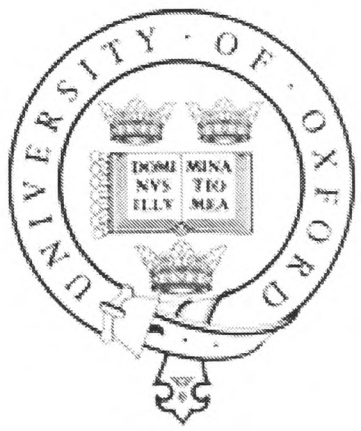
Essays in Labour and Behavioural Economics

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Abstract

The entire literature on adverse selection in the labour market spawned by Greenwald (1986, *Review of Economic Studies*, 63(3)) has been built, somewhat unwittingly, on the assumption that firms forget the type of a worker after the worker quits. In many contexts, this assumption is implausible. The first three chapters of this thesis therefore explore an alternative approach to modelling labour markets with asymmetric information by assuming firms will never forget a worker's type.

The first chapter turns the standard Greenwald result on its head by showing that if the worker knows her own type and productivity is unchanging, the possibility of competitive wage offers from fully-informed previous employers means that adverse selection will never persist. Job changing frictions can cause a semi-separating equilibrium where the more productive workers have their type revealed whilst the least productive workers receive a pooling payoff. But even where asymmetric information persists there is no adverse selection because job changing frictions shield potential employers from the winner's curse. The second chapter investigates the robustness of the non-persistence of adverse selection result where previous employers are asymmetrically informed. The result is found to be robust where firms bid for the worker under a closed but not an open auction. The third chapter finds that, if workers are not sure of their exact value to their employer, there will be an adversely selected stream of job changers in equilibrium, even as the probability of a worker quitting for exogenous reasons approaches zero. Less able workers are quickly revealed as such, whilst more able workers have their type revealed gradually.

The fourth substantive chapter of this thesis investigates the widely observed paradox that, despite what traditional economics would lead us to believe, there can be such a thing as too much choice. The model provides a formal theoretical explanation for this phenomenon using the regret theory of Loomes and Sugden (1982, *Economic Journal*, 92(368)). When options are few it is shown that enlarging the choice set improves welfare, but when options are many, a "less is more" phenomenon emerges. In some cases, excess search options can decrease search.

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Chapter 1

Introduction

Nobody is forgotten when it is convenient to remember him.

Miguel de Cervantes Saavedra
Don Quixote (pt. II, ch. LVII)

The first three essays of this thesis contribute to an area of economics which is still far from being fully understood: the functioning of markets with asymmetric information.

In the famous "market for lemons", Akerlof (1970) explained how a used car market could fail if only the seller is able to observe the quality of the car. This opened the floodgates on a whole new line of theoretical reasoning and vastly improved the understanding of how markets function in the real world. Breakthroughs were made in areas such as education (Spence, 1973), insurance (Rothschild and Stiglitz, 1976), moral hazard (Mirrlees, 1999; Holmstrom, 1999), labour (Jovanovic, 1979; Greenwald, 1979) and credit (Stiglitz and Weiss, 1981), that could explain the widespread evidence of market failure and inefficiency that was incompatible with what was, at the time, the default assumption of symmetric information.

When Joseph Stiglitz accepted his Nobel Prize for Economics jointly with

George Akerlof and Michael Spence, "for their analyses of markets with asymmetric information", he explained it as follows:

> When I began the study of economics some 41 years ago, I was struck by the incongruity between the models that I was taught and the world that I had seen... It was marred by poverty, periods of high unemployment, and massive racial discrimination. Yet the economic theories we were taught paid little attention to poverty, said that all markets cleared — including the labor market, so that unemployment must be nothing more than a phantasm — and claimed that the profit motive ensured that there could not be economic discrimination. As a graduate student, I was determined to try to create models with assumptions and conclusions closer to those that accorded with the world I saw, with all of its imperfections.[1]

The difficulty in modelling markets with asymmetric information, however, is that while there is only one way that all parties can be fully informed, there are an infinite number of ways one party can be partially informed. The early successes in theoretical modelling came by restricting attention to the class of models where information is fully asymmetric. So, for example, the seller of a car is assumed to know its quality exactly, whereas the potential buyer knows the quality of used cars in general but nothing at all about the specific car in question. Or a worker may know her productivity exactly, but a potential employer may know only the underlying distribution from which it is drawn.

This approach proved tractable, but in many cases yielded extreme results such as complete market failure or fully pooling equilibria that did not sit well with observations of the real world. Rather than relax the assumed degree of informational asymmetry, the approach more commonly adopted was to introduce a signalling aspect into the model. These signals, it was assumed, could be used to at least partially alleviate the informational disadvantage of the uninformed party. So, for example, Spence (1973) proposed that education could be used by an employee as an imperfect but credible signal to a potential employer. Akerlof (1970) proposed that a warranty could act as a signal of the quality of a car. This approach has

[1] Stiglitz (2002)

continued to be popular to the present day.

It is the contribution of this thesis to propose that the signalling approach to understanding markets with asymmetric information often overlooks a more fundamental approach that could similarly explain real world observations. It will be demonstrated that in some cases a "miniature" market with complete information can form endogenously out of the larger market with asymmetric information. The primary focus of the models is restricted to labour markets, although there are broad implications for other markets where information is asymmetrically distributed. In the labour context, this miniature market of complete information is made possible by the assumption that a firm that employs the worker and learns her type may remember it even after she quits. (Throughout the rest of this thesis, this assumption is sometimes referred to simply as "firm memory"). In this way, a worker with a number of informed previous employers could attract competitive offers, even where the larger market of potential employers remains uninformed.

1.1.2 Adverse Selection

In the labour economics literature, the classic model of asymmetric information with no signalling is Greenwald (1986), a paper which is partly based on his 1979 book. The paper is widely cited and provides one of the most elegant demonstrations of a common feature of markets with asymmetric information: adverse selection.

Greenwald starts with the assumption that information is fully asymmetric, with the current employer learning the worker's exact type after one period of employment, and potential employers knowing only the distribution of productivities in the population of workers. He assumes that the current employer has the ability to make a counter-offer after potential employers make their offer and shows that employers will then be more likely to retain their more productive workers. This in turn causes the stream of job changers to be predominantly composed of the less able.

In Greenwald's model, competing firms suffer from the winner's curse, that is, if a potential employer bids based on its expected productivity of the worker, conditional on the worker being recruited, it is probable that the wage offered is more than the worker is worth. The existence of the winner's curse leads potential employers to make low pooled wage offers. So in order to retain the worker, the employer need only offer a low pooling wage. This counter-offer assumption is at the heart of Greenwald's results and has since become common in labour market models, for example Lazear (1986) and Milgrom and Oster (1987).

The other of Greenwald's key assumptions is that workers may quit their current employer for exogenous reasons, regardless of the wage offer. Greenwald shows in a 2-period model that the chance of recruiting a high productivity worker for a low wage means that potential employers can afford to recruit some low productivity workers at a loss. Because all low type workers will be recruited but only some of the high types, the pool of job changers is adversely comprised of the less able.

Greenwald goes on to consider a 3-period model. Wage offers received in the final period are then contingent on whether or not the worker quit in period 2. He finds that the outside wage offers in period 3 are higher for those who stayed with the same employer in period 2 because a worker who was not retained by the current employer in period 2 is more likely to have been of poor productivity. This in turn makes it less attractive for workers to quit in period 2, because they know they will receive lower offers in period 3. As a result, the current employer can retain the workers it desires in period 2 by offering a wage less than the best outside offer. The 3-period model also shows that workers who stay with the first employer in the second period receive increased wages in the final period. Workers who stay with the first employer employer in period 2 are adversely comprised of the *more* able, which increases the amount potential employers are willing to offer in period 3. This in turn increases the amount the first employer must pay to retain them.

The strength of Greenwald's model is in its predictions about how the severity

of adverse selection varies with the exogenous quit rate and the distribution of productivities, and how adverse selection affects wage offers. But wage offers always pool in the model, so there is no mechanism by which even those workers who know themselves to be highly productive and therefore underpaid can reveal this to the market. The literature that attempts to address this failing commonly does so by weakening Greenwald's assumption that employers know everything about a worker's type and potential employers know nothing. These papers suggest that there are signals that potential employers can observe to infer something about the worker's ability. Some such papers are reviewed in Section 1.1.3.

The winner's curse effect, which is synonymous with adverse selection, is a typical outcome of competitive bidding over an item of common value where information is asymmetric. It was first noted by three engineers, Capen, Clapp and Campbell (1971), in the context of bidding over the drilling rights on a piece of land. The paper shows how, if multiple firms bid their expected value then, conditional on winning the auction, it is probable that too much has been paid. A rational agent will take this into account by making a lower bid than would otherwise be optimal, but as noted by Thaler (1988b), it is a common failure of rationality to underestimate the full effect of the winner's curse and bid too much.

Inderst (2005) looks at adverse selection in the labour market of the sort described by Rothschild and Stiglitz (1976). His approach differs from Greenwald (1986) and the models of this thesis by allowing contracting. The novelty of Inderst's approach is in assuming there is constant stream of new workers and firms that arrive in the labour market and then exit when a match is found. The benefit of this approach is that the composition of workers in the job changing pool is then endogenously determined jointly with the type of contract that firms will offer in equilibrium. Depending on the proportion of high and low type workers entering the labour market, contracts offered may be pooling or separating. Inderst shows that low type workers may take longer than high type workers to be offered a contract

that they are willing to accept. Like Greenwald, Inderst finds that the composition of workers in the labour market may be adversely comprised of the less able. In contrast to Greenwald, Inderst uses the composition of the stream of new workers as the exogenous parameter on which the composition of the stream of job changers is dependent, rather than the workers' propensity to quit.

Adverse selection is important not only in micro economic models of the labour market but also in macro economics. Montgomery (1999) shows how a labour market where poor quality workers are more likely to be fired can, if hiring costs are sufficiently large, exhibit cyclical unemployment rates in equilibrium. Riordan and Staiger (1993) and Smith (1989, 1995) show how the sectoral explanation of persistent unemployment is underpinned by the existence of adverse selection in the labour market.

Adverse selection has been shown to have analogous consequences in other types of markets. Stiglitz and Weiss (1981) examine the effect of imperfect information in the market for credit. If borrowers have differing probabilities of repaying a loan and banks are unable to distinguish this probability, Stiglitz and Weiss show that the bank may suffer from an adverse selection problem. Borrowers willing to pay high interest rates may, on average, present the greatest risk because they perceive their probability of actually repaying the loan as low. This screening effect of the interest rate may prevent the market from clearing and cause credit rationing in equilibrium. This has vast implications for the functioning of macro economic markets, especially in developing countries.

There is some empirical support for the existence of adverse selection. Greenwald and Glasspiegel (1983) find evidence in the market for slaves in pre-Civil War New Orleans. Genesove (1993) examines the used car market of Akerlof (1970) and concludes weakly in favour of the existence of adverse selection. Blinder and Choi (1990) carried out interviews with wage setters and report no awareness of adverse selection. Although the paper concludes against the adverse selection version of the

efficiency wage model, it does so on rather flimsy evidence. Wage setters are simply asked if they infer anything about the likely quality of a worker by the fact they are willing to accept a certain wage. In all cases the response was negative, but a lack of awareness of adverse selection cannot be construed as evidence against its existence.

Perhaps the most convincing empirical examination of theories of asymmetric information in the labour market is Gibbons and Katz (1991). They compare unemployment spells and post-displacement wages for workers who were laid off at the discretion of their employer to that of workers who were made redundant by plant closures. Their hypothesis is that if Greenwald's (1986) theory is correct, the post-displacement earnings of workers who were laid off should be lower and unemployment spells longer than those of the workers laid off for reasons unrelated to their ability. Using data from Current Population Surveys, they find strong support for Greenwald's predictions.

1.1.3 Asymmetric Information Models with Wage Separation

In response to the unrealistic prediction of Greenwald (1986) and others that asymmetric information will cause pooling wages, a subsequent stream of literature attempted to explain this as a result of it being too simplistic a model. They argue that the solution is to keep Greenwald's basic assumptions, but to introduce additional features that can act as a signal to potential employers of the worker's type.

One such approach is to assume that potential employers can observe a worker's task assignment and promotion. This concept was introduced by Waldman (1984) and has since been widely adopted. Waldman assumes a continuum of worker productivities and two types of task; one where output is dependent on productivity and the other where it is not. He finds that wage rates tend to be more closely related to the publicly observable task assignment than the privately observed ability.

Promotion signals a more able worker and so increases the wage that the current employer must pay to match outside offers. The current employer may therefore find it more profitable to under promote and under pay a worker, and this causes inefficiency in the market.

Task assignment as a signal of type has also been considered by Ricart I Costa (1988) who extends Waldman's model to allow workers to exploit knowledge about their own ability – a concept that also features in Chapter 2 of this thesis. Waldman finds that outside wage offers are pooled for a given task assignment because workers do not use their private information. Ricart I Costa's continuous time model allows managers to select a contract where expected utility is a function of first period output, thus allowing a degree of self-selection which gives outside firms additional information about worker ability beyond task assignment. He finds that, although wages for old managers are heavily dependent on task assignment, unlike Waldman there are also wage differentials within jobs that reflect worker ability. The model also finds that wages for able old managers are below their marginal product and wages for less able managers are above. Like Waldman's model, old managers are promoted less than is efficient because of what this reveals about type. Milgrom and Oster (1987) find similar results.

Similarly, Bernhardt (1995) assumes wage and promotion paths are observable. As a result there is an inefficiently low level of promotion since this reveals information about the worker's type to the market and hence increases the wage required to retain the worker. The most valuable contribution of the paper is the insight that a worker who has already been promoted will be more likely to be promoted again, even over a worker at the same level in the hierarchy and of the same ability. The employer of the worker on the slower career track has more private information relative to competing firms. This exacerbates the winner's curse and means the worker can be retained on a low wage for a longer period of time.

The strategic deception of potential employers is a theme of labour market

models with publicly observable task assignment. Employers trade off the benefit of efficient task assignment against the cost of making private information about the worker's type public. Koch and Peyrache (2005) show that this deception will be increased where there are several principals contracting with the same agents sequentially. Bernhardt and Scoones (1993) assume that, in addition to the information that can be gained from wage and promotion history it is also possible for potential employers to learn the worker's type by engaging in costly search. The model includes a firm-specific matching parameter, and this means that a potential employer that learns the worker's type may be willing to pay more than the current employer and so lure the worker away. This possibility can motivate the first employer to over-promote a low quality worker in order to deceive potential employers into thinking the worker already has a high match at the current employer and therefore refrain from investing in search.

In addition to task assignment, another approach to modelling labour markets with asymmetric information that is capable of producing wage separation is contingent contracts. Where output is observable by the employer but not outside firms, a worker who knows her ability to be high may select a contract where pay will increase in the second period if her performance reaches a specified level. But a moral hazard problem then exists because the employer would be tempted to report the worker's performance as low and retain her at a low wage. One way around this problem is "up-or-out" contracts, where the employer is legally bound to either promote or fire the worker within a specified time. It is then *ex post* optimal for the firm to promote good workers thus revealing something about their type to the market.

Kahn and Huberman (1988) find that such a contract can lead to involuntary unemployment though not in the Shapiro and Stiglitz (1984) sense where such unemployment is used as an incentive device. In up-or-out models, involuntary unemployment can only occur as a result of imperfect information, and even then,

it is only involuntary on the part of the worker *ex post*, not *ex ante*. Kahn and Huberman also explain how up-or-out contracts can give the necessary incentive to workers to invest in firm-specific human capital. This is extended by Waldman (1990) who finds the same results hold where workers can invest in general human capital.

Prendergast (1993) argues that promotion to a different task is an alternative and more commonly observed response to Kahn and Huberman's "two sided moral hazard problem" than up-or-out contracts. He finds that workers can then be induced to invest in firm-specific human capital if the firm can commit to a pay scale where wages are attached to jobs. The result further depends on the returns to training being higher in one position than the other, and the workers knowing their promotion chances are increased by investment in training. A further interesting result with some analogies to the model of Chapter 2 is that only the most talented workers invest in training and have their type revealed by promotion, while the rest are pooled.

1.1.4 A New Approach to Modelling Markets with Asymmetric Information

The theoretical models of the labour market discussed so far represent only a small cross section of the information-based labour market models. The vast majority of recent research either supports the findings of the foundational papers such as Akerlof (1970), Spence (1973), Rothschild and Stiglitz (1976), Jovanovic (1979) and Greenwald (1986), or takes these results as given and builds on their assumptions. To the economist attempting to add to the understanding of labour markets, it can appear that every approach or assumptional starting point has already been exhaustively examined. Much of the recent literature makes only small steps by fine tuning assumptions that can produce wage separation whilst assuming asymmetric

information.

In fact there is one very significant assumption that has, remarkably, never been investigated in this context. This is the assumption that a firm will forget the ability of a worker after she quits. The significance of this assumption can be easily overlooked since it is never made explicit in papers such as Greenwald (1986). If a worker were to quit, it is presumed that the old employer is of no significance in the rest of the game. Thus there would never be any reason for the worker to use quitting strategically.

But there is no intuitive reason why consideration of the alternative would not be interesting or workable. If anything, the assumption that firms will, in fact, remember the type of a worker after she quits seems a more obvious starting point than the antithesis. Perhaps the reason that this has never been pursued is due to its inherent technical complexity. Memory always carries a significant computational cost. In duration models for example, constant hazard rates are strongly favoured because the time at which previous events took place can be ignored, and this is far simpler than the alternative. In a similar fashion, firm memory adds complexity to an extensive form game. But adopting a particular assumption purely for the sake of computational simplicity is only a reasonable approach if the more complicated alternative would not yield any additional insights. And as the following chapters of this thesis show, the assumption of firm memory can reverse many of the results which labour economists have begun to take for granted.

By adopting the assumption of firm memory, the next three chapters of this thesis present a strong challenge to many of the widely accepted results of the papers discussed above. Of particular interest is, firstly, whether equilibrium wage offers will be pooling or separating and, secondly, whether adverse selection will in fact persist. As the point of reference in the established literature, the paper of Greenwald (1986) will be used most often. There are three reasons for this: 1) it is a prolifically cited, seminal paper that is widely accepted to the present day; 2) its

predictions regarding adverse selection and equilibrium wage offers are unambiguous and can therefore be readily challenged; 3) the model used by Greenwald can be readily modified to incorporate the assumption of firm memory.

Chapter 2 of this thesis makes the boldest claim and is the cornerstone of the three labour economics chapters of this thesis. The model assumes that productivity is constant and known by the worker, and that there is a switching cost borne by the worker who changes jobs. Under these conditions it is shown that there can be no persistence of adverse selection when firms remember the type of a worker even after she quits.

With this result being such a radical departure from the standard theoretical understanding of adverse selection, Chapter 3 of this thesis examines the robustness of this finding by introducing asymmetric information between previous employers. Where one of these previous employers has unambiguously better information, this can, in some circumstances, suppress competition and counteract the effect of firm memory. Two models are presented, the first where firms bid for the worker in an ascending-price auction and the second where a first-price auction is assumed. The models largely support the conclusions of Chapter 2 but show that the persistence of adverse selection depends on the specific assumptions regarding the bidding process.

Chapter 4 continues the investigation into the significance of firm memory of a worker's type. In contrast to Chapter 3 it is assumed that previous employers are symmetrically informed, but the worker does not know her own productivity. A 3-period model is presented and this is then extended to N-periods. The results again emphasise the significance of assumptions regarding firm memory, but are similar to Chapter 2 only in that wages are semi-separating. Other aspects of the results have more in common, surprisingly, with Greenwald (1986). In particular, the model exhibits endogenous turnover that is adversely selected, although the severity of adverse selection diminishes over time.

1.2 Behavioural Economics

The final substantive chapter of this thesis presents a model rooted in behavioural economics, one of the most rapidly growing branches of economics today. Literature specific to the model will be reviewed in the chapter itself, but here, a moment is taken to introduce the broader field of behavioural economics.

Behavioural economics draws upon the literature of psychology in an attempt to increase the explanatory power and realism of economic modelling. The extent to which this deflects the behavioural economist from the traditional neoclassical approach founded upon principles such as utility maximisation, effective use of information and stable, coherent preferences is highly variable, but often not as great as is commonly assumed.

Some behavioural models simply expand the utility function to include features that may have been considered before. It is natural in the development of economic thought that the simplest ideas with the broadest application be established first, and only when their explanatory power is exhausted are new approaches considered. For example, where a simple economic model may assume that individuals seek to maximise their own welfare, there is no intrinsic reason why a concern for fairness should not be included in the utility function, especially where this may explain a paradox.

The ultimatum game (Thaler, 1988a) is one such paradox. The game involves two players, one a proposer and one a responder. The proposer is allocated a sum of money and chooses a proportion of that sum to offer to the responder. The responder then chooses whether or not to accept the offer. If the responder accepts the offer each player keeps their share, but if the responder rejects the offer, both players receive nothing. With a traditional utility function, the equilibrium outcome of the game would see the proposer offering a penny and this being accepted by the responder. But laboratory experiments find that the proposer typically offers around a third of their allocation, and that the responder will sometimes reject this, even

when the allocation is large. (See Charness and Rabin (2002) for a full review of laboratory findings on social preferences). By expanding the agent's utility function to include a component for fairness, the findings of the ultimatum game can easily be accounted for. (See, for example, Fehr and Schmidt (1999)).

But behavioural economics goes further than merely expanding the utility function to add realism. Psychologists attempt to understand and explain human behaviour and as such they have no prejudice towards notions of rationality or stable preferences. Indeed psychologists find that humans are often highly irrational and unpredictable. So it is inevitable that by adopting psychological findings wholesale into economic theory there will be some clash with the neoclassical assumption of stable preferences. Hyperbolic discounting is a good example of this. Laboratory and field experiments on both humans and animals have led psychologists to conclude that discount rates are higher over the immediate future than over the more distant future (Ainslie, 1982). Strotz (1956) showed that if this result is accepted into economic models then dynamic inconsistency can occur, where an agent's revealed preferences are inconsistent over time. Given the choice between receiving $100 in a year or $101 in a year and a day, most people would prefer to wait the extra day for the extra dollar. But given the choice between receiving $100 immediately and $101 tomorrow people typically prefer the immediate payoff. The trade-off is identical in both cases but varying the time at which the decision is made reverses the outcome. This is inconsistent with the standard assumption of stable preferences, but to the behavioural economist is a gold mine of explanatory power. Understanding hyperbolic discount rates gives insight into related phenomena such as procrastination (Laibson, 1997; O'Donoghue and Rabin, 1999) and self-control (Gul and Pesendorfer, 2001).

On the effective use of information, not even the most ardently traditional economist would suggest that human beings have the ability to perform complex computations with complete accuracy in the way that economists traditionally

assume. But in most cases this assumption is a useful simplification and has the advantage of reducing an infinite array of possible optimisation criteria down to just one. There is no insight to be gained from admitting to the possibility that an agent presented with a difficult computation may be forced to estimate, and that estimate may sometimes be a little low and sometimes be a little high. But what is interesting are the cases where agents not only fail to act rationally, but fail to do so in a systematic way. The winner's curse for example, as explained in the previous section, should be accounted for by the rational agent. In reality however, people fail to adjust appropriately and the winner of an auction pays too much (Bazerman and Samuelson, 1983; Thaler, 1988b). The most interesting systematic failures are often in the ability of the agent to estimate how her own preferences will change in response to a change in circumstance or the passing of time. Projection bias, for example, explains the tendency for people to exaggerate the degree to which their future tastes will resemble their current tastes. A trip to the supermarket on an empty stomach may, as explained by Loewenstein et al. (2003), result in too much being purchased.

There remain a surprising number of vociferous critics of the behavioural economic approach. But surely the fact that so many of the so-called paradoxes of economics have been convincingly resolved by the behavioural approach justifies its validity. Admittedly where traditional neoclassical approaches provide adequate realism, there is no need for gratuitous overcomplication. Stigler (1965) suggests that economic theories should be based on three criteria: congruence with reality, generality and tractability. So long as a model is sufficiently congruent with reality, the simplest possible set of assumptions can only benefit generality and tractability. But generality and tractability are useless if real world observations cannot be explained, and this is where behavioural economics comes into its own.

A full survey of the behavioural literature is provided by Rabin (1998) and Camerer and Loewenstein (2004). Also enlightening is the series of "Anomalies"

papers due to Richard Thaler and others which began in the Journal of Economic Perspectives in 1987.[2]

Chapter 5 of this thesis draws from the behavioural economics toolkit in an attempt to explain a curious empirical observation of our time: that the abundance of choice available to us in everything from supermarket purchases to superannuation policies, rather than making us a happier, more content society is, in many cases, making us less happy and more anxious than ever before (Schwartz, 2004). How can it be that more choice equals less happiness when any traditional economic analysis would conclude that an expanded choice set is weakly preferred over the original?

The chapter uses the regret theory of Loomes and Sugden (1982). Regret theory proposes that agents may experience disutility from making choices that lead to an experienced outcome inferior to that which would have resulted from a different choice. So, after making a losing bet, a gambler would be worse off, not just as a direct result of the money lost, but also because of the feelings of culpability resulting from the inferior outcome.

In the model of Chapter 5, the utility function in a basic search model is extended to include a component for regret. It is shown that happiness could be single peaked in the number of search options available. Also explained by the model is the prospect of decision paralysis, where an excess of searchable options causes less search to be undertaken rather than more. In its final section, the chapter outlines a model of search where past options cannot be recalled. In this setting it is shown that there is a valuable distinction to be made between regret over options already forgone and regret over possible future outcomes.

[2]Some of the more influential papers in the series include Thaler (1988b, 1990), Dawes and Thaler (1988), Loewenstein and Thaler (1989), Tversky and Thaler (1990), Kahneman et al. (1991) and Siegel and Thaler (1997).

Chapter 2

Can Adverse Selection Persist in the Labour Market?

2.1 Introduction

This chapter presents an alternative to the widely accepted approach to modelling labour markets with asymmetric information as represented by Greenwald (1986). These models assume that employers have better information about a worker's ability than potential employers and find that, in the absence of any signal of productivity that can be observed by potential employers, adverse selection will persist and high productivity workers will receive low wages. But the model herein shows that this result is entirely dependent on the assumption that firms would forget the worker's type if she were to quit.

By reversing this assumption it is shown that the worker will be able to build a reputation amongst previous employers and then enjoy competitive wage offers. Job changing friction is assumed to exist, and this impedes the worker's ability to create a competitive market of previous employers. A worker of low ability gains little from having her type revealed to the market and so, if switching costs are sufficiently high, she can be retained by her first employer on a pooling wage. A more able worker, on the other hand, may be undeterred by the switching cost and require the competitive separating wage in order to be retained. So a semi-separating

equilibrium can occur where the threshold productivity level above which separation will occur is increasing in switching costs. But perhaps the most surprising result of the model is that, even if switching costs are such that asymmetric information persists, adverse selection never persists.

Asymmetric information has been a feature of micro foundations models of the labour market since papers such as Akerlof (1970), Spence (1973) and Rothschild and Stiglitz (1976) showed how it could explain the widespread evidence of market failure that was incompatible with existing models. The seminal work of Greenwald (1979, 1986) vividly explains how asymmetric information in the labour market can lead to adverse selection. If the current employer knows the worker's type but potential employers do not, Greenwald shows that the current employer will concentrate its efforts on retaining its best workers, and this in turn causes the market for job changers to be adversely comprised of the less able. This has two negative consequences: first, firms will be unwilling to recruit from the pool of job changers except at low wages, and second, workers will be reluctant to enter the pool of job changers because they will then be seen by potential employers as likely to be of low productivity.

Despite the intuitive appeal of Greenwald's assumptions and elegance of his results, many of his predictions are not observed in the real world. Whilst there is some empirical support for adverse selection (Greenwald and Glasspiegel, 1983; Genesove, 1993), it is clear that in reality wages are not fully pooling, and in many cases high productivity workers *can* change jobs without being marked as less able. There has been a vast body of literature subsequent to Greenwald's paper that has attempted to address this issue. The most common approach is to focus on the weakness of Greenwald's assumption that potential employers are able to infer nothing at all about the worker's type. As has been reviewed in Chapter 1 of this thesis, such papers typically add to Greenwald's basic model of asymmetric information an additional feature such as training (Chang and Wang,

1996), task assignment (Waldman, 1984; Ricart I Costa, 1988) or contracting (Kahn and Huberman, 1988; Waldman, 1989; Inderst, 2005) and argue that potential employers could observe these and update their beliefs about the worker's type accordingly, thereby facilitating wage separation.

In this chapter, a more fundamental approach is taken to explaining the aspects of Greenwald's model that do not agree with empirical observations. The approach is to simply reverse the assumption that firms forget the worker's type after she quits. There is, after all, no obvious intuitive reason why it ought to be assumed that a worker who approaches a previous employer for a job is treated as a complete unknown. In fact, it seems far more natural to assume that anything a previous employer learns about the worker's type would be remembered. Reversing the assumption that firms forget a worker's type after she quits turns out to have a radical effect on adverse selection. The model of this chapter finds that, under a set of assumptions otherwise similar to Greenwald's, adverse selection cannot be part of an equilibrium if firms remember type.

The traditional approach to modelling adverse selection may be appropriate in a one-off trade where one party is asymmetrically informed. But where the same good continues to be traded over a long period of time and its quality is unlikely to change, it seems only reasonable to consider the possibility that some kind of a reputation will be built amongst previous owners. This reputation could lead to bids that signal type, thus overcoming the problem of adverse selection. In the market for credit of Stiglitz and Weiss (1981) for example, a borrower who has demonstrated her ability to repay past loans is unlikely to have the same difficulty in obtaining a loan at the same bank as would a potential borrower whose likelihood of defaulting is unknown to the lender. When a good reputation has been built with a number of lenders, competition between these lenders could result in a competitive interest rate being offered. Likewise, in Akerlof's (1970) market for lemons, if a used car is sold not just once but multiple times, previous owners of the car would know something

of its current quality (assuming that a past knowledge of the quality of the car is informative of present quality) and may make competitive offers. If other buyers in the market can observe these offers, they too could learn about the quality of the car with no warranty required.

The significance of reputation building is particularly relevant to labour markets, since the worker's labour will almost certainly be traded at numerous stages during her career. Furthermore, the worker has a vested interest in selling her labour for more than the pooling amount, whereas in a used car market, the car itself has no concern about the price at which it is sold. (Of course, the owner has an interest in selling price, but this interest ends when the car changes hands). For reputation to play a role in a theoretical model of the labour market however, some notion of firm memory is essential. Otherwise a highly productive worker can never accumulate a sufficient number of informed previous employers because each will forget her type the instant she quits. Perhaps the reason that firm memory is never assumed despite its obvious intuitive appeal is that is that including memory tends to make for far more complicated. Or perhaps the reason is that the static model of Akerlof (1970) is so convincing that economists have been too easily persuaded that the same results should be expected in a dynamic setting.

Whatever the reason, the model of this chapter categorically shows that adverse selection cannot persist if firms remember the type of worker after she quits. The implications of this finding are far reaching for all models of asymmetric information in the labour market. It shows that the results of the entire existing literature can only be believed if one is also willing to accept the assumption that firms forget the type of a worker after she quits. It also opens a new line of enquiry for theoretical models trying to explain empirical labour market findings.

The result of this chapter also has an even broader implication, that *any* adverse selection result in a market for a good that is traded repeatedly — be it in insurance markets, credit markets in developing countries, or even used car markets — ought

to consider, or at the very least, only ignore explicitly, the possibility of reputation building that can come from memory.

Aside from the firm memory assumption, the model of this chapter also differs from Greenwald (1986) by including switching costs. These act as an imperfection in the labour market that makes it more difficult for a worker to accumulate the number of previous employers necessary to generate competitive offers. Switching costs have been used extensively in the modelling of consumer product markets and are reviewed by Klemperer (1995). In markets with switching costs, Klemperer finds that firms face a trade off between pricing low in order to profit from new customers and pricing high in order to extract maximum profit from those customers already loyal to the firm. If price discrimination is possible then both can be achieved at the same time. In the labour market context, switching costs can represent either the cost to a firm of firing a worker and hiring a replacement or the cost to a worker of moving between firms. Analogous to the consumer products market with price discrimination, firms can exploit employees by retaining them with wages lower than the market rate but will want to offer higher wages in order to win new recruits.

There are numerous models that consider the effect of hiring and firing costs (Bentolila and Bertola (1990), Hopenhayn and Rogerson (1993) to mention just two) but only a small subset of these do so in an adverse selection context with heterogeneous workers. Montgomery (1999) shows that hiring costs can lead to cyclical unemployment in a market with adverse selection. Canziani and Petrongolo (2001) find that the expected earnings of fired workers decreases with firing cost because it implies they are more likely to be unproductive. Firing costs also motivate the firm to raise its hiring standards to avoid costly mistakes. An empirical model using microdata on labour market transitions confirms the model's predictions. The result that job changers are likely to be less productive is consistent with Greenwald (1986) but is motivated by the "push" of the well-informed employer's determination to get rid of them rather than Greenwald's "pull" of the uninformed potential

employers offering more than the current employer is prepared to match.

Burguet et al. (2002) consider switching costs as a endogenous contracting device taking the form of a quit fee imposed on the worker or a buy-out fee imposed on the new employer. They find that quit fees can occur in equilibrium if the performance of the worker is observable by outside firms because it reduces the rents available to outsiders. When a worker is better suited to an outside job, quit fees are likely to be downwardly renegotiated and the split amicable. Where performance is not observable, buy-out fees will be used to prevent the endogenous turnover that results from adverse selection.

In both this model and that of Canziani and Petrongolo (2001), the pooling or separation of type is unrelated to switching cost. It is also worth noting that the switching costs which they describe are transfers between the one party and the other. The switching costs used in the model of this chapter represent real economics costs. The distinction is important because, where switching costs are just transfers, there is always the possibility that they can be renegotiated.

Malcomson et al. (2003) are interested primarily in the effect of adverse selection on the efficient provision of general on-the-job training, but other features of their model have a lot in common with the model of this chapter. They find that switching costs can give firms an incentive to invest in general training, where traditional human capital models show that such training must be paid for by the worker. Asymmetric information means that workers who quit do not receive their marginal products in the new firm but the external benefit conveyed to the new employer by the previous employer's investment in training means that training is provided at less than the efficient level.

Semi-separating equilibria are a relatively unusual outcome in game theoretic models. Fudenberg and Tirole (1983) develop a sequential bargaining model with incomplete information that has a semi-separating equilibrium with many properties in common with the model of this chapter. They use bargaining costs where the

model of this paper uses switching costs to enable the existence of pooling in equilibrium. Their model differs from that of this chapter by assuming that both the buyer's and the seller's valuation are private information. In the labour literature, Lazear (1998) derives a semi-separating equilibrium in a market with asymmetric information by assuming there are two types of firm – one where the worker's type will not be revealed and the other where the worker's type will be revealed both to the current employer and to potential employers. He assumes that a cost must be incurred to work for the firm that will learn type, and finds that only the most able workers find it worthwhile doing so. This is consistent with the findings of this chapter. The model is less concerned with adverse selection however, and the semi-separating outcome rests heavily on a number of specific assumptions that limit its generality.

In the next section, the details of the approach and specific assumptions are explained for the two models that follow. The first of these is a simple complete information model that introduces the principles of continuous time modelling and some basic results. Then the case of asymmetric information is introduced by assuming that initially no firm knows the worker's type, but that a firm may be able to learn the worker's type by employing her. The chapter finishes with a discussion of the results and their implications.

2.2 Preliminaries

A single, risk-neutral worker is drawn from a population of workers whose productivity distribution is given by $f(p)$ where $p \in P : [p^-, p^+]$, $p^+ > p^-$ and $f(p) > 0$ for all $p \in P$. The distribution is known to the worker and firms and the worker knows her own type. The worker lives for one unit of time $t \in T : [0, 1]$ and seeks to optimise lifetime earnings.

There are N firms with $i \in N$ indexing the firm who first employs the worker. $j \in N$ and $k \in N$ index the potential second and third employers respectively. Define

$W_i(p, \tau_1, \tau_2)$ as the present value of future payments, or the "payoff" for short, for a worker of type p who works for firm i from time τ_1 to τ_2. The worker can also opt to receive an unemployment benefit representing the dole plus the disutility of work. This is paid at rate $u \leq p^-$ leading to a payoff denoted $W_u(\tau_1, \tau_2)$. In all payoff notation, where $\tau_2 = 1$ the parameter is omitted for the sake of clarity.

In each period dt potential employers make simultaneous offers to the worker. These are observed by the current employer that then makes a counter-offer. The worker then chooses an employer or takes unemployment. In making an offer, firms commit to employing the worker for the period dt for an amount $w(p,t)dt$ where

$$w(p,t) = -\frac{dW(p,t)}{dt} \tag{2.1}$$

As $dt \to 0$ the model approaches continuous time and the amount that firms bid converges to the wage rate. Bids will be presented in terms of the payoff $W(p,t)$ that would be paid to the worker conditional on her staying with the offering firm for the rest of her lifetime. It is assumed there is no contracting in the model, so the offering firm is not actually committing to paying $W(p,t)$. If the worker accepts the offer at time t then the new employer will offer the profit maximising payoff $W(p, t+dt)$ at time $t+dt$, and similarly in each subsequent period. So the only payment that the offering firm is committed to paying in the period starting at time t is $W(p,t) - W(p,t+dt) = w(p,t)dt$. But the best response of a firm bidding at time t can only be solved with reference to the entire subgame beginning at t, hence the advantage of considering bids in terms of $W(p,t)$. Once best responses have been identified in terms of $W(p,t)$ it is then a simple matter to differentiate the payoff $W(p,t)$ in order to find the equilibrium wage rate $w(p,t)$.

In the event of a tie between payoffs it is assumed that the worker prefers to stay with the current employer if this is one of the tied offers, otherwise chooses randomly. It is also assumed the worker prefers employment over unemployment where these payoffs are identical. Offers are observable by all firms regardless of

whether or not they are accepted.

There is an exogenous quit rate μ which is the probability of the worker quitting her current employer regardless of the expected payoff in any period dt. For simplicity it is assumed $\mu \to 0$. (The exogenous quit rate must be non-zero for reasons that later become clear).

Job changing frictions are introduced into the model with the parameter s. This is a cost levied on the worker when she starts working for a new firm. It is assumed that switching costs are not incurred moving into unemployment but are incurred moving out of unemployment and when moving to a previous employer. This switching cost represents an aggregate of all the different costs incurred by the worker, the old employer and the new employer as a result of the job changing process. On the firm side, examples would be the cost of induction courses, days of low productivity while the worker gets started, and so on. On the worker side, costs from leaving one job and starting the next include the disutility of settling in to a new job, moving expenses, days without pay etc. Excluded from the switching cost are any transfers between worker and firm such as sign-on bonuses and severance pay. These have strategic effects which are not the concern of this model.

It is logically sound to model the worker as incurring all switching costs, even if some of these costs would in fact be borne by the new or old employer. Because the market is competitive, payoffs will adjust appropriately. For example, in a single period with symmetric information, if switching costs are incurred by the new employer then the zero profit offer of a potential employer would be $p - s$. In order to retain the worker, the current employer would have to match both this offer and unemployment with an offer of $\max[p - s, u]$. Comparing this to a model where switching costs are incurred by the worker, potential employers would offer p, but if the worker accepts this offer they incur switching cost s making the net payoff $p - s$. The current employer must match this and unemployment with $\max[p - s, u]$. So the payoff to the worker from quitting or staying with the current employer is the

same regardless of who bears the switching costs. The same is true with asymmetric information.

2.3 Complete Information Model

In the model of this section, it is assumed that all firms know the worker's type p at the start of the worker's lifetime. The productive output of the worker after $t = 0$ is $\int_0^1 p\,dt = p$. Competition between potential employers will ensure that the highest or equal highest offer will be $W_i(p, 0) = p$. It could not be a best response of potential employers to have a maximum offer less than p because this would lead to a profit and there would be an incentive for one of the potential employers to increase its offer. It could also not be a best response of potential employers to offer more than p, as this would generate a loss. So the worker chooses between the highest offer from a potential employer $W_i(p, 0) = p$ and unemployment. The unemployment payoff over the interval $[t, 1]$ is simply

$$W_u(t) = \int_t^1 u\,dt \tag{2.2a}$$

$$= u(1 - t) \tag{2.2b}$$

so at $t = 0$, $W_u(0) = u$. Since $p > u$ for all $p \in P$ by assumption, it must be that $W_i(p, 0) > W_u(0)$. So the worker will accept the offer of firm i.

Having commenced work at firm i, switching costs then come into play in the ensuing competition between the current employer and potential employers. At time $t > 0$ the worker will accept the offer which maximises lifetime income, which is

$$\max\left[W_i(p, t), W_j(p, t) - s, W_u(t)\right] \tag{2.3}$$

where

$$j = \text{argmax}_{l \neq i} W_l(p, t) \tag{2.4}$$

In order to retain the worker, the current employer must offer a payoff that matches the payoff from taking unemployment and the payoff from moving to a potential employer. From (2.3) this is

$$\max \left[W_j^{CI}(p,t) - s, W_u(t) \right] \tag{2.5}$$

The superscript CI indicates the complete information payoff. The current employer has the advantage of being able to make a counter-offer in response to the offers from potential employers and will not match firm j's offer if doing so would lead to a loss. Because the worker is worth the same to firm j as she is to firm i, it is clear there is no offer potential employers can make that will win the worker from the current employer without generating a loss. So any offer from a potential employer that will not recruit the worker must be a best response.

This outcome of non-unique best responses of potential employers is unsatisfactory however, because it means that the counter-offer of the current employer is undefined and a unique equilibrium cannot be found. This problem is avoided by introducing the exogenous quit rate. For any $\mu > 0$ there is some chance that the worker will leave the current employer even if doing so does not maximise her payoff. This possibility motivates potential employers to make competitive offers. As was the case at $t = 0$, the best response of potential employers will then lead to a unique maximum offer, that being the offer that would generate zero profit if accepted by the worker.

Introducing an exogenous quit rate adds significant complexity to the model however, and unlike Greenwald (1986) the primary purpose of this chapter is to investigate how adverse selection is affected by firm memory, not exogenous quits. So, for simplicity, it is assumed that $\mu \to 0$. This retains the advantage of creating a unique equilibrium without the undesirable complication. This equilibrium concept is similar to the "trembling hand" approach of Selten (1975) where there is some probability that the agent does not make the best move. Here there is some non-zero

probability that the worker does not choose the best offer.

With the $\mu \to 0$ assumption in place, the maximum offer of potential employers can now be determined as that which gives zero profit.

$$\int_t^1 p\,dt - W_j(p,t) = 0 \tag{2.6a}$$

$$W_j^{CI}(p,t) = p(1-t) \tag{2.6b}$$

Substituting this into equation (2.5) reveals that the current employer will always be willing to match the offers of potential employers with a counter-offer of $\max[p(1-t)-s, u(1-t)]$. Since $p \geq u$ there must exist a t at which the the payoff from moving to a potential employer and the payoff from taking unemployment are equal. This will be denoted $\tau(p)$.

$$p(1-\tau(p)) - s = u(1-\tau(p)) \tag{2.7a}$$

$$\tau(p) = 1 - \frac{s}{p-u} \tag{2.7b}$$

The best response of the current employer can now be written as

$$W_i^{CI}(p,t) = \begin{cases} p(1-t)-s & \text{if} \quad t < \tau(p) \\ u(1-t) & \text{otherwise} \end{cases} \tag{2.8}$$

So the worker will stay with firm i for her entire lifetime on a payoff that initially matches the offers from potential employers and then, towards the end of the worker's lifetime, matches the payoff from taking unemployment. If s is high relative to $p-u$ then $\tau(p)$ could be negative in which case the current employer can retain the worker by matching the unemployment payoff for all $t > 0$.

Suppose that the worker were to quit for exogenous reasons at time $t > 0$. It would then be the best response of the worker to accept offer $W_j^{CI}(p,t)$ from firm j. Firm j would then be the current employer and offer according to (2.8). Since

$W_j^{CI}(p,t) \neq W_i^{CI}(p,t)$, the difference between these payoffs defines an up front, one-off payment $x_j(p,t)$ paid by potential employers in the period dt starting at time t when the worker is hired.

$$x_j^{CI}(p,t) = W_j^{CI}(p,t) - W_i^{CI}(p,t)$$
$$= \begin{cases} s & \text{if} \quad t < \tau(p) \\ (p-u)(1-t) & \text{otherwise} \end{cases} \tag{2.9}$$

Because the exogenous quit rate is vanishingly small, the probability that this up front payment will ever be made approaches zero. There is a point on the equilibrium path where this upfront payment will be made however. When firm i first recruits the worker at $t = 0$ an up front payment will be made of $x_i^{CI}(p,0) = \min[s,(p-u)]$.

Figure 2.1 shows the payoffs as a function of time for worker productivity levels of $p = 1$ and $p = 2$ with $u = 0.2$. The time $\tau(p)$ at which the current employer switches from matching the offers of potential employers to the unemployment payoff is clear. $\tau(p)$ is also the time at which the up front payment drops below s. The slope of the payoffs represents the wage rate, which is p before $\tau(p)$ and u after.

2.4 Asymmetric Information Model

2.4.1 Further Assumptions and Notation

It is now assumed that at $t = 0$ no firm knows the worker's type, but the current employer learns the type of the worker after a period of time that is exponentially distributed with constant hazard rate λ. The time at which type is learned is observable by other firms but the productivity itself is learned only by the current employer. At this point the most significant assumption of this chapter also comes into play: that an employer that learns the type of a worker will remember it even after the worker quits.

The significance of this "firm memory" assumption is as follows. If the worker

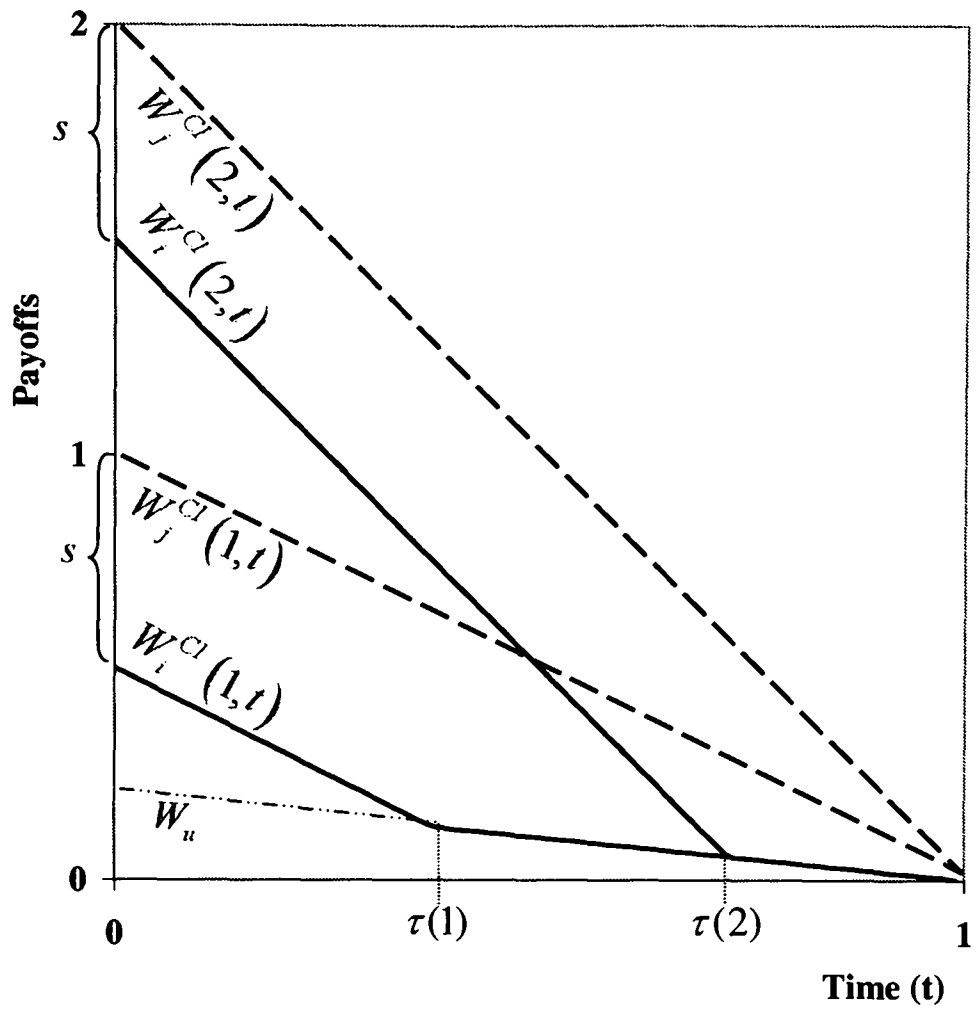


Figure 2.1: Complete information payoffs for $p = 1$ and $p = 2$ with $u = 0.2$ and $s = 0.5$.

were to quit firm i after the firm learns her type, and then quit firm j after that firm learns her type, then there would be two well-informed previous employers i and j competing to rehire the worker. This would lead to competitive offers that reveal the worker's type to all other firms. However, the potential for the worker to benefit from this possibility is hampered by the switching costs that would be incurred in changing employer. Also, if the time taken for a firm to learn the worker's type is likely to be long (that is, λ is low) then the time left in the worker's lifetime to benefit from competitive offers may be too short to justify the costs incurred in job changing. In the model that follows it will be shown that although the worker will be retained by the first employer in equilibrium, the existence of the off-equilibrium path occurrence described above may, in some cases, induce the first employer to reveal the worker's type.

The key to solving the model is in determining the payoff the worker could expect

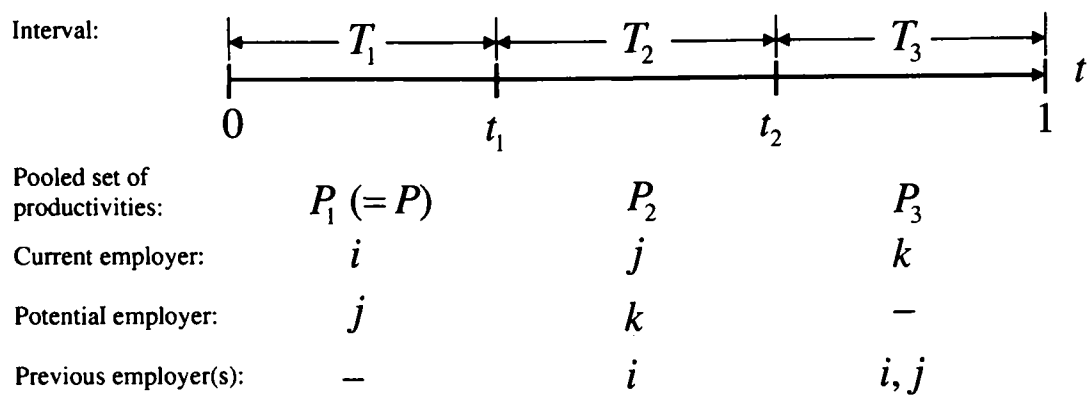


Figure 2.2: Timeline and notation.

to receive by quitting twice. This will be done by backwards induction over three distinct time intervals. Define time interval 1 as $t \in T_1 = [0, t_1]$, the interval when no firm knows the worker's type. The interval begins at $t = 0$ and ends when firm i learns the worker's type at $t = t_1$. Define interval 2 as $t \in T_2 = (t_1, t_2]$ during which there is exactly one firm that has learned the worker's type by employing them (as distinct from learning the worker's type from observing a separating wage offer). The interval ends at $t = t_2$ if firm j learns the worker's type. If this never occurs then interval 2 ends at the end of the worker's lifetime $t = 1$. Finally, define interval 3 as $t \in T_3 = (t_2, 1]$, the interval when two firms have learned the worker's type by employing them. Since the time taken for an employer to learn the worker's type is governed by a statistical process, t_1 and t_2 are non-deterministic.

For each of these three intervals, the set of pooled productivities is defined. During T_1 no firm knows the worker's type so the pooled set P_1 is simply P, the universal set of productivities. During T_2 and T_3 the set of pooled productivities will be defined $P_2 = [p_2^-, p_2^+]$ and $P_3 = [p_3^-, p_3^+]$ respectively. Time intervals and notation are clarified by Figure 2.2.

During interval 1 no firm knows the worker's type and so the payoff $W_i(P, 0, t_1)$ is pooling and will, in equilibrium, lead to zero expected profit for firm i over the lifetime of the worker. The strategies of the worker and firms start to get interesting at t_1 when firm i learns the worker's type. Then firm i gains an informational

advantage which means potential employers suffer from the winner's curse. t_1 is also the earliest point that the worker would quit if attempting to accumulate informed previous employers.

In the analysis that follows, an expression will be found for $\mathrm{E}^{t_2} W_j(p \in P_2, t_1)$. This is the payoff that the worker would expect to receive from t_1 onwards if moving to firm j at t_1. The E^{t_2} indicates that this payoff is based on the expectation of t_2 rather than a specific realisation of it. The parameter $p \in P_2$ indicates that this is the payoff the worker of type p will expect to receive conditional on the current employer's counter-offer signalling to firm j that the worker's type is in the set P_2.

2.4.2 Analysis and Results

Firm j's Best Response in Interval 3

Lemma 2.1. *The payoff the worker expects to receive in interval 3 conditional on moving to firm j at t_1 is given by*

$$\mathrm{E}^{t_2} W_j(p, t_2) = \int_{t_1}^{1} \max\left[p(1-t_2) - s, u(1-t_2)\right] g(t_2)\, dt_2 \tag{2.10}$$

Proof. *A proof is given in Appendix A.*

If the worker were to quit at t_1 and t_2 then at the start of interval 3 there would be two previous employers that know the worker's type. Competition between them would mean that the full information payoff $W_j^{CI}(p, t_2)$ would be offered. But at t_1, the time t_2 at which firm j would learn the worker's type is unknown. So the expected value in equation (2.10) is found by integrating over the possible values of $t_2 \in (t_1, 1]$. The term $g(t_2)$ is the probability density function for firm j learning the worker's type at $t_2 > t_1$. With constant hazard rate λ, $g(t_2)$ is given by the standard equation for the exponential distribution. The only important feature of the function $g(t_2)$ at this point in the analysis is that it is positive for all $t_2 \in (t_1, \infty)$. There is therefore some probability that firm j will not learn the worker's type before the

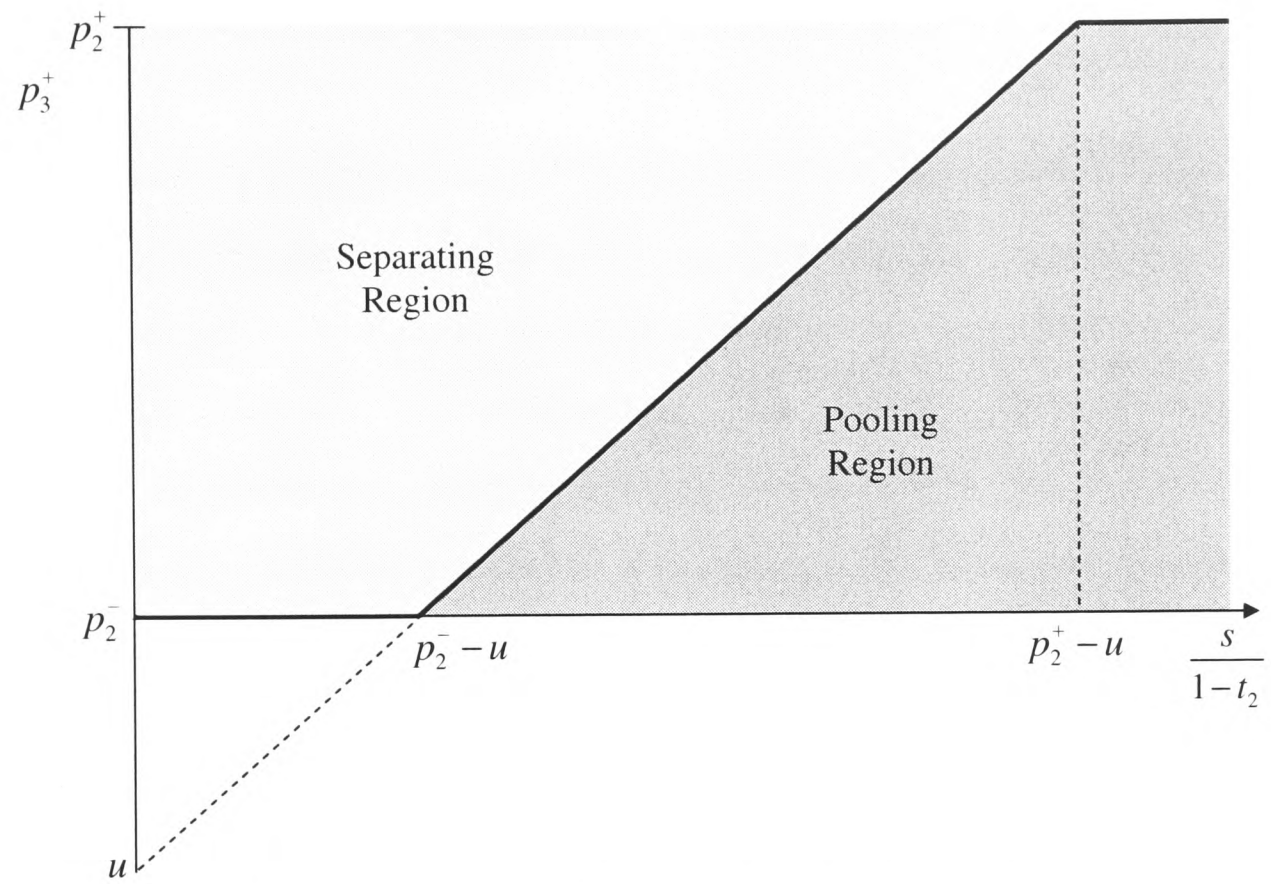


Figure 2.3: The pooling/separating threshold productivity level in interval 3 as a function of $\frac{s}{1-t_2}$.

the end of her lifetime. But since $\mathrm{E}^{t_2} W_j(p, t_2)$ is the expected payoff unconditional on interval 3 existing, there is no need to scale by the probability that $t_2 \in (t_1, 1]$.

For a given realisation of the random variable t_2, the pooling/separating threshold p_3^+ sets the payoff from quitting firm j $p(1-t_2) - s$ equal to the payoff from taking unemployment $u(1-t_2)$. Rearranging and applying appropriate limits since $p_3^+ \in [p_2^-, p_2^+]$

$$p_3^+ = \begin{cases} p_2^- & \text{if} \quad \frac{s}{1-t_2} < p_2^- - u \\ u + \frac{s}{1-t_2} & \text{if} \quad p_2^- - u \leq \frac{s}{1-t_2} \leq p_2^+ - u \\ p_2^+ & \text{if} \quad \frac{s}{1-t_2} \geq p^+ - u \end{cases} \tag{2.11}$$

This is plotted in Figure 2.3. At t_2, firm j will be willing to offer $p(1-t_2)$. But if switching costs are high then the net payoff to the worker from accepting this offer is reduced, and it is more likely that the worker can be retained on the pooling unemployment payoff. For a given switching cost, if the time remaining $1-t_2$ is small then pooling is also more likely because the difference between the separating

payoff $p(1-t_2)$ and the pooling payoff $u(1-t_2)$ is less likely to outweigh the cost of quitting.

There is one important feature of (2.10) that is not immediately obvious. If the worker received a separating payoff back at t_1 under complete information (that is, $p(1-t_1)-s > u(1-t_1)$) then the payoff the worker would expect after t_2 conditional on moving to firm j at t_1 is also separating. This is because there is always some probability, no matter how small, that having moved to firm j at t_1, firm j learns the worker's type in the next period dt and will then offer the separating complete information payoff $W_j^{CI}(p, t_2)$.

Firm j's Best Response in Interval 2

If the worker were to move to firm j at t_1, firm j's belief regarding the worker's type is that $p \in P_2$. This belief would take into consideration the information revealed by firm i's offer at t_1, and the fact that this offer was rejected by the worker. If firm i's counter-offer at t_1 is fully pooling, firm j would know nothing about the worker's type if its offer were accepted so P_2 would be P. If, on the other hand, firm i had made a separating offer, firm j would know the worker's type exactly and the set P_2 contains only the single value p.

Because firm j would learn no additional information about the worker's type between t_1 and t_2, the payoff the worker would receive from firm j between t_1 and t_2 must be pooling for all $p \in P_2$. At t_1 the worker forms an expectation of what this interval 2 payoff will be based on her beliefs about the possible value of t_2. So the expected value of the pooling interval 2 payoff will be denoted $\mathrm{E}^{t_2} W_j(P_2, t_1, t_2)$.

Lemma 2.2.

$$\mathrm{E}^{t_2} W_j(P_2, t_1, t_2) = \min\left[\mathrm{E}^{t_2} W_j^A(P_2, t_1, t_2), \mathrm{E}^{t_2} W_j^B(P_2, t_1, t_2)\right] \tag{2.12}$$

where the firm index j is given by

$$j = \text{argmax}_{l \neq i} \mathrm{E}^{t_2} W_l(p, t_1, t_2) \tag{2.13}$$

and

$$\mathrm{E}^{t_2} W_j^A(P_2, t_1, t_2) = p_2^-(1 - t_1) + s - \mathrm{E}^{t_2} W_j(p_2^-, t_2) \tag{2.14}$$

is firm j's expected interval 2 payoff if the possibility of adverse selection forms a binding constraint on firm j's best response. The interval 2 payoff $\mathrm{E}^{t_2} W_j^B(P_2, t_1, t_2)$ is firm j's best response at t_1 if adverse selection does not form a binding constraint, and is given by

$$\mathrm{E}^{t_2} W_j^B(P_2, t_1, t_2) = \mathrm{E}P_2(1 - t_1) - \mathrm{E}^p \mathrm{E}^{t_2} W_j(p, t_2) \tag{2.15}$$

where $\mathrm{E}P_2$ is the expected productivity of a worker in the set P_2, given by

$$\mathrm{E}P_2 = \frac{\int_{p_2^-}^{p_2^+} p\, f(p)\, dp}{\int_{p_2^-}^{p_2^+} f(p)\, dp} \tag{2.16}$$

and $\mathrm{E}^p \mathrm{E}^{t_2} W_j(p, t_2)$ is the interval 3 payoff as expected by firm j at t_1 when type is known only to be in the set P_2.

$$\mathrm{E}^p \mathrm{E}^{t_2} W_j(p, t_2) = \frac{\int_{p_2^-}^{p_2^+} \mathrm{E}^{t_2} W_j(p, t_2)\, f(p)\, dp}{\int_{p_2^-}^{p_2^+} f(p)\, dp} \tag{2.17}$$

Proof. *A proof is given in Appendix B.*

The intuition behind $\mathrm{E}^{t_2} W_j^A(P_2, t_1, t_2)$, or "Offer A", is as follows. With a negligible exogenous quit rate, any profit firm j can expect to make from exogenous hires is also negligible. So in order to make non-negative profit, firm j must avoid making an endogenous hire (that is, winning the worker by offering an expected payoff greater than the current employer is willing to match) since this will result

in a loss. So already it can be concluded that in equilibrium, with probability approaching 1, the first employer firm i will retain the worker, there will be no endogenous hires and no turnover. This is in contrast to the model of Greenwald (1986). There the exogenous quit rate can take any positive value and potential employers can afford to overpay the less able workers because of the profit made on able workers quitting for exogenous reasons.

Since it is this least productive worker in the set P_2 who is most likely to be "let go" by the current employer, naturally the most that firm j can offer with no chance of generating an exogenous quit $\mathrm{E}^{t_2}W_j^A(P_2, t_1, t_2)$ is a function of p_2^-. This will be referred to as the "adverse selection constraint" since what constrains firm j's best response from being higher is the fact that, conditional on the offer being accepted, the expected productivity of the worker would be inferior to the expected productivity of the set P_2.

The second upper bound on firm j's best response is $\mathrm{E}^{t_2}W_j^B(P_2, t_1, t_2)$, or "Offer B". This comes about because, even if firm j's offer is low enough to avoid endogenous recruitment, it must also be a best response conditional on the worker quitting for exogenous reasons. Even as $\mu \to 0$ there remains a non-zero probability of this occurring. Based on firm j's beliefs about the worker's type, the expected output of a worker who quits for exogenous reasons is $\mathrm{E}P_2(1 - t_1)$. Offer B reflects the fact that it cannot be a best response for firm j to make an interval 2 offer such that the expected payoff in intervals 2 and 3 across all the possible values of $p \in P_2$ is more than this expected output.

So it cannot be a best response for firm j to offer an expected payoff in interval 2 greater than $\mathrm{E}^{t_2}W_j^A(P_2, t_1, t_2)$ or $\mathrm{E}^{t_2}W_j^B(P_2, t_1, t_2)$, as this would lead to an expected loss. Equation (2.12) of Lemma 2.2 states that it will be a best response for firm j to offer the highest interval 2 payoff that satisfies the both these constraints. It could not be a best response for firm j to offer any less as this would lead to a positive expected profit, and competition from other potential employers means this cannot

be part of an equilibrium.

Lemma 2.3. *The possibility of adverse selection imposes a binding constraint on the best response of firm j at time t_1 if and only if*

$$s < s^A \tag{2.18}$$

where $s^A > 0$ if and only if the set P_2 contains more than one value (that is, $p_2^+ > p_2^-$).

Proof. *From equations* (2.12), (2.14) *and* (2.15), *the adverse selection constraint is binding if*

$$s < (\mathrm{E}P_2 - p_2^-)(1 - t_1) - \left(\mathrm{E}^p \mathrm{E}^{t_2} W_j(p, t_2) - \mathrm{E}^{t_2} W_j(p_2^-, t_2)\right) \tag{2.19}$$

Given the value of $\mathrm{E}^{t_2} W_j(p_2^-, t_2)$ in (2.10) *and $\mathrm{E}^p \mathrm{E}^{t_2} W_j(p, t_2)$ in* (2.17), *the right hand side of the inequality must be greater than zero if $\mathrm{E}P_2 > p_2^-$ which is true only when $p_2^+ > p_2^-$. If P_2 contains only one value then $p_2^- = p_2^+$, $\mathrm{E}P_2 = p_2^-$ and the right hand side of the inequality is zero.* ∎

Lemma 2.3 shows that if the current employer offers a pooling payoff, there must be a value of s small enough to mean that the adverse selection constraint will bind. Conversely, if the current employer offers a fully separating payoff then $s^A = 0$ and there is no value of $s \in [0, \infty)$ for which the adverse selection constraint will bind. This makes sense since, if the current employer offers a separating payoff, there is no asymmetric information and therefore can be no adverse selection.

With an expression for the expected interval 2 payoff established, the total payoff expected by the worker from moving to firm j at t_1 including the switching cost incurred can be given as

$$\mathrm{E}^{t_2} W_j(p \in P_2, t_1) = \mathrm{E}^{t_2} W_j(P_2, t_1, t_2) + \mathrm{E}^{t_2} W_j(p, t_2) - s \tag{2.20}$$

Firm i's Best Response at t_1

In order to retain the worker at t_1, firm i must match both the payoff the worker would expect to receive from moving to firm j and the payoff the worker could receive from taking unemployment. First the case will be considered where $s < s^A$. With the interval 2 payoff in (2.20) given by (2.14), the total expected payoff in intervals 2 and 3 from moving to firm j at t_1 including the switching cost incurred is

$$p_2^-(1-t_1) - \mathrm{E}^{t_2}W_j(p_2^-, t_2) + \mathrm{E}^{t_2}W_j(p, t_2) \tag{2.21}$$

The expected interval 3 payoff $\mathrm{E}^{t_2}W_j(p, t_2)$ is, by (2.10), weakly increasing in p. Given that a worker of any $p \in P_2$ must receive the same payoff by definition of P_2, firm i's lowest possible pooling payoff will be just enough to retain the highest productivity of worker in the set. So

$$\mathrm{E}^{t_2}W_i^A(P_2, t_1) = p_2^-(1-t_1) - \mathrm{E}^{t_2}W_j(p_2^-, t_2) + \mathrm{E}^{t_2}W_j(p_2^+, t_2) \tag{2.22}$$

If, on the other hand, $s \geq s^A$, then the payoff the worker of productivity $p \in P_2$ can expect to receive in intervals 2 and 3 from moving to firm j at t_1 including the switching cost incurred is, by (2.15) and (2.20),

$$\mathrm{E}P_2(1-t_1) - \mathrm{E}^p\mathrm{E}^{t_2}W_j(p, t_2) + \mathrm{E}^{t_2}W_j(p, t_2) - s \tag{2.23}$$

This is not necessarily greater than the unemployment payoff so in order to retain the worker firm i will offer

$$\begin{aligned}\mathrm{E}^{t_2}W_i^B(p \in P_2, t_1) = \max[&u(1-t_1),\\ &\mathrm{E}P_2(1-t_1) - \mathrm{E}^p\mathrm{E}^{t_2}W_j(p, t_2) + \mathrm{E}^{t_2}W_j(p, t_2) - s]\end{aligned} \tag{2.24}$$

From these equations comes the most important result of this chapter.

Proposition 2.1. *There exists no perfect Bayesian equilibrium in which adverse*

selection persists (that is, $s < s^A$).

Proof. *Suppose that it is a best response for firm i to offer a pooling payoff to a worker of any productivity $p \in P_2$, and that $s < s^A$. Then, in order to retain the least able worker in the set P_2, (2.22) shows that firm i's best response would satisfy* $\mathrm{E}W_i^A(P_2, t_1) > p_2^-(1 - t_1)$.

Compare this to the case where firm i offers the worker of type p_2^- a separating payoff. Then the set P_2 would be the single value p_2^- and by Lemma 2.3, $s^A = 0$. The adverse selection constraint will therefore not be binding for any value of s, and in order to retain the worker, (2.24) simplifies to show that firm i will offer $\mathrm{E}W_i^B(p_2^-, t_1) = \max[u(1 - t_1), p_2^-(1 - t_1) - s]$.

Comparing these payoffs, $W_i^B(p_2^-, t_1) < W_i^A(P_2, t_1)$ for any $s \geq 0$. So it follows that it would always be a best response for firm i to offer a separating payoff to the least productive worker in the set P_2 if adverse selection were to exist. The set P_2 can therefore only contain a single value and $s^A = 0$. There exists, therefore, no perfect Bayesian equilibrium in which $s < s^A$. ∎

This result completely reverses the conclusion of Greenwald (1986) by showing that there can be no persistence of adverse selection in equilibrium if firms remember the worker's type after she quits. Adverse selection can only exist if there is asymmetric information. Asymmetric information can only exist if the first employer offers a pooling payoff. Proposition 2.1 shows that it will never be a best response for the first employer to offer a pooling payoff if this would lead to the persistence of adverse selection.

Surprisingly, this non-persistence of adverse selection result is completely independent of the values of s and λ. If s is very large or λ very small, then one might expect that adverse selection is likely to persist because the worker is unlikely to be able to benefit from quitting in order to have type revealed, and therefore unlikely to receive a separating payoff from the first employer. As will soon be shown, this intuition is correct only insofar as regards the persistence of asymmetric information.

If s is very large or λ very small then this has the effect of relaxing adverse selection constraint (2.14). For example, if switching costs were extremely large, moving to firm j would not be a tempting offer for the worker and firm i could retain the worker by matching the unemployment payoff. Asymmetric information would therefore persist. Despite this, adverse selection would not persist since firm j can freely offer based on the expected productivity of set P_2 with no fear of endogenous recruitment precisely because s is large.

So, Proposition 2.1 has shown that it will never be adverse selection that determines the maximum offer made by potential employers, and that $s \geq s^A$ in equilibrium. This means it must be the best response of the first employer to offer according to (2.24) at t_1. From this equation the following results can be derived.

Proposition 2.2. *The payoff received by the worker after the first employer learns her type is given by*

$$W_i(p, t_1) = \begin{cases} p(1-t_1) - s & \text{if} \quad p > p_2^+ \\ u(1-t_1) & \text{otherwise} \end{cases} \tag{2.25}$$

The value of p_2^+ is weakly increasing in s, and there exists a value of $s \geq 0$ which will set p_2^+ to any value in P. The equilibrium payoff is therefore fully separating if s is sufficiently small, fully pooling if s is sufficiently large, or semi-separating for intermediate values of s.

Proof. *A proof is given in Appendix C.*

Consistent with Lazear (1998), a semi-separating equilibrium may be observed where only low ability workers receive a pooling payoff because the benefit they could expect from quitting twice to reveal type is outweighed by the switching cost that would be incurred. The higher the switching cost s, the more readily a worker of given productivity will accept the pooling payoff, so the higher the pooling/separating threshold p_2^+. If s is sufficiently large then even the most

productive worker could be retained on a pooling payoff. If s is sufficiently small the opposite is true: even a very unproductive worker would be willing to incur the switching cost in order to receive a separating payoff from firm j, so in order to retain the worker, the current employer must reveal her type.

The parameter λ that controls the amount of time that it is expected to take for an employer to learn the worker's type has the opposite effect on p_2^+ as the switching cost. The higher the value of λ, the faster the employer is likely to learn the worker's type. This reduces the time the worker requires to generate two previous employers that know her type, and therefore increases the amount of time that she can expect to enjoy the competitive payoff. So for a given t_1, quitting becomes a relatively more attractive option and it is a higher productivity of worker p_2^+ who will be indifferent between the separating and pooling payoffs. In contrast, if λ is very small then quitting does not provide a credible route to revealing type since the new employer is not likely to have time to learn the worker's type during their lifetime. Full pooling is then likely to occur.

The higher the unemployment wage u, the more likely a worker is, *ceteris paribus*, to be content to accept the unemployment payoff from the current employer than move to firm j, so the more likely pooling becomes. Furthermore, firm j's interval 3 offer is less likely to be separating if u is almost as high as p^- as shown by Figure 2.1. If $u = p^-$ then clearly the least able worker will have no incentive to have type revealed and the equilibrium wage offer will always include a pooling region.

So, Proposition 2.2 has shown that the worker will receive at t_1 either a separating offer $p(1-t_1)-s$ or a pooling offer $u(1-t_1)$. Unlike the complete information case however, is not necessarily the greater of these that will be paid.

Proposition 2.3. *In a semi-separating equilibrium the distribution of payoffs across type is discontinuous, with the smallest separating payoff being strictly greater than the pooling payoff.*

Proof. *If no discontinuity existed then a worker of type p_2^+ would be indifferent*

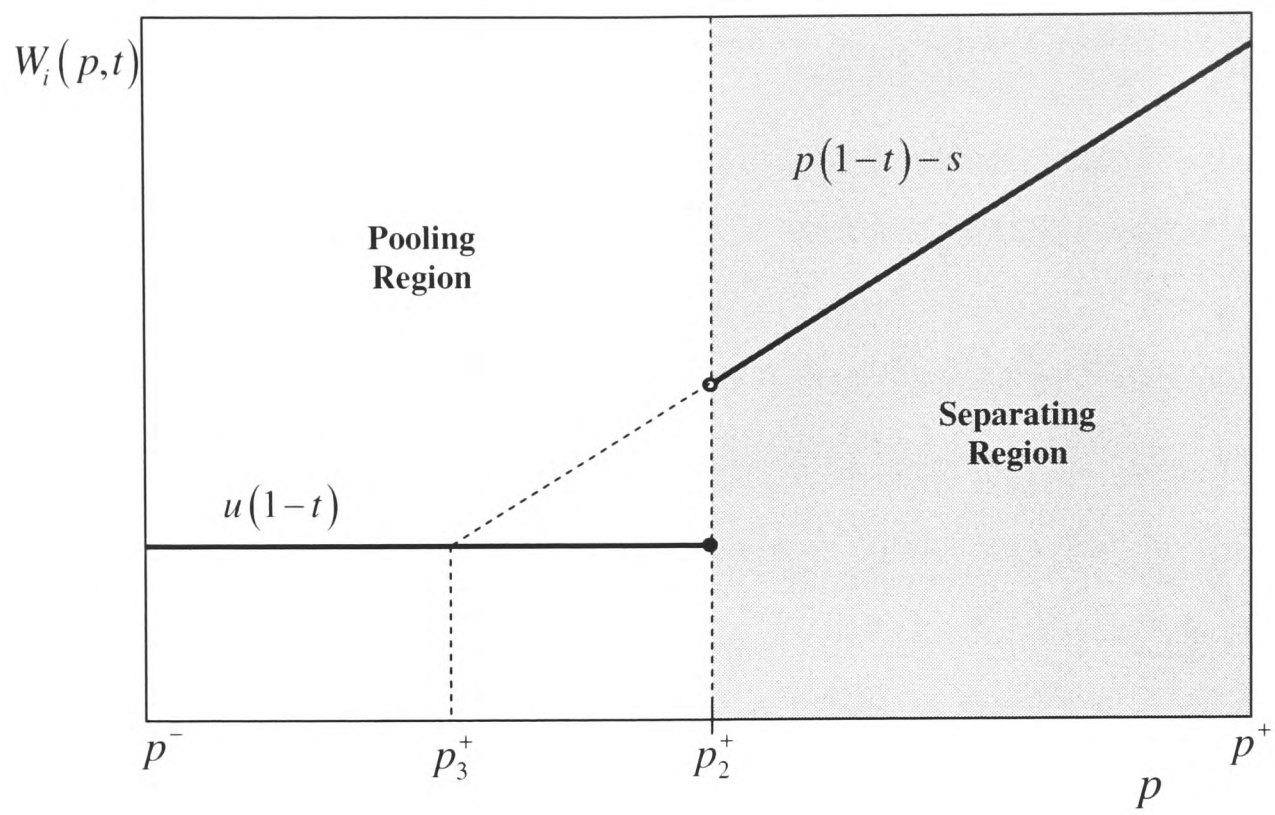


Figure 2.4: The payoff to the worker as a function of productivity for a combination of s and $t \geq t_1$ that gives rise to a semi-separating equilibrium.

between receiving the pooling payoff $u(1-t_1)$ and the separating payoff $p_2^+(1-t_1)-s$. But p_2^+ was defined as the worker indifferent between receiving $u(1-t_1)$ and the payoff from moving to firm j from (2.23)*. So a discontinuity must exist if*

$$\mathrm{E}P_2(1-t_1) - \mathrm{E}^p\mathrm{E}^{t_2}W_j(p,t_2) + \mathrm{E}^{t_2}W_j(p_2^+,t_2) - s < p_2^+(1-t_1) - s \qquad (2.26)$$

For any given value of $t_2 \in (t_1, 1]$, the term $\mathrm{E}P_2(1-t_1) - \mathrm{E}^p\mathrm{E}^{t_2}W_j(p,t_2)$ on the left hand side is the interval 2 payoff that would give zero profit based on the expectation of productivity being in the set P_2, and the term $\mathrm{E}^{t_2}W_j(p_2^+,t_2)$ is the payoff in interval 3 that would give zero profit for productivity p_2^+. The right hand side is a payoff that would give zero profit for productivity p_2^+ in both interval 2 and interval 3. Since $\mathrm{E}P_2 < p_2^+$ for any semi-separating equilibrium, the left hand side must be less than the right hand side. Therefore a discontinuity must exist. ■

Figure 2.4 shows this discontinuity by plotting the payoff as a function of type. The intuition behind this outcome is subtle, and will be explained in two stages. The

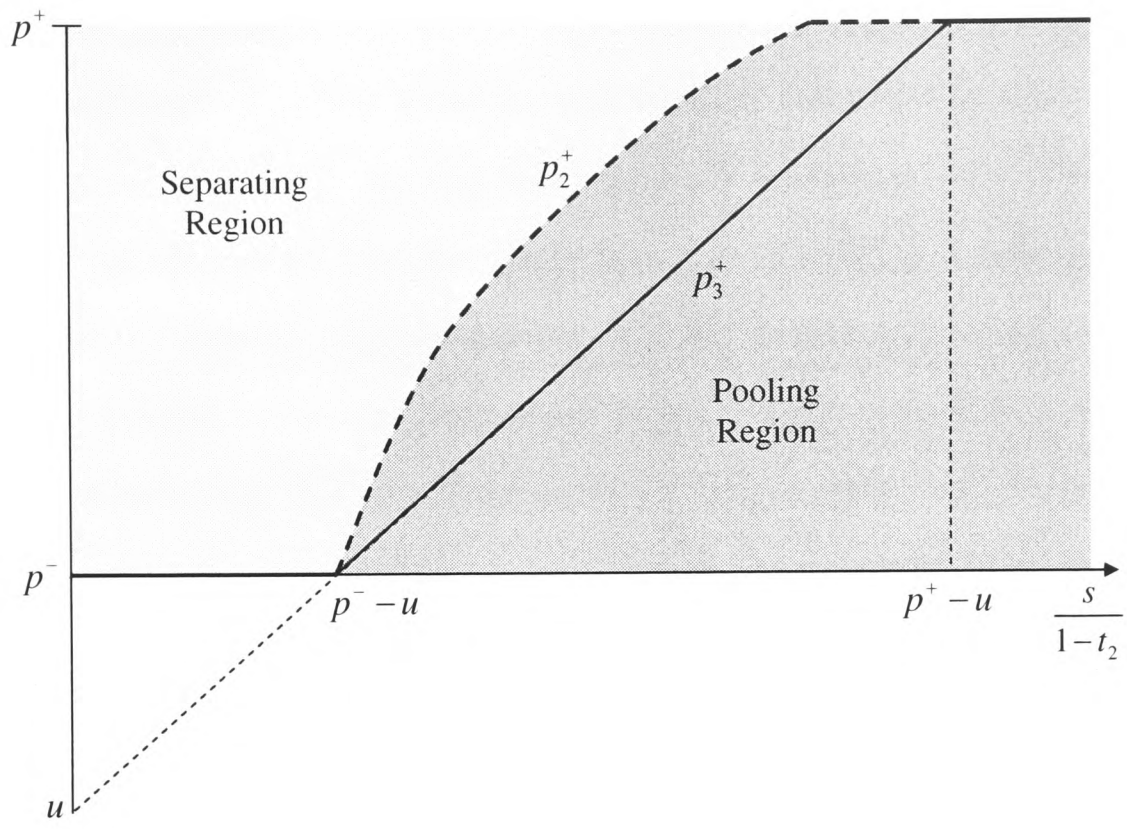


Figure 2.5: The pooling/separating threshold productivity levels p_2^+ and p_3^+ as a function of $\frac{s}{1-t_2}$. The curve for p_2^+ is only a representation; its exact shape will depend on the productivity distribution $f(p)$ and the value of λ. In the region $\frac{s}{1-t_2} \in (p^- - u, p^+ - u)$, $p_2^+ > p_3^+$, and outside the region $p_2^+ = p_3^+$.

first insight required is that if semi-separation occurs, the threshold productivity p_2^+ above which separation occurs in interval 2 is strictly greater than the threshold productivity in interval 3. This is shown in Figure 2.5. Recall that in the full information model, potential employer j would offer an upfront payment $x_j^{CI}(p,t)$ equal to the switching cost s provided $t < \tau(p)$, that is, when $p^- - u < \frac{s}{1-t}$ (Equation (2.9)). So the worker could effectively move to firm j "for free", because the switching cost incurred is exactly balanced by the one-off payment. Once at firm j however, it was the best response of firm j to offer $p(1-t) - s$, and since this is the same as firm i's offer, the worker never quit.

In the model of asymmetric information, an upfront payment may also be offered. However, in this case, the upfront payment drops below s not when $p - u > \frac{s}{1-t}$ as was the case in the complete information case, but when $p^- - u > \frac{s}{1-t}$. This is because the potential employer does not know the worker's type and so takes

an expectation across p. If the worker were to turn out to be of low productivity, an upfront payment of s may never be recouped by following it with a payoff of $\min[p(1-t_1)-s, u(1-t_1)]$. So the more that $\frac{s}{1-t}$ exceeds $p^- - u$, the smaller is firm j's upfront offer, since the less likely firm j is of making a subsequent profit of s. The reduced upfront payment means that the worker effectively has to "pay" at least some portion of the switching cost herself if moving to firm j. It will only be worthwhile incurring this cost if the worker's type is sufficiently in excess of the level p_3^+ that sets $p(1-t_1)-s = u(1-t_1)$. This explains why p_2^+ is greater than p_3^+ in the semi-separating region.

The second logical insight explains the discontinuity in Figure 2.4. Recall that p_2^+ was derived as the productivity of a worker indifferent between staying with firm i on a pooling payoff of $u(1-t_1)$ and moving to firm j. So why is it that, when firm j's offer is continuous in p, a worker of productivity just slightly superior to p_2^+ receives such a sudden increase in the payoff offered by firm i? The reason is that by offering any payoff in excess of $u(1-t_1)$, firm i reveals the worker's type to be greater than p_2^+. Since firm j's upfront payoff was less than s only because of the possibility that p could be as low as p^-, as soon as this information about the worker's type is revealed by firm i, the payoff the worker can expect from firm j increases in a discrete jump. Firm i must then increase its offer to match the payoff that the worker could receive by moving to firm j.

2.5 Conclusion

Despite the prevalent view in the literature that adverse selection will persist in the labour market if potential employers are unable to observe any signal of the worker's productivity, the model of this chapter shows that this conclusion is the result of an implicit assumption that is not necessarily appropriate – that firms forget the type of a worker after she quits. If this assumption is reversed, it has been shown that there can be no persistence of adverse selection even where job changing frictions

exist.

Job changing frictions can lead to a semi-separating payoff where the least able workers can be retained by the first employer on a pooling wage and the more able workers will none-the-less receive a separating wage. This separation result is remarkable because it occurs despite the fact that potential employers have no observable signals of the worker's type.

The implications of these findings are vast. It means that the results of the majority of models of asymmetric information in the labour market can only be believed if one is also willing to accept the assumption upon which they have been unwittingly founded. Beyond labour economics, the result also implies that in any market with asymmetric information where the good is traded multiple times, adverse selection may not persist if a reputation can be built amongst a number of past trading partners. Even in the absence of any observable signal, it is possible that price separation can occur.

2.6 Appendix

A Proof of Lemma 2.1

Suppose the worker has quit once at t_1 and the second employer, firm j, has just learned the worker's type at $t = t_2$. By assumption both the worker and potential employers know when this has occurred. If the worker were to quit again at t_2 and move into unemployment there would be two previous employers i and j that know the worker's type. It is a unique best response for both previous employers to offer $W_j^{CI}(p, t_2) = p(1 - t_2)$. It cannot be a best response for either previous employer to offer less than $p(1 - t_2)$, then it would be a best response of the other to exceed the offer by ϵ and so win the worker quitting for exogenous reasons and make a profit. It cannot be a best response for either previous employer to offer more than $p(1 - t_2)$ because if the worker were to quit for exogenous reasons and accept the offer, a loss would be made.

After a period of time dt in unemployment, the worker can then accept offer $p(1 - t_2 - dt)$ from a previous employer, incur a switching cost s and earn a payoff during interval 3 of $p(1 - t_2) - s$ as $dt \to 0$. Of course, if $p(1 - t_2) - s$ is less than unemployment, the worker will be better off staying unemployed for the rest of her career. So the payoff from this strategy of moving into unemployment at t_2 is $\max[p(1 - t_2) - s, u(1 - t_2)]$. In order to retain the worker after t_2 firm j must match this payoff from quitting with an offer

$$W_j(p, t_2) = \max[p(1 - t_2) - s, u(1 - t_2)] \tag{2.27}$$

It is noted that the worker could alternatively move to potential employer firm k when quitting at time t_2 rather than taking unemployment, but doing so leads to no better payoff than the strategy described above.

At time t_1, the realisation of t_2 is not known but it is assumed that the worker and firms know the underlying exponential distribution. With constant hazard rate θ,

the probability density function $g(t_2)$ for $t_2 > t_1$ is given by $g(t_2) = \theta e^{-\theta(t_2 - t_1)}$. This is a standard equation for the exponential distribution but also derived in Chapter 3 as equation (3.16b). The payoff $W_j(p, t_2)$ as expected at t_1 and unconditional on $t_2 < 1$ can then be found by integrating over the possible values of $t_2 \in (t_1, 1]$ giving (2.10).

B Proof of Lemma 2.2

In the derivation that follows, an expression for firm j's profit at t_1 is found as a function of the exogenous quit rate μ. Firm j's best response is then found by taking the limit as $\mu \to 0$.

The total payoff the worker expects from moving to firm j at t_1 is the pooling interval 2 payoff $\mathrm{E}^{t_2} W_j(P_2, t_1, t_2)$ added to the potentially separating interval 3 payoff $\mathrm{E}^{t_2} W_j(p, t_2)$, less the switching cost incurred. Firm i knows the worker's type exactly so will only be willing to match this payoff if it is no greater than $p(1 - t_1)$. So firm j will recruit only the worker quitting for exogenous reasons if

$$\mathrm{E}^{t_2} W_j(P_2, t_1, t_2) + \mathrm{E}^{t_2} W_j(p, t_2) - s \leq p(1 - t_1) \tag{2.28}$$

If this inequality is not satisfied, firm j will recruit the worker endogenously.

There exists some value p^* such that, for all values of $p \in P_2$, (2.28) is satisfied if $p \geq p^*$ and not satisfied if $p < p^*$. Given the expression for $\mathrm{E}^{t_2} W_j(p, t_2)$ in (2.10), the rate of change of the left hand side of (2.28) with respect to p is strictly less than that of the right hand side for all $p \in P_2$. So as p increases from p_2^- to p_2^+, the value of $\mathrm{E}^{t_2} W_j(P_2, t_1, t_2) + \mathrm{E}^{t_2} W_j(p, t_2) - s$ can cross the value of $p(1 - t_1)$ no more than once, and this must be from above.

Firm j's profit conditional on hiring the worker at t_1 is given by the output of the worker after t_1, $p(1 - t_1)$, less the interval 2 and 3 payoffs. So firm j's expected profit from making an offer of $\mathrm{E}^{t_2} W_j(P_2, t_1, t_2)$ can be found by integrating across all the values of $p \in P_2$, weighted according to whether there would be endogenous

or only exogenous recruitment. For $\mathrm{E}^{t_2}W_j(P_2, t_1, t_2)$ to be a best response it must lead to non-negative expected profit since a superior profit of zero would result from making no offer at all. So

$$\begin{aligned}\int_{p_2^-}^{p^*} p(1-t_1) - \left(\mathrm{E}^{t_2}W_j(P_2, t_1, t_2) + \mathrm{E}^{t_2}W_j(p, t_2)\right)\, f(p)\, dp \\ + \int_{p^*}^{p_2^+} \mu\left(p(1-t_1) - \left(\mathrm{E}^{t_2}W_j(P_2, t_1, t_2) + \mathrm{E}^{t_2}W_j(p, t_2)\right)\right)\, f(p)\, dp \geq 0\end{aligned} \tag{2.29}$$

The value of the second integral converges to zero as $\mu \to 0$. So for the inequality to hold it follows the first integral must also converge to zero as $\mu \to 0$. Since the value of $p(1-t_1) - \left(\mathrm{E}^{t_2}W_j(P_2, t_1, t_2) + \mathrm{E}^{t_2}W_j(p, t_2)\right)$ is negative for all $p \in [p_2^-, p^*]$ by definition of p^*, the only way the value of the first integral can converge to zero is if the upper bound is the same as the lower bound. Then $p^* = p_2^-$ and there are no endogenous hires.

By definition of p^*, (2.28) must be satisfied if $p = p^* = p_2^-$. Rearranging (2.28) with p set to p_2^- shows that the expected value of firm j's interval 2 must satisfy

$$\mathrm{E}^{t_2}W_j(P_2, t_1, t_2) \leq p_2^-(1-t_1) + s - \mathrm{E}^{t_2}W_j(p_2^-, t_2) \tag{2.30}$$

The maximum interval 2 payoff that satisfies this inequality is defined as $\mathrm{E}^{t_2}W_j^A(P_2, t_1, t_2)$. This is called the adverse selection constraint, since the reason that it cannot be a best response for firm j to offer more is that then $p^* > p_2^-$ and the expected productivity of a worker conditional on her being hired would be less than the expected productivity of the set P_2.

So firm j's best response must be low enough to avoid endogenous hires. But in addition, firm j's offer must also be a best response conditional on the worker quitting for exogenous reasons since, even as $\mu \to 0$, there remains a non-zero probability of this occurring. If the worker were to accept firm j's offer at t_1 then in interval 3 firm j will make a separating payoff $\mathrm{E}^{t_2}W_j(p \in P_2, t_2)$. But

during interval 2 firm j only knows that the worker's type is in P_2. Define $\mathrm{E}^p\mathrm{E}^{t_2}W_j(p \in P_2, t_1)$ as the expectation over $p \in P_2$ of $\mathrm{E}^{t_2}W_j(p \in P_2, t_1)$. Then

$$\mathrm{E}^p\mathrm{E}^{t_2}W_j(p, t_2) = \frac{\int_{p_2^-}^{p_2^+} \mathrm{E}^{t_2}W_j(p_2, t_2)}{\int_{p_2^-}^{p_2^+} f(p)\,dp} \tag{2.31}$$

Since the value of the first integral in (2.29) is non-positive, for inequality (2.29) to hold, the second integral must be non-negative. Then, with p^* set to p^-,

$$\int_{p_2^-}^{p_2^+} \mu\big(p - \mathrm{E}^{t_2}W_j(p \in P_2, t_1)\big)\, f(p)\,dp \geq 0 \tag{2.32}$$

Dividing through by μ, setting $\mathrm{E}^{t_2}W_j(p \in P_2, t_1) = \mathrm{E}^{t_2}W_j(P_2, t_1, t_2) + \mathrm{E}^{t_2}W_j(p, t_2)$ and rearranging gives

$$\mathrm{E}^{t_2}W_j(P_2, t_1, t_2)\int_{p_2^-}^{p_2^+} f(p)\,dp \leq \int_{p_2^-}^{p_2^+} p\, f(p)\,dp - \int_{p_2^-}^{p_2^+} \mathrm{E}^{t_2}W_j(p, t_2)\, f(p)\,dp \tag{2.33}$$

Dividing through by $\int_{p_2^-}^{p_2^+} f(p)\,dp$ and rearranging gives

$$\mathrm{E}^{t_2}W_j(P_2, t_1, t_2) \leq \mathrm{E}P_2 - \mathrm{E}^p\mathrm{E}^{t_2}W_j(p, t_2) \tag{2.34}$$

The maximum interval 2 payoff that satisfies this inequality is defined as $\mathrm{E}^{t_2}W_j^B(P_2, t_1, t_2)$.

Competition between potential employers drives their interval 2 offer up to the highest level that satisfies (2.30) and (2.34). Suppose this were not the case. Then there would be an incentive for another potential employer to offer ϵ more than firm j and make a profit should the worker quit for exogenous reasons. This contradicts the definition of firm index j in (2.13).

C Proof of Proposition 2.2

Since $s > s^A$ in equilibrium, the payoff the current employer firm i will offer at t_1 is given by (2.23). This is equal to the payoff the worker expects to receive from moving to firm j. Substituting in $s' = s - s^A$ sets this payoff to $p_2^-(1-t_1) - \mathrm{E}^{t_2}W_j(p_2^-, t_2) + \mathrm{E}^{t_2}W_j(p, t_2) - s'$. This offer is weakly increasing in p via the term $\mathrm{E}^{t_2}W_j(p, t_2)$ given by (2.10). So for firm i to retain any worker in P_2 with the lowest possible pooling payoff, firm i will offer the smallest payoff which would retain the worker of type p_2^+. This is

$$p_2^-(1-t_1) - \mathrm{E}^{t_2}W_j(p_2^-, t_2) + \mathrm{E}^{t_2}W_j(p_2^+, t_2) - s' \tag{2.35}$$

Since $s' < s$, this payoff is greater than the separating payoff $p_2^-(1-t_1)$ with which firm j could retain the worker of type p^-. So there can be no pooling payoff where firm i's offer is equal to firm j's payoff (2.23). So the only way a pooling payoff could be part of an equilibrium is if, in (2.24), $\mathrm{E}^{t_2}W_i(p \in P_2, t_1) = u(1-t_1)$. Since (2.23) is weakly increasing in p, conditional on a pooling region existing, the lowest productivity of worker in the set P must be in it, so $p_2^- = p^-$.

The highest productivity of worker in the set P_2 will be indifferent between accepting the pooling offer $u(1-t_1)$ and moving to firm j to receive an offer given by (2.23). So $u(1-t_1) = \mathrm{E}P_2(1-t_1) - \mathrm{E}^p\mathrm{E}^{t_2}W_j(p, t_2) + \mathrm{E}^{t_2}W_j(p_2^+, t_2) - s$. The right hand side of this expression is continuous and decreasing in s. And since, when $s = 0$ the right hand side is greater than the left, there must be some value of s for which the equation holds for any value of $p_2^+ \in P$.

Chapter 3

Adverse Selection With Asymmetry Between Previous Employers

3.1 Introduction

3.1.1 Objectives of This Chapter

On the issue of whether adverse selection persists in the labour market, the theoretical literature contains the full spectrum of predictions. At one extreme, Greenwald (1986) suggests that adverse selection will always persist if some quits take place for exogenous reasons and firms forget the type of a worker after she quits. A subsequent stream of papers such as Ricart I Costa (1988) and Waldman (1989) build on Greenwald's theory but incorporate further features which soften the result somewhat, finding that the worker may, in fact, be able to reveal their type to the market at least partially through observable signals such as task assignment or promotion.

The results of Chapter 2 of this thesis fall at the other end of the spectrum of adverse selection models. The model shows that if firms can remember the type of a worker after she quits and workers know their own type then adverse selection will never persist and a worker will always be paid the full value of their productivity by their first employer as soon as type is learned. This extreme prediction is tempered

by the existence of switching costs which may prevent some or all types of worker from having their type revealed to the market.

The contrasting results of Greenwald and Chapter 2 can be explained simply as a result of the differing assumptions regarding the symmetry of information between the current employer and previous employers. Greenwald's prediction of persistent adverse selection relies on the assumption that the current employer is perfectly informed about the worker's type but any previous employer is as ignorant about the type of their ex-employee as a firm who has never employed her. In contrast, the result from Chapter 2 that adverse selection will never persist relies on the assumption that a previous employer that learned the worker's type is, forever after, as well-informed as the current employer ever can be.

The objective of this chapter is to explore the intermediate case where previous employers know more about the worker's type than firms that have never employed the worker, but still suffer from some degree of informational asymmetry with respect to the current employer. There are a number of ways that this partial asymmetry of information could be introduced. One possibility would be to introduce noise into the firm's observation of the worker's type. Some degree of uncertainty would then be attached to any firm's knowledge of the worker's type and the degree of uncertainty would depend on the length of time the worker was employed. Previous employers that had employed the worker for different durations would then be asymmetrically informed, competition between them could be stifled and wage offers could pool.

Another approach would be to introduce a firm-specific matching parameter which is private information to the firm. This too could soften the degree of competition between previous employers and suppress separation. The best response of a previous employer would likely be to offer less than its true valuation because this would lead to a profit in the event that the firm with whom they are competing makes an even lower offer due to a poor match with the worker.

The problem with both these possible approaches is that the asymmetry of

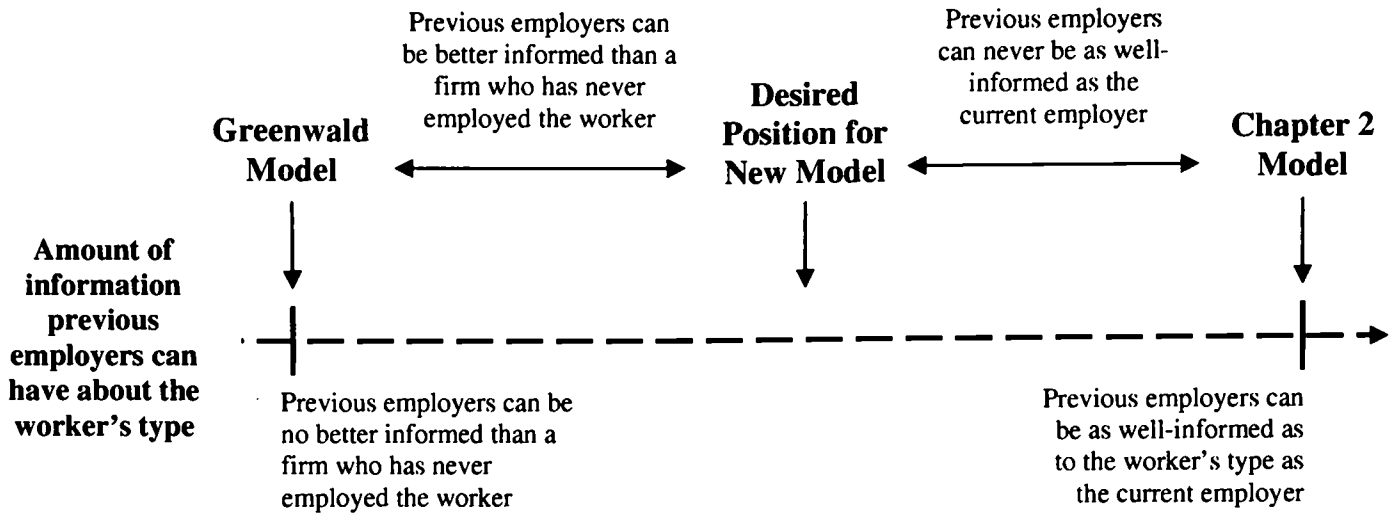


Figure 3.1: Diagram showing the amount of information a previous employer can have about the worker's type in relation to the models of Greenwald and Chapter 2.

information introduced does not necessarily favour the current employer. Whilst they succeed in reducing the ability of previous employers to make competitive offers, the current employer is similarly disadvantaged.[1] So these approaches would not satisfy the objective of exploring the effect of a partial informational disadvantage for previous employers relative to the current employer.

3.1.2 The Changing-Productivity Assumption

In this chapter the assumption that is instead adopted is that the productivity of the worker may change over time. Productivity changes are taken as exogenous rather than, for example, the result of a change in effort resulting from a change in the wage rate. In this way a past employer's knowledge of the worker's type becomes increasingly out-of-date the longer the time elapsed since their employment. Different previous employers thus have potentially different expectations of the worker's current type. Crucially, the current employer will always have the most up-to-date information about the worker's current productivity. Thus the model is

[1] If there were noise in the observation of type, it may be possible under certain assumptions about the statistical process generating the noise that the information learned by an employer about the worker's type could be represented by a sufficient statistic. If this sufficient statistic was revealed through the wage offer then a current employer would always inherit the information learned by any previous employer and so would, in fact, be at least as well-informed as any other firm. However, this case is of no interest because if all the information about a worker's type is revealed in the wage offer, wages are fully separating and there is no adverse selection issue to consider.

positioned in the desired space between the extremes of Greenwald and Chapter 2 as shown in Figure 3.1.

The changing-productivity assumption has considerable intuitive appeal. It is easy to imagine a situation where a change in circumstance would suddenly affect a person's productivity at work in a long term sense. For example, a worker who starts a family could find their focus shifts from being predominantly career-driven with the attendant long hours and aggressive climbing of the corporate ladder, to a preference for lower stress and regular hours even if this means less pay. In contrast, finding an inspiring mentor could allow a worker to better understand their best skills and, over time, select tasks at which they can excel.

The assumption of exogenous productivity changes is not a common one in the literature. In models where output is dynamic, the changes are usually explained endogenously as the result of human capital accumulation or a change in effort resulting from the worker's utility maximisation objective. For example, an incentive pay scheme may induce the worker to increase their rate of output, or career concerns may cause a worker to apply more effort early in their career (Holmstrom, 1999). The model of this chapter is not attempting to explain *why* a worker's productivity may change, but rather, *how* the possibility of a change in a worker's productivity may affect the results of Chapter 2. Traditionally, adverse selection models have paid no attention to the amount of information possessed by previous employers because, if the current employer forgets all the information it has about the worker after she quits, then a previous employer can then be treated in the same way as any other potential employer. So it is hardly surprising that, as this thesis opens up a new branch of models of adverse selection in the labour market, an assumption such as this changing-productivity can come into its own.

3.1.3 Auction Theory

The asymmetry of information between firms introduced by the changing-productivity assumption complicates the game theoretic analysis considerably beyond that of the fixed productivity case in Chapter 2. In order to determine the optimal strategies of firms bidding for the worker in this more complex setting, the process is framed as an auction, thus opening up the vast toolkit of the auction theory literature which has been extensively developed over the last 25 years.[2] In this section, a brief overview is provided only of the theory and literature that is relevant to the models that follow.

In the models of this chapter, there is no firm-specific matching parameter; the worker is assumed to be of identical value to whichever firm employers her. So an auction to employ the worker fits the description of a pure common-value auction, that is, where the object is of the same value to everyone, but each bidder may have different information about exactly what that value is. One of the main features of common-value auctions is the prevalence of the winner's curse. The winner's curse has already come up in Chapter 2 as the phenomenon which deterred the less well-informed firm from bidding up to the expected value of the worker because, conditional on this bid not being matched by the current employer, the winning bid would be more than the worker's actual value and a loss would be made. In auction theory, the winner's curse applies in exactly the same way, but a wider variety of informational possibilities are catered for than have been examined thus far.

There are numerous types of auctions used in practice and considered in theory. In some types of auction, information about a bidder's valuation of the object is revealed to other bidders as the auction takes place. If the object is of common value this information can be used by other bidders to update their own valuation, and hence bidding strategy, as the auction progresses. Such an auction is sometimes

[2]For a review of this literature, see Milgrom (1989), Klemperer (1999, 2003) and, on the empirical side, Klemperer (2004).

referred to as an "open" auction because of this transparency. Auctions where no such information is revealed may be referred to as "closed". In this chapter, two models will be developed, the first assuming the process by which firms compete to employ a worker can be represented by an open auction, and the second model assuming a closed auction. As noted by Wilson (1980), markets that suffer from adverse selection are far more sensitive to the specifics of the price setting process than are traditional markets with perfect information. Indeed the models of this chapter support Wilson's result, with the open and closed auctions leading to entirely different equilibrium wage offers.

The open auction that will be used is a type of ascending-price auction known as a Japanese auction. Here the price rises continuously and bidders exit the auction until only one remains. The Japanese auction is sometimes called a Button auction because of the way it is sometimes carried out by having bidders hold down a button as the price rises with the release of the button indicating the bidder's (irrevocable) exit from the auction. The Japanese auction is the most common variant of the ascending-price auction in the theoretical literature (Klemperer, 1999, page 229) as it prevents complications arising from the existence of the "jump-bidding" strategy described by Avery (1998). The theoretical underpinnings of the Japanese auction are given by Bikhchandani and Riley (1993).

The closed auction will be modelled as a first-price sealed-bid auction, referred to hereafter simply as a first-price auction. In terms of bidding strategies and the revenue generated, Vickrey (1961) shows this is identical to a descending-price auction.

In Chapter 2, the worker was assumed to be of fixed productivity, so all the firms that had learned the worker's type at any time were symmetrically informed when bidding to rehire the worker. Previous employers were assumed to make simultaneous offers for the worker, which in the parlance of auction theory is equivalent to a first-price auction. The model showed that competition between

previous employers drives the wage up to the level that gives zero expected profit. This possibility induces the first employer to offer a separating wage so long as switching costs are sufficiently small.

In this chapter though, changing productivity introduces informational asymmetry between previous employers and so it is no longer a certainty that competition between previous employers will lead to a separating wage. In fact, the models show that separation or pooling is possible, depending on what assumptions are made about the bidding process.

The literature on auctions with informational asymmetry began with Bikhchandani (1988) and was further examined by Klemperer (1998) and Bulow et al. (1999). The main finding of these papers is that, in common-value auctions, even very small informational differences can cause vast differences in the revenue raised by the auction (these auctions are sometimes described as "almost common values"). If one player has a slightly higher valuation than the others, that player can bid a little more aggressively. This exaggerates the winner's curse that is suffered by other players, since winning the auction against a more determined opponent increases the likelihood that too high a price has been paid. This in turn reduces the winner's curse suffered by the better-informed player, who can then afford to bid even more aggressively and so on.

A subset of the asymmetric information case is where one player has unambiguously superior information than the others. The changing-productivity assumption of this chapter introduces just such an informational advantage between previous employers bidding to rehire the worker. In this context, Milgrom (1981) shows for a second-price auction and Milgrom and Weber (1982) for a first-price auction that in a pure common-value setting, a bidder with no private information will make no profit in equilibrium. Where there are informational asymmetries, first-price auctions typically generate higher revenue than ascending-price auctions. This result is cited by numerous authors including Klemperer (1999). The models of this chapter

provide further evidence.

The papers described above are not directly drawn upon by the models of this chapter. All game theoretic results will be derived from basic assumptions, but the economist familiar with the auction theory literature will see many parallels between the results of this chapter and the papers mentioned here. The objective of this chapter is not to break new ground in auction theory, but to use auction theory to formalise the underlying bidding process in order to gain insight into wage rates and adverse selection in labour markets with asymmetric information under the assumption of firm memory.

3.1.4 Overview of This Chapter

Two models of the labour market will be presented. Section 3.2 begins with a technical introduction to the concept of changing productivity along with other assumptions that are common to both the models. Section 3.3 models the process by which firms bid for the worker as an ascending-price auction. This captures the "open" bidding process and allows general conclusions to be drawn about the situation where firms can update their bidding strategies based on the information that has already been revealed by other firms' bids. Section 3.4 models the case where firms bid for the worker using a first-price auction. This captures the "closed" bidding process where firms must make their bid based on private information and there is no opportunity to supplement that information by observing other bids. The chapter concludes with a comparison and general discussion of the two models.

3.2 Preliminaries

3.2.1 Assumptions

A single risk-neutral worker has unit lifespan, and productivity at time t of $p(t)$. The worker's initial productivity $p(0)$ is drawn from a distribution $P : [p^-, p^+]$ where

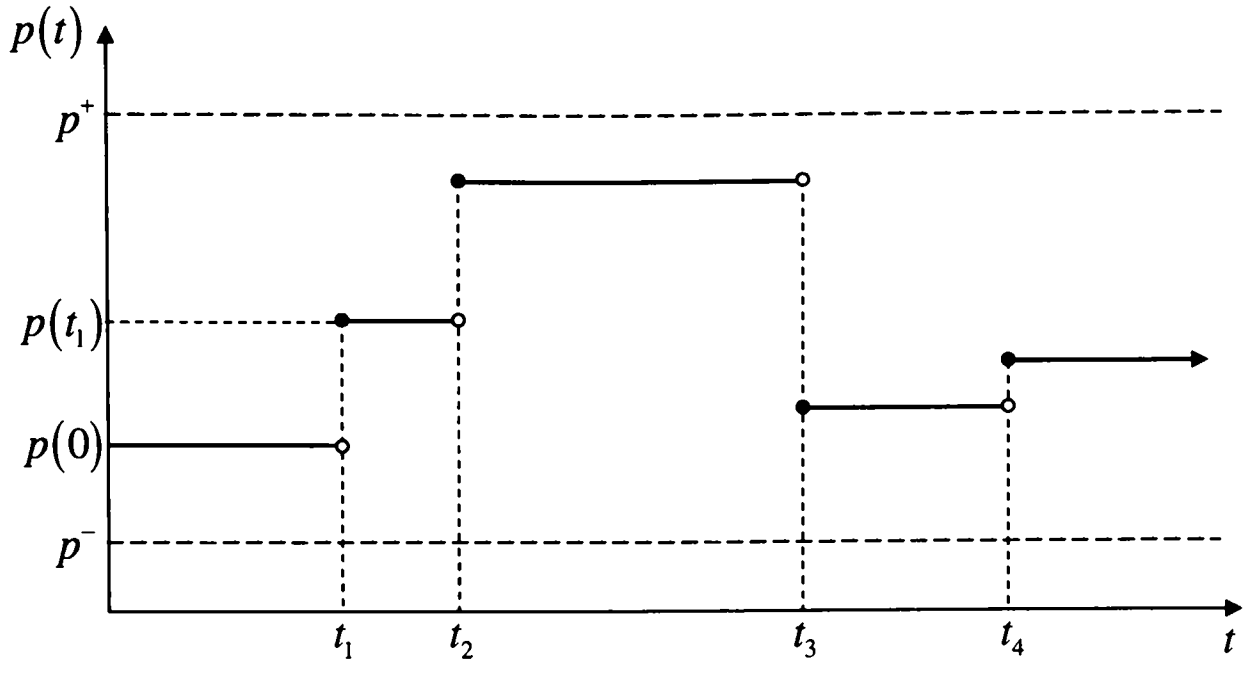


Figure 3.2: An example of the worker's productivity over time. The time between each change is exponentially distributed.

$p^+ > p^-$ and the probability density function is given by the function $f(p)$. This distribution is known to the worker and firms. It is assumed that the worker's productivity can change in discontinuous jumps, at times that are governed by an exponentially distributed statistical process with constant hazard rate ϕ. At each of these step changes, the new productivity level is a random draw from the distribution P, thus there is no correlation between the new productivity and the old. An example is given in Figure 3.2.

Since the exponential distribution is memoryless, the probability of type changing at any moment is independent of when it last changed. Moreover, since the new productivity is a random draw from the distribution P, the expectation of the worker's productivity after one, two or any number of changes is simply the expected value of the distribution P.

It is assumed that only the worker is able to observe when her productivity changes. The worker's productivity is not directly observable by any firm, but the probability of it being learned by the current employer is governed by a second exponentially distributed statistical process with constant hazard rate λ. As $\lambda \to \infty$

the interval between the current employer's updates approaches zero and the current employer becomes perfectly informed as to the worker's type. For lower values of λ, the current employer's updates are less frequent. This reduces the informational advantage it enjoys over other firms and allows the model to give further insights into the relationship between the information disparity and equilibrium wage rates.

It is assumed that other firms are able to observe *when* the current employer (re)learns the worker's type, but are not able to observe *what* is learned. Because the learning of type has constant hazard rate λ, the probability of type being learned in any moment of time is independent of when it was last learned, which firm the worker is currently employed by, and how long it has employed her.

There is no discounting by the worker or firms. It is further assumed there is no contracting, so firms cannot commit themselves to any future payment. At the start of each period dt, potential employers compete in an auction, the winner of which hires the worker for the period at the agreed wage. In order to capture the effect of adverse selection it is assumed that the current employer does not participate in the auction but instead has the opportunity to make a counter-offer to the winning bid after the auction has taken place. The worker then seeks to maximise expected lifetime payoff by either staying with the current employer for the period or moving to another firm. It is not assumed this will be the firm that won the auction, but it is shown that this would in fact be the case. There is only one exception to this strategy of the worker: it is assumed that there is an exogenous quit rate μ which is an instantaneous probability of the worker quitting her present employer regardless of the expected payoff. A worker quitting for exogenous reasons will then choose to work for the firm other than the current employer whose offer leads to the highest expected lifetime payoff. For simplicity the model lets $\mu \to 0$, but the non-zero probability of a potential employer recruiting the worker for exogenous reasons turns out to be crucial in defining a unique perfect Bayesian equilibrium in the same way as in Chapter 2.

Let t_n represent the time at which the worker's type is learned, or "sampled", by an employer for the nth time. Let $n(t)$ represent the number of times that the worker's type has been learned by any employer up to time t. Then $t_{n(t)}$ is the time at which type was most recently learned by any employer. Further, let $c(n)$ be a function which returns the index of the firm that learned the worker's type for the nth time. Then $c(n(t))$ represents the firm that most recently learned the worker's type.

With constant hazard rate λ, the expected number of times the worker's type will be sampled by any employer up to time t is λt and the (discrete) distribution of $n(t)$ is given by the standard Poisson formula

$$\text{prob}\,(n(t) = x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!} \tag{3.1}$$

where $x \in \{1, 2, 3 \ldots\}$. Note the close relationship between the exponential distribution, which describes the probability distribution of the time interval between occurrences, and the Poisson distribution, which models the number of occurrences that are expected in a specified interval. In both cases the probability of the event happening at a given moment in time, conditional on the trial continuing (that is, the hazard rate), is constant.

Attention is focussed on the period that begins at t_1, that is, after the first occasion on which an employer has learned the worker's type. Before t_1 there is no asymmetric information and therefore no adverse selection. All firms know nothing about the worker's type, and competition will motivate at least some of the competing firms to offer the wage that gives zero expected profit given prior beliefs about type. One might argue that the persistence of adverse selection is therefore irrelevant because the suppressing effect adverse selection may have on wages after t_1 is balanced out by higher wages before t_1. But this argument overlooks one important fact. Although the persistence or non-persistence of adverse selection will not affect the expected payoff for a worker of unknown productivity, it will affect

the expected payoff for a worker of *given* productivity. All workers will receive the same payoff up until t_1. Thereafter a high type worker will receive more than a low type worker if separation occurs, but the same amount if adverse selection persists.

After t_1 there is at least one firm that has private information about the worker's type and therefore some asymmetry of information between firms competing to hire the worker. To avoid confusion when referring to individual and sets of firms, the following definitions will be adhered to.

Current Employer: the firm that employs the worker at the moment in question, time t. This firm will be indexed with an i, so if the worker changes employer then the firm referred to as firm i also changes.[3]

Previous employer(s): a firm other than the current employer who has employed the worker before time t and, importantly, learned her type at least once, therefore possessing some private information. The index j will be used to refer to a specific previous employer.

Potential employers: the large number of firms that have never learned the worker's type up to time t and therefore have no private information, but that none-the-less will take part in the auction to attempt to recruit the worker. The current employer is excluded from this set.

Competing firms: this term will be used to refer to all the firms that participate in the auction, that is, potential employers and previous employers, but not the current employer. The index k will be used to refer to a competing firm.

The unemployment payment payoff is normalised to zero. By assumptions, $p^- > 0$, so there will always be an uninformed potential employer willing to exceed the unemployment payoff.

[3]Note that this system of firm indexes differs from the previous chapter. There the variables denoting firm indexes always referred to the same firm but in this chapter, index variables refer to the same *type* of firm

Define $W(\tau_1, \tau_2)$ as the present value of future payments to the worker, or the "payoff" for short, from time τ_1 conditional on staying at a given firm until at least τ_2. Where $\tau_2 = 1$ (as is most often the case) this parameter will be dropped for clarity. In each period dt an auction will take place with the winner committing to employ the worker for the period for the agreed amount $w(t)dt$ where

$$w(t) = -\frac{dW(t)}{dt} \tag{3.2}$$

As $dt \to 0$ the model approaches continuous time and the amount that firms bid in the auction approaches the wage rate.

As was the case in the previous chapter, bids in the auction will be presented in terms of the payoff $W(p,t)$ that would be paid to the worker conditional on her staying with the offering firm for the rest of her lifetime. Of course, since there is no contracting in the model, the offering firm is not actually committing to paying $W(p,t)$. If the worker accepts the offer at time t then the new employer will offer the profit maximising payoff $W(p, t+dt)$ at time $t+dt$. So the only payment that the offering firm is committed to paying in the period starting at time t is $W(p,t) - W(p,t+dt) = w(p,t)dt$. But the best response of a firm bidding at time t can only be solved with reference to the entire subgame beginning at t, hence the advantage of considering bids in terms of $W(p,t)$. Once best responses have been identified in terms of $W(p,t)$ it is then a simple matter to differentiate the payoff $W(p,t)$ in order to find the wage rate $w(p,t)$.

The worker who quits her current employer may choose to move to any firm that participated in the auction, and strategic reasons could mean this will not necessarily be the firm with the highest bid. So it is not necessary to specify which firm is assumed to win the auction in the event of a tie. An assumption regarding ties *is* required where the best possible expected payoff is offered by more than one firm. In this instance it is assumed that the worker has a slight preference to stay at the current employer rather than move to a competing firm, but amongst tied

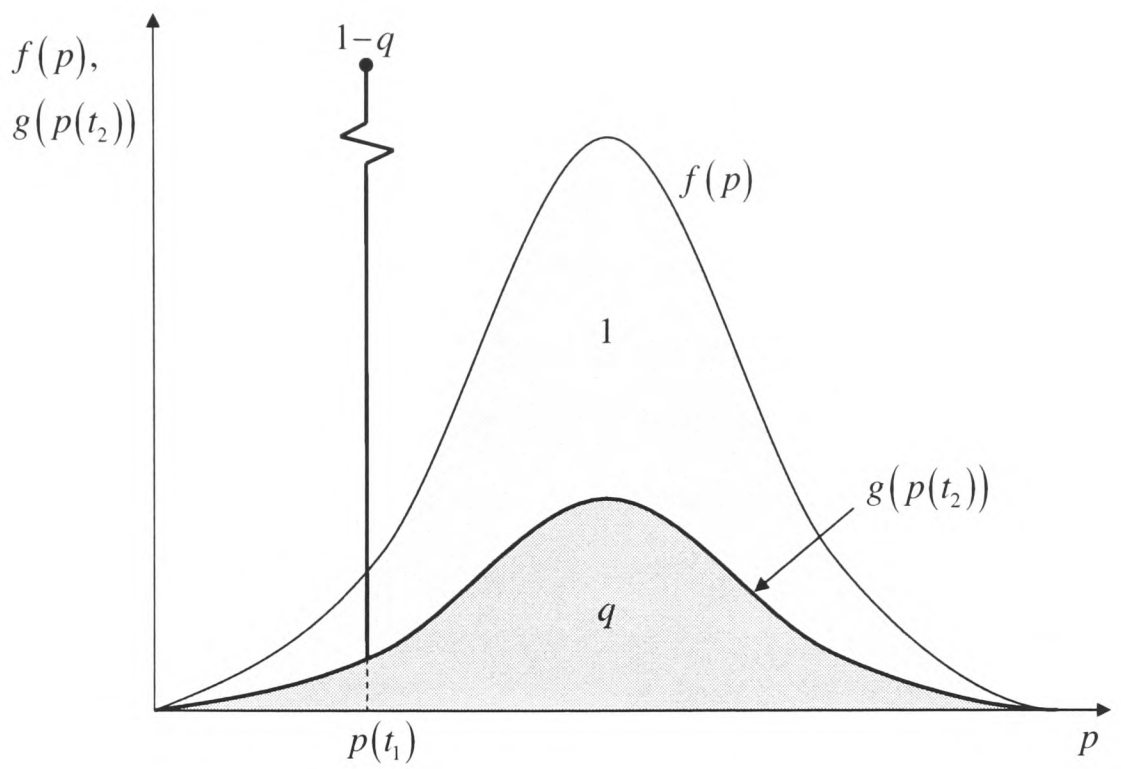


Figure 3.3: An example of a probability density function of $p(t_2)$ for a previous employer that knows $p(t_1)$.

competing firms chooses one at random.

3.2.2 Expected Output

Suppose the worker quits her first employer after it has learned her type at t_1 and stays with the second employer until it learns her type at t_2. There is some probability $q > 0$ that the worker's type has changed during the interval $[t_1, t_2]$. So the first employer is not certain of $p(t_2)$ but knows the probability distribution $g(p(t_2))$ from which it is drawn. This includes an atom at productivity $p(t_1)$ of probability $1 - q$ with the rest of the distribution taking the same shape as the population distribution $f(p)$ but scaled down so that the area under the curve equals q.[4] This is shown in Figure 3.3. Also of note is the fact that, while the second employer is better informed about $p(t_2)$ than the first, it does not know $p(t_1)$ and is therefore not certain of the first employer's beliefs about the worker's type at

[4]Strictly speaking, since the spike at $p(t_1)$ has no width it can not have any area either. To set the area to the desired $1 - q$, the Dirac Delta function $\delta(t_1)$ can be used with the appropriate weight. See Spanier and Oldham (1987) for a full description of this technique.

t_2.

Before proceeding further, an expression is presented that will be useful in both the models of this chapter that describes the expected value to a firm of employing the worker for a fixed amount of time, conditional on the information it has about the worker's type. This expected value is the integral of the worker's productivity over time, in other words, output. Unlike the fixed productivity model, calculating output is no longer a straightforward matter because the worker's productivity can change at any moment.

Lemma 3.1. *The output of a worker from τ_1 to $\tau_2 > \tau_1$ as expected at $\tau_0 \leq \tau_1$ where productivity at τ_0 is known to be $p(\tau_0)$ is given by*

$$W^*(p(\tau_0), \tau_0, \tau_1, \tau_2) = \mathrm{E}P(\tau_2 - \tau_1) + \frac{p(\tau_0) - \mathrm{E}P}{\phi}\left(e^{-\phi(\tau_1 - \tau_0)} - e^{-\phi(\tau_2 - \tau_0)}\right) \quad (3.3)$$

where $\mathrm{E}P$ is the expected productivity of the distribution P given by

$$\mathrm{E}P = \int_{p^-}^{p^+} p f(p)\, dp \quad (3.4)$$

Proof. *A proof is given in Appendix A.*

In the model that follows, τ_2 is nearly always 1, the end of the worker's lifetime. So for the sake of algebraic efficiency, τ_2 will be omitted from the parameter set of W^* in this instance.

The function W^* exhibits all the properties that would be expected. The more out-of-date the firm's knowledge of the worker's productivity, the more heavily its estimate of output is based on the expected productivity $\mathrm{E}P$ than the old known productivity $p(\tau_0)$. An increase in ϕ, the rate at which type changes, will further exaggerate this shift. The dynamics of the function will be further discussed once the model has been solved.

3.3 Open Auction Model

3.3.1 Analysis and Results

The open auction is investigated by adopting the Japanese style of ascending-price auction where the price is assumed to rise continuously with bidders dropping out until only one remains. Bidders can observe when other bidders quit the auction, and having quit, a bidder is not allowed to re-enter the auction. Bikhchandani and Riley (1993) give a full description of the Japanese auction. Under a sequential second-price auction such as this, the bidder with the highest valuation wins the auction but pays a price equal to ϵ more than the second highest bid. It is assumed that $\epsilon = \lim_{x\to\infty} \frac{1}{x}$ and that $\lim_{dt\to 0} \frac{\epsilon}{dt} \approx 0$. In this way, ϵ is sufficiently small to be inconsequential to wage outcomes, even over a very large number of periods.

The perfect Bayesian equilibrium outcome is solved by determining the best response of each firm conditional on its beliefs about the worker's type given her employment history, beginning with the current employer.

Lemma 3.2. *Conditional on the auction at time t being won or tied by competing firm k with a bid of $W_k(t)$ and the current employer firm i having the most up-to-date information about the worker's type (that is, $c(n(t)) = i$) it is a best response of the current employer to make a counter-offer of*

$$W_i(t) = \begin{cases} W_k(t) & \text{if} \quad W_k(t) \leq W^*(p(t_{n(t)}), t_{n(t)}, t) \\ 0 & \text{otherwise} \end{cases} \tag{3.5}$$

Proof. *Conditional on $W_k(t) > W^*(p(t_{n(t)}), t_{n(t)}, t)$ it cannot be a best response for firm i to offer $W_i(t) \geq W_k(t)$. No firm has more up-to-date information about the worker's type than firm i that knows $p(t_{n(t)})$. Since $W^*(p(t_{n(t)}), t_{n(t)}, t)$ is, by definition, the payoff that will lead to zero expected profit, to offer more than this would lead to an expected loss. A best response cannot lead to a negative expected profit since a superior profit of zero can be made by making no offer at all. Any*

offer less than $W_k(t)$ is a best response, causing the worker to quit and zero profit to be made. $W_i(t) = 0$ is one such best response.

Conditional on $W_k(t) < W^*(p(t_{n(t)}), t_{n(t)}, t)$, it is a unique best response for firm i to retain the worker by matching $W_k(t)$, leading to a positive and maximised expected profit.

Conditional on $W_k(t) = W^*(p(t_{n(t)}), t_{n(t)}, t)$, any offer $W_i(t) \leq W_k(t)$ will lead to zero expected profit. It is therefore a (non-unique) best response for firm i to offer $W_i = W_k(t)$ and so, by the assumption regarding ties, retain the worker. ■.

The lemma shows the typical adverse selection result – that when the current employer has superior information about the worker's ability (which is the case here since $c(n(t)) = i$), competing firms are subject to the winner's curse. The only way a competing firm can succeed in winning the worker away from firm i is by offering more than the worker is worth.

The statement in the proof of Lemma 3.2 that a best response cannot lead to a negative expected profit (because a superior profit of zero can be made by making no offer at all) holds at all times so will not be restated in the proofs that follow.

Lemma 3.3. *Conditional on the auction at time t being won or tied by competing firm k with a bid of $W_k(t)$ and firm k also having the most up-to-date information about the worker's type (that is, $c(n(t)) = k$) it is the best response of the current employer to make a counter-offer of*

$$W_i(t) = W_k(t) \tag{3.6}$$

Proof. *Conditional on $c(n(t)) = k$, it cannot be a best response for firm k to win the auction with an offer $W_k(t) > W^*(p(t_{n(t)}), t_{n(t)}, t)$ since this would give a negative expected profit if the worker was to quit the current employer for exogenous reasons and accept $W_k(t)$. With $W_k(t) \leq W^*(p(t_{n(t)}), t_{n(t)}, t)$ it is a unique best response for the current employer to match firm k's offer, thus retaining the worker and*

maximising expected profit. ∎

Lemma 3.3 shows that even if the current employer does not have the most up-to-date information about the worker's type, the ability to make counter-offers is sufficient to allow it to retain the worker. Note however that firm k is not subject to the winner's curse in this instance since the current employer will match *any* offer, not only those that lead to non-negative expected profit as in (3.5).

All firms other than the current employer compete to employ the worker in the Japanese auction. Define a *maximum bid* $\overline{W}(t)$ as the highest amount a competing firm would be willing to bid in the auction to hire the worker conditional on the information that has been revealed.

Within the auction that takes place at $t > t_1$ there could, in addition to the large number of potential employers, be zero, one or more previous employers bidding to recruit the worker depending on the worker's employment history. Each of these cases will be considered in turn.

Lemma 3.4. *Conditional on $t > t_1$ and there being zero previous employers amongst the competing firms, it is the unique best response of any competing firm k to have a maximum bid $\overline{W}_k(t)$ where*

$$\overline{W}_k(t) = W^*(p^-, t_{n(t)}, t) \tag{3.7}$$

Then in perfect Bayesian equilibrium all competing firms will simultaneously exit the auction with tied bids of

$$W_k(t) = W^*(p^-, t_{n(t)}, t) \tag{3.8}$$

Proof. *A proof is given in Appendix B.*

Lemma 3.4 is the standard adverse selection result described by Greenwald (1986). The current employer's superior knowledge of the worker's ability coupled with its ability to make counter-offers prevents uninformed potential employers from

bidding past the level that would give zero profit if the worker was of the lowest possible productivity. The point of this model is to investigate how firm memory of the worker's type may change this outcome by allowing the existence of partially-informed previous employers. It is this possibility that is now considered.

Lemma 3.5. *Whether the worker quits once, twice, or any number of times after* t_1*, the highest bid from competing firms will never exceed the payoff* $W^*(p^-, t_{n(t)}, t)$ *that, by* (3.8)*, is bid when there are no previous employers.*

Proof. *A proof is given in Appendix C.*

This is the crux of the model of adverse selection under an open auction. The existence of even a large number of partially-informed previous employers *does not* generate competitive offers. Even if the current employer does not know the worker's type, an asymmetry of information between previous employers is sufficient on its own to prevent competitive offers. The possibility of the worker's type changing makes all previous employers except for the one that most recently learned the worker's type victim to the winner's curse. Even though the best-informed previous employer cannot make counter-offers in the way that a current employer can, the open nature of the ascending-price auction allows the best-informed previous employer to exploit its informational advantage, no matter how small it may be, to prevent other previous employers from bidding past the level that would give zero profit if the worker was of the lowest possible productivity at the point when the best-informed previous employer last learned the worker's type.

With this result established, it is a simple matter to solve for the equilibrium wage offer.

Proposition 3.1. *If the worker's type can change over time with rate* $\phi > 0$ *and the bidding process for a worker is represented as an open auction, the first employer to learn the worker's type at time* t_1 *will, with probability approaching 1, retain the*

worker for the rest of her life on a pooling wage of

$$w_i(t) = \mathrm{E}P - (\mathrm{E}P - p^-)e^{-\phi(t-t_{n(t)})} \tag{3.9}$$

Proof. *Conditional on* $t > t_1$, *Lemmas 3.4 and 3.5 prove that the worker cannot improve the payoff she will receive from first employer to learn her type, firm* i, *by quitting any number of times. So the worker will stay with firm* i *for her lifetime, and competing firms will, by* (3.8), *all bid up to* $W^*(p^-, t_{n(t)}, t)$. *It is a unique best response for firm* i *to make an equal counter-offer after the auction closes, thus retaining the worker without revealing type. Substituting appropriate parameter values into equation* (3.3) *gives the payoff the worker would receive from staying with firm* i *for the rest of her lifetime conditional on no further updates of type being learned.*

$$W_i(t) = \mathrm{E}P(1 - t_{n(t)}) + \frac{p^- - \mathrm{E}P}{\phi}\left(e^{-\phi(t-t_{n(t)})} - e^{-\phi(1-t_{n(t)})}\right) \tag{3.10}$$

Differentiating this expression with respect to t *and negating according to* (3.2) *gives the wage rate* (3.9). ■

So the model has shown that there is no way the worker can succeed in forcing her type to be revealed to the market by quitting if there is even a small probability of type changing over time and the bidding process is open. This is in contrast to the model of Chapter 2 that showed if productivity is constant, the possibility of quitting forces the first employer to offer a separating wage (where switching costs are small). Here the pooling outcome under an ascending-price auction shows that the assumption of firm memory is not, on its own, sufficient to ensure wage separation will occur. The existence of any asymmetry of information between previous employers — and in this model the asymmetry has been introduced by assuming the worker's type can change over time — can allow adverse selection to persist and type never to be revealed.

Wage offers for the period up to t_1 are not solved for explicitly, but competition between the uninformed potential employers leads to maximum offers that give zero expected profit. After t_1, the first employer retains the worker on a wage less than EP and makes an expected profit. So wages before t_1 will reflect this and be always higher than EP. The wage before t_1 will decrease over time because of the diminishing of the amount of time after type is learned in which the offering firm could extract a profit.

3.3.2 Wage Dynamics

The equilibrium wage $w_i(t)$ given by equation (3.9) has some interesting properties. In particular, it gives insight into the relationship between the worker's expected wage and the distribution of information as controlled by ϕ and λ.

Firstly, the rate at which type changes, ϕ, controls how rapidly a firm's sample of the worker's productivity becomes out-of-date. For example, when the current employer learns the worker's type for the first time at t_1, setting $t_{n(t)} = t_1$ and $t = t_1$ in (3.9) shows that the current employer can retain the worker for a wage of p^- because competing firms are restricted by the winner's curse. As time advances there is an increasing probability that the worker's type has changed since t_1. If the current employer has not relearned the worker's type, competing firms can increase their offers because, even if the worker was of productivity p^- at t_1, at $t > t_1$ there is a chance that type has changed to an expected value of EP unbeknownst to the current employer. The higher the value of ϕ, the faster the expected productivity of the worker whose productivity at t_1 was p^- moves to EP and the faster competing firms can increase their offers. So, *ceteris paribus*, an increase in ϕ increases the worker's expected wage and lifetime payoff. An example is shown in Figure 3.4.

Another way to interpret ϕ is as a measure of the rate at which the current employer's informational advantage is eroded. At t_1 the current employer knows the worker's type exactly and competing firms know nothing of the worker's type.

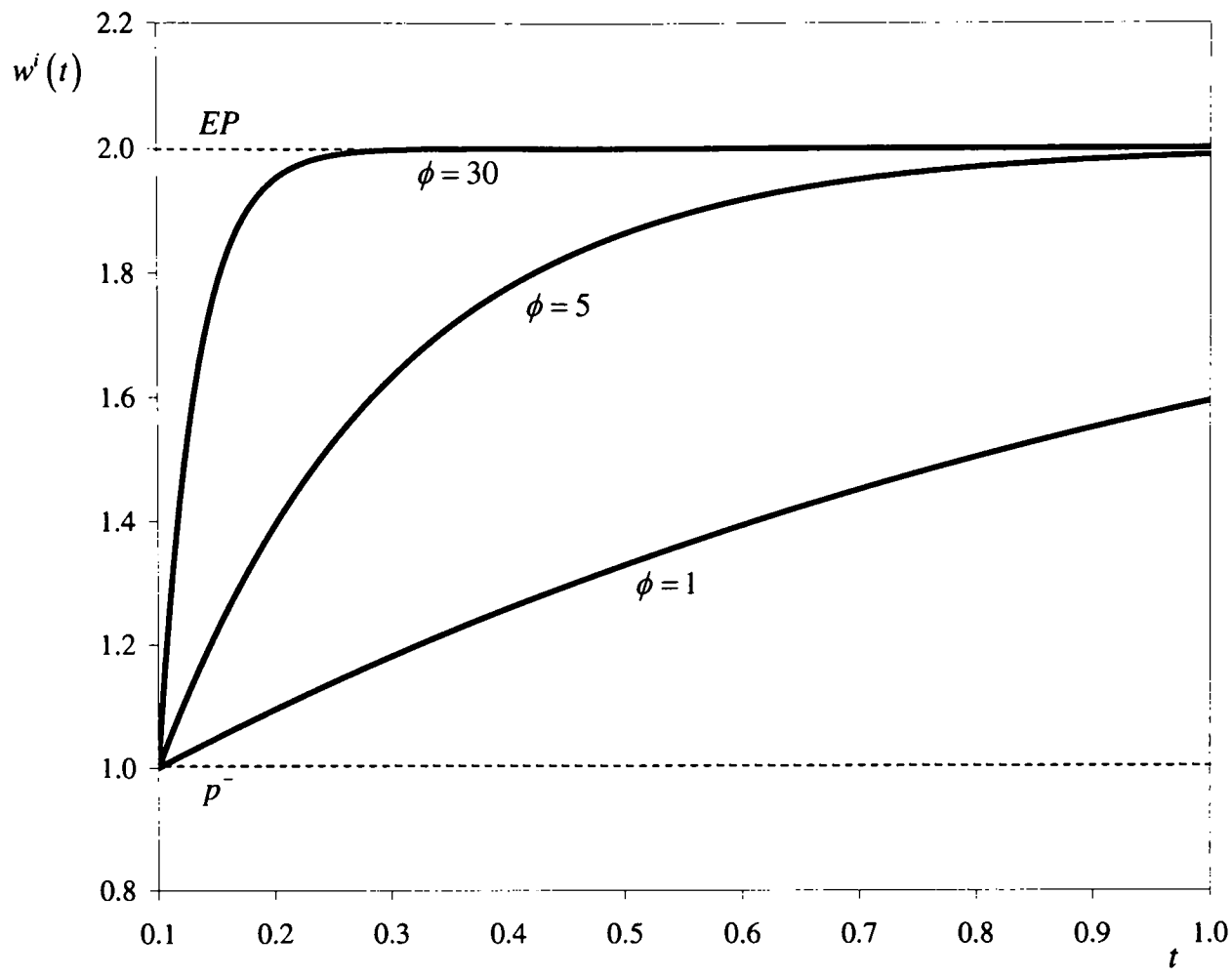


Figure 3.4: Plot of $w_i(t)$ for various values of ϕ for time interval $t \in [0.1, 1]$. Here $p^- = 1$, $EP = 2$, $t_1 = 0.1$ and type is not relearned by the current employer after t_1.

But as time advances, a high value of ϕ means the current employer's knowledge of the worker's type rapidly becomes out-of-date. This reduces its informational advantage, reduces the winner's curse, and allows competing firms to increase their offers towards EP.

The rate at which the employer samples the worker's type, λ, also plays a role in determining the worker's wages. Although λ does not enter equation (3.9) directly, it does implicitly affect the value of $t_{n(t)}$. When the current employer samples the worker's type, $t_{n(t)}$ is reset to the current value of t, the informational advantage of the current employer is returned to the maximum, and competing firms reduce their wage offers to p^-. The higher the value of λ the less time the wage rate $w_i(t)$ has (on average) to climb towards EP before returning to p^-. So λ has the opposite effect of ϕ. As λ increases, the current employer's informational advantage increases

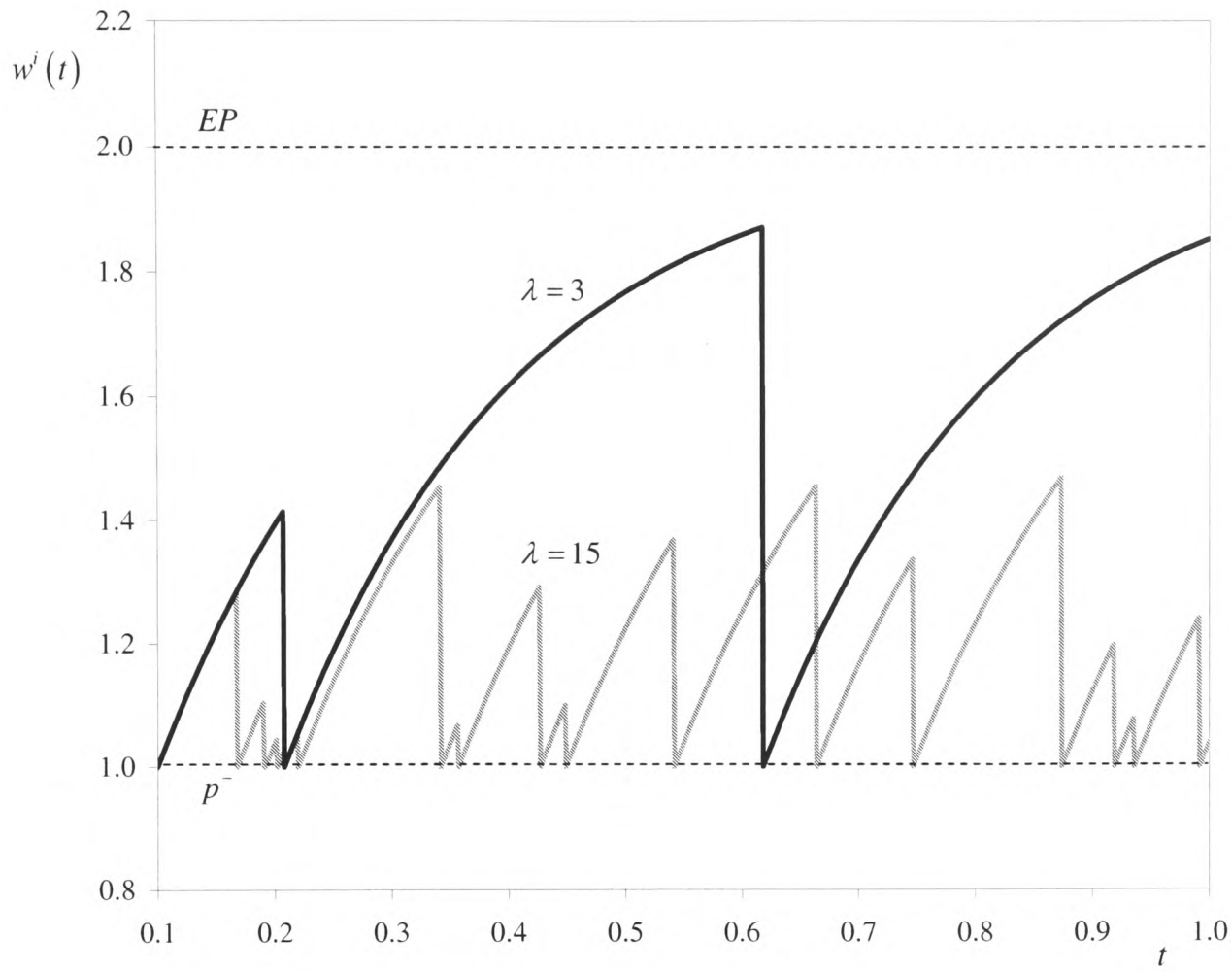


Figure 3.5: Representative plot of $w_i(t)$ for $\lambda = 3$ and $\lambda = 15$. Here $t_1 = 0.1$, $p^- = 1$, $EP = 2$ and $\phi = 3$.

and the worker's expected wage decreases. This is shown in Figure 3.5. As $\lambda \to \infty$ the current employer has a continuously updated knowledge of the worker's type. Then $w_i(t) = p^-$ for all t regardless of the value of ϕ.

This outcome as $\lambda \to \infty$ also describes what would happen if it were assumed that the time at which the current employer (re)learns the worker's type is not observable by potential employers. This would in one sense seem a more appropriate assumption but in actual fact reduces the richness of the model's results without adding anything new. If the sampling of type was unobservable then potential employers would never gain any informational advantage relative to the current employer. The winner's curse would then apply to the fullest extent possible at all times after t_1 and lead to wage offers of p^- from potential employers. This result is not especially surprising, and loses something of the real world since there are

always ways potential employers can infer something about the worker's type (for example by task assignment or promotion). With sampling of type being observable, the model has shown exactly how wage offers adjust to differing distributions of information between the current employer and potential employers, and this is an interesting result in itself.

3.3.3 Special Case: $\phi = 0$

In the model above it has been assumed that the probability ϕ of a worker's type changing is greater than zero. These results hold where $\phi \to 0$ but do not cover the case where $\phi = 0$, that is, type is fixed. Fixed type was considered in Chapter 2 but there the bidding process was modelled as a closed, first-price auction. So here the $\phi = 0$ case is considered under the open Japanese auction.

Proposition 3.2. *If the worker's type cannot change over time and the bidding process for a worker is represented as an open auction, the first employer to learn the worker's type at time t_1 will, with probability approaching 1, retain the worker for the rest of her life on a separating wage of $w_i(t) = p$.*

Proof. *A proof is given in Appendix D.*

The intuition behind Proposition 3.2 is straightforward. With $\phi = 0$, should the worker quit at t_1 and again at t_2 there would be two previous employers that are symmetrically informed about the worker's type. Competition between the previous employers to rehire the worker would lead to separating wage offers because the winner's curse does not affect the previous employer with the older sample. Although this occurrence is off the equilibrium path, its existence means that the first employer must offer a separating wage in order to retain the worker.

This result is interesting for two reasons. Firstly, when juxtaposed with the model of Chapter 2 the result proves that if previous employers are symmetrically informed then wage separation will occur regardless of whether an open or closed

auction is assumed. The pooling outcome of the $\phi > 0$ case in Section 3.3.1 must therefore be a result of the asymmetry of information that has been introduced between previous employers by the possibility of type changing, not solely the effect of the open auction.

Secondly, the $\phi = 0$ result is interesting when juxtaposed with the $\phi > 0$ case. Since the equilibrium at the limit as $\phi \to 0$ is pooling and the equilibrium at $\phi = 0$ is separating, this shows that the fully separating equilibrium of Chapter 2 is fragile if the bidding process is represented as an open auction. Any perturbation that causes even slight asymmetry between previous employers will undermine the separating outcome.

3.4 Closed Auction Model

3.4.1 Further Assumptions

Under a sealed-bid, first-price auction, competing firms make simultaneous offers for the worker in each period dt. The auction can be described as "closed" in the sense that firms cannot update their offers in response to the information revealed by the offers of other firms during the bidding process. Because of this, the strategy of a competing firm is simply a single offer $W(t)$, not, as was the case under the ascending-price auction, a maximum offer conditional on the information already revealed.

3.4.2 Analysis and Results

Consider the situation where there are two previous employers — firm k that knows $p(t_1)$ and firm j that knows $p(t_2)$ — and a current employer firm i that does not know the worker's type. Firms k and j make simultaneous offers along with potential employers and then firm i makes a counter-offer.

In this setting, the best responses of firms k and j are difficult to determine.

The winner's curse applies to firm k because firm j has more up-to-date information about the worker's type. If firms k and j were to each offer a wage that would give zero profit based on their private information about the worker's productivity, firm j would retain the worker in every case except where the worker's productivity had decreased between t_1 and t_2. Then firm k would win the worker by paying too much. This would give negative expected profit so cannot be a best response for firm k.

Although the previous employer with inferior information suffers from the winner's curse, unlike the open auction model of the previous section there is no pure strategy equilibrium where firms k and j both offer $W^*(p^-, t_2, t)$. The one-shot nature of the first-price auction means that firm k could deviate from this equilibrium by offering slightly more, firm j would have no ability to respond in that period, and a worker quitting for exogenous reasons would be hired by firm k. Of course, it could not be a best response for firm k to always offer this slightly increased wage because it would then be a best response for firm j to match the offer if the worker's productivity is sufficiently high. Firm k then makes negative expected profit because of the winner's curse.

Thinking through further iterations of the best response of one firm to a candidate best response of another, it quickly becomes apparent that there probably does not exist a perfect Bayesian equilibrium in pure strategies. Solving for the mixed strategy when productivity is continuous may be impossible. Even if it were possible, multiple equilibria would be likely. Fortunately some results can still be obtained without solving for the best responses of this subgame. Lemma 3.6 describes a property that any equilibrium must possess regardless of whether the best responses of previous employers are in mixed or pure strategies.

Lemma 3.6. *Where there is a current employer that does not know the worker's type, a previous employer j that learned the worker's type at t_1 and a previous employer k that learned the worker's type at t_2, for all $p(t_2) > p^-$, any perfect*

Bayesian equilibrium in pure or mixed strategies must satisfy

$$\mathrm{E}\max[W_j(t_2), W_k(t_2)] > W^*(p^-, t_2, t_2) \tag{3.11}$$

where E *is the expectation is over the potentially mixed strategy of firms* j *and* k.

Proof. *A proof is given in Appendix E.*

While the less well-informed previous employer is still affected by the winner's curse, it is not such a strong effect as under an open auction. The worker who quits at t_1 and t_2 can expect to receive higher wages in the remaining period than the pooling wage offer that would be received in the absence of previous employers. This result is not enough to define the perfect Bayesian equilibrium exactly, but an important feature property can be ascertained.

Proposition 3.3. *If the bidding process for a worker is represented as a closed auction, the wage offered by the employer that learns the worker's type at time* t_1 *will not be fully pooling. That is, for all* $p(t_{n(t)}) > p^-$,

$$w_i(t) > \mathrm{E}P - (\mathrm{E}P - p^-)e^{-\phi(t-t_{n(t)})} \tag{3.12}$$

Proof. *Suppose the worker were to quit at* t_1 *and again at* t_2. *Then the expected payoff after* t_2 *would be greater than the pooling payoff* $W^*(p^-, t_2, t_2)$ *by Lemma 3.6. Between* t_1 *and* t_2, *potential employers can offer no more than the amount that would give zero profit if the worker was of type* p^- *because of the winner's curse. Conditional on the worker quitting for exogenous reasons, her expected productivity is* $\mathrm{E}P > p^-$ *so the potential employer that hires the worker will make an expected profit. Competition over this expected profit will induce potential employer* k *to bid such that* $\mathrm{E}W_k(t_1, t_2) = \mathrm{E}[W^*(p^-, t_1, t_1) - W^*(p^-, t_1, t_2)]$ *where* E *is the expectation over the values of* $t_2 \in (t_1, 1]$.

So, for the interval t_1 *to* t_2, *the payoff the worker expects to receive from quitting*

is equal to the pooling payoff. But after t_2 the expected payoff from quitting is superior to the pooling payoff for any $p(t_1) > p^-$. Therefore, it cannot be a best response for firm i to offer the pooling payoff $W^*(p^-, t_1, t_1)$ to a worker of type $p(t_1) > p^-$ because this will not retain the worker. Any higher offer reveals the worker's type to be $p(t_1) > p^-$ and is therefore at least partially separating. ∎

So under a closed auction some wage separation will occur because the asymmetry of information between previous employers introduced by the possibility of type changing is not sufficient to hold a less well-informed previous employer to offering the pooling payoff.

3.4.3 Special Case: $\phi = 0$

A first-price auction with $\phi = 0$ is modelled in Chapter 2, the only difference being the existence of switching costs. With switching costs set to zero, the model of Chapter 2 has shown that the possibility of symmetrically informed previous employers makes it a best response for the first employer to reveal the worker's type as soon as it is learned with a separating wage offer. This is consistent with the findings of the model of this section where, for any $\phi > 0$, at least some degree of wage separation will occur. Combining the two models it can be said that under a first-price auction where there are no job changing frictions, some separation will occur for any $\phi \geq 0$.

Comparing the $\phi = 0$ case for the open and the closed auction, some separation occurs in both models. But only under the closed auction is non-pooling a robust result. Any perturbation in the value of ϕ under an open auction causes full pooling to occur.

	$\phi > 0$	$\phi = 0$
Open Auction	Full pooling	Full separation
Closed Auction	Some separation	Full separation (with $s = 0$)

Figure 3.6: Summary of possible wage outcomes under an open and closed auction, with productivity changing and constant.

3.5 Conclusion

In Greenwald's (1986) model of adverse selection, the ability of the current employer to make a counter-offer in response to the offer from a potential employer means potential employers make low wage offers because of the winner's curse. Wages pool and adverse selection persists. The model of Chapter 2 shows that adverse selection will not persist and wages may be separating if firms remember the type of a worker even after she quits.

This chapter has considered the intermediate case where previous employers know less about the worker's type than the current employer but more than firms that have never employed the worker. It has been shown that the separating wage result of Chapter 2 is robust to the introduction of some asymmetry of information between the current employer and previous employers, but only if the bidding process for the worker is closed. That is, firms other than the current employer cannot update their bidding strategy based on the information revealed by the bids of other firms. Then the existence of partially-informed previous employers would lead to a payoff superior to the pooling payoff. The first employer must therefore offer a payoff that is at least partially separating in order to retain the worker.

Under an open auction, the slight informational advantage of the most recent previous employer over other previous employers provides enough of a toehold (to use the parlance of Bulow et al. (1999)) for the most recent previous employer

to dominate the auction and win with a pooled wage offer. This undermines the worker's ability to generate competitive offers by accumulating partially-informed previous employers, and so allows the first employer to retain the worker for her lifetime on a pooling wage. If there is no chance that the worker's productivity can change over time, and therefore no asymmetry of information between previous employers, separation will occur under the open auction. But this result is fragile since pooling is the outcome even as the rate at which type changes approaches zero.

Whether pooling or separation occurs, the worker's wage at the current employer adjusts over time to reflect the amount of information about the worker's type possessed by competing firms relative to the current employer. Under a pooling equilibrium all competing firms suffer from the winner's curse so, after learning the worker's type, the current employer can retain the worker for a low wage. As the current employer's knowledge of the worker's type becomes out-of-date, the informational disadvantage of competing firms is reduced, the winner's curse becomes less severe and wage offers increase towards the mean productivity level. An increase in the rate at which type changes or a decrease in the rate at which type is learned has the effect of reducing the current employer's informational advantage over competing firms. This reduces the winner's curse effect, competing firms then increase their offers, and the expected wage of the worker at the first employer increases.

These results, though interesting, are of secondary importance compared to the conclusions regarding the robustness of the equilibrium. The most significant finding of this thesis is the result of Chapter 2, that, contrary to the predictions of most theoretical models, adverse selection will not persist if firms remember the type of a worker after she quits. This chapter has shown there is a caveat to that result. Where the bidding process between competing firms is open, even the smallest amount of asymmetric information between previous employers can lead to pooling wages and the persistence of adverse selection. Where the bidding process between competing

firms is closed however, the findings of Chapter 2 are robust.

3.6 Appendix

A Proof of Lemma 3.1

Suppose the current employer knows the worker's productivity at time τ_0 to be $p(\tau_0)$ and with no additional information available, wants to calculate the expected output of the worker over the subsequent interval $[\tau_1, \tau_2]$. All that matters to the current employer is the time at which productivity is expected to change for the first time after τ_0, because, for changes subsequent to the first, the resulting expected productivity is the same. So it is the density function describing the probability of type changing conditional on it not having already changed that needs to be derived.

The probability of the worker's type not changing between τ_0 and t is given by the survivor function $s(t)|_{t\geq\tau_0}$ which decays exponentially with slope $-\phi$.

$$\frac{ds(t)|_{t\geq\tau_0}}{dt} = -\phi s(t)|_{t\geq\tau_0} \tag{3.13a}$$

$$\frac{ds(t)|_{t\geq\tau_0}}{s(t)|_{t\geq\tau_0}} = -\phi dt \tag{3.13b}$$

$$\ln(s(t)|_{t\geq\tau_0}) = -\phi t + c \tag{3.13c}$$

$$s(t)|_{t\geq\tau_0} = e^{-\phi(t-c')} \tag{3.13d}$$

where c and c' are constants of integration. At $t = \tau_0$ there is zero probability that the worker's type has changed since τ_0. Solving for c' with $s(\tau_0) = 1$ gives

$$s(t)|_{t\geq\tau_0} = e^{-\phi(t-\tau_0)} \tag{3.14}$$

The cumulative density function is, for any parametric duration model, simply the complement of the survivor function

$$Z(t) = 1 - s(t) \tag{3.15a}$$

$$= 1 - e^{-\phi(t-\tau_0)} \tag{3.15b}$$

The density function $z(t)$ describing the probability of type changing at time t conditional on it not having changed since τ_0 is given by the derivative of the cumulative density function $Z(t)$ with respect to t.

$$z(t)|_{t\geq\tau_0} = \frac{dZ(t)}{dt} \tag{3.16a}$$

$$= \phi e^{-\phi(t-\tau_0)} \tag{3.16b}$$

Define t^* as the time at which the worker's type first changes after τ_0. At t^* the worker's productivity is redrawn from the set P. Since the new value is independent from the old, the productivity at $t \geq t^*$ as expected before t^* is simply the expected productivity of the set P. This is denoted EP where

$$\mathrm{E}P = \int_{p^-}^{p^+} pf(p)\,dp \tag{3.17}$$

Figure 3.7 shows the three possible cases for t^* relative to τ_1 and τ_2. In Case 1 the worker's expected productivity is EP across the entire interval τ_1 to τ_2. With the worker's output defined as the integral of her productivity over time then expected output conditional on $t^* < \tau_1$ is given by

$$W^*(p(\tau_0), \tau_0, \tau_1, \tau_2)|_{t^*\leq\tau_1} = \mathrm{E}P(\tau_2 - \tau_1) \tag{3.18}$$

In Case 2 the worker's productivity is $p(\tau_0)$ across the interval $[\tau_1, t^*)$ and expected productivity across the interval $[t^*, \tau_2]$ is EP. Expected output conditional on $\tau_1 < t^* \leq \tau_2$ is then given by

$$W^*(p(\tau_0), \tau_0, \tau_1, \tau_2)|_{\tau_1<t^*\leq\tau_2} = p(\tau_0)(t^* - \tau_1) + \mathrm{E}P(\tau_2 - t^*) \tag{3.19}$$

In Case 3 the worker's productivity is $p(\tau_0)$ across the entire interval τ_1 to τ_2.

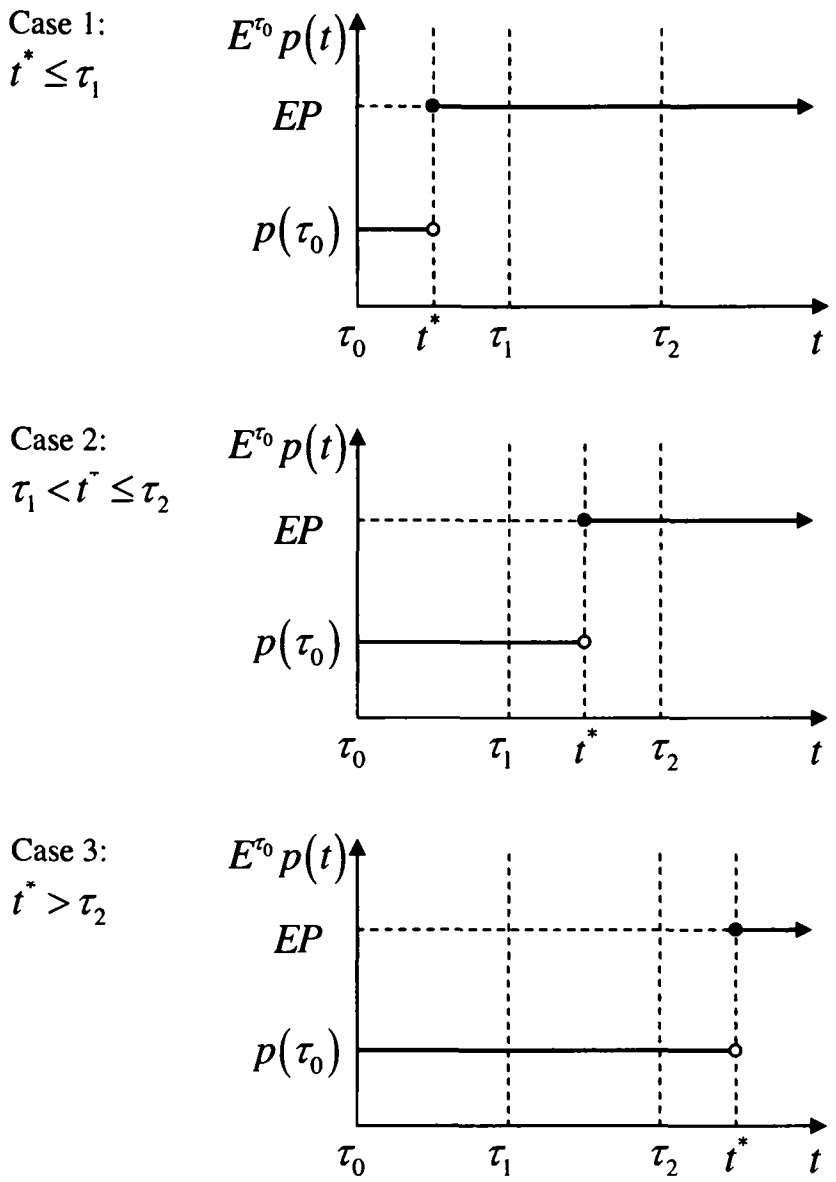


Figure 3.7: The three possible cases for t^* relative to τ_1 and τ_2. In this example $p(\tau_0)$ is less than EP.

Expected output conditional on $t^* > \tau_2$ is then given by

$$W^*(p(\tau_0), \tau_0, \tau_1, \tau_2)|_{t^* > \tau_2} = p(\tau_0)(\tau_2 - \tau_1) \tag{3.20}$$

The density function for t^* conditional on $t^* \geq \tau_0$ is given by equation (3.16b) with t set to t^*. The expression for $W^*(p(\tau_0), \tau_0, \tau_1, \tau_2)$ can then be found by integrating across the interval τ_0 to ∞ the product of $z(t^*)|_{t^* \geq \tau_0}$ and the appropriate expression from (3.18), (3.19) and (3.20).

$$
\begin{aligned}
W^*(p(\tau_0),\tau_0,\tau_1,\tau_2) &= \int_{\tau_0}^{\tau_1} z(t^*)|_{t^*\geq\tau_0} W^*(p(\tau_0),\tau_0,\tau_1,\tau_2)|_{t^*\leq\tau_1}\, dt^* \\
&+ \int_{\tau_1}^{\tau_2} z(t^*)|_{t^*\geq\tau_0} W^*(p(\tau_0),\tau_0,\tau_1,\tau_2)|_{\tau_1<t^*\leq\tau_2}\, dt^* \\
&+ \int_{\tau_2}^{\infty} z(t^*)|_{t^*\geq\tau_0} W^*(p(\tau_0),\tau_0,\tau_1,\tau_2)|_{t^*>\tau_2}\, dt^* \qquad (3.21a) \\
&= \mathrm{E}P(\tau_2-\tau_1)\int_{\tau_0}^{\tau_1} z(t^*)|_{t^*\geq\tau_0}\, dt^* \\
&+ \int_{\tau_1}^{\tau_2} z(t^*)|_{t^*\geq\tau_0}\left[p(\tau_0)(t^*-\tau_1)+\mathrm{E}P(\tau_2-t^*)\right] dt^* \\
&+ p(\tau_0)(\tau_2-\tau_1)\int_{\tau_2}^{\infty} z(t^*)|_{t^*\geq\tau_0}\, dt^* \qquad (3.21b)
\end{aligned}
$$

The integral of the density function $z(t)$ is simply the cumulative density function which from (3.15b) is $1-e^{-\phi(t-\tau_0)}$. So

$$
\begin{aligned}
W^*(p(\tau_0),\tau_0,\tau_1,\tau_2) &= \mathrm{E}P(\tau_2-\tau_1)(1-e^{-\phi(\tau_1-\tau_0)}) \\
&+ (\mathrm{E}P\tau_2 - p(\tau_0)\tau_1)(e^{-\phi(\tau_1-\tau_0)} - e^{-\phi(\tau_2-\tau_0)}) \\
&+ \phi(p(\tau_0)-\mathrm{E}P)\int_{\tau_1}^{\tau_2} t^* e^{-\phi(t^*-\tau_0)}\, dt^* \\
&+ p(\tau_0)(\tau_2-\tau_1)(e^{-\phi(\tau_2-\tau_0)})
\end{aligned} \qquad (3.22)
$$

Integrating by parts and simplifying further gives

$$
W^*(p(\tau_0),\tau_0,\tau_1,\tau_2) = \mathrm{E}P(\tau_2-\tau_1) + \frac{p(\tau_0)-\mathrm{E}P}{\phi}\left(e^{-\phi(\tau_1-\tau_0)} - e^{-\phi(\tau_2-\tau_0)}\right) \qquad (3.23)
$$

B Proof of Lemma 3.4

If $t > t_1$ and there are no previous employers, the current employer must be the only firm that knows the worker's type. This knowledge of type was last updated at $t_{n(t)}$, and this is known by competing firms. Suppose the auction is won by competing firm k with an offer of $W_k(t) = W^*(p^- + \Delta, t_{n(t)}, t)$ where $\Delta > 0$. Since $W^*(p, t_{n(t)}, t)$ is increasing in p it must be that $W^*(p^- + \Delta, t_{n(t)}, t) > W^*(p^-, t_{n(t)}, t)$. Conditional

on $p^- + \Delta > p(t_{n(t)})$ the current employer will not retain the worker given strategy (3.5). The worker could then be won by firm k for an expected loss. Conditional on $p^- + \Delta \leq p(t_{n(t)})$ the current employer will, by (3.5), match $W_k(t)$, retain the worker and firm k will make no profit. Since $p(t_{n(t)})$ is unknown to firm k, there is a positive probability that $p^- + \Delta > p(t_{n(t)}) \forall \Delta > 0$. Any $\Delta > 0$ will therefore generate an expected loss for firm k and so if follows that firm k's best response is to have a maximum bid that satisfies

$$\overline{W}_k(t) \leq W^*(p^-, t_{n(t)}, t) \tag{3.24}$$

Given (3.24) and the current employer's best response (3.5), the worker will be retained by the current employer with nothing revealed about the worker's type. Conditional on the worker quitting the current employer for exogenous reasons, her expected productivity must be EP since the propensity to quit for exogenous reasons is independent of type. So the expected output of the worker quitting for exogenous reasons is $W^*(\mathrm{E}P, t, t) > W^*(p^-, t_{n(t)}, t)$. The expected profit to any competing firm that hires the worker is therefore strictly positive. It cannot be a best response for all competing firms to bid less than $W^*(p^-, t_{n(t)}, t)$ because each firm would have an incentive to bid ϵ higher and win the auction outright. Conditional on one competing firm bidding $W^*(p^-, t_{n(t)}, t)$ it cannot be a best response for any other competing firm to bid less since this will never win the worker and so gives zero profit. Matching $W^*(p^-, t_{n(t)}, t)$ causes the worker quitting for exogenous reasons to choose randomly between tied competing firms, and this gives a positive probability of making a positive profit. So it is the unique best response of any competing firm k to have a maximum bid of (3.7).

Since there are, by assumption, a large number of competing firms participating in the auction, each with the same maximum bid, the price will rise until all competing firms exit the auction at price $W^*(p^-, t_{n(t)}, t)$. So firm k's actual bid when the auction closes is (3.8).

C Proof of Lemma 3.5

The proof is broken down into two parts, the first considering the case where there is exactly one previous employer, and the second considering the case where there is more than one previous employer.

One Previous Employer

Lemma 3.7. *Conditional on $t > t_1$ and there being exactly one previous employer, firm j, amongst the competing firms, the highest maximum bid from potential employers will satisfy*

$$\overline{W}_k(t) \leq W^*(p^-, t_{n(t)}, t) \tag{3.25}$$

where potential employer index k is given by

$$k = \text{argmax}_{l \neq i,j} \overline{W}_l(t) \tag{3.26}$$

and i is the index of the current employer. It is the unique best response of the previous employer to have a maximum bid of

$$\overline{W}_j(t) = \begin{cases} W^*(p(t_{n(t)}), t_{n(t)}, t) & \text{if} \quad c(n(t)) = j \\ W^*(p^-, t_{n(t)}, t) & \text{otherwise} \end{cases} \tag{3.27}$$

Then in perfect Bayesian equilibrium the auction closes with bids of

$$W_j(t) = W_k(t) \leq W^*(p^-, t_{n(t)}, t) \tag{3.28}$$

Proof. *Conditional on $c(n(t)) = j$ (that is, the previous employer firm j has the most up-to-date information about the worker's type), there is no way firm j can hire the worker not quitting for exogenous reasons given firm i's strategy described by Lemma 3.3. If the worker does quit firm i for exogenous reasons, it is profit maximising for firm j to win the worker with the lowest possible offer, providing*

doing so does not lead to an expected loss. Given firm j's information that the worker's type at $t_{n(t)}$ was $p(t_{n(t)})$, any offer greater than $W^*(p(t_{n(t)}), t_{n(t)}, t)$ would lead to an expected loss. This motivates the first line of equation (3.27).

Conditional on $c(n(t)) \neq j$ it must be that the current employer has the most up-to-date knowledge of the worker's type. Firm i's strategy is then given by Lemma 3.2, and there is no way firm j can hire the worker not quitting for exogenous reasons without making an expected loss. If the worker does quit firm i for exogenous reasons, firm j will again maximise profit by winning the worker with the lowest possible offer, providing doing so does not lead to an expected loss. Given firm j has less up-to-date information about the worker's type than the current employer, firm j suffers from the winner's curse and any bid greater than $W^*(p^-, t_{n(t)}, t)$ would lead to an expected loss by the same logic as was used to derive equation (3.24). *This motivates the second condition of equation* (3.27).

Given Lemmas 3.2 and 3.3, there is no bid a competing firm can make that will hire the worker not quitting for exogenous reasons without a loss being made. Conditional on the previous employer playing strategy (3.27), *there is also no bid a competing firm can make that will win the worker quitting for exogenous reasons without a loss being made. So any maximum bid that will never hire a worker is a best response. Strategy* (3.27) *shows that any bid higher than $W^*(p^-, t_{n(t)}, t)$ could hire the worker quitting for exogenous reasons if $p(t_{n(t)}) = p^-$. So the maximum bid from a competing firm must no greater than $W^*(p^-, t_{n(t)}, t)$ as given by* (3.25).

Given (3.25) *and* (3.27), *the auction will close when firm k quits the auction at price $\overline{W}_k(t) \leq W^*(p^-, t_{n(t)}, t)$. Firm j's final bid is then $W_j(t) = W_k(t) = \overline{W}_k(t)$ as given by* (3.28). ■

To summarise Lemma 3.7, if the previous employer has better information about the worker's type than the current employer, the previous employer's best response is to win the auction outright since a worker quitting for exogenous reasons will then be profitably recruited. If the worker does not quit for exogenous reasons there is no

way the previous employer can recruit the worker because of the current employer's ability to make counter-offers.

If, on the other hand, the previous employer does not have better information about the worker's type than the current employer then it suffer's from the winner's curse and cannot bid beyond the level that would give zero profit if the worker was of the lowest possible productivity.

The result described by Lemma 3.7 is not especially surprising. In order for the worker to benefit from the existence of partially-informed previous employers it would be a sensible prediction that at least two would be needed in order to induce the necessary competition. It is this most important case that is now considered.

Multiple Previous Employers

Lemma 3.8. *Conditional on $t > t_1$ and there being two or more previous employers amongst the competing firms, the highest maximum bid from competing firms other than the most recent previous employer firm j will satisfy*

$$\overline{W}_k(t) \leq W^*(p^-, t_{n(t)}, t) \tag{3.29}$$

where competing firm index k is given by

$$k = \text{argmax}_{l \neq i,j} \overline{W}_l(t) \tag{3.30}$$

and i is the index of the current employer. It is the unique best response of the most recent previous employer to have a maximum bid of

$$\overline{W}_j(t) = \begin{cases} W^*(p(t_{n(t)}), t_{n(t)}, t) & \text{if} \quad c(n(t)) = j \\ W^*(p^-, t_{n(t)}, t) & \text{otherwise} \end{cases} \tag{3.31}$$

Then in perfect Bayesian equilibrium the auction closes with bids of

$$W_j(t) = W_k(t) \leq W^*(p^-, t_{n(t)}, t) \tag{3.32}$$

The proof is identical to that of Lemma 3.7 , but with one extension. It remains only to prove that previous employers other than the most recent previous employer firm j will, like uninformed potential employers, bid no higher than $W^*(p^-, t_{n(t)}, t)$.

Proof. *Suppose a previous employer firm $l \neq j$ most recently learned the worker's type at $T(l,t)$ where $T(l,t)$ is a function that returns the time at which firm l most recently learned the worker's type before time t. Then $T(l,t) < T(j,t)$. Conditional on $c(n(t)) = i$ (that is, the current employer having the best information about the worker's type) the winner's curse means previous employer l will be unwilling to bid higher than $W^*(p^-, t_{n(t)}, t)$ in exactly the same way previous employer k was in Lemma 3.7.*

Conditional on $c(n(t)) \neq i$, it must be that $c(n(t)) = j$. Although the current employer will match any offer previous employers will be willing to make, the possibility of the worker quitting for exogenous reasons motivates potential employers to have the highest maximum bid that will not generate an expected loss. Conditional on the worker's type not having changed in between $T(l,t)$ and $T(j,t)$ it would be a best response for firms l and j to have a maximum bid of $\overline{W}_l(t) = \overline{W}_j(t) = W^*(p(T(j,t)), T(j,t), t)$. That is, both firms would be willing to bid up to the payoff that gives zero expected profit based on their (symmetric) information about the worker's type. Whether or not the worker is recruited, this leads to zero expected profit. Conditional on the worker's type having changed in between $T(l,t)$ and $T(j,t)$ (which is possible for any $\phi > 0$), firm l then has less up-to-date information about the worker's type than firm j and suffers from the winner's curse. If $\overline{W}_l(t) > \overline{W}_j(t)$ then firm l would win the worker quitting for exogenous reasons and make an expected loss. If $\overline{W}_l(t) \leq \overline{W}_j(t)$ then firm l would not win the worker and make no profit.

Since $\overline{W}_j(t)$ *could be as low as* $W^*(p^-, t_{n(t)}, t)$, *firm* l *must have a maximum bid* $\overline{W}_l(t) \leq W^*(p^-, t_{n(t)}, t)$ *to avoid an expected loss.* ∎

Together, Lemmas 3.7 and 3.8 prove that the worker cannot improve the payoff she will receive from first employer to learn her type, firm i, by quitting any number of times.

D Proof of Proposition 3.2

The expected output of a worker between τ_1 and τ_2 whose type was known at any time to be p is simply[5]

$$W^*(p, \tau_0, \tau_1, \tau_2) = (\tau_2 - \tau_1)p \tag{3.33}$$

Suppose the worker quits at t_1 and again at t_2. With symmetric information, competition between the previous employers over the worker quitting for exogenous reasons means that each will be willing to bid up to the worker's actual value over the remaining interval, so $\overline{W}_j(t_2) = \overline{W}_k(t_2) = (1 - t_2)p$, where j and k are the indexes of the two previous employers. When the auction ends, the final bids will be $W_j(t_2) = W_k(t_2) = (1 - t_2)p$.

When the first employer learns the worker's type at t_1, potential employers then suffer from the winner's curse so can bid no higher than the amount that would give zero profit if the worker was of productivity p^-. But the worker's expected productivity is $EP > p^-$, so if the worker were to quit for exogenous reasons, the potential employer that recruits the worker makes an expected profit. Competition for this expected profit makes it a unique best response for all potential employers to bid up to $(Et_2 - t_1)p^-$ where Et_2 is the value of t_2 as expected at t_1.[6]

[5]This is consistent with taking the limit as $\phi \to 0$ of (3.3) using L'Hôpital's rule. Then

$$\begin{aligned}\lim_{\phi \to 0} W^*(p, \tau_0, \tau_1, \tau_2) &= EP(\tau_2 - \tau_1) + \lim_{\phi \to 0} \frac{(p - EP)\frac{d}{d\phi}\left(e^{-\phi(\tau_1 - \tau_0)} - e^{-\phi(\tau_2 - \tau_0)}\right)}{\frac{d}{d\phi}\phi} \\ &= (\tau_2 - \tau_1)p\end{aligned}$$

[6]This does not necessarily mean that a wage of p^- will be paid by firm k at all times between t_1

So the first employer will not be able to retain the worker of type $p > p^-$ on a pooling wage $W_i(t_1) = (1 - t_1)p^-$ because the worker can expect to earn the same pooling payoff between t_1 and t_2 from quitting, and a superior separating payoff after t_2. For any $\lambda > 0$ there is a positive probability that $t_2 < 1$, that is, the second employer learns the worker's type before the end of her lifetime. So the best response of firm i is to retain the worker by offering some $W_i(t_1) > (1 - t_1)p^-$, provided that $p > p^-$.

In making this offer, firm i reveals to competing firms that $p > p^-$. Then in the period starting at $t_1 + dt$, competing firms can offer can increase their offers without suffering from the winner's curse, so that $W_k(t_1 + dt) > (1 - t_1 - dt)p^-$. The current employer must then further increase its offer to retain the worker. With each additional period dt that passes, the first employer's offer increases. This will continue until type has been fully revealed. Then the worker will not benefit further from receiving a separating payoff after t_2 and so the first employer can retain the worker without further wage increases. As $dt \to 0$ the time taken for type to be revealed after t_1 approaches zero and firm i's offer approaches $W_i(t_1) = (1 - t_1)p$. The equates to a wage rate of $w_i(t_1) = p$. The same logic applies for all $t > t_1$.

E Proof of Lemma 3.6

a) It cannot be a best response for previous employers to offer such that there is a non-zero probability that $\max[W_j(t_2), W_k(t_2)] < W^*(p^-, t_2, t)$. Then an uninformed potential employer could offer $W^*(p^-, t_2, t)$ and recruit a worker quitting for exogenous reasons whenever previous employers offer less.

b) It cannot be a best response for previous employers to offer such that $\max[W_j(t_2), W_k(t_2)] = W^*(p^-, t_2, t)$ with probability 1 when $p(t_2) > p^-$. Condi-

and t_2. After the worker has quit and there is one previous employer, Lemma 3.7 showed that there are multiple best responses for potential employers, and this argument applies equally when $\phi = 0$. The wage $w_k(t)$ actually paid by firm k is therefore undefined, although p^- is one equilibrium candidate. If a lesser wage were paid in equilibrium, this would be balanced out by a one-off payment at t_1 so that the overall expected payoff remains $(\mathrm{E}t_2 - t_1)p^-$.

tional on firm k offering $W_k(t_2) = W^*(p^-, t_2, t_2)$ with probability 1 it is a best response for firm j to offer ϵ more. This will win the worker quitting for exogenous reasons and maximise profit. There is no possibility that firm j will not be willing to offer $W_j(t) = W^*(p^-, t_2, t) + \epsilon$, even if firm j has the lowest possible impression of the worker's type. Then $p(t_1) = p^-$ and the zero profit offer is $W^*(p^-, t_1, t_2)$. But $W^*(p^-, t_2, t_2) + \epsilon$ is less than $W^*(p^-, t_1, t_2)$ as $\epsilon \to 0$ because $t_2 > t_1$ and $W^*(p^-, \tau_0, t_2)$ is decreasing in τ_0. So, provided the expected productivity of a worker at t_2 conditional on her having quit at t_1 and t_2 is greater than p^-, firm j will make positive expected profit from this strategy.

Conditional on firm j offering $W_j(t_2) = W^*(p^-, t_2, t_2)$ with probability 1 it is a best response for firm k to match this offer since in the event of a tie the worker has a slight preference for the most recent previous employer. But if $W_k(t_2) = W^*(p^-, t_2, t_2)$ it has already been established that firm j's best response is to offer more. So it cannot be a best response for firms j and k to offer such that $\max[W_j(t_2), W_k(t_2)] = W^*(p^-, t_2, t_2)$ with probability 1.

So from a), $\max[W_j(t_2), W_k(t_2)] \geq W^*(p^-, t_2, t_2)$ with probability 1, and from b), $\max[W_j(t_2), W_k(t_2)] > W^*(p^-, t_2, t_2)$ with non-zero probability. Therefore a perfect Bayesian equilibrium must satisfy $\mathrm{E}\max[W_j(t_2), W_k(t_2)] > W^*(p^-, t_2, t_2)$.

Chapter 4

Adverse Selection, Uninformed Workers and the Golden Ratio

All you need in this life is ignorance and confidence, and then success is sure.

Mark Twain

4.1 Introduction

With the emergence of information-based models of the labour market, significant attention has been given to the issue of how easily the productivity of a worker can be observed by her employer and potential employers. An asymmetric distribution of information about productivity between the employer and other firms can have enormous impact on wage rates and labour market efficiency as first explained by Akerlof (1970), Spence (1973) and Greenwald (1979). Considering the importance of employers' knowledge of the worker's productivity it is notable that the issue of the worker's knowledge of the value to her employer of her own productivity is rarely mentioned in the economics literature.

There is a good reason for this. In most models of the labour market, the worker's knowledge or otherwise of her own ability is irrelevant. In a simple model where supply and demand dictate wage rates, the worker chooses only whether or not she is willing to work at the prevailing wage and there are no strategic considerations. In information based models, knowledge of the worker's ability is important to

employers, but the worker's attempt to maximise utility involves a strategy that is rarely dependent on a knowledge of their exact worth to their employer.

This is certainly true of Greenwald's (1986) model of adverse selection in the labour market. In his model, the employer's superior knowledge of the worker's type relative to other potential employers means that all wages are pooling. The possibility of a worker quitting for exogenous reasons reduces the severity of the winner's curse experienced by potential employers and allows them to make offers higher than the level which would give zero profit if the worker was of the lowest possible productivity. Workers of low ability then find that the outside offer is more than their current employer is willing to match and so quit. Even though this endogenous turnover leads to a loss for the potential employer, it can still be part of an equilibrium because of the more able workers who quit for exogenous reasons. But whether the worker quits or stays with her first employer, the strategy is conditional only on the offers received: the workers' knowledge of their own type is irrelevant.

Chapters 2 and 3 of this thesis enhanced Greenwald's model to consider the possibility that firms remember the type of a worker even after she quits and have shown that in some cases, wage offers may be separating. In this context, the question of whether or not a worker knows her own type becomes important. Recall that a separating equilibrium results from the possibility of the off-equilibrium path action that a worker could quit twice in order to receive competitive offers from informed previous employers. This makes it necessary for the first employer to offer a high wage to a high ability worker specifically because this worker knows exactly what she will be paid when her type has been revealed. But it seems entirely plausible that, immersed in a complex organisational structure with numerous employees and unknown processes, a worker would be unsure exactly what effect her own labour has on bottom line profitability. So it is the task of this chapter to explore exactly what effect this assumption will have on labour market equilibrium.

Even before the formal modelling begins, it is possible to intuit some of the likely results. Firstly, if the worker does not know her own type then an equilibrium like that of Chapter 2 where the first employer offers a separating wage cannot exist. Suppose it were possible, and high type workers were offered high wages and low type workers low wages. Because the worker does not know her own type, it would be the employer's best response to "reveal" the type of the worker to be low in order retain her as cheaply as possible, even if the worker's type is in fact high. But if workers of any type are offered the same wage then this is not a separating equilibrium. In the model of Chapter 2, the separating wage offer rested entirely on the assumption that the worker knew her own type. High type workers knew they would benefit from quitting and having type revealed, so a high wage was necessary to retain them. Low type workers knew that there was no point in quitting to reveal type so would stay with the first employer on a low wage. So if the worker does not know her own type then the first employer's wage offer must be pooling.

Secondly, if the worker does not know her own type, a low offer from the current employer will not be accepted even by a low type worker because of the possibility of a high wage from quitting and having type revealed. The first employer will not be willing to offer a high wage to a low type worker, so it follows that low type workers will quit. This presents a rare opportunity to study the composition of the stream of job changes under "pure" endogenous turnover. By pure, it is meant that endogenous quit behaviour is not a function of an assumed exogenous propensity to quit. Models of the labour market that exhibit pure endogenous turnover are most commonly concerned with the efficiency of on-the-job training and firm-specific matching. In this context, turnover is the mechanism by which allocational efficiency is sought (For example, Acemoglu and Pischke (1998) and Moen and Rosen (2004)). The composition of the stream of job changers is of no particular interest as any worker could find herself with a bad match and attempt to change employers.

Greenwald (1986) was the first and remains the most important paper to show

that the stream of job changers may be disproportionately comprised of the less able if the worker's current employer has better information about her ability that potential employers. Potential employers are only willing to offer low wages because of the winner's curse. Job separation then can be seen as a signal of low productivity, as was further explored by Lazear (1986) and Gibbons and Katz (1991). But this and other models of adverse selection in the labour market that exhibit endogenous turnover such as Salop and Salop (1976) are unable to do so without reliance on a significant exogenous propensity to quit. The only way a potential employer can recruit a worker not quitting for exogenous reasons is by offering more than she is worth, and this can only occur in equilibrium if there is some probability of a high productivity worker quitting for exogenous reasons and being hired for less than she is worth. The reliance of these models on the exogenous propensity to quit is somewhat unsatisfactory when the composition of the stream of job changers is the primary interest. There are no papers the author is aware of at the time this thesis is being submitted that study the composition of the stream of job changers under adverse selection with pure endogenous turnover.

So the motivation for exploring the possibility that a worker may not know her value to her employer when firms remember the type of a worker even after she quits is already clear. In addition, the model also provides an opportunity to further investigate the robustness of the predictions of Chapter 2 that there will be no adverse selection if firms remember the type of the worker even after she quits. Chapter 3 has shown that this result depends on previous employers being symmetrically informed if the bidding process is modelled as an open auction. In this chapter it is shown that the assumption that the worker knows her own type is also crucial.

The chapter proceeds with a 3-period model for a general distribution of productivities and a discussion of the results. The model is then extended into N periods with a simplifying assumption of a unit rectangular productivity distribution. A

general expression is derived for the wage offers in any period of the N-period game. The effect of self-serving bias in the context of the model is then qualitatively explored. This is a psychological phenomenon whereby individuals making decisions under uncertainty do so with an unintentional bias in the direction that benefits themselves. The chapter then concludes.

4.2 3-period Model

4.2.1 Preliminaries

It is assumed there are a large number of firms and a single worker. The worker's productivity p is a random draw from a distribution $f(p)$ where $p \in [p^-, p^+]$, $p^- \geq 0$, $p^+ > p^-$ and $f(p) > 0$ for all $p \in [p^-, p^+]$. p is unknown to the worker and all firms, but is learned by an employer after one period. There are three periods in the model and at the start of each period potential employers make offers simultaneously, followed by a counter-offer from the current employer where one exists. It is assumed that wage offers are observable by the worker and all firms. The worker and firms share the same discount factor $\delta \in (0, 1]$.

When all firms have made their offers, the worker accepts the offer which gives the highest expected payoff over the remainder of the game. Where expected payoffs are tied it is assumed that the worker prefers to stay with the current employer. There is an exogenous quit rate μ, which is the probability of the worker quitting her present employer regardless of the expected payoff. It is assumed $\mu \to 0$. As has been explained in earlier chapters, a zero exogenous quit rate is problematic because it leads to non-unique best responses on the equilibrium path.

Let i index the employer in the first period, j index a potential second period employer and k index a potential third period employer. Let w represent a wage offer with subscript for the index of the offering firm and, in period 3, a superscript for the period 2 employer. Notation is clarified by Figure 4.1. W represents the

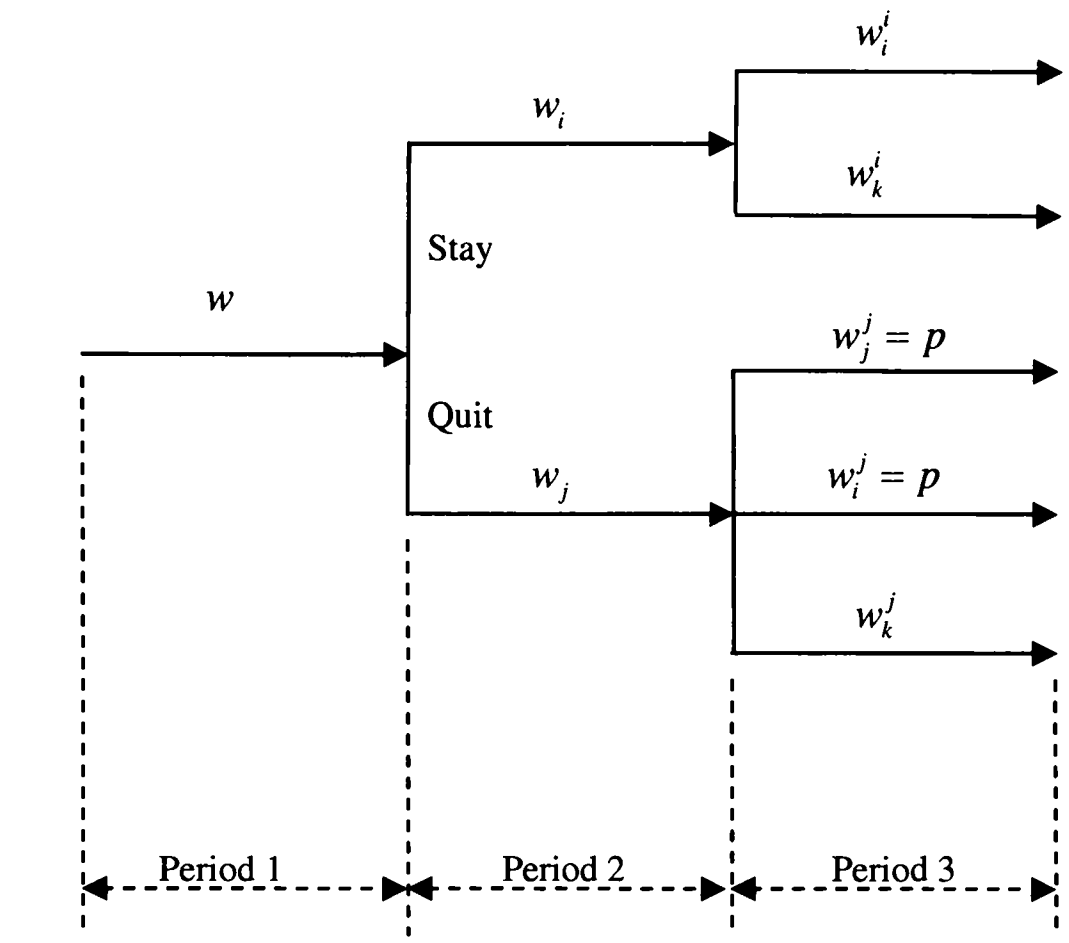


Figure 4.1: Game tree and wage notation for the 3-period model.

total payoff in all remaining periods and takes the same superscripts and subscripts as w.

The previous two chapters of this thesis have shown in continuous time that overcoming adverse selection requires the possibility of there being at least two previous employers that know the worker's type, neither of whom are the current employer. Only then would competition to recruit the worker quitting for exogenous reasons provide sufficient motivation for previous employers to make the highest affordable wage offer. In discrete time, the minimum number of periods required for this situation to arise is four. The worker can quit at the beginning of periods 2 and 3, then at the beginning of period 4 there are two previous employers in addition to the current employer. This is a somewhat inefficient approach for a discrete time

model to take however, because the sole purpose of period 3 is to allow the second employer to become a previous employer so that there can be symmetric competition between two previous employers in period 4. This is in contrast to the continuous time models where as soon as the second employer learns the worker's type it takes only dt for the worker to quit again and have two previous employers that reveal type. So in order to avoid unhelpful mathematical complication, two further assumptions will be made in the discrete time model that eliminate the need of an entire period only to allow the second employer to become a previous employer.

The first assumption is that a previous employer cannot recruit a worker quitting for exogenous reasons. This then leaves the previous employer to compete with the current employer to rehire the worker not quitting for exogenous reasons. The second assumption is that, where a previous employer has multiple best responses, a slight preference is given to the best response that maximises the payoff to the worker.

It will be shown in the model that follows that the combination of these assumptions means a worker who has quit at the start of period 2 will receive a separating wage offer from the previous employer at the start of period 3. Although the assumptions are somewhat artificial, they are valuable in that they allow a 3-period model to capture the salient features of what would otherwise require a vastly more complicated 4-period model.

4.2.2 Analysis and Results

Since a firm can learn the worker's type only after one period of employment, at the start of the first period no firm knows anything about the worker's type. The bidding process and first period wage offer is of little interest since there is no asymmetric information and therefore no adverse selection at this early stage. It suffices to say that competition between uninformed firms will lead to a maximum wage offer w that gives the offering firm i an expected profit of zero. At the start of period 2 the worker can either quit for potential employer firm j or stay with firm i. If the worker

quits in period 2, firms i and j know the worker's type at the start of period 3. Their wage offers are given by the following Lemma.

Lemma 4.1. *Conditional on the worker having quit in period 2, it is the unique best response in period 3 for previous employer firm i and the current employer firm j to offer*

$$w_j^j = w_i^j = p \tag{4.1}$$

In any perfect Bayesian equilibrium the worker is then retained by firm j.

Proof. *The current employer firm j offers last, and its unique best response will be to match the offer of firm i thus retaining the worker if this leads to a profit. If firm i offers $w_i^j > p$, any offer that will not retain the worker is a best response. If firm i offers p then firm j can match the offer, retain the worker and make no profit. Any offer less than p will mean the worker quits, but this also gives zero profit so must also be a best response. The complete set of best responses for firm j is then*

$$w_j^j = w_i^j \quad \textit{if} \quad w_i^j \leq p \tag{4.2a}$$

$$w_j^j \leq w_i^j \quad \textit{if} \quad w_i^j = p \tag{4.2b}$$

$$w_j^j < w_i^j \quad \textit{if} \quad w_i^j > p \tag{4.2c}$$

Conditional on firm j's best response satisfying equations (4.2a)-(4.2c) *there is no offer firm i can make that will lead to a profit. Any offer less than or equal to p will make zero profit. Firm i then prefers to maximise the worker's payoff by offering p.*

Conditional on firm i offering p, firm j will then, by 4.2b, offer $w_j^j \leq p$. But $w_j^j < p$ cannot be part of an equilibrium because it would then be a best response for firm i to offer $w_j^j + \epsilon$ rather than p. So in perfect Bayesian equilibrium firm j matches the offer of firm i, which gives (4.1). ■

Conditional on employing the worker in periods 1 and 2, firm i's best response in period 3 is to match the offers of potential employers and thus retain the worker,

provided doing so does not make a loss.

$$w_i^i = \begin{cases} w_k^i & \text{if} \quad w_k^i \leq p \\ 0 & \text{otherwise} \end{cases} \tag{4.3}$$

Although this equation has the first employer offering zero if the worker cannot be retained without making a loss, any $w_i^i < w_k^i$ is also a best response. w_k^i and w_i are now determined by the following Lemmas.

Lemma 4.2. *It is a mutual best response for the first employer's second period offer to satisfy*

$$w_i \leq p \tag{4.4}$$

and, conditional on this being accepted by the worker in period 2, the offer of potential employer k in period 3 to be

$$w_k^i = w_i \tag{4.5}$$

where

$$k = \text{argmax}_{l \neq i} w_l^i \tag{4.6}$$

Proof. *Conditional on firm k's strategy being given by* (4.5)*, it is not a best response for firm i to offer $w_i > p$ in period 2. Doing so would lead to an offer from firm k in period 3 of $w_k^i = w_i > p$. Firm i would not be willing to match this by* (4.3) *so the worker would quit and firm i would make an overall loss during periods 2 and 3. Conditional on firm i's period 2 strategy having property* (4.4)*, it cannot be a best response that the highest offer from a potential employer w_k^i is less than w_i. Were this the case, the possibility of an exogenous quit would create a positive expected profit for firm k. Another potential employer would then offer slightly more but this is inconsistent with* (4.6)*. $w_k^i > w_i$ can also be eliminated as a best response since by* (4.3) *it would mean the worker would be recruited by firm k in period 3 and a loss incurred if $w_k^i > p$, or, if $w_k^i \leq p$, firm i would match the offer and retain the*

worker. The expected profit from offering $w_k^i > w_i$ *is therefore negative and this cannot be a best response since making no offer would result in a superior profit of zero.*[1] *The unique best response for firm* k *to* (4.4) *is therefore* $w_k^i = w_i$. ∎

Lemma 4.3. *There exist no pure strategy equilibria in which it is a best response for firm* i *to offer a wage* w_i *in period 2 that does not satisfy* (4.4).

Proof. *A proof is given in Appendix A.*

Lemma 4.2 proves that $w_k^i = w_i$ is the unique best response in period 3 conditional on the current employer retaining the worker in period 2 with $w_i \leq p$. Lemma 4.3 proves that $w_i \leq p$ is the unique best response of firm i in period 2. So it follows that $w_k^i = w_i$ is unconditionally the unique best response of firm k in period 3.

Note that the Lemmas do not imply that *all* potential employers will match the offer w^i, only that the highest (or equal highest) offer from a potential employer will match w_i. Conditional on $w_k^i = w_i$, any offer less than or equal to w_i is a best response for other potential employers since all these offers will give the same expected profit of zero. But if all potential employers other than firm k were to offer less than w_i, it would not be a best response for firm k to offer w_i, it should instead offer ϵ more than the next highest offer from potential employers. So conditional on firm i retaining the worker with an offer of w_i in period 2, there must be at least two potential employers that offer w_i in period 3.

Lemma 4.3 has established that firm i's period 2 offer will not exceed the worker's productivity. As a corollary of this fact, both the worker and potential employers can infer that p is at least as great as the w_i they observe. Despite being armed with this knowledge, potential employers will still only offer $w_k^i = w_i$ in period 3 because of the winner's curse. Combining (4.4) and (4.5) reveals that $w_k^i \leq p$ which, when

[1] This argument relies on the assumption that $f(p) > 0$ for all $p \in [p^-, p^+]$. If $f(p) = 0$ for all $p \in [w_i, w_i + \epsilon)$ then firm k could offer as much as $w_k^i = w_i + \epsilon$ without risk of recruiting a worker at a loss.

substituted into (4.3), shows the worker will be retained by firm i in the final period for a wage

$$w_i^i = w_k^i = w_i \tag{4.7}$$

Define w^* as the minimum period 2 wage the first employer firm i can offer that will retain the worker.

Lemma 4.4.

$$w^* = \min\left\{w_i : w_i \geq \frac{w_j + \delta \mathrm{E}p|_{p \geq w_i}}{1+\delta}\right\} \tag{4.8}$$

where

$$j = \mathrm{argmax}_{l \neq i} w_l \tag{4.9}$$

Proof. *The total payoff to the worker in periods 2 and 3 from choosing to stay with firm i in period 2 is $W_i = w_i + \delta w_i^i$ which from (4.7) is*

$$W_i = (1+\delta)w_i \tag{4.10}$$

By quitting, the worker will receive the highest outside offer w_j in period 2 and then her productivity p in period 3. Productivity is unknown by the worker ex ante but offer w_i, which has already been made, reveals $p \geq w_i$ by Lemma 4.2. So the worker's expectation of the payoff from quitting is

$$\mathrm{E}W_j = w_j + \delta \mathrm{E}p|_{p \geq w_i} \tag{4.11}$$

For the first employer to retain the worker, the payoff from staying must be at least as high as the expected payoff from quitting, so

$$(1+\delta)w_i \geq w_j + \delta \mathrm{E}p|_{p \geq w_i} \tag{4.12a}$$

$$w_i \geq \frac{w_j + \delta \mathrm{E}p|_{p \geq w_i}}{1+\delta} \tag{4.12b}$$

In order to maximise profit, firm i will offer the lowest w_i satisfying (4.12b). *This gives* (4.8). ∎

Lemma 4.5. *There exists a w_i that satisfies equation* (4.12b) *with equality for any w_j that is a best response.*

Proof. *It cannot be a best response that $w_j > p^+$. p^+ is known to all firms, and conditional on recruiting a worker, such an offer would be sure to generate a loss in period 2 and zero profit in period 3 by* (4.1). *A superior profit of zero could be made by making no offer. So any best response satisfies $w_j \leq p^+$.*

Suppose that $w_i = w_j$. Then, given $w_j \leq p^+$, it follows that $\mathrm{E}p|_{p \geq w_i} \geq w_j$ and

$$w_i \leq \frac{w_j + \delta \mathrm{E}p|_{p \geq w_i}}{1+\delta} \tag{4.13}$$

Suppose that $w_i = p^+$. Then $\mathrm{E}p|_{p \geq w_i} = w_i$ and, given $w_j \leq p^+$,

$$w_i \geq \frac{w_j + \delta \mathrm{E}p|_{p \geq w_i}}{1+\delta} \tag{4.14}$$

$\mathrm{E}p|_{p \geq w_i}$ is not necessarily continuous in w_i, but it is strictly increasing in w_i given the assumption that $f(p) > 0$ for all $p \in [p^-, p^+]$. So $\frac{w_j + \delta \mathrm{E}p|_{p \geq w_i}}{1+\delta}$ cannot leap the 45° line from above. Therefore, for any feasible w_j it follows that there must exist a value of $w_i \in [w_j, p^+]$ which satisfies (4.12b) *with equality.* ∎

Figure 4.2 clarifies the existence of w^*. Lemma 4.5 can also be proven using Tarski's fixed point theorem (Tarski, 1955) which applies to non-decreasing discontinuous functions of any dimension.

There is no guarantee that there is only one w_i that satisfies (4.12b) with equality. As shown in Figure 4.3, $\frac{w_j + \delta \mathrm{E}p|_{p \geq w_i}}{1+\delta}$ can cut the 45° line more than once whilst strictly increasing in w_i and satisfying (4.13) and (4.14). Define $\tilde{w} = \{\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_M\}$ as the ordered set of wage offers w_i that satisfy (4.12b) with equality. Then $w^* = \tilde{w}_1$. For the distribution that leads to Figure 4.3 the set

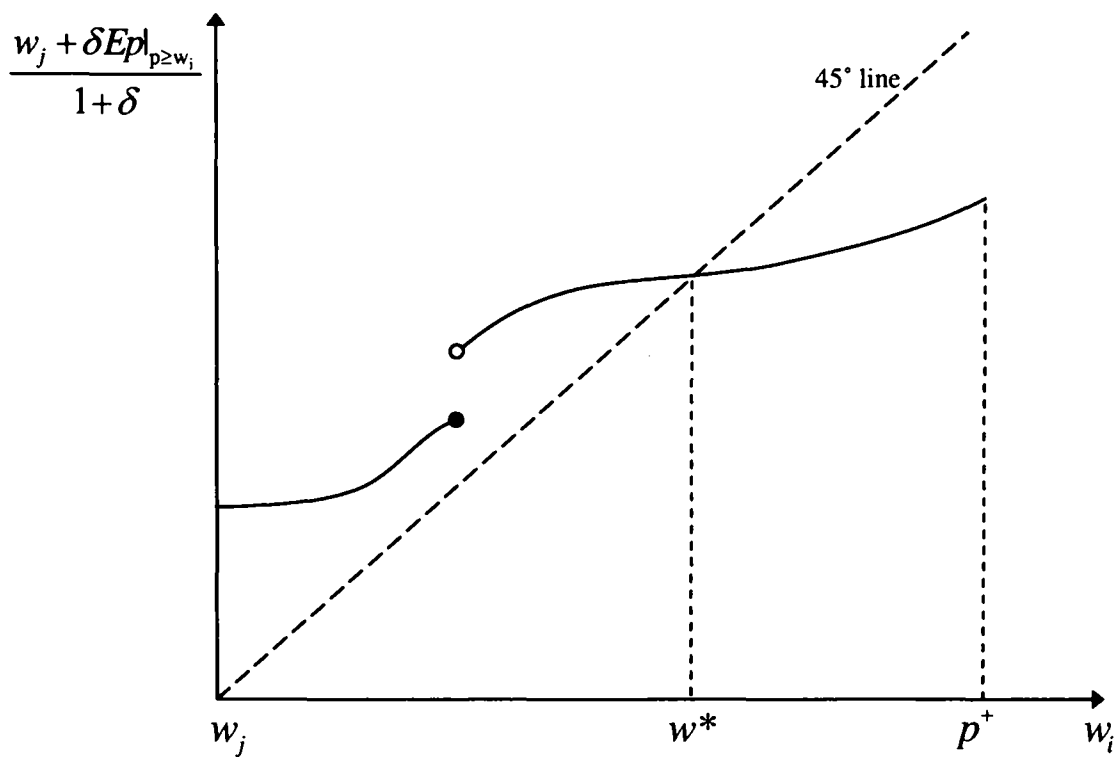


Figure 4.2: The existence of w^*.

$\tilde{w}$ would then contain three values with $M = 3$.

It is now possible to solve for firm i's best response in period 2, and subsequently that of firm k.

Proposition 4.1. *There exists a w^* such that, in period 2, it is a best response for the first employer to offer*

$$w^i = \begin{cases} w^* & \text{if} \quad p \geq w^* \\ 0 & \text{otherwise} \end{cases} \tag{4.15}$$

and the best response of potential employers to offer such that

$$w_j = \mathrm{E}p|_{p<w^*} \tag{4.16}$$

where j is given by (4.9). *Then in perfect Bayesian equilibrium*

$$w^* = \min \left\{ w_i : w_i = \frac{\mathrm{E}p|_{p<w_i} + \delta \mathrm{E}p|_{p \geq w_i}}{1+\delta} \right\} \tag{4.17}$$

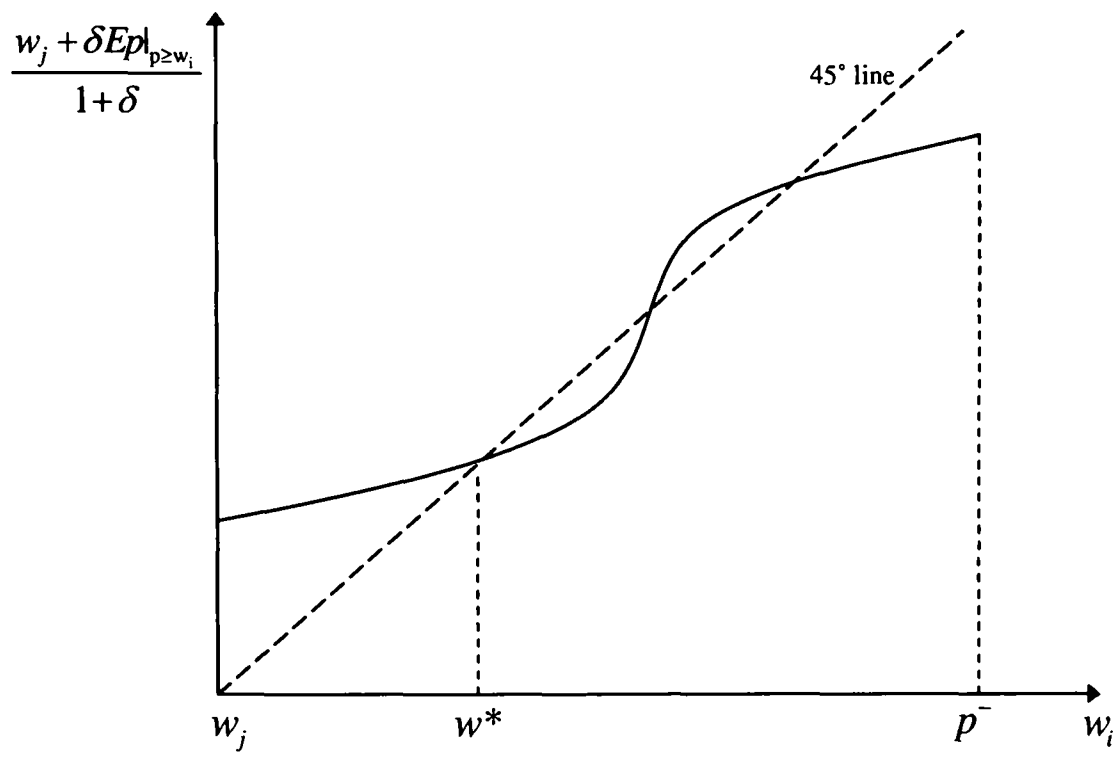


Figure 4.3: An example where there are multiple values of w_i that set $W_i = \mathrm{E}W_j$.

Proof. *Lemmas 4.4 and 4.5 combine to give*

$$w^* = \min\left\{w_i : w_i = \frac{w_j + \delta \mathrm{E}p|_{p \geq w_i}}{1+\delta}\right\} \tag{4.18}$$

Firm i will only be willing to offer w^* if (4.4) *is also satisfied. If $w^* > p$ then there is no offer firm i can make that will retain the worker and not lead to a loss. In this case $w_i = 0$ is a best response, although any offer less than or equal to p is also a best response. This gives* (4.15).

Conditional on firm i's reaction function as described by (4.15), *it is not a best response for firm j to offer $w_j > \mathrm{E}p|_{p<w^*}$. Firm j will be able to recruit a worker only when $w_i = 0$ since, by definition, w^* is the wage that will keep a worker at firm i. The expected productivity of a worker conditional on her being hired by firm j is therefore $\mathrm{E}p|_{p<w^*}$. Firm j will go on to make a profit of 0 in period 3 since $w_j^j = w_i^j = p$ from* (4.1). *So the expected profit to firm j in periods 2 and 3 from offering w_j conditional on hiring the worker in period 2 is $\mathrm{E}p|_{p<w^*} - w_j$.*

If $w_j > \mathrm{E}p|_{p<w^*}$ then expected profit is negative which cannot be a best response

since firm j can make zero profit from not making any offer. Furthermore $w_j <$ $\mathrm{E}p|_{p<w^*}$ cannot be a best response since this would make positive expected profit for firm j. Another potential employer would then make an offer ϵ higher which contradicts (4.9). *Setting $w_j = \mathrm{E}p|_{p<w^*}$ in equation* (4.18) *gives* (4.17). ∎

To summarise the equilibrium outcome: Firm i employs the worker for the first period and learns her productivity. In period 2, firm i offers a wage w^* to the worker if her productivity is greater than or equal to w^* where w^* is the weighted average of the expected productivity above and below the wage itself. This offer will be accepted by the worker who goes on to accept the same pooling wage in the final period. If the worker's productivity is less than w^* then firm i makes no offer in period 2. The worker quits and takes the highest offer from a potential employer. Then in period 3 the worker will remain with this employer on a separating wage. Conditional on a worker receiving the offer w^* from firm i in period 2, the expected payoff in the remaining two periods from quitting is identical to the payoff from staying with firm i, which is known with certainty.

Although simple, the model has some interesting features. Firstly, there is endogenous turnover in equilibrium. This is unusual. In models of adverse selection where employers can make counter-offers and the worker's type is known by herself and her employer, there is no way a potential employer can lure a worker from the current employer without making a loss. Moreover, the worker can be retained on a pooling wage because the winner's curse means potential employers do not make competitive offers.

Greenwald (1986) is able to generate turnover in equilibrium in spite of these features only by assuming that some workers will quit for reasons outside the model. His primary interest is in the composition of the pool of job changers, but these job changers would not exist at all without the assumption of exogenous quits. In the model of this chapter, endogenous quitting occurs even as the exogenous quit rate approaches zero, and there is still insight that can be gained about the composition

of the job changing pool under asymmetric information.

Under the assumption that firms remember the type of a worker after she quits as developed in Chapters 2 and 3, the worker is empowered to reveal her type to the market by quitting twice which induces competition between well-informed previous employers. This never actually happens in equilibrium but the possibility is important because it forces the first employer to reveal the type of the worker in order to retain her. When switching costs are introduced, less productive workers are retained on a pooling wage and only the more productive workers have their type revealed. But there is still no turnover in equilibrium.

In the 3-period model of this chapter, endogenous turnover occurs not as a direct result of the exogenous quit rate (which tends to zero) but rather because of the first employer's inability to credibly signal type to the uninformed worker whose ability is low. Although the first employer is always willing to match the period 2 offers of potential employers constrained by the winner's curse, this is not sufficient to retain the worker. Quitting is preferred by the worker because it confers the additional benefit of a separating wage offer in the final period as opposed to the pooling offer that the first employer will make. The first employer will only be willing to exceed the offers of potential employers by the margin required to retain the worker if the worker's productivity is sufficiently high.

For a worker of productivity $p < \mathrm{E}p|_{p<w_j}$, the first employer is willing to match the total payoff that the worker will receive by quitting yet is still unable to retain her. If, in making a low offer, the first employer could signal to the worker that her type truly was low, then the worker would be retained. But because the worker is not sure of her own type, such a signal is not credible. If accepted by the low productivity worker, the first employer would offer it to high productivity workers as well. And if all types of worker are offered the same pooling wage that potential employers are offering then in equilibrium the worker would prefer to quit and have her type revealed.

Not only is there turnover in equilibrium despite the exogenous quit rate tending to zero, but the second important result of the 3-period model is that there is adverse selection amongst the stream of job changers. That is, the expected productivity or a worker in the job changing pool is disproportionately comprised of the less able. This is identical to Greenwald's findings about the composition of the pool of job changers, thus capturing one of his key results without reliance on an exogenous quit rate.

Thirdly, in comparison to Greenwald (1986) where all wages are pooling and Chapter 2 of this thesis where only the high productivity workers have their type revealed, in this model it is only the low productivity workers who have their type revealed, with the more able retained by the first employer on a pooling wage. It is unsurprising that a worker who quits has her type revealed, as this follows directly from the firm memory assumption. But it is interesting that the high productivity workers who remain with the first employer do so on a pooling wage, thus forgoing the benefit that firm memory bestows.

The simplifying assumption that previous employers have a slight preference to allocate surplus to the worker over other firms turns out to be unimportant to many of these results. The assumption is used as an equilibrium selection device but any of the multiple equilibria that exist without the assumption still have some properties in common. For the worker who quit in period 2, any best response from potential employers in period 3 would lead to the worker being retained in period 3 by the second period employer for a wage that could be anywhere between p^- and p. But wage offers in period 2 are competitive and therefore take into account the wage required to retain the worker in period 3. The total payoff to the worker in periods 2 and 3 is therefore unaffected by the period 3 offer in equilibrium. This means the results described above regarding turnover in equilibrium, *ex ante* expected payoffs to all workers, and actual payoffs to workers retained by the first employer do not rest on the simplifying assumption. The only result that does rest on the simplifying

assumption is the separating wage offers for the less able workers. But the selection of this particular equilibrium can be justified on the basis that in four or more periods the possibility of competition between two previous employers would make separation the unique equilibrium outcome.

4.2.3 Discounting

The key to this model is the wage w^*, the minimum amount the first employer can offer that will retain the worker in period 2. This wage balances the worker's future benefit of having type revealed in the final period against the cost of doing so — accepting a lower wage in the current period from a potential employer than can be received by staying with the first employer. Because the costs and benefits being traded off are in different periods, the worker's rate of time preference affects the equilibrium wage offers. Equation (4.17) shows that the lower the discount factor (that is, the higher the discount rate $\frac{1-\delta}{\delta}$) the lower the minimum wage offer the worker will be willing to accept from the first employer in period 2. A higher rate of time preference causes the worker to put more weight on the cost of taking the lower offer from a potential employer in period 2 than the benefit of having type revealed in period 3 and less inclined to quit.

When the worker's rate of time preference is high, the first employer can not only retain the worker with a lower wage, but by (4.15) will also retain a greater proportion of a population of workers as w^* decreases. So discounting acts like a friction in the mechanism that eliminates adverse selection, and the more the worker discounts the future the less likely it is she will ever have her type revealed.

Firms also share the discount factor δ but this has no effect on their strategy. Potential employers make zero expected profit in every period so there are no intertemporal aspects to their strategy. The first employer never has to trade off profit in one period against profit in another so is also unaffected by the discount factor.

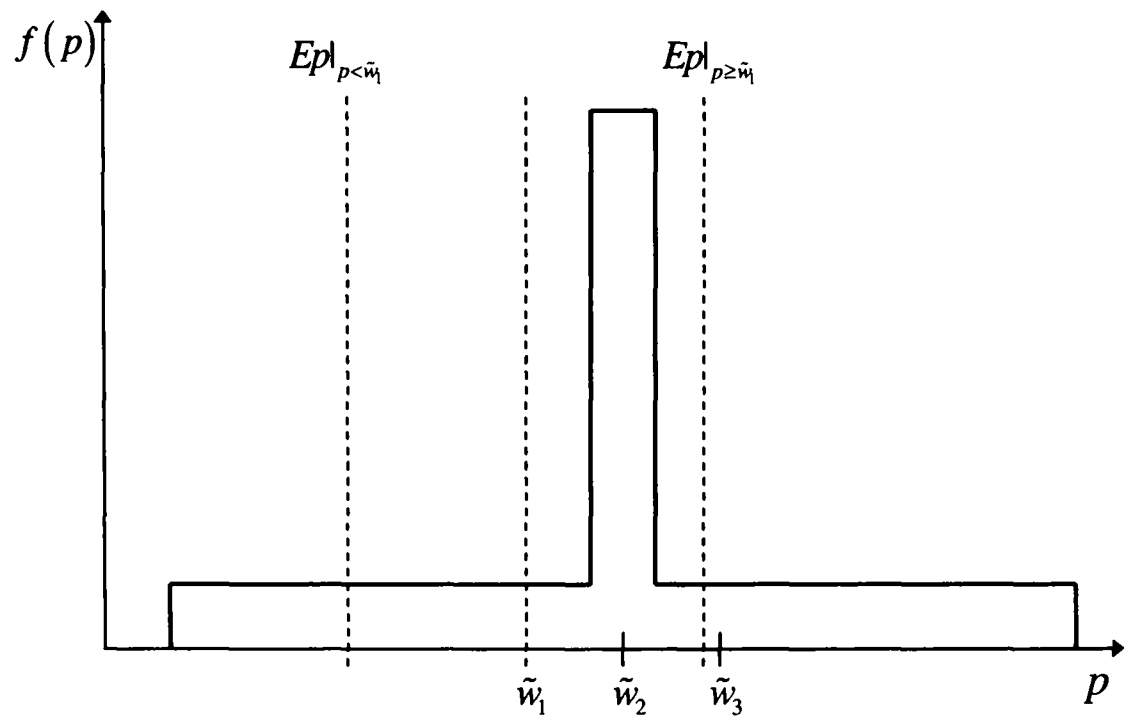


Figure 4.4: A distribution where there are three values of w_i that set the payoff from staying equal to the expected payoff from quitting. Here $\delta = 1$.

4.2.4 The Shape of the Productivity Distribution

Conjectures Regarding Possible Values of M

By Lemma 4.2, the RHS of $w_i = \frac{\mathrm{E}p|_{p<w_i} + \delta \mathrm{E}p|_{p \geq w_i}}{1+\delta}$ must start above the 45° line and finish below it. This implies $M \geq 1$. For a function qualified only as strictly increasing and discontinuous, M could indeed take any value greater than 1. But $\frac{\mathrm{E}p|_{p<w_i} + \delta \mathrm{E}p|_{p \geq w_i}}{1+\delta}$ cannot take the form of *any* strictly increasing discontinuous function. For any given $f(p)$ it is not clear how many values of M are possible.

Figure 4.4 shows a distribution where, with $\delta = 1$, $M = 3$. The distribution is symmetrical, and like any symmetrical distribution there will be a $\tilde{w}$ at Ep (again taking $\delta = 1$). But because of the long tails on the distribution there is also a $\tilde{w}$ to either side of Ep. By observation of Figure 4.4 it is easy to imagine a function where $M = 2$. Either there could be a discontinuity that allows the function to leap the 45° line from below, or the function could "kiss" the 45° line from either above or below.

It would appear that there is no productivity distribution for which $M > 3$. To

prove this it would be necessary to examine the expression $\mathrm{E}p|_{p<w_i} + \delta \mathrm{E}p|_{p\geq w_i}$ and show that there can be only one point of inflexion.

It would further appear that for any concave distribution, $M = 1$. This would require proving that the rate of change of $\mathrm{E}p|_{p<w_i} + \delta \mathrm{E}p|_{p\geq w_i}$ is non-increasing.

Spread

Imagine a productivity distribution where $w^* = \mathrm{E}p$ when $\delta = 1$. This is the case for the uniform distribution described later in Section 4.2.5. A mean preserving spread in $f(p)$ will cause $\delta \mathrm{E}p|_{p\geq w^*}$ to increase by the same amount as $\mathrm{E}p|_{p<w^*}$ decreases and w^* will remain unchanged. So the first employer will retain the same proportion of a population of workers as before, and for the same wage. The expected productivity of those retained is higher, so profit to the first employer after the first period increases with spread.

For the less able worker who is not made an offer by her first employer in period 2, the expected payoff from quitting decreases with the spread since $w_j = \mathrm{E}p|_{p<w^*}$. This is consistent with Greenwald (1986, Proposition 5, page 332) who finds that, because the stream of job changers is composed disproportionately from the lower tail of the distribution, increasing the spread has a net effect of lowering the expected productivity of job changers.

Skewness

For any distribution, increasing the spread of the part of the distribution that lies above w^* will cause w^* to increase. Let $\delta = 1$, w^*_{old} be the initial w^* shown in Figure 4.5 for a symmetrical distribution, and w^*_{new} be the w^* after the distribution has been modified to the dashed line. $\mathrm{E}p|_{p\geq w^*_{\text{old}}}$ increases when the distribution is modified, but $\mathrm{E}p|_{p<w^*_{\text{old}}}$ is unchanged. Since w^*_{new} is the average of these two expectations it follows that $w^*_{\text{new}} > w^*_{\text{old}}$. The intuition is simple. The longer tail means the worker's expected type is higher, and therefore the benefit from quitting

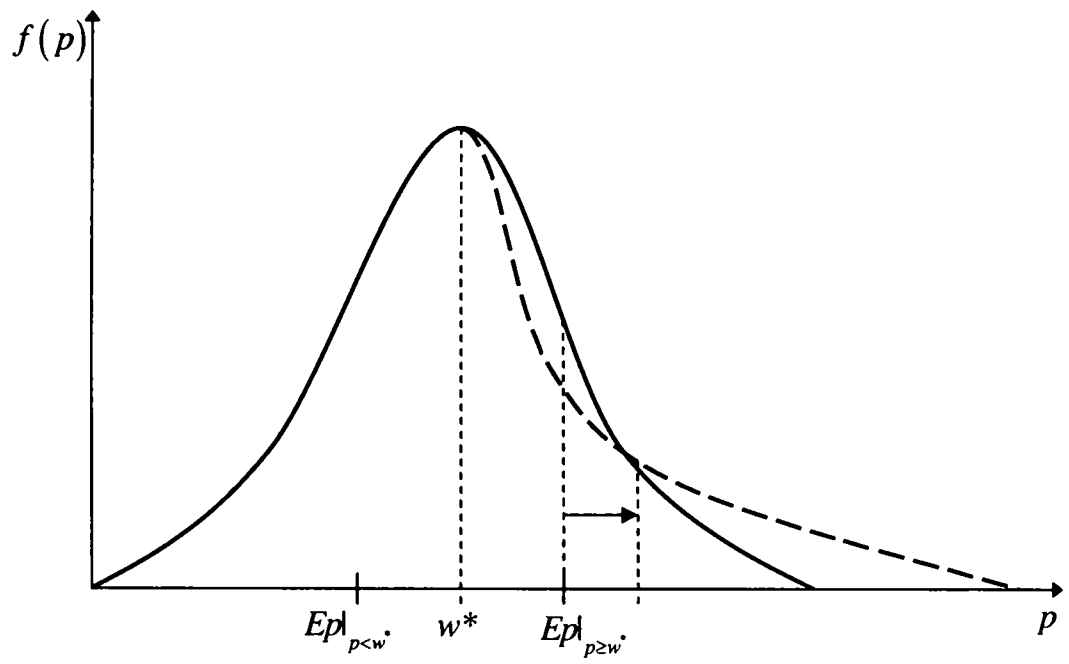


Figure 4.5: Increasing the productivity of more able workers ($\delta = 1$).

to have type revealed is higher. In order to retain the worker the first employer must increase its offer. But notice that w^* appears twice in equation (4.15). An increase in w^* means that conditional on being retained a worker will receive a higher wage, but also that in a population of workers, fewer of them will be retained. So increasing the tail of the distribution decreases the proportion of a population of workers that the first employer retains. This will not necessarily decrease the first employer's expected profit however, as the increased expected productivity of a retained worker may counteract the decreased probability of retention.

No general statements can be made about the effect of increasing the spread of the part of the distribution that lies below w^*. Any change in the distribution could eliminate one of the members of the set $\tilde{w}$. If it is the lowest value that is eliminated, the new w^* will be higher than the old, even though the distribution was shifted down on average.

4.2.5 Special Case: Uniform Productivity Distribution

If a uniform productivity distribution on $[p^-, p^+]$ is assumed then the expressions for the wage offers in Proposition 4.1 are greatly simplified. This can allow more

intuitive insight into the model and also forms the basis of the N-period model developed in Section 4.3.

Proposition 4.2. *If $f(p)$ is uniform on $[p^-, p^+]$*

$$w^* = \frac{p^- + \delta p^+}{1+\delta} \tag{4.19}$$

and

$$w_j = \frac{p^- + \delta p^+}{2(1+\delta)} \tag{4.20}$$

Proof. *For a uniform distribution $f(p)$ is constant for all p. Conditional expected productivities in* (4.17) *are therefore given by*

$$\mathrm{E}p|_{p \geq w_i} = \frac{\int_{w_i}^{p^+} p\, f(p)\, dp}{\int_{w_i}^{p^+} f(p)\, dp} \tag{4.21a}$$

$$= \frac{w_i + p^+}{2} \tag{4.21b}$$

and

$$\mathrm{E}p|_{p < w_i} = \frac{\int_{p^-}^{w_i} p\, f(p)\, dp}{\int_{p^-}^{w_i} f(p)\, dp} \tag{4.22a}$$

$$= \frac{p^- + w_i}{2} \tag{4.22b}$$

Substituting (4.21b) *and* (4.22b) *into* (4.17) *and solving for w_i gives*

$$w_i = \frac{\frac{p^- + w_i}{2} + \delta \frac{w_i + p^+}{2}}{1+\delta} \tag{4.23a}$$

$$w_i = \frac{p^- + \delta p^+}{1+\delta} \tag{4.23b}$$

This is the unique solution to (4.17) *and therefore represents w^*. Using* (4.21b) *to solve for* (4.16) *gives* (4.20). ■

If $\delta = 1$, $w^* = \mathrm{E}p$, that being equidistant between the expected productivities

of the lower and upper half of the distribution. Any worker of productivity greater than or equal to average productivity will be retained by the first employer for a wage in period 2 and 3 equal to average productivity. Furthermore, for a uniform distribution, the mean is the same as the median so in a population of workers, half of them will be retained by the first employer.

4.3 N-period Model

4.3.1 Preliminaries

By extending the model to N periods, it is possible to gain insight into dynamic features of the wage rate as N becomes large without the inherent lumpiness of the 3-period model. The simplifying assumption is made that $f(p)$ is uniformly distributed on $[0, 1]$. This simplifies the algebra and gives a clarity of result that cannot be achieved with an unspecified distribution. For the same reason discounting is not considered in this model.

Wage notation is changed from the 3-period model so that w_n is the offer made by the first employer in period n, and w_n^q is the offer made by the second employer in period n to the worker who quit in period q. Wage offers after multiple quits need not be considered because, having quit once, the worker receives p from the second employer in all subsequent periods so quitting again is off the equilibrium path. (The proof of this result is identical to Lemma 4.1 of the 3-period model so will not be restated here). The N-period game tree is shown in Figure 4.6.

The model shows that wages in each period follow a pattern based on the Fibonacci series and the Golden Ratio. The Fibonacci series, the significance of which was first noted by the 12th century Italian mathematician after whom it is named, starts 1, 1 and each subsequent value is the sum of the previous two. The Golden Ratio is the ratio of the nth and $n+1$th values of the series as n approaches infinity.

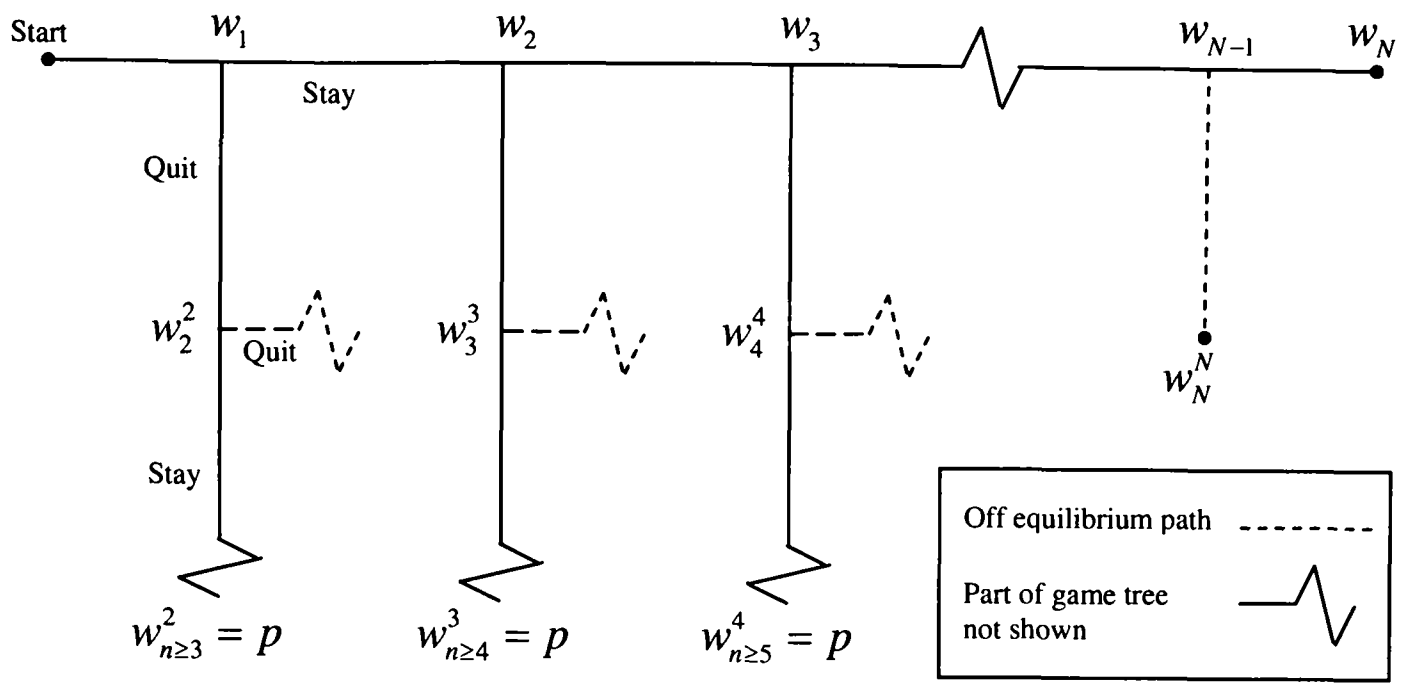


Figure 4.6: Game tree and wage notation for the N-period game.

4.3.2 Analysis and Results

In the 3-period model, Lemma 4.3 showed that the first employer would never offer a period 2 wage greater than the worker's productivity. In fact this result can be extended to any period $n \geq 2$ of the N-period game using backwards induction.

Lemma 4.6. *In the N-period game, a pure strategy perfect Bayesian equilibrium must satisfy*

$$w_n \leq p \quad for \quad n \in \{2 \dots N\} \tag{4.24}$$

Proof. *A proof is given in Appendix B.*

Define w_n^* as the minimum wage offered by the first employer in period n that will retain the worker. Then equilibrium wage offers are given by the following proposition.

Proposition 4.3. *In the N-period game with a unit rectangular productivity distribution it is a best response for the first employer to offer in the nth period a*

wage

$$w_{n\in\{2\ldots N\}} = \begin{cases} w_n^* & \text{if} \quad p \geq w_n^* \\ 0 & \text{otherwise} \end{cases} \tag{4.25}$$

where

$$w_n^* = 1 - \frac{\text{fib}(N-n+1)}{\text{fib}(N)} \tag{4.26}$$

and $\text{fib}(m)$ *is the* m*th value in the Fibonacci series. It is a best response of potential employers to offer*

$$w_n^n = \frac{w_{n-1}^* + w_n^*}{2} \tag{4.27}$$

Proof. *A proof is given in Appendix C.*

Like the 3-period model, the first employer will offer only enough to retain the worker if this amount is no greater than her productivity. For a worker who is retained, each time the first employer makes an offer it reveals by Lemma 4.6 that the worker's type must be at least as high as the wage itself. This information can then be exploited by both the worker and potential employers in the following period and so creates a "ratchet" effect, where each of the first employer's wage offers must be higher than the last because of the information unavoidably revealed about the worker's type.

One of the properties of the Fibonacci series is that the ratio of any two adjacent values in the series $\text{fib}(n)$ and $\text{fib}(n+1)$ approaches a constant as n becomes large. This constant φ, known as the Golden Mean or Golden Ratio is defined by the equation

$$\frac{1}{\varphi} = 1 + \varphi \tag{4.28}$$

So (4.26) can be approximated for large values of N as

$$w_n^* = 1 - \varphi^{n-1} \tag{4.29}$$

In each period the first employer's wage offer ratchets up approximately $100(1-\varphi)\%$

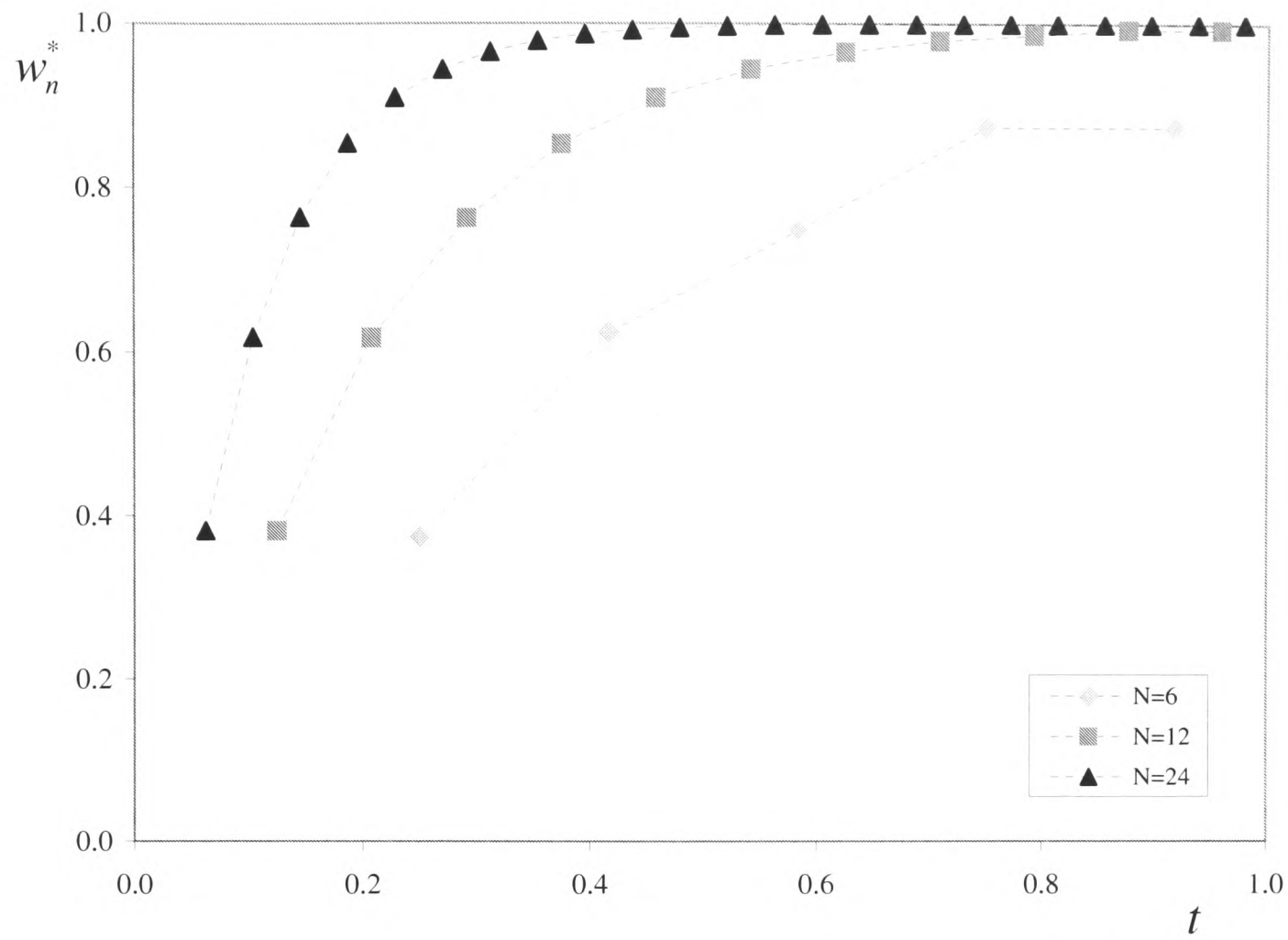


Figure 4.7: First employer wage offers $w^*_{n\in\{2\ldots N\}}$ for $N = 6$, $N = 12$ and $N = 24$ on a normalised time scale.

closer to the maximum level of productivity, provided the worker's type is sufficiently high to make such an increase profitable. This is shown in Figure 4.7 for three different values of N on a normalised time scale. Table 4.1 of Appendix D shows the values of w_n^* for values of N up to 15. The reason that the per-period rate of wage increase by the first employer is largely independent of N can be traced back to the proof of Proposition 4.3. Equation (4.46c) shows that the worker's expected wage after quitting in two periods' time is identical to the expected wage over the same period that results from quitting immediately. So when the first employer sets its period n wage offer at the level that matches the expected payoff from quitting equal to the expected payoff from staying, the expected wages in periods $n + 2$ onwards balance each other out. Thus w_n^* is largely independent of N. It is not completely independent because of the end game outcome that the first employer's wage offer

will not increase in the final period (This is because it is too late for the worker to quit and benefit from having type revealed). But moving far enough back in time, the end game payoffs become a smaller factor in the expected payoff and the percentage wage increase towards the upper bound of productivity rapidly converges to $100(1-\varphi)\%$.

Since the first employer is required to increase its wage offer toward the upper bound by roughly the same proportion each period in order to retain the worker, the more periods there are in a worker's lifetime, the faster information about her type will be revealed. This is not merely a mathematical phenomenon resulting from the decreasing lumpiness as N increases, but a meaningful result with an economic interpretation. An employer learns the worker's type after one period, so increasing N whilst holding the worker's lifetime at a constant length is effectively speeding up the process of type being learned. This is an advantage to the worker since as soon as two employers can learn her type, competitive wage offers must follow. Interpreted in this light, the explanation of the wage profiles in Figure 4.7 is clear: the faster type can be learned by a second employer, the faster the first employer must increase its offer in order to deter the worker from quitting.

The N-period model shows more clearly many of the results the 3-period model hinted at. Tenure at the first employer is correlated with productivity and only a highly productive worker will be retained by the first employer for her entire lifetime – one whose productivity is greater than w_N^*, which from equation (4.49) is $\frac{1}{\text{fib}(N)}$. As $N \to \infty$, $w_N^* \to 1$ and so a worker of any productivity will eventually quit the first employer and receive a fully separating wage at the second employer. This is consistent with the model of Chapter 2 where, as the rate at which an employer learns the worker's type $\theta \to 0$, the first employer offers a separating wage to a worker of any productivity.

Conditional on quitting once, a worker will never quit a second time. So as N increases, the proportion of a population of workers who will quit exactly once

in their lifetime approaches 1. Thus adverse selection decreases with N and the expected productivity of a worker conditional on her being in the job changing pool approaches the expected productivity of the population. This is consistent with intuition since the worker's solution to low wages that result from adverse selection is to quit in order to generate competition between previous employers, the cost of quitting is the amount of time that must be spent on a decreased wage at a second employer, and that amount of time approaches zero as N approaches infinity.

4.4 Self-Serving Bias

Throughout this paper it has been assumed that the worker does not know her own productivity, but that the underlying distribution of productivities is known. Following standard game theoretic logic it is assumed that the worker can accurately calculate expected productivity based on the information available to her at the time.

But how realistic is this assumption? Evidence from the psychology literature suggests strongly that when forming expectations under uncertainty, people unintentionally and unknowingly distort their estimation in a way that will benefit themselves. This is known as "self-serving bias" or the "above average" effect. For example, when asked whether they thought they were above or below average in their driving ability, Svenson (1981) found that some 80% of subjects rated themselves as above average. Similar bias was observed when subjects were asked about ethics (Baumhart, 1968) and health (Weinstein, 1980). Self-serving bias has also been used to explain economic behaviour. Babcock and Loewenstein (1997) show how bargaining impasse could be a result of each party's perception of a fair offer being unknowingly skewed to their own benefit.

In the labour economics domain, there is strong evidence that workers are prone to overestimate their own productivity (Cross, 1977), managerial ability (Larwood and Whitaker, 1977), and contribution to group tasks (Ross and Sicoly, 1979). In the context of the model of this chapter these findings are relevant and also quite

simple to integrate. Self-serving bias means workers will, on average, come up with an estimate of their productivity which is higher than the mathematical expectation used in the model. So the worker's estimate of the expected wage in period 3 from quitting in period 2 would be higher than the $Ep|_{p\geq w_i}$ which appears in (4.8). If such overestimates are systematic in nature then the employer would be able to predict what opinion the worker has of her own ability given her actual productivity. The worker's estimate of $Ep|_{p\geq w_i}$ could then be used to calculate the wage required to retain her instead of the mathematical estimate. By (4.17), w^* would then increase.

Not all workers would benefit from the increased offer by the first employer however. Some workers would find their inflated expectations are not accommodated by their employer however and they would be left to quit. Any worker who quits goes on to receive a competing wage, whereas the worker who stays with the first employer cannot receive competing offers from outside firms because of the winner's curse. The increased quit rate therefore increases average wages after the first period. So it can be concluded that the existence of self-serving bias in the context of the model of this chapter would increase turnover and average wages, and hasten the revelation of type.

These results hinge on the fact that agents are unaware of their own self-serving bias. Knowingly inflating the perception of one's own type is not a best response because quitting if the increased expectations of wage are not met is a non-credible threat.

Self-serving bias has the exact opposite effect to discounting, which decreases turnover and average wages, and causes a greater proportion of the workforce to receive pooling wages. Where self-serving bias makes the prospect of having type revealed in the future more appealing, discounting makes it less so.

It is interesting to note that if self-serving bias is taken to the extreme where all worker's assume they are of the very highest possible productivity, $w^* = p^+$ and all workers will quit after the first period. Full separation then occurs and there is no

adverse selection. This demonstrates that it is only the lack of a credible threat to quit that allows the first employer to retain any worker on a pooling wage and in so doing make a profit by exploiting adverse selection.

4.5 Conclusion

This chapter set out to achieve two objectives by assuming that the worker may not know her productivity. Firstly an investigation into the composition of the stream of job changers was sought that is not a function of the exogenous quit rate. Although Greenwald (1986) explains much about the composition of the job changing stream and equilibrium wage offers in markets with asymmetric information, both the size of the job changing stream and its composition are a function of the exogenous quit rate. Setting the exogenous quit rate to zero or approaching zero eradicates turnover and undermines Greenwald's results, and this is unsatisfactory. Secondly, this chapter explores the implications of firm memory for the persistence of adverse selection in the labour market. Chapter 3 showed that the result of Chapter 2 that firm memory of type can mean adverse selection will never persist is dependent on the assumption that the worker's productivity is constant. This chapter set out to investigate if there could be other assumptions that are crucial to Chapter 2's result.

Both the 3-period and N-period models of this chapter have shown that there is turnover in equilibrium even as the exogenous quit rate approaches zero and that the stream of job changers is adversely comprised of the less able. This is consistent with the results of Greenwald (1986) but puts the result on a stronger footing by reducing its dependence on an assumed exogenous quit rate.

The model also shows that if the worker is not aware of her own type then the more able workers will be retained on a pooling wage and asymmetric information will persist. The less able workers who leave their first employer ultimately have their type revealed. So adverse selection persists, but only in the sense that the pool of job changers is disproportionately comprised of the less able. The winner's curse,

a typical feature of labour markets with adverse selection, will persist only for the more able workers whose wages are pooled.

In addition to achieving the two primary objectives, the models of this chapter have also generated some other interesting results. In Chapter 2 it was only the low productivity workers who would ever receive pooling wages because the benefits of them quitting to have type were revealed could be outweighed by the switching cost. But here the opposite occurs. All workers are willing to stay with the first employer on the same pooling wage, but the first employer is not willing to offer this wage to the less productive workers because it is not profitable to do so. So the less productive workers quit, going on to receive separating wage offers because of the existence of firm memory. Where firms and the worker discount future payoffs, this reduces wage offers and counteracts the ability of firm memory of type to lead to wage separation.

In the N-period model the worker who stays with the first employer does so on an increasing wage, hence tenure is correlated with productivity. The worker's outside offers increase until such a time as the first employer is unwilling to match the payoff the worker expects to receive from quitting. The longer it takes for an employer to learn the worker's type the lower the worker's expected payoff after the first period and less likely it becomes that she will quit and have type revealed. The existence of a self-serving bias would increase the expected payoff after the first period, increase turnover, and increase the proportion of a population of workers that receive a separating wage.

4.6 Appendix

A Proof of Lemma 4.3

Lemma 4.2 has shown that $w_i < p$ is a best response conditional on $w_k^i = w_i$. There are two other possibilities for firm k's period 3 best response: $w_k^i > w_i$ or $w_k^i < w_i$, each of which will be considered in turn.

Suppose firstly that $w_k^i > w_i$. Then it remains a best response for firm i's period 2 offer to satisfy (4.4). It cannot be a best response that $w_i > p$ because a loss would be made in period 2 and in period 3, firm k would offer $w_k^i > w_i > p$. Firm i would not be willing to match this by (4.3) so the worker would quit and firm i would make an overall loss during periods 2 and 3. So, since it remains a best response for firm i's period 2 offer to satisfy (4.4), by Lemma 4.2 it cannot be a best response that $w_k^i > w_i$.

Turning to the possibility that $w_k^i < w_i$, let $w_k^i = w_i - \epsilon$ where $\epsilon > 0$. This will never recruit a worker not quitting for exogenous reasons. Given firm i's period 3 strategy in (4.3), firm i will retain the worker with $w_i^i = w_k^i$. Firm i's discounted profit in the final two periods as at the start of period 2 is then

$$\pi_i = (p - w_i) + \delta(p - w_i^i) \tag{4.30a}$$

$$= (p - w_i) + \delta(p - w_k^i) \tag{4.30b}$$

$$= (p - w_i) + \delta(p - (w_i - \epsilon)) \tag{4.30c}$$

$$= (1 + \delta)(p - w_i) + \delta\epsilon \tag{4.30d}$$

For w_i to be a best response for firm i, it must give non-negative profit since it is always possible to make zero profit by making no offer at all. So

$$(1 + \delta)(p - w_i) + \delta\epsilon \geq 0 \tag{4.31a}$$

$$p \geq w_i - \left(\frac{\delta}{1 + \delta}\right)\epsilon \tag{4.31b}$$

When choosing w_k^i, firm k has observed w_i so inequality (4.31b) provides a lower bound on p. It can therefore not be a best response for firm k to offer $w_k^i < w_i - \left(\frac{\delta}{1+\delta}\right)\epsilon$ because another potential employer would have an incentive to offer slightly more and be certain of hiring the worker who quits the first employer for exogenous reasons. This is inconsistent with the definition of firm k in (4.6). So $w_k^i \geq w_i - \left(\frac{\delta}{1+\delta}\right)\epsilon$, but this contradicts the original hypothesis that $w_k^i = w_i - \epsilon$. Both cannot be true for any $\epsilon > 0$ regardless of the value of δ. Therefore $w_k^i < w_i$ can be eliminated as a best response.

This leaves $w_k^i = w_i$ as the unique best response of firm k in period 3, so by Lemma 4.2, any best response for firm i in period 2 must satisfy (4.4).

B Proof of Lemma 4.6

Suppose Lemma 4.6 were not true. Then the first employer offers $w_n > p$ in some period $n \in \{2 \ldots N\}$. This will generate a loss in period n, so to be a best response, the first employer must retain the worker in some subsequent period at a lesser wage. Any subsequent periods during which the first employer retains the worker must be continuous starting at period $n+1$ since a worker who quits will stay with the second employer on a separating wage for the rest of her lifetime. Let $M \in \{n+1 \ldots N\}$ be the final period that the worker is employed by the first employer. In order to retain the worker for these $M - n$ periods after period n the first employer must match the highest offer from a potential employer w_m^m where

$$m \in \{n+1 \ldots M\} \tag{4.32}$$

Suppose the sum of the offers made by potential employers in periods $n+1$ to M is

$$\Sigma_{m=n+1}^{M} w_m^m = (M-n)w_n - \epsilon \tag{4.33}$$

where $\epsilon > 0$. Then the profit to the first employer in periods n to M is

$$\pi_{n...M} = \Sigma_{m=n}^{M}(p - w_m) \tag{4.34a}$$

$$= (p - w_n) + (M - n)p - \Sigma_{m=n+1}^{M} w_m^m \tag{4.34b}$$

$$= (M - n + 1)(p - w_n) + \epsilon \tag{4.34c}$$

For w_n to be a best response for the first employer, this profit must be non-negative for some M, so

$$(M - n + 1)(p - w_n) + \epsilon \geq 0 \tag{4.35a}$$

$$p \geq w_n - \frac{\epsilon}{M - n + 1} \tag{4.35b}$$

When determining their offers in periods $m \in \{n + 1 \ldots M\}$, potential employers have observed w_n so inequality (4.35b) provides a lower bound for p. It can therefore not be a best response for all potential employers to offer $w_m^m < w_n - \frac{\epsilon}{M-n+1}$ in any period m because one potential employer would have an incentive to offer slightly more and be certain of making a profit if the worker quits the first employer for exogenous reasons. So at least one potential employer will offer

$$w_m^m \geq w_n - \frac{\epsilon}{M - n + 1} \tag{4.36}$$

Then the sum of the wage offers from potential employers in periods $n + 1$ to M satisfies

$$\Sigma_{m=n+1}^{M} w_m^m \geq (M - n)\left(w_n - \frac{\epsilon}{(M - n + 1)}\right) \tag{4.37a}$$

$$\geq (M - n)w_n - \frac{(M - n)}{(M - n + 1)}\epsilon \tag{4.37b}$$

But by the original hypothesis of this proof, $\Sigma_{m=n+1}^{M} w_m^m$ is given by (4.33). Both (4.33) and (4.37b) cannot be true for any M, $\epsilon > 0$. So it must be that $\epsilon \leq 0$. Then

by (4.36) $w_m^m \geq w_n$. In other words, potential employers will never offer less than the first employer has offered in any previous period.

In order to retain the worker in periods $n+1$ to M, the first employer must match the offers of potential employers with $w_m \geq w_m^m \geq w_n$. So the first employer can then never retain the worker by offering less than it did in any previous period. It can therefore not be a best response for the first employer to offer $w_n > p$ because the loss can never be recouped.

C Proof of Proposition 4.3

In period n, potential employers make their offers, the highest of which is w_n^n, and this is followed by the first employer offering w_n and revealing $p \geq w_n$ by Lemma 4.6. The worker then chooses the offer which leads to the highest expected payoff in the remaining $N-n+1$ periods.

Conditional on the first employer offering according to (4.25), a worker leaving the first employer in period n must have been offered $w_n = 0$ since, by definition, w_n^* will retain the worker. If the worker is offered zero this signals her type to be $p < w_n^*$ by (4.25). If the worker is still at the first employer in period n then the worker must have been offered w_{n-1}^* in the previous period and is therefore of productivity $p \geq w_{n-1}^*$. So a wage offer from a potential employer of

$$w_n^n = Ep|_{p\in[w_{n-1}^*, w_n^*)} \tag{4.38}$$

would lead to zero expected profit. It cannot be a best response for all potential employers to offer less than this amount, otherwise there would be an incentive for one potential employer to offer slightly more and be sure of hiring any worker who is offered zero by her current employer. It can also not be a best response for any potential employer to offer more than (4.38), otherwise that firm would make an expected loss. So a potential employer who becomes the second employer must do so with an offer of (4.38). Conditional expectation for a uniform distribution is given

by

$$Ex|_{x\in[a,b]} = \frac{\int_a^b x\, g(x)\, dx}{\int_a^b g(x)\, dp} \tag{4.39a}$$

$$= \frac{a+b}{2} \tag{4.39b}$$

Using this to solve for (4.38) gives wage offer (4.27).

The first employer's best response is to offer the smallest wage that will retain the worker w_n^*, provided this does not lead to an expected loss being made. By Lemma 4.6, w_n^* will only be offered to a worker whose productivity is $p \geq w_n^*$. If the worker's productivity is less than w_n^* then any wage offer less than w_n^* will not retain the worker and is therefore a best response. Equation (4.25) shows the (non-unique) best response where this offer is zero.

The minimum wage what will retain the worker, w_n^*, will set the expected payoff from quitting equal to the expected payoff from staying with the first employer. The worker's expected payoff from quitting in period n conditional on w_n^* being offered by the first employer and w_n^n being offered by potential employers is

$$EW_n^n|_{w_n=w_n^*} = w_n^n + (N-n)Ep|_{p\geq w_n^*} \tag{4.40}$$

w_n^n is given by (4.27) and $Ep|_{p\geq w_n^*}$ by (4.39b). So

$$EW_n^n|_{w_n=w_n^*} = \frac{w_{n-1}^* + w_n^*}{2} + (N-n)\frac{w_n^*+1}{2} \tag{4.41}$$

The worker's expected payoff from staying with the first employer in period n conditional on w_n^* being offered is

$$EW_n|_{w_n=w_n^*} = w_n^* + E_{n+1}^n W \tag{4.42}$$

where $E_{n+1}^n W$ represents the payoff from period $n+1$ onwards as expected at period

n. This in turn depends on whether or not the first employer offers w^*_{n+1} in the next period, so

$$\begin{aligned} E^n_{n+1}W =& \text{prob}(p \geq w^*_{n+1})|_{p\geq w^*_n} \left(EW_{n+1}|_{w_{n+1}=w^*_{n+1}}\right) \\ &+ \text{prob}\left(p < w^*_{n+1}\right)|_{p\geq w^*_n} \left(EW^{n+1}_{n+1}|_{w_{n+1}=0}\right) \end{aligned} \tag{4.43}$$

w^*_{n+1} by definition sets the payoff from staying with the first employer equal to the payoff from quitting, so

$$EW_{n+1}|_{w_{n+1}=w^*_{n+1}} = EW^{n+1}_{n+1}|_{w_{n+1}=w^*_{n+1}} \tag{4.44}$$

Substituting this into (4.43) gives

$$\begin{aligned} E^n_{n+1}W =& \text{prob}\left(p \geq w^*_{n+1}\right)|_{p\geq w^*_n} \left(EW^{n+1}_{n+1}|_{w_{n+1}=w^*_{n+1}}\right) \\ &+ \text{prob}\left(p < w^*_{n+1}\right)|_{p\geq w^*_n} \left(EW^{n+1}_{n+1}|_{w_{n+1}=0}\right) \end{aligned} \tag{4.45}$$

The probability that $p \geq w^*_{n+1}$ conditional on $p \geq w^*_n$ is, by Bayes law, the probability that $p \geq w^*_{n+1}$ which is $1 - w^*_{n+1}$ divided by the probability that $p \geq w^*_n$, which is $1 - w^*_n$. The complement of this fraction is the probability $\left(p < w^*_{n+1}\right)|_{p\geq w^*_n}$. An expression for $EW^{n+1}_{n+1}|_{w_{n+1}=w^*_{n+1}}$ can be found by incrementing the index n in equation (4.41) by 1. The expression for $EW^{n+1}_{n+1}|_{w_{n+1}=0}$ is the same except that the expected payoff in the remaining $N-n-1$ periods is $\frac{w^*_n+w^*_{n+1}}{2}$ since an offer of $w_{n+1} = 0$ signals to competing firms that the worker's productivity is in $[w^*_n, w^*_{n+1})$. So equation (4.45) becomes

$$\begin{aligned} E^n_{n+1}W =& \left(\frac{1-w^*_{n+1}}{1-w^*_n}\right)\left(w^{n+1}_{n+1} + (N-n-1)\frac{w^*_{n+1}+1}{2}\right) \\ &+ \left(\frac{w^*_{n+1}-w^*_n}{1-w^*_n}\right)\left(w^{n+1}_{n+1} + (N-n-1)\frac{w^*_n+w^*_{n+1}}{2}\right) && \text{(4.46a)} \\ =& w^{n+1}_{n+1} + (N-n-1)\frac{1^2-(w^*_{n+1})^2+(w^*_{n+1})^2-(w^*_n)^2}{2(1-w^*_n)} && \text{(4.46b)} \\ =& w^{n+1}_{n+1} + (N-n-1)\frac{w^*_n+1}{2} && \text{(4.46c)} \end{aligned}$$

Substituting (4.46c) into (4.42), the expected payoff staying with the first employer in period n becomes

$$EW_n|_{w_n=w_n^*} = w_n^* + w_{n+1}^{n+1} + (N-n-1)\frac{w_n^*+1}{2} \tag{4.47}$$

w_n^* can now be found by setting (4.47) equal to (4.41). w_{n+1}^{n+1} can be found from (4.27) by incrementing n by 1.

$$w_n^* + w_{n+1}^{n+1} + (N-n-1)\frac{1+w_n^*}{2} = \frac{w_{n-1}^* + w_n^*}{2} + (N-n)\frac{1+w_n^*}{2} \tag{4.48a}$$

$$w_n^* + \frac{w_n^* + w_{n+1}^*}{2} - \frac{1+w_n^*}{2} = \frac{w_{n-1}^* + w_n^*}{2} \tag{4.48b}$$

$$w_{n-1}^* = w_n^* + w_{n+1}^* - 1 \tag{4.48c}$$

This must hold true for any N-period game including the 3-period game which revealed that the wages offered in the final two periods are equal. Setting $n = N-1$ and $w_{N-1}^* = w_N^*$ in (4.48c) gives

$$w_{N-2}^* = 2w_N^* - 1 \tag{4.49}$$

With repeated substitution into (4.48c) a pattern emerges:

$$w_{N-2}^* = 2w_N^* - 1 \tag{4.50a}$$

$$w_{N-3}^* = 3w_N^* - 2 \tag{4.50b}$$

$$w_{N-4}^* = 5w_N^* - 4 \tag{4.50c}$$

$$w_{N-5}^* = 8w_N^* - 7 \tag{4.50d}$$

$$w_{N-6}^* = 13w_N^* - 12 \tag{4.50e}$$

$$w_{N-m}^* = \text{fib}(m+1)(w_N^* - 1) + 1 \tag{4.50f}$$

In the first period there is nothing revealed about the worker's type since all potential employers offer the same zero profit wage. So for the sake of calculating subsequent

wages it can be said that $w_1^* = 0$, even though this is not in fact the wage that will retain the worker in period 1. Setting $m = N - 1$ and $w_1^* = 0$, (4.50f) can then be rearranged to give

$$w_N^* = 1 - \frac{1}{\text{fib}(N)} \tag{4.51}$$

Substituting this back into (4.50f) with $m = N - n$ gives

$$w_n^* = 1 - \frac{\text{fib}(N - n + 1)}{\text{fib}(N)} \tag{4.52}$$

D Table of N-Period Wage Offers

w_n^* n	N: 3	4	5	6	7	8	9	10	11	12	13	14	15	fib(n)
2	0.500	0.333	0.400	0.375	0.385	0.381	0.382	0.382	0.382	0.382	0.382	0.382	0.382	1
3	0.500	0.667	0.600	0.625	0.615	0.619	0.618	0.618	0.618	0.618	0.618	0.618	0.618	2
4		0.667	0.800	0.750	0.769	0.762	0.765	0.764	0.764	0.764	0.764	0.764	0.764	3
5			0.800	0.875	0.846	0.857	0.853	0.855	0.854	0.854	0.854	0.854	0.854	5
6				0.875	0.923	0.905	0.912	0.909	0.910	0.910	0.910	0.910	0.910	8
7					0.923	0.952	0.941	0.945	0.944	0.944	0.944	0.944	0.944	13
8						0.952	0.971	0.964	0.966	0.965	0.966	0.966	0.966	21
9							0.971	0.982	0.978	0.979	0.979	0.979	0.979	34
10								0.982	0.989	0.986	0.987	0.987	0.987	55
11									0.989	0.993	0.991	0.992	0.992	89
12										0.993	0.996	0.995	0.995	144
13											0.996	0.997	0.997	233
14												0.997	0.998	377
15													0.998	610

Table 4.1: First employer wage offers w_n^* for various values of N.

Chapter 5

Regret Theory and the Tyranny of Choice

5.1 Introduction

Most people, particularly economists, would accept the proposition that more choice can never make the decision maker worse off. Yet evidence from the psychology and economics literature suggests that this may not always be the case. It would seem, in fact, a more accurate proposition would be "*more choice is sometimes better but sometimes worse*". If too much choice can be damaging, greater paternalistic regulation of the form proposed by Camerer et al. (2003) is justifiable. For instance, Loewenstein (1999) asserts that consumers should not be offered the full range of social security products. Or perhaps, as Mirrlees (1987) argues, there are situations where regulation should be so strict that people have no choice whatsoever.

The empirical evidence for this phenomenon is persuasive. Iyengar and Lepper (2000) demonstrate experimentally that consumers are more likely to purchase exotic jams or chocolates when 6 options were available rather than 24 or 30 respectively. Similarly, students were more inclined to write an optional essay when they had to select from 6 possible topics over 24. Experiments by Tversky and Shafir (1992) found that decisions were delayed or deferred altogether when additional options were added to the choice set. For instance, 66% of one group of subjects would buy

a discounted CD player at a one-day clearance sale, but a different group faced with the choice of two different players were less likely to purchase at all: only 54% would purchase now, the remainder would seek more information. Similarly, 75% of one group of subjects chose to exchange cash winnings for a more valuable pen, yet only 47% of another group accepted the exchange when an inferior pen was offered as a third alternative. Tversky and Shafir (1992) report that even renowned economist Thomas Schelling is subject to this effect; his desire to purchase an encyclopedia was thwarted when he found two brands in the shop. While either one would have been satisfactory, he was unable to choose and came away empty-handed.

There are several theoretical explanations of why reducing choice might improve welfare. Firstly, if agents have self-control problems, more choice provides additional opportunity for self-damaging behaviour: the snooze button on an alarm clock makes the procrastinator late for work. Gul and Pesendorfer (2001) show that agents with self-control problems can be made worse off by the mere existence of a tempting option, even if that temptation is resisted. Agents with self-control problems may want to limit their future choice set to avoid temptation.

Secondly, if agents make decision-making errors, more choice can reduce welfare. Mirrlees (1987) demonstrates that an omniscient government should eliminate choice altogether unless individuals have a "rather high degree" of rationality. However, the model employs the "strict utility model" of Luce (1959), to which there are at least two objections. Firstly, over time an agent with this utility function should be able to deduce her true preferences and then improve her choices. Secondly, Debreu (1960) showed that the strict utility model yields some odd predictions. For instance, adding a second item identical or similar to the first increases the probability that the first option is chosen. Becker et al. (1963) found that such behaviour is not observed experimentally.

Bounded rationality *per se* does not provide an explanation for why too much choice can be damaging. For instance, Schwartz et al. (2002) argue that a satisficing

agent (as described by Simon (1955) and Simon (1959)) may have immunity to the negative effect of an enlarged choice set. When agents seek only an option that is "good enough", adding further options does not decrease utility.

It is argued that a more convincing theoretical foundation for the observed behaviour is to be found in regret theory proposed simultaneously by Bell (1982), Fishburn (1982) and Loomes and Sugden (1982). Regret theory supposes that agents desire to avoid consequences in which they appear *ex post* to have made the wrong decision, even if the decision was expected utility maximising given the information available *ex ante*. While regret theory is a non-transitive and therefore non-conventional preference model, it has strong empirical evidence in its support. It provides an explanation for the Allais (1953) paradox, the coexistence of insurance and gambling, and loss aversion identified by Kahneman and Tversky (1979). Loomes et al. (1989, 1991) show that it can also account for systematic violations of transitivity. Furthermore, Loomes et al. (1992) demonstrate that although regret theory, specifically "regret aversion", implies systematic violations of monotonicity, experimental evidence suggests that monotonicity is indeed violated in the manner predicted.

Regret theory has seen some interesting applications, ranging from the Cubitt and Sugden (1998) model of preference evolution to the work by Smith (1996) on the valuation of different health states. However, most applications have not presented formal economic models. Starmer (2000) concludes that

> there is a sense in which the formal models are lagging behind the less formal discussions. There is considerable evidence that considerations of regret do influence ordinary people in important decisions. At the same time, it seems clear that our existing models do not provide good formalization of such processes.

This chapter aims to employ regret theory to provide a formal theoretical underpinning for the result that more choice can decrease welfare. Sections 5.3-5.5 of the chapter develop formal search models where regret at failing to choose the best option enters the utility function. In Section 5.3 it is demonstrated that in a non-

sequential search model where the agent specifies in advance how many options will be searched, the optimal amount of search is concave and eventually decreasing in n. In Section 5.4 equivalent results are found when the agent searches sequentially, choosing at each round whether to stop or continue searching. The model assumes that options that are passed over can be returned to. Section 5.5 considers the alternate situation where options that are passed over cannot be returned to. Section 5.6 summarises the results of the chapter, the most important of these being that when agents suffer regret from counterfactuals, too much choice can reduce expected utility and lead to an inefficient amount of search.

5.2 Preliminaries

An agent chooses one of $n \in \mathbb{N}$ options, where option $i \in \mathbb{N}$ provides payoff $x_i \in \mathbb{R}$. Each payoff x_i is a realisation of an *iid* random variable X. The distribution of X is known by the agent to be unit rectangular, so that $x_i \in [0, 1]$, with density function $g(x_i) = 1$ and cumulative density function $G(x_i) = x_i$.

The agent can incur a utility cost c to reveal the utility payoff x_i of a particular option. This cost incorporates the time and effort required to both 1) procure the relevant information and 2) calculate the expected utility the option will yield based on personal preferences. It is assumed that the agent cannot accept an option without searching it first. This is often an appropriate assumption in real world settings, where "search" may be necessary to confirm that the item under examination is an option at all. Indeed, in some cases, a highly detailed "search" is a legal requirement before the option can be accepted, such as "due diligence" required by law before a corporate acquisition.

After x_i is revealed, the agent may accept option i, or any previously searched option, or invest in further search at cost c. In standard search terminology, this is a "search with recall" model.[1] In the event that the expected utility of continuing

[1] See, for example, McCall (1970).

search is the same as the utility of stopping it is assumed that agent will stop.

The model of this chapter differs from the standard search model because the agent's utility function includes a component for regret. Define

$$m(a,b) = \text{argmax}_i(x_a, x_{a+1}, \ldots, x_b) \tag{5.1}$$

as a function returning the index of the option with the highest payoff in the options indexed a though to b of the choice set. Where $a = 1$ the parameter will be dropped from the notation for simplicity, so $m(b)$ would signify the index of the highest option from the first b searches. Further define k as the index of the last option searched where $k \in 1, 2, \ldots, n$. Once the agent stops searching at option k, payoff $x_{m(k)}$ is received, but the agent forms an expectation of the payoff $x_{m(n)}$ that would have been received if all of the remaining options $x_{k+1}, \ldots, x_n$ had been searched. This expected maximum will be denoted $\mathrm{E}_k x_{m(n)}$ where $\mathrm{E}_k \equiv \mathrm{E}[\cdot \mid I_k]$ and where I_k is the information set available after draw k. The agent experiences regret equal to $\rho(\mathrm{E}_k x_{m(n)} - x_{m(k)})$, where $\rho \in [0, \infty)$ is a parameter representing the intensity of regret. It will be assumed that this regret is only experienced if the agent undertakes some search.

The agent's utility from stopping after searching k of the n options is therefore:

$$u(k,n) = x_{m(k)} - \rho(\mathrm{E}_k x_{m(n)} - x_{m(k)}) - kc \tag{5.2}$$

The term "payoff" will be used when referring to x_i and "utility" to denote the aggregate effect of the payoff received, search cost and regret. Although the notation suggests that the options must be searched in order, all options appear the same *ex ante* so the notation equally reflects the assumption that any option can be searched at any stage. This is important as it would be hard to account for an agent experiencing regret for not achieving payoff $x_{m(n)}$ if there were no way option $m(n)$ could have been selected without significant investment in further search.

Finally, it remains to derive an expression for $\mathrm{E}_k x_{m(j)}$, the maximum payoff from the first j draws as expected at round $k < j$. The first step is to derive the density function $f(\cdot)$ for $x_{m(a,b)}$ from the known density function of a single draw $g(x_i)$. Since each draw is *iid*, only the number of draws $b - a + 1$ affects the density of the maximum payoff, but for the sake of consistency with other parts of the model where $m(a, b)$ is meaningful, the over-specified notation will be carried. The probability density for a single draw x_i is $g(x_i)$ so the probability density for a single draw being x_i *and* the maximum of $b - a + 1$ draws is the product of $g(x_i)$ and the probability that the other $b - a$ draws are less than x_i, $G(x_i)^{b-a}$. The probability density for any of the $b - a + 1$ draws being $x_{m(a,b)}$ and the maximum is therefore

$$f(x_{m(a,b)}) = (b - a + 1)g(x_{m(a,b)})G(x_{m(a,b)})^{b-a} \tag{5.3a}$$

$$= (b - a + 1)x_{m(a,b)}^{b-a} \tag{5.3b}$$

The corresponding cumulative distribution function is given by:

$$F(x_{m(a,b)}) = \int_0^{x_{m(a,b)}} (b - a + 1)x_{m(a,b)}^{b-a}\, dx_{m(b-a)} \tag{5.4a}$$

$$= x_{m(a,b)}^{b-a+1} \tag{5.4b}$$

At period k there are $j - k$ unsearched options in the set $x_1, x_2, \ldots, x_j$ and the density function for the maximum of these $j - k$ searches from (5.3b) is $f\left(x_{m(k+1,j)}\right) = (j - k)x_{m(k+1,j)}^{j-k-1}$. $\mathrm{E}_k x_{m(j)}$ can now be found by integrating across the possible values of $x_{m(k+1,j)}$.

$$\mathrm{E}_k x_{m(j)} = \int_0^1 \max[x_{m(k)}, x_{m(k+1,j)}] f\left(x_{m(k+1,j)}\right)\, dx_{m(k+1,j)} \tag{5.5a}$$

$$= \int_0^{x_{m(k)}} x_{m(k)}(j - k)x_{m(k+1,j)}^{j-k-1}\, dx_{m(k+1,j)} \quad + \int_{x_{m(k)}}^1 x_{m(k+1,j)}(j - k)x_{m(k+1,j)}^{j-k-1}\, dx_{m(k+1,j)} \tag{5.5b}$$

$$= \frac{j - k + x_{m(k)}^{j-k+1}}{j - k + 1} \tag{5.5c}$$

At the start of the game, $k = 0$ and $x_{m(0)} = 0$ so the expected maximum of the first j draws is

$$\mathrm{E}_0 x_{m(j)} = \frac{j}{j+1} \tag{5.6}$$

5.3 Non-Sequential Search

Suppose the agent decides in advance how many options she will search. This non-sequential search model provides an elegant solution that has many of the characteristics of the full sequential search model examined in Section 5.4. Equation (5.2) describes the agent's utility $u(k, n)$ and $E_0 u(k, n)$ can be found by solving substituting values for $E_0 x_{m(n)}$ and $E_0 x_{m(j)}$ from equation (5.6). Then

$$\mathrm{E}_0 u(k, n) = \frac{k}{k+1} - kc - \rho\left(\frac{n}{n+1} - \frac{k}{k+1}\right) \tag{5.7}$$

If the agent searches $k + 1$ rounds instead of k rounds, the difference in expected utility is:

$$\mathrm{E}_0 u(k+1, n) - \mathrm{E}_0 u(k, n) = \frac{(1+\rho)}{(k+1)(k+2)} - c \tag{5.8}$$

Setting equation (5.8) to zero yields the value of $k \in \mathbb{R}$ beyond which a unit of additional search reduces utility.[2] The optimal rounds of search, k^*, is therefore the first integer beyond this level of k. This can be found by applying the ceiling function and noting that k is bounded above by n:

$$k^*(\rho, c) = \min\left(\left\lceil -\frac{3}{2} + \sqrt{\frac{1}{4} + \frac{(1+\rho)}{c}} \right\rceil, n\right) \tag{5.9}$$

[2]Note that the second order condition is satisfied by the concavity of $\mathrm{E}_0 u(k, n)$ in k, which follows from the fact that the difference in (5.8) is decreasing in k for all $k \in \mathbb{N}$. The second root is negative and so can be ignored since $k \in \mathbb{N}$.

where $\lceil x \rceil : \mathbb{R} \to \mathbb{Z}$ denotes the ceiling function that returns the smallest integer $\geq x$.

Equation (5.9) confirms the intuition that the optimal amount of search is increasing in regret intensity ρ and decreasing in search cost c. Indeed, as $c \to \infty$, $k^* \to 0$, and no search is optimal. Equally, as $c \to 0$, all available options will be searched. Setting $\rho = 0$ implies the basic result from the canonical search models such as Stigler (1962). Additionally, as $\frac{\partial^2}{\partial c \partial \rho}(k^*) < 0$, higher regret intensity implies a more rapid increase in search as the costs of search decrease. The central result of this model, however, is contained in Proposition 5.1.

Proposition 5.1. *There exists an optimal amount of choice, given by $n^* = k^*(0, c)$, such that increasing choice beyond this optimum reduces expected utility.*

Proof. *For $k^* < n$, it follows from (5.9) and (5.7) that increasing n reduces utility. For $k^* = n$ the agent searches the entire choice set. If an extra option is added to the choice set and searched,[3] the difference in expected utility is:*

$$\mathrm{E}_0 u(n+1, n+1) - \mathrm{E}_0 u(n, n) = \frac{1}{(n+1)(n+2)} - c \tag{5.10}$$

Solving for the optimal integer n employing the ceiling function gives:

$$n^* = \left\lceil -\frac{3}{2} + \sqrt{\frac{1}{4} + \frac{1}{c}} \right\rceil \tag{5.11}$$

It follows from equations (5.11) and (5.9) that $n^* = k^*(0, c)$. ∎

In other words, beyond a certain point, n^*, more choice reduces the utility of agents who experience regret. Moreover, the optimal amount of choice to provide is that which an agent without regret ($\rho = 0$) would elect to search. This simple result has several applications which will be explored in the conclusion.

[3] It is clear from (5.7) that if the extra option is not searched, then expected utility is reduced by the additional option.

5.4 Sequential Search with Recall

5.4.1 Optimal Search Strategy

The model of Section 5.3 is now extended to relax the assumption that the agent decides in advance how many options will be searched. The agent now decides at each round k whether to stop and accept payoff $x_{m(k)}$ or to continue searching. Under this assumption search models can be further classified as with or without recall. This section considers the case with recall, where options that are passed over can later be returned to if nothing better is found. Section 5.5 considers the case where options passed over cannot be returned to.

Define $r(k,n)$ as the reservation payoff so that search will stop at round k if $x_{m(k)} \geq r(k,n)$. The value of the n-period game at round k as expected at round k is

$$\mathrm{E}_k V(k,n) = \max\left[u(k,n), \mathrm{E}_k V(k+1,n)\right] \tag{5.12}$$

Search will stop whenever $u(k,n) \geq \mathrm{E}_k V(k+1,n)$. Recursively, the value function from $k+1$ searches is:

$$\begin{aligned}\mathrm{E}_k V(k+1,n) = \mathrm{E}_k \max\big[u(k+1,n), \mathrm{E}_{k+1} \max\big[u(k+2,n), \ldots \\ \max\big[u(n-1,n), \mathrm{E}_{n-1} u(n,n)\big] \ldots\big]\big]\end{aligned} \tag{5.13}$$

Suppose that the payoff from stopping immediately with utility $u(k,n)$ exceeds the expected utility from stopping exactly one period later $\mathrm{E}_k u(k+1,n)$. The latter can be found by incrementing the index k by 1 in equation (5.2).

$$\mathrm{E}_k u(k+1,n) = \mathrm{E}_k \left[x_{m(k+1)} - \rho\left(\mathrm{E}_{k+1} x_{m(n)} - x_{m(k+1)}\right) - (k+1)c\right] \tag{5.14a}$$

$$= (1+\rho)\mathrm{E}_k x_{m(k+1)} - \rho \mathrm{E}_k x_{m(n)} - (k+1)c \tag{5.14b}$$

The value of $\mathrm{E}_k x_{m(k+1)}$ can be found from equation (5.5c) as

$$\mathrm{E}_k x_{m(k+1)} = \frac{1 + x_{m(k)}^2}{2} \tag{5.15}$$

So if the utility from stopping immediately exceeds the expected utility from stopping one period later then

$$\begin{aligned} & x_{m(k)} - \rho(\mathrm{E}_k x_{m(n)} - x_{m(k)}) - kc \geq \\ & \qquad (1+\rho)\frac{1 + x_{m(k)}^2}{2} - \rho \mathrm{E}_k x_{m(n)} - (k+1)c & \text{(5.16a)} \\ & \qquad c \geq (1+\rho)\left(\frac{1 + x_{m(k)}^2}{2} - x_{m(k)}\right) & \text{(5.16b)} \end{aligned}$$

The left hand side of (5.16b) is the marginal cost of an extra search and on the right hand side the marginal benefit. The $(1-\rho)$ reflects how an increase in the expected maximum payoff after an extra search has a direct benefit to utility and also decreases the disutility of regret.

Proposition 5.2. *The minimum payoff the agent will be willing to accept in round k of an n round game $r(k,n)$ is constant, and given by*

$$r(k,n) = r = 1 - \sqrt{\frac{2c}{1+\rho}} \tag{5.17}$$

Proof. *Suppose that at period k, $x_{m(k)}$ is such that equation* (5.16b) *is satisfied. Then it must also be the case that* (5.16b) *is satisfied for any subsequent period $j \in \{k+1 \ldots n\}$ because $x_{m(j)} \geq x_{m(k)}$. This implies that $\max\left[u(j,n), \mathrm{E}_j u(j+1,n)\right] = u(j,n)$ which can be employed to simplify* (5.13) *recursively until it becomes:*

$$\mathrm{E}_k V(k+1,n) = \mathrm{E}_k u(k+1,n) \tag{5.18}$$

Setting (5.16b) *to hold with equality and solving for $x_{m(k)}$ yields two roots. Only the lesser of these is in the feasible range $[0,1]$. This root is given by* (5.17). ∎

Because the maximum of the searched options $x_{m(k)}$ is weakly increasing in k, the marginal benefit of search is weakly decreasing. So if it is not worth searching one more round then neither will it be worth searching one more round at any later stage. There is hence no "option value" of continuing to the next round, in the sense of Dixit and Pindyck (1994), and the reservation payoff at round k is that which sets the utility from stopping immediately equal to the expected utility of stopping in the next round.

So the optimal strategy is for the agent simply to search until a payoff greater than or equal to r has been found. The reservation payoff is not a function of k or n, hence under these conditions, maximising behaviour shows the features of satisficing behaviour of a Simon (1955) agent. The reservation payoff is decreasing in search costs c as one would expect. The higher the regret intensity ρ, the higher the reservation payoff as the agent is willing to invest more search in order to decrease the disutility resulting from regret.

For the agent who finds it extremely unsatisfactory to accept a payoff less than the expected maximum, it would be possible to set $\rho > 1$. This would be appropriate for the "perfectionist" – someone whose dissatisfaction from accepting an option that is not the best available is greater even than the difference between the best payoff and actual payoff. As $\rho \to \infty$, equation (5.17) shows that the reservation payoff approaches the maximum, or "perfect" payoff. Hence the probability of searching all options tends to 1 and the value of a game to the perfectionist plummets as n increases. This agrees with the findings of Schwartz et al. (2002) who suggest that happiness comes from accepting what is good enough rather than constantly seeking perfection.

Recognising that under the optimal strategy the probability of continuing in any given round is r, the expected rounds of search from the optimal strategy $\mathrm{E}_0 k$ is

given by

$$\begin{aligned} \mathrm{E}_0 k &= \sum_{j=1}^{n-1} j(1-r)r^{j-1} + nr^{n-1} \\ &= \frac{1-r^n}{1-r} \end{aligned} \tag{5.19}$$

The amount of search is increasing in r for all $n \geq 2$ so by considering equation (5.17) it can be concluded that, like the non-sequential case, the expected amount of search is increasing in ρ and decreasing in c. But unlike the non-sequential case, as n increases the expected amount of search also increases because of the possibility that the entire set be searched without the reservation payoff being met.

5.4.2 The Value of the Game

The implications from the optimal search strategy determined above are illuminated by Figures 5.1 and 5.2, which show simulated results of the value of an n-period game for various values of c and ρ.[4] As was the case in the non-sequential model, the existence of regret means that expected utility is decreasing in n when n is suitably large: providing agents with too much choice reduces welfare. The optimal size of the search set is when the marginal cost of increased regret induced by an extra option, if it is left unsearched, equals the marginal benefit of the extra option if all options are searched without r being met. The probability of r not being met in n rounds is r^n so as n becomes large the chances of all search options being utilised very rapidly approaches zero and the marginal cost dominates.

A higher value of ρ obviously decreases the value of any given game with more than one round (since regret enters the utility function negatively) and also reduces the optimal number of search options that a benevolent planner would provide (as indicated by Figure 5.2). A higher value of c also decreases the value of any given game and, beyond the optimal n, causes expected utility to decrease more rapidly, in the same way as ρ. The reason for this is that as c increases, the expected number

[4]The figures were produced in Microsoft Excel, where a nested function was employed to determine the expected value $V(0, n)$.

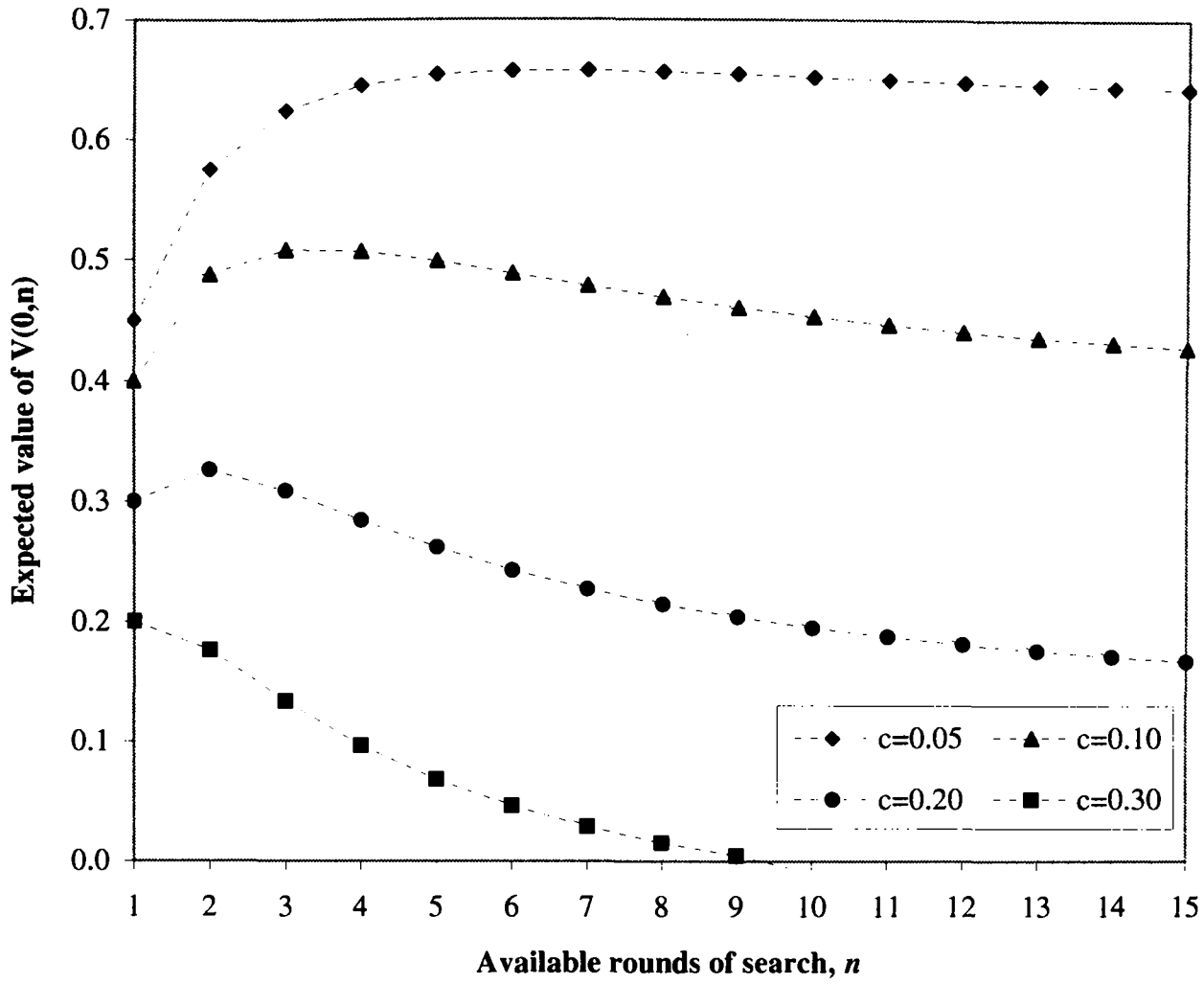


Figure 5.1: The expected value of the game as a function of n for various values of c. ($\rho = 0.5$).

of rounds of search decreases (by equation (5.19), as r is a function of c by equation (5.17)) and the larger number of options that are left unsearched increases regret. So there is an interaction effect between c and ρ. If either ρ or c are zero then the value of the game is non-decreasing in n. With $c = 0$ all options can be searched so there is no regret, and with $\rho = 0$ there may be unsearched options but they cause no regret.

Figures 5.1 and 5.2 also highlight an interesting feature of the model – the value of the game can be negative if $\rho > 0$ and n is sufficiently large. Recall that it is assumed that if $\mathrm{E}_0 V(0, n) < 0$ for any n, the agent can elect not to play the game and receive a zero payoff. Were this not the case, the mere existence of games never played would inflict disutility on a regretful agent even if the optimal strategy is adopted. This is clearly unrealistic, so it is assumed instead that the agent can

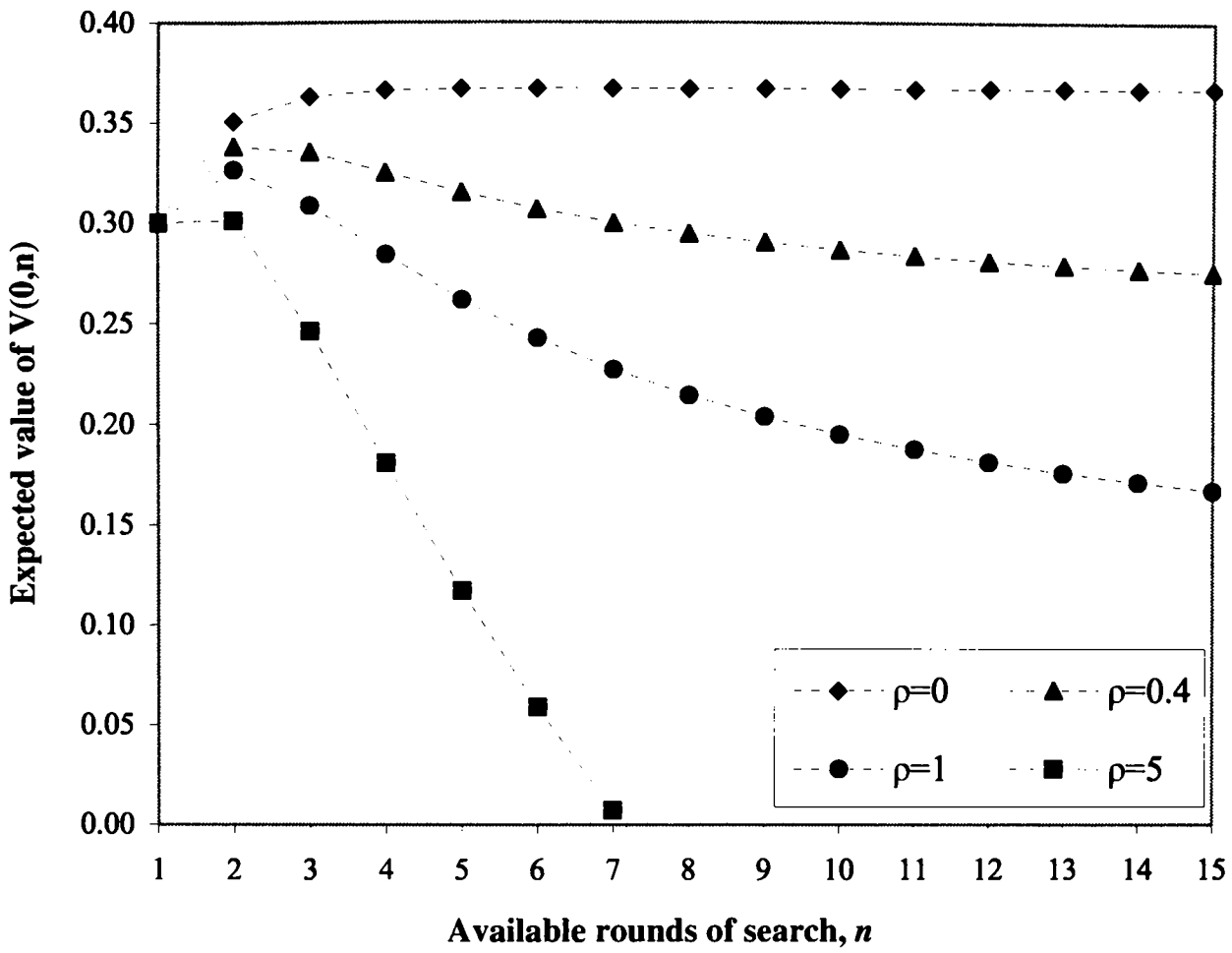


Figure 5.2: The expected value of the game as a function of n for various values of ρ. ($c = 0.2$).

"walk away" with reservation payoff of zero. Note that the value of the game can be negative even when the optimal strategy is to perform some search (that is, $r > 0$) if the agent experiences regret.

Although the general expression for the value of the game is complicated, there are two values of n for which the value of the game can easily be found. When $n = 1$, performing a single search will mean that no regret is experienced. The expected payoff of a single option is simply $\int_0^1 f(x_1)\,dx_1 = \frac{1}{2}$ so the value of an $n = 1$ game in which search is undertaken is, from (5.2), simply

$$\mathrm{E}_0 V(0,1) = \frac{1}{2} - c \tag{5.20}$$

A sufficient condition for search being undertaken where $n = 1$ is given by setting

(5.20) greater than zero, or

$$c < \frac{1}{2} \tag{5.21}$$

Because of the assumption that the agent will elect not to play the game if the expected utility is negative, this search condition $c < \frac{1}{2}$ applies for all ρ.[5]

The other special case is where $n \to \infty$. For any $c > 0$, the reservation payoff is $r < 1$ and as there are unlimited options this payoff will certainly be attained. The expected payoff x_k conditional on $x_k > r$ is simply $\int_r^1 f(x_k)\, dx_k = \frac{1+r}{2}$. The number of unsearched options is $n - k \to \infty$ so the expected maximum from (5.5c) tends to 1. The expected rounds of search from (5.19) is $\frac{1}{1-r}$ so the value of the game where search is undertaken and $n \to \infty$ is, from (5.2)

$$\mathrm{E}_0 V(0, n \to \infty) = \frac{(1+\rho)(1+r)}{2} - \rho - \frac{c}{1-r} \tag{5.22a}$$

$$= 1 - \sqrt{2c(1+\rho)} \tag{5.22b}$$

A sufficient condition for search where $n \to \infty$ is therefore given by setting (5.22b) greater than zero, or

$$c < \frac{1}{2(1+\rho)} \tag{5.23}$$

Putting together the results in equations (5.21) and (5.23) shows that it is possible for the agent to engage in search if there is only one option, yet avoid search altogether if there are infinite options. This holds whenever

$$\frac{1}{2(1+\rho)} < c \leq \frac{1}{2} \tag{5.24}$$

This leads to the second important result of this section.

Proposition 5.3. *For sequential search with recall, if the value of the game is*

[5] It would be inaccurate to rearrange (5.17), observe that the reservation payoff $r > 0$ whenever $c < \frac{1+\rho}{2}$, and then infer that a regretter may be willing to search for some $c > \frac{1}{2}$. This result appears because the utility of leaving the one option unsearched is negative; as the agent can accept a zero payoff this situation never applies.

concave in n, that is $[V(0,n+1)-V(0,n)]-[V(0,n)-V(0,n-1)]<0$, *then for* $\rho>0$ *and* $\frac{1}{2(1+\rho)}\leq c<\frac{1}{2}$, *the agent will only play the game (that is, invest in search and choose one of the available options) if n is sufficiently small.*

Proof. *Search will be undertaken when* $n=1$ *as* $c<\frac{1}{2}$ *from* (5.21) *and search will not be undertaken when* $n\to\infty$ *as* $2c<\frac{1}{1+\rho}$ *from* (5.23). *Assuming the value of the game is concave in n, it follows that there exists some critical number of options $n=m$ below which the agent will undertake search, and above which the agent will refuse to play the game.* ∎

So while regret may cause increased search relative to the non-regretter, in other cases it causes the regretter to avoid playing the game at all. This result explains the empirical observations of Iyengar and Lepper (2000) and Tversky and Shafir (1992) and captures the essence of the Schelling dictionary story described in Section 5.1. Had only one dictionary been available, it would have been purchased, but because two were available, no purchase was made, even though either would have been adequate. The model of this section has shown that the existence of regret can explain this seemingly irrational behaviour. The satisficing behaviour of the agent is supported by the empirical findings of Gigerenzer (2003) and Newell et al. (2003). They review a variety of different search rules and find that employing a stopping rule such as satisficing is the most commonly adopted approach to search problems of the nature considered in this section.

5.5 Sequential Search with No Recall

Suppose now that it is not possible for the agent to return to options that are passed over. At each round k, the agent must therefore decide whether to accept x_k (as opposed to $x_{m(k)}$ in the previous section) or continue search. It may appear at first that this new assumption will make no difference to the optimal search found for the model with recall of Section 5.5. There, search continued only until

$x_k > r(k,n)$ so the accepted option was always the most recently searched option anyway and the ability to recall an earlier choice was never exercised. The value of $r(k,n)$ however does depend on the recall assumption. The proof of Proposition 5.2 drew on the fact that $x_{m(k)}$ is non-decreasing, but the same cannot be said of x_k. With no recall, it is intuitively clear that searching the very last option is a risky situation to be in since the payoff will have to be accepted, even if it is very low. Therefore the reservation payoff in the second-to-last round should be low to reflect the undesirability of reaching the final round. Nearer the start of the game the probability of reaching the final round is considerably lower, so a higher reservation payoff would be expected. As such, it is observed that $r(k,n)$ is decreasing in n. It is worth noting however that as $n \to \infty$, $r(k,n)$ approaches the constant reservation payoff r.

However the objective of this section is not to develop another model where the only new assumption is related to recall. There is already a large body of literature that examines search both with and without recall. The aim of this chapter is to examine the effect of regret in a search model, with particular focus on how regret affects utility and the amount of search that will be carried out. Search without recall is interesting because it permits a distinction between "forwards" and "backwards" regret. Suppose that $r(k,n)$ is decreasing in k, and in round 1 the agent passes over an option because the payoff is just slightly below $r(1,n)$. Then in subsequent rounds a string of poor payoffs means that search continues until the final round, and that the payoff of that final round is $x_n < x_1$. This introduces the possibility of "backwards regret" over an option passed over. In contrast, "forwards regret" considered above describes the fear that the superior option has not yet been searched. Backwards regret is obviously not possible in the earlier model with recall because the agent could always return to accept the highest option searched so far.

The hypothesis of this section is that this backward-looking regret has a very different effect on search strategy than the forward-looking regret considered so

far. Backward looking regret could also provide an explanation for the empirically verified phenomenon that people have a tendency to refuse to accept sunk costs as sunk (Arkes and Blumer, 1985; Arkes, 1996). As an example, suppose two tour operators are offering similar skiing packages for the same price. After paying a £100 deposit on one of the holidays suppose the other operator introduces a last minute discount of £150. It is optimal to switch operator and treat the £100 as a sunk cost. But the type of person who refuses to accept sunk costs would instead stick with their original booking on the basis that abandoning the deposit would be a "waste" of money. A related phenomenon is where people behave sub-optimally in a forward-looking sense in an attempt to justify earlier decisions that would otherwise look bad. Staw and Ross (1989) cite as an example the United States' escalating commitment to the Vietnam War as an attempt at justifying the irreversible decision to go to war in the first place, even as the situation became hopeless.

Define ϕ as the parameter representing the strength of this backward-looking regret over the difference between the payoff received x_k and the maximum payoff found so far $x_{m(1,k)}$. The utility from stopping search at round k of n is now

$$u(k,n) = x_k - \rho(E_k x_{m(k,n)} - x_k) - \phi(x_{m(1,k)} - x_k) - kc \tag{5.25}$$

Under this more complicated formulation it is not especially interesting (or indeed possible) to re-examine the results of the previous chapters. Rather, a simple 2- and 3-period model will be developed to shed some light specifically on the effect of backward-looking regret.

5.5.1 2-Period Model

A 2-period model is the simplest possible context in which to consider the effect of backward-looking regret. The agent will stop searching in period 1 if the utility from stopping exceeds the utility from continuing. The utility from stopping after

period 1 is

$$u(1,2) = x_1 - \rho(E_1 x_{m(1,2)} - x_1) - \phi(x_{m(1,1)} - x_1) - c \tag{5.26}$$

The backwards regret component $\phi(x_{m(1,1)} - x_1)$ equals zero here because there are no previous payoffs to regret passing over. There is however the forwards regret over the unsearched option. The expected utility from continuing is

$$E_1 u(2,2) = E_1 x_2 - \rho(E_1 x_{m(2,2)} - E_1 x_2) - \phi(E_1 x_{m(1,2)} - E_1 x_2) - 2c \tag{5.27}$$

If the agent continues and searches both options, the forwards regret component $\rho(E_1 x_{m(2,2)} - E_1 x_2)$ is zero. The first period reservation payoff $r_{1,2}$ is the x_1 that leaves the agent indifferent between stopping and continuing. Then

$$x_1 - \rho(E_1 x_{m(1,2)} - x_1) = E_1 x_2 - \phi(E_1 x_{m(1,2)} - E_1 x_2) - c \tag{5.28}$$

$E_1 x_2 = \frac{1}{2}$ and the expected value of $x_{m(1,2)}$ as at period 1 when x_1 is known is given by (5.5c) as $\frac{1+x_1^2}{2}$. So

$$\frac{1}{2}(\rho - \phi)x_1^2 - (1+\rho)x_1 + \frac{1}{2}(1+\rho) - c = 0 \tag{5.29}$$

and

$$r_1 = \frac{(1+\rho) - \sqrt{(1+\rho)(1+\phi) + 2(\rho-\phi)c}}{\rho - \phi} \tag{5.30}$$

after discarding one of the two solutions by imposing the restriction that $r_2 \in [0,1]$. $\frac{dr_1}{d\phi}$ is negative for $c \in [0, \frac{1}{2}]$ which implies an increase in backward-looking regret decreases r_1 and reduces the amount of search. The anticipation of the regret that would be experienced if x_1 is passed over and x_2 turns out to be worse makes the agent more inclined to settle for a lower x_1. This is the exact opposite effect of forward-looking regret where the regret already experienced over the prospect of an

unsearched option being superior increases the amount of search.

5.5.2 3-Period Model

While a 2-period model captures the important result that backwards looking regret operates in the opposite direction to forwards looking regret, there are some interactions between the two types of regret that are not captured. As such, a 3-period search model with no recall is now examined, where the agent experiences both forwards and backwards regret. As before, in period 2, after the payoff of options x_1 and x_2 have been revealed, the agent will stop searching if the utility from stopping exceeds the utility from continuing. Utility from stopping is

$$u(2,3) = x_2 - \rho(E_2 x_{m(2,3)} - x_2) - \phi(x_{m(1,2)} - x_2) - 2c \tag{5.31}$$

The expected utility from continuing is given by

$$E_2 u(3,3) = E_2 x_3 - \phi(E_2 x_{m(1,3)} - E_2 x_3) - 3c \tag{5.32}$$

The reservation payoff r_2 is the payoff x_2 such that the agent is indifferent between stopping and continuing. Equating equations (5.31) and (5.32), and employing equation (5.5c) from which it follows that $E_2 x_{m(1,3)} = \frac{1}{2}(1 + x^2_{m(1,2)})$,

$$x_2 - \rho(\frac{1}{2}(1 + x_2^2) - x_2) - \phi(x_{m(1,2)} - x_2) - 2c = \frac{1}{2} - \frac{1}{2}\phi x^2_{m(1,2)} - 3c \tag{5.33}$$

Simplifying gives

$$\frac{1}{2}\rho x_2^2 - \frac{1}{2}\phi x^2_{m(1,2)} - (1 + \rho + \phi)x_2 + \phi x_{m(1,2)} + \frac{1}{2}(\rho + 1) - c = 0 \tag{5.34}$$

This equation for the reservation payoff in period 2 depends upon $x_{m(1,2)} = \max[x_1, x_2]$. But by period 2 it is known for certain whether $x_1 > x_2$ or vice versa. So each of the two cases will be considered in turn.

Firstly, suppose the current option is best. Then $x_{m(1,2)} = x_2$. Substituting this into equation (5.34) yields

$$\frac{1}{2}(\rho - \phi)x_2^2 - (1 + \rho)x_2 + \frac{1}{2}(\rho + 1) - c = 0 \tag{5.35}$$

from which it follows that that

$$r_2 = \frac{(1 + \rho) - \sqrt{(1 + \rho)(1 + \phi) + 2(\rho - \phi)c}}{\rho - \phi} \tag{5.36}$$

after discarding one of the two solutions by imposing the restriction that $r_2 \in [0, 1]$. Note, of course, that equation (5.36) is identical to equation (5.30), because when the current option is best at the second period in a 3-period model, the agent faces exactly the same decision as after the first period in a 2-period model.

In the second case, $x_{m(1,2)} = x_1$ and the first option was the best. As such, the agent is experiencing backwards regret when the decision is made. Solving equation (5.34) now yields

$$r_2 = \frac{(1 + \rho + \phi) - \sqrt{(1 + \rho + \phi)^2 - \rho(\rho + 1) + 2\rho(c - \phi x_1(1 - \frac{1}{2}x_1))}}{\rho} \tag{5.37}$$

after discarding one of the two solutions by imposing the restriction that $r_2 \in [0, 1]$. Further simplification shows that:

$$r_2 = \frac{(1 + \rho + \phi) - \sqrt{(1 + \phi)(1 + \rho + \phi) + \rho\phi(1 - x_1)^2 + 2\rho c}}{\rho} \tag{5.38}$$

This second case, where forwards and backwards regret are experienced simultaneously, appears to lead to some interesting interaction effects. Further work is required in this area, but as an illustration, Figure 5.3 shows one sample simulation. In this simulation, increasing forwards regret ρ always increases the reservation payoff. In contrast, the effect of increasing backwards regret ϕ depends upon the

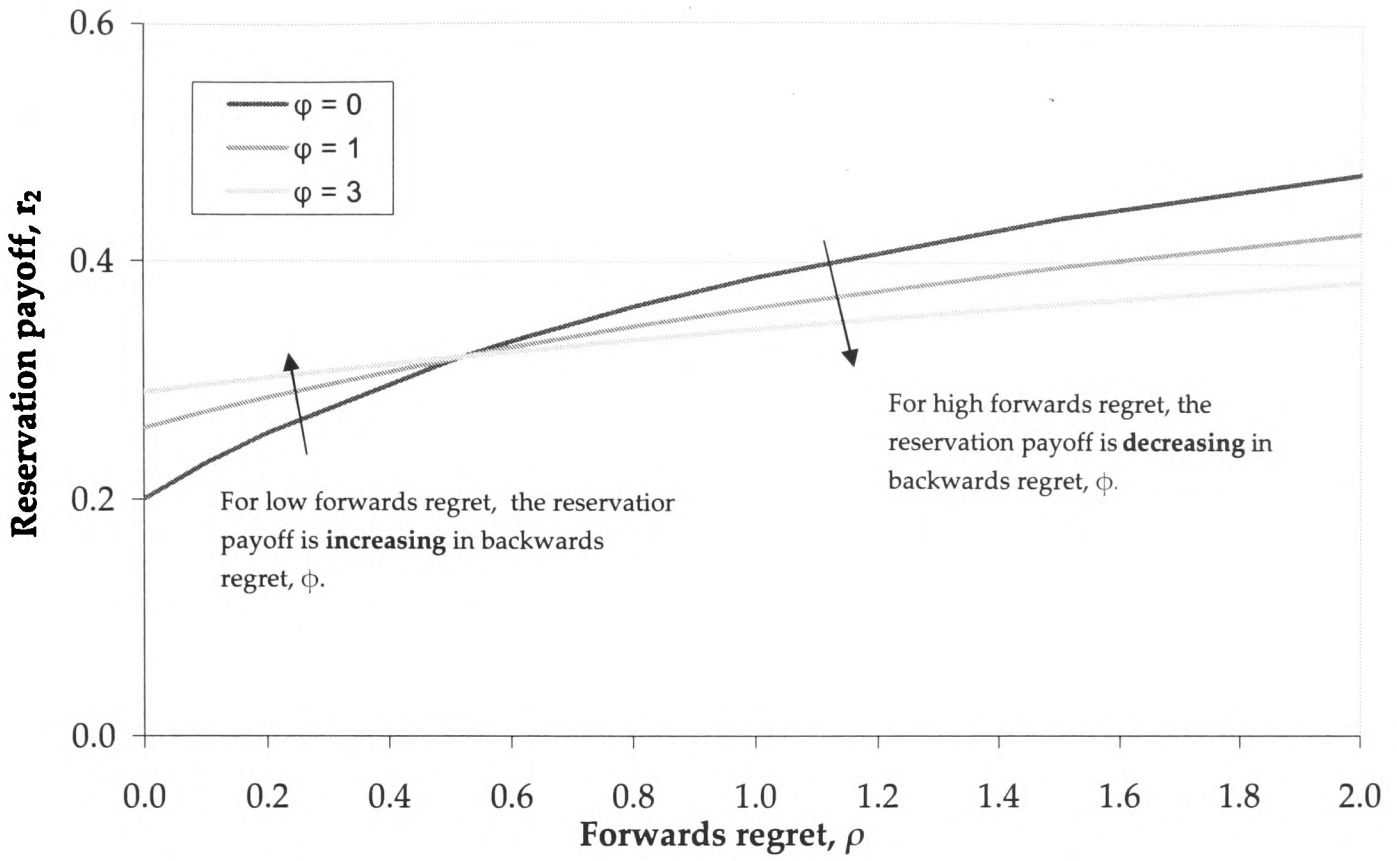


Figure 5.3: The effect of ρ and ϕ on the reservation payoff in the second case where $x_1 > x_2$. The curves were generated using parameters $c = 0.3$, $x_1 = 0.4$ and assuming that $x_2 < x_1$ so that if the agent stops she experiences both backwards and forwards regret.

strength of forwards regret. When forwards regret is strong, results are consistent with those obtained in the 2-period model. That is, increasing backwards regret reduces the reservation payoff, making the agent more likely to stop. Here, backwards regret leads the agent to "quit while they are ahead", rather than risk an even lower payoff in the following period.

However, when forwards regret is weak, the opposite occurs. Increasing backwards regret *increases* the reservation payoff, making the agent more likely to continue searching. Recall that the agent has just the misfortune to draw $x_2 < x_1$ and is already experiencing backwards regret. Here, the presence of this backward regret induces the agent to press on and keep searching, in a manner akin to a gambler who insists on going "double or nothing".

In sum, although rigorous proofs to complement these simulations are required, Figure 5.3 already suggests that full analysis of a 3-period model with no recall will yield some interesting results. Perhaps, for instance, the experimental observation

that many people exhibit risk-loving behaviour over losses might be explained by their efforts to eliminate backwards looking regret.

5.6 Conclusion

The model of this chapter has demonstrated that if agents suffer regret from counterfactuals, too much choice can lower welfare. This has implications for a wide range of policy. For example, consider the deregulation of the social security sector. Should the government permit free choice or place a limit on the number of pension funds offered to the public? Suppose there are n managed funds, where n is large, and that an agent incurs cost c to examine the merits of each fund. An agent who does not experience regret will examine k^* funds, at which point the marginal cost of additional research equals the marginal benefits. In contrast, *homo regreticus* will spend more time examining the options for fear of making a choice that may be sub-optimal in terms of payoff x_i. This additional search may exhaust all available options, so she can "sleep easy" knowing she has made the best possible decision. Alternatively, even after the additional search, some funds may remain unexamined. In this event the agent suffers from the knowledge that one of the unexamined options may turn out to have been better suited to her preferences than her selected option. In both cases, a sub-optimally high quantity of resources has been devoted to searching the different alternatives.

For non-sequential search, optimal policy would be for government to limit the number of approved funds to the level that a non-regretful agent would choose to search. This could be viewed as "asymmetric paternalism", to employ the language of Camerer et al. (2003), insofar as the policy provides net benefits to *homo regreticus* while imposing zero costs on other agents. For sequential search, the number of funds should also be restricted for welfare to be maximised. In this case, however, doing so will disadvantage non-regretful agents in the instance that they search all options and do not find one exceeding their reservation payoff. Nevertheless, sensible

government policy is likely to involve some level of protection for regretful agents, even if this comes at the expense of agents who do not experience regret.

The sequential search model has shown that an excess of search options may lead a regretful agent to reject the game outright, performing no search and accepting none of the available options. Under a traditional utility function this would be interpreted as irrational since the excess options could be ignored and search carried out on a subset of available options. It has been proven that if the agent experiences regret over options unsearched, rejecting the game under the enlarged choice set could, in fact, be rational.

Finally, the model of sequential search with no recall has shown that backward-looking regret could decrease the amount of search – the exact opposite effect of forward-looking regret. In a 3-period model backward-looking regret can explain the phenomenon of *ex post* rationalisation, the optimal strategy being a function of irreversible decisions.

A possible extension to the sequential search model with recall would be to make c endogenous. This could capture further empirical observations relating to search. In the "search with recall" model, it has been assumed that only the highest previous option need be remembered as search proceeds. This is realistic if the axiom of the irrelevance of independent alternatives holds, but there are many cases where it may not. Hsee et al. (1999) for example describe how separate evaluation of alternatives can give different ranking than joint evaluation. Participants were asked to value two dictionaries, one of 20000 words with a ripped cover and another of 10000 words in perfect condition. The latter was preferred in separate evaluation, but the former when both were considered at once. Such a concept could be modelled by allowing for the possibility that $m(2) = 2$ and $m(3) = 1$. As a result, *all* previously searched options would have to be revisited every time a new option is revealed. This increasing complexity of search could be represented by a search cost which increases with k. Thus something that was rejected in round 1 may be accepted in

round 5 because the individual is more eager to end the ever complicating search process.

Beyond the implications for policy, the model of this chapter may also shed light on seemingly paradoxical phenomena frequently observed in this modern age, such as those described by Schwartz (2000). As the internet or the hypermart "empower" individuals in their ability to shop beyond the corner store, this frequently leads to angst and frustration rather than the consumer euphoria once predicted. Similarly in job search, members of the so-called "Options Generation" — those young, well-educated, and determined — could fairly describe the world as their oyster. And yet this plethora of options appears to result in great distress. How can they possibly make the best decision when faced with so many uncertain options? Strangely, it is those people who have managed to narrow down their choice set, or who have had it narrowed down for them, who are more content.

Chapter 6

Conclusion

The first three substantive chapters of this thesis examined in detail the issue of adverse selection in the labour market and, in particular, the significance of the assumption that an employer may remember the type of a worker even after she quits. To the knowledge of the author, this assumption has never been considered in this context before, so the results of each of these chapters make a novel contribution to that literature.

Chapter 2 has shown that if productivity is constant and known to the worker, the existence of firm memory means that adverse selection will never persist, even where job changing frictions exist. This completely contradicts the widely cited result of Greenwald (1986) and shows that a large body of the labour market literature has been unwittingly built on an assumption that there is no reason to believe is reasonable. The model shows that switching costs may result in a semi-separating equilibrium where the more able workers receive separating wage offers and the less able receive pooled offers, but not as a result of adverse selection. So while adverse selection never persists if firms remember the type of a worker after she quits, asymmetric information may persist if job changing frictions are sufficiently large.

Chapter 3 relaxed the assumption of Chapter 2 that the worker's productivity

is constant. In so doing it was possible to examine the robustness of the results of Chapter 2 to any factor that may cause previous employers to have differing information about the worker's type. It was found that the non-persistence of adverse selection result is robust only if firms bidding to employ the worker are unable to update their strategies in response to the bids of other firms. Under an ascending-price auction, wage offers are pooling and adverse selection persists. Under a first-price auction the results of Chapter 2 are unaffected by the assumption that previous employers could be asymmetrically informed. The results of Chapter 2 are also unaffected by an open bidding process where productivity is constant.

Chapter 4 considered the possibility that workers may not know their own productivity. Under this assumption it was shown that endogenous turnover occurs and job changers are adversely comprised of the less able. This is consistent with Greenwald (1986) but is a stronger result because it does not depend on the worker's assumed exogenous propensity to quit. The result also shows that adverse selection may persist in some cases even if firms remember the type of a worker after she quits. On the issue of the persistence of the winner's curse, the results of the model of Chapter 4 are broadly consistent with the findings of Chapter 2. Less able workers have their type revealed and so the winner's curse does not persist. More able workers stay with the first employer on a pooling wage, and although the winner's curse persists, its severity diminishes over time.

The general conclusion that can be drawn from these three chapters is that adverse selection may not be as widespread or as severe as much of the labour literature would lead us to believe. The consequences of asymmetric information in markets — labour or otherwise — where the good is traded multiple times can be lessened by the possibility of a competitive market being formed by previous buyers with complete information.

Finally, in the field of behavioural economics, Chapter 5 has contributed to the literature the first theoretical attempt to explain the widely supported empirical

finding that there can be such a thing as too much choice. The model showed that if agents experience regret over forgone options and search is costly, expected utility is single-peaked in the size of the search set. In addition, the model gives a theoretical foundation to the concept of decision paralysis, where too much choice can lead to no decision being made at all.

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