The Mathematics Enthusiast

Volume 24
Number 1 *Number 1*

Article 9

2-2027

From words to worlds: Concluding reflections and emerging directions

Jenni Ingram

Núria Planas

David Pimm

Recommended Citation
Ingram, Jenni; Planas, Núria; and Pimm, David (2027) "From words to worlds: Concluding reflections and emerging directions," *The Mathematics Enthusiast*: Vol. 24 : No. 1 , Article 9.
DOI: https://doi.org/10.54870/1551-3440.1702
Available at: https://scholarworks.umt.edu/tme/vol24/iss1/9

From words to worlds: Concluding reflections and emerging directions

Jenni Ingram[1], Núria Planas and David Pimm
University of Oxford; Universitat Autònoma de Barcelona; Simon Fraser University

Abstract: In concluding the second series of eight articles in the double special issue on 'Engaging with communication and language in relation to mathematics and its education', we take the opportunity to reflect across the articles to prompt us all to consider potential and emerging directions for future research. This final article is overall a commentary inspired by the communication-centred and language-centred contributions to mathematics education across the entire collection of articles, that is, all those comments that situate mathematics and its education as language and communication praxis. We explore this relationship between mathematics and its education with communication and language through three themes: time, theories and methodologies, and the nuances of the term 'relation' in the title of this double special issue.

Keywords: Communication, language and mathematics; mathematics education; commentary

Not a trialogue, but three themes

In this article we conclude our double special issue by drawing connections between the articles published in volume 23 nos. 1&2 and the articles published in this issue, volume 24 no. 1. Although the articles in the first and second parts of this special issue, titled 'Engaging with communication and language in relation to mathematics and its education', are different in many ways, they all reflect issues and complexities around communication and language in mathematics education. They also highlight similarities that suggest the potential for three integrative themes. Perhaps we have heard at some point that a theme is what remains after a reading or analytical process has finished. Scholarly speaking, in some theories of communication and language, a theme refers to the information structure and the components of a message given by a clause; in others, a theme refers to the relationship between what is being said and what has gone before in the discourse and its organization into an act of communication; and in this way we could continue to report many other meanings for what a theme is. In this commentary, we consider themes as both personal and collective,

[1] Jenni.Ingram@education.ox.ac.uk

manifesting themselves in joint discussions of a work of art or image, an idea or a point of view, a collection of papers, a situation or an event, a proposition…

The three themes named and developed below are inherently personal, just as the introduction to the first part of the special issue and this commentary are personal too. They are tacit and indirect, not explicit or immediate, because they cannot be clearly traced in an article or a group of articles. None of these themes are directly discussed within any of the articles. Our themes are absent from the writing or thematising of the articles, yet present in both our individual and collective reading. Theme 1 represents a focus on time, drawing on notions of timescales and storylines, hence encouraging reflection across current work as a way to look forward. Theme 2 represents a focus on theories and methodologies, used within mathematics education but sometimes generated outside it. Theme 3 represents a focus on the different aspects of the title of this special issue, building on the discussion in our introductory article, but moving the focus to the deliberate separation of the terms ‘mathematics’ and ‘education’. The question of why these themes, as said, is personal is, at the same time, the question of why they are not personal only but communicational. We had to decide whether a theme that Jenni noticed, for example, was a theme that Núria and David noticed as well, and for this, we had to engage in three-sided communication, not without issues of meaning, form and translation. It would be interesting to find out whether other readers of the double special issue agree with us.

Theme 1 – a focus on timescales and storylines

As we conclude this pair of special issues, it strikes us that different storylines and timescales are at play (Herbel-Eisenmann et al., 2015). When we introduced the double special issue, we focused on the complexity of the challenges the authors in the first series of articles addressed in the context of thinking about the distinction between language and communication, not in general, but in mathematics education. To do this, we reflected on our own biographical timescales in the form of a trialogue, highlighting the similarities and differences in the contexts, theories and perspectives within our own careers. We reflected on different research, professional, and life experiences at different chronological times, and on how we experience communication not only with those groups or people present in time, but also with imagined groups or remembered people who influence our decisions. If we were secondary-school mathematics teachers decades ago, we could both imagine and remember the experiences with those students in the past and construct them in ways that influence our present actions, research, and teacher education. This reflection across diverse timescales and storylines helped us understand how different our experiences are, and have been, and how similar our sensitivity towards language and communication in mathematics education is.

The articles within the double special issue have operated across a range of timescales, ranging from what Lemke (2000) calls *Vocal articulation* (though in the context of communication as addressed in some of the articles, this articulation may not be vocal) taking around 10^{-1}seconds, to what he calls *Multi-year curriculum,* taking around 10^{8} seconds (3.2 years). Both Hewitt & Ingram's and Lin's articles in this second part illustrate the micro-temporal analyses that draw not only on speech, but also gestures, gaze and interactions with technologies. Kimber & Smith's, Skelton & Spiro's, and Nimkar & Gautham's articles align more closely with Lemke's lesson or lesson sequence scale, with a focus on tracing language and communication across lessons or sessions. With more of a focus on mathematics teacher education, González-Forte & Planas' and Tiong & Tan's articles also look across sessions, but with what Lemke describes as lifespan educational development by focusing on practices that develop over a teacher's career. Diaz-Rojas' article is interesting in that it combines a micro-temporal analysis at the vocal articulation timescale with an analysis that looks across a sequence of sessions.

These articles also draw on different storylines, some mathematical, some social or cultural, some systemic or political. The different contexts, theoretical perspectives and methodological approaches highlight these different storylines and timelines, which contribute to the richness and complexity of research in this area. Whereas context, space and time tend to be seen from a micro perspective, mostly limited to situations, theories and methodologies tend to be constructed differently, rather aligned with a macro representational perspective, and work at timelines spanning years, decades and even centuries. This is why we positioned reflections on theories and methodologies as our second theme.

Theme 2 – a focus on theories and methodologies

The maturity of research focusing on communication and language in mathematics education is reflected in how theories and perspectives initially borrowed from linguistics, sociolinguistics, and psychology, as well as mathematics, have evolved to address the particular concerns and contexts of mathematics education research. This evolution has involved the refinement, adaptation, and synthesis of theories that remain consistent with their original conceptual foundations while addressing the concerns in mathematics education research. Each article in this double special issue exemplifies a different evolution and synthesis of theories and methodologies. For example, the article by Skelton and Spiro (2027) in this new series synthesises linguistic theories, cognitive theories on the learning of division, and the pedagogic practices of translanguaging. Similarly, the article by Diaz-Rojas (2027) integrates the Discipline of Noticing (Mason, 2002), the sociological theory and methods of ethnomethodology (Garfinkel, 1967), and Fischbein's theories of intuition in the context of probability (Fischbein & Gazit, 1984). The evolution of theories to address the specific challenges in researching communication in mathematics and its education is

illustrated most clearly in the articles by Lin (2027)'s use of socio-materialism and by Tiong and Tan (2027)'s adaptation of the Knowledge Quartet (Rowland et al., 2005).

Looking forward, these theories will need to continue to evolve as the contexts and problems addressed in research continue to change. The rapid development of technologies and artificial intelligence, the social and cultural inequities that continue to exist around the world, the environmental challenges of climate change, depleting resources, and reducing biodiversity, and the new norms of multilingual, multicultural and globalised communities all require us to think differently and creatively to develop theories and methodologies that can capture and address the complexity of these challenges. The theories and methodologies need to span different timescales, as these challenges manifest themselves across these different timescales. Research focused on communication in relation to mathematics and its language is becoming increasingly important as many of these challenges affect not just what we are communicating, but also how we are communicating and the impact this has on the world in which we live.

The technological changes surrounding us, in particular, open up opportunities for new methods and methodologies. Multimodal and multilingual communication is becoming easier, but also more complex, as illustrated in the articles by Gíslason (2026) and Hewitt & Ingram (2027). However, we must take up these opportunities in responsible ways that develop solutions, improve the education of mathematics, and challenge existing inequalities rather than exacerbate them.

Theme 3 – a focus on the different aspects of the title

In the introduction, we explored the distinctions between communication and language in the title of this double special issue. We continue this exploration by turning our focus to the phrase "in relation to mathematics and its education" which highlights the unique and special context of mathematics, with its multi-semiotic formations and norms of communication (Pimm, 1987; Schleppegrell, 2007; Yackel & Cobb, 1996). This is most strongly illustrated in the articles by Farrugia et al. (2026), Ward-Penny et al. (2026), Rønning (2026), and El Mouhayar & Barwell (2026) in the first series of articles, and by Kimber & Smith (2027) and Hewitt & Ingram (2027) in this second series of articles. Yet the unique and special features of mathematics appear in all the articles in ways that have influenced the development and evolution of the theories used.

The title also refers to "its education". The separation of mathematics and education was deliberate as it emphasises the centrality of mathematics, which is key to research in this area. The education of mathematics cannot happen without communication and language, but how we communicate and interact affects the mathematics that we learn (Pimm, 1987; Planas et al., 2018; Radford & Barwell, 2016; Sfard, 2008). This is reflected in the increasing focus of

research addressing communication in the contexts of teacher knowledge (e.g., Tiong & Tan, 2027) and teacher education (e.g., González-Forte & Planas, 2027), teaching practices (e.g., Kimber & Smith, 2027; Nimkar & Gautham, 2027; Skelton & Spiro, 2027), and the engineering and design of learning situations (e.g., Diaz-Rojas, 2027; Hewitt & Ingram, 2027).

The power of the word 'its' in the title is undeniable. Such a tiny word unit emphasises relational meaning that goes with mathematics and education when they are put together. 'Mathematics and its education' could have been 'education that is mathematical', 'mathematical education', 'mathematics education', or possibly 'it is mathematics and it is education', but it would not be the same indeed. The process by which 'its' is placed and used in the title and in the double special issue can be called 'context'. The meaning of 'its' as critically relational, not possessive, becomes clearer in the process of reading the title and the collection of articles. The meaning of 'its', or 'its' meaning, is not limited by or reduced to the content meaning given in the dictionary, because we need the context. And context is another concept that shapes all the articles in the special issue, and the three themes in this commentary. If we can understand the meaning of words, it is because we imagine them in contexts, which are at the same time representations of a variety of worlds. So, we go 'from words to worlds', and back 'from worlds to words.'

Conclusion

In their introductory remarks to the second part of a book, Sriraman and Lee (2011) wrote that "a commentary cannot occur *in the void* meaning that it needs to be anchored in what is already existent in the literature" (p. 367). Luckily, in the research domain of mathematics, language and communication, there is abundant and very important written work. We have taken the contributions to the two parts of the special issue as the foundation and valuable literature supporting the current commentary.

The three themes in our commentary and the articles in the two parts of the double special issue aim to provide a coherent framework for the study (and concepts) of communication and language in mathematics education. The variety of timescales, storylines, theories, methodologies, contexts and foci helps us to notice that there is more than language(s) when studying communication, and that there is more than speaking when studying language; and this variety helps us to notice how distinctions are practised in interaction, in episodes or in and across lessons or sessions with participants and empirical data. In all this, we see an overlap between the more micro-perspectives linked to local situations at relatively small timescales and the more macro-perspectives at larger timescales and drawing on storylines linked to theorising and conceptualisation. Theories and methodologies are not operations of

‘pure’ abstracting, but they are signified by means of local practice and concrete participants engaged in worlds impacted by communication and language.

And now it is the end. We depart by noting one final theme of ‘three’. Three editors, who began with a trialogue, who identified three themes across the articles, and can imagine a third part to the special issue as we look to the future. We three have been fascinated by content, the data, and the arrival of the conclusions and contributions of the articles. A splendid event. Now it is finally done.

References

Diaz-Rojas, D. (2027). From uncertainty to dialogue: Visualising collaborative noticing and mathematical intuitions in students’ narratives within probability learning. *The Mathematics Enthusiast*, *24*(1),

El Mouhayar, R., & Barwell, R. (2026). Genre-based socialization practices in teaching and learning of geometric proof in language diverse classrooms. *The Mathematics Enthusiast*, *23*(1–2), 141–166. https://doi.org/10.54870/1551-3440.1684

Farrugia, M. T., Vella, L. A., Muscat, D., & Sammut, M. C. (2026). Mathematics in Maltese: The compilation of a bilingual English–Maltese glossary to support mathematics teaching and learning in Malta. *The Mathematics Enthusiast*, *23*(1–2), 15–36. https://doi.org/10.54870/1551-3440.1678

Fischbein, E., & Gazit, A. (1984). Does the teaching of probability improve probabilistic intuitions? An exploratory research study. *Educational Studies in Mathematics*, *15*(1), 1–24. https://doi.org/10.1007/BF00380436

Garfinkel, H. (1967). *Studies in ethnomethodology*. Prentice-Hall.

Gíslason, I. (2026). Learning about human mathematical dialogue from dialogue with chatbots: Babbling, gargling and funnelling. *The Mathematics Enthusiast*, *23*(1–2), 167–184. https://doi.org/10.54870/1551-3440.1685

González-Forte, J. M., & Planas, N. (2027). Pre-service teachers’ noticing of the function of mathematical naming in the teaching of fractions. *The Mathematics Enthusiast*, *24*(1).

Herbel-Eisenmann, B., Wagner, D., Johnson, K. R., Suh, H., & Figueras, H. (2015). Positioning in mathematics education: Revelations on an imported theory. *Educational Studies in Mathematics*, *89*(2), 185–204. https://doi.org/10.1007/s10649-014-9588-5

Hewitt, D., & Ingram, J. (2027). Multimodal interpretation of notation. *The Mathematics Enthusiast*, *24*(1).

Kimber, E., & Smith, C. (2027). Affordances for graphical shape thinking through vivid informal classroom talk. *The Mathematics Enthusiast, 24*(1).

Lemke, J. L. (2000). Across the scales of time: Artifacts, activities, and meanings in ecosocial systems. *Mind, Culture, and Activity*, *7*(4), 273–290. https://doi.org/10.1207/s15327884mca0704_03

Lin, Q. (2027). Affectivity as a form of communication during interactions between parent, child, and TouchCounts. *The Mathematics Enthusiast, 24*(1).

Mason, J. (2002). *Researching your own practice: The Discipline of Noticing*. Routledge. https://doi.org/10.4324/9780203471876

Nimkar, J., & Gautham, N. R. (2027). Co-creation of classroom language: Supporting learners to develop expressions of uncertainty and statistical concepts. *The Mathematics Enthusiast*, *24*(1).

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. Routledge & Kegan Paul.

Planas, N., Morgan, C., & Schütte, M. (2018). Mathematics education and language: Lessons and directions from two decades of research. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger, & K. Ruthven (Eds.), *Developing research in mathematics education. Twenty years of communication, cooperation and collaboration in Europe* (pp. 196-210). Routledge.

Radford, L., & Barwell, R. (2016). Language in mathematics education research. In A. Gutiérrez, G. C. Leder, & P. Boero (Eds.), *The second handbook of research on the psychology of mathematics education: The journey continues* (pp. 275–313). Sense Publishers.

Rønning, F. (2026). The role of the context for pupils' choice of solution strategies and representations when working with combinatorial problems. *The Mathematics Enthusiast*, *23*(1–2), 119–140. https://doi.org/10.54870/1551-3440.1683

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, *8*(3), 255–281. https://doi.org/10.1007/s10857-005-0853-5

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, *23*(2), 139–159. https://doi.org/10.1080/10573560601158461

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourse, and mathematizing*. Cambridge University Press.

Skelton, J., & Spiro, J. (2027). Linguistic and conceptual transfer: Crosslinguistic translanguaging in the English–French bilingual mathematics classroom in the UK. *The Mathematics Enthusiast, 24*(1).

Sriraman, B., & Lee, K. H. (2011). Commentary on Part II. In J. Cai, & E. Knuth (Eds.), *Early algebraization. Advances in mathematics education* (pp. 367–373). Springer. https://doi.org/10.1007/978-3-642-17735-4_19

Tiong, P., & Tan, S. (2027). Teachers' knowledge and use of mathematics register for teaching and communication. *The Mathematics Enthusiast*, *24*(1).

Ward-Penny, R., Ingram, J., Erath, K., Kimber, E., & Planas, N. (2026). Equals in the wild: How mathematical equality is talked about in lessons. *The Mathematics Enthusiast, 23*(1–2), 55–74, https://doi.org/10.54870/1551-3440.1680

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, *27*(4), 458–477. https://doi.org/10.5951/jresematheduc.27.4.0458