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ORIGINAL PAPER

Using vignettes to investigate mathematics teachers' beliefs for promoting cognitive engagement in secondary mathematics classroom practice

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Abstract

Promoting engagement is crucial for encouraging student participation, interest, and learning in mathematics. Student engagement has been conceptualized as interrelated types comprising behavioural, emotional, and cognitive characteristics. *Cognitive engagement*, our focus in this paper, relates to students' psychological investment in learning and practices used to enhance learning, such as self-regulatory strategies and metacognitive processes. Although crucial for students' learning, research suggests that teachers' practices for promoting students' cognitive engagement are not well understood. In this qualitative study, we investigated the beliefs of 40 secondary mathematics teachers across eight English schools concerning promoting cognitive engagement in mathematics classrooms, and whether teachers with different cognitive engagement beliefs differ in the features of classroom practice they attend to in relation to promoting student self-regulation and metacognition. We developed a Cognitive Engagement Framework (CEF) for the following purposes: (1) to develop vignettes that described the practices of two contrasting teachers (Teacher A and Teacher B), who differed in their use of specific self-regulation and metacognitive processes; and (2) to use as a tool for analysis. 17 participants identified with Teacher A who favoured a controlling style towards student strategy use such as activating knowledge, planning, and enacting and regulating strategies, and a passive approach towards students' use of self-reflection. 14 participants identified with Teacher B who favoured promoting student autonomy for planning and enacting and regulating strategies, self-reflection, and acknowledged affective elements. In addition to its findings, the paper makes a methodological contribution by using 'vignettes' as a new way of investigating teachers' beliefs, and a theoretical contribution through the development of the CEF.

Keywords Beliefs · Cognitive engagement · Metacognition · Self-regulation · Vignettes

1 Introduction

Engagement is a broad construct reflecting behavioural, emotional, and cognitive aspects and is associated with positive academic outcomes such as persistence and achievement (Fredricks et al., 2004). Behavioural engagement reflects forms of student participation and interactions. Emotional engagement refers to feelings, and positive or negative emotions. Cognitive engagement is concerned with strategic choices for planning, regulating, and reflecting on learning. The complexity of the engagement construct (Eccles, 2016) and the importance of the underlying motivational factors that drive different types of engagement are considered crucial for mathematics learning (Boekaerts, 2016).

At the classroom level all types of engagement are seen as being interrelated. In this study, however, we focused on cognitive engagement for two related reasons. The first is to make a contribution towards conceptual clarity for the field. Although cognitive engagement (including students' metacognitive knowledge, competencies, and strategy choices) is identified as substantially related to valuable educational outcomes (Michou et al., 2021) and performance (Lingel et al., 2019), it is the least clearly conceptualized aspect of engagement research (Skilling et al., 2016). The second reason is to investigate teacher beliefs about promoting

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students' cognitive engagement. Skilling et al., (2016) identified that teachers more readily noticed student behaviours and obvious emotions, whereas teacher reports of cognitive engagement were fewer in number and scope. Skilling et al. also noted that teachers' descriptions of cognitive engagement were variable, suggesting that recognition of cognitive engagement is not widespread, and perhaps not well understood. Promisingly, their results also indicated that teachers who were capable of identifying characteristics of cognitive engagement used the most effective practices for promoting student engagement in mathematics classrooms. These findings motivate further investigation into teachers' beliefs about promoting cognitive engagement in secondary mathematics classroom practice.

2 Elaboration on the scope

Definitions of cognitive engagement often refer to the psychological investment students apply to their learning. For example, Fredricks and McColskey (2012) described cognitively engaged students as being "thoughtful, strategic and willing to exert the necessary effort for comprehension of complex ideas or mastery of difficult skills" (p. 764). Cleary & Zimmerman (2012) described cognitive engagement as "the process through which students become cognitively and strategically invested in learning" (p. 238). Wolters & Taylor (2012), who draw on the self-regulation model proposed by Pintrich (2004), noted the conceptual overlap between self-regulation and cognitive engagement, and identified four similarities, as follows: cognitive control, which comprises regulation, monitoring, and selection and use of cognitive strategies; self-regulation of motivational beliefs, judgements about competency, perceptions of task difficulty, and regulating affect and emotions; behavioural regulation, which involves participation, organization, and managing one's time and effort for task completion; and, finally, regulating aspects of the learning environment.

Our study, which focuses on cognitive engagement, draws on the definition used by Cleary & Zimmerman (2012) that pays attention to students' cognitive and strategic investment in learning. Specifically, the extent to which students use cognitive strategies and metacognitive processes to self-regulate their learning, assists students with connecting new knowledge to existing knowledge, and for seeking meaning and reflecting on understanding (Vorhölter, 2019).

Although teachers report cognitive and emotional engagement less often than behavioural engagement, this does not diminish the need for teachers to understand this construct more clearly, particularly as teachers play a key role in establishing the learning environment and shaping instruction (Ader, 2019). This understanding involves moving beyond an awareness of the overarching aspects such as self-regulation and metacognition, towards elucidating finer grained aspects such as the following: goal setting; activating knowledge; making choices about and managing planning, enacting, regulating, and monitoring strategies; solving tasks; and fostering executive skills for monitoring and regulating cognitive activities (Lingel at al., 2019). By focusing on these specific practices, teachers are better placed instructionally to support and guide students' cognitive engagement in learning contexts and promote autonomous motivation (Ader, 2019). Failure to focus explicitly on students' self-regulation and metacognitive skills may result in less opportunities for students to take the initiative for making plans to regulate and monitor their learning, which can directly affect learning outcomes (Ader, 2019; Seufert, 2020) in the short and long term (De Corte, 2016).

Reeve and colleagues have highlighted the degree to which engagement can be promoted by teachers through autonomy supportive classrooms (as opposed to controlling classrooms) (Reeve, 2009; Reeve et al., 2014). It is argued that autonomy supportive environments can play an important role in supporting students' intrinsic motivation which underpins the drive for self-directed learning (Liu et al., 2019). Intrinsic motivation ultimately facilitates students' self-determination for valuing learning and establishing goals that are personally important (Deci & Ryan, 2000). Autonomy, which is likened to volition, concerns the desire to self-organize and experience integration and freedom of choice (Deci & Ryan, 2000). It has been shown that providing autonomy support (as compared to control) is associated with more positive outcomes, increased satisfaction, well-being, greater intrinsic motivation, and deeper learning (Vanstennkiste et al., 2009). This is because fulfilling one's innate psychological needs is crucial, and affects the degree to which an individual may satisfy their needs in terms of goal pursuits and attaining valued outcomes.

In educational settings, teachers who themselves are more supportive of autonomy (i.e., by understanding and supporting the students' perspective) are more likely to create autonomous learning environments by means of the following: providing choices (rather than making demands), being understanding of students' needs and interests, displaying patience, providing rationales, and acknowledging negative feelings (Reeve, 2009; Reeve & Cheon, 2021). Autonomy supportive environments have been shown to lead to greater student engagement (Stephanou et al., 2004) and can be promoted organizationally (e.g., through classroom rules and norms), procedurally (e.g., through offering choice for ways of working), or cognitively (e.g., through sharing and evaluating work). In contrast, teachers with controlling motivations are more teacher centred and use controlling styles (Hein et al., 2012). Their motivations tend to be derived from external regulations, where behaviour is driven by rewards, threats or deadlines, or introjected regulations such as shame and guilt (Liu, 2019).

It is important to acknowledge that the efforts teachers make to promote student engagement in classrooms are underpinned by their beliefs about their ability to engage students in learning (Hardré, 2011). Beliefs are considered to be "psychologically held understandings, premises, or propositions about the world that are felt to be true" (Philipp, 2007, p. 259), vary according to the individual, and have a context-specific nature (e.g., diSessa et al., 2002; Hoyles, 1992). Philipp (2007) suggested that what one perceives influences the construction of beliefs in terms of what an individual regards as 'knowledge' and 'values'. Therefore, teachers' beliefs have implications for their behaviours, including the engagement signs teachers notice in mathematics classrooms. As we mentioned previously, teacher reports of cognitive engagement are limited in number and scope, which suggests that recognition of cognitive engagement is not widespread and implies that its importance for student learning may not be well understood (Skilling et al., 2016). This, combined with the finding that teachers who were capable of identifying characteristics of cognitive engagement used the most effective practices for promoting student engagement in mathematics classrooms (Skilling et al., 2016), suggests that the consequences of not attending to cognitive engagement are reduced opportunities to plan, regulate, and monitor learning as these skills are unlikely to develop spontaneously (Desoete & De Craene, 2019).

These findings motivate further investigation into teachers' beliefs about promoting cognitive engagement in mathematics classroom practice. In the present paper we aim to contribute to this research area by reporting on an investigation of secondary mathematics teachers' responses to two specially constructed and contrasting vignettes that describe test preparation activities and with certain cognitive engagement practices intentionally embedded in them. In our investigation we used a qualitative approach to analyze what teachers discern as important or unimportant practices in the vignettes, and how their chosen practices correspond with features of a Cognitive Engagement Framework that we developed (see next section). Vignettes are particularly appropriate stimuli for representations of practice (Buchbinder & Kuntze, 2018) from which teacher awareness and beliefs can be elicited and analyzed. To conclude, the research questions we aimed to address in the research reported in this paper are the following:

1. What are teachers' beliefs about promoting student cognitive engagement in secondary mathematics classrooms?
2. In what ways do teachers with different cognitive engagement beliefs attend to features of classroom practice related to promoting student self-regulation and metacognition?

3 The Cognitive Engagement Framework

In Sect.3 we present the Cognitive Engagement Framework (CEF) and describe how we conceptualized it. In Sect.4 we explain how we operationalized the CEF to structure the two vignettes.

We propose that the CEF (summarised in Fig.1) is helpful for conceptualizing cognitive engagement through descriptions of phases and sub-phases relevant to self-regulation strategies and metacognitive processes. In conceptualizing the CEF we were cognizant of the role of teachers and that the extent to which they promote student engagement may vary. Drawing on the work of Reeve and colleagues (Reeve, 2009; Reeve et al., 2014), the CEF includes the higher-level construct, *autonomy supportive approaches*, which relate to students' innate psychological needs. The main body of the CEF draws on the work of Pintrich (2004) and Zimmerman (2008) on self-regulation strategies for learning and metacognitive processes, to describe the three main phases (forethought, performance, and reflection) and their sub-phases in which we operationalize the notion of Cognitive Engagement. We draw on the frameworks of Pintrich (2004) and Zimmerman (2008) because they take into account behavioural, affective, and motivational factors and include the dynamic and social interactions that take place in classroom settings (Chen & Bonner, 2020).

The *forethought phase* refers to activation of prior knowledge and other processes before initiating work on a task. This can include the following actions: setting learning goals; activating and assessing prior knowledge; planning for strategy use; considering affective elements such as the valuing of, interest in, and self-efficacy with regard to ones' ability to successfully reach the goals set; and goal adoption (e.g., intrinsic and/or extrinsic, immediate and/or distant) (Chen & Bonner, 2020; Zimmerman, 2000).

The *performance phase* includes two sub-processes. The enacting and regulating strategies sub-phase, involves students' actual use and management of the strategies and tactics intended to reach the desired goal (Pintrich et al., 2000). It also reflects learners' efforts to actively manage, modify, or change what they have been doing in order to maintain progress towards the goal, including volition practices (such as control) to prevent distractions that may hinder efforts to reach the goal (Wolters & Taylor, 2012). The monitoring strategies sub-phase is about self-observation and tracking task performance (Zimmerman, 2000). It describes students'

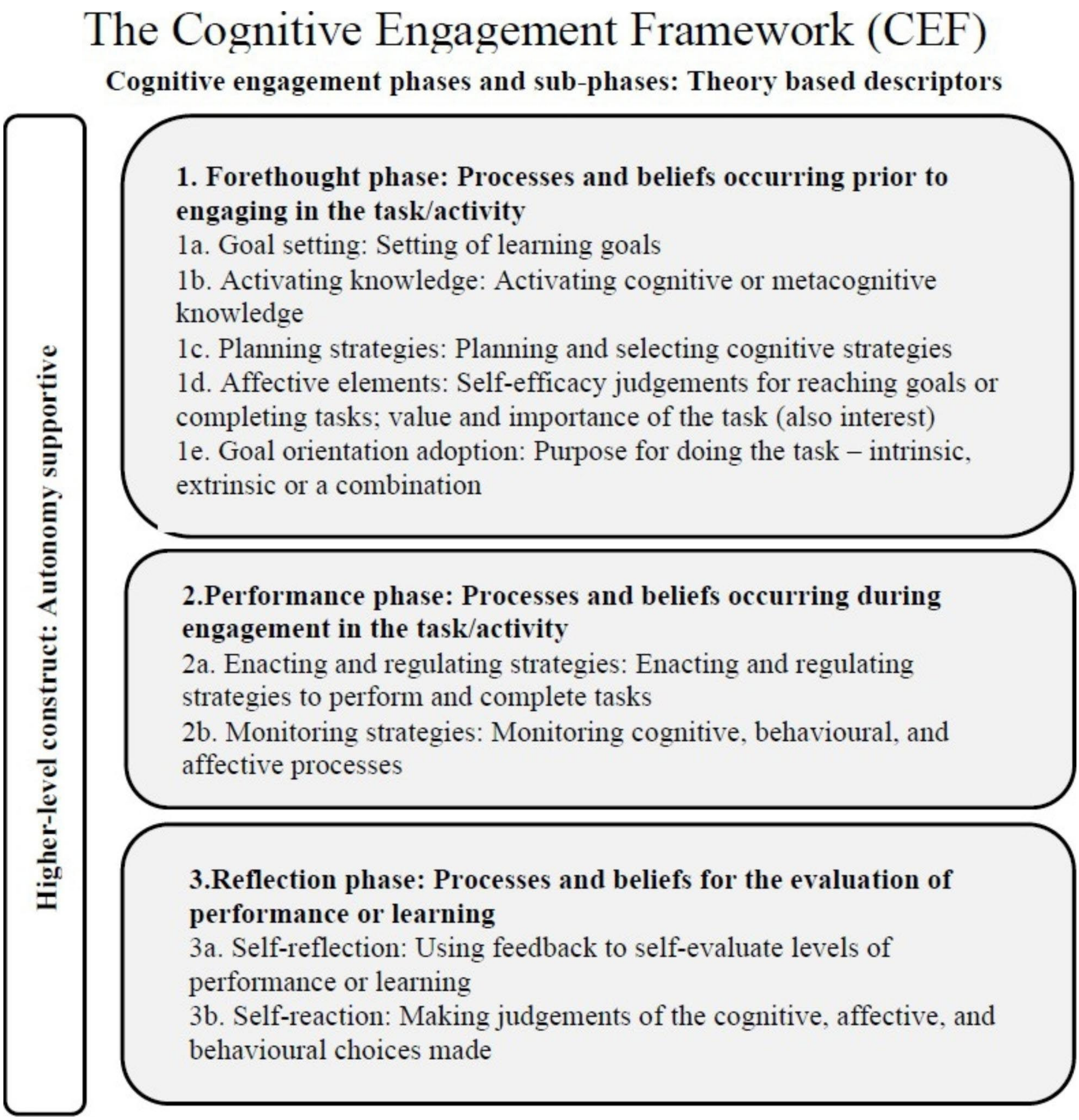

Fig. 1 The Cognitive Engagement Framework (CEF)

efforts to keep track of ongoing progress and performance on a task (Pintrich et al., 2000), which Winne & Hadwin (1998) described as 'metacognitive monitoring' including feedback about the rate of progress towards goals and the effectiveness of particular strategies (Wolters & Taylor, 2012).

The *reflection phase* includes the self-reflection and self-reaction sub-phases. Self-reflection includes learners' efforts to review and make judgements about their overall performance through the monitoring and feedback experiences with the task, as well as use of this information to self-evaluate their performance or learning (Ader, 2019; Wolters & Taylor, 2012). Self-reaction relates, but is not limited, to the generation of new metalevel knowledge about the tasks and strategies and their effectiveness. Here judgements can be made about the cognitive, affective, and behavioural choices made that explain discrepancies between actual performance and initial goals (Zimmerman, 2000).

Although we presented the three phases in a linear fashion, these phases are generally not seen to be time ordered (Pintrich, 2004). Self-regulated learners are expected to engage and re-engage in these phases in a "cyclical flexible and adaptive fashion so they can efficiently and successfully complete academic work" (Wolters & Taylor, 2012, p. 639). Therefore, it is the cycle of the self-regulation phases that is important, rather than a focus on one phase over another (Seufert, 2020). The CEF provides a structure for conceptualizing the specific phases and sub-phases for self-regulated learning and metacognitive process, which we suggest can provide conceptual clarity and have practical implications for teachers.

Not much is known about the practices teachers use to promote self-regulated learning (Ader, 2019) or the extent to which teachers explicitly teach students to self-regulate while they are involved in cognitive activities. There is also evidence that teachers lack metacognitive knowledge and are reluctant to promote these skills (Desoete & De Craene, 2019). It is important to understand whether teachers consistently and effectively promote the active use of self-regulatory strategies and metacognitive skills, because not all students will use these spontaneously. This may be because students become cognitively overloaded during learning and find it difficult to self-observe and reflect (Zimmerman, 2000), or as novices, they have yet to acquire more advanced self-regulation skills (De Corte, 2016). The CEF can be used as a tool to examine which strategies teachers might attend to and promote in mathematics classrooms in relation to students' cognitive engagement. To operationalize this affordance of the framework, we used the CEF to guide the design of the two vignettes that we used to elicit teachers' beliefs about promoting cognitive engagement in secondary mathematics classroom practice.

4 Methodology

4.1 Participants

We collected data from 40 mathematics teachers across eight secondary schools in England. The teachers' years of teaching experience spanned between 2 and 25, with an average of 12 years and 4 months. Eighteen schools that were local to the researchers were invited to participate, of which six declined, two did not respond and two of the schools that agreed to take part did not return any surveys, resulting in data from eight schools. The schools included five state (public) schools and three independent (private) schools. Information packs explaining the purpose of the study, consent forms, and surveys were sent to the Head of Mathematics in each school. These contained individual envelopes for participants and were pre-addressed so that they could be returned directly to the researchers once completed. The participants were asked to complete a survey with eight open-ended questions, which centred around two narrative vignettes (see Appendix) based on the theoretical frameworks of self-regulation, goal theory, and metacognition as those were captured in the CEF (Fig.1).

4.2 Vignettes and survey questions

In a previous paper (Skilling & Stylianides, 2020) we discussed the affordances of using vignettes for exploring complex issues, including beliefs, which as we mentioned previously can be context-specific (e.g., diSessa et al., 2002; Hoyles, 1992), and how vignettes make it possible for certain kinds of questions to be asked, allowing participants to convey their views in a way that is non-threatening to them (Torres, 2009). Vignettes are especially helpful for presenting realistic but hypothetical situations from which participants' beliefs and views about a topic can be elicited. In our study, a survey that posed eight open-ended questions about the two vignette narratives was the major data collection instrument. To guide the construction of vignettes so that they were credible and methodologically consistent with the aims of the research, we used the Vignette Framework (Skilling & Stylianides, 2020). Specifically, when developing the vignette narratives and survey, we followed the three key elements and associated characteristics in the Vignette Framework: (1) conception (capturing content, realistic and hypothetical portrayals, purpose/function); (2) design (presentation, length, settings and terminology, open or closed questioning, participant perspectives, piloting); and (3) administration (instructions, timing and responses, delivery mode and frequency).

The vignettes in the survey described two fictitious (albeit realistic) vignette narratives as to how Teacher A and Teacher B prepared their Year 7 students (11 years olds) for a mathematics test. In keeping with the 'conception' element of the Vignette Framework, each vignette drew on literature that guided the development of the CEF and embedded particular phrases as 'markers' to emphasize the different strategies and processes used by Teachers A and B.

In more detail, and in line with the CEF, the vignettes differed by the degree and nature to which Teachers A and B promoted student autonomy and active involvement (versus control), using particular strategies and processes when preparing their students for the test. For example, Teacher A handed students a list of topics for revision and suggested they revise at home by looking over their notebooks, whereas Teacher B asked students to reflect on how competent they felt about particular concepts and to develop an individual revision plan. Teacher A told students it was important to get a high grade in the test, to ask for help if needed, and offered a small number of practice questions in class for those who may not have been revising at home.

Table 1 How the two vignettes correspond to the different Cognitive Engagement Framework categories

Cognitive Engagement Framework	Teacher A vignette	Teacher B vignette	Comparison between the two vignettes
1. **Forethought phase**			
1a. Goal setting	preparing students for the test	preparing students for the test	Present and same
1b. Activating knowledge	handed students a list of concepts	asking students to recall likely concepts	Present but different
1c. Planning strategies	- asking students to practice each concept by looking over their notebooks - setting time to revise	- asking students to draw on self-assessment notes - creating class list and individual plans	Present but different
1d. Affective elements	no self-efficacy or confidence beliefs sought	asked students to assess competency beliefs	Absent vs. Present
1e. Goal orientation adoption	importance of high grade	focusing on mastery	Present but different
2. **Performance phase**			
2a. Enacting and regulating strategies	- encouraging students to ask questions in class if unsure of steps to solve tasks - being available to students during break for questions - doing 5 practice questions	- asking students to identify examples from the class list to practice - asking students to create an individual plan for revising at home	Present but different
2b. Monitoring strategies	asking students to seek help from teacher	asking students to revise at home and identify areas for improvement	Present but different
3. **Reflection phase**			
3a. Self-reflection	asking students to assess how many questions they got correct	asking students to focus on what they believed they needed to improve on	Present but different
3b. Self-reaction	advising students to study more if less than 3 practice questions are correct	checking affective aspects of preparation	Present but different

In contrast, Teacher B told students to focus on mastering concepts they did not understand, checked students' revision plans, and asked them how they felt about their test preparations. Overall, there were limited expectations by Teacher A about the range and depth of self-regulatory processes the students would use and there was an emphasis on performance rather than mastery goals. Teacher B, on the other hand, asked students to set their own goals based on their self-assessment, to make plans for mastering concepts, and to monitor and reflect on their revision preparation.

In the survey, the participants were asked to respond to open-ended questions about the fictitious scenarios by referring to information in each scenario that they felt provided evidence for their responses. To address research question 1 (RQ1) we used participants' responses to question 6 in the survey, whereas to address research question 2 (RQ2) we used participants' responses to questions 1–5. In more detail, participants' responses to question 6 ('Overall, which teacher do you identify with?') allowed us to group participants as part of our analysis for RQ1 into those who identified with Teacher A and those who identified with Teacher B, representing different cognitive engagement beliefs. We then used these two categories of participants as a frame for our analysis for RQ2 in order to examine how participants with different beliefs attended similarly or differently to features of classroom practice as depicted in the two vignettes.

4.3 Analysis and coding

The analysis of the surveys drew on the CEF, as components of this framework were reflected in the design of the two vignettes. Specifically, for each of the nine framework categories (1a, 1b, 1c, 1d, 1e, 2a, 2b, 3a, 3b) we examined whether, and if so how, it applied to the Teacher A or Teacher B vignettes. We summarize our analysis determinations in Table1. The first column in the table includes the nine framework categories of the CEF (1a to 3b). The second and third columns identify the nine categories relevant to the Teacher A and B vignettes, respectively. The final column indicates the extent to which the nine framework categories were present and the same, present but different, or one was present and the other absent. This preparatory work helped us operationalize the CEF as a coding scheme for our analysis. It also confirmed that the two vignettes were indeed suitable for our purposes as they both captured appropriately the various framework categories, albeit in their own distinct ways.

The responses to the six survey questions that are relevant to this paper were coded independently by at least two coders (the two authors and a research assistant) who then compared their codes and resolved disagreements. This analysis involved the individual researchers considering the participants' responses and assigning the responses to one or more of the categories of the coding scheme, which captured

Table 2 Summary of responses to survey question 6

Survey Question	Student characteristics (S)[1]					Structure/Control (SC)	Independence/ Autonomy (IA)	Time Considerations (T)	Not applicable
	S1	S2	S3	S4	S5				
6	2	3	2	1	4	6	17	6	8

Note 1: S1 - students' ability to be independent; S2 - students' age or maturity; S3 - students' individual needs; S4 - students' mathematical ability; S5 - other student characteristic.

the significant features of the vignettes with the embedded self-regulation and metacognitive sub-phases.

The fact that we did not include numerical measures of agreement between different coders is a limitation, which was mitigated by us following a systematic, explicit, and transparent process of analysis. In the approach we adopted, each coder was engaged in the textual meanings and interpretations of the responses as suggested by Krippendorf (2004). This included discussions between coders about the coding scheme, and as it was applied to the responses, reflecting the interpretive nature of qualitative research (O'Conner & Joffe, 2020) and demonstrated that a consensus for interpreting the data within a common conceptual framework could be reached.

In more detail, each response to questions 1–5 received one or more codes from the nine possible categories (1a to 3b) in the CEF (each code could be assigned at most once for each response). The responses to question 6 were coded at two levels. The first level related to which of the two teachers in the scenarios a participant identified with the most; four codes were used to code all the responses (Teacher A, Teacher B, both Teacher A and Teacher B, and neither Teacher A or B). The second level related to coding the reasons the participants provided for their responses. When coding the reasons we included the top level category of autonomy (versus control) for capturing the nature of the participants' responses; this approach to coding reflected an enriched and realistic consideration of participants' responses (Constas, 1992). As we explained earlier, although autonomy is a separate construct from cognitive engagement, it is closely connected to a number of motivational factors relevant in learning settings that influence the ways in which students engage, such as feeling a sense of belonging and competence (Reeve, 2009). The reasons provided by the participants in question 6 resonated with those connected to autonomy and its antithesis (control), and were categorized as follows: structure/control (providing more or less structure/control to the students); independence/autonomy (giving students more direction or information, or encouraging student independence/autonomy); time considerations; and student characteristics of which there were five categories (students' ability to be independent, students' age or maturity, students' individual needs, students' mathematical ability, and a final category that captured student characteristics where the particular characteristic was left unspecified). Each response received one or more these codes (each code could be assigned at most once per response). In Sect.5 we illustrate the codes using excerpts from participants' responses to the survey. Our selection of the excerpts aims to capture the breadth of the responses.

To conclude, our approach to the analysis of the data was qualitative for two key reasons. First, the research design used an open-ended survey centred around two vignette narratives from which we asked participants for written responses. These responses varied according to each participant and provided rich data which were attuned to a qualitative analysis approach. Second, although we recorded the frequencies of the participants' responses to categories 1a to 3b of the CEF, the number of cases for each category were variable and, for some of the open-ended questions, too small to allow application of statistical tests in the analysis. For example, when we compared the frequencies of the responses of participants who identified with Teacher A versus Teacher B for questions 3, 4, and 5, the number of cases for each category (1a to 3b) varied and did not meet the suggested threshold for using Chi-square tests, that 80% of the data categories have five cases or more (Cohen et al., 2017).

5 Results

5.1 Teachers' beliefs about promoting cognitive engagement

To address the first research question, we drew on participants' responses to question 6 of the survey, which asked them to state which of the two teachers (Teacher A or B) they identified with the most and why. Of the 40 participants, 17 identified with Teacher A (participants referred to as AT1 to AT17), 14 identified with Teacher B (referred to as BT18 to BT31), six identified with both (referred to as CT32 to CT37), and three identified with neither (referred to as DT38 to DT40).

Participants' explanations of the reasons they identified as they did resonated with autonomy and its antithesis (control). The categories we used for coding the responses, outlined in Sect.3.3, and the frequencies of codes, are summarized in Table2.

Table 3 Combined responses of Teacher As and Teacher Bs to survey questions 1 and 2

Survey Questions	1. Forethought					2. Performance		3. Reflection		
	1a. Goal setting	1b. Activating knowledge	1c. Planning strategies	1d. Affective elements	1e. Goal orientation adoption	2a. Enacting & regulating strategies	2b. Moni-toring strategies	3a. Self-reflection	3b. Self-reaction	not appli-cable
1	22	4	20	0	0	12	2	1	5	1
2	0	12	5	0	2	17	0	3	1	8

Table2 shows that responses citing student independence/autonomy (IA) as their reasons were greater than for the other categories. This reason was given 11 times by participants who identified with Teacher B, five times by those identifying with Teacher A, and once by a participant who identified with both. Some examples for the IA code are the following:

> "I try to get the students to identify their own areas for revision and how to use their exercise books to guide them." (BT20)
> "For students to achieve their potential they need to be self-critical & independent. Allowing students to identify their own areas to improve and implement their own plan is a step towards them improving because they want to, not because I'm telling them they need to." (BT24).

In comparison, all participants citing reasons of structure/control (SC) identified with Teacher A. Examples of responses are the following: "It leaves most control with me" (AT1) and "I tend to be more structured and lecture rather than give freedom to think. I don't trust my students to revise" (AT10).

Time considerations (T) were mentioned mainly by participants who identified with Teacher A. These included examples such as: "Time always seems to be too much a factor" (AT6), "Teacher A, as [they are] very much teacher led focused and less time consuming" (AT9), and "I would occasionally do some aspects but find it more time consuming" (AT11).

Several responses made references to particular student characteristics. For example, students' ability to be independent (S1) was evidenced by "students are often 'needy'" (AT8). Another characteristic related to student age or maturity (S2). For example, participant CT34 commented: "For me, it's all about time & age. I would be more like Teacher B for younger classes and/or when time is less of a pressure." The same participant then continued: "Another key factor is the maturity of the learner & how capable they are in preparing by themselves. […]" The individual needs of students (S3) were also mentioned in the responses: "not all students need to revise every topic" (BT29) and "different children and different classes need different approaches" (CT33). Students' mathematical ability (S4) was also identified as a reason for identifying with one of the two teachers: "Teacher A as I teach low ability sets and they need guidance as to how to revise" (AT12). Student characteristics not classified as S1 to S4 were categorized as 'Other Student Characteristics' (S5). For example, AT15 reported that their teaching approach is "slightly more with teacher A but it depends upon which students I am teaching as to how much I can use teacher B ideals".

5.2 Teachers' reports about features of classroom practice related to promoting cognitive engagement

To address the second research question, we focused on the 17 participants who identified themselves with Teacher A and the 14 participants who identified themselves with Teacher B (hereafter referred to as 'Teacher As' and 'Teacher Bs', respectively) as these two categories corresponded to 31 of the 40 participants. Specifically, we drew on those participants' responses to questions 1 to 5 of the survey that received one or more codes from the nine possible categories (1a to 3b) in the CEF. Because questions 1 and 2 asked participants to name similarities and differences between the two teacher scenarios, respectively, we summarize the combined responses of Teachers As and Teacher Bs in Table3.

Question 1 asked participants to report key similarities between the two scenarios. It can be seen from Table3 that the forethought phase, particularly the sub-phases of 'goal setting' (1a) and 'planning strategies' (1c), were perceived as being similar across Teacher A and B. The next key similarity was 'enacting and regulating strategies' (2a), which relates to the performance phase, followed by 'self-reaction' (3b) which is part of the reflection phase. In more detail, for 'goal setting', comments included informing, notifying, or reminding students about the test. For 'planning strategies', comments included: "both teachers giving lesson time to allow students to practice key concepts in preparation for the test" (AT16) and "revising using their exercise books as a revision guide" (BT19). Examples of 'enacting and regulating strategies' included the following responses: "Attempt to get students to identify their own problems" (AT16), "They both believe that practicing concepts is important"

(BT23), and "Expectations of individual preparation were given" (BT27). 'Self-reaction' is illustrated in these participants' comments: "Both teachers ensured students' work plans [were] checked" (AT12) and "Both [teachers] are checking students have done what is asked" (BT30).

Question 2 asked participants to identify differences between the two scenarios. It can be seen from Table3 that 'activating knowledge' (1b), 'planning strategies' (1c), and 'enacting and regulating strategies' (2a) were the most commonly reported. In more detail, regarding 'activating knowledge' (1b) example responses included the following: "Teacher A gave students guiding questions as a starter while teacher B provided self-assessment sheets" (AT14), and "Teacher A focused more on telling students what to do. Teacher B allowed students to decide for themselves" (BT18). For 'planning strategies' (1c), AT4 reported, "Teacher A supports students more directly and makes sure that all possibilities are covered. Teacher B allows more co-construction and individualised response to the problem". Examples of responses coded as 'enacting and regulating strategies' (2a) included the following: "[Teacher A vignette] is more directed with specific questions targeted to identify problems. [Teacher B vignette] left students to identify their own problems" (AT6); and "Scenario A directed the students whereas scenario B gave the students the opportunity to build their own revision" (BT20).

The results of survey questions 3 to 5 are summarized in Table4, which reports the participants' responses separately for Teacher As and Teacher Bs. Presenting the results in this way highlights the varying emphasis on sub-phases according to which teacher the participants identified more with in their response to question 6. Overall, it can be seen that, regardless of which teacher the participants identified with, both groups reported more characteristics of the 'forethought' phase than the other phases for each questions 3 to 5.

Survey question 3 asked participants to list up to two things that each teacher in the vignettes did that they believed were important. As can be seen in Table4, many of Teacher As' and Teacher Bs' responses related to sub-phases 'enacting and regulating strategies' (2a), 'activating knowledge' (1b), and 'planning strategies' (1c) even though Teacher As reported these sub-phases more frequently than Teacher Bs. It can also be seen that both groups' responses indicated that 'affective elements' (1d) were important and that none suggested that 'goal orientation adoption' (1e) was important. There were fewer mentions related to sub-phases of the 'reflection phase' (3a and 3b) for Teacher Bs than for Teacher As, even though both groups' responses suggested 'self-reflection' (3b) as being important. For example, BT22 noted, "Checked their revision plans to make them self-aware and in control of their revision".

Table 4 Teachers As' and Teacher Bs' responses to survey questions 3, 4, and 5

Survey Questions	Participants	1. Forethought						2. Performance			3. Reflection			
		1a. Goal setting	1b. Activating knowledge	1c. Planning strategies	1d. Affective elements	1e. Goal orientation adoption	sum	2a. Enacting & regulating strategies	2b. Monitoring strategies	sum	3a. Self-reflection	3b. Self-reaction	sum	not applicable
3	Teacher As	1	13	13	3	0	30	24	2	26	1	8	9	0
3	Teacher Bs	2	9	9	5	0	25	18	3	21	0	6	6	1
4	Teacher As	0	4	7	2	5	18	2	1	3	1	7	8	11
4	Teacher Bs	0	2	9	0	6	17	6	0	6	0	9	9	8
5	Teacher As	3	13	13	4	2	35	16	0	16	2	0	2	6
5	Teacher Bs	2	11	11	5	3	32	19	2	21	1	1	2	2

Question 4 asked participants to list up to two things that each teacher in the vignettes did that they perceived as *not* being important. As can be seen in Table4, overall, a number of Teacher As' and Teacher Bs' responses indicated that 'planning strategies' (1c) and 'goal orientation adoption' (1e) were not important. Regarding Teacher As, for 'planning strategies' (1c), AT15 reported that the two teachers "Encouraged students [to] 'look over notebooks' [as] maths needs to be an actively engaged in discipline". In addition, 'self-reaction' (3b) was not seen as important. For example, AT7 stated that "Checking individual revision plans [was] time wasting?!". Regarding Teacher Bs, for 'planning strategies' (1c), BT25 reported as not important to make "revision a 'suggestion' without clear guidance". Several Teacher Bs' responses suggested that 'enacting and regulating strategies' were not important, such as "Revising in class" (BT18). Regarding 'self-reaction'(3b), nine Teacher Bs' responses suggested this as not important. For example, BT24 reported that "Telling students 'they needed to study more' without students knowing what aspects to cover".

Finally, question 5 asked participants to list any of the strategies reported in the two vignettes that they used in their practice. As can be seen in Table4, the most commonly reported strategies, listed by both Teacher As and Teacher Bs, included 'activating knowledge' (1b), 'planning strategies' (1c), and 'enacting and regulating strategies' (2a). Responses to 'activating knowledge' included "Giving an outline of topics in tests" (AT10) or asking "students to develop revision notes" (BT18). Responses to 'planning strategies' included setting aside time for revision (AT10) and encouraging "students to self-assess and set targets" (BT24). For 'enacting and regulating strategies' responses included setting "practice questions to identify strengths and weaknesses" (AT8) and "Using student information to target key topics the students are struggling with" (BT24).

6 Discussion

Promoting students' engagement is crucial for their education, in general, and mathematical education, in particular, as it has been associated with encouraging participation, interest, and learning in mathematics. Yet teachers' practices for promoting students' cognitive engagement are currently not well understood (Skilling et al., 2016). In this paper we aimed to contribute to this area of research by investigating the beliefs of 40 secondary mathematics teachers across eight English schools about promoting cognitive engagement in mathematics classrooms, and whether teachers with different cognitive engagement beliefs differ in the features of classroom practice they attend to in relation to promoting student self-regulation and metacognition.

The lack of specialized research focusing on the specifics of cognitive engagement led us to develop the Cognitive Engagement Framework (CEF), which we used in the following different ways: as a theoretical frame, as a tool to guide the development of the vignettes, and as a tool to code our participants' responses to the survey questions. The design of the vignettes and the open-ended questions was purposeful, based on existing theories which were reflected in the structure of the CEF, and from which we could interpret the participants' rich understandings of self-regulation strategies and metacognitive processes at a micro-level. The rich data we collected illustrate the affordances of using vignettes for exploring complex issues such as participants' beliefs (Skilling & Stylianides, 2020).

Specifically, the two vignettes appeared to allow our study participants to convey their views, in ways that were non-threatening to them, in a complex classroom scenario related to preparing students for a test. The fact that there was relative parity between participants' identification with Teacher A and Teacher B (referred to as Teacher As and Teacher Bs, respectively) indicates that the vignette narratives comprehensively and fairly reflected the phenomenon they aimed to represent and teachers were able to identify with at least one of the two vignette narratives; this, we argue, demonstrates content validity (Cohen et al., 2017). The identification with either Teacher A or B indicates that the participants recognized and resonated with particular practices that we deliberately embedded within the two vignettes, albeit with different degrees of support for promoting cognitive and strategic investment in learning.

With respect to the first research question, which examined teachers' beliefs about promoting cognitive engagement in secondary mathematics classrooms, the findings revealed that substantially more teachers who identified with Teacher B gave reasons related to student independence/autonomy, whereas all teachers who identified with Teacher A gave reasons related to structure/control. More specifically, teachers who identified with independence/autonomy believed that students should be active in identifying areas for revision and improvement, that students should implement their own plans because they want to, and that they should be self-critical and use resources as guidance. These are characteristics of self-regulated learning and metacognition in terms of setting goals, making decisions about planning, enacting particular strategies, and being reflective (De Corte, 2016; Lingel et al., 2019). In contrast, teachers identifying with structure/control cited concerns about classroom time, as well as students' needs/maturity/ability as reasons for being in favour of more teacher-led approaches. This is of some concern as Reeve (2009) has identified that controlling teacher approaches lead to less motivated classes because students' innate needs for belonging and competency may not be met; this, in turn, can affect students' goals and values, and directly affect their academic outcomes (Wilson et al., 2021).

Moving now to the second research question, which examined the features of classroom practices to which participants reported attending. Overall, 'activating knowledge' and 'planning strategies' from the 'Forethought' phase, followed by 'enacting and regulating strategies' from the 'Performance' phase were the most frequently reflected in the participants' responses. That the participants attended to these sub-phases more than others was not unexpected. Perhaps setting goals, activating knowledge, and enacting and regulating strategies are more apparent and familiar phases and logical starting places for teachers for supporting self-regulation as compared to monitoring and reflection strategies or attention to affective elements. Teacher Bs considered 'goal-orientation' and 'self-reflection' as important, whereas Teacher As did not seem to consider 'self-reaction' as important. Teacher As also did not think that creating a class revision list or attending to students' affective elements were important and considered checking of individual plans not to be a good use of time. Yet supportive learning environments at the organizational, procedural, and cognitive level have been found to lead to greater student engagement (Stephanou et al., 2004).

When reporting about strategies the participants had previously used themselves, Teacher As reported the following three most frequently coded strategies in questions 1–5: 'activating knowledge' and 'planning strategies' from the 'Forethought' phase, followed by 'enacting and regulating strategies' from the 'Performance' phase. In contrast, Teacher Bs included more variety of strategies, including 'affective elements' and categories of the 'Reflection' phase. Specifically, Teacher Bs' responses tended to relate to 'goal setting', 'activating knowledge', 'enacting and monitoring strategies', and 'affective elements'. Many of these strategies are associated with autonomy supportive environments (Reeve, 2009), deeper learning (Vanstennkiste et al., 2009), and emphasize the importance of attending to all components of the self-regulation cycle (Seufert, 2020).

7 Conclusion

To conclude, this study uncovered the differing beliefs of teachers about autonomous (versus controlling) approaches and for promoting cognitive engagement in secondary mathematics classroom practice. The implications of supporting student autonomy have been well documented and seen to result in increased student engagement, by supporting students' intrinsic motivation and internalization (students' personal valuing), which drives self-directed learning behaviours (Liu et al., 2019; Reeve & Cheon, 2021). This is in contrast to controlling teaching styles that adopt an authoritarian attitude and prescribe what students should think and do, which results in students forgoing their own needs and goals to adhere to what they are told.

One limitation of our study is that a single teaching situation was depicted in the vignettes, that of test preparation. The selection of a specific situation, however, was consistent with the context-specific nature of beliefs (e.g., diSessa et al., 2002; Hoyles, 1992) and was purposefully chosen because we anticipated it would resonate with participants' school contexts and occasions for using self-regulation strategies and metacognitive processes to promote cognitive engagement. It would be important for future studies to investigate other teaching situations where phases and sub-phases of self-regulation and metacognition are explicitly guided by teachers (Ader, 2019; De Corte, 2016). For example, investigating the promotion of specific phases and sub-phases of self-regulation and metacognition for cognitively demanding activities (e.g., Doyle 1988), such as solving problems or non-routine mathematical tasks, would be valuable for understanding teachers' beliefs about the importance of attending equally to each of the CEF phases. Future studies could also investigate students' cognitive engagement in problem solving tasks by asking students to self-report their problem solving activity and using the various sub-phases of the CEF framework. Promoting an active approach for analyzing and developing self-regulatory practices not only supports student autonomy but encourages learners to take responsibility for their own progress, in the context of continuous and lifelong learning (De Corte, 2016).

Another limitation of the study relates to the small sample size, which as discussed earlier did not allow us to conduct any statistical tests in our analyses. Our rich data were more attuned to a qualitative analysis approach, which is how we decided to analyze them. We were cognizant of the limitations of self-reports for measuring teachers' promotion of students' self-regulation (Ader, 2019). To address these limitations, and to strengthen future research that might use vignettes as a key method, we took care to align the frameworks and constructs of cognitive engagement and embed the phases and sub-phrases outlined in the CEF in the two vignettes, in order to establish construct validity (Harrits & Møller, 2021). However, future research can build on the conceptual work we did in this paper to develop the CEF, as well as our methodological approach and instrument, in order to explore similar issues with a larger sample and to develop more generalizable findings.

The study has theoretical implications for specifying sub-phases of self-regulation and metacognition processes and framing these via the CEF. Specifically, we developed the CEF to operationalize cognitive engagement, and to some extent address the call for clarifying and conceptualizing engagement that reflects everyday language that is relevant to teachers' work (Fredricks et al., 2004). In addition, the CEF proposes 'Autonomy Supportive' approaches as a higher-level construct, which is emphasized by Reeve & Cheon (2021) as positioning the teacher to be responsive to and supportive of students' needs and underscores the importance of promoting their active participation in planning, monitoring, and regulating their own learning (Ader, 2019; De Corte, 2016).

From a practical perspective, the specific strategies discerned in the CEF present or capture those strategies that are important for promoting student cognitive engagement, and this framework can help teachers identify the ones that are more elusive (such as self-reflection) or difficult for them to use in their mathematics classrooms. Our findings showed that 'monitoring strategies' were rarely reflected in the participants' responses, suggesting that there is scope for drawing teachers' attention to the benefits of supporting learners to evaluate their own rate of progress towards reaching their goals and to assess the effectiveness of the strategies they are using. Teachers' reluctance to promote metacognition, particularly the *how* and *why* of using particular skills (Desoete & Craene, 2019), and the importance of attending to all phases of the self-regulation cycle is important to understand. Our study highlights that teachers with different beliefs about promoting students' cognitive engagement manifest different attention to relevant features of classroom practice. Future research might investigate whether this result indicates that some teachers do not think certain features are beneficial when planning approaches to instruction or, perhaps, that training and guidance are needed to enable teachers to support students more consistently and effectively.

A final implication is our use of vignettes as a tool for conveying hypothetical but realistic depictions of teaching and learning situations to investigate teachers' beliefs. Although other methods, such as surveys with Likert scale items and case studies and prompts have been used to investigate beliefs, vignettes can be constructed to approximate complex situations, and incorporate open-ended questions for investigating beliefs in an in-depth way (Finch, 1987). Vignettes were effective for stimulating the responses from the in-service teachers who participated in our study, and we see the potential this offers for pre-service teachers as well. Vignettes could include situations that pre-service teachers have yet to encounter and assist with both visualizing such situations and as a context for in-depth discussions in teacher preparation (Buchbinder & Kuntze, 2018).

8 Appendix: Teacher vignettes used in the study[1]

Context.

Two Year 7 mathematics classes at one school will complete a topic test during second term. The teachers of each class provided students with information a week before the test about ways they could prepare. Below are suggestions by Teacher A and Teacher B. Please read each scenario and respond to the questions at the end. The line numbers for each scenario have been included to help make references to the text.

Scenario involving Teacher A.

1. Teacher A reminded students about the upcoming topic test and handed out
2. a sheet with an outline of the key concepts that would likely be covered in
3. the test. The teacher suggested that the students set aside time for revision
4. and to make sure they practised each concept, by looking over their notebooks
5. as it was important for them to achieve a high grade on the test. The teacher
6. also mentioned that the students should ask questions in class if they were
7. unsure of the steps to solve questions. Alternatively, they could come and see
8. the teacher during break time to clarify any questions before the assessment.
9. In each lesson before the test the teacher set five practice questions in case
10. students had not been revising at home and students who got three or less
11. correct were advised they needed to study more.

Scenario involving Teacher B.

12. Teacher B also reminded students about their upcoming topic test. The
13. students were asked to look through their mathematics notebooks and
14. textbooks during the lesson and recall specific topic concepts that they
15. thought would likely to be included in the test. Based on their class
16. work, the students were then asked to record how competent they felt about
17. each concept. During the lesson, the teacher also asked the students to draw
18. on their self-assessment notes and contribute to the creation of a 'class'

[1] The survey included two further questions which we omitted here as we did not use them in our analysis.

19. revision list, from which examples could be revised during lessons before the
20. test. The teacher also told the class that it was expected that each student
21. would develop individual revision plans. Students would work on their
22. individual plans at home, making time to focus on mastering the concepts
23. they believed they needed to improve on. Throughout the week the teacher
24. checked the revision plans of each student and asked how they felt about their
25. preparations.

Open-ended questions:

1. What are some key similarities you perceive between the two teacher scenarios?
2. What are some key differences you perceive between the two teacher scenarios?
3. List up to two things that each of the teachers did that you believe were important for supporting the students' preparation for the test.
4. List up to two things that each of the teachers did that you **do not** believe were important for supporting students' test preparation.
5. Have you used any of the approaches used by Teacher A or B? If so, please list them.
6. Overall, which teacher do you identify with the most and why?

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