

The Development and Implementation of a Beam
Position Monitoring System for use in the FONT
Feedback System at ATF2.

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Abstract

Feedback On Nanosecond Timescales (FONT) is a feedback system being developed to correct the beam position jitter in the extraction line and final focus system at the Accelerator Test Facility (ATF2), Tsukuba, Japan. FONT5 is currently being tested and is used to correct the intra-train jitter for a 3-bunch train; the bunch spacing is 154ns. This system measures the position of an electron bunch, using beam position monitors (BPMs). From this measurement, digital feedback electronics calculate the required correction, and sends a pulse to a feedback kicker. The feedback kicker then deflects the next bunch in the train in order to correct its position. Stripline BPMs are used at ATF2, and analogue processors manipulate the BPM signals before they are interpreted by the feedback electronics.

The BPM system has been modelled and tested so that it can be parameterised and optimised. The BPMs are calibrated regularly, and the resolution of the system measured. Both of these properties have been analysed and modelled. This has allowed the resolution to be minimised. The resolution is an important factor which limits the achievable feedback correction. Several other factors have also been investigated; these include the feedback gain and the bunch-bunch correlation.

To allow the feedback electronics to be controlled remotely, several data acquisition systems (DAQs) have been developed to allow data flow both to and from the digital board. The DAQs have been designed specifically for the firmware on the FONT digital board.

I would like to dedicate this thesis to Judy Schiller and her late husband Klaus. She has been an important driving force for me over the past eight years, and has worked tirelessly to give whatever help and support I have needed over this time.

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Chapter 1

Introduction

1.1 The Standard Model

The Standard Model [1] is a theory describing the fundamental particles in particle physics; Table 1.1 shows the fundamental particles. It incorporates the unified electroweak theory and quantum chromodynamics. There are two types of fundamental particle; fermions and bosons. Fermions are spin $1/2$ particles which form two groups; quarks and leptons.

Quarks exist in six flavours; up, down, charm, strange, top and bottom. Up, charm and top are known as up-type quarks and have a charge of $+2/3$ (in units of elementary electric charge). Down, strange and bottom are known as down-type quarks and have a charge of $-1/3$. Lone (or naked) quarks are not observed in nature; instead they are either found as mesons or baryons. Mesons are comprised of a quark and an anti-quark, baryons are comprised of either three quarks or three anti-quarks. Mesons and baryons are the two observed groups of hadrons; tetra- or even penta-quarks may exist, but there is currently no observed evidence of their existence. Mesons and baryons are all observed to have integer charges. Quarks are subject to weak, electromagnetic and strong interactions.

Leptons exist as six different types; electron, muon, tau and their neutrino counterparts. Electrons, muons and taus all have a charge of -1 ; neutrinos have no charge and almost no mass. All leptons are subject to weak interactions and except for neutrinos, electromagnetic interactions; no lepton experiences strong interactions.

Fermions exist in three generations. For quarks each generation consists of an up-type and a down-type quark; for leptons generations consist of a lepton and its corresponding neutrino. The fermions have been organised in terms of their generations in Table 1.1.

The force carriers of the interactions are bosons. The force carriers are photons, W and Z bosons and gluons (Table 1.1). The weak force is mediated by the W and Z bosons, the electromagnetic force by photons and the strong force by gluons.

Fermions				Bosons
Quarks	u	c	t	W
	d	s	b	Z
Leptons	ν_e	ν_μ	ν_τ	γ
	e	μ	τ	g
				H

Table 1.1: The table of fundamental particles in the Standard Model (Higgs currently unobserved).

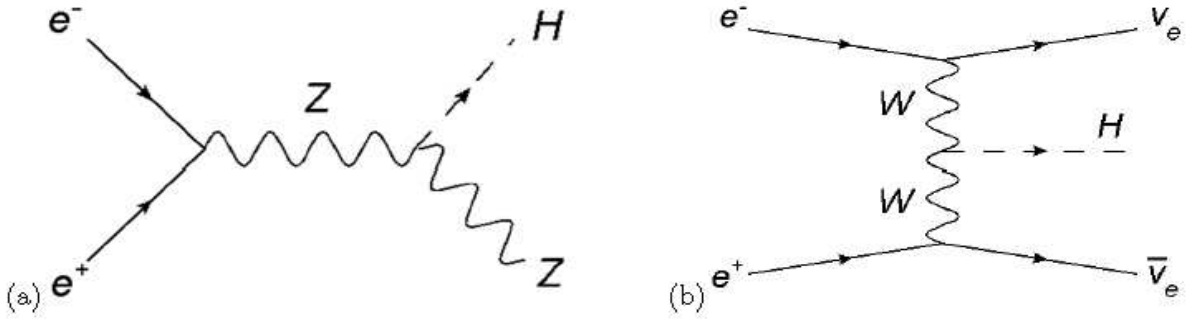


Figure 1.1: Feynman diagrams showing the dominant channels for Higgs production from e^+e^- annihilation.

1.1.1 Higgs Boson

The Standard Model has been very successful in accounting for the properties of the fundamental particles and has been amply verified by experimental data. Unfortunately, the Standard Model is unable to predict the mass of any of these particles, nor does it explain why different particles have different masses. A mechanism for mass generation was hypothesised by Peter Higgs and others in the 1960s [2]. The Higgs Mechanism describes the interaction of elementary particles with the postulated spinless Higgs Boson. In a similar manner to a charged particle distorting an otherwise uniform electromagnetic field, a massive particle locally distorts a uniform Higgs field. It is this distortion of the Higgs field which is perceived as mass. The strength of the particle's coupling to the Higgs field is directly proportional to its mass. Figure 1.1 shows the dominant channels for Higgs production from e^+e^- annihilation [3]. There are other theories that go beyond the Standard Model which offer alternatives to the Higgs Mechanism, or predict multiple Higgs particles.

1.1.2 Supersymmetry

Supersymmetry (SUSY) is a theoretical extension of the Standard Model [4]. The first realistic supersymmetric version of the Standard Model was developed in 1981 and is known as the Minimal Supersymmetric Standard Model (MSSM) [4]. It was proposed as a solution to the 'hierarchy problem'. The hierarchy problem in particle physics is the question as to

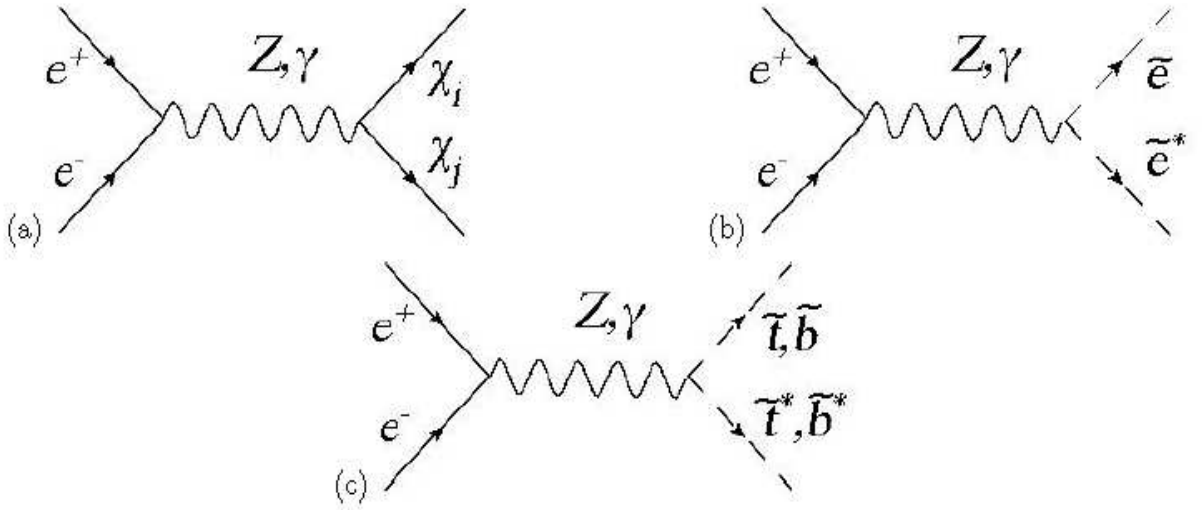


Figure 1.2: Feynman diagrams showing the production of possible supersymmetric particles from e^+e^- annihilation [7]. (a) Two charginos, (b) selectron-spositron pair and (c) squark-anti-squark pair.

why the masses of the W and Z Bosons are so much smaller than the Planck mass; the bosons' masses are approximately 10^{17} times smaller than the Planck mass. The Standard Model predicts that quantum corrections to the Higgs mass should make it many orders of magnitude larger unless extremely fine-tuned cancellations prevent this. The hierarchy problem also leads to the observed disparity between coupling strengths for the fundamental forces; for instance, the weak force is approximately 10^{32} times stronger than the gravitational force. SUSY would allow the unification of the weak, strong and electromagnetic interactions at high-energies. Each elementary particle has a 'superpartner'; bosons have fermions as their partners and fermions have bosonic partners. Particles and their superpartners differ in intrinsic spin by $1/2$. The lightest supersymmetric particles are subject to the weak interaction and are known as weakly interacting massive particles (WIMPs). Since WIMPs are the lightest SUSY particles, they are unable to decay (in some models); this makes them good candidates for dark matter [5]. In SUSY models the supersymmetric particles should have masses in the range of 100GeV to 1TeV [6]. Figure 1.2 shows some Feynman diagrams for the production of supersymmetric particles from e^+e^- annihilation [7].

1.1.3 Top quark Physics

The top quark is the most massive of the quarks and was the last one to be discovered in 1995 at the Tevatron accelerator at Fermilab. Its mass is measured to be $172.0 \pm 1.3 \text{ GeV}$ [8]. Relatively few precision measurements have been taken of its properties. Many particle accelerators currently being commissioned or designed are aiming to improve the precision of these measurements. The lifetime of the top quark is believed to be approximately $5 \times 10^{-25} \text{ s}$ [9]. The typical timescale for the strong interaction is approximately 10^{-24} s [10]; significantly longer than the lifetime of the top quark; because of this, the top does not hadronise and

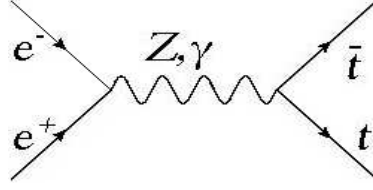


Figure 1.3: The leading order Feynman diagram for the production of top quarks from e^+e^- annihilation.

therefore is the only quark which normally exists as a lone quark before it decays. This allows a unique opportunity to investigate a quark directly, which may lead to evidence of physics beyond the Standard Model. Figure 1.3 shows the leading order Feynman diagram for the production of top quarks in e^+e^- annihilation.

1.2 Accelerators

The history of particle accelerators has been driven mainly by the study of fundamental particles and their interactions. As time has progressed, accelerators have increased in both energy and luminosity. Increased energy has been required for two reasons. The first is that the structures now observed are extremely small and in order to observe them, the resolution of the probe must be smaller than the structure. Visible light has a typical wavelength of $\sim 10^{-7}\text{m}$, which limits the spatial resolution. For a particle of momentum p , the De Broglie wavelength is given by Eq. 1.1.

$$\lambda_b = \frac{h}{p} \quad (1.1)$$

Eq. 1.1 shows that as the momentum increases (and hence the kinetic energy), the De Broglie wavelength decreases, and so smaller structures can therefore be observed.

The second reason for increasing the energy of particle accelerators is that some particles, such as the Higgs Boson, are believed to have a large mass. Einstein's famous equation, Eq. 1.2, shows the equivalence of energy and mass; where m_0 is the particle's rest mass.

$$E = m_0 c^2 \quad (1.2)$$

This suggests that in order to create these particles, larger energies are required.

1.2.1 Beam Dynamics

Twiss Parameters

As particles travel along an accelerator, their position and angle change; the transformation from a point S_0 to S_1 along the lattice can be described by a transfer matrix, M (Eq. 1.3). This is a simplified transfer matrix only considering a transformation in the vertical plane.

$$\begin{pmatrix} y_1 \\ y_1' \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} y_0 \\ y_0' \end{pmatrix} \quad (1.3)$$

The Twiss parameters are a means of parameterising the transfer matrix. Eq. 1.4 shows the transfer matrix for a non-periodic lattice in terms of the Twiss parameters (see chapter 5, section 5.2).

$$\mathbf{M} = \begin{pmatrix} \sqrt{\frac{\beta_1}{\beta_0}} [\cos \mu + \alpha_0 \sin \mu] & \sqrt{\beta_0 \beta_1} \sin \mu \\ -\frac{1+\alpha_0 \alpha_1}{\sqrt{\beta_0 \beta_1}} \sin \mu + \frac{\alpha_0 - \alpha_1}{\sqrt{\beta_0 \beta_1}} \cos \mu & \sqrt{\frac{\beta_0}{\beta_1}} [\cos \mu - \alpha_1 \sin \mu] \end{pmatrix} \quad (1.4)$$

Emittance

Emittance is a measure of the mean spread of particles in position-momentum phase space. If the position and angle of each particle in a beam is plotted, it would form an ellipse; the area of this ellipse is the emittance of the beam. As the beam becomes more relativistic, the emittance of the beam Lorentz contracts, therefore a more useful quantity is the normalised emittance (Eq. 1.5); this is now Lorentz invariant.

$$\epsilon^* = \beta \gamma \epsilon \quad (1.5)$$

The beam size is related to the Twiss parameters and the emittance and is $\sqrt{\epsilon \beta}$. In colliders, the luminosity of the beam is inversely proportional to the beam size (section 1.5.2). Therefore in order to maximise the luminosity, a small emittance is required.

Liouville's Theorem

Liouville's theorem states that 'the density in six-dimensional phase space of non-interacting particles, in a conservative dynamical system, measured along the trajectory of a particle, is invariant' [11]. This statement implies that the emittance of a beam is constant provided the system is not dissipating energy as this is generally velocity dependent.

The purpose of a damping ring is to reduce the transverse momentum of a beam and hence the transverse emittance. This is achieved by passing the beam through arc sections, which cause the particles to lose energy through synchrotron radiation. The bunches are then accelerated in the longitudinal direction in straight sections to compensate for the energy

lost in the arcs. As a result, the total momentum of the beam remains constant by the transverse momentum is reduced. In this case, since the particles are dissipating energy through synchrotron radiation, which is a non-conservative process, Liouville's theorem does not apply.

1.2.2 Stochastic Cooling

Stochastic cooling is a means of reducing the emittance of a beam in a circular machine with a feedback system. The layout of stochastic cooling feedback system is very similar to the FONT feedback system described in section 1.7. A beam position monitor is used to measure the beam position. This is used to calculate a correction for the beam position, which is transmitted along cables to a kicker further round the ring. The feedback signals travel slower than the beam, but the cable path length is shorter than the beam path length therefore the correction arrives before the beam. It is important that the feedback correction is delivered before synchrotron and betatron oscillations disrupt the beam; thus while the beam is still coherent.

In stochastic cooling the feedback system applies an average correction to the particles. Synchrotron and betatron oscillations then mix up the particles again and a new correction is given to the beam. Over many turns, particles which have large deviations from the nominal orbit receive larger corrections than particles on the nominal orbit. This has a net result of reducing the beam emittance.

1.3 Colliders

In order to create new particles in an accelerator, a particle interaction must occur. The most common way to achieve this is to collide particles. In the case of electron-positron annihilation, the easiest method of achieving an interaction would be to accelerate and collide a beam of positrons into a fixed target rich in electrons. The new particles created will move forward in the centre-of-mass frame of reference and a detector placed downstream is sufficient for detecting them.

There is a disadvantage with this collision method. The kinetic energy of the centre-of-mass frame cannot be used in the interaction, so this is wasted energy. The energy efficiency of this type of collision is given by Eq. 1.6. E^* is the energy available for particle interactions and E_i is the energy of the beam before collision.

$$\eta \equiv \frac{E^*}{E_i} = \sqrt{\frac{2m_0c^2}{E_i}} \quad (1.6)$$

As the beam energy increases, the energy efficiency, η , decreases; therefore the required E_i becomes very large. Therefore this is not a suitable method for achieving very high energy collisions. An alternative method is to accelerate a beam of particles and a beam of antiparticles, and collide them into each other. This method is more difficult since the

target is significantly smaller, and the timing is critical for collisions. In this scheme, the centre of mass of the system is stationary, and so no energy is wasted. A disadvantage with this method is that since the centre-of-mass frame has no kinetic energy, the newly created particles can travel in any direction provided the total momentum is zero. This means that the particle detector needs to completely surround the interaction point.

1.4 Modern Colliders

There are several theories in high energy physics (HEP) which predict the existence of particles that are currently undiscovered, although likely to exist. Some of these particles include the one or potentially more Higgs Bosons, supersymmetric partners to all currently known particles and many other possibilities. Many of these particles are postulated in order to fill ‘gaps’ in the current understanding of the Standard Model, such as the Higgs particle. Other particles are postulated by theories that go beyond the Standard Model, such as supersymmetry, in an attempt to achieve a more complete understanding of particle physics. With the recent commissioning of the Large Hadron Collider (LHC) at CERN, Geneva, Switzerland [12], it is hoped that many of these particles will be discovered.

1.4.1 Hadron Colliders

Quantum chromodynamics (QCD) predicts that hadrons consist of valence quarks and sea (virtual) quarks and gluons, all of which interact with each other. The total energy of the hadron is distributed amongst its constituents. When a collision occurs between two hadrons at the interaction point of a collider, typically only one of the many constituents within the hadron interacts with one constituent of the other hadron. Since the energy of a particular collision is unpredictable, the energy spread over many collisions is very wide. Hadron machines are therefore the ideal tool to search for new particles. The disadvantage is that the wide energy spread makes hadron machines unsuitable for precision measurements such as masses, lifetimes, spins and parities of the new particles.

1.4.2 Lepton Colliders

Leptons only exhibit electroweak interactions. Since leptons are fundamental particles, all of their energy is used in interactions. Due to this, lepton machines are typically used to perform precision measurements of particle masses, lifetimes, spins and parities.

1.5 Future Colliders

If the LHC is successful in discovering some of the postulated particles it is designed to find, a lepton collider is likely to be built in order to exploit the LHC’s findings. Two potential

Parameter		Minimum	Nominal	Maximum	Units
Luminosity	L	2	2	2	$\times 10^{34} \text{cm}^{-2} \text{s}^{-1}$
Repetition frequency	f_{rep}	5	5	5	Hz
Bunch population	N	1	2	2	$\times 10^{10}$
Number of bunches	n_b	1260	2526	5340	
Linac bunch interval	t_b	180	369	500	ns
RMS bunch length	σ_z	200	300	500	μm
Normalised horizontal emittance at IP	$\gamma\epsilon_x$	10	10	12	mm.mrad
Normalised vertical emittance at IP	$\gamma\epsilon_y$	0.02	0.04	0.08	mm.mrad
Horizontal beta function at IP	β_x	10	20	20	mm
Vertical beta function at IP	β_y	0.2	0.4	0.6	mm
RMS horizontal beam size at IP	σ_x	474	640	640	nm
RMS vertical beam size at IP	σ_y	3.5	5.7	9.9	nm
Fractional RMS energy loss to beamstrahlung	δ_{BS}	1.7	2.4	5.5	%

Table 1.2: Nominal and design range for beam parameters at the IP for the ILC [17].

accelerators currently being proposed are the International Linear Collider (ILC) [13], and the Compact Linear Collider (CLIC) [14]. Both of these colliders are described below.

1.5.1 The International Linear Collider

The ILC is a proposed electron-positron collider, with a centre of mass energy of 500GeV. The centre of mass energy is intended to be upgradable to 1TeV. Aside from being able to complement data from the LHC, an electron-positron machine of such high energy will be a feat of engineering, and will make advances in a range of other technologies. The total length of the ILC will be approximately 31km, and will have two damping rings, each with a circumference of approximately 6.7km. Figure 1.4 shows a schematic layout of the ILC.

There are two main technologies used for accelerating particle beams in modern colliders. Normal conducting radio-frequency (RF) cavities are a mature and well understood technology, although much of the RF power is lost through resistive losses in the material. An alternative technology employs superconducting RF cavities. This technology is less mature, but can accelerate particles with very high efficiency. The ILC design has adopted superconducting RF cavities to accelerate beams with a target accelerating gradient of 31.5MV/m.

Table 1.2 shows a list of the design ranges of beam parameters at the IP for the ILC. Luminosity is defined in Eq. 1.7. The mean energy lost per particle due to Beamstrahlung, δ_{BS} , is inversely proportional to the square of the horizontal and vertical beam size (σ_x , σ_y in Eq. 1.8) [16]. The beam parameters listed in table 1.2 are designed to maximise luminosity and minimise δ_{BS} . The difficulty is that in order to optimise the luminosity, σ_x and σ_y must be minimised; this will have the undesired effect of increasing δ_{BS} . In order to overcome this challenge, σ_x at the IP is made relatively large compared to σ_y ; this allows a compromise between luminosity and δ_{BS} .

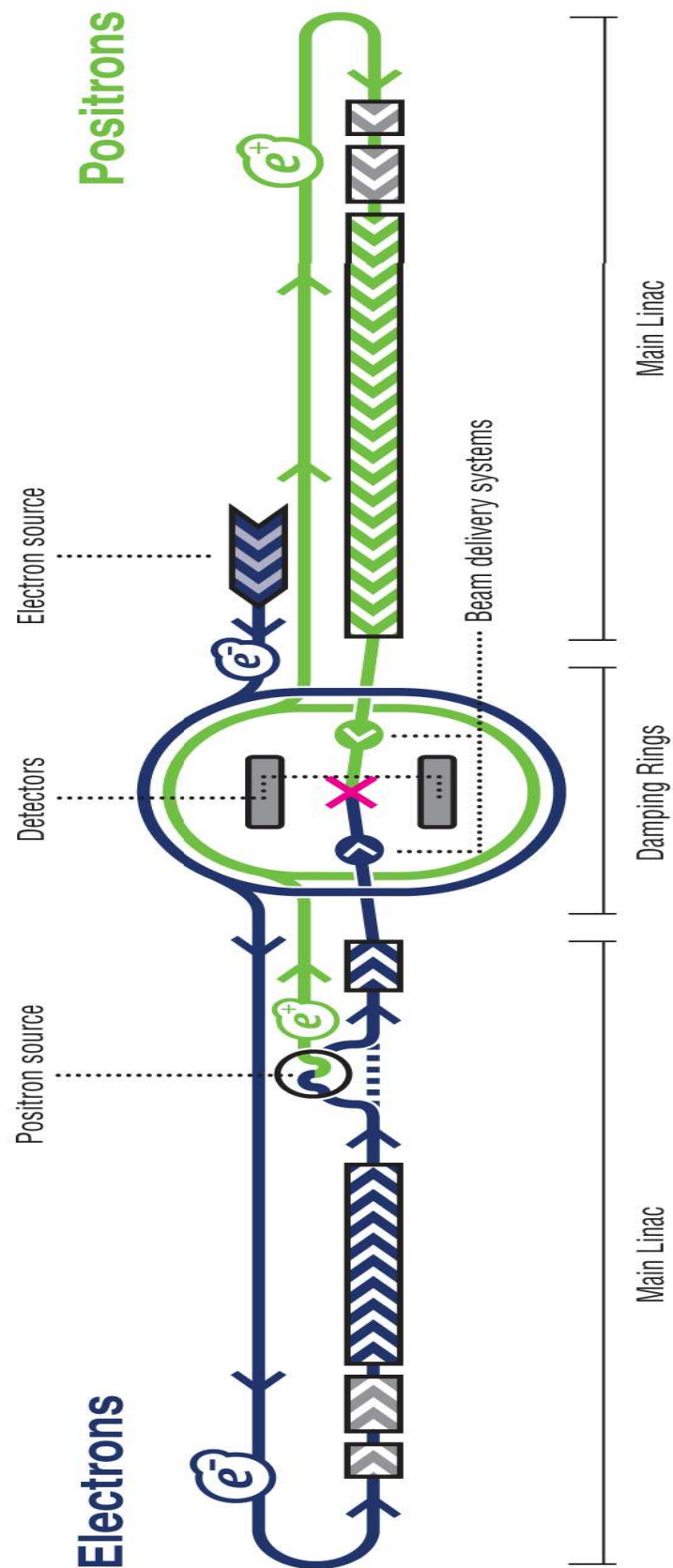


Figure 1.4: A schematic layout of the International Linear Collider [15].

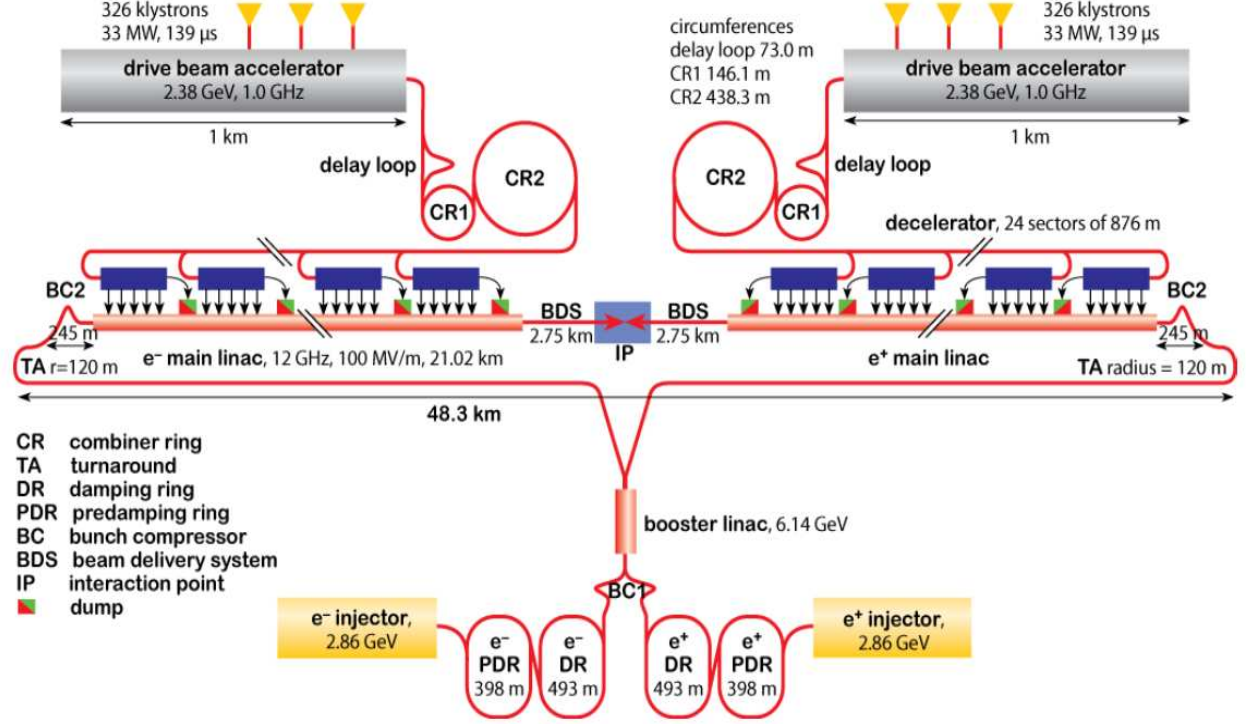


Figure 1.5: A schematic layout of the Compact Linear Collider [14].

$$L = f_{rep} n_b \frac{N^2}{4\pi\sigma_x\sigma_y} \quad (1.7)$$

$$\delta_{BS} \propto \frac{N^2 \epsilon_y}{\sigma_x^2 \sigma_y^2} \quad (1.8)$$

1.5.2 The Compact Linear Collider

CLIC is an alternative electron-positron collider currently being proposed. CLIC is designed to have a nominal centre of mass energy of 3TeV. This is a significantly higher energy than that proposed by the ILC design report, and will pose significantly more technological challenges. The total length of CLIC will be approximately 48km. Figure 1.5 shows a schematic layout of CLIC.

CLIC will accelerate the particles in a novel way. A beam of electrons will be accelerated using normal conducting RF cavities, this beam will then be decelerated, and the energy lost by the drive beam will be used to accelerate the main beam. CLIC is designed to be optimised for a centre-of-mass energy of 3TeV, although it will be capable of operating at centre-of-mass energies in the range of 500GeV to 5TeV. At a centre-of-mass energy of 3TeV, the initial drive beam will be a 2.4GeV 140 μ s bunch train, consisting of 24 $\times$ 24 sub-pulses with a bunch spacing of 2ns; the total beam current will be 4.2A. This bunch train would pass through a delay loop and two combiner rings to compress the bunch train (Figure 1.6).

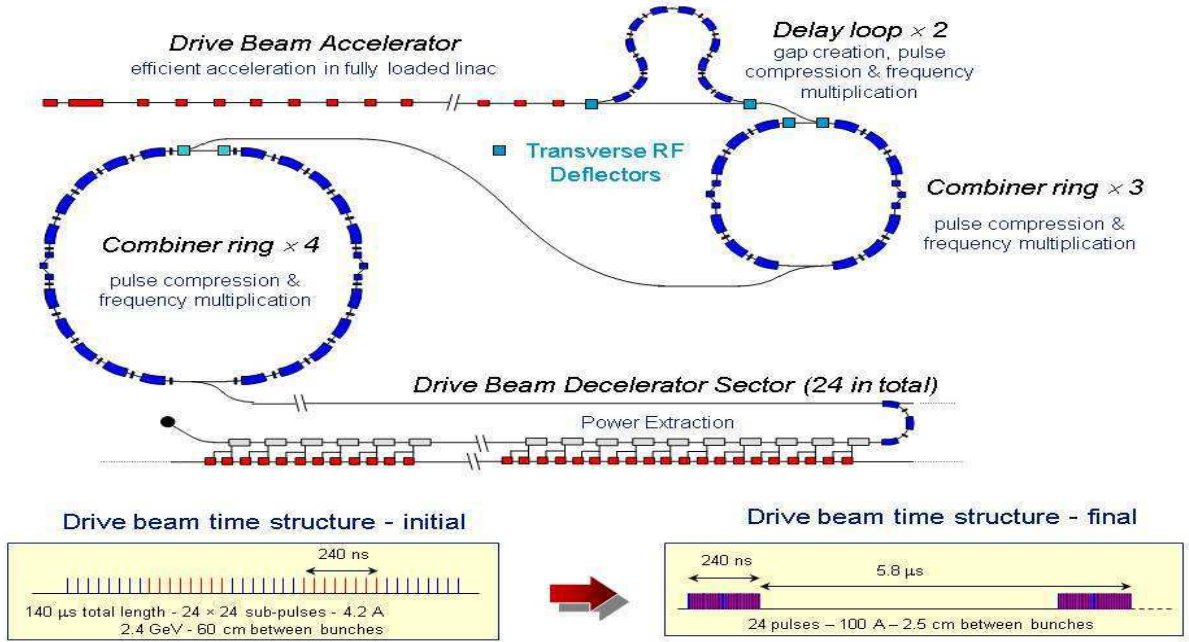


Figure 1.6: Schematic layout showing one of the CLIC RF power source complex [18].

The final drive beam would consist of 24 trains, 240ns long, with a train spacing of $5.8\mu\text{s}$; each train would contain 2904 bunches with a spacing of 82ps [18]. The beam current would be 100A. The drive beam would then be decelerated from 2.4GeV to 240MeV. The energy lost from the drive beam will be used to accelerate the main beam from 9GeV to 1.5TeV [14]. The main beam parameters are listed in Table 1.3.

1.6 Accelerator Test Facility (ATF/ATF2)

ATF2 is situated at KEK, Tsukuba, Japan [19]. Figure 1.7 shows the schematic layout of the ATF2. The original extraction line is shown for comparison to the extraction line and final focus system of ATF2. ATF was originally built to demonstrate the extremely low emittance beams required for the ILC. The measured vertical emittance at ATF was $0.04 \pm 0.01 \text{ mm.mrad}$ [20]. Having successfully achieved its goal, ATF was decommissioned and ATF2 built to replace the previous extraction line.

ATF2 was built to address two major challenges of the ILC. First to achieve a 37nm vertical beam size and secondly to achieve nanometre scale beam stability at the interaction point (IP) [22]. In order to address the second ATF2 goal, a position and angle feedback system has been developed to improve the stability of the beam (Section 1.7). The location of the FONT system (Feedback On Nanosecond Timescales) in the ATF2 extraction line is shown in Figure 1.8.

ATF2 consists (Figure 1.7) of a linear accelerator (Linac), which accelerates bunches of

Parameter		Nominal value	Units
Luminosity	L	5.9	$\times 10^{34} \text{cm}^{-2} \text{s}^{-1}$
Repetition frequency	f_{rep}	50	Hz
Bunch population	N	3.72	$\times 10^9$
Number of bunches	n_b	312	
Linac bunch interval	t_b	0.5	ns
RMS bunch length	σ_z	44	μm
Normalised horizontal emittance at IP	$\gamma\epsilon_x$	660	mm.mrad
Normalised vertical emittance at IP	$\gamma\epsilon_y$	10	mm.mrad
Horizontal beta function at IP	β_x	6.9	mm
Vertical beta function at IP	β_y	0.068	mm
RMS horizontal beam size at IP	σ_x	45	nm
RMS vertical beam size at IP	σ_y	0.9	nm
Fractional RMS energy loss to beamstrahlung	δ_{BS}	29	%

Table 1.3: Nominal (3TeV) beam parameters at the IP for the CLIC [18].

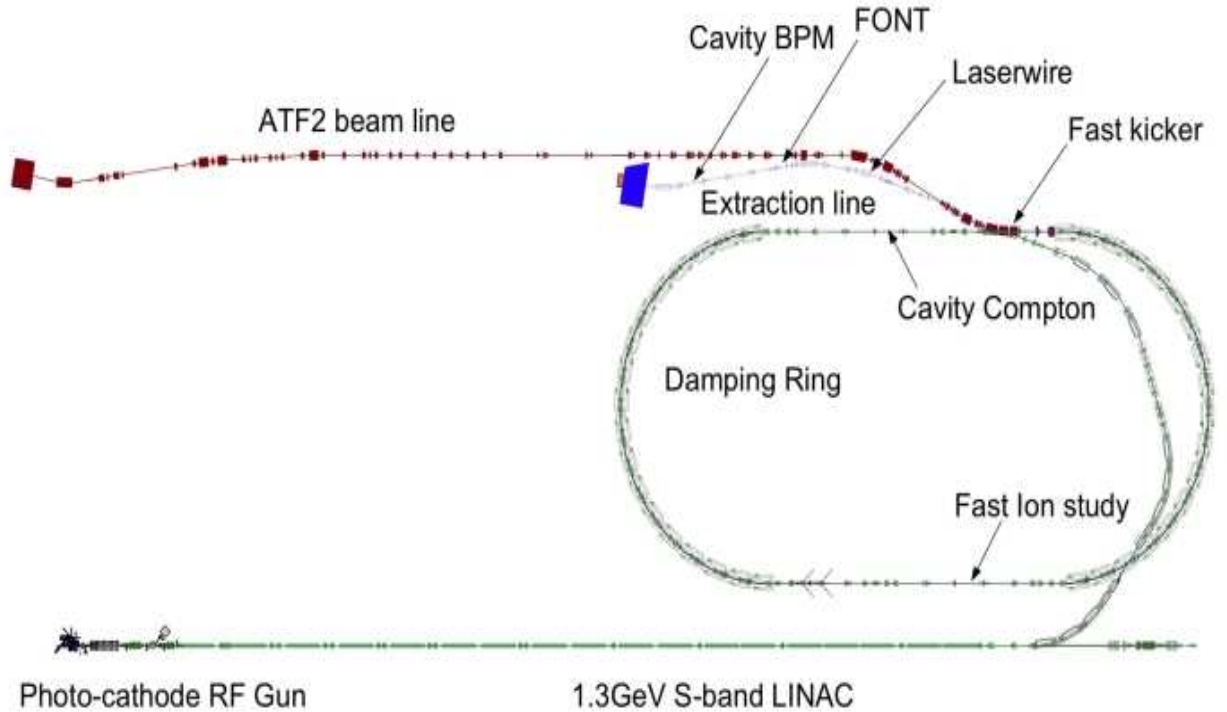


Figure 1.7: A schematic layout of the Accelerator Test Facility (ATF/ATF2) [21].

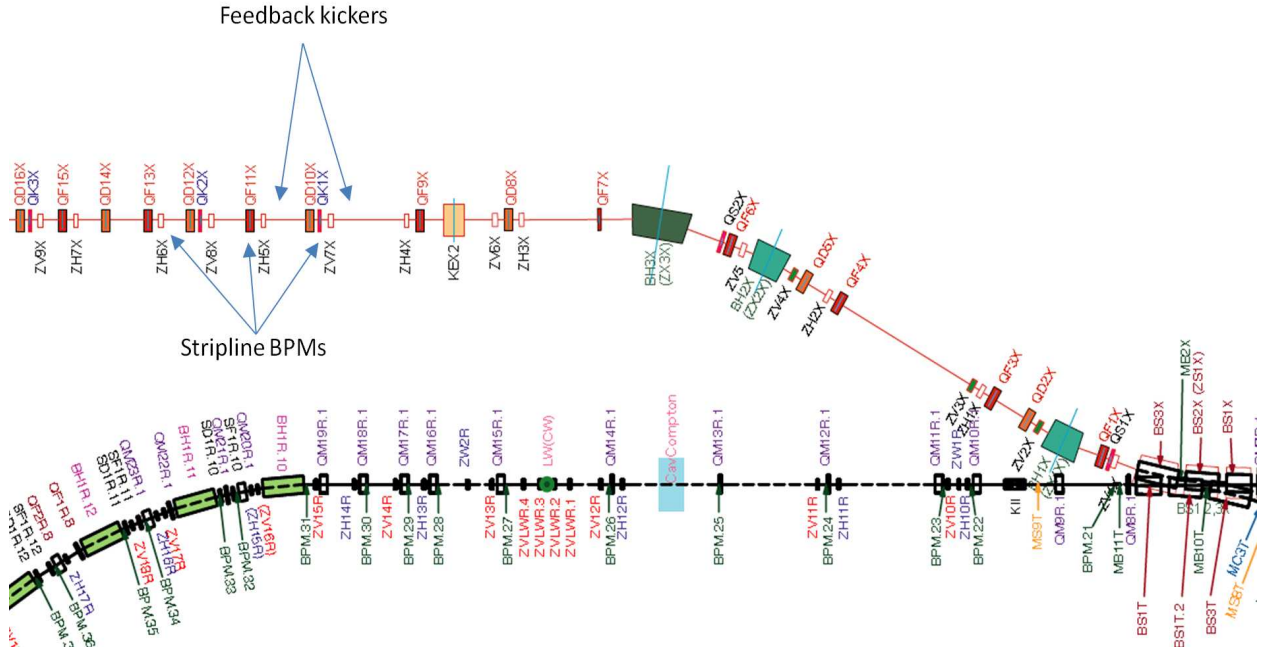


Figure 1.8: The FONT region in the ATF2 extraction line [23].

electrons up to 1.3GeV. The electron bunches then travel along a beam transfer line, and are injected into the damping ring at the location marked ‘fast kicker’. The electron bunches are then stored in the damping ring before being extracted into the ATF2 extraction line. The bunches are extracted at the same location as they are injected into the damping ring, although the injection and extraction kickers are separate devices.

A damping ring works by deflecting bunches of charged particles around arc sections. This causes the particles to emit synchrotron radiation, which causes the bunches to lose momentum radially. The bunches are then accelerated in the straight sections to compensate for the loss in longitudinal momentum. This has the overall effect of causing the bunch to lose transverse momentum while the kinetic energy remains constant. The bunches ‘damp’ onto a nominal orbit in the damping ring. Once the bunches are sufficiently damped they can be extracted from the damping ring. The ATF damping ring has two arc sections, each with a radius of curvature of 13.8m, and two straight sections each 25.9m long. The revolution period of the damping ring is 462ns. In normal operations three bunches with a spacing of 154ns are stored before extraction.

The standard extraction kicker at ATF2 has a kicker pulse width of approximately 300ns, the rise and fall times are approximately 50ns each. This is sufficient to extract the three bunches in the ring. The damping ring is capable of storing a maximum of 30 bunches. This consists of 3 bunch trains, which are separated in time by 103.6ns from the last bunch of one train to the first bunch of the next. Each train is comprised of 10 bunches, with a bunch spacing of 5.6ns. This mode was not used in this thesis.

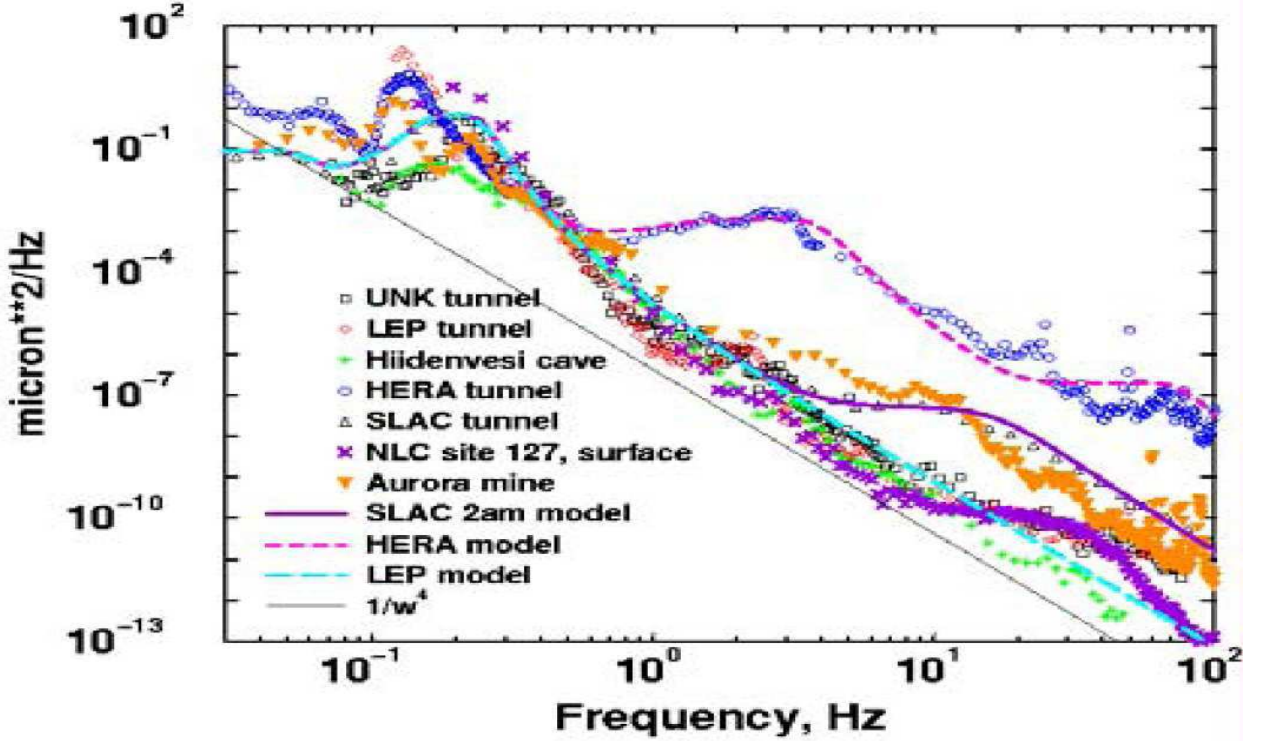


Figure 1.9: Ground motion vs. frequency at various sites around the world [25].

Therefore another extraction scheme, based on a fast extraction kicker [24] is being investigated. The fast kicker has a very narrow pulse width, which is capable of extracting one bunch from a train before the next bunch arrives 5.6ns later. In each turn of the ring the last bunch from one of the trains is extracted. Once the last bunch from each train has been extracted, the penultimate bunch is extracted from each train. This leads to a bunch spacing of 308ns, 308ns and 302.4ns, this pattern repeats until all the bunches in the damping ring have been extracted.

1.7 Feedback On Nanosecond Timescales (FONT)

The beam stability in an accelerator is governed by the factors affecting the beam position; this includes ground motion, mechanical vibrations and electrical noise on the magnets. Figure 1.9 shows ground motion measurements taken at various sites around the world. The ground motion follows a general trend of ω^{-4} . The deviations of the measured ground motion from the ω^{-4} trend are due to cultural noise, such as traffic; this is difficult to model and dependent on the location. In order to achieve a stable bunch train, it is important that the amplitude of ground motion is small on the timescale of the bunch spacing; 154ns at ATF2. Electrical noise is also a factor for the beam jitter. At ATF electrical noise on the extraction kicker pulse causes variations in the relative positions of the bunches in the extraction line.

Position offsets in a collider drastically reduce the luminosity at the IP. Figure 1.10 [26] shows a comparison of the luminosity loss between different colliders; the exact shape of the

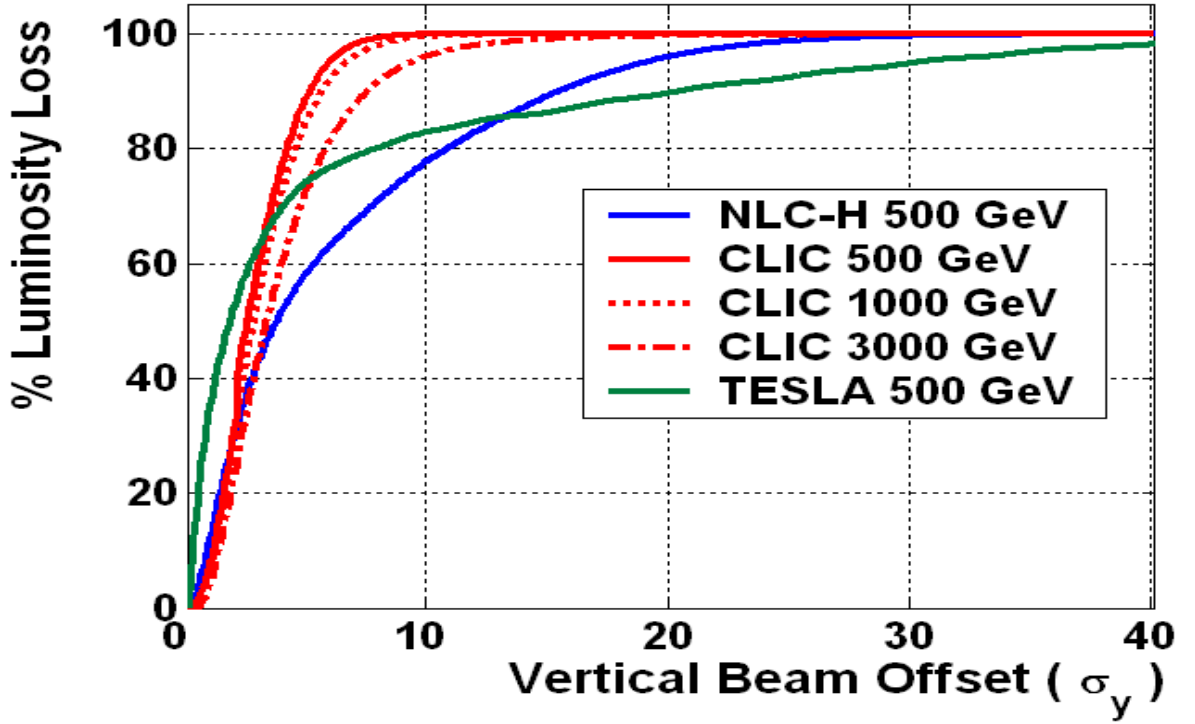


Figure 1.10: % luminosity loss vs. vertical beam offset [26]. The blue curve represents the luminosity loss for the Next Linear Collider (NLC) at 500GeV, the red curves for CLIC at 500GeV, 1000GeV and 3000GeV and the green curve for TESLA at 500GeV.

luminosity curve depends on factors such as σ_z and δ_{BS} . This steep drop in luminosity shows the importance in beam position stability and therefore the necessity for an IP feedback system. FONT is a fast feedback system, designed to correct the position and angle of electron and positron bunches. FONT is being developed for use on either the ILC or CLIC colliders. Currently FONT5 is being tested at ATF2, and is used to correct the beam jitter in the extraction line as it enters the final focus system.

The FONT feedback system works by using beam position monitors to determine the position of a bunch of electrons in a bunch train. This information is processed and interpreted by the FONT electronics. The FONT feedback board sends a signal to a kicker, which then deflects the trajectory of the next bunch in the train (Figure 1.11). For angle and position feedback, 2 feedback BPMs are used and 2 kickers deflect the bunches.

High frequency instabilities can reduce the bunch-bunch correlations along the train. The bunch-bunch correlation limits the optimum correction that can be achieved with a feedback system. Figure 1.12 shows a Monte Carlo simulation of normalised jitter versus bunch-bunch correlation. The normalised jitter represents the jitter after the feedback correction divided by the initial jitter. A random number generator was used to produce a set of correlated and uncorrelated jitters, from this two sets of bunch positions were calculated with a bunch-bunch correlation determined using Eq. 1.9. The plot shows that the feedback system can only reduce the beam jitter if the bunch-bunch correlation is greater than 50%; for a 0% bunch-bunch correlation, the feedback system would increase the jitter by a factor of $\sqrt{2}$.

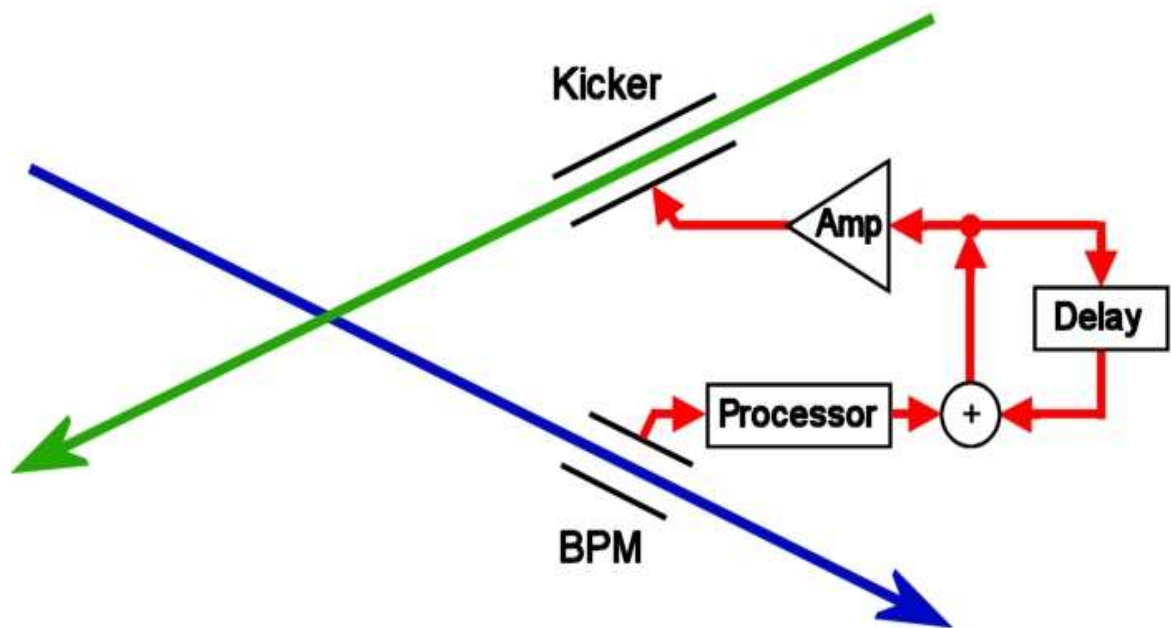


Figure 1.11: Schematic of an interaction point feedback installation for the ILC or CLIC [27].

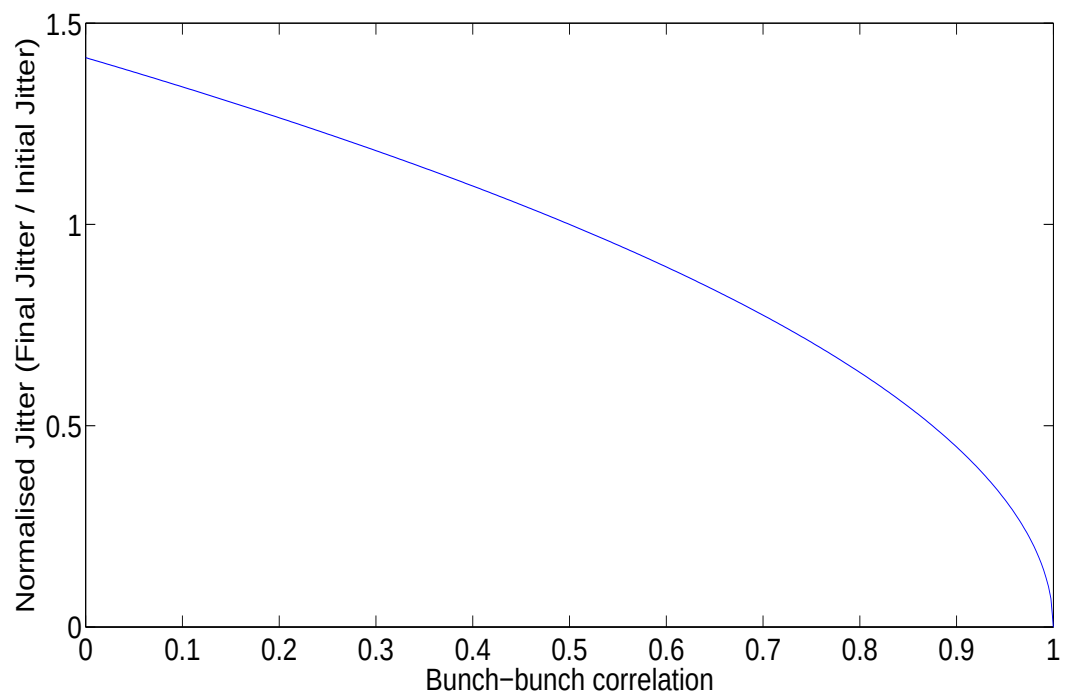


Figure 1.12: Monte Carlo simulation of normalised jitter vs. bunch-bunch correlation.

The bunch-bunch correlation is related to both the correlated and uncorrelated jitter components measured by a BPM. Eq. 1.9 shows how the bunch-bunch correlation, ρ_{corr} varies with the correlated (σ_{corr}) and uncorrelated (σ_{uncorr}) jitter components. The uncorrelated jitter is the factor which limits the maximum achievable correction; part of this uncorrelated jitter is due to the resolution of the BPM system. It is therefore important to minimise the resolution of the BPM processors in order to optimise the performance of the feedback system. In order to achieve sub-micron beam jitter at the IP at ATF2, FONT requires BPMs with a resolution of $1\mu\text{m}$ or better; this is discussed in later chapters.

$$\rho_{corr} = \frac{\sigma_{corr}^2}{\sigma_{corr}^2 + \sigma_{uncorr}^2} \quad (1.9)$$

Chapter 2

Introduction to Beam Position Monitors

This chapter introduces much of the underlying physics governing beam position monitors (BPM). There are several different types of BPM, as well as different methods of processing the data. These will be described briefly.

2.1 Types of Beam Position Monitor

A BPM is used to measure the position of the beam as it passes through the monitor. There are three main types of BPM in common use. Each type of BPM has different properties. The type used depends on the requirements of the BPM system.

Stripline BPMs

A stripline BPM consists of four electrode pickup strips, placed parallel to the direction of motion and electrically isolated from the beam pipe. The strips are arranged in pairs, on opposite sides of the beam pipe, typically two in the vertical axis and two in the horizontal axis; although the BPM can be orientated in any direction. As the beam passes through the BPM voltage signals are induced in the four striplines. By processing these signals a beam position can be calculated. Stripline BPMs produce high bandwidth signals, thus processing their signals typically involves filtering out a large portion of the frequencies. This reduces the signal-to-noise ratio, and hence affects the resolution of the BPM. Due to the high bandwidth, stripline BPMs have a short response time. FONT uses stripline BPMs. Each BPM is 14cm long, with a 12mm internal radius and each stripline has an angular width of 70° with respect to the centre of the BPM. Figure 2.1 shows one of the three FONT stripline BPMs used at ATF2.

Button BPMs

A button BPM is very similar to a stripline BPM, however the pickups are much shorter. Due to the short strips the induced voltage signals are much shorter in time. Due to the similarity to a stripline BPM, a button BPM output can be processed in the same way.

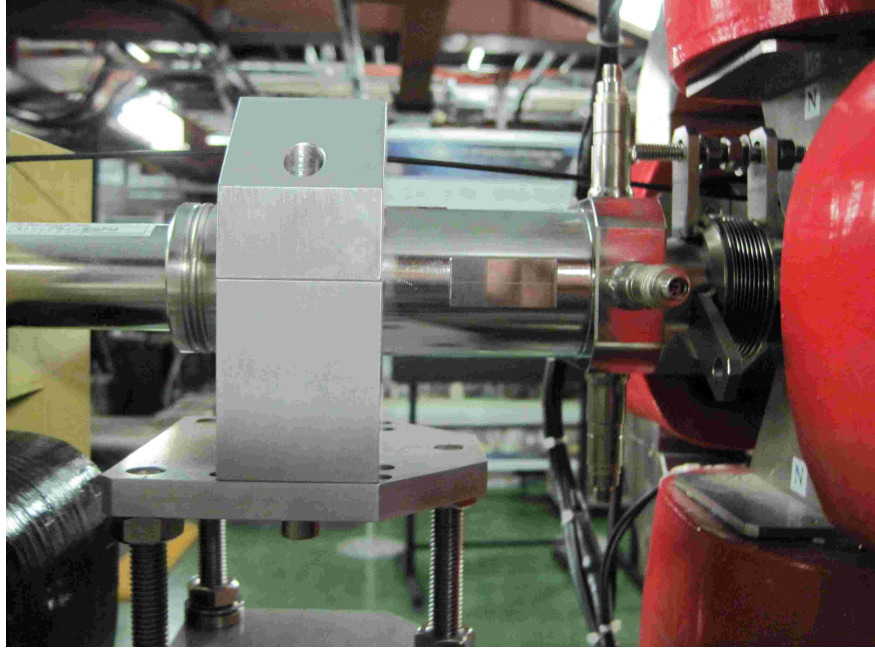


Figure 2.1: Photograph of the FONT P1 BPM at ATF2.

Cavity BPMs

A cavity BPM behaves in a very different way to stripline and button BPMs. As a bunch of charged particles passes through the RF cavity electromagnetic oscillations are excited. Figure 2.2 shows two excited EM modes; only the electric components are shown.

The TM₀₁₀ mode is known as the monopole mode and is proportional to the charge of the particle bunch. The TM₁₁₀ mode, known as the dipole mode, is proportional to the charge and position of the beam in the cavity. The two antennae are used to measure the phase and amplitude of these two modes. From this the beam position can be calculated.

The output of a cavity BPM has a very low bandwidth; as a result very little of the output is filtered out during processing, so the signal-to-noise ratio is not reduced. This allows a cavity BPM to achieve in principle much higher resolution than stripline or button BPMs. A cavity BPM can achieve $\sim 20\text{nm}$ resolution, whilst stripline and button BPMs are limited to $\sim 1\mu\text{m}$. However due to the low bandwidth of cavity BPMs, the response time is typically much longer than stripline and button BPMs.

2.2 Physics of Stripline and Button BPMs

The primary goal for FONT at ATF2 is to achieve, with low latency, sub-micron corrections to the beam jitter at the entrance of the final focus system. The FONT stripline BPMs are capable of resolving sub-micron changes in the beam position. Because of this, it is considered unnecessary to employ cavity BPMs; the improved resolution is not needed and would reduce the time available to process the BPM signals and calculate a correction for

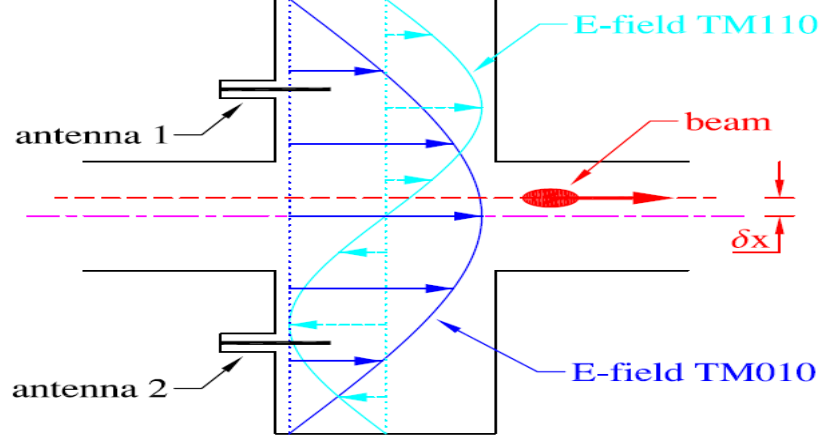


Figure 2.2: Sketch of the TM010 and TM110 modes of the electric field generated by a bunch of charged particles passing through a cavity BPM [28].

the kickers. Therefore the physics of cavity BPMs will not be described.

If it is assumed that a bunch of charged particles behaves as a point-like charge the electric field lines will be uniformly distributed radially in the rest frame of the particle bunch. At ATF2, electrons with an energy of 1.3 GeV are used. In the rest frame of a stripline BPM the field lines of the electron bunch are Lorentz-contracted into a very thin disc, perpendicular to the direction of motion (Figure 2.3). Eq. 2.1 shows the magnitude of an electric field a distance r from a point-like charge, in the rest frame of the charge, Q .

$$E(r) = \frac{Q}{4\pi\epsilon_0 r^2} \quad (2.1)$$

Eq. 2.2 shows how the electric field is transformed under a Lorentz transformation [29]; $\mathbf{E}$ is the electric field in the BPM rest frame and $\mathbf{E}'$ is the electric field in the bunch rest frame.

$$\mathbf{E}' = \gamma(\mathbf{E} - \boldsymbol{\beta} \times \mathbf{B}) - \frac{\gamma^2}{\gamma + 1} \boldsymbol{\beta}(\boldsymbol{\beta} \cdot \mathbf{E}) \quad (2.2)$$

If we define the z axis to lie along the direction of motion, then the electric field transforms as shown in Eq. 2.3. A magnetic field is observed due to the moving charged particles. R is the radius of the BPM and Q is the bunch charge.

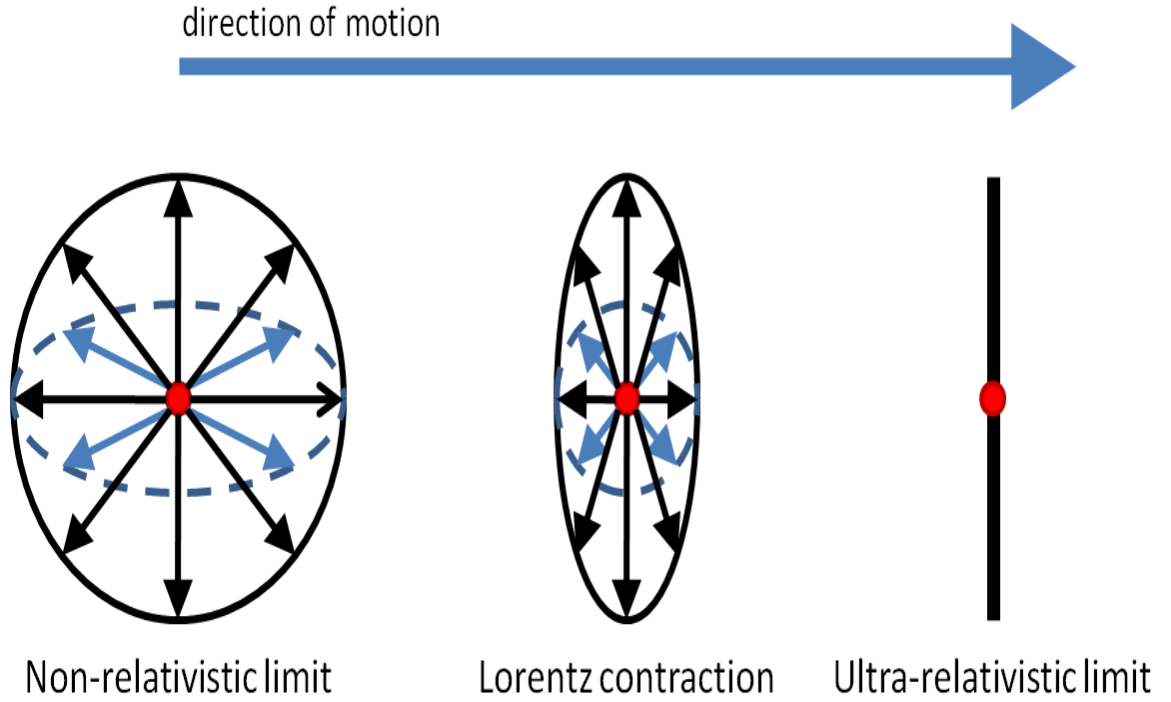


Figure 2.3: Diagram showing how the electric field differs between the non-relativistic and ultra-relativistic limits.

$$\begin{aligned}
 E_x = E'_x &= \frac{Q\gamma R}{(R^2 + \gamma^2 v^2 t^2)^{3/2}} \\
 B_y = \gamma\beta E'_x &= \beta E_x \\
 E_z = E'_z &= \frac{Q\gamma vt}{(R^2 + \gamma^2 v^2 t^2)^{3/2}}
 \end{aligned} \tag{2.3}$$

The image charge observed in one of the BPM striplines will be the electric field integrated over the surface projected onto the stripline (Eq. 2.4).

$$\int_A \mathbf{E} \cdot d\mathbf{S} = \frac{Q_{\text{image}}}{\epsilon_0} \tag{2.4}$$

$d\mathbf{S}$ is the element of the surface integral in the BPM frame of reference. Eq. 2.5 shows this element in terms of $d\mathbf{S}'$.

$$d\mathbf{S} = \begin{pmatrix} x' \\ y' \\ \gamma z' \end{pmatrix} \sqrt{dx'^2 + dy'^2} \frac{dz'}{\gamma} = \begin{pmatrix} x' \\ y' \\ \gamma z' \end{pmatrix} \frac{dS'}{\gamma} \tag{2.5}$$

From Eqs. 2.3 and 2.5, the integrand from Eq. 2.4 can be calculated (Eq. 2.6). The integrand is Lorentz invariant, thus so is the image charge.

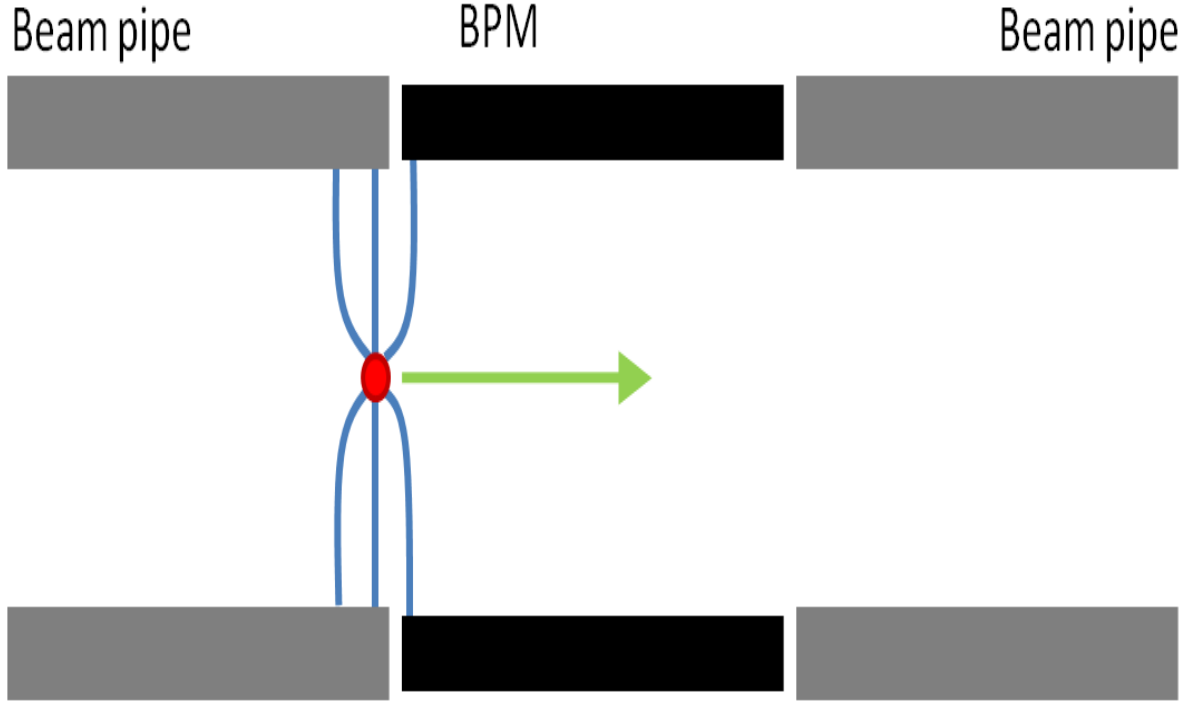


Figure 2.4: Diagram showing the field lines ‘jumping’ from the beam pipe wall to the BPM.

$$\mathbf{E} \cdot d\mathbf{S} = (\gamma E'_x x' + \gamma E'_y y' + \gamma E'_z z') \frac{d\mathbf{S}'}{\gamma} = \mathbf{E}' \cdot d\mathbf{S}' \quad (2.6)$$

As the electron bunch approaches the stripline BPM, the field lines ‘jump’ from the beam pipe wall to the BPM striplines; (Figure 2.4). This induces an image current in the striplines, which will have the opposite sign to the charge of the electrons. Because the electric field forms a thin disc in the lab frame, the induced voltage signal will be a narrow pulse. As the electron bunch approaches the end of the BPM, the field lines ‘jump’ from the stripline back to the beam pipe wall. This induces another narrow voltage pulse which will be equal in amplitude, but opposite in sign. The amplitude of the stripline signal is proportional to the image charge observed by the stripline which is proportional to the solid angle due to the angular width of the stripline (Figure 2.5).

Since the image charge is Lorentz invariant, it is easier to calculate in the rest frame of the bunch as the electric field radiates uniformly in all directions. The solid angle projected onto the stripline is proportional to the angular width of stripline i , θ_i (Eq. 2.7). The image charge is shown in Eq. 2.8.

$$\Omega_i = \frac{\theta_i^2}{\pi} \quad (2.7)$$

$$Q_{\text{image}} = -Q \frac{\Omega_i}{4\pi} = -\frac{Q\theta_i^2}{4\pi^2} \quad (2.8)$$

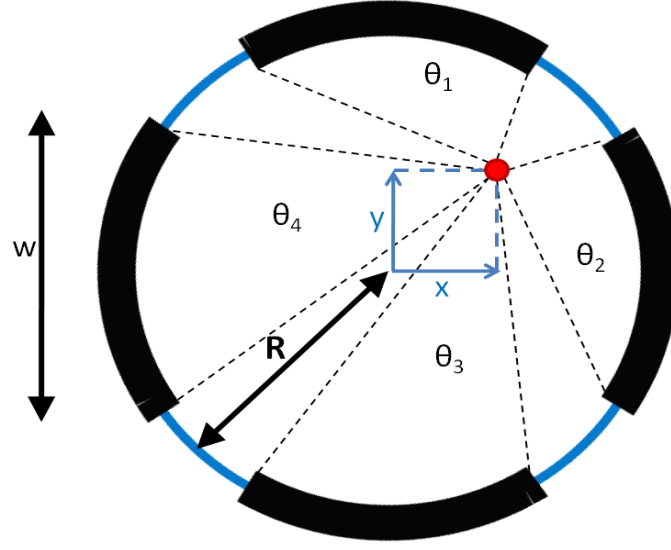


Figure 2.5: Sketch of the angular width of the striplines in a BPM as seen from the electron bunch.

If the bunch is displaced horizontally from the centre of the BPM by a distance x , and displaced vertically by a distance y , then the angular width of the top and bottom striplines are described by Eqs. 2.9 and 2.10; the approximation that $x \ll (R - y)$ is used. R is the radius of the BPM, w is the stripline width and θ_1 and θ_3 are the angular widths of the the top and bottom striplines in the BPM (Figure 2.5).

$$\theta_1 = 2 \arctan \left(\frac{w}{2 \sqrt{(R - y)^2 + x^2}} \right) \approx 2 \arctan \left(\frac{w}{2 (R - y)} \right) \quad (2.9)$$

$$\theta_3 = 2 \arctan \left(\frac{w}{2 \sqrt{(R + y)^2 + x^2}} \right) \approx 2 \arctan \left(\frac{w}{2 (R + y)} \right) \quad (2.10)$$

Where the Taylor expansion of an inverse tangent is given by

$$\arctan \left(\frac{w}{2 (R \pm y)} \right) = \frac{w}{2 (R \pm y)} - \frac{w^3}{24 (R \pm y)^3} + \frac{w^5}{160 (R \pm y)^5} - \dots \quad (2.11)$$

The resulting image charges are shown in Eqs. 2.12 and 2.13. Q_1^{image} and Q_3^{image} are the image charges observed on the top and bottom striplines respectively.

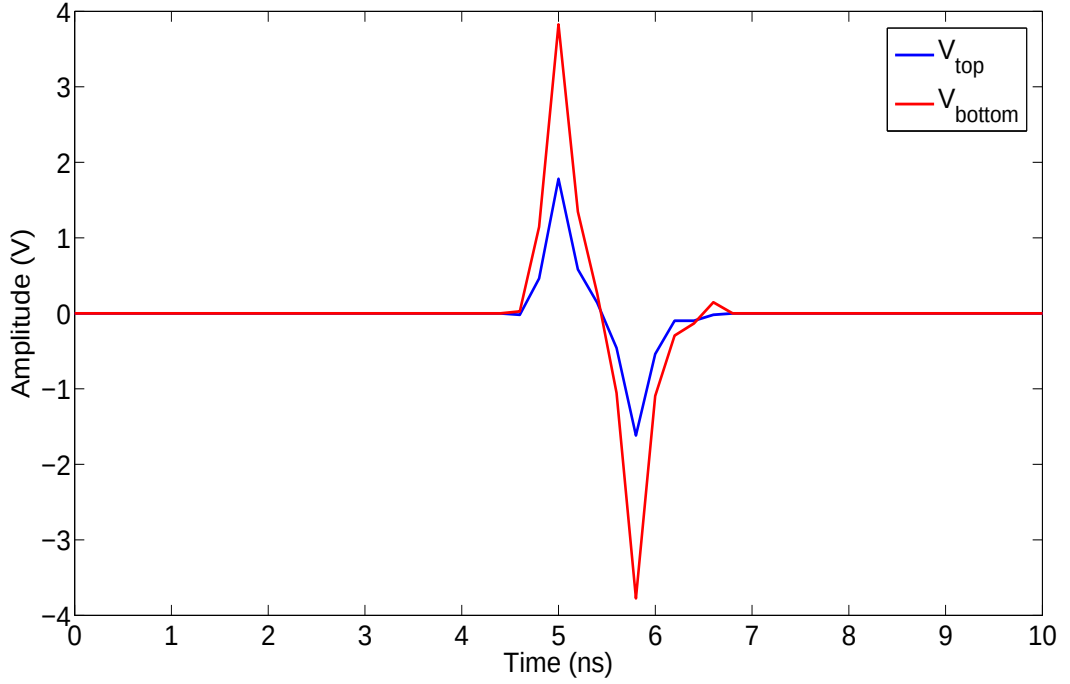


Figure 2.6: V_{top} (blue line) and V_{bottom} (red line) vs. time.

$$Q_1^{\text{image}} = -\frac{Q}{4\pi^2} \frac{w^2}{(R-y)^2} \quad (2.12)$$

$$Q_3^{\text{image}} = -\frac{Q}{4\pi^2} \frac{w^2}{(R+y)^2} \quad (2.13)$$

From Eqs. 2.12 and 2.13, the voltage outputs of the top and bottom striplines are given by Eqs. 2.14 and 2.15 respectively.

$$V_{top} = \rho \frac{w^2}{(R-y)^2} \left(\frac{dQ}{dt} \right) \quad (2.14)$$

$$V_{bottom} = \rho \frac{w^2}{(R+y)^2} \left(\frac{dQ}{dt} \right) \quad (2.15)$$

$\left(\frac{dQ}{dt} \right)$ is the image current induced by the bunch, ρ is the impedance of the measurement electronics and V_{top} and V_{bottom} are the voltage signals observed on the top and bottom stripline respectively. Figure 2.6 shows the V_{top} and V_{bottom} signals monitored. The stripline signal was measured using an oscilloscope at ATF; the low bandwidth of the oscilloscope broadens the real stripline signal, but the bipolar shape remains unaltered.

2.3 Processing Stripline BPM signals

2.3.1 Difference Over Sum

The sum and difference of the stripline outputs are shown in Eqs. 2.16 and 2.17 respectively.

$$\Sigma \equiv V_{top} + V_{bottom} = \rho \frac{2w^2 (R^2 + y^2)}{(R^2 - y^2)^2} \left(\frac{dQ}{dt} \right) \approx \rho \frac{2w^2}{R^2} \left(\frac{dQ}{dt} \right) \quad (2.16)$$

$$\Delta \equiv V_{top} - V_{bottom} = \rho \frac{4w^2 Ry}{(R^2 - y^2)^2} \left(\frac{dQ}{dt} \right) \approx \rho \frac{4w^2 y}{R^3} \left(\frac{dQ}{dt} \right) \quad (2.17)$$

Σ is only dependent on the bunch charge, whilst Δ is dependent on both bunch charge and bunch position. Dividing Δ by Σ gives a quantity that is only proportional to bunch position (Eq. 2.18).

$$\frac{\Delta}{\Sigma} = \frac{2y}{R} - \frac{w^2 y^3}{6R^5} + \frac{23w^4 y^5}{120R^9} - \dots \approx \frac{2y}{R} - \dots \quad (2.18)$$

2.3.2 Log Ratio

Another method of processing the output signals of a stripline BPM is to calculate the logarithms and subtract them, Eq. 2.19 [30].

$$\log(V_{top}) - \log(V_{bottom}) = \log\left(\frac{V_{top}}{V_{bottom}}\right) \quad (2.19)$$

Substituting Eqs. 2.14 and 2.15 into Eq. 2.19 gives the following expression.

$$\log\left(\frac{V_{top}}{V_{bottom}}\right) = 2 \log\left(\frac{R+y}{R-y}\right) = 2 \log\left(\frac{1+\frac{y}{R}}{1-\frac{y}{R}}\right) \quad (2.20)$$

A Taylor expansion for a logarithm, shown in Eq. 2.21, can be used to derive Eq. 2.22.

$$\log\left(1 \pm \frac{y}{R}\right) = \pm \frac{y}{R} - \frac{y^2}{2R^2} \pm \frac{y^3}{3R^3} - \dots \quad (2.21)$$

$$\log\left(\frac{V_{top}}{V_{bottom}}\right) = 4 \frac{y}{R} + 4 \frac{y^3}{3R^3} + 4 \frac{y^5}{5R^5} + \dots \approx 4 \frac{y}{R} + \dots \quad (2.22)$$

As with the difference over sum method, the log ratio method gives a linear approximation to the bunch position; both methods are linear up to similar ranges of position. In the FONT feedback system logarithmic amplifiers would be required for the log ratio method; these have a relatively long latency compared to the difference over sum processing scheme.

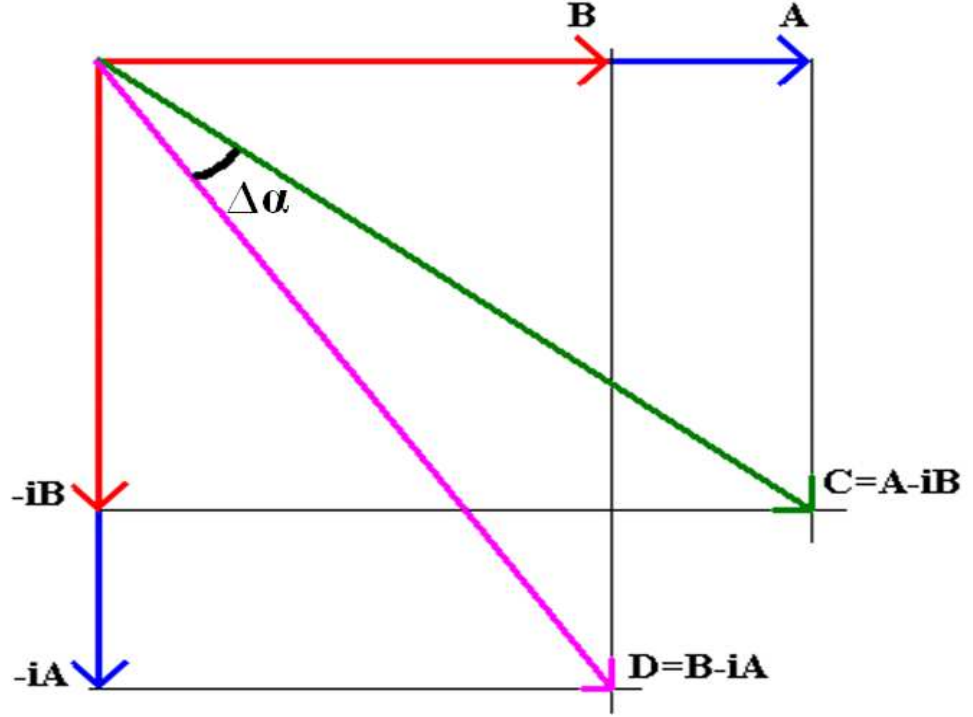


Figure 2.7: Diagram showing the amplitude to phase relations in the AM/PM scheme [31].

This method would reduce the available latency budget for the rest of the feedback system and is hence not used.

2.3.3 Amplitude Modulation to Phase Modulation (AM/PM)

In the previous two processing schemes, the beam position is calculated by using the signal amplitudes. In the AM/PM scheme, the signal amplitudes are converted into phase information and used to calculate the beam position.

Each stripline signal is split into two parts, one of these halves is delayed by 90° with respect to a reference frequency. The delayed half of the stripline signal is combined with the undelayed half of the other stripline signal. Figure 2.7 shows a diagram of the AM/PM scheme represented in complex space. A and B represent the BPM stripline signals V_{top} and V_{bottom} respectively.

In the complex representation, the stripline signals, V_{top} and V_{bottom} , will be transformed as shown in Eqs. 2.23 and 2.24. V_C and V_D are the manipulated signals as represented by C and D in Figure 2.7.

$$V_C = \sqrt{V_{top}^2 + V_{bottom}^2} \times e^{-i \arctan\left(\frac{V_{top}}{V_{bottom}}\right)} = \sqrt{V_{top}^2 + V_{bottom}^2} \times e^{-i\alpha_C} \quad (2.23)$$

$$V_D = \sqrt{V_{top}^2 + V_{bottom}^2} \times e^{-i \arctan\left(\frac{V_{bottom}}{V_{top}}\right)} = \sqrt{V_{top}^2 + V_{bottom}^2} \times e^{-i\alpha_D} \quad (2.24)$$

Eq. 2.11 can be used to find expressions for the phases of V_C and V_D , Eqs. 2.25 and 2.26.

$$\alpha_C = \frac{V_{top}}{V_{bottom}} = \left(\frac{R+y}{R-y}\right)^2 \quad (2.25)$$

$$\alpha_D = \frac{V_{bottom}}{V_{top}} = \left(\frac{R-y}{R+y}\right)^2 \quad (2.26)$$

From which the phase difference between the two signals can be calculated, Eq. 2.27.

$$\Delta\alpha = \left(\frac{R+y}{R-y}\right)^2 - \left(\frac{R-y}{R+y}\right)^2 = \frac{4Ry}{(R^2 - y^2)^2} \approx \frac{4y}{R^3} \quad (2.27)$$

The AM/PM scheme was not used because it is a more complicated scheme to use and the latency is larger than can be achieved with the difference over sum scheme.

2.4 Summary

FONT uses stripline BPMs. Each BPM is 14cm long, with a 12mm internal radius, and each stripline has an angular width of 70° with respect to the centre of the BPM.

FONT uses custom designed analogue BPM processors to manipulate the stripline signals. The difference over sum scheme is implemented and the Δ and Σ signals are produced by the analogue processors (Chapter 3). The log ratio scheme was not used because logarithmic amplifiers would be required, which have a large latency. The complexity of the AM/PM scheme as well as the larger latency than the difference over sum scheme renders it unsuitable for the FONT BPM system.

Chapter 3

BPM Processors

The frequency components of the raw signals from the stripline BPMs are too high for the FONT digital electronics to be able to process. In order to overcome this problem, analogue processors are used to limit the bandwidth and to mix down the frequency of the stripline signals. The FONT analogue processors were extensively modelled using SPICE [32], an electronics simulation software, and Matlab [33]. Each stage of the processor was simulated separately before being combined into a full simulation of the entire processor. These simulation results were compared and verified with measurements taken from the real processors.

3.1 Analogue Processor Design

The analogue processors consist of five main stages (Figure 3.1). At the inputs of the processor are low-pass filters, with a cut-off frequency of 1GHz. The components used were LFCN-1000, ceramic LPFs from Mini-Circuits [34]. The LPF was used to remove high frequency noise picked up on the cables, from other sources, whilst leaving the raw stripline signals mostly unaltered.

On the ‘Difference’ (Δ) channel of the processor, there is a 180° hybrid. The components used were SCPJ-2-9, from Mini-Circuits [35]. The SCPJ-2-9 applies a 180° phase shift to one of the two inputs with respect to the other input and then combines the two signals. This results in an analogue subtraction of the two input signals. The hybrid’s phase shift on the two inputs varies with frequency, which introduces a component to the Δ' signal that is 90° out of phase with respect to the expected signal; this is known as a quadrature component and will be discussed later in this chapter. Δ represents the Δ' signal at the output of the analogue processor (Figure 3.1).

The ‘Sum’ (Σ) channels are produced by using a resistive coupler network to combine the two input signals, as shown in Figure 3.2. Neglecting stray inductance and capacitance, the resistive coupler network will affect all frequencies equally, hence there will be no significant quadrature residuals on Σ' introduced here. Additionally, the quadrature component only becomes significant at very low signal levels, and Σ' is proportional to the bunch charge (Eq. 2.16). In practice, the bunch charge is maintained at a high enough level that any residual on Σ' would be negligible. Σ represents the ‘in-phase’ Σ' channel at the output of the analogue processor, while Σ_Q represents the ‘quadrature phase’ Σ' channel at the output of the analogue processor (Figure 3.1). Σ_Q is produced by using a 90° hybrid to shift the

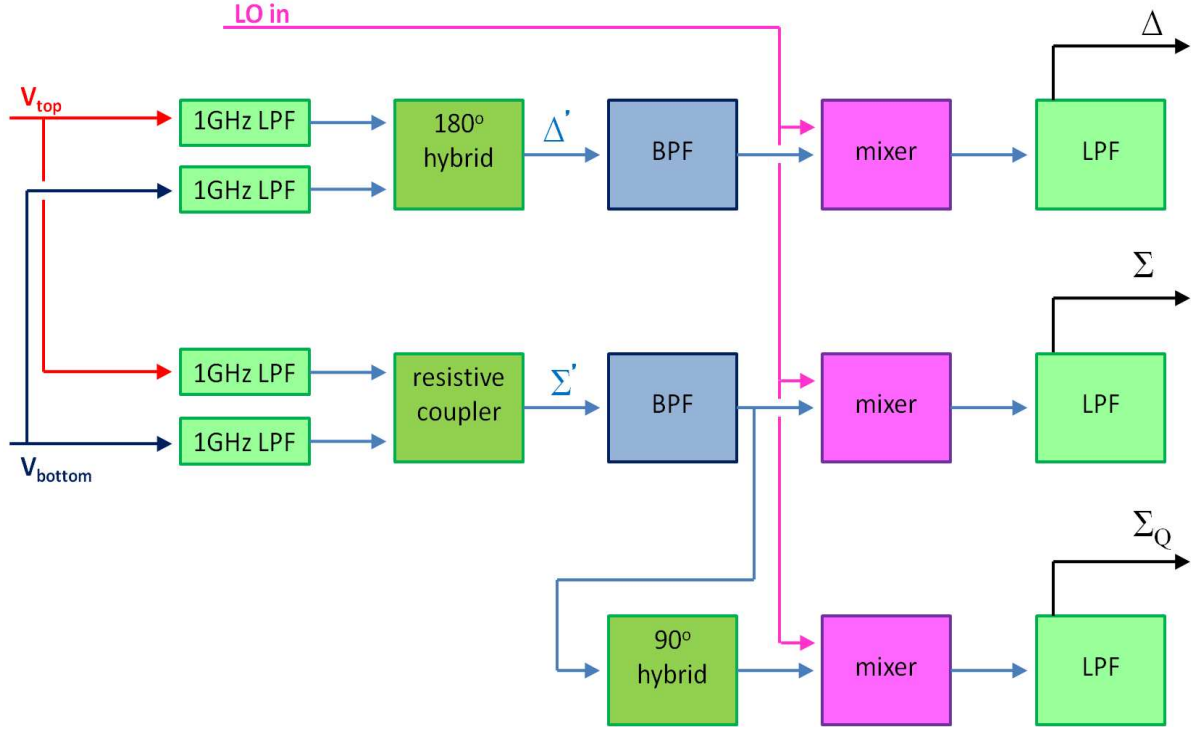


Figure 3.1: Schematic diagram of main parts of FONT BPM processor

phase of Σ' .

The remaining circuitry for the Σ and Δ channels is now identical. The band-pass filter (BPF) is used to reduce the bandwidth of the Σ and Δ signals; this reduces the bandwidth of the processor output signals. The BPF is also used to remove frequency components that would otherwise interfere with the Σ and Δ signals after the mixer. The BPF is designed to have a bandwidth of $\sim 200\text{MHz}$ and a central frequency of $\sim 714\text{MHz}$. The BPF bandwidth and central frequency are very sensitive to stray inductance and capacitance; this will be discussed later in this chapter. The band-pass filter design used is a Bessel filter [31].

A radio-frequency mixer is a semiconductor device that is used to output the product of two input signals. If the two input signals are sinusoidal, the product will be the sum of two sinusoidal signals. The frequency of one component is the sum of the two input signal frequencies, the other is the difference of the two frequencies; these are known as the up-mixed frequency and down-mixed frequency respectively. Due to this, the mixer can be used to down-mix the frequency of the BPF output. The mixers used for the FONT BPM processors are SYM-2 by Mini-Circuits [36]. A local oscillator (LO) drives the mixer; this is a $+14\text{dBm}$, 714MHz sinusoidal signal. The other input, known as the 'RF', is the output of the BPF; the amplitude of this signal is much smaller than the LO. If this were not the case, the RF would also drive the mixer, and would cause the output to become non-linear. In reality, the RF will always have some effect on the mixer. Indeed this is the most significant cause of non-linear effects in the processor. The output of the mixer is known as the intermediate frequency (IF) signal. Eq. 3.1 shows the mixer output when two sinusoidal signals are connected to the inputs; f_{RF} and f_{LO} are the frequencies of the RF and LO

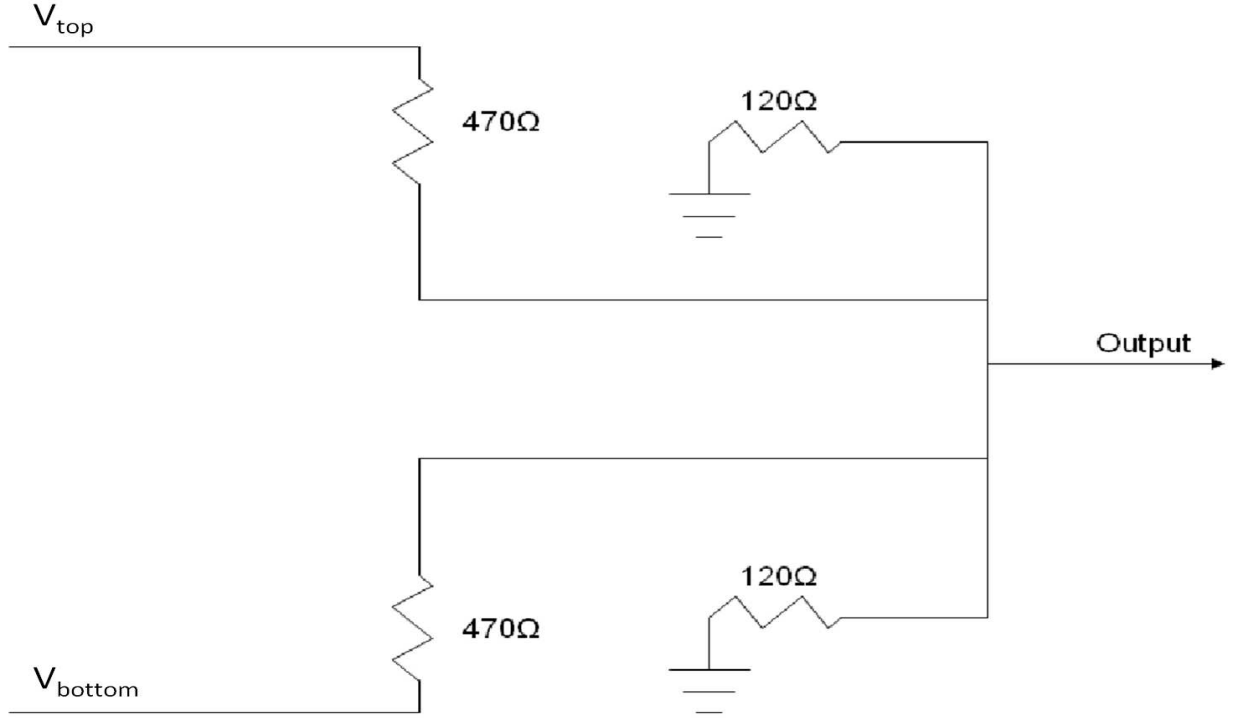


Figure 3.2: Circuit diagram of resistive coupler for ‘Sum’ channel of FONT BPM processor

signals respectively. In order to minimise the frequency of the low frequency (down-mixed) term, the LO and RF frequencies must be equal, reducing the down-mixed frequency to DC, thus the BPF is designed to have a central frequency of $\sim 714\text{MHz}$.

$$V_{IF} = \sin(2\pi f_{RF}t) \times \sin(2\pi f_{LO}t) = \frac{1}{2}[\cos(2\pi(f_{RF} - f_{LO})t) + \cos(2\pi(f_{RF} + f_{LO})t)] \quad (3.1)$$

If $f_{RF} \approx f_{LO}$ but the RF is given a phase, ϕ , with respect to the LO, Eq. 3.2 shows that in order to maximise the amplitude of the low frequency term of the IF, the phase ϕ between the RF and LO should be zero.

$$V_{IF} = \sin(2\pi f_{LO}t + \phi) \times \sin(2\pi f_{LO}t) = \frac{1}{2}[\cos(\phi) + \cos(4\pi f_{LO}t + \phi)] \quad (3.2)$$

The final stage in the processor is the low-pass filter, this is used to suppress the high frequency (up-mixed) term of the mixer output. A Chebyshev filter design [31] was used since this has a strong suppression for frequencies outside the passband. This is chosen at the expense of the flatness of the frequency response in the passband. An alternative filter would have been a Butterworth which is designed to be maximally flat in the passband, although this has a shallower roll-off outside the passband [37]. Finally it was decided that a steeper cut-off was more important than the flatness in the passband. Eq. 2.18 shows that as long as the time-dependent terms in the Σ' and Δ' signals are transformed equally, the position calculation will be unchanged. As a result, the flatness in the passband is not

as important as the out-of-passband suppression. The BPF is designed to have a central frequency of $\sim 714\text{MHz}$ and a bandwidth of $\sim 200\text{MHz}$. Since the LO is also 714MHz , the output from the mixer will have two spectra, one near DC, and one near 1400MHz . The cut-off frequency of the LPF is designed to suppress the high-frequency terms, thus only leaving the low-frequency terms. The output from the LPF is now at a low enough frequency for the digital electronics to be able to process it.

Figure 3.3 shows a photograph of one of the FONT BPM processors; the SMA connectors have been annotated using the notation defined in Figure 3.1.

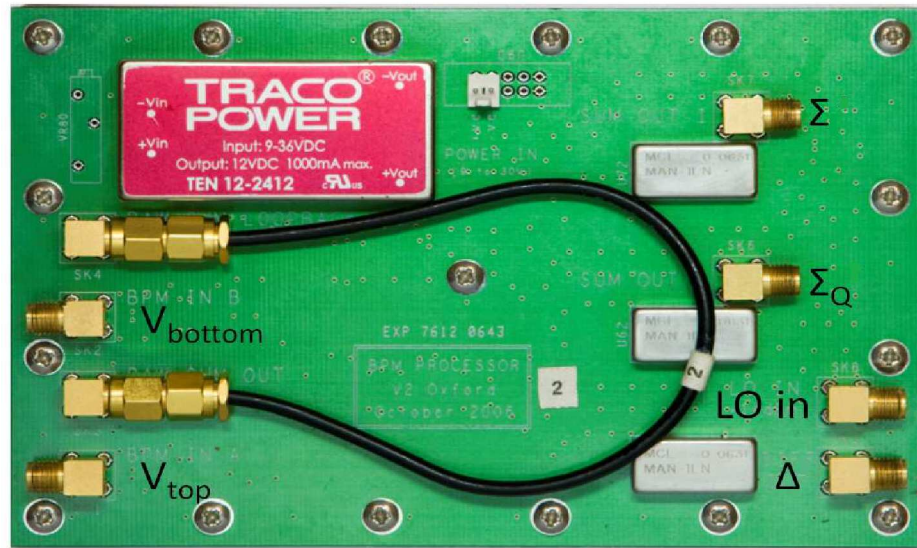


Figure 3.3: Photograph of a FONT BPM processor.

The raw stripline signals used in the simulation are actual signals taken from a stripline BPM at ATF (Figure 2.6). This allowed a direct comparison between the simulation and the real data. Measurements of signals at the different stages of the processor were also taken to verify the simulation of the constituent parts of the processor.

FONT has built 10 analogue processors in total, although processors 6 and 9 are not used due to irregularities in the filter responses. The other processors are regularly used at ATF2.

3.2 Analogue Processor Simulations

In order to fully understand the performance of the analogue BPM processors, the electronics were modelled using SPICE and Matlab. Initially each stage of the processor was modelled separately to ensure that its behaviour was similar to measurements. Once these models were complete, they were combined to simulate the entire processor; this was successful in modelling the real processors. Due to the success of the simulation, it became a valuable tool in optimising the performance of the real BPM system.

3.2.1 180° Hybrid

The ideal 180° hybrid would apply a 0° shift to one input, and a 180° shift to the other input, independently of the frequency of the signal, with no loss of signal. Then the two outputs would be combined to produce a signal equal to the difference between the two inputs. In reality the phase and amplitude responses of the input channels do depend on frequency. V_{top} was defined as the input that has a 0° phase shift applied, and V_{bottom} as the input that has a 180° phase shift applied. First, consider a case where V_{bottom} is shifted exactly 180° with respect to V_{top} , but a small constant phase shift, θ , is applied to both inputs. Next for a frequency, ω , V_{top} and V_{bottom} will be transformed as shown in Eq. 3.3 and 3.4 respectively; A_{top} and A_{bottom} are the amplitudes of V_{top} and V_{bottom} respectively.

$$V'_{top} = A_{top} \cdot \sin(2\pi\omega t + \theta) = A_{top} [\sin(2\pi\omega t) \cos(\theta) + \cos(2\pi\omega t) \sin(\theta)] \quad (3.3)$$

$$V'_{bottom} = -A_{bottom} \cdot \sin(2\pi\omega t + \theta) = -A_{bottom} [\sin(2\pi\omega t) \cos(\theta) + \cos(2\pi\omega t) \sin(\theta)] \quad (3.4)$$

When the two resulting signals are added together, Δ' will be shifted by a constant phase, θ , which can be considered as a time delay and therefore compensated for. Now consider the case where the relative phase between V'_{top} and V'_{bottom} varies slightly with frequency, neglecting any constant phase shift, but allowing a frequency dependent phase shift $\phi(\omega)$. V'_{bottom} will now vary with frequency as shown in Eq. 3.5, $\cos(\phi(\omega))$ and $\sin(\phi(\omega))$ have been substituted with $C(\omega)$ and $S(\omega)$ for simplicity.

$$V'_{bottom} \propto \sin(2\pi\omega t + \phi(\omega)) = C(\omega) \sin(2\pi\omega t) + S(\omega) \cos(2\pi\omega t) \quad (3.5)$$

Eq. 3.5 has a sine term, which is in phase with the original input, and a cosine term, which is orthogonally phased with respect to the original input; this term is known as the quadrature component. Figure 3.4 shows how a phase shift ϕ produces a quadrature component in the Δ' channel. Δ' varies with the relative amplitudes of the two inputs, and so when the amplitude of Δ' becomes very small, the quadrature component does not vanish. As a result, the quadrature component becomes significant at small amplitudes of Δ' , and will produce an apparent offset, as well as affect the pulse shape of the Δ' output. $\phi(\omega)$ is known as the phase unbalance.

Another consideration for the hybrid is that the amplitude of the hybrid channels varies with frequency. This will also produce an apparent offset, although it will not produce a contribution to the quadrature component since the phase is not affected by the amplitude of a sinusoidally varying signal. Figures 3.5 and 3.6 shows the frequency dependence of the phase and amplitude unbalance taken from the datasheet for the hybrid, SCPJ-2-9. The data were fitted to two polynomials, which are also shown. These polynomials were used to describe the frequency dependence of the phase and amplitude unbalance in the simulation.

The simulation for the hybrid was constructed in Matlab. Due to the frequency dependence of the hybrid, the simulation needed to be in the frequency domain, as opposed to the time

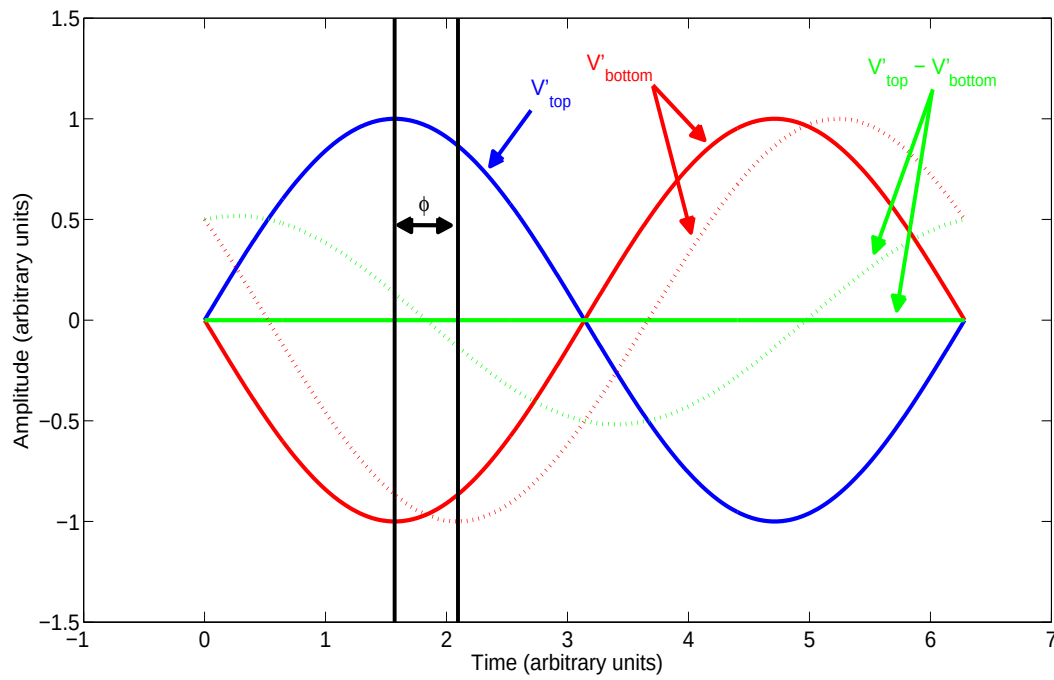


Figure 3.4: Sinusoidal voltage signal vs. time. The solid lines represent the signals without a phase shift; the dotted lines represent the signals with a phase shift.

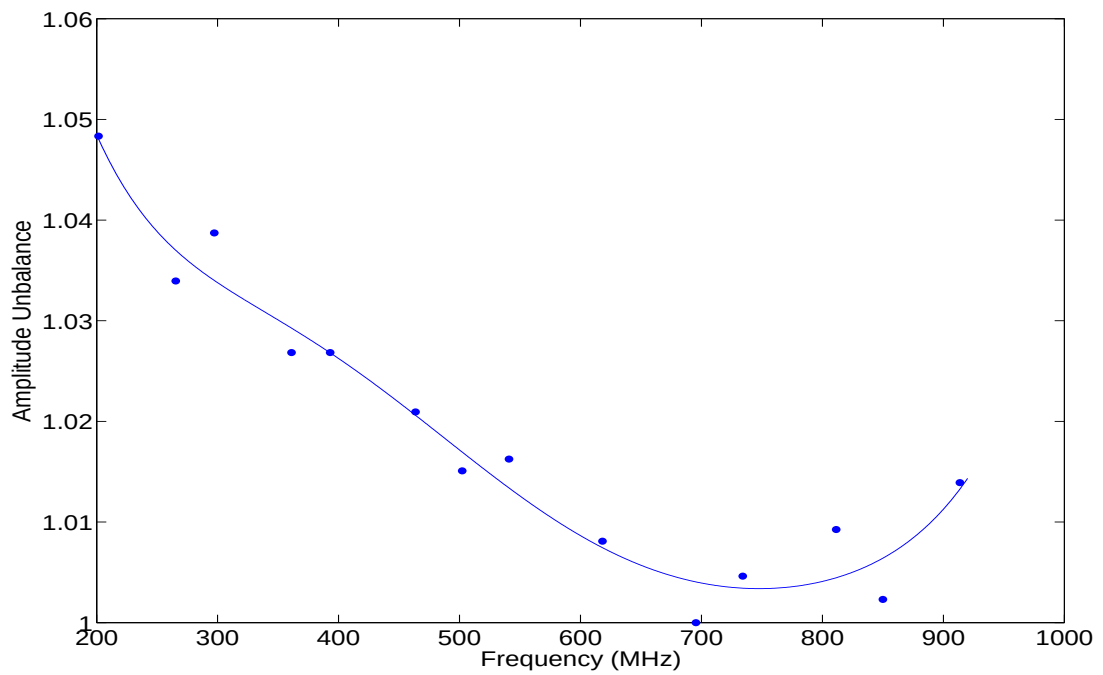


Figure 3.5: Amplitude unbalance vs. frequency from the hybrid datasheet. The line shows a polynomial fit.

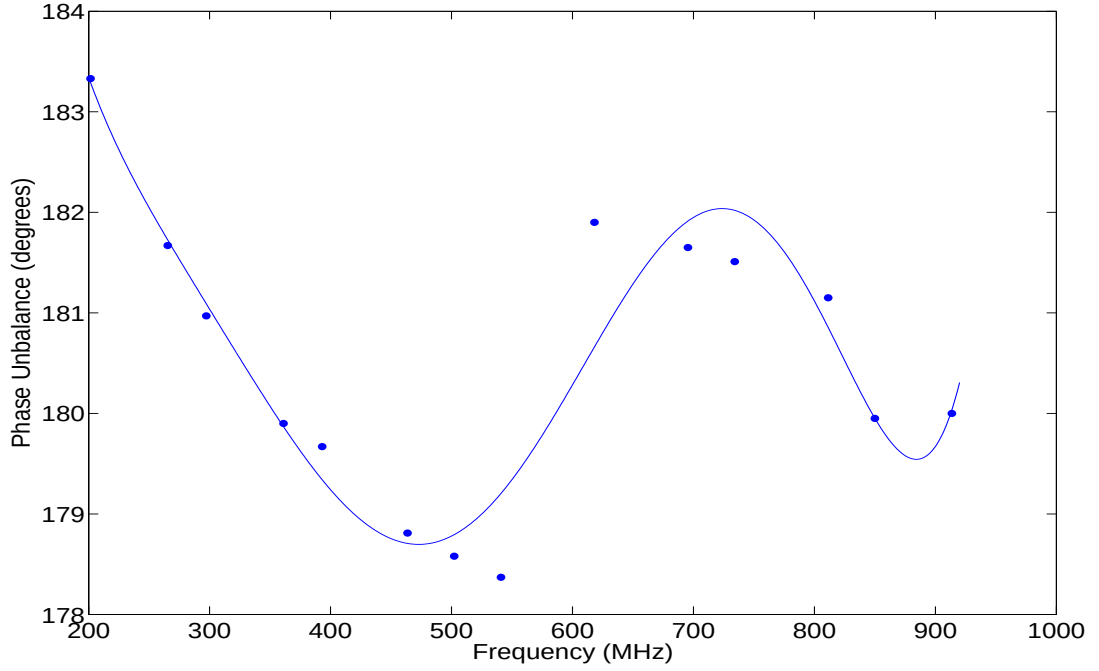


Figure 3.6: Phase unbalance vs. frequency from the hybrid datasheet. The line shows a polynomial fit.

domain. SPICE can either run simulations in the time domain or the frequency domain, but it cannot run simulations in both domains simultaneously. In order to convert the raw stripline signals into the frequency domain, a fast Fourier transform was applied. The Fourier transform is manipulated as shown by Eq. 3.6, where $f(t)$ is V_{bottom} , $F(\omega)$ is the Fourier transform of V_{bottom} , and $G(\omega)$ is the manipulated Fourier transform.

$$f(t) \rightarrow F(\omega) \rightarrow A(\omega) \times e^{i\phi(\omega)} \times F(\omega) = G(\omega) \rightarrow g(t) \quad (3.6)$$

$A(\omega)$ is the transformation due to the amplitude unbalance and $e^{i\phi(\omega)}$ is the transformation due to the phase unbalance. These transformations were derived from the polynomials fitted to the results from the hybrid datasheet. After this, the inverse Fourier transform was calculated, the result of which, is the simulated output of the 180° phase shift, $g(t)$. In order to simulate the output of the hybrid, waveforms of real stripline signals from ATF were used as the inputs to the simulation. These measurements were taken by connecting two stripline signals from a BPM to an oscilloscope and recording the waveforms as data files (Figure 2.6).

Figure 3.7 shows the simulated Δ' signal as well as the ideal Δ' signal; the ideal Δ' signal is the subtraction of the two stripline signals. The simulated Δ' signal uses the polynomial fits from Figures 3.5 and 3.6 and applies the transformation described in Eq. 3.6. Since the simulation performs a discrete fast fourier transform, the simulated signal is broadened slightly; this is the cause of the tails before and after the simulated Δ' signal. In practice

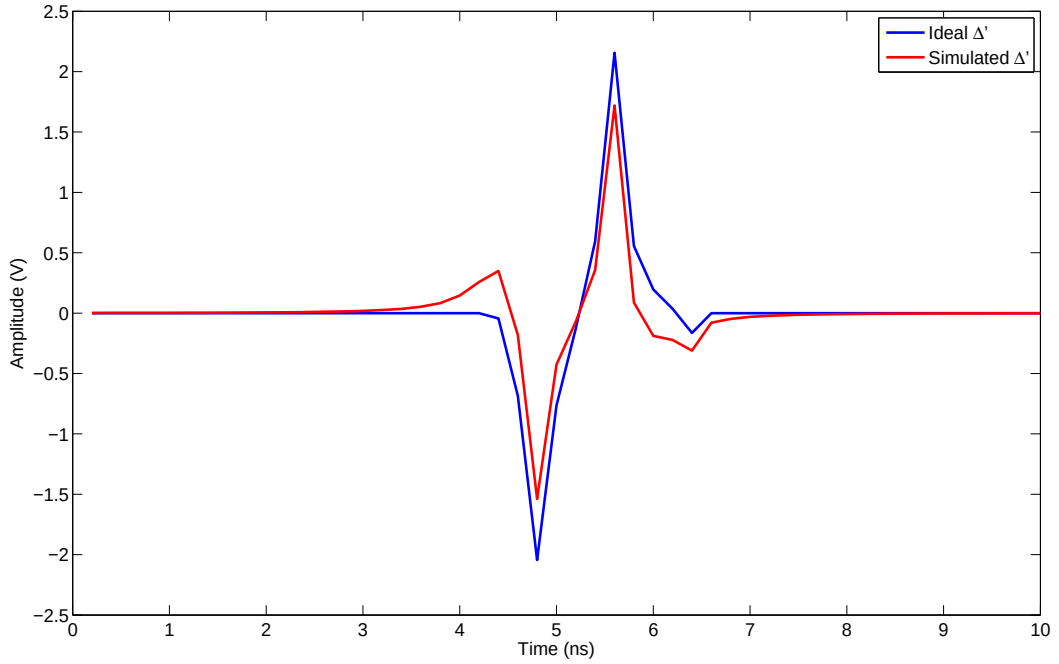


Figure 3.7: Ideal (blue) and simulated (red) Δ' signal vs. time.

these tails cause a negligible effect on the rest of the simulation; this is mostly because the band-pass filter operates in the frequency domain and so effects in the time domain are independent. The output of this simulation is loaded into SPICE for the remaining stages of the processor simulation.

3.2.2 Filter simulations

The band-pass (Figure 3.8) and low-pass (Figure 3.9) filters were simulated in SPICE. Since both of these filters are constructed from passive components, they will not introduce any non-linear effects to the signal amplitude. The filters will however transform the frequency spectra of the input signals, which will affect the pulse shape. Assuming that the filter responses are the same for the Σ and Δ channels of the processor, the calculated position will not be affected by the filters.

In the simulations, stray impedances were considered for the filters; the BPF frequency response is particularly sensitive to stray impedances. The filters were originally designed to take the strays into account.

Several sources of stray impedance were considered. First, the passive components used have an intrinsic stray impedance which is approximately constant for components of the same dimensions. The passive components were considered to have a stray inductance in series and a stray capacitance in parallel. The values of these stray impedances are governed mostly by the physical size of the passive component. For the components used on the

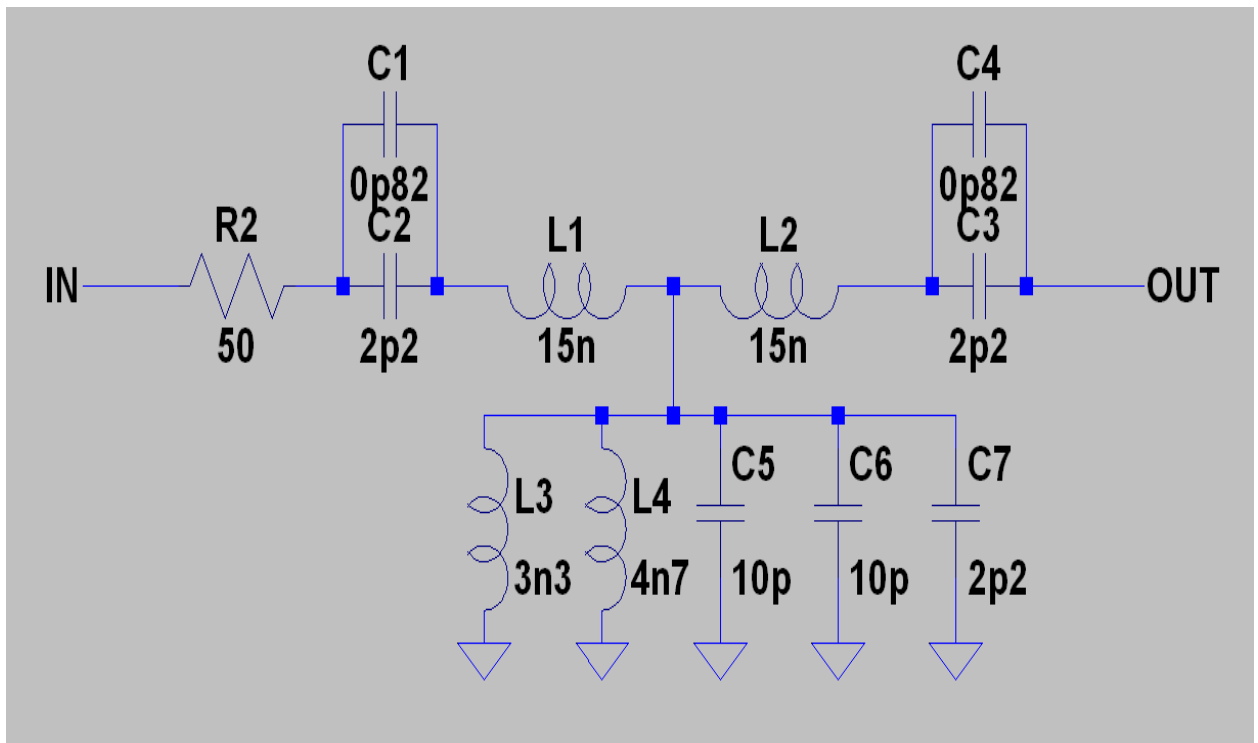


Figure 3.8: Circuit diagram of the band-pass filter used on the FONT analogue processors.

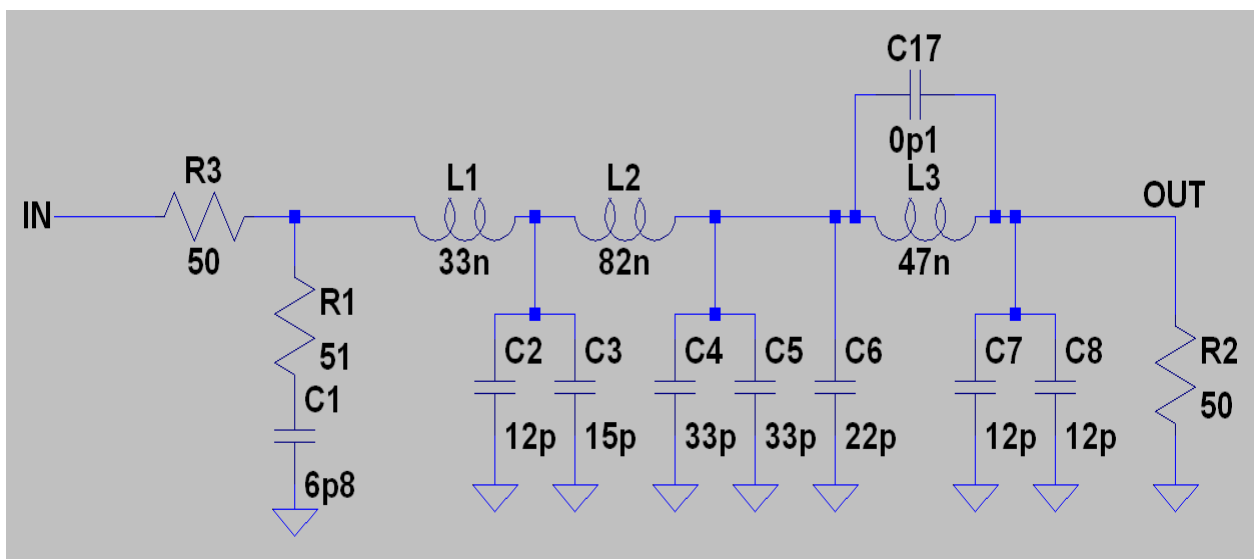


Figure 3.9: Circuit diagram of the low-pass filter used on the FONT analogue processors.

	Measured	Simulation with strays	Simulation without strays
Bandwidth	315MHz	289MHz	333MHz
Central frequency	670MHz	707MHz	788MHz

Table 3.1: Comparison between measured and simulated frequency response of the BPF.

FONT BPM processors a stray capacitance of 0.1pF and a stray inductance of 0.7nH are typical and thus were used in the simulation.

The other source of stray impedance considered was on the printed circuit boards (PCBs). The layout of each board is such that all the components are soldered onto one side of the board, and all the tracks are printed on this side. On the opposite side of the board is a grounding plane, a large thin sheet of copper covering most of the surface. Some of the larger electrical pads opposite the grounding plane contribute a significant capacitance. The PCB is made from a woven glass fibre and epoxy known as FR-4, this has a typical measured dielectric constant of 4.8. The geometry of the electrical pad and the grounding plate, with a dielectric material between, is the same as that of a parallel plate capacitor. Eq. 3.7 shows the capacitance of a parallel plate capacitor, where σ_A is the area of overlap between the two plates, which is the area of the electrical pad, d is the distance between the two plates, ϵ_r is the dielectric constant and ϵ_0 is the permittivity of free space.

$$C = \epsilon_0 \epsilon_r \frac{\sigma_A}{d} \quad (3.7)$$

On the BPF there are three pads and tracks that are electrically isolated from the grounding plane, and hence will produce a significant capacitance. Two of these have areas of approximately 12mm² and the other has an area of approximately 20mm². The distance between these pads and the grounding plane is known to be 1.6mm. Thus the calculated capacitance for the first two regions was ~ 0.33 pF, and ~ 0.55 pF for the second region. Figure 3.10 shows the frequency response of the simulated BPF with and without strays considered. Figure 3.11 shows the measured frequency response for the BPF on the Δ channel for processor 8. The probe used to take this measurement attenuates the signal by -26dB, hence the low signal level. Table 3.1 shows a comparison between measured and simulated frequency response for the BPF. The model with stray impedances considered has a central frequency that is significantly closer to the measured frequency than the model without stray impedances included. The bandwidth is similar for the two models and the measured value.

For the LPF, the stray capacitances from the board were not considered. These strays contribute $\sim 1\%$ to the capacitance values in the filter hence have a negligible effect on the filter response. Figure 3.12 shows the filter response for the LPF with and without the stray impedances considered. Figure 3.13 shows the measured frequency response for the LPF on the Δ channel for processor 8. The probe used to take this measurement attenuates the signal by -26dB, hence the low signal level. Table 3.2 shows a comparison between the measured and simulated cutoff frequency for the LPF. Compared to the BPF, the stray impedances have relatively small effect on the frequency response of the LPF. For both the BPF and LPF the simulations are similar to the measured frequency response.

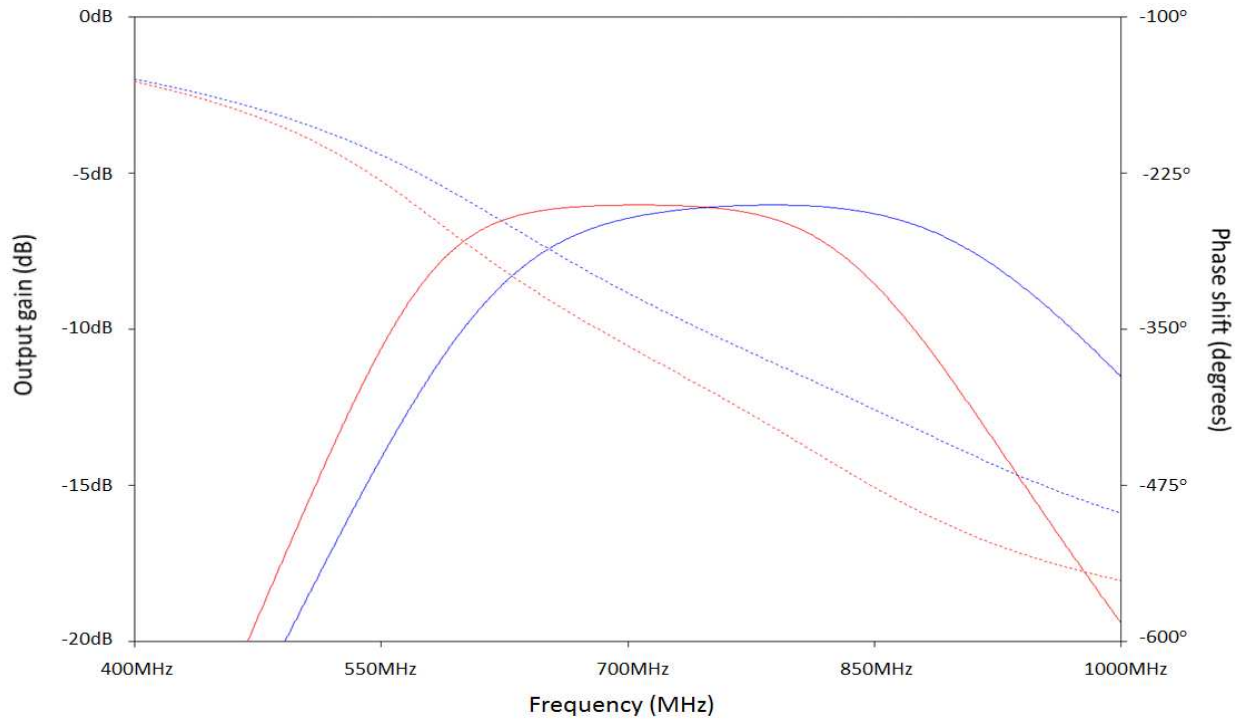


Figure 3.10: Simulated frequency response (dB) of the BPF vs. frequency. The BPF was modelled with (red line) and without (blue line) stray impedances considered. The dotted lines show the phase shift (right scale) and the solid lines show the amplitude response (left scale).

	Measured	Simulation with strays	Simulation without strays
Cutoff frequency	120MHz	105MHz	106.5MHz

Table 3.2: Comparison between measured and simulated frequency response of the LPF.

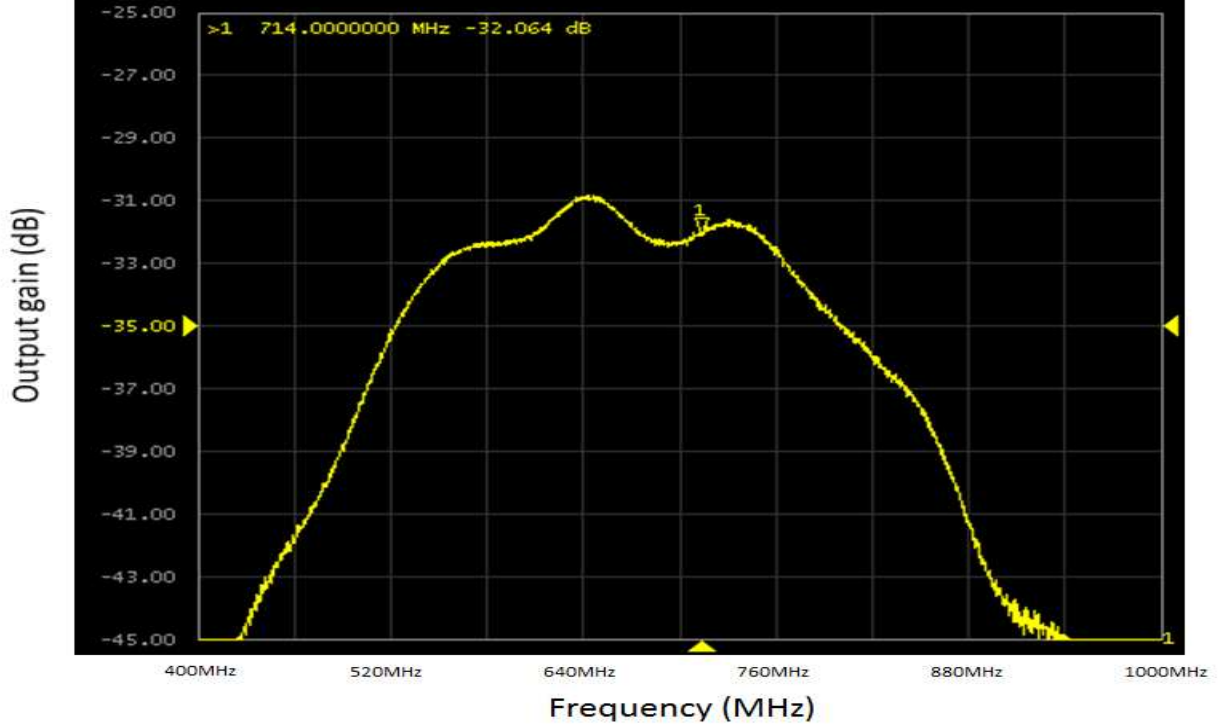


Figure 3.11: Measured frequency response (dB) of a BPF vs. frequency. The measurement was taken using a network analyser on the Δ channel of processor 8.

3.2.3 RF Mixer

The RF mixer is the main source of non-linear effects on the processor. Therefore, it was important to accurately describe the mixer in simulation to recreate these non-linearities. As shown in Figure 3.14, the mixer is a diode ring, driven by the LO signal. The mixers used for the FONT processors are SYM-2 by Mini-Circuits [36]. For the simulation the mixer diodes were modelled on the Agilent HSMS-2810 RF Schottky diode. Thus the HSMS-2810 has very similar properties to the mixer diodes. The diode parameters for the SPICE simulation are listed in table 3.3 [38]. The inductors forming the input transformers for the mixer were modelled as ideal inductors, each with a value of $1\mu\text{H}$ and perfect mutual inductance. These simplifications were made because the effects caused by considering more realistic inductors will be negligible in comparison to the effects of the BPF and LPF.

Figure 3.15 shows the mixer output versus RF input. The plot shows that the mixer output is reasonably linear for inputs in the region of 0-0.3V. For RF inputs $\geq 0.3\text{V}$, the mixer output flattens out because the RF input becomes comparable in amplitude to the LO, and so the RF begins to drive the diode ring. As a result, the mixer output becomes dependent on the LO amplitude, and not the RF amplitude and this saturation effect can severely alter the measured position of the beam.

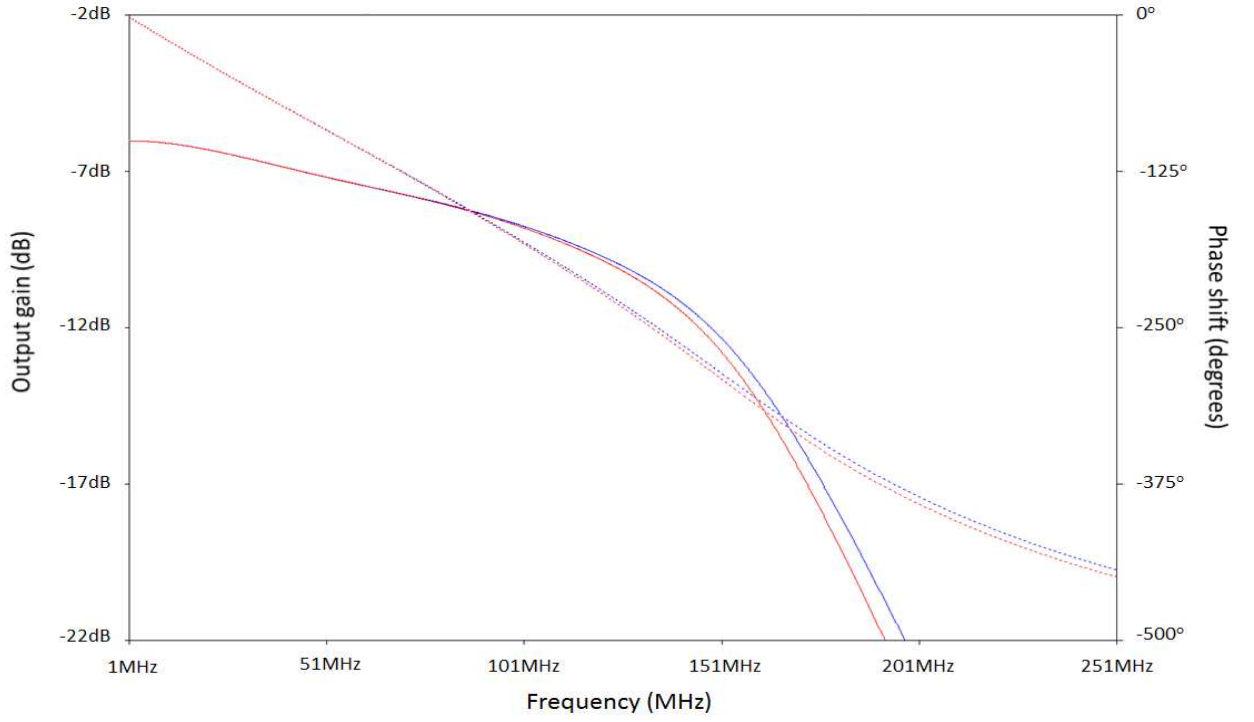


Figure 3.12: Simulated frequency response (dB) of the LPF vs. frequency. The LPF was modelled with (red line) and without (blue line) stray impedances considered. The dotted lines show the phase shift (right scale) and the solid lines show the amplitude response (left scale).

Parameter	Value
B_V	25V
C_{J0}	1.1pF
E_G	0.69eV
I_{bv}	10^{-5} A
I_s	4.8×10^{-9} A
N	1.08
R_S	10
P_b	0.65V
P_t	2
M	0.5

Table 3.3: SPICE parameters for Agilent HSMS-2810 RF Schottky diode

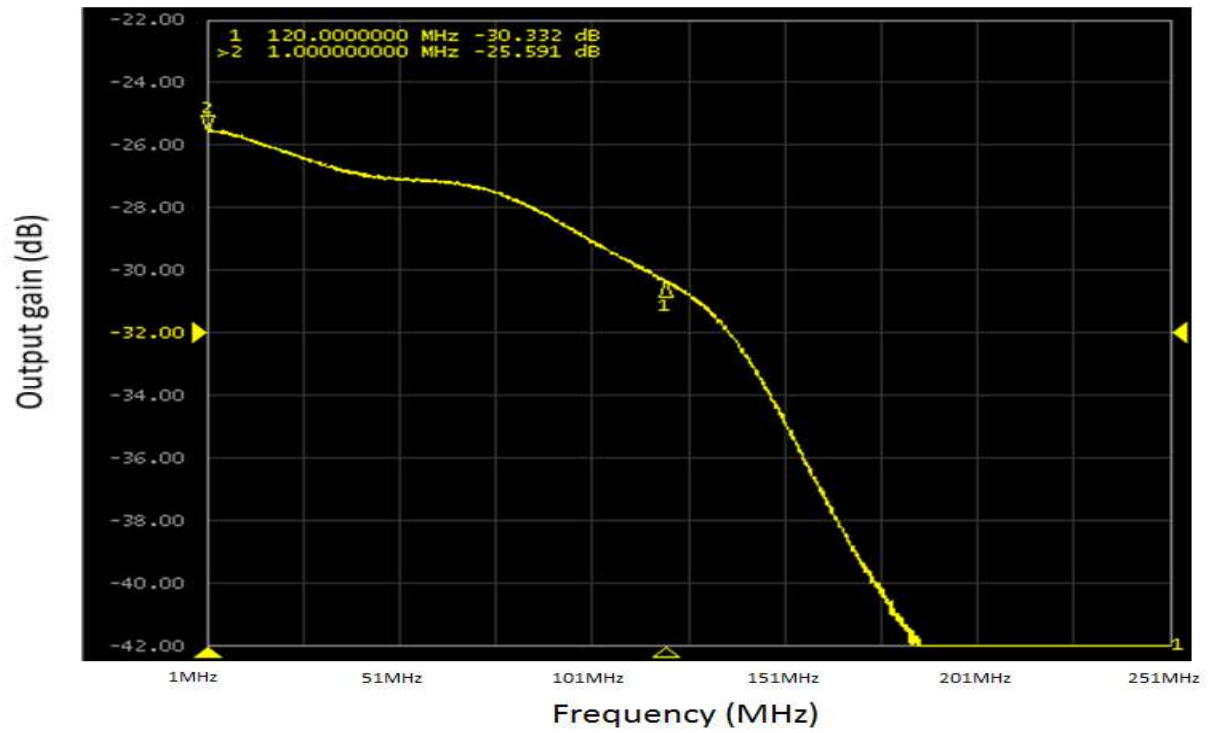


Figure 3.13: Measured frequency response (dB) of a LPF vs. frequency. The measurement was taken using a network analyser on the Δ channel of processor 8.

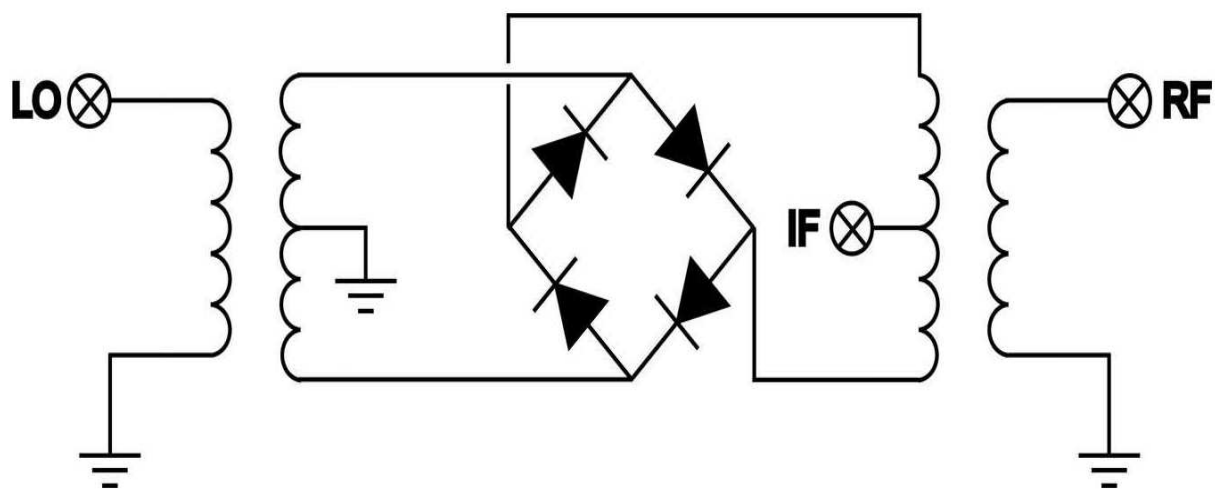


Figure 3.14: Schematic diagram of an RF mixer [39]

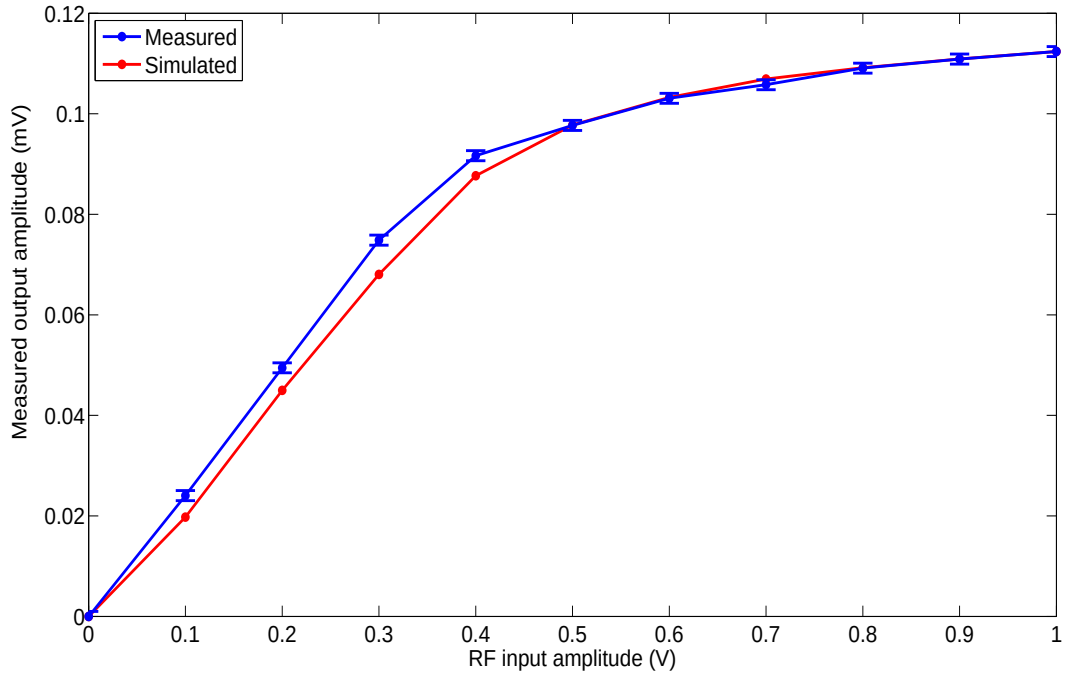


Figure 3.15: Measured (blue) and simulated (red) mixer output vs. RF input amplitude.

3.3 Complete Processor Simulation

Having simulated each stage of the processor, the entire processor was then simulated to observe the collective effects. Having adequately simulated the stray impedances of the system, the LO and raw stripline signals are the two amplitudes that were varied for this simulation.

3.3.1 Recording of Real Data

The raw stripline signals were split, so that one half could be used for the model inputs and the other half for the BPM processors; this gives a direct comparison of a processor output using the same input signals. This data was taken using a customised DAQ system in Matlab [40]. The raw stripline signals and processor output signals were connected to the four inputs of a Tektronix TDS7154B digital oscilloscope [41]. When monitoring the raw stripline signals and processor outputs, the scope records 500 samples on each channel in steps of 200ps for every acquisition. The DAQ saves this data in a structure, which is later analysed. Figure 3.16 shows a photograph of one of the digital oscilloscopes.



Figure 3.16: A photograph of one of the digital oscilloscopes used by FONT at ATF2.

3.3.2 Comparison of Data and Simulation

Figures 3.17 and 3.18 show comparisons of the simulated and real Δ and Σ channels respectively. The simulation closely matches the amplitude and frequency of the real data for both the Δ and Σ channels. The ringing tails observed after the main peaks of Δ and Σ are not as closely recreated by the simulation; this is due to the stray impedances in the BPFs. On the real BPFs, the stray impedances and values of the passive components vary with a tolerance of 5%; the shape of the ringing differs for each processor. The tails of the processor signals are not analysed by FONT; therefore it is not necessary to accurately reproduce this ringing.

Having made the comparison with real data, it was decided to use the simulation to investigate the effects of the LO further; this is described in the next section.

3.3.3 Simulation of LO-based Effects

First the saturation effects of the mixer were investigated. Figure 3.19 shows how Δ' varies with the amplitude of the input pulses for different LO amplitudes. For low LO amplitudes the LO cannot drive the mixer diodes until the RF is large enough to start driving them instead. Once the LO is large enough to start driving the mixer, the linearity of the processor is strongly dependent on the LO amplitude. These results show the importance of ensuring the LO amplitude is constantly kept at +14dBm (3.2V through a 50 Ω load) when taking measurements with the real processors. When the LO is connected to the processor, it is then split 4 ways (Figure 3.1) using a splitter. This is done to supply LO to the mixers on

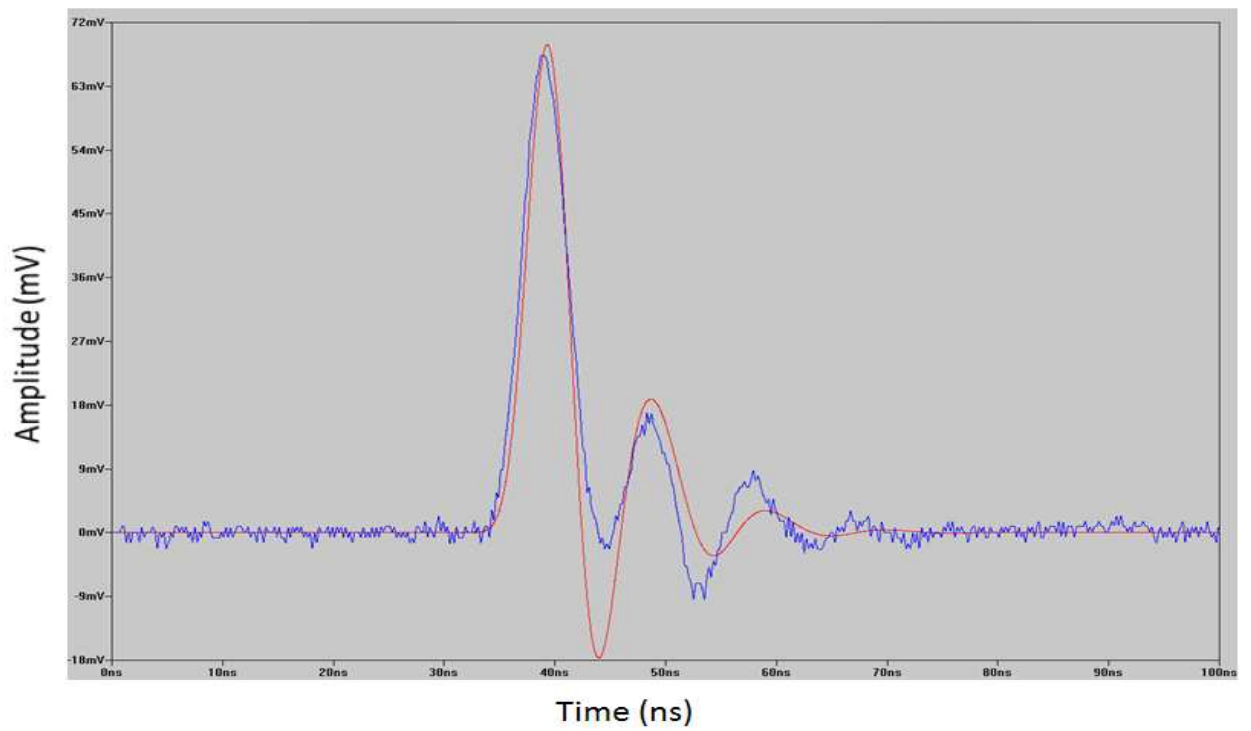


Figure 3.17: Amplitude of the Δ signal (mV) vs. time (ns). The blue line is a real Δ signal monitored at ATF; the red line is a simulated Δ signal using the same input stripline signals.

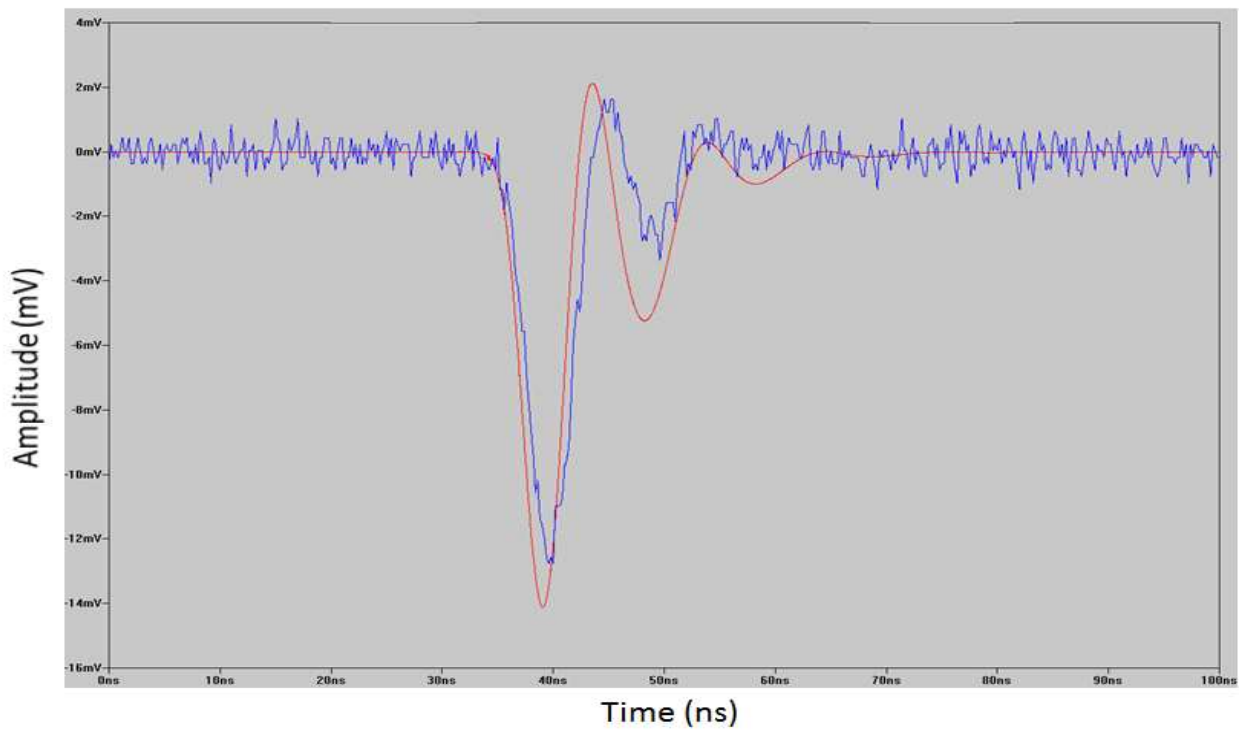


Figure 3.18: Amplitude of the Σ signal (mV) vs. time (ns). The blue line is a real Σ signal monitored at ATF; the red line is a simulated Σ signal using the same input stripline signals.

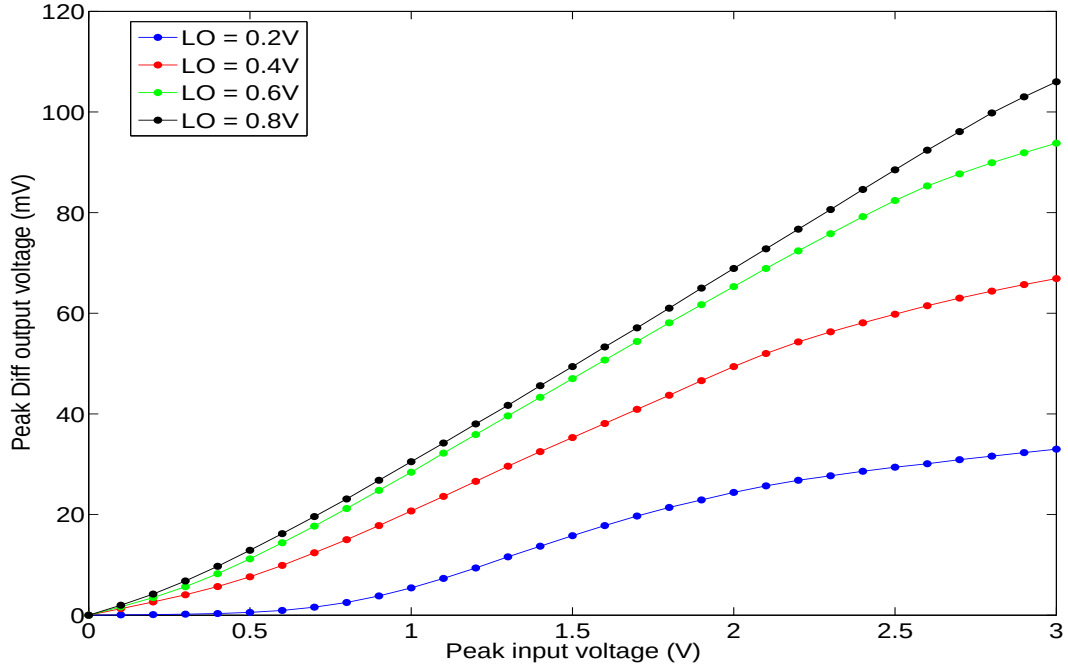


Figure 3.19: Simulated Δ (mV) vs. input amplitude (V) for different LO amplitudes (V).

the Δ , Σ and Σ_Q channels; the remaining output from the splitter is terminated. The LO amplitude at the mixer is +2dBm (0.7V) assuming no losses from the splitter.

The next phenomenon to investigate is the effect of the LO phase on the processor. Figure 3.20 shows the simulated Σ and Δ amplitudes integrated over time as a function of LO phase. The simulated results were fitted to a sinusoidal pattern to determine the optimal phase. The optimal phase is defined as the phase of the LO that maximises the amplitude of the Σ and Δ signals. The fit residuals are within 0.5%, which is a small enough effect to neglect. The simulation suggests that the Σ and Δ channels will have different optimal LO phases; they will differ by approximately 7° . The optimum phase of the Δ channel is 127.4° ; the optimum phase of the Σ channel is 120.4° . The reason for this is that the 180° hybrid contributes a quadrature component to the Δ' signal, which will change the phase of Δ' with respect to Σ' . On the real processor, a loopback cable is connected onto the Σ' channel in order to compensate for the phase difference on the two channels (Figure 3.3); these loopback cables have been fabricated such that the Σ and Δ channels are phased within a few degrees. This phase difference between Δ and Σ is accounted for by applying a phase shift on the LO for the Δ channel in the simulation.

The implications of the phase offset between the Σ and Δ channels is that the measured beam position will depend on the LO phase. The beam position can be calculated as a function of the LO phase and the phase offset between the Σ and Δ channels.

On the real processors, the LO is phased to optimise the Σ channel, to keep to this convention, the Δ channel was defined as having a phase ψ_Δ with respect to the Σ channel. In order to

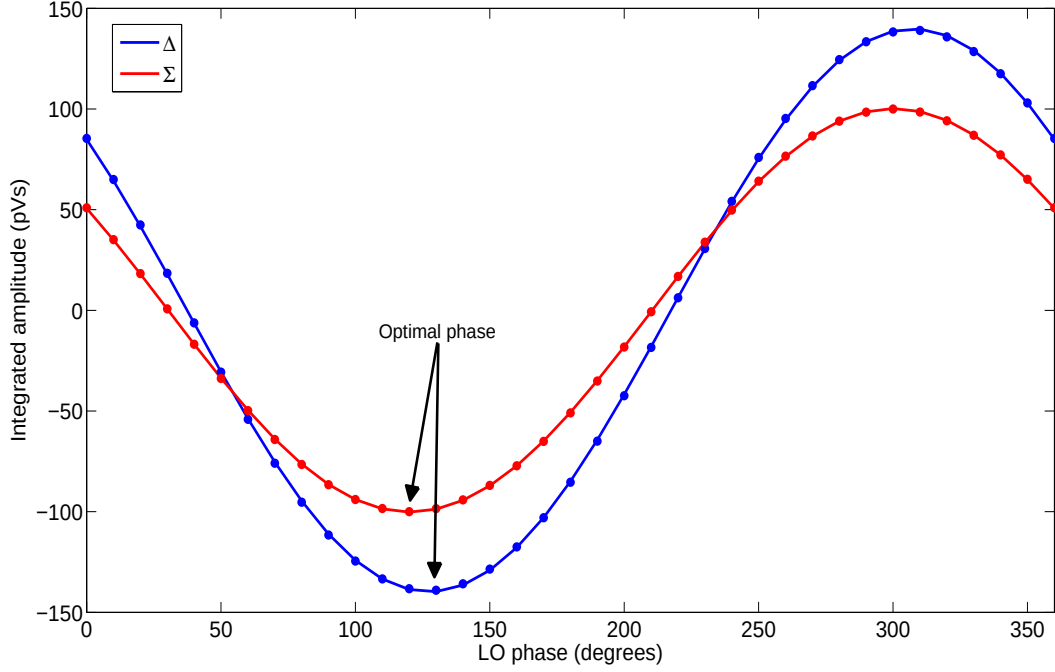


Figure 3.20: Simulated Σ (red) and Δ (blue) (pVs) vs. LO phase ($^\circ$).

account for the LO not being optimally phased, a phase θ_{LO} is defined as the phase the LO is from optimal.

$$y \propto \frac{\Delta}{\Sigma} \propto \frac{\cos(\theta_{LO} + \psi_{\Delta})}{\cos(\theta_{LO})} = \frac{\cos(\theta_{LO}) \cos(\psi_{\Delta}) - \sin(\theta_{LO}) \sin(\psi_{\Delta})}{\cos(\theta_{LO})} \quad (3.8)$$

If the assumption that the phase offset between the Σ and Δ channels is $< 10^\circ$, and that the LO is phased within the same limit, then Eq. 3.8 can be approximated as shown in Eq. 3.9, which shows that the measured position is linearly dependent on LO phase for small phase offsets.

$$y \propto [\cos(\psi_{\Delta}) - \sin(\psi_{\Delta}) \tan(\theta_{LO})] \approx [1 - \psi_{\Delta} \theta_{LO}] \quad (3.9)$$

Phasing the LO

It has just been shown that it is important to phase the LO signal for each processor in order to maximise the amplitude of the output signals. At ATF2, the LO signal is phased using a separate phase shifter for each processor. As previously explained, the Σ channel has a negligible quadrature component. The Σ' signal is passed through a 90° hybrid; half of the signal is subjected to a 90° phase shift, while the other half is not shifted at all. The unshifted part becomes the Σ signal while the other part becomes the Σ_Q signal. As the LO phase is altered in order to maximise the Σ amplitude, this will minimise the Σ_Q amplitude.

Near the optimum phase, the Σ amplitude is approximately constant, but the Σ_Q amplitude varies linearly with LO phase. The Σ_Q allows the maximum sensitivity to LO phase and is therefore a good method for phasing the LO.

Since the Σ channel uses a hybrid to obtain the quadrature signal, imperfections in the hybrid will mean that the Σ_Q is not exactly 90° out of phase relative to the Σ ; an improved method is now employed at ATF2. A two-way switch is now connected to the LO before it is split for all the processors. The switch has been calibrated so that the path length of one side of the switch is exactly 90° out of phase with the other side. In order to phase the LO for the processors, the switch is set one way and the LO is phased for each processor to minimise the Σ signal. Once this has been achieved, the switch is set to the other side so that the LO is exactly 90° out of phase; this is now the optimal phase. In simulation, the LO phase is determined in the same manner as the LO switch method used at ATF2.

3.3.4 Path Length Difference at Processor Inputs

An effect on the processor became apparent while attempting to optimise the resolution of the system; this will be described in Chapter 5. Figure 3.21 shows the effect on the measured position when the LO phase is changed by $\pm 9^\circ$. Since the beam position is set so that $\frac{\Delta}{\Sigma}$ is zero, the measured beam position should not change with LO phase in the absence of a path length difference. A small path length difference between the two inputs to each processor causes the measured position to be sensitive to the phase of the LO. A simulation of this effect was required so that the processors could be modified in order to eliminate the phase sensitivity of the measured position.

Assume that the two processor inputs are sinusoidal, with an angular frequency ω , Eqs. 3.10 and 3.11. ν is half the path length difference, $\frac{L}{2}$, represented as a phase of the LO frequency, Eq. 3.12; v_c is the signal speed in the cables.

$$V_{top} = A \cdot \sin(\omega t - \nu) \quad (3.10)$$

$$V_{bottom} = B \cdot \sin(\omega t + \nu) \quad (3.11)$$

$$\nu = \frac{2\pi\omega_{LO}L}{2v_c} \quad (3.12)$$

From these equations, the Σ' and Δ' signals can be calculated, as shown in Eqs. 3.13 and 3.14.

$$\Sigma' = (A + B) \sin(\omega t) \cos(\nu) - (A - B) \cos(\omega t) \sin(\nu) \quad (3.13)$$

$$\Delta' = (A - B) \sin(\omega t) \cos(\nu) - (A + B) \cos(\omega t) \sin(\nu) \quad (3.14)$$

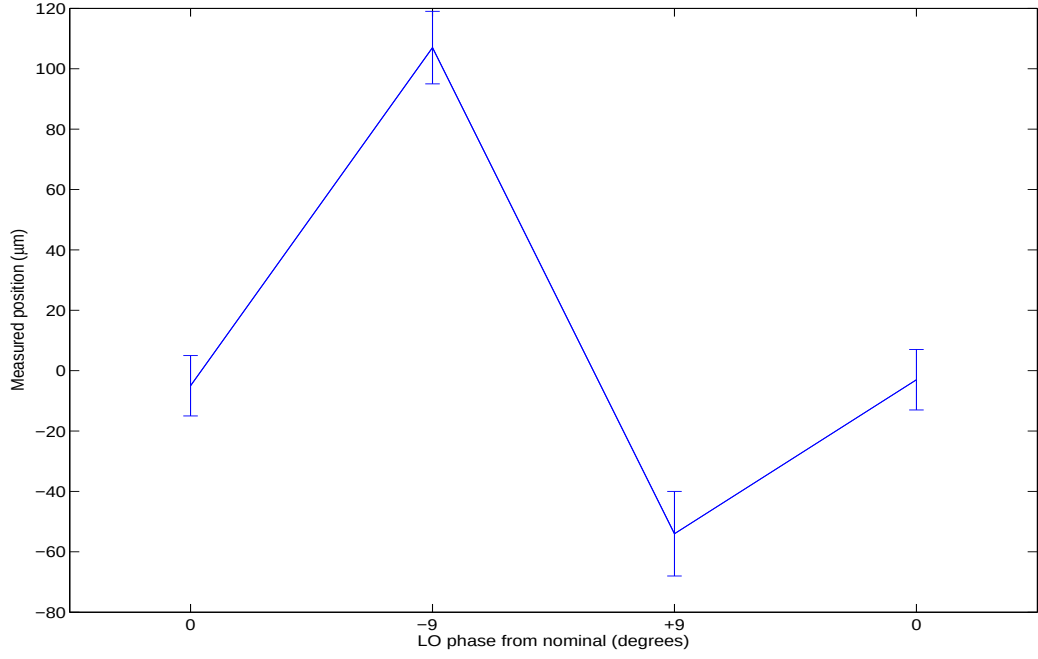


Figure 3.21: Measured position vs. LO phase from nominal.

These equations can be expressed as sinusoidal functions with an amplitude and a phase as shown in Eqs. 3.15 and 3.16.

$$\Sigma' = \sqrt{A^2 + B^2 + 2AB \cos(2\nu)} \sin(\omega t - \arctan \left[\frac{A - B}{A + B} \tan \nu \right]) \quad (3.15)$$

$$\Delta' = \sqrt{A^2 + B^2 - 2AB \cos(2\nu)} \sin(\omega t - \arctan \left[\frac{A + B}{A - B} \tan \nu \right]) \quad (3.16)$$

Now these signals are passed through the RF mixers; it is assumed that the Σ channel is optimally phased while the Δ channel has a phase setting difference ψ_Δ . Eqs. 3.17 and 3.18 show the Σ and Δ channels after the mixers. Only the DC components are shown because the high frequency terms are removed by the low-pass filter. $\delta\theta_{LO}$ is the phase jitter on the LO.

$$\Sigma = \frac{\sqrt{A^2 + B^2 + 2AB \cos(2\nu)}}{2} \cos(\delta\theta_{LO} - \arctan \left[\frac{A - B}{A + B} \tan \nu \right]) \quad (3.17)$$

$$\Delta = \frac{\sqrt{A^2 + B^2 - 2AB \cos(2\nu)}}{2} \cos(\psi_\Delta + \delta\theta_{LO} - \arctan \left[\frac{A + B}{A - B} \tan \nu \right]) \quad (3.18)$$

Parameter	Value
L	1-5mm
v_c	0.67c
ν	0.65 °-3.2 °
$\delta\theta_{LO}$	0.6 °
ψ_Δ	$\sim 3^\circ$
C_{cal}	~ 0.00225

Table 3.4: Parameters affecting the phase dependence of the measured position.

Simplifications

If the beam is centred in the BPM at ATF2, or the processors are being monitored on a test bench, the amplitudes of the two inputs are approximately equal. This approximation greatly simplifies the equations and allows the dependence of various parameters to be factorised. If $A \approx B$, Σ and Δ simplify:

$$\Sigma = A \cos(\nu) \cos(\delta\theta_{LO}) \quad (3.19)$$

$$\Delta = A \sin(\nu) \sin(\psi_\Delta + \delta\theta_{LO}) \quad (3.20)$$

The measured position, y' , can be calculated as shown in Eq. 3.21. To calculate the measured position jitter due to the LO phase jitter, the difference in position is calculated for an LO phase of one standard deviation either side of the optimal phase. This is shown in Eq. 3.22; C_{cal} is the calibration constant of the BPM system.

$$y' = \frac{\Delta}{C_{cal}\Sigma} = \frac{1}{C_{cal}} (\tan(\nu) \sin(\psi_\Delta) + \tan(\nu) \tan(\delta\theta_{LO}) \cos(\psi_\Delta)) \quad (3.21)$$

$$\Delta y = \frac{1}{C_{cal}} \tan(\nu) \tan(\delta\theta_{LO}) \cos(\psi_\Delta) \quad (3.22)$$

Table 3.4 shows the measured values of the parameters affecting Eq. 3.22. L is the measured path length difference for the processors before any correction is made; the path length difference varied between processors in the range of 1-5mm. v_c is the propagation speed of an electrical signal along the signal cables. ν is the path length difference, L , expressed as a phase of the 714MHz LO frequency. $\delta\theta_{LO}$ is the measured phase jitter on the LO signal. ψ_Δ is the phase setting difference between the Σ and Δ channels. C is a nominal calibration constant for the BPM system (Chapter 4). From these parameters, the position jitter contribution is calculated to be between 100 and 530nm. The path length differences have now all been corrected using matched cables so that they are within 0.5mm; ψ_Δ has been improved so that it is within 0.5°. With these improvements, the estimated position jitter contribution from the LO phase jitter is reduced to 50nm.

3.4 Summary

The analogue BPM processors have been extensively modelled using Matlab and Spice. Each of the main stages of the processor was tested and modelled individually, before modeling the whole processor as one unit. The model has been compared to the real processors to verify its validity. The model gives a good representation of the known features of the processors, and has been used to predict other features of the processors, such as the dependence of the output of the processors on the LO phase and amplitude. This has allowed the FONT group to take precautions to ensure that the LO amplitude and phase remain constant. The processor simulation has become an important tool in attempting to optimise the resolution of the BPM system.

Chapter 4

Calibration and Resolution Studies of the Analogue BPM Processors

The analogue processors convert the raw BPM stripline signals into Σ and Δ signals. Dividing the difference channel by the sum channel gives a quantity that is proportional to the position of the beam in the BPM. In order to convert this quantity into a position, the constant of proportionality needs to be measured, this is called the calibration constant. Once the calibration constants are known, the resolution of the processors can be calculated. In the feedback system, achievable correction to the beam jitter is dependent on the resolution of the BPMs and analogue processors.

4.1 BPM and Analogue Processor Calibration

4.1.1 Theoretical calibration constant

The raw stripline signals described by Eqs. 2.14 and 2.15 are converted into Σ and Δ signals by the analogue processors. In Eqs. 4.1 and 4.2 (see also Chapter 2), g_s and g_d represent the linear amplitude gains applied to the Σ and Δ signals respectively from the processors, and it is assumed that $y \ll R$.

$$\Sigma = g_s(V_{top} + V_{bottom}) = g_s \rho \frac{2w^2(R^2 + y^2)}{(R^2 - y^2)^2} \left(\frac{dQ}{dt} \right) \approx g_s \rho \frac{2w^2}{R^2} \left(\frac{dQ}{dt} \right) \quad (4.1)$$

$$\Delta = g_d(V_{top} - V_{bottom}) = g_d \rho \frac{4w^2 R y}{(R^2 - y^2)^2} \left(\frac{dQ}{dt} \right) \approx g_d \rho \frac{4w^2 y}{R^3} \left(\frac{dQ}{dt} \right) \quad (4.2)$$

Note that Σ is only dependent on the charge and the geometry of the BPM, whilst the difference signal is proportional to the charge, beam position and BPM geometry. Dividing the Δ signal by Σ gives a quantity that is proportional to the beam position:

$$\frac{\Delta}{\Sigma} \equiv C_{cal} y = \left(\frac{g_d}{g_s} \right) \frac{2y}{R} \quad (4.3)$$

Where C_{cal} is the calibration constant of the BPM system and is defined as:

$$C_{cal} = \left(\frac{g_d}{g_s} \right) \frac{2}{R} \quad (4.4)$$

The gain ratio is designed to be about 20 for the analogue processors and the internal radius of the stripline BPM is 14mm. From this, $C_{cal} \approx 2.9 \times 10^{-3} \mu\text{m}^{-1}$. In order to verify this theoretical value, the gain ratio for the processor was measured. The measurement of the gain ratio was achieved by connecting a 700MHz sinusoidal signal to one of the processor inputs and terminating the other input. Therefore the amplitude of the Δ and Σ outputs will depend directly on g_d and g_s respectively. Hence the gain ratio can be measured directly; this was measured to be 16.4 ± 1.1 . This result gives a calibration constant of $(2.7 \pm 0.2) \times 10^{-3} \mu\text{m}^{-1}$.

In Chapter 3, the effects of the phase offset between the Σ and Δ channels, and the imperfect LO phasing were considered. This can be taken into account for the calibration constant, from Eq. 3.9. The phase dependence of the calibration constant is shown in Eq. 4.5 where $\left(\frac{g_d}{g_s} \right)_0$ is the processor gain ratio if the phase offset between the Σ and Δ channels is zero and the LO is optimally phased. ψ_Δ is the phase offset between the Σ and Δ channels and θ_{LO} is the deviation of the LO phase from optimal.

$$C_{cal} = \left(\frac{g_d}{g_s} \right)_0 \frac{2 [\cos(\theta_{LO}) \cos(\psi_\Delta) - \sin(\theta_{LO}) \sin(\psi_\Delta)]}{R \cos(\theta_{LO})} \approx \left(\frac{g_d}{g_s} \right)_0 \frac{2 [1 - \psi_\Delta \theta_{LO}]}{R} \quad (4.5)$$

ψ_Δ is measured to be approximately 3° (0.05 rad); therefore C_{cal} varies by approximately 0.1% for every degree that θ_{LO} varies.

4.1.2 Calibration Measurement Techniques

The FONT BPMs and processors are calibrated by changing the position of the beam with respect to the centre of the BPM, and measuring $\frac{\Delta}{\Sigma}$ at each beam position. $\frac{\Delta}{\Sigma}$ is plotted against the beam position and a linear fit is applied to the results. The slope of the line is the calibration constant. The calibration constants are measured using two methods.

Magnet Based Calibration

The first calibration method makes use of a corrector magnet upstream of the FONT BPMs. This changes the downstream positions of the beam. In this method, the corrector magnet current is set, and the beam position in the FONT BPMs is calculated from these current values. Due to the construction of the corrector magnet, the magnetic field lines are perpendicular to the direction of motion of the beam, and are aligned either vertically or horizontally with respect to the beam pipe. From this Ampère's circuital law can be simplified to the form in Eq. 4.6. N is the number of windings of the coil around the iron core, L_{core} is the magnetic path length through the iron core and L_{gap} is the magnetic length through the air gap (Figure 4.1) and I is the current in the wire coil. The magnetic rigidity

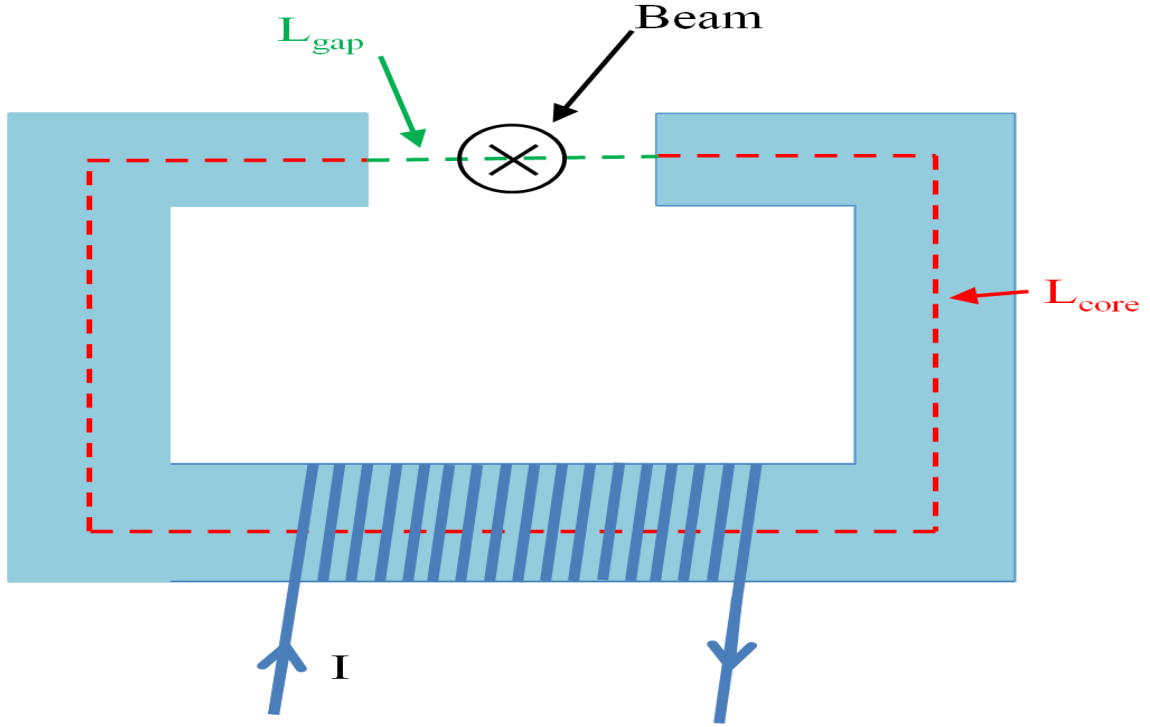


Figure 4.1: Diagram of a corrector used at ATF2.

(Eq. 4.7) can be used to calculate the angular deflection of the beam. This deflection is due to the magnetic field strength of the corrector, where ρ is the radius of curvature of the beam, p is the momentum of the beam and e is the charge of an electron.

$$\mathbf{B} = \int \frac{\mu_0 \mu}{4\pi} \frac{I d\mathbf{l} \times \hat{\mathbf{r}}}{|r|^2} = \frac{N}{4\pi \left(\frac{L_{core}}{\mu_0 \mu_r} + \frac{L_{gap}}{\mu_0} \right)} I \quad (4.6)$$

$$B\rho = \frac{p}{e} \quad (4.7)$$

Eq. 4.8 shows the angular deflection of the beam, θ_{corr} , as a function of the corrector magnet current, where W is the length of the corrector magnet along the beam line. Having calculated the angular deflection of the beam due to the corrector magnet the change in position in the downstream BPMs can be calculated, as shown in Eq. 4.9, where $\widehat{\mathbf{M}}$ is the transfer matrix between the corrector magnet and the BPM.

$$\theta_{corr} = \frac{W}{\rho} = \frac{eWB}{p} = \left(\frac{eW}{p} \right) \left(\frac{N}{4\pi \left(\frac{L_{core}}{\mu_0 \mu_r} + \frac{L_{gap}}{\mu_0} \right)} \right) I \quad (4.8)$$

$$\begin{pmatrix} y \\ y' \end{pmatrix}_{BPM} = \widehat{\mathbf{M}} \begin{pmatrix} 0 \\ \theta_{corr} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} 0 \\ \theta_{corr} \end{pmatrix} = \begin{pmatrix} M_{12}\theta_{corr} \\ M_{22}\theta_{corr} \end{pmatrix} \quad (4.9)$$

In Eq. 4.8, the first term in brackets is a property of the beam and the length of the corrector

Corrector Magnet	Gradient (mTA^{-1})	Offset (μT)
ZH4X	9.8 ± 0.1	16 ± 0.2
ZH5X	10.1 ± 0.1	4 ± 0.1
ZH6X	9.6 ± 0.1	22 ± 0.1
ZV6X	10.5 ± 0.1	11 ± 0.2
ZV7X	10.7 ± 0.1	-4 ± 0.1
ZV8X	10.4 ± 0.1	-8 ± 0.1

Table 4.1: Measured fit coefficients for the corrector magnets upstream of the FONT BPMs in the ATF2 extraction line [42].

and can be calculated, and the second term is a property of the corrector magnet and is measured. The momentum of the beam is known to be $1.3\text{GeV}/c$, and the length of the corrector magnet is 128mm , so the first term has a value of $2.95 \times 10^{-2} \text{T}^{-1} \text{rad}$. The second term was determined by measurement. Table 4.1 shows the list of measured fit coefficients for the correctors upstream of the FONT BPMs, the magnet currents were varied and the magnetic fields measured with a Hall probe; a linear fit was applied to the data. The offset column shows the measured residual magnetic field when the corrector currents are set to zero. These residual magnetic fields are likely to be due to magnetic hysteresis. An alternative possibility is that electrical offsets in the analogue to digital converters (ADCs) used to monitor the magnet currents would result in the readback monitors reporting a different magnet current to the actual value; thus a magnetic field would be measured when the magnet current is set to 0A . Typically ZH4X and ZV6X are used for the horizontal and vertical BPM calibrations respectively. Figure 4.2 shows one of the magnet calibrations taken on ZV6X.

The transfer matrices are calculated using MADX [43] and Flight Simulator [44]; this is done to ensure that both models give consistent transfer matrices. If this is not the case then a mover calibration is performed and used to compare the two sets of transfer matrices. The disadvantage with this method is that the beam position is determined indirectly, and requires a new set of transfer matrices to be calculated when the magnetic lattice is changed.

Mover Based Calibration

The second method is to calibrate the BPM system using horizontal and vertical movers. All three of the FONT BPMs have had mover systems designed, built and installed, and appropriate control software developed [45].

The BPM movers consist of two linear actuators which have a minimum step size of $1 \mu\text{m}$. One actuator changes the BPM position in the horizontal plane, while the other is connected to a metal wedge, which pushes another similar wedge, causing the vertical position of the BPM to change (Figure 4.3). The control software was developed in Labview, which has the advantage that the mover controls can be integrated into the data acquisition system, which has also been developed in Labview. Figure 4.4 shows a photograph of the mover attached to the P3 BPM at ATF2.

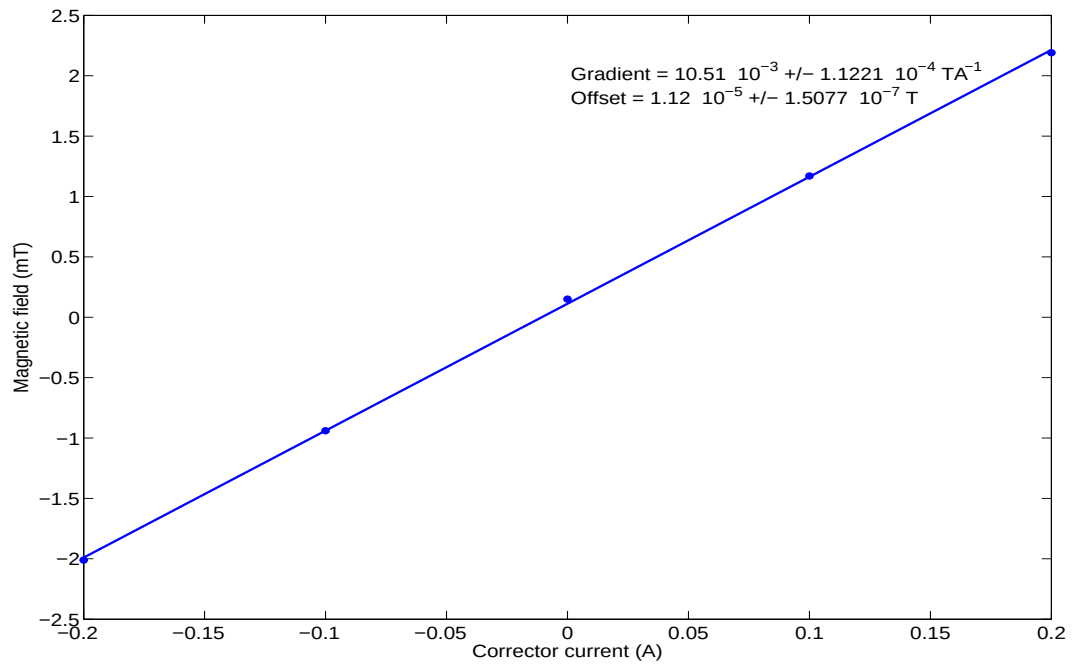


Figure 4.2: Magnetic field vs. current for ZV6X. The points represent the measurements and the line is a linear fit to the data.

Horizontal mover



Vertical mover



Figure 4.3: Diagram of the horizontal and vertical movers.

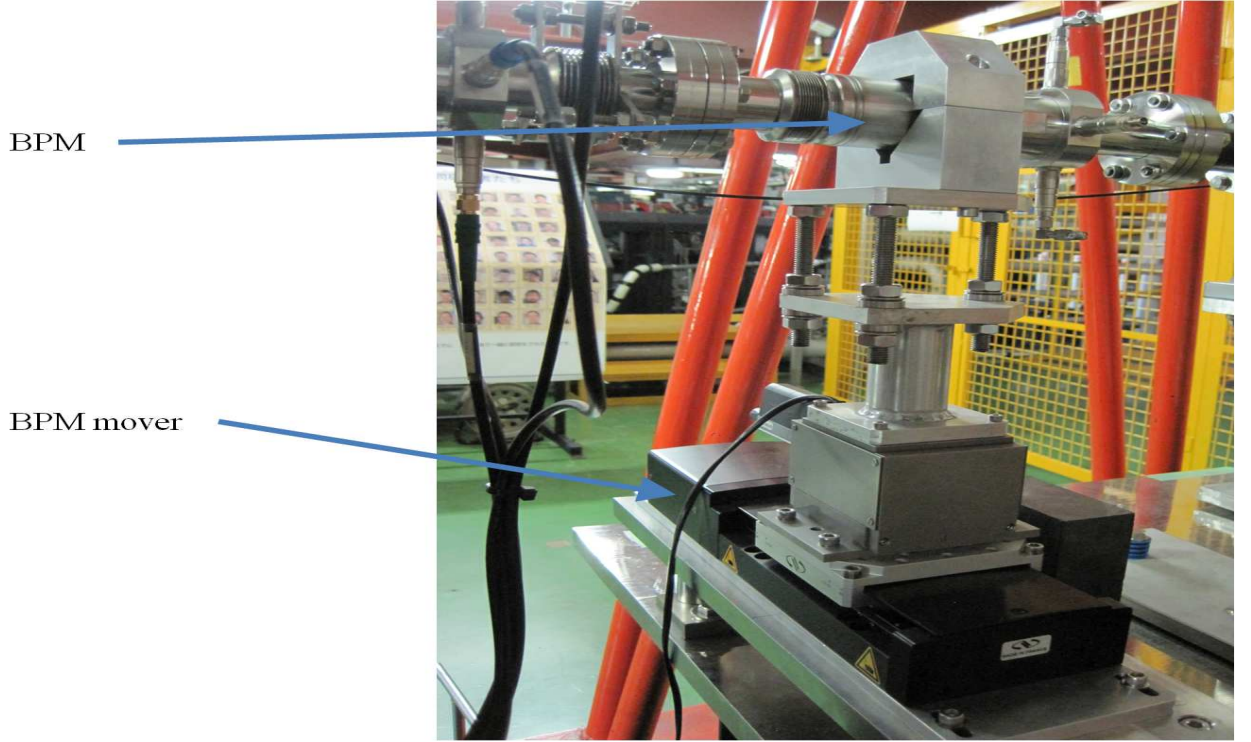


Figure 4.4: Photograph of the P3 mover system.

The mover calibration method works by changing the position of the BPMs by a known amount, and measuring the Σ and Δ signals from the analogue processors. The advantage this method has over the magnet calibration method is that the change in position is set directly, rather than calculated indirectly.

The theoretical value of the calibration constant calculated earlier does not consider the saturation effects of the analogue processors. Due to the non-linearity of the RF mixer, the amplitude of the peak of the Σ and Δ outputs will be suppressed slightly near their peaks. Due to the fact that the saturation is a continuous effect rather than a discrete one, even within the acceptable linear region of the processor there is a small saturation effect. For a signal amplitude of 10mV, the measured saturation effect is approximately 3%, while at a signal amplitude of 50mV, the saturation effect is measured to be approximately 20%.

In order to keep the BPM calibrations as consistent as possible, the beam is always centred in the BPM before a calibration scan is performed. This is either done by moving the beam or the BPM. The range of positions scanned is also kept constant. For the movers it is $\pm 100\mu\text{m}$ in $20\mu\text{m}$ increments; an equivalent is calculated for the corrector calibration. By keeping the measurement as consistent as possible, the suppression of the peak amplitude will be almost constant.

The analysis software minimises χ^2 to calculate the gradient and offset of a linear fit to the data (Eq. 4.10); the gradient is known as the calibration constant. μ_i is the mean measured beam position for the i^{th} position setting, σ_i is the standard deviation of the measurement, y_{set} is the set beam position and P_0 and P_1 are the offset and gradient of the fit respectively.

χ^2 is used to calculate the errors on the fit coefficients (Eq. 4.11); the derived errors on P_0 and P_1 are shown in Eqs. 4.12 and 4.13 respectively. The χ^2 per degree of freedom is calculated because this gives a quantitative measurement of the quality of the linear fit. The number of degrees of freedom is the number of data points minus the number of fit parameters; for a linear fit the number of fit parameters is two. The residuals are monitored because they are sensitive to any errors in the system. For example, if the residuals followed a cubic trend, this might suggest non-linearities in the system.

$$\chi^2 = \sum_i \frac{(\mu_i - (P_1 y_{set} + P_0))^2}{\sigma_i^2} \quad (4.10)$$

$$\sigma_{P_i} = \sqrt{\left(\frac{1}{2} \frac{\partial^2 (\chi^2)}{\partial P_i^2} \right)^{-1}} \quad (4.11)$$

$$\sigma_{P_0} = \sqrt{\left(\sum_i \frac{1}{\sigma_i^2} \right)^{-1}} \quad (4.12)$$

$$\sigma_{P_1} = \sqrt{\left(\sum_i \frac{y_{set}^2}{\sigma_i^2} \right)^{-1}} \quad (4.13)$$

1) Integration method

Before FONT4 was fully implemented, the Σ and Δ signals were monitored using oscilloscopes and a data acquisition system would convert the waveforms into data files for offline analysis. The amplitude of the Σ and Δ signals were measured in 500 samples at 200ps intervals. The Σ and Δ signals were integrated over time before $\frac{\int \Delta dt}{\int \Sigma dt}$ was calculated. This was done to minimise the effects of noise on the signals, and hence the measured position. The signals traveled through ~ 30 m of cable before they were connected to the oscilloscopes. As a result, considerable noise is picked up as well [46].

The Δ and Σ signals are each integrated (Eqs. 4.14 and 4.15) over a region around the peak signal (Figure 4.5). δt is the time interval between samples (200ps) and t_0 and t_1 are start and end of the integration window respectively. The measured calibration constant is dependent on the width of the integration window. This dependence is due to the fact that the Σ channel converges more rapidly than the Δ channel because of the absence of a quadrature residual. Figure 4.6 shows the Σ and Δ channels integrated about their peaks versus integration window and the apparent position this produces; the start of the integration window, t_0 is set to be the start of the signal pulse. As an example Figure 4.7 shows the corresponding calibration constant versus integration window. This data was taken in May 2008 using processor 1 on ‘BPM 11’ of the old ATF extraction line.

$$\int_{t_0}^{t_1} \Delta dt = \sum_{t_0}^{t_1} \Delta \delta t \quad (4.14)$$

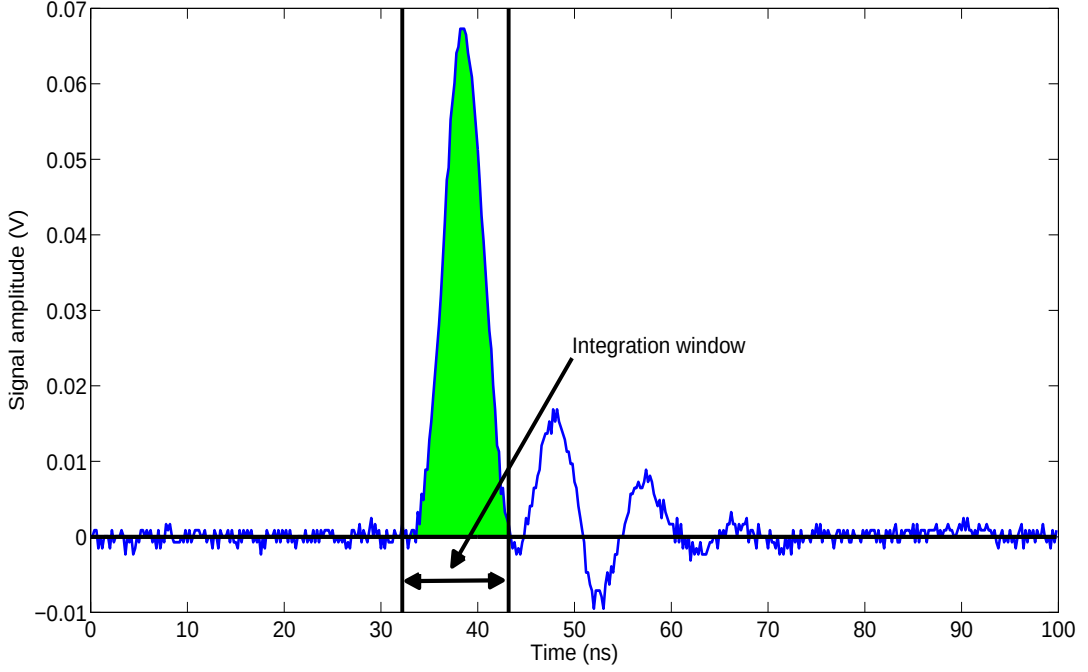


Figure 4.5: Δ signal vs. time. The integration window is shown.

$$\int_{t_0}^{t_1} \Sigma dt = \sum_{t_0}^{t_1} \Sigma \delta t \quad (4.15)$$

The mid-point of the integration window was set on the peaks of the Σ and Δ signals and the integration window varied about the signal peaks; for each integration window, the calibration constant was evaluated. Figure 4.7 shows that the calibration constant is at a minimum value of $\sim 2.3 \times 10^{-3} \mu\text{m}^{-1}$ when $\frac{\int \Delta dt}{\int \Sigma dt}$ is calculated with an integration window of one sample; as the integration window increases, the calibration constant reaches a steady value of $\sim 2.6 \times 10^{-3} \mu\text{m}^{-1}$. The sinusoidal pattern on the curve is due to the ringing in the tail of the Δ signal. The maximum value of the calibration constant at $\sim 2.7 \times 10^{-3} \mu\text{m}^{-1}$ is when the integration window is set to integrate the entire first peak of the Σ and Δ signals. The minimum value of the calibration constant is due to the non-linearity of the BPM processors, which suppress the amplitude of the peaks of the output signals (Figure 3.15).

In the peak sampling method (below), the electronics sample on the peaks of the signals, rather than over the entire signal in fine steps; thus the calibration can be expected to be $\sim 15\%$ lower than the analogue calibration constant. Figure 4.6 also shows that $\frac{\int \Delta dt}{\int \Sigma dt}$ varies by approximately 15% over the range of the integration window. The theoretical value of the calibration constant (Eq. 4.4) is $\sim 2.7 \times 10^{-3} \mu\text{m}^{-1}$ for the integration method and $\sim 2.3 \times 10^{-3} \mu\text{m}^{-1}$ for the peak sampling method.

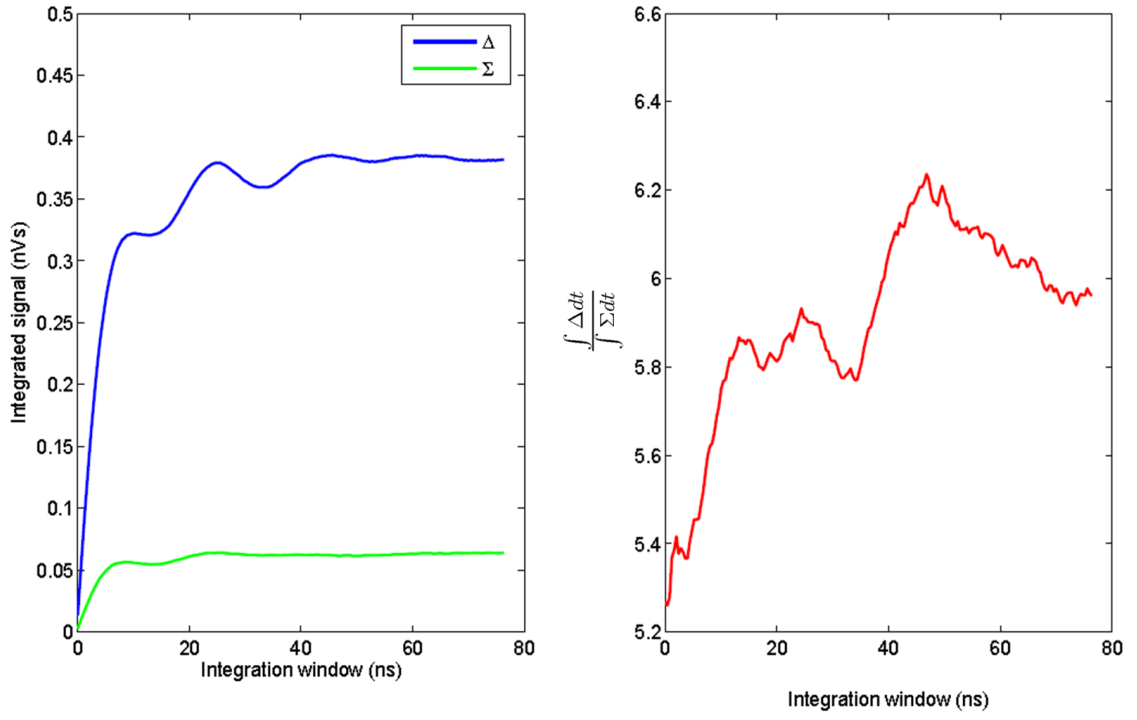


Figure 4.6: $\int \Sigma dt$ (green) and $\int \Delta dt$ (blue) signals integrated about their peak amplitudes (left) and $\frac{\int \Delta dt}{\int \Sigma dt}$ (right) vs. integration window.

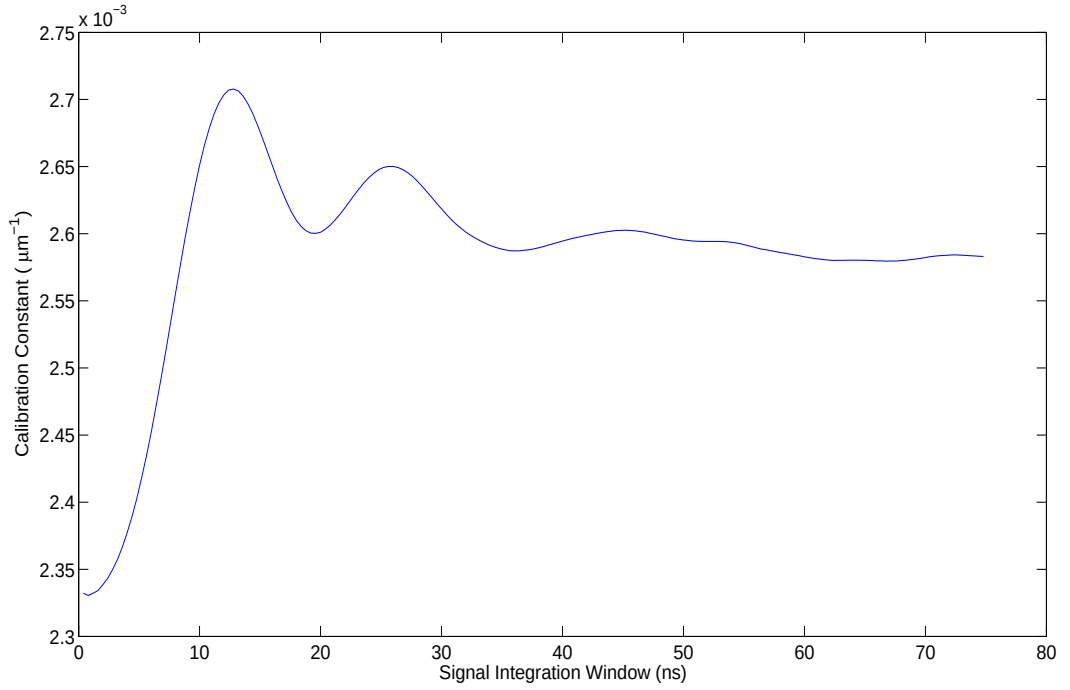


Figure 4.7: Calibration constant vs. integration window.

	Vertical Calibration		Horizontal Calibration	
	Gradient ($\times 10^{-3} \mu\text{m}^{-1}$)	χ_{dof}^2	Gradient ($\times 10^{-3} \mu\text{m}^{-1}$)	χ_{dof}^2
Corrector	2.142 ± 0.002	1.2	2.489 ± 0.004	3.8
Mover	2.144 ± 0.006	3.1	2.454 ± 0.014	10.7

Table 4.2: Measured calibration constants and χ_{dof}^2 for the P1 BPM system.

2) Peak Sampling Method

Once FONT4 and then FONT5 were implemented, the processor outputs were monitored with the digital electronics. In this regime, only the peaks of the Σ and Δ signals were sampled; $\frac{\Delta}{\Sigma}$ is therefore calculated from the signal peak samples. The signal to noise ratio is improved by a factor of approximately 3.5, mostly because the processor outputs are passed through a low-noise amplifier and are connected directly to the digital electronics; the required lengths of signal cables were significantly reduced.

Figures 4.8 and 4.9 show a horizontal and vertical magnet calibration respectively. Figures 4.10 and 4.11 show a horizontal and vertical mover calibration respectively. The calibration constants are consistent between the two calibration methods within the errors on the fits (Table 4.2); χ_{dof}^2 is the χ^2 per degree of freedom. It should be noted that the calibration constants are $\sim 2.4 \times 10^{-3} \mu\text{m}^{-1}$ vertically and $\sim 2.1 \times 10^{-3} \mu\text{m}^{-1}$ horizontally; this is compared to the value of $2.3 \times 10^{-3} \mu\text{m}^{-1}$ predicted from Figure 4.7. The values of χ_{dof}^2 are significantly larger for the mover calibration. This is because these calibration measurements were taken shortly after the movers had been installed and they had not received much use. Due to uneven lubrication, the mover would occasionally ‘stick’ and not stop at the required position. Since then, the movers have been cycled many times through their full range of positions and their motion is smoother and the χ_{dof}^2 are more consistent with the magnet calibrations.

Processor Dependence:

If the LO is properly phased, a BPM processor will give a consistent calibration constant. Care is taken to ensure consistency when setting up the BPM system and hence minimise variations in the calibration constants. Calibration constants may differ between processors due to variations in the measured gain ratios between processors and is due to errors on the values of component used to fabricate the BPM processors, especially the BPFs and RF mixers. The theoretical values of the calibration constants have been calculated from the gain ratios measured on each processor (Table 4.3) and are shown in Figure 4.12.

Figure 4.12 also shows the measured calibration constants for all the BPM processors used at ATF; both corrector and mover calibration constants are shown. There is a systematic difference between the mover and corrector constants; this is believed to be from measurement errors in the model used to calculate the transfer matrices.

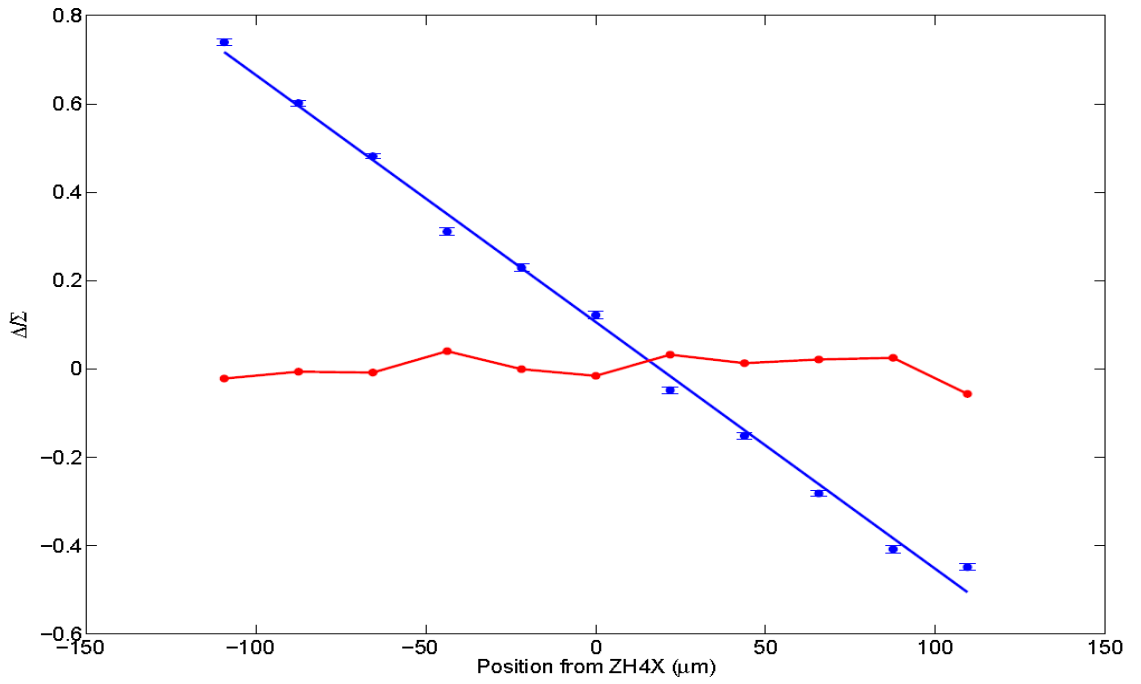


Figure 4.8: $\frac{\Delta}{\Sigma}$ vs. position from horizontal corrector ZH4X for the P1 BPM from December 2010. The blue points are the measurements, the blue line is a fit to the data and the red points show the residual of the fit.

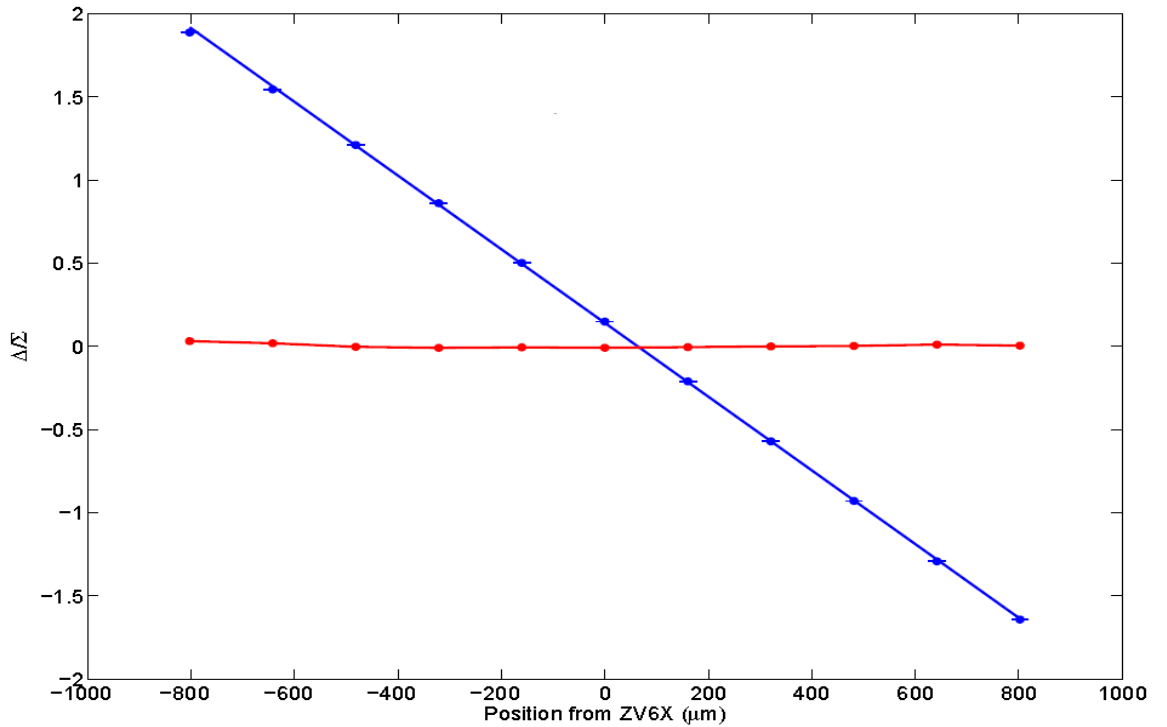


Figure 4.9: $\frac{\Delta}{\Sigma}$ vs. position from vertical corrector ZV6X for the P1 BPM from December 2010. The blue points are the measurements, the blue line is a fit to the data and the red points show the residual of the fit.

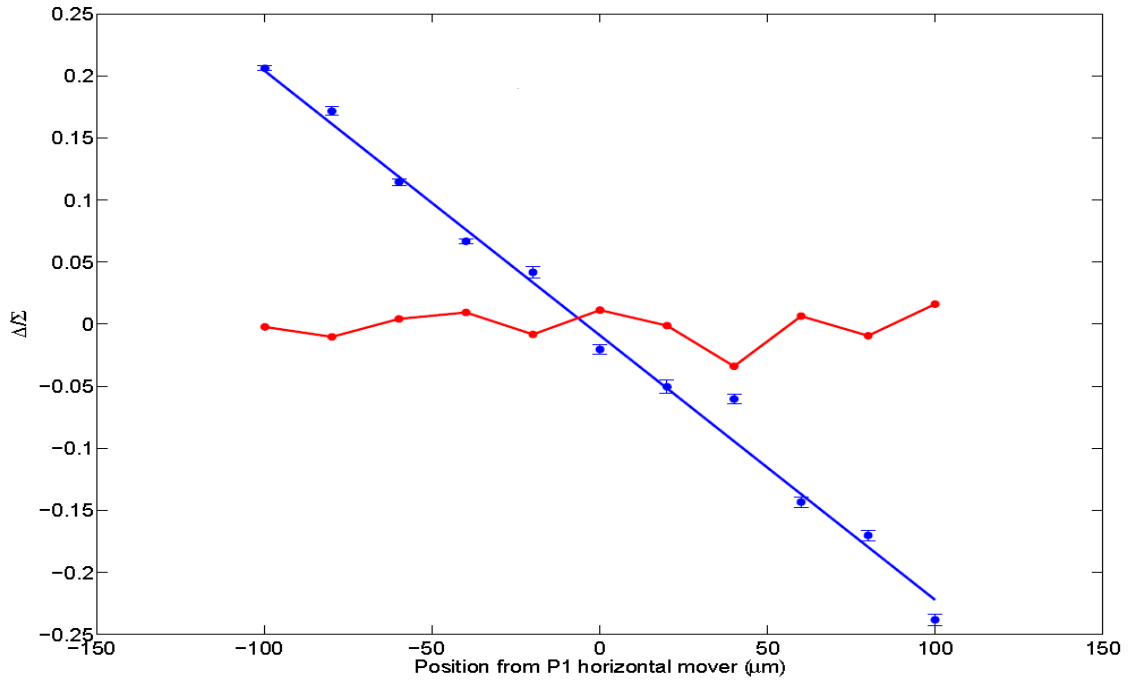


Figure 4.10: $\frac{\Delta}{\Sigma}$ vs. horizontal mover position, this is a calibration plot using the P1 BPM mover from December 2010. The blue points are the measurements, the blue line is a fit to the data and the red points show the residual of the fit.

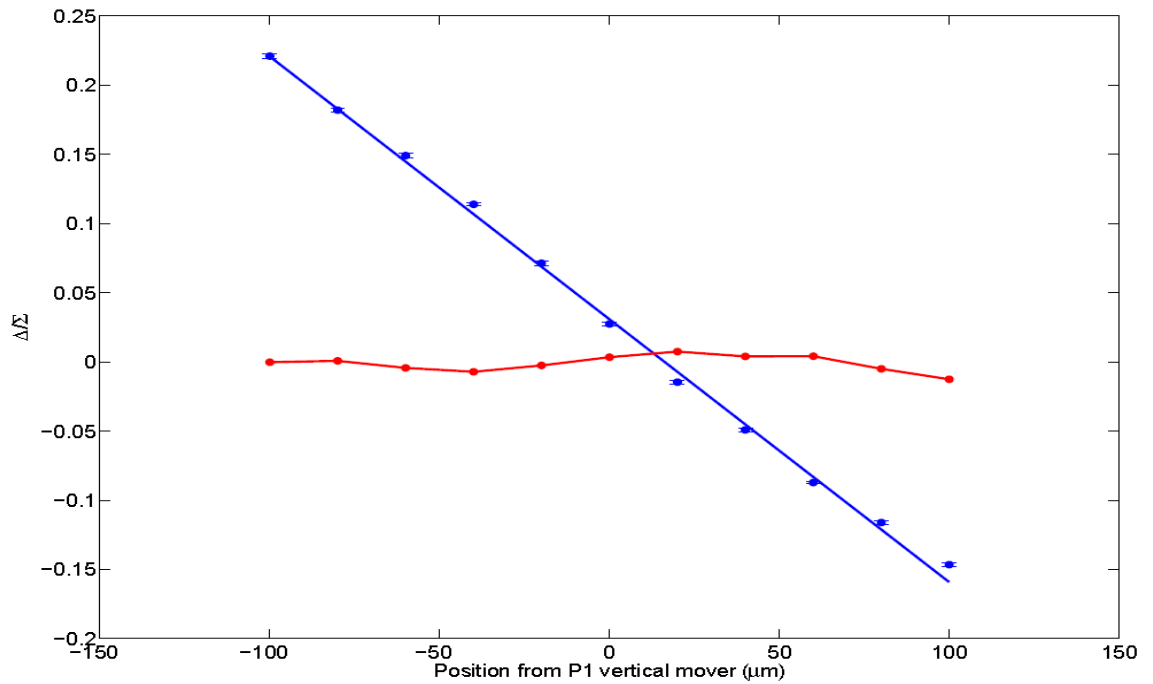


Figure 4.11: $\frac{\Delta}{\Sigma}$ vs. vertical mover position, this is a calibration plot using the P1 BPM mover from December 2010. The blue points are the measurements, the blue line is a fit to the data and the red points show the residual of the fit.

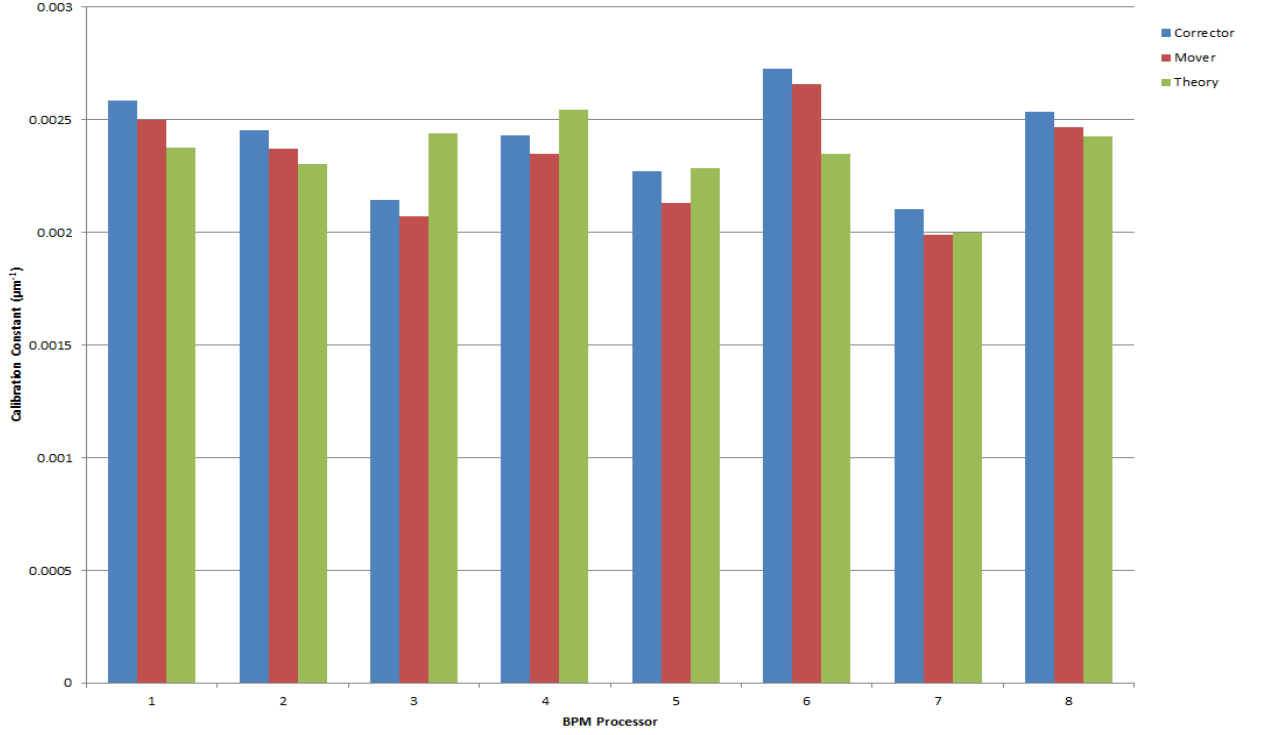


Figure 4.12: Plot showing the mover and corrector calibration constants for all the BPM processors used at ATF2. The theoretical calibration constant for each processor is also shown.

Comparison of Single and 3-Bunch Results

Table 4.4 shows the corrector calibration results for the first 4 processors between February and June 2010; with the exception of the calibration run taken in June, the calibrations were performed on a train of three bunches. Three-bunch mode (Chapter 5) is the mode required to operate the FONT feedback system. From these results, it can be seen that for any given processor, the measured calibration constants vary by up to $\sim 20\%$ over time but the measured values are consistent for all three bunches, as expected. The table only shows the gradients and errors of the linear fits; the offset does not alter the measurements. Table 4.5 show a comparison between the corrector and mover calibrations during May and June 2010. Overall, the two calibration methods are consistent; this verifies the validity of the transfer matrix model used in the FONT region. These results were all taken before improvements were made to the BPM processors; these improvements involve matching the input cables and changing the LO phasing scheme.

In November and December 2010, corrector and mover calibrations were performed with the improved processors; four more processors had also been produced. Table 4.6 shows the calibration results for November and December 2010. The measured values marked with ‘*’ denote that the processor output was in saturation. If the data are in saturation then the measured value will be smaller than the real calibration constant and the measured position will not depend linearly on the real position. For the 30th November, the processors were all connected onto the P2 y -axis striplines (first processors 1, 2, 3 and 4 and then processors 5,

Processor	Δ amplitude (mV)	Σ amplitude (mV)	$\frac{g_d}{g_s}$
1	259.6	15.62	16.62
2	263.6	16.36	16.11
3	265.6	15.56	17.07
4	273.6	15.36	17.81
5	261.6	16.36	15.99
7	285.6	17.36	16.45
8	243.6	17.42	13.98
10	277.6	16.34	16.99

Table 4.3: Measurements of the gain ratio for each BPM processor.

Processor	19 Feb	24 Feb	26 Feb	07 Apr	12 May	03 Jun
1 on P1(y -axis)	2.486 $\pm$ 0.002	2.370 $\pm$ 0.019	2.464 $\pm$ 0.002	2.462 $\pm$ 0.002	2.344 $\pm$ 0.001	2.455 $\pm$ 0.001
	2.463 $\pm$ 0.002	2.364 $\pm$ 0.015	2.474 $\pm$ 0.022	2.501 $\pm$ 0.003	2.334 $\pm$ 0.001	
	2.423 $\pm$ 0.002	2.358 $\pm$ 0.017	2.593 $\pm$ 0.003	2.486 $\pm$ 0.002	2.331 $\pm$ 0.001	
2 on P2(y -axis)	3.008 $\pm$ 0.004	3.072 $\pm$ 0.012	3.193 $\pm$ 0.004	2.492 $\pm$ 0.006	2.330 $\pm$ 0.001	2.351 $\pm$ 0.001
	2.954 $\pm$ 0.004	3.091 $\pm$ 0.012	3.244 $\pm$ 0.005	2.500 $\pm$ 0.006	2.313 $\pm$ 0.001	
	2.943 $\pm$ 0.003	3.032 $\pm$ 0.012	3.361 $\pm$ 0.006	2.489 $\pm$ 0.006	2.282 $\pm$ 0.002	
3 on P2(x -axis)						2.503 $\pm$ 0.001
4 on P3(y -axis)	2.514 $\pm$ 0.002	2.582 $\pm$ 0.013	2.787 $\pm$ 0.003	2.583 $\pm$ 0.002	2.618 $\pm$ 0.001	2.510 $\pm$ 0.001
	2.483 $\pm$ 0.002	2.610 $\pm$ 0.001	2.707 $\pm$ 0.003	2.581 $\pm$ 0.002	2.633 $\pm$ 0.001	
	2.464 $\pm$ 0.001	2.687 $\pm$ 0.013	2.801 $\pm$ 0.005	2.573 $\pm$ 0.002	2.593 $\pm$ 0.001	

Table 4.4: Corrector calibration results for the first 4 processors ($\times 10^{-3} \mu\text{m}^{-1}$). For each processor, where three numbers are shown, this represents the calibration constants for the first, second and third bunches.

	12 May 2010		03 June 2010	
Processor	Corrector	Mover	Corrector	Mover
1 on P1 (y -axis)	2.344 $\pm$ 0.001	2.342 $\pm$ 0.007	2.455 $\pm$ 0.001	2.517 $\pm$ 0.004
	2.334 $\pm$ 0.001	2.326 $\pm$ 0.007		
	2.331 $\pm$ 0.001	2.317 $\pm$ 0.007		
2 on P2 (y -axis)	2.330 $\pm$ 0.001	2.342 $\pm$ 0.007	2.351 $\pm$ 0.001	2.343 $\pm$ 0.005
	2.313 $\pm$ 0.001	2.434 $\pm$ 0.006		
	2.282 $\pm$ 0.002	2.442 $\pm$ 0.008		
3 on P2 (x -axis)			2.503 $\pm$ 0.001	2.505 $\pm$ 0.017
4 on P3 (y -axis)	2.628 $\pm$ 0.001	2.442 $\pm$ 0.009	2.510 $\pm$ 0.001	2.481 $\pm$ 0.004
	2.633 $\pm$ 0.001	2.441 $\pm$ 0.009		
	2.593 $\pm$ 0.001	2.429 $\pm$ 0.009		

Table 4.5: A comparison between corrector calibration data and mover calibrations ($\times 10^{-3} \mu\text{m}^{-1}$). For each processor, where three numbers are shown, this represents the calibration constants for the first, second and third bunches.

7, 8 and 10). For the dates after this, the processors were connected as shown in Table 4.7.

From these tables of results, it is clear that excluding the saturated data, the calibration constants of the 3-bunch data are consistent for all three bunches for each processor. It is also clear that the single bunch calibration constants are consistent with the 3-bunch calibration constants. Before improvements to the hardware were introduced, the calibration constants for each processor are stable within $\sim 15\%$; after these improvements were introduced, the calibration constants are stable within approximately 5%. This is regardless of the machine optics, number of bunches per train and the calibration method.

Variations in the calibration constants are due to changes in the experimental setup. These factors include:

- Phase of the LO signal
- Amplitude of the LO signal
- Setting the timing for the FONT5 board (Chapter 7)

Previously the LO phase was set by using the Σ_Q (Figure 3.1); this allowed the phase to be set with an accuracy of $\sim 15^\circ$, which introduces a shift on the measured calibration constant of approximately 15%. With the current system, the LO phase is shifted by 90° by changing the cable path length and then the Σ_I (Figure 3.1) is minimised; once the LO is set, the cable path length is changed back to the original length. Voltage dependent phase shifters are varied with analogue voltage supplies to phase the LO, which allows the LO phase to be set with an accuracy of $\sim 3^\circ$, causing a shift on the measured calibration constant of

Processor	30 Nov		06 Dec		08 Dec
	Corrector	Mover	Corrector	Mover	Mover
1	2.584±0.001	2.500±0.001			
2	2.454±0.001	2.372±0.092	2.489±0.004	1.738±0.028*	1.733±0.040* 2.454±0.014 2.257±0.015*
3	2.142±0.001	2.069±0.008		1.973±0.007	2.144±0.006 2.144±0.006 2.119±0.006
4	2.432±0.002	2.347±0.019	2.513±0.002	2.531±0.030	2.365±0.071* 2.132±0.021* 2.305±0.030*
5	2.271±0.001	2.132±0.009			
7	2.726±0.001	2.660±0.011		2.408±0.009	2.672±0.018* 2.443±0.010* 2.458±0.010*
8	2.101±0.001	1.988±0.010		1.980±0.005	1.907±0.005 1.905±0.004 1.884±0.005
10	2.533±0.003	2.469±0.024	2.472±0.008	2.516±0.051	2.903±0.055* 2.380±0.017* 2.615±0.023*

Table 4.6: A comparison between corrector and mover calibrations for the 8 FONT BPM processors ($\times 10^{-3} \mu\text{m}^{-1}$) [47]. For each processor, where three numbers are shown, this represents the calibration constants for the first, second and third bunches. Values marked with * denote that the processor output was in saturation.

BPM	x -axis	y -axis
P1	Processor 2	Processor 3
P2	Processor 4	Processor 7
P3	Processor 10	Processor 8

Table 4.7: A table showing the arrangement of processors connected to the FONT BPMs for December 2010.

approximately 3%; these analogue voltage supplies have been replaced with a digital voltage supply which should further improve the accuracy of the LO phasing.

The LO amplitude is checked regularly to ensure that the level is $+14 \pm 0.5$ dBm for all processors, which ensures that the linearity and performance of the processors is consistent.

The FONT5 is configured so that the digital electronics are sampling on the peaks of the processor signals. The FONT5 timing system employs three delays; the ‘trigger delay’, the ‘sample holdoff’ and the ‘scan delay’ (Chapter 7). The trigger delay uses a 2.16 MHz ring clock to set the timing system in increments of 462 ns; this is the revolution frequency of the ATF damping ring. The sample holdoff uses a 357 MHz clock, which is phase locked to the ring clock to alter the timing system in increments of 2.8 ns. The scan delay is used to set the timing in increments of 70 ps using a clock generated by the Xilinx Vertex5 FPGA digital processor [48]; which introduces an error on the Δ and Σ amplitudes of approximately 2%.

4.2 BPM Resolution Studies

4.2.1 Theoretical Resolution

As shown in Eqs. 4.1 and 4.2, the Δ signal is dependent on the bunch position and the bunch charge. The Σ signal is dependent only on the charge. Eq. 4.16 shows the measured beam position as a function of the real position, charge and electrical noise on the Σ and Δ channels. The parameters are defined in Table 4.8.

$$y_M = \frac{\alpha_\Delta Q y_R + V_\Delta}{C(\alpha_\Sigma Q + V_\Sigma)} \quad (4.16)$$

The variance of y_M can be calculated from the weighted sum of the variances of the parameters of y_M (Eq. 4.17) [49]. q_i represents the set of variables describing y_M ; these variables are Q , y_R , V_Δ and V_Σ .

$$\sigma_{y_M}^2 = \sum \left(\frac{\partial y_M}{\partial q_i} \right)^2 \sigma^2(q_i) \quad (4.17)$$

By applying this equation, calculating the partial derivatives, and assuming the electrical noise on Σ is of the same magnitude as the noise on Δ (i.e. $V_\Delta \approx V_\Sigma \equiv V$), the variance of

Parameter	Definition
y_M	Measured beam position
Q	Bunch charge
y_R	True beam position
C	BPM calibration constant
V_Δ	Electrical noise on the Δ channel
V_Σ	Electrical noise on the Σ channel
α_Δ	Constant relating Q and y_R to Δ
α_Σ	Constant relating Q to Σ

Table 4.8: A list of the parameters used to calculate the theoretical resolution.

y_M can be expressed as:¹

$$\sigma_{y_M}^2 = \frac{\alpha_\Delta^2 \sigma^2(y_R)}{C^2 \alpha_\Sigma^2} + \frac{\sigma^2(V)}{C^2 \Sigma^4} (\Sigma^2 + \Delta^2) \quad (4.18)$$

The $\sigma(y_R)$ term is the correlated term and represents the beam jitter. The $\sigma(V)$ term represents the resolution of the BPM. Thus from this derivation the theoretical resolution is:

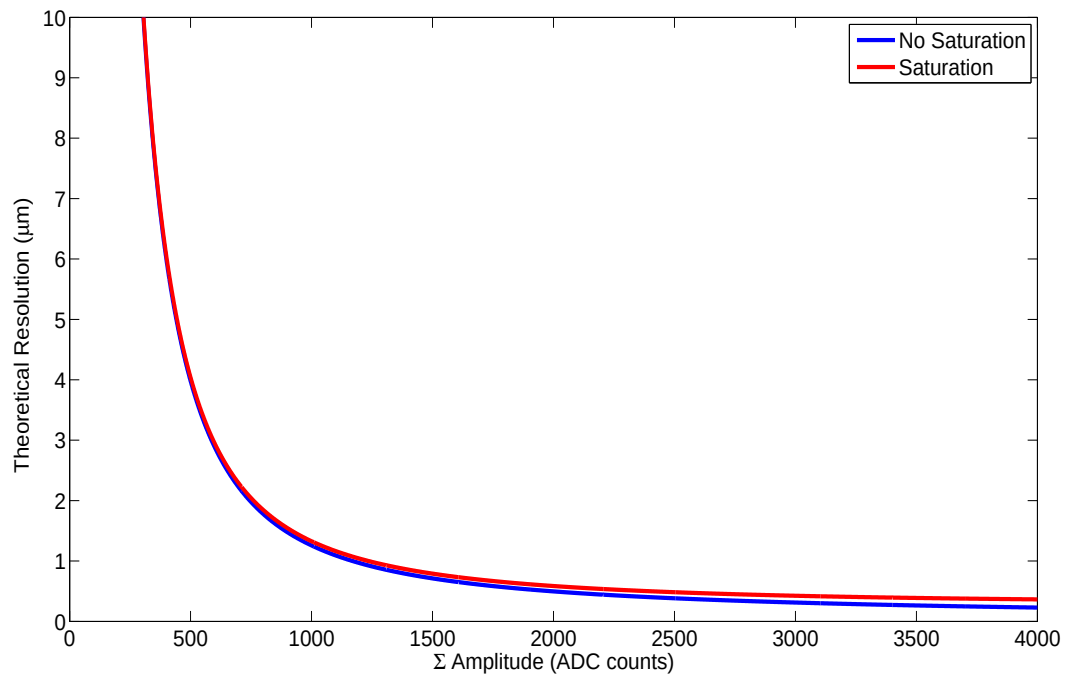
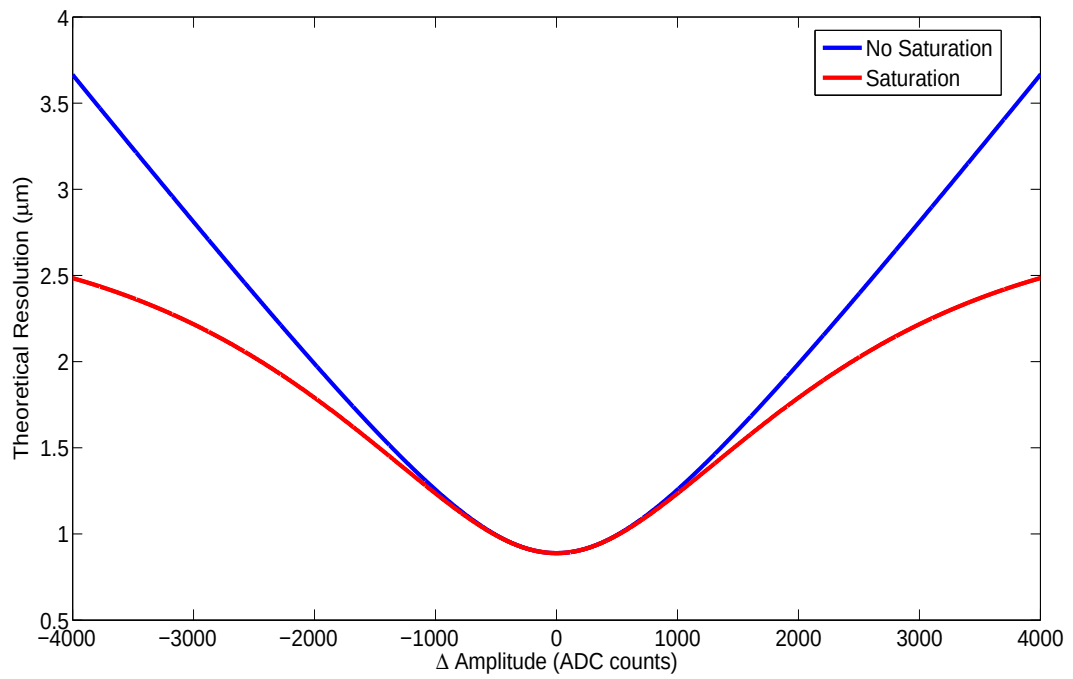
$$\sigma_{BPM} = \frac{\sigma(V)}{C \Sigma^2} \sqrt{\Sigma^2 + \Delta^2} \quad (4.19)$$

Eq. 4.19 does not take into account the effect of saturation from the BPM processors. Since the saturation effect of the processor is not an analytical function a hyperbolic tangent function was used to describe the qualitative effects on the resolution, Eq. 4.20. α_0 represents the amplitude at which the processor outputs begin to go into heavy saturation, this is measured to be ~ 3000 ADC counts.

$$\sigma_{BPM} = \frac{\sigma(V)}{C \alpha_0^2 \tanh\left(\frac{\Sigma}{\alpha_0}\right)^2} \sqrt{\alpha_0^2 \tanh\left(\frac{\Sigma}{\alpha_0}\right)^2 + \alpha_0^2 \tanh\left(\frac{\Delta}{\alpha_0}\right)^2} \quad (4.20)$$

Figures 4.13 and 4.14 are plots of the theoretical resolution as a function of the Σ and Δ amplitudes respectively. The blue lines show the resolution without saturation effects taken into account, the red lines show the resolution with saturation effects accounted for. For the plot where the Σ amplitude was varied, the Δ amplitude was kept constant at 1000 ADC counts, and conversely for the Δ plot. An electrical noise amplitude of 2 ADC counts was used; this is similar to the observed level.

¹See Appendix A for full derivation

Figure 4.13: Theoretical resolution vs. Σ signal.Figure 4.14: Theoretical resolution vs. Δ signal.

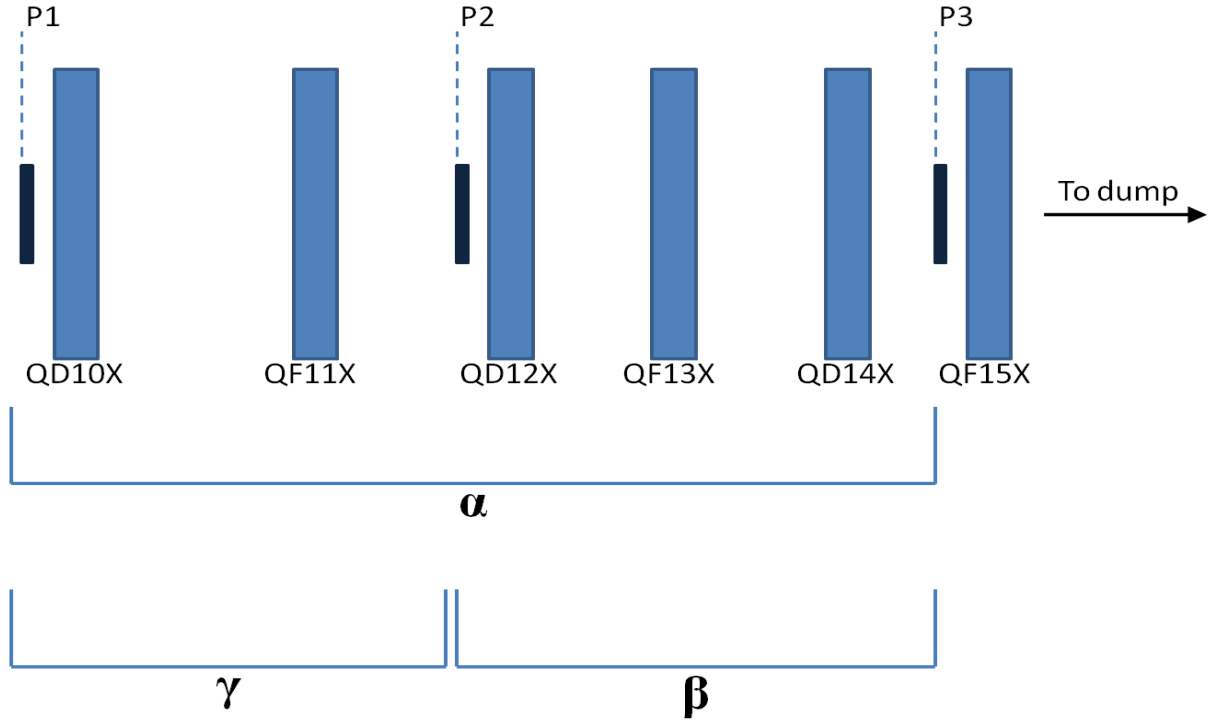


Figure 4.15: The locations of the FONT BPMs in the ATF2 extraction line. The transfer matrices between the BPMs are also shown.

4.2.2 Resolution Calculation

From the BPM signals, the beam position is calculated. The measured position from one bunch train to the next varies. There are two types of jitter contribution that need to be identified in order to calculate the resolution. First there is the beam jitter, which is the real variation in beam position and angle. This is caused by fluctuations in the accelerator, such as ground motion or variations in beam energy. The other contribution is not correlated between BPMs and is defined as the resolution of the BPM system.

In order to calculate the BPM resolution the correlated jitter needs to be removed from the measured jitter. Two methods have been used to calculate the correlated jitter.

Geometric Method

Neglecting the resolution of the FONT BPMs, then the correlation between the beam position measured in different BPMs is only dependent on the optics of the beam. Figure 4.15 shows the locations of the FONT BPMs in the ATF2 extraction line; the transfer matrices are also shown. In this case, the correlation coefficients between BPMs can be calculated from the transfer matrices between the BPMs. Eqs. 4.21, 4.22 and 4.23 are the general transfer matrices between the FONT BPMs.

$$\begin{pmatrix} y_3 \\ y_3' \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} y_1 \\ y_1' \end{pmatrix} \quad (4.21)$$

$$\begin{pmatrix} y_3 \\ y_3' \end{pmatrix} = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \begin{pmatrix} y_2 \\ y_2' \end{pmatrix} \quad (4.22)$$

$$\begin{pmatrix} y_2 \\ y_2' \end{pmatrix} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix} \begin{pmatrix} y_1 \\ y_1' \end{pmatrix} \quad (4.23)$$

By solving these equations simultaneously, the beam position in one BPM can be expressed in terms of the positions in the other two BPMs, as shown in Eq. 4.24 [50, 51]. y_i^{meas} is the measured beam position in the i^{th} BPM, $i = 1, 3$; while y_2^{calc} is the beam position in P2 calculated from the measured positions in P1 and P3. y_2^{calc} is described by a linear combination of y_1^{meas} and y_3^{meas} ; X_{21} and X_{23} are the coefficients of this combination.

$$y_2^{\text{calc}} = \left(\gamma_{11} - \frac{\alpha_{11}\gamma_{12}}{\alpha_{12}} \right) y_1^{\text{meas}} + \frac{\gamma_{12}}{\alpha_{12}} y_3^{\text{meas}} \equiv X_{21}y_1^{\text{meas}} + X_{23}y_3^{\text{meas}} \quad (4.24)$$

And cyclic permutations.

The residual for P2 is the difference between the measured position at P2 and the calculated position from the other two BPMs. The standard deviation of the residual, $\sigma_2^{\text{residual}}$, is shown in Eq. 4.25; $\sigma(y_i)$ is the standard deviation of the position in BPM i . Assuming the resolution is similar for all three BPMs, $\sigma_2^{\text{residual}}$ can be related to the BPM resolution σ_{BPM} (Eq. 4.26); the BPM resolution can thus be expressed as shown in Eq. 4.27.

$$\sigma_2^{\text{residual}} = \sqrt{X_{21}^2 \sigma^2(y_1) + \sigma^2(y_2) + X_{23}^2 \sigma^2(y_3)} \quad (4.25)$$

$$\sigma_2^{\text{residual}} = \sqrt{1 + X_{21}^2 + X_{23}^2} \sigma_{\text{BPM}} \quad (4.26)$$

$$\sigma_{\text{BPM}} = \sqrt{\frac{X_{21}^2 \sigma^2(y_1) + \sigma^2(y_2) + X_{23}^2 \sigma^2(y_3)}{1 + X_{21}^2 + X_{23}^2}} \quad (4.27)$$

Eq. 4.27 shows that the resolution estimate is a weighted mean of the squares of the resolutions of each BPM. If the assumption made that the resolutions are similar is not true, then the resolution estimate will be biased towards the largest resolution, this method gives a good estimate for the upper limit of the BPM resolution.

Least Squares Method

An estimate for the beam position in one of the FONT BPMs can be expressed in terms of the other two BPMs (Eq. 4.28). i, j and k represent the indices of the FONT BPMs, X_{ij} and

X_{ik} are the coefficients previously defined and X_{i0} is a constant offset. If these measurements are repeated many times, and the beam positions in each BPM are represented as a vector, then Eq. 4.28 can be represented in a matrix form (4.29); where each row represents one set of measurements.

$$y_i = X_{ij}y_j + X_{ik}y_k + X_{i0} \quad (4.28)$$

$$\begin{pmatrix} y_i \\ \dots \end{pmatrix} = \begin{pmatrix} y_j & y_k & 1 \\ \dots & \dots & \dots \end{pmatrix} \begin{pmatrix} X_{ij} \\ X_{ik} \\ X_{i0} \end{pmatrix} = \mathbf{Y}\mathbf{X} \quad (4.29)$$

Matrix $\mathbf{Y}$ is not a square matrix in general, and so in order to obtain the vector $\mathbf{X}$, the Moore-Penrose pseudo-inversion is implemented². This method gives the least squares estimate for the values of $\mathbf{X}$. Now if the BPM resolutions are considered again, the elements of $\mathbf{X}$ can be used to estimate the beam position in the i^{th} BPM (Eq. 4.30). From this the residual between the measured beam position and inferred beam position can be calculated. The residual is now independent of the correlated beam jitter; the standard deviation of the residual is the resolution of the BPM.

$$y_i^{\text{calc}} = X_{ij}y_j^{\text{meas}} + X_{ik}y_k^{\text{meas}} + X_{i0} \quad (4.30)$$

$$\sigma_i^{BPM} \equiv \sigma_i^{\text{res}} = \frac{\sigma(y_i^{\text{meas}} - y_i^{\text{calc}})}{\sqrt{1 + X_{ij}^2 + X_{ik}^2}} \quad (4.31)$$

In Eq. 4.31, the denominator is a normalisation factor. This method has the advantage that it is model independent, so it does not require any direct knowledge of the beam optics between the BPMs, and all three BPM resolutions can be calculated independently.

In order to understand the least-squares method, a simulation was constructed. Transfer matrices were obtained from Flight Simulator [44] and used to calculate the transformations of the beam from the extraction kicker to the three FONT BPMs. Initial position and angle jitters at the start of the extraction line were used to calculate position jitters for the FONT BPMs. A ‘true’ BPM resolution was defined for each BPM; for the results shown below, the true resolution was set to $1\mu\text{m}$. The least-squares method was used to calculate the resolution of the three BPMs as the jitter-resolution ratio, Π_i , was varied. Π_i is defined as the beam jitter measured at the i^{th} BPM divided by the BPM resolution, σ_i^{BPM} ; Π_0 is defined as the jitter-resolution ratio at the start of the extraction line (Eq. 4.32) and α_Π , β_Π and γ_Π are constants of proportionality.

$$\alpha_\Pi \Pi_1 = \beta_\Pi \Pi_2 = \gamma_\Pi \Pi_3 \equiv \Pi_0 \quad (4.32)$$

²See Appendix B

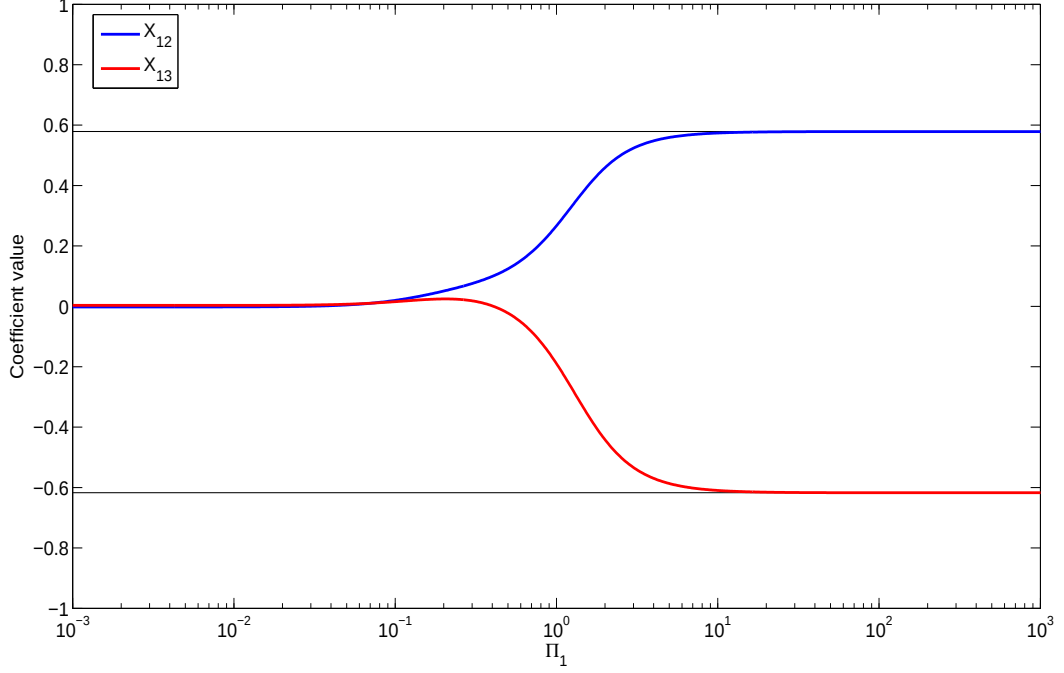


Figure 4.16: Simulated values of X_{12} (blue) and X_{13} (red) vs. Π_1 .

Figures 4.16, 4.17 and 4.18 show simulated values of the X coefficients between BPMs calculated with the least-squares method versus Π_i ; the nominal ATF2 optics were used for this simulation. When Π_i is very small, the resolution dominates, therefore the X coefficients tend to zero. Conversely, when Π_i is very large, the beam jitter dominates and so the X coefficients tend to the values calculated by beam optics. Between these two limits, it is evident that the values of these coefficients are strongly dependent on Π_i ; although the exact shape of the curve is dependent on the beam optics.

The X coefficients can be expressed in the form shown in Eq. 4.33. The variance and covariance terms can be expressed in terms of Π_0 (Eq. 4.34); σ_r is the true resolution of the BPM system. If $i = j$ then the equation represents a variance term, if $i \neq j$ then the equation represents a covariance term. a_{ij} and b_{ij} are constants related to the lattice parameters as well as the relationship between position and angle jitter at the start of the extraction line.

$$\begin{pmatrix} X_{ij} \\ X_{ik} \end{pmatrix} = \frac{1}{\sigma_j^2 \sigma_k^2 - cov_{jk}^2} \begin{pmatrix} \sigma_k^2 cov_{ij} - cov_{ik} cov_{jk} \\ \sigma_j^2 cov_{ik} - cov_{ij} cov_{jk} \end{pmatrix} \quad (4.33)$$

$$cov_{ij} = \frac{a_{ij}^2 \Pi_0^4}{b_{ij}^2 \Pi_0^2 + 1} \sigma_r^2 \quad (4.34)$$

From these equations the correlation coefficients can be expressed (Eq. 4.35). The A and B coefficients are constants dependent on the lattice parameters and the relationship between

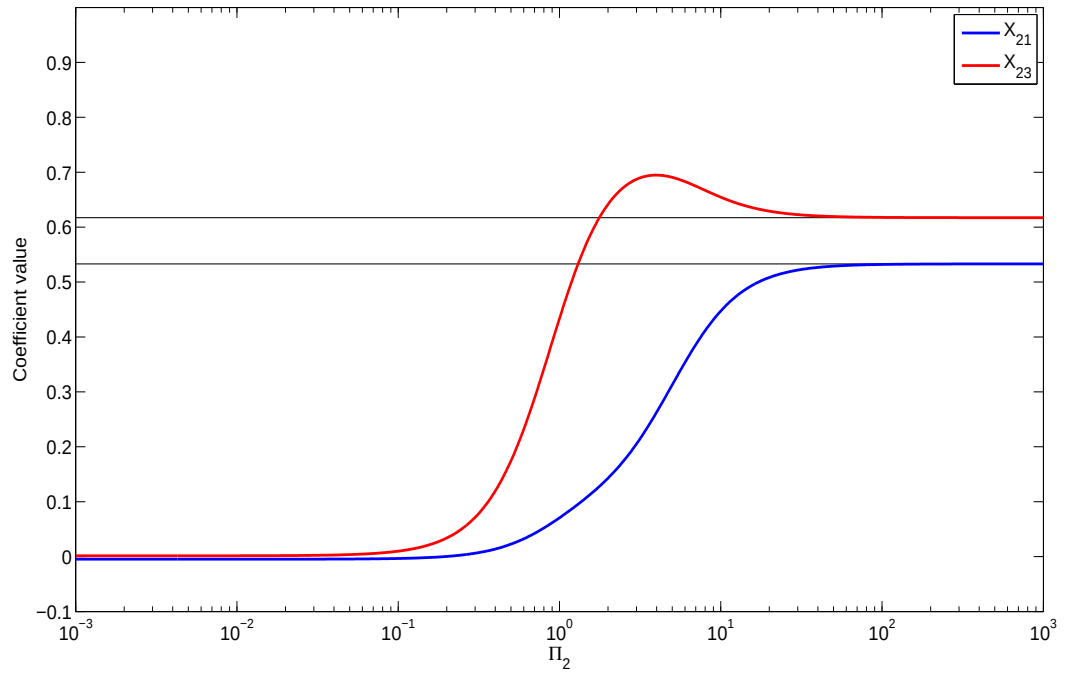


Figure 4.17: Simulated values of X_{21} (blue) and X_{23} (red) vs. Π_2 .

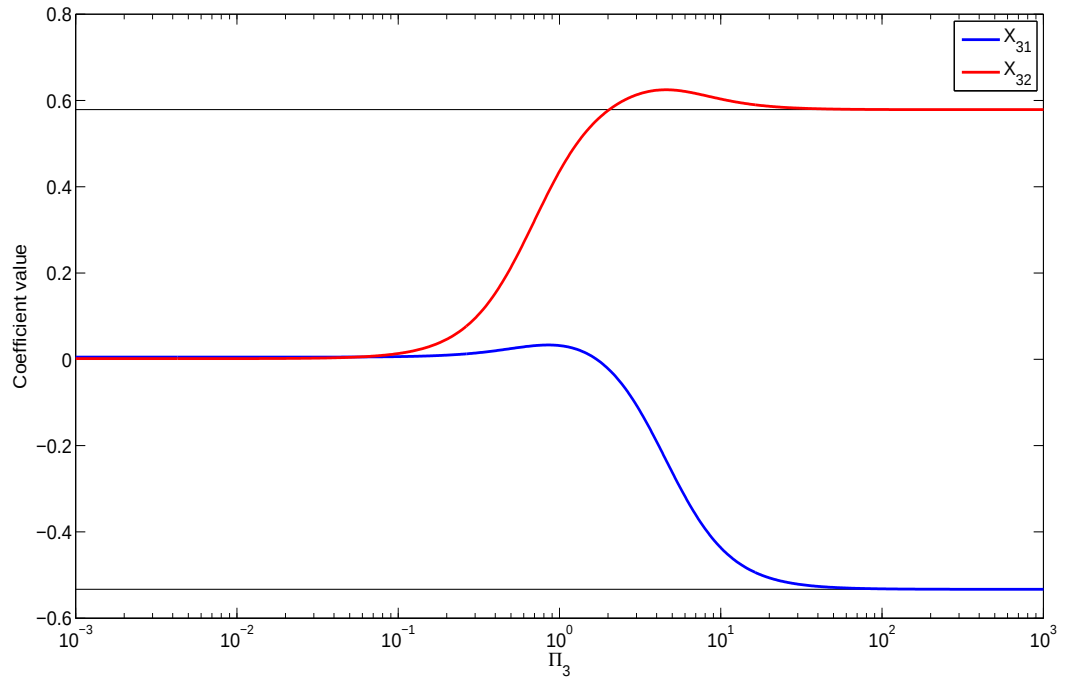


Figure 4.18: Simulated values of X_{31} (blue) and X_{32} (red) vs. Π_3 .

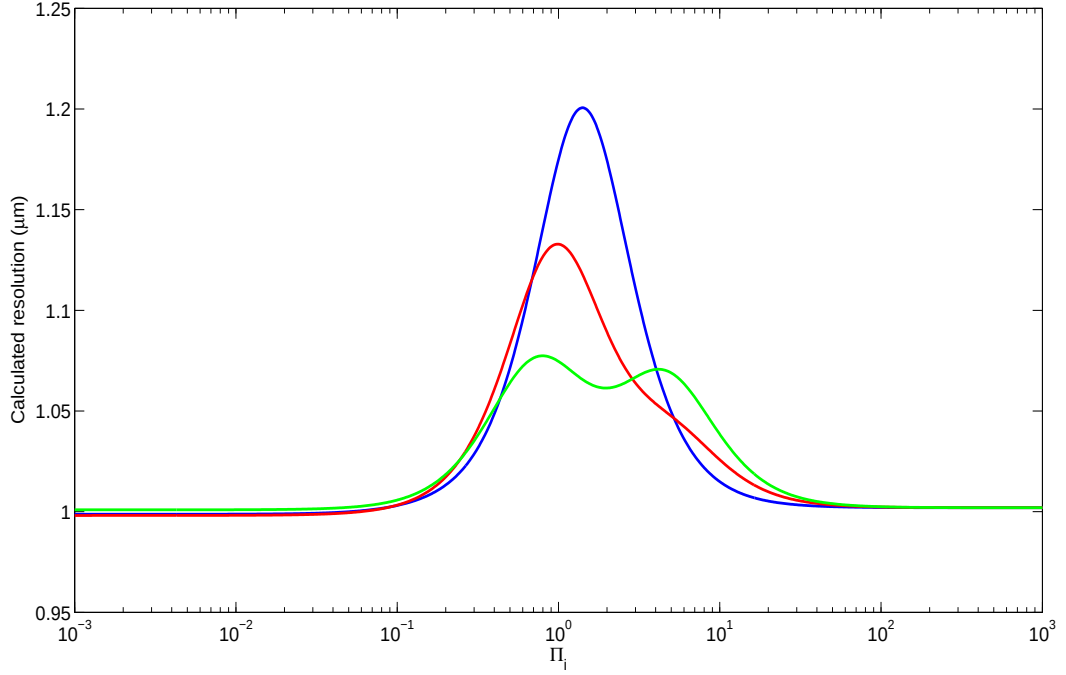


Figure 4.19: Calculated resolution for P1 (blue), P2 (red) and P3 (green) vs. Π_1 , Π_2 and Π_3 respectively.

the position and angle jitter at the start of the extraction line. In order for the X coefficients to be physical, the denominator cannot be zero for any value of Π_0 .

$$X_{ij} = \frac{A_1 \Pi_0^4 + A_2 \Pi_0^2}{B_1 \Pi_0^4 + B_2 \Pi_0^2 + B_3} \quad (4.35)$$

Figure 4.19 shows the estimated resolution for the three FONT BPMs from the simulation with the ‘true’ resolution of each BPM set to $1\mu\text{m}$. The peaks in this figure are due to the transition regions in the X coefficients, when these are combined to give an estimate of the BPM resolution, these transition regions form peaks; the plateaux regions produce a relatively stable estimate for the BPM resolution. A physical interpretation of this is that the transition regions represent regions with a greater uncertainty in the values of X_{ij} than the plateaux regions and therefore produce a greater uncertainty in the calculated resolution.

4.2.3 Factors Affecting Resolution

In the real FONT system, there have been several factors that have affected the measured resolution. One major factor is the phase jitter on the LO. In an ideal setup, the path lengths from the BPM striplines to the processor inputs would be identical for both striplines. On the real processors a difference in effective path length was measured; the value varied between processors and was measured to be between 0.5mm and 5mm. This causes the Δ signal

to be dependent on the LO phase. Thus, the apparent position of the beam will change with the LO phase. Since each processor's LO is phased independently, this will appear as an uncorrelated jitter between the BPMs, and all uncorrelated jitter is assumed to be due to the resolution. Therefore, the measured resolution will be increased because of the LO phase jitter. This problem was resolved by using cables with a compensating path length difference.

Another effect is that the beam position in the vertical plane couples into the horizontal plane, as well as the other way round. This means that the correlation in position from one BPM to the next BPM will be less than expected. This effectively increases the uncorrelated jitter. This problem is resolved by using skew quadrupole magnets around the FONT BPMs, which uncouple the horizontal and vertical jitter. The BPM to BPM correlation is not only important for resolution measurement, it also limits the minimum beam jitter achievable from the FONT feedback system, which will be discussed in Chapter 5.

4.3 Resolution Results

4.3.1 Charge Dependence

The theoretical resolution suggests that if Δ is small, the resolution should vary as the reciprocal of the charge (Eq. 4.19). Due to the saturation effects of the BPM processors, at high charge, the resolution will plateau; this is modelled as a hyperbolic tangent in the simulation. Figure 4.20 shows a series of resolution measurements, using the geometric method, taken at different charges at ATF2 in May 2009; these measurements were taken before intermediate amplifiers were employed. Curves showing the theoretical charge dependence of the resolution have been plotted for comparison. It is clear that the curve including the saturation effects is a good model to describe the resolution plateau at high charge.

Initially 6dB attenuators were placed on the inputs of the BPM processors and five resolution measurements were taken for different bunch charges. After this, the 6dB attenuators were replaced with 10dB attenuators and four resolution measurements were taken for different bunch charges. The attenuators were used to reduce the amplitudes of the BPM signals which is equivalent to reducing the bunch charge of the beam.

These measurements were not repeated at higher values of Σ because the resolution has reached the plateau by approximately 90 ADC counts. Without the intermediate amplifiers the resolution is increased by approximately a factor of 3.5. This is because the intermediate amplifiers increase the signal amplitude by a factor of 7, but it also introduces additional noise to the system as well as amplifying noise introduced before the amplifiers; the noise is measured to increase by approximately a factor of 2. Figure 4.21 shows resolution vs. Σ amplitude with the intermediate amplifiers connected; this data was taken in October 2011. The theoretical curves were derived from Eqs. 4.19 and 4.20 while the measurements were obtained from reference [47].

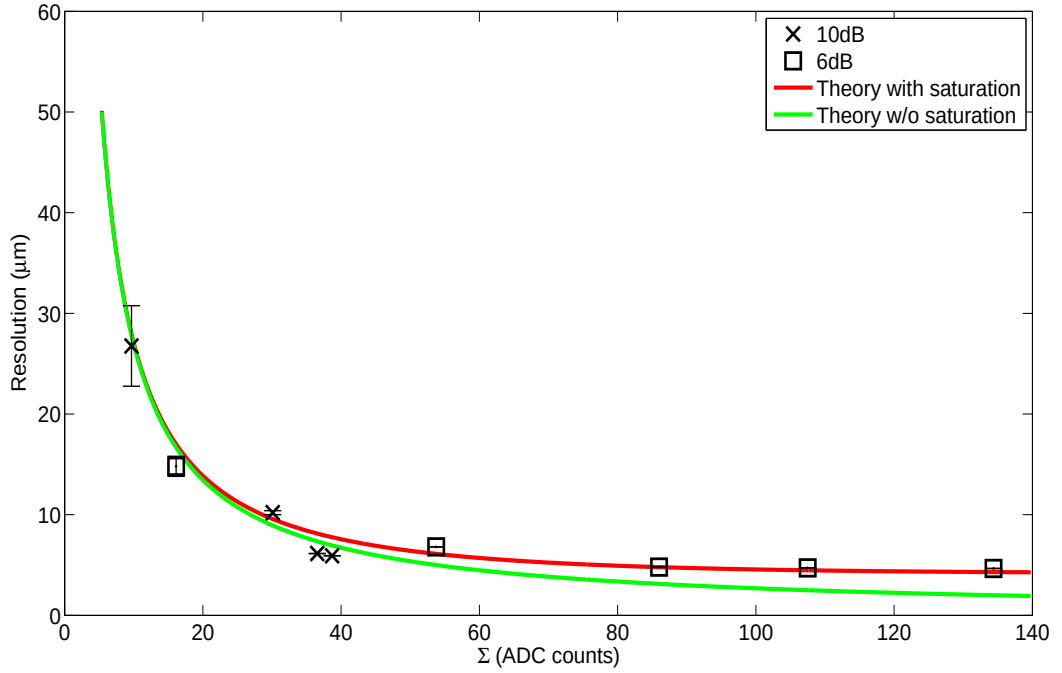


Figure 4.20: Resolution measurements vs. Σ amplitude. The green curve is a theoretical model of the resolution without saturation effects included; the red curve is a theoretical model of the resolution with saturation effects included.

4.3.2 Position Dependence

Figure 4.22 shows how the theoretical resolution varies with position for a Σ amplitude of 1000 ADC counts (blue) and 3000 ADC counts (red). At ATF2, the beam charge is tuned so that the BPM Σ amplitude is ~ 3000 counts; this is the maximum charge before the BPM processors begin to saturate. This figure shows that for high charge, the resolution is very weakly dependent on position. Therefore it was important to study the position dependence of the resolution at low bunch charge. Figure 4.23 shows the measured resolution versus Δ amplitude. This measurement was taken by splitting the stripline signals from the P1 y -axis 4 ways and connecting 3 processors to the splitters and terminators onto the unused connectors; the resolution was calculated using the least-squares method. The Δ amplitude was varied by changing the beam position with the BPM mover. Due to the low bunch charge, it was not possible to vary Δ enough to observe the effects of saturation.

4.3.3 Summary of Resolution Results

Table 4.9 shows some resolution measurements taken at ATF2 with the FONT BPMs [52]. Processors 1, 2 and 4 were connected to the P1, P2 and P3 y striplines respectively for all these measurements. In order to improve the resolution of the BPM processors, low-noise amplifiers were installed into the FONT system at the end of February 2010. Initially, the

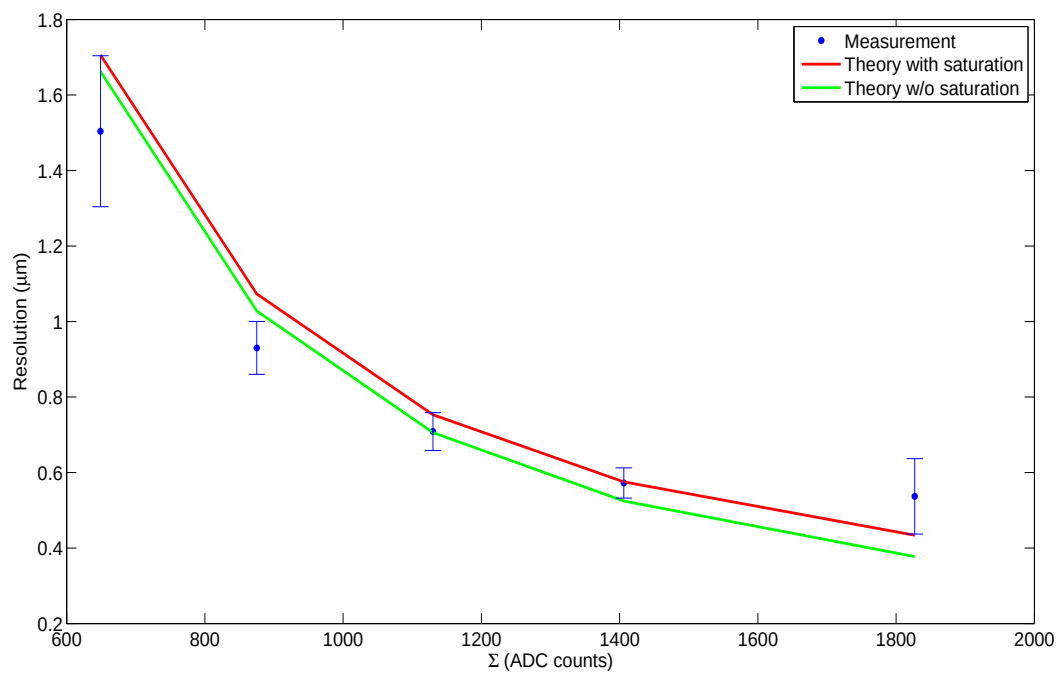


Figure 4.21: Resolution measurements vs. Σ amplitude with the intermediate amplifiers connected [47]. The green curve is a theoretical model of the resolution without saturation effects included; the red curve is a theoretical model of the resolution with saturation effects included.

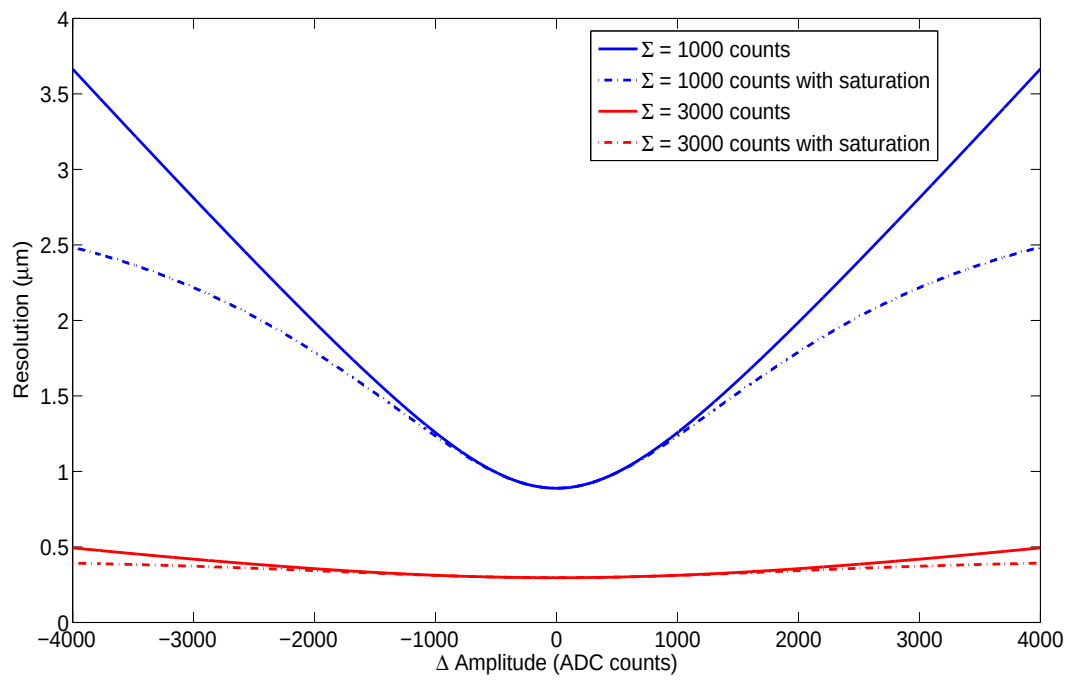


Figure 4.22: Theoretical resolution vs. Δ for different Σ amplitudes. The blue lines represent the resolution when $\Sigma = 1000$ counts, the red lines represent the resolution when $\Sigma = 3000$ counts. The solid lines represent the model without saturation effects included and the dotted lines represent the model with saturation effects included.

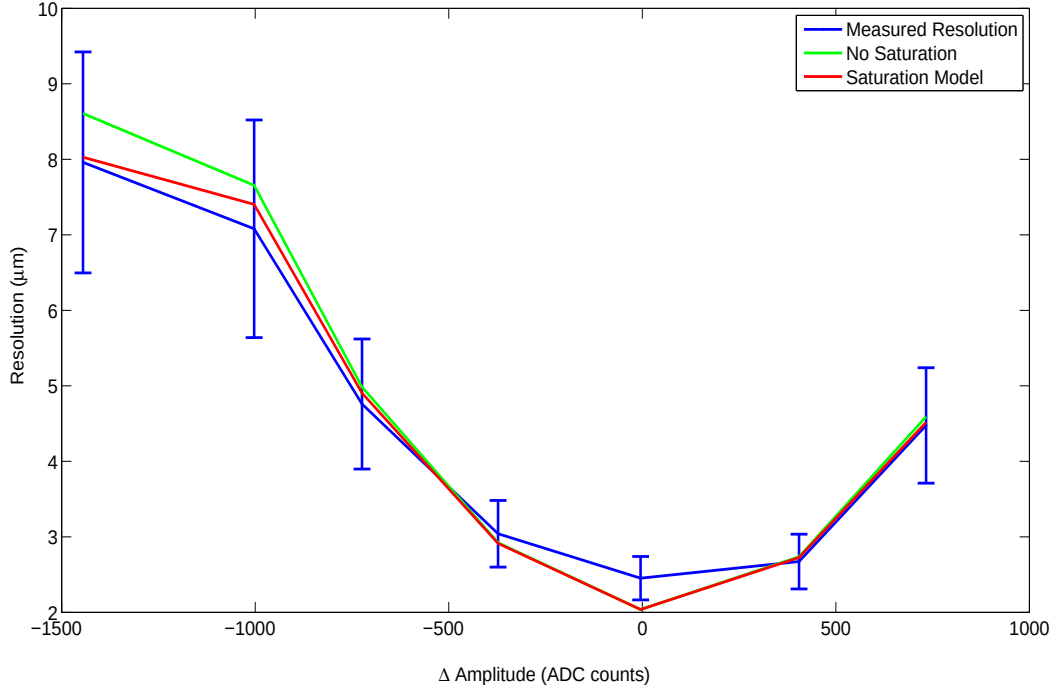


Figure 4.23: Measured resolution vs. Δ amplitude. The blue line and points represent the measured resolutions; the green and red lines are the models with and without saturation effects included respectively.

amplifiers were increasing the signal levels as well as the electrical noise from the extraction kicker pulse. The extraction kicker pulse was causing an uncorrelated jitter for the FONT system. Because the extraction kicker noise and the signal levels are amplified equally, there is no improvement in the resolution of the system. In order to overcome this problem, low-pass filters were connected to the inputs of the low-noise amplifiers; this almost entirely removed the noise from the extraction kicker. In April 2010, the improved BPM system was tested again; Table 4.9 shows that there is a significant improvement in the measured resolution of the system. The resolution measurements for the 16th April and 3rd June 2010 do not show a significant improvement in the resolution; this was due to unstable beam conditions which restricted the magnitude of the bunch charge to $\sim 2 \times 10^9$ electrons per bunch rather than 6×10^9 electrons per bunch. As a result, it was not possible to achieve optimal resolution measurements.

4.4 Summary

In this chapter, the calibration and resolution of the BPM processors were studied. The theoretical calibration constant was first calculated. Next, the two methods used by FONT to calibrate the BPMs were described, and a comparison of results between these methods was shown. The magnet based calibration gives slightly different results to the mover based calibration; this is due to errors in the model of the extraction line optics.

Date	Bunch 1	Bunch 2	Bunch 3
17 Dec 2009	4.40 ± 0.79	4.27 ± 0.75	3.49 ± 0.56
17 Feb 2010	3.56 ± 0.58	3.60 ± 0.59	3.60 ± 0.57
18 Feb 2010	3.46 ± 0.56	3.25 ± 0.50	3.52 ± 0.57
24 Feb 2010	2.80 ± 0.29	3.53 ± 0.55	3.41 ± 0.56
26 Feb 2010	3.18 ± 0.49	3.02 ± 0.46	5.96 ± 1.17
07 Apr 2010	1.59 ± 0.10	1.66 ± 0.12	1.34 ± 0.04
16 Apr 2010	2.44 ± 0.31	2.25 ± 0.26	2.42 ± 0.30
03 Jun 2010	2.33 ± 0.29	2.07 ± 0.22	2.10 ± 0.22

Table 4.9: A table showing the measured resolution (μm) at ATF2 with the FONT BPM system; the resolution was calculated using the geometric method [52].

The resolution of the BPM system was then investigated. The two methods for calculating the resolution were described first. Next the theoretical dependence of the resolution on Σ and Δ were investigated. The saturation effects of the BPM processors were also considered. This is important in understanding and minimising the resolution of the BPM system. The theoretical resolution was compared with measured data from ATF2 which are consistent with the theory.

Chapter 5

Position and Angle Feedback at ATF

As described in Chapter 1, the FONT feedback system is used to correct the angle and position jitter of the electron beam at ATF2. Firstly, a single-loop feedback system was developed in order to correct the bunch position at one point in the extraction line. After this, a dual feedback system was developed in order to correct both position and angle in the extraction line. In order to optimise the performance of the feedback system, factors affecting the feedback correction were investigated; this leads to a study of the ATF damping ring. The damping ring was then simulated in order to understand the features of the damping ring oscillations observed. The results of the damping ring simulation are shown in Chapter 6.

5.1 FONT Hardware

The primary goal of FONT is to correct the beam jitter at the entrance of the final focus system at ATF2 to within $1\mu\text{m}$. Figure 5.1 shows a schematic diagram of the FONT region of the ATF2 extraction line. A stripline BPM downstream of a FONT kicker is used to determine the bunch position. The stripline signals are connected to a BPM processor, the outputs of which are connected to inputs of ADCs (analogue to digital converter) on the FONT digital board. The firmware on the digital board then calculates the charge normalised position and multiplies this by a gain factor. This value is output from the board on a DAC (digital to analogue converter).

The gain factor is known as the feedback gain and is dependent on the beam optics. If the magnitude of the gain is too low, the beam position will be undercorrected, and vice versa. The DAC output is amplified and then connected to a kicker, which is used to deflect the following bunch to correct its position; in order to correct both position and angle in one plane, two feedback systems are required.

The previous generation of digital board, known as FONT4, was designed to control only one feedback loop. The FONT4 board could monitor the beam position from one BPM, and control one kicker. The current digital board, known as FONT5, is capable of monitoring x and y positions from three BPMs, a total of nine input channels. The FONT5 board is designed to control two feedback loops simultaneously, allowing both position and angle to be corrected in the y plane.

At ATF2, FONT uses 3 stripline BPMs, the one furthest upstream, known as P1, is used

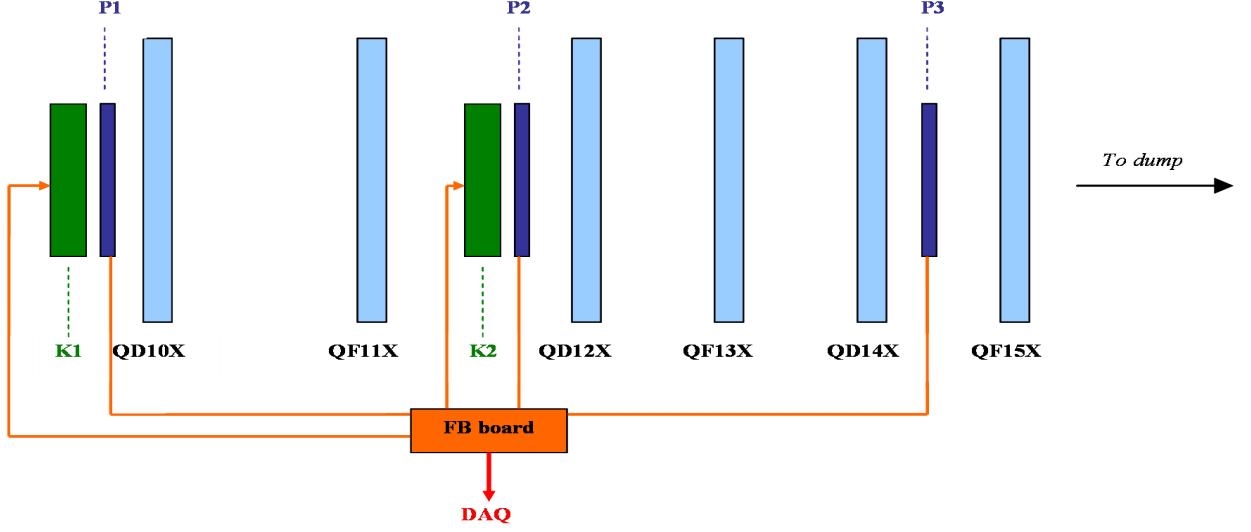


Figure 5.1: A schematic diagram of the FONT region of the ATF2 extraction line.

	Ideal	Flight Simulator
$\mu(K1 - P1)$	0°	17.46°
$\mu(K2 - P2)$	0°	20.88°
$\mu(K1 - K2)$	90°	94.5°
$\mu(P1 - P2)$	90°	97.7°
$\mu(P2 - P3)$	90°	63.36°

Table 5.1: Table showing the phase advances between kickers and BPMs along ATF2.

as a witness BPM. This is used to monitor the beam position before any kick has been applied to the bunches. The next BPM, P2, is positioned downstream of the first FONT kicker, known as K1. The phase advance from K1 to P2 is designed to be approximately 90° . K1 is slightly upstream of P1, although the kick to the beam is very small in P1. K2 is positioned slightly upstream of P2, and P3 is approximately 90° downstream of K2 (Figure 5.1). Positioning the kickers and BPMs approximately 90° apart maximises the effect on position in the BPMs due to an angular deflection from the kickers. This is shown from the $\mathbf{M}_{12}$ element ($\sqrt{\beta_0\beta_1}\sin\mu$) in Eq. 5.1 (see Section 5.2) which represents the contribution to a downstream position from an angular kick upstream. BPMs P2 and P3 are known as the feedback BPMs because they are used for the feedback system.

In reality, the kickers and BPMs are not positioned at optimal locations; Table 5.1 shows the phase advances, μ , between the kickers and BPMs. The phase advances were calculated using Flight Simulator [44], where the nominal beam optics for ATF2 have been used. The K notation represents the kickers, and the P notation represents the BPMs (Figure 5.1).

Figure 5.2 shows a schematic layout of the FONT hardware at ATF2 [47]. The cable map shows two digital boards being used to monitor the FONT BPMs as well as three downstream BPMs. Currently, FONT only uses one digital board in the extraction line, although the scheme described in the diagram is intended to be tested in the near future. Table 5.2 shows

Abbreviation	Description
A1/A2	TMD kicker amplifiers
K1/K2	FONT feedback kickers
P1/P2/P3	FONT BPMs
Q1/Q2/Q3	Downstream BPMs
M1/M2/M3	BPM movers
S1/S2	4-port serial servers (RS-232 to Ethernet)
F1/F2	FONT5 digital boards

Table 5.2: A table of abbreviations for the FONT cable map.

a list of the abbreviations used in the diagram. The green numbered squares represent the BPM processors.

The FONT5 board has 9 ADC input channels; each set of 3 ADCs is clocked by its own 357MHz clock. This is done because the signals from different BPMs arrive at different times, so the clock timings can all be changed to allow the FONT5 board to sample on peak. To ensure that all the signals from one BPM arrive at the FONT5 board at the same time, matched cables are used.

The FONT5 board is capable of controlling two feedback systems simultaneously. The FONT5 board outputs two trigger signals which are used to arm the kicker amplifiers; two DAC signals are also output. The DAC output is the feedback signal which is used as the input to the kicker amplifier.

5.1.1 FONT5 Digital Board

The FONT5 digital board uses a Xilinx Virtex-5 FPGA [48] as a processor. The processor runs on two clock domains; the critical latency processes run on the 357MHz domain, all other processes run on the 40MHz domain (Figure 5.3). The 357MHz is an external clock signal which originally used to be derived from the 714MHz LO signal directly. Due to instabilities with the frequency divider, the 357MHz signal would occasionally jump by a phase of 180°; this would cause the FONT5 board to sample the BPM signals off-peak. This method of producing the 357MHz clock was later abolished; now the clock signal is produced by a signal generator. There are several disadvantages with generating the 357MHz clock rather than deriving it directly. The first is that the phase of the 357MHz clock drifts slightly relative to the 714MHz signal; typically, this is only a few degrees per day and does not pose a significant problem for the FONT system. The other main disadvantage is that as the air temperature changes over the course of the day as well as seasonally, the LO frequency (as well as all harmonics) is varied slightly. When the LO frequency is changed, the 357MHz clock must also be changed otherwise the FONT5 board cannot sample on peak.

The 40MHz clock is produced on the FONT5 board by a quartz crystal oscillator. A 2.16MHz clock and a trigger are also connected to the FONT5 board. The 2.16MHz clock is the revolution frequency of the electron bunches in the damping ring and is known as the ring

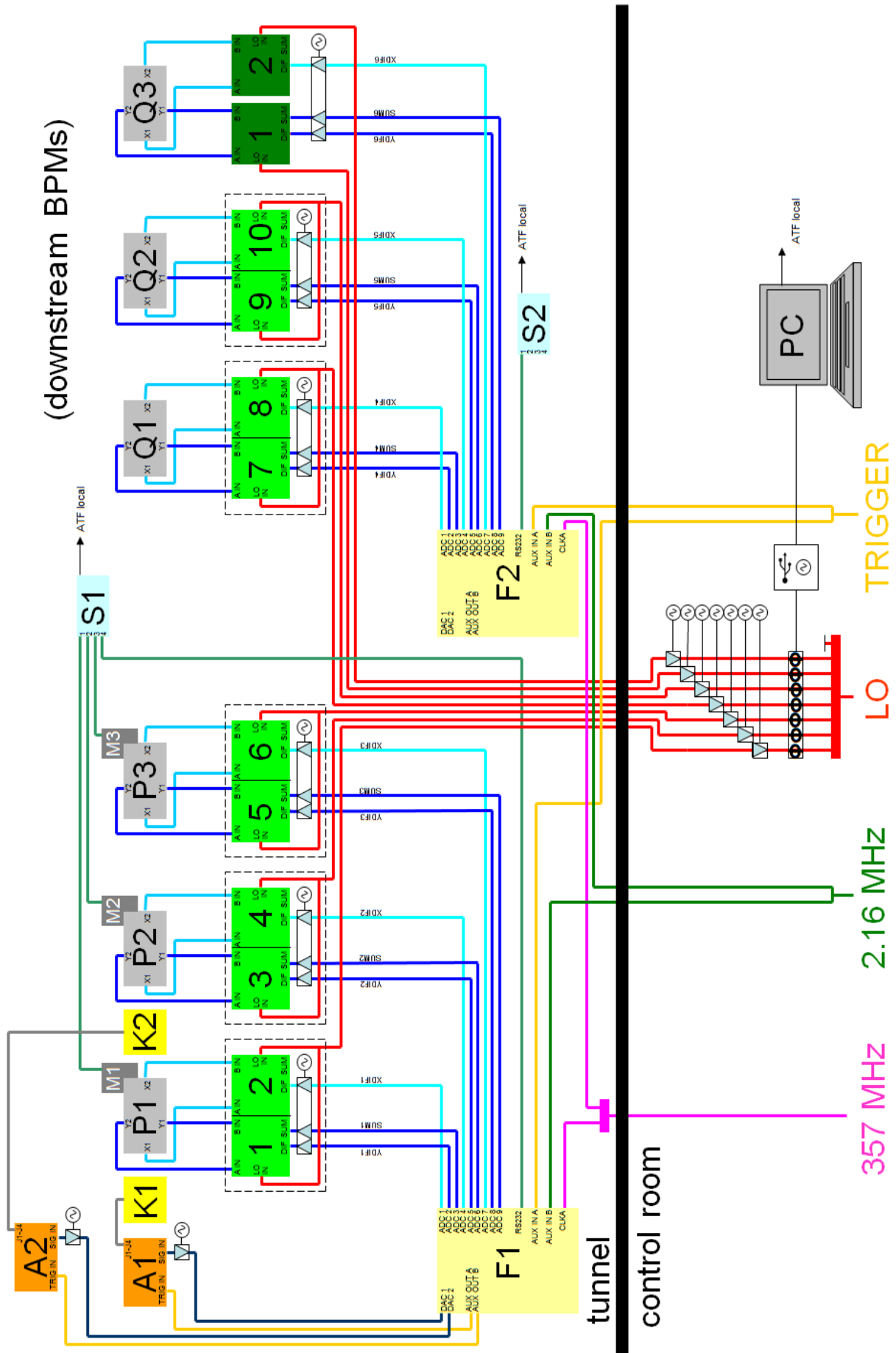


Figure 5.2: A cable map of the FONT extraction line hardware at ATF2.

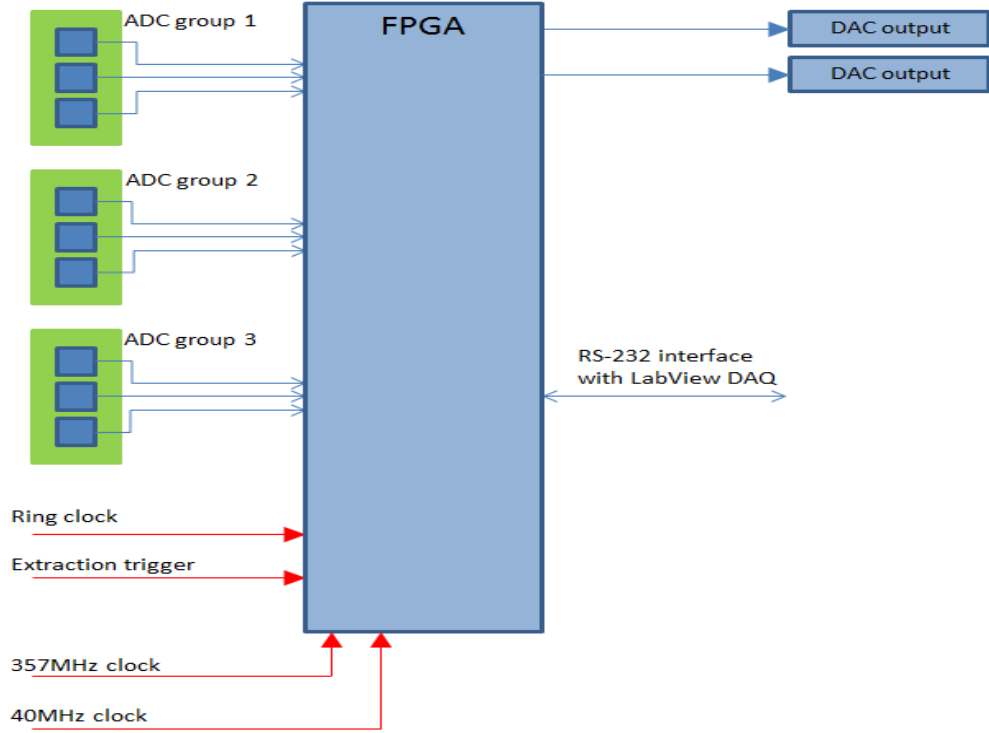


Figure 5.3: Schematic diagram of the functionality of the FONT5 digital board.

clock; this is therefore phase locked to the bunch timing. In order to sample on the peaks of the BPM signals, the clock timings are configured on the FONT5 board; the ring clock is used to make coarse adjustments, while the 357MHz clock makes fine adjustments. In order to sample accurately on the signal peaks, a scan delay is set; the timing can be set with a step size of 70ps.

Figure 5.4 shows a photograph of the FONT5 digital board in its metal casing. Figure 5.5 shows the internal electronics of the FONT5 board.

5.1.2 Kicker Amplifier

The kicker amplifiers were developed and built by TMD Technologies [53]. The amplifier requires a minimum of 15ms to charge the capacitors, therefore a trigger is sent to the amplifier $\sim 15\text{-}20\text{ms}$ before the feedback signal arrives. The amplifier has a design latency of 35ns once the feedback signal arrives at the amplifier input. The feedback signal is initially passed through a pre-amplifier before it is connected to the TMD amplifier. The pre-amplifier used is a ZPUL-21 [54].

The amplifiers are powered with a 24V DC power supply and are designed to deliver a maximum output of approximately 300V. Because the amplifier uses charged capacitors to deliver the power, the charge decays exponentially over time; the decay time is approximately $1\mu\text{s}$. Thus in order to give the correct kick to bunch 3, which arrives $\sim 300\text{ns}$ after bunch 1, the feedback gain must be increased by 10-20% for this bunch. Figure 5.6 shows a photograph



Figure 5.4: A photograph of the FONT5 board in its metal casing.

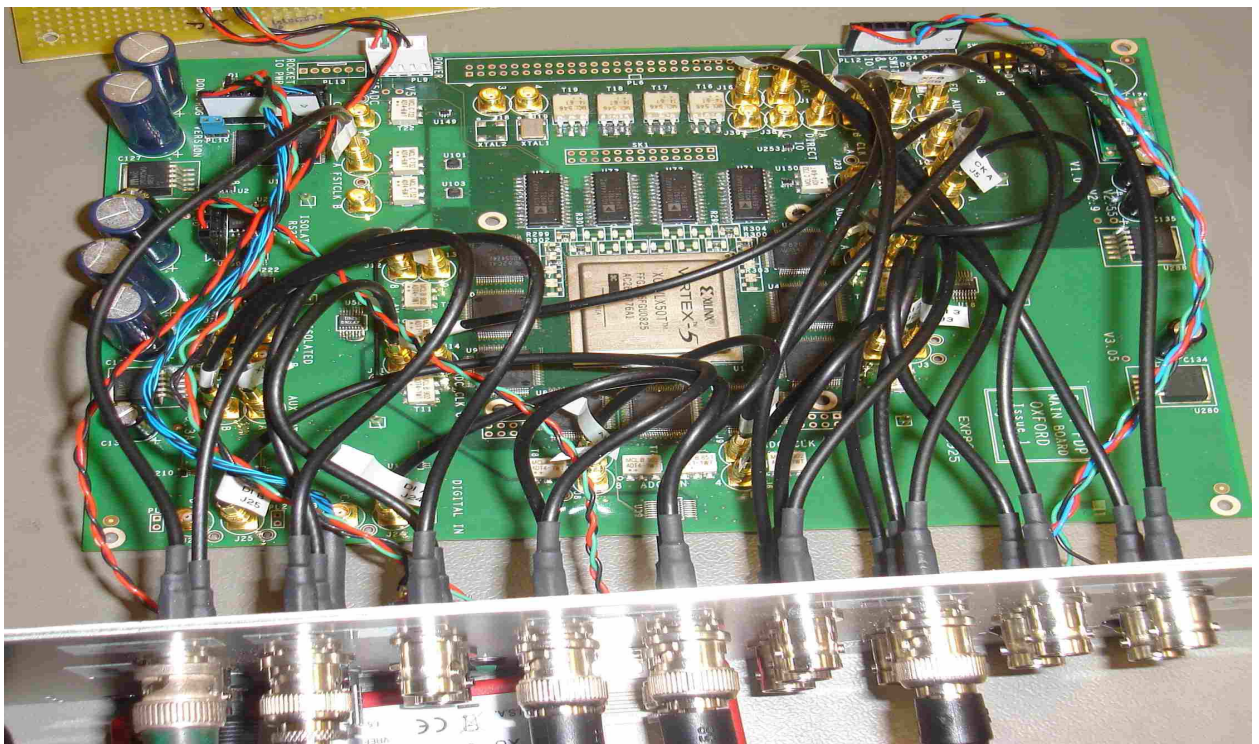


Figure 5.5: A photograph of the internal electronics of the FONT5 board.



Figure 5.6: Photographs of the TMD kicker amplifier.

of the kicker amplifier.

5.1.3 Feedback Kicker

The kickers consist of two metal strips which are charged by the kicker amplifier; the kickers were donated by SLAC [55]. The fill time of the kicker is 3ns and over the linear range of the kicker amplifier, the kickers can give a maximum deflection of $\pm 20\mu\text{Rad}$. Figure 5.7 shows a photograph of the K2 FONT kicker situated at ATF2; Figure 5.8 shows the inside of one of the kickers.

5.1.4 BPM mover System

As described in Chapter 4, BPM movers were installed on the FONT BPMs at ATF2, and control software developed [45]. The movers are controlled remotely with a control system developed in LabView. The movers are used to centre the electron beam in each BPM with a step size of $1\mu\text{m}$.

When using the corrector magnets in the extraction line to alter the beam position in the BPMs, the beam position in all downstream BPMs is also changed. This makes the task of centring the beam difficult and slow. The movers allow the beam to be centred quickly and efficiently.

5.1.5 Serial Server

In order to control the BPM movers and the FONT5 boards remotely, FONT uses serial servers. The serial servers used are quatech QSE-100D [56]. This model has 4 RS-232 ports,

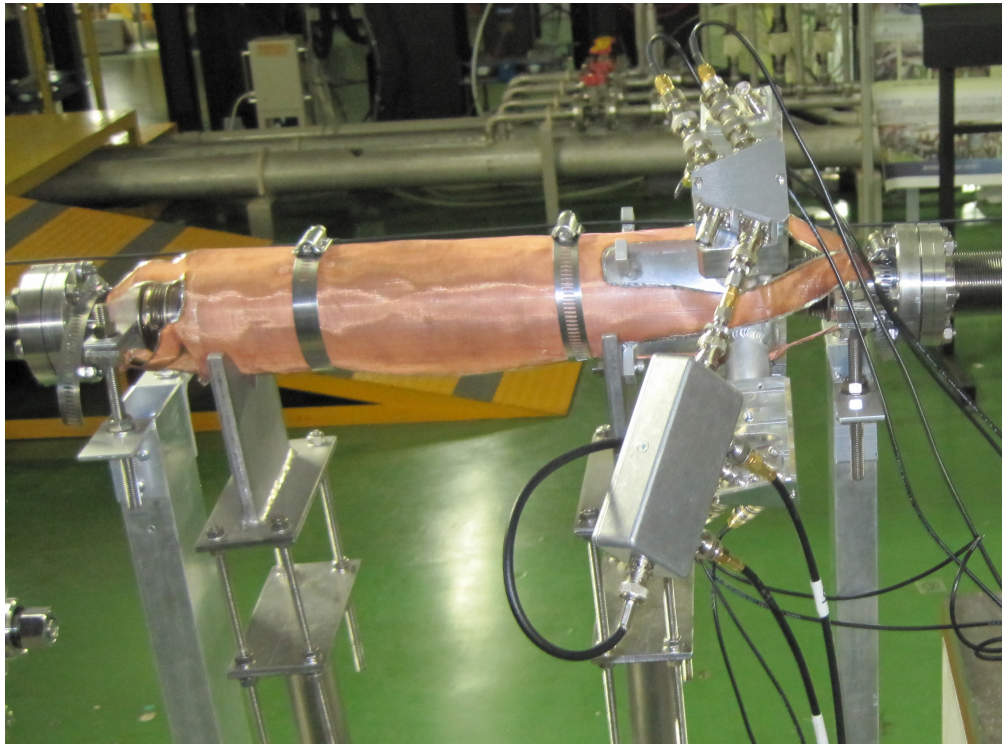


Figure 5.7: A photograph of the K2 FONT kicker.

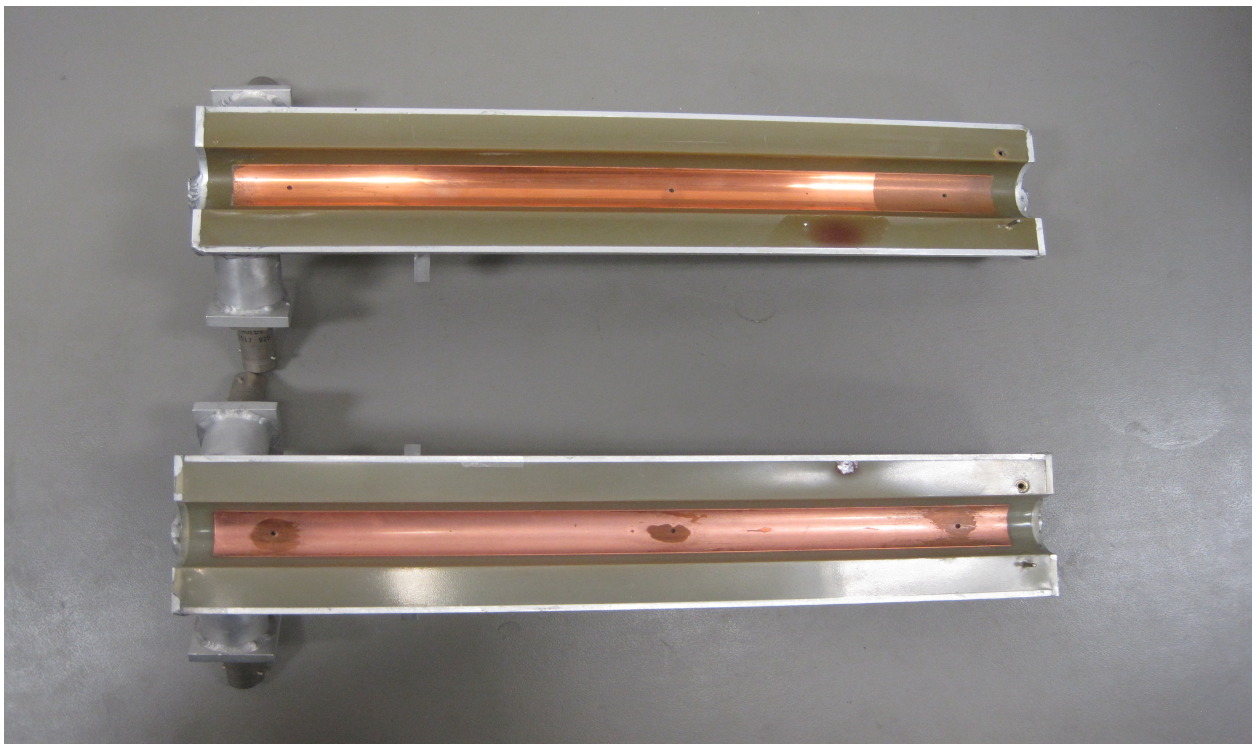


Figure 5.8: A photograph of the inside of one of the FONT kickers.

which can run at a maximum baud rate of 460800 bits per second in the normal operating mode. The data are then transmitted via Ethernet onto a local area network and received by a computer which is also connected.

Currently, the three BPM movers and the FONT5 board are all connected to one serial server in the ATF2 extraction line. In the future, it is planned that FONT will install another serial server and FONT5 board downstream of the FONT region so that the effect of the feedback system can be monitored away from the FONT region.

5.1.6 LO Distribution

At ATF2, the LO signal is initially at 0dBm, this signal is amplified with a low noise amplifier up to ~ 23 dBm. The model of amplifiers used are ZRL-1200 [57]. The amplified LO is then connected to an 8-way splitter; the model used is ZN8PD1-53+ [58]. This reduces the signal level on each output of the splitter to ~ 13 dBm. Each output from the splitter is then connected to a phase shifter, JSPHS-1000+ [59]; this attenuates the LO to ~ 11 dBm. The LO is then further amplified with another ZRL-1200 before being transmitted to the extraction line along approximately 30m of heliax cable (Figure 5.2). The length of cable introduces approximately 3dB of attenuation; when the LO signal reaches the extraction line, the amplitude is typically 24-30dBm. The BPM processors require an LO amplitude of 14dBm, therefore attenuators are connected to the LO cable before reaching the analogue processors.

Phase Shifters

Originally, the phase shifters were controlled with a variable voltage supply. This required the use of variable resistors; once the optimum phase was found, a switch locked the variable resistor in place. The disadvantage with this method is that occasionally the variable resistors would slip, causing the phase to shift slightly. The variable voltage power supplies have been replaced with a analogue output USB bus, DT9854 [60]. This allows the phase shifter voltage to be set with 16-bit resolution. Control software has been developed using Matlab [33] [61].

5.2 FONT Feedback System

5.2.1 Dual Feedback

Eq. 5.1 is the Twiss matrix for a non periodic lattice. α_i and β_i are the Twiss parameters used to describe the beam at point S_i along the beam line; μ is the phase advance between two points S_0 and S_1 . The Twiss matrix represents the change in beam trajectory from a point S_0 to a point S_1 along the beam line. If we consider a case where the phase advance of this transfer matrix, is 90° , then the Twiss matrix is represented by Eq. 5.2.

$$\mathbf{M} = \begin{pmatrix} \sqrt{\frac{\beta_1}{\beta_0}} [\cos \mu + \alpha_0 \sin \mu] & \sqrt{\beta_0 \beta_1} \sin \mu \\ -\frac{1+\alpha_0 \alpha_1}{\sqrt{\beta_0 \beta_1}} \sin \mu + \frac{\alpha_0 - \alpha_1}{\sqrt{\beta_0 \beta_1}} \cos \mu & \sqrt{\frac{\beta_0}{\beta_1}} [\cos \mu - \alpha_1 \sin \mu] \end{pmatrix} \quad (5.1)$$

$$\mathbf{M} = \begin{pmatrix} \alpha_0 \sqrt{\frac{\beta_1}{\beta_0}} & \sqrt{\beta_0 \beta_1} \\ -\frac{1+\alpha_0 \alpha_1}{\sqrt{\beta_0 \beta_1}} & -\alpha_1 \sqrt{\frac{\beta_0}{\beta_1}} \end{pmatrix} \quad (5.2)$$

$$\alpha = -\frac{1}{2} \frac{d\beta}{ds} \quad (5.3)$$

In order for the diagonal elements of the Twiss matrix to be 0, α_0 and α_1 must also be 0. As shown in Eq. 5.3, α can only be 0 if β is at a minimum or a maximum [62], which does not occur for the kickers nor the BPMs are inside the quadrupole magnets, α cannot be 0.

The implications of this are that angle and position are coupled together. This means that in order to perform feedback corrections on both position and angle, the two feedback systems need to be coupled together. If two independent feedback systems were implemented, they would compete against each other and would not minimise the position and angle jitter.

Coupled Feedback System

Eq. 5.4 represents the transfer matrix from kicker i to BPM j . If kicker i applies a kick of $\Delta\theta$, then the change in trajectory at BPM j is shown in Eq. 5.5; the subscript indices identify the element of the matrix.

$$\mathbf{M}_{ij} = \begin{pmatrix} \mathbf{M}_{11}^{ij} & \mathbf{M}_{12}^{ij} \\ \mathbf{M}_{21}^{ij} & \mathbf{M}_{22}^{ij} \end{pmatrix} \quad (5.4)$$

$$\begin{pmatrix} \Delta y_j \\ \Delta y_j' \end{pmatrix} = \begin{pmatrix} \mathbf{M}_{11}^{ij} & \mathbf{M}_{12}^{ij} \\ \mathbf{M}_{21}^{ij} & \mathbf{M}_{22}^{ij} \end{pmatrix} \begin{pmatrix} 0 \\ \Delta\theta \end{pmatrix} = \begin{pmatrix} \mathbf{M}_{12}^{ij} \Delta\theta \\ \mathbf{M}_{22}^{ij} \Delta\theta \end{pmatrix} \quad (5.5)$$

If kicker 1 gives a kick $\Delta\theta_1$ and kicker 2 gives a kick of $\Delta\theta_2$, then Eqs. 5.6 and 5.7 represent the observed positions in BPMs 2 and 3 respectively.

$$y_2 (\text{new}) = y_2 (\text{old}) + \mathbf{M}_{12}^{12} \Delta\theta_1 + \mathbf{M}_{12}^{22} \Delta\theta_2 \quad (5.6)$$

$$y_3 (\text{new}) = y_3 (\text{old}) + \mathbf{M}_{12}^{13} \Delta\theta_1 + \mathbf{M}_{12}^{23} \Delta\theta_2 \quad (5.7)$$

These equations can be solved simultaneously to determine the required kicks from K1 and K2 to zero the beam position in P2 and P3 respectively (Eqs. 5.8 and 5.9).

$$\Delta\theta_1 = \frac{-M_{12}^{23}}{M_{12}^{12} M_{12}^{23} - M_{12}^{22} M_{12}^{13}} y_2 + \frac{M_{12}^{22}}{M_{12}^{12} M_{12}^{23} - M_{12}^{22} M_{12}^{13}} y_3 = G_{12} y_2 + G_{13} y_3 \quad (5.8)$$

		χ^2_{dof}	Constant
P2	Bunch 2	2.99	75.3±0.5
	Bunch 3	3.73	68.3±0.6
P3	Bunch 2	23.9	29.4±1.7
	Bunch 3	23.5	26.5±1.7

Table 5.3: K1 calibration constants and χ^2 per degree of freedom (χ^2_{dof}) for BPMs P2 and P3. The calibration constants are measured in nm per DAC count.

		χ^2_{dof}	Constant
P2	Bunch 2	6.49	21.5±0.8
	Bunch 3	6.66	17.3±0.8
P3	Bunch 2	13.2	86.3±1.5
	Bunch 3	20.5	69.0±1.7

Table 5.4: K2 calibration constants and χ^2 per degree of freedom (χ^2_{dof}) for BPMs P2 and P3. The calibration constants are measured in nm per DAC count.

$$\Delta\theta_2 = \frac{M_{12}^{13}}{M_{12}^{12}M_{12}^{23} - M_{12}^{22}M_{12}^{13}}y_2 + \frac{-M_{12}^{12}}{M_{12}^{12}M_{12}^{23} - M_{12}^{22}M_{12}^{13}}y_3 = G_{22}y_2 + G_{23}y_3 \quad (5.9)$$

The four coefficients G_{ij} are the coupling coefficients for the two coupled feedback systems. Another method for determining the coupling of the two feedback systems was developed. The difference between the methods is that rather than using transfer matrices to calculate the coupling, the kickers are calibrated with respect to the BPMs [27]. The method involving kicker calibrations has the advantages that no knowledge about the beam optics is required to calculate the gains and the gains are calibrated for the kicker amplifiers. The method involving transfer matrices assumes that the beam optics are well known and the gains are given in units of $\mu\text{m.rad}^{-1}$, therefore calibration factors are required to convert the gains into the correct units. Gains are calculated using both methods to ensure that the gains are as expected.

5.2.2 Kicker Calibration

In order to do the kicker calibration, the DAC output is set to a constant value. This will cause the kicker to deliver a constant kick. The amplitude of the kick is varied across the entire range of the DAC output. The kicker is only fired on alternate bunch trains. This is done so that the unkick position can be subtracted from the kicked position, giving an estimate of the deflection. This also removes any low frequency position drift. The uncalibrated kick (charge normalised position) is plotted against the DAC output (in digital counts), and a line is fitted to the data (Figures 5.9 and 5.10); the calibration results are shown in Tables 5.3 and 5.4.

In a single feedback loop, the feedback gain is the reciprocal of the gradient of the fitted line.

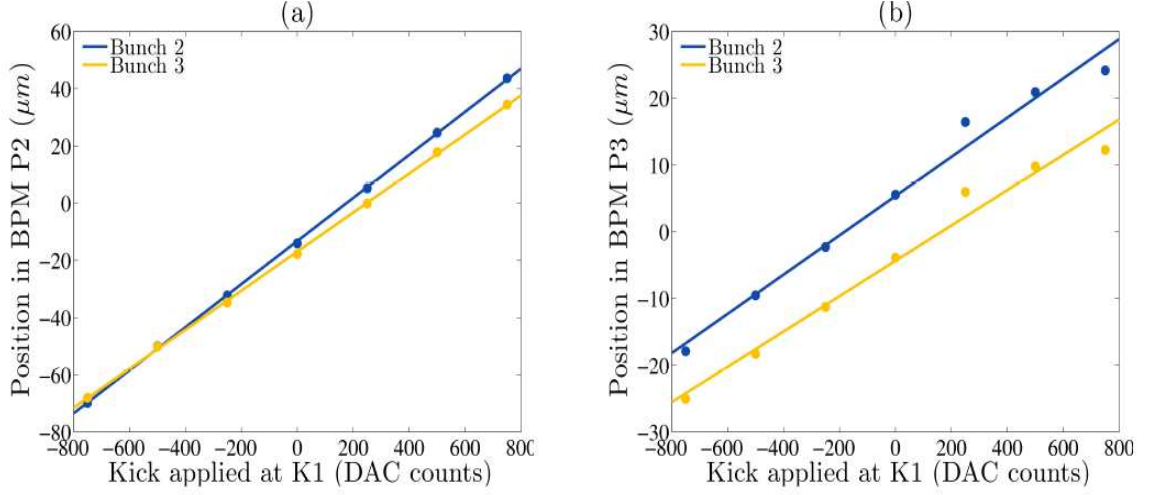


Figure 5.9: Positions of bunches 2 (blue) and 3 (orange) (μm) vs. applied kick (DAC counts). Kicker calibration for K1 observed in P2 (a) and P3 (b). The points represent the data and the lines are a linear fit [27].

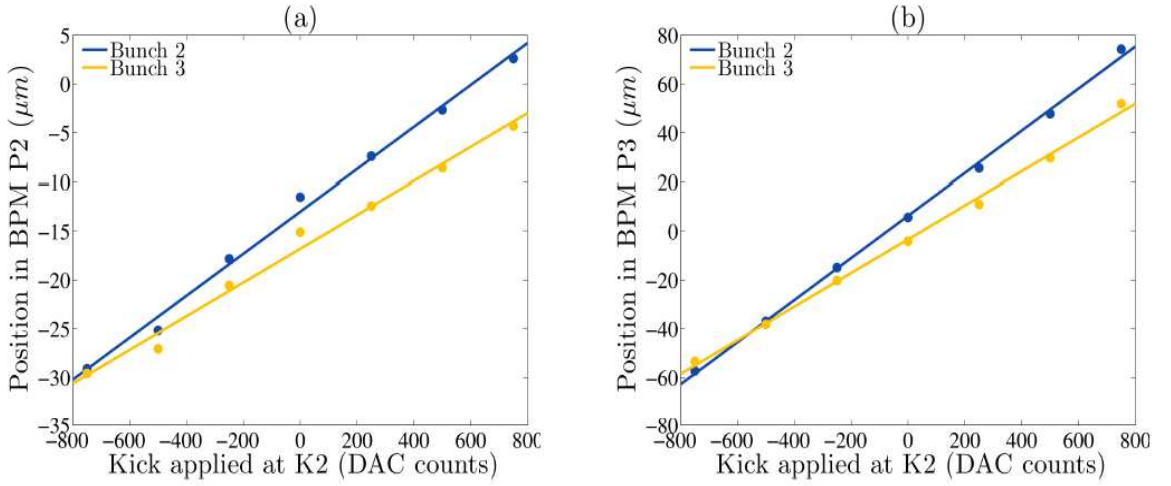


Figure 5.10: Positions of bunches 2 (blue) and 3 (orange) (μm) vs. applied kick (DAC counts). Kicker calibration for K2 observed in P2 (a) and P3 (b). The points represent the data and the lines are a linear fit [27].

In a coupled feedback system, the four feedback gains are defined by Eq. 5.10. G_{ij} represent the feedback gains, and g_{ij} represent the gradients of the lines fitted to the data.

$$\begin{pmatrix} G_{12} & G_{13} \\ G_{22} & G_{23} \end{pmatrix} = \begin{pmatrix} g_{12} & g_{13} \\ g_{22} & g_{23} \end{pmatrix}^{-1} \quad (5.10)$$

5.2.3 Latency Measurements

In order to be able to perform bunch to bunch corrections, the feedback system must be able to calculate the correction and drive the kickers before the next bunch arrives. The latency of the feedback system can be separated into two types, reducible and irreducible latency. All the feedback electronics are considered reducible because they can be changed or modified to reduce their overall latency. The time of flight of the beam through the FONT region and the time taken by signals to propagate are considered irreducible.

In the coupled feedback system, there are four latency paths to consider. These are the latencies for the two feedback BPMs to the two kickers. The critical latency will always be the longest latency path, which is the latency from P3 to K1. If the latency of this path is less than the bunch spacing, then all other paths will also be less than the bunch spacing.

Originally, the cables used for the feedback system were mostly RG58, which had two important disadvantages. First, the signal propagation speed of RG58 is typically $\sim 0.66c$, and not fast enough for the P3 to K1 loop. The other disadvantage is that RG58 has shielding from external noise of $\sim -30\text{dB}$. It was decided that the cables on the critical feedback path should be replaced with LMR-200. This has a propagation speed of $\sim 0.83c$, which reduced the latency on the P3 to K1 path by approximately 10ns. The shielding from external noise for LMR-200 is $\sim -90\text{dB}$, which significantly reduces the noise picked up in the cables. The latency was further reduced by approximately 10ns due to modifications to the firmware on the FONT5 board [27]. Table 5.5 shows the measured latency after the changes to the feedback system for each of the critical latency paths [63]. The measured latency is defined as the time taken for the feedback system to provide 90% of its total kick. The time difference between the 90% point, $t_{90\%}$, and the following bunch is known as the timing slack, t_{slack} . The system latency, t_{lat} is defined as the bunch spacing, t_{bunch} , minus the timing slack (Eq. 5.11).

$$t_{lat} = t_{bunch} - t_{slack} \quad (5.11)$$

Table 5.6 gives latency estimations for each component in the feedback system. In coupled feedback mode, the P3 signals are processed on a low-latency path; this is done so that the coupled feedback system does not exceed the latency budget. This makes the latency of the FONT5 board 9.8ns (3.5 clock cycles of 357MHz) shorter for P3 than P2, but only in coupled feedback mode. In uncoupled feedback mode, the P3 signals are processed on the normal latency path; therefore P2 and P3 signals have the same latency. This low-latency path only affects the K1-P3 path.

Latency Path	Measured	Estimated
P2-K1	132.6ns $\pm$ 7.8ns	131
P3-K1	141.9ns $\pm$ 7.7ns	140
P3-K2	129.3ns $\pm$ 3.3ns	131
P2-K2	110.2ns $\pm$ 9.3ns*	112

Table 5.5: Comparison between the measured and estimated latency for the critical feedback paths. P2-K2 latency has been measured indirectly.

Component	Estimated Latency (ns)
Reducible Latency	
Analogue Processor	10
FONT5 Board (normal/low latency)	39/29
Kicker Amplifier	35
Kicker Fill Time	3
Total Reducible Latency	97/87
Irreducible Latency	
P2-K1 (Cables + Beam TOF)	44
P3-K1 (Cables + Beam TOF)	63
P3-K2 (Cables + Beam TOF)	44

Table 5.6: The estimated latency for the components in the FONT feedback system.

Figure 5.11, 5.12 and 5.13 show the latency scans for the K1-P2, K2-P3 and K1-P3 feedback loops respectively. The latency of the K2-P2 loop is not measured because it is the shortest feedback path and can be calculated implicitly from the other latencies (Eq. 5.12). t_{K2-P2} , t_{K1-P2} , t_{K2-P3} and t_{K1-P3} are the latencies of the 4 coupled feedback loops; the additional 9.8ns is to account for the low-latency path used in the K1-P3 loop.

$$t_{K2-P2} = t_{K1-P2} + t_{K2-P3} - t_{K1-P3} - 9.8ns \quad (5.12)$$

5.2.4 Factors Affecting Feedback Performance

There are several factors that affect the performance of the FONT feedback system. The most fundamental factor being BPM resolution. For a single-loop feedback system, neglecting all other factors, the optimum correction observed in the feedback BPM will be equal to the BPM resolution. In a coupled feedback system, the best achievable correction is dependent on the coupling coefficients, and therefore the beam optics. Eqs. 5.13 and 5.14 show the best possible correction in P2 and P3 respectively ρ_i is the bunch-bunch correlation in BPM i .

$$\sigma_{P2} = \sqrt{2(1 - \rho_{P2}^2)}\sigma_{res} \quad (5.13)$$

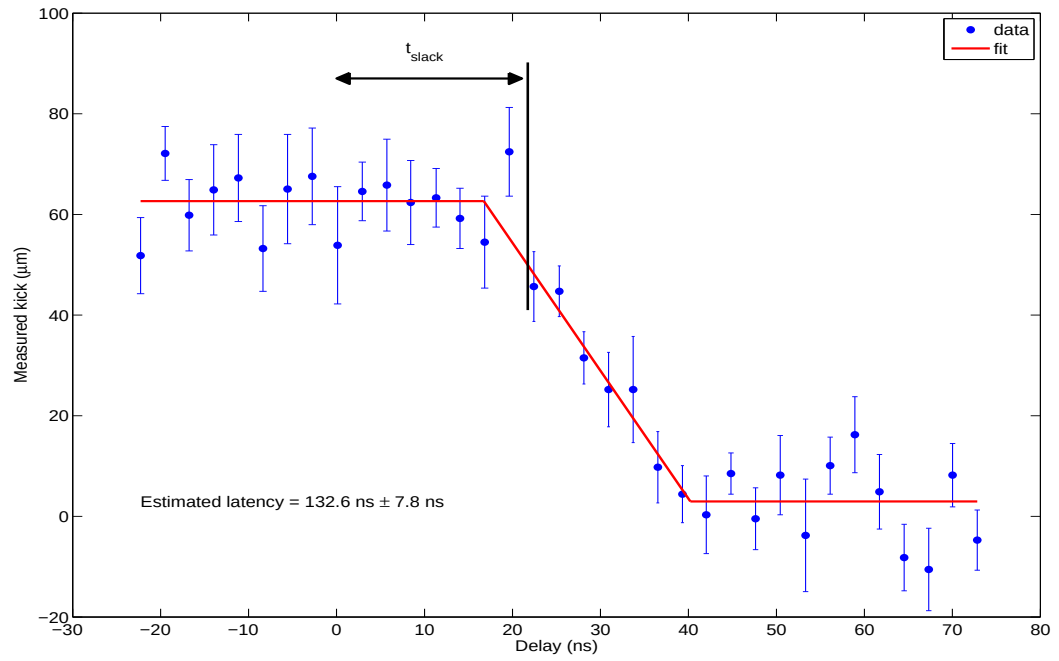


Figure 5.11: Measured kick vs. delay for the K1-P2 feedback loop at ATF2. The blue points are the measurements, the red line is a fit to the data.

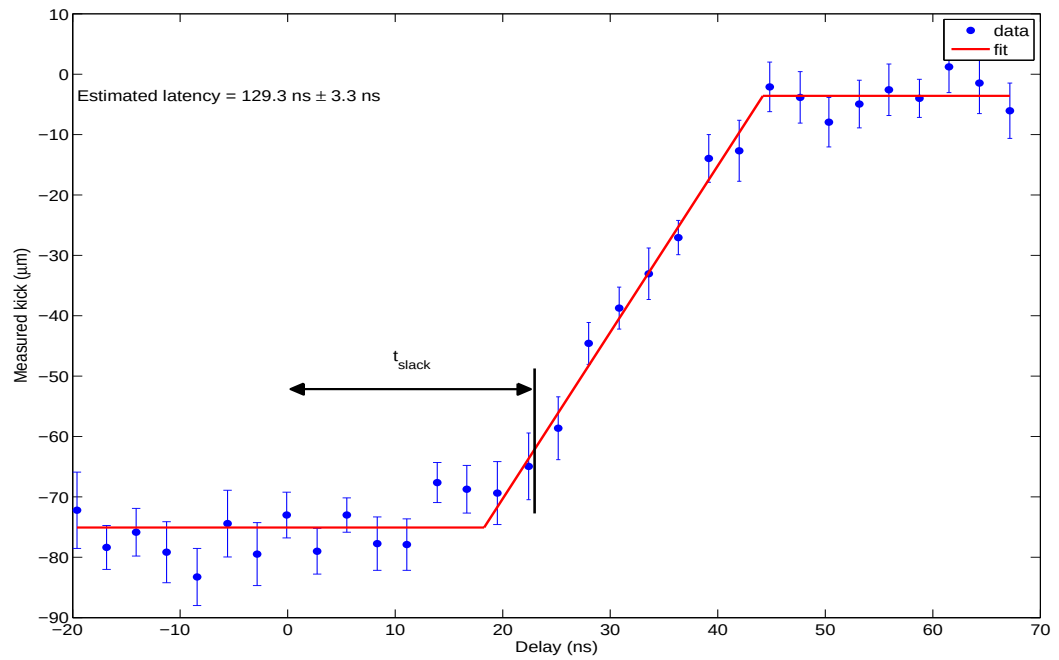


Figure 5.12: Measured kick vs. delay for the K2-P3 feedback loop at ATF2. The blue points are the measurements, the red line is a fit to the data.

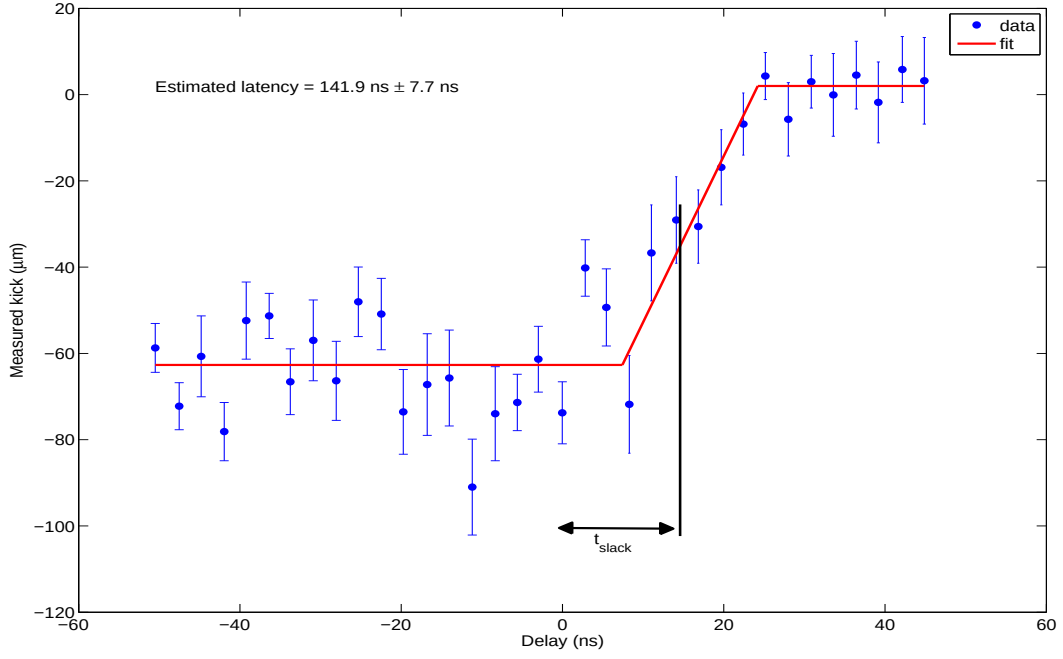


Figure 5.13: Measured kick vs. delay for the K1-P3 feedback loop at ATF2. The blue points are the measurements, the red line is a fit to the data.

$$\sigma_{P3} = \sqrt{2(1 - \rho_{P3}^2)}\sigma_{res} \quad (5.14)$$

This shows the importance of minimising the BPM resolution. The factors affecting the resolution were discussed in Chapter 4.

Another important element is the bunch to bunch correlation. If the bunch to bunch correlation is low, then bunch 1 does not give an accurate prediction for the position of bunch 2. This results in a poor feedback correction. If the bunch to bunch correlation falls below 50% then the FONT feedback system will cause the beam jitter to grow rather than decrease. It is suspected that the damping ring extraction may cause a reduction in bunch to bunch correlation.

5.3 Feedback Results

5.3.1 Single Loop Feedback

Figure 5.14 shows histograms for the operation of the K1 to P2 feedback loop taken in April 2010; the blue histograms are the uncorrected position, while the red histograms show the feedback correction. Figure 5.16 shows the same histograms but for the K2 to P3 loop [27]. Figure 5.15 shows the histograms for the K1-P2 loop corrections observed in P3, while Figure

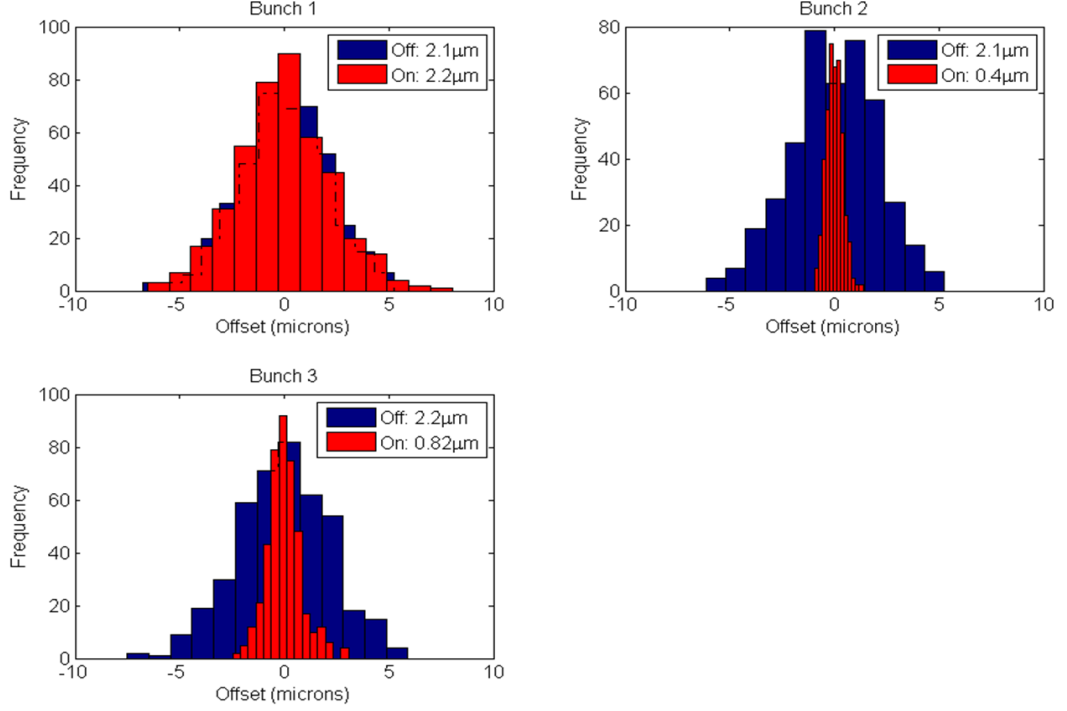


Figure 5.14: Distributions of corrected (red) and uncorrected (blue) beam positions observed in P2 for the operation of the K1-P2 feedback loop [27].

5.17 shows the histograms for the K2-P3 loop corrections observed in P2. These plots show that with the single loop feedback, the position jitter can only be corrected in one BPM; this is because the angle jitter is not also corrected and this becomes position jitter further downstream.

By using Eq. 1.9 from Chapter 1, the optimum corrected beam position, σ_{uncorr} , is shown in Eq. 5.15; ρ_{corr} is the correlation between two consecutive bunches with the feedback system off; σ_{corr} represents the beam jitter. This equation can be used to calculate the maximum corrections for the feedback loops and these results are shown in Table 5.7. This table clearly shows that the single loop feedback system is able to achieve corrections very close to the best possible correction.

$$\sigma_{uncorr} = \sigma_{corr} \sqrt{\frac{1 - \rho_{corr}}{\rho_{corr}}} \quad (5.15)$$

Figures 5.18 and 5.19 show how the performance of the K1-P2 and K2-P3 feedback loops vary with feedback gain [47]. The blue points and line represent the positions with the feedback system running; the feedback position is linearly dependent on gain as expected; where the blue line crosses the x axis represents the optimum feedback gain.

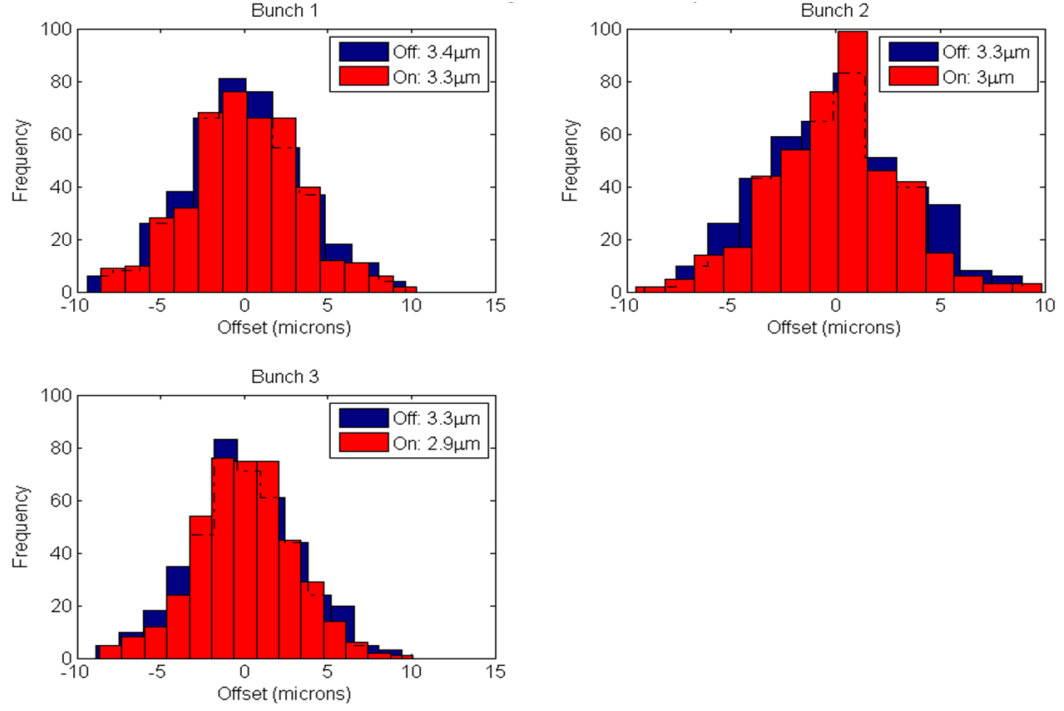


Figure 5.15: Distributions of corrected (red) and uncorrected (blue) beam positions observed in P3 for the operation of the K1-P2 feedback loop [27].

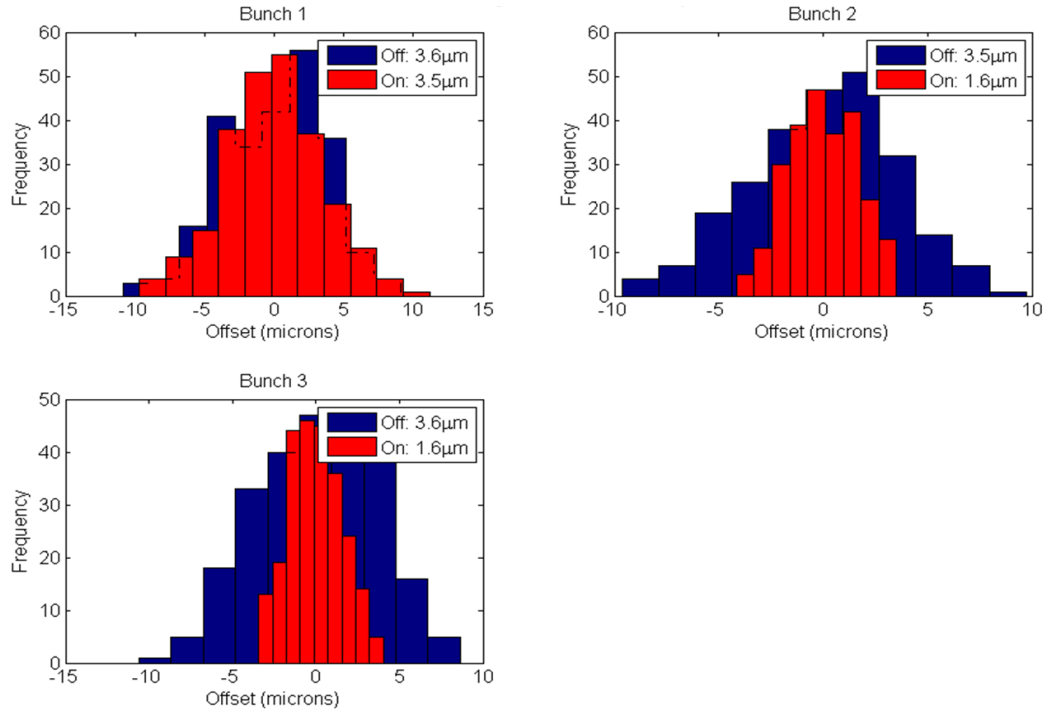


Figure 5.16: Distributions of corrected (red) and uncorrected (blue) beam positions observed in P3 for the operation of the K2-P3 feedback loop [27].

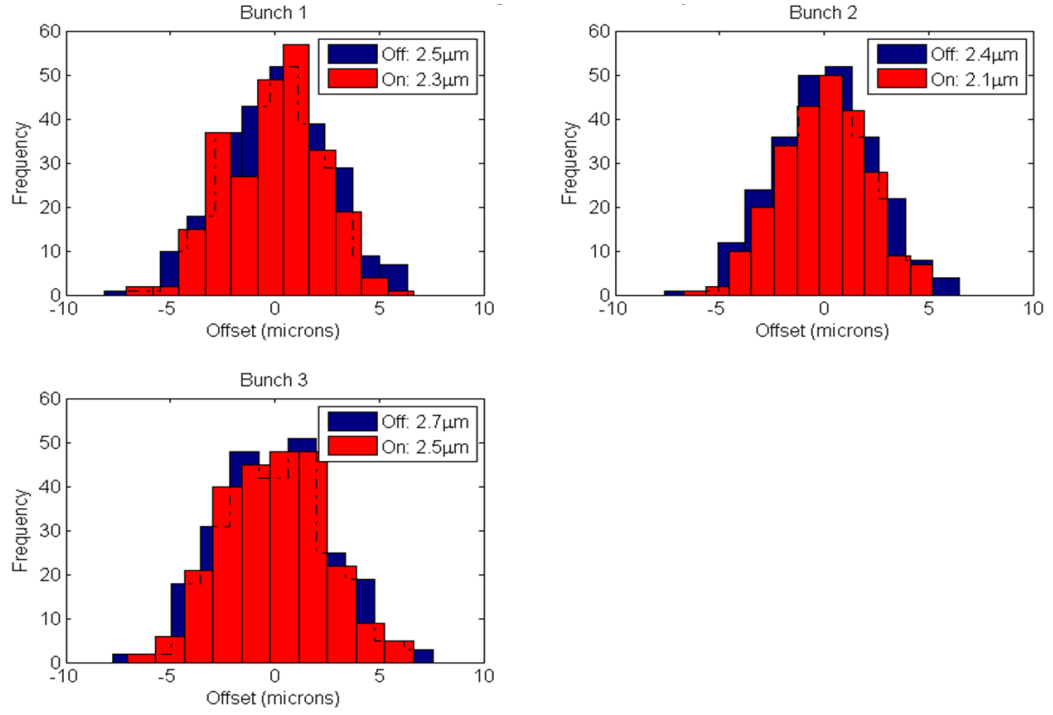


Figure 5.17: Distributions of corrected (red) and uncorrected (blue) beam positions observed in P2 for the operation of the K2-P3 feedback loop [27].

Feedback Loop	Bunch-Bunch Correlation	Optimal Correction	Measured Correction
K1-P2 Bunch 2	98%	$0.30\mu\text{m}$	$0.40\mu\text{m}$
K1-P2 Bunch 3	90%	$0.73\mu\text{m}$	$0.82\mu\text{m}$
K2-P3 Bunch 2	87%	$1.35\mu\text{m}$	$1.60\mu\text{m}$
K2-P3 Bunch 3	92%	$1.06\mu\text{m}$	$1.60\mu\text{m}$

Table 5.7: Table showing a comparison between the optimum and measured feedback corrections for single loop feedback at ATF2.

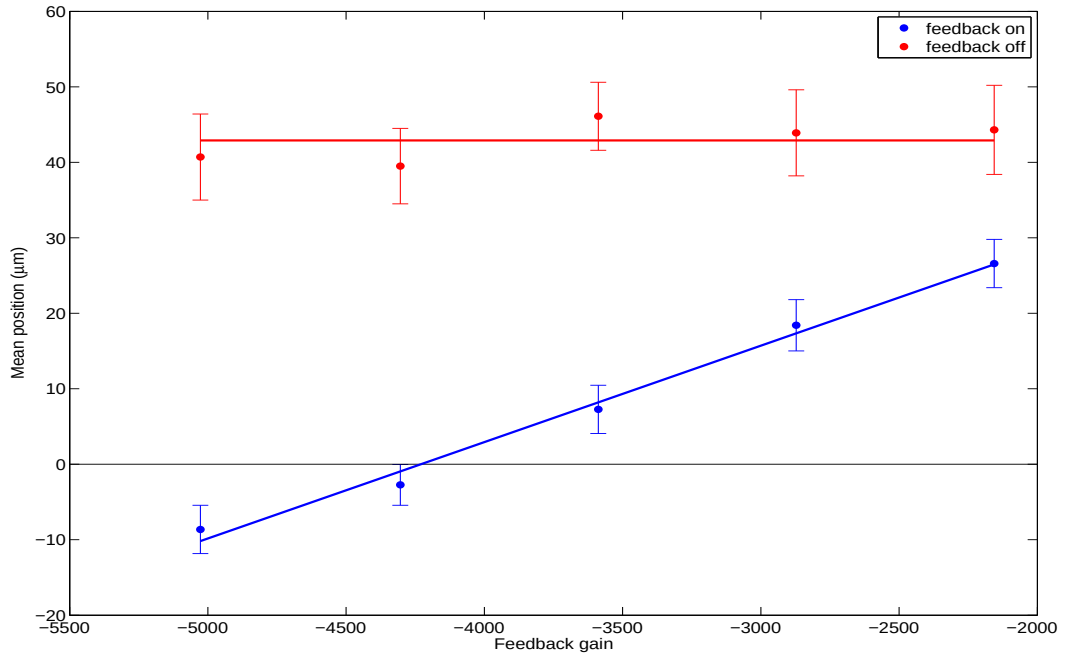


Figure 5.18: Mean beam position vs. gain for the K1-P2 feedback loop for bunch 2 in P2. The blue points are the ‘feedback on’ data and the red points are ‘feedback off’ data; the blue and red lines are fits to the feedback on and off data respectively.

5.3.2 Uncoupled Dual Feedback

In the uncoupled dual feedback mode, K1 and K2 are simultaneously used to correct the beam positions in P2 and P3 respectively. However K1 and K2 alter the beam position in both P2 and P3, therefore the two feedback loops effectively compete against each other. Hence it is not possible to achieve the optimal feedback correction; these results are shown in Table 5.8. It is clear that the feedback corrections are significantly larger than the optimal correction. Figure 5.20 shows histograms for the uncoupled feedback corrections in P2. Figure 5.21 shows the histograms for the uncoupled feedback corrections in P3.

Figures 5.22 and 5.23 show the K1 gain scans observed in P2 for bunches 2 and 3 respectively for the uncoupled feedback system. Figures 5.24 and 5.25 show the K2 gain scans observed

Feedback BPM	Bunch-Bunch Correlation	Optimal Correction	Measured Correction
P2 Bunch 2	98%	$0.30\mu\text{m}$	$0.91\mu\text{m}$
P2 Bunch 3	86%	$0.89\mu\text{m}$	$1.20\mu\text{m}$
P3 Bunch 2	84%	$1.40\mu\text{m}$	$2.00\mu\text{m}$
P3 Bunch 3	91%	$1.01\mu\text{m}$	$1.80\mu\text{m}$

Table 5.8: A comparison between the optimum and measured feedback corrections for uncoupled dual feedback at ATF2.

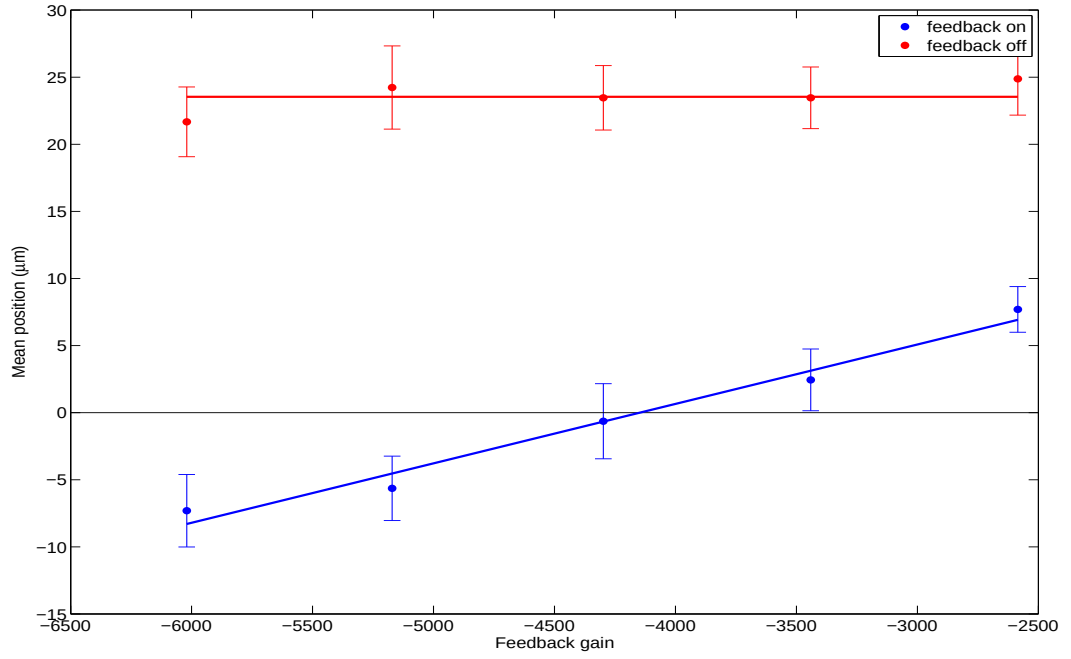


Figure 5.19: Mean beam position vs. gain for the K2-P3 feedback loop for bunch 2 in P3. The blue points are the ‘feedback on’ data and the red points are ‘feedback off’ data; the blue and red lines are fits to the feedback on and off data respectively.

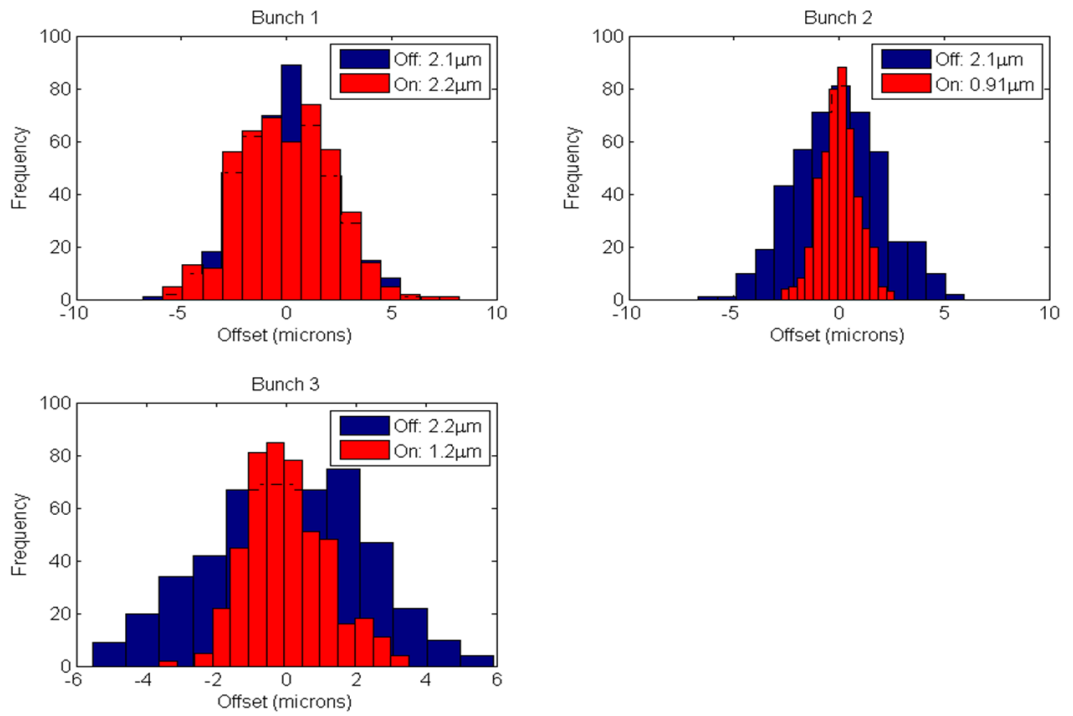


Figure 5.20: Distributions of corrected (red) and uncorrected (blue) beam positions observed in P2 for the operation of the uncoupled feedback system [27].

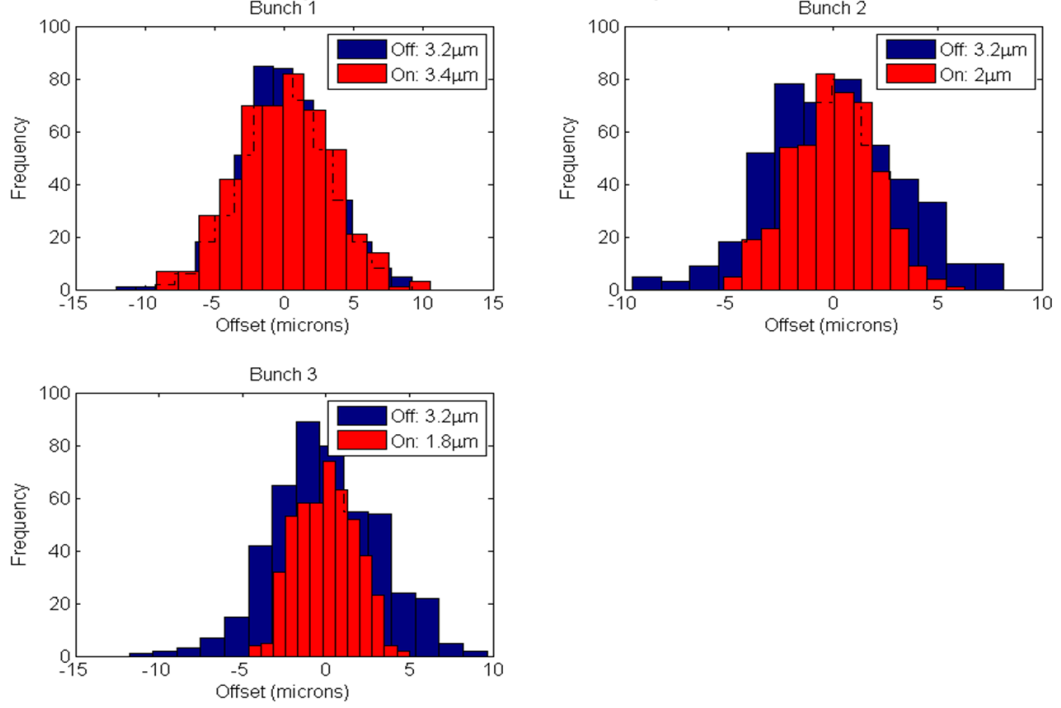


Figure 5.21: Distributions of corrected (red) and uncorrected (blue) beam positions observed in P3 for the operation of the uncoupled feedback system [27].

in P3 for bunches 2 and 3 respectively. Figure 5.24 shows that there is very little correction in P3 for bunch 2; this is because the K1 and K2 feedback loops compete against each other, adversely affecting the P3 correction. For bunch 3, the additional P2 correction from K1 is relatively small, therefore the K2 loop can greatly improve the P3 correction (Figure 5.25).

As shown in Eqs. 5.16 and 5.17, the gain scans for bunch 2 should vary linearly with gain, while the gain scans for bunch 3 should vary quadratically with gain. $y_{i,j}^c$ and $y_{i,j}^u$ represent the corrected and uncorrected bunch positions for the i^{th} bunch respectively observed in the j^{th} BPM. G_j represent the sum of the gains for BPM j , $(G_{12} + G_{22})$ and $(G_{13} + G_{23})$ respectively. Therefore the gain scans for bunch 2 should vary linearly with gain (Figures 5.22 and 5.24), while the bunch 3 gain scans should vary quadratically (Figures 5.23 and 5.25). As expected, the fit coefficients from Eq. 5.16 are consistent with the fit coefficients from Eq. 5.17; the errors on the fit are $\frac{\sigma(y_{i,j}^u)}{\sqrt{n}}$.

$$y_{2,j}^c = y_{2,j}^u - y_{1,j}^u G_j \quad (5.16)$$

$$y_{3,j}^c = y_{3,j}^u - y_{2,j}^u G_j + y_{1,j}^u G_j^2 \quad (5.17)$$

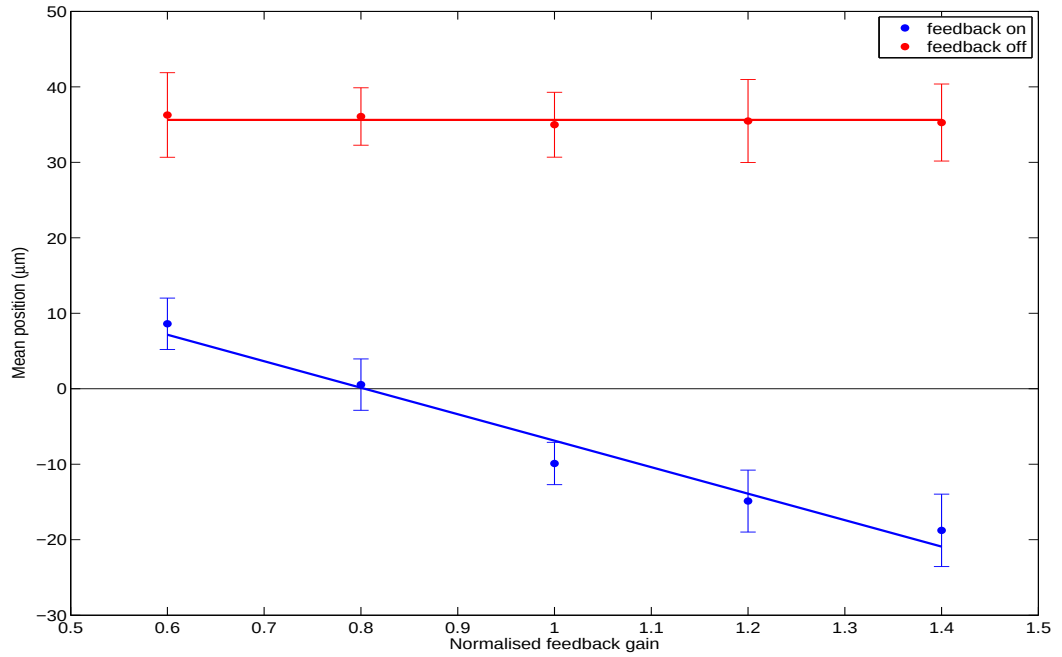


Figure 5.22: Position vs. gain for the K1 loop for bunch 2 observed in P2 in the uncoupled feedback system. The blue points are the ‘feedback on’ data and the red points are ‘feedback off’ data; the blue and red lines are fits to the feedback on and off data respectively.

5.3.3 Coupled Dual Feedback

In the coupled dual feedback mode, K1 and K2 contribute to the corrections in P2 and P3; the two do not compete against each other. Figure 5.26 shows histograms for the coupled feedback corrections in P2, taken in April 2010. Figure 5.27 shows the histograms for the coupled feedback corrections in P3. It is clear that the coupled feedback system is able to produce better corrections to the beam jitter than the uncoupled system. The single loop feedback is capable of achieving the same quality correction as the coupled feedback system, but only in one BPM; the coupled system is able to maintain the beam quality over an extended distance. Table 5.9 shows the comparison between the optimum feedback correction and the measured feedback for the coupled feedback system.

Feedback BPM	Bunch-Bunch Correlation	Optimal Correction	Measured Correction
P2 Bunch 2	98%	$0.33\mu\text{m}$	$0.40\mu\text{m}$
P2 Bunch 3	85%	$1.05\mu\text{m}$	$1.10\mu\text{m}$
P3 Bunch 2	84%	$1.44\mu\text{m}$	$1.80\mu\text{m}$
P3 Bunch 3	92%	$0.97\mu\text{m}$	$1.60\mu\text{m}$

Table 5.9: A comparison between the Optimum and measured feedback corrections for coupled dual feedback at ATF2.

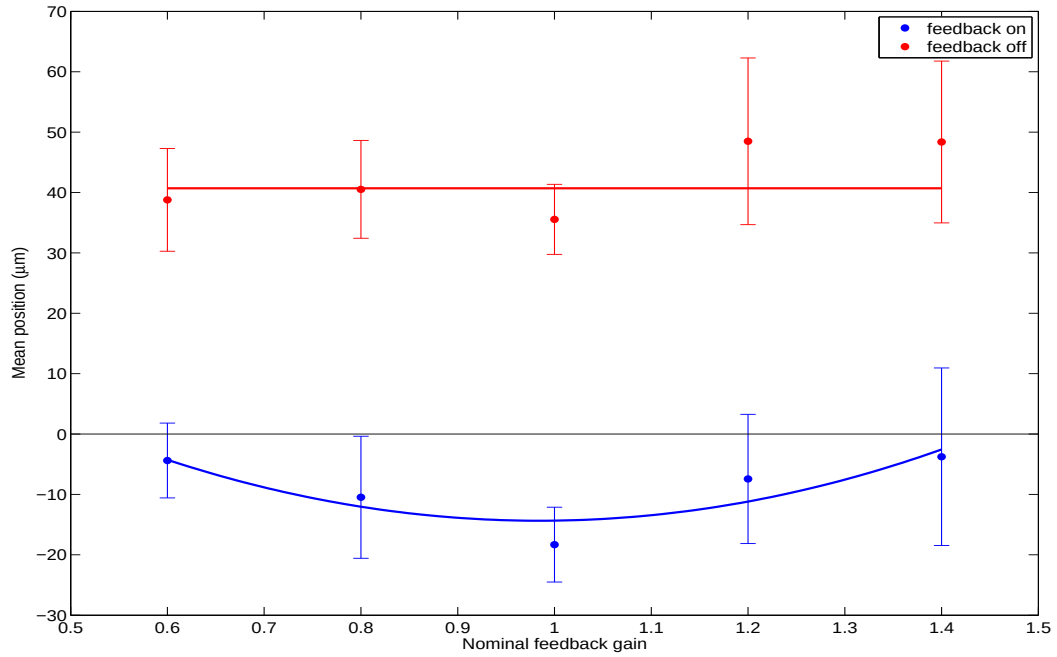


Figure 5.23: Position vs. gain for the K1 loop for bunch 3 observed in P2 in the uncoupled feedback system. The blue points are the ‘feedback on’ data and the red points are ‘feedback off’ data; the blue and red lines are fits to the feedback on and off data respectively.

Figures 5.28 and 5.29 show a gain scan of K1 in the coupled feedback system for bunches 2 and 3 respectively. Figures 5.30 and 5.31 show a gain scan of K2 in the coupled feedback system for bunches 2 and 3 respectively.

5.4 Summary

The FONT feedback system is used to correct correlated beam jitter in the ATF2 extraction line. In order to correct the beam position at one point in the extraction line, a single loop feedback system is required; this consists of one BPM monitoring the beam position, and one kicker to correct the position upstream of the BPM. One witness BPM is also used to monitor the beam jitter with no feedback correction applied.

In order to correct both position and angle jitter in the extraction line a dual feedback system is necessary. This requires two feedback BPMs and two kickers to correct the beam. Although the BPMs and kickers are designed to have phase advances of 90° between them, the actual phase advances are rather different. Even in the ideal case where the phase advances were 90° , position would not generally convert directly into angle and vice versa. In order to account for this, the dual feedback system must be coupled together.

The FONT feedback system has successfully demonstrated consistent submicron beam jitter at the entrance of the final focus system; this is the primary goal for the feedback system.

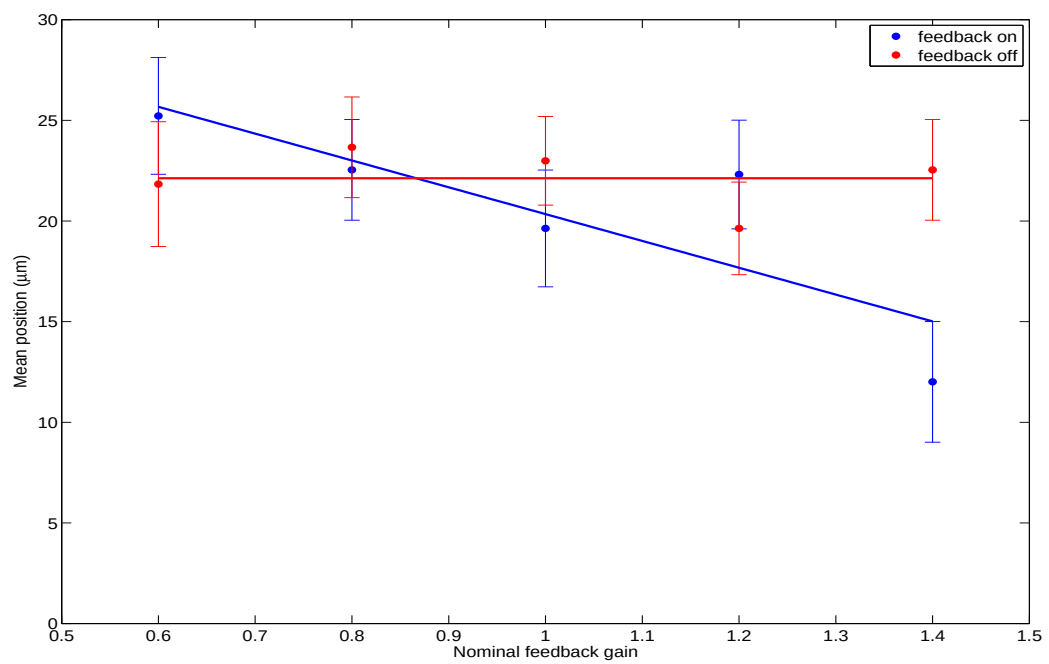


Figure 5.24: Position vs. gain for the K2 loop for bunch 2 observed in P3 in the uncoupled feedback system. The blue points are the ‘feedback on’ data and the red points are ‘feedback off’ data; the blue and red lines are fits to the feedback on and off data respectively.

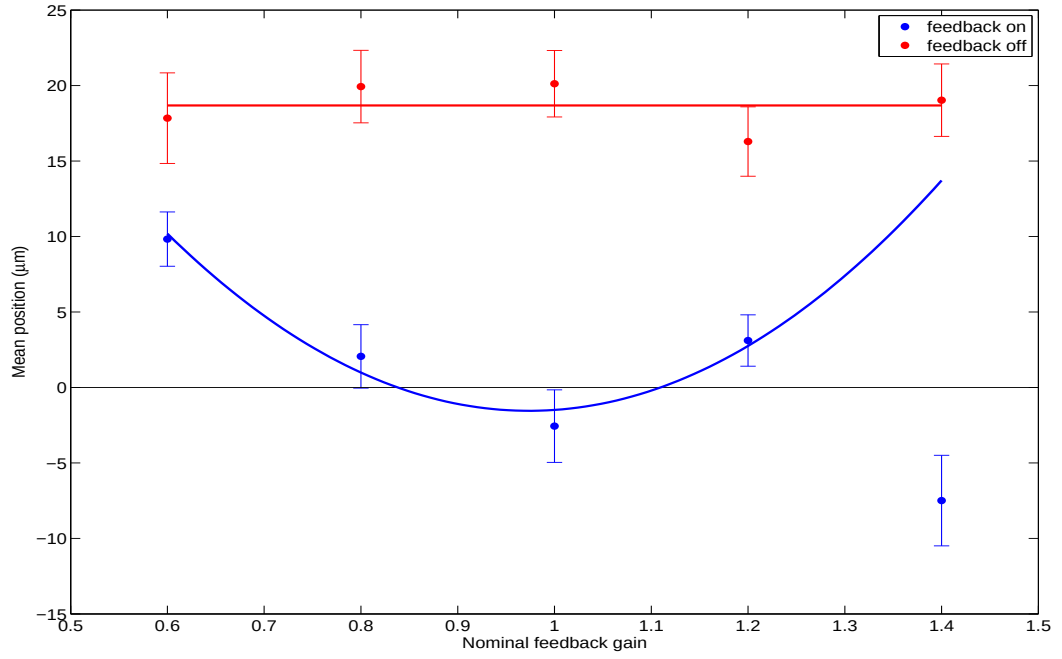


Figure 5.25: Position vs. gain for the K2 loop for bunch 3 observed in P3 in the uncoupled feedback system. The blue points are the ‘feedback on’ data and the red points are ‘feedback off’ data; the blue and red lines are fits to the feedback on and off data respectively.

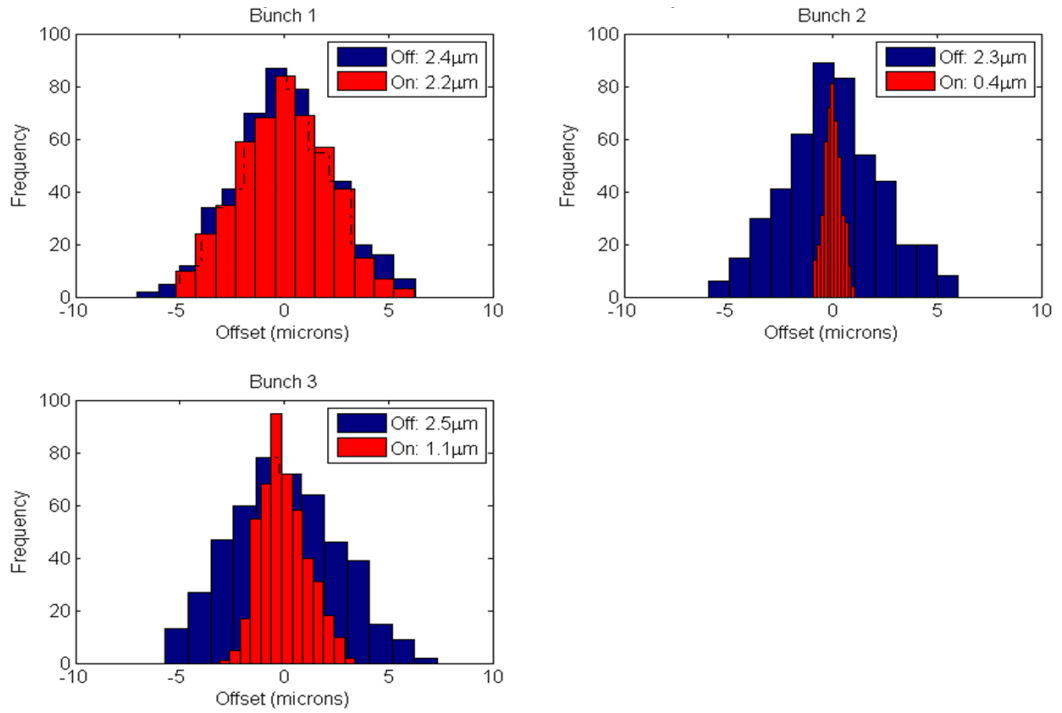


Figure 5.26: Distributions of corrected (red) and uncorrected (blue) beam positions observed in P2 for the operation of the coupled feedback system [27].

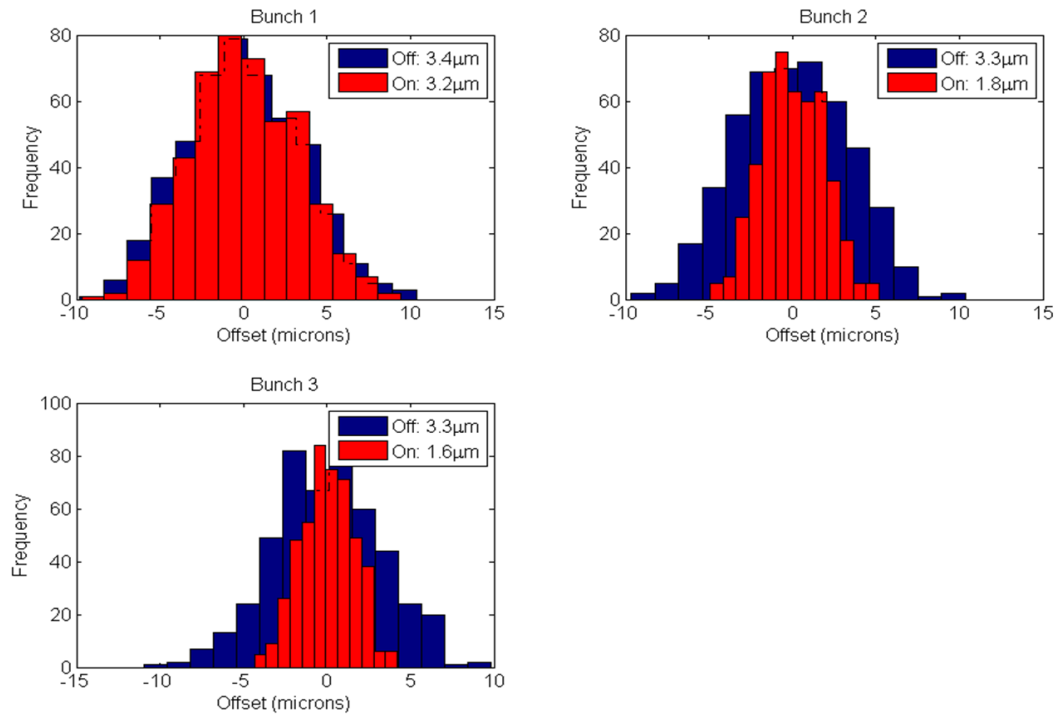


Figure 5.27: Distributions of corrected (red) and uncorrected (blue) beam positions observed in P3 for the operation of the coupled feedback system [27].

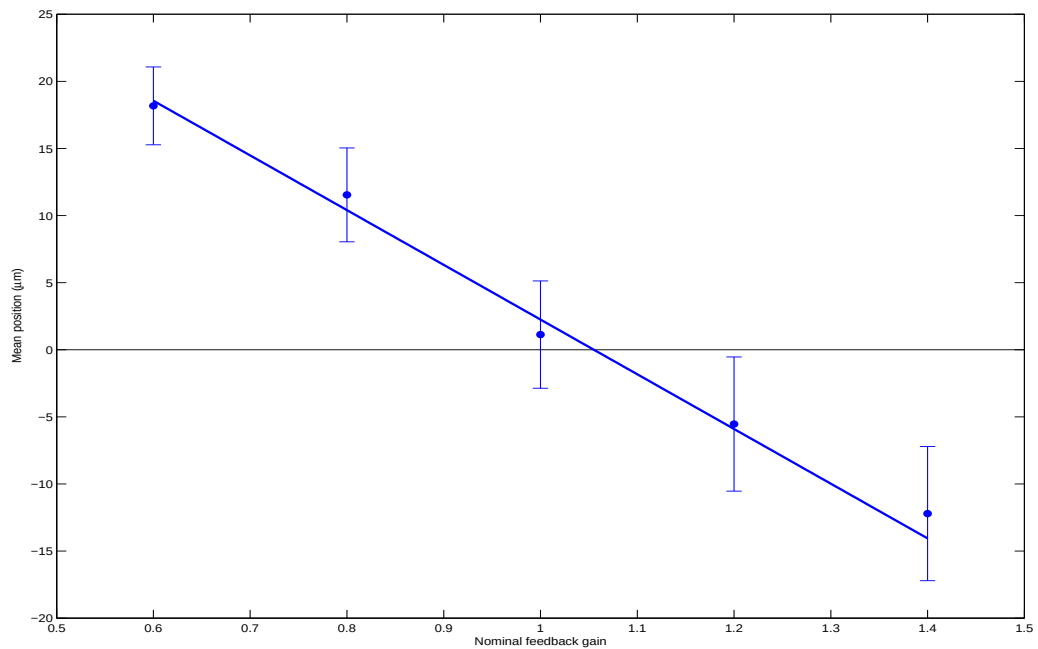


Figure 5.28: Position vs. gain for the K1 loop for bunch 2 in the coupled feedback system observed in P2. The points represent the measurements and the line is a fit to the data.

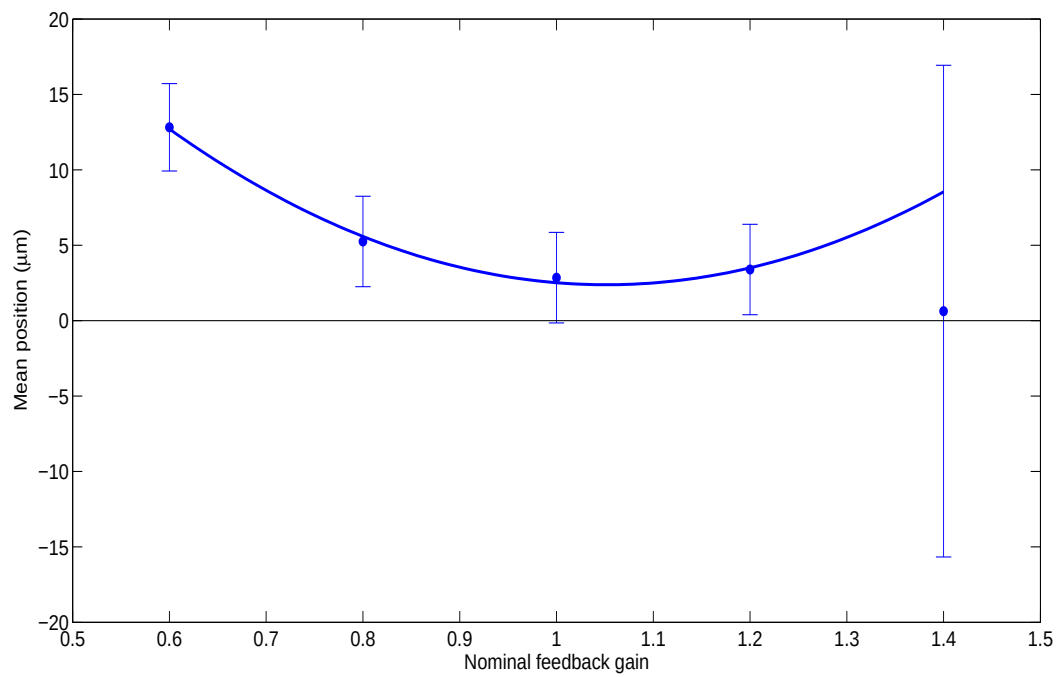


Figure 5.29: Position vs. gain for the K1 loop for bunch 3 in the coupled feedback system observed in P2. The points represent the measurements and the line is a quadratic fit to the data.

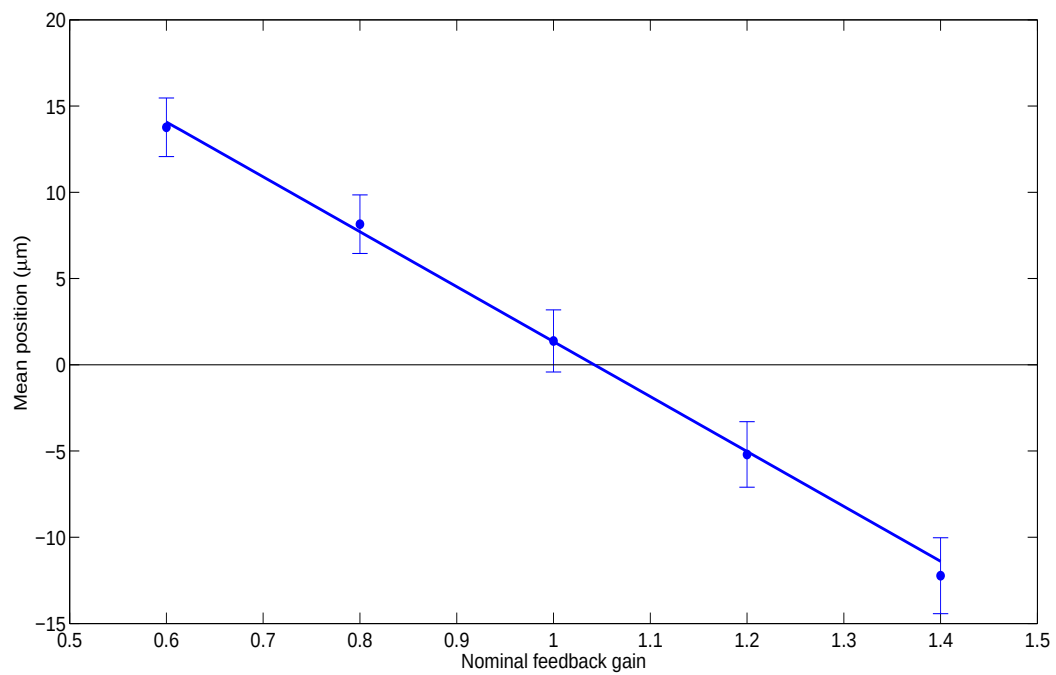


Figure 5.30: Position vs. gain for the K2 loop for bunch 2 in the coupled feedback system observed in P3. The points represent the measurements and the line is a fit to the data.

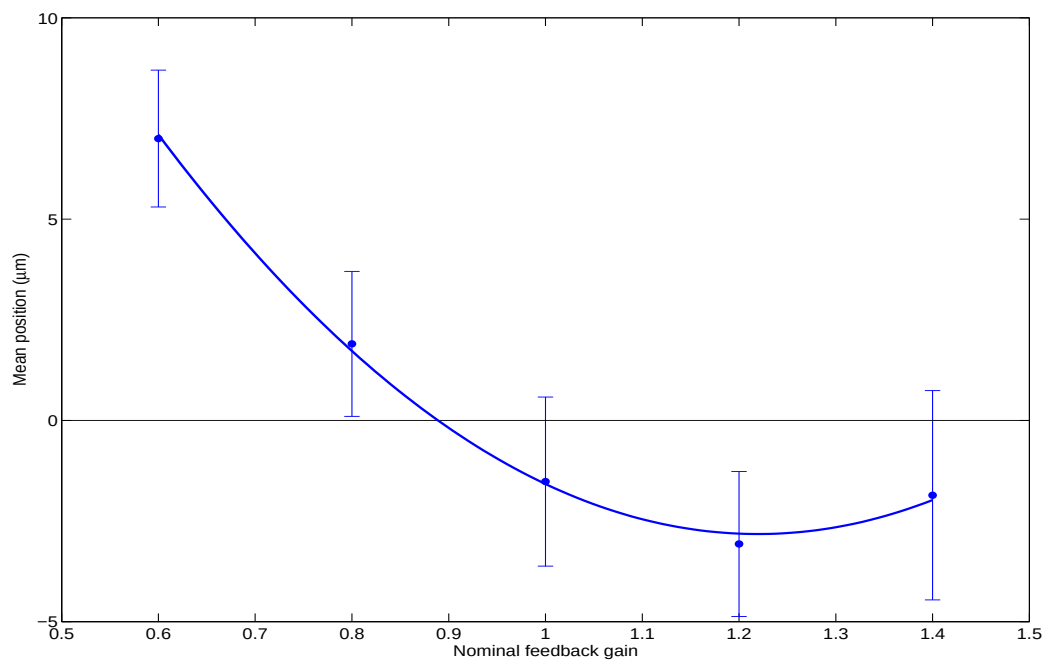


Figure 5.31: Position vs. gain for the K2 loop for bunch 3 in the coupled feedback system observed in P3. The points represent the measurements and the line is a quadratic fit to the data.

Chapter 6

Damping Ring Studies at ATF

In order to optimise the correction for the FONT feedback system in the ATF2 extraction line, the bunch-bunch correlation in the extraction line must be maximised; as explained in Chapter 1. Beam oscillations in the damping ring were investigated as a possible cause for reduced bunch-bunch correlations observed in the extraction line. A simulation of the damping ring oscillations was developed and allowed the observed features to be identified and explained.

6.1 Damping Ring Studies

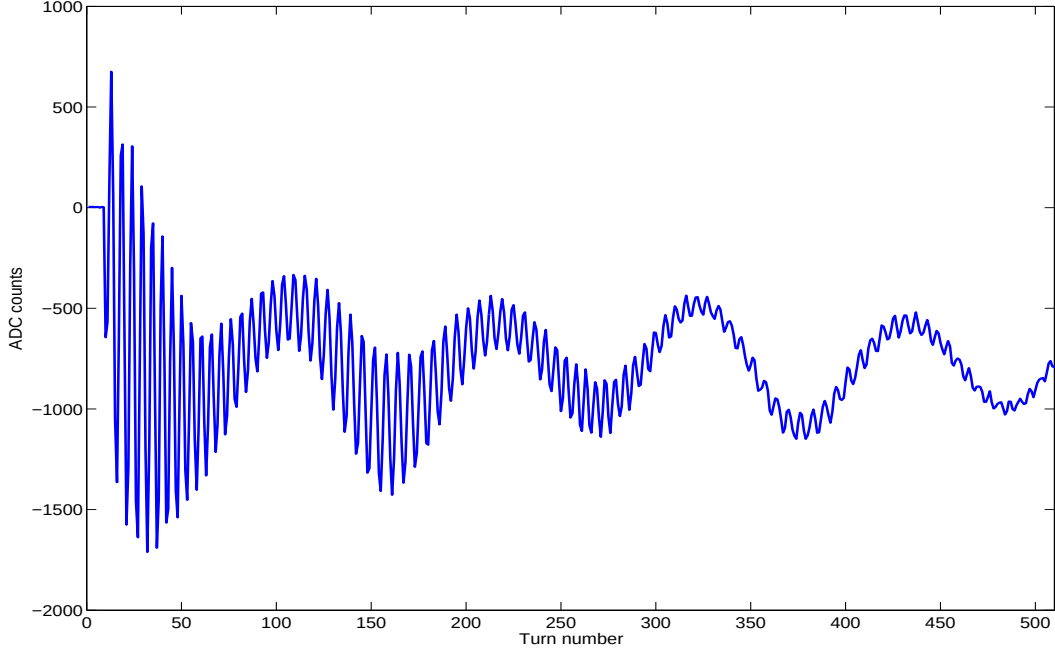
In a damping ring, there are two main types of oscillation that can occur. Firstly, synchrotron oscillations are due to the beam's orbit length varying from one turn to the next. This is due to oscillations of the beam energy causing variations on the radius of curvature of the beam in the arc sections of the damping ring; hence a varying orbit length. This will have the effect of causing the bunches to arrive early or late with respect to the ring clock; this is a longitudinal oscillation. The ring clock, as described in Chapter 4, is a 2.16MHz clock locked to the nominal revolution frequency of the ATF damping ring. The second main type of oscillation is known as betatron oscillation. This is a transverse oscillation where the beam position oscillates about the nominal orbit.

The FONT digital electronics are designed to sample on the peak of the Σ and Δ signals. If a synchrotron oscillation is present in the damping ring, it will cause the FONT5 board to sample off peak. This will be observed as a small oscillation which is visible on both the Σ and Δ signals. In the case of betatron oscillations, since Σ is independent of position, this type of oscillation will only be observed in the Δ signal.

Figure 6.1 shows a bunch in the ATF damping ring for the first 500 turns after injection. The high frequency oscillation is due to the horizontal betatron oscillation while the lower frequency oscillation is due to the synchrotron oscillation.

6.1.1 Hardware Modifications

The BPMs in the damping ring used for this study were BPMs 26 and 27 and are situated downstream of the start of the extraction line (Figure 6.2); these BPMs are not mounted on

Figure 6.1: Δ signal vs. turn number.

movers. The bunches can hence pass through the BPMs with a large position offset. In order to reduce the Δ signal to a level where it did not saturate the BPM processor, attenuation was attached to the inputs of the processor. Different value attenuators were put on the two inputs to mimic a position offset. The value of an attenuator can be converted into a ratio by using Eq. 6.1. A_{dB} is the value of the attenuation expressed in decibels (dB) and a_{factor} is the value of the attenuation expressed as a factor.

$$a_{\text{factor}} = 10^{A_{dB}/10} \quad (6.1)$$

As the bunches need to be monitored for many thousands of turns in the damping ring, the FONT5 board required modified firmware. In order to maximise the amount of data that could be recorded, the firmware was modified to maximise the amount of RAM on the FONT5 board that could be used to store the data [64]. A new data acquisition system was also developed in LabView (Chapter 7).

6.1.2 Σ and Δ Coupling

The attenuation factor on inputs 1 and 2 of the processor are defined as a_1 and a_2 respectively. Eqs. 6.2 and 6.3 show the amplitudes of the stripline signals at the inputs of the processor.

$$V_1 = w\rho \frac{a_1}{(R-y)^2} \left(\frac{dQ}{dt} \right) \quad (6.2)$$

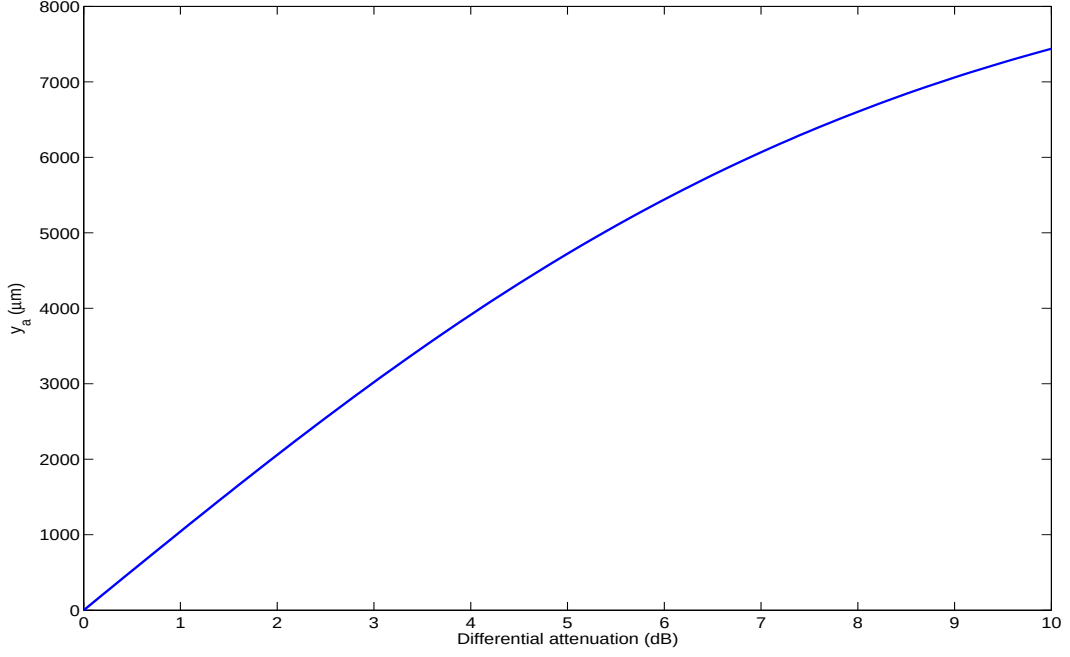


Figure 6.3: Apparent position offset vs. differential attenuation.

From Eqs. 6.4 and 6.5 the asymmetric attenuation gives an apparent position offset, as shown in Eq. 6.8. The apparent offset, y_a , is shown in Eq. 6.9 where C is the calibration constant for the BPM system. The coefficient of Δ_0 in Eq. 6.4 is very small, thus there is a negligible coupling of Δ into Σ . Figure 6.3 shows how the apparent position offset varies with the differential attenuation on the processor inputs. The differential attenuation is the difference in attenuation on the two processor inputs expressed in decibels.

$$\frac{\Delta}{\Sigma} = \frac{\Delta_0}{\Sigma_0} + \left(\frac{g_d}{g_s} \right) \left(\frac{a_1 - a_2}{a_1 + a_2} \right) \quad (6.8)$$

$$y_a \equiv \left(\frac{g_d}{g_s C} \right) \left(\frac{a_1 - a_2}{a_1 + a_2} \right) \quad (6.9)$$

Figures 6.4 and 6.5 show a comparison between the coupled (Σ and Δ) and uncoupled (Σ_0 and Δ_0) signals respectively [65]. Σ and Δ are said to be coupled signals because the differential attenuation couples Σ_0 and Δ_0 together to produce Σ and Δ (Eqs. 6.4 and 6.5).

6.1.3 Simulation of the damping ring oscillations

In order to study and understand the observed effects in the damping ring, a simulation was developed in Matlab. Identifying the effects of the synchrotron and betatron oscillation allowed more detailed effects to be studied. Initially, the synchrotron and betatron oscillations

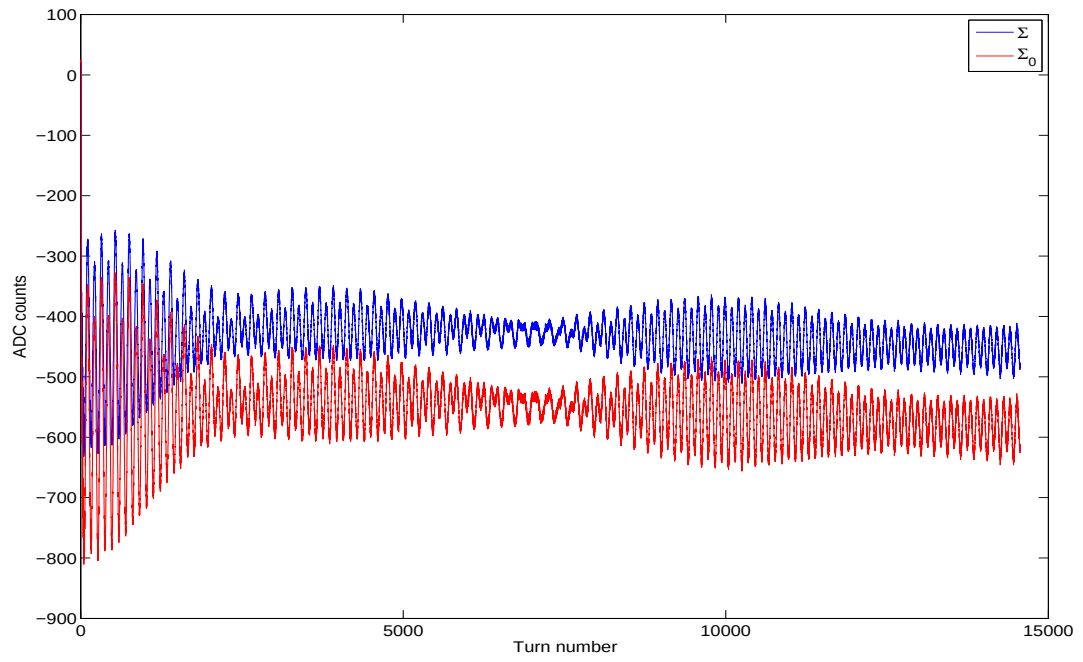


Figure 6.4: Coupled (Σ) and uncoupled (Σ_0) signals vs. turn number.

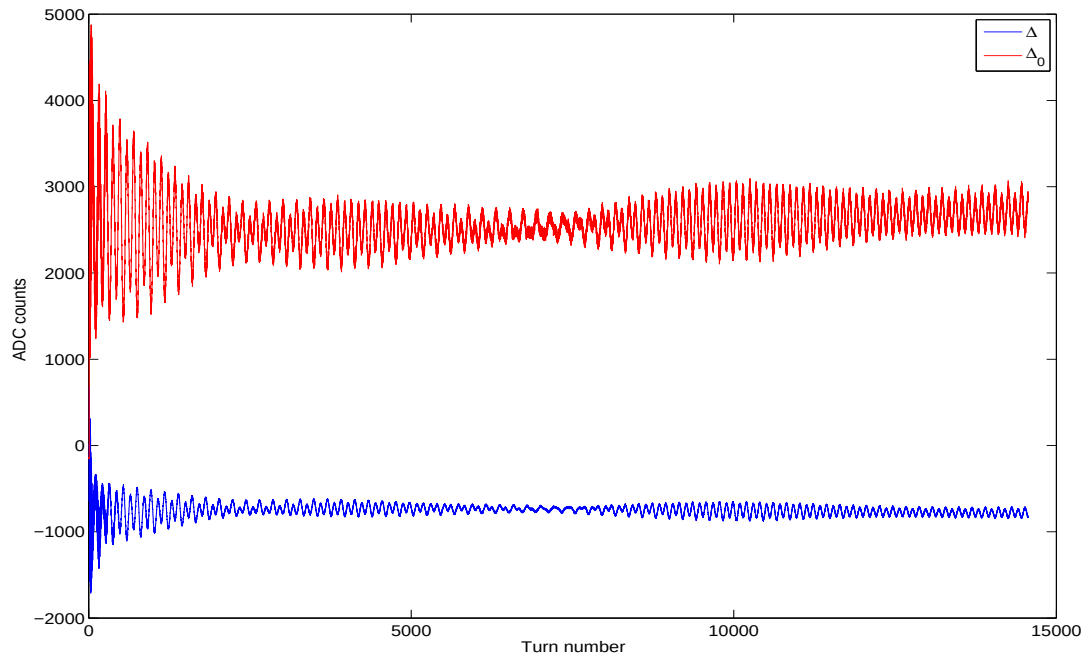


Figure 6.5: Coupled (Δ) and uncoupled (Δ_0) signals vs. turn number.

were simulated.

Synchrotron Oscillation

Synchrotron oscillations in the ATF damping ring cause the revolution period of the electron bunches to oscillate about a nominal value. In the FONT beam position monitoring scheme, this will affect the phase of the LO with respect to the bunches, as well as the timing of the FONT5 board. For a given turn number, the electron bunch will arrive with a phase ϕ_{bunch} (Eq. 6.10). $D_S(T)$ is the synchrotron damping term, T is the turn number and ω is the synchrotron frequency. The maximum phase shift before the oscillation damps is $\sim \pm 57^\circ$, or $\sim \pm 0.99$ radians [37]. This amplitude of this phase oscillation was measured by connecting a BPM signal from the damping ring to one of the FONT oscilloscopes as well as an LO signal. The scope was set onto persistence mode to plot all waveforms and triggered off the BPM signal. The trigger was gated so that it would only record the first 500 triggers; the spread of the LO phase was then measured from the scope. The synchrotron frequency of the ATF damping ring has been measured to be 10.8kHz [66].

$$\phi_{\text{bunch}} = 0.99 \times D_S(T) \cos(\omega T) \quad (6.10)$$

The phase oscillation will affect the amplitude of the outputs of the BPM processors as shown in Eq. 6.11. ϕ_{LO} is a term to account for errors phasing the LO and A_{LO} is the effect on amplitude of the output signals.

$$A_{LO} \equiv \cos(\phi_{\text{bunch}} + \phi_{LO}) \quad (6.11)$$

The FONT5 board is set up to measure the value of the processor outputs on the signal peaks. If the bunch phase is oscillating about the nominal timing, the FONT5 electronics will typically measure the signal off peak. Assuming that the sampling point is near the peak, the signal can be approximated as a quadratic function. For this effect, ϕ_{bunch} is converted into a time, t_{bunch} , in nanoseconds, rather than a phase (Eq. 6.12); 1.4ns is the period of the 714MHz LO signal. The effect on amplitude due to the timing oscillation is shown in Eq. 6.13; t_{Sample} is the timing error of the FONT5 sampling point.

$$t_{\text{bunch}} \equiv \frac{1.4 \times \phi_{\text{bunch}}}{2\pi} \quad (6.12)$$

$$A_{\text{Sample}} \equiv 1 - 0.5 \times (t_{\text{bunch}} + t_{\text{Sample}})^2 \quad (6.13)$$

The total effect due to the synchrotron oscillation on the FONT BPM system is shown in Eq. 6.14. A_{Peak} is the value of the peak of the processor output signal. This result contains sinusoidal terms up to and including cubic terms; thus the first three harmonics of the synchrotron oscillation will be observed.

	Q_β	f_β
x	0.181524	392.91kHz
y	0.264319	1.22MHz

Table 6.1: Table showing the nominal betatron tunes and frequencies for the ATF damping ring.

$$A_{\text{Synch}} = A_{\text{Peak}} \times A_{\text{LO}} \times A_{\text{Sample}} \quad (6.14)$$

Betatron Oscillations

The betatron oscillation is observed as a position oscillation, and therefore will only be observed on the Δ signals from the BPM processors. The betatron frequency, f_β , is defined in Eq. 6.15, where Q_β is the betatron tune and f_{rev} is the revolution frequency of the damping ring. The nominal horizontal and vertical betatron tunes and frequencies are shown in Table 6.1. The vertical betatron frequency is above the Nyquist Frequency, and is therefore not observed to have a large amplitude in the processor signals. The Nyquist frequency is half the sampling frequency (2.16MHz) and is the highest frequency that can be monitored without aliasing affecting the measured signals; the Nyquist frequency is therefore 1.08MHz. The horizontal betatron frequency is observed to have a large initial amplitude, and decays quickly.

$$f_\beta = Q_\beta \times f_{\text{rev}} \quad (6.15)$$

In the simulation of the damping ring oscillations, only the horizontal betatron oscillation was considered. This is because the observed vertical oscillation has a very small amplitude compared to the horizontal oscillation. Eq. 6.16 shows the betatron oscillation term in the simulation. D_β is the betatron damping term, ω_β is the betatron frequency and A_β is the amplitude of the oscillation on the Δ signal due to the betatron oscillation.

$$A_\beta = D_\beta \cos(\omega_\beta T) \quad (6.16)$$

Synchrotron-Betatron Coupling

Synchrotron-betatron coupling was also considered in the simulation; firstly the synchrotron oscillation coupling into betatron oscillations. Since synchrotron oscillations cause variations in the radius of curvature in the arc sections of the damping ring, this will also cause variations in the horizontal position in the straight sections. Therefore the synchrotron oscillations excite betatron oscillations in the horizontal plane.

As the bunch travels around the arc sections, the change in horizontal position can be expressed in terms of the ϕ_{bunch} (Eq. 6.17); δr is the variation of the radius of curvature in

the arc sections.

$$\frac{2\pi\delta r}{c} = t_{\text{bunch}} = \frac{1.4 \times \phi_{\text{bunch}}}{2\pi} \quad (6.17)$$

Eq. 6.18 shows $\delta\Delta$, the change in Δ due to the synchrotron oscillation, expressed in terms of δr ; C is the calibration constant for the BPM system. Substituting Eq. 6.18 into Eq. 6.17, the effect on the Δ signal due to the synchrotron oscillation can be expressed. Eq. 6.19 has been calculated numerically using a calibration constant of $2.2 \times 10^{-3} \mu\text{m}^{-1}$.

$$\delta\Delta = (C\Sigma) \delta r \quad (6.18)$$

$$\delta\Delta = \frac{1.4cC\Sigma}{4\pi^2} \phi_{\text{bunch}} = 23.41 \times \phi_{\text{bunch}} \Sigma \quad (6.19)$$

In order to calculate the synchrotron-betatron coupling from this, the convolution between Eq. 6.19 and Eq. 6.16 is calculated:

$$A_{S-B} = \delta\Delta * A_{\beta} \quad (6.20)$$

Another form of synchrotron-betatron coupling that was considered was the betatron oscillations coupling into synchrotron oscillations. If the beam position is slightly further away from the centre of a focusing magnet, it would be deflected more and takes a longer overall path than a bunch traveling through the centre of the focusing magnets; thus triggering a synchrotron oscillation. Since the total position variations due to the betatron oscillation are very small ($\sim 10\mu\text{m}$), the maximum path length difference will also be very small ($\sim 0.1\text{nm}$). Therefore, the synchrotron oscillations excited by the betatron oscillation will be negligible, and thus were not included in the damping ring simulation.

Beating Effect On ATF Damping Ring Data

After considering all the aforementioned phenomena, the real damping ring data still showed a beating effect on the signal envelopes which the simulation could not explain (Figures 6.6 and 6.7). The beating does not have the same phase from one damping ring injection to the next, which would be expected if it were indeed due to an oscillation in the damping ring.

It was originally proposed that the beating could be caused by interference between higher order harmonics of the horizontal and vertical betatron oscillations. All combinations of the first 10 harmonics for the vertical and horizontal betatron frequencies were calculated, but no combination produced a frequency low enough to account for the observed beating.

Another postulated cause for this beating is that it is caused by a higher order harmonic of the 50Hz mains voltage. In this case, the 50Hz would not be phase locked to the damping ring timing; which would account for the arbitrary phase of the observed beating. The observed beating has a frequency of $\sim 200\text{Hz}$. An oscillation due to the mains power would cause an

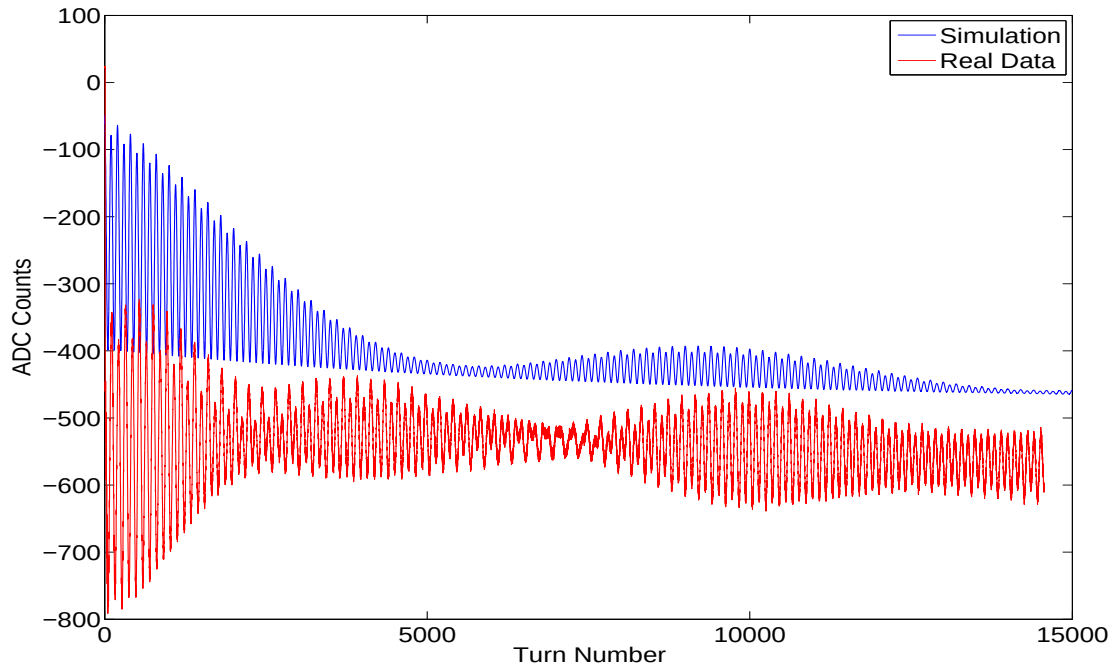


Figure 6.6: Σ vs. turn number for simulated and real data.

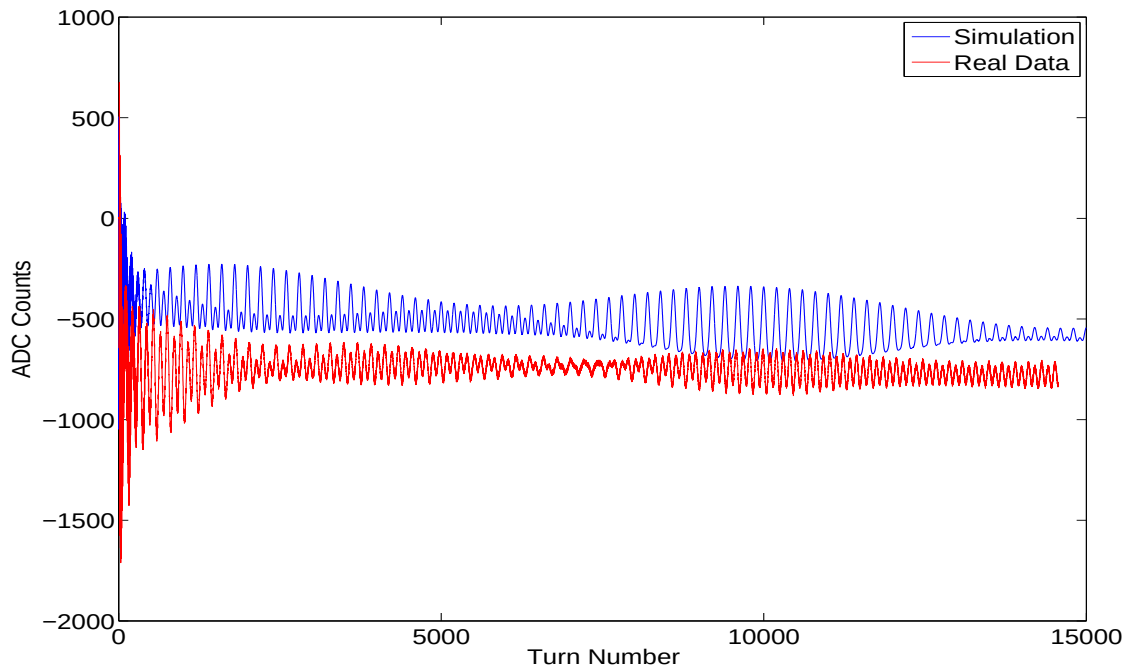


Figure 6.7: Δ vs. turn number for simulated and real data.

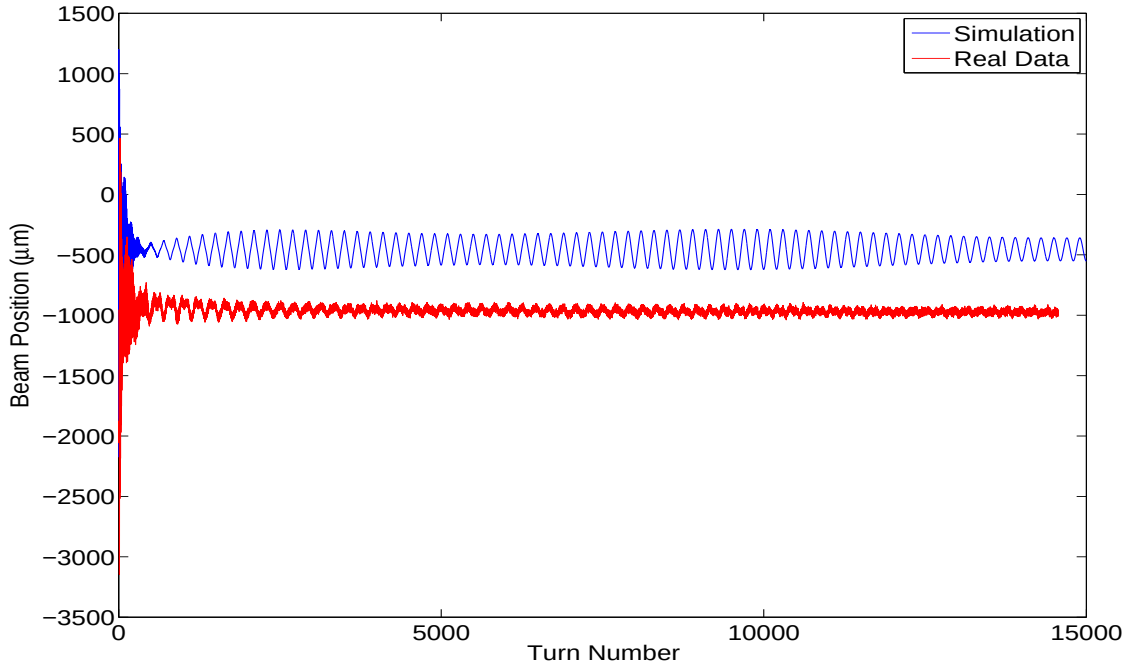


Figure 6.8: Beam position vs. turn number for simulated and real data.

oscillation in the power for the magnetic components in the damping ring. In this case, the nominal orbit of the damping ring would be changing; which would allow the position and path length of the orbit to change. This accounts for the beating being observed on both the Σ and Δ channels.

6.1.4 Conclusions Of The Damping Ring Simulation

Figure 6.8 shows a comparison of the simulated and real $\frac{\Delta}{C\Sigma}$ measured in the damping ring. The simulation was only required for a qualitative study of the damping ring oscillations; the plots show that many of the main features of the oscillations are successfully identified by the simulation. The simulation does exhibit disparities when compared to the real data. Figure 6.6 shows that the oscillations in the simulation damp more quickly over a large timescale than the real data. This is due to the simplistic model used for the synchrotron damping term, $D_S(T)$, which was assumed to be an exponential decay. Similarly Figure 6.8 shows that in this case the simulated position oscillations do not damp as quickly as the real data. It is also evident that the envelope of the simulated oscillation in Figure 6.6 does not match the real oscillation; unfortunately this is not currently understood.

One concern about this BPM system for the damping ring is that the timing effects are visible in the observed data. This means that it is very difficult to differentiate between real position effects in the damping ring and effects due to the bunch timing. The simulation was used to calculate the beam position, only taking into account real position effects. This was compared to the beam position calculated by considering all damping ring effects. Figure

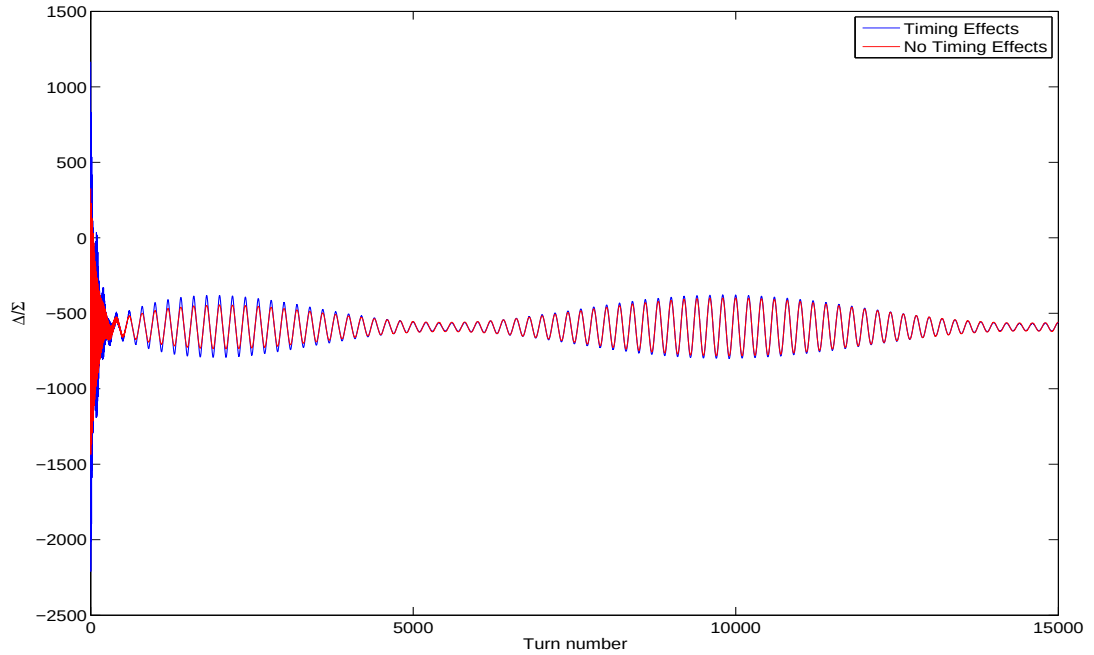


Figure 6.9: $\frac{\Delta}{\Sigma}$ vs. turn number for simulations with and without timing effects included.

6.9 shows the comparison between the two simulated effects. The figure shows that there is a very small difference in position; although the Σ and Δ signals are heavily dependent on timing effects, the calculated beam position is not.

6.2 Summary

The standard FONT hardware was modified in order to investigate possible instabilities in the ATF damping ring. A new firmware was developed and loaded onto the FONT5 digital board and a new DAQ system developed in order to record the damping ring data.

When data was taken with the modified setup, oscillations were observed in the damping ring. In order to understand the qualitative behaviour of the electron bunches in the ring, a simulation was developed in Matlab. The simulation identifies many of the features observed from data in the damping ring. The simulation suggests that a higher order harmonic of the mains power frequency may be causing the nominal orbit of the damping ring to oscillate. Since this is not phase locked to the timing system of the damping ring, the oscillation would have an arbitrary phase. This could potentially lead to a reduction in bunch-bunch correlation in the ATF2 extraction line.

Chapter 7

Data Acquisition Systems

The Σ and Δ outputs from the analogue processors and the feedback system are all controlled and monitored by the digital electronics. FONT4 and FONT5 both have the capability to transmit data via RS-232. Data acquisition systems (DAQs) were developed in order to control the FONT system.

For FONT4, a DAQ system was created using C++ [27]. When the FONT5 digital board was introduced, the FONT4 DAQ was modified in order to allow the early versions of FONT5 firmware to be controlled. As the FONT5 firmware became more complex, exploiting the increased computing power of the new digital electronics, the C++ DAQ became more complex. It was decided to develop a new DAQ in LabView [67]. LabView is specifically designed to develop data acquisition systems and easier to maintain than a DAQ in C++.

7.1 Extraction Line DAQ

For the extraction line, FONT has developed two different versions of firmware. The short-train firmware is used to record data when the normal extraction kicker is in operation. This firmware can record 164 samples over one ring clock cycle (462 ns) for each ADC in increments of 2.8 ns (357MHz). The short-train firmware is used when ATF2 is operating in single-bunch or three-bunch mode. In single-bunch mode one bunch is extracted for each extraction trigger; in three-bunch mode three bunches are extracted every third extraction trigger. A fast extraction kicker is being tested at ATF2 which is capable of extracting up to 30 bunches from the damping ring over 20 ring clock cycles; as described in Chapter 1. The short-train firmware is unable to record 20 ring clock cycles; therefore a new firmware was required. The long-train firmware records 3280 samples in increments of 2.8 ns for 20 ring clock cycles for each ADC [47]. The extraction line DAQ has been developed so that it can communicate with both versions of firmware.

Figure 7.1 shows a schematic diagram of the extraction line firmware. The three ADC groups are sampled and clocked by the timing and synchronisation module (TSM); the TSM is clocked with the 357MHz clock. The ADC samples and the control readbacks are output by the TSM and read into the DAQ RAMs; the ADC samples are also read into the feedback module. Data is read into the DAQ RAMs on the 357MHz domain but read out on the 40MHz domain. This is done so that the output from the DAQ RAMs can be read by the UART, which is clocked on the 40MHz domain. The Feedback module calculates the

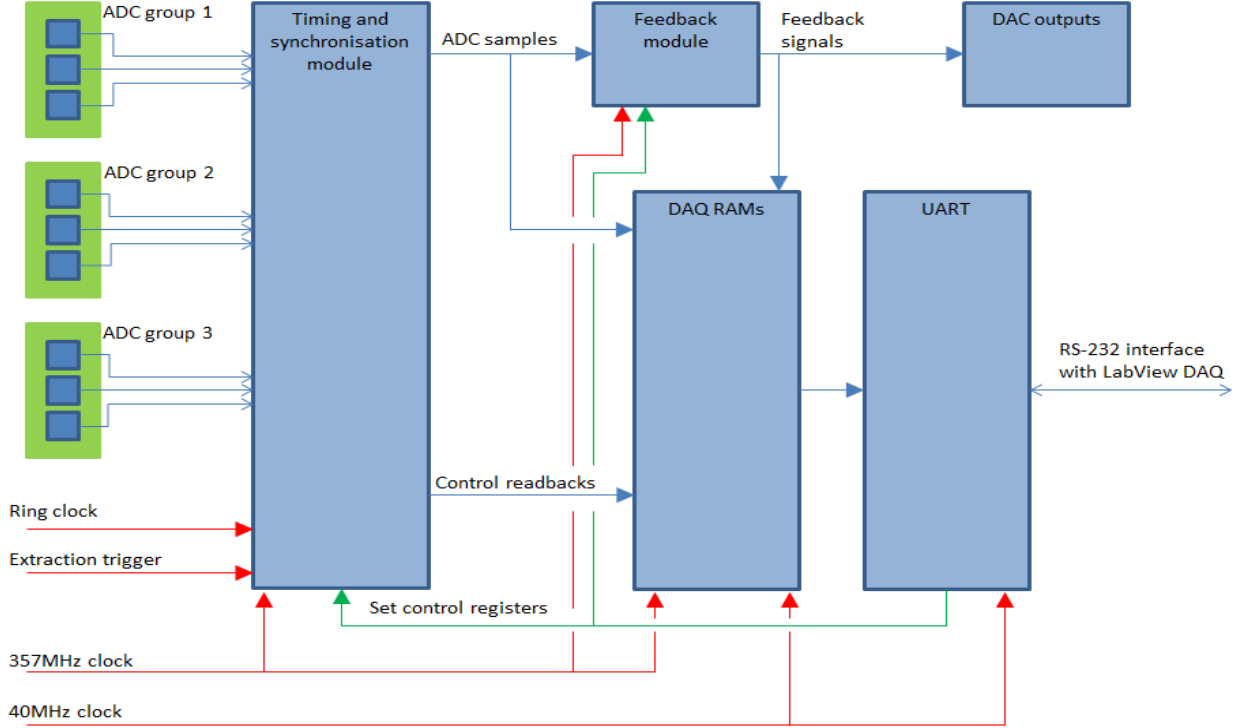


Figure 7.1: Schematic diagram of the extraction line firmware.

required feedback correction from the ADC samples and outputs the results onto the DAC outputs as well as the DAQ RAMs. The UART communicates with DAQ system on a PC via RS-232. Commands are transmitted from the PC to the UART, which are loaded into control registers on the FPGA.

Control Registers

The control registers for the FONT5 system are on two clock domains. All the controls which are on the critical latency path are clocked on the 357MHz domain. Table 7.1 shows the control registers on the 357MHz and 40MHz clock domains. In the cases where two addresses are given, the control register also occupies two addresses. The first byte is the most significant byte (MSB) and the second is the least significant (LSB). The addresses of the control registers on the 357MHz domain are identified by having the 6th bit set high. Therefore the 357MHz control registers occupy addresses 64 to 95. The 40MHz addresses are identified by setting the 5th and 6th bits high, hence they occupy addresses 96 to 127. All of these control registers are loaded using the LabView DAQ.

In order to send a command to a control register, an address byte corresponding to a specific control register is transmitted, followed by a data byte. Data bytes are identified by having the 7th bit set to 1. In the case where the control register occupies more than one address, the first address byte is sent, followed by the data byte for that address. Then the second address byte is followed by its corresponding data byte. There are several instances where the data sent from the DAQ to the FONT5 board does not follow this pattern. First, in

order to send a ‘Full Reset’ command to the firmware, a byte with the value zero is sent.

Secondly where loading a look-up table (LUT). In this case, the address byte for the start of the LUT is transmitted and then a series of data bytes. Thirdly for loading the trim DAC LUT, a trigger byte is also transmitted at the end of the data stream. Finally when changing the scan delays or the offset delays. For these controls, an address byte is sent, followed by the data byte and then a trigger byte. A strobe is transmitted for control registers that are not clocked, therefore a trigger is required before the control register can change state; these are known as asynchronous controls. All asynchronous control registers are shown in Table 7.2.

357MHz Domain		40MHz Domain	
Address	Control Register	Address	Control Register
0	Unused	0	P1 Channel Select
1	Trig Out Delay	1	P2 Channel Select
2	Trig Out Enable	2	P3 Channel Select
4:3	P1 Bunch 1 Position	3	P1 Offset Delay
6:5	P1 Bunch 2 Position	4	P2 Offset Delay
8:7	P1 Bunch 3 Position	5	P3 Offset Delay
10:9	P2 Bunch 1 Position	6	P1 Scan Delay
12:11	P2 Bunch 2 Position	7	P2 Scan Delay
14:13	P2 Bunch 3 Position	8	P3 Scan Delay
16:15	P3 Bunch 1 Position	9	Master 357 Delay
18:17	P3 Bunch 2 Position	11:10	K1 Bunch 2 Offset
20:19	P3 Bunch 3 Position	13:12	K1 Bunch 3 Offset
21[0]	K1 Feedback Enable	14	Ring Clock Threshold
21[1]	K2 Feedback Enable	15	Trigger Threshold
21[2]	K1 Delayloop Enable	16	K1 Droop Correction
21[3]	K2 Delayloop Enable	18:17	K2 Bunch 2 Offset
21[4]	K1 Constant DAC Enable	20:19	K2 Bunch 3 Offset
21[5]	K2 Constant DAC Enable	21	K2 Droop Correction
23:22	K1 Constant DAC Output	22	Coupled FB Select
25:24	K2 Constant DAC Output	31:23	Unused
26	Ring Clock Edge Select		
27	Sample Holdoff		
29:28	Trig Delay		
31:30	Unused		

Table 7.1: Table showing the addresses assigned to the control registers on the 357MHz and 40MHz clock domains in the extraction line firmware.

Reading Data

In the extraction line DAQ (Ext DAQ), the data is transmitted and received from the FONT5 board asynchronously. This was important because no data is received from the FONT5 board until the ring clock and trigger thresholds have been set. As a result, the

Address	Control Register
0	Full Reset
1	P1 Delay Trigger
2	P2 Delay Trigger
3	P3 Delay Trigger
4	Trim DAC LUT
5	357 Master Delay Trigger
6	Trim DAC Trigger
32	K1P2 LUT
33	K1P3 LUT
35	K2P2 LUT
36	K2P3 LUT

Table 7.2: Table showing the asynchronous control registers in the extraction line firmware.

thresholds would not be set until data was received from the FONT5 board, this would cause the DAQ to timeout.

The data received from the FONT5 board is always in the same sequence. Firstly a timestamp byte is received, followed by 368 bytes from each of the nine ADC channels. After this, the DAC output values are received and finally the control register readbacks. The data stream is separated by using framing bytes as this allows the type of data to be identified. Table 7.3 shows the list of framing bytes in the order in which they are received.

Once the data are read into the DAQ, they are separated into different subsets. Since the data are identified by the 7th bit being set to 1, this bit is stripped from all data, leaving 7-bit words. Some of the data are 14-bit words, such as the data from the 9 ADC channels and the DAC readbacks. These data take values in the range of -4096 to 4095. These negative numbers are represented as a two's complement in the binary representation and are converted into a signed integer in the DAQ.

Once all the data has been converted into the appropriate format, it is saved into data files. The data from the ADC channel are saved in files with a '.dat' format. The control register readbacks are saved to '.rb' files, and the DAC outputs are saved to '.dac' files. The readbacks are also displayed on the DAQ front panel next to the corresponding control. This is done to confirm that all control registers are set with the desired value.

Figure 7.2 shows the extraction line DAQ taking data at ATF2 in December 2010. The plots give a real-time display of the Δ and Σ signals from the BPM processors connected to the FONT5 ADCs. Every control register can be changed using the DAQ user interface. The FONT5 board is connected to the PC using an RS-232 cable to one of the PC's external COM ports; the COM port used must be defined before communication can occur between the two devices. Before starting the DAQ, the COM port is defined using the control labeled 'VISA resource name', the data flow rate is defined using the 'Baud rate' control and the number of samples per ADC is defined to determine which version of firmware is being used. Once the DAQ has started, the ring clock and trigger thresholds must be configured; now the PC and FONT5 board are able to communicate. The DAQ can change any of the

Value	Framing Byte
31	Timestamp
16	P1 XDiff
18	P1 YDiff
20	P1 Sum
21	P2 XDiff
22	P2 YDiff
23	P2 Sum
24	P3 XDiff
25	P3 YDiff
26	P3 Sum
29	K1 DAC Output
30	K2 DAC Output
27	357MHz Readbacks
28	40MHz Readbacks
15	Monitor Readbacks
14	Termination Byte

Table 7.3: Table showing the framing bytes in the standard extraction line firmware.

control registers and display all the data received from the FONT5 board. The samples from all 9 ADC channels are displayed on plots on the main screen. Features of the DAQ include a function to load the feedback look-up tables (LUTs), 3-bunch mode to veto empty extractions, interleaved feedback which toggles the feedback system on and off, automated file saving and a reload control switch. All these features will now be described in more detail.

Loading the Feedback LUTs

The feedback correction transmitted from the DAC outputs, V_{DAC} , is shown in Eq. 7.1; G_{FB} is the feedback gain. Performing a digital division is a relatively slow procedure and so a look-up table is used instead. The value of Σ defines an address in the LUT which is loaded with the value $\frac{G_{FB}}{\Sigma}$. This is then multiplied by Δ to give the feedback correction, V_{DAC} . This is known as a feedback LUT and there are 4 which need to be loaded in the extraction line firmware.

$$V_{DAC} = G_{FB} \frac{\Delta}{\Sigma} \quad (7.1)$$

The LUTs are loaded with 23-bit words representing the reciprocal of the Σ multiplied by a gain. In order to change the gain in one of the feedback LUTs, the DAQ calculates the values for all addresses in the LUT and sends the data to the FONT5 board. This algorithm involves several steps to prevent incorrect values being loaded onto the LUT. It is important to ensure that the LUTs are loaded correctly for the feedback corrections.

Figure 7.3 is a block diagram of the LUT loading algorithm, showing the key stages in the



Figure 7.2: Display of the extraction line DAQ being used to record data at ATF2 in December 2010.

procedure. The gain is multiplied by 4096 on the FONT5 board; the LUT entry is divided by 4096 to account for this. This allows the LUT entry for a given Σ value to have 23-bit resolution rather than 7-bits. This value is then divided by every possible value of Σ and rounded to the nearest integer. The resulting array of numbers is then converted into two's complement. At this point, values of the array that are outside the range of -1048576 to 1048575 are changed to be at the closer limit. This is required because numbers outside this range will be larger than 23 bits. Therefore only the least significant 23 bits will be read, which will load a drastically different value into the LUT. At this point, the values are all separated into three 7-bit words. An 8th bit on each word is set to 1 to state that these are data bytes and the bytes are then loaded to the LUT. As stated earlier, only the first address on the LUT is transmitted.

3-Bunch Mode

At ATF2, when running in 3-bunch mode, the beam will only be in the accelerator for one in three triggers; depending on when the DAQ is initiated, the beam could be in the 1st, 2nd or 3rd trigger. When looking at the ADC signals on the DAQ, for two of the extraction triggers, only the baseline signal is observed. An algorithm has been implemented so that in 3-bunch mode, only the correct trigger will be plotted on the DAQ. Figure 7.4 shows a block diagram to show how this algorithm works.

Every time data is received by the DAQ, a while loop increments a counter. A selection

control is used to select single-bunch mode or the 1st, 2nd or 3rd extraction trigger in 3-bunch mode. If single-bunch mode is selected then the ADC data for every trigger is plotted. Alternatively, if the control is set to the first, second or third extraction triggers in 3-bunch mode then 1, 2 or 3 is added to the counter value. Every time the modified counter value is a multiple of 3, the data is plotted on the DAQ front panel.

Interleaved Feedback

At ATF2, the beam can drift very slowly over time. If the feedback system is running, it is not possible to determine whether the beam is drifting. In order to overcome this, an algorithm was developed to toggle the feedback system on and off. Since the drift occurs over the course of many minutes, it is very useful for analysis to observe the beam drift. The algorithm works by turning feedback on for one train and switching it off for the next train and repeating the process.

The algorithm is related to the function to veto empty extractions. The modified counter is divided by 3 and rounded down to the nearest integer. If this value is divisible by 2, then the interleaved function switches feedback on. Conversely, if the value leaves a remainder of 1, feedback is switched off.

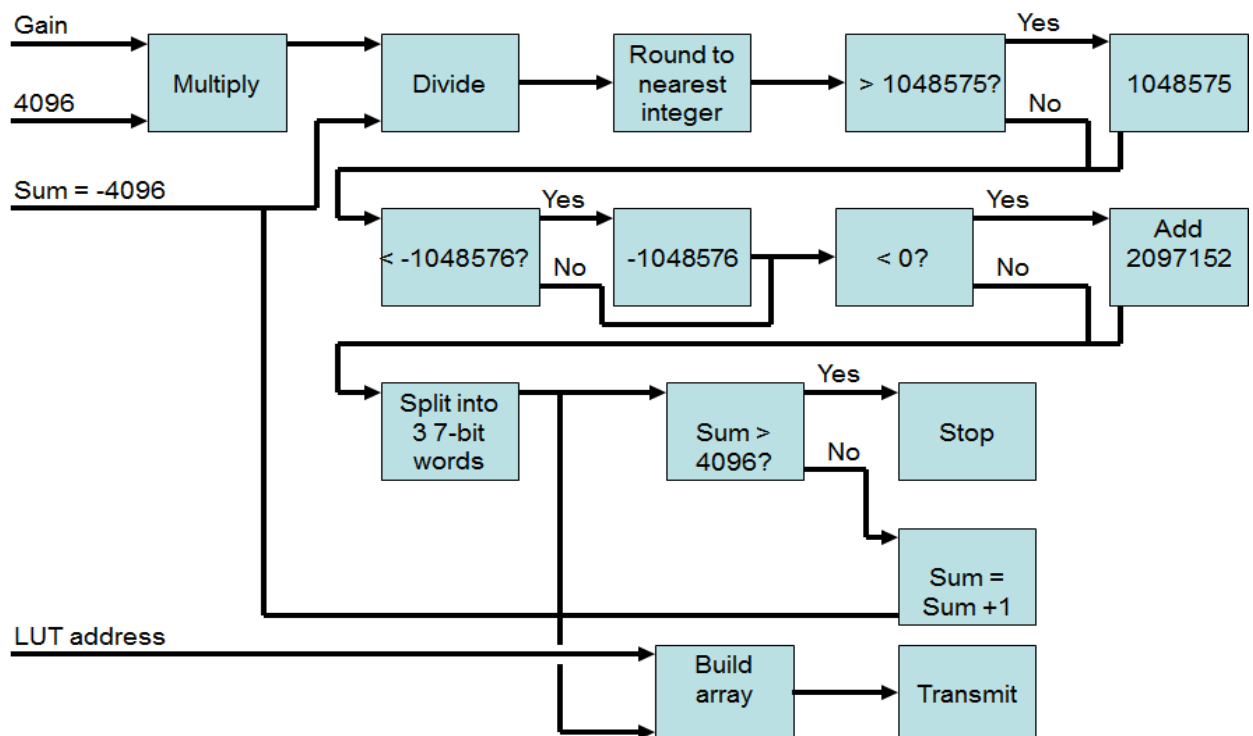


Figure 7.3: Block diagram of the LUT loading algorithm.

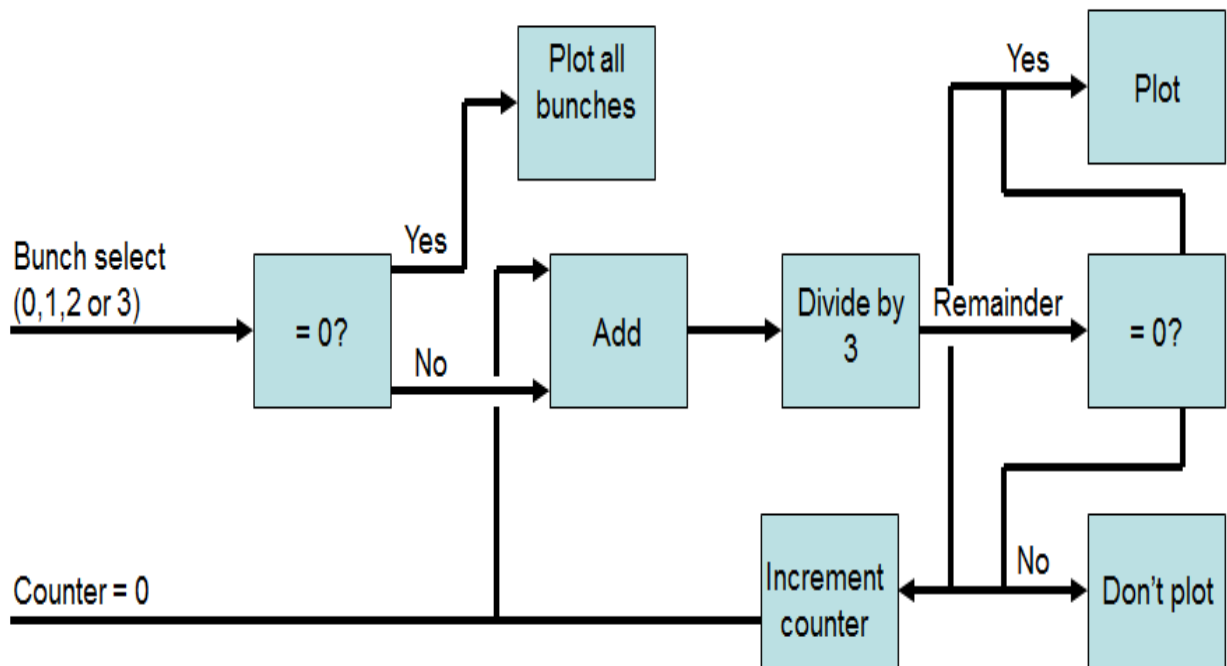


Figure 7.4: Block diagram showing the algorithm to veto empty extraction triggers in 3-bunch mode.

Automated File Saving

When saving data, a counter increments so that the total number of recorded extractions can be seen. When another data file is created, the counter starts at 0 again. This is done by using a case structure, so that if data is not being saved to file, the value for the counter is 0. When data is recorded, the counter increments.

A control has also been implemented so that a specific number of extractions can be recorded. This is done by comparing the value of the save counter to the specified number of extractions. If the value of the counter is greater than the specified number, the DAQ will stop saving data. If the required number of extractions is left unspecified then the DAQ will continue recording data indefinitely.

Reload Controls

When a full reset command is transmitted to the FONT5 board, all the control registers are loaded with their default values. In order to prevent the user manually reloading the value for every control register, a 'reload controls' command has been implemented in the DAQ. This transmits the value of every control on the DAQ to the FONT5 board; hence restoring the FONT5 system back to its previous state before the full reset.

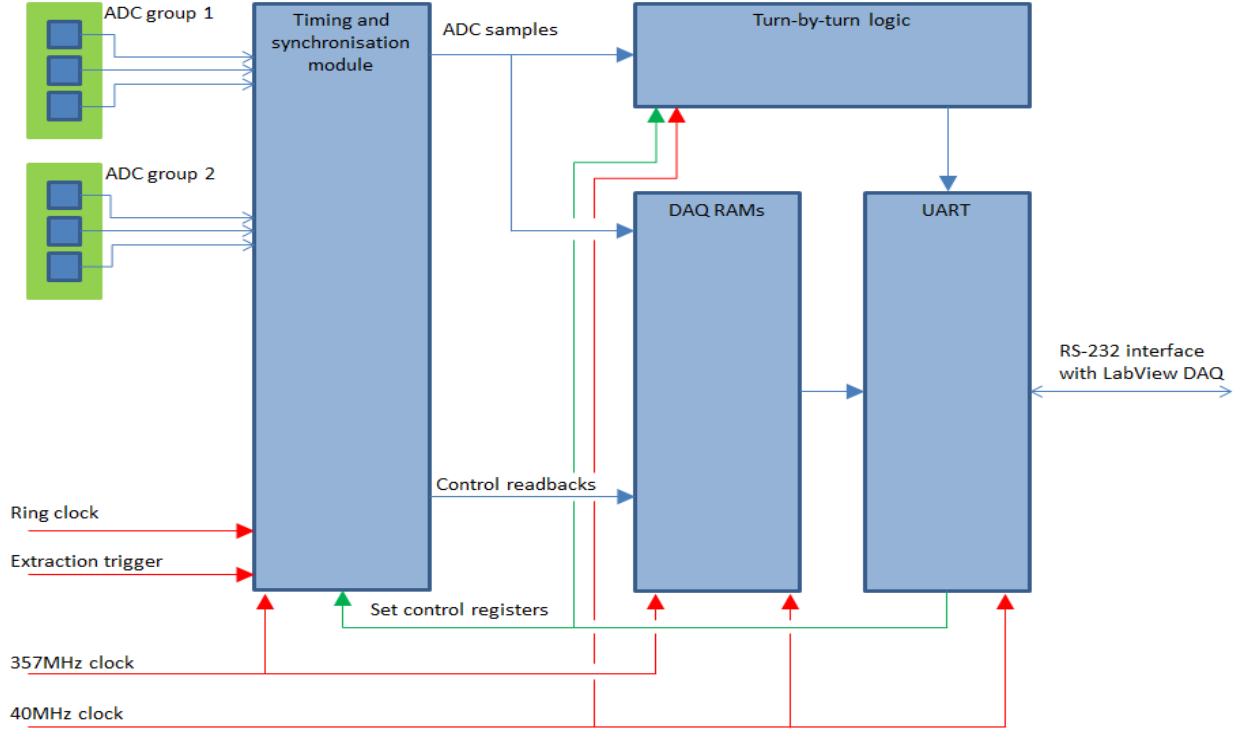


Figure 7.5: Schematic diagram of the damping ring firmware.

7.2 Damping Ring DAQ

In order to observe oscillations in the damping ring, it was important to maximise the amount of data that could be recorded from the damping ring BPMs. Firmware was developed to monitor the damping ring, and is capable of recording a total of 131071 turns of the damping ring for 1 bunch, 1 ADC channel. The data is sent to the DAQ system via RS-232 at a baud rate of 460800 bits per second. It takes approximately 5 seconds to transmit the data from the FONT5 board for each acquisition. A DAQ system was developed in order to minimise the time taken to process the data and hence maximise the data acquisition rate.

Figure 7.5 shows a schematic diagram of the damping ring firmware. The architecture of the firmware is very similar to that of the extraction line firmwares. The main differences being that only two of the ADC groups are used and rather than the feedback module, there is a turn-by-turn module. This module stores the peak values of the ADCs for every turn of the damping ring until all the available memory is filled, at which point all the data is transmitted to the DAQ via the UART.

Figure 7.6 shows a screenshot of the damping ring DAQ system being used at ATF2 in December 2010. The damping ring DAQ is configured in the same manner as the extraction line DAQ. The COM port and baud rate are defined before initiating the DAQ and then the ring clock and trigger thresholds are configured. The control readbacks and ADC samples for the last turn in the damping ring are displayed for each acquisition. In order to reduce the time required by the DAQ to process data read from the FONT5 board, no additional features were implemented such as automated file saving.

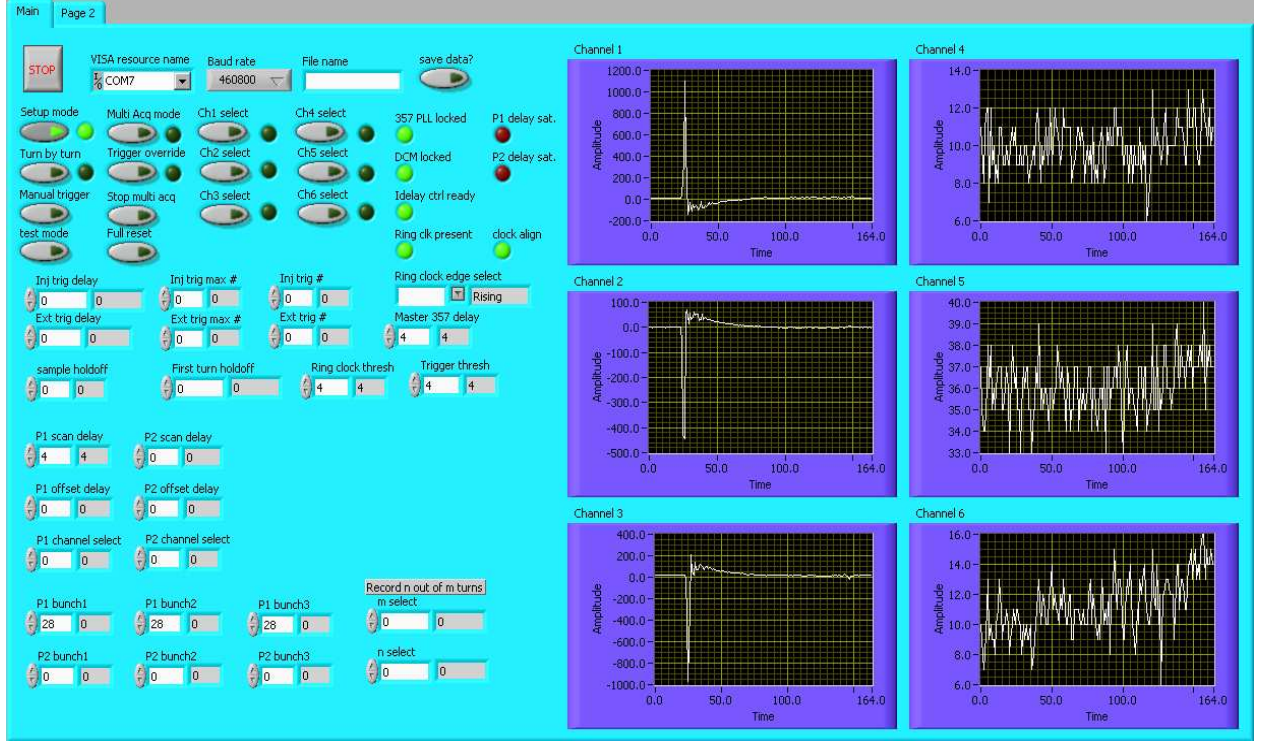


Figure 7.6: A display of the damping ring DAQ system being used at ATF2 to record data in December 2010.

Data Acquisition Modes

There are two main modes of data acquisition, firstly Setup mode. Setup mode records 164 samples for one ring clock cycle, like the 3-bunch extraction line firmware. This is plotted on the DAQ's display and is used to set up the timing so that the ADCs are sampling on the peaks of the Σ and Δ signals. Setup mode is processed in the same way as in the 3-bunch DAQ.

The other acquisition mode is Turn-By-Turn Mode. A total of 131071 samples are recorded, although the total number of turns this corresponds to depends on the number of bunches in the ring and the number of ADC channels being used. In order to maximise the total number of turns, only the peaks of the ADC signal inputs are recorded. In the DAQ, this data is saved directly to file as ascii characters, in order to minimise processing time. Due to the amount of data being recorded, each Turn-By-Turn acquisition is saved to a separate file. The data files are then analysed offline using Matlab.

Due to the large quantities of data, the DAQ would not be able to process data from every single machine cycle. Instead the DAQ sends a command to the FONT5 board to arm the trigger, ready for the next machine cycle. The trigger override control will make the firmware ignore all triggers, even when the trigger enable signal has been sent. In this mode, the firmware can be triggered manually. This can be used if the damping ring is in storage mode rather than extraction mode; a multi-acquisition control can also be used. The multi-acquisition mode is used to record data as quickly as possible, so the firmware will not wait

for a trigger. Instead, the Trigger Enable command is used as the trigger.

Channel and Bunch Selection

Since the number of turns of the damping ring that can be recorded depends on the number of ADC channels recording data, it is important to select only the ADC channels that are required. There are two BPMs in the damping ring that are available for use for the FONT damping ring studies, so a maximum of 6 ADC channels can be used. They are Σ , Δ_x and Δ_y for each BPM. A toggle switch for each of the 6 channels allows any combination of channels to be selected. Each switch represents one bit of a byte which makes up the channel select command that is sent to the FONT5 board. The number of bunches also affects the number of turns that can be recorded. As with the 3-bunch DAQ, there are controls to input the sample number of the peaks of the bunch signals.

Control Registers

Many of the control registers from the 3-bunch firmware are not required. In order to maximise the total available RAM to record data, all superfluous control registers were either removed or reassigned. Table 7.4 records the addresses assigned to control registers in the damping ring firmware. Table 7.5 records the asynchronous control registers in the damping ring firmware [64].

7.3 Summary

In this chapter, the various DAQ systems used by FONT have been discussed. The DAQ systems are developed in LabView, which is specifically designed for creating DAQ systems. The DAQ has been specifically designed around the FONT5 firmware in a modular fashion. This allows changes to the DAQ to be made quickly and easily as the firmware is developed.

For the extraction line, two different firmwares are used depending on whether ATF2 is running in single-bunch, 3-bunch, or in long-train mode. These two firmwares are both controlled using the same DAQ system. The damping ring firmware is operated in a different way, therefore another DAQ system was developed to control this. Both the extraction line and damping ring DAQs have been described in detail in this chapter.

357MHz Domain		40MHz Domain	
Address	Control Register	Address	Control Register
2:0	Unused	0	P1 Channel Select
4:3	P1 Bunch 1 Position	1	P2 Channel Select
6:5	P1 Bunch 2 Position	2[0]	Channel 1 Select
8:7	P1 Bunch 3 Position	2[1]	Channel 2 Select
10:9	P2 Bunch 1 Position	2[2]	Channel 3 Select
12:11	P2 Bunch 2 Position	2[3]	Channel 4 Select
14:13	P2 Bunch 3 Position	2[4]	Channel 5 Select
15[0]	Setup Mode Disable	2[5]	Channel 6 Select
15[1]	Turn-By-Turn Mode Enable	3	P1 Offset Delay
15[2]	Multi-Acquisition Mode	4	P2 Offset Delay
15[3]	Manual Trig Override	5	Unused
15[4]	Manual Trig	6	P1 Scan Delay
15[5]	Stop Multi-Acquisition	7	P2 Scan Delay
16	Trigger Enable	8	Unused
17[1:0]	Injection Trig Max	9	357 Master Delay
17[3:2]	Injection Trig Select	13:10	First Turn Holdoff
18[5:4]	Extraction Trig Max	14	Ring Clock Threshold
18[7:6]	Extraction Trig Select	15	Trigger Threshold
22:19	Injection Trig Delay	16	n-in-m Turns, m Select
25:23	Unused	17	n-in-m Turns, n Select
26	Ring Clock Edge Select	31:18	Unused
27	Sample Holdoff		
29:28	Extraction Trig Delay		
31:30	Unused		

Table 7.4: Table showing the addresses assigned to the control registers on the 357MHz and 40MHz clock domains with the damping ring firmware.

Address	Control Register
0	Full Reset
1	P1 Delay Trigger
2	P2 Delay Trigger
3	P3 Delay Trigger
4	Trim DAC LUT
5	357 Master Delay Trigger
6	Trim DAC Trigger

Table 7.5: Table showing the asynchronous control registers in the damping ring firmware.

Chapter 8

Conclusions

8.1 Summary

In order to make precision measurements of the new particles and physical processes discovered at the LHC, a machine of unprecedented precision and stability will be required. CLIC and ILC are the two leading candidates for providing the high energy lepton collisions required. Although the two design proposals rely on different technologies, they both face the same technical challenges; namely they require nanoscale bunches at the interaction point and high luminosity. In order to maintain high luminosity, it is necessary to achieve nanometre stability on the bunch positions at the interaction point. To maintain this level of beam stability in the presence of dynamic disturbances such as ground motion, a fast feedback system is essential.

To demonstrate the feasibility of a feedback system capable of achieving the required level of beam stability for CLIC and ILC, the FONT5 intra-train feedback system has been developed. After the construction of ATF2 was completed, the three stripline BPMs and two kickers were installed for the FONT group to test the FONT5 prototype; along with the FONT electronics.

Calibration and resolution measurements are taken as a routine part of FONT shifts at ATF2. If the calibration constants differ by more than $\sim 5\%$ from their nominal, this is an indication that there is a fault in the system. Similarly, if the position resolution measurements produce values significantly larger than $1\text{-}2\mu\text{m}$, then either there is a fault or the beam optics are not properly adjusted. In order to optimise the feedback system, it is critical to minimise the BPM resolution; indeed this is one of the limiting factors of the FONT5 feedback system.

The calibration and resolution of the system were modelled and compared to measurements; this has allowed important factors to be identified such as the Δ and Σ dependence of the resolution. Modelling the calibration and resolution has led to operational changes being made, such as to the LO phasing system, which have improved the consistency of measurements.

Measurements have been taken of the system's latency; this is to ensure that the latencies of all feedback loops are less than the 154 ns bunch spacing. The P2 to K1 loop and the P3 to

K2 loop are both measured to have latencies of approximately 135 ns. The longest feedback loop, P3 to K1, has a measured latency of 147 ns.

8.2 Simulation of the BPM System

Since the resolution of the BPM system limits the optimum feedback performance, a detailed understanding of the system is essential. Initially theoretical models were developed to describe the calibration and resolution of the system. These models have been compared to measurements taken at ATF2 and agree.

In order to further understand the BPM system, the analogue BPM processors were simulated using SPICE and Matlab (Chapter 3); the simulation has been verified with measurements taken from the real BPM processors. This simulation has become an important tool in understanding factors affecting the calibration and resolution of the BPM system. The flexibility of the simulation allows any parameter to be varied, thus new effects can be studied. This is crucial in allowing the FONT group to optimise the performance of the system.

8.3 Demonstration of an Interaction Point Feedback System for ILC or CLIC

The FONT5 digital intra-train feedback system has been successful in demonstrating sub-micron corrections to beam jitter. An interaction point position feedback system requires a single degree of freedom correction. In order to achieve the required beam stability at CLIC and ILC, the feedback system must be capable of nanometre position corrections at ~ 1 TeV energy scales. The beam deflection from a kicker scales with its energy; therefore micron-level corrections are required for a beam energy of ~ 1 GeV such as at ATF. Using the P2 to K1 single loop feedback system, the measured feedback corrections for bunches 2 and 3 were $0.4 \mu\text{m}$ and $0.82 \mu\text{m}$ respectively.

In order to provide a feedback correction, the system's latency must be smaller than the bunch spacing. For a single loop feedback system, FONT5 has a latency of 135 ns. The nominal bunch spacing for ILC would be 369 ns, although this could be reduced to 184.5 ns for certain modes of operation; thus the latency of the FONT5 system is suitable for the ILC.

A single loop feedback system can only correct the position of the beam at one point along the lattice. In order to provide a feedback correction over an extended range, both position and angle need to be corrected; hence two degrees of freedom need to be corrected. In general, it is necessary to implement two coupled feedback loops to correct two degrees of freedom (section 5.2.1). Single loop, uncoupled and coupled feedback has been tested at ATF2 and demonstrated consistent sub-micron corrections. With appropriate modifications to the BPM system, a sustained sub-micron jitter stability downstream of the feedback

should be achievable.

8.4 Outlook

The next step for the FONT5 feedback system is to further improve the BPM system. Recent modifications to the signal processing hardware and improvements to data analysis software have suggested that the resolution of the BPM system is ~ 500 nm; this would be consistent with the feedback corrections observed and must therefore be investigated further.

Following an investigation into the BPM resolution, the aim would be to investigate the effects of the FONT5 feedback system downstream of the feedback region. In order to achieve this, two FONT5 boards will operate in tandem; one will be used to monitor the FONT stripline BPMs and operate the feedback system, the other will be connected to downstream striplines. A possible extension to this would be to monitor all BPMs in the extraction line. In order to successfully achieve this goal, several technical challenges must be overcome. The most prominent challenge would be to develop a DAQ system which is integrated into the ATF control system; which uses a software environment known as EPICS (Experimental Physics and Industrial Control System) [68].

It is intended that the FONT group will develop phase and position feedback and feedforward systems for the CLIC Test Facility (CTF3) [69]. These systems will require new BPM processors as the BPFs would not be suitable for the different LO frequency; although the FONT5 board would be reused with new firmware.

The next stage of the FONT5 system's development will be documented in subsequent theses [47] and [70].

Appendix A

Derivation of the Theoretical Resolution for the BPM System

As shown in Eq. 4.16, the measured beam position can be expressed in terms of real position, charge and electrical noise. The measured beam jitter can be expressed in terms of the partial derivatives of Eq. A.3, as shown in Eq. A.4. Σ and Δ are defined in Eqs. A.1 and A.2.

$$\Sigma = \alpha_{\Sigma} Q \quad (\text{A.1})$$

$$\Delta = \alpha_{\Delta} Q y_R \quad (\text{A.2})$$

$$y_M = \frac{\alpha_{\Delta} Q y_R + V_{\Delta}}{C (\alpha_{\Sigma} Q + V_{\Sigma})} \quad (\text{A.3})$$

$$\sigma_{y_M}^2 = \sum \left(\frac{\partial y_M}{\partial q_i} \right)^2 \sigma^2(q_i) \quad (\text{A.4})$$

The next step is to explicitly calculate these terms. Eqs. A.5, A.6, A.7 and A.8 are the four partial derivatives of Eq. A.3. The mean value for the electrical noise is assumed to be 0.

$$\frac{\partial y_M}{\partial y_R} = \frac{\alpha_{\Delta} \hat{Q}}{C (\alpha_{\Sigma} \hat{Q} + \hat{V}_{\Sigma})} = \frac{\alpha_{\Delta}}{\alpha_{\Sigma} C} \quad (\text{A.5})$$

$$\frac{\partial y_M}{\partial Q} = \frac{\alpha_{\Delta} \hat{y}_R}{C (\alpha_{\Sigma} \hat{Q} + \hat{V}_{\Sigma})} - \frac{\alpha_{\Sigma} (\alpha_{\Delta} \hat{Q} \hat{y}_R + \hat{V}_{\Delta})}{C (\alpha_{\Sigma} \hat{Q} + \hat{V}_{\Sigma})^2} = \frac{\alpha_{\Delta} \hat{y}_R}{C \alpha_{\Sigma} \hat{Q}} - \frac{\alpha_{\Delta} \hat{y}_R}{\alpha_{\Sigma} \hat{Q}} = 0 \quad (\text{A.6})$$

$$\frac{\partial y_M}{\partial V_d} = \frac{1}{C (\alpha_{\Sigma} \hat{Q} + \hat{V}_{\Sigma})} = \frac{1}{C \Sigma} \quad (\text{A.7})$$

$$\frac{\partial y_M}{\partial V_s} = -\frac{\alpha_\Delta \hat{Q} \hat{y}_R + \hat{V}_\Delta}{C (\alpha_\Sigma \hat{Q} + \hat{V}_\Sigma)^2} = -\frac{\Delta}{C \Sigma^2} \quad (\text{A.8})$$

Substituting these equations into Eq. A.9, the beam jitter can be derived. It is assumed that the magnitude of the electrical noise on the Σ and Δ channels are equal.

$$\sigma^2(y_M) = \left(\frac{\alpha_\Delta}{\alpha_\Sigma C} \right)^2 \sigma^2(y_R) + \frac{\sigma^2(V)}{C^2 \Sigma^4} (\Sigma^2 + \Delta^2) \quad (\text{A.9})$$

Appendix B

Moore-Penrose Pseudo-Inversion

The inverse of a square matrix is trivial to calculate, provided it is not a singular matrix. For a non-square matrix, a pseudo-inverse can be calculated. $\mathbf{x}$ is an m -dimensional column vector, $\mathbf{y}$ is an n -dimensional column vector and $\mathbf{M}$ is an m -by- n matrix. $\mathbf{M}$ can be re-written in terms of m n -dimensional column vectors, $\mathbf{M}_i$.

$$\mathbf{y} = \mathbf{M}\mathbf{x} = \begin{pmatrix} \mathbf{M}_1 & \mathbf{M}_2 & \cdots & \mathbf{M}_m \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_m \end{pmatrix} \quad (\text{B.1})$$

By performing a left multiplication of $\mathbf{M}^T$ to both sides of Eq. (B.1), the matrix becomes square.

$$\mathbf{M}^T \mathbf{y} = \mathbf{M}^T \mathbf{M} \mathbf{x} = \begin{pmatrix} \mathbf{M}_1 \cdot \mathbf{M}_1 & \mathbf{M}_1 \cdot \mathbf{M}_2 & \cdots & \mathbf{M}_1 \cdot \mathbf{M}_m \\ \mathbf{M}_2 \cdot \mathbf{M}_1 & \mathbf{M}_2 \cdot \mathbf{M}_2 & \cdots & \mathbf{M}_2 \cdot \mathbf{M}_m \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{M}_m \cdot \mathbf{M}_1 & \mathbf{M}_m \cdot \mathbf{M}_2 & \cdots & \mathbf{M}_m \cdot \mathbf{M}_m \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_m \end{pmatrix} \quad (\text{B.2})$$

$\mathbf{M}^T \mathbf{M}$ is a m -by- m square matrix. If this is non-singular, then it can be inverted in the same manner as a any other square matrix. Eq. (B.3) shows the result of this inversion.

$$\left(\mathbf{M}^T \mathbf{M} \right)^{-1} \mathbf{M}^T \mathbf{y} = \mathbf{x} \quad (\text{B.3})$$

Thus the pseudo-inversion, $\mathbf{M}^+$ is defined as shown in Eq. (B.4).

$$\mathbf{M}^+ = \left(\mathbf{M}^T \mathbf{M} \right)^{-1} \mathbf{M}^T \quad (\text{B.4})$$

B.1 Interpretation of the Result

The Moore-Penrose pseudo-inversion is used in the least squares calculation of the BPM resolution. If n is greater than m for an m -by- n matrix, then the system of simultaneous equations is over-constrained. Therefore the pseudo-inversion gives a least squares estimate for the elements of column vector $\mathbf{x}$. The least squares method is a model independent approach to calculating the resolution of the BPM system; this makes the least squares method a very robust means of calculating the resolution.

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