as opposed to η , arises from the directional dependence of spontaneous emission³. An ion undergoing a change in motional state along a given axis couples more strongly to axial vacuum fluctuations than to fluctuations at an angle to this axis. Hence the matrix element is decreased: $\alpha = 2/5$ [109]. Note that, for $\delta_1 > 0$, $A_+ < A_-$ i.e. the ion is cooled.

The principle of detailed balance, applied to any pair of levels, yields for the population ρ_m of level m

$$A_- \rho_{m+1} = A_+ \rho_m \quad (4.30)$$

$$\therefore \rho_m = (1-s)s^m, \text{ where } s = A_+/A_-. \quad (4.31)$$

This solution is the same as that given by a Boltzmann distribution of temperature $T = -\hbar\nu_z/k_B \ln s$. The mean energy of the solution is

$$E_{\text{fin}} \equiv \langle E \rangle_T = \hbar\nu_z \left[\frac{1}{s^{-1}-1} + \frac{1}{2} \right] \quad (4.32)$$

$$= \hbar\nu_z \left[\frac{A_+}{A_- - A_+} + \frac{1}{2} \right], \quad (4.33)$$

³For isotropic motion $\alpha = 1/3$; this would require a transition between S-states in an atom and is thus dipole-forbidden.

and the mean phonon number is

$$\langle m \rangle_T = \frac{1}{s^{-1} - 1}. \quad (4.34)$$

This thermal mean is plotted in figure 4.3 against the detuning of the cooling laser, for different values of the cooling laser intensity, and with and without the cooling repumper. In this figure it is clear that the minimum temperature is obtained with a low-intensity cooling laser detuned to the half-width of the transition. We also observe that, with detunings greater than $\sim 10^4$ half-linewidths (~ 100 GHz), the ion temperature is independent of the presence or absence of the cooling repumper. We will show later how this relates to an effective linewidth of the unrepumped transition.

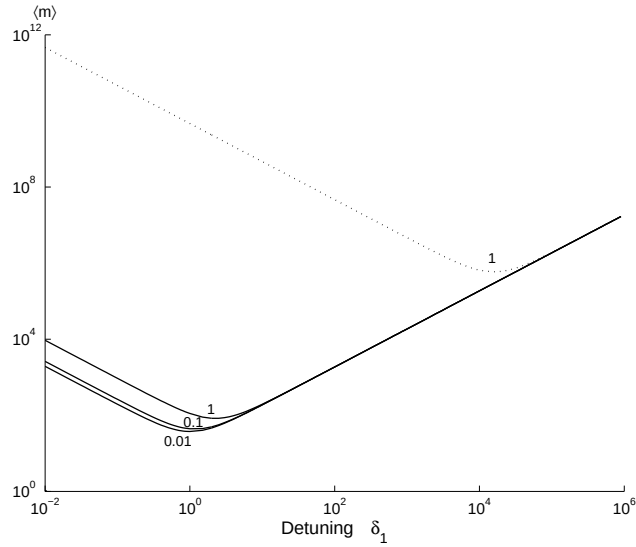


Figure 4.3: Graph of the thermal mean of the vibrational quantum number $\langle m \rangle_T$, plotted against dimensionless detuning δ_1 (measured in units of the half-linewidth Γ_2). The solid lines indicate the presence of the cooling repumper; the dotted line is the temperature of the ion without the repumper. The dimensionless intensity of the cooling laser I_1 is indicated beside each plot.

Let us consider the case $|A_+ - A_-| \ll A_+$. This occurs when for example $\delta_1 \gg 2\nu_z/\Gamma_2$. In this situation we can approximate

$$A_{\pm} = A_0(\alpha + 1)[1 \mp \epsilon]. \quad (4.35)$$

Hence

$$k_B T \approx h\nu_z/2\epsilon. \quad (4.36)$$

I. ION ADDRESSED BY BOTH COOLING LASERS We obtain the equilibrium temperature when δ_1 is large. If the 866 laser is illuminating the ion correctly the excitation probabilities in equations (4.29) are:

$$A_{\pm} = \frac{\eta^2 A_{21} r_1 I_1}{\delta_1^2} \left(\alpha + 1 \mp 2 \left[\frac{2\nu_z}{\Gamma_2 \delta_1} \right] + 3 \left[\frac{2\nu_z}{\Gamma_2 \delta_1} \right]^2 + \dots \right). \quad (4.37)$$

In the case where $\delta_1 \gg 2\nu_z/\Gamma_2$ this becomes equation (4.35) with

$$A_0 = \frac{\eta^2 A_{21} r_1 I_1}{\delta_1^2}, \quad \epsilon = \frac{4\nu_z}{\Gamma_2 \delta_1 (\alpha + 1)}. \quad (4.38)$$

Assuming no extra heating rates this gives us a storage temperature T where

$$k_B T \approx \frac{1}{8} h \delta_1 \Gamma_2 (\alpha + 1). \quad (4.39)$$

At a laser detuning of $\delta_1 \Gamma_2 / 2 = 4$ GHz, and assuming no other heating rates, the ion temperature is

$$T = 67 \text{ mK}. \quad (4.40)$$

II. EFFECT OF LOSS OF 866 NM LIGHT If the 866 laser is no longer functional we have a different expression for $A_{\pm}$. As a preliminary insight into this effect let us examine the rate of 397 nm fluorescence in both cases. We use the stable-state equation (4.23) to describe the system with the 866 laser operational. With $\Gamma_2 = 22$ MHz and the 866 laser on resonance, one term in equation (4.23) dominates. The population and hence blue-photon scattering rate are:

$$\begin{aligned} n_2 &\simeq \left\{ \frac{\delta_1^2 + 1}{r_1 I_1} \right\}^{-1} \\ &\simeq 7.56 \cdot 10^{-6} \\ \therefore n_2 A_{21} &\simeq 1030 \text{ s}^{-1}. \end{aligned}$$

The branching ratio of decays to $|S_{1/2}\rangle$ and $|D_{3/2}\rangle$ is approximately 94:6. This means that, after around 16 blue photons have been scattered, the ion will on average have decayed to $|3\rangle$, where it will remain for around 1.2 s. A scattering rate of 1030 s^{-1} is far greater than the rate implied by this simple argument, and this also implies that optical pumping out of $|D_{3/2}\rangle$ by the 866 laser is integral to this result.

When the 866 laser is not addressing the transition, the level of fluorescence on the 397 nm transition is given by equation (B.9):

$$\begin{aligned} n_2 A_{21} &= a_{31} \frac{A_{21}}{A_{23}} \left\{ 1 + \frac{a_{31}}{A_{23}} \left(1 + \frac{1}{r_1} + \frac{\delta_1^2 + 1}{r_1 I_1} \left(1 + \frac{A_{23}}{A_{21}} \right) \right) \right\}^{-1} \\ &\approx a_{31} \frac{A_{21}}{A_{23}} \left\{ 1 - \frac{a_{31}}{A_{23}} \frac{\delta_1^2 + 1}{r_1 I_1} \left(1 + \frac{A_{23}}{A_{21}} \right) \right\} \end{aligned} \quad (4.41)$$

$$\approx 12.9 \text{ s}^{-1}. \quad (4.42)$$

The second term in the braces contributes approximately 1.4% to the final answer. This result can be understood as a burst of, on average, 13 photons scattered before the ion is pumped to $|D_{3/2}\rangle$, where it remains for on average 1.2 s; the small second term is due to the fact that the 397 laser pumps population into $|P_{1/2}\rangle$ at a finite rate.

Returning to equation (4.41), we expand the dependence on the frequency of 397 nm light once again in powers of ν_z/Γ_2 :

$$n_2(\delta_1 \pm 2\nu_z/\Gamma_2) = n_2(\delta_1) \mp \left(\frac{a_{31}}{A_{23}}\right)^2 \left(1 + \frac{A_{23}}{A_{21}}\right) \frac{2\delta_1}{r_1 I_1} \frac{2\nu_z}{\Gamma_2} + \mathcal{O}(\nu_z/\Gamma_2)^2. \quad (4.43)$$

Then $A_{\pm}$ become

$$A_{\pm} \approx A'_0(\alpha + 1)(1 \mp \epsilon') \quad (4.44)$$

$$\text{where } A'_0 = \eta^2 A_{21} n_2(\delta_1) \quad , \quad \epsilon' = \frac{\left(\frac{a_{31}}{A_{23}}\right)^2 \left(1 + \frac{A_{23}}{A_{21}}\right) \frac{2\delta_1}{r_1 I_1} \frac{2\nu_z}{\Gamma_2}}{(\alpha + 1)n_2(\delta_1)}. \quad (4.45)$$

We calculate that

$$n_2(\delta_1) = 9.75 \cdot 10^{-8} \Rightarrow \epsilon' = 1.04 \cdot 10^{-6}, \quad (4.46)$$

and this gives a storage temperature of:

$$T' \approx h\nu_z/(2k_B\epsilon') \approx 4.8 \text{ K}. \quad (4.47)$$

This means that, in the absence of other heating mechanisms, the repumper laser is not required to trap the ion for long periods.

4.3.4.2 Comparison with simple model

A semiclassical model of laser cooling is available which relates the temperature of the ion to its fluorescence spectrum [114]. Let the scattering rate for photons of angular frequency ω be $R(\omega)$. The rate for an ion moving at speed v_z is then

$$R(\omega) \rightarrow R(\omega) - \frac{dR(\omega)}{d\omega} \frac{(v_z)\omega}{c}. \quad (4.48)$$

The scattering force is $R\hbar\omega c$. From this we obtain the equation of motion of the ion, ignoring heating:

$$m \frac{d^2 z}{dt^2} + \frac{dR(\omega)}{d\omega} \frac{\hbar\omega^2}{c^2} \frac{dz}{dt} + m\omega_z^2 z = 0. \quad (4.49)$$

The mean rate of doing work on the ion (cooling rate only) is therefore

$$\frac{dE}{dt} = - \frac{dR(\omega)}{d\omega} \frac{\hbar\omega^2}{c^2} \frac{E}{m}, \quad (4.50)$$

We now consider the photon heating rates. These arise from: the directional randomness of spontaneous photon emission; and the temporal randomness of the laser intensity fluctuations (shot noise). In steady state, the total rate of change of temperature from these two terms [76] and the cooling rate given above must be zero i.e.

$$- \frac{dR(\omega)}{d\omega} \frac{\hbar\omega^2}{c^2} \frac{E}{m} + \frac{\hbar^2 k^2}{4m} \alpha R(\omega) + \frac{\hbar^2 k^2}{4m} R(\omega) = 0. \quad (4.51)$$

The steady state temperature of the ion is therefore

$$T = \frac{\hbar}{2k_B} \frac{R(\omega)}{dR(\omega)/d\omega} \frac{\alpha + 1}{2}. \quad (4.52)$$

At the Doppler limit, with an unbroadened transition, this reduces to $T = T_{\text{Dopp}} \hbar \Gamma / 2k_B$. More generally, for a homogeneously-broadened transition of linewidth Γ' , equation (B.2) yields in terms of cyclical laser frequencies:

$$T = \frac{(\nu - \nu_0)^2 + \Gamma'^2/4}{\nu - \nu_0} \frac{\hbar}{4k_B} (\alpha + 1). \quad (4.53)$$

In our case, the scattering rate $R = n_2 A_{21}$. We note from equations (B.7), (B.9) that n_2 is Lorentzian both with and without the 866 laser. In the first case the laser is far-detuned with respect to the transition width. The ion temperature is, from equation (B.3),

$$T \approx \frac{\hbar \delta_1 \Gamma_2}{8k_B} (\alpha + 1) = 67 \text{ mK}. \quad (4.54)$$

This is the same as the result in equation (4.39). In the second case, with no repumper, the transition has shape

$$R = n_2 A_{21} \approx A_{21} \frac{I_1}{I_1(A_{23}/a_{31}) + \delta_1^2}. \quad (4.55)$$

This corresponds to a transition approximately 60 GHz wide, with the laser detuned a small frequency from line centre relative to this width. We recall in figure 4.3 that the temperature of the ion both with and without the repumper is identical at detunings much greater than this linewidth. This is because the excitation rate into $|2\rangle$ is becoming comparable with the spontaneous decay from $|3\rangle$, and therefore repumping is no longer the limiting factor on cooling the ion.

In this limit equation (4.53) becomes

$$\begin{aligned} T &= \frac{\hbar \Gamma'}{8k_B} \frac{(\nu - \nu_0)^2 + \Gamma'^2/4}{(\nu - \nu_0) \Gamma'} (\alpha + 1) \\ &\approx \frac{\hbar \Gamma_2}{8k_B \delta_1} I_1(A_{23}/a_{31}) \left(1 - \frac{A_{23}}{A_{21}}\right) (\alpha + 1) \\ &\approx 4.8 \text{ K}. \end{aligned} \quad (4.56)$$

This result is again the same as that obtained from our more detailed theory in equation (4.47).

4.3.4.3 The heating rate constant κ

Let us now make the following assumption about heating rates in our trap. The processes we have already considered—creation and annihilation of phonons by the atom-laser interaction—have yielded rates proportional to the number of phonons in that particular oscillation mode. With this in mind we also assume that other, unwanted heating rates have this bosonic property [115]. We shall discuss this assumption below.

Let us consider the heating rate ma_+ to be incorporated into the existing photon-scattering rate i.e. $mA_+ \rightarrow mA'_+ = mA_+ + ma_+$. We define the heating rate constant κ by

$$a_+ = A_0(\alpha + 1)\kappa. \quad (4.57)$$

This addition therefore increases the steady-state temperature to T_κ , where

$$k_B T_\kappa \approx \frac{h\nu_z}{2\epsilon - \kappa}. \quad (4.58)$$

Let the maximum phonon number for an ion to remain trapped be $m_{\text{trap}} = k_B T_{\text{trap}} / h\nu_z$. The requirement that we do not exceed this phonon number yields

$$\kappa < 2\epsilon - \frac{1}{m_{\text{trap}}}. \quad (4.59)$$

If we assume we only trap for a finite time t_f , rather than indefinitely, then this permits an increase in κ ; indeed the ion may be heated during this time so long as it remains within the trap. We therefore consider the rate of change of the mean phonon number $\langle m \rangle$ (see [113, eq. (5.60)]):

$$\frac{d}{dt} \langle m \rangle = \langle m \rangle (A'_+ - A_-). \quad (4.60)$$

In this equation we have assumed $\langle m \rangle \gg 1$ i.e. a hot ion. This amounts to neglecting a spontaneous emission term A_+ on the right-hand side. The solution to the resultant equation is exponentially increasing; the neglected term provides a linear contribution which can indeed be dropped⁴.

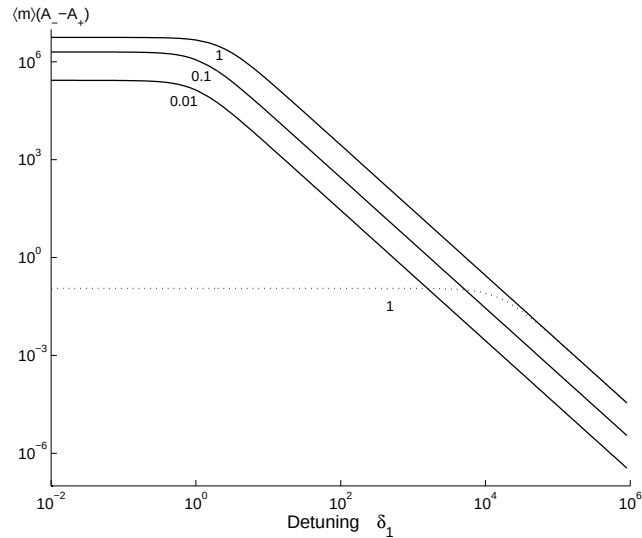


Figure 4.4: Graph of the cooling rate provided by the laser, plotted against dimensionless detuning δ_1 (measured in units of the half-linewidth Γ_2). As with figure 4.3, the dotted line indicates cooling without the repumper. I_1 is indicated beside each plot.

Equation (4.60) with no extraneous heating ($A'_+ = A_+$) is plotted in figure 4.4 against the detuning δ_1 . This provides us with an estimate of the cooling rate which the lasers can provide as a rate of phonon removal from the trap. The cooling rate increases with greater cooling laser intensity: this is just a function of the increased scattering rate of photons and is accompanied by an increase in ion temperature (see figure 4.3). As before, with cooling laser detunings $\gtrsim 60$ GHz, the cooling power of the lasers is dependent only on δ_1 and not on the presence or absence of the cooling

⁴Inclusion of the linear term yields equation (4.34) as a steady-state solution when $A_- > A_+$.

repumper. For large δ_1 the cooling rate decreases as δ_1^{-3} : this is consistent with equations (4.35) and (4.38).

Let the ion have an initial phonon number of $\langle m \rangle_i$. Over ten hours it can be heated ($A'_+ > A_-$) so long as its final phonon number is less than m_{trap} . With this condition we obtain

$$\kappa - 2\epsilon < \frac{\ln \frac{m_{\text{trap}}}{\langle m \rangle_i}}{A_0(\alpha + 1)t_f}. \quad (4.61)$$

κ is related to the additional heating rate (as a rate of change of temperature) by

$$\frac{dT}{dt} = \langle m \rangle A_0(\alpha + 1)\kappa h\nu_z/k_B \equiv Ta_+. \quad (4.62)$$

Our initial assumption—that the heating rate is proportional to $m\kappa$ —has resulted in $\langle m \rangle$ having an exponential time dependence. This is in approximate agreement with the conclusions of other studies [103, 116]. In these references, exponential heating arises from fluctuations in the spring constants of a Paul trap and optical FORT [9] respectively. There is an additional linear increase of the phonon number with time arising from fluctuations in the position of trap centre. We neglect this term for two reasons: the symmetry of our trap voltages implies a preference for symmetrical noise in the potential, unlike the beam-steering discussed in a FORT; over the timescales we are considering (several hours), the exponential heating dominates, and in equation (4.60) we have already discarded a linear heating contribution for this reason.

We proceed as follows. Using our bosonic heating model, we determine an upper limit on κ implied by our successful overnight storage, both with and without the repumper laser. We discuss the (phonon-number dependent) absolute heating rates implied by this upper limit, when the 397 nm light is red-detuned firstly 4 GHz and then to the half-width of the transition.

I. THE HEATING RATE CONSTANT WITH BOTH LASERS We calculate the maximum κ by considering the maximum cooling rate available from the laser cooling. In the steady state this is equal to the maximum heating rate from other processes which can be withstood without losing the ion. Finally we show that the non-steady state case is a small correction to this result.

Consider equation (4.59). From the definition of ϵ (in presence of the 866 laser) in equation (4.38), $\epsilon = 1.04 \cdot 10^{-4}$. In addition, from table 4.1, $m_{\text{trap}} = 1.17 \cdot 10^{10}$. Our upper limit on κ is therefore

$$\kappa < 2.07 \cdot 10^{-4}. \quad (4.63)$$

Equations (4.38),(4.57) yield

$$A_0 = 54800 \text{ s}^{-1} \rightarrow a_+ < 15.9 \text{ s}^{-1}. \quad (4.64)$$

Let us briefly consider the effect of finite time, where we do not have a steady state. Equation (4.61) yields

$$\kappa - 2\epsilon < \frac{14.4}{(54800 \text{ s}^{-1})(7/5)(36000 \text{ s})}. \quad (4.65)$$

This is a small correction.

II. THE HEATING RATE CONSTANT WITH NO 866 NM LIGHT When the 866 laser does not address the relevant transition, $\epsilon \rightarrow \epsilon'$ and $A_0 \rightarrow A'_0$ as shown earlier. We find that we can no longer assume the steady-state case with the adjustment for finite confinement time as a ‘small correction:’ indeed, the reverse is the case. Let us begin at $\langle m \rangle_T = 6750$ ($T = 67$ mK) and turn the 866 laser off. Equations (4.45, 4.61) give

$$\kappa' \lesssim 2.08 \cdot 10^{-6} + 1.45 \cdot 10^{-4} \approx 1.5 \cdot 10^{-4}. \quad (4.66)$$

The dependence on trapping time is because the laser-induced cooling and heating rates are very small: over ten hours’ confinement the upper limit on extraneous heating is almost equivalent to the physical situation where there is no laser cooling at all.

III. HEATING OF THE ION Using κ and κ' we determine heating of the ion as a rate of change of temperature and phonon number. We first consider the situation where both cooling lasers are illuminating the ion, using κ . Let us determine the heating rate from our successful overnight storage i.e. at $\delta_1 = 364$. We must consider the possibility that extraneous heating a_+ has increased the mean phonon number of the ion to less than or equal to the trap phonon number: $\langle m \rangle_T \lesssim 10^{10}$. In this situation the scattered photons cool very strongly. The maximum possible heating rate that permits indefinite containment of an ion, with the 397 laser detuned 4 GHz, is

$$\frac{dT}{dt} = 1.6 \text{ MK s}^{-1}, \quad (4.67)$$

from equation (4.63).

We now move to the situation where there is no 866 laser, using κ' . The upper limit on the new heating rate is given by:

$$\kappa' = 1.5 \cdot 10^{-4} \quad , \quad A'_0 = 1.85 \quad (4.68)$$

$$\Rightarrow a'_+ = 3.9 \cdot 10^{-4} \text{ s}^{-1}. \quad (4.69)$$

This quantity is difficult to relate to an absolute heating rate, because the phonon number and hence heating rate is not assumed constant over the ten hours’ storage. Consider instead the ion cooled to the Doppler limit, and the heating rates implied at this limit. Knowledge of heating at this limit is useful, because it is in this regime that we will perform coherent manipulation of ions, and the heating may have a detrimental effect on the fidelity of these operations [103]. From κ, A_0 (i.e. assuming a well-behaved 866 laser) we have a maximum heating rate at $\langle m \rangle_T \sim 50$ of

$$\left. \frac{dT}{dt} \right|_{\text{Dopp}} = 8.4 \text{ mK s}^{-1}. \quad (4.70)$$

or 800 phonons per second. At $\langle m \rangle_T \lesssim 1$ (i.e. cooled down to the ground state of motion) the heating rate is ~ 15 phonons per second. If, on the other hand, we assume no 866 nm light on the ion during overnight storage then we obtain from κ', A'_0

$$\left. \frac{dT}{dt} \right|_{\text{Dopp}}' = 0.21 \text{ } \mu\text{K s}^{-1}. \quad (4.71)$$

or only 0.021 phonons per second; at $\langle m \rangle_T \lesssim 1$ this corresponds to less than $4 \cdot 10^{-4}$ phonons per second.

In order to determine how this might affect the fidelity of quantum gate operations, consider the following. The NIST elliptical Paul trap [117] has a heating rate of around 19^{+40}_{-13} phonons per second when two ions (qubits) have been cooled to the ground state of motion. The ‘NIST gate’ is a phase shift of the internal state of one qubit, dependent on the state of the other qubit, performed via the COM vibrational mode [118]. At the quoted heating rate the fidelity of this gate is around 90% [103].

We note that our ion trap has different trapping parameters, and that linear rather than exponential heating will dominate over the short timescales involved in quantum logic gates. Nonetheless, we have observed ions remaining trapped overnight even when the 866 laser went multimode at some unknown time during the night. This implies that the ion remained trapped for several hours with little or possibly negligible repumping from this laser. Therefore the heating rate in the trap could be above the limit set in equation (4.69) but it must be substantially below that set in equation (4.64). Overall, this implies the exponential part of the heating in our trap is small enough that it will not be the dominant limit on fidelity of quantum gate operations.

4.3.5 Discussion

From rate equation methods we have developed a description of the thermal behaviour of a trapped ion when the cooling 397 laser is far-red detuned. This gives the temperature of the ion, and the maximum heating rate permitted in the system. It also encompasses the expected behaviour when the repumping 866 laser malfunctions in some fashion and is no longer addressing the transition in the ion. Monitoring of the behaviour of this laser overnight, or simply turning the laser off would confirm the lower bound on the heating rate.

Our predictions are based on several assumptions. We assume that the cooling laser is well-behaved all night. We also assume that the ion is not subject to any sudden shocks. This assumption is questionable, as we have seen sudden losses of fluorescence during long-term ion storage. These losses are consistent with collisions with the background gas of the trap. Such sudden events occur on average once per hour. They will reduce our maximum limits on heating from other sources, by an amount that depends on the energy they impart and the frequency of the collisions. From consideration of these assumptions, we conclude that our upper limits on the non-collisional heating rates are pessimistic.

Also, however, it is not clear how our recent ion storage problems agree with our previous observations. Since we stopped using the 794M/794S frequency-doubled system and moved to a blue laser diode at 397 nm we have had great difficulty storing an ion overnight. This may be due to problems with the 397 laser lock being compounded by the laser’s high intensity: if the laser wanders temporarily to the blue of the transition then it very quickly heats the ion out of the trap.

4.4 Conclusions

Most discussions of laser cooling treat—effectively—a 2-level atom. We have extended this to a 3-level system, and have considered a bound absorber instead of a free particle. We have also developed a system wherein bosonic heating can be easily introduced.

If the repumper laser in our 3-level Λ -system is absent, we have shown in this chapter that we are still able to confine an ion indefinitely. We have also shown that the greatest cooling rate is obtained at near detunings of the cooling laser. In addition we have placed an upper bound on one form of heating. This exponential heating is at such a level that it will not be the limiting factor in later quantum-computation experiments.

Chapter 5

Improvement of the radio-frequency potential in a Paul trap

5.1 Introduction

As we have seen in section 4.2 the motion of an ion in a Paul trap of given geometry depends on the frequency and amplitude of the radio-frequency voltage on the Paul electrodes. We would like our new RF supply to satisfy three requirements:

1. The pseudopotential must be tighter.
2. It must produce a single-frequency output (i.e. sinusoidal without harmonics).
3. Random noise on the output must be minimized.

Harmonics and noise in the RF contribute to heating of the ions [103]. We will use passive, reactive impedors rather than active electronics in our tuned circuit, to avoid anharmonic distortion. A resonant circuit provides low-noise amplification at the required RF frequency Ω_{RF} , while filtering other frequencies. It is especially important to avoid noise at the secular trap frequency $\omega_r = q\Omega_{\text{RF}}/2\sqrt{2}$. The circuit will generate a tighter pseudopotential which is still stable when confining particles with $^{40}\text{Ca}^+$'s charge-to-mass ratio. It is driven by a low-noise, low-distortion commercial supply. This combination of high-quality tuned circuit driven by low-noise commercial components of guaranteed specification will hopefully fulfil our three criteria for the supply.

In this chapter we present the theory and results of a series of experiments on resonant circuits. These experiments have been designed in order to amplify a moderate RF signal, producing capacitive voltages on the electrodes. We begin by proposing an increase in the RF frequency Ω_{RF} , which requires a corresponding increase in the RF amplitude $V_{\text{old}} \rightarrow V_{\text{new}}$. We then discuss the theory of passive, reactive circuits, in order to provide insights into the experiments. Components for the circuits were constructed to requirements and we give the specifications of these home-made impedances. We then present results which provide a complete description of the behaviour of a replica trap under the application of RF fields at the frequency and power required to trap ions. Finally we report successful generation of the ion trap using a helical resonator-based RF supply.

5.2 Proposal for the new RF supply

5.2.1 Requirements

We wish to tighten the trap i.e. increase the vibrational frequency of the secular motion. At a given ion temperature a tighter trap leads to a smaller orbit, and hence smaller micromotive velocity excursions (see section 4.2). It will also permit Doppler cooling to put our ion in the Lamb-Dicke regime. This is defined as the regime where the wave function's spatial extent is smaller than $\lambda/2\pi$, where λ is the wavelength of light addressing the ion. In other words, the ion orbit is small enough that the phase of the light field varies by less than one radian over it. We are well within the regime when

$$\eta\sqrt{\langle m \rangle_T + \frac{1}{2}} \ll 1. \quad (5.1)$$

We wish to put the ion in this regime primarily for practical reasons. Although having the ion localized in this way does not necessarily remove the need to cool the ion further e.g. with sideband or Sisyphus cooling, these methods have been found to be more successful [18] if given the ‘head start’ of the ion already in the Lamb-Dicke regime.

Table 5.1 shows empirically verified parameters of the existing ‘trap electrodes and voltages. This shows that the radial trapping is currently not tight enough for the ions to be strictly within the Lamb-Dicke regime. At $\gtrsim 80$ V the existing supply shows an unacceptable amount of higher harmonics.

Parameter	Symbol	Radial value	Axial value
Geometrical factor	α_{AC}, α_z	1.1	0.0237
Characteristic distance/mm	ρ_0, z_0	1.22	3.6
Characteristic voltage/V	V_{AC}, V_{DC}	<u>80</u>	<u>35</u>
RF frequency/ Ms^{-1}	Ω_{RF}	<u>$2\pi 6.29$</u>	—
Mathieu parameter	q, a	0.365	$-4.0 \cdot 10^{-5}$
Secular frequency/kHz	ω_r, ω_z	$2\pi 825$	$2\pi 88$
Lamb-Dicke parameter	η_r, η_z	0.196	0.598
Temperature / mK	$\hbar\Gamma_{1/2}/2k_B$	0.54	
Mean thermal occupation	$\langle m \rangle_T$	6.18	61.7
Doppler limit wave-function extent	$\eta\sqrt{\langle m \rangle_T + \frac{1}{2}}$	0.506	4.72

Table 5.1: Existing trap parameters. See section 4.2 for definitions. The first four sets of parameters are empirical.

We wish to tighten the trap while maintaining its stability. Stability is related to the Mathieu q parameter. For constant q equation (4.8) implies that $\omega_r \propto \Omega_{\text{RF}}$. If we increase the RF frequency $\Omega_{\text{RF}} \rightarrow r\Omega_{\text{RF}}$ then we must increase the trapping voltage $V_{AC} \rightarrow r^2 V_{AC}$ by equation (4.4).

The AC potentials in the existing trap are as follows. One diagonal pair of the four AC electrodes carries an RF voltage; the other pair carries the same voltage in anti-phase. With our new supply we wish to avoid distortion harmonics in the RF waveform by using only reactive, not active, components. The circuit is simplified if one diagonal pair of electrodes is grounded while the other carries the signal. This means that, for the same V_{AC} , twice the voltage as before must be applied to the signal electrode pair. This is clarified in appendix A.3.

The main empirical limitation on indefinite tightening of the trap is the vacuum feedthrough. Although this feedthrough is specified at a maximum DC voltage of 1 kV, its AC behaviour is not known [119]. Let us consider a signal of amplitude 800 V AC on the AC electrodes i.e. $V_{\text{new}} = 400$ V. We assume we can achieve this without discharges in the feedthrough ceramic. This increase implies a corresponding frequency increase to $\Omega_{\text{RF}} \sim 2\pi 14$ MHz. Table 5.2 details the expected behaviour of the ion with such a trapping voltage.

Parameter	Symbol	Radial value	Axial value
Geometrical factor	$\alpha_{\text{AC}}, \alpha_z$	1.1	0.0237
Characteristic distance/mm	ρ_0, z_0	1.22	3.6
Characteristic voltage/V	$V_{\text{AC}}, V_{\text{DC}}$	<u>400</u>	<u>175</u>
RF frequency/ Ms^{-1}	Ω_{RF}	<u>$2\pi 14.06$</u>	—
Mathieu parameter	q, a	0.365	$-4.0 \cdot 10^{-5}$
Secular frequency/kHz	ω_r, ω_z	$2\pi 1820$	$2\pi 198$
Lamb-Dicke parameter	η_r, η_z	0.132	0.4
Temperature / mK	$\hbar\Gamma_{1/2}/2k_B$	0.54	
Mean thermal occupation	$\langle n \rangle_T$	2.55	27.3
Doppler limit wave-function extent	$\eta\sqrt{\langle m \rangle_T + \frac{1}{2}}$	0.23	2.11

Table 5.2: Proposed trap parameters. V_{AC} has been raised by a factor of 5; in order to preserve q , Ω_{RF} has also been increased by a factor of $\sqrt{5}$.

5.2.2 Existing supply

Our existing supply was built by D. T. Smith and the Central Electronics Group (circuit EW1233). It operates fully loaded at 6.29(3) MHz, and was tuned with different known capacitances to obtain this. The supply has two outputs which operate in antiphase. Thus one diagonal pair of electrodes has a voltage $V_{\text{old}} \sin \Omega_{\text{RF}} t$ while the other has $-V_{\text{old}} \sin \Omega_{\text{RF}} t$. The amplitude V_{old} can be set at 0–200 V. A capacitive divider provides a 1/200 monitor output V_{MON} .

Typically V_{MON} is not increased beyond 400 mV ($V_{\text{old}} = 80$ V) as it is evident that the monitor voltage is no longer sinusoidal at higher amplitudes. The circuit contains an active component, a transistor—it is based on a Colpitts oscillator [120]—and this is believed to be the source of the distortion.

5.2.3 New supply

We have decided that the new supply shall be built from reactive, rather than active, components. This entails using the resonant properties of inductor/capacitor pairs to magnify a small input signal to a high voltage across the reactive components. As with the tuned Colpitts oscillator the trap will act predominantly as a capacitance with a small stray resistance. To investigate the properties of such a system we constructed resonant circuits as per the theory of section 5.3. Series-*LCR* circuits were subjected to RF signals; the capacitive element in the circuit was the AC electrodes of a ‘dummy’ ion trap, or occasionally a commercial capacitor of known behaviour. With these results a new supply can be designed.

5.3 Resonance in reactive electric circuits

5.3.1 Overview

This section details the circuit theory specific to the electrical components used in this chapter. Resonant reactive circuits are in general based on either the series- or parallel- LCR circuit, as shown in figure 5.1. Such circuits are useful as amplitude or phase filters respectively.

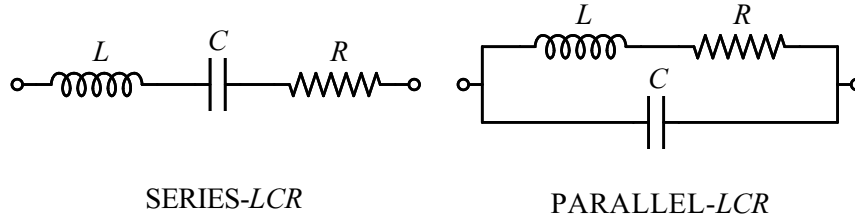


Figure 5.1: Series- and parallel- LCR circuits. The inductance L and capacitance C give the circuits a complex impedance $\mathbf{Z}(\omega)$ as a function of frequency ω .

The discussion follows the construction of a circuit starting with the capacitive trap, adding an inductive element, and then adding a transmission line and a measuring device. It begins with a review of LCR circuits. Then we introduce a transformer and a transmission line between these circuits and the voltage generator. The conditions for good impedance matching discussed. We note ‘anomalous broadening’ of the circuit resonance in this situation, and discuss using the L of the LCR as the secondary coil in a transformer. We also discuss the effect of poorly-coupled transformer coils on the resonance. Finally we relate our findings to the experimental situation, and discuss the most favourable circuits in this light.

5.3.2 Voltage standing-wave ratio and power ratios

VSWR [121], or voltage standing-wave ratio, is a measure of power typically quoted at radio frequencies. VSWR is defined as the ratio of the antinodal to nodal amplitudes on a transmission line preceding the circuit under test (this line will have an effect on the VSWR for which we must account). If a power supply applies a running-wave voltage of amplitude V_0 , and the reflected wave has amplitude V_1 , then the standing wave ratio is

$$\text{VSWR} = \frac{V_0 + V_1}{V_0 - V_1} \quad (5.2)$$

$$\Rightarrow \frac{V_1}{V_0} = \frac{\text{VSWR} - 1}{\text{VSWR} + 1}. \quad (5.3)$$

VSWR is always greater than unity, and V_1/V_0 less than unity.

The RS VSWR meter¹, component number 612-221, uses only a small fraction of incident and reflected powers; thus the total power in the circuit must be large to accurately determine this small fraction: we require $\gtrsim 1$ W.

¹This meter will be referred to as such even though it can take measurements other than VSWR—absolute incident power, reflected power etc.

5.3.3 The series- LCR circuit

We begin our circuit analysis with the series- LCR circuit in figure 5.1. Its impedance is

$$\mathbf{Z} = R + j\omega L + 1/(j\omega C). \quad (5.4)$$

We define the centre of a circuit resonance as being the frequency where this impedance is real i.e. $\omega_0 = 1/\sqrt{LC}$. On resonance all of the voltage appears across the resistor: $V_0 = V_R = IR$. The source sees no reactance in the circuit. The voltage across the capacitor $V_C = I/j\omega_0 C$ is balanced by an equal and opposite e.m.f. in the inductor, and we define

$$Q_F = V_{C\max}/V_{R\max} = 1/(\omega_0 CR) \quad (5.5)$$

$$\Rightarrow Q_F = \frac{\omega_0 L}{R} \equiv \frac{1}{\omega_0 CR}. \quad (5.6)$$

Defining Q_F in terms of a voltage ratio rather than a ratio of centre frequency to bandwidth firstly yields an exact expression, and secondly implies we are treating Q_F as a ‘circuit magnification factor’ i.e. the primary motivation for building a resonant circuit is to magnify the applied voltage V_R for a given power dissipation. The filtering capacity of such a high- Q_F circuit is of secondary importance.

Note that this definition assumes resonant frequencies under an external driving voltage. There are equivalent definitions Q_N in terms of the natural, undriven oscillations of the circuit [122] at frequency $\omega_N = \omega_0(1 - 1/4Q_F^2)^{1/2}$. For large Q_F the natural frequency is approximately the same as the resonant frequency, and the quality factors are then equivalent.

In this approximation we may determine the width of the resonance in terms of the quality factor. The average power absorbed by a reactive circuit over a cycle of the AC signal is given by

$$\langle P(\omega) \rangle_{2\pi/\omega} = \frac{1}{2} V_0^2 \Re \{ \mathbf{Y} \}. \quad (5.7)$$

where the admittance $\mathbf{Y} \equiv \mathbf{Z}^{-1} = (R - jX)/(R^2 + X^2)$, and the reactance $X = \omega L - 1/(\omega C)$. For high-quality circuits we expect the resonance to be narrow and we expand equation (5.7) in powers of the small detuning from resonance $\delta\omega = \omega - \omega_0$ [123]. With this approximation the power absorbed by the circuit is Lorentzian with FWHM $\Gamma_\omega \approx \omega_0/Q_F$. Note that the exact expression for power absorption is asymmetric [124]. For low Q_F we will fit to data using the exact expression.

5.3.4 The parallel- LCR circuit

The parallel- LCR circuit is also depicted in figure 5.1. A resonance of $\Im(\mathbf{Z}) = 0$ exists in this circuit at a frequency close to that of the series- LCR circuit. The admittance of this circuit is $\mathbf{Y}(\omega) = 1/(R + j\omega L) + j\omega C$. As before, we define the resonant frequency as the point where the impedance—or, more conveniently, the admittance—becomes real, and we call this frequency $\omega_0^{(p)}$ to distinguish it from the series- LCR frequency:

$$\Im(\mathbf{Y}) = \Im \left(j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2} \right) = 0 \quad (5.8)$$

$$\Rightarrow R^2 + (\omega_0^{(p)})^2 L^2 = L/C. \quad (5.9)$$

From this we determine the resonant frequency to be $\omega_0^{(p)} = \omega_0 \sqrt{1 - 1/Q_F^2}$. Note this difference means that the two expressions for Q_F are no longer equivalent, although they are a good approximation to each other for $Q_F \gg 1$. On resonance the impedance is a pure resistance $Z = Q_F^2 R$. From equation (5.7) the power absorbed is:

$$\langle P(\omega) \rangle_\tau = \frac{1}{2} V_0^2 \frac{R}{R^2 + \omega^2 L^2}.$$

whereas the power incident is always $\frac{1}{2} V_0^2 / 50 \Omega$. In this circuit the power absorption no longer has a sharply defined peak around its reactive resonance. Instead, the sudden drop in reflected power is centred on $\omega = 0$, at DC frequencies because of the existence of a low-resistance DC path in the circuit. This feature is only dependent on L and R .

5.3.5 Introduction of a transformer

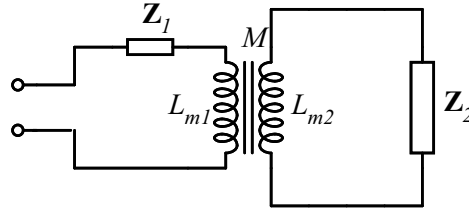


Figure 5.2: General transformer circuit: the secondary impedance Z_2 is linked through the mutual inductance M of two coils, L_{m1} and L_{m2} , to a primary circuit with its own (usually negligible) impedance Z_1 .

It would be inefficient for us to match a 50Ω coaxial cable to the circuit by introducing 50Ω of resistance. This would give us a low Q_F : for $\omega_0 = 2\pi \times 14 \text{ MHz}$ and $C \sim 20 \text{ pF}$, $Q_F \sim 11$. This would require a power dissipation of 70 W for the proposed capacitance voltage $V_C = 400 \text{ V}$. Instead we can use a transformer to match the circuit impedance to the coaxial impedance, as shown in figure 5.2.

The general transformer formula $Z_L = Z_2 n_1^2 / n_2^2$ is an approximation, and we therefore start from first principles to ensure we are within the limits of the formula. The transformer consists of two coils, of inductance L_1 and L_2 in the primary and secondary circuits respectively. They are coupled such that the mutual inductance is $M = \kappa \sqrt{L_1 L_2}$, where $\kappa \leq 1$. We apply Kirchhoff's 2nd Law to both sides of the transformer and consider the total e.m.f.'s in each:

$$V_1 = I_1 Z_1 + I_2 j\omega M \quad (5.10)$$

$$0 = I_2 (j\omega L_2 + Z_2) + I_1 j\omega M. \quad (5.11)$$

$Z_1(Z_2)$ is the impedance in the primary (secondary) circuit that does not include the self- and mutual-inductances of the transformer. The load impedance Z_L that the power supply perceives is given by elimination within these two equations:

$$Z_L = \frac{V_1}{I_1} = \frac{j\omega L_1}{j\omega L_2 + Z_2} Z_2. \quad (5.12)$$

We have assumed in this equation that the transformer coupling is perfect i.e. $\kappa = 1$ and $M = \sqrt{L_1 L_2}$. In the limit that the secondary coil's self-inductance is much greater than any other

impedance in the secondary circuit, we recover the general transformer formula. It will be useful to work experimentally in this regime.

For small resistance R the series- LCR circuit satisfies this requirement over a wider frequency range than the resonance itself: $\delta\omega \gtrsim \omega_0/Q_F$. The reason for this is as follows. If we examine the impedance $\mathbf{Z}_2$ when the absorbed power is $P_{\text{abs}} = \Pi_{\text{abs}} P_{\text{in}}$ we obtain

$$\mathbf{Z}_2 \approx R \left(1 + j\sqrt{1/\Pi_{\text{abs}} - 1} \right)$$

$$|\mathbf{Z}|_2 \approx R/\sqrt{\Pi_{\text{abs}}},$$

e.g. at the half-maxima $|\mathbf{Z}|_2 \approx R\sqrt{2}$. It is therefore possible to use a circuit with a transformer whose secondary inductance L_2 is much less than the tuned LCR 's inductance, as long as both $\omega_0 L_2 \gg R$ and the circuit is operated close to resonance.

5.3.6 Tuned inductor as secondary coil

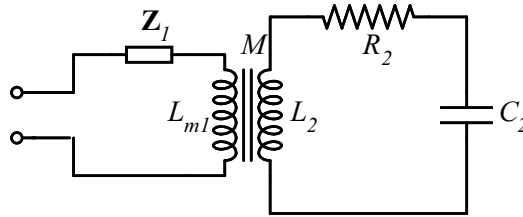


Figure 5.3: Transformer coupling into the tuned inductor, L_2 , rather than a stand-alone secondary coil L_{m2} . The remaining CR circuit in the secondary is shown.

One possibility to minimize the pickup (i.e. the number of antenna-like components) in the circuit is to couple the tuned inductor to the primary circuit i.e. L also acts as secondary coil L_{m2} in the transformer. In this situation we do not satisfy the requirements of the ideal transformer formula, as an LCR circuit on resonance has similar impedances in both the inductive and the capacitive components. We must therefore return to the transformer formula in equation (5.12). This gives for the admittance of the total circuit

$$\mathbf{Y} = \frac{1}{\mathbf{Z}_2} \left(\frac{L_2}{L_1} + \frac{Z_2}{j\omega L_1} \right) \quad (5.13)$$

$$= \left[\frac{L_2}{L_1} \frac{R}{R^2 + 1/(\omega C)^2} \right] + j \left[\frac{L_2}{L_1} \frac{1/(\omega C)}{R^2 + 1/(\omega C)^2} - \frac{1}{\omega L_1} \right] \quad (5.14)$$

$$\Rightarrow \Im(\mathbf{Y}) = 0 \quad \text{when} \quad \omega^{-1} = \omega_0^{-1} \sqrt{1 - 1/Q_F^2}. \quad (5.15)$$

This circuit is similar in behaviour to the parallel- LCR circuit. At the reactive resonance there is no clear signature in the absorbed power: this time the signature is at high frequencies $\omega C \rightarrow \infty$ instead of DC frequencies.

5.3.7 Transmission lines

In our experiments we separate the apparatus—resonant circuit, vacuum can, feedthrough—from the voltage source by wires of sufficient length that they act like a transmission line. Generally this

will be a $50\ \Omega$ coaxial cable, and the output of the amplifier will be impedance-matched to such a cable. Let a line of known, real impedance $\mathbf{Z}_{co} \equiv Z_{co}$ be terminated by a load of impedance $\mathbf{Z}_L = Z_L \exp j\phi$. At the point of reflection the phase angle between V_0 and V_1 is δ . The voltage of the signal that continues into the load is V_2 , and we equate the voltages either side of this point to obtain [123]

$$\text{Reflected wave:} \quad \frac{V_1}{V_0} = \frac{\mathbf{Z}_L - \mathbf{Z}_{co}}{\mathbf{Z}_L + \mathbf{Z}_{co}} \quad (5.16)$$

$$\text{Absorbed power fraction:} \quad \Pi_{\text{abs}} = \frac{P_{\text{abs}}}{P_{\text{in}}} = 1 - \left| \frac{V_1}{V_0} \right|^2 = \frac{4\Re\{\mathbf{Z}_L\} \mathbf{Z}_{co}}{|\mathbf{Z}_L + \mathbf{Z}_{co}|^2}. \quad (5.17)$$

The relative amplitude and phase of the reflected wave are given by

$$\frac{V_1}{V_0} = \left(\frac{Z_L^2 - Z_{co}^2 - 2Z_L Z_{co} \cos \phi}{Z_L^2 - Z_{co}^2 + 2Z_L Z_{co} \cos \phi} \right)^{\frac{1}{2}} \quad (5.18a)$$

$$\tan \delta = \frac{Z_L Z_{co} \sin \phi}{Z_L^2 - Z_{co}^2}. \quad (5.18b)$$

For no incident power to be reflected back to the amplifier we therefore have in equations (5.18) two conditions:

- **Zero reactance:** there must be no imaginary component to the load impedance.
- **Matching:** the real component must be equal to the output resistance of the voltage source.

In a sense they are the same condition: $\mathbf{Z}_L = \mathbf{Z}_{co}$. However, in practice we achieve each condition separately. We first tune the *LCR* circuit until the reactance drops to zero. Then it is transformed by coupling two inductive coils in order to match the second condition.

5.3.8 Short transmission line as capacitance

A short length of transmission line acts as a capacitance in parallel with its load [125]. We can see this intuitively as two coaxial conductors with a (nearly) constant voltage along them, and that same voltage applied to the load $\mathbf{Z}_L$. Let us analyse the problem using the transmission line formula. The voltage in the line consists of an incident signal of amplitude A and a reflected signal of amplitude A' ; the current forward signal has amplitude $\mathbf{Z}_{co} A$ and the return signal $-\mathbf{Z}_{co} A'$. We take the terminated end of the line as our origin ($z = 0$), and at this point on the line

$$\frac{V}{I\mathbf{Z}_{co}} = \frac{A + A'}{A - A'} = \frac{\mathbf{Z}_L}{\mathbf{Z}_{co}}. \quad (5.19)$$

Let us now examine the effective impedance at the power supply, $-z = l \ll \lambda$. In the approximation $e^{\pm jkz} = 1 \pm jkz$ we obtain:

$$\mathbf{Z} \approx \mathbf{Z}_L (1 - j\omega l C \mathbf{Z}_L) \approx (\mathbf{Z}_L^{-1} + j\omega C)^{-1}. \quad (5.20)$$

We have also assumed that $\omega L \ll Z_L, 1/\omega C$, which is feasible for both short wavelengths and the specifications of the components we use. Hence for low frequencies the impedance of the terminated transmission line is equivalent to $\mathbf{Z}_L$ and C in parallel. We shall see later the effect this has on the resonant frequency of the circuit.

5.3.9 Anomalous width in the series- LCR resonance

Consider a series- LCR circuit coupled by a transformer of turns ratio $N = n_1/n_2 > 1$ to a transmission line of characteristic, real impedance $\mathbf{Z}_{co} = Z_0$. This cable is then connected to a generator, whose output impedance is also $\mathbf{Z}_{co}$. At a point on the transmission line we place a VSWR meter to monitor the incident and reflected powers. This meter is impedance-matched to the line. From equation (5.17) we obtain for the fractional power absorbed

$$\Pi_{\text{abs}} = \frac{4\Re\{\mathbf{Z}_L\}\mathbf{Z}_{co}}{|\mathbf{Z}_L + \mathbf{Z}_{co}|^2} = \frac{4RN^2Z_0}{|(RN^2 + Z_0) + jN^2(\omega L - 1/\omega C)|^2}. \quad (5.21)$$

Again we consider frequencies close to the resonant frequency and this expression becomes

$$\Pi_{\text{abs}} \approx \frac{4RN^2Z_0}{(N^2R + Z_0)^2 + (2\delta\omega N^2L)^2}. \quad (5.22)$$

The absorption resonance the power meter records is Lorentzian, such that its maximum approaches unity as matching tends towards perfect ($N^2R \rightarrow Z_0$) and its FWHM Γ_ω is

$$\Gamma_\omega = \frac{R}{L} + \frac{Z_0}{N^2L} = \omega_0/Q_F + \frac{Z_0}{N^2L}. \quad (5.23)$$

We have a wider absorption profile owing to the addition of the (lossless) transmission line. At perfect matching the total width becomes $2\omega_0/Q_F$. The extra losses may be considered to be owing to the impedance matching at the input end of the transmission line i.e. the output impedance of the signal generator/amplifier. The reflected signal passes through this impedance—equal to Z_0 —before being earthed.

5.3.10 Theory of imperfectly-coupled transformers

When the two coils of a transformer are poorly coupled, equation (5.12) is no longer true. The impedance of the primary and secondary circuits is now

$$\frac{V_1}{I_1} = \frac{\omega^2 L_1 L_2 (\kappa^2 - 1) + j\omega L_1 Z_2}{j\omega L_2 + Z_2} \quad (5.24)$$

We will now make the approximation of the ideal transformer, that the impedances in the secondary are dominated by the transformer coil's self inductance ($L_2 \gg Z_2$) i.e. that we are close to resonance. The absorbed power fraction when a transmission line has been connected (equation (5.22)) is therefore

$$\Pi_{\text{abs}} \approx \frac{4RN^2Z_0}{(N^2R + Z_0)^2 + (2\delta\omega N^2L + \omega L_1(1 - \kappa^2))^2}. \quad (5.25)$$

This is a Lorentzian of equal height to the well-coupled circuit, but with the resonance shifted:

$$\omega'_0 = \omega_0 \frac{N^2L}{N^2L + \frac{1}{2}L_1(1 - \kappa^2)} \quad , \quad \Gamma'_\omega = \Gamma_\omega \frac{N^2L}{N^2L + \frac{1}{2}L_1(1 - \kappa^2)}.$$

This presents a problem for interpreting measurements of the circuit. The only impact that our transformer has on the measured signal is to decrease the resonant frequency and increase the circuit magnification by the same fraction. This is identical to circuit behaviour with an increased

inductance, and it will be impossible to deduce the inductance and the transformer coupling independently from fits. However, if we use the exact expression of equation (5.24) for the transformer's behaviour then we find that poor coupling does indeed reduce the height of the peak. This behaviour

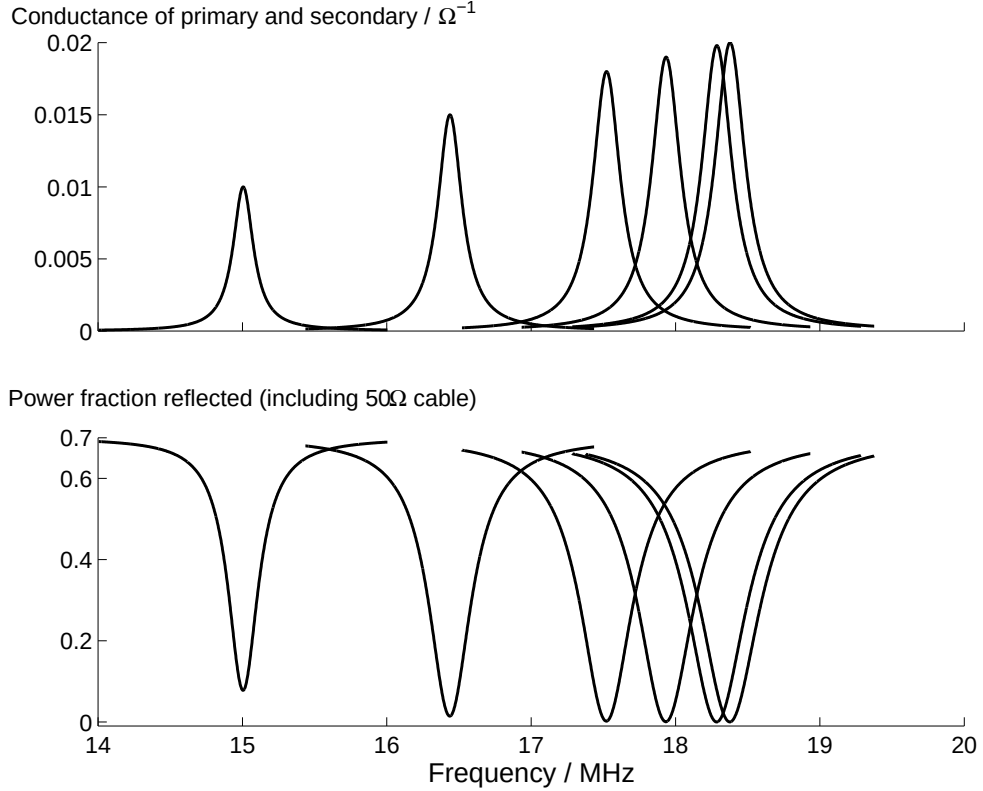


Figure 5.4: Effect of a poorly coupled transformer. A series- LCR circuit is connected across the secondary and a transmission line across the primary. The mutual inductance of the transformer is $M = \kappa\sqrt{L_1L_2}$, where from left to right $\kappa^2 = 0.5, 0.75, 0.90, 0.95, 0.99, 1$. In the upper graph we see the conductance of the transformed, series- LCR circuit. The peak heights are approximately $\kappa^2/50 \Omega$. In the lower figure we see the effect of hanging this circuit off a 50 Ω coaxial cable. Our observable, the power fraction reflected, is only modestly affected by very poor coupling. Other parameter values: $L = 5 \mu\text{H} = L_2$, $L_1 = (50/8)L_2$, $C = 15 \text{ pF}$, $R = 8 \Omega$.

is shown in figure 5.4. The upper figure is the real part of the transformed series- LCR circuit's admittance. In the absence of a transmission line we see the power absorbed proportional to κ^2 , the fraction of power reflected into the secondary. In the lower figure a transmission line has been added. Here we plot the observable Π_{abs} for a circuit with 30% background losses. This situation is similar to our experimental results as we shall show later.

5.3.11 Practical considerations

In order to discover a resonance experimentally, we need the circuit to exhibit on resonance a clear experimental signature. In our case we must be able to see a large change in the VSWR or the reflection/absorption of the circuit, as these are the easiest methods of measuring the circuit's response. Measuring VSWR is a useful method of 'zeroing in' on the resonance, as it increases monotonically from a minimum at the circuit resonance. However, the VSWR meter must be re-

calibrated whenever the frequency is changed. It was found to be easier to characterize a resonance by instead measuring P_{in} and P_{ref} over the feature.

The parallel- LCR circuit has no obvious feature in the real part of the admittance at resonance. This also occurs when using the L of an LCR circuit as one half of the transformer (see section 5.3.6). It may be possible to make careful measurements of phase using an antenna or a capacitive divider close to the trap, but it is far easier to observe the Lorentzian profile of the series circuit, transformed by an independent coil pair to match to a coaxial cable, and then connected to a high-power amplifier. One loses the high impedance of a resonant parallel circuit (RQ_F^2) in favour of useful diagnostics.

We note in passing that one model for the helical resonator (discussed in the next section) in terms of bulk impedances consists of a parallel- LCR circuit, with very weak coupling into the inductor [123]. The weakly-coupled regime of the circuit in figure 5.3 is different from the strongly-coupled regime, and does produce a Lorentzian diagnostic in the absorbed power fraction. However, as we wish our bulk LCR circuits to avoid the problems associated with weak coupling we will only deal with strongly-coupled bulk circuits.

5.4 Components for RF experiments

5.4.1 Introduction

In order to carry out exploratory experiments we constructed apparatus to our own specifications. We compared the performance of these components to that of commercial products to standardize results.

We built a replica of the ion trap electrode arrangement to allow us to try different resonant circuits. Inductors and transformers of various specifications—some tunable—were built in order to reach the desired resonant frequency and Q_F in our circuits. A helical resonator (a quasi- $\lambda/4$ coaxial device) was constructed and tested. The resonator was connected to the real trap, and then an LCR circuit also connected.

5.4.2 Supplies and probes

We use the Stanford DS345 signal generator to provide frequencies up to 30 MHz. Spurious and subharmonic noise from this generator is suppressed to less than -50 dBc, and harmonically related signals less than -25 dBc. In later experiments this is amplified using the Frankonia FLL25 RF amplifier. This amplifier can provide a maximum of 60 W power. It is specified at a maximum of 25 W ‘without distortion’ i.e. distortion harmonics less than noise. Noise is specified as less than -25 dBc at all powers [126].

Connections between DS345, FLL25 and the experiment are effected with coaxial $50\ \Omega$ double-shielded cables. The apparatus intended to mimic the trap is typically kept in a sealed, grounded copper shield. This reduces RF noise in the rest of the laboratory and ensures compliance with NRPB safety recommendations [127]. Access through the shield is via coaxial bulkhead connectors.

5.4.3 The helical resonator

Although a resonant circuit can be made from discrete reactive components, a conceptually simple design results from consideration of an inductive/capacitive transmission line. Consider the appa-

ratus shown in figure 5.5. The central axis of a coaxial cable is connected to the conducting sheath at one end. An oscillating voltage is applied via a voltage tap to the central axis, a short distance from the short-circuit. Slowly-varying voltages are conducted directly to ground via the short, and fast oscillations pass via a displacement current from the central axis to the sheath. At a certain frequency, however, the speed of voltage oscillations (close to, but less than, c) is such that the transmission line becomes a $\lambda/4$ element. The applied voltage V_{in} close to the shorted end of the cable causes a much greater voltage V_{out} to be observed at the open end. Such coaxial resonators have been used e.g. on a high-RF Paul trap [128] operating at 240 MHz.

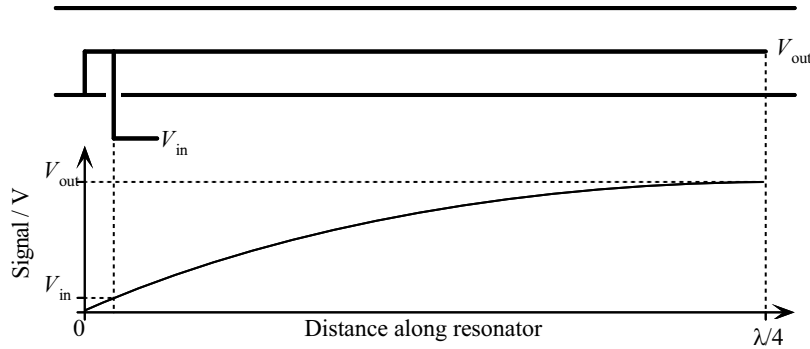


Figure 5.5: Principle of the $\lambda/4$ amplifier. A coaxial conducting element is shorted at one end. The sheath is earthed and a voltage applied close to the short-circuit.

Because of the necessary length, a coaxial resonator designed for frequencies much lower than this would be unwieldy. Consider applying signals to our trap at $\Omega_{\text{RF}} = 2\pi 20 \text{ MHz}$: $\lambda/4 = \pi c / 2\Omega_{\text{RF}} = 3.75 \text{ m}$ in vacuo. Not only would a resonator of this size be difficult to store, but there would also be the possibility of antenna-like pickup and emission from the device.

One alternative is the helical resonator [129]. This is an application of the above concept; in essence it is a quarter-wavelength transmission line where the axial cable has been wound into a helix. Such resonators have been used to provide signals for ion traps [130], and have also been successful in RF discharge applications [131] and as travelling-wave amplifiers. They can have high quality factors, although they also permit odd harmonics owing to their wavelength selectivity [132]. A helical resonator was built to the design in figure 5.6. The sheath and coil were constructed from copper. It was not necessary to polish the surfaces at the frequencies we are using here, but at higher frequencies this would be required. To remove the insulating copper oxide surface the resonator was sandblasted. The coil was wound onto a nylon former inscribed with a groove at the correct pitch.

Calculation of the resonator's properties is semi-empirical. This calculation has been performed elsewhere [133], and we reproduce the essential design choices here. In the subsequent equations, frequencies are measured in MHz. We consider a copper sheath of inside diameter $D \approx 100 \text{ mm}$. At a frequency of 20 MHz our quality factor is expected to be

$$Q_{\text{F}} = 1.97D\sqrt{\Omega_{\text{RF}}/2\pi} \approx 890. \quad (5.26)$$

The helical diameter d and length b are constrained by $d = 0.55D$ and $b/d = 1.5$ respectively, and the sheath length B is such as to enclose the helical length with $\sim D/4$ extra sheath at each end. Finally the total number of turns is

$$N = 48300/(D\Omega_{\text{RF}}/2\pi) \approx 24. \quad (5.27)$$

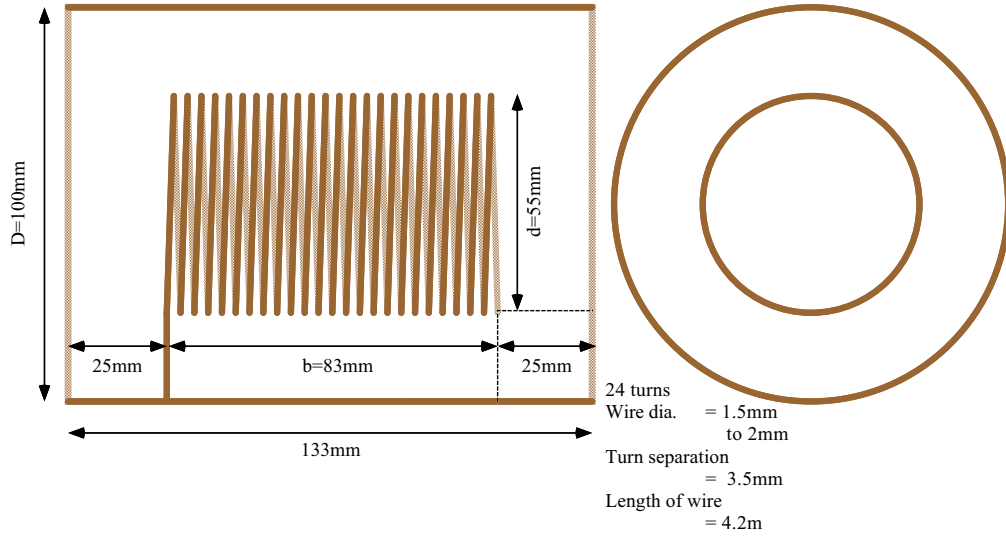


Figure 5.6: Dimensions of a helical resonator designed to oscillate at 20 MHz.

From these values we are able to compute all other dimensions—pitch etc.—and in principle electrical properties such as the inductance and capacitance:

$$L = 0.98 \left(\frac{Nd}{b} \right)^2 \left[1 - \left(\frac{d}{D} \right)^2 \right] \mu\text{H m}^{-1} \quad , \quad C = \frac{29.5}{\log_{10}(D/d)} \text{pF m}^{-1}. \quad (5.28)$$

This gives $L = 14.4 \mu\text{H}$, $C = 9.53 \text{ pF}$. An equivalent LC circuit would have resonant frequency 14 MHz.

5.4.4 The model trap

In order for these experiments to run alongside ion trapping experiments, a model trap was built with AC electrodes only, of the same dimensions as the real trap as documented in reference [76]. An engineering diagram is shown in figure 5.7.

The axes of the four AC electrodes, each of dia. 1.7 mm, lie 2.5 mm from the axis of the trap in this reference. These are reported to be constructed from steel [76], but comparison of their dimensions with material used during the trap construction suggests they are a copper/beryllium alloy. For ease of construction nylon formers² were used instead of Macor, and only the AC electrodes were built. The ends of these electrodes were spot-welded to a ceramic vacuum feedthrough; although this feedthrough was not the same model as that used in our vacuum can it was intended as an approximate substitute. In addition the 1.7 mm Cu/Be rods taken from vacuum feedthroughs and used in constructing the real trap were not available. Instead 1.0 mm steel rods were positioned so that the distance from the centre of the trap to their nearest surface was kept constant i.e. centred 0.35 mm closer to the trap axis.

²Experience with the helical resonator implied that there were no absorptive problems with nylon at our frequencies.

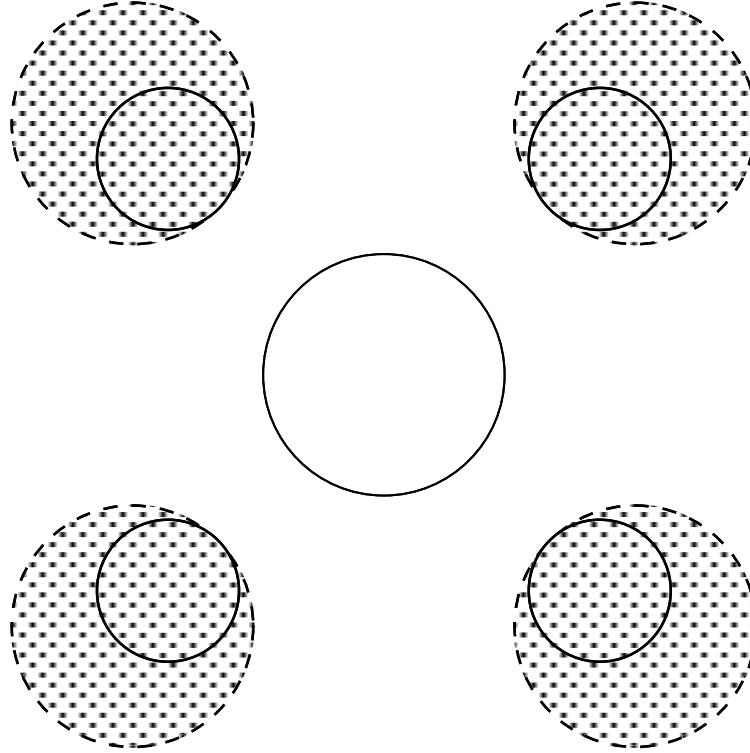


Figure 5.7: Schematic of the model trap, viewed along the trap axis. The white circle denotes the position of the endcaps. The dotted, shaded circles are the positions of the dia. 1.7 mm electrodes as given by reference [76], but we were unable to obtain the same diameter of material and used dia. 1.0 mm stainless steel, positioned closer to the trap axis. This position is given by the solid, shaded circles. See text for more details.

5.4.5 Inductors

We are able to add capacitance to a circuit, in order to shift the resonant frequency. However, this is not favourable because, for a given resistance, the quality factor Q_F decreases (see equation (5.6)). Instead we constructed a tunable inductor. If an inductor is solenoidal in shape it can be tuned by moving a core in and out of the centre of the solenoid: we designed the component with a low inductance and used the core to concentrate the flux, increasing the inductance.

It is possible to use the perfect solenoid formula and adapt it with numerical factors [134] to compensate for the finite length of our coil. However, empirical formulas exist for certain ranges of coil dimensions and these are as accurate as our engineering tolerances. Stephan's formula [135] can be used for multilayered coils of mean radius a , length l and turn depth c all of similar, modest magnitude:

$$L = \frac{aN^2}{343} \log_{10} \left(\frac{91.4}{\sqrt{l^2 + c^2}} \right). \quad (5.29)$$

Here L is in microHenrys, N is the total number of turns and all lengths are in millimetres. The GQ inductor (see results, section 5.5) was designed using Stephan's formula to give an inductance of $4.7 \mu\text{H}$, and experimentally its inductance is $4.3(2) \mu\text{H}$. These values agree within design error. Another formula used to predict coil inductances is

$$L = \frac{1}{25.4} \frac{a^2 N^2}{[9 - (a/5l)]a + 10l}. \quad (5.30)$$

This was used in designing the GQ transformers. It is accurate within $\pm 2\%$ for $3.5 < 2a/l < 20$.

When building components for use at radio-frequencies the component's precise specification is strongly affected by the materials employed. Many plastics exhibit strong absorption of RF signals and can therefore reduce Q_F . Therefore, where the coil was not built free-standing or core-wound, the former was constructed from Tufnol, a resin-bonded plastic.

With the skin depth in mind the coil was constructed with low resistance. Predicted RF resistances are given in table 5.3. Finally, generic magnetic cores have high losses at $\gtrsim 1$ MHz. Special cores from MMG Neosid³ were employed in the toroidal coils to minimize losses.

The following is a list of transformers employed in the RF experiments:

Axial inductor The Ferroperm 1582 series choke. This component has specified inductance of $4.7 \mu\text{H}(\pm 10\%)$, and is available from RS or Farnell.

Trial transformer This auto-transformer was wound on a nylon former which was then removed. This made it structurally weak and liable to pick-up that proved problematic with the FLL25. It was solenoidal, with length 100 mm, diameter 30 mm and 20 turns (tapped at the 4th turn).

GQ transformer This transformer was initially wound by Graham Quelch with 20 turns of dia. 0.71mm (22 swg) copper wire on a resin former. The turns were wound tightly together and the resulting coil had diameter $d = 50$ mm. The coil was tapped at 4 turns to give a 1:5 auto-transformer. On the advice of John Siertsema and Peter Hirst the number of turns was reduced to 5, and tapped at 1, 2 or 3 turns.

GQ inductor This inductor consists of 15 turns of dia. 1mm wire, wound on a small resin former. Using the terminology of equation (5.29) its dimensions are: $l = 5$ mm, $a = 6$ mm, $c = 3$ mm, $N = 15$.

NeoSID transformer Two ring cores were acquired from MMG NeoSID. One of them was wound with five turns in the primary coil and two in the secondary (i.e. not tapped as an auto-transformer).

NeoSID inductor The second toroid from MMG NeoSID was wound with one or two turns of wire in a choke configuration to give an inductive element for the LCR circuits.

In later sections of this chapter we will discuss circuits involving these inductors, using the names in bold as a reference.

5.4.6 Resistances and reactances of components

We have used different components in a variety of circuits. It is therefore important to have an idea of the impedances that have been deduced from one application of a component, in order to predict how the component will behave in another circuit.

In table 5.3 we list the properties of numerous components. If these have been calculated by indirect methods (e.g. skin effect) then the data is given in italics.

We note the following with respect to the table:

³High-frequency soft ferrite, grade F25: inner dia. 6.5mm; outer dia. 14mm; length 9mm.

Description	Specification	R at DC / Ω	R at AC / Ω	L / μH	C / pF	Notes
Inductors						
GQ	4.7	<i>0.016</i>	<i>0.27</i>	4.3(2)	—	
Axial	4.7(5) μH , 0.04 Ω	<i>0.07</i>	<i>0.285</i>	4.9(3)	—	
NeoSID	3 turns	<i>0.004</i>	<i>0.06</i>	?	—	
Transformers						
GQ	20 turns	<i>0.13</i>	<i>1.61</i>	27.5(5)	—	3.08 m long
	5 turns	<i>0.034</i>	<i>0.40</i>	2.88(6), 3.6(2)	—	
NeoSID	5 turns	<i>0.006</i>	<i>0.10</i>	2.7(2)	—	
Capacitors						
10 pF	($\pm 10\%$); $V \leq 2 \text{ kV}$	0	0	0	10.0(5)	Assumed just C
Model trap		0	0	—	14(2)	Varies
Resistors						
Commercial	56(1)	55.6	42(10)	—	—	see sec. 5.5.2
NiCr wire	dia. 0.11mm $l=1 \text{ mm}$	<i>0.114</i>	0.11(1)	—	—	see sec. 5.5.4

Table 5.3: Table of specified and measured impedances for components used in the RF experiments. Question marks by zero values indicate that no addition to the circuit impedance was observed from the component, within error margins. Italicized entries are either specified by the manufacturer or calculated *ab initio*, rather than obtained from fits to data. Dashes indicate that the component was assumed to have zero impedance of that type, or the impedance was dependent on the geometry of the whole circuit.

1. Measurements of inductance were made relative to the 10 pF commercial capacitor. We endeavoured to separate out spurious inductive and resistive properties of this component by redundancy—using multiple copies of the component and attempting to observe an unexpected change in L or R . We know that $R_C \ll 50 \Omega$ and $L_C \ll 4 \mu\text{H}$.
2. Using equation (5.38) we calculate the AC resistance at a cyclical frequency f to be:

$$R_{AC} \approx \rho l / \pi d d_0 \quad (5.31)$$

$$\approx \frac{l}{d} \sqrt{\frac{f \mu \rho}{\pi}}. \quad (5.32)$$

For the above resistances $f = 20 \text{ MHz}$. l and d depend on component geometries.

3. The skin depth of the nichrome alloy (last line of the table) is calculated to be approximately $120 \mu\text{m}$, larger than the diameter. AC resistance is therefore presumed to be approximately the same as DC.
4. The resonance that calibrated the NeoSID transformer had a line centre greater than the maximum frequency available from the DS345. We fit to one half of the resonance, giving an approximate result.

5.5 Results

5.5.1 Impedance of the traps

5.5.1.1 Preliminary measurement of model trap capacitance

Without the FLL25 amplifier, the DS345 signal generator was connected directly to a series- LCR circuit consisting of the model trap and two commercial inductors. Although the DC resistance was $\sim 0.4 \Omega$, good coupling of the circuit to a 50Ω coaxial cable implied that its AC resistance was much higher. A minimum in reflected power was observed at $\Omega_{\text{RF}} = 2\pi 14.8 \text{ MHz}$. This corresponds to a trap capacitance of $C_T \approx 12.3 \text{ pF}$. The model trap had no shielding for this experiment and the low powers prohibited a measurement of the resonance's FWHM and hence Q_F .

5.5.1.2 Prediction of real trap capacitance

The existing ion trap, connected to the old supply, has an apparently high capacitance. The unloaded supply resonates at $\omega_{\text{un}} = 2\pi 11.2 \text{ MHz}$. With $C' = 2 \times 52 \text{ pF}$ capacitive loads connected this frequency is reduced to $\omega' = 2\pi 8.2 \text{ MHz}$. With the load of the trap (plus its connecting leads) the resonant frequency is $\omega_T = 2\pi 6.29 \text{ MHz}$.

Let us model the circuit as a simple LCR resonator with $C = C_0 + C_L$ i.e. the total capacitance in the circuit is that of the supply's capacitance, in parallel with the load capacitance. It is debatable whether we can decompose the Colpitts oscillator this way. However, if we do so then it implies that the capacitance of the trap, plus feedthrough and connecting leads, is approximately 260 pF . Two metre-lengths of UR76 cable are specified at a capacitance $\sim 200 \text{ pF}$. The remaining 60 pF will be as a result of the trap and feedthrough, plus the cases of two high-pass (inductive) filters used to attenuate low-frequency noise from the old RF supply. We therefore expect the capacitance of the trap and feedthrough to be $\lesssim 60 \text{ pF}$.

5.5.2 Measurement of home-made inductors

To measure the inductance of an unknown inductor, we constructed an LCR circuit using discrete components. Initially we set $R = 50 \Omega$. Any real resistor has some stray capacitance and inductance: if we use a loop of resistance wire it will cut flux in the circuit and add to the inductance; if we use a discrete resistor it will potentially have both inductance and capacitance. We therefore made experiments with different component values and observed the resonance behaviour, in order to isolate any stray extra reactances and determine the impedance of the inductor alone.

For fixed L , $R = 25, 50, 100 \Omega$ were used in order to see the expected change in the resonance: if R adds no reactance then the centre of the resonance should not shift, but the peak height and width will vary. Then the capacitance was changed. Because the resonance width is $\omega_0^2 CR$, the capacitance must still be of order the trap capacitance; otherwise the resonance will be broad, and an accurate estimate of the resonant frequency will be difficult. 10 pF capacitors were used, with a tolerance of $\pm 5\%$. This gave circuits with $Q_F \sim 12$. Since this quality factor is low we fitted the data with the exact, non-Lorentzian expression for the resonance given in equation (5.17).

Figure 5.8(a) shows the resonance with the GQ inductor and $C = 10 \text{ pF}$, $R = 50 \Omega$. The fit to the data yields $L = 4.3(2) \mu\text{H}$. The majority of the error comes from the $\pm 5\%$ tolerance in the capacitor construction. The resistance from the fit is 42Ω . However, the fit is insensitive to the precise value of the resistance: because the FWHM depends on L, C and R then the independent information about R comes from the depth of the peak. When $R = Z_0 = 50 \Omega$ equation 5.22

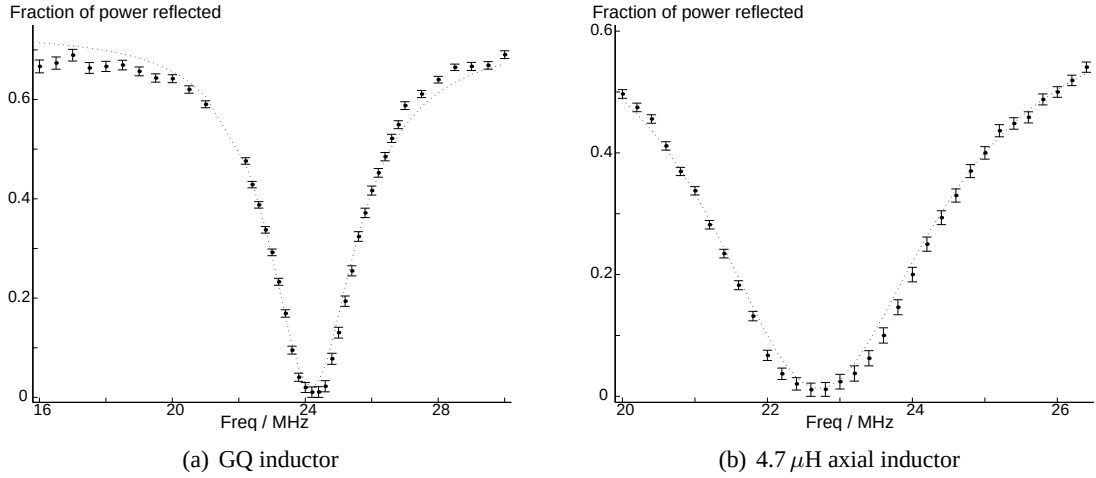


Figure 5.8: Series- LCR resonances using a 10 pF capacitor and $50\ \Omega$ resistance. The circuit is at the end of a transmission line, and the resonances are used to calibrate the (a) GQ and (b) commercial $4.7\ \mu\text{H}$ inductors.

gives a resonant reflected fraction of 0%; at $R = 42\ \Omega$ the reflected fraction has risen to 0.76%. A resonance using the axial inductor is shown in figure 5.8(b). The inductor is specified at $4.7(5)\ \mu\text{H}$, and the fit gives $4.9(3)\ \mu\text{H}$.

5.5.3 Series- LCR circuits

5.5.3.1 Apparent losses at all frequencies

All the measurements using the VSWR meter showed a frequency-independent loss of 30–35%. This cannot be explained as an effect of the cables. Using Minicircuits directional couplers before and after the VSWR meter, it was determined [136] that the loss is an artefact of the meter: the real reflected powers are scaled by this factor. The data shown in the figures is as given by the VSWR meter without rescaling.

5.5.3.2 Resonances with intermediate matching

In figure 5.9 we see a resonance arising from the GQ inductor and 10 pF capacitor with 2.8 cm NiCr resistance wire. This was coupled to a $50\ \Omega$ transmission line with the GQ transformer, tapped at a turns ratio of 1:5. We believe from later measurements that this transformer has good coupling, and thus attribute the shallowness of the resonance to impedance mismatching.

We fit to the data with the inductance $L = 4.9\ \mu\text{H}$ fixed. This yields a capacitance of 12 pF and a turns ratio of 1:4.9. The total resistance of the secondary circuit is $13.5\ \Omega$. The commercial capacitor was then replaced with the model trap. The observed resonance is shown in figure 5.10. Again the inductance is fixed and the fitted capacitance is 13 pF. The resistance is slightly lower, but still surprisingly high: $10.5\ \Omega$. The deduced turns ratio is now to 1:5.3.

5.5.4 Variable resistance

In order to understand the resistive part of the circuit better, we used lengths of resistive wire to alter the dissipation in the secondary circuit. We used dia. 0.11 mm nichrome resistance wire, of

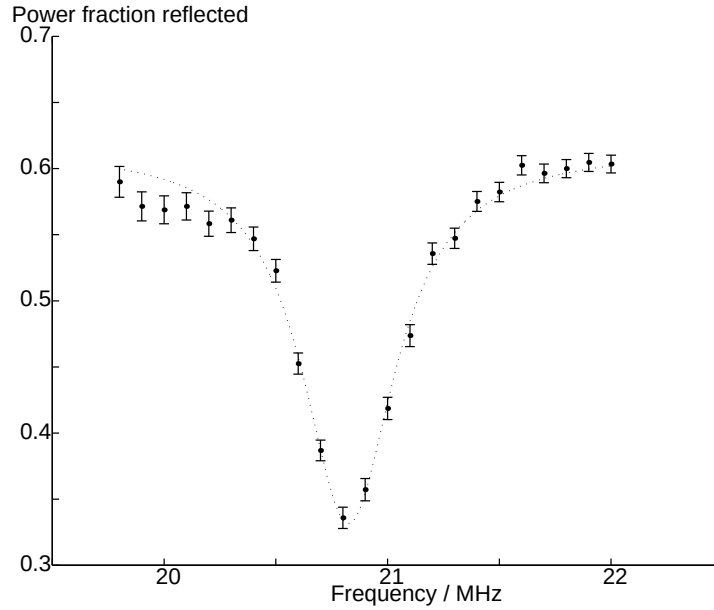


Figure 5.9: Partially-coupled resonance with 10 pF capacitor. On resonance 28% of the power is still reflected, compared to background losses of 38%.

Length l_i/mm	Fitted resistance R_i/Ω	Fitted capacitance C_i/pF
11	15.2	14.1
23	8.15	14.7
28	8.85	14.8
31	9.26	14.6
48	10.5	14.8
68	9.95	15.0
80	11.2	14.9
97	19.2	14.5

Table 5.4: Individual fits to series- LCR resonances, where the resistance R is varied by changing the length of resistive wire in the circuit. Note the anomalous fits for the shortest and longest wires.

resistivity $1.08 \cdot 10^{-6} \Omega \text{ m}$. As shown in table 5.3 a DC calculation of resistance per unit length gives $0.114 \Omega \text{ mm}^{-1}$. We compare our AC result to this value because of the large skin depth of nichrome at these frequencies.

As we shall show in section 5.5.5 we can assume a perfectly-coupled transformer for all fits. The GQ inductor's specification was assumed constant at $4.3 \mu\text{H}$. Thus we must float the capacitance between fits, because the centre frequency of the resonance is very sensitive to $\sqrt{LC}$. We shall see this gives a small variation in C . Eight different lengths of wire were used, generating a resonance data set each. Initially these sets were fitted separately, and the results of these fits are shown in table 5.4. The first and last resonances have the highest resistances and the lowest capacitances. The mean capacitance of all data is $\langle C \rangle_8 = 14.7(1) \text{ pF}$; with the extreme fits excluded the six remaining fits give $\langle C \rangle_6 = 14.81(5)$. The two excluded values are many standard deviations away from $\langle C \rangle_6$. This anomaly is reflected in the individual fits in figure 5.11: poor impedance matching yields inaccurate data sets with a strong asymmetry in the 11 mm data.

Discarding the two doubtful fits, the six remaining independent fits are shown in figure 5.12.

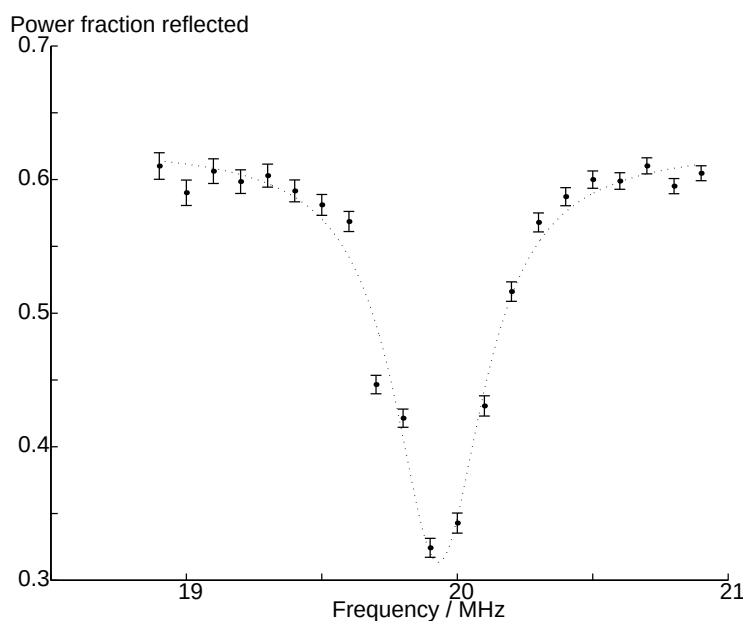


Figure 5.10: Partially-coupled resonance with model trap. As with figure 5.9 a large fraction of power—31% as opposed to a 38% background—is reflected on resonance.

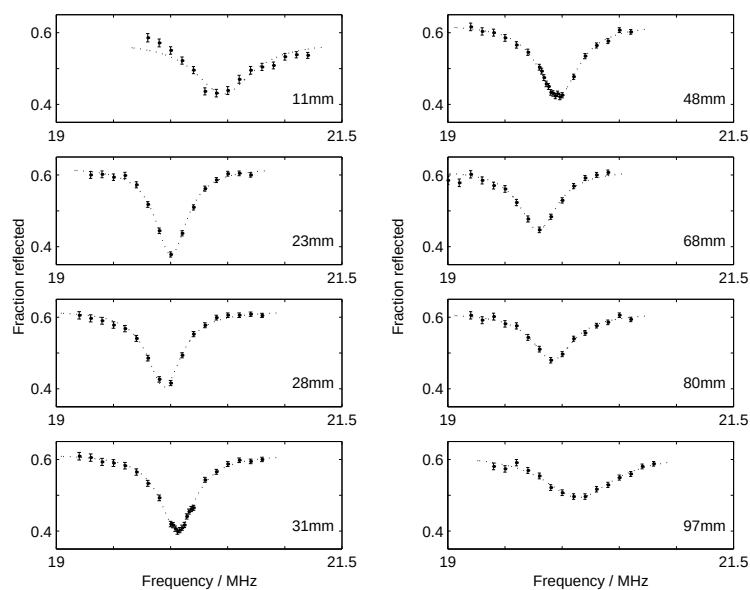


Figure 5.11: Fits to the eight resonances with different lengths of resistance wire. The length is shown above each fit.

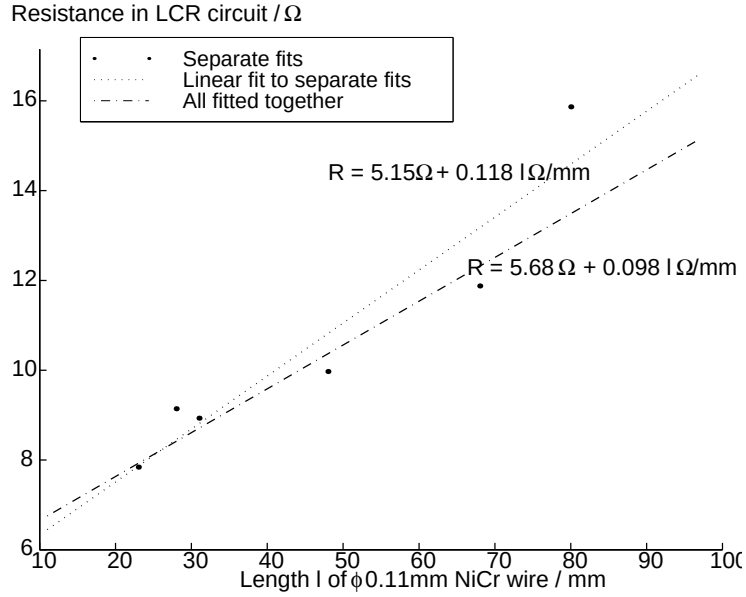


Figure 5.12: Linear fits to the resistance R of six lengths of wire. The dots are data points for individual fits, and a linear fit to these is given by the dotted line. The dot-dashed line is the result of parametrizing the initial resonance fitting $R = R_0 + rl$.

Also on the graph is a dotted line showing a least-squares linear fit to these six data points. This fit gives a resistance per unit length of $0.118 \Omega \text{ mm}^{-1}$.

Let us return to fitting the six remaining resonances, and parametrize the resistance not by $R_i, i \in \{2 \dots 7\}$ but by $R_i = R_0 + rl_i$. This is a physically realistic assumption: the resistance of a length of wire of cross-section A and resistivity ρ is

$$R = \rho l / A.$$

This reduces the number of free parameters in our system and makes the fitting more robust. It is still not possible to parametrize C_i more robustly.

The result of this alternative fitting method is also shown on the graph. The resistance per unit length is now $0.098 \Omega \text{ mm}^{-1}$. The two fitting methods therefore imply a value $0.11(1) \Omega \text{ mm}^{-1}$, compared to the specified value of $0.114 \Omega \text{ mm}^{-1}$. Therefore the resistance of the LC circuit alone is 5.15Ω for the independent fits and 5.68Ω for the parametrized single fit. The origin of this resistance is not clear.

5.5.5 Well-coupled resonances

There is very low reflection from a circuit on resonance when it is well-coupled ($\kappa \approx 1$) and well-matched. Both of these conditions are necessary for no reflected power from a transformed, series- LCR circuit hanging from a transmission line. The resonances were observed using the NeoSID transformer wound with a turns ratio of 5:2 and GQ inductor. For good coupling the dependence of the fit on the inductances of the transformer coils is very low. It was difficult therefore to fit the data in this subsection to the exact transformer formula: the confidence intervals on the transformer parameters were high. We therefore fitted the observed reflected fraction $1 - \Pi_{\text{abs}}$ using the form

$$1 - \Pi_{\text{abs}} = 2R(1 - B)(1 - \Re\{\mathbf{Y}_{L,C,2R}\}). \quad (5.33)$$

This functional form assumes perfect matching, making the anomalous width formula (equation (5.23)) equivalent to the absorption of a series- LCR' circuit with $R' = 2R$. From equation (5.7) this is proportional to the admittance of the circuit. We then normalize the admittance to unity less background losses B . The results of this fit necessarily give imprecise data on the turns ratio of the inductor, and will in general overestimate the resistance.

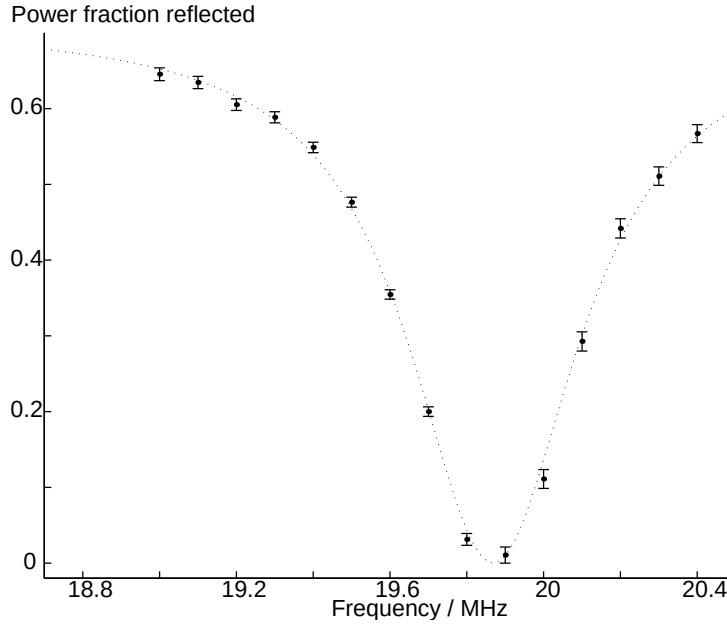


Figure 5.13: Fully-coupled resonance. Exploratory Lorentzian fits to the data implied $>100\%$ absorption on resonance. Because of this strong coupling a simplified formula was used for fitting. See text for details.

The data in figure 5.13 were fit using this expression. The fit yielded background losses of 28.8%, and component values of:

$$L_{71} = 4.3(2) \mu\text{H} \quad , \quad C_{71} = 14.9(7) \text{ pF} \quad , \quad 2R_{71} = 14.5(7) \Omega.$$

The inductance was fixed from earlier measurements, and the dominant error in the other component values is the propagation of the error in L .

From this data we can deduce what power is needed for $V_C = 800 \text{ V}$. Let the resonant frequency $\omega_0 = 2\pi 14.1 \text{ MHz}$ and $Q_F = 74$. An input signal of amplitude V_0 is dissipated entirely in the residual resistance:

$$V_0 \equiv V_R = \sqrt{2RP_{\text{abs}}} \quad (5.34)$$

This leads to a capacitance voltage of

$$V_C = Q_F \sqrt{2R(P_{\text{in}})} \quad (5.35)$$

$$= 334 \sqrt{P_{\text{in}}}, \quad (5.36)$$

where P_{in} is in Watts and V_C in Volts. We therefore require

$$P_{\text{in}} = 5.75 \text{ W.} \quad (5.37)$$

5.5.6 Helical resonator circuits

5.5.6.1 Resonator behaviour over time

We now turn to the helical resonator described in section 5.4.3. A signal generator was connected between the resonator shield, and a fraction of the first turn of the helix. Because of the small capacitances in the resonator it was impossible to observe the output voltage directly; any probe connected to the open end of the resonator altered its behaviour drastically. Instead a small inductive loop was placed near the resonator in order to couple a small fraction of the stored flux. By measuring the width of the frequency resonance, one determined Q_F and hence the amplification factor on resonance. Because this is a direct measure of the resonant circuit there was no extra width from e.g. delivery of the signal via a transmission line.

With a direct connection to the resonator a resonance was observed at 7 MHz. Using the small loop we observed a resonance at approx. 18.1 MHz, with FWHM varying between 30 and 62 kHz. This corresponds to $Q_F=290\text{--}610$. At such high qualities the variation could be accounted for by variations in the flux linkage of the aerial used to measure the resonance. The resonant frequency was only 10% from the design frequency. We also attempted to verify the properties of the resonator independently using a Q -meter, without success.

The apparatus seemed to have strong antenna-like properties. The resonant frequency, and hence Q_F , depended strongly on the positions of other contents of the room: when a hand was waved over one end of the resonator on resonance its magnification dropped almost to zero. This implied a shift of the resonant frequency Ω_{RF} of $\Delta\Omega \gg \Omega_{\text{RF}}/Q_F$. The resonator behaviour was improved using copper endcaps to create a sealed cavity. A small loop of wire, not physically connected to the resonator coil, was used to couple weakly into the cavity mode. This method was the most successful and the most stable way of operating the resonator [136]. Nonetheless the Q_F varied considerably, especially over time. After eighteen months the resonator quality had dropped to $Q_F \sim 60$. Sandblasting the resonator's inside surface raised the unloaded quality factor to ~ 200 . The cavity continued to deteriorate after sandblasting, however. We can explain this as follows.

As the copper tarnishes the conductivity of the resonator surface is reduced. For a good conductor such as copper the electric field penetrates the surface exponentially as $E \propto E_0 \exp(-d/d_0)$, where the skin depth d_0 is given by [123]

$$d_0 = \left(\frac{1}{2}\sigma\omega\mu\right)^{-\frac{1}{2}}. \quad (5.38)$$

The current flows in this surface region. At 293 K the conductivity of copper is $\sigma \approx 5.96 \cdot 10^7 \text{ } \Omega\text{m}^{-1}$; its relative permeability $\mu_r = \mu/\mu_0 \approx 0.98$ [123], giving a skin depth of $15 \text{ } \mu\text{m}$. One possibility for future work is to coat the resonator with a thin layer of a good conductor that does not tarnish: silver, for example.

5.5.6.2 Effect of loading the resonator with transmission lines and capacitances

We expect from equation (5.20) that a short length of transmission line will act like a capacitance. However, we must be certain that we are in a regime where this approximation will work. In

addition we need to consider how a helical resonator will behave when loaded with transmission lines and capacitances e.g. commercial capacitances or the model trap.

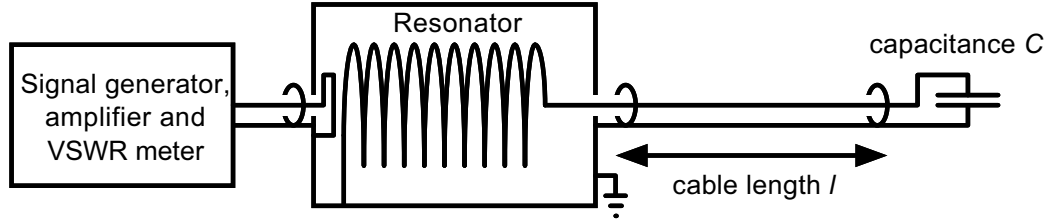


Figure 5.14: Electrical circuit for testing the effect of short transmission lines and bulk capacitances on the helical resonator. Signals from the DS345/FLL25 are coupled in to the resonator using a small input loop.

To test the behaviour of the loaded resonator we connected varying lengths l of coaxial cable, terminated with different commercial capacitors C , to the output of the helical resonator. The circuit is shown in figure 5.14. We applied radio-frequency signals to the circuit and measured the resonant frequency using the VSWR meter to search for minimum reflected power. The results of this experiment are shown in figure 5.15. We model this data as a resonant LCR circuit, where C is the capacitances of the resonator, the cable and the terminating capacitor: all in parallel. Fits based on this model are also shown in the figure.

Fitting yields a capacitance per unit length of the coaxial cable of $102(1) \text{ pF m}^{-1}$. This is close to the specified value of 100 pF m^{-1} . The effective capacitance of the resonator is $11.3(3) \text{ pF}$. We calculated in subsection 5.4.3 that the shield of the resonator as designed has capacitance 9.53 pF ; the discrepancy can be explained by the resonator not being a simple LCR circuit. We have therefore shown that short lengths of transmission line do indeed behave like parallel capacitors, and the effect this has on the resonator is equivalent to that predicted from simple LCR circuit theory.

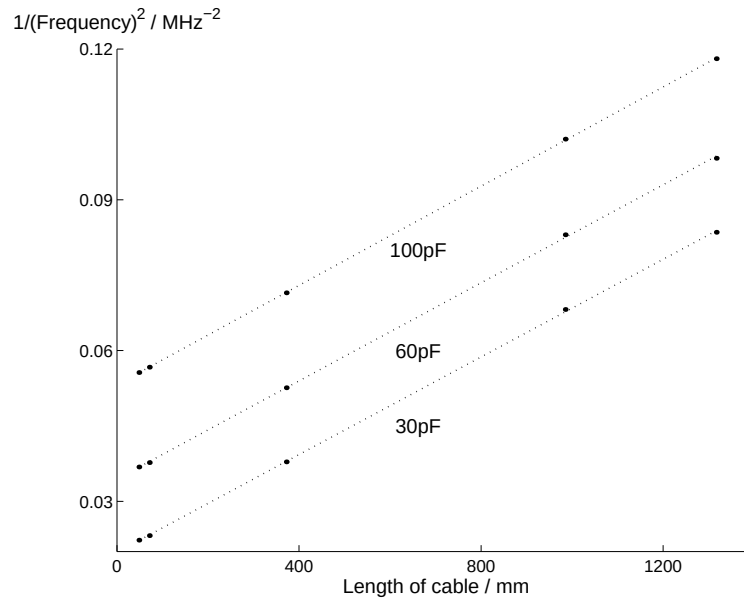


Figure 5.15: Results of loading the helical resonator with coaxial cables and capacitances C . $1/f^2 \propto C$ is plotted against l , where f is the resonant frequency of the loaded circuit and l is the length of the cable. The data points ($\bullet$) and the fits ($\cdots$) are shown for all three load capacitances: 30 pF, 60 pF and 100 pF.

5.5.7 Trapping ions with the new supplies

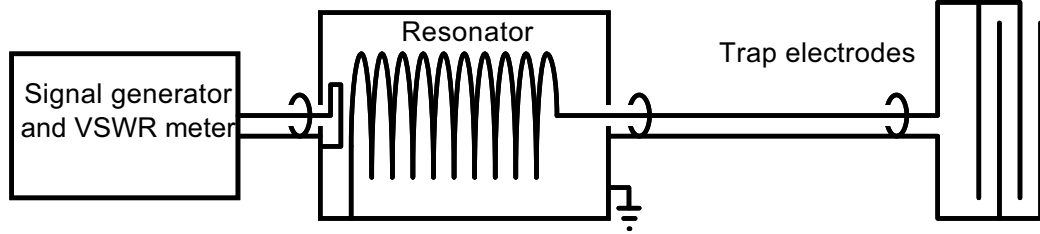


Figure 5.16: Low-noise circuit with reactive, passive amplification, used to successfully trap ions at 4.05 MHz. The signal generator is fed directly into the helical resonator, and this is loaded with 800 mm of coaxial cable and the feedthrough/trap electrodes. See text for more details.

Further work, carried out by Jonathan Home with the aid of the author, has resulted in trapping ions using the new RF supply [136]. A schematic of this circuit is shown in figure 5.16. The helical resonator was connected to the trap using approximately 800 mm of coaxial cable. This resulted in a resonant frequency of 4.05 MHz and we were able to trap ions at this frequency. From this measurement we were able to infer that the trap and feedthrough had capacitance ~ 30 pF. The circuit had a quality factor $Q_F \sim 190$ and was able to provide sufficient voltages using only the signal generator (no active amplification: this was partly because of the reduced trapping frequency necessitating lower voltages for the same q). However, although the coupling of power to the bare helical resonator supply is high (over 85% on resonance), measurements of the ion's secular frequency implied that the RF voltage at the ion is lower than one would expect from the input power and the circuit magnification factor, by a factor of ~ 8 .

5.6 Conclusions

We have examined two approaches to generating the RF requirement outlined at the beginning of this chapter: that is, a low-noise AC voltage at the electrodes oscillating with amplitude 800 V and frequency approximately 20 MHz. The first approach was a resonant circuit based on separate capacitors and inductors with a transformer for impedance matching. The second was a helical resonator, with adjustable input coupling to achieve impedance matching. Although ‘on paper’ both methods appear equally viable, in practice we found the resonant circuits never produced as high a Q_F as was expected. This meant not only that more RF power would be needed to operate them, but also that they provide a less good filter against electrical noise at the secular motion frequency. Therefore the main conclusion is that the helical resonator is the better choice. However, the analysis of the resonant circuits is a necessary background to understanding the operation of either method, so could not have been avoided.

We found the helical resonator was in practice well described by the empirical formulæ quoted in section 5.4.3. To achieve a high Q_F and stable behaviour, care had to be given to close the resonator ends and prevent tarnishing. The experiments also enabled us to deduce the joint capacitance of the trap electrodes and vacuum feedthrough (30 pF), and to understand the influence of the cables. This information has subsequently allowed a new resonator to be designed to achieve the desired resonant frequency when it is connected to the trap.

Chapter 6

Control of the magnetic field at the ion trap

6.1 Introduction

This chapter presents work on controlling the magnetic field in the trapping region.

The magnetic field magnitude must be controllable in order to split the Zeeman sublevels of the ground state of a calcium ion by a known (and tunable) amount; its direction must be controllable in order that applied laser beams of given polarization will address fixed, known transitions in the ion.

The most stringent conditions on the magnetic field stability come from the desire to reduce decoherence to a minimum in a quantum information experiment (see section 2.8). To keep the field stable, the optimal solution is clearly to leave it at a fixed magnitude and direction throughout the experiment. However, in the course of moving towards this goal it has been necessary to be able to steer the field around and vary its magnitude from zero to hundreds of Gauss. This has required several moderately complicated sets of field coils, the design, construction and performance of which will be described in detail.

An important theme of this work has been the ability to perform a ‘read-out’ experiment: that is, to prepare the spin state of the calcium in its ground state, and then observe a signal which can reveal which spin state was prepared with good signal-to-noise ratio. The read-out method we adopted consists of two steps. First, a spin-state selective ‘shelving’ step transfers population from one Zeeman sublevel of $|S_{1/2}\rangle$ to the metastable $|D_{5/2}\rangle$. Next, the cooling lasers (397 with 866 repumper) are switched on. Depending on whether we observe fluorescence, we are able to deduce if the ion was shelved, and hence in which Zeeman sublevel the ion was originally. Spin state-selective shelving is the difficult part of this process, and was the motivation behind the direction taken by experiments described in this chapter.

One way to achieve the shelving is direct excitation of the quadrupole transition $|S_{1/2}\rangle \rightarrow |D_{5/2}\rangle$. However, this requires a narrow-linewidth and reasonably powerful laser. We wished to avoid the difficulties associated with construction and operation of such a device. We instead adopted methods to excite population to $|P_{3/2}\rangle$ whence it could decay spontaneously to the shelf. Initially we wished to do this directly on the 393 nm transition, and this required a large magnetic field (~ 200 G) to split the Zeeman levels much more than the transition linewidth $\Gamma_{3/2} = 22$ MHz. This field was to be switched on only for the measurement stage of the experiment; cooling and state preparation were performed at fields $\ll 10$ G. We used an ion that was optically pumped by

the σ -polarized 397σ beam into one or other qubit state as a test of the shelving method.

The method successfully transferred the population [137]; however, remanence in the can from this large field changed the low field direction and magnitude unpredictably. This was sufficient to reduce the population preparation to 85–95%, and represents a field uncertainty much too large for quantum computing experiments. We therefore moved to a static, intermediate field regime of ~ 5 G. This avoided the problems associated with switching between the two field extremes, but necessitated a new shelving method based on narrower transitions. After consideration of several different multiphoton schemes we decided on a method inspired by dark resonances [138] or electromagnetically induced transparency (EIT) [139]. We have obtained preliminary results from the scheme and experiments are underway to perfect it [88].

The structure of this chapter is as follows. We begin by stating the requirements for each field regime in which we wish to run an experiment and how these dictate the magnetic coil construction. This construction is then described, including the dimensions of each coil and its former. The properties of these coils were measured with Hall probes, both away from all magnetic material and near the vacuum can. Finally we discuss the results of experiments using transitions in a single ion as a precise field probe at the centre of the trap.

6.2 Requirements

6.2.1 Magnitude and direction

Several sets of coils will be discussed: low-, high- and intermediate-field coils.

Low-field coils must be capable of applying $\gtrsim 1$ G. This field must both cancel out the earth's magnetic field and allow a resultant field to be applied in any direction. Eventually we will conduct experiments with this field along one of the laser beam directions (see section 2.5).

The high-field coils must apply > 200 G. We can position our coils closest to the trap along the vertical axis. Therefore we used this geometry, although it is not the most convenient for laser beam access. On the horizontal low-field coils turns were wound to steer the high field. This improves alignment of the field. A field of 5 G will steer the large 200 G field by a couple of degrees. Before the extent of field-direction coupling (owing to remanence in the can) was known, it was felt that these steering fields would be sufficient.

The intermediate-field coils need apply ~ 5 G, with the same requirements on direction control as the low-field coils.

6.2.2 Stability and homogeneity

The field at a given ion will depart from our 'ideal' applied low field by some unknown amount δB . This amount includes transient instability and positional variation owing to inhomogeneous field. We require the field to be sufficiently stable for a single ion over a given timescale (although the field could be monitored and laser AOMs adjusted to follow the splitting, this is difficult). Similarly the field at any two ions must be identical to within a particular margin of error; as we steer the laser from one ion to the next we might in theory change its frequency but this is again not favourable. We discuss below what is considered sufficient stability and homogeneity for different field regimes.

6.2.2.1 Low and intermediate fields

Manipulation of the qubit's state during computation (low or intermediate field) is to be accomplished by a Raman process on the 397 nm transition, far detuned and hence not scattering photons from $|P_{1/2}\rangle$.

To determine an upper tolerance on δB we consider a pulse sequence of length T_{pul} . Let us assume that this time is dominated by transitions that transfer quantum information to the centre-of-mass 'data bus' oscillations of the ion string. We shall see that this assumption is warranted shortly. Let us consider how quickly we may effect this transfer.

Let Ω be the Rabi frequency characterizing the interaction strength of the Raman lasers on the fundamental transition between qubit states. This is the angular frequency of Rabi flops between the states with no change in the motion of the ion string. If the Raman lasers are now tuned to a sideband in order to change the motional state, then the coupling on this transition is $\eta\Omega$ (see section 4.2). Off-resonant excitation still occurs on the carrier transition: owing to the detuning of the laser by the trap frequency ω_z the Rabi flopping on the carrier gives an unwanted population of size $\Omega^2/(\Omega^2 + \omega_z^2) \approx \Omega^2/\omega_z^2$. Therefore the sideband transition cannot be excited arbitrarily quickly. The condition $\Omega^2/\omega_z^2 \ll 1$ yields:

$$\eta\Omega = \eta\omega_z \sqrt{\Omega^2/\omega_z^2} \ll \sqrt{\omega_z\omega_R}, \quad (6.1)$$

where $\omega_R = 2\pi 32 \text{ kHz}$ is the recoil frequency for emission or absorption of one photon by the $^{40}\text{Ca}^+$ ion (this recoil also corresponds to a Raman two-photon absorption for lasers propagating at 60° to the trap axis¹, the experimental arrangement that we intend to implement). Hence the rate of data-bus operations in our quantum processor must be much less than the geometric mean of trap frequency and recoil frequency. Consider a sequence of ten data-bus operations, each taking approximately 0.1 ms. The cumulative error in the precession of the qubit phase is:

$$\delta\phi = \frac{g_J\mu_B\delta B}{\hbar} \times 1 \text{ ms}. \quad (6.2)$$

We require this error to be much less than a phase flip i.e:

$$\begin{aligned} \delta\phi &\ll \pi \\ \Rightarrow \delta B &\ll 0.2 \text{ mG}. \end{aligned}$$

At $B = 1 \text{ G}$ this implies

$$\delta B/B \ll 10^{-4}. \quad (6.3)$$

In the light of mathematical modelling of our coils in later sections this implies that the current supplies to our low-field coils will, over the timescale of a single experimental sequence, need to have a current stability of micro-Amps.

6.2.2.2 High field

Shelving at high field is effected by a short pulse of the 393 UV shelving laser. The optimal duration of this pulse τ_{opt} depends on the Rabi frequency Ω_{393} of the atom-laser interaction and the laser detuning. We wish to minimize the total shelving error² ϵ . If the product $\Omega_{393}\tau_{\text{opt}}$ is too low then

¹ $2\hbar k \cos 60^\circ \equiv \hbar k$.

²See appendix A.2.

ϵ_- increases; if it is too high then ϵ_+ increases. Since τ_{opt} is controlled accurately (by the PC's input to the relevant AOM) this gives us an estimate on the maximum instability in the magnetic field that we can tolerate.

Consider the desired shelving transition to be $|S_{1/2}^{+1/2}\rangle \rightarrow |P_{3/2}^{+3/2}\rangle$. This transition shifts under a magnetic field B by $\mu_B B$ (see table 6.3). We tune the laser to the centre of this transition to minimize ϵ . We need the uncertainty in the Zeeman splitting δB to shift the line centre much less than a fraction of the transition linewidth; specifically

$$\mu_B \delta B \ll h (\Gamma_{3/2} + \Gamma_{\text{las}}). \quad (6.4)$$

We have no direct measure of the laser linewidth. However, when we lock the 393 laser to its cavity we observe excursion noise on the lock signal consistent with $\Gamma_{\text{las}} \sim 1$ MHz. This implies we can neglect the contribution of the laser. If the field is large enough to split the transition and the field uncertainty contributes a shift much less than the transition width, then

$$\left. \begin{array}{l} \mu_B B \gg h \Gamma_{3/2}, \\ \mu_B \delta B \ll h \Gamma_{3/2} \end{array} \right\} \implies \delta B/B \ll 1. \quad (6.5)$$

This is a much less stringent requirement than for the low/intermediate field's stability. We can therefore use different current supplies for the two cases. One set must be stable; the other capable of developing higher powers.

6.2.3 Switch-on time

There are both upper and lower bounds to the switch-on time of the high field. Typically the pulse sequence of the shelving experiment takes approximately 70 ms with shutters. We must wait for the high field to stabilize to within $\delta B/B \lesssim 0.08$. If we consider the rise time τ_{LR} of the inductive L/R circuit which the coil comprises, then this stability requirement equates to a time delay of over $2.5\tau_{LR}$. We do not wish this switch-on time to dominate the time of an experiment. This yields an upper limit on the time constant of the coil: $\tau_{LR} \lesssim 70 \text{ ms}/2.5 = 28 \text{ ms}$, which is easily achieved with our coil geometries.

The lower limit on switch-on time is dictated by the need for the qubit's state to follow the changing field direction adiabatically without flipping i.e. switching time long compared to the precession rate associated with the low field Zeeman effect.

6.2.4 Coil configurations

To generate an arbitrary field, it is necessary to place coils on a set of three linearly independent axes. Symmetrical pairs of coils are preferred (if possible in Helmholtz symmetry [123]) to reduce inhomogeneities. It is possible to wind coils on a large, cuboid cage, many times greater than the region over which we require field application. This increases the volume within which the field is homogeneous at the expense of reducing the field strength for a given current-turns in the coils. Such an arrangement is unwieldy, though, and it was difficult to see how it might fit around our existing apparatus. The vacuum can and its retaining bolts strongly distort the field and this distortion was clear from high-field experiments on a single ion. The magnetic field from a given pair of coils is diverted partially away from the ion. A close-fitting coil pair 'channels' field directly through the glass windows. These considerations lead us to design sets of coils of dimensions similar to the vacuum can, mounted close to the vacuum can.

6.3 Coil design and construction

6.3.1 Overview

Three pairs of coils were initially constructed for the low-field regime. The upper vertical coil was put on the trap first. Then, when it was clear that alignment along other directions was essential, the other coils were added.

The vertical coils were next replaced with high-field coils inside brass formers, which had a second set of low-field turns on each former. The formers were sealed with glue, and water cooling passed through them. Unfortunately the seals failed and the water leaked. However, a low duty cycle permitted us to perform a readout experiment on a single ion at a high, switched field.

The horizontal low-field coils were then also replaced. New coils, constructed by the Clarendon Workshop (WS) and Graham Quelch (GQ), had integral water cooling, designed with the failure of the high-field cooling in mind. Because the vertical high-field coils were so close to the trap they could be used in the intermediate-field regime at safe currents that required only air cooling.

6.3.2 Mathematical modelling

Coils		B/I at centre/ G A^{-1}		Distance to ion/ mm	B/I at ion/ G A^{-1}	
		Inner	Outer		Inner	Outer
Air-cooled	Top & bottom $\times 2$	2.63	7.90	50	2.49	7.46
	Close conflat $\times 2$	—	13.8	90	—	2.08
	Far coils $\times 2$	16.0	21.5	195	0.43	0.56
	High-field $\times 2$	15.3	2.53	47	17.9	2.97
Water-cooled	WS coil A	—	12.8	92	—	5.26
	WS coil B	—	12.2	92	—	
	GQ $\times 2$	—	401.0	168	—	18.6

Table 6.1: Fields produced by coils discussed in the text, as calculated in free space. The dimensions of all coils are given in table 6.2. The fields at coil centre are for one coil; the fields at the ion have contributions from two coils. ‘Inner’ and ‘outer’ refer to two sets of turns wound on the same former. Note there is an asymmetry in the number of turns in the WS coils. The result quoted is the centre of the homogeneous field region, offset from the trap centre by 1.4 mm.

Initial estimates from simple formulæ [123] implied that a current-turns product of 10 A-turns in close horizontal coils would give fields of ~ 1 G at the ion. We therefore considered coils of many tens of turns in the low field regime (many hundreds in the intermediate- and high-field regimes).

An analytical expression is available for the field produced by a circular coil, along the axis of the coil. For a more general result mathematical modelling of each coil was done by the program `coils.m` over a cube of side 10 mm. This program performed an integration of the Biot-Savart law for the magnetic field around a current element. All coils were approximated to a circle of radius equal to the mean radius of the windings, except for the GQ coils. These have a strongly solenoidal geometry, and hence the model of one coil consisted of seven circular elements. This is shown in figure 6.4.

The results of the modelling are in table 6.1. For coils with two sets of turns the results are strictly in the ratio of the number of current turns (because of the approximation to a single circular element). The integration was numerical and consisted of a sum over n arcs, each of angle $360^\circ/n$.

For coarse integration $n \gtrsim 10$, the result was within a few percent of a much more accurate calculation ($n = 100, 1000$). Integrations were performed for the centre of one coil, and for the position that the ion would occupy when both coils in any pair were *in situ*.

6.3.3 Design principles

The structure of most coil formers is very similar. The former consists of a toroid of either Dural or brass. Being metals their mechanical and thermal properties are more favourable than plastics, which are also difficult to machine. The cross-section of a former consists of a recess for the coils. In all but the high-field water-cooled coils this was on the outside edge. For the high-field design the former was constructed so that, if it were lying flat on the table, the high-field coil could be dropped into it and a lid glued in place. These cross-sections are shown in figure 6.2.

The vertical coil pairs, and one set of the horizontal coils, were attached to conflat on the can. For low and high fields the third coil pair was placed on another set of conflat, at 60° to the horizontal axis already occupied. These conflat were further from the can, separated by vacuum pipes: see figure 6.5. When the intermediate coils were mounted they were placed at the points of the hexagonal vacuum can instead. This was slightly closer to the ion, although we expected the can to provide more shielding along this axis.

Table 6.2 details the dimensions of all formers constructed. Most coil formers follow the same generic construction given in figure 6.1. Where the design has been altered to accommodate e.g. other apparatus, then this has been noted in the table. Cross sections of all coils are shown in figure 6.2.

6.3.3.1 Low-field coils

All coils discussed in this chapter were wound with varnished 1 mm dia. wire. The low-field coils are wound on six Dural formers. Windings are held in place with vinyl tape. The coils were screwed to typical optical-table Dural mounts and positioned as close as possible to the design configuration in figure 6.3.

6.3.3.2 High- and intermediate-field coils

The arrangement of these coils is shown in figure 6.4. Each former consists of a body and a lid, both brass. The body was lathed so that a pre-formed coil of wire could be dropped into the body lying flat, leaving space in the channel for water cooling. Each coil was wound on a disposable separate former, and packed with resin so that the turns could support their own weight. The body has six tapped holes like the low-field coils, and six pairs of radially drilled holes. The lid design is nearly flat, but with a lip to increase surface area contact with the body. Each lid was glued to its respective body with Araldite. When this seal burst attempts were made to strengthen it with hoops of plastic, glued over the seals in order to increase the surface area. Unfortunately this did not stop the leaks.

Intermediate-field coils were build by the Clarendon Laboratory Workshop (WS) and by the group laboratory technician Graham Quelch (GQ). The WS coils, constructed from brass, were an adaptation of the high-field designs. On the inside of the U-shape was a channel, made from a second piece of brass welded to the former: see figure 6.2. Cooling water was pumped under less than mains pressure through this channel. Water cooling was connected to the inside rim of the GQ coils by means of two pipes, bent into a zigzag and welded to the formers. Neither intermediate pairs

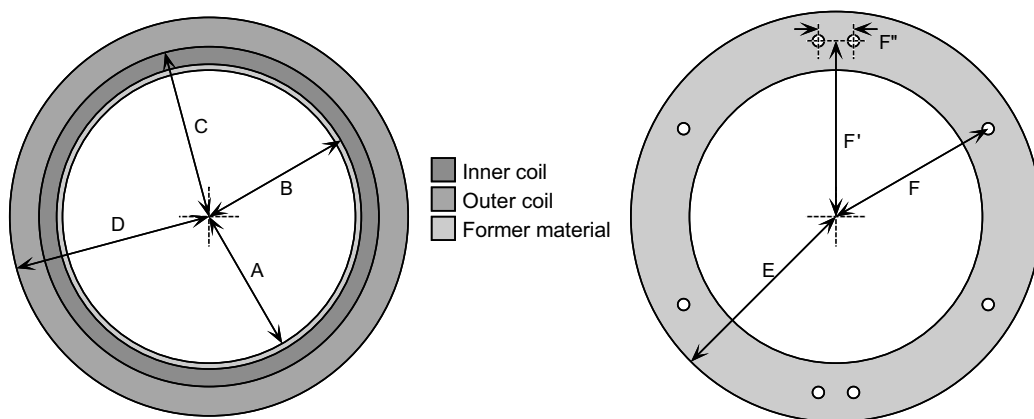


Figure 6.1: Schematic of generic coil dimensions. See text for details, and table 6.2 for the values of the above dimensions for all coils.

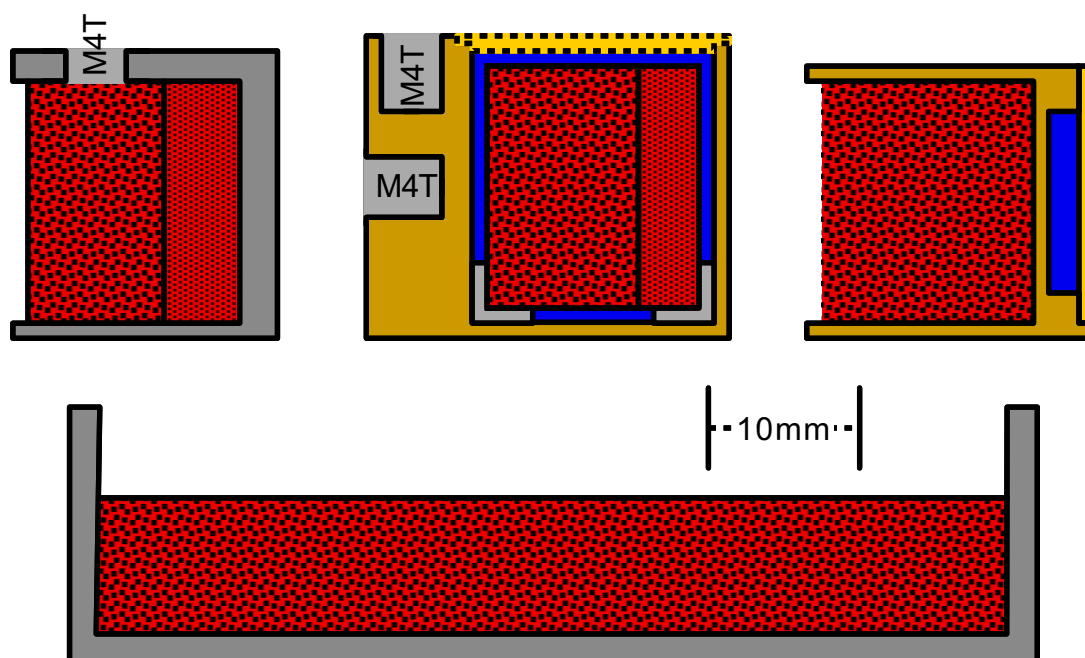


Figure 6.2: Cross-sections of the coils in this thesis. Clockwise from top left: low-field coil (representative); high-field coils; WS coils; GQ coils. All diagrams are to scale. Grey formers are made from Dural or steel; brown formers from brass. Blue is cooling water and each pattern of red is a different set of turns.

Coils		A	B	C	D	E	F	F'	F''	Inner	Outer
Air-cooled	Top & bottom $\times 2$	57.2	58	60	66	66.0	61	61	9	26	78
	Close conflat '③'	36.1	38	—	<47	47.1	n/a — holes radial $\times 6$			—	92
	Close conflat '④'	36.1	37	—	45	47.1	n/a — holes radial $\times 6$			—	92
	Far coils $\times 2$	40.0	41	47	<55	55.0	46	46	9	129	171
	High-field $\times 2$	56.9	59	61	73	80.9	78.0	—	20.0	175	29
Water-cooled	WS coil A	57	61	—	69	76	n/a — internal 'grip'			—	139
	WS coil B	57	61	—	69	76	n/a — internal 'grip'			—	132
	GQ $\times 2$	41.3	43.8	—	52.3	54.8	n/a — mounted in cradles			—	500

Table 6.2: Dimensions in mm of various coils built by the author, the Clarendon Laboratory Workshop (WS) or Graham Quelch (GQ) to apply magnetic fields at the trap position. Columns A to F'' show distances as indicated in figure 6.1. The final two columns give the number of turns. The high-field coils have six holes as shown on the diagram, and six pairs of radial holes on the edges: hence no measurement of F'. Coils with no measurement of C have only one set of turns wound on each former. All mounting holes are tapped M4.

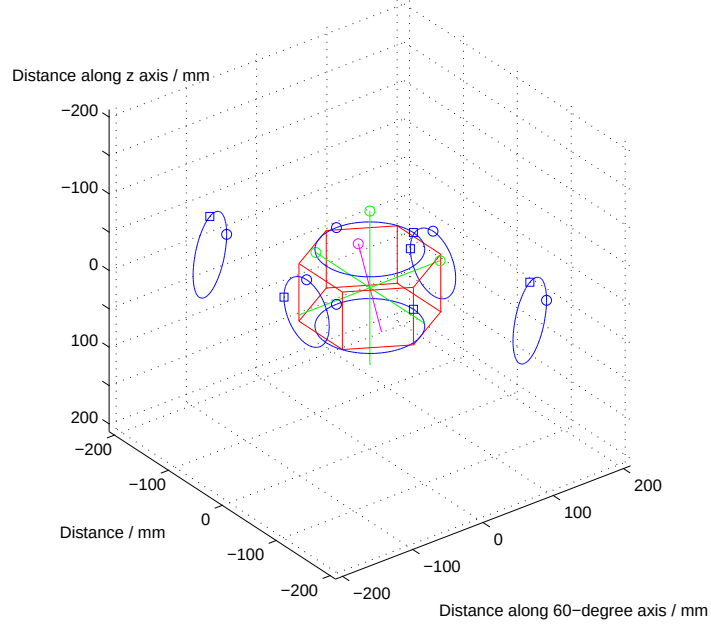


Figure 6.3: Locations of the low-field coils. The red hexagonal prism is the main chamber of the vacuum can. Blue circles mark the centre turn of each coil. The magenta line is the trap axis, and the three green lines correspond to the B_{60} , $\perp B_{60}$ and B_v -axes (see figure 6.5).

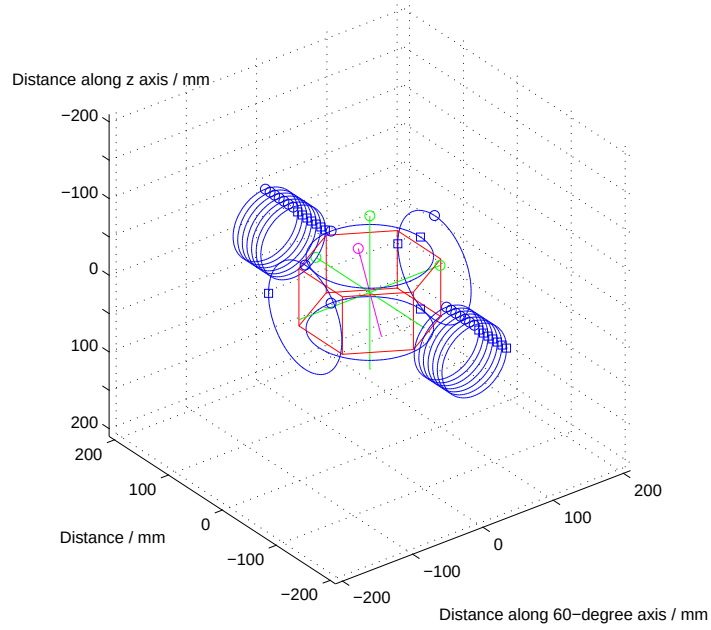


Figure 6.4: Locations of the high- and intermediate-field coils. The conventions of this diagram are identical to those in figure 6.3. The GQ coils are modelled with seven single-turn coils each, to account for their solenoidal aspect ratio.

of coils had mounting holes on the formers, but clamps were constructed to enclose the formers and mount them in position.

6.4 Methods of field measurement

6.4.1 Principle

We measure the field with two methods. Away from the apparatus, and close to the vacuum can we measure it by the Hall effect, in a 3-axis probe. When we wish to measure fields within the vacuum can we use the Zeeman effect and optical pumping in the ion.

6.4.2 Field conventions and directions

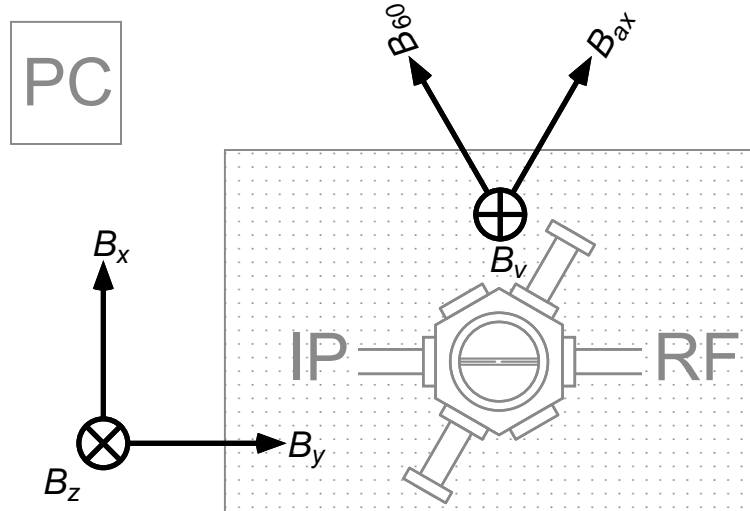


Figure 6.5: Conventions for recording magnetic fields. This is a reproduction of the schematic shown in figure 2.6.

Conventions of field directions are shown in figure 6.5. When field measurements are made along orthogonal axes relative to the orientation of the optical table this is indicated thus: $\mathbb{B} = \{B_x, B_y, B_z\}$. To discuss fields with horizontal magnetic coils on axes at 60° to each other, we use the non-orthogonal basis $\mathbf{B} = \{B_{60}, B_{ax}, B_v\}$. This basis is better suited to aligning fields with respect to the cooling beams $\mathbf{k}_{866} \parallel B_{ax}$, and the 397σ σ -polarized cooling laser beam $\mathbf{k}_{397\sigma} \parallel B_{60}$. Currents in the coils $\mathbf{I} = \{I_{60}, I_{ax}, I_v\}$ generate a theoretical free-space field $\mathbf{B}$; this is deflected by the apparatus to give a real field $\mathbf{B}'$. Thus the primed quantities represent the field at the ion, and the unprimed quantities represent the fields which would be produced by the coils in free space.

6.4.3 Hall probe

The Hall probe used was a Bartington 3-axis Mag-03-OIE assembly, serial number 027. This produces a voltage of 10 V G^{-1} , within the range $\pm 10 \text{ V}$. Its bandwidth is quoted at 3 kHz. The three probe heads were mounted in the x , y and $-z$ directions on a Dural post. The bandwidth of each probe was $\sim 2.5 \text{ kHz}$.

6.4.4 Observation of optical pumping

When two levels have the same multiplicity of sublevels, population can be optically pumped into the highest or lowest M_J sublevel of the lower level using either σ^+ or σ^- light only³. When the lower level has a higher degeneracy either σ^+ , σ^- or π alone will optically pump into the extended state. If the lower level is of lower degeneracy it is not possible to optically pump into an unaddressed sublevel: transitions from all lower sublevels are addressed by the laser. In the lowest five levels of the $^{40}\text{Ca}^+$ manifold there are three obvious candidates for optical pumping: σ^+ - or σ^- -polarized light on the 397 nm transition, and π -polarized light on either the 866 nm or 854 nm transitions. The first two of these are shown in figure 6.6. We use the first to align the magnetic field along the 397 nm laser direction and the second to align the field perpendicular to the 397 nm, 866 nm cooling lasers, typically vertically.

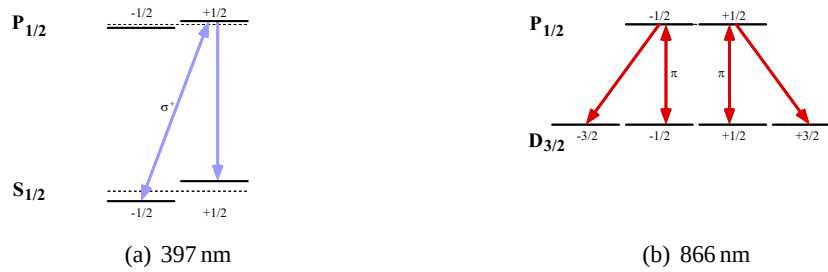


Figure 6.6: Examples of optical pumping in the $^{40}\text{Ca}^+$ manifold.

We have used optical pumping entirely as a tool for alignment of magnetic field, and not as a diagnostic for field magnitude. However, there is a dependence on magnitude. Let us restrict the discussion to pumping with π 866 nm light. A $\lambda/2$ -waveplate permits rotation of the 866 cooling repumper's linear polarization to any angle. If there is a high magnetic field then $\mathbf{B}$ defines the most convenient axis of projection for the M_J states. On the other hand, in low field the 866 laser will pump continuously into the dark states while the field will cause the state to slowly precess into the manifold of bright states: the laser polarization is in this situation *de facto* π with a small impurity, and defines the axis of atomic polarization along its own polarization vector. As a result at low enough fields the ion is always dark at any laser polarization. In general the fluorescence of an ion as a function of $\lambda/2$ -waveplate angle depends on the ratio of the rates of 866 nm pumping into the dark state (the intensity of the laser) and precession out of the dark state (the strength of the magnetic field), when the latter is not an energy eigenstate.

6.4.5 Single-photon resonance: shelving method

The most natural way to use the trapped ion as a magnetic field probe is through the Zeeman effect. One might imagine that this could be done, for example, by observing the fluorescence as a function of 397 laser frequency in the simplest situation where the cooling lasers illuminate the ion. However, in such an experiment, one does not observe well-defined single-photon resonances separated by the Zeeman splittings, because optical pumping continuously changes the distribution of population in the ground state as the laser is scanned. The result is a single broadened Lorentzian function, rather than a set of two or more separate peaks. Since broadening can also result from

³This is only true for half-integer J [140]

poor cooling or micromotion compensation, this is not an accurate way to measure the magnetic field.

To obtain a more precise measurement we used the $|S_{1/2}\rangle \rightarrow |P_{3/2}\rangle$ transition. Since this has cycling transitions when illuminated with either σ^+ or σ^- light⁴, the optical pumping does not ‘blur away’ the peaks of the spectrum. It is possible to observe continuous fluorescence excited by the 393 UV shelving laser. However, since we were interested in developing our shelving readout experiments as well as in calibrating the coils, we used a shelving method instead.

The timing for one shelving pulse sequence is shown in figure 6.7. Firstly the ion is cooled and then the cooling lasers are switched off. Then a preparatory step ‘X’ is activated. In the case of the first (high-field) readout method this consists of: using σ^+/σ^- 397 nm and 866 nm light to prepare the atom in one ground-state Zeeman sublevel; then the high field is ramped on. Once the field is stable a 393 nm shelving pulse is then switched on for a time τ_{393} . Fields are then reduced to the low-field regime.

Next, fluorescence counts A are observed⁵, typically for 10 ms. If A is above a threshold set in the PC then the ion is considered to be fluorescing: not shelved. If it is below the threshold then a shelving event has occurred. Then the 854 laser is introduced; it repumps the shelved population into the cooling manifold. Fluorescence is again recorded and logged as signal B . If fluorescence counts B fall below the threshold then there has been an experimental error, and this is recorded as a ‘failure’ in the data files.

This pulse sequence was embedded into an experiment as follows. The 393 laser was tuned across the transition using a DAC from the PC that controls the piezo in the cavity to which the laser is locked. At each 393 laser frequency a large number of shelving sequences was conducted to determine a statistical average of the shelving efficiency. In this way we measured the spectral response of the ion shelving, dependent on its prepared state. The results of such an experiment are shown in figure 6.8. In this figure we see three traces. Each corresponds to the probability of shelving the ion at different detunings of the 393 laser. In the black trace we maintain a low field on the ion, and we see excitation of the ion on the 23 MHz-wide 393 nm transition. In the red and blue traces we apply a field of approximately 200 G on the ion during the shelving pulse. This separates the components of the transition. We use circularly-polarized 397 nm light at low field to prepare the ion in one or other Zeeman sublevel: the red and blue traces correspond to different handedness of this light. The lack of 100% discrimination between the components is owing to imperfect preparation of the state at low field.

In order to discriminate between the two qubit states we required a Zeeman splitting on the 393 nm transition much greater than the natural linewidth $\Gamma_{3/2}$ [69]. The discrimination did not need to be 100%. However as the shelving failure probability $\epsilon = \epsilon_+ + \epsilon_-$ (defined in appendix A.2) increases the experiment must be repeated more times for the same final accuracy.

Let us take the qubit state we wish to shelve as the $S_{1/2}^{-1/2}$ sublevel⁶ of the ground state. The energy shift of a state of angular momentum projection M_J is $g_J M_J \mu_B B$. The Landé g_J factors for $^2S_{1/2}$ and $^2P_{3/2}$ are 2 and $\frac{4}{3}$ respectively, which gives the Zeeman splittings shown in table 6.3. $|P_{3/2}\rangle$ decays with a branching ratio such that we expect to scatter on average eighteen 393 nm photons before the ion is shelved. With this in mind we excite the σ^- transition, so that excited population can only decay to the original $S_{1/2}^{-1/2}$ state and we do not therefore mix the qubit state populations. A field change δB will shift this transition by $\mu_B \delta B$.

⁴Such cycling of population also requires two repumping lasers: the 854 shelving repumper, and either the 866 laser or the 850 IR shelving lasers.

⁵See section 3.2.

⁶See appendix A.2 for notation.

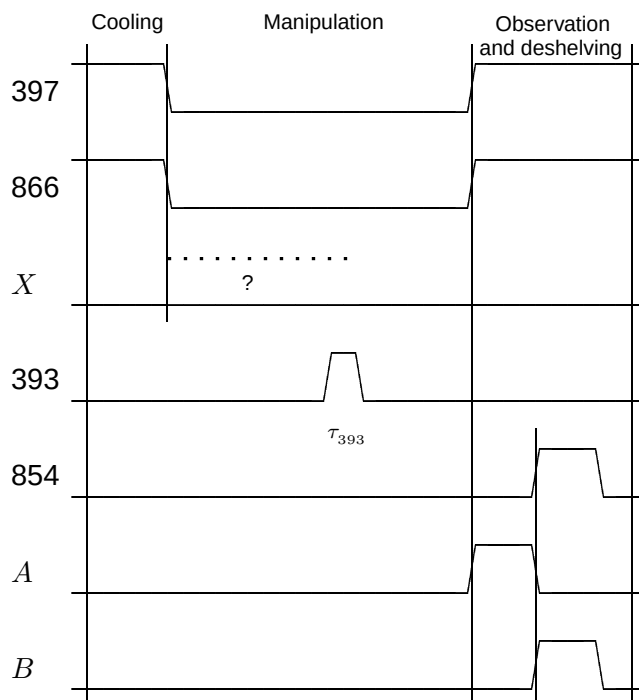


Figure 6.7: Timing diagram for a sequence of shelving pulses. The time on the horizontal axis is split into three main periods: cooling of the ion; manipulation of its internal state (involving some parameter X e.g. a magnetic field change or optical pumping with σ^+ 397 nm light, followed by shelving on the 393 nm transition); and finally observation of fluorescence and deshelling. Where the relative timing of two events is important then a vertical line is shown. X indicates state preparation laser pulses, and ramping of the magnetic field on and off.

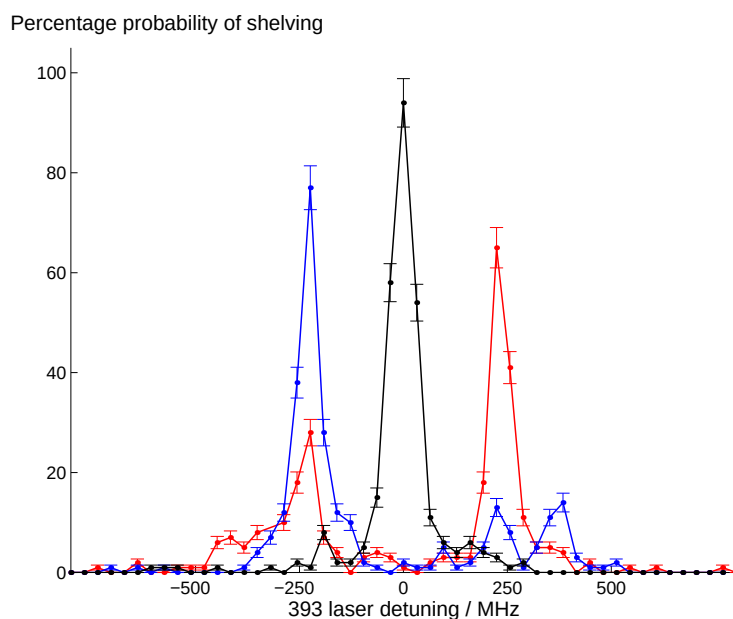


Figure 6.8: Results of a shelving scan. At each point on the frequency axis, the results of 100 shelving sequences (shown in figure 6.7) were averaged. This gives a measure of the mean population fraction shelved. The data is coloured depending on the nature of the preparatory step X : red, ground state preparation with σ^- 397 nm light followed by increasing field to 200 G; blue, preparation with σ^+ 397 nm light followed by 200 G; black, no field increase.

Transition	Required polarization	Detuning / $\mu_B B$
$ +\frac{1}{2}\rangle \rightarrow +\frac{3}{2}\rangle$	σ^+	+1
$ -\frac{1}{2}\rangle \rightarrow +\frac{1}{2}\rangle$		+5/3
$ +\frac{1}{2}\rangle \rightarrow -\frac{1}{2}\rangle$	σ^-	-5/3
$ -\frac{1}{2}\rangle \rightarrow -\frac{3}{2}\rangle$		-1

Table 6.3: Zeeman splittings of the 393 nm transition. The state labels refer to the M_J quantum number of the initial $S_{1/2}$ state and the final $P_{3/2}$ state. Only the $\sigma^\pm$ components are shown.

In practice we use linearly polarized light with the polarization vector perpendicular to the magnetic field. This permits us to use the strong vertical field coils while addressing the ion with a laser at a non-vertical angle. The light from such a laser is an equal superposition of σ^+ and σ^- polarizations. However, the closest unwanted transition is in any case another σ^- transition from the other ground state; therefore the presence of the σ^+ component is not the main source of failure.

If the atomic response is a Lorentzian $\mathcal{L}(\nu)$ then the observed shelving rate as a function of detuning ν is [137]

$$R = 1 - \exp(-c\mathcal{L}(\nu)I_{393}\tau_{393}). \quad (6.6)$$

The exponential factor broadens the resonance. Before fitting to resonances this effect was accounted for by taking the logarithm of the data. In addition one can analyse shelving in the case of two or more ions. In the absence of fluorescence noise there are n distinct levels of fluorescence for n ions. Let the probability of not shelving any ion be $p_{1,1} = 1 - P_{1,1}$. In the absence of correlated events [68], the probability of shelving at least one of n ions is

$$P_{n,>1} = 1 - (p_{1,1})^n = 1 - \exp(-nc\mathcal{L}(\nu)I_{393}\tau_{393}). \quad (6.7)$$

This still has the form of exponential decay, and can therefore be analysed by taking the logarithm.

The information about magnetic fields deduced from these experiments is discussed in section 6.5.

6.4.6 Two-photon resonance

As the repumper 866 laser is tuned over the $|D_{3/2}\rangle \leftrightarrow |P_{1/2}\rangle$ transition, with the 397 laser also addressing the ion, a fluorescence profile is observed in which the ion goes dark at certain combinations of 866 and 397 laser detunings. These dark features correspond to the detuning conditions for Raman two-photon transitions, where the ion's state is optically pumped into a coherent superposition of sublevels of $D_{3/2}$ and $S_{1/2}$ such that the transition amplitudes to $|P_{1/2}\rangle$ destructively interfere [138].

To find the field strength alone it is sufficient to use an approximate fit. We subtract eight Lorentzian profiles (for the eight two-photon transitions) of different weights from the Lorentzian of the broad transition:

$$F = \mathcal{L}_0(\nu) \left[1 - \sum_{n=1}^8 a_n \mathcal{L}'_n(\nu) \right]. \quad (6.8)$$

This does not take into account all of the quantum-mechanical behaviour of the ion of course. Further information is available from a fit using the full Bloch equations. This yields information

about the laser intensities, polarizations and combined linewidth, as well as the magnetic field. We used both approaches.

Automating the fitting from the full Bloch equations was difficult, and the data was such that minimizing χ^2 often resulted in spurious ‘best’ fits. Therefore the predominant method of fitting was ‘chi-by-eye’ [141].

6.5 Results

6.5.1 Introduction

In this section we give the results of experiments with magnetic fields and magnetic coils. Firstly the ambient field is discussed, and then the effects of each set of coils in turn. Where relevant we discuss the heating and time constants of our coils. Measurements were made both with macroscopic probes and with the ion. These methods of measurement are described in the previous section. The results are both a calibration of our coils and an investigation of some methods to manipulate and measure the $^{40}\text{Ca}^+$ ion.

6.5.2 Ambient field

The ambient laboratory field was measured with the Hall probe. Ambient fields within the can were also measured using the ion: we shall discuss these measurements later. The precise direction of the field varied around the lab, owing to the presence of the optical tables and other equipment. The field was measured over the table, but away from any other apparatus. This occurred before any coils were positioned on, or switched on near, the table. The measurement yielded

$$|\mathbb{B}_{\text{amb}}| \approx |\{0.156 \text{ G}, -0.52 \text{ G}, 0.94 \text{ G}\}| = 1.08 \text{ G predominantly downwards}, \quad (6.9)$$

in the notation of section 6.4.2. This field measurement had a noise of $\lesssim 0.8 \text{ mG}$ on all axes. Given the probe’s bandwidth as measured in section 6.4, this noise includes any effects from 50 Hz mains AC and likely harmonics thereof. Because of the large DC offset the recording oscilloscope was used in AC mode and there will therefore be some filtering of low frequencies.

The field at the head of one of the bolts on the vertical conflat was $\sim 2 \text{ G}$. At the junction of the can and ion pump (IP) the field rose to nearly 5 G. The IP was considered a source of strong magnetic field, and a μ -metal box was designed to enclose the pump’s permanent magnets. However as we shall see this offset field was a small effect compared to our ability to magnetize and remagnetize the vacuum can. The probe was put in the μ -metal enclosure for calibration. In this enclosure the AC noise fields were $\lesssim 0.1 \text{ mG}$.

With sufficient 866 nm power (approximately 50 saturation intensities) we were able to perform a preliminary optical pumping experiment in ambient field. Before we applied any fields to the trap, the 866 cooling repumper’s polarization was rotated until a minimum in fluorescence of 110 counts was seen. This corresponded to a laser polarization at 14° to the vertical. The maximum fluorescence was 6060 counts. Hence a figure of merit for the optical pumping achieved is 98.2% reduction of the fluorescence. The remaining fluorescence indicates the presence of a component of the magnetic field along the propagation axis of the laser.

6.5.3 Low-field coils

6.5.3.1 Heating of coils

The low-field coils were designed so as to apply sufficient fields at currents of a few Amps. However, we were interested in how much we could increase the current before the coil became too hot. Our main limitation is the seals on our vacuum chamber. These are copper deformed on a steel ring, and hence their elastic flexure is low. We do not wish to alternately heat and cool the apparatus. For the purposes of our experiment, therefore, temperatures are too high if they exceed the benchmark figure of 50°C . All low-field coils are of similar dimensions and construction. The dense, far coils will not be attached to the trap and so, although their heat dissipation will be poorer, they can get hotter without affecting conflat.

Current in coil/ A	Temperature of coil/ $^\circ\text{C}$
0	24
1	30
2	37
2.5	41
3	49

Table 6.4: Effect of current on the steady-state temperature of a close-conflat coil

Temperature measurements at several different currents are given in table 6.4. These were made with a thermocouple on one ‘close-conflat’ coil (see figure 6.3 and table 6.2 for coil nomenclature). These measurements were made with the coil clamped in free space. When the coil was attached to the can, this acted as a heat sink and temperatures were reduced. The coil resistance was $0.67\ \Omega$ at room temperature.

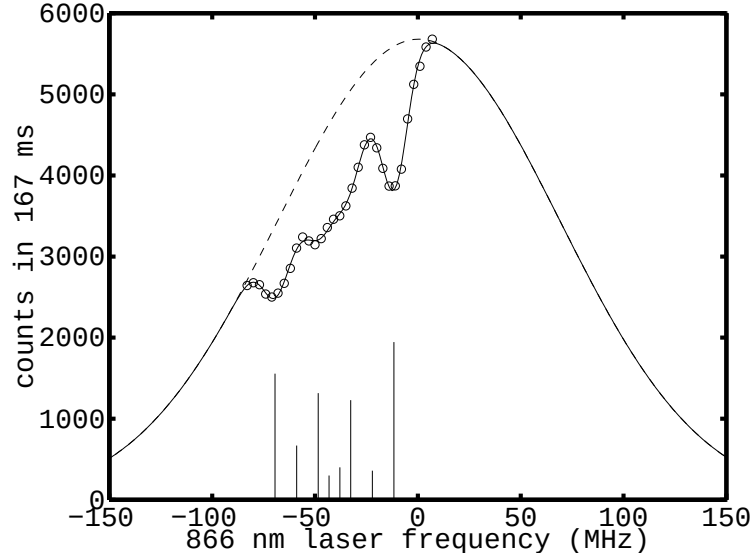
6.5.3.2 Probe measurements of the low-field coils

The effect of the vacuum can on horizontal fields was investigated initially with one close-conflat coil. This coil was placed on a horizontal conflat, then on a corner of the hexagonal can. A field probe diametrically opposite measured the field along the coil’s axis, which was compared to theory for free space. Between the conflat there was no noticeable field loss or deflection. However, between the corners of the hexagon there was a loss of the axial field from $33\ \text{mG A}^{-1}$ to $27\ \text{mG A}^{-1}$: 17% shielding.

We shall see from measurements using the ion that there is very strong field reduction and deflection within the trapping region—the field is reduced to 30–35% of its theoretical value—and this must be reconciled with the probe measurements. The difference in results can be explained as follows. The vacuum can and its components direct a large fraction of the flux within them, but this fraction returns to free space again at the opposite side of the can. The field is deflected much more strongly by the points of the hexagon than by the glass conflat, leading to a departure from theory at the opposite corner.

A measurement of the upper vertical coil’s field was made. With the coil *in situ* this yielded $9\ \text{G A}^{-1}$. The measurement was made with the probe directly over the top conflat, close to the window. Hence the probe head extended from 9 mm from the centre of the coil to 35 mm. The predicted value from this uncertain probe geometry was $(8.5 \pm 1)\ \text{G A}^{-1}$.

6.5.3.3 Low-field alignment and measurement using the ion

Figure 6.9: First quantitative analysis of the two-photon dark resonance in Ca^+ . See text for details.

The vertical coil was calibrated using the ion. This result is documented elsewhere [37] and is reproduced in figure 6.9. With both turns on the one vertical coil it was possible to apply a sufficiently large field to split the 866nm/397nm dark resonance in Ca^+ into its eight components. Both the 397D cooling laser and 866 repumper beams were horizontally polarized. If the field were wholly vertical at the ion then this polarization would result in $\sigma^\pm$ light in both beams and only four components to the dark resonance. Although these are the dominant components in the resonance there is clearly still some off-axis field in the can. This is a combination of the ambient field and distortion of the applied field by the can.

If we assume the splitting is entirely due to the applied field we obtain a calibration for just the top vertical coil (low turns) of $4.70(4) \text{ G A}^{-1}$. Theoretical calculations for free space in table 6.1 put this calibration at 4.98 G A^{-1} . Unfortunately, as we shall see later, we cannot extrapolate this calibration to the lower of the two vertical coils: the nearness of the optical table to the lower coil strengthens the field at the ion.

All six low-field coils were positioned on the vacuum can in the arrangement shown in figure 6.3. An optical pumping experiment was carried out with π 866 nm light. By zeroing in on the minimum fluorescence we align the field in the trap with the laser polarization. This provides fine tuning of the currents. By obtaining such a fluorescence minimum for all directions of the laser polarization, the field can be nulled. In figure 6.10 we see optical pumping at three different 866 laser intensities. The width of the feature is approximately proportional to intensity, because the rate of pumping of population into the dark state is also proportional to laser intensity.

Minimum fluorescence in these experiments was observed at approximately

$$\mathbf{I}_{\text{amb}} = \{1.40 \text{ A}, \sim 0 \text{ A}, \sim 0 \text{ A}\}. \quad (6.10)$$

Other experiments were conducted on the trap, including magnetic field experiments. These induced remanence (see later) and a similar measurement on a different day gave

$$\mathbf{I}_{\text{amb}} = \{0.67 \text{ A}, -0.156 \text{ A}, 0.186 \text{ A}\}. \quad (6.11)$$

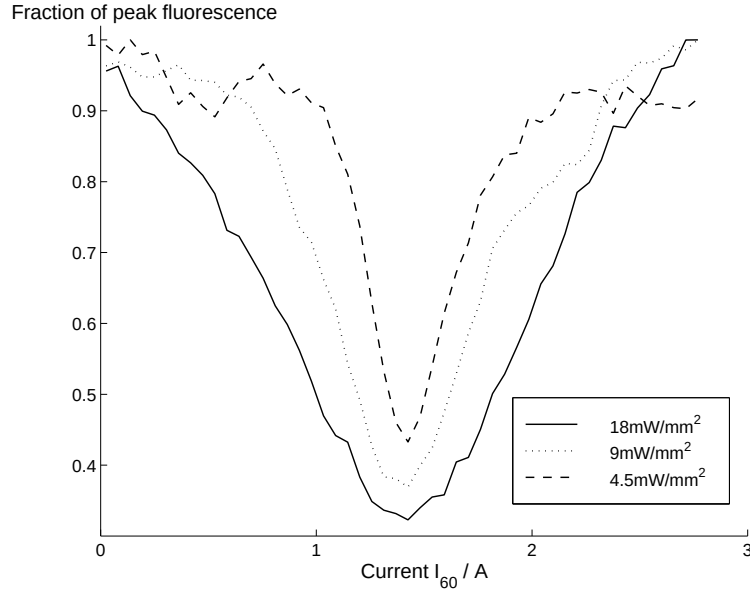


Figure 6.10: Fluorescence on the $\pi 866$ nm transition while the current I_{60} is scanned through the point of minimum fluorescence. Because of the different powers used, all scans are normalized with respect to their maximum fluorescence.

Starting from this field we increased the current I_{60} in the close-conflat coils. Using the 397σ laser, which propagated along the axis of these coils, we varied the currents in the other two pairs of coils until we once again observed optical pumping, this time on the $\sigma^+ 397$ nm transition. This was achieved for several values of current I_{60} , and the results are shown in figure 6.11. Note that this graph plots current against current, not field against field. Theoretically in free space the field conversion factors for I_{60} , I_{ax} and I_v are 2.08 , 0.99 and 2.49 G A^{-1} respectively (see table 6.1). With the experimental results this implies field coupling such that

$$\left. \begin{aligned} B_v &= -0.15B_{60}, \\ B_{ax} &= -0.57B_{60} \end{aligned} \right\} \Rightarrow \mathbf{B}' = \{B'_{60}, 0, 0\}. \quad (6.12)$$

We have omitted an offset field which varied over time owing to remanence. The notation is as set out in subsection 6.4.2.

Unfortunately from optical pumping data we are unable to obtain the magnitude of the resultant field B'_{60} . To determine field magnitudes we obtained further dark resonance scans. The low-field vertical coils were replaced by the high-field vertical coils, and we used the smaller set of turns. We concentrated on calibrating these vertical coils first. The current in the smaller set of turns on these coils was set at 1.5 A , 3 A and 5 A in several experiments, with several different ions over some weeks. At each vertically applied field B_v optical pumping was used with vertically polarized 866 nm light to bring the resultant field $\mathbf{B}'$ to the vertical. Hence our calibrations of the vertical coils include some field coupled into the vertical direction from the other coils.

The results of forty-four dark resonance fits, at three different values of the vertical current are summarized in table 6.5. A linear fit to the data is given in figure 6.12. Two sample dark resonances with their fits are given in figure 6.13. This figure also shows histograms of the data distribution within the data sets at different currents. The data set is consistent with a linear relation between applied vertical current (i.e. applied vertical field) and resultant vertical field. The result of the linear fit is a field of 3.68 G A^{-1} from the small turns on both of the vertical high-field coils. This

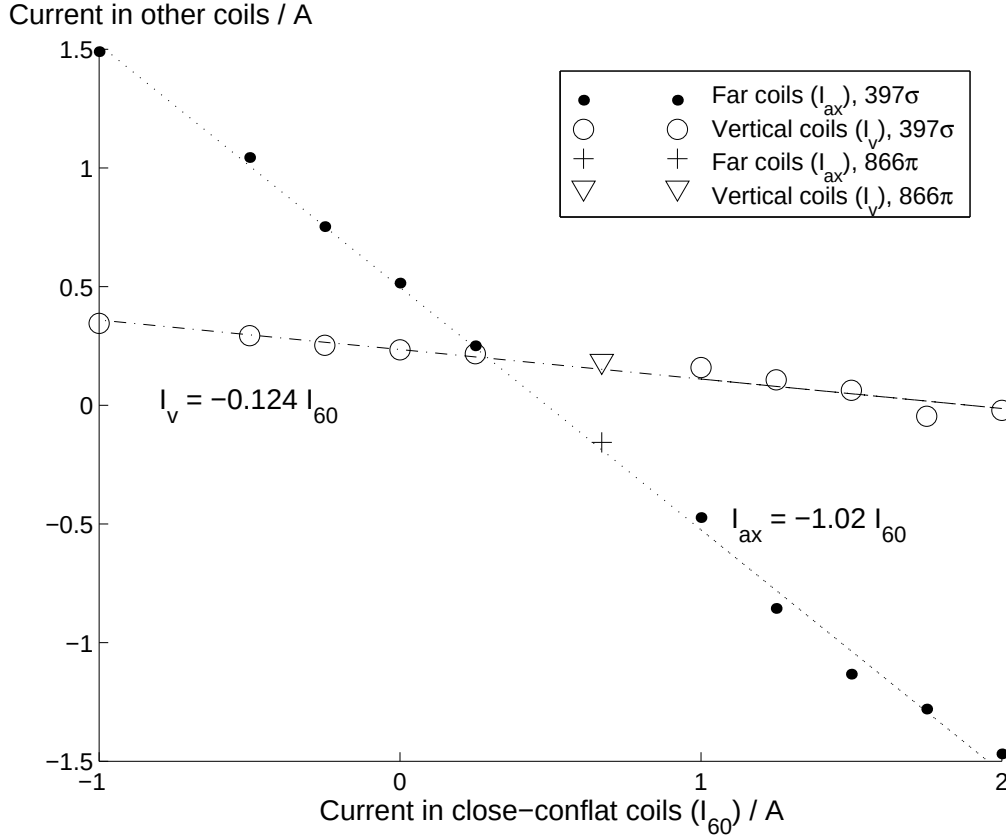


Figure 6.11: Currents in the low-field coils that result in good optical pumping. Circles ($\circ$) and dots ($\bullet$) represent the fields for σ^+397 nm pumping, i.e. field along B_{60} ; the cross (+) and triangle (∇) represent a zero-field measurement, nulling the ambient field in the can.

is greater than the theoretical ‘free space’ value in table 6.1 by approximately 24%.

6.5.4 ‘Extra’ set of low-field coils

We were at first unable to resolve dark resonances with our horizontal low-field coils, because the B_{ax} coils could not be run at high enough currents without getting too hot. However, consider the following. For linear polarizations of the cooling lasers it is impossible to align the field along the B'_{ax} direction by optical pumping. We therefore cannot directly calibrate fields along this direction, and are limited to experiments with B'_v and B'_{60} field directions. We can think of the close-conflat coils applying a field, and the far coils as just being compensation coils: we calibrate the former but do not worry about calibration of the latter.

We therefore strengthened the far coils by adding more turns. When the vertical high-field (see table 6.2) coils were put on the trap, they replaced the vertical low-field coils. The latter were then used to increase the magnitude of B_{60} . Each coil was connected to a low-field ‘far’ coil using a flat ‘washer’ of inner radius equal to that of the far coil and outer radius equal to that of the added coil. The combination was positioned so that the far coils were in the same place that they had previously occupied, and the added coils as close to the trap as possible.

With these new coils dark resonance scans were performed at various values of I_{60} , with currents in the other two coil axes set to achieve σ^+397 nm optical pumping. The 397D and 866 laser

Current I_v / A	1.5	3	5
Num. dark resonance fits	2	39	3
Mean field B'_v / G	7.3(2)	12.29(4)	19.93(7)

Table 6.5: Summary of data on low-field vertical coil current, for the data given in figure 6.12.

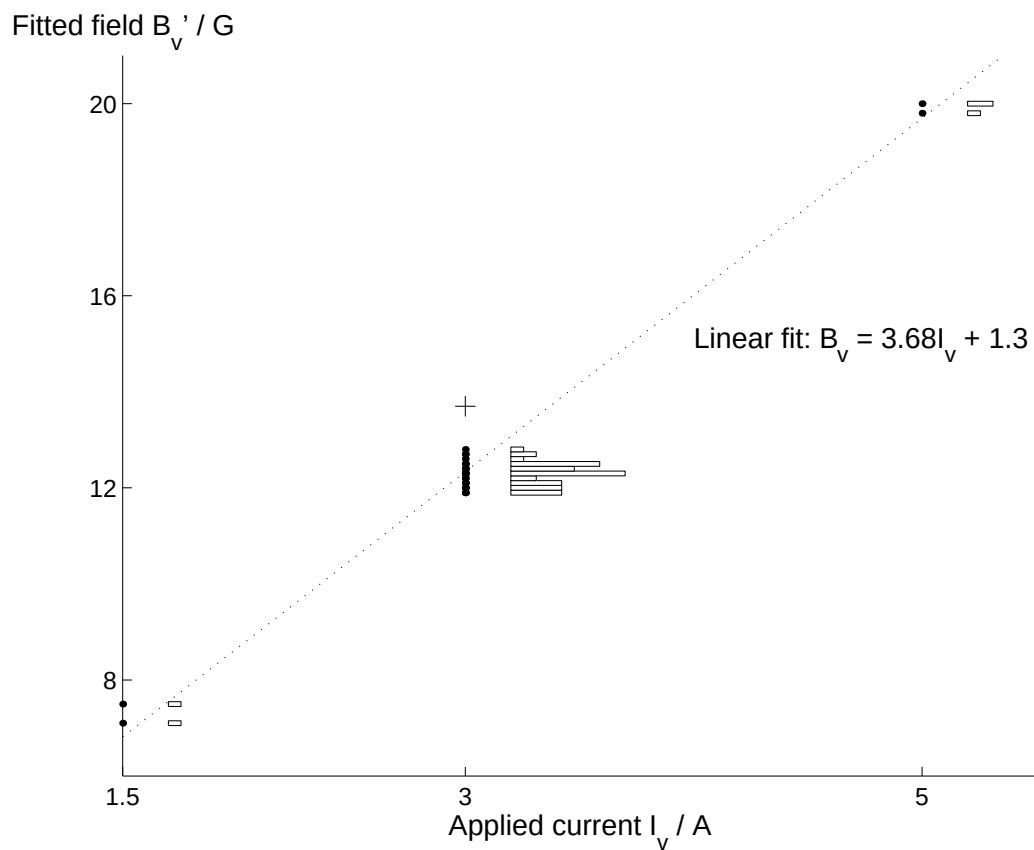


Figure 6.12: Calibration of the low-field vertical coils. The dots ($\cdot$) are the results of fits to dark resonance scans at three values of the current. The resolution of the fitting program was $\pm 0.1 \text{ G}$ and the histograms by each of the three data sets shows the distribution of data within the set. The cross (+) indicates a dubious fit.

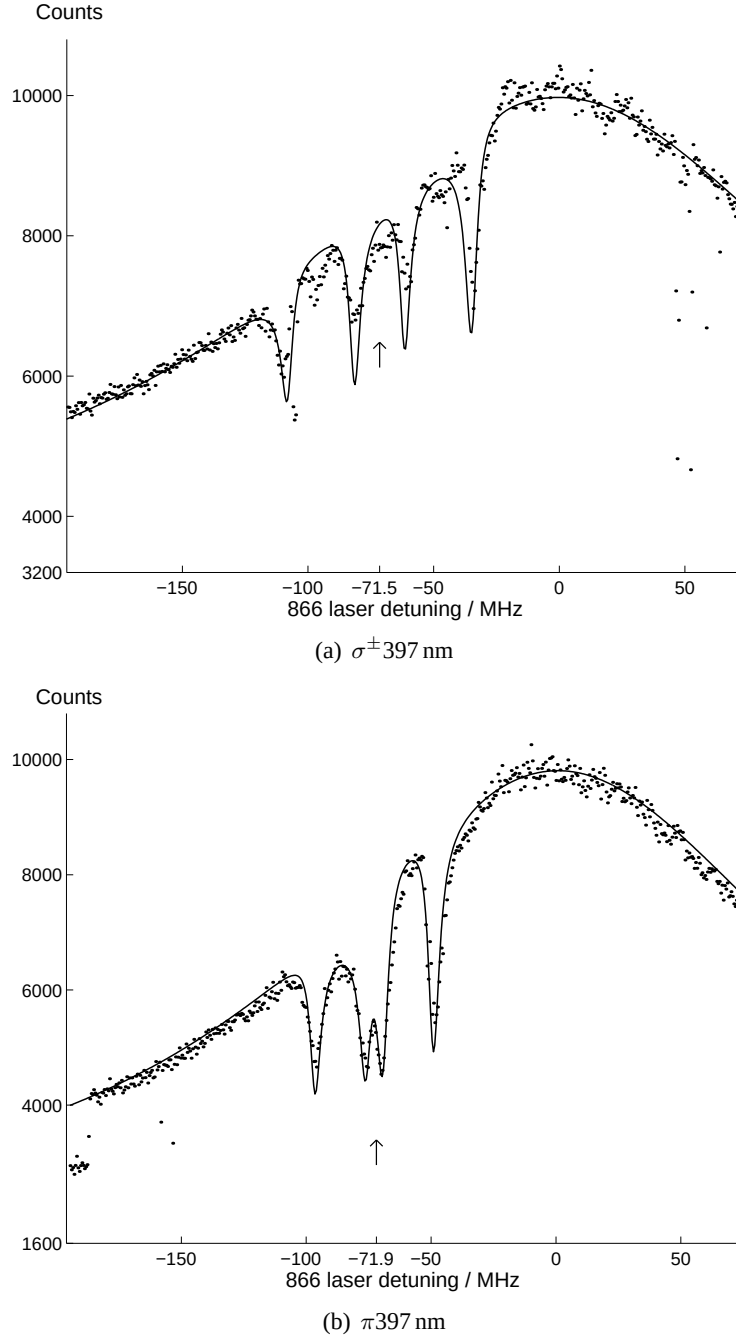


Figure 6.13: Examples of dark resonance fits in the vertical-field experiments with different polarizations in the 397 nm light. The vertical axes begin at the fitted baselines; the detuning of the 397 nm light (from the 794 lasers) deduced from the fit is marked on the horizontal axis and by arrows ($\uparrow$). The fitted fields, 397 nm and 866 nm intensities are (a) 11.9 G, $1.2I_{s397}$, $10I_{s866}$ (b) 12.1 G, $1.4I_{s397}$, $10I_{s866}$. Fitted 866 nm intensities were on average $\sim 60\%$ of that calculated from the spot size expected at the trap; 397 nm intensities were approximately as expected. Both scans were at $I_v = 3 \text{ A}$. The total fitted laser linewidth is approximately 0.5 MHz.

polarizations were vertical, to give $\sigma^\pm$ polarization during the scans. The fitted field results from these dark resonances are shown in figure 6.14. They show a linear dependence of field B'_{60} on current I_{60} of 0.63 G A^{-1} , approximately 30% of the theoretical value. We should stress at this point that this is a calibration of the new horizontal coils in the presence of ‘steering fields’ from the other two pairs of coils. These steering fields can be of similar magnitude to the fields produced by the horizontal coils themselves.

The vector sum of B_{60} and B_{ax} is not parallel to the resultant B'_{60} . This implies that the can not only shields the ion from the applied magnetic fields, but also deflects the magnetic field. This may be a result of the coils being placed asymmetrically around the trap axis. In figure 6.16 we represent the action of the can as a B -field vector $\Delta B \equiv (\mathbf{B}' - \mathbf{B})$. This vector diagram shows the horizontal fields only for the 4.5 A scan, and we can see that the can acts to deflect the field onto the 60° axis, at the same time reducing the field. The vertical field is a small adjustment, as we have already shown in figure 6.11.

Because the fields that the low coils are able to apply are still relatively low, the resonances are not as distinct as those with vertical current. In fitting to the profiles we concentrated on the dark resonances: therefore the 397 laser detuning and the magnetic field will be more reliable fit parameters than others e.g. laser intensities. A sample fit is shown in figure 6.15. In fitting we attempted to determine the field primarily, rather than any other parameters of the fit. The fit is reasonable around the dark resonances, given the noise on the data and the effect of dark resonance heating [142] (which is not accounted for in the fit function). In particular the ‘plateau’ seen between the dark resonance and the centre of the single-photon resonance is a feature of dark resonance heating [143].

6.5.5 High-field coils

6.5.5.1 Heating rates of coils

Cooling water was passed slowly through the brass jackets of the coils, and DC current passed through the coils themselves. Because it was impossible to measure the temperature of the coils directly—a temperature gradient was set up between the outside at 18°C and the centre of the coil—we measured the voltage and current from the power supply. We now determine the power being dissipated in the coil and the resistance of the coil. The temperature coefficient for copper is

$$\gamma = \frac{\Delta R}{R_0 \Delta T} = \sim 4 \cdot 10^{-3} \text{ K}^{-1}. \quad (6.13)$$

We therefore infer a ‘mean temperature’ of the heated coil. The results are shown in figure 6.17. On average the coil temperature increases 67°C kW^{-1} .

6.5.5.2 Switch-on time of high field

The rise time of the coils was found to be dependent on the permeability of their surroundings. A current of 20 mA was used in the coils to avoid saturating the probe. This was switched at 10 Hz, 50% duty cycle. The results are in table 6.6 and we summarize them here.

The general trend of field rise times was to increase with increasing volume of current-carrying apparatus. We discuss here only one coil, but with two sets of turns. Initially the two sets were tested without their brass former, as they were wound in resin which kept their shape. The 29 turns had a short rise time, until the 175 turns with which it was wound was short-circuited. Then eddy currents could pass through the larger set of turns, and the resultant rise time was the same as for when the

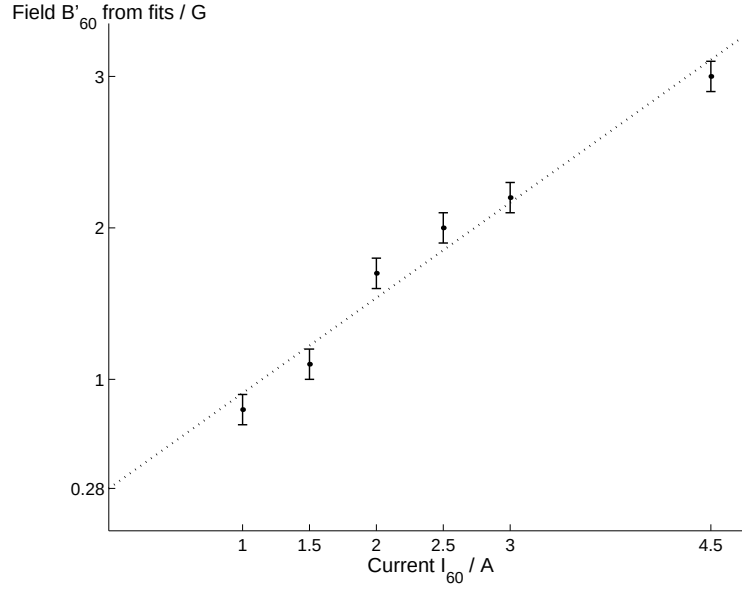


Figure 6.14: Calibration of the new low-field coils (extra turns on the B_{ax} axis; see text). Fields in all coils were varied to give $\sigma^+ 397 \text{ nm}$ optical pumping i.e. $|\mathbf{B}'| = B'_{60}$. Then scans were performed with lasers along the B_{ax} direction. The fit gives $B'_{60} = 0.63I_{60} + 0.28$, with field in G and current in A.

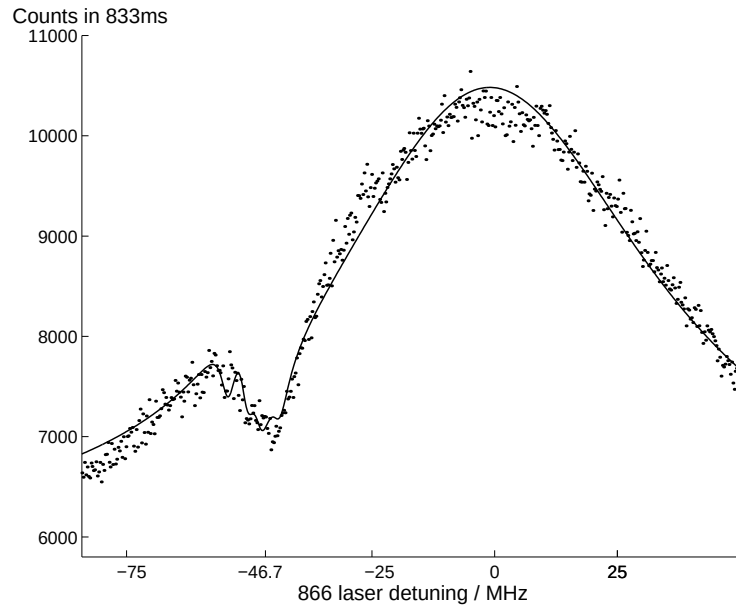


Figure 6.15: Example of a dark resonance fit in the experiments involving the new low-field coils. The coils carried currents $\mathbf{I} = \{2.0 \text{ A}, -1.3 \text{ A}, 0.05 \text{ A}\}$. The fitted field was 1.7 G.

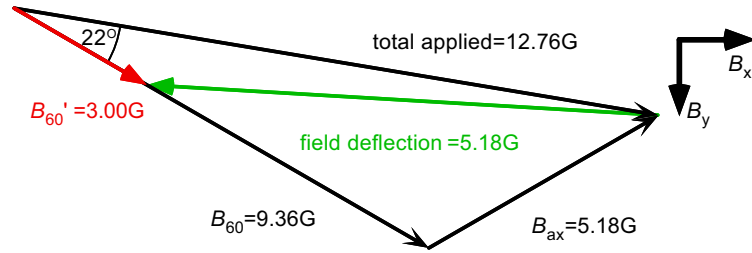


Figure 6.16: Example of the applied and resultant fields from one of the dark resonance scans. This arrangement corresponds to the 4.5 A data point in figure 6.14. The applied fields (black) are reduced and deflected by the can ‘field’ (green) to produce the resultant field (red).

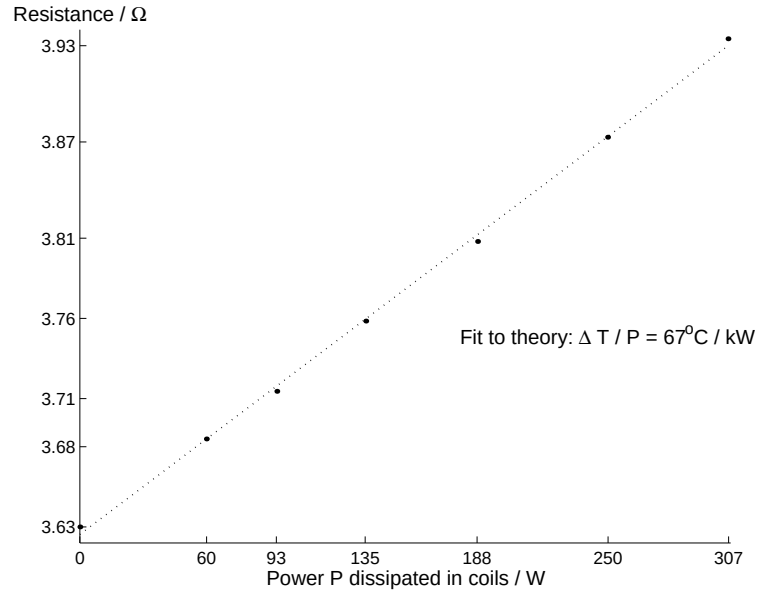


Figure 6.17: Effect of power dissipation on the resistance of the water-cooled, vertical low-field coils. Using the temperature coefficient of copper the fit yielded the temperature increase per power dissipation $\Delta T/P$ as shown.

two sets were connected in series. When the sets were connected in anti-series—i.e. the field from one coil antiparallel to the field in the other sets—then the time constant was decreased somewhat. When formers and the optical table were brought close to the coil, they acted to concentrate the flux and hence increase the back e.m.f. which slowed down the current’s rise time (in agreement with Lenz’s law).

We calibrated the vertical coils using shelving on the 393 nm single-photon resonance. After preparing the ion in either qubit state ($|S_{1/2}^{+1/2}\rangle$ or $|S_{1/2}^{-1/2}\rangle$) using $\sigma^{\pm}$ 397 nm optical pumping, the vertical, high field was switched on. This tilted the states until they were the eigenstates of the vertically-polarized ion. Simple two-level calculations implied that the rise time of the coils was long enough to guarantee adiabaticity [115] and not to mix the populations of the qubit states. Then the 393 UV shelving laser was pulsed on to shelve the ion (recall the sequence shown in figure 6.7 the ion). We explored the shelving spectrum as a function of 393 laser frequency at different vertical fields $B'_{v,hi}$, and determined the peak position. This peak corresponded to the transition $|S_{1/2}^{-1/2}\rangle \rightarrow |P_{3/2}^{-3/2}\rangle$. From table 6.3 we see that, in a field $B'_{v,hi}$, this component shifts by $\mu_B B'_{v,hi}$. The current was measured as a monitor voltage across a small wire: its calibration is in

29-turn coil	175-turn coil	Former	Rise time / ms
✓	open-circuit	×	1.8
	shorted	×	5.0
✓	✓	×	4.94
-✓	✓	×	3.6
open-circuit	✓	×	3.4
open-circuit	✓	✓	4.6
open-circuit	✓	✓+opt	5.2

Table 6.6: Summary of rise-time experiments for the coils on the high-field formers. The symbol ✓ denotes the coil through which current was run; -✓ means the current was in the opposite sense to other coils. The third column details the presence or absence of the brass former, and the optical table ('opt').

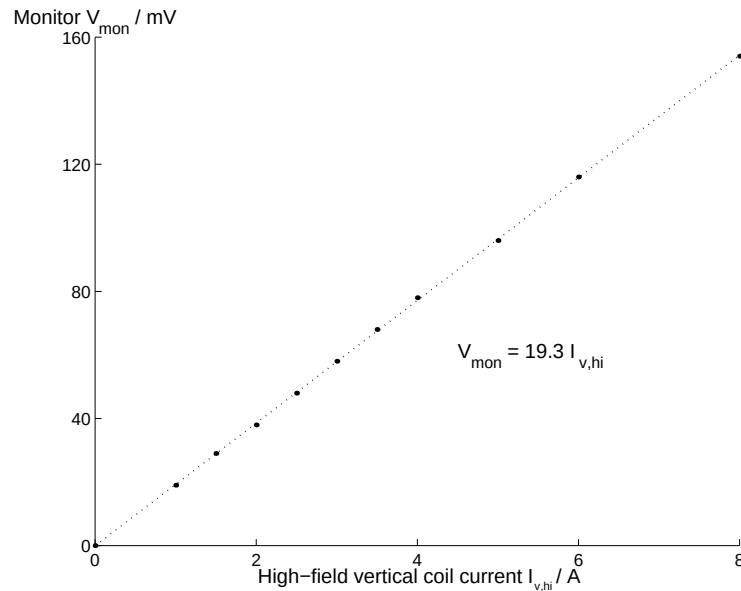


Figure 6.18: Calibration of the monitor voltage for the high-field vertical coils. Coil heating causes a small deviation from the straight-line fit.

figure 6.18.

The results of the experiment are shown in figure 6.19. The vertical high-field coils are calibrated from the data at $20.9(4) \text{ G A}^{-1}$ for the pair. As with the previous vertical coils this is greater than the free-space prediction of 17.9 G A^{-1} by approximately 20% owing to magnetization of the optical table.

6.5.6 Remanence and permanent magnets

The data in figure 6.11 was not taken in left-to-right order. Nonetheless the points are close to their straight line fits, implying that such low fields do not change the magnetization of the apparatus in a permanent or metastable way. However, when we apply fields of $\sim 200 \text{ G}$, we see remanence back in the low-field regime. Fluorescence on the 397 nm transition, which was reduced to negligibility by optical pumping using circularly-polarized light, increased by a few percent of the ion's maximum fluorescence. This partial recovery of fluorescence implied that high-field remanence

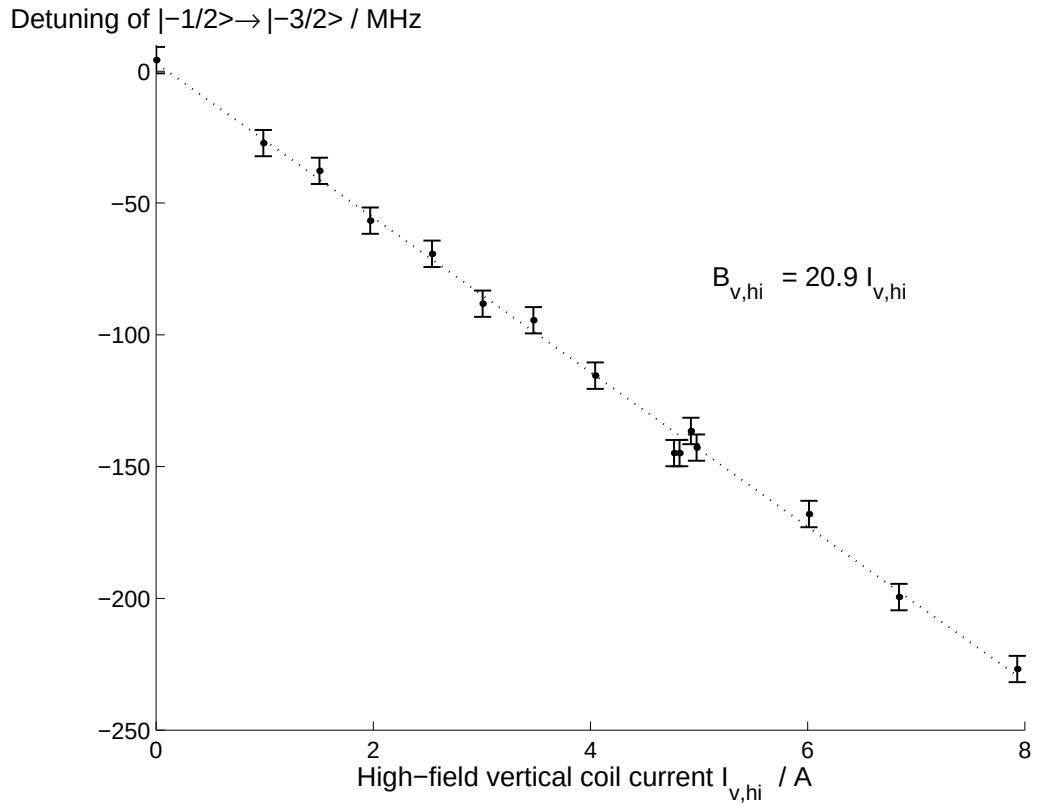


Figure 6.19: Zeeman shift of the $|S_{1/2}^{-1/2}\rangle \rightarrow |P_{3/2}^{-3/2}\rangle$ transition at different coil currents. From the shift we determine a calibration for the coils as shown, where the constant of proportionality is in G A^{-1} .

generates a random low field of $\lesssim 0.1$ G superposed on the applied low field. We conducted further experiments on remanence fields in the can as follows.

The field was minimized in the can by $\pi 866$ nm optical pumping. An observed ion fluorescence of approximately 5800 s^{-1} was reduced to 15 s^{-1} with the 866 laser polarization horizontal, and $\sim 0 \text{ s}^{-1}$ with the polarization vertical. Several Unbrako bolts, made of soft iron, were then brought close to the can. The fluorescence increased to 200 s^{-1} and did not return to zero when the bolts were removed. A spare vacuum conflat was brought to the can with no effect. A soft iron bar had the effect of increasing the fluorescence permanently to 500 s^{-1} . Stronger permanent magnets increased the fluorescence to its unpumped maximum.

Next, with all coils at zero currents, we introduced a magnet from a Faraday rotator. We moved it close to the can at such an angle as to give good optical pumping from the 397σ beam. The fluorescence dropped and, when the permanent magnet was removed, the fluorescence did not return to the value it previously had. We infer the can had been magnetized, with a long relaxation time constant.

6.5.7 Intermediate-field coils

The WS coils and GQ coils were positioned on the trap, with the vertical high-field coils providing field control on the third axis. The resistances of these pairs of coils were 3.5Ω and 7.2Ω respectively. These coils could be run at 10 A (175 W and 360 W) with modest cooling water flow rates, and the rate of heat transfer to the cooling water was sufficient to keep the surfaces of the coils at room temperature. At 4.33 A the mean temperature of a WS coil increases by $\sim 40^\circ \text{C}$.

Three dark resonances were recorded: two were at the same fields but with a different range of scan of the 866 laser frequency; one was at close to zero field. Fitting to these dark resonances was easier than with the low-field coils, as the components were better resolved. Nonetheless there was still some departure from theory, possibly owing to heating of the ion. This ‘dark resonance heating’ [142] typically causes recovery of fluorescence at lower frequencies than predicted by the positions of the dark resonances in the theoretical fit.

Two distinct fits were obtained with these coils, at fitted fields B'_{60} of 6.0 G and 0.2 G. These are shown in figure 6.20. As with earlier low-field fits the two data sets exhibit dark resonance heating. During fitting we concentrate on the red-detuned side of each dark resonance, and attempt to fit best to the most red-detuned dark resonances. At such detunings of the laser, the ion suffers least from heating [143]. In the lower figure the heating effect is worse, and causes a loss of fluorescence both before and after the dark resonance. This may be owing to the reduced fluorescence at low fields. The error in the fitted field here is as much as 50%: however, this contributes little to the calibration error because the field itself is so low.

If we can assume linear behaviour (which we have seen so far at these low fields) then the fitting implies a coil calibration of 2.1 G A^{-1} . This is approximately 39% of the theoretical free-space dependence, and hence agrees with the results from the low-field horizontal coils. We must repeat the caveat from subsection 6.5.4: this is a calibration of B'_{60} as a function of I_{60} , with the assumption that the other two pairs of coils have ‘steering fields.’ The currents required in these coils are $I_v = -0.28 I_{WS}$, $I_{GQ} = 1.48 I_{WS}$, which give fields of:

$$\left. \begin{aligned} B_v &= -0.20 B_{WS}, \\ B_{GQ} &= 14.7 B_{WS}. \end{aligned} \right\} \Rightarrow \mathbf{B}' = \{B'_{60}, 0, 0\}. \quad (6.14)$$

where we have used the measured vertical field calibration of $B_v = 3.68 I_v$ rather than the theoret-

ical, free-space field.

Let us compare these results to the low-field case. We note that, while the field required from the vertical coils is very similar to that in the low-field case (see figure 6.11 and surrounding text), the field required from the GQ coils is much higher than the field from the far conflat coils. This may be the result of increased shielding of these coils owing to their location on the corners of the can's hexagonal geometry.

6.6 Conclusions

We have designed and built magnetic field coils, and used these to manipulate the internal state of a single ion.

The ambient field in the laboratory is approximately 1 G with AC noise in a bandwidth less than 3 kHz of $\lesssim 0.8$ mG. Within a μ -metal enclosure the noise was reduced to $\lesssim 0.1$ mG.

In free space, measured with a Hall probe, the coils produced magnetic fields in accordance with expectations from the Biot-Savart law. The air-cooled coils had moderate heating at around 3–4 A, whereas the water cooling was effective in carrying all the heat away from the surface of the other coils. The water cooling on the high-field coils unfortunately broke irreparably at the start of *in situ* experiments.

Coils		Field/ G A^{-1}	Percentage of free-space theoretical
Air-cooled	Top & bottom $\times 2$	3.68	124%
	High-field $\times 2$	20.9	117%
Water-cooled	Close conflat $\times 2$	0.63	30%
	WS $\times 2$	1.8	39%

Table 6.7: Summary of field calibrations from dark resonances. Only B'_v and B'_{ax} were calibrated. The middle column shows dB'/dI where B' is the field at the ion, and I is the current in the relevant coil, in conditions where the other coils had current adjusted to make B' point along the desired direction.

Inside the vacuum can the strength of the magnetic field was changed by an amount which depended on the field direction. We were able to magnetize the can predictably using permanent magnets. The field was also deflected, primarily in the horizontal direction. We observed zero field in the can and were then able to increase the field strength along either the vertical axis or the axis at 60° to the cooling lasers. From observations of the behaviour of ion fluorescence we were able to calibrate field strengths for the vertical and close-conflat coils in all field regimes. These calibrations are summarized in table 6.7

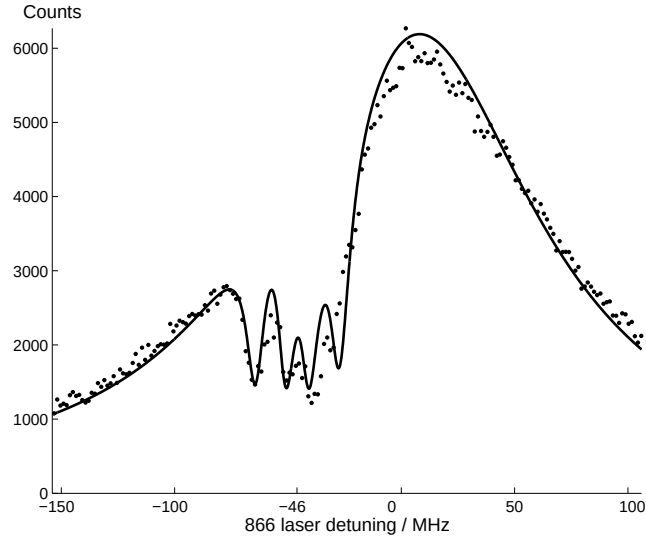
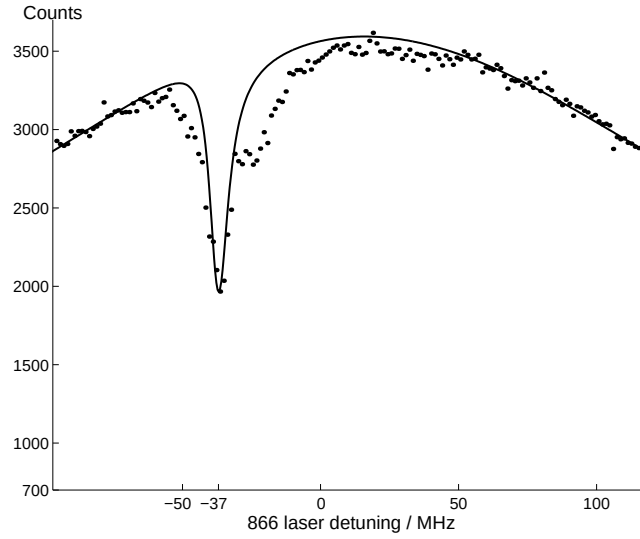
(a) $I_{60}=2.26$ A, $B'_{60}=6.0$ G(b) $I_{60}=-0.55$ A, $B'_{60}=0.2$ G

Figure 6.20: Examples of dark resonance fits with the intermediate-field coils. The WS coils are on the B'_{60} axis and their currents are shown in the captions. The vertical axis begins at the background count level, whilst on the horizontal axis is marked the fitted detuning of the 397 laser and the (arbitrary) centre of the 866 nm transition. Departures from the fit owing to dark resonance heating are discussed in the text.

Chapter 7

Saturated absorption and isotope-selective photoionization of Calcium I

7.1 Introduction

In this chapter we present spectroscopy of the 423 nm transition in neutral calcium, and photoionization of calcium atoms in the ion trap. We show that we are able to selectively trap many different isotopes of calcium.

We begin by discussing the principle of photoionization in the ion trap, and previous experiments in the field. We then crudely calibrate the 389 ionizing laser and the 423 neutral fluorescence lasers involved in the two-stage photoionization process that we decided to use. Because control of the 423 laser requires much greater precision we perform absorption and saturated absorption spectroscopy in a hollow-cathode cell.

Both lasers are then aligned into the trap and fluorescence on the 423 nm transition in calcium effusing from the oven is observed. We analyse the resultant spectra using a kinetic theory model. From this we optimize the crossed-beam geometry of the photoionization system. We make predictions about the ability of our system to discriminate fluorescence from different isotopes, and to trap isotopically pure ion crystals. Finally, we describe the demonstration of such isotopically pure ion trapping, and show a two-species ion crystal.

7.2 Background and previous experiments

7.2.1 Principle of two-stage photoionization

The lowest lying level of neutral calcium, $4s^2\ ^1S_0$, is 40304.80 cm^{-1} below the ionization continuum [144]. There are various ways to ionize the atom. The single-photon direct route corresponds to a threshold wavelength of 248 nm. Because the first (and only) transition is to the continuum, it is very broad and hence not species-selective. Alternatively, a three-stage process can be utilized whereby two 272 nm photons ionize the atom so that one still needs only a single laser. The first photon transfers population to the $4s5p\ ^1P_1$ level, from where it decays to the metastable $4s3d\ ^1D_2$ level. The second photon then links this state to the far continuum. Such UV wavelengths are

comparatively difficult to obtain. Instead we implement a two-stage process using readily-available blue laser diodes. This is similar to that used by Gulde et al [145].

The energy levels used are shown in figure 2.3. First we excite atoms using 423 nm light into the intermediate $4s4p\ ^1P_1$ level, which is 25653 cm^{-1} below the continuum. Experimentally we were able to saturate this transition, thus giving an upper-state population close to 50%. A second laser that broadly covers the 389 nm transition to the continuum ionizes atoms in the intermediate state. The first transition has a natural linewidth of 35 MHz and thus the 423 laser addressing it must have appropriate frequency control and stability; the second transition is very broad and it is sufficient for one of the several modes of the 389 ionizing laser [82] to be close to the transition.

7.2.2 Isotopic effects in Calcium I

7.2.2.1 Abundance

The natural abundances [73] of different isotopes of Ca are shown in figure 7.1. This diagram also shows the positions of the isotopic components in the $|^1S_0\rangle \leftrightarrow |^1P_1\rangle$ transition at 423 nm. The lowest-mass isotope, ^{40}Ca , dominates naturally-occurring calcium. The next abundant isotope is ^{44}Ca , comprising $\sim 2\%$ of the isotope mixture. We note that only 0.14% of the natural mixture is the isotope ^{43}Ca .

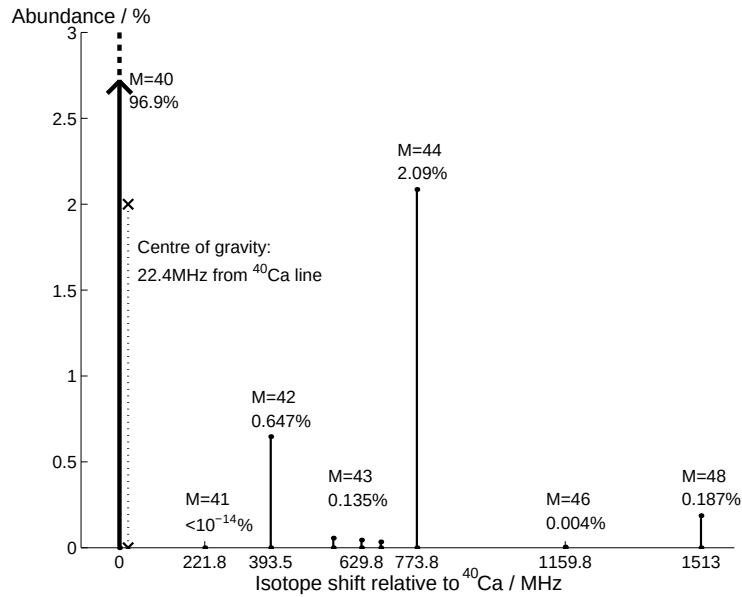


Figure 7.1: Abundances and isotope shifts for the known isotopes of calcium, for the 423 nm $|^1S_0\rangle \leftrightarrow |^1P_1\rangle$ transition. The shift of only the central component in the ^{43}Ca multiplet is given; the $F = \frac{9}{2}$ component is at 556.5 MHz, and the $F = \frac{5}{2}$ component is at 679.9 MHz.

One possible future direction for the Oxford ion trap group is to use hyperfine states in ^{43}Ca as qubit states [146]. Although samples enriched in ^{43}Ca are available, the Oxford ion trap is loaded from a source containing the natural mixture, because initial interest centred on the most abundant isotope. Even enriched samples only contain approximately 20% ^{43}Ca , so an isotopically selective method of ionization would still be necessary. Therefore one goal of the present investigation is to find whether it is feasible to ionize ^{43}Ca with sufficient selectivity to load it from a non-enriched

source.

7.2.2.2 Isotopic components and nuclear spin

The isotope shifts and splittings in the 423 nm transition in neutral calcium are shown in figure 7.1 for all radioactively stable isotopes. Other isotope shifts in neutral and singly-ionized calcium are tabulated elsewhere [147]. The even isotopes of calcium have even numbers of both protons and neutrons: hence no spin. Thus transitions in these isotopes undergo an isotope shift but do not split into hyperfine components [1]. The odd isotope, ^{43}Ca , has a nuclear spin $I = \frac{7}{2}$. Although the ground state has zero electron angular momentum, and hence no splitting, $|^1\text{P}_1\rangle$ has $J = 1$, yielding three hyperfine components with $F = \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$. All shifts are towards the continuum relative to ^{40}Ca i.e. the laser must be blue detuned to match them. Some references e.g. [145] use a different convention.

7.2.3 Earlier experiments

The spectra of the alkaline earth metals have been researched extensively. This data gives important information on parity nonconservation, nuclear charge distributions and electron-electron interactions and provides good tests of atomic many-body theory. In addition, the mass spectrometry that the isotope shifts permit has a wide range of applications in other fields, including radiochemical dating and medical tracer applications.

Early experiments concentrated on improving the spectroscopic apparatus, while the sample source was a hollow-cathode lamp or similar gas cell [148, 149]. Atomic beam experiments concentrated on mass spectrometry, especially for the rarest isotopes, and utilizing two-stage photoionization as a tool to deflect the detected species out of the atomic beam [150]. Most recently the research has concentrated on ion trap experiments. These have resulted in measurements of the previously unmeasured shifts [151] and the isotopic selectivity of the process has been used to load different isotopes controllably into RF traps [145, 152].

7.3 Preliminary experiments

7.3.1 Spectroscopy and modes of the 389 ionizing laser

The construction, operation and characterization of this laser is discussed more fully elsewhere [82]. We present a brief overview of the laser in section 2.5, and summarize details of its operation and spectroscopic properties here.

The laser is temperature-stabilized at 20 °C. Its threshold current is 49 mA. However, photoionization is sufficiently favourable that we run the laser well below threshold, in what is effectively a light-emitting diode mode at approximately 35 mA. At this current the power is partitioned between ~ 10 frequency modes, each separated by 59 GHz [153]. Above threshold the power is predominantly in two or three modes, but there is still 10–20% power in the other modes. This is similar to the behaviour of the 397 cooling laser discussed in subsection 3.4.5. One effect of this multimode behaviour was to confuse the wavemeter's acquisition system. The Burleigh wavemeter consists of an interferometer which counts fringes: if the laser consists of several equally-spaced modes then the beats fringe pattern will give errors in the fringe count.

7.3.2 Alignment and tuning of the 423 laser

Before the 423 neutral fluorescence laser was shipped to the Clarendon lab it had been tuned using its grating to 422.8 nm at the set temperature of 20 °C. The effect of transit was to shift its frequency to 422.6 nm. Retuning the laser using the grating was difficult. At certain current detunings this laser caused the Burleigh wavemeter to have difficulty measuring the frequency, as with the 389 laser. It is believed that the same effect—multimode operation of the laser—was behind the measurement difficulty. The vertical alignment of the grating is very sensitive to changes in the horizontal alignment, and optical feedback from elements further down the beam path may have caused the mode instability.

A combination of temperature and grating tuning was used, and the temperature was eventually set at 22 °C, with current at approximately 79 mA. Once this mode was found the laser could be switched off, then on again, and would return to the same mode.

7.3.3 Calibration of the 423 laser tuning

The frequency tuning of the 423 laser was initially calibrated as follows. The laser was aligned into the blue spectrum analyser described in section 2.6.3. The piezo on the spectrum analyser was not scanned; instead the laser frequency was scanned using its own piezo. In order to minimize the effect of etalon drift the laser frequency was scanned at 5–10 Hz. The laser’s control unit provided a feed-forward to the current to ensure internal and external cavity modes moved synchronously.

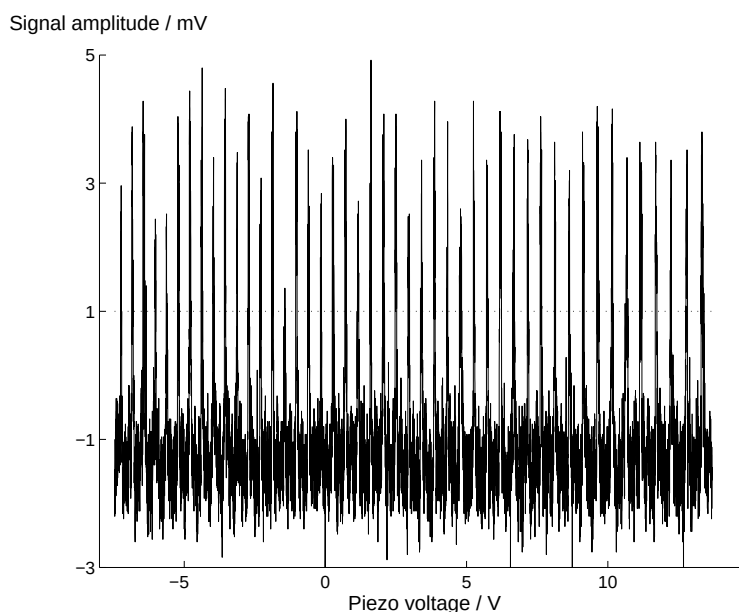


Figure 7.2: Transmission of 423 laser power through an etalon as the laser piezo voltage is scanned. We plot the signal on the photodiode against the voltage on the grating piezo. The data above the dotted line is used for centre-of gravity fitting to the transmission peaks. See text for more details.

As the laser was scanned, its transmission through this etalon varied as an Airy function convolved with the laser’s spectrum [154]. The analyser’s photodiode recorded the power transmitted at different values of the piezo voltage. The data are shown in figure 7.2. Given that the etalon’s fsr is 300 MHz, then by counting the number of orders (approximately forty-six) we can estimate that

the laser can be scanned phase-continuously over approximately 14 GHz i.e. without jumping to a new mode.

For a more accurate calibration we fit a piezo calibration curve to the centre of gravity (cog) of each peak. We only use intensity measurements above a certain threshold in the cog calculation: this threshold is shown in the figure. Thus the noisy background has a minimum influence on the peak centre. If, in calculating the position of the n th order, we only use intensity data $I_n(V)$ for piezo voltages $V_n^- < V < V_n^+$ then the cog piezo voltage for this order is

$$V_{\text{cog}} = \frac{\sum_{V=V_n^-}^{V_n^+} V I_n(V)}{\sum_{V=V_n^-}^{V_n^+} I_n(V)}, \quad (7.1)$$

This is a simplification of the principle of local M-estimates [141]. We then assume that each order corresponds to a frequency which is a multiple of the fsr of the etalon. This conversion yields the graph in figure 7.3. A quadratic fit

$$f = 0.6879V - (4.858 \cdot 10^{-3})V^2 \quad (7.2)$$

(f in GHz, V in volts) is shown¹ along with the departure from this fit if we drop the quadratic term. This is shown preferentially to a straight line fit because frequency calibration over short ranges was performed using just the first term in equation (7.2).

Later, during photoionization experiments, it was observed that the isotope shifts deduced from the above calibration were no longer close to the theoretical values. We therefore recalibrated the piezo using the wavemeter. This calibration is less accurate, owing to the limit in accuracy of the wavemeter reading. A graph of the calibration is shown in figure 7.4. Because the 423 laser was now controlled by an analogue output from the PC, only voltages in the range 0–5V were used. The fitted dependence of frequency on voltage is

$$f_{\text{quad}} = 0.236V + 9.09 \cdot 10^{-3}V^2, \quad (7.3)$$

$$f_{\text{lin}} = 0.286V. \quad (7.4)$$

These are the quadratic and linear fits respectively. For small frequency excursions we use the second fit. This calibration change is confirmed by the isotope shifts measured in experiments conducted at different times, as we shall see later. A possible explanation for the change in calibration is that the laser was run at a different current mode for the later experiments, although it is surprising that this caused a factor of > 2 change in the piezo calibration. The quadratic term has a different sign in this case, possibly because of the large errors in the data points: we generally therefore only use the linear fit. We shall see in figure 7.20 the effect of this.

7.3.4 Problems with the digital oscilloscopes

We use the Tektronix TDS 3014B and TDS 5104B digital phosphor oscilloscopes. These are high bandwidth and can record for a long period of time (100 seconds). They are therefore ideal for the slow scans required of the phase sensitive detection method described later.

However, the following issue was noted with the DPOs. When a trace has been recorded and held by the DPO, it has some quantization noise. Although flipping between two adjacent bins is to

¹The frequency offset in this fit is arbitrary. This is feasible because the data is centred round $V = 0$.

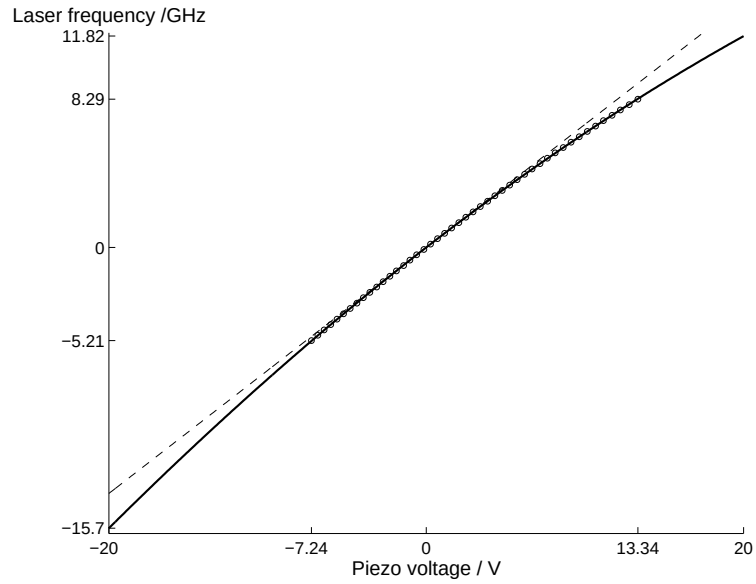


Figure 7.3: Fit to the centres of the lines in the 423 laser's transmission spectrum. Two fits are shown: the quadratic fit which we typically use (hence in bold); and the effect of ignoring the quadratic term (dashed line).

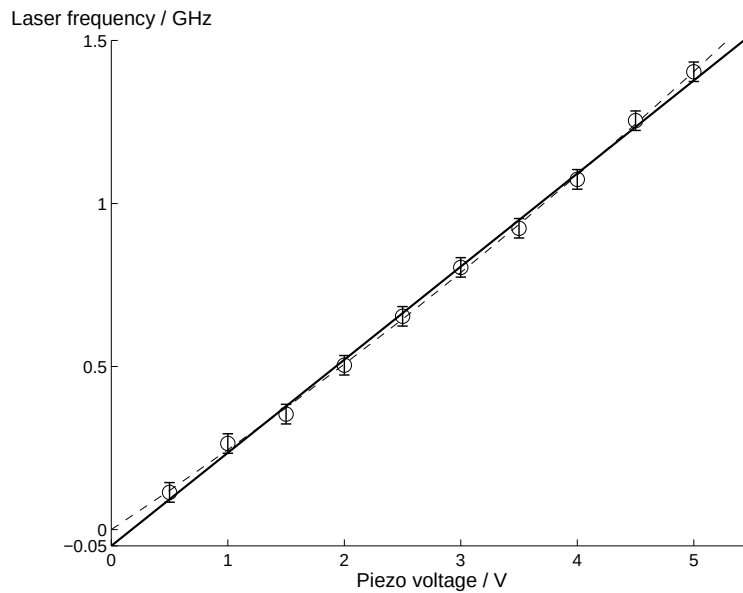


Figure 7.4: Fit to the wavemeter reading of the 423 laser frequency. This was a later calibration of the laser piezo, which differs by over a factor of two from the earlier results. We show the linear fit (bold) and an independent quadratic fit (dashed).

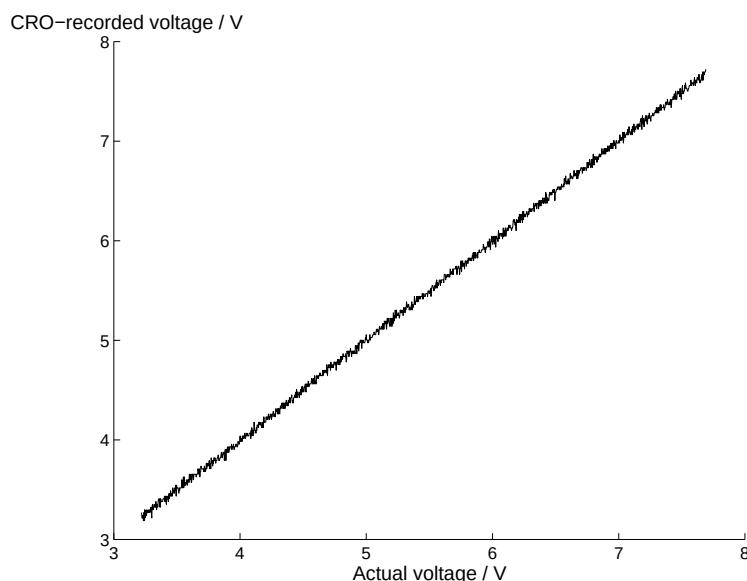


Figure 7.5: Comparison of voltage recorded by the DPO with voltage recorded by a source without observable jitter.

be expected, the oscilloscopes show a greater level of noise than that. This is illustrated in figure 7.5. A linear ramping voltage is measured. Measurements with other devices verified that the noise on this supply was much less than 10 mV. However, the DPO trace in the figure shows quantization noise extending over perhaps seven bins. This noise was periodic with both the time and signal quantizations of the oscilloscopes and was thus believed to be an artefact of the sampling procedure of the scope. Although averaging over many traces removed the effect, this was undesirable for e.g. PSD scans, where the 423 laser will drift strongly over the course of e.g. 16×100 s.

This noise is undesirable but it was felt that, for traces which have inherent noise of a similar order of magnitude or greater, then it would not be a problem. For traces where we have knowledge of the underlying signal e.g. triangular-wave voltages, then we have substituted e.g. a straight-line fit for the noisy data.

7.4 Saturated absorption spectroscopy of the 423 nm transition

7.4.1 Introduction

We investigate the 423 nm transition in a hollow-cathode lamp before beginning photoionization experiments. The purpose of this was to understand the interaction of our 423 neutral fluorescence laser with atomic calcium. It is useful to determine beforehand whether our photoionization scheme is feasible. We therefore give the details of experiments which require progressively greater control over the 423 laser.

Firstly the laser was aligned once through the glass cell containing calcium vapour. The 423 laser was capable of being scanned repeatably over the inhomogeneously broadened (2 GHz) absorption resonance. Qualitative differences in the absorption signal were seen as the current was varied in the lamp. The laser was then used in a Doppler-free experiment. This placed more stringent requirements on the laser behaviour. This experiment yielded rudimentary mass spectrometry of calcium: a small peak corresponding to the isotope ^{44}Ca was observed at the side of the main

peak from ^{42}Ca . The broadening of the saturated absorption signal is consistent with predictions from theory.

7.4.2 Absorption signal

7.4.2.1 Optical thickness broadening

We assume that the laser emits light at a monochromatic frequency ω . The intensity transmitted by a lamp with a discharge region length l_0 is I where²

$$-\ln(I/I_0) = l_0 \sigma_{ik}(\nu) \langle N_i - N_k g_i / g_k \rangle_l, \quad (7.5)$$

where we assume the populations of the atomic states to be independent of the intensity of the laser. I_0 is the incident radiation, and the population difference between the ground state $|i\rangle \equiv |^1\text{S}_0\rangle$ and the excited state $|k\rangle \equiv |^1\text{P}_1\rangle$ is averaged over the distance l of the discharge region. The cross-section σ_{ik} is related to the atomic response function by [155]

$$\begin{aligned} \sigma_{ik}(\nu) &= \frac{c^2}{4\nu_0^2} A_{ki} \int_{-\infty}^{\infty} \mathcal{L}(\nu') \mathcal{G}(\nu - \nu') d\nu' \\ \text{where } \int_{-\infty}^{\infty} \mathcal{L}(\nu) d\nu &= \int_{-\infty}^{\infty} \mathcal{G}(\nu) d\nu = 1, \end{aligned} \quad (7.6)$$

the centre of the atomic transition is ν_0 , and A_{ki} is the Einstein A coefficient for the transition. The functions $\mathcal{L}$ and $\mathcal{G}$ are defined in appendices A.4.1 and A.4.2 respectively. σ_{ik} is proportional to the Voigt profile $\mathcal{V} \equiv \mathcal{L} \star \mathcal{G}$. We take logarithms of the normalized transmission data to obtain data in the form of equation (7.5). Because the hollow-cathode gas is so hot, we can assume as a first approximation that there is no Lorentzian linewidth. In this case equation (7.5) becomes

$$-\ln(I/I_0) = l_0 \langle N_i - N_k g_i / g_k \rangle_l \frac{c^2}{4\nu_0^2} A_{ki} \mathcal{G}(\nu). \quad (7.7)$$

7.4.2.2 Absorption signals

The 423 laser was aligned through the Hamamatsu galvatron (hollow-cathode lamp) discussed in section 2.5. We illuminated the whole of the discharge region within the hollow-cathode lamp. The fibre's output collimator was slightly displaced axially from its optimum position, and this resulted in a diverging emergent beam. This was approximately collimated using a 50 mm lens after the output. The beam was large enough for the illumination of the hollow cathode to be approximately uniform. The input beam was nearly Gaussian, as shown in figure 7.11; however, the cathode clipped the beam, yielding a non-Gaussian exit cross-section. The discharge region had diameter $d_0 \sim 5$ mm and length $l_0 \sim 10$ mm.

A typical scan is shown in figure 7.6. The piezo was scanned using a triangle-wave control voltage. We observed a linear variation in laser power with the lamp switched off; this was a result of the feed-forward from the piezo control circuit to the laser current. As the 423 laser frequency was scanned across the Ca resonance (with the lamp on) the intensity of the transmitted beam was seen to drop by approximately 50% compared to the signal with lamp off, whilst the discharge region glowed violet. Fitting to the logarithm of the normalized data gave a FWHM of 2.2 GHz, consistent with both earlier experiments [76] and later data. This corresponds to a gas temperature

²Here we follow the notation of section 4.3.

of 750 K. However, because of the scan speed, hysteresis was observed and the width of the same feature, with the piezo scanning in the opposite direction, differed by approximately 10%.

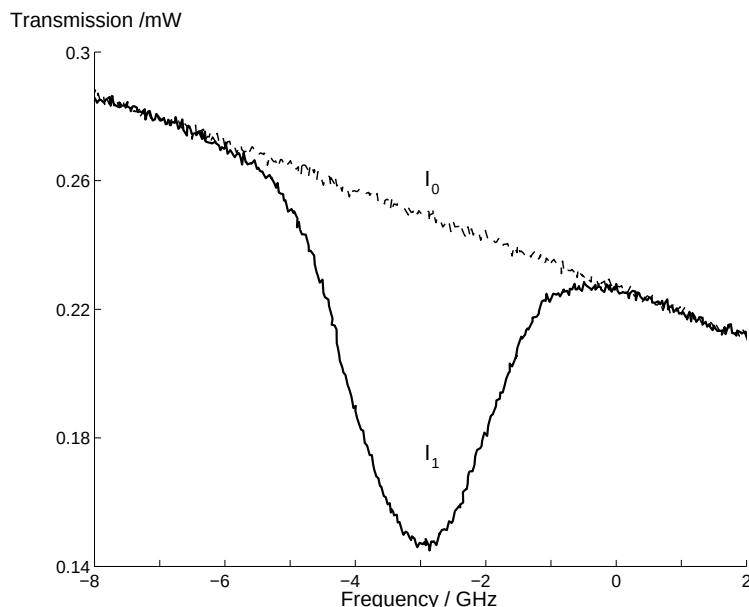


Figure 7.6: A typical absorption feature. The graph shows the transmission of laser power (corrected for background light) both with (I_1) and without (I_0) a current passing through the lamp.

We varied the current in the galvatron. At higher currents the buffer gas is ionized more quickly and hence the concentration of vaporized calcium increases. This deepens the absorption feature. We see this in figure 7.7. Here we plot the normalized logarithm data $-\ln(I/I_0)$, with the background subtracted. The data in figure 7.7 are shown with Gaussian fits.

In order to obtain more quantitative data—in the light of the hysteresis observed so far—the frequency scale of further scans was calibrated using the blue spectrum analyser during the scan [86]. Each scan takes of order a minute. A cubic curve was fit through the centres of gravity of the etalon transmission peaks. The fits to the data were found to be strongly dependent on the assumed background light levels, both with the lamp off and the lamp on. With these levels fixed at the measured values we found that the inhomogeneous width of the absorption feature varied approximately linearly from 1.63 GHz ($T = 412$ K) at a lamp current of 5 mA, to 2.77 GHz ($T = 1190$ K) at 20 mA. Most importantly, by permitting the background light levels to vary in the fitting procedure, we obtained upper limits on the homogeneous component of the absorption feature. Using the trace at 20 mA we fitted a Voigt profile to the data, and minimized the cost function χ^2 to obtain an optimum Lorentzian width of $\sim 50(50)$ MHz and Gaussian width of $\sim 2.0(1)$ GHz. This limits the homogeneous broadening, owing to collisional effects and spontaneous decay, that we expect to see in the saturated absorption traces.

7.4.3 Saturated absorption spectroscopy

Saturated absorption spectroscopy is based on the selective saturation of an inhomogeneously broadened transition [156]. An intense pump beam excites a large fraction of the population of a certain velocity class of atoms, effectively removing the fraction from the scattering state. The velocity class is such that the Doppler shift the atoms experience tunes the pump beam frequency

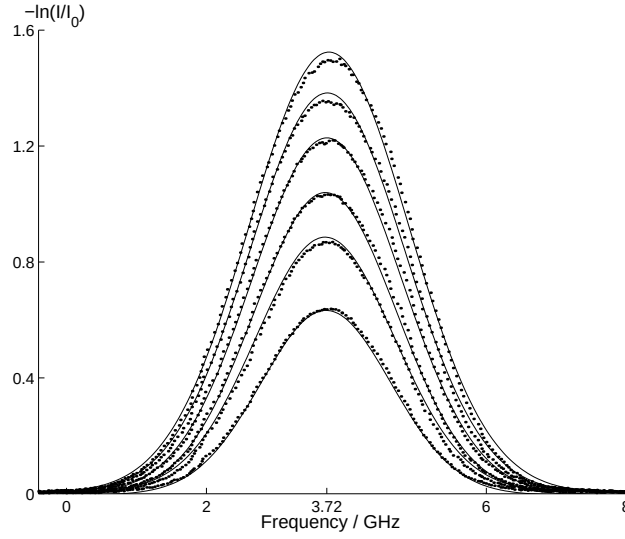


Figure 7.7: Variation in the absorption signal with lamp current, plotted logarithmically to show underlying Voigt dependence. From bottom to top the current in the lamp was 10,12,14,16,18,20 mA, and Gaussian fits (-) are shown through the data (•) for each current. At high currents almost 80% (i.e. $1 - e^{-1.6}$) of the laser power is absorbed.

to resonance. A counter-propagating probe beam then interrogates the atoms. If the probe beam is also in resonance with the excited velocity class, then it is not as attenuated as it would otherwise be. This can happen for one of two reasons:

1. Both counter-propagating beams are resonant with the same transition frequency ν_0 . The two Doppler shifts are equal: $-\nu_0 v/c = +\nu_0 v/c = 0$. The velocity class that is excited is therefore the stationary i.e. zero-velocity class.
2. One beam is shifted to be resonant with one transition frequency ν_0 , whereas the other is shifted to be resonant with a different transition frequency $\nu_0 + \delta\nu_0$. The Doppler shift is $\nu_0 v/c = \frac{1}{2}\delta\nu_0$. This crossover resonance only occurs if both transitions have the same lower (scattering) state i.e. such resonances will only be present within the ^{43}Ca spectrum.

One therefore has a much narrower profile subtracted from the wider, normal absorption profile. Ideally this narrower profile has no inhomogeneous broadening, although we shall see later that the profile shape is more complicated than this. Components of the transition separated by more than the homogeneous linewidth can be distinguished and analysed.

7.4.3.1 Preparation of pump and probe beams

The pump beam must be moderately intense in the active region i.e. within the hollow cathode. The more intense this beam, the greater the saturation of the velocity class with which it interacts, and the lesser the population remaining to scatter the probe beam. The saturation intensity on the 423 nm transition is, from equation (4.19):

$$I_s = 3.6 \text{ mW mm}^{-2}. \quad (7.8)$$

The alignment into the fibre was such that only approximately 1–2 mW of power emerged from the output. A Gaussian beam³ of spot size w and power P has a peak intensity of $2P/(\pi w^2)$. The saturation power P_s for a 1 mm×1 mm beam is 5.7 mW. Hence we must focus the beam down to much less than a millimetre in diameter to obtain many saturation intensities at the centre of the beam.

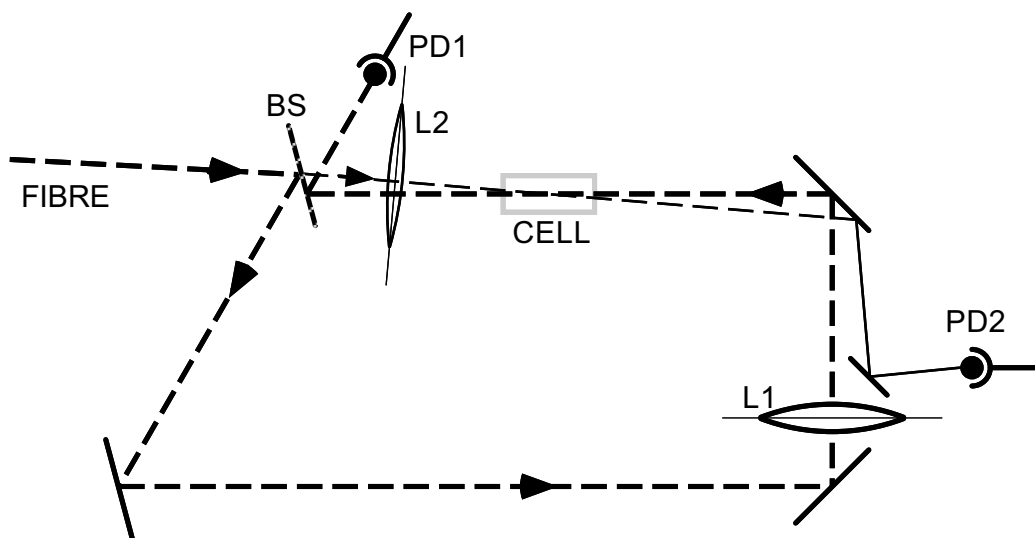


Figure 7.8: Schematic of optics in saturated absorption experiment. From the fibre, a beam splitter BS reflects most of the power into a pump beam which is focussed using the lens L1 into the hollow cathode cell. After the cell this beam is directed onto a photodiode PD1. The weaker probe beam is transmitted through BS and focussed using the lens L2 to a waist in the cell. The emergent probe beam is directed onto the photodiode PD2.

The fibre collimator head was mounted on a tiltable Thorlabs mount, and the beam focussed down into the cell. The optics after the fibre and collimating lens are shown schematically in figure 7.8. A beam splitter BS reflected the more intense pump beam. This beam was focussed down and its waist was approximately at the centre of the hollow-cathode cell. Because of the intensity of this beam, the Pulnix camera saturated even with the laser well below threshold, so the laser was run at < 1 mA during alignment. The probe beam was also focussed down and passed through the cell in the opposite direction, at an angle of approximately 0.05 rad to the pump beam.

7.4.3.2 Shape of the saturated absorption signal

The lineshape of a saturated absorption signal is modified from the ideal homogeneously-broadened Lorentzian. At line centre the pump and probe beams interact with the same velocity class of atoms, and the line is broadened homogeneously. However, consider the following. The pump and probe are detuned from the line centre and interact with different velocity classes. Velocity-changing collisions with calcium and the neon buffer gas are continuously transferring atoms between all velocity classes. A significant fraction of the class excited by the pump beam will be moved to resonance with the probe beam: in the excited, transparent state, if the collisions occur on a timescale comparable with the radiative decay of the upper level. The presence of the pump beam thus reduces the absorption even of atoms not in resonance with it. This leads to a ‘pedestal’ or a broad

³See appendix A.1.

background underneath the sharply peaked saturated absorption signal. This pedestal will be inhomogeneous but narrower than the absorption feature and also not Gaussian. This is because the perturbed calcium atoms do not fully rethermalize before they spontaneously decay.

Let us consider the shape of the sharp peak. A particular velocity class experiences a Doppler shift of $\delta\nu_D$ in its rest frame. In appendix B.2 we solve the Bloch equations for the transition. The population inversion $\delta\rho$ on the transition is a Lorentzian in $\delta\nu_L + \delta\nu_D$, where $\delta\nu_L$ is the laser's detuning from the transition centre. The width of this Lorentzian is given by equation (B.20). This is the 'hole' burnt in the population by the pump laser. The probe is detuned from the velocity class under consideration by $\delta\nu_L - \delta\nu_D$. In order to determine the total absorption of the probe we integrate over the whole velocity distribution, with the assumption that the Doppler width is much greater than the homogeneous width (i.e. the velocity distribution is flat over the homogeneous linewidth). Hence the absorption coefficient for the probe can be expressed as a convolution of the population inversion $\delta\rho$ with the atomic response function

$$\alpha \propto \int \delta\rho \left(\frac{1}{(\delta\nu_D - \delta\nu_L)^2 + \beta^2} \right) d(\delta\nu_D), \quad (7.9)$$

where $2\beta = \Gamma_c + \Gamma_{5p} + \Gamma_{\text{las}}$ is the sum of all homogeneous widths. The convolution of two Lorentzians is itself a Lorentzian. However, as we scan the probe beam then we also scan the pump beam in the opposite direction. This has the effect of halving the width of the resultant convolution. The profile therefore has linewidth

$$\Gamma'' = \beta + \beta \sqrt{1 + 3 \frac{\Gamma_{5p}}{\beta} \frac{I}{I_s} \frac{\Gamma_{5p} + 2\gamma}{\Gamma_{5p} + 3\gamma}}. \quad (7.10)$$

I_s is the saturation intensity on the transition, as defined in equation (4.19). This expression is only valid for weak probe beams.

We can estimate the collisional linewidth Γ_c in the lamp as follows. The majority species in the lamp is neon, and collisions between the comparatively rare calcium atoms can be ignored. At 862 K the collisional broadening per unit neon density, on the 423 nm transition, is $14.9 \cdot 10^{-24} \text{ mK m}^3$ [157]. At a constant density broadening decreases with temperature, approximately proportional to $T^{0.3}$ for van der Waals forces. We expect therefore:

$$T = 400 \text{ K}, \Gamma_c \approx 50 \text{ MHz}$$

$$T = 600 \text{ K}, \Gamma_c \approx 38 \text{ MHz}.$$

i.e. at 600 K

$$2\beta \approx 35 \text{ MHz} + 38 \text{ MHz} + \Gamma_{\text{las}} \gtrsim 73 \text{ MHz}.$$

We relate Γ_c to the population transfer rate in the optical Bloch equations 2γ as follows. Collisional broadening is a result of all perturbations to the phase of the dressed atoms. Even very distant collisions will cause this phase to be interrupted. However, in order to flip the atomic orientation (i.e. redistribute population among the M_J levels) a much closer collision is required, to provide a stronger perturbation. The collisions comprising 2γ are therefore a subset of those comprising Γ_c ; for our purposes

$$2\gamma \ll \Gamma_c. \quad (7.11)$$

The power broadening is more difficult to quantify. We can determine the peak intensity of the pump beam from its cross-section. However, the intensity varies over its width, and the two beams are only superimposed for a short length of the cathode cell. We therefore expect the average saturation broadening to be much less extreme than that at the centre of the pump beam.

A final source of broadening of the sharp peak is inhomogeneous. Because the pump and probe beams counter-propagate at a small angle θ to each other, then they do not sample just the stationary velocity class. The signal is therefore spread over a Doppler distribution corresponding to the component of velocity along the propagation directions i.e. it will have a Doppler width scaled by $\sin(\theta/2)$. At 20 mA in the hollow-cathode cell the Doppler width of the absorption feature is ~ 2 GHz; if the beams are at an angle of 0.05 rad then the Doppler broadening of the saturated feature will be ~ 50 MHz.

We do not analyse the pedestal, although we note that a Gaussian function fits it well. The pedestal only gives us information on the rethermalization rates of the gas, and will not yield any information about the 423 laser not present in the fits to the main peak.

7.4.3.3 Observation using phase-sensitive detection

In order to improve our data, and observe signatures from the other isotopes of calcium, we used the technique of phase-sensitive detection [158]. The laser frequency is swept by a slow triangle wave of frequency ν_{sc} . The pump intensity is modulated at a much higher frequency, ν_{psd} and the signal in the probe transmission at this frequency is detected by a lock-in amplifier. This includes a low-pass filter (with 3 dB width ν_{fi}).

We installed a chopper wheel to modulate the pump beam with 50% duty cycle. The pump intensity was recorded by the photodiode and normalized into a sequence of TTL reference pulses. A Thorlabs LIA100 lock-in amplifier produced a PSD signal from the reference pulses and the probe signal. The laser frequency was scanned using a 0.01 Hz triangle wave, and the piezo voltage and PSD signal were acquired using the DPOs, although acquisition was sometimes done using the PC in order to check the DPO's behaviour. We require

$$\nu_{sc} \ll \nu_{fi} \ll \nu_{psd}. \quad (7.12)$$

In practice we have

$$\nu_{sc} = 0.01 \text{ Hz}, \nu_{fi} = 63 \text{ Hz}, \nu_{psd} = 500 \text{ Hz}. \quad (7.13)$$

The signal from the PSD circuit is shown in figure 7.9. It is plotted against frequency, calibrated using equation (7.2) and including the quadratic term. The data shows the saturated absorption feature of the calcium isotopes in the lamp. For this scan the peak intensity of the pump beam was approximately $70I_s$ in a spot size of $62 \times 66 \mu\text{m}$, from cross-section and power measurements. The ^{40}Ca feature has been fitted with a Lorentzian and a Gaussian pedestal, neglecting the data corresponding to the small peak of ^{44}Ca . The fit parameters are given in table 7.1. The Gaussian pedestal is less broad than the normal absorption signal, owing to incomplete thermalization. The Lorentzian fit to the peak (strictly a Voigt profile) includes approximately 30 MHz from the 50 MHz inhomogeneous component to the linewidth.

It was not possible to resolve the ^{42}Ca isotope, owing to the broad Gaussian pedestals of other peaks. However, ^{44}Ca was observed in the residuals and we plot the difference between data and the ^{40}Ca fit in figure 7.10. The dotted line is the result of translating the ^{40}Ca curve by the expected

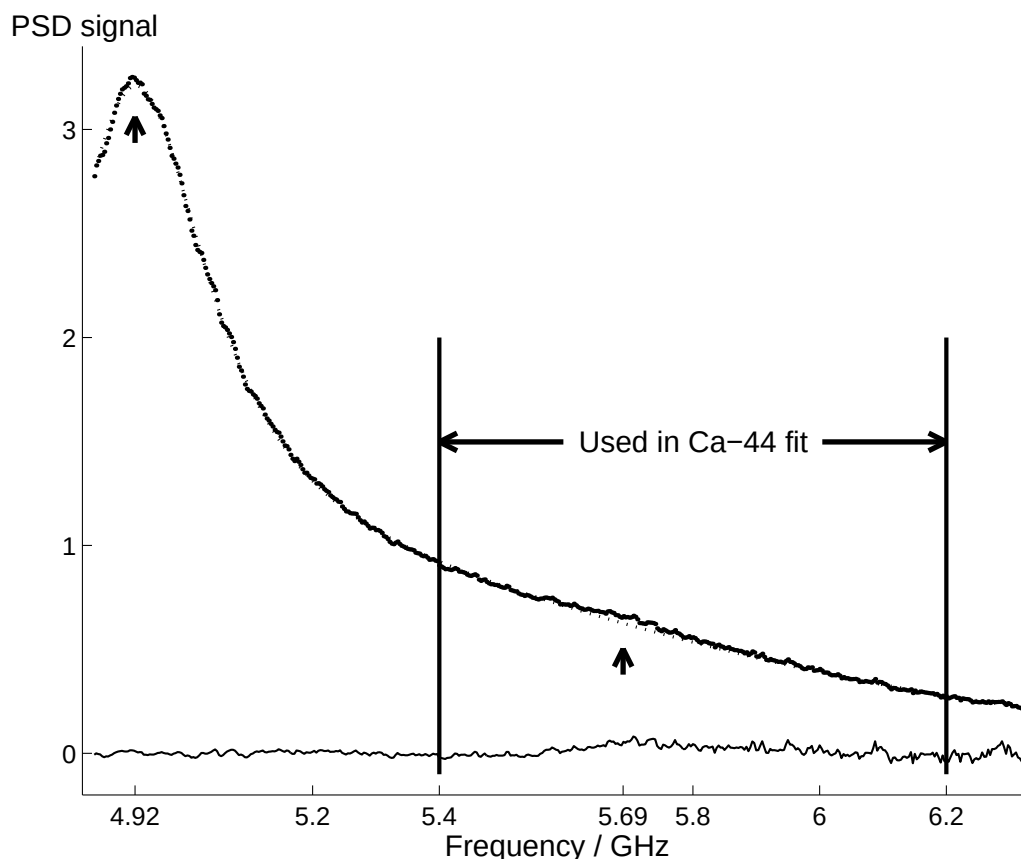


Figure 7.9: Saturated absorption spectrum of the calcium vapour, obtained using phase-sensitive detection. This was fitted with a Lorentzian on a Gaussian ‘pedestal’ (dotted line). The data corresponding to the peak of ^{44}Ca was ignored during the fit. The residuals are displayed below the fit, and the peak centre (and expected position of the ^{44}Ca peak) are indicated on the frequency axis and with arrows.

isotope shift (774 MHz) and reducing its height by the relative abundance of the isotope. We attempted to float this pedestal fit, but the noise on the data made this impossible. Instead we used a Lorentzian with a linear baseline. The result of this fit is the solid line, and the fit parameters are also in table 7.1.

The observed value of the isotope shift is 800(40) MHz. This is consistent with the known value of 774 MHz. The measured abundance ratio 0.018 is also consistent with the known value. The width of the Lorentzian gives information about the collisional and power broadening; however, these are not independent. We therefore reduced the intensity of the pump beam. Also we scanned the laser frequency while observing the signal on the spectrum-analyser etalon to ensure that there was no hysteresis in the frequency scan. It was not necessary to use phase-sensitive detection for these scans.

Again, we only fit Lorentzians to the results. They provide a satisfactory fit, within the signal-to-noise of the data. The data was taken at a lamp current of 20 mA, and a peak pump intensity of several hundred saturation intensities. Despite such a high peak intensity the mean linewidth of the fitted Lorentzians was 119(6) MHz. We believe that, as before, the low power broadening is explained by the low cross-over of the two beams: for these experiments the spot sizes of the two beams were reduced to $\sim 20\ \mu\text{m}$. Halving the intensity decreased the linewidth to 102(11) MHz.

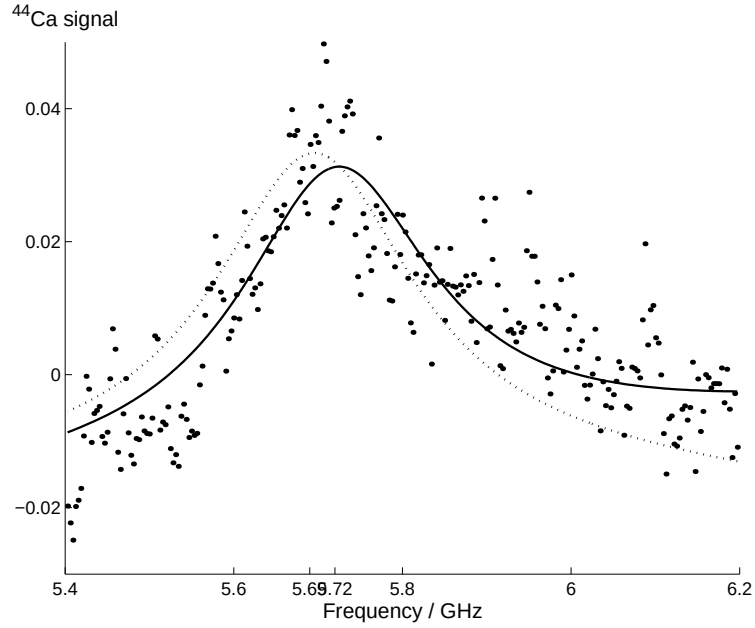


Figure 7.10: Fitting to the ^{44}Ca data. The solid line is a Lorentzian with a linear baseline, all parameters floated. The dotted line is an unfitted translation of the ^{40}Ca peak by the isotope shift, with its height modified to take into account the difference in isotope abundances.

Parameter	^{40}Ca	^{44}Ca
Peak width $\Gamma_{\mathcal{L}}/\text{MHz}$	284	278
Pedestal width $\Gamma_{\mathcal{G}}/\text{MHz}$	1.79	—
Centre frequency ν_0/GHz	4.92	5.72
Isotope shift / MHz	800(40)	
Abundance ratio	0.018	

Table 7.1: Comparison of the results from fitting to the ^{40}Ca and ^{44}Ca PSD data. The fit to the ^{44}Ca peak did not include a Gaussian pedestal

Extrapolating to zero laser intensity it is clear that the total linewidth is < 100 MHz. Let us consider what homogeneous and inhomogeneous linewidths give a Voigt profile with this linewidth. By trial and error we find that a 100 MHz-wide Lorentzian fit is consistent with a Voigt of 80 MHz Lorentzian width and 50 MHz Gaussian width. These are reasonable values and do not suggest any further line-broadening mechanisms are present.

7.4.4 Summary

Using absorption and saturated absorption spectroscopy we have shown that it is possible for the 423 laser to be tuned over the corresponding transition in neutral calcium. We have obtained data consistent with the known isotope shift and abundance of ^{44}Ca relative to ^{40}Ca . By optimizing the saturated absorption system we found that the linewidth of the saturated absorption feature is consistent with a slightly elevated homogeneous linewidth (80 MHz as opposed to 73 MHz) convolved with the expected 50 MHz inhomogeneous broadening from residual Doppler effects. This is consistent with the homogeneous broadening of the normal absorption signals. The extra homogeneous

linewidth can be explained by laser linewidth, as we shall show later in figure 7.16.

7.5 Implementation of the trapping of photoionized calcium

7.5.1 Introduction

In this section we discuss the selective ionizing and trapping of different isotopes of calcium. First we introduce the two photoionizing lasers—423 and 389—into the trap through the vertical conflat using the fibre from the 423 laser. We observe strong reflection from the conflat into the imaging system, dependent on laser alignment. We observe fluorescence on the 423 nm transition at different oven currents.

From preliminary scans of the 423 laser frequency over the transition we are able to obtain a measure of the relative isotope shift of ^{42}Ca . In addition we fit to the asymmetric inhomogeneous broadening using a kinetic theory model, which implies that the lasers and atomic beam are not orthogonal. We use the results of these fits to optimize the crossed-beam geometry and obtain high-resolution scans across the transition. This yields our first observation of ^{43}Ca in the ion trap.

We predict the isotope selectivity of the ion trap under different geometries, and observe the success of selectivity so far. We give data of multi-species crystals in the ion trap, produced using the new photoionization method.

7.5.2 Fibre alignment and transmission

7.5.2.1 Alignment into the fibre of the 389 ionizing laser

The 423 laser was prealigned by the manufacturer into an optical fibre (details in section 2.5). The transmission was around 10–15%, instead of the quoted $\sim 30\%$. We improved this using translation and tilting of the 423 beam, finally yielding a transmission of 28% of the 5.4 mW input power. The poor transverse quality of the beams from the Nichia/Toptica lasers—although they have improved since the earlier 39x nm diodes [37]—restricted the maximum coupling into the fibre. The fibre input was placed approximately half a metre from the laser. Thus we were able to superpose the 389 beam on the 423 beam in between the laser box and fibre input, as follows.

With a 1 m lens the 389 laser beam has a horizontal waist of $107\ \mu\text{m}$ and vertical waist of $186\ \mu\text{m}$ with negligible astigmatism. The fibre is specified for a waist $170\ \mu\text{m}$ at 1.2 m from its end. This translates to a spot size at the fibre of $900\ \mu\text{m}$ and a radius of curvature of 1.25 m. We concentrated on fine control of the curvature of the beam rather than the spot size, as the overlap of any laser mode with the fibre’s waveguide mode is high for spot sizes different by even a factor of two from the optimum.

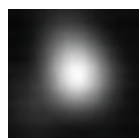


Figure 7.11: Cross-section of the 423 laser beam after the fibre. This was taken with the Pulnix CCD camera. The unfocused beam fits convincingly to a Gaussian, implying a good output mode from the fibre. The image is approximately $830\ \mu\text{m}$ wide, although the beam was resized for different experiments.

Two lenses, $f50\ \text{mm}$ and $f300\ \text{mm}$, were used as a beam expander with 290 mm between them. Calculations of Gaussian beam propagation imply a spot size in the vertical of $913\ \mu\text{m}$ and a ra-

dius of curvature of 1.24 m. By tilting the $f300$ mm lens we can increase the focal length in the horizontal plane to 315 mm, as required to give the same radius of curvature but with a spot size of approximately $600\text{ }\mu\text{m}$. Optimization of the tilt of this lens resulted in approximately 10% transmission of the 1.7 mW input power. After the fibre the beam has a cross-section which overlaps well with a Gaussian, as shown in figure 7.11.

7.5.2.2 Alignment into trap

The photoionization lasers were initially aligned along the path of a beam known to intersect with the trapping region: the fibre output beams were directed along the vertical beam given in figure 2.7. We discuss in subsection 7.5.4.1 how this affects the spectral profiles observed.

The fibre collimator produced a diverging beam. A $f50$ mm lens was positioned 48 mm after the fibre. Chromatic aberration resulted in a 423 spot width of $40\text{ }\mu\text{m}$ at 320 mm from the lens, and a 389 spot width of $36\text{ }\mu\text{m}$ at 305 mm from the lens. At the focal point of the 389 laser beam, the 423 beam had width $\sim 80\text{ }\mu\text{m}$. This corresponds to a peak intensity per unit power in the beam of $\sim 99.4\text{ mW mm}^{-2}/\text{mW}$, or $28I_s/\text{mW}$. A camera placed approximately at a position optically equivalent to the trap recorded spot sizes of $\sim 100\text{ }\mu\text{m}$ in both beams⁴. The differences between the spot sizes and the estimated waists are probably due to the divergence of such tightly-focussed beams over short distances.

7.5.2.3 Light scattering into the imaging system

When the 423 and 389 lasers are on, there is a reflection from the top conflat window. As with the 397 cooling laser we attempted to block this using a small ‘flag’: a piece of metal foil on a thin support, painted black and placed over the primary reflection. The flag reduced the background laser light on the PMT considerably.

The imaging system above the trap contains a removable beam splitter which, when in the optical path to the PMT, directs a fraction of the light to the camera. Around 70% of ionic fluorescence was diverted by the beam splitter. Ambient light, e.g. from room lights, scattered into the system was blocked more fully by the beam splitter: approximately 90% of the light was either deflected to the camera or absorbed by the mount (the beam splitter is small and we surmise that, although focussed fluorescence easily passed through the aperture, the unfocussed background light did not).

7.5.3 Observation of temperature changes in the oven

When the current in the calcium oven is turned on, the oven begins to warm up. As it does so, the vapour pressure and calcium gas density increase, and the mean speed of the calcium atoms in the vapour increases. These two effects increase the rate of effusion from the oven. We see in figure 7.12 evidence of a strong dependence of atom density on oven temperature. Five observations were performed at different oven currents. At the beginning of each observation all lasers are blocked and the oven is off. The sudden rise at a time of ~ 15 s indicates switching on of both the oven and the 423 laser. After a further ~ 100 s the 423 nm fluorescence is rising rapidly, but begins to level off towards the end. Eventually the oven and laser light are switched off (the sharp drop to almost zero in the data), and then the laser light switched back on to show the rapid decrease in atom density as the oven cools down. We note

⁴It was calculated that dispersion in the glass conflat of the vacuum can lead to negligible displacement between the two beams.

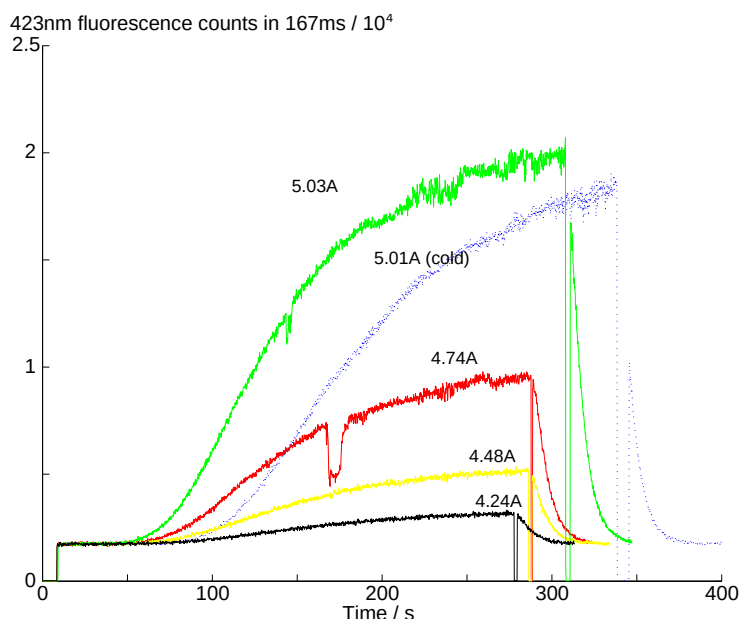


Figure 7.12: Fluorescence of calcium atoms observed using the trap imaging system. The oven is switched on at different currents, as shown in the legend, and we plot fluorescence counts against elapsed time. Counts are summed over $20 \times 8333 \mu\text{s}$ (see section 3.2).

1. As the current increases from 4.24 A to just over 5 A, the steady-state value of the fluorescence increases by a factor of ~ 3 each time the oven current is increased by ~ 0.25 A.
2. The dotted graph at 5.01 A was the first one to be taken. The fluorescence begins later for this, and also levels off later. Once the oven was warm from this scan, the other four scans were performed and their timings were more consistent with each other.

The sudden losses of fluorescence in the 5.03 A and 4.74 A scans were caused by necessary adjustments to the 423 laser's current as the laser began to exhibit multimode behaviour.

7.5.4 In-trap mass spectrometry and loading of different isotopes

7.5.4.1 Geometry of the atomic beams

The geometry of the oven and trap is shown in figure 7.13. In this figure, the 389 beam is in the yz -plane, entering from below the plane of the figure at 25° from the $-z$ -axis. The oven consists of a 2 mm stainless steel tube, crimped at both ends and filled with granulated calcium. A hole was filed at a point on the tube such that, when the oven was positioned in the vacuum chamber, there was a line of sight between the hole and the centre of the trap. The vertical position was not carefully controlled, and we will show that we believe the hole to be about 1.4 mm higher than the centre of the trap. To compensate for this, the fibre output had to be translated in the $-x$ direction, changing the laser beam a angle at the trap.

A foil shield was wrapped around the support rods closest to the oven, to prevent build-up of calcium elsewhere. A hole cut in this, of height ~ 6 mm, permits the effused calcium beam (and the horizontal laser beams) to enter the trapping region. The atomic beam is collimated by the AC electrodes. The angle of the beam to the vertical is approximately 25° : thus the effective

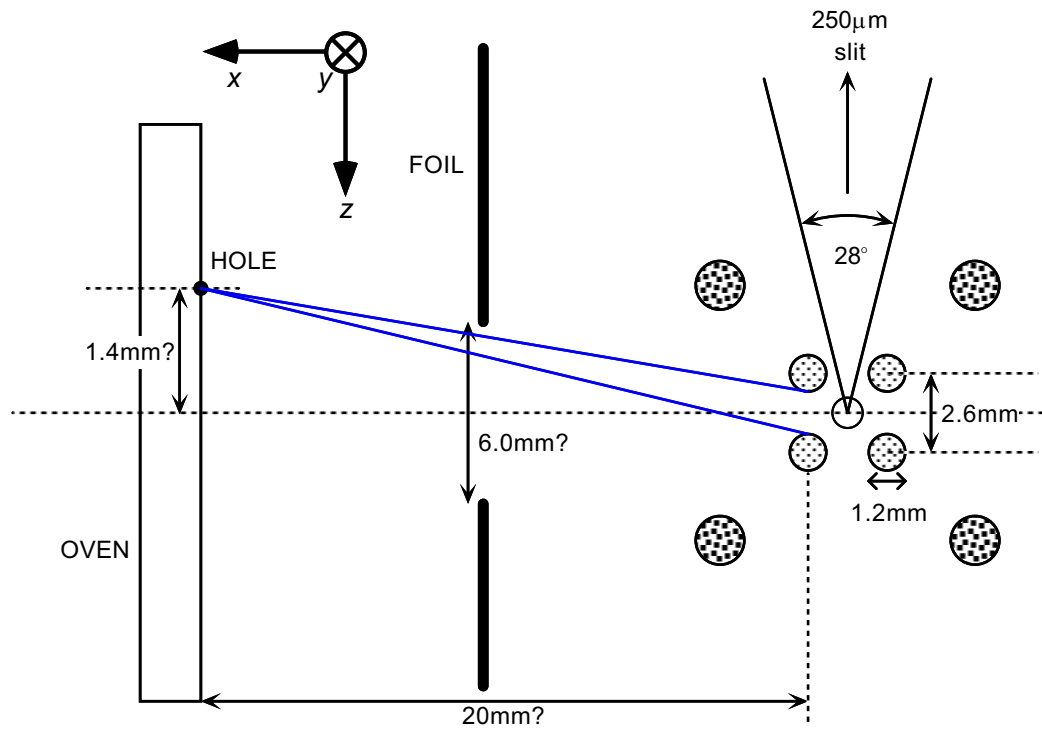


Figure 7.13: Geometry of the oven and trap electrodes. The trap support rods have been omitted, but the foil is wrapped closely around these rods. The oven is also displaced into the plane of the paper, which necessitated realigning of the photoionization lasers to minimize inhomogeneous broadening. For more details and dimensions see figure 2.5. Dimensions marked ‘?’ are uncertain, since they were not measured carefully before the vacuum system was assembled.

collimation width of the electrodes is $1.4 \text{ mm} / \cos 25^\circ \approx 1.56 \text{ mm}$. Also, the region of the 423 nm beam from which fluorescence is collected is limited by the imaging slit. The electrodes provide the widest collimation slit in system for trapping selectivity; the imaging slit defines the collimation for 423 nm spectroscopy.

7.5.4.2 Preliminary scans and analysis

To observe the shape of the 423 nm fluorescence profile, we illuminated the calcium beam with this laser only. The oven was run at 4.48 A and we waited several minutes for the steady-state calcium flux to be reached. Once there was no further increase in fluorescence at a fixed laser detuning, the frequency of the laser was scanned across the atomic resonance. In figure 7.14 we see the results of one of these scans. The main peak was fitted with a Voigt profile as shown. The smaller peak, which appears in the residuals, is fitted in figure 7.15.

The isotope shift is 391(5) MHz, very close to the expected value. However, noise on the fluorescence gives a large uncertainty in the height ratio between the two isotopes: the relative abundance of ^{42}Ca is 3(2)%. The width of the Lorentzian is 161(3) MHz. If $I = 28I_s$ then we expect a homogeneous linewidth of approximately 240 MHz. However, this is based on the peak intensity of the 423 laser beam, and the actual shape of the resonance when the calcium is illuminated by a beam of finite width will therefore be not Lorentzian but a sum of Lorentzians of different linewidths $\Gamma(I)$. It is reasonable that the resultant curve can be fit by a Lorentzian of reduced linewidth.

The beam from the oven was not well collimated, as it was not considered necessary when the vacuum can was originally assembled. The atomic and laser beams are also not currently at right angles to each other. Hence the velocity distribution along the laser beam axis is not Gaussian, as we shall see later. The distribution has a sharp peak, with asymmetric quasi-Gaussian wings. This will contribute to both Lorentzian and Gaussian widths in the case of a simplified Voigt fit. The Gaussian component of the Voigt fit was 107(2) MHz.

With reference to the high dependence of atomic fluorescence on oven current that we saw in figure 7.12, laser frequency scans were repeated with identical conditions apart from the oven current and we present a qualitative analysis here. Although the velocity distribution is non-Gaussian, a substantial increase in vapour temperature will still increase the inhomogeneous broadening of the line. However, analysis of the data at different currents showed no trend in the width of the resonances. This implies that the increase in effusion at higher currents is dominated by the increase in vapour pressure rather than the increase in the average speed of ions leaving the oven. This is to be expected since the vapour pressure is an exponential function of temperature [159].

7.5.4.3 Laser jitter observed in atomic fluorescence

Once we have calibrated the frequency scale of a scan across the 423 nm resonance in the trap, we can use the atomic response to measure the jitter and wander of the 423 laser. The laser was tuned to the half-height of a fluorescence feature to provide maximum sensitivity to frequency jitter. For small excursions the fluorescence varied by 82.7 counts/MHz and we converted the fluorescence excursions to frequency excursions using this simple linear form. In figure 7.16 we see the excursions observed during $\sim 8 \text{ s}$. On the right is a histogram of this data, summed in 100 bins. A Lorentzian fit to this histogram gives an excursion FWHM of 6.3 MHz; a Gaussian yields a slightly better fit with a FWHM of 7.2 MHz, but the residuals are high in both cases.

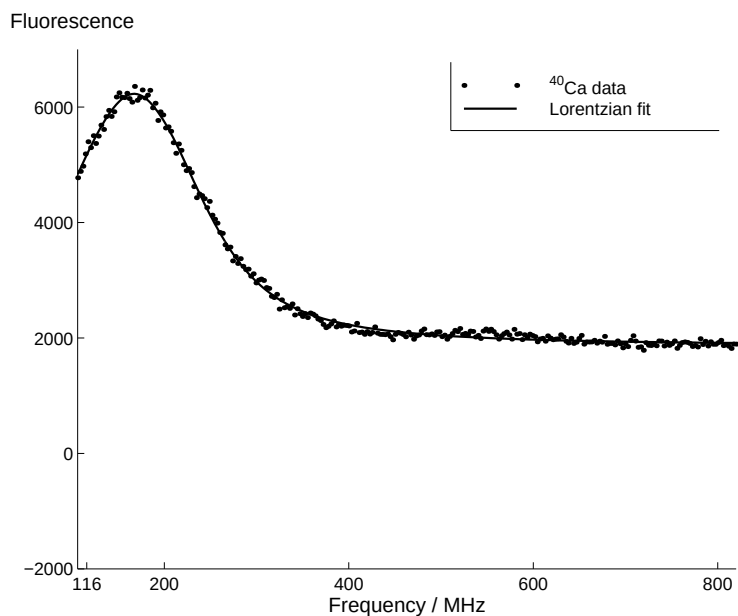


Figure 7.14: Scan of the 423 laser over the atomic resonance. The oven was run at 4.48 Å for several minutes until the fluorescence at a given laser detuning was constant. A Voigt fit to the data is shown, along with the residuals (the difference between data and fit, divided by the fit) magnified 20,000 times.

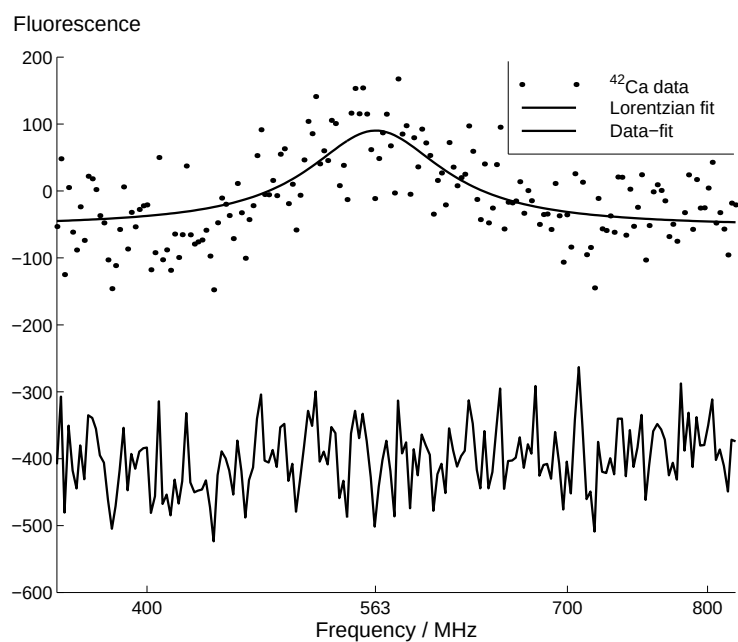


Figure 7.15: Fitting to the smaller ^{42}Ca peak from figure 7.14. The lower graph shows the difference between data and fit.

7.5.4.4 Model of inhomogeneous broadening in effusive beam

Let us consider the general situation in figure 7.17. The axes have been rotated from our usual convention. This is necessary to effect an analytical expression. The laser beam is aligned along the $-z'$ -direction, and the x' -axis is antiparallel to the x -axis. Along the z' -axis⁵ the slit has width $c_2 - c_1$. Along the y' -axis the slit extends from $-b$ to b .

Consider an atom of known v_z effusing from the oven. For it to pass through a small area $\delta y \delta z$ its other velocity components must satisfy:

$$\frac{s}{z + \delta z} v_z < v_x < \frac{s}{z} v_z \quad (7.14)$$

$$\frac{y + \delta y}{z} v_z > v_y > \frac{y}{z} v_z, \quad (7.15)$$

to first order in small quantities. The probability of an atom effusing with a given velocity is

$$\delta P(\mathbf{v}) \propto \exp\left(-\frac{v_x^2 + v_y^2 + v_z^2}{\zeta^2}\right) v_x \delta^3 v, \quad (7.16)$$

where ζ is characteristic of the Boltzmann distribution in the oven. However, for atoms to reach the laser, they must travel from the aperture to the interaction region. Thus the beam density is

$$\delta D(\mathbf{v}) = \delta P(\mathbf{v}) / v_x, \quad (7.17)$$

and thus atoms passing through a small area of the slit contribute an atomic density:

$$\delta D(v_z, \delta y, \delta z) \propto \exp\left(-\frac{v_z^2}{\zeta^2} \left[1 + \frac{y^2}{z^2} + \frac{s^2}{z^2}\right]\right) \frac{v_z^2}{z^3} s \delta y \delta z. \quad (7.18)$$

Moving to infinitesimals we integrate over the area of the slit. Recalling that, in our experiment, $s^2 \gg y^2$, we neglect the integral over y to yield density

$$D(v_z) \propto \int_{z=c_1}^{z=c_2} \exp(-u/\zeta^2) du \text{ where } u = v_z^2 \left[1 + \frac{s^2}{z^2}\right] \quad (7.19)$$

$$\propto \exp(-v_z^2/\zeta^2) \left\{ \exp\left(-\frac{v_z^2 s^2}{c_2^2 \zeta^2}\right) \exp\left(-\frac{v_z^2 s^2}{c_1^2 \zeta^2}\right) \right\}. \quad (7.20)$$

The thermal distribution of speeds is asymmetric with a non-zero mean. In addition we have lost sign information for the velocities: for $c_1, c_2 > 0$ the above is only valid for $v_z > 0$. All atoms with velocity $v_z \leq 0$ are cut off by the slit and $D(v_z \leq 0) = 0$.

7.5.4.5 Optimization of the 423 nm spectrum

The preliminary scans show both homogeneous and inhomogeneous broadening in excess of what we expect from an ideal crossed-beam experiment. We therefore aimed to make the spectrum narrower, and improve signal-to-noise, until we were able to distinguish resonances from all the major isotopes, and fit well to the asymmetric inhomogeneous broadening. The major broadening mechanisms, and estimates of their contribution, are as follows:

⁵A slit at any angle can be projected onto this axis without loss of generality (e.g. the ‘slit’ defined by the AC electrodes).

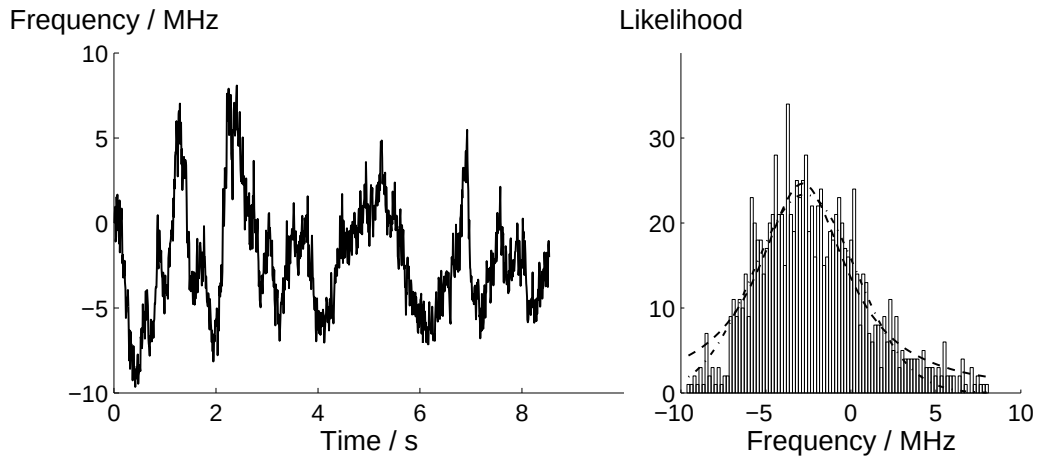


Figure 7.16: Jitter of the 423 laser, calibrated using the fluorescence of ^{40}Ca in the trap. On the left is the frequency excursion during several seconds; on the right is a histogram of the excursion probabilities.

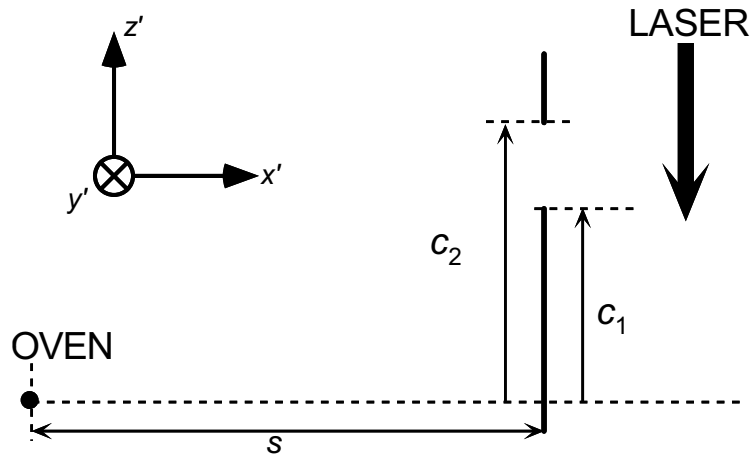


Figure 7.17: Schematic of effusion geometry. The axes have been rotated to simplify the calculation. See text for details.

Homogeneous broadening The intensity of the 423 laser causes the transition to saturate. We do not expect any collisional broadening given the low density of calcium in the beam. To minimize power broadening we misaligned the 423 laser into the fibre input, yielding a peak intensity of $11 \mu\text{W}$ in the trap. This maximum corresponds to $0.3I_s$ on the 423 nm transition. At zero laser linewidth the power-broadened FWHM is then $< 39 \text{ MHz}$.

Inhomogeneous broadening Initially photoionization lasers were aligned along the path previously occupied by the vertical 397 beam. This geometry is shown in figure 2.7. We improved matters by translating the fibre head out of the plane of this figure—in the $-x$ direction—to compensate for the elevated position of the hole in the oven wall. When the beams were realigned through the trap centre, they were now perpendicular to the axis of effusion and inhomogeneous broadening is minimized.

Consider the realignment to compensate for the elevated position of the hole in the oven as follows. Fits to the data in figure 7.14 using the asymmetric inhomogeneous function implied that we had to translate the beam by 33 mm. The position of the beam on the last mirror was then 197 mm from the trap centre, resulting in an angular displacement of $\tan^{-1} \frac{33}{197} = 9.5^\circ$, in a plane P at 25° to the xz -plane. Once translated, the beam strikes the last mirror in figure 2.7 at $\mathbf{r} = \{-33 \text{ mm}, 83 \text{ mm}, -176 \text{ mm}\}$ (where the origin is at trap centre). If we assume the oven to be $\sim 20 \text{ mm}$ from the trap, then the deduced displacement of the oven above the xy -plane is 1.4 mm. Projecting the imaging slit onto the new beam vector gives an effective fluorescence collimation slit of 0.59 mm; projecting the slit defined by the electrodes leaves this effective collimation slit unchanged at 1.56 mm.

PMT saturation Broadening also occurs when the 8254 counter that receives the PMT signal saturates. The photon counting system has a 100 ns dead time following the collection of a single photon (see section 3.2). Photon arrival follows the Poisson distribution: if on average μ photons arrive in a unit time, then the probability of observing n photons is⁶ $\mathcal{P}_\mu(n)$. If $n = 0, 1$ then the counting system records the correct count. However, if $n > 0$ then the arrival of only one photon is recorded. Thus the apparent mean photon arrival is

$$\begin{aligned} \bar{\mu} &= \sum_{n=1}^{\infty} \mathcal{P}_\mu(n) = 1 - \mathcal{P}_\mu(0) \\ &= 1 - e^{-\mu}. \end{aligned} \quad (7.21)$$

The size of this correction is $\sim 5\%$ for a typical peak PMT count rate of 10^6 s^{-1} .

Transit time broadening Consider a laser beam whose intensity profile is a ‘top hat’ function of width $100 \mu\text{m}$. If the ions pass at right angles to this beam then the transition is stimulated for a finite time t : at a temperature of $\sim 600 \text{ K}$ the thermal velocity is $\sim 350 \text{ m s}^{-1}$ and the transit time over $100 \mu\text{m}$ is $\sim 0.29 \mu\text{s}$. The frequency response of the ions is then [156] of the form $\sin^2(2\pi[\nu - \nu_0]t/2)/(2\pi[\nu - \nu_0])^2$, which, when convolved with the spectrum, increases the FWHM by $\sim 2.78/(2\pi t) = 3.2 \text{ MHz}$.

However, our laser’s intensity profile is a Gaussian of spot width $100 \mu\text{m}$. In this case the Fourier transform of the intensity is also a Gaussian, yielding an extra quasi-inhomogeneous linewidth of 1.3 MHz.

⁶See appendix A.4.4.

7.5.4.6 Results after optimization

We are able to observe the angle of the laser beam on the Andor camera. The 423 laser causes fluorescence in the wide atomic beam, and an image of this fluorescence is shown in figure 7.18. On the camera the image of the beam is at $\sim 23^\circ$ to the camera axis. We expect an angle of $\tan^{-1}(33/83) = 21.7^\circ$, although the camera has only been oriented relative to the trap axis to within a degree. From the half-maxima points on the cross-section of the laser we infer a spot width of $w \sim 100 \mu\text{m}$, close to the expected $80 \mu\text{m}$.

In order to determine how well the laser beam has been aligned perpendicular to the axis of effusion, we compare the in-trap fluorescence feature of ^{40}Ca with the saturated absorption signal from the hollow-cathode lamp. A scan of laser frequency across the ^{40}Ca resonance is shown in figure 7.19. Both features fit well to Voigt profiles: all isotopes are included in the in-trap fluorescence fit; only the centre of the saturated absorption peak is considered, and hence only the ^{40}Ca isotope need be fitted. From three scans we determine a blue Doppler shift of $4(2)$ MHz. If we also consider the collisional shift in the hollow-cathode lamp at 6 torr, then the Doppler shift from the unperturbed transition is $-9(2)$ MHz.

Once the system was optimized we scanned the laser slowly over the 423 nm resonance. The results of this scan are shown in figure 7.20. This figure shows the logarithm of the observed fluorescence from the atomic beam at different detunings of the 423 laser. The theoretical fit includes the effect of the asymmetric velocity distribution. In this fit we fix the isotope shifts, and the isotopic abundances for natural calcium. We are approaching the limit of shot noise and thus expect our error in a count N to be $\sqrt{N}$. Therefore the residuals below the data are weighted by the square root of the count rate. The inset to the upper figure shows the result of neglecting the ^{43}Ca isotope in the best fit: the presence of the odd isotope is necessary to fit well to the data.

The frequency axis is calibrated linearly using the wavemeter. We see in the peak of ^{48}Ca at ~ 1500 MHz an error of approximately 2% in this calibration. The fitted homogeneous linewidth was 42.5 MHz: without considering the laser linewidth we expect $\lesssim 39$ MHz. The ‘slit width’ is fixed. This is defined by the aperture in the imaging system, rather than the separation of electrodes. This is because the imaging system only samples a small interaction region. The effective length of this region is 0.6 mm.

If we float the temperature in the fits then we obtain an unphysical temperature of 4200 K, with an associated thermal width of ~ 46 MHz. At a physically realistic temperature⁷ of approximately 600 K we expect an inhomogeneous FWHM of $\Gamma_{in} \sim 18$ MHz. Thus there is ~ 28 MHz of non-Lorentzian width unaccounted for in our model. A partial explanation of the elevated temperature in the fits is that the laser jitter can contribute around 7 MHz of Gaussian linewidth (see figure 7.16).

7.6 Isotope selectivity in the trap

7.6.1 Predictions of selectivity

We wish to be able to observe and trap only ^{43}Ca ions using the photoionization system. In figure 7.21 we show a theoretical fluorescence profile based on our fit parameters, both with and without the odd isotope present. We neglect the effect of scattered light i.e. the baseline has been

⁷The peak fluorescence on the transition implies a beam number density of $1.6 \cdot 10^{12} \text{ m}^{-3}$. From kinetic theory calculations with an oven hole of diameter 0.1–1 mm, and using vapour pressure data [160], we obtain an oven temperature of approximately 600(40) K for all oven currents used.

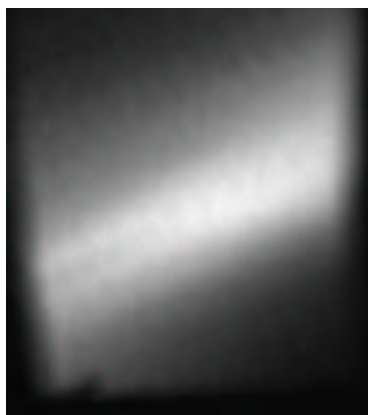


Figure 7.18: 423 nm fluorescence imaged on the Andor camera, showing the orientation of the laser beam.

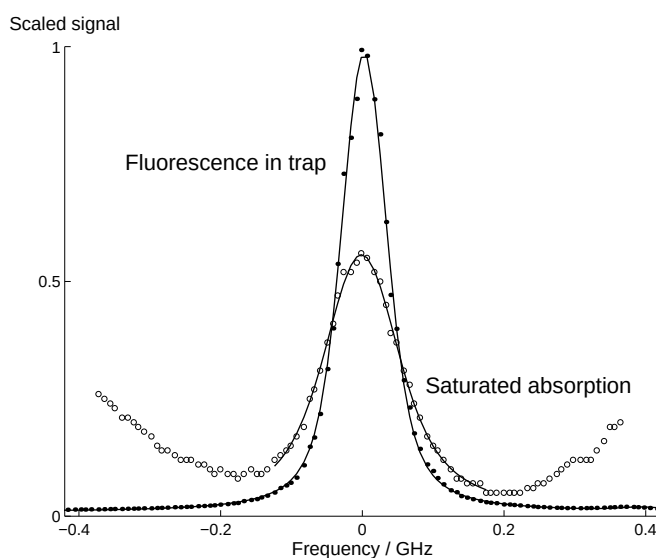


Figure 7.19: Spectroscopy of the 423 nm transition in ^{40}Ca in the trap and in a glass cell using saturated absorption methods. The lower, broader data with elevated wings ($\circ$) is the saturated absorption feature, whereas the sharper, taller feature ($\bullet$) is the fluorescence of ions in the trap. The 423 beam was translated in order to compensate for the angular displacement implicit in the fits to early data. At this translation it was found that we had reduced the Doppler shift between the two signals to approximately 9 MHz.

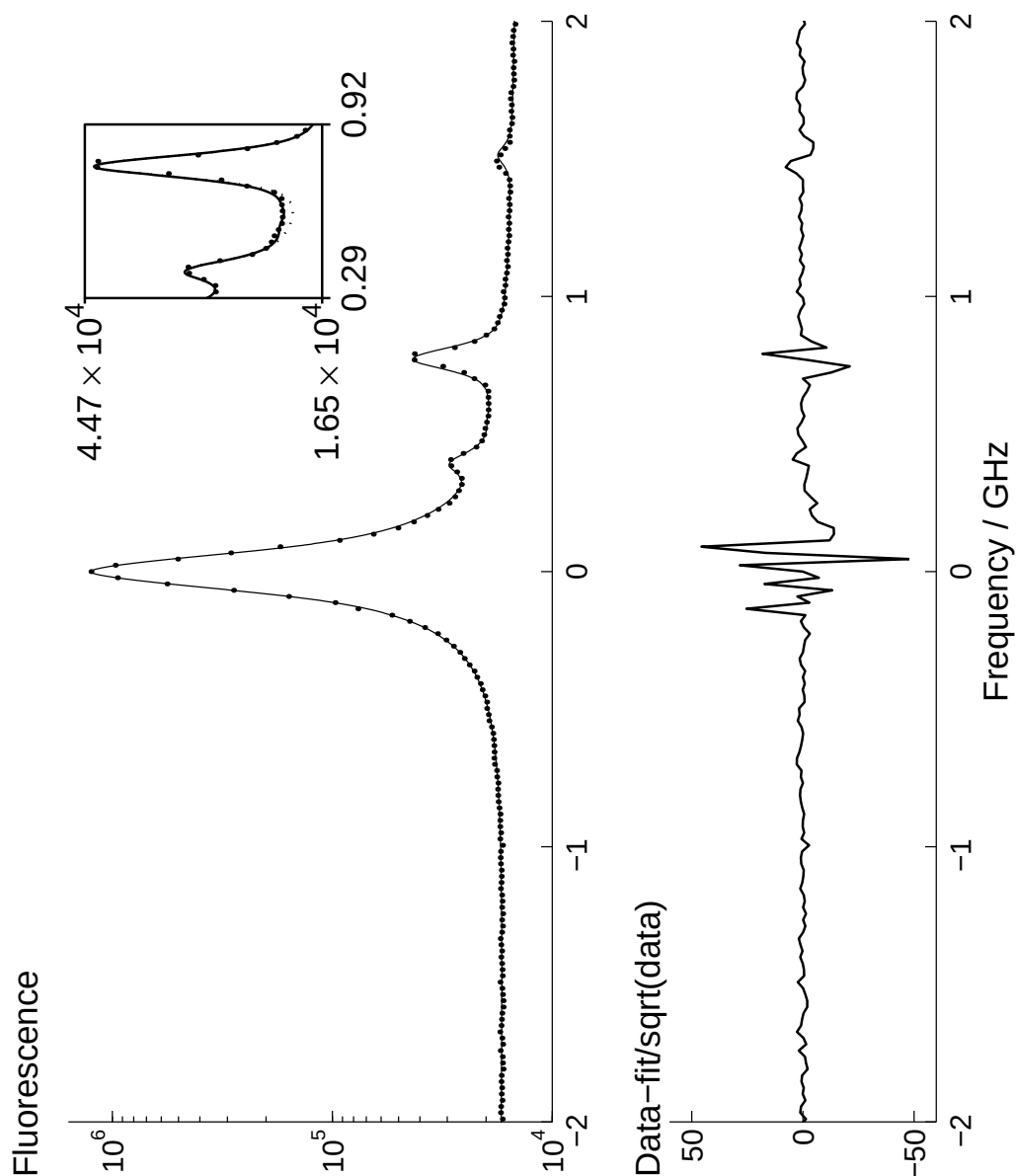


Figure 7.20: Optimized spectrum of the 423 nm transition in atomic calcium. In the upper figure we see the fit to the spectrum including all isotopic components. By neglecting the quadratic term in the frequency calibration there is an error in the frequency of the ^{48}Ca peak of $\sim 2\%$. In the inset we see the best fits both including and excluding the ^{43}Ca peaks: the dotted line is the result of excluding the isotope and is a less good fit to the data. In the lower figure we plot the residuals: the difference between data and fit, divided by the square root of the data.

subtracted from the theoretical profile. The maximum ^{43}Ca signal is approximately 29.3 dB, compared to 35.3 dB from all other isotopes. Hence with a ‘slit width’ limited by the imaging system, the ratio of ^{43}Ca fluorescence to other isotope fluorescence is 1/4.

Whereas the fluorescence discrimination is limited by the (relatively wide) slit in the imaging system, photoionization selectivity depends on the width of the region of interaction between the atomic and laser beams. Let us first consider such a region collimated only by the electrodes as a ‘slit.’ Since the photoionization lasers are at an angle of 25° to the vertical, this ‘slit’ has effective width $1.4 \text{ mm} / \cos 25^\circ$. The isotope photoionization discrimination under these conditions is shown in the upper subfigure of figure 7.22: for every ^{43}Ca atom ionized, we expect to ionize 6.4 ions from the other isotopes.

We can improve photoionization selectivity further by aligning the 389 laser horizontally into the trap. This laser defines the region within which both lasers are able to photoionize any given atom: the width of the 389 laser defines the ‘slit.’ In the lower subfigure we show the expected ionization rates for such a ‘slit’ of width of $100 \mu\text{m}$. The ratio of trapping of $^{43}\text{Ca}^+$ to that of trapping other isotopes is then 1/3. This is shown in the lower subfigure of figure 7.22. With such a narrow slit, the width of the components in the trapping spectrum is dominated by the natural width of the 423 nm transition and we will therefore not improve selection of the ^{43}Ca isotope by focussing the 389 laser any further.

7.6.2 Loading and trapping of isotopically pure crystals

We conclude with a brief discussion of the loading of the ion trap with different isotopes.

In figure 7.23 we show the predicted loading fractions of all isotopes separately. This implies that both ^{44}Ca and ^{48}Ca can be loaded as isotopically pure samples, relative to ^{42}Ca and ^{43}Ca .

We can also tune the frequency of the 423 laser to trap mixed crystals with desired isotopic fractions. In figure 7.24 we see two isotopes loaded into the same trap. In the upper subfigure we see fluorescence on the cooling transition in $^{40}\text{Ca}^+$. From the distorted shape of the crystal it is clear that there are dark ions present. Indeed, we have observed mixed-species crystals before, but did not identify the other species. When the lasers were retuned to the equivalent transitions in $^{44}\text{Ca}^+$ we observed the crystal in the lower subfigure: all the previously dark ions are now fluorescing. Because we have been able to form a cold crystal with cooling of either isotope, it is clear that we have been able to provide sympathetic cooling of the dark isotope in both cases [161]. This opens up the possibility of performing quantum computation on the dark ions, while simultaneously cooling the bright ions to keep the ion crystal at a low temperature [162].

The spatial separation of isotopes in the trap may be explained by a number of possible mechanisms. The separation is in the horizontal plane (the crystal is viewed from above) so it is not an effect of differing isotopic weight. One possibility is that, when the laser is detuned to the cooling transitions of one isotope, it exerts a dipole force on the ions of the other isotope. Because the sign of the dipole force is related to the sign of the laser detuning, and the cooling lasers enter from bottom left in these images, then it pushes the $^{40}\text{Ca}^+$ ions away to the top. This effect is most likely not dynamic, owing to the small dipole force at only $\sim 200 \text{ MHz}$ laser detuning. Instead such pressure probably dictates the shape of the crystal during its formation.

However, radiation pressure as a method of isotopic separation also seems unlikely, in the light of other groups’ results [163]. In these results the predominant mechanism for isotope separation is the dependence of the pseudopotential on the charge-mass ratio. For large crystals ($\gtrsim 100$ ions) this leads to $^{40}\text{Ca}^+$ lying entirely along the trap axis, surrounded by $^{44}\text{Ca}^+$. As our results involve relatively small crystals, the isotopic separation in figure 7.24 is most likely a combination of a

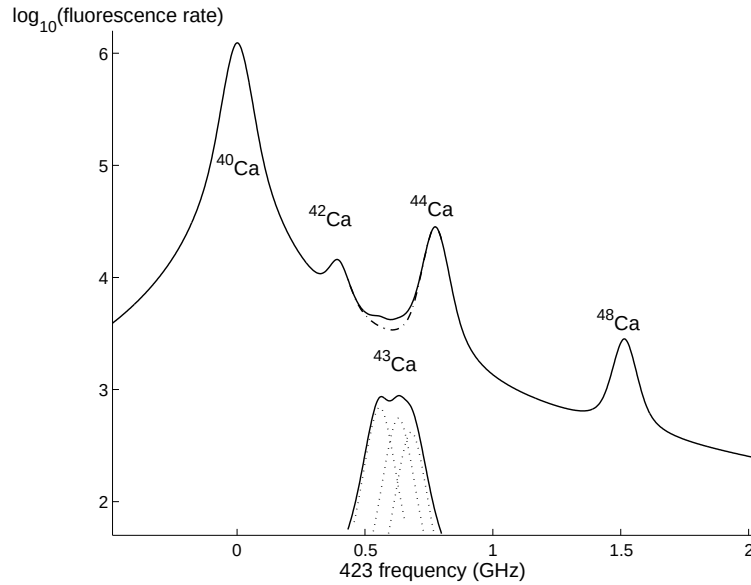


Figure 7.21: Predicted discrimination of fluorescence from different isotopes. The parameters from the fit in figure 7.20 are used to examine the relative fluorescence from ^{43}Ca and other isotopes. Frequencies are measured relative to the ^{40}Ca peak. The solid line in the larger graph is the fluorescence from all isotopes; the dash-dotted line shows the contribution from the even isotopes. Below the graph, on the same scale, we show the three hyperfine components of ^{43}Ca (dotted) and the sum of all these components (solid line).

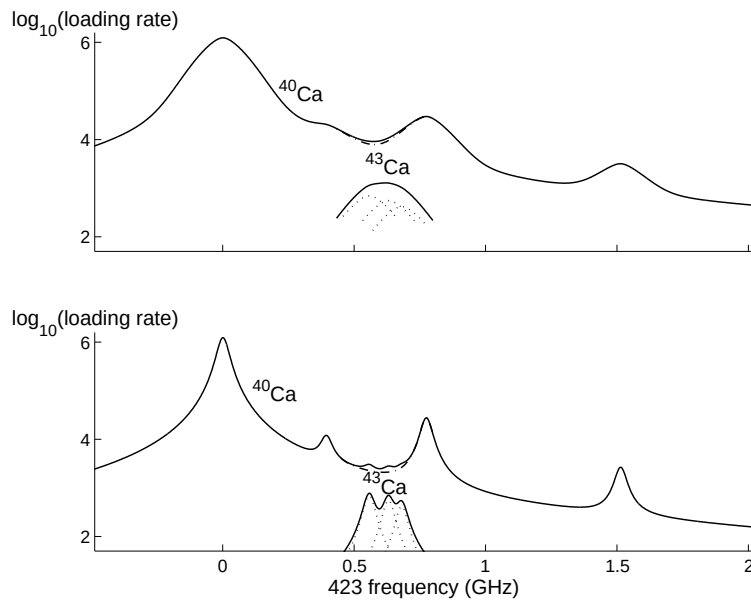


Figure 7.22: Predicted discrimination of isotopes during ionization/trapping. In the upper figure the velocity discrimination is limited by the electrodes acting as a slit between the trap region and the oven: it is the worst-case scenario. In the lower figure we assume a horizontal 389 laser beam, focussed to a 'slit width' of $100\ \mu\text{m}$, defines the vertical extent of the interaction region. The vertical scales of both graphs are arbitrary.

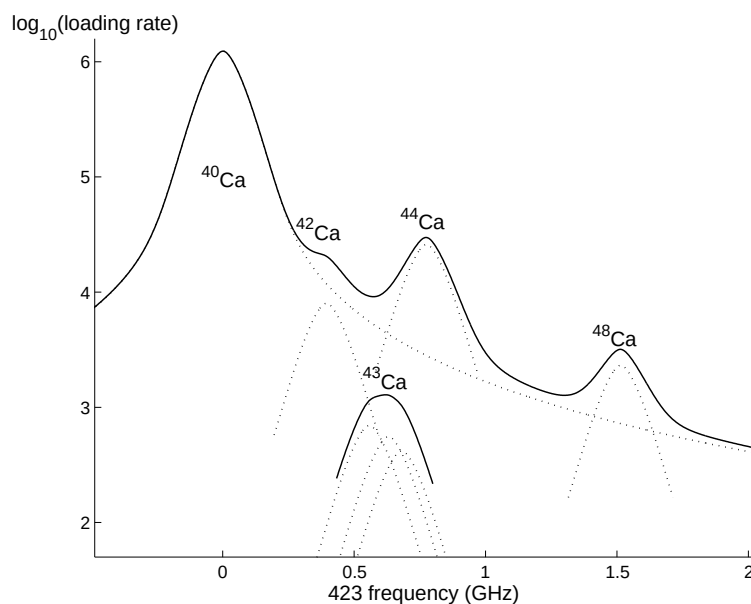


Figure 7.23: Predicted loading rates of all isotopes, with velocity selection by the electrodes. This is modelled on the current beam geometry in the Oxford Ion Trap and shows each isotope explicitly as dotted lines.

purely random effect and some residual imperfections in the pseudopotential. With both photoionization beams entering along the vertical beam path we have been able to trap all of the stable isotopes of calcium with no impurities present. Single ions of the odd isotope have been trapped, although it was often necessary to discard impure crystals. Trapping of e.g. pure ^{42}Ca implies that the isotopic discrimination is considerably better than that implied by figure 7.23. A possible explanation for this is that, although the electrodes collimate the beam, only ions produced in a much narrower region—determined by the shape of the radio-frequency trapping potential—are retained after ionization. As shown in the lower half of figure 7.22 we are then limited in our selectivity by the natural linewidth.

7.7 Conclusions

We have performed spectroscopy of the 423 nm transition in neutral calcium, and used the method of photoionization to ionize and capture selected isotopes of calcium in the ion trap. We began by using the 423 neutral fluorescence laser to perform absorption spectroscopy in a hollow-cathode cell. Using a counter-propagating probe beam we then performed saturated absorption spectroscopy with phase-sensitive detection, and could detect the two most abundant isotopes in neutral calcium. These experiments, combined with *a priori* knowledge of isotope shifts and abundances, provided us with information about the control we had over 423 laser.

Using both the 423 and 389 lasers we were able to photoionize calcium in the ion trap. Fluorescence on the 423 nm transition yielded information about the effusion rates from the oven, and also permitted us to optimize the geometry of the atomic and laser beams, so that the beams were perpendicular. At this point we performed frequency scans across the 423 nm transition for all isotopes, and fitted the data using a kinetic theory model for the effusive beam. After optimization we observed approximately 9 MHz of residual Doppler shift between the in-trap beams and a saturated absorption experiment, and were able to use the 423 nm fluorescence to determine the extent of the



Figure 7.24: Image of a two-species ion crystal. In the upper image we tune the Doppler cooling lasers to the relevant transitions in $^{40}\text{Ca}^+$; in the lower image these are tuned to the equivalent transitions in $^{44}\text{Ca}^+$. The scale is given above.

423 laser's jitter and longer-term wander.

Using the new photoionization method we have been able to load pure crystals of all even isotopes. We have provided limits on the discrimination of the rare odd isotope ^{43}Ca which predict that we can photoionize one ion of $^{43}\text{Ca}^+$ for every three ions of all other isotopes. This is limited primarily by the natural linewidth of the 423 nm transition. However, by choosing the correct laser for cooling, we should be able to produce pure crystals of $^{43}\text{Ca}^+$ as follows. Firstly the 423 laser is detuned such that it is resonant with the transition in the required $^{43}\text{Ca}^+$. Then the 393 UV shelving laser is tuned to the $S_{1/2}^4 \leftrightarrow P_{3/2}^5$ transition in the same isotope. Because of the hyperfine structure in calcium, this detuning is such that all even isotopes are heated while the $^{43}\text{Ca}^+$ ions are cooled. Although we nonetheless expect some sympathetic cooling to trap other isotopes, we can expect a much higher purity of trapping of the odd isotope.

Chapter 8

Conclusions

This thesis describes work undertaken to carry forward the quantum information project in the Oxford Ion Trap Group. The main developments discussed are: the new laser, the AOM system and photon correlation system; a study of heating rates and laser cooling; a new RF supply for the Paul trap; the magnetic field coils and their use in readout experiments; and spectroscopy of neutral calcium and the use of photoionization to load ions into the trap.

The apparatus development described in chapters 3, 5 and 6 has greatly improved the speed and reliability of experiments in the ion trap. The new electronics for correlating photon arrival times have permitted stray field compensation to be carried out rapidly and more precisely. The 850 IR shelving laser and the AOMs are essential components to all the readout experiments and to future work on quantum information.

The study of laser cooling of a trapped atom in chapter 4 treated the case of a three-level atom, giving various insights and in particular allowing us to calculate the cooling rate when the repumper laser at 866 nm is turned off. Our observations of long confinement times in the trap were interpreted to yield upper bounds for heating rates. Parametric heating, which gives an exponential increase in temperature with time, was deduced to be at a negligible level with respect to limitations imposed by the minimum necessary fidelity in quantum-computational operations. However, this method does not place tight constraints on any heating process which simply gives a constant heating rate (linear increase in temperature with time).

The study of high-voltage, radio-frequency resonant circuits in chapter 5 provided a thorough analysis of resonant circuits based on separate inductive and capacitive elements, and introduced the helical resonator. Although in theory the parameter regime we require should be accessible with ‘traditional’ bulk-element circuits, in practice we found the quality factors of these circuits were lower than expected. A helical resonator was built and was found to give a higher quality factor; this was therefore used to trap ions. This is already an improvement on our previous RF supply because it is less noisy. However, the resonant frequency was low (4 MHz) owing to capacitance in the cables, the vacuum feedthrough and the ion trap itself. The present study has shown where the various contributions to capacitance in the system are coming from. The success of the approach has since led to the design and construction of a new resonator to give the desired RF frequency with this information taken into account.

The magnetic field at the trapped ion is crucial to all our experiments involving the Zeeman sublevels of the ground state, and to the dark resonance experiments which provide diagnostics on the laser parameters as well as on the magnetic field itself. The field at the ion turned out to be a non-trivial function of the fields provided by the coils: this was owing to complicated magnetization

properties of the vacuum chamber. Vertical fields were higher than we expected, whereas horizontal fields were both strongly deflected and reduced in magnitude. Clearly it would be advantageous in a future experiment to avoid magnetic materials in the chamber completely. For the present system, the coils had to be made considerably more bulky than would have been the case in a non-magnetic system; they had to be carefully designed to fit in the space available and admit both air and water cooling; and the field had to be calibrated using the ion itself as the ‘detector.’ All these problems were solved and the resulting well-controlled field has allowed two new sets of experiments on readout of the spin state of the ion to be performed. The first involved a transient ‘high’ (200 G) field in the vertical direction, to allow a large Zeeman splitting and hence frequency discrimination of the spin state. The second involved a horizontal field of order 7 G which had to be well-aligned along a laser beam direction; two-photon coherence effects (EIT) were used to allow discrimination of the spin states. The details of the latter experiments are described elsewhere [88].

The experiments on saturated absorption spectroscopy of neutral calcium at 423 nm were found to be in good agreement with a theoretical model which took account of saturation broadening, collisional broadening and collisional redistribution of velocities. The isotope shifts and abundances of other isotopes relative to ^{40}Ca were detectable in the vapour cell experiments, and found to be consistent with known values. Crossed-beam spectroscopy, using the oven in our ion trap vacuum chamber as the source and the trap electrodes as the collimating aperture, gave higher resolution and we were able to detect all the stable isotopes of calcium. The results were consistent with the known relative abundances and isotope shifts. When combined with a second laser at 389 nm this permitted us to photoionize selected isotopes of calcium and thus load the trap.

This method has several important advantages over electron-beam ionization. First, we can preferentially load a desired isotope, and in particular the crossed-beam experiments imply that we will be able to load the rare isotope ^{43}Ca with modest efficiency even without an unenriched source. Secondly, the selectivity greatly reduces the probability of loading undesired so-called ‘dark’ ions, which saves considerable time and will become essential when larger (pure) groups of ions are desired. Thirdly, by avoiding the electron beam we do not charge the insulating parts of the trap structure, which greatly reduces stray electric field effects and hence drift of the compensation. Fourthly, we find the photoionization requires much lower oven temperatures (i.e. currents) and so reduces the tendency of the oven to deposit calcium on the trap structure. Such deposits are associated with stray field drift and electric field noise.

All this work has been part of an ongoing project to achieve quantum information processing in trapped ions. The most natural next steps are to investigate single-qubit gates by observing Rabi flopping between the Zeeman sublevels of the ground state, and to improve the laser cooling and temperature diagnostics. Experiments are under way to do the former by magnetic resonance and the latter by using a two-photon resonance driven by π and σ^+ beams at 397 nm. After this a two-qubit logic gate will be investigated, with the major goal of demonstrating controlled entanglement of massive particles. The latter has up till now only been achieved in two laboratories, and methods to achieve the logic gate in a robust and rapid manner are highly sought-after.

Appendix A

Conventions and simple formulæ

A.1 The Gaussian beam formula

There is occasionally confusion over certain definitions within the Gaussian beam formula. In order to be precise about e.g. the beam width we reproduce the formulæ here:

$$b = \frac{\pi w_0^2}{\lambda} \quad (\text{A.1})$$

$$w_x(z) = w_{x0} \sqrt{1 + z^2/b^2} \quad (\text{A.2})$$

$$I(x, y, z) = I_0(z) \exp(-2\{x^2/w_x^2 + y^2/w_y^2\}) \quad (\text{A.3})$$

$$P = \int I(r, z) r dr d\theta \quad (\text{A.4})$$

$$= \frac{1}{2} \pi w^2 I_0 \quad (\text{A.5})$$

I_0 has a z -dependence in order to conserve energy in the beam. This definition of the beam width $w(z)$ above implies it is best described as an amplitude measure: it is the distance from beam centre to the 1/e amplitude heights.

A.2 Shelving probabilities

In the discussion of shelving techniques, this thesis uses the conventions given in reference [164], as follows. A coherent superposition of our two qubit states $\{|S_{1/2}^{+1/2}\rangle, |S_{1/2}^{-1/2}\rangle\}$ is split into two incoherent populations and one of these populations shelved i.e. transferred to a metastable level. Let $P_{\pm}(\delta)$ be the population of the shelf when all the population was originally in the state $|S_{1/2}^{\pm 1/2}\rangle$. The ‘failure’ probabilities are defined as

- $\epsilon_+ = 1 - P_+$, ‘failure to shelve what ought to be shelved.’
- $\epsilon_- = P_-$, ‘failure to retain population in the ground state.’
- $\epsilon = \epsilon_+ + \epsilon_-$, ‘total probability of measurement failure.’

We can relate this to a visibility of Rabi oscillation fringes. Let us assume we perform an ideal rotation of a qubit from $|0\rangle$ to $|1\rangle$ and back using a Raman pulse of variable length. Readout will

yield a sinusoidal dependence of the shelved fraction, which will vary between P_- and P_+ . The visibility of these fringes is then

$$\Upsilon = \frac{P_+ - P_-}{P_+ + P_-}. \quad (\text{A.6})$$

A.3 AC voltages on the Paul trap electrodes

In a Paul trap the ion motion can be described by the Mathieu equation, which has two variables a and q which determine the stability and tightness of the confinement. Previously [37, 76] the relationship between monitors of our RF voltage, and the voltage V_{AC} that appears in the Mathieu parameter

$$q = \frac{4e\alpha_{\text{AC}} V_{\text{AC}}}{m\Omega_{\text{RF}}^2 \rho_0^2} \quad (\text{A.7})$$

has been unclear. This section is intended to clarify the relationships in our existing apparatus, and extend the definitions to our new RF design. All voltages mentioned here are measured as *amplitudes* of the RF fundamental signal.

The existing electrical connections set one diagonal pair of electrodes oscillating in anti-phase to the other pair. The power supply can provide a voltage to any one electrode of amplitude $V_{\text{old}} = 0\text{--}120\text{ V}$, and so the potential difference between one diagonal pair and the other has amplitude $V_{\Delta} = 0\text{--}240\text{ V}$. During initial exploratory trapping V_{AC} was varied between 50 V and 120 V: $V_{\text{AC}} \equiv V_{\text{old}}$. The monitor voltage V_{MON} is provided by capacitive division of V_{old} such that $V_{\text{MON}} = V_{\text{old}}/200$. When the new system is in place, with one pair of electrodes grounded, then $V_{\Delta} = V_{\text{new}}$ and $V_{\text{AC}} = \frac{1}{2}V_{\text{new}}$.

In summary, therefore:

1. $V_{\text{old}} = 200V_{\text{MON}}$
2. $V_{\Delta} = 2V_{\text{old}}$
3. $V_{\Delta} = V_{\text{new}}$
4. $V_{\text{AC}} \equiv \frac{1}{2}V_{\Delta}$ always.

The voltage in the motional equations of the ion V_{AC} is therefore precisely half of the amplitude of the voltage swing between electrodes. This is a convenient measure for our old power supply as it is equivalent to the voltage on an electrode. Its convenience is less clear for the new supply, where one pair of electrodes will be grounded. However, in order to ensure consistency with reference [37] we maintain this convention.

A.4 Mathematical functions

A.4.1 The Lorentzian curve

The Lorentzian $\mathcal{L}$ used in all models is parametrized as follows:

$$L = \mathcal{L}(\nu) = \left(\frac{A\Gamma}{2\pi} \right) \frac{1}{(\nu - \nu_0)^2 + (\Gamma_\nu/2)^2} + B. \quad (\text{A.8})$$

B is the baseline; A is a scaling factor for the Lorentzian; ν_0 is the centre frequency; and Γ is the FWHM of the curve. The peak height is therefore given by $\mathcal{L}(\nu_0) \equiv \mathcal{L}_0 = B + 2A/\pi\Gamma$, and the integral over all frequency space is

$$\int_{-\infty}^{\infty} (\mathcal{L}(\nu) - B) d\nu = A.$$

For computation, parameters are passed to the MATLAB program `lorentzian.m` as a row vector $[B, A, \nu_0, \Gamma]$. This program is in turn used by the cost function program `lorfun.m` to give a χ^2 cost for best fit procedures [165].

A.4.2 The Gaussian curve

The Gaussian $\mathcal{G}$ is generally parametrized during fitting by

$$G = \mathcal{G}(\nu) = A \exp\left(-\frac{(\nu - \nu_0)^2}{2\sigma^2}\right) + B. \quad (\text{A.9})$$

As with the Lorentzian B is the baseline and ν_0 is the centre frequency of the distribution. The FWHM is given by

$$\Gamma = \sigma\sqrt{8\ln 2}.$$

The scaling factor A in this case is the amplitude of the centre of the function rather than the integral over the whole function. The MATLAB program `gaussian.m` takes a parameter vector $[B, A, \nu_0, \sigma]$, and the cost function `gaussfun.m` computes the χ^2 cost for best fit.

A.4.3 The Voigt profile

If one convolves a Gaussian with a Lorentzian then the resultant spectral feature is a Voigt profile:

$$\mathcal{V}(\nu) = \int_{-\infty}^{\infty} \mathcal{L}(\nu') \mathcal{G}(\nu - \nu') d\nu'. \quad (\text{A.10})$$

The real part of the transcendental Faddeeva function

$$F(u, a) = \Re \left\{ \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{\exp(-t^2)}{u + ia - t} dt \right\} \quad (\text{A.11})$$

is related to the Voigt profile by

$$\mathcal{V}(\nu, \{A, B, \nu_0, \Gamma_{\mathcal{L}}, \Gamma_{\mathcal{G}}\}) = AF(u, \sqrt{\ln 2}(\Gamma_{\mathcal{L}}/\Gamma_{\mathcal{G}})) + B, \quad (\text{A.12})$$

where $u = 2\sqrt{\ln 2}(\nu - \nu_0)/\Gamma_{\mathcal{G}}$. In this notation the Voigt profile is parametrized with a baseline B and amplitude A like the Gaussian curve, and $\Gamma_{\mathcal{L}}$ and $\Gamma_{\mathcal{G}}$ are the FWHMs of the underlying Lorentzian and Gaussian curves respectively.

We do not compute Voigt profiles by a convolution method, as this is time consuming and the in-built MATLAB function `conv.m` has stringent requirements on its input data, including evenly-spaced points in the frequency domain. Instead we use approximate numerical methods. Two algorithms were immediately available to us: the ‘Reichel¹ routine’ with quoted accuracy of 1 in 10^8 ; and the ‘Humlíček routine’ [167] which has accuracy quoted as only 1 in 10^4 . The MATLAB function `voigt.m` can be made to call either of the two routines: these subroutines are parametrized

¹This was developed by both Reichel and Chiarella [166].

as per the Faddeeva function. The main routine takes five parameters: $[B, A, \nu_0, \Gamma_{\mathcal{L}}, \Gamma_{\mathcal{G}}]$. Following earlier nomenclature, `voigtfun.m` computes the best-fit cost. We find that, over a few linewidths, both routines have residuals from the convolution fit of less than 3 in 10^6 . Also, there is little difference in the time required for computation with either routine; we therefore consider them interchangeable for the purposes of our calculations.

A.4.4 The Poissonian distribution

The Poissonian $\mathcal{P}$ distribution yields the probability of observing a certain number n of events in a particular region of space or time, with the proviso that the events do not overlap. It is given by [168]

$$\mathcal{P}_{\mu}(n) = \frac{\mu^n}{n!} e^{-\mu}, \quad (\text{A.13})$$

where over very many samples the mean number of events $\langle n \rangle \rightarrow \mu$.

A.5 Samples, populations and errors

If n samples of a measure x are taken from a population of $N \gg n$ then the mean and variance of this sample $\bar{x}$, s_x are given by

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = \langle x_i \rangle_i \quad (\text{A.14})$$

$$s_x^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu_x)^2 \equiv \langle x_i^2 \rangle_i - \langle x_i \rangle_i^2, \quad (\text{A.15})$$

where $\langle a \rangle_b$ is the mean of a averaged over parameter b . The sample mean is the best estimate [168] of the underlying population mean μ_x i.e. $\bar{x} = \hat{\mu}_x$. The best estimate of the underlying population's variance σ_x^2 is

$$\hat{\sigma}_x^2 = \frac{n}{n-1} s_x^2. \quad (\text{A.16})$$

The error in $\bar{x}$ as an estimate is

$$\sigma_{\bar{x}} = \sigma / \sqrt{n}. \quad (\text{A.17})$$

Appendix B

Calculations for quantum systems

B.1 Rate-equation calculations

B.1.1 The two-level system and saturation intensity

Following the conventions of section 4.3, the rate equations for a two-level system are:

$$\frac{dn_1}{dt} = -\frac{dn_2}{dt} = n_2 A_{21} + (n_2 A_{21} I - r_1 n_1 A_{21} I) \mathcal{L}(\nu), \quad (\text{B.1})$$

where $|1\rangle$ has a lower energy than $|2\rangle$, and I is measured in units of the saturation intensity I_s (see equation (4.19)). In the steady state this equation is equal to zero. If the total population is $n_1 + n_2 = 1$ then the rate of fluorescence from the excited state is then:

$$n_2 A_{21} = A_{21} \frac{r_1 I \Gamma^2 / 4}{(\nu - \nu_0)^2 + (1 + [1 + r_1] I) \Gamma^2 / 4}. \quad (\text{B.2})$$

Thus we see the definition for the saturation intensity is not simply one of doubling the transition linewidth. The intensity-broadened linewidth is

$$\Gamma' = \Gamma \sqrt{1 + [1 + r_1] I}; \quad (\text{B.3})$$

if $I = 1$ and both levels have equal degeneracies then the linewidth increases by a factor of $\sqrt{3}$.

B.1.2 The three-level system with decay

B.1.2.1 Derivation

The three-level system with decay is shown in figure 4.1(b), and we use the conventions discussed in that section: intensities I_i of the lasers addressing each transition of lower level i are measured in units of the saturation intensity of that transition; laser detunings δ_i in units of the HWHM ($\Gamma_0/2$). In order of increasing energy the states are $|1\rangle$, $|3\rangle$, $|2\rangle$. We begin with rate equations from the Einstein A and B coefficients:

$$\frac{dn_1}{dt} = n_2 A_{21} + n_2 A_{21} \mathcal{L}(\nu_1) I_1 - r_1 n_1 A_{21} \mathcal{L}(\nu_1) I_1 + n_3 a_{31} \quad (\text{B.4a})$$

$$\frac{dn_3}{dt} = n_2 A_{23} + n_2 A_{23} \mathcal{L}(\nu_3) I_3 - r_3 n_3 A_{23} \mathcal{L}(\nu_3) I_3 - n_3 a_{31}, \quad (\text{B.4b})$$

and we add the third condition, that population be conserved:

$$1 = n_1 + n_2 + n_3. \quad (\text{B.4c})$$

The steady state solutions correspond to setting the first two of these equations to zero. Immediately equation (B.4b) leads to a relation between n_2 and n_3 :

$$\begin{aligned} n_3/n_2 &= (1 + \mathcal{L}(\nu_3)I_3) / \left(r_3 \mathcal{L}(\nu_3)I_3 + \frac{a_{31}}{A_{23}} \right) \\ &= \left[\frac{\delta_3^2 + 1 + I_3}{r_3 I_3 + \frac{a_{31}}{A_{23}}(\delta_3^2 + 1)} \right], \end{aligned} \quad (\text{B.5})$$

which may then be substituted into equation (B.4a) to eliminate n_3 :

$$\begin{aligned} r_1 \mathcal{L}(\nu_1)I_1 n_1 &= n_2 \left\{ 1 + \mathcal{L}(\nu_1)I_1 + \frac{a_{31}}{A_{21}}[\dots] \right\} \\ n_1 &= n_2 \left\{ \frac{1 + \mathcal{L}(\nu_1)I_1}{r_1 \mathcal{L}(\nu_1)I_1} + \frac{a_{31}}{A_{21}} \frac{1}{r_1 \mathcal{L}(\nu_1)I_1} [\dots] \right\} \\ &= n_2 \left\{ \frac{1}{r_1} + \frac{\delta_1^2 + 1}{r_1 I_1} \left(1 + \frac{a_{31}}{A_{21}} [\dots] \right) \right\} \end{aligned} \quad (\text{B.6})$$

where we have used the shorthand $[\dots]$ to refer to the right-hand side of equation (B.5). The third of the set of simultaneous equations (B.4) may now be expressed entirely in terms of the population of $|2\rangle$, the excited state:

$$n_2 = \left\{ \frac{1}{r_1} + \frac{\delta_1^2 + 1}{r_1 I_1} \left(1 + \frac{a_{31}}{A_{21}} [\dots] \right) + 1 + [\dots] \right\}^{-1}. \quad (\text{B.7})$$

B.1.2.2 The stable state equations

If we imagine $|3\rangle$ to be stable i.e. in the limit of $\frac{a_{31}}{A_{21}}, \frac{a_{31}}{A_{21}} \rightarrow 0$, then we immediately recover the results of section 4.3.3.1. Equation (B.5) becomes (4.24); equation (B.7) becomes (4.23).

B.1.2.3 Absence of the repumping laser

We want to understand the ion's behaviour overnight, should the 866 cooling repumper undergo some drastic change in mode structure. This effect may be modelled in a number of ways, which are equivalent within the rate equations. One may set $I_3 \rightarrow 0$, or $\delta_3 \rightarrow \infty$: both of these are summarized in the limit $I_3 \mathcal{L}(\nu_3) \rightarrow 0$. Equation (B.5) immediately reduces to:

$$\lim_{\delta_3 \rightarrow \infty} \frac{n_3}{n_2} = \left[\frac{A_{23}}{a_{31}} \right], \quad (\text{B.8})$$

and consideration of population conservation then gives us the fluorescence rate on the transition $|1\rangle \leftrightarrow |2\rangle$:

$$\begin{aligned}
n_2 A_{21} &= A_{21} \left\{ \frac{A_{23}}{a_{31}} + 1 + \frac{1}{r_1} + \frac{\delta_1^2 + 1}{r_1 I_1} \left(1 + \frac{A_{23}}{A_{21}} \right) \right\}^{-1} \\
&= a_{31} \frac{A_{21}}{A_{23}} \left\{ 1 + \frac{a_{31}}{A_{23}} \left(1 + \frac{1}{r_1} + \frac{\delta_1^2 + 1}{r_1 I_1} \left(1 + \frac{A_{23}}{A_{21}} \right) \right) \right\}^{-1}.
\end{aligned} \tag{B.9}$$

We see this expression has all the correct physical dependencies. The branching ratio A_{21}/A_{23} gives the number of photons scattered on average before population is trapped in $|3\rangle$. The rate of decaying from the trap state is a_{31} . Finally there is an extra factor dependent on the pumping rate to the fluorescing level, because each decay on either the fluorescent transition or the trapping/metastable transitions puts the system in the ground state $|1\rangle$, from which the laser must pump it back to $|2\rangle$.

B.2 Bloch equations of the $|J = 0\rangle \leftrightarrow |J = 1\rangle$ transition

We present here a solution to the four-state Bloch equations describing transitions between $|J = 0, M_J = 0\rangle$ and $|J = 1, M_J = 0, \pm 1\rangle$ [169], where the level with higher J has the higher energy. Specifically we consider the $|^1S_0\rangle \leftrightarrow |^1P_1\rangle$ transition in neutral calcium having linewidth Γ_{5p} . Let the light field addressing the transition be detuned from the unshifted line centre by $\delta\nu_L$ and have linewidth Γ_{las} . We assume that the laser is polarized along an axis which defines the angular momentum states. Hence excitation can only occur on $|0, 0\rangle \rightarrow |1, 0\rangle$. Collisions act both to broaden the transitions (through phase interruptions) and to distribute population between the upper states. In principle the ambient magnetic field ~ 1 G will cause precession at $\sim \mu_B/h \equiv 1.4$ MHz in the excited state, but for the transition we are considering (the 423 nm transition in neutral calcium), the Einstein A coefficient $A \gg 1.4$ MHz. Hence precessive coherence is rapidly destroyed.

The Bloch equations comprise [107]

$$\frac{d\rho_{jj}}{dt} = \left(\sum_l i[\Omega_{jl}\rho_{lj} - \rho_{jl}\Omega_{lj}] + A_{lj}\rho_{ll} \right) - \Gamma_j \rho_{jj} \tag{B.10}$$

$$\frac{d\rho_{jk}}{dt} = \left(\sum_l i[\rho_{lk}\Omega_{jl} - \rho_{jl}\Omega_{lk}] \right) - \rho_{jk} \left(i\delta\nu_L + \frac{\Gamma_j + \Gamma_k + \Gamma_c + \Gamma_{\text{las}}}{2} \right). \tag{B.11}$$

where Ω_{jl} is the Rabi frequency characterizing the stimulated transitions between $|j\rangle$ and $|l\rangle$, $\delta\nu_L$ is the detuning of the laser from the $\Delta M_J = 0$ transition and Γ_c (Γ_{las}) is the line broadening owing to collisions (laser linewidth). Subscripts j, k, l vary over $\{g, -1, 0, +1\}$ for the ground state and the three excited states. The steady-state equations reduce to

$$0 = i\Omega(\rho_{0g} - \rho_{g0}) + A(\rho_{00} + \rho_{ee}) \tag{B.12}$$

$$0 = 2\pi\gamma(2\rho_{00} - \rho_{ee}) - A\rho_{ee} \tag{B.13}$$

$$0 = i\Omega(\rho_{00} - \rho_{gg}) - \rho_{g0}(i\delta\nu_L + \frac{\Gamma_c + \Gamma_{5p} + \Gamma_{\text{las}}}{2}) \tag{B.14}$$

$$1 = \rho_{00} + \rho_{ee} + \rho_{gg}, \tag{B.15}$$

where the density matrix elements are now

$$\begin{aligned}\rho_{00} &= |1, 0\rangle\langle 1, 0|, \rho_{ee} = |1, -1\rangle\langle 1, -1| + |1, 1\rangle\langle 1, 1|, \\ \rho_{gg} &= |0, 0\rangle\langle 0, 0|, \rho_{g0} = |0, 0\rangle\langle 1, 0| \equiv \rho_{0g}^*,\end{aligned}\tag{B.16}$$

the linewidth of the transition is Γ_{5p} and the J, M_J state is $|J, M_J\rangle$. Spin-flip collisions will in general not occur with the same frequency as phase-interruptive collisions. Let the collisional transfer rate between any two excited states¹ be $2\pi\gamma$. We define

$$2\beta = \Gamma_c + \Gamma_{5p} + \Gamma_{\text{las}}$$

and the steady-state solutions to the equations reduce to

$$\rho_{ee} = \frac{2\gamma\rho_{00}}{\Gamma_{5p} + \gamma}\tag{B.17}$$

$$\rho_{gg} = 1 - \frac{\rho_{00}(\Gamma_{5p} + 3\gamma)}{\Gamma_{5p} + \gamma}\tag{B.18}$$

$$\rho_{00} = \frac{1}{2} \frac{\beta(\Gamma_{5p} + \gamma)(\Omega^2/\pi^2)}{(\delta\nu_L^2 + \beta^2)\Gamma_{5p}(\Gamma_{5p} + 3\gamma) + \beta(\Gamma_{5p} + 2\gamma)(\Omega^2/\pi^2)}.\tag{B.19}$$

The population inversion $\delta\rho = \rho_{gg} - \rho_{00}$ is thus reduced by a Lorentzian in the laser's detuning from the transition centre. The linewidth of the inversion is

$$\Gamma' = 2\beta\sqrt{1 + 3I\frac{\Gamma_{5p}}{\beta}\left(\frac{\Gamma_{5p} + 2\gamma}{\Gamma_{5p} + 3\gamma}\right)},\tag{B.20}$$

where, as in the previous section, the laser intensity I is measured in units of the saturation intensity on the transition, defined in equation (4.19). In the absence of collisions the above simplifies to

$$\Gamma' = (\Gamma_{5p} + \Gamma_{\text{las}})\sqrt{1 + 3I\frac{\Gamma_{5p}}{\Gamma_{5p} + \Gamma_{\text{las}}}}.\tag{B.21}$$

¹The rate of transfer $|0, 0\rangle \leftrightarrow |1, \pm 1\rangle$ will generally not be the same as that for $|1, 1\rangle \leftrightarrow |1, -1\rangle$. However, we only consider the population ρ_{ee} by symmetry, and the assumption that the rates are the same will not affect the final answer.

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Colophon

This thesis was typeset in 11 point Times, using the MiKTeX distribution of L^AT_EX. WinEdt provided a front-end and editor. Schematics and diagrams were drawn with Canvas v3.52, and graphs with MATLAB v5.1.0. Timing diagrams were typeset using the L^AT_EX `timing` package.

Ambiguities of spelling and meaning were clarified using the online *Oxford English Dictionary* [170]. Where encyclopædic knowledge of technical terms was required, *Chambers' Science and Technology Dictionary* [171] was used.

The work for this thesis was conducted in the basement of the Clarendon Laboratory, Parks Road, Oxford, with the exception of section 4.3. This section was thrashed out in two sessions: one on the train from Oxford to Edinburgh, and one on a bicycle travelling through Barton and Headington.

Chapters 1–8 contain approximately 50,000 words.

The rainbow in a sunny shower may be called a mere appearance, and the rain the thing itself. This is correct, if the latter concept be taken in a mere physical sense. Rain will then be viewed only as that which, in all experience and in all its various positions relative to the senses, is determined thus (and not otherwise) in our intuition.

But if we take this empirical object in its general character and ask, without considering whether or not it is the same for all human sense, whether it represents an object in itself (and by that we cannot mean the drops of rain, for these are already, as appearances, empirical objects), the question as to the relation of the representation to the object at once becomes transcendental.

We then realise that, not only are the drops of rain mere appearances, but even their round shape—nay even the space in which they fall—are nothing in themselves, but merely modifications or fundamental forms of our sensible intuition, and that the transcendental object remains unknown to us.

Critique of Pure Reason,
Immanuel Kant, transl. Norman Kemp Smith