Networks and Search for Goods in Macroeconomics



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Wadham College

University of Oxford

A thesis submitted for the degree of

Doctor of Philosophy in Economics

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Abstract

This dissertation consists of three self-contained chapters which discuss topics related to production networks and goods market search in macroeconomics. Chapter 1 offers novel econometric evidence on the contribution of production networks to the effect of monetary shocks on real macroeconomic variables. Using a highly disaggregated monthly dataset on US final sectoral consumption it is estimated that 30% of the effect of monetary shocks on aggregate consumption comes from amplification through input-output linkages. Such network amplification rises in the frequency of price non-adjustment and intermediates intensity. Chapter 2 develops a general theory of state-dependent fiscal multipliers in a framework featuring two empirically relevant frictions: idle capacity and unsatisfied demand. The key novel finding is that the source of economic fluctuations determines the cyclicality of multipliers. Demand-side policies, such as government spending, have multipliers that are large in demand-driven recessions, but small in supply-driven downturns. Conversely, supply-side policies, such as cuts in payroll taxes, are ineffective in demand-driven recessions, but powerful if the downturn is supply-driven. The theoretical findings are supported by empirical evidence using US data. Finally, Chapter 3 develops a sticky-price model, where input-output linkages are formed endogenously and in a manner consistent with sectoral and firm-level data. The model simultaneously rationalizes the following observed non-linearities in monetary transmission. First, the model produces cycle dependence: the magnitude of real GDP's response to a monetary shock is procyclical. Second, there is path dependence: non-neutrality of real GDP is higher following previous periods of loose monetary policy. Third, there is size dependence: larger monetary expansions make the network denser and have a disproportionally larger effect on GDP than smaller expansions.

Supervisor This thesis was supervised by Professor Guido Ascari and Dr Francesco Zanetti.

Statement of authorship The work in this thesis is based on my work at the University of Oxford between April 2017 and May 2022. Chapter 1 is single-authored work, whose earlier version was submitted in partial fulfillment of the requirements for the degree of Master of Philosophy in Economics at the Department of Economics, University of Oxford. Chapter 2 is co-authored with Francesco Zanetti. Chapter 3 is single-authored work.

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Glory to Ukraine! Glory to the Heroes!

Mishel Ghassibe
Oxford, May 2022

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Chapter 1

Monetary Policy and Production Networks: an Empirical Investigation

Abstract This paper offers novel econometric evidence on the contribution of production networks to the effect of monetary shocks on real macroeconomic variables. In particular, we construct a highly disaggregated monthly dataset on US final sectoral consumption to estimate that at least 30% of the effect of monetary shocks on aggregate consumption comes from amplification through input-output linkages, which facilitate downstream propagation of price rigidity. At the sectoral level, we find that the network effect rises in the frequency of price non-adjustment and intermediates intensity. Moreover, the network effect is highly concentrated: sectors that jointly account for 17% of our sample aggregate consumption account for 98% of the amplification. In order to develop our econometric specification, we obtain novel analytical sector-level solutions to a forward-looking New Keynesian model with asymmetric input-output linkages.

1.1 Introduction

How different economic sectors are interlinked and affect each other has long been a question of interest to economists – at least since Wassily Leontief's invention of input-output analysis in the 1950s. Nevertheless, it is only in recent years that input-output linkages, or production networks, have firmly established their presence in neoclassical macroeconomics. Indeed, production networks have recently shed light on a range of fundamental economic questions. Most notably, Acemoglu et al. (2012) build on the multi-sector framework of Long and Plosser (1983), and theoretically show that when certain sectors are disproportionally large suppliers of inputs to the rest of the economy, sectoral shocks can get amplified by input-output linkages, causing aggregate business cycle fluctuations. Such role for input-output linkages in creating aggregate fluctuations has since found support in the empirical literature (Foerster et al., 2011; Acemoglu et al., 2016).

On the side of aggregate nominal shocks, production networks have been studied as a mechanism that enables New Keynesian models to simultaneously match microeconomic evidence on frequency of price adjustment and macroeconomic evidence on strength of monetary transmission to real variables.[1] Basu (1995) theoretically establishes that monetary policy shocks get amplified in presence of production networks, as the latter create strategic complementarities in firms' price setting. Intuitively, the latter makes firms inherit price stickiness from their suppliers, thus creating extra short-run money non-neutrality. The above theoretical mechanism has since been studied in fully specified multi-sector quantitative models (Nakamura and Steinsson, 2010; Pasten et al., 2020; Carvalho et al., 2021). However, econometric evidence on the ability of production networks to amplify the effects of monetary policy shocks on the real economy remains almost non-existent. In what follows we make a first step towards filling this gap in the literature.

[1] As discussed in Mackowiak and Smets (2008), in order to match VAR-based estimates of the effect of monetary shocks on real variables, a New Keynesian model with no additional real rigidities needs to be calibrated with a frequency of price adjustment much lower than that observed in microeconomic studies.

Our paper is the first study to offer econometric evidence on the role of production networks in creating short-run monetary non-neutrality of real macroeconomic variables. In particular, our empirical contribution is in establishing the following results. First, at least 30 per cent of the effect of monetary policy shocks on US real aggregate final consumption is due to production networks, which facilitate downstream propagation of price rigidity. This result is robust to varying the degree of sectoral disaggregation and the type of monetary shocks used. Second, we document substantial heterogeneity in network amplification across sectors and further examine its dimensions. Our cross-sectional analysis suggests that the network effect is, *ceteris paribus*, larger for sectors that adjust prices less frequently and those that rely more on intermediate inputs; moreover, we rationalize such results through the lens of our theoretical model. Third, we show that the aggregate network contribution is highly concentrated in the sense of being driven by a relatively small number of sectors; quantitatively, sectors that jointly account for 17 per cent of our sample aggregate consumption account for 98 per cent of the amplification by production networks.[2]

Our econometric specification is derived from a multi-sector New Keynesian model with asymmetric input-output linkages, and we make two theoretical contributions that allow us to form a tractable econometric strategy. First, we formally show that the response of sectoral final consumption to a monetary shock can be decomposed into the *direct effect*, driven by that sector's own frequency of price non-adjustment, and the *downstream effect*, driven by persistence in price setting inherited from supplier sectors and the intensity with which intermediates are used. It is the downstream effect that represents network amplification of monetary shocks. Second, we consider a finite-time version of our model, where we use backward induction in order to find novel analytical solutions for sectoral final consumptions. Those solutions directly imply a simple econometric strategy based on linear regressions that allows to consistently estimate a lower bound for

[2]Specifically, those broad sectoral groups are "Transportation and warehousing", "Retail trade", "Information", "Agriculture, forestry, fishing and hunting" and "Professional and business services".

the downstream effect. Our analytical solutions introduce a novel concept of the *Forward-looking Leontief Inverse*, which can be used to find analytical solutions to other problems featuring production networks and nominal rigidities in a forward-looking setting. Previous studies, such as Pasten et al. (2020), instead rely on myopic firms or other deviations from forward-looking behavior in order to obtain sector-level closed-form results.

Estimating our econometric specification requires sectoral final consumption time series whose classification matches that used in input-output accounts. Such time series are not readily available for the US, and our paper makes a further contribution by constructing a novel monthly dataset for sectoral final consumptions for up to 161 sectors of the US economy, whose classification matches that in the US Bureau of Economic Analysis (BEA) input-output accounts. In particular, we take monthly product-level consumption and consumer price time series contained in NIPA tables and use PCE Bridge Tables to match NIPA product definitions to sectoral classifications of the US BEA. In this way, our sectoral final consumption series are consistent in their classification with the sectoral classification used by BEA for their input-output-tables, and can be used together in our econometric analysis. To the best of our knowledge, we are the first ones to construct a monthly dataset for sectoral final consumption that uses classification consistent with BEA input-output accounts.

Contribution to the literature. Our results should be viewed in the context of at least two strands of existing literature.[3] First, we contribute to the empirical literature that studies the extent to which production networks amplify the effects of economic shocks. On the side of sector-specific shocks, Acemoglu et al. (2016) identify sector-specific fiscal, imports and productivity shocks and show that the vast majority of their non-negligible impact on US sectoral value added comes from the amplification via input-output linkages. Barrot and Sauvagnat (2016) identify even more disaggregated firm-level shocks via the occurrence of natural disasters and similarly report large spillovers from suppliers of inputs to their customers.

[3]See Carvalho and Tahbaz-Salehi (2019) for a comprehensive review of the literature on production networks in macroeconomics.

The empirical literature on amplification of aggregate nominal shocks remains extremely thin. One exception is Ozdagli and Weber (2017) who use spatial econometric techniques to show that 50 to 85 per cent of the effect of monetary policy shocks on US stock returns comes from the amplification through production networks. Our work makes a contribution in at least two substantial ways. First, we focus on the effect of monetary shocks on real macroeconomic variables, most specifically final consumption, whereas Ozdagli and Weber (2017) study stock market responses. Second, we empirically assess a completely different theoretical mechanism: our specification estimates contribution of production networks that comes from complementarities in price setting and downstream propagation of price rigidity, whereas Ozdagli and Weber (2017) build their specification in a flexible-price framework and consider upstream propagation of demand created by monetary shocks and the effect on stock market returns.

Secondly, our results add to the quantitative theory literature on complementarities in price setting and monetary policy under production networks. Following the seminal study by Basu (1995), Carvalho (2006) shows in a DSGE setting that when the degree of price stickiness is allowed to vary across sectors, complementarities in price setting significantly increase the degree of money non-neutrality. Nakamura and Steinsson (2010) develop a multi-sector menu cost model that they calibrate for up to 14 US sectors and report large increases in short-run money non-neutrality coming from differential price stickiness across sectors coupled with intermediate inputs. Pasten et al. (2020) develop a multi-sector model with sector-specific probabilities of price adjustment that they calibrate to 341 sectors of the US economy and confirm large amplifications of monetary policy shocks coming from strategic complementarities in price setting. As for optimal monetary policy under production networks, it is studied in La'O and Tahbaz-Salehi (2019).

Several recent studies have used a mixture of aggregate and sectoral data to explicitly estimate multi-sector New Keynesian models with roundabout production and heterogeneous price stickiness. Most notably, Smets et al. (2019) use Bayesian techniques to estimate a 35-sector model on US data, and

subsequently study the role of production networks for inflation dynamics; Carvalho et al. (2021) estimate sector-specific probabilities of price adjustment for 27 major sectors of the US economy, and show that the estimated model successfully replicates sectoral price responses to both idiosyncratic and aggregate shocks from Boivin et al. (2009). In addition, Bouakez et al. (2014) estimate a 30-sector menu cost model with roundabout production, and show that the implied sectoral frequencies of price adjustment match those from micro-based studies (Nakamura and Steinsson, 2008).

The rest of the paper is structured as follows. In Section 1.2 we build a theoretical framework and derive analytical solutions for sectoral consumptions that informs our econometric specification. Section 1.3 outlines the construction of our dataset. Our econometric strategy is detailed in Section 1.4. Sections 1.5 and 1.6 discuss the main empirical results of this paper, whose robustness is then tested in Section 1.7. Section 1.8 concludes and outlines our strategy for future work.

1.2 Model

In this section, we formalize our ideas in the context of a multi-sector New Keynesian model with production networks. Our theoretical contribution is twofold. First, we formally show that the response of sectoral consumption to a monetary shock can be decomposed into the *direct effect*, driven by sector's own price stickiness, and the *downstream effect*, driven by stickiness of suppliers and the intensity with which intermediates are used. Second, we find novel analytical solutions for sectoral final consumptions by considering a version of our model in finite time; all proofs are given in Appendix 1.B.

1.2.1 Firms

On the production side, our economy consists of K sectors, indexed by $k \in \{1, 2, ..., K\}$, and a continuum of monopolistically competitive firms, indexed by j, that each belongs to one sector; let Φ_k denote the set of all

firms in sector k. The production function of firm j that operates in sector k is given by:

$$Y_{kt}(j) = N_{kt}(j)^{1-\delta_k} Z_{kt}(j)^{\delta_k} - \Xi_k(j), \tag{1.1}$$

where $Y_{kt}(j), N_{kt}(j)$ and $Z_{kt}(j)$ are, respectively, total output, labor input and intermediate inputs used, δ_k is intermediates intensity that is allowed to be sector-specific, $\Xi_k(j)$ is a fixed cost which is set to equalize steady-state profits to zero and rule out entry and exit.

Total intermediate inputs used is an aggregate of intermediate goods purchased from all the different sectors in the economy, so that $Z_{kt}(j) \equiv \prod_{r=1}^{K} \omega_{kr}^{-\omega_{kr}} Z_{krt}^{\omega_{kr}}(j)$, where $Z_{krt}(j)$ is the intermediate inputs purchased by firm j in sector k from all firms in sector r; ω_{kr} is the relative intensity with which firms in sector k use goods produced in sector r as inputs, so that $\sum_{r=1}^{K} \omega_{kr} = 1 \quad \forall k$. In turn, $Z_{krt}(j)$ is also an aggregator for all the inputs purchased from the different firms that belong to sector r, namely $Z_{krt}(j) \equiv \left(\int_{\Phi_r} Z_{krt}(j,j')^{\frac{\theta-1}{\theta}} dj' \right)^{\frac{\theta}{\theta-1}}$, where $Z_{krt}(j,j')$ denotes total inputs purchased by firm j in sector k from firm j' in sector r.

Given the sectoral nominal wage W_{kt} and the input price index $P_t^k, \quad P_t^k \equiv \prod_{r=1}^{K} P_{rt}^{\omega_{kr}}$, firms choose $N_{kt}(j)$ and $Z_{kt}(j)$ in order to minimize their total inputs cost subject to the production function in (1.1). The latter delivers the the optimal labor/intermediate inputs mix condition $\frac{Z_{kt}(j)}{N_{kt}(j)} = \frac{\delta_k}{1-\delta_k} \frac{W_{kt}}{P_t^k}$ and for the nominal marginal cost, common to all firms within a given sector:

$$MC_{kt}(j) = MC_{kt} = \frac{\delta_k^{-\delta_k}}{(1-\delta_k)^{1-\delta_k}} W_{kt}^{1-\delta_k} \left[\prod_{r=1}^{K} P_{rt}^{\omega_{kr}} \right]^{\delta_k}. \tag{1.2}$$

One can see that the marginal cost for any firm is effectively a weighted average of its labor input cost, given by the wage, and its sector-specific inputs price index, whose relative importance increases in the share of intermediate inputs δ_k. It can also be seen how $\delta_k > 0$ makes the marginal cost of any firm sensitive to prices set by its suppliers. This is the mechanism

through which production networks create complementarities in price setting, which in turn amplifies the degree of monetary non-neutrality in our economy.

Price stickiness in our economy is modeled as in Calvo (1983), although the probability of price adjustment is allowed to vary across sectors. More precisely, in any period a firm in sector k has probability $(1-\alpha_k)$ of setting its price equal to its optimal value. The optimal price at time t is chosen to maximize expected future discounted nominal profits:

$$\max_{P_{kt}(j)} \mathbb{E}_t \sum_{s=0}^{\infty} \alpha_k^s F_{t,t+s}(P_{kt}(j)Y_{k,t+s}(j) - W_{k,t+s}N_{k,t+s}(j) - P_{t+s}^k Z_{k,t+s}(j)), \quad (1.3)$$

subject to the production function in (1); $F_{t,t+s}$ is the stochastic discount factor between periods t and $t+s$ and is defined in the next subsection. After imposing the optimal inputs mix condition and the marginal cost equation from (1.2), the first order condition for the optimal reset price for any firm in sector k, P_{kt}^* is given by $P_{kt}^* = \frac{\theta}{\theta-1}\frac{\mathbb{E}_t \sum_{s=0}^{\infty} \alpha_k^s F_{t,t+s} P_{k,t+s}^{\theta} Y_{k,t+s} MC_{k,t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \alpha_k^s F_{t,t+s} P_{k,t+s}^{\theta} Y_{k,t+s}}$. Given that the optimal reset price is identical for all firms within a sector, sectoral price index can be obtained by simple aggregation: $P_{kt} = \left[\alpha_k P_{k,t-1}^{1-\theta} + (1-\alpha_k)(P_{kt}^*)^{1-\theta}\right]^{\frac{1}{1-\theta}}$.

1.2.2 Households

A continuum of infinitely lived households populates our economy and owns all the firms. Markets are assumed to be complete, so a full set of Arrow-Debreu securities is available. The representative household makes choices to maximize the lifetime utility:

$$\max_{\{C_{t+s},\{N_{k,t+s}\}_{k=1}^K, B_{t+s+1}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\ln(C_{t+s}) - \sum_{k=1}^{K} g_k N_{k,t+s}\right] \quad (1.4)$$

subject to

$$P_t^c C_t + \mathbb{E}_t\left[F_{t,t+1}B_{t+1}\right] \leq B_t + \sum_{k=1}^{K} W_{kt}N_{kt} + \sum_{k=1}^{K} \Pi_{kt}, \quad \forall t \quad (1.5)$$

where C_t and P_t^c are the composite consumption good and the consumption price index (defined below) respectively, N_{kt} is sectoral labor supply, B_{t+1} is the stochastic payoff of securities purchased at time t, $F_{t,t+1}$ is the stochastic discount factor to price those securities at time t and Π_{kt} denotes aggregate nominal profits of firms in sector k; β is the discount factor for future utility and g_k is a parameter capturing the relative disutility of sector-specific labor supply.

The composite consumption index C_t is an aggregator for the final consumption of goods produced in the different sectors of our economy, so that $C_t \equiv \prod_{k=1}^{K} \omega_{ck}^{-\omega_{ck}} C_{kt}^{\omega_{ck}}$, where ω_{ck} is the relative weight consumers put on goods produced in sector k, $\sum_{k=1}^{K} \omega_{ck} = 1$; C_{kt} is in turn an aggregator for the final consumption of goods produced by firms that belong to sector k: $C_{kt} \equiv \left(\int_{\Phi_r} C_{kt}(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}$.

The households' maximization problem delivers standard first-order conditions, namely an equation for the stochastic discount factor $F_{t,t+s} = \beta^s \left(\frac{C_{t+s}}{C_t} \right)^{-1} \frac{P_t^c}{P_{t+s}^c}$, and equations for consumption-sectoral labor supply choice $C_t^{-1} = g_k \frac{P_t^c}{W_{kt}}, \quad \forall k$, where the consumption price index is given by $P_t^c = \prod_{k=1}^{K} P_{kt}^{\omega_{ck}}$.

1.2.3 Monetary policy

Following Nakamura and Steinsson (2010), we assume that the monetary authority acts in a way that ensures that nominal GDP, given by $M_t \equiv P_t^c C_t$, follows an exogenous process:

$$\ln M_t = \ln M_{t-1} + r_t, \tag{1.6}$$

where $r_t = \rho_r r_{t-1} + \varepsilon_t^r$ defines an AR(1) process for monetary policy shocks, and innovations $\{\varepsilon_t^r\}_t$ constitute a zero-mean white noise process with variance σ_r^2. The above can be rationalized by imposing a cash-in-advance constraint on aggregate final consumption, and assuming a constant velocity of money. As discussed in Nakamura and Steinsson (2013),

under such assumptions the process in (1.6) is equivalent to a rule for money supply.[4]

For the remainder of the main text we focus on the simplified case of non-persistent monetary shocks ($\rho_r = 0$); however, in Appendix 1.B we provide formal analytical results for the case of persistent shocks ($|\rho_r| \in (0, 1)$).

1.2.4 Market clearing

In addition to the optimality conditions, budget constraints and the policy rule above, equilibrium in our economy is also characterized by market-clearing conditions in the asset market: $B_t = 0$; the labor markets: $N_{kt} = \int_{\Phi_k} N_{kt}(j)dj, \quad \forall k$; and the goods markets: $Y_{kt}(j) = C_{kt}(j) + \sum_{r=1}^{K} \int_{\Phi_r} Z_{rkt}(j', j)dj', \quad \forall k, \forall j \in \Phi_k$.

1.2.5 Log-linearized equilibrium

We log-linearize the equilibrium conditions from the previous subsections around a symmetric zero-inflation steady-state. In Appendix 1.A we fully characterize both the steady state and the log-linearized system; here we instead focus on the resulting sectoral pricing equations (lower case letters denote log-deviations from the steady state):

$$p_{kt} = \alpha_k p_{k,t-1} + (1-\delta_k)(1-\alpha_k)m_t + \delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\omega_{kr}\mathbb{E}_t p_{r,t+s}, \tag{1.7}$$

for $k = 1, 2, ..., K$. One can clearly see that every sectoral price is now positively related to every other sectoral price in the economy, both current and those expected in future periods, weighted by the input-output entry between the respective sectors and increasing in the share of intermediate inputs δ_k. As a result, persistence in price adjustment can travel from supplier sectors to customer sectors, leading to slower adjustment of sectoral prices.

[4] An alternative would be to close our model with a Taylor rule for the nominal interest rate, which comes at the cost of analytical tractability. In Section 1.7 we report numerical exercises with a Taylor rule.

In addition, given the slower adjustment of sectoral prices, the degree of short-run money non-neutrality rises and the effect on real variables, such as consumption, becomes larger in the short run. The following proposition formalizes the above statement for the sectoral consumption:

Proposition 1.1. *The response of final consumption in sector k c_{kt} to a monetary shock r_t is given by:*

$$c_{kt} = \overbrace{\alpha_k c_{k,t-1} + \alpha_k r_t}^{\textbf{Direct effect}} + \underbrace{\delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\omega_{kr}\mathbb{E}_t c_{r,t+s}}_{\textbf{Downstream effect}}, \quad (1.8)$$

for $k = 1, 2, ..., K$. It consists of the the direct effect, unaffected by the intermediate inputs share δ_k, and the downstream effect, which fully vanishes under $\delta_k = 0$ and increases in δ_k, representing amplification through input-output linkages.

Proposition 1.1 establishes that the effect of a monetary shock r_t on sectoral final consumption can be decomposed into the *direct effect*, caused by that sector's own price stickiness and hence independent of δ_k, and the *downstream effect*, caused by price stickiness inherited from that sector's suppliers and hence increasing in δ_k and disappearing under $\delta_k = 0$. It is therefore the downstream effect that represents the amplification of the effect of monetary shocks through input-output linkages. Intuitively, the direct effect strictly increases in sector's own probability of price non-adjustment α_k. As for the downstream effect, two opposing channels of own price stickiness are at work. On the one hand, higher α_k reduces sector k's price sensitivity to its current and expected future marginal costs, making it harder to inherit stickiness from suppliers, thus dampening the downstream effect; on the other hand, higher α_k makes sector $k's$ optimal price more sensitive to expected future marginal costs (as any price set is more likely to persist in the future), thus amplifying the downstream effect.

A natural next step is to find a solution for sectoral consumption that can then be used to empirically disentangle the direct and downstream effects. Due to a number of asymmetries, however, such as heterogeneous input-output weights $\{\omega_{kr}\}_{k,r}$ and Calvo parameters $\{\alpha_k\}_k$ one cannot easily find

a sector-level analytical solution to the above problem. However, in the next subsection we show how a slight modification to our model, featuring finite time, enables us to find analytical solution for sectoral consumption.[5]

1.2.6 Analytical solution in finite time

In this subsection we propose a slight modification to our original model that allows to find analytical solutions for sectoral price indices and final consumptions. In particular, we assume that the economy exists for a finite deterministic number of time periods $T > 1$, and firms face a Calvo lottery between periods 1 and $T-1$, whereas in the final period T all firms get to adjust their prices. The assumptions are summarized below:

Assumption 1.1. *The economy exists for a finite deterministic number of periods* $T > 1$.

Assumption 1.2. *In periods t=1,2,...,T-1 every firm faces a Calvo lottery with a sector-specific probability of price non-adjustment given by* α_k*; in period* T *all firms get to set their prices optimally.*

Note that the above two assumptions have no implications for the households' problem. In particular, the intratemporal consumption-labor supply choice equations remain unchanged for all $t = 1, 2, ..., T$, and the equation for the stochastic discount factor remains valid for any $1 \leq t, t+s \leq T-1$.

Slight modifications happen on the firms' side, however. In period $t = T$, the representative firm in sector k solves a static problem of choosing price $P_{kT}(j)$ to maximize contemporaneous profits $\Pi_{kT}(j) = (P_{kT}(j)Y_{kT}(j) - W_{kT}N_{kT}(j) - P_T^k Z_{kT}(j))$, subject to the production function in (1); given no restrictions on price setting, ever firm sets the same optimal price at a constant mark-up to the marginal cost, so that $P_{kT} = P_{kT}^* = \frac{\theta}{\theta-1} MC_{kT}, \quad \forall k$, or $p_{kT} = p_{kT}^* = mc_{kT} = m_T$ in log-linear terms. Consequently, money is neutral at $t = T$, and $c_{kT} = c_T = 0, \forall k$.

[5] In Appendix 1.C we numerically solve a simple calibrated three-sector ($K = 3$) example in the infinite-horizon case, allowing us to gain further intuition behind monetary transmission and the role of production networks.

In period $1 \leq t \leq T-1$, the representative firm in sector k chooses its price to maximize expected future discounted nominal profits between periods t and $T-1$, as in T it will get to re-set prices with certainty. The latter delivers a familiar expression for the optimal price, shared by all firms in sector k: $P_{kt}^{*} = \frac{\theta}{\theta-1}\frac{\mathbb{E}_t \sum_{s=0}^{T-t-1} \alpha_k^s F_{t,t+s} P_{k,t+s}^{\theta} Y_{k,t+s} MC_{k,t+s}}{\mathbb{E}_t \sum_{s=0}^{T-t-1} \alpha_k^s F_{t,t+s} P_{k,t+s}^{\theta} Y_{k,t+s}}, \quad 1 \leq t \leq T-1, \quad \forall k$, or, when written in log-linear terms: $p_{kt}^{*} = \frac{1-\beta\alpha_k}{1-(\beta\alpha_k)^{T-t}} \sum_{s=0}^{T-t-1} (\beta\alpha_k)^s \mathbb{E}_t mc_{k,t+s}, \quad 1 \leq t \leq T-1, \quad \forall k$. In every period $1 \leq t \leq T-1$ every firm is still subject to the Calvo lottery with a sector-specific probability of price non-adjustment α_k. Given the optimal sectoral price p_{kt}^{*}, the sectoral price index is then given by $p_{kt} = \alpha_k p_{k,t-1} + (1-\alpha_k) p_{kt}^{*}$, $1 \leq t \leq T-1, \forall k$.

One can now obtain the finite-time equivalent of the sectoral price equation (7), highlighting complementarities in price setting; for compactness, we let $\mathbb{p}_t \equiv [p_{1t}, p_{2t}, ..., p_{Kt}]'$, $\mathbb{m}_t \equiv [m_t, m_t, ..., .m_t]'$, $A \equiv diag[\alpha_1, \alpha_2, ..., \alpha_K]'$, $\Delta \equiv diag[\delta_1, \delta_2, ..., \delta_K]'$, $G_{T-t} \equiv (I - \beta A)(I - (\beta A)^{T-t})^{-1}$ and write it in matrix form as:

$$\mathbb{p}_t = A\mathbb{p}_{t-1} + (I-\Delta)(I-A)\mathbb{m}_t + \Delta(I-A)G_{T-t} \sum_{s=0}^{T-t-1} (\beta A)^s \Omega \mathbb{E}_t \mathbb{p}_{t+s} \quad (1.9)$$

for $1 \leq t \leq T-1$. Further, recall that $c_{kt} = p_t^c + c_t - p_{kt} = m_t - p_{kt}$, and letting $\mathbb{c}_t \equiv [c_{1t}, c_{2t}, ..., c_{Kt}]', \mathbb{r}_t \equiv [r_t, r_t, ..., .r_t]'$ we can write the finite-time equation for sectoral consumption as:

$$\mathbb{c}_t = A\mathbb{c}_{t-1} + A\mathbb{r}_t + \Delta(I-A)G_{T-t} \sum_{s=0}^{T-t-1} (\beta A)^s \Omega \mathbb{E}_t \mathbb{c}_{t+s} \quad (1.10)$$

for $1 \leq t \leq T-1$; which is just the finite-time version of the direct-downstream effect decomposition from Proposition 1.1.

It should now be clear how equations (1.9) and (1.10) allow us to use backward induction to find analytical solutions for sectoral prices and consumptions respectively. Here we focus on the case of consumptions, although the same procedure can be applied to prices. According to (1.10), at $t = T-1$ sectoral consumptions vector is given by:

$$\mathbb{c}_{T-1} = A\mathbb{c}_{T-2} + A\mathbb{r}_{T-1} + \Delta(I-A)G_1 \Omega \mathbb{c}_{T-1}, \quad (1.11)$$

and it follows that $\mathbb{c}_{T-1} = [I - \Delta(I - A)G_1\Omega]^{-1}A\mathbb{c}_{T-2} + [I - \Delta(I - A)G_1\Omega]^{-1}A\mathbb{r}_{T-1}$. Similarly, at $t = T - 2$:

$$\mathbb{c}_{T-2} = A\mathbb{c}_{T-3} + A\mathbb{r}_{T-2} + \Delta(I - A)G_2[\Omega\mathbb{c}_{T-2} + (\beta A)\Omega\mathbb{E}_{T-2}\mathbb{c}_{T-1}], \tag{1.12}$$

but note that we have already solved for $\mathbb{c}_{T-1}$ above, so $\mathbb{E}_{T-2}\mathbb{c}_{T-1}$ can be easily found, allowing to find a solution for $\mathbb{c}_{T-2}$. One can then repeat this procedure all the way back to $\mathbb{c}_1$, and find analytical solutions along the way. All in all, one can use the strong induction principle to show that analytical solution for generic $\mathbb{c}_t, 1 \leq t \leq T-1$, can be written down using the notion of the *Forward-looking Leontief Inverse*, as formally introduced in the following proposition.

Proposition 1.2. *The response of final sectoral consumption c_{kt} to a monetary policy shock r_t is*

$$c_{kt} = \sum_{r=1}^{K} l_{kr,T-t}\alpha_r c_{r,t-1} + \tilde{\alpha}_{k,T-t}r_t, \qquad 1 \leq t \leq T-1 \tag{1.13}$$

where $\tilde{\alpha}_{k,T-t}$ is

$$\tilde{\alpha}_{k,T-t} \equiv \alpha_k + (l_{kk,T-t} - 1)\alpha_k + \sum_{r \neq k} l_{kr,T-t}\alpha_r, \tag{1.14}$$

and $\{l_{kj,T-t}\}_{kj}$ are entries of the Forward-looking Leontief Inverse:

$$L_{T-t} = \underbrace{\left[I - \Delta(I - A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s\Omega\left\{\Pi_{j=T-t-s}^{T-t-1}L_jA\right\}\right]^{-1}}_{\text{Forward-looking Leontief Inverse}}, \tag{1.15}$$

where $A \equiv diag[\alpha_1, \alpha_2, ..., \alpha_K]'$, $\Delta \equiv diag[\delta_1, \delta_2, ..., \delta_K]'$, $G_{T-t} \equiv (I - \beta A)(I - (\beta A)^{T-t})^{-1}$ and Ω is the input-output matrix.

Note that for $\delta_k = 0, \forall k$, the Forward-looking Leontief Inverse collapses to the identity matrix, so that $L_{T-t} = I, \forall 1 \leq t \leq T-1$, and $c_{kt} = \alpha_k c_{k,t-1} +$

$\alpha_k r_t$, which is exactly the *direct effect* defined earlier. However, for $\delta_k > 0$ the coefficients on lagged consumption and monetary policy shocks are instead given by a weighted sum of a sector's own Calvo parameter and the Calvo parameters of its suppliers.

1.2.7 Analytical Impulse Response Functions

Having obtained analytical solutions for sectoral consumptions, we can now solve for the horizon-specific impulse responses of sectoral consumptions to a monetary shock:

Corollary 1.1. *[Full IRF] Suppose the economy is in steady state until the beginning of period t; the impulse response of final sectoral consumption c_{kt}, $k = 1, 2, ..., K$ at horizon $H \geq 0$, following a one-time monetary policy shock r_t is given by:*

$$c_{k,t+H} = \tilde{\tilde{\alpha}}^{H}_{k,T-t} r_t, \tag{1.16}$$

where $1 \leq t, t+H \leq T-1$, $\tilde{\tilde{\alpha}}^{H}_{k,T-t} \equiv \sum_{r=1}^{K} \left[\Pi_{j=T-t-H}^{T-t} L_j A\right]_{kr}$, L_j is period-j Forward-looking Leontief Inverse, $A \equiv diag[\alpha_1, \alpha_2, ..., \alpha_K]'$. Therefore, $\frac{dc_{k,t+H}}{dr_t} = \tilde{\tilde{\alpha}}^{H}_{k,T-t}$.

In other words, in a linear projection of final sectoral consumption at $t+H$ on a monetary policy shock at t, the coefficient on the latter gives the horizon-H impulse response.

Moreover, one can derive an alternative representation of the horizon-specific impulse response of sectoral final consumption, which allows to separate out an upper bound of the *direct effect*, as defined in Proposition 1.1. The next corollary formalizes this statement:

Corollary 1.2. *[Upper Bound of Direct IRF] Suppose the economy is in steady state until the beginning of period t; the impulse response of final sectoral consumption c_{kt}, $k = 1, 2, ..., K$ at horizon $H \geq 1$, following a one-time monetary policy shock r_t can be written as follows:*

$$c_{k,t+H} = \left\{\alpha_k^{H+1} + \delta_k q_{T-t-H,H}^k\right\} r_t + \sum_{\tau=1}^{H} \delta_k s_{T-t-H+\tau}^k \sum_{r=1}^{K} \omega_{kr} c_{r,t+H-\tau}, \quad (1.17)$$

where $1 \leq t, t+H \leq T-1$, $q_{T-t-H,H}^k \equiv \sum_{j=1}^{K} [\mathbb{Q}_{T-t-H,H}]_{kj} \geq 0$, $s_{T-t-H+\tau}^k \equiv \alpha_k^\tau (1-\alpha_k) g_{T-t-H+\tau}, g_{T-t-H+\tau} \equiv [G_{T-t-H+\tau}]_{kk}, \mathbb{Q}_{T-t,\tau} \equiv \left\{(I-A)G_{T-t}\Omega\left\{\Pi_{j=T-t}^{T-t+\tau} L_j A\right\} + \sum_{l=0}^{\tau} A^l \mathbb{R}_{T-t+l,\tau-l}\right\}, \mathbb{R}_{x,y} \equiv (I - A)G_x\left[\sum_{s=1}^{x-1}(\beta A)^s \Omega\left\{\Pi_{j=x-s}^{x+y} L_j A\right\}\right]$. *Hence,* $\frac{\partial c_{k,t+H}}{\partial r_t} = \alpha_k^{H+1} + \delta_k q_{T-t-H,H}^k \geq \alpha_k^{H+1}$.

Verbally, Corollary 1.2 establishes that in a linear projection of sectoral final consumption at $t+H$ on a monetary policy shock at t *and* a weighted average of all other sectors' final consumptions between t and $t+H-1$, the coefficient on the monetary policy shock $\left(\alpha_k^{H+1} + \delta_k q_{T-t-H,H}^k\right)$ is an upper bound for the true horizon-H direct effect $\left(\alpha_k^{H+1}\right)$.

Importantly, since in Corollary 1.1 $\tilde{\tilde{\alpha}}_{k,T-t}^H$ gives the sectoral full effect, and in Corollary 1.2
$\left(\alpha_k^{H+1} + \delta_k q_{T-t-H,H}^k\right)$ gives an *upper bound* for the sectoral direct effect, the difference between the two, given by $\left[\tilde{\tilde{\alpha}}_{k,T-t}^H - \left(\alpha_k^{H+1} + \delta_k q_{T-t-H,H}^k\right)\right]$, gives a *lower bound* for the sectoral downstream effect.[6]

In Section 1.4 we use the analytical expressions in Corollaries 1.1 and 1.2 to directly inform our econometric specification. However, before that, we outline the construction of our dataset.

1.3 Data

This section contains an overview of the data used in our empirical analysis. More details are given in the Appendix.

1.3.1 Sectoral consumption

We use the underlying detail tables of NIPA accounts published by the United States Bureau of Economic Analysis (BEA) to obtain monthly sectoral nominal Personal Consumption Expenditure (PCE) and PCE Price In-

[6] Appendix 1.B formally extends Corollaries 1.1 and 1.2 for the case where $|\rho_r| \in (0,1)$.

dex data from 1976:1 until 2015:12, where the start date was chosen to maximize the number of sectors available. In order to construct real PCE series, we combine nominal PCE and PCE Price Index data.

However, the classification used in the NIPA tables does not match that from the BEA Input-Output (IO) accounts that is used to construct our input-output weights. In order to match NIPA series to BEA IO accounts we use PCE Bridge Tables also published by the BEA. The Bridge Tables are available at two levels of aggregation: Summary and Detail that roughly correspond to 3-digit and 6-digit NAICS codes precision levels respectively. In particular, we use the PCE Bridge Tables in order construct weights $\{\zeta_{ki}\}_{ki}$, which give the share of sector k from BEA IO accounts in sector i from NIPA accounts. Then, given the time series of final sectoral consumptions based on NIPA accounts, we construct time series of final consumptions consistent with BEA IO sector definitions:

$$c_{kt}^{BEA} = \sum_{i=1}^{K^{NIPA}} \zeta_{ki} c_{it}^{NIPA}, \qquad k = 1, 2, ..., K \tag{1.18}$$

We construct BEA IO Accounts consistent sectoral consumption series at both levels of aggregation. The full list of sectors at each level of aggregation is available in Appendix 1.E.

At the Detail level, which is our baseline sample, we have constructed data for 161 sectors going from 1987:1 until 2015:12 where some sectors had to be deleted due to lack of data and the starting point had to be shifted forwards to 1987:1 to minimize the number of sectors to be deleted. Apart from the advantage of a much more detailed sectoral data, the disadvantage of the Detail level accounts is that the PCE Bridge Table is only available for the year 2007, which is the table we use to match series for the entire duration of the sample under the assumption that the matching is remaining stable throughout. Overall, over the sample, the sectoral consumption series constructed account for around 83 per cent of aggregate real Personal Consumption Expenditure (PCE).

At the less disaggregated Summary level, we have only constructed data for 53 sectors, but the advantage of this level of disaggregation is the

longer time series available (1976:1 until 2015:12). The consumption series constructed account for around 84 per cent of aggregate real Personal Consumption Expenditure (PCE) over the sample.[7]

1.3.2 Input-output weights, consumption shares and intermediates intensities

In equilibrium, the input-output weight ω_{kr} is the proportion of the total expenditure of a firm in sector k on intermediate inputs that goes to intermediate inputs produced in sector r. Similarly, the final consumption share ω_{ck} is the share of households' final consumption expenditure that goes to firms in sector k. As for the intermediates intensity δ_k, we know that in equilibrium $\delta_k/(1-\delta_k)$ is given by $P_t^k Z_{kt}(j)/W_{kt}N_{kt}(j)$. Following aggregation over all firms in a given sector, we can write ω_{kr}, ω_{ck} and δ_k as:

$$\omega_{kr} = \frac{P_{rt}Z_{krt}}{P_t^k Z_{kt}}, \quad \forall k, r \qquad \omega_{ck} = \frac{P_{kt}C_{kt}}{P_t^c C_t}, \quad \forall k \qquad \frac{\delta_k}{1-\delta_k} = \frac{P_t^k Z_{kt}}{W_{kt}N_{kt}}, \quad \forall k, \tag{1.19}$$

We use data from the 2007 annual "$Make$" and "Use" tables, published by the US Bureau of Economic Analysis (BEA) as part of its Input-Output accounts, in order to evaluate ω_{kr}, ω_{ck} and δ_k. The exact details behind the evaluation are given in Appendix 1.F.

1.3.3 Sectoral Calvo paramaters

We measure sectoral Calvo parameters as one minus the sectoral frequency of price adjustment, as constructed by Pasten et al. (2020). Their measure is based on confidential firm-level pricing data from the US Bureau of Labor Statistics (BLS).[8]

[7] We report results using the less disaggregated Summary level accounts in Appendix 1.K. Despite the lower degree of disaggregation, estimation at Summary level allows us to extend our estimation sample back to 1976:1, thus benefiting from a higher number of identified monetary shocks. When estimated sector-by-sector and aggregated, the downstream effect contribution is at least 47 per cent of the cumulative full effect, with panel fixed effects estimation yielding a contribution of around 60 per cent.

[8] We are grateful to Michael Weber for sharing sectoral data on frequency of price adjustment.

1.3.4 Monetary policy shocks

Our baseline measure of US monetary policy shocks comes from Romer and Romer (2004), available between 1969:1 and 1996:12, and extended by Wieland and Yang (2016) until 2007:12.

We also consider monetary policy shocks identified using different strategies.[9] First, we consider shocks recursively identified from a structural vector autoregression (SVAR) in Coibion (2012).[10] Second, we consider monetary policy shocks identified using event studies and high frequency data, from Kuttner (2001).[11] In Appendix 1.H, we provide summary statistics and time series plots for all three monetary policy shocks measures.

1.4 Econometric strategy

In this section we develop our econometric specification, based on the analytical solutions for sectoral consumption obtained in Section 1.2.

1.4.1 Baseline specification

Our baseline econometric specification aims to estimate both full and (an upper bound of) the direct effect of a monetary shock on final consumption, and is based on horizon-specific analytical IRFs given in Corollaries 1.1 and 1.2. In particular, we adapt the local projection methodology of Jordà (2005a) in order to estimate both full and direct effect at every horizon without assuming an underlying dynamic system.

First, in order to estimate the horizon-specific *full effect* of a monetary shock, we consider the following baseline specification:

[9] We report results using alternative measures of monetary policy shocks in Appendix 1.L.

[10] SVAR features industrial production, unemployment, consumer price index (CPI), commodity price index and the effective federal funds rate. The recursive Cholesky identification strategy is used with the effective federal funds rate ordered last. We estimate our SVAR using monthly data between 1969:1 and 2007:12 with twelve lags.

[11] We use the publicly available data from Kuttner (2001) on monthly shocks identified as changes in current month's federal funds futures within daily windows around monetary policy announcements; the series is available on Kenneth Kuttner's website: https://econ.williams.edu/faculty-pages/research/. One disadvantage of this approach is that these shocks are only available from 1989:06 (and until 2007:12 for our purposes).

$$\hat{c}_{k,t+H} = \phi^F_{k,H} r_t + \varepsilon^F_{k,t+H}, \qquad H = 1, 2, ..., \overline{H} \tag{1.20}$$

where $\hat{c}_{k,t+H} \equiv \left[c_{k,t+H} - c^*_{k,t+H}\right]$, $\{c_{kt}\}_t$ being series of (log) final consumption in sector k, and $\{c^*_{kt}\}_t$ is trend of (log) final consumption in sector k [12]; $\{r_t\}_t$ is the monetary policy shock process; $\varepsilon^F_{k,t+H} \equiv \left[\rho^F_{k,H} + e^F_{k,t+H}\right]$, where $\rho^F_{k,H}$ absorbs all unconditional means and unobserved time-invariant sectoral heterogeneity, whereas $\{e^F_{kt}\}_t$ is the sector-specific error process; the coefficient $\phi^F_{k,H}$ represents the horizon-H full effect of a monetary policy shock, and can be mapped back to the structural parameters of our model using Corollary 1.1.

Second, in order to estimate a horizon-specific (upper bound of) the *direct effect*, we use the following specification:

$$\hat{c}_{k,t+H} = \phi^D_{k,H} r_t + \sum_{\tau=1}^{H} \psi^\tau_{k,H} \sum_{r=1}^{K} \omega_{kr} \hat{c}_{r,t+H-\tau} + \varepsilon^D_{k,t+H}, \qquad H = 1, 2, ..., \overline{H} \tag{1.21}$$

where ω_{kr} is the share of sector k's expenditure on intermediate inputs that goes to sector r, corresponding to the $[k, r]$ entry of the input-output matrix Ω; the rest of the notation follows that in (1.20). The coefficient $\phi^D_{k,H}$ represents an upper bound of horizon-H direct effect of a monetary policy shock, whereas $\psi^\tau_{k,H}$ measures the sensitivity of sectoral consumption at $(t + H)$ to that of its suppliers at $(t + H - \tau)$; both coefficients can be mapped back to the structural parameters of our model using Corollary 1.2.

Now given that $\phi^F_{k,H}$ and $\phi^D_{k,H}$ represent, respectively, full and (upper bound of) direct effect of a monetary policy shock, it follows that $\left[\phi^F_{k,H} - \phi^D_{k,H}\right]$ gives a *lower bound* of the *downstream effect* at horizon H, representing the contribution of production networks to the effect of a monetary policy shock on final consumption.

[12] We extract trends of sectoral final consumption by HP-filtering.

1.4.2 Coefficient estimation

Our specification in (1.20) and (1.21) allows for different estimation strategies, depending on the degree of cross-sectional coefficient heterogeneity one is willing to consider.

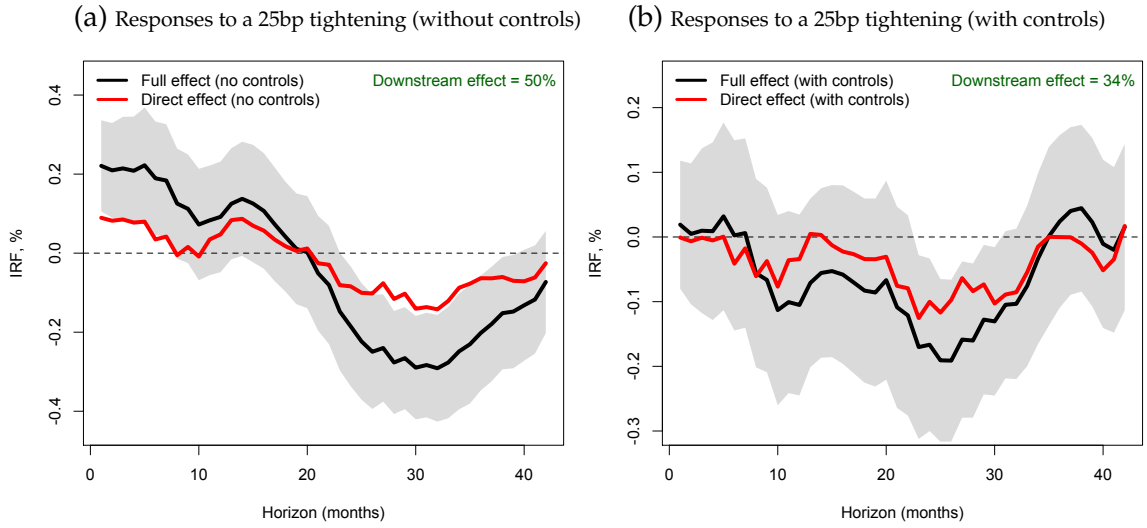
On the one hand, we consider the case where one imposes $\phi^i_{k,H} = \phi^i_H, k = 1, 2, ..., K, \quad i \in \{F, D\}$, leaving $\rho^i_{k,H}, i \in \{F, D\}$, as a sector fixed effect. We then obtain $\hat{\phi}^F_H$ and $\hat{\phi}^D_H$ by Fixed Effects estimation, separately for every horizon, interpreting $\left[\hat{\phi}^F_H - \hat{\phi}^D_H\right]$ as a lower bound of the downstream effect at horizon H, on average.[13] We can also allow for *specific* dimensions of cross-sectional heterogeneity under fixed effects estimation by adding interaction terms of the monetary shock with sector-specific characteristics; in particular, in Section 1.5.2 we add interactions with sectoral Calvo parameters and intermediates intensities, and compare the results with model-based predictions regarding cross-sectional heterogeneity in responses.

On the other hand, we also consider the case with full cross-sectional coefficient heterogeneity, which amounts to estimating (1.20) and (1.21) separately for every sector by OLS. Having obtained $\hat{\phi}^F_{k,H}$ and $\hat{\phi}^D_{k,H}$, we then interpret $\left[\hat{\phi}^F_{k,H} - \hat{\phi}^D_{k,H}\right]$ as a lower bound of the downstream effect in sector k at horizon H. This allows us to see whether or not the contribution of production networks to the effect of a monetary policy shock is disproportionally larger for some sectors. We also aggregate the sectoral responses using the empirical consumption weights $\{\omega_{ck}\}_k$, thus evaluating aggregate full, (upper bound of) direct and (lower bound of) downstream effects.

Our specifications in (1.20) and (1.21) are based on theoretical expressions for IRFs, derived under the assumption of a one-time exogenous monetary shock hitting a system, which is originally in steady state. In the data, however, shocks can arrive in any period and are hitting the system that is not necessarily in steady state. In order to more closely mimic the theoretical environment in our estimation, we add a vector of controls $\mathbb{X}^H_t$ to the right-hand side of both (1.20) and (1.21). The vector of controls includes a

[13] For inference purposes, we construct heteroskedasticity-and-autocorrelation consistent (HAC) standard errors, additionally allowing for clustering at both sector- and month-level.

Figure 1.1: Final consumption response to 25bp tightening: full and direct effects (Detail level, 161 sectors)



***Note**: panel (a) shows results of panel fixed effects estimation of (1.20) and (1.21), without adding controls, reported as IRFs following a 25bp monetary tightening; panel (b) reports results of panel fixed effects estimation of (1.20) and (1.21), with a vector of controls $\mathbb{X}_t^H = \left\{\{r_{t+\tau}\}_{\tau=1}^H, \{\hat{c}_{k,t-s}\}_{s=1}^1, \{ffr_{t-q}\}_{q=1}^4\right\}$ added, reported as IRFs following a 25bp monetary tightening. Shaded areas denote 90% confidence bands around the full effect. The downstream effect contribution is constructed as the difference between cumulative full and direct effects as a percentage of the cumulative full effect (all cumulation is over 42 months). All reported IRFs have been smoothed with a 3-period centered moving average filter, with the original non-smoothed impulse responses reported in Appendix 1.I (downstream effect contribution is constructed using non-smoothed IRFs.)*

vector of leads of the monetary shock $\{r_{t+\tau}\}_{\tau=1}^H$, a vector of lags of the dependent variable $\{\hat{c}_{k,t-s}\}_{s=1}^S$, as well as a vector of lags of the federal funds rate $\{ffr_{t-q}\}_{q=1}^Q$.[14]

1.5 Baseline results: panel fixed effects estimation

1.5.1 Average full and direct effects

In Panel (a) of Figure 1.1 we report the horizon-specific full effect and (an upper bound of) the direct effect on sectoral consumption of a 25 basis points monetary tightening, obtained by panel fixed effects estimation of (1.20) and

[14] Adding a vector of leads of the shock additionally corrects for possible autocorrelation in monetary shock series, as shown in Alloza et al. (2019); adding lags of the dependent variable and the federal funds rate parsimoniously controls for initial conditions before the shock arrives, and we set $S = 1$ and $Q = 4$.

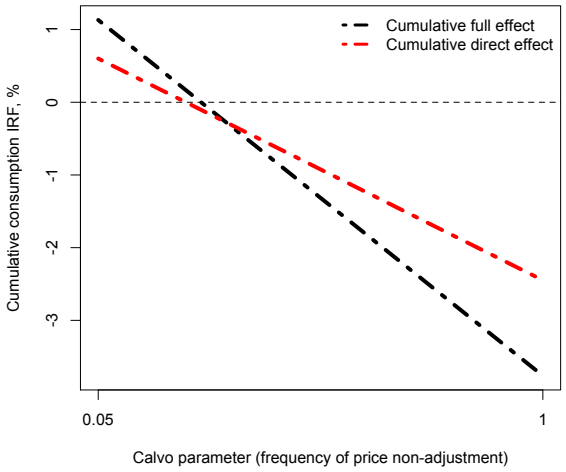
(1.21), without adding control variables. The black line represents the full effect impulse response function; as one can see, consumption actually rises at early horizons following the tightening, before reaching a trough of around -0.30 percentage points decline just after 30 months. As for the red line, which represents (an upper bound of) the direct effect impulse response function with no controls in estimation, it similarly exhibits a small increase immediately after the tightening, but eventually reaches a trough of approximately -0.15 percentage points. As argued earlier, the difference between the two impulse response functions gives a lower bound for the downstream effect coming from input-output linkages. We summarize the downstream effect contribution by looking at the difference between cumulative full and direct effects over 42 months following the shock, and conclude that input-output linkages contribute to at least *50 per cent* of the cumulative full effect.

Panel (b) of Figure 1.1 reports impulse response functions estimated with a vector of controls $\mathbb{X}_t^H$ added to both (1.20) and (1.21). As can be seen from the black line, adding controls essentially removes the "consumption puzzle" observed earlier, with the full impulse response function being close to and not significantly different from zero, before turning negative; as for the trough of the consumption response, it is reached at around 26 months and is -0.20 percentage points in magnitude. Turning to the direct effect, its "consumption puzzle" is also removed by adding controls, and the peak drop achieved is at -0.12 percentage points. Summarizing the downstream effect contribution by the difference between cumulative full and direct effects over 42 months following the shock, the input-output linkages contribute to at least *34 per cent* of the cumulative full effect.

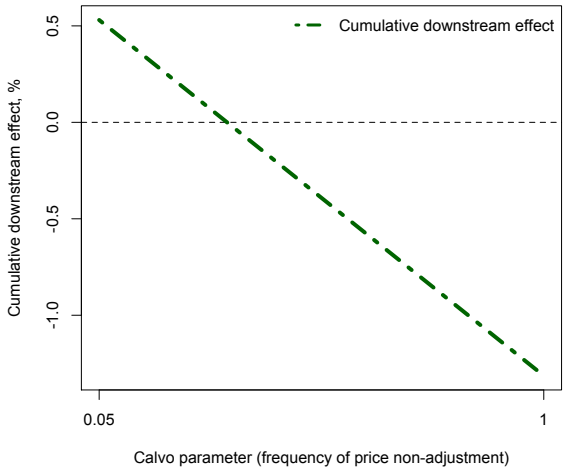
Given that adding controls helps to address the theoretically implausible "consumption puzzles" in average responses, for the rest of the paper we add the vector of controls in all exercises.

Figure 1.2: Response heterogeneity to 25bp tightening across frequencies of price adjustment and intermediates intensities

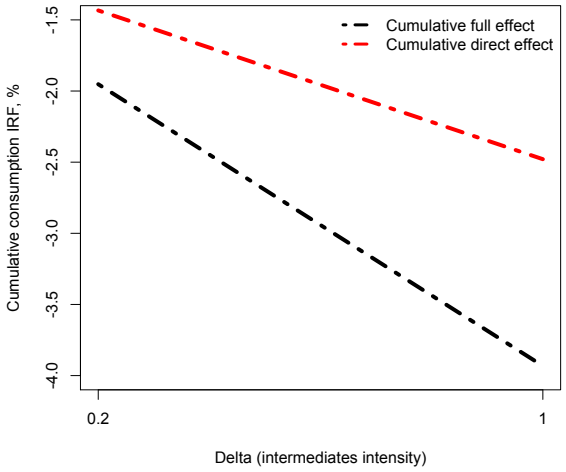
(a) Cumulative full and direct effects across Calvos



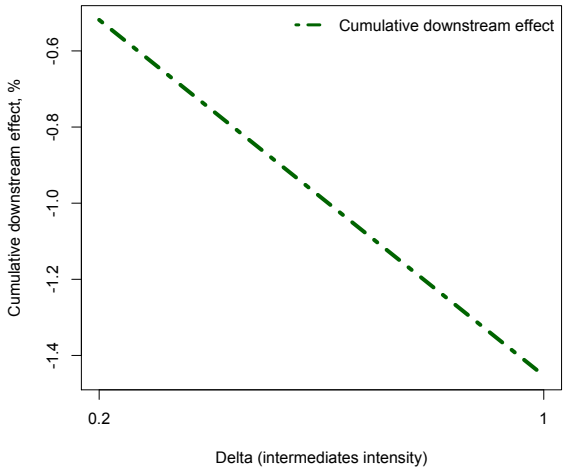
(b) Cumulative downstream effects across Calvos



(c) Cumulative full and direct effects across deltas



(d) Cumulative downstream effects across deltas



***Note**: Figure 1.2 shows results of panel fixed effects estimation of (1.20) and (1.21) with interaction terms $r_t \times \alpha_k$ and $r_t \times \delta_k$ added, as well as vector of controls $\mathbb{X}_t^H = \{\{r_{t+\tau}\}_{\tau=1}^H, \{\hat{c}_{k,t-s}\}_{s=1}^1, \{ffr_{t-q}\}_{q=1}^4\}$; panels (a) and (b) show cumulative full, direct and downstream effects for various values of the Calvo parameter, holding intermediates intensity fixed at its sample median value; panels (c) and (d) show cumulative full, direct and downstream effects for various values of intermediates intensity, holding Calvo parameter fixed at its sample median value. All cumulative effects are over 42 months.*

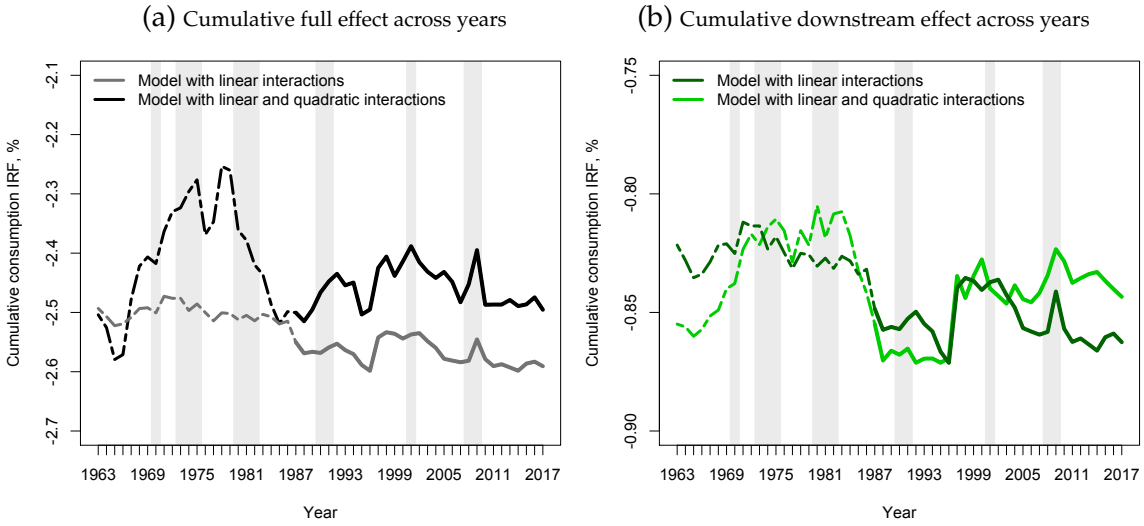
1.5.2 The role of heterogeneous Calvo parameters and intermediates intensity

In this subsection we report results obtained when interactions of monetary shock with sectoral Calvo parameters $(r_t \times \alpha_k)$ and sectoral intermediates intensities $(r_t \times \delta_k)$ are added to the right-hand side of both (1.20) and (1.21). This allows us to test theoretical predictions regarding cross-sectional heterogeneities in the downstream effect. In particular, recall that Proposition 1.1 establishes that the downstream effect is, *ceteris paribus*, unambiguously larger for sectors with larger intermediates intensity; on the other hand, the degree of price rigidity has an ambiguous effect on the magnitude of the downstream effect.

Panel (a) of Figure 1.2 shows estimated cumulative full and direct effects (over 42 months) following a 25bp tightening, for various degrees of price rigidity (while intermediates intensity is held fixed at its sample median value). As one can see, both cumulative full and direct effects become more negative as the frequency of price non-adjustment rises, although the full effect is estimated to be more sensitive to the Calvo parameter. As a result, the downstream effect, given by the difference between cumulative full and direct effects, is estimated to be larger in magnitude as the frequency of price non-adjustment rises, as is shown graphically in Panel (b) of Figure 1.2. The latter suggests that empirically, the effect of higher price rigidity on being more forward-looking, and hence being more sensitive to expectations of future prices set by suppliers, dominates the effect of adjusting prices less frequently and hence being less sensitive to current and expected future marginal costs.

Panel (c) of Figure 1.2 shows estimated cumulative full and direct effects (over 42 months) following a 25bp tightening, for various intermediates intensities (while the Calvo parameter is held fixed at its sample median value). The results suggest that both cumulative full and direct effects become more negative as the intermediates intensity rises, although the full effect is estimated to be more sensitive to intermediates intensity. As a result,

Figure 1.3: Cumulative response to 25bp tightening across years: the role of time variation in intermediates intensity



***Note**: panels (a) and (b) show time series of cumulative (over 42 months) full and downstream effects, aggregated using sectoral consumption shares, under sectoral values of intermediates intensities across the years, as constructed by Ghassibe (2020), using estimation of (1.20) and (1.21) with controls either just linear interactions $r_t \times \alpha_k$ and $r_t \times \delta_k$ added, as well as with the addition of quadratic interaction terms $r_t \times \alpha_k^2, \quad r_t \times \delta_k^2, \quad r_t \times \alpha_k \delta_k$; solid lines use true intermediates intensities available between 1987-2017, dotted lines show results using interpolated values of intermediates intensities constructed for 1963-1986 under the additional assumption of sectoral labor shares of value added fixed at their values in 1987. Shaded bars represent NBER recession dates.*

the downstream effect is estimated to be larger in magnitude as intermediates intensity rises, as is shown graphically in Panel (d) of Figure 1.2. The latter is fully consistent with our theoretical predictions.

In this subsection we have examined *linear* relationships between consumption responses and the degrees of price rigidity and intermediates intensity. However, Proposition 1.2 suggests that the relationship may be non-linear. In order to account for potential non-linearities, in Appendix 1.J we report results under additional non-linear interaction terms $r_t \times \alpha_k^2, \quad r_t \times \delta_k^2$ and $r_t \times \alpha_k \delta_k$ added. The results suggest our findings regarding cross-sectional heterogeneity remain true even once non-linearities have been allowed for.

1.5.3 Time variation in intermediates intensity

Results in the previous subsection use values of sectoral intermediates intensities δ_k based on the 2007 BEA "Use" tables at the Detail level. Accounts at such high level of disaggregation are not available at annual (or higher) frequency over time, but Ghassibe (2020) constructs annual measures of intermediate intensity for less disaggregated 65 sectors of the US economy between 1987-2017, with a possibility of extending the measures back to 1963 under additional assumptions regarding the labor share of value added.[15] Further, it is shown that intermediates intensities are strongly procyclical.

Estimates in Panels (c) and (d) of Figure 1.2 give us the (cumulative) full, direct and downstream effect for various values of intermediates intensity, holding fixed the degree of price rigidity. In this subsection we combine those estimates with the annual data on intermediates intensities to perform the following exercise: for every sector, we compute the implied (cumulative) full, direct and downstream effect under its intermediates intensity in year t, $\quad t = 1963, 1964, ..., 2017$. We then aggregate the sectoral effects to gauge the aggregate (cumulative) full, direct and downstream effects following a 25bp tightening in a particular year.

Figure 1.3 shows cumulative (over 42 months) full and downstream effects following a 25bp tightening, between 1963-2017. We report results based both on the specification with only linear interactions with Calvo parameter and intermediates intensity, as well as the specification that additionally includes second-order interaction terms. First, note that in both specifications, the magnitude of both full and downstream effects is procyclical, which in term comes from the procyclicality of intermediates intensity. Such results provide a possible rationalization for the procyclical magnitude of GDP response to a monetary shocks documented by Tenreyro and Thwaites (2016) among others. Second, the observed procyclicality is substantially more pronounced in the non-linear specification, particularly

[15] Prior to 1987, BEA KLEMS accounts do not report sectoral employee compensation separately from sectoral value added; hence in order to construct sectoral intermediates intensities between 1963-1986 is it assumed the sectoral labor share of value added is fixed at its 1987 level.

in the Volcker disinflation episode, which witnessed a substantial drop in intermediates intensity; such empirical result suggests that the strength of monetary transmission to GDP can fall especially quickly in strong recessions which are associated with much smaller reliance on intermediate inputs.

1.6 Baseline results: sector-by-sector OLS estimation

1.6.1 Sectoral responses and aggregation

Figure 1.4 summarizes the results of sector-by-sector OLS estimation of both (1.20) and (1.21), yielding sectoral full and (upper bound of) direct impulse response functions. The latter are then aggregated in order to gauge economy-wide effects.

Panel (a) of Figure 1.4 shows the full effect impulse response functions to a 25 basis points tightening for all 161 sectors in our baseline sample. Several aspects are worth highlighting. First, there is significant heterogeneity in the responses; given that our model predicts that sectoral responses are pinned down by, among other things, sectoral Calvo parameters and intermediates intensities, such dispersion in magnitudes of responses in consistent with large cross-sectional differences in frequencies of price adjustment, as documented by Nakamura and Steinsson (2008) and Gorodnichenko and Weber (2016) among others, as well as large heterogeneity in intermediates intensities, as is documented in Appendix 1.G. Second, the aggregated response obtained by weighing individual responses by their respective consumption weights (solid black line) follows the aggregate real consumption response (dashed purple line) very closely, and considerably more closely than the response obtained under aggregation using equal weights (dashed blue line).This suggests that the aggregate response may be driven by several large sectors, and hence that sectoral responses should indeed be aggregated with sectoral consumption weights, as opposed to equal weights.

Panel (b) reports the aggregated sectoral full effect (black line), as well as the aggregated (upper bound of) sectoral direct effects (red line), follow-

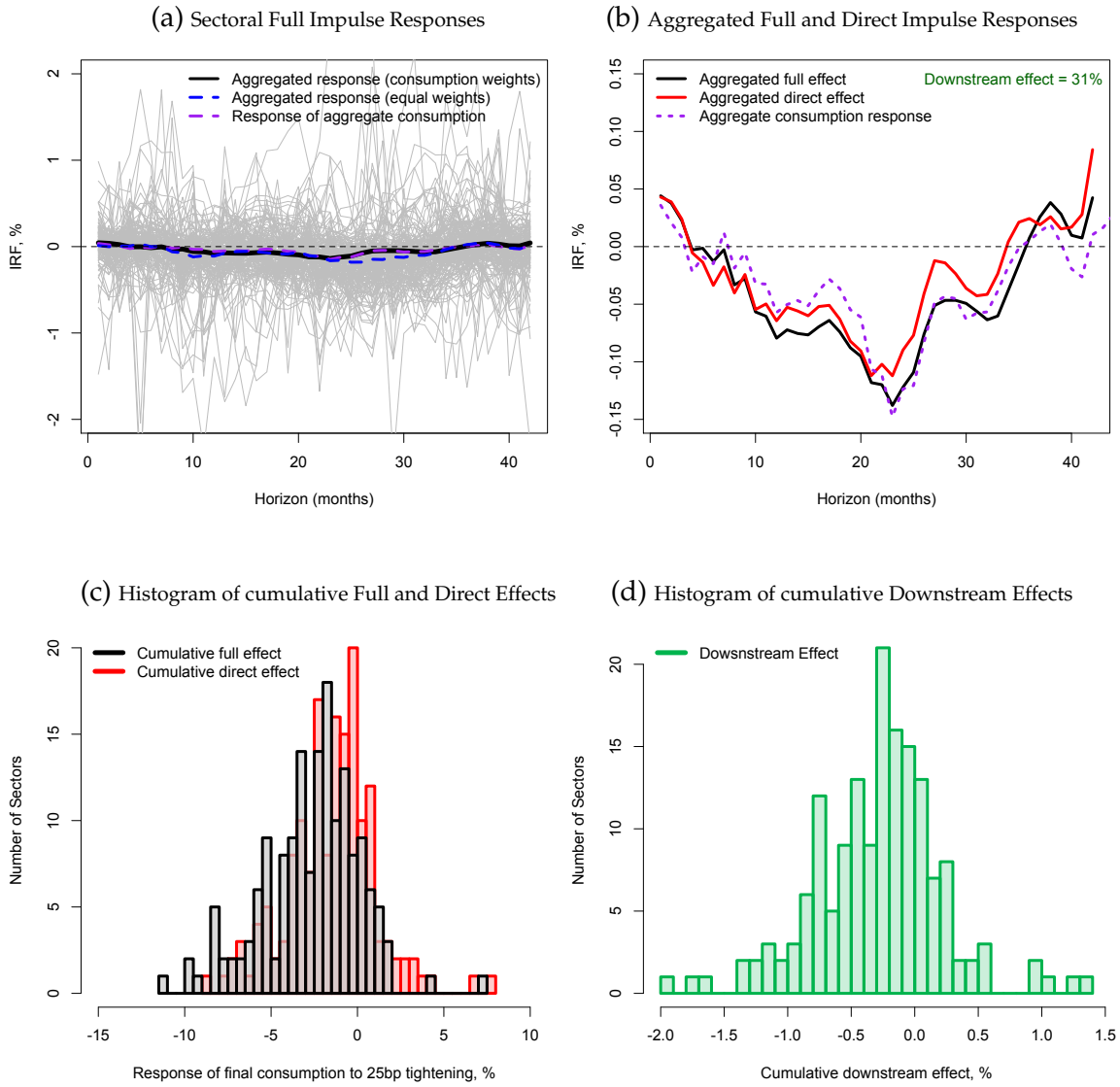
ing a 25 basis points monetary tightening, where the aggregation performed with empirical consumption weights. As can be seen, the aggregated sectoral full effect is slightly positive at early horizons, but eventually reaches a trough of around -0.14 percentage points. Importantly, the aggregated full response follows very closely the impulse response of aggregate consumption, providing extra confidence in the new sectoral dataset that we have constructed. As for the aggregated (upper bound of) direct effect, it similarly starts with a slightly positive response, but then experiences a drop down to around -0.10 percentage points at the trough. When measured by the difference between cumulative aggregated full and direct effects (after 42 months), the downstream effect is estimated to contribute to (at least) 31 per cent of the cumulative full effect. The latter is very similar to the findings in the previous section under panel fixed effects estimation.

In Panels (c) and (d) we report histograms of sectoral cumulative full and (upper bounds of) cumulative direct effects (after 42 months), as well as sector-level cumulative downstream effects. One can see that the distribution of the sectoral full effects is shifted in the direction of more negative values compared to that of sectoral direct effects, which is consistent with the notion that the effect of monetary shocks on sectoral consumption is amplified by production networks; as for the sectoral downstream effects, those are estimated to be negative for the vast majority of sectors, confirming the finding that production networks amplify the effects of monetary shocks.

Note that although most of the results in this subsection are qualitatively in line with the predictions of our theory outlined in Section 1.2, two features cannot be reconciled using our baseline model. First, as can be seen in Panel (a) of Figure 1.4, quite a number of sectors appear to be experiencing an increase in their consumption at certain horizons following a monetary tightening. [16] Second, as can be seen in Panel (d) of Figure 1.4, though sector-level downstream effects are negative for the vast majority of sectors, it is actually positive for some sectors and thus production networks seem to actually at-

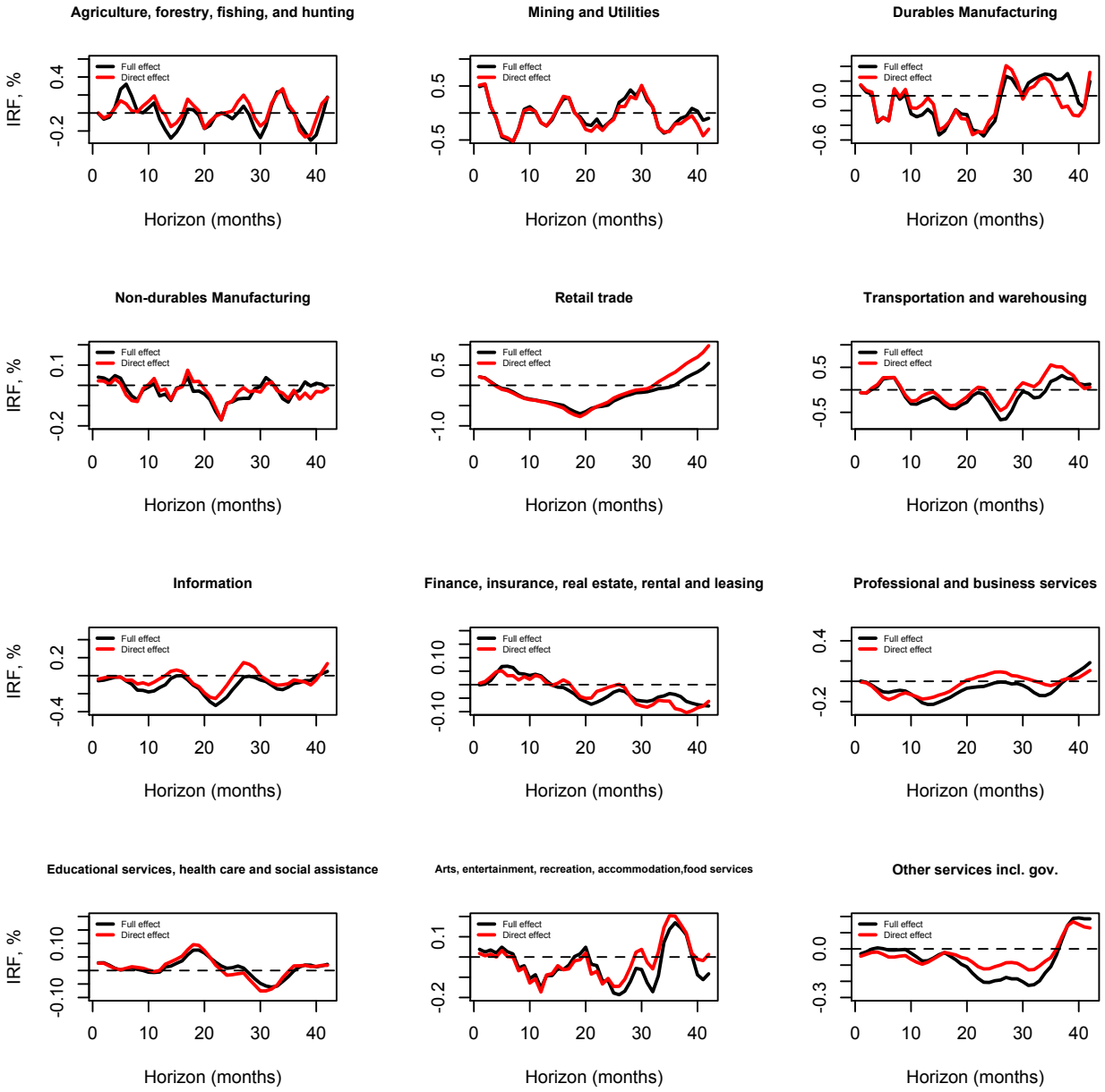
[16]The latter is also found by Boivin et al. (2009) in their FAVAR study of product-level consumption responses.

Figure 1.4: Sector-level consumption responses to 25bp tightening (Detail level, 161 sectors)



***Note**: panel (a) shows results of sector-by-sector OLS estimation of (1.20) with controls, reported as sector-level IRF following a 25bp monetary tightening; panel (b) reports results of aggregation of sector-level full and direct effects using empirical final consumption shares $\{\omega_{ck}\}_{k=1}^{161}$; panels (c) and (d) report histograms of cumulative (over 42 months) sectoral full and direct effects, as well as the corresponding sectoral cumulative downstream effects. In panel (b) the reported impulse responses have been smoothed with a 3-period centered moving average filter, with the original non-smoothed impulse responses reported in Appendix 1.I.*

Figure 1.5: Consumption responses to a 25bp tightening across broad sectoral groups



***Note**: Figure 1.5 shows results of sector-by-sector OLS estimation of (1.20) and (1.21) with controls, reported as sector-level direct and full IRFs following a 25bp monetary tightening that have been aggregated into broad sectoral groups using empirical consumption shares $\{\omega_{ck}\}_{k=1}^{161}$.The reported impulse responses have been smoothed with a 3-period centered moving average filter, with the original non-smoothed impulse responses reported in Appendix 1.I.*

tenuate, rather than amplify, the effect of monetary policy shocks on those sectors' consumptions.

To address this mismatch, in Appendix 1.D we show that relaxing the assumption of unit elasticity of substitution across sectors can help jointly reconcile both of the two puzzles documented above (however, at the cost of analytical tractability at sector-level). This is because under non-unit elasticity of substitution, a sector's final consumption response is pinned down by the speed of its price response *relative* to the speed of adjustment of the aggregate consumption price index. Hence, if the sectoral goods are substitutes, following a monetary tightening the price index of the sectors with the highest speed of price adjustment drops much faster than the aggregate consumption price index, and hence those sectors can "steal" customers from other sectors and see an *increase* in their consumption; at the same time, sectors with the lowest speed of price adjustment can have their negative consumption response *dampened* by networks, as price complementarities make their price index more aligned with the aggregate consumption price index, thus making them "lose" fewer customers to the other sectors. Crucially, however, the *aggregate* consumption response is fully independent of the elasticity of substitution across sectors; hence, although sectoral responses can exhibit different properties under non-unit elasticity of substitution, the aggregate consumption response is always negative following a monetary tightening and always amplified by networks regardless of the elasticity of substitution across sectors.

1.6.2 Full and direct effects in broad sectoral groups

In order to gain further understanding of cross-sector heterogeneities in full and direct effects, in Figure 1.5 we aggregate individual responses into 12 broad sectoral groups.

As before, we summarize the downstream effect by the difference between cumulative full and cumulative direct effects (over 42 months). By this metric, the group with the largest downstream effect is "Transportation and warehousing", followed by "Retail trade", "Information", "Agriculture,

forestry, fishing and hunting" and “Professional and business services" as other groups with a substantial downstream effect. In fact, although those sectoral groups account for only 17 per cent of the aggregate consumption basket, their combined downstream effect equals 98 per cent of the aggregate downstream effect .

At the same time, “Other services including government" and “Arts, entertainment, recreation, accommodation, and food services" also exhibit downstream effects suggestive of amplification through production networks, though smaller in magnitude. As for “Educational services, health care, and social assistance", “Non-Durables manufacturing" and “Finance, insurance, real estate, rental and leasing" their downstream effect is essentially zero.

Notably, two sectoral groups with some of the largest full effects, namely “Mining and utilities" and “Durables manufacturing", exhibit a downstream effect of an inverted sign, implying that production networks actually slightly dampen their response to a monetary shock.

1.7 Further exercises and robustness checks

1.7.1 Monte Carlo experiments

As argued earlier, our econometric specification allows to estimate the full effect of a monetary shock, as well as an *upper bound* for the direct effect, thus giving us a *lower bound* for the downstream effect. As an additional exercise, we apply our econometric specification to synthetic data generated by our model, calibrated to the same 161 sectors used in our baseline sample, and compare our estimated (lower bound of) downstream effect, to the true

known contributions of linkages in the model.[17] Panel (a) of Figure 1.6 shows that based on means across simulations, our estimated downstream effect accounts for 41.02 per cent of the cumulative full effect (over 42 months), while the true contribution of linkages in the model equals 41.29 per cent; hence, the lower bound that our econometric strategy recovered from simulated data is very close to the true contribution.

Importantly, our econometric specification is based on sector-level analytical solutions to our model, which requires simplifying assumptions such as monetary policy based on a money supply rule, unit elasticity of substitution across sectors and linear disutility of labor. Relaxing the above assumptions comes at the cost of sector-level analytical tractability. However, we can still assess the robustness of econometric strategy to model misspecification by applying it to synthetic data generated by alternative models.[18] In Panel (b) of Figure 1.6 we show that when data is generated by a model with a Taylor rule[19], we estimate the downstream effect contribution at 35.81 per cent of the cumulative full effect, while the true contribution is 30.38 per cent. In Panel (c) of Figure 1.6 we additionally allow for elasticity of substitution across sectors equal to 2, the true contribution of linkages remains at 30.38 per cent, while we estimate it at 36.98 per cent. Finally, in Panel (d) of Figure 1.6 we generate data additionally allowing for finite Frisch elasticity of labor supply equal to 0.5, which gives a true contribution of linkages equal to 43.54 per cent, while our specification estimates it at 41.86 per cent.

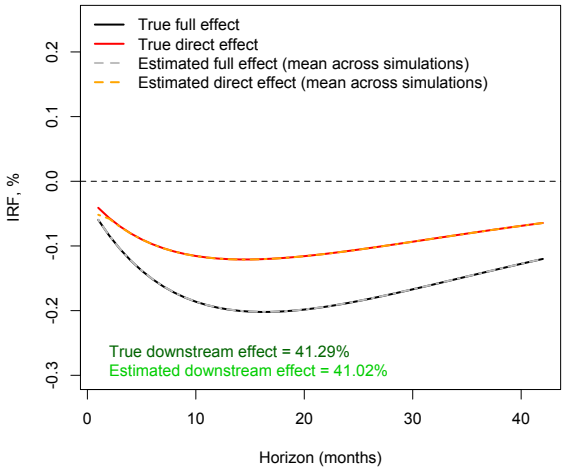
[17] We simulate 1,000 histories of log-linearized version of our model set out in Appendix 1.A for 2,000 periods, discarding the first 1,748 periods as burn-in and only using the last $T = 252$ time periods (matching the time dimension of our sample), and apply specifications (1.20) and (1.21) (with controls) to estimate full and direct effects sector-by-sector with subsequent aggregation using empirical consumption weights. We calibrate the model at monthly frequency, setting $\beta = 0.9975, \rho_r = 0.95, \theta = 6$ and $\sigma_r = 0.001$, with input-output weights, sectoral Calvo parameters and intermediates intensities calibrated using data described in Section 1.3. In order to avoid possible singularities in the the simulated data, we include sector-specific i.i.d. productivity shocks $\{\ln \mathcal{A}_{kt}\}_t \sim^{iid} (0, \sigma_a^2)$, where we set $\sigma_a = 10^{-6} \times \sigma_r$.

[18] Appendix 1.A outlines a generalized version of our model that relaxes the simplifying assumptions.

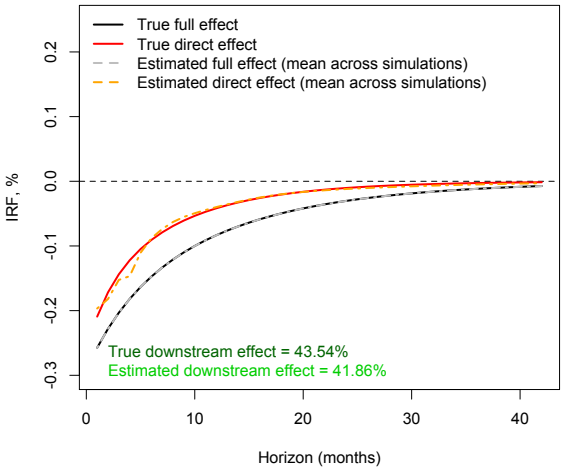
[19] We consider a Taylor rule of the form $\frac{I_t}{I} = \left(\frac{I_{t-1}}{I}\right)^{\rho_i} \left[\left(\frac{P_t^c}{P_{t-1}^c}\right)^{\phi_\pi} \left(\frac{C_t}{C}\right)^{\phi_c} \left(\frac{C_t}{C_{t-1}}\right)^{\phi_{gc}}\right]^{1-\rho_i} e^{\mu_t}, \quad \mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t^\mu, \quad \{\varepsilon_t^\mu\}_t \sim^{iid} (0, \sigma_\mu^2)$, where we set $\rho_i = 0.9, \phi_\pi = 1.24, \phi_c = 0.33/12, \phi_{gc} = 1.5, \rho_\mu = 0$ and $\sigma_\mu = 0.000225$.

Figure 1.6: Theoretical full and direct effects, and those estimated from model-generated data

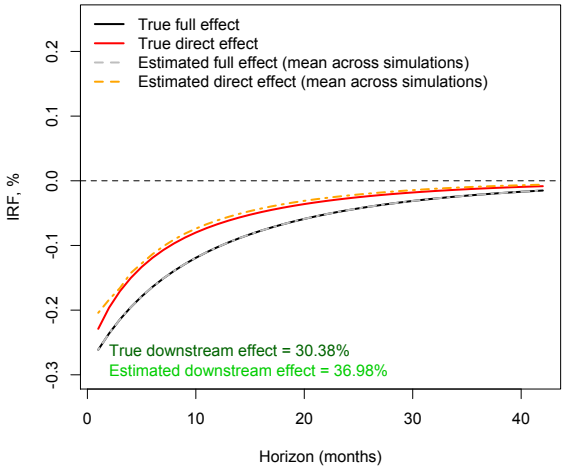
(a) Baseline model with money supply rule



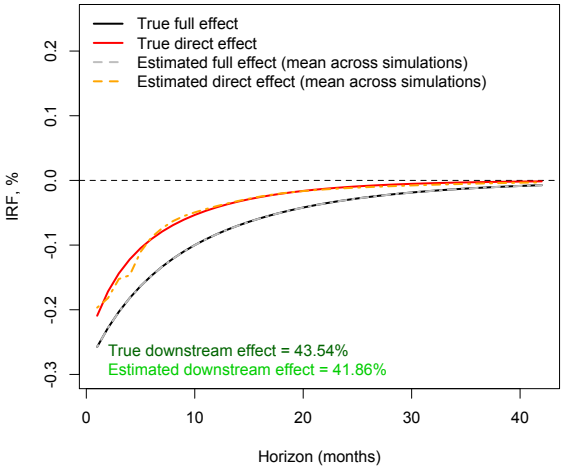
(b) Model with Taylor rule



(c) Model with Taylor rule + CES aggregation



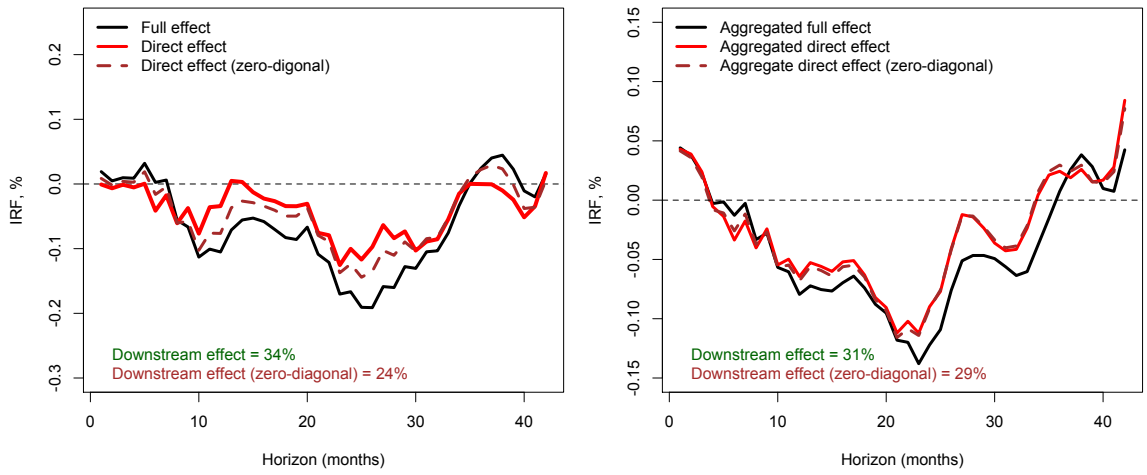
(d) Model with Taylor rule + CES agg. + finite Frisch elasticity



Note: *Figure 1.6 shows results of Monte Carlo experiments using simulations of log-linearized models fully outlined in Appendix 1.A. In each exercise, we simulate 1,000 histories of calibrated model for 2,000 periods, discarding the first 1,748 periods as burn-in and only using the last $T = 252$ time periods (matching the time dimension of our sample), for the same 161 sectors used in our baseline sample, and apply specifications (1.20) and (1.21) (with controls) to estimate full and direct effects sector-by-sector with subsequent aggregation using empirical consumption weights; we then use the estimates to measure the downstream contribution to cumulative aggregated full effect, and compare it to the true known contribution in the model. In panel (a) we use our baseline model with money supply rule; in panel (b) we replace the money supply rule with the Taylor rule for the nominal interest rate; in panel (c) we additionally allow for CES aggregation across sectors with elasticity above one; in panel (d) we additionally allow for finite Frisch elasticity of labor supply.*

Figure 1.7: Final consumption response to 25bp tightening: full and direct effects under zero-diagonal (Detail level, 161 sectors)

(a) Responses after a 25bp tightening: panel estimation (b) Responses after a 25bp tightening: sector-by-sector



***Note**: Figure 1.7 shows additional estimation of the direct effect using (1.21) where we force the diagonal elements of the input-output matrix to be equal to zero. The new estimates are reported alongside estimation of (1.20) and (1.21) without further restrictions. Panel (a) shows results using panel fixed effects estimation, whereas panel (b) shows results using sector-by-sector OLS estimation with subsequent aggregation using empirical consumption shares $\{\omega_{ck}\}_{k=1}^{161}$. The reported impulse responses have been smoothed with a 3-period centered moving average filter, with the original non-smoothed impulse responses reported in Appendix 1.I.*

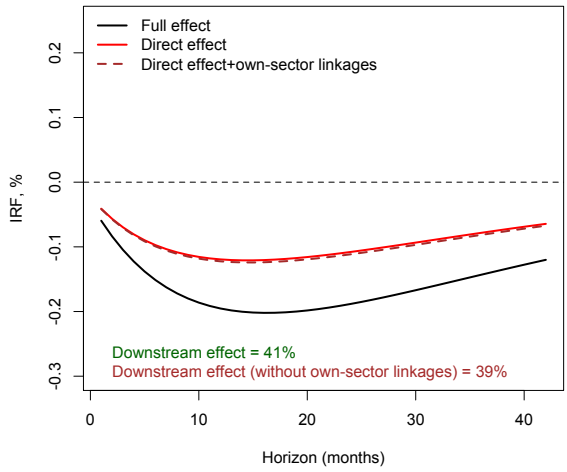
1.7.2 Within-sector linkages vs. across-sector linkages

Our model allows for both within-sector and across-sector input-output linkages; however, our baseline econometric specification does not specifically estimate the downstream effect coming from across-sector linkages. In Figure 1.7 we report results of an additional exercise which attempts to separate out the effects coming from the two types of linkages. In particular, we estimate (1.21) forcing diagonal elements of the input-output matrix to be equal to zero ($\omega_{kk} = 0, \forall k$). If it was true that the majority of the downstream effect came from within-sector linkages, then the estimated direct effect under zero-diagonal matrix would coincide with the full effect; instead, we see in Figure 1.7 that the downstream effect under zero-diagonal matrix is close to that under the true matrix. Our results therefore suggest that the majority of the downstream effect comes from across-sector linkages.

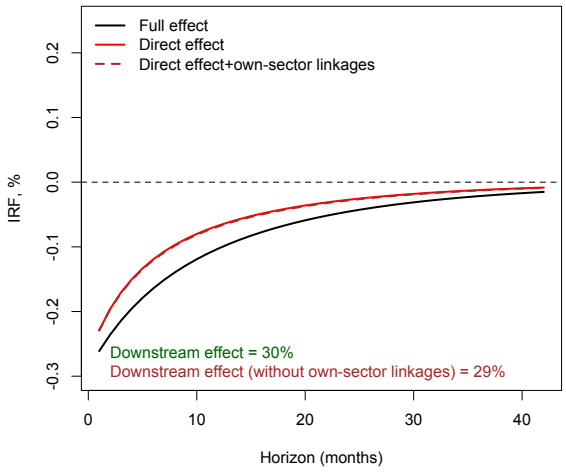
Importantly, however, in the previous exercise we forced a parameter in-

Figure 1.8: Theoretical full and direct effects, and direct effects combined with own-sector linkages

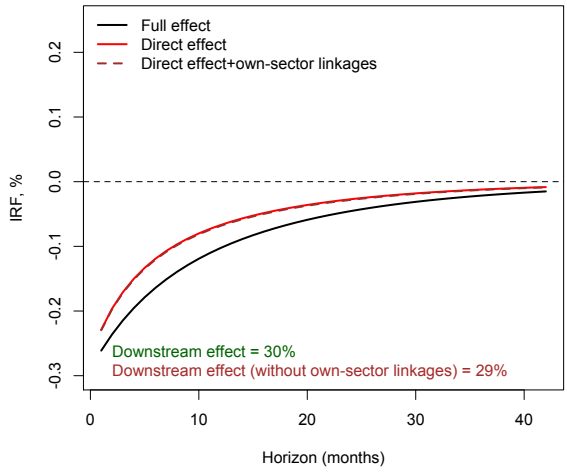
(a) Baseline model with money supply rule



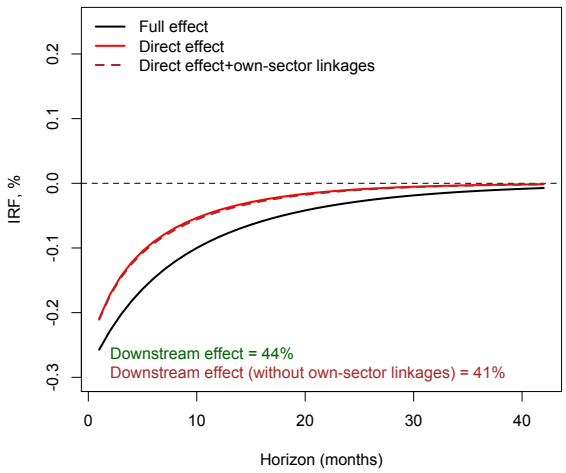
(b) Model with Taylor rule



(c) Model with Taylor rule + CES aggregation



(d) Model with Taylor rule + CES agg. + finite Frisch elasticity



***Note**: Figure 1.8 shows aggregated full and direct effects from our calibrated theoretical model, as well effects from a calibration which only allows for within-sector input-output linkages, calibrated so that the sectoral intermediates intensities are equal to the empirical share of total variable costs that goes to intermediates bought from own sector (interpreted as direct effect+own linkages). The difference between the latter and the full effect is used to measure the (cumulative) downstream contribution without own-sector linkages. In panel (a) we use our baseline model with money supply rule; in panel (b) we replace the money supply rule with the Taylor rule for the nominal interest rate; in panel (c) we additionally allow for CES aggregation across sectors with elasticity above one; in panel (d) we additionally allow for finite Frisch elasticity of labor supply.*

side a reduced-form econometric model, which is why our results should be treated with caution. As an additional robustness exercise, we separate out effects of within-sector versus across-sector linkages using our calibrated structural theoretical model.[20] In Figure 1.8 we report aggregate consumption responses under three configurations: first, under input-output linkages completely switched off ($\delta_k = 0, \forall k$), giving us the theoretical direct effect; second, under input-output matrix set equal to the identity matrix, so that we only allow for within-sector linkages, with alternative sectoral intermediates intensities ($\overline{\delta}_k$) given by the empirical share of variable costs that are paid to firms within own sector ($\overline{\delta}_k \equiv \delta_k \omega_{kk}, \forall k$), giving us aggregate direct effect combined with the effect coming from own-sector linkages; third, under true δ_k and Ω for all sectors, giving us the theoretical full effect. As one can see, both in our baseline theoretical model, and in extensions featuring a Taylor rule, non-unit elasticity of substitution across sectors and finite Frisch elasticity, the direct effect combined with the effect of own-sector linkages is essentially indistinguishable from the direct effect. Therefore, our results using calibrated models also suggest that the majority of the downstream effect comes from across-sector linkages.

1.8 Conclusion and future extensions

A large theoretical literature outlining the way in which production networks create complementarities in price setting and thus amplify the effect of monetary policy shocks exists. However, there has been hardly any econometric assessment of the above mechanism. In this paper we make a first step towards filling this large gap in the literature. In particular, we offer novel empirical evidence that at least 30 per cent of the effect of monetary policy shocks on US aggregate consumption comes from the amplification through input-output linkages. We also find that the network effect is larger, *ceteris paribus*, for sectors that adjust prices less frequently and those that rely more on intermediate inputs. Our results also suggest substantial concentration: sectors that jointly account for 17 per cent of our sample aggre-

[20] We use the same calibrations as outlined in Section 1.7.1

gate consumption account for 98 per cent of the amplification by production networks.

Apart from the novel econometric evidence, we make two additional contributions. First, we obtain novel sector-level analytical solutions to a multi-sector New Keynesian models with asymmetric input-output linkages, and thus form a tractable econometric strategy. Second, we construct a new highly disaggregated monthly dataset for sectoral final consumptions in the US that can is consistent with sectoral classification used in input-output accounts of the US Bureau of Economic Analysis.

Our results create space for future research. First, our theoretical framework has only considered time-dependent pricing schedules following Calvo (1983); it would be an interesting agenda to additionally allow for state-dependent pricing and study, both theoretically an empirically, how firms with state-dependent and time-dependent pricing interact with each other in a production network, and what are the implications of such interaction for the response of aggregate variables to shocks of different magnitudes. Second, our paper has focused on production networks in a closed economy setting, and it would be an interesting extension to consider international production networks and their role in creating cross-border spillovers of monetary policy shocks.

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Appendices to Chapter 1

1.A Full model details

In this section we outline a more general version of our model, where we additionally allow for sector-specific exogenous productivity process ($\mathcal{A}_{kt}$), finite Frisch elasticity of labor supply (given by $1/\varphi$), CES aggregation across sectors (with elasticity of substitution given by λ) and monetary policy implemented through a Taylor rule. Naturally, the model considered in the main text is nested as a special case where $\mathcal{A}_{kt} = 1, \varphi = 0, \lambda = 1$ and monetary policy is implemented as a money supply rule.

1.A.1 Full non-linear system (at firm level)

(a) Firms

$$Y_{kt}(j) = \mathcal{A}_{kt} N_{kt}(j)^{1-\delta_k} Z_{kt}(j)^{\delta_k} - \Xi_k(j), \qquad \{\ln \mathcal{A}_{kt}\}_t \sim^{iid} (0, \sigma_a^2) \tag{1.22}$$

$$Z_{kt}(j) \equiv \left[\sum_{r=1}^{K} \omega_{kr}^{\frac{1}{\lambda}} Z_{krt}(j)^{\frac{\lambda-1}{\lambda}} \right]^{\frac{\lambda}{\lambda-1}} \tag{1.23}$$

$$Z_{krt}(j) \equiv \left(\int_{j' \in \Phi_r} Z_{krt}(j,j')^{\frac{\theta-1}{\theta}} dj' \right)^{\frac{\theta}{\theta-1}} \tag{1.24}$$

$$Z_{krt}(j) = \omega_{kr} \left(\frac{P_{rt}}{P_t^k} \right)^{-\lambda} Z_{kt}(j) \tag{1.25}$$

$$Z_{krt}(j,j') = \left(\frac{P_{rt}(j')}{P_{rt}} \right)^{-\theta} Z_{krt}(j) \tag{1.26}$$

$$P_t^k = \left[\sum_{r=1}^{K} \omega_{kr} P_{rt}^{1-\lambda} \right]^{\frac{1}{1-\lambda}} \tag{1.27}$$

$$MC_{kt}(j) = MC_{kt} = \frac{1}{\mathcal{A}_{kt}} \frac{1}{1-\delta_k} \left(\frac{\delta_k}{1-\delta_k} \right)^{-\delta_k} W_{kt}^{1-\delta_k} \left(P_t^k \right)^{\delta_k} \tag{1.28}$$

$$\frac{Z_{kt}(j)}{N_{kt}(j)} = \frac{\delta_k}{1-\delta_k} \frac{W_{kt}}{P_t^k} \tag{1.29}$$

$$P_{kt}^*(j) = P_{kt}^* = \frac{\theta}{\theta-1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \alpha_k^s F_{t,t+s} P_{k,t+s}^{\theta} Y_{k,t+s} MC_{k,t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \alpha_k^s F_{t,t+s} P_{k,t+s}^{\theta} Y_{k,t+s}} \tag{1.30}$$

$$P_{kt} = \left[\alpha_k P_{k,t-1}^{1-\theta} + (1-\alpha_k)(P_{kt}^*)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{1.31}$$

(b) Households

$$F_{t,t+s} = \beta^s \frac{P_t^c C_t}{P_{t+s}^c C_{t+s}} \tag{1.32}$$

$$g_k N_{kt}^{\varphi} C_t = \frac{W_{kt}}{P_t^c} \tag{1.33}$$

$$C_t \equiv \left[\sum_{r=1}^{K} \omega_{ck}^{\frac{1}{\lambda}} C_{kt}(j)^{\frac{\lambda-1}{\lambda}} \right]^{\frac{\lambda}{\lambda-1}} \tag{1.34}$$

$$C_{kt} \equiv \left(\int_{j \in \Phi_k} C_{kt}(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \tag{1.35}$$

$$C_{kt} = \omega_{ck} \left(\frac{P_{kt}}{P_t^c} \right)^{-\lambda} C_t \tag{1.36}$$

$$C_{kt}(j) = \left(\frac{P_{kt}(j)}{P_{rt}} \right)^{-\theta} C_{kt} \tag{1.37}$$

$$P_t^c = \left[\sum_{k=1}^{K} \omega_{ck} P_{kt}^{1-\lambda} \right]^{\frac{1}{1-\lambda}} \tag{1.38}$$

(c) Monetary policy

Either

$$P_t^c C_t = M_t, \quad \ln M_t = \ln M_{t-1} + r_t, \quad r_t = \rho_r r_{t-1} + \varepsilon_t^r, \quad \{\varepsilon_t^r\}_t \sim^{iid} (0, \sigma_r^2) \tag{1.39}$$

or

$$\frac{I_t}{I} = \left(\frac{I_{t-1}}{I} \right)^{\rho_i} \left[\left(\frac{P_t^c}{P_{t-1}^c} \right)^{\phi_\pi} \left(\frac{C_t}{C} \right)^{\phi_c} \left(\frac{C_t}{C_{t-1}} \right)^{\phi_{gc}} \right]^{1-\rho_i} e^{\mu_t}, \quad \mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t^\mu, \quad \{\varepsilon_t^\mu\}_t \sim^{iid} (0, \sigma_\mu^2) \tag{1.40}$$

$$I_t = \frac{1}{\mathbb{E}_t F_{t,t+1}} \tag{1.41}$$

(d) Market clearing

$$Y_{kt}(j) = C_{kt}(j) + \sum_{r=1}^{K} \int_{j' \in \Phi_r} Z_{rkt}(j', j) dj' \tag{1.42}$$

$$N_{kt} = \int_{j \in \Phi_k} N_{kt}(j) dj \tag{1.43}$$

$$B_t = 0 \tag{1.44}$$

1.A.2 Steady state

We consider a symmetric steady state, so that $P_k(j) = P_k = P^k = P^c = P, \quad \forall k, \forall j \in \Phi_k$.

$$\frac{W_k}{P} = \left[\frac{\theta}{\theta - 1} \frac{\delta_k^{-\delta_k}}{(1 - \delta_k)^{1-\delta_k}} \right]^{\frac{1}{\delta_k - 1}} \tag{1.45}$$

$$\Xi_k(j) = \frac{1}{\theta - 1} Y_k(j) \tag{1.46}$$

$$N_k(j) = (1 - \delta_k) \left(\frac{W_k}{P} \right)^{-1} Y_k(j) \tag{1.47}$$

$$Z_k(j) = \delta_k Y_k(j) \tag{1.48}$$

$$[Y_1, ..., Y_K]' = [I - \Omega' diag(\delta)]^{-1} [C_1, ..., C_K]' \tag{1.49}$$

$$C_k(j) = C_k = \omega_{ck} C \tag{1.50}$$

$$Y_k(j) = Y_k \tag{1.51}$$

$$N_k(j) = N_k \tag{1.52}$$

$$C = \sum_{k=1}^{K} \frac{W_k}{P} N_k \tag{1.53}$$

1.A.3 Log-linearized system (at sectoral level)

(a) Firms

$$y_{kt} = \frac{\theta}{\theta - 1}[a_{kt} + (1 - \delta_k)n_{kt} + \delta_k z_{kt}] \tag{1.54}$$

$$z_{kt} = \sum_{r=1}^{K} \omega_{kr} z_{krt} \tag{1.55}$$

$$z_{krt} = \lambda(p_{rt} - p_t^k) + z_{kt} \tag{1.56}$$

$$p_t^k = \sum_{r=1}^{K} \omega_{kr} p_{rt} \tag{1.57}$$

$$mc_{kt} = -a_{kt} + (1 - \delta_k)w_{kt} + \delta_k p_t^k \tag{1.58}$$

$$z_{kt} = n_{kt} + w_{kt} - p_t^k \tag{1.59}$$

$$p_{kt}^* = \beta\alpha_k \mathbb{E}_t p_{k,t+1}^* + (1 - \beta\alpha_k)mc_{kt} \tag{1.60}$$

$$p_{kt} = \alpha_k p_{k,t-1} + (1 - \alpha_k)p_{kt}^* \tag{1.61}$$

(b) Households

$$c_t = w_{kt} - p_t^c - \varphi n_{kt} \tag{1.62}$$

$$c_t = \sum_{k=1}^{K} \omega_{ck} c_{kt} \tag{1.63}$$

$$c_{kt} = -\lambda(p_{kt} - p_t^c) + c_t \tag{1.64}$$

(c) Monetary policy

Either

$$p_t^c + c_t = m_t, \quad m_t = m_{t-1} + r_t, \quad r_t = \rho_r r_{t-1} + \varepsilon_t^r \tag{1.65}$$

or

$$c_t = \mathbb{E}_t c_{t+1} - (i_t - \mathbb{E}_t p_{t+1}^c + p_t^c) \tag{1.66}$$

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)[\phi_\pi(p_t^c - p_{t-1}^c) + \phi_c c_t + \phi_{gc}(c_t - c_{t-1})] + \mu_t, \quad \mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t^\mu \tag{1.67}$$

(d) Market clearing

$$y_{kt} = \frac{\omega_{ck} C}{Y_k} c_{kt} + \sum_{r=1}^{K} \omega_{rk} \frac{\delta_r Y_r}{Y_k} z_{rkt} \tag{1.68}$$

1.B Proofs

In this section we present proofs of formal results in the baseline version of our model, corresponding to the model in 1.A under $\mathcal{A}_{kt} = 1, \varphi = 0, \lambda = 1$ and monetary policy implemented as a money supply rule. Unlike in the main text, we assume persistent monetary shocks, so that $r_t = \rho_r r_{t-1} + \varepsilon_t^r, \quad |\rho_r| \in (0,1)$. Trivially, proofs of results in main text follow as a special case under $\rho_r = 0$.

Proposition 1.1´. *The response of final consumption in sector k c_{kt} to a monetary shock r_t, where $r_t = \rho_r r_{t-1} + \varepsilon_t^r$ and $\{\varepsilon_t^r\}_t$ is a zero-mean i.i.d. process, is given by:*

$$c_{kt} = \overbrace{\alpha_k c_{k,t-1} + \frac{\alpha_k - \beta\alpha_k\rho_r}{1-\beta\alpha_k\rho_r} r_t}^{\textbf{Direct effect}} + \underbrace{\delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\omega_{kr}\mathbb{E}_t c_{r,t+s}}_{\textbf{Downstream effect}},$$

for $k = 1, 2, ..., K$. It consists of the the direct effect, unaffected by the intermediate inputs share δ_k, and the downstream effect, which fully vanishes under $\delta_k = 0$ and increases in δ_k, representing amplification through input-output linkages.

Proof. From the sectoral pricing equation we know that:

$$\begin{aligned} p_{kt} &= \alpha_k p_{k,t-1} + (1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \mathbb{E}_t mc_{k,t+s} \\ &= \alpha_k p_{k,t-1} + (1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \mathbb{E}_t \left[(1-\delta_k)m_{t+s} + \delta_k \sum_{r=1}^{K}\omega_{kr}p_{r,t+s}\right] \end{aligned}$$

Given that cash-in-advance constraint implies that $p_{kt} = m_t - c_{kt}$:

$$p_{kt} = \alpha_k p_{k,t-1} + (1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \mathbb{E}_t\left[(1-\delta_k)m_{t+s} + \delta_k \sum_{r=1}^{K}\omega_{kr}[m_{t+s} - c_{r,t+s}]\right]$$

$$= \alpha_k p_{k,t-1} + (1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \mathbb{E}_t m_{t+s} - \delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\omega_{kr}\mathbb{E}_t c_{r,t+s}$$

$$= \alpha_k p_{k,t-1} + (1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \left[m_t + r_t\rho_r\frac{1-\rho_r^s}{1-\rho_r}\right]$$
$$- \delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\omega_{kr}\mathbb{E}_t c_{r,t+s}$$

$$= \alpha_k p_{k,t-1} + (1-\alpha_k)m_t + (1-\alpha_k)(1-\beta\alpha_k)r_t\frac{\rho_r}{1-\rho_r}\sum_{s=0}^{\infty}(\beta\alpha_k)^s(1-\rho_r^s)$$
$$- \delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\omega_{kr}\mathbb{E}_t c_{r,t+s}$$

Similarly replace p_{kt} and $p_{k,t-1}$ using the cash-in-advance constraint:

$$m_t - c_{kt} = \alpha_k(m_{t-1} - c_{k,t-1}) + (1-\alpha_k)m_t + (1-\alpha_k)r_t\rho_r\frac{\beta\alpha_k}{1-\beta\alpha_k}$$
$$- \delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\mathbb{E}_t c_{r,t+s}$$

$$c_{kt} = \alpha_k c_{k,t-1} + \left[\alpha_k - \frac{(1-\alpha_k)\beta\alpha_k\rho_r}{1-\beta\alpha_k\rho_r}\right]r_t + \delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\mathbb{E}_t c_{r,t+s}$$

$$c_{kt} = \alpha_k c_{k,t-1} + \frac{\alpha_k - \beta\alpha_k\rho_r}{1-\beta\alpha_k\rho_r}r_t + \delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\mathbb{E}_t c_{r,t+s}.$$

□

Proposition 1.2´. *The response of final sectoral consumption c_{kt} to a monetary policy shock r_t, where $r_t = \rho_r r_{t-1} + \varepsilon_t^r$ and $\{\varepsilon_t^r\}_t$ is a zero-mean i.i.d. process, is*

$$c_{kt} = \sum_{j=1}^{K} l_{kj,T-t}\alpha_j c_{j,t-1} + \tilde{\alpha}_{k,T-t}r_t, \qquad 1 \le t \le T-1$$

where $\tilde{\alpha}_{k,T-t}$ is

$$\tilde{\alpha}_{k,T-t} \equiv \sum_{j=1}^{K} l_{kj,T-t}\left(\sum_{j'=1}^{K}\kappa_{jj',T-t}\right),$$

and $\{l_{kj,T-t}\}_{kj}$ *are entries of the Forward-looking Leontief Inverse:*

$$L_{T-t} = \underbrace{\left[I - \Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s\Omega\left\{\Pi_{j=T-t-s}^{T-t-1}L_jA\right\}\right]^{-1}}_{\text{Forward-looking Leontief Inverse}},$$

and $\{\kappa_{jj',T-t}\}_{jj'}$ *are entries of the following matrix:*

$$K_{T-t} = \left[(\underline{A}+\Gamma_{T-t}) + \Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s\Omega\sum_{\tau=0}^{s-1}\rho_r^{s-\tau}\left\{\Pi_{j=T-t-s}^{T-t-s+\tau-1}L_jA\right\}\left\{L_{T-t-s+\tau}K_{T-t-s+\tau}\right\}\right]$$

where $\underline{A} \equiv diag[\underline{\alpha}_k, \underline{\alpha}_2, ..., \underline{\alpha}_K]'$, $\underline{\alpha}_k \equiv \frac{\alpha_k - \beta\alpha_k\rho_r}{1-\beta\alpha_k\rho_r}$ $\forall k$, $\Gamma_{T-t} \equiv diag[\gamma_{1,T-t}, \gamma_{2,T-t}, ..., \gamma_{K,T-t}]'$,
$\gamma_{k,T-t} \equiv \frac{\rho_r}{1-\rho_r}\frac{(1-\alpha_k)(1-\beta\alpha_k)}{1-\beta\alpha_k\rho_r}\left[\frac{1-(\beta\alpha_k\rho_r)^{T-t}}{1-(\beta\alpha_k)^{T-t}} - 1\right]$ $\forall k$, $A \equiv diag[\alpha_1, \alpha_2, ..., \alpha_K]'$, $\Delta \equiv diag[\delta_1, \delta_2, ..., \delta_K]'$,
$G_{T-t} \equiv (I-\beta A)(I-(\beta A)^{T-t})^{-1}$ *and* Ω *is the input-output matrix.*

Proof. For convenience, rewrite the equation for sectoral consumption in the proposition in matrix form:

$$\mathbb{c}_t = L_{T-t}A\mathbb{c}_{t-1} + L_{T-t}K_{T-t}\mathbb{r}_t, \qquad 1 \le t \le T-1$$

where $\mathbb{c}_t \equiv [c_{1t}, c_{2t}, ..., c_{Kt}]'$, $\mathbb{r}_t = [r_t, r_t, ..., r_t]'$.

Note that from Proposition 1′ there follows a finite-time direct-downstream decomposition:

$$\mathbb{c}_t = A\mathbb{c}_{t-1} + [\underline{A}+\Gamma_{T-t}]\mathbb{r}_t + \Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s\Omega\mathbb{E}_t\mathbb{c}_{t+s}$$

where $\underline{A} \equiv diag[\underline{\alpha}_k, \underline{\alpha}_2, ..., \underline{\alpha}_K]'$, $\underline{\alpha}_k \equiv \frac{\alpha_k - \beta\alpha_k\rho_r}{1-\beta\alpha_k\rho_r}$ $\forall k$, $\Gamma_{T-t} \equiv diag[\gamma_{1,T-t}, \gamma_{2,T-t}, ..., \gamma_{K,T-t}]'$,
$\gamma_{k,T-t} \equiv \frac{\rho_r}{1-\rho_r}\frac{(1-\alpha_k)(1-\beta\alpha_k)}{1-\beta\alpha_k\rho_r}\left[\frac{1-(\beta\alpha_k\rho_r)^{T-t}}{1-(\beta\alpha_k)^{T-t}} - 1\right]$ $\forall k$.

The case for $t = T-1$ is trivial and follows from the finite-time direct-downstream decomposition above:

$$\mathbb{c}_{T-1} = A\mathbb{c}_{T-2} + [\underline{A}+\Gamma_1]\mathbb{r}_{T-1} + \Delta(I-A)G_1\Omega\mathbb{c}_{T-1},$$
$$\mathbb{c}_{T-1} = \underbrace{[I-\Delta(I-A)G_1\Omega]^{-1}}_{L_1}A\mathbb{c}_{T-2} + \underbrace{[I-\Delta(I-A)G_1\Omega]^{-1}}_{L_1}\underbrace{[\underline{A}+\Gamma_1]}_{K_1}\mathbb{r}_{T-1}.$$

As for $1 \leq t \leq T-2$, we will build our proof by strong induction. First, we will show that our conjecture holds for $t = T-2$; then we will show that it holds for any $1 \leq t \leq T-2$, if we suppose that it holds for all $t+1 \leq \tilde{t} \leq T-2$. For $t = T-2$, the finite-time direct-downstream decomposition is given by:

$$\mathbb{c}_{T-2} = A\mathbb{c}_{T-3} + [\underline{A} + \Gamma_2]\mathbb{r}_{T-2} + \Delta(I-A)G_2[\Omega\mathbb{c}_{T-2} + (\beta A)\Omega\mathbb{E}_{T-2}\mathbb{c}_{T-1}],$$

but we know that $\mathbb{c}_{T-1} = L_1 A\mathbb{c}_{T-2} + L_1 K_1 \mathbb{r}_{T-1}$, and hence

$$\begin{aligned}
\mathbb{c}_{T-2} &= A\mathbb{c}_{T-3} + [\underline{A} + \Gamma_2]\mathbb{r}_{T-2} + \Delta(I-A)G_2[\Omega\mathbb{c}_{T-2} + (\beta A)\Omega\mathbb{E}_{T-2}\{L_1 A\mathbb{c}_{T-2} + L_1 K_1 \mathbb{r}_{T-1}\}] \\
&= A\mathbb{c}_{T-3} + [\underline{A} + \Gamma_2]\mathbb{r}_{T-2} + \Delta(I-A)G_2\Omega\mathbb{c}_{T-2} + \Delta(I-A)G_2(\beta A)\Omega[L_1 A\mathbb{c}_{T-2} + \rho_r L_1 K_1 \mathbb{r}_{T-2}] \\
&= A\mathbb{c}_{T-3} + [(\underline{A} + \Gamma_2) + \Delta(I-A)G_2(\beta A)\Omega\rho_r L_1 K_1]\mathbb{r}_{T-2} + \Delta(I-A)G_2[\Omega + (\beta A)\Omega L_1 A]\mathbb{c}_{T-2}
\end{aligned}$$

which allows us to solve for $\mathbb{c}_{T-2}$:

$$\begin{aligned}
\mathbb{c}_{T-2} &= \underbrace{[I - \Delta(I-A)G_2(\Omega + (\beta A)\Omega L_1 A)]^{-1}}_{L_2} A\mathbb{c}_{T-3} \\
&\quad + \underbrace{[I - \Delta(I-A)G_2(\Omega + (\beta A)\Omega L_1 A)]^{-1}}_{L_2} \underbrace{[(\underline{A} + \Gamma_2) + \Delta(I-A)G_2(\beta A)\Omega\rho_r L_1 K_1]}_{K_2} \mathbb{r}_{T-2}, \\
\mathbb{c}_{T-2} &= L_2 A\mathbb{c}_{T-3} + L_2 K_2 \mathbb{r}_{T-2}.
\end{aligned}$$

as required to prove our conjecture for $t = T-2$.

Now let us write the direct-downstream decomposition for a generic period $1 \leq t \leq T-2$, and further suppose that our conjecture holds for all

$t+1 \leq \tilde{t} \leq T-1$:

$$\begin{aligned}
\mathbb{c}_t &= A\mathbb{c}_{t-1} + [\underline{A} + \Gamma_{T-t}]\mathbb{r}_t + \Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s \Omega \mathbb{E}_t \mathbb{c}_{t+s} \\
&= A\mathbb{c}_{t-1} + [\underline{A} + \Gamma_{T-t}]\mathbb{r}_t + \Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s \Omega \mathbb{E}_t \left[L_{T-t-s}A\mathbb{c}_{t+s-1} + L_{T-t-s}K_{T-t-s}\mathbb{r}_{t+s}\right] \\
&= A\mathbb{c}_{t-1} + [\underline{A} + \Gamma_{T-t}]\mathbb{r}_t + \Delta(I-A)G_{T-t}\times \\
&\times \sum_{s=0}^{T-t-1}(\beta A)^s \Omega \mathbb{E}_t \left[L_{T-t-s}AL_{T-t-s+1}A\mathbb{c}_{t+s-2} + L_{T-t-s}AL_{T-t-s+1}K_{T-t-s+1}\mathbb{r}_{t+s-1} + L_{T-t-s}K_{T-t-s}\mathbb{r}_{t+s}\right] \\
&= ... \\
&= A\mathbb{c}_{t-1} + [\underline{A} + \Gamma_{T-t}]\mathbb{r}_t + \Delta(I-A)G_{T-t}\times \\
&\times \sum_{s=0}^{T-t-1}(\beta A)^s \Omega \mathbb{E}_t \left[\left\{\Pi_{j=T-t-s}^{T-t-1} L_j A\right\}\mathbb{c}_t + \sum_{\tau=0}^{s-1}\left\{\Pi_{j=T-t-s}^{T-t-s-+\tau-1} L_j A\right\} L_{T-t-s+\tau}K_{T-t-s+\tau}\mathbb{r}_{t+s-\tau}\right] \\
&= A\mathbb{c}_{t-1} + [\underline{A} + \Gamma_{T-t}]\mathbb{r}_t + \Delta(I-A)G_{T-t}\times \\
&\times \sum_{s=0}^{T-t-1}(\beta A)^s \Omega \left[\left\{\Pi_{j=T-t-s}^{T-t-1} L_j A\right\}\mathbb{c}_t + \sum_{\tau=0}^{s-1}\rho_r^{s-\tau}\left\{\Pi_{j=T-t-s}^{T-t-s-+\tau-1} L_j A\right\} L_{T-t-s+\tau}K_{T-t-s+\tau}\mathbb{r}_t\right]
\end{aligned}$$

where we can now solve for $\mathbb{c}_t$:

$$\begin{aligned}
\mathbb{c}_t = &\underbrace{\left[I - \Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s \Omega\left\{\Pi_{j=T-t-s}^{T-t-1} L_j A\right\}\right]^{-1}}_{L_{T-t}} A\mathbb{c}_{t-1} + \\
&\underbrace{\left[I - \Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s \Omega\left\{\Pi_{j=T-t-s}^{T-t-1} L_j A\right\}\right]^{-1}}_{L_{T-t}} \times \\
&\times \underbrace{\left[(\underline{A} + \Gamma_{T-t}) + \Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s \Omega \sum_{\tau=0}^{s-1}\rho_r^{s-\tau}\left\{\Pi_{j=T-t-s}^{T-t-s-+\tau-1} L_j A\right\} L_{T-t-s+\tau}K_{T-t-s+\tau}\right]}_{K_{T-t}} \\
\mathbb{c}_t = &\, L_{T-t}A\mathbb{c}_{t-1} + L_{T-t}K_{T-t}\mathbb{r}_t.
\end{aligned}$$

□

Corollary 1.1´. *[Full IRF] Suppose the economy is in steady state until the beginning of period t; the impulse response of final sectoral consumption c_{kt}, $k =$*

$1, 2, ..., K$ *at horizon* $H \geq 0$*, following a one-time monetary shock innovation* ε_t^r*, where* $r_t = \rho_r r_{t-1} + \varepsilon_t^r \quad \forall t$*, is given by:*

$$c_{k,t+H} = \tilde{\tilde{\alpha}}_{k,T-t}^{H} \varepsilon_t^r, \qquad 1 \leq t, t+H \leq T-1$$

where $\tilde{\tilde{\alpha}}_{k,T-t}^{H} \equiv \sum_{r=1}^{K} \left[\sum_{h=0}^{H} \rho_r^{H-h} \left\{ \Pi_{j=T-t-H}^{T-t} L_j A \right\} \left\{ L_{T-t-H+h} K_{T-t+H-h} \right\} \right]_{kr}$*,* L_j *is period-*j *Forward-looking Leontief Inverse, matrix* K *is defined in Proposition 2´ and* $A \equiv diag[\alpha_1, \alpha_2, ..., \alpha_K]'$*. Therefore,* $\frac{dc_{k,t+H}}{d\varepsilon_t^r} = \tilde{\tilde{\alpha}}_{k,T-t}^{H}$*.*

Proof. Take the solution in Proposition 1.2´ written in matrix form, and iterate backwards:

$$\begin{aligned}
\mathbb{c}_t &= L_{T-t} A \mathbb{c}_{t-1} + L_{T-t} K_{T-t} \mathbb{r}_t, \quad 1 \leq t \leq T-1 \\
&= L_{T-t} A L_{T-t+1} A \mathbb{c}_{t-2} + L_{T-t} A L_{T-t+1} K_{T-t+1} \mathbb{r}_{t-1} + L_{T-t} K_{T-t} \mathbb{r}_t \\
&= L_{T-t} A L_{T-t+1} A L_{T-t+2} A \mathbb{c}_{t-3} + L_{T-t} A L_{T-t+1} L_{T-t+2} K_{T-t+2} \mathbb{r}_{t-2} + L_{T-t} A L_{T-t+1} K_{T-t+1} \mathbb{r}_{t-1} + L_{T-t} K_{T-t} \mathbb{r}_t \\
&= ... \\
&= \left\{ L_{T-t} A L_{T-t+1} A ... L_{T-1} A \right\} \mathbb{c}_0 + \\
&\qquad \left\{ L_{T-t} A L_{T-t+1} A ... L_{T-1} K_{T-1} \right\} \mathbb{r}_1 + ... + \left\{ L_{T-t} A L_{T-t+1} K_{T-t+1} \right\} \mathbb{r}_{t-1} + L_{T-t} K_{T-t} \mathbb{r}_t \\
\mathbb{c}_t &= \left\{ L_{T-t} A L_{T-t+1} A ... L_{T-1} A \right\} \mathbb{c}_0 + \sum_{h=0}^{t-1} \left\{ \Pi_{j=T-t}^{T-t+h-1} L_j A \right\} L_{T-t+h} K_{T-t+h} \mathbb{r}_{t-h}.
\end{aligned}$$

The expression above evaluated at a generic period $1 \leq t+H \leq T$:

$$\mathbb{c}_{t+H} = \left\{ L_{T-t-H} A L_{T-t+H+1} A ... L_{T-1} A \right\} \mathbb{c}_0 + \sum_{h=0}^{t+H-1} \left\{ \Pi_{j=T-t-H}^{T-t-H+h-1} L_j A \right\} L_{T-t-H+h} K_{T-t-H+h} \mathbb{r}_{t+H-h}.$$

Now suppose the economy is in steady state until the beginning of period t, so that $\mathbb{c}_0 = \mathbb{r}_1 = ... = \mathbb{r}_{t-1} = \mathbb{0}_{K \times 1}$, and there is a one-time monetary innovation ε_t^r hitting the economy, which does not repeat in the future; then $r_{t+H} = \rho_r^H \varepsilon_t^r, \quad H \geq 0$, and the impulse response of sectoral consumption H periods after the innovation is given by:

$$\mathbb{c}_{t+H} = \sum_{h=0}^{H} \rho_r^{H-h} \left\{ \Pi_{j=T-t-H}^{T-t-H+h-1} L_j A \right\} L_{T-t-H+h} K_{T-t-H+h} \varepsilon_t^r,$$

or when written in scalar form:

$$c_{k,t+H} = \tilde{\tilde{\alpha}}^{H}_{k,T-t}\varepsilon^{r}_{t},$$

where $\tilde{\tilde{\alpha}}^{H}_{k,T-t} \equiv \sum_{r=1}^{K}\left[\sum_{h=0}^{H}\rho_r^{H-h}\left\{\Pi_{j=T-t-H}^{T-t}L_jA\right\}\left\{L_{T-t-H+h}K_{T-t+H-h}\right\}\right]_{kr}, \forall k = 1,2,...,K; 1 \leq t, t+H \leq T-1.$ □

Corollary 1.2´. *[Upper Bound of Direct IRF] Suppose the economy is in steady state until the beginning of period t; the impulse response of final sectoral consumption c_{kt}, $k = 1,2,...,K$ at horizon $H \geq 1$, following a one-time monetary shock innovation ε^r_t, where $r_t = \rho_r r_{t-1} + \varepsilon^r_t \quad \forall t$, can be written as:*

$$c_{k,t+H} = \left[\sum_{\tau=0}^{H}\rho_r^{H-\tau}\left\{\alpha_k^{\tau}(\underline{\alpha}_k + \gamma_{k,T-t-H+\tau}) + \delta_k q^k(T-t-H,\tau)\right\}\right]\varepsilon^r_t + \sum_{\tau=1}^{H}\delta_k s^k_{T-t-H+\tau}\sum_{r=1}^{K}\omega_{kr}c_{r,t+H-\tau},$$

$1 \leq t, t+H \leq T-1$, *where* $q^k(T-t-H,\tau) \equiv \sum_{j=1}^{K}\left[\mathbb{Q}(T-t-H,\tau)\right]_{kj} \geq 0$, $\underline{\alpha}_k \equiv \frac{\alpha_k - \beta\alpha_k\rho_r}{1-\beta\alpha_k\rho_r}$, $\gamma_{k,T-t} \equiv \frac{\rho_r}{1-\rho_r}\frac{(1-\alpha_k)(1-\beta\alpha_k)}{1-\beta\alpha_k\rho_r}\left[\frac{1-(\beta\alpha_k\rho_r)^{T-t}}{1-(\beta\alpha_k)^{T-t}} - 1\right]$, $s^k_{T-t-H+\tau} \equiv \alpha_k^{\tau}(1-\alpha_k)g_{T-t-H+\tau}$, $g_{T-t-H+\tau} \equiv [G_{T-t-H+\tau}]_{kk}$. *Hence,* $\frac{\partial c_{k,t+H}}{\partial r_t} = \left[\sum_{\tau=0}^{H}\rho_r^{H-\tau}\left\{\alpha_k^{\tau}(\underline{\alpha}_k + \gamma_{k,T-t-H+\tau}) + \delta_k q^k(T-t-H,\tau)\right\}\right] \geq \left[\sum_{\tau=0}^{H}\rho_r^{H-\tau}\left\{\alpha_k^{\tau}(\underline{\alpha}_k + \gamma_{k,T-t-H+\tau})\right\}\right]$.

Proof. Substitute the expression for $\mathbb{c}_t$ obtained in Corollary 1.1´ into the finite-time direct-downstream decomposition (let $\mathbb{c}_0 = \mathbb{0}_{K\times 1}$):

$$\mathbb{c}_t = A\mathbb{c}_{t-1} + [\underline{A} + \Gamma_{T-t}]\mathbb{r}_t + \Delta(I-A)G_{T-t}\Omega\mathbb{c}_t + \Delta(I-A)G_{T-t}\sum_{s=1}^{T-t-1}(\beta A)^s\Omega\left[\sum_{h=0}^{t+s-1}\left\{\Pi_{j=T-t-s}^{T-t-s+h-1}L_jA\right\}L_{T-t-s+h}K_{T-t-s+h}\mathbb{E}_t\mathbb{r}_{t+s-h}\right],$$

and separating out past and future shocks:

$$\mathbb{c}_t = A\mathbb{c}_{t-1} + [\underline{A} + \Gamma_{T-t}]\mathbb{r}_t + \Delta(I-A)G_{T-t}\Omega\mathbb{c}_t + \Delta(I-A)G_{T-t}\sum_{s=1}^{T-t-1}(\beta A)^s\Omega\left[\sum_{h=0}^{s}\left\{\Pi_{j=T-t-s}^{T-t-s+h-1}L_jA\right\}L_{T-t-s+h}K_{T-t-s+h}\mathbb{E}_t\mathbb{r}_{t+s-h}\right] + \Delta(I-A)G_{T-t}\sum_{s=1}^{T-t-1}(\beta A)^s\Omega\left[\sum_{h=s+1}^{t+s-1}\left\{\Pi_{j=T-t-s}^{T-t-s+h-1}L_jA\right\}L_{T-t-s+h}K_{T-t-s+h}\mathbb{r}_{t+s-h}\right],$$

$$\mathbb{c}_t = A\mathbb{c}_{t-1} + \Delta(I-A)G_{T-t}\Omega\mathbb{c}_t +$$
$$\left[\underline{A} + \Gamma_{T-t} + \Delta(I-A)G_{T-t}\sum_{s=1}^{T-t-1}(\beta A)^s\Omega\left[\sum_{h=0}^{s}\rho_r^{s-h}\left\{\Pi_{j=T-t-s}^{T-t-s+h-1}L_jA\right\}L_{T-t-s+h}K_{T-t-s+h}\right]\right]\mathbb{r}_t +$$
$$\Delta(I-A)G_{T-t}\sum_{s=1}^{T-t-1}(\beta A)^s\Omega\left[\sum_{h=s+1}^{t+s-1}\left\{\Pi_{j=T-t-s}^{T-t-s+h-1}L_jA\right\}L_{T-t-s+h}K_{T-t-s+h}\mathbb{r}_{t+s-h}\right]$$

For convenience, let $\tau \equiv (h-s)$:

$$\mathbb{c}_t = A\mathbb{c}_{t-1} + \Delta(I-A)G_{T-t}\Omega\mathbb{c}_t +$$
$$\left[\underline{A} + \Gamma_{T-t} + \Delta(I-A)G_{T-t}\sum_{s=1}^{T-t-1}(\beta A)^s\Omega\left[\sum_{h=-s}^{0}\rho_r^{-h}\left\{\Pi_{j=T-t-s}^{T-t-s+h-1}L_jA\right\}L_{T-t-s+h}K_{T-t-s+h}\right]\right]\mathbb{r}_t +$$
$$\Delta(I-A)G_{T-t}\sum_{s=1}^{T-t-1}(\beta A)^s\Omega\left[\sum_{\tau=1}^{t-1}\left\{\Pi_{j=T-t-s}^{T-t+\tau-1}L_jA\right\}L_{T-t+\tau}K_{T-t+\tau}\mathbb{r}_{t-\tau}\right],$$
$$\mathbb{c}_t = A\mathbb{c}_{t-1} + [\underline{A} + \Gamma_{T-t} + \Delta\tilde{\mathbb{R}}(T-t,\rho_r)]\mathbb{r}_t + \sum_{\tau=1}^{t-1}[\Delta\mathbb{R}(T-t,\tau)]\mathbb{r}_{t-\tau} + \Delta(I-A)G_{T-t}\Omega\mathbb{c}_t,$$

where $\tilde{\mathbb{R}}(x,\rho_r) \equiv (I-A)G_x\left[\sum_{s=1}^{x-1}(\beta A)^s\Omega\sum_{h=-s}^{0}\rho_r^{-h}\left\{\Pi_{j=x-s}^{x+h-1}L_jA\right\}L_{x+h}K_{x+h}\right]$,
$\mathbb{R}(x,y) \equiv (I-A)G_x\left[\sum_{s=1}^{x-1}(\beta A)^s\Omega\left\{\Pi_{j=x-s}^{x+y-1}L_jA\right\}L_{x+y}K_{x+y}\right]$.

Now iterate the above expression backwards:

$$\begin{aligned}
\mathbb{c}_t &= A^2\mathbb{c}_{t-2} + [\underline{A} + \Gamma_{T-t} + \Delta\tilde{\mathbb{R}}(T-t,\rho_r)]\mathbb{r}_t + \\
&\quad [A(\underline{A} + \Gamma_{T-t+1}) + A\Delta\tilde{\mathbb{R}}(T-t+1,\rho_r) + \Delta\mathbb{R}(T-t,1)]\mathbb{r}_{t-1} + \\
&\quad \sum_{\tau=2}^{t-1} \left[\Delta\mathbb{R}(T-t,\tau) + A\Delta\mathbb{R}(T-t+1,\tau-1)\right]\mathbb{r}_{t-\tau} + \\
&\quad \Delta(I-A)G_{T-t}\Omega\mathbb{c}_t + A\Delta(I-A)G_{T-t+1}\Omega\mathbb{c}_{t-1} \\
\mathbb{c}_t &= A^3\mathbb{c}_{t-2} + [\underline{A} + \Gamma_{T-t} + \Delta\tilde{\mathbb{R}}(T-t,\rho_r)]\mathbb{r}_t + \\
&\quad [A(\underline{A} + \Gamma_{T-t+1}) + A\Delta\tilde{\mathbb{R}}(T-t+1,\rho_r) + \Delta\mathbb{R}(T-t,1)]\mathbb{r}_{t-1} + \\
&\quad [A^2(\underline{A} + \Gamma_{T-t+2} + \Delta\tilde{\mathbb{R}}(T-t+2,\rho_r)) + A\Delta\mathbb{R}(T-t+1,1) + \Delta\mathbb{R}(T-t,2)]\mathbb{r}_{t-2} + \\
&\quad \sum_{\tau=3}^{t-1} \left[A^2\delta\mathbb{R}(T-t+2,\tau-2) + A\Delta\mathbb{R}(T-t+1,\tau-1) + \Delta\mathbb{R}(T-t,\tau)\right]\mathbb{r}_{t-\tau} + \\
&\quad \Delta(I-A)G_{T-t}\Omega\mathbb{c}_t + A\Delta(I-A)G_{T-t+1}\Omega\mathbb{c}_{t-1} + A^2\Delta(I-A)G_{T-t+2}\Omega\mathbb{c}_{t-2}. \\
&= ... \\
\mathbb{c}_t &= A^t\mathbb{c}_0 + \sum_{\tau=0}^{t-1}\left[A^\tau(\underline{A} + \Gamma_{T-t+\tau} + \Delta\tilde{\mathbb{R}}(T-t+\tau,\rho_r)) + \sum_{l=0}^{\tau-1} A^l\Delta\mathbb{R}(T-t+l,\tau-l)\right]\mathbb{r}_{t-\tau} + \\
&\quad \Delta(I-A)G_{T-t}\Omega\mathbb{c}_t + \sum_{\tau=1}^{t-1} A^\tau\Delta(I-A)G_{T-t+\tau}\Omega\mathbb{c}_{t-\tau} \\
\mathbb{c}_t &= \sum_{\tau=0}^{t-1}\left[A^\tau(\underline{A} + \Gamma_{T-t+\tau} + \Delta\tilde{\mathbb{R}}(T-t+\tau,\rho_r)) + \sum_{l=0}^{\tau-1} A^l\Delta\mathbb{R}(T-t+l,\tau-l)\right]\mathbb{r}_{t-\tau} + \\
&\quad \Delta(I-A)G_{T-t}\Omega\left[\sum_{\tau=0}^{t-1}\left\{\Pi_{j=T-t}^{T-t+\tau-1}L_jA\right\}L_{T-t+\tau}K_{T-t+\tau}\mathbb{r}_{t-\tau}\right] + \sum_{\tau=1}^{t-1} A^\tau\Delta(I-A)G_{T-t+\tau}\Omega\mathbb{c}_{t-\tau} \\
\mathbb{c}_t &= \sum_{\tau=0}^{t-1}\left[A^\tau(\underline{A} + \Gamma_{T-t+\tau}) + \Delta\mathbb{Q}(T-t,\tau)\right]\mathbb{r}_{t-\tau} + \sum_{\tau=1}^{t-1} A^\tau\Delta(I-A)G_{T-t+\tau}\Omega\mathbb{c}_{t-\tau},
\end{aligned}$$

$$\mathbb{Q}(T-t,\tau) \equiv \left[A^\tau\tilde{\mathbb{R}}(T-t+\tau,\rho_r) + \textstyle\sum_{l=0}^{\tau-1} A^l\mathbb{R}(T-t-l,\tau-l) + (I-A)G_{T-t}\Omega\left\{\Pi_{j=T-t}^{T-t+\tau-1}L_jA\right\}L_{T-t+\tau}K_{T-t+\tau}\right]$$

The expression above evaluated at a generic period $1 \leq t+H \leq T$:

$$\mathbb{c}_{t+H} = \sum_{\tau=0}^{t+H-1}\left[A^\tau(\underline{A} + \Gamma_{T-t-H+\tau}) + \Delta\mathbb{Q}(T-t-H,\tau)\right]\mathbb{r}_{t+H-\tau} + \sum_{\tau=1}^{t+H-1} A^\tau\Delta(I-A)G_{T-t-H+\tau}\Omega\mathbb{c}_{t+H-\tau}.$$

Now suppose the economy is in steady state until the beginning of period t, so that $\mathbb{c}_0 = \mathbb{r}_1 = ... = \mathbb{r}_{t-1} = \mathbb{0}_{K\times 1}$, and there is a one-time monetary

innovation ε_t^r hitting the economy, which does not repeat in the future; then $r_{t+H} = \rho_r^H \varepsilon_t^r, \quad H \geq 0$, and the impulse response of sectoral consumption H periods after the innovation is given by:

$$\mathbb{c}_{t+H} = \sum_{\tau=0}^{H} \rho_r^{H-\tau} \left[A^\tau (\underline{A} + \Gamma_{T-t-H+\tau}) + \Delta \mathbb{Q}(T-t-H,\tau) \right] \varepsilon_t^r + \sum_{\tau=1}^{H} A^\tau \Delta (I-A) G_{T-t-H+\tau} \Omega \mathbb{c}_{t+H-\tau}.$$

or when written in scalar form:

$$c_{k,t+H} = \left[\sum_{\tau=0}^{H} \rho_r^{H-\tau} \left\{ \alpha_k^\tau (\underline{\alpha}_k + \gamma_{k,T-t-H+\tau}) + \delta_k q^k (T-t-H,\tau) \right\} \right] \varepsilon_t^r + \sum_{\tau=1}^{H} \delta_k s^k_{T-t-H+\tau} \sum_{r=1}^{K} \omega_{kr} c_{r,t+H-\tau}$$

where $q^k(T-t-H,\tau) \equiv \sum_{j=1}^{K} \left[\mathbb{Q}(T-t-H,\tau) \right]_{kj} \geq 0, \quad s^k_{T-t-H+\tau} \equiv \alpha_k^\tau (1-\alpha_k) g_{T-t-H+\tau}, \quad g_{T-t-H+\tau} \equiv [G_{T-t-H+\tau}]_{kk}$, so that $\frac{\partial c_{k,t+H}}{\partial r_t} = \left[\sum_{\tau=0}^{H} \rho_r^{H-\tau} \left\{ \alpha_k^\tau (\underline{\alpha}_k + \gamma_{k,T-t-H+\tau}) + \delta_k q^k (T-t-H,\tau) \right\} \right] \geq \left[\sum_{\tau=0}^{H} \rho_r^{H-\tau} \left\{ \alpha_k^\tau (\underline{\alpha}_k + \gamma_{k,T-t-H+\tau}) \right\} \right], \forall k = 1,2,...,K; 1 \leq t, t+H \leq T-1. \quad \square$

1.C Three-sector economy example ($K = 3$)

1.C.1 Model with money supply rule

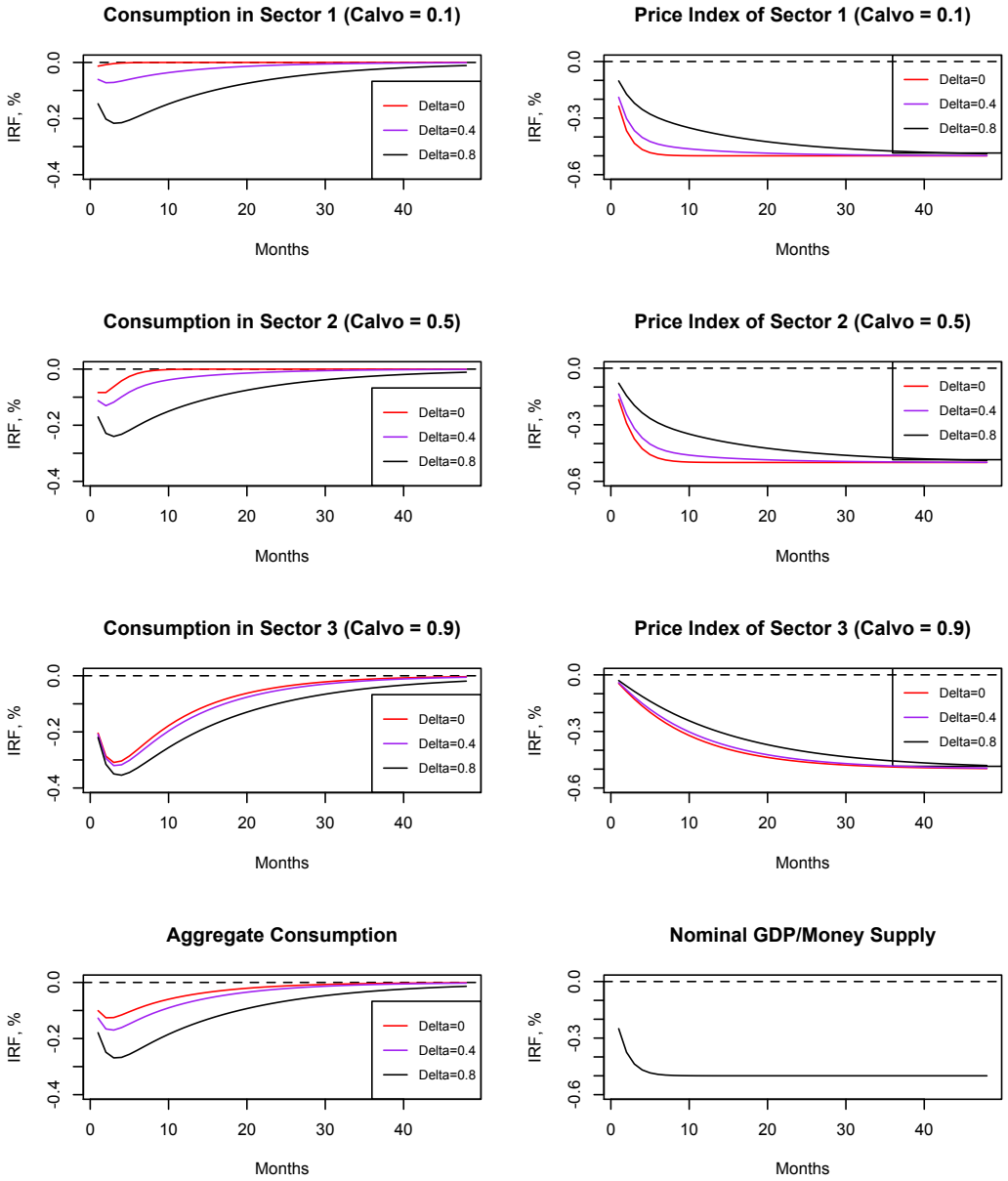
In this subsection we focus on a three-sector ($K = 3$) version of our economy, with the baseline setup considered in the main text. Its equilibrium conditions are summarized in 1.A under $\mathcal{A}_{kt} = 1, \varphi = 0, \lambda = 1$ and monetary policy implemented through a money supply rule. We calibrate our model to monthly data by setting $\beta = 0.9975, \rho_r = 0.5, \sigma_r = 0.0025$, and the consumption shares are set to be equal ($\omega_{c1} = \omega_{c2} = \omega_{c3} = 1/3$); for simplicity, we also assume $\delta_1 = \delta_2 = \delta_3 = \delta$. We perform two exercises that consider the response of our system to a monetary contraction. In particular, we subject our system to one-time one-standard deviation negative innovation to the monetary policy shock variable r that translates into a permanent contraction of nominal GDP/money supply.

In our first exercise, Sector 1 is assumed to be almost, but not completely, flexible ($\alpha_1 = 0.1$), Sector 2 is semi-flexible ($\alpha_2 = 0.5$) and Sector 3 is almost completely rigid ($\alpha_3 = 0.9$). The input-output matrix is set to be completely symmetric, so that $\omega_{kr} = 1/3 \quad \forall k, r \in \{1,2,3\}$. Figure 1.C.1 shows the responses of such system to the permanent contraction in money supply for

three different values of the intermediate inputs share $\delta \in (0, 0.4, 0.8)$. We can see that in the economy with no intermediate inputs ($\delta = 0$), the magnitude of the reduction in consumption is proportional to own probability of price non-adjustment α_k, which is a standard result in models with no intermediate inputs and represents the *direct effect* in the notation from Proposition 1.1. As we increase the share of intermediate inputs, the magnitude of reduction in sectoral consumption becomes larger for all sectors, representing the addition of the *downstream effect*. Other things equal, we can see that the *degree of amplification increases in the economy's share of intermediate inputs* δ. This is because higher δ means that the sensitivity of marginal cost to suppliers' prices is higher and hence allowing for stronger "inheritance" of stickiness.

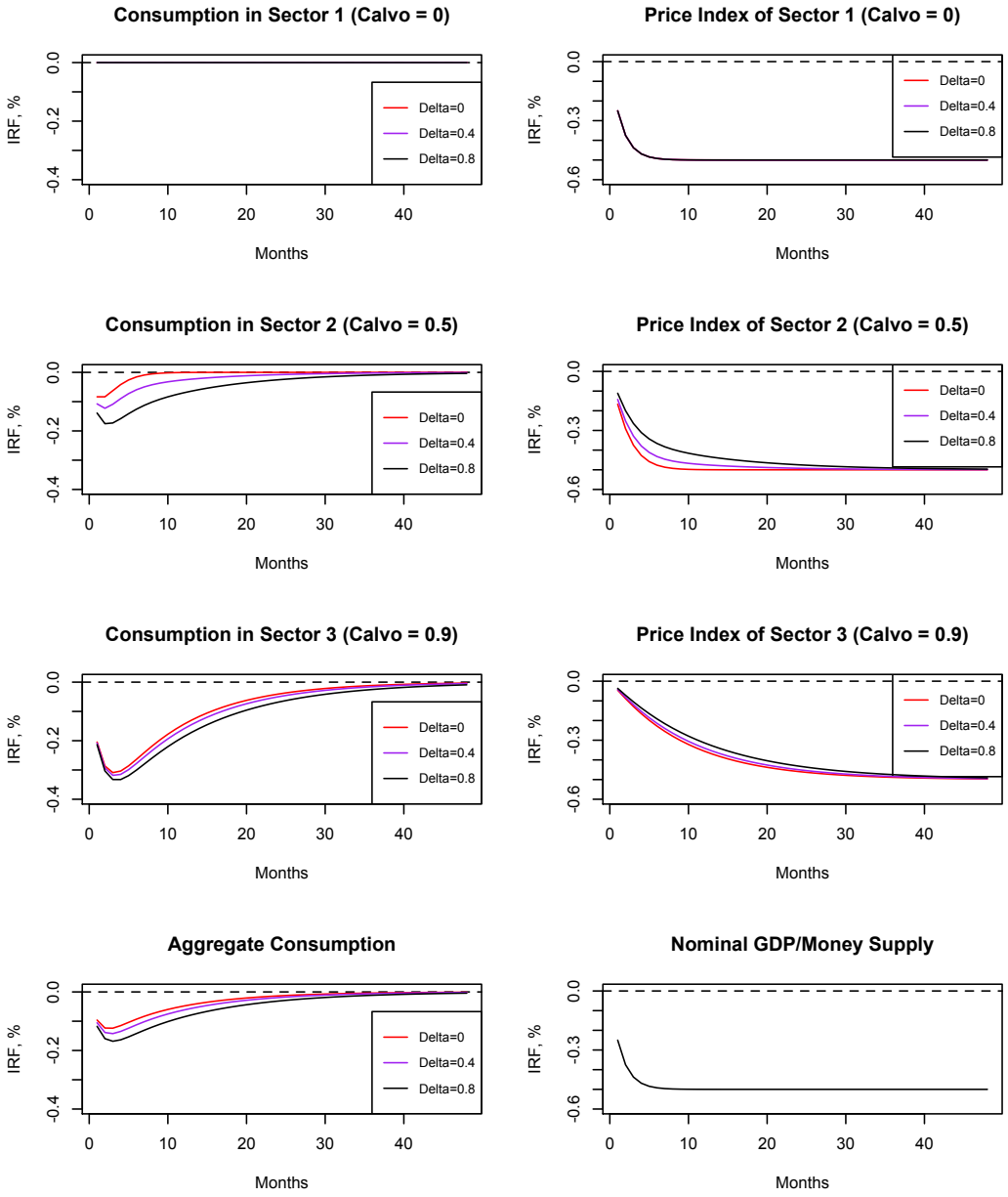
In our second exercise, Sector 1 is completely flexible ($\alpha_1 = 0$) with the other sectors remaining unchanged from the previous exercise ($\alpha_2 = 0.5, \alpha_3 = 0.9$). In addition, firms in Sector 1 are assumed to be buying inputs only from other firms in Sector 1 ($\omega_{11} = 1, \omega_{12} = \omega_{13} = 0$), whereas the input-output weights for the other two sectors are as in the previous exercise ($\omega_{kr} = 1/3 \quad k = 2, 3; r = 1, 2, 3$). Figure 1.C.2 shows the responses to the monetary contraction. As before, for $\delta = 0$, the magnitude of sectoral consumption fall is proportional to the degree of price stickiness, with the fully flexible Sector 1 exhibiting no change in sectoral consumption, as its price index adjusts one-for-one with the money supply. Most importantly, as we increase δ, Sector 2 and Sector 3 experience greater decrease in consumption, whereas Sector 1 still showing no response. The latter highlights the fact that the amplification of consumption response is due to *downstream propagation* of price stickiness, as it travels from suppliers firms to customer firms. Indeed, firms in Sector 1 do not purchase inputs from the sticky Sectors 2 and 3 and only from other fully flexible firms in their own sector. Hence, firms in Sector 1 do not inherit any stickiness from the other sectors and experience no amplification.

Figure 1.C.1: Responses of sectoral consumptions and price indices to a monetary contraction



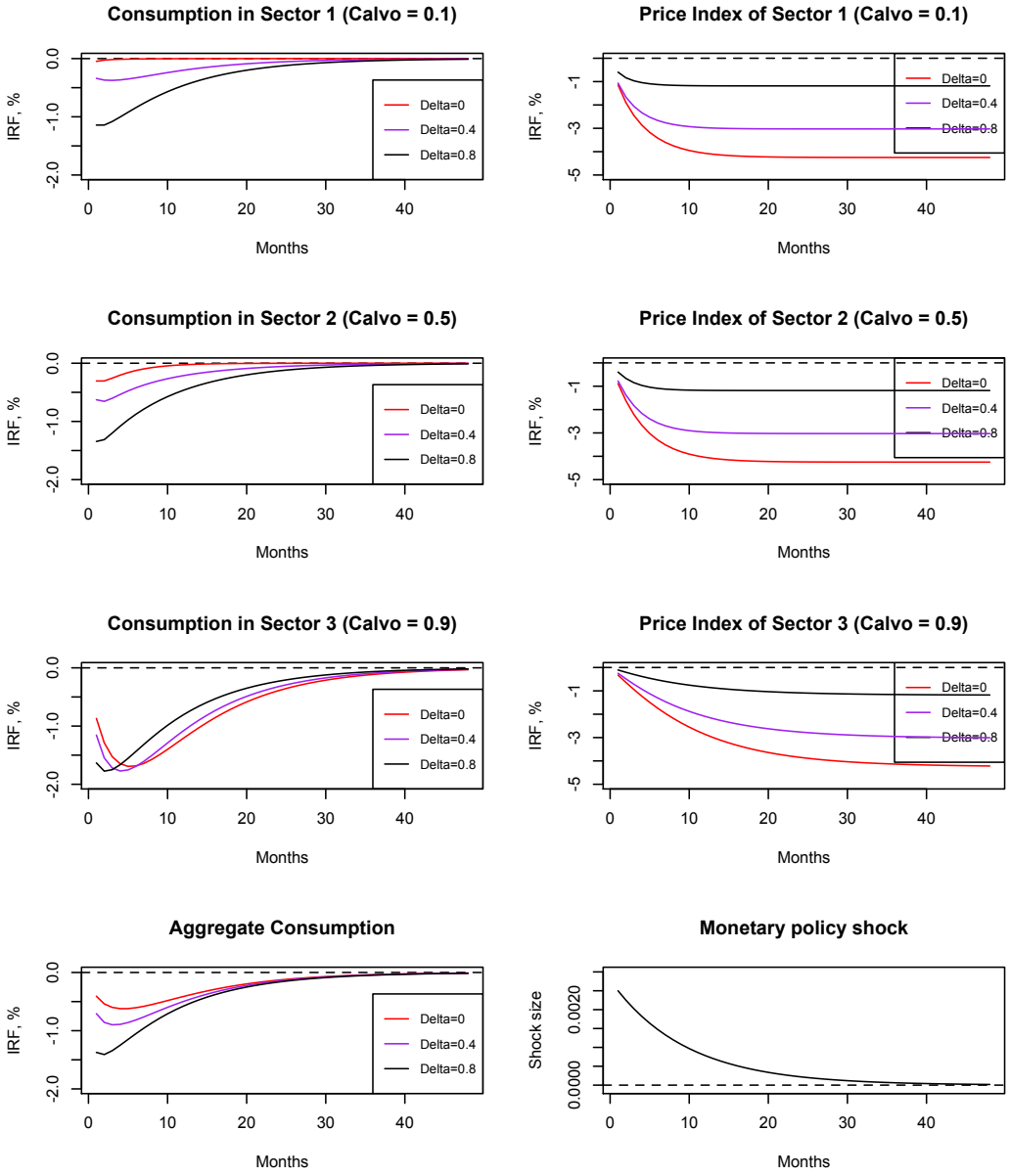
***Note**: The figure is constructed by calibrating the log-linearized model in 1.A with money supply rule and considering a one-time one standard deviation negative innovation to the monetary policy shock variable that generates a permanent contraction in money supply. The impulse responses are found numerically using Dynare for the following calibration:* $\lambda = 1, \varphi = 0, \beta = 0.9975, \rho_r = 0.5, \sigma_r = 0.0025, \alpha_1 = 0.1, \alpha_2 = 0.5, \alpha_3 = 0.9, \omega_{kr} = \omega_{cr} = 1/3 \quad \forall k, r \in \{1, 2, 3\}$.

Figure 1.C.2: Responses of sectoral consumptions and price indices to a monetary contraction



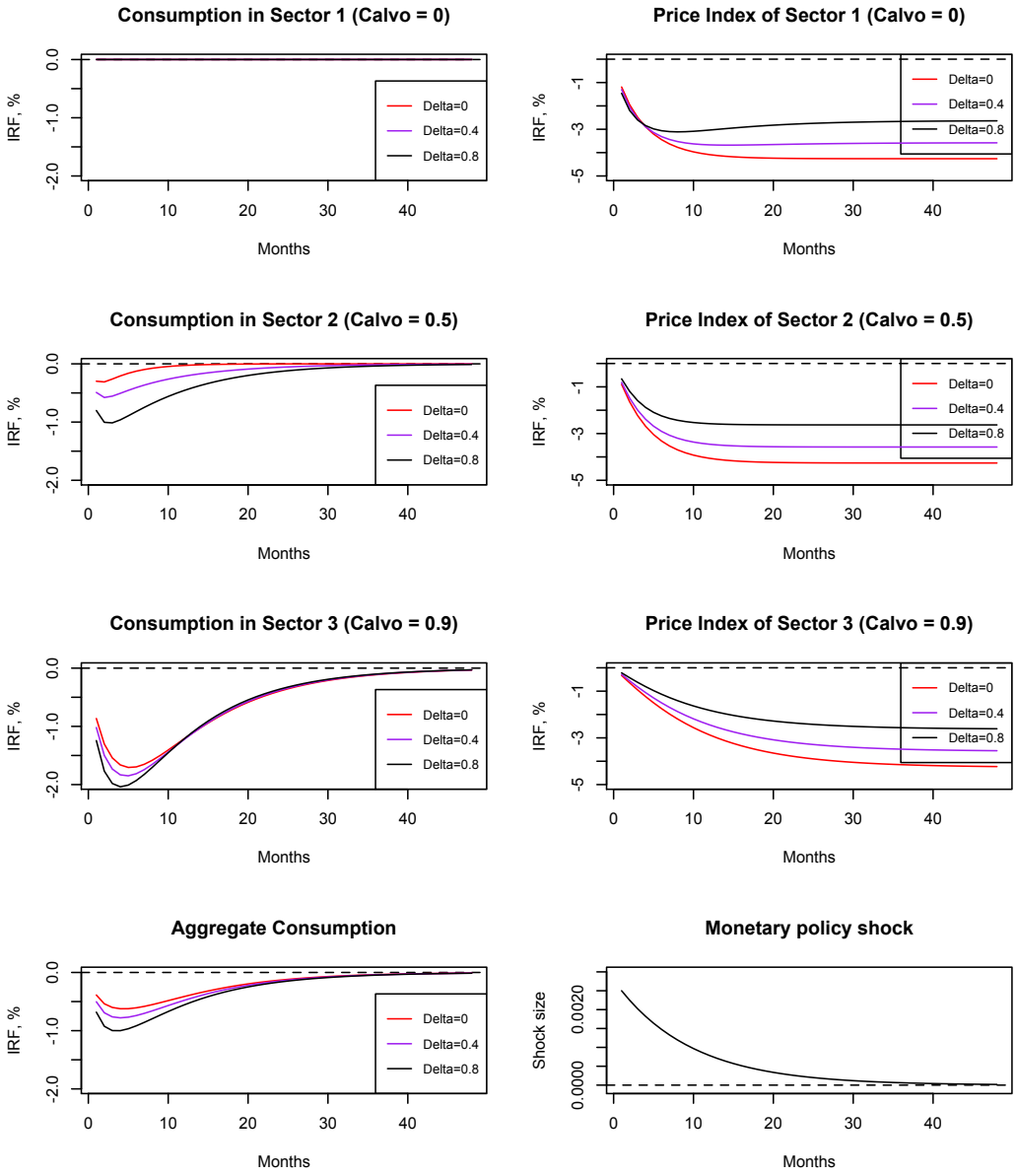
Note: *The figure is constructed by calibrating the log-linearized model in 1.A with money supply rule for* $K = 3$ *and considering a one-time one standard deviation negative innovation to the monetary policy shock variable that generates a permanent contraction in money supply. The impulse responses are found numerically using Dynare for the following calibration:* $\lambda = 1, \varphi = 0, \beta = 0.9975, \rho_r = 0.5, \sigma_r = 0.0025, \alpha_1 = 0, \alpha_2 = 0.5, \alpha_3 = 0.9, \omega_{11} = 1, \omega_{12} = \omega_{13} = 0, \omega_{kr} = \omega_{cr} = 1/3 \quad k = 2, 3; r = 1, 2, 3.$

Figure 1.C.3: Responses of sectoral consumptions and prices to a monetary policy shocks (Taylor rule)



***Note**: The figure is constructed by calibrating the log-linearized model in 1.A with Taylor rule for $K = 3$ and considering a one-time one standard deviation positive innovation to the monetary policy shock variable that generates a monetary contraction. The impulse responses are found numerically using Dynare for the following calibration: $\lambda = 1, \varphi = 0, \beta = 0.9975, \rho_i = 0, \rho_\mu = 0.9, \sigma_\mu = 0.0025, \phi_\pi = 1.24, \phi_{gc} = 0, \phi_c = 0.33/12, \alpha_1 = 0.1, \alpha_2 = 0.5, \alpha_3 = 0.9, \omega_{kr} = \omega_{cr} = 1/3 \quad \forall k, r \in \{1, 2, 3\}$.*

Figure 1.C.4: Responses of sectoral consumptions and prices to a monetary policy shocks (Taylor rule)



***Note**: The figure is constructed by calibrating the log-linearized model in 1.A with Taylor rule for $K = 3$ and considering a one-time one standard deviation positive innovation to the monetary policy shock variable that generates a monetary contraction. The impulse responses are found numerically using Dynare for the following calibration: $\lambda = 1, \varphi = 0, \beta = 0.9975, \rho_i = 0, \rho_\mu = 0.9, \sigma_\mu = 0.0025, \phi_\pi = 1.24, \phi_{gc} = 0, \phi_c = 0.33/12, \alpha_1 = 0, \alpha_2 = 0.5, \alpha_3 = 0.9, \omega_{11} = 1, \omega_{12} = \omega_{13} = 0, \omega_{kr} = \omega_{cr} = 1/3 \quad k = 2, 3; r = 1, 2, 3.$

1.C.2 Model with Taylor rule

Consider an arguably more realistic setting, where we model the monetary authority as setting the level of nominal interest I_t in the economy, as described in 1.A, under Taylor rule and $\mathcal{A}_{kt} = 1, \varphi = 0, \lambda = 1$. Once again, we focus on a three-sector $(K = 3)$ version of our economy and perform two exercises that consider the response of our system to a monetary contraction. In particular, we subject our system to a one-time one-standard deviation positive innovation to the monetary policy shock ε^{μ} that generates a monetary tightening. We calibrate our model to monthly data by setting $\beta = 0.9975, \rho_i = 0, \rho_\mu = 0.9, \sigma_\mu = 0.0025, \phi_\pi = 1.24, \phi_{gc} = 0$ and ϕ_c=0.33/12, and also additionally assume for simplicity that $\delta_1 = \delta_2 = \delta_3 = \delta$.

Just like in the previous subsection model, in our first exercise Sector 1 is assumed to be almost, but not completely flexible, $(\alpha_1 = 0.1)$, Sector 2 is semi-flexible $(\alpha_2 = 0.5$ and Sector 3 is almost completely rigid $(\alpha_3 = 0.9)$. The input-output matrix is set to be completely symmetric, so that $\omega_{kr} = 1/3 \quad \forall k, r \in \{1, 2, 3\}$. Figure 1.C.3 shows the responses of our system to the monetary contraction for different values of $\delta \in \{0, 0.4, 0.8\}$. As one can see, the results are qualitatively identical to those obtained under money supply rule in our original model. Firstly, as before, under $\delta = 0$, the magnitude of contraction in sectoral consumption increases in sector's own price stickiness. Secondly, other things equal, the degree of amplification increases in δ.

In our second exercise, we assume that Sector 1 is completely flexible $(\alpha_1 = 0)$, with the other sectors remaining unchanged from the previous exercise $(\alpha_2 = 0.5, \alpha_3 = 0.9)$. Just as before, firms in Sector 1 buy inputs only from other firms in Sector 1 $(\omega_{11} = 1, \omega_{12} = \omega_{13} = 0)$, whereas the input-output weights for the other two sectors are as in the previous exercise $(\omega_{kr} = 1/3, \quad \forall k, r \in \{2, 3\})$. Figure 1.C.4 shows the response of such a system to a monetary contraction. We can see that, just as with the money supply rule in the original model, Sector 1 does not experience any change in consumption under $\delta = 0$, with $\delta > 0$ leaving the zero consumption response in place due to the fact that the propagation of price stickiness still goes

downstream. As before, since firms in Sector 1 do not purchase any inputs from firms in sticky sectors 2 and 3, the do not inherit any stickiness.

1.D Reconciling sectoral “consumption puzzles"

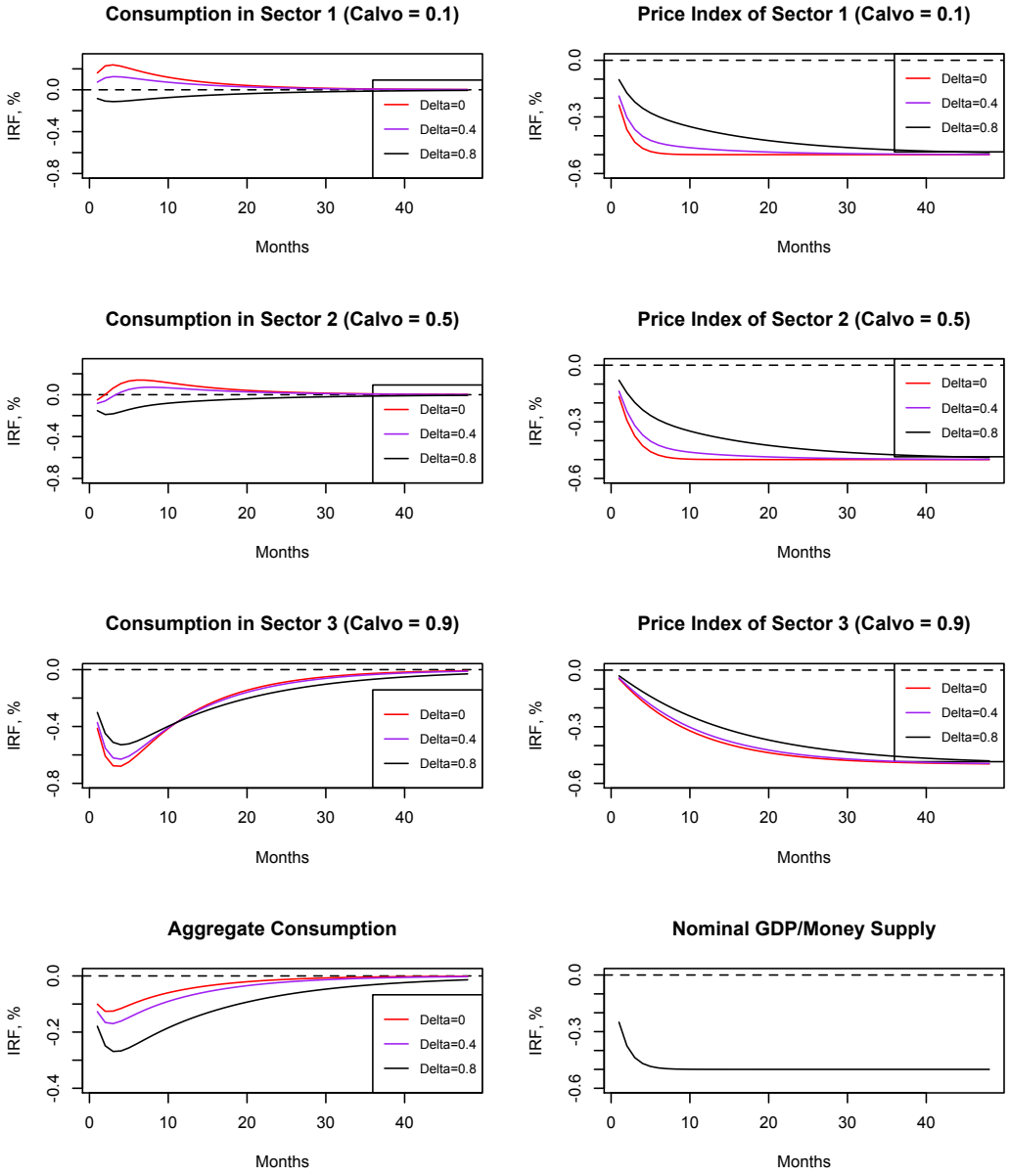
In this section we show how a slight modification to our baseline model in the main text can reconcile the “inverted" responses of sectoral consumption to monetary policy shocks, as well as dampening downstream effect contributions, that we documented in Section 1.6. Our baseline model assumed unit elasticity of substitution across sectors; however, in 1.A we allow for a general constant elasticity of substitution (CES) across sectors λ, which nests unit elasticity of substitution under $\lambda = 1$.

The log-linearized expression for sectoral consumption under CES aggregation (while keeping the money supply rule and infinite Frisch elasticity $\varphi = 0$) becomes:

$$c_{kt} = m_t - p_{kt} + (\lambda - 1)(p_t^c - p_{kt}). \tag{1.69}$$

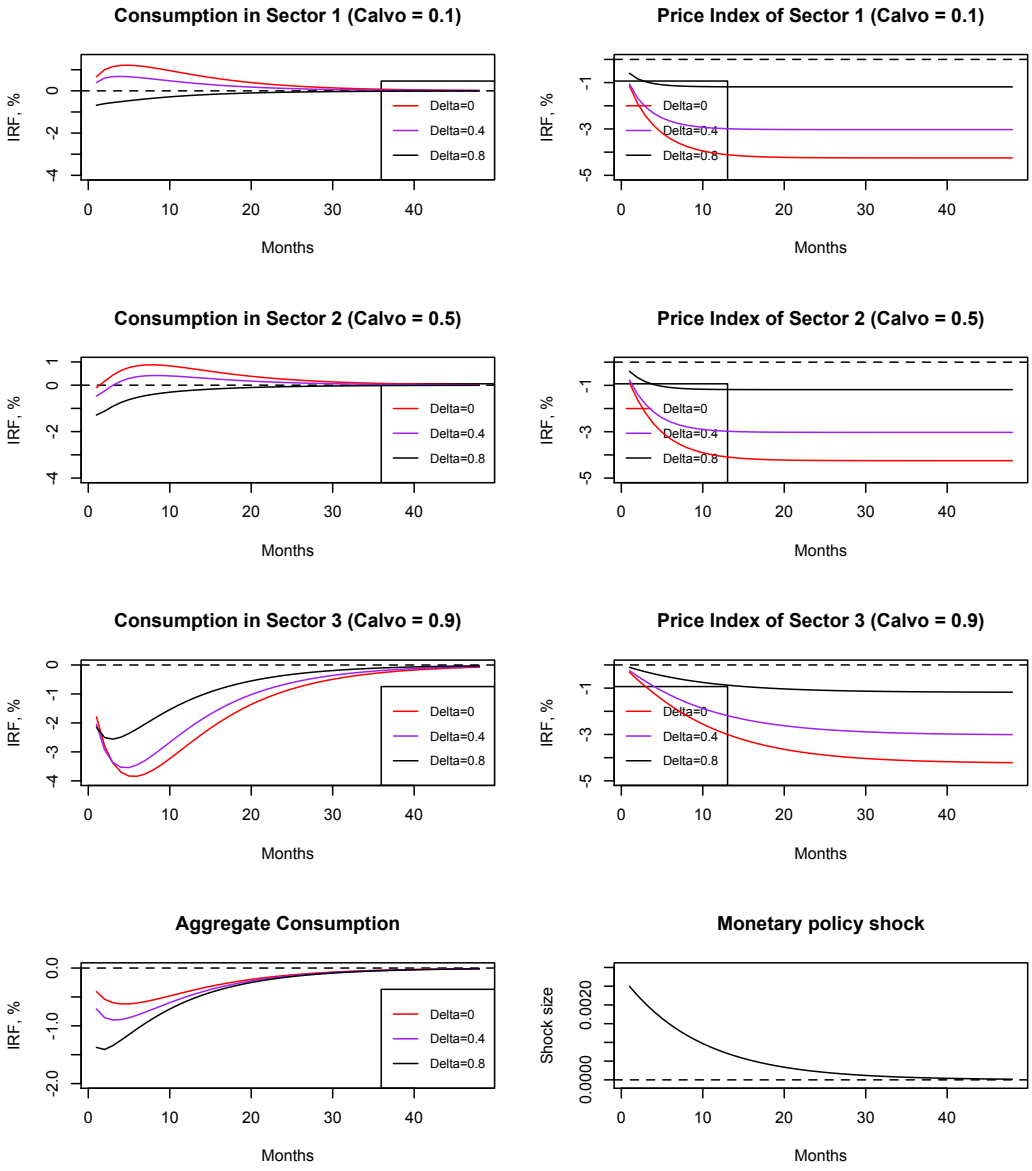
Under the benchmark case of $\lambda = 1$ we can see that “inverted" responses in consumption are impossible. This is because following a contraction (fall in m_t), sectoral price falls at most as quickly (happens under full flexibility) as m_t, implying a fall in sectoral consumption (and vice versa for an expansion). However, under $\lambda > 1$, for example, “inverted" sectoral consumption responses are possible. Consider a monetary contraction and let us focus on the most flexible sector in our economy. Following the contraction, the sectoral price index optimally adjusts downward faster than the overall consumption price index of the economy, so that the second term in the sectoral consumption equation above becomes positive and possibly outweighs the negative $(m_t - p_{kt})$ term, hence generating a positive sectoral consumption response. Intuitively, firms in flexible industries lower their prices faster than firms in more rigid industries and “steal" customers from the latter. Naturally, we should observe less of this behavior as the downstream effect becomes stronger as the degree of strategic complementarity in price setting becomes larger, eliminating the relative pricing effects.

Figure 1.D.5: Responses of sectoral consumptions and price indices to a monetary contraction



***Note**: The figure is constructed by calibrating the log-linearized model in 1.A with money supply rule for $K = 3$ and considering a one-time one standard deviation negative innovation to the monetary policy shock variable that generates a permanent contraction in money supply. The impulse responses are found numerically using Dynare for the following calibration: $\lambda = 3, \varphi = 0, \beta = 0.9975, \rho_r = 0.5, \sigma_r = 0.0025, \alpha_1 = 0, \alpha_2 = 0.5, \alpha_3 = 0.9, \omega_{11} = 1, \omega_{12} = \omega_{13} = 0, \omega_{kr} = \omega_{cr} = 1/3 \quad k = 2, 3; r = 1, 2, 3.$*

Figure 1.D.6: Responses of sectoral consumptions and prices to a monetary policy shocks (Taylor rule)



Note: *The figure is constructed by calibrating the log-linearized model in 1.A with Taylor rule for* $K = 3$ *and considering a one-time one standard deviation positive innovation to the monetary policy shock variable that generates a monetary contraction. The impulse responses are found numerically using Dynare for the following calibration:* $\lambda = 3, \varphi = 0, \beta = 0.9975, \rho_i = 0, \rho_\mu = 0.9, \sigma_\mu = 0.0025, \phi_\pi = 1.24, \phi_{gc} = 0, \phi_c = 0.33/12, \alpha_1 = 0.1, \alpha_2 = 0.5, \alpha_3 = 0.9, \omega_{kr} = \omega_{cr} = 1/3 \quad \forall k, r \in \{1, 2, 3\}$.

To gain further intuition, consider again the three sector $(K = 3)$ version of our economy, with calibration just like in the our first exercise with money supply rule in 1.C, with the exception that we set $\lambda = 3$, so that goods produced in different sectors are now stronger substitutes. Figure 1.D.5 shows impulse responses to a one-time one standard deviation monetary policy shock that generates a permanent contraction in nominal GDP/money supply.

We can see that in the case with no intermediate inputs $(\delta = 0)$ and hence no strategic complementarities in price setting, the most flexible Sector 1 shows an "inverted" response in consumption over the entire horizon, with Sector 2 also having an "inverted" response after 5 months. One can see that this is because they lower their prices much faster than firms in the rigid Sector 3, thus stealing some of the customers of firms in Sector 3. As we increase δ, the extent of "inverted" consumption responses becomes lower, with $\delta = 0.8$ generating no positive consumption responses. This is because higher share of intermediate inputs makes the degree of complementarity in price setting higher, as firms' marginal costs are more sensitive to their suppliers' prices, making the timing of price changing more uniform across sectors. Note that the move from $\delta = 0$ to $\delta = 0.4$ corresponds to a dampening contribution of downstream effect in framework we documented in Section 1.6.

Interestingly, although higher δ makes the consumption response more negative for the more flexible Sectors 1 and 2, the magnitude of peak drop in consumption for the rigid Sector 3 falls for higher shares for intermediate inputs. Hence, for Sector 3 the downstream effect does not lead to amplification, but rather to a contraction of the consumption response. Again, the latter corresponds to negative contribution of the downstream effect in our earlier notation.

We would also like to test whether such property would also be preserved under the Taylor rule specification for monetary policy. In Figure 1.D.6 we consider a monetary tightening for calibration identical to the first exercise with a Taylor rule in 1.C, with the exception that we set $\lambda = 3$. As

we can see, for $\delta = 0$ and $\delta = 0.4$ the more flexible sectors 1 and 2 indeed exhibit “inverted" consumption responses, whereas for $\delta = 0.8$ all “inverted" responses are eliminated. Hence, in this respect, our model with Taylor rule behaves just like the original one with money supply rule. In addition, just as in the original model. we can observe that the magnitude of peak drop in consumption for the most rigid Sector 3 falls for higher values of δ.

1.E List of sectors used

The definition of sectors here follows that used by the US Bureau of Economic Analysis (BEA) in their Input-Output accounts. The figures in parentheses are shares of corresponding real final sectoral consumption in aggregate real personal consumption expenditure (expressed in per cent). The consumption shares are constructed using the 2007 Use Tables for Detail and Summary accounts.

1.E.1 Detail level (161 sectors)

1. Abrasive product manufacturing (0.0019)
2. Accommodation (1.1866)
3. Accounting, tax preparation, bookkeeping, and payroll services (0.1914)
4. Air transportation (1.0285)
5. All other food and drinking places (0.8536)
6. All other food manufacturing (0.1801)
7. All other miscellaneous manufacturing (0.1879)
8. All other wood product manufacturing (0.0449)
9. Animal (except poultry) slaughtering, rendering, and processing (0.6849)
10. Animal production, except cattle and poultry and eggs (0.0438)
11. Audio and video equipment manufacturing (0.4104)
12. Automobile manufacturing (0.8281)
13. Automotive equipment rental and leasing (0.6185)
14. Automotive repair and maintenance (1.9310)
15. Beef cattle ranching and farming, including feedlots and dual-purpose ranching and farming (0.0013)

16. Book publishers (0.2493)
17. Bread and bakery product manufacturing (0.4550)
18. Breweries (0.2989)
19. Broadcast and wireless communications equipment (0.0658)
20. Business support services (0.0540)
21. Carpet and rug mills (0.0966)
22. Cheese manufacturing (0.1452)
23. Child day care services (0.5177)
24. Civic, social, professional, and similar organizations (0.4754)
25. Clay product and refractory manufacturing (0.0281)
26. Coal mining (0.0002)
27. Coffee and tea manufacturing (0.0562)
28. Commercial and industrial machinery and equipment rental and leasing (0.0046)
29. Community food, housing, and other relief services, including rehabilitation services (0.4243)
30. Computer terminals and other computer peripheral equipment manufacturing (0.1554)
31. Consumer goods and general rental centers (0.3190)
32. Couriers and messengers (0.0208)
33. Curtain and linen mills (0.1412)
34. Cut stone and stone product manufacturing (0.0446)
35. Cutlery and handtool manufacturing (0.0635)
36. Dog and cat food manufacturing (0.1561)
37. Doll, toy, and game manufacturing (0.2437)
38. Dry-cleaning and laundry services (0.1608)
39. Electric lamp bulb and part manufacturing (0.0156)
40. Electric power generation, transmission, and distribution (1.8453)
41. Electromedical and electrotherapeutic apparatus manufacturing (0.0259)
42. Electronic and precision equipment repair and maintenance (0.0502)
43. Electronic computer manufacturing (0.2082)
44. Elementary and secondary schools (0.4815)

45. Employment services (0.0188)
46. Fabric mills (0.0181)
47. Fats and oils refining and blending (0.0486)
48. Fiber, yarn, and thread mills (0.0013)
49. Fishing, hunting and trapping (0.0654)
50. Flour milling and malt manufacturing (0.0364)
51. Fluid milk and butter manufacturing (0.2456)
52. Frozen food manufacturing (0.2550)
53. Fruit and tree nut farming (0.1780)
54. Full-service restaurants (2.2680)
55. Funds, trusts, and other financial vehicles (1.5434)
56. Gambling industries (except casino hotels) (1.0675)
57. Glass and glass product manufacturing (0.0376)
58. Grain farming (0.0136)
59. Grantmaking, giving, and social advocacy organizations (0.4974)
60. Greenhouse, nursery, and floriculture production (0.1574)
61. Hardware manufacturing (0.0055)
62. Home health care services (0.8074)
63. Hospitals (8.0865)
64. Housing (18.401)
65. Independent artists, writers, and performers (0.0012)
66. Individual and family services (0.7272)
67. Industrial gas manufacturing (0.0008)
68. Insurance carriers (3.5857)
69. Investigation and security services (0.0955)
70. Iron and steel mills and ferroalloy manufacturing (0.0004)
71. Jewelry and silverware manufacturing (0.3222)
72. Junior colleges, colleges, universities, and professional schools (2.0402)
73. Lawn and garden equipment manufacturing (0.0101)
74. Leather and allied product manufacturing (0.4185)
75. Legal services (1.1241)
76. Light truck and utility vehicle manufacturing (1.4458)

77. Limited-service restaurants (2.9901)
78. Manufacturing and reproducing magnetic and optical media (0.0095)
79. Medical and diagnostic laboratories (0.3585)
80. Monetary authorities and depository credit intermediation (2.1384)
81. Motion picture and video industries (0.2758)
82. Motor home manufacturing (0.0632)
83. Motor vehicle body manufacturing (0.0006)
84. Motorcycle, bicycle, and parts manufacturing (0.1007)
85. Museums, historical sites, zoos, and parks (0.1523)
86. Natural gas distribution (0.7686)
87. News syndicates, libraries, archives and all other information services (0.0418)
88. Newspaper publishers (0.1222)
89. Nondepository credit intermediation and related activities (0.4368)
90. Nonferrous metal (except copper and aluminum) rolling, drawing, extruding and alloying (0.0003)
91. Nonferrous metal foundries (0.0024)
92. Nursing and community care facilities (1.6924)
93. Office machinery manufacturing (0.0038)
94. Office supplies (except paper) manufacturing (0.0319)
95. Offices of dentists (1.2069)
96. Offices of other health practitioners (0.7623)
97. Offices of physicians (4.3123)
98. Ophthalmic goods manufacturing (0.0916)
99. Optical instrument and lens manufacturing (0.0002)
100. Other concrete product manufacturing (0.0003)
101. Other crop farming (0.0033)
102. Other educational services (0.6037)
103. Other fabricated metal manufacturing (0.0268)
104. Other financial investment activities (1.3466)
105. Other furniture related product manufacturing (0.1166)
106. Other nonmetallic mineral mining and quarrying (0.0012)

107. Other personal services (0.6149)
108. Other petroleum and coal products manufacturing (0.0615)
109. Other retail (7.2663)
110. Other textile product mills (0.0496)
111. Outpatient care centers (0.9844)
112. Performing arts companies (0.2462)
113. Personal and household goods repair and maintenance (0.2106)
114. Personal care services (0.7316)
115. Petroleum refineries (2.6296)
116. Photographic services (0.0906)
117. Postal service (0.1233)
118. Poultry and egg production (0.0521)
119. Poultry processing (0.3536)
120. Power-driven handtool manufacturing (0.0252)
121. Printing (0.0307)
122. Promoters of performing arts and sports and agents for public figures (0.1101)
123. Radio and television broadcasting (0.0564)
124. Rail transportation (0.0813)
125. Religious organizations (0.9615)
126. Residential mental retardation, mental health, substance abuse and other facilities (0.4264)
127. Satellite, telecommunications resellers, and all other telecommunications (0.1364)
128. Scenic and sightseeing transportation and support activities for transportation (0.1168)
129. Scientific research and development services (0.0576)
130. Search, detection, and navigation instruments manufacturing (0.0004)
131. Securities and commodity contracts intermediation and brokerage (0.5098)
132. Services to buildings and dwellings (0.1895)
133. Small electrical appliance manufacturing (0.1161)

134. Software publishers (0.3259)
135. Sound recording industries (0.1035)
136. Soybean and other oilseed processing (0.0092)
137. Spectator sports (0.1946)
138. Sporting and athletic goods manufacturing (0.1699)
139. Steel product manufacturing from purchased steel (0.0057)
140. Storage battery manufacturing (0.0241)
141. Support activities for agriculture and forestry (0.0020)
142. Surgical and medical instrument manufacturing (0.0023)
143. Surgical appliance and supplies manufacturing (0.0983)
144. Telephone apparatus manufacturing (0.0078)
145. Textile and fabric finishing and fabric coating mills (0.0048)
146. Tire manufacturing (0.1232)
147. Tobacco product manufacturing (0.5646)
148. Transit and ground passenger transportation (0.3535)
149. Travel arrangement and reservation services (0.1784)
150. Truck transportation (1.0641)
151. Upholstered household furniture manufacturing (0.1503)
152. Vegetable and melon farming (0.2041)
153. Vending, commercial laundry, and other commercial and service industry machinery manufacturing (0.0002)
154. Waste management and remediation services (0.1925)
155. Watch, clock, and other measuring and controlling device manufacturing (0.0583)
156. Water transportation (0.1871)
157. Water, sewage and other systems (0.3766)
158. Wet corn milling (0.0061)
159. Wineries (0.1746)
160. Wired telecommunications carriers (1.9818)
161. Wireless telecommunications carriers (except satellite) (1.0132)

1.E.2 Summary level (53 sectors)

1. Accommodation (1.1568)
2. Administrative and support services (0.5408)
3. Air transportation (1.3167)
4. Ambulatory health care services (8.182)
5. Amusements, gambling, and recreation industries (1.5836)
6. Apparel and leather and allied products (2.7165)
7. Broadcasting and telecommunications (2.9244)
8. Chemical products (2.0157)
9. Computer and electronic products (0.8209)
10. Data processing, internet publishing, and other information services (0.0674)
11. Educational services (2.734)
12. Electrical equipment, appliances, and components (0.4607)
13. Fabricated metal products (0.1543)
14. Farms (0.7729)
15. Federal government enterprises (0.1824)
16. Federal Reserve banks, credit intermediation, and related activities (2.5918)
17. Food and beverage and tobacco products (6.6865)
18. Forestry, fishing, and related activities (0.1002)
19. Furniture and related products (0.5656)
20. Hospitals (7.257)
21. Housing (18.038)
22. Insurance carriers and related activities (3.6451)
23. Legal services (1.1564)
24. Machinery (0.129)
25. Mining, except oil and gas (0.008)
26. Miscellaneous manufacturing (1.1988)
27. Miscellaneous professional, scientific, and technical services (0.5387)
28. Motion picture and sound recording industries (0.4514)
29. Motor vehicles, bodies and trailers, and parts (2.8586)

30. Nonmetallic mineral products (0.0957)
31. Nursing and residential care facilities (1.9923)
32. Other retail (7.4421)
33. Other services, except government (6.3469)
34. Other transportation and support activities (0.0773)
35. Other transportation equipment (0.2029)
36. Paper products (0.2591)
37. Performing arts, spectator sports, museums, and related activities (0.5057)
38. Petroleum and coal products (1.3304)
39. Plastics and rubber products (0.327)
40. Primary metals (0.0145)
41. Printing and related support activities (0.065)
42. Publishing industries, except internet (includes software) (1.0897)
43. Rail transportation (0.075)
44. Rental and leasing services and lessors of intangible assets (1.3618)
45. Securities, commodity contracts, and investments (1.5601)
46. Social assistance (1.3046)
47. Textile mills and textile product mills (0.3438)
48. Transit and ground passenger transportation (0.3278)
49. Truck transportation (0.9186)
50. Utilities (3.167)
51. Waste management and remediation services (0.1879)
52. Water transportation (0.1175)
53. Wood products (0.0333)

1.F Input-output matrix construction

The "$Make$" tables show how much of a given product is produced by a particular sector, measured in nominal (US dollar) terms. The "Use" tables show how much a given product is used as an intermediate input by a given sector (as well as its use by final users), again measured in nominal (US dollar) terms. Assume that products in the "$Make$" and "Use" tables are

indexed by $q \in \{1, 2, ..., Q\}$. To calculate the $P_{rt}Z_{kt}(r)$ component, or the nominal value of inputs sold by sector r to sector k, we first work out the market share of sector r in the production of product q, given by $Share_{r,q}$, using the "$Make$" table:

$$Share_{r,q} = \frac{Make_{r,q}}{\sum_{r'=1}^{K} Make_{r',q}}, \tag{1.70}$$

where $Make_{r,q}$ denotes the nominal value of sector r's production of product q. Later, we take the nominal expenditure of sector k on every product q as an intermediate input, given by $Use_{k,q}$, and find out how much of this expenditure goes to sector r by multiplying it by r's respective market share. The latter gives us the empirical counterpart of $P_{rt}Z_{kt}(r)$, or total nominal sales of intermediate inputs from sector r to sector k:

$$Sales_{r,k} = \sum_{q=1}^{Q} Share_{r,q}Use_{k,q}. \tag{1.71}$$

Finally, the empirical counterpart of $P_t^k Z_{kt}$, or the total expenditure of sector k on intermediate inputs, is given by $\sum_{r'=1}^{K} Sales_{r,k}$ and hence ω_{kr} can be evaluated as:

$$\omega_{kr} = \frac{Sales_{r,k}}{\sum_{r'=1}^{K} Sales_{r',k}}, \qquad \forall k, r. \tag{1.72}$$

Note that the number of sectors considered in "$Make$" and "Use" tables is generally larger than that available in PCE Bridge Tables and can be matched to sectoral consumption in NIPA Tables. This is because PCE Bridge Tables exclude sectors that either don't sell anything to final consumers or those whose final consumption only accounts for a very small proportion of total output. For example, at Detail level, "$Make$" and "Use" tables consider 389 sectors, whereas PCE Bridge Tables only include data on 257 of them. After organizing our input-output weights into an input-output matrix Ω, so that $\omega_{kr} = [\Omega]_{k,r}$, we delete rows and columns for sectors that are not represented in PCE Bridge Tables and therefore don't sell a large enough proportion of their output to final consumers. In order to satisfy our theoretical requirement that the input-output weights add up to one for any sector, we normalize the rows of the reduced input-output matrix.

In addition we also use the "Use" tables to construct the consumption weights $\{\omega_{ck}\}_{k=1}^{K}$ and intermediates intensities $\{\delta_k\}_{k=1}^{K}$. Recall that in equilibrium of our baseline model those are given by (aggregating across all firms within a sector):

$$\omega_{ck} = \frac{P_{kt}C_{kt}}{P_t^c C_t}, \quad \forall k \qquad\qquad \frac{\delta_k}{1-\delta_k} = \frac{P_t^k Z_{kt}}{W_{kt}N_{kt}}, \quad \forall k \tag{1.73}$$

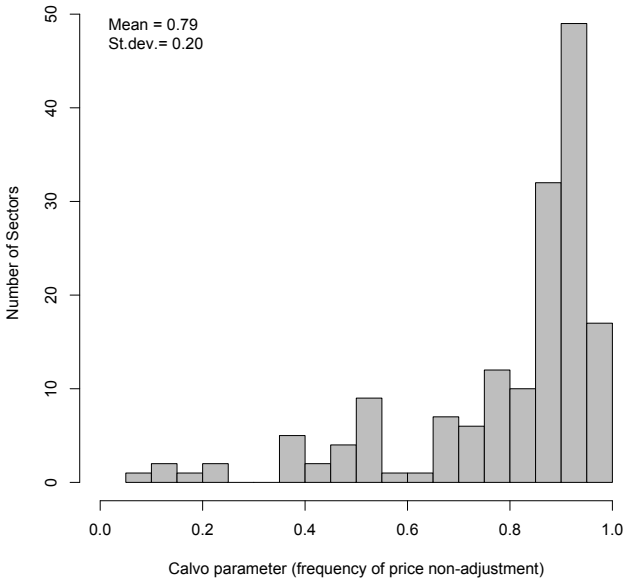
The "Use" tables contain information on Private Consumption Expenditure (PCE) for every sector that year (the empirical equivalent of $P_{kt}C_{kt}$), aggregate nominal Private Consumption Expenditure (PCE) that year (the empirical equivalent of $P_t^c C_t$), total expenditure of intermediates by every sector (the empirical equivalent of $P_t^k Z_{kt}$) as well as total compensation of employees by every sector (the empirical equivalent of $W_{kt}N_{kt}$).

At Detail level, "$Make$" and "Use" tables are only available for the year 2007, and so we construct both input-output and consumption weights based on the 2007 tables. At Summary level, those tables are available every year between 1997 and 2015; we have estimated input-output and consumption weights for every year those tables are available. However, upon inspection, the constructed weights show very little variation over time. As a result, for Summary level we have decided to use weights for the year 1997.

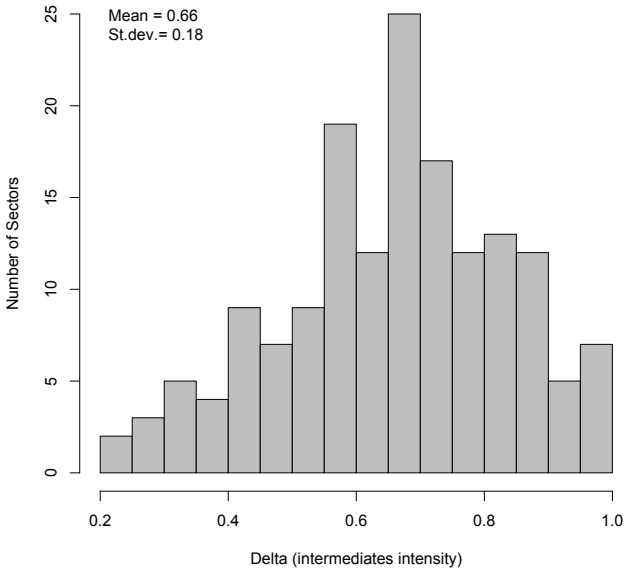
1.G Calvo parameters and intermediates intensity

Figure 1.G.7: Calvo parameters and intermediates intensities (161 sectors)

(a) Histogram of sectoral Calvo parameters



(b) Histogram of sectoral intermediates intensities



***Note**: panel (a) shows a histogram of sectoral Calvo parameters, given as one minus sectoral frequencies of price adjustment constructed by Pasten et al. (2020); panel (b) shows a histogram of sectoral intermediates intensities estimated using 2007 BEA "Use" table.*

1.H Monetary policy shocks

Figure 1.H.8: Monetary policy shocks series (1969:1-2007:12)

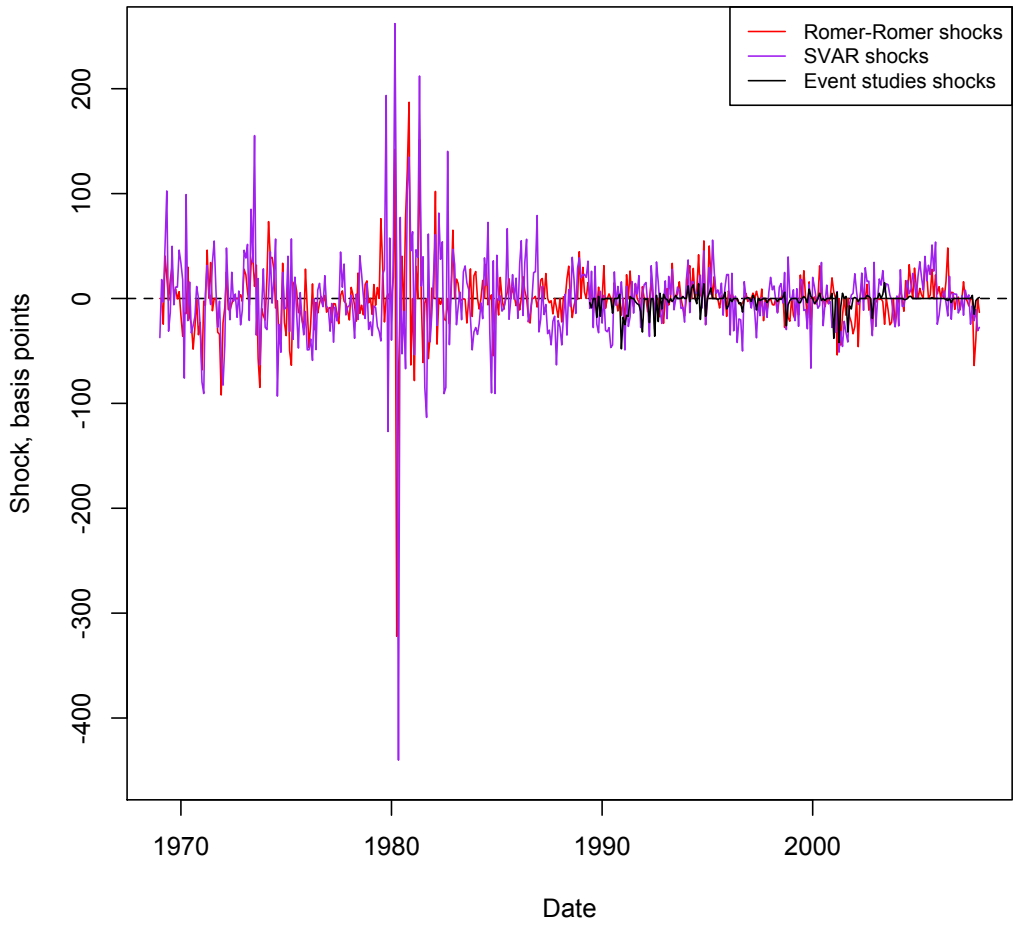


Table 1.H.1: Correlation matrix and summary statistics of shocks

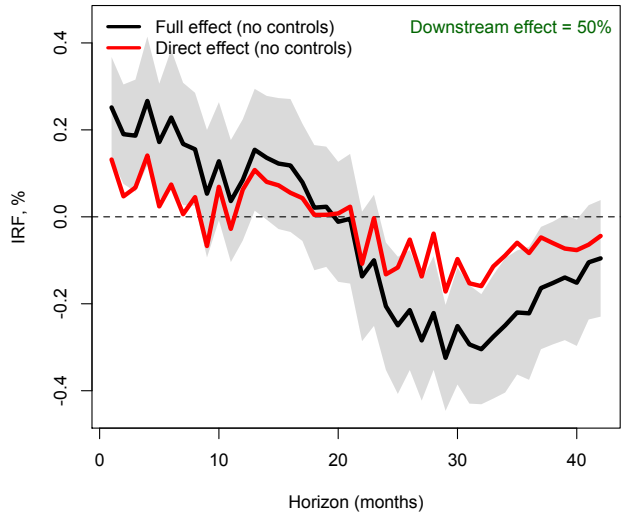
	RR	SVAR	ES
RR	1	0.34	0.19
SVAR	0.34	1	0.29
ES	0.19	0.29	1

	RR	SVAR	ES
Mean (bp)	-0.68	0.02	-2.27
St. Dev. (bp)	29.8	43.5	8.48

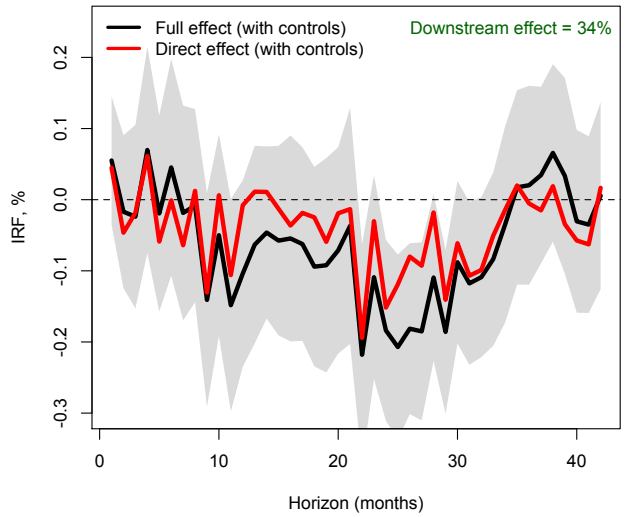
1.I Non-smoothed IRFs reported in the main text

Figure 1.I.9: Final consumption response to 25bp tightening: full and direct effects (Detail level, 161 sectors)

(a) Full and direct effects after a 25bp tightening (with controls)

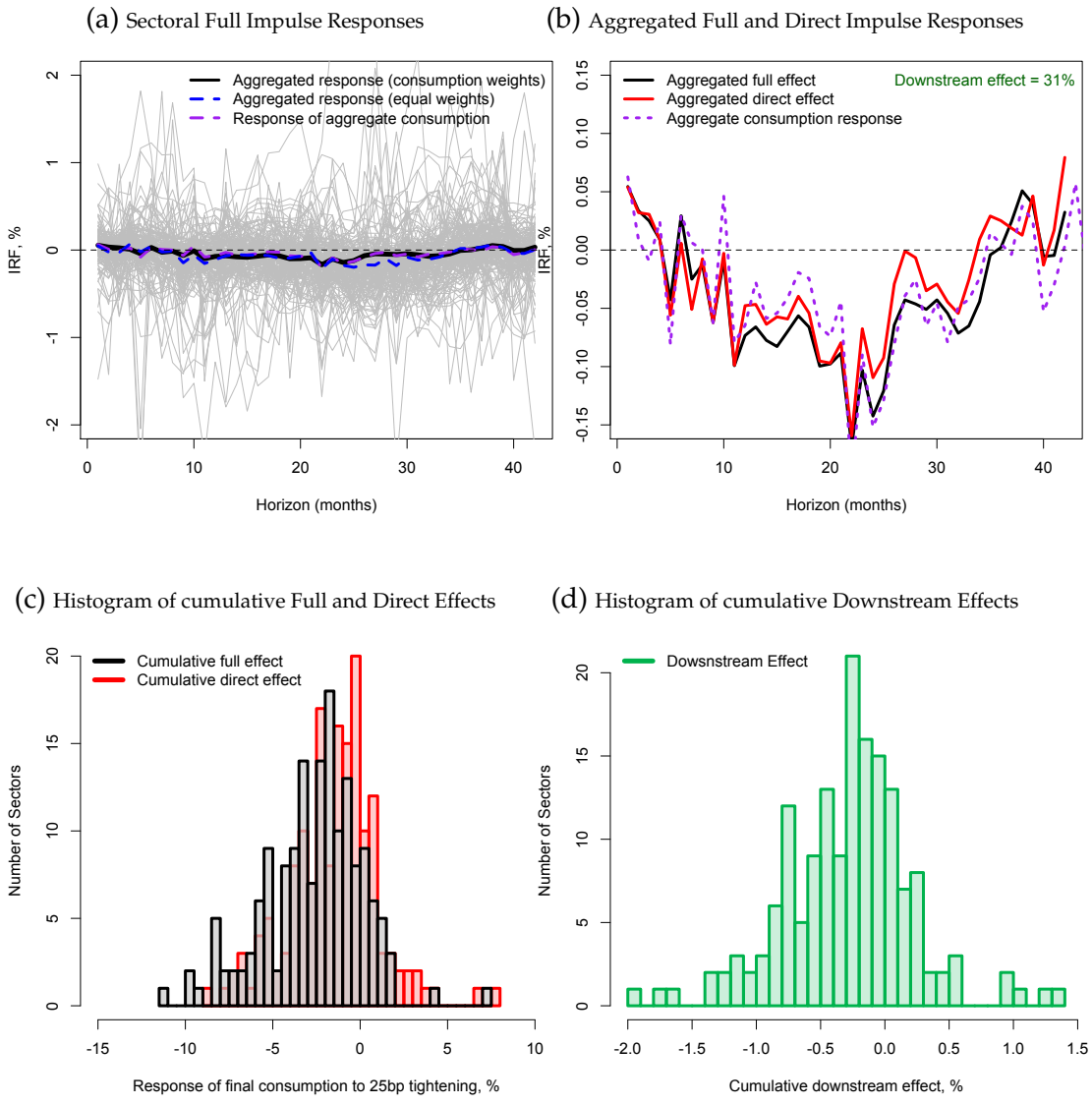


(b) Full and direct effects after 25bp tightening (with controls)



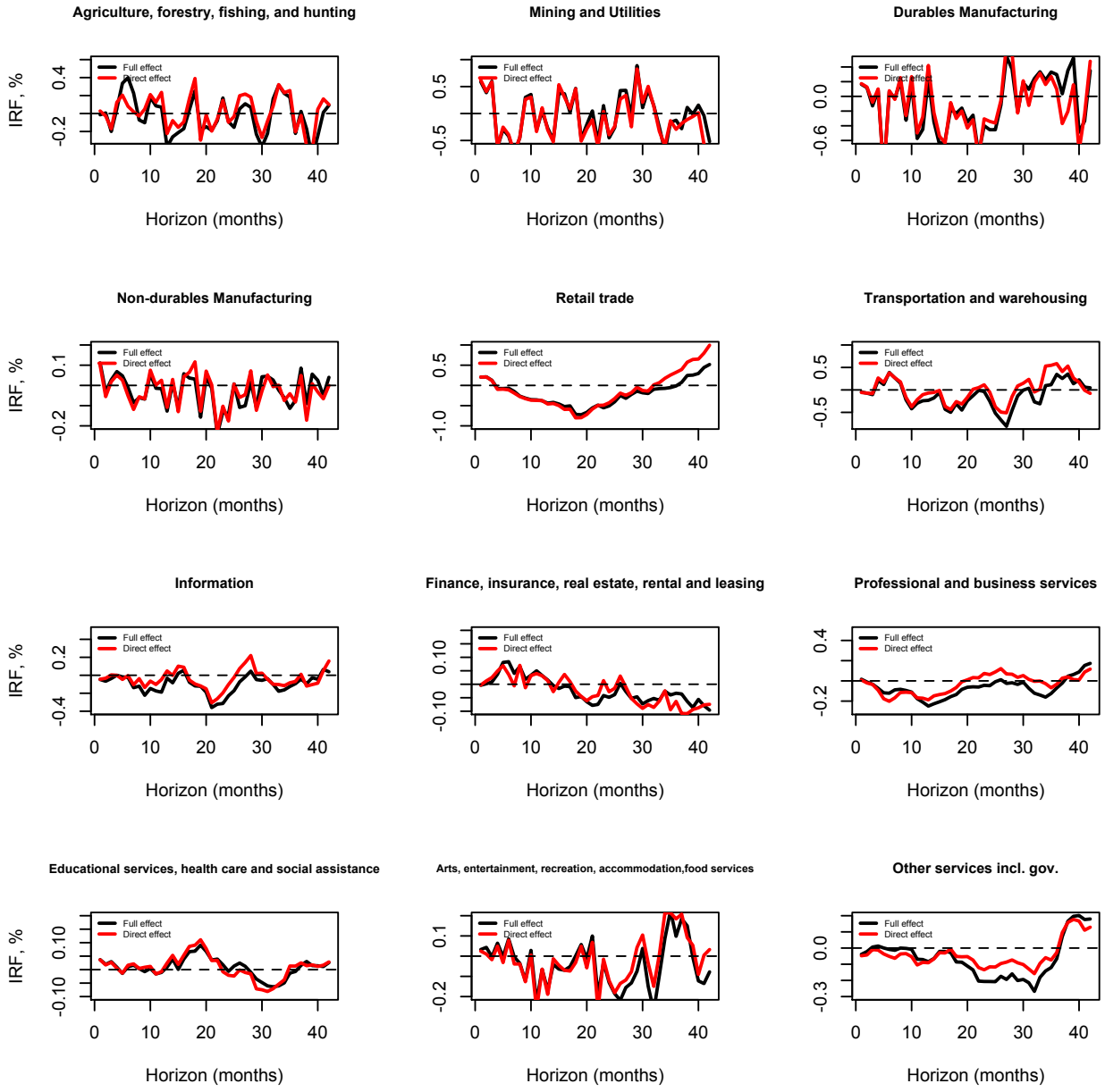
***Note**: panel (a) shows results of panel fixed effects estimation of (1.20) and (1.21), without adding controls, reported as IRFs following a 25bp monetary tightening; panel (b) reports results of panel fixed effects estimation of (1.20) and (1.21), with a vector of controls $\mathbb{X}_t^H = \left\{\{r_{t+\tau}\}_{\tau=1}^H, \{\hat{c}_{k,t-s}\}_{s=1}^1, \{ffr_{t-q}\}_{q=1}^4\right\}$ added, reported as IRFs following a 25bp monetary tightening. Shaded areas denote 90% confidence bands around the full effect. The downstream effect contribution is constructed as the difference between cumulative full and direct effects as a percentage of the cumulative full effect (all cumulation is over 42 months).*

Figure 1.I.10: Sector-level consumption responses to 25bp tightening (Detail level, 161 sectors)



***Note**: panel (a) shows results of sector-by-sector OLS estimation of (1.20) with controls, reported as sector-level IRF following a 25bp monetary tightening; panel (b) reports results of aggregation of sector-level full and direct effects using empirical final consumption shares $\{\omega_{ck}\}_{k=1}^{161}$; panels (c) and (d) report histograms of cumulative (over 42 months) sectoral full and direct effects, as well as the corresponding sectoral cumulative downstream effects.*

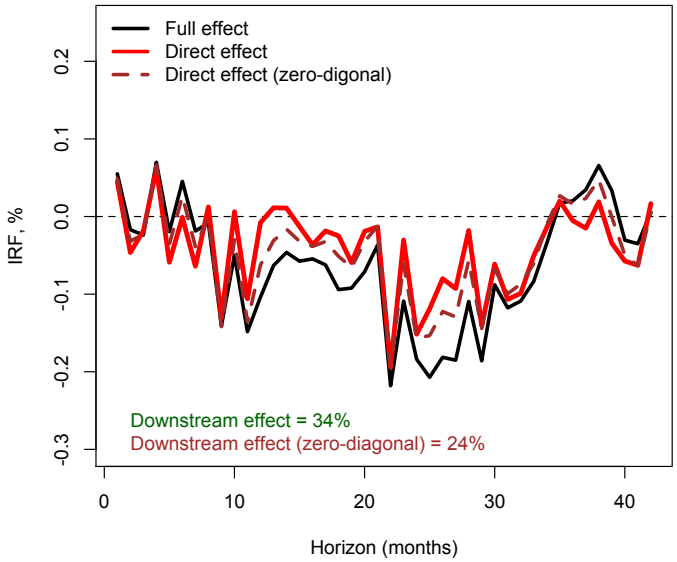
Figure 1.I.11: Consumption responses to a 25bp tightening across broad sectoral groups



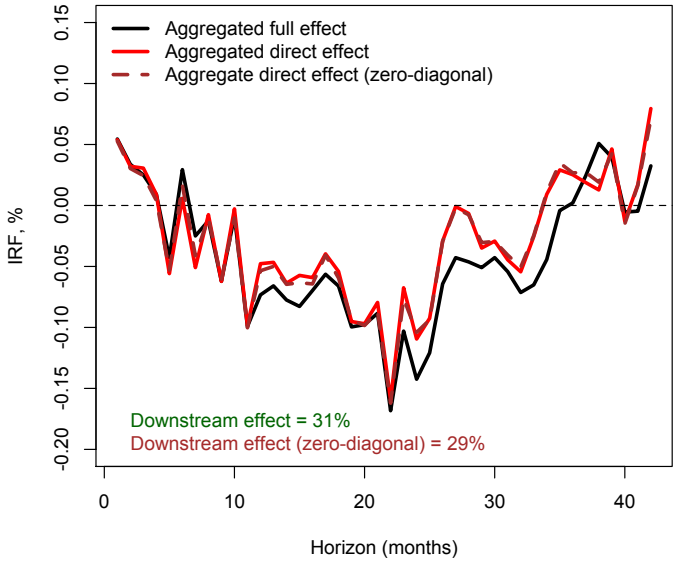
Note: *Figure 1.I.11 shows results of sector-by-sector OLS estimation of (1.20) and (1.21) with controls, reported as sector-level direct and full IRFs following a 25bp monetary tightening that have been aggregated into broad sectoral groups using empirical consumption shares* $\{\omega_{ck}\}_{k=1}^{161}$.

Figure 1.I.12: Final consumption response to 25bp tightening: full and direct effects under zero-diagonal (Detail level, 161 sectors)

(a) Aggregated full and direct effects after a 25bp tightening: panel estimation



(b) Full and direct effects after a 25bp tightening 25bp tightening: sector-by-sector estimation

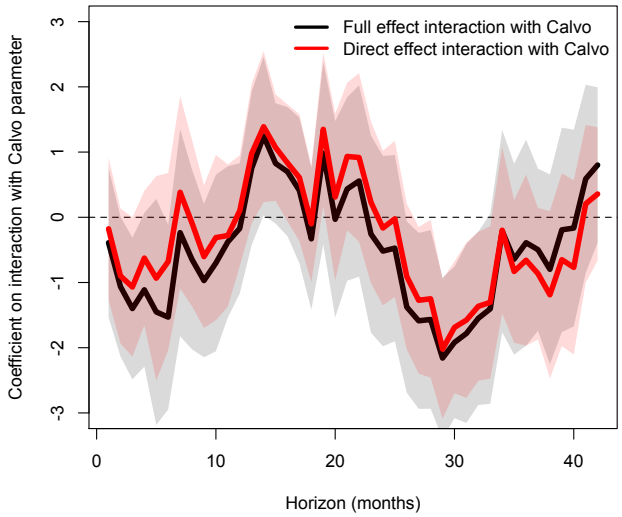


***Note**: Figure 1.I.12 shows additional estimation of the direct effect using (1.21) where we force the diagonal elements of the input-output matrix to be equal to zero. The new estimates are reported alongside estimation of (1.20) and (1.21) without further restrictions. Panel (a) shows results using panel fixed effects estimation, whereas panel (b) shows results using sector-by-sector OLS estimation with subsequent aggregation using empirical consumption shares* $\{\omega_{ck}\}_{k=1}^{161}$.

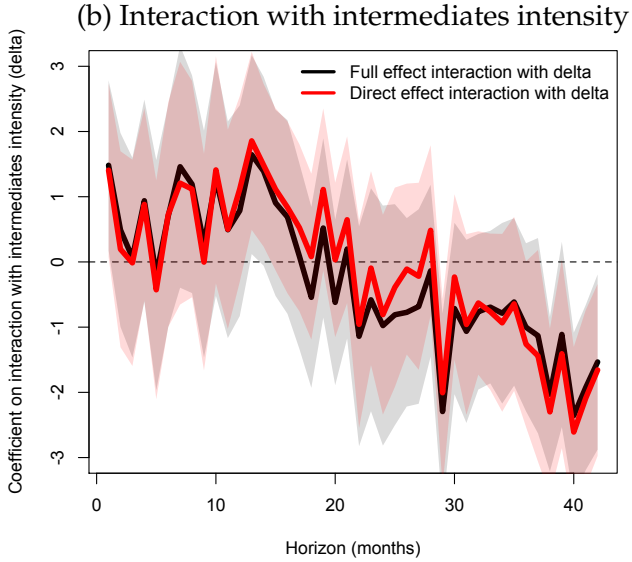
1.J The role of Calvo and delta heterogeneity

Figure 1.J.13: Confidence bands around estimates of interaction terms

(a) Interaction with the Calvo parameter

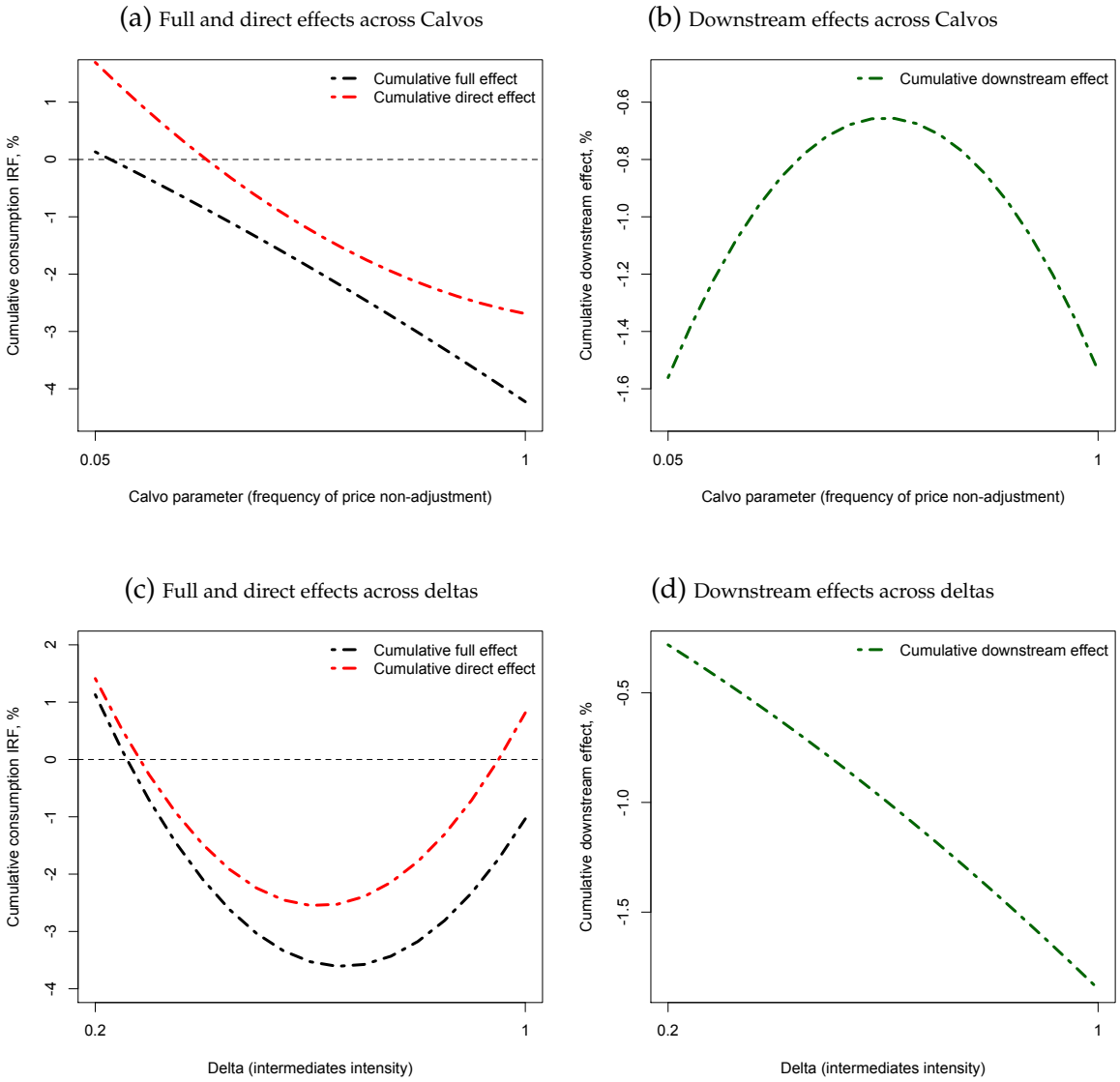


(b) Interaction with intermediates intensity



Note: *Figure 1.J.13 shows results of panel fixed effects estimation of (1.20) and (1.21) with interaction terms* $r_t \times \alpha_k$ *and* $r_t \times \delta_k$ *added, as well as vector of controls* $\mathbb{X}_t^H = \left\{\{r_{t+\tau}\}_{\tau=1}^H, \{\hat{c}_{k,t-s}\}_{s=1}^1, \{ffr_{t-q}\}_{q=1}^4\right\}$*; panel (a) shows point estimates and 90% confidence bands around interaction term* $r_t \times \alpha_k$ *for full and direct effects for every horizon; panel (a) shows point estimates and 90% confidence bands around interaction term* $r_t \times \delta_k$ *for full and direct effects for every horizon.*

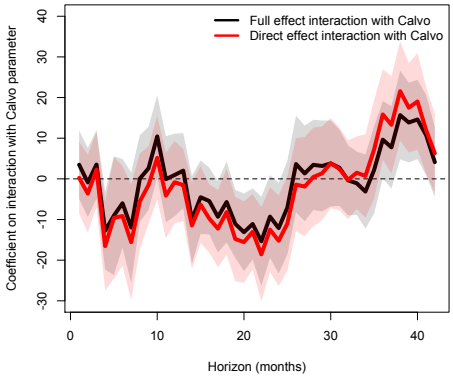
Figure 1.J.14: Response heterogeneity across frequencies of price adjustment and intermediates intensity



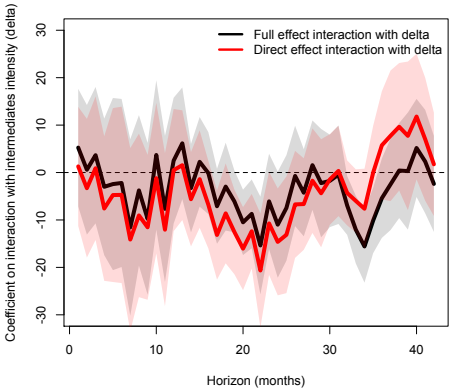
***Note**: Figure 1.J.14 shows results of panel fixed effects estimation of (1.20) and (1.21) with interaction terms $r_t \times \alpha_k, r_t \times \delta_k, r_t \times \alpha_k^2, r_t \times \delta_k^2$ and $r_t \times \alpha_k \delta_k$ added, as well as vector of controls $\mathbb{X}_t^H = \{\{r_{t+\tau}\}_{\tau=1}^H, \{\hat{c}_{k,t-s}\}_{s=1}^1, \{ffr_{t-q}\}_{q=1}^4\}$; panels (a) and (b) show cumulative full, direct and downstream effects for various values of the Calvo parameter, holding intermediates intensity fixed at its sample median value; panels (c) and (d) show cumulative full, direct and downstream effects for various values of intermediates intensity, holding Calvo parameter fixed at its sample median value. All cumulative effects are over 42 months.*

Figure 1.J.15: Confidence bands around estimates of interaction terms

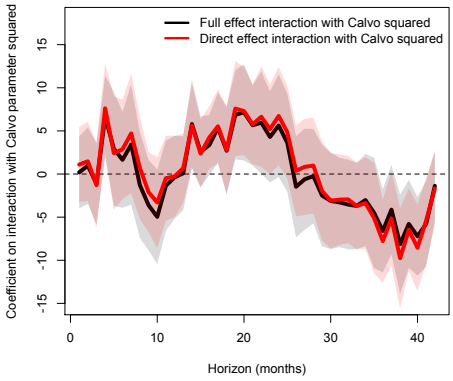
(a) Interaction with the Calvo parameter



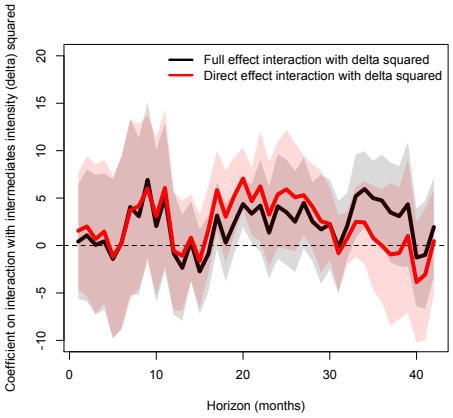
(b) Interaction with intermediates intensity



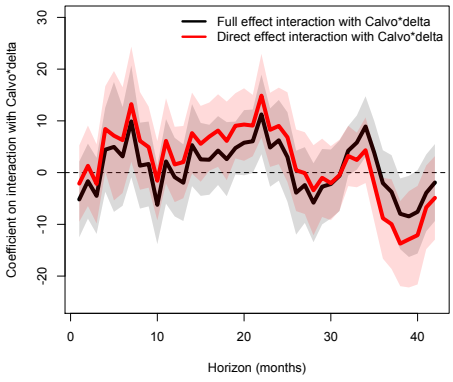
(c) Interaction with Calvo parameter squared



(d) Interaction with intermediates intensity squared



(e) Interaction with Calvo parameter×intermediates intensity

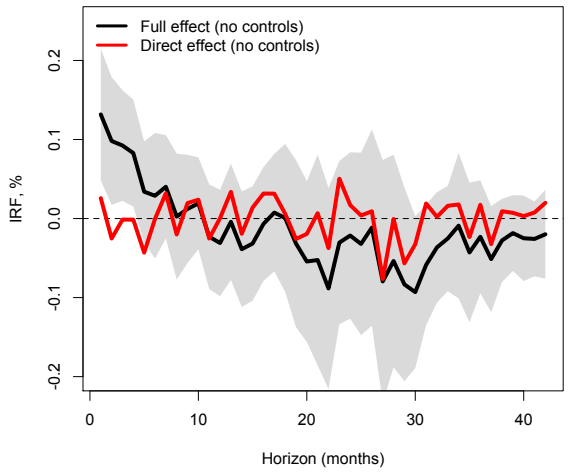


Note: *Figure 1.J.15 shows results of panel fixed effects estimation of (1.20) and (1.21) with interaction terms* $r_t \times \alpha_k, r_t \times \delta_k, r_t \times \alpha_k^2, r_t \times \delta_k^2$ *and* $r_t \times \alpha_k\delta_k$ *added, as well as vector of controls* $\mathbb{X}_t^H = \left\{\{r_{t+\tau}\}_{\tau=1}^H, \{\hat{c}_{k,t-s}\}_{s=1}^1, \{ffr_{t-q}\}_{q=1}^4\right\}$*; panels (a)-(e) show point estimates and 90% confidence bands around interaction term* $r_t \times \alpha_k, r_t \times \delta_k, r_t \times \alpha_k^2, r_t \times \delta_k^2$ *and* $r_t \times \alpha_k\delta_k$*, respectively, for full and direct effects for every horizon.*

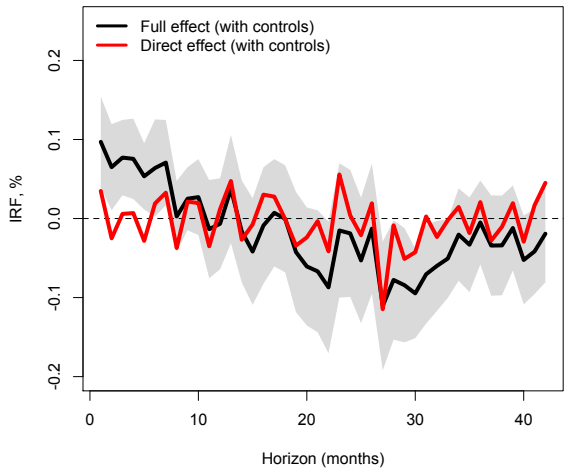
1.K Summary level of disaggregation (53 sectors)

Figure 1.K.16: Final consumption response to 25bp tightening: full and direct effects (Summary level, 53 sectors)

(a) Full and direct effects after a 25bp tightening (with controls)



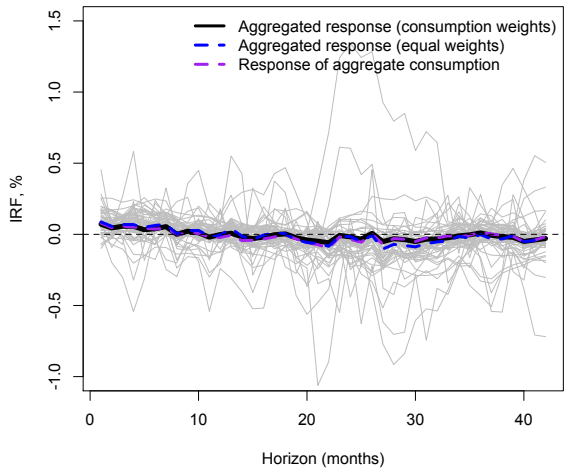
(b) Full and direct effects after a 25bp tightening (with controls)



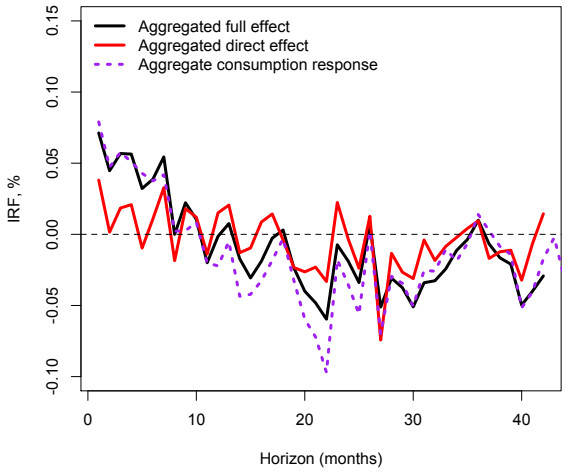
***Note**: panel (a) shows results of panel fixed effects estimation of (1.20) and (1.21), without adding controls, reported as IRFs following a 25bp monetary tightening; panel (b) reports results of panel fixed effects estimation of (1.20) and (1.21), with a vector of controls $\mathbb{X}_t^H = \{\{r_{t+\tau}\}_{\tau=1}^H, \{\hat{c}_{k,t-s}\}_{s=1}^1, \{ffr_{t-q}\}_{q=1}^4\}$ added, reported as IRFs following a 25bp monetary tightening. Shaded areas denote 90% confidence bands around the full effect. The downstream effect contribution is constructed as the difference between cumulative full and direct effects as a percentage of the cumulative full effect (all cumulation is over 42 months).*

Figure 1.K.17: Sector-level consumption responses to 25bp tightening (Summary level, 53 sectors)

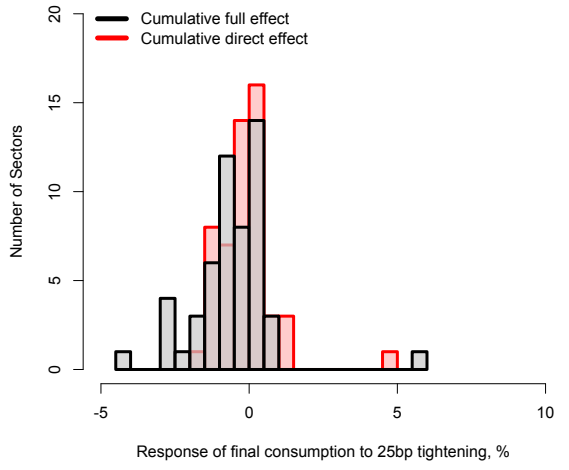
(a) Sectoral Full Impulse Responses



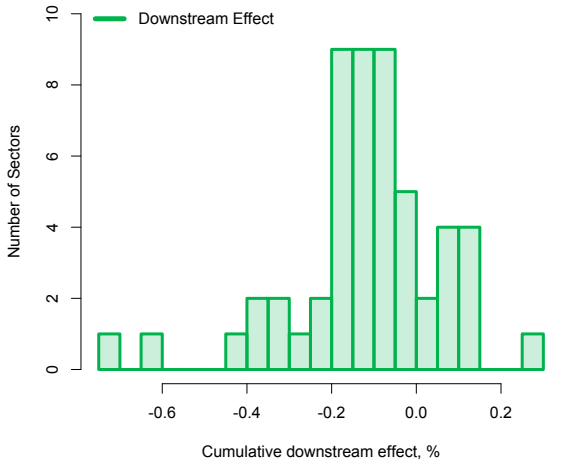
(b) Aggregated Full and Direct Impulse Responses



(c) Histogram of cumulative Full and Direct Effects



(d) Histogram of cumulative Downstream Effects

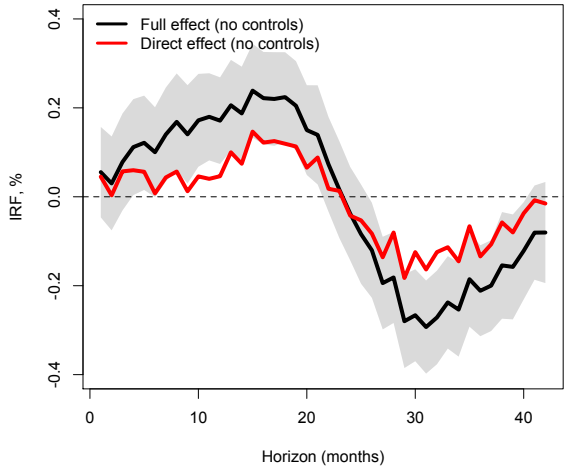


***Note**: panel (a) shows results of sector-by-sector OLS estimation of (1.20) with controls, reported as sector-level IRF following a 25bp monetary tightening; panel (b) reports results of aggregation of sector-level full and direct effects using empirical final consumption shares $\{\omega_{ck}\}_{k=1}^{53}$; panels (c) and (d) report histograms of cumulative (over 42 months) sectoral full and direct effects, as well as the corresponding sectoral cumulative downstream effects.*

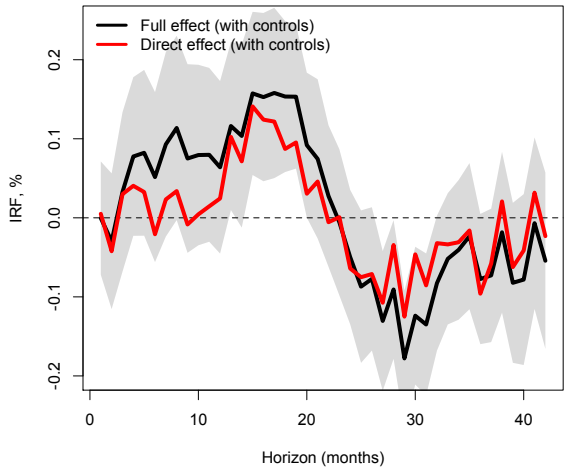
1.L Alternative monetary shocks

Figure 1.L.18: Final consumption response to 25bp tightening: full and direct effects (Detail level, 161 sectors) – Coibion (2012) SVAR shocks

(a) Full and direct effects after a 25bp tightening (with controls)



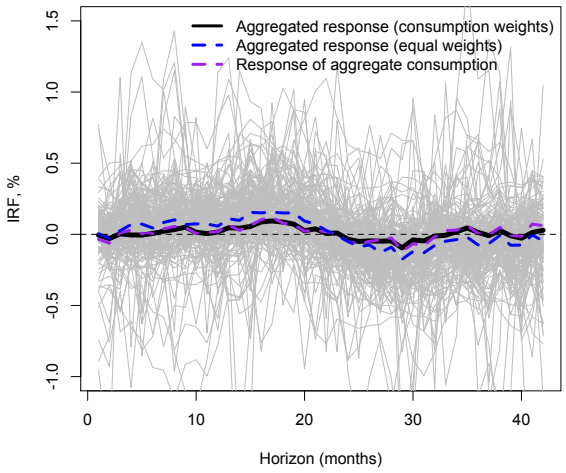
(b) Full and direct effects after a 25bp tightening (with controls)



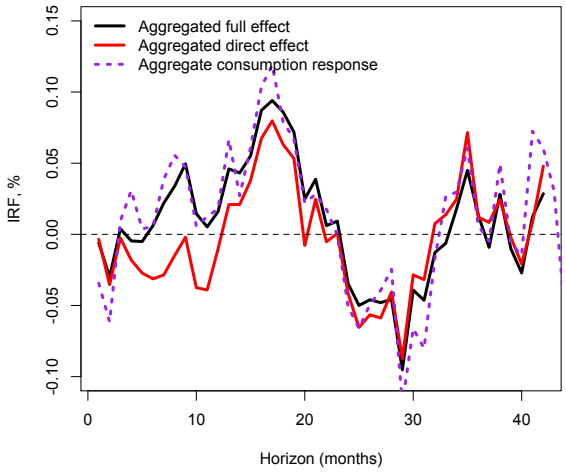
***Note**: panel (a) shows results of panel fixed effects estimation of (1.20) and (1.21), without adding controls, reported as IRFs following a 25bp monetary tightening; panel (b) reports results of panel fixed effects estimation of (1.20) and (1.21), with a vector of controls $\mathbb{X}_t^H = \{\{r_{t+\tau}\}_{\tau=1}^H, \{\hat{c}_{k,t-s}\}_{s=1}^1, \{ffr_{t-q}\}_{q=1}^4\}$ added, reported as IRFs following a 25bp monetary tightening. Shaded areas denote 90% confidence bands around the full effect. The downstream effect contribution is constructed as the difference between cumulative full and direct effects as a percentage of the cumulative full effect (all cumulation is over 42 months).*

Figure 1.L.19: Sector-level consumption responses to 25bp tightening (Detail level, 161 sectors) – Coibion (2012) SVAR shocks

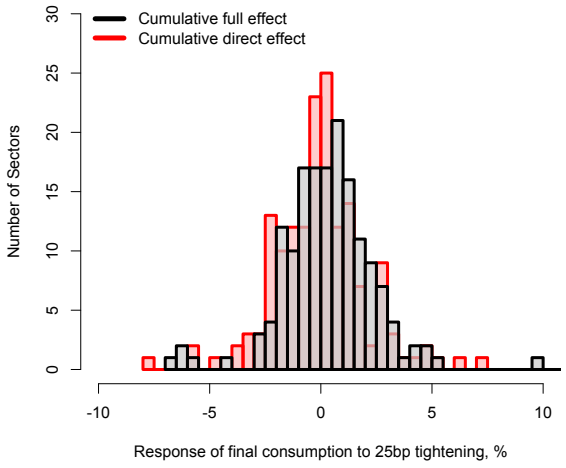
(a) Sectoral Full Impulse Responses



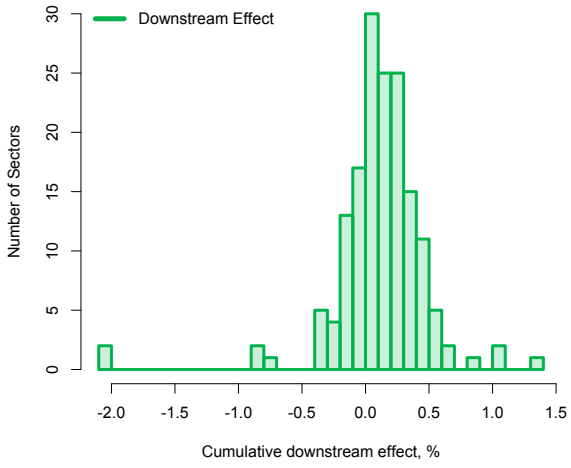
(b) Aggregated Full and Direct Impulse Responses



(c) Histogram of cumulative Full and Direct Effects



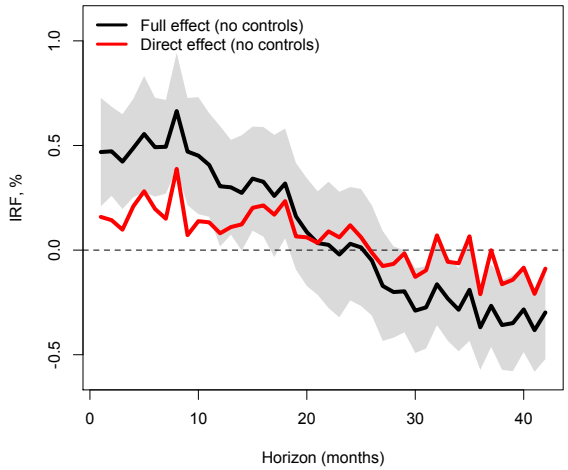
(d) Histogram of cumulative Downstream Effects



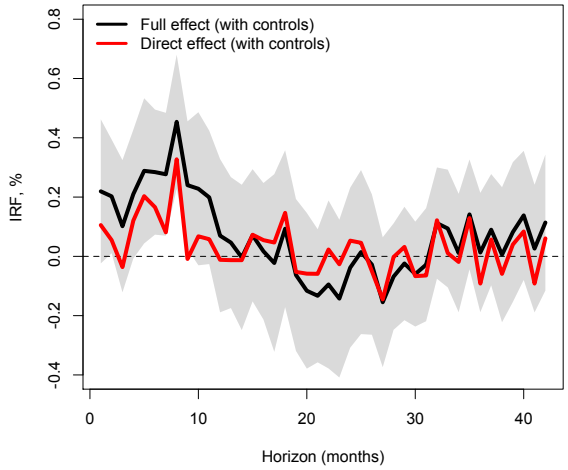
***Note**: panel (a) shows results of sector-by-sector OLS estimation of (1.20) with controls, reported as sector-level IRF following a 25bp monetary tightening; panel (b) reports results of aggregation of sector-level full and direct effects using empirical final consumption shares $\{\omega_{ck}\}_{k=1}^{161}$; panels (c) and (d) report histograms of cumulative (over 42 months) sectoral full and direct effects, as well as the corresponding sectoral cumulative downstream effects.*

Figure 1.L.20: Final consumption response to 25bp tightening: full and direct effects (Detail level, 161 sectors) – Kuttner (2001) event study shocks

(a) Full and direct effects after a 25bp tightening (with controls)



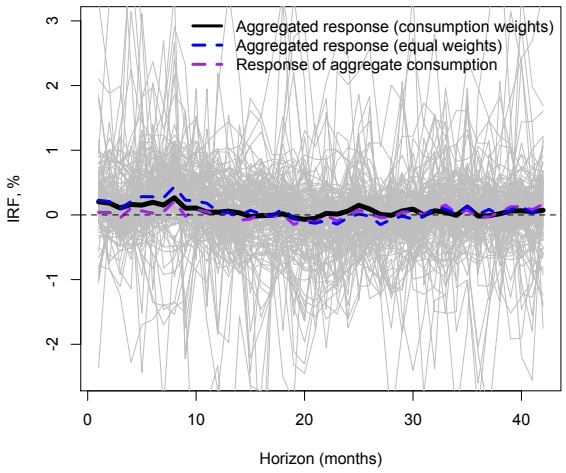
(b) Full and direct effects after a 25bp tightening 25bp tightening (with controls)



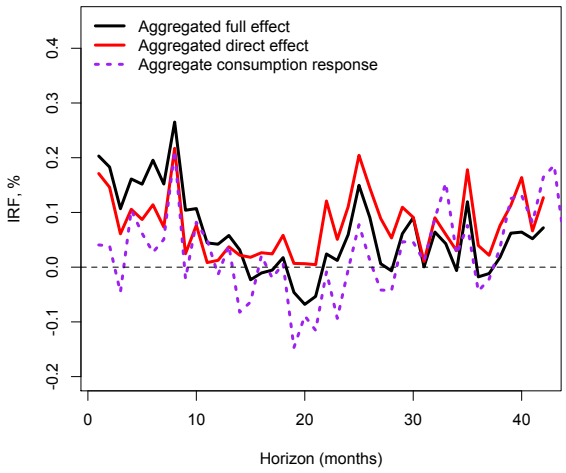
***Note**: panel (a) shows results of panel fixed effects estimation of (1.20) and (1.21), without adding controls, reported as IRFs following a 25bp monetary tightening; panel (b) reports results of panel fixed effects estimation of (1.20) and (1.21), with a vector of controls $\mathbb{X}_t^H = \left\{\{r_{t+\tau}\}_{\tau=1}^H, \{\hat{c}_{k,t-s}\}_{s=1}^1, \{ffr_{t-q}\}_{q=1}^4\right\}$ added, reported as IRFs following a 25bp monetary tightening. Shaded areas denote 90% confidence bands around the full effect. The downstream effect contribution is constructed as the difference between cumulative full and direct effects as a percentage of the cumulative full effect (all cumulation is over 42 months).*

Figure 1.L.21: Sector-level consumption responses to 25bp tightening (Detail level, 161 sectors) – Kuttner (2001) event study shocks

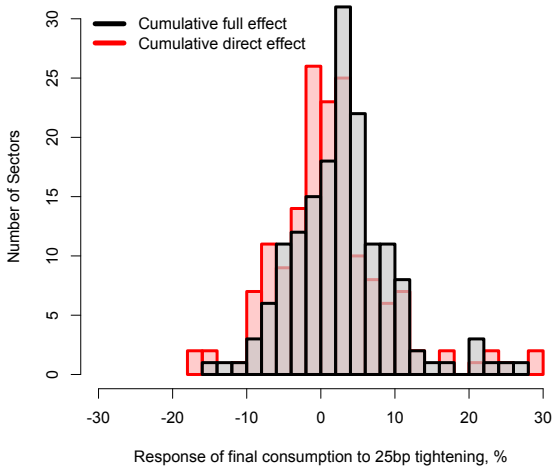
(a) Sectoral Full Impulse Responses



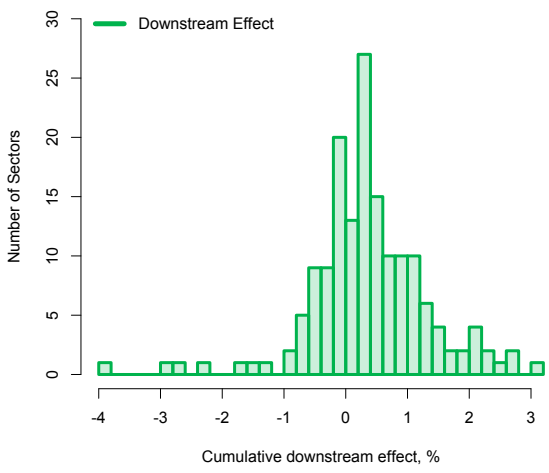
(b) Aggregated Full and Direct Impulse Responses



(c) Histogram of cumulative Full and Direct Effects



(d) Histogram of cumulative Downstream Effects



***Note**: panel (a) shows results of sector-by-sector OLS estimation of (1.20) with controls, reported as sector-level IRF following a 25bp monetary tightening; panel (b) reports results of aggregation of sector-level full and direct effects using empirical final consumption shares $\{\omega_{ck}\}_{k=1}^{161}$; panels (c) and (d) report histograms of cumulative (over 42 months) sectoral full and direct effects, as well as the corresponding sectoral cumulative downstream effects.*

Chapter 2

State Dependence of Fiscal Multipliers: the Source of Fluctuations Matters

Abstract We develop a general theory of state-dependent fiscal multipliers in a framework featuring two empirically relevant frictions: idle capacity and unsatisfied demand. Our key novel finding is that the *source of fluctuations* determines the cyclicality of multipliers. Policies that stimulate demand, such as government spending, have multipliers that are large in demand-driven recessions, but small and possibly negative in supply-driven downturns. Conversely, policies that boost supply, such as cuts in payroll taxes, are ineffective in demand-driven recessions, but powerful if the downturn is supply-driven. Austerity, implemented by a reduction in government consumption, can be the policy with the largest multiplier in supply-side recessions and demand-driven booms, provided elasticities of labor demand and supply are sufficiently low. We obtain model-free empirical support for our predictions by developing a novel econometric specification that allows to estimate spending and tax multipliers in recessions and expansions, *conditional* on those being either demand- or supply-driven.[1]

[1]This chapter is co-authored with Francesco Zanetti.

2.1 Introduction

A long tradition in economics, starting with the general theory of Keynes (1936), envisages the possibility that the effect of fiscal policy on output is different in times of economic contractions and expansions.This notion of *state dependence* of fiscal multipliers has received renewed attention in recent years, when nominal interest rates reached the effective lower bound in a number of advanced economies, granting fiscal policy a chief stabilizing role. Despite the long history and recent revival, there is still no comprehensive theoretical framework to study the sources, magnitudes and policy implications stemming from state dependence of different fiscal instruments. Our study aims to fill this gap in the literature, and develops a general analytic theory of state-dependent fiscal multipliers for a broad range of spending and taxation policies. A key novel prediction of our theory is that fiscal multipliers' variation over the business cycle is pinned down by the *source of economic fluctuations*, a result that we prove in closed-form. Further, we perform model-free econometric assessment of our novel theoretical predictions and find strong empirical support in US data.

Our theoretical framework accounts for empirically relevant frictions in the goods market, which manifest themselves in idle productive capacity on the firms' side and unsatisfied demand on the side of households. We track congestion in the goods market by looking at the ratio of households' shopping visits to firms' productive capacity; intuitively, whenever the goods market is congested, there is little idle capacity and large amount of unsatisfied demand, and *vice versa*. In our model, the effectiveness of fiscal policy is pinned down by the degree of goods market congestion. Demand-side stimuli that raise the number of visits are ineffective whenever congestion is already high, as they strongly crowd out private consumption. Supply-side stimuli that expand productive capacity are ineffective whenever congestion is already low, as they weakly crowd in private consumption. In demand-side recessions, visits drop and the goods market becomes less congested; by contrast, supply-side recessions witness shrunk capacity and hence stronger congestion. Therefore, the cyclical properties of fiscal multipliers are pinned

down by the *type of shocks* that drive the business cycle, and we establish the following properties for a range of spending and taxation instruments.

First, multipliers associated with fiscal instruments that stimulate aggregate demand, such as government consumption spending and consumption tax cuts, are *countercyclical* under demand-driven fluctuations and *procyclical* under supply-driven fluctuations. A recession originated by a lack of demand generates a reduction in the number of household visits, thus *lowering congestion* in the goods market. In such environment, a demand-side fiscal stimulus that boosts aggregate demand and increases the number of visits leads to an increase in production without raising congestion. Consequently, the crowding out of private consumption is small, leading to a high value of the multiplier. By contrast, a supply-driven recession, originated by a fall in productivity, generates a contraction in capacity and hence an *increase in congestion*. A demand-side fiscal stimulus that leads to an increase in the number of visits results in a further increase in congestion that crowds out private consumption and generates a small and possibly negative multiplier.

Second, multipliers associated with interventions that stimulate aggregate supply, such as reductions in taxes on firms' payroll, sales and households' labor income, are *countercyclical* under supply-driven fluctuations and *procyclical* under demand-driven fluctuations. A supply-driven recession is associated with a drop in capacity and hence a surge in congestion; in such an environment, a tax cut that expands capacity leads to a substantial drop in congestion, which leads to strong crowding in of private consumption, and hence a high multiplier. Instead, in a demand-side recession, where visits and congestion drop, expanding capacity through tax cuts leads to a further drop in congestion, thus generating a weak crowding in of private consumption, and hence a low multiplier.

Third, our theoretical framework assigns an important role to fiscal austerity, implemented by a reduction in government consumption, in severe *supply-driven recessions* and *demand-driven booms*. In particular, we show that states of the world exist where goods market congestion is sufficiently high so that a demand-driven stimulus crowds out private consumption at a ra-

tio of more than one-to-one, and the multiplier becomes negative. Moreover, provided elasticities of labor supply and labor demand are sufficiently low so that supply-side stimuli generate a very small drop in congestion, a government consumption austerity, which reduces visits and thus crowds in private consumption, becomes the policy with the highest multiplier. Our results provide a theoretical rationale for empirical findings in Alesina et al. (2015) on the preferential properties of spending-based austerity programs, as well as an alternative justification for austerity that does not rely on government credibility to avoid default (Reinhart and Rogoff, 2010).

Fourth, we develop and estimate an econometric specification that allows for model-free testing of our novel theoretical predictions. We build on the local projections approach of Jordà (2005b) and estimate spending and tax cut multipliers in recessionary and expansionary episodes, *conditional* on those being demand- or supply-driven in nature. We determine the nature of each episode by looking at the co-movement between cyclical components of economic activity and inflation. A positive co-movement is taken to be indicative of demand-driven fluctuations, whereas negative co-movement corresponds to supply-driven fluctuations. Empirical studies as early as Bayoumi and Eichengreen (1992) have widely used this approach to pinning down the source of fluctuations, but we are the first study to exploit such co-movement in a state-dependent local projections setting. In accordance with our theory, we find (cumulative) spending multipliers to be high in demand- and low in supply-side recessions, especially at horizons shorter than two years; the opposite patterns hold for tax cut multipliers.

Contribution to the literature. Our study contributes to the growing literature on theories of fiscal state dependence.[2] Early studies focus on fiscal policy at the effective lower bound, showing that fiscal multipliers rise substantially when nominal interest rates are close to zero (Christiano et al., 2011; Coenen et al., 2012; Fernández-Villaverde et al., 2015, Rendahl, 2016 and Roulleau-Pasdeloup, 2020), although more recent studies challenge such

[2]See Ramey (2019) for a comprehensive review of recent developments in the fiscal policy literature.

findings under fully non-linear solutions (Boneva et al., 2016; Lindé and Trabandt, 2018), under market incompleteness (Hagedorn et al., 2019) or when the liquidity trap is driven by a self-fulfilling expectations shock (Mertens and Ravn, 2014). As for multipliers away from the effective lower bound, Michaillat (2014) establishes that government employment multipliers increase in times of high unemployment, Canzoneri et al. (2016) and Faria-e-Castro (2019) show that the widening of credit spreads caused by financial frictions increases government spending multipliers during recessions, Shen and Yang (2018) show that spending multipliers become countercyclical under downward nominal wage rigidity, Boehm and Pandalai-Nayar (2020) show that firm-level capacity constraints lead to fiscal multipliers that vary with utilization and Cloyne et al. (2020) find large fiscal multipliers when monetary policy is less activist.[3] Fernández-Villaverde et al. (2019) show that search complementarities between producing firms generate multiple equilibria and that the fiscal multiplier becomes state dependent if fiscal policy is sufficiently powerful to move the economy across equilibria. In the study most related to ours, Michaillat and Saez (2019) conduct a normative analysis in a model with search frictions to show that the *socially optimal stock* of government spending can vary with unemployment.[4] Ziegenbein (2017) and Sims and Wolff (2018) show that multipliers out of tax cuts vary significantly across the business cycle and are larger in states in which output is high.

[3]Hall (2009) emphasizes the importance of introducing search-driven unemployment to deliver realistic multipliers out of government consumption. Our work models search in the goods market, and considers both government consumption and a range of tax cut instruments.

[4]Though the conceptual framework used is similar, our analysis substantially differs from that in Michaillat and Saez (2019) in a number of ways. First, Michaillat and Saez (2019) do not study the role played by the source of fluctuations in determining cyclicality of fiscal multipliers. Second, the supply side of our economy is fully endogenous and allows us to study multipliers from both government consumption, which is the exclusive focus of Michaillat and Saez (2019), as well as a wider range of fiscal instruments, including government employment, distortionary taxation on consumption, labor income, and firms' sales. Third, the dynamic version of our model is set in discrete time and features transition dynamics, which allow us to study multipliers at different time horizons that can be mapped to empirical estimates in the literature. Fourth, we provide model-free econometric assessment of the predictions of our model, whereas Michaillat and Saez (2019) only perform model-based simulations.

Compared to the aforementioned studies, we are the first study to *jointly* rationalize state dependence in a broad range of spending and taxation multipliers and to develop a tractable model with closed-form solutions. To the best of our knowledge, we are also the first study to link the state dependence of fiscal multipliers to the source of economic fluctuations. Our theoretical findings offer direct guidance for the conduct of fiscal policy, particularly to establish the effectiveness of alternative fiscal instruments in a given phase of the business cycle.

Our study also contributes to the empirical literature on fiscal state dependence, and our econometric findings offer a resolution to the debate on the degree of variation of fiscal multipliers over the business cycle. Early studies find government spending multipliers to be substantially larger in recessions compared to expansions both in the US (Auerbach and Gorodnichenko, 2012; Fazzari et al., 2014) and internationally (Auerbach and Gorodnichenko, 2013). However, more recently, Ramey and Zubairy (2018) construct a comprehensive historical dataset for government spending in the US, and find almost acyclical spending multipliers. Moreover, empirical studies do not find spending multipliers to be substantially larger at the effective lower bound, either in the UK (Crafts and Mills, 2013), the US (Ramey and Zubairy, 2018), or Japan (Miyamoto et al., 2018). Ziegenbein (2017) and Eskandari (2019) find that tax cut multipliers are highly procyclical.

Our key empirical contribution is in showing that once the estimation controls for the source of economic fluctuations, both spending and tax cut multipliers exhibit significant state dependence and that their variation over phases of the business cycle is consistent with our theory. Note that our estimation of state-dependent spending multipliers uses the same dataset as Ramey and Zubairy (2018), and our specification nests theirs as a special case where the source of fluctuations is irrelevant. We therefore offer a resolution to the empirical debate on state dependence of spending multipliers on both empirical and theoretical grounds. Barnichon et al. (2021) propose an alternative resolution to recover state dependence by controlling for the *sign* of the spending shock.

The rest of the paper is structured as follows. Section 2.2 develops the theoretical framework. Section 2.3 derives and discusses the key results on the state dependence of spending and taxation multipliers. Section 2.4 studies the relative effectiveness of spending and taxation multipliers and investigates the role for fiscal austerity. Section 2.5 quantifies the state dependence of multipliers in a dynamic version of the model. Section 2.6 develops an econometric model that supports our theoretical results. Section 2.7 concludes and outlines possible directions for future research.

2.2 The theoretical framework

To motivate our framework, we begin by providing evidence of idle productive capacity on the firms' side and unsatisfied demand on the households' side. We then introduce these features to the model by embedding search-and-matching frictions into the goods market in an otherwise standard production economy. By default, proofs of results in main text are in Appendix 2.A.

2.2.1 Market clearing, idle capacity and unsatisfied demand

The textbook definition of goods market clearing in a closed economy with fixed capital is:

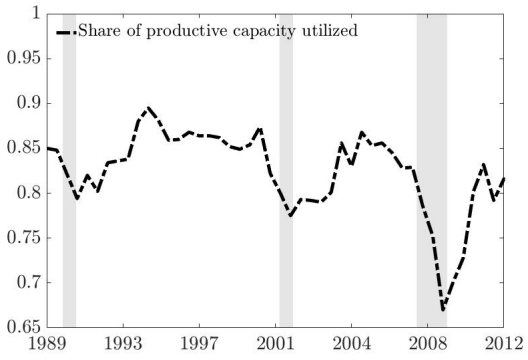
$$Y = C + G, \tag{2.1}$$

where Y is the productive capacity of the economy and $C + G$ represents aggregate demand coming from households and the government. In most standard models, equation (2.1) implies that firms sell off their entire capacity; otherwise prices fall sufficiently to clear any excess supply. At the same time, aggregate demand is generally assumed to be satisfied frictionlessly, with no resources spent on completing the purchases.

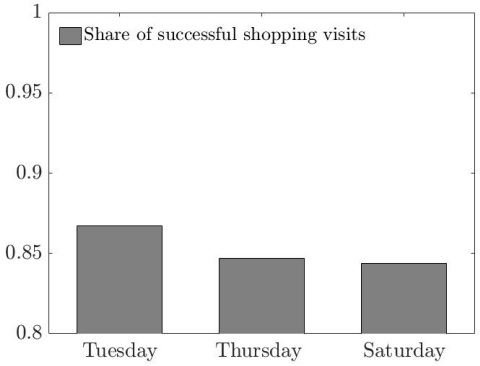
Despite the common assumption of a frictionless goods market in macroeconomic models, the data strongly endorses the presence of frictions that generate idle productive capacity and unsatisfied demand. Panel (a) of Figure 2.1 uses firm-level US data collected by the Institute for Supply Management (ISM) to show that, on average, firms in manufacturing sectors sell

Figure 2.1: Evidence on frictions in the United States goods market

(a) Share of productive capacity utilized



(b) Share of successful visits to retails stores



***Notes**: panel (a) shows a time series of the share of current capacity utilized by US firms, as calculated by the Institute for Supply Management (ISM) with NBER recessions denoted by grey shaded areas, as reported by Michaillat and Saez (2015); panel (b) shows the share of successful visits to retail stores on different days of the week, as reported by Taylor and Fawcett (2001), where Tuesday-Thursday and Tuesday-Saturday differences are statistically significant at 5% level.*

around 80 percent of their current productive capacity, and the proportion of utilized capacity is subject to regular business cycle fluctuations, with much limited fractions utilized in recessions.

Similar frictions are present on the aggregate demand side. Several studies in the fields of business logistics and marketing research document that around 15 per cent of visits to US retail stores are unsuccessful due to stock-outs (Taylor and Fawcett, 2001). Panel (b) of Figure 2.1 shows that demand frictions are also cyclical, with visits to stores being on average more successful on weekdays, as opposed to weekends, when shops tend to be more congested.Such frictions also are encountered in online stores, where as many as 25 per cent of online orders cannot be fulfilled due to out-of-stock items (Jing and Lewis, 2011). Finally, evidence from the American Time Use Survey (ATUS) shows that unsuccessful shopping visits are indeed costly for households, with an average American spending roughly one hour per day on queuing and searching for products.

Workhorse macroeconomic models do not jointly account for idle productive capacity and unsatisfied demand, despite their clear empirical relevance. In the rest of the section, we outline a theoretical framework

that jointly models those features in an otherwise standard macroeconomic model.

2.2.2 A model with search frictions in the goods market

We begin our analysis with a static model that features a goods market with search-and-matching frictions in spirit of Michaillat and Saez (2015).[5] We assume a competitive labor market.[6] The economy is composed of households, firms, and the government. Firms hire labor in order to manufacture an endogenous *productive capacity* (k), and consumers and the government make a total of v *visits* in order to purchase goods. Due to search-and-matching frictions, part of productive capacity remains idle and not all visits are successful, as encapsulated by the *matching function* that maps productive capacity (k) and visits (v) into *sales* (y):

$$y = (k^{-\delta} + v^{-\delta})^{-\frac{1}{\delta}}, \tag{2.2}$$

where $\delta > 0$ ensures that $y < \min\{k, v\}$. We define goods market *tightness* (x) as the ratio of visits to capacity:

$$x \equiv \frac{v}{k}. \tag{2.3}$$

Abstracting from aggregate uncertainty, each unit of productive capacity is sold with probability:

$$f(x) \equiv \frac{y}{k} = (1 + x^{-\delta})^{-\frac{1}{\delta}}, \tag{2.4}$$

where $f(0) = 0, \lim_{x\to+\infty} f(x) = 1$, and $f'(x) > 0, \forall x \in [0, +\infty)$. Intuitively, the probability of selling a unit of productive capacity is higher in a tighter goods market, and *vice versa*. Similarly, a purchasing visit is successful with probability:

$$q(x) \equiv \frac{y}{v} = (1 + x^{\delta})^{-\frac{1}{\delta}}, \tag{2.5}$$

[5] We develop and present a dynamic version of our model in Section 2.5. Our framework builds on the general-disequilibrium model by Barro and Grossman (1971), whose application to fiscal policy is considered in van Wijnbergen (1987). Recent studies with goods market search frictions include Bai et al. (2012), Den Haan (2013), Gourio and Rudanko (2014), Brzustowski et al. (2018) and Roldan-Blanco and Gilbukh (2020).

[6] Adding search frictions to the labor market with flexible wages leaves our results for fiscal multipliers unchanged.

where $q(0) = 1, \lim_{x \to +\infty} q(x) = 0$ and $q'(x) < 0, \forall x \in [0, +\infty)$. The probability of a successful visit is lower in a tighter goods market, and *vice versa*. [7]

Within this framework, it is useful to think of productive capacity (k) as the size of the "store" and visits (v) as the length of the "queue" comprising private and government consumers. Goods market tightness (x) can be interpreted as the number of queuing consumers per square meter of the store or as a measure of congestion in the goods market. This interpretation will be helpful to develop the intuition behind our results.

2.2.3 Households

There is a continuum of identical households of size one. Households make v^c visits in order to purchase and consume c units of the produced good. There is a cost $\rho \in (0, 1)$ of the produced good per visit.[8] Total sales of the produced good to households (y^c) comprise household consumption (c) and search costs (ρv^c):

$$y^c = c + \rho v^c. \tag{2.6}$$

Since each visit is successful with probability $q(x)$, total sales are equal to $y^c = q(x)v^c$, and the consumption of c units of the produced good requires $c/(q(x) - \rho)$ visits. Therefore, the total number of goods that need to be purchased (inclusive of the cost of search) in order to consume c units is given by:

$$[1 + \gamma(x)]c, \tag{2.7}$$

[7] Petrosky-Nadeau et al. (2016) provide evidence that the average time spent shopping fell in the demand-deficient period of 2008-2010, which is consistent with the property that the probability of a successful visit rises in a less congested goods market.

[8] The visiting cost can be interpreted as the cost of time spent queuing in shops, or alternatively the (expected) cost of returning a purchased good that the customer did not like. This interpretation is supported by empirical evidence. According to the American Time Use Survey (ATUS), the average American spends approximately one hour per day on shopping and queueing. Our formulation for search costs allows us to treat government and private customers symmetrically, as we discuss in section 2.2.5. An alternative way to model search costs for the households is to assume that there is a *utility* cost per visit. In Appendix 2.F, we show that results of the analysis continue to hold with this alternative approach to modelling search costs.

where $\gamma(x) \equiv \frac{\rho x}{f(x)-\rho x}$ represents the wedge introduced by search-and-matching frictions that strictly rises in goods market tightness, such that $\gamma'(x) > 0, \forall x \in (0, x_m)$.[9] Intuitively, a tighter goods market diminishes the probability of a successful visit, increasing the expected number of visits required for a successful purchase, thus raising total search costs.

The representative household gains utility from consumption of the produced good (c), the non-produced good (m) that is in fixed exogenous supply $(\bar{m})$, and gains disutility from supplying labor (l). The non-produced good is traded in a frictionless competitive market, and we use it as the numeraire by normalizing its price to one.[10] Every household is small relative to the size of the market, and therefore takes the price (p), wage (w), goods market tightness (x) and hence the search wedge $[1 + \gamma(x)]$ as given. The representative household maximizes utility function:

$$\max_{c,m,l} \left[\chi \frac{c^{1-\sigma}}{1-\sigma} + \zeta(m) - \frac{l^{1+\psi}}{1+\psi} \right] \tag{2.8}$$

subject to the budget constraint

$$p[1 + \gamma(x)]c + m \leq wl + \Pi + \bar{m} - T, \tag{2.9}$$

where $\chi > 0$ determines the relative preference for the produced good, ψ is the inverse Frisch elasticity of labor supply, p is the price of the produced good, $[1 + \gamma(x)]$ is the search wedge, w is the wage received per unit labor supplied, Π is profits from firms owned by the representative household, and T is a lump-sum tax collected by the government. The representative household also receives an endowment of the non-produced good equal to its (fixed) supply $\bar{m}$. Finally, $\zeta(.)$ is an increasing differentiable function.

Lemmas 2.1 and 2.2 report the consumption function, $c(p, x)$, and labor supply function, $l(w)$, respectively, that solve the representative household's

[9] We restrict the admissible values of tightness to $(0, x_m)$, where x_m is given by the condition $f(x_m) = \rho x_m$; this approach ensures that the aggregate supply of the produced good, net of search costs, remains positive.

[10] Introducing the numeraire good m allows us to separately pin down both the price of the produced good p, as well as the wage w, in our static framework.

maximization problem.[11]

Lemma 2.1. *The consumption function $c(p, x)$ is the optimal consumption choice in the representative household's problem evaluated under non-produced goods market clearing ($m = \bar{m}$) and is equal to:*

$$c(p, x) = \frac{\chi}{p[1 + \gamma(x)]}, \tag{2.10}$$

where $\frac{\partial c}{\partial p} < 0$, $\frac{\partial c}{\partial x} < 0$ and $\frac{\partial c}{\partial \chi} > 0$.

Equation (2.10) shows that, *ceteris paribus*, higher preference parameter χ increases consumption since it generates larger utility for every unit of consumption; higher price (p) and tightness (x) increase the relative price of consumption and hence decrease consumption.

Lemma 2.2. *The labor supply function $l(w)$ is the optimal labor supply choice in the representative household's problem evaluated under non-produced goods market clearing ($m = \bar{m}$), and is equal to:*

$$l(w) = w^{\frac{1}{\psi}}, \tag{2.11}$$

where $\frac{\partial l}{\partial w} > 0$.

Equation (2.11) shows that, *ceteris paribus*, higher wage (w) increases the supply of labor. The elasticity of labor supply is equal to $\epsilon^s \equiv \frac{1}{\psi}$, and if $\psi \to \infty$, labor supply becomes perfectly inelastic and fixed at one.

2.2.4 Firms

The economy is populated by a continuum of identical, perfectly competitive firms that manufacture an identical good that is sold in a goods market characterized by search-and-matching frictions. Firms have access to a production technology that transforms labor input (n) into *productive capacity* (k):

$$k(n) = an^{\alpha}, \tag{2.12}$$

[11] For the remainder of the main text we assume log utility of consumption ($\sigma = 1$). Our choice of log utility of consumption is consistent with mean estimates of the coefficient of relative risk aversion in microeconomic studies (Chetty, 2006). Appendix 2.E provides closed-form solutions for a generic CRRA utility of consumption. We also normalize the supply of the non-produced good so that $\zeta'(\bar{m}) = 1$.

where $\alpha \in (0, 1]$ is returns to labor and $a > 0$ is the level of productivity. In the presence of search-and-matching frictions, every unit of productive capacity is utilized with probability $f(x)$. Abstracting from uncertainty, the representative firm achieves the following level of *sales* $y(x, n)$:

$$y(x, n) = f(x)k(n) = f(x)an^{\alpha}. \tag{2.13}$$

Each firm is small relative to the size of the market and thus takes the price (p) and wage (w) as well as goods market tightness (x) and probability $f(x)$ as given. The profit maximization problem of the representative firm is:

$$\max_{n} \Pi = \left[pf(x)an^{\alpha} - wn(1 + \tau) \right], \tag{2.14}$$

where $\tau \in [0, 1)$ is a payroll tax from the government, which is a fraction τ of the representative firm's wage bill.

Lemma 2.3 reports the labor demand function $n(w; p, x, \tau)$ that solves the representative firm's profit-maximization problem.

Lemma 2.3. *The labor demand function $n(w; p, x, \tau)$ is the solution to the representative firm's profit maximization problem and is equal to:*

$$n(w; p, x, \tau) = \left[\frac{\alpha p f(x) a}{w(1 + \tau)} \right]^{\frac{1}{1-\alpha}}, \tag{2.15}$$

where $\frac{\partial n}{\partial p} > 0$, $\frac{\partial n}{\partial x} > 0$, $\frac{\partial n}{\partial w} < 0$, $\frac{\partial n}{\partial a} > 0$ and $\frac{\partial n}{\partial \tau} < 0$.

Equation (2.15) shows that, *ceteris paribus*, a higher price (p) increases the revenue from every unit sold and hence incentivizes production and labor demand. Higher tightness (x) increases the probability of selling each unit produced and hence also incentivizes production and labor demand. Labor demand decreases with the cost of hiring labor, given by the wage (w). A payroll tax $\tau > 0$ increases the cost of hiring and thus lowers labor demand. Finally, higher productivity (a) increases the marginal product for each unit of labor hired, increasing labor demand. The (absolute) elasticity of labor demand is given by $|\epsilon^d| \equiv \frac{1}{1-\alpha}$, with the case of constant returns ($\alpha = 1$) corresponding to a perfectly elastic labor demand.

2.2.5 Government

The government consumes an exogenous quantity G of the privately produced good and is subject to the same search-and-matching frictions as private consumers. Thus, for a given desired government consumption of the produced good G, the government must purchase $[1 + \gamma(x)]G$ units, where $[1 + \gamma(x)]$ is the same wedge as that faced by private consumers.[12,13] Given the rate of payroll tax τ and hence the tax revenue $wn\tau$ collected, the government imposes a lump-sum tax T on the households to balance the public budget:

$$T = p[1 + \gamma(x)]G - wn\tau. \tag{2.16}$$

There are alternative ways of including government spending into our model. For example, government spending on defense or other public goods may be introduced in the model as government spending on employing labor to produce public goods that are offered to private consumers. Appendix 2.C.1 extends the baseline model to allow for government employment and production of public goods, and it shows that the cyclical properties of the multiplier out of public employment are identical to those of the government consumption multiplier.

We focus on distortionary payroll taxes in the baseline model. Appendix 2.C.2 considers alternative distortionary taxation on firms' sales and on households' labor income, showing that the cyclical properties of multipliers out of cuts to sales tax and labor income tax are identical to those of cuts to the payroll tax. In addition, Appendix 2.C.2 also considers distortionary taxation of households' consumption and shows that cyclical properties of

[12]Our assumption of identical costs per visit for households and the government, and hence of identical search wedges, is made for algebraic simplicity and is inessential for our results. Our results hold even if we assume that the government faces a smaller ($\rho^G < \rho$) or zero ($\rho^G = 0$) visiting cost.

[13]Our model features a single production sector, so we abstract from any differences in sectoral compositions of government and household consumption, as documented by Ramey and Shapiro (1998) and Cox et al. (2020). However, our qualitative mechanism may still hold even if sectoral compositions are different: higher government consumption may draw factors of production away from households' sectors, thus increasing good market tightness in the latter. This mechanism is even more apparent if the government directly competes for factors of production with the private sector; we consider such extension in Appendix 2.C.1.

the multiplier out of consumption tax cuts are identical to those of the government consumption multiplier considered in the baseline model.

2.2.6 Market clearing

The economy is composed of three distinct markets for the produced good (c), the non-produced good (m), and labor (n). By Walras' Law, we obtain equilibrium allocations by determining market clearing in any two markets. We focus on the markets for the produced good and labor.

Aggregate demand consists of the households' demand $c(p, x)$ and the government's exogenous demand G. Aggregate supply is given by the fraction of the firm's sales $y(x; n) = f(x)k(n; \tau)$ that is not spent on the cost of search, and hence is given by $f(x)k(x; \tau)/[1 + \gamma(x)]$. The market clearing condition in the goods market is:

$$\underbrace{\frac{f(x)}{1+\gamma(x)}k(n;\tau)}_{\text{Aggregate supply}} = \underbrace{c(p,x)+G}_{\text{Aggregate demand}} . \tag{2.17}$$

From Lemma 2.1 we know that aggregate demand is downward-sloping in tightness-quantity space. As for the aggregate supply curve, it is backward-bending: it rises in tightness between $(0, x^*)$ and falls in tightness for values (x^*, x_m). For a given productive capacity k, goods market tightness exerts two counteracting effects on aggregate supply. On the one hand, higher x increases the probability $f(x)$ of selling each produced good, thus raising aggregate supply; on the other hand, search costs, encapsulated by the wedge $[1+\gamma(x)]$, increase in tightness. The increase in the selling probability outweighs the increase in search costs for tightness below the efficient level ($x < x^*$). On the other hand, aggregate supply decreases for tightness above the efficient level ($x > x^*$). Aggregate supply is maximized at $x = x^*$, where the two effects offset each other, which corresponds to the social planner's allocation in our economy, as we formally show in Appendix 2.G.1.

Market clearing in the labor market is achieved by equating labor demand, $n(w; p, x, \tau)$, to labor supply, $l(w)$:

$$\underbrace{l(w)}_{\text{Labor supply}} = \underbrace{n(w;p,x,\tau)}_{\text{Labor demand}} . \tag{2.18}$$

The labor supply function is upward sloping in the wage-employment space, as established in Lemma 2.2, since higher wages encourage households to work more, and the labor demand function is downward sloping in the wage-employment space, as established in Lemma 2.3.

Equilibrium is described by price, wage, tightness, and allocations that satisfy the optimality conditions of households, firms, the government budget constraint, and the market clearing conditions. The system is indeterminate because once optimality and market clearing conditions have been combined, we are left with two equations in three unknowns (p, x, w). We therefore need a selection mechanism to choose a specific equilibrium across the infinitely many admissible combinations.[14]

2.2.7 Comparative statics: two polar equilibria

In the baseline model, we resolve indeterminacy by considering two polar equilibrium cases. First, in a *competitive* equilibrium, tightness is fixed at its socially efficient level x^*, formally defined in Appendix 2.G.1, and p and w adjust fully flexibly to satisfy optimality and market clearing conditions.[15] Second, in a *fixprice* equilibrium, price p is fixed at a constant value p_0, and x and w adjust to satisfy optimality and market clearing conditions. Appendix 2.B extends our analysis to more general equilibrium cases.

Comparative statics: competitive equilibrium

A competitive equilibrium is formally defined as follows:

Definition 2.1. *A **competitive** equilibrium is a pair (p^*, w^*) and associated allocations, such that the agents' optimality conditions and the market clearing conditions are satisfied with tightness at its efficient level $(x = x^*)$.*

[14] Goods market tightness (x) is taken as given by agents in a decentralized economy, and indeterminacy is a common feature of search-and-matching models. A standard approach to resolve indeterminacy in the search-and-matching framework of the labor market is adding Nash bargaining over the wage.

[15] The name *competitive equilibrium* is a reference to the directed search literature and in particular, the competitive search equilibrium in Moen (1997), where full adjustment via prices and wages leads to efficient allocations.

Following a positive demand shock, parameterized as an increase in χ, households choose to consume more, hence increasing the number of visits; since tightness is to remain fixed at x^*, the price p has to rise to discourage further consumption and expand capacity, until markets clear with more sales in the new equilibrium.[16] On the other hand, after a positive supply shock, represented by a rise in a, capacity expands. In order to keep tightness fixed, the price has to fall, thus encouraging more consumption and more visits until markets clear with higher sales in equilibrium. Lemma 2.4 formally summarizes the comparative statics for the competitive equilibrium:

Lemma 2.4. *In a competitive equilibrium, the comparative statics of tightness (x), sales (y) and the price (p):*

$$\frac{dx}{d\chi} = 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} > 0; \qquad \frac{dx}{da} = 0, \frac{dy}{da} > 0, \frac{dp}{da} < 0. \tag{2.19}$$

Figure 2.D.2 in Appendix 2.D provides graphical representation of comparative statics following demand- and supply-side shocks in a competitive equilibrium.

Comparative statics: fixprice equilibrium

A fixprice equilibrium is formally defined as follows:

Definition 2.2. *A **fixprice** equilibrium is a vector (p_0, x, w) and associated allocations, which satisfy the agents' optimality conditions and the market clearing conditions with fixed price p_0 ($p = p_0$).*

After a positive demand shock, consumption and visits rise. Since the price p is fixed, the only way to clear such excess demand is for the tightness x to rise, which increases the cost of search, thus discouraging any further consumption until markets clear with higher sales in the new equilibrium. At the same time, following a positive supply shock, capacity expands, and

[16] In Appendix 2.G.1, we show that the socially efficient level of tightness is given by the condition $f'(x^*) = \rho$ and hence x^* is invariant to changes in either χ or a.

the only way for such excess supply to be cleared is through a fall in tightness, which encourages consumption until markets clear.[17] Lemma 2.5 below formally summarizes the comparative statics in a fixprice equilibrium:

Lemma 2.5. *In a fixprice equilibrium, the comparative statics of tightness* (x)*, sales* (y)*, and the price* (p) *are:*

$$\frac{dx}{d\chi} > 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} = 0; \qquad \frac{dx}{da} < 0, \frac{dy}{da} = 0, \frac{dp}{da} = 0. \tag{2.20}$$

Figure 2.D.3 in Appendix 2.D provides graphical representation of comparative statics following demand- and supply-side shocks in a competitive equilibrium.

2.3 Fiscal multipliers: key analytical results

In this section, we establish our key novel analytical results regarding cyclical properties of fiscal multipliers. In particular, we consider the *demand-side* multiplier, associated with government consumption, and the *supply-side* multiplier, associated with payroll tax cuts.[18] We show that in a competitive equilibrium, where tightness is fixed and price and wage are fully flexible, the two multipliers are *identical* and *acyclical*, and they are pinned down exclusively by the elasticities of labor supply and demand. In a fixprice equilibrium, where the price is fixed and tightness and wages clear markets, the demand-side multiplier is *countercyclical* under *demand-side fluctuations* and *procyclical* under *supply-side fluctuations*. On the other hand, the supply-side multiplier is *procyclical* under *demand-side fluctuations* and *countercyclical* under *supply-side* fluctuations. Finally, we provide evidence that equilibria featuring price rigidity and fluctuations in tightness provide a better descrip-

[17] Note that following a positive supply shock, the level of sales remains unchanged in a fixprice equilibrium. This is because, on the one hand, capacity k expands, but on the other, tightness the utilization rate $f(x)$ drops. The two effects cancel each other out only as long as prices remain completely fixed: as we show in Lemma 2.9 in Appendix 2.B.2 In equilibria featuring rigid but not completely fixed prices, sales increase following a positive supply shock, while tightness still falls.

[18] In this section, we limit our attention to the two polar equilibria: competitive and fixprice. Appendix 2.B studies fiscal multipliers under more general equilibrium types whereas Appendix 2.C extends the analysis to multipliers out of government employment, as well as consumption, sales, and labor income tax cuts.

tion of the US economy at business cycle frequencies, thus endorsing equilibria featuring state-dependent fiscal multipliers, whose cyclicality depends on the source of economic fluctuations.

2.3.1 Definitions

A fiscal multiplier measures the effect of a marginal change in the fiscal instrument, be it government spending or a tax rate, on GDP *in equilibrium*. The following definition introduces the demand-side fiscal multiplier, associated with increases in government consumption spending: [19]

Definition 2.3. *The demand-side fiscal multiplier, $\varphi^d(x)$, is given by:*

$$\varphi^d(x) \equiv \frac{d\{c+G\}}{dG} = \frac{dc}{dG} + 1. \tag{2.21}$$

Similarly, the following definition introduces the supply-side fiscal multiplier, associated with payroll tax cuts, respectively.

Definition 2.4. *The supply-side fiscal multiplier, $\varphi^s(x)$, is given by:*

$$\varphi^s(x) \equiv \frac{1}{c+G}\frac{d\{c+G\}}{d[-\tau]} = -\frac{1}{c}\frac{dc}{d\tau}. \tag{2.22}$$

2.3.2 Competitive equilibrium multipliers

We first derive the fiscal multipliers in a competitive equilibrium, where p and w are fully flexible, and adjust in response to shocks to maintain tightness at the efficient level ($x = x^*$) to satisfy the equilibrium conditions:

Proposition 2.1. *In a competitive equilibrium, the demand- and supply-side fiscal multipliers are equal and given by:*

$$\varphi^* \equiv \frac{\alpha}{1+\psi} = \frac{1-\frac{1}{|\epsilon^d|}}{1+\frac{1}{\epsilon^s}}, \tag{2.23}$$

where $\alpha \in (0,1]$ and $\psi > 0$ are, respectively, returns to labor and inverse Frisch elasticity, whereas $|\epsilon^d| = \frac{1}{1-\alpha}$ and $\epsilon^s = \frac{1}{\psi}$ are (absolute) elasticities of labor demand and labor supply. Hence $\varphi^* \in (0,1]$, and it is pinned down by elasticities of labor demand and labor supply.

[19] For simplicity and to retain direct comparability with related studies, the multipliers are evaluated at the point where there is no government intervention, so that $G = 0$ and $\tau = 0$.

Proposition 2.1 outlines several important results. First, in a competitive equilibrium, the demand- and supply-side multipliers are equal, implying that either of the two fiscal interventions generates the same effect on GDP. However, in accordance with Definitions 2.3 and 2.4, although φ^* is identical and between zero and one for both fiscal policy instruments, consumption is crowded out ($\frac{dc}{dG} < 0$) under demand-side fiscal policy, and it is crowded in ($-\frac{dc}{d\tau} > 0$) under supply-side fiscal policy.

Second, the competitive equilibrium multiplier in Proposition 2.1 coincides with the government spending multiplier derived in Woodford (2011) for a New Keynesian model considered in the limit of fully flexible prices and wages, parameterized for our preferences and technology specifications. One can therefore treat φ^* as a benchmark for multipliers under fully flexible prices, in either our model with goods market search frictions or in a New Keynesian model.

Third, the competitive equilibrium multiplier is determined exclusively by the relative elasticities of labor supply and demand, equal to $\epsilon^s \equiv \frac{1}{\psi}$ and $|\epsilon^d| \equiv \frac{1}{1-\alpha}$, respectively. As labor supply becomes perfectly inelastic ($\psi \to \infty$), the multiplier decreases ($\varphi^* \to 0$). The intuition is straightforward. When the ratio of "queue length" to "store size" is to be kept at the efficient level and the "store size" is fixed, the only way to accommodate additional government customers in the queue is for the price to increase to the point where private customers in the queue are crowded-out one-for-one. Similarly, payroll tax cuts that lead to higher employment and larger "store size" result in higher wages with no change in employment and consumption when labor supply is perfectly inelastic and tightness is kept at the efficient level.

On the other hand, when the demand and supply of labor are perfectly elastic ($\alpha = 1$ and $\psi = 0$), the fiscal multiplier reaches the maximum value of one ($\varphi^* = 1$). In this case, any additional queue length from government customers generates an increase in employment and enlarges the capacity of the store, without crowding out consumption and retaining tightness at the efficient level. Any payroll tax cuts leave wages unchanged and increase

the supply of labor and hence the production of goods, which enlarges the capacity of the store. The only way to retain tightness at the efficient level is for the price to fall, leading to a one-for-one crowd-in of private customers into the queue.

Given the mapping between the elasticities of labor supply and labor demand and the competitive equilibrium multipliers, we interpret φ^* as a measure of flexibility of the labor market. The more elastic the labor demand (larger α) and labor supply (smaller ψ), the larger the multiplier in a competitive equilibrium.

It is now straightforward to show that both demand- and supply-side fiscal multipliers are *acyclical* in a competitive equilibrium, as outlined formally below:

Corollary 2.1. *In a competitive equilibrium, both demand- and supply-side multipliers are acyclical.*

Proof. A trivial consequence of Proposition 2.1: in a competitive equilibrium, both multipliers are equal to $\varphi^* = \frac{\alpha}{1+\psi}$ and do not change when either preference χ or technology a varies. □

In a competitive equilibrium, prices and wages are fully flexible and adjust to ensure that tightness remains at the efficient level (x^*) in response to demand- and supply-side shocks. Hence, both the utilization probability, $f(x)$, and the search wedge, $[1 + \gamma(x)]$, remain unchanged over the business cycle, leading to a fixed level of private consumption crowding out and hence a constant multiplier.

2.3.3 Fixprice equilibrium multipliers

Demand-side fiscal multiplier

Before studying the demand-side fiscal multiplier in a generic fixprice equilibrium, we make an intermediate step and derive the multiplier in the special case of *fixed capacity*, which arises under a perfectly inelastic labor supply ($\psi \to \infty$). The properties of such *fixed capacity fiscal multiplier* will be helpful

for studying fiscal multipliers in a generic fixprice equilibrium as well as for deriving a link between demand- and supply-side fiscal multipliers.

Lemma 2.6. *Let the fixed capacity fiscal multiplier, $\theta(x)$, be the demand-side fiscal multiplier in a fixprice equilibrium under fixed labor supply, such that*

$$\theta(x) \equiv \frac{d\{c+G\}}{dG}|_{\psi\to\infty}. \tag{2.24}$$

It can be shown that $\theta(x)$ has the following properties:

$$\theta(x) = \begin{cases} (0,-\infty), & \text{if} \quad x \in (x^*, x_m) \\ 0, & \text{if} \quad x = x^* \\ (0,1), & \text{if} \quad x \in (0, x^*) \end{cases} \tag{2.25}$$

$$\theta'(x) < 0, \qquad \forall x \in (0, x_m),$$

where x_m is given by $f(x_m) = \rho x_m$.

Lemma 2.6 outlines several results. First, in an efficient fixprice equilibrium, where $x = x^*$, the First Welfare Theorem applies and the fixed capacity fiscal multiplier is equal to zero, just like the competitive equilibrium multiplier under perfectly inelastic labor supply ($\psi \to \infty$), such that $\theta(x^*) = \varphi^*_{\psi\to\infty} = 0$. Just as in a competitive equilibrium under perfectly inelastic labor supply, $\psi \to \infty$, the only way additional government spending can be accommodated under fixed labor supply and a fixed price is by crowding out private consumption, which is achieved by an increase in tightness. Moreover, in an efficient fixprice equilibrium, the crowding out of consumption is exactly one-for-one, as the increase in tightness fails to increase supply in the goods market, which is already at its maximum, given the fixed capacity.

Second, whenever $x \in (0, x^*)$, additional demand from government spending is accommodated via higher tightness, which crowds out private consumption *less than one-for-one*. This is because under under $x \in (0, x^*)$, the effect of higher tightness on increasing the fraction of capacity utilized $f(x)$ dominates the effect of increasing the search wedge $[1+\gamma(x)]$, so that aggregate supply increases following the government consumption increase.

Third, whenever $x \in (x^*, x_m)$, higher government spending crowds out private consumption *more than one-for-one*. This is driven by the fact that under $x \in (x^*, x_m)$ the effect of higher tightness on increasing the cost of search $[1 + \gamma(x)]$ dominates the effect on increasing the fraction of capacity utilized $f(x)$, and aggregate supply falls following the rise in government consumption.

Having established the properties of $\theta(x)$, we can now provide a convenient expression for the demand-side fiscal multiplier in a generic fixprice equilibrium:

Proposition 2.2. *In a fixprice equilibrium, the demand-side fiscal multiplier, $\varphi^d(x)$, is given by:*

$$\varphi^d(x) = \underbrace{\varphi^*}_{\text{State-invariant component}} + \underbrace{\theta(x) \times (1 - \varphi^*)}_{\text{State-dependent component}}, \tag{2.26}$$

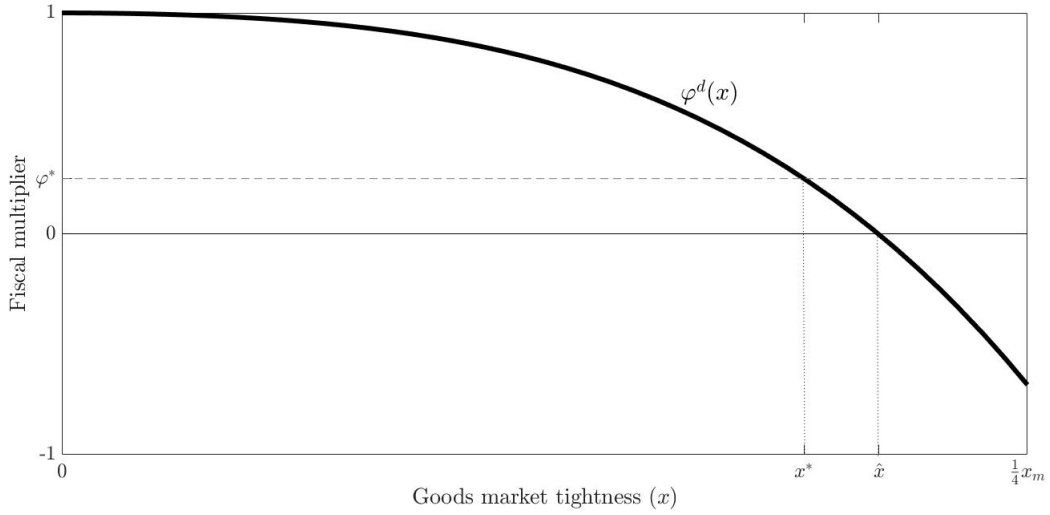
where $\varphi^* = \frac{\alpha}{1+\psi}$ is the competitive equilibrium multiplier. Hence, $\varphi^d(x) \in (-\infty, 1)$ and $\frac{d\varphi^d(x)}{dx} < 0, \forall x \in (0, x_m)$.

Proposition 2.2 establishes several important results. First, the demand-side fiscal multiplier can be represented as the sum of a state-invariant component, given by the competitive equilibrium multiplier φ^*, and a state-dependent component that is a function of the underlying goods market tightness. Moreover, in the special case of an efficient fixprice equilibrium, where $x = x^*$, the state-dependent component disappears since $\theta(x^*) = 0$, and the fixprice demand-side multiplier collapses to the multiplier in a competitive equilibrium, $\varphi^d(x^*) = \varphi^*$.

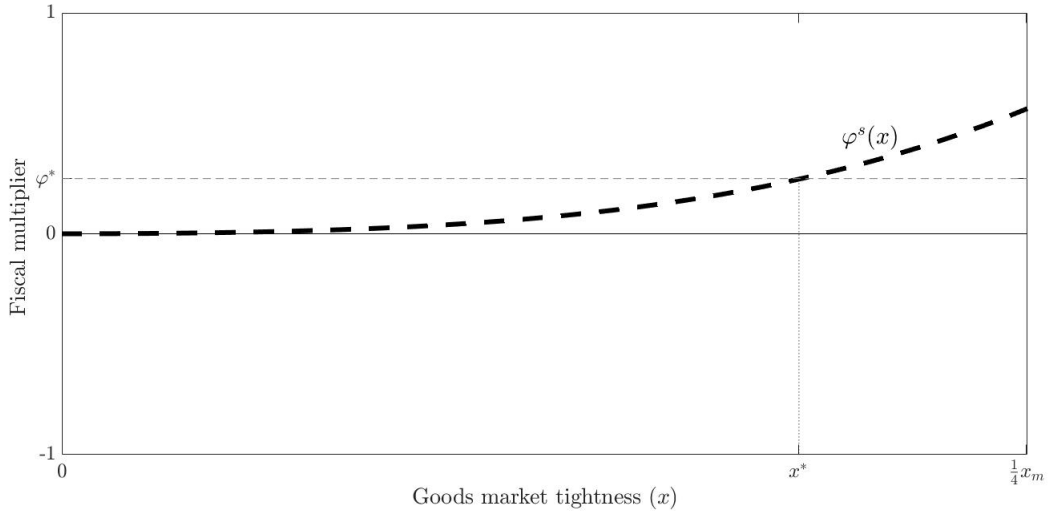
Second, from the properties of $\theta(x)$, it follows that in a fixprice equilibrium, the demand-side fiscal multiplier lies between one and negative infinity and strictly falls in tightness on the whole domain. Hence, government expenditure always crowds out private consumption, and the crowding-out effect is stronger whenever the goods market tightness is higher, as shown graphically in Panel (a) of Figure 2.2. To outline the intuition behind this finding, consider our narrative of "store size" and "queue length". A low goods market tightness environment is akin to a short queue for a given size

Figure 2.2: Fiscal multipliers in a fixprice equilibrium

(a) Demand-side fiscal multiplier, $\varphi^d(x)$



(b) Supply-side fiscal multiplier, $\varphi^s(x)$



***Notes**: Panels (a) and (b) show demand-side and supply-side fiscal multipliers in a fixprice equilibrium of a calibrated version of our model ($\alpha = 0.3, \delta = 2, \rho = 0.1, \psi = 0.2$) for values of goods market tightness in the range $(0, \frac{1}{4}x_m)$ (to avoid extreme values as x gets closer to x_m); Panel (a) shows that the demand-side fiscal multiplier $\varphi^d(x)$ starts at one when $x = 0$, then strictly falls in goods market tightness, and turns negative after $x = \hat{x}$; Panel (b) shows that the supply-side fiscal multiplier φ^s starts at zero when $x = 0$, then strictly rises in tightness, tending to infinity as $x \to x_m$.*

of the store. When the government implements demand-side fiscal expansion by sending its customers to join a short queue, it does not make the shop excessively crowded and hence achieves a relatively high multiplier since the crowding out effect is limited. Instead, in a high tightness environment, the government sends customers to join a crowded store, which results in strongly crowding out private customers and delivering a low multiplier.

Third, as seen in Panel (a) of Figure 2.2, under sufficiently high tightness $\hat{x}$, the demand-side fiscal multiplier turns negative, implying that private consumption gets crowded out more than one-for-one. This is despite the fact that in a generic fixprice equilibrium higher tightness increases labor demand and, *ceteris paribus*, gives a boost to capacity and aggregate supply. Moreover, as we establish in the next corollary, such threshold $\hat{x}$ always exists, regardless of the elasticities of labor demand and labor supply:

Corollary 2.2. *There always exists tightness $\hat{x} \in (x^*, x_m)$, such that $\varphi^d(\hat{x}) = 0$ and $\varphi^d(x) < 0, \forall x \in (\hat{x}, x_m)$, and it is equal to:*

$$\hat{x} = \theta^{-1}\left(-\frac{\varphi^*}{1-\varphi^*}\right), \tag{2.27}$$

where $\frac{d\hat{x}}{d\varphi^*} > 0$.

To interpret this finding, recall that Lemma 2.5 establishes that in a fixprice equilibrium tightness increases following a positive demand-side or a negative supply-side shock. Thus, Corollary 2.2 implies that in a fixprice equilibrium the demand-side multiplier can become negative either under a sufficiently strong demand-driven overheating or a sufficiently severe supply-side contraction.

Most importantly, results in Proposition 2.2 imply well-defined, cyclical properties of the demand-side fiscal multiplier, outlined in the next corollary:

Corollary 2.3. *In a fixprice equilibrium, the demand-side fiscal multiplier, $\varphi^d(x)$, is countercyclical under demand-side fluctuations and procyclical under supply-side fluctuations.*

Proof. From Lemma 2.5, we know that in a fixprice equilibrium $\frac{dx}{d\chi} > 0, \frac{dx}{da} < 0$; further, from Proposition 2.2, we know that in a fixprice equilibrium $\frac{d\varphi^d(x)}{dx} < 0, \forall x \in (0, x_m)$. Hence, $\frac{d\varphi^d(x)}{d\chi} = \frac{d\varphi^d(x)}{dx}\frac{dx}{d\chi} < 0, \forall x \in (0, x_m)$ and $\frac{d\varphi^d(x)}{da} = \frac{d\varphi^d(x)}{dx}\frac{dx}{da} > 0, \forall x \in (0, x_m)$. □

Corollary 2.3 establishes that the demand-side multiplier, associated with government consumption, is large in *demand-driven recessions* and *supply-driven expansions*, but small in *supply-driven recessions* and *demand-side expansions*. Intuitively, a demand-driven recession lowers the length of the "queue", whereas a supply-driven expansion increases the size of the "store"; in both cases, congestion of the store falls. In such cases, an increase in government consumption adds government customers to an uncongested store, leading to a small amount of private consumption crowding out, and hence a higher multiplier.[20] Conversely, both a supply-driven recession and a demand-driven expansion make the "store" more congested, leading to strong crowding out of private consumption, and hence a lower multiplier, following an increase in government consumption.

Supply-side fiscal multiplier

The next proposition provides an expression for the supply-side fiscal multiplier in a generic fixprice equilibrium:

Proposition 2.3. *In a fixprice equilibrium, the supply-side fiscal multiplier, $\varphi^s(x)$, is given by:*

$$\varphi^s(x) = \underbrace{\varphi^*}_{\text{State-invariant component}} - \underbrace{\theta(x) \times \varphi^*}_{\text{State-dependent component}}, \tag{2.28}$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the competitive equilibrium multiplier. Hence, $\varphi^s(x) \in (0, +\infty)$ and $\frac{d\varphi^s(x)}{dx} > 0, \forall x \in (0, x_m)$.

Proposition 2.3 shows that the supply-side multiplier is the sum of a state-invariant component, equal to the competitive multiplier φ^*, and a

[20] Michaillat (2014) shows that a similar result holds in a labor market with search-and matching-frictions following shocks to public employment.

state-dependent component that depends on goods market tightness. Similarly to the demand-side multiplier, the supply-side fiscal multiplier collapses to its competitive equilibrium value in the special case of an efficient fixprice equilibrium with $x = x^*$, so that $\varphi^s(x^*) = \varphi^*$

The supply-side fiscal multiplier is always positive and strictly increases in goods market tightness on the whole domain, shown graphically in Panel (b) of Figure 2.2. Hence, supply-side fiscal policy always crowds in private consumption and does so more strongly in a tighter goods market. Supply-side fiscal policy in the form of a payroll tax cut encourages more labor demand for a given wage, which in turn increases capacity and reduces goods market tightness, lowering the search wedge and encouraging higher consumption; the latter positive effect on consumption through capacity expansion is stronger whenever capacity is already low, relative to the number of visits. Intuitively, the payroll tax cut increases the size of the "store" and hence reduces its congestion, thus crowding in private consumption; moreover, such crowding in of private consumption is stronger whenever the store already is very congested.

The next corollary uses the results from Proposition 2.3 to establish the cyclicality properties of the supply-side fiscal multiplier in a fixprice equilibrium:

Corollary 2.4. *In a fixprice equilibrium, the supply-side fiscal multiplier, $\varphi^s(x)$, is procyclical under demand-side fluctuations and countercyclical under supply-side fluctuations.*

Proof. From Lemma 2.5, we know that in a fixprice equilibrium $\frac{dx}{d\chi} > 0, \frac{dx}{da} < 0$; further, from Proposition 2.3, we know that in a fixprice equilibrium $\frac{d\varphi^s(x)}{dx} > 0$. Hence, $\frac{d\varphi^s(x)}{d\chi} = \frac{d\varphi^s(x)}{dx}\frac{dx}{d\chi} > 0, \forall x \in (0, x_m)$ and $\frac{d\varphi^s(x)}{da} = \frac{d\varphi^s(x)}{dx}\frac{dx}{da} < 0, \forall x \in (0, x_m)$. □

Corollary 2.4 establishes that the supply-side multiplier, associated with payroll tax cuts, is high in *supply-driven recessions* and *demand-driven expansions*, but low in *demand-driven recessions* and *supply-driven expansions*. Intuitively, a supply-driven recession makes the size of the "store" smaller,

whereas a demand-driven expansion increases the "queue" length; in either case, the store becomes more congested. In such cases, a supply-side policy that generates an increase in capacity produces large fiscal multipliers by lowering the congestion of the "store" and strongly crowding in private consumption. On the other hand, in demand-driven recessions and supply-driven expansions the "store" becomes much less congested, so that any further increases in capacity generate only modest decreases in the cost of search, and hence a very modest crowding in of private consumption.[21]

2.3.4 Robustness of results

Robustness I: more general equilibrium types

In Appendix 2.B, we solve for demand-side and supply-side multipliers under much more general equilibrium types and establish their cyclical properties, which we summarize here.

First, we show that results obtained under a competitive equilibrium fully extend to a class of *flexible* equilibria, where tightness is fixed at an arbitrary level $x^L \in (0, x_m)$ and does not respond to shocks, instead letting fully flexible prices and wages (p^L, w^L) accommodate any disturbances. Such equilibria could be obtained, for example, when the price is established by Nash bargaining between firms and households, or when the price is set at a fixed mark-up over the marginal cost. We show that any such equilibrium will have both demand- and supply-side multipliers fixed at $\varphi^* = \frac{\alpha}{1+\psi}$ and acyclical.

Second, we show that the cyclicality properties established under a fix-price equilibrium extend to a more general class of *frictional* equilibria, where prices are partially rigid. Such equilibria could occur, for example, if the price is set at an intermediate level between a fixed parameter (p_0) and the price under a flexible equilibrium (p^L), so that $p = (p_0)^{\varepsilon}(p^L)^{1-\varepsilon}$ and $\varepsilon \in (0, 1]$ pins down the degree of price rigidity. More generally, we show that in any

[21] Landais et al. (2018) show that the effect of unemployment insurance on employment is weaker if recessions are driven by labor demand shocks. The finding can be interpreted as special case of the present result that supply-side multipliers are procyclical under demand-driven cycles.

frictional equilibrium, where the elasticity between p and p^L lies in $[0, 1)$, the demand- and supply-side multipliers still fall and rise in tightness, respectively, and hence preserve the cyclicality properties established under a fixprice equilibrium.

Robustness II: alternative fiscal policy instruments

In Appendix 2.C, we extend our analysis to alternative fiscal instruments, such as government employment, as well as distortionary taxation of households' consumption, labor income, and firms' sales. Here we briefly summarize the results.

First, we show that multipliers out of consumption tax cuts and government employment strictly fall in tightness and hence their cyclicality properties are identical to those of the government consumption multiplier established earlier. In any flexible equilibrium, both multipliers are still acyclical. In any frictional equilibrium, however, a cut in the rate of consumption tax encourages higher consumption and more visits by private households; the latter lengthens the "queue", making the shop more congested and thus raising the search cost, crowding out some of the initial increase in private consumption. Consumption tax cuts implemented under high goods market congestion are associated with stronger crowding out, and hence a lower value of the multiplier. As for government employment, whenever the government hires a fraction of the labor supply, it removes labor resources from the private sector, which shrinks the capacity of privately produced goods, making the goods market more congested and crowding out private consumption. Whenever the government employs labor under already very high tightness, congestion rises even more strongly and private consumption becomes more crowded out.

Second, we show that multipliers out of cuts in the rate of labor income tax and the rate of firms' sales tax are both identical to the multiplier out of a cut in the rate of firms' payroll tax. Hence both of them also strictly rise in goods market tightness and their cyclical properties also are just like those of the supply-side multiplier that we considered earlier. Indeed, in any flexible equilibrium, both multipliers are acyclical. In any frictional equilibrium,

however, a cut in the rate of labor income tax increases labor supply, *ceteris paribus*, whereas lowering the rate of the sales tax encourages more labor demand. Either of those policies leads to higher equilibrium employment and hence higher capacity. The latter lowers goods market tightness and crowds in private consumption, doing so more strongly whenever goods market tightness is more congested in the first place.

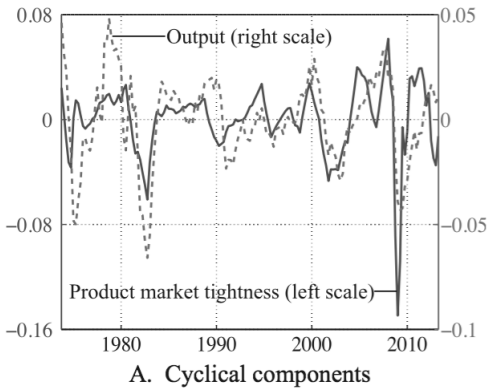
2.3.5 Evidence on the equilibrium type

Our analysis shows that the joint dynamics of price and tightness adjustment pin down the cyclical properties of fiscal multipliers. On one hand, whenever tightness is fixed over the business cycle, and prices and wages are fully flexible, fiscal multipliers are acyclical. On the other hand, whenever tightness varies over the business cycle and prices are rigid, fiscal multipliers are state-dependent with cyclicality determined by the source of economic fluctuations.

Figure 2.3 plots time series for (the cyclical component of) US goods market tightness, as constructed by Michaillat and Saez (2015). It is immediately apparent that goods market tightness varies significantly over time, suggesting that equilibria that feature cyclical variations in tightness provide a better description of the US economy at business cycle frequencies. The latter also implies that equilibria featuring state-dependent fiscal multipliers, with their cyclicality determined by the source of economic fluctuations, are more empirically relevant.

Figure 2.3 also shows a strong co-movement between the cyclical components of goods market tightness and sales, which, according to Lemma 2.5, reflects the dominance of demand shocks as the primary source of fluc-

Figure 2.3: US goods market tightness and sales (Michaillat and Saez, 2015)



tuations.[22,23] Combined with the cyclicality properties established in Corollary 2.3, the latter suggests that the demand-side multiplier is, *on average*, countercyclical. This finding is consistent with some empirical literature (Auerbach and Gorodnichenko, 2012, 2013, Fazzari et al., 2014), although Ramey and Zubairy (2018) estimate spending multipliers to be mildly countercyclical at best. The predominance of demand shocks, combined with cyclicality properties established in Corollary 2.4, suggest that the supply-side multiplier is, *on average*, procyclical. Such a finding is consistent with the econometric findings in Ziegenbein (2017) and Eskandari (2019), who estimate multipliers out of tax cuts to be much lower in recessions than in expansions.

The fact that spending and tax cut multipliers are, respectively, countercyclical and procyclical, *on average*, gives us no indication on their relative sizes in a *particular* recessionary or expansionary episode. Indeed, our analytical results show that depending on the *type of shock* that generates the episode in the first place, the magnitudes of both multipliers may be substantially different. In order to gain further understanding of the behavior

[22]Strictly speaking, Lemma 2.5 only applies whenever prices remain completely fixed over the business cycle. In Lemma 2.9 in Appendix 2.B.2 we show that in any equilibrium where prices are partially rigid, but not completely fixed, tightness and sales co-move under demand-driven fluctuations and counter-move under supply-driven fluctuations.

[23]Figure 2.I.5 in Appendix 2.I.1 reports estimated impulse responses of output, inflation and goods market tightness to an identified productivity shock based on Fernald (2014a). Consistently with our theory, a positive productivity shock leads to a statistically significant increase in output and a statistically significant drop in inflation and goods market tightness.

of the *relative* magnitudes of demand- and supply-side multipliers, in the next section, we provide further analytical results regarding the particular states of the world in which the size of multipliers is different. In Section 2.5 we develop and calibrate a quantitative dynamic version of our model, and use a non-linear solution method to evaluate both spending and taxation multipliers in shock-specific recessionary and expansionary episodes. Subsequently, in Section 2.6 we develop an econometric specification that allows to estimate spending and tax cut multipliers in recessionary and expansionary episodes, *conditional* on those being either demand- or supply-driven.

2.4 Fiscal multipliers: additional analytical results

We now provide further analytical results that describe how the *relative* size of demand- and supply-side multipliers in a fixprice equilibrium varies with goods market tightness. First, we establish that the demand-side multiplier is lower than the supply-side multiplier whenever goods market tightness is above the socially efficient level, and *vice versa*. Second, we show that for sufficiently low elasticities of labor supply and demand, there always exists a sufficiently high level of tightness that makes *spending austerity*, implemented by a reduction in government consumption, the policy with the largest multiplier.

2.4.1 Link between demand- and supply-side multipliers

One can combine Propositions 2.2 and 2.3 to conveniently link the demand- and supply-side multipliers in a fixprice equilibrium:

Corollary 2.5. *In a fixprice equilibrium, the demand-side and supply-side fiscal multipliers are related as:*

$$\underbrace{\varphi^d(x)}_{\text{Demand-side multiplier}} = \underbrace{\theta(x)}_{\text{Fixed capacity multiplier}} + \underbrace{\varphi^s(x)}_{\text{Supply-side multiplier}} . \tag{2.29}$$

Hence, the demand-side multiplier is lower whenever $x \in (x^*, x_m)$, higher whenever $x \in (0, x^*)$, and equal to the supply-side multiplier when $x = x^*$.

Corollary 2.5 establishes that in the special case of an efficient fixprice equilibrium, where $x = x^*$ and $\theta(x^*) = 0$, the size of demand- and supply-side multipliers is the same, $\varphi^d(x^*) = \varphi^s(x^*) = \varphi^*$, in accordance with Propositions 2.2 and 2.3. When $x \in (x^*, x_m)$ and hence $\theta(x) \in (-\infty, 0)$, the demand-side multiplier is smaller than the supply-side multiplier, $\varphi^d(x) < \varphi^s(x)$, since enlarging capacity by stimulating supply lowers tightness and search costs in the inefficiently congested goods market, thus crowding in private consumption and making supply-side policies more effective. The opposite result holds when $x \in (0, x^*)$ and $\theta(x) \in (0, 1)$, making the demand-side multiplier higher than the supply-side multiplier.

2.4.2 Austerity multipliers

So far, we have focused on policies that either increase government consumption or cut the rate of payroll tax. However, policymakers also have reverse options at their disposal, namely spending austerity, implemented as a reduction in government consumption, and also an increase in the tax rate.

In our framework, the multiplier from austerity implemented by a reduction in government consumption is the mirror image of the demand-side multiplier, and is equal to $-\varphi^d(x)$. Similarly, the multiplier from an increase in the rate of payroll tax is equal to $-\varphi^s(x)$. From Proposition 2.3, we know that $\varphi^s(x) \in (0, +\infty), \forall x \in (0, x_m)$, which implies that the multiplier from an increase in the rate of payroll tax is *negative* on the whole domain of tightness, and hence in no state of the world can it be the policy option with the highest multiplier. However, the latter is not the case when it comes to spending policies. We already know from Corollary 2.5 that whenever $x \in (x^*, x_m)$ the supply-side multiplier exceeds the demand-side multiplier; moreover we know from Corollary 2.2 that there exists $\hat{x} \geq x^*$ such that whenever $x \in (\hat{x}, x_m)$, the spending multiplier is in fact negative, implying that the austerity multiplier is positive. Yet, is there an admissible level of tightness such that the austerity multiplier is sufficiently positive to exceed the supply-side multiplier from tax cuts? The next corollary establishes that

the answer is yes, as long as the elasticities of labor demand ($|\epsilon^d|$) and labor supply (ϵ^s) are sufficiently low, as encapsulated by $\varphi^* = \frac{\alpha}{1+\psi} = \frac{1-\frac{1}{|\epsilon^d|}}{1+\frac{1}{\epsilon^s}}$.

Corollary 2.6. *For sufficiently low elasticities of labor demand and labor supply such that $\varphi^* < 0.5$, an **Austerity Threshold** $\tilde{x} \in [\hat{x}, x_m)$ exists such that:*

$$-\varphi^d(x) > \varphi^s(x) > \varphi^d(x), \quad \forall x \in (\tilde{x}, x_m). \tag{2.30}$$

Furthermore, $\tilde{x}$ is given by:

$$\tilde{x} = \theta^{-1}\left(-\frac{2\varphi^*}{1-2\varphi^*}\right), \quad \varphi^* < 0.5 \tag{2.31}$$

and hence $\frac{d\tilde{x}}{d\varphi^*} > 0$.

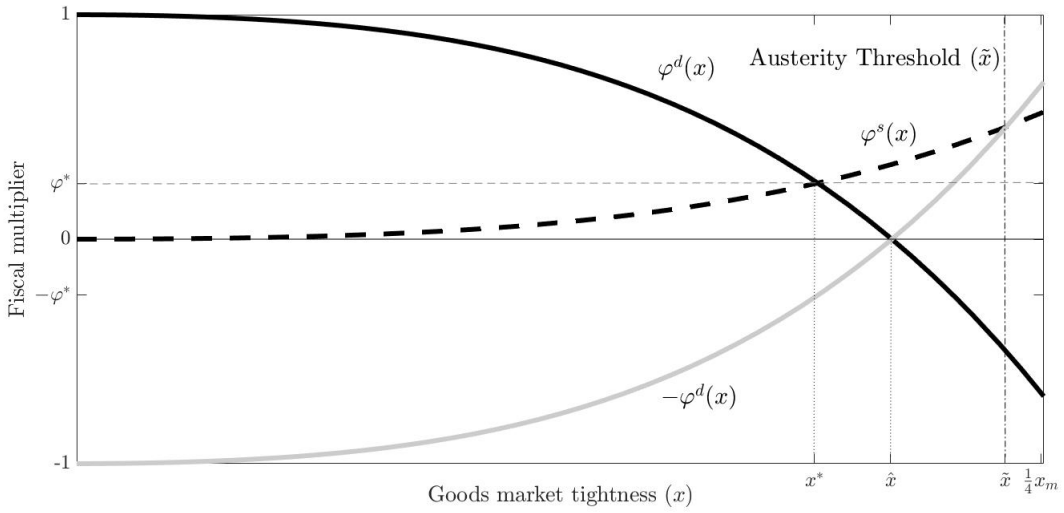
Panel (a) of Figure 2.4 compares the austerity multiplier against demand- and supply-side multipliers for an inelastic labor market ($\varphi^* < 0.5$). It shows that the multiplier associated with government consumption austerity exceeds the supply-side multiplier (dashed line) *and* demand side multiplier (dark-solid line), provided that tightness is larger than $\tilde{x}$.[24] Panel (b) reports that case for a flexible labor market ($\varphi^* > 0.5$), showing that if the labor market is sufficiently flexible, the supply-side multiplier is always lager than the austerity multiplier, and the Austerity Threshold does not exist.

In intuitive terms, Corollary 2.6 states that the store can be so congested, that decreasing tightness by removing government customers from the queue could be more effective at crowding in private consumption than enlarging the store by tax cuts. In particular, this could only be the case when the elasticities of labor demand and labor supply are sufficiently low, so that tax cuts that encourage more labor demand are not effective at increasing equilibrium employment and capacity. Based on our results about cyclical fluctuations in tightness in Lemma 2.5, the Austerity Threshold could be reached in an economy with sufficiently inelastic labor markets, which is hit by either a very strong positive demand-side shock or following a severe negative supply-side shock.

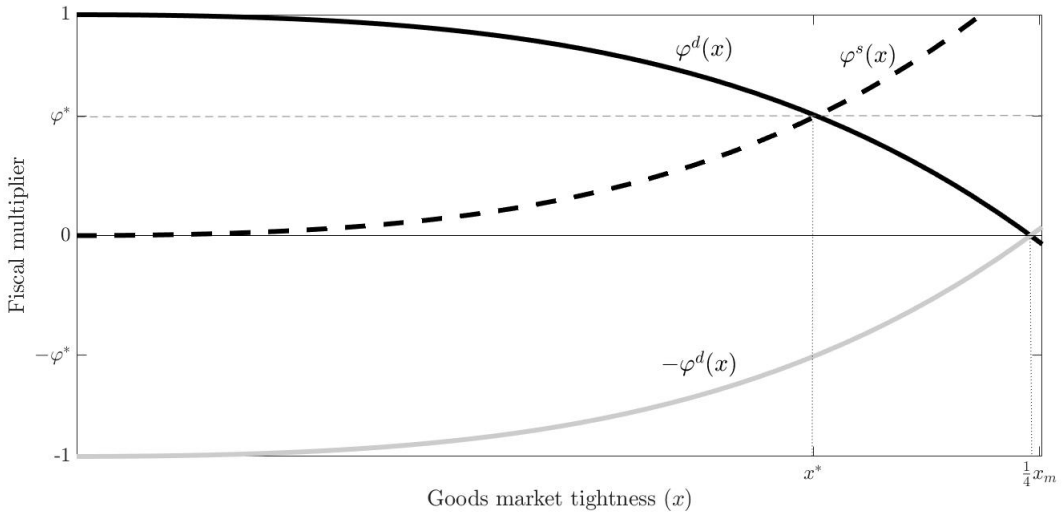
[24]Note that $\tilde{x} \geq \hat{x}$. Reaching the Austerity Threshold requires tightness to be larger than the threshold that makes the government spending multiplier negative, defined in Corollary 2.2.

Figure 2.4: Comparing fiscal multipliers in a fixprice equilibrium

(a) Inelastic labor market ($\varphi^* < 0.5$)



(b) Elastic labor market ($\varphi^* > 0.5$)



***Notes**: Panels (a) and (b) show demand-side, supply-side and spending austerity multipliers in a fixprice equilibrium of a calibrated version of our model ($\delta = 2, \rho = 0.1, \psi = 0.2$); in Panel (a), we set $\alpha = 0.3$, so that the elasticity of labor demand is relatively low and $\varphi^* = 0.25 < 0.5$ – in this case one can see that Austerity Threshold $\tilde{x}$ exists, and for all $x \in (\tilde{x}, x_m)$ spending-austerity is the policy with the highest multiplier; in Panel (b), we set $\alpha = 0.65$ so that labor demand is relatively elastic and $\varphi^* = 0.57 > 0.5$ – in this case, Austerity Threshold does not exist and spending austerity is never the policy with the highest multiplier.*

Recent studies by Alesina et al. (2015) provide evidence that austerity programs based on spending reductions are more powerful than programs based on tax increases when it comes to stimulating GDP during recessions. Our results provide a theoretical underpinning for such findings, suggesting that differences between spending- and tax-based austerity programs are especially pronounced in countries with inflexible labor markets and under high goods-market tightness, arising in supply-side side recessions, or in demand-side expansions.

2.5 Fiscal multipliers in a quantitative dynamic model

In this section, we develop and calibrate a discrete-time dynamic version of our model and use a non-linear solution method in order to quantitatively assess cyclical properties of both spending and taxation multipliers, conditional on different sources of fluctuations.[25] Relative to our static model, we make the additional assumption of *long-term customer relationships* between households and firms, which is empirically relevant and it allows us to map our matching process to realistic goods market frictions.[26] The dynamic model corroborates the finding in our static model and shows substantial state dependence *conditional* on a particular type of shock that drives the business cycle, especially for impact multipliers.

2.5.1 Goods market with long-term customer relationships

Sales materialize through *long-term customer relationships* between firms and consumers from private and government sectors that are subject to an exogenous destruction rate η per period. The total number of long-term customer relationships at the end of period t is y_t. At the beginning of each period

[25] In this section we limit our attention to multipliers out of government consumption and payroll tax cuts. In Appendix 2.H.1, we study cyclical properties of multipliers out of cuts in taxes on consumption and labor supply in our dynamic model.

[26] Michaillat and Saez (2015) offer cross-country evidence that long-term customer relationships are prevalent in goods markets; for example, they report that in the US around 77 per cent of sales go to long-term customers. Moreover, Gourio and Rudanko (2014) show theoretically how such desire to accumulate long-term customers can be microfounded.

t, firms inherit $(1-\eta)y_{t-1}$ relationships that have survived destruction in the previous period $t-1$, and they hire labor n_t to yield current capacity $k_t = a_t n_t^{\alpha}$, which they utilize through the relationships carried over from last period, leaving $[k_t - (1-\eta)y_{t-1}]$ as unutilzed capacity. Households and the government make v_t visits to form new relationships that fill the unutilzed capacity. However, not every purchasing visit is successful. The number of new customer relationships formed in each period is tracked by the matching function:

$$\left[(k_t - (1-\eta)y_{t-1})^{-\delta} + v_t^{-\delta}\right]^{-\frac{1}{\delta}}, \tag{2.32}$$

where $\delta > 0$ ensures that not every unit of unitilized capacity is filled and not every visit is successful. Goods market tightness is defined as: $x_t \equiv \frac{v_t}{k_t-(1-\eta)y_{t-1}}$, and the probability of filling a unit of unutilized capacity is given by $f(x_t) \equiv (1+x_t^{-\delta})^{-\frac{1}{\delta}}$, $f' > 0$, whereas the probability of a given visit yielding a new relationship is given by $q(x_t) \equiv (1+x_t^{\delta})^{-\frac{1}{\delta}}$, $q' < 0$.

2.5.2 Households

As in the static version of our model, households face a cost $\rho \in (0,1)$ of consumption goods per visit. They form long-run customer relationships both to consume and purchase goods that go towards satisfying the total cost of visits. At the beginning of period t, households have $(1-\eta)y_{t-1}^c$ relationships that survived from the previous period, and the number of new relationships formed in period t is: $y_t^c - (1-\eta)y_{t-1}^c$. Since every visit is only successful with probability $q(x_t)$, the total number of visits required to form new relationships in period t is $(y_t^c - (1-\eta)y_{t-1}^c)/q(x_t)$, yielding the following expression for y_t^c:

$$y_t^c = c_t + \rho\left[\frac{y_t^c - (1-\eta)y_{t-1}^c}{q(x_t)}\right], \tag{2.33}$$

which can be rearranged to obtain a more familiar-looking expression for y_t^c:

$$y_t^c = [1+\gamma(x_t)]c_t - (1-\eta)\gamma(x_t)y_{t-1}^c, \tag{2.34}$$

where, as before, $\gamma(x_t) \equiv \frac{\rho x_t}{f(x_t)-\rho x}$ is the wedge introduced by search-and-matching frictions.

There is a continuum of identical infinitely lived households. Markets are assumed to be complete, so a full set of Arrow-Debreu securities is available. The representative household is small relative to the size of the market, and maximizes expected discounted lifetime utility, taking prices, wages and goods market tightness as given:

$$\max_{\{c_{t+s},y^c_{t+s},m_{t+s},B_{t+s+1},l_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\chi_{t+s} \frac{c_{t+s}^{1-\sigma}}{1-\sigma} + \zeta(m_{t+s}) - \nu \frac{l_{t+s}^{1+\psi}}{1+\psi} \right]$$

subject to the budget constraint and the equation of motion of their customer relationships:

$$p_t y_t^c + m_t + \mathbb{E}_t \left[F_{t,t+1} B_{t+1} \right] \leq w_t l_t + \bar{m}_t + B_t + \Pi_t - T_t, \qquad \forall t \geq 0 \tag{2.35}$$

$$y_t^c = [1 + \gamma(x_t)] c_t - (1-\eta)\gamma(x_t) y_{t-1}^c, \qquad \forall t \geq 0 \tag{2.36}$$

where $\ln \chi_t = \rho_\chi \ln \chi_{t-1} + \varepsilon_t^\chi, \varepsilon_t^\chi \sim^{iid} (0, \sigma_\chi^2)$, is an exogenous process for the relative preference for consumption, β is the utility discount factor, ν captures relative labor disutility, and the rest of the notation carries over from the static version of the model. The exogenous supply of the non-produced good is assumed to be constant over time ($\bar{m}_t = \bar{m}, \forall t \geq 0$) and as before, we normalize it so that $\zeta'(\bar{m}) = 1$.

The optimization problem yields the following first-order conditions for the intertemporal choice of consumption:

$$\underbrace{\chi_t c_t^{-\sigma} + \beta(1-\eta)\mathbb{E}_t \left[\chi_{t+1} c_{t+1}^{-\sigma} \frac{[1+\gamma(x_t)]}{[1+\gamma(x_{t+1})]} \gamma(x_{t+1}) \right]}_{\text{Expected marginal benefit}} = \underbrace{p_t [1+\gamma(x_t)]}_{\text{Marginal cost}}, \tag{2.37}$$

and for the labor supply and the stochastic discount factor:

$$l_t = [w_t/\nu]^{\frac{1}{\psi}}, \qquad F_{t,t+s} = \beta^s. \tag{2.38}$$

Note that for $\eta = 1$, the consumption function in (2.37) nests equation (2.10) in the static model, with the comparative statics intuition preserved. However, when $\eta \in (0,1)$, the marginal utility from consuming an extra unit of the good consists of both the contemporaneous component ($\chi_t c_t^{-\sigma}$) as well

as a forward-looking component, which stems from the fact that a fraction $(1-\eta)$ of relationships in period t will be preserved in period $t+1$, leading to further consumption. The intuition behind the labor supply function in (2.38) remains unchanged from the static case.

2.5.3 Firms

There is a continuum of identical perfectly competitive producing a homogenous good. At the beginning of each period t, firms have $(1-\eta)y_{t-1}$ customer relationships that have survived from the previous period $t-1$. Firms hire labor n_t that yields current capacity $k_t = a_t n_t^{\alpha}$, leaving $[a_t n_t^{\alpha} - (1-\eta)y_{t-1}]$, a fraction $f(x_t)$ of which is then utilized. The latter gives the following equation of motion for firms' sales:

$$y_t = (1-\eta)y_{t-1} + f(x_t)\left[a_t n_t^{\alpha} - (1-\eta)y_{t-1}\right], \qquad \forall t \geq 0 \tag{2.39}$$

where $\ln a_t = \rho_a \ln a_{t-1} + \varepsilon_t^a$, and $\varepsilon_t^a \sim^{iid} (0, \sigma_a^2)$, is an exogenous process for productivity.

The representative firm is small relative to the size of the market, and therefore maximizes its lifetime discounted profits taking prices, wages, and tightness as given:

$$\max_{\{y_{t+s}, n_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} F_{t,t+s}\left[p_{t+s} y_{t+s} - w_{t+s} n_{t+s}(1+\tau_{t+s})\right]$$

subject to equation (2.39). The optimization problem yields the following optimality condition:

$$\underbrace{p_t + (1-\eta)\mathbb{E}_t\left[F_{t,t+1}\frac{w_{t+1}(1+\tau_{t+1})}{\alpha f(x_{t+1}) a_{t+1} n_{t+1}^{\alpha-1}}[1-f(x_{t+1})]\right]}_{\text{Expected marginal benefit}} = \underbrace{\frac{w_t(1+\tau_t)}{\alpha f(x_t) a_t n_t^{\alpha-1}}}_{\text{Marginal cost}}. \tag{2.40}$$

Note that under $\eta = 1$, the above expression collapses to the labor demand function in the static problem given in equation (2.15). However, under $\eta \in (0,1)$ the marginal benefit from selling an extra unit in period t consists both of the contemporaneous marginal increase in revenue (p_t) as well as the (expected) marginal increases in future revenue that come from retained customer relationships.

2.5.4 Government

In the baseline version of our model, government consumption is isomorphic to private consumption. Given a sequence of government spending $\{G_t\}_{t=0}^{\infty}$, the government's customer relationships evolve according to:

$$y_t^G = G_t + \rho \left[\frac{y_t^G - (1-\eta) y_{t-1}^G}{q(x_t)} \right], \tag{2.41}$$

or alternatively

$$y_t^G = [1 + \gamma(x_t)] G_t - (1-\eta)\gamma(x_t) y_{t-1}^G. \tag{2.42}$$

The government levies a lump-sum tax T_t on households to run balanced budgets every period, given a sequence of payroll tax rates $\{\tau_t\}_{t=0}^{\infty}$:

$$T_t = p_t y_t^G - w_t n_t \tau_t. \tag{2.43}$$

In the baseline version of our model, we assume exogenous autoregressive paths for government spending and the payroll tax rates:

$$G_t = (1-\rho_G) g + \rho_G G_{t-1} + \varepsilon_t^G, \qquad \forall t \geq 0 \tag{2.44}$$

$$\tau_t = (1-\rho_\tau)\tau + \rho_\tau \tau_{t-1} + \varepsilon_t^\tau, \qquad \forall t \geq 0, \tag{2.45}$$

where $\{\varepsilon^G\}_t \sim^{iid} (0, \sigma_G^2)$ and $\{\varepsilon^\tau\}_t \sim^{iid} (0, \sigma_\tau^2)$ are exogenous government spending and payroll taxation shocks, respectively.

2.5.5 Market clearing

The equilibrium of the system is described by optimality conditions (37), (38), (40), the equations of motion for customer relationships in (34), (39) and (42) as well as market clearing conditions in the goods market:

$$y_t = y_t^c + y_t^G, \qquad \forall t \geq 0 \tag{2.46}$$

in the labor market:

$$l_t = n_t, \qquad \forall t \geq 0, \tag{2.47}$$

and the market for the non-produced good:

$$m_t = \bar{m}, \qquad \forall t \geq 0. \tag{2.48}$$

With the market clearing in asset market ($B_t = 0$) omitted by Walras Law, they give ten equations in eleven endogenous variables $\{y_t, y_t^c, y_t^G, l_t, n_t, c_t, m_t, x_t, p_t, w_t, F_{t,t+1}\}_{t=0}^{\infty}$. Just like in the static model, this indeterminacy is intrinsic in any model with search-and-matching frictions. The next subsection outlines our strategy for closing the model.

2.5.6 Closing the model: pricing rule

Recall that in the main text we introduced broad classes of flexible and frictional equilibria, which differ in the way the model is closed. We are going to follow a similar approach in our dynamic model. As an example of a flexible equilibrium, one could consider our equilibrium conditions, augmented by a sequence of prices $\{\tilde{p}_t\}_{t=0}^{\infty}$ that would ensure that resulting equilibrium tightness is at the same level that would be chosen by a social planner.[27] Hence, closing our model with the pricing equation $p_t = \tilde{p}_t, \forall t \geq 0$ gives an example of a flexible equilibrium.

As an example of a frictional equilibrium, one could close the model with a pricing equation that describes persistent adjustment to the price $\tilde{p}_t$:

$$p_t = p_{t-1}^{\varepsilon} \tilde{p}_t^{1-\varepsilon}, \qquad \forall t \geq 0 \tag{2.49}$$

where $\varepsilon \in (0, 1]$ pins down the degree of price rigidity. Given the evidence in Subsection 2.3.5 that equilibria featuring price rigidity are a better description of the US economy at business cycle frequencies, we assume a degree of price rigidity instead of fully flexible prices.

2.5.7 Calibration

We calibrate the model on US data at annual frequencies. Most of the calibration is standard. We set $\beta = 0.96$ (to produce a real interest rate of 4 per cent in steady state), $\sigma = 1.00$ (log utility of consumption, based on mean estimates in Chetty, 2006), $\psi = 0.50$ (Frisch elasticity of labor supply equal to 2) and $\alpha = 0.60$ (labor share of income equal to 0.6). The labor disutility parameter $\nu = 2.33$ is set to target a steady-state employment rate of $1/3$ ($l = 1/3$).

[27] Appendix 2.G.2 provides the solution to the social planner's problem in the dynamic model.

Table 2.1: Parameter Calibrations (United States, annual frequency)

Parameter	Description	Value	Source/Target
Household parameters			
β	Time discount factor	0.96	Annual real rate of 4 per cent
σ	Relative risk aversion	1.00	Chetty (2006)
ψ	Inverse Frisch elasticity of labor supply	0.50	Standard
ν	Disutility of supplying labor	2.13	Target $l = 1/3$
Firm parameters			
α	Returns to labor	0.60	Standard
ε	Price rigidity	0.70	Standard
Goods market parameters			
η	Rate of destruction of customer relationships	0.40	Mattersion (2001)
ρ	Goods cost per visit	0.41	Target $q(x) = 0.77$
δ	Elasticity parameter of the matching function	3.62	Target $\left[\frac{f(x)}{\eta+f(x)(1-\eta)}\right]^{\frac{1}{\alpha}} = 0.91$
Fiscal policy parameters			
g	Steady state government spending	0.07	Target $g/(c+g)$ = 0.18
τ	Steady state firms' payroll tax rate	0.20	—
Exogenous processes parameters			
$\rho_\chi = \rho_a = \rho_G = \rho_\tau$	Persistence of exogenous processes	0.90	Standard

The degree of price rigidity is calibrated at $\varepsilon = 0.70$. We set $\eta = 0.40$, which yields the rate of destruction of customer relationships equal to 40 per cent per year (based on US customer attrition evidence in Mattersion, 2001).

The cost per visit ρ and elasticity of marching function δ are non-standard parameters. We calibrate them by targeting the steady-state rate of current labor utilization $\left[\frac{f(x)}{\eta+f(x)(1-\eta)}\right]^{\frac{1}{\alpha}}$ (estimated at 0.91 as the long-run average of labor utilization rate, reported by the Institute for Supply Management)[28] and the steady-steady probability of a successful shopping visit $q(x)$ (estimated at 0.77 as one minus the average stock-out rate, reported by Taylor and Fawcett (2001) and Jing and Lewis (2011).

Steady state of government spending parameter g is set to match the spending-to-GDP ratio equal to 18% in steady state. The steady-state payroll tax rate τ is calibrated to be equal to 0.20. Finally, all autoregressive parameters are set equal at 0.9, so that $\rho_\chi = \rho_a = \rho_G = \rho_\tau = 0.9$. Table 2.1 summarizes the calibration.

2.5.8 *Conditional* state-dependent fiscal multipliers

To quantitatively assess the degree of state-dependence of fiscal multipliers and its variation with the source of business cycle fluctuations, we compute

[28] From the equation of motion of firms' sales in (39) it follows that in steady state $y = \frac{f(x)}{\eta+f(x)(1-\eta)} a n^\alpha = a\left\{\left[\frac{f(x)}{\eta+f(x)(1-\eta)}\right]^{\frac{1}{\alpha}} n\right\}^\alpha$, so that $\left[\frac{f(x)}{\eta+f(x)(1-\eta)}\right]^{\frac{1}{\alpha}}$ is the steady-state labor utilization rate.

spending and tax cut multipliers in recessionary and expansionary episodes, conditioning on whether a particular episode was generated by a demand or supply shock. We consider a fully non-linear solution to our model under perfect foresight.

We first compute the impulse response of GDP to a one-time preference/technology shock, $\{GDP_j^{shock}\}_{j=0}^{H}$, where $shock \in \{\varepsilon^{\chi}, \varepsilon^{a}\}$; we then compute the impulse response of GDP subject to the same shock, combined with either a spending shock $\{\varepsilon^{G} > 0\}$ or a tax cut shock $\{-\varepsilon^{\tau} < 0\}$, to obtain time series that embed the interaction between the fundamental shock driving the business cycle and the shock related to the expansion in government spending, $\{GDP_j^{shock+\varepsilon^{G}}\}_{j=0}^{H}$, or reduction in taxes, $\{GDP_j^{shock-\varepsilon^{\tau}}\}_{j=0}^{H}$. We construct the *conditional* government spending multiplier as follows:

$$\varphi^{G}(shock) = \frac{\sum_{j=0}^{H}\left[GDP_j^{shock+\varepsilon^{G}} - GDP_j^{shock}\right]}{\sum_{j=0}^{H}\left[G_j^{\varepsilon^{G}} - g\right]}, \tag{2.50}$$

where $\left\{G_j^{\varepsilon^{G}}\right\}_{j=0}^{H}$ is the impulse response of government spending to the spending shock $\{\varepsilon^{G} > 0\}$ and H is the horizon of the impulse response, so that $\sum_{j=0}^{H}\left[G_j^{\varepsilon^{G}} - g\right]$ denotes a cumulative increase in government spending compared to its steady-state value.

Similarly, we construct horizon-H *conditional* tax cut multipliers out of cutting the rate of tax τ_t as:

$$\varphi^{\tau}(shock) = \frac{\left[GDP_H^{shock-\varepsilon^{\tau}} - GDP_H^{shock}\right]/\overline{GDP}}{\varepsilon^{\tau}} \tag{2.51}$$

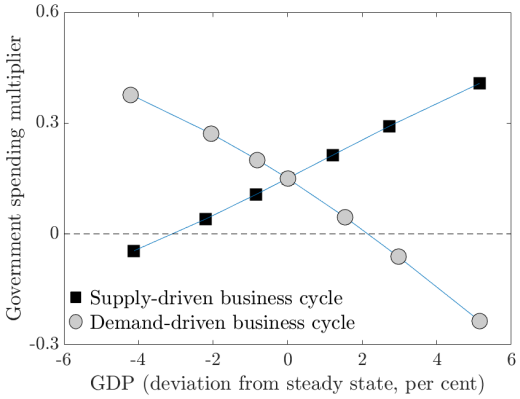
where $\overline{GDP}$ is steady-state level of GDP.

We calibrate $\varepsilon^{G} = 0.01\overline{GDP}$ to consider a one-period spending shock equal to 1 per cent of steady-state GDP; further, we set $\varepsilon^{\tau} = 0.01$, so that we consider a 1 percentage point cut in the rate of the payroll tax.

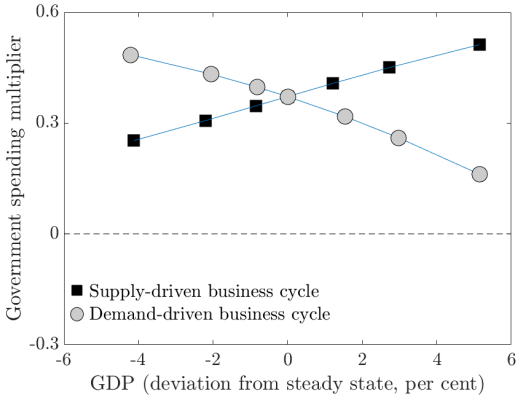
To investigate the link between state dependence and the horizon of the response, we distinguish between the *impact* multipliers $(H = 0)$, and *cumulative* 2-year $(H = 2)$ and 4-year $(H = 4)$ multipliers, following the convention in the empirical literature that considers a two- and four-year horizons (Ramey and Zubairy, 2018).

Figure 2.5: Conditional state-dependent fiscal multipliers

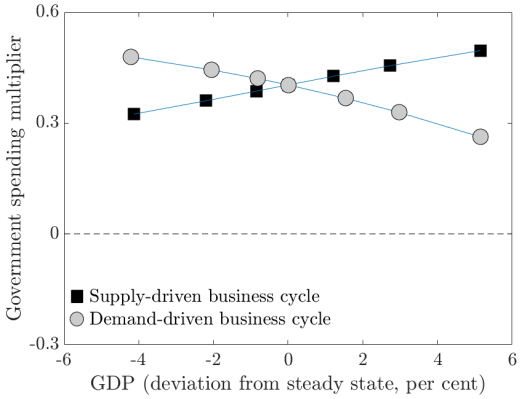
(a) Impact multiplier: spend 1% of $\overline{GDP}$



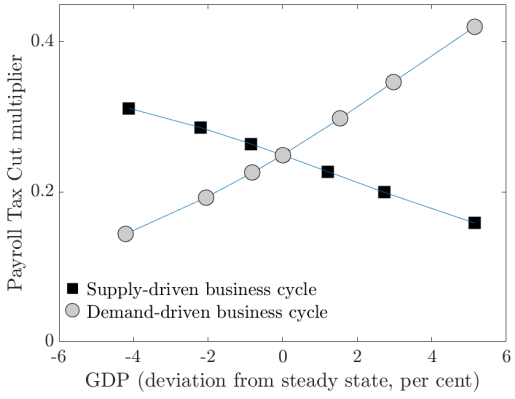
(b) Cumulative 2-year multiplier: spend 1% of $\overline{GDP}$



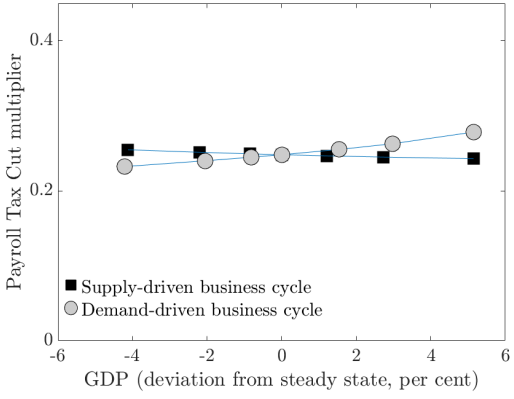
(c) Cumulative 4-year multiplier: spend 1% of $\overline{GDP}$



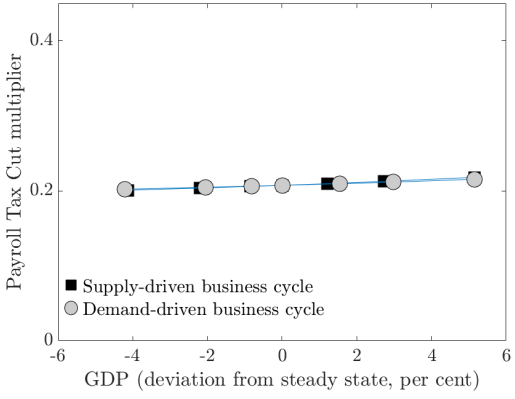
(d) Impact multiplier: cut payroll tax by 1%



(e) 2-year horizon multiplier: cut payroll tax by 1%



(f) 4-year horizon multiplier: cut payroll tax by 1%



***Notes**: Panel (a) shows impact multipliers following a one-time innovation to the government spending process equal to 1% of steady-state GDP, in recessionary and expansionary episodes caused by different types of shocks; Panels (b) and (c) repeat the exercise for cumulative multipliers, computed over a 2-year and 4-year horizon, respectively.*
Panel (d) shows impact multipliers following a one-time innovation to the payroll tax rate process equal to negative 1 percentage point, in recessionary and expansionary episodes caused by different types of shocks; Panels (e) and (f) repeat the exercise for the 2-year and 4-year horizon tax cut multipliers, respectively.

Figure 2.5 shows our constructed *conditional* government spending and payroll tax cut multipliers. Panel (a) plots the impact of the government spending multiplier, which is equal to around 0.15 when GDP is at the steady-state value. A strong demand-driven recession that takes GDP 4 per cent below the steady state, raises the spending multiplier to 0.40. On the other hand, a demand-driven expansion that raises output 4 per cent above the steady state, decreases the multiplier to -0.2. Under supply-driven fluctuations, the cyclicality of spending multipliers is reversed. In a supply-driven recession, where GDP drops 4 per cent below the steady state, the spending multiplier drops to around -0.05, and in a 4 per cent supply-driven expansion the multiplier increases to around 0.30. From Panels (b) and (c), the above properties are preserved for cumulative spending multipliers, although the degree of state dependence is weaker. This is because price rigidity is crucial for state dependence in our model, and over the five-year horizon a higher fraction of firms gets to adjust prices.

Panel (d) of Figure 2.5 shows that the impact multiplier out of payroll tax cuts is close to 0.25 in steady state, but almost doubles in size in a 4 per cent demand-driven expansion, and increases to 0.30 in a 2 per cent supply-side recession. However, it drops to almost 0.15 in a 4 per cent demand-side recession and a 4 per cent supply-side expansion. As before, Panel (e) and (f) show that the two- and four-year horizon multipliers out of payroll tax cuts preserve the properties of their impact counterparts, although state dependence is significantly muted.

2.5.9 Implications for the empirical debate on fiscal state dependence

Results in Figure 2.5 offer an interpretation to the recent empirical debate on the degree of fiscal state dependence. In particular, studies such Auerbach and Gorodnichenko (2012, 2013) and Fazzari et al. (2014) point to much larger spending multipliers in recessions compared to expansions, whereas Ramey and Zubairy (2018) construct a longer historical dataset and estimate the degree of state dependence to be much more modest. All of the

aforementioned studies distinguish between recessionary and expansionary episodes with unconditional thresholds, such as unemployment, GDP growth, or the output gap. However, as shown in Panels (a)-(c) of Figure 6, a given level of GDP could be consistent with both low and high levels of the government spending multiplier, depending on the *type of shock* that caused that level of GDP in the first place. As a result, econometric techniques that distinguish between recessions and expansions with unconditional thresholds can estimate spending multipliers to be either countercyclical or procyclical, depending on whether the estimation sample covers periods with predominantly demand-driven fluctuations or supply-driven fluctuations.

In the next section, we develop and estimate an econometric model that allows us to evaluate *conditional* state-dependent fiscal multipliers, where one explicitly accounts for the source of economic fluctuations.

2.6 Econometric evidence

This section develops and estimates a novel econometric model that allows to perform reduced-form estimation of *conditional* state-dependent fiscal multipliers, controlling for either the demand- or supply-driven nature of a given recessionary or expansionary state. We find strong empirical support to our theoretical predictions: the estimated spending multipliers in demand-side recessions are substantially higher than those in supply-side recessions, particularly at shorter horizons. At the same time we find tax cut multipliers to be significantly higher in supply-side recessions compared to the demand-side ones.

2.6.1 Conditional state-dependent fiscal multipliers

Our theory establishes that fiscal multipliers are state-dependent and cyclicality is determined by the source of economic fluctuations. Conditional on demand-driven fluctuations, demand-side multipliers are countercyclical and supply-side multipliers are procyclical. Conditional on supply-driven fluctuations, the exact opposite hold.

We estimate two econometric models using the local projection methodology in Jordà (2005b) and evaluate spending and tax cut multipliers in recessionary and expansionary episodes, *conditional* on those being demand- or supply-driven in nature. Our approach identifies demand- and supply-driven fluctuations using observed co-movement between cyclical components of economic activity and inflation. In accordance with the insights of a wide range of models, a demand-side recession is characterized by a joint fall in economic activity and inflation while a supply-driven recession is characterized by a fall in economic activity and a rise in inflation.[29] This study is the first one to estimate state-dependent fiscal multipliers controlling for the source of fluctuations.[30]

Conditional state-dependent spending multipliers

We extend the one-step IV procedure from Ramey and Zubairy (2018) to account for the source of economic fluctuations. Instead of distinguishing between expansions and recessions using an unconditional unemployment threshold $\bar{U}$, we split recessionary states, where $U_t \geq \bar{U}$, into those where inflation is below its trend value, $\pi_t < \bar{\pi}_t$, corresponding to demand-side recessions, and those where inflation is above trend, $\pi_t \geq \bar{\pi}_t$, corresponding to supply-side recessions. We could additionally split expansionary states, $U_t < \bar{U}$, into those where inflation is below its trend value, $\pi_t < \bar{\pi}_t$, corresponding to supply-side expansions, and those where inflation is above trend, $\pi_t \geq \bar{\pi}_t$, corresponding to demand-side expansions; we perform this exercise in Appendix 2.I.

[29] Recent work by Guerrieri et al. (2020) introduces the notion of "Keynesian supply shocks", which can lead to demand-deficiency, and hence create a positive co-movement between inflation and activity, unlike traditional supply shocks. Though such shocks are not a feature of our model, the demand-deficiency they create is akin to a state with low goods market tightness. In this sense, even if Keynesian supply shocks are present in the data, our strategy of using co-movement between inflation and activity still allows to identify periods with high and low goods market tightness, and test our theory.

[30] Ramey and Zubairy (2018) and Ziegenbein (2017) distinguish between recessions and expansions with an unconditional unemployment threshold whereas Auerbach and Gorodnichenko (2013) employ smooth state transitions based on the unconditional rate of economic growth.

Our baseline specification to estimate cumulative spending multipliers at horizon H is:

$$\begin{aligned}\sum_{s=t}^{t+H}\left(\frac{GDP}{GDP^*}\right)_s =& \mathbf{1}\{U_{t-1}<\bar{U}\}\left[\alpha_H^E+\beta_H^E\sum_{s=t}^{t+H}\left(\frac{G}{GDP^*}\right)_s+\gamma_H^E\mathbf{z}_{t-1}\right]+\\ &\mathbf{1}\{U_{t-1}\geq\bar{U};\pi_{t-1}<\bar{\pi}_{t-1}\}\left[\alpha_H^{DR}+\beta_H^{DR}\sum_{s=t}^{t+H}\left(\frac{G}{GDP^*}\right)_s+\gamma_H^{DR}\mathbf{z}_{t-1}\right]+\\ &\mathbf{1}\{U_{t-1}\geq\bar{U};\pi_{t-1}\geq\bar{\pi}_{t-1}\}\left[\alpha_H^{SR}+\beta_H^{SR}\sum_{s=t}^{t+H}\left(\frac{G}{GDP^*}\right)_s+\gamma_H^{SR}\mathbf{z}_{t-1}\right]+\varepsilon_{t+H},\end{aligned} \tag{2.52}$$

where $\sum_{s=t}^{t+H}\left(\frac{GDP}{GDP^*}\right)_s$ and $\sum_{s=t}^{t+H}\left(\frac{G}{GDP^*}\right)_s$ are cumulative real GDP and real government expenditures, both normalized by trend real GDP (GDP^*), [31] $\mathbf{z}_{t-1}$ is a vector of controls, [32] and $\mathbf{1}\{.\}$ is the indicator variable. The above equation is estimated by 2SLS, where the instrument set includes exogenous government spending shocks, such as narrative military spending news shocks from Ramey and Zubairy (2018) or VAR-based shocks from Blanchard and Perotti (2002) , interacted with the state-specific indicator variable.

An advantage of this approach is that our estimates for β_H^E, β_H^{DR}, and β_H^{SR} directly give us values for horizon-H cumulative spending multipliers in, respectively, expansions (E), demand-side recessions (DR), and supply-side recessions (SR). Our theory predicts that spending multipliers in demand-side recessions are higher that those in supply-side recessions, so that $\beta_H^{DR} > \beta_H^{SR}$, and we can test this prediction.

Conditional state-dependent tax cut multipliers

In a similar fashion, we use local projections to estimate *conditional* state dependence for tax cut multipliers. We extend the approach in Eskandari

[31] Following Ramey and Zubairy (2018), this normalization is to ensure cumulative GDP and government spending are measured in the same units, which avoids the need to covert estimates in logs to levels.

[32] The precise set of variables used as controls is outlined in description to the relevant regression tables.

(2019) that distinguishes between recessionary and expansionary episodes using an unconditional unemployment threshold by further differentiating between demand-side and supply-side recessions. One also can split expansionary states, $U_t < \bar{U}$, into those where inflation is below its trend value, $\pi_t < \bar{\pi}_t$, representing supply-side expansions, and those where inflation is above trend, $\pi_t \geq \bar{\pi}_t$, representing demand-side expansions; we perform this exercise in Appendix 2.I.

Our baseline specification to estimate tax cut multipliers at horizon H is given by:

$$\begin{aligned}\ln GDP_{t+H} - \ln GDP_{t-1} =& \mathbf{1}\{U_{t-1} < \bar{U}\}\left[\alpha_H^E + \beta_H^E \tau_t + \gamma_H^E \mathbf{z}_{t-1}\right] + \\ & \mathbf{1}\{U_{t-1} \geq \bar{U}; \pi_{t-1} < \bar{\pi}_{t-1}\}\left[\alpha_H^{DR} + \beta_H^{DR} \tau_t + \gamma_H^{DR} \mathbf{z}_{t-1}\right] + \\ & \mathbf{1}\{U_{t-1} \geq \bar{U}; \pi_{t-1} \geq \bar{\pi}_{t-1}\}\left[\alpha_H^{SR} + \beta_H^{SR} \tau_t + \gamma_H^{SR} \mathbf{z}_{t-1}\right] + \varepsilon_{t+H},\end{aligned} \tag{2.53}$$

where τ_t is an exogenous shock to the average tax rate in the economy, and the rest of the notation carries over from the spending multiplier regressions. Given a time series for exogenous tax rate shocks, such as narrative tax shocks from Romer and Romer (2010), the above specification is estimated by OLS.

The estimates for β_H^E, β_H^{DR}, and β_H^{SR} directly provide values for horizon-H tax cut multipliers in, respectively, expansions (E), demand-side recessions (DR), and supply-side recessions (SR). Our theory predicts that tax cut multipliers in demand-side recessions are lower than those in supply-side recessions, so that $\beta_H^{DR} < \beta_H^{SR}$, and we test the prediction.

2.6.2 Data

We estimate the model using quarterly US data. We use the series for real GDP (GDP), civilian unemployment (U), and government consumption and fixed capital formation (G) data that extend back to 1889 by Ramey and Zubairy (2018). Trend GDP (GDP^*) is measured as sixth-order polynomial exponential trend of real GDP, following Gordon and Krenn (2010).

We measure quarterly inflation (π_t) as year-on-year change in (log) GDP deflator, and trend inflation $(\bar{\pi}_t)$ is obtained by HP-filtering the raw inflation series with a smoothing parameter $\lambda = 1600$ for quarterly data. The baseline unemployment threshold is set at $\bar{U} = 6.5\%$, consistent with Ramey and Zubairy (2018).

Our baseline measure of the government spending shock is the narrative military spending news shocks in Ramey and Zubairy (2018), and for tax rate shocks, we use the narrative measure in Romer and Romer (2010). Appendix 2.I shows that results hold using VAR-based spending and tax shocks constructed following Blanchard and Perotti (2002).

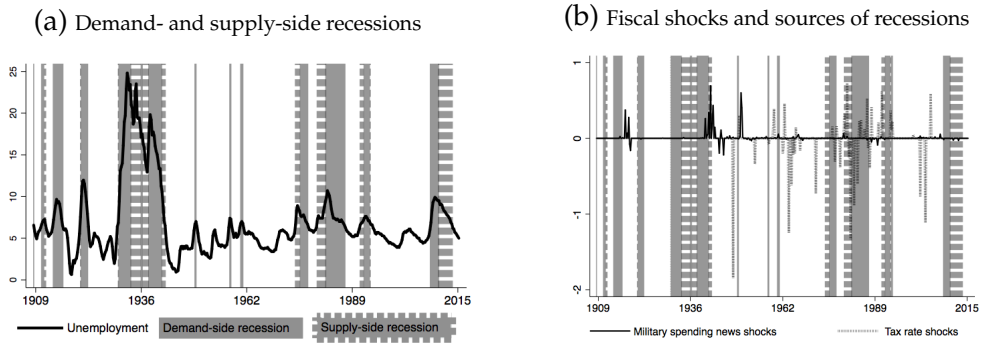
For spending multipliers, the sample period is 1909:Q1-2015:Q4; for tax cut multipliers, we use the shorter time sample 1947:Q1-2007:Q4, with the shorter sample driven by the available time series of tax shocks in Romer and Romer (2010).[33]

2.6.3 Demand-side and supply-side recessions: a closer look

Panel (a) in Figure 2.6 shows historical periods of demand-side recessions characterized by a negative co-movement between unemployment and the cyclical component of inflation (solid shaded area), and supply-side recession characterized by positive comovement between these variables (striped shaded area). The majority of the US Great Depression is identified as a demand-side recession; the oil shocks of the 1970s start off as a supply-side recession, evolving into a demand-side recession. In the case of the late 1970s/early 1980s recession, this could be due to Volcker disinflation that immediately followed the second wave of oil shocks. The Great Recession, on the other hand, originates as a demand-side recession, evolving into a supply-side recession. One explanation is that initial negative effect on households wealth and income evolved into a supply-side constraint as firms were unable to access capital due to the distorted financial system.

[33] In principle, the narrative military spending news shocks series from Ramey and Zubairy (2018) goes back to 1889:Q1, but we exclude the first twenty years of their sample due to excessive volatility of inflation in that time period, as the latter complicates our strategy of separating out episodes with inflation above and below trend; however, our baseline results are robust to considering the full sample and are available upon request.

Figure 2.6: Demand- and supply-side recessions and fiscal shocks



***Notes**: Panel (a) shows the unemployment rate in the US between 1909-2015, as well as demand-side recessions, identified by the indicator variable $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$, and supply-side recessions, identified by the indicator variable $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$; Panel (b) additionally plots time series of military spending news shocks from Ramey and Zubairy (2018) and narrative tax rate shocks from Romer and Romer (2010).*

Our identification strategy relies on having enough spending and taxation shocks in each of the three states of the world considered in the baseline specification. Panel (b) in Figure 2.6 plots the time series for military spending news shocks from Ramey and Zubairy (2018) and narrative tax rate shocks from Romer and Romer (2010) against our definition of states. The figure shows that spending and tax rate shocks are spread fairly evenly across expansions, demand- and supply-side recessions.[34] Formally, 28% of quarters identified as an expansion, 16% of quarters identified as a demand-side recession and 16% of quarters identified as a supply-side recession contain a non-zero military spending news shock. Similarly, 14% of quarters identified as an expansion, 31% of quarters identified as a demand-side recession and 32% of quarters identified as a supply-side recession contain a non-zero narrative tax shock.

[34] A bulk of variation in our spending and tax rate shocks overlaps with periods of price controls around World War II and the Korean War. In order to check that such price controls do not pose a challenge to our strategy of using co-movement between inflation and activity, Figure 2.I.6 in Appendix 2.I.2 reports estimated responses of inflation to our spending and tax rate shocks. Reassuringly, we find that both a positive spending shock and a positive tax rate shock produce a statistically significant rise in inflation.

2.6.4 Empirical results

Conditional state-dependent spending multipliers

Table 2.2 shows baseline estimation results for spending multipliers. Column (1) shows that the 2-year cumulative spending multiplier is equal to 0.70 without any conditioning on the source of fluctuations. Column (2) replicates the exercise in Ramey and Zubairy (2018), by distinguishing between recessions and expansions based on an unconditional unemployment threshold. The 2-year cumulative multiplier is equal to 0.68 in expansions, which is larger than the estimated recession multiplier equal to 0.54, although the difference is not statistically significant. These estimates blend demand- and supply-driven episodes, while our theory shows that the source of fluctuations is crucial to establish an estimate for the spending multiplier. To test our theoretical prediction, column (3) separately estimates 2-year cumulative spending multipliers in demand- and supply-driven recessions. Consistent with our theoretical findings, the spending multiplier in demand-driven recessions is equal to 0.86, which is larger than the multiplier in supply-driven recessions, which equals 0.32.

Columns (4)-(6) repeat the exercise for 4-year cumulative multipliers. As before, conditioning on recessions and expansions delivers spending multipliers that are slightly higher in expansions (0.76) than in recessions (0.65), although the difference is not statistically significant. Instead, controlling for whether recessions are generated by demand- or supply-side shocks corroborates our theory: spending multipliers are higher in demand-side recessions (0.71) than in supply-side recessions (0.63), although the difference is smaller than in the case of 2-year multipliers. The finding that conditional state dependence becomes weaker at longer horizons is consistent with our theory, as verified in the quantitative dynamic model. At longer time horizons, prices adjust to shocks and tightness plays a smaller role in business cycle adjustment, bringing the multiplier closer to its value under flexible prices, determined by the elasticities of labor demand and labor supply.

To investigate the relationship between the degree of conditional state dependence and the horizon of cumulation, Figure 2.7 repeats the exercise

for horizons from 4 to 20 quarters. In Panel (a), we do not condition on the source of fluctuations and simply distinguish between recessions and expansions; consistently with Ramey and Zubairy (2018), very limited state dependence is detected, with formal statistical tests showing no significant differences at any horizon. However, when we condition on the source of fluctuations in Panel (b), we find that, consistent with our theory, spending multipliers in demand-side recessions are higher than those in supply-side recessions at all horizons. Moreover, the degree of such conditional state dependence is strongest at shorter horizons: 4 quarters after a spending shock, the cumulative spending multiplier is close to one in demand-driven recessions, but close to zero in supply-side recessions. On the other hand, and again consistent with our theory, multipliers in demand- and supply-side recessions become very similar after 12 quarters. We formally test the restriction $\beta_H^{DR} = \beta_H^{SR}$, which implies that the source of fluctuations does not matter as in Ramey and Zubairy (2018), and can reject it at the 10% level at 6- and 7-quarter horizons, and at 32% level for all horizons between 4 and 11 quarters.

Conditional state-dependent tax cut multipliers

Table 2.3 shows baseline estimation results for tax cut multipliers. Column (1) shows that 2-year tax cut multiplier is 1.50 without any conditioning on the state of the economy, and not significantly different from zero. The lack of significance could be explained by the fact that tax cuts affect GDP through expansions in capacity, a very gradual process that is difficult to detect within 2 years. Column (3) conditions the estimates on recessions, and it shows that the tax cut multiplier is equal to 1.81 in expansions and 0.98 in recessions, although again neither are significantly different from zero. As before, the recessionary states blend demand- and supply-driven episodes, and our theory predicts that tax cut multipliers are larger in supply-side recessions. Results in column (3), derived by controlling for the source of fluctuations, support our theoretical prediction: in demand-driven recessions, the tax cut multiplier is 1.49 and not significantly different from zero, whereas in supply-side recessions it is 4.29, and statistically significant at 10% level.

Columns (4)-(6) repeat the tax-cut estimation exercise for the 4-year horizon. The unconditional tax cut multiplier in column (4) is 1.71 and statistically significant at 5% level. The fact that the multiplier is significant at 4-year horizon, and not at 2-year horizon, is consistent with the fact that capacity expansion is considered to be a gradual process. Column (5) reports that the tax cut multiplier is equal to 2.37, and significant at 5 per cent level, in expansions, but lower and equal to 1.24 and insignificant in recessions. Column (6) shows estimates that control for demand- and supply-side recessions, showing that in supply-side recessions the multiplier is higher at 1.80, and significant at 10 per cent, whereas it is negative at -1.98 and highly insignificant in demand-side recessions.

Figure 2.8 investigates the relationship between conditional state dependence of tax cut multipliers and the horizon considered. In Panel (a), we do not condition on the source of fluctuations and simply distinguish between recessions and expansions: as one can see, very limited state dependence is detected in this case. However, once we condition on the source of fluctuations in Panel (b), the tax cut multiplier in supply-side recessions is consistently higher than the multiplier in demand-side recessions, as our theory predicts, except for 4- and 5-quarter horizons. Unlike the spending multiplier, conditional state dependence of tax cut multipliers is not at its maximum at shorter horizons and instead is close to uniform after approximately 8 quarters. We formally test the restriction $\beta_H^{DR} = \beta_H^{SR}$, which implies that the source of fluctuations does not matter, and can reject it at the 10% level at 11- and 13-quarter horizons, and at 32% level for all horizons between 8 and 16 quarters, as well as at the 20-quarter horizon.

Table 2.2: Conditional state-dependent spending multipliers ($\bar{U} = 6.5\%$; US military spending news shocks

US data: 1909:Q1-2015:Q4	2-year horizon			4-year horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)
β_H: Linear	**0.70*** **			**0.75*** **		
	(0.06)			*(0.06)*		
β_H^E: $\mathbf{1}\{U_t < \bar{U}\}$		**0.68*** **	**0.68*** **		**0.76*** **	**0.76*** **
		(0.10)	*(0.09)*		*(0.13)*	*(0.12)*
β_H^R: $\mathbf{1}\{U_t \geq \bar{U}\}$		**0.54*** **			**0.65*** **	
		(0.13)			*(0.08)*	
β_H^{DR}: $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			**0.86*** **			**0.72*** **
			(0.33)			*(0.12)*
β_H^{SR}: $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			**0.32*** **			**0.63*** **
			(0.11)			*(0.09)*
$\beta_H^E = \beta_H^R$ (p-value)		0.37			0.44	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.14			0.54
T	416	416	416	408	408	408

***Notes**: HAC standard errors are reported in parentheses, with ***(**,*) denoting statistical significance at 1%(5%, 10%) level; all regressions include a set of controls, consisting of four lags of real GDP, real government spending and military spending news shocks, all normalized by trend real GDP as well as a constant (coefficients on controls are allowed to be state-specific).*

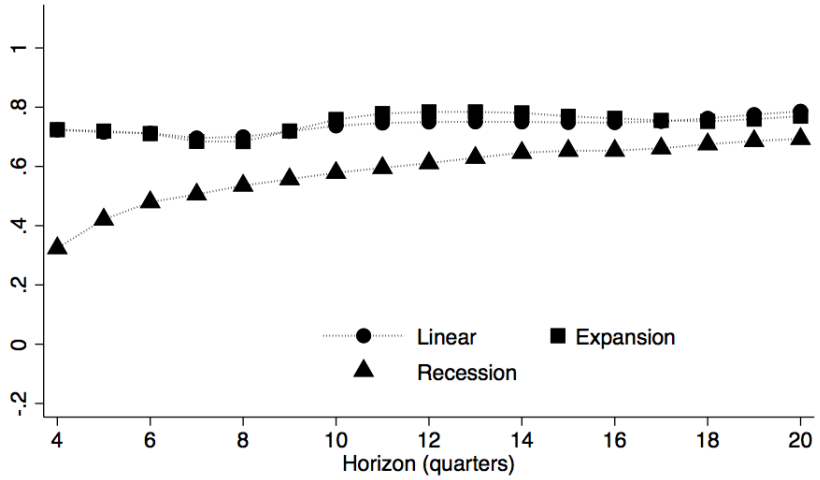
Table 2.3: Conditional state-dependent tax cut multipliers ($\bar{U} = 6.5\%$; US Romer-Romer narrative tax shocks)

US data: 1947:Q1-2007:Q4	2-year horizon			4-year horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)
β_H: Linear	1.50			**1.71** **		
	(1.14)			*(0.82)*		
β_H^E: $\mathbf{1}\{U_t < \bar{U}\}$		1.81	1.81		**2.37** **	**2.37** **
		(1.17)	*(1.12)*		*(0.99)*	*(0.99)*
β_H^R: $\mathbf{1}\{U_t \geq \bar{U}\}$		0.98			1.24	
		(1.07)			*(0.87)*	
β_H^{DR}: $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			1.49			-1.98
			(1.04)			*(2.75)*
β_H^{SR}: $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			**4.29***			**1.80***
			(2.18)			*(1.00)*
$\beta_H^E = \beta_H^R$ (p-value)		0.48			0.39	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.25			0.20
T	240	240	240	240	240	240

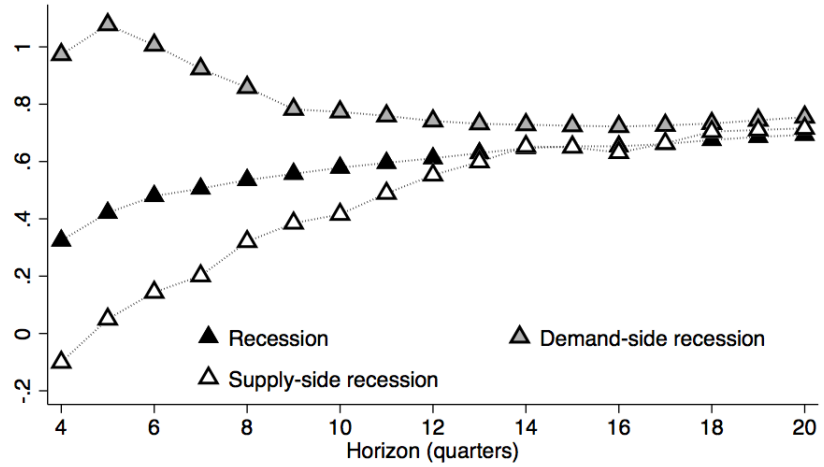
***Notes**: HAC standard errors are reported in parentheses, with ***(**,*) denoting statistical significance at 1%(5%, 10%) level; all regressions include a set of controls, consisting of four lags of (log) real GDP as well as a constant (coefficients on controls are allowed to be state-specific).*

Figure 2.7: Government Spending Multipliers across Horizons (US military spending news shocks, 1909-2015)

(a) Cumulative government spending multipliers in recessions and expansions



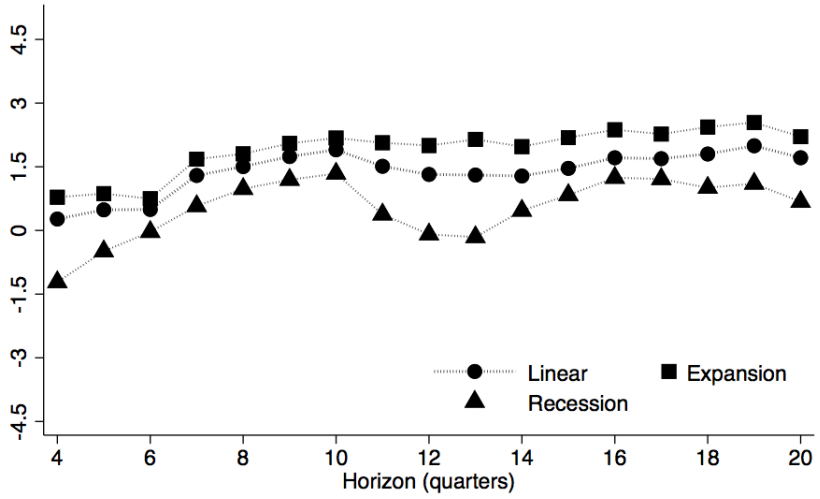
(b) Cumulative government spending multipliers in demand-side and supply-side recessions



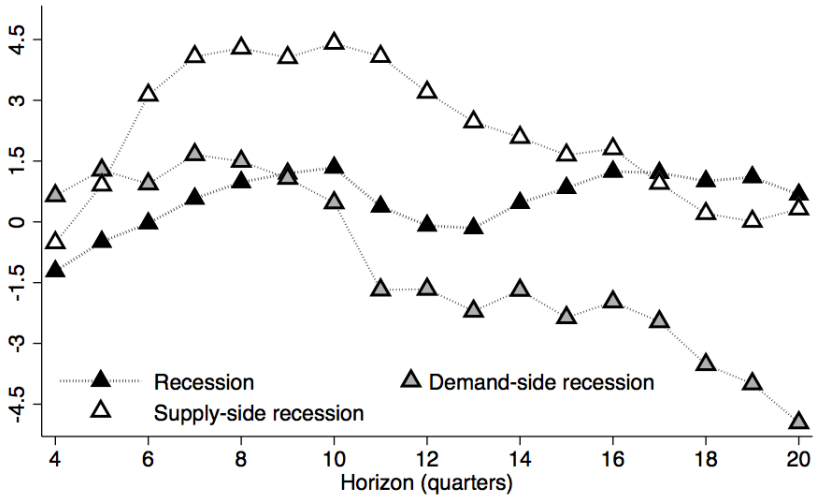
***Notes**: Panel (a) shows cumulative government spending multipliers estimated in recessionary $\mathbf{1}\{U_t \geq \bar{U}\}$ and expansionary $\mathbf{1}\{U_t < \bar{U}\}$ episodes as well as linear benchmarks for different cumulation horizons $4 \leq H \leq 20$;*
Panel (b) shows cumulative government spending multipliers estimated in demand-side recessionary episodes $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$ and supply-side recessionary episodes $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$ as well as unconditional recessions $\mathbf{1}\{U_t \geq \bar{U}\}$ for different horizons $4 \leq H \leq 20$; we set $\bar{U} = 6.5\%$ in all estimations.

Figure 2.8: Tax Cut Multipliers across Horizons (US Romer-Romer narrative tax shocks, 1947-2007)

(a) Tax cut multipliers in recessions and expansions



(b) Tax cut multipliers in demand-side and supply-side recessions



Notes: *Panel (a) shows tax cut multipliers estimated in recessionary* $\mathbf{1}\{U_t \geq \bar{U}\}$ *and expansionary* $\mathbf{1}\{U_t < \bar{U}\}$ *episodes as well as linear benchmarks for different horizons* $4 \leq H \leq 20$*;*
Panel (b) shows tax cut multipliers estimated in demand-side recessionary episodes $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$ *and supply-side recessionary episodes* $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$ *as well as unconditional recessions* $\mathbf{1}\{U_t \geq \bar{U}\}$*, for different horizons* $4 \leq H \leq 20$*; we set* $\bar{U} = 6.5\%$ *in all estimations.*

Robustness checks

In Appendix 2.I, we perform further robustness checks, briefly outlined here. In particular, in Appendix 2.I.3, we show that once one further distinguishes between demand- and supply-side expansions, our theory receives further empirical support: spending multipliers are higher in supply-side expansions, whereas multipliers out of tax cuts are larger in demand-driven economic upturns. In Appendix 2.I.4, we show that our results are robust to using VAR-based fiscal shocks, following Blanchard and Perotti (2002). Finally, in Appendix 2.I.5 we provide results where instead of measuring economic activity using unemployment, we are using detrended real GDP, a measure more consistent with our theory.

2.7 Conclusion

This paper develops a general theory of state-dependent fiscal multipliers for a broad range of spending and taxation policies. The framework accounts for empirically relevant goods market frictions by incorporating idle productive capacity and unsatisfied households' demand into an otherwise standard general equilibrium setup. Our key novel finding is that cyclicality of fiscal multipliers is pinned down by the *source of economic fluctuations*, and we provide model-free econometric evidence that strongly supports our predictions.

Crucially, we establish that multipliers associated with fiscal instruments which stimulate aggregate demand, such as government spending and consumption tax cuts, are *countercyclical* under demand-driven fluctuations and *procyclical* under supply-driven fluctuations. On the other hand, multipliers associated with interventions that stimulate aggregate supply, such as reductions in taxes on firms' payroll, sales and households' labor income, are *countercyclical* under supply-driven fluctuations and *procyclical* under demand-driven fluctuations. In addition, our theoretical results establish a relevant role for fiscal austerity, implemented by a reduction in government consumption in severe *supply-driven recessions* and *demand-driven booms*, provided elasticities of labor supply and labor demand are sufficiently low.

Further, we develop and estimate a novel econometric specification that allows us to perform model-free evaluation of both spending and taxation multipliers in recessionary and expansionary episodes, *conditional* on those being either demand- or supply-driven in nature. Our empirical results de-

tect substantial state dependence, *conditional* on the source of fluctuations, which is in line with the predictions of our theory. Such findings offer a resolution to the debate on state dependence of fiscal policy, on both empirical and theoretical grounds, and they provide guidance for the conduct of fiscal policy in the different phases of the business cycle.

Our analysis opens fruitful avenues for future research. First, in our dynamic framework, current changes in fiscal policy determine the future path of goods market tightness and thus constrain the effectiveness of policy in the future. Our framework can therefore be extended to study the intertemporal trade-offs and the *path dependence* of fiscal policy. Second, by extending the model to include heterogeneity in the goods markets, one can study how composition of spending and taxation policies may generate spillover effects of fiscal policy across sectors and the socially optimal distribution of such policies. We plan to investigate these issues in future research.

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Appendices to Chapter 2

2.A Proofs of results in main text

Lemma 2.1. *The consumption function $c(p, x)$ is the optimal consumption choice in the representative household's problem evaluated under non-produced goods market clearing($m = \bar{m}$) and is given by:*

$$c(p, x) = \frac{\chi}{p[1 + \gamma(x)]},$$

where $\frac{\partial c}{\partial p} < 0$, $\frac{\partial c}{\partial x} < 0$ and $\frac{\partial c}{\partial \chi} > 0$.

Proof. The Lagrangian of the representative household's problem is given by:

$$\mathcal{L} = \left[\chi \frac{c^{1-\sigma}}{1-\sigma} + \zeta(m) - \frac{l^{1+\psi}}{1+\psi} + \lambda(wl + \Pi + \bar{m} - T - p[1 + \gamma(x)]c - m)\right]. \tag{2.54}$$

First-order conditions with respect to consumption of the produced and non-produced good, as well as the labor supply are given by:

$$\frac{d\mathcal{L}}{dc} = \chi c^{-\sigma} - \lambda p[1 + \gamma(x)] = 0, \tag{2.55}$$

$$\frac{d\mathcal{L}}{dm} = \zeta'(m) - \lambda = 0, \tag{2.56}$$

$$\frac{d\mathcal{L}}{dl} = -l^{\psi} + \lambda = 0. \tag{2.57}$$

Combining the first-order conditions for consumption of the produced and the non-produced good, and evaluating it under $m = \bar{m}$ and our baseline assumption of $\zeta'(\bar{m}) = 1$ delivers the following consumption function:

$$c(p, x)^{\sigma} = \frac{\chi}{p[1 + \gamma(x)]}. \tag{2.58}$$

Further, in the main text we assume $\sigma = 1$:

$$c(p, x) = \frac{\chi}{p[1 + \gamma(x)]}. \tag{2.59}$$

It then follows that $\frac{\partial c}{\partial p} = -\frac{\chi}{p[1+\gamma(x)]}\frac{1}{p} < 0$, $\frac{\partial c}{\partial x} = -\frac{\chi}{p[1+\gamma(x)]}\frac{1}{[1+\gamma(x)]} < 0$, $\frac{\partial c}{\partial \chi} = \frac{1}{p[1+\gamma(x)]} > 0$. □

Lemma 2.2. *The labor supply function l(w) is the optimal labor supply choice in*

the representative household's problem evaluated at $m = \bar{m}$, and is given by:

$$l(w) = w^{\frac{1}{\psi}},$$

where $\frac{\partial l}{\partial w} > 0$.

Proof. Combing the first-order conditions for consumption of the non-produced good and labor supply , and evaluating it under $m = \bar{m}$ and our baseline assumption of $\zeta'(\bar{m}) = 1$ delivers the following labor supply function

$$l(w) = w^{\frac{1}{\psi}}.$$

It then follows that $\frac{\partial l}{\partial w} = \frac{1}{\psi} w^{\frac{1}{\psi}-1} > 0$. □

Lemma 2.3. *The labor demand function $n(p, x, w)$ is the solution to the representative firm's profit maximisation problem and is given by:*

$$n(p, x, w) = \left[\frac{\alpha p f(x) a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}},$$

where $\frac{\partial n}{\partial p} > 0$, $\frac{\partial n}{\partial x} > 0$, $\frac{\partial n}{\partial w} < 0$, $\frac{\partial n}{\partial a} > 0$ and $\frac{\partial n}{\partial \tau} < 0$.

Proof. The first-order condition of the representative firm's profit maximization problem is given by:

$$\frac{\partial \Pi}{\partial n} = \alpha p f(x) a n^{\alpha-1} - w(1+\tau) = 0. \tag{2.60}$$

Solving for n from the above first-order condition gives the following labor demand function:

$$n(p, x, w) = \left[\frac{\alpha p f(x) a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}}.$$

It then follows that $\frac{\partial n}{\partial p} = \frac{1}{1-\alpha}\left[\frac{\alpha p f(x) a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}-1} \frac{\alpha f(x) a}{w(1+\tau)} > 0$, $\frac{\partial n}{\partial x} = \frac{1}{1-\alpha}\left[\frac{\alpha p f(x) a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}-1} \frac{\alpha p f'(x) a}{w(1+\tau)} > 0$, $\frac{\partial n}{\partial w} = -\frac{1}{1-\alpha}\left[\frac{\alpha p f(x) a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}-1} \frac{\alpha p f(x) a}{w(1+\tau)} \frac{1}{w} < 0$, $\frac{\partial n}{\partial a} = \frac{1}{1-\alpha}\left[\frac{\alpha p f(x) a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}-1} \frac{\alpha p f(x)}{w(1+\tau)} > 0$, $\frac{\partial n}{\partial \tau} = -\frac{1}{1-\alpha}\left[\frac{\alpha p f(x) a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}-1} \frac{\alpha p f(x) a}{w(1+\tau)} \frac{1}{(1+\tau)} < 0$. □

Lemma 2.4. *In a competitive equilibrium, the following are the comparative statics*

of tightness (x)*, sales* (y) *and the price* (p)*:*

$$\frac{dx}{d\chi} = 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} > 0; \qquad \frac{dx}{da} = 0, \frac{dy}{da} > 0, \frac{dp}{da} < 0$$

Proof. Combining the labor supply function $l(w)$ in Lemma 2.2 and the labor demand function $n(p, x, w)$ in Lemma 3 with the labor market clearing condition delivers the following expression for equilibrium employment:

$$l = n = (\alpha p f(x) a)^{\frac{1}{1-\alpha+\psi}} (1+\tau)^{-\frac{1}{1-\alpha+\psi}}. \tag{2.61}$$

Inserting equilibrium employment level into goods market clearing condition:

$$\frac{f(x)}{1+\gamma(x)} a \left[(\alpha p f(x) a)^{\frac{1}{1-\alpha+\psi}} (1+\tau)^{-\frac{1}{1-\alpha+\psi}} \right]^{\alpha} = c(p,x) + G, \tag{2.62}$$

$$\frac{f(x)}{1+\gamma(x)} a \left[(\alpha p f(x) a)^{\frac{1}{1-\alpha+\psi}} (1+\tau)^{-\frac{1}{1-\alpha+\psi}} \right]^{\alpha} = \frac{\chi}{p[1+\gamma(x)]} + G, \tag{2.63}$$

$$p f(x) a \alpha^{\frac{\alpha}{1-\alpha+\psi}} (p f(x) a)^{\frac{\alpha}{1-\alpha+\psi}} (1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \chi + p[1+\gamma(x)]G \tag{2.64}$$

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} (p f(x) a)^{\frac{1+\psi}{1-\alpha+\psi}} (1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \chi + p[1+\gamma(x)]G. \tag{2.65}$$

By definition of the competitive equilibrium, $x = x^*$, and so $\frac{dx}{d\chi} = \frac{dx}{da} = 0$. The latter implies the following comparative statics for p (for simplicity, evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} (f(x)a)^{\frac{1+\psi}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} p^{\frac{\alpha}{1-\alpha+\psi}} \frac{dp}{d\chi} = 1 \tag{2.66}$$

$$\frac{dp}{d\chi} = \alpha^{-\frac{\alpha}{1-\alpha+\psi}} (f(x)a)^{-\frac{1+\psi}{1-\alpha+\psi}} \frac{1-\alpha+\psi}{1+\psi} p^{-\frac{\alpha}{1-\alpha+\psi}} = \frac{1-\alpha+\psi}{1+\psi} \frac{p}{\chi} > 0. \tag{2.67}$$

$$\tag{2.68}$$

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} (f(x))^{\frac{1+\psi}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} (pa)^{\frac{\alpha}{1-\alpha+\psi}} (p + a\frac{dp}{da}) = 0 \tag{2.69}$$

$$\frac{dp}{da} = -\frac{p}{a} < 0. \tag{2.70}$$

Finally, the above implies the following comparative statics for sales $y =$

$[1+\gamma(x)](c(p,x)+G) = \chi/p + G[1+\gamma(x)]$ (also evaluated at $G = \tau = 0$):

$$py = \chi + p[1+\gamma(x)]G \tag{2.71}$$

$$\frac{dp}{d\chi}y + p\frac{dy}{d\chi} = 1 \tag{2.72}$$

$$\frac{dy}{d\chi} = \frac{1}{p}\left(1 - \frac{dp}{d\chi}y\right) = \frac{1}{p}\frac{\alpha}{1+\psi} > 0. \tag{2.73}$$

$$\frac{dp}{da}y + p\frac{dy}{da} = 0 \tag{2.74}$$

$$\frac{dy}{da} = -\frac{dp}{da}\frac{y}{p} > 0. \tag{2.75}$$

□

Lemma 2.5. *In a fixprice equilibrium, the following are the comparative statics of tightness* (x)*, sales* (y) *and the price* (p)*:*

$$\frac{dx}{d\chi} > 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} = 0; \qquad \frac{dx}{da} < 0, \frac{dy}{da} = 0, \frac{dp}{da} = 0$$

Proof. Condition (65) remains unchanged in a fixprice equilibrium. However, now the price is a parameter, so that $\frac{dp}{d\chi} = \frac{dp}{da} = 0$. The latter implies the following comparative statics for x (for simplicity, evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}(pa)^{\frac{1+\psi}{1-\alpha+\psi}}\frac{1+\psi}{1-\alpha+\psi}f(x)^{\frac{\alpha}{1-\alpha+\psi}}f'(x)\frac{dx}{d\chi} = 1 \tag{2.76}$$

$$\frac{dx}{d\chi} = \alpha^{-\frac{\alpha}{1-\alpha+\psi}}(f(x)a)^{-\frac{1+\psi}{1-\alpha+\psi}}\frac{1-\alpha+\psi}{1+\psi}f(x)^{-\frac{\alpha}{1-\alpha+\psi}}\frac{1}{f'(x)} > 0. \tag{2.77}$$

(2.78)

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}(p)^{\frac{1+\psi}{1-\alpha+\psi}}\frac{1+\psi}{1-\alpha+\psi}(f(x)a)^{\frac{\alpha}{1-\alpha+\psi}}(f(x)+af'(x)\frac{dx}{da}) = 0$$

$$\frac{dx}{da} = -\frac{f(x)}{a}\frac{1}{f'(x)} < 0. \tag{2.79}$$

The above implies the following comparative statics for sales $y = [1+$

$\gamma(x)](c(p,x)+G) = \chi/p + G[1+\gamma(x)]$ (also evaluated at $G = \tau = 0$):

$$py = \chi + p[1+\gamma(x)]G \tag{2.80}$$

$$p\frac{dy}{d\chi} = 1 \tag{2.81}$$

$$\frac{dy}{d\chi} = \frac{1}{p} > 0. \tag{2.82}$$

$$p\frac{dy}{da} = 0 \tag{2.83}$$

$$\frac{dy}{da} = 0. \tag{2.84}$$

□

Proposition 2.1. *In a competitive equilibrium, the demand-side and the supply-side fiscal multipliers are equal and given by:*

$$\varphi^* \equiv \frac{\alpha}{1+\psi} = \frac{1-\frac{1}{|\epsilon^d|}}{1+\frac{1}{\epsilon^s}},$$

where $\alpha \in (0,1]$ and $\psi > 0$ are, respectively, returns to labor and inverse Frisch elasticity, whereas $|\epsilon^d| = \frac{1}{1-\alpha}$ and $\epsilon^s = \frac{1}{\psi}$ are (absolute) elasticities of labor demand and labor supply. Hence $\varphi^* \in (0,1]$ and it is pinned down by elasticities of labor demand and labor supply.

Proof. First differentiate (2.65) with respect to G (evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1} \left[\frac{dp}{dG} f(x)a + pf'(x)\frac{dx}{dG}a\right] = p[1+\gamma(x)],$$

(2.85)

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}} \left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = p[1+\gamma(x)],$$

(2.86)

$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{p[1+\gamma(x)]}{\frac{1+\psi}{1-\alpha+\psi} \underbrace{\alpha^{\frac{\alpha}{1-\alpha+\psi}} (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}}_{=\chi \text{(by (2.67) under } G=\tau=0)}},$$

(2.87)

$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{p[1+\gamma(x)]}{\frac{1+\psi}{1-\alpha+\psi}\chi}$$

(2.88)

$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)}.$$

(2.89)

From definition of demand-side fiscal multiplier:

$$\varphi^d = \frac{d\{c+G\}}{dG} = \frac{dc}{dG} + 1 = \frac{\partial c}{\partial p}\frac{dp}{dG} + \frac{\partial c}{\partial x}\frac{dx}{dG} + 1 \tag{2.90}$$

In a competitive equilibrium, $x = x^*$, so that $\frac{dx}{dG} = 0$, which combined with (2.88) implies the following:

$$\varphi^d = \frac{\partial c}{\partial p}\frac{dp}{dG} + 1 = -\frac{\chi}{p[1+\gamma(x)]}\frac{1}{p}\frac{dp}{dG} + 1 = -c(p,x)\frac{1}{p}p\frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)} + 1$$

(2.91)

$$= -\frac{1-\alpha+\psi}{1+\psi} + 1 \tag{2.92}$$

$$= \frac{\alpha}{1+\psi} \equiv \varphi^*. \tag{2.93}$$

Similarly, differentiate (2.65) with respect to τ (evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left[\frac{1+\psi}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1}\left(\frac{dp}{d\tau}f(x)a+pf'(x)\frac{dx}{d\tau}a\right)-\frac{\alpha}{1-\alpha+\psi}(-1)(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}\right]=0 \tag{2.94}$$

$$\left[\frac{1}{p}\frac{dp}{d\tau}+\frac{f'(x)}{f(x)}\frac{dx}{d\tau}\right]=\frac{\alpha}{1+\psi}. \tag{2.95}$$

From definition of supply-side fiscal multiplier:

$$\varphi^s=\frac{d\{c+G\}/\{c+G\}}{d[-\tau]}=-\frac{1}{c}\frac{dc}{d\tau}=-\frac{1}{c}\left[\frac{\partial c}{\partial p}\frac{dp}{d\tau}+\frac{\partial c}{\partial x}\frac{dx}{d\tau}\right]. \tag{2.96}$$

In a competitive equilibrium, $x = x^*$, so that $\frac{dx}{d\tau} = 0$, which combined with (95) implies the following:

$$\varphi^s=-\frac{1}{c}\frac{\partial c}{\partial p}\frac{dp}{d\tau}=-\frac{1}{c}\left[-\frac{\chi}{p[1+\gamma(x)]}\frac{1}{p}\right]\frac{dp}{d\tau}=\frac{1}{c}c\frac{1}{p}p\frac{\alpha}{1+\psi} \tag{2.97}$$

$$=\frac{\alpha}{1+\psi}=\varphi^d=\varphi^*. \tag{2.98}$$

□

Lemma 2.6. *Define the fixed capacity fiscal multiplier $\theta(x)$ to be the demand-side fiscal multiplier in a fixprice equilibrium under fixed labor supply, so that*

$$\theta(x)\equiv\frac{d\{c+G\}}{dG}|_{\psi\to\infty}$$

then $\theta(x)$ has the following properties:

$$\theta(x)=\begin{cases}(-\infty,0), & \text{if} \quad x\in(x^*,x_m)\\ 0, & \text{if} \quad x=x^*\\ (0,1), & \text{if} \quad x\in(0,x^*)\end{cases}$$

$$\theta'(x)<0, \qquad \forall x\in(0,x_m),$$

where x_m is given by $f(x_m)=\rho x_m$.

Proof. Under $\psi \to \infty$, (2.89) can be written as:

$$\lim_{\psi\to\infty}\left[\frac{1}{p}\frac{dp}{dG}+\frac{f'(x)}{f(x)}\frac{dx}{dG}\right]=\lim_{\psi\to\infty}\frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)}. \tag{2.99}$$

$$\lim_{\psi\to\infty}\left[\frac{1}{p}\frac{dp}{dG}+\frac{f'(x)}{f(x)}\frac{dx}{dG}\right]=\lim_{\psi\to\infty}\frac{1}{c(p,x)}=\frac{1}{\lim_{\psi\to\infty}c(p,x)}. \tag{2.100}$$

From definition of $\theta(x)$:

$$\theta(x)\equiv\frac{d\{c+G\}}{dG}|_{\psi\to\infty}=\lim_{\psi\to\infty}\frac{dc}{dG}+1=\lim_{\psi\to\infty}\left[\frac{\partial c}{\partial p}\frac{dp}{dG}+\frac{\partial c}{\partial x}\frac{dx}{dG}\right]+1 \tag{2.101}$$

In a fixprice equilibrium $p = p_0$ is a parameter, so that $\frac{dp}{dG} = 0$, which combined with (2.101) implies the following:

$$\theta(x)=\lim_{\psi\to\infty}\frac{\partial c}{\partial x}\frac{dx}{dG}+1=\lim_{\psi\to\infty}\frac{\partial c}{\partial x}\lim_{\psi\to\infty}\frac{dx}{dG}+1 \tag{2.102}$$

$$=-\lim_{\psi\to\infty}\frac{\chi}{p[1+\gamma(x)]}\frac{\gamma'(x)}{[1+\gamma(x)]}\frac{f(x)}{f'(x)}\frac{1}{\lim_{\psi\to\infty}c(p,x)}+1 \tag{2.103}$$

$$=1-\frac{\gamma'(x)}{[1+\gamma(x)]}\frac{f(x)}{f'(x)}\frac{\lim_{\psi\to\infty}c(p,x)}{\lim_{\psi\to\infty}c(p,x)} \tag{2.104}$$

$$=1-\frac{\gamma'(x)}{[1+\gamma(x)]}\frac{f(x)}{f'(x)}. \tag{2.105}$$

Recall that $\gamma(x)\equiv\frac{\rho x}{f(x)-\rho x}$, so that $\gamma'(x)=\frac{\rho(f(x)-\rho x)-(f'(x)-\rho)\rho x}{(f(x)-\rho x)^2}=\frac{\rho(f(x)-f'(x)x)}{(f(x)-\rho x)^2}$, and $\theta(x)$ may be rewritten as:

$$\theta(x)=1-\frac{\frac{\rho(f(x)-f'(x)x)}{(f(x)-\rho x)^2}}{\frac{f(x)}{f(x)-\rho(x)}}\frac{f(x)}{f'(x)}=1-\frac{\rho(f(x)-f'(x)x)}{(f(x)-\rho x)f(x)}\frac{f(x)}{f'(x)} \tag{2.106}$$

$$=1-\frac{\rho(f(x)-f'(x)x)}{f'(x)(f(x)-\rho x)}=\frac{f'(x)f(x)-\rho f(x)}{f'(x)f(x)-f'(x)\rho x}=\frac{1-\frac{\rho}{f'(x)}}{1-\frac{\rho x}{f(x)}}. \tag{2.107}$$

We can now show that $\theta(x)$ possesses several convenient properties. Firstly, $\theta'(x) < 0, \quad \forall x \in (0, x_m)$, where x_m is given by $f(x_m) = \rho x_m$. In order to show this, notice that $q(x) = \frac{f(x)}{x}$ and $f'(x) = q(x)^{1+\delta}$, which allows us to rewrite $\theta(x)$ as follows:

$$\theta(x)=\frac{1-\frac{\rho}{q(x)^{1+\delta}}}{1-\frac{\rho}{q(x)}}=\frac{q(x)^{1+\delta}-\rho}{q(x)^{1+\delta}-\rho q(x)^{\delta}}, \tag{2.108}$$

and $\theta'(x)$ is now given by:

$$\theta'(x) = \frac{(1+\delta)q(x)^{\delta}q'(x)[q(x)^{1+\delta} - \rho q(x)^{\delta}] - [(1+\delta)q(x)^{\delta}q'(x) - \delta\rho q(x)^{\delta-1}q'(x)](q(x)^{1+\delta} - \rho)}{(q(x)^{1+\delta} - \rho)^2}. \tag{2.109}$$

Given that $q(x) > 0, q'(x) < 0, (q(x)^{1+\delta} - \rho)^2 > 0, \quad \forall x \in (0, \infty)$, a sufficient condition for $\theta'(x) < 0$ is:

$$(1+\delta)q(x)^{\delta}[q(x)^{1+\delta} - \rho q(x)^{\delta}] - [(1+\delta)q(x)^{\delta} - \delta\rho q(x)^{\delta-1}](q(x)^{1+\delta} - \rho) > 0 \tag{2.110}$$

$$-\rho q(x)^{2\delta} + \rho(1+\delta)q(x)^{\delta} - \delta\rho^2 q(x)^{\delta-1} > 0 \tag{2.111}$$

$$\rho q(x)^{\delta-1}[q(x) - q(x)^{\delta+1}] + \delta[q(x) - \rho] > 0. \tag{2.112}$$

Finally, $q(0) = 1$ and $q(x_m) = \rho$, and since $q'(x) < 0, \forall x \in (0, \infty)$ it follows that $q(x) \in (\rho, 1), \quad \forall x \in (0, x_m)$; it is clear that for all $q(x) \in (\rho, 1)$ the sufficient condition above is satisfied. Hence, $\theta'(x) < 0, \quad \forall x \in (0, x_m)$.
Secondly, it follows directly from (107) that $\theta(x^*) = 0$, since $f'(x^*) = \rho$. At the extremes:

$$\theta(0) = \frac{q(0)^{1+\delta} - \rho}{q(0)^{1+\delta} - \rho q(0)^{\delta}} = \frac{1^{1+\delta} - \rho}{1^{1+\delta} - \rho 1^{\delta}} = 1, \tag{2.113}$$

$$\lim_{x \to x_m^-} \theta(x) = \lim_{h \to 0} \frac{q(x_m - h)^{1+\delta} - \rho}{q(x_m - h)^{1+\delta} - \rho q(x_m - h)^{\delta}} \tag{2.114}$$

$$= \frac{\rho^{1+\delta} - \rho}{\rho^{1+\delta} - \rho\rho^{\delta}} = \frac{\rho(\rho^{\delta} - 1)}{0} = -\infty. \tag{2.115}$$

Since $\theta(0) = 1, \theta(x^*) = 0$ and $\lim_{x \to x_m^-} \theta(x) = -\infty$, and $\theta'(x) < 0, \forall x \in (0, x_m)$ it follows that $\theta(x) \in (0, 1), \forall x \in (0, x^*)$ and $\theta(x) \in (-\infty, 0), \forall x \in (x^*, x_m)$. □

Proposition 2.2. *In a fixprice equilibrium, the demand-side fiscal multiplier $\varphi^d(x)$ is given by*

$$\varphi^d(x) = \underbrace{\varphi^*}_{\text{State-invariant component}} + \underbrace{\theta(x) \times (1 - \varphi^*)}_{\text{State-dependent component}},$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the competitive equilibrium multiplier. Hence, $\varphi^d(x) \in (-\infty, 1)$

and $\frac{d\varphi^d(x)}{dx} < 0, \forall x \in (0, x_m)$.

Proof. From (2.89) we know that:

$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)} = (1-\varphi^*)\frac{1}{c(p,x)}. \tag{2.116}$$

Further, in a fixprice equilibrium $p = p_0$ is a parameter, so that $\frac{dp}{dG} = 0$ and it follows that:

$$\frac{dx}{dG} = (1-\varphi^*)\frac{f(x)}{f'(x)}\frac{1}{c(p,x)}. \tag{2.117}$$

From the definition of the demand-side fiscal multiplier:

$$\varphi^d(x) = \frac{d\{c+G\}}{dG} = \frac{dc}{dG} + 1 = \frac{\partial c}{\partial x}\frac{dx}{dG} + 1 = -\frac{\chi}{p[1+\gamma(x)]}\frac{\gamma'(x)}{[1+\gamma(x)]}(1-\varphi^*)\frac{f(x)}{f'(x)}\frac{1}{c(p,x)} + 1, \tag{2.118}$$

$$= 1 - (1-\varphi^*)\underbrace{\frac{\gamma'(x)}{[1+\gamma(x)]}\frac{f(x)}{f'(x)}}_{1-\theta(x)}\frac{c(p,x)}{c(p,x)} = 1 - (1-\varphi^*)(1-\theta(x)), \tag{2.119}$$

$$= \varphi^* + \theta(x) \times (1-\varphi^*). \tag{2.120}$$

Since $\frac{d\varphi^d(x)}{dx} = \theta'(x)(1-\varphi^*)$ and $\theta'(x) < 0, \forall x \in (0, x_m)$ it follows that $\frac{d\varphi^d(x)}{dx} < 0, \forall x \in (0, x_m)$. Further, $\varphi^d(0) = \varphi^* + \theta(0) \times (1-\varphi^*) = 1$ and $\lim_{x\to x_m^-}\varphi^d(x) = \varphi^* + \lim_{x\to x_m^-}\theta(x) \times (1-\varphi^*) = -\infty$, so that $\varphi^d(x) \in (-\infty, 1), \forall x \in (0, x_m)$. □

Corollary 2.1. *There always exists tightness* $\hat{x} \in (x^*, x_m)$ *such that* $\varphi^d(\hat{x}) = 0$ *and* $\varphi^d(x) < 0, \forall x \in (\hat{x}, x_m)$*, and it is given by:*

$$\hat{x} = \theta^{-1}\left(-\frac{\varphi^*}{1-\varphi^*}\right),$$

where $\frac{d\hat{x}}{d\varphi^*} > 0$.

Proof. Suppose there exists $\hat{x} \in (0, x_m)$, such that $\varphi^d(\hat{x}) = 0$; then it should

satisfy the following condition:

$$\varphi^d(\hat{x}) = \varphi^* + (1-\varphi^*) \times \theta(\hat{x}) = 0, \tag{2.121}$$

$$\theta(\hat{x}) = -\frac{\varphi^*}{1-\varphi^*}. \tag{2.122}$$

We know that $\theta(x)$ lies between $(-\infty, 1)$ and is differentiable and strictly decreasing on $(0, x_m)$; hence the inverse function $\theta^{-1}(.)$ exists on $(-\infty, 1)$ and returns values in $(0, x_m)$. Moreover, since $-\frac{\varphi^*}{1-\varphi^*} \in (-\infty, 1), \forall \varphi^* \in (0,1)$, then $\hat{x} \in (0, x_m)$ always exists and is given by:

$$\hat{x} = \theta^{-1}\left(-\frac{\varphi^*}{1-\varphi^*}\right). \tag{2.123}$$

Since $\varphi^d(x^*) = \varphi^* \in (0,1)$ and $\frac{d\varphi^d(x)}{dx} < 0, \forall x \in (0, x_m)$ it must be that $\hat{x} \in (x^*, x_m)$; further, since $\frac{d\varphi^d(x)}{dx} < 0, \forall x \in (0, x_m)$ and $\varphi^d(\hat{x}) = 0$, it follows that $\varphi^d(x) < 0, \forall x \in (\hat{x}, x_m)$. It is also true that:

$$\theta'(\hat{x})\frac{d\hat{x}}{d\varphi^*} = \frac{d}{d\varphi^*}\left(-\frac{\varphi^*}{1-\varphi^*}\right) = -\frac{1}{(1-\varphi^*)^2}, \tag{2.124}$$

$$\frac{d\hat{x}}{d\varphi^*} = -\frac{1}{\theta'(\hat{x})}\frac{1}{(1-\varphi^*)^2} > 0. \tag{2.125}$$

□

Proposition 2.3. *In a fixprice equilibrium, the supply-side fiscal multiplier $\varphi^s(x)$ is given by*

$$\varphi^s(x) = \underbrace{\varphi^*}_{\text{State-invariant component}} - \underbrace{\theta(x) \times \varphi^*}_{\text{State-dependent component}},$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the competitive equilibrium multiplier. Hence, $\varphi^d(x) \in (0, +\infty)$ and $\frac{d\varphi^d(x)}{dx} > 0, \forall x \in (0, x_m)$.

Proof. From (2.95) we know that:

$$\left[\frac{1}{p}\frac{dp}{d\tau} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau}\right] = \frac{\alpha}{1+\psi} = \varphi^*. \tag{2.126}$$

In a fixprice equilibrium, $p = p_0$ is a parameter, so that $\frac{dp}{d\tau} = 0$, and it follows

that:

$$\frac{dx}{d\tau} = \varphi^* \frac{f(x)}{f'(x)}. \tag{2.127}$$

From the definition of the supply-side multiplier:

$$\varphi^s(x) = \frac{d\{c+G\}/\{c+G\}}{d[-\tau]} = -\frac{1}{c}\frac{dc}{d\tau} = -\frac{1}{c}\frac{\partial c}{\partial x}\frac{dx}{d\tau} = \frac{1}{c}\frac{\chi}{p[1+\gamma(x)]}\underbrace{\frac{\gamma'(x)}{[1+\gamma(x)]}\frac{f(x)}{f'(x)}}_{1-\theta(x)}\varphi^* \tag{2.128}$$

$$= \frac{c}{c}(1-\theta(x))\varphi^* = \varphi^* - \theta(x) \times \varphi^*. \tag{2.129}$$

Since $\frac{d\varphi^s(x)}{dx} = -\theta'(x)\varphi^*$ and $\theta'(x) < 0, \forall x \in (0, x_m)$ it follows that $\frac{d\varphi^s(x)}{dx} > 0, \forall x \in (0, x_m)$. Further, $\varphi^s(0) = \varphi^* - \theta(0) \times \varphi^* = 0$ and $\lim_{x \to x_m^-} \varphi^s(x) = \varphi^* - \lim_{x \to x_m^-} \theta(x) \times \varphi^* = \infty$, so that $\varphi^s(x) \in (0, \infty), \forall x \in (0, x_m)$. □

Corollary 2.5. *In a fixprice equilibrium, the demand-side and supply-side fiscal multipliers are related as*

$$\underbrace{\varphi^d(x)}_{\text{Demand-side multiplier}} = \underbrace{\theta(x)}_{\text{Fixed capacity multiplier}} + \underbrace{\varphi^s(x)}_{\text{Supply-side multiplier}},$$

so that the demand-side multiplier is higher in slack equilibria, lower in tight equilibria and exactly equal to the supply-side multiplier in an efficient fixprice equilibrium.

Proof. From the expression for the demand-side fiscal multiplier is a fixprice equilibrium in Proposition 2.2:

$$\varphi^d(x) = \varphi^* + \theta(x)(1-\varphi^*) = \theta(x) + \underbrace{\varphi^* - \theta(x)\varphi^*}_{\varphi^s(x)}, \tag{2.130}$$

$$\varphi^d(x) = \theta(x) + \varphi^s(x). \tag{2.131}$$

□

Corollary 2.6. *Suppose that elasticities of labor demand and labor supply are sufficiently low so that $\varphi^* < 0.5$; then there always exists tightness $\tilde{x} \in [\hat{x}, x_m)$ such that:*

$$-\varphi^d(x) > \varphi^s(x) > \varphi^d(x), \quad \forall x \in (\tilde{x}, x_m).$$

Furthermore, $\hat{x}$ is given by:

$$\tilde{x} = \theta^{-1}\left(-\frac{2\varphi^*}{1-2\varphi^*}\right), \quad \varphi^* < 0.5$$

and hence $\frac{d\tilde{x}}{d\varphi^*} > 0$.

Proof. It is apparent that the austerity threshold for tightness cannot be below $\hat{x}$, as in that case $\varphi^d(x) > 0 > -\varphi^d(x), \forall x \in (0, \hat{x})$. However, suppose there exists $\tilde{x} \in (\hat{x}, x_m)$ such that $-\varphi^d(\tilde{x}) = \varphi^s(\tilde{x}) > \varphi^d(\tilde{x})$. Then it must satisfy the following:

$$-\varphi^* - \theta(\tilde{x})(1-\varphi^*) = \varphi^* - \theta(\tilde{x})\varphi^*, \tag{2.132}$$

$$\theta(\tilde{x}) = -\frac{2\varphi^*}{1-2\varphi^*}. \tag{2.133}$$

As established earlier, $\theta(x)$ is differentiable and strictly decreasing on $(0, x_m)$, taking values in $(-\infty, 1)$. Therefore, the inverse function $\theta^{-1}(.)$ exists on the domain $(-\infty, 1)$. Hence, as long as $\varphi < 0.5$, $-\frac{2\varphi^*}{1-2\varphi^*} \in (-\infty, 1)$, the austerity threshold $\tilde{x}$ exists and is given by:

$$\tilde{x} = \theta^{-1}\left(-\frac{2\varphi^*}{1-2\varphi^*}\right), \quad \varphi^* < 0.5. \tag{2.134}$$

Further, if $\varphi^* < 0.5$, $\frac{d[-\varphi^d(x)-\varphi^s(x)]}{dx} = -\theta'(x)(1-2\varphi^*) > 0$, so that $-\varphi^d(x) > \varphi^s(x) > \varphi^d(x), \forall x \in (\tilde{x}, x_m)$. It also follows that:

$$\theta'(\tilde{x})\frac{d\tilde{x}}{d\varphi^*} = \frac{d}{d\varphi^*}\left[-\frac{2\varphi^*}{1-2\varphi^*}\right], \tag{2.135}$$

$$\frac{d\tilde{x}}{d\varphi^*} = -\frac{1}{\theta'(\tilde{x})}\frac{2}{(1-2\varphi^*)^2} > 0. \tag{2.136}$$

□

2.B Fiscal multipliers: (more) general cases

In this section we show that the results derived earlier hold in much more general settings. In particular, we introduce the class of *flexible* equilibria, which is a superset of the competitive equilibria. We then show that in any flexible equilibrium that has tightness fixed over the business cycle, both demand-side and supply-side multipliers are equal and acyclical, just like in

the competitive equilibrium. On the other hand, we show that the cyclicality results established under fixprice equilibria extend to the more general class of *frictional* equilibria, where part of the adjustment happens via tightness.

2.B.1 Flexible equilibria multipliers

In the previous section we started off be considering a competitive equilibrium, where tightness was fixed at the efficient level x^* and all adjustment happened via prices and wages. However, this is not the only way to pin down tightness. Below we consider two common alternatives found in search-and-matching literature (Nash bargaining, fixed markup pricing), before introducing a much more general *Tightness Determination Mapping (TDM)*.

Nash bargaining

One alternative, very common in the search-and-matching literature, is to consider Nash bargaining over the price between consumers and firms in order to get an extra equilibrium condition needed to close the model. In our case, the surplus to consumers from buying an additional unit of the produced good at price $\tilde{p}$ after a match is made is given by:

$$\mathcal{B}(\tilde{p}) = \frac{\chi}{c} - \tilde{p}, \tag{2.137}$$

whereas the firms' surplus from selling an extra unit at price $\tilde{p}$ is

$$\mathcal{S}(\tilde{p}) = \tilde{p} - pf(x). \tag{2.138}$$

Assuming the consumers' bargaining power is given by $\beta \in (0, 1)$, the solution to Nash bargaining is given by:

$$(1 - \beta)\mathcal{S}(p) = \beta\mathcal{B}(p). \tag{2.139}$$

Combining the above with agents' optimality conditions obtained earlier, one gets:

$$\frac{1-\beta}{\beta} = \frac{\gamma(x^L)}{1 - f(x^L)}, \quad \frac{dx^L}{d\beta} < 0. \tag{2.140}$$

As one can see, the condition above pins down tightness at $x = x^L$, and we can even get the equivalent of the Hosios (1990) condition for the bargaining

power β^* that delivers the socially efficient allocation, $\beta^* = \frac{1}{1+\frac{\gamma(x^*)}{1-f(x^*)}}$.

Fixed markup pricing

An alternative way to pin down tightness is to assume that the equilibrium price p is set as a fixed markup over the marginal cost, so that:

$$p = \mu \times mc, \tag{2.141}$$

where $\mu \geq 1$ is a markup parameter and mc is the marginal cost. From firms' optimisation problem one gets that the effective selling price $pf(x)$ is set equal to the marginal cost:

$$pf(x) = mc. \tag{2.142}$$

Combining the above two equations one gets the following condition for pinning down the level of tightness:

$$f(x^L) = \frac{1}{\mu}, \quad \frac{dx^L}{d\mu} < 0. \tag{2.143}$$

As before, the equivalent of the Hosios (1990) condition here is the markup μ^* that delivers the socially efficient allocation, n amely $\mu^* = \frac{1}{f(x^*)}$.

Generalization: Tightness Determination Mapping

In fact, the above approaches to pinning down tightness can be generalized by introducing the notion of a Tightness Determination Mapping (TDM):

Definition 2.5. *A Tightness Determination Mapping (TDM) $\mathcal{M}$ is given by:*

$$\mathcal{M}: \quad \left\{\Omega^M, \Omega^S, \Omega^T\right\} \to x^L, \tag{2.144}$$

*where $\Omega^M = \{\rho, \gamma, \psi, \alpha\}$ is the set of model structural parameters, $\Omega^S = \{\chi, a, G, \tau\}$ is the set of shock parameters, Ω^T is the set of parameters specific to the TDM and x^L is the resulting tightness. Further, a TDM $\mathcal{M}$ is said to be **shock invariant** if and only if*

$$\frac{d\mathcal{M}\left(\Omega^M, \Omega^S, \Omega^T\right)}{d\tilde{s}} = 0, \quad \forall \tilde{s} \in \Omega^S. \tag{2.145}$$

so that changes in shock parameters do not affect the determination of tightness.

It is easy to see that the TDM used in the competitive equilibrium is simply $x^L = x^*$. Also, Nash bargaining is a particular TDM with $\Omega^T = \{\beta\}$, which pins down tightness according to $\frac{1-\beta}{\beta} = \frac{\gamma(x^L)}{1-f(x^L)}$; similarly, fixed markup pricing is a TDM with $\Omega^T = \{\mu\}$, which pins down tightness according to $f(x^L) = \frac{1}{\mu}$. Note that all of the above are also shock invariant TDMs, as none of the shock parameters enter the conditions that pin down the level of tightness.

We can now define a class of *flexible* equilibria that is a superset of the competitive equilibrium considered before:

Definition 2.6. *A **flexible** equilibrium is a vector $(p^L, w^L, \mathcal{M})$, and associated allocations, such that the agents' optimality conditions and the market clearing conditions are satisfied with tightness pinned down at a level $x^L = \mathcal{M}\left(\Omega^M, \Omega^S, \Omega^T\right)$.*

Clearly, the competitive equilibrium considered earlier is just a flexible equilibrium with $x^L = x^*$ as the TDM. In fact, it can be shown that all of the comparative statics results established for the competitive equilibrium in Lemma 2.4 hold in the exact same way for any flexible equilibrium with a shock invariant TDM:

Lemma 2.7. *In any flexible equilibrium generated by a shock-invariant TDM, the following are the comparative statics of tightness (x), sales (y) and the price (p):*

$$\frac{dx}{d\chi} = 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} > 0; \qquad \frac{dx}{da} = 0, \frac{dy}{da} > 0, \frac{dp}{da} < 0 \tag{2.146}$$

Proof. Note that in this more generalized setting, condition (2.65) still holds:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \left(pf(x)a\right)^{\frac{1+\psi}{1-\alpha+\psi}} (1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \chi + p[1+\gamma(x)]G. \tag{2.147}$$

In a flexible equilibrium, $x = x^L = \mathcal{M}(\Omega^M, \Omega^S, \Omega^T)$; further, since the TDM $\mathcal{M}$ is shock-invariant it follows that $\frac{dx^L}{d\chi} = \frac{dx^L}{da} = 0$. The latter implies the

following comparative statics for p (for simplicity, evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} (f(x)a)^{\frac{1+\psi}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} p^{\frac{\alpha}{1-\alpha+\psi}} \frac{dp}{d\chi} = 1 \tag{2.148}$$

$$\frac{dp}{d\chi} = \alpha^{-\frac{\alpha}{1-\alpha+\psi}} (f(x)a)^{-\frac{1+\psi}{1-\alpha+\psi}} \frac{1-\alpha+\psi}{1+\psi} p^{-\frac{\alpha}{1-\alpha+\psi}} = \frac{1-\alpha+\psi}{1+\psi} \frac{p}{\chi} > 0. \tag{2.149}$$

$$\tag{2.150}$$

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} (f(x))^{\frac{1+\psi}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} (pa)^{\frac{\alpha}{1-\alpha+\psi}} (p + a\frac{dp}{da}) = 0$$

$$\frac{dp}{da} = -\frac{p}{a} < 0. \tag{2.151}$$

Finally, the above implies the following comparative statics for sales $y = [1+\gamma(x)](c(p,x)+G) = \chi/p + G[1+\gamma(x)]$ (also evaluated at $G = \tau = 0$):

$$py = \chi + p[1+\gamma(x)]G \tag{2.152}$$

$$\frac{dp}{d\chi} y + p\frac{dy}{d\chi} = 1 \tag{2.153}$$

$$\frac{dy}{d\chi} = \frac{1}{p}\left(1 - \frac{dp}{d\chi} y\right) = \frac{1}{p}\frac{\alpha}{1+\psi} > 0. \tag{2.154}$$

$$\frac{dp}{da} y + p\frac{dy}{da} = 0 \tag{2.155}$$

$$\frac{dy}{da} = -\frac{dp}{da}\frac{y}{p} > 0. \tag{2.156}$$

□

Of our particular interest, however, is the fact that all the results that we established for demand-side and supply-side fiscal multipliers under the competitive equilibrium remain true for any flexible equilibrium with a shock invariant TDM:

Proposition 2.4. *In any flexible equilibrium generated by a shock-invariant TDM, the demand-side fiscal multiplier and the supply-side fiscal multiplier are equal and given by:*

$$\varphi^* \equiv \frac{\alpha}{1+\psi} = \frac{1 - \frac{1}{|\epsilon^d|}}{1 + \frac{1}{\epsilon^s}}, \tag{2.157}$$

where $\alpha \in (0,1]$ and $\psi > 0$ are, respectively, returns to labor and inverse Frisch elasticity, whereas $|\epsilon^d| = \frac{1}{1-\alpha}$ and $\epsilon^s = \frac{1}{\psi}$ are (absolute) elasticities of labor demand and labor supply. Hence $\varphi^* \in (0,1]$ and it is pinned down by elasticities of labor

demand and labor supply.

Proof. Note that in this more generalized setting, condition (2.88) still holds, so that:

$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)}. \tag{2.158}$$

Further, the definition of demand-side fiscal multiplier also remains unchanged:

$$\varphi^d = \frac{d\{c+G\}}{dG} = \frac{dc}{dG} + 1 = \frac{\partial c}{\partial p}\frac{dp}{dG} + \frac{\partial c}{\partial x}\frac{dx}{dG} + 1 \tag{2.159}$$

In a flexible equilibrium, $x = x^L = \mathcal{M}(\Omega^M, \Omega^S, \Omega^T)$; further, since the TDM $\mathcal{M}$ is shock-invariant it follows that $\frac{dx^L}{dG} = 0$, which combined with (159) implies the following:

$$\varphi^d = \frac{\partial c}{\partial p}\frac{dp}{dG} + 1 = -\frac{\chi}{p[1+\gamma(x)]}\frac{1}{p}\frac{dp}{dG} + 1 = -c(p,x)\frac{1}{p}p\frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)} + 1 \tag{2.160}$$

$$= -\frac{1-\alpha+\psi}{1+\psi} + 1 \tag{2.161}$$

$$= \frac{\alpha}{1+\psi} \equiv \varphi^*. \tag{2.162}$$

Similarly, condition (2.95) also holds in this more generalised setting

$$\left[\frac{1}{p}\frac{dp}{d\tau} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau}\right] = \frac{\alpha}{1+\psi}. \tag{2.163}$$

And the definition of supply-side fiscal multiplier also stays the same:

$$\varphi^s = \frac{d\{c+G\}/\{c+G\}}{d[-\tau]} = -\frac{1}{c}\frac{dc}{d\tau} = -\frac{1}{c}\left[\frac{\partial c}{\partial p}\frac{dp}{d\tau} + \frac{\partial c}{\partial x}\frac{dx}{d\tau}\right]. \tag{2.164}$$

In a flexible equilibrium, $x = x^L = \mathcal{M}(\Omega^M, \Omega^S, \Omega^T)$; further, since the TDM $\mathcal{M}$ is shock-invariant it follows that $\frac{dx^L}{d\tau} = 0$, which combined with (163) implies the following:

$$\varphi^s = -\frac{1}{c}\frac{\partial c}{\partial p}\frac{dp}{d\tau} = -\frac{1}{c}\left[-\frac{\chi}{p[1+\gamma(x)]}\frac{1}{p}\right]\frac{dp}{d\tau} = \frac{1}{c}c\frac{1}{p}p\frac{\alpha}{1+\psi} \tag{2.165}$$

$$= \frac{\alpha}{1+\psi} = \varphi^d = \varphi^*. \tag{2.166}$$

□

In other words, any equilibrium where tightness remains fixed over the business cycle will have demand-side and supply-side fiscal multipliers both fixed at φ^* and acyclical.

2.B.2 Frictional equilibria multipliers

As an alternative to the competitive equilibrium, last section considered a fixprice equilibrium, where all adjustment is happening via tightness and wages. Here we start off by considering a slightly more general *rigid price* equilibrium, that allows for an arbitrary degree of price rigidity and nests fixprice equilibrium as a special case. Subsequently, we introduce a much more general notion of a *Frictional Mapping (FM)*.

Rigid price equilibrium

A rigid price equilibrium is formally introduced as:

Definition 2.7. *A **rigid price** equilibrium is a vector $(p_0, x, w, \varepsilon, \mathcal{M})$, and associated allocations, such that the agents' optimality conditions and the market clearing conditions are satisfied with price given by:*

$$p = (p_0)^{\varepsilon}(p^L)^{1-\varepsilon}, \quad \varepsilon \in (0, 1] \tag{2.167}$$

where ε is the degree of price rigidity and p_0 is a parameter and p^L is the price from the flexible equilibrium $(p^L, w^L, \mathcal{M})$.

Clearly, the fixprice equilibrium is just a special case under $\varepsilon = 1$. In fact, a non-fixprice rigid price equilibrium $(p_0, x, w, \varepsilon, \mathcal{M})$ where $\mathcal{M}$ is a shock invariant TDM shares a lot in common with the corresponding fixprice equilibrium. Particularly, the comparative statics to demand-side and supply-side shocks are given by:

Lemma 2.8. *In a non-fixprice ($\varepsilon \in (0, 1)$) rigid price equilibrium $(p_0, x, w, \varepsilon, \mathcal{M})$ where $\mathcal{M}$ is a shock invariant TDM, the following are the comparative statics of tightness (x), sales (y) and the price (p):*

$$\frac{dx}{d\chi} > 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} > 0; \qquad \frac{dx}{da} < 0, \frac{dy}{da} > 0, \frac{dp}{da} < 0 \tag{2.168}$$

Proof. Special case of Lemma 9 under $\mathcal{T}(p^L; \{p_0, \varepsilon\}) = (p_0)^{\varepsilon}(p^L)^{1-\varepsilon}$. □

As one can see, the only difference compared to a fixprice equilibrium is that the price co-moves with tightness and supply-side shocks have an effect on the level of sales.

Of a greater interest to us, however, are the properties of fiscal multipliers under rigid price equilibria. The following proposition establishes the demand-side multiplier is a rigid price equilibrium:

Proposition 2.5. *In a rigid price equilibrium $(p_0, x, w, \varepsilon, \mathcal{M})$, where $\mathcal{M}$ is a shock invariant TDM, the demand-side fiscal multiplier $\varphi^d(x)$ is given by*

$$\varphi^d(x) = \varphi^* + \theta(x) \times \left[(1-\varphi^*)\{1-(1-\varepsilon)g(x, x^L)\}\right] \tag{2.169}$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the flexible equilibrium multiplier and the function $g(x, x^L)$ is given by:

$$g(x, x^L) = \frac{f(x) - \rho x}{f(x^L) - \rho x^L}. \tag{2.170}$$

Hence, $\varphi^d(x) \in (-\infty, 1]$ and $\frac{d\varphi^d(x)}{dx}|_{x=x^L} < 0$.

Proof. Special case of Proposition 2.7 under $\mathcal{T}(p^L; \{p_0, \varepsilon\}) = (p_0)^\varepsilon (p^L)^{1-\varepsilon}$. □

Note that for $\varepsilon = 1$ the expression above collapses back to the fixprice equilibrium demand-side multiplier from Proposition 2.2. We can also see that the rigid price equilibrium demand-side multiplier above maintains a lot of the properties of its fixprice equilibrium counterpart. In particular, it also collapses back to φ^* if the equilibrium tightness happens to coincide with the socially efficient one $(x = x^*)$, it also lies between $-\infty$ and one, so that consumption always gets crowded out; it is also guaranteed to fall in tightness, although only in the neighbourhood of the corresponding flexible equilibrium allocation. The role played by ε here is in determining the relative magnitude of the state-dependent component. The upper panel of Figure 9 (drawn for simplicity under $x^L = x^*$) shows that as the degree of price rigidity ε falls, the multiplier becomes flatter around x^* at the level equal to φ^* suggesting that the degree of state-dependence falls as well.

Similarly, one can derive the supply-side multiplier under rigid price equilibrium:

Proposition 2.6. *In a rigid price equilibrium $(p_0, x, w, \varepsilon, \mathcal{M})$, where $\mathcal{M}$ is a shock invariant TDM, the supply-side fiscal multiplier $\varphi^s(x)$ is given by*

$$\varphi^s(x) = \varphi^* - \theta(x) \times \varepsilon\varphi^*, \tag{2.171}$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ *is the long-run equilibrium multiplier. Hence,* $\varphi^d(x) \in (0, +\infty)$ *and* $\frac{d\varphi^d(x)}{dx} > 0, \forall x \in (0, x_m)$.

Proof. Special case of Proposition 2.8 under $\mathcal{T}(p^L; \{p_0, \varepsilon\}) = (p_0)^\varepsilon (p^L)^{1-\varepsilon}$. □

Again, under $\varepsilon = 1$ the expression above collapses back to the fixprice equilibrium supply-side multiplier. One can see that the expression above shares every single property with the fixprice equilibrium counterpart, the only difference being the magnitude of state-dependence, which increases in the degree of price rigidity ε, as shown in the bottom panel of Figure 9 (again, for simplicity drawn for $x^L = x^*$).

Generalization: Frictional Mapping

One can in fact show that the properties of fiscal multipliers that we have established for the rigid price equilibrium hold more generally, and not for the particular parametric form of frictions that we have considered so far. We generalize our findings by introducing the notion of a Frictional Mapping:

Definition 2.8. *For a given flexible equilibrium* $(p^L, w^L, \mathcal{M})$*, a Frictional Mapping (FM)* $\mathcal{T}$ *is given by:*

$$\mathcal{T}: \quad \left\{p^L, \Omega^F\right\} \to p^F, \tag{2.172}$$

where Ω^F *is the set of parameters specific to the FM and* p^F *is the resulting price. Moreover, the Frictional Mapping* $\mathcal{T}(p^L; \Omega^F)$ *is said to be* ***contractionary*** *if and only if*

$$\frac{d \ln p^F}{d \ln p^L} = \frac{d\mathcal{T}(p^L; \Omega^F)}{dp^L} \frac{p^L}{p^F} \in [0, 1). \tag{2.173}$$

Having defined a Frictional Mapping (FM), one can now define a *frictional* equilibrium:

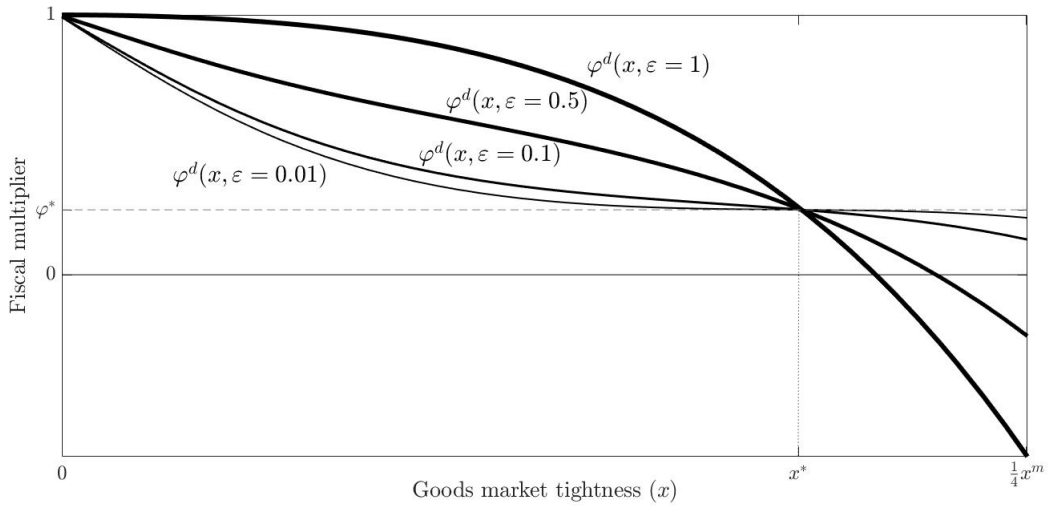
Definition 2.9. *A* ***frictional*** *equilibrium is a vector* $(p^F, x^F, w^F, \mathcal{T}, \mathcal{M})$*, and associated allocations, such that the agents' optimality conditions and the market clearing conditions are satisfied with price given by:*

$$p^F = \mathcal{T}(p^L) \tag{2.174}$$

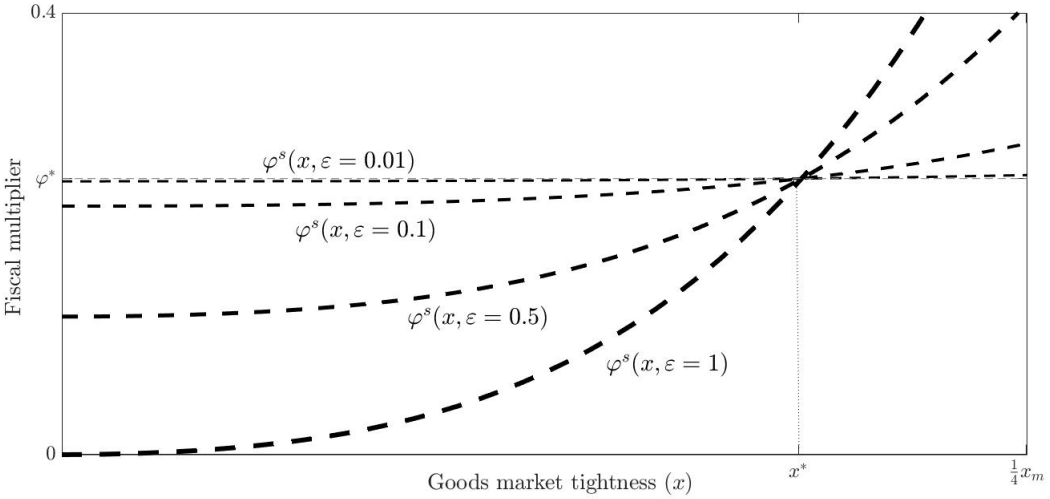
where $\mathcal{T}$ *is the Frictional Mapping and* p^L *is the price from the flexible equilibrium* $(p^L, w^L, \mathcal{M})$.

Figure 2.B.1: Fiscal multipliers in a rigid price equilibrium

(a) Demand-side fiscal multiplier $\varphi^d(x)$



(b) Supply-side fiscal multiplier $\varphi^s(x)$



***Notes**: Panels (a) and (b) show demand-side and supply-side fiscal multipliers in a rigid price equilibrium of a calibrated version of our model ($\alpha = 0.3, \delta = 2, \rho = 0.1, \psi = 0.2, x^L = x^*$). Panel (a) shows demand-side fiscal multipliers for different values of the price rigidity parameter ε – one case see that $\varphi^d(x)$ strictly falls in tightness for all considered values of ε, but the degree of state-dependence rises in the degree of price rigidity; in Panel (b) we can see that the supply-side fiscal multiplier strictly rises in tightness for all values of ε considered, but again the degree if state-dependence falls as we allow for more price flexibility.*

Rigid price equilibrium is a special case of a frictional equilibrium for $\mathcal{T}(z) = (p_0)^{\varepsilon}(z)^{1-\varepsilon}$, $\Omega^F = \{p_0, \varepsilon\}$, $\varepsilon \in (0,1]$. Further, the above frictional mapping associated with a rigid price equilibrium is indeed contractionary, since

$$\frac{d\mathcal{T}(z;\Omega^F)}{dz}\frac{z}{p^F} = (1-\varepsilon) \in [0,1), \tag{2.175}$$

as $\varepsilon \in (0,1]$.

We can now derive and discuss properties of demand-side and supply-side multipliers in a generic frictional equilibrium. Firstly, note that the comparative statics to demand-side and supply-side shocks established in a rigid price equilibrium extend to a generic frictional equilibrium generated by a contractionary frictional mapping:

Lemma 2.9. *In a non-fixprice* $\left(\frac{d\mathcal{T}(p^L;\Omega^F)}{dp^L}\frac{p^L}{p^F} \neq 0\right)$ *frictional equilibrium* $(p^F, x^F, w^F, \mathcal{T}, \mathcal{M})$ *where* $\mathcal{T}$ *is a contractionary FM and* $\mathcal{M}$ *is a shock invariant TDM, the following are the comparative statics of tightness* (x)*, sales* (y) *and the price* (p)*:*

$$\frac{dx}{d\chi} > 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} > 0; \qquad \frac{dx}{da} < 0, \frac{dy}{da} > 0, \frac{dp}{da} < 0 \tag{2.176}$$

Proof. Note that condition (2.65) still holds in this more generalized setting:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left(pf(x)a\right)^{\frac{1+\psi}{1-\alpha+\psi}}(1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \chi + p[1+\gamma(x)]G. \tag{2.177}$$

Differentiate both sides with respect to χ (evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\frac{1+\psi}{1-\alpha+\psi}\left[pf(x)a\right]^{\frac{1+\psi}{1-\alpha+\psi}}\left[\frac{1}{p}\frac{dp}{d\chi} + \frac{f'(x)}{f(x)}\frac{dx}{d\chi}\right] = 1, \tag{2.178}$$

$$\frac{dx}{d\chi} = \frac{f(x)}{f'(x)}\left[\alpha^{-\frac{\alpha}{1-\alpha+\psi}}\frac{1-\alpha+\psi}{1+\psi}\left[pf(x)a\right]^{-\frac{1+\psi}{1-\alpha+\psi}} - \frac{1}{p}\frac{dp}{d\chi}\right], \tag{2.179}$$

$$\frac{dx}{d\chi} = \frac{f(x)}{f'(x)}\left[\frac{1-\alpha+\psi}{1+\psi}\frac{1}{\chi} - \frac{1}{p}\frac{dp}{d\chi}\right]. \tag{2.180}$$

Since $p = \mathcal{T}(p^L)$, it follows that $\frac{dp}{d\chi} = \frac{d\mathcal{T}(p^L)}{dp^L}\frac{dp^L}{d\chi}$; further, from Lemma 7 we

know that $\frac{dp^L}{d\chi} = \frac{1-\alpha+\psi}{1+\psi}\frac{p^L}{\chi}$ and it follows that:

$$\frac{dp}{d\chi} = \underbrace{\frac{d\mathcal{T}(p^L)}{dp^L}\frac{p^L}{p}}_{\in(0,1) \text{ as } \mathcal{T} \text{ contractionary FM}} \frac{1-\alpha+\psi}{1+\psi}\frac{p}{\chi} > 0. \tag{2.181}$$

$$\frac{dx}{d\chi} = \frac{f(x)}{f'(x)}\left[\frac{1-\alpha+\psi}{1+\psi}\frac{1}{\chi}\underbrace{\left(1-\frac{d\mathcal{T}(p^L)}{dp^L}\frac{p^L}{p}\right)}_{\in(0,1) \text{ as } \mathcal{T} \text{ contractionary FM}}\right] > 0 \tag{2.182}$$

Given that $py = \chi$, it follows that $\frac{dy}{d\chi} = \frac{1}{p}\left[1 - \frac{dp}{d\chi}\frac{\chi}{p}\right]$ and hence:

$$\frac{dy}{d\chi} = \frac{1}{p}\left[1 - \underbrace{\frac{d\mathcal{T}(p^L)}{dp^L}\frac{p^L}{p}}_{\in(0,1) \text{ as } \mathcal{T} \text{ contractionary FM}}\frac{1-\alpha+\psi}{1+\psi}\right] > 0. \tag{2.183}$$

Similarly, differentiate both sides of (2.177) with respect to a (evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\frac{1+\psi}{1-\alpha+\psi}\left[pf(x)a\right]^{\frac{1+\psi}{1-\alpha+\psi}}\left[\frac{1}{p}\frac{dp}{da} + \frac{f'(x)}{f(x)}\frac{dx}{da} + \frac{1}{a}\right] = 0, \tag{2.184}$$

$$\frac{dx}{da} = -\frac{f(x)}{f'(x)}\left[\frac{1}{a} + \frac{1}{p}\frac{dp}{da}\right] = 0. \tag{2.185}$$

Since $p = \mathcal{T}(p^L)$, it follows that $\frac{dp}{da} = \frac{d\mathcal{T}(p^L)}{dp^L}\frac{dp^L}{da}$; further, from Lemma 2.7 we know that $\frac{dp^L}{da} = -\frac{p^L}{a}$ and it follows that:

$$\frac{dp}{da} = -\underbrace{\frac{d\mathcal{T}(p^L)}{dp^L}\frac{p^L}{p}}_{\in(0,1) \text{ as } \mathcal{T} \text{ contractionary FM}}\frac{p}{a} < 0. \tag{2.186}$$

$$\frac{dx}{da} = -\frac{f(x)}{f'(x)}\left[\frac{1}{a}\underbrace{\left(1-\frac{d\mathcal{T}(p^L)}{dp^L}\frac{p^L}{p}\right)}_{\in(0,1) \text{ as } \mathcal{T} \text{ contractionary FM}}\right] < 0 \tag{2.187}$$

Given that $py = \chi$, it follows that:

$$\frac{dy}{da} = -\frac{dp}{da}\frac{y}{p} > 0. \tag{2.188}$$

□

Moreover, one can also solve for the demand-side fiscal multiplier under a generic frictional equilibrium and see that under certain properties it also falls in tightness:

Proposition 2.7. *In a frictional equilibrium $(p^F, x^F, w^F, \mathcal{T}, \mathcal{M})$, where $\mathcal{T}$ is a contractionary FM and $\mathcal{M}$ is a shock invariant TDM, the demand-side fiscal multiplier $\varphi^d(x)$ is given by*

$$\varphi^d(x) = \varphi^* + \theta(x) \times \left[(1-\varphi^*)\left\{1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}g(x, x^L)\right\}\right] \tag{2.189}$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the flexible equilibrium multiplier and the function $g(x, x^L)$ is given by:

$$g(x, x^L) = \frac{f(x) - \rho x}{f(x^L) - \rho x^L}. \tag{2.190}$$

Hence, $\varphi^d(x) \in (-\infty, 1)$ and $\frac{d\varphi^d(x)}{dx}|_{x=x^L} < 0$.

Proof. Note that (2.89) still holds in this more general setting:

$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)} = (1-\varphi^*)\frac{1}{c(p,x)}. \tag{2.191}$$

From the definition of the demand-side fiscal multiplier:

$$\varphi^d(x) = \frac{d\{c+G\}}{dG} = \frac{dc}{dG} + 1 = \frac{\partial c}{\partial p}\frac{dp}{dG} + \frac{\partial c}{\partial x}\frac{dx}{dG} + 1 = \tag{2.192}$$

$$= -c(p,x)\left[\frac{1}{p}\frac{dp}{dp^L}\frac{dp^L}{dG}\right] - c(p,x)\underbrace{\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}_{1-\theta(x)}\left[(1-\varphi^*)\frac{1}{c(p,x)} - \frac{1}{p}\frac{dp}{dp^L}\frac{dp^L}{dG}\right]. \tag{2.193}$$

From Proposition 2.4 we know that $\frac{dp^L}{dG} = p^L(1-\varphi^*)\frac{1}{c(p^L,x^L)}$, hence:

$$\varphi^d(x) = -c(p,x)\frac{dp}{dp^L}\frac{p^L}{p}(1-\varphi^*)c(p^L,x^L) - c(p,x)(1-\theta(x))\left[(1-\varphi^*)\frac{1}{c(p,x)} - \frac{dp}{dp^L}\frac{p^L}{p}(1-\varphi^*)\frac{1}{c(p^L,x^L)}\right] \tag{2.194}$$

$$= 1-(1-\varphi^*)(1-\theta(x)) - \theta(x)(1-\varphi^*)\frac{dp}{dp^L}\frac{p^L}{p}\frac{c(p,x)}{c(p^L,x^L)} \tag{2.195}$$

$$= \varphi^* + \theta(x)(1-\varphi^*)\left[1-\frac{dp}{dp^L}\frac{p^L}{p}\frac{c(p,x)}{c(p^L,x^L)}\right] \tag{2.196}$$

$$= \varphi^* + \theta(x)(1-\varphi^*)\left[1-\frac{dp}{dp^L}\frac{p^L}{p}\frac{p^L[1+\gamma(x^L)]}{p[1+\gamma(x)]}\right] \tag{2.197}$$

$$= \varphi^* + \theta(x)(1-\varphi^*)\left[1-\frac{dp}{dp^L}\frac{p^L}{p}\frac{\frac{p^Lf(x^L)}{f(x^L)-\rho x^L}}{\frac{pf(x)}{f(x)-\rho x}}\right] \tag{2.198}$$

$$= \varphi^* + \theta(x)(1-\varphi^*)\left[1-\frac{dp}{dp^L}\frac{p^L}{p}\frac{f(x)-\rho x}{f(x^L)-\rho x^L}\right] \tag{2.199}$$

$$= \varphi^* + \theta(x)(1-\varphi^*)\left[1-\frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}g(x,x^L)\right], \tag{2.200}$$

where $g(x,x^L) = \frac{f(x)-\rho x}{f(x^L)-\rho x^L}$. Further, notice that:

$$\frac{d\varphi^d(x)}{dx}|_{x=x^L} = \theta'(x^L)(1-\varphi^*)\left[1-\frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\right] - \theta(x^L)(1-\varphi^*)\frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\frac{f'(x^L)-\rho}{f(x^L)-\rho x^L} < 0, \forall x^L \in (0,x_m) \tag{2.201}$$

since $\theta'(x^L) < 0, \theta(x^L)(f'(x^L)-\rho) > 0, \forall x \in (0,x,)$. Also, it follows that $\varphi^d(0) = \varphi^* + \theta(0)(1-\varphi^*)[1-0] = 1$, and $\lim_{x\to x_m^-}\varphi^d(x) = \varphi^* + \lim_{x\to x_m^-}(1-\varphi^*)[1-\frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}] = -\infty$, so that $\varphi^d(x) \in (-\infty,1), \forall x \in (0,x_m)$. □

Similarly, one can also solve for the supply-side fiscal multiplier in a generic frictional equilibrium and establish its properties:

Proposition 2.8. *In a frictional equilibrium $(p^F,x^F,w^F,\mathcal{T},\mathcal{M})$, where $\mathcal{T}$ is a contractionary FM and $\mathcal{M}$ is a shock-invariant TDM, the supply-side fiscal multiplier $\varphi^s(x)$ is given by*

$$\varphi^s(x) = \varphi^* - \theta(x)\times\left(1-\frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\right)\varphi^*, \tag{2.202}$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the flexible equilibrium multiplier. Hence, $\varphi^d(x) \in (0,+\infty)$ and $\frac{d\varphi^d(x)}{dx} > 0, \forall x \in (0,x_m)$.

Proof. Note that (2.95) still holds in this more general setting:

$$\left[\frac{1}{p}\frac{dp}{d\tau} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau}\right] = \varphi^*. \tag{2.203}$$

From the definition of the supply-side fiscal multiplier:

$$\varphi^s(x) = \frac{d\{c+G\}/\{c+G\}}{d[-\tau]} = -\frac{1}{c(p,x)}\frac{dc}{d\tau} = -\frac{1}{c(p,x)}\left[\frac{\partial c}{\partial p}\frac{dp}{d\tau} + \frac{\partial c}{\partial x}\frac{dx}{d\tau}\right] \tag{2.204}$$

$$= \frac{1}{c(p,x)}\left[c(p,x)\frac{1}{p}\frac{dp}{d\tau} + c(p,x)\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}\left(\varphi^* - \frac{1}{p}\frac{dp}{d\tau}\right)\right] \tag{2.205}$$

$$= (1-\theta(x))\varphi^* + \frac{1}{p}\frac{d\mathcal{T}(p^L)}{dp^L}\frac{dp^L}{d\tau}\theta(x). \tag{2.206}$$

From Lemma 4 we know that $\frac{dp^L}{d\tau} = p^L\varphi^*$:

$$\varphi^s(x) = \varphi^* - \theta(x)\left(1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\right)\varphi^*. \tag{2.207}$$

Further, $\frac{d\varphi^s(x)}{dx} = -\theta(x)\left(1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\right)\varphi^* > 0, \forall x \in (0, x_m)$ since $\theta'(x) < 0, \forall x \in (0, x_m)$. Also, $\varphi^s(0) = \varphi^*\frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)} \in [0,1)$, and $\lim_{x\to x_m^-}\varphi^s(x) = \varphi^* - \lim_{x\to x_m}\theta(x)\left(1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\right) = +\infty$, so that $\varphi^s(x) \in (0, +\infty), \forall x \in (0, x_m)$. □

2.B.3 Cyclicality of fiscal multipliers

Firstly , note that our result of equal and acyclical demand-side and supply-side multipliers in a competitive equilibrium extends more generally to *any* flexible equilibrium generated by a shock-invariant TDM:

Corollary 2.10. *In any flexible equilibrium generated by policy-invariant TDM both demand-side and supply-side multipliers are acyclical.*

Proof. Trivial consequence of Proposition 2.4: in any flexible equilibrium generated by a shock-invariant TDM, both multipliers are equal to $\varphi^* = \frac{\alpha}{1+\psi}$ and do not change as either preference χ or technology a varies. □

In the words, any equilibrium that sees tightness fixed over the business cycle, will see both multipliers fixed at φ^* and hence acyclical.

Further, our state-dependence result for the demand-side multiplier in a fixprice equilibrium still holds in *any* frictional equilibrium as long as the

elasticity between frictional and flexible price is in $[0, 1)$ and the flexible equilibrium around which the friction is defined is generated by a shock-inariant TDM:

Corollary 2.11. *In any frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM, in the local neighbourhood of the flexible equilibrium allocation, the demand-side multiplier is countercyclical under demand-driven fluctuations, and procyclical under supply-driven fluctuations.*

Proof. From Lemma 2.9 we know that in any frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM, $\frac{dx}{d\chi} > 0, \frac{dx}{da} < 0$; further, from Proposition 2.7 we know that in a frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM $\frac{d\varphi^d(x)}{dx}|_{x=x^L} < 0$. Hence, $\frac{d\varphi^d(x)}{d\chi}|_{x=x^L} = \frac{d\varphi^d(x)}{dx}|_{x=x^L}\frac{dx}{d\chi}|_{x=x^L} < 0$ and $\frac{d\varphi^d(x)}{da}|_{x=x^L} = \frac{d\varphi^d(x)}{dx}|_{x=x^L}\frac{dx}{da}|_{x=x^L} > 0$. □

Similarly for the supply-side multiplier:

Corollary 2.12. *In any frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM, the supply-side multiplier is procyclical under demand-driven fluctuations, and countercyclical under supply-driven fluctuations.*

Proof. From Lemma 2.9 we know that in any frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM, $\frac{dx}{d\chi} > 0, \frac{dx}{da} < 0$; further, from Proposition 2.8 we know that in a frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM $\frac{d\varphi^s(x)}{dx} > 0$. Hence, $\frac{d\varphi^s(x)}{d\chi} = \frac{d\varphi^s(x)}{dx}\frac{dx}{d\chi} > 0$ and $\frac{d\varphi^s(x)}{da} = \frac{d\varphi^s(x)}{dx}\frac{dx}{da} < 0$. □

2.C Alternative fiscal instruments

2.C.1 Government employment

Let the government employ a fraction $h \in [0, 1)$ of the households' labor supply, so that government labor demand is given by $n^G = hl$ and the government collects additional lump-sum taxes to finance public sector wages, so that $T = p[1 + \gamma(x)]G - wn\tau - wn^G$. Then labor market clearing condition

becomes:

$$n(w;p,x,\tau) + n^G = l(w) \tag{2.208}$$
$$n(w;p,x,\tau) = (1-h)l(w) \tag{2.209}$$

Without loss of generality, assume $\tau = 0$, and substitute our solutions for n and l:

$$[\alpha p f(x) a]^{\frac{1}{1-\alpha}} w^{-\frac{1}{1-\alpha}} = (1-h) w^{\frac{1}{\psi}} \tag{2.210}$$
$$[\alpha p f(x) a]^{\frac{1}{1-\alpha}} (1-h)^{-1} = w^{\frac{1-\alpha+\psi}{\psi(1-\alpha)}} \tag{2.211}$$
$$w = [\alpha p f(x) a]^{\frac{\psi}{1-\alpha+\psi}} (1-h)^{-\frac{\psi(1-\alpha)}{1-\alpha+\psi}} \tag{2.212}$$
$$n = (1-h)l(w) = (1-h)w^{\frac{1}{\psi}} = [\alpha p f(x) a]^{\frac{1}{1-\alpha+\psi}} (1-h)^{\frac{\psi}{1-\alpha+\psi}}. \tag{2.213}$$

Substituting the solution for n above into the goods market clearing condition:

$$\frac{f(x) a n^\alpha}{1+\gamma(x)} = c(p,x) + G \tag{2.214}$$
$$p f(x) a n^\alpha = \chi + p[1+\gamma(x)]G \tag{2.215}$$
$$p f(x) a [\alpha p f(x) a]^{\frac{\alpha}{1-\alpha+\psi}} (1-h)^{\frac{\alpha\psi}{1-\alpha+\psi}} \tag{2.216}$$
$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} [a p f(x)]^{\frac{1+\psi}{1-\alpha+\psi}} (1-h)^{\frac{\alpha\psi}{1-\alpha+\psi}} = \chi + p[1+\gamma(x)]G \tag{2.217}$$

Differentiating with respect to h (for simplicity, where $G = h = 0$):

$$\frac{1+\psi}{1-\alpha+\psi} [a p f(x)]^{\frac{1+\psi}{1-\alpha+\psi}-1} \left(a \frac{dp}{dh} f(x) + a p f'(x) \frac{dx}{dh} \right) (1-h)^{\frac{\alpha\psi}{1-\alpha+\psi}} = [a p f(x)]^{\frac{1+\psi}{1-\alpha+\psi}} \frac{\alpha\psi}{1-\alpha+\psi} (1-h)^{\frac{\alpha\psi}{1-\alpha+\psi}-1} \tag{2.218}$$

$$\frac{1+\psi}{1-\alpha+\psi} [a p f(x)]^{\frac{1+\psi}{1-\alpha+\psi}} \left(\frac{1}{p}\frac{dp}{dh} + \frac{f'(x)}{f(x)}\frac{dx}{dh} \right) = [a p f(x)]^{\frac{1+\psi}{1-\alpha+\psi}} \frac{\alpha\psi}{1-\alpha+\psi} \tag{2.219}$$

$$\frac{1}{p}\frac{dp}{dh} + \frac{f'(x)}{f(x)}\frac{dx}{dh} = \frac{\alpha\psi}{1+\psi}. \tag{2.220}$$

Define the government employment multiplier:

$$\varphi^h(x) \equiv \frac{d\{c+G\}/\{c+G\}}{dh} = \frac{1}{c}\frac{c}{h}. \tag{2.221}$$

In a competitive equilibrium $x = x^*$ so that:

$$\varphi^h = \frac{1}{c}\left[-\frac{\chi}{p[1+\gamma(x)]}\frac{1}{p}\frac{dp}{dh}\right] = -\frac{1}{p}p\frac{\alpha\psi}{1+\psi} = \frac{\alpha\psi}{1+\psi}. \tag{2.222}$$

Therefore, just like the multipliers studied in the main text, the government employment multiplier is acyclical in the competitive equilibrium and is pinned down exclusively by the relative elasticities of labor demand and labor supply.

In a fixprice equilibrium, $p = p_0$ so that:

$$\varphi^h(x) = -\frac{\gamma'(x)}{1+\gamma(x)}\frac{dx}{dh} = -\underbrace{\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}_{1-\theta(x)}\frac{\alpha\psi}{1+\psi} = (\theta(x)-1)\frac{\alpha\psi}{1+\psi}. \tag{2.223}$$

Note that $\frac{d\varphi^h(x)}{dx} = \theta'(x)\frac{\alpha\psi}{1+\psi} < 0, \forall x \in (0, x_m)$; therefore, in a fixprice equilibrium, the government employment multiplier strictly falls in tightness and has the same cyclicality properties as the government consumption spending multiplier considered in the main text.

2.C.2 Distortionary taxes on consumption, labor income and firms' sales

We introduce taxes on households' consumption and labor income, so that the budget constraint of the representative households is given by:

$$p(1+\tau^c)[1+\gamma(x)] + m \leq w(1-\tau^l)l + \bar{m} + \Pi - T, \tag{2.224}$$

where τ^c is the consumption tax rate, τ^l is the labor income tax rate. The consumption function and the labor supply function become:

$$c(p,x) = \frac{\chi}{p(1+\tau^c)[1+\gamma(x)]}, \qquad l(w) = [w(1-\tau^l)]^{\frac{1}{\psi}}. \tag{2.225}$$

Further, we introduce taxes on firms' payroll, so that firms' profits are given by:

$$\Pi = p(1-\tau^s)f(x)an^\alpha - wn(1+\tau), \tag{2.226}$$

where τ^s is the rate of tax on firms' sales. The labor demand function resulting from profit maximisation is then given by:

$$n(w;p,x,\tau,\tau^s) = \left[\frac{\alpha p(1-\tau^s)f(x)a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}}. \tag{2.227}$$

Combining the labor demand function with labor market clearing condition delivers the following equilibrium employment:

$$n = [\alpha p f(x)a]^{\frac{1}{1-\alpha+\psi}}(1-\tau^s)^{\frac{1}{1-\alpha+\psi}}(1+\tau)^{-\frac{1}{1-\alpha+\psi}}(1-\tau^l)^{\frac{1}{1-\alpha+\psi}}. \tag{2.228}$$

Using the goods market clearing condition:

$$\frac{f(x)an^\alpha}{1+\gamma(x)} = c(p,x) + G \tag{2.229}$$

$$pf(x)an^\alpha = \frac{\chi}{1+\tau^c} + p[1+\gamma(x)]G \tag{2.230}$$

$$\alpha^{\frac{\alpha}{1\alpha+\psi}}[pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}}(1-\tau^s)^{\frac{\alpha}{1-\alpha+\psi}}(1-\tau)^{-\frac{\alpha}{1-\alpha+\psi}}(1-\tau^l)^{\frac{\alpha}{1-\alpha+\psi}} = \frac{\chi}{1+\tau^c} + p[1+\gamma(x)]G. \tag{2.231}$$

Consumption tax cut multiplier

Differentiate with respect to τ^c (at $\tau = \tau^c = \tau^l = \tau^s = G = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left[\frac{1+\psi}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1}\left\{\frac{dp}{d\tau^c}f(x)a + pf'(x)\frac{dx}{d\tau^c}\right\}\right] = -\frac{\chi}{(1+\tau^c)^2} \tag{2.232}$$

$$\underbrace{\alpha^{\frac{\alpha}{1-\alpha+\psi}}[pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}}}_{\chi}\left\{\frac{1}{p}\frac{dp}{d\tau^c} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau^c}\right\} = -\chi\frac{1-\alpha+\psi}{1+\psi} \tag{2.233}$$

$$\frac{1}{p}\frac{dp}{d\tau^c} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau^c} = \varphi^* - 1. \tag{2.234}$$

Define the consumption tax cut multiplier:

$$\varphi^{\tau^c}(x) \equiv \frac{d\{c+G\}/\{c+G\}}{d[-\tau^c]} = -\frac{1}{c}\frac{dc}{d\tau^c}. \tag{2.235}$$

In a competitive equilibrium $x = x^*$, so that:

$$\varphi^{\tau^c} = -\frac{1}{c}\left[-c\frac{1}{p}\frac{dp}{d\tau^c} + \frac{\partial c}{\partial \tau^c}\right] = -\frac{1}{c}\left[-c\frac{1}{p}p(\varphi^* - 1) - c\right] = \varphi^*, \tag{2.236}$$

so that in a competitive equilibrium the consumption tax cut multiplier is acyclical and pinned down exclusively by elasticities of labor supply and labor demand.

In a fixprice equilibrium $p = p_0$, so that:

$$\varphi^{\tau^c}(x) = -\frac{1}{c}\left[-c\frac{\gamma'(x)}{1+\gamma(x)}\frac{dx}{d\tau^c} + \frac{\partial c}{\partial \tau^c}\right] = -\frac{1}{c}\left[-c\underbrace{\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}_{1-\theta(x)}(\varphi^* - 1) - c\right] = \varphi^* + \theta(x)(1-\varphi^*) = \varphi^d)(x), \tag{2.237}$$

so that in a fixprice equilibrium the consumption tax cut multiplier is identical to the government consumption spending multiplier, and thus shares all of the properties of the latter.

Labor income tax cut multiplier

Differentiate the goods market clearing condition with respect to τ^l (at $\tau = \tau^c = \tau^l = \tau^s = G = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left[\frac{1+\psi}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1}\left\{\frac{dp}{d\tau^l}f(x)a + pf'(x)\frac{dx}{d\tau^l}a\right\} - \frac{\alpha}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}\right] = 0 \tag{2.238}$$

$$\frac{1}{p}\frac{dp}{d\tau^l} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau^l} = \frac{\alpha}{1+\psi} = \varphi^*. \tag{2.239}$$

Define the labor income tax cut multiplier:

$$\varphi^{\tau^l}(x) \equiv \frac{d\{c+G\}/\{c+G\}}{d[-\tau^l]} = -\frac{1}{c}\frac{dc}{d\tau^l}. \tag{2.240}$$

In a competitive equilibrium $x = x^*$, so that:

$$\varphi^{\tau^l} = -\frac{1}{c}\frac{\partial c}{\partial p}\frac{dp}{d\tau^l} = -\frac{1}{c}[-c\frac{1}{p}]p\varphi^* = \varphi^*, \tag{2.241}$$

so that in a competitive equilibrium the labor income tax cut multiplier is acyclical and pinned down exclusively by the elasticities of labor demand and labor supply.

In a fixprice equilibrium $p = p_0$, so that:

$$\varphi^{\tau^l}(x) = -\frac{1}{c}\frac{\partial c}{\partial x}\frac{dx}{d\tau^l} = -\frac{1}{c}[-c\underbrace{\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}_{1-\theta(x)}]\varphi^* = \varphi^* - \theta(x)\varphi^* = \varphi^s(x), \tag{2.242}$$

so that in a fixprice equilibrium the labor income tax cut multiplier is identical to the payroll tax cut multiplier considered in the main text, and shares all of its properties.

Firms' sales tax cut multiplier

Differentiate the goods market clearing condition with respect to τ^s (at $\tau = \tau^c = \tau^l = \tau^s = G = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left[\frac{1+\psi}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1}\left\{\frac{dp}{d\tau^s}f(x)a + pf'(x)\frac{dx}{d\tau^s}a\right\} - \frac{\alpha}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}\right] = 0 \tag{2.243}$$

$$\frac{1}{p}\frac{dp}{d\tau^s} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau^s} = \frac{\alpha}{1+\psi} = \varphi^*. \tag{2.244}$$

Define the sales tax cut multiplier:

$$\varphi^{\tau^s}(x) \equiv \frac{d\{c+G\}/\{c+G\}}{d[-\tau^s]} = -\frac{1}{c}\frac{dc}{d\tau^s}. \tag{2.245}$$

In a competitive equilibrium $x = x^*$, so that:

$$\varphi^{\tau^s} = -\frac{1}{c}\frac{\partial c}{\partial p}\frac{dp}{d\tau^s} = -\frac{1}{c}[-c\frac{1}{p}]p\varphi^* = \varphi^*, \tag{2.246}$$

so that in a competitive equilibrium the sales tax cut multiplier is acyclical and pinned down exclusively by the elasticities of labor demand and labor supply.

In a fixprice equilibrium $p = p_0$, so that:

$$\varphi^{\tau^s}(x) = -\frac{1}{c}\frac{\partial c}{\partial x}\frac{dx}{d\tau^s} = -\frac{1}{c}[-c\underbrace{\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}_{1-\theta(x)}]\varphi^* = \varphi^* - \theta(x)\varphi^* = \varphi^s(x), \tag{2.247}$$

so that in a fixprice equilibrium the sales tax cut multiplier is identical to the payroll tax cut multiplier considered in the main text, and shares all of its properties.

2.D Comparative statics

2.D.1 Competitive equilibrium

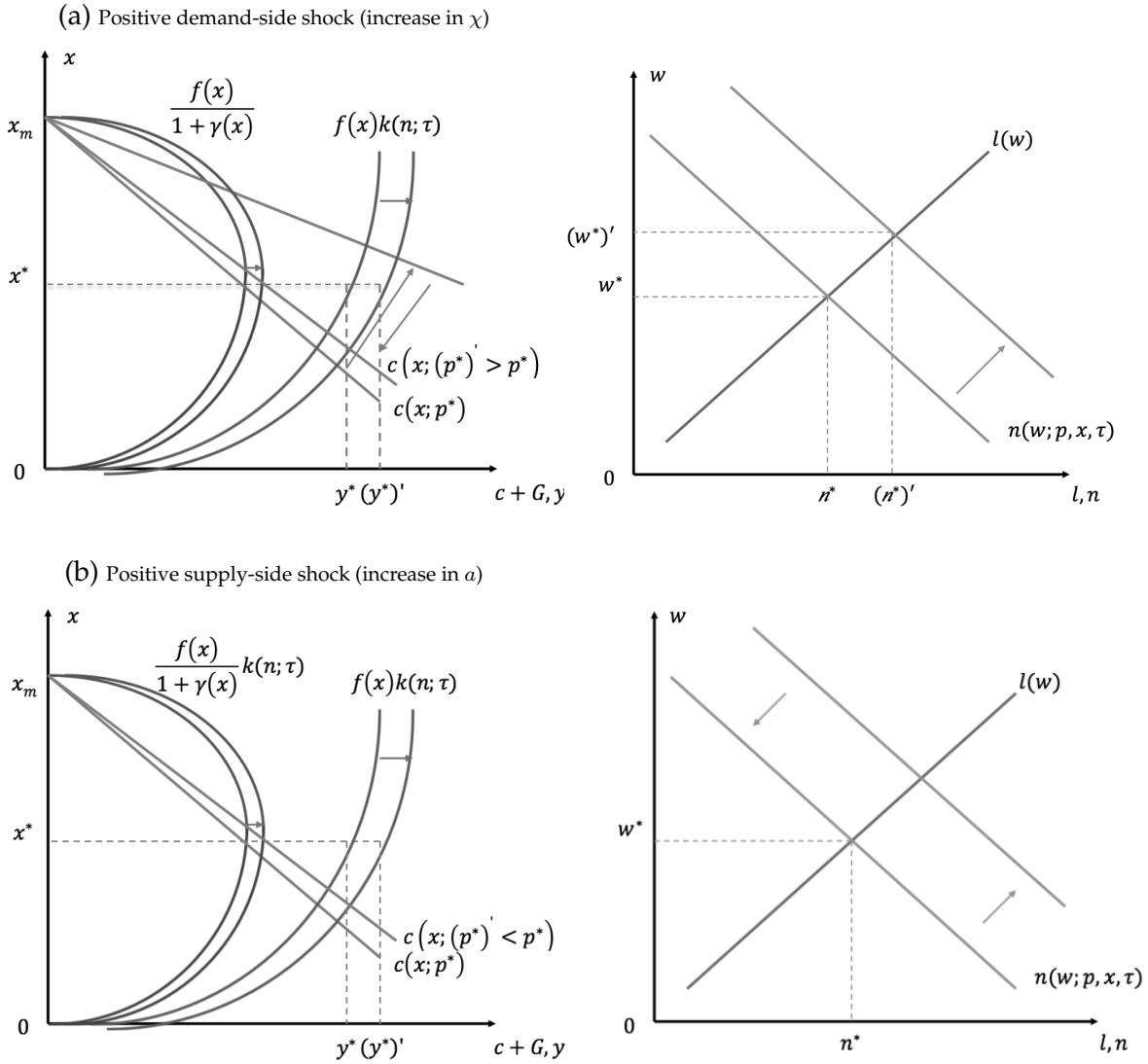
Panel (a) of Figure 2.D.2 shows comparative statics following a positive demand-side shock, parameterized as a permanent increase in χ. The aggregate demand curve shifts out, exercising upward pressure on goods market tightness. In order to retain tightness at the socially efficient level, the price must increase to lower private consumption to offset the rise in aggregate demand. Higher price increases labor demand, which expands capacity until the goods market reaches the efficient level of tightness (x^*). The new equilibrium features tightness at the efficient level, with higher price and sales compared to the original equilibrium.

Panel (b) of Figure 2.D.2 shows comparative statics following a positive supply shock, parameterized as a permanent increase in a. In response to the shock, aggregate supply curve shifts out, putting downward pressure on goods market tightness. In order to retain tightness at the socially efficient level, the price decreases in order to increase private consumption, until there are no more pressures on tightness to deviate from x^*. Eventually, tightness remains at the socially efficient level, sales increase and price falls.

2.D.2 Fixprice equilibrium

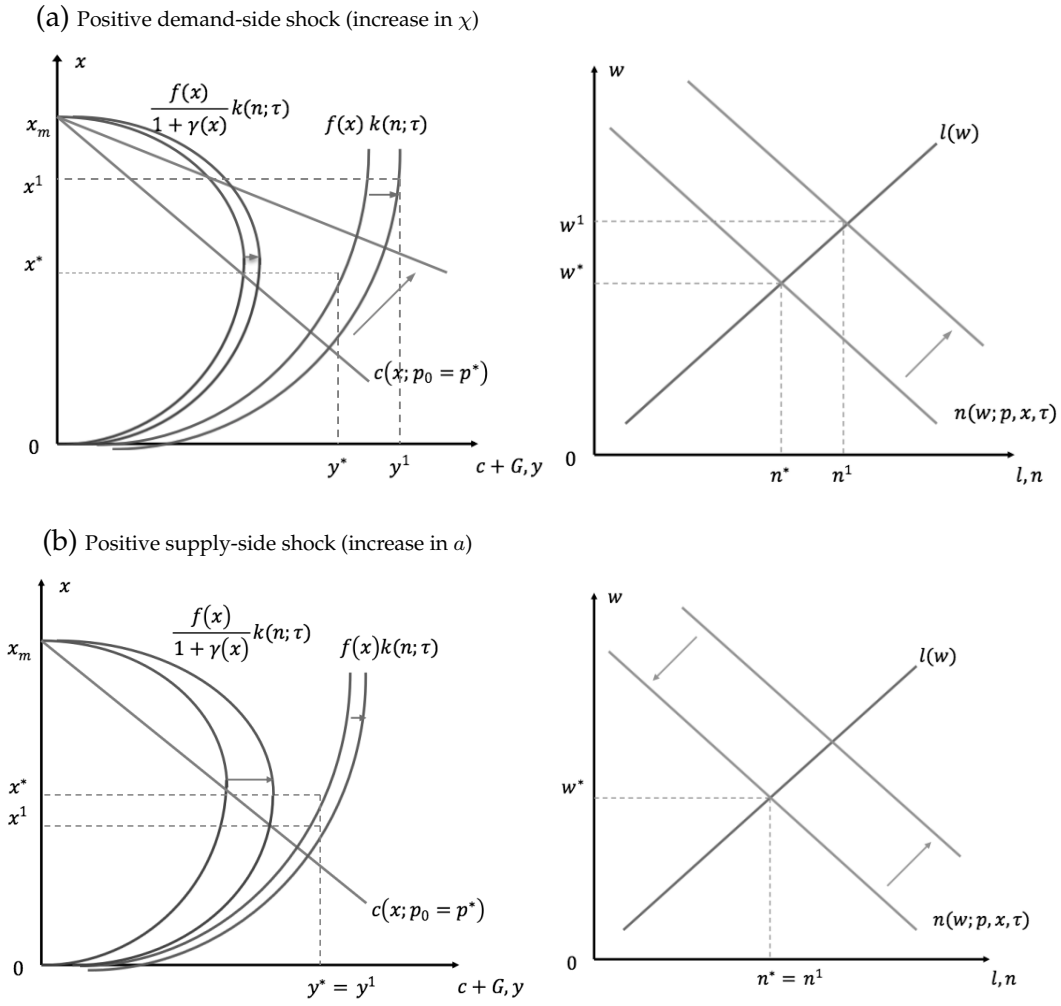
Panel (a) of Figure 2.D.3 shows comparative statics following a positive demand-side shock, parameterized as a permanent increase in χ. The aggregate demand curve shifts out, creating excess demand that under the fixed price is cleared out by rising tightness increase the cost of search and de-

Figure 2.D.2: Comparative statics in a competitive equilibrium



***Notes**: Panel (a) shows comparative statics in a competitive equilibrium, following a positive demand-side shock, parameterised as an increase in the preference parameter χ; following the shock, price increases to clear excess demand created by the shock and keep tightness at x^*, and equilibrium labor and sales also increase.*
Panel (b) shows comparative statics in a competitive equilibrium, following a positive supply-side shock, parameterised as an increase in the technology parameter a; following the shock, price falls to clear excess supply caused by the shock and keep tightness at x^*, equilibrium labor remains unchanged and sales increase, as every unit of labor is now more productive, leading to higher capacity and higher sales, due to unchanged level of tightness.

Figure 2.D.3: Comparative statics in a fixprice equilibrium



***Notes**: Panel (a) shows comparative statics in a fixprice equilibrium that initially coincides with the socially efficient allocation, and is hit by a positive demand-side shock, parameterised as an increase in the preference parameter χ; following the shock, goods market tightness increases to clear excess demand created by the shock, and equilibrium labor and sales also increase.*
Panel (b) shows comparative statics in a fixprice equilibrium that initially coincides with the socially efficient allocation, and is hit by a positive supply-side shock, parameterised as an increase in the technology parameter a; following the shock, goods market tightness falls to clear excess supply caused by the shock, whereas equilibrium labor and sales remain unchanged, since the effects of lower tightness and higher level of technology exactly offset each other.

creasing private consumption; higher tightness also encourages more labor demand as the effective price from the firms' perspective increases. The latter effect causes an outward shift of the aggregate supply curve, but tightness remains above the initial level, similarly to sales. By construction, the price remains unchanged.

Panel (b) of Figure 2.D.3 shows comparative statics for a positive supply-side shock, parameterized as a permanent increase in a. The aggregate supply curve shifts out, putting downward pressure on tightness via excess supply, and under the fixed price tightness falls to clear the market by lowering the cost of search for households and increasing private consumption. In equilibrium, tightness fall and sales remain unchanged.[35]

2.E Results under general CRRA utility of consumption

2.E.1 Comparative statics

Goods market clearing under general CRRA utility is given by:

$$\frac{f(x)an^{\alpha}}{1+\gamma(x)} = \frac{\chi^{\frac{1}{\sigma}}}{(p[1+\gamma(x)])^{\frac{1}{\sigma}}} + G. \tag{2.248}$$

Combining with the equilibrium labor expression:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}(1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \chi^{\frac{1}{\sigma}}(p[1+\gamma(x)])^{1-\frac{1}{\sigma}} + p[1+\gamma(x)]G. \tag{2.249}$$

Competitive equilibrium

In a competitive equilibrium $x = x^*$, so that $\frac{dx}{d\chi} = 0$; differentiate (2.249) (at $G = \tau = 0$) with respect to χ to find the comparative statics to a demand

[35]Note that sales remain the same due to two countervailing forces: on the one hand, productivity increases, expanding capacity and leading to more sales, *ceteris paribus*; on the other hand, tightness falls, lowering $f(x)$, which decreases sales, *ceteris paribus*. In the special case of log utility of consumption and a fixprice equilibrium these two effects exactly offset each other. However, once one considers equilibria with rigid, but not fully fixed prices, as we do in Appendix 2.B, the first effect dominates and sales rise following a positive technology shock.

shock in a competitive equilibrium:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\frac{1+\psi}{1-\alpha+\psi}\left[pf(x)a\right]^{\frac{1+\psi}{1-\alpha+\psi}-1}\frac{dp}{d\chi}f(x)a=\frac{1}{\sigma}\chi^{\frac{1}{\sigma}-1}(p[1+\gamma(x)])^{1-\frac{1}{\sigma}}+\left(1-\frac{1}{\sigma}\right)\chi^{\frac{1}{\sigma}}(p[1+\gamma(x)])^{-\frac{1}{\sigma}}\frac{dp}{d\chi}[1+\gamma(x)] \tag{2.250}$$

$$\frac{dp}{d\chi}=\left[\frac{\alpha}{1-\alpha+\psi}+\frac{1}{\sigma}\right]^{-1}\frac{p}{\sigma\chi}>0. \tag{2.251}$$

$$\frac{dc}{d\chi}=\frac{d}{d\chi}\left[\chi^{\frac{1}{\chi}}p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}}\right]=\frac{1}{\sigma}\chi^{\frac{1}{\sigma}-1}p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}}-\frac{1}{\sigma}\chi^{\frac{1}{\sigma}}p^{-\frac{1}{\sigma}-1}\frac{dp}{d\chi}[1+\gamma(x)]^{-\frac{1}{\sigma}} \tag{2.252}$$

$$\frac{dc}{d\chi}=\frac{c}{\sigma\chi}\left[\frac{1}{\sigma}\left(\frac{\alpha}{1+\psi}\right)^{-1}+\left(1-\frac{1}{\sigma}\right)\right]^{-1}>0. \tag{2.253}$$

$$\frac{dy}{d\chi}=\frac{d}{d\chi}(c[1+\gamma(x)])=[1+\gamma(x)]\frac{dc}{d\chi}>0. \tag{2.254}$$

Similarly, differentiate (249) (at $G=\tau=0$) with respect to a to find the comparative statics to a supply shock in a competitive equilibrium:

$$\frac{1+\psi}{1-\alpha+\psi}\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left[pf(x)a\right]^{\frac{1+\psi}{1-\alpha+\psi}-1}\left[\frac{dp}{da}f(x)a+pf(x)\right]=\left(1-\frac{1}{\sigma}\right)\chi^{\frac{1}{\sigma}}\left(p[1+\gamma(x)]\right)^{-\frac{1}{\sigma}}\frac{dp}{da}[1+\gamma(x)] \tag{2.255}$$

$$\frac{dp}{da}=-\frac{p}{a}\frac{\sigma(1+\psi)}{1+\psi+(\sigma-1)\alpha}<0. \tag{2.256}$$

(2.257)

$$\frac{dc}{da}=\frac{d}{da}\left[\chi^{\frac{1}{\chi}}p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}}\right]=-\frac{1}{\sigma}\chi^{\frac{1}{\sigma}}p^{-\frac{1}{\sigma}-1}\frac{dp}{da}[1+\gamma(x)]^{-\frac{1}{\sigma}} \tag{2.258}$$

$$\frac{dc}{da}=\frac{c}{a}\frac{\frac{1}{\sigma}}{\frac{1}{\sigma}+\left(1-\frac{1}{\sigma}\right)\frac{\alpha}{1+\psi}}>0. \tag{2.259}$$

(2.260)

$$\frac{dy}{da} = \frac{d}{da}(c[1+\gamma(x)]) = [1+\gamma(x)]\frac{dc}{da} > 0. \tag{2.261}$$

Fixprice equilibrium

In a fixprice equilibrium $p = p_0$ is a parameter, so that $\frac{dp}{d\chi} = 0$; differentiate (249) (at $G = \tau = 0$) with respect to χ to find the comparative statics to a demand shock in a fixprice equilibrium:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi}[pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}-1} pf'(x)\frac{dx}{d\chi}a = \frac{1}{\sigma}\chi^{\frac{1}{\sigma}-1}(p[1+\gamma(x)])^{1-\frac{1}{\sigma}} + \left(1-\frac{1}{\sigma}\right)\chi^{\frac{1}{\sigma}}(p[1+\gamma(x)])^{-\frac{1}{\sigma}}p\gamma'(x)\frac{dx}{d\chi} \tag{2.262}$$

$$\left[\frac{1+\psi}{1-\alpha+\psi} - \left(1-\frac{1}{\sigma}\right)\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}\right]\frac{f'(x)}{f(x)}\frac{dx}{d\chi} = \frac{1}{\sigma\chi} \tag{2.263}$$

$$\frac{dx}{d\chi} = \frac{1}{\sigma\chi}\left[\frac{1+\psi+(\sigma-1)\alpha}{\sigma(1-\alpha+\psi)} + \left(1-\frac{1}{\sigma}\right)\theta_{\sigma=1}(x)\right]^{-1}\frac{f(x)}{f'(x)} \tag{2.264}$$

$$\frac{dx}{d\chi}|_{x=x^*} = \frac{1}{\sigma\chi}\left[\frac{1+\psi+(\sigma-1)\alpha}{\sigma(1-\alpha+\psi)}\right]^{-1}\frac{f(x^*)}{f'(x^*)} > 0. \tag{2.265}$$

$$\frac{dc}{d\chi} = \frac{d}{d\chi}\left[\chi^{\frac{1}{\chi}}p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}}\right] = \frac{1}{\sigma}\chi^{\frac{1}{\sigma}-1}p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}} - \frac{1}{\sigma}\chi^{\frac{1}{\sigma}}p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}-1}\gamma'(x)\frac{dx}{d\chi} \tag{2.266}$$

$$\frac{dc}{d\chi} = \frac{c}{\sigma\chi}\left[\frac{1}{\sigma}[\varphi^d_{\sigma=1}(x)]^{-1} + \left(1-\frac{1}{\sigma}\right)\right]^{-1}. \tag{2.267}$$

$$\frac{dc}{d\chi}|_{x=x^*} = \frac{c}{\sigma\chi}\frac{\sigma\alpha}{1+\psi+(\sigma-1)\alpha} > 0. \tag{2.268}$$

$$\frac{dy}{d\chi} = \frac{d}{d\chi}(c[1+\gamma(x)]) = \frac{dc}{dx}[1+\gamma(x)] + c\gamma'(x)\frac{dx}{d\chi} \tag{2.269}$$

$$\frac{dy}{d\chi}|_{x=x^*} = \frac{dc}{dx}|_{x=x^*}[1+\gamma(x^*)] + c\gamma'(x^*)\frac{dx}{d\chi}|_{x=x^*} > 0. \tag{2.270}$$

Similarly, differentiate (2.249) (at $G = \tau = 0$) with respect to a to find the comparative statics to a supply shock in a fixprice equilibrium:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} [pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}-1} \left[pf'(x)\frac{dx}{dx}a + pf(x) \right] = \left(1 - \frac{1}{\sigma}\right) \chi^{\frac{1}{\sigma}} (p[1+\gamma(x)])^{-\frac{1}{\sigma}} p\gamma'(x)\frac{dx}{da} \tag{2.271}$$

$$\frac{dx}{da} = -\frac{1}{a}\frac{1+\psi}{1-\alpha+\psi}\frac{f(x)}{f'(x)} \left[\frac{1+\psi+(\sigma-1)\alpha}{\sigma(1-\alpha+\psi)} + \left(1 - \frac{1}{\sigma}\theta_{\sigma=1}(x)\right) \right]^{-1} \tag{2.272}$$

$$\frac{dx}{da}|_{x=x^*} = -\frac{1}{a}\frac{1+\psi}{1-\alpha+\psi}\frac{f(x^*)}{f'(x^*)} \left[\frac{1+\psi+(\sigma-1)\alpha}{\sigma(1-\alpha+\psi)} \right]^{-1} < 0. \tag{2.273}$$

$$\frac{dc}{da} = \frac{d}{da}\left[\chi^{\frac{1}{\chi}} p^{-\frac{1}{\sigma}} [1+\gamma(x)]^{-\frac{1}{\sigma}} \right] = -\frac{1}{\sigma}\chi^{\frac{1}{\sigma}} p^{-\frac{1}{\sigma}} [1+\gamma(x)]^{-\frac{1}{\sigma}-1}\gamma'(x)\frac{dx}{da} \tag{2.274}$$

$$\frac{dc}{da} = \frac{c}{a}\frac{1+\psi}{\alpha}\frac{\frac{1}{\sigma}\varphi^s_{\sigma=1}(x)}{\frac{1}{\sigma} + \left(1-\frac{1}{\sigma}\right)\varphi^d_{\sigma=1}(x)} \tag{2.275}$$

$$\frac{dc}{da}|_{x=x^*} = \frac{c}{a}\left[1 + (\sigma-1)\frac{\alpha}{1+\psi} \right]^{-1} > 0. \tag{2.276}$$

$$\frac{dy}{da} = \frac{dc}{da}[1+\gamma(x)] + c\gamma'(x)\frac{dx}{da} \tag{2.277}$$

$$\frac{dy}{da} = (1-\sigma)\frac{dc}{da}[1+\gamma(x)] \tag{2.278}$$

$$\frac{dy}{da}|_{x=x^*} = (1-\sigma)\frac{dc}{da}_{x=x^*}[1+\gamma(x^*)]. \tag{2.279}$$

2.E.2 Demand-side fiscal multiplier

Differentiate (2.249) with respect to G (at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} \left(\frac{dp}{dG}f(x)a + \frac{f'(x)}{f(x)}\frac{dx}{dG}pa \right) (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1} = \tag{2.280}$$

$$\chi^{\frac{1}{\sigma}} (p[1+\gamma(x)])^{-\frac{1}{\sigma}} (1-\frac{1}{\sigma}) \left([1+\gamma(x)]\frac{dp}{dG} + p\gamma'(x)\frac{dx}{dG} \right) + p[1+\gamma(x)]. \tag{2.281}$$

In a competitive equilibrium $x = x^*$, so that $\frac{dx}{dG} = 0$; the demand-side multiplier is thus given by:

$$(\varphi_\sigma^d)^* = 1 + \frac{\partial c}{\partial p}\frac{dp}{dG} = \frac{\frac{1+\psi}{1-\alpha+\psi} - 1}{\frac{1+\psi}{1-\alpha+1} - (1-\frac{1}{\sigma})} = \frac{\frac{\alpha}{1+\psi}}{\frac{1}{\sigma} + (1-\frac{1}{\sigma})\frac{\alpha}{1+\psi}} = \left[\frac{1}{\sigma} \times \left(\frac{\alpha}{1+\psi}\right)^{-1} + \left(1-\frac{1}{\sigma}\right) \times 1^{-1}\right]^{-1} \tag{2.282}$$

$$\tag{2.283}$$

or just a weighted harmonic average between $\frac{\alpha}{1+\psi}$ and one, with weights given by $\left(\frac{1}{\sigma}, 1-\frac{1}{\sigma}\right)$. Hence, in a competitive equilibrium the demand-side multiplier remains acyclical under a general CRRA utility of consumption.

In a fixprice equilibrium $p = p_0$, so that $\frac{dp}{dG} = 0$; the demand-side multiplier is thus given by:

$$\begin{aligned}\varphi_\sigma^d(x) = 1 + \frac{\partial c}{\partial x}\frac{dx}{dG} = \frac{\frac{1+\psi}{1-\alpha+\psi} - \frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}{\frac{1+\psi}{1-\alpha+\psi} - \left(1-\frac{1}{\sigma}\right)\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}} &= \frac{\varphi_{\sigma=1}^d(x)}{\frac{1}{\sigma} + \left(1-\frac{1}{\sigma}\right)\varphi_{\sigma=1}^d(x)} \\ &= \left[\frac{1}{\sigma} \times \left\{\varphi_{\sigma=1}^d(x)\right\}^{-1} + \left(1-\frac{1}{\sigma}\right) \times 1^{-1}\right]^{-1}\end{aligned} \tag{2.284}$$

or just a weighted harmonic average between $\varphi_{\sigma=1}^d(x)$, which is the demand-side multiplier under $\sigma = 1$ considered in the main text, and one, with weights given by $\left(\frac{1}{\sigma}, 1-\frac{1}{\sigma}\right)$. Hence, $\varphi_\sigma^d(0) = 1$ and $\frac{d\varphi_\sigma^d(x)}{dx} = \frac{\frac{1}{\sigma}\frac{d\varphi_{\sigma=1}^d(x)}{dx}}{\left[\frac{1}{\sigma}+\left(1-\frac{1}{\sigma}\right)\varphi_{\sigma=1}^d(x)\right]^2} < 0, \quad \forall x \in (0, x_m)$, so that $\frac{d\varphi_\sigma^d(x)}{d\chi}|_{x=x^*} = \frac{d\varphi_\sigma^d(x)}{dx}|_{x=x^*}\frac{dx}{d\chi}|_{x=x^*} < 0$ and $\frac{d\varphi_\sigma^d(x)}{da}|_{x=x^*} = \frac{d\varphi_\sigma^d(x)}{dx}|_{x=x^*}\frac{dx}{da}|_{x=x^*} > 0$, establishing that the cyclicality properties of the demand-side multiplier found in the main text are preserved under a general CRRA utility of consumption in the local neighborhood of the efficient allocation.

2.E.3 Supply-side fiscal multiplier

Differentiate (2.249) with respect to τ (at $G = \tau = 0$):

$$\left[\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)} - \left(1-\frac{1}{\sigma}\right)\frac{\gamma'(x)}{1+\gamma(x)}\right]\frac{dx}{d\tau} + \left[\frac{1+\psi}{1-\alpha+\psi}\frac{1}{p} - \left(1-\frac{1}{\sigma}\right)\frac{1}{p}\right]\frac{dp}{d\tau} = \frac{\alpha}{1-\alpha+\psi}. \tag{2.285}$$

In a competitive equilibrium $x = x^*$, so that $\frac{dx}{d\tau} = 0$; the supply-side multiplier is thus given by:

$$(\varphi_\sigma^s)^* = -\frac{1}{c}\frac{\partial c}{\partial p}\frac{dp}{d\tau} = \frac{\frac{1}{\sigma}\frac{\alpha}{1-\alpha+\psi}}{\frac{1+\psi}{1-\alpha+\psi} - \left(1-\frac{1}{\sigma}\right)} = \frac{\frac{1}{\sigma}\frac{\alpha}{1+\psi}}{\frac{1}{\sigma} + \left(1-\frac{1}{\sigma}\right)\frac{\alpha}{1+\psi}} = \frac{1}{\sigma}(\varphi^d)^*, \quad (2.286)$$

so that in a competitive equilibrium the supply-side multiplier remains acyclical under a general CRRA utility of consumption.

In a fixprice equilibrium $p = p_0$, so that $\frac{dp}{d\tau} = 0$; the supply-side multiplier is thus given by:

$$\varphi_\sigma^s(x) = -\frac{1}{c}\frac{\partial c}{\partial x}\frac{dx}{d\tau} = \frac{\frac{1}{\sigma}\frac{\alpha}{1-\alpha+\psi}\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}{\frac{1+\psi}{1-\alpha+\psi} - \left(1-\frac{1}{\sigma}\right)\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}} = \frac{\frac{1}{\sigma}\varphi_{\sigma=1}^s(x)}{\frac{1}{\sigma} + \left(1-\frac{1}{\sigma}\right)\varphi_{\sigma=1}^d(x)}, \quad (2.287)$$

where $\varphi_{\sigma=1}^s(x)$ is the supply-side multiplier under $\sigma = 1$ considered in the main text. Hence, $\varphi_\sigma^s(0) = 0$ and $\frac{d\varphi_\sigma^s(x)}{dx} = -\frac{\frac{1}{\sigma}\frac{\alpha}{1+\psi}\theta'(x)}{\left[\frac{1}{\sigma}+\left(1-\frac{1}{\sigma}\right)\varphi_{\sigma=1}^d(x)\right]^2} > 0, \quad \forall x \in (0, x_m)$, so that $\frac{d\varphi_\sigma^s(x)}{d\chi}|_{x=x^*} = \frac{d\varphi_\sigma^s(x)}{dx}|_{x=x^*}\frac{dx}{d\chi}|_{x=x^*} > 0$ and $\frac{d\varphi_\sigma^s(x)}{da}|_{x=x^*} = \frac{d\varphi_\sigma^s(x)}{dx}|_{x=x^*}\frac{dx}{da}|_{x=x^*} < 0$, establishing that the cyclicality properties of the supply-side multiplier found in the main text are preserved under a general CRRA utility of consumption in the local neighborhood of the efficient allocation.

2.F Results under utility cost per visit

2.F.1 Household optimization

In this version of the model, the setup is remains unchanged compared to the baseline case in main text, except households now face a *utility* cost $\iota > 0$ per visit; given that every visit is successful with probability $q(x)$, the total number of visits required to purchase c units of the produced good is $v = c/q(x)$, and the total utility cost of search is $\iota \times v = \iota \times c/q(x)$. Households' optimization problem is now given by:

$$\max_{c,m,l}\left[\chi\frac{c^{1-\sigma}}{1-\sigma} + \zeta(m) - \frac{l^{1+\psi}}{1+\psi} - \iota\frac{c}{q(x)}\right] \qquad \text{s.t.} \quad (2.288)$$

$$pc + m \leq wl + \bar{m} + \Pi - T. \quad (2.289)$$

As before, here we normalize $\bar{m}$ so that $\zeta'(\bar{m}) = 1$, and focus on the special case of log utility of consumption ($\sigma = 1$). The solution to the above problem delivers a labor supply function identical to the one in the baseline model; however, the consumption function now takes a different form:

$$c(p, x) = \frac{\chi}{p + \varsigma(x)}, \tag{2.290}$$

where $\varsigma(x) \equiv \frac{\iota x}{f(x)} = \frac{\iota}{q(x)} > 0, \quad \varsigma'(x) > 0, \forall x \in (0, +\infty)$ summarizes the total cost of search, which is now additive to the price, and strictly increases in tightness on the whole domain. It thus follows that $\frac{\partial c}{\partial p} = -\frac{\chi}{[p+\varsigma(x)]^2} < 0$ and $\frac{\partial c}{\partial x} = -\frac{\chi \varsigma'(x)}{[p+\varsigma(x)]^2} < 0$.

The firms' problem remains unchanged, hence labor market equilibrium remains unaffected; goods market clearing can now be written as:

$$f(x) a n^{\alpha} = c(p, x) + G \tag{2.291}$$

$$f(x) a \left[\alpha p f(x) a\right]^{\frac{\alpha}{1-\alpha+\psi}} (1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \frac{\chi}{p + \varsigma(x)} + G \tag{2.292}$$

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \left[p f(x) a\right]^{\frac{1+\psi}{1-\alpha+\psi}} \left[p + \varsigma(x)\right] = p\chi + pG[p + \varsigma(x)]. \tag{2.293}$$

Given the evidence that fixprice equilibrium is more empirically relevant at business cycle frequencies, we will continue our analysis in this section under the assumption of fixprice equilibrium, so that $p = p_0$ is a parameter.

2.F.2 Comparative statics

Differentiate (2.293) with respect to χ (at $G = \tau = 0$) in order to find comparative statics after a demand shock:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left[\frac{1+\psi}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1}pf'(x)\frac{dx}{d\chi}a[p+\varsigma(x)]+(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}\varsigma'(x)\frac{dx}{d\chi}\right]=p \tag{2.294}$$

$$\frac{dx}{d\chi}=\frac{1}{\chi}\left[\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)}+\frac{\varsigma'(x)}{p+\varsigma(x)}\right]^{-1}>0. \tag{2.295}$$

$$\frac{dc}{d\chi}=\frac{1}{p+\varsigma(x)}-\frac{\chi\varsigma'(x)}{[p+\varsigma(x)]^2}\frac{dx}{d\chi} \tag{2.296}$$

$$\frac{dc}{d\chi}=\frac{\frac{1+\psi}{1-\alpha+\psi}}{\left[\frac{1+\psi}{1-\alpha+\psi}+\frac{\varsigma'(x)}{p+\varsigma(x)}\right][p+\varsigma(x)]}>0. \tag{2.297}$$

Similarly, differentiate (2.293) with respect to a (at $G = \tau = 0$) in order to find comparative statics after a supply shock:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left[\frac{1+\psi}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1}(pf'(x)\frac{dx}{da}a+pf(x))[p+\varsigma(x)]+(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}\varsigma'(x)\frac{dx}{da}\right]=0 \tag{2.298}$$

$$\frac{dx}{da}=-\frac{1+\psi}{1-\alpha+\psi}\frac{1}{a}\left[\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)}+\frac{\varsigma'(x)}{p+\varsigma(x)}\right]^{-1}<0. \tag{2.299}$$

$$\frac{dc}{da}=-\frac{\chi\varsigma'(x)}{[p+\varsigma(x)]^2}\frac{dx}{da} \tag{2.300}$$

$$\frac{dc}{da}=\frac{\frac{\chi\varsigma'(x)}{[p+\varsigma(x)]^2}\frac{1+\psi}{1-\alpha+\psi}\frac{1}{a}}{\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)}+\frac{\varsigma'(x)}{p+\varsigma(x)}}>0. \tag{2.301}$$

2.F.3 Demand-side fiscal multiplier

Differentiating (2.293) with respect to G (at $G = \tau = 0$) delivers the following:

$$\frac{dx}{dG} = \frac{1}{c}\left[\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)}\frac{\varsigma'(x)}{p+\varsigma(x)}\right]^{-1}. \tag{2.302}$$

From the definition of the demand-side fiscal multiplier:

$$\varphi^d(x) = 1 + \frac{\partial c}{\partial x}\frac{dx}{dG} \tag{2.303}$$

$$= \left[1 + \frac{1-\alpha+\psi}{1+\psi}\frac{\varsigma'(x)}{p+\varsigma(x)}\frac{f(x)}{f'(x)}\right]^{-1}. \tag{2.304}$$

After some algebra it can be shown that:

$$\frac{d\varphi^d(x)}{dx} = \frac{\iota\frac{1-\alpha+\psi}{1+\psi}q(x)^\delta q'(x)(\delta[p+\varsigma(x)] + p(1-q(x)^\delta))}{[[p+\varsigma(x)]q(x)^{1+\delta} + \frac{1-\alpha+\psi}{1+\psi}\iota(1-q(x)^\delta)]^2} < 0, \tag{2.305}$$

since $\varsigma(x) > 0, q(x) \in (0,1), q'(x) < 0, \forall x \in (0,+\infty)$ and $\delta > 0$. Hence, $\frac{\varphi^d(x)}{d\chi} = \frac{d\varphi^d(x)}{dx}\frac{dx}{d\chi} < 0$ and $\frac{d\varphi^d(x)}{da} = \frac{d\varphi^d(x)}{dx}\frac{dx}{da} > 0$, so the cyclicality properties of the demand-side fiscal multiplier found in the main text are preserved.

2.F.4 Supply-side fiscal multiplier

Differentiating (2.293) with respect to τ (at $G = \tau = 0$) delivers the following:

$$\frac{dx}{d\tau} = \frac{\alpha}{1-\alpha+\psi}\left[\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)} + \frac{\varsigma'(x)}{p+\varsigma(x)}\right]^{-1}. \tag{2.306}$$

From the definition of the supply-side fiscal multiplier:

$$\varphi^s(x) = -\frac{1}{c}\frac{\partial c}{\partial x}\frac{dx}{d\tau} \tag{2.307}$$

$$= \frac{\alpha}{1+\psi}\varsigma'(x)f(x)\left[1 + \frac{1-\alpha+\psi}{1+\psi}\frac{\varsigma'(x)}{p+\varsigma(x)}\frac{f(x)}{f'(x)}\right]^{-1}. \tag{2.308}$$

After some algebra it can be shown that:

$$\frac{\varphi^s(x)}{dx} = -\frac{\iota\frac{\alpha}{1+\psi}q(x)^\delta q'(x)(\delta(p+\varsigma(x)) + p(1-q(x)^\delta))}{[[p+\varsigma(x)]q(x)^{1+\delta} + \frac{1-\alpha+\psi}{1+\psi}\iota(1-q(x)^\delta)]^2} > 0, \tag{2.309}$$

since $\varsigma(x) > 0, q(x) \in (0,1), q'(x) < 0, \forall x \in (0,+\infty)$ and $\delta > 0$. Hence, $\frac{\varphi^s(x)}{d\chi} = \frac{d\varphi^s(x)}{dx}\frac{dx}{d\chi} > 0$ and $\frac{d\varphi^s(x)}{da} = \frac{d\varphi^s(x)}{dx}\frac{dx}{da} < 0$, so the cyclicality properties of the supply-side fiscal multiplier found in the main text are preserved.

2.G Social planner's allocation

2.G.1 Static model

The social planner's problem is given by:

$$\max_{c,l,v,m} \left[\chi \frac{c^{1-\sigma}}{1-\sigma} + \zeta(m) - \frac{l^{1+\psi}}{1+\psi}\right] \qquad \text{s.t.} \tag{2.310}$$

$$c + G + \rho v = \left[(al^{\alpha})^{-\delta} + v^{-\delta}\right]^{-\frac{1}{\delta}}, \qquad m = \bar{m}. \tag{2.311}$$

Inserting $m = \bar{m}$, the associated Lagrangian becomes:

$$\mathcal{L} = \left[\chi \frac{c^{1-\sigma}}{1-\sigma} + \zeta(\bar{m}) - \frac{l^{1+\psi}}{1+\psi}\right] + \lambda\left(\left[(al^{\alpha})^{-\delta} + v^{-\delta}\right]^{-\frac{1}{\delta}} - (c + G + \rho v)\right) \tag{2.312}$$

The first-oder conditions are:

$$\frac{\partial \mathcal{L}}{\partial c} = \chi c^{-\delta} - \lambda = 0 \tag{2.313}$$

$$\frac{\partial \mathcal{L}}{\partial v} = \lambda\left(-\frac{1}{\delta}\left[(al^{\alpha})^{-\delta} + v^{-\delta}\right]^{-\frac{1}{\delta}-1}(-\delta)v^{-\delta-1} - \rho\right) = 0 \tag{2.314}$$

$$\frac{\partial \mathcal{L}}{\partial v} = -l^{\psi} + \lambda\left(-\frac{1}{\delta}\left[(al^{\alpha})^{-\delta} + v^{-\delta}\right]^{-\frac{1}{\delta}-1}(-\delta)\alpha a l^{\alpha-1}\right) = 0 \tag{2.315}$$

Note that $x \equiv \frac{v}{al^\alpha}$ and $f'(x) = (1+x^\delta)^{-\frac{1}{\delta}-1}$, the social planner's allocation $\{c^*, l^*, v^*, m^*, x^*\}$ is given by:

$$f'(x^*) = \rho \tag{2.316}$$

$$x^* = \frac{v^*}{a(l^*)^\alpha} \tag{2.317}$$

$$\chi(c^*)^{-\sigma} = \frac{\left[(a(l^*)^\alpha)^{-\delta} + (v^*)^{-\delta}\right]^{-\frac{1}{\delta}-1} \alpha a (l^*)^{\alpha-1}}{(l^*)^\psi} \tag{2.318}$$

$$c^* + G + \rho v^* = \left[(a(l^*)^\alpha)^{-\delta} + (v^*)^{-\delta}\right]^{-\frac{1}{\delta}} \tag{2.319}$$

$$m^* = \bar{m}. \tag{2.320}$$

2.G.2 Dynamic model

The social planner's problem is given by:

$$\max_{\{c_{t+s}, l_{t+s}, m_{t+s}, v_{t+s}, y_{t+s}\}_{s=0}^\infty} \mathbb{E}_t \sum_{s=0}^\infty \beta^s \left[\chi_{t+s} \frac{c_{t+s}^{1-\sigma}}{1-\sigma} + \zeta(m_{t+s}) - \nu \frac{l_{t+s}^{1+\psi}}{1+\psi}\right] \qquad \text{s.t.} \tag{2.321}$$

$$y_t = (1-\eta) y_{t-1} + \left[v_t^{-\delta} + (a_t l_t^\alpha - (1-\eta) y_{t-1})^{-\delta}\right]^{-\frac{1}{\delta}}, \qquad \forall t \geq 0 \tag{2.322}$$

$$y_t = c_t + G_t + \rho v_t, \qquad m_t = \bar{m}, \qquad \forall t \geq 0. \tag{2.323}$$

The associated Lagrangian is given by:

$$\mathcal{L}_t = \mathbb{E}_t \sum_{s=0}^\infty \beta^s [\chi_{t+s} \frac{c_{t+s}^{1-\sigma}}{1-\sigma} + \zeta(\bar{m}) - \frac{l_{t+s}^{1+\psi}}{1+\psi} + \lambda_{t+s} \left(y_{t+s} - (1-\eta) y_{t+s-1} - \left[v_{t+s}^{-\delta} + (a_{t+s} l_{t+s}^\alpha - (1-\eta) y_{t+s-1})^{-\delta}\right]^{-\frac{1}{\delta}}\right)$$
$$+ \mu_{t+s}(y_{t+s} - c_{t+s} - G_{t+s} - \rho v_{t+s})]. \tag{2.324}$$

The first order conditions are given by:

$$\lambda_t + \frac{\nu l_t^{\psi}}{\left[x_t^{-\delta}+1\right]^{-\frac{1}{\delta}-1}\alpha a_t l_t^{\alpha-1}} = 0, \qquad \forall t \geq 0 \tag{2.325}$$

$$\lambda_t[1+x_t^{\delta}]^{-\frac{1}{\delta}-1} + \rho\chi_t c_t^{-\sigma} = 0, \qquad \forall t \geq 0 \tag{2.326}$$

$$\lambda_t + \chi_t c_t^{-\sigma} + \beta(1-\eta)\mathbb{E}_t\left[\lambda_{t+1}\left((x_{t+1}^{-\delta}+1)^{-\frac{1}{\delta}-1}-1\right)\right] = 0, \qquad \forall t \geq 0 \tag{2.327}$$

which together with the definition of tightness $x_t = \frac{v_t}{a_t l_t^{\alpha} - (1-\eta)y_{t-1}}$ and the feasibility constraints describe the social planner's allocation.

2.H Dynamic model: further results and steady state

2.H.1 Alternative fiscal instruments in the dynamic model

In this subsection we consider two additional fiscal instruments in the context of the our dynamic model: distortionary taxation on consumption (τ_t^c) and households' labor income (τ_t^l). Compared to the baseline model in the main text, the representative household's per-period budget constraint becomes:

$$p_t(1+\tau_t^c)y_t^c + m_t + \mathbb{E}_t\left[F_{t,t+1}B_{t+1}\right] \leq w_t(1-\tau_t^l)l_t + \bar{m}_t + B_t + \Pi_t - T_t, \qquad \forall t \geq 0, \tag{2.328}$$

and the first order conditions for the choice of consumption and labor supply become:

$$\chi_t c_t^{-\sigma} + \beta(1-\eta)\mathbb{E}_t\left[\chi_{t+1}c_{t+1}^{-\sigma}\frac{[1+\gamma(x_t)]}{[1+\gamma(x_{t+1})]}\gamma(x_{t+1})\right] = p_t(1+\tau_t^c)[1+\gamma(x_t)], \tag{2.329}$$

$$l_t = [w_t(1-\tau_t^l)/\nu]^{\frac{1}{\psi}}. \tag{2.330}$$

We further assume that the two additional tax rates follow exogenous autoregressive processes:

$$\tau_t^i = (1-\rho_\tau)\tau^i + \rho_\tau\tau_{t-1}^i + \varepsilon_t^{\tau^i}, \qquad \forall t \geq 0, \quad \tau_t^i \in \{\tau_t^l, \tau_t^c\}, \tag{2.331}$$

and the lump sum tax raised by the government is now given by:

$$T_t = p_t y_t^G - w_t n_t \tau_t - w_t n_t \tau_t^l - p_t y_t^c \tau_t^c. \tag{2.332}$$

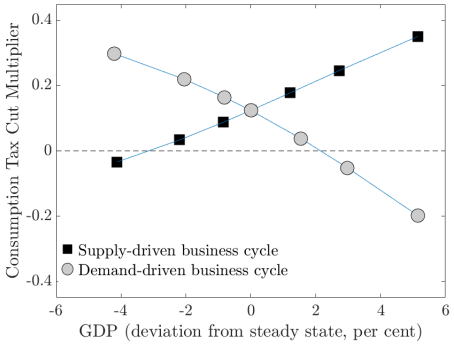
Steady state consumption and labor income taxes ($\tau^c = 0.05, \tau^l = 0.28$) follow calibrations in Trabandt and Uhlig (2011) and Zanetti (2012). The rest of the model remains unchanged.

We compute horizon-specific conditional state-dependent multipliers out of cuts in taxes on consumption and labor income, following the methodology used for multipliers out of payroll taxes, as detailed in the main text. Figure 2.H.4 shows the results for impact, 2-year and 4-year horizon multipliers. As one can see, multipliers out of cuts in consumption taxes exhibit cyclicality properties that are similar to those of government consumption multipliers, as detailed in the main text; in particular, compared to the steady state, consumption tax cut multipliers rise in demand-side recessions and supply side recessions, but fall in demand-side expansions and supply-side recession, with the magnitude of state dependence falling at further horizons.

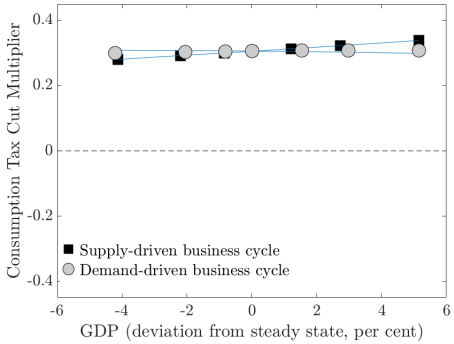
As for multipliers out of cuts in taxes on labor income, those have cyclicality properties identical to those of payroll tax cut multipliers. Indeed, compared to steady state, labor income tax cut multipliers are high in supply-side recession and demand-side expansions, whereas they are low in demand-side recessions and supply-side expansions. As before, the magnitude of state-dependence falls with the horizon considered.

Figure 2.H.4: Conditional state-dependent fiscal multipliers

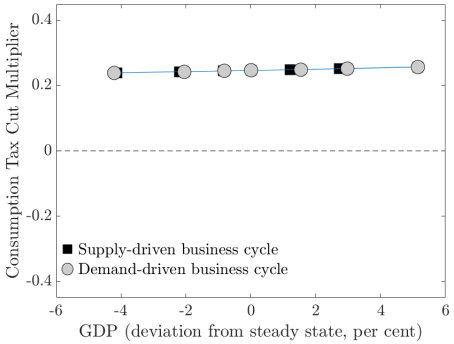
(a) Impact multiplier: cut consumption tax by 1%



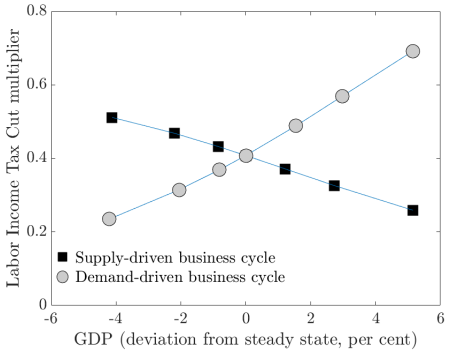
(b) 2-year hor. multiplier: cut consumption tax by 1%



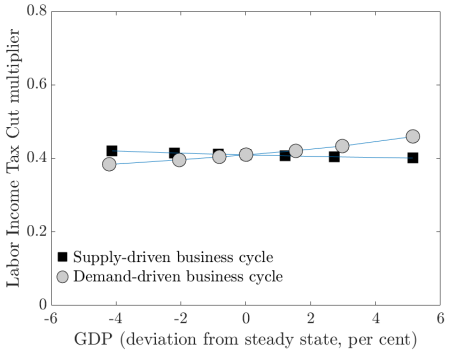
(c) 4-year hor. multiplier: cut consumption tax by 1%



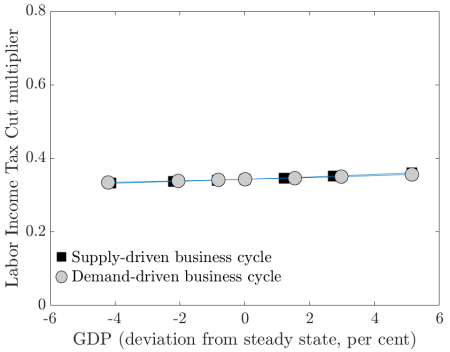
(d) Impact multiplier: cut labor income tax by 1%



(e) 2-year hor. multiplier: cut labor income tax by 1%



(f) 4-year hor. multiplier: cut labor income tax by 1%



***Notes**: Panel (a) shows impact multipliers following a one-time innovation to the consumption tax rate process equal to negative 1 percentage point, in recessionary and expansionary episodes caused by different types of shocks; Panels (b) and (c) repeat the exercise for the 2-year horizon and the 4-year horizon tax cut multipliers, respectively.*
Panel (d) shows impact multipliers following a one-time innovation to the labor income tax rate process equal to negative 1 percentage point, in recessionary and expansionary episodes caused by different types of shocks; Panels (e) and (f) repeat the exercise for the 2-year horizon and the 4-year horizon tax cut multipliers, respectively.

2.H.2 Decentralized equilibrium: steady state

$$c^{-\sigma} = \frac{p(1+\tau^c)[1+\gamma(x)]}{1+\beta(1-\eta)\gamma(x)} \tag{2.333}$$

$$y^c = \frac{[1+\gamma(x)]c}{1+(1-\eta)\gamma(x)} \tag{2.334}$$

$$l^{\psi} = w(1-\tau^l)/\nu \tag{2.335}$$

$$y = \frac{f(x)l^{\alpha}}{1-(1-\eta)(1-f(x))} \tag{2.336}$$

$$p = \frac{w(1+\tau)}{\alpha f(x) l^{\alpha-1}}[1-(1-\eta)\beta(1-f(x))] \tag{2.337}$$

$$y^G = \frac{[1+\gamma(x)]g}{1+(1-\eta)\gamma(x)} \tag{2.338}$$

$$y = y^c + y^G \tag{2.339}$$

$$x = \frac{v}{l^{\alpha}-(1-\eta)y} \tag{2.340}$$

$$m = \bar{m} \tag{2.341}$$

$$\gamma(x) = \frac{\rho x}{f(x)-\rho x} \tag{2.342}$$

$$f(x) = (1+x^{-\delta})^{-\frac{1}{\delta}}. \tag{2.343}$$

2.H.3 Social planner's allocation: steady state

$$c^{-\sigma} = \frac{\nu l^{1+\psi-\alpha}}{\rho}\left[\frac{1+x^{\delta}}{1+x^{-\delta}}\right]^{-\frac{1}{\delta}-1} \tag{2.344}$$

$$y = \frac{1}{\eta}\left[v^{-\delta} + (l^{\alpha}-(1-\eta)y)^{-\delta}\right]^{-\frac{1}{\delta}} \tag{2.345}$$

$$\frac{1}{\rho}[1+x^{\delta}]^{-\frac{1}{\delta}-1} = 1+\beta(1-\eta)[(1+x^{-\delta})^{-\frac{1}{\delta}-1}-1] \tag{2.346}$$

$$y = c + g + \rho v \tag{2.347}$$

$$x = \frac{v}{l^{\alpha}-(1-\eta)y} \tag{2.348}$$

$$m = \bar{m}. \tag{2.349}$$

2.I Econometric evidence: additional results and robustness checks

2.I.1 Responses to an identified productivity shock

In this subsection we would like to test whether following an identified productivity shock, output, tightness and inflation move in the same direction as predicted by our theory. We run the following sequence of local projections:

$$variable_{t+H} = \alpha_H + \beta_H \times a_t + \gamma_H \mathbf{z}_{t-1} + \varepsilon_{t+H}, \tag{2.350}$$

with $variable_t \in \left\{ \frac{GDP_t}{GDP_t^*}, x_t, \pi_t \right\}$, where $\frac{GDP_t}{GDP_t^*}$ is real GDP over its polynomial trend, x_t is goods market tightness series from Michaillat and Saez (2015), π_t is inflation based on GDP deflator, and $a_t \equiv \ln TFP_t - \ln TFP_{t-1}$, where TFP is utilization-adjusted TFP series from Fernald (2014a); the set of controls $\mathbf{z}_{t-1}$ is specified in the description to the results figure.

In Figure 2.I.5 one can see that consistently with our theory, following a positive productivity shock, one obtains a statistically significant increase in (cyclical) output, as well as a statistically significant reduction in inflation and goods market tightness.

2.I.2 Inflation responses to fiscal shocks

A bulk of variation in our spending and tax rate shocks overlaps with periods of price controls around World War II and the Korean War. In order to check that such price controls do not pose a challenge to our strategy of using co-movement between inflation and activity to identify sources of fluctuations, we estimate the response of inflation to our fiscal shocks:

$$\pi_{t+H} = \alpha_H + \beta_H \times shock_t + \gamma_H \mathbf{z}_{t-1} + \varepsilon_{t+H}, \tag{2.351}$$

where π_t is inflation based on GDP deflator, $shock_t$ is either military spending news shock from Ramey and Zubairy (2018) or tax rate shocks from Romer and Romer (2010); the set of controls $\mathbf{z}_{t-1}$ is specified in the description to the results figure.

As can be seen in Figure 2.I.6, both a positive spending shock and a positive shock to the tax rate produce a statistically significant increase in inflation. In this sense, even though the bulk of variation in our spending and tax

rate shocks overlaps with periods of price controls, they produce a response in inflation that is consistent with our theory. The latter gives extra support to our strategy of using co-movement of inflation and activity to identify periods driven by either demand or supply shocks.

2.I.3 Demand-side and supply-side expansions

In Table 2.I.1 we repeat estimation of conditional state-dependent spending multipliers, but extend our baseline exercise by further splitting expansionary states, where $U_t < \bar{U}$, into those where inflation is above trend, $\pi_t \geq \bar{\pi}_t$, corresponding to demand-side expansions, and those where inflation is below trend, $\pi_t < \bar{\pi}_t$, corresponding to supply-side expansions. Consistently with our theory, we find the 2-year horizon cumulative spending multiplier in supply-side expansions (0.77) to be higher than in demand-side expansions (0.64); however, the 4-year horizon spending multiplier is very imprecisely estimated in supply-side expansions, making it hard to test our predictions. In Figure 2.I.7 we report conditional state-dependent spending multipliers at horizons ranging from 4 to 20 quarters; Panel (b) confirms our earlier finding: our prediction of higher multipliers in supply-side expansions finds confirmation only at shorter horizons, up to 8 quarters.

In Table 2.I.2 we lower the unemployment threshold down to $\bar{U} = 4.5\%$, so that our expansionary states, where $U_t < \bar{U}$ now pick up more severe overheating episodes, potentially making our identification sharper and helping test our theoretical predictions regarding spending multipliers in demand- and supply-side expansions. Once again, we find strong confirmation of our theory at the 2-year horizon: in supply-side expansions the multiplier is at 1.12, as opposed to 0.85 in demand-side expansions; at the 4-year horizon we still find supply-side expansion multipliers to be higher, although the demand-side expansion multiplier is very imprecisely estimated. Figure 2.I.8 confirms that most robust confirmation of our theory for expansions is indeed found at shorter horizons, up to 8 quarters.

Table 2.I.3 extends our analysis of conditional state-dependent tax cut multipliers to demand- and supply-side expansions. Our theory predicts that tax cut multipliers should be higher in demand-side recessions, and we find empirical support for this at the 4-year horizon, but not at the 2-year horizon; moreover Figure 2.I.9 shows that our prediction for expansions holds at longer horizons, above 10 quarters, but not at shorter ones. One reason behind this could be income effects associated with tax cuts that our

model does not capture very well.

2.I.4 Blanchard and Perotti (2002) shocks

In Tables 2.I.4 and 2.I.5 as well as Figures 2.I.10 and 2.I.11 we repeat the conditional state-dependent spending multiplier estimation using VAR-based spending shocks following Blanchard and Perotti (2002), for both demand- and supply-side recessions and expansions. Overall, when we set $\bar{U} = 6.5\%$, we find confirmation to our theory for both expansions and recessions at shorter horizons, up to 5-quarters, whereas the results at longer horizons are less precise and deliver mixed evidence; when we set $\bar{U} = 4.5\%$, the results are consistent with our theoretical predictions across all horizons, but still most quantitatively significant at shorter horizons, up to 8 quarters. Therefore, our theoretical predictions for spending multiplier find most robust econometric confirmation at shorter cumulation horizons, regardless of whether one performs estimation with military spending news shocks or Blanchard-Perotti (2002) shocks; this horizon-dependence is in fact consistent with our dynamic simulations: at longer horizons more firms set prices optimally, less adjustment happens via tightness, state-dependence of multipliers is weaker and hence harder to detect econometrically.

2.I.5 Economic activity threshold based on detrended real GDP

Our baseline analysis uses unemployment as the measure of economic activity, which is done to be consistent with Ramey and Zubairy (2018). However, our theoretical model does not feature (involuntary) unemployment, and a measure of activity most consistent with our model is the cyclical component of real GDP. In this subsection we describe the results of performing estimation with an activity threshold based on detrended real GDP. [36]

In Table 2.I.6 we show results for 2- and 4-year horizon cumulative spending multipliers, where we define a recession as an episode where real GDP drops more than 3% below trend. Our classification of demand- and supply-driven recessions and expansions based on cyclical component of inflation remains unchanged. Consistently with our theory, we find that spending multipliers in demand-driven recessions are larger than spend-

[36] We use the same polynomial trend as in Gordon and Krenn (2010)

ing multipliers in supply-driven recessions: 0.55 vs. 0.11 at the 2-year horizon, and 0.60 vs. 0.48 at the 4-year horizon. In panel (c) of Figure 2.I.12 we show that the pattern of higher multipliers in demand-driven recessions holds consistently across horizons, with the effect most pronounced at earlier horizons, again in line with our theory.

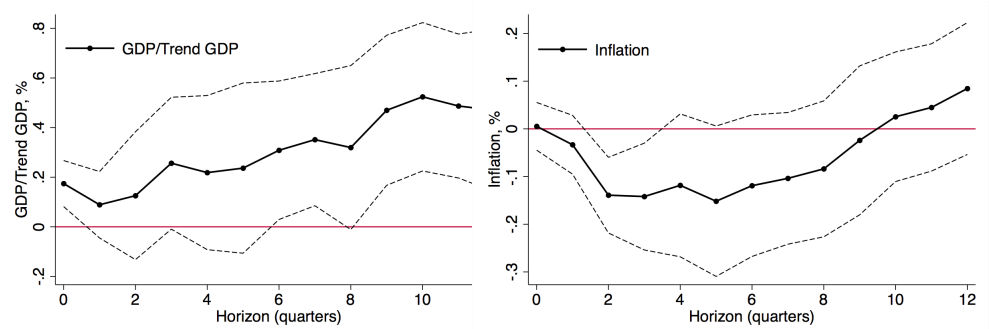
In Table 2.I.7 we change the threshold, so that only episodes where real GDP is more than 3% above trend counts as an expansion (and anything else counts as a recession). In this way we can focus on the most substantial episodes of overheating and have more power to test our theoretical predictions for expansions. Consistently with our theory we find that spending multipliers are higher in supply-driven expansions relative to demand-driven expansions: 0.68 vs 0.38 at the 2-year horizon and 0.69 vs 0.40 at the 4-year horizon. In panel (b) of Figure 2.I.13 we show that the pattern of higher multipliers in supply-driven expansions holds consistently across horizons, with the effect most pronounced at earlier horizons, again in line with our theory.

In Table 2.I.8 we again define a recession as an episode where real GDP drops for than 3% below trend, but this time estimate our specification for tax shocks. Further, panels (b) and (c) of Figure 2.I.14 exhibit estimation results for a broader set of horizons. At horizons beyond 8 quarters we find results consistent with our theory: tax cut multipliers are larger in demand-side expansions and supply-side recessions.

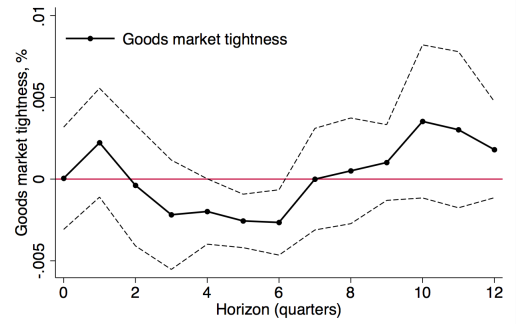
Figure 2.I.5: Impulse response functions to a +1% productivity shock (utilization-adjusted shocks from Fernald (2014a))

(a) Response of cyclical component of GDP

(b) Response of inflation (based on GDP deflator)



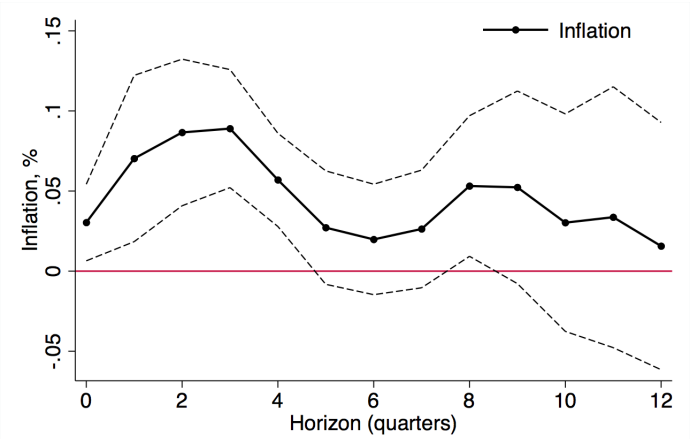
(c) Response of goods market tightness measure of Michaillat and Saez (2015)



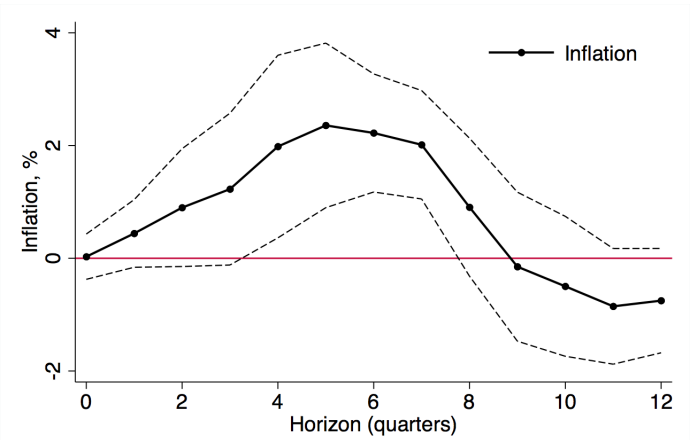
***Notes**: the figure presents estimation using local projections based on econometric specification outlined in Appendix 2.I.1 over the sample period 1973:Q4-2013:Q2; the set of controls includes one lag of cyclical GDP, inflation and goods market tightness. The dotted lines represent 90% confidence bands based on standard errors robust to autocorrelation and heteroskedasticity.*

Figure 2.I.6: Impulse response functions of inflation to government spending and tax shocks

(a) Response to +1% military spending shock of Ramey and Zubairy (2018)



(b) Response to +1% tax rate shock of Romer and Romer (2010)



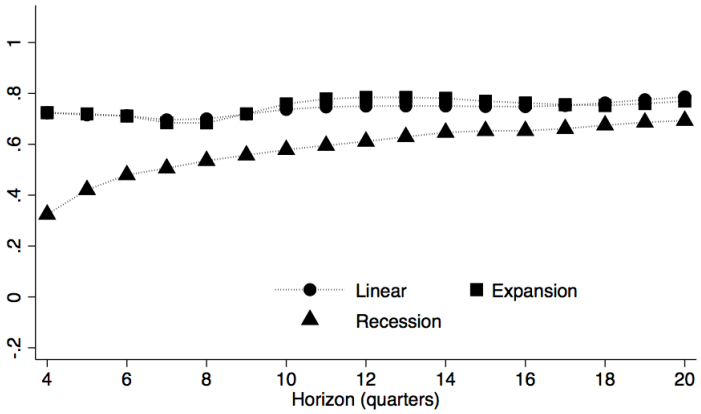
***Notes**: the figure presents estimation using local projections based on econometric specification outlined in Appendix 2.I.2 over the sample period 1909:Q1-2015:Q4 (panel (a)) and 1947:Q1-2007:Q4 (panel (b)) ; the set of controls includes four lags of cyclical GDP, inflation and spending shock (panel (a)) and four lags of cyclical GDP, inflation and average tax rate (panel (b)). The dotted lines represent 90% confidence bands based on standard errors robust to autocorrelation and heteroskedasticity.*

Table 2.I.1: Conditional state-dependent spending multipliers ($\bar{U} = 6.5\%$; US military spending news shocks)

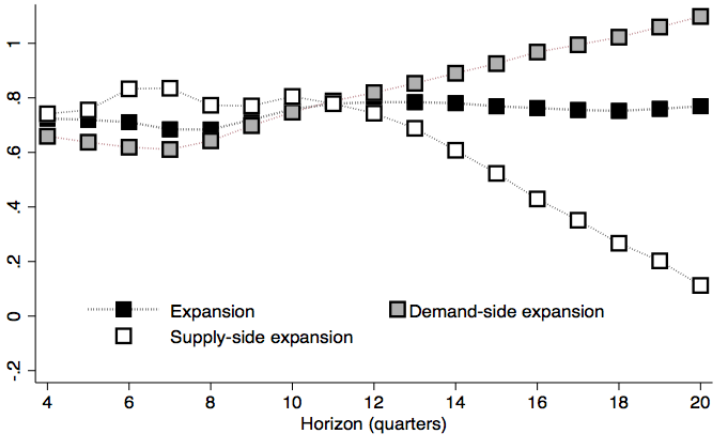
US data: 1909-2015			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	0.70***				0.75***			
	(0.06)				*(0.06)*			
β_H^E : $\mathbf{1}\{U_t < \bar{U}\}$		0.68***	0.68***			0.76***	0.76***	
		(0.10)	*(0.09)*			*(0.13)*	*(0.12)*	
β_H^R : $\mathbf{1}\{U_t \geq \bar{U}\}$		0.54***				0.65***		
		(0.13)				*(0.08)*		
β_H^{DE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t \geq \bar{\pi}_t\}$				0.64***				0.97***
				(0.06)				*(0.20)*
β_H^{SE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t < \bar{\pi}_t\}$				0.77***				0.43
				(0.29)				*(0.47)*
β_H^{DR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			0.86***	0.86***			0.72***	0.72***
			(0.33)	*(0.33)*			*(0.12)*	*(0.12)*
β_H^{SR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			0.32***	0.32***			0.63***	0.63***
			(0.11)	*(0.11)*			*(0.09)*	*(0.09)*
$\beta_H^E = \beta_H^R$ (p-value)		0.37				0.44		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.62				0.81	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.01				0.40	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.14	0.14			0.54	0.54
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.63				0.29
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure 2.I.7: Government Spending Multipliers across Horizons (US military spending news shocks, 1909-2015)

(a) Government spending multipliers in recessions and expansions across horizons ($\bar{U} = 6.5\%$)



(b) Government spending multipliers in demand-side and supply-side expansions across horizons



(c) Government spending multipliers in demand-side and supply-side recessions across horizons

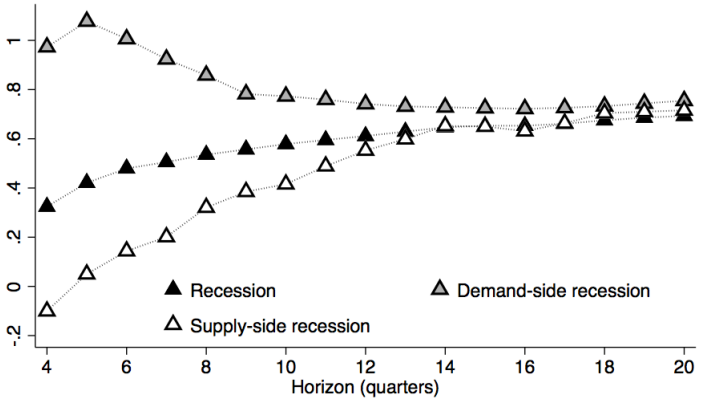
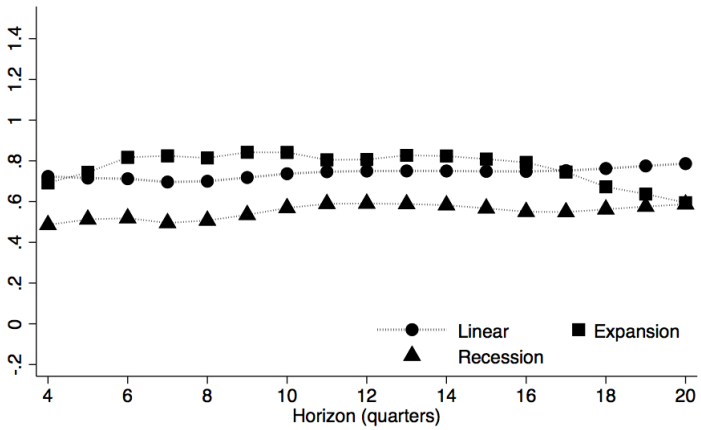


Table 2.I.2: Conditional state-dependent spending multipliers ($\bar{U} = 4.5\%$; US military spending news shocks)

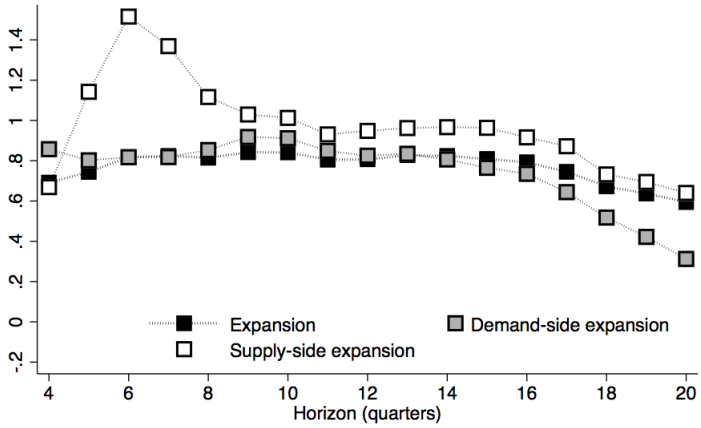
US data: 1909-2015		2y horizon				4y horizon			
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	0.70***				0.75***			
	(0.06)				*(0.06)*			
β_H^E : $\mathbf{1}\{U_t < \bar{U}\}$		0.81***	0.81***			0.79***	0.79***	
		(0.20)	*(0.20)*			*(0.28)*	*(0.30)*	
β_H^R : $\mathbf{1}\{U_t \geq \bar{U}\}$		0.51***				0.55***		
		(0.12)				*(0.12)*		
β_H^{DE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t \geq \bar{\pi}_t\}$				0.85**				0.74
				(0.34)				*(0.66)*
β_H^{SE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t < \bar{\pi}_t\}$				1.12*				0.92*
				(0.61)				*(0.55)*
β_H^{DR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			0.69**	0.69**			0.51**	0.51**
			(0.32)	*(0.32)*			*(0.23)*	*(0.25)*
β_H^{SR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			0.31***	0.31***			0.55***	0.55***
			(0.09)	*(0.09)*			*(0.10)*	*(0.10)*
$\beta_H^E = \beta_H^R$ (p-value)		0.11				0.41		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.73				0.43	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.02				0.48	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.22	0.22			0.88	0.88
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.72				0.83
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure 2.I.8: Government Spending Multipliers across Horizons (US military spending news shocks, 1909-2015)

(a) Government spending multipliers in recessions and expansions across horizons ($\bar{U} = 4.5\%$)



(b) Government spending multipliers in demand-side and supply-side expansions across horizons



(c) Government spending multipliers in demand-side and supply-side recessions across horizons

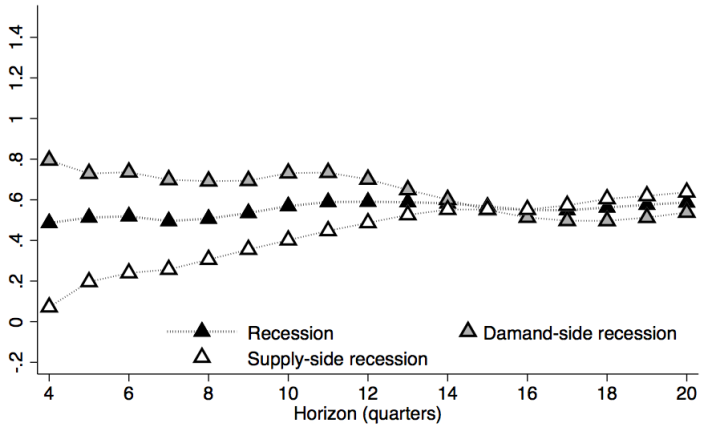
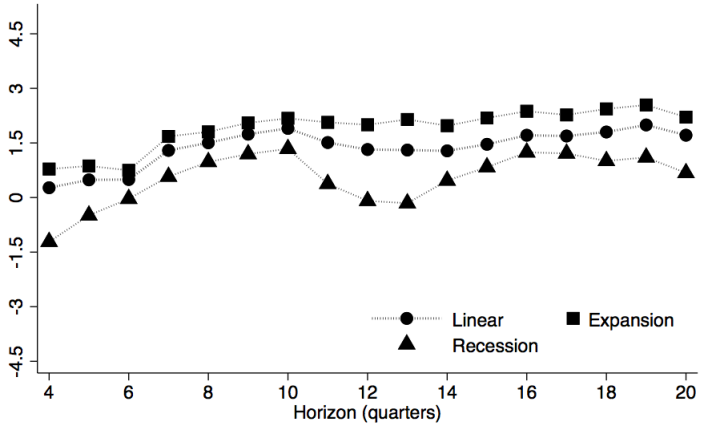


Table 2.I.3: Conditional state-dependent tax cut multipliers ($\bar{U} = 6.5\%$; US Romer-Romer narrative tax shocks)

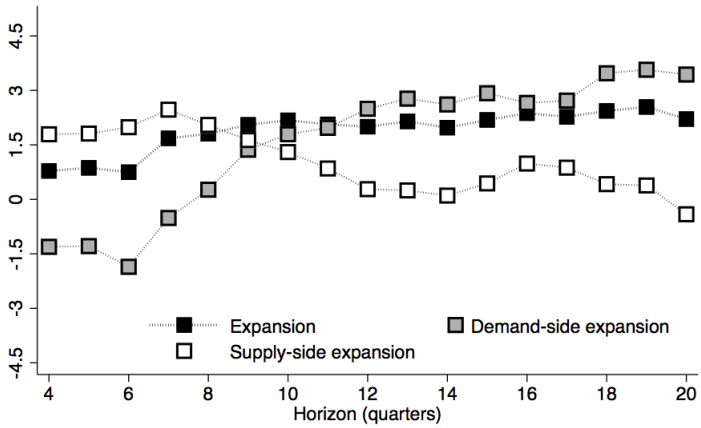
US data: 1947-2097			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	1.50				1.71**			
	(1.14)				*(0.82)*			
β_H^E : $\mathbf{1}\{U_t < \bar{U}\}$		1.81	1.81			2.37**	2.37**	
		(1.17)	*(1.16)*			*(0.99)*	*(0.99)*	
β_H^R : $\mathbf{1}\{U_t \geq \bar{U}\}$		0.98				1.24		
		(1.07)				*(0.87)*		
β_H^{DE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t \geq \bar{\pi}_t\}$				0.27				2.65*
				(1.08)				*(1.50)*
β_H^{SE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t < \bar{\pi}_t\}$				2.05				0.99
				(1.34)				*(1.89)*
β_H^{DR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			1.49	1.49			-1.98	-1.98
			(1.04)	*(1.04)*			*(2.75)*	*(2.77)*
β_H^{SR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			4.29*	4.29*			1.80*	1.80*
			(2.18)	*(2.18)*			*(1.00)*	*(1.00)*
$\beta_H^E = \beta_H^R$ (p-value)		0.48				0.39		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.84				0.12	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.28				0.70	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.25	0.25			0.20	0.20
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.32				0.51
Paap-Kleibergen LM-test								
T	240	240	240	240	240	240	240	240

Figure 2.I.9: Tax Cut Multipliers across Horizons (US Romer-Romer narrative tax shocks, 1947-2007)

(a) Tax cut multipliers in recessions and expansions across horizons ($\bar{U} = 6.5\%$)



(b) Tax cut multipliers in demand-side and supply-side expansions across horizons



(c) Tax cut multipliers in demand-side and supply-side recessions across horizons

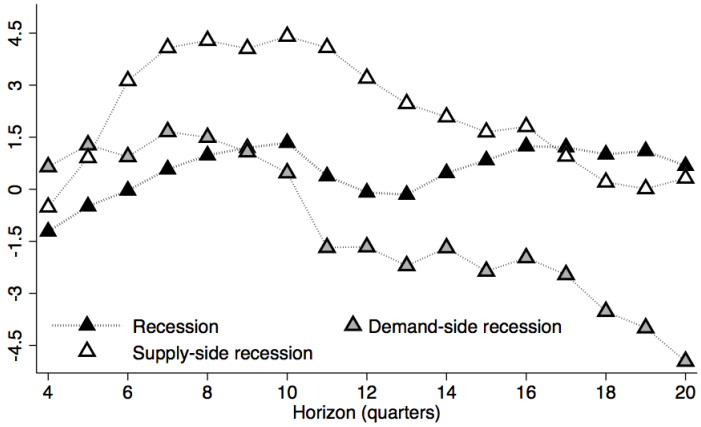
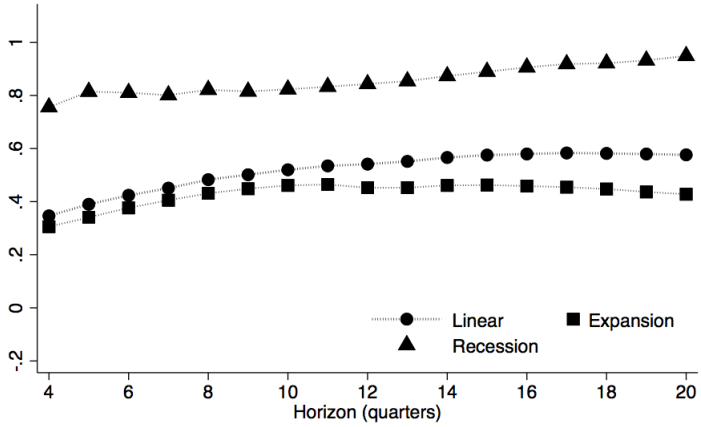


Table 2.I.4: Conditional state-dependent spending multipliers ($\bar{U} = 6.5\%$; US Blanchard-Perotti spending shocks)

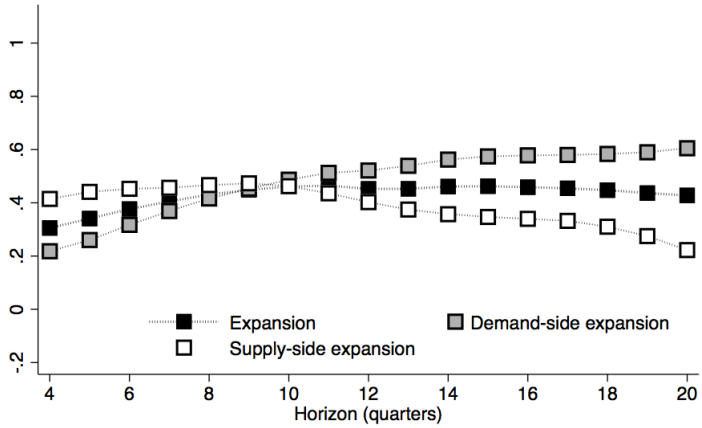
US data: 1909-2015			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	0.48***				0.58***			
	(0.09)				*(0.10)*			
β_H^E : $\mathbf{1}\{U_t < \bar{U}\}$		0.43***	0.42***			0.46***	0.44***	
		(0.08)	*(0.09)*			*(0.12)*	*(0.13)*	
β_H^R : $\mathbf{1}\{U_t \geq \bar{U}\}$		0.82***				0.91***		
		(0.11)				*(0.07)*		
β_H^{DE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t \geq \bar{\pi}_t\}$				0.42***				0.58***
				(0.08)				*(0.14)*
β_H^{SE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t < \bar{\pi}_t\}$				0.47**				0.34
				(0.19)				*(0.25)*
β_H^{DR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			0.54***	0.56***			0.62***	0.65***
			(0.16)	*(0.15)*			*(0.13)*	*(0.14)*
β_H^{SR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			0.84***	0.84***			0.92***	0.91***
			(0.14)	*(0.14)*			*(0.21)*	*(0.21)*
$\beta_H^E = \beta_H^R$ (p-value)		0.00				0.00		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.50				0.36	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.01				0.05	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.19	0.22			0.34	0.37
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.81				0.35
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure 2.I.10: Government Spending Multipliers across Horizons (US Blanchard-Perotti spending shocks, 1909-2015)

(a) Government spending multipliers in recessions and expansions across horizons ($\bar{U} = 6.5\%$)



(b) Government spending multipliers in demand-side and supply-side expansions across horizons



(c) Government spending multipliers in demand-side and supply-side recessions across horizons

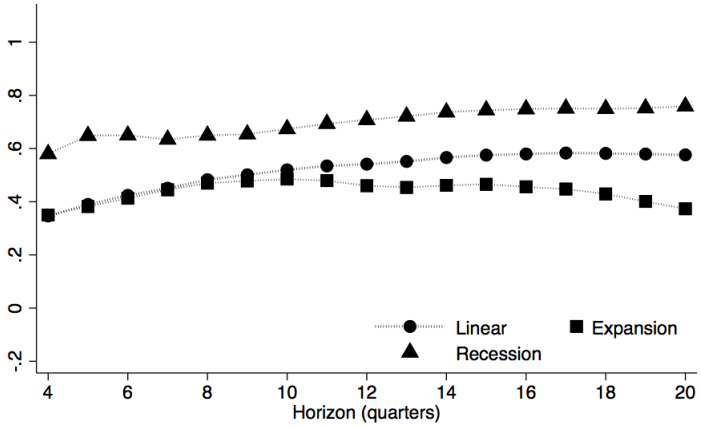


Table 2.I.5: Conditional state-dependent spending multipliers ($\bar{U} = 4.5\%$; US Blanchard-Perotti spending shocks)

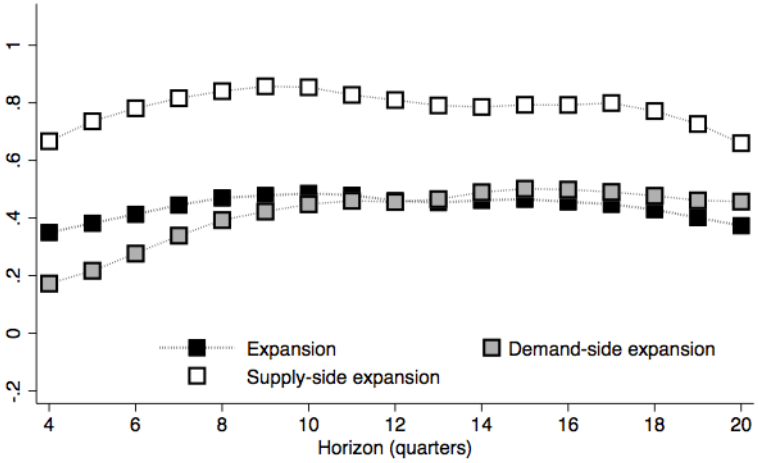
US data: 1889-2015			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	0.48***				0.58***			
	(0.09)				*(0.10)*			
β_H^E : $\mathbf{1}\{U_t < \bar{U}\}$		0.47***	0.47***			0.46**	0.46**	
		(0.15)	*(0.15)*			*(0.18)*	*(0.18)*	
β_H^R : $\mathbf{1}\{U_t \geq \bar{U}\}$		0.65***				0.75***		
		(0.10)				*(0.10)*		
β_H^{DE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t \geq \bar{\pi}_t\}$				0.39***				0.50***
				(0.09)				*(0.14)*
β_H^{SE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t < \bar{\pi}_t\}$				0.84***				0.79***
				(0.11)				*(0.26)*
β_H^{DR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			0.75***	0.76***			0.74***	0.76***
			(0.16)	*(0.16)*			*(0.10)*	*(0.09)*
β_H^{SR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			0.54***	0.51***			0.73***	0.70***
			(0.15)	*(0.15)*			*(0.12)*	*(0.12)*
$\beta_H^E = \beta_H^R$ (p-value)		0.25				0.08		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.22				0.12	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.70				0.09	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.38	0.33			0.99	0.68
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.00				0.31
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure 2.I.11: Government Spending Multipliers across Horizons (US Blanchard-Perotti spending shocks, 1909-2015)

(a) Spending multipliers in recessions and expansions across horizons ($\bar{U} = 4.5\%$)



(b) Spending multipliers in demand-side and supply-side expansions across horizons



(c) Spending multipliers in demand-side and supply-side recessions across horizons

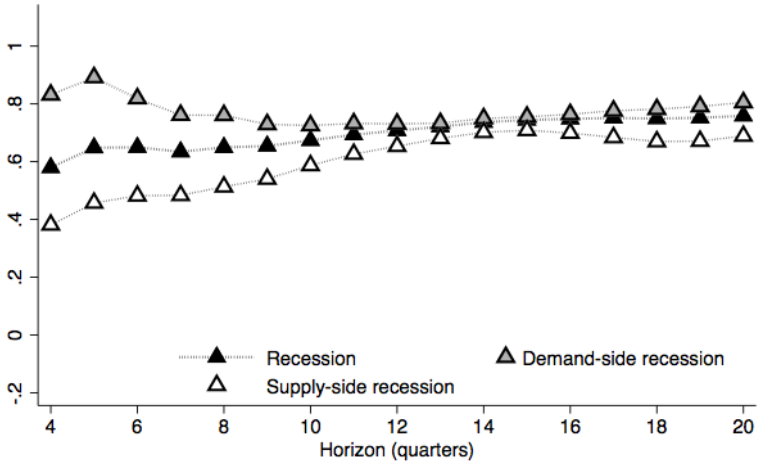
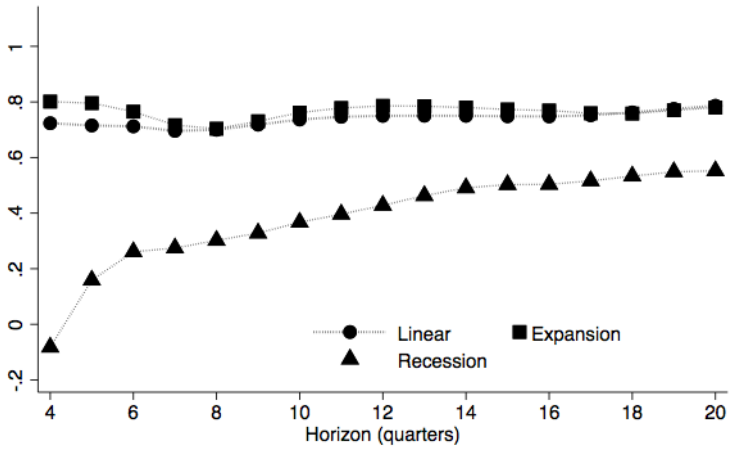


Table 2.I.6: Conditional state-dependent spending multipliers (cyclical GDP-based threshold $G\hat{D}P_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP; US military spending news shocks)

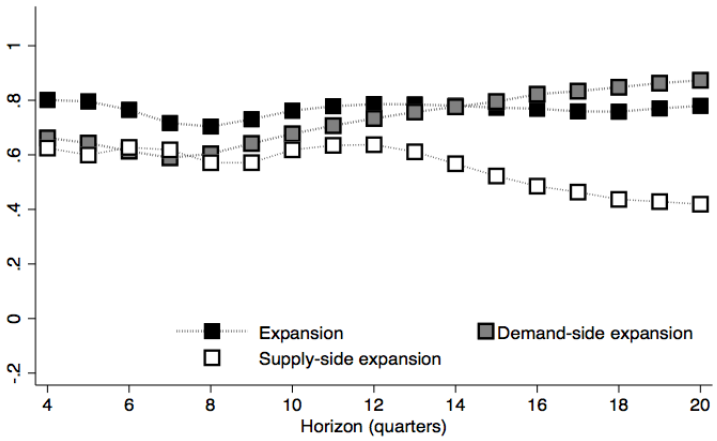
US data: 1909-2015			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	0.70***				0.75***			
	(0.06)				*(0.06)*			
β_H^E : $\mathbf{1}\{G\hat{D}P_t \geq -3\%\}$		0.70***	0.70***			0.77***	0.77***	
		(0.11)	*(0.11)*			*(0.08)*	*(0.08)*	
β_H^R : $\mathbf{1}\{G\hat{D}P_t < -3\%\}$		0.30			0.50***			
		(0.24)			*(0.17)*			
β_H^{DE} : $\mathbf{1}\{G\hat{D}P_t \geq -3\%; \pi_t \geq \bar{\pi}_t\}$				0.60***				0.82***
				(0.07)				*(0.11)*
β_H^{SE} : $\mathbf{1}\{G\hat{D}P_t \geq -3\%; \pi_t < \bar{\pi}_t\}$				0.57***				0.49**
				(0.18)				*(0.19)*
β_H^{DR} : $\mathbf{1}\{G\hat{D}P_t < -3\%; \pi_t < \bar{\pi}_t\}$			0.55**	0.55**			0.60***	0.60***
			(0.25)	*(0.24)*			*(0.12)*	*(0.12)*
β_H^{SR} : $\mathbf{1}\{G\hat{D}P_t < -3\%; \pi_t \geq \bar{\pi}_t\}$			0.11	0.11			0.48***	0.48***
			(0.29)	*(0.31)*			*(0.15)*	*(0.15)*
$\beta_H^E = \beta_H^R$ (p-value)		0.13				0.20		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.56				0.28	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.06				0.13	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.26	0.28			0.48	0.48
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.85				0.12
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure 2.I.12: Government Spending Multipliers across Horizons (US military spending news shocks, 1909-2015, cyclical GDP-based threshold $\hat{GDP}_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP, threshold of -3%)

(a) Spending multipliers in recessions and expansions (recession if GDP more than 3% below trend)



(b) Spending multipliers in demand-side and supply-side expansions



(c) Spending multipliers in demand-side and supply-side recessions

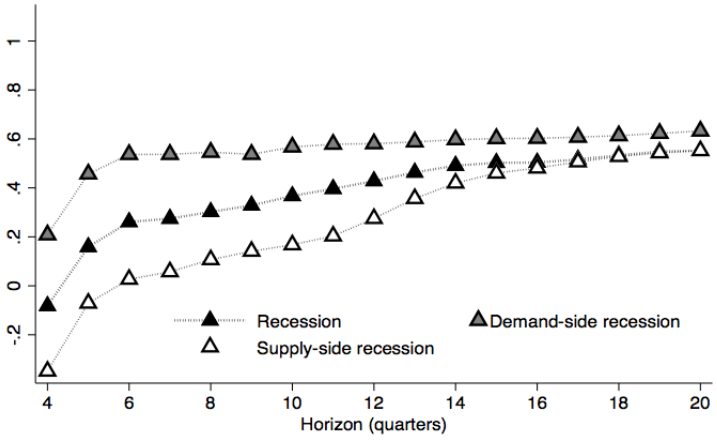
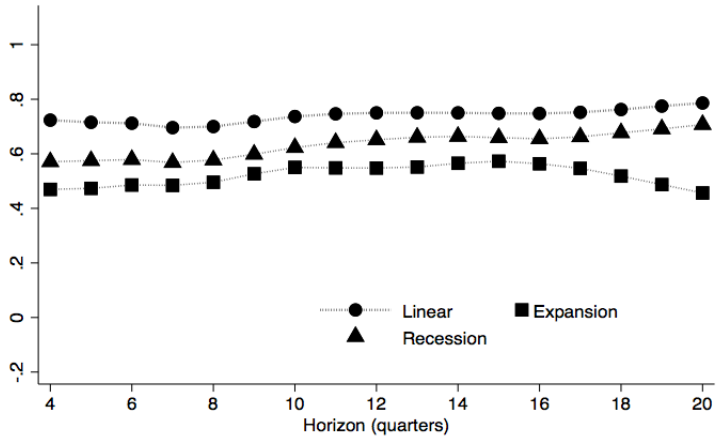


Table 2.I.7: Conditional state-dependent spending multipliers (cyclical GDP-based threshold $\hat{GDP}_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP; US military spending news shocks)

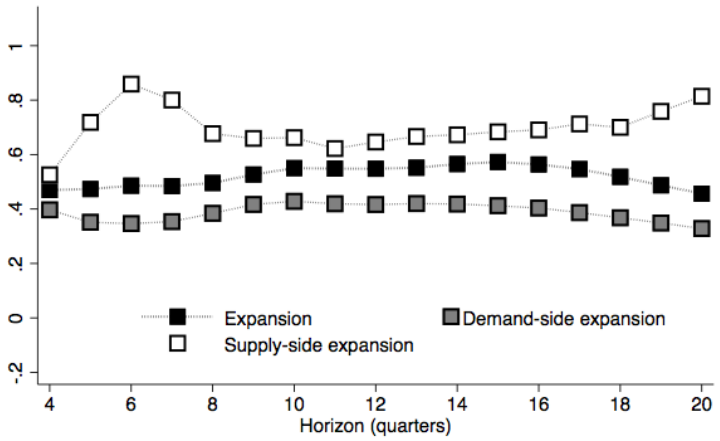
US data: 1909-2015			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	0.70***				0.75***			
	(0.06)				*(0.06)*			
β_H^E : $\mathbf{1}\{\hat{GDP}_t \geq 3\%\}$		0.50***	0.50***			0.56***	0.56***	
		(0.08)	*(0.08)*			*(0.09)*	*(0.10)*	
β_H^R : $\mathbf{1}\{\hat{GDP}_t < 3\%\}$		0.58***				0.66***		
		(0.07)				*(0.07)*		
β_H^{DE} : $\mathbf{1}\{\hat{GDP}_t \geq 3\%; \pi_t \geq \bar{\pi}_t\}$				0.38***				0.40***
				(0.08)				*(0.12)*
β_H^{SE} : $\mathbf{1}\{\hat{GDP}_t \geq 3\%; \pi_t < \bar{\pi}_t\}$				0.68**				0.69**
				(0.30)				*(0.32)*
β_H^{DR} : $\mathbf{1}\{\hat{GDP}_t < 3\%; \pi_t < \bar{\pi}_t\}$			0.33	0.33			0.32	0.32
			(0.28)	*(0.27)*			*(0.26)*	*(0.40)*
β_H^{SR} : $\mathbf{1}\{\hat{GDP}_t < 3\%; \pi_t \geq \bar{\pi}_t\}$			0.57***	0.57***			0.76***	0.76***
			(0.07)	*(0.06)*			*(0.12)*	*(0.10)*
$\beta_H^E = \beta_H^R$ (p-value)		0.42				0.48		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.54				0.33	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.50				0.28	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.43	0.39			0.11	0.28
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.36				0.41
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure 2.I.13: Government Spending Multipliers across Horizons (US military spending news shocks, 1909-2015, cyclical GDP-based threshold $\hat{GDP}_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP, threshold of 3%)

(a) Spending multipliers in recessions and expansions (expansion if GDP more than 3% above trend)



(b) Spending multipliers in demand-side and supply-side expansions



(c) Spending multipliers in demand-side and supply-side recessions

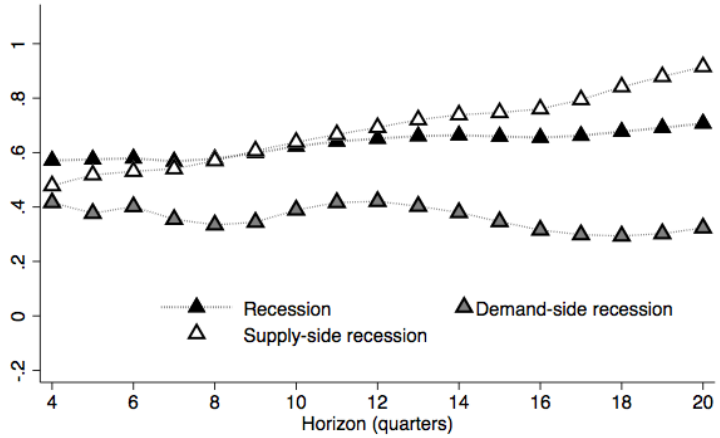
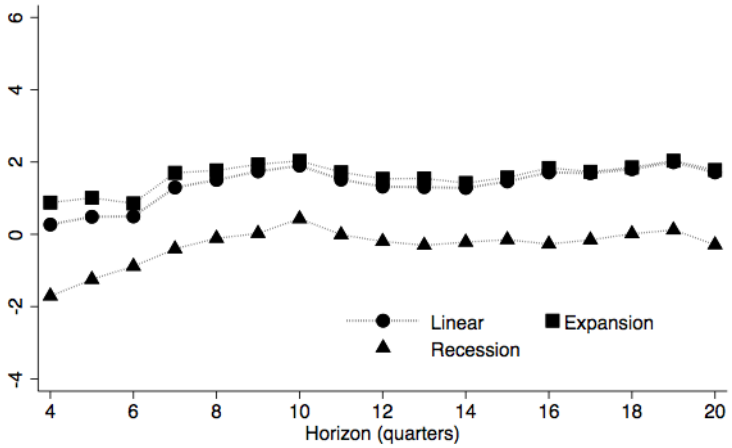


Table 2.I.8: Conditional state-dependent tax cut multipliers (cyclical GDP-based threshold $\hat{GDP}_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP; US Romer-Romer narrative tax shocks)

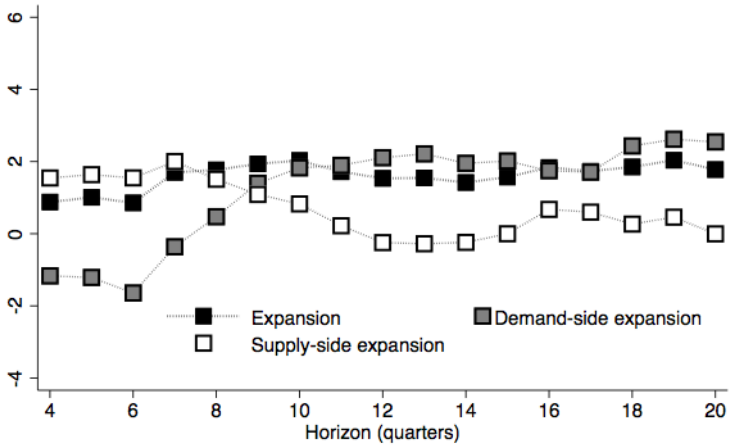
US data: 1909-2015			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	1.50				1.71**			
	(1.14)				*(0.82)*			
β_H^E : $\mathbf{1}\{\hat{GDP}_t \geq -3\%\}$		1.77*	1.77*			1.84*	1.84*	
		(1.01)	*(1.01)*			*(0.95)*	*(1.01)*	
β_H^R : $\mathbf{1}\{\hat{GDP}_t < -3\%\}$		-0.11				-0.27		
		(0.60)				*(0.45)*		
β_H^{DE} : $\mathbf{1}\{\hat{GDP}_t \geq -3\%; \pi_t \geq \bar{\pi}_t\}$				0.47				1.75
				(1.56)				*(1.25)*
β_H^{SE} : $\mathbf{1}\{\hat{GDP}_t \geq -3\%; \pi_t < \bar{\pi}_t\}$				1.51				0.67
				(1.16)				*(2.13)*
β_H^{DR} : $\mathbf{1}\{\hat{GDP}_t < -3\%; \pi_t < \bar{\pi}_t\}$			1.05	1.05			1.51	1.51
			(1.45)	*(1.45)*			*(1.35)*	*(1.38)*
β_H^{SR} : $\mathbf{1}\{\hat{GDP}_t < -3\%; \pi_t \geq \bar{\pi}_t\}$			0.81	0.81			2.27*	2.27*
			(0.89)	*(0.89)*			*(1.30)*	*(1.30)*
$\beta_H^E = \beta_H^R$ (p-value)		0.04				0.04		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.70				0.85	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.45				0.78	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.88	0.88			0.69	0.69
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.61				0.66
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure 2.I.14: Tax Cut Multipliers across Horizons (US Romer-Romer narrative tax shocks, 1947-2007, cyclical GDP-based threshold $G\hat{D}P_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP, threshold of -3%)

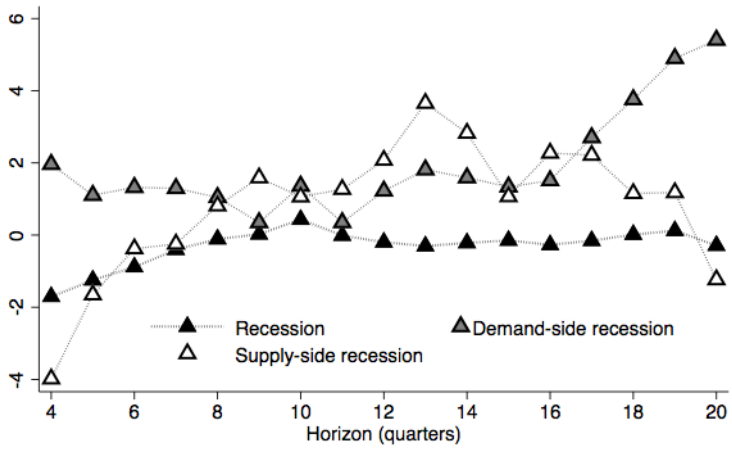
(a) Tax cut multipliers in recessions and expansions (recession if GDP more than 3% below trend)



(b) Tax cut multipliers in demand-side and supply-side expansions



(c) Tax cut multipliers in demand-side and supply-side recessions



Chapter 3

Endogenous Production Networks and Non-Linear Monetary Transmission

Abstract We develop a tractable sticky-price model, where input-output linkages are formed endogenously. The model delivers cyclical properties of networks that are consistent with those estimated using sectoral and firm-level data, conditional on identified real and nominal shocks. A novel source of state dependence in nominal rigidities arises: the strength of complementarities in price setting and monetary non-neutrality increase in the number of suppliers that firms optimally choose to have. As a result, the model simultaneously rationalizes the following observed non-linearities in monetary transmission. First, the model produces *cycle dependence*: the magnitude of real GDP's response to a monetary shock is procyclical. This occurs because in expansions the level of productivity is high, encouraging cost-minimizing firms to connect to more suppliers, which makes pricing decisions more co-ordinated and monetary non-neutrality stronger. Second, there is *path dependence*: non-neutrality of real GDP is higher following previous periods of loose monetary policy. This happens as under nominal rigidities higher supply of money makes prices charged by suppliers cheap relative to the cost of in-house labor, encouraging more connections and strengthening pricing complementarities. Third, there is *size dependence*: larger monetary expansions make the network denser and have a disproportionally larger effect on GDP than smaller expansions. On the other hand, larger monetary contractions shrink the network and generate a disproportionally smaller decrease in GDP. Such size dependence holds even if the probability of price adjustment is state-independent.

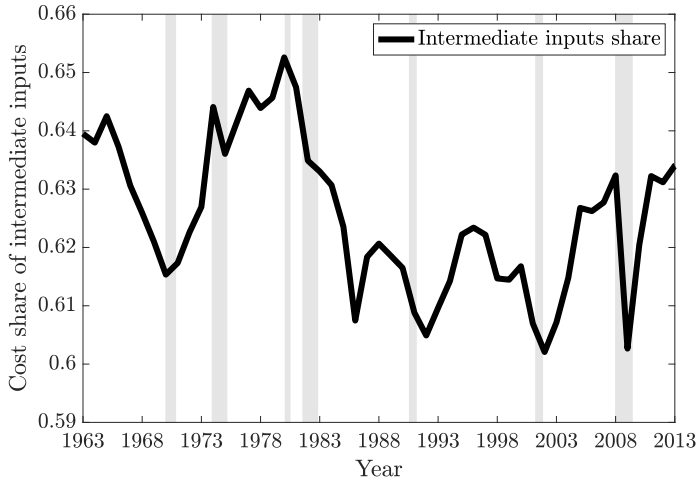
3.1 Introduction

Intermediate inputs, and production networks that facilitate their trade, are a cornerstone of modern economies' production structures. According to the Bureau of Economics Analysis, in the United States more than 50 per cent of all produced goods are processed as intermediate inputs. Despite intermediates being a key part of production, their relative importance falls in recessions: Figure 3.1 shows that the aggregate cost share of intermediates drops sharply in periods of economic slack. Moreover, one can observe that the reliance on intermediates falls both in recessions that originated in the real sector, as well as in those caused by monetary contractions, such as the Volcker Disinflation. Such network cyclicality is missed by models that take input-output linkages as given. We fill this gap by developing a sticky-price general equilibrium model with endogenous network formation. The model delivers cyclical properties of networks that are consistent with those we estimate using sectoral and firm-level data, conditional on both real and nominal shocks.

Accounting for the observed network cyclicality allows the model to rationalize multiple observed non-linearities in the transmission of monetary shocks to real variables. This is because the model links the density of the network, given by the number of suppliers each firms optimally chooses to have, with the degree of complementarities in price setting created by the roundabout production structure. Specifically, in states of the world where firms endogenously connect to more suppliers, their unit cost becomes dependent on a larger number of prices set by other firms, which strengthens pricing complementarities and amplifies monetary non-neutrality. Naturally, in states with few linkages across firms the opposite occurs: pricing complementarities weaken and monetary non-neutrality diminishes. This novel mechanism makes the strength of monetary transmission depend on the *phase of the business cycle*, *past monetary stance* and the *size of the shock* even if the probability of price adjustment is state-independent. Below we explain each of the three non-linearities step-by-step.

First, the magnitude of real GDP response to monetary shocks is *cycle-dependent*, since the degree of short-run monetary non-neutrality is stronger in expansionary states. The mechanism that generates this effect is as follows: in expansions, the level of productivity is high, which incentivizes firms to connect to more suppliers by lowering their unit costs directly

Figure 3.1: The relative importance of intermediate inputs (United States)



***Notes**: the figure documents the ratio of total expenditure on intermediate inputs and total production costs in the United States (1963-2013), based on data published by the US Bureau of Economic Analysis (BEA)*

through higher productivity and indirectly through lower prices charged by suppliers. The latter creates stronger non-neutrality of money due to amplified complementarities in price setting. On the other hand, in recessions firms optimally decide to disconnect from some suppliers, making the economy more money-neutral, as complementarities in price setting weaken. These theoretical findings are consistent with the recent model-free econometric studies that find a weaker response of GDP to monetary shocks in recessions (Tenreyro and Thwaites, 2016; Alpanda et al., 2019; Jordà et al., 2019).

Second, there is *path dependence* in the strength of transmission, since following a monetary loosening any further monetary intervention has a stronger effect on GDP. Due to rigidities in price setting, a monetary loosening makes prices charged by supplier firms cheaper relative to the cost of in-house labor, which incentivizes firms to connect to more suppliers, hence expanding the network and strengthening complementarities in price setting. As a result, any subsequent monetary intervention has a stronger effect on GDP. Of course, the converse also holds: any monetary shocks have a weaker effect on real GDP whenever they are preceded by episodes of tight monetary policy. Such theoretical findings are consistent with the econometric evidence of stronger monetary transmission to GDP under an already

loose monetary stance (Alpanda et al., 2019) and additional evidence that finds GDP to be more sensitive to monetary interventions whenever credit is loose (Jordà et al., 2019).

Third, the model produces *size dependence*, as the response of GDP does not change linearly with the size of monetary shock, even if the probability of price adjustment is state-independent. Larger monetary expansions have a disproportionally *larger* positive effect on real GDP compared to smaller monetary expansions, as the former make the network denser. As a result, achieving a given degree of real GDP expansion requires a (weakly) smaller monetary easing. On the other hand, larger monetary contractions have a disproportionally *smaller* negative effect on real GDP compared to smaller monetary contractions, as the former shrink the network. A consequence of the above is that achieving a given degree of real GDP contraction requires a (weakly) larger monetary tightening. Econometric findings in Alvarez et al. (2017) and Ascari and Haber (2021), find that large nominal shocks have a disproportionally smaller effect on real variables, compared to small nominal shocks, which is consistent with our predictions for monetary contractions.

Fourth, we provide novel model-free econometric evidence on network cyclicality, conditional on identified productivity and monetary shocks, which corroborates the theoretical mechanism in our model. As a first exercise, we use sectoral data from US Bureau of Economic Analysis (BEA) to construct annual time series of intermediates intensities for 65 sectors of the US economy. Consistent with the theoretical prediction, we find that intermediates intensity rises following a positive productivity shock, and following a monetary easing. Moreover, we find evidence that the effect is asymmetric: expansionary shocks, both productivity and monetary, lead to disproportionally larger magnitudes of network responses. We compare estimated responses with those generated by a calibrated version of our model, and find them to be close, both in magnitudes and in the asymmetry of responses. One limitation of using sectoral data is that it does not allow to disentangle intensive and extensive margins of network adjustment, whereas it is the latter that is predicted by our model. We therefore provide additional evidence using data on firm-level linkages in Compustat, as constructed by Atalay et al. (2011). our findings using firm-level data confirm the evidence obtained using sectoral data, both in terms of directions of responses to productivity and monetary shocks, and in terms of the asymmetries detected.

Contribution to the literature. This paper is related to three strands of the literature. First, it contributes to the relatively recent literature on endogenous production networks in macroeconomics, by developing the first general equilibrium model featuring endogenous network formation under nominal rigidities. It also provides novel model-free econometric evidence on network cyclicality, both unconditionally and conditional on productivity and monetary shocks, which the model successfully replicates. Previous macroeconomic studies have focused on environments with flexible prices (Carvalho and Voigtländer, 2015; Oberfield, 2018; Acemoglu and Azar, 2020) or on finding the social planner's solution to the network formation problem (Taschereau-Dumouchel, 2019).

Second, it contributes to the literature on non-linearities in monetary transmission. In particular, it develops a tractable framework that can rationalize multiple non-linearities through a novel theoretical channel, which is supported by both aggregate and disaggregated data. Therefore, although this is not the first paper to rationalize non-linearities in monetary transmission, it does so using a completely new mechanism. In related literature, Santoro et al. (2014) study asymmetric transmission of monetary policy under loss aversion; McKay and Wieland (2019) rationalize path dependence in a framework with lumpy durable consumption demand; Alpanda et al. (2019) explain non-linearities in a model with constraints on household borrowing and refinancing; Eichenbaum et al. (2018) show how the effect on monetary policy depends on the distribution of savings from refinancing mortages; Berger et al. (2021) investigate path-dependence under pre-payable mortgages; Bernstein (2021) shows that presence of occasionally binding borrowing constraints and household heterogeneity makes responses to monetary transmission stronger in expansions and contractionary shocks more powerful than expansionary ones.

Third, the paper contributes to the literature on monetary transmission in multi-sector New Keynesian models with roundabout production and asymmetric input-output linkages. Specifically, it provides novel sector-level closed-form characterization of monetary transmission under endogenous network formation in a simplified version of the model, and develops a novel numerical algorithm for finding sector-level solutions to a forward-looking version of the model with an arbitrary number of sectors. Previous studies quantify monetary transmission under fixed exogenous production networks (Nakamura and Steinsson, 2010; Pasten et al., 2020; Ghassibe, 2021)

or aim to explicitly estimate New Keynesian models with exogenous networks (Smets et al., 2019; Carvalho et al., 2021). As for optimal monetary policy under fixed exogenous networks, it has been studied by (n.d.) and Rubbo (2020).

The remainder of the paper is structured as follows. Section 3.2 sets out the general theoretical framework. Section 3.3 develops an analytically tractable version of the model and presents the key theoretical results. Section 3.4 develops the numerical algorithm for solving a dynamic forward looking version of the model and quantifies the key effects. Section 3.5 provides novel econometric evidence that corroborates the key mechanisms of the model. Section 3.6 concludes.

3.2 Theoretical framework

My theory unifies two environments that have so far been treated as separate in the literature. On the one hand, the model features a multi-sector roundabout production structure, with firms' pricing subject to sector-specific nominal rigidities, in spirit of Pasten et al. (2020) and Ghassibe (2021), but without specifying functional forms. On the other hand, input-output linkages are formed *endogenously* by firms that optimize their production costs, in a manner introduced by Acemoglu and Azar (2020) in a static, flexible-price environment. As a result, the equilibrium production network responds to changes in real *and* monetary conditions, both realized and anticipated, allowing for the degree of complementarity in price setting to vary across states of the world. The latter is our key novel mechanism.

3.2.1 Model overview

Time is discrete, with outcomes in $t = 0$ exogenously given, and outcomes in $t \geq 1$ determined by agents' decisions. There are three types of agents in our model. First, a continuum of infinitely lived households. Second, a continuum of monopolistically competitive firms, owned by the households, where each firm belongs to one, and only one, of the K sectors; let the set of all firms in sector k be $\Phi_k, \forall k = 1, 2, ..., K$. Third, a government, comprising of a central bank which conducts monetary policy, and a fiscal authority which collects taxes from firms and rebates them to households.

A crucial feature of our economy is presence of a roundabout production structure across sectors, where the input-output linkages are formed

endogenously through each firm's choice of set of suppliers, denoted by $S_k \subseteq \{1, 2, ..., K\}, \forall k, \forall j \in \Phi_k$. For every choice of supplier sectors, there is a given level of productivity pinned down by a predetermined time- and sector-specific mapping $\{\mathcal{A}_{kt}(S_{kt})\}_{t=1}^{\infty}$, and $\mathcal{A}_t \equiv [\mathcal{A}_{1t}(.), \mathcal{A}_{2t}(.), ..., \mathcal{A}_{Kt}(.)]', \forall t$. Importantly, the entire path $\{\mathcal{A}_t\}_{t=1}^{\infty}$ is known to the agents at $t = 0$, and it is expected to remain unchanged forever. For any two mappings $\underline{\mathcal{A}}$ and $\overline{\mathcal{A}}$, the convention is that $\overline{\mathcal{A}} \geq \underline{\mathcal{A}}$ if and only if $\overline{\mathcal{A}}_k(S_k) \geq \underline{\mathcal{A}}_k(S_k), \forall S_k, \forall k$.

As for the nominal side of the economy, at $t = 0$ the agents know the initial level of money supply $\mathcal{M}_0$, and expect it to remain unchanged forever. At the beginning of the first period $t = 1$, they discover the future path of money supply $\{\mathcal{M}_t\}_{t=1}^{\infty}$, and there is no uncertainty from there onwards. The central bank credibly commits to a set of policies that maintain the equilibrium quantity of money at the exogenous level $\mathcal{M}_t$ in every period $t \geq 1$.

3.2.2 Firms: production and endogenous choice of suppliers

On the production side, there are K sectors, indexed by $k = 1, 2, ..., K$ with a measure one of firms in each sector; let Φ_k denote the set of all firms in sector k. The production function of firm $j \in \Phi_k$ is given by:

$$Y_{kt}(j) = \mathcal{F}_k \left[S_{kt}, \mathcal{A}_{kt}(S_{kt}), N_{kt}(j), \{Z_{krt}(j)\}_{r \in S_{kt}} \right] \tag{3.1}$$

where $S_{kt} \subseteq \{1, 2, ..., K\}$ is the set of sectors, whose firms supply inputs to firms in sector k at time t, $\mathcal{A}_{kt}(.)$ is a mapping from the chosen set of suppliers to the associated level of productivity at time t, $N_{kt}(j)$ is the labor input of firm j in sector k at time t, whereas $Z_{krt}(j)$ denotes purchases of intermediate inputs from sector r, which is in turn is an aggregator of purchases from all firms in that sector: $Z_{krt}(j) \equiv \left(\int_{j' \in \Phi_r} Z_{krt}(j, j')^{\frac{\theta-1}{\theta}} dj' \right)^{\frac{\theta}{\theta-1}}$, $\theta > 1$. We impose the following regularity conditions on the production function:

Assumption 3.1 (Production function)**.** *For every sector $k = 1, 2, ..., K$, the production function satisfies the following conditions: (a) $\mathcal{F}_k$ is strictly quasi-concave, exhibits constant returns to scale in $(N_{kt}(j), \{Z_{krt}(j)\}_{r \in S_{kt}})$, is increasing and continuous in $\mathcal{A}_{kt}(S_{kt})$, $N_{kt}(j)$ and $\{Z_{krt}(j)\}_{r \in S_{kt}}$, and is strictly increasing in $\mathcal{A}_{kt}(S_{kt})$ when $N_{kt}(j) > 0$ and $\{Z_{krt}(j)\}_{r \in S_{kt}} > 0$; (b) labor is an essential factor of production: $\mathcal{F}_k(\cdot, \cdot, 0, \cdot) = 0$; (c) $\mathcal{A}_{kt}(\emptyset) > 0$.*

Conditional on a particular set of suppliers S_{kt}, each firm's total cost of production at time t is given by $\left[W_t N_{kt}(j) + \sum_{r \in S_{kt}} P_{rt} Z_{krt}(j) \right]$, where W_t is

the nominal wage and P_{rt} is price index of sector r. Taking as given S_{kt}, W_t and $\{P_{rt}\}_{r \in S_{kt}}$, each firm chooses labor and intermediate input quantities to minimize the total cost, subject to the production function in (1). The latter delivers the following unit cost function:

$$\mathcal{Q}_{kt}(j) = \mathcal{Q}_k \left[S_{kt}, \mathcal{A}_{kt}(S_{kt}), W_t, \{P_{rt}\}_{r \in S_{kt}} \right]. \tag{3.2}$$

Three properties of the unit cost function should be noted. First, the unit cost function is common for all firms within a given sector. Second, given the properties of the production function, $\mathcal{Q}_k$ is decreasing and continuous in $\mathcal{A}_{kt}(S_{kt})$ and is increasing and homogenous of degree one in $(W_t, \{P_{rt}\}_{r \in S_{kt}})$. Third, as the set of supplier sectors S_{kt} expands, the unit cost function becomes function of a larger set of sectoral prices. The latter property is crucial for delivering state-dependent degree of complementarity in price setting, which is what ultimately drives the key novel results in our model.

Finally, the set of suppliers is chosen optimally to minimize the unit cost of production in every period:

$$S_{kt} \in \arg\min_{S_{kt}} \mathcal{Q}_k \left[S_{kt}, \mathcal{A}_{kt}(S_{kt}), W_t, \{P_{rt}\}_{r \in S_{kt}} \right], \quad \forall k \tag{3.3}$$

The above minimization problem highlights the trade-off faced by firms when choosing the optimal set of suppliers: firms would like to purchase inputs from sectors whose combination delivers a high level of productivity, while at the same time avoiding those that charge high prices for their output.

3.2.3 Firms: pricing under nominal rigidities

Price stickiness in our economy is modeled as a modified finite-horizon version of Calvo (1983). In particular, there exists a finite, deterministic time period $T > 1$, such that in periods $1 \leq t \leq (T-1)$ firms face a constant and sector-specific probability of price non-adjustment $\alpha_k \in (0, 1)$, whereas in periods $t \geq T$ firms face no nominal rigidities. The cut-off period $T > 1$ is known by all agents in the economy from $t = 0$ and they expect it to stay fixed forever; naturally, as $T \to \infty$ the price setting problem collapses back to the standard Calvo (1983) pricing.

More precisely, in any period $1 \leq t \leq (T-1)$ a firm in sector k has probability $(1-\alpha_k)$ of setting its price equal to its optimal value. The optimal

reset price at time $1 \leq t \leq (T-1)$ is chosen to maximize expected future discounted nominal profits:

$$\max_{\hat{P}_{kt}(j)} \sum_{s=0}^{T-t-1} \alpha_k^s F_{t,t+s} \left[\hat{P}_{kt}(j) \tilde{Y}_{k,t+s}(j) - (1+\tau_k) \mathcal{Q}_{k,t+s} \tilde{Y}_{k,t+s}(j) \right], \tag{3.4}$$

where $\tilde{Y}_{k,t+s}(j) = \left[\frac{\hat{P}_{kt}(j)}{P_{t+s}} \right]^{-\theta} Y_{k,t+s}$, $F_{t,t+s}$ is the stochastic discount factor between periods t and $t+s$ and is defined in the next subsection; τ_k is a tax imposed by the government, revenue from which is rebated to households as a lump-sum transfer. The first order condition for the optimal reset price for any firm in sector k, $\hat{P}_{kt}$ is given by:

$$\hat{P}_{kt}(j) = \hat{P}_{kt} = (1+\mu_k) \frac{\sum_{s=0}^{T-t-1} \alpha_k^s F_{t,t+s} P_{k,t+s}^{\theta} Y_{k,t+s} \mathcal{Q}_{k,t+s}}{\sum_{s=0}^{T-t-1} \alpha_k^s F_{t,t+s} P_{k,t+s}^{\theta} Y_{k,t+s}}, \quad 1 \leq t \leq T-1, \quad \forall k \tag{3.5}$$

where $(1+\mu_k) \equiv (1+\tau_k)\frac{\theta}{\theta-1}$ is the steady-state markup. On the other hand, in any period $t \geq T$ there are no nominal rigidities, firms' maximize contemporaneous profits, and optimally set $P_{kt} = (1+\mu_k)\mathcal{Q}_{kt}, t \geq T$. Given that the optimal price is identical for all firms within a sector, sectoral price index can be obtained by aggregation using the ideal sectoral price index $P_{kt} \equiv \left(\int_{j \in \Phi_k} P_{kt}(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \forall k$:

$$P_{kt} = \begin{cases} \left[\alpha_k P_{k,t-1}^{1-\theta} + (1-\alpha_k)(\hat{P}_{kt})^{1-\theta} \right]^{\frac{1}{1-\theta}}, & 1 \leq t \leq (T-1); \\ \\ (1+\mu_k)\mathcal{Q}_{kt}, & t \geq T. \end{cases} \tag{3.6}$$

I use such modified version of Calvo (1983) pricing for two reasons. First, the special case where $T = 2$ is analytically tractable at the sector-level, as we show in Section 3.3. Second, given that money is neutral for all $t \geq T$, such formulation of pricing allows to obtain a numerical sector-level solution for the forward-looking problem with endogenous network formation at $t < T$ by backward induction, as is performed in Section 3.4.

3.2.4 Households

A continuum of infinitely lived households populates our economy and owns all the firms. Markets are complete, and a full set of Arrow-Debreu

securities is available. The representative household makes choices to maximize the lifetime utility:

$$\max_{\{C_{t+s},N_{t+s},B_{t+s+1}\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} \beta^s \left[\ln C_{t+s} - N_{t+s}\right] \tag{3.7}$$

subject to

$$P_t^c C_t + [F_{t,t+1} B_{t+1}] \leq B_t + W_t N_t + \sum_{k=1}^{K} \Pi_{kt} + T_t, \quad \forall t \tag{3.8}$$

where C_t is aggregate consumption, P_t^c is consumption price index (defined below), N_t is labor supply, B_{t+1} is the payoff of securities purchased at time t, $F_{t,t+1}$ is the price of those securities at time t, Π_{kt} denotes aggregate nominal profits of firms in sector k, β is the discount factor for future utility and T_t are lump-sum transfers from the government.[1]

The households' maximization problem yields standard aggregate first-order conditions, namely an equation for the stochastic discount factor $F_{t,t+s} = \beta^s \left(\frac{C_{t+s}}{C_t}\right)^{-1} \frac{P_t^c}{P_{t+s}^c}$, and the labor supply equation $C_t^{-1} = \frac{P_t^c}{W_t}$.

The composite consumption index C_t is an aggregator for the final consumption of goods produced in the different sectors of our economy:

$$C_t \equiv u(C_{1t}, C_{2t}, ..., C_{Kt}), \tag{3.9}$$

where C_{kt} is in turn an aggregator for the final consumption of goods produced by firms in that sector: $C_{kt} \equiv \left(\int_{j\in\Phi_r} C_{kt}(j)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}, \forall k$. We impose the following regularity conditions on the aggregator u:

Assumption 3.2 (Consumption aggregation)**.** *The consumption aggregator u is continuous, differentiable, increasing, strictly quasi-concave and homogeneous of degree one in $(C_{1t}, C_{2t}, ..., C_{Kt})$, and all sectoral consumption goods are normal.*

Households choose their sectoral consumption levels by minimizing the total cost of purchases $\sum_{k=1}^{K} P_{kt} C_{kt}$ subject to the consumption aggregator in (9). The latter also delivers the consumption price index $P_t^c = P_t^c(P_{1t}, P_{2t}, ..., P_{Kt})$, as the minimal cost of assembling such a basket.

[1]The assumption of log utility of consumption and linear disutility of labor follows the indivisible labor model introduced by Hansen (1985). Such formulation is made for analytical tractability and is the baseline choice in other workhorse models of monetary policy and production networks, such as Nakamura and Steinsson (2010).

3.2.5 Monetary policy

Purchases of final goods are subject to a cash-in-advance constraint, so that $P_t^c C_t = \mathcal{M}_t, \forall t \geq 1$. Agents are aware of the initial level of money supply $\mathcal{M}_0$, and at $t = 0$ anticipate it to stay at that level forever. In period $t = 1$ they discover the future path of money supply $\{\mathcal{M}_t\}_{t=1}^{\infty}$ and therefore any $\mathcal{M}_t \neq \mathcal{M}_0$ constitutes a monetary shock at time t to the agents. We assume that the central bank makes a credible commitment at $t = 1$ to ensure the equilibrium quantity of money follows the exogenous path $\{\mathcal{M}_t\}_{t=1}^{\infty}$.

3.2.6 Market clearing and equilibrium

In addition to the optimality conditions, budget constraints and the policy rule above, equilibrium in our economy is characterized by market-clearing conditions in the asset market: $B_t = 0$; the the labor market: $N_t = \sum_{k=1}^{K} \int_{j \in \Phi_k} N_{kt}(j) dj$; and the goods markets: $Y_{kt}(j) = C_{kt}(j) + \sum_{r=1}^{K} \int_{j' \in \Phi_r} Z_{rkt}(j', j) dj', \quad \forall k, \forall j \in \Phi_k$. All in all, equilibrium in our economy can be summarized as follows:

Definition 3.1 (Equilibrium)**.** *Equilibrium is a collection of prices* $\{P_{kt}(j) | j \in \Phi_k\}_{k=1}^{K}$*, wage* W_t*, allocations* $\left\{Y_{kt}(j), N_{kt}(j), C_{kt}(j), \{Z_{krt}(j, j') | j' \in \Phi_r\}_{r=1}^{K} | j \in \Phi_k\right\}_{k=1}^{K}$ *and supplier choices* $\{S_{kt}\}_{k=1}^{K}$*, which given the exogenous path of productivity mapping* $\{\mathcal{A}_t\}_{t=1}^{\infty}$*, the exogenous series of money supply* $\{\mathcal{M}_t\}_{t=0}^{\infty}$ *and the exogenous initial prices* $\{P_{k0}\}_{k=1}^{K}$*, satisfy agent optimization and market clearing in every time period* $t \geq 1$*.*

3.3 An analytically tractable version

In this section we consider an analytically tractable version of the model obtained when nominal rigidities are only present in the first period. Such simplification allows to formally characterize propagation of monetary shocks to real variables, under different states of productivity and initial levels of money supply. Formal propositions establish that small monetary shocks, which do not affect the shape of the network, have an impact on GDP that is larger whenever productivity mapping improves or initial money supply rises. Further, large monetary expansions have more than proportional positive effect on GDP than small monetary expansions; on the other hand, large

monetary contractions have less than proportional negative effect on GDP than small monetary contractions. All proofs are given in Appendix 3.A.

3.3.1 Equilibrium in the simplified version

In this section we focus on the version of our model with $T = 2$, so that nominal rigidities are only present at $t = 1$. In particular, we show that such simplification allows to both formally establish equilibrium existence and uniqueness properties, as well as to analytically characterize transmission of monetary shocks under different baseline productivity mappings and levels of money supply. The assumption is formally documented below:

Assumption 3.3 (Horizon of stickiness)**.** *In the firms' pricing problem $T = 2$, so that nominal rigidities are only present at $t = 1$, and prices are fully flexible for all $t \geq 2$.*

One implication of the above assumption is that the optimal reset price at $t = 1$ is determined by $\hat{P}_{k1} = (1 + \mu_k)\mathcal{Q}_{k1}, \forall k$, delivering a tractable expression for the sectoral price index in the first period, namely $P_{k1} = \left[\alpha_k P_{k0}^{1-\theta} + (1 - \alpha_k)\left\{(1 + \mu_k)\mathcal{Q}_{k1}\right\}^{1-\theta}\right]^{\frac{1}{1-\theta}}, \forall k$. At the same time, prices are flexible after the first period, so that $P_{kt} = (1 + \mu_k)\mathcal{Q}_{kt}, \forall k, t \geq 2$. Moreover, notice that the intratemporal consumption-labor supply condition and the cash-in-advance constraint jointly imply that $W_t = M_t$, so that the nominal wage equals money supply in every period. This exogeneity of the nominal wage, combined with the fact that the only endogenous component in the pricing equations above is the unit cost function, jointly imply that prices in this simplified setting are pinned down exclusively by the exogenous supply-side factors – productivity mapping $\mathcal{A}_t$ and markups $\{\mu_k\}_{k=1}^K$ – and by the exogenous level of money supply. It is the latter property which allows to represent equilibrium sectoral prices as a lattice, which in turn delivers equilibrium existence and uniqueness properties summarized below.

Proposition 3.1 (Equilibrium)**.** *Suppose Assumptions 3.1-3.3 hold. Then the equilibrium introduced in Definition 3.1 entails the following properties: (a) it exists; (b) the equilibrium sectoral prices and final sectoral consumptions are unique; (c) the equilibrium supplier choices and remaining sectoral allocations are generically unique.*

Since Assumption 3.3 implies that money is neutral for $t \geq 2$ and given that our interest is in the transmission of monetary shocks to real variables,

in the rest of this section we am going to focus exclusively on outcomes at $t = 1$. For notational simplicity, we drop time subscripts for variables at $t = 1$ for the remainder of this section.

The theoretical results in the remainder of this section are going to be presented in two steps. Before establishing every key finding, we explain it in a stylized setting featuring only two sectors. The simplified two-sector setting is summarized below.

Two-sector setting. *Consider our economy with two sectors* $(K = 2)$*, where for simplicity firms are not allowed to buy inputs from other firms in their own sector* $(k \notin S_k, k = 1, 2)$*, and the production function is Cobb-Douglas with the functional form:* $Y_k(j) = e(k)\mathcal{A}_k(S_k)N_k(j)^{(1-\omega_{k,-k})}Z_{k,-k}(j)^{\omega_{k,-k}}$*, and* $e(k) = (1-\omega_{k,-k})\omega_{k,-k}^{-\omega_{k,-k}}$*. The input-output shares are calibrated such that* $\omega_{k,-k} = 0.5$ *if* $-k \in S_k$ *and* $\omega_{k,-k} = 0$ *otherwise, for* $k = 1, 2$*. Price stickiness is calibrated such that Sector 1 is price flexible* $(\alpha_1 = 0)$ *and Sector 2 features nominal rigidities* $(\alpha_2 = 0.5)$*. The productivity mapping is given by* $a_k(\varnothing) \equiv \ln A_k(\varnothing) = 1, k = 1, 2$ *and* $a_k(\{-k\}) \equiv \ln A_k(\{-k\}) = \overline{a}, k = 1, 2$*. The exogenous initial prices are set at* $P_{k0} = 1, k = 1, 2$*. Finally, assume that* $\tau_k = -1/\theta$*, so that all pricing distortions are removed, and* $\theta \to 1^+$*, which allows to obtain closed-form expressions for equilibrium sectoral prices and quantities. In this setting, there are only four possibilities for supplier choices* (S_1, S_2)*, namely* $(\varnothing, \varnothing), (\varnothing, \{1\}), (\{2\}, \varnothing)$ *and* $(\{2\}, \{1\})$*. Letting* $m \equiv \ln \mathcal{M}$*, one can summarize (log) unit costs* (q_1, q_2) *associated with the four possible supplier choices as follows*:

	$S_2 = \varnothing$	$S_2 = \{1\}$
$S_1 = \varnothing$	$(m-1, m-1)$	$(m-1, -\overline{a} + m - \frac{1}{2})$
$S_1 = \{2\}$	$(-\overline{a} + \frac{3}{4}m - \frac{1}{4}, m-1)$	$\left(-\frac{10}{7}\overline{a} + \frac{5}{7}m, -\frac{12}{7}\overline{a} + \frac{6}{7}m\right)$

Another consequence of money neutrality for all $t \geq 2$ is that the only change in money supply relevant for real variables at $t = 1$ is that between money supply at $t = 1$ and its baseline level $\mathcal{M}_0$. For the ease of presentation of our results in this section, we am therefore introducing the following definition of a *monetary shock*:

Definition 3.2 (Monetary shock)**.** *Let* $\varepsilon^m \equiv \ln(\mathcal{M}/\mathcal{M}_0)$*, where* $\mathcal{M}$ *is money supply at* $t = 1$ *and* $\mathcal{M}_0$ *is the baseline level of money supply, be the monetary shock in our economy.*

In the next subsection, we study properties of our economy's baseline at $t = 1$, which occurs under zero monetary shock ($\varepsilon^m = 0$). The subsequent section studies propagation of a monetary shock $\varepsilon^m \neq 0$ conditional on different baselines.

3.3.2 Baseline ($\varepsilon^m = 0$)

The baseline is given by the equilibrium evaluated under $\varepsilon^m = 0$. Baseline prices, allocations and supplier choices are pinned down by the productivity mapping $\mathcal{A}$, baseline money supply $\mathcal{M}_0$ and exogenous initial prices $\{P_{k0}\}_{k=1}^K$. In this subsection we am going to investigate how, holding the initial prices fixed, changes in the baseline pair $(\mathcal{A}, \mathcal{M}_0)$ affect the equilibrium.

First, we am going to use the two-sector setting introduced earlier to build intuition on how changes in the productivity mapping affect the baseline, *ceteris paribus*.

Example 3.1 (Productivity mapping and baseline)**.** *In Panel (a) of Figure 3.2 we use the two-sector setting where, holding baseline money supply fixed at $m_0 = 0$, we consider three different productivity mappings by varying the parameter $\overline{a}$, which represents the productivity associated with using the other sector as a supplier. When $\overline{a} = 0$, neither sector finds it optimal to buy inputs from the other sector, and the equilibrium network is empty. As we increase $\overline{a}$ to 0.65, the sticky-price Sector 2 finds it optimal to purchase inputs from the flexible-price Sector 1, but not vice versa. This is because Sector 2 finds it optimal to lower its unit cost by purchasing inputs from Sector 1, whose price flexibility is associated with lower prices. Finally, as we further increase $\overline{a}$ to 0.8, both sectors find it optimal to connect to each other. This is because the productivity associated with the connection is now sufficiently high to convince the flexible-price Sector 1 to purchase from Sector 2, whose price stickiness prevents it from fully lowering the price. In addition, notice that as we increase the productivity parameter, equilibrium unit costs and hence sectoral prices drop, which lowers the consumption price index and through the cash-in-advance constraint implies that baseline GDP rises.*

I now use the two-sector setting to build intuition on the link between initial money supply and baseline equilibrium:

Example 3.2 (Money supply and baseline)**.** *In Panel (b) of Figure 3.2 we use the two-sector setting where, holding productivity mapping fixed at $\overline{a} = 0$, we consider three different initial money supplies by varying the parameter m_0. When $m_0 = 0$, neither sector finds it optimal to buy inputs from the other sector, and the equilibrium network is empty. As we increase m_0 to 4, the flexible-price Sector*

1 finds it optimal to purchase inputs from the sticky-price Sector 2, but not vice versa. This is because the nominal wage rises one-for-one with money supply, and Sector 1 finds it cheaper to substitute in-house labor for inputs bought from Sector 2, whose price increases less than one-for-one, even though such connection lowers 1's productivity. Finally, as we further increase m_0 to 8, both sectors find it optimal to connect to each other. This is because the increase in money supply/nominal wage is now so large that even the sticky-price Sector 2 wishes to substitute its in-house labor for inputs from Sector 1, whose price now does not track money supply since it inherits price stickiness from Sector 2, even though such connection lowers 2's productivity. In addition, notice that as we increase money supply, the gap between money supply and equilibrium unit costs and prices, both sectoral and the aggregate consumption index, is growing, which through the cash-in-advance constraint implies that baseline GDP rises.

Therefore, in our simple examples, baseline GDP and the number of suppliers of each sector (weakly) rise, *ceteris paribus*, both as one improves the productivity mapping and as one increases initial money supply. However, the mechanisms through which linkages are added are, in fact, diagonally opposite under the two scenarios. As one improves the productivity mapping, extra sectors that get adopted as suppliers are those that *adjust* their prices downwards by enough to lower the buyer's unit cost for the level of productivity they bring. On the contrary, as one increases money supply, sectors that receive additional intermediate customers are those that *do not adjust* their prices upwards enough, so that they are cheaper relative to labor for the level of productivity they bring.[2]

Having built intuition in the two-sector setting, we now formalize the relationship between the baseline pair $(\mathcal{A}, \mathcal{M}_0)$, GDP and supplier choices in equilibrium. As for the effect on GDP, no further assumptions, over and above the ones already made, are required:

Lemma 3.1 (Baseline GDP)**.** *Suppose Assumptions 3.1-3.3 hold. Consider any two baseline pairs $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ such that either $\overline{\mathcal{A}} \geq \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ or $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geq \underline{\mathcal{M}}_0$. Then, $C(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \geq C(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$.*

[2]The mechanism would also be different if one were to consider contractionary changes in the baseline, namely "deteriorations" in the productivity mapping or decreases in money supply. In the former scenario, sectors that get dropped as suppliers are those that adjust their prices upwards by enough to increase the buyer's unit cost for the level of productivity they bring. On the contrary, as one decreases money supply, sectors that lose intermediate customers are those that do not adjust their prices downwards by enough, so that they are expensive relative to labor for the level of productivity they bring.

Figure 3.2: Equilibrium supplier choices under different baseline conditions

(a) Choices of suppliers under different baseline productivity mappings ($m_0 = 0$)

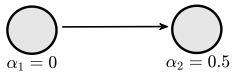
Recession: $\overline{a} = 0$

	∅	$\{1\}$
∅	**(-1,-1)**	$\left(-1, -\frac{1}{2}\right)$
$\{2\}$	$\left(-\frac{1}{4}, -1\right)$	$(0, 0)$



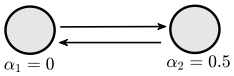
Normal: $\overline{a} = 0.65$

	$\varnothing$	**{1}**
∅	$(-1, -1)$	$\left(\mathbf{-1}, \mathbf{-\frac{23}{20}}\right)$
$\{2\}$	$\left(-\frac{9}{10}, -1\right)$	$\left(-\frac{13}{14}, -\frac{39}{35}\right)$



Expansion: $\overline{a} = 0.80$

	$\varnothing$	**{1}**
$\varnothing$	$(-1, -1)$	$\left(-1, -\frac{13}{10}\right)$
{2}	$\left(-\frac{21}{20}, -1\right)$	$\left(\mathbf{-\frac{8}{7}}, \mathbf{-\frac{48}{35}}\right)$



(b) Choices of suppliers under different baseline levels of money supply ($\overline{a} = 0$)

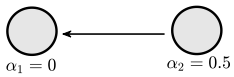
Tight money: $m_0 = 0$

	∅	$\{1\}$
∅	**(-1,-1)**	$\left(-1, -\frac{1}{2}\right)$
$\{2\}$	$\left(-\frac{1}{4}, -1\right)$	$(0, 0)$



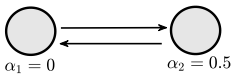
Normal money: $m_0 = 4$

	∅	$\{1\}$
$\varnothing$	$(3, 3)$	$\left(3, \frac{5}{2}\right)$
{2}	$\left(\mathbf{\frac{11}{4}}, \mathbf{3}\right)$	$\left(\frac{20}{7}, \frac{24}{7}\right)$



Loose money: $m_0 = 8$

	$\varnothing$	**{1}**
$\varnothing$	$(7, 7)$	$\left(7, \frac{13}{2}\right)$
{2}	$\left(\frac{23}{4}, 7\right)$	$\left(\mathbf{\frac{40}{7}}, \mathbf{\frac{48}{7}}\right)$



***Notes**: the figure uses the analytically tractable version of our model under $T = 2$, calibrated for $K = 2, \omega_{kk} = 0, \forall k$, $Y_k(j) = e(k)\mathcal{A}_k(S_k)N_k(j)^{(1-\omega_{k,-k})}Z_{k,-k}(j)^{\omega_{k,-k}}$ and $e(k) = (1 - \omega_{k,-k})\omega_{k,-k}^{-\omega_{k,-k}}$, $a_{k,0} \equiv \log \mathcal{A}_{k,0}, \forall k$ and $m_0 = \log \mathcal{M}_0$, the production technology be given by $a_{1,0}(\varnothing) = 0, \quad a_{1,0}(\{2\}) = \overline{a}, a_{2,0}(\varnothing) = 0, \quad a_{2,0}(\{1\}) = \overline{a}$, $P_{10} = P_{20} = 1$, the sectoral shares are given by $\omega_{12} = \omega_{c1} = 0.5 = \omega_{21} = \omega_{c1} = 0.5$, and Calvo parameters by $\alpha_1 = 0, \alpha_2 = 0.5$. Finally, assume that $\tau_k = -1/\theta$, and $\theta \to 1^+$.*

The overall mechanism behind the above result is as follows. Holding initial money supply fixed, an improvement in the productivity mapping lowers each unit cost function, which in turn lowers every sectoral price. Firms re-optimize their set of suppliers, which delivers lower new unit costs, since the original set of suppliers remains available. The latter further lower sectoral prices. The mechanism repeats until a new equilibrium is reached, which features lower sectoral prices and hence a lower consumption price index, which through the cash-in-advance constraint implies a larger GDP. Similarly, holding productivity mapping fixed, larger initial money supply decreases the ratios of sectoral prices to money supply due to sticky prices. Firms re-optimize their set of suppliers, which delivers lower new unit costs, since the original set of suppliers remains available. The latter further lowers the ratios of sectoral prices to money supply. The mechanism repeats until a new equilibrium is reached, which features lower ratios of sectoral prices to money supply and hence a lower ratio of consumption price index to money supply, which through the cash-in-advance constraint implies a larger GDP.

The link between the baseline pair $(\mathcal{A}, \mathcal{M}_0)$ and supplier choices in equilibrium can be formalized using monotone comparative statics theorems of Milgrom and Shannon (1994). Those, however, require imposing further regularity conditions. The first one puts a restriction on the relationship between the unit cost function, the choice of suppliers and the productivity mapping, holding sectoral prices and wage fixed:

Assumption 3.4. *For all* $W, \{P_k\}_{k=1}^K$, *the unit cost function* $\mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), W, \{P_r\}_{r\in S_k}\right]$ *is quasi-submodular in* $(S_k, \mathcal{A}_k(S_k))$, *for all* $k = 1, 2, ..., K$.

The assumption of quasi-submodularity ensures that, holding wage and prices fixed, an improvement in the productivity mapping does not encourage any sector to shrink its number of suppliers. However, prices also vary across baselines, which brings in the need for a sector regularity condition. The additional restriction is a single-crossing property on the unit cost function, relating joint variations in prices, wages and the set of suppliers:

Assumption 3.5. *Let* $\mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), \{\tilde{P}_r\}_{r\in S_k}\right] \equiv \frac{1}{W}\mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), 1, \{\frac{P_r}{W}\}_{r\in S_k}\right], \forall k$. *For all* $S_k \subseteq S_k'$ *and for all* $\{\tilde{P}_r, \tilde{P'}_r\}_{r=1}^K$ *such*

that $\tilde{P'}_r \leq \tilde{P}_r, \forall r \neq k$*:*

$$\mathcal{Q}_k\left[S'_k, \mathcal{A}_k(S'_k), \{\tilde{P}_r\}_{r\in S'_k}\right] - \mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), \{\tilde{P}_r\}_{r\in S_k}\right] \leq 0$$
$$\implies \quad \mathcal{Q}_k\left[S'_k, \mathcal{A}_k(S'_k), \{\tilde{P'}_r\}_{r\in S'_k}\right] - \mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), \{\tilde{P'}_r\}_{r\in S_k}\right] \leq 0, \quad \forall k. \tag{3.10}$$

The above single crossing property ensures that a reduction in all sectoral price to wage ratios does not discourage the adoption of a larger number of suppliers by every sector. Such an assumption immediately holds under a range of widely used production functions, most notably Cobb-Douglas with Hicks-neutral technology.[3]

The additional assumptions above allow to formalize the link between the baseline pair $(\mathcal{A}, \mathcal{M}_0)$ and supplier choices in equilibrium:

Lemma 3.2 (Baseline supplier choices)**.** *Suppose Assumptions 3.1-3.5 hold. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ *such that either* $\overline{\mathcal{A}} \geq \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ *or* $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geq \underline{\mathcal{M}}_0$*. Then,* $S_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \supseteq S_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$*, for all* $k = 1, 2, ..., K$*.*

The intuition behind the above result is as follows. An improvement in the productivity mapping, *ceteris paribus*, incentivizes firms to connect to more suppliers, as it lowers their unit costs directly through higher productivity and indirectly through lower prices charged by suppliers. Similarly, an increase in initial money supply, *ceteris paribus*, leads to a reduction in sectoral price to wage ratios, implying that it is cost-reducing to substitute in-house labor for intermediates bought from other sectors, thus leading to an increase in the number of suppliers.

3.3.3 Propagation of a monetary shock

Having established properties of the baseline, we now consider deviations from the baseline, driven by a monetary shock ε^m. The mechanics of a monetary shock in terms of its effect on the equilibrium allocations and supplier choices are isomorphic to those for changes in baseline money supply estab-

[3]This is formally shown in Acemoglu and Azar (2020). See their work for a fuller characterization of families of production functions under which the single-crossing property in Assumption 3.5 holds.

lished in the previous subsection. We can therefore immediately formalize comparative statics following a monetary shock

Lemma 3.3 (Comparative statics following a monetary shock)**.** *Consider a baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, which is perturbed by a monetary shock* $\varepsilon^m > 0$*. Suppose Assumptions 3.1-3.3 hold, then following the monetary shock equilibrium GDP rises (falls) relative to its baseline level:* $C(\mathcal{A}, \mathcal{M}) > C(\mathcal{A}, \mathcal{M}_0)$*. Further, suppose that in addition Assumption 3.4-3.5 also hold, then following the monetary shock the set of suppliers for each sector weakly expands:* $S_k(\mathcal{A}, \mathcal{M}) \supseteq S_k(\mathcal{A}, \mathcal{M}_0)$*, for all* $k = 1, 2, ..., K$*.*

Note that although the above lemma is stated for an expansionary monetary shock $\varepsilon^m > 0$, it naturally extends to a contractionary shock $\varepsilon^m < 0$, which leads to a reduction in GDP and a weak fall in the number of supplier for every sector.

From Lemma 3.3 it follows that a non-zero monetary shock can either change the set of suppliers relative to the baseline, or leave them unchanged. In light of this it is useful to formally distinguish between two such types of monetary shocks, as is done below.

Definition 3.3 (Small monetary shock)**.** *Define a monetary shock* ε^m *to be small with respect to the baseline* $(\mathcal{A}, \mathcal{M}_0)$ *if and only if it leaves the equilibrium supplier choices unchanged for all sectors relative to the baseline:* $S_k(\mathcal{A}, \mathcal{M}) = S_k(\mathcal{A}, \mathcal{M}_0), \ \forall k$*. Otherwise, define the monetary shock to be large with respect to the baseline* $(\mathcal{A}, \mathcal{M}_0)$*.*

Crucially, the above definition helps classify every monetary shock as either small or large *with respect to* a specific baseline pair $(\mathcal{A}, \mathcal{M}_0)$. In principle, as one changes the baseline pair, a shock of a given size can switch from being small to large, or *vice versa*. Moreover, the definition is not symmetric: for a given baseline, an expansionary shock $\varepsilon^m > 0$ being small does not necessarily imply that a contractionary shock of an equal absolute value is small with respect to the same baseline.

Small monetary shocks

In this section we study how responses of prices and allocations to a small monetary shock depend on the baseline productivity mapping and initial

money supply. As before, we begin by building intuition in the simplified two-sector setting before establishing general results.

First, we use the two-sector setting consider the same small monetary expansion occurring under different baseline productivity mappings.

Example 3.3 (Small monetary shocks across productivity mappings)**.** *Consider the two-sector setting with the additional assumption on consumption aggregation:* $C = C_1^{\omega_{c1}} C_2^{\omega_{c2}}, \omega_{ck} = 0.5, k = 1, 2$. *Panel (a) of Figure 3.4 considers three baselines associated with different productivity mappings, as in Example 1. We perturb each of the baselines with the same small monetary expansion, which, by definition, does not change the equilibrium network. As one can see, in the low productivity baseline with the empty network, only the sticky-price Sector 2 responds to the shock; in the medium-productivity baseline the situation is unchanged, as Sector 1 does not buy inputs from Sector 2, and hence inherits no stickiness. However, in high-productivity state, where both sectors buy from each other, both sectors see their final consumption rise. We can see that the magnitude of both sectoral and aggregate final consumption's response to a monetary shock is weakly larger in baselines with higher productivity.*

Similarly, we use the two-sector setting to consider the same small monetary expansion occurring under different levels of initial money supply.

Example 3.4 (Small monetary shocks across initial money supplies)**.** *Consider the two-sector setting with the additional assumption on consumption aggregation:* $C = C_1^{\omega_{c1}} C_2^{\omega_{c2}}, \omega_{ck} = 0.5, k = 1, 2$. *Panel (b) of Figure 3.6 considers three baselines with different levels of initial money supply, as in Example 2. We perturb each of the baselines with the same small monetary expansion, which, by definition, does not change the equilibrium network. One can see that in the tight money baseline only the sticky-price Sector 2 responds to the shock; in the normal money baseline flexible-price Sector 1 buys from Sector 2 and inherits stickiness, hence responding to the shock. Finally, in the loose money baseline both sectors buy from each other and respond by even more than in the normal money state. Overall, one can see that both sectoral and aggregate consumption respond weakly stronger under baselines with higher initial money supply.*

One can see that in the examples above, following a small monetary shock of the same size, consumption responds more strongly in states that feature more linkages across sectors. This is because whenever a sector increases the number of suppliers, its marginal cost becomes function of a larger number of other sectoral prices, which in turn strengthens complementarities in price setting and the degree of money non-neutrality.

The above mechanism can be formalized by noticing that the existence of a small monetary shock around a specific baseline implies that there is a neighborhood within which variations in money supply do not affect the equilibrium set of supplier choices. The latter in turn means that such a neighborhood features no discontinuities created by formation or destruction of linkages, and one can appeal to local properties around the baseline. In order to establish analytically tractable local properties, we make several further assumptions. First, a vital required additional assumption is differentiability of the production function in labor and intermediate inputs:

Assumption 3.6. *For every sector, the production function $\mathcal{F}_k$ is differentiable in $\left(N_{kt}(j), \{Z_{krt}(j)\}_{r\in S_{kt}}\right)$.*

The second additional assumption concerns the initial sectoral prices, which have so far been assumed to be completely exogenous:

Assumption 3.7 (Initial sectoral prices)**.** *For every sector $k = 1, 2, ..., K$, the initial sectoral price is given by $P_{k0} = (1 + \mu_k)g(\mathcal{M}_0)\mathcal{Q}_k[S_k, \mathcal{A}_k(S_k), \mathcal{M}_0, \{P_r(\mathcal{A}, \mathcal{M}_0)\}_{r\in S_k}]$, where $(\mathcal{A}, \mathcal{M}_0)$ is the baseline pair, $\mu_k = (1 + \tau_k)\frac{\theta}{\theta-1}$ and $g : \mathbb{R}^+ \to (0, 1)$ and strictly decreasing on the whole domain.*

The above assumption states that for every sector the initial price is set at a fixed markup over the baseline (steady-state) unit cost, with the markup falling in the initial money supply. In this way, we tractably capture the idea that whenever the initial money supply is high, a monetary shock is occurring in an environment with low markups, representing the interaction of price stickiness with past loose money supply.[4] Crucially, baseline comparative statics properties established in Lemmas 3.1 and 3.2 continue to hold under this additional assumption about initial prices.

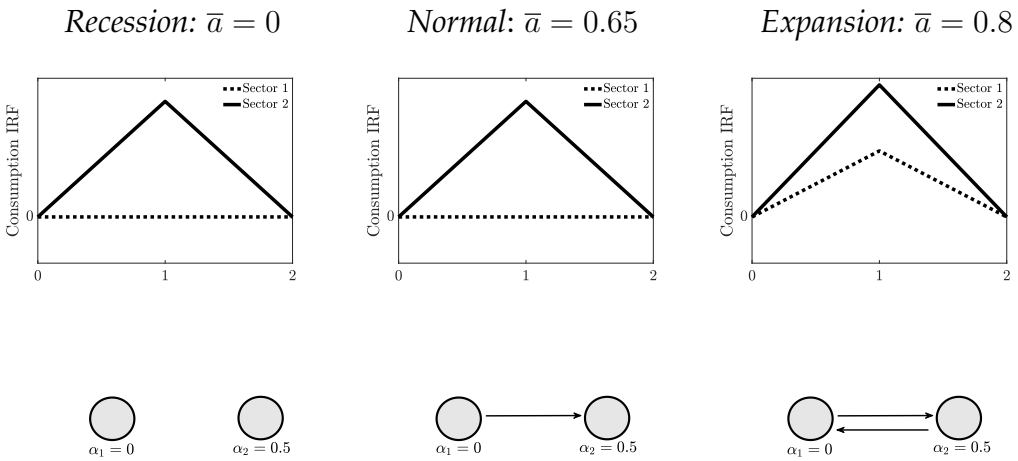
Armed with the two additional assumptions, we can now begin to formalize the differences in responses to a small monetary shock that arrives under different baselines. As a first step, the lemma below documents baseline-specific local responses of sectoral prices to a monetary shock:

Lemma 3.4. *Suppose Assumptions 3.1-3.3 and 3.6-3.7 hold. For any baseline pair $(\mathcal{A}, \mathcal{M}_0)$, let $p_k(\mathcal{A}, \mathcal{M}_0)$ be a first order approximation of $\ln P_k(\mathcal{A}, \mathcal{M})$ around*

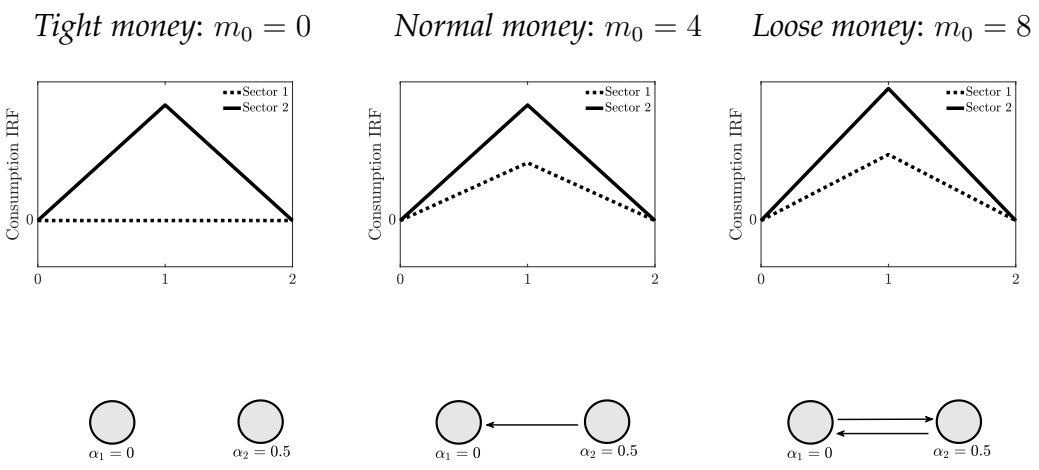
[4]With this additional assumption on initial prices, the baseline sectoral markups are given by $(1 + \mu_k)[\alpha_k g(\mathcal{M}_0))^{1-\theta} + (1 - \alpha_k)]^{\frac{1}{1-\theta}}, \forall k$, which fall in $\mathcal{M}_0$.

Figure 3.3: Final consumption following a small monetary expansion

(a) Responses of consumption to a small monetary expansion under recessions and expansions ($m_0 = 0$)



(b) Responses of consumption to a small monetary expansion under tight and loose initial money ($\overline{a} = 0$)



***Notes**: the figure uses the analytically tractable version of our model under $T = 2$, calibrated for $K = 2, \omega_{kk} = 0, \forall k$, $Y_k(j) = e(k)\mathcal{A}_k(S_k)N_k(j)^{(1-\omega_{k,-k})}Z_{k,-k}(j)^{\omega_{k,-k}}$ and $e(k) = (1 - \omega_{k,-k})\omega_{k,-k}^{-\omega_{k,-k}}$, $a_{k,0} \equiv \log \mathcal{A}_{k,0}, \forall k$ and $m_0 = \log \mathcal{M}_0$, the production technology be given by $a_{1,0}(\varnothing) = 0, \quad a_{1,0}(\{2\}) = \overline{a}, a_{2,0}(\varnothing) = 0, \quad a_{2,0}(\{1\}) = \overline{a}$, $P_{10} = P_{20} = 1$, the sectoral shares are given by $\omega_{12} = \omega_{c1} = 0.5 = \omega_{21} = \omega_{c1} = 0.5$, and Calvo parameters by $\alpha_1 = 0, \alpha_2 = 0.5$. Finally, assume that $\tau_k = -1/\theta$, and $\theta \to 1^+$.*

$\ln P_k(\mathcal{A}, \mathcal{M}_0)$, *given* $S_k = S_k(\mathcal{A}, \mathcal{M}_0), \forall k$. *Consider any two baseline pairs* $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ *and a monetary shock* ε^m *which is small with respect to both baselines, then:*

$$\mathbb{p}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathbb{p}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) = -\left[\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathcal{L}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)\right] \mathcal{E}^m \tag{3.11}$$

where $\mathbb{p} = [p_1, p_2, ..., p_K]'$, $\mathcal{E}^m = [\varepsilon^m, \varepsilon^m, ..., \varepsilon^m]'$ *and* $\mathcal{L}$ *is a Leontief inverse given by:*

$$\mathcal{L}(\mathcal{A}, \mathcal{M}_0) = \left[I - (I - A)\Gamma(\mathcal{M}_0)\Omega(\mathcal{A}, \mathcal{M}_0)\right]^{-1} \left[I - (I - A)\Gamma(\mathcal{M}_0)\right] \tag{3.12}$$

where $A = diag(\alpha_1, ..., \alpha_K)$, $\Gamma(\mathcal{M}_0) = diag(\gamma_1(\mathcal{M}_0), ..., \gamma_K(\mathcal{M}_0))$, $\gamma_k = \frac{1}{\alpha_k (g(\mathcal{M}_0))^{1-\theta} + 1 - \alpha_k}$ *and*
$\Omega(\mathcal{A}, \mathcal{M}_0)$ *is a matrix with entries given by* $[\Omega(\mathcal{A}, \mathcal{M}_0)]_{kr} = \frac{\partial \ln \mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), W, \{P_{k'}\}_{k' \in S_k}\right]}{\partial \ln P_r}|_{\mathcal{M}=\mathcal{M}_0}, \forall k, r.$

The above result details channels through which changes in the baseline affect local properties of price responses to a monetary shock. In particular, a baseline with a higher money supply, *ceteris paribus*, features lower markups and delivers smaller increases in sectoral prices following a small monetary expansion. At the same time, baselines with either a better productivity mapping or a higher money supply, *ceteris paribus*, feature more suppliers for each sector and hence more positive elements in the $\Omega(\mathcal{A}, \mathcal{M}_0)$ matrix, whose entries denote cross-elasticities of a sector's unit cost to sectoral prices. The latter affects price responses through two margins. At the extensive margin, having more suppliers strengthens complementarities in price setting and unambiguously delivers smaller increases in sectoral prices following a small monetary expansion. At the intensive margin, however, the effect of a change in the baseline on the unit cost elasticity to prices of already existing suppliers is ambiguous. Therefore, the effect of a change in baseline on the responses of sectoral prices to small monetary shock depends on the relative quantitative importance of the three channels described above.

However, a sufficient condition exists under which the ambiguity described above is resolved. In particular, it occurs in the special case when the intensive margin adjustment described above is absent. The latter hap-

pens when, conditional on a supplier relationship existing, the elasticity of a sector's unit cost to that supplier's price is fixed across baselines, which is the case under Cobb-Douglas production function with Hicks-neutral technology:

Assumption 3.8. *For every sector* $k = 1, 2, ..., K$ *the production function is Cobb-Douglas with Hicks-neutral technology:* $Y_{kt}(j) = \mathcal{A}_{kt}(S_{kt})N_{kt}(j)^{1-\sum_{r\in S_{kt}}\omega_{kr}}\prod_{r\in S_{kt}}Z_{krt}(j)^{\omega_{kr}}, \quad \omega_{kr} \geq 0, \quad \sum_{r=1}^{K}\omega_{kr} < 1.$

Adding the above assumption ensures that under a baseline with either a better productivity mapping or a higher money supply, the same small monetary expansion delivers smaller increases in all sectoral prices, as is formalized below:

Proposition 3.2. *Suppose Assumptions 3.2-3.5 and 3.7-3.8 hold. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ *such that either* $\overline{\mathcal{A}} \geq \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ *or* $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geq \underline{\mathcal{M}}_0$*. For a monetary shock* $\varepsilon^m > 0$ *that is small with respect to both baselines, it follows that* $p_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \leq p_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$ *for all* $k = 1, 2, ..., K$*.*

Importantly, though the above proposition is formulated for a small monetary expansion $\varepsilon^m > 0$, it trivially extends to a small monetary contraction $\varepsilon^m < 0$. In particular, it would follow that under a baseline with either a better productivity mapping or a higher money supply, the same small monetary contraction delivers smaller decreases in all sectoral prices.

I now use properties of price responses established above to study how changes in the baseline affect consumption responses to small monetary shocks. From the cash-in-advance constraint, it follows that $[\ln C(\mathcal{A}, \mathcal{M}) - \ln C(\mathcal{A}, \mathcal{M}_0)] = \varepsilon^m + [\ln P^c(\mathcal{A}, \mathcal{M}) - \ln P^c(\mathcal{A}, \mathcal{M}_0)]$. It follows that local properties of GDP around the baseline can be inferred from local properties of the consumption price index. For a monetary shock ε^m that is small with respect to a baseline pair $(\mathcal{A}, \mathcal{M}_0)$ one can write the following local approximation:

$$\ln P^c(\mathcal{A}, \mathcal{M}) - \ln P^c(\mathcal{A}, \mathcal{M}_0) \approx \sum_{k=1}^{K} \frac{\partial \ln P^c(\mathcal{A}, \mathcal{M}_0)}{\partial \ln P_k} \left[\ln P_k(\mathcal{A}, \mathcal{M}) - \ln P_k(\mathcal{A}, \mathcal{M}_0)\right]. \tag{3.13}$$

From Proposition 3.2 we know local properties of (log-)deviations of sectoral prices as one varies the baseline. However, the variation in elasticities

of the consumption price index with respect to sectoral prices as one varies the baseline is ambiguous. In this sense, the local properties of aggregate consumption/GDP around different baselines remain on relative quantitative properties of movements in sectoral prices and the elasticities that are used to aggregate them.

However, a sufficient condition exists under which the ambiguity described above is resolved. In particular, it occurs in the special case when the elasticities of the consumption price index with respect to sectoral prices remain fixed across all baselines. The latter occurs when the consumption aggregator takes the Cobb-Douglas form, as is detailed below:

Assumption 3.9. *The consumption aggregator* $u(.)$ *takes Cobb-Douglas form:* $C_t = \prod_{k=1}^{K} C_{kt}^{\omega_{ck}}, \omega_{ck} \geq 0, \quad \sum_{k=1}^{K} \omega_{ck} = 1.$

With the additional assumption above, we can now formalize non-linear transmission of a small monetary shock to both sectoral and aggregate consumption, the latter being equivalent to GDP in our economy. First, whenever a small monetary expansion arrives under a baseline with a better productivity mapping, it triggers a consumption increase of a larger magnitude:

Theorem 3.1 (Cycle dependence)**.** *Suppose Assumptions 3.3-3.5 and 3.7-3.9 hold. For any baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, let* $c_k(\mathcal{A}, \mathcal{M}_0) \equiv \ln C_k(\mathcal{A}, \mathcal{M}) - \ln C_k(\mathcal{A}, \mathcal{M}_0), \forall k$*. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \mathcal{M}_0)$*,* $(\overline{\mathcal{A}}, \mathcal{M}_0)$ *such that* $\overline{\mathcal{A}} \geq \underline{\mathcal{A}}$*. For a monetary shock* $\varepsilon^m > 0$ *which is small with respect to both baselines it follows that* $c_k(\overline{\mathcal{A}}, \mathcal{M}_0) \geq c_k(\underline{\mathcal{A}}, \mathcal{M}_0)$ *for all* $k = 1, 2, ..., K$*, and* $c(\overline{\mathcal{A}}, \mathcal{M}_0) \geq c(\underline{\mathcal{A}}, \mathcal{M}_0)$*.*

Three points should be noted. First, although the theorem is stated for small monetary expansions, it trivially extends to small monetary contractions. In particular, it follows that if a small monetary contraction arrives in a state with, *ceteris paribus*, higher productivity, the resulting fall in GDP is (weakly) smaller. Second, the additional assumption of Cobb-Douglas consumption aggregation implies cycle dependence not only at the level of aggregate GDP, but also at the level of final consumptions of individual sectors. Third, our cycle dependence result provides a theoretical rationale for empirical finds of procyclical magnitude of impulse response of GDP to monetary shocks (Tenreyro and Thwaites, 2016, Alpanda et al., 2019, Jordà et al., 2019).

In a similar way, we can formalize that whenever a small monetary expansion arrives under a baseline with higher money supply, it triggers a consumption increase of a larger magnitude:

Theorem 3.2 (Path dependence)**.** *Suppose Assumptions 3.3-3.5 and 3.7-3.9 hold. For any baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, let* $c_k(\mathcal{A}, \mathcal{M}_0) \equiv \ln C_k(\mathcal{A}, \mathcal{M}) - \ln C_k(\mathcal{A}, \mathcal{M}_0), \forall k$*. Consider any two baseline pairs* $(\mathcal{A}, \underline{\mathcal{M}}_0)$*,* $(\mathcal{A}, \overline{\mathcal{M}}_0)$ *such that* $\overline{\mathcal{M}}_0 \geq \underline{\mathcal{M}}_0$*. For a monetary shock* $\varepsilon^m > 0$ *which is small with respect to both baselines it follows that* $c_k(\mathcal{A}, \overline{\mathcal{M}}_0) \geq c_k(\mathcal{A}, \underline{\mathcal{M}}_0)$ *for all* $k = 1, 2, ..., K$*, and* $c(\mathcal{A}, \overline{\mathcal{M}}_0) \geq c(\mathcal{A}, \underline{\mathcal{M}}_0)$*.*

As before, the theorem trivially extends to small monetary contractions: if a small monetary contraction arrives in a state with, *ceteris paribus*, lower money supply, the resulting fall in GDP is (weakly) smaller. our path dependence result provides a theoretical rationale for empirical finds of monetary transmission to GDP being stronger under already loose monetary stance (Alpanda et al., 2019) and additional evidence that finds that GDP is more sensitive to monetary interventions in states of the world with already loose credit (Jordà et al., 2019).

The intuition behind both theorems is very similar. Baselines with a better productivity mapping or higher initial money supply, *ceteris paribus*, feature more suppliers for every sector in equilibrium. The latter strengthens complementarities in price setting, and hence deliver more money non-neutrality and consumption response of a larger magnitude.

Large monetary shocks

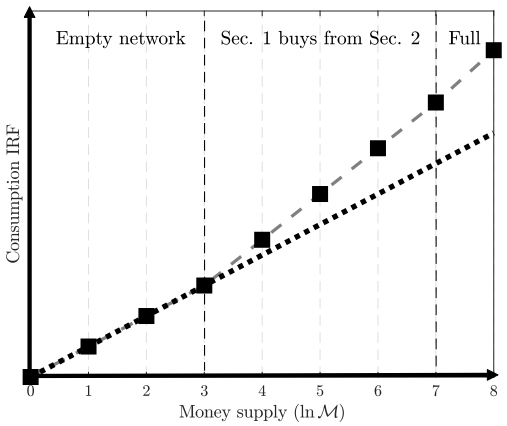
I now move to studying properties of large monetary shocks that are able to change the equilibrium set of suppliers relative to the baseline. As before, we begin with building intuition using the two-sector setting before establishing formal results.

First, we consider larger monetary expansions in the two-sector setting:

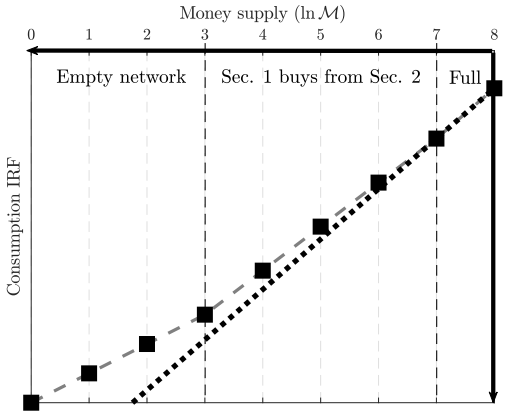
Example 3.5 (Large monetary expansions)**.** *Panel (a) of Figure 3.4 considers large monetary shocks in the context of our earlier two-sector example with an additional assumption on consumption aggregation:* $C = C_1^{\omega_{c1}} C_2^{\omega_{c2}}, \omega_{ck} = 0.5, k = 1, 2$*. Stating with an initial state with* $\overline{a} = m_0 = 0$*, we subject our economy with progressively larger monetary shocks. One can see that shocks smaller than* $\varepsilon^m = 3$ *keep the initially empty network unchanged, and the changes in the aggregate consumption impulse response are exactly proportional to the size of the shock. How-*

Figure 3.4: Aggregate consumption response to large monetary shocks

(a) Large monetary expansions



(b) Large monetary contractions



***Notes**: the figure uses the analytically tractable version of our model under $T = 2$, calibrated for $K = 2, \omega_{kk} = 0, \forall k$, $Y_k(j) = e(k)\mathcal{A}_k(S_k)N_k(j)^{(1-\omega_{k,-k})}Z_{k,-k}(j)^{\omega_{k,-k}}$ and $e(k) = (1 - \omega_{k,-k})\omega_{k,-k}^{-\omega_{k,-k}}$, $a_{k,0} \equiv \log \mathcal{A}_{k,0}, \forall k$ and $m_0 = \log \mathcal{M}_0$, the production technology be given by $a_{1,0}(\varnothing) = 0, \quad a_{1,0}(\{2\}) = \overline{a}, a_{2,0}(\varnothing) = 0, \quad a_{2,0}(\{1\}) = \overline{a}$, $P_{10} = P_{20} = 1$, the sectoral shares are given by $\omega_{12} = \omega_{c1} = 0.5 = \omega_{21} = \omega_{c1} = 0.5$, and Calvo parameters by $\alpha_1 = 0, \alpha_2 = 0.5$. Finally, assume that $\tau_k = -1/\theta$, and $\theta \to 1^+$. Throughout exercises we set $\overline{a}_0 = 0$.*

ever, as we consider larger expansions, they turn out to be large enough to expand the network by encouraging the flexible-price Sector 1 to buy from the sticky-price Sector 2, also increasing the slope of the relationship between the monetary shock size and consumption response.

Similarly, we use the two-sector setting to study large monetary contractions:

Example 3.6 (Large monetary contractions)**.** *Panel (b) of Figure 3.4 considers large monetary contractions in the context of the same two-sector example with an additional assumption on consumption aggregation: $C = C_1^{\omega_{c1}}C_2^{\omega_{c2}}, \omega_{ck} = 0.5, k = 1, 2$ we from a baseline with $\overline{a} = 0$ and $m_0 = 8$. Initial reductions in money supply leave the baseline full network unchanged and the magnitude of aggregate consumption contraction is exactly proportional to the size of the monetary shock. However, larger contractions break the network, in this case by discouraging Sector 2 from buying from Sector 1, and lower the slope of the relationship between the monetary shock size and consumption response.*

One can see that in the first example above, large monetary expansions deliver an increase in GDP that is larger than the one that would occur under

fixed networks. In this sense, large monetary expansions deliver more than proportional increases in GDP by expanding the set of suppliers for every sector. On the other hand, in the second example above, large monetary contraction deliver reductions in GDP that are smaller in magnitude that those under exogenous networks. Therefore, large monetary contractions break supplier relationships and in this way deliver less than proportional decreases in GDP, relative to the outcome under exogenous networks.

In order to formalize the above intuition, we introduce several new concepts. First, we let the exact impulse response of a sectoral price to monetary shock ε^m be given by $\tilde{P}_k(\varepsilon^m) \equiv P_k(\mathcal{A}, \mathcal{M})/P_k(\mathcal{A}, \mathcal{M}_0), \forall k$. Second, in order to facilitate comparison with a setting where supplier choices are exogenous, we let $\tilde{P}^e_k(\varepsilon^m; S) \equiv P_k(\mathcal{A}, \mathcal{M}, S)/P_k(\mathcal{A}, \mathcal{M}_0, S), \forall k$ denote exact impulse responses of sectoral prices to monetary shock ε^m in a version of our economy with an *exogenous* production network S. We am now ready to state a proposition which establishes bounds on how the exact impulse responses of sectoral prices grow with the size of the monetary shock:

Proposition 3.3. *Suppose Assumption 3.1-3.5 hold. For a baseline pair $(\mathcal{A}, \mathcal{M}_0)$, let $\varepsilon^m > 0$ be a small monetary shock and $E^m > 0$ be a large monetary shock. Under the large monetary shock, the equilibrium set of supplier choices is $S^L_k \supset S_k(\mathcal{A}, \mathcal{M}_0) = S^0_k, \forall k$. The ratios of exact impulse responses of sectoral prices satisfy:*

$$\frac{\tilde{P}^e_k(E^m; S^L)}{\tilde{P}^e_k(\varepsilon^m; S^L)} \leq \frac{\tilde{P}_k(E^m)}{\tilde{P}_k(\varepsilon^m)} \leq \frac{\tilde{P}^e(E^m; S^0)}{\tilde{P}^e_k(\varepsilon^m; S^0)}, \tag{3.14}$$

for all $k = 1, 2, ..., K$.

Intuitively, as one moves from a small monetary expansion to a large monetary expansion, the rate at which the exact responses of sectoral prices increase is smaller than what would occur if the network stayed fixed at its baseline level. This is because the extra linkages created by the large monetary expansion induce additional pricing complementarities, thus reducing the rate at which prices grow. At the same time, this drag on price increases is bounded from below by what would occur if the network was fixed at the level delivered by the large monetary expansion. Crucially, although the above proposition is stated for monetary expansions it trivially extends to monetary contractions by reversing the signs of inequalities. In particular, as one moves from a small monetary contraction to a large monetary contrac-

tion, the rate at which the exact magnitudes of prices drops change is larger than what would occur if the network stayed fixed at its baseline level. This is because the large monetary contraction destroys some of the supplier relationships and hence weakens pricing complementarities, allowing prices to drop faster.

Since the above proposition applies to all sectoral prices, it implies corresponding properties for the aggregate consumption price index. The latter combined with the cash-in-advance in turn delivers properties of aggregate consumption, which also represents GDP in our model:

Theorem 3.3 (Size dependence)**.** *Suppose Assumption 3.1-3.5 hold. For a baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, let* $\varepsilon^m > 0$ *be a small monetary shock and* $E^m > 0$ *be a large monetary shock. Under the large monetary shock, the equilibrium set of supplier choices is* $S_k^L \supset S_k(\mathcal{A}, \mathcal{M}_0) = S_k^0, \forall k$*. The ratios of exact impulse responses of GDP satisfy:*

$$\frac{\tilde{C}^e(E^m; S^0)}{\tilde{C}^e(\varepsilon^m; S^0)} \leq \frac{\tilde{C}(E^m)}{\tilde{C}(\varepsilon^m)} \leq \frac{\tilde{C}^e(E^m; S^L)}{\tilde{C}^e(\varepsilon^m; S^L)}. \tag{3.15}$$

The theorem above formally establishes size-dependence in the response of GDP to monetary shocks in our model. In particular, as one moves from a small monetary expansion to a large monetary expansion, the rate at which the exact impulse response of GDP rises is larger than what would occur if the set of suppliers was fixed at the baseline level. This is due to additional pricing complementarities created by the extra linkages formed by the large monetary expansion. At the same time, the above theorem also implies that following a small monetary contraction the rate at which the magnitude of exact impulse response of GDP changes is smaller than what would occur if the set of suppliers was fixed at the baseline level. This is because the large monetary contraction destroys some of the linkages, which weakens pricing complementarities, allowing prices to drop faster and consumption to fall by less.

Crucially, we have established our size dependence results under purely time-dependence pricing, without appealing to menu costs or any other state-dependencies in the probability of price adjustment. Moreover, notice that both theorems above do not require specifying the functional form of either the production function or the consumption aggregator, and do not even impose that the production function is differentiable.

3.4 A quantitative dynamic version

After establishing the key properties of our model in a simplified static version, we now quantify the effects in a forward-looking setting calibrated to 389 sectors of the US economy. We develop a numerical algorithm for solving a dynamic version of our multi-sector New Keynesian model with endogenous network formation and it use to determine how changes in baseline productivity and money supply affect equilibrium supplier choices, and how the latter creates non-linearities in transmission of monetary shocks.

3.4.1 Forward-looking setting

I now quantify the effects established analytically in the previous section using a dynamic forward-looking version of our model. In particular, in this section we do not make the simplifying assumptions needed to obtain closed-form results, and solve the model numerically. Numerical simulations still require functional form assumptions and we therefore maintain the assumption of Cobb-Douglas production function and Cobb-Douglas consumption aggregation. Moreover, we need to specify the functional form of the productivity mapping, where we augment the mapping of Acemoglu and Azar (2020) with an aggregate productivity term:

Assumption 3.10 (Productivity mapping)**.** *For every sector $k = 1, 2, ..., K$ the productivity mapping $\mathcal{A}_{kt}(S_{kt})$ takes the following form:*

$$\mathcal{A}_{kt}(S_{kt}) = \mathcal{Z}_t e(S_{kt}) B_0 \prod_{r \in S_{kt}} B_{kr}, \tag{3.16}$$

where $\mathcal{Z}_t$ is aggregate productivity which follows an AR(1) process in logs: $\ln \mathcal{Z}_t = \rho_z \ln \mathcal{Z}_{t-1} + \zeta_t$, $e(S_{kt})$ is a normalization term given by $e(S_{kt}) \equiv (1 - \sum_{r \in S_{kt}} \omega_{kr})^{-(1-\sum_{r \in S_{kt}} \omega_{kr})} \prod_{r \in S_{kt}} \omega_{kr}^{-\omega_{kr}}$, and B_0, $\{B_{kr}\}_{kr}$ are parameters.

Two points should be noted regarding the productivity mapping above. First, it delivers an equilibrium unit cost function given by $\mathcal{Q}_k = \left[\mathcal{Z}_t B_0 \prod_{r \in S_{kt}} B_{kr}\right]^{-1} W_t \prod_{r \in S_{kt}} \left(\frac{P_{rt}}{W_t}\right)^{\omega_{kr}}, \forall k$, or
$-z_t - b_0 + w_t + \sum_{r \in S_{kt}} [\omega_{kr}(p_{rt} - w_t) - b_{kr}]$ in logs. The latter implies a tractable rule for whether or not it is optimal to buy inputs from a particular supplier.

Namely, sector k should only buy inputs from sector r if $\omega_{kr}(p_{rt} - w_t) < b_{kr}$. Second, the entire path of aggregate productivity is known to the agents, so that they are aware of $\mathcal{Z}_0$ and $\{\zeta_t\}_{t=1}^{\infty}$ at $t = 0$.

I also need to specify the functional form of the money supply process, which we assume to follow an AR(1) process in log-differences:

Assumption 3.11 (Money supply)**.** *For a given initial money supply $\mathcal{M}_0$, the money supply in $t \geq 1$ takes the following form:*

$$\Delta \ln \mathcal{M}_t = \rho_m \Delta \ln \mathcal{M}_{t-1} + \varepsilon_t^m. \tag{3.17}$$

The agents are aware of $\mathcal{M}_0$ at $t = 0$ and at the beginning of $t = 1$ discover the entire future path of monetary shocks $\{\varepsilon_t^m\}_{t=1}^{\infty}$, facing no uncertainty beyond that point.

My modified finite-horizon version of Calvo (1983) pricing, combined with the simple rule for inclusion of suppliers described above, allows to use backward induction to solve our model at the sector-level. Most, specifically, we am interested in solving for equilibrium for periods $t < T$, which feature nominal rigidities and hence real variables responding to monetary shocks. For $t \geq T$, the problem is static from firms' pricing perspective, and money is neutral, which can therefore be used as a terminal condition in the backward induction algorithm. Here we outline our method, which numerically solves for sectoral prices, supplier choices and allocations:

Algorithm 3.1 (New Keynesian Model with Endogenous Networks)**.** *Start from a guess for sectoral prices, supplier choices and allocations; let $\mathcal{X}_t^-$, $\mathcal{X}_t$ and $\mathcal{X}_t^+$ be, respectively, the full set of past, present and future prices, supplier choices and allocations at time t for any $1 \leq t \leq T-1$. Let $\{\{P_{kt}^0\}_{k=1}^K\}_{t=1}^{T-1}$ be sectoral prices from the initial guess. Taking as given exogenous paths for money supply and the productivity mapping, follow the steps below, starting from $t = T-1$*

(i) Taking as given sectoral supplier choices, as well as past and future variables $\mathcal{X}_t^-, \mathcal{X}_t^+$, solve for prices $\{P_{kt}\}_{k=1}^K$;

(ii) Given prices $\{P_{kt}\}_{k=1}^K$, update supplier choices according to the following rule: sector k should only buy inputs from sector r if $\omega_{kr}(p_{rt} - m_t) < b_{kr}$,

where $p_{rt} \equiv \ln P_{kt}$ *and* $m_t \equiv \ln \mathcal{M}_t$*;*

(iii) Taking as given $\mathcal{X}_t^-, \mathcal{X}_t^+$, $\{P_{kt}\}_{k=1}^K$ *and* $\{S_{kt}\}_{k=1}^K$*, update* $\mathcal{X}_t$*;*

(iv) Repeat (i)-(iii) until convergence within the time period;

(v) If $t > 1$*, decrease* t *by one and go back to (i). Otherwise, compare* $\{\{P_{kt}^0\}_{k=1}^K\}_{t=1}^{T-1}$ *with* $\{\{P_{kt}\}_{k=1}^K\}_{t=1}^{T-1}$*; if they are equal, stop the algorithm; if they are not equal, set* $P_{kt} = P_{kt}^0, \forall k, 1 \leq t \leq T-1$*, set* $t = T-1$ *and return to (i).*

3.4.2 Calibration

I calibrate our model for the United States an annual frequencies. Calibration of aggregate parameters is standard. We set $\beta = 0.99$ to target annualized real interest rate of 1% in steady state. The within-sector elasticity of substitution is set at $\theta = 6$. The persistence parameters of productivity and money supply growth are set at $\rho_a = 0.86$ and $\rho_m = 0.80$, respectively. As for the threshold beyond which there are no nominal rigidities, we choose $T = 50$, so that after fifty years all firms can adjust their prices with certainty. Finally, we normalize $\mathcal{Z}_0 = 1$.

As for sector-specific parameters, those are selected for the US Bureau of Economic Analysis (BEA) Detail level of disaggregation featuring $K = 389$ sectors. Sector-specific Calvo parameters $\{\alpha_k\}_{k=1}^K$ are set as one minus the sector-specific frequency of price adjustment from Pasten et al. (2020).[5] The long-run markups in our economies are given by $\{(1+\tau_k)\frac{\theta}{\theta-1}\}_{k=1}^K$ and sectoral tax rates $\{\tau_k\}_{k=1}^K$ are chosen to match the estimated sectoral markups from De Loecker et al. (2020). The observed input-output shares $\{\omega_{kr}\}_{kr}$ are calibrated using the 2007 BEA Input-Output table, whereas input-output shares for linkages not observed in the data are imputed following Acemoglu and Azar (2020) by setting each unobserved share equal to 0.95 of the cost share of labor of the buying sector divided by the number of potential additional suppliers for that sector. Finally, B_0 and $\{B_{kr}\}_{kr}$ are estimated to ensure the steady-state equilibrium of our economy under $\mathcal{M}_t = \mathcal{Z}_t = 1, \forall t$ simultaneously matches observed input-output linkages and real GDP in 2007.

[5] We are grateful to Michael Weber for sharing sectoral frequencies of price adjustment.

3.4.3 Simulation results

Baseline economies

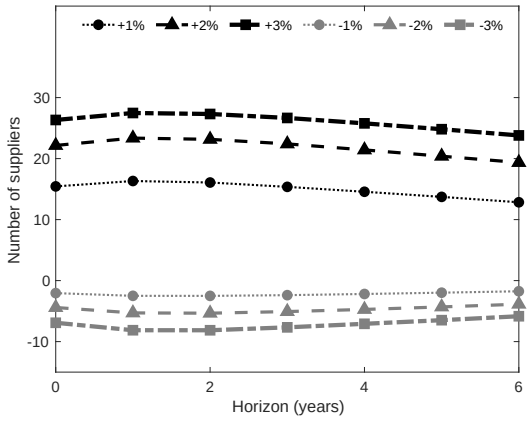
Variations in the baseline of our economy are driven by changes in the initial money supply $\mathcal{M}_0$ and changes in the sequence $\{\zeta_t\}_{t=1}^{\infty}$, which pins down the path of aggregate productivity. We consider two sets of variations in the baseline. First, we hold $\mathcal{M}_0 = 1$ and $\zeta_t = 0, \forall t \geq 2$, and consider values of $\zeta_1 = \{-3\%, -2\%, -1\%, 0, 1\%, 2\%, 3\%\}$. In this way, we look at baselines with both low and high productivity paths, all of which eventually converge to the initial value of $\mathcal{Z}_0 = 1$; the case of $\zeta_1 = 0$ where aggregate productivity remains fixed at the initial value is set as a benchmark against which the other baselines are compared. Second, we hold $\zeta_t = 0, \forall t \geq 1$ and consider different values of $\mathcal{M}_0$. In particular, we assume that in the unmodelled past before $t = 0$ money supply starts from $\mathcal{M}_{-\infty} = 1$ and follows the process in (17), experiencing a one-time never repeating shock $\varepsilon^m_{-\infty}$, eventually converging to the value of $\mathcal{M}_0$. We consider values of $\varepsilon^m_{-\infty} = \{-6\%, -4\%, -2\%, 0, 2\%, 4\%, 6\%\}$, where the case of $\varepsilon^m_{-\infty} = 0$ is treated as a benchmark against which all the other baselines are compared.

Panels (a) and (b) of Figure 3.5 consider baselines with different aggregate productivity paths and their associated paths of average number of suppliers and average intermediates intensity, which measures the average share of total costs that goes to suppliers. Relative to the fixed-productivity baseline ($\zeta_1 = 0$), letting $\zeta_1 = 1\%$ is associated with a long-lived increase by around 15 in the average number of suppliers per sector, which translates into an increase in average intermediates intensity by around 0.04. Similarly, setting $\zeta_1 = 3\%$, which delivers a baseline with an even better aggregate productivity, increases the average number of suppliers per sector by around 27, or an approximately 0.06 increase in average intermediates intensity. Importantly, the results suggest that the effect of changing the baseline productivity path is not symmetric, which is a quantitative result specific to the parameters of the productivity mapping $\{B_{kr}\}_{kr}$ we have estimated. Specifically, a low-productivity baseline with $\zeta_1 = -1\%$ decreases the number of suppliers only by approximately 2 per sector, or a small decrease of 0.002 in average intermediates intensity. Under an even worse productivity baseline with $\zeta_1 = -3\%$, the average number of suppliers falls by around 8 per sector, equivalent to a 0.01 reduction in average intermediates intensity.

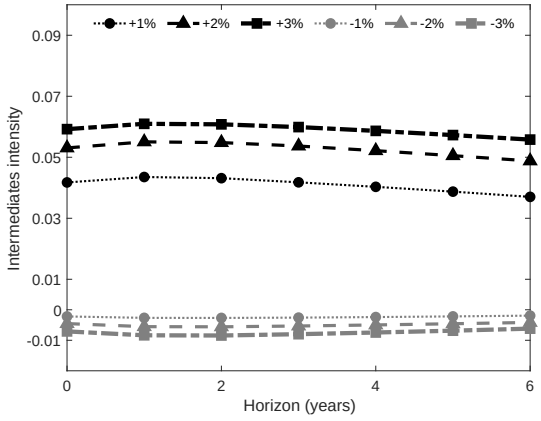
As for results under baselines with different initial levels of money sup-

Figure 3.5: Number of suppliers and intermediates intensities across baselines

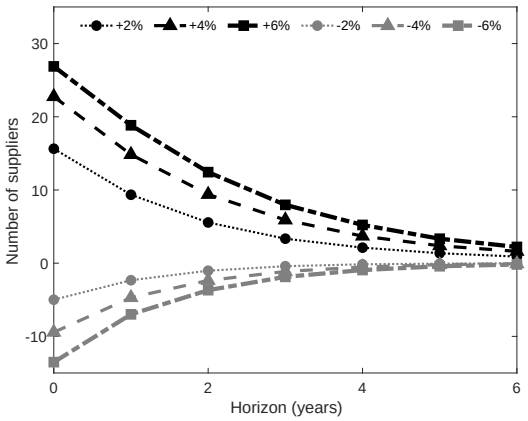
(a) Average number of suppliers



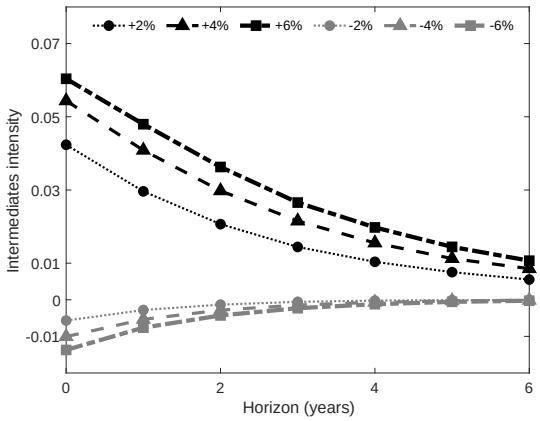
(b) Average intermediates intensity



(c) Average number of suppliers



(d) Average intermediates intensity



***Notes**: Panels (a) and (b) show the average number of suppliers and average intermediates intensities across baselines where $\mathcal{M}_0 = 1$ and $\zeta_t = 0, \forall t \geq 2$, and consider values of $\zeta_1 = \{-3\%, -2\%, -1\%, 1\%, 2\%, 3\%\}$, relative to $\zeta_1 = 0$; Panels (c) and (d) show the average number of suppliers and average intermediates intensities across baselines where $\zeta_t = 0, \forall t \geq 1$ and values of $\varepsilon^m_{-\infty} = \{-6\%, -4\%, -2\%, 2\%, 4\%, 6\%\}$, relative to the case where $\varepsilon^m_{-\infty} = 0$.*

ply, those are reported in Panels (c) and (d) of Figure 3.5. Relative the benchmark case ($\varepsilon^m_{-\infty} = 0, \mathcal{M}_0 = 1$), a looser money supply baseline under $\varepsilon^m_{-\infty} = 2\%$ features an initial rise in the average number of suppliers by around 16, or an increase in the average intermediates intensity by around 0.045. A baseline with an even looser initial money supply ($\varepsilon^m_{-\infty} = 6\%$) is associated with an initial increase in the average number of suppliers by around 28, equivalent to a 0.06 increase in average intermediates intensity. Just like with variations in baseline aggregate productivity, the effect of variations in initial money supply is not symmetric. In particular, a baseline with tighter money supply ($\varepsilon^m_{-\infty} = -2\%$) has a initial drop in the average number of suppliers by 15, or a 0.005 reduction in average intermediates intensity. An even tighter initial money supply ($\varepsilon^m_{-\infty} = -6\%$) features a drop of 15 in average number of supplier, or a 0.012 reduction in average intermediates intensity.

Small monetary shocks

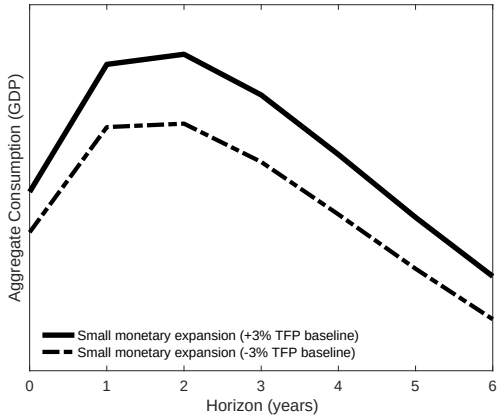
I now perturb each of the baselines considered in the previous subsection with the same monetary shock, which is small in the sense that it leaves the equilibrium set of suppliers unchanged relative to the baseline in every period. The aim is to study how variations in the baseline affect the magnitude of monetary transmission to GDP.

Figure 3.6(a) shows IRFs of GDP to the same small monetary expansionary shock under baselines with low ($\zeta_1 = -3\%$) and high ($\zeta_1 = 3\%$) aggregate productivity. One can see that under high productivity the response of GDP is persistently higher across horizons, despite the fact that the size of the shock is the same across the two baselines. This is because, as seen in Figure 3.5(a), under the high productivity baseline the average number of suppliers is higher, which strengthens complementarities in price setting and amplifies monetary non-neutrality. Moreover, Figure 3.5(a) also shows that under the high productivity baseline the increase in the average number of suppliers is persistent, which is what creates persistently higher non-neutrality of money, and explains why the gap between the two IRFs holds across horizons.

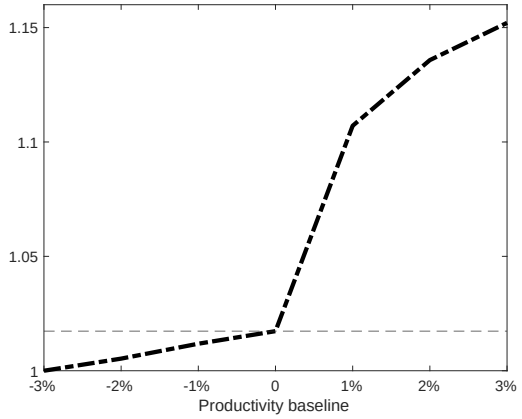
In order to further quantify such interplay between baseline productivity and monetary non-neutrality, Figure 3.6(b) plots peaks of IRFs of GDP to the same small monetary expansion across baselines with different aggregate productivities. For the ease of interpretation, the peak under the

Figure 3.6: Impulse responses of GDP to a small monetary expansion across baselines

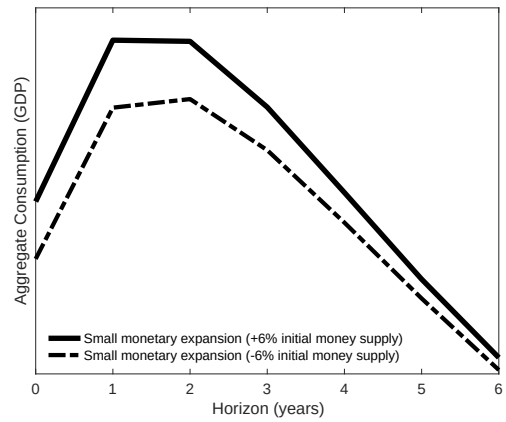
(a) IRFs of GDP: expansion and recession



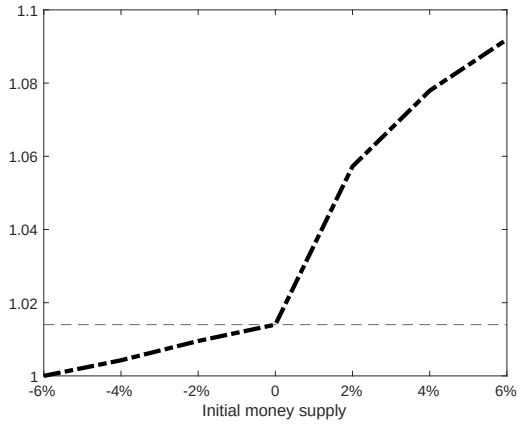
(b) Peaks of IRFs across productivities



(c) IRFs of GDP: tight and loose money



(d) Peaks of IRFs across money supp.



***Notes**: Panel (a) shows IRFs of GDP to the same small monetary expansion around two baselines, with ($\mathcal{M}_0 = 1, \zeta_1 = -3\%$) and ($\mathcal{M}_0 = 1, \zeta_1 = 3\%$); Panel (b) shows peaks of IRFs to the same small monetary expansion around two baselines, with $\mathcal{M}_0 = 1$ and $\zeta_1 = \{-3\%, -2\%, 0, -1\%, 1\%, 2\%, 3\%\}$, where that peak for $\zeta_1 = -3\%$ is normalized to one. Panel (c) shows IRFs of GDP to the same small monetary expansion around two baselines, with ($\zeta_t = 0, \forall t \geq 1, \varepsilon^m_{-\infty} = \{-6\%\}$) and ($\zeta_t = 0, \forall t \geq 1, \varepsilon^m_{-\infty} = \{6\%\}$); Panel (d) shows peaks of IRFs to the same small monetary expansion around two baselines, with $\zeta_t = 0, \forall t \geq 1$ and $\varepsilon^m_{-\infty} = \{-6\%, -4\%, -2\%, 0, 2\%, 4\%, 6\%\}$, where that peak for $\varepsilon^m_{-\infty} = -6\%$ is normalized to one.*

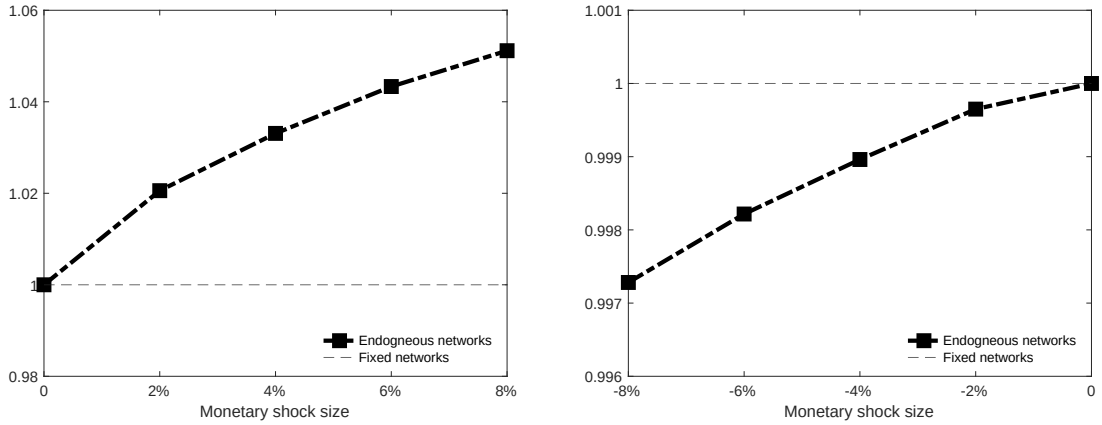
lowest productivity baseline ($\zeta_1 = -3\%$) is normalized to one. One can see that peak response of GDP under the highest productivity baseline is almost one fifth larger than under the lowest productivity baseline. However, the rate at which monetary non-neutrality varies with baseline productivity is highly non-linear. In particular, the change in non-neutrality between the highest productivity baseline ($\zeta_1 = 3\%$) and the benchmark case ($\zeta_1 = 0$) is almost seven times larger than the change between the benchmark case and the lowest productivity baseline ($\zeta_1 = -3\%$). In this sense, in our quantitative model monetary policy becomes powerful in expansions relative to normal times, but its effectiveness does not wane by nearly as much in recessions. The latter is explained by the asymmetry in the relationship between the average number of suppliers and baseline productivity, as documented in Figure 3.5(a).

I now repeat the above exercise for baselines that differ in their initial money supply. Figure 3.6(c) shows IRFs of GDP to the same small monetary expansionary shock under baselines with tight ($\varepsilon^m_{-\infty} = -6\%$) and loose ($\varepsilon^m_{-\infty} = 6\%$) initial money supply. One can see that under loose initial money supply the response of GDP is higher across horizons. As before, this is because under the loose initial money supply baseline the average number of suppliers is higher, which strengthens complementarities in price setting and amplifies monetary non-neutrality. Notice that according to Figure 3.5(c), the change in average number of suppliers triggered by change in initial money supply is much shorter lived that that caused by changes in baseline productivity. As a result, additional monetary non-neutrality created by high initial money supply is shorter lived and, as can be seen in Figure 3.6(c), IRFs of GDP under different baseline money supplies converge to each other faster than in Figure 3.6(a).

In Figure 3.6(d) we plot peaks of GDP responses to the same small monetary expansion across baselines with different initial money supply. One can see that peak response of GDP under the baseline with most loose money supply ($\varepsilon^m_{-\infty} = 6\%$) is almost one tenth larger than under the baseline with tightest money supply ($\varepsilon^m_{-\infty} = -6\%$). As in Figure 3.6(b) the relationship between peak GDP and changes in the baseline is highly non-linear: the drop in potency of monetary policy under tight initial money supply is much smaller than the rise in potency under loose initial money. And just as before, this is explained by the asymmetry in the relationship between the average number of suppliers and baseline money supply, as documented in Figure

Figure 3.7: IRFs of GDP to large monetary shock relative to a fixed network benchmark

(a) Large monetary expansions (b) Large monetary contractions



***Notes**: Panel (a) shows peaks of IRFs to large monetary expansions around the baseline with $\zeta_t = 0, \forall t \geq 1$ and $\mathcal{M}_0 = 1$, for shock values $\varepsilon_1^m = \{0, 2\%, 4\%, 6\%, 8\%\}$, where that peak for $\varepsilon_1^m = 0$ is normalized to one; Panel (b) shows peaks of IRFs to large monetary contractions around the baseline with $\zeta_t = 0, \forall t \geq 1$ and $\mathcal{M}_0 = 1$, for shock values $\varepsilon_1^m = \{-8\%, -6\%, -4\%, -2\% 0\}$, where that peak for $\varepsilon_1^m = 0$ is normalized to one.*

3.5(c).

Large monetary shocks

Having studied properties of small monetary shocks, we now turn to monetary shocks that are large enough to affect equilibrium supplier choices. Recall that in the previous section we formally established that large monetary expansions expand the set of suppliers which leads to a larger GDP response relative to the case where networks are fixed. On the contrary, large monetary contractions reduce the number of linkages and thus deliver a smaller contraction in GDP relative to what would happen under exogenous networks.

In Figure 3.7 we report results, where we fix the baseline at $\mathcal{M}_0 = \mathcal{Z}_t = 1, \forall t$, and subject our economy to one-time large monetary expansions $\varepsilon_1^m = \{2\%, 4\%, 6\%, 8\%\}$ and large monetary contractions $\varepsilon_1^m = \{-8\%, -6\%, -4\%, -2\%\}$. For each shock size, we normalize GDP IRFs with the response of GDP that would occur if the same shock hit an otherwise

identical economy with fixed exogenous input-output linkages.

Panel (a) reports normalized peaks of GDP IRFs to large monetary expansions. Relative to an economy with fixed networks, the peak response of GDP to a 2% monetary expansion is larger by 2 percent, whereas for an 8% shock the response is 5 percent larger. Consistently with our formal results in the previous section, large monetary expansions deliver more than proportional response of GDP, despite fully time-dependent probabilities of price adjustment,

Results for large monetary contractions are reported in Panel (b). Relative to an economy with exogenous production networks, an 8% monetary contraction delivers a drop in GDP that is 0.3 percent smaller at the trough, with the gap shrinking down to 0.1 percent for a 4% contraction. Overall, the effects are consistent with formal results from the previous section, though their quantitative relevance is much smaller than under large monetary expansions. This is a consequence of asymmetric sensitivity of equilibrium supplier changes to changes in money supply that was documented in Figures 3.5(c)-(d), where reductions in money supply do not eliminate as many linkages as what is added under increase in money supply of the same magnitude.

3.5 Empirical evidence

Having established and quantified our theoretical results, we now turn to empirical assessment of the key mechanisms. In this section we use both sectoral and firm-level input-output data to estimate network cyclicality conditional on identified technology and monetary shocks. We show that our novel empirical results strongly support the theoretical predictions of our model.

3.5.1 Evidence using sector-level data

Data

In this subsection we use sector-level data on intermediates intensity in order to perform model-free evaluation of the predictions of our model regarding business cycle fluctuations in the shape of the production network. In particular, we use US data on sector-level employee compensation and

expenditure on intermediate inputs, published by the Bureau of Economics Analysis (BEA), to construct the share of total costs going to intermediate inputs:

$$\delta_{kt} = \frac{\text{Intermediate inputs costs}_{kt}}{\text{Compensation of Employees}_{kt} + \text{Intermediate inputs costs}_{kt}} \tag{3.18}$$

I construct such measure of intermediates intensity for 65 three-digit sectors of the US economy at annual frequency between 1987-2017. In the case of a Cobb-Douglas production function, such measure maps directly into the model-based measure of sector-specific intermediates intensity in a particular time period, given by $\sum_{r \in S_{k,t}} \omega_{kr}, \forall k$.

Econometric strategy

Once those measures have been constructed, we use the local projection approach of Jordà (2005a) in order to estimate horizon-specific impulse responses of sectoral intermediates intensity to productivity and monetary shocks:

$$\delta_{k,t+H} = \alpha_{k,H} + \beta_H s_t + \gamma_H x_{k,t-1} + \varepsilon_{k,t+H}, \tag{3.19}$$

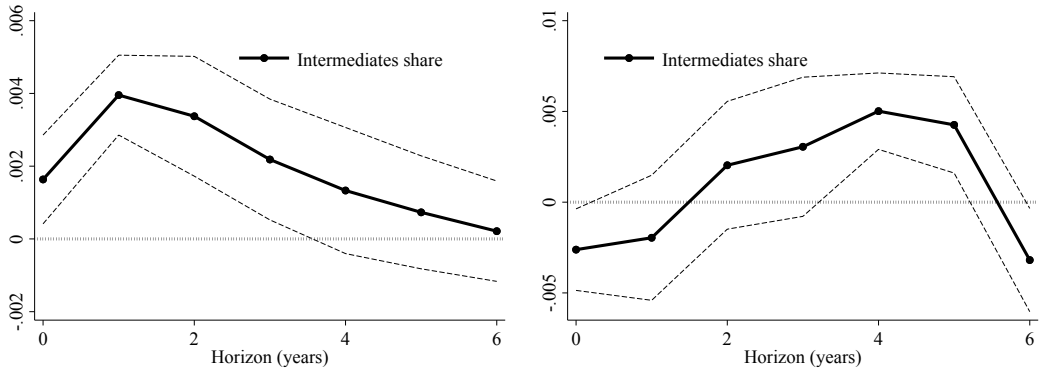
for $H = 0, 1, ..., \overline{H}$, where $\delta_{k,t}$ is intermediate intensity of sector k, s_t is an identified exogenous shock, $x_{k,t-1}$ is a vector of control variables and $\alpha_{k,H}$ represents horizon-specific sectoral fixed effects.[6] The estimated value of β_H gives the horizon-specific response of intermediate inputs intensity to an identified exogenous shock; according to our theory, intermediates intensity should, *ceteris paribus*, rise following expansionary productivity and monetary shocks, and *vice versa*. The specification above allows to test such predictions in a transparent model-free way using horizon-by-horizon fixed-effects regressions.

A feature of our theory is that changes in productivity and monetary conditions can affect the equilibrium set of suppliers in a non-linear fashion. For example, in the previous section we documented how improvements in productivity and money supply affect the use of intermediates much more strongly than deteriorations of the same magnitude. In order to test for presence of such non-linearities in the data, we consider the following aug-

[6]The exact set of controls is specified in descriptions to figures presenting each of the estimation results.

Figure 3.8: Linear IRFs of intermediates intensity to productivity and monetary shocks

(a) Productivity expansion (+1%) (b) Monetary expansion (-100bp)



***Notes**: Panel (a) shows estimated IRFs of intermediates intensity to a productivity shock based on Fernald (2014b), using specification in (3.19); the set of controls includes one lead of productivity shock and one lag of intermediates intensity, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows estimated IRFs of intermediates intensity to a monetary shock based on Romer and Romer (2004), using specification in (3.19); the set of controls includes one lead of monetary shock and one lag of intermediates intensity, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence bands.*

mented local projection specification:

$$\delta_{k,t+H} = \alpha_{k,H} + \beta_H^{lin} s_t + \beta_H^{sign} s_t \times \mathbf{1}\{s_t > 0\} + \beta_H^{size} s_t \times |s_t| + \gamma_H x_{k,t-1} + \varepsilon_{k,t+H}, \tag{3.20}$$

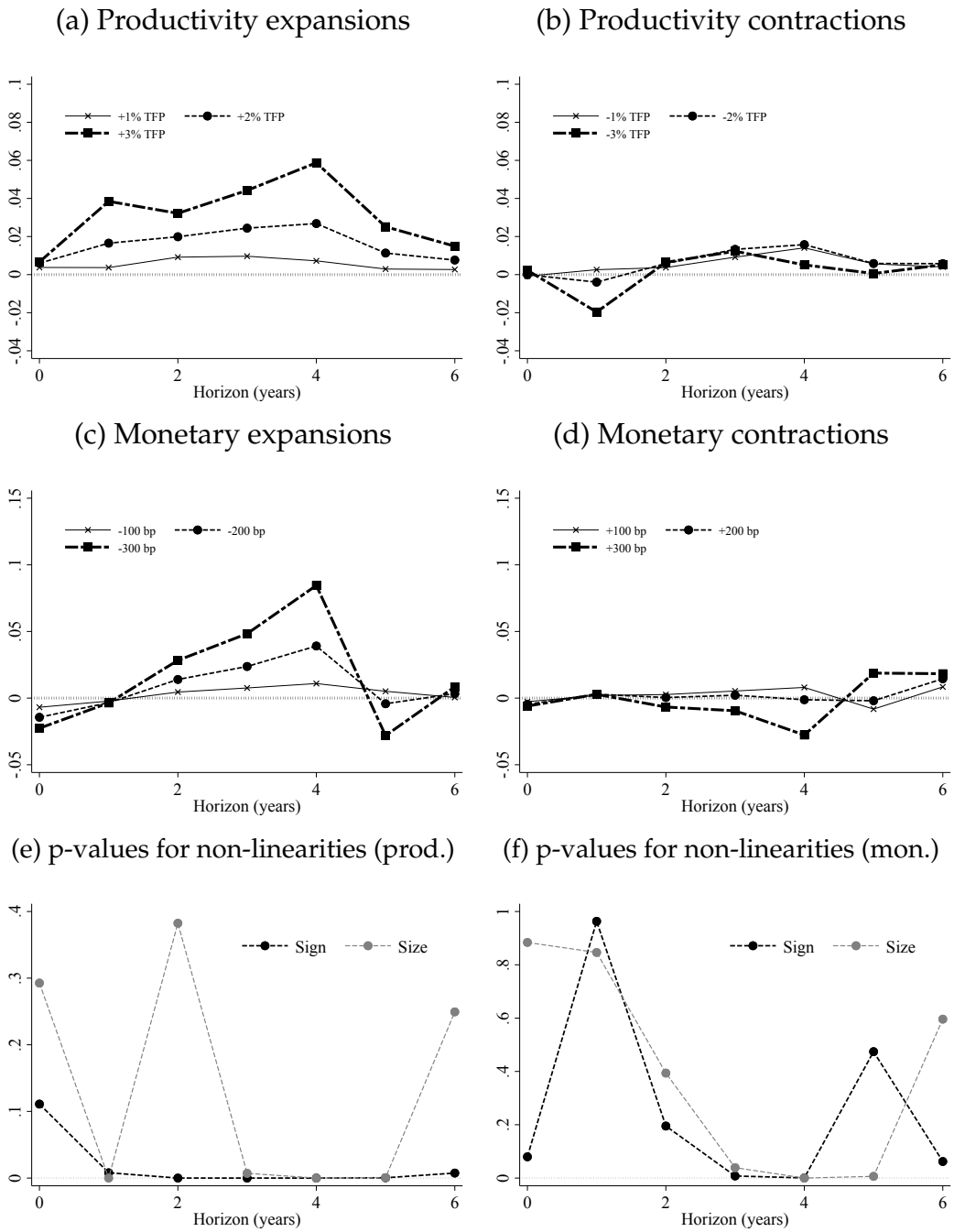
for $H = 0, 1, ..., \overline{H}$, where it follows that $\beta_H^{sign} > 0$ indicates that positively-valued shocks make the impulse response more positive, whereas $\beta_H^{size} > 0$ indicates that shocks which are large in magnitude make the response more positive.

In the baseline results presented in this subsection, we use identified annual productivity shocks from Fernald (2014b) and identified annual monetary shocks from Romer and Romer (2004) that have been extended by Wieland and Yang (2016).

Estimation results

Figure 3.8 shows estimation results using linear local projections in equation (3.19). As one can see in Panel (a), following a positive 1% shock to aggregate productivity, the intermediates intensity responds positively on impact,

Figure 3.9: Non-linear IRFs of intermediates intensity to productivity and monetary shocks



***Notes**: Panels (a)-(b) show estimated non-linear IRFs of intermediates intensity to a productivity shock based on Fernald (2014b), using specification in (3.20); the set of controls includes one lead of productivity shock and one lag of intermediates intensity, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panels (c)-(d) show estimated non-linear IRFs of intermediates intensity to a monetary shock based on Romer and Romer (2004), using specification in (3.20); the set of controls includes one lead of monetary shock and one lag of intermediates intensity, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Panels (e) and (f) shock p-values for the test that non-linear terms are zero.*

reaching peak increase of 0.004 after one year, and then gradually declines back to zero; such positive response is consistent with our theory. As for a monetary expansion, in Panel (b) shows that a surprise one-time 100bp easing leads to a slight decline in the intermediates intensity on impact, after which it gradually increases, reaching a peak of 0.005 increase after four years; the response at longer horizons is consistent with theoretical predictions of our model.

In Figure 3.9 we report results of estimation using non-linear local projections in (3.20). As one can see in Panel (a), following a 1% productivity expansion, intermediates intensity rises by around 0.01 at the peak, whereas a 3% productivity expansion increases intermediates intensity by around 0.06 at the peak. Importantly, this means that responses estimated allowing for non-linearities are not equal to linear IRFs scaled by the size of the shock, suggesting an important role for size effects. Moreover, Panel (e) shows that the coefficient representing size effects is statistically significant at 5% level at 1-,3-,4- and 5-year horizons. Another important aspect is that the responses estimated allowing for non-linearities match the magnitudes of responses generated by the quantitative model; recall that in Figure 3.5(b) we reported that a baseline generated by a one-time 3% aggregate productivity improvement also generates a rise in average intermediates intensity of around 0.06.

Figure 3.9(b) considers productivity contractions. A 1% productivity contraction does not generate a drop in intermediates intensity, whereas a 3% contraction generates a modest drop in intermediates intensity of 0.02 at the trough. The latter suggest presence of sign effects: productivity contractions do not produce drops in intermediates intensity of the same magnitude as the rises in intensity produced by productivity expansions. Panel (e) suggests such sign effects are statistically significant at 5% level at all horizons beyond impact. Importantly, the quantitative version of our model produces size effects of the same direction and magnitude; as can be seen in Figure 3.5(b), a baseline generated by a one-time 3% aggregate productivity contraction has average intermediates intensity drop by just around 0.01 at the trough.

In Figure 3.9(c) we turn to the effect of monetary expansions on intermediates intensity. A 100bp monetary easing raises intermediates intensity by around 0.01 at the peak, whereas a 300bp easing produces a rise of close to 0.08. Once again, size effects are present and Panel (f) confirms that they

are statistically significant at 5% level at 3-,4- and 5-year horizons. As for monetary contractions, Panel (d) shows that a 100bp tightening produces close to no drop in intermediates intensity, while a larger 300bp tightening delivers a drop of around 0.02 at the trough. Just like in the case of productivity changes, sign effects are detected: monetary contractions produce drops in intermediates intensity that are much smaller in magnitude than the increases produced by easings of the same size. Moreover, Panel (f) suggests such sign effects are statistically significant at 3- and 4-year horizons, exactly where peaks and troughs of responses occur. Finally, recall that in Figure 3.5(d) we documented presence of similar sign effects in the quantitative version of our model as one varies money supply.

Further results and robustness checks

In Appendix 3.B we perform further econometric exercises and robustness checks, which we briefly summarize here.

First, we re-estimate the effect of identified shocks on intermediates intensity, allowing for non-linearity, at the level of six broad sectoral groups, to determine if the observed network cyclicality could be driven by specific parts of the US economy. Results suggest that largest sensitivities of intermediates intensities to changes in productivity are detected in "Trade, transportation and warehousing" and "Durable manufacturing goods", whereas very little responsiveness is found in "Information, Finance-Insurance-Real Estate and other services" and "Education, healthcare and other services". As for the effect of monetary interventions, by far the largest response is found in "Non-manufacturing goods", whereas the smallest effect is found in "Education, healthcare and other services".

Second, we consider an alternative approach to modeling non-linearity in our local projections setting. In particular, we re-estimate (20), where instead of adding interactions with a sign dummy and absolute value of the shock, we add quadratic and cubic shocks, which similarly capture possible sign and size effects. Reassuringly, results remain virtually unchanged, both in terms of magnitudes of effects, as well as when it comes to sign and size effects and their statistical significance.

3.5.2 Evidence using firm-level data

Data

My model predicts that adjustments in the reliance of sectors on intermediate inputs happen on the extensive margin. However, results in the previous subsection give no guidance on the specific margin of adjustment. In this subsection we use firm-level data on the number of suppliers of US publicly listed firms in order to perform model-free evaluation of the *extensive margin* of network adjustment over the business cycle. In particular, we use the dataset constructed by Atalay et al. (2011) based on US Compustat data, which contains the *indegree*, or simply the number of supplier firms in the dataset, for a large number of US publicly listed firms.

Econometric strategy

Using the indegree measure, we once again follow the local projection approach of Jordà (2005a) in order to estimate horizon-specific impulse responses of a firm's number of suppliers to productivity and monetary shocks:

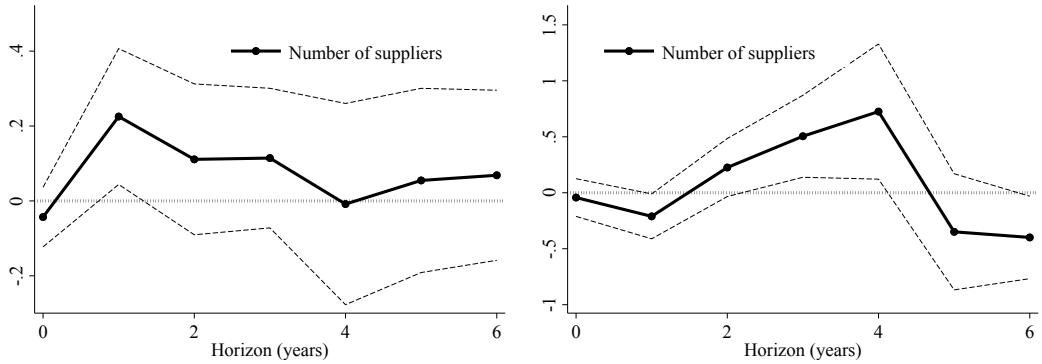
$$indeg_{j,t+H} = \alpha_{j,H} + \beta_H s_t + \gamma_H x_{j,t-1} + \varepsilon_{j,t+H}, \tag{3.21}$$

for $H = 0, 1, ..., \overline{H}$, where $indeg_{j,t}$ is the indegree (number of suppliers) of firm j in year t, s_t is an identified exogenous shock, $x_{j,t-1}$ is a vector of control variables and $\alpha_{j,H}$ represents horizon-specific firm fixed effect. The estimated value of β_H gives the horizon-specific response of the number of suppliers to an identified exogenous shock; according to our theory, the number of suppliers should, *ceteris paribus*, rise following expansionary productivity and monetary shocks, and *vice versa*. The specification above allows to test such predictions in a transparent model-free way using horizon-by-horizon fixed-effects regressions.

As discussed earlier, our model predicts a non-linear relationship between changes in productivity and monetary conditions, and the equilibrium set of suppliers. In order to test for such non-linear adjustment at the extensive margin, we once again extend the previous specification to include

Figure 3.10: Linear IRFs of number of suppliers to productivity and monetary shocks

(a) Productivity expansion (+1%) (b) Monetary expansion (-100bp)



***Notes**: Panel (a) shows estimated IRFs of the number of suppliers to a productivity shock based on Fernald (2014b), using specification in (3.21); the set of controls includes four lags of the number of suppliers, one lag of productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panel (b) shows estimated IRFs of the number of suppliers to a monetary shock based on Romer and Romer (2004), using specification in (3.21); the set of controls includes one lead of monetary shock, four lags of the number of suppliers, one lag of monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Dotted lines represent 95% confidence bands.*

additional terms:

$$indeg_{j,t+H} = \alpha_{j,H} + \beta_H^{lin} s_t + \beta_H^{sign} s_t \times \mathbf{1}\{s_t > 0\} + \beta_H^{size} s_t \times |s_t| + \gamma_H x_{j,t-1} + \varepsilon_{j,t+H}, \tag{3.22}$$

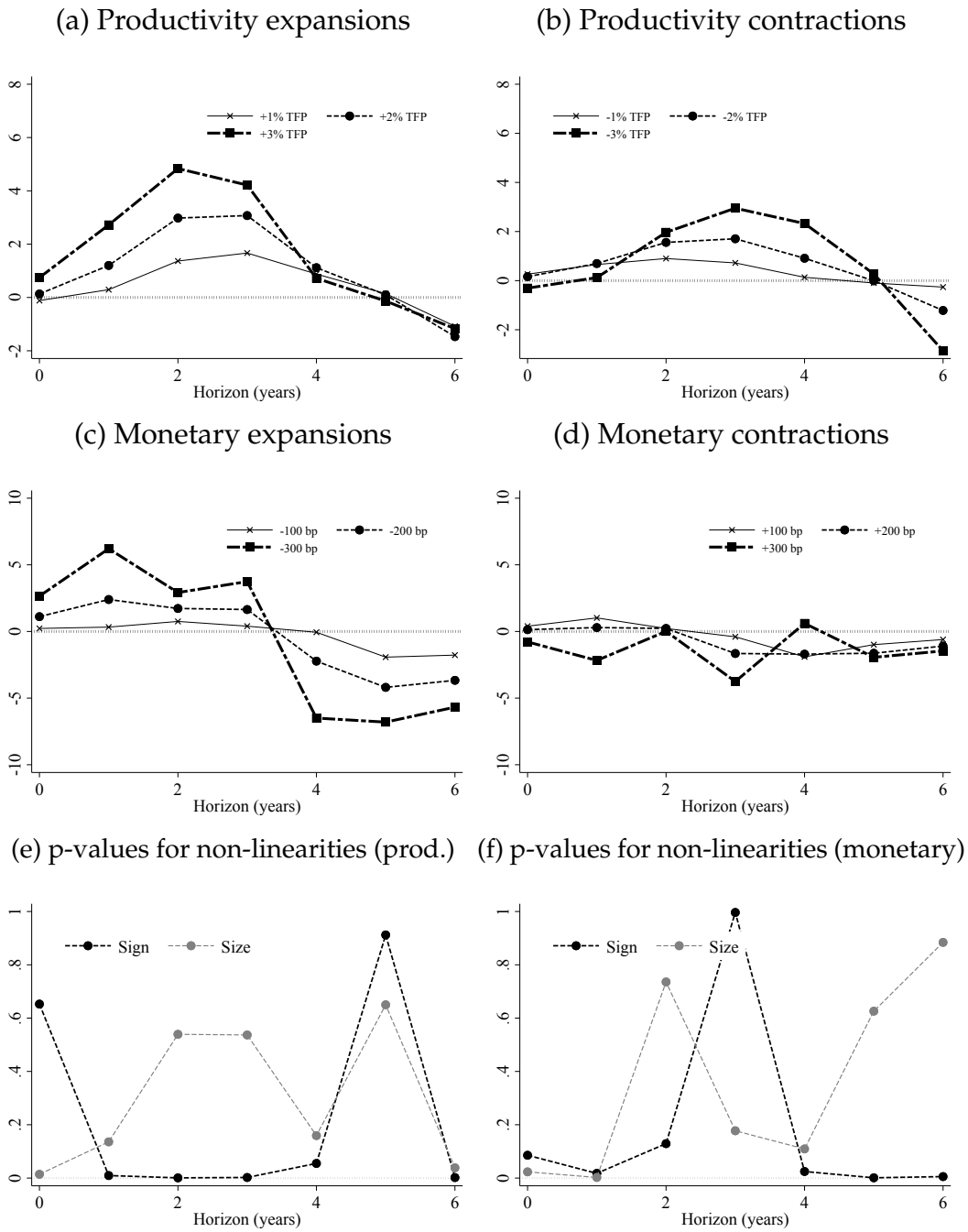
for $H = 0, 1, ..., \overline{H}$, where it follows that $\beta_H^{sign} > 0$ indicates that positively-valued shocks make the impulse response more positive, whereas β_H^{size} indicates that larger shocks make the response more positive.

In the baseline results presented in this subsection, we once again use identified annual productivity shocks from Fernald (2014b) and identified annual monetary shocks from Romer and Romer (2004) that have been extended by Wieland and Yang (2016).

Estimation results

Figure 3.10 shows estimation results using linear local projections in (3.21). As one can see, following a positive 1% shock to aggregate productivity, the (average) number of suppliers responds positively on impact, reaching peak increase of around 0.20 after one year, and then gradually declines back to

Figure 3.11: Non-linear IRFs of number of suppliers to productivity and monetary shocks



***Notes**: Panels (a)-(b) show estimated non-linear IRFs of the number of suppliers to a productivity shock based on Fernald (2014b), using specification in (3.22); the set of controls includes one lead of productivity shock, four lags of the number of suppliers, one lag of productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panels (c)-(d) show estimated non-linear IRFs of the number of suppliers to a monetary shock based on Romer and Romer (2004), using specification in (3.22); the set of controls includes one lead of monetary shock, four lags of the number of suppliers, one lag of monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Panels (e) and (f) shock p-values for the test that non-linear terms are zero.*

zero; such positive response is consistent with our theory. As for a monetary expansion, a surprise one-time 100bp easing first leads to a slight insignificant decline, after which in gradually increases, reaching a peak of 0.85 increase after four years; once again, the response at longer horizons is both statistically significant and consistent with theoretical predictions.

In Figure 3.11 we report results obtained using non-linear local projections in (3.22). Panel (a) shows that following a 1% productivity expansion, the (average) number of suppliers goes up by almost 2 at the peak, whereas a 3% productivity expansion increases the number of suppliers per firm by approximately 5. At the same time, Panel (b) suggest that productivity contractions are unable to deliver drops in the number of suppliers. The latter suggests that sign effects, documented both in estimation using sectoral data and in the quantitative version of our model, are similarly found in firm-level data. Moreover, Panel (e) suggests such sign effects are significant at 1-,2- and 3-year horizons.

Similarly, Figure 3.11(c) shows that following monetary easings the (average) number of suppliers per firm rises: by around 1 following a 100bp easing, an and by close to 6 after a 300bp loosening. And once again, monetary contractions are unable to generate a drop in the number of suppliers per firm of a comparable magnitude, giving another evidence in favor of sign effects, previously documented for monetary interventions both using estimation with sectoral data, as well as using a quantitative version of our theoretical model. Panel (f) confirms that such sign effect is highly significant at the 1-year horizon, precisely where largest effect of monetary easings on the number of suppliers is documented.

Further results and robustness checks

In Appendix 3.B we once again consider the alternative approach to modeling non-linearity in our local projections setting, this time applied to the firm-level data on the number of suppliers. In particular, when we instead add quadratic and cubic shocks to the specification, results remain virtually unchanged, both in terms of magnitudes of effects, as well as in when it comes to sign effects and their statistical significance.

3.6 Conclusion

I develop a novel multi-sector New Keynesian model with roundabout production, where input-output linkages are formed endogenously through firms' optimizing decisions. We provide novel empirical evidence on the cyclical behavior of production networks and show that the model replicates the observed variation, conditional on either productivity or monetary shocks driving the cycle. In particular, in the data the reliance on intermediates rises following productivity improvements and monetary easings. In the model, technological improvements also increase the reliance on intermediates, as expanding the set of suppliers lowers firms' unit costs directly through higher productivity and indirectly through lower prices charged by suppliers. Similarly, nominal easings also expand the production network in the model, as they make prices charged by supplier firms cheaper relative to the cost of in-house labor.

The model delivers a novel mechanism of state-dependence in monetary transmission, which is generated by cyclical variation in the strength of complementarities in price setting. This mechanism makes the strength of monetary transmission depend on the *phase of the business cycle*, *past monetary stance* and the *size of the shock* even if the probability of price adjustment is state-independent. In particular, in periods of high productivity and loose monetary policy, firms optimally decide to connect to more suppliers, which strengthens pricing complementarities and amplifies monetary non-neutrality. On the contrary, episodes of low productivity and tight monetary policy lead to sparse networks and weaker complementarities in price setting, which diminishes monetary non-neutrality. At the same time, larger monetary expansions have a disproportionally *larger* positive effect on GDP compared to smaller monetary expansions, as the former expand the production network. Larger monetary contractions instead have a disproportionally *smaller* negative effect on GDP, as they shrink the network. The nonlinearities in the transmission of monetary shocks in the theoretical model are consistent with the econometric evidence (Tenreyro and Thwaites, 2016; Jordà et al., 2019; Alpanda et al., 2019; Ascari and Haber, 2021).

The analysis offers multiple avenues for future research. First, optimal choice of suppliers can be studied in an open economy setting, where firms minimize their production costs by deciding whether to purchase intermediate inputs from domestic producers or to import them instead. Such a

mechanism would pin down the optimal degree of participation in global value chains, with important implications for both monetary and trade policies. Second, the analysis focused on positive aspects of monetary transmission under endogenous production networks, while future work could study the optimal conduct of monetary policy. Third, a deeper quantitative assessment of the non-linearities generated by endogenous network formation would require enriching the setting with medium-scale features, such as capital accumulation, habits-in-consumption and trend inflation to name a few.

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Appendices to Chapter 3

3.A Proofs of results in main text

Proposition 3.1 (Equilibrium)**.** *Suppose Assumptions 3.1-3.3 hold. Then equilibrium introduced in Definition 3.1 possesses the following properties: (a) equilibrium exists; (b) equilibrium sectoral prices and final sectoral consumptions are unique; (c) equilibrium supplier choices and remaining sectoral allocations are generically unique.*

Proof. (a) Existence;

In periods $t \geq 2$ prices are flexible, and firms' problem is static and collapses to that studied by Acemoglu and Azar (2020). Hence, it suffices to show existence at $t = 1$, where prices are sticky. For convenience, let $\overline{P}_k \equiv \frac{P_k}{\mathcal{M}}$ and $\overline{\mathcal{Q}}_k \equiv \frac{\mathcal{Q}_k}{\mathcal{M}}$ be ratios of sectoral prices and unit costs to money supply.

As a first step, I am going to establish existence of equilibrium prices and supplier choices. Subsequently, I am going to show how existence of equilibrium prices and supplier choices implies existence of equilibrium quantities.

Rewrite equation for sectoral price aggregation, using the fact that every sectoral production function exhibits constant returns to scale:

$$P_k^{1-\theta} = \alpha_k P_{k,0}^{1-\theta} + (1-\alpha_k)\left[\mathcal{M}(1+\mu_k)\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k})\right]^{1-\theta} \tag{3.23}$$

$$\overline{P}_k = \left[\alpha_k \left[\frac{P_{k,0}}{\mathcal{M}}\right]^{1-\theta} + (1-\alpha_k)\left[(1+\mu_k)\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k})\right]^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{3.24}$$

or $\overline{P}_k = f_k[\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k}); \mathcal{M}]$, where $f_k' > 0, f_k'' < 0 \forall k$, and $f_k(.; \overline{\mathcal{M}}) < f_k(.; \underline{\mathcal{M}}), \forall k : \alpha_k > 0$ for any $\overline{\mathcal{M}} > \underline{\mathcal{M}}$. Define the following function:

$$\kappa(\{\overline{P}_k\}_{k=1}^K) = \left[f_1[\min_{S_1} \overline{\mathcal{Q}}_1(S_1, \mathcal{A}_1(S_1), \{\overline{P}_r\}_{r\in S_1}); \mathcal{M}], ..., f_K[\min_{S_K} \overline{\mathcal{Q}}_K(S_K, \mathcal{A}_K(S_K), \{\overline{P}_r\}_{r\in S_K}); \mathcal{M}]\right]' \tag{3.25}$$

One can show that $\kappa(.)$ has a fixed point, and that its fixed point

corresponds to equilibrium ratios of sectoral prices to money supply. This follows from the fact that the set $\mathbb{L} = \{\overline{P}_k \geq 0, \forall k : \overline{P}_k = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r \in S_k}); \mathcal{M}]\}$ is a complete lattice with respect to operations $\mathbb{P} \bigwedge \mathbb{Q} = [\min(P_1, Q_1), ..., \min(P_K, Q_K)]'$ and $\mathbb{P} \bigvee \mathbb{Q} = [\max(P_1, Q_1), ..., \max(P_K, Q_K)]'$. I now formally establish the latter point.

First, a producer in any sector k can produce using labor only, incurring a unit cost of $\overline{\mathcal{Q}}_k(\varnothing, \mathcal{A}_k(\varnothing), \varnothing)$, which delivers the price to money supply ratio of $\overline{P}'_k \equiv f_k[\overline{\mathcal{Q}}_k(\varnothing, \mathcal{A}_k(\varnothing), \varnothing); \mathcal{M}], \forall k$, which does not depend on sectoral prices, and so $\kappa(\{\overline{P}_k\}_{k=1}^K) \leq [\overline{P}'_1, ..., \overline{P}'_K]'$ for all $\{\overline{P}_k\}_{k=1}^K$. Since labor is an essential input by assumption, it follows that at zero prices the unit cost is positive for any choice of suppliers: $\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), 0_{|S_k| \times 1}) > 0, \forall S_k, k$. Let $\overline{P}''_k \equiv \kappa_k(0_{|S_k| \times 1}) = f_k[\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), 0_{|S_k| \times 1}); \mathcal{M}], \forall k$. Since the unit cost function is increasing in sectoral prices and $f'_k > 0, \forall k$, it follows that $\kappa(\{\overline{P}_k\}_{k=1}^K) \geq \kappa(0_{|S_k| \times 1}) = [\overline{P}''_1, ..., \overline{P}''_K]'$ for all $\{\overline{P}_k\}_{k=1}^K$. Hence, $\mathbb{O} \equiv \times_{k=1}^K [\overline{P}'_k, \overline{P}''_k]$ is a complete lattice and κ maps $\mathbb{O}$ to $\mathbb{O}$.

Second, for any $\{\overline{P}_k\}_{k=1}^K$ and $\{\overline{\overline{P}}_k\}_{k=1}^K$ such that $\overline{P}_k \leq \overline{\overline{P}}_k \forall k$ we have $\overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r \in S_k}) \leq \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{\overline{P}}_r\}_{r \in S_k}), \forall S_k, k$ and hence $\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r \in S_k}) \leq \min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{\overline{P}}_r\}_{r \in S_k}), \forall k$, which further implies $f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r \in S_k})] \leq f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{\overline{P}}_r\}_{r \in S_k})], \forall k$ and $\kappa(\{\overline{P}_k\}_{k=1}^K) \leq \kappa(\{\overline{\overline{P}}_k\}_{k=1}^K)$. Hence, κ is an order-preserving function which maps a complete lattice $\mathbb{O}$ onto itself. By Knaster-Tarski theorem, it follows that κ has a minimal fixed point, which yields equilibrium ratios of sectoral prices to money supply $\{\overline{P}_k\}_{k=1}^K$ and supplier choices $\{S_k\}_{k=1}^K$. Trivially, equilibrium sectoral prices are then recovered as $P_k = \mathcal{M} \times \overline{P}_k, \forall k$. As for equilibrium firm-level prices within a particular sector, those are given by $P_{k,0}$ for non-adjusters at $t = 1$ and by $(1 + \mu_k)\mathcal{Q}_k(S_k, \mathcal{A}_k(S_k), \mathcal{M}, \{P_r\}_{r \in S_k})$ for adjusters at $t = 1$.

Having established existence of equilibrium prices and supplier choices, I now show that their existence allows to recover equilibrium allocations. First, the cash-in-advance constraint allows to recover aggregate final con-

sumption $C_t = \mathcal{M}_t/P_t^c$, where $P_t^c = P_t^c[P_{1t}, ..., P_{Kt}]'$ is the consumption price index. Given aggregate final consumption, sectoral consumptions $C_{kt} = C_{kt}(\{P_{kt}\}_{k=1}^K, C_t)$ are pinned down uniquely from properties of the aggregator $u(.)$. As for firm-level consumptions, those are pinned down as $C_{kt}(j) = \left(\frac{P_{kt}(j)}{P_{kt}}\right)^{-\theta} C_{kt}$. Let $z_{krt} \equiv \frac{Z_{krt}(j)}{Y_{kt}(j)}$ and $n_{kt} \equiv \frac{N_{kt}(j)}{Y_{kt}(j)}$ be firm-level ratios of factor demand to output; given assumed properties of the production function, those are common to all firms within a particular sector, and independent of output. From goods market clearing condition:

$$C_{kt}(j) + \sum_{r=1}^{K} \int_{j' \in \Phi_r} Z_{rkt}(j', j) dj' = Y_{kt}(j), \quad \forall k, \forall j \in \Phi_k \tag{3.26}$$

$$C_{kt} + \sum_{r=1}^{K} z_{rkt} \int_{j' \in \Phi_r} \left(\frac{P_{rt}(j')}{P_{rt}}\right)^{-\theta} Y_{rt} dj' = Y_{kt}, \quad \forall k, \forall j \in \Phi_k \tag{3.27}$$

$$C_{kt} + \sum_{r=1}^{K} z_{rkt} \Delta_{rt} Y_{rt} = Y_{kt}, \quad \forall k, \forall j \in \Phi_k \tag{3.28}$$

where $\Delta_{rt} = \int_{j \in \Phi_k} \left(\frac{P_{rt}(j)}{P_{rt}}\right)^{-\theta} dj$ is price dispersion in sector r. The expression above allows to recover sectoral outputs:

$$[Y_{1t}, ..., Y_{Kt}]' = (I - \Omega_t')^{-1} [C_{1t}, ..., C_{Kt}]', \tag{3.29}$$

where Ω_t is a matrix whose entries are given by $[\Omega_t]_{kr} = z_{rkt}\Delta_{rt}$. Given sectoral outputs, firm-level outputs can be recovered as $Y_{kt}(j) = \left(\frac{P_{kt}(j)}{P_{kt}}\right)^{-\theta} Y_{kt}$, further allowing to pin down factor demands: $Z_{krt}(j) = z_{krt} Y_{kt}(j)$, $N_{kt}(j) = n_{kt} Y_{kt}(j)$.

(b) Uniqueness of sectoral prices and final consumptions;

As before, uniqueness of sectoral prices under no nominal rigidities ($t \geq 2$) is shown in Acemoglu and Azar (2020). Here I only need to establish uniqueness of equilibrium prices under nominal rigidities in $t = 1$.

Let $\{\overline{P}_k\}_{k=1}^K$ be the minimal element of complete lattice $\mathbb{L}$ introduced in (a); suppose $\{\overline{P}_k^*\}_{k=1}^K$ is another set of equilibrium ratios of sectoral prices to money supply; then it must be that $\overline{P}_k^* > \overline{P}_k, \forall k$. Below I show by contradiction that the latter implies $\{\overline{P}_k\}_{k=1}^K$ is the unique equilibrium set of ratios of sectoral prices to money supply.

Given assumed properties of the production function, it follows that $\overline{\mathcal{Q}}_k$ is concave in $\{\overline{P}_k\}_{k=1}^K$. Moreover, minimum of a collection of functions is also concave, so $\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k})$ is also concave. Recall that $f_k' > 0, f_k'' < 0, \forall k$ so that f_k is increasing and concave, and hence $\kappa_k(\{\overline{P}_k\}_{k=1}^K) = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r\}_{r\in S_k})]$ is also concave for all $k = 1, ..., K$.

Let $\nu \in (0, 1)$ be such that $\nu\overline{P}_k^* = \overline{P}_k, \forall k$ with at least one $r = 1, 2, ..., K$ such that $\overline{P}_k^* = \overline{P}_k$. Then:

$$\kappa_r(\overline{P}_r) - \overline{P}_r \geq \kappa_r(\nu\overline{P}_r^*) - \nu\overline{P}_r^* \geq \kappa_r(0)(1-\nu) + \underbrace{\kappa_r(\overline{P}_r^*)\nu - \nu\overline{P}_r^*}_{=0} \geq (1-\nu)\kappa_r(0) > 0.$$

The final inequality implies $\kappa_r(\overline{P}_r) > \overline{P}_r$, which contradicts that $\{\overline{P}_k^*\}_{k=1}^K$ is a fixed point. Hence, $\{\overline{P}_k\}_{k=1}^K$ is the unique fixed point, and equilibrium sectoral prices are unique.

Uniqueness of sectoral prices implies uniqueness of the aggregate consumption price index, and through the cash-in-advance constraint, it follows that equilibrium aggregate final consumption is also unique. Finally, properties of the consumption aggregator u imply that given unique sectoral prices and aggregate consumption, sectoral consumptions are also unique.

(c) Generic uniqueness of supplier choices and remaining quantities;

Generic uniqueness of sectoral supplier choices follows directly from the proof in Acemoglu and Azar (2020), since the possibility of multiplicity in

equilibrium networks is unrelated to the degree of nominal rigidities.

From (29) one can see that equilibrium sectoral outputs are function of the amount of inputs sectors are buying from each other. Since supplier choices are generically unique, so are sectoral outputs in equilibrium. Finally, since factor demand are pinned down, among other things, by sectoral outputs, sectoral labor demands and desired purchases of intermediate inputs are also generically unique.

□

Lemma 3.1 (Baseline GDP)**.** *Suppose Assumptions 3.1-3.3 hold. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$*,* $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ *such that either* $\overline{\mathcal{A}} \geq \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ *or* $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geq \underline{\mathcal{M}}_0$*. Then,* $C(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \geq C(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$*.*

Proof. (a) $\overline{\mathcal{A}} > \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0 = \mathcal{M}_0$*;*

Let $\{\overline{P}_k^0\}_{k=1}^K$ be equilibrium ratios of sectoral prices to money supply under the baseline pair $(\underline{\mathcal{A}}, \mathcal{M}_0)$. Naturally, they satisfy the following fixed point condition:

$$\overline{P}_k^0 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \underline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}); \mathcal{M}_0], \quad \forall k \tag{3.30}$$

where functions f_k were introduced in (24).

Suppose the productivity mapping changes to $\overline{\mathcal{A}} > \underline{\mathcal{A}}$, while baseline money supply remains unchanged. Define $\{\overline{P}_k^1\}_{k=1}^K$ such that they satisfy:

$$\overline{P}_k^1 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r \in S_k}); \mathcal{M}_0], \quad \forall k \tag{3.31}$$

Since $\overline{\mathcal{Q}}_k$ is decreasing in $\mathcal{A}_k$ and $f_k' > 0$, it follows that:

$$\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}) < \min_{S_k} \overline{\mathcal{Q}}_k(S_k, \underline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}), \quad \forall k \tag{3.32}$$

$$\Rightarrow f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \mathcal{M}_0] < f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \underline{\mathcal{A}}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \mathcal{M}_0], \quad \forall k \tag{3.33}$$

$$\Rightarrow \overline{P}_k^1 < \overline{P}_k^0, \quad \forall k. \tag{3.34}$$

As before, define $\kappa_k(\{\overline{P}_k\}_{k=1}^K) = f_k[\min_{S_k} \overline{\mathcal{Q}_k}(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_k\}_{k=1}^K; \mathcal{M}_0], \forall k$. The new equilibrium ratios of sectoral prices to money supply are the minimal fixed point of $\kappa(.)$. For $\tau \geq 1$, define $\overline{P}_k^\tau = \kappa_k(\{\overline{P}^{\tau-1}\}_{k=1}^K)$. Since $\kappa(.)$ is increasing, it follows that:

$$\overline{P}_k^1 < \overline{P}_k^0, \forall k \Rightarrow \kappa(\{\overline{P}^1\}_{k=1}^K) < \kappa(\{\overline{P}^0\}_{k=1}^K) \tag{3.35}$$

$$\Rightarrow \overline{P}_k^2 < \overline{P}_k^1, \forall k \Rightarrow \kappa(\{\overline{P}^2\}_{k=1}^K) < \kappa(\{\overline{P}^1\}_{k=1}^K) \tag{3.36}$$

$$\Rightarrow \overline{P}_k^3 < \overline{P}_k^2, \forall k \Rightarrow ... \Rightarrow \overline{P}_k^t < \overline{P}_k^{t-1}, \forall k \tag{3.37}$$

$$\Rightarrow \lim_{t\to\infty} \overline{P}_k^t < \overline{P}_k^0, \forall k. \tag{3.38}$$

Since $\kappa(.)$ is continuous, $\lim_{t\to\infty} \overline{P}_k^t$ is a fixed point of $\kappa(.)$. Further, since the new equilibrium is a minimal fixed point, it must be that $\overline{P}_k(\overline{\mathcal{A}}, \mathcal{M}_0) \leq \lim_{t\to\infty} \overline{P}_k^t < \overline{P}_k^0 = \overline{P}_k(\underline{\mathcal{A}}, \mathcal{M}_0), \forall k$.

From the definition of the aggregate consumption price index it follows that $\overline{P}^c(\overline{\mathcal{A}}, \mathcal{M}_0) = \frac{P^c(\overline{\mathcal{A}}, \mathcal{M}_0)}{\mathcal{M}_0} = P^c(\frac{P_1(\overline{\mathcal{A}}, \mathcal{M}_0)}{\mathcal{M}_0}, ..., \frac{P_K(\overline{\mathcal{A}}, \mathcal{M}_0)}{\mathcal{M}_0}) = P^c(\overline{P}_1(\overline{\mathcal{A}}, \mathcal{M}_0), ..., \overline{P}_K(\overline{\mathcal{A}}, \mathcal{M}_0)) < P^c(\overline{P}_1(\underline{\mathcal{A}}, \mathcal{M}_0), ..., \overline{P}_K(\underline{\mathcal{A}}, \mathcal{M}_0)) = \overline{P}^c(\underline{\mathcal{A}}, \mathcal{M}_0)$. From the cash-in-advance constraint, $C(\mathcal{A}, \mathcal{M}_0) = 1/P^c(\mathcal{A}, \mathcal{M}_0)$, and hence $C(\overline{\mathcal{A}}, \mathcal{M}_0) > C(\underline{\mathcal{A}}, \mathcal{M}_0)$.

(b) $\overline{\mathcal{A}} = \underline{\mathcal{A}} = \mathcal{A}, \overline{\mathcal{M}}_0 > \underline{\mathcal{M}}_0$;

Similarly, let $\{\overline{P}_k^0\}_{k=1}^K$ be equilibrium ratios of sectoral prices to money

supply under the baseline pair $(\mathcal{A}, \underline{\mathcal{M}}_0)$. Suppose $\underline{\mathcal{M}}_0$ rises to $\overline{\mathcal{M}_0} > \underline{\mathcal{M}}_0$, and define $\{\overline{P}_k^1\}_{k=1}^K$ such that they satisfy:

$$\overline{P}_k^1 = f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \overline{\mathcal{M}_0}], \quad \forall k \tag{3.39}$$

Since $f_k[\quad . \quad ; \overline{\mathcal{M}_0}] < f_k[\quad . \quad ; \underline{\mathcal{M}}_0], \forall k$, it follows that:

$$f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \overline{\mathcal{M}_0}] < f_k[\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r^0\}_{r\in S_k}); \underline{\mathcal{M}}_0], \quad \forall k \tag{3.40}$$

$$\Rightarrow \overline{P}_k^1 < \overline{P}_k^0, \quad \forall k. \tag{3.41}$$

The rest of the proof follows from (a), where it is similarly established that $\overline{P}_k(\mathcal{A}, \overline{\mathcal{M}}_0) < \overline{P}_k(\mathcal{A}, \underline{\mathcal{M}}_0), \forall k$, and hence $\overline{P}^c(\mathcal{A}, \overline{\mathcal{M}}_0) < \overline{P}^c(\mathcal{A}, \underline{\mathcal{M}}_0)$ and $C(\mathcal{A}, \overline{\mathcal{M}}_0) > C(\mathcal{A}, \underline{\mathcal{M}}_0)$. □

Lemma 3.2 (Baseline supplier choices)**.** *Suppose Assumptions 3.1-3.5 hold. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ *such that either* $\overline{\mathcal{A}} \geq \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ *or* $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geq \underline{\mathcal{M}}_0$*. Then,* $S_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \supseteq S_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$*, for all* $k = 1, 2, ..., K$.

Proof. (a) $\overline{\mathcal{A}} > \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0 = \mathcal{M}_0$; Let $S_k^0 = S_k(\underline{\mathcal{A}}, \mathcal{M}_0), \forall k,$ be the original set of supplier choices in equilibrium, which satisfy:

$$S_k^0 \in \arg\min_{S_k} \mathcal{Q}_k(S_k, \underline{\mathcal{A}}_k(S_k), \mathcal{M}_0, \{P_r(\underline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k}) = \arg\min_{S_k} \mathcal{M}_0 \overline{\mathcal{Q}}_k(S_k, \underline{\mathcal{A}}_k(S_k), \{\overline{P}_r(\underline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k}) \tag{3.42}$$

$$= \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \underline{\mathcal{A}}_k(S_k), \{\overline{P}_r(\underline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k}), \quad \forall k. \tag{3.43}$$

Suppose $\underline{\mathcal{A}}$ improves to $\overline{\mathcal{A}} > \underline{\mathcal{A}}$, and define S^1 such that $S_k^1 \in \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r(\underline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k}), \forall k$. From Theorem 4 of Milgrom and Shannon (1994) it follows that $S_k^1 \supseteq S_k^0, \forall k$.

From the proof of Lemma 3.1 it is known that $\overline{P}_k(\overline{\mathcal{A}}, \mathcal{M}_0) <$

$\overline{P}_k(\underline{\mathcal{A}}, \mathcal{M}_0), \forall k$, and hence

$$\arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r(\overline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k}) \supseteq \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \overline{\mathcal{A}}_k(S_k), \{\overline{P}_r(\underline{\mathcal{A}}, \mathcal{M}_0)\}_{r\in S_k}), \quad \forall k \tag{3.44}$$

$$\Rightarrow S_k(\overline{\mathcal{A}}, \mathcal{M}_0) \supseteq S_k^1 \supseteq S_k(\underline{\mathcal{A}}, \mathcal{M}_0), \quad \forall k. \tag{3.45}$$

(b) $\overline{\mathcal{A}} = \underline{\mathcal{A}} = \mathcal{A}, \overline{\mathcal{M}}_0 > \underline{\mathcal{M}}_0$;

Let $S^0 = S(\mathcal{A}, \underline{\mathcal{M}}_0)$ be the original equilibrium network, which satisfies

$$S_k^0 \in \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r(\mathcal{A}, \underline{\mathcal{M}}_0)_{r\in S_k}\}), \quad \forall k. \tag{3.46}$$

Suppose $\underline{\mathcal{M}}_0$ changes to $\overline{\mathcal{M}}_0 > \underline{\mathcal{M}}_0$. From the proof of Lemma 3.1 it is known that $\overline{P}_k(\mathcal{A}, \overline{\mathcal{M}}_0) < \overline{P}_k(\mathcal{A}, \underline{\mathcal{M}}_0), \forall k$, and hence by Theorem 4 of Milgrom and Shannon (1994):

$$\arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r(\mathcal{A}, \overline{\mathcal{M}}_0)\}_{r\in S_k}) \supseteq \arg\min_{S_k} \overline{\mathcal{Q}}_k(S_k, \mathcal{A}_k(S_k), \{\overline{P}_r(\mathcal{A}, \underline{\mathcal{M}}_0)\}_{r\in S_k}), \quad \forall k \tag{3.47}$$

$$\Rightarrow S_k(\mathcal{A}, \overline{\mathcal{M}}_0) \supseteq S_k^1 \supseteq S_k(\mathcal{A}, \underline{\mathcal{M}}_0), \quad \forall k. \tag{3.48}$$

□

Lemma 3.3 (Comparative statics following a monetary shock)**.** *Consider a baseline pair $(\mathcal{A}, \mathcal{M}_0)$, which is perturbed by a monetary shock $\varepsilon^m > 0$. Suppose Assumptions 1-3 hold, then following the monetary shock equilibrium GDP rises (falls) relative to its baseline level: $C(\mathcal{A}, \mathcal{M}) > C(\mathcal{A}, \mathcal{M}_0)$. Further, suppose that in addition Assumption 3.4-3.5 also hold, then following the monetary shock the set of suppliers for each sector weakly expands: $S_k(\mathcal{A}, \mathcal{M}) \supseteq S_k(\mathcal{A}, \mathcal{M}_0)$, for all $k = 1, 2, ..., K$.*

Proof. Proof follows directly from Lemmas 3.1 and 3.2, where variation in money supply is driven by the monetary shock, as opposed to changes in baseline money supply, which is instead kept fixed. □

Lemma 3.4. *Suppose Assumptions 3.1-3.3 and 3.6-3.7 hold. For any baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, let* $p_k(\mathcal{A}, \mathcal{M}_0)$ *be a first order approximation of* $\ln P_k(\mathcal{A}, \mathcal{M})$ *around* $\ln P_k(\mathcal{A}, \mathcal{M}_0)$*, given* $S_k = S_k(\mathcal{A}, \mathcal{M}_0), \forall k$*. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$*,* $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ *and a monetary shock* ε^m *which is small with respect to both baselines, then:*

$$\mathbb{p}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathbb{p}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) = -\left[\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathcal{L}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)\right] \mathcal{E}^m \tag{3.49}$$

where $\mathbb{p} = [p_1, p_2, ..., p_K]'$*,* $\mathcal{E}^m = [\varepsilon^m, \varepsilon^m, ..., \varepsilon^m]'$ *and* $\mathcal{L}$ *is a Leontief inverse given by:*

$$\mathcal{L}(\mathcal{A}, \mathcal{M}_0) = \left[I - (I - A)\Gamma(\mathcal{M}_0)\Omega(\mathcal{A}, \mathcal{M}_0)\right]^{-1} \left[I - (I - A)\Gamma(\mathcal{M}_0)\right] \tag{3.50}$$

where $A = diag(\alpha_1, ..., \alpha_K)$*,* $\Gamma(\mathcal{M}_0) = diag(\gamma_1(\mathcal{M}_0), ..., \gamma_K(\mathcal{M}_0))$*,* $\gamma_k = \frac{1}{\alpha_k (g(\mathcal{M}_0))^{1-\theta} + 1 - \alpha_k}$ *and*
$\Omega(\mathcal{A}, \mathcal{M}_0)$ *is a matrix with entries given by* $[\Omega(\mathcal{A}, \mathcal{M}_0)]_{kr} = \frac{\partial \ln \mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), W, \{P_{k'}\}_{k' \in S_k}\right]}{\partial \ln P_r}|_{\mathcal{M}=\mathcal{M}_0}, \forall k, r.$

Proof. First, notice that under the assumptions used in this lemma, if a shock $\varepsilon^m > 0$ is small with respect to a baseline pair $(\mathcal{A}, \mathcal{M}_0)$, then any other monetary shock $\overline{\varepsilon}^m \in (0, \varepsilon^m)$ is also small with respect to the baseline pair $(\mathcal{A}, \mathcal{M}_0)$. The proof of that follows by contradiction. Suppose there exists $\overline{\varepsilon}^m \in (0, \varepsilon^m)$ such that $S_k(\mathcal{A}, \mathcal{M}_0 \exp(\overline{\varepsilon}^m)) \supset S_k(\mathcal{A}, \mathcal{M}_0)$ for at least one k. Then for those k it follows $S_k(\mathcal{A}, \mathcal{M}_0 \exp(\overline{\varepsilon}^m)) \supset S_k(\mathcal{A}, \mathcal{M}_0 \exp(\varepsilon^m)) = S_k(\mathcal{A}, \mathcal{M}_0)$ where $\varepsilon^m > \overline{\varepsilon}^m$, which implies that for those sectors the set of suppliers shrinks as money supply increases, which contradicts Theorem 4 of Milgrom and Shannon (1994). Similarly, suppose there exists $\overline{\varepsilon}^m \in (0, \varepsilon^m)$ such that $S_k(\mathcal{A}, \mathcal{M}_0 \exp(\overline{\varepsilon}^m)) \subset S_k(\mathcal{A}, \mathcal{M}_0)$ for at least one k. Then for those k it follows $S_k(\mathcal{A}, \mathcal{M}_0 \exp(\overline{\varepsilon}^m)) \subset S_k(\mathcal{A}, \mathcal{M}_0)$ where $\varepsilon^m > 0$, which once again implies that for those sectors the set of suppliers shrinks as money supply increases, which contradicts Theorem 4 of Milgrom and Shannon (1994).

Hence, starting from the baseline $(\mathcal{A}, \mathcal{M}_0)$, the solution to my model for any monetary shock $\overline{\varepsilon}^m \in (0, \varepsilon^m]$ coincides with the solution to a restricted version of my model where the set of suppliers is kept fixed at $S(\mathcal{A}, \mathcal{M}_0)$. In the rest of this proof I consider such restricted version of my model for an unrestricted value of monetary shock, and ultimately evaluate solution to that model under $\overline{\varepsilon}^m \in (0, \varepsilon^m]$. Write sectoral price aggregation in such restricted model:

$$P_k^{1-\theta} = \alpha_k P_{k,0}^{1-\theta} + (1-\alpha_k)\left[(1+\mu_k)\mathcal{Q}_k(S_k, \mathcal{A}_k(S_k), \mathcal{M}, \{P_r\}_{r\in S_k}\right]^{1-\theta} \quad (3.51)$$

For a given baseline $(\mathcal{A}, \mathcal{M}_0)$ and model parameters, equilibrium prices are pinned down by the value of the monetary shock ε^m. The model with a fixed network features no discontinuities and since the unit cost function is differentiable by assumption, one can write a first order Taylor approximation around $\varepsilon^m = 0$:

$$\ln P_k(\varepsilon^m) = \ln P_k(0) + \frac{(1-\alpha_k)\left[(1+\mu_k)\mathcal{Q}_k(0)\right]^{1-\theta}}{\alpha_k P_{k,0}^{1-\theta} + (1-\alpha_k)\left[(1+\mu_k)\mathcal{Q}_k(0)\right]^{1-\theta}}[\ln \mathcal{Q}_k(\varepsilon^m) - \ln \mathcal{Q}_k(0)] \quad (3.52)$$

By assumption, $P_{k,0} = (1+\mu_k)g(\mathcal{M}_0)\mathcal{Q}_k(0)$:

$$\ln P_k(\varepsilon^m) = \ln P_k(0) + \frac{(1-\alpha_k)}{\alpha_k g(\mathcal{M}_0)^{1-\theta} + (1-\alpha_k)}[\ln \mathcal{Q}_k(\varepsilon^m) - \ln \mathcal{Q}_k(0)]. \quad (3.53)$$

$$= (1-\alpha_k)\gamma_k(\mathcal{M}_0)\left[\varepsilon^m + \ln \overline{\mathcal{Q}}(\varepsilon^m) - \ln \overline{\mathcal{Q}}(0)\right] \quad (3.54)$$

where $\gamma_k(\mathcal{M}_0) \equiv \frac{1}{\alpha_k g(\mathcal{M}_0)^{1-\theta} + 1 - \alpha_k}, \forall k$. Further,

$$\ln \overline{P}_k(\varepsilon^m) - \ln \overline{P}_k(0) + \varepsilon^m = (1-\alpha_k)\gamma_k(\mathcal{M}_0)\varepsilon^m + (1-\alpha_k)\gamma(\mathcal{M}_0)\sum_{r\in S_k}\frac{\partial \ln \mathcal{Q}_k(0)}{\partial \ln P_r}\left[\ln \overline{P}_r(\varepsilon^m) - \ln \overline{P}_r(0)\right]. \quad (3.55)$$

Letting $\overline{p}_k \equiv [\ln \overline{P}_k(\varepsilon^m) - \ln \overline{P}_k(0)]$ and $\overline{\mathbb{p}} = [\overline{p}_1, \overline{p}_2, ..., \overline{p}_K]'$, one can re-write

the above conveniently in matrix form:

$$\overline{\mathbb{p}} = [(I - A)\Gamma(\mathcal{M}_0) - I]\mathcal{E}^m + (I - A)\Gamma(\mathcal{M}_0)\Omega(\mathcal{A}, \mathcal{M}_0)\overline{\mathbb{p}} \tag{3.56}$$

$$\overline{\mathbb{p}} = -[I - (I - A)\Gamma(\mathcal{M}_0)\Omega(\mathcal{A}, \mathcal{M}_0)]^{-1}[I - (I - A)\Gamma(\mathcal{M}_0)]\mathcal{E}^m \tag{3.57}$$

where $A = diag(\alpha_1, ..., \alpha_K)$, $\Gamma(\mathcal{M}_0) = diag(\gamma_1(\mathcal{M}_0), ..., \gamma_K(\mathcal{M}_0))$, $\mathcal{E}^m = [\varepsilon^m, \varepsilon^m, ..., \varepsilon^m]'$ and $\Omega(\mathcal{A}, \mathcal{M}_0)$ is a matrix with entries given by $[\Omega(\mathcal{A}, \mathcal{M}_0)]_{kr} = \frac{\partial \ln \mathcal{Q}_k\left[S_k, \mathcal{A}_k(S_k), W, \{P_{k'}\}_{k' \in S_k}\right]}{\partial \ln P_r}|_{\mathcal{M}=\mathcal{M}_0}, \forall k, r$

Return to the original version of my model, where network formation is endogenous. For a monetary shock $\varepsilon^m > 0$ that is small with respect to two baselines $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$, the expression in (57) holds local around both baselines. Further, for those two baselines $\overline{p}_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \overline{p}_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) = p_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) + \varepsilon^m - p_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}) - \varepsilon^m = p_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - p_k(\underline{\mathcal{A}}, \underline{\mathcal{M}})$, and so $\overline{\mathbb{p}}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \overline{\mathbb{p}}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) = \mathbb{p}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathbb{p}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$. Combining this with (57) delivers the final result:

$$\mathbb{p}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathbb{p}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) = -\left[\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathcal{L}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)\right] \mathcal{E}^m \tag{3.58}$$

where $\mathcal{L}(\mathcal{A}, \mathcal{M}_0) = [I - (I - A)\Gamma(\mathcal{M}_0)\Omega(\mathcal{A}, \mathcal{M}_0)]^{-1} [I - (I - A)\Gamma(\mathcal{M}_0)]$. □

Proposition 3.2. *Suppose Assumptions 3.2-3.5, 3.7-3.8 hold. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ *such that either* $\overline{\mathcal{A}} \geq \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ *or* $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geq \underline{\mathcal{M}}_0$. *For a monetary shock* $\varepsilon^m > 0$ *that is small with respect to both baselines, it follows that* $p_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \leq p_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$ *for all* $k = 1, 2, ..., K$.

Proof. Let $D \equiv \Omega(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \Omega(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, then one can rewrite the difference between the Leontief Inverses as follows:

$$\begin{aligned}\mathcal{L}(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)-\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0) =&(I-A)\Gamma(\underline{\mathcal{M}}_0)\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)\mathcal{L}(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)\\ &-(I-A)\Gamma(\overline{\mathcal{M}}_0)\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)\\ &-(I-A)\Gamma(\overline{\mathcal{M}}_0)D\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)\\ &+(I-A)(\Gamma(\overline{\mathcal{M}}_0)-\Gamma(\underline{\mathcal{M}}_0)).\end{aligned} \tag{3.59}$$

Let $\Xi \equiv diag\left(\frac{\gamma_1(\overline{\mathcal{M}}_0)}{\gamma_1(\underline{\mathcal{M}}_0)},...,\frac{\gamma_K(\overline{\mathcal{M}}_0)}{\gamma_K(\underline{\mathcal{M}}_0)}\right)$, then the above can be re-written as:

$$\begin{aligned}&\mathcal{L}(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)-\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0) = -[I-(I-A)\Gamma(\underline{\mathcal{M}}_0)\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)]^{-1}\times\\ &\times[(I-A)\Gamma(\overline{\mathcal{M}}_0)D\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)+(I-A)(I-\Xi)\Gamma(\underline{\mathcal{M}}_0)(I-\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0))].\end{aligned} \tag{3.60}$$

From the definition of $\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)$:

$$\begin{aligned}\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0) &= [I-(I-A)\Gamma(\overline{\mathcal{M}})\Omega(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)]^{-1}[I-(I-A)\Gamma(\overline{\mathcal{M}}_0)] =\\ &I+[I-(I-A)\Gamma(\overline{\mathcal{M}})\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)]^{-1}(I-A)\Gamma(\overline{\mathcal{M}}_0)(\Omega(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)-I)\\ \Rightarrow I-\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)\mathcal{L}(\overline{\mathcal{A}},\overline{\mathcal{M}}_0) &= [I-\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)]+\\ &\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)[I-(I-A)\Gamma(\overline{\mathcal{M}}_0)\Omega(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)]^{-1}(I-A)\Gamma(\overline{\mathcal{M}}_0)(I-\Omega(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)\end{aligned} \tag{3.61}$$

Suppose we have any two baselines pairs $(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)$ such that either $\overline{\mathcal{A}} \geq \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ or $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geq \underline{\mathcal{M}}_0$. Under Cobb-Douglas production function entries of the Ω matrix in the Leontief Inverse are given by shares in the production function, so that $[\Omega]_{kr} = \omega_{kr}$ if sector k buys inputs from sector r and $[\Omega]_{kr} = 0$ otherwise. As a result, $D = \Omega(\overline{\mathcal{A}},\overline{\mathcal{M}}_0)-\Omega(\underline{\mathcal{A}},\underline{\mathcal{M}}_0)$ is a $K \times K$ matrix with non-negative entries, since from Lemma 3.2 the first baseline will have (weakly) more suppliers for every sector. Further, from the properties of γ_k, it follows that all entries of the Ξ matrix are positive and less than or equal to one. Finally, since labor is an essential input, it follows

that sum of rows of $(I - \Omega(\mathcal{A}, \mathcal{M}))$ for any baseline pair are positive and less than or equal to one.

Combining the above observations with the expressions in (60) and (61), it follows that for any two baselines pairs $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ such that either $\overline{\mathcal{A}} \geq \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ or $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geq \underline{\mathcal{M}}_0$, and a monetary shock $\varepsilon^m > 0$ that is small with respect to both baselines, it holds that:

$$\mathbb{p}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathbb{p}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) = -\left[\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathcal{L}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)\right] \mathcal{E}^m \leq 0_{K\times 1}. \tag{3.62}$$

or $p_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \leq p_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0), \forall k$ in scalar form. □

Theorem 3.1 (Cycle dependence)**.** *Suppose Assumptions 3.3-3.5, 3.7-3.9 hold. For any baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, let* $c_k(\mathcal{A}, \mathcal{M}_0) \equiv \ln C_k(\mathcal{A}, \mathcal{M}) - \ln C_k(\mathcal{A}, \mathcal{M}_0), \forall k$*. Consider any two baseline pairs* $(\underline{\mathcal{A}}, \mathcal{M}_0)$*,* $(\overline{\mathcal{A}}, \mathcal{M}_0)$ *such that* $\overline{\mathcal{A}} \geq \underline{\mathcal{A}}$*. For a monetary shock* $\varepsilon^m > 0$ *which is small with respect to both baselines it follows that* $c_k(\overline{\mathcal{A}}, \mathcal{M}_0) \geq c_k(\underline{\mathcal{A}}, \mathcal{M}_0)$ *for all* $k = 1, 2, ..., K$*, and* $c(\overline{\mathcal{A}}, \mathcal{M}_0) \geq c(\underline{\mathcal{A}}, \mathcal{M}_0)$*.*

Proof. Under Cobb-Douglas aggregation of sectoral consumption it follows that sectoral consumption demand takes the following form:

$$C_k = \omega_{ck} \left[\frac{P_k}{P^c}\right]^{-1} C = \omega_{ck} \frac{P^c C}{P_k} = \omega_{ck} \frac{\mathcal{M}}{P_k}. \tag{3.63}$$

From the above it follows that $\ln C_k = \ln \omega_{ck} + \ln \mathcal{M} - \ln P_k$ and evaluating the latter under baseline and following the shock yields:

$$\begin{aligned} \ln C_k(\mathcal{A}, \mathcal{M}) - \ln C_k(\mathcal{A}, \mathcal{M}_0) &= \ln \mathcal{M} - \ln P_k(\mathcal{A}, \mathcal{M}) - \ln \mathcal{M}_0 + \ln P_k(\mathcal{A}, \mathcal{M}_0) \\ c_k(\mathcal{A}, \mathcal{M}_0) &= \varepsilon^m - p_k(\mathcal{A}, \mathcal{M}_0). \end{aligned} \tag{3.64}$$

For the two baselines $(\underline{\mathcal{A}}, \mathcal{M}_0)$, $(\overline{\mathcal{A}}, \mathcal{M}_0)$ such that $\overline{\mathcal{A}} \geq \underline{\mathcal{A}}$, and a monetary shock $\varepsilon^m > 0$ that is small with respect to both baselines it follows:

$$c_k(\overline{\mathcal{A}}, \mathcal{M}_0) - c_k(\underline{\mathcal{A}}, \mathcal{M}_0) = -\left[p_k(\overline{\mathcal{A}}, \mathcal{M}_0) - p_k(\underline{\mathcal{A}}, \mathcal{M}_0)\right] \geq 0, \quad \forall k \tag{3.65}$$

where the last part follows from Proposition 3.2.

Consider deviation of aggregate consumption from baseline:

$$c_k(\overline{\mathcal{A}}, \mathcal{M}_0) - c_k(\underline{\mathcal{A}}, \mathcal{M}_0) = \sum_{k=1}^{K} \omega_{ck} \left[c_k(\overline{\mathcal{A}}, \mathcal{M}_0) - c_k(\underline{\mathcal{A}}, \mathcal{M}_0) \right] \geq 0. \tag{3.66}$$

□

Theorem 3.2 (Path dependence)**.** *Suppose Assumptions 3.3-3.5, 3.7-3.9 hold. For any baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, let* $c_k(\mathcal{A}, \mathcal{M}_0) \equiv \ln C_k(\mathcal{A}, \mathcal{M}) - \ln C_k(\mathcal{A}, \mathcal{M}_0), \forall k$*. Consider any two baseline pairs* $(\mathcal{A}, \underline{\mathcal{M}}_0)$*,* $(\mathcal{A}, \overline{\mathcal{M}}_0)$ *such that* $\overline{\mathcal{M}}_0 \geq \underline{\mathcal{M}}_0$*. For a monetary shock* $\varepsilon^m > 0$ *which is small with respect to both baselines it follows that* $c_k(\mathcal{A}, \overline{\mathcal{M}}_0) \geq c_k(\mathcal{A}, \underline{\mathcal{M}}_0)$ *for all* $k = 1, 2, ..., K$*, and* $c(\mathcal{A}, \overline{\mathcal{M}}_0) \geq c(\mathcal{A}, \underline{\mathcal{M}}_0)$*.*

Proof. Recall from the proof of Theorem 1 it is true that $c_k(\mathcal{A}, \mathcal{M}_0) = \varepsilon^m - p_k(\mathcal{A}, \mathcal{M}_0), \forall k$. For the two baselines $(\mathcal{A}, \underline{\mathcal{M}}_0)$, $(\mathcal{A}, \overline{\mathcal{M}}_0)$ such that $\overline{\mathcal{M}}_0 \geq \underline{\mathcal{M}}_0$, and a monetary shock $\varepsilon^m > 0$ that is small with respect to both baselines it follows:

$$c_k(\mathcal{A}, \overline{\mathcal{M}}_0) - c_k(\mathcal{A}, \underline{\mathcal{M}}_0) = - \left[p_k(\mathcal{A}, \overline{\mathcal{M}}_0) - p_k(\mathcal{A}, \underline{\mathcal{M}}_0) \right] \geq 0 \tag{3.67}$$

where the last part follows from Proposition 3.2.

Consider deviation of aggregate consumption from baseline:

$$c_k(\mathcal{A}, \overline{\mathcal{M}}_0) - c_k(\mathcal{A}, \underline{\mathcal{M}}_0) = \sum_{k=1}^{K} \omega_{ck} \left[c_k(\mathcal{A}, \overline{\mathcal{M}}_0) - c_k(\mathcal{A}, \underline{\mathcal{M}}_0) \right] \geq 0. \tag{3.68}$$

□

Proposition 3.3. *Suppose Assumption 3.1-3.5 hold. For a baseline pair* $(\mathcal{A}, \mathcal{M}_0)$*, let* $\varepsilon^m > 0$ *be a small monetary shock and* $E^m > 0$ *be a large monetary shock. Under the large monetary shock, the equilibrium set of supplier choices is* $S_k^L \supset$

$S_k(\mathcal{A}, \mathcal{M}_0) = S_k^0, \forall k$. *The ratios of exact impulse responses of sectoral prices satisfy:*

$$\frac{\tilde{P}_k^e(E^m; S^L)}{\tilde{P}_k^e(\varepsilon^m; S^L)} \leq \frac{\tilde{P}_k(E^m)}{\tilde{P}_k(\varepsilon^m)} \leq \frac{\tilde{P}^e(E^m; S^0)}{\tilde{P}_k^e(\varepsilon^m; S^0)}, \tag{3.69}$$

for all $k = 1, 2, ..., K$.

Proof. Starting from a baseline pair $(\mathcal{A}, \mathcal{M}_0)$, following the large monetary shock the original network S^0 is still available, but is not chosen, hence:

$$\mathcal{Q}_k(S_k^L, \mathcal{A}_k(S_k^L), \mathcal{M}^{\mathcal{L}}, \{P_k(\mathcal{M}^{\mathcal{L}}, S^L)\}_{r=1}^K) \leq \mathcal{Q}_k(S_k^0, \mathcal{A}_k(S_k^0), \mathcal{M}^{\mathcal{L}}, \{P_k(\mathcal{M}^{\mathcal{L}}, S^0)\}_{r=1}^K) \tag{3.70}$$

$$\Rightarrow P_k(S_k^L, \mathcal{M}^L) \leq P_k(S_k^0, \mathcal{M}^L) \tag{3.71}$$

$$\Rightarrow \frac{P_k(S_k^L, \mathcal{M}^L)}{P_k(S_k^0, \mathcal{M}_0)} \leq \frac{P_k(S_k^0, \mathcal{M}^L)}{P_k(S_k^0, \mathcal{M}_0)} \tag{3.72}$$

$$\Rightarrow \tilde{P}_k(E^m) \leq \tilde{P}_k^e(E^m; S^0) \tag{3.73}$$

$$\Rightarrow \frac{\tilde{P}_k(E^m)}{\tilde{P}_k(\varepsilon^m)} \leq \frac{\tilde{P}_k^e(E^m; S^0)}{\tilde{P}_k^e(\varepsilon^m; S^0)} \tag{3.74}$$

Starting from a baseline pair $(\mathcal{A}, \mathcal{M}_0)$, following the small monetary shock the network S^L is available, but is not chosen, hence:

$$\mathcal{Q}_k(S_k^0, \mathcal{A}_k(S_k^0), \mathcal{M}^{\int}, \{P_k(\mathcal{M}^{\int}, S^0)\}_{r=1}^K) \leq \mathcal{Q}_k(S_k^L, \mathcal{A}_k(S_k^L), \mathcal{M}^{\int}, \{P_k(\mathcal{M}^{\int}, S^L)\}_{r=1}^K) \tag{3.75}$$

$$\Rightarrow P_k(S_k^0, \mathcal{M}^s) \leq P_k(S_k^L, \mathcal{M}^s) \tag{3.76}$$

$$\Rightarrow \frac{1}{P_k(S_k^0, \mathcal{M}^s)} \geq \frac{1}{P_k(S_k^L, \mathcal{M}^s)} \tag{3.77}$$

$$\Rightarrow \frac{P_k(S_k^L, \mathcal{M}^L)}{P_k(S_k^0, \mathcal{M}^s)} \geq \frac{P_k(S_k^L, \mathcal{M}^L)}{P_k(S_k^L, \mathcal{M}^s)} \tag{3.78}$$

$$\Rightarrow \frac{P_k(S_k^L, \mathcal{M}^L)/P_k(S_k^0, \mathcal{M}^0)}{P_k(S_k^0, \mathcal{M}^s)/P_k(S_k^0, \mathcal{M}^0)} \geq \frac{P_k(S_k^L, \mathcal{M}^L)//P_k(S_k^L, \mathcal{M}^0)}{P_k(S_k^L, \mathcal{M}^s)/P_k(S_k^L, \mathcal{M}^0)} \tag{3.79}$$

$$\Rightarrow \frac{\tilde{P}_k(E^m)}{\tilde{P}_k(\varepsilon^m)} \geq \frac{\tilde{P}_k^e(E^m; S^L)}{\tilde{P}_k^e(\varepsilon^m; S^L)} \tag{3.80}$$

Combining (3.74) and (3.80):

$$\frac{\tilde{P}_k^e(E^m;S^L)}{\tilde{P}_k^e(\varepsilon^m;S^L)} \leq \frac{\tilde{P}_k(E^m)}{\tilde{P}_k(\varepsilon^m)} \leq \frac{\tilde{P}^e(E^m;S^0)}{\tilde{P}_k^e(\varepsilon^m;S^0)}, \tag{3.81}$$

for all $k = 1, 2, ..., K$. □

Theorem 3.3 (Size dependence)**.** *Suppose Assumption 3.1-3.5 hold. For a baseline pair $(\mathcal{A}, \mathcal{M}_0)$, let $\varepsilon^m > 0$ be a small monetary shock and $E^m > 0$ be a large monetary shock. Under the large monetary shock, the equilibrium set of supplier choices is $S_k^L \supset S_k(\mathcal{A}, \mathcal{M}_0) = S_k^0, \forall k$. The ratios of exact impulse responses of GDP satisfy:*

$$\frac{\tilde{C}^e(E^m;S^0)}{\tilde{C}^e(\varepsilon^m;S^0)} \leq \frac{\tilde{C}(E^m)}{\tilde{C}(\varepsilon^m)} \leq \frac{\tilde{C}^e(E^m;S^L)}{\tilde{C}^e(\varepsilon^m;S^L)}. \tag{3.82}$$

Proof. Recall from (71) that $P_k(S_k^L, \mathcal{M}^L) \leq P_k(S_k^0, \mathcal{M}^L) \forall k$, which implies that $\overline{P}_k(S_k^L, \mathcal{M}^L) \leq \overline{P}_k(S_k^0, \mathcal{M}^L) \forall k$. Hence,

$$\overline{P}^c(S^L, \mathcal{M}^L) = P^c(\overline{P}_1(S_1^L, \mathcal{M}^L), ..., \overline{P}_K(S_K^L, \mathcal{M}^L)) \leq P^c(\overline{P}_1(S_1^0, \mathcal{M}^L), ..., \overline{P}_K(S_K^0, \mathcal{M}^L)) = \overline{P}^c(S^0, \mathcal{M} \tag{3.83}$$

From the cash-in-advance constraint. combined with (83):

$$\frac{C(S^L, \mathcal{M}^L)}{C(S^0, \mathcal{M}^L)} = \frac{\overline{P}^c(S^0, \mathcal{M}^L)}{\overline{P}^c(S^L, \mathcal{M}^L)} \geq 1 \tag{3.84}$$

$$\Rightarrow C(S^L, \mathcal{M}^L) \geq C(S^0, \mathcal{M}^L) \tag{3.85}$$

$$\Rightarrow \frac{C(S^L, \mathcal{M}^L)}{C(S^0, \mathcal{M}^s)} \geq \frac{C(S^0, \mathcal{M}^L)}{C(S^0, \mathcal{M}^s)} \tag{3.86}$$

$$\Rightarrow \frac{C(S^L, \mathcal{M}^L)/C(S^0, \mathcal{M}_0)}{C(S^0, \mathcal{M}^s)/C(S^0, \mathcal{M}_0)} \geq \frac{C(S^0, \mathcal{M}^L)/C(S^0, \mathcal{M}_0)}{C(S^0, \mathcal{M}^s)/C(S^0, \mathcal{M}_0)} \tag{3.87}$$

$$\Rightarrow \frac{\tilde{C}(E^m)}{\tilde{C}(\varepsilon^m)} \geq \frac{\tilde{C}^e(E^m;S^0)}{\tilde{C}^e(\varepsilon^m;S^0)}. \tag{3.88}$$

Recall from (3.76) that $P_k(S_k^0, \mathcal{M}^s) \leq P_k(S_k^L, \mathcal{M}^s) \forall k$, which implies that

$\overline{P}_k(S_k^0, \mathcal{M}^s) \leq \overline{P}_k(S_k^L, \mathcal{M}^s) \forall k$. Hence,

$$\overline{P}^c(S^0, \mathcal{M}^s) = P^c(\overline{P}_1(S_1^0, \mathcal{M}^s), ..., \overline{P}_K(S_K^0, \mathcal{M}^s)) \leq P^c(\overline{P}_1(S_1^L, \mathcal{M}^s), ..., \overline{P}_K(S_K^L, \mathcal{M}^s)) = \overline{P}^c(S^L, \mathcal{M}^s) \tag{3.89}$$

From the cash-in-advance constraint. combined with (3.89):

$$\frac{C(S^0, \mathcal{M}^s)}{C(S^L, \mathcal{M}^s)} = \frac{\overline{P}^c(S^l, \mathcal{M}^s)}{\overline{P}^c(S^0, \mathcal{M}^s)} \geq 1 \tag{3.90}$$

$$\Rightarrow \frac{1}{C(S^0, \mathcal{M}^s)} \leq \frac{1}{C(S^L, \mathcal{M}^s)} \tag{3.91}$$

$$\Rightarrow \frac{C(S^L, \mathcal{M}^L)}{C(S^0, \mathcal{M}^s)} \leq \frac{C(S^L, \mathcal{M}^L)}{C(S^L, \mathcal{M}^s)} \tag{3.92}$$

$$\Rightarrow \frac{C(S^L, \mathcal{M}^L)/C(S^0, \mathcal{M}_0)}{C(S^0, \mathcal{M}^s)/C(S^0, \mathcal{M}_0)} \leq \frac{C(S^L, \mathcal{M}^L)/C(S^L, \mathcal{M}_0)}{C(S^L, \mathcal{M}^s)/C(S^L, \mathcal{M}_0)} \tag{3.93}$$

$$\Rightarrow \frac{\tilde{C}(E^m)}{\tilde{C}(\varepsilon^m)} \leq \frac{\tilde{C}^e(E^m; S^L)}{\tilde{C}^e(\varepsilon^m; S^L)}. \tag{3.94}$$

Combine (3.88) and (3.94):

$$\frac{\tilde{C}^e(E^m; S^0)}{\tilde{C}^e(\varepsilon^m; S^0)} \leq \frac{\tilde{C}(E^m)}{\tilde{C}(\varepsilon^m)} \leq \frac{\tilde{C}^e(E^m; S^L)}{\tilde{C}^e(\varepsilon^m; S^L)}. \tag{3.95}$$

□

3.B Additional econometric exercises

3.B.1 Estimation in broad sectoral groups

In Figures 3.B.1-3.B.6 I re-estimate the effect of identified shocks on intermediates intensity, allowing for non-linearity as in specification (20), at the level of six broad sectoral groups, to determine if the observed network cyclicality could be driven by specific parts of the US economy. Results suggest that largest sensitivities of intermediates intensities to changes in productivity are detected in "Trade, transportation and warehousing" and "Durable manufacturing goods", whereas very little responsiveness is found in "Information, Finance-Insurance-Real Estate and other services" and "Education, healthcare and other services". As for the effect of monetary interventions, by far the largest response is found in "Non-manufacturing goods", whereas the smallest effect is found in "Education, healthcare and other services".

3.B.2 Alternative non-linear estimation using sectoral data

Here I consider an alternative approach to modeling non-linearity in my local projections setting. In particular, I re-estimate (3.20), where instead of adding interactions with a sign dummy and absolute value of the shock, I add quadratic and cubic shocks, which similarly capture possible sign and size effects:

$$\delta_{k,t+H} = \alpha_{k,H} + \beta_H^{lin} s_t + \beta_H^{sign} s_t^2 + \beta_H^{size} s_t^3 + \gamma_H x_{k,t-1} + \varepsilon_{k,t+H}, \tag{3.96}$$

for $H = 0, 1, ..., \overline{H}$, where it follows that $\beta_H^{sign} > 0$ indicates that positively-valued shocks make the impulse response more positive, whereas β_H^{size} indicates that larger shocks make the response more positive. Figure 3.B.7 reports estimation results using this alternative specification. Reassuringly, results remain virtually unchanged relative to the non-linear specification in (20), both in terms of magnitudes of effects, as well as in when it comes to sign and size effects and their statistical significance.

3.B.3 Alternative non-linear estimation using firm-level data

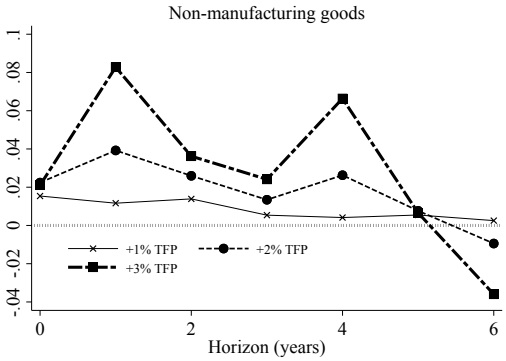
Here I once again consider the alternative approach to modeling non-linearity in my local projections setting, this time applied to the firm-level data on the number of suppliers.

$$indeg_{j,t+H} = \alpha_{j,H} + \beta_H^{lin} s_t + \beta_H^{sign} s_t^2 + \beta_H^{size} s_t^3 + \gamma_H x_{j,t-1} + \varepsilon_{j,t+H}, \qquad (3.97)$$

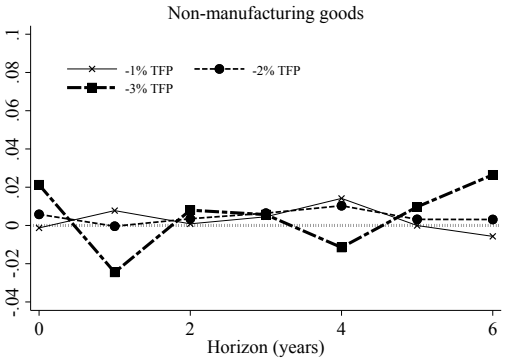
for $H = 0, 1, ..., \overline{H}$. Figure 3.B.8 reports the estimation results. One can see that results relative to the specification (22), results remain virtually unchanged, both in terms of magnitudes of effects, as well as in when it comes to sign effects and their statistical significance.

Figure 3.B.1: Non-linear IRFs of intermediates intensity to productivity and monetary shocks

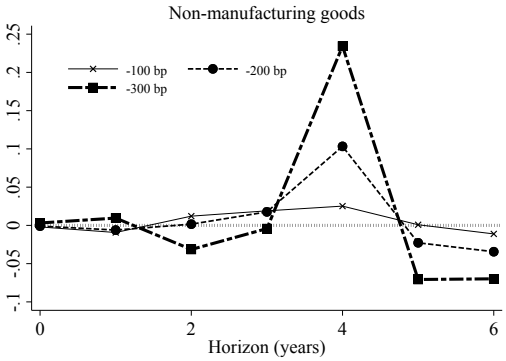
(a) Productivity expansions



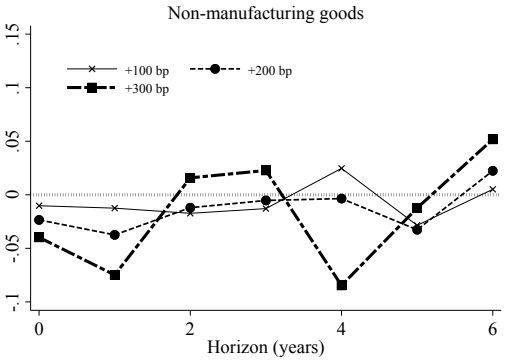
(b) Productivity contractions



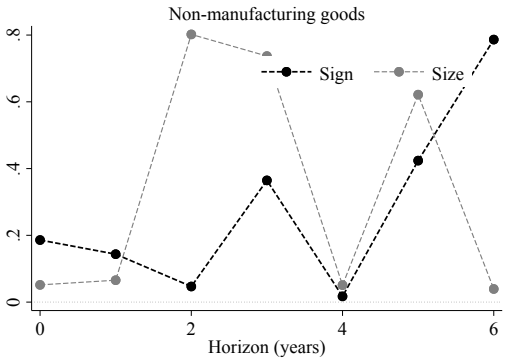
(c) Monetary expansions



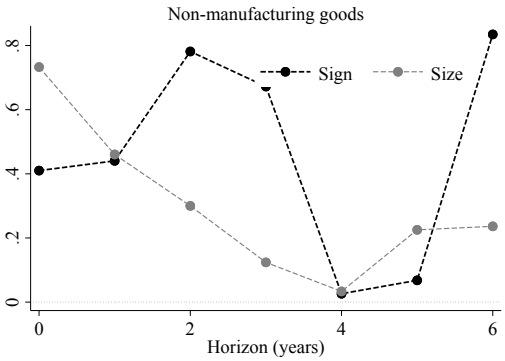
(d) Monetary contractions



(e) p-values for non-linearities (prod.)



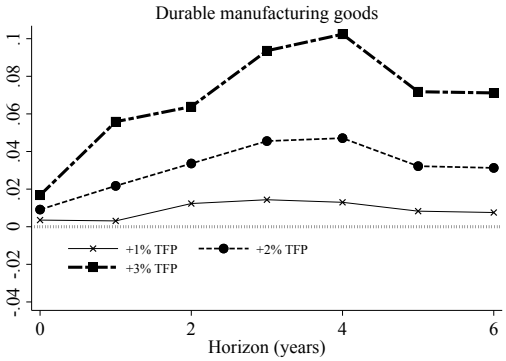
(f) p-values for non-linearities (mon.)



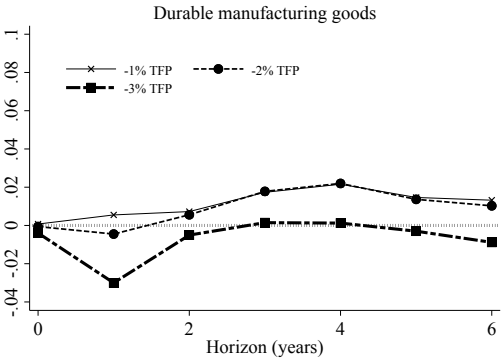
***Notes**: Panels (a)-(b) show estimated non-linear IRFs of intermediates intensity to a productivity shock based on Fernald (2014b), using specification in (3.20); the set of controls includes one lead of productivity shock and one lag of intermediates intensity, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panels (c)-(d) show estimated non-linear IRFs of intermediates intensity to a monetary shock based on Romer and Romer (2004), using specification in (3.20); the set of controls includes one lead of monetary shock and one lag of intermediates intensity, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Panels (e) and (f) shock p-values for the test that non-linear terms are zero.*

Figure 3.B.2: Non-linear IRFs of intermediates intensity to productivity and monetary shocks

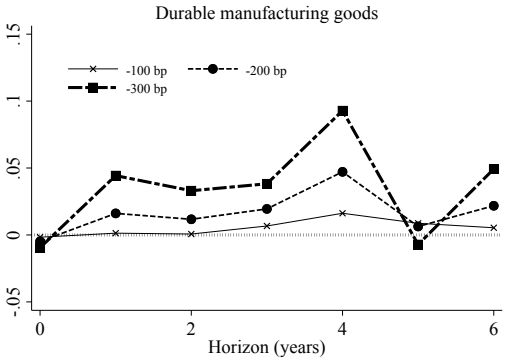
(a) Productivity expansions



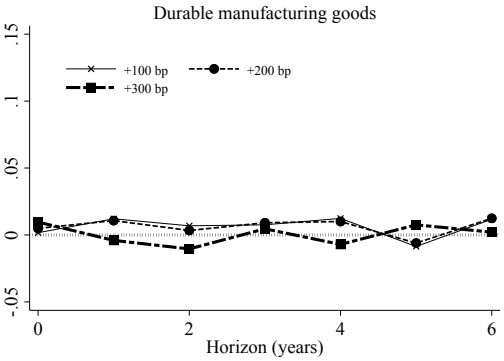
(b) Productivity contractions



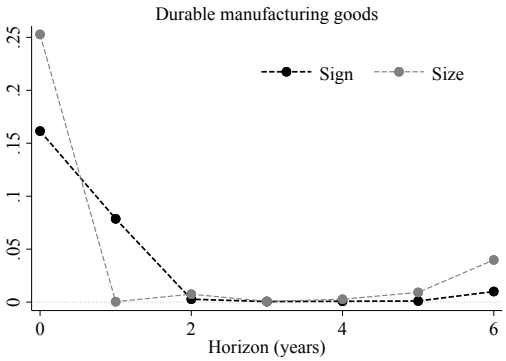
(c) Monetary expansions



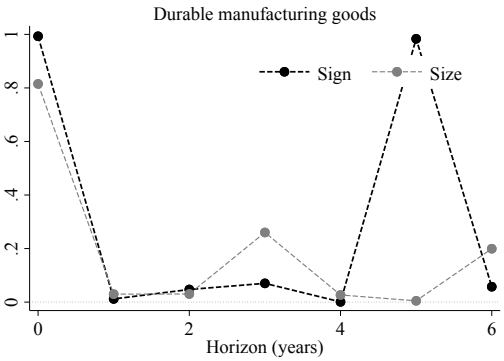
(d) Monetary contractions



(e) p-values for non-linearities (prod.)



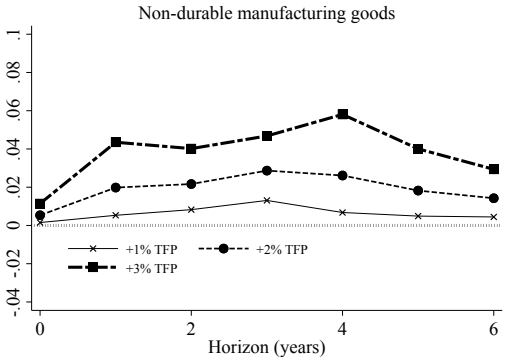
(f) p-values for non-linearities (mon.)



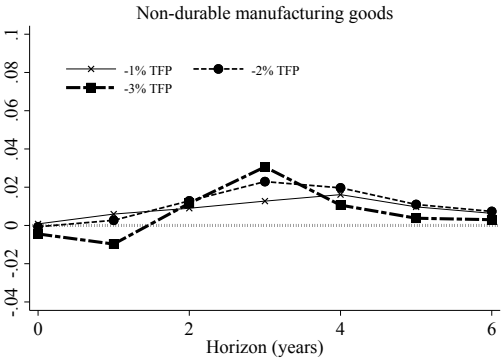
***Notes**: Panels (a)-(b) show estimated non-linear IRFs of intermediates intensity to a productivity shock based on Fernald (2014b), using specification in (3.20); the set of controls includes one lead of productivity shock and one lag of intermediates intensity, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panels (c)-(d) show estimated non-linear IRFs of intermediates intensity to a monetary shock based on Romer and Romer (2004), using specification in (3.20); the set of controls includes one lead of monetary shock and one lag of intermediates intensity, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Panels (e) and (f) shock p-values for the test that non-linear terms are zero.*

Figure 3.B.3: Non-linear IRFs of intermediates intensity to productivity and monetary shocks

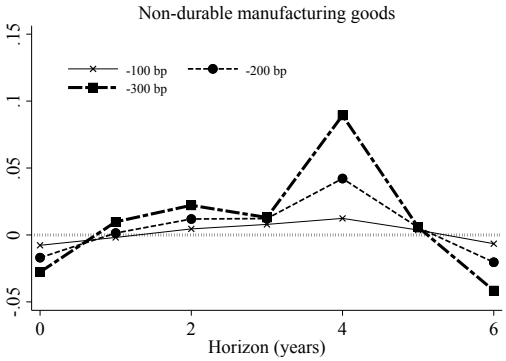
(a) Productivity expansions



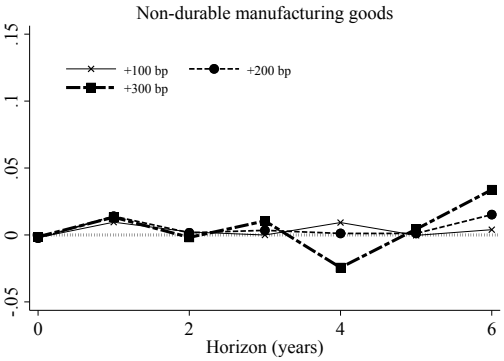
(b) Productivity contractions



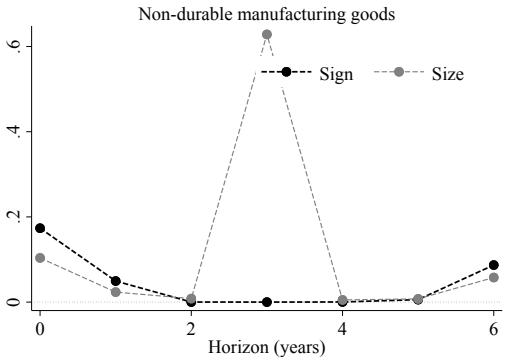
(c) Monetary expansions



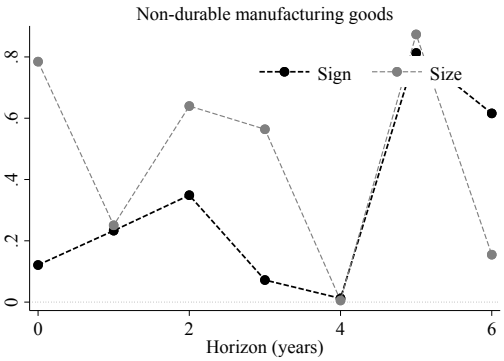
(d) Monetary contractions



(e) p-values for non-linearities (prod.)



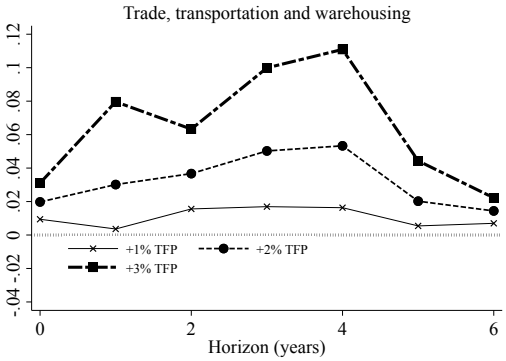
(f) p-values for non-linearities (mon.)



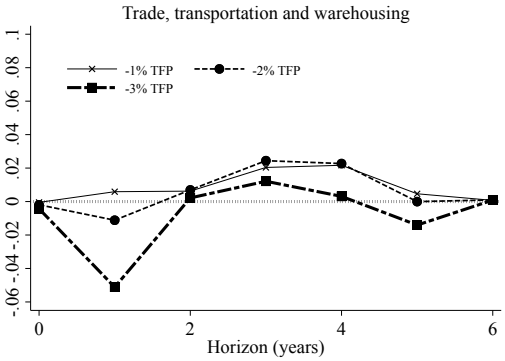
***Notes**: Panels (a)-(b) show estimated non-linear IRFs of intermediates intensity to a productivity shock based on Fernald (2014b), using specification in (3.20); the set of controls includes one lead of productivity shock and one lag of intermediates intensity, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panels (c)-(d) show estimated non-linear IRFs of intermediates intensity to a monetary shock based on Romer and Romer (2004), using specification in (3.20); the set of controls includes one lead of monetary shock and one lag of intermediates intensity, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Panels (e) and (f) shock p-values for the test that non-linear terms are zero.*

Figure 3.B.4: Non-linear IRFs of intermediates intensity to productivity and monetary shocks

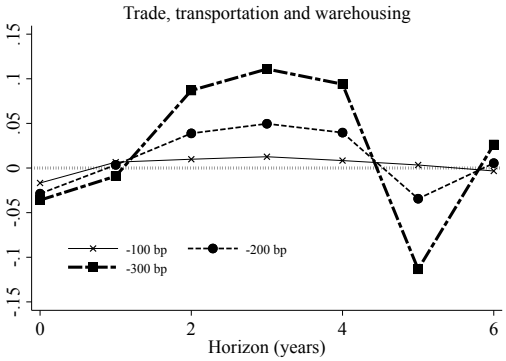
(a) Productivity expansions



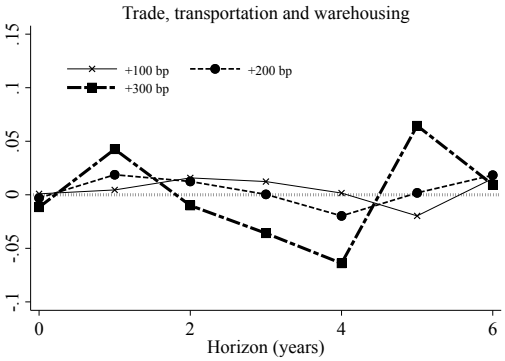
(b) Productivity contractions



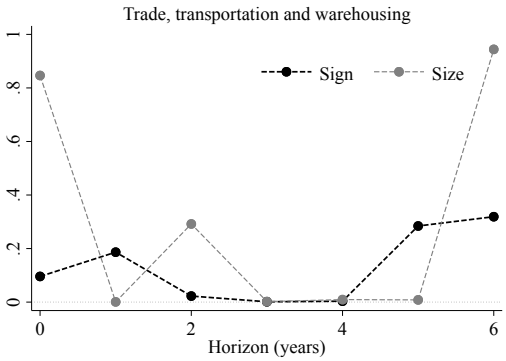
(c) Monetary expansions



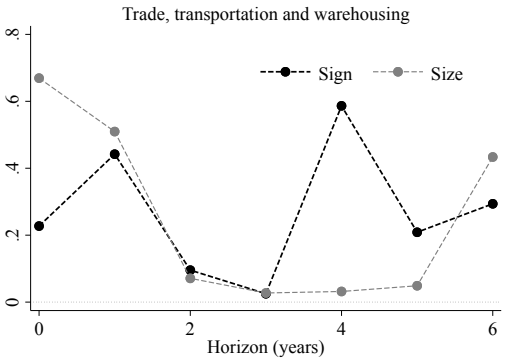
(d) Monetary contractions



(e) p-values for non-linearities (prod.)



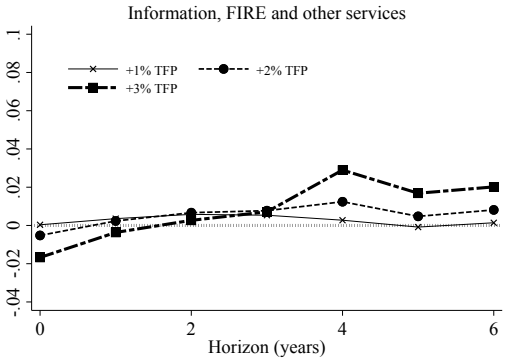
(f) p-values for non-linearities (mon.)



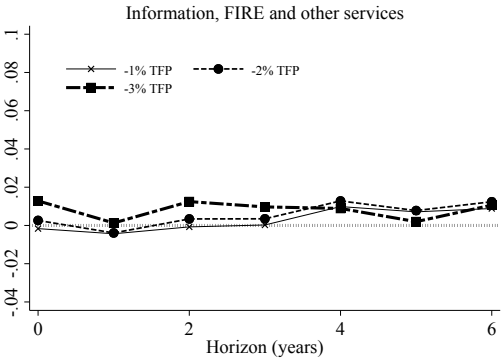
***Notes**: Panels (a)-(b) show estimated non-linear IRFs of intermediates intensity to a productivity shock based on Fernald (2014b), using specification in (3.20); the set of controls includes one lead of productivity shock and one lag of intermediates intensity, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panels (c)-(d) show estimated non-linear IRFs of intermediates intensity to a monetary shock based on Romer and Romer (2004), using specification in (3.20); the set of controls includes one lead of monetary shock and one lag of intermediates intensity, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Panels (e) and (f) shock p-values for the test that non-linear terms are zero.*

Figure 3.B.5: Non-linear IRFs of intermediates intensity to productivity and monetary shocks

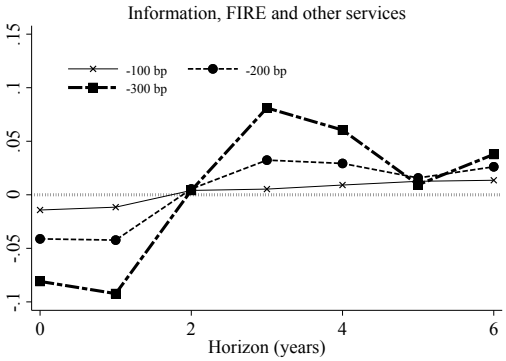
(a) Productivity expansions



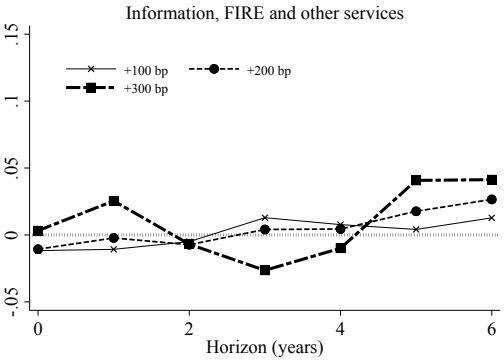
(b) Productivity contractions



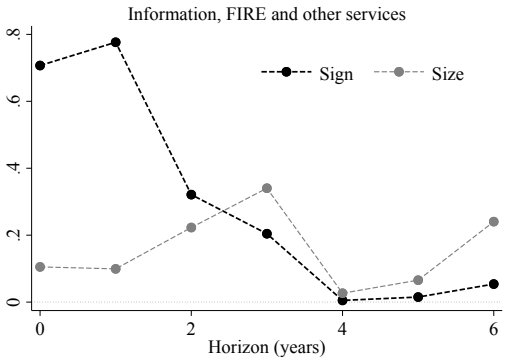
(c) Monetary expansions



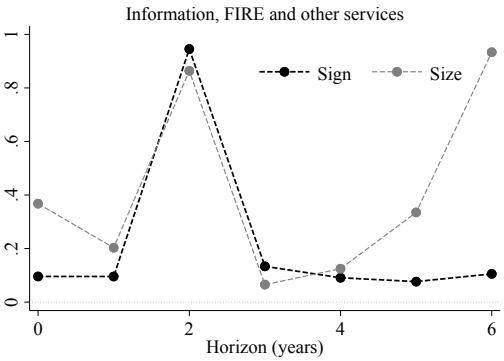
(d) Monetary contractions



(e) p-values for non-linearities (prod.)



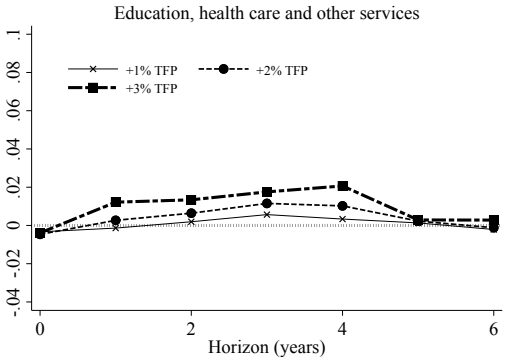
(e) p-values for non-linearities (prod.)



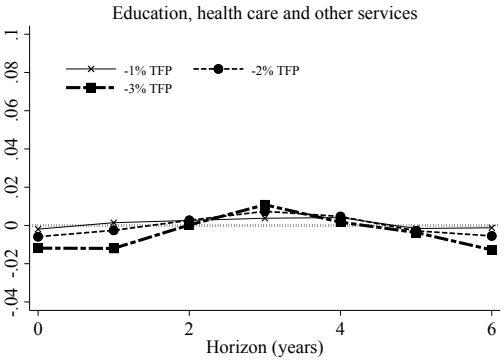
***Notes**: Panels (a)-(b) show estimated non-linear IRFs of intermediates intensity to a productivity shock based on Fernald (2014b), using specification in (3.20); the set of controls includes one lead of productivity shock and one lag of intermediates intensity, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panels (c)-(d) show estimated non-linear IRFs of intermediates intensity to a monetary shock based on Romer and Romer (2004), using specification in (3.20); the set of controls includes one lead of monetary shock and one lag of intermediates intensity, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Panels (e) and (f) shock p-values for the test that non-linear terms are zero.*

Figure 3.B.6: Non-linear IRFs of intermediates intensity to productivity and monetary shocks

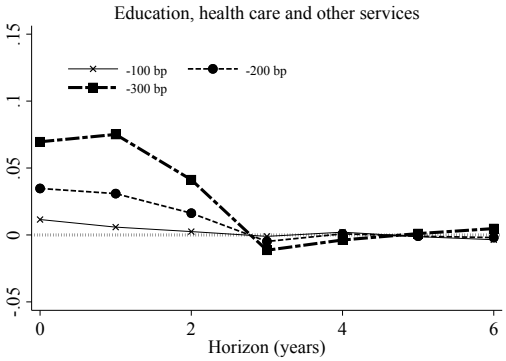
(a) Productivity expansions



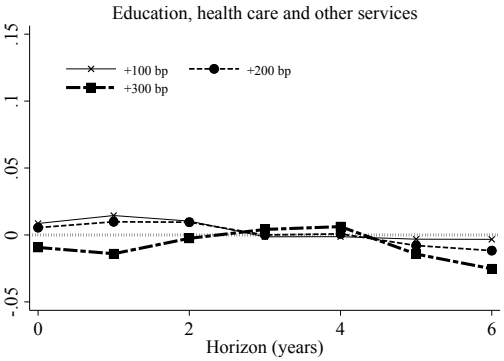
(b) Productivity contractions



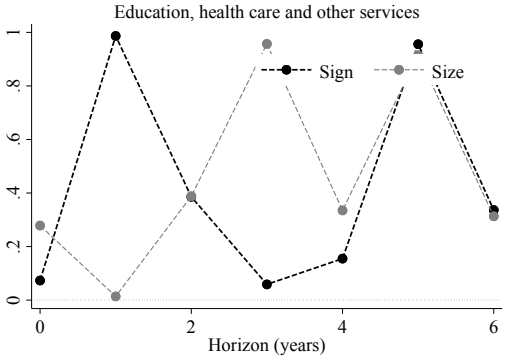
(c) Monetary expansions



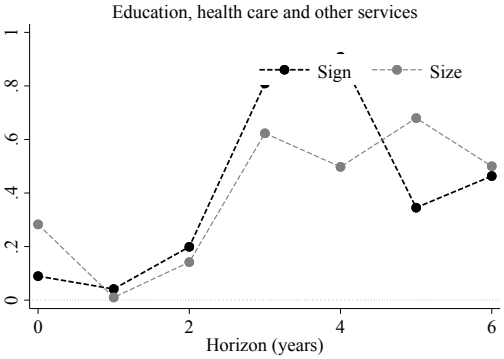
(d) Monetary contractions



(e) p-values for non-linearities (prod.)

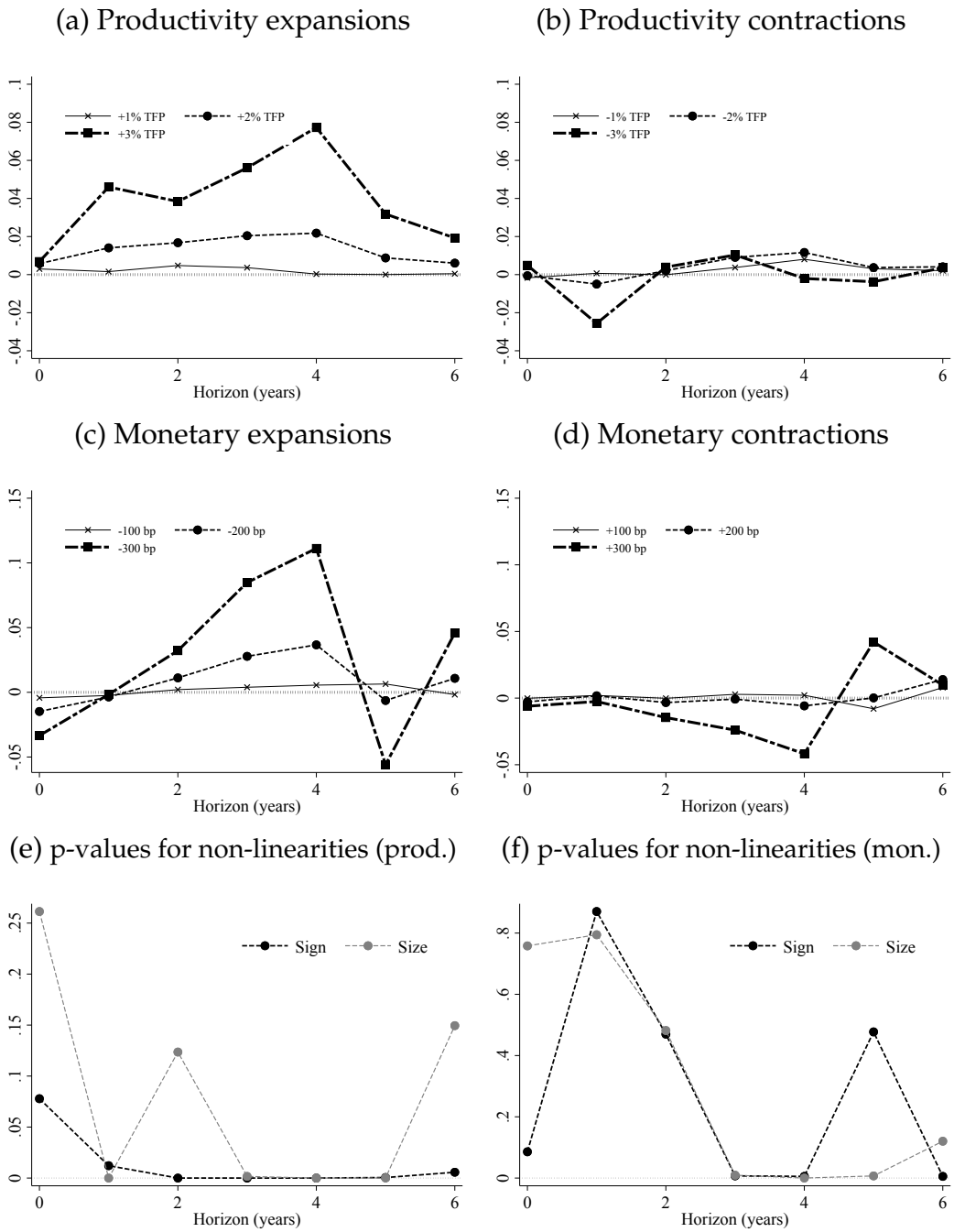


(f) p-values for non-linearities (mon.)



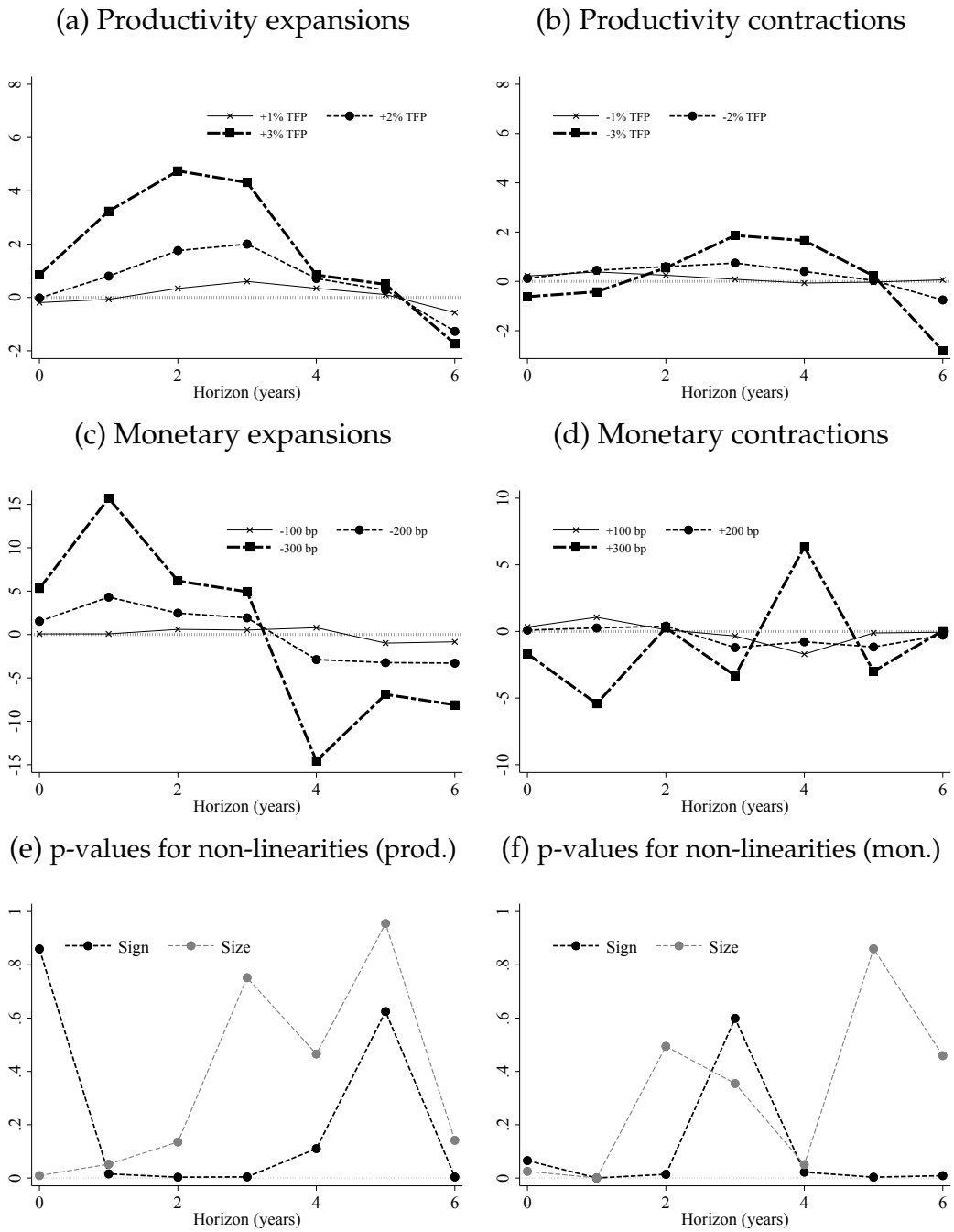
***Notes**: Panels (a)-(b) show estimated non-linear IRFs of intermediates intensity to a productivity shock based on Fernald (2014b), using specification in (3.20); the set of controls includes one lead of productivity shock and one lag of intermediates intensity, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panels (c)-(d) show estimated non-linear IRFs of intermediates intensity to a monetary shock based on Romer and Romer (2004), using specification in (3.20); the set of controls includes one lead of monetary shock and one lag of intermediates intensity, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Panels (e) and (f) shock p-values for the test that non-linear terms are zero.*

Figure 3.B.7: Non-linear IRFs of intermediates intensity to productivity and monetary shocks



***Notes**: Panels (a)-(b) show estimated non-linear IRFs of intermediates intensity to a productivity shock based on Fernald (2014b), using specification in (3.96); the set of controls includes one lead of productivity shock and one lag of intermediates intensity, productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panels (c)-(d) show estimated non-linear IRFs of intermediates intensity to a monetary shock based on Romer and Romer (2004), using specification in (3.96); the set of controls includes one lead of monetary shock and one lag of intermediates intensity, monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Panels (e) and (f) shock p-values for the test that non-linear terms are zero.*

Figure 3.B.8: Non-linear IRFs of number of suppliers to productivity and monetary shocks



***Notes**: Panels (a)-(b) show estimated non-linear IRFs of the number of suppliers to a productivity shock based on Fernald (2014b), using specification in (3.97); the set of controls includes one lead of productivity shock, four lags of the number of suppliers, one lag of productivity shock, (log) real GDP, (log) total factor productivity, as well as a time trend. Panels (c)-(d) show estimated non-linear IRFs of the number of suppliers to a monetary shock based on Romer and Romer (2004), using specification in (3.97); the set of controls includes one lead of monetary shock, four lags of the number of suppliers, one lag of monetary shock, (log) real GDP, federal funds rate, as well as a time trend. Panels (e) and (f) shock p-values for the test that non-linear terms are zero.*

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