SOUND SCATTERING FROM MICROBUBBLE DISTRIBUTIONS NEAR THE SEA SURFACE

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ABSTRACT. Backscatter from the sea surface is thought to be governed by the roughness of the surface and subsurface bubble distributions. At low frequencies, due to the paucity of large bubbles, scattering results primarily from coherent and/or collective scatter from bubbles entrained by the subsurface vorticity or carried to depth by the Langmuir circulation and thermal convection. It is shown that scattering from compact regions is a function of the volume fraction of air and to first order can be described by a Minnaert formula modified with the volume fraction. Measurement of sound scattering from a submerged cloud of bubbles produces low-frequency peaks with large low-frequency target strength consistent with this theory.

1. INTRODUCTION

Recent experimental evidence shows that when waves break, microbubble layers and plumes are produced, and these bubbly assemblages are convected to depth by the Langmuir circulation as well as the thermal convection found in the oceanic mixed layer [1,2,3,4]. The fundamental question to be answered is what role, if any, do microbubble layers and plumes play in the near-surface radiation and scattering of sound in the low-to mid-frequency range (20 Hz to 2 kHz). There exists ample evidence that sound scattering at these frequencies has a different characteristic than expected from Bragg scattering from gravity waves [5,6]. It is our hypothesis that if these microbubble clouds and plumes have structures with volume fractions exceeding 10^{-4} , then these structures can radiate as well as scatter sound through a dampened resonant oscillation [6,9,10,11,

12,13]. Recent analytical studies of low-frequency sound scattering from geometrically shaped bubble distribution beneath the sea surface [7,8] indicate that these types of bubble distributions may also coherently scatter sound.

This paper discusses these issues in terms of the classical theories of sound scattering from complex but compact distributions of bubbles. In particular, our intention is to show, with the Morse and Ingard formulation [19], the relationship between the Born approximation [19], continuum theory, and single-bubble incoherent scattering. We show that when the bubbles are treated as scatters in the Lighthill method that one proceeds from single-bubble scattering, to coherent scattering, and finally to the scattering from the distribution as if it were a continuum. In fact, the continuum theory is shown to provide a bound on the region of validity of the Born approximation. The problem is also scaled by the volume fraction and its effect on the liquid and gas mixture compressibility. Finally, we present the results of a free bubble cloud scattering experiment conducted at Lake Seneca. These measurements clearly illustrate the complex nature of the scattering cross section of a compact bubble cloud.

2. A REVIEW OF SCATTERING OF SOUND FROM COMPACT BUBBLE DISTRIBUTIONS

The classical theory of sound radiation and scattering from compact regions containing bubbly liquids is a mature subject and several texts [14-21] contain very useful overviews. In recent years, the scattering of sound from bubble distributions near the sea surface has received renewed interest[6]. This is in part due to the work of Thorpe [1] and his observation of subsurface bubble plumes and clouds. McDaniel [5] has shown that high frequency (>1 kHz) sea surface scattering can be explained using bubble distributions near the sea surface coupled with composite rough surface scattering. However, the bubble size distributions [4,5] measured at sea are too small to explain scattering at frequencies less than 1 kHz using a single bubble scattering formalism. The purpose here is to discuss this low-frequency bubble scattering from bubble plumes and clouds in light of classical scattering theories as well as to place current analytical work [7,8,9,10,11,12,13] in perspective. Consequently, we closely follow and use material found in these texts, most notably the Morse and Ingard text [19].

Fig. 1 shows the geometry for a compact bubble cloud. The primary focus is on distributions of salt water bubbles with radii less than 1 mm, resonant frequencies greater than 3 kHz, and excitation frequencies less than 1 kHz. Since we are treating a compact source, the sea surface will be incorporated using the method of images.

The Fourier transform of the wave equation is

Figure 1. The geometry of the general scattering problem

$$\nabla^2 P_\omega(r) + k^2 P_\omega(r) = -F_\omega(r), \quad (1)$$

where $F_\omega(r)$ represents a general source distribution. Since the problem is a farfield radiation one and since our source region is compact, the more general retardation notation has not been used. The Green's equation is

$$\nabla^2 g_\omega(r|r_o) + k^2 g_\omega(r|r_o) = -\delta(\vec{r}-\vec{r}_o), \quad (2)$$

where $g_\omega(r|r_o) = \exp(i\vec{k} \cdot (\vec{r}-\vec{r}_o))/4\pi|\vec{r}-\vec{r}_o|$ and will be used in this form throughout this paper. The standard procedure is to multiply 1) by g_ω , 2) by P_ω , subtract, use reciprocity to interchange r and r_o , and then use the divergence theorem to yield:

$$P_\omega(r) = \iiint g_\omega(\vec{r}|\vec{r}_o) F_\omega(r_o) dV_o + \iint \left[g_\omega \frac{\partial P_\omega}{\partial n_o} - P_\omega \frac{\partial g_\omega}{\partial n_o} \right] ds_o. \quad (3)$$

This equation describes both the radiation and scattering of sound from a region V_o . The surface integral in the radiation problem goes to zero as r_o increases due to imposition of the radiation condition. This integral in the case of the scattering problem becomes the incident wave coming in from infinity. Thus both the radiation and scattering of sound require the evaluation of the volume integral, the difference being the source of excitation. In this paper we will simply use a plane wave, consequently

$$P_\omega(r) = P_{i\omega}(r) + \iiint g_\omega(\vec{r}|\vec{r}_o) P_{i\omega}(r_o) f_\omega(r_o) dV_o \quad (4)$$

$$F_\omega(r) = P_{i\omega}(r_o) f_\omega(r_o), \quad P_{i\omega}(r) = P_o e^{i\vec{k}_o \cdot \vec{r}}$$

The various approaches to scattering can be considered as consisting of the method by which $f_\omega(r_o)$ is determined coupled with the means by which the volume integral can be solved. Traditional treatments employed at higher frequencies consider bubble densities, the number of bubbles per wavelength volume, are small and multiple scattering is therefore not a problem. However, individual bubble resonance is important, thus:

$$f_\omega(r_o) = f_n(\omega_o, \theta, r_n) \delta(\vec{r}_o - \vec{r}_n), \quad (5)$$

where $f_n(\omega_o, \theta, r_n)$ represents the response function of the n th bubble. Substitution of this expression for f_ω , and after some algebraic manipulation, we obtain for a uniform distribution of N bubbles the following expression for the scattered intensity

$$I_s(r) = \frac{|P_{s\omega}|^2}{2\rho c} = \frac{P_o^2}{2\rho c r^2} |\mathcal{A}(\omega_o, \theta)|^2 \left[N + \sum_{m \neq n} \exp(i\vec{\mu} \cdot \vec{R}_{mn}) \right], \quad (6)$$

where $\vec{\mu} = \vec{k}_o - \vec{k}$, $|\mu| = 2k \sin(\theta/2)$. When the summation within the brackets is zero, the scattered intensity is seen to be proportional to the number of bubbles N ; i.e., the incoherent sum of scattering from individual bubbles. For the case of an acoustically compact spherical volume of radius a , the bracketed terms can be shown to be [18]

$$I_s(r) \rightarrow N^2 \left[1 - \frac{2(N-1)}{5} \frac{k^2 a^2 \sin^2(\theta/2)}{N} \cdot \dots \right]. \quad (7)$$

This is the coherent scattering case where the scattered intensity is directional and proportional to N^2 . The distinguishing factor between these two expressions is the number of bubbles within V_o or the volume fraction.

The incoherent, proportional to N , scattering particularly applies to non-compact distributions of bubbles which are sparse. This is the case treated by McDaniel [5] and has been shown to describe several sea surface scattering measurements at frequencies >1 kHz and at low grazing angles.

The work of Foldy [14,16] was based on the above case of $f_\omega(r)$, except it included the effects of scattering from neighboring bubbles (here it is assumed that they are mono-dispersed in radius with resonance frequency ω_o):

$$f_{\omega_n}(r_o) = P_{\omega_n}(r_n) f_n(\omega_o, \theta_n, r_n) \delta(\vec{r}_o - \vec{r}_n) \text{ and} \quad (8)$$

$$P_\omega(r) = P_{i\omega}(r) + \sum_{n=1}^N \frac{e^{ikr_n}}{r_n} P_{\omega_n}(r_n) f_n. \quad (9)$$

The problem when the interaction between bubbles must be considered is the choice of $P_{\omega n}(r_n)$. Foldy [16] recognized that this quantity depends on the incident field and the sum of the scattered field from all the bubbles and could be written as

$$P_{\omega n} = P_{i\omega}(r_n) + \sum_{m \neq n}^N \frac{e^{ikr_{mn}}}{r_{mn}} f_m P_{\omega m}(r_m) = P_\omega(r_n) - \frac{f_n}{r_n} P_{\omega n}(r_n) e^{ikr_n}. \quad (10)$$

This result when substituted in equation 9 yields

$$P_\omega(r) \approx P_{i\omega}(r) + \sum_{n=1}^N P_\omega(r_n) f_n \frac{e^{ikr_n}}{r_n}, \quad (11)$$

$$\xrightarrow{N \rightarrow \infty} P_{i\omega}(r) + \int P_\omega(r_n) f_n \frac{e^{ikr_n}}{r_n} dr_n.$$

Foldy then recognized that the configurational average of equation (9) is equivalent to the solution of

$$\nabla^2 \langle P_\omega(r) \rangle + k_s^2 \langle P_\omega(r) \rangle = 0, \quad k_s^2 = k^2 + 4\pi \langle f_r \rangle, \quad (12)$$

which is a wave equation with a different wave number or index of refraction. This index of refraction is determined by the continuum properties of the bubbly volume [11,22,23].

The concept that sound scattering from a cloud of air bubbles in water is described as a progression from incoherent, coherent to continuum scattering was fully developed in references [14-21] prior to the mid-nineteen sixties. Morse and Ingard [19] have provided an extension of these results based on the Lighthill method.

The use of Lighthill's approach requires the determination of $f_\omega(r_0)$ from the two-component conservation equations. Retaining only second order terms and neglecting the Lighthill stress tensor yields

$$f_\omega(r_0) = k^2 \gamma_k P_\omega(r_0) + \bar{\nabla}_o (\gamma_\rho \bar{\nabla}_o P_\omega(r_0)), \quad (13)$$

where γ_k and γ_ρ are the compressibility and density contrasts given by

$$\gamma_k = \frac{\kappa_e - \kappa}{\kappa}, \gamma_\rho = \frac{\rho_e - \rho}{\rho} \text{ for } r_0 < a \text{ and } \gamma_k = \gamma_\rho = 0 \text{ for } r_0 > a. \quad (14)$$

Substitution of this form into equation (4) yields

$$P_\omega(r) = P_{i\omega}(r) + \iiint [k^2 \gamma_k P_\omega g_\omega - \bar{\nabla}_o (\gamma_\rho \bar{\nabla}_o P_\omega) g_\omega] dV_o. \quad (15)$$

This equation is extremely interesting, because substitution of the mean compressibility contrast ($\bar{\gamma}_k$) and mean density contrast ($\bar{\gamma}_\rho$) results in a wave equation that is similar to Foldy's with an effective wavenumber given by $[1 + \bar{\gamma}_\rho] \bar{\nabla}_o^2 P_\omega(r_0) + k^2 (1 - \bar{\gamma}_k) P_\omega(r_0)$ where $k^2 (1 - \bar{\gamma}_k) / (1 + \bar{\gamma}_\rho)$ is the effective wavenumber squared. This is another way of showing that the region may be described by its mixture density and compressibility. This also provides a means by which the scattering regimes may be distinguished. To show this we integrate equation 15 by parts and use our free space Green's function:

$$P_\omega(r) = P_o e^{ik_s r} + \frac{P_o e^{ik_s r}}{r} \phi(k_s), \text{ where} \quad (16)$$

$$\phi(k_s) = \frac{k^2}{4\pi P_o} \iiint \left[\gamma_k P_\omega - i \gamma_\rho \frac{k_s}{k^2} \cdot \nabla_o P_\omega \right] e^{-ik_s r_o} dV_o.$$

This quantity is referred to as the angle distribution factor and is recognized as a three-dimensional transform of the compressibility and density contrasts. Since Fourier methods apply, the function $\phi(\vec{k}_s) = F(\phi(\vec{r}_o))$ can also be determined by the convolution of the spatial transforms of P_o and the γ 's. Since the scattered intensity is proportional to $|\phi(k_s)|^2$, the full statistical description of the power spectral density and correlation properties can thus be used. This is an important observation, for the description of non-compact near-surface bubble layers can be treated in this manner. Gilbert [24] has employed these techniques to characterize low-frequency sound scattering from subsurface bubble distributions. The case of a compact distribution is presented here.

The problem is still to specify P_o within the volume integral [15]. The method of successive approximations has been used to solve this equation. This method approximates the term P_o by a sum of successively higher order terms $P_o(r_o) = P_{io}(r_o) + P_1 + P_2 + \dots$. When $P_{io}(r_o) > P_1$ throughout the source volume, we simply allow $P_o(r_o) = P_{io}(r_o)$. This approximation is referred to as the Born approximation. This approach has been applied to bubble scattering in underwater acoustics for over 30 years. When we assume the internal field to be given by the incident field, the function $\phi_s(k_s, k_o)$ reduces to (Morse and Ingard [19]):

$$\begin{aligned}\phi(k_s, k_o) &= \frac{k^2}{4\pi} \iiint [\gamma_k + \gamma_p \cos\theta] e^{-i\vec{\mu} \cdot \vec{r}_o} dV_o \\ &= 2\pi^2 k^2 [\Gamma_\kappa(\vec{\mu}) + \Gamma_\rho(\vec{\mu}) \cos\theta].\end{aligned}\quad (17)$$

This expression, when applied to a sphere of radius a , yields the well known result [19]:

$$\phi_s(\vec{\mu}) = 1/3k^2 a^3 \left(\left\langle \frac{\kappa_e - \kappa}{\kappa} \right\rangle + \left\langle \frac{\rho_e - \rho}{\rho} \right\rangle \cos\theta \right) \quad (18)$$

This result yields a scattered intensity proportional to the number of bubbles squared, equivalent to our previous result of coherent scattering.

This result from the use of the classical Born approximation can be shown to be limited to the treatment of the bubbly liquid in V_o as a continuum with different ρc^2 . To show this result we return to our schematic (Fig. 1) and treat the source volume as being bounded by a surface integral, S_a , the region of r would thus be between S_a and the surface at distance S_o . For the inner surface S_a , $\vec{a}$ is the outward directed normal into volume V_a . The resulting pressure field is

$$P_o(r) = P_{io}(r) + \iint \left[P_o \frac{\partial g_o}{\partial a} - g_o \frac{\partial p}{\partial a} \right] dS_a. \quad (19)$$

The radiation and scattering problems are now reduced to specifying $P_o J(\partial P / \partial a)$ on the surface S_a bounding the medium with properties ρ_m , C_m . The material in the

sphere will determine this ratio, as this is equivalent of requiring continuity of velocity and pressure at the boundary. The procedure is to use spherical harmonics to expand the field within the sphere, the field external to the sphere, and the incident plane wave (Anderson [21], also Carey [12]). The mathematical manipulation is straightforward but tedious. Morse and Ingard show that to the lowest order in ka the result is

$$\phi_s = 1/3k^2a^3 \left(\frac{\kappa_m - \kappa}{\kappa} + \frac{3\rho_m - 3\rho}{2\rho_m - \rho} \cos\theta \right). \quad (20)$$

This expression allows us to define the region of applicability of the Born approximation and the beginning of the region determined by the volume fraction in which the bubbly liquid should be treated as a continuum. The case of air bubbles in water when $\kappa_e > \kappa$ and $\rho_e \sim \rho$ we find

$$\phi_s \sim (1/3)k^2a^3 \left(\frac{\rho c^2}{\rho_m c_m^2} - 1 \right) \sim 1/3k^2a^3 \left(\frac{c^2}{c_m^2} - 1 \right). \quad (21)$$

This theoretical result, which, we stress, is taken from the classical literature, has attempted to place the question of sound scattering from compact bubble distributions in context. In particular it was our intention to show the relationship between the Born approximation, continuum theory, and single incoherent bubble scattering. The validity of these approximations was shown to depend on the quantity $(c^2/c_m^2 - 1)$, which is proportional to χ , the volume fraction of the free air in the water volume.

The discussion has focused on a sphere of radius a . We have used this geometry because of its simplicity, but our conclusions for both the radiation and scattering from a complex region do not depend on the geometry. For an arbitrary complex yet compact distribution of air bubbles in water the radiated field can be shown to consist of monopole, dipole, quadrupole, and higher order terms. In our application the dipole and higher terms are not important. The monopole term corresponds to the volume pulsation; hence this treatment of a spherical source should also apply to any complex distribution radiating or scattering in the volumetric mode. This is similar to the observations made by Strasberg concerning single-bubble radiation [25].

3. SOUND PROPAGATION IN BUBBLY MIXTURES

In a bubbly mixture, in which the of bubbles possess resonance frequencies which are much greater than the frequencies of interest, then the propagation of sound in the mixture can be described by an effective wave equation with mixture density and sound speed. These effective wave equations have been studied by several investigators (i.e., Foldy [14,16], Crighton [26], Van Wijngaarden [24]) and confirmed by Karplus (1958) at frequencies < 1 kHz (See Carey [11] as well as [12,13] and Prosperetti [9]; also Ruggles [29]).

A. B. Wood [22] showed that at low frequencies in an air-water mixture that when the size of the bubbles and spacing between bubbles were small compared to a wavelength, that for a given volume fraction (χ) the mixture density (ρ_m) and compressibility (κ_m) are under equilibrium conditions given by

$$\rho_m = (1 - \chi)\rho_l + \chi\rho_g \text{ and } \kappa_m = (1 - \chi)\kappa_l + \chi\kappa_g, \quad (22)$$

where l represents the liquid and g the gaseous component. The low-frequency limiting sonic speed, C_{mif} , is seen to be

$$C_{mif}^2 = \frac{dP}{d\rho} = (\rho_m \kappa_m)^{-1}. \quad (23)$$

In the case of the air bubble-water mixture, the process can be considered either adiabatic or isothermal. Since the controlling factor is the rate of heat transfer during bubble compression to the surrounding fluid which is rapid in water due to its large thermal capacity, the bubble oscillations at low frequencies may be considered isothermal. The following equations result:

$$C_{mif}^{-2} = \frac{(1-\chi)^2}{C^2} + \frac{\chi^2}{C_g^2} + (\chi)(1-\chi) \frac{\rho_g^2 C_g^2 + \rho^2 C^2}{\rho_g C_g^2 \rho C^2} \quad (24)$$

$$\chi \rightarrow 0, C_m^{-2} \rightarrow C^{-2} \text{ and } \chi \rightarrow 1, C_m^{-2} \rightarrow C_g^{-2}$$

and when $0.002 < \chi < 0.94$, $C_{mif}^2 = P/\rho\chi(1-\chi)$.

These equations show the large effect on sonic speed as a function of void fraction and relative bulk modulus. In the low-frequency range (<1 kHz) Karplus used an acoustic tube to determine the standing wave pattern as a function of air volume fraction and verified these results. Ruggles (1987) [29] has extended Karplus's technique to study a wide range of volume fractions. This behavior is due to the mixture's mass being primarily due to the liquid while its compressibility is due to the gas. Since we are dealing with the low-frequency response, much less than any individual bubble resonant frequency, only the volume fraction is important and not the bubble size distribution.

This equation clearly shows the dispersive character of the mixture speed of sound. In Fig. 2, we show several curves for a cloud of bubbles with individual resonant frequencies near 9.18 kHz. Below the resonance range we see the low-frequency minimum predicted by Wood's equation and above the resonance range we see the curves approach the speed of sound determined by the liquid compressibility. The phase velocity reaches a minimum at the resonance frequency and goes supersonic for a range of frequencies slightly above f_0 . This is due to the fact that when driven above resonance bubbles pulsate out of phase with the driving force. Thus the bubbles "appear" to be stiff and the phase velocity subsequently increases dramatically. These strong dispersive

effects need to be considered when describing sea surface scattering.

Equation (24) does not show the dispersive nature of this sonic speed variation. If one considers a bubbly liquid with a uniform bubble size whose corresponding resonant frequency is ω_0 and dampening $\delta(\omega)$, then one [23,26] can solve the component conservation equations to obtain the following frequency-dependent sonic speed:

$$\frac{1}{C_m^2} = \frac{(1-\chi)^2}{C^2} + \frac{1}{C_{mf}^2(1 - \omega^2/\omega_0^2 + 2i\delta\omega/\omega_0)}. \quad (25)$$

This expression reduces to our previous equation for C_{mf}^2 when $\omega \ll \omega_0$. The equation does show a dispersive character as illustrated in Fig. 2, where we have used the explicit expression from Prosperetti [23]. The dispersive character of the phase velocity is the reason that volume fraction measurements are necessary to determine the low-frequency sound speed changes near the surface of the sea.

Figure 2. Dispersion curves for a cloud of 9.18 kHz bubbles

4. THE OSCILLATION OF A BUBBLE SPHERICAL VOLUME

The generation and scattering of sound from a compliant sphere immersed in a fluid can be found in classical texts on the theory of acoustics (see Reschevkin, or Morse). In this particular case, the bubbly sphere has no well-defined boundary, but nonetheless is localized by perhaps a vortex or some other circulatory feature beneath the breaking wave. Since this analysis is somewhat standard [20,12,29], only a brief outline will be presented.

Here we assume that the bubbly region (Fig. 3) is compact with an arbitrary radius r_0 and the region is composed of microbubbles with resonance frequencies far above the

frequency of excitation. Buoyancy forces and restoring forces such as surface tension are not important. The properties of the bubbly region are described by the mixture speed c_m and density ρ_m with a resulting compressibility $1/\rho c^2$. The microbubbles supply the compressibility and the liquid supplies the inertia.

Fig. 3 shows the random collection of microbubbles within a radii r_0 from the origin, where (P_i = incoming wave or excitation, ρ_m, c_m = the properties of the mixtures, ρ, c = the properties of water, and P_s = the scattered sound.)

The source of excitation is assumed to be global compared to the dimensions of the compact sphere. This assumption means we can consider a plane wave expanded in terms of spherical harmonics. The physical reasoning is that the properties of the bubbly region determine its ability to radiate provided there is a source of excitation; i.e.,

Figure 3. The random collection of micro bubbles.

$$P_i = P_o \exp(i\omega t - ikr) = P_o \exp(i\omega t) \sum_{m=0}^{\infty} i^m (2m+1) P_m(\theta) J_m(kr). \quad (26)$$

We require continuity of velocity and pressure at the generalized radius, r_0 . Furthermore, a radiation condition is imposed at large r and the field is required to remain finite within the bubbly region.

$$\text{The particle velocity } V_i = (-1/i\omega\rho) \partial P_i / \partial r, \quad (27)$$

$$\text{Continuity of Pressure } P_i(a) + P_s(a) = \bar{P}(a), \quad (28)$$

$$\text{Continuity of Velocity } V_i(a) + V_s(a) = \bar{V}_n(a). \quad (29)$$

The procedure is to assume the scattered wave or radiated wave is a sum of outward propagating spherical waves.

$$P_s = \exp(i\omega t) \sum_{m=0}^{\infty} A_m \bar{P}_m(\theta) H_m(kr) \exp(-i\epsilon_m kr). \quad (30)$$

The pressure field inside the volume is expanded in terms of spherical Bessel function of the first kind; i.e.,

$$\bar{P} = \exp(i\omega t) \sum_{m=0}^{\infty} \bar{A}_m P_m(\theta) J_m(kr). \quad (31)$$

When the region is compact $\lambda > 2\pi r_o$, we can solve for the coefficients A_m and $\bar{A}_m$ by using the boundary conditions (27, 28, and 29) and equating each m order term. To $O(k^3 r^3)$, we find for A_0 and A_1 the following:

$$A_0 = \frac{iP_o(k_o r_o)^3 (1 - y/\bar{y})/3}{\left[(1 - y/\bar{y}) \frac{(kr_o)^2}{3} \right] + i(kr_o) \left[y/\bar{y} \frac{(kr_o)^2}{3} \right]} \text{ and} \quad (32)$$

$$A_1 = \frac{P_o(kr_o)^3 (\rho_m - \rho)}{(2\rho_m + \rho)}, \quad (33)$$

where $y = \rho c^2$ and $\bar{y} = \rho_m c_m^2$. The term A_0 becomes interesting when the real part of the denominator is equal to zero, we may say a monopole resonance has occurred. This occurs when

$$(kr_o)^2 = \left(\frac{2\pi f_o r_o}{C} \right)^2 = \frac{3\rho_m C_m^2}{\rho C^2} \text{ and } f_o = \frac{C}{2\pi r_o} \sqrt{\frac{3\rho_m C_m^2}{\rho C^2}}, \quad (34)$$

$$\text{since } \rho_m \sim \rho_e \text{ and } C_m^2 \sim \gamma P / \rho_i \chi (1 - \chi), \quad f_o = \frac{1}{2\pi r_o} \sqrt{\frac{3\gamma P}{\rho \chi}}, \quad (35)$$

which we recognize as a modified Minnaert formula for the resonance frequency of a volume oscillation of a gas bubble (Carey and Fitzgerald [12]). (Note that we have inserted a factor γ , the ratio of specific heats. This factor applies to individual bubble pulsations and for the cloud we set $\gamma = 1$ corresponding to isothermal conditions.) This result follows because the resonance angular frequency is proportional to the compressibility of the bubbly region, characterized by the stiffness $(4\pi r_o \rho C^2)$ and the inertia $(4\pi r_o^3 \rho / 3)$.

5. THE MEASUREMENT OF BACKSCATTER FROM A SUBMERGED BUBBLE CLOUD

We report results from an experiment designed to measure low-frequency acoustic backscattering from a bubble cloud in fresh water under known propagation conditions. At issue is the scattering cross section for a bubble cloud. Is it determined by the bubble sizes and number density (as in the case of coherent or incoherent resonance and off-

resonance scattering from individual bubbles) and discussed previously, or is it related to the free-gas volume fraction and the length scales for the cloud proper? In this effort we did not set out to duplicate "realistic" saltwater bubble plumes, but rather to obtain data to test bubble cloud scattering theories.

5.1 Experimental Procedures

The Naval Undersea Warfare Center (NUWC) Seneca Lake, New York test facility consists of two moored barges in 130 m of water, with the smaller barge (10.7 m x 42.7 m) serving as the platform for our test range. Equipment was deployed with the use of davits, a cable meter, and the edge of the barge, with the resulting vertical geometry shown in Fig. 4. Parametric and conventional transmitters and conventional receivers were orientated colinearly, with the axis of the range intersecting the path of a rising bubble cloud. Though the actual test consisted of measurements for frequencies ranging from 300 Hz to 14 kHz, only the low-frequency (<1.6 kHz) results will be presented here.

The parametric source (PS) was driven with a 22-kHz carrier signal that was up- and down-shifted by one half the difference frequency. The repetition frequency was fixed at either 1 Hz or 2 Hz and the pulse length varied from 5 to 15 msec. Calibration of the source was performed *in situ*. At 500 Hz, the beamwidth and source level was 8.5° and 167 dB *re* 1 μ Pa respectively. Although parametric sources have the advantage of low-frequency directionality, bubbly liquids are highly non-linear and the possibility of enhanced parametric interaction within the cloud needed to be addressed. Therefore, a conventional source (CS) (Honeywell HX-29) was also utilized. The working depth of 87.6 m was chosen to minimize reverberation.

The receivers consisted of four spherical hydrophones (ITC, 15.2 cm), situated as indicated in Fig. 4 (see H1 through H4). All hydrophone outputs were band-pass filtered,

Figure 4. Test range for the backscattering experiment.

preamplified, and recorded on analog tape. Sound speeds were measured daily. At depths greater than 70 m, isovelocity conditions prevailed, with speeds of 1421.7 m/sec and 1420.8 m/sec at 70 m and 91.4 m respectively. For the calculations that follow, a speed of 1421.5 m/sec was used. We aligned the range using the 22-kHz signal to measure time-of-flight and verify alignment and slant ranges. We then inserted a 1.12-m diameter, hollow, steel spherical target in order to test our ability to measure target strength (TS). Assuming a perfectly reflected surface, this sphere possessed a theoretical

TS of -11.1 dB at 5 kHz. Our measured TS was -12.3 dB.

Bubble clouds were produced at 91.4-m depth using a pressurized steel enclosure, vented to the lake via a concentric circular array of 48 22-gauge hypodermic needles connected to a bank of 24 solenoid valves. A 3.5-sec burst of air at 68.95 kPa (10 psi) overpressure yielded a roughly cylindrically shaped cloud length $\approx 1.7 \pm 0.3$ m, radius $\approx 0.24 \pm 0.01$ m, with a void fraction of $\approx 0.40 \pm 0.007\%$. The bubble sizes, which were measured with a video camera at depth as well as in a pressurized tube, were normally distributed about a peak at 1.1 ± 0.2 mm radius, which corresponds to $f_0 \approx 9.2 \pm 1.7$ kHz at a depth of 87.6 m.

Figure 5. The evolution of the TS as the cloud traverses the beam of the parametric source operating at a pulse repetition frequency of 1 Hz

The clouds rose at ≈ 0.3 m/sec, yielding a time-varying echo that reached a maximum level as the cloud crossed the range axis. Fig. 5 depicts the evolution of the backscattered target strength as the cloud rises through the beam. This was obtained by time-gating the sequence of band-pass filtered echo returns (the filter bandwidth was set to equal the signal bandwidth; i.e., 200 Hz). Rms voltages were computed for both the main bang signal (mv) and for each echo return (ev). These values were inserted, along with the transmission loss factor (TLF), into the following expression for the target strength:

$$TS = 20 \log \left[\frac{ev}{mv} \right] - TLF; \quad TLF = TL_{sr} - TL_{st} - TL_{tr}, \quad (36)$$

where the TLF accounts for source-receiver, source-target, and target-receiver transmission losses, assuming spherical spreading of the propagating waves.

5.2 Experimental Results

Figure 6. Backscatter TS measurements for conventional (squares) and parametric (circles) sources.

Peak target strength measurements obtained using conventional and parametric sources are plotted in Fig. 6. The target strengths are very high; at 300 Hz, the TS is about 12 dB greater than one would expect from a 1-m diameter perfectly reflecting sphere at 5 kHz. Each point corresponds to a mean value computed over a population of 4-5 clouds run at each frequency. The uncertainty corresponds to the standard error of the mean. There are apparent resonance peaks at 300 Hz and 1.3 kHz. If these peaks were due to single-bubble resonance scattering, the bubbles would possess radii of 3.4 cm and 7.8 mm respectively, which is highly unlikely in light of our revised observations of the bubbles generated at the needles and of the cloud as a whole.

It is probable that these peaks are the result of resonance scattering due to collective oscillations of the bubble cloud. The resonance frequency for a spherical bubble cloud is given by equation 35, where χ , γ , P_0 , ρ , and a are, respectively, the void fraction, the polytropic exponent, the ambient pressure, the water density, and the cloud radius. We assume that the spherical cloud which has the same volume as the cylindrical cloud will describe the resonance frequency. Such a cloud, with $\chi = 0.40 \pm 0.07\%$ and

$a = 0.42 \pm 0.04$ m, has an isothermal resonance frequency of 324 ± 53 Hz. Moreover, the target strength is given by:

$$TS = 10 \log \left[\frac{I_s}{I_i} \right]_{r=1m}, \quad (37)$$

where I_s and I_i are the scattered and incident intensities respectively. From equations 30 and 32 this is given by

$$\frac{I_s}{I_i} \approx \frac{k^4 a^6}{9} \frac{\left[1 - \frac{y}{\bar{y}} \right]^2}{\left[\left(1 - \left(\frac{y}{\bar{y}} \right) \frac{z_o^2}{3} \right)^2 + \left(\left(\frac{y}{\bar{y}} \right) \frac{z_o^3}{3} \right)^2 \right]}, \quad r = 1m, \quad (38)$$

where $z_o = ka$, $y = \rho c^2$, and $\bar{y} = \rho_m c_m^2$; the subscript denotes the effective properties of the air/water mixture. For the cloud in question, Equations 37 and 38 yield a calculated TS of -3.9 ± 0.8 dB at approximately 309 Hz. Although the measurements and calculations of the fundamental resonance frequency of the bubble cloud agree to within the stated uncertainty, the calculated target strengths are about 4 dB too low. All uncertainties on these results are based on either statistical analysis or *ad hoc* assumption and are propagated through the calculations by Taylor expanding relevant quantities to first order in the perturbed parameters. One such assumption was that the incident wave from the parametric source decreased as $1/r$ between the hydrophone and the target. This could lead us to underestimate the true P_i and subsequently overestimate the target strength. Consequently, the difference between our measurements and calculations may be attributed to the assumption of propagating spherical wavefronts for the parametric source and/or an underestimate of the damping within the cloud.

Parametric and conventional measurements exhibit "reasonably" good agreement despite the fact that reverberation made it difficult to obtain CS data for frequencies less than 1 kHz. It appears that enhanced parametric excitation in the region occupied by the bubbly fluid did not serve to appreciably bias the TS measurements.

6. SUMMARY AND CONCLUSIONS

We have reviewed the classical theory of sound radiation and scattering from compact bubble clouds. The theory shows that the important factor is the compressibility of the mixture which is characterized by the volume fraction at the lower frequencies. The volume pulsation of the cloud is the important mechanism whereby the cloud radiates or scatters sound; Strasberg has shown for the single bubble that shape may not be important. Since any compact radiator may be expanded in terms of a distribution of simple sources such as monopoles, dipoles, and quadrupoles. An expansion of the

radiation field in terms of spherical harmonics identifies the monopole as the oscillation of a spherical source. We have shown that in this limit the cloud of bubbles, radiates with a resonance frequency described by a Minnaert equation modified to include the volume fraction of the gas.

Secondly, we have presented experimental results on the backscatter from a cloud of bubbles in the absence of boundaries. The results were interesting because of the high target strengths and the low-frequency resonance behavior. The experimental and theoretical results were shown to agree in a qualitative manner, we estimated the resonance frequency correctly, but were several dB off with respect to target strength. These results should be considered preliminary as analysis of the data is continuing. Nevertheless, they clearly show the effect of low-frequency resonant scatter.

The presence of the sea surface at these low frequencies may to first order be treated by the method of images. Hence a compact source at a depth below the pressure release surface will have a dipole characteristic and a magnitude which scales as $(z/\lambda)^2$ whereas a scatter below this surface will have a dipole pattern squared and a $(z/\lambda)^4$ dependence. Therefore, the large target strengths observed here may be offset by the proximity of the rough sea surface; the deeper plumes are the most important.

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NOMENCLATURE

$A_m, \bar{A}_m$	series coefficients
C	speed of sound
$H_m(kr)$	Hankel function
I	intensity
$J(kr)$	Bessel function
N	number
$P_\omega, P_{i\omega}, P_\omega$	pressure transform
P	ambient pressure

P_o	source pressure
$P_m(\theta)$	Legendre polynomial
R	radial distance
S, S_o, S_a	surfaces
TS	target strength (dB)
TL	transmission loss (dB)
V, V_o, V_a	volumes

a	radial distance	t	time
$\hat{a}$	normal to S_a	v, v_i, v_o	normal velocities
$c_m, c_{mi}, c_g,$ $c_o, \bar{c}_a$	sonic speeds	$\gamma, \bar{\gamma}$	bulk moduli
ev	echo voltage	γ	gas constant
f_o	natural frequency	$\gamma_\kappa, \gamma_\rho$	compressibility, density ratio
$f_n(\omega_o, \theta, r_n)$	bubble response	δ	dampening
$f_\omega(r)$	source function	ϵ	small quantity
$g_\omega(r r_o)$	Green's function	θ	angular measure
g	subscript for gas	κ	compressibility
i	subscript incident	λ	wave length
k	wave number	μ	wave number
l	liquid	π	pi
m	subscript mixture or index	ρ	density
mv	main bang voltage	χ	void fraction
$\hat{n}_o$	normal to S_o	ω	angular frequency
r	radial distance	Γ	gamma function
s	subscript, scatter		

REFERENCES

1. Thorpe, S. A. (1982), "On the clouds of bubbles formed by breaking wind-waves in deep water and their role in air-sea gas transfer," Phil. Trans, Roy. Soc. A 304, 155-185.
2. Su, M. Y., Green, A.W., and Bergin, N.T. (1984), "Experimental studies of surface wave breaking and air entrainment," in W. Brutsaert and G. Jirka (eds.), Gas Transfer at Water Surfaces, Reidel Press, pp. 211-219.
3. Monahan, E. C. and MacNiocaill (eds.) (1986), Oceanic Whitecaps and Their Role in Air-Sea Exchange Processes, D. Reidel, Boston.
4. Monahan, E. C. and Lu, M. (1990), "Acoustically relevant bubble assemblages and their dependence on meteorological parameters," IEEE Jour. Ocean Engr. 15(4), 340-349.
5. McDaniel, S. (1988), "High-frequency sea surface scattering: Recent Progress," J. Acoust. Soc. Am 84(S1), S121 (also: ARL/PSU T.M. 88-134-21, July 1988)
6. Carey, W., Goodman, R., McDaniel, S., Jackson, D., Deferrare, H., and Briscoe, M. (August 1985), "Sea surface scatter experiments, a review," Memo to M. Orr, ONR (11250A).

7. McDonald, B. E. (1991), "Echoes from vertically striated subresonant bubble clouds: A model for ocean surface reverberation," *J. Acoust. Soc. Am* 89(2), 617-622.
8. Henyey, F. (1991), "Acoustic scattering from ocean microbubble plumes in the 100 Hz to 2 kHz region," *J. Acoust. Soc. Am* 90(1), 399-405.
9. Prosperetti, A. (1988), "Bubble related ambient noise in the ocean," *J. Acoust. Soc. Am.* 84(3), 1042-1054.
10. Prosperetti, A. (1988), "Bubble dynamics in ocean ambient noise," in B. R. Kerman (ed.), *Sea Surface Sound*, Kulwer Acad., Boston, MA, pp. 171-171.
11. Carey, W. M. and Browning, D. G. (1988), "Low frequency ocean ambient noise: Measurements and theory," in B. R. Kerman (ed.), *Sea Surface Sound*, Kulwer Acad., Boston, MA.
12. Carey, W. M. and Fitzgerald, J. W. (1987), "Low frequency noise and bubble plume oscillations," *J. Acoustic soc. Am.* 82(51), p. 362 (paper 001) (also available NUSC TD 8495, 24 February 1989; DTIC AD206537)
13. Carey, W. M., Fitzgerald, J. W., and Browning, D. G. (in press 1990), "Low-frequency noise from breaking waves," in B. R. Kerman (ed.), *Natural Physical Sources of Underwater Sound*, Kluwer Acad., Dordrecht, Netherlands, (also: NUSC Tech. Rep. 8783, 5 oct., 1990, avail. DTIC AD227969).
14. Foldy, L. (1944), "Propagation of sound through a liquid containing bubbles," paper, Columbia Univ. NRDC DIV 6.1. (Available DTIC).
15. Spitzer, L. (1943), "Acoustic properties of gas bubbles in a liquid," paper, NRDC DIV 6.1, (available DTIC).
16. Foldy, L. (February 1945), "Multiple scattering of waves," *Phys. Rev.* 67(3&4) 107-119.
17. Carstensen, E. L. and Foldy, L. (May 1947), "propagation of sound through a liquid containing bubbles," *J. Acous. Soc.* 19(3), 481-501.
18. Morse, P. M. and Feshback, H. (1943), *Methods of Theoretical Physics, Part II*, McGraw Hill Book Co., N.Y.
19. Morse, P. M. and Ingard, K. (1968), *Theoretical Acoustics*, McGraw Hill Book Co., N.Y., pp. 413-414.
20. Rschevkin, S. N. (1963), *The Theory of Sound*, Pergamon Press Ltd., N.Y.

21. Anderson, V. (1950), "Sound scattering from a fluid sphere," J. Acous. Soc. Am. 22(4), 426-431.
22. Wood, A. B. (1932/1955), A Textbook of Sound, G. Bell and Sons Ltd., London, pp. 360-364.
23. Commander, K. and Prosperetti, A. (1989), "Linear pressure waves in bubbly liquids: Comparison between theory and experiments," J. Acoust. Soc. Am. 85(2), 732-746.
24. Gilbert, K., Wang, L., and Goodman, R. (1991), "A stochastic model for scattering from near-surface oceanic bubble layers," J. Acoust. Soc. Am. 90(s4), 2301.
25. Strasberg, M. (1956), "Gas Bubbles as Sources of Sound in Liquids," J. Acoust. Soc. Am. 28(1), 20-26.
26. Crighton, D. G. and Williams, J. E. (1969), "Sound generation by turbulent two-phase flow," J. Fluid Mech. 36, 585-603.
27. Van Wijngaarden, L. (1958), "On the equation of motions for mixtures of liquid and gas bubbles," J. Fluid Mech. 133(P3), 465-474.
28. Karplus, H. B. (1958), "The velocity of sound in a liquid containing gas bubbles," paper, Armour Research Foundation, University of Illinois, Proj. c00-248, TID-4500.
29. Ruggles, A. (1987), "The Propagation of Pressure Perturbations in Bubbly Air/Water Flows," Ph.D. Thesis, Dept. of Mech. Eng., Rensselaer Polytechnic Institute, Troy, NY.