

Optimal Cross-Tier Power Allocation for D2D Multi-Cell Networks

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Abstract—Efficient transmission power control is indispensable for cellular networks. It not only provides a high energy efficiency, but also maintains reliable connections. With the emergence of 5G mobile technology, the presence of device-to-device (D2D) communications within the cellular network has stimulated research on radio resource sharing. In this paper, we consider an underlay D2D network operating in a Rayleigh fading channel and propose a power allocation method that assigns transmit power levels to D2D UEs (DUEs) and cellular UEs (CUEs) such that the joint connection probability of DUEs and CUEs is maximized. The approach is formulated as an optimization problem, and we prove that the problem is log concave. Hence, the optimum powers for active UEs can be found easily using modern computational methods. Both the theoretical and simulated results show that the joint connectivity probability is improved by one to two orders of magnitude by applying the optimization procedure compared to conventional LTE open loop power allocation. This dramatic improvement comes at the cost of an increase in UE average transmit power. Thus, the proposed technique is well suited to 5G public safety and disaster relief communication modes where enhanced connectivity is the top priority.

Index Terms—D2D, heterogeneous networks, power allocation, convex optimization.

I. INTRODUCTION

Device-to-device (D2D) communication enables mobile devices in close proximity to communicate directly to each other without the intermediate transmission to an evolved Node B (eNB). The D2D mode was included in Release 12 of the 3GPP LTE physical layer specification in an effort to provide proximity based services (ProSe) [1]. Enabling robust D2D network would benefit future 5G public safety communications [2]. D2D user equipment (DUE) can coexist with cellular user equipment (CUE) by sharing the overall spectrum resources. In the literature, spectrum underlay and overlay are two widespread spectrum sharing approaches. In underlay mode, D2D transmissions reuse spectrum resources utilized by cellular transmitters, while in overlay mode spectrum is used opportunistically as it becomes available [1]. More efficient spectrum utilization is expected to be achieved in cellular systems incorporating D2D. In spite of these potential benefits, achieving reliable connections may be problematic when the cellular network is severely affected by the interference arising from the coexistence of D2D and cellular communications in the same spectrum.

Network connectivity is an important requirement to guarantee reliable communication among mobile devices and/or eNBs. Bettstetter [3] studied the connectivity of wireless multi-hop networks in a shadow fading environment. Coon et al [4] developed a cluster expansion model for the full connectivity probability of random geometric networks. An upper bound on connectivity of the network was studied by [5]. In [6], the probability of k -connectivity in random networks was discussed in the context spatially confined networks. As a typical measurement of communication network robustness, k -connectivity brings resilience to random faults or attacks for smooth functionality of systems. This is because even if any $k - 1$ nodes are randomly chosen and removed, the remaining nodes are still fully connected. When the total number of users is $k + 1$, a k -connected network is equivalent to a complete graph. For D2D communications, it is desirable that any DUE can connect with its neighbours directly. In such a case, the DUEs form a complete graph. The results reported in the references given above consider network connectivity without power allocation. It is well known that the appropriate selection of transmit powers at different nodes can lead to an improved connectivity performance.

Power allocation is an effective approach to reduce interference and enhance the desired performance metrics of a wireless network such as connectivity and throughput. However, simply reducing one UE's power or increasing another cannot ensure that the metric is improved. Recently, optimization procedures have been developed that tackle this problem efficiently. In [7], the author proposed a non-convex power allocation problem to maximize the total system throughput metric under quality of service (QoS) constraints in the medium-to-low signal-to-interference ratio (SIR) regime. The authors of [8] considered the optimal power allocation of interference-limited wireless networks constrained by outage probability conditions. The work in [9] investigated robust power allocation methods to achieve completion time minimization for wireless packet networks. In spite of these efforts, the study of joint successful connection probability optimization in an underlay D2D multi-cell network is still an open topic.

In this paper, we propose a cross-tier power allocation framework for maximizing the joint successful connection probability of the network where the underlay DUEs coexist

with CUEs in a Rayleigh fading environment. Here, only uplink resources are allocated for the D2D communications. Both the theoretical and simulated results show that the robustness of the network characterized by the joint successful connection probability is improved by one to two orders of magnitude with the proposed optimal procedure compared to the conventional LTE uplink open loop power allocation technique at the expense of more power consumption. As a result, the method details in this paper will be useful in 5G (and beyond) public safety and disaster relief modes, where the priority first priority is connectivity.

The rest of the paper is structured as follows. Section II begins with a description of the system model and addresses the connectivity between two devices. Section III focuses on the joint successful connection probability of the network and its maximization through power allocation. Section IV gives simulation results, and section V concludes the paper.

II. SYSTEM MODEL

A. Network Layout

A cellular system with K neighboring cells is shown in Fig. 1. We assume that UEs work in half-duplex mode and are distributed randomly within every operating cell. The DUEs transmit in the uplink channel, and a group of DUEs in close proximity can communicate with each other with the same channel resource as a CUE in the underlay scheme. The proximity distance is the maximum allowed pair distance between DUEs [10], [11]. Time division duplexing is used to allow one pair of DUEs within the group to communicate at one time. We also assume that the neighboring eNBs are interconnected by backhaul and the eNBs work cooperatively to share locations and adjust the power levels of UEs in a centralized fashion. Focusing on D2D communication reusing the uplink [12], the receiving signal from a CUE at an eNB is affected by cross-tier DUE interference and neighboring CUEs who are active on the same channel. Conversely, DUEs suffer from cross-tier interference arising from CUEs active on the same channel resource.

B. Received Signal

For simplicity of notation, we give unique node indices $\{i\}$ to all the UEs and eNBs. A node i has a transmitting power P_i , and the received signal power $P_{r_{ij}}$ from node i at node j is

$$P_{r_{ij}} = G_{ij} F_{ij} P_i, \quad (1)$$

where G_{ij} is the path gain¹ which is modeled as proportional to the attenuation factor $d_{ij}^{-\beta}$ [13] where d_{ij} is the pair distance between node i and j , and β is the pathloss exponent [7]. The antenna gain and coding gain are also assumed to be included in G_{ij} . F_{ij} models the power of Rayleigh fading which obeys an exponential distribution with unit mean. The

¹In this convention, we read the indices from left to right as the direction of the interaction while in some publications such as [7] and [9], the opposite convention is followed.

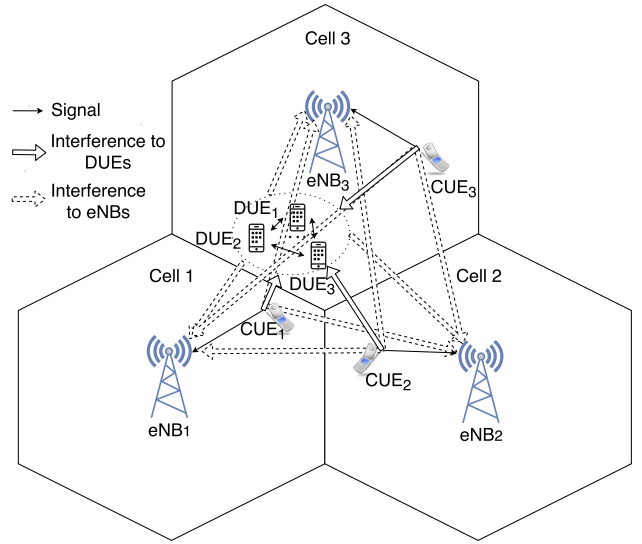


Fig. 1. Existing signals and interference in the cellular uplink.

channels are unchanged over the block time and independently and identically distributed from one block to the next [14]. The communication channel gain is $W_{ij} = G_{ij} F_{ij}$. We assume in this work that channels are reciprocal, i.e., $W_{ij} = W_{ji}$.

C. Signal to Interference-Plus-Noise Ratio (SINR) and Connection Probability

In the receiving mode, a UE or an eNB not only receives the signal but also endures the background noise and interference. The interference consists of unwanted signals coming from the UEs who are sending the messages simultaneously.

Using L_c to denote the set of indices belongs to CUEs and L_d to denote the set of indices belongs to DUEs, the interference at a node j while transmitting signal from a node i is given by

$$I_{ij} = \begin{cases} \sum_{k \in L_c} I_{kj}, & \text{if } j \text{ belongs to a DUE,} \\ \sum_{k \in L_d \cup L_c \setminus \{i\}} I_{kj}, & \text{if } j \text{ belongs to an eNB,} \end{cases} \quad (2)$$

where

$$I_{kj} = M_k G_{kj} F_{kj} P_k \quad (3)$$

and M_k is a given normalized traffic demand [15], it simplifies the analysis on the contribution of interference from DUE to eNB, because it is hard to foresee which DUE is active concurrently with the CUEs without knowing the traffic pattern, MAC and routing protocol. $M_k = 1$ when the interference is from a CUE node k to a DUE node j , while $0 \leq M_k \leq 1$ and $\sum_{k \in L_d} M_k = 1$ when the interference is received at an eNB node j from a DUE node k .

The signal to interference-plus-noise ratio (SINR) of receiving signal from node i at node j is given by

$$\text{SINR}_{ij} = \frac{G_{ij} F_{ij} P_i}{N_0 + I_{ij}}, \quad (4)$$

where N_0 is the thermal noise power at each node, and it can be written as [16]

$$N_0 = 10^{NF/10} k T_0 B, \quad (5)$$

where NF is the noise figure of the receiver, $k = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant, $T_0 = 290$ K is the room temperature and B is the transmission bandwidth.

The connection probability of a link is the likelihood that the actual SINR of the link is higher than a predetermined SINR threshold $S = 2^t - 1$ with link spectrum efficiency t . For a given set of traffic demand M_k and transmission power P_i , following the Theorem 1 in [17], the connection probability from node i to node j can be written as

$$p_{ij} = \mathbb{P}\{SINR_{ij} > S\} \\ = \exp\left(-\frac{SN_0}{G_{ij}P_i}\right) \prod_{k \in L} \left(1 + \frac{M_k S G_{kj} P_k}{G_{ij} P_i}\right)^{-1}, \quad (6)$$

where L is either L_c or $L_d \cup L_c$ depending on the type of receiving node j as in (2).

III. CONNECTIVITY AND OPTIMIZATION

A. Connectivity

A network is described as *connected* if there is a direct or multi-hop communication link between any two nodes. In a fading environment, the connectivity of a link is characterized by the probabilistic link connection while the connectivity of the network is described by full connectivity probability. It is the likelihood that a network is 1-connected. The relationship between the full connectivity probability and the connection probability of a link is discussed in [4].

The concept of *k-connectedness* describes that N nodes network can still preserve its connectivity even after removing any set of $k - 1$ nodes. Let $p_{fc}^N(k)$ denote the probability that the network is k -connected, and $p_{md}^N(k)$ denote the probability that it has minimum degree k . It is obvious that a k -connected network has minimum degree k , i.e. each node has at least k neighboring nodes.

The network is said to be *complete* if each pair of nodes within the network is connected directly. By using (6), the complete network probability of a D2D group can be obtained as

$$p_d = \prod_{i < j, i, j \in L_d} p_{ij}. \quad (7)$$

B. Optimization

In this subsection, we are interested in optimizing the joint successful connection probability of the network, so that both cellular communications and D2D communications can operate concurrently towards a given link spectrum efficiency. This goal is achieved through the optimal selection of UE power levels. The probability that the CUEs successfully connect to their respective eNBs in the uplink is given by

$$p_c = \prod_{i \in L_c} p_{ij_i}, \quad (8)$$

where j_i is the index of the eNB that node i is connected to. Hence, the joint successful connection probability is defined as

$$p_s = p_d p_c. \quad (9)$$

Based on this, we form the following optimization problem to maximize the joint successful connection probability by assigning different transmission powers to different UEs:

$$\begin{aligned} & \text{maximize } p_s(\mathbf{P}) \\ & \text{subject to } P_{min} \leq P_i \leq P_{max}, \end{aligned} \quad (10)$$

where P_{min} is the minimum output power of a UE, P_{max} is the maximum output power of a UE as described in [18], and \mathbf{P} is a vector of length $|L_c| + |L_d|$ that denotes the transmit powers for the CUEs and the DUEs. To make progress, we require the following theorem, which forms a significant contribution of this paper.

Theorem 1: The optimization problem given by (10) is log concave in $\tilde{\mathbf{P}}$, where the i th element of $\tilde{\mathbf{P}}$ is $\tilde{P}_i = \ln P_i$.

Proof: Consider a node i that wishes to transmit a message to node j . \mathbf{W}_{-j} are all the interfering channel gains at node j , $\mathbf{W}_j = [W_{ij}, \mathbf{W}_{-j}]$ represents all the channel gains. Let the \sim symbol over a scalar or a vector denote the element-wise log operation on the original content [9]. For a realization $\tilde{\mathbf{w}}_j$ of \mathbf{W}_j , the marginal probability density function (PDF) of \mathbf{W}_{-j} is given by integrating out \tilde{w}_{ij}

$$f_{\tilde{\mathbf{W}}_{-j}}(\tilde{\mathbf{w}}_{-j}) = \int f_{\tilde{\mathbf{W}}_j}(\tilde{\mathbf{w}}_j) d\tilde{w}_{ij}. \quad (11)$$

Given $\tilde{\mathbf{W}}_{-j} = \tilde{\mathbf{w}}_{-j}$ has occurred, the connection probability is written as

$$p_{ij}(\tilde{\mathbf{P}}) = \int \bar{F}_{\tilde{W}_{ij}}(g_{ij}(\tilde{\mathbf{P}}, \tilde{\mathbf{w}}_{-j})) f_{\tilde{\mathbf{W}}_{-j}}(\tilde{\mathbf{w}}_{-j}) d\tilde{\mathbf{w}}_{-j}, \quad (12)$$

where

$$\begin{aligned} \bar{F}_{\tilde{W}_{ij}}(\tilde{w}_{ij}) &= 1 - F_{\tilde{W}_{ij}}(\tilde{w}_{ij}), \\ g_{ij}(\tilde{\mathbf{P}}, \tilde{\mathbf{w}}_{-j}) &= \ln\{N_0 \exp(\tilde{S} - \tilde{P}_i) \\ &\quad + \sum_{k \in L} \exp(\tilde{S} + \tilde{M}_k + \tilde{w}_{kj} + \tilde{P}_k - \tilde{P}_i)\}, \end{aligned} \quad (13)$$

and $F_{\tilde{W}_{ij}}(\tilde{w}_{ij})$ is the cumulative distribution function (CDF) of \tilde{W}_{ij} , $\bar{F}_{\tilde{W}_{ij}}(\tilde{w}_{ij})$ is the complementary CDF (CCDF). $f_{\tilde{\mathbf{w}}_j}(\tilde{\mathbf{w}}_j)$ is log concave in $\tilde{\mathbf{w}}_j$ [9], then its CCDF is log concave in $\tilde{\mathbf{w}}_j$ [19]. $g_{ij}(\tilde{\mathbf{P}}, \tilde{\mathbf{w}}_{-j})$ is a convex function, and $\bar{F}_{\tilde{W}_{ij}}(g_{ij}(\tilde{\mathbf{P}}, \tilde{\mathbf{w}}_{-j}))$ is log concave [20]. The integration in (11) and (12), and multiplication between CCDF and PDF in (12) preserve log concavity. Thus, the product of the connection probabilities that yields the joint successful connection probability p_s is log concave in $\tilde{\mathbf{P}}$. \square

As a result of Theorem 1, the optimization problem can be reformulated as

$$\begin{aligned} & \text{maximize } \ln p_s(\tilde{\mathbf{P}}) \\ & \text{subject to } \ln P_{min} \leq \tilde{P}_i \leq \ln P_{max}. \end{aligned} \quad (14)$$

TABLE I
SYSTEM PARAMETERS.

Parameter	value
Layout	Hexagonal grid
Cell radius	500 m
Height of the trapezium	86.6 m
Spectrum allocation (UL/DL)	20 MHz
Duplex mode	Half duplex
Thermal noise power density	-174 dBm/Hz
Maximum distance between DUEs d_{DUEs}	25 m, 50 m
Number of cell towers	3
Number of CUEs shares the channel with the DUEs	1 per cell tower
Maximum UE TX power	23 dBm
Minimum UE TX power	-40 dBm
Coding gain	0 dB
UE antenna gain	0 dBi
eNB Rx antenna gain	18 dBi
UE/eNB noise figure	9 dB
UE link spectrum efficiency	0.05 bps/Hz
Carrier frequency	2 GHz
Pathloss exponent	2.7
Simulation iterations	5000

Numerous efficient convex optimization algorithms exists for solving such problems [20]. Hence, the computational complexity of the power allocation solution given in (10) would not pose a difficulty in practice.

IV. SIMULATION

In this section, we provide simulation results to validate the power allocation method detailed above.

A. Simulation Configuration

In the simulations, we consider three neighboring cellular hexagons as illustrated in Fig. 2. Inside each hexagon, an eNB is fixed at the center. CUEs are randomly located within every hexagon, and DUEs are arbitrarily located inside a circle. The diameter of the circle is equivalent to the proximity distance, and the center of the circle is randomly located within a trapezium region of the upper hexagon so that the DUEs are active near a cellular boundary. We assume that, at a certain time, three CUEs that belong to three different cellular hexagons communicate to their respective eNBs with the same channel resource. Meanwhile, DUEs within a proximity distance are active on the identical channel resource in a fair weighted time division manner. Therefore, the normalized traffic demand M_k is inverse to the number of DUEs in the group. The positions of the UEs are known and shared by the neighboring eNBs. The optimization procedure used to allocate powers to UEs runs on the upper eNB since it contains the D2D group in the same cell.

The benchmarks in the simulations are full power transmission and conventional open loop power allocation for UEs as described in [18], [21], [22]. The UE transmit power for the physical uplink shared channel (PUSCH) transmission is given by

$$P_{PUSCH} = \min\{P_{max}, 10 \log_{10} M + P_0 + \alpha l\} \text{ [dBm]} \quad (15)$$

where M is the number of assigned resource blocks (RBs), $P_0 = \{SNR_t + P_N\}$ [dBm/RB] with the open loop target signal

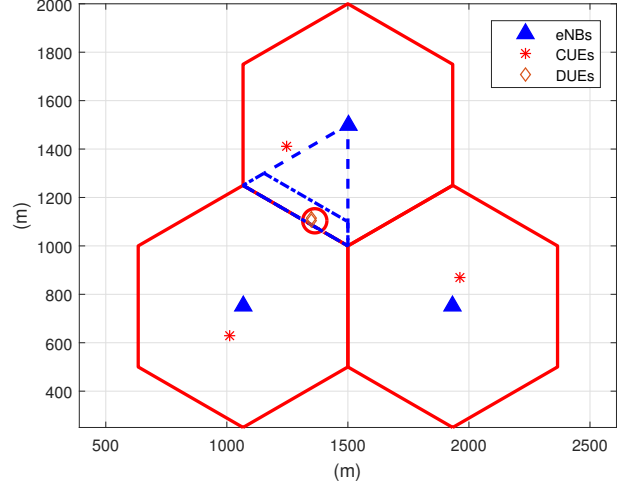


Fig. 2. The active regions of DUEs and CUEs.

to noise ratio (SNR) being SNR_t and P_N denoting the noise power per RB, α is the cell-specific path-loss compensation factor, and l is the downlink path loss at the UE [22]. In the simulations, we adopt conventional settings for open loop power allocation where $\alpha = 1$. $l = 10 \log_{10}[(4\pi/\lambda)^2 d^\beta]$, where d is similar to d_{ij} when it represents the distance between the CUE and eNB, while not the same for D2D communications where it is the distance from a DUE to its furthest DUE neighbour. The latter makes sure that a DUE could connect to any neighbor within the proximity distance, and thus naturally form a complete network among the DUEs. A trivial notice is that the transmission power should be larger than P_{min} . The parameters used in the simulation are given in Table I [10], [11], [21], [23]. The simulation results are obtained by averaged over 5000 independent Monte Carlo trials.

B. Simulation Results

Fig. 3 gives the optimization results for the joint successful connection probability. One is clearly able to see that the joint successful connection probability of the network decreases as the number of DUEs increases. Comparing with maximum power transmission and the optimal method, the performance of the open loop power allocation performs the worst. This can be explained by the fact that the open loop power allocation gives the amount of power to UEs based on different path losses and DUEs could be heavily affected by the nearby CUEs. As a result of this, the interference from CUEs to DUEs causes a huge performance drop, as seen in the figure. The value of the proximity distance affects the joint successful connection probability significantly in the open loop system, while it has little impact on the optimal approach. To conclude, the proposed method achieves up to two orders of magnitude of improvement in the joint successful connection probability.

Fig. 4 presents the results related to power savings. Compared to the full transmit power case, the optimal method reduces the CUE power between 3 and 6 dB and the DUE

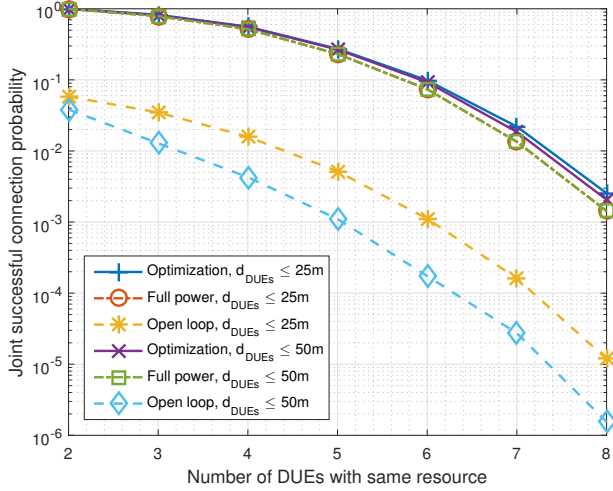


Fig. 3. Optimization result for different D2D maximum proximity distances and CUE to eNB distances.

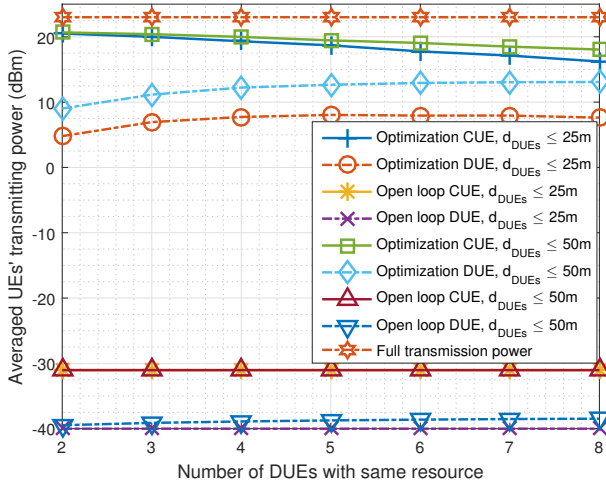


Fig. 4. Average transmit power of UEs after optimization and open loop power allocation.

power between 11 and 18 dB. The open loop power allocation scheme conserves energy very well. As discussed earlier, path loss is the major factor in open loop power allocation; taking interference into consideration as is done in the proposed method, more power must be assigned. From Fig. 4, we also note that the average transmission power of the DUEs is affected by the proximity distance. As the DUEs' pair distance grows larger, more power is consumed to achieve a similar joint successful connection probability.

It is apparent that there is a power cost associated with obtaining a significant improvement in cross-tier connectivity performance. As a result of this connectivity-power trade-off, one can surmise that the proposed method is well suited to 5G (and beyond) public safety and disaster relief modes, where emphasis is placed on connectivity.

V. CONCLUSION

In this paper, we proposed a power allocation procedure that maximizes the joint (cross-tier) successful connection probability of a network where underlay DUEs share radio resources with CUEs. The power allocation technique was formulated as an optimization problem, which we proved to be log concave. Hence, the problem can be solved efficiently using standard algorithms in practice. We provided detailed simulation results to compare the proposed method with conventional open loop power allocation and full power allocation benchmarks. The simulations confirmed that the connectivity performance can be improved by up to two orders of magnitude compared to the open loop power allocation scheme. Thus, our method could form a useful robust solution for 5G public safety or disaster relief communication modes.

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