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**Dialoguing about engaging with communication and language
in relation to mathematics and its education**

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Abstract: This is our introduction to the series of eight articles in the first part of the double special issue on the challenges of ‘Engaging with communication and language in relation to mathematics and its education’. In the current article, we think together about ‘communication’ and ‘language’ in relation to mathematics and its education, by turning to reflections and examples from our decades of work as researchers and teacher educators, and in response to the articles in this first part. Following the preamble, most reflections are presented in the form of a fresh dialogue, through which we share some perspectives and anticipate several distinct approaches to communication and to language in contexts of mathematics teaching, learning and teacher education. In the epilogue, we concentrate on some of the reasons for reading the papers in this first part and the coming papers in the second part of the special issue.

Keywords: Communication, language and mathematics; mathematics education

Preamble

The word *preamble* comes from Latin, as a significant number of English words did, including English mathematics words. We could certainly have chosen the word *prologue* instead, which comes from Greek, as another significant number of English mathematics words did, again including English mathematics words. Either way, this preamble opens the introduction to a double special issue on the challenges of ‘Engaging with communication and language in relation to mathematics and its education’. We have divided the issue into two parts due to the large number of contributions, but any of the articles could go in one part or the other. In this introduction, rather than summarising each article and the themes crossing them, we consider some aspects of the complexity involved in many and varied challenges.

Complexity is the term that possibly best defines the conjunction of ‘language’ and ‘communication’ in respect to mathematics. Regardless of this complexity, or perhaps precisely

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because of it, the study of communication and language has attracted continuous attention from many mathematics education researchers across the world, including the three of us. One recent collaboration for the purpose of a survey work (Planas & Pimm, 2024) has allowed us to start discussing some of the presumptions that remain unaddressed, or even unknown to some extent, regarding interactions between understanding of communication and understanding of language, both in mathematics and in mathematics education. The aim of this double special issue is to bring together researchers with a variety of scholarly backgrounds at the intersection of mathematics education and issues of communication and language. The first of the two issues presents contributions, including some collaborative articles, by authors from Canada, England, Germany, Iceland, Ireland, Lebanon, Malta, Norway, Spain, and Turkey. These countries' cultural and linguistic diversity strengthens the value of this special issue project.

The distinction between language and communication is not so simple, despite some thinking that language is simply a subset of communication. 'Communication' is commonly described as a process of exchanging information between individuals, including signs, symbols and gestural behaviour. It is described as sending and receiving information, even digitally. It also makes meaning-making more than with words. Even gestures are presumed to be paralinguistic phenomena. But individual students and teachers also use language by themselves to work on mathematics and mathematics education (both writing and self-speaking). Thus, we come to 'language' and how it is commonly described as a means of communication and taken to be a system. Aspects of all this certainly come up in the articles that follow, in a variety of ways.

To open this first part of the double special issue, we think together about 'communication' and 'language' in relation to mathematics and its education, by turning to reflections and examples from our decades of work as researchers and teacher educators, and in response to the articles in this first part of the special issue. All these reflections and examples, around communication and language in mathematics education, suggest issues of complementarity and raise some further lessons that are discussed in the *epilogue*. This discussion is expanded and advanced by the other article contributions to this special issue (and the next one). Moreover, what makes this double special issue special compared with other special issues on language and communication in mathematics education research journals (e.g., Barwell et al., 2005; Morgan et al., 2014; Planas & Schütte, 2018) is the explicit invitation to authors to reflect upon the distinction between communication and language. Our reflections are in the form of a trialogue.

Triologue(s)

The choice of a triologue form, rather than regular prose for example, has several advantages. It permits to show fresh and more ambulatory thinking, at least metaphorically. This form also permits us to show that each of us is a careful reader/listener of the other two. The initial prompt given to David was to reflect freshly on communication and language in mathematics and its education. Then, David's reflection served as the prompt for Jenni's first contribution, and the reflections of David and Jenni were the prompts for Núria's subsequent contribution, and in this way, prompts accumulated. This triologue took time, with about one or two weeks between two consecutive entries, and online meetings for discussion in-between as well (see Figure 1).

Figure 1

Screen capture from one of our online meetings



The process followed permits us to discover and read subtrialogues on more specific topics that arose within *the* triologue. There are many coherent combinations of reading and meaning with which to experiment, made of any three entries, including the three entries of each of us. These different interconnected levels of structure are built up as the triologue progresses. The following triologue can therefore be approached in many active ways, but mainly through a diversity of subtrialogues. The communication exercise of what is emphasized or what is absent by different

readings of the triad is itself a representation of the complexity of meaning around ‘language’ and ‘communication’ in mathematics and its education.

David: I started and focused working on a single language, namely English, in relation to mathematics and mathematics education. To me, back then (nearly forty years ago: Pimm, 1987), ‘speaking’ and ‘communicating’ were equivalent, or at least ‘speaking’ was a subset of ‘communicating’. With regard to communication, I predominantly focus on more than a single person (after all, apart perhaps from practicing teaching, the self-communication is generally inside the brain and silent outside, except when writing to oneself). When teaching and learning mathematics, communication includes both speaking and writing (including writing extensive mathematical symbols, even though many symbols are not generally global, rather than using words for writing symbols: $\frac{3}{5}$ rather than ‘three-fifths’, Hewitt & Pimm, 2021). I think the mathematical symbols need to be turned into languages (mostly when spoken), and not just because of (English plus) letters being used mathematically (e.g., $ab + c$), or punctuation (such as comma, semicolon, colon, full stop, apostrophe, quotation, question mark, exclamation mark, bracket, hyphen, ...). There are also drawing diagrams and employing gestures (particularly concept gestures – see Marmur & Pimm, 2023) and making physical actions, and more recently involving computers. Gestures are predominantly silent (mostly apart from digital ones, as they actually involve some physical contact: and, by the way, ‘digital gesture’ is a metaphoric expression which, as very often in this case, alters the noun not the adjective). Nathalie Sinclair has engaged strongly with gestures, talk and diagrams (e.g., Sinclair & Gol Tabaghi, 2010), as well as digital gestures (e.g., Sinclair & de Freitas, 2014). But language is not simply a subset of communication, either. Because, among other reasons, there is more to come.

Jenni: My own start was an interest in interaction, which began while I was teaching mathematics in school. I found and still find the reflexive nature of classroom interaction intriguing, particularly in relation to how mathematics is shaped, built, and communicated within these interactions. The communication of mathematical ideas, concepts, practices, and so on is not just influenced by the words, symbols, gestures, etc. that we use, but also by how things are said. A pause in a particular place, emphasis on a word, how symbols

are written and spoken: do you write ‘5’ then ‘-’ and then ‘3’ when writing $\frac{3}{5}$ or ‘3’ then ‘-’ then ‘5’, or something else, and what do you say as you write them? Does the distinction between these orders place an emphasis on the role of the numerator, the role of the denominator, or the relationship between them for those watching, listening, and learning? David’s (1987) book *Speaking Mathematically*, along with teaching resources developed by Dave Hewitt, and questions and prompts developed by Anne Watson and John Mason, which I all encountered as a school teacher, inspired me to think about not only what I said, but also how I said or represented mathematics in my classrooms. These features often go unnoticed or unremarked upon by many, or even dismissed as unimportant when attention is drawn to them, and this is what has prompted me to investigate classroom interactions further. I was interested in the implicit communication within these interactions (Ingram, 2020). I have always found it challenging to define communication and language as each definition seems to place boundaries and limits on what is included. The term *language* also carries a lot of baggage for me, as I taught at a time where pedagogic advice for teachers either focused on named languages (and debates around whether students should be allowed to use their home language in the classroom) or where language was treated synonymously with vocabulary. This was highly relevant to me at the time, as I taught in a school where the vast majority of students were multilingual and often more comfortable communicating in a language unfamiliar to me, and the advice and support offered to me as a new teacher generally focused on vocabulary and translating this vocabulary. This was not enough for me, and it was not enough for many of the authors in this special issue that highlight communication and language in multilingual contexts (e.g., Morell & Planas, and Ní Ríordáin & Neville, this issue) as learning mathematics is more than learning words and symbols and routines for using them. It is about ways of thinking, working, and making sense of the world around us (Mason, 2006).

Núria: I am a teacher educator and researcher at the university now, but I also started as a mathematics teacher in a school where most of the students were recent migrants. There were other issues going on, like poverty and marginalisation. It was fascinating to see how those students made mathematical meaning out of the classroom tasks through

creative uses of their languages. Teaching was challenging, because there was one official language of instruction, namely Catalan, and because I could not understand some of the languages that supported the students' mathematical reasoning and its communication. Teaching the notions of least common multiple, *mínim comú múltiple*, and highest common factor, *màxim comú divisor*, of two or more numbers was challenging indeed. One day, it seemed to me that two girls from Bangladesh were unclear about the meanings for the words, *màxim* and *mínim*. In the pair work, I could listen to them talking about *el màxim* (i.e., the highest), *el mínim* (i.e., the least), *el mínim comú divisor* (i.e., the least common factor) and *el màxim comú múltiple* (i.e., the highest common multiple) of two and four, and much more in a language that I could not follow. The least common factor is not very interesting, because it is always one, and the highest common multiple can never be reached! At that time, I do not think I was aware of how fundamental language and languages are in mathematics teaching and learning, and I had never heard about translanguaging indeed. I could not see the opportunities raised by non-conventional mathematical labelling, nor could I simply imagine the possibility of the two girls translanguaging around the common divisors of two and four, identifying the lowest and then generalizing that result for any set of numbers. Some years later, I would find the Spanish translation of the splendid book by David (*El lenguaje matemático en el aula*), but multilingual mathematics was not an area touched. Multilingual mathematics and multilingual mathematics education might be a distinct story, perhaps away from the canon, which might well be worth a move into research. Of course, a strong tradition of multilingual mathematics education had been initiated. The teaching dilemmas of Jill Adler (2001) helped me to think about my teaching and early research, as much as the ideas of David had done.

David: As I mentioned above, for me, back then (nearly forty years ago), 'speaking' and 'communicating' were equivalent, or at least 'speaking' was a subset of 'communicating'. In the book, the subtitle 'communication in mathematics classrooms' included speaking, drawing and writing (both English words and mathematical symbols, as well as diagrams), but I did not focus on gesturing, as I do now. Written non-language (in particular, symbols) when spoken, or drawn non-language (diagrams) when described, have to be added into a language (in particular, more about a 'mathematics register',

which for me still is a subset of a singular language – Pimm, 2021 – and should be mentioned with a language (“the English mathematics register”) when more than one language is around). And, in passing, this is a common research author error, for, when a teacher’s or a student’s speech is transcribed into text, the symbols spoken in a specific mathematics register are often written down as symbols. But the mathematical symbols are mostly not part of any language (although some of them, including language semi-specific letters, get used in mathematics more generally). And even the differences between speech and writing in respect to mathematics are often insufficiently attended. And, even more, it is worth exploring distinctions between language and communication, as well as varying languages when students and teachers involve more than one (either publicly or privately). As for me, I am now retired from teaching mathematics and mathematics education at universities (age 70 in 2022), locations in three countries where I have had presence even since age eighteen (and starting teaching at age 21). So, all my texts about mathematics education are not specifically focused on self-school teaching. It is terrific that both of you have and are.

Jenni: I now *try* to avoid making distinctions between communication, interaction or language as the boundaries are fuzzy and open, but also mean different things to teachers and researchers, as well as between researchers. However, these distinctions are often necessary in communicating ideas. I focus my own research on classroom interactions, as I love the messiness and complexity in what is being communicated both explicitly and implicitly. But it is the implicit that intrigues me the most, because it can be about something that is taken-as-shared, something that relates to the norms of interacting in that particular classroom (Yackel & Cobb, 1996), or simply because those involved are unaware of what is being communicated. It can also be in tension with what is being communicated explicitly. The implicit aspects can influence what we understand mathematics as (Ingram & Riser, 2019), the understanding of mathematical ideas we develop (see Rønning, and Ward-Penny et al. in this issue), but also how we feel about mathematics. Alongside the implicit communication that arises in interactions around mathematics, there is also ambiguity around something that is treated as taken-as-shared. So much of communication relies on treating concepts, ideas, objects, images, etc. as shared – we would be here all day if we didn’t! But when we make mathematical

mistakes, is this due to a shortcoming in our understanding or a shortcoming in the communication? When we communicate about something that is taken-as-shared, some aspects about that thing are foregrounded in our communication, but our communication will not include all aspects and will not be precise – yet, as mathematicians, we pride ourselves on our precision.

Núria: Implicitness and explicitness, and the moves between these two in mathematics teaching, learning, and thinking, also intrigue me, especially the over-expectations that we often have on explicitness. As far as I am concerned, I know that we must be cautious about implicitness, but the impact of explicitness is also unclear. Making something explicit is not necessarily evidence of some understanding or reasoning being enacted. This is very much the case with connectives. When a word like ‘because’ is explicit or present, for example, we often assume that some explanation is involved. But there are many types of ‘because’ sentences being said in the mathematics classroom. One of my doctoral students, who is a mathematics teacher and who is an author (see Morell & Planas, this issue), was concerned with the pupils’ misuse of logical connectives, which, for him, was not supporting mathematics reasoning. He then had the idea of asking pupils to complete sentences such as, ‘two is the first prime number because ...’, and obtained completions such as, ‘... because two is a prime number and because it is first’, which is true but does not explain much. The teacher did not expect this repetitive reasoning. Only enacting students to use ‘because’, or making ‘because’ explicit, had not been sufficient. Michael Halliday’s (2002) *On Grammar* shows that, alongside the presence of certain words and phrases, the communication of the theme (which may be number two, or the concept of prime number) is critical in any linguistic practice of reasoning. I would add that any linguistic practice of reasoning needs discursive support, including paralinguistic cues such as shifts in tone.

David: And so, a triple trio begins – a third sizeable paragraph by each of we three in this section, termed the triologue. The fact that we have authors coming from a significant number of countries, and certainly not just in initial English authors, can also involve language specification and diversion when engaging with mathematics and mathematics education. English is not global, but it has some elements (including letters used in algebra, for

instance) which have significant use in mathematical symbols broadly across the world. But there are also significant diagrams, which are not necessarily language-related (though they certainly would be described so). And computers are relatively recent, not least in regard to mathematics and mathematics education, and while not specifically human, they produce everything that humans do, and more. Although describing computer gestures is intriguing, because it is not yet possible not to touch the computer.

Jenni: Decades of research have illustrated the complexities in the relationships between communication, language, mathematics, and mathematics education. The articles in this double special issue exemplify some of this complexity and highlight the diversity in framings, in focus, and in the aspects of communication that are considered. So too does this triologue. Each of us draws on different theories, experiences, and literature, in mathematics education but not only, as we examine communication in our research and our teaching. All of these authors, including ourselves, have our own interests and foci, and this drives us to pay attention and foreground different aspects and features of communication, including different aspects and features of language. This, in turn, leads to a richness in collections such as this double special issue where the whole is more than the sum of its parts.

Núria: This experience of a triologue has been new to me! What is becoming less and less new to me is the experience of collaborative diverse teams with mathematics education researchers and language, and language education researchers. I now realise that the first part of this double special issue does not show any team of this discipline-diverse type. The second part perhaps will do. We need to learn so much about communication and language, but, after all, this is not our original area of knowledge or expertise. We struggle with notions such as translanguaging that colleagues in language and language education research may understand with specific theoretical depth. We need their knowledge and expertise. So, we should ask ourselves where are the language and language education experts in our own studies and in general in the domain of mathematics education research on communication and language. This question is more than critical. In my case, the language education department is physically next to the mathematics education department, same building, same flat, and same toilet. Although it has been like this for

many years, it was only recently that I introduced myself to Emilee Moore as a language person too – I was moved by the reading of one of her collaborative studies on translanguaging practices (Bradley et al., 2017) – and started discussions with her, and even collaborations (Planas et al., 2025). I am not that sure that we, mathematics education researchers on communication and language, should be the only ones driving the bus of the domain. Although a focus on language and on communication is not a focus on mathematics or mathematics education, of course.

Epilogue

While a *preamble* tends to designate an initiation, an *epilogue* – which is now a word that comes from Greek – tends to designate what merely concludes a work or the final part of something. This section, however, is not an epilogue in any strict sense. It is at most an epilogue-like section, because its content is associated with the triologue and, at the same time, with the story that the rest of the articles following this in the special issue tell us. There is possibly and hopefully a triologue continuing among the three of us, the writers of the articles, and the readers. That said, we particularly reflect on our triologue and the articles that follow in this first part of the special issue to consider what we learn about the relationships, interplay, and distinctions around ‘communication’ and ‘language’ in relation to mathematics and its education. Not only do these reflect the variety and diversity of research currently taking place focused on communication and language in mathematics education, but also the variety and diversity of modes (Rønning, this issue) and paralinguistic elements (Okumus, this issue) we use to communicate mathematics and mathematically. In this first instalment, the focus is largely on the relationships among symbolic, written, and spoken communication modes, in a range of languages and contexts, including emerging contexts involving AI agents (Gíslason, this issue). Interactions not only between people or computers, but also across and between languages (El Mouhayar & Barwell, Farrugia et al., Morell & Planas, and Ní Ríordáin & Neville, this issue), discourses and registers (Ward-Penny et al., this issue), and traditions (mathematical and research). Our own meanings attached to words like *communication* and *language* have evolved over time as the research evolves and as we gain richer experiences of the complexity of communication and language in the processes and contexts of teaching and learning mathematics. The articles that follow this trio piece capture much of this

complexity through the intense focus of each study. We learn much from each subsequent article and from looking across the articles and those that will follow.

Importantly, the complexity entailed in the study of communication and language in mathematics education is better understood today because multilingualism in mathematics education is better understood. Mathematics teachers and educators are learning to take advantage of the opportunities that all learners, including multilingual learners, bring to the teaching contexts, and researchers are seriously considering the study of these opportunities and of how to develop them further. What multilingual learners, educators, and researchers can teach us about language and communication in mathematics education is much, and not totally explored, but is visible in the articles in this special issue, both those with a focus on multilingual contexts and those without this focus. Nonetheless, the tracing and the mashing of multimodal–multilingual practices remain uncommon across sites of mathematics education and mathematics education research. Multilingualism is one resource within a sophisticated semiotic repertoire for communication that incorporates images, gestures, and even tastes and smells, as well as languages. Communication cannot be understood solely from a linguistic point of view, the (multilingual) characterisation of communication is necessarily a linguistic and a non-linguistic composition. In all this, as we proceed to develop theoretical and analytical frameworks to consider practices of communication, including those that are multilingual and multimodal, we may need to consider further engagement and collaborations with communication, language and linguistics experts so that we can understand what they understand. Many of the articles in this special issue draw on research and literature beyond mathematics education and this has been fundamental to our growing understanding of communication and language in mathematics education.

As for the mathematical gestures, which are significant of communication but not necessarily of language, these too will appear and will be discussed in at least a couple of articles in the second part of the special issue. There is much to be said about gestures, the meaning they convey, the set of uses, and the thinking they enact, sometimes along with language, speech, and other gestures. As Marmur and Pimm (2023) stated, “Gestures, whether derived from teachers/lecturers or students in relation to mathematics education, are becoming more and more present in our field. Our focus here is on concept gesture, which can also be seen as a body metaphor, where (part of) the body attempts to enact or depict the mathematical concept dynamically, both somewhat idiosyncratically yet also, to some extent, generically.” (p. 323). Finally, as semi-mentioned at the

beginning of this final section, the term *epilogue* actually comes from the Greek word *epilogos*, which means “conclusion word” and is always at the end. It is therefore, as mentioned at the first section, the opposite of prologue (the Greek word *prologos*), right at the beginning of this article (though we used *preamble* instead). And for we three, at least, and without any gesture, the last word of all is *final*.

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