Measurement of electron-phonon interaction time of Niobium using heating effect in SIS tunnel junction

Boon Kok Tan, Ghassan Yassin, Phichet Kittara and Jamie Leech

***Abstract*—The heating of SIS tunnel junctions by local oscillator (LO) power and bias voltage is well known and has been reported previously. In this paper, we present a novel method for recovering the heating parameters from the experimental pumped I-V curves of an SIS device at 700 GHz, together with the coupled LO power and the embedding impedance. Since this is obtained without assuming a particular power law between LO power and junction temperature, we will be able to find τ_{eph}, the electron-phonon interaction time of the superconducting material at various bath temperatures. We would deduce a power law that describes the dependence of the heat flow equation on temperature.**

***Index Terms*—Superconductor-insulator-superconductor devices, electron-phonon interaction time, heat flow, power law.**

I. INTRODUCTION

NIOBIUM based superconductor-insulator-superconductor (SIS) tunnel junctions are currently the most promising heterodyne mixers for submillimetre radio astronomy below $\sim$ 1 THz. The performance and characteristic of an SIS mixer near its energy gap is itself an interesting subject to study. Here radiation near the energy gap is strong enough to break the Cooper pairs. A strong dependency of gap voltage on the local oscillator (LO) pumping level can clearly be observed when the mixer is operating in this frequency range. This is commonly related to the heating effects taking place in the device [1], [2]. The implication of the effect on mixer performance is worth studying since accurate determination of the energy gap is central to the performance characteristic of the mixer. In this paper, we present a simple model for studying the heating effects in SIS tunnel junctions near the energy gap, and demonstrate that the electron-phonon interaction time, τ_{eph}, of the superconducting material can be directly recovered from the dc pumped I-V curves of an SIS device.

II. METHOD OF ANALYSIS

Our SIS device consists of a Nb-AlO$_x$-Nb junction, sandwiched between the Nb wiring layers and ground plane deposited on quartz substrate across a microstrip line. The junction area is typically 1μm $\times$ 1μm with the Nb thickness about 200 nm. The I-V curves measured at different pump levels are shown in Figure 1 and the gap depression with increasing pump level is clearly noted, which is typically in the $\sim$ 0.1 mV range. The critical temperature of Nb used throughout this paper is assumed to be 9.3 K. The measured curves used in our calculations were obtained using a niobium finline mixer in the frequency range of 650-720 GHz. The Josephson current is suppressed by magnetic field during measurement.

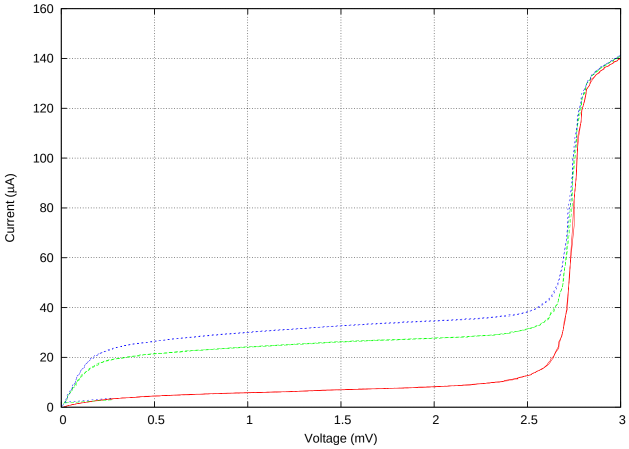

Fig. 1. The measured I-V curve for different LO pump levels.

Although the heating effect by RF signals for hot electron bolometer (HEB) has been widely studied, there is relatively limited information reported on similar effects in SIS junctions. Leone et. al. [3] has previously reported electron heating of a niobium SIS mixer, using Niobium Titanium Nitrate (NbTiN) stripline with Nb-Al-AlO$_x$-Nb junction. They explained that the mechanism responsible for the heating is the energy gap discontinuity at Nb/NbTiN interface. We, however, believe the heating effect in SIS mixers can result from different mechanisms, although there are some similarities to HEB.

As the photon excites quasiparticles to tunnel through the junction barrier, power is dissipated in the junction. In steady state, this power is transfered by thermal conduction through the superconducting ground film to the substrate. For simplicity, we assume that the entire junction is essentially at the same temperature. The heat transfer from the junction to the bath can be described phenomenologically using an effective heat transfer coefficient, α, which may be written as

$$P_{total} = \alpha(T_e)wlt(T_e^n - T_b^n) \tag{1}$$

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where P_{total} is the total power coupled the junction, T_e is the effective electron temperature (junction temperature), T_b is the bath temperature, wlt is the junction volume and n is an index which depends on the model used for heat transfer. Equation 1 is a rather general equation to describe the heat transfer process. The heat transfer coefficient $\alpha(T_e)$, is a temperature dependent parameter defined as

$$\alpha(T_e) = \frac{C_e}{nT_e^{n-1}\tau_{eph}} \tag{2}$$

where C_e is the electron heat capacity and τ_{eph} is the electron-phonon interaction time. In order to incorporate the temperature dependence effect of the heat capacity into the final equation, the low temperature limit heat capacity equation is used to calculate C_e [4].

$$C_e = \frac{2N(\xi_F)\Delta^2}{T_e} \exp\frac{-\Delta}{k_BT_e} \tag{3}$$

Recovery of the embedding parameters, including coupled LO power and embedding impedance, are obtained by matching the measured IV-curves with theoretical models based on Werthamer [5] and BCS theory [6]. We have used CalTech's SuperMix package for this purpose. In recovering the embedding impedance, we have assumed that the heating effect is negligible, which is a reasonable assumption, since we are working with the narrow range of the first photon step. From the BCS equations, the DC and RF power dissipated in the junction at each bias point are given by,

$$P_{rf} = \sum_{n=-\infty}^{n=+\infty} \frac{n\hbar\omega}{e} J_n^2(eV_{LO}/\hbar\omega) I_{dc}(V + n\hbar\omega/e) \tag{4}$$

$$P_{dc} = V_{bias} \times I_{dc} \tag{5}$$

where J_n is the n^{th} order Bessel function of the first kind and V_{LO} the voltage across the junction associated with the LO power, which is calculated from the recovered LO power from SuperMix. V_{bias} is the bias voltage and I_{dc} is given by the standard BCS unpumped I-V relation

$$I_{dc} = \frac{1}{eR_n} \int_{-\infty}^{\infty} N(\xi)N(\xi - eV)[f(\xi - eV) - f(\xi)]d\xi \tag{6}$$

where R_n is the junction normal state resistance, $f(\xi)$ is the Fermi-Dirac distribution function and $N(\xi)$ describes the density of states. Note that both functions, $f(\xi)$ and $N(\xi)$, are explicit functions of the voltage-dependence electron temperature, in our case,

$$N(\xi) = \frac{\xi}{\sqrt{\xi^2 - \Delta^2(T_e)}} \tag{7}$$

$$f(\xi) = \frac{1}{\exp\frac{\xi}{k_BT_e} + 1}. \tag{8}$$

We start with an initial guess of τ_{eph} and index n, and use equation 5 and equation 4 to give P_{total} into equation 1. By substituting equation 2 and equation 3 in equation 1, we can then solve for T_e. The new gap voltage Δ is then numerically re-computed using the standard temperature dependent expression,

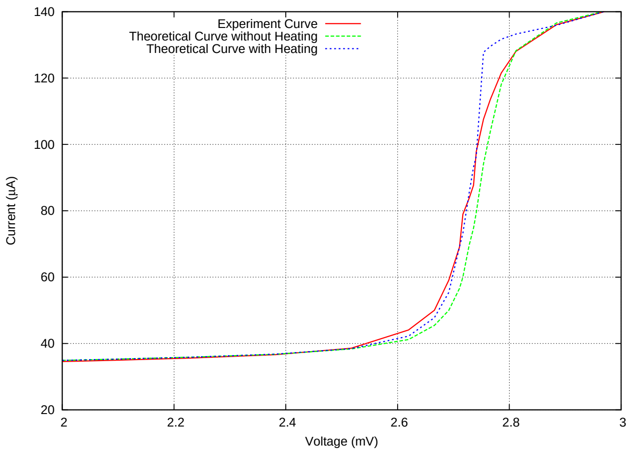

Fig. 2. The matching of simulated I-V curves with and without the heating effect with the experimental I-V curve.

$$\frac{1}{N(\xi_F)V} = \int_0^{\hbar\omega_c} \frac{\tanh(\frac{1}{2k_BT_e}\sqrt{\xi^2 + \Delta^2})}{\sqrt{\xi^2 + \Delta^2}} d\xi \tag{9}$$

where $N(\xi_F)$ is the density of states at Fermi level, V is the attractive potential, and $\hbar\omega_c$ is the Debye frequency. It should be noted that the above equation gives a specific value of Δ at a certain electron temperature, where in reality the I-V curve near the transition is rounded.

Since all the junction parameters found using SuperMix are normalized to the gap voltage, the theoretical current calculated using the previously recovered LO power and embedding impedance, is simply the product of the normalized current from SuperMix and the gap current given as V_{gap}^2/R_n. This entire process is repeated throughout the whole range of bias voltages. The generated I-V curve is then compared to the measured curve, and the parameters are optimized until both theoretical and measured curves are matched.

III. RESULT AND DISCUSSION

Figure 2 shows the I-V response of our SIS junction using the highest pumping level curve shown in figure 1. Also in the figure we show the theoretical I-V curves calculated using embedding parameters recovered by the SuperMix model with and without the heating effect. The curve with heating is simulated using the value of τ_{eph} = 7.8 ns and index $n = 4$. The glitch in the heated curve near the knee of the gap voltage is probably due to the electron temperature being calculated without the rounding of the gap voltage in BCS equation, as noted previously. This can potentially be corrected by incorporating the imaginary lifetime-broadening parameter in the density of states equation [7], [8]. Unfortunately, the value of this parameter for Nb is difficult to obtain from the literature. This sudden step change is also seen in figure 3, where T_e, gap voltage and power across the junction are plotted as a function of bias voltage.

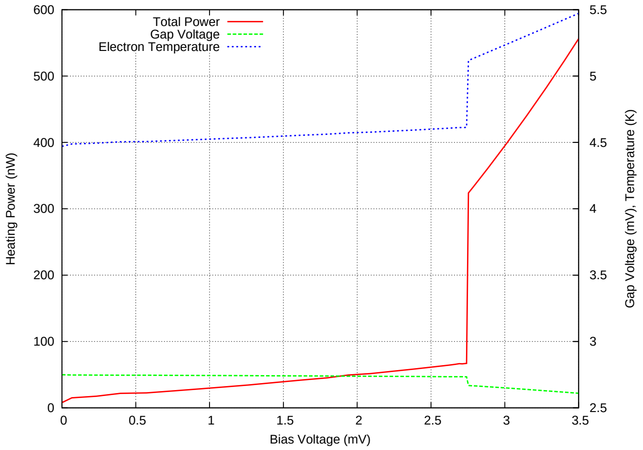

Fig. 3. The heating power coupled to the junction as a function of bias voltage. Shown together are the junction temperature and the gap voltage change corresponding to the bias voltage.

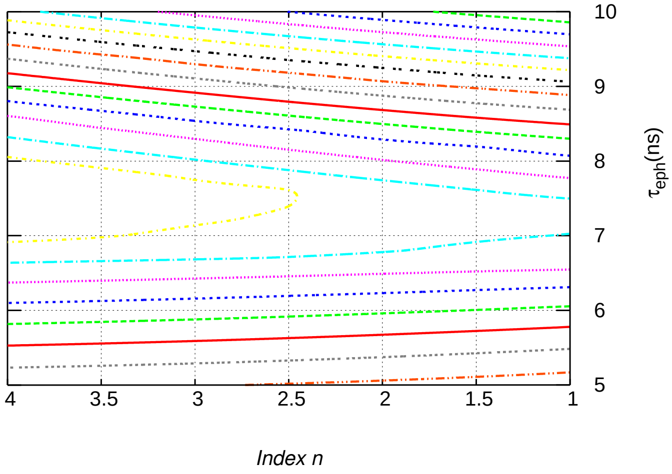

Fig. 4. Contour of the error surface with different guess value of τ_{eph} and index n.

Figure 4 shows the error surface contour of different guess value of n and τ_{eph}. It is clear from the figure that the change in n had little effect on the error surface. In other words, the index n is not as significant as τ_{eph}. This can easily be explained if we assume the difference between T_e and T_b is small compared to T_e and T_b. The term $(T_e^n - T_b^n)/nT_e^{n-1}$ can then be approximated as $T_e - T_b$ without index n in the final equation.

It should be noted that heating parameters can also be recovered solely from the unpumped I-V curve. A procedure similar to the one described above can be employed, with the junction model replaced by a modified temperature dependence polynomial equation. The results obtained in this method can be compared with those obtained using the pumped I-V curve to give better constrains on the heating parameters recovered.

IV. CONCLUSION

We present a simple model for measuring parameter of a superconductor by recovering the change in the electron temperature from the experimentally pumped I-V curve. Our results also show that the heat transfer equation does not depend significantly on the index n.

REFERENCES

[1] B. Leone, B. D. Jackson, J. R. Gao, and T. M. Klapwijk, “Geometric heat trapping in niobium superconductor-insulator-superconductor mixers due to niobium titanium nitride leads,” *Applied Physics Letters*, vol. 76, no. 6, pp. 780–782, 2000.

[2] P. Dieleman, T. M. Klapwijk, S. Kovtonyuk, and H. van de Stadt, “Direct current heating in superconductorinsulatorsuperconductor tunnel devices for thz mixing applications,” *Applied Physics Letters*, vol. 69, no. 3, p. 418, 1996.

[3] B. Leone, J. R. Gao, T. M. Klapwijk, B. D. Jackson, and W. M. L. amd G. de Lange, “Hot electron effect in terahertz hybrid devices,” *IEEE Transactions on Applied Superconductivity*, vol. 11, no. 1, pp. 649–652, 2001.

[4] T. V. Duzer and C. W. Turner, *Principles of superconductive devices and circuits*. New York, Oxford: Elsevier, 1981.

[5] N. R. Werthamer, “Nonlinear self-coupling of josephson radiation in superconducting tunnel junctions,” *Physical Review*, vol. 147, pp. 255–263, 1966.

[6] J. Bardeen, L. Cooper, and J. Schrieffer, “Theory of superconductivity,” *Physical Review*, vol. 108, no. 5, pp. 1175–1204, 1957.

[7] R. C. Dynes, V. Narayanamurti, and J. P. Garno, “Direct measurement of quasiparticle-lifetime broadening in a strong-coupled superconductor,” *Phys. Rev. Lett.*, vol. 41, no. 21, pp. 1509–1512, 1978.

[8] B. Mitrovi and L. A. Rozema, “On the correct formula for the lifetime broadened superconducting density of states,” *Journal Physics: Condense Matter*, vol. 20, pp. 15 215–15 218, 2008.