Abstract of

Sunspot Velocity Fields

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Research in sunspot velocity fields was initiated by Evershed in 1909. He observed that, in general, Fraunhofer lines crossing a sunspot appeared to be equally displaced in wavelength in opposite directions on either side of the spot, and subsequent measures of sunspot spectra enabled him to deduce a radial outflow from the spot centre over the surface of the sun, the maximum value of which approached $2 \text{ km/s sec}$. This sunspot feature has since been referred to as the Evershed effect. In addition, Evershed noticed that the velocity increased outwards from the umbra to the penumbra, where it reached the maximum value, and appeared to cease abruptly at the outer edge of the penumbra. Further work by St. John in 1913 confirmed Evershed's observations, but St. John maintained that the motion continued beyond the penumbra into the photosphere. Extending the investigation to chromospheric regions, St. John was also able to show that lines of calcium and hydrogen exhibited an instreaming motion.

The recent investigations of sunspot velocity fields, notably by Kinman, Servajean and Bumba, have attempted more detailed analyses of the sunspot motions. Based on a scheme suggested by H.H. Plaskett, both Kinman and Servajean, after a series of detailed visual measures of sunspot spectra, attempted solutions for three sunspot velocity components viz. a radial component $u$ along the surface of the sun, a
vertical component $w$ at right angles to the sunspot plane along a solar radius, and a tangential component $v$ at right angles to $u$ and $w$ and in the plane of the spot. Their solutions confirmed Evershed's initial hypothesis, that the material flow in sunspot penumbrae is predominantly radial. In addition, both Kinman and Servasjean deduced a variation in the maximum radial velocity $U_{\text{max}}$. Kinman suggested $U_{\text{max}}$ increased linearly with umbral radius, while Servasjean concluded that in the same spot, $U_{\text{max}}$ decreased as the spot proceeded from the disk centre to the limb.

In all such Evershed effect investigations, care must be taken to distinguish between line-shifts due to velocities in the sunspot along the line-of-sight, and shifts which have their origin in line-splitting produced by the magnetic field associated with the spot. The relation between magnetic lines of force and material motion is a matter of considerable importance in sunspot theory, and is the basis of a recent sunspot investigation by Bumba, who concludes that motion in sunspot penumbrae is along the lines of force of a fan-shaped magnetic field located outside the umbra. Verification of these results is desirable.

The present work was designed to investigate velocity fields in sunspots from detailed measures of an Fe I line at $\lambda 5576$, a Fraunhofer line whose Landé splitting factor $g$ is zero, i.e. it is unaffected by the sunspot magnetic field. Using
high dispersion sunspot spectra obtained by Professor Plaskett we again followed his suggested scheme to determine penumbral velocity components in one large and one small sunspot. A further investigation was concerned with the analysis of sunspot spectra taken during the passage of one and the same spot across the solar disk. This latter work was especially designed to determine the umbral velocity components and to investigate the variation of the penumbral velocity components with disk position. Two main series of results have emerged. First, the sunspot penumbral motion is confirmed to be predominantly radial, while the umbral motion is characterized by a small, descending vertical component. Also from the spectra of the same spot, we deduce that \( u_{\text{max}} \) decreases from centre to limb and find that simultaneously, the radial velocity pattern is considerably broadened. Light scattering suggests an explanation for this feature and a preliminary investigation of the effect of scattered light on the sunspot velocity field was attempted. The radial velocity corrections due to scattered light were greater than had been previously estimated and it is suggested that the centre-to-limb change in \( U_{\text{max}} \) may be due to this cause. An alternative explanation is provided by the possible existence of a radial velocity variation with depth in the sunspot. The second observational result concerns a phenomenon revealed by the high-dispersion
spectra, and referred to as line-flare. This is a diffuse widening of the spectral line at the edge of the penumbra in the direction of the measured velocity. (It was simultaneously observed by Servazean and Bumba.) In this thesis, a detailed photometric study of the line-flare is undertaken to investigate intensity variations and to determine its influence on the visual measures of velocity fields in sunspots.

The final survey of both our velocity field and photometric results leads us to the conclusion that no satisfactory interpretation of sunspot velocity fields can be suggested until two major issues have been decided vis. (a) The problem of a sunspot model to describe the variations of pressure and temperature with geometric and optical depths. (The determination of line equivalent widths and central intensities is a step in this direction.) If any depth variations in the sunspot are to be confirmed, the question of a sunspot model is essential. (b) The effect of scattered light on sunspot velocity and magnetic fields needs to be more carefully considered. Only when these problems have been decided can we attempt to relate our sunspot velocity field observations with our knowledge of umbral granulation, penumbral filaments and sunspot magnetic fields, in order to visualize the true pattern of the motions of material in a sunspot.
THESIS SUBMITTED FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

IN THE UNIVERSITY OF OXFORD

SUNSPOT VELOCITY FIELDS

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# TABLE OF CONTENTS

**INTRODUCTION**

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.</td>
<td>Previous investigations of sunspot velocity fields.</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td>Analysis of component velocities.</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>Recent sunspot velocity field determinations.</td>
<td>24</td>
</tr>
</tbody>
</table>

**CHAPTER II.**

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>II.1</td>
<td>Redetermination of sunspot velocity fields using a magnetically undisturbed line.</td>
<td>42</td>
</tr>
<tr>
<td>II.2</td>
<td>The choice of solar line for investigation.</td>
<td>43</td>
</tr>
<tr>
<td>II.3</td>
<td>1959 Observations and measurements.</td>
<td>46</td>
</tr>
<tr>
<td>II.4</td>
<td>1959 Reductions and results.</td>
<td>53</td>
</tr>
<tr>
<td>II.5</td>
<td>Discussion of 1959 results.</td>
<td>57</td>
</tr>
<tr>
<td>II.6</td>
<td>1960 Observations and measurements.</td>
<td>60</td>
</tr>
<tr>
<td>II.7</td>
<td>1960 Reductions and results.</td>
<td>67</td>
</tr>
<tr>
<td>II.8</td>
<td>Discussion of velocity field results.</td>
<td>71</td>
</tr>
</tbody>
</table>

**CHAPTER III.**

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>III.1</td>
<td>The effect of scattered light on sunspot velocity field measures. Introduction.</td>
<td>77</td>
</tr>
<tr>
<td>III.2</td>
<td>Scattered light corrections for previous Evershed effect measures.</td>
<td>82</td>
</tr>
<tr>
<td>III.3</td>
<td>A new determination of the effect of scattered light.</td>
<td>83</td>
</tr>
<tr>
<td></td>
<td>a) Method</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>b) Basic data.</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>c) Effect of scattering on sight-line velocities.</td>
<td>95</td>
</tr>
<tr>
<td>III.4</td>
<td>Discussion of results.</td>
<td>97</td>
</tr>
</tbody>
</table>
CHAPTER IV. Photometry of the sunspot FeI absorption line \( \lambda 5576.101 \).

IV.(1) Experimental procedure.

IV.(2) Results of photometric analysis.
   (a) Line profile asymmetries.
   (b) Determination of the effective photographic density of measured sight-line velocities.
   (c) Equivalent widths.
   (d) Central intensities.

IV.(3) Discussion of photometric results.

APPENDIX CHAPTER IV. The spectroscope vertical apparatus function.

Appendix IV(1) Introduction and Method.

Appendix IV(2) Details of Method.
   (a) Determination of vertical apparatus function.
   (b) The true intensity distribution across the spot.
   (c) Determination of the true sight-line velocity pattern and conclusion.

CHAPTER V. Discussion of Observations and Conclusion.

V.(1) Summary of results.

V.(2) Discussion of results.

V.(3) Conclusion.

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REPRINT OF PUBLICATION.
<table>
<thead>
<tr>
<th>FIGURES AND PLATES</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intro. FIG. 1. The &quot;Butterfly&quot; diagram showing change in spot latitude during the 11 year cycle.</td>
<td>4</td>
</tr>
<tr>
<td>Intro. PLATE 1. A high altitude photograph of a sunspot showing penumbral structure.</td>
<td>6</td>
</tr>
<tr>
<td>FIG. I. 1. Definition of x- y- and z- axes.</td>
<td>20</td>
</tr>
<tr>
<td>FIG. I. 2. Definition of velocity components at a point P in penumbra.</td>
<td>22</td>
</tr>
<tr>
<td>FIG. I. 3. Determination of co-ordinates of P.</td>
<td>22</td>
</tr>
<tr>
<td>FIG. I. 4. Kinman's measurement positions.</td>
<td>24</td>
</tr>
<tr>
<td>FIG. I. 5. Kinman's velocity components u and w as a function of distance from spot centre.</td>
<td>27</td>
</tr>
<tr>
<td>FIG. I. 6. Servajean's measurement positions in his investigation of the variation of Evershed effect with depth.</td>
<td>30</td>
</tr>
<tr>
<td>FIG. I. 7. Comparison of Mattig's and Richard's sunspot models, illustrating variation of optical depth ( t ) with geometric depth ( h ).</td>
<td>32</td>
</tr>
<tr>
<td>FIG. I. 8. Servajean's maximum radial velocity as a function of ( \sin \theta )</td>
<td>54</td>
</tr>
<tr>
<td>FIG. I. 9. Bumba's variation of maximum radial velocity ( V^+ ) with ( \theta )</td>
<td>38</td>
</tr>
<tr>
<td>FIG. I. 10. Bumba's variation of maximum radial velocity ( V^- ) with ( \theta )</td>
<td>38</td>
</tr>
<tr>
<td>PLATE II. 1. Zeeman splitting in Fe ( \lambda 6302 ).</td>
<td>44</td>
</tr>
<tr>
<td>PLATE II. 2. Line-flare in a large and small spot.</td>
<td>44</td>
</tr>
<tr>
<td>FIG. II. 1. Comparison of visual and photoelectric settings on ( \lambda 5576 ) in the region of a sunspot.</td>
<td>50</td>
</tr>
</tbody>
</table>
FIG. II. 2 (a) Observed velocities and correction line for small spot of Plate E 28.

FIG. II. 2 (b) Observed velocities and correction line for large spot of Plate E 22.

FIG. II. 3 (a) Radial velocity for small spot of Plate E 28, assuming $v = w = 0$.

FIG. II. 3 (b) Radial velocity for large spot of Plate E 22, assuming $v = w = 0$.

FIG. II. 4. Least squares solution for the velocity field of small spot.

FIG. II. 5. Adjustable vee for sunspot umbral exposures.

FIG. II. 6. 1960 Sunspot positions and slit sections.

FIG. II. 7 (a) True sight-line velocities for Plate Ho 40, taken when spot is nearest limb.

FIG. II. 7 (b) True sight-line velocities for Plate Ho 58, taken when spot is nearest disk centre.

FIG. II. 8. Solution for velocity components $u$ and $w$ in umbra.

FIG. II. 9. Variation of penumbral radial velocity $u$ (smoothed) with distance from spot centre for two disk positions.

FIG. II. 10. A comparison of Servajean's least squares solution for $U_{max}$ at varying disk position, with the present investigation.

FIG. III. 1. Wormell's sunspot intensity distribution after correcting for scattered light.

FIG. III. 2. Wander's variation of umbral intensity as a function of disk position, corrected and uncorrected for scattered light.
FIG. III. 3. Circular spot at two disk positions and sub-divisions considered for scattering calculations. (Details shown for one sector only.)

FIG. III. 4. Assumed radial velocity distributions for scattered light calculations.

FIG. III. 5. Comparison of calculated energy distribution at limb using accepted scattering function, with observed distribution.

FIG. IV. 1. Bumba's velocity distribution in a vertical section of a sunspot.

FIG. IV. 2. Marking the measured plates for the line-flare photometry.

FIG. IV. 3. Comparison of $\lambda$ 5576 profiles showing maximum asymmetry for 3 sunspot positions on disk.

FIG. IV. 4. Comparison of photometric and visually determined sight-line velocity shifts at selected $I/15$ values for Plate Ho 40.

FIG. IV. 5. Equivalent width of $\lambda$ 5576 versus distance from spot centre for three disk positions.

FIG. IV. 6. Variation of central intensity of $\lambda$ 5576 with distance from spot centre for three disk positions.

FIG. Ap. IV. 1. The observed wire and ghost profiles and the predicted profile using the determined apparatus function.

FIG. Ap. IV. 2. The observed and true sight-line velocity patterns across the sunspot of Plate Ho 62.

FIG. V. 1. Variation of the sunspot radial velocity component and equivalent line width of $\lambda$ 5576 as a function of distance from spot centre for two disk positions.
INTRODUCTION
Velocity Fields in Sunspots

Introduction

"......Our knowledge of the sun's action is but fragmentary, and the publication of speculations on the nature of his spots would be a very precarious venture".

R.C.Carrington. 1858. (10)

In spite of intensive research on sunspots since Carrington's day, there is still some truth in the warning he issued to the Royal Astronomical Society over one hundred years ago. Every aspect of the nature of sunspots has since been considered, but a satisfactory solution to even the most basic problem, that of their origin (2) (3) (4) (12) (13) (41) has yet to reconcile the conflicting evidence of observation and theory. For example, the most plausible theory of spot coolness (4) is that a spot arises because the magnetic field inhibits the convection of energy up to the surface below the spot. This leaves unexplained the observed flow of material in the spot (the Evershed effect) and the reason for the existence of a surrounding penumbra distinct from the central umbra. This introduction to a thesis based on one major problem, that of sunspot velocity fields, aims
to give a survey of the most important sunspot features, if not to attempt the "precarious venture" of their explanation.

The earliest reference to a sunspot as yet discovered dates from the mid-fourth century B.C., and up to the time of the invention of the telescope in 1608, there are numerous recordings of keen-sighted observers having seen 'freckles' in the sun. Scheiner's 17th Century telescopic observations, recorded in detailed drawings taken from a projected disk image, show the distinction between the dark central umbra of a sunspot and its lighter surrounding penumbra. His systematic observations led him to realize that the sun must be rotating on an axis with a period of about 27 days. Two centuries later, Schwabe, observing with a small refractor, announced that sunspot activity fluctuated within a period of about 10 years (for which he was awarded the Gold Medal of the Royal Astronomical Society) and a periodicity of 11 years has since been well accepted. A remarkable periodicity is also exhibited by the distribution of sunspots in solar latitudes. Practically all the spots occur within two zones parallel to the equator and within ± 45° latitude. The first spots of a cycle occur at about 30° N and S, at sunspot maximum the zones reach ± 15°, while the last spots of a cycle appear at about ± 8°. The regularity of this pattern is best seen from the "butterfly diagram" (illustrated in Fig.1) which gives the latitude distribution over a complete
INTRO. Fig. 1. The "butterfly" diagram showing change in spot latitude during the 11-year cycle.
sunspot cycle.

The characteristic appearance of a sunspot is shown in the photograph (reproduced in PLATE 1) taken during high altitude balloon research in 1959 (14). A typical spot, which may vary in size from umbral diameter 4" (about 3000 km) to about 50", has usually two clearly defined regions. The darkest central portion, or umbra, is surrounded by a lighter penumbra, whose respective intensities were found by Wormell (42) to be 0.3 and 0.8 respectively of the photospheric intensity. Corroborating lines of evidence from Pettitt and Nicholson (32), Richardson (33), Moore (31) and Howard (23) have shown that the umbra has a temperature of approximately 4000°K, compared with the photospheric temperature of 6000°K. Danielson's (14) high definition time sequences of sunspot photographs clearly resolved the penumbra into an array of predominantly radial bright filaments. Such filamentary structure had, in fact, been seen in photographs taken from the earth's surface by Janssen (24) before 1900, as well as more recent photographs taken at Mount Wilson (Kiepenheuer (25) 1953), Meudon (Naoris (28) 1953), and Pic du Midi (Rösch (34) 1959). However, the high altitude work confirmed two basic properties of penumbral filaments, namely that they are long narrow structures having an apparent width of 300 km, and lengths exceeding 5000 km., and that the filaments live roughly five times longer than granules. In spite of its apparently uniform appearance, the umbra is not a homogeneous
INTRO. PLATE I A high altitude photograph of a sunspot showing penumbral structure.
region, as Bray and Loughhead (6), (7) have recently shown. In their high resolution observations of sunspots in 1958, they confirmed the existence of umbral granulation, resembling photospheric granulation, but differing in lifetime and cell size. They concluded that umbral granules are longer lived than photospheric granules i.e. lifetimes greater than 10 minutes, and that the mean cell size was about 2.0 sec. of arc compared with a probable value of 2.6 sec. for the photospheric granulation cell size.

The three-dimensional shape of a sunspot has been in dispute since Wilson first observed in 1769, that there is an apparent displacement of umbra relative to the penumbra towards the centre of the disk, as a round spot approached the western limb (i.e. the so-called Wilson effect). The countless explanations have ranged from the theory of sunspots as relatively shallow formations of average depth 1000 km., to the assumption that they are cloud formations above the photosphere. To reconcile these opposing views, it was further suggested that sunspots can have the form of depressions or humps. Bray and Loughhead (5) claimed to have demonstrated the reality of the Wilson effect by repeatedly photographing a regularly-shaped sunspot over a period of 11 days, on its passage from the East to the West limb, but pointed out that a geometrical interpretation is inadequate, and that the principal factor causing the Wilson effect is the greater
transparency of the spot compared with the surrounding photosphere. If this is so, it means that a line of sight penetrates further into the spot than the surrounding photosphere, but this solution faces the theoretical difficulty of accepting a sunspot model by which to compare optical depths in the photosphere and spot. (This difficulty will be emphasized in Chapter I in a discussion of Servajean's work.) A recent analysis by Chistyakov (11) confirmed the existence of the Wilson effect and eliminated all geometrical hypotheses except that in which the sunspot is a conical funnel with a flat bottom, but the problem is still by no means solved.

The first successful high dispersion photograph of the sunspot spectrum was made by Hale and Adams in 1906 (20) at Mount Wilson. They observed that there was a change in intensity in certain lines of metals, from the spot to the photospheric spectrum, and consequently suggested that the sunspot was a region of lower temperature than the photosphere. This was further confirmed by the discovery of molecular bands in the sunspot spectrum. Another important feature was the widening, and even complete splitting, of many of the sunspot spectral lines. Hale (18) (19) was the first to interpret this in terms of the Zeeman effect, indicating the existence of a magnetic field associated with each spot. The investigations of the magnitude and direction of sunspot magnetic fields as a function of position and depth in a spot have since promoted the development of varied techniques.
To determine field strengths, the observations consist essentially of the measurement of the splitting of a Zeeman triplet, from which the total magnetic field in which the line originates is given by

\[ \Delta \lambda = gCH\lambda^2 \]

\( \lambda \) = wavelength
\( g \) = splitting factor
\( C \) = constant
\( H \) = field strength

The practical difficulty in this basically simple measurement is due to the fact that the splitting for average spot fields is of the same order of magnitude as the line width, so that even at high dispersion and resolving power, the components are only properly separated at the centre of large spots. The following methods illustrate the various ingenious attempts to solve this problem.

Hale's method was to view the Zeeman triplet through a quarter-wave plate and Nicol, and such a procedure forms the basis of a long series of field measures by Hale and Nicholson (21) at Mount Wilson. They deduced that the field strength varies with the area of the spot from a maximum of about 4000 gauss to values of the order of 100 gauss in the smallest spots measurable. Houtgast and van Sluiters (22) give as a relation between the maximum field strength \( H \) and area \( A \) :-
\[ H = 3700 \frac{A}{A+10^{-6}} \text{ Gauss. (}A\text{ in }10^{-6}\text{ visible hemisphere)} \]

Von Klüber (26) refined the method of Hale and his techniques form the basis of a programme of maximum field measures now in progress at Potsdam (27). In 1942 Broxon (8) published his parabolic law of field intensity.

\[ H(r) = H_0 \left(1 - \frac{r^2}{b^2}\right) \]

Where \( H(r) \) = field strength at a distance \( r \) from spot centre.

\( H_0 \) = central field.

Mattig's (29) measures indicate a more gradual variation given by

\[ H(r) = H_0 \left(1 - \frac{r^4}{b^4}\right) e^{-2\left(\frac{r}{b}\right)^2} \]

While Bumba (9) has shown the variation of field strength can be represented by

\[ H(r) = \frac{a}{r^3} \]

and \( \log_{10} H_0 = 0.27 \log_{10} a - 4.82 \)

where \( a \) = magnetic moment of the field.

Of these Bumba's work was done with higher resolving power and shorter exposure times, and on this account should be more reliable. In all magnetic field determinations a limitation is imposed by the difficulty of instrumental polarization.
The work on the determination of the inclination of the lines of force was developed by Nicholson (21) using the quarter-wave plate and Nicol method, and comparing the intensities of the Zeeman components of a triplet. Searles (35) gave the intensity of these components by:

\[ \sigma_v = \frac{1}{4} (1 - \cos \chi_0)^2 \]

\[ \pi = \frac{1}{2} \sin^2 \chi_0 \]

\[ \sigma_r = \frac{1}{4} (1 + \cos \chi_0)^2 \]

where \( \chi_0 \) is the inclination of the magnetic vector to the line of sight. (A more elaborate treatment of the problem has been proposed by Stepanov (36) who suggests replacement of Searles' law by a more complicated expression derived from considerations of the transfer problem.) When \( \chi_0 = \frac{\pi}{2} \)

\( \sigma_r = \sigma_v \) so by observing the point in a spot at which these components were of equal intensity and repeating the observation as the spot moved towards the limb, Nicholson's investigation of the directional properties of the lines of force gave the relation

\[ \chi_0 = 0.75 \frac{\pi}{2} \frac{r}{b} \]

where \( r = \) distance from spot centre

\( b = \) penumbral radius.

A method more recently developed by Treanor (40) minimizes a complicating effect in such measures, namely that of instrumental polarization. By means of a Babinet compensator,
he produced polarization fringe systems in the components of a normal Zeeman triplet. From the phase difference of the $\sigma$-component fringes, the constants of elliptical polarization at each point in the spot were determined, and the orientation of the lines of force deduced. His attempts to extend measures to the edge of the spot were hampered by failing resolution of the components due to their intrinsic width, and the presence of a central component. The results he obtained for the variation of field strength over a spot were found to be adequately represented by Broxon's parabolic formula, but the determinations of the field direction enabled him to calculate that 40 per cent of the total flux of the spot is funnelled into unit solid angle, compared with less than 20 per cent calculated using Nicholson's measurements. This means that Treanor's results indicate a more strongly radially directed field than those of Nicholson. This work is at present being continued by Miss Adam at Oxford.

Investigations of the change in magnetic field with depth in a spot were based on either a comparison of field strengths determined from weak and strong lines (i.e. lines formed at different depths in the spot) a centre-limb variation in the field intensity, or alternatively, from the divergence of the lines of force. Houtgast and van Sluieters (22) attempted all three methods and arrived at the somewhat indefinite estimates ranging from 0.5 to 5 oersted/km. Tanaka and Tagaki (39) however, in an investigation of lines of different strengths ranging in intensity from -3 to 28, found a barely
detectable gradient. Treanor's observations on centre-limb field strengths gave no evidence for a variation of field with depth, although an upper limit of 10000 ergs/Km was suggested. Treanor's results were the first obtained by the Babinet method and were based on measures of only four spots.

Velocity fields also play an important part in sunspot investigations. This is the problem of the Evershed effect, so called at the beginning of the century after Evershed (15) observed the displacement of Fraunhofer lines in the penumbral regions of sunspot spectra, and deduced that the material at photospheric level was flowing radially away from the spot axis with a velocity of the order of one or two Km/sec. The strongest lines originating in the chromospheric layers (Ca\(^+\), H\(\alpha\) etc) exhibited an instreaming motion up to three Km/sec. He was also able to show that the radial velocity increased outwards from the umbra to a maximum at the outer edge of the penumbra, where it appeared to cease abruptly. In his later work (16) (17) there were also indications of tangential components, and a velocity of descent in the umbra. St.John (37)(38) confirmed Evershed's hypothesis from measures on 506 different lines (\(\lambda\lambda\) 3624-6644) but contended that the motion continued beyond the penumbra into the photosphere. In addition, he claimed larger velocities for weak lines than for strong, indicating a velocity variation with depth in the spot. In 1932, Abetti (1) confirmed the existence of radial
velocities varying from 0.2 to 5.8 km/sec, and also found irregular tangential velocities of from 0 to 5.0 km/sec, concluding that the motions must vary markedly from spot to spot. Evershed, St. John and Abetti however, compared their measured spectra with spectra taken at the disk centre, a method which would introduce considerable error in the use of a spectrograph with high resolving power and dispersion because of irregularities in the solar lines produced by local photospheric velocity fields. Their observations (which were only in rough agreement) also refer to, at most, four points in the spot, and do not determine the complete velocity field.

In 1950, Michard (30) analysed the results obtained by Abetti, to examine the variation of radial velocity with spot size and magnetic field. For each group of spots considered he determined a mean radial velocity (he fails to define "la vitesse moyenne") and a mean \( \sin \Theta \) value (\( \Theta \) = angle between the line-of-sight and the normal to the solar surface) and deduced a linear relationship:

\[
V = V_0 \left( 1 - 0.8 \sin \Theta \right)
\]

where \( V_0 \) = the velocity which would be observed at the disk centre. He explained this apparent decrease from centre to limb as an increase in radial velocity with depth in the spot, the largest velocities being observed at the disk centre where the line-of-sight penetrates further into the spot. The effects of scattered light, he argues, are probably insufficient to account for the observed variation. Such an analysis
which estimates a so-called "mean" velocity for a group of
spots at a "mean" position, irrespective of their size, age
and magnetic field, could not produce very meaningful results.
The hypothesis of a velocity variation with depth can only be
justified by comparing the velocity fields obtained from
following the same spot across the disk, and applying the
appropriate correction for scattered light at each disk
position. Even so, such an investigation would be limited by
the assumption that the spot velocity field was unchanged in
its passage across the disk.

The comparatively small scale of sunspot features requires
that in all investigations concerning sunspots, due considera-
tion must be paid to the factor which is always present in
solar observations, namely, the effects of scattered light.
In Evershed measures, this will produce distortion in the true
velocity field, and in magnetic field measures, distortion of
the Zeeman components.

An understanding of the nature of sunspots is essentially
a problem in magnetohydrodynamics, and to solve such a problem
it is essential to have both an observational picture of the
magnitude and direction of magnetic and velocity fields in
the sunspot. The more recent investigations discussed in
Chapter I and the contents of this thesis as a whole, present
the results of the work on velocity fields in sunspots.
References to Introduction

(10) R. G. Carrington, M.N. 19, 1, 1858.

'The Sun' ed. G.P. Kuiper.


CHAPTER I

Previous investigations of sunspot velocity fields.
Chapter I

Previous investigations of Sunspot Velocity Fields

I. (1) Analysis of component velocities.

To determine the complete velocity field in a sunspot, it is necessary to relate the measured sight-line velocity at any point to velocity components within the spot. Professor Plaskett (15) has suggested a scheme, followed by both Kinman (8) (9) and Servajean (16) in their velocity field determinations, which will be described in this first section.

Consider (Fig.I.1) a point Q on the solar surface which is the origin of axes x, y, z. Qx is parallel to the direction of solar rotation, Qy is tangent to meridian to the sun's north pole of solar rotation, and Qz points radially outwards. Let the velocity at Q be represented by three components \( p, q \) and \( r \) along these axes. If the angles between these axes and the line of sight are \( \sigma_1, \sigma_2, \sigma_3 \) respectively

\[
p \cos \sigma_1 + q \cos \sigma_2 + r \cos \sigma_3 = V \tag{I.1}
\]

where \( V \) = corrected sight-line velocity at Q

\( V \) is related to the measured sight-line velocity \( V^* \) by

\[
V = V^* - C - \Delta_1 - \omega \cos \sigma_1 \tag{I.2}
\]

where

\( C \) = correction due to observer's motion.

\( \Delta_1 \) = correction due to Limb Effect.

\( \omega \) = correction for solar rotation at Q.
C may be calculated directly from a knowledge of

1. The observed epoch of exposure.
2. The equatorial co-ordinates of \( Q(\alpha, \delta) \)
3. The geocentric equatorial rectangular co-ordinates of the sun \((x, y, z)\).

The resultant correction is given by equation (8) in Professor Plaskett's paper.

FIG I: Definition of \( x, y \), and \( z \)-axes.
The limb effect may be determined from previous observations (1) and the solar rotation correction at a latitude \( B \) calculated from a Faye type formula (1)

\[ R_B = R_o \cos B \left( 1 - 0.229 \sin^2 B \right) \text{ km sec}^{-1} \]

where \( R_o \) = linear velocity at equator.

If we now consider any point \( P (r, \phi) \) (Fig.1.2) in the penumbra of a sunspot centred at \( Q \) which is also the origin of the cylindrical coordinate system \( r, \phi, z \) the actual motion in the spot may be resolved into 3 components, a radial motion outwards \( u = \dot{r} \), a tangential motion \( v = \dot{\phi} \) and a vertical motion \( w = \dot{z} \). The direction cosines for these components are

\[
\begin{align*}
\cos \phi, & \quad \sin \phi, \quad 0 \\
\sin \phi, & \quad \cos \phi, \quad 0 \\
0, & \quad 0, \quad 1
\end{align*}
\]

for \( u, v, w \) respectively.

Then

\[
\begin{align*}
p & = u \cos \phi - v \sin \phi \\
q & = u \sin \phi + v \cos \phi \\
r & = w
\end{align*}
\]

and our equation of condition I.1 may be written

\[
V = (u \cos \phi - v \sin \phi) \cos \chi_1 + (u \sin \phi + v \cos \phi) \cos \chi_2 + w \cos \chi_3
\]

i.e.

\[
V = u (\cos \phi \cos \chi_1 + \sin \phi \cos \chi_2) + v (\cos \phi \cos \chi_2 - \sin \phi \cos \chi_1) + w \cos \chi_3
\]

= corrected sight-line velocity. I.3.
**FIG 1-2** Definition of velocity components at a point P in penumbra.

**FIG 1-3** Determination of co-ordinates of P.
Hence a knowledge of corrected sight-line velocities \( V \), position angles \( \phi \) and direction cosines \( \xi_1, \xi_2, \xi_3 \) will provide a set of equations in terms of the required sunspot velocity components \( u, v, w \). Since it is possible to determine the co-ordinates of the sunspot centre on the projected disk at the time of observation, it is a simple matter to calculate \( \phi \) for any point along the spot-section provided by the spectroscope slit. Assuming the motion of photospheric matter to have cylindrical symmetry about the origin, so that the velocity components \( u, v \) and \( w \) are functions of \( \rho \) alone, the foregoing equation of condition may be solved by least squares. The direction cosines are given by (15)

\[
\begin{align*}
\cos \xi_1 &= \sin (L-L_0) \cos \theta \_0 \sin (\theta + \theta') \cos \sec \theta \\
\cos \xi_2 &= \left[ \sin \theta \cos \theta \_0 \cos (L-L_0) - \cos \theta \sin \theta \_0 \right] \sin (\theta + \theta') \cos \sec \theta \\
\cos \xi_3 &= - \cos (\theta + \theta')
\end{align*}
\]

where \( \theta' = \rho / \rho_0 \) 

\[
\sin (\theta + \theta') = \theta' / \theta
\]

\( S = \) angular semi-diameter of sun.

\( \theta = \) angle subtended at centre of sun \( C \) by \( Q \) and sub-terrestrial point \( E \). (Fig. I.1)

The co-ordinates \((\rho, \phi)\) of any point \( P \) in the spot may be determined from (Fig. I.3)

\[
\rho = R_0 \left[ (\Delta \lambda \cos \beta)^2 + (\Delta \beta)^2 \right]^{1/2}
\]

where \( R_0 = \) radius of sun in kilometres

\[
\tan \phi = \frac{\Delta \beta}{\Delta \lambda \cos \beta}
\]
Where $\Delta B$ and $\Delta \lambda$ are the differential heliographic co-
ordinates measured from the centre of the umbra (heli-
graphic co-ordinates $B$, $L$).

I. (2) Recent sunspot velocity field determinations.

The main investigations of a complete sunspot velocity
field have been performed by Kinman at Oxford in 1952 and
1953 (8) (9), Servajean at Pic du Midi in 1960 (16) and Bumba
at the Crimea during the years 1957-1960 (6). The details of
their work are summarised in Table I.1. Kinman's research
was initially devoted to the analysis of the velocity field
of a regular spot of umbral diameter 20". Fig 1.4 shows
the spot sections covered by the 15 spectra he measured, and
his selected points of measurement. His 15 measured lines
were of Fe I and Ti I in the $\lambda$ 5900 region (Rowland Nos. 0-10),
and 22

\begin{center}
\includegraphics[width=0.8\textwidth]{fig1.png}
\end{center}

\textbf{Fig} I.4 Kinman's measurement positions.
<table>
<thead>
<tr>
<th>Author</th>
<th>Observatory</th>
<th>Solar Image Diameter</th>
<th>Spectroscopic Dispersion $\AA$/mm</th>
<th>Spectral Region</th>
<th>Exposure Time</th>
<th>Method of Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1952</td>
<td>Kinman</td>
<td>Oxford 18 cm</td>
<td>1.5</td>
<td>$\lambda 5900$</td>
<td>5–6 secs.</td>
<td>Visually</td>
</tr>
<tr>
<td>1953</td>
<td></td>
<td></td>
<td>1.5</td>
<td>(15 lines)</td>
<td>2.5–5 secs.</td>
<td>Positive–on Negative</td>
</tr>
<tr>
<td>1960</td>
<td>Servajean</td>
<td>Pic du Midi 20 cm</td>
<td>1.0</td>
<td>$\lambda 5691.508$</td>
<td>1–12 secs</td>
<td>Visually</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\lambda 5040$–$5404$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1957–60</td>
<td>Bumba</td>
<td>Crimea 30 cm</td>
<td>0.29</td>
<td>$\lambda 5900$</td>
<td>1–2 secs.</td>
<td>Photoelectric Spectro-comparator</td>
</tr>
</tbody>
</table>
Neighbouring atmospheric lines (Rowland Nos. 1-7) were used as standards to remove any instrumental defects. Following the scheme outlined at the beginning of this chapter, from least squares solutions for 237 points, he showed that the penumbral motion has cylindrical symmetry. It was found to consist of an outward horizontal radial velocity which increases from approximately 1 km/sec at the edge of the umbra to 2 km/sec in the middle zone of the penumbra, finally falling to zero in the photosphere well outside the spot, the form of this motion being illustrated in Fig. 1.5. He concluded that the tangential and vertical velocity components were less than the errors of measurement. The following year, his work was extended to investigations of velocity fields in four spots whose umbral diameters were 4", 6", 8" and 20". Graphically derived corrections for scattered light (see Chapter III) were applied to the velocities obtained. The motion in the large spot confirmed the previous result of a purely radial horizontal flow which was of the form illustrated in Fig.1.5. In addition, from a comparison of the velocity fields obtained from the four measured spots, Kinman showed that the maximum radial velocity increased linearly with the umbral radius. However, the range in his spot size does not seem adequate to truely justify any such prediction, and moreover, spots as small as 4" will be very considerably affected by bad seeing. There is also the
FIG 1.5 Kinman's velocity components \( u \) and \( w \) as a function of distance from spot centre.
effect of sunspot age to be considered. Small spots are probably in process of growing while large spots may be on the point of disintegration, so it would be unwise to correlate velocity and size without considering age. But a more important factor lies in the selection of the measured solar lines. It has already been emphasized in the introduction that the connection between material flow and magnetic fields is one of considerable importance and consequently, to ensure that the measured Doppler shifts are completely free from the complicating factor of magnetic field, it is wise to select lines for measurement whose Zeeman splitting factor is zero. Having measured lines with \( g \neq 0 \), Kinman's comparison of velocities in spots of different sizes is a dangerous one, for the larger spot will have a larger associated magnetic field, producing a larger Zeeman splitting, which would appear as a "false" Doppler shift. However, his careful and detailed measures and reductions leave little doubt about the general form of the sunspot velocity field.

Servajean's investigation of sunspot kinematics (16) was carried out with the 9 metre solar spectrograph at Meudon and Pic du Midi, and his first consideration was the possible variation of Evershed effect with depth. This investigation may be attempted in two ways. The possible variation in Evershed velocity may be obtained either from measurements of one sunspot spectral line, during the passage of the spot from centre to limb, or from measurements of a number of lines
formed at different depths in the same spot, at any one disk position. Servajean attempted the latter method. He selected a regular spot for the investigation, and from 2nd order sunspot spectra obtained, the maximum wavelength change $\Delta \lambda$ across the spot spectrum was determined for 124 lines of Fe I and Ti I, of Rowland intensity 1 to 7 in the region $\lambda \lambda 5040-5404$. $\Delta \lambda$ was measured by considering two sets of 6 points, one on either side of the penumbra, 3 of which were in the centre of the penumbra and 3 of which were situated at the extremity of the visible velocity field, as indicated in Fig 1.6. The measures $A$ and $B$ of points $a_n$ and $b_n$ relative to $x_n$ resulted in determinations of the quantity $E = A - B$, which is proportional to the so-called 'Evershed velocity vector' $V_E$ (Viz. the resultant of $u$, $v$ and $w$). In this way, Servajean tabulated 124 selected solar lines and their corresponding $V_E$'s. The optical depth $\tau$ at which each line was formed, was then determined using the theory of Minnaert (12), Claas (7) and Pocker (13), which expresses equivalent width $W$ for weak absorption lines in terms of (among other calculable quantities) optical depth $\tau$. The equivalent width values (see Chapter IV) were taken from Bell (3) and Allen (2). $\tau$ was in turn related to geometric depth $h$ in the penumbra by means of a modified umbral sunspot model developed by Michard (11), so that Servajean was finally able to relate geometric depth in the penumbra with the measured Evershed velocity. In this way, he was able to show that the Evershed velocity increased with increasing depth in the spot, a change
FIG. 1-6 Servajean's measurement positions in his investigation of the variation of Evershed effect with depth.
of about 1.5 km/sec being observed over a geometric depth of about 350 km.

This result is in immediate quantitative doubt from a comparison of Michard's (11) and Mattig's (10) sunspot models, between which there is basic disagreement as is obvious from Fig.I.7. Both Michard and Mattig determined variations of temperature, electron and gas pressure with optical depth, and hence found a variation of these quantities with geometric depth in the sunspot. Mattig's determination was theoretical and Michard's was empirical. The basic disagreements in the two models obtained are firstly on the question of sunspot gas pressure, where Mattig predicts a pressure in the sunspot continually greater than in the photosphere, while Michard obtains a smaller value in the sunspot. Secondly, Mattig predicts sunspots as regions of where the opacity coefficient is greater than the photosphere i.e. the sunspots are less transparent than the photosphere, while Michard maintains the opposite. Michard's model is doubtful because not only did he use in his determination the observational material of ten Bruggen\textregistered and von Klüber (4), which applied essentially to spots of different size and temperature from those of Michard, but he assumed a pure hydrogen atmosphere, taking the abundance ratio of Hydrogen:Metals as $10^4$. Wiedemann's work (18) indicates that this is too small by a factor 2. More recently, Van't Veer(17) developed a sunspot model which
FIG 1-7 Comparison of Mattig's and Michard's sunspot models, illustrating variation of optical depth $\tau$ with geometric depth $h$. 
qualitatively agrees with Michard, although his investigation was based on only one large and one small sunspot. Until the question of a sunspot model is settled, there seems little point in pursuing any such detailed geometrical depth analysis.

As well as the work on depth dependence of velocity, Servajean's second consideration was the complete investigation of a sunspot velocity field, for which he obtained 6 series each of 50 spectra of the same circular sunspot over a period of 8 days, as it proceeded from the East to the West limb. In this case, the line selected for measurement was the Fe Ni \( \lambda 5691.508 \), chosen because of its small Landé splitting factor, and its proximity to atmospheric lines. 13360 measures of sight-line velocities were made from these spectra, and reduced by least squares solutions to \( u, v, w \) components using the methods described at the beginning of the chapter. The results obtained showed that the tangential component \( v \) was insignificant, the vertical component \( w \) was small (\( \sim 0.25 \, \text{km/sec} \)) but consistently negative, (indicating a downwards flow) and the radial velocity \( u \) was the predominant component, the form of which was in agreement with that obtained by Kinman. A new feature in the radial velocity results, arising from following the spot across the disk, was the variation of the maximum value of \( u \) with \( \sin \theta \). \( \dot{U}_{\text{max}} \) appeared to decrease from centre to limb as indicated in Fig. 1.8. Servajean considered this a confirmation of his
FIG. 1.8 Servajean's maximum radial velocity as a function of $\sin \theta$. 
hypothesis of a velocity variation with depth in the spot. Near the centre of the disk, the line-of-sight penetrates to the lower spot regions of higher velocity. However, no mention was made of the correction of measured velocities due to scattered light. Such a correction obviously increases as the spot approaches the limb, when the foreshortening becomes more pronounced (see Chapter III).

The suggested increase in radial velocity with depth was also used to explain the observed diffuse widening of the spectral lines, in the direction of the measured velocities, at the outer limits of the penumbra. The widening was dependent on disk position, being pronounced for spots near the limb and weak for spots near the disk centre. The explanation for such a phenomenon suggested by Servajean is that the profile of the line is the resultant of summations of Gaussian elements of different widths, produced at different levels in the spot. The deeper and hotter levels give rise to the broadest profiles, and are thus chiefly responsible for the observed diffuse widening of the lines. If the velocity variation with depth is not regarded as established, such an explanation must be considered invalid. This point will be considered in the thesis conclusion.

Bumba's sunspot investigations were aimed at connecting magnetic fields (5) and motion in a spot. We shall consider
here only his measurement of the Evershed effect and the study of velocity variation with spot position on the disk. The magnetic field results were referred to in the Introduction.

For each spot, spectra were taken of at least five spot sections, with the slit parallel and perpendicular to the solar radius. The main observational details are given in Table I.1. All the velocity measures were made with a photoelectric spectrocomparator, and the 7 measured lines in the λ 5900 region were selected because of their small magnetic splitting and their nearness to atmospheric lines which were used as standards. However, Bumba did not correct his measured velocities for the effects of solar rotation, observer's motion and limb effect, and then proceed to reduce his corrected sight-line velocities to the u, v and w components, in the manner of Hinman and Servajean. He maintained that by averaging statistically a large number of measures of the observed motions over the entire region of the sunspot, an "average line shift" was obtained, which could be taken as a reference "level of zero velocity". The velocities obtained with respect to this reference level he transformed into the radial velocity component, by knowing the sunspot disk position. Apart from the fact that this method saves a little time and computation, it has no advantages and must be less accurate. The results given from slit sections
obtained by previous workers. No indication of the results obtained from sections perpendicular to the solar radius is given. Bumba also attempted to investigate the variation in maximum radial velocity with disk position, from a consideration of spots whose disk position was in the range $14^\circ < \theta < 74^\circ$. These results are shown in Figs. I.9 and 10, which illustrate the variation in spot maximum radial velocity with disk position, on opposite sides of the spot. Fig. I.9 shows the variation in $V^+$, which is the spot velocity directed towards the observer, while Fig. I.10 shows the variation in $V^-$, the spot velocity directed away from the observer. (The definition of $V^+$ and $V^-$ as given here is dubious. $V^+$ and $V^-$ are defined in opposite terms in Bumba's abstract and discussion of results.) The relationship between the maximum radial velocity and disk position was found by Bumba empirically, and gives approximately:

\[
V^+ = 2v\sin\theta \quad \text{I.4}
\]

\[
V^- = v(\sin\theta + \cos\theta) \quad \text{I.5}
\]

where $V = 1 \text{ km/sec}$.

This analysis was based on the measurements of different solar lines for 22 spots, presumably with a fair range in spot size, (spot sizes were not given) and consequently Equations I.4 and I.5 represent an "average spot" velocity variation. Such a generalization could not be expected to compare favourably with Servajean's analysis of one spot across the disk.
FIG. I.9 Bumba's variation of maximum radial velocity $V^+$ with $\theta$.

FIG. I.10 Bumba's variation of maximum radial velocity $V^-$ with $\theta$. 
with disk position, as an effect of velocity variation with depth in the spot. Bumba's results do not give any justification for this hypothesis. However, Bumba, unlike Servajean, does mention the effect of scattered light on velocity measures, quoting Kinman's maximum error of 10% as an upper limit.

In addition to the velocity investigations, Bumba attempted a photometric analysis of his measured lines to examine the penumbral diffuse widening mentioned earlier. This feature of line asymmetry was referred to by Bumba as "flag". Bumba, like Servajean, observed that the line asymmetry increased as the spot distance from the disk centre increased. An increase in asymmetry was also noted for a decrease in line intensity, which Bumba suggests is an indication of velocity variation with depth. In the faintest lines, the asymmetry was observed as almost a separately defined line, and was then given the name "satellite". The distance of satellites from the undisturbed line represented a velocity of about 5 km/sec.

Bumba further investigated the influence of the asymmetry on the line core. In the weak lines (Rowland intensity ~ 2) practically the whole line was displaced, whereas in medium and strong absorption lines (Rowland intensity ~ 6) the asymmetry appeared to have only a slight influence on the line core.
There seems little doubt from these investigations that the results from Evershed measures at the photospheric level indicate a material flow radially along the spot surface, away from the spot axis. A maximum radial velocity of the order of 2 Km/sec is observed near the outer edge of the penumbra. Whether or not this maximum is a function of disc position is still uncertain. There is some indication of a small, vertically downward component of velocity and some rather doubtful evidence about a velocity variation with depth in a spot. The existence of the high dispersion feature of penumbral line-asymmetry is well established, although its effect on velocity measures and its physical interpretation have still to be considered. The remainder of the thesis is concerned with a detailed investigation of the sunspot velocity field in an attempt to solve some of these remaining problems. Chapter II is concerned with sunspot kinematics from visual measures and Chapter IV describes the detailed photometric analysis of the line asymmetry, referred to as "line-flare". Chapter III is devoted to the problem which complicates all sunspot investigations, namely the effect of scattered light.
References for Chapter I.

(4) P. ten Bruggencate and H.von Kluber, Zs.f. Ap.18, 284, 1938
CHAPTER II

Redetermination of sunspot velocity fields using a magnetically undisturbed line.
Chapter II

Redetermination of sunspot velocity fields using a magnetically undisturbed line.

II. (1). The choice of solar line for investigation.

In all investigations of the Evershed effect, it is important to distinguish between line shifts due to actual velocities along the line of sight, and shifts which have their origin in the line splitting produced by the magnetic field of the sunspot. The Introduction and Chapter I have already shown that sunspot velocity fields and magnetic fields are both observed to change in a more or less regular fashion across the penumbra, and the relation between magnetic lines of force and material motion is a matter of considerable importance in sunspot theory. Plate II.1. shows the splitting of the Fe line $\lambda 6302$ across a sunspot. This clearly shows that the partial suppression of one or other of the two $\sigma$ components of the pattern due to instrumental polarization can easily lead to an apparent line displacement. A line such as $\lambda 6302$ is an extreme case and would not be used for velocity measures, but such effects exist on a smaller scale for all lines, except those for which the Zeeman splitting factor is zero. It is only by using such a magnetically unaffected line that we can be sure that the velocities we measure are free from magnetic field effects.
PLATE I: Zeeman splitting in Fe $\lambda$6302.

PLATE II: Line flare in a large and small sunspot.
For this reason we have selected the Fe line $\lambda 5576.101$ for our present study of sunspot velocity fields. This is one of the lines first suggested by von Klüber (7) for which the Landé splitting factor $g = 0$. But we now have to contend with the observational difficulties presented by such a choice. There are no terrestrial standards in this region, such as the oxygen lines which appear in PLATE II.1, or the water vapour lines used by Kinman (5) (6), but the difficulty has been overcome here by introducing absorption lines due to iodine vapour into the solar spectrum. These could not be introduced directly into the sunspot spectrum because the iodine line density is too high but, instead, two exposures were made in rapid succession; the first for the spot spectrum alone and the second for the spot spectrum plus iodine. The iodine lines could then be used to show slit curvature and possible slit inclination, and consequently these effects could be allowed for in the solar exposure. We then assume that at the top and bottom of the slit, far removed from the spot, the line shows only the normal photospheric velocity shift due to solar rotation, plus a possible limb effect. (We take the effect of the photospheric velocity fields to be uniform across the slit.) After calculating and allowing for these known effects at selected points across the slit, we have the true Evershed effect.

Observations for this thesis were made on spots in two consecutive years. The 1959 work was based on one large
and one small spot, and the 1960 work was planned to follow one and the same spot across the disk. The details of observations, measurements and results for both of these years will be separately described in successive sections of this chapter.

II. (2). 1959 observations and measurements.

The sunspot spectra for the investigation were obtained by Professor Plaskett in September 1959. The image radius \( r \) as given by the new solar telescope (12) was 165mm, and the slit height of 25mm corresponds to approximately 145 seconds of arc or 105,500 km on the solar surface at the disk centre. The Babcock grating was used in the 5th order with the 40 ft. spectrograph, resulting in a linear dispersion of 0.17 \( \text{Å/mm} \). Since this telescope produces a rotating image, the direction of the slit on the solar surface constantly changes. In each exposure, the slit passed through the spot centre. Details of the measured plates are given in TABLE II.1. In this table, \( r \) = radial distance of the spot from the disk centre and B, L are the spot heliographic latitude and longitude.

Measurements were made using the Hilger micrometer with a transverse motion for the plate stage (11). The field of view, perpendicular to the dispersion, was restricted to 0.3mm by a diaphragm in the microscope eyepiece, so that this width of spectrum can be measured at any desired height. Since the sunspot spectra contain no atmospheric lines
TABLE II. 1. 1959 Measured sunspot plates.

Measured line $\lambda 5576.101$ $25^\circ P^0 - e5^D$ FeI.
Rowland no. of $\lambda 5576.101$ in spot = 4.
Slit height = 25mm.
Slit width = 0.045mm.
Dispersion = 0.171/\(\text{mm}\).

Ilford R40 plates

<table>
<thead>
<tr>
<th>PLATE</th>
<th>DATE 1959</th>
<th>u.T.</th>
<th>EXPOSURE TIME</th>
<th>P/(P_0)</th>
<th>B</th>
<th>L</th>
<th>UMBRAL DIAMETER</th>
</tr>
</thead>
<tbody>
<tr>
<td>E 22</td>
<td>Sept.7</td>
<td>16$^h$ 02$^m$ 16$s$</td>
<td>40 sec.</td>
<td>0.793</td>
<td>$+17^\circ 02'$</td>
<td>$15^\circ 54'$</td>
<td>46&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16$^h$ 12$^m$ 35$s$</td>
<td>60 sec.</td>
<td></td>
<td></td>
<td></td>
<td>34000 Km.</td>
</tr>
<tr>
<td>E 28</td>
<td>Sept.9</td>
<td>08$^h$ 41$^m$ 57$s$</td>
<td>30 sec.</td>
<td>0.597</td>
<td>-18$^\circ 10'$</td>
<td>274$^\circ 50'$</td>
<td>17&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>08$^h$ 49$^m$ 38$s$</td>
<td>31 sec.</td>
<td></td>
<td></td>
<td></td>
<td>12000 Km.</td>
</tr>
<tr>
<td>E 29</td>
<td>Sept.10</td>
<td>08$^h$ 28$^m$ 16$s$</td>
<td>30 sec.</td>
<td>0.471</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>09$^h$ 17$^m$ 06$s$</td>
<td>30 sec.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
or lines of zero velocity, a ruled glass plate was used to provide a fiduciary line. This was permanently attached to the microscope stage and check settings were made on the ruled line throughout all the micrometer measurements of sunspot and iodine lines. Provided the sunspot spectra and iodine spectra are closely aligned at the same angle with respect to the fiduciary line, the iodine line readings give a correction curve as a function of slit height, by means of which all instrumental shifts may be removed from the observed displacements of the solar line. The six iodine correction curves were identical within the errors of measurement and were combined to give a single curve by means of a least squares solution.

All spectra were measured in the direct and reversed directions to the nearest micron (= 0.01 mm. Sec.-1) and mean readings obtained. The sunspot spectra were measured at over 100 points over the 25mm. slit height at intervals as small as 0.1mm. in the penumbral region, and wider intervals in the outer parts, where we suspect the Evershed velocities are changing less rapidly. A complete measurement of the plates was made in February and the whole set of readings repeated in April as a check, final readings being taken as the mean of these two sets. Although the high dispersion is a great advantage in this work, the solar lines are now so broad as to make visual measurements a difficult matter,
especially over the restricted height our diaphragm imposes. A.D. Petford has devised a new photo-electric method of measurement (8) which could be used here to great advantage. In fact, the visual and photo-electric measurements made on plate E 28 of our TABLE II.1. are in very good agreement, as his Fig. 2 shows. This figure is reproduced in Fig. II.1. The photoelectric measurements however show considerably less scatter in repeated settings and are indisputably quicker and easier to obtain. The comparison of photoelectric and visual measurements on one plate indicated a velocity difference of about 0.15 Km/sec. in the penumbra (the most difficult place to make a setting) and the discrepancy was considerably less in the region away from the penumbra. We can thus consider a line setting error of 0.15 Km/sec. throughout as a generous estimate. The error in setting visually on the solar lines after taking direct and reversed readings and the mean of two sets of readings, is approximately 10 microns, which is equivalent to a velocity error of 0.1 Km/sec.

The solar line displacements corrected for instrumental effects, as explained above, were then expressed as Doppler shifts, (1 mm displacement = 9.19 Km/sec.) and the results are shown in Fig. II.2. Fig. II.2 (a) is the mean of the direct and reversed readings for the February and April measurements, and gives the observed curve of the small spot of Plate E 28, while Fig. II.2 (b) refers to a similar mean
FIG II:1 Comparison of visual and photoelectric settings on \( \lambda 5576 \) in the region of a sunspot.
FIG. II·2a. Observed velocities and correction line for small spot of plate E28.
FIG. II2b. Observed velocities and correction line for large spot of plate E22.
of the large spot of E 22.

The ordinates in Fig.II.2. are arbitrarily set equal to zero for the photosphere on the solar centre side of the spot and expressed in \( \text{km sec}^{-1} \), while the abscissa is given by the spectrum height in mm. The approximate extent of the umbra and penumbra is indicated for both spots. The positions of the umbra and penumbra were estimated from the spectra using the Hilger microscope, and the spot sizes which could thus be calculated were checked with estimations obtained from the Greenwich photographs.

II (3). 1959 Reductions and Results.

In the reduction of the observed velocities, we have followed the scheme proposed by Professor Pleskett, which was described in Chapter I. The first step in deducing the motion characteristic of the spot is to remove the effect of the observer's motion and to allow for the displacements which we know will arise from solar rotation and limb effect. The Mercury Computer of the Oxford University Computing Laboratory was used to calculate the corrections at representative points up the slit height, using a programme written by L.A. Higgs (4) and interpolations were made for the remaining measured points. The limb effect corrections (1) were found to be negligible for the disk positions considered. We have then the velocity shifts which would occur across the slit height in the undisturbed photosphere. The lines in
Figs. II.2 (a) and (b) show the run of these velocity corrections across the slit and are positioned for the small and the large spots to give zero photospheric velocity on either side of the spot. It is the shifts above and below these lines which we believe represent the true sight-line velocities \( V \) in the spot.

Ideally, we should have liked to make a complete least squares solution for the three unknowns \( u, v, w \), (see Equation I.3) but our observational material was insufficient to provide a satisfactory set of observational equations. We therefore based our calculations on Kinman's conclusions, which were obtained from 12 least squares solutions for \( u, v, w \) at four values of \( \tau \) for each of three plates. These showed that there is no evidence for a tangential component \( v \) and that the vertical component \( w \), relative to the photosphere, is zero.

As a first description of the velocity field, we have then assumed that the motion is entirely a radial flow outward from the centre of the umbra \( (v = 0, w = 0) \). Fig II.3 (a) shows the results of the calculation for the small spot of E 28, and Fig. II.3 (b) is a similar plot for the large spot of E 22. Positive velocities are now shown for both sides of the spot, since both refer to outflow velocities. For the small spot, where we have four sections of the same spot, we may go further and attempt a solution for another component of motion. Assuming that the spot has radial symmetry about
FIG II:3a. Radial velocity for small spot of plate E28, assuming \( v = w = 0 \).
FIG II.3b. Radial velocity for large spot of plate E22

assuming \( v = w = 0 \).
the centre of the umbra, the two sides of the spot on four plates lead to eight observational equation for the state of motion at any one \( r \) value. Still assuming \( \psi = 0 \) (5) (13) we have used these equations to obtain a least squares solution for finding components \( u \) and \( w \). The solutions were carried out for eight \( r \) values and lead to the results shown in Fig. II.4.

We find as Kinman did, that \( w \) is zero within the limits of error. We see also that the values of the radial velocity thus obtained for the four plates are quite similar to those shown in Fig. II.3 (a) for plate \# 28 only.

II (4) Discussion of 1959 results.

The general form of motion in the small spot is very like that obtained by Kinman, and within the limits of our observational material, it again appears that the flow is wholly radial. The maximum radial velocity (mean uncorrected value of 1.1 Km/sec.) is achieved towards the edge of the penumbra, and we see that the flow extends well beyond the penumbral region, falling to zero at approximately 30,000 km. from the spot centre. Considering Kinman's linear relation (6) between maximum radial velocity and spot size, we see that there is fair agreement for the small spot, but our maximum radial velocity for the large spot (mean uncorrected value approx. 0.6 Km/sec.) falls well below the value (approx. 3.5 Km/sec.) which may be estimated from the linear relation. In addition, the velocity field for the large spot
FIG. 3.4 Least squares solution for velocity field of small spot.
is of quite a different form from that of the small spot. The large spot pattern is much more spread out, falling to zero at approximately 70,000 Kms from the spot centre, and it does not show such well defined maxima in the penumbra.

The high dispersion spectra (0.17 Å/mm) we are now using clearly reveal the phenomenon of the penumbral diffuse widening which was referred to in Chapter I. This feature, which appears in PLATE II.2, for the large and the small spot, will be referred to as "line-flare". The appearance of the plate indeed suggests that the main line is undisturbed and that the measured shift is caused by this diffuse "flare". The line-flare observed extends over some 120 m Å from the centre of the undisturbed line, corresponding to a sight-line velocity up to 6 Km./sec. In low dispersion work, such as Kinman's, the line-flare and the undisturbed line would be blended, and consequently the amalgamated flare would contribute in a different way to the measured Doppler shift. This phenomenon could well account for the differences observed in velocity displacements in high and low dispersion investigations. In measuring the high dispersion plates, it is difficult to say exactly to what part of the line the micrometer setting refers, but presumably (Plate II.2) it is closer to the undisturbed line than would be possible with lower dispersion.

The results obtained from these 1959 observations
suggested some further investigations that were needed.
The 1959 plates were not calibrated for photometry and so it was planned to obtain high dispersion calibrated plates for a full-scale study of the intensity and extent of line-flare, and to investigate more fully the part it plays in our measurement of sunspot velocity fields. (Chapter IV) Plates were also obtained with the aim of investigating umbral velocities and the variation of sunspot velocity components with disk position. This work is described in the remaining sections of the chapter.

II (5). 1960 observations and measurements.

The observations were made in May and June of 1960 with the 65 foot solar telescope (9) during the passage of one small spot across the disk. The image radius \( \rho \) was 90 mm. and the slit height of 25 mm. corresponds to approximately 260 seconds of arc or 193,200 Kms. on the solar surface at the disk centre. The Babcock grating was used in the 5th order and the resultant linear dispersion was 0.20 \( \AA/mm \).

The following spectra were taken for each spot position considered:

(1) An iodine spectrum, in order to remove any instrumental defects of slit curvature and inclination from the measured spot spectrum. (See II (1))

(2) A penumbral spectrum, taken with the spectroscope slit passing through the spot centre.
(3) Wedge and disk centre spectra, in order to obtain calibrations for the penumbral line-flare plates.

(4) An umbral spectrum, again taken with the spectroscope slit passing through the spot centre, but for which we needed to make special arrangements. (On the 40 sec. spot spectra previously investigated, the umbra is underexposed.) This involved the use of a moveable metal vee (Fig. II.5), which could be raised and lowered by a flexible cable to uncover or cover the spectroscope slit. The transverse position of the vee was adjusted before exposures according to spot size, so that it just exposed the umbra. The procedure for taking an umbral exposure was then as follows. The exposure was begun with the vee in position, but raised to expose the whole spot and photospheric slit section. Then throughout the exposure period, the vee was alternately lowered (thus exposing only the spot umbra) and raised for measured periods, to provide simultaneously an acceptable umbral and photospheric exposure. This amounted to an exposure time of about 40 secs. for the photosphere and 2 minutes for the umbra. The spot was guided throughout the umbral and penumbral exposures by means of a Guiding plate (10) on the optical bench in front of the spectroscope slit. The details of the plates selected for measurement, taken in following one spot across the disk, are given in TABLE II.2. Fig. II.6 shows the position of the spot and spectroscope slit on these days. The spot heliographic latitudes and
FIG. II.5 Adjustable vee for sunspot umbraal exposures.

FIG. II.6 1960 Sunspot positions and slit sections.
### TABLE II. 2. 1960 Measured sunspot plates

Measured line $\lambda$ 5576.101 FeI.
Slit height = 25 mm.
Slit width = 0.045 mm.
Dispersion = 0.20 $\lambda$/mm.

**Ilford R40 plates**

<table>
<thead>
<tr>
<th>PLATE</th>
<th>DATE 1960</th>
<th>EXPOSURE TIME (spot)</th>
<th>EXPOSURE TIME (umbra)</th>
<th>$R/R_0$</th>
<th>$B$</th>
<th>$L$</th>
<th>UMBRAL DIAMETER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ho 40</td>
<td>May 30</td>
<td>40 secs.</td>
<td>2 min.</td>
<td>0.74</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ho 52</td>
<td>May 31</td>
<td>40 secs.</td>
<td>3 min.</td>
<td>0.55</td>
<td>-8°.5</td>
<td>+12°.2</td>
<td>17° = 12,200 km</td>
</tr>
<tr>
<td>Ho 58</td>
<td>June 2</td>
<td>60 secs.</td>
<td>2½ min.</td>
<td>0.17</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ho 62</td>
<td>June 4</td>
<td>40 secs.</td>
<td>3 min.</td>
<td>0.36</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
longitudes were determined from the \((\varphi, \phi)\) coordinates (10) measured for each spot position by means of a polar coordinate disk, which is also mounted on the optical bench in front of the spectroscopic slit.

The details of measurement and corrections for instrumental effects were given in II (2), although no photoelectric measurement checks were made on the 1960 plates. However, a check was provided by comparing the two sets of measures obtained from the umbral and complete spot exposures. (When the umbral measures were made, the regions on either side of the vee edge were omitted to avoid any distortions due to diffraction.) By overlapping these measures, when they had been corrected for slit inclination and curvature, we obtained a mean sight-line velocity curve across the complete spot section and surrounding photosphere. Figs. II 7 (a) and (b) show the sight-line velocities obtained when the spot was in the positions nearest the limb and the disk centre. The ordinates are expressed in \(\text{Km} \cdot \text{sec}^{-1}\) (1 mm displacement = 11.13 \(\text{Km/sec.}\)) and the abscissa is given by the spectrum height in mm. The sight-line velocities indicated in Fig. II.7 are true sight-line velocities, i.e. they have been corrected for the effects of observer's motion, solar rotation and limb effect by the procedure described in II (3). The approximate extent of the umbra and penumbra are again indicated and also a predominant bright ring, which was observed at the outer edge.
FIG. II·7 a. True sight-line velocities for Plate HO 40, taken when spot is nearest limb.
FIG II·7b. True sight-line velocities for Plate HO58, taken when spot is nearest disk centre.
of the penumbra for each spot position. (This latter feature was not obvious on the Greenwich photographs.)

II (6). 1960 Reductions and Results.

Having thus obtained the true sight-line velocities for the 4 spot positions (two of which are shown in Fig. II.7), we could attempt solutions for the velocity field. Two solutions were made for the velocity components \( u, v \) and \( w \)
(a) in the sunspot umbra and (b) in the penumbra and surrounding photosphere. If we assume radial symmetry about the sunspot centre, our 4 measured plates give 8 observational equations at each distance \( r \) from the spot centre. This is insufficient for a satisfactory solution for 3 unknowns, so we have therefore assumed (a) in the umbra, that the tangential component \( v \) is zero and (b) in the penumbra, that both \( v \) and \( w \) (the vertical component) are zero. The assumption of \( w = 0 \) follows from the results of both Kinman and Servajean (see Chapter I (2)) and the earlier work in this present investigation.

(a) The solution for \( u \) and \( w \) in the umbra.

Our 8 observational equations with the two unknowns \( u \) and \( w \), were solved by the method of least squares. (A short programme for a least squares solution was written for the Mercury computer, to facilitate the computations.) The results and mean errors from the least squares solution for the \( u \) and \( w \) components are illustrated in Fig. II.8. It is seen that the component \( u \) increases with distance from the
**FIG II.8** Solution for velocity components $u$ and $w$ in umbra.
centre of the umbra and indicates an out-flow from the spot centre. The component \( w \) has the approximately constant value of about 0.1 km/sec across the umbra, and indicates a motion of descent.

(b) The solution for \( u \) in the penumbra.

Although a solution was attempted for the penumbra on both sides of the spot, we have only considered the results from the "following" side of the spot, as the "preceeding" spot side was observed on the Greenwich sunspot photographs to have an irregular penumbra. This irregularity was revealed as a double-peak in the sight-line velocity (Fig. II.7) and it seemed wiser and simpler to give it no further consideration. The results of this solution for the spot nearest the limb (Ho 40 plate) and the spot nearest the disk centre (Ho 58 plate) are shown in Fig. II.9. The ordinate represents the radial velocity component in \( \text{km sec}^{-1} \) and the abscissa the distance from the spot centre. The results for the other two plates lie between those illustrated in Fig. II.9 but have been omitted from the diagram for the sake of simplicity. The most obvious features of this solution are

1. an apparent decrease in the maximum value of the radial velocity from centre to limb. When the spot is at \( \sin \theta = 0.172 \), \( u_{\text{max}} = 3.5 \text{ km sec}^{-1} \), and this value has fallen to

\[ 0.9 \text{ km/sec} \] when the spot is at \( \sin \theta = 0.737 \). If we compare the true sight-line velocities for these spot positions (Fig. II.7), it is seen that the maximum penumbral
FIG II.9 Variation of penumbral radial velocity $u$ (smoothed), with distance from spot centre for two disk positions.
velocity is approximately the same (0.6 Km/sec) for both spots. Therefore, this large decrease in $u_{max}$ only arises from geometrical considerations. Since the eight-line velocity $V$, the same values of $V$ give rise to the larger radial-velocity components $u$ for the smaller values. (ii) the radial velocity pattern spreads out as the spot position moves from centre to limb. For the spot at the centre, the radial velocity reaches a maximum at a point near the edge of the penumbra, about 10,000 Kms. from the spot centre, and has fallen to zero at a distance of 50,000 Kms. from the spot centre, whereas for the spot nearest the limb, the maximum radial velocity is attained outside the penumbra in the bright ring region, at a distance of about 25,000 Kms. from the spot centre, and has not fallen to zero until a distance of 70,000 Kms. from the spot centre.

II (7). Discussion of Velocity Field results.
(a) Umbral Velocities.

Evershed had earlier attempted to detect a vertical motion in the umbra (3) by measuring spot spectra taken near the disk centre. He claims to have detected a motion of descent of the order of 0.4 Km/sec$^{-1}$. The only mention of umbral velocities to be found in recent Evershed investigations is in Bumba's work (2). (He gives no indication of measurement difficulties or special procedures for umbral spectra with exposure times of 1-2 secs, but presumably the photoelectric spectro comparator is not faced with the
problems of the human eye.) Bumba did not attempt to solve for the velocity components \( u, v \) and \( w \) (see Chapter I (2)), but the sight-line velocity he measured was translated immediately into a radial velocity component \( u \). He thus claims to have observed a radial motion of the order of 0.2 to 0.25 \( \text{Km/sec} \) "away from the observer". Servajean and Kinman (Chapter I (2)) found a descending component of \( w \) in the penumbra, of the order of 0.2 \( \text{Km/sec} \), but were both of the opinion that their results were within the limits of experimental error.

Although our velocity components in the umbra have been obtained with the assumptions of the preceding section, Fig. II.8 gives a consistent and fairly convincing demonstration of the reality of the vertical component \( w \), coupled with the gradually developing radial component \( u \). It is difficult to imagine a photospheric velocity field of such a nature as to produce in 4 different disk positions such a system of velocities. It could be argued that the hypothesis of a variation of velocity components with disk position would invalidate the results obtained from the least squares solution. This problem will be dealt with in the next chapter.

(b) Penumbral Velocities.

The variation in the pattern of the radial velocity component from centre to limb has not been remarked upon before. Only Servajean (Chapter I (2)) has previously
followed a spot across the disk to investigate the variation in velocity field with disk position, and the resultant curve he obtained for the radial velocity variation with distance from spot centre seemed very much the same for his six spot positions, except for the previously noted decrease in the maximum value of the radial velocity $v$. The change observed in the present work may have escaped his notice, due to the fact that he does not appear to have worked out his distances from the spot centre accurately. In his penumbral regions, these distances are expressed as fractions of the penumbral size, and he does not indicate how they were obtained. In the present investigation, the distances from the spot centre were calculated precisely for 8 points across the spot (using the results if Chapter 1) and the intermediate distances obtained by interpolation.

However, we find we are in agreement with Servajean's result that $u_{\text{max}}$ decreases from centre to limb. A comparison of our two sets of results giving the variation of $u_{\text{max}}$ with disk position is given in Fig. II.10. The theoretical implications of this result are considered in the Conclusion.
FIG II.10. A comparison of Servajean's least squares solution for $U_{\text{max}}$ at varying disk position, with the present investigation.
References Chapter II.


(3) J. Evershed, M.N. 70, 217, 1910.


CHAPTER III

The effect of scattered light on sunspot velocity field measures.
Chapter III

The effect of scattered light on sunspot velocity field measures.

Introduction.

A perfect optical telescope, in the absence of atmospheric turbulence, throws the light from a point source into a central diffraction disk (into which most of the light falls) surrounded by diffraction rings. However, the theoretical diffraction pattern can only be observed with telescopes of small aperture and under good conditions of seeing. For perfect seeing, with a 12" telescope, the diameter of the diffraction disk is of the order of 0.75, but in practice with a reflecting telescope the image is usually scattered into a circular "tremor" disk with extended wings, the disk varying from 1" in diameter under very good seeing conditions, up to 5" for poor conditions. It is this effect of seeing, produced by light scattering (in the atmosphere and instrumentally) which ultimately limits the definition attainable in solar photography. All observations of the physical properties of sunspots are beset by this problem of scattered light. This is not surprising when we consider that average sized sunspots with umbral diameters of 5" - 20" near the centre of the solar disk, appear comparable in size with the
telescope tremor disk in their foreshortened position near the solar limb. In addition, both sunspot intensity and velocity distributions exhibit sharp variations across distances in the spot of the order of the tremor disk diameter and therefore cannot fail to be distorted by seeing. Wormell (11) was able to show that the observed apparent intensity distribution for a nearly circular spot near the disk centre, with umbral diameter 14" and penumbral diameter 40", when corrected for scattered light, may be completely accounted for by an umbra of constant intensity 0.3 and diameter 10", surrounded by a penumbra of constant intensity 0.6 and diameter 30". (Fig. III.1). In recent years, observers have become increasingly aware of the importance of correcting solar observations for the effects of scattered light. As an example, we may quote the results obtained from investigations of sunspot intensities.

One of the earliest quantitative observations of the intensity of radiation from sunspots was that of Wilson (10), using a Boys radiomicrometer. He concluded that the ratio of the intensity of a sunspot umbra to that of the surrounding photosphere varied, on the average, from about 0.4 near the centre of the disk to about 0.8 near the limb. Schwarzschild and Villiger (7) also concluded from their photometric observations of sunspots, that the ratio of umbral intensity to that of the neighbouring photosphere
FIG III.1 Wormell's sunspot intensity distribution after correcting for scattered light.
increased from the centre of the disk to the limb. No correction was applied for scattered light in either of these investigations.

In a later investigation by Wanders (8) (9), the distribution of the intensity across the spot and neighbouring photosphere was determined in various spectral regions and a careful correction applied for the effect of the light scattered. No evidence for the centre-to-limb variation in intensity was then observed. Wanders attributed any such apparent effect to the increase in the effect of scattered light, when the spot is foreshortened near the limb. His results, corrected and uncorrected for scattered light are shown in Fig. III.2. These results were in agreement with those obtained earlier by Richardson (6). Richardson had made no correction for scattered light, but this effect was probably minimized because he had only considered the plates (17 out of 4800) taken under optimum seeing conditions. Hormell's results (11) also carefully corrected for scattered light, showed no significant change from centre to limb.

This brief account of the investigation of sunspot intensities is given to emphasize what misleading results can arise when the problem of scattered light is not given fullest consideration.
FIG III-2 Wander's variation of umbral intensity as a function of disk position, corrected and uncorrected for scattered light.
III (1). Scattered light corrections for previous Evershed measures.

The importance of a scattered light correction in sunspot velocity field measures was appreciated later than in the intensity measures. In Michard's analysis of Abetti's Evershed results, (Introduction) a centre-to-limb variation in the "mean radial velocity" was observed and a linear decrease from centre to limb was deduced. Michard remarked that scattered photospheric light "ne joue donc probablement pas de rôle appréciable dans le phénomène". The only justification he gives for this remark is in saying that if we attribute the variation to scattered light, then the observed decrease in the region of $\sin \theta = 0.5$ to 0.6 does not seem sensible, for at this disk position, the apparent area of the spot only varies by a few per cent. This implies that if the spot area appears unchanged, any effects of light scattered into the spot are unchanged, and so no velocity decrease would be expected. But this is not so. Also, since Michard analysed results from sunspots of varying sizes to obtain this velocity variation, his argument is scarcely legitimate.

Kinman (Chapter I (2)) emphasized the need for the application of a scattering correction in velocity measures and attempted a graphical method to correct his measured sunspot velocities. He estimated an error of $10\%$ for the effect of scattered light on the maximum radial velocity.
determined (4). However, Kinman did not consider the effect of foreshortening as the spot approached the limb, (circular spots appear markedly elliptical at disk positions \( \theta \approx 45^\circ \)) so it is probable that his correction of 10' is an underestimate, at distances where foreshortening is appreciable. In addition, he considered all the scattered light from the near penumbral and photospheric regions as light of zero velocity, arguing that it came from regions exhibiting both positive and negative velocity shifts, whose net effect is consequently zero. This was a first approximation in a pioneer treatment of the problem.

The remaining Evershed investigations (Chapter I (2)) have little to add to the question of a scattered light correction. Servajean makes no mention of it whatsoever, when he considers his final Evershed velocity results, and Bumba quotes Kinman's correction of 10%. However, Bumba again emphasized that to obtain good observational material for such investigations, the conditions of a good quality image exhibiting minimum vibration are essential. Consequently, his plates were taken over periods of two hours in early spring mornings, when observational conditions were optimum.

III (2). A new determination of the effect of scattered light.

Previous considerations of the effects of light scattered by the sky and instruments in Evershed velocity measures
The only real efforts to make any corrections to velocity measures introduced by scattered light have been by Kinman and we have already indicated that even these appear to be underestimates. The solution to the problem needs a more exact and careful treatment.

In the present investigation, the observed spreading of the radial velocity pattern from centre to limb (Fig. II. 9) suggests straight away that scattered light plays a part in sunspot velocity field measures. In the 1960 work, where we followed the same spot across the disk, the pattern near the disk centre appeared sharp and narrow. The maximum radial velocity (3.5 Km/sec) was attained about 18,000 Kms. from the spot centre and fell to zero at a distance of 50,000 Kms. At the limb, it was observed to be both considerably flattened and broadened and the maximum radial velocity (0.9 Km/sec) was not attained until a distance of 25,000 Kms. from the spot centre, falling to zero at 70,000 Kms. (Chapter II (6)). This is exactly what would be expected if, as the spot moved from the centre of the disk to the limb, the influence of the scattered light became more apparent. Although the 1959 investigations were based on two spots of different sizes, and are therefore not so convincing in centre-to-limb comparisons, exactly the same trend in the radial velocity pattern was observed. (Chapter II (4)).
These considerations convinced us that an attempted solution to the problem of scattered light in velocity field investigations was essential. We are not now attempting a detailed correction of our own results, but using typical measures, we are attempting to estimate the magnitude of scattering effects on observed sunspot velocities.

(a) Method.
Our method will be to work numerically on the following lines. For a circular sunspot, with radially symmetrical properties, we start with a given true sunspot intensity distribution $B(r)$, a true sunspot velocity distribution $V(r)$ and a scattering function $U(r)$, where $r = \text{distance from spot centre}$. The scattering function is to be used in the form of a redistribution function i.e. the total light $U_0$ in unit area at a point $P$ is redistributed over a region centred at $P$ according to an intensity variation $U(r)$. In limb work with large images, it is possible to consider this intensity variation in one direction only (viz. along a solar radius) and we can thus use a linear redistribution function, but for sunspot work, we must use a circular distribution. The function $U(r)$ in its circular form is normalised so that

$$\int_0^\infty U(r) 2\pi r \, dr = U_0$$
Our procedure to obtain this circular scattering function has been to use a linear function as a guide and to guess at a circular function which we could test on a true limb darkening intensity distribution and modify to obtain satisfactory agreement with observed limb and scattered sky light intensity distributions.

Our theory of the effect of scattering velocities is based on the assumption that addition to a spectrum of a small quantity of a spectrum differing velocity results in a fractional change in velocity varying linearly as the fractional change in intensity. (2) The scattering function is expressed as a step function in \( P \) concentric annular zones. Suppose the sunspot is divided into \( n \) zones and that within each zone, \( B \) and \( V \) are constant. To find the effect of scattering at any point in the spot, we centre the scattering function at this point and calculate the observed sight-line \( V^* \) due to scattered light from:

\[
V^* = \frac{\sum \left( \sum_p \sigma_{p,n} U_p \right) B_n V_n}{\sum \left( \sum_p \sigma_{p,n} U_p \right) B_n}
\]
where

\[ \sigma_{pn} = \text{area common to } p\text{th scattering zone and } n\text{th sunspot zone.} \]

\[ U_p = \text{average value of scattering function in } p\text{th zone.} \]

\[ B_n = \text{average brightness of } n\text{th sunspot zone.} \]

\[ V_n = \text{characteristic velocity of } n\text{th sunspot zone.} \]

The \( n \) sunspot zone divisions were constructed by the following procedure. A typical spot (the full details of which are given in the next section) was drawn to scale at a selected position near the disk centre and on the solar equator, and divided into 2" annular zones. The first position was selected as \( \sin \theta = 0.2 \) (this being the nearest position to the disk centre at which sunspot velocity measures have been attempted) and here the spot appeared circular as any foreshortening is negligible. Then the spot was considered at a second disk position \( \sin \theta = 0.7 \) and allowing for foreshortening, the same zones were constructed. The shape of the spot at this position appeared markedly elliptical. In both of these positions, the spot was further divided into 12 equal sectors, and to the zones in each sector, an appropriate mean sight-line velocity \( V_n \) was assigned. (Fig.III.3). The value of \( V_n \) was calculated from

\[ V_n = \mu \sin \theta \cos \phi \]

The

\[ 87 \]
FIG III·3 Circular spot at two disk positions and subdivisions considered for scattering calculations. (Details shown for one sector only.)
where \( u \) = the assumed radial velocity.

\[ \Theta = \text{the angle between the line-of-sight and a true solar radius at the point of observation.} \]

\[ \phi = \text{angle measured in spot plane from the direction of solar rotation.} \]

Equation III.1. shows that a knowledge of a sunspot intensity distribution \( B(r) \), a sunspot velocity distribution \( V(r) \) and a scattering function \( S(r) \) suffices to determine a correction for observed sunspot velocities due to scattered light. The details of these functions adopted for the present investigation are given in the next section. This calculation to determine the observed sight-line velocity due to scattered light at any point in the spot has not considered the effect of introducing a distant photospheric velocity component due to solar rotation. The effect of this component would be to increase the peak sight-line velocity in the penumbra on one side of the spot and diminish it on the opposite side, (The overshed sight-line velocity shifts are in opposite directions on the two sides of the spot.) but the mean peak velocity is unaffected. We therefore considered the omission of the solar rotation component permissible.
(b) Basic data.

The sunspot intensity distribution $B(r)$

We considered a typical average sized circular spot with the following dimensions:

- **Umbral radius** = 10"
- **Outer penumbral radius** = 25"

and an intensity distribution:

- **Photosphere** = 100%  
- **Penumbra** = 80%  
- **Umbra** = 30%

This intensity distribution was determined by Wormell (11) for a spot whose true umbral and penumbral radii were 5" and 15" respectively. It has been assumed that the distribution is not very different for a spot of the size considered here, for which no intensity distribution has been determined.

The sunspot velocity distribution $V(r)$

The radial velocity ($u$) distribution deduced by Kinman (3) for a spot of this size (Fig. III.4) was adopted as a fair
Fig III-4. Assumed radial velocity distributions for scattered light calculations.
representation of a sunspot velocity field. The appropriate sight line velocity $V$ at any point in the spot can then be determined by equation III.\ldots, and we can thus build up the required distribution $V(r)$. (We can only really get the true velocities by determining the corrections to be applied to an observed curve and iterating, but Kinman's observed curve suffices as a true distribution for our purpose here.)

The scattering function $U(r)$.

Since the procedure adopted for the correction due to scattered light is based on a typical rather than specific spot size and velocity distribution, we have also used a typical scattering function, obtained from velocity measures of scattered light plates taken at Oxford.

The actual linear function used as a guide to obtain the circular scattering function required was one obtained for the Oxford North Tower Telescope during centre-to-limb wavelength investigations. \footnote{1} (The final linear scattering function values accepted were found later to compare favourably with a linear scattering function derived for the Oxford South Tower Telescope in 1960. Details of this latter function are directly applicable to our results but were not available to us at the time of the scattering investigation.) The zone values of the "guide" linear scattering function were assumed to apply over circles having
radii of the corresponding zones, and this trial circular scattering function was then tested on the true limb darkening curve (1). A modification of the two outermost zones of the original linear scattering function produced a satisfactory agreement between the computed limb intensity distribution and the observed limb intensity distribution of Hollow, Burger and van der Bilt (5) and at the same time give an acceptable distribution for the scattered sky light, the distribution observed by Kinman being used as a comparison. (Fig.III.5) The final values of the adopted circular scattering function \( U(r) \) (unnormalized) which we used are given in TABLE III.1.

**TABLE III.1.**

<table>
<thead>
<tr>
<th>Range</th>
<th>( U(r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0&quot; - 1&quot;</td>
<td>0.6</td>
</tr>
<tr>
<td>1&quot; - 3&quot;</td>
<td>0.5</td>
</tr>
<tr>
<td>3&quot; - 5&quot;</td>
<td>0.27</td>
</tr>
<tr>
<td>5&quot; - 7&quot;</td>
<td>0.15</td>
</tr>
<tr>
<td>7&quot; - 9&quot;</td>
<td>0.09</td>
</tr>
<tr>
<td>9&quot; - 11&quot;</td>
<td>0.05</td>
</tr>
<tr>
<td>11&quot; - 20&quot;</td>
<td>0.02</td>
</tr>
<tr>
<td>20&quot; - 30&quot;</td>
<td>0.001</td>
</tr>
<tr>
<td>30&quot; - 100&quot;</td>
<td>0.0002</td>
</tr>
</tbody>
</table>
FIG III.5 Energy distribution at limb.

- - - Calculated using accepted scattering function.

- - o -- Moll, Burger and Van der Bilt for λ6000 (inside), Kinman (outside).
(c) **Effect of scattering on sight-line velocities.**

The adopted circular scattering function (TABLE III.1) was drawn on a sheet of kodatrace on the same scale as the sun-spot diagrams. (Fig. III.3). To calculate the observed sight-line velocity due to scattered light at any point in the spot, the scattering function was centred at this point and the summation over all the zones of the relevant products (equations III.1) was computed. Ideally, we should like to have calculated the sight-line velocities due to scattered light at a number of points across the spot, but since the computation was somewhat lengthy and time was limited, we restricted our attention to the most interesting positions viz. the centre of the penumbra, which is the region where the maximum radial velocity is observed, and the edge of the velocity field, i.e. where the radial velocity is zero.

We simplified the procedure further by selecting for the velocity determination the symmetrical positions on the spot equator. We calculated the velocity at the point in the photosphere where the sight-line velocity fell to zero, to see what effect the scattered light has on the width of the velocity pattern. The results of these calculations are given in TABLE III.2. Table 2 (a) refers to the results for the peak velocity at the centre of the penumbra and Table 2 (b) for the point in the photosphere where the sight-line velocity has fallen to zero.
### TABLE III 2 (a). Effect of scattered light on peak velocity at centre of penumbra, using the Kinman velocity distribution.

<table>
<thead>
<tr>
<th>SPOT POSITION</th>
<th>BEFORE SCATTERING</th>
<th>AFTER SCATTERING</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>True sight line velocity Km/sec</td>
<td>True maximum radial velocity $U_{true} = V_{true}/\sin\theta$</td>
</tr>
<tr>
<td>$\sin\theta = 0.2$ (Centre)</td>
<td>0.378</td>
<td>1.89</td>
</tr>
<tr>
<td>$\sin\theta = 0.7$ (Limb)</td>
<td>1.32</td>
<td>1.89</td>
</tr>
</tbody>
</table>

### TABLE III 2 (b). Effect of scattered light on point in photosphere where velocity has fallen to zero.

<table>
<thead>
<tr>
<th>SPOT POSITION</th>
<th>BEFORE SCATTERING</th>
<th>AFTER SCATTERING</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>True sight line velocity Km/sec</td>
<td>True radial velocity Km/sec</td>
</tr>
<tr>
<td>$\sin\theta = 0.7$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
III (3) Discussion of results.

Table III 2 (a) shows that the correction to the observed sight-line velocities due to scattered light at the two disk positions considered is of the order of 40%. In addition, as the spot moves from the centre ($\sin \theta = 0.2$) to the limb ($\sin \theta = 0.7$) there is a decrease in the observed maximum radial velocity of 0.15 km/sec, although the true maximum radial velocity for this change of disk position does not vary. (This was a basic assumption viz. that the Kinman radial velocity distribution assumed is the same at any disk position.) In other words, the effect of scattered light on the maximum radial velocity is to introduce an apparent centre-to-limb decrease. We also see from Table III. 2 (b) that the effect of scattered light, at the disk position $\sin \theta = 0.7$, is to broaden the true velocity pattern.

The centre-to-limb decrease in the maximum radial velocity, predicted from the scattered light calculations, is exactly the effect that was observed in Servajean's investigation (Chapter II (2)), although his observational estimate gave a drop of approximately 0.9 km/sec in the maximum radial velocity for this change in disk position. Since the calculated decrease in the maximum radial velocity is dependent on the assumed true radial velocity distribution adopted for the calculation, (we have already noted that Kinman's distribution, which we used here as true was,
in fact, an observed distribution) we therefore considered
the effect of modifying our initial Kinman velocity dis-
tribution and repeating the calculations. The second
radial velocity distribution, which was sharper and narrower
than Kinman's, is shown in Fig. III.4, and the results
obtained using this distribution are given in TABLE III.3.
In this case, we can see that as the spot moves from near
the centre of the disk to the limb, the effect of scattered
light is to produce an apparent decrease in the maximum
radial velocity of 0.32 km/sec.

There is definite qualitative agreement between our
predictions of a centre-to-limb decrease in the maximum
radial velocity, and Servajean's observations, although we
are not in quantitative agreement. We have made the com-
parison with Servajean because his results were the only
observations (apart from our own) available, and it is
probably due to the necessary limitations of our prototype
investigation (the main purpose of which was to estimate a
typical scattered light correction) that we have not obtained
data.

The ideal investigation for the effects of scattered
light would involve following a circular spot across the
disk in optimum observing conditions. The spot intensity
distribution, corrected for scattered light, would need to
be determined by careful photometric analysis. We could
also take the velocity distribution determined near the disk
TABLE III. 3. Effect of scattered light on the peak velocity at the centre of the penumbra using the second velocity distribution.

<table>
<thead>
<tr>
<th>SPOT POSITION</th>
<th>BEFORE SCATTERING</th>
<th>AFTER SCATTERING</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>True sight line velocity ( V_{\text{true}} ) ( \text{Km/sec.} )</td>
<td>True maximum radial velocity ( U_{\text{true}} = \frac{V_{\text{true}}}{\sin \theta} )</td>
</tr>
<tr>
<td>( \sin \theta = 0.20 ) (Centre)</td>
<td>0.650</td>
<td>3.25</td>
</tr>
<tr>
<td>( \sin \theta = 0.70 ) (limb)</td>
<td>2.27</td>
<td>3.25</td>
</tr>
</tbody>
</table>
centre (where the scattered light has a minimum effect) as the true distribution, and follow through the procedure described in this section, to compare the observed and calculated change in the sunspot radial velocity component as the disk position changed. We should need to determine for such an investigation, the scattering function applicable for the observational period. Only then could we be sure that we were justified in comparing the observed and calculated velocity changes across the disk. This is quite a lengthy investigation, but the necessary material is available in the present instance and awaits our measurement and reduction.

In spite of the fact that our present investigation falls short of this ideal, we have nevertheless shown that the effect of scattered light on measured velocities is considerably greater than had previously been predicted and may partly, if not wholly, account for the observed centre-to-limb decrease in the sunspot maximum radial velocity.
References for Chapter III

(10) W.E.Wilson, M.N. 55, 457, 1895.
CHAPTER IV

Photometry of the sunspot FeI absorption line $\lambda 5576.101$
Chapter IV

Photometry of the sunspot FeI absorption line \( \lambda 5576.101 \).

Introduction.

Evershed (6) first observed that sunspot spectral lines showed a diffuse widening in the direction of the velocity displacement near the penumbral limits, but remarked that this phenomenon might well be involved with Zeeman widening. However, the high dispersion spectra taken for the present investigation revealed the same feature (to be referred to here as 'line-flare') in the Fe I line at \( \lambda 5576.101 \), which was specifically selected for velocity measures because the Landé splitting factor \( g = 0 \) for this line. This immediately dispels Evershed's theory that line-flare is a phenomenon associated with the sunspot magnetic field. Other recent investigations (Servajean (13)), Bumba (2)) also noted the same feature in spectral lines with small \( g \) values and we may conclude that line-flare is a feature of all high dispersion sunspot spectral lines.

Bumba's conclusions from his investigation of line-flare features ("flags" in his notation) in spectral lines are listed below:

1. The line-flare is a result of Doppler line shifts.
2. As a rule, the line as a whole is not displaced but the
Doppler shift produces a pronounced asymmetry which results in a displacement of the centre of gravity of the line. At the maximum Evershed effect, the asymmetric line often becomes a clearly defined line (a "satellite") whose distance from the centre of the undisturbed line represents a velocity of approximately 5 km/sec.

3. The asymmetry increases with decreasing line intensity, actually becoming a satellite in the weakest lines.

4. The asymmetry depends on the spot position on the disk. Close to the disk centre (θ = 10°-20°) the asymmetry is small but becomes more pronounced at greater distances.

5. The magnitude of the asymmetry depends on the side of the spot considered. At distances of about θ = 30°, the asymmetry is greater on the side of the spot nearer the disk centre.

Bumba explained the variation of "flag" shape by a velocity gradient in the spot (2). He maintained that the velocity of motion in layers lying close to the photosphere/chromosphere boundary is almost zero. The velocity vector is directed outwards below this boundary and inwards above it, the flow being very nearly radial in the spot along the surface of the sun. This radial velocity component increases with distance from the photosphere/chromosphere boundary both above and below the boundary. Bumba's velocity results were
based on an investigation of different strength metallic lines in the photosphere and the CaII H and K lines in the chromosphere. His scheme of motions is illustrated in a vertical section of a sunspot in Fig. IV.1. Bumba proposes that the undisplaced line core is formed in the region of zero velocity and that the observed asymmetrical metallic line wings are formed in the deeper, higher velocity layers in the spot, the extent of the asymmetry depending on the direction of the line of sight. He suggests that this proposed scheme of motions is verified by the appearance of "flags" in the chromosphere lines of H and K, which are displaced in the opposite direction to those of the photospheric lines.

A theory of the depth of line formation and the relation between geometrical and optical depth in a sunspot, depends critically upon a sunspot model. Bumba used Michard's model for the calculation of optical and geometrical depths in the spot, but as we showed in Chapter I (2) there is still some doubt over its acceptance.

In Servajean's investigation of sunspot spectral line profiles, there was no evidence of a weak secondary line, but apart from this feature, Servajean seems to be in agreement with Bumba about the properties of line-flare. He remarked in addition, that the feature was rarely observed in the umbra, but was confined to the penumbra, even when the Evershed
FIG IV.1 Bumba's velocity distribution in a vertical section of a sunspot.
velocity extended beyond the penumbra. The line-flare was also restricted to the dark parts of the fine penumbral structure. (Introduction) (It is difficult to understand how Servajean was able to restrict the region of spectral line formation to a dark penumbral filament, considering that filaments are \( \leq 300 \text{ km} \) wide (5) which must surely be approaching his theoretical resolving limit.) The explanation he proposes for the line-flare phenomenon also arises from a possible velocity variation with depth in the spot, similar to that of Bumba. The uncertainties in the methods attempted to verify the existence of such a velocity gradient have been remarked upon earlier. (Chapter I (2))

The general characteristics of line-flare in the sunspot spectrum have thus been fairly well established by Bumba and Servajean. However, neither of these workers attempted to investigate its intensity variation or the part it plays in the measurement of Evershed velocities. This latter problem is one of great importance. If we are to attempt to construct any accurate physical picture of the motions in a sunspot, it is not sufficient to measure the magnitudes of line shifts across a sunspot. This will only give a "mean" velocity for the region in which the line is formed. In discussing the Evershed effect, we must also consider and explain the shape and variation of the spectral line profile.

The 1960 observational work was planned to take high dispersion calibrated plates for a full-scale study of the
intensity and extent of line-flare, and to investigate more fully the part it plays in the measurement of sunspot velocity fields. This work is described in the following sections, which deal respectively with the experimental procedure, the photometric analysis to examine the asymmetries, equivalent widths and central intensities of the line and a discussion of the results.

IV (1). Experimental procedure.

The details of obtaining the calibrated sunspot spectra for line-flare were given in Chapter II (5). All the plates for this investigation were taken in May and June of 1960, with the Oxford South Tower Telescope and the microphotometry was performed with the observatory microphotometer (10) connected to a 'spot-following' pen and ink recorder.

The first step in any photometric investigation using photographic plates is the construction of the characteristic response curve of the plate used. This relates the intensity of the light incident on the plate with the resultant plate density. In order to construct such a curve, we obtained in addition to the sunspot spectra, spectra taken through a previously calibrated (1) rhodium step wedge, which allowed seven exposures of equal duration but different intensities to be taken simultaneously. These spectra were corrected for any non-uniformity of slit illumination by taking spectra of the solar centre through the clear glass plate of the
wedge. A 63,' filter was used for these latter plates to obtain an acceptable plate density within the limits of the wedge spectrum density. Microphotometer tracings through each spectrum perpendicular to the dispersion, in a region of continuous background, then gave a record of the plate densities produced by light of known relative intensities. These were used to obtain a smooth characteristic curve applicable to the sunspot plates under investigation.

Since we wished to compare the results of the visual measures with those obtained photometrically, three of the four measured sunspot plates (No 40, No 58, No 62. Chapter II (5)) were selected for the photometric analysis. The fourth plate (No 52) gave a saturated trace. Each plate was prepared for the microphotometry as follows: (Fig. IV.2)
(a) A nylon thread was fastened across the plate, roughly parallel to and about 10 mm. away from λ 5576, in a region of the plate scraped clear of emulsion. Its purpose was to act as a fiduciary to which all the tracings could be referred. (The slope of the thread was allowed for by a series of subsequent measures on the thread, using the Hilger microscope.)
(b) The plate was carefully marked with a marking machine at 1 mm. intervals perpendicular to the direction of dispersion, on either side of the measured line λ 5576. These served to act as a guide for the position of the required microphotometer tracings.
FIG IV-2 Marking the measured plates for the line-flare photometry.

(Positions of traces indicated.)
With the nylon thread in position and the markings made, we were ready to take a series of line-flare tracings. For every trace, a direct and reverse recording was made across the nylon thread and $\lambda$ 5576 i.e. in the direction of the dispersion, using a microphotometer magnification of 50. Two reference tracings were obtained well out in the photosphere on either side of the spot. These were selected (by considering the visual measure plots) to be well away from any large photospheric velocity fields. Six line-flare traces were made at 1 mm. intervals in the region of the umbra and penumbra. As for the visual measures (Chapter II (6)), we again restricted the investigation to the side of the spot showing the regular velocity field. By measuring the traces and applying the characteristic curve, we thus obtained a series of line profiles of $\lambda$ 5576 in the photosphere and across the spot penumbra, for three different disk positions.

IV (2). Results of photometric analysis.

(a) Line profile asymmetries.

For each microphotometer trace obtained across $\lambda$ 5576, a plot of the $I/I_0$ variation (i.e. ratio of line intensity to continuous background intensity) was constructed using the calibration curve. The $I/I_0$ plots for the photospheric traces on either side of the spot were symmetrical and showed no marked differences in form for the three disk positions. In the region of the penumbra, however, the profile appeared
asymmetrical in the direction of the measured velocity, and became more asymmetrical as the trace position across the penumbra approached the position where the maximum sight-line velocity was observed. The degree of asymmetry also depended on the spot position on the solar disk, the greatest asymmetry being observed for the spot nearest the limb. A comparison of three traces showing the maximum asymmetry for each of the three disk positions considered is shown in Fig. IV.3. The ordinate gives the $I/I_0$ ratio and the abscissa is in milliangströms. In Fig. 3 (a), for the spot nearest the limb, the marked asymmetry towards red wavelengths is indicated by the dotted median line. For the spot nearest the disk centre in Fig. 3 (b) the asymmetry is less pronounced, but still towards red wavelengths. In Fig. 3 (c), where the spot has crossed the meridian the asymmetry is fairly pronounced towards violet wavelengths.

As far as we could estimate, the position in the penumbra of the maximum asymmetry in the line profile coincided with the position of the maximum sight-line velocity, for each spot position. The traces however were only taken every $0.5$ mm on the plate, while the velocities were measured at $0.1$ mm intervals and consequently the position of the measured maxima were known with greater precision.
FIG IV-3 Comparison of $\lambda$ 5576 profiles showing maximum asymmetry for 3 sunspot positions on disk.
(b) Determination of the effective photographic density of the measured sight-line velocities.

In direct visual measures of Doppler shifts, we have to make the assumption that the eye sets on the position of the centre of gravity of the line. This is feasible when we measure symmetrical profiles, but in high dispersion penumbral velocity measures, it is difficult to say what is the influence of the pronounced asymmetry. The present investigation was instigated by this problem, and we have here attempted to correlate the measured sight-line velocities with a line profile $I/I_0$ ratio i.e. to determine exactly to which point on the profile median-line our visual setting refers.

The plots of visually measured sight-line velocities we had previously constructed, and with which we wished to compare the velocities determined for different $I/I_0$ values, had been corrected by the iodine line measures for instrumental defects of slit curvature and inclination. We had thus to allow for this correction in the photometric traces. Since we were using the nylon thread as a reference line, we also needed to correct for any slope of the nylon thread on the photographic plate. For each of the three plates considered, we therefore constructed a correction curve which would allow for both the effects of the solar line curvature and the slope of the nylon thread, and which would thus enable
us to compare directly our sight-line velocities measured visually and those determined from the microphotometer tracings. We then set out to determine the velocity shifts indicated by the median line at different $I/I_c$ ratios viz. the line core ($\sim 0.35$), 0.4, 0.5 and 0.6. To determine the velocity shift indicated by the median line at a selected $I/I_c$ ratio, at any one trace position across $\lambda 5576$, the procedure was as follows:

1. Measure the distance between the nylon fiduciary and the selected position on the median line. This was given directly in time marks by the pen trace of the microphotometer, and was measured with great care with a glass rule to $\pm 0.05$ of a time mark.

2. Correct the measured time mark interval for slit defects and the nylon thread slope using the determined correction curve (as previously described).

3. Convert the corrected time mark into a sight-line velocity (in Km/sec.) (this was done using the microphotometer magnification and plate dispersion values) measured relative to one of the photospheric traces.

The relative sight-line velocity obtained in this way could then be compared with the visually measured sight-line velocity, and the procedure repeated for a different $I/I_c$ ratio until agreement was obtained between the two, i.e. until we had determined the position on the median line at which the visual measurement was made.
The density level of the visual measurement appeared to depend slightly on the plate density. For the faintest plate, (Ho 40) the visual settings were estimated to have been made at \( I/I_0 = 0.5 \), while the densest plate (Ho 62) gave equality at \( I/I_0 = 0.6 \). The results of the comparison of visual and photometric sight-line velocities for the four selected \( I/I_0 \) ratios for Ho 40 are given in Fig. IV.4, and are typical of the three plates considered. Fig. IV.4 (a), for the minimum value of \( I/I_0 \) indicates a movement of the line-core, amounting to a sight line velocity shift of approximately 0.4 Km/sec. The other two plates Ho 58 and Ho 62 gave line-core shifts of the same order. Fig. IV. 4 (c) shows the final correlation between the measured and calculated velocity shifts at \( I/I_0 = 0.5 \).

(c) Equivalent widths.

Equivalent widths were determined to \( \pm 3 \)m \( \AA \) by repeated planimeter tracings of each line profile of \( \lambda 5576 \) obtained. The results for the three disk positions (Ho 40, \( \sin \theta = 0.74 \); Ho 58, \( \sin \theta = 0.17 \); Ho 62, \( \sin \theta = 0.36 \)) are given in Fig. IV.5. The ordinate gives the equivalent width in m\( \AA \) and the abscissa indicates the distance from the spot centre in units of 10,000 Kms. The results for all three spot positions clearly illustrate that the equivalent width of the line decreases as the distance from the spot centre increases. For the two plates nearest the centre (Ho 58 and Ho 62) there is a minimum value of the equivalent width in the region of the bright ring feature of the spot. Comparing the results
FIG IV.4 Comparison of photometric and visually
determined sight-line velocity shifts at selected
$I_{Ic}$ values for plate HO 40 (continued overleaf).
FIG IV.4 (continued from overleaf)
FIG IV-5 Equivalent width of \( \lambda 5576 \) versus distance from spot centre for 3 disk positions.
of Ho 40 and Ho 58 i.e. the limb and centre plates, we see that for corresponding positions in the spot, the equivalent width increases with the distance of the spot from the disk centre. The results for Ho 62 are puzzling in that they are consistently smaller than those for Ho 58, although Ho 62 is further from the disk centre than Ho 58, but the spot has crossed the meridian between these two positions.

Vinnaert (8) gives a photospheric equivalent width of 113 mÅ for 5576 measured at the solar centre. For our spot nearest the disk centre (Ho 58) the photospheric trace gives an equivalent width of (116 ± 3) mÅ.

(d) Central intensities.

An examination of the line profiles constructed from the microphotometer traces revealed a central intensity variation of the 5576 profile with distance from the spot centre. The observed variation is illustrated in Fig.IV. 6 for the three disk positions. There is no conclusive change in the central intensity with disk position, as it may be seen that the variation for the centre plate Ho 58 is very nearly identical to that for the limb plate Ho 40. However, within the spot itself, the central intensity is seen to increase with distance from the spot centre. The photospheric central intensity of 5576 from this investigation is estimated as 0.37, while the Utrecht Atlas (9) gives a central intensity of 0.38 for the same line in a solar centre spectrum.
FIG IV.6 Variation of central intensity of λ5576 with distance from spot centre for 3 disk positions.
IV (3) Discussion of photometric results.

The present photometric work on the sunspot line asymmetries has confirmed Bumba's and Servajean's conclusion that the observed line asymmetry is more pronounced in the sunspot at greater distances from the solar centre. We see no evidence of Bumba's satellite line at any disk position, nor from an examination of other fainter Fraunhofer lines on the measured photographic plates. For each spot position, the maximum line asymmetry was observed at approximately the position of the measured maximum sight-line velocity. The possible causes of the line-flare phenomenon are discussed in more detail in the next and final chapter.

The influence of the line-flare on sunspot velocity measures was not estimated by Bumba or Servajean. It seems certain, and it is not surprising, that the eye does not set on the line core, but at a point towards the asymmetry. The actual position of measurement varies slightly with plate density, but is in the neighbourhood of an $I/I_0$ ratio of 0.5. At this position, we measured the actual Evershed velocity, but we have detected, in addition, a movement of the line core of the order of 0.3 km/sec. in the sight-line velocity, indicating that the material in the spot is moving as a whole.

This is the first detailed work on sunspot umbral and penumbral equivalent line-widths and central intensities and we can do no more here than to quote the results obtained for
the Fe I line at $\lambda 5576$. We see that the equivalent width of this line varies both with the position in the spot itself and with the position of the spot on the disk. It increases with the distance of the spot from the disk centre and decreases within the spot with increasing distance from the centre of the umbra. (See Footnote) The minimum value of the equivalent width observed in the region of the bright ring is an additional feature of interest. The central intensity of the line profile was also observed to increase with increasing distance from the spot centre. These problems of sunspot equivalent widths and central intensities are not within the scope of the thesis as originally planned. Any further analysis and detailed discussion of the results here obtained must be left for the future, and will present some further evidence in the problem of the construction of a reliable sunspot model.

Footnote: Servajean (Chapter I (2) and Chapter V (2)) did not consider any possible variation of sunspot line equivalent widths when he attempted his sunspot velocity analysis.
The spectroscope vertical apparatus function.
Appendix Chapter IV

The spectroscope vertical apparatus function.

Appendix IV (1). Introduction and Method.

The step wedge used for the calibrated spectra is crossed by two fine, parallel reference wires, perpendicular to the spectroscope slit. Their function is primarily for positioning the wedge and solar centre spectra but they can also be used to produce test images in these spectra which can be used to ensure that each spectrum is free from any vertical ghost pattern produced by instrumental effects, which must otherwise be allowed for. These instrumental effects, such as diffraction arising from the spectroscope aperture and imperfections in the optical system due to the prisms, gratings or lenses, are characterized by an apparatus function, which is the profile recorded by the spectrometer for a point on the slit. An examination of the wedge and centre spectra taken for the 1960 investigation showed that the sharp wire image was accompanied by a faint ghost. It was therefore necessary to examine the effect of such a vertical apparatus function on the true sunspot photometric and visual observations. This section gives a description of the investigation undertaken. As an introduction, we give a brief account of the theory on which it was based.
If \( T(x') \) be a true profile, \( O(x) \) the observed profile and \( S(x-x') \) the vertical apparatus function (where \( x \) is measured in the direction of the spectroscope slit for this problem)

\[
O(x) = \int_{0}^{\infty} T(x') S(x-x') \, dx.
\]

This is an integral equation expressing the result that the observed profile is the fold of the true profile and the apparatus function. Numerical solutions to this integral equation are difficult, but if, for example, we know \( S(x) \) and \( O(x) \) an approximate solution for \( T(x) \) may be found by an iterative method of Burger and van Cittert (3) (4).

For such a solution, the observed function is adopted as a first approximation to the true function and corrections to this approximation are successively derived until a satisfactory agreement is obtained between the calculated and observed profiles. The solutions of equation Ap. IV.1. for different unknowns formed the basis of this appendix.

The procedure here was greatly simplified by the use of a programme written for the University Mercury Computer by L.A. Higgs, for the solution of the integral equation Ap. IV.1. by the method of Burger and van Cittert.

A discussion of this method and details of the programming are given in his thesis (7). It was suggested by J.S. Rollett (11) that a Fourier transform solution for unfolding
line profiles (i.e. solving Equation Ap. IV.1 for $T(x)$ or $T(x)$) would overcome the limitations of the method of Burger and van Cittert. (The disadvantages of this latter method are the large amplitude errors. Any irregularities that are present in $O(x)$ are enhanced in $T(x)$ and troublesome irregularities can arise.) Consequently, a further programme was written, based on the Fourier analysis method. Its spectroscopic application and a full discussion of the method are given by Higgs and Rollett (12).

The present investigation, based on Solutions to Equation Ap. IV.1, is outlined in points (a) - (c) given below. Each point is considered in detail in the following paragraph:-

(a) From a knowledge of the true and observed wedge wire profiles, $T_w(x)$ and $O_w(x)$, we determined the vertical apparatus function $S(x)$.

(b) Using the determined apparatus function $S(x)$ and the observed intensity distribution $O(x)$ across the sunspot region where our velocity measures were made, we found the true intensity distribution $T(x)$ across the sunspot.

(c) Then knowing the vertical apparatus function $S(x)$, the true and observed sunspot intensities across the region of velocity measures ($T(x)$ and $O(x)$) and the observed sight-line velocities $O_v(x)$ in this same region, it only remained to determine the true sight-line velocities $T_v(x)$. This
final step was based on the assumption (see Chapter III (2) that

\[ O_v(x) O_I(x) = \int_\infty^0 T_v(x') T_I(x') S(x-x') \, dx \quad \text{Ap. IV.1.} \]

which is the original problem of the solution of equation Ap. IV.1. In this case, we can form the products of the observed velocity and intensity \(O_v(x) O_I(x)\) and from the solution \(T_v(x) T_I(x)\) i.e. the product of the true velocity and intensity) we can finally determine the true sight-line velocity \(T_v(x)\), as we know the true intensity \(T_I(x)\). We were thus finally able to compare the observed and true sight-line velocity patterns across the spot.

Appendix IV (2). Details of method.

This investigation was based, as a preliminary measure, on the calibrated plates and sunspot velocity measures of plate Ho 62 only. If it had been found necessary to correct the photometric and visual observations for this plate, we could have gone on to determine the corrections for the three remaining plates. Fortunately, however, we found (as will now be shown) that corrections to the measured velocity plates, for the effect of the vertical apparatus function, were unnecessary.

(a) Determination of the vertical apparatus function \(S(x)\).

We have shown that we could determine \(S(x)\) from a
knowledge of the true and observed intensity profiles of the wedge wire, and then solving equation Ap. IV.1. The true intensity profile of the wire is, in fact, rectangular and 0.13mm wide, but it was considered here in the form of its Rayleigh diffraction pattern corresponding to the spectroscope aperture, which we calculated to have a half-intensity width of 0.138mm. This negligible change in the form of the true intensity profile has little effect on the deduced $S(x)$. The observed intensity profile was determined by applying the characteristic curve to a series of microphotometer traces, taken in a direction perpendicular to the dispersion across the calibrated spectrum. We were then able to solve for the vertical apparatus function using the computer programme to unfold line profiles. Both the Burger van Cittert and the Fourier Transform methods were used for unfolding and although the results they gave were in good agreement, the Fourier Transform solution gave the smoothest apparatus function, which was finally adopted. We tested this accepted function by a further programme written by L.A. Higgs to fold line profiles i.e. to operate on the true intensity profile with the apparatus function. A comparison of the observed intensity pattern of the wire and its ghost image and the pattern obtained from this folding procedure is given in Fig. Ap. IV.1. It can be seen that the agreement is satisfactory.
FIG APP.IV.1 The observed wire and ghost profiles and the predicted profile using the determined apparatus function.
(b) **The true intensity distribution across the spot.**

The observed intensity distribution across the sunspot was determined from a series of microphotometer traces across the sunspot plates under investigation. A further application of the Fourier Transform programme using the deduced apparatus function allowed us to determine the true intensity distribution across the same region. The observed and true intensity distributions were identical in shape but displaced by 0.1 mm approximately.

(c) **Determination of the true sight-line velocity pattern and conclusion.**

Having determined the observed velocity and intensity distributions across the sunspot, we were able to form the products for the left-hand side of the integral equation Ap. IV.2 (The actual observed sight-line velocity measures of plate Ho 62 were smoothed to facilitate the computations.) A final application of the Fourier Transform programme for unfolding line profiles enabled us to determine the product of the true velocities and intensities. Having previously found the true intensity distribution \((b)\), we could straight away determine the true velocity distribution by a series of simple divisions. The final results comparing the observed (smoothed) and true sight-line velocity patterns are given in Fig. Ap. IV.2. From this figure it is apparent that the resultant effect of the existing vertical apparatus
FIG APP. IV.2 The observed and true sight-line velocity patterns across the sunspot of plate HO 62.
function on the true sight-line velocity pattern is simply to displace the pattern by approximately 0.2 mm without introducing any observable distortion. We have therefore shown that the only effect of the vertical apparatus function on the visual investigation of the sunspot velocity distribution and the photometric investigation of the sunspot intensity distribution is to shift the true pattern by approximately 0.2 mm. The remaining photometric investigation for this thesis was the examination of line asymmetry. Since we have previously shown (Chapter IV (2)) that the observed asymmetry closely follows the observed velocity, there seems no reason to expect that the existing vertical apparatus function will produce any more severe effect on the line asymmetry, than that already indicated.
References Chapter IV


CHAPTER 7

Discussion of observations and conclusion
Chapter V.

Discussion of observations and conclusion.

Introduction.

It is over a century since Richard Carrington warned the Royal Astronomical Society that our knowledge of the sun's action is but fragmentary and that speculations on the nature of sunspots would be a very precarious venture. To-day, we can be encouraged by the fact that, through improved and accurate techniques, our present astrophysical knowledge is by no means fragmentary; it is expanding rapidly, amassing details of new and already established phenomenon, so that we must still heed the advice on the "precarious venture of speculation". This final chapter will first give a summary of our results from the sunspot velocity fields investigation and then attempt to show what remaining problems must be considered to ensure that our subsequent sunspot speculations are indeed well founded.

V (1). Summary of results.

The detailed observational results of the present sunspot velocity field investigation are given in Chapter III, and here we will merely summarize the most important characteristics for the clarity of the discussion which follows. The main result obtained from our investigation is in agreement with all previous Evershed workers viz. that the velocity
field in the sunspot penumbra at photospheric level consists of a predominantly radial component $u$, directed outwards from the sunspot axis along the solar surface. The general form of motion is illustrated for two spot positions on the disk in Fig. V.1 (a) which shows that the radial component attains a maximum value at some point near the outer edge of the penumbra and falls to zero well out in the photosphere surrounding the spot. This same figure also illustrates the new observation (to be confirmed) that the maximum value of the radial velocity component $u_{\text{max}}$, shows an apparent decrease as the spot position moves from the centre of the disk to the limb, and at the same time, the radial velocity pattern is observed to spread out.

A solution for the radial ($u$) and vertical ($w$) velocity field components in the umbra (assuming the tangential component $v = 0$) has revealed a small, but consistent, vertical motion of descent into the umbra and a radial component which gradually increases with distance from the umbral centre.

The photometric work on the measured sunspot plates (Chapter IV) has investigated the properties of the high dispersion penumbral phenomenon of line-flare. The position in the penumbra of the maximum line asymmetry in the measured FeI line at $\lambda 5576$ was the same (within the errors of measurement) as the maximum measured sight-line velocity. With the
FIG V.1 Variation of sunspot radial velocity component \( u \) and equivalent line width of \( \lambda 5576 \), as a function of distance from spot centre, for 2 disk positions.
spot near the disk centre, $\lambda$ 5576 shows a maximum asymmetry amounting to a displacement of approximately $2^\circ \text{m} \lambda$ in the profile median line between the core and the wing. This asymmetry is dependent on disk position and the displacement increases to approximately $3^\circ \text{m} \lambda$ at the position near the limb. By comparing photometric and visual measures, we find that the visual setting is not made on the line core, but at a point towards the asymmetry in the neighbourhood of an $I/I_c$ ratio of 0.5.

This was the first detailed investigation of equivalent line widths and central intensities in a sunspot. Ten Bruggencate and von Klüber, in their determination of sunspot temperatures (4) compared the equivalent widths of Ti and Fe lines formed in the photosphere at the disk centre with the same lines formed in sunspot umbrae. The spot equivalent widths were observed to be consistently greater than the corresponding photospheric values, and it has been suggested that this feature is due to magnetic intensification, produced by the sunspot magnetic field. In the present work, the increase in the equivalent width of $\lambda$ 5576 observed from photosphere to spot cannot be due to this cause, as the FeI line was specifically selected because of its non-Zeeman property. The variation of the equivalent width for two positions of the same spot (Ho 40 nearest the limb and Ho 58 nearest the disk centre) is illustrated in Fig. V.1 (b).
It would appear that the change in equivalent width is not closely correlated with the radial velocity component.

**V. (2). Discussion of results.**

In the previous chapter, we emphasized that measures of the magnitudes of line shifts across a sunspot are alone not sufficient in any attempt to construct an accurate physical picture of sunspot motions. In all velocity field investigations, we are only measuring a "mean" sight-line velocity for the region in which the line is formed. (In the penumbra, the existence of narrow (<300 Km) dark and bright penumbral filaments will complicate the problem even further, especially (as seems not unreasonable) if dark and bright filaments have different radial velocities.) Bearing this limitation in mind, we shall now discuss the ideas and physical interpretations suggested by our results.

There seems little doubt from previous investigations (10) and the present work, that the maximum radial velocity attained in sunspot penumbrae exhibits an apparent decrease as the spot moves from the disk centre to the limb. At present, it seems that there are two possible interpretations of this result. Either (a) the variation is true, in which case as Servajean suggested (10) an increase in radial velocity with depth in the spot is implied or (b) the variation is only apparent and may be partly, if not wholly, accounted for by the effects of scattered light. We shall
consider the two hypotheses (a) and (b) in turn.

**Hypothesis (a).** The variation of radial velocity with depth.

A depth variation of radial velocity can be investigated in two ways.

(i) At the same disk position, either from an analysis of weak and strong lines or from different $I/I_c$ values in the same line.

(ii) At different disk positions, from an investigation based on one and the same line.

St. John (11) considered only weak and strong lines in his Evershed effect research, whereas Servajean attempted both these investigations to determine the depth dependence of radial velocity. Both these workers concluded that the radial velocity component increased with depth in a sunspot. In neither case was any correction made for the effects of scattered light.

Any determination of a depth variation of velocity depends critically on the adopted sunspot model. Servajean used Richard's model, which we have already shown to be in contradiction with Mattig's. (Chapter 1) However, even if we assume Richard's sunspot model to be true, Servajean overlooked any possible variation of line equivalent width across a sunspot, for his analysis, and accepted the photospheric values of Allen and Bell. The marked variation in the equivalent width of the FeI line at $\lambda 5576$ is illustrated in Fig. V.1 (b), and is most probably typical of other Fe
and Ti lines on which Servajean based his analysis.

If, in fact, the suggested velocity gradient exists, it has been pointed out (1) that it is difficult to understand why the penumbral filaments, in view of the great length compared with their width, do not disintegrate under the shearing force.

Hypothesis (b). The effect of scattered light.

(See Chapter III)

One immediate reason for considering this hypothesis is simply due to the fact that any possible significance of scattered light in previous sunspot velocity measures has been neglected. It has been shown that the corrections introduced by scattered light to sunspot intensity determinations are not negligible (12). Hattig (9) has similarly shown that even for a large spot, a scattered light correction suffices to change a true limb increase of the maximum sunspot magnetic field strength into an observed decrease to the limb. This is sufficient proof that we are not justified in ignoring the effect of scattered light on sunspot velocities.

The observed variation in the radial velocity pattern viz. a decrease in the maximum and a broadening of the pattern from center to limb, is exactly what would be expected if the scattered light does affect the velocity field. Einman (8) observed that the position of the maximum radial velocity
moves out from the penumbra to the photosphere with decreasing spot size, which again substantiates the scattered light theory, although Hinman suggests this was simply due to an under-estimate of the penumbral widths for the smaller spots. Our preliminary determination of the sunspot velocity corrections due to scattered light (Chapter III) has indeed shown the significance of this factor. We have so far only been able to relate such an investigation to typical sunspot results.

At the present moment, we do not have enough information to be decisive in selecting one of these hypotheses in favour of the other. However, the final choice or compromise between the two will simultaneously provide an explanation for the observed line asymmetries and their variation from centre to limb, for the explanation of this feature again depends on the two hypotheses suggested for the variation in the maximum radial velocity. Servajean has suggested that the penumbral line-flare is a consequence of a velocity variation with depth, but we can here question this argument in the light of our knowledge of penumbral filamentary structure. If we consider two adjacent penumbral filaments, one dark filament along which material flows with a small velocity, and one bright filament along which the material flows with a larger velocity, the line profile observed will
be the resultant of two superimposed profiles separated by a distance corresponding to the difference in their velocities. This would produce the characteristic line asymmetry in the direction of the measured velocity. (Danielson (5) has estimated the maximum width of penumbral filaments to be about 300 Km. Even with our 32 cm. diameter image, a 2″ (≈ 1/3 mm.) tremor disk would result in an integration over approximately 1,400 Km. in the penumbra, so that all the sunspot lines would be formed in a region of the penumbra occupied by more than one filament.) The widening of the line profile as the spot approaches the limb could then be explained by the increasing effect of scattered light at limb positions. So we are effectively back to the original discussion for the explanation of line flare viz. the hypothesis of a velocity variation with depth versus that of an effect of scattered light.

Our result of a velocity component of descent in the sunspot umbra has only been remarked upon previously by Evershed (6). Any physical picture of a "sinking" sunspot umbra, which this result suggests, must also explain the bright umbra granules (presumably hot, rising elements) which are sometimes observed (2) (3). In view of this down flow of material in the umbra, we are now faced with the problem of suggesting a source of supply for the observed material outflow in the penumbra. It seems that the apparently obvious explanation of a material upflow in the umbra,
bending over with the lines of force (assuming the lines of force do follow this pattern) to provide the outflow in the penumbra is unacceptable. Danielson's recent suggestion of penumbral convection rolls (5) is a novel possibility. It is interesting to contemplate the possible connection between a velocity of descent in the umbra at photospheric level, and the observed inflow of material in the penumbra in chromospheric regions (11). In addition, the descending umbral velocity suggests a further consideration in the question of the Wilson effect. (Introduction)

V (3). Conclusion.

Any interpretation of our results of sunspot velocities must be consistent with modern knowledge of sunspot structure i.e. the existence of both penumbral filaments and umbral granules. In addition, the conclusions drawn must not be inconsistent with the results of the sunspot magnetic field. It has long been recognized that the physical processes taking place in sunspots present a complex problem in magneto-hydrodynamics, but unfortunately there is still some conflict over the fundamental question of direction of magnetic lines of force and the variation of field strength with depth in the spot. This latter problem also depends critically on the sunspot model.

We must therefore conclude that no physical sunspot interpretations can be made with safety until;
(a) A whole new field of investigation is opened to determine a satisfactory sunspot model. (Apparently the same conclusion was reached in a discussion of velocity fields in the solar atmosphere at the 1960 I.A.U. Symposium in Italy (7).) Our preliminary work on equivalent line widths in sunspot umbrae and penumbrae is the first step in such an investigation.

(b) Satisfactory conclusions are reached on the effect of scattered light on sunspot velocity and magnetic field measures.
References Chapter V

Acknowledgements

I am indebted to Dr. W.C. Adam for her unfailing help and inspiration throughout the course of this investigation. The 1960 observations were made with her help and the ideas developed to attempt the scattered light problem (Chapter III) are due to her. My thanks are also due to Professor Plaskett for obtaining the 1959 sunspot spectra and to Dr. L.A. Higgs for the use of his computing programmes and instruction in their application. I am grateful to the Director of the Oxford University Computing Laboratory for his permission to use the Mercury Computer. Finally, I should like to express my gratitude to the Department of Scientific and Industrial Research for the provision of a maintenance grant during this research period.

J.H.

University Observatory.

Trinity Term 1962.
A STUDY OF SUNSPOT VELOCITY FIELDS USING A MAGNETICALLY UNDISTURBED LINE

by

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Summary

The relation between magnetic lines of force and material motion is a matter of considerable importance in sunspot theory, and this investigation of sunspot velocity fields uses a line for which the Landé splitting factor $g = 0$ (Fe I $\lambda 5576 \cdot 101$). Doppler displacements were measured at over 100 points in the region of both a large and small sunspot. At the high dispersion value of 0.17 Å/mm, measurement difficulties are presented by the broad solar lines, but a check on visual accuracy was provided by a new photo-electric method of measurement. The measured velocities were corrected for observer's motion and solar rotation and reduced to velocity components, with origin at the spot centre. Using Kinman's conclusion that the spot flow is wholly radial, we have obtained fair agreement for the maximum radial velocity in the penumbra of the small spot but found a value much smaller than that predicted by Kinman for the large spot.

The differences from the earlier measures have been attributed to the fact that we are now able to measure the true Doppler shifts, freed from the complicating effects of Zeeman splitting, and also to the "line-flare", which appears in the penumbral region of high dispersion spectra, but was not resolved in the earlier work on Evershed effect.

Introduction.—Motion in the neighbourhood of sunspots was first investigated by Evershed in 1901, and its general form is now well established (1, 5, 6, 7, 8, 9, 18, 19). Detailed measurements were made by Kinman at Oxford in 1952 (10, 11) and again by Servajean at Meudon in 1958 (17). What has been neglected to date, however, is the distinction between line shifts due to velocity along the line of sight and shifts which have their origin in the line splitting produced by the magnetic field of the sunspot. The sunspot velocity fields and magnetic fields are both observed to change in a more or less regular fashion across the penumbra, and the relation between magnetic lines of force and material motion is a matter of considerable importance in sunspot theory. Plate 6 (a) shows the splitting of the Fe line $\lambda 6302$ across a sunspot. This clearly shows that the partial suppression of one or other of the two $\sigma$ components of the pattern due to instrumental polarization can easily lead to an apparent line displacement. A line such as $\lambda 6302$ is an extreme case and would not be used for velocity measures, but such effects exist on a smaller scale for all lines, except those for which the Zeeman splitting factor is zero. It is only by using such a magnetically unaffected line that we can be sure that the velocities we measure are free from magnetic field effects.

For this reason we have selected the Fe line $\lambda 5576 \cdot 101$ for our present study of sunspot velocity fields. This is a line first suggested by Von Klüber (20), for Servajean's work (Ann. d'Astrophys., 24, 1, 1961), using the line $\lambda 5691 \cdot 508$ (Fe Ni) for which $g = 0$, has been published while the present paper was in the press.
which the Landé splitting factor $g = 0$. But we have now to contend with the observational difficulties presented by such a choice. There are no terrestrial standards in this region, such as the oxygen lines which appear in Plate 6 (a), or the water vapour lines used by Kinman, but the difficulty has been overcome here by introducing absorption lines due to iodine vapour into the solar spectrum. These could not be introduced directly into the sunspot spectrum because the iodine line density is too high but, instead, two exposures were made in rapid succession; the first for the spot spectrum alone and the second for the spot spectrum plus iodine. The iodine lines could then be used to show slit curvature and possible slit inclination, and consequently these effects could be allowed for in the solar exposure. We then assume that at the top and bottom of the slit, far removed from the spot, the line shows only the normal photospheric velocity shift due to solar rotation, plus a possible limb effect, if we take the effect of the photospheric velocity fields to be uniform across the slit. After calculating and allowing for such effects at selected points across the slit, we have the true Evershed effect.

Observations have so far been made on one small and one large spot. Central meridian passages (kindly supplied by the Royal Greenwich Observatory) were September 11.30 and September 3.25 respectively. The observational material, together with its measurement and reduction, are described in the first two sections of the paper and we conclude with a short discussion of the velocity fields now obtained.

1. Observations and measurements.—The sunspot spectra for the investigation were obtained by Professor Plaskett in September 1959. The image radius $p_0$ as given by the new solar telescope was 165 mm and the slit height of 25 mm corresponds to approximately 145 seconds of arc or approximately 105 500 km on the solar surface at the disk centre. The Babcock grating was used in the 5th order with the 40 ft spectrograph, resulting in a linear dispersion of 0.17 A/mm. In each exposure, the slit passed through the spot centre and details of the measured plates are given in Table I. In this table, $p =$ radial distance of the spot from the disk centre and $B$, $L$ are the heliographic latitude and longitude of the spots.

<table>
<thead>
<tr>
<th>Plate</th>
<th>Date 1959</th>
<th>U.T.</th>
<th>Exposure time</th>
<th>B</th>
<th>L</th>
<th>$p/p_0$</th>
<th>Umbral diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>E22</td>
<td>Sept. 7</td>
<td>16h 02m 16s</td>
<td>60 sec</td>
<td>+17° 02'</td>
<td>15° 54'</td>
<td>0.793</td>
<td>46° 34000 km.</td>
</tr>
<tr>
<td>E23</td>
<td></td>
<td>16h 12m 35s</td>
<td>40 sec</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E28</td>
<td>Sept. 9</td>
<td>08h 41m 57s</td>
<td>30 sec</td>
<td></td>
<td></td>
<td>0.597</td>
<td>17° 12000 km.</td>
</tr>
<tr>
<td>E29</td>
<td></td>
<td>08h 49m 38s</td>
<td>31 sec</td>
<td>-18° 10'</td>
<td>274° 50'</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E36</td>
<td>Sept. 10</td>
<td>08h 28m 16s</td>
<td>30 sec</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E37</td>
<td></td>
<td>09h 17m 06s</td>
<td>30 sec</td>
<td></td>
<td></td>
<td>0.471</td>
<td></td>
</tr>
</tbody>
</table>

Measurements were made using the Hilger micrometer with a transverse motion for the plate stage (16). The field of view, perpendicular to the dispersion,
was restricted to 0.3 mm by a diaphragm in the microscope eyepiece, so that this width of spectrum can be measured at any desired height. Since the sunspot spectra contain no atmospheric lines or lines of zero velocity, a ruled glass plate was used to provide a fiduciary line. This was permanently attached to the microscope stage and check settings were made on the ruled line throughout all the micrometer measurements of sunspot and iodine lines. Provided the sunspot spectra and iodine spectra are closely aligned at the same angle with respect to the fiduciary line, the iodine line readings give a correction curve as a function of slit height, by means of which all instrumental shifts may be removed from the observed displacements of the solar line. The six iodine correction curves were identical within the errors of measurement and were combined to give a single curve by means of a least squares solution.

All spectra were measured in the direct and reversed directions and mean readings obtained. The sunspot spectra were measured at over 100 points over the 25 mm slit height at intervals as small as 0.1 mm in the penumbral region, and wider intervals in the outer parts, where we suspect the Evershed velocities are changing less rapidly. A complete measurement of the plates was made in February and the whole set of readings repeated in April as a check, final readings being taken as the mean of these two sets. Although the high dispersion is a great advantage in the work, the solar lines are now so broad as to make visual measurements a difficult matter, especially over the restricted height our diaphragm imposes. A. D. Petford has devised a new photo-electric method of measurement (14) which could be used here to great advantage. In fact, the visual and photoelectric measurements made on plate E28 of our Table I are in very good agreement, as his Fig. 3 shows. The photoelectric measurements however show considerably less scatter in repeated settings and are indisputably quicker and easier to obtain.

The solar line displacements corrected for instrumental effects, as explained above, were then expressed as Doppler shifts, (1 mm displacement = 9.19 km/sec) and the results are shown in Fig. 1. Fig. 1 (a) is the mean of the direct and reversed readings for the February and April measurements and gives the observed curve of the small spot of Plate E28, while Fig. 1 (b) refers to a similar mean of the large spot of E22.

The ordinates in Fig. 1 are arbitrarily set equal to zero for the photosphere on the solar centre side of the spot and expressed in km sec\(^{-1}\), while the abscissa is given by the spectrum height in mm. The approximate extent of the umbra and penumbra is indicated for both spots.

2.1. Reduction to sunspot velocity fields.—In the reduction of the observed velocities, we have followed the scheme proposed by Professor Plaskett (15) and followed by T. D. Kinman (10, 11). The first step in deducing the motion characteristic of the spot is to remove the effect of the observer's motion and to allow for the displacements which we know will arise from solar rotation and limb effect. The Mercury computer of the Oxford University Computing Laboratory was used to calculate the corrections at representative points up the slit height, and interpolations were made for the remaining measured points. The corrections for the observer's motion may be calculated immediately from the observed epoch of exposure and the polar coordinates of the slit. For solar rotation at latitude \(B\), we have used (2)

\[
R_b = 2\left(1 - 0.229 \sin^2 B\right) \cos B \text{ km sec}^{-1}
\]
The limb effect corrections (2) were found to be negligible for the disk positions considered. These corrections show what velocity shift would occur across the slit height in the undisturbed photosphere. The lines in Fig. 1 show the run of these velocity corrections across the slit and are positioned for the small and large spots to give zero photospheric velocity on either side of the spot. It is the shifts above and below these lines which we believe represent the true sight-line velocities in the spot.
To deduce the spatial motion in the spot region from the sight-line velocities, we transform the coordinates of the measured points on the projected disk to polar coordinates \((r, \phi)\) on the solar surface and origin at the centre of the umbra. In this system, \(r\) is the horizontal radius from the centre of the umbra and \(\phi\) is the position angle measured counter clockwise from the direction of solar rotation. If \(\Delta B\) and \(\Delta A\) are the differential heliographic coordinates measured from the centre of the umbra (heliographic coordinates \(B, L\)) then:

\[
r = R_0 \left[ (\Delta \lambda \cos B)^2 + (\Delta B)^2 \right]^{1/2} \quad \text{and} \quad \tan \phi = \frac{\Delta B}{\Delta \lambda \cos B}
\]

where \(R_0\) = radius of the Sun in kilometres.

Motion in the sunspot is conveniently considered in terms of the cylindrical velocity components \(u = \dot{r}, v = r \dot{\phi}, w = \dot{z}\), which are the radial, tangential and vertical velocity components respectively. It can be shown (11) that the sight-line velocity \(V\) is given in terms of these components by the linear relation

\[V = u (\cos \phi \cos \gamma_1 + \sin \phi \cos \gamma_2) + v (\cos \phi \cos \gamma_2 - \sin \phi \cos \gamma_1) + w \cos \gamma_3\]

where \(\cos \gamma_1\) is a direction cosine which depends upon the position of the points of measurement on the disk and is defined (15) as follows:

\[
\cos \gamma_1 = \sin (L - L_0) \cos B_0 \sin (\theta + \theta_1) \cosec \theta
\]

\[
\cos \gamma_2 = [\sin B \cos B_0 \cos (L - L_0) - \cos B \sin B_0] \sin (\theta + \theta_1) \cosec \theta
\]

\[
\cos \gamma_3 = - \cos (\theta + \theta_1)
\]

where \(\theta_1 \approx \frac{\rho - S}{\rho_0}\)

\(S = \) angular semi-diameter of Sun

and

\[
\sin (\theta + \theta_1) \approx \frac{\theta_1}{S}.
\]

Ideally, we should have liked to make a complete least squares solution for the three unknowns \(u, v, w\), but our observational material was insufficient to provide a satisfactory set of equations. (It is hoped to attempt such a complete solution in future investigations.) We have therefore based our calculations on Kinman’s conclusions, which were obtained from 12 least squares solutions for \(u, v, w\), at four values of \(r\) for each of three plates. These showed that the tangential component \(v\) is entirely due to random error and that the vertical component \(w\), relative to the photosphere, is zero.

As a first description of the velocity field, we have then assumed that the motion is entirely a radial flow outward from the centre of the umbra \((v = 0, w = 0)\). Fig. 2 (a) shows the results of the calculation for the small spot of E28, and Fig. 2 (b) is a similar plot for the large spot of E22. Positive velocities are now shown for both sides of the spot, since both refer to outflow velocities. For the small spot, where we have four sections of the same spot, we may go further and attempt a solution for another component of motion. Assuming that the spot has radial symmetry about the centre of the umbra, the two sides of the spot on four plates lead to eight observational equations for the state of motion at any one \(r\) value. Still assuming \(v = 0\) (16, 17), we have used these equations to obtain a least squares solution for finding components \(u\) and \(w\). These solutions were carried out for eight \(r\) values and lead to the results shown in Fig. 3.

We find as Kinman did, that \(w\) is zero within the limits of error. We see also that the values of the radial velocity thus obtained for the four plates are quite similar to those shown in Fig. 3 (a) for plate E28 only.
2.2. Errors.—The error in setting on the solar lines after taking direct and reversed readings and the mean of two sets of readings is approximately 10 microns, which is equivalent to a velocity error of 0.1 km/sec. The comparison of photo-electric and visual measurements on one plate indicated a velocity difference of about 0.15 km/sec in the penumbra (the most difficult place to make a setting) and
the discrepancy was considerably less in the region away from the penumbra. We can thus consider a line setting error $0.15 \text{ km/sec}$ throughout as a generous estimate. No correction has been applied to the observed sight-line velocities for scattered light, but Kinman (10) gives an estimate for a reduction in the observed penumbral velocities of about 9 per cent due to this effect. However, as Plate 6 (a) shows, the definition of our spectra is good and observations have shown that the instrumental scatter for the 35 m telescope is very similar to that for the 19 m telescope used by Kinman.

![Fig. 3.—Least squares solution for the velocity field of small spot.](image)

**Ordinate:** Velocity component in $\text{km sec}^{-1}$ (Radial velocity $u$ above; vertical velocity $w$ below.)
**Abscissa:** Mean distance (for the 4 small spot plates) from spot centre in units of 10,000 km.

**Discussion.**—The general form of motion in the small spot is very like that obtained by Kinman, and within the limits of our observational material, it again appears that the flow is wholly radial. The maximum radial velocity (mean uncorrected value of $1.1 \text{ km sec}^{-1}$) is achieved towards the edge of the penumbra, and we see that the flow extends well beyond the penumbral region, falling to zero at approximately 30,000 km from the spot centre. Considering Kinman's linear relation (11) between maximum radial velocity and spot size, we see that there is fair agreement for the small spot, but our maximum radial velocity for the large spot (mean uncorrected value approx. $0.6 \text{ km sec}^{-1}$) falls well below the value (approx. $3.5 \text{ km sec}^{-1}$) which may be estimated from the linear relation. In addition, the velocity field for the large spot is of quite a different form from that of the small spot. The large spot pattern is much more spread out, falling to zero at approximately 70,000 kms from the spot centre, and it does not show such well defined maxima in the penumbra.

The high dispersion spectra ($0.17 \text{ A/mm}$) reveal a phenomenon (discovered at the McMath-Hulbert Observatory (12)) which is unobservable in the low
dispersion spectra. This feature, which appears in Plate 6 (b) for the large and the small spot, will be referred to as “line-flare”. The appearance of the plate indeed suggests that the main line is undisturbed and that the measured shift is caused by this diffuse “flare”. The line-flare now observed extends over some 120 mA from the centre of the undisturbed line, corresponding to a sight-line velocity up to 6 km sec\(^{-1}\). Evershed (9) first observed that spectral lines showed diffusive widening in the direction of displacement near the penumbral limits, but he remarked that this phenomenon might well be involved with Zeeman widening. The line-flare cannot be due to Zeeman splitting here, but is probably a Doppler effect due to turbulence, with a mean motion of matter away from the umbra (as shown by the flare appearing on opposite sides of the line on either side of the umbra). The same feature has been recently noted in Bumba’s investigation (3) of spot velocity distributions. A complete account of his work (4) has come to our notice during the preparation of this paper. He also suggests that the line-flares (“flags” in his notation) are a result of Doppler shifts, and that the line as a whole is not shifted. In low dispersion work, such as Kinman’s, the line-flare and the undisturbed line would be blended and consequently the amalgamated flare would contribute in a different way to the measured Doppler shift. This phenomenon could well account for the differences observed in velocity displacements in high and low dispersion investigations. In measuring the high dispersion plates, it is difficult to say exactly to what part of the line the micrometer setting refers, but presumably (Plate 6 (b)) it is closer to the undisturbed line than would be possible with lower dispersion.

The plates used here for radial velocities are not calibrated for photometry, but high dispersion calibrated plates have been obtained in the early summer of this year for a full-scale study of the intensity and extent of line-flare, and to investigate more fully the part it plays in our measurement of sunspot velocity fields. Further possible considerations in such investigations are the effects of spot age and disk position (13, 17) on the sunspot velocity components.

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References

A study of sunspot velocity fields

(a) Zeeman splitting in Fe λ 6302.
(b) "Line-flare" in large and small spots.

J. Holmes, A study of sunspot velocity fields using a magnetically undisturbed line.