

Radiocarbon dating of the Bakhshālī manuscript

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The Bakhshālī Manuscript (MS. Sansk. d. 14)

This manuscript written on birch-bark was discovered in May 1881 in Bakhshālī, a village 80 kilometers north-east from Peshawar, in what is now Pakistan and not far from the site of the ancient trading centre of Taxila. Recovered by an agricultural labourer in either a field or a ruin (unfortunately the accounts of its discovery are not precise), it was sent first to the Lieutenant Governor of the Panjab. It was subsequently passed on to the German scholar A.F.R. Hoernle to be examined and was eventually published as ‘The Bakhshālī manuscript’ (Kaye and Hoernle 1927-1933). In 1902 Hoernle presented the manuscript to the Bodleian Library in Oxford, where it is now stored under the shelfmark MS. Sansk. d. 14.

The manuscript is fragmentary, consisting of 70 extremely badly damaged folios with substantial loss of text. It is written in the *Śāradā* script and contains a mathematical work in a form of Classical Sanskrit influenced by features corresponding to medieval Northwestern Middle Indo-Aryan (and partly New Indo-Aryan) languages. Consisting of algorithms and sample problems in verse, it is notable for the number of zeros it contains. Its exact age has long been the subject of academic debate with scholars having proposed dates ranging from as early 200 CE to 1100 CE. It is, however, almost certainly the oldest extant witness of a South Asian mathematical work.

This report presents radiocarbon determinations on five of the Bakhshālī folios (Folio 15, 16, 23, 33, and 55).

Radiocarbon Dating

Radiocarbon dating can be applied using most organic material formed during the last 50,000 years. Radiocarbon is a naturally occurring radioactive isotope of the element carbon which is continually produced in the upper atmosphere. Once formed, radiocarbon is rapidly distributed throughout the atmosphere as carbon dioxide. Primary producers incorporate carbon dioxide via photosynthesis and use the carbon in organic polymers, such as the lignin and cellulose which formed the birch bark on which the Bakhshālī manuscript was written. As soon as this carbon is contained in

the structure of the bark, it is isolated from the atmosphere and no new radiocarbon can be added. Radiocarbon decays with a half-life of about 5,730 years, so once the bark is formed the amount of radiocarbon within the bark decreases but the amount of stable carbon isotopes remain constant.

By measuring the ratio of radiocarbon to stable carbon in the bark it is possible to calculate how long ago the bark was formed. However, to do so, past variations in the atmospheric concentration of radiocarbon must be corrected for. These variations in the atmospheric concentration of radiocarbon can be caused, among other factors, by a change in the production rate of radiocarbon in the upper atmosphere. To deal with this complication, radiocarbon measurements are calibrated using a calibration curve constructed from measurements on material of known age (such as tree rings independently dated using dendrochronology) to give calibrated age ranges, which are usually presented as 95.4% confidence intervals.

A further possible complication of radiocarbon dating is the effect of contamination, for example that from an ink which may contain carbon of a different age to the bark on which it is written. The easiest way to deal with potential contamination is through careful sampling. For this reason, areas of ink in the Bakhshālī manuscript were avoided when taking samples. As well as avoiding contamination through sampling, material to be radiocarbon dated routinely undergoes chemical pretreatment to remove as much potential contamination as possible while still leaving enough material on which to make a radiocarbon determination. The Bakhshālī samples were selected ensuring that the writing media was not affected, finding the most inobtrusive areas for their extraction. The samples were very small (less than 2 mg of bark) so were treated with a mild chemical pretreatment, which included washes using organic solvents to remove any oils or grease on the manuscript and a sequential acid-base-acid wash to remove any potential carbonates or humics absorbed from soil during reported possible burial.

Methods

Samples of bark (*Betula sp.*) from five folios of the Bakhshālī manuscript were taken (Folio 15, 1.95 mg; Folio 16, 1.60 mg; Folio 23, 1.58 mg; Folio 33, 1.39 mg and Folio 55, 1.80 mg), Sampling locations were selected so that no ink was sampled and to cause as little damage to the manuscript as possible. The sampled bark was pretreated with sequential washes of organic solvents (acetone, 50°C, 1 hour; methanol, 50°C, 1 hour; chloroform, room temperature, 1 hour) and three rinses of ethanol (room temperature) and four of ultrapure water (Millipore MilliQ, room temperature), before sequential washes of 1 M hydrochloric acid, 0.2 M sodium hydroxide and 1 M hydrochloric acid. After each acid and base wash the samples were rinsed three times with ultrapure water. (ORAU pretreatment code VV*; Brock et al., 2010).

After chemical pretreatment the bark samples were converted to carbon dioxide, using an elemental analyzer. The carbon dioxide was graphitized at 560°C in the presence of hydrogen gas and an iron catalyst (Dee and Bronk Ramsey, 2000) and the radiocarbon content of the graphite determined using the Oxford HVEE 2.5 MV accelerator mass spectrometer (Bronk Ramsey et al., 2004). Radiocarbon measurements were calibrated against the IntCal 20 curve (Reimer, 2020) using OxCal v4.4 (Bronk Ramsey, 2021). The R_combine function and simple phase model from same program were used for data analysis.

Initial measurements were made on Folio 16, 17 and 33 during 2017. The data from the determinations on Folio 16 and 17 produced unexpected ages so a subsequent round of dating was performed in 2018 on Folio 23 and 55, and, later during the same year, on Folio 15 and a second sample from Folio 16. The second determination on Folio 16 demonstrated that the initial determination was inaccurate. The measured age of Folio 17 is an outlier (OxCal outlier analysis) compared to the determinations made on the other folios. For this reason, the data from the determination on Folio 17 has not been included in this report. The determination on Folio 33, included here, was made at the same time as the erroneous determination on Folio 16 but as the data produced are consistent with the those from the other folios, the data from Folio 33 has been included in this report.

Results

Radiocarbon determinations and calibrated age ranges are shown in Table 1 and Figures 1 to 5. The data associated with each measurement can be accessed using the OxA laboratory code at <https://c14.arch.ox.ac.uk/database/db.php?page=checkDate>

Table 1: Laboratory codes, radiocarbon determinations, stable carbon isotope composition, and calibrated ages for the birch bark from the Bakhshālī manuscript. Reported uncertainties in the radiocarbon determination are one standard deviation. Calibrated ages were produced using the IntCal20 calibration curve (Reimer et al., 2020) and OxCal 4.4 (Bronk Ramsey, 2021).

Sample	ORAU laboratory code	Radiocarbon determination / BP	$\delta^{13}\text{C}$ / ‰	Calibrated age, 95.4% confidence interval / cal AD
Bakhshālī Folio 15	OxA-37,300	1145 ± 23	-26.4	773 - 986
Bakhshālī Folio 16	OxA-37,301	1047 ± 23	-25.3	901 - 1032
Bakhshālī Folio 23	OxA-36,758	1096 ± 24	-26.6	890 - 1014
Bakhshālī Folio 33	OxA-35,407	1108 ± 26	-24.6	885 - 995
Bakhshālī Folio 55	OxA-36,759	1137 ± 23	-25.0	774 - 991

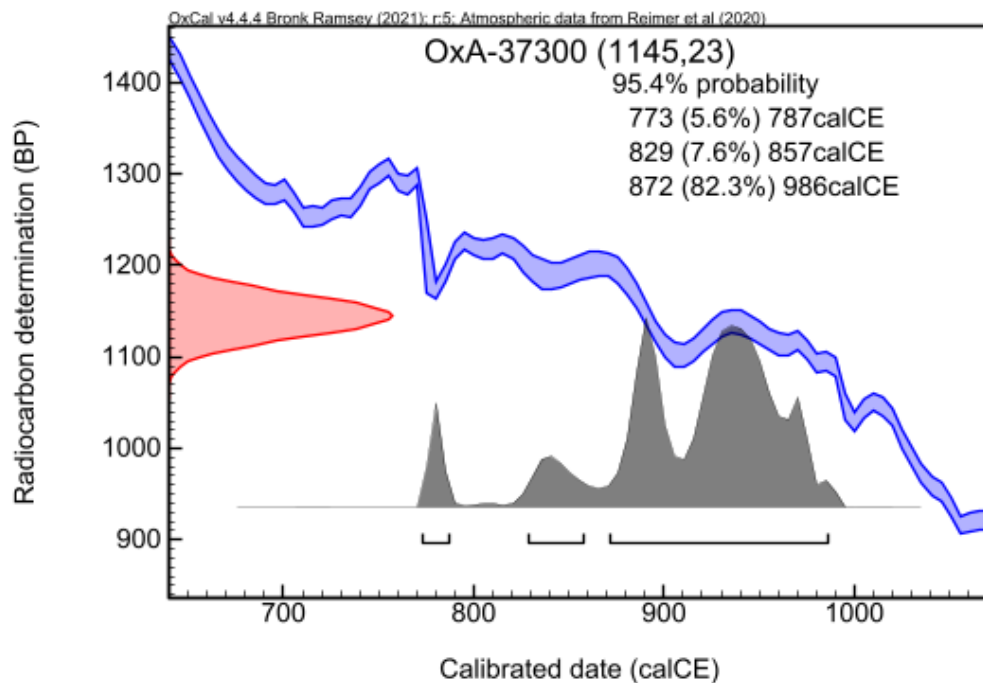


Figure 1. Radiocarbon determination and 95.4% calibrated age range for Folio 15.

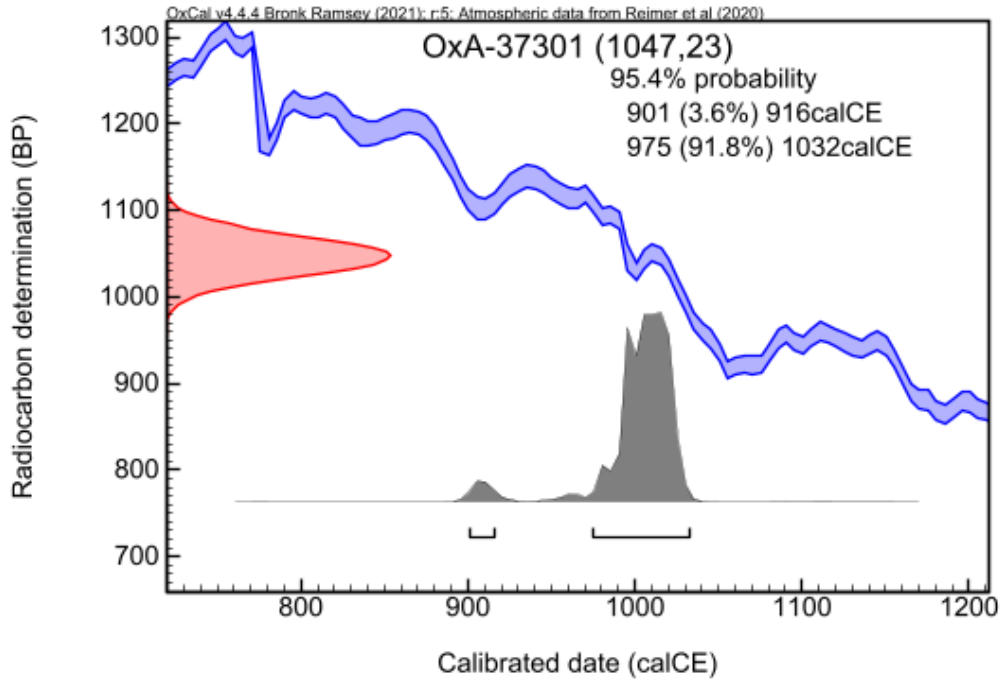


Figure 2. Radiocarbon determination and 95.4% calibrated age range for Folio 16.

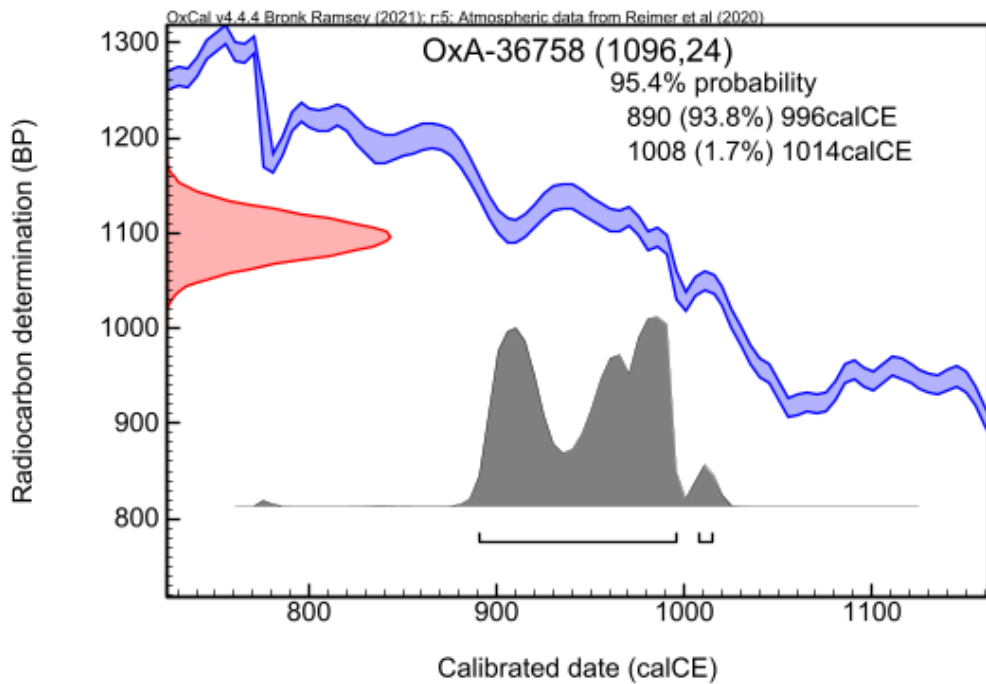


Figure 3. Radiocarbon determination and 95.4% calibrated age range for Folio 23.

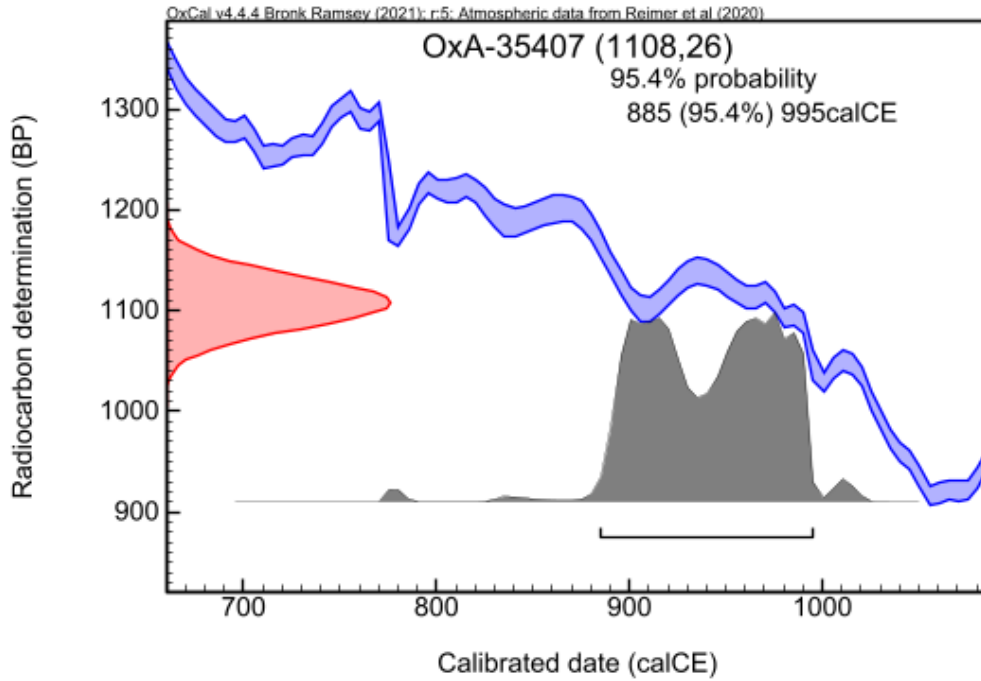


Figure 4. Radiocarbon determination and 95.4% calibrated age range for Folio 33.

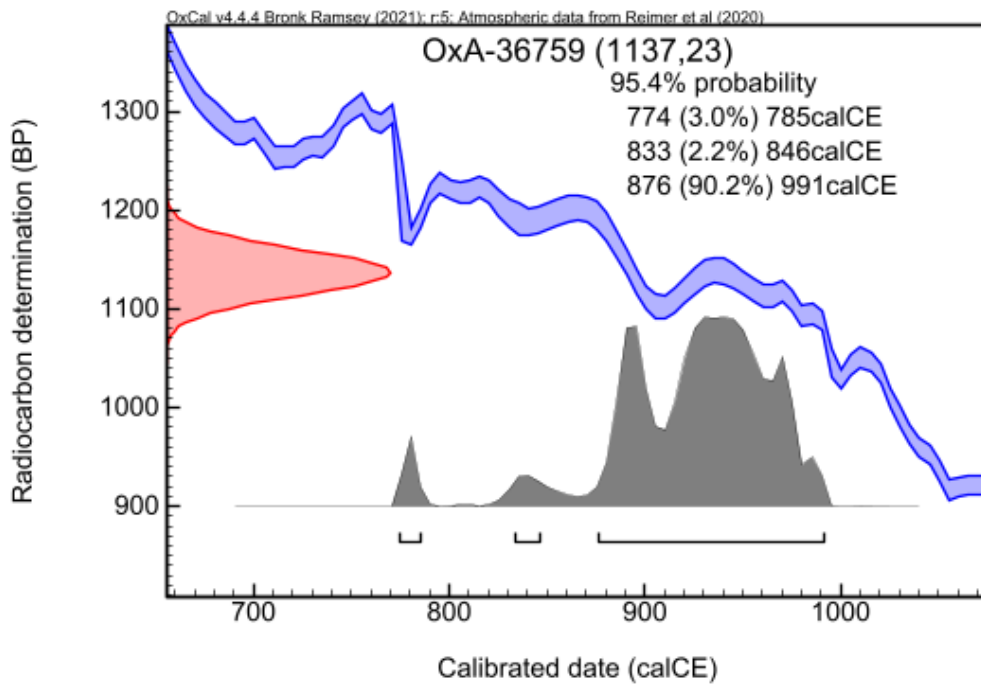


Figure 5. Radiocarbon determination and 95.4% calibrated age range for Folio 55.

Discussion

When interpreting these radiocarbon determinations, it is important to remember that the radiocarbon age relates to the formation of the bark on which the manuscript was written and, although likely to be related, not the time at which the manuscript was written. Of course, it is not possible for the manuscript to be written before the bark is formed, so these dates provide a *terminus post quem* for the construction of the manuscript.

Assuming that the bark in each of the five folios was formed at the same time allows a weighted average age to be calculated from the measurements on all of the folios (OxCal R_Combine function). This gives a calibrated calendar age of 893 – 993 calCE (95.4%). However, the five measured ages fail a chi-squared test ($T=11.416$; where the 5% threshold is 9.5) which means that we can't say that all the folios contain carbon of the same age. This may reflect either the route of carbon into the birch bark and the duration of turnover of bark on each tree, the difficulty of radiocarbon dating such small samples, or that the pages of the folio were written on bark of different ages. Meanwhile, a weighted average of the measured ages for folios 15, 23, 33, and 55 does pass a chi-squared test ($T = 2.9$; 5% threshold 7.8) and gives a calibrated age of 889 – 979 calCE (95.4%).

To provide an age for the manuscript without making any assumptions about the relationship between the ages of the different folios, the radiocarbon ages for the individual folios can be used to populate an OxCal phase model, which can provide a probability distribution for the start and end of a phase which consists of the formation of the bark from each of the dated folios. The output of such a model is displayed in Figure 6 and Table 2; the model code is displayed in the appendix. The phase model provides an age of 799 – 892 calCE (95.4%) for the start of the production of the Bakhshālī manuscript and 900 – 1102 calCE (95.4%) for the end of production of the manuscript. Repeat dates may increase the precision of the modelled start and end of the phase but, because of the destructive nature of the radiocarbon technique, it is not expected that further dating on the manuscript will be carried out.

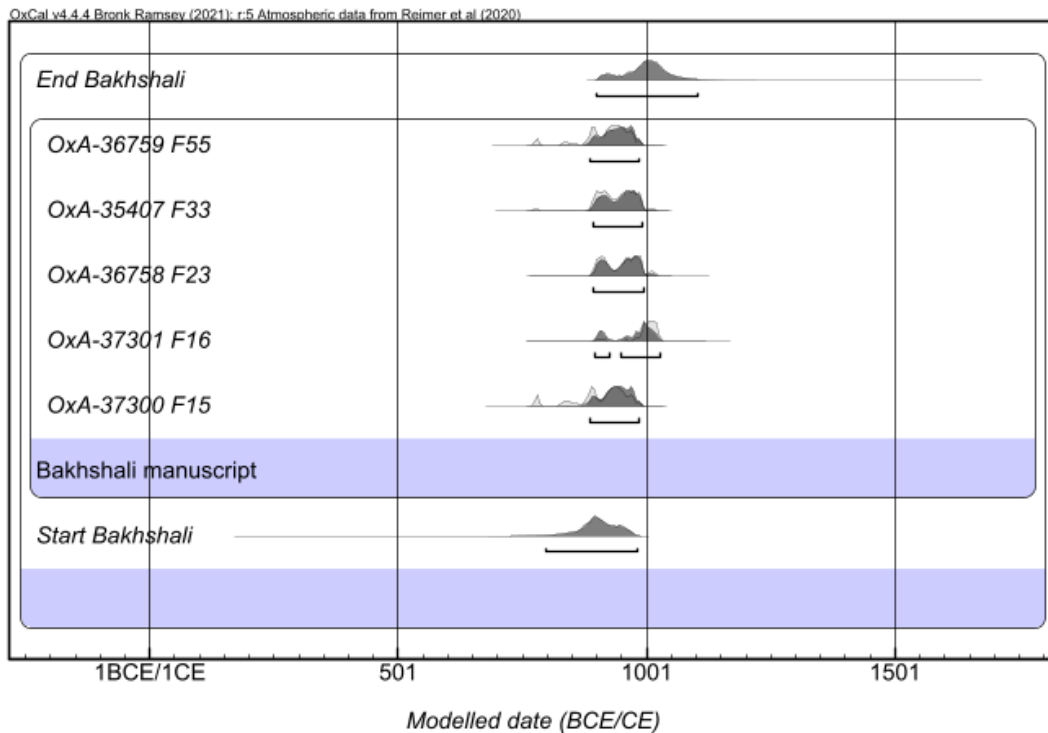


Figure 6. Output from an OxCal phase model for the Bakhshālī manuscript. The light grey areas show the probability distributions for the individual ages, used as priors in the model, and the darker grey areas show the modelled posterior ages.

References

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doi:10.2458/azu_js_rc.55.16947

Appendix

OxCal phase model code

```
Plot()
{
  Outlier_Model("RScaled",T(5),U(0,4),"r");
  Sequence()
  {
    Boundary("Start Bakhshali");
    Phase("Bakhshali")
    {
      R_Date("OxA-35407",1108,26)
      {
        Outlier("RScaled", 0.05);
      };
      R_Date("OxA-36758",1096,24)
      {
        Outlier("RScaled", 0.05);
      };
      R_Date("OxA-36759",1137,23)
      {
        Outlier("RScaled", 0.05);
      };
      R_Date("OxA-37300",1145,23)
      {
        Outlier("RScaled", 0.05);
      };
      R_Date("OxA-37301",1047,23)
      {
        Outlier("RScaled", 0.05);
      };
    };
    Boundary("End Bakhshali");
  };
};
```