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WELFARE-INCREASING MONOPOLIZATION*

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The conditions for monopolization to be good for social welfare are examined. Social welfare can be higher when a monopoly sells to a monopoly, with double margins, than when a competitive industry sells to a downstream Cournot oligopoly with differing efficiency levels. This requires inverse demand to be sufficiently concave, and cannot hold when demand is convex. When there are no vertical issues conditions are found for elimination of an inefficient firm to raise welfare, building on Lahiri and Ono (1988). In general greater demand concavity increases the relative importance of the benefit of redistributing output to the efficient firm.

I. INTRODUCTION

I provide conditions for monopolization to raise social welfare because of the benefits of the redistribution of output to the most efficient firm. The shape of the inverse demand function matters. The more concave is inverse demand the more likely it is that monopolization will increase welfare. Two related issues are covered. First, I consider a perfectly competitive upstream industry selling to downstream oligopolists with different efficiency levels. Upstream monopolization causes the downstream industry to become a monopoly and double margins are introduced. Nevertheless a monopolist selling to a monopolist may be better for social welfare than a competitive industry selling to an oligopoly. Second, I abstract from vertical issues and consider a simple question: when will eliminating an inefficient firm from an oligopoly be better for social welfare than retaining that firm? Lahiri and Ono [1988] show that eliminating a minor firm can raise welfare. I re-examine their model, with the particular focus on how such a possibility depends on the shape of demand.

The conditions under which upstream monopolization raises welfare are special but not paradoxical. Inverse demand must be strictly concave. The more concave is inverse demand the greater is the benefit from the reallocation of output. At the same time more concavity reduces the effect of an increased

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wholesale price on the retail price, that is, the pass-through coefficient. The resulting impact on consumers of the retail price increase induced by upstream monopolization is reduced. For a general discussion of demand curvature in the context of welfare analysis see Kang and Vasserman [2022].

In a recent paper Ghosh *et al.* [2022] consider a related model to the first one in this paper. An upstream industry whose firms have different efficiency levels supplies a downstream retail sector. There is Cournot competition at each stage. A merger in the downstream sector can raise social welfare if demand is sufficiently concave. The merger induces the most efficient upstream firm to increase its output, while total output falls.

For an oligopoly with no vertical issues elimination of an inefficient firm can be welfare improving if demand is strictly log-concave. This is a weaker condition than strict concavity which is necessary in the vertical model. I also show that with duopoly and log-convex demand (for example when there is a constant elasticity) welfare is always higher when both firms produce.

Section II contains the standard analysis of Cournot oligopoly when marginal costs differ. Section III provides conditions for a marginal increase in the wholesale price to have a positive effect on social welfare and shows that upstream monopolization can raise welfare. Section IV reconsiders the model of Lahiri and Ono [1988] and provides conditions for the elimination of a firm to raise welfare when there are no vertical issues. Conclusions are in Section V.

II. DOWNSTREAM COURNOT COMPETITION

The downstream market has $n \geq 1$ retailers who are Cournot players. Retailer i buys quantity q_i and sells this to final consumers, paying the wholesale price w and incurring a marginal retail cost of $c_i \geq 0$. The retail costs differ across retailers. The simple average is $\bar{c} = \frac{1}{n}\sum_{i=1}^{n} c_i > 0$ and the variance is $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(c_i - \bar{c})^2 > 0$. There are no fixed costs. Inverse demand for the downstream industry, $p(Q)$, where $Q = \sum_{i=1}^{n} q_i$ is total output, is strictly decreasing and twice differentiable. Assume that $p(0) > c_i$ for all i. Define $E(Q) \equiv -\frac{Qp''(Q)}{p'(Q)}$ as the curvature of inverse demand. Demand is concave if $E(Q) \leq 0$ and convex if $E(Q) \geq 0$. It is log-concave if $E(Q) \leq 1$ and log-convex if $E(Q) \geq 1$.

The profit of retailer i is $\pi_i(q_i, Q_{-i}) = (p(q_i + Q_{-i}) - c_i - w)q_i$ where $Q_{-i} = \sum_{j \neq i}^{n} q_j$ is the output of the other retailers. This is assumed to be strictly quasi-concave in q_i so that the second-order conditions hold: a sufficient (but not necessary) condition for this is that $E(Q) < 2$ so that industry marginal revenue is decreasing.[1]

[1] An alternative sufficient condition for strict quasi-concavity is that direct demand curvature, $-pQ''(p)/Q'(p)$, is strictly below 2. This implies that the second-order condition holds whenever the first-order condition holds with equality.

The first-order condition for firm i, taking others' outputs as given and assuming at this stage that $q_i > 0$ for all i, is

$$p(Q) + q_i p'(Q) - c_i - w = 0. \tag{1}$$

The market share of firm i is $s_i \equiv \frac{q_i}{Q} \leq 1$. The Nash equilibrium in the retail market is characterized by the n first-order conditions of form (1). Adding these gives

$$np(Q) + Qp'(Q) = n(\bar{c} + w). \tag{2}$$

The left-hand side of (2) is strictly decreasing in Q if $n + 1 - E(Q) > 0$, which is assumed to hold and is Seade's stability condition (Seade [1980]). A sufficient condition is that $E(Q) < 2$. It follows that the Cournot equilibrium is unique—see Kolstad and Mathiesen [1987], Gaudet and Salant [1991] and Vives [2001]. The right-hand side of (2) shows that total output is a function of the average of the retail costs plus the wholesale price, and does not depend on the variation in retail costs. Bergstrom and Varian [1985] discuss this feature of Cournot oligopoly with different costs, which applies when the number of firms is unchanged.

This framework is now used in two different applications: upstream monopolization when the downstream firms have different efficient levels; and the effect on welfare of a cost increase for an inefficient firm when there are no vertical issues. In the latter case the analysis above applies with $w = 0$.

III. UPSTREAM MONOPOLIZATION

I first show the effect on social welfare, at the margin, of an increase in the wholesale price that a downstream industry faces, before considering upstream monopolization (which will increase the wholesale price above cost to the profit-maximizing level). Initially the upstream industry is perfectly competitive. The wholesale price is equal to upstream marginal cost, which is assumed to be constant and, without loss of generality, equal to zero. When the upstream industry is monopolized the wholesale price is raised above zero, and allowance needs to be made for inefficient retailers to leave the market when their margins become negative. Assume that there is a single downstream firm with the lowest cost technology.

How does an increase in the wholesale price affect social welfare at the margin? The question is approached in stages. From (2) the effect of w on total output is negative,

$$\frac{dQ}{dw} = \frac{n}{p'(Q)(n + 1 - E)} < 0 \tag{3}$$

and the effect on the retail price, the cost pass-through coefficient, is positive,

$$\frac{dp}{dw} = p'(Q)\frac{dQ}{dw} = \frac{n}{n+1-E} > 0. \tag{4}$$

Corchón [2001], Proposition 2.7, derives the same formula as (4). From now on the dependence of E on Q is understood. As demand becomes more concave, that is E is a more negative number, the pass-through coefficient falls.

To examine the welfare effect of the wholesale price increase we need to find the effect on each individual retailer. The effect of the wholesale price on the output of an individual retailer is written here as the derivative of total output, $\frac{dQ}{dw}$, which is negative, multiplied by the fraction of the total output reduction that is attributable to firm i:

$$\frac{dq_i}{dw} = \frac{dQ}{dw}\left(\frac{1}{n} + E\left(s_i - \frac{1}{n}\right)\right). \tag{5}$$

Equation (5) implies that if demand is linear ($E = 0$), or if there is no retail cost variation so $s_i = \frac{1}{n}$, each firm reduces output by the same amount. When, however, demand is nonlinear and there is retail cost variation so that market shares differ the effect of the wholesale price increase on a firm's output depends on whether demand is convex or concave. If demand is strictly convex then more efficient firms, with $s_i > \frac{1}{n}$, reduce their outputs by more than the less efficient firms, as $\frac{1}{n} + E\left(s_i - \frac{1}{n}\right) > \frac{1}{n}$. Inefficient firms may even raise their outputs if $E > 1$.[2] The redistribution of output effect goes the wrong way when demand is strictly convex. Seade [1985] notes that when $E > 0$ a common cost increase, such as an excise tax, penalizes efficient firms—see also Fevrier and Linnemer [2004] and Van Long and Soubeyran [1997]. With strictly concave demand, however, efficient firms reduce their outputs by less than inefficient firms. In fact an efficient firm will raise its output if its market share is large enough.

Proposition 1. An increase in the wholesale price induces retail firm i to raise its output if $s_i > \frac{1-E}{-En}$ while demand is strictly concave.

Proof. The condition implies that $\frac{1}{n} + E\left(s_i - \frac{1}{n}\right) < 0$ and thus that (5) is positive. ■

For example if $E = -5$ and $n = 5$ firm i raises its output in response to a wholesale price increase if $s_i > 0.24$. Market shares with equally efficient firms

[2] A firm increases its output when demand is strictly log-convex if its market share is small enough with $s_i < \frac{E-1}{En}$.

would equal 0.2 in this example, so it is plausible that a firm could be efficient enough to increase its output. If, counter-factually, the other firms were to keep their outputs constant then firm i would reduce q_i. This is the direct effect of the wholesale price increase. The equilibrium response of the other firms, however, is to cut their joint output, and firm i responds to this by raising its output. With concave demand outputs are strategic substitutes. The derivative in (5) can be written as

$$\frac{dq_i}{dw} = \frac{1}{p'(Q)(2 - s_i E)} - \frac{(1 - s_i E)}{2 - s_i E}\frac{dQ_{-i}}{dw},$$

where the first term is the direct effect and $-\frac{(1-s_i E)}{2-s_i E} = \frac{dq_i}{dQ_{-i}} < 0$ is the slope of i's best-response to the other firms' output. The more concave inverse demand is, other things equal, the larger the increase in a firm's output when the other firms reduce their collective output. Proposition 1 holds when the strategic substitutes effect offsets the direct effect.

When a wholesale price increase induces firm i to raise its output the combined retail and wholesale profit associated with firm i, $(p(Q) - c_i - w)q_i + wq_i = (p(Q) - c_i)q_i$, increases. The wholesale price increase causes $p(Q)$ to rise, so both $p(Q) - c_i$ and q_i increase. This profit measure increases even if q_i falls as long as the proportional fall in q_i is smaller than the proportional increase in $p(Q) - c_i$.

The effect of the wholesale price increase on total costs, $\sum_i c_i q_i$, using (5), is

$$\text{(6)} \qquad \sum_i c_i \frac{dq_i}{dw} = \frac{dQ}{dw}\left(\bar{c} - E\left(\bar{c} - \sum_i c_i s_i\right)\right) = \frac{dQ}{dw}\left(\bar{c} - \frac{E\sigma^2}{p(Q) - \bar{c} - w}\right).$$

The expression $\bar{c} - \sum_i c_i s_i$ is the difference between the simple average and the weighted average of the marginal costs and is positive because lower-cost retailers have higher market shares. Appendix A shows that $\bar{c} - \sum_i c_i s_i = \frac{\sigma^2}{p(Q)-\bar{c}-w}$. Call $\frac{dQ}{dw}\frac{E\sigma^2}{p(Q)-\bar{c}-w}$ the benefit from the redistribution of output.

Social welfare, W, is consumer surplus plus producer surplus or, equivalently, direct utility, $U(Q)$, less total costs: $W = U(Q) - \sum_i c_i q_i$. Over the relevant outputs $U(Q)$ is increasing and concave, and its derivative equals the price. Direct utility falls as the wholesale price rises because total output falls. The effect on direct utility of a wholesale price increase is $U'(Q)\frac{dQ}{dw} = p(Q)\frac{dQ}{dw} < 0$. The effect on welfare will be positive if the reduction in total costs offsets this. Subtracting the change in costs, given in (6), yields the marginal welfare effect as

$$\text{(7)} \qquad \frac{dW}{dw} = \frac{dQ}{dw}(p(Q) - \bar{c}) + \frac{dQ}{dw}\frac{E\sigma^2}{p(Q) - \bar{c} - w}.$$

The marginal welfare effect has two parts. The first part, $\frac{dQ}{dw}(p(Q)-\overline{c})$, is negative. This captures the standard feature that an output reduction when price exceeds marginal cost reduces welfare. The second part, $\frac{dQ}{dw}\frac{E\sigma^2}{p(Q)-\overline{c}-w}$, is the benefit of the redistribution of output and is positive if and only if $E<0$.

If $E<1$ then the retail margin, $p-w-c_i$, falls as w increases. Using (4), the effect on the margin is $\frac{dp}{dw}-1=\frac{E-1}{n+1-E}$. For such demand functions the expression for the marginal welfare effect in (7) still applies when the wholesale price has risen far enough that only the most efficient retailer remains. At that point the variance of marginal costs is zero, $\overline{c}$ equals the marginal cost of the most efficient retailer, and the welfare effect is negative.[3] When demand is log-convex, so $E\geq 1$, each retail margin, however, rises or remains constant as w increases and the retailers would not exit.[4] We assume that there is no possibility of entry of new retailers in this case. The first welfare result may now be stated.

Proposition 2. If demand is convex everywhere then the marginal effect on welfare is negative for all wholesale prices, so monopolization of the upstream industry reduces social welfare.

Proof. With $E\geq 0$ and $n\geq 2$ the marginal welfare effect is the sum of a negative term and a non-positive term and so is negative. When $n=1$, so only the most efficient retailer is in the market, the welfare effect is $\frac{dQ}{dw}(p(Q)-\overline{c})<0$ where $\overline{c}$ is now the lowest marginal cost. ■

Proposition 2 implies that a necessary condition for welfare to rise with upstream monopolization is that demand is not convex everywhere. In particular if $E<0$ the marginal welfare effect in (7) is the sum of a negative term and a positive term, giving the possibility that the marginal welfare effect is positive for $n\geq 2$.

Assume now that $E<0$ everywhere. Three factors, curvature, E, and the variance and simple average of the marginal costs, σ^2 and $\overline{c}$, determine the sign of the marginal welfare effect. In each case start with a marginal welfare effect that is negative.

First, the marginal welfare effect will be positive for inverse demand that is sufficiently concave. Suppose that at the equilibrium total output and price there is a concave transformation of inverse demand. The position and slope of inverse demand at the initial total output are preserved, so each firm's output and the equilibrium price are unchanged by the transformation.

[3] At wholesale prices where less efficient retailers drop out the welfare function may have sharp points and so would not be differentiable at those points. The welfare function is, however, continuous and integrable.

[4] I am grateful to the editor, Andrew Rhodes, for this point.

Malueg [1994] shows how curvature affects monopoly outcomes using the same device. For a sufficiently negative value of E the marginal welfare effect in (7) will be positive. The average margin $p(Q) - \overline{c}$ is unaffected by the concave transformation, while $\frac{E\sigma^2}{p(Q)-\overline{c}-w}$ becomes more negative.

Second, suppose that the variance of marginal costs, σ^2, increases while $\overline{c}$ remains constant. For this effect to work the number of retailers is assumed to be unchanged. The average margin, $p(Q) - \overline{c}$, remains constant, because total output does not depend on the variance of costs, by the result of Bergstrom and Varian [1985], and $E(Q)$ is also unchanged. A high enough value of σ^2 ensures that the redistribution of output effect becomes sufficiently positive that the marginal welfare effect is positive.

Third, suppose that $\overline{c}$ increases while σ^2 remains constant. Again assume that the number of firms remains unchanged. Now assume also that E is constant. The retail price increases as $\overline{c}$ rises, with the same pass-through coefficient as in (4). Because demand is concave pass-through is below 1, and the average margin, $p(Q) - \overline{c}$, falls. Thus $p(Q) - \overline{c}$ is a smaller positive number, and $\frac{E\sigma^2}{p(Q)-\overline{c}-w}$ is a more negative number. The marginal welfare effect in (7) has the opposite sign to $p(Q) - \overline{c} + \frac{E\sigma^2}{p(Q)-\overline{c}-w}$. At a high enough value of $\overline{c}$ this expression falls below zero and the welfare effect becomes positive.

Upstream monopolization can increase welfare. I use an example with a concave demand function to show this. Full details of the example are in Appendix B. Inverse demand has constant curvature of −8 and is $p(Q) = 1.2 - (Q^9 - 1)/9$. There is one efficient retailer with cost $c_1 = 0.8$ and there are four inefficient retailers with cost $c_2 = 1.05$. The simple average of the costs is $\overline{c} = 1$ and the variance is $\sigma^2 = 0.01$. The Cournot total output is 1, with the output and share of the efficient firm being 0.4. The Cournot price is 1.2. At the initial wholesale price of zero the marginal effect of a higher wholesale price, given in (7), is positive.

Upstream monopolization induces the inefficient retailers to leave the market. The upstream monopolist sets a wholesale price of $w = 0.46$. The downstream monopolist sets a retail price of $p(Q) = 1.306$ with monopoly output of 0.7103. The ratio of the downstream margin to the upstream margin, $(p(Q) - w - c_1)/w$, equals 1/10. Adachi and Ebina [2014] show generally that this ratio equals $1/(2 - E)$. Monopolization induces double margins but the effect of the additional margin is small because inverse demand is very concave and thus cost pass-through is low.

Total welfare is 0.35 with upstream competition and downstream oligopoly. With upstream monopolization total welfare is higher at 0.3627.

Example 1. Suppose that $p(Q) = 1.2 - (Q^9 - 1)/9$, the efficient retailer has cost $c_1 = 0.8$ and the four inefficient retailers each have cost $c_i = 1.05$. Monopolization of the upstream industry entails that only the efficient retailer survives. Social welfare is higher with a monopoly upstream and

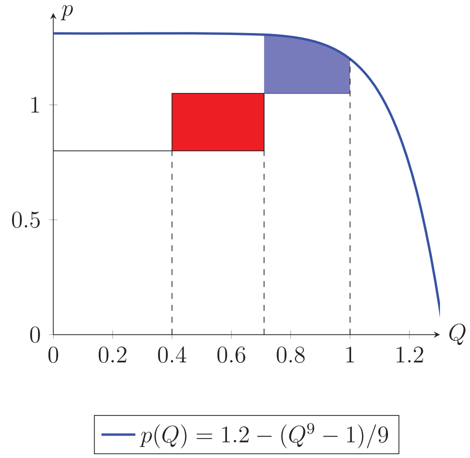

Figure 1
Welfare-Increasing Upstream Monopolization

Notes: [Colour figure can be viewed at wileyonlinelibrary.com]

a monopoly downstream than with competition upstream and a Cournot equilibrium downstream.

The welfare change can be divided into two parts. First, because monopolization reduces total output there is a loss of welfare. Total output falls by the difference between oligopoly output and monopoly output (here by 1 – 0.7103). Utility is lower but the lost output was relatively expensive to supply under oligopoly. This loss of welfare is illustrated by the blue (or lightly-shaded) area in Figure 1. Second, there is an increase in welfare because the extra units of output that the most efficient retailer sells cost less than those units did when the inefficient retailers sold them. This cost saving is represented by the red (or dark shaded) rectangle in Figure 1. This is the increase in output of the efficient firm, 0.7103 – 0.4, times the reduction in the marginal cost of producing those units, 1.05 – 0.8. The second effect is stronger than the first.

The functions are chosen so that a concave transformation of inverse demand, represented by a more negative value of E, does not affect the outputs or the price in the Cournot equilibrium. As E becomes more negative the loss, represented by the blue area, becomes smaller for two reasons: inverse demand becomes even flatter up to the Cournot output of 1, and the monopoly output increases. The increase in monopoly output also increases the width of the welfare gain represented by the red area. In the limit as inverse demand approaches a rectangle the loss term becomes close to zero while the gain from reallocating output approaches its upper bound, which is the cost saving if the monopoly output were to equal 1.

IV. ELIMINATING A FIRM

In this section, I find conditions that determine when the elimination of a firm from an asymmetric oligopoly raises social welfare. There are no vertical issues. This is a classic 'market power versus efficiency' question. The framework of Lahiri and Ono [1988] is used and developed. A simple formula gives the effect of a cost increase in one firm on social welfare. The formula depends on inverse demand curvature at the equilibrium, the Herfindahl Index of market concentration, the number of firms, and the market share of the firm whose cost increases. I explore the role of curvature in determining the effect of the cost increase on welfare.

Lahiri and Ono [1988] show that if a minor firm has a cost reduction social welfare can fall. The reason is that other firms with higher margins reduce their outputs in response. The framework is developed here for a general n-firm oligopoly. Suppose that firm k's marginal cost increases while its rivals have unchanged costs. The general expression for the effect of this on social welfare, which we look to be positive, is

$$\frac{dW}{dc_k} = \sum_{i=1}^{n} (p(Q) - c_i)\frac{dq_i}{dc_k} - q_k. \tag{8}$$

The direct effect of the cost increase, before any responses, is negative and is $-q_k$ as the initial output of firm k costs more to produce. The summation term is the output response of each firm, including k, multiplied by that firm's margin. In summary, firm k's output costs more, and it produces less output, both of which reduce welfare, while there are gains in welfare from increases in the outputs of the other firms (if outputs are strategic substitutes).

Lahiri and Ono [1988] assume that outputs are strategic substitutes, which in our notation is $1 - s_i E > 0$ for all i. This implies that a firm's marginal revenue falls as the output of the other firms increases, and is also the Hahn stability condition (Hahn [1962]). The analysis that follows does not require the strategic substitutes assumption to hold for each firm.

The effects of the cost increase on firm k's output and the other firms' outputs may be written as

$$\frac{dq_k}{dc_k} = -\frac{(1 - s_k E)}{p'(Q)(n + 1 - E)} + \frac{1}{p'(Q)};$$

$$\frac{dq_j}{dc_k} = -\frac{(1 - s_j E)}{p'(Q)(n + 1 - E)}, \quad j \neq k.$$

The Herfindahl Index, defined as the sum of the squared market shares, $H \equiv \sum_{i=1}^{n} s_i^2$, is the standard measure of concentration. The output derivatives and the fact that the margin is $p(Q) - c_i = -q_i p'(Q)$ imply that (8) is

$$\frac{dW}{dc_k} = Q\left(\frac{1 - EH}{n + 1 - E} - 2s_k\right). \tag{9}$$

Expression (9) is new: it is consistent with the analysis of Lahiri and Ono [1988] but is not given in their article. It is positive if the expression in large brackets is positive, for which a necessary condition is that $1 - EH = \sum_{i=1}^{n} s_i(1 - s_i E) > 0$ so on 'average' outputs are strategic substitutes.

Two existing results can be shown immediately using (9). First, if demand is linear, so $E = 0$, welfare rises with a cost increase for a firm whose share is below $1/2(n + 1)$. Zhao [2001] obtains this result for linear demand by direct calculation. Note that it also follows that eliminating such a firm will raise welfare: the welfare derivative remains positive for all cost increases for the inefficient firm until it exits the market. Second, Lahiri and Ono [1988] show that a cost increase reduces welfare when the firms are identical. With symmetry $H = \frac{1}{n}$ and $s_k = \frac{1}{n}$. The welfare effect has the same sign as $-(n + 2 - E)$, which is negative by the stability condition.

To simplify the notation let the possible cost of the inefficient firm be c and its share be s. Its actual cost is $\underline{c}$ and the cost level at which it would exit the market is $\overline{c}$. In the case of duopoly $\overline{c}$ is the monopoly price of the rival. We are interested in the sign of (9) for all values of $c \in \left[\underline{c}, \overline{c}\right]$. If (9) is positive at $\underline{c}$ and this remains positive as c increases to $\overline{c}$ then eliminating this firm raises welfare. Alternatively if the derivative is negative over this domain then eliminating this firm reduces welfare. The case of linear demand is illustrated in the next example.

Example 2. Suppose that $n = 2$, inverse demand is $p(Q) = 1 - Q$, the cost of the efficient firm is zero and the inefficient firm's cost is $c \leq 0.5$ where 0.5 is the efficient firm's monopoly price. The outputs of the efficient and inefficient firms are $\frac{1+c}{3}$ and $\frac{1-2c}{3}$ respectively and $Q = \frac{2-c}{3}$. The welfare derivative is $\frac{dW}{dc} = \frac{Q}{3} - 2\frac{(1-2c)}{3} = \frac{-4+11c}{9}$.

The marginal welfare effect in Example 2 is illustrated in Figure 2, where the horizontal axis is c. The marginal welfare effect at $c = 0$ is negative, because the firms are equally efficient. The welfare derivative increases linearly, is negative until $c = \frac{4}{11}$ and thereafter is positive, and reaches its maximum at $c = 0.5$. If $\underline{c} \geq \frac{4}{11}$ then elimination of firm 2 raises welfare by the integral of $\frac{dW}{dc}$ over $[\underline{c}, \overline{c}]$. Note that the sufficient condition for welfare to rise is not necessary: from the diagram there is a range of values of c below $\frac{4}{11}$ for which the integral will be positive. In fact if $\underline{c} = \frac{8}{11} - \frac{1}{2} = 0.227$ then the welfare change from

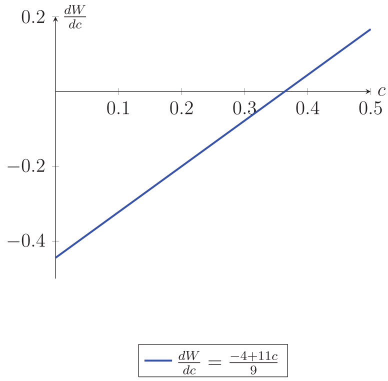

Figure 2
The marginal welfare effect with linear demand

Notes: [Colour figure can be viewed at wileyonlinelibrary.com]

eliminating the inefficient firm will be zero and for $\underline{c}$ higher than this the welfare effect is positive.

The first new result is a negative one and applies when there is a duopoly. With log-convex demand, and industry marginal revenue that is decreasing, the move from an asymmetric duopoly to an efficient monopoly reduces social welfare. The efficient firm's share is $1-s$ and the Herfindahl is $H = 1 - 2s(1-s) > 0$. The assumption that $E \geq 1$ provides an upper bound to the welfare derivative expression that is negative because $E < 2$.

Proposition 3. Suppose there is a duopoly and that demand is log-convex with curvature $E \in [1, 2)$. Monopolization reduces social welfare.

Proof. The expression in large brackets in (9) is

$$\frac{1 - E(1 - 2s(1-s))}{3 - E} - 2s.$$

Because $(E-1)(1 - 2s(1-s)) \geq 0$ given $E \geq 1$, the numerator of the first term is bounded above,

$$2s(1-s) \geq 1 - E(1 - 2s(1-s))$$

with equality when $E = 1$. So

$$2s\left(\frac{1-s}{3-E} - 1\right) \geq \frac{1 - E(1 - 2s(1-s))}{3 - E} - 2s.$$

For $s > 0$ the upper bound is negative as $1 - s < 1$ and $3 - E > 1$, implying that the welfare derivative is negative for all $c < \overline{c}$. At $c = \overline{c}$ firm 2's share is

$s = 0$ and the left-derivative of welfare has the same sign as $1 - E \leq 0$. Thus the welfare derivative is negative for all $c \in [\underline{c}, \overline{c})$ and non-positive at $\overline{c}$ and monopolization reduces welfare. ■

Proposition 3 applies to exponential demand, which has $E = 1$, and to constant elasticity demand with an elasticity above 1. The utility loss from the output reduction that monopolization induces is particularly large when demand is log-convex. The possible cost savings from the reallocation of output cannot offset the loss of utility under duopoly.

Lahiri and Ono [1988] state (in Proposition 2) that "Under Cournot oligopoly, national welfare increases if a firm with a sufficiently low share is removed from the market." An implication of Proposition 3 here is that Lahiri and Ono's result does not hold under the conditions stated. They did not allow for log-convex demand. Our proposition complements their analysis.

Proposition 3 applies to duopoly. When there are more than two firms elimination of an inefficient firm can raise welfare even with log-convex demand if the inefficiency is large enough. The next example illustrates.

Example 3. There are two efficient firms with cost of zero, and an inefficient one with cost $c \in (0, 0.5)$. Inverse demand is $p(Q) = -\ln Q$ for $Q \leq 1$ so $E = 1$. The inefficient firm's share is $\frac{1-2c}{3}$. Welfare is $\frac{2}{3}(2 + c^2)e^{-\frac{(1+c)}{3}}$ with three firms and $\frac{3}{2}e^{-\frac{1}{2}}$ with two firms. Welfare is higher with the two efficient firms for $c \in [0.22, 0.5)$.

To understand the role of curvature in determining whether the welfare derivative can be positive suppose there is a concave transformation of inverse demand that preserves the price, output and slope of inverse demand. The initial equilibrium, the market shares and the Herfindahl are unchanged. The derivative of $\frac{1-EH}{n+1-E}$ with respect to E, with H constant, is

$$\frac{1 - (n + 1)H}{(n + 1 - E)^2} < 0$$

because $Hn \geq 1$. Thus as demand becomes more concave, that is, E falls, the welfare derivative rises. Intuitively, as demand becomes more concave the demand function becomes more rectangular. The loss of utility from lower output becomes smaller, as the demand function is flatter in the region where the monopoly output is and because monopoly output itself is higher. This increases the relative importance of the benefit of redistributing output to the efficient firm: this is unaffected by the concave transformation.

In a duopoly the more concave is inverse demand the greater the positive output response of the other firm when the inefficient firm's cost increases, when demand is log-concave. In the case of an n-firm oligopoly the change in total output as the inefficient firm's cost increases is $\frac{dQ}{dc} = \frac{1}{p'(Q)(n+1-E)}$, which goes to zero as E goes to minus infinity.

I now consider when elimination of an inefficient firm can be good for welfare generally. I use the fact that the Herfindahl is bounded below and above: $\frac{1}{n} \le H \le 1$. In general $H = \sum_{i=1}^{n} \left(s_i - \frac{1}{n}\right)^2 + \frac{1}{n}$ and with a monopoly $H = 1$.

The initial result is an existence one, that is parallel to Proposition 2 in Lahiri and Ono [1988]. This applies when the actual cost $\underline{c}$ is just below $\overline{c}$.

Proposition 4. Consider an n-firm oligopoly. Suppose that demand is strictly log-concave. Elimination of the inefficient firm raises welfare if its cost is very close to the maximum feasible cost.

Proof. We first show that $\frac{dW(\overline{c})}{dc} > 0$ (formally, this is the left-derivative). Because $s = 0$ when $c = \overline{c}$, the left-derivative of welfare has the same sign as $1 - EH > 0$. This is strictly positive because $H \le 1$ and $E < 1$. Define the difference between welfare with $n - 1$ firms and with n firms as $D(c) = W(\overline{c}) - W(c)$. By construction $D(\overline{c}) = 0$ as the inefficient firm just chooses to exit at that cost level. The left-derivative of $D(c)$ at $\overline{c}$ is $-\frac{dW(\overline{c})}{dc} < 0$. Thus in the neighborhood to the left of $\overline{c}$ the welfare difference falls, from a positive number to zero, and there is a region where the welfare difference is positive. ■

This argument does not work when $E \ge 1$ and $n = 2$, which is why Proposition 3 holds. At $\overline{c}$ the Herfindahl equals 1 as the efficient firm is a monopolist. Thus $1 - EH = 1 - E \le 0$. When $n > 2$ the Herfindahl is strictly below 1 so elimination can raise welfare even with log-convex demand, as in Example 3.

More generally, if the actual cost is not close to $\overline{c}$ we can use the bounds on the Herfindahl and the assumption that demand is strictly log-concave to show that the welfare derivative can be bounded below.

Proposition 5. Consider an n-firm oligopoly. Suppose that demand is strictly log-concave. A small increase in the inefficient firm's cost raises social welfare if its share is sufficiently small.

Proof. Suppose first that $E \le 0$. The expression in large brackets in (9) is bounded below because $H \ge \frac{1}{n}$:

$$\frac{1 - EH}{n + 1 - E} - 2s \ge \frac{n - E}{n(n + 1 - E)} - 2s.$$

Concavity of demand implies that $n - E > 0$ so $\frac{n-E}{n(n+1-E)} > 0$. Thus if $s < \frac{n-E}{2n(n+1-E)}$ the welfare derivative is positive. When $0 \le E < 1$ we have $1 - EH \ge 1 - E > 0$ because $H \le 1$ and $E < 1$. The lower bound to the

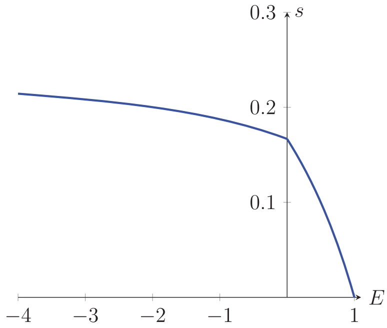

Figure 3

Bounds on the market share and equilibrium curvature

Notes: [Colour figure can be viewed at wileyonlinelibrary.com]

welfare derivative expression is $\frac{1-E}{n+1-E} - 2s$. Thus if $s < \frac{1-E}{2(n+1-E)}$ the welfare derivative is positive. ■

Suppose there is a duopoly. A share below $\frac{2-E}{4(3-E)}$ when demand is concave, or below $\frac{1-E}{2(3-E)}$ when $E \in [0, 1)$, implies that the derivative is positive. Figure 3 illustrates these upper bounds to the market share against equilibrium curvature. They coincide at $E = 0$ and $s = \frac{1}{6}$. As E goes towards minus infinity the bound tends to 0.25, and as E goes towards 1 the bound goes to 0. The lower is equilibrium curvature the higher the set of market shares that ensures that the welfare derivative is positive.

Proposition 5 gives conditions for the welfare derivative to be positive locally. The derivative remains positive as the cost increases further if curvature is non-decreasing. It follows that elimination of the firm raises welfare.

Proposition 6. Consider an n-firm oligopoly. Suppose that demand is strictly log-concave and that the welfare derivative is positive because one of the conditions in Proposition 5 holds. If curvature is weakly increasing in output, $E'(Q) \geq 0$, then elimination of the inefficient firm raises welfare.

Proof. The effect of higher output on $\frac{n-E(Q)}{2n(n+1-E(Q))}$ has the same sign as $-E'(Q) \leq 0$, as does the effect on $\frac{1-E(Q)}{2(n+1-E(Q))}$. As total output falls with a cost increase it follows that the relevant bound is higher or remains constant. Thus for all $c \in [\underline{c}, \overline{c}]$ the sufficient condition holds for the welfare derivative to be positive, and elimination of the inefficient firm raises welfare. ■

Proposition 6 has a graphical interpretation. Suppose in Figure 3 that initially the share is at or below one of the bounds. As the cost of the inefficient firm rises further the curvature-share point moves either vertically downwards or to the south-west, so the share remains below the bounds.

Curvature is constant for many commonly used demand functions. It is increasing in output if direct demand curvature is constant and above 0. There is, though, a class of demand functions that is strictly log-concave and which has curvature that is strictly decreasing in output. These are demands based on log-concave distributions of consumer valuations such as logistic or normal distributions (Fabinger and Weyl [2018]). These are concave for high values of output and convex for low values of output. Proposition 6 does not apply to these demand functions, but Propositions 4 and 5 do.

Consider a duopoly and suppose that curvature is constant with $E \leq 1$. The exact value of the market share at which the welfare derivative is zero is the solution to

$$\frac{1 - E(1 - 2s(1 - s))}{3 - E} = 2s.$$

For $E = 0$ the solution is $s = \frac{1}{6}$. For $E \neq 0$ the solution is

$$s = \frac{1}{2E}\left(2E - 3 + \sqrt{(3 - 2E)(3 - E) - E}\right). \tag{10}$$

When E goes to minus infinity the critical value of the market share is $s = 0.293$. As E rises the critical value falls, going to $s = 0$ when $E = 1$.

With constant curvature and duopoly the welfare effect of eliminating a firm can be calculated directly. In Appendix C I compare monopoly and duopoly welfare for constant curvature, with the feature that the duopoly equilibrium does not depend on curvature, and allowing for different curvature values. The result of Lahiri and Ono [1988] for linear demand that the critical share is 0.308 is reproduced. For $E = 0.95$ the critical value of the share is 0.043, while for the demand function used in Section III, with $E = -8$, the critical value is 0.464.

V. CONCLUSION

The reallocation of output effect is important in determining the sign of welfare effects in oligopoly. This paper provides a comprehensive analysis of the conditions for elimination of an inefficient firm to raise welfare. The key is demand curvature. With highly concave demand the cost savings from the reallocation of output can imply that a monopoly selling to a monopoly with double margins yields higher social welfare than a competitive industry selling to an oligopoly. With strictly log-concave demand social welfare can be higher when an inefficient firm is eliminated when there are no vertical issues. The more concave inverse demand is the more likely it is that such elimination will be beneficial.

Throughout we have considered situations where total output falls. Suppose, instead, that total output increases, perhaps because an industry-wide tax is reduced or a production subsidy is offered. The analysis suggests that it will be when demand is strictly convex that there will be a positive reallocation of output effect, and this will reinforce the positive effect of higher total output.

There are several reasons for being cautious about applying the analysis of this paper to policy contexts. First, antitrust policymakers often have the objective of maximizing consumer surplus and in the models in this paper consumer surplus is always lower with monopoly, or with the elimination of a firm from an oligopoly, because total output falls. Second, arguments for monopolization might be improperly used by special interests seeking to entrench unwarranted market power.

APPENDIX A

RETAIL MARKET SHARES AND THE COST OF RETAILING

The first-order condition (1) implies that the output of firm i is

$$q_i = \frac{p(Q) - c_i - w}{-p'(Q)}$$

and its market share is

$$s_i = \frac{p(Q) - c_i - w}{-Qp'(Q)} = \frac{p(Q) - c_i - w}{n(p(Q) - \bar{c} - w)}$$

using (2). Subtracting and adding $\bar{c}$ within the numerator gives

$$s_i = \frac{1}{n} + \frac{\bar{c} - c_i}{n(p(Q) - \bar{c} - w)}.$$

It follows that the share-weighted average of the costs is

$$\sum_i c_i s_i = \frac{\sum_i c_i}{n} + \frac{\sum_i c_i(\bar{c} - c_i)}{n(p(Q) - \bar{c} - w)} = \bar{c} - \frac{\sigma^2}{p(Q) - \bar{c} - w},$$

where $\sigma^2 = \frac{\sum_i (c_i - \bar{c})^2}{n}$ is the variance of costs.

APPENDIX B

THE UPSTREAM MONOPOLIZATION EXAMPLE

Inverse demand is $p(Q) = 1.2 - \frac{Q^9 - 1}{9}$, and the costs are $c_1 = 0.8$ and $c_2 = c_3 = c_4 = c_5 = 1.05$. Note that $Qp'(Q) = -Q^9 = -1$ when $Q = 1$. The first-order condition for firm 1 is

$$p(Q) + s_1 Qp'(Q) - 0.8 = 0$$

and the first-order conditions for the other firms, $i = 2, 3, 4, 5$, are

$$p(Q) + s_i Qp'(Q) - 1.05 = 0.$$

The solution is $Q = 1$, $p(1) = 1.2$, $s_1 = 0.4$ and $s_i = 0.15$ for firms 2 to 5. Aggregate revenue is 1.2 and total cost is $0.8 \times 0.4 + 1.05 \times 0.6 = 0.95$ so industry profit is 0.25. Consumer surplus with constant curvature is $\frac{1}{2-E}Q^{2-E}$ in general and equals 0.1 here. Social welfare in the oligopoly equilibrium is thus 0.35.

Suppose first that with upstream monopolization all five retailers remain in the market. The first-order condition for firm 1 is $p(Q) + s_i Qp'(Q) - 0.8 - w = 0$, and for each inefficient firm it is $p(Q) + s_i Qp'(Q) - 1.05 - w = 0$. Adding the five first-order conditions and dividing by 5, gives $p(Q) + \frac{Qp'(Q)}{5} - 1 - w = 0$. The resulting upstream demand function, conditional on the five retailers being in the market is $Q(w) = \left(1 - \frac{45}{14}w\right)^{\frac{1}{9}}$ and upstream profit is $wQ(w)$. These apply for $w < 0.24$, and upstream profit is increasing over the range $w = [0, 0.24)$. At that point the profit function has a sharp point as the inefficient firms exit and the downstream industry becomes a monopoly. The retailer's first-order condition $p(Q) + Qp'(Q) - 0.8 - w = 0$ determines downstream output as a function of the wholesale price for $w \geq 0.24$, $Q(w) = (0.46 - 0.9w)^{\frac{1}{9}}$. The optimal wholesale price maximizes $wQ(w)$ and is 0.46, which implies that output is 0.7103 and the retail price is $p(0.7103) = 1.306$. Aggregate profit is $(1.306 - 0.8) \times 0.7103 = 0.3594$, which itself is higher than welfare in the oligopoly equilibrium. Adding consumer surplus of $\frac{0.7103^{10}}{10} = 0.00327$ gives welfare with upstream and downstream monopoly of 0.3627.

APPENDIX C

CALCULATING THE WELFARE EFFECT

Consider a duopoly. Curvature is constant and below 1. Inverse demand is $p(Q) = a - \frac{Q^{1-E}-1}{1-E}$. Firm 1 has cost $c_1 = c - x$ and 2's cost is $c_2 = c + x$ where $c \geq 0.5$ and $x \in [0, 0.5]$. The variance of costs is x^2. Total output in duopoly does not depend on x because the sum of the marginal costs is constant (Bergstrom and Varian [1985]).

Assume that $a = c + 0.5$, which implies that the duopoly price is $a = c + 0.5$, which is independent of E, and duopoly total output is 1. The market shares are $s_1 = 0.5 + x$ and $s_2 = 0.5 - x$ and the Herfindahl is $H = \frac{1}{2} + 2x^2$. Faulí-Oller [2002] shows that with constant curvature, and constant marginal costs, social welfare can be written as

$$W(Q, H) = \frac{1}{2-E}Q^{2-E} + Q^{2-E}H,$$

where the first term is consumer surplus and the second is industry profit, which, for a given total output, is proportional to the Herfindahl. Duopoly welfare is thus

$$W^d(x) = \frac{1}{2-E} + \frac{1}{2} + 2x^2.$$

The monopoly price is $p^m(x) = c + \frac{1.5-0.5E-x}{2-E}$. Monopoly welfare, with $H = 1$, is

$$W^m(x) = \left(\frac{3-E}{2-E}\right)(Q(p^m(x)))^{2-E}.$$

TABLE C1
CRITICAL MARKET SHARES AND CURVATURE

E	s
−8	0.464
−7	0.461
−6	0.455
−5	0.449
−4	0.440
−3	0.428
−2	0.409
−1	0.375
0	0.308
0.5	0.228
0.95	0.043

Using the direct demand function $Q(p) = ((1-E)(a-p)+1)^{\frac{1}{1-E}}$ monopoly output is

$$Q(p^m(x)) = \left(\frac{3-E+2(1-E)x}{2(2-E)}\right)^{\frac{1}{1-E}}.$$

Thus the difference between monopoly welfare and that with duopoly is:

$$\Delta(x) = \left(\frac{3-E}{2-E}\right)\left(\frac{3-E+2(1-E)x}{2(2-E)}\right)^{\frac{2-E}{1-E}} - \left(\frac{1}{2-E}\right) - \frac{1}{2} - 2x^2.$$

This is defined for $x \in [0, 0.5]$. The welfare difference depends only on curvature and the cost-difference factor x.

Welfare with two equally efficient duopolists is higher than with a monopoly with the same cost, so $\Delta(0) < 0$. When x is at its maximum value $\Delta(0.5) = 0$, because the duopoly equilibrium is the same as the monopoly outcome when the inefficient firm's share drops to zero. The function $\Delta(x)$ is strictly concave, is increasing at $x = 0$ and decreasing at $x = 0.5$, so there exists a range of values of x within which monopolization yields higher welfare. The critical market shares for different curvature levels are found by determining the unique value of $x \in (0, 0.5)$ at which $\Delta(x) = 0$ and using the fact that $s_2 = \frac{1}{2} - x$. For example monopolization raises welfare when the share is below 0.464 for $E = -8$, below 0.308 for $E = 0$, and below 0.043 for $E = 0.95$. Table C1 gives more critical values of the market share for different curvature values.

REFERENCES

Adachi, T. and Ebina, T., 2014, 'Double Marginalization and Cost Pass-through: Weyl–Fabinger and Cowan Meet Spengler and Bresnahan–Reiss,' *Economics Letters*, 122(2), pp. 170–175.

Bergstrom, T. C. and Varian, H. R., 1985, 'Two Remarks on Cournot Equilibria,' *Economics Letters*, 19(1), pp. 5–8.

Corchón, L., 2001, *Theories of Imperfectly Competitive Markets* (Springer, Berlin).

Fabinger, M. and Weyl, E. G., 2018, Functional Forms for Tractable Economic Models and the Cost Structure of International Trade, *CIRJE F-Series CIRJE-F-1092* (Faculty of Economics, University of Tokyo, CIRJE).

Faulí-Oller, R., 2002, ‘Mergers between Asymmetric Firms: Profitability and Welfare,’ *The Manchester School*, 70, pp. 77–87.
Fevrier, P. and Linnemer, L., 2004, ‘Idiosyncratic Shocks in an Asymmetric Cournot Oligopoly,’ *International Journal of Industrial Organization*, 22(6), pp. 835–848.
Gaudet, G. and Salant, S. W., 1991, ‘Uniqueness of Cournot Equilibrium: New Results from Old Methods,’ *The Review of Economic Studies*, 58(2), pp. 399–404.
Ghosh, A., Morita, H. and Wang, C., 2022, ‘Welfare Improving Horizontal Mergers in Successive Oligopoly,’ *The Journal of Industrial Economics*, 70(1), pp. 89–118.
Hahn, F. H., 1962, ‘The Stability of the Cournot Oligopoly Solution,’ *The Review of Economic Studies*, 29(4), pp. 329–331.
Kang, Z. Y. and Vasserman, S., 2022, Robustness Measures for Welfare Analysis, Working Paper 29656 (National Bureau of Economic Research, Cambridge, MA).
Kolstad, C. D. and Mathiesen, L., 1987, ‘Necessary and Sufficient Conditions for Uniqueness of a Cournot Equilibrium,’ *The Review of Economic Studies*, 54(4), pp. 681–690.
Lahiri, S. and Ono, Y., 1988, ‘Helping Minor Firms Reduces Welfare,’ *The Economic Journal*, 98(393), pp. 1199–1202.
Malueg, D. A., 1994, ‘Monopoly Output and Welfare: The Role of Curvature of the Demand Function,’ *The Journal of Economic Education*, 25(3), pp. 235–250.
Seade, J. K., 1980, ‘On the Effects of Entry,’ *Econometrica*, 48(2), pp. 479–489.
Seade, J. K., 1985, Profitable Cost Increases and the Shifting of Taxation: Equilibrium Response of Markets in Oligopoly, the Warwick Economics Research Paper Series (TWERPS) 260 (University of Warwick, Warwick).
Van Long, N. and Soubeyran, A., 1997, ‘Cost Heterogeneity, Industry Concentration and Strategic Trade Policies,’ *Journal of International Economics*, 43(1), pp. 207–220.
Vives, X., 2001, *Oligopoly Pricing: Old Ideas and New Tools* (MIT Press, Cambridge, MA).
Zhao, J., 2001, ‘A Characterization for the Negative Welfare Effects of Cost Reduction in Cournot Oligopoly,’ *International Journal of Industrial Organization*, 19(3), pp. 455–469.