Segregated Ice Growth in a Suspension of Colloidal Particles

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Abstract

We study the freezing of a dispersion of colloidal silica particles in water, focusing on the formation of segregated ice in the form of ice lenses. Local temperature measurements in combination with video microscopy give insight into the rich variety of factors that control ice lens formation. We observe the initiation of the lenses, their

growth morphology and their final thickness and spacing, over a range of conditions, in particular the effect of the particle packing and the cooling rate. We find that increasing the particle density drastically reduces the thickness of lenses, but has little effect on the lens spacing. Therefore, the fraction of segregated ice formed reduces. The effect of the cooling rate, which is the product of the temperature gradient and the pulling speed across the temperature gradient, depends on which parameter is varied. A larger temperature gradient causes ice lenses to be initiated more frequently, while a lower pulling speed allows for more time for ice lenses to grow: both increase the fraction of segregated ice. Surprisingly, we find that the growth rate of a lens does not depend on its undercooling. Finally, we have indications of pore ice in front of the warmest ice lens, which has important consequences for the interpretation of the measured trends. Our findings have important consequences for ice segregation occurring in a wide range of situations, ranging from model lab experiments and theories, to geological and industrial processes, like frost heave and frozen food production.

1 Introduction

Ice segregation is of key importance in determining the microstructure and bulk properties of materials in many different fields. Notably, it occurs in cold regions where soils and rocks which are often damaged by the formation of bands of segregated (particle-free) ice lenses – a process called ‘frost heave’. Processes that involve faster cooling rates also experience ice segregation. Freeze casting uses ice growth in particle suspensions to form porous ceramics, with controlled, anisotropic microstructure^{1–4}. During the freezing of foods, the partitioning of colloidal scale particles from ice affects their texture and rheological properties^{5,6}. In biology, ice segregation can cause disruption during the cryopreservation of biological matter^{7,8} or in protein crystal annealing⁹.

Segregated ice growth is complicated, as evidenced by the wide community actively working to understand the physics that underlies this process (see e.g. 10,11 for reviews specifically

on frost heave). This is compounded by the fact that experiments on soils and engineering materials are hard to interpret, due to their heterogeneity and polydispersity. Thus it is necessary to work with model systems, such as suspensions of colloidal particles, to carefully control the freezing process. Previous work has shown that, under slow freezing, ice growth in such media is determined by a number of factors. In particular, cooling rate¹² and particle size¹³ both play a role in determining the final structures, due to their effect on the interaction of the particle with the moving ice interface¹⁴, and the effect curvature has on freezing, i.e. the Gibbs–Thomson effect¹⁵.

Here, we systematically study the process of ice lens growth in a simple model system, using a directional solidification method (adapted from 16,17), which gives a fixed, well defined rate of cooling. We freeze a suspension of colloidal silica particles in water unidirectionally. Our setup allows for microscopic inspection of the freezing process, measuring the initiation and evolution of the lenses under a range of conditions. We focus on varying several factors, which are thought to affect ice lens formation, but which have not yet been fully scrutinized in experiment: (i) the particle packing in the colloidal suspension and (ii) the cooling rate, as controlled separately by both the temperature gradient, and the speed of sample motion across the gradient.

We start by describing the experimental methods in section 2. In section 3, we present our results: the general case (section 3.1), and the effect of changing the conditions (section 3.2). We discuss our results in section 4, and conclude in section 5.

2 Experimental methods

A directional solidification apparatus was used to observe the formation of segregated ice by freezing a thermally one-dimensional suspension of silica particles. A sample was pulled across a temperature gradient, G , at a fixed pulling speed, V , where both could be independently controlled. A diagram of the setup is shown in figure 1.

Figure 1: Schematics of the experimental setup (not to scale). The sample is placed on two temperature controlled plates, at temperatures T_C and T_H , where T_m , the melting temperature of pure bulk water, is between the plates. The positive direction of the velocity of the motor, V , is shown. (a) The sample is contained within two glass walls and has depth h . The phases of the sample are shown above their corresponding x position, and the length of the gap between the plates, x_g is also shown. (b) The initial and final positions of two thermocouples, represented by square and triangle symbols, are shown. The thermocouples are placed inside the sample and measure temperature as a function of position, as they move with velocity V .

2.1 Sample cell

The cell was prepared by gluing (UV glue NOA 81 with 60 s cure) two glass slides together with two glass slide ‘spacers’ in between. All glass was washed with ethanol, then plasma cleaned for 1 minute. The internal volume of the cell was $45 \times 12 \times 0.28 \text{ mm}^3$ (length $l \times$ width $w \times$ depth h). The l and w dimensions of the cells could vary by a few mm, and h by up to 0.02 mm. The area ($l \times w$) was not thought to affect the ice lens formation, as $l, w \gg h$.

2.2 Colloidal model system

The silica particles were obtained as a powder from JGC Catalysts and Chemicals. They were polydisperse, with a documented average diameter of $\sim 1.5 \text{ }\mu\text{m}$ (with 90% particles ranging from 1 – 3 μm in size). Particle suspensions were made up with solutions of ultra

pure water at two different initial particle volume fractions, $\phi_{p,i} = 0.31$ and 0.41 , in order to study the effect of particle packing, with initial concentrations determined by mass. The particles were washed with water 3 times, by repeated centrifugation and redispersion. The suspension was dispersed by ultra-sonic pulsation before being pipetted into the sample cell. The top of the cell was left open during the course of the freezing step. The particles were charge stabilised, and have a zeta potential of -40 ± 5 mV (Malvern Zetasizer Nano Z).

In some cases, the particles were packed by centrifugation. Values of acceleration between $39\ g$ and $435\ g$ were used, with g Earth’s acceleration, for 30 minutes. The centrifugation step changed the initial packing of the particle suspension. The smallest acceleration produced a diffuse interface between a layer of colloidal suspension and nearly pure water, which was close to the top of cell. With increasing acceleration, the interface moved down the cell and became sharper. A simple estimate for the effective packing fraction, ϕ_p , is given by $\phi_{p,i} \times l/l_c$, where l_c is the length of the suspension after centrifuging. In the samples we tested, ϕ_p was calculated to be between 0.32 and 0.68 , for the lowest and highest acceleration.

2.3 Directional freezing

The sample was pulled across a fixed temperature gradient at a constant velocity. A Linkam GS350 stage was employed, which uses electrical resistance heaters and a pump-controlled flow of liquid nitrogen to control the temperature of two plates. The temperature gradient was created by independently fixing the temperature of the plates, which were separated by a gap of length $x_g \approx 2$ mm. The temperatures of the cold and hot plates, T_C and T_H , were held such that $T_C < T_m < T_H$, where T_m is the melting temperature of pure bulk water. The sample was observed in the gap. A stepper motor held the sample cell in contact with the plates and moved it along the gradient, G , at a velocity, V , with $1\ \mu\text{m}$ increments. The sample depth was small so a constant temperature along the z -direction was assumed¹⁷. The apparatus depicted in figure 1 was contained within a sealed box to prevent the condensation of water vapour, which would hamper observations.

First, ice was nucleated by setting $T_C = -30^\circ\text{C}$, and holding T_H at room temperature. Several millimetres of the suspension was therefore ‘pre-frozen’, with rapid unstable growth of large ice crystals. This created irreversible changes in ϕ_p ¹⁸ and therefore this region was discarded during analysis. After the nucleation event, a temperature gradient was established by setting the plate temperatures: $T_C = -2^\circ\text{C}$ and $T_H = 0.5^\circ\text{C}$. The gradient was left to stabilise for a minimum of 15 minutes before the motor was started at a speed of $V=1.5\text{ }\mu\text{m s}^{-1}$. The duration of the directional freezing period was about 3 hours.

2.4 Microscopy and image analysis

The sample was illuminated from below using a home-built Köhler lens microscopy setup. An image of the sample was projected onto a CCD camera, which had a field of view spanning nearly the entire length of the gap between the two plates. The suspension could be observed at temperatures close to and including T_m . Movies were recorded for the duration of sample freezing, typically at one frame every five seconds. After an experiment, the whole frozen sample was imaged to see the final structure. In all images presented, the samples were frozen from left to right.

The optical contrast between the areas of ice and the areas of particle suspension (frozen and unfrozen) was large. Therefore a simple image analysis technique was employed, where a threshold brightness was set, above which ice in a lens was determined to be present. In all images presented, segregated ice is light in colour, as light is transmitted from the source below, and the frozen particle suspension is dark. The average thickness (x-dimension) of lenses was calculated by measuring the thickness of a lens along its width (y-dimension), then taking the mean thickness of all the lenses along the length (l) of the cell. In addition, the growth of single lenses was also measured using the same method.

2.5 Temperature measurements

A standpipe was used to measure temperature at a fixed point in the cell. A standpipe is a cell containing pure water, which is built in parallel to the sample cell that contains the suspension¹⁹. The standpipe cell experiences to the best approximation the same conditions as the sample cell. Therefore the position of the ice–water interface at the temperature of bulk melting, T_m , can be compared to the positions of interfaces in the parallel suspension–containing cell. The standpipe cell was made using the method described in section 2.1, but by placing an extra glass slide spacer along the centre of a cell, to divide it into two discrete sections. To determine the effect of non particulate impurities on the bulk melting temperature, a sample cell containing the supernatant of the suspension was also frozen under the same conditions.

The actual temperature gradient, G , within the sample cell, was measured using two thermocouples, fixed at different initial positions along x inside the cell, as shown in figure 1 (b). The cell used was different to those described in section 2.1 only in its larger depth, 1.1 mm, in order to accommodate the thermocouples. The two independent thermocouples are used to verify each other’s measurement, and to enhance the range of the sweep. However, we note that this makes the assumption of the one-dimensionality of the temperature gradient less certain.

3 Results

In all samples regions of segregated ice were formed during directional freezing. The process of ice lens formation is presented in detail in section 3.1, and in section 3.2 we show the effects of changing conditions.

Figure 2: (a) Typical image of segregated ice in the form of ice lenses (labelled 6 – 10); segregated ice appears light, and frozen suspension appears dark. The scale bar denotes 2 mm. (b) A time series showing the nucleation of a lens; above the first image the relative positions of the cold and hot plates, T_C and T_H , and the direction of the pulling speed, V , is shown. The sample has moved across a distance of $(V \times \text{time})$ in each subsequent frame. Thick arrows point towards the growing lens (9), and the surrounding lenses (8 and 10) are labelled. The image colour has been processed using ImageJ (Fiji, ‘Red Hot’ LUT)^{20,21} to highlight the interface between the frozen suspension, which appears dark, and the unfrozen suspension, which appears red (lighter original colour); thin arrows point towards this interface. The temperature range from left to right in each image is $-0.8 \rightarrow -0.3^\circ\text{C}$. The scale bar denotes 500 μm . The time is shown with respect to the approximate nucleation time of lens 9 at $t = 0$ s.

3.1 Segregated ice formation

Figure 2 (a) shows a typical example of the macroscopically formed patterns. We first discuss the dynamics of ice lens growth (section 3.1.1), and then the final structure of the frozen

suspension (section 3.1.2).

3.1.1 Growth of segregated ice

The emergence of segregated ice lenses perpendicular to the temperature gradient was observed. A time series showing the formation and growth of a lens (labelled lens 9) is seen in figure 2 (b). Once a region of segregated ice is initiated within the suspension, it spreads quickly along the width (the y-direction) of the cell to form a lens.

This lens then grows in thickness (parallel to the temperature gradient) due to flow of water through the suspension towards the ice lens. The motor speed, V , is faster than the growth rate of the lens. Therefore, the growing lens is translated towards colder temperatures and the particle suspension ahead of the lens, in the direction of freezing, is cooled below T_m . A new lens nucleates within the suspension ahead of the growing lens, in the direction of freezing, when a critical temperature is reached. In the final image in figure 2 (b), a new lens (labelled lens 10) nucleates at almost the same relative position as the previous two lenses (8 and 9). As a consequence, regularly spaced lenses emerge.

The lens must initiate from existing inter-particle ice, as homogeneous nucleation does not occur at the temperatures inside the cell²². It is therefore probable that ice exists at the point where the warmest lens nucleates. We indeed detect an optically darker region (DR) of the particle suspension ahead of the warmest lens (see thin arrows in figure 2 (b)). The edge of the DR remains at the same position along the temperature gradient, and thus is consistent with the idea that pore ice exists in the DR. If the temperature of the hot or cold plate is adjusted, we see movement of the DR edge corresponding to whether the overall temperature is increased/decreased. When water freezes, the refractive index (RI) changes from 1.33 to 1.31, while the silica particles have an RI of 1.54. The RI difference between the surrounding medium and the particle will therefore increase when ice is present. The enhanced refractive index difference on freezing (ice-particle) and also the new ice-water RI difference, will result in more scattering, and therefore less transmission of light, hence

rationalising the DR. We note that because lens growth can compact the particles, this too can reduce the intensity of light transmission. However, if the extent of the DR would indeed be due to changes in the particle packing, we would expect an effect of particle concentration; in all cases (including the random close packing sample) in section 3.2.1, we see a DR.

In order to establish the temperature at which an ice lens nucleates, two methods were used (see section 2.5). First, the temperature was measured by two thermocouples as a function of position, x , as the cell travelled with velocity V . Secondly, use was made of a standpipe to determine the absolute temperature. The results are shown in figure 3 (a) and (b). One cell contained just pure water and ice, and another cell contained the silica particles as well. The temperature gradient, G , is nearly constant between the plates. There is some heat transfer from the surroundings, but the two data sets overlap, and within error they are very nearly the same. The temperature gradient measurements are limited by both the precision of the thermocouples and the position determination. For the particle-containing cell, $G = 0.28 \text{ K mm}^{-1}$; this value is used in all subsequent calculations.

In figure 3 (b) the position of the ice-water interface for pure water in the standpipe corresponds to $T = 0^\circ\text{C}$ on the graph. The ice-water interface in the cell containing the supernatant of the particle suspension is displaced to colder temperatures, $x_{T_m} - x_{T_m(c \neq 0)} = 200 \pm 100 \text{ }\mu\text{m}$, which corresponds to an undercooling of $60 \pm 30 \text{ mK}$, using $\Delta T = G\Delta x$. Here, x_{T_m} and $x_{T_m(c \neq 0)}$ are the positions of the bulk melting temperature in the pure water and supernatant samples, and c is the impurity concentration. Considering the colligative effect, where the cryoscopic constant for water is²³ $K_f \approx 1.9 \text{ K M}^{-1}$, the undercooling of the supernatant relative to the pure water corresponds to an impurity concentration of $\sim 30 \pm 14 \text{ mM}$ which we expect to be similar between different experiments. We note that the impurities are concentrated at the interface, therefore the bulk impurity concentration will be lower than this value, see e.g. 24.

The position of the dark region in the suspension contained in the standpipe cell, which was parallel to the pure water cell, has a larger undercooling, at a displacement of $\Delta x_{DR} =$

Figure 3: Methods of relating x positions to temperature. (a) Temperature, T , measured by the two thermocouples (square and triangle symbols) *vs* the distance along the cell, x , where $x = 0$ is the centre between the plates. Two data sets are plotted: pure water + ice (filled symbols); and water and silica particles + ice (empty symbols), in which ice lenses form. Linear fits are plotted and labelled. (b) Three images placed at the corresponding x values of the graph (at the same scale), showing freezing in (top) a particle suspension, (centre) pure water, and (bottom) the supernatant of the suspension. The ice–water interface, x_{T_m} , and the position of the edge of the dark region (also the lens nucleation position), x_{DR} , are marked by labelled arrows. x_{DR} is found by eye using a plot of the average pixel intensity over the y -direction *vs* x (shown above the image). The bottom two images are overexposed because a high light intensity was required to see the interface, which was beyond the warm plate edge. Note that gaseous impurities in the ‘normal’ cell led to the formation of bubbles.

Figure 4: (a) Schematic diagram showing the final structure of the experiment, with definitions of the thickness, L_{th} , and spacing, s , of ice lenses; (b) the mean thickness, L_{th} , of consecutive ice lenses plotted *vs* time, t ; and (c) the mean thickness, L_{th} , of two lenses plotted *vs* the time after the motor has stopped, $(t - t_{stop})$. The inset shows the spacing between lenses L1 and L2, s , *vs* $(t - t_{stop})$. For (a) and (c), before the motor stops, linear fits are used to measure the growth rate.

$x_{T_m} - x_{DR} = 1050 \pm 150 \mu\text{m}$, which corresponds to $300 \pm 40 \text{ mK}$. Here, x_{DR} is the position of the edge of the dark region, which is also where the ice lenses nucleate. The position was measured by eye, using the plot of pixel intensity as a guide (figure 3 (b)). The position is

taken to be where there is a discontinuity corresponding to the change in light transmission, before the intensity reduces due to the presence of the warm plate edge and microscope pinhole. Note that the DR is most easily established on a computer screen, where one can manipulate the image contrast. The implications of this undercooling will be discussed in section 4.

To quantify the growth process of ice lenses, the lens thickness, L_{th} , defined in figure 4 (a), was measured as a function of time, t , see figure 4 (b). The growth rate of a lens starts off being constant, and only slows down as a new lens begins to form, until it stops completely. Eventually it leaves the field of view. This basic pattern is repeated for consecutive lenses. The average growth rate of a lens during the constant growth period is $0.3 \pm 0.03 \text{ } \mu\text{m s}^{-1}$.

In order to probe the permeability of the frozen parts in the suspension, and to investigate the ice lens growth rate after progressive cooling, the motor was stopped after the formation of several ice lenses in succession, and directly after the nucleation of a new lens. We followed the growth of the lenses within the field of view, shown schematically in figure 4 (a), and in figure 4 (c), the lens thickness is plotted *vs* time. Lens 2 (L2) nucleates and grows linearly. Its growth then slows down as lens 1 (L1) forms, much the same as in figure 4 (b). The motor is then stopped, and L1 continues to grow, eventually overtaking L2 in size. However, L2 continues to grow, as well albeit more slowly, whilst being colder than the newest ice lens. Because it grows from its warm edge, water must be supplied through the band between the lenses. Since the band, which is the region between the two lenses, stays of a roughly constant width (see the inset in figure 4 (c)), it suggests that either melting occurs at the cold edge of L1 or the unfrozen fraction in the band, or both supply this water. The lenses that were colder than L2 (e.g. L3) did not grow after the motor stopped, indicating that ice was now completely blocking the pores in the colder band. L1 may continue to grow until the macroscopic ice is in equilibrium with the suspension, i.e. the warmest edge is close to the position of T_m .

3.1.2 Final structures

The frozen suspension captures key features of frost heave, characterised by the average final thickness, L_{th} , and spacing, s , of the ice lenses (see figure 4 (a)). From this, we calculate the ratio of ice lenses *vs* the frozen particle suspension, equal to L_{th}/s , which is a measure of the total heave of the suspension. For the example in figure 2, the average thickness of the lenses was $90 \pm 20 \text{ }\mu\text{m}$, and the average spacing was $450 \pm 35 \text{ }\mu\text{m}$, giving the proportion of ice lenses as 0.20. It is useful to compare these parameters after a change in experimental conditions: this is done in section 3.2. Finally, experiments where we deliberately doubled the height of the cell did not lead to any changes in the lens thickness or spacing. This strongly suggests that the third dimension (the z-direction) does not play a significant role.

3.2 The effect of changing conditions

Final structures of ice lenses formed under different particle packing conditions, and under different cooling rates, are shown in figures 5 (a)–(c).

3.2.1 Particle packing

The influence of particle concentration is of key interest in the study of ice lens formation, affecting the permeability, ice fraction and the compressibility of the suspension. Here, we change the particle concentration either by starting with (a) different overall packing fractions or (b) by using different centrifugation speeds. The table below shows, with reference to each final structure presented in figure 5 (a) and (b), the conditions under which they were prepared, including the initial particle concentration, $\phi_{p,i}$, the centrifugation acceleration and the estimate for the final particle concentration, ϕ_p , directly before freezing was started. Note that method (a) provides a direct measure of ϕ_p . For method (b), ϕ_p is based on an estimate as explained in section 2.2.

The plot in figure 5 (d) shows how the thickness, spacing of lenses, and average total

Table 1: List of experiments with different particle packings, where ϕ_p after centrifugation is calculated as explained in section 2.2.

Figure 5	$\phi_{p,i}$	acceleration	ϕ_p
(a) 1	0.31	-	0.31
2	0.41	-	0.41
3	0.31	435 g	0.68
(b) 1	0.31	39 g	0.32
2	0.31	156 g	0.37
3	0.31	278 g	0.48

heave, change with packing conditions. Our results show that the spacing of ice lenses does not change significantly with ϕ_p . The lens thickness however varies considerably with ϕ_p , and hence so too does L_{th}/s . For lower particle concentrations, the average lens thickness can be double that of a dense particle suspension, allowing heave to occur to a much greater extent. Importantly, the ice lenses formed with no centrifugation step (acceleration = 0), are less regular in thickness and shape than those formed with centrifugation, showing the role of heterogeneous packing. These lenses decrease in thickness over the course of the experiment (from left to right). As stated in section 2.2, the particles will be jammed into the cell by the centrifugation step, therefore producing a more regularly and more densely packed network of particles, in a shorter suspension length. The suspension is not at random close packing, except for in experiment (a) 3, nor is it density matched. When the cell is placed perpendicular to gravity before the start of freezing, this particle network is less likely to vary over time due to sedimentation.

3.2.2 Growth conditions

The growth conditions, controlled by the temperature gradient, G , and the pulling speed, V , are important parameters in ice lens formation and frost heave, since they can have a pronounced effect on the proportion of segregated ice formed in the particle suspension. Final structures are shown in figure 5 (c), and figure 5 (e) shows how the thickness, spacing of lenses, and average total heave, change with the product VG . In a single sample cell,

Figure 5: (a) and (b) Final structures of ice lenses formed under different particle packing conditions, as stated in table 1. (c) Final structures of ice lenses formed under different cooling conditions. (d) Plot of the thickness, L_{th} , spacing, s , and L_{th}/s of ice lenses *vs* the estimated particle volume fraction, ϕ_p . (e) Plot of L_{th} , s , and L_{th}/s of ice lenses *vs* the cooling rate, VG . The scale bars all denote 1 mm.

the effect of changing V under a constant G was investigated ($G = 0.28 \text{ K mm}^{-1}$, the first and third images), and in separate sample cells, G was changed, while V remained the same ($V = 1.5 \mu\text{m s}^{-1}$, the second and forth images).

When V was increased, the average thickness of ice lenses reduced significantly and there was a slight reduction in the spacing. The final proportion of ice lenses (heave) decreased.

The spacing only depends on the nucleation position of a new lens, relative to the edge of the previous one, and theory suggests this to be independent of V ²⁵. Therefore the change in s can be accounted for mainly by the large change in L_{th} , and hence L_{th}/s reflects the variation in L_{th} .

Upon increase of G (circle data point, to be compared with 2nd data point, at the same V), the lens thickness decreased slightly, but the decrease in spacing was more significant, and therefore the proportion of ice lenses, L_{th}/s , increased greatly. Here, the spacing changes significantly, because the initiation position of new lenses relative to the previous lens decreases with an increase of G , as expected in a steady temperature field²⁵.

4 Discussion

4.1 Lens formation

Our system demonstrates the appearance of multiple ice lenses within a slowly freezing, closely packed porous suspension. The growth rate is between two extremes: ice rejecting particles indefinitely (primary frost heave) and the engulfment of particles by a fast freezing front. Within our system, both these regimes occur, within a mixed phase, where segregated ice grows as an ice lens, by rejecting all particles. After further cooling, ice then grows into the suspension phase, in the pores between the particles. The ice effectively engulfs the particles within this phase until a new lens opens at a warmer temperature, and the old lens can become effectively frozen in place. However, at warm enough temperatures, we have found that this colder, old lens can still grow, suggesting that a water supply still exists. Certain aspects of previous theories are observed in our system, and we will discuss our results in light of them below, as well as highlighting any currently unexplained observations.

Firstly, the presence of liquid water between frozen lenses and in the DR can be explained due to the solid/liquid interfacial curvature and/or colligative effects. The Gibbs–Thomson effect prevents ice from growing into the small pores, which allows water to continue to flow

Figure 6: Plot of Δx_F (left hand axis), which relates to the Gibbs–Thomson undercooling, ΔT_{GT} , by the temperature gradient $G = 0.28 \text{ K mm}^{-1}$ (right hand axis), *vs* the radius of the pores, R_p .

to a growing ice lens, even when it is below the bulk freezing temperature of water. This occurs because equilibrium between ice and water at a curved interface with temperature T requires that

$$\Delta T_{GT} = T - T_m = -\frac{T_m \gamma_{sl}}{L_m \rho_s} \kappa, \quad (1)$$

where $T_m = 273.15 \text{ K}$, the bulk melting temperature, $\gamma_{sl} = 3 \times 10^{-2} \text{ J m}^{-2}$, the solid-liquid (ice-water) surface tension, $L_m = 3.34 \times 10^5 \text{ J kg}^{-1}$, the latent heat of fusion per kg of ice, $\rho_s = 917 \text{ kg m}^{-3}$, the density of the solid, and a pore typically has a surface curvature of $\kappa \sim 2/R_p$, where R_p is the pore radius. Figure 6 shows how $\Delta x_F = x_{T_m} - x_F$, the positional shift corresponding to the change in melting temperature, varies with R_p . Here, x_F is the position up to which the pores are frozen and x_{T_m} is the position of the bulk melting temperature, and hence $\Delta x_F = \Delta T_{GT}/G$ according to equation 1.

Assuming that the pores are frozen up to the end of the DR, one can set $\Delta x_F = \Delta x_{DR}$, which is $850 \pm 100 \text{ μm}$, taking the colligative effect into account (see figure 3 (b)). From this, one can infer a pore radius of $R_p = 0.23 \text{ μm}$. Furthermore, the ratio of pore radius to particle radius is $R_p/R = 0.3$. This is reasonable, based on theoretical predictions for random close packed spheres with a log-normal size distribution^{26,27}. Alternatively, the measured x_{DR} may not be the limit of the presence of pore ice. For low particle packings, we would expect

some pores to be much larger than the particle size, and therefore a fraction of ice would be present very near the melting point (e.g. 28). However, our observations are consistent with the idea of having a reasonably sharp distinction between pore ice and pore water, and this measured interface position is consistent with theoretical predictions. The edge of the dark region is also where ice lenses nucleate, which agrees with frozen fringe theories. Models involving the presence of this frozen fringe, the region between a growing ice lens and the position where pores are frozen, successfully describe how intermittent ice lenses may form, e.g. 25,29–32, and a frozen fringe has been observed in other recent experiments using directional freezing apparatus²⁷.

The fast propagation (figure 2 (b)) and subsequent wavy lens-structure along the y -direction (figure 2 (a)) can also be rationalised by the geometrical supercooling model proposed by Style *et al.*²⁸. They suggest a mechanism by which an ice-filled crack would propagate continuously along y , fracturing the suspension after its nucleation at the point with the largest geometrical supercooling, which is defined as the undercooling with respect to a spontaneous ice lens nucleation temperature. This supercooling is caused by a region of low permeability ahead of the first lens. The low permeability region could be caused by both spontaneous fluctuations in the concentration of particles ahead of the lens, and/or ice presence in the pores. However, the ragged edge of a lens has other possible causes. For example, an inhomogeneity in particle packing along the y -direction will result in a variation in the position of the edge of the frozen fringe, and therefore the exact lens nucleation position.

4.2 Lens growth

Our experiments show that the rate of ice lens growth is independent of the temperature of the ice lens growth surface. This surface is translated towards colder temperatures over time, and as the frozen fringe boundary is at a constant temperature throughout, the frozen fringe increases in thickness with time. It might seem plausible that as the fringe width

increases, the ability to suck water towards the lens would decrease, therefore slowing the growth. Alternatively, if there was no fringe, it is expected that the water pressure next to the ice lens gradually decreases, according to the Clausius-Clapeyron equation, therefore causing an acceleration in the suction of water, see e.g., 33. However, we see a period of constant growth. This is followed by an abrupt slowing down of the growth when a new ice lens is nucleated in front of it.

The permeability as a function of temperature may be measured by considering the change in growth rate in the stationary situation (figure 4 (b) and (c)). L1 continues to grow, after $t = 0$ the motor is stopped ($t = 0$), its growth surface moving towards warmer temperatures, where the pore ice concentration will reduce. It slows down when the pressure gradient in the water diminishes. Water flux occurs across the bands between ice lenses: a warmer lens shrinks on its cold side, and a colder lens grows on its warm side, consistent with thermomolecular pressure driven ice growth. This causes these interfaces to creep towards warmer temperatures. This behaviour will occur until the permeability decreases such that pores are blocked by ice, at low enough temperatures. Hence, our observation that at least one colder lens grows (figure 4 (c)), with the spacing between it and the warmer lens remaining constant, shows the importance of permeability in ice lens growth. This shows that segregated ice can evolve beyond dynamic cooling, therefore leading to further disruption of the particle suspension.

4.3 Changing the particle packing and growth conditions

The experimental results presented in figure 5 (a),(b) and (d) show that the particle packing is a key parameter in frost heave. Our observations show that ice lens formation occurs across a range of different packing fractions, and furthermore that the particle packing has an effect on the growth rate of a lens. Some models of ice lens formation and frost heave take into account the effects of particle packing^{34,35}. Though the effect of spacial inhomogeneity in the particle packing is a challenging problem not captured by most theories, while some

models simply fix the particle volume fraction, see e.g., 30. Several factors may explain the reduction in thickness of ice lenses with increase in particle volume fraction. Firstly, when particles are more closely packed, the suspension is less compressible and the ice lens will not be able to rupture the matrix. Secondly, the permeability of the suspension will be affected by the change in ϕ_p , reducing as a function of volume fraction. This reduction will cause the water flux towards the ice lens to slow down, resulting in thinner ice lenses. Thirdly, a more concentrated particle suspension will mean there will be less water in the vicinity of the ice lens, and therefore a smaller supply to the growing lens. Further experimental or theoretical work is required to determine the relative importance of these two effects.

Figure 5 (b) shows that increasing the time spent by the suspension at each temperature leads to an increase in lens thickness but does not affect the spacing. This is in agreement with other experiments, see e.g., 27,36. The effect of temperature gradient on our system is also clear from figure 5 (b). The spacing changes with frozen fringe thickness, which will depend on the temperature gradient. The thickness of ice lenses can change by either the time allowed for the ice lens to grow (changing V), or the frozen fringe thickness (changing G). If the frozen fringe is thicker, the suction of water towards the growing surface of the lens will be smaller. We here show that the individual terms control the ice lens formation, and the data does not collapse (figure 5 (e)). Our results are qualitatively consistent with the theory of Rempel *et al.*²⁵, and the experiments of Taber on clay³⁷, where soil was frozen in an undefined temperature gradient, but the gradient could be inferred. The most frequent scenario in natural or industrial settings will be a change in temperature gradient, since most situations involve cooling by lowering the temperature from one end. Our experiments have shown that the change in heave in this case is significant.

5 Conclusions

We have performed a series of experiments to observe segregated ice growth in the freezing of particle suspensions, as a model for frost heave. Our results are relevant to many fields in which ice segregation is a significant issue: from the manufacture of frozen foods to the preservation of biotissue, as well as in freezing soils. We find that regularly spaced ice lenses nucleate at a temperature where ice is likely to be present in the pores between the particles, and they grow at a rate which is independent of the undercooling.

We have shown that several factors are important in determining lens structures. In particular, variations in particle packing density strongly affect the final thickness of a lens, and consequently the heave of the suspension. This will have implications for situations where changes in packing can occur over time, such as in cyclical processes like frost heave occurring in periglacial ground. Ice segregation itself causes regions of low and high particle concentrations in a suspension. Upon melting, this suspension will be more susceptible to ice lens growth in the less concentrated regions, therefore reinforcing the pattern formation process. Our model system also demonstrates some theoretically expected properties of lens formation, such as the changes in thickness and spacing of lenses upon a decrease in cooling rate, and the crack-like propagation of a lens. This indicates a strong practical need to control these effects for industrial process that include a particle suspension freezing step.

Finally, we have shown that suspensions containing colder ice lenses are not completely frozen, and water can still flow through it. This raises the possibility that a particle suspension need not be progressively cooled for it to be influenced by ice lens growth. Certain factors described in this paper, such as the effect of many of the spatio-temporal variations in particle concentrations, have not yet been accounted for in theories on frost heave and we hope our experimental data may stimulate further theoretical work along these lines.

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Figure 7