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Gödelian platonism and mathematical intuition

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Abstract

This paper has two key aims. The first is to clarify the nature of Gödel's platonism. I offer an interpretation of Gödel's remarks on realism and intuition from which a clarified Gödelian position, called *conceptual platonism*, can be extracted. The second aim is to assess the philosophical merits of this theory. I defend it from charges of mysticism and theology, arguing instead that conceptual platonism is problematic for a more familiar reason. Given the familiarity of this problem, there are implications for a broad family of views in the philosophy of mathematics.

1 | INTRODUCTION

Gödel's platonism is a difficult position to assess. His remarks on the subject are scattered over a large number of writings, and much of the material we have is from reported conversations, rough notes for talks and lectures, and draft manuscripts. Moreover, Gödel recorded in an undated manuscript from around 1975 that he had subscribed to some form of platonism since 1925 (Gödel, 1974–6, p. 447). Five decades is a long time to hold a view, and Gödel's position was far from static. Most notably, mathematical intuition, a concept that has come to be intimately associated with Gödel, plays little to no explicit role in his support for platonism in the major articles (1944), (1947), and (1951). But later, and especially in (1964), it takes on a leading role. The result is that there is no one unique view that can be called “Gödel's platonism”; there are only various forms of platonism that have a fair claim to the title.

In light of these considerations, it is difficult to make progress in assessing Gödel's position and the prominent criticisms of it in the literature. Given the lack of a comprehensive statement of Gödel's view, some reconstruction is necessary. I will outline (Sections 2–7) a promising version of platonism that can be extracted from his various writings on the subject. I will call this view *conceptual platonism*. Along the way, I will also sketch what I take to be a Gödelian account of mathematical intuition (Sections 3–6). I will argue that Gödel deploys this concept in two distinct ways. Moreover, the significance of the less well-developed *objectual* sense of intuition is unclear, and only the comparatively clear *propositional* sense is a crucial moving part of conceptual platonism. In Sections 8 and 9 I will argue

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that Gödel's view, understood along these lines, involves no “mysticism” or “theology.” Accusations of such, from Chihara and others, are based on a feature of Gödel's view which I call *existential internalism*, a feature that his platonism shares with a wide range of respectable views in the philosophy of mathematics. Although I do not think conceptual platonism is acceptable as it stands, the familiarity of its failure means that there are important lessons to be learned for the philosophy of mathematics more broadly.

2 | TWO KINDS OF PLATONISM

In the Gibbs lecture, Gödel characterizes platonism or realism¹ in mathematics as the view “that mathematical objects and facts (or at least *something* in them) exist objectively and independently of our mental acts and decisions” (Gödel, 1951, p. 311). Such comments are similar to traditional “textbook” characterizations of platonism or mathematical realism, but they do not reveal what is distinctive about Gödel's thought on the subject. Additional comments, such as that platonism amounts to the view that mathematical objects are not located in the mind or the natural world (Gödel, 1951, p. 312, fn. 17), are similar in this respect. In particular, platonism is formulated in these remarks with respect to mathematical *objects*. But a striking feature of Gödel's view is that his platonism is not *primarily* a kind of realism about mathematical objects, but is rather a realism about (the content of) mathematical *concepts*. At several places in his writings, Gödel makes it clear that he is a platonist about mathematical objects, in the more traditional sense, but his remarks elsewhere indicate that platonism about concepts is explanatorily prior in his view. Indeed, at times, he simply *equates* mathematical platonism with realism about mathematical concepts (Gödel, 1951, p. 314), while at others, he takes it that platonism about concepts *implies* platonism about objects:

[I]f the meanings of the primitive terms of set theory as explained[...] are accepted as sound, it follows that the set-theoretical concepts and theorems describe some well-determined reality (Gödel, 1964, p. 260).

In Section 9, I will address the issue of whether this platonism is “a kind of theology,” as critics like Chihara (1991, p. 21) claim. Of course, concepts so conceived are presumably abstract, acausal, and so forth, just like the objects of traditional platonism, so there is no question of this account being satisfactory to a nominalist. But a commitment to abstracta does not on its own constitute theology, and indeed, the remark above gives us reason to think that the central commitments of Gödel's view are consistent with a secular conception of set theory. His view is that a grasp of the concept *set* is the proper source of platonism about sets in general. And according to him, our grasp of the concept *set* is explained primarily in terms of our apprehending the truth of increasingly strong axiomatic theories of sets (Gödel, 1964, pp. 260–261). Hence, what this platonism requires is an account of how we apprehend the truth of mathematical axioms, and we should hope to be able to provide such an account without recourse to mystical means.

Viewed this way, Gödel's platonism is relatively minimalistic. In the context of arithmetic, Potter argues that “Gödel thinks that once we have grasped the concept ‘natural number’ there is nothing further involved in the claim that natural numbers exist, because he thought the concept ‘natural number’ itself has a real content, namely the existence of the natural numbers themselves” (Potter, 2001, p. 343). Understanding Gödel's view this way makes clear an analogy between his platonism and Quine's (despite their radical disagreements elsewhere): for Gödel, it is quite proper to say that admitting sets into our ontology demands nothing more than our acceptance (as “sound”) of some axiomatization of the concept *set* which implies the existence of sets, such as the axioms of **ZFC**.² According to the picture so sketched, the pressing philosophical question is not how we have “access” to causally inert abstract objects, but rather how it is that we apprehend the truth of axioms.

In pursuing the justification of axioms, Gödel takes two relatively distinct routes. One is the justification of axioms by *extrinsic* methods, based on an analogy Gödel draws between mathematics and natural science. This kind of justification is a major topic in its own right, and discussion of it lies beyond the scope of this paper. The other focuses on what is known today as *intrinsic* justification, and the concept of mathematical intuition is at the heart of Gödel's account of such justification.

Gödel is perhaps more infamous for his belief in the existence of a faculty of mathematical intuition than he is for his platonism. And as with platonism, it is not entirely clear what Gödel takes mathematical intuition to be; there is no well-developed *theory* of intuition in his writings. This lack of clarity certainly contributes to the sense that Gödel's view might be somewhat mystical. But I think that with some effort, we can clarify what Gödel takes intuition to be, and more importantly, we can see that the “moving parts” of Gödel's view are not at all mystical.

I will argue that we can make the most sense of Gödel's written commitments by understanding him as having a *dual* view of intuition. As Gödel is a platonist about both mathematical objects and mathematical concepts, there are correspondingly two notions of intuition that appear in his philosophy of mathematics. The first notion is *objectual intuition*, which is directed toward mathematical objects. The second is *propositional intuition*, which is directed toward mathematical truths. Each of these notions will be examined in turn.

3 | OBJECTUAL INTUITION

The notion of objectual intuition is perhaps the more alien to philosophers working in the analytic tradition today. According to Gödel, we can have experience of the objects of mathematics via a faculty of intuition in a manner analogous the experience we have of physical objects via the faculty of perception. His most well-known remarks to this effect appear in (Gödel, 1964), (Gödel, 1953/9) and (Gödel, 1972). The oft-cited remark from the former is that “we do have something like a perception also of the objects of set theory” (Gödel, 1964, p. 268). In the second article, he refers to the “*intuitive character*” of mathematical objects (Gödel, 1953/9, p. 361, Gödel's emphasis) and to the “direct perceptibility of mathematical objects” (Gödel, 1953/9, p. 354, fn. 45). The latter remark, however, must be taken with a pinch of salt: Gödel is abundantly clear elsewhere that talk of perceiving sets is not to be taken literally. The remark from (1964) above is preceded by “despite their remoteness from sense experience” and in the earlier Gibbs lecture, Gödel again claims that our “perception” of mathematical objects is nothing to do with the spatio-temporal world (Gödel, 1951, p. 320), and that mathematical objects “must be totally different from sensual objects” (Gödel, 1951, p. 312, fn. 18). So although objectual intuition is supposed to be *perception-like*, it is not supposed to literally *be* perception.

Gödel's remarks in (Gödel, 1972) are much less direct, as he is there discussing Hilbert's conception of mathematical intuition, rather than his own. He refers to this as *concrete intuition* and takes it to be object-directed, in particular, to finite combinations of discrete concrete objects, such as symbols (Gödel, 1972, p. 272 and fn. b). Gödel's discussion heavily implies that he accepts the existence of this faculty.³ It seems certain that Gödel's own notion of objectual intuition went significantly further than Hilbert's, given that his remarks in the other papers extend the reach of this faculty to abstract objects like sets. But what is significant about these remarks for present purposes, however, is that Gödel endorses a faculty of mathematical intuition that, in virtue of being Hilbertian, is directed toward mathematical objects, rather than concepts or propositions.

So it is clear that Gödel accepts the existence of objectual intuition. What is significantly less clear is how he thinks it works. He says remarkably little about this notion, beyond the analogy with perception. Given that Gödelian objectual intuition is supposed to extend Hilbertian intuition, we might think to look at Gödel's account of the latter for clues. Gödel reads Hilbert as thinking of concrete intuition as being, roughly, Kantian spatio-temporal intuition with a restricted domain of concrete objects (Gödel, 1972, p. 272, fn. b). But this is a very scant clue for the interpreter, since it is not at all clear how Gödel thought that Kantian intuition works. Moreover, it is deeply unclear the extent to which Gödelian objectual intuition can be taken to be Kantian. While this is not the place to go into detail

on the issue, it is perhaps worth mentioning that the majority of commentators are sceptical of the strength of the relation between Kantian and Gödelian intuition. Parsons (1995, p. 68) and Hallett (2006, p. 127) both emphasize Gödel's disparaging remarks about the "subjectivism" of Kant's notion and cite several instances of disagreement between Gödel's thought and Kant's. Tait (2010, p. 97) rejects the idea that there is anything much in common between Kant's notion and Gödel's, although Tait is thinking only of propositional Gödelian intuition (see below). Folina (2014) is a notable dissenting voice; she defends the view, roughly, that Kant and Gödel's notions are epistemologically similar and ontologically distinct. But even according to her quasi-Kantian reading of Gödel, the role of objectual intuition is deeply unclear, since, as she emphasizes, Gödel is not sufficiently careful to distinguish intuition of objects from intuition of the concepts which they fall under (Folina, 2014, p. 52).

In the *Dialectica* paper (Gödel, 1972), Gödel interprets Hilbert's finitary mathematics as the fragment of mathematics that is made evident by spatio-temporal intuition of finite numbers of discrete concrete objects. On this basis, Tait suggests that Gödel had a notion of *abstract* intuition, which is more extensive than Hilbert's concrete intuition but performs a similar epistemological function in securing the cogency of (a part of) mathematics (Tait, 2010, p. 94). This suggestion strikes me as fascinating and plausible, but nevertheless, it does not give us a very detailed picture of how objectual intuition is supposed to work for Gödel. As Tait stresses, Gödel never worked out this notion to his satisfaction, and this particular notion of intuition seems absent from Gödel's other writings. So once again, it is almost completely unclear how we should understand Gödel's remarks on objectual intuition.

This lack of clarity is, I think, one explanation for the widespread mistrust of intuition in readers of Gödel. Objectual intuition goes largely unexplained by Gödel and hence is apt to make his platonism appear mysterious. Worse still, it is deeply unclear how any account of intuition which is even remotely Kantian or Hilbertian could do the work that Gödel assigns to it. As Guyer and Wood emphasize, Kant understood intuition as giving "singular and immediate representations of particular objects" (Kant, 1787, p. 36). According to Kant, such intuitions (temporal and spatial, respectively) play a foundational role in our knowledge of arithmetic and geometry (Kant, 1783, 4:283). The role it plays in Hilbert's philosophy is more restricted, being used to account only for a finitary fragment of arithmetic.⁴ The most prominent modern account of intuition is due to Parsons, and on his account, objectual intuition does not even extend to natural numbers (Parsons, 2008, Section 37). By contrast, Gödel claims that mathematical intuition can deliver much stronger verdicts, such as the existence of Mahlo cardinals (Gödel, 1964, pp. 260–261 and fn. 20).⁵ These are large infinite sets, the existence of which is demonstrably independent from the standard axioms of set theory, so why would anybody think that we stood in a quasi-perceptual relation to such remote objects? In the next three sections, I will argue that Gödel does not need to be understood as making such extravagant claims about the power of intuition. The first step in solving the puzzle is to distinguish the role of *propositional intuition* in Gödel's philosophy.

4 | PROPOSITIONAL INTUITION

According to Gödel, propositional intuition is delivered by the grasp we have of a concept. When our grasp of a concept is sufficiently clear, this will deliver knowledge of the truth of axioms. For example, our clear grasp of the concept *natural number* might yield knowledge of the axioms of **PA**, on this picture. As in the case of his platonism, Gödel's view of propositional intuition can be interpreted as involving fairly minimal commitments. Thanks largely to the incompleteness theorems, Gödel argues that mathematics cannot be the result of syntactical or semantic convention and stipulation; in other words, he thinks that mathematics is importantly *nontrivial*, a view that few would take issue with today. Potter (2001, p. 340) argues that from this view, Gödel's terminological conventions with respect to thought make the existence of something fulfilling the role of mathematical intuition immediate. Gödel thinks that "by our thinking we cannot create any qualitatively new elements, but only reproduce and combine those that are given" (Gödel, 1964, p. 268). If thinking is, by definition, essentially combinatorial and mathematics is not

reducible to combinatorics of syntax, then it follows that mathematical thought involves something “extra,” and *this*, whatever it may be, is intuition.

Parsons comes to a similar conclusion about the role of intuition in Gödel's thought, namely that “the deliverances of mathematical intuition are just those mathematical propositions and inferences that we take to be evident on reflection and do not derive from others, or justify on a posteriori grounds, or explain away by a conventionalist strategy” (Parsons, 1995, p. 59).

I think this minimalistic interpretation of Gödelian propositional intuition is essentially correct and bears no resemblance to any kind of mystical insight or to perception. When considered as a source of evidence for the truth of mathematical propositions, intuition can be an imperfect tool. In particular, the credence intuition lends to a particular proposition need not be particularly strong and, in some cases, can be outright misleading. Gödel claims, for instance, that insufficiently clear intuition is responsible for the set-theoretic paradoxes (Gödel, 1951, p. 321). He is of course confident that many imperfect intuitions can be corrected over time; in a similar passage (Gödel, 1964, p. 268), he claims that the paradoxes “are hardly any more troublesome for mathematics than deceptions of the senses are for physics.”

So for Gödel, intuition is not *factive* (i.e., having an intuition that P does not in and of itself entail P). Parsons makes a related point in emphasizing that there is a noticeable gap between acknowledging the existence of intuition and giving any credence to it (Parsons, 1995, p. 70). Moreover, intuitive knowledge of an axiom need not, according to Gödel, be independent of our knowledge of other axioms. He speaks of “an intuition which is sufficiently clear to produce the axioms of set theory and an open series of extensions” (Gödel, 1964, p. 268) and says that the new axioms serve to “unfold the content of the concept of set” (Gödel, 1964, p. 261). The idea that an initial intuition justifies the basic theory of sets collectively is an important one; the view that these axioms are intuitively justified would be deeply implausible if intuition had to give knowledge of each axiom independently of the others, since it is a conception of a *universe* of sets that makes a *theory* like **ZFC** so intuitively powerful. So on the Gödelian picture, there is no requirement that the various deliverances of intuition have to be independent of one another (as perception of physical objects might plausibly be taken to be). It is similarly important that, by this account, ever stronger axioms can “unfold” a mathematical concept successively, with certain principles becoming apparent only on the basis of earlier ones. This means that axioms known intuitively need not be immediately apparent, in whatever way (if any) one might think of perceptual knowledge as immediate.

So, on Gödel's account, we have intuitive knowledge of an axiom if we know it, but we do not have a proof of it from other known axioms or from syntactic conventions, and nor does our knowledge of it rest on quasi-empirical considerations. The latter is not precisely defined but includes familiar considerations about the desirable consequences of the axiom, its lack of undesirable consequences, its facilitation of the speed-up of proofs, and so forth.⁶ Such intuitive knowledge need not be infallible, obvious, or immediate and, hence, differs crucially both from a naïve picture of axioms as self-evident truths and from a naïve analogy with perceptual knowledge.

Though there is no reason to buy into Gödel's terminological convention of calling only combinatorial thoughts “thoughts,” accepting that mathematics is nontrivial is almost sufficient for the existence of intuition in Gödel's sense. Given the explicit juxtaposition Gödel makes between intuitive sources of justification in set theory and those that we would today call “extrinsic” or “quasi-empirical” (Gödel, 1964, p. 269), the only means of getting by without Gödelian intuition would be to make the extreme claim that the soundness of *all* acceptable mathematical axioms can be established exclusively by methods analogous to those employed in the sciences.

This is not the place to engage fully with this kind of radical empiricism about mathematics, but it is perhaps worth raising a point against it. A primary use of propositional intuition for Gödel is the justification of undecidable statements formalizing syntactic information about sound theories. For example, the canonical consistency statement of **PA** follows straightforwardly from the soundness of the theory, and hence is an acceptable mathematical axiom. As Gödel would put it, the stronger system **PA** + *Con*_{PA} is “exactly as evident” (1937, p. 164) as **PA**. But *Con*_{PA} is not provable in **PA** and is certainly not trivial, so our apprehension of the truth of this axiom is a prime example of

intuition at work. Even if extrinsic methods do have an important role to play in the epistemology of mathematics, it is implausible to claim that such methods are the *only* source of verification we have for propositions of this kind. This consideration is not conclusive of course, but it does at least to show quite how radical this kind of scientism really is.

If we reject this radical empiricism, then Potter and Parsons are correct to take propositional intuition in Gödel's thought as involving fairly minimal commitments: mathematical intuition is simply that which enables us to have non-deductive, nontrivial, and nonextrinsic mathematical knowledge. In the context of Gödel's platonism, in particular, the deliverances of propositional intuition are simply what we know in virtue of the grasp we have of mathematical concepts.⁷ For intuition to yield knowledge of objects, the concept grasped must have the right kind of "objective" content. Cases where a concept is not of the right kind might fail to yield mathematical knowledge in any non-trivial sense.⁸ As Parsons highlights (Parsons, 1995, p. 70), this does not entail anything as strong as Gödel's platonism. One could, for example, accept that we have an intuition that the axioms of **PA** are true, owing to a grasp of the concept *number*, but deny that we have a similar grasp of the concept *set*, perhaps on grounds related to the paradoxes. But for those of us who do not find the concept *set* to be inherently flawed on such grounds, Gödel's account of propositional intuition is simply the unproblematic claim that a grasp of that concept can yield nontrivial mathematical knowledge without recourse to extrinsic methods.

5 | OTHER DISTINCTIONS

Before continuing, I would like to clarify my interpretation of Gödelian intuition by situating it in the broader literature on the subject. I have argued that Gödel posits two distinct kinds of intuition, one objectual and one propositional. This is not to be confused with a related distinction, drawn by Parsons, between *intuition of* and *intuition that* (Parsons, 1995, p. 59). Parsons observes that Gödel uses the expression "intuition" in two distinct senses, one object-relational and one indicating a propositional attitude. But to mark that distinction is *not* to endorse the dual interpretation of Gödel that I have been urging. For instance, according to Tait, Gödel's notion of intuition is just propositional (Tait, 2010, p. 95), and of course, Tait is well aware that Gödel uses the expression "intuition" with an object-relational grammar. The interpretative trouble arises because Gödel's object-relational form of expression does not on its own imply a belief in objectual intuition as I have construed it, because he frequently uses such language when discussing *concepts*. To use a famous example, he claims that mathematical "concepts form an objective reality of their own, which we cannot create or change, but only perceive and describe" (Gödel, 1951, p. 320). Gödel does not insist on the Fregean terminology of distinguishing objects from concepts, and this means that the objectual/propositional distinction does not coincide with the *of*/*that* distinction.

When I speak of mathematical *objects*, I mean to refer only to familiar mathematical objects such as sets and numbers, and not to concepts.⁹ Hence, if we have *intuition of* a concept that ultimately yields intuitive knowledge of an axiom, such intuition would, by my standards, count as propositional, despite the object-relational grammar. This is because the resultant knowledge is yielded by our grasp of a mathematical concept, not by a representation of a mathematical object. Further evidence that the *of*/*that* distinction does not track the objectual/propositional distinction is that Parsons is rather reserved on the matter of whether Gödel endorses objectual intuition (in my sense). Despite his own thorough discussion of intuition *of*, he claims that Gödel neither affirms nor denies that we have objectual intuition (Parsons, 1995, p. 65). Elsewhere, Parsons elaborates that he does "not believe that Gödel thinks we have 'something like a perception' of the sets that are of concern to set theorists other than by way of concepts that define them" (Parsons, 2014, p. 139, fn. 11). Depending on what sets Parsons thinks set theorists are concerned with, this might be something compatible with my interpretation, since I do not think that Gödel takes us to have objectual intuition of large sets such as Mahlo cardinals.

Despite Parsons' cautious assessment of the source material, I have offered what I consider to be sound textual evidence above for Gödel's belief in an objectual mode of intuition. But I must acknowledge that any direct

interpretation is extremely difficult; for not only can concepts be the objects of intuition in Gödel's terminology, he also uses the term “concept” on occasion to refer to (what I would take to be) objects. As Goldfarb highlights (Goldfarb, 1995, pp. 332–333), this usage is especially rife in the drafts of “Is mathematics syntax of language?” But it should be noted that even in that paper, Gödel does refer to the perceptibility or intuitability of *objects* while explicitly contrasting them with concepts (Gödel, 1953/9, p. 354, fn. 45). So, despite Gödel's confusing terminological and grammatical decisions, I must disagree with Parsons' assessment: there is textual evidence across writings from the 1950s, 1960s, and 1970s that Gödel thought we had a perception-like intuitive relation to some of the *objects*, as opposed to just the concepts, of mathematics.

Certain other interpreters have adopted a similar view. Føllesdal interprets Gödel as believing in intuition of objects (Føllesdal, 1995, p. 370). Although he reaches his conclusion via a different route to myself (namely, a comparative examination of Gödel and Husserl), it is certainly congenial to my view. Burgess draws a more heterogeneous picture of the situation (Burgess, 2014, p. 31). Of the many kinds of intuition he ascribes a belief in to Gödel, one is an objectual faculty of intuition, namely the “geometrical intuition” of a particular Euclidean structure of geometric objects (Burgess, 2014, p. 14). But in the realm of set theory, he ascribes to Gödel only intuition of the relevant concepts (Burgess, 2014, p. 20). Hallett is also a dualist of some kind, but not in the same sense as myself. He sees Gödel as operating with both the intuition of concepts which ultimately deliver knowledge of axioms and with a vaguer sense of intuition as an insight into plausibility (Hallett, 2006, p. 121).¹⁰

Given Gödel's variegated thinking and terminology, there are many dimensions along which to draw such distinctions, and I am not in the business here of calling any of the above into question. All I am emphasizing is that a perfectly legitimate distinction which can be made among the kinds of intuition to which Gödel appeals is the objectual/propositional distinction. And I'd also like to emphasize that this distinction is not to be confused with the of/that distinction. With the objectual/propositional distinction in hand, we can now more carefully examine Gödel's account of intrinsic justification in set theory to see what sense we can make of some of his more puzzling remarks about the scope of mathematical intuition.

6 | INTUITION IN SET THEORY

We have seen that Gödel's platonism extends to both mathematical objects and mathematical concepts. Corresponding to this metaphysical picture, there are two kinds of Gödelian intuition, objectual and propositional. But there are many questions remaining about the epistemological significance of these notions. Propositional intuition, as we have seen, is an important notion for Gödel; although the deliverances of propositional intuition are neither immediate nor infallible, they are the parts of mathematics which are not deduced from other known axioms, trivial, or known via quasi-empirical methods. Importantly, some axioms which can be known intuitively imply the existence of mathematical objects; hence, conceptual reflection is a sufficient basis for Gödel's more traditional platonism about mathematical objects. The significance of objectual intuition is significantly less clear. My contention in this section will be that, although Gödel subscribed to the existence of an objectual faculty of intuition, it plays no significant role in his epistemology of mathematics in general and of set theory in particular. As it were, all the heavy lifting is done by propositional intuition. Furthermore, when the rest of Gödel's views are shorn away from the obscure notion of objectual intuition, the result, which I call *conceptual platonism*, is a perfectly respectable and non-mystical view.¹¹

Although the notion of intuition may appear to be somewhat arcane to analytic philosophers today, the idea that reflection on the concept *set* can yield set-theoretical knowledge independent of quasi-empirical considerations is relatively mainstream. A well-known classic addressing the question of which axioms follow from the concept *set* is (Boolos, 1989).¹² There Boolos identifies three distinct “thoughts” behind the concept *set* from which the axioms of ZFC follow. The first is *analyticity*, from which the axiom of extensionality follows: since it is the criterion of identity for sets, if what you are talking about does not satisfy extensionality, then they are trivially nonsets.¹³ The

second thought is the *iterative* conception, according to which sets are formed in successive stages, such that at each stage every combinatorially possible set is formed, given what was “present” at the previous stage. According to Boolos, this validates all the remaining axioms with the exception of choice and (the instances of) replacement. These, he claims, follow from the third thought, *limitation of size*: any things form a set unless there are *too many* of them, in a sense which can be made precise.¹⁴

Gödel similarly considers which axioms follow from the concept *set*, though he takes less care than Boolos does to distinguish the various thoughts behind the concept. With respect to analyticity, Gödel's view is that a wide range of axioms “might fittingly be called analytic” (Gödel, 1951, p. 321). The thesis is not as radical as it might at first seem, since Gödel operates with a dual notion of analyticity. He distinguishes between what we might call *narrowly* and *broadly* analytic propositions (Gödel, 1951, p. 321). The former are characterized by Gödel as “true owing to our definitions,” or as “tautologies,” and as such are recognizably analytic in the traditional sense. Gödel is quite explicit that the axioms of mathematics as a whole are *demonstrably* not analytic in this sense, as there is no decision procedure for arithmetic (Gödel, 1944, p. 139 and fn. 44). On the other hand, a proposition can be broadly analytic, in the sense that it is true “owing to the meaning of the concepts occurring in it” (Gödel, 1944, p. 139) or the “nature” of the relevant concepts (Gödel, 1951, p. 321). As Parsons highlights, this is why Gödel takes certain propositions to be knowable on the basis of conceptual reflection (Parsons, 2014, p. 139).

The broad conception is quite removed from the traditional conception of analyticity, since Gödel goes so far as to claim that an analytic proposition, in the broad sense, might be undecidable (Gödel, 1951, p. 321).¹⁵ Although Gödel does not go into enormous detail on the distinction between kinds of analyticity, the basic idea seems to be that narrowly analytic propositions are reducible to explicit tautologies by purely syntactic methods, whereas broadly analytic propositions can be seen to be true by semantic reflection on the primitive terms which appear therein. But it is in *this* sense that Gödel claims that the discussed axioms of mathematics might be called analytic; hence, the thesis is closer to being a restatement of his platonism, rather than a radically new account of analyticity. In any case, Gödel gives us no hint that analyticity, conceived of in this manner, distinguishes the axiom of extensionality from other mathematical axioms.

The iterative concept of *set* does, however, play an important role in Gödel's analysis, as it is from this concept that our justification of set-theoretic axioms follows (other than for those justified by extrinsic means). According to him, the primitive concepts involved in set theory are that of sets as “arbitrary multitudes” (Gödel, 1964, p. 262) and the concept *property of set* (Gödel, 1964, p. 260, fn. 18).¹⁶ Gödel's concept of sets as arbitrary multitudes seems to be what we would today think of as the iterative conception, namely that of sets obtained from the urelements by iterated application of the “set of” operation (Gödel, 1964, p. 259). At stages of the process where the set-forming operator is the powerset operation, this is “by definition” the full powerset (Gödel, 1951, p. 306, fn. 5).

The limitation of size does not play a direct role in Gödel's analysis of the concept *set*, but a similar role is played by a different maximality principle built into the iterative conception, namely that the cumulative hierarchy of sets is *inexhaustible*.¹⁷ The idea plausibly has its origin in Cantor's distinction between the transfinite and the “absolutely” infinite, but the mature statement of this principle is given by Gödel in terms of the concept *set*, rather than the hierarchy itself (as one might expect given his platonism about concepts). Moreover, it is stated in terms of the axiomatization of this concept, rather than its extension (also to be expected on the account of intuition given above). Comments suggesting that Gödel took the axiomatization of set theory, as well as the hierarchy itself, to be inexhaustible appear at least as early as (Gödel, 1933, p. 47). But the mature statement is:

[T]he axioms of set theory by no means form a system closed in itself, but, quite on the contrary, the very concept of set on which they are based suggests their extension by new axioms which assert the existence of still further iterations of the operation “set of” (Gödel, 1964, p. 260).

In other words, *the concept set is not exhausted by our ability to axiomatize it*. It is in these terms that we should understand Gödel's more cryptic comments such as that “concepts form an objective reality of their own, which we

cannot create or change” (Gödel, 1951, p. 320). The concept set, in particular, is not simply invented by us. If it were, then it would in some sense be up to us what axioms were valid with respect to it. But the nonarbitrariness of both the axioms of set theory (evidenced by the possibility of substantive debate about what follows from the concept) and the open series of extensions of the axioms (by ever stronger axioms of infinity), shows that we do not enjoy the sort of freedom a creator would have.

Just as for Gödel platonism about concepts leads to platonism about objects, the notion of the inexhaustibility of the axioms of set theory has an objectual counterpart in the conception of the set-theoretic hierarchy that it gives us. The cumulative iterative hierarchy is, on this conception, *absolutely* infinite, in being such that it cannot be *characterized from below*. There is a lot of metaphorical talk here, which is worth unpacking. A “bottom-up” approach to set theory is broadly one in which the hierarchy of sets is described in terms of its members or levels and without explicit mention of V or classes over V (as occurs, for instance, in the limitation of size axioms). In practical terms, a bottom-up selection of axioms is motivated by considerations about what sets are like and how they should behave, as opposed to considerations about how the universe as a whole (or the models of the theory) should look.¹⁸ So, to say that the hierarchy cannot be characterized from below is to say that in set-theoretic terms (and perhaps in a stronger sense), the height of the hierarchy is indescribable.¹⁹

Axioms which, in some form or another, state that the hierarchy is inexhaustible in this sense are called *reflection principles*. Such principles form a key part of Gödel's epistemology of set theory, as they are the articulation of the inexhaustibility principle which he takes to be intrinsically justified by our grasp of the concept set. Indeed at one time, Gödel appeared to think that *all* principles of set theory could be derived from some form of reflection principle (Wang, 1996, p. 283). Intuitively speaking, a reflection principle says that no formula in the language of set theory, ϕ , can correctly and uniquely characterize the hierarchy. One consequence is that if ϕ *does* characterize the hierarchy, it does not do so uniquely; rather it characterizes an initial segment. Hence if ϕ is true, then it is true when reinterpreted to be just about that initial segment, rather than the hierarchy as a whole. The formula ϕ is “reflected downward” and fails to characterize the hierarchy uniquely. The idea of a reflection principle is therefore thoroughly informal, and the strength of any formalized axiom will vary enormously depending on the language of which ϕ is a sentence and the order of the parameters which occur in ϕ . One such axiom is the *Bernays Reflection Principle*. This axiom schema consists of every sentence of the form

$$\phi(A_1, \dots, A_k) \rightarrow \exists \alpha \phi^\alpha(A_1^\alpha, \dots, A_k^\alpha)$$

where $\phi(A_1, \dots, A_k)$ is a formula in the language of second-order set theory with parameters A_1, \dots, A_k of order ≤ 2 , α is an ordinal and ϕ^α is the result of restricting the quantifiers of ϕ to V_α , a level of the standard von Neumann universe.²⁰ For the parameters, $A_i^\alpha = A_i$ if A_i is a first-order (set) parameter, and $A_i^\alpha = A_i \cap V_\alpha$ if A_i is a second-order (class) parameter.

Bernays' principle implies the existence of Mahlo cardinals; indeed, a proper class of them (Incurvati, 2016, pp. 166–167), meaning that reading Gödel as I have suggested allows us to make sense of his unusually strong claims about the power of intuition. He does not claim that we have an objectual intuition, or something like a perception, of large cardinals such as Mahlo cardinals; indeed, it is hard to see how this could be possible if we understand intuition along broadly Kantian (and hence spatio-temporal) lines. Rather, Gödel claims that an axiom implying the existence of such a large cardinal follows from the concept set (Gödel, 1964, pp. 260–261 and fn. 20). This is just as well; it is much easier to digest the idea that we have propositional knowledge of some appropriate axiom, than it is to give credit to the idea that we could stand in a quasi-perceptual relation to such a cardinal.

This point is primarily addressed toward the textual puzzle about why Gödel ascribes such *prima facie* implausible powers to mathematical intuition, and I take it that the puzzle is solved by properly distinguishing propositional and objectual intuition in Gödel's thought. There is a great deal that could (but will not) be said here about whether the concept set really does imply this axiom and others like it. The issue is a contentious one. But even those who disagree with Gödel's particular claim about intuition delivering the existence of

Mahlo cardinals can understand it as a relatively mainstream claim about the concept *set*, rather than an implausibly strong claim about the power of objectual intuition, and I take this to be clear evidence in favor of the interpretation given here.

7 | CONCEPTUAL PLATONISM

At the start of this paper, I expressed scepticism about the idea that there was one unique theory that could be said to be Gödel's platonism. What is certain, however, is that he was committed to a platonistic account of mathematics grounded in intuition. I have attempted to sketch a Gödelian theory which, as we will see, debunks the claims of mysticism surrounding Gödel's work. I will call this position *conceptual platonism*. Its core commitments are:

1. **Platonism About Concepts:** Certain mathematical concepts, including the concepts *set* and *natural number*, have real or objective content.
2. **Axioms:** The content of such concepts is partially expressed in theories which axiomatize them. Such axiomatic systems can be inconsistent or incomplete; hence, our grasp of the concepts involved can be insufficiently clear or not completely articulated.
3. **Platonism About Objects:** Certain mathematical objects exist, namely those that are quantified over by theories which axiomatize a concept with objective content.
4. **Propositional Intuition:** Certain parts of our mathematical knowledge, including the result of reflection on concepts, are neither extrinsically justified, nor tautological, nor the conclusion of a proof from other known axioms.

The theory as it stands is clearly short of a full articulation; most pressingly, there is a lack of clarity around the notion of objective content, an issue to which we will turn in the next sections. The position as I have reconstructed it is one that I think is ultimately unacceptable, but my only claims for now are that it is a sensible reconstruction of Gödel's view, and that it is tolerably clear what mathematical intuition is here, and what its role in the theory amounts to.

The account sketched above omits entirely the discussion of objectual intuition, that is, “something like a perception” of mathematical objects. My view is that this element of Gödel's thought is cleanly separable from the four elements above in a way that those four are not separable from each other. The reasons for this are both textual and philosophical. As we saw above, it is almost completely unclear what Gödel takes objectual intuition to be, how he takes it to work, and what its epistemological significance is. So, whatever cogent position we can extract from Gödel's writings will not be one in which objectual intuition has a serious role to play.

Aside from the lack of textual clarity, several of Gödel's philosophical claims make the role of objectual intuition especially puzzling. In Section 2, I cited Gödel's remark from (1964) to the effect that there is a conceptual or meaning-theoretic justification for platonism about mathematical objects, implying that (at the time of writing at least), Gödel did not take objectual intuition to be the primary basis of platonism about sets. A similar remark in the same paper implies that Gödel is “someone who considers mathematical objects to exist independently of our constructions and of our having an intuition of them individually, and who requires *only* that the general mathematical concepts must be sufficiently clear for us to be able to recognize their soundness and the truth of the axioms concerning them” (Gödel, 1964, p. 258, my emphasis). Indeed, in a frequently cited remark from the same paper, Gödel claims that “we do have something like a perception also of the objects of set theory, *as is seen from the fact* that the axioms force themselves upon us as being true” (Gödel, 1964, p. 268, my emphasis). This once again indicates the explanatory primacy of propositional over objectual intuition in Gödel's thought, as we are encouraged to accept the existence of objectual intuition on the basis of its propositional counterpart.²¹ We have also seen above that it is *propositional*, rather than objectual, intuition that provides whatever evidence we have for the existence of certain

infinite cardinals. But if objectual intuition is not necessary for platonism about mathematical objects, then what, on Gödel's account, could it possibly be for?

Quasi-empirical methods of justification in set theory are of course another major element of Gödel's thought, and undoubtedly the most well-received by later philosophers. It is possible that the notion of objectual intuition has an interesting role to play here. Gödel writes:

It seems to me that the assumption of such objects [classes and concepts] is quite as legitimate as the assumption of physical bodies and there is quite as much reason to believe in their existence. They are in the same sense necessary to obtain a satisfactory system of mathematics as physical bodies are necessary for a satisfactory theory of our sense perceptions (Gödel, 1944, p. 128).

So one possible role for objectual intuition is the provision of “sense data” to be accounted for by mathematical theories, fueling the quasi-empirical justification of axioms which lie beyond the reach of propositional intuition. Unfortunately, a full discussion of these issues is out of the question here and must be set aside for future work. But since the extrinsic/quasi-empirical justification of axioms is clearly an independent strand of Gödel's thought, in that one could adopt conceptual platonism without acknowledging the legitimacy of *any* quasi-empirical methods in set theory (and certainly vice versa), it suffices to say for now that objectual intuition does not have a significant role to play in Gödel's platonism as it relates to the intrinsic justification of axioms. With our reconstruction in hand, we can now move on to consider the merits of Gödel's view, understood as conceptual platonism.

8 | EXISTENTIAL INTERNALISM

To sum up where we have got to so far: Gödel's platonism concerns both mathematical objects and concepts, and its epistemology is grounded in mathematical intuition. Gödel posits the existence of (at least) two kinds of mathematical intuition, the objectual and the propositional. When the problematic (yet unimportant) notion of objectual intuition is shorn away, this view, which I have called *conceptual platonism*, combines a novel ontology of mathematical concepts and an equally novel account of their role in mathematical thought with elements of various philosophical views that are more-or-less mainstream. The central such elements are that our knowledge of mathematics is sometimes nondeductive, nonextrinsic, and nontrivial; that we should believe that sets exist because they are quantified over by axioms that we accept; and that the content of the concept *set* is partially expressed in the axioms of set theory, which can (in the case of at least some of the axioms) be justified by reflection on the concept *set*.

Before proceeding to assess the philosophical merits of this view, it is of some general interest to make sense of its place among other views in the philosophy of mathematics. As we have seen, a cornerstone of conceptual platonism is the idea that we gain knowledge of the truth of axioms via reflection on concepts with sound or objective content. Since some of the axioms which can be so verified by propositional intuition have existential implications, the existence of mathematical objects, and of sets in particular, follows. So according to Gödel, simply by mathematical thought (in a broad sense that is not restricted merely to the production of mathematical proofs), we can establish the existence of a whole structure of mathematical objects satisfying the axioms of set theory (or at least, the portion of the axioms which are intuitable). And in this aspect of his thought, Gödel has many close and respectable neighbors in the philosophy of mathematics.

For example, Gödel's platonism incorporates elements strikingly similar to Hilbert's philosophical thinking about axiomatic theories, as discussed in his correspondence with Frege. Hilbert's view of such theories is neatly expressed in his claim that “[i]f the arbitrarily given axioms do not contradict one another, then they are true, and the things defined by the axioms exist” (Hilbert, 1899, p. 42). Gödel's view of axiomatic systems is certainly not identical to Hilbert's, and care must be taken in ensuring that the views are not conflated. The crucial points of difference

between Hilbert and Gödel are that Hilbert takes the primitive terms appearing in his axioms to be *meaningless* in isolation, whereas Gödel explicitly speaks of the meanings of the primitive terms of set theory (Gödel, 1964, p. 260). Similarly, Hilbert takes the axioms of a theory to be partially interpreted syntax, but Gödel thinks they express part of the content of the concept that they axiomatize. Nonetheless, the similarity is clear; both are willing to infer the existence of certain things on the basis of mathematical reasoning alone. Hilbert is willing to infer the existence of a mathematical system satisfying a theory on the basis of a consistency proof for it, and Gödel is willing to infer the existence of mathematical objects that are quantified over by a theory which axiomatizes a concept which has been shown to have objective content.

Of course, it is to some extent a matter of taste whether or not an analogy holds good. I find myself impressed by the similarity of Gödel's view with Hilbert's, but perhaps others will be less so. But for interpretative purposes, what is significant is not whether the views are “similar” in some vague overall sense, but rather that there is an important respect in which they are similar. To make this particular respect clear, I would like to canvas a broad distinction between kinds of views with respect to the ontology of mathematics; or better, with respect to the kinds of evidence that might support the supposition that some mathematical objects of a particular kind exist. Some new terminology will be helpful here:

A view is *existentially internalist* with respect to a particular mathematical concept if, and only if, according to that view, establishing that the concept is instantiated, or that a theory axiomatizing the concept is satisfied, is a *broadly mathematical matter*.

Likewise, a view is *existentially externalist* with respect to a particular mathematical concept if, and only if, establishing that the concept is instantiated, or that a theory axiomatizing the concept is satisfied, requires the use of methods that *are not properly mathematical*.

Admittedly, this distinction falls far short of any kind of formal precision. This is, however, deliberate; given that I am trying to cast a very wide net over a selection of diverse positions in the philosophy of mathematics, the significance of the distinction hinges on its ability to classify views which are distinct in respect of numerous details as being of the same kind. Positions will differ, for instance, on what they take to be a mathematical concept and what it is for a theory to axiomatize a concept. (Do **ZFC** and **ZFC** + $V = L$ axiomatize two different concepts, or do they axiomatize the same concept differently?) They will also differ on what it is for a concept to be instantiated. (Is the concept set satisfied by particular objects, namely the sets, or is it instantiated by a mathematical structure, or a kind thereof?) But we have, I think, a firm enough grasp on the informal notion of mathematical thought, in a broad sense, to make sense of the distinction between internalist and externalist views with the help of a few examples.

In the first instance, we can see that Hilbert and Gödel both sit firmly in the internalist camp (at least with respect to the particular concepts that they individually discuss). For Hilbert, establishing the consistency of a theory is sufficient to ensure the existence of the things it implicitly defines, and giving a proof of the consistency of a theory is a broadly mathematical matter (if we construe “mathematical matters” broadly enough to include logic). Likewise, for Gödel, the instantiation of a concept is guaranteed by the existence of an intuition which is of sufficient clarity to produce a forceful and nonarbitrary theory which partially expresses its content (see Section 9 below). The exercise of mathematical intuition is, of course, a broadly mathematical activity in the intended sense.

Having identified an internalist as somebody who thinks that the instantiation of a mathematical concept can be established on the basis of broadly mathematical considerations alone, we can see that there are many other examples of internalist views among central figures in both the history of analytic philosophy and the discipline as it stands at present. For example, the various kinds of logicism qualify as internalist in my sense; Frege maintained that numbers exist simply as a matter of logic (Frege, 1884), and neo-logicists maintain that the existence of numbers follows from an analytic or conceptual truth, namely Hume's Principle (e.g., Hale & Wright, 2001). Since logical reasoning

and the identification of conceptual (mathematical) truths are both broadly mathematical matters, the classification of these approaches is straightforward.

Similarly, this kind of internalism seems to appear with relative frequency in the structuralist vein of thought. Dedekind, for instance, thought that the numbers were “free creations” of mathematical thought (Dedekind, 1888, p. 203), and mathematical thought is certainly a mathematical matter in the broad sense intended here. Indeed Dedekind thought that the existence of a simply infinite system could be demonstrated by a simple proof (Dedekind, 1888, pp. 217–218). Amongst more recent structuralists, we find Shapiro positing that certain axiomatic theories have a property he calls “coherence.” This is a “primitive, intuitive notion, not [to be] reduced to something formal” (Shapiro, 2000, p. 135), that does for theories in general what syntactic consistency does for first-order theories (thanks to the completeness theorem). That is, if a set of axioms has this property, then there is a mathematical structure which satisfies it (Shapiro, 2000, p. 133). Given Shapiro's platonistic understanding of the existence of structures, his view is, in this respect, remarkably close to Gödel's as I have understood it, differing mainly in his focus on theories rather than the concepts which they axiomatize.

A similar internalist tendency is exhibited in Balaguer's *full-blooded* platonism, according to which the consistency of a mathematical theory is sufficient for its truth (Balaguer, 1995, p. 315). Although this statement makes Balaguer's view look strikingly like Hilbert's, the two are not identical. Balaguer is noncommittal on exactly what consistency amounts to (Balaguer, 1995, p. 305), but presumably he *does not* mean syntactic consistency; the view encompasses second-order theories, and there are consistent second-order theories which are unsatisfiable (such as $PA_2 + \neg Con_{PA_2}$). He seems comfortable with the idea that the consistency of a theory is a primitive notion (Balaguer, 1995, p. 319).

So we can see that a number of seemingly respectable positions are correctly described as being existentially internalist. Further examples could be given of course, but perhaps the intended sense of “broadly mathematical considerations” can be illustrated more helpfully at this point by considering the kinds of views which qualify instead as externalist in the above classification.

An example of an externalist platonist is Quine. According to his naturalistic holism, “[o]ur ontology is determined once we have fixed upon the over-all conceptual scheme which is to accommodate science in the broadest sense” (Quine, 1948, pp. 16–17). We determine the ontological commitments of the conceptual scheme by examining the range of the bound variables appearing in some favored formulation of it (Quine, 1948, p. 15). So on this kind of picture, the only way to establish that some mathematical objects exist is to see if they are in the range of the first-order quantifiers of our best overall scientific theory; if, and only if, we find the objects in question in the range of our quantifiers, then we ought to believe in their existence. Such a view does not qualify as internalist with respect to any mathematical concept: the objects of mathematics must, according to the Quinean picture, earn their keep as part of the best overall scientific theory. But it is not a broadly mathematical matter whether the best overall formalization of science quantifies over mathematical objects, for a panoply of nonmathematical considerations are relevant to determining what our best overall scientific theory is.

An example of an anti-platonist externalist position is Field's fictionalism. A crucial part of making good on Field's proposal is providing nominalistically acceptable alternatives to standard scientific theories which help themselves to the full apparatus of classical mathematics. Where such a nominalization can be provided, Field sees “no reason to regard this part of mathematics as *true*” (Field, 2016, p. 2). Contraposing, what it would take for Field to regard a mathematical theory as true (and hence the concept axiomatized by it as instantiated) would be our *failure* to provide a nominalized alternative to a scientific theory (in sufficiently good standing) which deployed the mathematical theory in question. Since it is not a mathematical matter whether or not there is some such theory which would, by Field's lights, vindicate belief in the relevant mathematical objects, he does not qualify as an internalist either.²²

An important point to note is that whether a view counts as internalist or externalist is primarily an *epistemological* matter. Internalist and externalist alike will agree that any concept which has the property *axiomatized by a true theory* is instantiated by objects or a structure. The crucial difference between an internalist and an externalist is that

for the internalist, *establishing* the truth of the theory is a broadly mathematical matter, whereas the externalist requires that nonmathematical considerations be brought to bear.²³

Given this epistemological dimension, the distinction between internalism and externalism crosses various traditional boundaries in the philosophy of mathematics. Platonists and anti-platonists alike are to be found in each camp. We have already seen an example of both in the externalist camp; on the internalist side, Gödel, of course, is a platonist, but an intuitionist who regards mathematical objects as mental constructions still qualifies as an internalist. This is because the construction of a proof validating an existential claim is an exercise of properly mathematical thought. It also worth noting that an internalist is not automatically someone who thinks a justification for belief in the existence of mathematical objects must be provided a priori; that will depend on whether one thinks any properly mathematical justification can be a posteriori. Unfortunately, it is beyond the scope of this article to delve more deeply into these issues, and the many other questions that could be asked about the distinction. For now, it is sufficient for my purposes that the place of Gödel's thought in the literature has been clarified; as we will see, the notion of an internalist view, and the fact that such views are widespread in the philosophy of mathematics, will play a crucial role in responding to some of Gödel's most prominent critics.²⁴

9 | INTERNALISM, MYSTICISM, AND THEOLOGY

By now, I have said plenty in defence of Gödel's conceptual platonism. For all that, I think it has at its heart a serious philosophical issue, and to see why I will examine several prominent criticisms of Gödel's view, starting with Chihara's charge that Gödel's position is mysterious and overly theological. As put, I think the charge is misplaced, but nevertheless the commitments of Gödel's view here render it untenable, at least pending further analysis.

The charge of mysticism is, I think, flatly false, and can be seen to be so on the basis of textual evidence. Chihara claims that “Gödel's appeal to mathematical perceptions to justify his belief in sets is strikingly similar to the appeal to mystical experiences that some philosophers have made to justify their belief in God” (Chihara, 1991, p. 21). If Chihara were right and Gödel's platonism was based primarily on having “visions” of sets, this charge would have some force. But as we have seen, *propositional* intuition, rather than its perception-like counterpart, is the driving force behind Gödel's platonism, and the experience of an axiom irresistibly striking one as true is far from mystical; indeed, it is a commonplace experience in the learning of mathematics. Chihara's failure to observe the distinction between objectual and propositional intuition is the downfall of his interpretation on this point. Moreover, we have seen that various mathematical and philosophical writers engage in the practice of conceptual reflection as a means of validating axioms, and there is nothing obviously mysterious about this practice, even if it is not the whole story about the justification of axioms.

The charge of *theology*, however, is in my view a more serious one, although ultimately ill-founded. In the 11th century, Anselm of Canterbury wrote in the *Proslogium* “So truly, therefore, do you exist, O Lord, my God, that you can not be conceived not to exist” (Anselm of Canterbury, 1077/8, Section 3). This is the conclusion of his *ontological argument*; that the very concept *God* guarantees the existence of an object falling under it. The premises being (roughly speaking) that the concept *God* subsumes all perfections and that a being which did exist would be more perfect than one which did not.

The connection between the ontological argument and Gödelian platonism is explicitly remarked upon by Chihara (1991, p. 20), and despite his misreading of Gödel at certain junctures, his complaint expresses an understandable concern, even for the cleaned-up conceptual platonism I have presented here. The conceptual platonist might, for instance, reason along the following lines: merely by reflecting on the concept *set*, we can determine that the axiom of infinity (for instance) expresses some of its content. Since that concept has real content, the axiom of infinity is *true*, and therefore an infinite set exists. Hence, we might think that conceptual platonism is founded upon a mathematical version of the ontological argument, for the platonist, just like Anselm, has deduced the existence of objects out of mere concepts.

Of all such charges made against Gödel, this, I think, has the most force. However, the philosopher of mathematics should look closer to home than theology to find an analogue of Gödel's view. Indeed, Chihara's charge of theology could seemingly be leveled equally well against *any* of the internalist views discussed in the previous section, and while Anselm may be a theologian *par excellence*, we should not want to say the same of Frege, Hilbert, Shapiro, and the rest.²⁵

My point is not, of course, that we should adopt conceptual platonism merely because it sits in such illustrious company. Rather, the point is that we should not charge Gödel's view with mysticism or theology *simply* because it takes the existence of objects to be verifiable by reflection on the relevant theories and concepts. The basic pattern of Gödel's inference, from concepts to objects, is shared by a number of views which are clearly nontheological by any reasonable standard. So Chihara's attempt to portray Gödel as lacking a serious philosophical position to offer is found wanting.

There does, however, remain the question of the viability of the specific form of existential internalism offered by conceptual platonism. Wright emphasizes that the ontological argument is mistaken in detail, rather than in style (Wright, 1990, p. 163); whether or not we agree with him on the theological point, it is clear that not just any old internalist view is automatically a good one. We may still judge conceptual platonism negatively, even if the charges of mysticism and theology are misplaced. In assessing this, it is helpful to draw on a distinction sketched by Martin (2005, pp. 209 and 212) between two differing interpretations of Gödel's remarks on the concept set. The first is to interpret Gödel's remarks in the "straightforward" sense, according to which the concept set is what sorts sets from other objects. In Fregean terminology, this is the first-level concept under which all and only sets fall. The second interpretation offered by Martin is to read Gödel as talking about the concept set in a structural sense.²⁶ In Fregean terminology, this is the second-level concept under which all and only first-level concepts of set fall. The concept set in this sense characterizes the properties that structures of objects satisfying a first-level concept must have if that concept is to genuinely qualify as a set-concept, rather than some other kind of concept.

Martin's central criticism of Gödel is that reflection on the concept set can only give us structural or second-level information. We might think, for instance, that reflection on the iterative conception of set could deliver the verdict that the axiom of pairs is true of it. At each level of the hierarchy, we have *all possible* sets given what has come before. So if a and b both appear in V_α , the iterative conception dictates that the set $\{a, b\}$ appears at $V_{\alpha+1}$. Hence, the axiom of pairs is satisfied. This axiom irresistibly strikes us as being true at the second level, insofar as we think no first-level concept could be a set-concept if there were structures of objects satisfying it which did not validate the axiom of pairs.

What Martin rejects, however, is the idea that reflection upon the concept set can yield similar knowledge in the straightforward sense. The complaint is particularly vivid when we consider explicitly existential axioms, such as the axiom of infinity. Given that the iterative conception dictates that the sequence of stages goes on as far as possible, we might be able to convince ourselves that any set structure must have a stage indexed by ω . We already have the means to form the stages indexed by finite ordinals, so the result of the closure of the operation of forming such stages must also be a set. Even if convincing, however, such reflection cannot show (according to Martin) that *there* is such an infinite set, only that any set structure must contain such a set. Martin's criticism is thus a much-clarified version of an earlier one offered by Chihara. The latter distinguishes between the *ontological* platonist (a platonist in the straightforward sense) and the *mythological* platonist (Chihara, 1973, pp. 62–63). The mythological platonist regards the axioms as true "in the story" of set theory, while viewing the story itself as myth. For such a platonist, the axiom of infinity is (or at least might be) *literally false* despite being true of the concept set. The mythological platonist and Martin therefore seem to be in broad agreement, though the latter dispenses with the vocabulary of story and myth.

Gödel's position, however, is that reflection on the concept set allows us to determine that axioms are true of the concept set in the straightforward sense. The meaning of the primitive terms of set theory do not *just* determine that any set structure must contain an infinite set, though according to Gödel, they do determine this. Much more than this, they determine that there is such a structure satisfying the axiom of infinity. Gödel's "criterion of truth in

set theory” is the existence of a sufficiently forceful axiomatization of the concept set and a nonarbitrary series of extensions of that axiomatization (Gödel, 1964, pp. 268–269). Since (according to Gödel) our intuition of the concept set is sufficiently clear to produce the axioms of set theory, which “force themselves upon us as being true” (Gödel, 1964, p. 268), and since these axioms can be extended non-arbitrarily by further principles (e.g., successively stronger axioms of infinity) which serve to “unfold the content of the concept” (Gödel, 1964, p. 261) using further appeals to intuition, the criterion of truth is satisfied. So according to Gödel, there is no gap between the truth of the axioms in the structural sense and the straightforward sense, just as for Hilbert there is no gap between the consistency of a theory and the existence of a system which is characterized by it. It is the objectivity of the content of the concept set which bridges the division between structural and straightforward truths about sets, and it is in terms of this objectivity that the conceptual platonist must answer Martin's argument.

Is this a plausible line of defence? Martin is essentially rejecting the internalist aspect of Gödel's position, that some concepts guarantee their own satisfaction by a structure. As he puts it:

I don't see why the consistency or coherence or “soundness” of the concept of set implies that it is instantiated[...] Suppose the nominalists are right and there are no abstract objects, and suppose there are only finitely many concrete ones. Would this mean that the concept of natural number (in the sense of the concept of a number sequence) was inconsistent or incoherent? Wouldn't it still make sense to consider what a number sequence would be like if there were one? (Martin, 2005, p. 220)

Whether or not this criticism can be adequately responded to strikes me as being of very general interest for internalist views of all kinds, far beyond the relatively narrow confines of Gödel scholarship. The internalist view has great historical pedigree in the philosophy of mathematics and is a mainstream position for analytic philosophers today, as shown by the examples discussed in Section 8. The examples could certainly be multiplied further, but that is not the important point: the current and historical popularity of internalist views means that the widespread rejection of Gödel's version of it has something important to teach us about the philosophy of mathematics more generally, and should prompt us to think very specifically about *why* we might reject it.

Martin's criticism is certainly compelling, but we can imagine the kinds of thing an internalist might say in response. For one thing, it is not completely clear how an internalist could make sense of the idea of *what a number sequence would be like if there were one*, under the supposition that there is not. I presume that for most internalists, if the concept *natural number* is instantiated then it is necessarily instantiated, in some strong mathematical or logical sense of necessity (though any physical instantiations it might have do not, I presume, exist of necessity under any of the views canvassed above). Such counterpossibles are difficult to make much pretheoretic sense of, so it is not clear to me that the internalist must take the criticism seriously without further elaboration.

Glossing over this difficulty, however, the real force of Martin's complaint against internalism is still not apparent to me. As I remarked above, the disagreement between internalists and their detractors is best construed as a disagreement over whether the instantiation of a mathematical concept can be established by broadly mathematical considerations. And this reading accords well with Martin's complaint; as he says, “[a]ppplied mathematics is concerned with instantiations of mathematical concepts. But why are instantiations needed for pure mathematics?” (Martin, 2005, p. 220, Martin's emphasis). So clearly Martin does not think that it is a mathematical matter whether or not a concept is instantiated. But what evidence would he consider in favor of the instantiation of a concept, other than finding a physical instantiation of it? Physical instantiations are not of particular concern in the platonism–nominalism debate; what is relevant there is the instantiation (or otherwise) of concepts by *abstract* objects or structures. And this, I think, is what the Gödelian platonist, and the existential internalist in general, will seize on.

If all the *mathematical* evidence points to the coherence of a concept via an intuitively compelling axiomatization of it, of which we are confident of the consistency, then, the internalist will ask, *what further* evidence could be relevant? Of course, Martin's answer suggests that further evidence is to be sought in applications, but note that it is not

the concern of applied mathematics whether a mathematical concept is instantiated by *abstracta*, or a structure thereof. Clearly Martin is demanding evidence beyond the broadly mathematical, and the same is true of Quine, Field, and other externalists. But what has not been made sufficiently clear, to my way of thinking, is whether this demand for further evidence is legitimate.

We have seen one way of portraying the internalist which puts them in an uncomfortable light: as effectively trying to *define* the objects of mathematics into existence, a position to which we might be reluctant to give much credit. But likewise, to the internalist, their opponent might resemble a philosophical character of equal ill-repute: the radical sceptic. The sceptic demands, even once all the overwhelming *perceptual* evidence for the existence of chairs and tables is in, that their opponent supply *further* evidence of some kind, to the effect that their perceptual evidence is not systematically misleading, as it would be if their opponent were dreaming or were a brain in a vat. Likewise, the externalist demands, even once all of the mathematical evidence is in to the effect that a concept such as *natural number* is coherent, objective, and axiomatized in the right way, some further evidence that the numbers (or the number structure) *really* exist(s). If the antisceptic's position is reasonable with respect to the external world, why is the internalist's not similarly respectable regarding the existence of the "mathematical world"?

None of this particularly vindicates Gödel's version of internalism, of course. Any internalist still owes us a compelling account of which concepts have the right features to guarantee their instantiation, and an account of why it is that those features do so guarantee the instantiation of concepts which have them. But on the other side of the coin, an externalist similarly owes us an account of *what* nonmathematical evidence is relevant to assessing whether a concept is instantiated, and *why* the *prima facie* relevant evidence, of a broadly logical and mathematical kind, is insufficient to settle the issue.

Now, I do not think that this challenge is insurmountable for the externalist, and in fact, some externalists have already discharged their duty on this front. Consider Quine, for instance. His holism explains precisely why he does not take mathematical matters alone to be sufficient to settle questions of mathematical ontology, and his naturalism explains why he *does* take scientific considerations to be relevant. But here is the significant point for present purposes: even if the ontological argument is nothing more than mysticism, the same can clearly *not* be said of rejecting Quine's holistic naturalism. And more generally, taking questions of mathematical ontology to be broadly mathematical in nature is also not mysticism, nor is it theology. It is, as far as the general outline goes, a perfectly respectable philosophical position that takes mathematical thought to be a legitimate and autonomous means of settling questions about its own objects.

Of course, a great deal more could be said in general about the philosophical merits of internalism and externalism, broadly construed. As Martin himself notes, it is an advantage of a philosophical account of mathematics if it can take the existential sentences of mathematics at face value (Martin, 2005, p. 220), which an ontologically generous internalist like Gödel will typically be in a position to do. On the other hand, generous internalist views will typically score poorly on the issue of ontological parsimony as a consequence. But these are deep waters, and I cannot hope to resolve any of these issues here. It is unclear, for instance, how much of an advantage a face-value interpretation of mathematical discourse is,²⁷ and equally whether ontological parsimony is relevant at all to theory choice in mathematics. For now, all we can hope to do is make some progress on the assessment of Gödel's internalism specifically.

As I see it, there are three options. One option is to insist that any internalist view is unacceptable *simply because it is such a view*. This option might appeal to one who thought that all views of this kind resemble the ontological argument to an uncomfortable degree, and hence all need to be rejected. But I have given some reason above why I think this line of thought should be resisted, and in any case, this does not seem to be a response that tempts many philosophers, even concerning recent debates. The platonistic theories of Balaguer, Shapiro, and others have been taken seriously, and criticism of these theories has typically focused on the specifics of the views, rather than on general accusations of mysticism. These accounts certainly deserve to be taken seriously, and so a wholesale rejection of internalist theories regardless of the specifics of each view would be ill-advised.

Another option, of course, is to accept the tidied-up version of Gödel's platonism that I have presented in this paper, or a variant thereof. My expectation, of course, is that this will not be a popular course of action. But if we are going to reject Gödelian platonism without doing so simply because it is a kind of internalism, we must identify something specifically wrong with Gödel's version of this view, and this is the third option. The interesting point is that it is not straightforward to say exactly what is wrong with it.

One worry might be that it is unclear what it means to ascribe objective content to a concept. Hence it is unclear *why* a concept with this kind of content is guaranteed to be satisfied by some objects (unless that guarantee is simply what the having of such content consists in). But the lack of a definition (or similar) of the objectivity of a concept's content is not (terminology aside) a problem which is unique to Gödelian platonism. A number of internalist theories discussed above take the relevant property to be primitive (e.g., Shapiro's notion of *coherence*), so why should the objectivity of a concept's content not be taken as such? Of course, it would be preferable to have a more substantive explanation to hand, but any philosophical position will have to take some notions as primitive. So if this is the central problem with Gödelian platonism, then it seems to me that it is one which we could perhaps tolerate.

But even if we take the notion of objective or real content to be theoretically primitive, the account suffers from a serious lack of clarity. In particular, Gödel does not give a tolerably clear criterion of application for this notion, so it is deeply unclear *which* theories are supposed to axiomatize concepts with objective content. And if we lack a means by which to distinguish the concepts which have real content from those which lack it, then there can be little justification for taking the notion to be primitive. After all, the whole point of deploying the notion was to explain the existence of certain mathematical objects in terms of mathematical concepts, but no explanation will be forthcoming if we cannot distinguish the concepts which have objective content from those which lack it.

One of the hallmarks of a concept with objective content is that its axiomatization fails to settle all of the relevant mathematical questions, necessitating further appeals to intuition (Gödel, 1964, p. 269). This principle sorts the wheat from the chaff, to some extent, since no concept with objective content can be properly axiomatized by an inconsistent theory, or even by a negation-complete theory, since such theories are not extendible by further axioms that can be gleaned from conceptual reflection. But since most theories of central philosophical interest are neither inconsistent nor negation-complete, this still leaves an overly wide field; $\text{PA} + \neg\text{Con}_{\text{PA}}$ for instance, is neither inconsistent nor negation-complete, but presumably does not characterize a concept with objective content, since one of the axioms is false.

The other signifier of objectivity for Gödel is that the “axioms force themselves upon us as being true.” This is a deeply unhelpful means of sorting the theories which axiomatize a concept with objective content from those theories which do not. Presumably, the axioms of PA (with the quantifiers interpreted as ranging over \mathbb{N}) would pass muster on this front; their intuitive clarity is sufficiently evidenced by their near-universal acceptance by those familiar with them. But in the case of the concept *set* specifically, the ongoing controversy about the justification of ZFC , as well as its extension by large cardinal principles, ought to be a primary cause for concern. The axioms strike *many* people as true, but this acceptance is by no means universal.²⁸ So how can we tell whether the concept *set* has objective content or not?

Without some tolerably clear means of distinguishing concepts with objective content from those which lack it, it is simply *pointless* to claim that if a concept has such content, then it follows that some objects satisfy it. And this is ultimately why we should reject conceptual platonism: a crucial part of Gödel's theory is left insufficiently developed, meaning that the theory cannot provide an informative account of the phenomena it seeks to explain. Even if there is no harm in taking the notion of real content to be a primitive one, any theoretical utility that the notion might have is conditional upon giving it sufficiently clear criteria of application, and Gödel does not provide us with such.

This rejection comes only to agnosticism on my part, however. I do not currently see how an adequate version of Gödel's internalism can be developed, but perhaps such a development is nevertheless possible. Indeed it does not seem (to me at least) that no clarification could be given on this point, especially considering that the experience of finding axioms to be sufficiently forceful is a relatively well-documented one. And more relevantly, it is not clear

that Gödelian platonism is significantly worse-off than its internalist competitors in this respect. Would anybody claim to have a fully developed account of exactly which concepts guarantee their own instantiation or of which theories guarantee their own satisfaction, and how we can distinguish those which do from those which do not?

If correct, this point is of broad significance for the philosophy of mathematics; as I have argued, existentially internalist views are relatively common both historically and currently in the philosophy of mathematics. It is typical of such views that the feature of a concept or theory which guarantees the existence of the relevant objects is a primitive, or else informal, one which is not to be characterized in precise terms. Hilbert might be thought an exception here, given that the notion of syntactic consistency can be made sufficiently precise. But even so, his account is only properly applicable in the limited context of first-order theories, since the syntactic consistency of a higher-order theory is not sufficient for its satisfiability. Now, given that internalism is, like any other philosophical view, typically formulated with less than complete precision, if this is the central complaint against Gödel and if it is legitimate, then it is damaging to a large swath of views. Of course, imprecision comes in degrees, and perhaps Gödel might be thought worse than the typical offender in this regard. But it is nevertheless important that, having dealt with the accusations of mysticism and theology and having articulated more carefully what is actually wrong with conceptual platonism, all we have come across is a problem that is applicable to a wide range of positions.

In summary, my sense is that somebody has been treated unfairly: either Gödel has been treated much too harshly or many other internalist views have been treated much too generously. I believe the former is the case, and that a sympathetic attempt to fill in the missing details of Gödelian platonism is warranted. Despite that, it is not currently viable. Unless absolutely necessary, the notion of a concept having objective content should not be taken to be primitive, and in any case, the conceptual platonist owes us a more detailed account of how we can determine which theories axiomatize the right kind of concept. Although I do not currently see how Gödel's version of existential internalism can be made to work, the fact that his account has an explanatory gap can hardly qualify it as mysticism. Gödel has an *argument* as to why the existence of sets follows from the nature of the concept set, albeit an unsuccessful one. But that is a rather mundane philosophical problem.

10 | CONCLUSION

The central goals of this paper were to get clear about what Gödel's platonism amounts to and to assess the merits of the view, particularly in relation to charges of mysticism and theology. Although I am sceptical of claims by any one position to represent Gödel's views uniquely, I outlined a position called *conceptual platonism* reconstructed from salient remarks made by Gödel in several major works. According to this position, what matters for establishing platonism about mathematical objects is not an account of how we “interact” with acausal objects or similar, but rather an account of how we intuitively apprehend the truth of axioms which express the content of the concepts under which those objects fall. In particular, sets are thought to exist because they are quantified over in the axioms of set theory, the intuitive forcefulness and nonarbitrariness of which suffice to establish the objectivity of the iterative concept of set. The notion of mathematical intuition is prominent in the account, but I have argued that it should be construed purely propositionally. The notion of objectual intuition does appear in Gödel's writings but does not play a philosophically significant role, in any clear respect.

I also argued that the central flaw in this position is the lack of a developed account as to how we can distinguish, even in principle, between those theories which axiomatize a concept with objective content and those which do not. Although I do not think we can accept the view in light of that serious omission, I do think it is a sophisticated position which cannot be fairly accused of mysticism or theology. Indeed, the view shares crucial points of contact with other views in the philosophy of mathematics, namely those which are existentially internalist. Despite its flaws, conceptual platonism stands as a serious philosophical position, and not a series of mystical insights.

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ENDNOTES

- ¹ Here and elsewhere, Gödel uses the terms “platonism” and “realism” interchangeably.
- ² Though a first-order formulation of the theory is standard today, there are good reasons to read Gödel as having second-order axioms in mind in many of his remarks. Firstly, Gödel's conception of set theory owes much to Zermelo's, whose system (Zermelo, 1930) is most naturally taken to be second-order. Further, Gödel claims that set theory involves the primitive term *property of sets* (Gödel, 1947, p. 181 fn. 17), which suggests a second-order axiomatization with properties as the values of the second-order variables.
- ³ The details, being somewhat technical, are condemned to an endnote. Gödel claims that the validity of recursion up to ω^2 can be made “immediately evident” in the context of a discussion about the extent of finitary evidence (Gödel, 1972, p. 273). Since he takes finitary evidence to be the deliverance of *concrete intuition*, this strongly implies that he followed Hilbert in believing in the existence of this faculty. So Gödel's remarks here may be less direct than in the other papers cited, but are nonetheless extremely suggestive.
- ⁴ Hilbert is not clear about the extent of intuitively justified arithmetic, but for a classic analysis of the extent of finitary arithmetic, see (Tait, 1981).
- ⁵ A cardinal κ is (strongly) inaccessible iff it is uncountable, regular, and such that $2^\lambda < \kappa$, for all $\lambda < \kappa$ (Jech, 2003, p. 58). A cardinal κ is (strongly) Mahlo iff the set of strongly inaccessible cardinals $< \kappa$ is stationary in κ (Kanamori, 2003, p. 21).
- ⁶ Since it presumably could, in at least some cases, be established a priori that an axiom candidate has such virtues, I prefer the term “quasi-empirical” to Parsons' “a posteriori.” My thanks to an anonymous referee for highlighting the issue.
- ⁷ Martin (2005, p. 222) asserts that Gödel would no doubt reject an account of intuition which equates intuition with the understanding of a concept. This appears to be for reasons that relate to the perception of mathematical objects. But note that the reading I have urged of Gödel is *not* that intuition and the understanding of concepts are to be equated, but that there are two distinct notions of intuition, the epistemologically significant of which is roughly the understanding of concepts.
- ⁸ An example of a flawed concept of this sort might be the concept of a Fregean extension.
- ⁹ I do this without taking a stand on whether there is any good sense in which concepts are a kind of object.
- ¹⁰ The latter might be something like Burgess' notion of “heuristic intuition” (Burgess, 2014, p. 27).
- ¹¹ Which is not to say that I think it's *correct*. See Section 9.
- ¹² See also Boolos (1971) and Paseau (2007) for more detail. Parsons also lists Tait and Koellner as examples of philosophers who have at least taken the idea that axioms can be conceptually justified in this way seriously (Parsons, 2014, p. 149). In addition, you are likely to find a conceptual argument for at least some of the axioms in any set theory textbook.
- ¹³ This argument is offered by Boolos only tentatively, due to Quinean anxiety about the notion of analyticity.
- ¹⁴ The limitation of size is usually given as a second-order axiom, that any class is a set iff either there is no bijection between the class and V , the universe of sets (von Neumann Limitation of Size) or there is no bijection between it and On , the class of ordinals (Cantor Limitation of Size). The idea is certainly independent of the iterative conception; as Parsons emphasizes, these axioms are often understood as sorting principles for dividing sets and proper classes in a given second-order setting, motivated by the need to avoid paradox (Parsons, 2008, p. 133). Under either axiom, choice and replacement do not imply the existence of sets which are too big.

- ¹⁵ Though unusual, Gödel's view here is not completely without precedent. Ramsey argues that certain set-theoretic propositions might be both unprovable and tautological (Ramsey, 1925, p. 224). Of course, Ramsey is operating with a very particular notion of a tautology here.
- ¹⁶ The notion of arbitrary multitude here is meant as opposed to that of a *definable* multitude. A truly arbitrary concept of multitude would perhaps include non-well-founded sets, but Gödel does not discuss this issue here.
- ¹⁷ Parsons (2008, p. 133) identifies the inexhaustibility principle as intimately linked to the limitation of size principle. However, this seems to me to obscure a connection between the inexhaustibility principle and the iterative conception of set which does not hold between that conception and the standard versions of the limitation of size idea.
- ¹⁸ All of this is merely heuristic, of course, and the bottom-up approach to set theory is not precisely defined. But for an example of the approach in action, see Tait's work, (Tait, 2005) in particular. As he emphasizes there (Tait, 2005, p. 134), the approach is somewhat out of fashion, with most set-theoretic work today being top-down, thanks to the focus on models of set theory.
- ¹⁹ Of course, ZFC has models whose height can be described in set-theoretic terms as a strongly inaccessible cardinal. But for the advocate of inexhaustibility, this just shows that no such model could contain the whole hierarchy.
- ²⁰ Where $\alpha = 0$, V_α is the (possibly empty) set of urelements. Where α is a successor ordinal, $V_\alpha = P(V_{\alpha-1})$. Where α is a limit, $V_\alpha = \bigcup_{\beta < \alpha} V_\beta$.
- ²¹ Incidentally, Chihara's interpretation of Gödel, according to which his view is decidedly mystical, places a lot of emphasis on this passage. It is indeed an important passage, but Chihara's interpretation, that Gödel thought of experiences of the forcefulness of axioms as themselves being perception-like experiences of sets (Chihara, 1982, p. 214) is, to my mind, a failure to observe the objectual/propositional distinction in Gödel's writing. Notably, Chihara's later interpretation of this passage is significantly weakened (Chihara, 1991, p. 16); there he understands Gödel as merely offering the propositional experiences as a justification for the claim that we stand in a quasi-perceptual relation to sets.
- ²² Both Quine and Field locate the external factors relevant to mathematical ontology in the application of mathematics to science. I do not take this to be a necessary feature of externalism, though it seems to be a feature of its most prominent forms.
- ²³ A related point is made by Wright when he characterizes the disagreement between himself and Field as to whether neo-logicism is theological as a disagreement about what is required to vindicate a platonist conception of numbers (Wright, 1990, p. 167).
- ²⁴ I am very grateful to an anonymous referee at the *European Journal of Philosophy* for pressing me to be clearer about the nature and extent of the distinction between internalist and externalist views.
- ²⁵ That said, perhaps it is no coincidence that Gödel thought that some version of the ontological argument could be made to work (Gödel, 1970). Furthermore, the connection between Hilbert's view of mathematical theories and the ontological argument was highlighted by Frege (1900, p. 47) merely eight days after Hilbert outlined his views on the subject. The neo-logicians have also, like Gödel, been accused of deploying a mathematical analogue of the ontological argument (Field, 1984, pp. 659–660).
- ²⁶ Martin calls this the concept set in “my sense.” The terminology has been altered to avoid confusion.
- ²⁷ Martin describes it as being of some, but not much consequence (Martin, 2005, p. 220). On the other hand, Wright seems to put relatively weighty emphasis on the matter (Wright, 1990, p. 160).
- ²⁸ The full axioms of ZFC will of course be unacceptable to most mathematicians and philosophers of a constructivist stripe. But even closer to the mainstream, the axiom scheme of replacement and the axiom of choice are both controversial to some extent. For example, Potter's favoured set theory does not include either axiom (Potter, 2004, p. 76). Johnstone also describes the “controversial nature” of the axiom of choice in the mathematical community, and even goes so far as to claim that “mathematicians generally prefer not to regard the axiom of choice as one of the basic axioms of set theory” (Johnstone, 1987, p. 79). So although ZFC may be widely accepted (and is certainly widely used), it is not clear that its axioms are as “forceful” as those of PA.

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