

Stacking Revenues from Flexible DERs in Multi-Scale Markets using Tri-Level Optimization

Ángel Paredes , *Student, IEEE*, José A. Aguado  *Member, IEEE*, Chaimaa Essayeh ,
Yuxin Xia, *Student, IEEE*, Iacopo Savelli , Thomas Morstyn , *Senior, IEEE*,

Abstract—Rapid proliferation of flexible Distributed Energy Resources (DERs) as a result of Net Zero Emissions objectives entails a profound shift in the paradigm of local and national energy systems. Currently, DERs’ simultaneous participation in multiple markets is generally restricted, which undermines their profitability. With the aim of increasing the number of business cases for them, a tri-level optimization problem that seeks the maximisation of revenues from DERs is proposed. The optimization problem considers simultaneous participation of different flexible DERs, such as, Electric Vehicles (EVs), Battery Energy Storage Systems (BESSs) and Heating, Ventilation and Air Conditioning (HVACs), in national and local markets. Markets are cleared sequentially, and the model is recast into a tractable single-level problem using its dual formulation and strong duality condition. Results from a case study based on the IEEE 14 bus transmission network, a realistic distribution network and SimBench dataset demonstrate the effectiveness of the proposed approach in increasing profits compared with a baseline scenario.

Index Terms—Distributed Energy Resources, Duality, Local Markets, Profit Maximisation, Sequential Markets, Stacking Revenues, Tri-Level Optimization.

NOMENCLATURE

Parameters are in upper case letter and variables in lower case letter. $|\Omega|$ denotes the cardinality of the set Ω . Vector and matrices are denoted by lower and upper case **bold** letters.

Acronyms

BESS	Battery Energy Storage System.
DAM	Day-Ahead Market.
DER	Distributed Energy Resource.
DSO	Distribution System Operator.
EV	Electric Vehicle.
FG	Flexible Generator.
FL	Flexible Load.
FSP	Flexibility Service Provider.
HVAC	Heating, Ventilation and Air Conditioning.
LEM	Local Energy Market.
LFM	Local Flexibility Market.
LMO	Local Market Operator.

MO	Market Operator.
RM	Reserve Market.
SOC	State of Charge.
TSO	Transmission System Operator.

Indices and sets

a, Ω_a	Index and Set for agent a , $a \in \Omega_a$.
f, Ω_f	Index and Set for FLs, $f \in \Omega_f$.
g, Ω_g	Index and Set for FGs, $g \in \Omega_g$.
s, Ω_s	Index and Set for BESSs, $s \in \Omega_s$.
b, Ω_b	Index and Set for HVACs $b \in \Omega_b$.
t, Ω_t	Index and Set for time periods, $t \in \Omega_t$.
i, Ω_n	Index and Set for network nodes, $i \in \Omega_n$.
i, j, Ω_l	Indices and Set for network branches, $(i, j) \in \Omega_l$.

Parameters

$G_{i,j}, B_{i,j}$	Conductance and susceptance of the line (i, j) (S).
$\pi_{a,t}^p$	Price of product p of agent a in time period t (€/kW, €/kWh).
η_s^C, η_s^D	Charging (C) and discharging (D) efficiencies for BESS s .
$P_{a,t}^{ref}$	Baseline power of agent a in time period t (kW).
$\underline{P}_a, \overline{P}_a$	Lower and upper power bounds of agent a (kW).
$\underline{SOC}_s, \overline{SOC}_s$	Lower and upper State of Charge bounds of BESS s (kWh).
P_s^{conv}	Power converter rating of BESS s (kW).
$\overline{S}_{i,j}$	Thermal limit of branch (i, j) (kVA).
$\tau_{b,t}^{out}$	Outdoors <i>out</i> temperature of the building b in time period t (°C).
C_b, R_b	Thermal constants of the building b .
R_t^u, R_t^d	Upward u and downward d reserves in time period t (kW).
$\overline{P}_b^{he}, \overline{P}_b^{co}$	Heating he and cooling co rating of HVAC system b (kW).
η_b^{he}, η_b^{co}	Heating he and cooling co efficiencies of HVAC system b .
A_m, B_m	Matrix of coefficients of market m .
b_m	Vector of independent terms of market m .

Variables

$\omega_{a,t}^p$	Energy product p from agent a at time period t (kWh).
$\nu_{a,t}^p$	Capacity product p from agent a at time period t (kW).
$soc_{a,t}$	SOC of agent a in time period t (kWh).
$\tau_{b,t}$	Temperature of building b in time period t (°C).

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- $p_{i,j,t}, q_{i,j,t}$ Active and reactive power flow through line (i, j) at time period t (kW, kVAR).
- $v_{i,t}, \theta_{i,t}$ Voltage magnitude and phase angle of node i at time period t (V, rad).
- \mathbf{x}_m General vector of variables for market m .
- λ, μ Dual variables associated to equality and inequality constraints (€/kW, €/kWh).

I. INTRODUCTION

THE ever-increasing penetration of Distributed Energy Resources (DERs), the decarbonisation, sustainability and electrification of mobility and heating are motivating a profound change in the power landscape [1]. The proliferation of clean DERs is mainly driven by the Net Zero emissions objectives, which increases the penetration of technologies such as household and factory energy arbitrage systems, utility scale batteries, Electric Vehicles (EVs) and PV systems [2]. Distribution systems will be characterized by a strong renewable and uncontrollable foundation, which eventually would lead load to follow generation. [3] In this context, Distribution System Operators (DSOs) will face new challenges concerning those new consumption and generation patterns. Flexibility and Local Energy Markets (LEMs) are key elements of modern renewable energy systems that enable DSOs to deal with this change of paradigm [4]. Timescale is of paramount importance as generation and demands must remain in equilibrium at every instant. Moreover, given the intrinsic uncertainty associated to renewable generation and its hurried deployment, the risk of contingencies increases [5]. In this context, energy and flexibility markets offer resilient and cost-effective solutions at a national and local level to system operators, which previously could only resort to grid reinforcements to meet these challenges. Extensive research has been conducted regarding the design of these short-term Local Flexibility Markets (LFMs) [6], and numerous innovative industry-lead projects have been developed to test them [7]. Moreover, system operators could capitalise on the particular suitability of DERs for providing multiple types of services simultaneously at national and local scale [8]. Despite of this, individual DERs are limited in size to directly participate in national markets [9]. Then, although aggregation allows their participation in individual markets at a national scale, the determination of the optimal stacking of revenue across multiple scales remains a major challenge.

Concurrent participation in several markets enables adding worth to management techniques of DERs by stacking revenues. However, being too small, their participation is limited when they are operated independently. Flexibility Service Provider (FSP) figure naturally arises as a market facilitator for DERs aiming to maximise their profitability. Thus, there is a desire of improving the business cases for those resources as they provide low-cost solutions and promotes energy independent societies while accelerating transition to sustainable energy systems [10]. In this context, there is a strong motivation to study a business model for DERs with revenue stacking from the provision of flexibility services in multiple markets.

Various methodologies have been studied to manage DERs. Markov Decision Process was used in [11] to stack revenues

from energy arbitrage and frequency regulation in PV-Battery Energy Storage System (BESS) systems. Deep-Learning was used in [12] to accelerate the solution of the energy management problem of a community of DERs under uncertainty. Those techniques only address the problem of DERs providing transmission or local services, but not both at the same time.

Optimization techniques were also used to manage simple business cases. Scalar indices are used in [13] and [14] to manage the participation of BESSs in national markets. Authors in [15] use Particle Swarm Optimization to co-optimize BESS size along wind systems to maximise profits. Reference [16] stacks revenues streams for BESSs in microgrids, with the aim of making them financially viable using linear programming. Long term BESSs bidding strategy in day-ahead and frequency markets is investigated at national scale by [17]. Energy, capacity and ancillary services were stacked in [18] considering different DER technologies over a monthly planning horizon for national energy and capacity markets. Nevertheless, hierarchical structures cannot be modelled by these single-level approaches.

Multi-level optimization is a well-suited method for the modelling of leader-follower problems, in which the results of the lower level problem depends on the variables upper level problem. In this sense, multiple time scales were considered in [19], stacking revenues from energy arbitrage and residuals unit commitments. Virtual Power Plant services were co-optimized in [9] using a multi-level framework. Multi-level optimization also captures strategic decisions made by a profit maximising operator. Bi-level optimization was used in [20], [21] to model the maximisation of revenues of BESSs in day-ahead and reserve markets. Microgrids bidding strategy in national day-ahead and real-time markets were investigated in [22], also considering their possible reconfiguration [23]. Besides of these assets, clusters of buildings with BESSs were used for the stacking of flexibility benefits in national markets [24]. Clusters of BESSs were used at national [25] and local markets [26] to stack flexibility revenues using two-stage programming. However, the main limitation of these proposals is that they do not address the sequential market clearing as all of them includes market with different timescales in the same level of the problem. To overcome this, Stackelberg games were used in [27] and [28] for multi-time scale allocation of energy and reserves. Equilibrium models were proposed in [29] and [30] to determine the optimal bidding strategy of virtual power plants participating in national day-ahead and real-time markets. Both Multi-Level and Equilibrium Problems are usually recast into single-level optimization using KKTs conditions. These MINLP suffer from tractability issues, which hinders the proliferation of new business cases. To address this issue, authors in [31] proposed a bi-level approach that address the sequential market clearing of heat and electricity markets. Nevertheless, in this proposal short-term decision-making were placed over day-ahead decisions, which is unrealistic. Reference [32] overcome this issue using a tri-level optimization model for sequential clearing of heat and electricity markets. However, how to deal with the sequential market clearing of national and future local electricity markets inside a revenues-maximization strategy has not been addressed.

TABLE I: Summary of the literature review.

Reference	Markets	Agents	Time Scale	Spatial Scale	Problem
[17]	Energy arbitrage, Frequency regulation	BESS	Monthly	National	QCP
[13]	Energy arbitrage, Fast Frequency Response	BESS	Monthly	National	Scalar
[14]	Energy arbitrage	BESS	Monthly	National	MILP
[16]	Energy arbitrage	BESS, PV	Hourly	Local	NLP
[15]	Energy arbitrage	BESS, Wind	Monthly	National	MINLP
[11]	Energy arbitrage, Frequency regulation	BESS, PV	Monthly	National	Heuristics
[9]	Energy arbitrage, Fast Frequency Response, Ancillary Services	Virtual Power Plant	Multiple	National	MILP
[19]	Energy arbitrage, Reserve provision, Residual unit-commitment	CHP, BESS	Multiple	National	MILP
[27]	Energy arbitrage, Reserve provision	EV	Hourly	National	MIQCP
[18]	Energy arbitrage, Reserve provision	PV, BESS, EV, Wind	Monthly	National	MILP
[29]	Energy arbitrage, Real-time markets	Virtual Power Plant	Hourly	National	MPEC
[30]	Energy arbitrage, Real-time markets	Virtual Power Plant	Hourly	National	MPEC
[22]	Energy arbitrage, Reserve provision, Real-time markets	Microgrids	Hourly	National	MILP
[23]	Energy arbitrage, Reserve provision, Real-time markets	Microgrids	Hourly	National	MINLP
[24]	Energy arbitrage, Reserve provision, Regulation services	Buildings with BESS	Hourly	National	MILP
[25]	Energy arbitrage, Reserve provision, Real-time markets	BESS	Hourly	National	MINLP
[26]	Energy arbitrage, Reserve provision, LFM	BESS	Hourly	National, Local	MINLP
This paper	DAM, RM, LEM, LFM	BESSs, HVACs, FLs, FGs	Multiple	National, Local	NLP

A summary of the conducted literature review is shown in Table I, where the number of markets, DER technologies, time and spatial scales and type of problem are compared. In light of the above, the identified gaps in knowledge are,

- G1 The sequential clearing of national and local markets has not been well addressed by literature. Research to date has not yet determined how to strategically manage DERs participating in national and local markets that are cleared sequentially.
- G2 Aggregation of disparate DER technologies, considering the expected grow in Heating, Ventilation and Air Conditioning (HVAC) systems and the electrification of the mobility, when they participate in national and local markets, is not properly addressed. Future local flexibility markets will be another source of incomes to DERs, which could unlock new business studies and boost their deployment.
- G3 Few studies have investigated the need of considering the influence of strategic decisions made by a FSP. This requires a multi-level optimisation approach, and the number of methodologies to address it is scarce.

To fill these gaps, this paper proposes a tri-level optimization problem for the maximisation of stacked revenues from flexible DERs. Disparate DERs technologies are managed to provide both national and local services, taking into account that markets are cleared sequentially. A tri-level optimization problem is recast into a single-level NLP using the methodology based on duality proposed in [32]. Contributions of the proposed work are,

- C1 A novel framework that supports simultaneous participation of DERs in local and national markets, providing both energy and capacity services. This methodology can deal with the physical interface of national and local markets and with their sequential clearance.
- C2 Integration of multiple DER technologies including EVs and electric HVACs providing spatial and temporal flexibility coverage. The proposed framework can deal with multiple sources of flexibility providing several services

to national and local markets.

- C3 A tri-level optimization problem that models the stacking of flexibility revenues of DERs participating in sequential national and local markets. This tri-level problem is then converted into a tractable single-level problem which captures strategic decisions made by a FSP when dealing with temporal and spatial scales.

The remaining sections are organised as follows. Section II presents the problem formulation. Section III proposes a method for solving the tri-level optimization problem that maximise stacked revenues. Results of the case study are depicted in Section IV. Lastly, Section V concludes the paper.

II. PROBLEM FORMULATION

In this section, the national and local market problems, the agent constraints and the profit maximiser objective are described. The scheme of the problem formulation is depicted in Fig. 1. A FSP manages DERs connected to the distribution grid. Those assets participate in two markets at national level, i.e., the Day-Ahead Market (DAM) and Reserve Market (RM), and other two at local level LEM and LFM. Those markets are linked by power flows $p_{i,j,t}$ between transmission and distribution networks. Besides, the deployment of the FSP's bids in national and local market scales are mutually affected by each other. This setting provides a new framework for strategic decision-making, using a tri-level optimisation problem. This methodology captures both the sequential market clearing of national and local markets and the strategic behaviour of the FSP, which is reflected in the modification of the bids when the FSP is participating in multiple markets. Then, we use duality theory and the method proposed by [32] to find an equivalent single-level optimisation problem.

The FSP aims to stack flexibility revenues from all markets. National markets are cleared first on an hourly basis, then, local markets are run with a timescale of 15 minutes. Clearing price of DAM is λ_t^{DA} , RM are μ_t^{ru} and μ_t^{rd} , LFM is λ_t^{LFM} and LEM is λ_t^{LEM} . FSP decides the price of the bids π_t and the limits of the bids of energy $\bar{\omega}_t$, and capacity $\bar{\nu}_t$ products for each time period t .

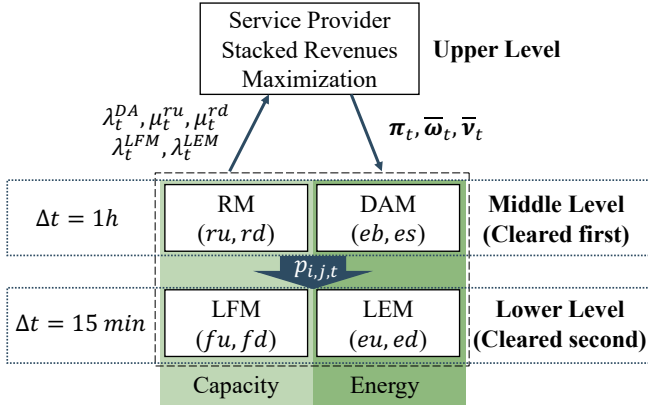


Fig. 1: General scheme of the proposed structure. FSP sets bids prices π_t and limits for energy $\bar{\omega}_t$ and capacity \bar{v}_t products, for the participation in national DAM and RM, and LEM and LFM, which set prices for energy λ_t^{DA} , upward μ_t^{ru} and downward μ_t^{rd} reserve, and local energy λ_t^{LEM} and flexibility λ_t^{LFM} products.

The timeline of the market is presented in Fig. 2. The DAM, which is managed by the Transmission System Operator (TSO), is the market for clearing energy trading at transmission level. Market Operator (MO) receives asks and bids from agents at transmission level and the FSP, and network information from the TSO. Then, the DAM is cleared and agents are dispatched. After that, RM is cleared on an hourly timescale. In this market, TSO asks for reserves to the MO, which receives bids from agents connected to the transmission network. Before local markets are cleared, network constraints at the interface are sent to the DSO.

Each distribution network has its own LEM and LFM

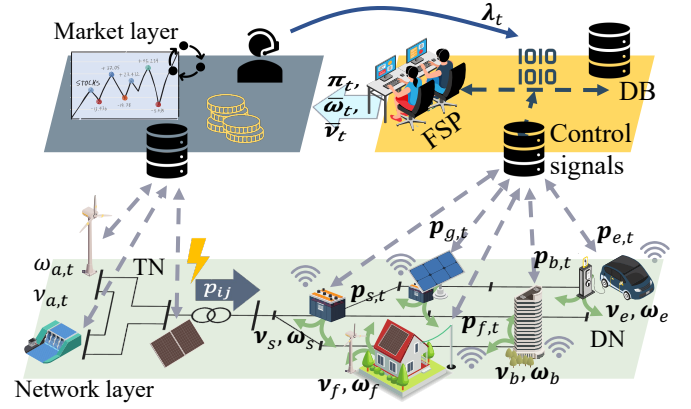


Fig. 3: Conceptual diagram of the data exchanged in the proposed approach within the market/network layer and the FSP. Control signals are represented by grey dashed arrows while products dispatched are presented by green arrows

running in parallel with a 15 min timescale. The LEM supplies the local energy mismatch between DAM clearing and real scheduling of the assets. Energy bids are submitted to the Local Market Operator (LMO) and the market is cleared maximizing the social welfare of participants. Then, a LFM is cleared in case any congestion or imbalance appears near real-time operation in the distribution network. Flexibility bids are sent by independent DERs and by the FSP.

The proposed approach aims to maximize revenue stacking from DERs in the market layer, the FSP serves as an interface between the network and control layers, leveraging historical data and sending control signals to ensure efficient market participation $\pi_t, \bar{\omega}_t, \bar{v}_t$. FSP collects information about past results to forecast future market states, and the flexibility

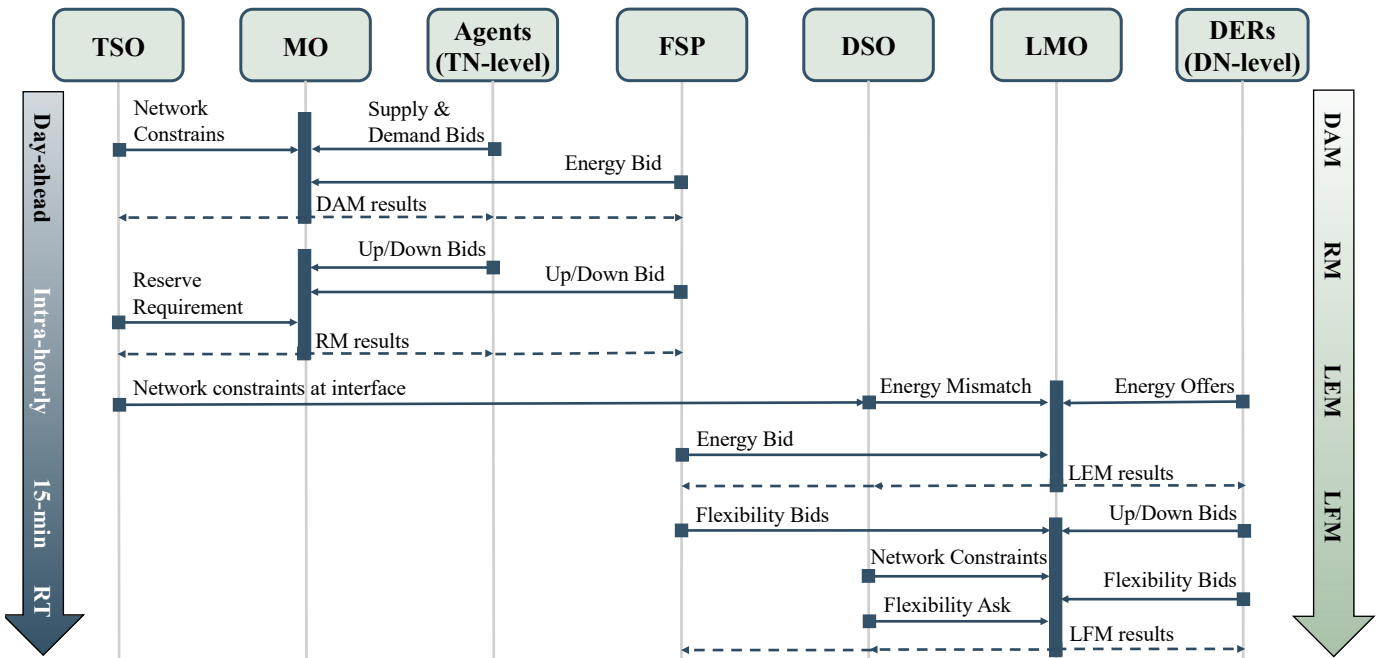


Fig. 2: Timeline diagram of the market interactions among participants.

availability of its DERs. Figure 3 shows how the FSP manages this data to compute the optimal bid strategy and the control signals $p_{a,t}$ it sends to the DERs once the markets are cleared and the products ω, ν are dispatched. Note that no sensitive information about the DERs leaves the domains of the FSP. Thus, privacy is ensured.

A. Day-Ahead Market

Problem (1) represents the DAM problem, where the social welfare of the participating agents is maximised. Let $\pi_{a,t}^{eb}$ and $\pi_{a,t}^{es}$ be the offer prices and $\omega_{a,t}^{eb}$ and $\omega_{a,t}^{es}$ be the quantity of the energy bought eb and energy sold es of the agent a in time period t in the DAM.

$$\max_{\omega_{a,t}^{eb}, \omega_{a,t}^{es}, \theta_{i,t}^{TN}} \sum_{t \in \Omega_t} \sum_{a \in \Omega_a} (\pi_{a,t}^{eb} \omega_{a,t}^{eb} - \pi_{a,t}^{es} \omega_{a,t}^{es}) \quad (1a)$$

Subject to,

$$\underline{\omega}_{a,t}^{eb} \leq \omega_{a,t}^{eb} \leq \bar{\omega}_{a,t}^{eb} \quad \forall a \in \Omega_a, \forall t \in \Omega_t \quad (1b)$$

$$\underline{\omega}_{a,t}^{es} \leq \omega_{a,t}^{es} \leq \bar{\omega}_{a,t}^{es} \quad \forall a \in \Omega_a, \forall t \in \Omega_t \quad (1c)$$

$$\sum_{a \in \Omega_a} \omega_{a,t}^{eb} = \sum_{a \in \Omega_a} \omega_{a,t}^{es} : \lambda_t^{DA} \quad \forall t \in \Omega_t \quad (1d)$$

$$\sum_{j \in \Omega_n^{TN}} B_{i,j}^{TN} (\theta_{i,t}^{TN} - \theta_{j,t}^{TN}) = p_{i,t}^{TN} \quad \forall i \in \Omega_n, \forall t \in \Omega_t \quad (1e)$$

$$\|B_{i,j}^{TN} (\theta_{i,t}^{TN} - \theta_{j,t}^{TN})\| \leq \bar{P}_{i,j} \quad \forall (i,j) \in \Omega_l, \forall t \in \Omega_t \quad (1f)$$

$$-\pi \leq \theta_{i,t}^{TN} \leq \pi \quad \forall i \in \Omega_n, \forall t \in \Omega_t \quad (1g)$$

Equations (1b) and (1c) represent the limit of the offers for the products traded in DAM. Demand is matched with generation through (1d). A DC power flow model is considered in (1e) – (1g) to characterize the node power balance as the DAM takes place in the transmission network. $p_{i,t}^{TN}$ represents the nodal injections of the agents connected to the grid. The prices of the products traded in the market are settled on a marginal basis using dual variables λ_t^{DA} associated with (1d).

B. Reserve Market

The RM is described by (2), where the cost of acquiring upward ru and downward rd reserves are minimised.

$$\min_{\nu_{a,t}^{ru}, \nu_{a,t}^{rd}} \sum_{t \in \Omega_t} \sum_{a \in \Omega_a} (\pi_{a,t}^{ru} \nu_{a,t}^{ru} + \pi_{a,t}^{rd} \nu_{a,t}^{rd}) \quad (2a)$$

Subject to,

$$\nu_{a,t}^{ru} \leq \bar{\nu}_{a,t}^{ru} \quad \forall a \in \Omega_a, \forall t \in \Omega_t \quad (2b)$$

$$\nu_{a,t}^{rd} \leq \bar{\nu}_{a,t}^{rd} \quad \forall a \in \Omega_a, \forall t \in \Omega_t \quad (2c)$$

$$\sum_{a \in \Omega_a} \nu_{a,t}^{ru} \geq R_t^u : \mu_t^{ru} \quad \forall t \in \Omega_t \quad (2d)$$

$$\sum_{a \in \Omega_a} \nu_{a,t}^{rd} \geq R_t^d : \mu_t^{rd} \quad \forall t \in \Omega_t \quad (2e)$$

Upward ru and downward rd offer blocks are described by (2b) and (2c). TSO asks for upward R_t^u and downward R_t^d reserves in the market in prevision of future eventualities in the grid. The price of the upward and downward products are settled by dual variables μ_t^{ru} and μ_t^{rd} , respectively.

C. Local Energy Market

LEM is organized at the local level to adjust for the lack of demand or generation (i.e. ΔE_t) considering what has been previously settled up in the DAM. This market aims to maximise social welfare of participants as (3) states.

$$\max_{\omega_{a,t}^{eu}, \omega_{a,t}^{ed}} \sum_{t \in \Omega_t} \sum_{a \in \Omega_a} (\pi_{a,t}^{eu} \omega_{a,t}^{eu} - \pi_{a,t}^{ed} \omega_{a,t}^{ed}) \quad (3a)$$

Subject to

$$\omega_{a,t}^{eu} \leq \bar{\omega}_{a,t}^{eu} \quad \forall a \in \Omega_a, \forall t \in \Omega_t \quad (3b)$$

$$\omega_{a,t}^{ed} \leq \bar{\omega}_{a,t}^{ed} \quad \forall a \in \Omega_a, \forall t \in \Omega_t \quad (3c)$$

$$\sum_{a \in \Omega_a} (\omega_{a,t}^{eu} - \omega_{a,t}^{ed}) = \Delta E_t : \lambda_t^{LEM} \quad \forall t \in \Omega_t \quad (3d)$$

Equations (3b) and (3c) represent the upward eu and downward rd energy offer limits. Then, the mismatch is compensated by those energy products in (3d). Dual variable λ_t^{LEM} associated to (3d) represents the price of the products traded.

D. Local Flexibility Market

LFM is organized to mitigate congestions in the distribution grid. DSO minimizes the cost of acquiring flexibility in (4).

$$\min_{\nu_{a,t}^{fu}, \nu_{a,t}^{fd}, \theta_{i,t}^{DN}, v_{i,t}^{DN}, p_{i,j,t}, q_{i,j,t}} \sum_{t \in \Omega_t} \sum_{a \in \Omega_a} (\pi_{a,t}^{fu} \nu_{a,t}^{fu} + \pi_{a,t}^{fd} \nu_{a,t}^{fd}) \quad (4a)$$

Subject to

$$\nu_{a,t}^{fu} \leq \bar{\nu}_{a,t}^{fu} \quad \forall a \in \Omega_a, \forall t \in \Omega_t \quad (4b)$$

$$\nu_{a,t}^{fd} \leq \bar{\nu}_{a,t}^{fd} \quad \forall a \in \Omega_a, \forall t \in \Omega_t \quad (4c)$$

$$\sum_{j \in \Omega_n} [G_{i,j}^{DN} v_{j,t}^{DN} - B_{i,j} \theta_{j,t}^{DN}] = p_{i,t}^{DN} \quad \forall i \in \Omega_n, \forall t \in \Omega_t \quad (4d)$$

$$\sum_{j \in \Omega_n} [-B_{i,j}^{DN} v_{j,t}^{DN} - G_{i,j} \theta_{j,t}^{DN}] = q_{i,t}^{DN} \quad \forall i \in \Omega_n, \forall t \in \Omega_t \quad (4e)$$

$$p_{i,j,t} = G_{i,j} v_{i,j,t} - B_{i,j} \theta_{i,j,t} \quad \forall (i,j) \in \Omega_l, \forall t \in \Omega_t \quad (4f)$$

$$q_{i,j,t} = -B_{i,j} v_{i,j,t} - G_{i,j} \theta_{i,j,t} \quad \forall (i,j) \in \Omega_l, \forall t \in \Omega_t \quad (4g)$$

$$p_{i,j,t}^2 + q_{i,j,t}^2 \leq \bar{S}_{i,j}^2 \quad \forall (i,j) \in \Omega_l, \forall t \in \Omega_t \quad (4h)$$

$$\sum_{a \in \Omega_a} \nu_{a,t}^{fu} = \sum_{a \in \Omega_a} \nu_{a,t}^{fd} : \lambda_t^{LFM} \quad \forall t \in \Omega_t \quad (4i)$$

$$-\pi \leq \theta_{i,t}^{DN} \leq \pi \quad \forall i \in \Omega_n, \forall t \in \Omega_t \quad (4j)$$

$$\underline{V}_i^{DN} \leq v_{i,t}^{DN} \leq \bar{V}_i^{DN} \quad \forall i \in \Omega_n, \forall t \in \Omega_t \quad (4k)$$

Upward fu and downward fd limits are represented by (4b) and (4c), respectively. Let $p_{i,t}^{DN}$ and $q_{i,t}^{DN}$ be nodal injections at node i and time period t in the distribution network. Let $v_{i,j,t}$ and $\theta_{i,j,t}$ be the voltage magnitude and phase angle difference between nodes i and j in time period t . We consider a linear approximation of the active and reactive node balance of the distribution grid in (4d) and (4e) as in [33]. Then, power flows are computed in (4f) and (4g). Thermal limit of the branch is computed in conic constraint (4h). Equation (4i) ensures that the total amount of upward and downward products are the same, so the solution of the LFM market is compatible with

previously settled markets. Lastly, voltage phase angle and magnitude limits are described by (4j) and (4k). Price of the products are settled by dual variable λ_t^{LFM} associated to (4i).

E. FSP objective

The FSP seeks the maximization of the revenues obtained from all the markets previously described, as (5) presents.

$$\begin{aligned} \max_{\pi_t, \bar{\omega}_t, \bar{\nu}_t} \sum_{t \in \Omega_t} & \left[\lambda_t^{DA} (\omega_t^{es} - \omega_t^{eb}) + \mu_t^u \nu_t^{ru} + \mu_t^d \nu_t^{rd} \right. \\ & \left. + \lambda_t^{LEM} (\omega_t^{ed} - \omega_t^{eu}) + \sum_{a \in \Omega_a} \lambda_t^{LFM} (\nu_{a,t}^{fu} + \nu_{a,t}^{fd}) \right] \end{aligned} \quad (5a)$$

Subject to

$$\begin{aligned} \pi_t^{eb}, \pi_t^{es}, \pi_t^{ru}, \pi_t^{rd}, \pi_t^{eu}, \pi_t^{ed}, \\ \bar{\omega}_t^{eb}, \bar{\omega}_t^{es}, \bar{\nu}_t^{ru}, \bar{\nu}_t^{rd}, \bar{\omega}_t^{eu}, \bar{\omega}_t^{ed}, \geq 0 \quad \forall t \in \Omega_t \end{aligned} \quad (5b)$$

$$\pi_{a,t}^{fu}, \pi_{a,t}^{fd}, \bar{\nu}_{a,t}^{fu}, \bar{\nu}_{a,t}^{fd} \geq 0 \quad \forall a \in \Omega_a, \forall t \in \Omega_t \quad (5c)$$

$$\omega_t^{es} = \sum_{a \in \Omega_a} \omega_{a,t}^{es}, \quad \omega_t^{eb} = \sum_{a \in \Omega_a} \omega_{a,t}^{eb} \quad \forall t \in \Omega_t \quad (5d)$$

$$\nu_t^{ru} = \sum_{a \in \Omega_a} \nu_{a,t}^{ru}, \quad \nu_t^{rd} = \sum_{a \in \Omega_a} \nu_{a,t}^{rd} \quad \forall t \in \Omega_t \quad (5e)$$

$$\omega_t^{eu} = \sum_{a \in \Omega_a} \omega_{a,t}^{eu}, \quad \omega_t^{ed} = \sum_{a \in \Omega_a} \omega_{a,t}^{ed} \quad \forall t \in \Omega_t \quad (5f)$$

$$(12) - (16) \quad (5g)$$

FSP stacks revenues from the markets it participates in by defining the optimal bid, i.e., price and quantity. The objective described in (5a) maximises the revenues obtained from the DAM, RM, LEM and LFM. Prices and offers are positive as (5b) and (5c) defines. Aggregation of the bids for the DAM is described in (5d), for RM in (5e) and for LEM in (5f). The agents constraints for Flexible Loads (FLs), Flexible Generators (FGs), BESSs, EVs and HVACs systems are explained in Appendix A and included in the model in (5g).

III. TRI-LEVEL OPTIMIZATION FOR REVENUES MAXIMIZATION

The maximization problem of the FSP is subject to the sequential market clearing of the DAM, RM, LEM and LFM. We propose a tri-level optimization problem for stacking revenues of DERs managed by a FSP in those markets. The sequential clearing of the national and local markets is represented by the mid and lower-level problem, respectively, following [32].

A. Sequential problem formulation

We use matrix notation to represent the tri-level optimization problem. Let \mathbf{x}_m , λ_m be the vector of primal and dual decision variables for a given market m . Constraints of the markets are represented without loss of generality with inequalities for the sake of readability. Let \mathbf{A}_m and \mathbf{B}_m be the matrices of coefficients and \mathbf{b}_m the vector of independent terms for market m . \mathbf{B}_m is the matrix associated to FSP agents, and \mathbf{A}_m is the matrix of the rest of agents. Lastly,

vector \mathbf{c}_m represents the price of the offers in the market m . Thus, the full tri-level optimization is presented in (6).

$$\min -\lambda_{DA}^T \mathbf{x}_{DA}^{FSP} - \lambda_{RM}^T \mathbf{x}_{RM}^{FSP} - \lambda_{LEM}^T \mathbf{x}_{LEM}^{FSP} - \lambda_{LFM}^T \mathbf{x}_{LFM}^{FSP} \quad (6a)$$

$$\text{s.t. } \mathbf{A}^{FSP} \mathbf{x}^{FSP} \leq \mathbf{b}^{FSP} \quad (6b)$$

$$\min -\mathbf{c}_{DA}^T \mathbf{x}_{DA} - \mathbf{c}_{DA}^{FSP^T} \mathbf{x}_{DA}^{FSP} \quad (6c)$$

$$+ \mathbf{c}_{RM}^T \mathbf{x}_{RM} + \mathbf{c}_{RM}^{FSP^T} \mathbf{x}_{RM}^{FSP}$$

$$\text{s.t. } \mathbf{A}_{DA} \mathbf{x}_{DA} + \mathbf{B}_{DA} \mathbf{x}_{DA}^{FSP} \leq \mathbf{b}_{DA} : \lambda_{DA} \quad (6d)$$

$$\mathbf{A}_{RM} \mathbf{x}_{RM} + \mathbf{B}_{RM} \mathbf{x}_{RM}^{FSP} \leq \mathbf{b}_{RM} : \lambda_{RM} \quad (6e)$$

$$\min \mathbf{c}_{LEM}^T \mathbf{x}_{LEM} + \mathbf{c}_{LEM}^{FSP^T} \mathbf{x}_{LEM}^{FSP} \quad (6f)$$

$$+ \mathbf{c}_{LFM}^T \mathbf{x}_{LFM} + \mathbf{c}_{LFM}^{FSP^T} \mathbf{x}_{LFM}^{FSP}$$

$$\text{s.t. } \mathbf{A}_{LEM} \mathbf{x}_{LEM} \quad (6g)$$

$$+ \mathbf{B}_{LEM} \mathbf{x}_{LEM}^{FSP} \leq \mathbf{b}_{LEM} : \lambda_{LEM}$$

$$\mathbf{A}_{LFM} \mathbf{x}_{LFM} \quad (6h)$$

$$+ \mathbf{B}_{LFM} \mathbf{x}_{LFM}^{FSP} \leq \mathbf{b}_{LFM} : \lambda_{LFM}$$

Equation (6a) and (6b) represents the maximization problem of the FSP, which is subject to the mid and lower level problems. Mid-level problem objective (6c) represents the joint clearing of the DAM and RM. Equations (6d) and (6e) represents the DAM and RM constraints. Lower level problem (6f) jointly minimize costs for LEM and LFM, which constraints are represented by (6g) and (6h).

B. Equivalent problem

In this section, the tri-level problem in (6) is converted into a single-level problem using lexicographic optimization and duality theory. Note that variables from the mid-level problem (6c) – (6e) do not depend on the variables of the low-level problem (6f) – (6h). This enables to reformulate (6c) – (6h) as a single-level problem using the following lexicographic function [34].

$$\min \begin{pmatrix} -\mathbf{c}_{DA}^T \mathbf{x}_{DA} - \mathbf{c}_{DA}^{FSP^T} \mathbf{x}_{DA}^{FSP} \\ + \mathbf{c}_{RM}^T \mathbf{x}_{RM} + \mathbf{c}_{RM}^{FSP^T} \mathbf{x}_{RM}^{FSP} \\ \mathbf{c}_{LEM}^T \mathbf{x}_{LEM} + \mathbf{c}_{LEM}^{FSP^T} \mathbf{x}_{LEM}^{FSP} \\ + \mathbf{c}_{LFM}^T \mathbf{x}_{LFM} + \mathbf{c}_{LFM}^{FSP^T} \mathbf{x}_{LFM}^{FSP} \end{pmatrix} \quad (7a)$$

$$\text{s.t. } \mathbf{A}_{DA} \mathbf{x}_{DA} + \mathbf{B}_{DA} \mathbf{x}_{DA}^{FSP} \leq \mathbf{b}_{DA} : \lambda_{DA} \quad (7b)$$

$$\mathbf{A}_{RM} \mathbf{x}_{RM} + \mathbf{B}_{RM} \mathbf{x}_{RM}^{FSP} \leq \mathbf{b}_{RM} : \lambda_{RM} \quad (7c)$$

$$\mathbf{A}_{LEM} \mathbf{x}_{LEM} + \mathbf{B}_{LEM} \mathbf{x}_{LEM}^{FSP} \leq \mathbf{b}_{LEM} : \lambda_{LEM} \quad (7d)$$

$$\mathbf{A}_{LFM} \mathbf{x}_{LFM} + \mathbf{B}_{LFM} \mathbf{x}_{LFM}^{FSP} \leq \mathbf{b}_{LFM} : \lambda_{LFM} \quad (7e)$$

This lexicographic problem can be asymptotically approximated by the linear problem (8) when $\gamma \rightarrow 1$ [32]. Let $f(y)$ and $g(z)$ be the objective of the mid and lower level problems. The resulting lexicographic function is $l(y, z) = \gamma f(y) + (1-\gamma)g(z)$. The term $(1-\gamma)g(z)$ becomes negligible when $\gamma \rightarrow 1$, so the objective first find the optimal value of y that minimises $f(y)$, and then optimises $g(z)$, approximating

the sequential clearing behaviour. The sequential clearing is then approximated by,

$$\min \gamma [-\mathbf{c}_{DA}^T \mathbf{x}_{DA} - \mathbf{c}_{DA}^{FSP^T} \mathbf{x}_{DA}^{FSP} + \mathbf{c}_{RM}^T \mathbf{x}_{RM} + \mathbf{c}_{RM}^{FSP^T} \mathbf{x}_{RM}^{FSP}] + (1 - \gamma) [\mathbf{c}_{LEM}^T \mathbf{x}_{LEM} + \mathbf{c}_{LEM}^{FSP^T} \mathbf{x}_{LEM}^{FSP} + \mathbf{c}_{LFM}^T \mathbf{x}_{LFM} + \mathbf{c}_{LFM}^{FSP^T} \mathbf{x}_{LFM}^{FSP}] \quad (8a)$$

s.t. (7b) - (7e)

At this point, the tri-level problem has been converted into a bi-level problem. To tackle this, the lower-level problem will be replaced by its set of primal and dual constraint. After that, the resulting bi-level optimization is solved by replacing the inner problem for its set of primal (9c) – (9f), dual (9g) – (9n) and strong duality constraint (9o). The optimality of this problem will be guaranteed by also including the strong duality condition into the single-level problem [35]. Strong duality condition guarantees that any feasible solution of the proposed single level problem is an optimal solution of the lower level problem. The objective function is composed by two terms: FSP profit maximisation in national and local markets.

However, the previous approximation affects to the scale of the dual variables of the inner problem. Thus, the original scale of the dual variables of the inner problem should be recovered by dividing mid-level dual variables by γ and lower-level dual variables by $(1 - \gamma)$ in the final objective function (9a). The final result is presented in (9).

$$\min - \frac{1}{\gamma} [\lambda_{DA}^T \mathbf{x}_{DA}^{FSP} + \lambda_{RM}^T \mathbf{x}_{RM}^{FSP}] - \frac{1}{1 - \gamma} [\lambda_{LEM}^T \mathbf{x}_{LEM}^{FSP} + \lambda_{LFM}^T \mathbf{x}_{LFM}^{FSP}] \quad (9a)$$

Subject to,

$$\mathbf{A}^{FSP} \mathbf{x}^{FSP} \leq \mathbf{b}^{FSP} \quad (9b)$$

$$\mathbf{A}_{DA} \mathbf{x}_{DA} + \mathbf{B}_{DA} \mathbf{x}_{DA}^{FSP} \leq \mathbf{b}_{DA} \quad (9c)$$

$$\mathbf{A}_{RM} \mathbf{x}_{RM} + \mathbf{B}_{RM} \mathbf{x}_{RM}^{FSP} \leq \mathbf{b}_{RM} \quad (9d)$$

$$\mathbf{A}_{LEM} \mathbf{x}_{LEM} + \mathbf{B}_{LEM} \mathbf{x}_{LEM}^{FSP} \leq \mathbf{b}_{LEM} \quad (9e)$$

$$\mathbf{A}_{LFM} \mathbf{x}_{LFM} + \mathbf{B}_{LFM} \mathbf{x}_{LFM}^{FSP} \leq \mathbf{b}_{LFM} \quad (9f)$$

$$\lambda_{DA}^T \mathbf{A}_{DA} \leq -\gamma \mathbf{c}_{DA}^T \quad (9g)$$

$$\lambda_{DA}^T \mathbf{B}_{DA} \leq -\gamma \mathbf{c}_{DA}^{FSP^T} \quad (9h)$$

$$\lambda_{RM}^T \mathbf{A}_{RM} \leq \gamma \mathbf{c}_{RM}^T \quad (9i)$$

$$\lambda_{RM}^T \mathbf{B}_{RM} \leq \gamma \mathbf{c}_{RM}^{FSP^T} \quad (9j)$$

$$\lambda_{LEM}^T \mathbf{A}_{LEM} \leq -(1 - \gamma) \mathbf{c}_{LEM}^T \quad (9k)$$

$$\lambda_{LEM}^T \mathbf{B}_{LEM} \leq -(1 - \gamma) \mathbf{c}_{LEM}^{FSP^T} \quad (9l)$$

$$\lambda_{LFM}^T \mathbf{A}_{LFM} \leq (1 - \gamma) \mathbf{c}_{LFM}^T \quad (9m)$$

$$\lambda_{LFM}^T \mathbf{B}_{LFM} \leq (1 - \gamma) \mathbf{c}_{LFM}^{FSP^T} \quad (9n)$$

$$\gamma \left[-\mathbf{c}_{DA}^T \mathbf{x}_{DA} - \mathbf{c}_{DA}^{FSP^T} \mathbf{x}_{DA}^{FSP} + \mathbf{c}_{RM}^T \mathbf{x}_{RM} + \mathbf{c}_{RM}^{FSP^T} \mathbf{x}_{RM}^{FSP} \right] + (1 - \gamma) \left[-\mathbf{c}_{LEM}^T \mathbf{x}_{LEM} - \mathbf{c}_{LEM}^{FSP^T} \mathbf{x}_{LEM}^{FSP} + \mathbf{c}_{LFM}^T \mathbf{x}_{LFM} + \mathbf{c}_{LFM}^{FSP^T} \mathbf{x}_{LFM}^{FSP} \right] = \quad (9o)$$

$$\mathbf{b}_{DA} \lambda_{DA}^T + \mathbf{b}_{RM} \lambda_{RM}^T + \mathbf{b}_{LEM} \lambda_{LEM}^T + \mathbf{b}_{LFM} \lambda_{LFM}^T$$

Equation (9b) represents FSP constraints, primal constraints of each market are depicted from (9c) to (9f). Equations (9g) - (9n) represent dual constraints of the sequential markets clearing considering the previously lexicographic function. Lastly, (9o) ensures that strong duality condition of the sequential markets clearing is satisfied.

C. Stochastic formulation

In this section, a stochastic formulation of the problem is described using a scenario tree, as Fig. 5 depicts. We assume that the FSP have access to historical data of previous market clearing results. This enables the FSP to build forecast tools which can predict the future values of the bids $\pi_{e,a,t}$, the quantities $\bar{\omega}_{e,a,t}$, $\bar{\nu}_{e,a,t}$ for all agent $a \in \Omega_a$, and time period $t \in \Omega_t$ for a set of scenarios $e \in \Omega_e$. Following authors in [36], the errors of the forecasting tools are characterized using a normal distribution, with a standard deviation of 25% and a mean value equal to the forecasted value. To avoid computational burden, we use a scenario reduction technique to include the most representative scenarios into the final problem [37].

The stochastic version is described as follows. Let \hat{X} be the mean of the uncertain parameter X and $\Delta \tilde{e}_e^X$ be the forecast error in the scenario e for the parameter X . Thus, the bids $\pi_{e,a,t}$ and their quantities $\bar{\omega}_{e,a,t}$, $\bar{\nu}_{e,a,t}$ are decomposed as follows,

$$\pi_{e,a,t} = \hat{\pi}_{a,t} + \Delta \tilde{e}_e^{\pi_{a,t}} \quad \forall e \in \Omega_e, \forall a \in \Omega_a, \forall t \in \Omega_t \quad (10a)$$

$$\bar{\omega}_{e,a,t} = \hat{\omega}_{a,t} + \Delta \tilde{e}_e^{\bar{\omega}_{a,t}} \quad \forall e \in \Omega_e, \forall a \in \Omega_a, \forall t \in \Omega_t \quad (10b)$$

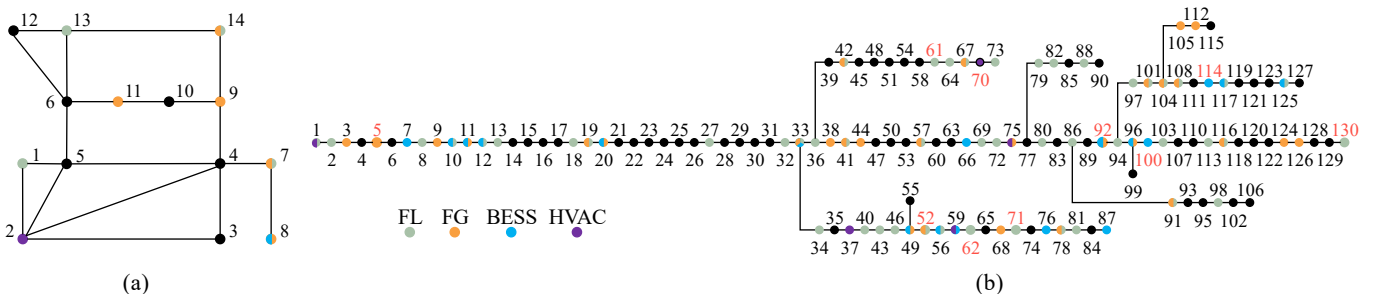


Fig. 4: (a) IEEE 14 network (b) N5_1_DSS network. Assets managed by the FSP are noted in red.

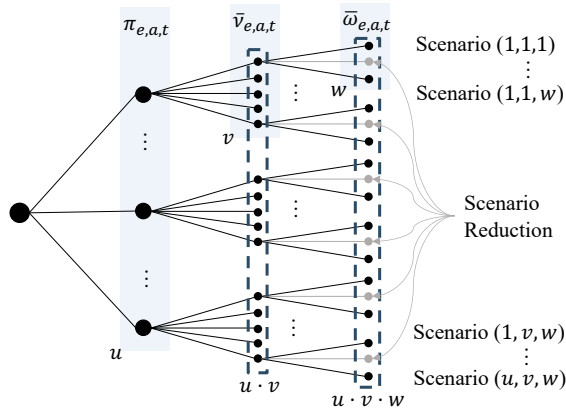


Fig. 5: Scenario tree of the stochastic version of the problem. Unrepresentative scenarios are deleted using a scenario reduction technique.

$$\bar{v}_{e,a,t} = \hat{v}_{a,t} + \Delta \tilde{e}_{e,a,t} \quad \forall e \in \Omega_e, \forall a \in \Omega_a, \forall t \in \Omega_t \quad (10c)$$

Then, the expected value of the profits is maximised for all scenario e in the set of possible realizations Ω_e . Let \mathcal{P}_e the probability of the scenario e , the stochastic formulation of the problem stand as follows,

$$\min \sum_{e \in \Omega_e} \mathcal{P}_e \left[-\frac{1}{\gamma} \left[\lambda_{e,DA}^T \mathbf{x}_{e,DA}^{FSP} + \lambda_{e,RM}^T \mathbf{x}_{e,RM}^{FSP} \right] - \frac{1}{1-\gamma} \left[\lambda_{e,LEM}^T \mathbf{x}_{e,LEM}^{FSP} + \lambda_{e,LFM}^T \mathbf{x}_{e,LFM}^{FSP} \right] \right] \quad (11a)$$

$$\text{s.t. (9b) - (9o)} \quad \forall e \in \Omega_e \quad (11b)$$

IV. CASE STUDY AND SIMULATION RESULTS

In this section, results based on a case study that builds on IEEE 14 bus network, acting as transmission network, and N5_1_DSS [38], acting as distribution network, are shown. Transmission and distribution networks are depicted in Fig. 4 (a) and (b), respectively. Bus 1 of the distribution network is connected to bus 14 of the transmission network. A realistic dataset for generation and load profiles is used from [39]. Bids of the different market participants are randomly generated following Spanish markets average prices. 11 agents are connected to the transmission network as depicted in Fig. 4 (a), with a power of 3,85 MW. There are 105 agents connected to the distribution network as Fig. 4 (b) shows, with a power of 1,26 MW. FSP manages 4 FLs, 3 FGs, 2 BESSs and 1 HVACs connected to the distribution grid, they are noted with red text in Fig. 4 (b). National markets, i.e., DAM and RM are cleared at transmission level, while local markets, i.e., LEM and LFM, are cleared at distribution level. Simulations are carried out using PYOMO [40] and large-scale non-linear solver CONOPT v3.17A using an Apple M1, 3.2 GHz processor with 16 GB of RAM. CONOPT solver was used as it outperforms heuristics techniques to solve NLP problems, obtaining consistent solutions and computational times more than 30 times lower [41]. The optimization problem has 557,437 variables and 100,608 constraints. Time until

convergence was 323.9165 seconds, which is compliant with the clearing timeframe of the short-term markets. The number of internal solver iterations until convergence was 3,769.

A. Stacked revenues from market participation

In this section, simulation results are presented for the case study to show how the FSP stacks profits by participating in different markets simultaneously. Figure 6 presents the products traded in the markets. Energy products eb , es , eu and ed are depicted in Fig. 6 (a), while capacity products ru , rd , fu and fd are depicted in Fig. 6 (b). What stands out in this figure is the multi-temporal scale of the products. Local products are traded with 15 min time granularity, while national products are traded on an hourly basis.

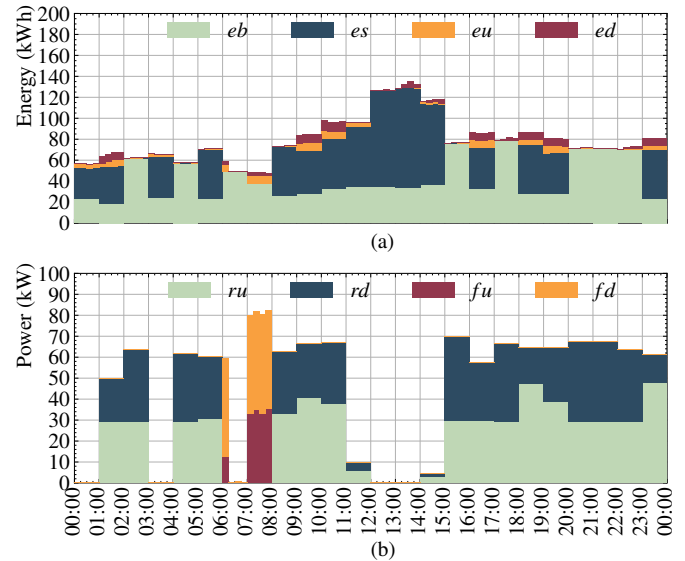


Fig. 6: Stacked energy (a) and capacity (b) products traded by FSP when it participates in national and local markets.

Energy products are traded in the DAM to maximise profits. Minimum demand is ensured for FLs and HVAC systems, while maximising the injection of the FGs. A summary of the traded products can be found in Table II. Trading in the DAM represents a 45.7% of the total trades of the case study. Then, national reserve trading accounts for a 28.6% while LEM and LFM have a 15.2% and 10.5%, respectively. Trading shares among agents are 34.4% for FLs, 37.4% for FGs, 20.2% for BESSs and 8% for HVAC systems.

Temporal distribution of the profits obtained in each market is depicted in Fig. 7. 86.22% of the energy profits obtained in Fig. 7 (a) are due to energy sold es in the DAM. Revenues obtained from the participation in capacity markets are shown in Fig. 7 (b), where benefits from RM and LFM obtained. Total profits for this case study add up to 336.04 € for one day of operation. A summary of the revenues obtained by product, and by technology, is presented in Table II. Energy trade in the DAM is specially profitable for BESSs, obtaining 72.46 € of profits. In the case of the HVAC system, its participation in RM and LFM allows it to recover a part of its energy costs.

Figure 8 represents the evolution of the variables of the HVAC system controlled by the FSP. It buys energy from the

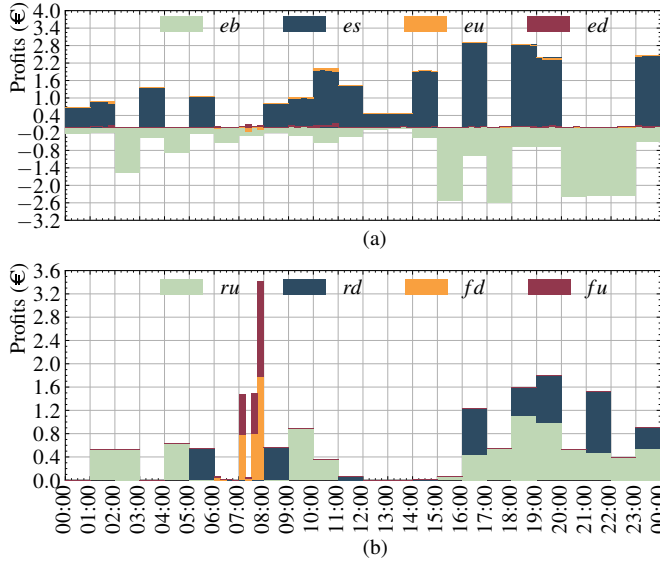


Fig. 7: Stacking energy (a) and capacity (b) revenues for the products traded by the FSP.

TABLE II: Summary of the products quantities and profits obtained by DER technology.

	FLs	FGs	BESSs	HVACs	Total	
Profits	eb (€)	-64.11	-	-13.76	-6.62	-84.50
	es (€)	-	277.30	86.22	-	363.52
	eu (€)	-0.43	-0.19	-0.28	-0.19	-1.09
	ed (€)	0.18	0.74	0.00	0.06	0.98
	ru (€)	0.05	27.67	4.43	-	32.15
	rd (€)	7.42	4.79	3.07	3.41	18.70
	fu (€)	1.16	-	1.25	0.58	2.99
	fd (€)	0.01	2.05	1.24	-	3.29
Products	eb (kWh)	857.640	-	141.184	74.052	1,072.88
	es (kWh)	-	615.064	165.220	-	780.284
	eu (kWh)	135.863	37.247	88.844	61.440	323.393
	ed (kWh)	40.518	239.180	-	11.960	291.659
	ru (kW)	38.586	415.061	96.526	18.793	568.967
	rd (kW)	244.465	63.686	170.092	115.191	593.434
	fu (kW)	77.603	-	70.592	41.140	189.335
	fd (kW)	0.387	148.124	88.240	-	236.751

DAM and LEM, and takes advantage of its thermal inertia to obtain profits from RM and LFM. Evolution of the temperature is depicted in Fig. 8 (b), which is maintained inside of the comfort limits. Figure 9 represents the evolution of the variables of one of the BESS systems controlled by the FSP. Energy profits are obtained in the short term, for example, energy is bought before 03:00 to sell it in the following period of time when energy price increases. Moreover, it also participates in the LEM by selling energy that was bought in the DAM. Trading of reserve and flexibility services increase profits, as Fig. 9 (d) shows.

B. Profitability comparison with baseline scenarios

In this section, the proposed approach is compared with four different baselines where the FSP maximise profits in one single market at a time. The baselines are computed as a bi-level program where the FSP problem is subject to the market clearing. These bi-level problems are converted into a single-level problem by incorporating the lower-level primal,

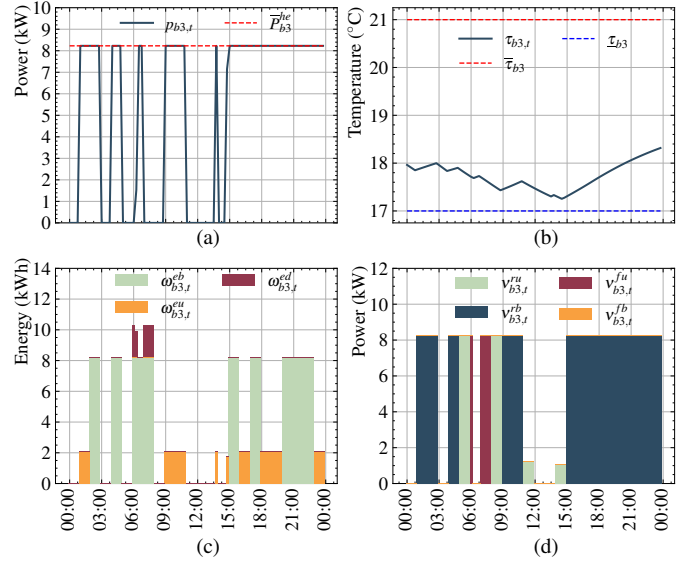


Fig. 8: Evolution of the variables of the HVAC system managed by FSP, Power (a) temperature (b) energy products (c) capacity products (d).

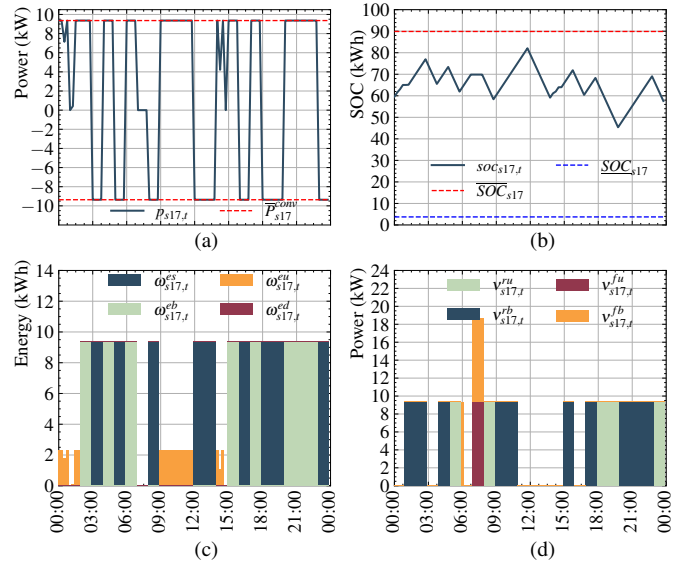


Fig. 9: Evolution of the variables of the BESS managed by FSP, Power (a) State of Charge (SOC) (b) energy and (c) capacity products (d).

dual, and strong-duality equations into the upper-level problem [35].

The quantity of the products traded, and the profits obtained are compared in Fig. 10. The total profits obtained by the proposed strategy are 878.59 €. This supposes an increase in profits compared with the individual baselines, as Fig. 10 (a) presents. The quantity of products exchanged are compared in Fig. 10 (b). Products exchanged over the markets are lower individually, as the proposed method calculate the optimal bidding strategy to maximise profits, as the flexibility managed by the FSP is limited.

Figure 11 presents a comparison of one of the FLs man-

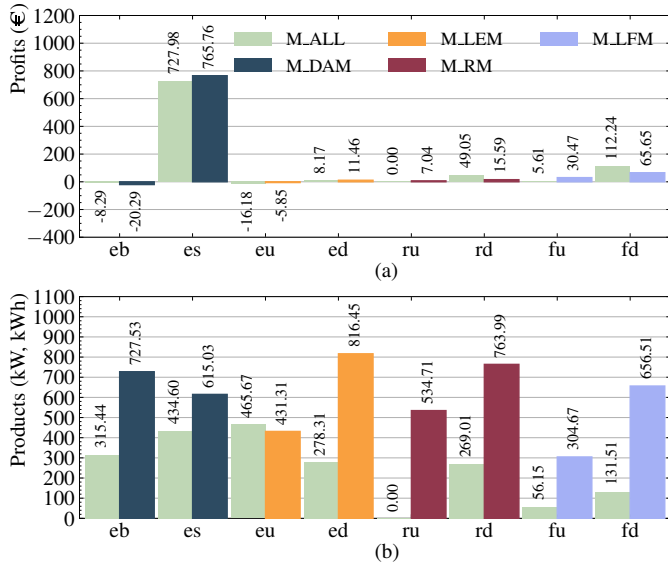


Fig. 10: Comparison of the profits (a) and products (b) traded by the FSP in the baselines and using the proposed strategy.

aged by the FSP under the baselines (right column) and the proposed strategy (left column). The demand in the DAM strategy is displayed in Fig. 11 (a) for the DAM baseline, where the consumption is at minimum level for most of the day. In the LEM baseline (depicted in lilac) the strategy tries to obtain short term profits for its participation in the market, taking advantage of short variations in price. Lastly, the baselines for the RM and LFM are depicted in Fig. 11 (f), where the participation of the FL is maximised for the sake of benefits. Meanwhile, the profit maximiser strategy identifies the most profitable strategy. In this sense, it combines the short-term profit strategy of the LEM with the minimization of participation of consumption from DAM, while also providing products in the RM and LFM. Note that the products showed in the right column of the figure are not stacked together, while those of the left column are.

Market clearing results are compared in Fig. 12. A closer inspection to the figure shows that the proposed strategy increases the price variability in the markets, however, it does lower mean prices in LEM and LFM markets. Energy cleared in the DAM and the LEM is lower, this is since if the energy price is low, generators do not get cleared in the market, and if it is high, demands are no longer willing to pay the price. Reserve quantities are the same in both cases, as it is assumed that the TSO will not modify its asking. Then, to maximize its participation, the FSP sends bids with lower prices to maximise its profits. Lastly, LFM market reduces its operation costs, spreading the products all over the day.

Following this comparison, Table III presents the results that the strategic behaviour of the FSP has in the markets. As shown, social welfare is reduced in the DAM, while increased in the LEM. In term of the costs of the reserve markets, RM costs increase, while LFM costs decrease. Then, although the DAM and LEM profits are reduced with the proposed approach, the overall balance is positive, as the net

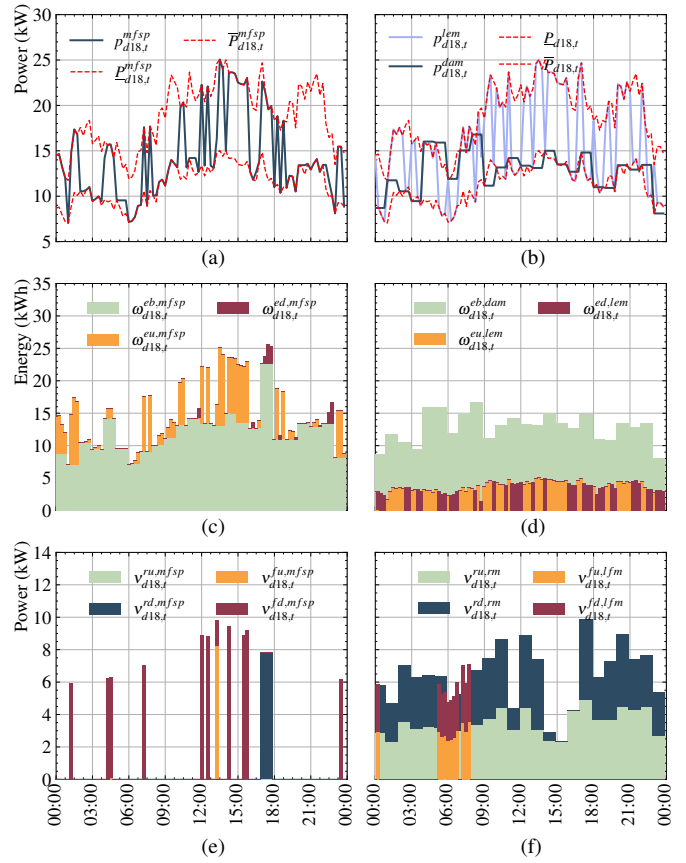


Fig. 11: Comparison of the behaviour of a FL between profit maximiser strategy and the baselines (left and right column). Power: (a) and (b), energy products (c) and (d), capacity products (e) and (f).

profits are 878.59€, a 17.86% more than the most profitable single-market strategy, i.e., DAM maximisation strategy with 745.47€.

C. Impact of the uncertainty in the stacking of revenues

In this section, the impact that the uncertainty in the prices and in the dispatch have in the stacking of revenues is assessed. Considering a normal distribution of the forecast error, 50 values per uncertain parameter are considered, originating 125,000 possible scenarios. Nevertheless, after the scenario reduction only the 125 most representative scenarios were simulated in parallel.

TABLE III: Comparison of the market objectives and profits in the FSP maximization and the baselines.

	Baselines	Profit max.	Diff. (%)
DAM Social Welfare (€)	31.18	27.44	-13.62
RM Costs (€)	19.37	53.62	63.88
LEM Social Welfare (€)	14.16	63.66	77.75
LFM Costs (€)	13.65	10.27	-32.87
FSP DAM Profits (€)	745.47	719.69	-3.58
FSP RM Profits (€)	22.63	49.05	53.86
FSP LEM Profits (€)	5.62	-8.01	-170.16
FSP LFM Profits (€)	96.12	117.86	18.45

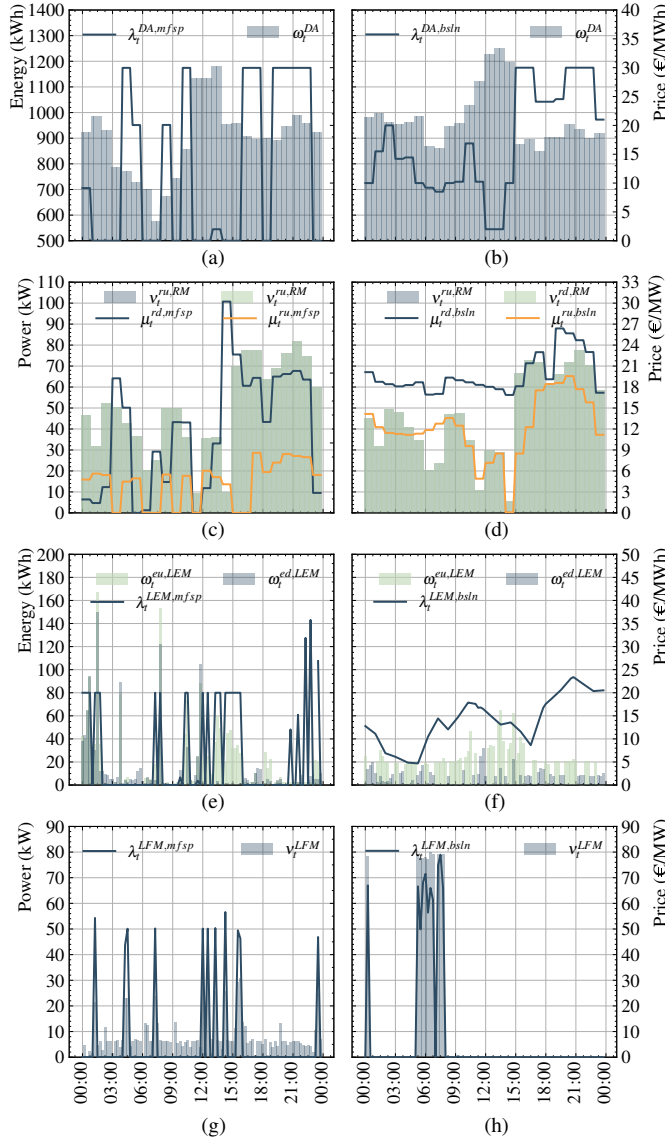


Fig. 12: Comparison of the market clearings between the FSP maximization (left column) and the baselines (right column). DAM: (a) and (b), RM: (c) and (d), LEM: (e) and (f), LFM: (g) and (h). Prices are shown in lines, and traded quantities in bars.

The probability density function of the products that the FSP exchange in the market is shown in Fig. 13. This representation extends what has been obtained in Fig. 10 (b), giving information to the FSP regarding the likelihood of its offers being matched in the market, and the possible outcomes of a determined strategy. The expected profits are depicted in Fig. 14 along with the 95% Interval Confidence, which demonstrate the feasibility of the proposed approach to obtain benefits under uncertainty.

V. CONCLUSION

This paper proposed a tri-level optimization problem for the maximisation of stacked revenues of DERs participating to multiple sequential markets. This approach responds to

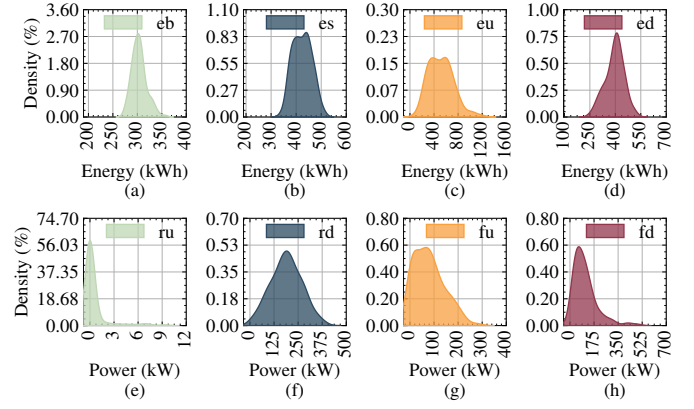


Fig. 13: Probabilistic Density Function (%) of the total products quantities exchanged in the stochastic formulation.

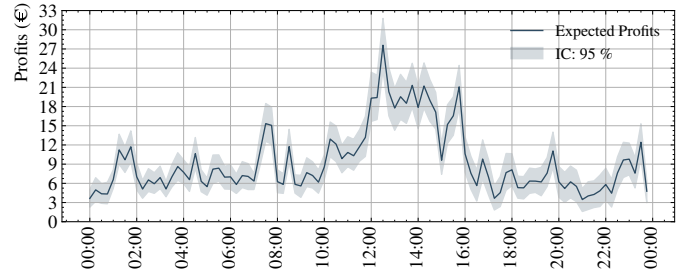


Fig. 14: Expected FSP profits and 95% Interval Confidence for the stochastic formulation.

the necessity of increasing the number of business cases for flexible distributed technologies such as FLs, BESSs, EVs and HVACs systems, with the aim of achieving long-term Net Zero Emissions objectives. Using duality theory and strong duality condition, the sequential tri-level optimization is recast into a tractable single-level problem, which maximises the profits of DERs for their participation in different markets through a FSP. A case study based on the IEEE-14 transmission network, N5_1_DSS distribution network and a realistic dataset demonstrates the feasibility of the approach. Profitability of flexibility procurement is enhanced when the FSP adopts the proposed strategy, compared with four different baselines, where profits were maximised for one market at a time. The proposed model increases the profits of the FSP as it takes a holistic view of the market participation of the FSP. In the case study, profits were increased a 17.86% compared with the most profitable single-market strategy, while reducing 32.87% the costs in the LFM and increasing a 77.75% the social welfare of the LEM. Future works will assess the market participation of the FSP considering only its partial access to the market information.

APPENDIX A AGENT CONSTRAINTS

With the aim of modelling disparate DER technologies, constraints of FLs, FGs, BESSs, EVs and HVACs are described in this appendix. FLs are modelled as elastic demands which can modify their consumption $p_{d,t}$ between an upper and lower bound $\underline{P}_d \leq p_{d,t} \leq \overline{P}_d$ [42]. Let $P_{d,t}^{ref}$ be the static

consumption of the FL d in time period t , it can offer upward $\omega_{d,t}^u$ and downward $\omega_{d,t}^d$ products following,

$$p_{d,t} = P_{d,t}^{ref} + (\omega_{d,t}^u - \omega_{d,t}^d)/\Delta t \quad \forall d \in \Omega_d, \forall t \in \Omega_t \quad (12)$$

In addition, FLs can also offer upward $\nu_{d,t}^u$ and downward $\nu_{d,t}^d$ capacity products restricting their limits of demand, as $\underline{P}_d + \nu_{d,t}^d \leq p_{d,t} \leq \bar{P}_d - \nu_{d,t}^u$. Similarly, FGs can modify their energy production $p_{g,t}$ between an upper and lower bound while providing upward and downward capacity products $\underline{P}_g + \nu_{g,t}^d \leq p_{g,t} \leq \bar{P}_g - \nu_{g,t}^u$. Then, considering $P_{g,t}^{ref}$ its generation,

$$p_{g,t} = P_{g,t}^{ref} + (\omega_{g,t}^u - \omega_{g,t}^d)/\Delta t \quad \forall g \in \Omega_g, \forall t \in \Omega_t \quad (13)$$

Let s be a storage system with a SOC $soc_{s,t}$ that could charge at power $p_{s,t}^{ch} \geq 0$ and discharge at power $p_{s,t}^{dis} \geq 0$, with efficiencies charging η_s^{ch} and discharging η_s^{dis} [42]. The internal constraints that model storage agents are the following,

$$soc_{s,t+1} = soc_{s,t} + \Delta t \left(\eta_s^{ch} p_{s,t}^{ch} - \frac{p_{s,t}^{dis}}{\eta_s^{dis}} \right) \quad \forall s \in \Omega_s, \forall t \in \Omega_t \quad (14a)$$

$$\underline{SOC}_s \leq soc_{s,t} \leq \overline{SOC}_s \quad \forall s \in \Omega_s, \forall t \in \Omega_t \quad (14b)$$

$$0 \leq p_{s,t}^{ch} \leq \bar{P}_{s,t}, \quad 0 \leq p_{s,t}^{dis} \leq \bar{P}_{s,t} \quad \forall s \in \Omega_s, \forall t \in \Omega_t \quad (14c)$$

Using this model, is possible to include the behaviour of EVs. Let t_e^{arr}, t_e^{dep} be arrival and departure times of the EV e , considering it must leave at departure time with SOC_e^{OBJ} , two additional constraints must be added [43],

$$\bar{P}_{e,t} = \begin{cases} 0, & \forall t \notin [t_e^{arr}, t_e^{dep}] \\ p_e^{EV} & \forall t \in [t_e^{arr}, t_e^{dep}] \end{cases} \quad \forall e \in \Omega_e, \forall t \in \Omega_t \quad (15a)$$

$$soc_{e,t_e^{dep}} = SOC_e^{OBJ} \quad \forall e \in \Omega_e \quad (15b)$$

Thermal characteristics of the HVAC systems are modelled by a first order discrete temperature model in (16). Let $\tau_{b,t}$ be the temperature of the building, $\tau_{b,t}^{out}$ be the ambient temperature, $p_{b,t}^{he}, p_{b,t}^{co}$ be the heating and cooling power, R_b, C_b , be thermal constants and η_b^{he}, η_b^{co} are efficiencies of the heating and cooling [44]. Then, the HVAC system is modelled by,

$$\begin{aligned} \tau_{b,t+1} = & \tau_{b,t} + \frac{\Delta t}{R_b C_b} [\tau_{b,t}^{out} - \tau_{b,t}] \\ & + \frac{\Delta t}{C_b} [\eta_b^{he} p_{b,t}^{he} - \eta_b^{co} p_{b,t}^{co}] \quad \forall b \in \Omega_b, \forall t \in \Omega_t \end{aligned} \quad (16a)$$

$$\underline{\tau}_{b,t} \leq \tau_{b,t} \leq \bar{\tau}_{b,t} \quad \forall b \in \Omega_b, \forall t \in \Omega_t \quad (16b)$$

$$0 \leq p_{b,t}^{he} \leq \bar{P}_b^{he}, \quad 0 \leq p_{b,t}^{co} \leq \bar{P}_b^{co} \quad \forall b \in \Omega_b, \forall t \in \Omega_t \quad (16c)$$

REFERENCES

- [1] N. K. Dhaliwal, F. Bouffard, and M. J. O'Malley, "A Fast Flexibility-Driven Generation Portfolio Planning Method for Sustainable Power Systems," *IEEE Trans. Sustain. Energy*, no. 1, pp. 368–377, 2021.
- [2] G. Tsaousoglou, J. S. Giraldo, and N. G. Paterakis, "Market Mechanisms for Local Electricity Markets: A review of models, solution concepts and algorithmic techniques," *Renew. Sustain. Energy Rev.*, vol. 156, no. 754462, pp. 82–96, 2022.
- [3] P. Pinson, "What may future electricity markets look like?" 2023. [Online]. Available: arXiv:2302.02833
- [4] J. Villar, R. Bessa, and M. Matos, "Flexibility products and markets: Literature review," *Electr. Power Syst. Res.*, vol. 154, pp. 329–340, 2018.
- [5] F. Escobar, J. M. Viquez, J. Garcia, P. Aristidou, and G. Valverde, "Coordination of DERs and Flexible Loads to Support Transmission Voltages in Emergency Conditions," *IEEE Trans. Sustain. Energy*, vol. 13, no. 3, pp. 1344–1355, 2022.
- [6] X. Jin, Q. Wu, and H. Jia, "Local flexibility markets: Literature review on concepts, models and clearing methods," *Appl. Energy*, vol. 261, pp. 114387–114422, 2020.
- [7] T. Schittekatte and L. Meeus, "Flexibility markets: Q&A with project pioneers," *Util. Policy*, vol. 63, pp. 101017–101028, 2020.
- [8] K. Zhang, S. Troitzsch, and X. Han, "Distributionally robust co-optimized offering for transactive multi-energy microgrids," *Int. J. Electr. Power Energy Syst.*, vol. 143, pp. 108451–108462, 2022.
- [9] J. Naughton, H. Wang, M. Cantoni, and P. Mancarella, "Co-Optimizing Virtual Power Plant Services under Uncertainty: A Robust Scheduling and Receding Horizon Dispatch Approach," *IEEE Trans. Power Syst.*, vol. 36, no. 5, pp. 3960–3972, 2021.
- [10] Y. Zhou, J. Wang, C. Wei, and Y. Li, "Active energy interaction between multiple distributed energy systems with flexible decentralization operation: Energy bank," *Sustain. Cities Soc.*, pp. 104071–104085, 2022.
- [11] B. Huang and J. Wang, "Deep-Reinforcement-Learning-Based Capacity Scheduling for PV-Battery Storage System," *IEEE Trans. Smart Grid*, vol. 12, no. 3, pp. 2272–2283, 2021.
- [12] G. Tsaousoglou, K. Mitropoulou, K. Steriotis, N. G. Paterakis, P. Pinson, and E. Varvarigos, "Managing Distributed Flexibility Under Uncertainty by Combining Deep Learning With Duality," *IEEE Trans. Sustain. Energy*, vol. 12, no. 4, pp. 2195–2204, 2021.
- [13] P. V. Brogan, R. Best, J. Morrow, R. Duncan, and M. Kubik, "Stacking battery energy storage revenues with enhanced service provision," *IET Smart Grid*, vol. 3, no. 4, pp. 520–529, 2020.
- [14] P. Zamani-Dehkordi, H. Chitsaz, L. Rakai, and H. Zareipour, "A price signal prediction method for energy arbitrage scheduling of energy storage systems," *Int. J. Electr. Power Energy Syst.*, vol. 122, pp. 106122–106131, 2020.
- [15] F. Fan, G. Zorzi, D. Campos-Gaona, G. Burt, O. Anaya-Lara, J. Nwobu, and A. Madariaga, "Sizing and coordination strategies of battery energy storage system co-located with wind farm: The uk perspective," *Energies*, vol. 14, no. 5, pp. 1439–1460, 2021.
- [16] A. O. Gbadegehin, Y. Sun, and N. I. Nwulu, "Stacked value streams of hybrid energy storage systems in prosumer microgrids," *J. Eng. Des. Technol.*, vol. 19, no. 5, pp. 1063–1079, 2020.
- [17] Y. Tian, A. Bera, M. Benidris, and J. Mitra, "Stacked Revenue and Technical Benefits of a Grid-Connected Energy Storage System," *IEEE Trans. Ind. Appl.*, vol. 54, no. 4, pp. 3034–3043, 2018.
- [18] A. A. Mohamed, C. Sabillon, A. Golriz, and B. Venkatesh, "Value-stack aggregator optimal planning considering disparate DERs technologies," *IET Gener. Transm. Distrib.*, vol. 15, no. 18, pp. 2632–2644, 2021.
- [19] A. W. Dowling, R. Kumar, and V. M. Zavala, "A multi-scale optimization framework for electricity market participation," *Appl. Energy*, vol. 190, pp. 147–164, 2017.
- [20] E. Nasrolahpour, J. Kazempour, H. Zareipour, and W. D. Rosehart, "A Bilevel Model for Participation of a Storage System in Energy and Reserve Markets," *IEEE Trans. Sustain. Energy*, vol. 9, no. 2, pp. 582–598, 2018.
- [21] H. Pandžić, Y. Dvorkin, and M. Carrión, "Investments in merchant energy storage: Trading-off between energy and reserve markets," *Appl. Energy*, vol. 230, pp. 277–286, 2018.
- [22] P. Fazlalipour, M. Ehsan, and B. Mohammadi-Ivatloo, "Risk-aware stochastic bidding strategy of renewable micro-grids in day-ahead and real-time markets," *Energy*, vol. 171, pp. 689–700, 2019.
- [23] M. A. Mirzaei, M. Hemmati, K. Zare, M. Abapour, B. Mohammadi-Ivatloo, M. Marzband, and A. Anvari-Moghaddam, "A novel hybrid two-stage framework for flexible bidding strategy of reconfigurable micro-grid in day-ahead and real-time markets," *Int. J. Electr. Power Energy Syst.*, vol. 123, pp. 106293–106309, 2020.
- [24] H. Tang and S. Wang, "Multi-level optimal dispatch strategy and profit-sharing mechanism for unlocking energy flexibilities of non-residential building clusters in electricity markets of multiple flexibility services," *Renew. Energy*, vol. 201, no. P1, pp. 35–45, 2022.
- [25] M. Farahani, A. Samimi, and H. Shateri, "Robust bidding strategy of battery energy storage system (BESS) in joint active and reactive power of day-ahead and real-time markets," *J. Energy Storage*, vol. 59, pp. 106520–106540, 2023.

- [26] K. Steriotis, K. Sepetanc, K. Smpoukis, N. Efthymiopoulos, P. Makris, E. Varvarigos, and H. Pandzic, "Stacked Revenues Maximization of Distributed Battery Storage Units Via Emerging Flexibility Markets," *IEEE Trans. Sustain. Energy*, vol. 13, no. 1, pp. 464–478, 2022.
- [27] T. Zhao, Y. Li, X. Pan, P. Wang, and J. Zhang, "Real-Time Optimal Energy and Reserve Management of Electric Vehicle Fast Charging Station: Hierarchical Game Approach," *IEEE Trans. Smart Grid*, vol. 9, no. 5, pp. 5357–5370, 2018.
- [28] T. Jiang, C. Y. Chung, P. Ju, and Y. Gong, "A Multi-Timescale Allocation Algorithm of Energy and Power for Demand Response in Smart Grids: A Stackelberg Game Approach," *IEEE Trans. Sustain. Energy*, vol. 13, no. 3, pp. 1580–1593, 2022.
- [29] M. Rahimiyan and L. Baringo, "Strategic Bidding for a Virtual Power Plant in the Day-Ahead and Real-Time Markets: A Price-Taker Robust Optimization Approach," *IEEE Trans. Power Syst.*, vol. 31, no. 4, pp. 2676–2687, 2016.
- [30] M. Shafiekhani, A. Badri, M. Shafie-khah, and J. P. Catalão, "Strategic bidding of virtual power plant in energy markets: A bi-level multi-objective approach," *Int. J. Electr. Power Energy Syst.*, vol. 113, pp. 208–219, 2019.
- [31] S. Pineda and J. M. Morales, "Capacity expansion of stochastic power generation under two-stage electricity markets," *Comput. Oper. Res.*, vol. 70, pp. 101–114, 2016.
- [32] L. Mitridati, P. Van Hentenryck, and J. Kazempour, "Electricity-Aware Bid Format for Heat Commitment and Dispatch," 2020. [Online]. Available: arXiv:2005.03120
- [33] J. Yang, N. Zhang, C. Kang, and Q. Xia, "A State-Independent Linear Power Flow Model with Accurate Estimation of Voltage Magnitude," *IEEE Trans. Power Syst.*, vol. 32, no. 5, pp. 3607–3617, 2017.
- [34] G. Byeon and P. Van Hentenryck, "Unit Commitment with Gas Network Awareness," *IEEE Trans. Power Syst.*, no. 2, pp. 1327–1339, 2020.
- [35] I. Savelli and T. Morstyn, "Electricity prices and tariffs to keep everyone happy: A framework for fixed and nodal prices coexistence in distribution grids with optimal tariffs for investment cost recovery," *Omega (United Kingdom)*, vol. 103, pp. 102450–102467, 2021.
- [36] M. MansourLakouraj, M. Shahabi, M. Shafie-khah, and J. P. Catalão, "Optimal market-based operation of microgrid with the integration of wind turbines, energy storage system and demand response resources," *Energy*, vol. 239, pp. 122156–122170, 2022.
- [37] J. Morales, S. Pineda, A. Conejo, and M. Carrion, "Scenario Reduction for Futures Market Trading in Electricity Markets," *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 878–888, 2009.
- [38] A. Navarro-Espinosa and P. Mancarella, "Probabilistic modeling and assessment of the impact of electric heat pumps on low voltage distribution networks," *Appl. Energy*, vol. 127, pp. 249–266, 2014.
- [39] S. Meinecke, D. Sarajlić, S. R. Drauz, A. Klettke, L. P. Lauven, C. Rehtanz, A. Moser, and M. Braun, "SimBench-A benchmark dataset of electric power systems to compare innovative solutions based on power flow analysis," *Energies*, vol. 13, no. 12, pp. 3290–3309, 2020.
- [40] W. E. Hart, J.-P. Watson, and D. L. Woodruff, "Pyomo: modeling and solving mathematical programs in python," *Mathematical Programming Computation*, vol. 3, no. 3, pp. 219–260, 2011.
- [41] M. P. Čalasan, L. Nikitović, and S. Mujović, "CONOPT solver embedded in GAMS for optimal power flow," *J. Renew. Sustain. Energy*, vol. 11, no. 4, pp. 046301–046316, 2019.
- [42] P. Olivella-Rosell, F. Rullan, P. Lloret-Gallego, E. Prieto-Araujo, R. Ferrer-San-José, S. Barja-Martinez, S. Bjarghov, V. Lakshmanan, A. Hentunen, J. Forström, S. O. Øttesen, R. Villafila-Robles, and A. Sumper, "Centralised and Distributed Optimization for Aggregated Flexibility Services Provision," *IEEE Trans. Smart Grid*, vol. 11, no. 4, pp. 3257–3269, 2020.
- [43] T. Morstyn, K. A. Collett, A. Vijay, M. Deakin, S. Wheeler, S. M. Bhagavathy, F. Fele, and M. D. McCulloch, "OPEN: An open-source platform for developing smart local energy system applications," *Appl. Energy*, vol. 275, pp. 115397–115408, 2020.
- [44] C. Zhong, A. P. S. Meliopoulos, B. Xie, J. Xie, and K. Liu, "Multi-Stage Quadratic Flexible Optimal Power Flow With a Rolling Horizon," *IEEE Trans. Smart Grid*, vol. 12, no. 4, pp. 3128–3137, 2021.

he is a visiting researcher at the University of Edinburgh, UK. His research interests include operation of energy systems and development of bidding strategies for market participation.

Jose A. Aguado (M'01) received the Ingeniero Eléctrico and Ph.D. degrees from the University of Málaga, Málaga, Spain, in 1997 and 2001, respectively. Currently, he is full Professor and Head of the Department of Electrical Engineering at the University of Málaga. His research interests include operation and planning of smart grids and numerical optimization techniques.

Chaimaa Essayeh Research associate, Market design for scaling up local clean energy systems at School of engineering, University of Edinburgh. She holds an MSc in communication systems and a PhD in computer science. Her research is focused on AI for smart grids with interests in optimal energy management strategies, advanced methods for energy and consumption forecasting and renewable sources integration with the grid. This involves the application of advanced methods from multi-agent control, stochastic optimization and game theory.

Yuxin Xia received the BEng degree in Industrial Electronics and Control Engineering from Liverpool John Moores University, Liverpool, UK and the MSc degree in Control Systems from Imperial College London, UK. She is currently working toward the Ph.D. degree with the Institute for Energy Systems, the University of Edinburgh, UK. Her research interests include power system expansion planning, power system economics and the integration of distributed energy resources into power system operations.

Iacopo Savelli He holds a PhD in Engineering, an MS in Finance, and a BS in Economics. He was a postdoc at the University of Edinburgh and the University of Oxford, working on Energy Market Design. He presented his works in several forums, including the European Commission's Joint Research Centre in Ispra. He received the 1st Prize Best Conference Paper award at the 17th European Energy Market conference in 2020. He is currently a postdoctoral researcher (RTDa) in Applied Economics at the GREEN centre since January 2023. His current research interests include power system economics, with major focus on electricity markets.

Thomas Morstyn (Senior Member, IEEE) received the B.Eng. degree (Hon.) in electrical engineering from the University of Melbourne in 2011, and the Ph.D. degree in electrical engineering from the University of New South Wales in 2016. He is a Lecturer of Power Electronics and Smart Grids with the School of Engineering, University of Edinburgh. He is also the Deputy Champion of Energy Distribution and Infrastructure for the Scottish Energy Technology Partnership. His research interests include multi-agent control and market design for integrating distributed energy resources into power system operations. He is an Associate Editor of the IEEE TRANSACTIONS ON POWER SYSTEMS.

Ángel Paredes (S'21) is a PhD candidate at the Department of Electrical Engineering of the University of Málaga (UMA), Spain. He received the B.Sc. (Hons) in Mechanical and Electrical Engineering and the M.Sc. in Industrial Engineering from the UMA, Spain, in 2019 and 2021, respectively. Currently,