

Next-Generation Neutrino Interaction Measurements and Beyond-Standard-Model Physics with the Upgraded T2K Near Detector

Weijun Li

The Queen's College
University of Oxford

A thesis submitted for the degree of
Doctor of Philosophy

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Abstract

Lord Kelvin identified two “clouds” over 19th-century physics in 1900, which were resolved relatively promptly, yet additional “clouds” have since emerged and dissipated. One of the most prominent remaining “clouds” is Charge-Parity violation (CPV) in the lepton sector. The T2K experiment is dedicated to measure CPV via neutrino oscillation. Although significant progress has been made, its potential is limited by an incomplete understanding of neutrino–matter interactions. To address this shortfall, the T2K near detector (ND) has been upgraded, and I had the privilege of participating in its assembly and research. This thesis focuses on the new techniques and physics potential enabled by the ND upgrade. Specifically, I have adopted the Elastically Scattered and Contained proton selection and developed the pion trackless reconstruction. These novel techniques are illustrated with Monte Carlo (MC) studies and the first glimpse of a preliminary comparison between MC and data collected by the upgraded ND. The upgrade also enables measurements of Transverse Kinematic Imbalance (TKI) variables with unprecedented precision. Furthermore, a new set of centre-of-momentum variables has been devised to uniquely probe final-state interactions and to provide a complementary handle to TKI for neutrino–hydrogen event selection. The usefulness of these novel variables is demonstrated in a tuning project where existing TKI measurements are used to constrain the current model, significantly improving the agreement between simulation and data. Finally, this thesis concludes with a sensitivity estimation for a heavy neutral lepton search at the upgraded ND280, exploring the elusive “clouds” in an ever clearer sky of physics.

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Abstract

Lord Kelvin identified two “clouds” over 19th-century physics in 1900, which were resolved relatively promptly, yet additional “clouds” have since emerged and dissipated. One of the most prominent remaining “clouds” is Charge-Parity violation (CPV) in the lepton sector. The T2K experiment is dedicated to measure CPV via neutrino oscillation. Although significant progress has been made, its potential is limited by an incomplete understanding of neutrino–matter interactions. To address this shortfall, the T2K near detector (ND) has been upgraded, and I had the privilege of participating in its assembly and research. This thesis focuses on the new techniques and physics potential enabled by the ND upgrade. Specifically, I have adopted the Elastically Scattered and Contained proton selection and developed the pion trackless reconstruction. These novel techniques are illustrated with Monte Carlo (MC) studies and the first glimpse of a preliminary comparison between MC and data collected by the upgraded ND. The upgrade also enables measurements of Transverse Kinematic Imbalance (TKI) variables with unprecedented precision. Furthermore, a new set of centre-of-momentum variables has been devised to uniquely probe final-state interactions and to provide a complementary handle to TKI for neutrino–hydrogen event selection. The usefulness of these novel variables is demonstrated in a tuning project where existing TKI measurements are used to constrain the current model, significantly improving the agreement between simulation and data. Finally, this thesis concludes with a sensitivity estimation for a heavy neutral lepton search at the upgraded ND280, exploring the elusive “clouds” in an ever clearer sky of physics.

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List of Abbreviations

CP	Charge-Parity
LBL	long-baseline
T2K	Tokai-to-Kamioka
Super-K	Super Kamiokande
Hyper-K	Hyper Kamiokande
MSW	Mikheyev-Smirnov-Wolfenstein
SNO	Sudbury Neutrino Observatory
NO	Normal ordering
IO	Inverted ordering
PDG	Particle Data Group
JUNO	JiangMen Underground Neutrino Observatory
LArTPC	Liquid Argon Time Projection Chamber
DUNE	Deep Underground Neutrino Experiment
ND	Near detector
FD	Far detector
HNL	Heavy neutral lepton
1-D, 2-D	One- or two-dimensional, referring in this thesis to spatial dimensions in an image.
TKI	Transverse Kinematic Imbalance
COM	Centre-of-momentum
IA	Impulse approximation
IS	Initial state
FSI	Final state interaction
RPA	Random Phase Approximation
P0D	π^0 -detector

FGD	Fine Grained Detector
TPC	Time Projection Chamber
ECAL	Energy calorimeters
SFGD	Super Fine Grained Detector
TOF	Time-of-flight detector
HAT	High angle TPC
BDT	Boosted decision tree
PGUN	Particle gun
MIP	Minimal ionizing particle
ESC	Elastically scattered and contained
TL	Tracklesskless
SPK	Single particle kinematics
RES	Resonance

A deep question is one where either a yes or no answer is interesting.

— Niel Bohr F. Reines, Nobel Lecture 1995

1

Prologue

Neutrinos are a type of elementary particle that form our material world. They are crucial in addressing one of the most fundamental questions in physics: the existence of our universe. Cosmological observations have established that our universe is dynamic, implying that it has evolved into its present state through certain physical processes since its inception. In nature, physical processes follow certain rules of symmetry, and in particular, Charge-Parity (CP) symmetry dictates that particles and antiparticles should interact in the same way. If the universe began with an equal number of particles and antiparticles, and if all physical processes obey CP symmetry, the universe should still contain the same number of particles and antiparticles today. This presents a problem because particles would annihilate with their antiparticle counterparts to release energy, leading to a universe full of energy but devoid of matter, and hence no stars or planets, let alone life. Clearly, this cannot be true because we do exist, so either the universe did not begin with the same number of particles and antiparticles, or some physical processes violate CP symmetry such that an excess of particles can accumulate over time. While there is no evidence for the former, CP violation is indeed observed in certain processes. However, the presently measured extent of CP violation is insufficient to account for the vast amount of matter in our universe, and the only known source of potential CP violation yet to be measured lies in the neutrino sector.

Hence, a precise measurement of CP violation in neutrinos is a significant step towards answering the matter-antimatter asymmetry mystery and will be a strong indicator of new physics beyond the Standard Model.

CP violation in neutrinos is measured by analysing neutrino oscillations, i.e., the phenomenon that neutrinos can switch between the three known types, namely electron neutrino, muon neutrino, and tau neutrino, as they travel through space. Long-baseline (LBL) neutrino experiments are built to measure the difference between neutrino oscillations and antineutrino oscillations to quantify CP violation, for example, the Tokai-to-Kamioka (T2K) experiment. The T2K experiment utilises the Super Kamiokande (Super-K) detector as the far detector (FD) to measure the oscillated neutrino spectrum, while the neutrino spectrum near the beam source is measured by a near detector (ND), ND280, located 280 meters from the source. It has produced the first constraint on CP violation [1], but due to limited statistics, the measurement still has a large range with considerable uncertainties. Super-K will be replaced by Hyper Kamiokande (Hyper-K) [2] in 2027, which is much larger than Super-K and will increase statistics many-fold. The uncertainties in the measurement of CP violation will be dominated by systematic uncertainties. One of the most significant systematic uncertainties lies in the neutrino-nucleus interaction modelling. Although the neutrino source beam energy spectrum is well studied, the energy reconstruction of each neutrino is not directly measurable as it leaves no visible track in detectors. Thus, we have to rely on approximate neutrino-nucleus interaction models to reconstruct its energy from the interaction products, of which the hadrons and charged leptons could be detected and their energy could be measured if energetic enough. Hence, reducing particle energy measurement uncertainty and developing a more sophisticated neutrino-nucleus interaction model are crucial for future CP violation measurements. The ND280 has been upgraded to achieve this goal, and the research of my thesis centres around this upgrade.

This thesis is structured as follows: Ch. 2 will provide a brief history of neutrino physics, and Ch. 3 lays the theoretical foundation necessary for subsequent discussion. The following chapter, Ch. 4, will describe the T2K experiment and its upgrade

in detail. Ch. 5 elaborates on the development of multiple selections using novel techniques, paving the foundation for physical analyses presented in Ch. 6. As cross-section measurements using the upgraded ND are not possible yet, Ch. 7 presents one important application of cross-section analysis—tuning—using existing measurements. Besides Standard Model physics, the upgraded ND also holds promising potential for exotic searches. Ch. 8 presents a sensitivity study for the upgraded ND using the heavy neutral lepton (HNL) simulation package, **BeamHNL**, in the **GENIE** [3] event generator. The last chapter summarises this thesis and suggests possible extensions of this work.

Leider kann ich nicht persönlich in Tuebingen erscheinen,
da ich infolge eines in der Nacht vom 6. zum 7. Des.
in Zuerich stattfindenden Balles hier unabkoemmlich
bin.

Unfortunately, I cannot personally appear in Tübingen
since I am indispensable here in Zürich
because of a ball on the night from December 6 to 7.

— Wolfgang Pauli [4], translation: Kurt Riesselmann

2

The History of Neutrinos

Contents

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The field of neutrino physics is relatively young and has developed rapidly over the years. This chapter will go back in time to the birth of neutrino physics and note some of the important milestones. As this only serves as a prologue to the thesis, it is by no means a complete historical account. Let us dive into the fruitful history of the field.

2.1 The Past

The field of neutrino physics has a dramatic opening that many know of—in a letter [4] from Wolfgang Pauli to the Radioactivity Conference in 1930, which he did not attend in person as he was indispensable at a ball in Zürich. The letter proposed a solution to the then very puzzling problem of β decay. It was thought of as a two-body decay, where the heavier nucleus decays to a lighter nucleus and an electron, which carries away a well-defined amount of energy due to the mass

difference between the parent and daughter nucleus. However, the measured electron energy spectrum was a wide distribution rather than the fixed value expected. In his letter, Pauli postulated a third, but undetected, particle in the decay that shared the energy budget with the electron so that the sum of the third particle's energy and the electron's energy is equal to the fixed value expected, but not the electron energy alone. Originally, he coined this particle the “neutron”, but this name did not stick and was given to the now-known neutron discovered by James Chadwick in 1932. The present name, “neutrino”, was later coined by Edoardo Amaldi in a conversation with Enrico Fermi, who later published the seminal paper on his theory of beta decay in 1934 [5], laying the theoretical foundation for subsequent experimental work. However, the cross-section for neutrino interaction is estimated to be so small, making it extremely difficult to detect. Even Pauli lamented, “I have done a terrible thing. I have postulated a particle that cannot be detected.”

It was such a formidable task that Frederick Reines even proposed using the atomic bomb as the neutrino source to have an intense source of neutrinos for detection. Fortunately, Reines and his collaborator, Clyde Cowan, discovered that the delay between the signal of neutron capture and the signal of annihilation between electron and positron, where the neutron and the positron are the products of the antineutrino interacting with a proton, is distinct and could be utilised to significantly reduce backgrounds, allowing a less intense (and less dangerous) source of antineutrinos to be used. It was not until about 26 years after the postulation of the neutrino that it was directly detected in the Cowan-Reines experiment at the Savannah River Plant in 1956 [6]. The detector was mainly composed of 1400 litres of liquid scintillator and 200 litres of cadmium-loaded water as target and neutron capturer. This remarkable achievement was recognised with a Nobel Prize in 1995.

After the Cowan-Reines experiment, the existence of the neutrino was beyond doubt, and the field of neutrino physics developed rapidly. There were two naturally complementary directions for development. One was to build larger and more advanced detectors to measure the existing neutrino sources, for example, solar neutrinos, reactor neutrinos, atmospheric neutrinos, and others. The other was to

include neutrino production as part of the experiment, which became the field of accelerator neutrinos.

Roughly about the same time, Raymond Davis was also working at the Savannah River Plant, trying to use the inverse beta decay of chlorine to argon, suggested by Pontecorvo [7], to detect neutrinos. His experiment measured at least a 20 times smaller cross-section for antineutrinos than for neutrinos in the inverse beta decay turning chlorine into argon [8].

Later, he applied the same technique to measure the solar neutrino flux with a detector of the size of 378,000 litres at the Homestake Mine [9], sufficient to detect solar neutrinos based on the Standard Solar Model calculation done by John Bahcall [10]. It was to everyone's surprise that the Homestake experiment only observed about one-third of the expected flux [11], a discrepancy known as the solar neutrino problem. Davis and Bahcall did meticulous checks on their results and found no errors large enough to explain the discrepancy.

This phenomenon sparked many efforts, both experimentally and theoretically, to resolve the observed discrepancy. The idea of neutrino oscillation was actually proposed even before the solar neutrino problem by Pontecorvo in 1957 [12] and completed by Maki, Nakagawa, and Sakata in 1962 [13]. However, it was just one of the possible solutions back then among many other theories. Papers on the other piece of the solution, the Mikheyev–Smirnov–Wolfenstein (MSW) matter effect [14, 15], were published in the 1980s. It is the phenomenon that the neutrino propagation is affected by matter density. More details are provided in the next chapter. The combined solution, the large-mixing-angle MSW solution, provided an attractive answer to the solar neutrino problem. Subsequent experiments, such as Kamiokande, a 3,000,000-litre water Cherenkov experiment led by Masatoshi Koshiba [16], GALLEX [17], and SAGE [18], confirmed the solar neutrino deficit. Another profound discovery at that time was the first measurement of supernova neutrino, from SN1987A, by the Kamiokande collaboration [19]. The definitive evidence for solar neutrino oscillation came in 2001, when the Sudbury Neutrino Observatory (SNO) experiment, with a 1,000,000-litre heavy water Cherenkov

detector, published its result [20], directly showing that the observed deficit of electron neutrinos was due to oscillation into other flavours. The following year, Davis and Koshiba received a Nobel Prize in physics. Meanwhile, in 1998, the upgrade of Kamiokande, the Super-Kamiokande (Super-K) (50,000-tonne water Cherenkov detector), reported the observation of neutrino oscillation in atmospheric neutrinos [21]. Takaaki Kajita (Super-K) and Arthur McDonald (SNO) shared a Nobel Prize for their work on neutrino oscillation in 2015.

Along the other development route, Pontecorvo suggested the possibility of using accelerated protons hitting targets to produce mesons that decay to produce a beam of neutrinos in 1959 [22]. A year later, a similar proposal was made independently by Mark Schwartz [23] to use accelerator-produced neutrinos to address another puzzle at that time, whether the muon neutrino is the same as the electron neutrino. Schwartz and his collaborators, which included Leon Lederman and Jack Steinberger, used the Alternating Gradient Synchrotron at the Brookhaven National Laboratory to produce the first accelerator neutrino beam. The neutrino interaction was recorded by a 10-tonne spark chamber, which was able to reconstruct clear muon tracks from the interaction. In 1962, the collected data proved that muon neutrinos are distinct from electron neutrinos [24]. Schwartz, Lederman, and Steinberger were awarded the Nobel Prize in 1988.

For the subsequent decades, accelerator neutrinos were used to help progress the more general field of particle physics due to the unique nature of neutrino scattering. For instance, neutral current interaction was first observed by the Gargamelle bubble chamber at CERN using a beam of neutrinos [25]. High-energy neutrino beams were also deployed to test the electroweak theory and study the nucleon structure by the CDHS experiment at CERN [26].

During the 1990s, in light of the evidence of neutrino oscillations observed in solar neutrinos and atmospheric neutrinos, accelerator neutrinos were first proposed to be used to study neutrino oscillations in two long-baseline (LBL) experiments, namely the K2K experiment in Japan [27] and the Main injector neutrino oscillation search (MINOS) experiment in the U.S. [28]. Both of these LBL experiments deployed

two detectors, similar in design, a small one near the neutrino source and a large one far away, to compare the neutrino fluxes at the two locations to measure the oscillation. The K2K experiment used Super-K as the far detector at approximately 250 km away, while MINOS uses a 6,000-tonne steel detector at 735 km away. Their results [29, 30] in 2000s after a few years of data collection corroborated the Super-K results, confirming the oscillation of muon neutrinos into other flavours.

The field has entered a phase of precision measurement that will be illustrated in the next subsection. The two complementary development routes in the past have combined in long-baseline experiments to elucidate the few remaining questions in the field.

2.2 The Present

With neutrino oscillation finally confirmed, the field of neutrino physics has a fruitful past and a clear theoretical framework laid out. One present task for the field is the precise measurement of the parameters in the theoretical framework, namely the neutrino masses and elements of the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix [12, 13]. The PMNS matrix describes the mixing between the three mass eigenstates of neutrinos to form the three flavour eigenstates, namely electron neutrino, muon neutrino, and tau neutrino. It can be parameterised by three mixing angles, namely θ_{12} , θ_{13} and θ_{23} , and a CP violation phase, δ_{CP} . The specific meaning of the parameters in the PMNS matrix will be illustrated later in Sec. 3.1, while the respective measurements are first given here without breaking the flow.

The mass of the electron neutrino can be studied by investigating the shape of the high-energy tail of the electron spectrum in tritium beta decay, a method proposed in Fermi’s theory paper on β -decay [5] and independently by Francis Perrin [31]. The KATRIN experiment employs this method and produces one of the most stringent upper bounds, 0.8 eV with 90% confidence level [32]. There are other methods to extract the neutrino mass, such as from cosmological observations [33], but as this thesis focuses on neutrino oscillation, these will not be elaborated for conciseness.

As neutrino oscillation depends on the mass difference squared as well, oscillation experiments are able to measure the neutrino masses indirectly through the mass differences in addition to the PMNS matrix elements. Contrary to the electron neutrino mass measured from β -decay, the mass differences measured in oscillation experiments are the differences between the neutrino mass eigenstates. The general structure of the mass eigenstates is that there is a small difference between m_2 and m_1 , but m_3 is either much smaller or larger than m_2 and m_1 . The relation between m_3 and the others is the mass hierarchy problem. The case where m_3 is the largest is called the Normal Ordering (NO), and the case where m_3 is the smallest is the Inverted Ordering (IO).

In 2013, the KamLAND collaboration made one of the most precise measurement of Δm_{21}^2 , $\Delta m_{21}^2 = 7.53 \pm 0.18 \times 10^{-5} \text{ eV}^2$ [34]. Note that this difference is not the absolute value as it is known that $m_2 > m_1$. As for Δm_{32}^2 , the estimation is produced from a global fit of accelerator, reactor, and atmospheric data by PDG [35], giving $\Delta m_{32}^2 = 2.455 \pm 0.028 \times 10^{-3} \text{ eV}^2$ (NO) and $\Delta m_{32}^2 = -2.529 \pm 0.029 \times 10^{-3} \text{ eV}^2$ (IO).

As for the mixing angles, the current best estimate of θ_{12} is given by a fit of the KamLAND measurement and solar measurements in 2016, producing $\sin^2(\theta_{12}) = 0.307^{+0.013}_{-0.012}$. The estimation of θ_{23} is done by the PDG using accelerator and atmospheric neutrino data [35], leading to $\sin^2(\theta_{23}) = 0.553^{+0.016}_{-0.024}$ (IO) and $\sin^2(\theta_{23}) = 0.558^{+0.015}_{-0.021}$ (NO). The last mixing angle, the average of θ_{13} , is obtained by PDG using accelerator and reactor data, giving $\sin^2(\theta_{13}) = 2.19 \pm 0.07$.

The statistics of δ_{CP} measurements are still limited, so no definitive conclusion on CP violation in neutrinos can be drawn yet. The T2K experiment has produced the first constraint on δ_{CP} [1], which is $-3.41 < \delta_{\text{CP}} < -0.03$ (NO) or $-2.53 < \delta_{\text{CP}} < -0.32$ (IO) in radians at 3σ confidence interval, while the NOvA experiment does not exclude any values of δ_{CP} at 3σ confidence level [36].

From being completely unknown to precisely measured, neutrino oscillation is now a well-understood phenomenon except for the mass ordering and the extent of CP violation, which is the pressing goal of the present and future neutrino experiments.

2.3 The Plans

To elucidate the remaining questions in neutrino physics, new experiments are planned or under construction and existing experiments are undergoing upgrades. This sub-section will only briefly describe some of them and it is not meant to be an exhaustive review.

The successor of the Daya Bay experiment, which produced one of the most precise measurements of θ_{13} , the Jiangmen Underground Neutrino Observatory (JUNO), is set to start operation in summer 2025 [37]. Located near the Yangjiang and Taishan nuclear power plants, JUNO has a 20,000-tonne liquid scintillator. It is projected to determine the mass ordering with 3 (3.1) σ significance for NO (IO) in approximately 7 years of data taking [38].

The successor of the Super-K experiment, the Hyper-Kamiokande (Hyper-K) [2], is under construction and is expected to operate in 2027. Hyper-K is a 258,000-ton water Cherenkov detector, about 8 times the size of its predecessor, Super-K. It uses the same beamline as the T2K experiment and measures the electron neutrino appearance to determine δ_{CP} . With 10 years of data taking, it is projected to exclude CP conservation to 5 σ for more than 60% of true δ_{CP} values and to 3 σ for 75% of δ_{CP} values assuming the systematic uncertainties are controlled at 2.7% [39].

Another large-scale LBL neutrino experiment is the Deep Underground Neutrino Experiment (DUNE) [40–44]. Its far detector consists of four 17,000-ton Liquid Argon Time Projection Chambers (LArTPC) and it has a beamline of 1300 km, which enables it to be highly sensitive to the mass ordering as well. With the novel LArTPC technology, it is expected to reach 5 σ for the mass ordering with 3 years of data taking and reach 3 σ for 75% of δ_{CP} values with about 15 years of data taking [45]. There is no official date for DUNE yet, but the test experiment for its far detector, the ProtoDUNE experiment, has operated for more than 2 years at CERN.

In the projected sensitivity of these future experiments, the importance of reducing systematic uncertainties cannot be over-emphasised as results obtainable with years of data taking would need decades instead if the systematic uncertainties do not reach the required level. This is exactly why T2K is upgrading its near

detector to better understand neutrino interactions to reduce systematics, which is exactly what this thesis is about.

Besides the measurement of the known parameters, there are also a plethora of beyond the Standard Model searches both at existing and future experiments. Moreover, there has been impressive progress on utilising neutrinos for cosmological observation by experiments, such as IceCube [46]. Although they are not elaborated in this thesis, they represent the highly exciting development of the field.

From the 1400-litre liquid scintillator detector in the Cowan-Reines experiment to the 1,000,000-litre heavy water Cherenkov detector of SNO and to the 258,000,000-litre water Cherenkov detector of Hyper-K, the field of neutrino physics has grown rapidly. From being the most elusive particle, which was thought never possible to detect, to a sophisticated understanding of neutrino oscillation, our understanding of the properties of neutrinos has improved drastically. With the incredible experiments running and in plan, it is promising that a definitive measurement of all the neutrino properties is reachable, and its results will be profound and shed light on new physics directions.

Well, you shouldn't believe everything you read in the papers.

— Hans Bethe F. Reines, Nobel Lecture 1995

3

The Theory of Neutrinos

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The main goal of neutrino experiments is to measure all neutrino properties, including the mass, mixing angles, and the CP violation parameter, δ_{CP} . The mixing angles and δ_{CP} fully describes neutrino oscillation, while the mass is the reason why neutrino oscillation occurs. Hence, these parameters will be better illustrated in Sec. 3.1 in the context of the theoretical framework of neutrino oscillation. As only products from neutrino interactions are detected in experiments, achieving this goal requires a good understanding of neutrino interactions. Hence, a brief overview of neutrino interactions is given in Sec. 3.2.

3.1 Oscillation

Neutrinos come in three flavours: the electron neutrino (ν_e), the muon neutrino (ν_μ), and the tau neutrino (ν_τ). The different flavours of neutrinos will only produce the corresponding lepton in weak interactions, so they are eigenstates of the weak interaction. However, neutrinos propagate through space as mass eigenstates, ν_1 , ν_2 , and ν_3 . The reason for neutrino oscillation to occur is twofold. One is because the mass eigenstates do not have a simple one-to-one correspondence with the weak eigenstates. They are superpositions of the weak eigenstates. The other reason is that these mass eigenstates have different masses. Neutrinos are created as flavour eigenstates in weak interactions, but they propagate as a linear combination of mass eigenstates, which propagate with different phase velocities due to their different masses. Hence, throughout the propagation, the linear combination of mass eigenstates continually changes, leading to a varying superposition of flavour eigenstates, and thus, neutrino oscillation.

To be more specific, the matrix describing the mixing of mass eigenstates into flavour eigenstates is the PMNS matrix. It is conventionally parametrised as

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\text{CP}}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\text{CP}}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (3.1)$$

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$, and δ_{CP} is the CP violation phase. The angles θ_{ij} are the mixing angles. Using the PMNS matrix, the flavour eigenstates can be expressed in terms of the mass eigenstates as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \quad (3.2)$$

The derivations of the following results can be obtained readily with pen and paper, so to keep a close focus on the development of the theoretical idea, detailed derivations are not included here. Due to the small neutrino masses, a mass eigenstate with mass m_i acquires the following phase approximately:

$$|\nu_i(L)\rangle = e^{-i\frac{m_i^2 L}{2E}} |\nu_i(0)\rangle, \quad (3.3)$$

after travelling a distance L with energy E . Hence, the probability that a neutrino of flavour α created at the source is detected as a neutrino of flavour β after travelling a distance L is given by

$$P_{(\alpha \rightarrow \beta)} = |\langle \nu_\beta | \nu_\alpha(L) \rangle|^2 = \left| \sum_i U_{\alpha i} U_{\beta i}^* e^{-i \frac{m_i^2 L}{2E}} \right|^2 \quad (3.4)$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re} \left(U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} \right) \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) \quad (3.5)$$

$$+ 2 \sum_{i>j} \text{Im} \left(U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} \right) \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right), \quad (3.6)$$

where $\delta_{\alpha\beta}$ is the Kronecker delta, $U_{\alpha i}$ is the element of the PMNS matrix, and $\Delta m_{ij}^2 = m_i^2 - m_j^2$. This result shows that the oscillation probability depends on the mixing angle through the PMNS matrix elements and on the mass difference and neutrino energy through the sine terms. For brevity, the dependence on the sine terms can be characterised by a phase angle,

$$\Delta_{ij} = \frac{\Delta m_{ij}^2 L}{2E} \approx 1.27 \frac{\Delta m_{ij}^2 [\text{eV}^2] L [\text{km}]}{E [\text{GeV}]}, \quad (3.7)$$

where the oscillation is maximal when $\Delta_{ij} = \pi/2$. These results can be used to better understand how the various parameters are measured in different types of neutrino experiments. The different sensitivities of these experiments arise from the relatively large differences in both the mixing angles and the mass differences. The latest values for the neutrino parameters based on Ref. [35, 47] are provided in Table 3.1, which shows that the magnitude of Δm_{21}^2 is much smaller than that of $|\Delta m_{31}^2|$. Unlike the CKM matrix, where all mixing angles are small, only θ_{13} is small here, while the other two angles are considerably larger, giving rise to significant mixing.

Parameter	Value	Sensitive Experiment
Δm_{21}^2 (eV ²)	$7.36_{-0.15}^{+0.16} \times 10^{-5}$	Reactor and solar
$ \Delta m_{31}^2 $ (eV ²)	$2.448_{-0.031}^{+0.023} \times 10^{-3}$	Atmospheric
θ_{12} (°)	$33.40_{-0.82}^{+0.80}$	Reactor and solar
θ_{23} (°)	$42.4_{-0.9}^{+1.0}$	Atmospheric
θ_{13} (°)	$8.59_{-0.12}^{+0.13}$	Reactor and solar
δ_{CP} (°)	223_{-23}^{+32}	Accelerator and atmospheric

Table 3.1: The latest values for the neutrino parameters.

3.1.1 Atmospheric neutrinos

Atmospheric neutrinos are mostly muon neutrinos produced from the decay of mesons created by cosmic rays interacting with the atmosphere. The neutrino energy is typically of the order of 1 GeV, while the distance travelled ranges from $O(10^2)$ to $O(10^4)$ km. Substituting these numbers into Eq. 3.7, Δ_{23} is of the order of $O(10^{-2})$ to $O(1)$, whereas Δ_{21} is of the order of $O(10^{-3})$ to $O(10^{-1})$. Hence, oscillation is much more prominent in the $\nu_\mu \rightarrow \nu_\tau$ channel than in the $\nu_\mu \rightarrow \nu_e$ channel. This is why atmospheric neutrino experiments are sensitive to θ_{23} and Δm_{32}^2 , sometimes referred to as the atmospheric mixing angle and mass difference, respectively.

3.1.2 Reactor neutrinos

Reactor neutrinos are mostly electron antineutrinos produced from nuclear fission in power plants. Their energy is typically of the order of 1 MeV. Unlike atmospheric neutrino measurements, detectors can be placed at various locations to measure different parameters. Thus, it is more convenient to discuss reactor neutrino oscillation by introducing the oscillation length variable given as:

$$L_{\text{osc}} = \frac{4\pi E}{\Delta m^2} = 2.48 \frac{E[\text{MeV}]}{\Delta m^2[\text{eV}^2]} \text{m}, \quad (3.8)$$

which corresponds to one complete period of neutrino oscillation. Substituting the reactor neutrino energy into Eq. 3.8, one obtains $L_{\text{osc}} = O(10^2)$ km for Δm_{21}^2 and $L_{\text{osc}} = O(1)$ km for $|\Delta m_{31}^2|$. Hence, by placing detectors kilometres away from the power plants, reactor neutrino experiments offer the unique opportunity to measure θ_{13} and $|\Delta m_{31}^2|$, sometimes referred to as the reactor mixing angle and mass difference, respectively. Additionally, placing detectors at about $O(100)$ km from the power plants allows measurements of Δm_{21}^2 .

3.1.3 Solar neutrinos

Solar neutrinos are mostly electron neutrinos produced from nuclear fusion in the Sun. Most solar neutrinos have energies below 1 MeV, except those from ^8B decay, which can reach 15 MeV. The distance travelled by solar neutrinos is of the order

of $O(10^8)$ km. Substituting these values into Eq. 3.8, one obtains $L_{\text{osc}} = O(10^4)$ m for Δm_{21}^2 and $L_{\text{osc}} = O(10^3)$ m for $|\Delta m_{31}^2|$. Both are much smaller than the average distance travelled, so observing the clear oscillatory pattern (as in other neutrino experiments) would require knowing each neutrino's travel distance and starting position with extremely high precision, which is beyond current detection capabilities. Fortunately, the average oscillation result can still be measured by the total survival rate of electron neutrinos.

Beyond the usual neutrino oscillation, there is another important phenomenon in solar neutrino measurements: the Mikheyev–Smirnov–Wolfenstein (MSW) matter effect [14, 15]. A detailed discussion of the MSW effect is beyond the scope of this thesis. In short, the MSW effect modifies the mixing parameters due to the presence of electrons in matter. This modification can enhance or suppress the oscillation probability depending on the mass difference and the electron density. Roughly speaking, the required electron density is proportional to the mass difference squared and inversely proportional to the neutrino energy. Hence, lower neutrino energy or a larger mass difference necessitates a higher electron density to modify the oscillation probability. Because $|\Delta m_{31}^2|$ is considerably larger, for nearly all solar neutrino energies, the electron density in the Sun is insufficient to modify the mixing between ν_1 and ν_3 significantly. As for the relatively smaller Δm_{21}^2 , the electron density is high enough to substantially modify the mixing between ν_1 and ν_2 for the high-energy neutrinos from ^8B decay. The outcome for these high-energy electron neutrinos is a rearrangement of their mass-eigenstate composition such that, upon leaving the Sun's surface, they are mostly ν_2 , a considerably greater fraction than in the unmodified scenario for the low-energy electron neutrinos. The MSW effect is seen when different experiments measure solar neutrinos at different energies and observe varying electron-neutrino survival rates. Hence, the absolute survival rates are particularly sensitive to θ_{12} , which is often referred to as the solar mixing angle.

3.1.4 Accelerator neutrinos

In accelerator neutrino experiments, there is relatively greater flexibility than in other types of neutrino experiments because both the neutrino source and the detector can be designed to optimise oscillation studies. This section uses T2K as an example, but the underlying principles apply to all LBL experiments. More details about the T2K experiment are presented in Chapter 4.

Neutrinos are produced in accelerators by bombarding protons onto a target (carbon in T2K), creating numerous mesons. The charged mesons can be steered to point in the required direction using sophisticated magnetic horns. Besides directionality, the magnetic horns can also select mesons of a desired charge. For instance, positive (negative) pions can be focused if a beam of muon (anti)neutrinos is needed. It is this control over the neutrino source that enables accelerator experiments to measure δ_{CP} . Since the neutrino energy from meson decay depends on the emission angle, varying the angle between the detector and the beam direction yields different energy distributions. T2K places its detectors 2.5° off-axis to obtain a neutrino beam with a narrow energy peak at about 0.6 GeV. According to Eq. 3.7, for maximal oscillation with the larger mass difference (i.e. $|\Delta m_{31}^2| \approx 2.5 \times 10^{-3} \text{ eV}^2$), the far detector should be placed around 300 km from the source. Hence, accelerator measurements are sensitive to θ_{13} . Furthermore, by analysing muon neutrino disappearance, accelerator experiments can also measure θ_{23} and Δm_{32}^2 . By analysing oscillations in neutrino and antineutrino modes, T2K can extract a best-fit value of δ_{CP} that describes the observed data [1]. However, the current measurement is severely limited by statistics and thus suffers large uncertainties. Due to the significant increase in the fiducial volume of Hyper-K, statistics are expected to increase significantly, and reducing systematic uncertainties proportionally will be paramount for fully exploiting that potential. This leads naturally to the discussion of one of the major sources of systematic uncertainty in neutrino experiments: neutrino interactions.

3.2 Interaction

The underlying theory for neutrino oscillation is straightforward, and the parameters to be measured are clear, but the experimental determination of these parameters is challenging. Due to the weak interaction of neutrinos, they cannot be detected directly. All neutrino measurements are based on detecting the particles produced in neutrino interactions. Hence, to extract the desired neutrino oscillation parameters with high precision, a good understanding of neutrino interactions is crucial. This chapter provides the basic understanding of neutrino interactions necessary for interpreting the analyses presented in the subsequent chapters.

3.2.1 ν -quark

The most fundamental interaction between a neutrino and matter is the weak interaction between the neutrino and a quark, whose Feynman diagrams are shown in Fig. 3.1.



Figure 3.1: Feynman diagrams for neutrino interactions with a quark.

More specifically, the interaction shown in Fig. 3.1a is mediated by the W boson, and the neutrino is converted into a charged lepton, while Fig. 3.1b is mediated by the Z boson, and the neutrino remains a neutrino. The former is the charged current (CC) interaction, while the latter is the neutral current (NC) interaction. Following the Feynman diagram, it is straightforward to write down the amplitude for the interaction. The amplitude for the CC interaction is given by:

$$\mathcal{M}_{\text{CC}} = \frac{g^2}{2} \bar{u}_\ell(p') \gamma^\mu (1 - \gamma^5) u_\nu(p) \frac{-ig_{\mu\nu}}{q^2 - M_W^2} \bar{u}_q(k') \gamma^\nu (1 - \gamma^5) u_q(k), \quad (3.9)$$

where g is the weak coupling constant, u_ℓ and u_ν are the spinors for the outgoing lepton and incoming neutrino respectively, u_q are the spinors for the quarks, q is

the momentum transfer, and M_W is the mass of the W boson. Due to confinement, this interaction does not occur freely in nature. Instead, this interaction occurs only when the neutrino has sufficiently high energy to probe inside the nucleon and interact with the quarks. The product quark will hadronise, producing a jet of particles. This type of interaction is referred to as deep inelastic scattering (DIS). DIS is highly complicated and not so relevant for this thesis, so it will not be discussed further. More details can be found in reviews such as Ref. [48].

3.2.2 ν -nucleon

At the T2K beam energy, most neutrinos do not have enough energy to probe within the nucleon but rather interact with the nucleon as a whole. Depending on the specific neutrino energy, the interaction can be classified as quasi-elastic (QE), resonance, or deep inelastic scattering (DIS).

QE

Although at the quark level the interaction appears the same as in Fig. 3.1, the interacted quark cannot be treated as independent but rather as part of a nucleon. Hence, the effective ν -nucleon interaction Feynman diagram is shown in Fig. 3.2.



Figure 3.2: Feynman diagrams for neutrino interactions with a nucleon.

The leptonic current remains the same, but the hadronic current is now a nucleon current instead of a quark current. Hence, the hadronic current is much more complicated, comprising several form factors that parametrise our understanding of the nucleon structure. For instance, the amplitude for the CC interaction

in Fig. 3.2a is given by:

$$\mathcal{M}_{\text{CC}} = \frac{G_F}{\sqrt{2}} \bar{u}_\ell(p') \gamma^\mu (1 - \gamma^5) u_\nu(p) \quad (3.10)$$

$$\times \bar{u}_N(k') \left[F_1(q^2) \gamma_\mu + F_2(q^2) \frac{i \sigma_{\mu\nu} q^\nu}{2M} + F_A(q^2) \gamma_\mu \gamma^5 + F_P(q^2) \frac{q_\mu \gamma^5}{m_\pi} \right] u_N(k), \quad (3.11)$$

where G_F is the Fermi constant, M is the nucleon mass, m_π is the pion mass, and the hadronic current is parametrised by the form factors, F_1 , F_2 , F_A , and F_P . More details can be found in Ref. [49]. It is important to note that F_1 and F_2 are the vector form factors, which can be extracted from electron scattering measurements, and F_P is the pseudoscalar form factor, which can be related to F_A , the axial form factor, through the Partially Conserved Axial Current (PCAC) hypothesis. Hence, the axial form factor is unique to neutrino experiments and can only be extracted from past measurements. Fig. 3.3 shows an event display of a candidate CC0 π event, likely a QE event, in the upgraded ND280 during the beam run in June 2024.

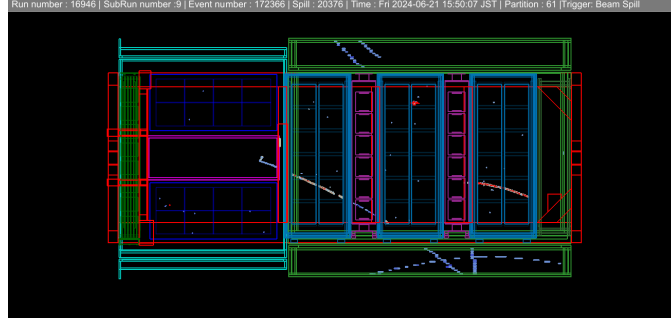


Figure 3.3: A CC0 π candidate event in upgraded ND280, where the long track is the primary muon and the short track is the primary proton.

Resonance

When the neutrino energy is high enough to excite the nucleon to a higher energy state (e.g. $\Delta(1232)$), the interaction is classified as a resonance interaction. The excited nucleon decays to produce a pion. Hence, resonance modelling is sometimes called single pion production modelling. One of the most common models used today is the Berger–Sehgal model, which is an improvement on the earlier Rein–Sehgal model by accounting for the effect of the lepton mass.

The Rein–Sehgal model is based on the approximate relativistic quark model in Ref. [50]. Subsequent developments, such as the MK model [51–53], provide more sophisticated calculations.

Fig. 3.4 shows an event display of a candidate $CC1\pi$ event, likely a resonance event, in the upgraded ND280 during the beam run in June 2024.

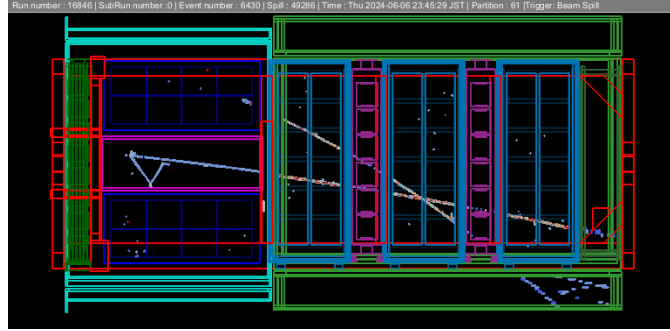


Figure 3.4: A $CC1\pi$ candidate event in upgraded ND280, where the long track is the primary muon and the V-shaped track is identified as a pion track by the short delayed track, identified as the Michel electron, attached to its end.

3.2.3 ν -nucleus

Modern neutrino experiments use heavier elements, such as hydrocarbon and argon, to increase event rates. This further complicates the neutrino interaction. In the simplest case (relatively high neutrino energy), the Impulse Approximation (IA) can be applied. The neutrino sees each nucleon as independent, so the interaction reduces to the ν -N interaction. Even in this simplest case, the presence of the nuclear medium still affects the interaction in two ways: the initial state (IS) effect and the final state interaction (FSI).

As the neutrino energy decreases, IA is no longer valid, and the nucleus can only be resolved as a whole. In this case, Random Phase Approximation (RPA) corrections are needed. Interested readers can refer to Ref. [48] for more details. Instead, the two nuclear effects are elaborated below.

Initial state

Nuclear effects manifest in two ways. Firstly, the nucleon in the interaction is in a bound state, meaning it cannot have arbitrary energy and momentum as a

free nucleon would. Instead, the nucleon's energy distribution is parameterised by the so-called Spectral Function (SF). The simplest form of SF is the Fermi gas model, which treats nucleons as a Fermi gas. Hence, the nucleon momentum is filled up to the Fermi momentum, given by

$$k_F = \left(3\pi^2\rho/2\right)^{1/3}, \quad (3.12)$$

where ρ is the nuclear density, approximately $2.3 \times 10^{-17} \text{ kg/m}^3$, leading to a Fermi momentum of about $250 \text{ MeV}/c$. This model oversimplifies the nuclear structure. A more realistic approach is the local Fermi gas (LFG) model, which accounts for the varying nuclear density. Further improvements include short-range correlations between nucleons, which increase the fraction at high momentum [54].

Final state interactions

The second nuclear effect is the final state interactions (FSI). Regardless of how the neutrino interacts with the nucleon, the products remain inside the nucleus and must propagate through the nuclear medium to be detected. These interactions with the nuclear medium are classified as FSI. They are more pronounced for hadrons than for leptons, as the latter are less likely to interact in the nuclear medium. There are several types of FSI, including elastic scattering, charge exchange, and absorption.

1. Charge exchange (CEX) involves changing the charge of the participating particles; for example,

$$\pi^+ + n \rightarrow \pi^0 + p, \quad (3.13)$$

or vice versa. This rescattering type is crucial for event topologies requiring the presence of a pion; depending on the signal pion's charge, CEX could migrate events between signal and background.

2. Inelastic scattering (INEL) is the case where the nucleus is left in an excited state after rescattering. This category applies only to situations where a single additional nucleon is emitted or knocked out after rescattering. Since it does not affect the number of pions, it will not convert a pionless topology into a

pion-production topology. However, it can change the number of final-state nucleons and affect the kinematics of the leading particle.

3. Absorption (ABS) refers to processes where the pion is effectively absorbed. For instance, π^+ can interact with multiple nucleons, initially forming a baryon resonance that subsequently emits multiple nucleons rather than a pion. Thus, the π^+ no longer emerges from the nucleus.
4. Pion production (PIPD) can occur for energetic particles where an extra pion is generated through rescattering, for example:

$$p + p \rightarrow p + n + \pi^+. \quad (3.14)$$

Such an interaction significantly alters the event topology, adding an extra pion.

Transverse Kinematic Imbalance

Due to the heavy target nucleus in current neutrino experiments, nuclear effects are almost inextricable from conventional single-particle kinematic measurements. This greatly impedes model development because nuclear effects can mimic different underlying ν -N processes. For instance, if the proton from a QE ν -N interaction has sufficient energy to produce an additional pion during FSI, that pion may emerge from the nucleus. Thus, a QE event could mimic resonance production. Hence, accurate descriptions of nuclear effects are crucial to reduce model systematic uncertainties.

To gain insights into nuclear effects, observables that are sensitive to these effects must be measured. Transverse Kinematic Imbalance (TKI) variables are exactly such observables [55, 56]. The TKI variables are illustrated in Fig. 3.5.

In the simplest case, there are only two final particles after the neutrino–nucleon interaction: a muon and a proton. Without nuclear effects, these two particles should have momenta of equal magnitude but opposite direction in the plane transverse to the neutrino beam. The TKI variables quantify the deviation from this ideal scenario, thereby probing nuclear effects. Because of the initial state

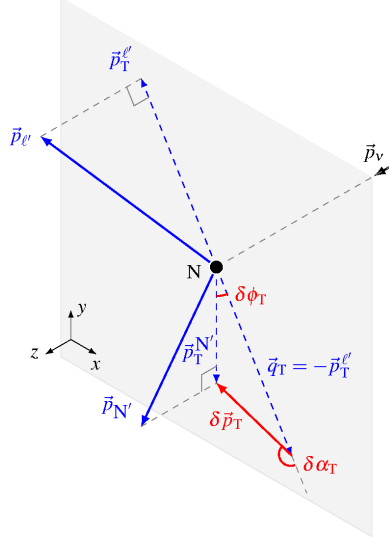
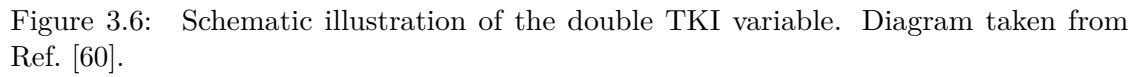


Figure 3.5: Schematic illustration of the TKI variables. Diagram taken from Ref. [56].

(IS), the nucleon's momentum distribution causes the sum of the muon and proton transverse momenta to deviate from zero, quantified by δp_T , and their directions need not be precisely opposite, quantified by $\delta\phi_T$. Moreover, if the nucleus is initially at rest and no additional particles (beyond the muon and proton) are knocked out, the nucleon initial momentum p_N can be estimated with an $\mathcal{O}(20\%)$ correction [57] following methods in Refs. [58, 59]. Hence, δp_T and p_N serve as good probes of IS models. While FSI tends to smear these distributions, the general shape and peak location mainly reflect the IS model. Since most IS models assume no preferred direction for nucleon motion, one expects a uniform distribution in $\delta\alpha_T$, the angle between the initial nucleon momentum and the lepton momentum in the transverse plane. Any deviation from flatness in $\delta\alpha_T$ is thus attributed to FSI, making it an excellent probe of FSI.

In pion-production events, a double-transverse variable, δp_{TT} , can be constructed by projecting $\delta\vec{p}_T$ onto the axis perpendicular to the lepton scattering plane, defined by $\vec{p}_l$ and $\vec{p}_\nu$ (see Fig. 3.6).

In the absence of nuclear effects, δp_{TT} should be zero. Its distribution width reveals the role of nuclear effects in pion production [60, 61]. Analogous variables, δp_{Tx} and δp_{Ty} , have been proposed and studied in MINERvA [62] for pionless



There has been a wealth of TKI-based measurements from multiple neutrino experiments, including T2K [60, 64], MINERvA [61, 62, 65, 66], and MicroBooNE [67–71]. Because the TKI concept applies equally to electron scattering, experiments such as CLAS [72] have also produced TKI measurements, demonstrating the wide applicability of TKI.

おれは海賊王になる男だ!!!

I'll be the one to measure δ_{CP} .

— Monkey D. Luffy

4

The T2K Experiment

Contents

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The T2K experiment is a long-baseline neutrino experiment with the aim of measuring CP violation in the lepton sector through neutrino oscillation measurements. This chapter begins by describing the experimental setup and then provides a concise overview of the software, thereby laying the groundwork for the subsequent analyses.

4.1 Hardware

As a long-baseline experiment, T2K encompasses a near detector (ND280) and a far detector (Super-Kamiokande, often referred to as Super-K). Both detectors must discriminate between ν_μ and ν_e to measure neutrino oscillations. Naturally, detecting neutrinos directly is impossible. Instead, each detector must be able to identify the different particles produced when neutrinos interact with its active material. Most commonly, such neutrino–nucleus interactions generate the corresponding leptons (i.e., electrons or muons) and various hadrons, including protons, neutrons, and light mesons such as pions.

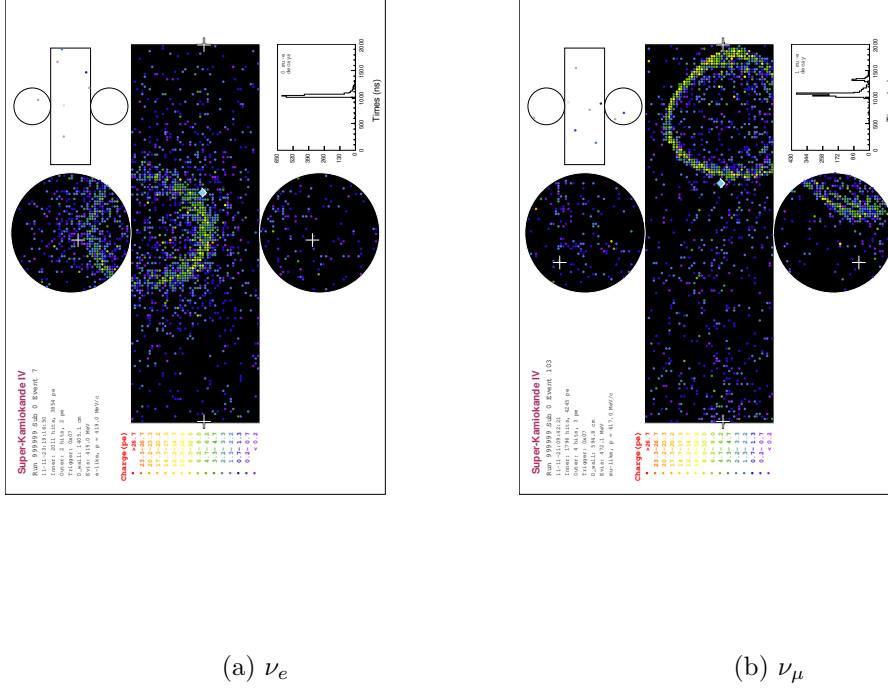


Figure 4.1: SK event displays.

The far detector, SK, is a large water Cherenkov detector. It detects particles by collecting the Cherenkov light emitted when they travel faster than the speed of light in water. Hadrons resulting from neutrino interactions typically lack sufficient energy to emit Cherenkov radiation, whereas leptons often exceed this threshold. Consequently, SK primarily observes electrons and muons, whose Cherenkov radiation forms ring-like patterns transverse to their direction of flight. Because the electron is lighter than the muon, its path is more prone to scattering, causing the electron's Cherenkov ring to appear more diffuse than the muon's, as illustrated in Fig. 4.1.

In contrast, the near detector, ND280, consists of multiple specialized subdetectors designed to measure as many neutrino interaction products as well as possible. The classic ND280 configuration is shown in Fig. 4.2a. Its central detector assembly contains a π^0 -detector (P0D), three vertical Time Projection Chambers (TPCs), and two Fine Grained Detectors (FGDs) positioned between the TPCs. Surrounding these devices are various electromagnetic calorimeters (ECALs)—the P0D ECAL, the Barrel ECAL, the Upstream ECAL, and the Downstream ECAL—all enclosed

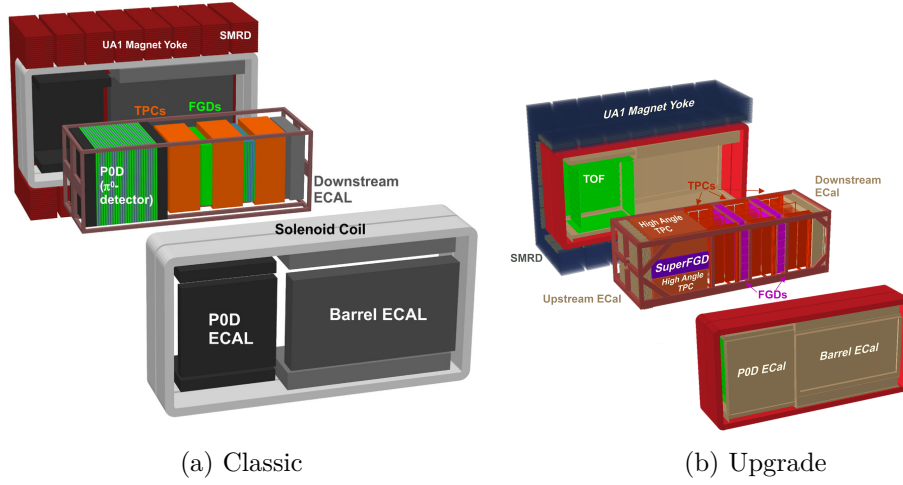


Figure 4.2: ND280 detector diagram.

by the UA1 magnet comprising a solenoid coil and a yoke.

The neutrino beam passes through the P0D, TPCs, and FGDs. Each FGD consists of scintillator bars with a cross section of $1.0 \text{ cm} \times 1.0 \text{ cm}$. Every layer of bars is oriented orthogonally to the layer above it. As a particle traverses these layers, it produces photons in the scintillator; these photons are collected by wavelength-shifting fibres embedded in each bar and ultimately read out by end electronics. Hence, the bar recording the signal determines the track position transverse to the neutrino beam, while the layer index specifies the longitudinal position of the energy deposition.

Originally, FGDs served as the active target for neutrino interactions. However, their bar geometry offers limited acceptance for particles travelling at large angles to the beam direction—i.e., along the bar’s long axis—because such particles intersect only a few bars, making them difficult to reconstruct. At the far detector, the large Cherenkov detector offers near-isotropic acceptance. This mismatch in phase space between the near and far detectors increases uncertainties in oscillation analyses. Moreover, having sparse signals along the particle trajectory degrades reconstruction resolution. These limitations motivated the ND280 upgrade.

In the upgraded ND280, the π^0 -detector is replaced by several new subsystems (Fig. 4.2b): the Time-of-Flight detector (TOF), two High-Angle TPCs (HATs), and the Super Fine Grained Detector (SFGD). The TOF comprises six layers of

scintillator bars with sub-nanosecond timing resolution, which help veto crossing particles and enhance sample purity. The HATs incorporate a redesigned field cage and new Micromegas detectors, expanding the tracking volume and improving resolution relative to the original vertical TPCs. Serving as the new active target, the SFGD consists of about two million 1 cm^3 scintillator cubes. Its finer segmentation markedly increases acceptance for high-angle tracks, better aligning with the near-isotropic acceptance at the far detector. Additionally, the improved tracking, enabled by enhanced energy deposition measurements, lowers detection thresholds and sharpens resolution, facilitating new reconstruction strategies (e.g., trackless pion reconstruction) and innovative variables (e.g., centre-of-mass variables), which will be introduced in the subsequent chapters. Construction for the upgrade was completed in April 2024, and data taking with the upgraded ND280 commenced in June 2024.

4.2 Software

This thesis focuses primarily on improving reconstruction performance; thus, the following overview of the software used for ND280’s upgraded system aims to clarify and contextualize the methodologies employed in later chapters.

Each scintillator cube in the SFGD contains three passing optical fibres, each is connected to electronics at one of the ends. As a charged particle traverses these cubes, scintillation photons travelled along the optical fibres and are measured by readout boards located on three orthogonal faces of the SFGD. After calibration, these signals are digitized and assembled into three-dimensional “Hits”, each corresponding to a specific cube position and including a measure of the deposited charge. The reconstruction software then sorts these Hits into either clusters or tracks using clustering and track-fitting algorithms. Clusters consist of fewer than three Hits, whereas tracks typically contain many more, because objects containing too few Hits are not long enough to be considered as tracks. Additionally, due to imperfect optical isolation between cubes, light may leak into neighbouring cubes not traversed by the particle. To account for this, the reconstruction merges nearby Hits into “Nodes”, which more accurately capture the particle’s trajectory.

Energy deposition at each Node is then estimated from the associated Hits and refined according to the expected continuous energy loss along the path. Finally, a Boosted Decision Tree (BDT), trained on Particle Gun simulations, uses these smoothed energy values and other measured parameters for particle identification and momentum estimation.

银鞍照白马，飒沓如流星。

With great techniques come great measurements.

— Bai Li Xia Ke Xing

5

New Techniques in Selection Development

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The upgraded detector has enhanced measuring capabilities, which is especially important for hadrons, As hadrons tend to have much shorter tracks and are conventionally harder to reconstruct in scintillator detectors compared to muons, the previously explored hadronic kinematic phase space has been relatively restricted, and the reconstructed hadronic variables have relatively large uncertainties. Since more exclusive event topologies, such as $\nu_\mu \text{CC}1\pi1p$, allow the measurement of

high-level variables like the Transverse Kinematic Imbalance (TKI) variables, which provide invaluable insights on neutrino-nucleus interactions, it is highly desirable to reconstruct hadrons, measure their kinematic properties as precisely as possible, and explore their phase space as extensively as possible.

Two of the most common product hadrons from neutrino-nucleus interactions are protons and pions. The default particle identification and momentum reconstruction have been developed by colleagues at T2K using the Boosted Decision Tree (BDT) algorithm, which is a machine learning algorithm that takes the energy deposit as the input and outputs the particle identity and momentum. The BDT algorithm has good performance, but it is still lacking in some aspects. For instance, as it uses only single-track information, it cannot effectively distinguish between pions and muons. It can reconstruct proton momentum with an excellent resolution of about 3.5%, but it is insufficient for TKI analysis, which is highly sensitive to hadron kinematics.

To address these gaps, I have developed and implemented new techniques for both hadrons, exploiting the precise position and dE/dx measurements and timing resolution. For pions, I have invented the pion trackless reconstruction, a novel technique to reconstruct pions without requiring the presence of a reconstructed track, thereby lowering the detection threshold and significantly increasing reconstruction efficiency for low-momentum pions. As for protons, I have adapted the Elastically Scattered and Contained (ESC) protons selection technique [73] to SFGD, showing significant improvement in proton momentum reconstruction resolution. These new techniques are applied in the development of two signal sample selections, namely the $\nu_\mu\text{CC}0\pi1\text{p-ESC}$ selection and the $\nu_\mu\text{CC}1\pi\text{-TL}$ selection, where the “TL” refers to the use of the trackless reconstruction technique. Additionally, to prepare for the data-MC comparisons and future cross-section analysis, I have also developed a stopping pion control sample selection.

5.1 The Elastically Scattered and Contained (ESC) protons technique

5.1.1 Working Principles of the ESC selection

The subsection elaborates on the underlying principles of the ESC selection technique. Protons coming to rest deposit a large amount of energy within a short distance just before stopping. Hence, for these protons, if the energy deposited per distance, dE/dx , is plotted along the particle trajectory, a peak, the so-called Bragg Peak, appears near the track end. This feature can be exploited to select protons that come to rest without undergoing secondary interactions, which are thus called "elastically scattered" protons by the original developer of the technique [73]. To identify the Bragg peak, the entire proton track must be contained within the scintillation detector such that the dE/dx can be measured at the end of the track. Thus, this technique is called the elastically scattered and contained technique. These ESC protons are of particular interest because their range is strongly correlated with their momentum, which could be exploited by the machine learning algorithm. Hence, even though all proton momenta are reconstructed by the BDT algorithm, the ESC protons tend to better momentum reconstruction resolution.

In the SFGD, having the Bragg peak implies large dE/dx for the last few nodes of the proton track. For illustration, the fractional difference between the reconstructed and true proton momentum is plotted against the energy deposited at the third last node of the proton track in Fig. 5.1 with column normalization. More specifically, all bins with the same x value are considered to be in the same column, and the bin height is normalized by the largest bin height in the column. Thus, the maximum value in each column is 1, thereby visualising the comparison between columns. For instance, in Fig. 5.1, the red bins (the bin with the largest value) concentrate around $y = 0$, i.e. small differences between reconstructed and true proton momentum, for columns having dE/dx above 1000 arbitrary units¹, while for columns with dE/dx below 1000, the red bins deviate from $y = 0$ with larger

¹The energy deposit output from the SFGD reconstruction does not have a physical unit, but its magnitude is an algorithmic estimate of the energy deposited by the particle at each node.

fluctuations. Thus, selecting protons with large dE/dx for the last few nodes of the proton track can produce a sample with good proton momentum reconstruction resolution. This is the underlying principle of the ESC technique.

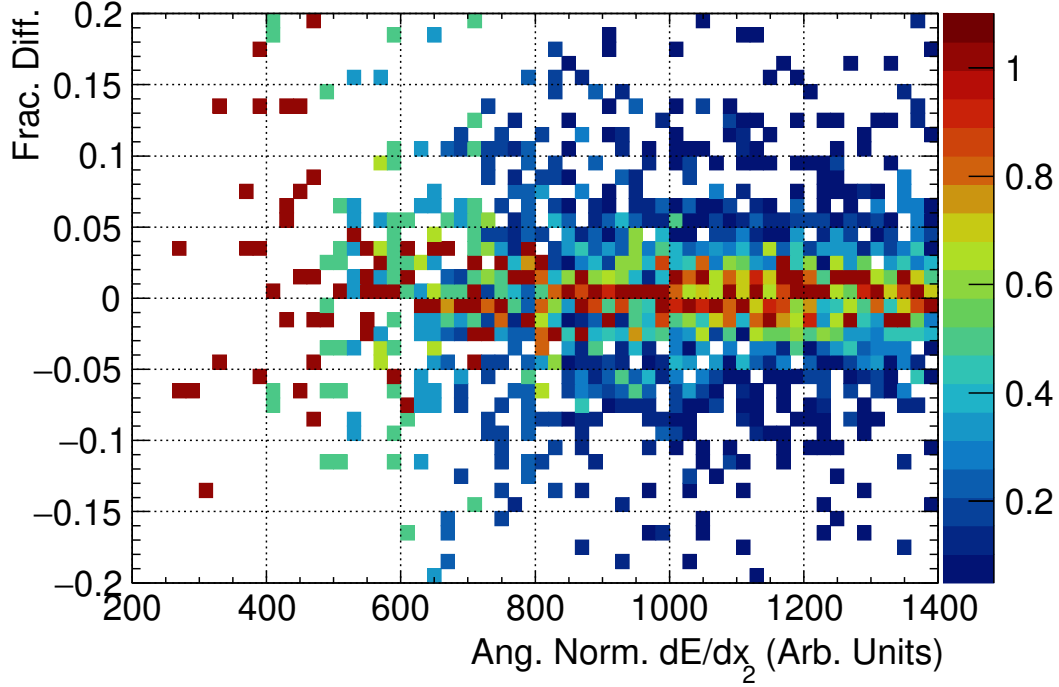


Figure 5.1: Proton momentum fractional difference against the angular normalized dE/dx (detailed explanation of this quantity is given in the next subsection) at the second last node. This plot is column-normalized. All bins with the same x value are considered to be in the same column, and the bin height is normalized by the largest bin height in the column, thereby setting the maximum value in each column to be 1. The red bins show much larger deviation from $y = 0$, when the dE/dx_2 falls below 800.

5.1.2 Implementation

As the ESC selection is based on dE/dx at the end of the proton track, accurate reconstruction of dE/dx must first be obtained. The ND280 reconstruction software provides a smoothed energy deposit at each node as an estimate of dE/dx . To investigate the quality of this estimate of dE/dx , I have simulated multiple particle gun (PGUN) samples with protons travelling in different directions in the SFGD. More specifically, three samples travel along the x -, y -, and z -axes, and five other samples travel at 15° , 30° , 45° , 60° , and 75° to the x -axis in the x - z

plane. As the proton direction should not affect its energy deposition, distinct characteristics of dE/dx along the proton trajectory should be consistent for the different proton samples. This consistency can be an indicator of the quality of the reconstructed dE/dx .

The first feature is the Bragg peak due to the ESC protons as elaborated in Sec. 5.1.1. Another distinct feature of dE/dx is the minimal ionizing particle (MIP) region, where particles at high momentum traverse the scintillator depositing a minimal amount of energy, E_{MIP} , per unit distance. This is due to the fact that the energy loss of a particle travelling through matter is proportional to the square of its charge and inversely proportional to its velocity, as described by the Bethe-Bloch formula. For protons, which are charged particles, the dE/dx is relatively constant at high momentum, as they are fast enough that their energy loss per unit distance is dominated by ionization losses rather than other processes like nuclear interactions or radiation losses. Hence, for protons travelling at high momentum, the dE/dx remains at a relatively constant and small value for all such particles. Hence, the dE/dx remains at a relatively constant and small value for all such particles. When a proton undergoes a secondary interaction while still possessing high momentum, its track ends prematurely, and the dE/dx values near the end of this track would be close to E_{MIP} per distance. Thus, when the dE/dx values for each node near the end of the proton tracks are plotted, besides the Bragg peak at large dE/dx corresponding to ESC protons, there should be another peak at smaller dE/dx corresponding to protons that undergo secondary interactions, which will be referred to as the MIP peak for subsequent discussion.

Fig. 5.2a plots the smoothed energy deposits for the third to sixth nodes from the end for all PGUN samples travelling at non-orthogonal angles. It is reassuring to indeed observe the two peaks, the MIP peak and the Bragg peak, as hypothesized. However, there are still some discrepancies. The MIP peaks are close to each other, but their positions differ noticeably. This is due to the different grouping of hits into nodes for protons travelling in different directions. The different numbers of hits in a node lead to different distances between nodes, so using the smoothed

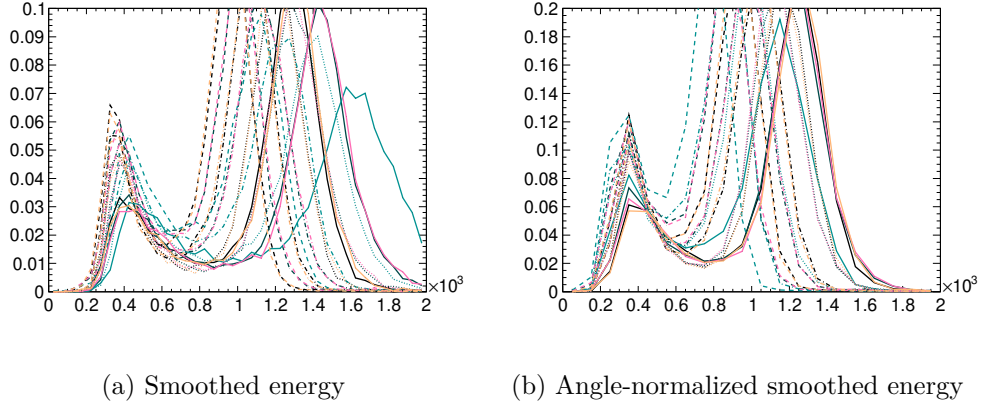


Figure 5.2: dE/dx estimation. Each colour represents a proton sample going in different directions. Each line style represents a different node from the end of the track. Altogether, there are 5 samples, each plotting for 4 nodes, resulting in 20 curves. Due to the large number of curves, the legend is omitted. The individual differences between the curves are not of interest here, but rather the overall agreement of the MIP peak positions across all curves can validate the quality of the estimation of dE/dx . Fig. 5.2a shows considerable spread in the MIP peak positions across different curves, while the MIP peak positions in Fig. 5.2b are much closer across different curves. Therefore, the angle-normalized smoothed energy is a better estimate of dE/dx than the smoothed energy.

energy deposited per node as an estimation of dE/dx is not strictly valid. To account for the different propagation directions, I divided the energy by the sine (or cosine, whichever is larger) of the angle between the trajectory and the z -axis. The angle-normalized energy is plotted in Fig. 5.2b, which demonstrates that the angle-normalization step has achieved its goal—all MIP peaks have the similar position. Hence, the angle-normalized energy is used as a good reconstruction of dE/dx for the ESC selection, and it will be referred to as dE/dx in the following discussion.

While the MIP peaks serve as a validation of the quality of the dE/dx estimation, the Bragg peaks are used to determine specific dE/dx thresholds for the ESC selection. To have a larger sample for threshold determination, the multiple PGUN samples are combined into one large sample. While plots like Fig. 5.1 show the underlying idea of the ESC selection, profile plots are better suited for determining proper thresholds. Hence, the profile plots of dE/dx for the last six nodes at the end of the proton track are collected in Fig. 5.3. To simplify the discussion, the dE/dx at the last node is referred to as dE/dx_1 , the second last node as dE/dx_2 , and so on, up to the sixth last node as dE/dx_6 . For example, Fig. 5.3a shows considerably

larger fluctuations in reconstruction quality when dE/dx_1 falls below 200, where a cut should be placed to exclude these events. Similarly, the threshold values from dE/dx_2 to dE/dx_6 are extracted to be 500, 1000, 900, 840, and 740 respectively.

Furthermore, based on the 6 distributions of dE/dx , the average Bragg peak profile can be obtained by fitting the peaks of these distributions. Fig. 5.4 shows the dE/dx distributions for the last six nodes of the proton track. The specific dE/dx values constituting the Bragg peak are extracted by fitting the distributions with a Cauchy function near the peak region. Note that the fit is used to extract the peak value, not to model the entire distribution, so the fit quality is not of primary concern. From dE/dx_1 to dE/dx_6 , the fitted Bragg peak values are 989.5, 1080, 1234, 1131, 990.0, and 887.2, respectively.

The dE/dx for the last six nodes for each track can then be compared with this average profile using a χ^2 ,

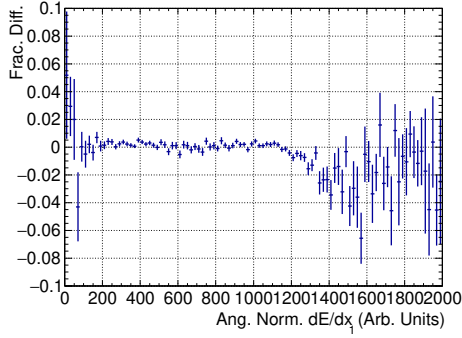
$$\chi^2 = \sum_{i=1}^6 \frac{(dE/dx_i - \overline{dE/dx_i})^2}{\overline{dE/dx_i}} \quad (5.1)$$

A proton coming to rest should have a similar profile, so an upper cut on χ^2 could further select protons with good momentum resolution. Similar to the determination of the threshold for the individual dE/dx , the momentum fractional difference is plotted against χ^2 , as shown in Fig. 5.5. The upper cut on χ^2 is implemented at 800, above which considerably larger fluctuations are observed.

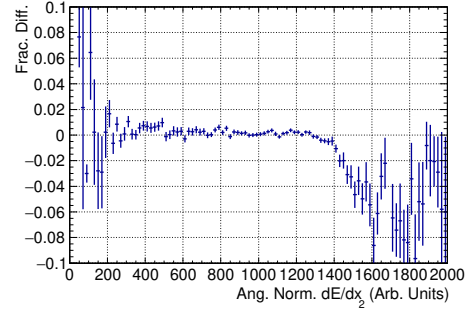
5.1.3 Momentum reconstruction bias assessment

In Ref. [73], reconstruction bias in the transverse component of the momentum is observed after the ESC implementation. Hence, such bias is also checked for the signal sample in SFGD. However, unlike in Ref. [73], no apparent bias in proton momentum reconstruction is present, while such bias is observed in muon momentum reconstruction as shown in Fig. 5.6.

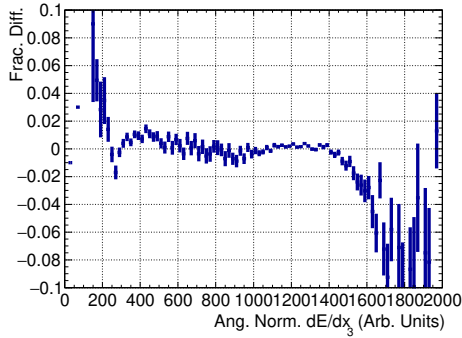
Further investigation shows that there is no bias in the muon angle, as shown in Fig. 5.7a. Additionally, such bias is shown to be present even before the implementation of the ESC step in Fig. 5.7b, while there is no appreciable bias in



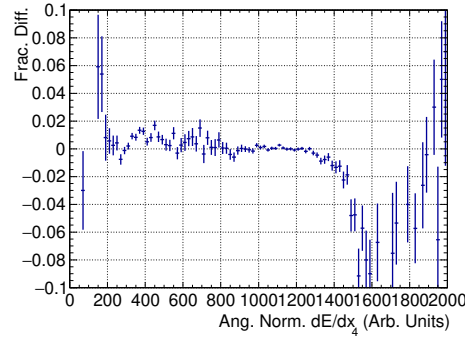
(a) dE/dx_1 , considerably larger fluctuations in reconstruction quality when dE/dx_1 falls below 200.



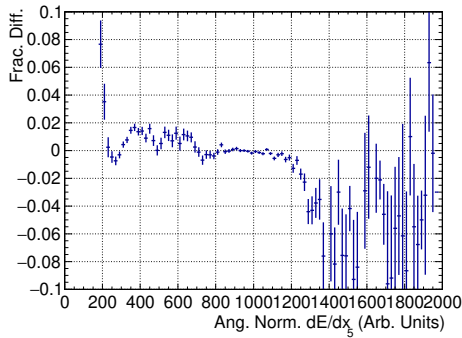
(b) dE/dx_2 , considerably larger fluctuations in reconstruction quality when dE/dx_2 falls below 500.



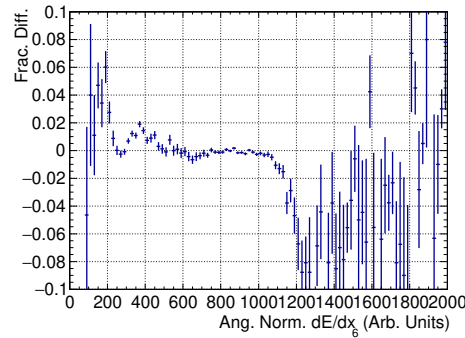
(c) dE/dx_3 , considerably larger fluctuations in reconstruction quality when dE/dx_3 falls below 1000.



(d) dE/dx_4 , considerably larger fluctuations in reconstruction quality when dE/dx_4 falls below 900.



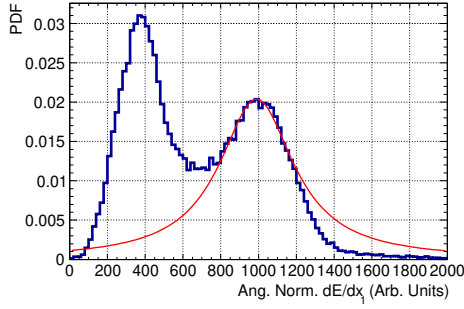
(e) dE/dx_5 , considerably larger fluctuations in reconstruction quality when dE/dx_5 falls below 840.



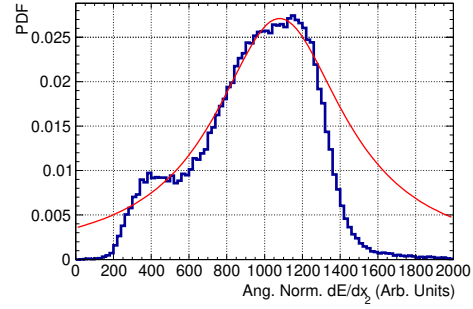
(f) dE/dx_6 , considerably larger fluctuations in reconstruction quality when dE/dx_6 falls below 740.

Figure 5.3: Proton momentum reconstruction fractional difference against dE/dx for determination of dE/dx thresholds. The corresponding thresholds values are extracted in each subfigure.

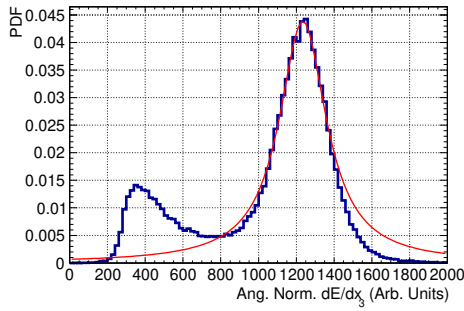
the full muon momentum reconstruction. This could be due to a bias in the neutrino direction reconstruction. However, such a direction bias should have affected the



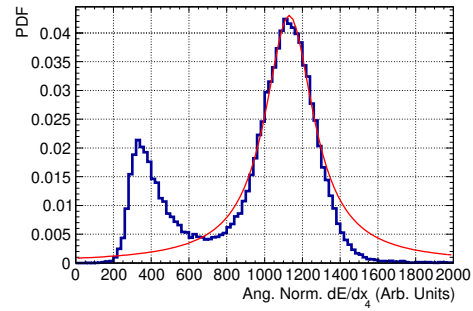
(a) dE/dx_1 Bragg peak value fit. The fitted peak value is 989.5.



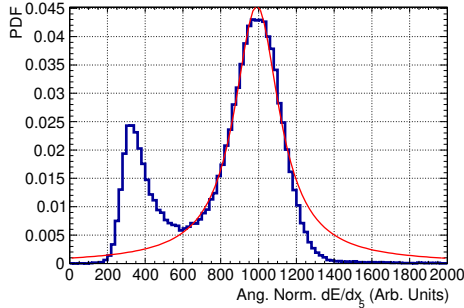
(b) dE/dx_2 Bragg peak value fit. The fitted peak value is 1080.



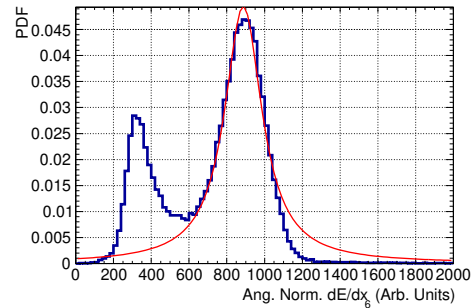
(c) dE/dx_3 Bragg peak value fit. The fitted peak value is 1234.



(d) dE/dx_4 Bragg peak value fit. The fitted peak value is 1131.



(e) dE/dx_5 Bragg peak value fit. The fitted peak value is 990.0.



(f) dE/dx_6 Bragg peak value fit. The fitted peak value is 887.2.

Figure 5.4: Cauchy fits to extract the peak values of dE/dx at each node to estimate the average Bragg peak values. The corresponding fitted peak values are shown in each subfigure.

proton transverse momentum as well. As the impact is not pronounced and the simulation software is not the final stable version yet, a correction is done solely on the muon transverse momentum for the calculations of TKI variables, and further investigations on the root cause for this bias should be carried out after a stable simulation software version becomes available.

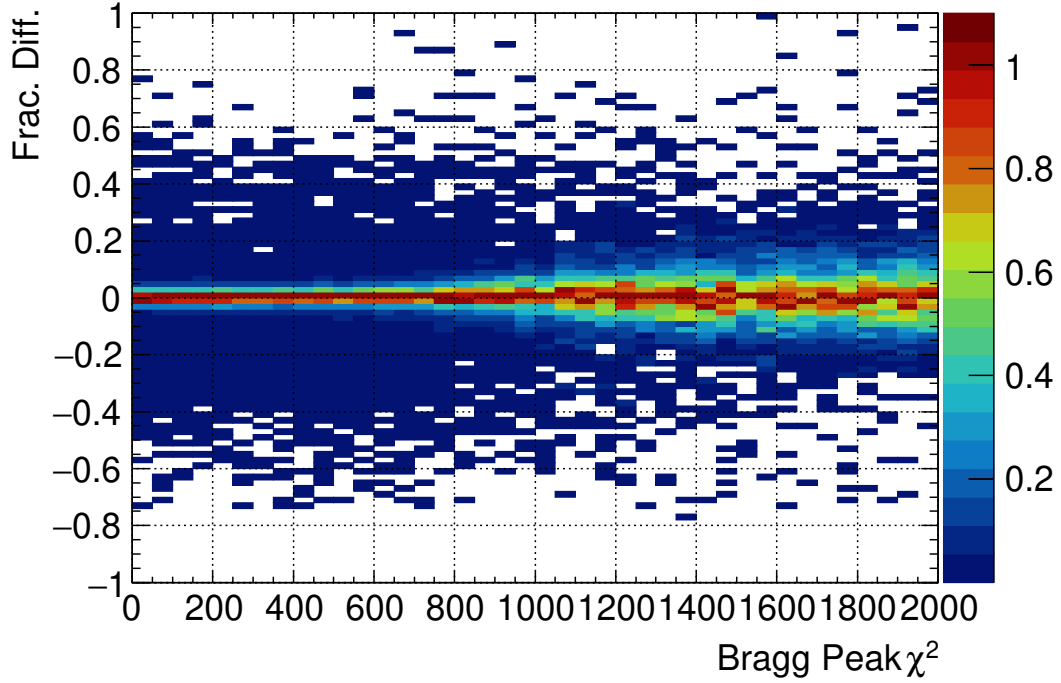


Figure 5.5: Proton momentum reconstruction fractional difference against χ^2 for determination of χ^2 thresholds. Considerably larger fluctuations in reconstruction quality when χ^2 is above 800.

The muon correction is applied through the inverse of the linear fit of the bias region between 0.5 GeV/ c and 1.1 GeV/ c . The corrected muon transverse momentum is shown in Fig. 5.8, which demonstrates that the bias has been rectified.

5.1.4 Impacts on Single Particle Kinematics

The ESC technique is completed with the transverse momentum bias correction. The overall impact of the technique on the reconstructed single particle kinematics (SPK), i.e. reconstructed proton kinematics in this case, is shown in Fig. 5.9. It has significantly reduced off-diagonal entries, which represent badly reconstructed events. Moreover, the conspicuous line with a negative slope in the θ_p has mostly disappeared. These could be due to a mis-reconstruction of the track direction for short protons, which are excluded by the first requirement of the ESC step: the number of nodes has to be at least 6.

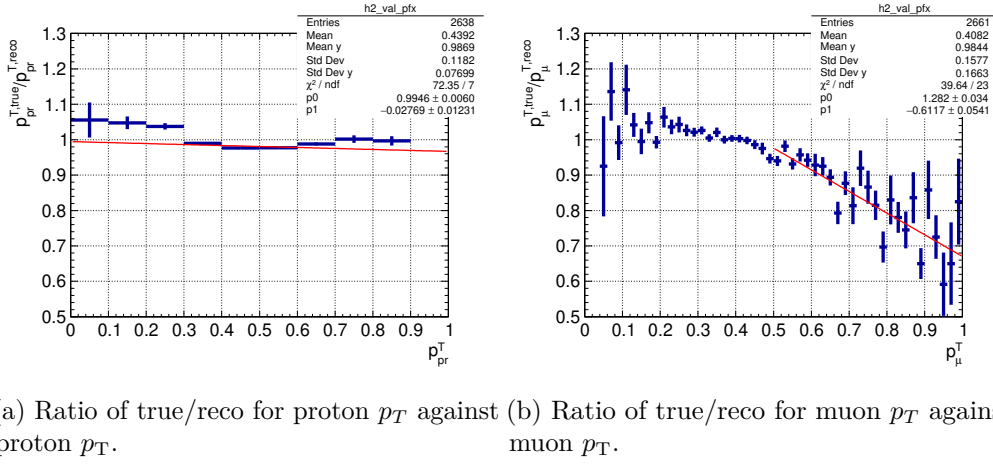


Figure 5.6: Ratios of true/reco for the transverse component of the momentum for proton and muon are plotted against the transverse component for transverse momentum reconstruction bias assessment. No bias is observed for proton, while a linear bias is present for muon.

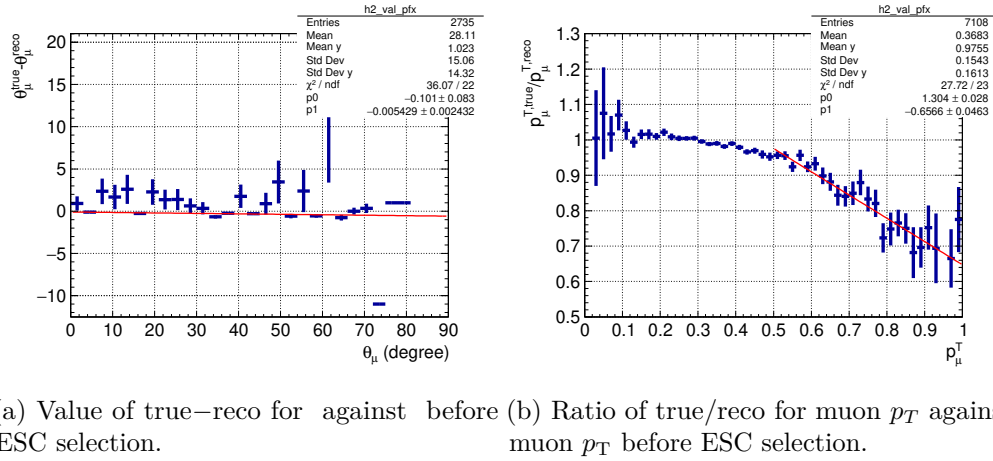


Figure 5.7: Muon transverse momentum reconstruction bias investigation plots. There is bias in the muon angle reconstruction, while the transverse momentum bias is present even before the ESC selection.

The overall improvement in the SPK is best illustrated with a Cauchy fit to the proton momentum reconstruction resolution histogram, as shown in Fig. 5.10a and Fig. 5.10c, which shows a more than 50% reduction in the fitted half width at half maximum (HWHM) of the distribution. This improvement is achieved at the cost of a two-thirds reduction in efficiency, as shown in Fig. 5.10b and Fig. 5.10d. The drop in efficiency might seem drastic, but it proves to be necessary for the TKI analysis, as shown later in Sec. 6.1, which showcases the significant improvement in the evaluation of TKI variables due to the ESC technique.

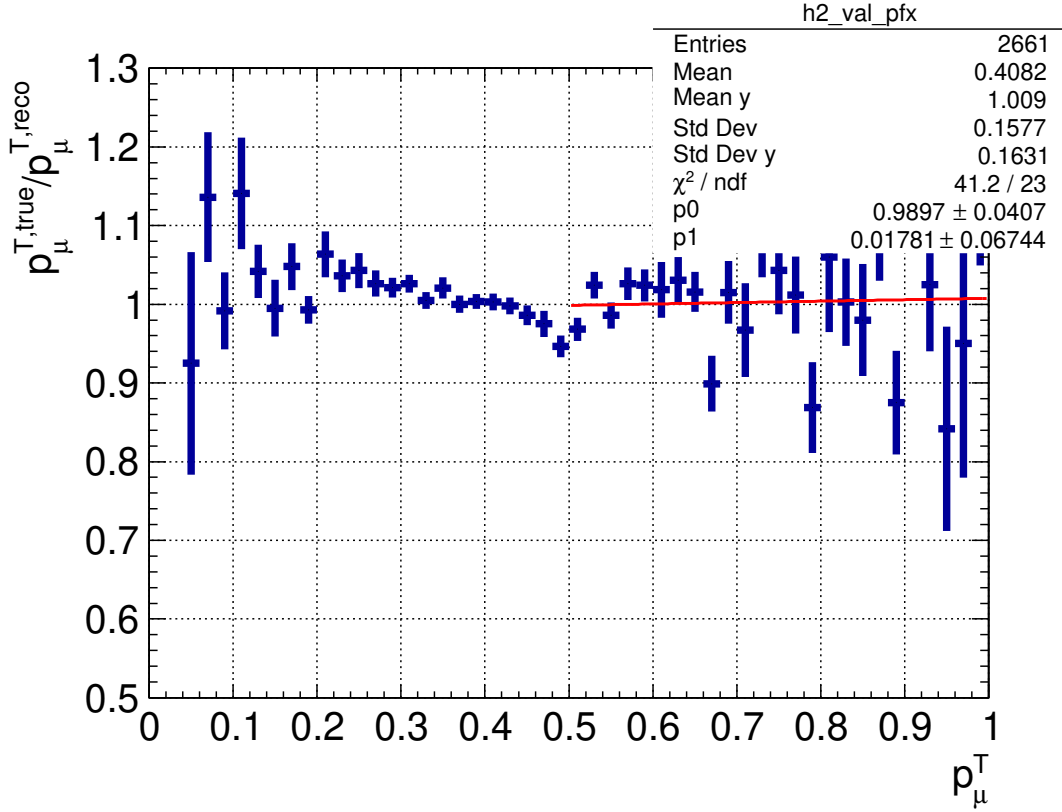


Figure 5.8: Ratio of true/reco for muon p_T against muon p_T after correction. The nearly flat line suggests that the bias has been rectified.

5.2 Pion Trackless Reconstruction

As briefly elaborated in Sec. 4.2, in SFGD, particle identification and momentum reconstruction are only performed on reconstructed tracks, which have a minimum length of about 30 mm. Hence, low-momentum particles, which travel only a short distance, could not be reconstructed at all. When these low-momentum pions are not reconstructed, these $\text{CC}1\pi$ events are misidentified as $\text{CC}0\pi$ events, both decreasing the signal for the $\text{CC}1\pi$ samples and increasing the background for the $\text{CC}0\pi$ sample. Thus, it would be highly beneficial if we could reconstruct these pions. The pion trackless reconstruction technique I have developed can address this gap well.

5.2.1 Working Principles

The trackless technique is an advancement of previous methods. It was first attempted in MINER ν A [74] and in the Fine Grain Detectors of ND280 [74]. In

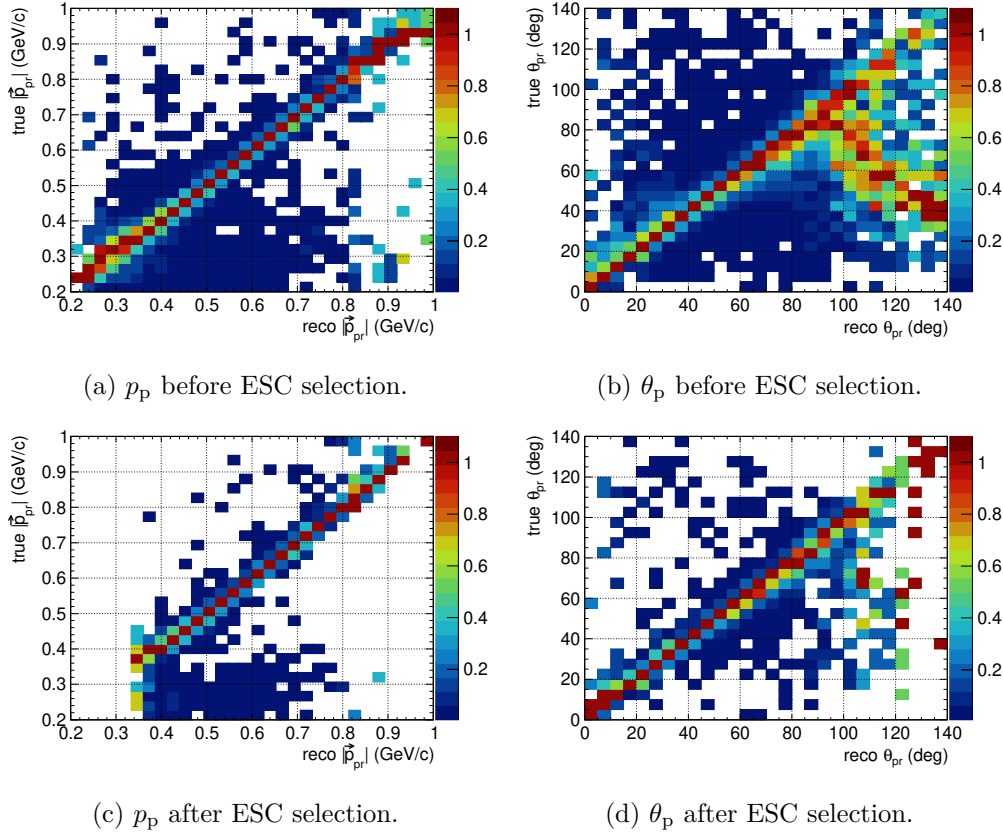
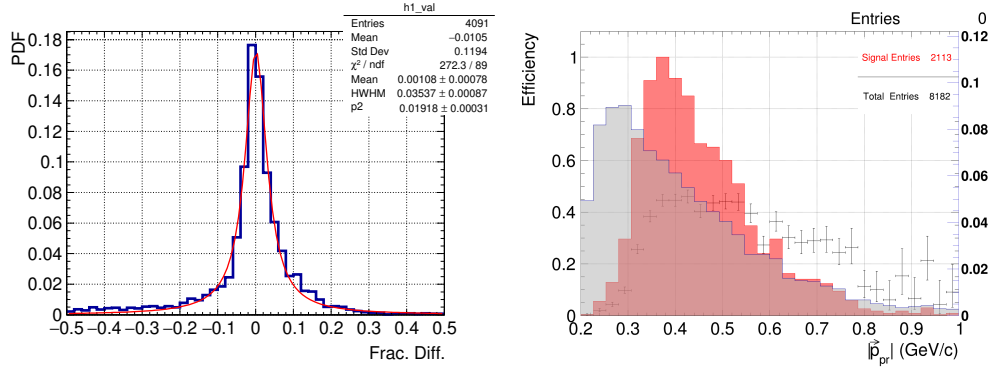


Figure 5.9: Reconstruction quality plots of truth against reconstruction values for SPK variables before (top) and after (bottom) selection. A significant reduction in the off-diagonal entries for both p_p and t_{pi} is observed after the ESC selection, indicating a better reconstruction quality. The conspicuous line with a negative slope for θ_p has mostly disappeared.

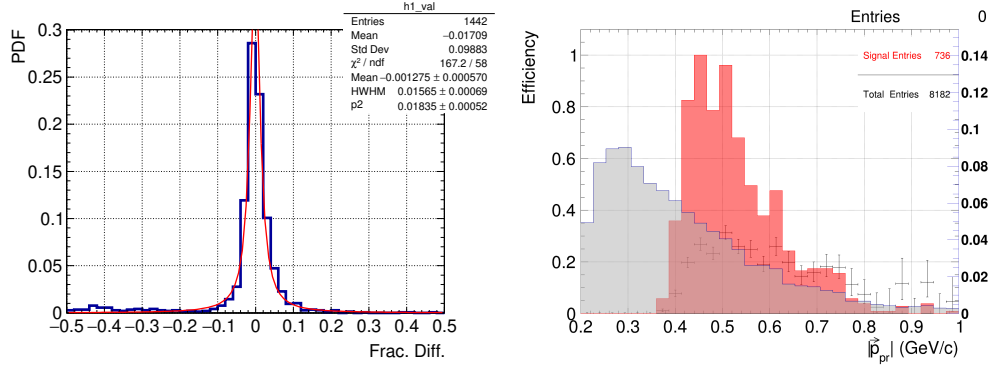
MINER ν A, a reconstructed cluster must be identified as a pion candidate. In contrast, ND280 includes a prototype of the trackless reconstruction; however, it is integrated with other pion reconstruction methods, and its full potential has not yet been explored.

This reconstruction technique is based on two key aspects of the pion decay chain, $\pi \rightarrow \mu \rightarrow e$:

1. Delayed Michel Electron (ME) Signal: The ME, as the grandchild in the decay chain, produces a delayed signal that occurs on a much longer timescale compared to other processes, such as π^0 decay. Consequently, the presence of the primary pion can be inferred from the delayed ME signal in the SFGD.



(a) p_p resolution before ESC selection. The fitted HWHM is about 3.5%. (b) Selection efficiency before ESC selection. The overall efficiency is $2113/8182 \approx 25.8\%$.



(c) p_p resolution after ESC selection. The fitted half width is about 1.6%. (d) Selection efficiency after ESC selection. The overall efficiency is $736/8182 \approx 9.0\%$.

Figure 5.10: Proton ESC selection results. The grey histograms are the true probability distribution, while the pink ones are those that get reconstructed. The selection efficiency has dropped from 25.8% to 9.0%, while the proton momentum reconstruction resolution has improved from a fitted HWHM of 3.5% to 1.6%.

2. Kinematic Determination via Energy-Momentum Conservation: The pion decay, $\pi \rightarrow \mu + \nu$, is a two-body process. The kinematics of the daughter particles are entirely determined by energy-momentum conservation. When a pion decays at rest, the daughter muon moves only about 0.12 cm. Therefore, the starting point of the ME provides a reliable estimate of the pion's endpoint.

Using the distance between the primary neutrino interaction vertex and the ME starting point, the pion momentum can be reconstructed by range. Note that the pion must decay at rest for this technique to be applicable. For pions that undergo secondary interactions, their range does not correlate with their momentum, so their momentum cannot be reconstructed by range. For a pion

decaying in flight, its daughter muon will have a non-negligible momentum and hence travel an appreciable distance before decaying into the ME. Hence, the ME starting point is no longer a good estimate of the pion's endpoint. Alternatively, the muon starting point can be used to estimate the pion's endpoint, but the range of the pion does not correlate with the pion starting momentum well as the ending momentum is non-zero. It might be possible to reconstruct the pion's ending momentum from the muon momentum due to the two-body decay kinematics, and then to reconstruct the pion's starting momentum from the pion's range and its ending momentum. This will be an interesting avenue to explore in the future, while the current analysis focuses on pions decaying at rest.

For these target pions, an empirical relationship between the pion's kinetic energy and its range was established using a PGUN sample, as shown in Fig. 5.11. The PGUN sample initiates pions from the centre of SFGD and propagates them isotropically. It comprises 500,000 events with kinetic energies uniformly distributed between 20 MeV and 200 MeV. The $\log_{10}$ of the true kinetic energy is plotted against the true pion length in Fig. 5.11. The central region of the plots is fitted with a fourth-order polynomial. Although the χ^2/NDF of the fit is relatively large, the fitted line matches the data points well. Since the simulation is not yet final, this fit is sufficient for the current analysis and should be updated once the simulation is finalized.

5.2.2 Preliminary Study on Pion Fates

Before implementing the pion trackless reconstruction technique, it is essential to assess its potential significance, as this technique can only reconstruct pions that decay at rest. If only a limited number of pions decay in this manner, the technique would be inapplicable. Therefore, a preliminary study on pion fates relevant to T2K was conducted.

The first step involved examining the primary pion momentum distribution in SFGD as produced by the T2K neutrino beam. For a rough estimation, the first 100,000 events in the T2K input flux file were processed through the T2K ND simulation, and events containing a primary pion were recorded. The true

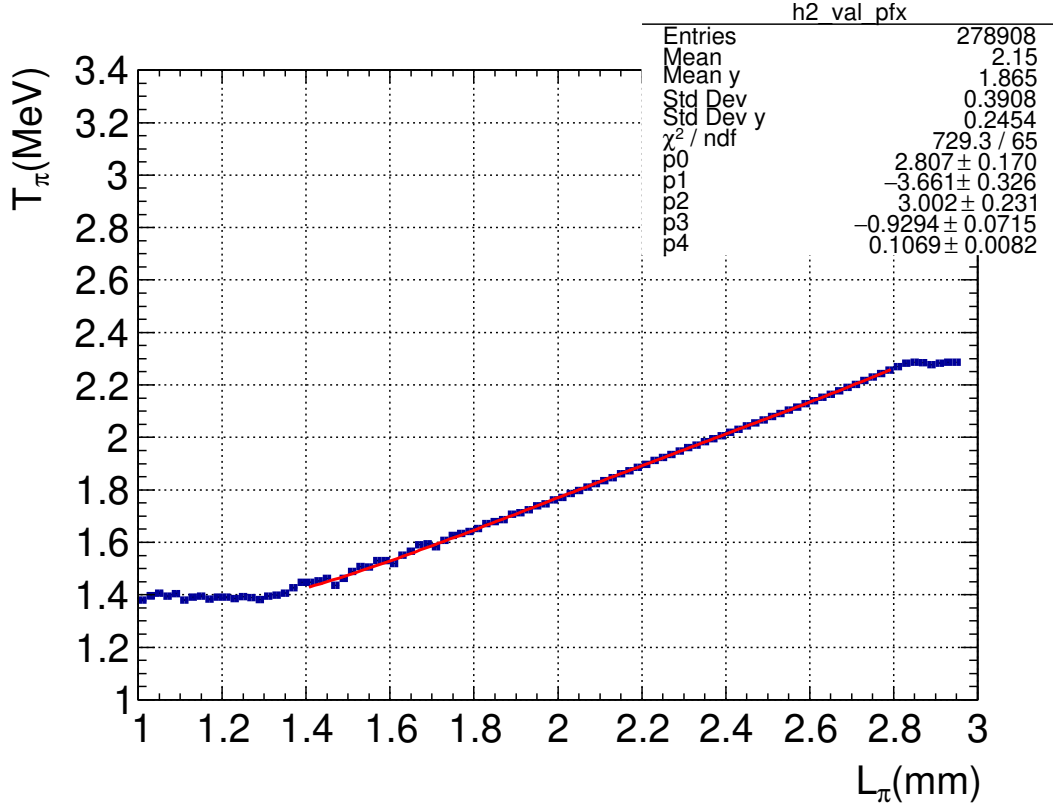
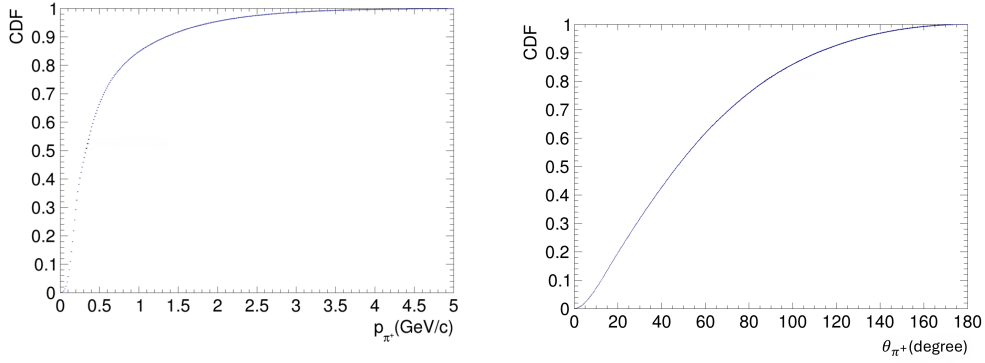


Figure 5.11: A degree 4 polynomial fit of $\log_{10} T_\pi$ against $\log_{10} L_\pi$, where T_π and L_π are the kinetic energy and the range of the pion, respectively. The p_i 's are the corresponding coefficients of the term of order i . The fit matches the data points well.

pion momentum cumulative distribution is shown in Fig. 5.12a, indicating that the majority of pions have momentum below 1000 MeV/ c .

The median momentum is approximately 300 MeV/ c , with about 30% of pions having momentum below 200 MeV/ c and 80% below 800 MeV/ c . Similarly, the pion angle distribution, depicted in Fig. 5.12b, shows that approximately 80% of pions have an angle below 87° .

The second step is to simulate pion propagation in SFGD with the relevant kinematics. A total of six samples, each containing 10,000 events, were simulated with pions traveling at momenta of 200 MeV/ c , 300 MeV/ c , and 800 MeV/ c , and angles of 0° and 70° . Since SFGD reconstruction is largely independent of pion direction, the 70° samples serve as consistency checks. For each sample, the fate of each pion was determined using the truth information.



(a) Primary pion momentum cumulative distribution. The median momentum is about 300 MeV/c. (b) Primary pion angle cumulative distribution. The 80% angle is 87° .

Figure 5.12: Cumulative density functions of primary pion kinematics in SFGD. The relevant values are extracted for subsequent investigation of pion fates.

A total of ten fate topologies are defined as follows:

1. DK@rest (Good) – The pion decays at rest, and the total number of true trajectories is three, corresponding to π^+ , μ , and the ME exactly. This represents a clean pion decay-at-rest event.
2. DK@rest (Dirty) – Similar to the previous fate, but the total number of true trajectories exceeds three. The additional trajectories are mostly electromagnetic showers due to the ME.
3. DK@flight – The pion decays with non-zero momentum, resulting in a relatively large starting momentum for the daughter muon.
4. Elastic Knock-out – The pion undergoes secondary interactions (SI), leading to the liberation of an additional proton. The pion continues to propagate and decay, but its range is no longer well correlated with its starting momentum.
5. Deflection – The pion undergoes SI. No additional particles are produced, but its direction changes.
6. Absorption – Another type of SI where the pion does not emerge from its interaction with the nucleus, so no ME is present.

7. Charge Exchange – The π^+ is converted to a π^0 via a charge exchange interaction with the nucleus. Since π^0 decays much faster, no ME is present.
8. π^- – While there is no single interaction that can convert a π^+ to a π^- , this fate—where a π^- appears as one of the daughters of the π^+ —is observed, albeit rarely, due to complex pion SI within the nucleus.
9. Escaped – The pion escapes SFGD before decaying. Regardless of its subsequent decay, the potential ME cannot be detected in SFGD.
10. Others – This category includes all uncategorised events, such as cases where the pion produces delta ray electrons or undergoes bremsstrahlung. These events are relatively rare and do not require special treatment.

Fates 1 through 3 involve pions that decay before any secondary interactions, i.e., pions decay via the chain $\pi \rightarrow \mu \rightarrow e$. Thus, these pions can be tagged by the presence of the ME. However, as illustrated in Sec. 5.2.1, the trackless technique relies on the extremely short distance travelled by the daughter muon. Pions decaying in flight result in a longer distance travelled by the daughter muon, making them unsuitable for the trackless technique.

Fates 4 through 8 involve pions that undergo secondary interactions, resulting in the absence of the ME, except for those that experience deflection, which also renders them unsuitable for the trackless technique. The range of a deflected pion does not correlate well with its momentum, further disqualifying it from being reconstructed by the trackless technique. In summary, the trackless technique is most applicable to Fates 1 and 2.

The fate distributions are shown in Fig. 5.13. For the 200 MeV/ c samples, both at 0° and 70° , more than 70% of the pions fall into Fates 1 and 2. In the 300 MeV/ c samples, this fraction drops to 30% for the 0° angle and about 12% for the 70° angle. This trend is expected because as pion momentum increases, the decay length increases, and the probability of secondary interactions rises. The further decrease observed at 70° is attributed to an increased escape fraction. The geometry of SFGD,

being much shorter in the vertical direction, facilitates the escape of high-angle pions. For 800 MeV/ c pions, the decay length is too long for SFGD, resulting in a significant increase in the escape fraction—approximately 30% at 0° and 50% at 70° . The unescaped pions are those that have undergone secondary interactions, rendering them not reconstructable by the trackless technique at this high momentum.

This preliminary study demonstrates that a considerable fraction of the primary pions produced in SFGD have relatively low momentum (< 200 MeV/ c). For these pions, the majority decay at rest, producing an ME that can be utilized in the trackless technique. Thus, the trackless technique is applicable to approximately 30% of the total pion events in SFGD, making it a significant reconstruction method to develop.

5.2.3 Implementation

The pion trackless reconstruction is integrated into my $\nu_\mu\text{CC}1\pi$ selection, which builds upon the existing $\nu_\mu\text{CC}$ -inclusive selection. Therefore, it is appropriate to discuss the trackless technique within the context of the $\nu_\mu\text{CC}1\pi$ selection.

The $\nu_\mu\text{CC}$ -inclusive selection identifies the primary vertex position and the primary muon, providing the trackless reconstruction with a reliable approximation of the pion's starting point. The following steps are implemented to identify suitable ME candidates and reconstruct the pion momentum in the $\nu_\mu\text{CC}1\pi$ selection:

- Step 0 - $\nu_\mu\text{CC}$ -inclusive Selection
- Step 1 - Find Suitable ME Candidates
- Step 2 - One Trackless Pion Cut
- Step 3 - Kink Cut
- Step 4 - Background Reduction

Find Suitable ME Candidates – Iterate through all delayed reconstructed objects with more than one hit. If an object occurs 30.0 ns after the primary vertex time, it is saved as a potential ME candidate. However, ME candidates may trigger

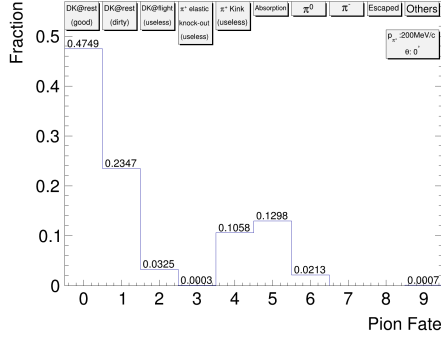
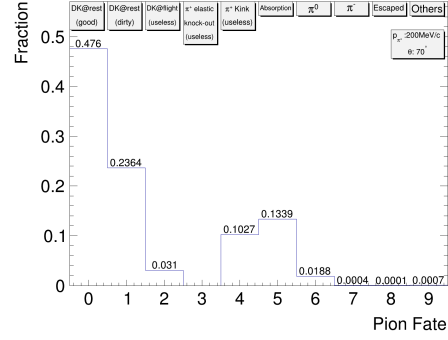
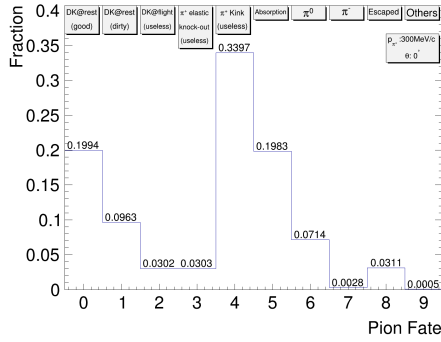
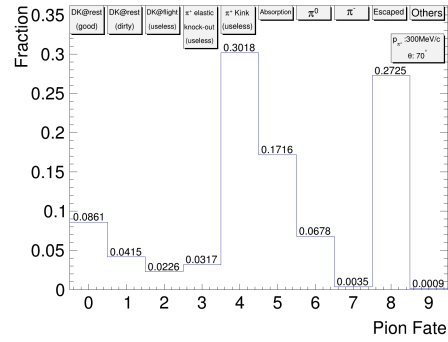
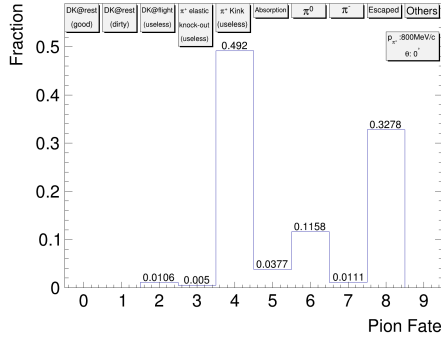
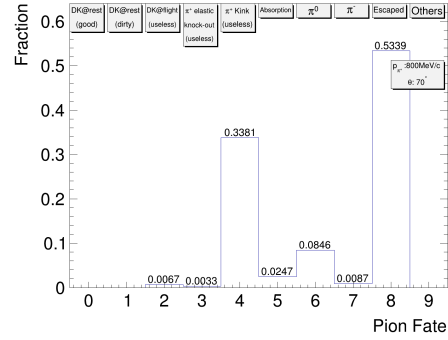
(a) $p_\pi = 200\text{MeV}/c$ and $\theta_\pi = 0^\circ$.(b) $p_\pi = 200\text{MeV}/c$ and $\theta_\pi = 70^\circ$.(c) $p_\pi = 300\text{MeV}/c$ and $\theta_\pi = 0^\circ$.(d) $p_\pi = 300\text{MeV}/c$ and $\theta_\pi = 70^\circ$.(e) $p_\pi = 800\text{MeV}/c$ and $\theta_\pi = 0^\circ$.(f) $p_\pi = 800\text{MeV}/c$ and $\theta_\pi = 70^\circ$.

Figure 5.13: Pion fates in the SFGD for different combinations of pion kinematics. A significant fraction, approximately 80% of pion decays at rest at $p_\pi = 200\text{MeV}/c$ regardless of the angle. The fraction drops to about 30% at $p_\pi = 300\text{MeV}/c$ for low angles and 10% for high angles, and further decreases to 0% at $p_\pi = 800\text{MeV}/c$.

showers, resulting in additional delayed reconstructed objects. To distinguish the true ME from these secondary objects, the "prompt energy" technique used by MINERvA [75] is employed. Prompt energy refers to the energy deposited by the short muon and pion. Although these particles may have insufficient energy to

form a reconstructable track, they still deposit energy much earlier than the ME onset. Therefore, by examining the energy deposition within 30 mm around both ends of an ME candidate, it can be determined if energy was deposited 30 ns prior, confirming the suitability of the ME candidate. Additionally, if the primary muon is contained within SFGD, it may also decay and produce an indistinguishable ME based solely on time delay. To address this, an additional check excludes ME candidates near the end of the identified primary muon.

One Trackless Pion Cut – Select events that have exactly one proper ME candidate.

Kink Cut – Since the trackless reconstruction does not require the ME to be adjacent to a primary track, it cannot differentiate pions travelling directly from the primary interaction to decay at rest from those that have been deflected before decaying at rest. To ensure accurate momentum-by-range reconstruction, events with deflected pions are rejected. Specifically, any event with an ME connected to non-primary tracks is excluded.

Background Reduction – Multiple background reduction steps are implemented, including an existing π^0 rejection developed by colleagues at T2K. However, these measures are insufficient, as evidenced by the significant portion of CC-other backgrounds in Figs. 5.14a, 5.14c, 5.14e, and 5.14g. Therefore, based on a logical understanding of different event topologies, I developed a series of track number cuts and introduced a new concept called the "track family tree."

An event topology resembles a family tree sideways. The first generation consists of the primary tracks, while the second generation includes tracks connected to the ends of the primary tracks. The family tree tracing step involves looping through all reconstructed tracks to build the family tree starting from the primary vertex. One useful variable is the number of generations. A $\nu_\mu \text{CC}1\pi$ event should not have a large number of generations, as the primary pion can have an ME attached to its end without producing showers or multiple scatterings. Therefore, the number of generations is required to be fewer than three. Another associated variable is the number of lone tracks, which are not connected to any tracks in the family tree.

A clean $\nu_\mu\text{CC}1\pi$ event should have a limited number of lone tracks, which could result from noise or low-energy scattering of the ME, leading to clusters isolated from the ME track. Further details on the other track number cuts are provided in Appendix. A.1. The overall impact of the track number cuts is illustrated by the topological decomposition of the pion kinematic distributions in Fig. 5.14.

As shown in Figs. 5.14a and 5.14b, the purity of the $\nu_\mu\text{CC}1\pi$ selection improved from 74.1% to 87.5% after applying the track number cuts for the TPC- μ sub-sample. Similarly, Figs. 5.14e and 5.14f show an improvement in selection purity from 65.9% to 80.7%. Although the cuts result in an 18% and 14% decrease in the number of signal events, these results demonstrate that the track number cuts effectively remove backgrounds. Since the HAT- μ sub-sample still suffers from a global matching problem, its results are not included here.

For events passing all selection steps, the presence of a primary pion is inferred from the existence of a proper ME candidate, and its momentum is reconstructed by range. A quick consistency check involves plotting the distribution of the time difference between the ME and the primary vertex, which should equal the pion travel time plus the muon decay time. Since the pion travel time is on the order of nanoseconds, the resulting distribution should closely follow the muon exponential decay. The distributions of the ME time delay for the TPC- μ and SFGD- μ sub-samples are shown in Fig. 5.15.

Fig. 5.16 displays a candidate CC1 π event passing the $\nu_\mu\text{CC}1\pi\text{-TL}$ selection in the upgraded ND280 during the beam run in June 2024. The second-longest track is the ME, whose starting point is very close to that of the primary muon, indicating the extremely short distance traveled by the primary pion. This short distance makes reconstruction using a track-based method practically unfeasible.

5.2.4 Comparison with Conventional Method

One key advantage of the trackless technique is its improved reconstruction in the low-momentum phase space. To verify this claim, I compared pion momentum reconstruction using the conventional method, where the pion is identified by a

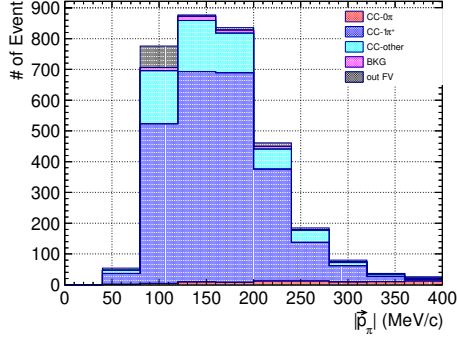
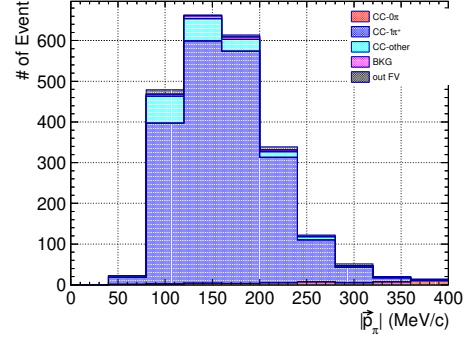
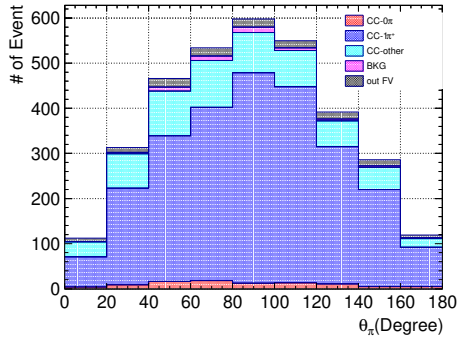
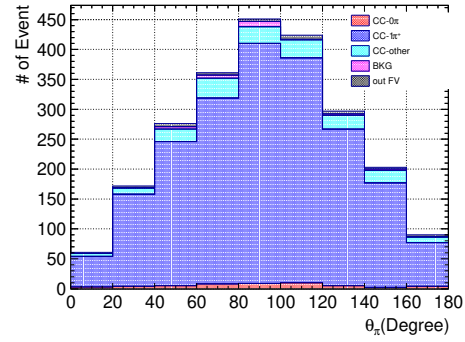
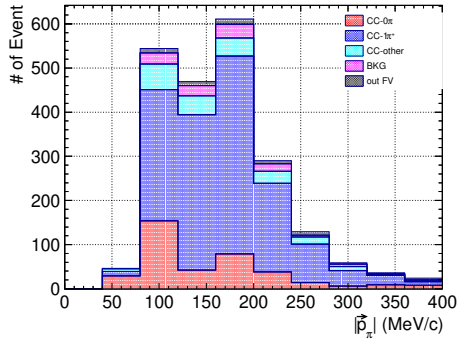
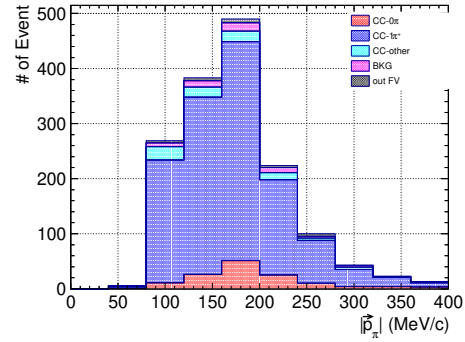
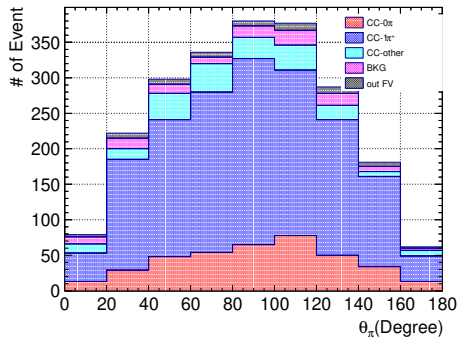
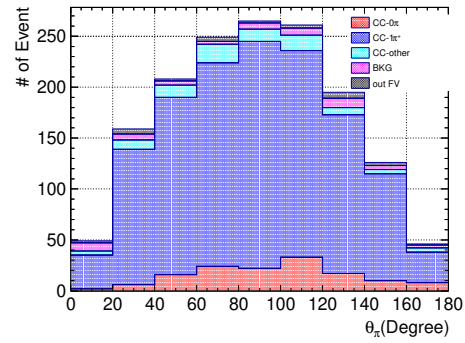
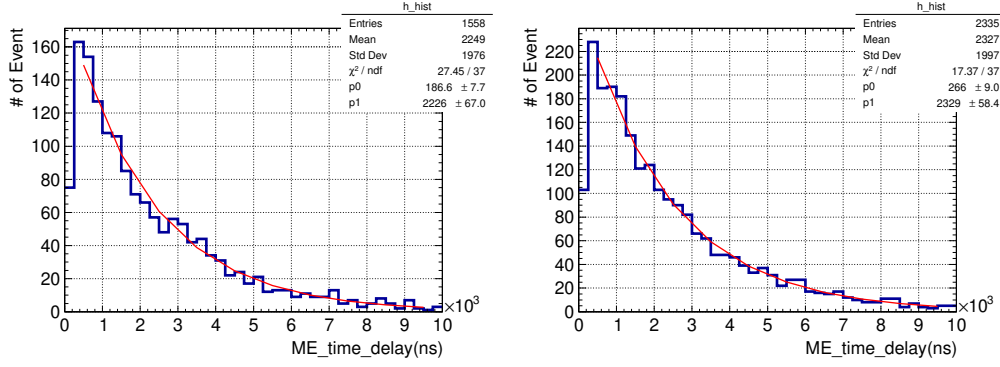
(a) TPC- μ : p_π before track number cut.(b) TPC- μ : p_π after track number cut.(c) TPC- μ : θ_π before track number cut.(d) TPC- μ : θ_π after track number cut.(e) SFGD- μ : p_π before track number cut.(f) SFGD- μ : p_π after track number cut.(g) SFGD- μ : θ_π before track number cut.(h) SFGD- μ : θ_π after track number cut.

Figure 5.14: Reconstructed pion kinematics before (left) and after (right) track number cuts. An appreciable reduction in the background is observed.



(a) SFGD- μ . The fitted lifetime is 2226 ± 67 ns. (b) TPC- μ . The fitted lifetime is 2329 ± 58 ns.

Figure 5.15: Time difference between ME and the end point of the pion track for the subsamples where the primary muon is contained in the SFGD or escapes into the TPC. Both distributions are fitted with an exponential decay function, and the fitted lifetimes are consistent with the expected muon lifetime of approximately 2200 ns.

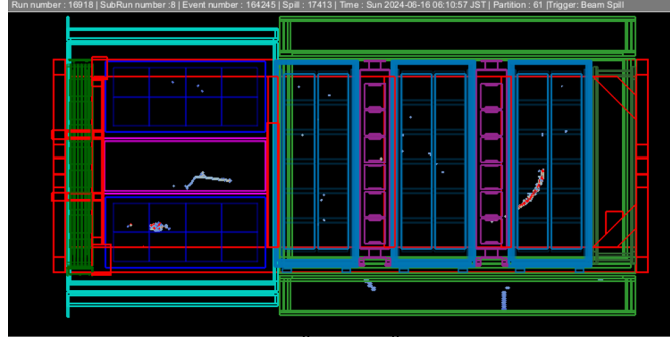


Figure 5.16: A CC1 π -TL candidate event in upgraded ND280, where the long track is the primary muon and the other track is a delayed ME, which implies the existence of a very low momentum primary pion.

reconstructed track with an attached ME. A preliminary selection based on the conventional method was developed for this comparison. To differentiate between the two pion selections, the trackless technique selection is referred to as ν_μ CC1 π -TL, while the conventional method selection is referred to as ν_μ CC1 π -TR.

The ν_μ CC1 π -TR selection is based on the ν_μ CC-inclusive selection and employs the same background reduction steps as the ν_μ CC1 π -TL. The difference lies in the reconstruction of the primary pion. A reconstructed primary track with an attached ME is required. No further optimization is performed, ensuring that this selection encompasses the maximally possible phase space. The results of the

comparison are shown in Fig. 5.17². The efficiencies for pions with momentum higher than 100 MeV/ c are similar for both methods. Below 100 MeV/ c , the improvement brought by the trackless technique is twofold. Firstly, the efficiencies below 100 MeV/ c are significantly higher for the trackless technique. The efficiency for $\nu_\mu\text{CC}1\pi\text{-TR}$ drops drastically below 80 MeV/ c . Secondly, the momentum threshold in $\nu_\mu\text{CC}1\pi\text{-TL}$ extends below 50 MeV/ c , which is impossible for the conventional method. More encouragingly, this improvement in low momentum efficiency and lower threshold does not compromise selection purity, as shown in Figs. 5.17a and 5.17b.

5.2.5 Impacts on SPK

The results of the $\nu_\mu\text{CC}1\pi$ selection using the trackless pion technique are summarized in Fig. 5.18. As shown in Fig. 5.18a, the trackless selection has achieved its goal of reconstructing low-momentum pions. A considerable number of events contain pions with momentum below 100 MeV/ c , corresponding to a track length of approximately 50 mm. Moreover, pions with momentum below 80 MeV/ c have also been reconstructed; these pions travel about 30 mm and cannot be reconstructed as tracks. Furthermore, the momentum-by-range calculation has achieved an excellent HWHM of approximately 2%, as shown in Fig. 5.18b. More encouragingly, the overall efficiencies remain relatively high. The step-by-step efficiencies for each selection step are shown in Fig. 5.18c. The signal definition used in the calculation requires one true muon that has travelled to the vertical TPC and one true pion that is contained in SFGD. Steps 1 to 7 correspond to the $\nu_\mu\text{CC}$ -inclusive selection. The large drop in efficiency at Step 8 is expected, as it requires exactly one pion to be reconstructed tracklessly. All pions undergoing secondary interactions, except for deflection, cannot be reconstructed via this method. Hence, the relative efficiency of $0.37/0.66 \approx 56\%$ is considerably high. A further significant drop in efficiency occurs

²Note that the result for $\nu_\mu\text{CC}1\pi\text{-TL}$ in this figure does not include the implementation of the early energy deposit in choosing the suitable ME candidate, as this comparison was conducted during the early stage of development. Since the main purpose of this comparison is to demonstrate the limitations of the conventional method, the final reconstruction threshold for the $\nu_\mu\text{CC}1\pi\text{-TL}$ is presented in Fig. 5.18.

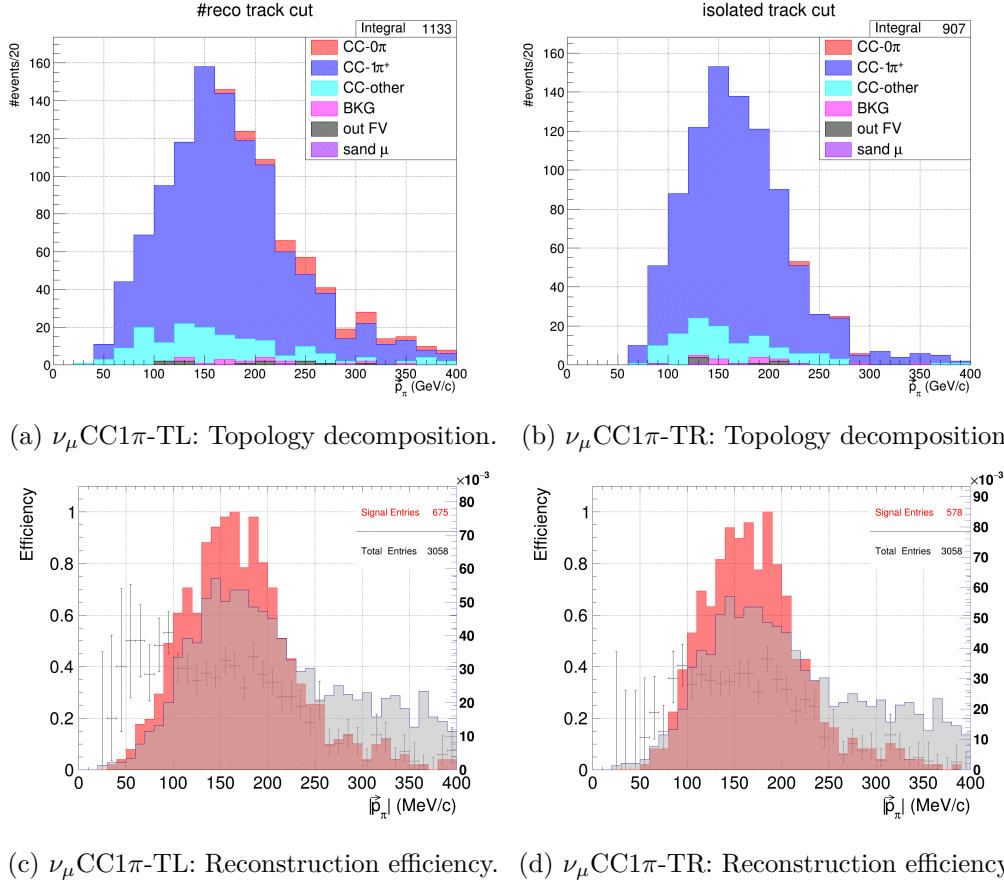


Figure 5.17: Comparison between the pion trackless reconstruction (TL) and the conventional track-based reconstruction (TR). Both the decomposition in event topology and the efficiency are plotted. The TL method achieves a significantly higher efficiency in the low-momentum region and a lower threshold, as suggested by the non-empty bin at 50 MeV/c in Fig. 5.17a.

in the next step, the kink cut, which is reasonable as a considerable portion of pions undergo deflection. The gradual drops are subsequent cuts to remove backgrounds, resulting in a high purity of $\nu_\mu \text{CC}1\pi$, as shown in Fig. 5.18a.

5.3 Development of stopping pion control sample

5.3.1 Implementation

Conventionally, selections are used for cross-section analysis, which requires a sufficient portion of all detector systematics to be correctly evaluated. However, due to time constraints, this is unlikely to happen within the timeframe of my doctoral study. Therefore, only preliminary data–MC comparisons are feasible.

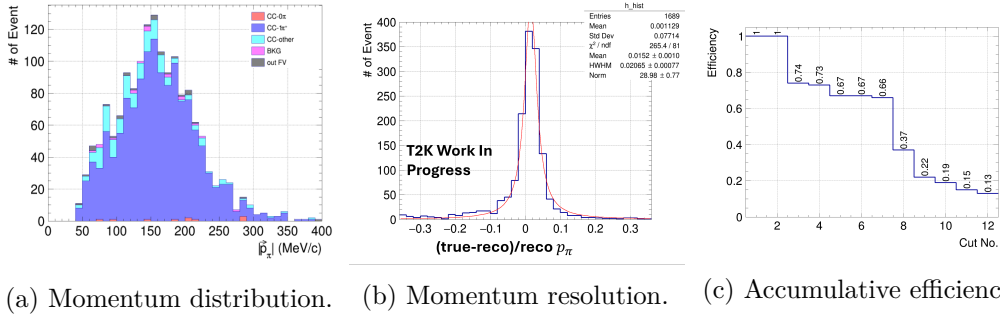


Figure 5.18: Pion trackless reconstruction results. A significant number of low-momentum pions are reconstructed, with a threshold as low as 50 MeV/ c . The fitted HWHM for the momentum reconstruction is excellent at 2.1%. The overall efficiency remains acceptable at 13%.

Nonetheless, because systematic evaluation is an important aspect of cross-section analysis—crucial for improving our understanding of neutrino–nucleus interactions—I implemented the propagation of the momentum-by-range reconstruction systematics for pions tagged by ME. To evaluate this systematic, I developed the stopping pion control sample with the following steps:

1. Find all tracks that contain an SFGD segment and segments in the other sub-detectors.
2. Check that one end of this track is stopped in SFGD.
3. Check the PID of this track provided by the other sub-detectors, i.e., HAT or TPC.
4. Check that there are no other (except ME) near the track end in SFGD.
5. A final cut requiring the presence of an ME near the track end in SFGD.
6. Keep only the event with a single stopping pion candidate.

The preliminary result suggests that the control sample consists predominantly of tracks passing HAT and stopping in SFGD, while there are few events propagating backwards from TPCs to SFGD. As the current PID in HAT is not fully developed but indispensable for the control sample development, I developed a temporary PID method for HAT for demonstration only. The HAT PID method will need to

be updated in the future with a sophisticated version developed by HAT experts, but the other steps can remain intact.

As HAT has fundamentally the same working principle as the TPC, I followed closely the TPC PID method. Different particles can be distinguished by their specific energy loss, i.e. dE/dx , along the tracks in the TPC. The expected dE/dx for a given type of particle, like a proton, at a given momentum can be obtained numerically. The measured dE/dx of a reconstructed track is compared with the expected dE/dx to determine the particle type. This comparison is quantified by the "pulls", which are conventionally defined as the difference between the measured and expected dE/dx divided by the uncertainty of the measurement. The existing TPC PID is based on the logarithms of the pulls. In practice, this value is constructed such that the closer the value is to 1, the more likely the particle is of the corresponding type. For subsequent discussion, this value will simply be referred to as the pull as well.

There are four pulls for each reconstructed track, corresponding to the four particle types: muon, electron, proton, and pion. For the existing TPC PID, the pion pull was set to be larger than 0.3 and the sum of the muon and the pion pull was set to be larger than 0.8 for pion PID, due to the similarities between muons and pions. The distributions of the pulls at HAT for positively charged tracks before any cuts are shown in Fig. 5.19.

A significant fraction of pions possess considerably large muon pulls, which indicates that a significant fraction of muons will remain after the TPC pull cuts. Hence, stricter cuts are required to obtain a pion sample with higher purity. In particular, a direct upper on the muon pull is necessary. To determine the precise cut value, a further zoom of the first pull is shown in Fig. 5.20, suggesting a cut to exclude events with the muon pull larger than 0.04. After the cut on the muon pull, the other pull distributions do not show a clear separation between the muon background and the pion signal, as illustrated in Fig. 5.21.

Instead, I used a concept suggested by my colleague, Xingyu Zhao, from his FGD analysis, where he defined a variable called the heavy length—the portion

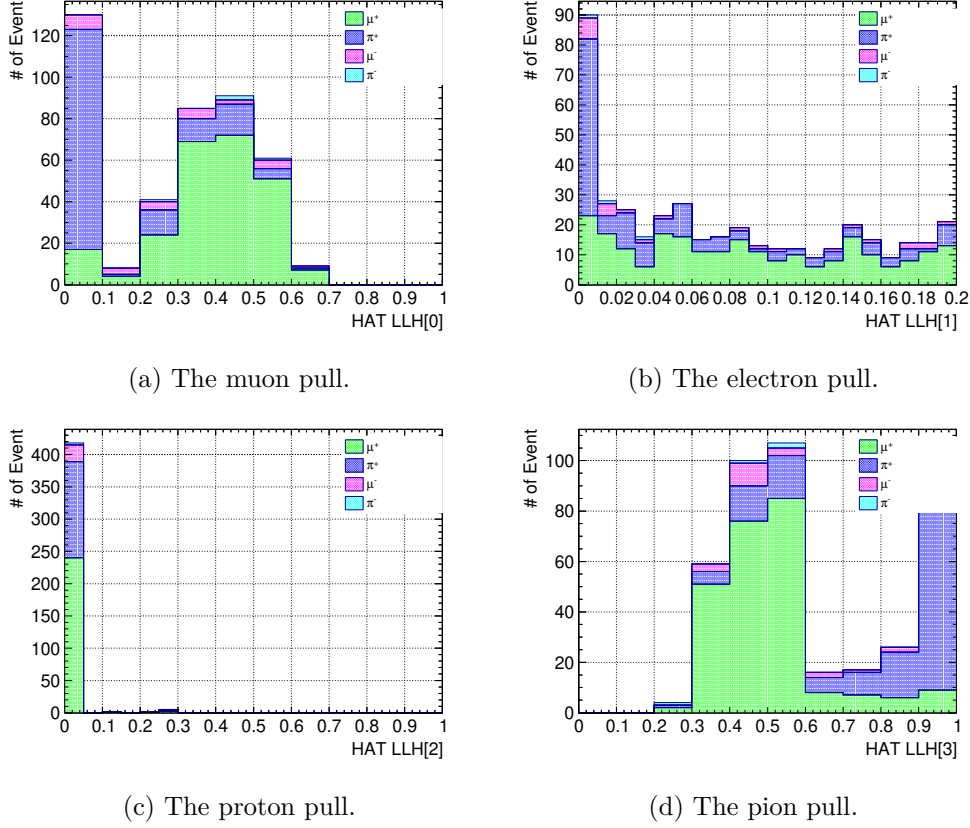


Figure 5.19: Pull distributions at HAT before any PID cuts for positive reconstructed tracks. Most pion tracks have large pion pulls, small electron pulls, and small proton pulls. However, a sizeable fraction of pions have non-negligible muon pulls, which can lead to a significant muon background in the stopping pion sample.

of the particle inside a dense detector (e.g., SFGD). This length can distinguish between leptons and hadrons because hadrons are much more likely to interact with dense material, thereby having shorter lengths within such detectors. For the stopping pion sample, the heavy length is dominated by the portion in the SFGD, as the ECAL and TOF contribute only marginally. As shown in Fig. 5.21d, a cut on the heavy length to keep events below 150 mm can remove a considerable portion of the muon background remaining after the cut on the muon pull.

The pull distributions shown are based on the sub-sample where the pion passes through the Top HAT. The Bottom HAT has similar performance, and the same cuts are applied. For brevity, the pull distributions for the Bottom HAT are collected in Sec. A.2 in the Appendix. The final selection results for the two subsamples are shown in Fig. 5.22. The Top HAT subsample has achieved a purity of $94.8 \pm 2.3\%$

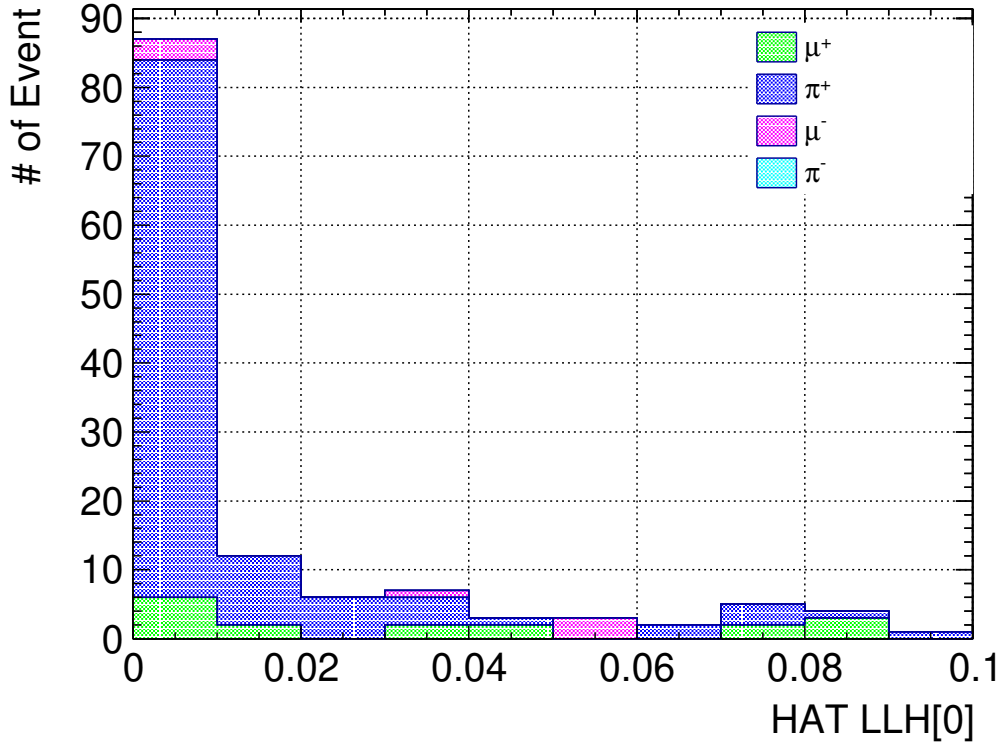
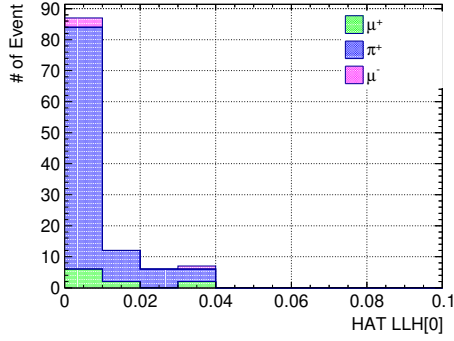


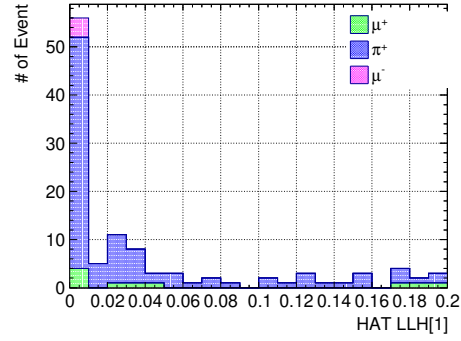
Figure 5.20: Zoom on the muon pull distribution at HAT. A cut at 0.04 strikes a good balance between removing muon background and keeping pion signal.

as shown in Fig. 5.22a and Fig. 5.22b, which has 91 π^+ signal events, 3 μ^- , and 2 μ^+ background events. Similarly, the Bottom HAT subsample has achieved a purity of $87.5 \pm 1.8\%$ as shown in Fig. 5.22c and Fig. 5.22d, which has 293 π^+ signal events, 15 μ^- , 26 μ^+ , and 1 π^- background event.

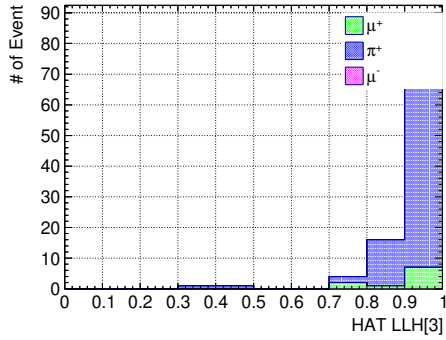
As this HAT PID method is temporary, the achieved purity is sufficient to demonstrate the usage of the stopping pion sample for systematics evaluation. Before proceeding to systematics evaluation, a good consistency check of using the ME method to select the stopping pion is to check the time difference between the ME and the end point of the stopping track. This distribution should closely follow the muon exponential decay curve. The result is shown in Fig. 5.23.



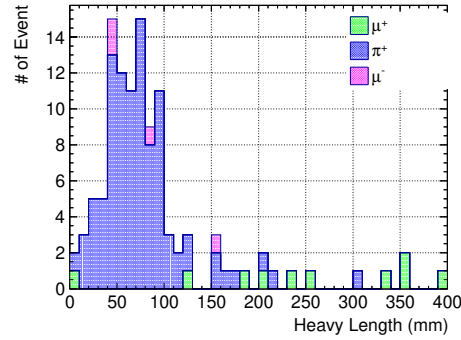
(a) The muon pull.



(b) The electron pull.



(c) The pion pull.



(d) Heavy length.

Figure 5.21: Pull distributions and a distribution of heavy length at HAT after the cut on the muon pull. The pull distributions can no longer provide a good separation between muons and pions, while a cut on long heavy length can still remove some background events.

5.3.2 Systematics evaluation

The control sample is developed with evaluation of the pion momentum by range reconstruction in mind. Although the selection is not finalized as the HAT reconstruction and global track matching are still subjected to changes, it is still valuable to develop the systematics evaluation framework. One is to learn the method of performing systematics evaluation, the other is to get a first glimpse of the performance of the pion momentum reconstruction in SFGD.

The systematics evaluation follows that in the FGD analysis as outlined in Ref. [76]. While it is straightforward to examine the reconstruction resolution with MC studies, there is no truth information when analysing data. As the momentum reconstruction in TPCs has good resolution and is well validated, i.e. there is

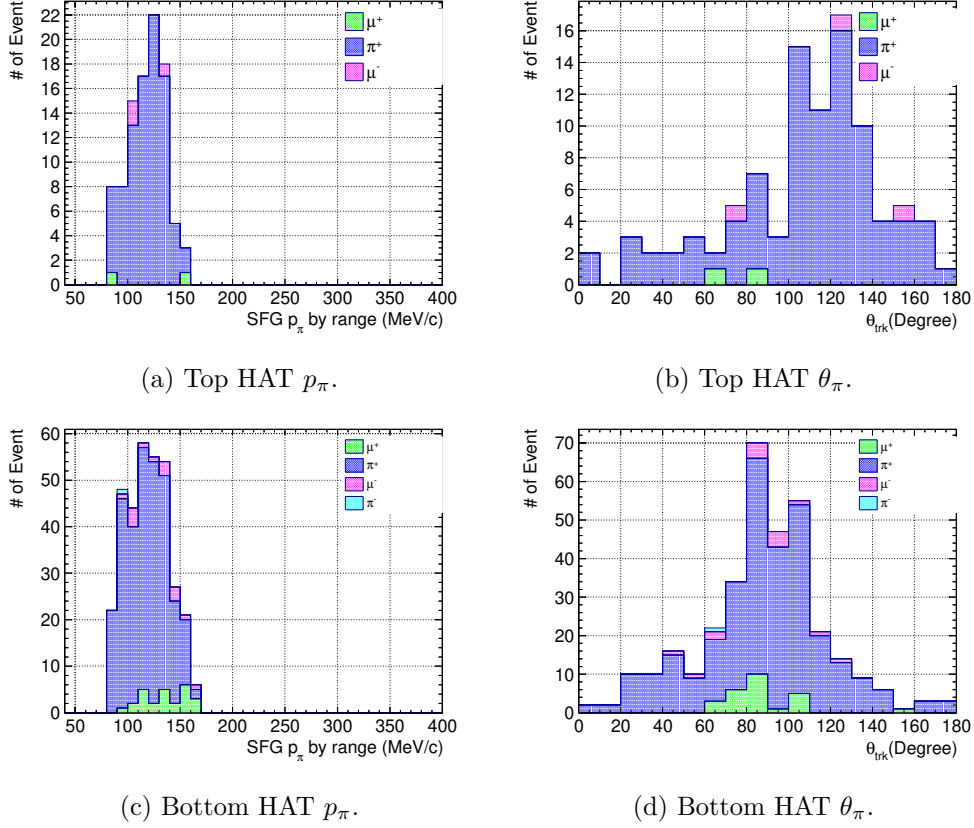
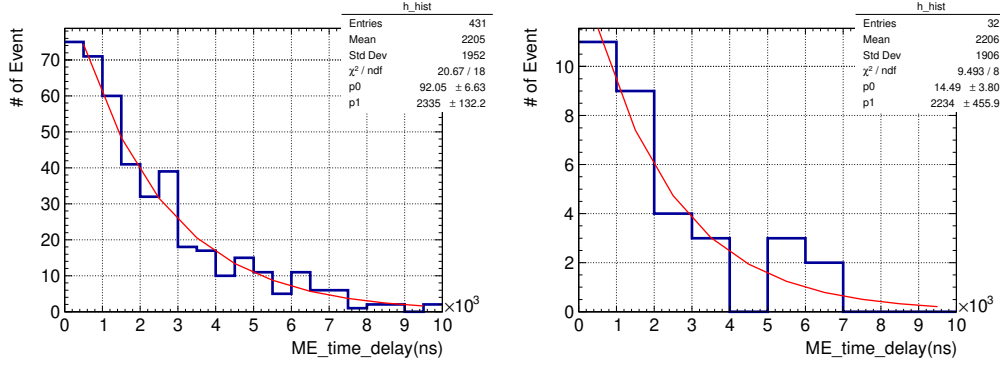


Figure 5.22: Stopping pion kinematic distributions decomposed according to true particle type. Overall high purity is achieved.

good agreement between MC and data, instead of comparing the reconstructed momentum to the true momentum, it can be used as a good estimation of the true momentum. Moreover, as the control sample selects pion tracks that travel from HAT into SFGD and the pion loses an infinitesimal amount of energy in the HAT, the momentum obtained from the HAT reconstruction is a good approximation of the true enter momentum in SFGD. The deviations of the SFGD reconstructed momentum from the HAT reconstructed momentum can be computed for both MC and data. On one hand, this deviation can be regarded as an approximation of the SFGD momentum resolution. On the other hand, the difference between the deviations of MC and data can assess the difference in the reconstruction performance between MC and data in SFGD, i.e. the systematics related to the SFGD momentum by range reconstruction.

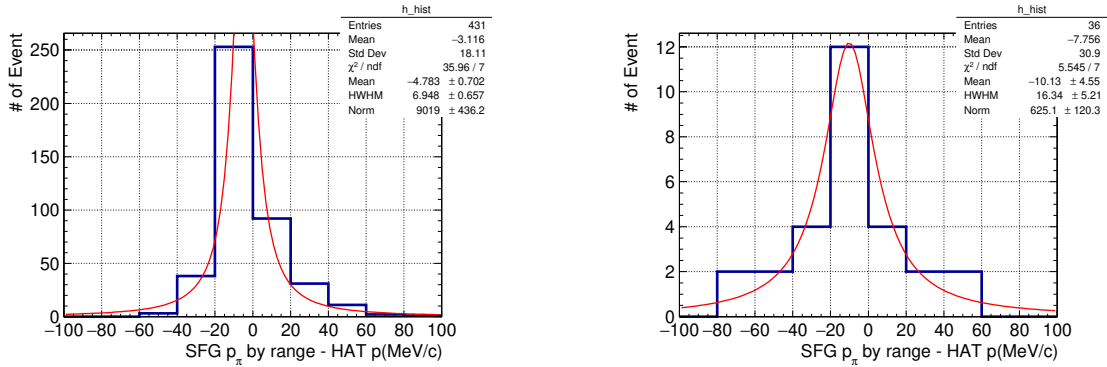
The distributions of the deviations for MC and data are shown in Fig. 5.24



(a) MC. The fitted lifetime is 2335 ± 132 ns. (b) Data. The fitted lifetime is 2234 ± 456 ns.

Figure 5.23: Time difference between ME and the end point of the stopping pion track. Both distributions follow the muon exponential decay curve, suggesting that the selected particles are likely stopping pions.

and the fitting parameters, the mean (x_0) and the HWHM (γ), are summarised in Table. 5.1. Both widths are significantly smaller than those in the FGD analysis presented in Ref. [76], in the FGD study. However, due to the low data statistics, the numerical results here are for illustration only, and a more reliable evaluation of the reduction in the pion momentum-by-range reconstruction systematic error will be performed when the HAT reconstruction is finalized and more data are collected.



(a) Momentum difference between SFGD and HAT for MC.

(b) Momentum difference between SFGD and HAT for data.

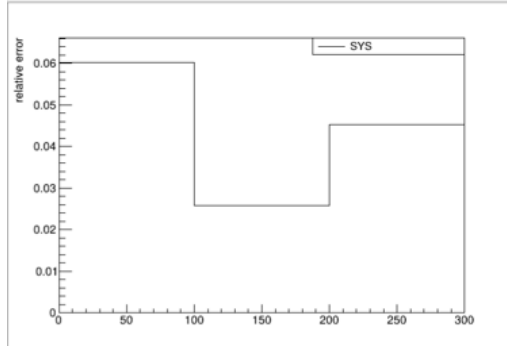
Figure 5.24: Pion momentum reconstruction resolution estimation for MC and data. Both momentum deviations have shown small widths, 6.9 ± 0.7 MeV/c for MC and 16 ± 5 MeV/c for data. The statistics for data is low, so the numerical results are for illustration only.

The systematic error parameters from Table 5.1 are then passed to the High-LAND framework, the selection development and systematics propagation software

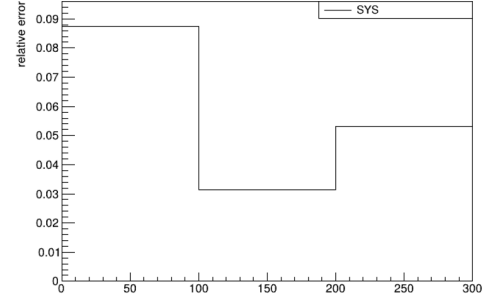
	MC- x_0 (MeV/c)	MC- γ (MeV/c)	Data- x_0 (MeV/c)	Data- γ (MeV/c)
FGD	-25.03 ± 0.52	85.38 ± 1.20	-19.43 ± 1.41	93.57 ± 4.15
SFGD	-5.29 ± 0.73	7.49 ± 0.65	-10.14 ± 4.55	16.34 ± 5.21

Table 5.1: Fit parameters for the momentum deviation distributions in the SFGD compared to those in the FGD. Both MC and data in the SFGD have shown significant improvement in both the bias and the resolution. However, the data statistics in the SFGD is still low, so the numerical results are for demonstration of the systematic propagation process only.

used at T2K, to propagate the uncertainty through the $\nu_\mu\text{CC1}\pi\text{-TL}$ selection. The result is shown in Fig. 5.25. As shown in Fig. 5.25, due to the limited amount



(a) Relative error for the TPC- μ sub-sample.



(b) Relative error for the SFGD- μ sub-sample.

Figure 5.25: Relative errors for the $\nu_\mu\text{CC1}\pi\text{-TL}$ selection. This shows the successful propagation of the systematic error. The specific values of the relative error are subject to change due to the low data statistics.

of data, the difference between the data and MC is still relatively large that the magnitude of the average systematic error, about 3% to 5%, is slightly larger than that in the FGD analysis, about 2%. Due to the incompleteness of global reconstruction and the limited amount of data, this error propagation is performed with only 20 toys, which can only serve as a proof of the successful propagation of the systematics. Even though no physics conclusion can be drawn with this limited amount of data, it is promising to observe a considerable decrease in the error for the low momentum pion, comparing to the FGD analysis, showcasing the strength of SFGD and the trackless reconstruction technique.

5.4 Data-MC comparisons

Conventionally, kinematic distributions from these selections will be extracted and passed through the unfolding process which maps the reconstructed variables to the truth space. These unfolded distributions can then be used to compare directly with output from MC event generators. However, the unfolding process needs to account for all major detector effects, and unfortunately, a full coverage of systematics for the new sub-detectors is not possible within my doctoral study timeline. Hence, I have to resort to a reconstruction-level Data-MC comparisons instead. Although the comparison result will only be valid for the specific configurations of interaction model used to produce the MC sample, it is the only possible way to utilize the collected data to produce physical results.

With the development of the pion momentum by range reconstruction systematics, it is possible to take a first glimpse of data-MC comparisons for the $\nu_\mu\text{CC}1\pi\text{-TL}$ selection. As the global reconstruction is yet to be finalized and only one preliminary systematics is propagated, this data-MC comparison is merely a demonstration of how the data can be used in the future analysis, and no physics conclusion should be drawn. The data-MC comparisons are shown in Fig. 5.26. It is reassuring to observe no significant discrepancy between data and MC in the $\nu_\mu\text{CC}1\pi\text{-TL}$ selection. The data-MC comparison workflow is thus complete. Official analysis results can be readily obtained once the various systematics are in place by putting both MC and data through the $\nu_\mu\text{CC}1\pi\text{-TL}$ selection.

5.5 Discussion

The two new techniques elaborated in this chapter, the ESC selection and the pion trackless reconstruction, have shown significant improvement in the reconstruction of hadrons in SFGD. One leads to a 50% improvement in the momentum resolution of protons, while the other significantly improves the low momentum pion reconstruction efficiency and extends the pion reconstruction threshold to 50 MeV/ c , which is impossible for the conventional method. Both the $\nu_\mu\text{CC}1\pi$ and ESC proton

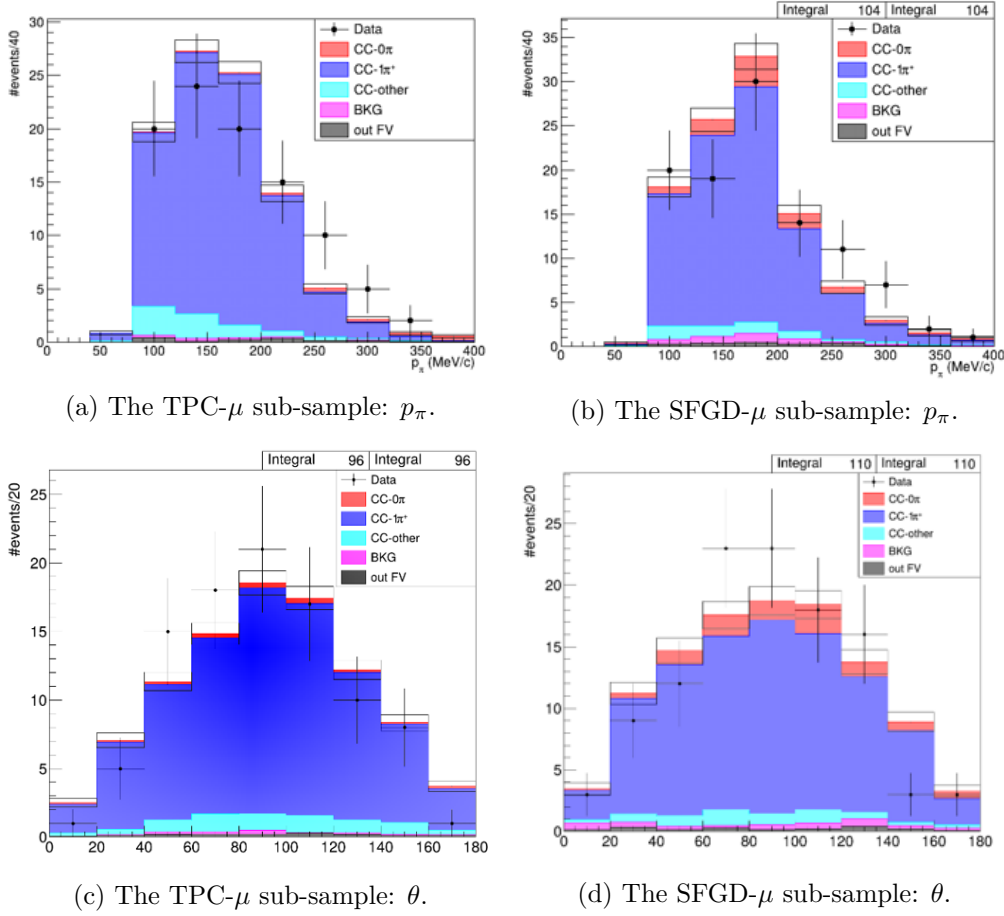


Figure 5.26: Data-MC comparisons for the $\nu_\mu\text{CC}1\pi\text{-TL}$ selection. Data and MC appears to be in good agreement. However, due to the low data statistics, this comparison is merely a demonstration of the data-MC comparison workflow, and no physics conclusion should be drawn.

selections are finished for primary muons going into the vertical TPC and SFGD. The sub-sample for muons going into HAT is missing as the global reconstruction matching for tracks passing through HAT is still under development. However, as the two new techniques in this chapter are developed for hadrons, the performance should not depend on the ending position of the muons. Hence, reconstruction improvement showcased in this chapter should be applicable to the muons going into HAT as well. With this improvement in the single particle kinematics, it is exciting to see the performance of the reconstruction of the higher level variables, such as the TKI variables, in the next chapter.

真実はいつも一つ。

There is always only one truth.

— Edogawa Conan

6

High-Level Variables

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As elaborated in Sec. 3.2.3, the neutrino-nucleus interaction is a complex process involving multiple components, including the neutrino-nucleon interaction, the nucleon initial states (IS), and final state interactions (FSI). Hadronic single particle kinematics (SPK) are easy to measure and interpret, but they are affected by all components of the neutrino-nucleus interaction. Hence, changes in different components can lead to similar changes in the SPK, thereby making it difficult to use them to improve neutrino–nucleus interaction models. To match the statistical improvement of the next generation long-baseline neutrino experiments, it is imperative to refine neutrino interaction models to significantly reduce the systematic

uncertainties. To address this issue, high-level variables are cleverly constructed to probe specific components of the neutrino–nucleus interaction. This chapter focuses on the potential improvements in measuring these high-level variables with the upgraded ND280 detector. Besides the well-established Transverse Kinematic Imbalance (TKI) variables elaborated in Sec. 3.2.3, the centre-of-momentum (COM) variables are introduced for the first time to probe FSI with strong robustness against changes in IS. The first section provides a brief introduction to these constructed variables, while the following section presents the MC study results. Although it would be highly valuable to perform data–MC comparisons of these variables, the current state of the global simulation and reconstruction does not permit them. Because the global simulation and reconstruction are still under development, the ESC technique is not yet implemented in the latest analysis chain. As proton kinematics are required for these high-level constructed variables, data–MC comparisons for these variables are not possible at this stage, and improvements for these variables will only be demonstrated with MC studies.

6.1 The TKI Variables

With the improved hadronic SPK offered by the novel reconstruction techniques introduced in the previous chapter, the TKI variables are expected to be measured with higher precision. As in the previous chapter, the impacts of the new techniques on the TKI variables will be illustrated separately for the $\nu_\mu\text{CC}0\pi1\text{p}$ and $\nu_\mu\text{CC}1\pi1\text{p}$ samples.

6.1.1 $\nu_\mu\text{CC}0\pi1\text{p}$ selection

To demonstrate the benefits of the ESC technique, the TKI reconstruction in two samples is compared: the $\nu_\mu\text{CC}0\pi1\text{p}$ and the $\nu_\mu\text{CC}0\pi1\text{p}$ -ESC sample. The $\nu_\mu\text{CC}0\pi1\text{p}$ (-ESC) sample is selected by requiring the presence of exactly one contained SFGD (ESC) proton, in addition to the $\nu_\mu\text{CC}0\pi$ -inclusive selection. As discussed in Sec. 5.1, the ESC technique comprises the proton ESC cut and the muon momentum bias correction for TKI calculations. Hence, TKI reconstruction

comparisons are presented step by step. Note that the result shown in this section for each selection is the sub-sample where the primary muon escapes SFGD and passes through the vertical TPC. For the SFG muon subsample, the improvement is similar, and the figures are collected in Appendix B.

The impact of the proton ESC cut on the four TKI variables is best illustrated by the response matrices in Fig. 6.1. All TKI variables exhibit significant improvements in reconstruction after the ESC cut. Most notably, the dense off-diagonal cluster in the lower-right corner of the $\delta\phi_T$, δp_T , and p_N response matrices is appreciably reduced. The $\delta\alpha_T$ response matrix entries also align more closely along the diagonal following the ESC cut.

The impact of the muon momentum bias correction is best illustrated with a resolution fit in Fig. 6.2. This correction has a negligible impact on the angular variables, but it helps reduce bias in the momentum variables. As shown in Fig. 6.2e and Fig. 6.2f, the fitted mean for the truth to reconstruction ratio for δp_T improves from 0.92 to 0.95 after the muon bias correction. Similarly, in Fig. 6.2g and Fig. 6.2h, the fitted mean for the truth to reconstruction ratio for p_N improves from 0.94 to 1.01. This reduction in bias can also be observed in the decreased overestimated tails in the δp_T and p_N distributions.

In Sec. 5.1, it was shown that the ESC technique improves proton momentum reconstruction resolution but diminishes efficiency by about 60%. The results in this section further highlight the necessity of the ESC technique for precise TKI measurements. With the ESC technique, the upgraded T2K ND can achieve high-resolution TKI measurements with minimal bias. Recently, an improved reconstruction algorithm enabled an updated TKI measurement at the pre-upgrade ND280. This yields a standard deviation of about 27% for the p_N reconstruction resolution. The resolution of 30% for the TPC- μ sub-sample is comparable to this improvement, whereas the resolution of about 17% for the SFGD- μ sub-sample is noticeably better as shown in Fig. 6.3. There remains room for additional improvement in the upgraded measurement as the global reconstruction matures.

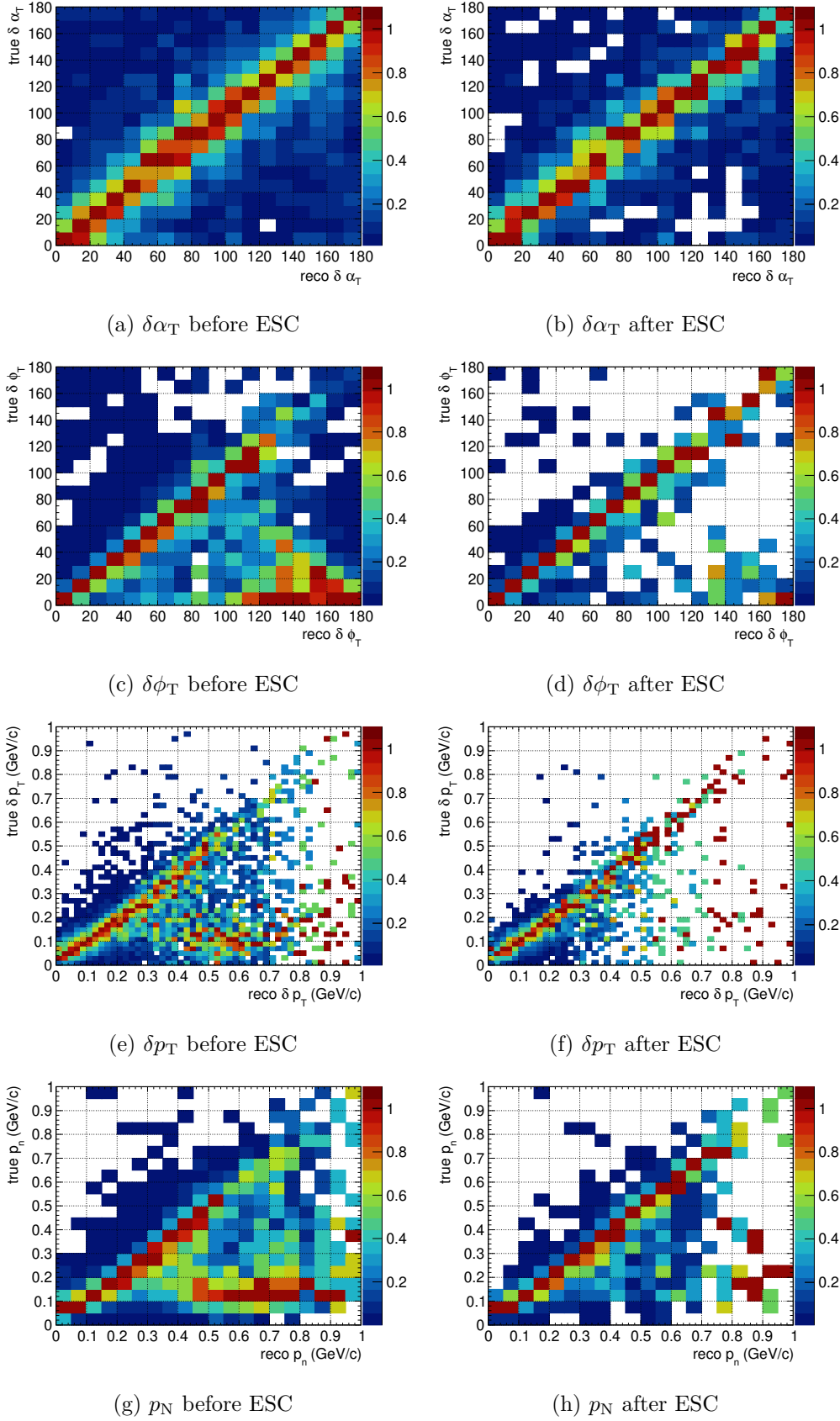


Figure 6.1: Reconstruction quality plots of truth against reconstruction values for TKI variables before (left) and after (right) ESC for the $\nu_\mu\text{CC}0\pi1p$ selection. The ESC selection has significantly removed the off-diagonal entries for all TKI variables.

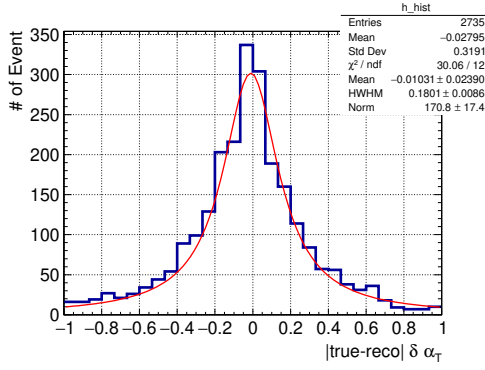
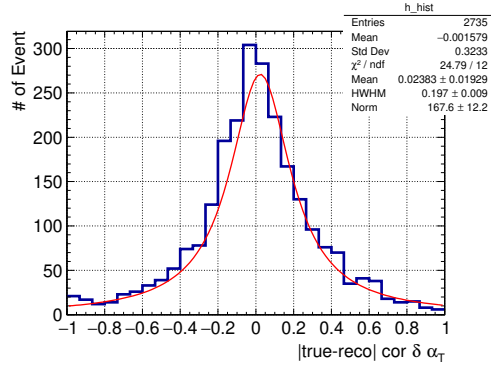
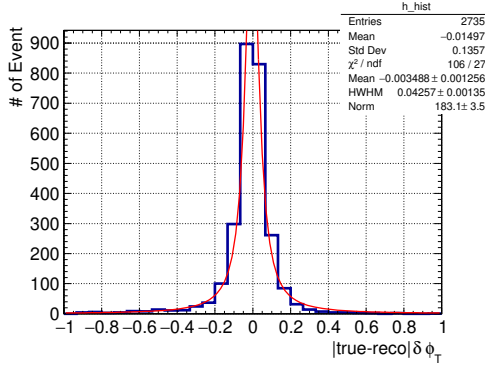
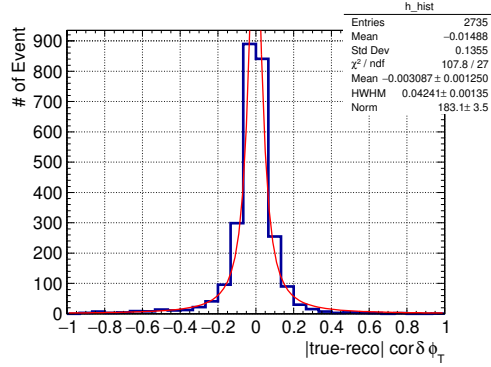
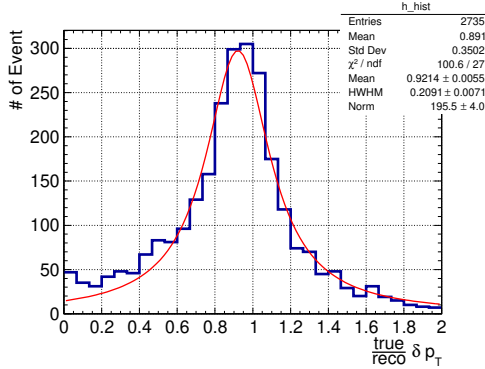
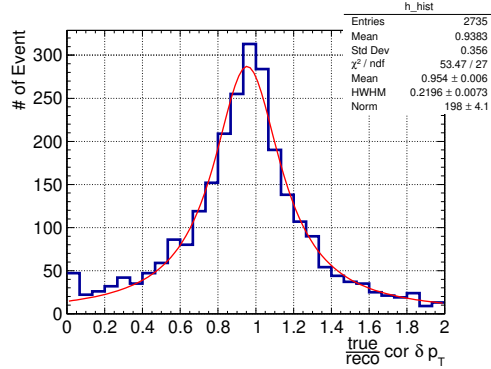
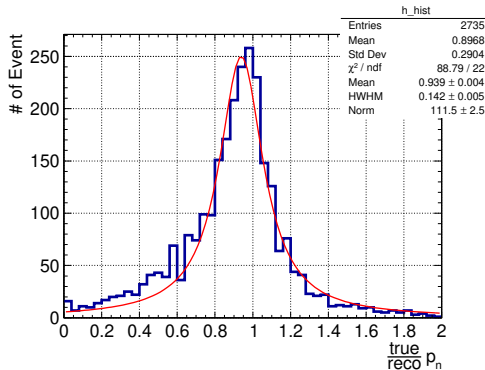
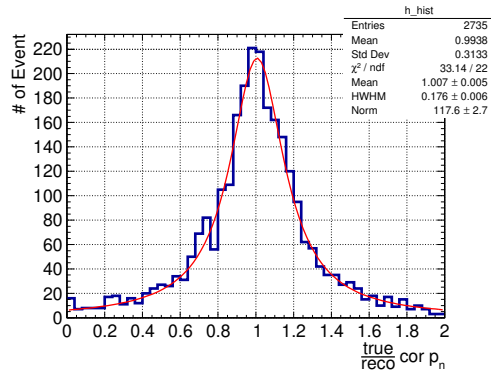
(a) $\delta\alpha_T$ before muon bias correction(b) $\delta\alpha_T$ after muon bias correction(c) $\delta\phi_T$ before muon bias correction.(d) $\delta\phi_T$ after muon bias correction.(e) δp_T before muon bias correction(f) δp_T after muon bias correction(g) p_N before muon bias correction(h) p_N after muon bias correction

Figure 6.2: Fits of reconstruction resolution for TKI variables before (left) and after (right) muon bias correction for the $\nu_\mu CC0\pi 1p$ selection. The fitted HWHM remains at similar values, while the fitted means for both δp_T and p_N both changed to be closer to 1, indicating a reduced bias.

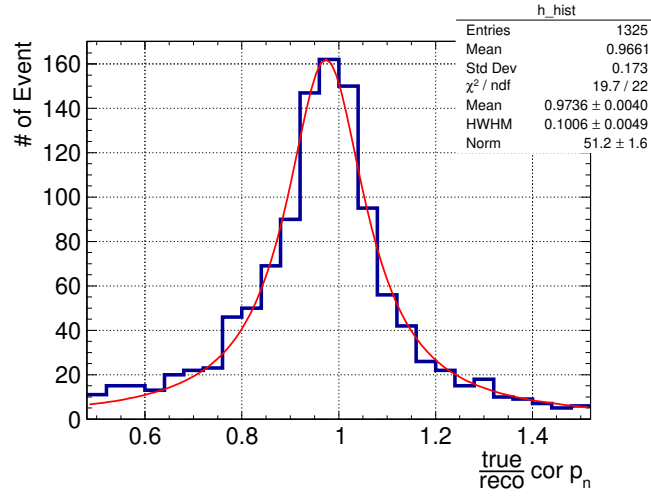


Figure 6.3: p_N distribution for the SFGD- μ sub-sample after ESC selection. It shows an excellent resolution of 10%.

6.1.2 $\nu_\mu \text{CC}1\pi1p$ selection

The $\nu_\mu \text{CC}1\pi1p$ selection uses the pion trackless reconstruction technique as elaborated in Sec. 5.2 for the $\nu_\mu \text{CC}1\pi\text{-TL}$ selection. Additionally, it requires the reconstruction of one ESC proton. Hence, this sample uses both novel techniques in Chapter 5, showcasing the potential of the upgraded ND280 detector in hadronic measurements.

The TKI variables for the $\nu_\mu \text{CC}1\pi1p$ selection are shown in Fig. 6.4, with even better resolution than the $\nu_\mu \text{CC}0\pi1p\text{-ESC}$ selection. For example, Fig. 6.4d shows a fitted HWHM of 12%, significantly better than either of the sub-samples shown in the previous section. This improvement could be due to a purer sample due to the requirement of the presence of the extra pion. However, considerable underestimation is observed for δp_T in Fig. 6.4c, which is surprising given the good performance of the other TKI variables. This could be due to a bug in the reconstruction, but as the current reconstruction software has become obsolete, this issue will be further investigated when the new and stable reconstruction becomes available. Nevertheless, the excellent resolutions shown in other TKI variables demonstrate a potential for the upgraded ND280 detector to achieve high-precision TKI measurements. The same workflow of the ESC technique and muon momentum bias correction can be applied

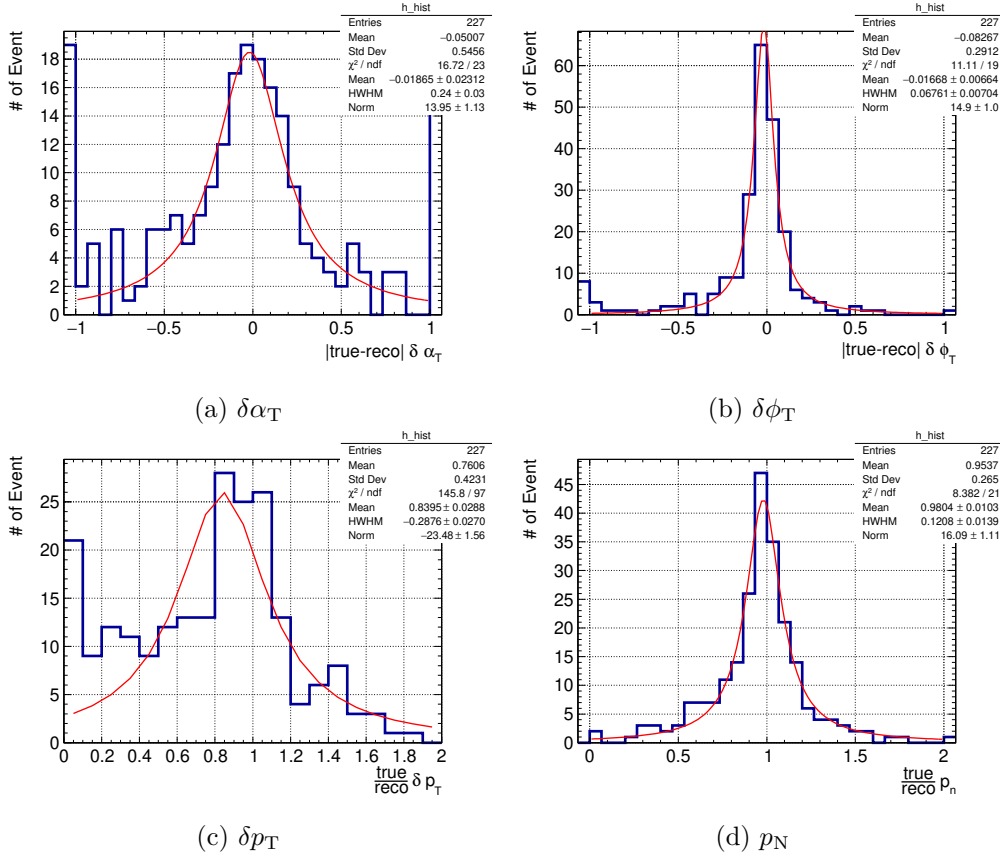


Figure 6.4: Fits of reconstruction resolution for TKI variables for the $\nu_\mu\text{CC}1\pi1p$ selection. All TKI variables except δp_T show good resolutions, for example, the fitted HWHM for p_N is 12%. The δp_T variable shows an unexpected underestimation, which will be further investigated in the future with a stable version of the reconstruction software.

to the future software, while keeping in mind of the unexpected behaviour in δp_T .

In addition to the single transverse variables presented above, the $\nu_\mu\text{CC}1\pi1p$ sample can also be used to measure the double transverse variable, δp_{TT} , the net momentum transverse to both the neutrino momentum and the muon momentum. Without FSI, δp_{TT} should be 0. This special property can be exploited to select ν -H events amidst ν -C events as suggested in Ref. [55]. Hence, the main figure of merit for δp_{TT} measurement is the width of its distribution for ν -H events, where the hydrogen events are picked by an additional check on the MC truth information. As shown in Fig. 6.5, the fitted half width is about 12 MeV/c, which is significantly improved compared to 21 MeV/c, the pre-upgrade ND280 result [55], suggesting a greater potential to apply it for a hydrogen sample selection, which will be discussed at the end of this chapter in Sec. 6.3. However, the final sample size of the hydrogen

sample, 31 events, is statistically limited, so the practical improvement of the δp_{TT} measurement needs to be further assessed with more simulation and data.

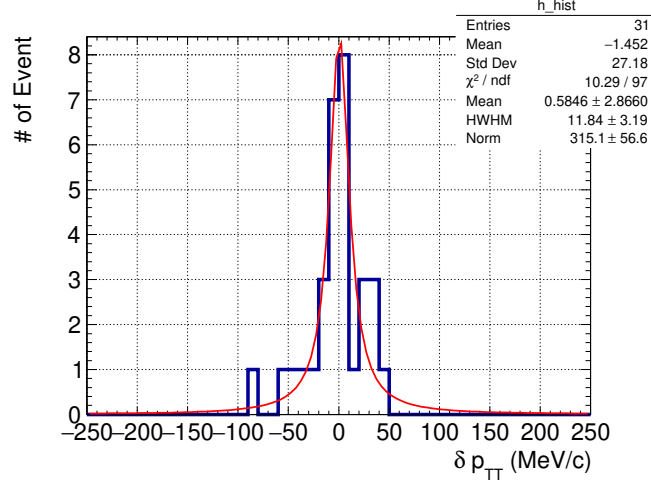


Figure 6.5: δp_{TT} distribution for ν -H events selected from the $\nu_\mu \text{CC}1\pi1\text{p}$ sample based on true information. The resolution is approximately 12 MeV/c, a considerable improvement from the pre-upgrade ND280 result of 21 MeV/c [55]. However, the sample size is statistically limited, so the concrete improvement requires further investigation with higher statistics.

6.2 The COM Variables

When a neutrino has sufficient energy, it can excite a nucleon into a resonance state, for example, the Δ^{++} . This process is referred to as a resonance (RES) interaction. Since Δ^{++} is one of the most commonly observed resonances in neutrino experiments, this work focuses on it as an example. Nevertheless, the concept presented here is equally applicable to other resonances, and the methodology can be easily generalized.

The resonance decays rapidly, before leaving the nucleus, via the process

$$\Delta^{++} \rightarrow \pi^+ + \text{p}. \quad (6.1)$$

In the Δ^{++} rest frame, as illustrated in the top left of Fig. 6.6, the kinematics of this two-body decay are well-defined, and the proton and pion are emitted back-to-back. The pion decay angle, $\theta_{\pi\Delta}$, is defined as the angle between $\vec{p}_\pi^{(0)}$ and the x -axis, which is taken to align with $\vec{p}_\Delta$, the momentum of Δ^{++} in the

lab frame. $\theta_{\pi\Delta}$ is a resonance property that follows an underlying distribution defined by various models [51–53, 77].

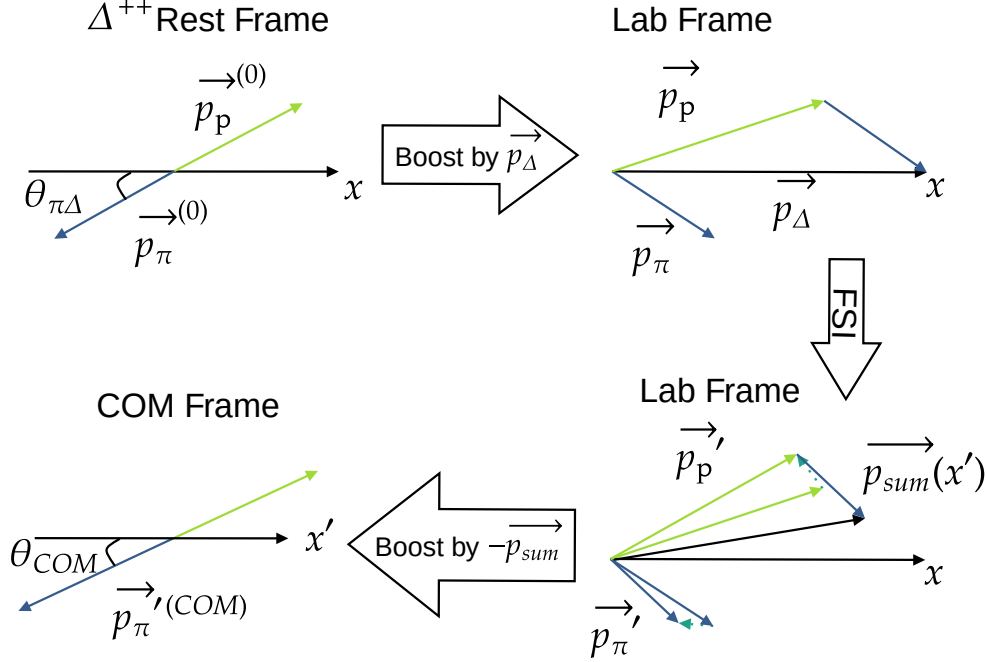


Figure 6.6: Schematic illustration of the COM angle. Without FSI, $\vec{p}_{\text{sum}} = \vec{p}_{\Delta}$ and the lab frame and the Δ^{++} rest frame can be transformed into each other by $\vec{p}_{\Delta}$. With FSI, $\vec{p}_{\text{sum}} \neq \vec{p}_{\Delta}$, the Δ^{++} rest frame is not accessible, but the lab frame can be boosted into the COM frame using $\vec{p}_{\text{sum}}$. Hence, the major difference of the COM frame from the Δ^{++} rest frame is caused by FSI.

The kinematics in the lab frame are related to those in the Δ^{++} rest frame by a boost with $\vec{p}_{\Delta}$, as depicted in the top right of Fig. 6.6. Without FSI, $\vec{p}_{\Delta}$ equals the sum of $\vec{p}_p$ and $\vec{p}_{\pi}$. However, the target materials of modern day detectors are mainly comprised of nuclei with multiple nucleons and FSI alters the kinematics of the hadrons considerably, as illustrated by the dotted arrows in the bottom right of Fig. 6.6.

The altered momenta, $\vec{p}'_p$ and $\vec{p}'_{\pi}$, are the ones measured by detectors. Their sum generally differs from $\vec{p}_{\Delta}$. Thus, the Δ^{++} rest frame, with its simple kinematic relations, becomes inaccessible. Nevertheless, the system can be boosted to the proton-pion COM frame using $\vec{p}_{\text{sum}}$, as depicted in the bottom left of Fig. 6.6, where the x' -axis is aligned with the $\vec{p}_{\text{sum}}$ direction in the lab frame. Similarly, a pion

decay angle, θ_{COM} , can be defined between $\vec{p}_{\pi}^{(\text{COM})}$, the pion momentum in the COM frame, and the x' -axis. Due to FSI, θ_{COM} typically differs from $\theta_{\pi\Delta}$.

The COM frame coincides with the Δ^{++} rest frame only in the absence of FSI. Thus, the strength of FSI dictates the deviation of θ_{COM} from $\theta_{\pi\Delta}$. In practice, the measured θ_{COM} serves as a probe for studying FSI. $\theta_{\pi\Delta}$, being a rest-frame property of Δ^{++} , is independent of the resonance's momentum, neutrino energy, and initial state (IS). In neutrino event generators, θ_{COM} deviates from $\theta_{\pi\Delta}$ only due to FSI, which is independent of neutrino energy and IS. Therefore, θ_{COM} retains these important independencies. As different resonances could have different decay properties, $\theta_{\pi R}$, the similarly defined pion decay angle of a higher resonance R , generally differs from $\theta_{\pi\Delta}$. Hence, θ_{COM} , a superposition of $\theta_{\pi\Delta}$ and all possible $\theta_{\pi R}$, will deviate from $\theta_{\pi\Delta}$, when more resonances become energetically possible with an increase in neutrino energy. This deviation will be further elaborated in Sec. 6.2.2.

Moreover, the total energy in the COM frame, E_{COM} , will be equal to the mass of the resonance in the absence of FSI. Cutting on events with total energy far from the rest mass peak of the resonance will select events with minimal FSI effects, such as ν -H events. Also, an upper cut on E_{COM} , e.g. $E_{\text{COM}} < 1330$ GeV, can help to select events with a Δ^{++} resonance, as the Δ^{++} rest mass is 1232 MeV. The application of E_{COM} for hydrogen event selection will be discussed in Sec. 6.3.

Up to this point, the discussion of resonance decay has been entirely general and can thus be applied to and validated by various experiments, including hadron scattering measurements. However, there are features specific to neutrino experiments. First, the resonance is generated through neutrino-nucleon interactions, which involve both vector and axial currents. Second, the interaction occurs within the nuclear medium. This section focuses on the latter, with the application of COM variables to the former is better elaborated in Sec. 6.3.

While the COM angle may appear conceptually similar to the reconstructed Adler angle, θ_{Adt} , in neutrino experiments [78], there are significant differences. Notably, the COM frame is reconstructed exclusively from hadronic kinematics, whereas the reconstructed Adler frame relies on leptonic kinematics and assumes

stationary nucleons. Consequently, the reconstruction of the Adler frame is implicitly influenced by IS effects, whereas the COM frame’s hadronic variables are impacted by final state interactions (FSI). Therefore, θ_{Adt} , being a hadronic variable in the Adler frame, is influenced by both IS and FSI. Additionally, the necessity of reconstructing the neutrino energy renders θ_{Adt} sensitive to neutrino flux uncertainties, which are among the largest systematic uncertainties in neutrino measurements [60, 79–82].

6.2.1 Analysis result

In the absence of existing measurements of the COM angle, MC studies were conducted to evaluate its potential advantages—namely, robustness against IS effects and sensitivity to FSI effects. MC samples were generated using **GENIE** [83, 84] for the T2K beam and target, unless otherwise noted. Several **GENIE** tunes are employed in this section, and their model details are summarized in Table 6.1. Each sample consists of 600,000 muon neutrino events. For the T2K flux on carbon, approximately 70% of the events are charge-current (CC) events, of which on average 16% are CC single pion ($\nu_\mu \text{CC}1\pi$) events and 9% are CC single pion single proton ($\nu_\mu \text{CC}1\pi1p$) events. A selection of $\nu_\mu \text{CC}1\pi1p$ events was used to generate the plots. Based on preliminary MC studies of the T2K upgraded near detector, approximately 1.8×10^{21} POT will yield this number of events, corresponding to less than one year of data taking in the ν_μ mode with the J-PARC beam upgrade [85]. A few plots were generated using the MINERvA low-energy (LE) flux, which has a neutrino energy peak around 3.5 GeV. The collected MINERvA LE data contain approximately 300,000 ν_μ events—half the size of the simulated event sample used in this analysis—and are therefore used only as a proof of concept. Although the MINERvA flux used in this analysis is the LE flux, similar performance is expected for the medium-energy flux, which has a neutrino energy peak at 6 GeV and provides more than 10 times the data.

The most stringent test of θ_{COM} with respect to IS effects is to compare the θ_{COM} distributions for ν -H and ν -C events when FSI is disabled. The ν -H interaction is free from any nuclear effects, whereas ν -C events without FSI reflect only the

CMC	IS	QE	2p2h	RES	FSI
G18-01a	RFG	LS	Dytman	RS	hA
G18-02a	RFG	LS	Dytman	BS	hA
G18-10a	LFG	Valencia	Nieves	BS	hA
G18-10b	LFG	Valencia	Nieves	BS	hN
G24-0	SF-CFG	Valencia	SuSAv2	BS	hA
G24-c	SF-CFG	Valencia	SuSAv2	BS	hA(tuned)

Table 6.1: The full tune names are: 1) G18-01a: G18_01a_02_11b; 2) G18-02a: G18_02a_02_11b; 3) G18-10a: G18_10a_02_11b; 4) G18-10b: G18_10b_02_11b; 5) G24-0: G24_20i_00_000; 6) G24-c: G24_20i_06_22c. The four nuclear IS models are relativistic Fermi gas (RFG), local Fermi gas (LFG), spectral-function-like CFG (SF-CFG) [84, 86, 87]. The respective cross-section models are: 1) Quasielastic (QE) - Llewellyn-Smith (LS) [88] and Valencia [89]; 2) 2 particle 2 hole (2p2h) - Dytman [90], Nieves [91] and SuSAv2 [92]; 3) Resonance(RES) - Rein-Sehgal (RS) [93] and Berger-Sehgal (BS) [94]. The FSI models used are the built-in GENIE models [3]: INTRANUKE hA and hN. Note that in G24_20i_06_22c [95], the hA model has been tuned using TKI data, resulting in parameters that differ from the default version of hA.

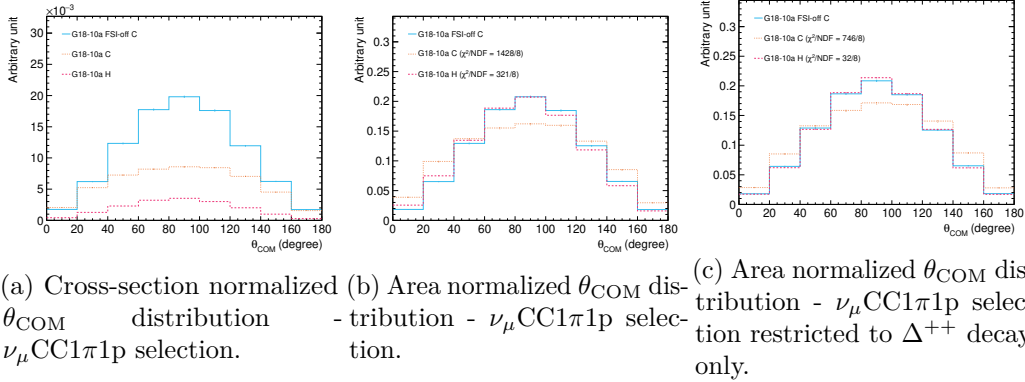


Figure 6.7: Cross-section normalized (6.7a) and area normalized (6.7b and 6.7c) comparisons for FSI on/off for carbon and hydrogen with G18-10a for θ_{COM} using the T2K flux. (6.7c) has a more stringent selection to restrict the resonance to be Δ^{++} .

impact of IS, including effects such as carbon removal energy and Fermi motion. FSI effects can be examined by comparing the θ_{COM} distributions of ν -C events obtained with FSI enabled versus disabled. When FSI is enabled, two of the major nuclear effects—both IS and FSI—is present. The cross-section-normalized θ_{COM} distributions for all three types of events are shown in Fig. 6.7a.

As illustrated in Fig. 6.7a, enabling or disabling FSI has a significant effect on the θ_{COM} cross-section. The θ_{COM} distributions of the two ν -C (FSI-on and FSI-off) samples are considerably larger than that of ν -H, owing to the larger number of

nucleons. Given that the event selection is based on the topology (i.e. $\nu_\mu\text{CC}1\pi1\text{p}$), enabling FSI can lead to a considerable decrease in the cross section via pion absorption and via pion-induced pion production. Pion charge exchange can also convert π^+ to π^0 , thereby reducing the cross section; however, this type of FSI has a negligible impact for the T2K flux [95]. Readers can refer to the literature (e.g. Ref. [96]) for further demonstration of the impact of FSI on the cross section.

To examine the detailed effect of IS on θ_{COM} , the cross-section shape of $\nu\text{-C}$ events (with FSI enabled and disabled) is compared to that of $\nu\text{-H}$ in Fig. 6.7b. To better quantify the shape differences, χ^2/NDF values (where NDF stands for the number of degrees of freedom) are calculated for all shape-comparison plots. Each χ^2/NDF value is calculated with respect to the first curve in its respective plot, accounting only for statistical uncertainties. As shown by the “G18-10a FSI-off C” and “G18-10a C” curves in Fig. 6.7b, enabling FSI leads to a drastic change in the cross-section shape. In contrast, the $\nu\text{-H}$ and $\nu\text{-C}$ (FSI-off) curves are much closer to each other. This bolsters the claim that FSI has a much stronger impact on the θ_{COM} distribution than do IS effects. Nonetheless, the FSI-off $\nu\text{-C}$ distribution remains considerably different from the $\nu\text{-H}$ distribution, with a large χ^2/NDF value of 321/8. If the comparison is restricted to events from Δ^{++} decay only (based on true information), as shown in Fig. 6.7c, the difference between the FSI-off $\nu\text{-C}$ and $\nu\text{-H}$ θ_{COM} distributions is significantly reduced, with a χ^2/NDF of 32/8, suggesting that these distributions are almost statistically compatible. However, the large χ^2/NDF observed for $\nu_\mu\text{CC}1\pi1\text{p}$ indicates that even without FSI, using carbon instead of hydrogen as the target introduces complexities beyond a simple change in the nucleon kinematic distribution. It is challenging to pinpoint the exact cause of this large difference, which should be reserved for investigation in future studies.

Because comparing FSI-off carbon with FSI-off hydrogen appears to be too aggressive a test to isolate the impact of IS modelling, a more reasonable approach is to compare different IS models while keeping other factors fixed. The simulation results for the default G18-10a and its variant using the correlated Fermi gas (CFG) model are shown in Fig. 6.8a. The χ^2/NDF value of 18/8 between G18-10a

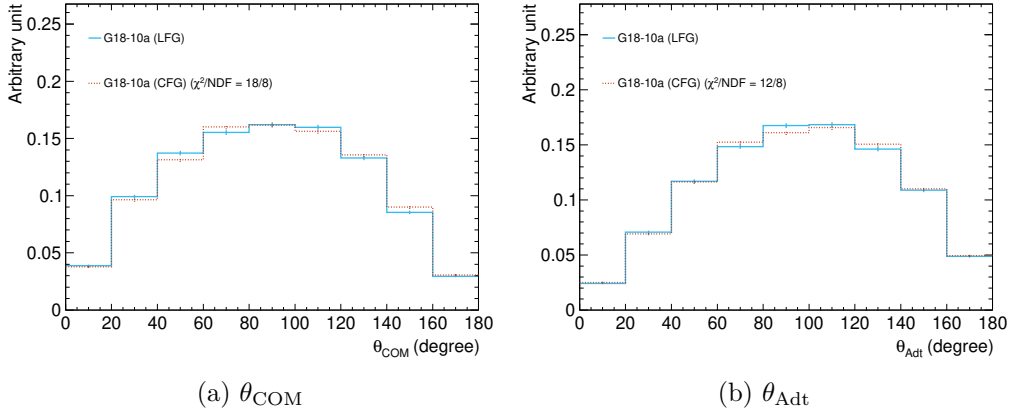


Figure 6.8: Area normalized distributions for different IS models, LFG (default) and CFG, with the T2K flux on carbon. The nominal flux is G18-10a.

(LFG) and G18-10a (CFG) is indeed small, suggesting that θ_{COM} is robust against changes in IS models. Since θ_{Adt} is conceptually similar to θ_{COM} but exhibits a larger dependence on IS, it is imperative to compare the θ_{Adt} distributions for G18-10a (LFG) and G18-10a (CFG) as well, as shown in Fig. 6.8b, to assess the impact of the IS model change. Unexpectedly, an even smaller difference between the two curves for θ_{Adt} is observed in Fig. 6.8b compared to Fig. 6.8a, indicating that despite its dependence on IS, changing from LFG to CFG does not distort the shape of θ_{Adt} appreciably.

This observation could challenge the claimed IS insensitivity of θ_{COM} , as the agreement shown in Fig. 6.8a might be due to the relatively minor influence of IS. A further stress test was conducted by varying the removal energy of carbon across a wide range—even to unphysical levels—to assess its effect on the shapes of θ_{COM} and θ_{Adt} . In GENIE, the removal energy is a parameter that describes the energy required to liberate a nucleon from the nucleus. For a given incoming neutrino, a higher removal energy results in less of the energy transferred to the nuclear reaching the final state hadronic system. The results of this comparison are presented in Fig. 6.9. As shown in Fig. 6.9a, variations in the removal energy (E_{rm}) have very limited impact on θ_{COM} . The θ_{COM} distribution remains statistically compatible with the default removal energy of 10 MeV, even when E_{rm} is increased to 90 MeV. When E_{rm} is further increased to 180 MeV, the θ_{COM} distribution

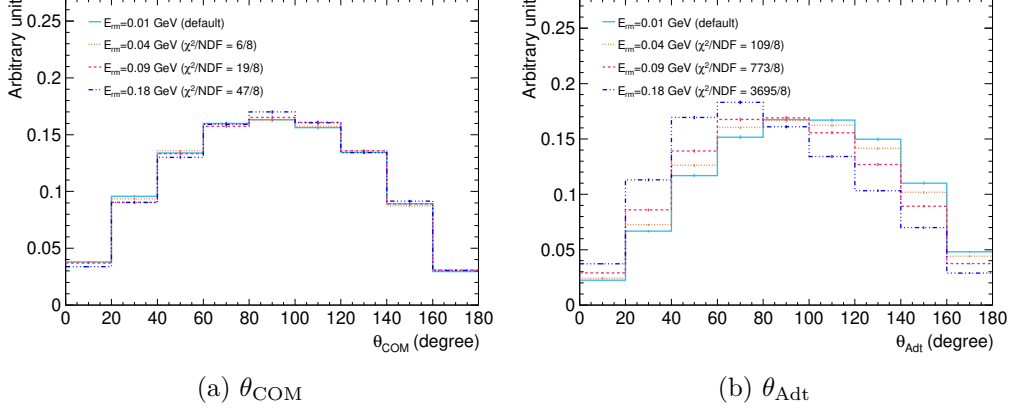


Figure 6.9: Area normalized comparisons for different removal energies, E_{rm} , with the T2K flux on carbon. The nominal tune is G24-0.

begins to deviate from the default case, although the χ^2/NDF remains relatively small at 47/8. In contrast, the θ_{Adt} peak exhibits a noticeable shift accompanied by a gradual change in its shape as E_{rm} varies. For θ_{Adt} , the χ^2/NDF rises to 109/8 when E_{rm} is increased to 40 MeV—a deviation much larger than that observed for θ_{COM} —and further increases to 3695/8 when E_{rm} is raised to 180 MeV, which is unphysically high. These results confirm that θ_{COM} is robust against IS effects to a large extent, whereas θ_{Adt} is more susceptible, as predicted. With the strong robustness of θ_{COM} against IS effects demonstrated relative to θ_{Adt} , the subsequent investigation will focus solely on θ_{COM} .

Another important property of θ_{COM} to verify is its sensitivity to FSI effects. Similar to the investigation of IS effects, the θ_{COM} distributions for different FSI models are shown in Fig. 6.10. There are two commonly used FSI models in GENIE, namely hA and hN. Thus, two GENIE tunes—G18-10a (hA) and G18-10b (hN), which differ only in FSI modeling—are compared in Fig. 6.10a. The θ_{COM} distributions for these two tunes are statistically incompatible, with a large χ^2/NDF value of 64/8, a difference that is significantly more pronounced than the fluctuation induced by changing the IS model from LFG to CFG, as shown in Fig. 6.8a. To further verify the sensitivity of θ_{COM} to FSI effects, a comparison between G24-0 (hA) and G24-c (tuned hA) is presented in Fig. 6.10b. In G24-c [95], the hA model has been tuned to maintain good agreement with the T2K TKI $\nu_\mu \text{CC}1\pi1\text{p}$ data [60],

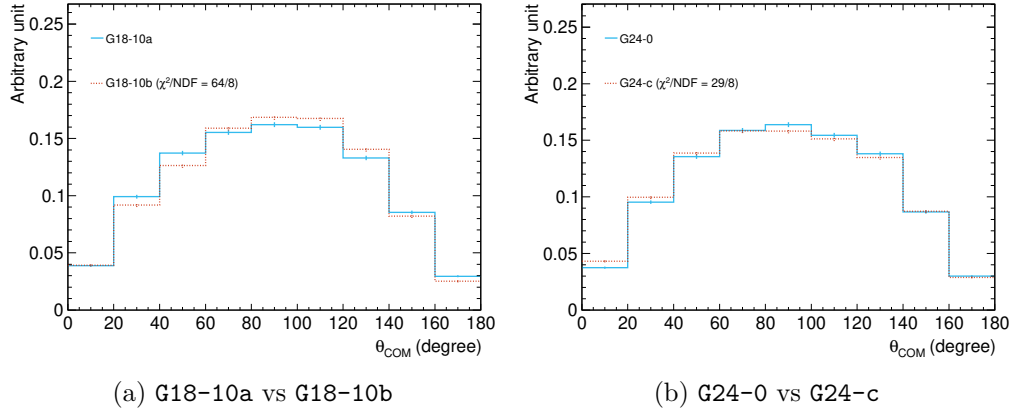


Figure 6.10: Area normalized comparisons for different FSI models, 6.10a for G18-10a (hA) and G18-10b (hN) and 6.10b for G24-0 (hA) and G24-c (tuned hA) with the T2K flux on carbon.

so the considerable difference— $\chi^2/\text{NDF} = 29/8$ —between the two distributions in Fig. 6.10b further demonstrates the sensitivity of θ_{COM} to FSI effects.

The two major advantages of θ_{COM} have been demonstrated, namely its robustness against IS effects and its sensitivity to FSI effects. The next step is to investigate the impact of other factors in neutrino–nucleus interactions on θ_{COM} . Since θ_{COM} is calculated from the decay products of resonances, it can be affected by changes in RES modeling, which dictate the production and decay of these resonances. As a proxy for investigating the effect of a change in the RES model on θ_{COM} , the axial mass parameter, M_A , in the G24-0 tune was varied to an exaggerated—and most likely unphysical—extent, as shown in Fig. 6.11. As anticipated, Fig. 6.11a demonstrates that varying M_A alters the cross-section, while the shape of θ_{COM} remains almost unchanged—as confirmed by the small χ^2/NDF values in Fig. 6.11b. This is likely because Δ^{++} is the dominant resonance for the T2K flux, and changes in M_A do not appreciably alter its decay properties—thereby leaving the θ_{COM} distribution largely unaffected.

To complement this investigation, a similar comparison conducted using the MINERvA flux on the MINERvA target is shown in Fig. 6.12. Since the MINERvA flux is more energetic, higher resonances also make a significant contribution. Changes in M_A can affect the resonances differently, as demonstrated by the W distribution, which represents the invariant mass of the pion-nucleon system

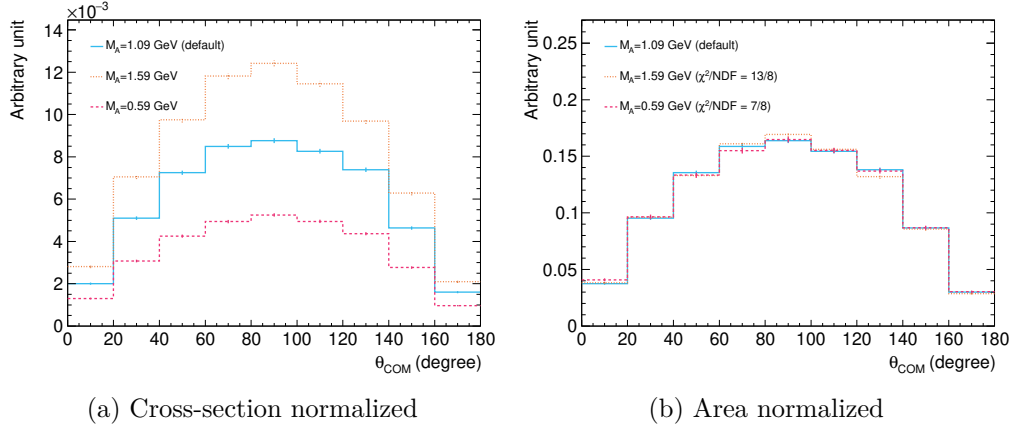


Figure 6.11: Comparisons for different M_A values with the T2K flux on carbon. The nominal tune is G24-0.

estimated from leptonic kinematics. The derivation of W is readily available in the literature, such as in Ref. [97]. As evident in Fig. 6.12a, when M_A decreases to 0.59 GeV, the relative contribution of resonances above 1.8 GeV increases, while the contribution of those below decreases appreciably. In contrast, when M_A increases to 1.59 GeV, the relative contribution of resonances above 1.8 GeV decreases, while the contribution of those below either increases or remains relatively unchanged. Consequently, these changes in M_A lead to considerable differences in the θ_{COM} distributions, with $\chi^2/\text{NDF} = 28/8$ for $M_A = 1.59$ GeV and $\chi^2/\text{NDF} = 44/8$ for $M_A = 0.59$ GeV, as shown in Fig. 6.12c. To minimize the influence of higher resonances, a practical cut, $E_{\text{COM}} < 1330$ MeV, aimed at selecting Δ^{++} events, is imposed on the selection in Fig. 6.12d. As anticipated, the higher resonances in the W distribution have largely disappeared in Fig. 6.12b, and the θ_{COM} distributions for different M_A values in Fig. 6.12c become statistically compatible. This suggests that for an energetic flux such as the MINERvA flux—which produces multiple resonances—the θ_{COM} distribution can still be used to study FSI with minimal impact from RES modelling by imposing an E_{COM} cut to select only Δ^{++} events.

Besides varying a parameter in the RES model, a more aggressive test is to compare two different RES models directly. This comparison is achieved by contrasting two GENIE tunes—G18-01a (employing the RS model) and G18-02a (employing the BS model)—as shown in Fig. 6.13. The results using the T2K flux

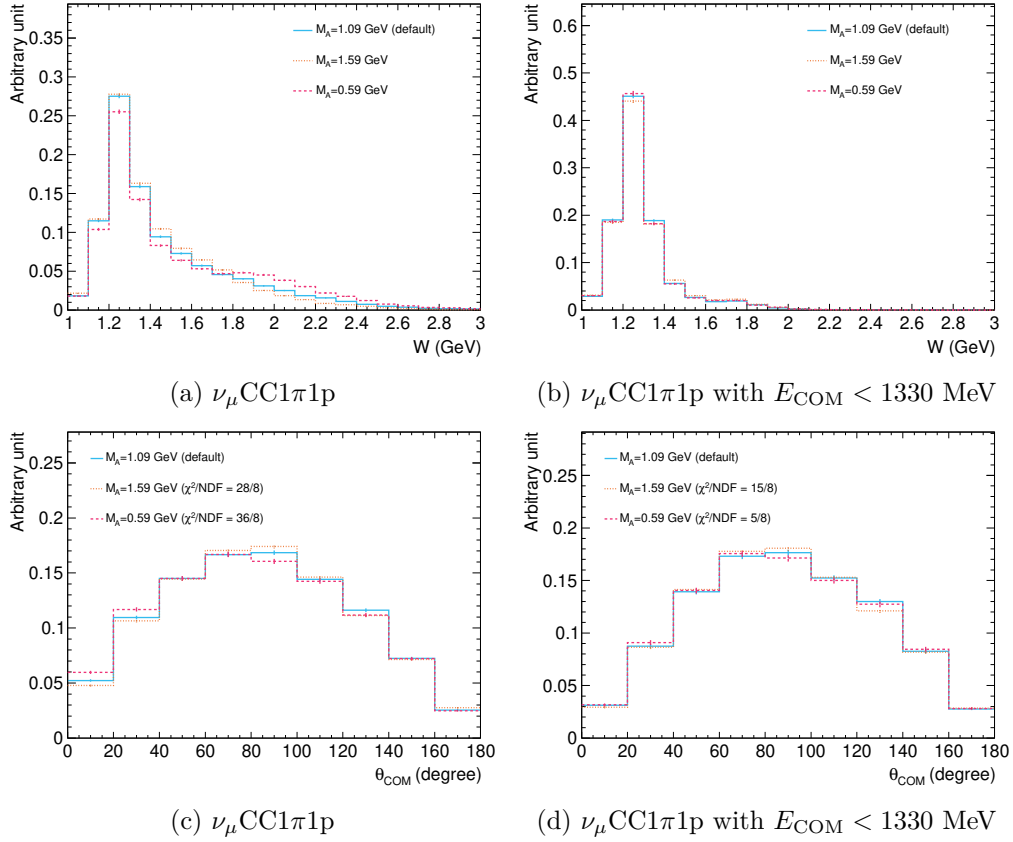


Figure 6.12: Area normalized comparisons of W and θ_{COM} for different M_A values with the MINERvA flux on the MINERvA target. The nominal tune is G24-0.

and the MINERvA flux are presented in Fig. 6.13a and Fig. 6.13b, respectively. Both χ^2/NDF values are low—specifically, 6/8 and 8/8—suggesting that this change in the RES model is far less extreme than the unphysically large modification of M_A shown in Fig. 6.11 and that such a reasonable change in RES modelling does not lead to a noticeable alteration of the θ_{COM} distribution.

On one hand, restricting the analysis to Δ^{++} events—by imposing an E_{COM} cut—can minimize the impact of RES modeling on θ_{COM} even for an energetic beam like the MINERvA flux. On the other hand, this behaviour implies that θ_{COM} is sensitive to the onset of higher resonances. Given that the robustness of θ_{COM} against both IS effects and considerable changes of RES modelling have been demonstrated, the difference between the θ_{COM} distributions with and without the E_{COM} cut (as shown in Fig. 6.12) is most likely attributable to the onset of higher resonances. This advocates employing θ_{COM} to study both the production of

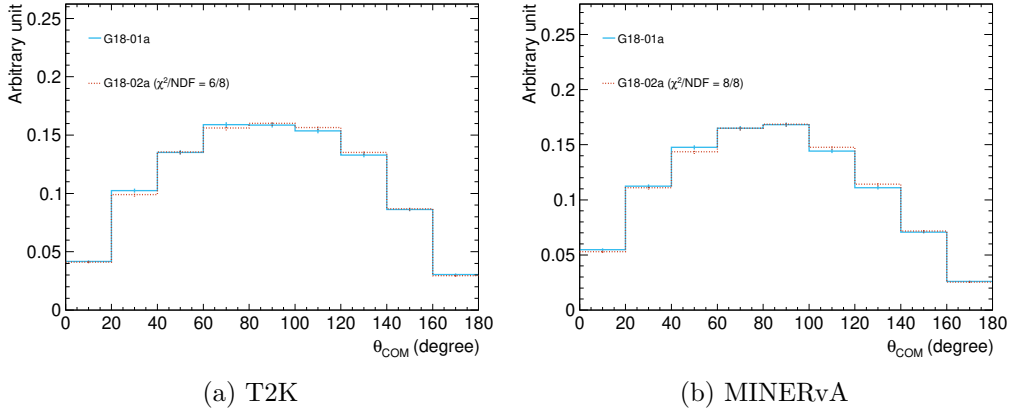


Figure 6.13: Area normalized comparisons for different RES models with the T2K (6.13a) and MINERvA (6.13b) fluxes using G18-01a and G18-02a. Both select $\nu_\mu \text{CC}1\pi1p$ events, but the MINERvA sample has a additional cut of $E_{\text{COM}} < 1330$ MeV.

higher resonances and the correlations among their production. Since the BS model in **GENIE** does not implement resonance correlations, this potential application of θ_{COM} is reserved for future studies—when more advanced models, such as the MK model [51–53], are reliably implemented and validated.

Besides its appealing sensitivity and robustness, θ_{COM} has the practical advantage that its reconstruction does not require determining the kinematics of the incoming neutrino. This decouples the reconstruction of θ_{COM} from the systematic uncertainties associated with neutrino energy reconstruction—which are among the largest in neutrino measurements [60, 79–82]. However, the reconstruction of θ_{COM} still depends on the accurate reconstruction of proton kinematics, which is itself a significant source of systematic uncertainty. The overall impact on precision for θ_{COM} measurements will be investigated in future studies for specific experiments.

To assess the impact of E_ν on θ_{COM} , mono-energy neutrino beams are used to simulate ν -H events using tune G24-0, as shown in Fig. 6.14. Hydrogen is chosen as the target to isolate the impact solely due to E_ν , while complications arising from FSI will be discussed later.

In Fig. 6.14a, although the shapes of the θ_{COM} distributions appear largely consistent for E_ν values between 0.5 GeV and 2.0 GeV, they are statistically incompatible—with χ^2/NDF values on the order of 100/8. This inconsistency is likely due to the onset of higher doubly positive resonances. While θ_{COM} for a

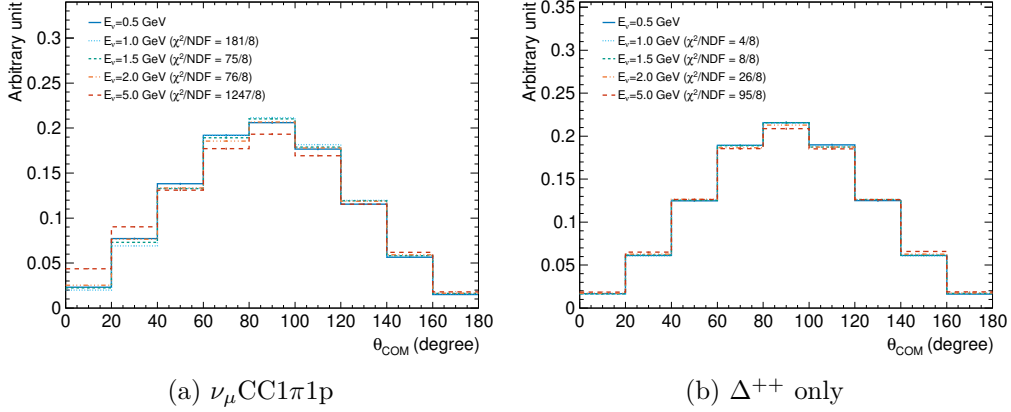


Figure 6.14: Area normalized comparisons for different E_ν fluxes on hydrogen for θ_{COM} . The nominal tune is G24-0.

single resonance is largely independent of the production mechanism—unless a strong correlation exists between production and decay—it will be affected when the relative contributions of different resonances vary due to their distinct decay properties. If the analysis is restricted to Δ^{++} events (based on true information), as in Fig. 6.14b, it is encouraging to observe that all E_ν distributions—except for $E_\nu = 5.0$ GeV—become compatible with much smaller χ^2/NDF values, thereby confirming the independence of θ_{COM} from E_ν for a single resonance.

In the case of $E_\nu = 5.0$ GeV, much more energetic Δ^{++} resonances are produced, which enhance the influence of Δ^{++} kinematics on their decay and tend to favor either very small or very large pion angles. Since such highly energetic neutrinos contribute only marginally to actual cross-section measurements, this effect is not expected to pose a significant challenge for the practical application of θ_{COM} in neutrino analyses. A comprehensive explanation of this observation is beyond the scope of this work; instead, a more detailed investigation of the correlation between resonance production and decay is warranted in future studies.

In the resonance rest frame, the pion decay angle is an intrinsic property of the resonance and should be largely independent of its kinematics—and consequently, of E_ν . However, in the lab frame the boost experienced by the decay products results in different hadronic kinematic distributions. Since FSI depends on the hadronic kinematics, the measured kinematics—and hence θ_{COM} —are indirectly

affected by E_ν . Therefore, although the reconstruction of θ_{COM} for an individual event is independent of E_ν , the overall θ_{COM} distribution may vary due to changes in the neutrino energy spectrum when FSI is present. To assess this impact in practice, mono-energetic neutrino beams are used to simulate ν -C events using tune G24-0, and the results are compared to those obtained with the T2K flux, as shown in Fig. 6.15.

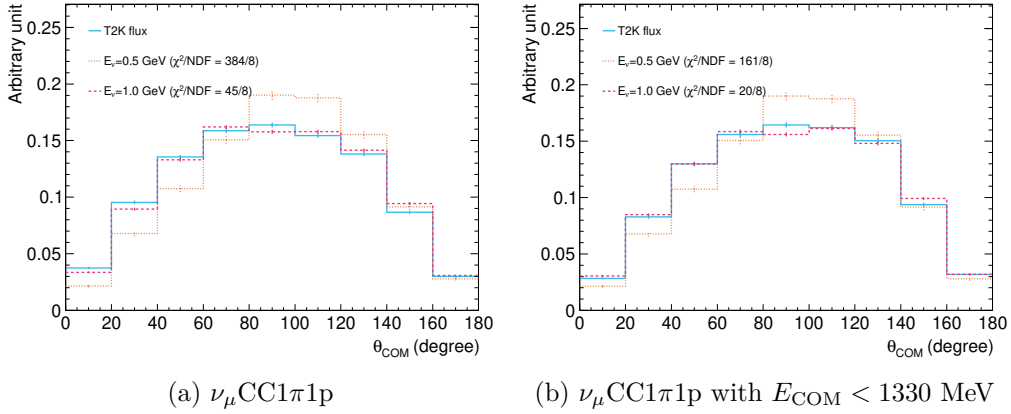


Figure 6.15: Area normalized comparisons for different E_ν fluxes and the T2K flux on carbon for θ_{COM} . The nominal tune is G24-0.

As expected, variations in hadronic kinematics caused by different E_ν values lead to noticeable changes in the θ_{COM} distribution, as illustrated in Fig. 6.15a. This effect is more pronounced for $E_\nu = 0.5$ GeV, where low-momentum pions—the decay products of low-momentum Δ^{++} resonances resulting from low-energy neutrinos—are more likely to be absorbed through FSI. Thus, even with an $E_{\text{COM}} < 1330$ MeV cut imposed, the θ_{COM} distribution for $E_\nu = 0.5$ GeV remains incompatible with that for the T2K flux, whereas the $E_\nu = 1.0$ GeV distribution becomes relatively compatible with the T2K flux, exhibiting a χ^2/NDF value of 20/8. This implies that even in the extreme case where the T2K neutrino energies are misestimated to be uniformly 1 GeV, the resulting θ_{COM} distribution would remain consistent. This finding strongly corroborates the practical robustness of θ_{COM} against E_ν variations, provided that an $E_{\text{COM}} < 1330$ MeV cut is applied for the T2K flux. A similar level of robustness should be attainable for other fluxes, although flux-specific cuts may be necessary; this remains an avenue for future investigation.

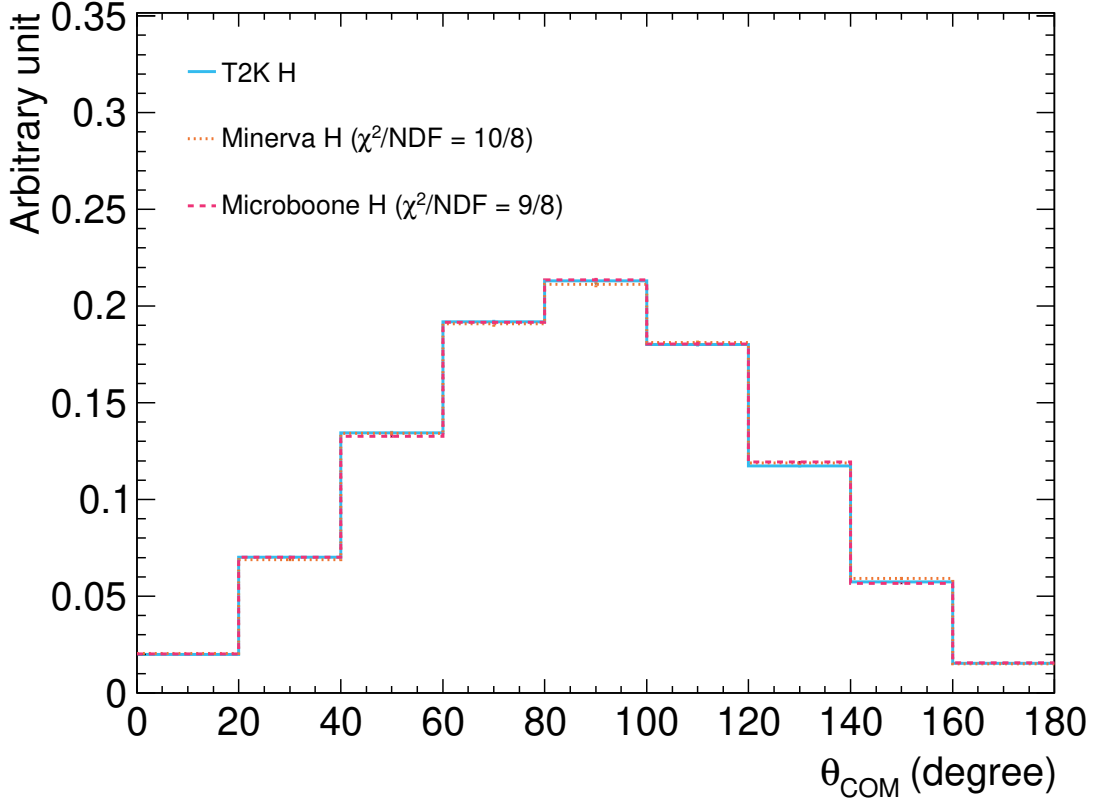


Figure 6.16: Area normalized comparisons for different fluxes on hydrogen with the $E_{\text{COM}} < 1330$ MeV cut imposed for MINERvA. The nominal tune is G24-0.

The observed robustness of θ_{COM} against E_ν variations in ν -H events—across a wide range of E_ν with an $E_{\text{COM}} < 1330$ MeV cut, as seen in Fig. 6.14—opens up a new avenue for cross-experiment comparisons. As depicted in Fig. 6.16, the θ_{COM} distributions for the T2K, MINERvA, and MicroBooNE¹ fluxes on a hydrogen target using tune G24-0 are highly statistically compatible, with χ^2/NDF values of 10/8 and 9/8, respectively. The advantage of a ν -H selection is that it avoids the complications introduced by FSI affecting the hadronic kinematics in the lab frame, as illustrated in Fig. 6.15. Consequently, despite differing energy profiles, the neutrino fluxes from these experiments yield nearly identical θ_{COM} distribution shapes. In practice, MINERvA [98] has already conducted a ν -H selection, and the T2K collaboration is planning similar analyses [55], thereby demonstrating the feasibility of such cross-experiment comparisons in the future.

¹There is no hydrogen in the target for MicroBooNE, but such a simulation is possible and plotted for reference only.

Given a high-purity hydrogen sample, the θ_{COM} cross section can be directly compared across different experiments.

Thus far, the discussion has primarily focused on the insensitivity of θ_{COM} to variations in one aspect (excluding FSI) of the complex neutrino–nucleus interactions. To further demonstrate the robustness of θ_{COM} , a comparison among different **GENIE** tunes is presented in Fig. 6.17, in which multiple components of the neutrino–nucleus interactions vary concurrently.

Notably, the configurations differing in their FSI model—**G24-c** (tuned hA) and **G18-10b** (hN)—exhibit the largest χ^2/NDF values, 29/8 and 58/8, respectively. **G24-c** differs from **G24-0** only in its FSI model, as discussed in Fig. 6.10b. In addition to differing in the FSI model, **G18-10b** also differs from **G24-0** in its IS and 2p2h models. In contrast, **G18-10a**—which shares the same FSI model as **G24-0** but differs in IS and 2p2h models in a manner similar to **G18-10b**—exhibits a much smaller χ^2/NDF value of 13/8, providing strong evidence that FSI is the primary reason for the large deviation observed in **G18-10b** relative to **G24-0**.

Furthermore, **G18-01a** and **G18-02a** differ from **G24-0** in additional model components, yet both exhibit relatively small χ^2/NDF values. In particular, **G18-02a** is statistically compatible with **G24-0**—with a χ^2/NDF of 13/8—despite differences in its IS, QE, and 2p2h models. While a change in the RES model alone does not impact θ_{COM} , as illustrated by the comparison between **G18-01a** and **G18-02a** in Fig. 6.13, the combined modifications in QE, IS, and 2p2h models cause **G18-01a** to deviate further from **G24-0**, yielding a χ^2/NDF of 24/8.

This comparison between different **GENIE** tunes demonstrates that θ_{COM} is most sensitive to changes in FSI while remaining robust against variations in other model components. A change in the RES model combined with modifications in other components can result in a noticeable alteration of the θ_{COM} distribution, whereas a change in the RES model alone does not appear to have a significant impact.

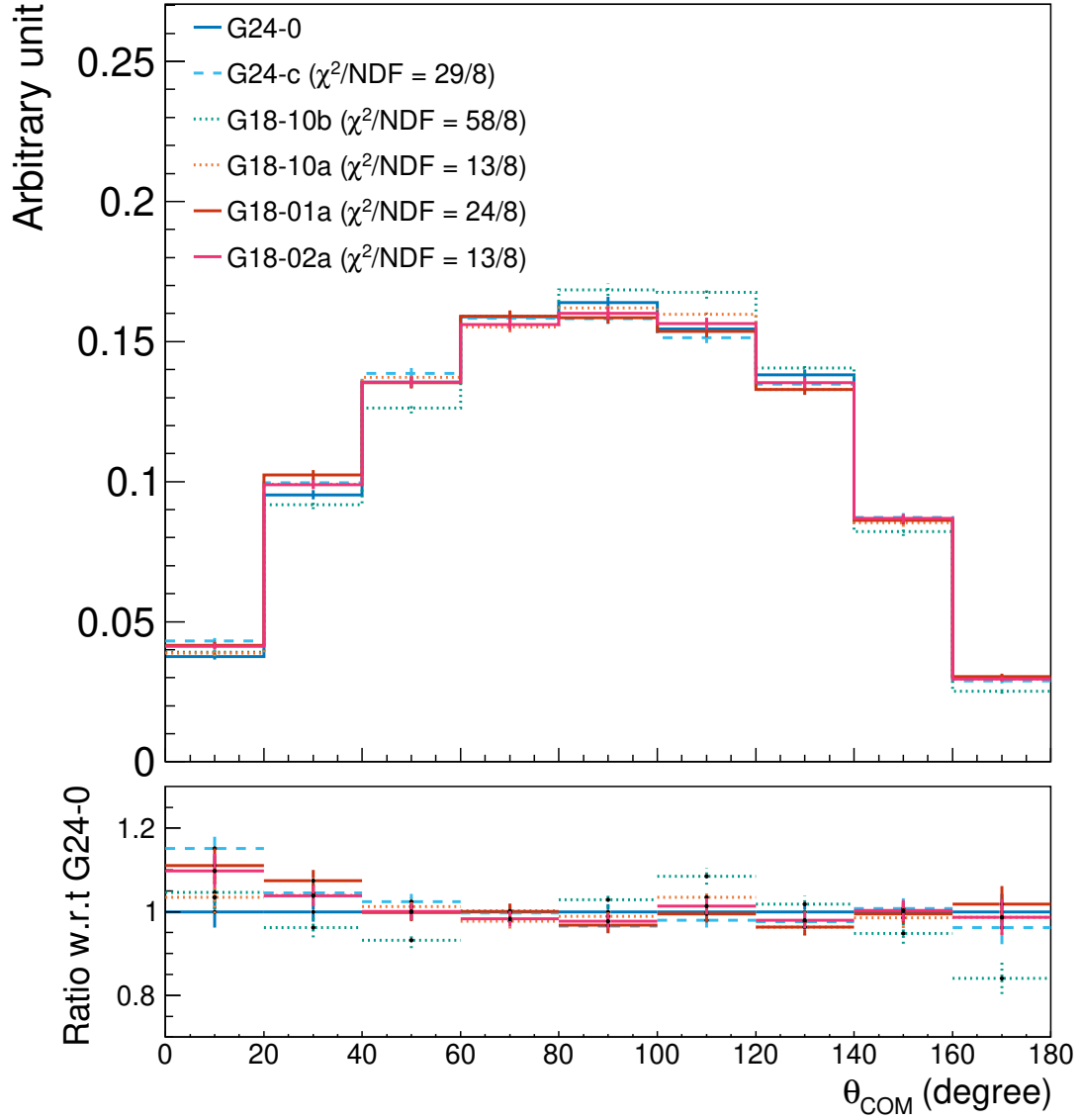


Figure 6.17: θ_{COM} distribution comparisons using the T2K flux on carbon with multiple GENIE configurations.

6.2.2 Discussion

The simulation studies in this work demonstrate that the COM angle is a novel variable with multiple advantages, and its measurement will be valuable for a focused study on FSI with minimal influence from other aspects of neutrino–nucleus interactions. Models constrained by θ_{COM} measurements—e.g., following the methodology in Ref. [99]—will remain relevant even if other aspects of neutrino–nucleus

interactions, in particular the IS model, are updated in the future. This is an advantage that variables—such as θ_{Adt} , which depend on multiple processes of neutrino–nucleus interactions—do not possess. Furthermore, the analyses presented are based on true information without realistic fluctuations, meaning that challenges related to reconstruction—particularly those involving neutrino energy for the Adler angle—are not accounted for, whereas θ_{COM} exhibits minimal dependence on neutrino energy reconstruction for T2K, as shown in Fig. 6.15b. Therefore, the full potential of the COM angle will be better realized in a cross-section measurement that accounts for these complexities.

Although the focus of the θ_{COM} analysis is on nuclear effects, it is inherently affected by resonance interaction modelling. Fortunately for T2K, resonance production is dominated by the Δ^{++} , thereby rendering the Δ^{++} measurement at T2K resistant to changes in RES modelling, as shown in Fig. 6.11 and Fig. 6.13a. In contrast, the more energetic MINERvA flux produces multiple resonances, and the θ_{COM} measurement is more sensitive to changes in RES modelling, as shown in Fig. 6.12. This sensitivity to RES modelling can be managed or even exploited in different ways.

Firstly, imposing a cut on E_{COM} to select Δ^{++} events can reduce this sensitivity, as shown in Fig. 6.12d and Fig. 6.13b, and under this condition, the θ_{COM} measurement can be used for focused studies of FSI, similar to the T2K case. Secondly, comparing the θ_{COM} distributions with and without the E_{COM} cut can be used to study the onset of higher resonances and the correlations among different resonances.

As noted at the beginning of Sec. 6.2, the θ_{COM} distribution is influenced by the production of higher resonances that decay via the same channels as Δ^{++} . Due to the general difference between $\theta_{\pi\text{R}}$ and $\theta_{\pi\Delta}$, θ_{COM} will start to deviate from $\theta_{\pi\Delta}$ (smeared by FSI), as shown in Fig. 6.14a. Although θ_{COM} is inspired by $\theta_{\pi\Delta}$, it is not restricted to the reconstruction or estimation of $\theta_{\pi\Delta}$. Rather, by definition, θ_{COM} is a superposition of all energetically accessible $\theta_{\pi\text{R}}$, with $\theta_{\pi\Delta}$ as the dominant component at low energy. If all $\theta_{\pi\text{R}}$ values are relatively well understood, a simple, theoretically

motivated expectation of θ_{COM} can be obtained by summing the contributions from all possible resonances according to their relative production ratios.

This approach is complicated further by the fact that even with a complete understanding of the FSI model, the predicted θ_{COM} distribution may deviate from measurements due to correlations among different resonance production modes; that is, the overall θ_{COM} is not simply a linear combination of the individual $\theta_{\pi\text{R}}$ contributions. Modelling of such correlations is currently absent in the BS model but is accounted for in the MK model [51–53]. Therefore, when the MK model is implemented and validated in the future, θ_{COM} can be used to study these resonance correlations. This is particularly important for the DUNE experiment, where resonance production plays a major role.

One limitation of using θ_{COM} is that the FSI effects causing variations in the measured hadronic kinematic distributions are entangled with resonance effects. Hence, for the most sensitive study of resonance correlations, ν -H selections are most suitable, as shown in Fig. 6.14.

Furthermore, θ_{COM} measurements based on ν -H selections can be compared directly between different fluxes, as shown in Fig. 6.16. If high-purity ν -H selections are achieved, θ_{COM} measurements can provide a valuable pivot for cross-experiment comparison.

The importance of a high-purity ν -H sample cannot be overstated. For example, such events provide a unique opportunity for precise measurements of the axial current—a property unique to neutrinos. These advantages have driven significant efforts to develop techniques for selecting a high-purity ν -H sample, as demonstrated in Refs. [55, 98, 100]. While Refs. [100] and [98] focus on neutrino and antineutrino pion-less events, respectively, Ref. [55] pertains to the same event topology as that used for the COM total energy. With new or future detectors—e.g., the SFGD and the 3DST of DUNE [44]—there is greater potential to implement novel techniques for selecting a high-purity ν -H sample. To this end, the sensitivity of the COM frame to FSI can be leveraged not only to study FSI but also to select events with minimal FSI.

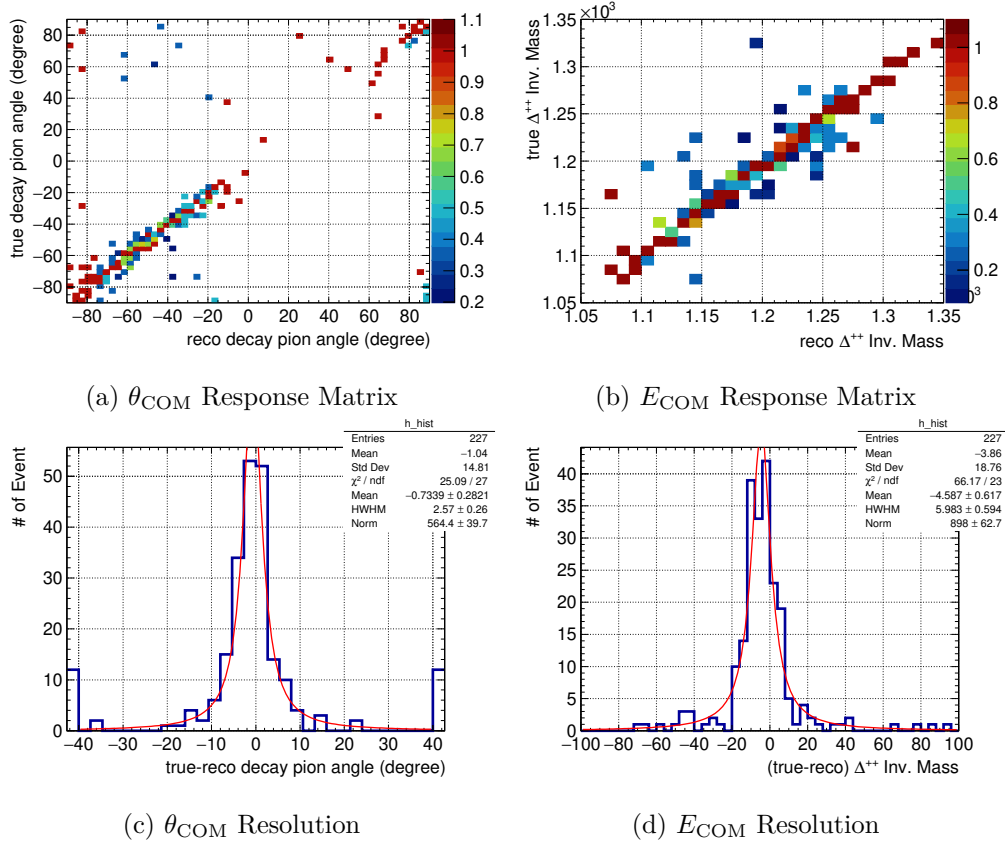


Figure 6.18: Reconstruction quality plots of truth against reconstruction values and fits of reconstruction resolution for COM variables for the $\nu_\mu \text{CC}1\pi1p$ selection. Both variables show excellent resolutions. In particular, the fitted HWHM of E_{COM} is approximately 6.0 MeV, which will be crucial for hydrogen sample selection, which will be discussed later in Sec. 6.3.

6.2.3 Reconstruction Quality

The above discussion assumes true information, with ideal particle kinematics and selection criteria. It is important to assess the practicality of this approach by evaluating the uncertainties in reconstructing the COM variables. Therefore, I calculated the COM variables using the $\nu_\mu \text{CC}1\pi1p$ selection, identical to that used for the TKI calculation above. The reconstruction results are presented in Fig. 6.18. The analysis presented in Sec. 6.2.1 uses a binning of 20° for θ_{COM} . In contrast, Fig. 6.18c demonstrates that the reconstruction has an excellent half-width of 2.6° , significantly smaller than the analysis binning. This suggests that reconstruction will not impede the model differentiation capability discussed in Sec. 6.2.1 and may even enhance it due to the superb reconstruction quality of the SFGD. The fitted

half-width of E_{COM} is approximately 6.0 MeV, which is also exceedingly precise. It will be more relevant for hydrogen sample selection, as discussed in the next section.

6.2.4 Conclusion

This study introduces a novel set of variables—the COM angle and total energy. These variables possess several advantageous properties, which motivate their use in future cross-section measurements. Such measurements have the potential to significantly enhance our understanding and modelling of FSI processes in neutrino-nucleus interactions, thereby contributing to more accurate neutrino cross-section predictions. The reconstruction quality of the COM variables has been shown to be excellent, and it is promising that actual measurements can outperform the potential demonstrated in the true MC study.

6.3 Hydrogen Sample Selection

The sensitivity of the COM frame to FSI can be leveraged not only to study FSI but also to select events without FSI. These events are primarily composed of ν -H interactions, which provide a unique opportunity for precise measurements of the axial current—a feature specific to neutrinos. Additionally, a ν -H sample allows for the improvement of ν -H modelling without the complications of IS and FSI, facilitating accurate neutrino energy reconstruction. These advantages have driven significant efforts to develop techniques for selecting a high-purity ν -H sample, as seen in Ref. [55, 98, 100]. While Ref. [100] and Ref. [98] focuses on neutrino and antineutrino pion-less events respectively, Ref. [55] applies to the same event topology as the COM total energy. They complement each other well. The proposal in Ref. [55] is to use δp_{TT} to select ν -H events. It is challenging to achieve high efficiency and purity with a one-dimensional cut. Therefore, a COM total energy cut is proposed to complement δp_{TT} , and this section demonstrates its effectiveness.

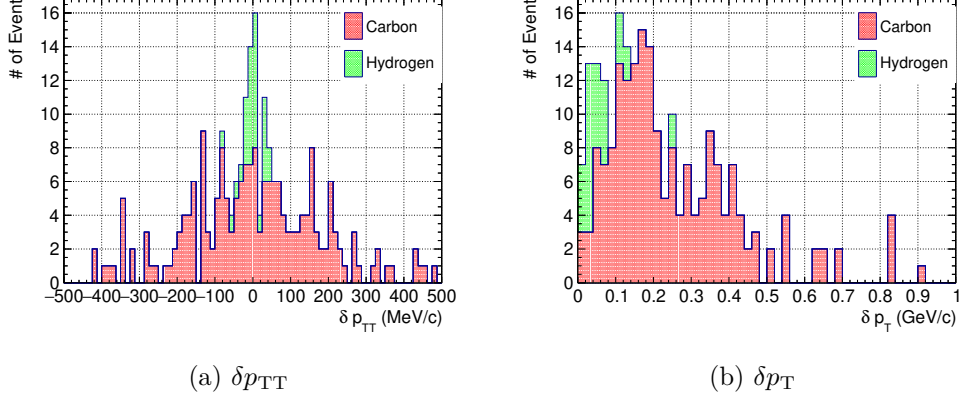


Figure 6.19: Decomposed distributions for δp_{TT} and δp_T for hydrogen and carbon events. It suggests that carbon backgrounds can be rejected by selecting large δp_{TT} and δp_T values.

6.3.1 MC Study

The key to removing carbon backgrounds from hydrogen events is to identify variables that effectively differentiate between the two. Therefore, distributions decomposed by target nucleus type are shown in Fig. 6.19 for the proposed variables.

As shown in Fig. 6.19a Based on δp_{TT} alone, the most effective cut is $-10 \text{ MeV}/c \leq \delta p_{TT} \leq 30 \text{ MeV}/c$. This cut yields 16 hydrogen events and 16 carbon events, for a purity of $(50 \pm 8.9)\%$, where the uncertainty is statistical only. In comparison, Ref. [98] reported “5,580(180) signal events over an estimated background of 12,500 events,” which corresponds to a purity lower than 50% but a much larger total number of events. Hence, for the T2K sample to be most useful, maximizing purity is essential to minimize systematic uncertainties in parameter measurements.

Because SFGD excels in 3D tracking, a cut on δp_T may further reduce carbon backgrounds. A similar decomposition is shown in Fig. 6.19b for δp_T . It is observed that δp_T and δp_{TT} for hydrogen events cluster near 0, while carbon events exhibit a wider spread. Since the variables are not entirely independent (because $\delta p_{TT} \leq \delta p_T$), identifying a suitable two-dimensional cut is simpler by plotting them together, as shown in Fig. 6.20 for hydrogen and carbon separately.

Fig. 6.20 suggests the cuts, $\delta p_T < 80 \text{ MeV}/c$ and $-40 \text{ MeV}/c \leq \delta p_{TT} \leq 30 \text{ MeV}/c$, which lead to a sample of 19 hydrogen events and 10 carbon events,

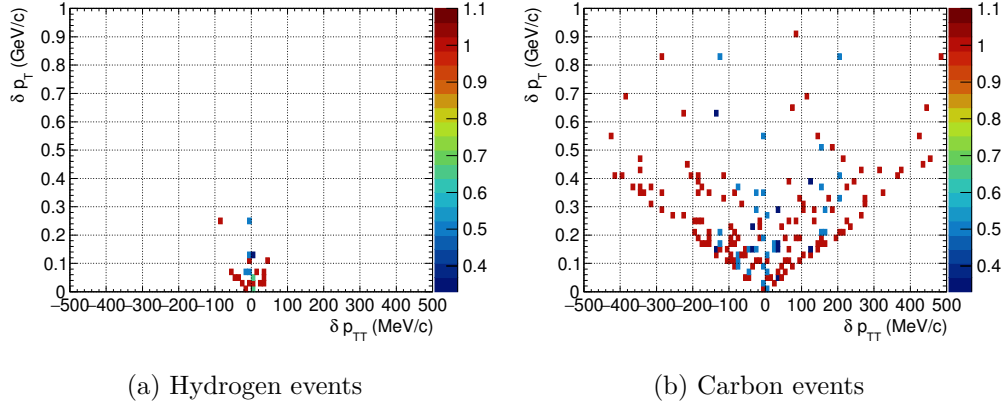


Figure 6.20: Two-dimensional distributions of δp_T and δp_{TT} for hydrogen and carbon events. They display distinct distributions, suggesting that a two-dimensional cut can effectively separate hydrogen and carbon events.

with a purity of $(65.5 \pm 8.9)\%$. This is the optimal purity and efficiency achievable by using TKI variables only. To further improve the purity, an additional cut on the COM variables is proposed. Similar to Fig. 6.19, the decompositions for the COM variables are shown in Fig. 6.21. Although FSI alters the θ_{COM} distributions, leading to distinct distributions between hydrogen and carbon, θ_{COM} for hydrogen is expected to be uniform across the entire range; therefore, the cut on θ_{COM} to select hydrogen is not expected to be effective. Note that to show the wide range of θ_{COM} for hydrogen, the colours for hydrogen and carbon are switched in Fig. 6.21b respectively. In contrast, without FSI, E_{COM} is expected to centre around the mass of the resonance, which is 1232 MeV for Δ^{++} , while the carbon events are expected to have a much wider distribution, as shown in Fig. 6.21a. Hence, an additional cut, $1168 \text{ MeV} < E_{\text{COM}} < 1257 \text{ MeV}$ is implemented to further reduce the carbon backgrounds, achieving a sample of 18 hydrogen events and 5 carbon events, with an impressive purity of $(78.3 \pm 8.6)\%$.

To demonstrate the effectiveness of these cuts, the δp_{TT} distributions are plotted step-wise in Fig. 6.22. Comparing the δp_{TT} distributions before and after each cut, namely the cuts on δp_T and E_{COM} , it is evident that these cuts successfully remove carbon backgrounds in the low δp_{TT} region, which is impossible to remove based on δp_{TT} alone. Furthermore, by comparing Fig. 6.22b and Fig. 6.22c, it is demonstrated that the cut on E_{COM} is capable of removing carbon backgrounds

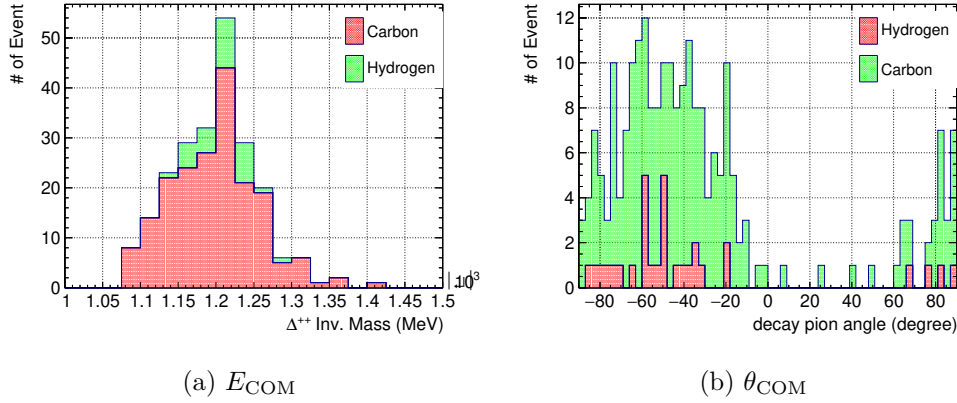


Figure 6.21: Decomposed distributions for E_{COM} and θ_{COM} for hydrogen and carbon events. Rejecting events with E_{COM} deviating significantly from the Δ^{++} mass can remove considerable carbon backgrounds, while θ_{COM} does not show a differentiating feature.

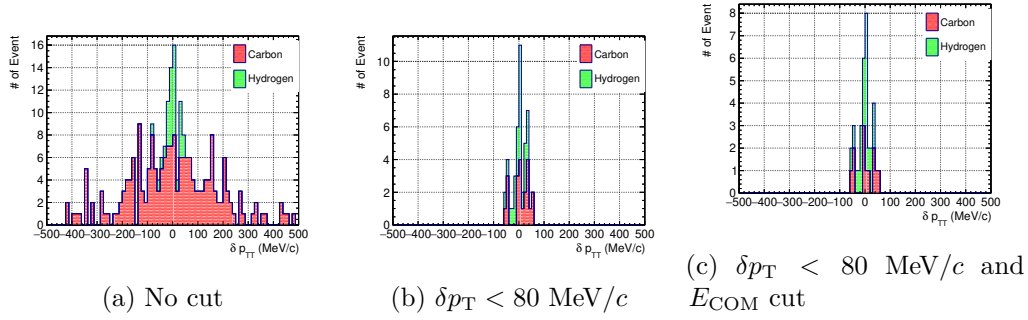


Figure 6.22: Step-wise δp_{TT} distributions for hydrogen and carbon events. The cut on δp_T has removed a significant portion of carbon backgrounds as shown in Fig. 6.22b. The cut on E_{COM} is able to further remove carbon backgrounds in the centre region of the δp_{TT} distribution, as shown in Fig. 6.22c, which cannot be removed by the transverse variables alone.

that cannot be removed by the transverse variables. Hence, all three variables are indispensable for achieving the optimal purity and efficiency.

6.3.2 Discussion

Both the transverse variables and E_{COM} have limitations. The true distributions of the transverse variables on hydrogen are delta functions, but the dependence on an accurate neutrino direction reconstruction limits the reconstruction resolution of the transverse variables. The impact of the resolution is amplified as the cuts are placed to select events with small values when reconstruction uncertainties could relatively

easily move the signal events out of the cut. On the other hand, while E_{COM} has excellent reconstruction quality as it is independent of neutrino direction, its true distribution has a natural half-width of about 60 MeV [35]. Hence, even if E_{COM} can be reconstructed perfectly, it cannot separate hydrogen events from carbon events completely. Due to the limited amount of MC events available and the relatively small fraction of resonance events in the T2K beam, the statistics of the hydrogen selection are too small to be conclusive. Given the relatively large statistical uncertainties in the purity estimates, approximately 9% for all the selections, the realistic improvement of utilizing the COM variables needs to be further investigated with larger MC samples. The results presented in this sub-section is for illustration of the idea only. Nevertheless, the reduction of carbon background events after adding the E_{COM} cut does emphasize the existence of carbon events that cannot be removed by the transverse variables alone, thereby bolstering the argument that the COM variables are complementary to the transverse variables. Hence, it is hopeful that using the two kinds of variables together, the upgraded ND280 data can produce a hydrogen sample with high purity.

まだまだだね。

We are still far from getting a perfect model.

— Ryoma Echizen

7

TKI Tuning

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Current neutrino interaction models are effective models, i.e. they have variable parameters, aiming to parameterize our ignorance of the true underlying physics. These parameters can be constrained by comparing model predictions to data, which is the so-called tuning process. The data used in tuning are the cross section measurements from various neutrino experiments. A successfully tuned model will have a better agreement with the data used in tuning than the original model, and it could also shed light on the direction of future model development. The validity of the tuned model can be tested against new data. In view of the incoming data collected by the upgraded ND280, it is the perfect time to tune the current model to the existing data to maximize the utility of the new data. The GENIE collaboration

has developed a comprehensive tuning framework, and has collected an extensive database of neutrino, charged lepton, and hadron scattering measurements. Hence, we have performed the tuning of an effective model in **GENIE** using this framework. The procedures and the results are presented in this chapter¹.

7.1 Tuning methodology

This study employs the tuning procedure delineated in Ref. [101], utilizing N_{par} model parameters. The primary objective is to determine the best fit—that is, the optimal set of parameter values—within the parameter space by minimizing the χ^2 between the model predictions and the experimental data. Due to the high dimensionality of the parameter space, a brute-force, grid-based point-by-point scan is impractical. Therefore, a sufficiently large number of points in the model parameter space is sampled at random, with each point initiating a full simulation. Subsequently, the simulation output for each data observable bin is parametrized using Professor² [102] with a polynomial of the model parameters of a given order, N_{ord} . The minimal number of points (N_s) required for tuning N_{par} parameters with a polynomial of order N_{ord} is determined by the combinatorial formula,

$$N_s = \binom{N_{\text{par}} + N_{\text{ord}}}{N_{\text{ord}}}. \quad (7.1)$$

Since N_s increases factorially, it is imperative to exercise caution when increasing either the number of parameters or the polynomial degree. Given that our tuning incorporates all 2 SF-CFG and 12 hA parameters (see Sec. 7.2.1), a fourth-order parametrisation necessitates over 6,000 simulation generations, which are already computationally expensive. Thus, only polynomials up to order 4 are considered, a choice that has been shown to reproduce the MC predictions satisfactorily, as illustrated in Fig. 7.1.

¹This chapter is based on the work published in Ref. [95].

²A software capable of approximating and extrapolating cross-section outputs in terms of polynomials of chosen model parameters.

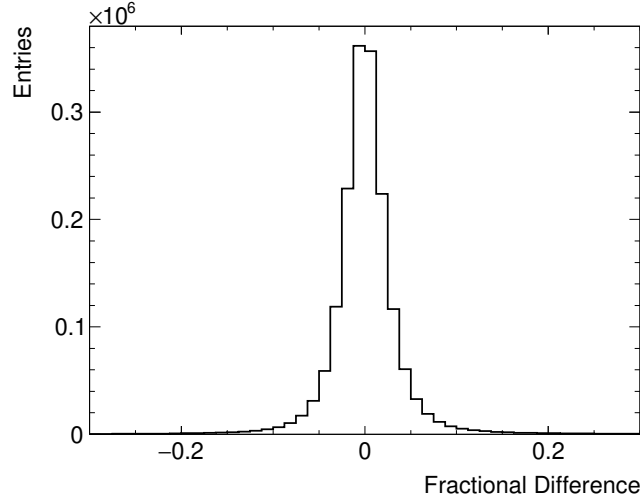


Figure 7.1: Fractional difference for the bin-by-bin cross sections between MC truth and the parametrised approximation using fourth-order polynomials, both in the Norm-Shape (NS) [103, 104] space with `AllPar`. See Table 7.2 for the definition of `AllPar`. The residual exhibits a mean of 0.003 and a standard deviation of 0.073.

Furthermore, to circumvent Peelle’s Pertinent Puzzle³ [105, 106], the Norm-Shape (NS) transformation prescription [103, 104] is adopted. Thereafter, the extremal point is determined by minimizing χ^2 between the NS-transformed polynomial approximation and the NS-transformed data. During the minimisation process, priors—typically derived from systematic uncertainties—are imposed on each parameter to prevent them from deviating excessively from their nominal values. The following subsections elaborate on the specific measurement observables and model parameters to be tuned.

7.2 Inputs

The first step towards tuning is to identify the models to be tuned and the data to be used by spotting the lacking area of the current model, which is manifested by the data-MC discrepancy. One such example is illustrated in Fig. 7.2 and Fig. 7.3, which shows the data-MC comparison using `G24_20i_00_000`, one of the latest Comprehensive Model Configurations (CMC) in `GENIE`, for four TKI measurements, namely $\text{T2K-}0\pi$ [64], $\text{T2K-}\pi^+$ [60], $\text{MINERvA-}0\pi$ [62, 65], and $\text{MINERvA-}\pi^0$ [61].

³This is a phenomenon where unknown correlations between observables can skew the χ^2 minimisation.

This data-comparison shows that the model fails to describe the MINERvA π^0 TKI data, despite that its prediction matches other TKI data well.

As the neutrino-nucleus interaction is a complicated process, it requires multiple models to simulate the whole process, and the CMC in **GENIE** specifies the combination of models to be used. Tuning the entire CMC is extremely computationally expensive, and this is the first tuning on TKI data across event topology, i.e. on both $CC0\pi$ and $CC1\pi$. Hence, before embarking on this ultimate step, it is important to first perform a proof-of-concept study to explore the model flexibility and to identify the most relevant model parameters. As TKI data are sensitive to the nuclear IS and FSI, this study focuses on the tuning of these two models. To lay the ground for better understanding of this tuning effort, I will provide a more detailed description of the inputs, i.e. the data and the models, in the following subsections.

7.2.1 The models

The CMCs in **GENIE** are formatted strings, and they are sometimes also called tunes, regardless of whether they are obtained from an actual tuning. The tune, **G24_20i_00_000**, mentioned above is improved on the tune, **G18_10a_02_11b**, which is obtained from the free nucleon tuning effort in Ref. [99], and describes bubble chamber data better. As **G24_20i_00_000** is widely used in the paper, it will be referred to as **G24-0** for simplicity.

The IS in **G24-0** is modelled by the spectral-function-like Correlated Fermi Gas model (SF-CFG) [84, 86, 87]. SF-CFG has more adjustable parameters for tuning and improved physics compared to the local fermi gas (LFG) model used in **G18_10a_02_11b**. The improvement arises from two aspects. “Spectral-function-like” refers to the implementation of removal energy as a varying function rather than a fixed value, while “Correlated” highlights the incorporation of the high-momentum tail above the Fermi momentum due to nucleon-nucleon short-range correlations (SRC), as evidenced by electron-scattering data [107].

Both enhancements are incorporated into the Valencia Model [89]). Consequently, SF-CFG more closely reproduces the Valencia initial state than previous LFG

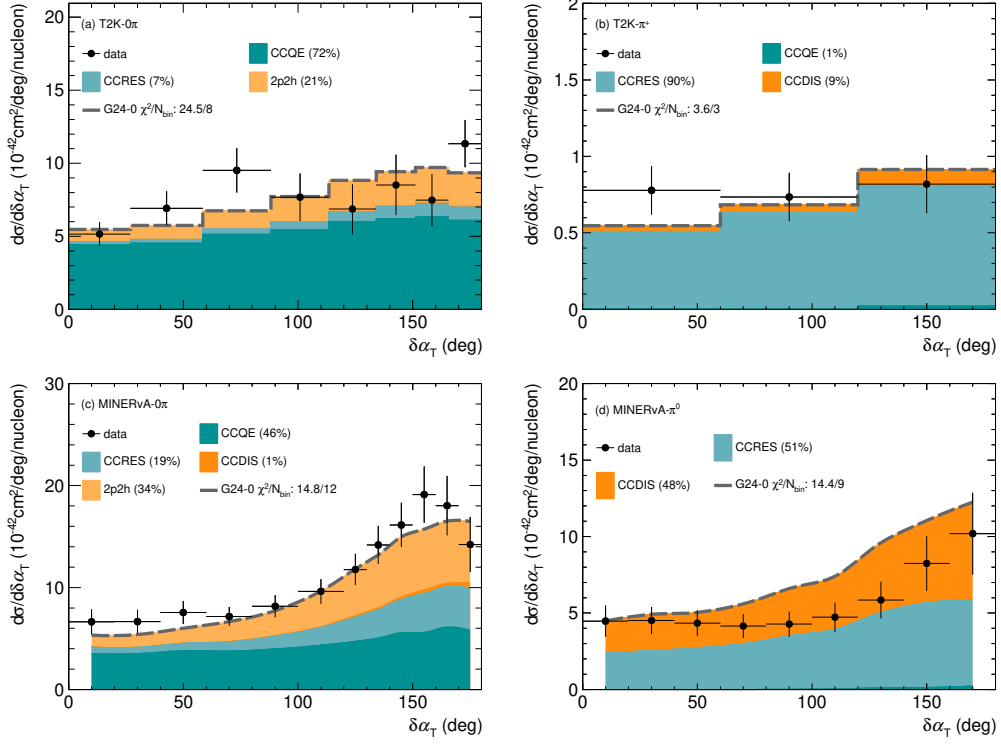


Figure 7.2: $\delta\alpha_T$ measurements decomposed in interaction types, compared to G24-0 prediction.

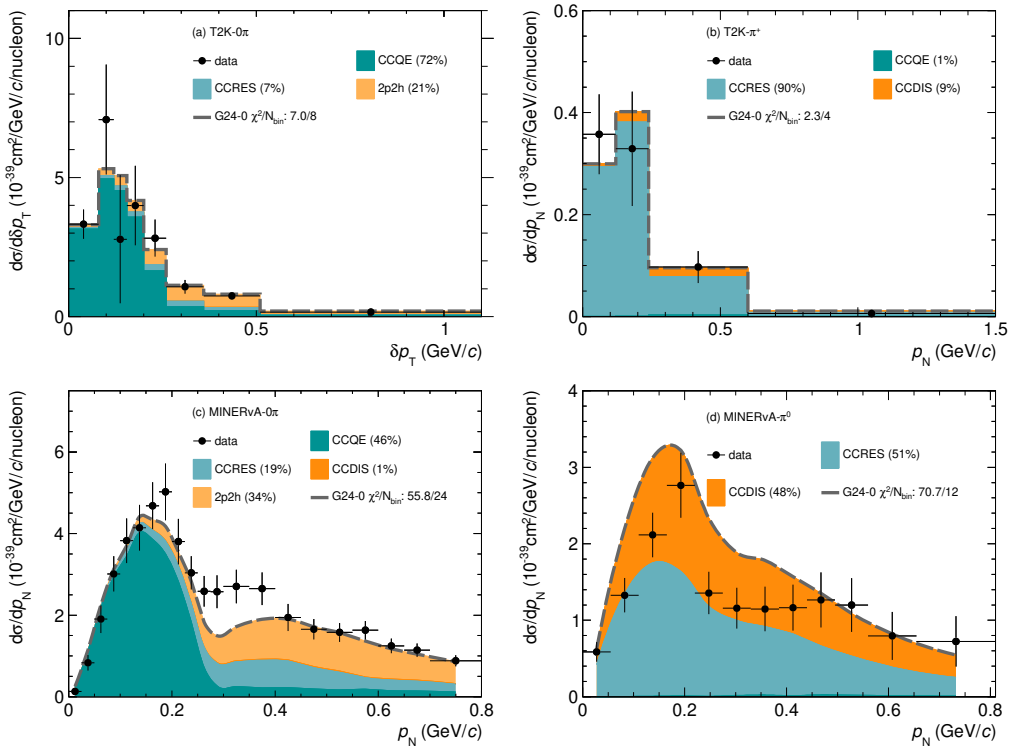


Figure 7.3: Similar to Fig. 7.2 but for the p_N (T2K- π^+ , MINERvA- 0π , and MINERvA- π^0) and δp_T (T2K- 0π) measurements.

implementations. For further details on SF-CFG, see Ref. [84]. Note that the original Valencia Model employs its own approach to modelling FSI. However, since FSI is factorized from other processes in the **GENIE** implementation, that method is not applied here. Instead, GENIE has developed the INTRANUKE hA FSI model (hA for short), which is chosen as the FSI candidate for the tuning performed in this work for its simplicity and interpretability.

Additional improvements of **G24-0** include: 1) adopting a z -expansion axial-vector form factor [108] in lieu of the Valencia dipole form factor for QE processes [89]; and 2) substituting the Valencia model for 2p2h processes [91] with the SuSAv2 Model [92], which covers a broader region of the q^0 and q^3 phase space. The complete list of model components is provided in Table 7.1. Having selected the

Simulation component	Model
Nuclear state	SF-CFG [84, 86, 87]
QE	Valencia [89]
2p2h	SuSAv2 [92]
QE $\Delta S = 1$	Pais [109]
QE $\Delta C = 1$	Kovalenko [110]
Resonance (RES)	Berger-Sehgal [94]
Shallow/Deep inelastic scattering (SIS/DIS)	Bodek-Yang [111]
DIS $\Delta C = 1$	Aivazis-Tung-Olness [112]
Coherent π production	Berger-Sehgal [113]
Hadronization	AGKY [114]
FSI	INTRANUKE hA [3]

Table 7.1: Model components of **G24-0**. Processes with non-trivial ΔS and ΔC are those with strangeness and charm production, respectively.

G24-0 CMC as the tuning starting point, a closer look into the IS and FSI models, the models to be tuned, is necessary. The SF-CFG and hA model contains a total of 14 tuneable parameters of SF-CFG and hA.

The SF-CFG model behaviour is chiefly specified by two parameters: R_{SRC} , which quantifies the fraction of the high-momentum tail from SRC above the Fermi surface, and E_{RM}^{C} , which defines the onset of the nuclear removal energy distribution for carbon. A higher R_{SRC} suggests that the initial nucleons possess greater energy,

whereas a larger E_{RM}^{C} implies that more energy is required to liberate a nucleon, resulting in lower-energy final-state particles. Due to the innovative yet loosely constrained nature of this spectral-function-like implementation, this study adopts relatively lenient priors of 0.12 ± 0.12 for R_{SRC} and 0.01 ± 0.005 GeV for E_{RM}^{C} .

As for the hA model [3], its widespread use is primarily due to its straightforward reweighting capability. Unlike cascade models such as the hN model [3], the hA model assigns the FSI type for each hadron exactly once, without simulating further propagation of rescattered hadrons. This single-rescattering approach is critical for interpreting tuning results. The hA model parameters are described as follows.

1. Mean Free Path (MFP) Scaling Factors: The model computes the MFP, λ , as a function of the hadron's energy, E , and its radial distance, r , from the nucleus centre:

$$\lambda(E, r) = \frac{1}{\sigma_{\text{hN,tot}}(E)\rho(r)}, \quad (7.2)$$

where $\sigma_{\text{hN,tot}}$ is the total hadron–nucleon cross section and $\rho(r)$ is the local nucleon density. Once produced within the nucleus, a hadron travels a distance λ before a probabilistic decision on rescattering is made. If no rescattering occurs, the hadron is propagated by another λ , and the process repeats until either a rescattering happens or the hadron escapes the nucleus. When rescattering occurs, its type is chosen based on the relevant cross sections; the relative probabilities of these types are extracted from stored hadron scattering data [115–121]. Once a specific rescattering type is determined, the resulting products are generated and considered final, with no further rescattering simulated. The MFPs for pions and nucleons are adjustable via scaling factors ($S_{\lambda}^{\pi^{\pm}}$, $S_{\lambda}^{\pi^0}$, and S_{λ}^{N}) as detailed in Table 7.2. Due to isospin symmetry, both charged pions, π^+ and π^- , share the same scaling factor $S_{\lambda}^{\pi^{\pm}}$, while the π^0 MFP is computed from the charged pion values and adjusted separately by $S_{\lambda}^{\pi^0}$. A scaling factor greater than unity increases λ , which in turn reduces the number of propagation steps necessary for escape and diminishes the average probability of rescattering. Owing to the high precision of total

hadron–nucleus cross section measurements [115–119], strict Gaussian priors with a standard deviation (σ) of 0.2 (matching the systematic uncertainties shown in Table 7.2) are applied to the MFP scaling factors. Notably, varying $S_{\lambda}^{\pi^0}$ by $\pm 2.5 \sigma$ significantly affects the MINERvA- π^0 $d\sigma/dp_N$, as illustrated in Fig. 7.4. In summary, a reduced MFP naturally increases rescattering, leading to fewer π^0 s escaping the nucleus and a decreased cross section.

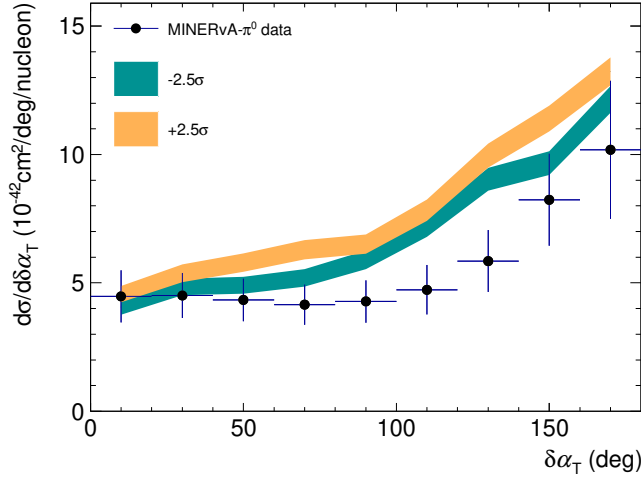


Figure 7.4: Effect of varying $S_{\lambda}^{\pi^0}$ by $\pm 2.5 \sigma$ compared to the MINERvA- π^0 measurement. Each band’s width indicates the GENIE prediction’s statistical uncertainty from 10^5 events.

2. Rescattering Type Scaling Factors: The relative probability for each rescattering type is adjusted by a scaling factor. Four rescattering types are available for both nucleons and pions: charge exchange (CEX), inelastic scattering (INEL), absorption (ABS), and pion production (PIPD). A detailed discussion of these tunable FSI types is provided in Sec. 3.2.3. The default energy-dependent probability distributions, derived from hadron data tuning [115–119], are modified by corresponding scaling factors (e.g., S_{CEX}^{π} for the CEX of all pions) as listed in Table 7.2. After scaling, the probabilities are renormalized so that they sum to one. For instance, if the initial absorption fraction, f_{ABS} , is 0.5 (with the remaining fraction, $f_{\text{other}} = f_{\text{INEL}} + f_{\text{CEX}} + f_{\text{PIPD}}$, also 0.5), scaling f_{ABS} by $S_{\text{ABS}} = 2$ yields

$$f'_{\text{ABS}} = \frac{f_{\text{ABS}} \cdot S_{\text{ABS}}}{f_{\text{ABS}} \cdot S_{\text{ABS}} + f_{\text{other}}} = \frac{2}{3}, \quad (7.3)$$

illustrating that the effective increase in f_{ABS} is less than the scaling factor alone due to normalization with the other FSI types. Here, f_{ABS} is effectively scaled by a factor of 1.3 rather than 2. Therefore, a relatively relaxed Gaussian prior with a σ of 0.5 is imposed on the FSI fate scales, slightly exceeding the systematic uncertainties in Table 7.2. A side-effect due to this normalization is that changing one scaling factor inherently affects the others. In practice, plotting the cross section by rescattering type clarifies the net changes.

Table 7.2 summarizes the tuneable parameters and their ranges imposed in tuning. Nominal values and associated uncertainties for the hA model are taken from Table 17.3 in Ref. [3].

7.2.2 The data

Not only the MINERvA- π^0 data set, where the discrepancy lies, is used for tuning, but the other three data sets are also included, as it is important to maintain the good data-MC agreement for the existing data sets, while improving the agreement for the MINERvA- π^0 data set.

The common requirement of the four measurements is the presence of one CC muon and at least one proton in the final state. The 0π data sets, i.e. T2K- 0π and MINERvA- 0π , require the absence of any pions, while T2K- π^+ requires exactly one π^+ , and MINERvA- π^0 requires at least one π^0 . The four data sets further differ slightly in the kinematic cuts, as summarized in Table 7.3, and they also contain slightly different combinations of the TKI variables, as detailed in Table 7.4.

As not all TKI observables are linearly independent, such as δp_T and p_N , and some might have strong dependence outside IS and FSI, such as $\delta\phi_T$ displaying a strong dependence on neutrino beam energy [56], it might be redundant and ineffective to include all of them in the tuning. In order to discern the most sensitive observables, a systematic evaluation of various combinations of observables was undertaken for the tuning process. A total of 26 combinations, as shown in Table 7.4, are examined. These combinations are formed according to the following patterns:

Parameter	Nominal (G24-0)	Range In Tuning	RedPar(G24-c)	AllPar(G24-t)
SF-CFG				
R_{SRC}	0.12	(0.0, 0.5)	0.09 ± 0.08	0.17 ± 0.10
E_{RM}^{C}	0.01	(0.0, 0.2)	0.01	0.06 ± 0.000004
hA				
$S_{\lambda}^{\pi^{\pm}}$	1.0 ± 0.2	(0.0, 3.0)	1.0	1.21 ± 0.15
$S_{\lambda}^{\pi^0}$	1.0 ± 0.2	(0.0, 3.0)	0.34 ± 0.11	0.91 ± 0.000004
S_{λ}^{N}	1.0 ± 0.2	(0.0, 3.0)	1.0	0.89 ± 0.000382
S_{CEX}^{π}	1.0 ± 0.5	(0.0, 3.0)	0.27 ± 0.28	0.86 ± 0.000056
$S_{\text{CEX}}^{\text{N}}$	1.0 ± 0.5	(0.0, 3.0)	1.39 ± 0.33	0.92 ± 0.001178
S_{INEL}^{π}	1.0 ± 0.4	(0.0, 3.0)	1.0	0.90 ± 0.00008
$S_{\text{INEL}}^{\text{N}}$	1.0 ± 0.4	(0.0, 3.0)	1.0	0.92 ± 0.001218
$S_{\text{ABS}}^{\pi^{\pm}}$	1.0 ± 0.2	(0.0, 3.0)	1.0	1.36 ± 0.38
$S_{\text{ABS}}^{\pi^0}$	1.0 ± 0.2	(0.0, 3.0)	1.0	0.92 ± 0.000024
$S_{\text{ABS}}^{\text{N}}$	1.0 ± 0.2	(0.0, 3.0)	0.48 ± 0.33	1.02 ± 0.413668
S_{PIPD}^{π}	1.0 ± 0.2	(0.0, 3.0)	1.0	1.11 ± 0.342484
$S_{\text{PIPD}}^{\text{N}}$	1.0 ± 0.2	(0.0, 3.0)	1.90 ± 0.71	1.12 ± 0.57198
χ^2 for combi				
	untuned		231.75	129.61
	tuned		168.67	131.46
	diff		-63.08	1.85
χ^2 for vald				
	untuned		229.5	331.64
	tuned		232.3	304.4
	diff		2.8	-27.24
χ^2 for combi+vald				
	untuned		461.25	461.25
	tuned		400.97	435.86
	diff		-60.28	-25.39

Table 7.2: Parameters in G24-0, G24-c, and G24-t. Lower section: **combi** indicates that the following χ^2 sums are calculated for the tuned measurements, while **vald** indicates that the respective validation sets are used; **untuned** means that the χ^2 calculation uses the nominal values of the parameters, while **tuned** denotes the tuned ones. **diff** displays the improvement achieved by the respective tuning.

- **Combi-1** through 5 are cross-experiment selections of a single observable. For example, **Combi-3** uses only $\delta\alpha_{\text{T}}$ from all four data sets. If a chosen observable is absent from a dataset, that dataset is excluded from the specific combination. For example, T2K- 0π is not used in **Combi-1** due to the absence of p_{N} .

Variables	Cuts (p in GeV/ c)
T2K- 0π [64]	
$\vec{p}_\mu$	$0.25 < p_\mu, \cos \theta_\mu > -0.6$
$\vec{p}_p$	$0.45 < p_p < 1.0, \cos \theta_p > 0.4$
T2K- π^+ [60]	
$\vec{p}_\mu$	$0.25 < p_\mu < 7, \theta_\mu < 70^\circ$
$\vec{p}_p$	$0.45 < p_p < 1.2, \theta_p < 70^\circ$
$\vec{p}_\pi$	$0.15 < p_\pi < 1.2, \theta_\pi < 70^\circ$
MINERvA- 0π [62, 65]	
$\vec{p}_\mu$	$1.5 < p_\mu < 10, \theta_\mu < 20^\circ$
$\vec{p}_p$	$0.45 < p_p < 1.2, \theta_p < 70^\circ$
MINERvA- π^0 [61]	
$\vec{p}_\mu$	$1.5 < p_\mu < 20, \theta_\mu < 25^\circ$
$\vec{p}_p$	$0.45 < p_p$

Table 7.3: Kinematic cuts for the samples of the TKI measurements.

- **Combi-6** through **9** incorporate all variables from a single measurement. For example, **Combi-9** uses MINERvA- π^0 only.
- **Combi-10** to **13** uses two out of four measurements according to the experiment or topology. For example, **Combi-10** uses only the two data sets from T2K, and **Combi-13** only with pion production.
- **Combi-14** to **17** are cross-experiment selections of two observables. For example, **Combi-14** uses all $\delta\alpha_T$ and δp_{TT} measurements across all data sets, while **Combi-17** uses $\delta\alpha_T$ and δp_T .
- **Combi-18** through **22** are cross-experiment selections of three observables. For example, **Combi-18** uses all available $\delta\alpha_T$, δp_{TT} , and p_N , while **Combi-22** uses δp_T , $\delta\phi_T$, and p_N .
- **Combi-23** uses all observables except $\delta\phi_T$ from all data sets.
- **Combi-24** encompasses all variables, acting as the superset.
- **Combi-25** and **26** are the same as **Combi-21** and **23**, respectively, except that δp_T in MINERvA- 0π is removed to avoid correlation with p_N in the same data set.

Combi-	# of bins	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15*	16	17	18	19	20	21	22	23	24	25	26†
T2K- 0π																											
$\delta\alpha_T$	8			✓			✓				✓		✓		✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
δp_T	8		✓				✓				✓		✓		✓	✓		✓		✓	✓	✓	✓	✓	✓	✓	✓
$\delta\phi_T$	8				✓		✓				✓		✓							✓		✓		✓	✓	✓	✓
T2K- π^+																											
$\delta\alpha_T$	3			✓			✓				✓		✓		✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p_N	4	✓					✓				✓		✓		✓	✓		✓		✓	✓	✓	✓	✓	✓	✓	✓
δp_{TT}	5					✓		✓			✓		✓		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓
MINERvA- 0π																											
$\delta\alpha_T$	12			✓			✓				✓	✓	✓		✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p_N	24	✓					✓				✓	✓	✓		✓	✓		✓		✓	✓	✓	✓	✓	✓	✓	✓
δp_T	24		✓				✓				✓	✓	✓		✓				✓		✓	✓	✓	✓	✓	✓	✓
$\delta\phi_T$	23				✓		✓				✓	✓	✓		✓				✓		✓	✓	✓	✓	✓	✓	✓
MINERvA- π^0																											
$\delta\alpha_T$	9			✓			✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p_N	12	✓					✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
δp_{TT}	13				✓		✓			✓	✓	✓	✓		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓

Table 7.4: Specifications of observable combinations. **Combi-15*** is **Best-AllPar**, **Combi-24** is **Superset**, and **Combi-26†** is **Best-RedPar**.

For each combination, variables omitted from tuning—including proton momentum, p_p (25 bins), and angle, θ_p (26 bins), [65], as well as δp_{Tx} (32 bins) and δp_{Ty} (33 bins) [62] in MINERvA- 0π —were reserved solely for validation; the expectation is that the model will accurately reproduce these observables following tuning.

7.3 Results

A priori, it is difficult to ascertain which parameters are most relevant. Hence, all 14 parameters are used in the initial tuning, denoted as **AllPar**, and a reduced set, denoted as **RedPar**, is subsequently employed after identifying the most sensitive parameters. In order to perform the tuning with a degree 4 polynomial parametrization using **AllPar**, 6,600 points are randomly sampled from the parameter space, and the corresponding event generation is performed. Note that this is considerably larger than the minimal number of points required in Sec. 7.1 to accommodate failed event generation and to provide more points for better parametrization.

7.3.1 AllPar tune

After all simulations are completed, a 4th order polynomial parametrization of MC predictions for all observable bins in terms of **AllPar** is obtained from Professor. The χ^2 between the data and the parametrised MC predictions for each combinations of observables can be calculated and minimized to find the optimal set of **AllPar**.

By construction, the χ^2 over the observables used in the tuning should decrease after the tuning process. However, a successful model should also make good predictions for the observables not used in the tuning, i.e. the validation observables. Hence, to identify the optimal combination, this study employs a criterion based on the total χ^2 calculated over the complete (tuned plus validation) observable set.

The blue bars in figure 7.5 shows the change in total χ^2 for all observable combinations after the tune. The largest reduction in χ^2 is -25.39 for **Combi-13**,

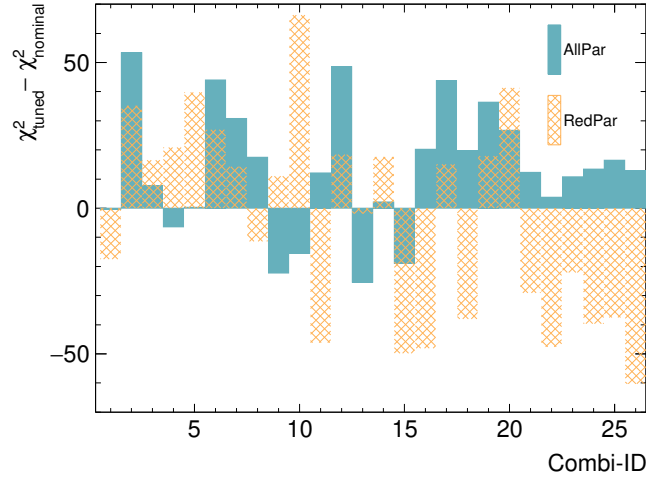


Figure 7.5: Change of χ^2 calculated for the full (i.e., tuned plus validation) observable set as a function of the tuned combination (cf. Table 7.4). The two model parameter sets (**AllPar** and **RedPar**, see Table 7.2 for definitions) are compared and it can be seen that the respective minima happen at **Combi-13** and **26**.

denoted as **Combi-Best-AllPar** for subsequent discussion. The parameters resulted from this tune are referred to as **G24-t** for conciseness and are summarized in Table 7.2, in which the upper part contains the parameter values and the lower section illustrates χ^2 improvements for selected observables and their corresponding validation sets.

MC predictions using **G24-t** are compared to the data in Fig. 7.6 and Fig. 7.7 for the $\delta\alpha_T$ and p_N measurements, respectively. It is interesting that the decrease in χ^2 for **G24-t** mainly comes from the validation set, as evident in the small χ^2 for p_N of MINERvA- 0π in Fig. 7.7.

The most prominent change of **G24-t** is the noticeable increase of R_{SRC} from 0.12 to 0.17, while all other FSI fates are close to the nominal values. This increases

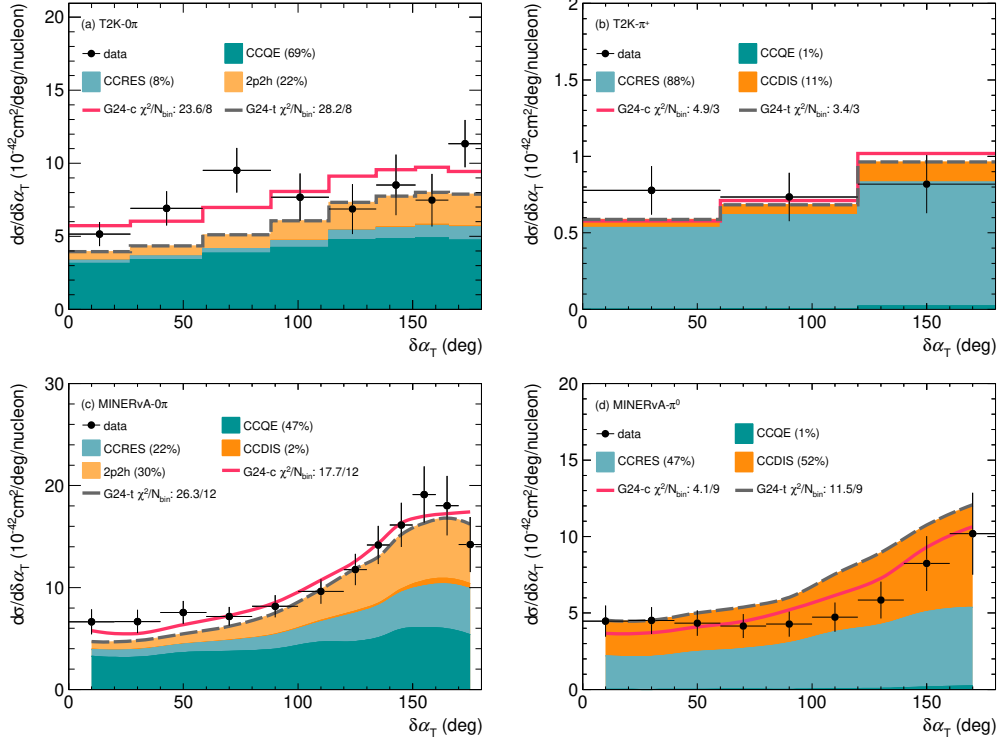


Figure 7.6: Similar to Fig. 7.2 but with G24-t. The G24-c, the best tune obtained (see next subsection), prediction is also plotted for comparison.

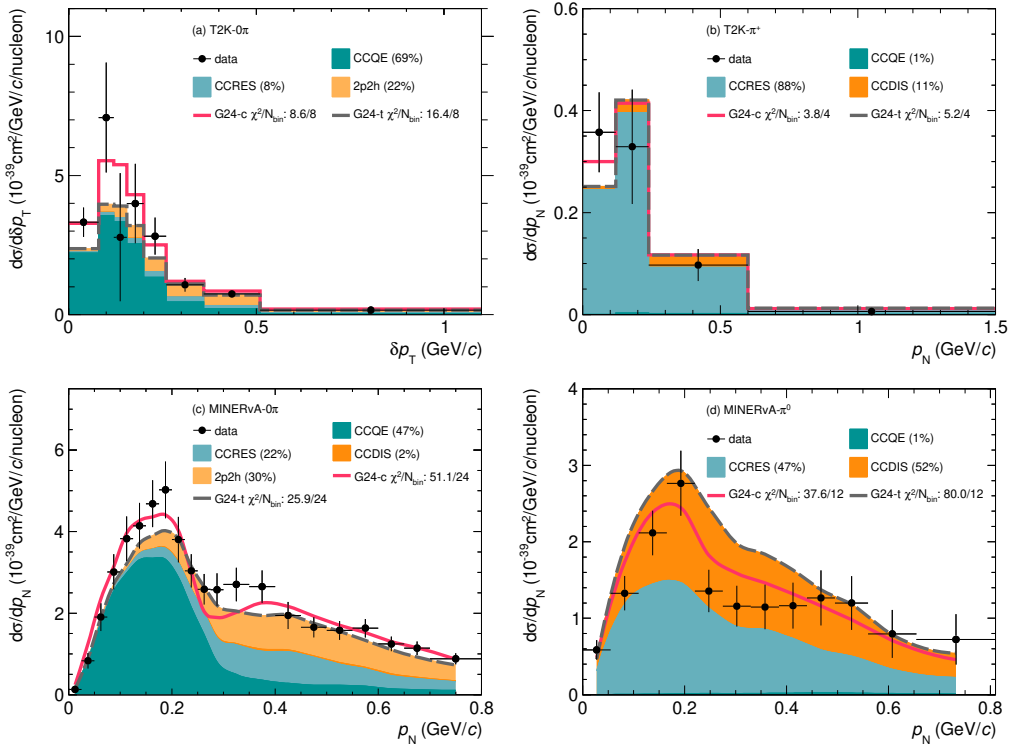


Figure 7.7: Similar to Fig. 7.6 but for the p_N (T2K- π^+ , MINERvA-0 π and MINERvA- π^0) and δp_T (T2K-0 π) measurements.

the high energy tail in the p_N distribution, raising the dip in the MC prediction of the p_N distribution between 250 MeV/ c and 300 MeV/ c and moving the distribution peak to the appropriate place, thereby closing the gap between the data and the MC prediction in the high energy region. However, as the MINERvA- π^0 p_N prediction using **G24-0** is already higher than data, this increase in R_{SRC} does not improve the data-MC agreement for MINERvA- π^0 . Overall, this leads to a moderate reduction of 25.39 in χ^2 (Table 7.2), as well as a relatively improved data-MC agreement across all data sets. Regarding the other combinations of data sets, there does not seem to be a consistent pattern regarding to observable combinations with respect to the χ^2 reduction. For example, as MINERvA- π^0 has the largest deviation, one might expect that the combinations containing MINERvA- π^0 would have larger reduction in χ^2 . This is true for **Combi-13**, which includes only the pion data, i.e. T2K- π^+ and MINERvA- π^0 , but not for **Combi-11**, which includes only MINERvA data, i.e. MINERvA- 0π and MINERvA- π^0 . Unexpectedly, the majority of the combinations yield an increase in χ^2 . This could be due to the intricate correlations among model parameters, so correlation between **AllPar** in the tuning using **Combi-Best-AllPar** is plotted in Fig. 7.8a. Indeed, considerable correlation and anti-correlation are observed between different FSI fates for the same particle, e.g. $S_{\text{ABS}}^{\pi^\pm}$ and $S_{\text{PIPD}}^{\pi^\pm}$.

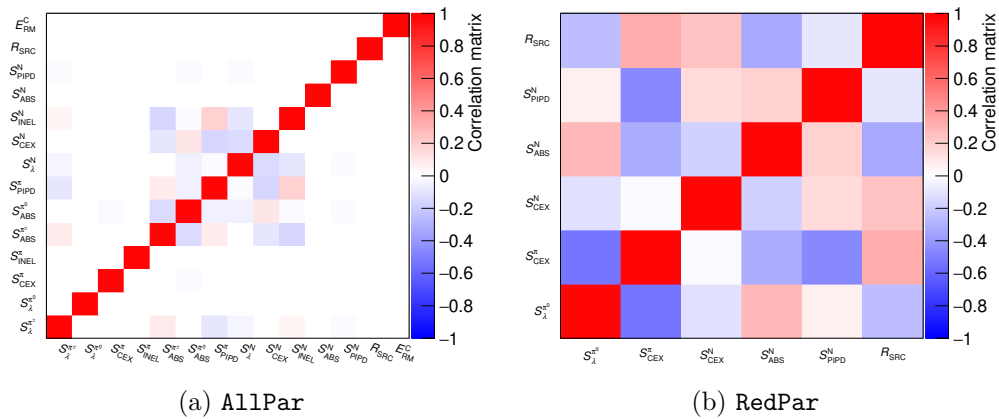


Figure 7.8: Post-fit Correlation coefficient on **Combi-26**.

Moreover, it was observed that for most combinations, several parameters stayed near their nominal values—typically within one standard deviation of the imposed priors. For instance, $S_{\text{PIPD}}^{\pi^\pm}$ remained largely unchanged since no data set

significantly constrains the PIPD contribution of pions, indicating that excluding these parameters from the tuning has little effect on the results. Consequently, we constructed a reduced set consisting only of R_{SRC} , $S_{\lambda}^{\pi^0}$, S_{CEX}^{π} , $S_{\text{CEX}}^{\text{N}}$, $S_{\text{ABS}}^{\text{N}}$, and $S_{\text{PIPD}}^{\text{N}}$ (denoted as **RedPar** in Table 7.2) and applied the tuning procedure to all 26 observable combinations using this parameter subset. Note that only one of the set of considerably correlated parameters, shown by the central square in Fig. 7.8a, namely $S_{\text{CEX}}^{\text{N}}$ is present in **RedPar**.

7.3.2 RedPar tune

The **RedPar** tuning proved more stable, with the majority of combinations showing a more negative χ^2 change than the **AllPar** tuning, as shown in Fig. 7.5. The most significant reduction in χ^2 is observed for **Combi-26** (referred to as **Combi-Best-RedPar** in Table 7.4). This combination selects all TKI observables except for $\delta\phi_{\text{T}}$ and δp_{T} —namely, $\delta\alpha_{\text{T}}$, p_{N} (or δp_{T} if p_{N} is missing), and δp_{TT} —from both pionless and pion-production measurements in T2K and MINERvA.

The χ^2 improvement for **Combi-Best-RedPar** is -60.28, which is significantly better than the improvement obtained with **Combi-Best-AllPar** tuning. The parameters obtained from this tuning are referred to as **G24-c** for conciseness and are summarized in Table 7.2. The main reduction in χ^2 for **G24-c** comes from the tuned observables, which is as expected, and it is also encouraging that the validation observables have a negligible change of 2.8 in χ^2 . The MC predictions using **G24-c** are compared to the data in Fig. 7.9 and Fig. 7.10 for the $\delta\alpha_{\text{T}}$ and p_{N} measurements, respectively.

For the **G24-c** tune, the modifications to the SF-CFG model are moderate (with R_{SRC} decreasing from 0.12 to 0.09), whereas in the hA model both $S_{\lambda}^{\pi^0}$ and S_{CEX}^{π} are substantially suppressed, resulting in a nucleon FSI with increased CEX and PIPD and reduced ABS. For the T2K- 0π , T2K- π^+ , and MINERvA- 0π datasets, the new χ^2 values remain comparable to those obtained with **G24-0**, with variations smaller than the degree of freedom. In the case of MINERvA- π^0 , **G24-c** clearly outperforms **G24-0**, demonstrating that an accurate simultaneous description

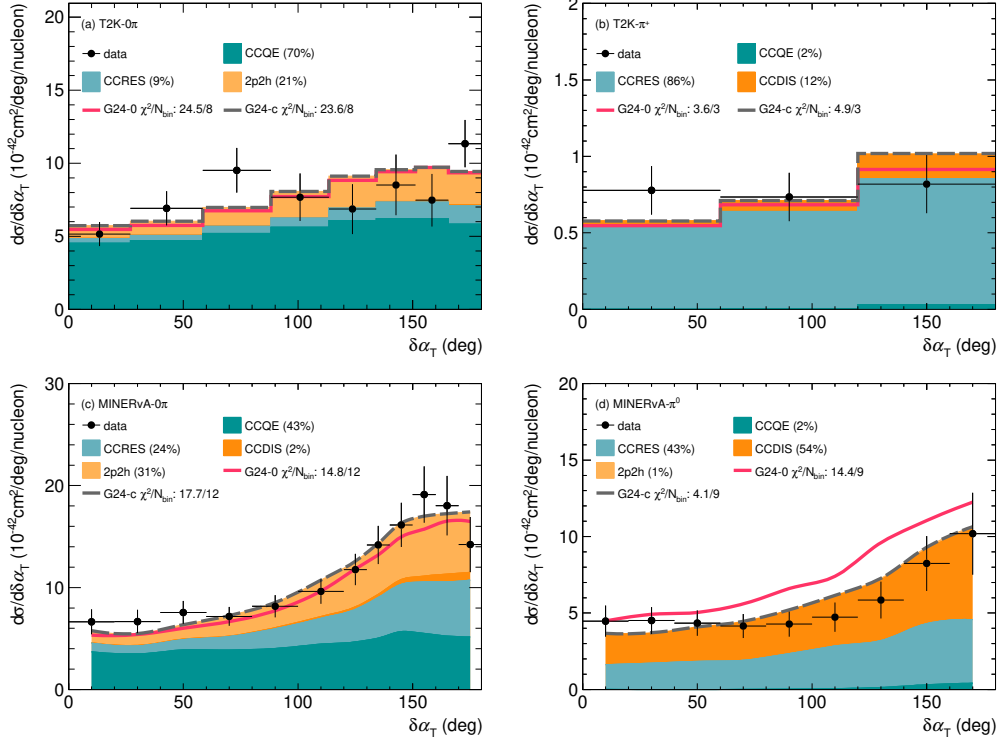


Figure 7.9: Similar to Fig. 7.2 but with G24-c. The G24-0 prediction is also plotted for comparison.

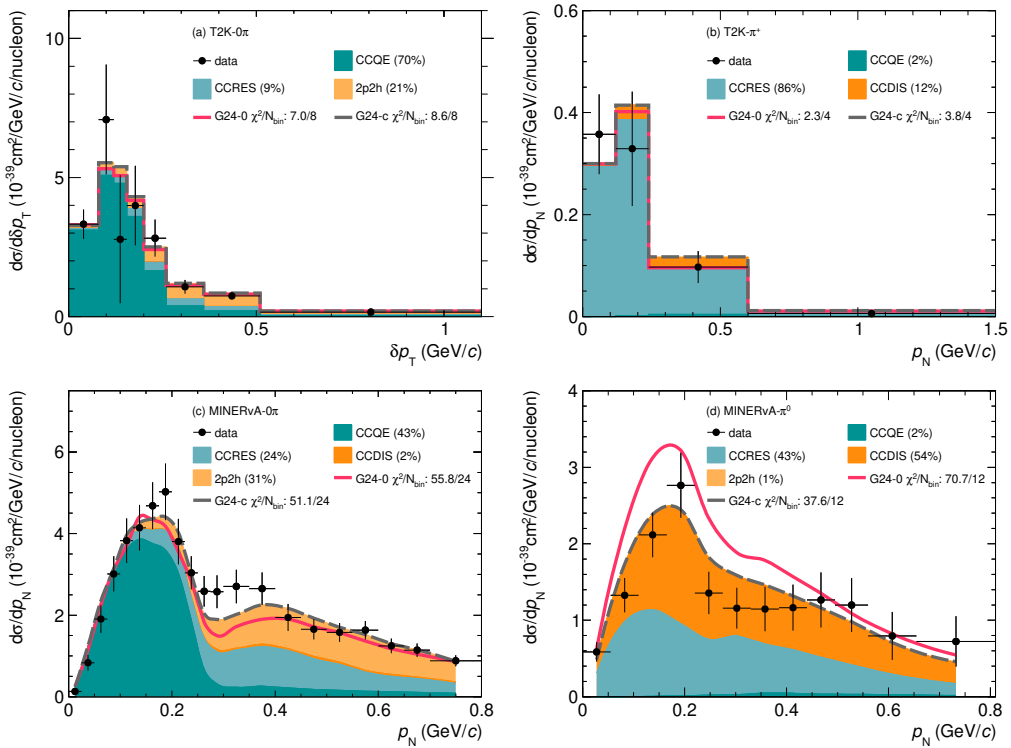


Figure 7.10: Similar to Fig. 7.9 but for the p_N (T2K- π^+ , MINERvA- 0π and MINERvA- π^0) and δp_T (T2K- 0π) measurements.

of both pionless and pion production samples is achievable using constrained parameters from cross-topology TKI tuning. It should be noted that in the region of $p_N \sim 0.3$ GeV/ c , the model deficit previously reported in Ref. [65] (and shown in Fig. 7.3c) persists even after the fit. This deficit may be attributed to the strength of the 2p2h contribution [65], although its exact origin remains a subject of considerable discussion within the community.

A detailed examination of individual datasets sheds light on the source of the improvement. Figure 7.11 presents the decomposition of the $\delta\alpha_T$ cross section according to the FSI fates of the π^0 . In the hA model, primary interaction products undergo only a single rescattering—recorded using a specific rescattering code—and the resulting final-state particles are stored as daughters of the primary products. Thus, the FSI fate responsible for the final particles can be determined by inspecting the rescattering code of their first parent. More specifically, we loop through all final-state particles to identify the leading π^0 and examine the rescattering code of its initial parent.

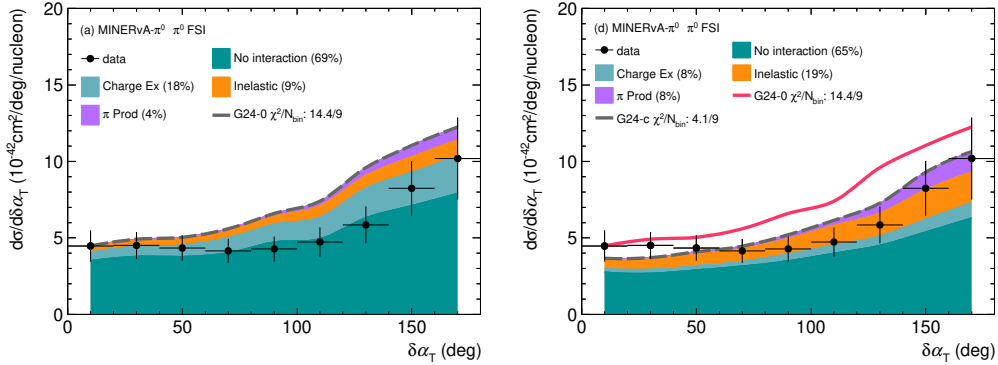


Figure 7.11: MINERvA- π^0 $\delta\alpha_T$ measurement compared to GENIE predictions decomposed in π^0 FSI fates with (a) G24-0 and (b) G24-c.

In MINERvA- π^0 , the number of π^0 s that experience no FSI (“No Interaction”, as shown in the figures) is controlled by the $S_\chi^{\pi^0}$ parameter. Reducing $S_\chi^{\pi^0}$ diminishes the fraction of “No Interaction” events, as illustrated by the comparison between Fig. 7.11b and Fig. 7.11a. In contrast, an enhancement in π^0 rescattering appears in pionless measurements only via ABS. Although there is an increase in ABS for π^0 in T2K-0 π and MINERvA-0 π , the overall fraction is low, resulting in a limited impact.

While the increased π^0 rescattering could boost T2K- π^+ through CEX (as discussed subsequently), the substantial suppression of CEX ensures that its effect on T2K- π^+ remains minimal (for a detailed breakdown, see Figs. C.1–C.4 in Appendix C).

As shown in Fig. 7.11a, the nominal MINERvA- π^0 prediction contains significant contributions from CEX governed by S_{CEX}^π . Since CEX merely alters the pion type without removing them, it does not affect T2K- 0π and MINERvA- 0π , where events with pions in the final state are rejected regardless of the pion charge. In principle, CEX can also transfer events between the signal and background definitions for T2K- π^+ when a π^0 is converted to a π^+ and vice versa. However, given the initially negligible CEX fraction in T2K- π^+ , adjustments in CEX have little impact on its prediction. Thus, suppressing CEX represents an effective approach to reducing the MINERvA- π^0 cross section prediction without significantly altering other measurements; indeed, S_{CEX}^π is markedly suppressed, as indicated in Table 7.2 and by the contrast between Fig. 7.11b and Fig. 7.11a. Nevertheless, due to the complex correlations among FSI fates in the hA implementation (discussed in Sec. 7.2.1), even though S_{INEL}^π and S_{PIPD}^π are not explicitly modified, both the INEL and PIPD components for MINERvA- π^0 exhibit a considerable increase.

Suppressing both the “No Interaction” channel and CEX for π^0 brings the MINERvA- π^0 prediction into proper agreement while exerting minimal effects on other datasets. The influence of the remaining parameter modifications is more evident from the comparison of the p_N distributions in MINERvA- π^0 presented in Figs. 7.12a and 7.12b. A pronounced increase in S_{CEX}^N and S_{PIPD}^N shifts events away from the Fermi motion peak at $p_N \leq 0.25$ GeV/ c , an effect that can also be realized through an elevated R_{SRC} .

Furthermore, the larger S_{PIPD}^N gives rise to contributions from 2p2h (about 1%) and CCQE (about 2%) processes in the MINERvA- π^0 cross section, as depicted in Fig. 7.10d. Although these neutrino–nucleon interactions do not inherently produce pions, the resulting nucleons may generate pions via rescattering within the nucleus, thereby contributing to topologies that include pions. Nonetheless, the magnitude of this contribution remains small.

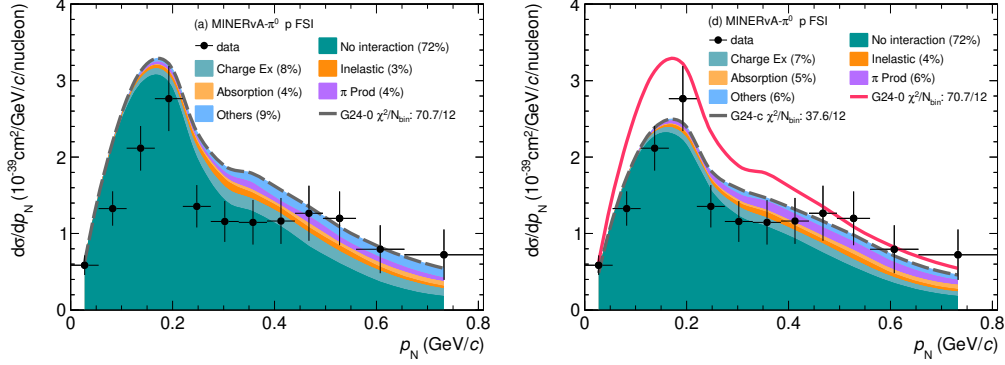


Figure 7.12: Similar to Fig. 7.11 but for p_N with proton FSI fate decomposition. Note that the “Absorption” of the proton is referring to the absorption of the π^+ from the decay of Δ^{++} , which could lead to emission of nucleons as discussed in Sec. 7.2.1.

Although the interaction models (e.g., 2p2h) are not tuned explicitly, their fractional contributions show modest adjustments in Fig. 7.9 and Fig. 7.10 compared to Fig. 7.2 and Fig. 7.3. This change is driven by the increased R_{SRC} , which results in a higher number of events with elevated initial nucleon momenta, thereby enhancing interactions that require greater initial state energy, such as RES and DIS. For example, the RES component rises from 19% in Fig. 7.3c to 24% in Fig. 7.10c for MINERvA-0 π . Consequently, the relative contributions of the other interactions are slightly reduced; in particular, the 2p2h fraction decreases from 31% to 30% for MINERvA-0 π .

7.4 Discussion

Among the 26 observable combinations considered for the AllPar and RedPar tunes, combination **Combi-Best-RedPar** in the RedPar tune yields the best result. This result offers valuable insights for future tuning efforts—both the selection of observables and the choice of parameters are critical for a successful tuning. Increasing the number of parameters and observables does not necessarily improve the outcome. Instead, it is preferable to include only those parameters that significantly affect the selected observables and only those observables that are highly sensitive to the chosen parameters while exhibiting minimal dependence on others. For example, $\delta\phi_T$ is excluded from **Combi-Best-RedPar** because it is

strongly influenced by the neutrino energy, which is not modified by any of the tuning parameters. Admittedly, practical limitations in our understanding of the model and the data can preclude this ideal. Nevertheless, optimizing the sensitivity matching between observables and model parameters is essential for achieving the best results.

The **G24-c** fit results imply extreme parameter values, yet their impact on observables is less pronounced than the magnitude of the parameter changes might suggest. For individual FSI fate cross sections, the renormalization applied after scaling causes the effective change for a given fate to be smaller than the scaling parameter alone, as detailed in Sec. 7.2.1. Regarding the total FSI cross sections, only that of π^0 is altered. Figure 7.13 displays the total π^0 -C scattering cross section as a function of the pion kinetic energy. The overall shape remains similar and consistent with Fig. 2.23 (for π^+ -C reactions) in Ref. [3], although the peak increases from 380 to 489 mb—a 29% rise—which is significantly less than the scaling factor $S_{\lambda}^{\pi^0} = 0.34$ might imply. Moreover, the default π^0 parameter values are derived from charged pion data under the assumption of isospin symmetry rather than being directly extracted from experimental measurements. Consequently, this modification does not conflict with existing agreements based on hadron scattering data. Furthermore, the overall agreement of this tune with most non-TKI neutrino

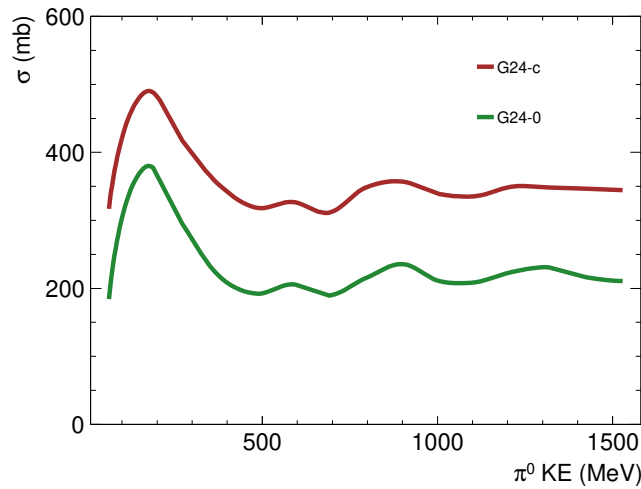


Figure 7.13: Change in MC prediction for π^0 cross section between **G24-0** and **G24-c**.

datasets remains virtually unchanged before and after tuning, thereby corroborating

the physicality of the model.

While exclusive electron scattering data would offer excellent constraints on FSI models, most current electron scattering measurements are inclusive [122] and are not ideally suited for FSI tuning. The “Electrons for Neutrinos” (e4nu) collaboration uses data from the CLAS6 spectrometer to measure exclusive final states, such as the $1p0\pi$ cross-section on carbon as a function of TKI variables [72]. However, in the context of electron-scattering pion production, **GENIE** tends to over-predict the data due to the absence of dedicated tunes based on free nucleon data. Large uncertainties in the pion production model thus obscure any underlying nuclear or FSI dependence. To resolve these effects, one must first tune electron scattering to free nucleon data. The current tuning study can be extended in the future when dedicated electron-pion production measurements become available.

In summary, this tune constitutes a valid and effective model that serves as a robust starting point for further analyses. Future refinements will be achievable as additional data, including non-TKI observables and electron-scattering measurements, are incorporated into the fit.

7.5 Summary and outlook

This work represents the first global tuning effort on TKI data. Our partial tune of the SF-CFG and hA models, **G24_20i_06_22c** (**G24-c**), yields an effective model that improves the description of both neutrino-hydrocarbon pionless and pion production interactions. The most significant modification in the model is driven by the MINERvA- π^0 TKI measurement [61], which was considerably overestimated by **GENIE** as shown in Fig. 7.2d and Fig. 7.3d. This improvement is crucial for the next generation of precision GeV neutrino experiments. This tuning configuration has been integrated into the **master** branch of **GENIE** and is scheduled for inclusion in the upcoming release.

To develop an effective model, we concentrated on the parameters most sensitive to the data, namely R_{SRC} , $S_{\lambda}^{\pi^0}$, S_{CEX}^{π} , $S_{\text{CEX}}^{\text{N}}$, $S_{\text{ABS}}^{\text{N}}$, and $S_{\text{IPD}}^{\text{N}}$, thereby reducing the total number of adjustable parameters from 14 to 6. The optimal combination of

observables comprises $\delta\alpha_T$, p_N (or δp_T when p_N is unavailable), and δp_{TT} , drawn from both pionless and pion-production measurements in T2K and MINERvA. In this approach, the pion production model remains fixed, so that the tuned values of $S_{\lambda}^{\pi^0}$, S_{CEX}^{π} , S_{ABS}^N , and S_{PIPD}^N diverge from the constraints established by hadron scattering data. A different pion production model might partially compensate for these tuning effects [123]. We also opted for the hA FSI model due to its simplicity and numerical efficiency, as noted in Ref. [101]. Future studies might consider more sophisticated alternatives. With the current set of four TKI measurements from T2K and MINERvA, alternative effective tunes, such as `G24_20i_fullpara_alt` (G24-t, have also been derived. The degeneracy among these tunes can be resolved with additional data, particularly from argon-based measurements [67–71, 124] and e4nu experiments [72].

Everything comes to him who knows how to wait.

— Wolfgang Pauli

8

Heavy Neutral Lepton Sensitivity Estimation at the SFGD

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8.1 Introduction

As a detailed review of heavy neutral lepton (HNL) would exceed the scope of this report, only a highly simplified introduction to HNL is given here. The origin of the neutrino mass is still a mystery. One intuitive solution is to extend the neutrino family to include heavy members, which are SM singlets and mix with the existing SM neutrinos via an extended PMNS matrix, thereby giving them the small masses observed via the see-saw mechanism [125]. In a simplified model, the HNL, N , is augmented to the SM neutrino family. The neutrino flavour eigenstates, ν_α , can then be expressed as a linear combination of the SM neutrino

mass eigenstates and the HNL:

$$\nu_\alpha = \sum_{i=1}^3 U_{\alpha i} \nu_i + U_{\alpha N} N, \quad (8.1)$$

where $\alpha = e, \mu, \tau$, ν_i are the SM neutrino mass eigenstates, the $U_{\alpha i}$ are the elements of the PMNS matrix¹, and $U_{\alpha N}$ is the new matrix entry describing the mixing of the HNL with the SM neutrinos to form the flavour eigenstates. Due to this mixing with the SM neutrinos, when an SM neutrino is produced, e.g. ν_μ , it has a probability of $|U_{\alpha N}|^2$ to propagate as N . Hence, the intense neutrino beam in the long-baseline neutrino experiments can produce an intense flux of HNL, so the near detector of these experiments is a good place to look for HNL. As N is an SM singlet, it does not interact with the target materials in the detectors. Its presence can only be inferred from its decay into SM particles, for example, $N \rightarrow l + \pi$, where l is a lepton, such as an electron or a muon. The probability of the HNL decaying into a lepton, α , is also proportional to the mixing element, $|U_{\alpha N}|^2$. These decay products can be detected by the sophisticated near detectors. Moreover, as the HNL signals come only from the HNL decay instead of interactions with the target materials, the detectable signal rate is proportional to the detector volume, while the SM neutrino background rate is proportional to the target mass. Hence, the ideal detector for HNL searches will have good detection capabilities with a large volume and a low density. This is why T2K has searched for HNL [126] using the gaseous TPCs, a combined volume of about 7 m³, but not the scintillation detector, FGD, which has a considerably higher density and thus a significant background rate. As the ND280 upgrade replaces the P0D with SFGD and HAT, a combined volume of 8 m³, which are much better trackers than the P0D, they could be used in an HNL search as well. Since the HATs are gaseous TPCs as well, they are expected to have at least comparable performance to the TPCs. Although the SFGD is a scintillation detector, it has improved reconstruction capabilities. If it is capable of sufficiently suppressing the background rate, it can also be used in an HNL search, thereby improving the

¹They should have values close to the SM PMNS matrix elements, which has unitarity imposed.

overall sensitivity of the T2K HNL search. Hence, the goal of this chapter is to demonstrate the feasibility of performing an HNL search using the SFGD.

To perform an HNL search, a complicated chain of simulations is required, including the generation of an HNL flux based on the SM neutrino flux, the propagation of HNL and the decay of HNL. The previous T2K search was based on a dedicated tool-set, `nd280HNLSim`, which was built only to search for $N \rightarrow l + \pi$ decay channels, where l is a lepton. It would require a large amount of efforts to extend it to include other production and decay channels. Hence, I adapt the recently published general HNL package, `BeamHNL` [127], to the T2K ND280 simulation to evaluate the sensitivity improvement brought by the upgrade.

As the global reconstruction and systematics for the upgraded ND280 is not yet available, the result presented in this section is to demonstrate the successful implementation of the complete sensitivity estimation workflow, including the HNL simulation and selection, the background estimation, and the sensitivity extraction. For this purpose, the $N \rightarrow \mu + \pi$ channel is chosen due to its similar event topology to the $\nu_\mu \text{CC}1\pi1p$ selection presented in Sec. 5.2. Moreover, the MC statistics used are relatively low for a quick demonstration of the implementation rather than a comprehensive sensitivity study, which does require higher statistics but would only be possible when the ND280 upgrade software is sophisticated enough to include as many decay channels as possible and to allow a comprehensive background estimation.

8.2 Implementation

The simulation of HNL production and decay using `BeamHNL` is performed in several steps. Since `BeamHNL` is designed to be a cross-experiment package, it requires a standardized flux input in the `dk2nu` format. The `dk2nu` file is a simple flat-tree ROOT file that must contain the minimally sufficient information for `BeamHNL` to function correctly. As the T2K flux is available in its own specific format, a conversion is necessary. The required `dk2nu` variables and the corresponding conversion from variables in a T2K flux file are provided in Appendix D.1.

Using this input, the desired HNL production channels can be implemented by specifying a few additional parameters in a configuration `xml` file, `CommonHNL.xml`, located within the `GENIE` configuration directory. These parameters are experiment-specific and must be set by the user. Details on how each parameter is set specifically for the T2K ND upgrade are provided in Appendix D.2.

Before proceeding to subsequent steps, we validate the setup by feeding the T2K `dk2nu` file to `BeamHNL` and setting the HNL mass, M_N , to a small value, such as 1 keV. The output HNL flux should closely mimic the SM flux. The result is shown in Fig. 8.1, where the orange dashed line ($M_N = 1$ keV) closely overlaps with the black solid line, which is directly plotted from the T2K input flux. After validation,

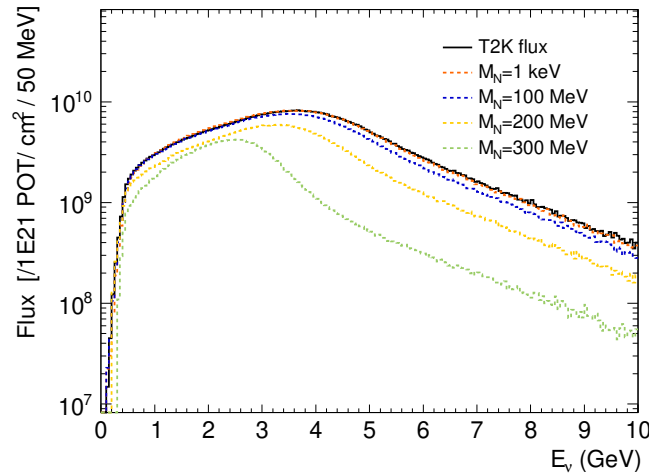


Figure 8.1: HNL fluxes produced by `BeamHNL` for different M_N . The solid line is directly plotted from T2K input flux files. The dashed lines are `BeamHNL` outputs for different M_N . The $M_N = 1$ keV curve agrees well with the T2K flux, validating the output. The curves for larger M_N displaying apparent shape differences.

the HNL mass is set to target values, e.g., 300 MeV, to generate HNL events, which are then passed through the T2K simulation chain for selection development.

With the successful interfacing between `BeamHNL` and the ND280 simulation, I proceeded to develop the selection for the $N \rightarrow \mu + \pi$ channel. The steps are as follows:

1. Basic T2K Event Quality Checks
2. Find SFGD Primary Vertex

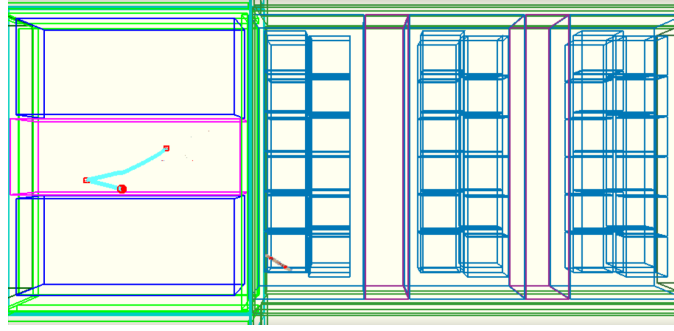


Figure 8.2: A $N \rightarrow \mu + \pi$ event display in the upgraded ND280.

3. Primary Track Count Cut
4. Escaping Muon Cut
5. Single Trackless Pion Cut
6. Kink Cut
7. μ - π Angle and Invariant Mass Cut
8. μ - π δp_T Cut

Basic T2K Event Quality Checks - A quality check ensures the event contains at least one reconstructed track.

Find SFGD Primary Vertex - Identify a primary vertex in the SFGD. Since the primary vertex for an HNL event does not necessarily include a muon, as in the ν_μ CC selection, the primary vertex should be identified independently from the primary muon. All vertices are iterated through to record the number of tracks connected to each vertex, n_{trk} , and the length of the longest track connected to each vertex, L_{max} . The vertex with the longest L_{max} is selected as the primary vertex. If more than one such vertex exists, the vertex with the largest n_{trk} is selected. If there is still more than one vertex, the vertex with the earliest time is selected.

Primary Track Count Cut - For the target channel $N \rightarrow \mu + \pi$, there should be at most two primary tracks. Therefore, events with $n_{trk} > 2$ are rejected.

Escaping Muon Cut - This step identifies muons that escape the SFGD.

Single Trackless Pion Cut and Kink Cut - These cuts are identical to those in the $\nu_\mu\text{CC1}\pi$ -Trackless selection, selecting a primary pion that has not undergone secondary interactions to ensure accurate reconstruction of its kinematics.

μ - π Angle and Invariant Mass Cut - These are kinematic cuts that exploit the fact that SM events with one muon and one pion are mostly resonance events that have one more proton than HNL decays. Due to the additional proton, the opening angle between the μ track and the π track, $\theta_{\mu\pi}$, can vary widely, whereas in HNL decays, the angle is more collimated. Similarly, the invariant mass of the μ - π system, $M_{\mu\pi} = (p_\mu + p_\pi)^2$, has a larger range compared to the HNL, which centres around the input value of 300 MeV, as shown in Fig. 8.3a. The stark contrast is apparent in Fig. 8.3. To preserve the largest number of HNL events, the specific cuts applied are: $30^\circ < \theta_{\mu\pi} < 90^\circ$ and $270 \text{ MeV} < M_{\mu\pi} < 320 \text{ MeV}$. For a full sensitivity study over a wide range of HNL masses, the cut on $M_{\mu\pi}$ needs to be relaxed, and the associated impact will be investigated in the future. Note that the number of HNL events reaching this cut as shown in Fig. 8.3a is relatively small, as this selection effectively includes only muons escaping from the SFGD and travelling into the vertical TPC. The statistics will increase when other sub-samples including muons in the other subdetectors are included when the global reconstruction is available.

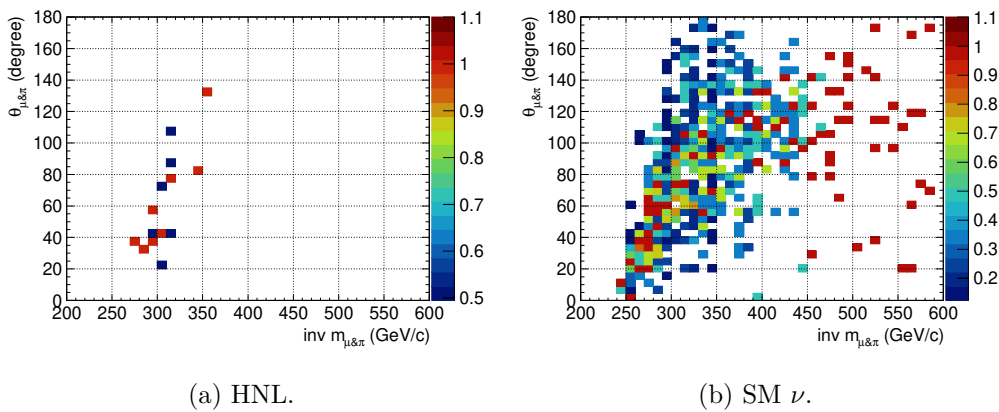


Figure 8.3: $\theta_{\mu\pi}$ plotted against $M_{\mu\pi}$. The HNL events are concentrated near the rest mass of the HNL, while the SM events are more widely distributed.

μ - π δp_T Cut - Similar to previous kinematic cuts, the net transverse momentum of μ and π should be small, while for SM events it has a much larger range. To

determine the appropriate cut value, the histograms of the net transverse momentum, $\delta p_{T,\mu\&\pi}$, are plotted for both HNL and SM events, as shown in Fig. 8.4. As the goal is to select HNL events while rejecting SM backgrounds, the histograms are only plotted for small $\delta p_{T,\mu\&\pi}$ values, i.e., $\delta p_{T,\mu\&\pi} < 50$ MeV/ c . The majority of SM events, with large $\delta p_{T,\mu\&\pi}$, is thus not plotted in Fig. 8.3b. In this particular case, applying the cut $\delta p_{T,\mu\&\pi} < 15$ MeV/ c will retain eight HNL events and reject all SM backgrounds. A more realistic background estimation is given in the next subsection.

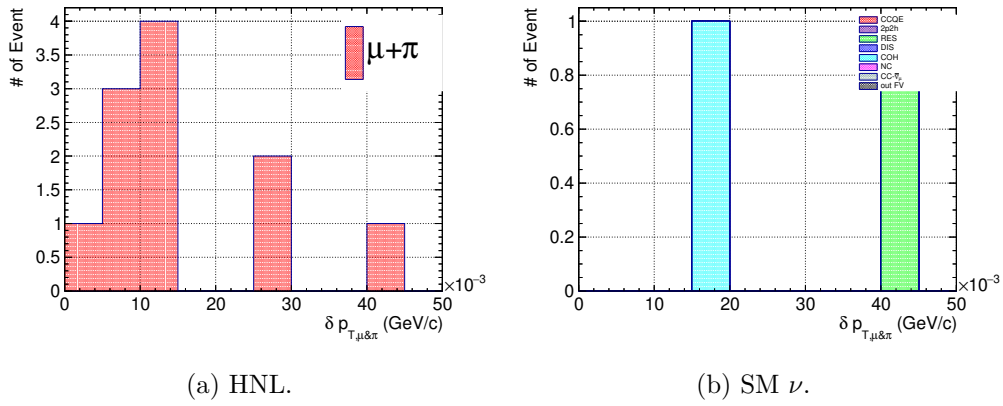


Figure 8.4: $\delta p_{T,\mu\&\pi}$ distributions for HNL and SM ν . HNL has significantly more events at small $\delta p_{T,\mu\&\pi}$ values compared to SM ν . The cut at 15 MeV/ c retains eight HNL events and rejects the remaining SM backgrounds. However, this does not imply that this selection is background-free. A more realistic background estimation is given in the Sec. 8.3.

8.3 Background estimation

The previous T2K HNL search found that the coherent pion production is the dominant source for the $\mu\pi$ channel [126]. Hence, this background source is chosen in this subsection to demonstrate the background estimation procedure, while a comprehensive background estimation will be performed in the future when the upgrade software is ready.

The $\delta p_{T,\mu\&\pi}$ cut shown in Fig. 8.4b removes all coherent background. However, the zero event is partly due to the small statistics used in this preliminary study. To properly estimate the coherent background, the full distribution of $\delta p_{T,\mu\&\pi}$ before the last cut is fitted with a Cauchy distribution as shown in Fig. 8.5. The number of

expected background events is obtained by integrating the fitted curve from 0 to the cut value, i.e. 15 MeV/ c , which is evaluated to be 0.33. Although the fit quality is not perfect due to the low statistics, the procedure is still valid and can be applied to the full statistics when available. This background estimation procedure has not taken into account the systematic uncertainties, which needs to be evaluated by running toy simulations when the nd280 upgrade software is ready.

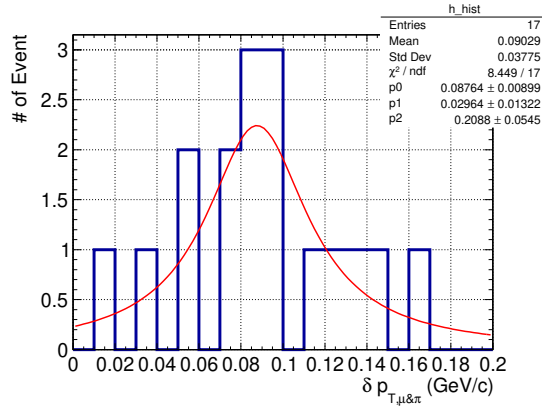


Figure 8.5: distribution fit for coherent pions surviving until the cut. The background is estimated by integrating the fitted curve from 0 to the cut value, 15 MeV/ c , which is evaluated to be 0.33.

8.4 Sensitivity extraction

Sensitivity is extracted using the CL_s method [128]. CL_x is defined as $\text{Prob}(N \leq N_{\text{obs}} | \mu = x)$, i.e., the probability of predicting a number of events smaller than or equal to the number of observed events assuming model x . When $x = b$ or $x = s + b$, it corresponds to the background-only model or the signal plus background model, respectively. Then, CL_s is defined as

$$CL_s = \frac{CL_{s+b}}{CL_b} = \frac{\text{Prob}(N \leq N_{\text{obs}} | \mu = s + b)}{\text{Prob}(N \leq N_{\text{obs}} | \mu = b)} = \alpha, \quad (8.2)$$

where $1 - \alpha$ is the confidence level.

For a target confidence level, f , one starts from $s = 0$ and iterates with increasing s to find the largest s_{up} such that $\alpha < 1 - f$. Then, s_{up} is the upper limit for the number of signal events at confidence level f such that the background-only

model is not rejected. As for the value of b , it is conventional to use the median number of background events to evaluate s_{up} . After the s_{up} is obtained, the limit on the desired parameter is extracted from its relation to s_{up} . In HNL searches, the parameter of interest is the mixing element $U_{\alpha N}$, where α is the lepton flavour of the decay products, i.e., μ or e . The production and decay of HNL are both proportional to the square of the mixing element. As the T2K flux is dominated by muon neutrinos and the decay channel explored is $N \rightarrow \mu + \pi$, the number of expected signal events is proportional to $|U_{\mu N}|^4$. Since only the squared magnitude of the mixing element can be extracted from the number of events, it is conventional to define $U_\alpha = |U_{\alpha N}|^2$ as the parameter of interest.

Specifically, for demonstration, the background is taken to be $b = 0.33$ as evaluated in the previous subsection, leading to $s_{up} = 2.3$ following the above CL_s method. The number of observed events is $N_{\text{obs}} = 8$, which is the number of HNL events surviving all cuts in Sec. 8.2. However, one caveat is that the number of protons on target (POT) used to produce the HNL events is calculated from the BeamHNL output, which is approximately 0.81×10^{21} , while the background is estimated from a POT of 1.0×10^{21} . Hence, the number of expected selected signal events will be $8/0.81 \approx 9.9$ for 10^{21} POT. This number of HNL events is generated with a mixing element of $U_\mu = 10^{-7}$, so the corresponding limit, U_{up} , on the mixing element, U_μ , for observing $s_{up} = 2.3$ events is calculated as

$$\left(\frac{U_{up}}{10^{-7}}\right)^2 = \frac{2.3}{9.9} \quad (8.3)$$

$$U_{up} = 10^{-7} \times \sqrt{\frac{2.3}{9.9}} = 4.8 \times 10^{-8}. \quad (8.4)$$

8.5 Discussion

This chapter has demonstrated the framework for the HNL sensitivity estimation using the SFGD has been successfully implemented. The crude sensitivity extracted from SFGD alone is approximately one order less sensitive the limit placed by the previous search, $U_{up} \approx 4 \times 10^{-9}$, for $M_N = 300$ MeV. There are two potential reasons for the lower sensitivity. Firstly, the current result includes only HNL decaying in

SFGD, which has only about $1/3$ of the volume compared to the vertical TPCs, leading to at least a factor of $\sqrt{3} \approx 1.73$ less sensitivity. Secondly, the selection only includes the muons escaping into the vertical TPC, while a considerable number of muons can escape into the HAT, so the sensitivity will increase when the HAT is included in the global reconstruction for selection development. While the background estimation appears optimistic, it is the signal to background ratio that is crucial for the sensitivity. As the MC statistics used is relatively low, the estimated number of signal events after the selection is susceptible to large statistical fluctuations. Hence, the overall impact of including the SFGD to the HNL search can only be reliably assessed with a larger sample and a full-scale background estimation when the ND280 upgrade software is ready.

The next step is to extend the selection to the entire upgraded ND280 when the HAT reconstruction becomes available and to investigate the overall improvement in HNL sensitivity brought by the upgrade. Fig. 8.6 shows a $N \rightarrow \mu + \pi$ event display in the Top HAT, demonstrating that the simulation of production and decay in HAT is successful. Unfortunately, the global reconstruction is not yet stable as

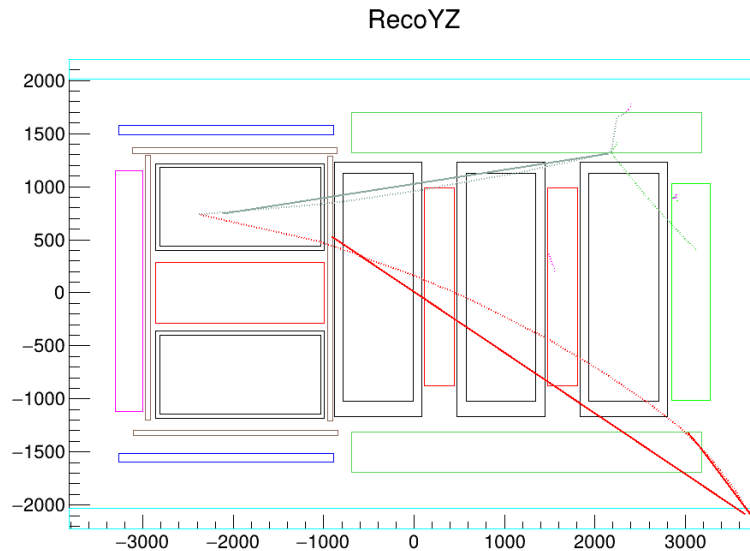


Figure 8.6: A $N \rightarrow \mu + \pi$ event display in the Top HAT. The solid lines are reconstructed tracks, and the dashed lines are true tracks. The red tracks correspond to a muon, and the grey tracks correspond to a pion. The pion is reconstructed satisfactorily, whereas the muon starting point is mis-reconstructed between the Top HAT and the first vertical TPC.

shown by the mis-reconstruction of the muon in Fig. 8.6, so the sensitivity study of the full upgraded ND280 has to be reserved for future studies.

回首向来萧瑟处，归去，也无风雨也无晴。

After all that has been through, at the end of the day,
nothing matters, and everything will be alright.

— Shi Su Ding Feng Bo

9

Epilogue

Back in October 2022, I traveled to Tokai to join the task force responsible for assembling the SFGD. We began by constructing the outer box to hold the cubes together, and the first layer was installed on October 28th. For approximately two months, we meticulously stacked the scintillator cubes layer by layer, aligning each new layer with great effort. As the number of layers increased, maintaining vertical alignment became progressively more challenging due to the growing friction between the accumulating cubes. Initially, vertical alignment was achieved using thin metal rods approximately 10 cm in length, which soon proved too short. The original plan was to utilize fishing lines for subsequent alignments; however, it quickly became evident that they lacked the necessary strength to shift the cubes in alignment. Fortunately, we were able to purchase longer metal rods in time, preventing the need to undertake an impossible task. The final layer was added in December just before the new year.

After a short break, we proceeded to the next task: fibre insertion. With more team members joining, the workforce became sufficiently large and the workflow sufficiently independent to allow for a clear division of labour. Some team members were responsible for removing the fishing lines that held the cubes together during layer installation, others pushed the optical fibres through the designated holes where the fishing lines had been removed, and some maintained a constant supply

of fibres for the fibre pushers. The fibre insertion process took an additional two months. Afterward, I returned to focus on my analysis work while the electronics were diligently added and calibrated. The assembly was finally completed in October 2023 and was ready to be installed in the ND280 pit. In June 2024, the entire ND280 upgrade was completed and began taking data for the first time. The data event display images presented in this thesis were collected during the June 2024 run.

As a doctoral student, assembling a detector and witnessing its functionality was a truly unique and fortunate experience. Although it is unfortunate that there was not enough time to perform a full cross-section measurement using the new data, my Monte Carlo (MC) analysis and the preliminary data-MC comparisons demonstrate the far-reaching physics potential of the upgraded ND280. The upgraded detector will be capable of measuring the TKI and COM variables with unprecedented precision. My tuning project, utilizing existing TKI measurements, already shows promising improvements in neutrino interaction modelling. It is thus exciting to anticipate the advancements in interaction modelling that future TKI and COM measurements of ND280 will bring. Beyond advancing SM physics, my HNL sensitivity study of the SFGD demonstrates that it possesses comparable HNL detection capabilities to the gaseous TPCs. Consequently, the entire upgraded ND280 boasts a detection volume nearly twice its previous size. Combined with the forthcoming beamline upgrade, T2K has the potential to conduct the most sensitive searches for HNLs produced from kaon and pion decays.

New and upgraded experiments, such as JUNO and SBND, around the world have either commenced operations or are about to do so. Hyper-K and DUNE are also under active construction. It is optimistic that we will achieve a sufficient understanding of neutrino interactions, enabling next-generation LBL experiments to make precise measurements of δ_{CP} within our lifetime. I hope that my work can contribute to shortening this timeline, even if only slightly, and when that day arrives, I will be gratified.

Appendices

行至水穷处，坐看云起时。

Stroll to the end of the river, sit down, and watch
the clouds rise.

— Wei Wang Zhong Nan Bie Ye



New Reconstruction Technical Details

Contents

A.1 Track number cuts in pion trackless selection	136
A.2 Pull distributions for the stopping pion bottom HAT sub-sample	140

A.1 Track number cuts in pion trackless selection

There are a few track number variables that are helpful in reducing background events for the $\nu_\mu \text{CC}1\pi$ selection. These are:

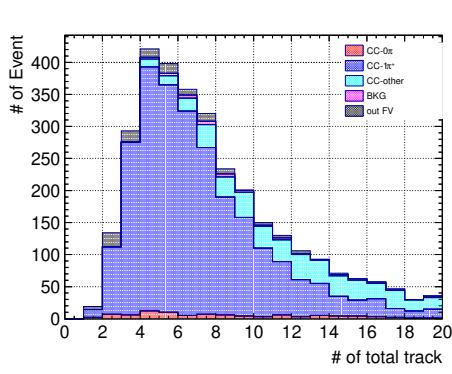
1. N_{tot} , The total number of reconstructed tracks.
2. $N_{\text{S-tot}}$, The total number of reconstructed tracks that have a segment in SFGD.
3. N_{delayed} , The number of long delayed tracks.
4. N_{pri} , The number of primary tracks.
5. $N_{\text{non-pri}}$, The number of non-primary tracks.
6. N_{lone} , The number of lone tracks.
7. N_{gen} , The number of generations.

A common feature for these track numbers is that they should not be too large, which should correspond to DIS events. Hence, an upper cut is implemented on them. Furthermore, a lower cut, $N_{\text{S-tot}} > 2$, is useful in cutting away out-of-fiducial volume (OOFV) backgrounds for the TPC- μ sample. This is understandable as a $\nu_\mu\text{CC}1\pi$ event should have the primary muon, the primary pion and most likely, the primary proton as well, thereby leading to at least 3 reconstructed tracks in SFGD. As for the SFGD- μ event, additional lower cuts on N_{delayed} , N_{pri} and N_{tot} are effective in removing CC0 π backgrounds. The presence of the CC0 π background could be due to the mis-identification of a proton track as a muon track and the ME from the muon is mis-identified as the ME from the pion. These events are likely to have only fewer primary tracks and total number of tracks, so lower cuts on these track number could help to remove such backgrounds.

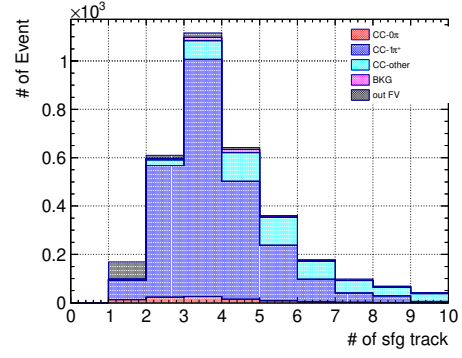
Fig. A.1 shows the distributions of the track number variables for the TPC- μ sub-sample before making any track number cuts. The selection purity is shown in the topological decomposition of the p_π distribution in Fig. A.1h. After some trials and errors and investigating the impact of cuts on a single variable stepwise, the following cuts for the TPC- μ sub-sample are applied:

1. $N_{\text{tot}} < 12$.
2. $1 < N_{\text{S-tot}} < 7$.
3. $N_{\text{pri}} < 5$.
4. $N_{\text{non-pri}} < 5$.
5. $N_{\text{lon}} < 5$.

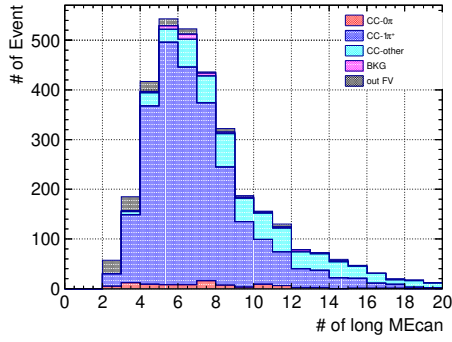
The results of these cuts are shown in Fig. A.2. It turned out that after making the above cuts, further cuts on N_{delayed} and N_{gen} are not necessary. As shown in Fig. , the purity of the $\nu_\mu\text{CC}1\pi$ selection has improved from 74.1% to 87.5% after making the track number cuts, while the number of signal events decreases from 2496 to 2044. Although the decrease in the signal number is about 18%, the



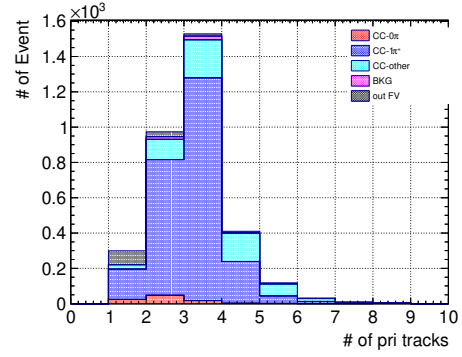
(a) The total number of tracks in the event.



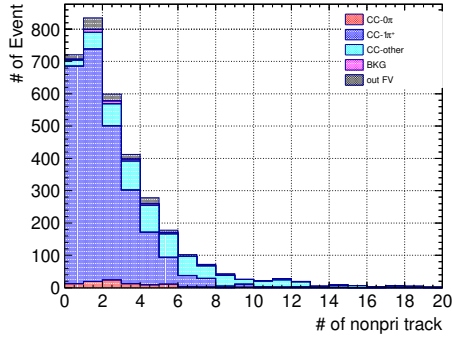
(b) The total number of tracks with a segment in SFGD.



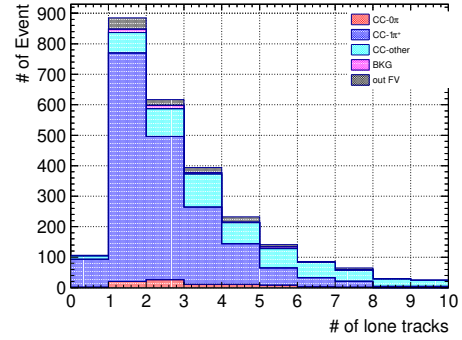
(c) The number of delayed tracks.



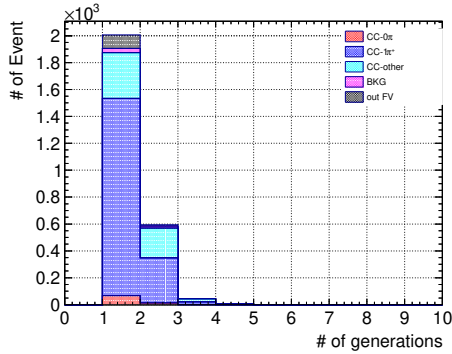
(d) The number of primary tracks.



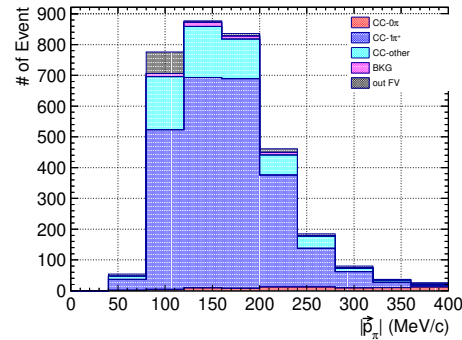
(e) The number of non-primary tracks.



(f) The number of lone tracks.

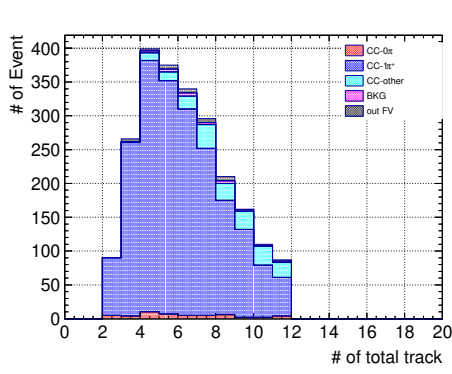


(g) The number of generations.

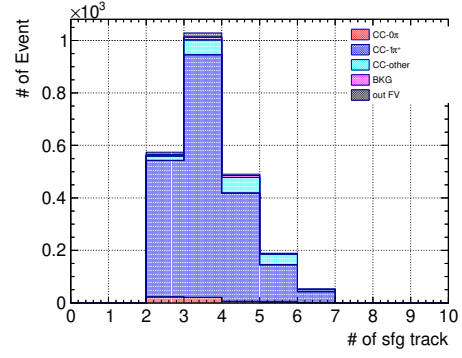


(h) The total number of tracks in the event.

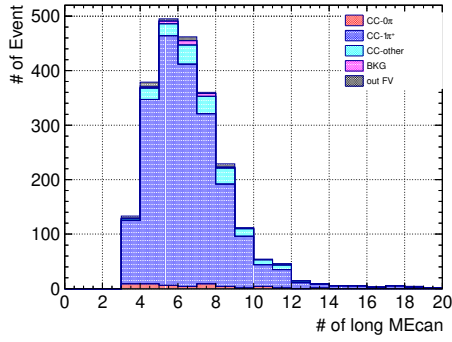
Figure A.1: Track number distributions for the TPC- μ sample.



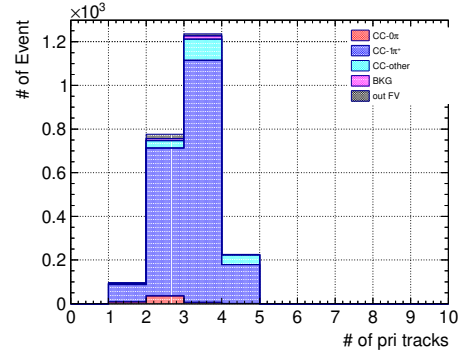
(a) The total number of tracks in the event.



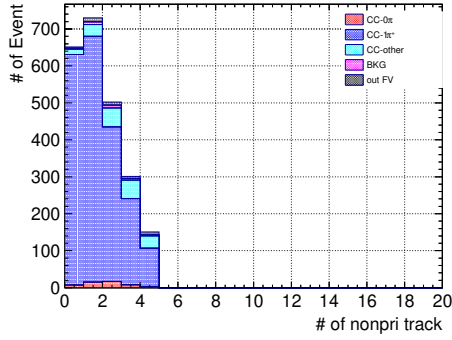
(b) The total number of tracks with a segment in SFGD.



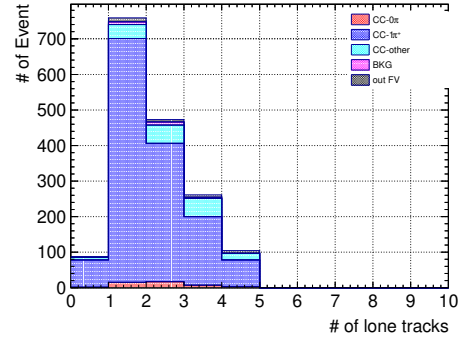
(c) The number of delayed tracks.



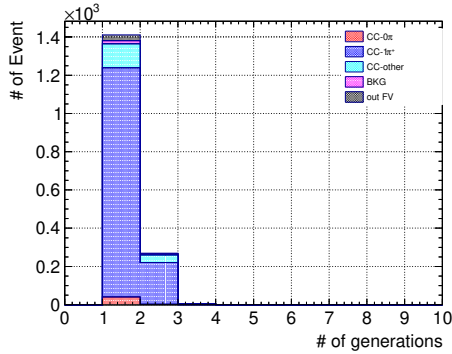
(d) The number of primary tracks.



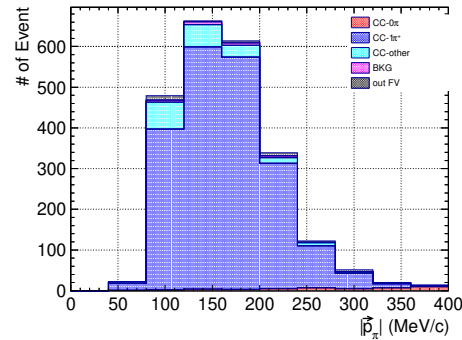
(e) The number of non-primary tracks.



(f) The number of lone tracks.



(g) The number of generations.



(h) The total number of tracks in the event.

Figure A.2: Track number distributions for the TPC- μ sample after making the track number cuts.

remaining number of events is still large enough for the analysis. The background events are reduced by 66.7% from 874 to 291.

A similar investigation is also performed for the SFGD- μ sub-sample. The figures before and after the track number cuts are shown in Fig. A.3 and Fig. A.4.

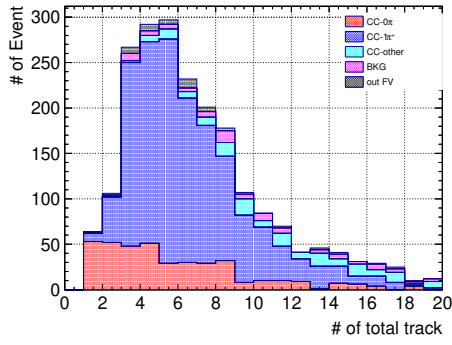
The cuts for the SFGD- μ sub-sample are:

1. $1 < N_{\text{tot}} < 15$.
2. $1 < N_{\text{S-tot}} < 7$.
3. $N_{\text{delayed}} > 4$.
4. $1 < N_{\text{pri}} < 6$.
5. $N_{\text{non-pri}} < 6$.
6. $N_{\text{lonc}} < 6$.
7. $N_{\text{gen}} < 3$.

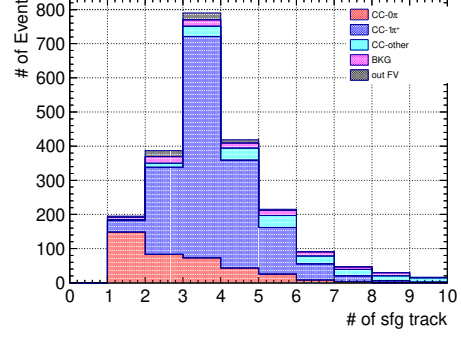
The purity for this sub-sample has improved from 65.9% to 80.7% after making the track number cuts, while the number of signal events decreases from 1464 to 1257, corresponding to a 14.1%. The background events are reduced by 60.3% from 758 to 301.

A.2 Pull distributions for the stopping pion bottom HAT sub-sample

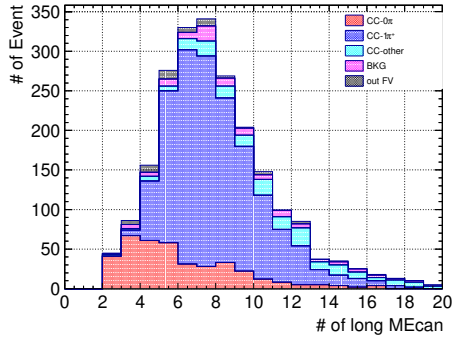
The final distributions of the pull variables and the heavy length for the stopping pion bottom HAT sub-sample are shown in Figure A.5. It demonstrates that the applied cuts for these variables are near optimal. A stricter cut could be put on the heavy length, but this finer cut is not necessary at this stage as the HAT reconstruction and the global matching are still being improved.



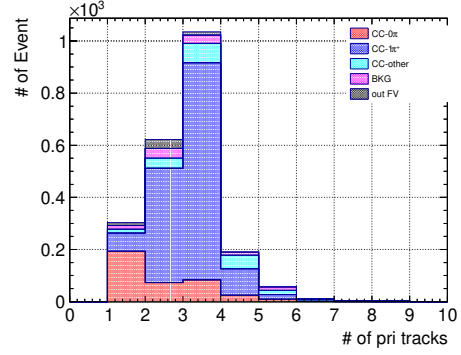
(a) The total number of tracks in the event.



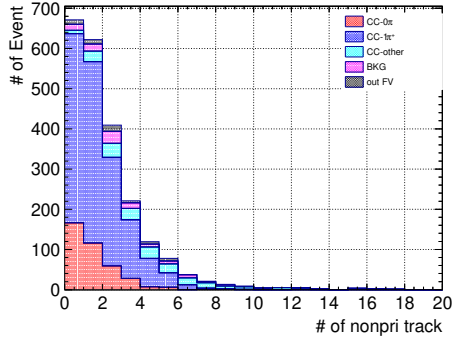
(b) The total number of tracks with a segment in SFGD.



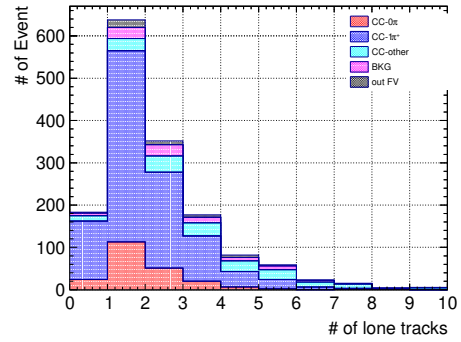
(c) The number of delayed tracks.



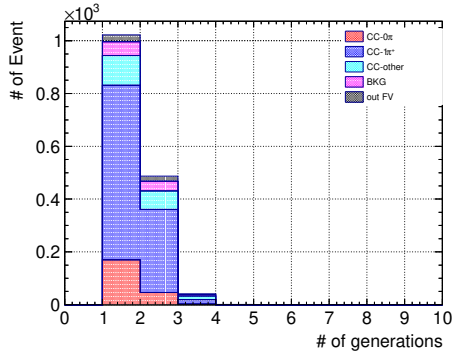
(d) The number of primary tracks.



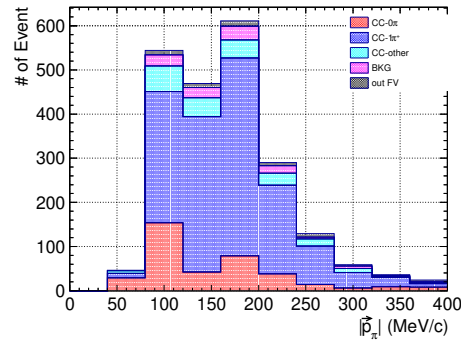
(e) The number of non-primary tracks.



(f) The number of lone tracks.

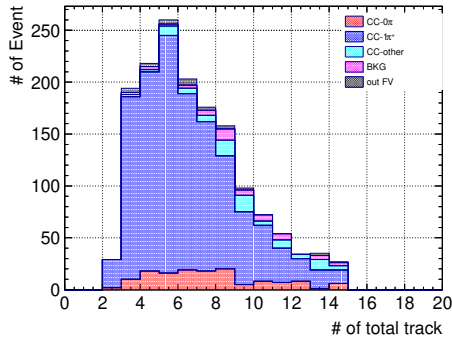


(g) The number of generations.

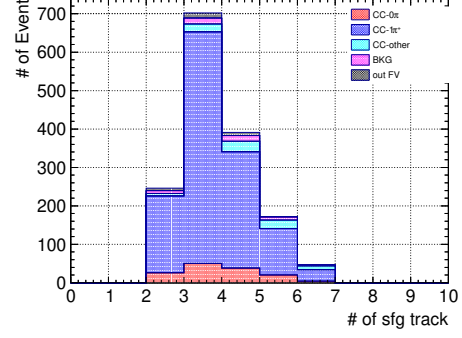


(h) The total number of tracks in the event.

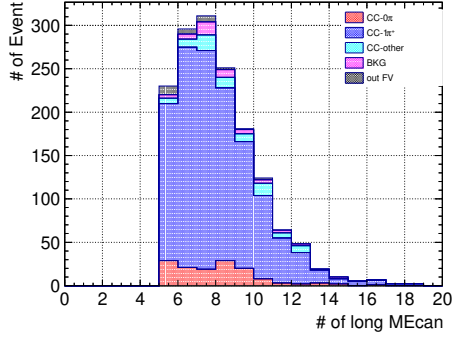
Figure A.3: Track number distributions for the SFGD- μ sample.



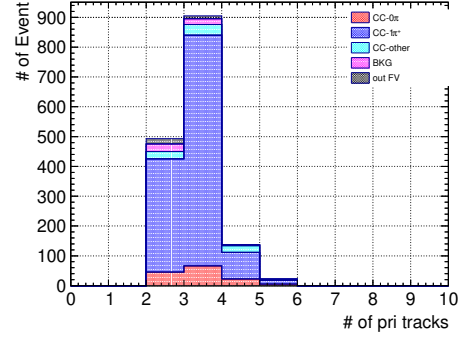
(a) The total number of tracks in the event.



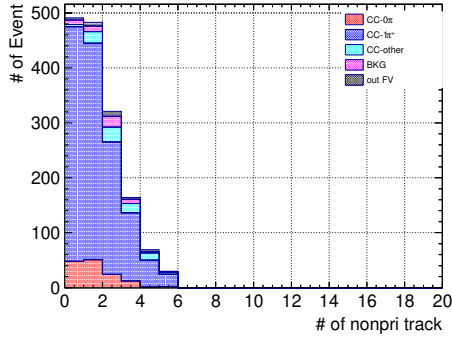
(b) The total number of tracks with a segment in SFGD.



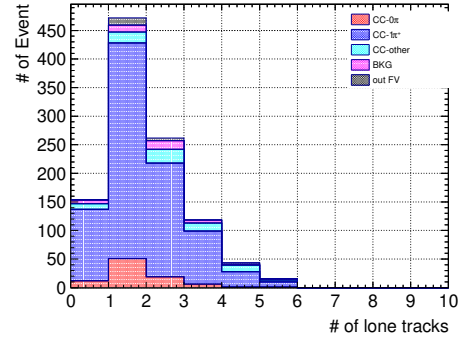
(c) The number of delayed tracks.



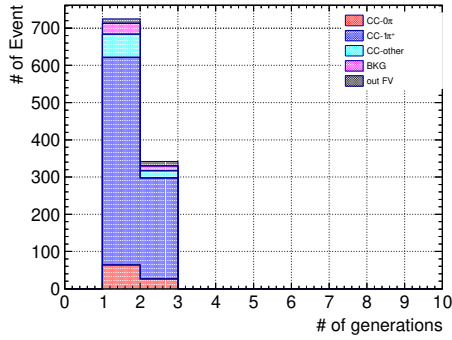
(d) The number of primary tracks.



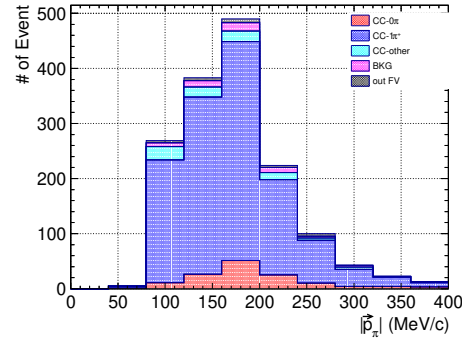
(e) The number of non-primary tracks.



(f) The number of lone tracks.

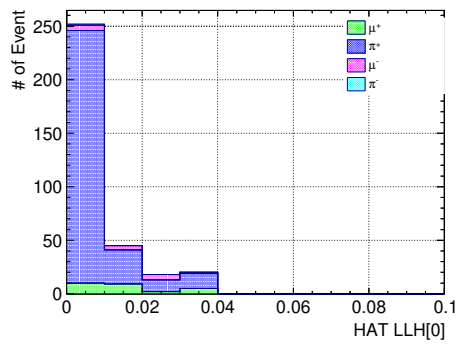


(g) The number of generations.

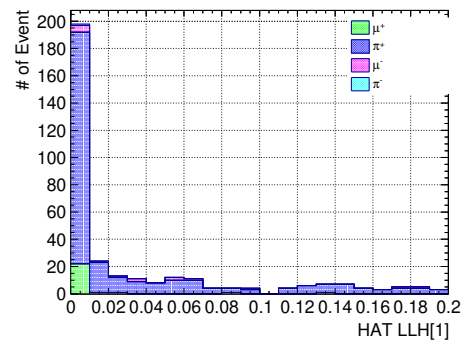


(h) The total number of tracks in the event.

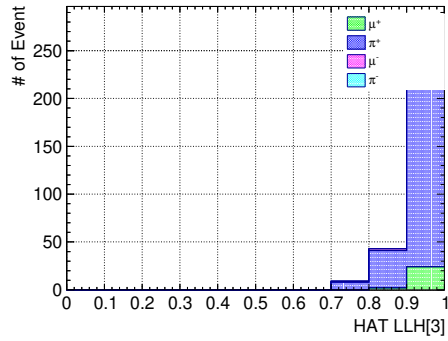
Figure A.4: Track number distributions for the SFGD- μ sample after making the track number cuts.



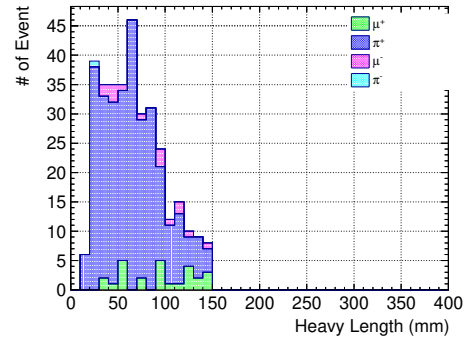
(a) The first pull.



(b) The second pull.



(c) The fourth pull.



(d) Heavy length.

Figure A.5: Pull distributions and a distrution of heavy length at HAT with a cut on the first pull.

Der Mensch kann tun was er will; er kann aber nicht
wollen was er will

Man can do what he wills but he cannot will what
he wills.

— Arthur Schopenhauer On The Freedom Of The
Will

B

TKI comparisons for the SFGD- μ sub-sample for the ν_μ CC0 π 1p selection

Figs. B.1 and B.2 provide the response matrix plots and the resolution plots for the ν_μ CC0 π 1p TKI variables of the SFGD- μ sub-sample respectively. The comparison is between the performance before the ESC technique and that after the ESC technique, including the Bragg peak cut and the transverse momentum bias correction.

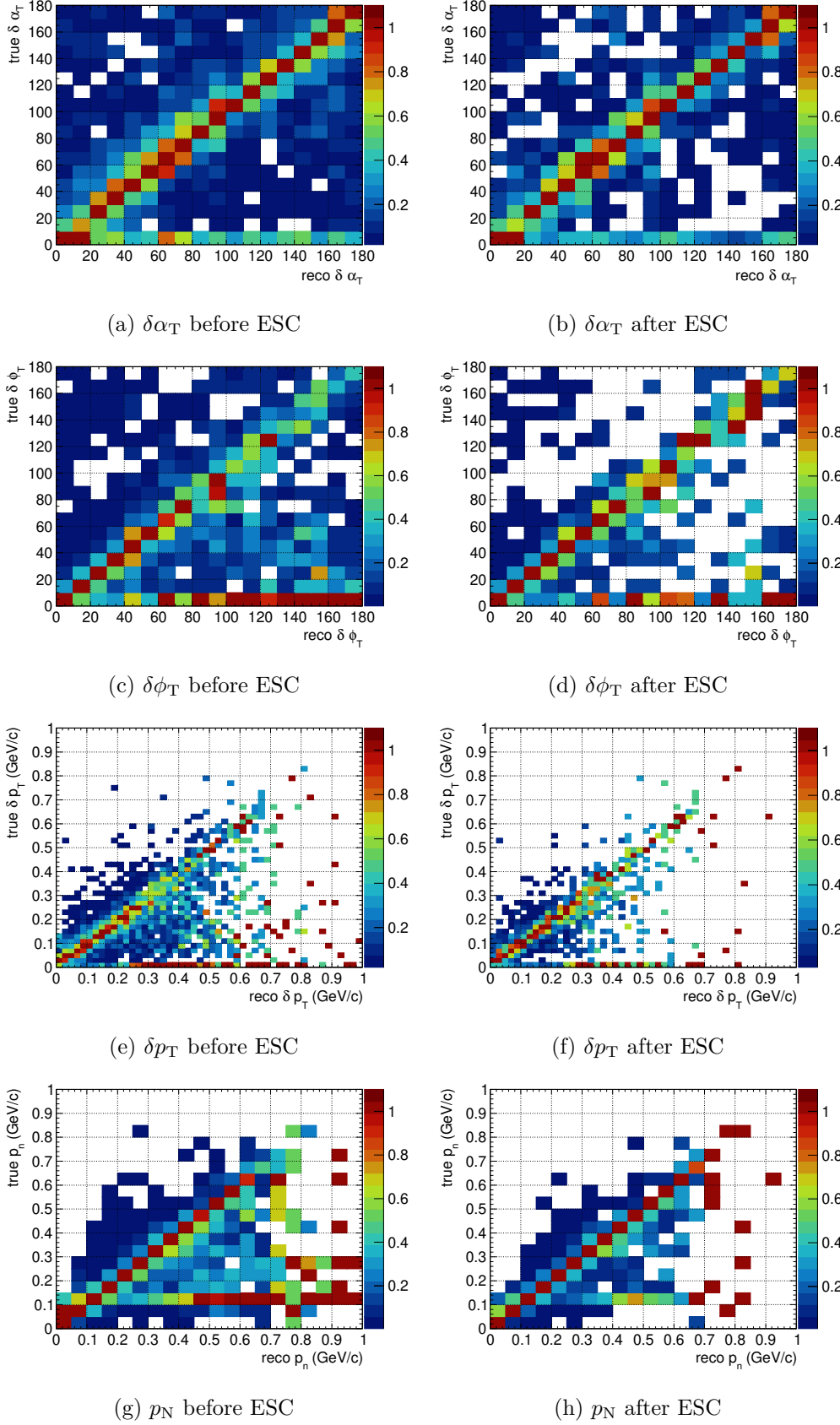


Figure B.1: TKI variables before and after ESC for the ν_μ CC0 π 1p selection for the SFGD- μ sub-sample.

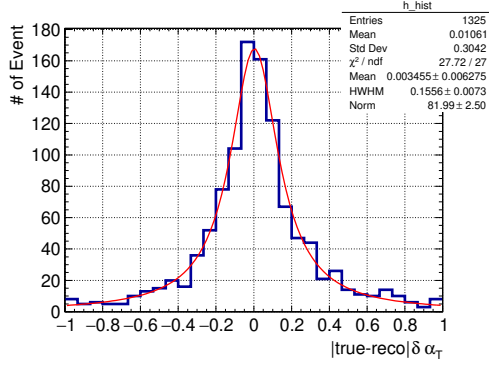
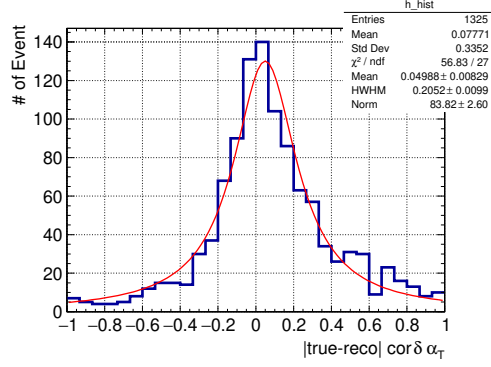
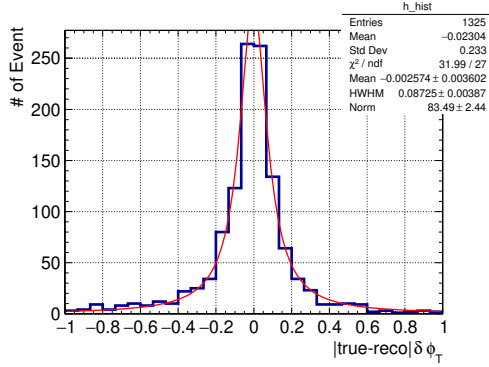
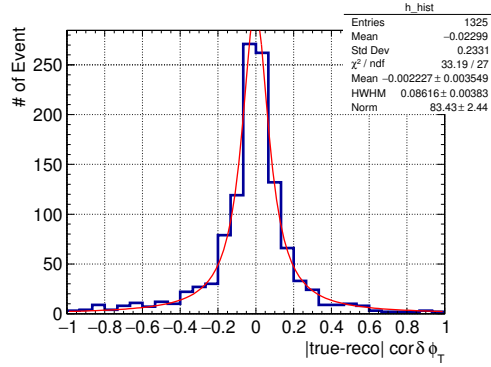
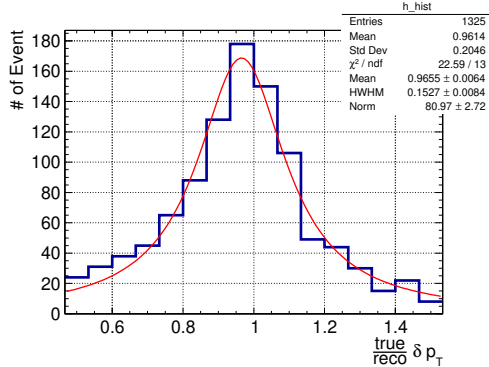
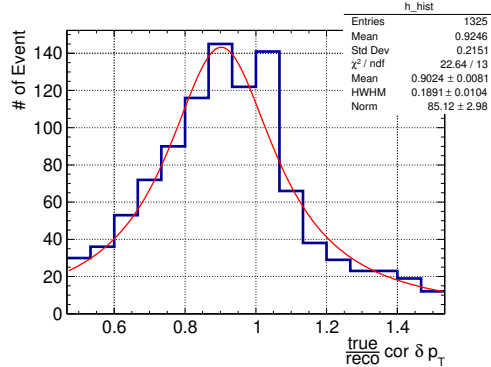
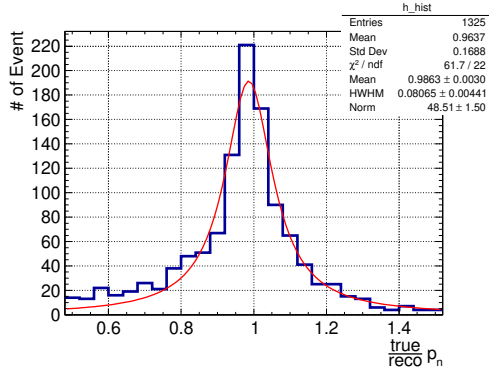
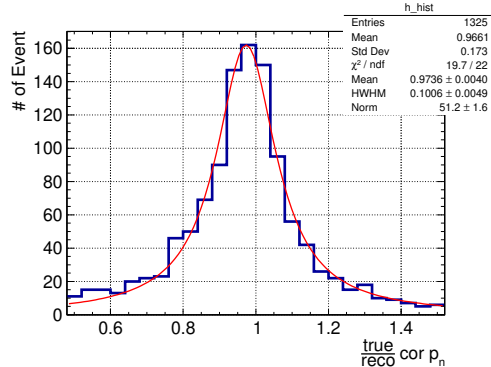

 (a) $\delta\alpha_T$ before muon bias correction

 (b) $\delta\alpha_T$ after muon bias correction

 (c) $\delta\phi_T$ before muon bias correction

 (d) $\delta\phi_T$ after muon bias correction

 (e) δp_T before muon bias correction

 (f) δp_T after muon bias correction

 (g) p_N before muon bias correction

 (h) p_N after muon bias correction

 Figure B.2: TKI variables before and after muon bias correction for the ν_μ CC0 π 1p selection for the SFGD- μ sub-sample.

欲说还休，却道天凉好个秋。

Life is ... like ... well ... the weather here.

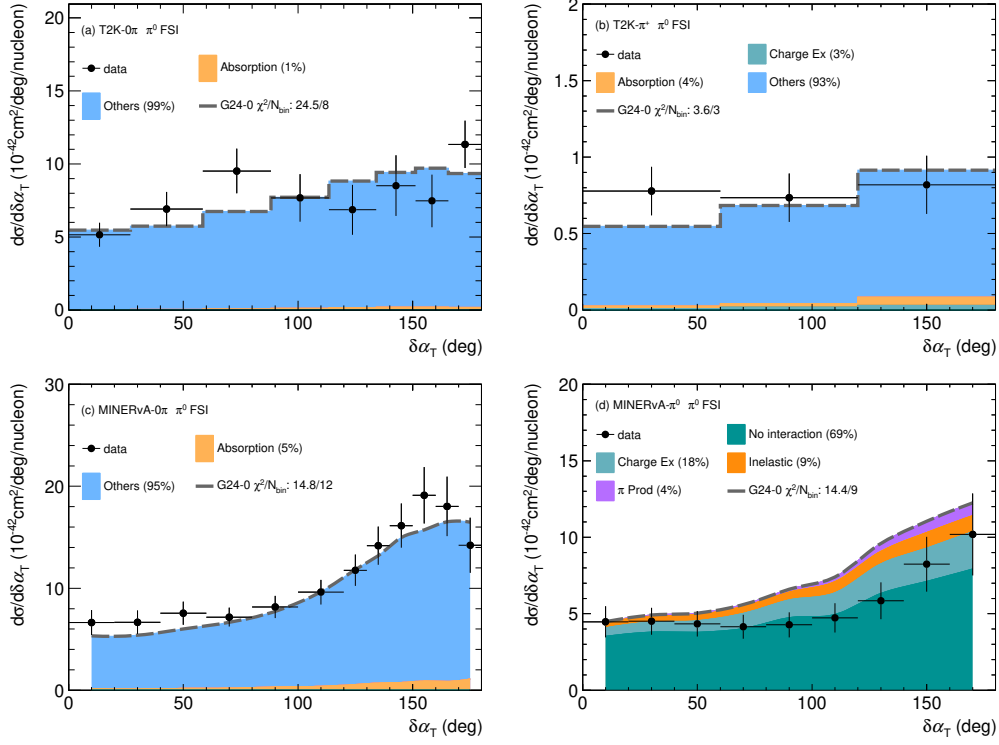
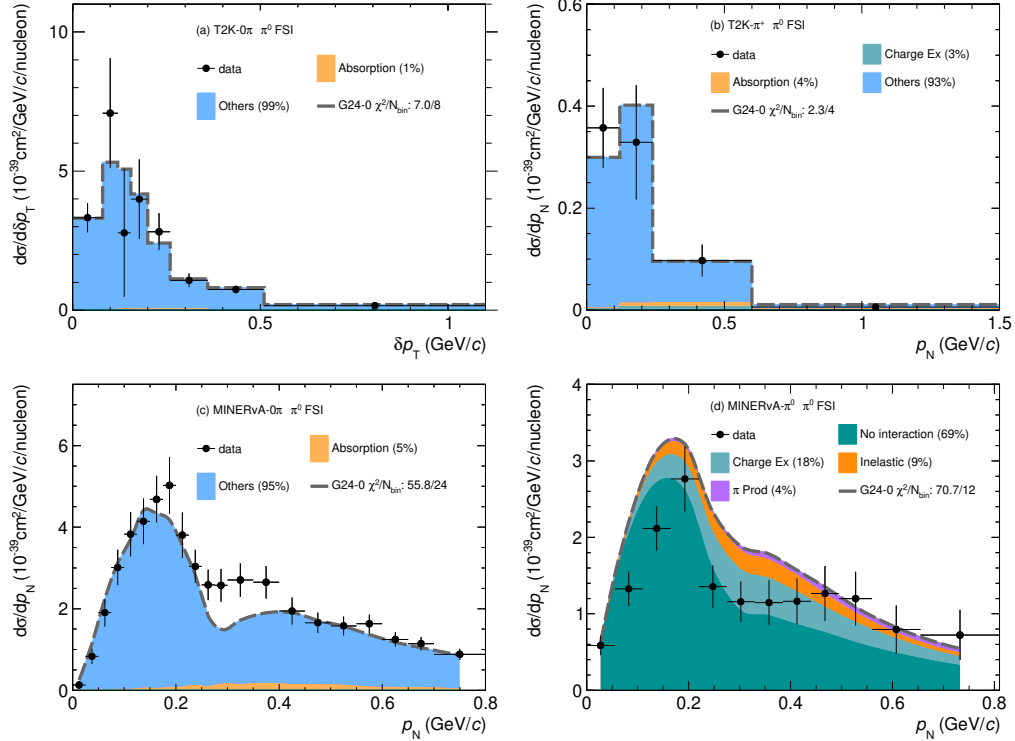
— Qiji Xin Chou Nu Er

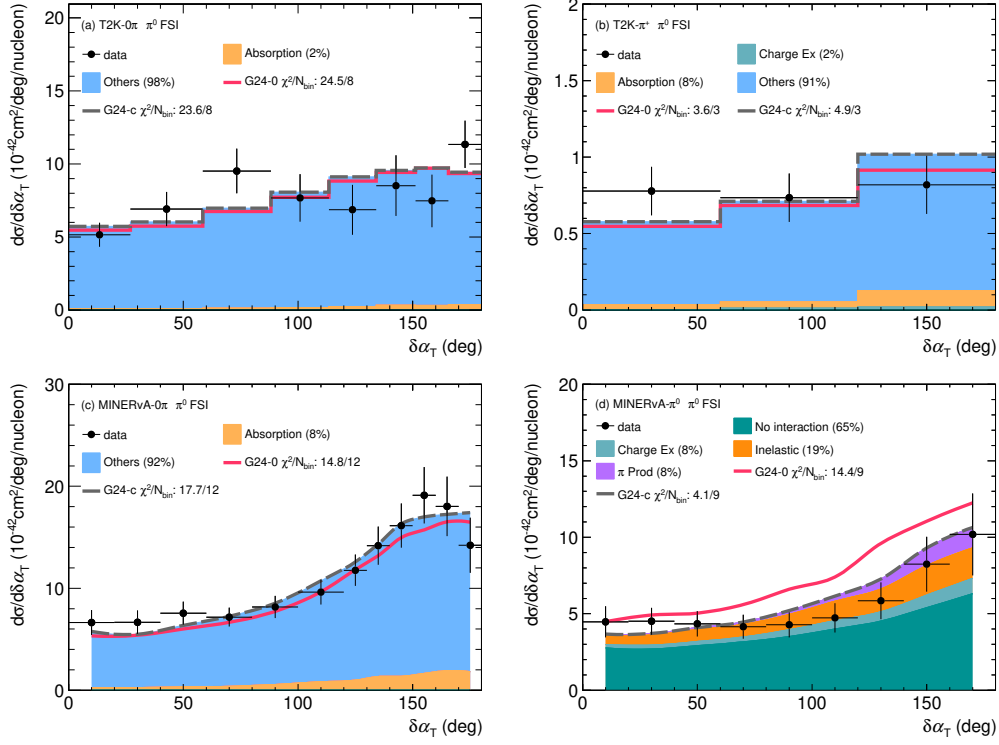
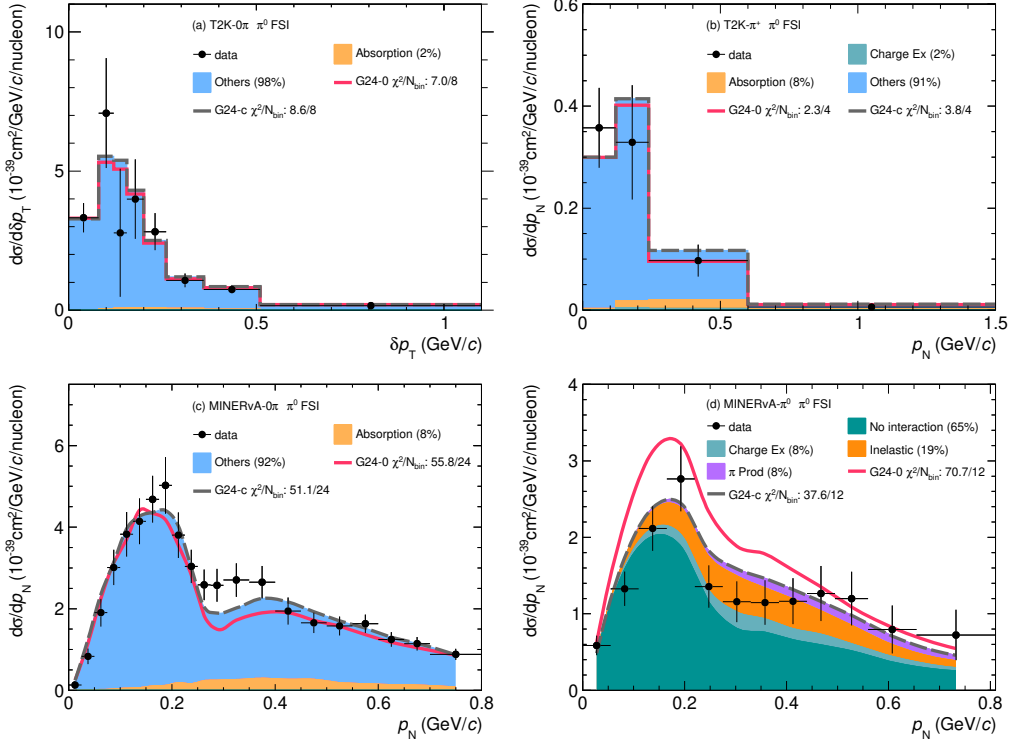
C

Tuning supplementary plots

π^0 FSI fate decomposition for **G24-0** and **G24-c**

Figures C.1-C.4 display comparisons of **G24-0** and **G24-c** predictions to data, detailing the composition by π^0 FSI fate.

Figure C.1: Extension of figure 7.11a to all four $\delta\alpha_T$ measurements.Figure C.2: Extension of figure 7.11a but to all four p_N measurements.

Figure C.3: Extension of figure 7.11b to all four $\delta\alpha_T$ measurements.Figure C.4: Extension of figure 7.11b but to all four p_N measurements.

日啖荔枝三百颗，不辞长作岭南人。

Keep calm and eat lychees.

— Dongpo Su Zhu Hai Yi Jue

D

BeamHNL Technical Details

Contents

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D.1 dk2nu conversions

Quantity	GENIE (dk2nu)	T2K
parent PDG	decay_ptype	ppid
ν parent decay coordinates [cm]	decay_vx,vy,vz	Xpi[0],Xpi[1],Xpi[2]
parent momentum in lab frame [GeV/c]	decay_pdp _{x,y,z}	ppi*(npi[0],npi[1],npi[2])
E_ν in centre-of-mass frame of parents [GeV]	decay_necm	calculated
relative weight of each event entry	decay_nimpwt	norm ¹

Table D.1: Conversion of variables from T2K flux file to **dk2nu**.

D.2 HNL Input Parameter Calculations

This section elaborates on the relevant parameters in the **GENIE BeamHNL** configuration file, **CommonHNL.xml**. The parameters present in the configuration file but not mentioned are set to default values.

D.2.1 Common parameters

There are several parameters that are common for HNL searches independent of experimental setups under the parameter set, `ParameterSpace`. `InterestingChannels`.

1. `HNL-Mass`, mass of the HNL. As the hadrons produced by the T2K beam are predominantly pions and kaons, the M_N range that can be explored from hadron decay is between 140 MeV to 390 MeV. Furthermore, as the decay channel investigated in this work are the two-body decay channels, namely $\pi+e$ and $\pi+\mu$, the chosen range for M_N is from 140 MeV (250 for the $\mu\pi$ decay channel only) to 390 MeV.
2. `HNL-LeptonMixing`, the mixing coefficient of the extended PMNS matrix between the HNL and the electron neutrino and muon neutrino respectively. U_e and U_μ are chosen to be equal to 10^{-7} . They are chosen to be small such that the first order approximation of the proportionality between the number of observed events and the fourth power of the mixing elements are valid.
3. `HNL-2B_x_y`, the switch for a particular HNL decay channel, where x and y are the two decay products, for example `mu` and `pi`. If one wants to investigate all decay channels without having to worry about accounting for the proper decay widths, one could set all the channels to be true, as they account for the vast majority of the decay channels.

D.2.2 T2K-specific parameters

The parameters need to be modified for the T2K ND upgrade are contained in `CoordinateXForm` and `FluxCalc`. The `CoordinateXForm` set specifies the geometric relation between the beam and the detector. In order to specify the relations between the target position, the beam direction and the detector position accurately, three coordinate systems are used, namely NEAR, BEAM and USER. The NEAR frame has the origin inside the target with its y axis parallel to the floor of the target hall. This frame is also the global frame used by the NEUT generator. The BEAM frame has the z axis parallel to the beam direction, which is typically

rotated from the NEAR frame. The USER frame is the coordinate system setup at ND280 for event selection and analysis. Parameters describing the frame transformation are as follows.

1. **Near2Beam_R**, the rotation of the BEAM frame with respect to the NEAR. According to the beam centre coordinates given in the T2K flux file, the variables **Bx_{fd}** and **By_{fd}**, the beam centre has a (x, y) -coordinate, $(0, -1780.400)$ cm at the centre of ND280, which is located at $z = 28010$ cm. This correspond to a rotation of $\arctan(-1780.400/28010) = -0.06348$ rad around the x -axis. Hence, this parameter should be set to `0.0 ; 0.0 ; -0.06348`.
2. **Near2User_T**, position of the centre of USER in the NEAR frame. For T2K, the ND centre is located at `(-3.222 ; -8.146 ; 280.1)` m.
3. **DetCentre_User**, position of the detector centre in the USER frame. As origin of the USER frame is the centre of the whole ND280, the centre of the upgrade part, namely SFGD and HAT, is located at `(0.0 ; 0.029985 ; -1.94394)` in the USER frame.

Moreover, the **FluxCalc** set contains additional flux-related specifications:

1. **ParentPOTScalings**, the average number of HNL parents produced per POT for all, muons, pions and K^L .
2. **IsParentOnAxis**, this should be set to `false`, as ND280 is located off axis.
3. **InputFluxesInBEAM**, this should be set to `false`, as the T2K flux input is given in the NEUT frame, i.e. the NEAR frame, instead of the BEAM frame.
4. **CollectionRadius**, the radius of the detector in the USER frame, which is set to `6.0` m. It should be large enough to cover the whole ND280.

If I see further, that is because I am on the shoulders
of giants.

— Sir Issac Newton

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