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DEMAND SHIFTS AND IMPERFECT COMPETITION

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Demand shifts and imperfect competition

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Abstract

The effects of demand shifts on output, price and profits in imperfectly competitive industries with no entry or exit are derived. Four types of demand shift are modelled: additive and multiplicative shifts of the demand and inverse demand functions. Necessary and sufficient conditions for output, price and profits to increase with an outward demand shift are derived for each case. Either output or the price, but not both, may fall with a positive demand shift. Profits may also fall for a Cournot oligopoly. Log-concavity of the direct demand function is sufficient for the standard results to go through.

JEL classification: D42, L12, L13
Keywords: comparative statics, monopoly, oligopoly, log-concavity.

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1. Introduction

How does a shift in the demand function affect prices, quantities and profits in imperfectly competitive industries? The standard expectation for a monopoly with constant marginal cost is that both output and profits will increase when demand shifts out while the price rises if and only if the price elasticity falls. Unless, however, some structure is imposed on the way that demand shifts, and the shape of the demand function is known, we cannot say more about the price effect, and the output effect remains unclear. Two facts about demand shifts are well known. First, if the demand function is subject to a multiplicative shift the price elasticity of demand does not change and thus the price remains constant. Second, when a linear inverse demand function is subject to a parallel shift both the price and industry output increase.

In this paper we analyze the effects of demand shifts in general, and apply the framework to additive and multiplicative shifts of both the inverse demand function and direct demand. In each case necessary and sufficient conditions are derived for the signs of the output and price effects. Standard comparative statics techniques are used. The focus is on the role of the curvature of the demand function and on the effects the demand shift has on inverse demand and on industry marginal revenue. With some standard functional forms the price falls and, more surprisingly, in other cases output can fall with an outward shift of demand. When the industry is monopolised profits always increase, but we find a condition under which a Cournot oligopoly suffers a reduction in its profits as demand shifts out and this condition is applied to additive and multiplicative shifts of inverse demand. Generally when the demand function is log-concave the price, output and profit effects have the expected signs, while demand functions that are log-convex, such as iso-elastic ones, can produce unusual results.

The fact that a demand increase may be associated with a price fall has implications for the analysis of both collusion and the welfare effects of advertising and quality choice. Rotemberg and Saloner (1986) argued that a cartel may drop its price in the face of a demand increase, not because cartel discipline has broken down, but because this reduces the incentive for firms to cheat. The cartel puts up with profits below the short-run maximal level in order to deter cheating. Our model offers a simple alternative explanation. A price drop may maximize short-run profits for the cartel. Dixit and Norman (1978) examine the welfare economics of advertising and argue that if the monopoly price increases with advertising then the level of

advertising is socially excessive at the margin. Shapiro (1980) points out that a price increase would typically occur only with persuasive advertising (rather than informative advertising). In our model an upward shift in the inverse demand function causes a fall in the price when demand is log-convex, which implies that persuasive advertising is under-supplied.

The possibility of a perverse output effect is potentially of some importance to environmental and natural resource policy. Suppose that oil is supplied by a cartel, the demand for oil shifts as temperature changes and that oil consumption itself affects global temperature positively. A rise in global temperature would shift the demand curve for oil for heating purposes inwards. When the perverse output effect operates the result would be an increase in the amount of oil bought, which would then reinforce rather than offset the climate change problem.

Three related papers are Quirmbach (1988), Hamilton (1999) and Baldenius and Reichelstein (2000). Quirmbach uses a conjectural variations model of oligopoly and shows how the effect of a demand shift varies with the degree of competition. He shows that a necessary condition for oligopoly profits to fall is that the price falls, and finds an example where this holds. Baldenius and Reichelstein (2000) examine the effects on a monopolist of an additive shift to inverse demand and find sufficient conditions to predict both the sign and the size of the price change. Our analysis generalizes their results. We find a condition that is both necessary and sufficient for the price to increase, and also show that when this condition is not satisfied profits may fall when there is an oligopoly. Hamilton (1999) examines the implications of demand shifts when there is free entry and firms differ in their efficiency levels.

Section 2 presents the analysis of monopoly. Section 3 examines the case of Cournot oligopoly, where the focus is on the profits effect. Section 4 contains brief conclusions.

2. The monopoly model

2.1 Comparative statics for general demand functions

The inverse demand (willingness-to-pay) function is $P = P(Q, S)$ where P is the price, Q is output and S is the shift factor. Inverse demand is twice continuously differentiable in both arguments and strictly decreasing in Q, i.e. $P_Q < 0$, where the subscript denotes a partial derivative. A rise in S unambiguously shifts the whole

inverse demand function out ($P_S > 0$).[1] The demand function is $Q(P, S)$ and we call this direct demand. The price elasticity of demand is $\varepsilon \equiv - P/QP_Q$. The relative curvature of inverse demand is $E \equiv - QP_{QQ}/P_Q$, which is also the elasticity of the slope of inverse demand or the slope elasticity for short. Since demand slopes down E has the same sign as P_{QQ}, so E is positive (negative) when demand is strictly convex (concave) and zero for linear demand. The relative curvature coefficient may also be expressed as a function of direct demand and its first and second derivatives: $E = QQ_{PP} / Q_P^2$. This follows from differentiating $P_Q = 1/Q_P(P(Q, S), S)$ for a fixed S to give $P_{QQ} = -Q_{PP}P_Q / Q_p^2$. See Vives (1999, pp. 104-5) for a summary of the role of E in the analysis of marginal cost changes in Cournot models.

Marginal cost, c, is constant and positive in general, though at times the implications of zero marginal cost will be mentioned. Marginal revenue is $MR \equiv P + QP_Q$ and its slope is $MR_Q = 2P_Q + QP_{QQ}$. The difference between the slope of marginal revenue and that of inverse demand is $MR_Q - P_Q = P_Q(1 - E)$. Thus marginal revenue is steeper than inverse demand (in absolute terms) when $E < 1$. E equals 1 when inverse demand has the logarithmic form $P = a - b\log(Q)$ where a or b may be functions of S. Direct demand in this case is exponential. Relative curvature of inverse demand is related to the log-concavity of direct demand. A function is log-concave if and only if its logarithm is concave. The second derivative of $\log(Q)$ is

$$\frac{d^2 \log(Q)}{dP^2} = \frac{QQ_{PP} - Q_P^2}{Q^2} = \left(\frac{Q_P}{Q}\right)^2 (E - 1)$$

since $E = QQ_{PP} / Q_P^2$. Thus direct demand is log-concave in the price if and only if $E \leq 1$. Many standard demand functions are log-concave, including those that are strictly concave, linear and exponential. Essentially log-concavity means that demand is no more convex than an exponential function. A standard demand function that is log-convex everywhere is the iso-elastic one, $Q = P^{-\alpha}$, where the elasticity is $\alpha > 0$. For this function $E = 1 + 1/\alpha$. The direct demand function $Q = - \log(P)$ for $P \leq 1$ has $E = Q$, so demand is log-convex for $Q > 1$ and log-concave otherwise.

[1] See Johnson and Myatt (2004) for an analysis of monopoly pricing where the firm is able, for example by changing product design, to rotate the demand curve because the dispersion of customers' valuations changes.

The firm chooses output to maximize its profits $\Pi = [P(Q, S) - c]Q$. The first-order condition requires marginal revenue to equal marginal cost.

$$P(Q^*, S) + Q^* P_Q(Q^*, S) - c = 0\,, \tag{1}$$

where Q^* denotes the profit-maximizing output level. Standard theory tells us that at an interior solution demand must be elastic, i.e. $\varepsilon(Q^*, S) > 1$. It would pay a monopolist facing inelastic demand to cut output since this would lower costs and raise revenue. Rearranging the first-order condition, (1), gives the monopolist's price-cost mark-up: $(p^* - c)/p^* = 1/\varepsilon$. The mark-up and the price are inversely related to the price elasticity.

The sufficient second-order condition is

$$2P_Q(Q^*, S) + Q^* P_{QQ}(Q^*, S) < 0\,,$$

that is marginal revenue should be decreasing in output at the optimum.[2] The second-order condition can be rewritten using the relative curvature coefficient as $MR_Q = P_Q(2 - E) < 0$ or $2 - E > 0$. This certainly holds if demand is concave, but otherwise the degree of convexity must not be too large. Log-concavity of demand is sufficient. With iso-elastic demand, which has $E = 1 + 1/\alpha$, the second-order condition holds as long as $\alpha > 1$(which in any case must be assumed for the first-order condition to hold).

The output effect is (the star superscripts are mostly dropped from now on)

$$\frac{dQ^*}{dS} = -\frac{MR_S}{MR_Q} = -\frac{P_S + QP_{QS}}{P_Q(2 - E)}\,. \tag{2}$$

This expression has the same sign as $MR_S = P_S + QP_{QS}$. Intuitively if a rise in S increases marginal revenue at the initial output level the firm wants to raise the quantity sold. When, however, $P_{QS} < 0$ a rise in S may reduce marginal revenue, because the inverse demand function becomes steeper and thus the revenue gained

[2] If this holds everywhere, i.e. for all values of Q and not just Q^*, then the solution to (1) is unique.

from an extra unit is lower than otherwise. A sufficient condition for marginal revenue to rise with S is that the slope of inverse demand is non-decreasing in S.

An increase in S, for a given quantity, raises the price directly. It can also alter marginal revenue and thus induce the firm to change its quantity, which means that the price adjusts along the new demand curve. The total effect of S on the optimal price, P^*, is

$$\frac{dP^*}{dS} = P_S + P_Q \frac{dQ^*}{dS} = P_S - P_Q \frac{MR_S}{MR_Q} = \frac{P_S(1 - E - QP_{QS}/P_S)}{(2 - E)}. \qquad (3)$$

It follows that P_S/MR_S must be positive and greater than $P_Q/MR_Q = 1/(2 - E)$ for the price derivative to be negative.[3] Thus it is only when the "normal" case applies, i.e. a demand shift raises marginal revenue, that an unusual price effect is possible. On reflection this is intuitive. If the increase in S has a strong positive effect on marginal revenue then the firm wants to increase output significantly. This may cause the price on the new demand curve to be below the one set before the shift. The final version of (3) is also instructive. The term QP_{QS}/P_S is the elasticity of P_S with respect to Q, and may be called the shift elasticity. Thus (3) gives a type of Marshall-Lerner condition for the price effect: a demand increase raises the price if and only if the sum of the slope and shift elasticities is below 1.

An immediate result that comes from inspection of the first version of (3) is that the total effect of the demand shift on price is below the partial effect, i.e. $dP^*/dS < P_S$, if and only if the output effect is positive. This generalizes part (i) of the theorem in Baldenius and Reichelstein (2000), which applies to the special case where $P_S = 1$.

The main general result is that the output and the price effects for a monopoly cannot both be negative. Formally (3) is negative only if MR_S is positive, but this implies that dQ^*/dS is positive. Intuitively if the price falls then output must have increased since the new demand curve is above the old one, while an output reduction must similarly be associated with a price increase. We can also use a revealed preference argument, at least in the monopoly case. If both quantity and price were to

[3] Edwards (1950) in his analysis of price discrimination calls P_Q/MR_Q the "slope ratio". It can be shown that the effect of a change in marginal cost on the monopoly price – the cost-pass-through coefficient – is $dP^*/dc = 1/(2 - E)$, which equals the slope ratio. This is below one if and only if the demand function is strictly log-concave.

fall then profits would be reduced, but an outward demand shift always gives the firm the opportunity to increase profits by keeping either the price or the quantity fixed. By the envelope theorem the total effect on maximized profit of a rise in S is the partial effect, i.e. $d\Pi^*/dS = QP_S > 0$. We now apply (2) and (3) to the four types of demand shift.

2.2 Additive shifts of the direct demand function

There are several possible causes of additive shifts to the direct demand function. The demand for energy used for heating may be modelled with the external temperature as an additive shift factor (see Friedman, 1987, and Cowan, 2004). A durable goods monopolist who sells more in the current period reduces the next period's demand function by the same amount. In international trade an import quota entails that the residual demand facing a domestic monopolist is the market demand curve less the quota. A change in the quota would then shift the residual demand curve additively. In a Cournot oligopoly model a reduction in the output expected to be produced by a firm's rivals shifts that firm's residual demand curve additively (and it then acts a monopolist against the residual demand).

The demand function is $Q = S + q(P)$ where $q(P)$ is a decreasing function. Inverse demand is $P(Q, S) = p(Q - S)$ with $p' < 0$. Marginal revenue is $MR = p(Q - S) + Qp'(Q - S)$. The effect of a change in S on marginal revenue is $MR_S = -[p' + Qp''] = -p'(1 - E)$. Substituting into equation (2) the output effect is

$$\frac{dQ^*}{dS} = \frac{1-E}{2-E}, \tag{4}$$

so output increases if and only if $E < 1$, i.e. demand is strictly log-concave. Using (4) in (3) the price effect is

$$\frac{dP^*}{dS} = \frac{-p'}{2-E},$$

which is positive. This is intuitive: at the initial price the increase in S raises the quantity demanded but does not change the slope of demand, so the price elasticity falls and the firm wants to raise the price.

Proposition 1. When the direct demand function shifts additively the monopoly price rises. Output increases if and only if the relative curvature of inverse demand is below 1, i.e. demand is strictly log-concave.

With log-convex demand an additive shift cuts output because the inverse demand function becomes so steep at the initial quantity that marginal revenue falls. We illustrate log-convexity with three examples.

Example 1. Let demand be $Q = S + P^{-\alpha}$ for $\alpha > 1$ and initially $S = 0$. The value of E is $(1 + 1/\alpha)Q/(Q - S)$ so $E > 1$ for all non-negative values of S and in this region increases in S always cut output. When S falls below zero, however, there comes a point at which $E = 1$, defined by $S = -Q/\alpha$, and as S falls further E goes below 1 so in this region demand is log-concave and a demand shift raises output. A related point for the case where S is negative is made by Bulow *et al*. (1985). They analyze a Cournot duopoly model with an iso-elastic demand function and show that the reaction function is upward sloping close to the monopoly output level but for larger outputs of the rival the reaction function becomes negatively sloped.

For the next two examples we relax the usual assumption and suppose that marginal cost is zero to ensure that the first-order conditions can be solved analytically.

Example 2. Let $Q = S - \beta\log(P)$ for $0 < S < \beta$. Inverse demand is $P = e^{-(Q - S)/\beta}$. The profit-maximizing output, $Q^* = \beta$, is independent of S. In general $E = Q/\beta$ and when output is at the profit-maximizing level $E = 1$.[4]

Example 3. Let $P = e^{-\sqrt{Q-S}}$ and assume that $S \leq 1$. This is a version of the demand function discussed by Martin (2002, pp. 22-24) and attributed originally to Stackelberg (1934, pp. 126-30). The profit-maximizing output is $Q^* = 2 + 2\sqrt{1 - S}$, which is decreasing in S for all feasible values of S.

[4] Amir and Lambson (2000, p 242, Example 3) use a similar exponential demand function in a Cournot model to show that each firm has the dominant strategy of setting a constant output level.

2.3 Additive shifts of the inverse demand function

Persuasive advertising, a quality improvement, a change in the price or specification of a related good, or a reduction in a specific tax may increase willingness to pay by a constant amount.[5] The inverse demand function can be written as $P = S + p(Q)$ where $p'(Q) < 0$. Marginal revenue is $MR = S + p(Q) + Qp'(Q)$ so $MR_S = P_S = 1$. With no effect on the slope of inverse demand a rise in S increases marginal revenue by the same amount and optimal output goes up. The change in output is

$$\frac{dQ^*}{dS} = \frac{-1}{p'(Q)(2-E)}. \tag{5}$$

The price derivative, (3), becomes

$$\frac{dP^*}{dS} = \frac{1-E}{2-E}. \tag{6}$$

Proposition 2. When inverse demand shifts additively the effect on output is positive. The monopoly price rises if and only if the relative curvature of inverse demand is below 1, i.e. demand is strictly log-concave.

With linear demand $E = 0$ so $dP^*/dS = ½$. When $p(Q) = Q^{-1/\alpha}$ for $\alpha > 1$ and $c > S$, $E = 1 + 1/\alpha$ and a demand shift reduces the price.[6] The price is $P^* = (\alpha c - S)/(\alpha - 1)$. With logarithmic inverse demand, i.e. $p(Q) = a - b\log(Q)$ for $b > 0$, we have $E = 1$ and $P^* = c + b$ so the price stays constant in the face of a demand increase.

Baldenius and Reichelstein (2000) analyse this case and find that the price rises if $p(Q)$ is log-concave, the shift in inverse demand is bounded above by c and the starting level of S is 0. It can be shown that this set of jointly sufficient conditions implies that $\varepsilon E < 1$ where εE is evaluated at the starting level of S, i.e. 0. Since $\varepsilon > 1$ an implication is that $E < 1$ so the sufficient condition in Proposition 2 is satisfied. The condition in Proposition 2 has the advantage that it is both necessary and sufficient.

[5] Fluet and Phaneuf (1997) use an inverse demand function with an additive shift term in a menu costs model.

[6] Baldenius and Reichelstein (2000) note that the price falls when an iso-elastic demand curve shifts upwards additively.

2.4 Multiplicative shifts of the direct demand function

This case is standard so the discussion here is brief. Multiplicative shifts in the demand function may occur because of income shocks or changes in the number of customers (caused, for example, by informative advertising). The monopoly price stays fixed. Formally demand is $Q = Sq(P)$ for $q(P) > 0$, $q'(P) < 0$ and $S > 0$, and inverse demand is $P = p(Q/S)$. Some manipulation confirms that the price derivative in (3) is indeed zero. It is more natural in this case to work with the direct demand function. The elasticity is $\varepsilon = -PQ_p/Q = -Pq'(P)/q(P)$ which is independent of S. Output, of course, increases and $dQ^*/dS = q(P^*) > 0$.

2.5 Multiplicative shifts of inverse demand.

The inverse demand function that is subject to multiplicative shocks may be written as $P = Sp(Q)$ for $S > 0$ and $p(Q) > 0$ with $p'(Q) < 0$. This may be because the monopolist sells an input to a competitive industry that is subject to multiplicative shocks to either the price of downstream output or to marginal productivity. Green and Porter (1984) use this type of inverse demand function in their analysis of collusion with imperfect price information.[7] Dasgupta and Stiglitz (1980) also use a version of this function in their model of R&D and oligopoly. Another interpretation has the product being subject to an *ad valorem* tax. In this case the consumer price is $p(Q)$ and the producer price is $P = (1 - v)p(Q)$ where v is the tax rate.

Marginal revenue is $Sp(Q) + SQp'(Q)$. An increase in S raises marginal revenue proportionally so output certainly rises: $MR_S = MR/S = p(Q)(1 - 1/\varepsilon)$.[8] Using this in (3) gives the price derivative

$$\frac{dP^*}{dS} = \frac{p}{(2-E)}\left[1+\frac{1}{\varepsilon}-E\right].$$

Proposition 3. With a multiplicative shift in inverse demand the output effect is positive. The price rises if and only if $E < 1 + 1/\varepsilon$. A sufficient condition for the price

[7] In their empirical work on the Green and Porter model, however, Porter (1983) and Ellison (1994) use a direct demand function with multiplicative uncertainty.

[8] Note the importance here of the assumption that marginal cost is positive. When marginal cost is zero the profit function is proportional to S and thus the profit-maximizing output is independent of S.

to rise is that demand is less convex than an iso-elastic function, for example a log-concave function.

With iso-elastic demand, i.e. $P(Q, S) = SQ^{-1/\varepsilon}$, $E = 1 + 1/\varepsilon$ so the price remains constant. The direct demand function is multiplicative in a function of S so the price elasticity is not affected. Only when the inverse demand function has relative curvature that exceeds that of an iso-elastic function is the price effect negative. Essentially demand has to be very convex (but note that the second-order condition must still hold) for the price to fall.

Example 4. Let inverse demand be $P = S(1 - 3Q + 6Q^2)$ for $Q < 0.25$ and marginal cost be 1. The first-order condition is a quadratic equation and the solution that satisfies the second-order condition is the smaller root $Q^* = \left(1 - \sqrt{2c/S - 1}\right)/6$. When $S \in [1.949, 2)$ an increase in S within this range causes the price to be cut. For example a move from $S = 1.95$ to $S = 1.96$ reduces the monopoly price from 1.360375 to 1.36.

2.6 Summary for monopoly

Table 1 summarizes the results for monopoly. For the three cases of interest (the two types of additive shock, and the multiplicative shock to inverse demand) strict log-concavity of demand implies that both the output and price effects are positive. There is duality between the two additive shock cases.

One point to note is that a demand function may fit into two of the categories of Table 1. Thus the inverse demand function in Example 2, $P = e^{-(Q - S)/\beta}$, is a case of both an additive shift to direct demand and a multiplicative shift of inverse demand since P is multiplicative in a function of S. Similarly with linear demand an additive shift to direct demand is also an additive shift of inverse demand. Finally the direct demand $Q = Sp^{-\alpha}$ is multiplicative in S while its inverse demand is multiplicative in a function of S.

We have assumed constant marginal cost, but it is straightforward to allow for rising or falling marginal cost. The *sign* of the output effect is unaffected, since this depends on the effect S has on marginal revenue and not on the properties of the cost function (though the *size* of the output effect does depend on the rate at which

marginal cost changes). The price effect, though, is altered in an intuitive way. The price is more likely to rise when marginal cost increases with output than when it is constant. For example the price effect will be strictly positive when the cost function is convex and there are multiplicative shocks to direct demand.

	Direct demand function	Inverse demand	Effect on output	Effect on price
Additive shift of direct demand	$Q = S + q(P)$	$P = p(Q - S)$	Positive iff $E < 1$	Positive
Additive shift of inverse demand	$Q = q(P - S)$	$P = S + p(Q)$	Positive	Positive iff $E < 1$
Multiplicative shift of direct demand	$Q = Sq(P)$	$P = p(Q/S)$	Positive	Zero
Multiplicative shift of inverse demand	$Q = q(P/S)$	$P = Sp(Q)$	Positive	Positive iff $E < 1 + 1/\varepsilon$

Table 1: Output and Price Effects for Monopoly

3. Demand shifts and oligopoly

We extend the analysis of the previous section to Cournot oligopoly. The main focus is on the impact of a demand shift on oligopoly profits. A standard symmetric Cournot model is used. The number of firms, n, is fixed and exceeds 1. Each firm has a constant marginal cost of c. Products are homogeneous. Industry output is Q, the output of firm i is q_i and its profits are $\Pi_i = [P(Q, S) - c]q_i$. The first-order condition is

$$\frac{d\Pi_i}{dq_i} = P(Q,S) + q_i P_Q(Q,S) - c = 0. \qquad (7)$$

The Nash equilibrium in outputs is characterized by (7) holding for each firm. With symmetry $q_i = q \equiv Q/n$ for all i. The second-order condition is

$$\frac{d^2\Pi}{dq^2} = 2P_Q + qP_{QQ} < 0,$$

which can be written as $2n - E > 0$. For $n > 1$ the set of demand functions that satisfies the second-order condition is larger than under monopoly. For example when the demand function is iso-elastic with elasticity ε and $E = 1 + 1/\varepsilon$ the second-order condition holds for $\varepsilon > 1/(2n - 1)$, which allows demand to be inelastic. The higher n goes the smaller is the lower bound on the elasticity. It is standard in oligopoly models to assume stability (see Dixit, 1986). The most general stability condition is $n + 1 - E > 0$ (Seade, 1980a and 1980b).[9] Schlee (1993) proves that this condition also implies both uniqueness and symmetry of the Cournot equilibrium. If the stability condition holds then the second-order condition also automatically holds (for $n > 1$), since $2n - E > n + 1 - E > 0$.

The effect of a shift in S on each firm's output comes from total differentiation of equation (7) and is

$$\frac{dq}{dS} = \frac{-[P_S + qP_{QS}]}{P_Q[n+1-E]}. \tag{8}$$

The denominator of (8) is negative when the stability condition holds. The term $P_S + qP_{QS}$ is the effect of a rise in S on the firm's perceived marginal revenue. With an additive shift to demand the expression $P_S + qP_{QS}$ has the same sign as $1 - E/n$, so the output effect is negative when $n < E$, i.e. when the Hahn condition does not hold. For example let demand be iso-elastic ($Q = S + P^{-\alpha}$, with initially $S = 0$) and assume a duopoly. The stability condition requires that $3 > E$, which implies $\alpha > 0.5$, and the output effect is negative if in addition $\alpha < 1$ (i.e. demand is inelastic at $S = 0$).

Substituting (8) into $dP/dS = P_S + P_Q n dq/dS$ gives the effect of a demand shift on the price of a Cournot industry:

[9] A stronger stability condition is that each firm's perceived marginal revenue should be decreasing in the other firms' total output. This requires that $n - E > 0$ and is known as the Hahn condition (Hahn, 1962).

$$\frac{dP}{dS} = \frac{P_S[1 - E - QP_{QS} / P_S]}{n + 1 - E}. \tag{9}$$

The numerator of equation (9) is the same as that of the final version of (3), so the sign of the price effect is the same as under monopoly. Again since a fall in output increases the price we cannot have both the price and output falling.

How are profits affected? When both output and the price rise (or one is constant) profits increase. The two remaining cases are when output falls (and thus the price rises), and when the price falls (and output rises). The first result is that if output falls then profits must increase. To see this note that the profit effect is

$$\frac{d\Pi}{dS} = q\frac{dP}{dS} + (P - c)\frac{dq}{dS} = qP_S + \frac{dq}{dS}\left(P + QP_Q - c\right). \tag{10}$$

The first expression in equation (10) accounts for the fact that a demand shift affects profits through the change in the margin, dP/dS, on sales q and the change in the volume of sales at the existing margin. The second expression in (10) uses the fact that the total price effect is $dP/dS = P_S + P_Q n dq/dS$. The term $P + QP_Q - c$ is industry marginal profitability evaluated at the Cournot equilibrium and is negative (this can be shown by adding the first-order conditions across the n firms and is intuitive since a Cournot industry produces more than a collusive one). Thus when $dq/dS < 0$ expression (10) is certainly positive. It follows that a necessary condition for the profit effect to be negative is that the price falls.

The general expression for the change in profits with a demand shift is

$$\frac{d\Pi}{dS} = \frac{qP_S[2 - E + (1 - n)qP_{QS} / P_s]}{n + 1 - E} \tag{11}$$

where we have used (7), (8) and (9) in (10). We now consider the two types of demand shift that admit price reductions: additive and multiplicative shifts to inverse demand. With an additive shift to inverse demand the slope remains constant, so $P_{QS} = 0$. Thus the profit effect is negative if and only if $E > 2$. This could not hold with log-concave demand, but when $p(Q)$ is iso-elastic, i.e. $P = S + Q^{-1/\alpha}$, $E = 1 + 1/\alpha$ so the perverse case occurs if and only if $\alpha < 1$ (so demand is inelastic at $S = 0$). To

explore this further note that stability requires $n > 1/\alpha$. The full set of conditions for the equilibrium to be stable and for profits to fall is $n > 1/\alpha > 1$.

Seade (1985) showed for marginal cost increases that there is a perverse profits effect, i.e profits rise, when $E > 2$. The mathematical structure of the two problems is the same. In both case profits are $[S + p(Q) - c]q$. With the demand shift interpretation inverse demand is $S + p(Q)$, while in the case of a cost shift S is the cost shift parameter and $p(Q)$ is inverse demand. Either way the comparative statics depend on the curvature of $p(Q)$.

With a multiplicative shift to inverse demand the square-bracketed term in the numerator of (11) becomes $2 - E + (n - 1)/n\varepsilon$ and the profit effect has the same sign as this expression. Again a negative profit effect cannot occur when demand is log-concave because $2 - E > 0$ and $n > 1$. With iso-elastic demand the price stays constant as S changes and thus profits increase. The possibility of a profit fall remains, however, for demand functions that locally are more convex than iso-elastic functions.

Example 5. We use the same functions as in Example 4 and assume a duopoly. The symmetric equilibrium is characterized by the first-order condition $P + 0.5QP_Q - c = 0$, which is a quadratic equation in Q. Industry output is the smaller root of this equation $Q^* = \left(4.5 - \sqrt{48c/S - 27.75}\right)/24$. For values of S above 1.719 profits fall. For example when S rises from 1.72 to 1.725 the Cournot price falls from 1.139 to 1.135 and profits are cut from 0.02384 to 0.023724. The stability condition is $3 > E$, which holds because $E = 2.16$ at the lower value of S and 2.38 at the higher value.

The oligopoly results are summarized in the following proposition.

Proposition 4. For a symmetric Cournot oligopoly with a fixed number of firms the qualitative effects of a demand shift on the price are the same as for monopoly. An additive shift to inverse demand cuts profits if and only if $E > 2$, for example with a constant elasticity that is below unity. With a multiplicative shift to inverse demand profits fall if and only if $E > 2 + (n - 1)/n\varepsilon$.

4. Conclusion

This paper has assessed the price, output and profit effects of a demand shift on oligopoly and monopoly equilibria. The importance of both the type of demand shift and the value of the relative curvature coefficient has been highlighted. By imposing some structure on the types of demand shift we have been able to characterize the comparative statics effects fully. For the three cases of interest (both types of additive shift and the multiplicative shift of inverse demand) all the results that hold with a linear demand function also hold for strictly log-concave functions. The equilibrium is unique and stable, the cost-pass-through coefficient is less than one, the price rises with a demand shift, the output effect is positive, the price rise is less than the shift in inverse demand, the second-order condition automatically holds and oligopoly profits rise. Meanwhile demand functions that are log-convex, such as iso-elastic functions, produce unusual results. Output falls when there is an additive shift to demand, the price falls with an additive shift to inverse demand and oligopoly profits can fall with either type of shift of inverse demand.

One limitation of the analysis has been the assumption of differentiability of inverse demand with respect to the shift factor. While this assumption is both common and useful it is somewhat restrictive. For large shifts in demand monotone comparative statics techniques that use lattice programming will be required (see Amir and Lambson, 2000, and Vives, 1999, Chapter 2, for application of these techniques to oligopoly).

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