# Monitoring Indirect Contagion*

Rama Cont[†] Eric Schaanning[‡]

Revised version: March 2019

**Abstract**

We propose two indicators for quantifying the potential exposure of financial institutions to indirect contagion arising from deleveraging of assets in stress scenarios. The first indicator, the Endogenous Risk Index (ERI) captures spillovers across portfolios arising from deleveraging in stress scenarios. The second indicator, the Indirect Contagion Index (ICI) measures the systemic importance of a bank by quantifying the loss its distressed liquidation would inflict on other institutions. Both are computable from portfolio holdings of financial institutions and measures of market depth for the assets held in the portfolio. We discuss the micro-foundation of these indicators and apply them to the analysis of the vulnerability of the European banking system to indirect contagion.

Using data on portfolio holdings of European banks, we show that our indicators correlate to the magnitude of fire-sales losses in simulated stress scenarios, thus providing a simple to compute proxy for the outcome of stress tests. We also show that the information provided by our indicators on the systemic importance of banks is different from indicators based on size, thereby providing a measure of interconnectedness complementary to those currently used by supervisors.

**JEL classification:** C63, E58, G17, G18, G28.

**Keywords:** Financial stability, price-mediated contagion, macro prudential regulation, systemic risk measurement.

*The views expressed are those of the authors and do not necessarily reflect those of Norges Bank, the European Systemic Risk Board or the European Central Bank.

†Mathematical Institute, University of Oxford. Rama.Cont@maths.ox.ac.uk.

‡ETH Zurich, RiskLab, Department of Mathematics, Rämistrasse 101, 8092 Zürich, and Norges Bank, Bankplassen 2, 0151 Oslo. eric.schaanning@esrb.europa.eu.

# Contents

# 1 Indirect contagion

Fire sales and portfolio deleveraging by financial institutions in distress have been identified, in theoretical and empirical studies, as possible mechanisms for market instability (Shleifer and Vishny, 1992) and contagion (Kyle and Xiong, 2001; Ellul et al., 2011). The market impact of asset sales arising from the deleveraging of institutional portfolios may potentially lead to mark-to-market losses and trigger destabilizing feedback loops. In contrast to direct contagion mechanisms arising from counterparty exposures or funding relations (Battiston et al., 2012), this type of contagion is *indirect* and arises from common asset holdings, which are a natural product of diversification.

This has led to a need for indicators for quantifying the vulnerability of the financial system to such indirect contagion and identifying institutions which are systemically important for this channel of contagion. One approach to assessing such vulnerabilities is to perform systemic stress tests which account for indirect contagion. This approach, applied to the US and EU banking system (Greenwood et al., 2015; Duarte and Eisenbach, 2013; Cont and Schaanning, 2016), has identified common asset holdings as the main driver of exposures to indirect contagion, confirming earlier theoretical studies (Cont and Wagalath, 2013; Caccioli et al., 2014). However, these studies leave open the question of designing appropriate institution-level indicators for quantifying the contribution of each institution to this channel of contagion.

**Contributions.** Based on the insights derived from these studies, we introduce two indicators to quantify the potential contribution of financial institutions to indirect contagion arising from deleveraging of assets in stress scenarios.

1. the **Endogenous Risk Index** ($ERI$), defined as a measure of (eigenvector) centrality in the network defined by liquidity-weighted portfolio overlaps, is shown to be a good proxy for the amplitude of losses in fire sales scenarios.

2. the **Indirect Contagion Index** ($ICI$) of a financial institution quantifies the loss inflicted on other financial institutions by the deleveraging of this institution's portfolio. Differently from $ERI$, it excludes self-inflicted losses resulting from market impact of liquidation. We argue that the $ICI$ is useful as an additional indicator in the framework of the Basel GSIB methodology to quantify the interconnectedness of systemically important institutions.

We discuss the micro-foundation of these indicators and show them to be good proxies for the losses due to fire sales in systemic stress tests such as those described in (Greenwood et al., 2015; Duarte and Eisenbach, 2013) and Cont and Schaanning (2016).

We illustrate the usefulness of our indicators for the analysis of the vulnerability of the European banking system to indirect contagion using data from the 2016 EBA stress test. Through extensive robustness checks, we show the predictive power of our indicators is robust to a wide range of modeling assumptions such as the market depth, the liquidation strategies employed by institutions to delever portfolios, or the scenarios that trigger these distressed liquidations.

**Link to previous literature.** Our contributions complement the previous literature on the role of diversification and commonalities in asset holdings in financial stability and systemic risk.

Loss contagion via fire sales in a multi-asset setting was examined in Kyle and Xiong (2001). As underlined by (Ibragimov et al., 2011) and (Beale et al., 2011), diversification may be optimal from an institution's individual point of view but can increase exposure to systemic risk, leading to externalities. (Cont and Wagalath, 2013) analyse cross-asset contagion and loss amplification in a model with multiple assets and emphasise the role of common asset holdings as measured by *portfolio overlaps*. (Caccioli et al., 2014) show that deleveraging cascades arising from common asset holdings lead to a "stable yet fragile" behaviour: contagion has a small likelihood to occur, but may be catastrophic if it does. (Braouezec and Wagalath, 2018) and (Coen et al., 2017) study the optimal deleveraging of bank portfolios in response to stress. (Wagner, 2011) shows that, in the presence of liquidation risks, it is rational for investors to forego some diversification benefits, so as to reduce their exposure to crowding risk.

Many empirical studies confirm these theoretical insights. (Boyer et al., 2006) and (Jotikasthira et al., 2012) provide empirical evidence for the spillover effects of portfolio rebalancing by large institutional investors. (Ellul et al., 2011) study the forced divestment of downgraded bonds by insurance companies and their impact on prices and emphasise the role of portfolio constraints, which was pointed out in (Shleifer and Vishny, 1992). The effects on prices are all the more important when the buyers (hedge funds in (Ellul et al., 2011)) are constrained. (Khandani and Lo, 2011; Pedersen, 2009) analyse the role of indirect contagion in the Quant crash of August 2007; (Cont and Wagalath, 2016) analysed the role of price-mediated contagion in the Quant Crash of 2007 and proposed a statistical methodology for the forensic analysis of fire sales events. (Greenwood et al., 2015) (Duarte and Eisenbach, 2013) and Cont and Schaanning (2016) use systemic stress tests to study the exposure of the banking system to indirect contagion.

These studies underline the role played by common asset holdings and the associated notion of *crowding risk* (Pedersen, 2009). (Cont and Wagalath, 2013) elaborate this connection by clarifying the role played by (liquidity-weighted) *portfolio overlaps*, a measure of commonality in asset holdings, in the transmission of mark-to-market losses. Many empirical studies confirm the importance of portfolio overlaps. (Guo et al., 2015) analyse the topology of common holdings in the US fund industry and find that greater overlap contributes to greater negative excess returns in the presence of liquidations. (Braverman and Minca, 2016) use a related network of portfolio overlaps to propose a measures of vulnerability to stress. (Getmansky et al., 2016) show that insurance companies with a higher measure of commonality tend to engage in larger common sales, regardless of portfolio size. (Blei and Bakhodir, 2014) measure the degree of commonality between large portfolios and relate it to the build-up of systemic risk prior to the 2008 crisis. (Capponi and Larsson, 2015) propose a global indicator of the potential for contagion in a network of interlinked balance sheets in terms of the spectral radius of a matrix whose components depend on the price elasticity of assets, portfolio sizes and holdings. (Feinstein and El-Masri, 2017) embed a model of fire sales in multiple asset classes into the standard Eisenberg & Noe framework and derive sufficient conditions for the existence of an equilibrium liquidation strategy with corresponding clearing payments and liquidation prices. (Markose et al., 2012) consider the central eigenvector of a net-CDS exposure matrix to compute a super-spreader tax to mitigate socialised losses. (Billio et al., 2012) compare different econometric measures to quantify the increase of interconnections between the banks, broker-dealers, insurers and hedge-funds.

Our study complements these approaches by proposing *institution-level* risk indicators, as opposed to *global* risk indicators, which have a micro-foundation in terms of a loss contagion mechanism which we describe in detail.

By exploiting the information contained in portfolio holdings of financial institutions, our indicators quantify the potential for future losses not necessarily reflected in market data, thereby complementing the information contained in market-based indicators of systemic risk such as ($\Delta$-)CoVaR"(Adrian and Brunnermeier, 2016), SRISK (Brownlees and Engle, 2016), and the Marginal Expected Shortfall "(Acharya et al., 2017).

**Structure.** The paper is organised as follows: Section 2 defines the Endogenous Risk Index and the Indirect Contagion Index and discusses their micro-foundation in terms of a model of portfolio deleveraging in response to stress. Sections 3.1 and 3.2 apply our methodology to a dataset of European Banks. A detailed sensitivity analysis to parameters is given in Section 3.3. Alternative deleveraging assumptions are discussed in Section 3.4. Section 3.5 compares the $ERI$ and $ICI$ to other measures of portfolio similarity. Section 3.6 suggests an application of the indicators to monitor Global Systemically Important Banks (GSIBs) in the Basel Committee on Banking Supervisions' framework. Section 4 concludes.

# 2 Monitoring indirect contagion

## 2.1 Fire-sales losses and liquidity-weighted overlaps

We consider a financial network consisting of $i = 1..N$ institutions and $\mu = 1..M$ securities. The institutions are subject to regulatory, market- or self-imposed constraints, which may depend on their capital, leverage or liquidity levels. Let $\Pi^{i,\mu}$ denote the value of institution $i$'s holdings in asset class $\mu$ in monetary units. The portfolio holdings of the entire system are given by the matrix $\Pi \in \mathbb{R}^{N \times M}$. This matrix corresponds to a bi-partite network of institutions and asset classes, as shown in Figure 1.

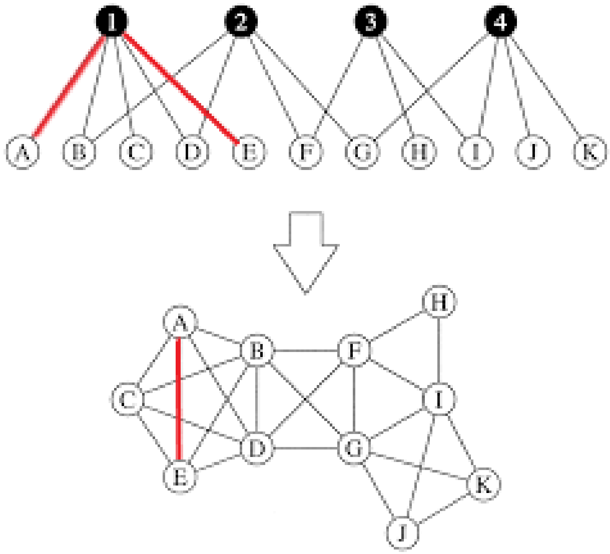


Figure 1: Bipartite network of institutions A-K and asset classes 1-4, which gives rise to a network of indirect contagion between institutions.

Such a network can give rise to *indirect* contagion: Even though institutions A through E may have no connection, institutions B to E may suffer losses if for some reason A is forced to liquidate its position in asset class 1, thereby depressing its price. Specifically, two types of losses can arise: (i) mark-to-market losses on remaining holdings, suffered by all parties that hold the asset undergoing the distressed liquidation and (ii) implementation costs which the liquidating institution suffers on the position it is trying to liquidate. Moreover, if, as a result of the forced deleveraging by A, institutions B or D need to liquidate a part of their portfolio, the price of asset class 2 may drop and cause losses to institutions F and G, who were previously unaffected. This illustrates how overlapping portfolios can become a driver of *cross-asset class contagion*, even when the asset classes are economically and geographically essentially independent.

In order to quantify this phenomenon properly, we need to specify: (i) *when* institutions react to a shock, (ii) *how* they react when forced to, and (iii) *how* prices respond to forced liquidations. Using these three building blocks, we show that an endogenous risk index naturally arises when one quantifies the liquidation losses in such a network of constrained and overlapping portfolios.

The presentation follows the model discussion in Cont and Schaanning (2016), to which we refer the reader for further details:

1. At time $k = 1$, in response to a stress scenario, parametrised by percentage shocks

to asset classes $\epsilon \in [0,1]^M$, some institutions $j$ deleverage their portfolio by selling a proportion $\Gamma^j_k(\epsilon) \in [0,1]$ of marketable assets. In Section 3.4, we assume that banks do not sell assets proportionately, but choose $\Gamma^j$ so as to minimise the liquidation losses they suffer. This leads to an aggregate amount $q^\mu_k = \sum_{j=1}^N \Pi^{j,\mu}_{k-1}\Gamma^j_k(\epsilon)$ of sales in asset class $\mu$. In general, the deleveraging proportion, $\Gamma^j_k(\epsilon)$ can depend on the capital buffer, the liquidity reserves and other balance sheet components of institution $j$. In particular, when $j$ is well-capitalised and has a prudent liquidity buffer, it will be able to withstand the shock without resorting to distressed liquidations, in which case $\Gamma^j(\epsilon) = 0$. We specify $\Gamma$ precisely in the empirical section and will drop the $\epsilon$ in $\Gamma(\epsilon)$ for notational convenience.

2. The market price of asset class $\mu$ is denoted by $S^\mu_k$. The impact of asset sales results in a decline of the market price, moving it to

$$S^\mu_k = S^\mu_{k-1}\left(1 - \frac{q^\mu_k}{D_\mu}\right), \tag{1}$$

where the **market depth** $D_\mu(\tau) \propto \frac{ADV_\mu}{\sigma_\mu}\sqrt{\tau}$ is proportional to the ratio of the average daily volume (ADV) and the volatility of the asset class times the liquidation horizon, $\tau$, which is assumed to be $\tau = 20$ days in our calibration. This corresponds to a linear price impact function $\frac{\Delta S}{S} = -\Psi_\mu(x) := \frac{x}{D_\mu}$, as used in ((Obizhaeva, 2012), (Kyle and Obizhaeva, 2016), (Amihud, 2002), (Cont and Wagalath, 2016)). For a more general discussion on non-linear price impact functions in the context of stress testing, we refer to (Cont and Schaanning, 2016). Section 3.3 includes a detailed sensitivity analysis with respect to assumptions on market depth.

3. For any institution $i$, the combined effect of its own deleveraging (if it occurs) and the impact of other forced sales changes the market value of its holdings in asset class $\mu$ to

$$\Pi^{i,\mu}_k = \underbrace{\left(1 - \Gamma^i_k\right)}_{\text{Remainder after deleveraging by } i} \overbrace{\Pi^{i,\mu}_{k-1}}^{\text{Previous value}} \underbrace{\left(1 - D^{-1}_\mu \sum_{j=1}^N \Gamma^j_k \Pi^{j,\mu}_{k-1}\right)}_{\text{Price impact on remaining holdings}}. \tag{2}$$

Importantly, this shows that in general the value of institution $i$'s holdings under stress cannot simply be inferred from historically observed returns and covariances but may depend on the stress scenario and the network of overlapping portfolios.

The price move generates two types of losses for portfolio $i$: First, it causes *mark to* *market losses* on the remaining part of the portfolio given by

$$M_k^i = \sum_{\mu=1}^{M} \left((1-\Gamma_k^i)\Pi_{k-1}^{i,\mu} - \Pi_k^{i,\mu}\right) = (1-\Gamma_k^i)\sum_{\mu=1}^{M} \Pi_{k-1}^{i,\mu} D_\mu^{-1} \left(\sum_{j=1}^{N} \Gamma_k^j \Pi_{k-1}^{j,\mu}\right)$$
$$= (1-\Gamma_k^i)\sum_{j=1}^{N}\sum_{\mu=1}^{M} \frac{\Pi_{k-1}^{i,\mu}\Pi_{k-1}^{j,\mu}}{D_\mu}\Gamma_k^j. \tag{3}$$

A second source of loss stems from the fact that assets are not liquidated at the current market price but at a discount. We assume for simplicity that the deleveraging institutions suffer the same price impact on the liquidated part of the portfolio as on their remaining part, yielding the *realised loss*

$$R_k^i = \sum_{\mu=1}^{M} \left(\Gamma_k^i \Pi_{k-1}^{i,\mu} - \Gamma_k^i \Pi_{k-1}^{i,\mu}\left(1 - \frac{q_k^\mu}{D_\mu}\right)\right) = \Gamma_k^i \sum_{j=1}^{N}\sum_{\mu=1}^{M} \frac{\Pi_{k-1}^{i,\mu}\Pi_{k-1}^{j,\mu}}{D_\mu}\Gamma_k^j. \tag{4}$$

Summing (4) and (3) yields the total loss of portfolio $i$ at the $k$-th round of deleveraging:

$$L_k^i = \sum_{j=1}^{N} \underbrace{\sum_{\mu=1}^{M} \frac{\Pi_{k-1}^{i,\mu}\Pi_{k-1}^{j,\mu}}{D_\mu}}_{\Omega^{i,j}(\Pi_{k-1})} \Gamma_k^j = \sum_{j=1}^{N} \Omega_{k-1}^{i,j}\Gamma_k^j,$$

which shows that the magnitude of fire sales spillovers from institution $i$ to institution $j$ is proportional to the *liquidity-weighted overlap* $\Omega^{i,j}$ between portfolios $i$ and $j$ (Cont and Wagalath, 2013):

$$\Omega^{i,j}(\Pi_k) := \sum_{\mu=1}^{M} \frac{\Pi_k^{i,\mu}\Pi_k^{j,\mu}}{D_\mu}.$$

or, in matrix form

$$\Omega(\Pi_k) = \Pi_k D^{-1} \Pi_k^\top, \tag{5}$$

where $D := \mathrm{diag}(D_1, \ldots, D_M)$

can be viewed as a (liquidity-)weighted adjacency matrix of the network linking portfolios through their common holdings.

## 2.2 The Endogenous Risk Index.

From the derivation above, it follows that in the first round, the fire-sales losses for all banks are given by the vector

$$FLoss = \Omega\Gamma. \tag{6}$$

As $\Omega$ is symmetric positive semi-definite by construction, we know that there exists an orthonormal basis of eigenvectors with real eigenvalues for it. We further assume that $\Omega$ is an irreducible and non-negative matrix. This is equivalent to the network of overlapping portfolios being strongly connected (i.e. it is not a union of disjoint sub-networks) and that there are no pairs of portfolios such that $\sum_{\mu=1}^{M} \frac{\Pi^{i,\mu}\Pi^{j,\mu}}{D_\mu} < 0$ respectively. The European banking network which we use in our empirical analysis satisfies these assumptions. Under these conditions, the Perron-Frobenius theorem ensures that the components of the principal eigenvector are positive.

Furthermore, if the first eigenvalue dominates, as observed in the EBA data, the network of asset holdings is well represented aby a one-factor model

$$\Omega \approx \lambda_1 u u^\top, \tag{7}$$

where $u$ is the principal eigenvector corresponding to the largest eigenvalue $\lambda_1$. Using this approximation, the fire-sales loss can be written as

$$FLoss^i = \lambda_1 u_i \sum_{j=1}^{N} u_j \Gamma^j(\epsilon).$$

Taking logarithms, we get

$$\begin{aligned} \log(FLoss^i) &= \log(\lambda_1 u_i \sum_{j=1}^{N} u_j \Gamma^j(\epsilon)) \\ &= \underbrace{1\times}_{\text{slope}} \log(u_i) + \underbrace{\log(\lambda_1) + \log(u^\top \Gamma(\epsilon))}_{\text{Intercept}}, \end{aligned} \tag{8}$$

which implies that the logarithm of the principal eigenvector should be a good predictor of fire-sales losses. Moreover, we should expect a slope of approximately one when regressing log-fire-sales losses on it. Equation (8) motivates the following definition:

**Definition 1** (Endogenous Risk Index)**.** *Let $\Omega$ be the matrix of liquidity-weighted portfolio overlaps, defined in* (5) *and $u = (u_i, i \in I)$ the (normalised) principal eigenvector of $\Omega$, corresponding to the largest eigenvalue, where the label $i$ corresponds to financial institution $i$ and $\|u\| = 1$.*

*The* **Endogenous Risk Index (**$ERI$**)** *of a financial institution is defined as its component in the principal eigenvector:*

$$ERI(i) = u_i. \tag{9}$$

$ERI$ provides a measure of centrality of the node $i$ in the network whose links are weighted by the overlap matrix $\Omega$. The $ERI$ is constructed as follows:

1. Collect portfolio holdings $\Pi^{i,\mu}$ by asset class for each financial institution in the network, at the granularity level corresponding to bank stress tests.

2. Estimate a market depth parameter $D_\mu \propto \frac{ADV_\mu}{\sigma_\mu}$ for each asset class.

3. Check that $\Omega^{i,j} \geq 0$ and that $\Omega$ is irreducible.

4. Compute the largest eigenvalue and the corresponding eigenvector (the "principal eigenvector") $u = (u_i, i = 1...N)$ of the matrix of liquidity-weighted overlaps $\Omega(\Pi) = \Pi D^{-1}\Pi^\top$.

5. The Endogenous Risk Index is the principal eigenvector, $ERI = u$. An institution's ERI is its component in $u$.

## 2.3 The Indirect Contagion Index

Before turning to the empirical application, we propose a modification relative to the $ERI$ that discounts the fire-sales losses that a distressed portfolio liquidation would inflict on itself, and only take into consideration the spillover-losses that are generated to *other* portfolios. We call this measure the **Indirect Contagion Index** ($ICI$):

**Definition 2** (Indirect Contagion Index)**.** *Let $\Omega$ be the matrix of liquidity-weighted portfolio overlaps, defined in* (5) *and*

$$E = \Omega - diag(\Omega_{11}, \ldots, \Omega_{NN})$$

*its non-diagonal component. Let $v = (v_i, i \in I)$ be the (normalized) principal eigenvector of $E$, corresponding to the largest eigenvalue, normalized via $\|v\| = 1$.*

*The* **Indirect Contagion Index** ($ICI$) *of a financial institution is defined as its component in the principal eigenvector:*

$$ICI(i) = v_i. \tag{10}$$

To compute $ICI$ we

1. Compute the matrix of liquidity-weighted portfolio overlaps $\Omega = \Pi D^{-1}\Pi^\top$.

2. Compute the largest eigenvalue and the corresponding principal eigenvector $v$ of $E := \Omega - \mathrm{diag}(\Omega_{11}, \ldots, \Omega_{NN})$.

3. The Indirect Contagion Index is given by the components of $v$.

We illustrate the difference between the $ERI$ and the $ICI$, in a simple financial network of 7 banks and 2 asset classes given by:

$$\Pi = \begin{pmatrix} 1000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 100 & 1100 & 100 & 100 & 100 & 100 & 100 \end{pmatrix}^{\top}$$

$$D = (1000, 2000)^{\top}.$$

Banks 1 and 2 are thus large (and equal in size), while banks 3 - 7 are smaller. Asset class 1 is twice as *illiquid* as asset class 2. The left panel of Figure 2 shows the network, where the blue squares denote banks, the red circles denote asset classes and the size of the edge denotes the magnitude of the holding (not to scale). The bigger banks are denoted by larger squares; the more illiquid asset class is depicted by a larger circle. The right panel of Figure 2 shows the $ICI$ (blue bars) and the $ERI$ (black crosses) for this network. Bank 1 clearly dominates the $ERI$ ranking. This is due to its main holdings being less liquid compared to Bank 2's holdings, and the amount of self-contagion that this could trigger. In contrast, Bank 2 has a large position in asset class 2, which all the medium-sized banks hold as well. Consequently, even though the same liquidation volume for Bank 2 in asset class 2 would cause a smaller loss than it would for Bank 1 in asset class 1, if Bank 2 delevers, the rest of the system also suffers significant fire-sales losses. The $ICI$ discounts the self-inflicted fire-sales loss and only accounts for system externalities, which is why in the $ICI$ ranking, Bank 2 dominates, and Bank 1 is equally ranked as the medium-sized banks.

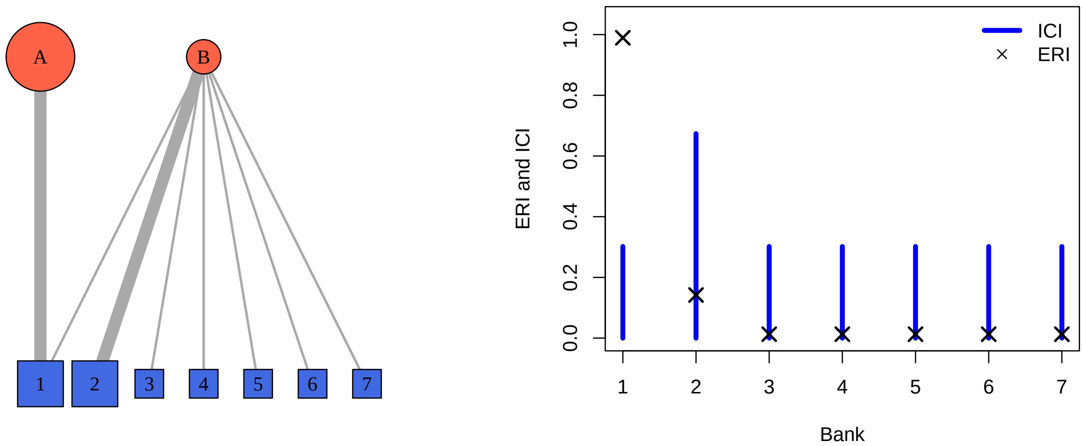


Figure 2: Illustrative example showing how the $ICI$ discounts self-inflicted losses compared to the losses caused to other participants relative to the $ERI$.

We will further discuss the difference between the $ERI$ and the $ICI$ for European banks in the sections below.

# 3 Vulnerability to indirect contagion of the European banking system

## 3.1 Data

We use data collected by the European Banking Authority (EBA) for the 2016 supervisory stress test. This public dataset provides information on the balance sheet holdings of 51 European banks in various asset classes as well as their capital.[1] The left column of Table 1 lists the balance sheet quantities that we collect from the EBA dataset and provides, for ease of reference, for each quantity its item and exposure code in the EBA dataset.

| **Assets** | **EBA Item** | **EBA Exposure** |
|---|---|---|
| *Marketable assets* $\Pi$ | | |
| Corporate exposures | 1690201, 1690301 | 1100, 1200, 1300, 1400<br>1500, 1700, 2100, 2200<br>3000, 6100, 6200, 6500 |
| Sovereign exposures | 1690503, 1690506<br>1690507, 1690508 | not applicable |
| Direct sovereign exposures in derivatives | 1690511 | not applicable |
| | | |
| *Illiquid assets* $\Theta$ | | |
| Residential exposures ($\epsilon_\kappa \geq 0$) | 1690201, 1690301 | 4120, 4320, 4700, 5000 |
| Commercial exposures ($\epsilon_\kappa \geq 0$) | 1690201, 1690301 | 3100, 3200, 4110, 4200<br>4310, 4500, 6600 |

| **Liabilities** | **EBA Item** | **EBA Exposure** |
|---|---|---|
| Tier 1 capital | 1690110 | not applicable |
| Deposits and debt | - | - |

| **Other information** | **EBA Item** | **EBA Exposure** |
|---|---|---|
| Total loans | 1690903 | not applicable |
| Total marketable assets | 1690902 | not applicable |
| Total leverage ratio exposure | 1690111 | not applicable |

Table 1: Model balance sheet items and their EBA data reference codes.

In the EBA dataset, holdings are specified by asset type and geographical region. We refer to Cont and Schaanning (2016) for a more detailed description of the data.

We subdivide the balance sheet into marketable assets (securities), which may be liquidated in a stress scenario, and illiquid assets. Marketable assets are further divided

[1] See `https://eba.europa.eu/risk-analysis-and-data/eu-wide-stress-testing/2016/results`

into corporate and sovereign bonds. Other asset classes are classified as illiquid and assumed to be unavailable for short-term liquidation.

Ignoring asset classes for which exposure is below 1M EUR leaves us with 93 asset classes, yielding a $51 \times 93$ matrix of liquid holdings $\Pi$. The most important regions for corporate exposures, covering over 75% of the total, are France (21.0%), U.S. (14.1%), Germany (11.7%), Italy (6.5%), Spain (4.6%), Netherlands (4.4%) and Belgium (3.2%). The most important regions for sovereign exposures, covering more than 75% of the total, are Germany (13.8 %), France (13.3%), U.S. (12.8%), Italy (9.2%), U.K. (8.4%), Spain (6.3%), Netherlands (4.6%), Belgium (4.2 %) and Japan (3.4%). Through a similar procedure, we obtain a $51 \times 98$ matrix of illiquid holdings, which we denote $\Theta$. This corresponds to 97 asset classes for commercial and residential mortgage exposures respectively in the various regions. We insert a 98th entry consisting of all remaining illiquid asset holdings (intangible goods, defaulted exposures etc) such that the total asset size corresponds to the total exposures given in the data.

Collectively, the 51 banks hold assets totalling 26.3 trillion EUR, of which 54.6 % (14.3 tn EUR) are in loans and advances, 31.1 % (8.2 tn EUR) are in marketable assets, and 14.3 % (3.8 tn EUR) are in "residual" assets classes that are not of relevance for our model.

The market depth parameter $D_\mu$ defined in (1) is computed following the methodology in (Obizhaeva, 2012) and (Cont and Schaanning, 2016). The estimation procedure is explained in more detail in Section 3.3, where we perform a sensitivity analysis on this parameter.

**Empirical analysis.** As the portfolio holdings from the 2016 EBA data fulfil all our assumptions, we compute $\Omega$ as well as the $ERI$ and $ICI$ as detailed above. The left-hand panel of Figure 3 shows that the distribution of liquidity-weighted overlaps $\Omega^{i,j}$ displays considerable heterogeneity. The right panel shows the first twenty ranked eigenvalues of $\Omega$. The first eigenvalue accounts for about 65% of the total variation and clearly dominates the remaining ones. We thus expect the $ERI$ to be a good predictor of fire-sales losses in the sequel.

Figure 4 shows the Endogenous Risk Index for the European Banking network in 2016, where we have labelled the banks with the highest $ERI$: Crédit Agricole, BNP Paribas, Deutsche Bank, Société Générale and Barclays. All banks that are identified are either Global Systemically Important Banks (GSIBs) or Domestic Systemically Important Banks (DSIB). It is important to note that the banks are ranked by the amount of contagion they could trigger should they engage in distressed liquidations. This is *not* the same as ranking them according to their size – an issue we illustrate in detail in the next section.

Finally, Figure 5 depicts the indirect contagion network between European banks as implied by the EBA data. Node sizes are proportional to the balance sheet size, and

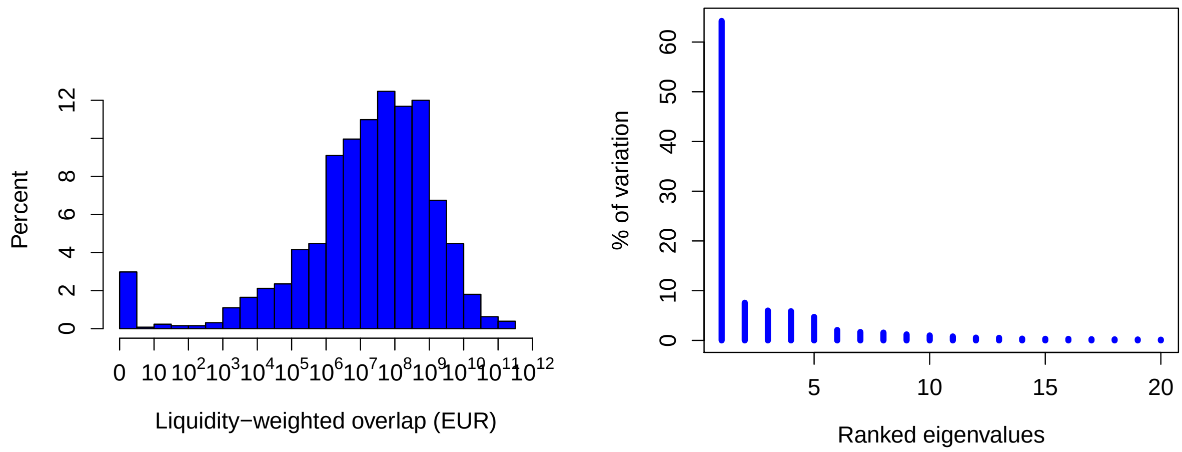


Figure 3: Distribution of liquidity-weighted overlaps (left) and the ranked eigenvalues of the indirect contagion network.

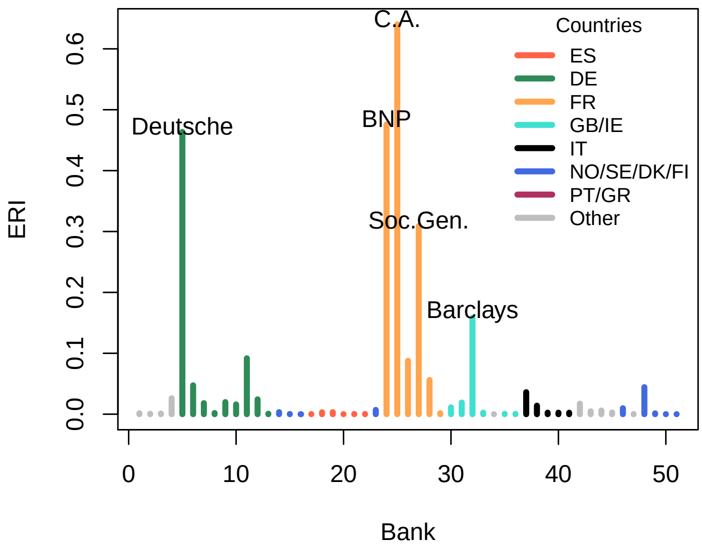


Figure 4: The Endogenous Risk Index for the European Banking system. (Data: EBA. Calculations: Authors).

the edge width between bank $i$ and $j$ is proportional to $\log(\Omega^{i,j})$. Orange edges denote the portfolio overlaps for banks with the highest values of $ERI$.

### 3.2 The Endogenous Risk Index as a proxy for fire-sales losses under stress

Using the official loss rates of the EBA stress test as starting point, we estimate deleveraging that may occur as a result and the spillover losses that this may cause. The left panel of Figure 6 shows the EBA scenario losses for the individual banks in percent of their capital. The 10 most severely hit banks are Banca Monte dei Paschi di Siena (MdPdS), Allied Irish Banks (AIB), Commerzbank (Com), The Royal Bank of Scotland (RBS),

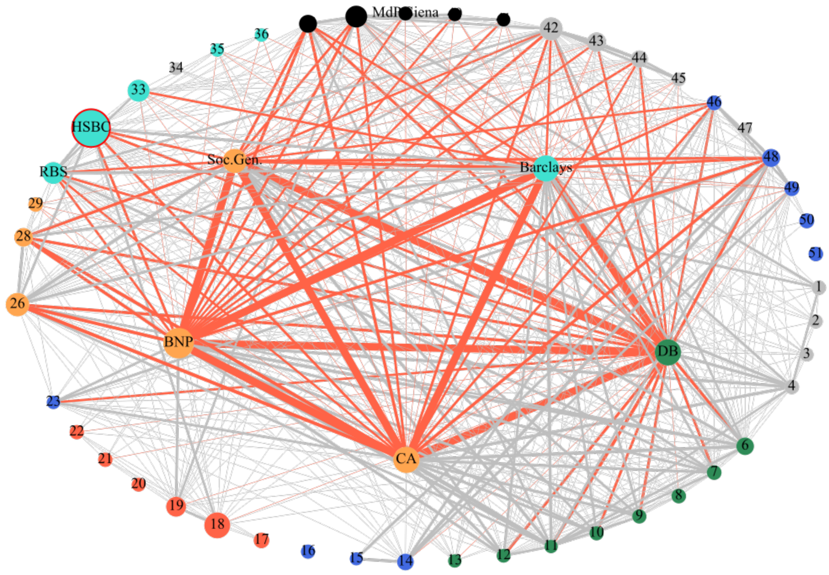


Figure 5: The EU indirect contagion network.

Bayerische Landesbank (BL), Raiffeisen-Landesbanken-Holding (RLH), Banco Popular Español (BPE), The Governor and Company of the Bank of Ireland (GCBI), Cooperatieve Centrale Raiffeisen-Boerenleenbank (CCRB)and Landesbank Baden-Württemberg (LBBW). As the EBA dataset does not provide information on the liquidity, or the risk weights of individual assets on the balance sheet, we focus on a leverage constraint only. Using supervisory data from the Bank of England, Coen et al. (2017) show how the model can be adapted to include risk-weighted capital and liquidity constraints. The initial lever-

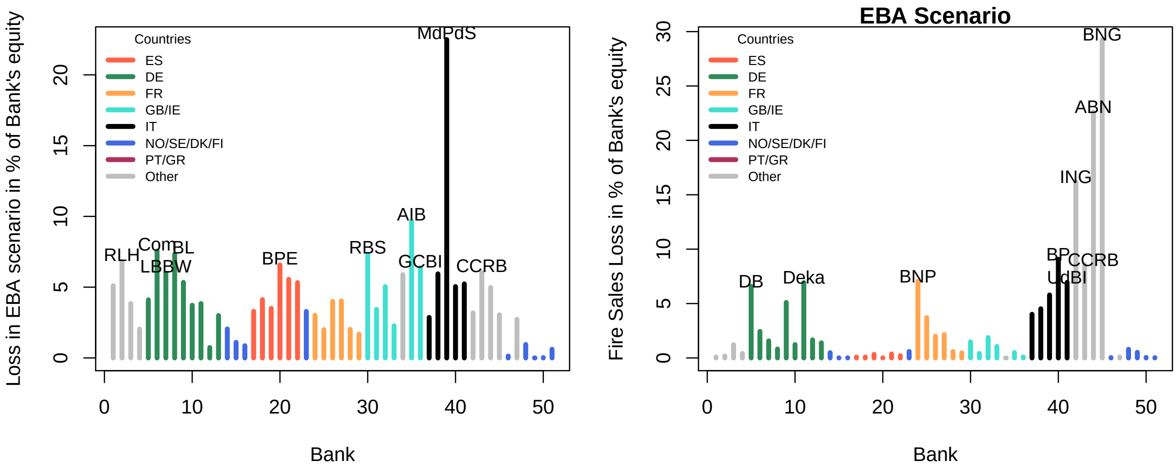


Figure 6: Left: The losses in percent of bank equity that are estimated in the 2016 EBA adverse scenario. Right: Fire-sales losses in response to the EBA stress scenario.

age of bank $i$ is $\lambda^i = (\sum_{\mu=1}^{M} \Pi^{i,\mu} + \sum_{\kappa=1}^{K} \Theta^{i,\kappa})/C^i$, where $C^i$ is the capital of bank $i$. After a loss $l_i$ the leverage increases to

$$\lambda^i(\Pi, \Theta, C, l) = \frac{\sum_{\mu=1}^{M} \Pi^{i,\mu} + \sum_{\kappa=1}^{K} \Theta^{i,\kappa} - l_i}{(C^i - l_i)_+}. \tag{11}$$

If as a result of the shock the leverage $\lambda^i$ exceeds the regulatory constraint $\lambda_{\max} = 33$, then the bank will liquidate a portion, $\Gamma^i \in [0, 1]$, of its marketable assets in order to become compliant again with the leverage constraint. The bank thus solves

$$\frac{(1 - \Gamma^i) \sum_{\mu=1}^{M} \Pi^{i,\mu} + \sum_{\kappa=1}^{K} \Theta^{i,\kappa} - l_i}{(C^i - l_i)_+} = \lambda_b,$$

for $\Gamma^i$, where $\lambda_b \leq \lambda_{\max}$ is a buffer leverage. This yields

$$\Gamma^i(\Pi, C, \Theta, l) = \left( \frac{\sum_{\mu=1}^{M} \Pi^{i,\mu} + \sum_{\kappa=1}^{K} \Theta^{i,\kappa} - \lambda_b(C^i - l_i)}{\sum_{\mu=1}^{M} \Pi^{i,\mu}} \wedge 1 \right) \mathbf{1}_{\lambda^{\mathbf{i}}(\mathbf{\Pi},\mathbf{\Theta},\mathbf{C},\mathbf{l}) > \lambda_{\max}}. \tag{12}$$

In Section 3.4, banks will not simply sell assets proportionally to its current holdings but choose individual assets so as to minimise the liquidation losses they suffer. On the one hand if the fire-sales losses from the distressed liquidations of the first round cause other (or the same banks) to breach their leverage constraint (again), then one or more further rounds of deleveraging occur. On the other hand if the initial loss is not too severe and/or banks are well capitalised, they may absorb the losses without any distressed sales occurring at all. Such a threshold-type reaction is typical of the asymmetric reaction of market participants to downside risk(Ang et al., 2006), sudden and non-linear event-driven deleveraging (Khandani and Lo, 2011), (Cont and Wagalath, 2016), trading-desk VaR constraints (Danielsson et al., 2012), and characteristic of systems of complex systems (Granovetter, 1978), (Duffie, 2010). This threshold-type reaction differs from the leverage targeting assumption of (Greenwood et al., 2015). Cont and Schaanning (2016) show that the leverage targeting assumption may lead to counter-intuitive results when used as reaction function for simulating fire sales.

We follow the estimation procedure in (Obizhaeva, 2012) and (Cont and Schaanning, 2016), and estimate the market depth as

$$D_\mu(\tau) = c \frac{ADV_\mu}{\sigma_\mu} \sqrt{\tau}, \tag{13}$$

where $c = 0.3$ (Ellul et al., 2011) and $\tau = 20$ days. For the median asset class, this yields an estimated impact of 440 basis points per 10 bn EUR liquidated but there is considerable variation of depth across asset classes, as expected.

**Remark.** While (Obizhaeva, 2012) estimated the value of $c$ for the equity market, we note that (Kyle and Obizhaeva, 2016) successfully apply the same approach to quantify the price impact in the treasuries market. Moreover, (Ellul et al. (2011), Fig. 2) estimate, based on Amihud's illiquidity measure, cumulative negative returns of up to 8% when insurance companies are divesting bonds that have been downgraded, showing that price impacts can be quite substantial.

As quantifying the price impact for hypothetical distressed sales is very difficult, we conduct a rigorous sensitivity analysis of the market depth in the next section. Note that given (13) above, it is equivalent to conduct a sensitivity analysis in $c$ or in $\tau$. To make these quantities more tangible, we therefore phrase the sensitivity analysis in terms of the basis point impact per 10 bn EUR volume liquidated.

To compute the price impact during the fire sales cascade, $\frac{\Delta S_k^\mu}{S_k^\mu} = -\Psi_\mu(q_k^\mu)$, we use the non-linear two parameter market impact function

$$\Psi_\mu(q_k, S_k) = \left(1 - \frac{B_\mu}{S_k^\mu}\right)\left(1 - \exp(\frac{-q_k}{\delta_\mu})\right), \tag{14}$$

where $\delta_\mu = D_\mu(1 - B_\mu/S_0^\mu)$ ensures that the impact of small-volume sales matches the observed impacts that are captured well by a linear model, and where $B_\mu$ is a floor below which the price does not fall. The floor accounts for the arrival of large institutional value-investors when prices drop far below fundamentals. This two-parameter impact model for stress testing was first used in (Cont and Schaanning, 2016) to which we refer for further details.

To be clear, we thus simulate the deleveraging cascade with a more realistic non-linear impact model, and use the linear approximation to compute the Endogenous Risk Index as detailed above. Table 2 shows the results from the regression specified in (8) for the estimated market depth (which results in a median impact of 440 bps per 10bn of assets liquidated).

Under these assumptions, the EBA scenario is sufficiently severe to trigger a first round of deleveraging by some banks. In reaction to these fire-sales losses other banks also need to deleverage their portfolio resulting in a total loss of 42 bn EUR. The right panel of Figure 6 plots the fire-sales losses for all banks in the model as percent of their initial equity. This corresponds to 3% percent of total system equity. However, as the right panel of Figure 6 shows, these losses are distributed quite unevenly: While for some banks the losses are negligible relative to their capital base, other banks, such as N.V. Bank Nederlandse Gemeenten (BNG), ABN Amro, ING suffer fire-sales losses on the order of 30%, 25% and 20% of their capital.[2] The results are encouraging: When regressing the log of these

[2] The other banks are: Banco Popolare (BP), Cooperatieve Centrale Raiffeisen-Boerenleenbank

fire-sales losses on the $ERI$, the coefficient of the slope is indeed close to 1, lying between 0.73 and 0.80 for the first three rounds. The $R^2$ ranges between 0.43 and 0.79 for the first four rounds. Even when considering the total fire-sales loss across all rounds, the regression still achieves an $R^2$ of 0.73 and has a slope of 0.62. All estimates are highly significant, with $p < 10^{-4}$. The first empirical results corroborate the theoretical suggestion that the $ERI$ may be well suited to predict fire-sales losses. Additionally, Figure 17 in the Appendix shows the data and regression lines.

Table 2: Regression of bank-level fire-sales losses on the Endogenous Risk Index for all banks, as specified in (8).

| | Round 1 | Round 2 | Round 3 | Round 4 | All rounds |
|---|---|---|---|---|---|
| Slope | 0.730*** | 0.795*** | 0.752*** | 0.516*** | 0.623 *** |
| | (0.072) | (0.060) | (0.068) | (0.112) | (0.055) |
| Intercept | 10.5*** | 10.9*** | 10.7*** | 9.76*** | 11.1*** |
| | (0.190) | (0.151) | (0.164) | (0.326) | (0.143) |
| n | 51 | 49 | 41 | 30 | 51 |
| $R^2$ | 0.68 | 0.79 | 0.76 | 0.43 | 0.73 |

**Is bank size sufficient to measure contagion?** Next, we address the question whether the $ERI$ captures information that is not already included when considering the size of banks. The vector of the banks' normalised (marketable) portfolio sizes is

$$Size = \frac{(\Pi^1, \ldots, \Pi^N)}{||(\Pi^1, \ldots, \Pi^N)||_2}, \tag{15}$$

where $\Pi^i := \sum_{\mu=1}^{M} \Pi^{i,\mu}$. Table 3 shows the $R^2$ and root-mean squared error (RMSE) when regressing log fire-sales losses on either log size, log $ERI$ and on both variables.[3] In this configuration, the $ERI$ and size vector have essentially the same predictive power ($R^2$ of 0.77 and 0.79 respectively). More importantly, when regressing log fire-sales losses on both variables, they do both turn out highly significant ($p < 10^{-4}$), and the $R^2$ increases to 0.86, while the RMSE decreases from 0.33 to 0.27. Overall, the $ERI$ thus clearly captures information above and beyond the mere size of bank portfolios. Next, we show that this result is robust against a wide range of different model assumptions and for all relevant parameter values.

(CCRB), BNP Paribas (BNP), Union Di Banche Italiane (UdBI), Deka Bank (Deka), Deutsche Bank (DB).

[3]When size in (15) is defined based on total assets rather than securities holdings, the $ERI$ has a higher $R^2$ and lower RMSE than Size.

Table 3: The $ERI$ captures information beyond the portfolio size.

| | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Size | 0.98 ($p = 0.0000$) | – | 0.58 ($p = 0.0000$) |
| $ERI$ | – | 0.54 ($p = 0.0000$) | 0.27 ($p = 0.0000$) |
| $R^2$ | 0.79 | 0.77 | 0.86 |
| RMSE | 0.32 | 0.33 | 0.27 |
| n | 51 | 51 | 51 |

### 3.3 Sensitivity analysis and robustness checks

In order to exclude the possibility that the encouraging results for the $ERI$ are due to a specific choice of the stress scenario or other model parameters such as the market depth and the shock intensity, we perform sensitivity analyses with respect to each of these three factors.

**Stress scenarios.** To assess the sensitivity of our results with respect to the choice of stress scenario, we consider as in (Cont and Schaanning, 2016) three further scenarios, giving a list of four scenarios:

1. **2016 EBA stress scenario:** We take the bank-level losses directly from the loss estimates of the 2016 EBA stress test.
2. **"Brexit" scenario:** Losses materialise on British and Irish residential exposures, as well as on the commercial exposures of the UK itself and its main European trading partners: Ireland, Germany, Norway and Italy;
3. **Southern European scenario:** Losses materialise on commercial and residential exposures in Italy, Spain and Portugal;
4. **Eastern European scenario:** Losses materialise on commercial exposures in Eastern Europe (Regions: Bulgaria, Czech Republic, Estonia, Hungary, Lithuania, Latvia, Poland, Romania and Slovakia);

The composition of the scenarios is arbitrary, with the intention to have different subsets of banks shocked in each scenario.

**Stress intensity.** In order to assess the sensitivity of the results with respect to the severity of the shock, we vary the shock intensity for all asset classes from 0 to 20% in each scenario.[4]

[4]The EBA scenario is calibrated such that a 10% shock corresponds exactly to the losses predicted in the official EBA stress test.

**Market depth.** The market depth $D_\mu$ for each asset class $\mu$ determines the magnitude of the the price impact. We perform a detailed analysis of the sensitivity of results to this parameter by scaling the median impact from its estimated impact of 440 bps per 10 Bn EUR up to 2000 bps and down to 50 bps. The upper half of Table 4 shows the estimated market depths for a number of selected corporate and sovereign asset classes for a 1bn and a 10 bn liquidation volume under the original estimate, as well as minimum and maximum impacts. The lower half of the table shows the median, average and weighted average (weighted by holdings) of the market depths. As mentioned above, these results may also be viewed as a sensitivity analysis with respect to the liquidation horizon $\tau$. In terms of basis points per 10 bn EUR liquidated in the median asset class, this corresponds to varying the liquidation horizon between half a day and 4.5 years.

Table 4: Impact in basis points in selected asset classes as for 1 bn and 10 bn in sales.

| Asset | Impact in bps for **1 bn** sale | | | Impact in bps for **10 bn** sale | | |
|---|---|---|---|---|---|---|
| class | Estimate | Minimum | Maximum | Estimate | Minimum | Maximum |
| US gov | 0.04 | 0 | 0.23 | 0.41 | 0.04 | 2.27 |
| UK gov | 0.52 | 0.06 | 2.87 | 5.18 | 0.56 | 28.63 |
| DE gov | 0.93 | 0.1 | 5.18 | 9.34 | 1.02 | 51.53 |
| FR gov | 1.73 | 0.19 | 9.58 | 17.27 | 1.89 | 94.97 |
| IT gov | 3.18 | 0.35 | 17.62 | 31.75 | 3.47 | 173.43 |
| US corp | 0.21 | 0.02 | 1.19 | 2.15 | 0.23 | 11.88 |
| UK corp | 2.68 | 0.29 | 14.84 | 26.74 | 2.92 | 146.4 |
| DE corp | 4.85 | 0.53 | 26.83 | 48.31 | 5.29 | 261.94 |
| FR corp | 9 | 0.98 | 49.68 | 89.28 | 9.81 | 475.19 |
| IT corp | 16.5 | 1.8 | 90.78 | 162.59 | 17.99 | 837.07 |
| **Median** | 46 | 5 | 249 | **440** | **50** | **2000** |
| Mean | 499 | 136 | 1086 | 1410 | 518 | 2591 |
| Weighted av. | 60 | 9 | 174 | 247 | 63 | 700 |
| Median sov | 26 | 3 | 142 | 253 | 28 | 1251 |
| Mean sov | 153 | 19 | 586 | 848 | 165 | 1855 |
| Wghtd av. sov | 11 | 1 | 53 | 90 | 11 | 358 |
| Median corp | 152 | 17 | 787 | 1327 | 166 | 4062 |
| Mean corp | 796 | 238 | 1516 | 1892 | 821 | 3224 |
| Wghtd av. corp | 100 | 16 | 274 | 378 | 107 | 984 |

**Fire-sales losses.** Figure 7 shows the fire-sales losses in percent of total bank equity as function of the shock severity and the market depth for the four different scenarios. As the banks' exposures to Eastern Europe are smaller, Scenario 4 requires a lower market depth and/or larger shock in order to generate significant fire-sales losses compared to the other scenarios.

**Results.** We run the regression (8) for all combinations of scenarios, market impacts and shock intensities. Each panel in Figure 8 shows the $R^2$ of the regression in each of

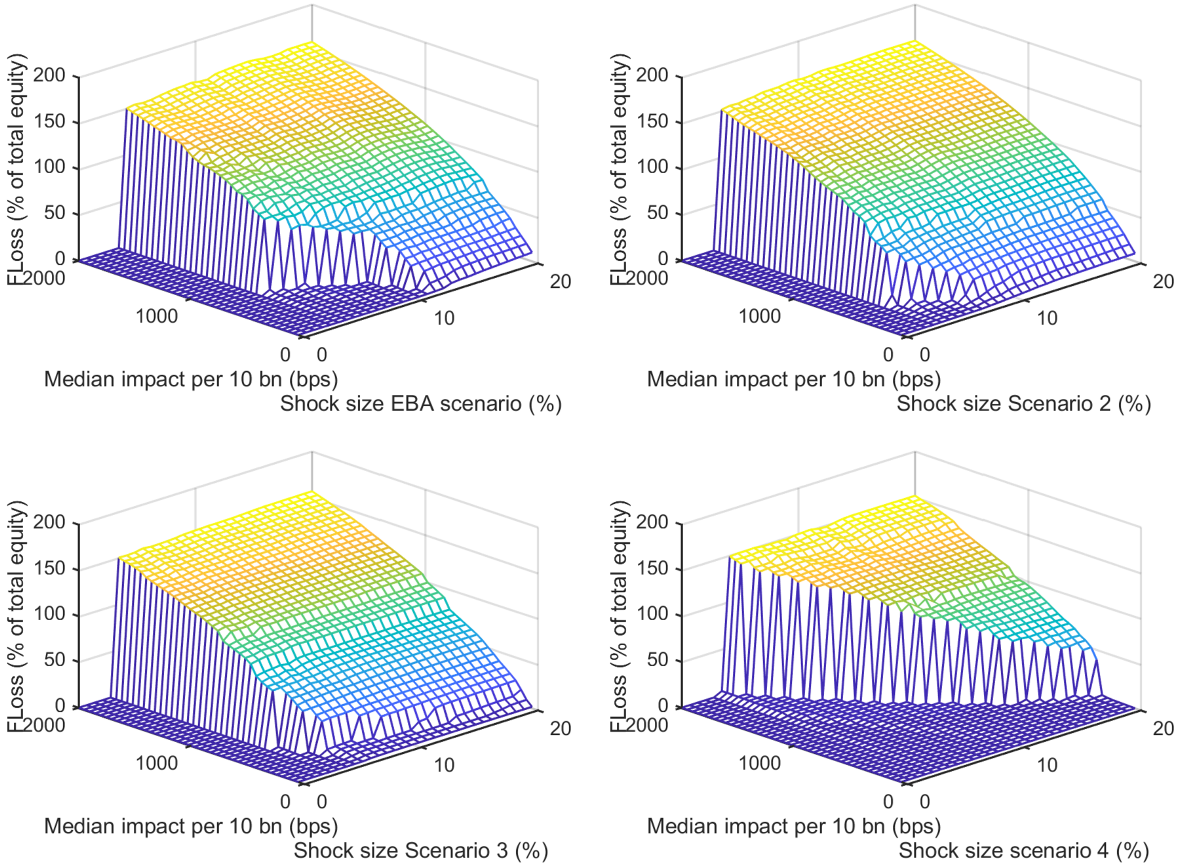


Figure 7: Fire-sales losses as a function of the shock intensity and the market-impact in each of the four scenarios.

the four scenarios as a function of the market depth and the shock intensity. The graphs show that across all combinations of scenarios, shock intensities and market depths, the regression is robust. For combinations of market depths and shock intensities where large-scale contagion occurs (see Figure 7), the $R^2$ is particularly predictive, with values above 0.75. However, for combinations of shock intensities and market depths that lead to localised fire sales(e.g. scenario 4), the $R^2$ is lower at around 0.4 - 0.5. Figure 9 illustrates that the slopes of the regressions range between 0.5 and 0.85 as a function of the market impact and shock intensity across the four scenarios.

Figure 7 illustrates that the cliffs in Figures 8 and 9 correspond to the boundaries between moderate versus considerable fire-sales losses.

As a rule of thumb, when contagion spreads far throughout the system, the predictive value of the $ERI$ – as measured by the $R^2$ of the regression – increases substantially. For instance in Scenario 1, when the median impact is close to 2000 bps, the $R^2$ ranges between 0.7 and 0.9, while the slope lies between 0.76 and 0.96. Table 5 shows the results of these regressions in this high-impact region.

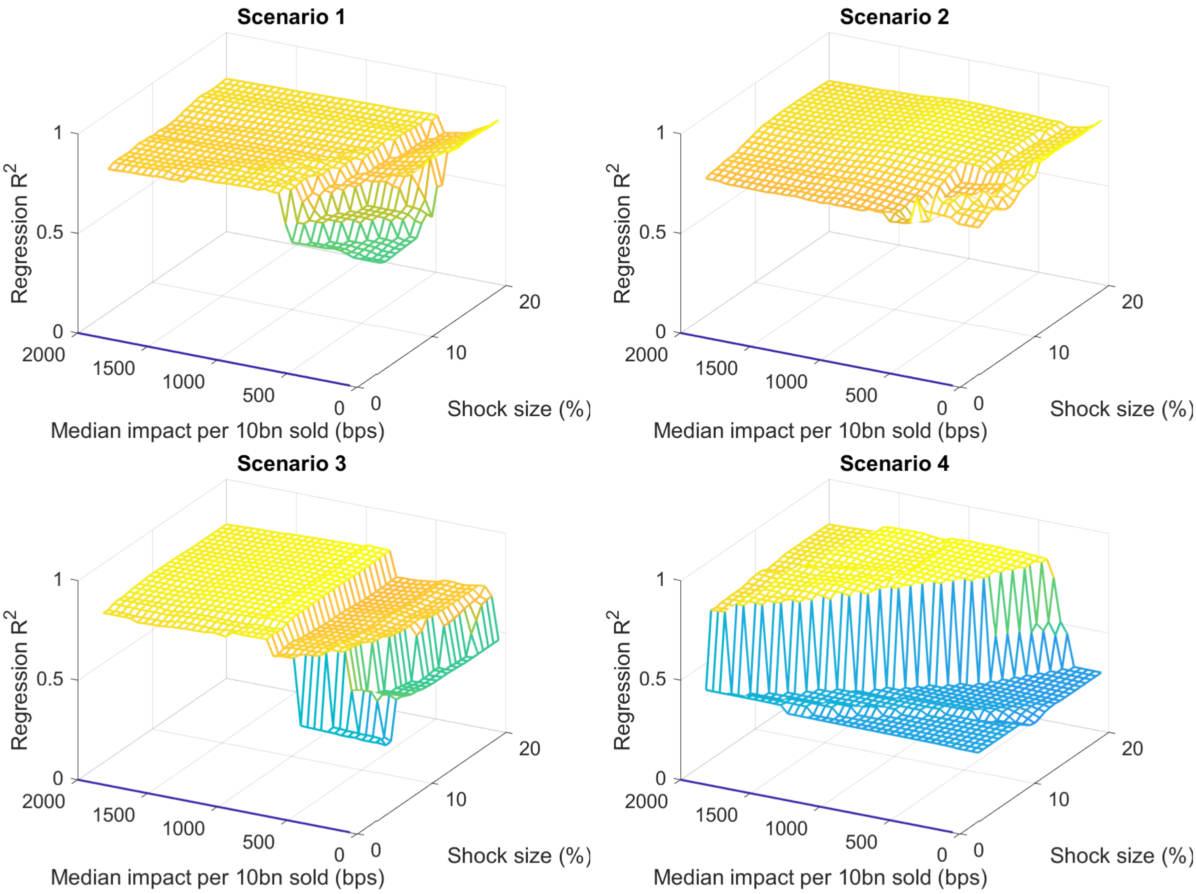


Figure 8: $R^2$ of the regression of fire-sales losses on the ERI as a function of the shock size and market depth in each scenario.

Table 5: Regression of bank-level fire-sales losses on the Endogenous Risk Index for all banks, as specified in (8) for the high-impact region.

| | Round 1 | Round 2 | Round 3 | Round 4 | All rounds |
|---|---|---|---|---|---|
| Slope | 0.758*** | 0.898*** | 0.957*** | 0.954*** | 0.774 *** |
| | (0.072) | (0.050) | (0.049) | (0.062) | (0.051) |
| Intercept | 9.96*** | 10.2*** | 10.1*** | 9.81*** | 10.6*** |
| | (0.188) | (0.134) | (0.118) | (0.148) | (0.135) |
| n | 51 | 49 | 46 | 46 | 51 |
| $R^2$ | 0.70 | 0.87 | 0.90 | 0.84 | 0.82 |

**Sensitivity analysis on relative market depths.** As a final robustness check, we assume that we have erred on the *relative* liquidity of the assets. As robustness check we therefore randomize the *order* of the market depths for given correlations to the initial vector. To do so, we follow a simple idea of (i) creating a random vector of market depths that is orthogonal to the estimated market depth and, (ii) taking a linear combination of the estimated market depth and the orthogonal vector that results precisely in the desired correlation between the two.[5] In order to make results more comparable, we scale the

[5] For an implementation by caracal and further details see:`https://stats.stackexchange.com/questions/15011/generate-a-random-variable-with-a-defined-correlation-to-an-existing-variables`.

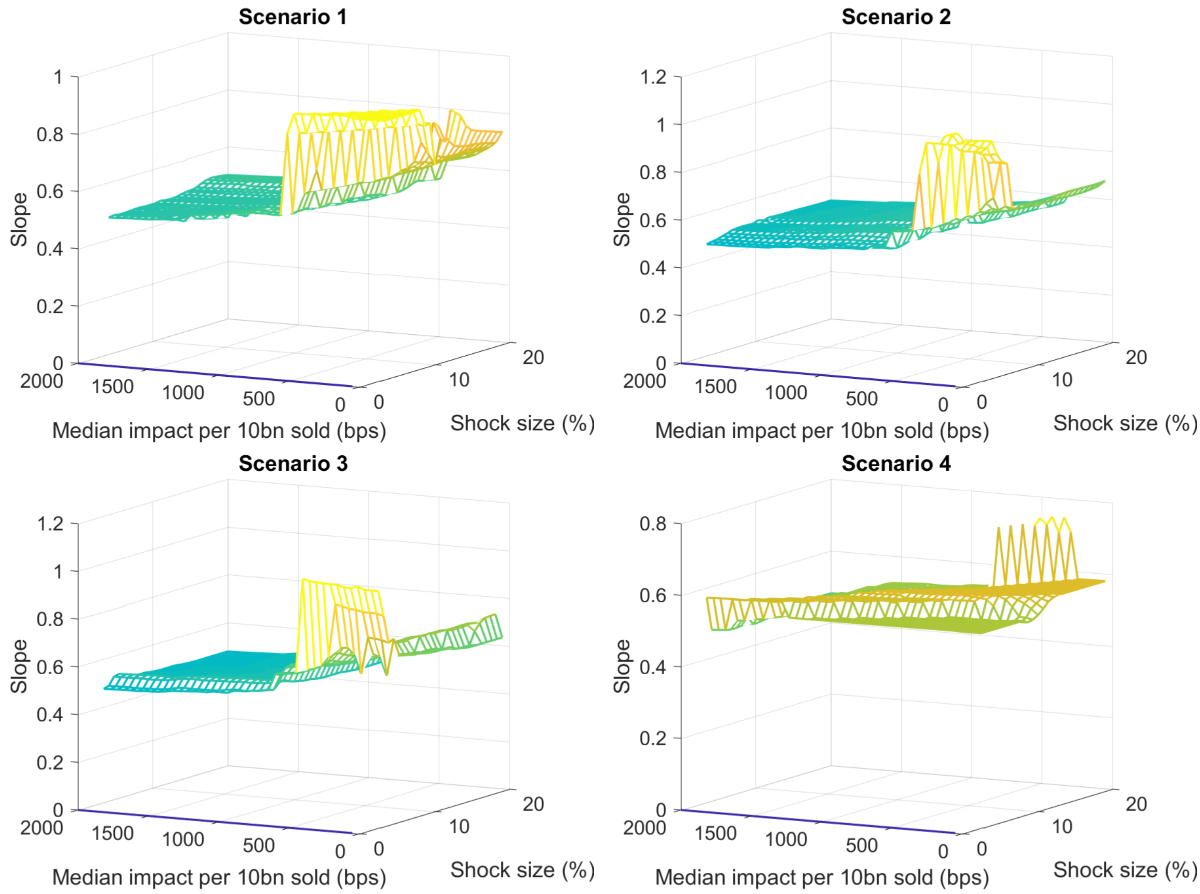


Figure 9: Slopes of the regression of the fire-sales losses on the $ERI$ as a function of the shock size and market depth in each scenario.

mean and standard deviation of this generated set of market depths to be the same as the initial market depths. For correlation $\rho = 1$, the generated market depths are identical to the initial market depths. A negative correlation implies that we have tended to estimate that most liquid assets have the lowest market depth and vice versa, which is scarcely realistic, we focus on the range $\rho\,[0, 1]$. Indeed, all else equal, it is not plausible that, e.g. US treasuries should have a lower market depth than the corporate bonds from a small European country.

Figure10 plots the randomised market depths following the above procedure against the initially estimated market depths for two correlations of $\rho = 0.4$ (left) and $\rho = 0.8$ (right) respectively.

For each value of $\rho$, we produce 20 samples and compute the average fire-sales loss across these samples (for a given shock size and median impact). The left panel of Figure 11 plots these average fire-sales losses as a function of the shock size and the correlation between the randomised and initial market depths. The original estimated fire-sales loss is plotted at $\rho = 1$. The graph illustrates that the average fire-sales loss increases when the correlation between the vector of originally estimated market depths and randomised market depths decreases. The key driver behind this result is that as the correlation

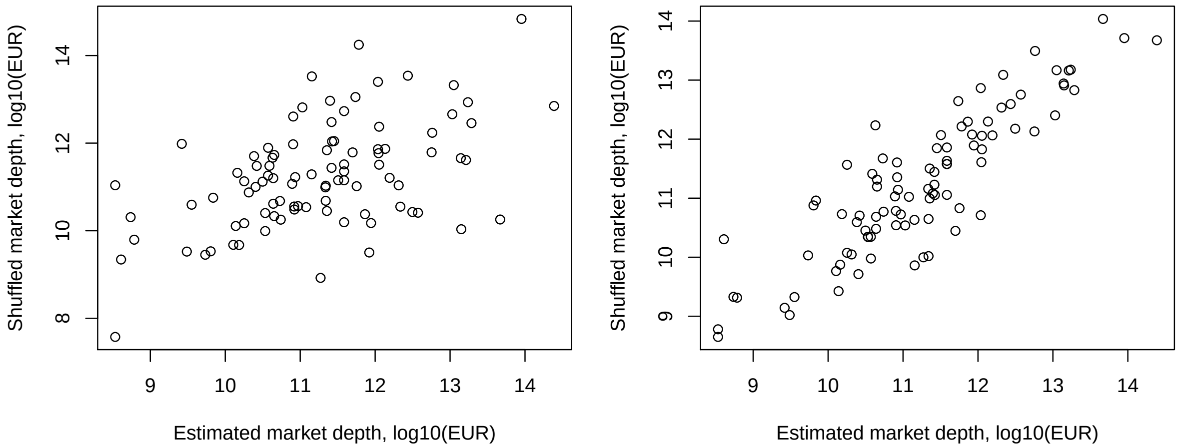


Figure 10: Two examples of shuffled market depths with correlation 0.4 and 0.8 (left and right respectively).

decreases, large holdings that were associated with large market depths become associated with lower market depths, thus increasing the fire-sales losses. The right panel of Figure 11 shows the average fire-sales loss as a function of $\rho$ and the median market impact per 10 bn for a given shock of 10%. The graph shows the same behaviour as the left panel: as the correlation decreases, the average fire-sales loss (across 20 samples) increases.

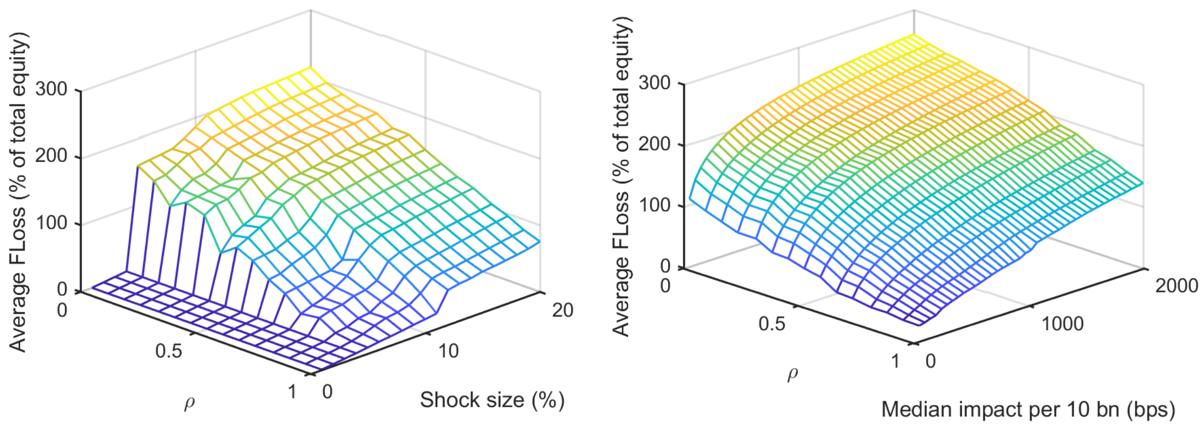


Figure 11: Two examples of shuffled market depths with correlation 0.4 and 0.8 (left and right respectively).

## 3.4 Loss-minimising portfolio deleveraging

Section 3.3 showed that our findings are robust to stress scenario composition, market depth and shock intensity. In this section, we extend the robustness check to the behaviour of banks. Previously, we assumed that deleveraging occurs proportionally across all assets. In the sequel, we assume, similar to the approach taken in (Braouezec and Wagalath, 2018), (Coen et al., 2017) that a bank $i$ needing to liquidate a part of its portfolio will do so by trying to minimise the fire-sales losses that it incurs:

$$\min_{\Gamma^i \in [0,1]^M} \sum_{\mu=1}^{M} \frac{\Pi^{i,\mu}\Pi^{i,\mu}}{\delta_\mu} \Gamma^{i,\mu} \tag{16}$$

subject to the bank complying with its leverage constraint

$$\frac{\sum_{\mu=1}^{M}(1-\Gamma^{i,\mu})\Pi^{i,\mu} + \sum_{k=1}^{K}\Theta^{i,k} - l_i - \sum_{\mu=1}^{M}\frac{\Pi^{i,\mu}\Pi^{i,\mu}}{\delta_\mu}\Gamma^{i,\mu}}{C^i - l_i - \sum_{\mu=1}^{M}\frac{\Pi^{i,\mu}\Pi^{i,\mu}}{\delta_\mu}\Gamma^{i,\mu}} \leq \lambda_{\max}. \tag{17}$$

As before, due to the absence of information on liquidity and risk weights in the public EBA data, we limit the exercise to the leverage constraint.

The following proposition (Baes and Schaanning, 2019) shows that in the case of a leverage constraint and under a linear market impact assumption, the solution to this problem is given by the following strategy.

**Proposition 1.** *Under a leverage constraint and assuming market impact is linear, the loss-minimizing deleveraging strategy according to the optimization criterion* (16) *and* (17)*, is to sell assets sequentially in decreasing order given by the ratios* $\frac{\delta_{\mu_1}}{\Pi^{i\mu_1}} > \ldots > \frac{\delta_{\mu_M}}{\Pi^{i\mu_M}}$ *until the constraint is satisfied or the bank runs out of marketable assets to delever (and defaults).*

Next, we analyse the predictive value of the $ERI$ for fire-sales losses given that banks engage in such optimal sequential deleveraging. Table 6 reports the results for the estimated market depths (median impact of 440 bps per 10 bn). The slope lies between 0.60 and 0.66 in the first three rounds, and the $R^2$ lies between 0.42 and 0.60. In comparison to Table 2, the predictive power has become somewhat weaker but coefficients are still highly significant and the slope remains close to 1. We performed the same sensitivity analyses as in the previous section on the scenarios, market depths and shock intensities when banks liquidate assets optimally. We find that across all scenarios, market depths and shock intensities the regression slope varies between 0.6 and 0.85, while the $R^2$ varies between 0.3 and 0.6. As results are similar to the case of proportional deleveraging, we relegate the corresponding figures to the Appendix (Figures 18, 19, 20, 21, and 22). Overall, the main contrast between the optimised versus the proportional deleveraging assumption is thus the magnitude of fire-sales losses, which is smaller when banks liquidate their portfolio in this way.[6]

Results are similar for non-parallel shifts of the market depths when banks deleverage optimally as at the end of the previous section, where banks deleverage proportionally (See Figure 23 in the Appendix).

Table 6: Regression of bank-level fire-sales losses on the Endogenous Risk Index for all banks, as specified in (8) for optimised bank responses.

| | Round 1 | Round 2 | Round 3 | Round 4 | All rounds |
|---|---|---|---|---|---|
| Slope | 0.614*** | 0.658*** | 0.600*** | 0.554*** | 0.613 *** |
| | (0.072) | (0.105) | (0.107) | (0.111) | (0.073) |
| Intercept | 10.2*** | 8.75*** | 8.23*** | 7.76*** | 10.2*** |
| | (0.190) | (0.281) | (0.288) | (0.301) | (0.191) |
| n | 51 | 46 | 46 | 46 | 51 |
| $R^2$ | 0.60 | 0.47 | 0.42 | 0.36 | 0.59 |

## 3.5 Comparison with other measures of overlap.

We have established that the $ERI$ captures information above and beyond the size of bank portfolios. In this final section, we compare the ERI to two further measures of portfolio similarity and overlap and analyse the predictive power of ERI for various scenarios, shock intensities and market depths. The measures that we consider are:

1. **Cosine Similarity.** We follow (Getmansky et al., 2016), and define the weights of a portfolio as the relative proportion of wealth invested in the various assets

$$w_i := \frac{1}{\sum_{\mu=1}^{M} \Pi^{i,\mu}} (\Pi^{i,1}, \ldots, \Pi^{i,M})^\top . \tag{18}$$

   (Getmansky et al., 2016) proceed to define a measure of similarity, called "cosine similarity" between two portfolios via

$$\Omega^{i,j}_{C.S.} = \frac{< w_i, w_j >}{||w_i||_2 ||w_j||_2} \in [-1, 1] \, . \tag{19}$$

   Our cosine-similarity measure is computed as the first eigenvector of $\Omega_{C.S.}$.

2. **Nominal Overlap.** The Nominal overlap measure is derived in the same manner as the $ERI$, except for the difference that the portfolio overlap is *not* adjusted for

[6]However two caveats apply: Firstly, when banks liquidate assets in an order close to the ranking from the most liquid to the least liquid (as normally $\delta_\mu >> \Pi^{i,\mu}$), it is not clear that the most liquid asset ex-ante should still turn out to be the most liquid ex-post. Including such considerations requires a dynamic model for the market depth, which is outside the scope of this paper. Secondly, when banks are constrained by risk-weighted capital, they have an incentive to liquidate assets with higher risk-weights first (Braouezec and Wagalath (2018)). As assets with higher risk weights usually are less liquid, this would incentivise banks to liquidate assets from the *least* liquid to the most.

| | *ERI* | Nom. Ov. | Cos. Sim. | Size |
|---|---|---|---|---|
| *ERI* | 1 | **0.68** (0.85) | **-0.13** (- 0.22) | **0.60** (0.80) |
| Nom. Ov. | | 1 | **-0.14** (-0.22) | **0.78** (0.92) |
| Cos. Sim. | | | 1 | **-0.17** (-0.27) |
| Size | | | | 1 |

Table 7: Similarity between the various overlap measures: The bold numbers are rank-correlations (Spearman's correlation), while the numbers in brackets are linear correlations (Pearson's $\rho$).

the liquidity of the underlying assets:

$$\Omega_{Nominal} = \Pi\Pi^{\top}. \tag{20}$$

In analogy to the $ERI$, the Nominal overlap measure is defined as the principal eigenvector of $\Omega_{Nominal}$.

By deffnition, these measures are also given by a vector of unit size, in which the value of the individual components can thus be interpreted as a measure of systemic importance. We begin by analysing the correlation between the $ERI$, Size, Cosine Similarity and the Nominal Overlap in Table 7. The bold numbers refer to rank-correlations (Spearman's) while the figures in parentheses refer to classical linear correlations (Pearson's $\rho$). There is a positive correlation between the $ERI$, the Nominal Overlap measure and the Size measure, while the correlation is negative between the Cosine Similarity and all other measures. Intuitively, we find the highest correlation between the Size and Nominal Overlap measures. While the $ERI$ is positively correlated to these measures the correlation is lower (0.60 and 0.68).

Figure 12 shows the ranking of the individual banks according to the four measures. The high correlation between the Nominal overlap and the Size measures is clearly visible in these graphs. All of the banks that are identified are either GSIBs or DSIBs, with the difference that the importance of banks such as Santander, ING or UniCredit decreases in the Nominal overlap measure relative to the others. In the $ERI$, the French banks Crédit Agricole (CA), BNP Paribas, Société Générale stand out, while UK banks, except for Barclays are significantly less systemic. The German Deutsche Bank is approximatively equally systemic according to all measures.

In contrast, the Cosine Similarity measure provides a completely different picture: The banks that dominate the first eigenvector of the Cosine Similarity matrix are "Erste Group Bank AG" (Erste), "Belfius Banque SA" (Bel), "KBC Group NV" (KBC), "Landesbank Hessen-Thüringen Girozentrale" (LHT), "Banco Popular Españ ol S.A." (BPE), "Banco de Sabadell S.A." (BS), "Groupe Crédit Mutuel" (GCM), "OTP Bank Nyrt." (OTP), "Intesa Sanpaolo S.p.A." (Int), "N.V. Bank Nederlandse Gemeenten" (BNG), "Svenska

Handelsbanken - group" (SHB), "Swedbank - group" (SBG). The similarity measure thus allows to identify clusters of similarly composed portfolios that would be affected simultaneously by market downturns.[7] However, whether portfolios are similarly composed (and thus exposed to the same types of market shocks) is a different question from whether portfolios can cause large price-mediated contagion to each other. While for the former issue size is indeed irrelevant, it is highly relevant for the latter. The Cosine Similarity measure thus captures a different type of information relating to portfolio overlaps.

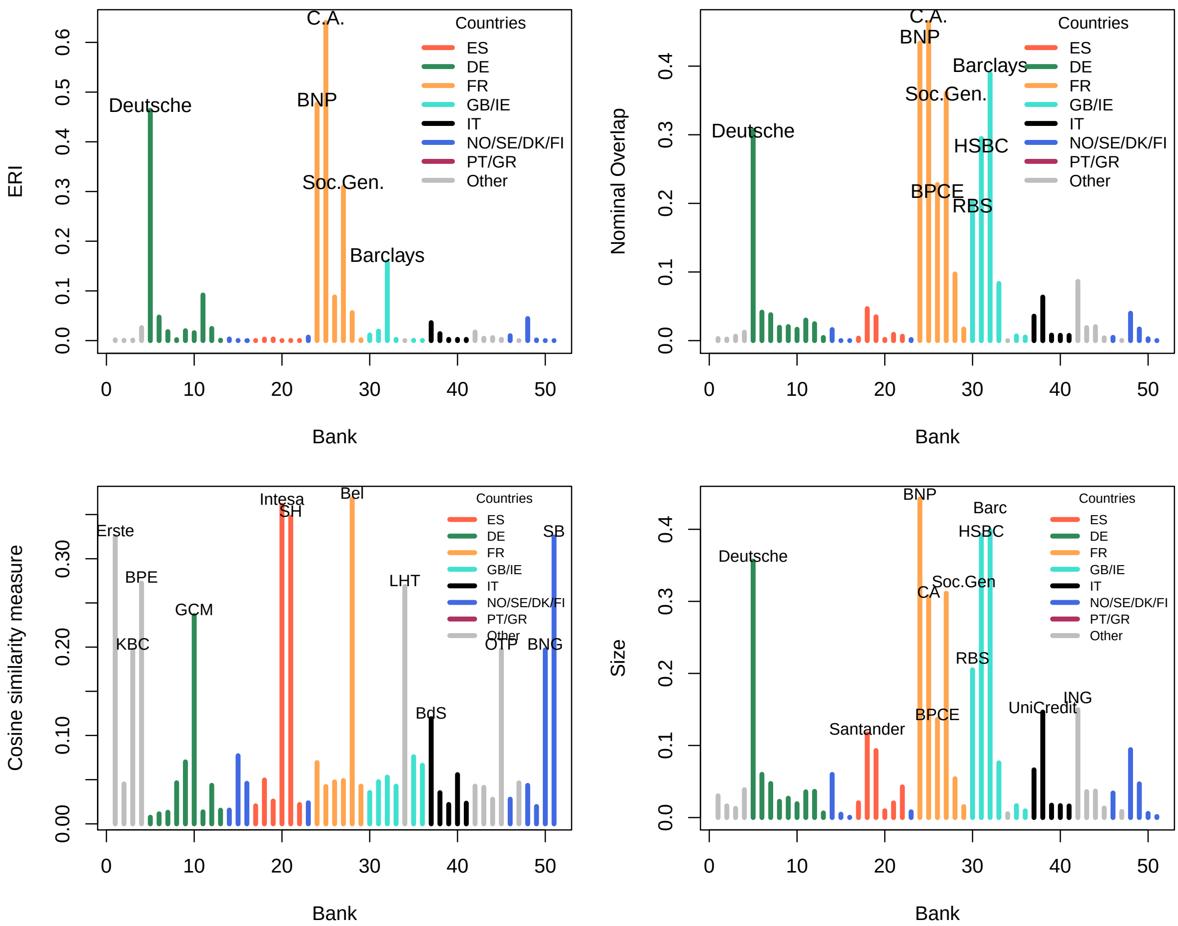


Figure 12: Various overlap and contagion measures. Endogenous Risk Index (top left), Nominal overlap (top right), Cosine Similarity (bottom left), size (bottom right).

As a last exercise in this subsection, we run stepwise regressions to analyse whether the similarity and nominal overlap measures improve the prediction of fire-sales losses over and above the $ERI$ and Size, as we did in Table 3. Table 8 shows that the Nominal overlap eigenvector performs relatively well compared to the $ERI$, while the Cosine Similarity eigenvector performs worst in predicting fire-sales losses. Performing a model selection where we start with (i) no predictors, and (ii) all variables, leads us in both cases to Model 3, where Size and the $ERI$ are the only final variables that are included, while

[7]The second eigenvector of the matrix built from cosine similarity loads higher on the international banks, captured by the other measures.

the Nominal Overlap and Cosine Similarity are not. When forcing all four indicators into the model, we see that the Nominal overlap and the Cosine Similarity measure can be discarded as they are not statistically significant.

Table 8: Size and $ERI$ are retained as predictors for fire-sales losses.

| | Model 1 | Model 2 | **Model 3** | Model 4 |
|---|---|---|---|---|
| Size | – | – | 0.58 ($p = 0.0000$) | 0.72 ($p = 0.0000$) |
| $ERI$ | – | – | 0.27 ($p = 0.0000$) | 0.31 ($p = 0.0000$) |
| Nom. Ov. | 0.59 ($p = 0.0000$) | – | – | -0.14 ($p = 0.1946$) |
| Cos. Sim. | – | -0.47 ($p = 0.0326$) | – | -0.01 ($p = 0.9177$) |
| $R^2$ | 0.70 | 0.09 | 0.86 | 0.86 |
| RMSE | 0.38 | 0.66 | 0.27 | 0.27 |
| n | 51 | 51 | 51 | 51 |

Finally, we perform this stepwise regression for all combinations of shock intensities and market depths in each of the four scenarios introduced earlier. Figure 13 shows the inclusion (yellow) or exclusion (blue) of the four measures in Scenario 1 for different combinations of shock size and market depth as previously. Clearly, across the different parameter values ERI and Size are incorporated most often. Note that shock values below 4% are not relevant, as no bank deleverages in this case and thus no fire-sales losses arise.

When averaging across shock sizes, market depths and scenarios, the Size is included in 96.51 % of the cases as predictor. Second comes the $ERI$, which is included in 91.92% of the cases. The nominal overlap $ERI$ is included in 51.96% of the cases, while the cosine similarity is only included in 6.54% of the cases.[8] Figures 24, 25, 26 and 27 in the appendix, from which we have computed the above percentages, show the inclusion/exclusion of the individual measures across all scenarios for all market impact & shock intensity combinations.

Finally, we run the stepwise regression model for all scenarios and shock size/market-impact combinations, assuming that banks *liquidate optimally.* In this case we find that the Size variable is included in 78.54% of the cases, the $ERI$ in 42.10 % of the cases, the Nominal Overlap in 64.11 % of the cases, while the Cosine Similarity measure is included in less than 0.1 % of the cases. In this particular case, the Nominal Overlap measure has on average a higher predictive value than the Endogenous Risk Index. This has an intuitive reason: liquidating more liquid assets first, obviously leads to larger sales and (relatively speaking) larger impacts in the more liquid asset classes, which to some extent counters the liquidity-weighting that enters the $ERI$.

[8] If in (15), one defines size starting from the total assets, rather than the liquid portfolio, $\Pi$ only, results change a little. In this case the different variables are included as follows $ERI$ 94.74%, Size 65.77% (significant drop!), Nominal overlap $ERI$ 34.84%, while Similarity is never included.

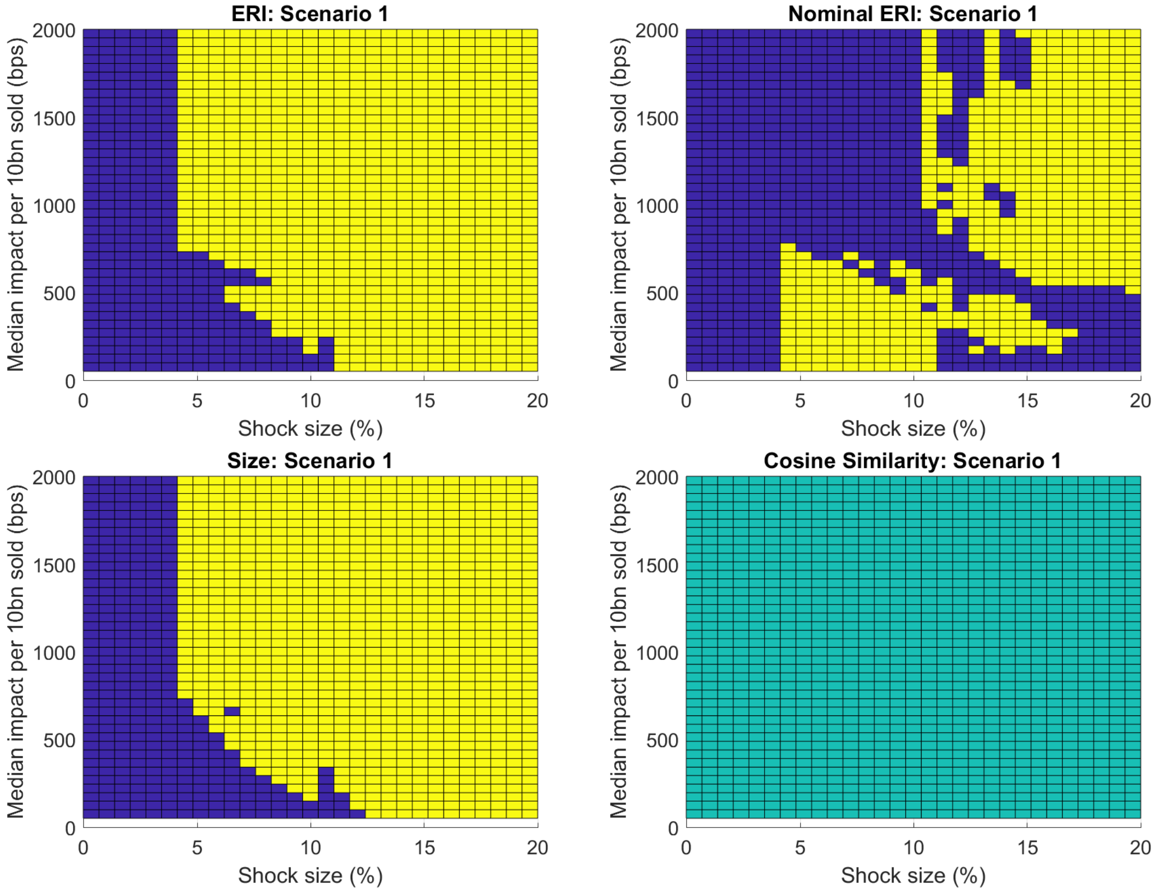


Figure 13: Inclusion (yellow) or exclusion (blue) of the four measures ERI (top left), Nominal overlap (top right), Cosine similarity (bottom left) and Size (bottom right) in Scenario 1 for different combinations of shock size and market depth as previously.

## 3.6 Quantifying interconnectedness for GSIBs

Given the robust performance of the $ERI$, we propose in this final section how our indicators could be used for quantifying the interconnectedness between systemically important financial institutions and Global Systemically Important Banks in particular.

First, we compare the ranking of European banks according to the Indirect Contagion Index ($ICI$) and the Endogenous Risk Index (ERI). Recall that the Indirect Contagion Index is the principal eigenvector of $E = \Omega - \text{diag}(\Omega_{11}, \ldots, \Omega_{NN})$, which thus disregards self-inflicted contagion and solely accounts for contagion to *other* portfolios. Figure 14 shows the $ICI$ (coloured bars) and the $ERI$ (black crosses) for the largest European banks. The most systemic banks, according to our analysis of the 2016 public EBA data, are Crédit Agricole, Deutsche Bank and BNP Paribas, as well as on a slightly lower tier, Société Générale and Barclays. The main difference between the $ERI$ and $ICI$ ranking of the banks is that, relative to the $ERI$, Barclays and Société Générale become more important, while Crédit Agricole becomes less significant. Ranking a bank via the $ERI$ versus the $ICI$ boils down to the question whether one would like to focus on systemic externalities to other market participants only, or also include self-contagion.

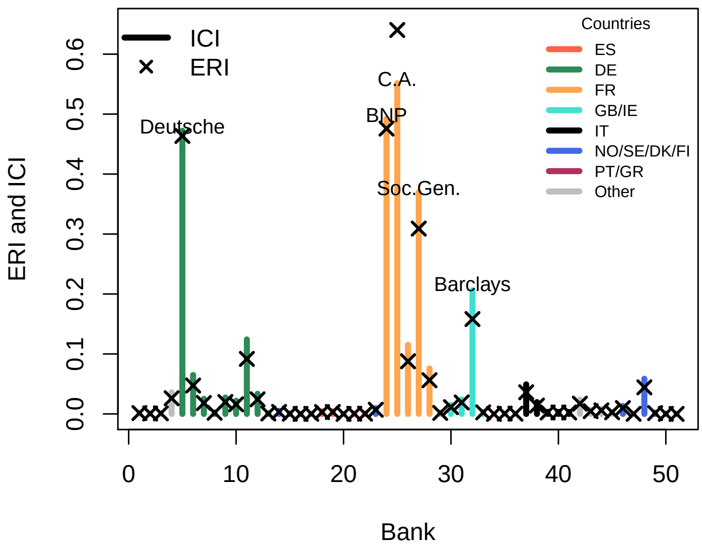


Figure 14: *ICI* (coloured bars) which discounts losses inflicted on the own portfolio relative to the losses inflicted on other portfolios, and the *ERI* (black crosses) for the European banking system. Data source: EBA 2016, Calculations: Authors.

Next, we turn to the the Basel Committee on Banking Supervision's methodology to quantify the systemic importance of Global Systemically Important Banks which is summarised in Figure 15.[9] Depending on which total score a bank achieves in this rating, its Common Equity Tier 1 (CET1) capital ratio requirement increases in 0.5% steps from 1% to 3.5%. This buffer is called the "additional loss absorbency requirement" or sometimes less formally the "GSIB buffer".[10] Currently, "interconnectedness", which contributes 20% to the overall score, is only assessed through total intra-financial system assets and liabilities as well as the amount of securities outstanding. These quantities only depend on the size of banks' securities holdings and are insensitive to *how* portfolios overlap. Moreover, as Figure 16 shows, these measures are closely related to the total size of the banks' balance sheet (captured by the "Size" component of the BCBS methodology).

These results suggest that the Endogenous Risk Index and the Indirect Contagion Index contain information complementary to measures of portfolio size or exposure size.

# 4 Conclusion

We have proposed two indicators for monitor exposure to indirect contagion: the Endogenous Risk Index (*ERI*) and the Indirect Contagion Index (*ICI*) and disucssed their micro-foundation based on a model of loss contagion arising from delevraging of financial

[9]Basel Committee on Banking Supervision, Global Systemically Important Banks: Updated Assessment Methodology and the Higher Loss Absorbency Requirement. URL: `https://www.bis.org/publ/bcbs255.htm`

[10]See: `https://www.bis.org/bcbs/gsib/cutoff.htm`.

| Indicator-based measurement approach | | Table 1 |
|---|---|---|
| Category (and weighting) | Individual indicator | Indicator weighting |
| Cross-jurisdictional activity (20%) | Cross-jurisdictional claims | 10% |
| | Cross-jurisdictional liabilities | 10% |
| Size (20%) | Total exposures as defined for use in the Basel III leverage ratio | 20% |
| Interconnectedness (20%) | Intra-financial system assets | 6.67% |
| | Intra-financial system liabilities | 6.67% |
| | Securities outstanding | 6.67% |
| Substitutability/financial institution infrastructure (20%) | Assets under custody | 6.67% |
| | Payments activity | 6.67% |
| | Underwritten transactions in debt and equity markets | 6.67% |
| Complexity (20%) | Notional amount of over-the-counter (OTC) derivatives | 6.67% |
| | Level 3 assets | 6.67% |
| | Trading and available-for-sale securities | 6.67% |

Figure 15: BCBS GSIB Indicator measurement approach. Source: Basel Committee on Banking Supervision (2013).

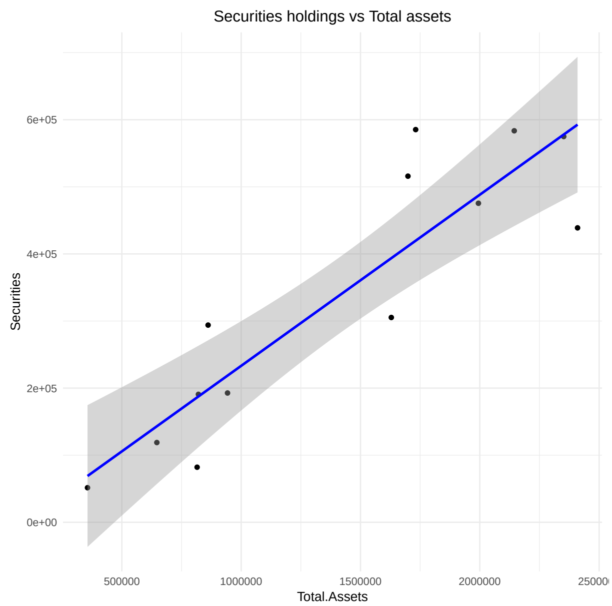


Figure 16: Strong correlation between total assets and securities holdings of the largest global banks. Amounts taken from annual reports 2015. ($R^2 = 0.8$, slope $= 0.255$, $p$-value $= 3.5 \times 10^{-5}$).

institutions in stress scenarios. The $ERI$, which is defined as the principal eigenvector of the matrix of liquidity-weighted portfolio overlaps was shown to be a good proxy for exposure to fire-sales losses, possess an intuitive interpretation, and outperformed other measures of contagion in pinpointing the most systemic institutions across a wide range of different modeling assumptions and parameters. We have further suggested how the $ERI$ can be modified to discount self-inflicted fire-sales losses and focus solely on spillover losses inflicted on other institutions, which defines the Indirect Contagion Index, $ICI$.

As suggested in Section 3.6, $ERI$ and $ICI$ can improve the quantification of interconnectedness of global systemically important banks beyond current measures. These indicators may be readily computed in jurisdictions such as the EU or the US using granular regulatory datasets such as the Securities Holdings Statistics (SHS) in Europe. On a global level the need to build the full portfolio matrix $\Pi$ of asset holdings raises questions regarding availability and confidentiality of holdings data.

# 5 Acknowledgements

Rama Cont's research was funded by the Royal Society APEX Award for Excellence in Interdisciplinary Research (AX160182). This work was initiated as part of Eric Schaanning's PhD at Imperial College London, funded by the Fonds National de la Recherche Luxembourg under the AFR PhD scheme, grant no. 5028140. We thank Michel Baes, Martin Summer and seminar participants at the Bank of England, National bank of Austria, CRM Workshop on Systemic Risk (Univ Montreal), the Vienna University of Economics and Business, the BIS and Banca d'Italia, the Bachelier Financial Society World Congress, Columbia University and the European Systemic Risk Board. An earlier version of this paper appeared under the title "*Measuring systemic risk: The indirect contagion index.*"

# References

Acharya, V. V., Pedersen, L. H., Philippon, T., and Richardson, M. (2017). Measuring systemic risk. *Review of Financial Studies*, 30(1):2–47.

Adrian, T. and Brunnermeier, M. K. (2016). CoVaR. *American Economic Review*, 106(7):1705–41.

Amihud, Y. (2002). Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets*, 5(1):31 – 56.

Ang, A., Chen, J., and Xing, Y. (2006). Downside risk. *Review of Financial Studies*, 19(4):1191–1239.

Baes, M. and Schaanning, E. (2019). Reverse stress testing. *Working Paper.*

Battiston, S., Puliga, M., Kaushik, R., Tasca, P., and Caldarelli, G. (2012). Debtrank: Too central to fail? financial networks, the Fed and systemic risk. *Scientific reports*, 2.

Beale, N., Rand, D. G., Battey, H., Croxson, K., May, R. M., and Nowak, M. A. (2011). Individual versus systemic risk and the regulator's dilemma. *Proceedings of the National Academy of Sciences*, 108(31):12647–12652.

Billio, M., Getmansky, M., Lo, A. W., and Pelizzon, L. (2012). Econometric measures of connectedness and systemic risk in the finance and insurance sectors. *Journal of Financial Economics*, 104(3):535 – 559. Market Institutions, Financial Market Risks and Financial Crisis.

Blei, S. and Bakhodir, E. (2014). Asset commonality and systemic risk among large banks in the United Sates. *SSRN Working paper.*

Boyer, B. H., Kumagai, T., and Yuan, K. (2006). How do crises spread? evidence from accessible and inaccessible stock indices. *The Journal of Finance*, 61(2):957–1003.

Braouezec, Y. and Wagalath, L. (2018). Risk-based capital requirements and optimal liquidation in a stress scenario. *Review of Finance*, 22:747–782.

Braverman, A. and Minca, A. (2016). Networks of common asset holdings: Aggregation and measures of vulnerability. *Statistics and Risk Modeling, Forthcoming.*

Brownlees, C. and Engle, R. F. (2016). SRISK: A conditional capital shortfall measure of systemic risk. *Review of Financial Studies.*

Caccioli, F., Shrestha, M., Moore, C., and Farmer, J. D. (2014). Stability analysis of financial contagion due to overlapping portfolios. *Journal of Banking and Finance*, 46:233 – 245.

Capponi, A. and Larsson, M. (2015). Price contagion through balance sheet linkages *. *Review of Asset Pricing Studies.*

Coen, J., Lepore, C., and Schaanning, E. (2017). Taking regulation seriously: Fire sales under solvency and liquidty constraints. *Working paper.*

Cont, R. and Schaanning, E. (2016). Fire sales, indirect contragion and systemic stress testing. *Norges Bank Working paper.*

Cont, R. and Wagalath, L. (2013). Running for the exit: Distressed selling and endogenous correlation in financial markets. *Mathematical Finance*, 23:718–741.

Cont, R. and Wagalath, L. (2016). Fire sales forensics: Measuring endogenous risk. *Mathematical Finance*, 26:835–866.

Danielsson, J., Shin, H. S., and Zigrand, J.-P. (2012). Procyclical leverage and endogenous risk. *Working Paper.*

Duarte, F. and Eisenbach, T. M. (2013). Fire sale spillovers and systemic risk. *Federal Reserve Bank of New York Staff Report*, 645.

Duffie, D. (2010). *How Big Banks Fail and What to Do About It?* Princeton University Press.

Ellul, A., Jotikasthira, C., and Lundblad, C. T. (2011). Regulatory pressure and fire sales in the corporate bond market. *Journal of Financial Economics*, 101(3):596 – 620.

Feinstein, Z. and El-Masri, F. (2017). The effects of leverage requirements and fire sales on financial contagion via asset liquidation strategies in financial networks. *Statistics & Risk Modeling*, 34:113–139.

Getmansky, M., Girardi, G., Hanley, K. W., Nikolova, S., and Pelizzon, L. (2016). Portfolio similarity and asset liquidation in the insurance industry. *SSRN Working paper.*

Granovetter, M. (1978). Threshold models of collective behavior. *American Journal of Sociology*, 83:489–515.

Greenwood, R., Landier, A., and Thesmar, D. (2015). Vulnerable banks. *Journal of Financial Economics*, 115(3):471 – 485.

Guo, W., Minca, A., and Wang, L. (2015). The topology of overlapping portfolio networks. *Statistics and Risk Modeling*, 33(3-4):139–155.

Ibragimov, R., Jaffee, D., and Walden, J. (2011). Diversification disasters. *Journal of Financial Economics*, 99(2):333 – 348.

Jotikasthira, C., Lundblad, C. T., and Ramadorai, T. (2012). Asset fire sales and purchases and the international transmission of funding shocks. *Journal of Finance*, 67(6):2015–2050.

Khandani, A. E. and Lo, A. W. (2011). What happened to the quants in august 2007? evidence from factors and transactions data. *Journal of Financial Markets*, 14(1):1–46.

Kyle, A. and Obizhaeva, A. (2016). Trading liquidity and funding liquidity in fixed income markets: Implications of market microstructure invariance. *Working Paper.*

Kyle, A. S. and Xiong, W. (2001). Contagion as a wealth effect. *The Journal of Finance*, 56(4):1401–1440.

Markose, S., Giansante, S., and Rais, S. A. (2012). "too interconnected to fail" financial network of us cds market: Topological fragility and systemic risk. *Journal of Economic Behaviour and Organization.*

Obizhaeva, A. (2012). Liquidity estimates and selection bias. *Working paper.*

Pedersen, L. H. (2009). When Everyone Runs for the Exit. *The International Journal of Central Banking*, 5:177–199.

Poledna, S., Martínez-Jaramillo, S., Caccioli, F., and Thurner, S. (2018). Quantification of systemic risk from overlapping portfolios in the financial system. *arxiv Working Paper*.

Shleifer, A. and Vishny, R. W. (1992). Liquidation values and debt capacity: A market equilibrium approach. *The Journal of Finance*, 47(4):1343–1366.

Wagner, W. (2011). Systemic liquidation risk and the Diversity – Diversification trade-off. *The Journal of Finance*, 66(4):1141–1175.

# A Online Appendix

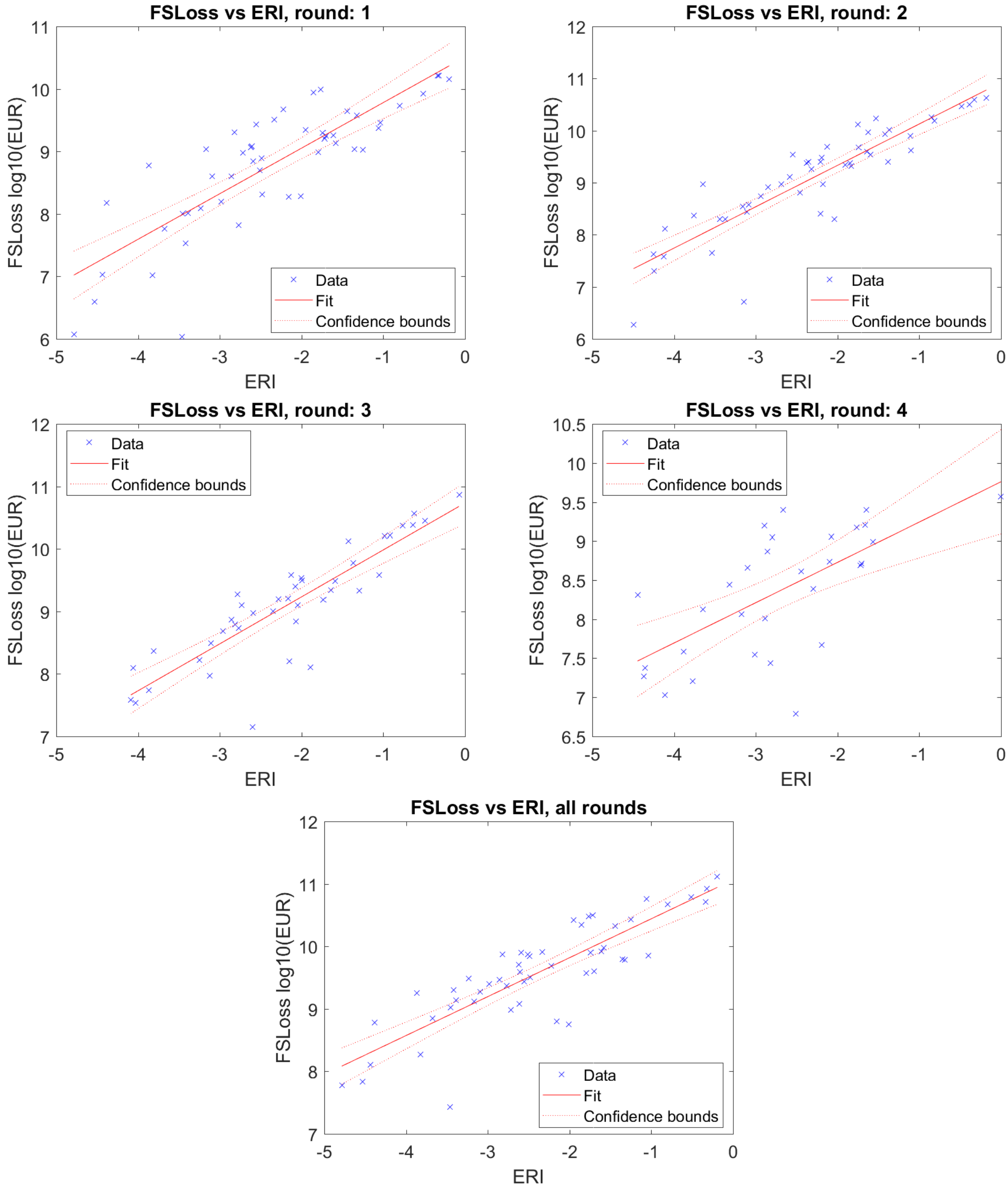


Figure 17: Bank-level fire-sales losses regressed on the *ERI* under proportional deleveraging.

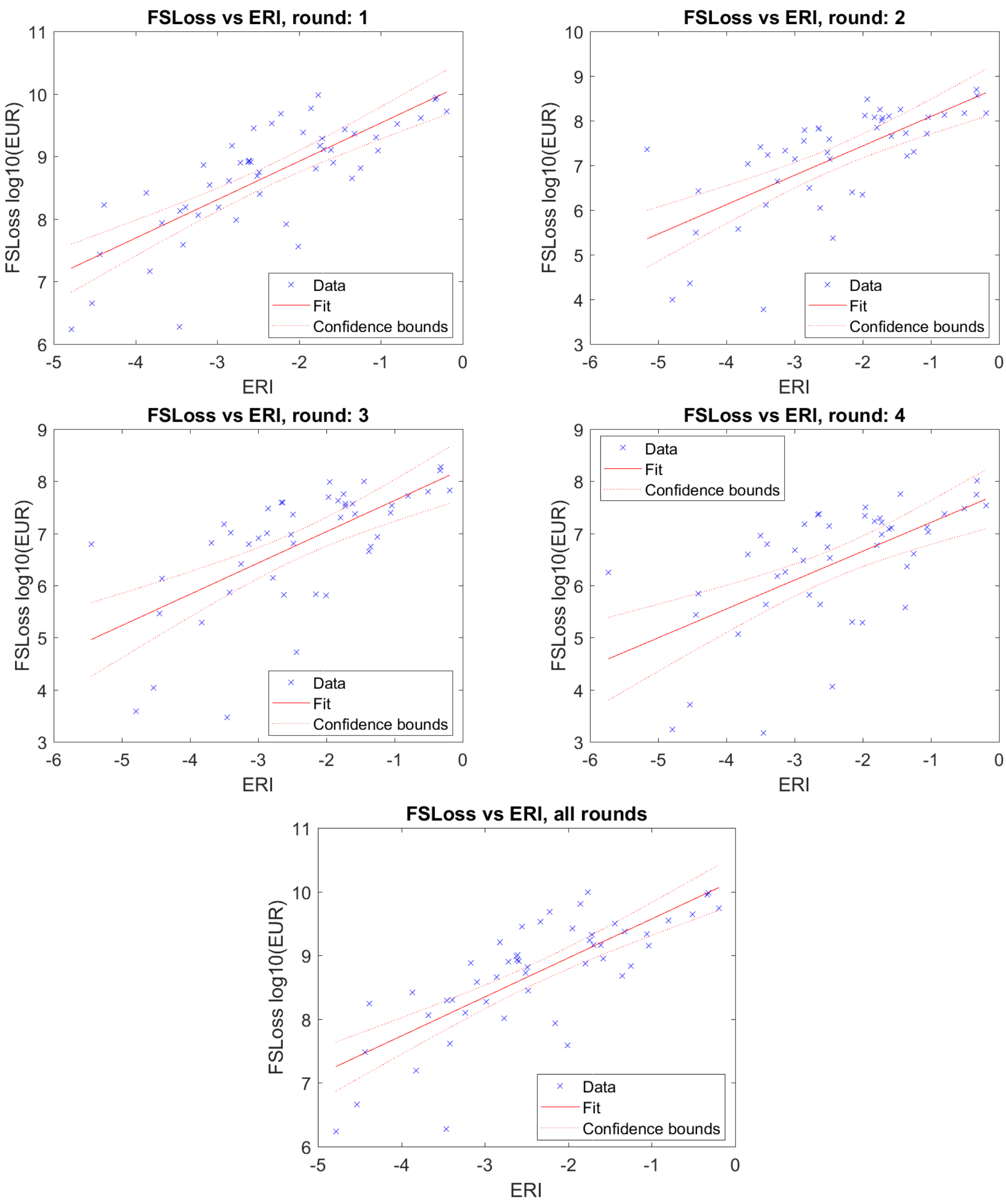


Figure 18: Bank-level fire-sales losses regressed on the $ERI$ under optimised deleveraging.

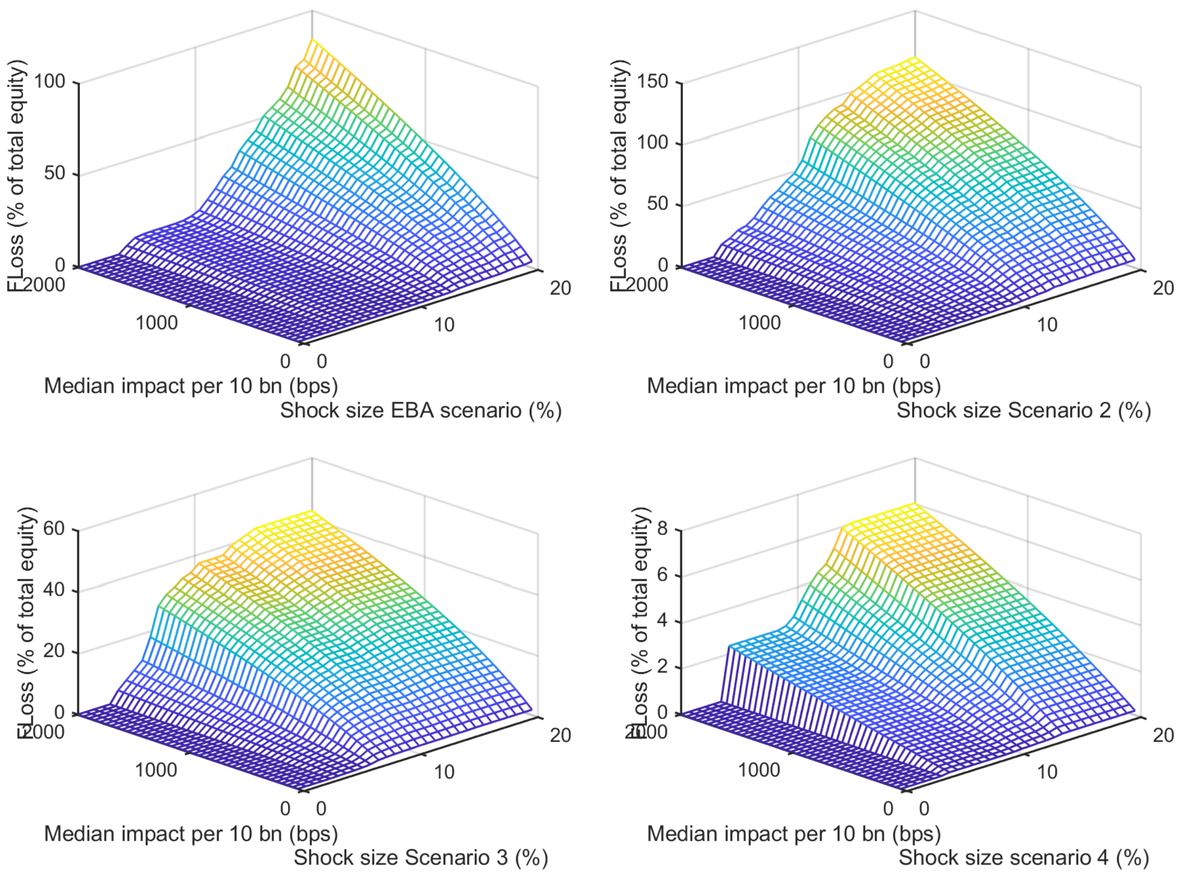


Figure 19: Fire-sales losses as a function of the shock intensity and the market impact for the four scenarios under optimised deleveraging (note the different $z$-axis scales!).

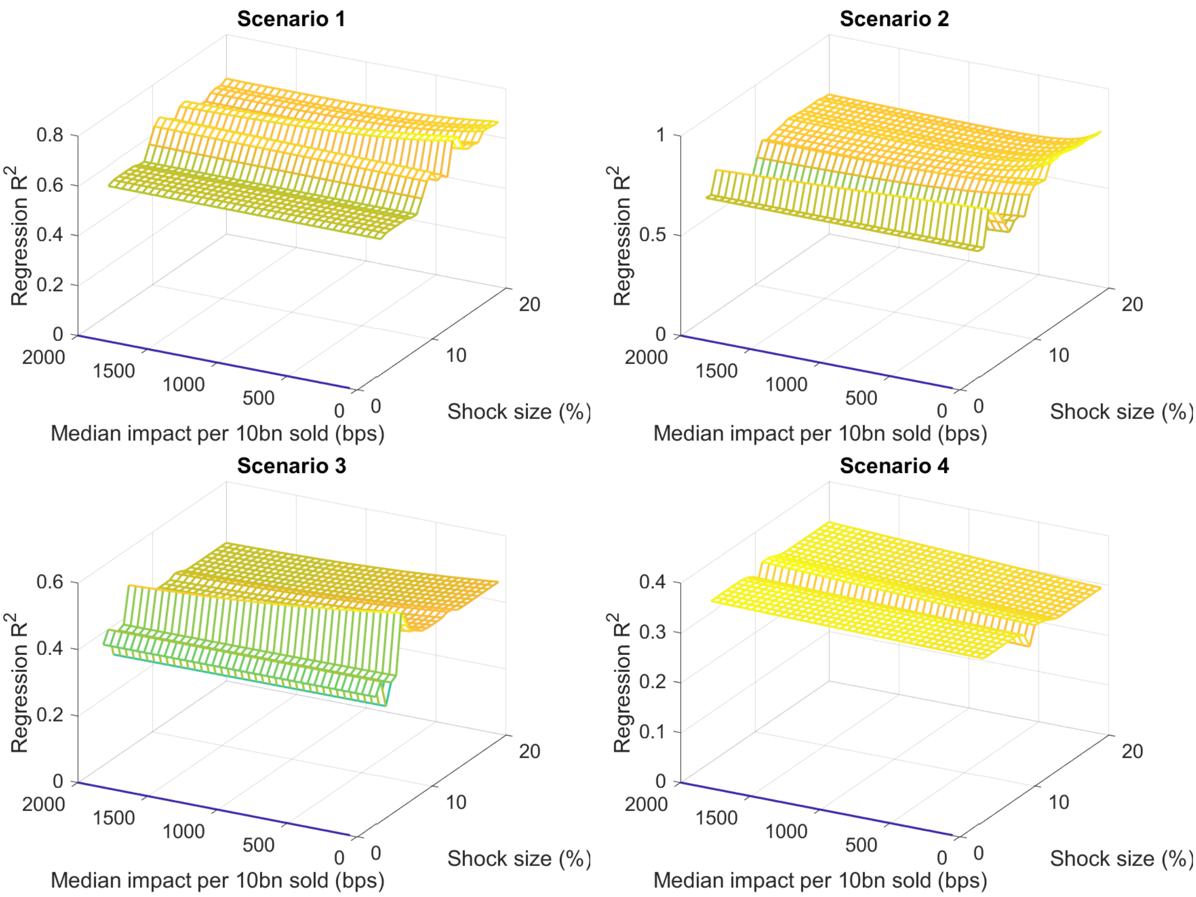


Figure 20: $R^2$ from regressing the fire-sales losses on the $ERI$ in the optimal deleveraging case for all four scenarios and combinations of market impact and shock intensity.

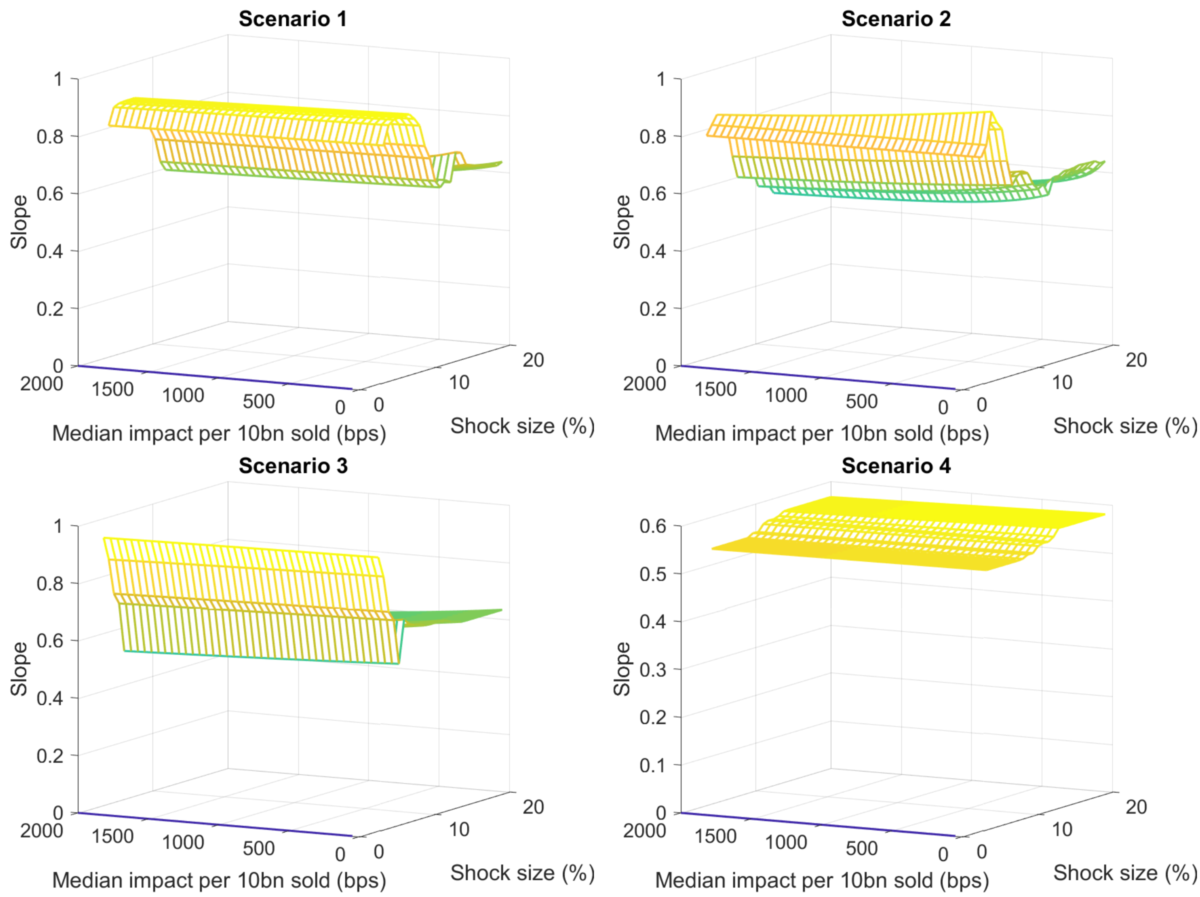


Figure 21: Slopes of the regression of the fire-sales losses on the $ERI$ under optimised deleveraging.

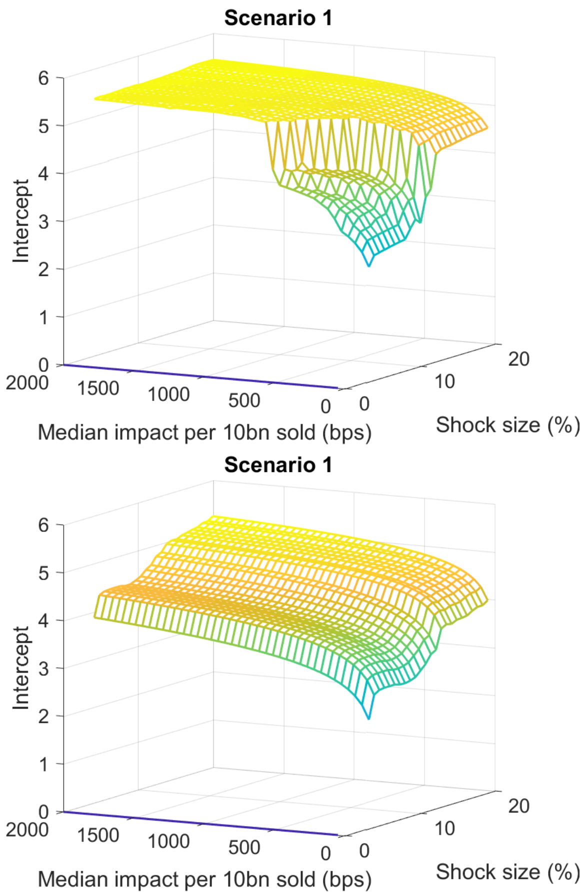


Figure 22: Intercepts of the regression of fire-sales losses on $ERI$ for scenarios 1 and 2 under proportional deleveraging (top) and optimised deleveraging (bottom).

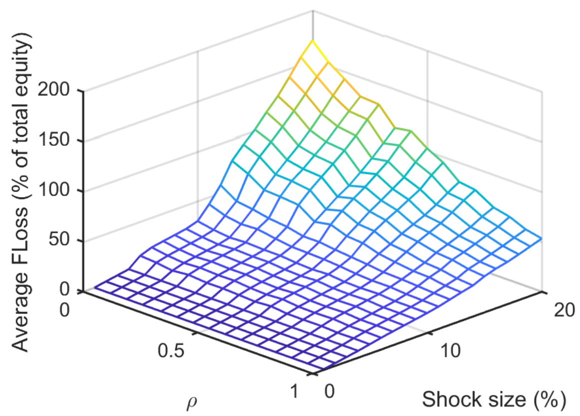


Figure 23: Non-parallel shifts of market depths under optimal deleveraging: As the correlation between the original market depth estimate and its randomised version decreases, fire-sales losses increase.

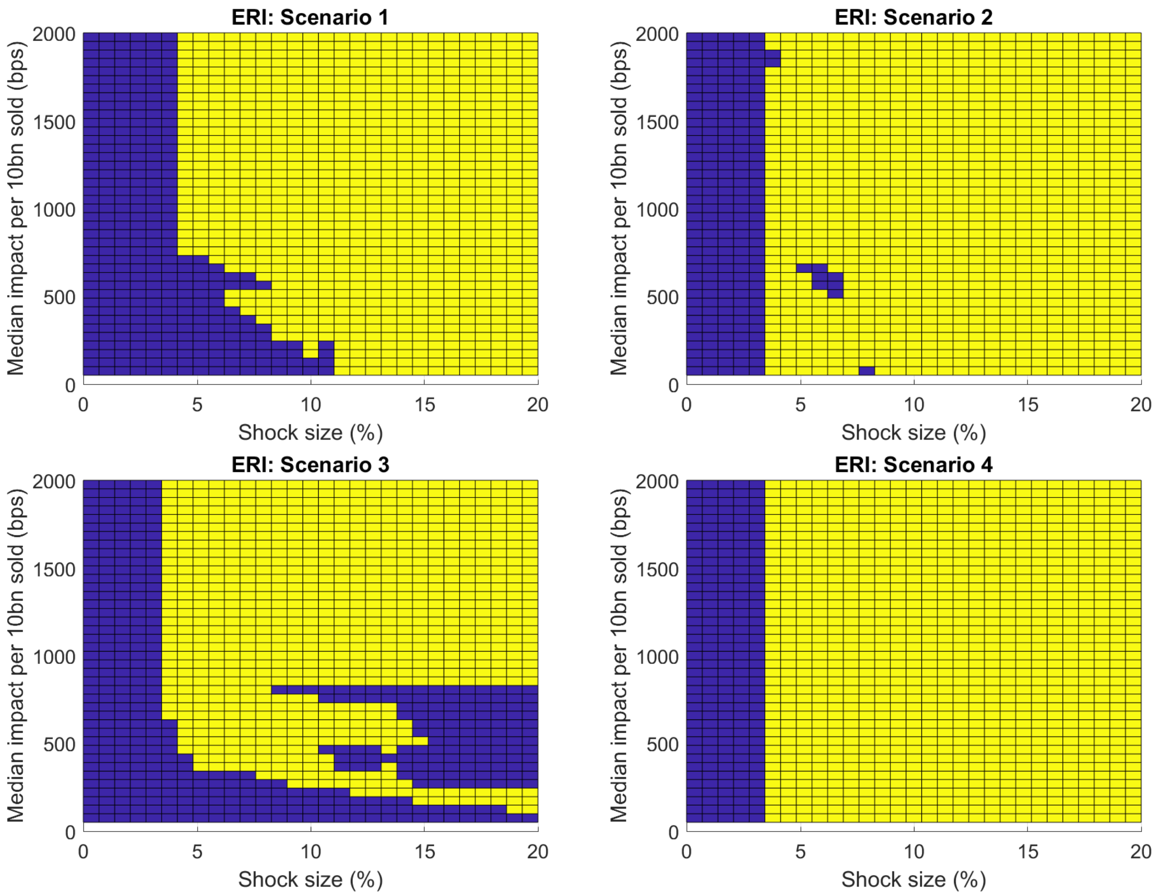


Figure 24: Inclusion (yellow) or exclusion (blue) of the $ERI$ as a predictor for fire-sales losses in a stepwise regression model for all scenarios and market impact & shock size combinations.

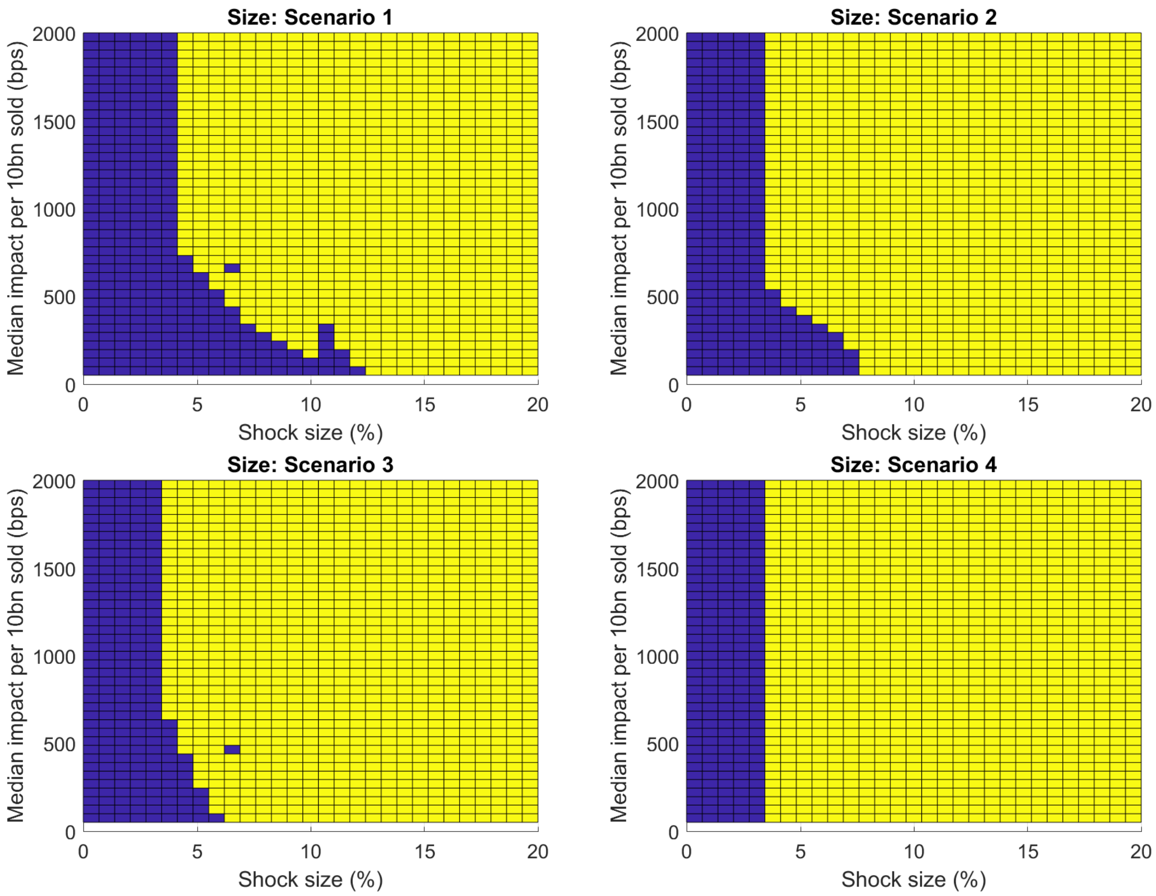


Figure 25: Inclusion (yellow) or exclusion (blue) of the Size as a predictor for fire-sales losses in a stepwise regression model for all scenarios and market impact & shock size combinations.

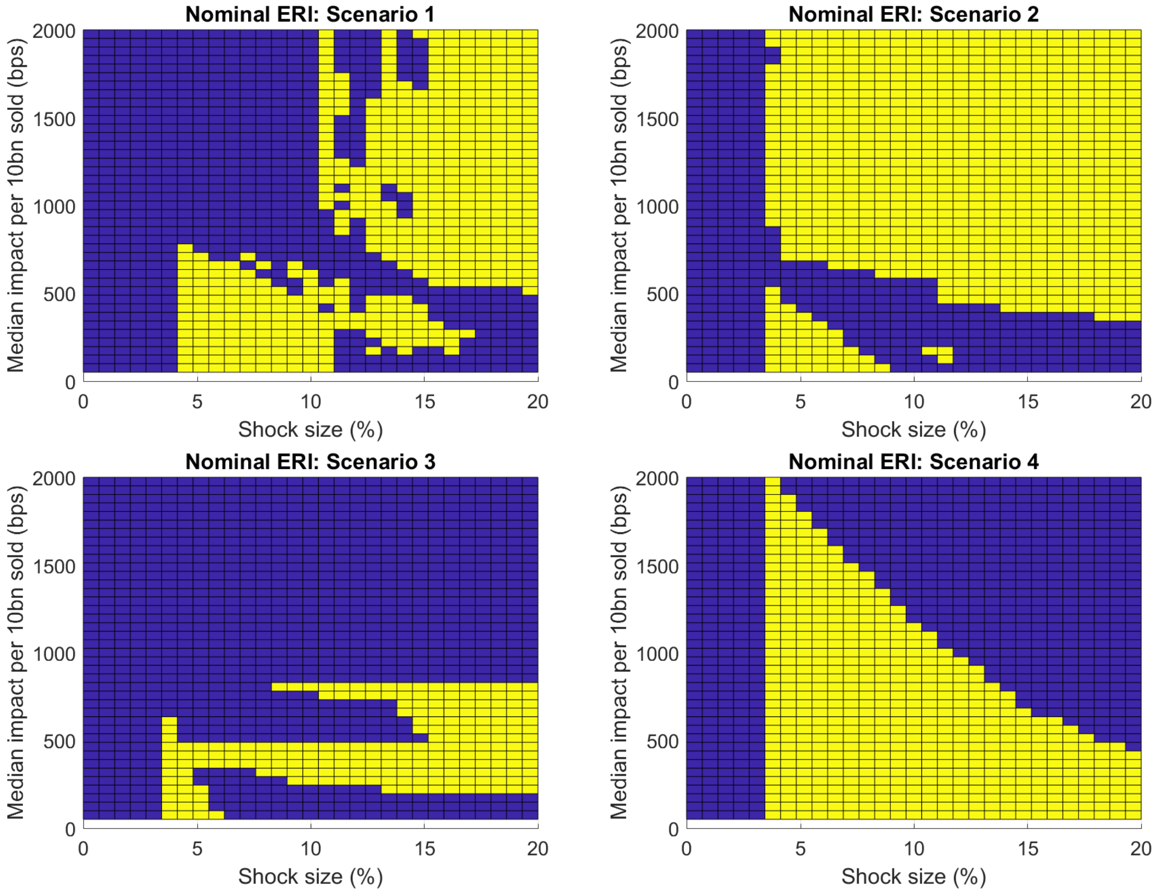


Figure 26: Inclusion (yellow) or exclusion (blue) of the Nominal-$ERI$ as a predictor for fire-sales losses in a stepwise regression model for all scenarios and market impact & shock size combinations.

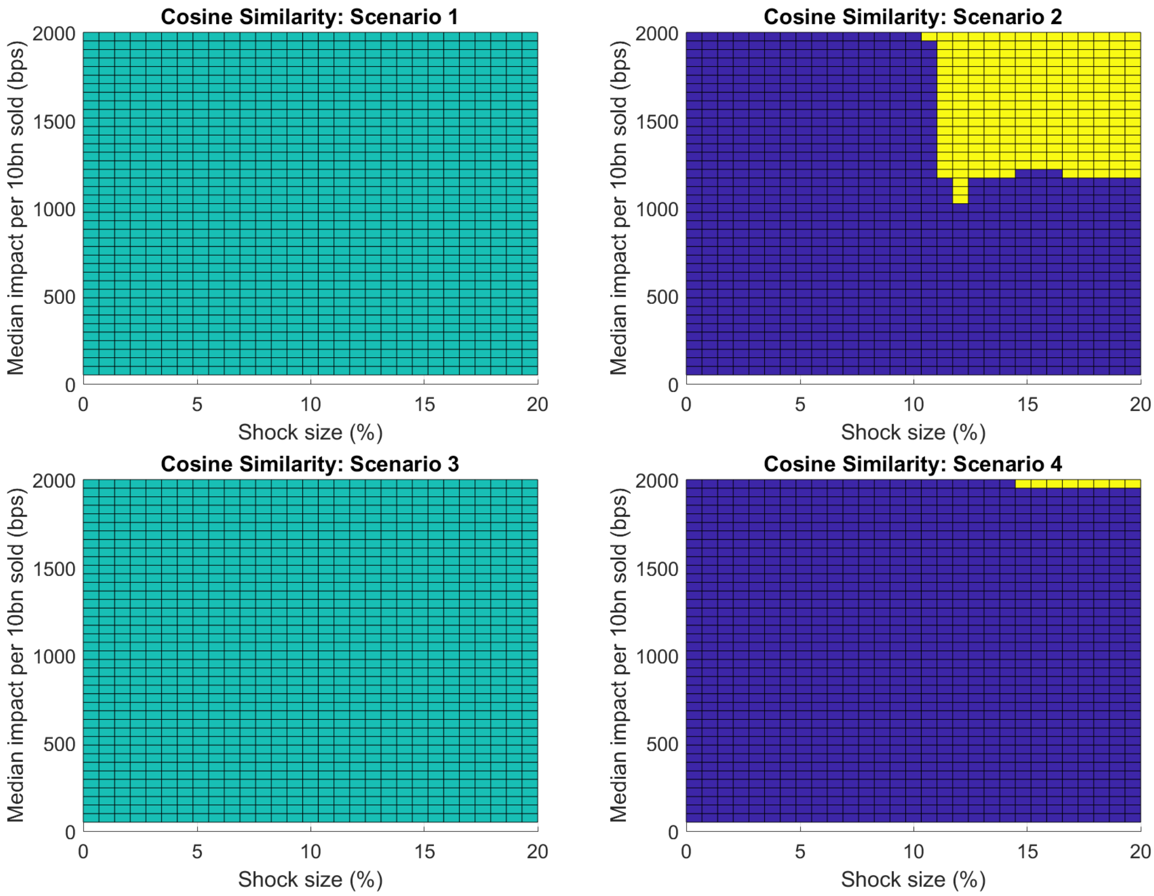


Figure 27: Inclusion (yellow) or exclusion (blue) of the Cosine Similarity as a predictor for fire-sales losses in a stepwise regression model for all scenarios and market impact & shock size combinations.