

Resource Funds: Stabilising, Parking, and Inter-generational Transfer[†]

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Abstract

The paper explores strategies for managing revenue from natural resources, focusing on the balance between domestic and foreign asset accumulation. It suggests that domestic asset accumulation is the priority in developing countries, while there are three motives for accumulating foreign assets; inter-generational transfer, temporary ‘parking’ of funds, and stabilisation. The paper argues that the first of these is inappropriate for low income countries. The second is required if it is difficult to absorb extra spending in the domestic economy and takes time to build up domestic investment. The third is important, and depends on the extent to which the economy has other ways of adjusting to shocks.

Key words: resource curse, managing windfalls, fiscal rules, volatility, absorptive capacity, Dutch disease, public investment

JEL classification: E60, F34, F35, F43, H21, H63, O11, Q33

1. Introduction

Failure to save and invest a sufficiently high fraction of resource revenues is a key aspect of the poor performance of many resource rich economies. This is particularly apparent at the end of a commodity super-cycle, when many countries find that they have little to show for the years of high resource prices. Yet at the same time there has been a steady increase in the number of Sovereign Wealth Funds (SWFs) that have been established, many of them by resource rich economies. Some of these are large and have played a useful role; others have unclear objectives and have been used with little economic logic.¹

1 At its most extreme, we see governments placing resources in low-earning foreign funds and at the same time issuing sovereign bonds at higher rates of interest.

This suggests that fundamental thinking is needed about the uses of revenues from non-renewable resources, such as oil, natural gas and minerals. There are two central questions. How much revenue should be used for current spending and how much used for capital spending, i.e., for accumulating assets? And what sort of assets should be acquired? In economic terms the first choice is that between consuming and saving. The second choice is, at the most aggregate level, between accumulation of foreign and domestic assets. Foreign assets can be held in different forms, which we will refer to collectively as SWFs.² Domestic asset accumulation means investment in the domestic economy; this may be capital spending by the public or the private sector, and includes expenditures on education and health that build human capital.

This paper lays out the arguments, summarising and integrating both the micro- and macro-economic literature on resource funds and revenue management.³ Drawing on these literatures, we argue that, in capital-poor developing economies, saving rates out of resource revenues should be high, and the priority should be investment in the domestic economy. High savings rates are motivated both by the temporary nature of revenues from exhaustible resources and by lack of capital in developing economies. Capital scarcity also motivates investment in the domestic economy; there is a sizeable opportunity cost to placing funds offshore in SWFs, and this should be done only to meet well-defined objectives. We identify three such objectives. One is for long-run saving (inter-generational transfer): this objective, we argue, is better met by investing in and growing the domestic economy than by accumulating assets abroad. The second we refer to as ‘parking’; the efficient time profile of domestic investment will not coincide with the time path of resource revenues, so revenues should be held in SWFs until they can be used most productively in the domestic economy. The third objective is for economic stabilisation; resource revenues are volatile and it is prudent to establish a buffer against price fluctuation.

We make the case using economic principles. We start (Section 2) with an analytical framework which captures trade-offs between alternative uses of resource revenues. This involves a simple model of an inter-temporal economy which, when applied to a developing country, starts with a low capital stock and consequently low income. The economy grows as it builds up its domestic capital stock by saving, and perhaps also by foreign borrowing. A resource discovery enables the economy to accelerate this process. It can use the revenue to increase consumption, to finance extra domestic investment, and to improve its foreign asset position (cutting debt or building a SWF). What is the best balance between these alternatives?

In Sections 3 and 4 we look at the long-run saving decision, making the case that in many circumstances resource revenues are better used for investment in the domestic economy than invested in a SWF. Section 5 qualifies this argument, on the grounds that many economies are not – in the short run – in a position to undertake high return domestic investments. Furthermore, resource rich economies have experienced boom

2 We break this down into separate possibilities in Section 7 of the paper.

3 References to relevant parts of this literature are given throughout the paper. For a wider view of the challenges of using natural resources for development see Venables (2016).

and bust as domestic spending has moved in line with resource revenues, often rising and falling too abruptly. It is crucial that the domestic investment path is efficient, and this requires uncoupling domestic investment (and spending more broadly) from current revenue flows. To achieve this, revenue needs to be parked in off-shore funds until productive domestic investments can be found and funds can be spent without destabilising the domestic economy. Section 6 turns to the issues surrounding the volatility of resource revenues. This volatility impacts government revenues and the overall macro-economy, and there is a case for building a stabilisation fund to cushion these impacts. Section 7 discusses some of the details involved in operating SWFs, and Section 8 concludes.

2. Analytical framework: inter-temporal efficiency and the budget constraint

Analysis of the economic principles underlying allocation of resource revenues between alternative uses – consumption versus saving, and domestic investment versus accumulation of foreign assets – requires an intertemporal model. In this section we sketch such a model, giving more technical detail in the appendix. The model is applied to the issues in following sections.

The model has two key ingredients. One is the economy's intertemporal budget constraint, and the other the efficiency conditions that shape choices within this budget constraint. To analyse the *intertemporal budget constraint* we define the wealth of the economy at date t , W_t , as the value of all assets held by the economy,

$$W_t \equiv F_t + PVY_t + PVN_t. \quad (1)$$

The right hand side of this equation has three components. The first is foreign assets, F_t (negative if foreign debt); the rate of return on such assets is r which, for the moment, we take to be constant and refer to as the world rate of interest.⁴ The second is the present value of income generated by domestic primary factors, which we write as

$$PVY_t = \int_t^\infty \{Y(K_\tau) - \dot{K}_\tau\} e^{-r(\tau-t)} d\tau. \quad (2)$$

The economy's capital stock is K and the production function is $Y(K)$ (depending also on labour, which we hold constant and suppress in the notation).⁵ The integral is therefore the present value of the economy's output net of investment \dot{K} , discounted at rate r to date t .⁶ The return on domestic capital is the marginal product, $Y'(K)$, decreasing in K .

4 We endogenise the rate of return on foreign assets in Section 4. Notice that, if it is time varying, then discount factors in (2) and (3) become $\exp[-\int_t^\tau r(z)dz]$.

5 All variables can be re-interpreted per capita, to cover the effects of a growing labour force.

6 We ignore depreciation of capital in the text, and include it in the appendix.

The third term in equation (1) is the value of sub-soil assets, i.e., the expected present value of the resource revenues, net of extraction costs. Expected net resource revenue at date τ is denoted N_τ so the present value at date t is

$$PVN_t = \int_t^\infty N_\tau e^{-r(\tau-t)} d\tau. \quad (3)$$

Discovery of a resource is a shock that opens up a path of revenues, N_τ , taken to be exogenous, given by the size and nature of the resource deposit and by future prices. The discovery – or any change in expected future revenues, e.g., due to a change in the resource price – shifts the expected present value of resource revenues, PVN_t , and hence shifts the wealth of the economy.

The inter-temporal budget constrains consumption, C_t , by the economy's wealth. This too we express as a present value constraint, so

$$W_t = PVC_t \equiv \int_t^\infty C_\tau e^{-r(\tau-t)} d\tau. \quad (4)$$

This says that, at each date, the present value of consumption, PVC_t , should not exceed (and will in general equal) the wealth of the economy.

Inter-temporal efficiency conditions characterise both the efficient allocation of wealth between assets and the efficient rate of asset accumulation, i.e., the division of income between consumption and saving. The condition for efficient allocation of wealth is simply to hold assets yielding the highest return and, where multiple assets are held, choose quantities of each such that their marginal rates of return are equalised. In our framework, this means dividing domestic and foreign assets, $K + F$, such that $Y'(K) = r$.⁷ Later sections of the paper will explore factors that influence r and $Y'(K)$; Section 4 looks at the way in which r depends on a country's access to capital markets and its wealth (in particular whether it is borrowing or lending, $F < 0$ or $F > 0$); Section 5 discusses how $Y'(K)$ depends on a country's stage of development, and on its ability to absorb a rapid increase in capital investment.

Inter-temporal efficiency in the saving/consumption choice depends on the return to investment (r , equal to $Y'(K)$ if wealth is allocated efficiently) and the rate at which society trades-off present consumption for future consumption, as measured by the consumption rate of interest. This is usually thought of as containing two elements, written as $\rho + g/\sigma$. The first term, ρ , is the rate of pure time preference, capturing inherent impatience – the fact that we prefer to consume now rather than to postpone gratification. For social decision taking, it is often argued, this should be extremely small; the impatience of the present generation gives it no right to consume at the expense of future generations. The second term, g/σ , captures the fact that future generations may be richer than the present generation, so a marginal unit of consumption is less valuable to them than to people currently alive. The variable g is the rate of growth of consumption, and parameter σ captures the

7 This analysis is easily extended to give the optimal rate of depletion of the resource, i.e., the Hotelling rule.

rate at which the marginal value of consumption diminishes as individuals become richer.⁸ In a developing country context this can be thought of as saying that, if income is growing ($g > 0$), poverty reduction needs are greater now than they will be in future, meaning that greater weight is placed on the present, i.e., the future is discounted more heavily (like an increase in the rate of pure time preference, ρ). Typical values of the parameters ρ and σ might be $\rho = 2\%$, $\sigma = 0.5$, so if consumption is growing at 3% p.a., then the consumption rates of interest, $\rho + g/\sigma = 8\%$.

Efficient intertemporal saving/consumption decisions will equate the rate of return on investment with the consumption rate of interest (the Euler equation, see appendix), so

$$r = \rho + g/\sigma, \quad \text{or} \quad g = \sigma(r - \rho). \quad (5)$$

One way to interpret this equation is that, given values of r and ρ , the *rate of growth* of consumption, g , should be set to satisfy the equation. Thus, given the budget constraint, if the rate of return on assets, r , is high consumption should be relatively low today and high in future, i.e., should grow fast, $g = \sigma(r - \rho)$. Intuitively, a lot is saved today in order to make the most of the high rate of return. If the budget constraint is relaxed (e.g., a windfall of resource revenue becomes available) and values of r , ρ and σ happen to remain unchanged, then the overall level of consumption is shifted up but the subsequent rate of change of consumption is unaffected.

3. Long run savings: high income countries and the permanent income hypothesis

The benchmark application of the framework sketched above is the permanent income hypothesis (PIH), and this has frequently been used as a basis of advice to resource rich economies. In its simplest form, the PIH contains two assumptions. One is that the economy can borrow or lend internationally at a constant (and exogenous) world rate of interest, r , and consequently the domestic capital stock has adjusted such that $Y'(K) = r$. This is tantamount to assuming that the economy is fully developed. It can borrow or lend on world capital markets at a relatively low rate and has no shortage of domestic capital, having invested up to the point where there are no investment opportunities that yield more than the return on foreign assets. The second assumption is that the world rate of interest is equal to the rate of pure time preference, so $r = \rho$. The condition for inter-temporal efficiency, $r = \rho + g/\sigma$, is therefore satisfied if consumption is constant through time, i.e., $g = 0$.⁹

In the absence of resource revenues, an economy satisfying these assumptions is stationary, with consumption, capital stock, and wealth constant through time. It follows that

$$C = rW = Y(K) + rF \quad (6)$$

8 σ is the elasticity of the marginal utility of consumption, $\sigma = -CU''(C)/U'(C)$.

9 Including depreciation, labour force growth, and labour-augmenting technical change, is straightforward, the last of these implying that per capita consumption increases through time. See Acemoglu (2009) Chapter 8.

The first of these equations comes from the inter-temporal budget constraint, (4), the right-hand side of which is C/r . The second is the flow budget constraint (consumption equals income) given a constant capital stock.

What is the effect of a resource discovery? The discovery occurs at date $t = 0$, and has present value PVN_0 . To see its effects notice first that, with the assumption that r is exogenous, the domestic capital stock, K , (and hence the level of domestic non-resource output) is fixed by the equation $Y'(K) = r$. It follows that a resource windfall has no effect on the domestic capital stock or on investment. The answer to the second question we posed – how much resource revenue should be invested in the domestic economy – follows directly from these assumptions; it is zero.

Second, the assumption that $r = \rho$ means that consumption adjusts at the date of discovery, but is constant along the optimal consumption path thereafter. At date of discovery wealth jumps by the value of the resource discovery, $\Delta W = PVN_0$, where Δ denotes the change at date of discovery. The jump in consumption is therefore

$$\Delta C = r\Delta W = rPVN_0. \quad (7)$$

This says that consumption increases by an amount equal to the annuity value of the resource discovery, and this increase in consumption occurs at the date of discovery and is maintained in perpetuity.

These are two strong results, but they are derived from two strong assumptions – that the domestic capital stock is such that the marginal product of capital equals the world interest, and that $r = \rho$, so the consumption path is flat. Neither of these assumptions is appropriate for developing economies and, in the next section, we look at the consequence of relaxing both of them. Despite this, the PIH gives some important messages for the fraction of resource revenues that should be saved and the path of asset accumulation, and we devote the rest of this section to drawing out these messages.

The time profiles of the stocks of assets and flows of income and consumption are shown in Figure 1 (with time on the horizontal and, for simplicity, assuming that the economy has no foreign assets or debt at date of discovery, $F_0 = 0$). The bottom panel is assets. As noted above, wealth jumps up by PVN_0 at date of discovery, and is constant thereafter. This means that as the resource follows its depletion path (assumed to be exogenous) and PVN falls, so foreign assets, F , increase to exactly offset this, $F = \Delta W - PVN$, as illustrated.

The upper panel gives flows of income and spending. Incremental consumption is constant, $\Delta C = rPVN_0$, resource income N declines (exogenously, the dashed line), and interest earned on other assets, rF , increases. The sum of these is the post-discovery increment to income, $N + rF$, which, by the flow budget constraint, must equal saving, S , plus the increment to consumption, i.e., $N + rF = \Delta C + S$, as illustrated by the solid line. These savings all go into foreign assets, $S = \dot{F}$, driving the increase in F illustrated on the lower panel.

It is convenient to write these relationships one further way. Since $\Delta C = r\Delta W$, $\Delta W = F + PVN$ and $S = N + rF - \Delta C$, savings must satisfy

$$S = N - rPVN, \quad \text{or} \quad S/N = 1 - r \cdot PVN/N. \quad (8)$$

This last equation gives the rate of saving out of the flow of resource revenue, S/N , a key policy variable. This depends on PVN/N , the ratio of the present value of resource

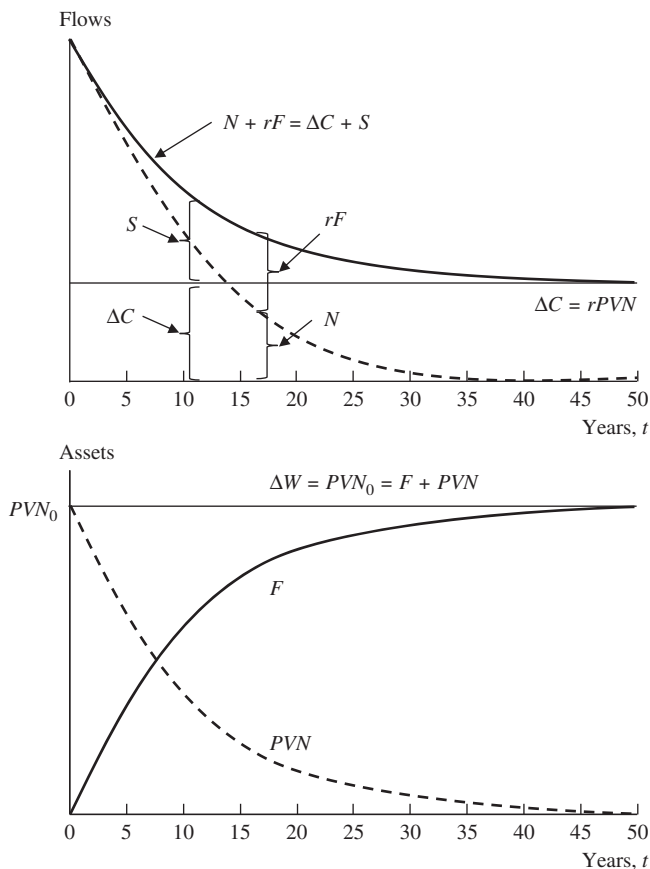


Figure 1: Permanent income hypothesis.

revenue remaining to the current flow of resource revenue. For an extremely long-lived resource, the present value of resource revenue remaining, PVN is large relative to current resource revenue, so the savings rate is low. The shorter lived is a resource discovery (the faster the decline in revenues) the smaller is this ratio and the higher the rate of saving. Some insights can be derived from looking at particular examples.

The simplest example is that in which the value of resource revenues declines exponentially at the constant rate κ (as illustrated in Figure 1). In this case the share of resource revenue saved is

$$\frac{S}{N} = \frac{\kappa}{(r + \kappa)}. \quad (9)$$

(Since the revenue flow is $N_t = N_0 e^{-\kappa t}$ and the present value of resource remaining is $PVN_t = N_0 e^{-\kappa t} / (r + \kappa)$). This demonstrates that the savings rate should be higher the faster the rate of decline of resource revenues, κ . Thus, if revenues are expected to decline at twice the rate of interest, 2/3rds of resource revenue should be saved. The savings rate is constant

during the life of the resource, although this is not generally the case, as illustrated in the next example.

Suppose alternatively that revenues from depletion are constant until the point of exhaustion is reached at date T . Then N is constant at $N = \bar{N}$ for $t < T$, and then drops to zero. Evidently, as the resource is depleted PVN_t falls, so the savings rate rises. The present value is $PVN_t = \bar{N}[1 - e^{-r(T-t)}]/r$ so the savings rate is

$$\frac{S}{N} = 1 - [1 - e^{-r(T-t)}] = e^{-r(T-t)}. \quad (10)$$

This means that the fraction of revenue which is saved increases during the life-time of the resource and goes to unity at the day of exhaustion. The reason is that, as time progresses, the stock of the resource left becomes smaller relative to the flow, and thus the windfall becomes more temporary, necessitating more saving.

Figure 2 illustrates this for two resource discoveries of equal size, but which are assumed to have different (and exogenous) time profiles of revenue. For both, the flow revenue from extraction is assumed to be constant over the life of the resource, but one depletes fast and the other slowly. The left hand panel of the figure shows the two revenue flows, with revenue, N , on the vertical and time on the horizontal. Both profiles, shown by the solid and dashed lines, have the same initial present value, $PVN_0 = 1$ (with $r = 0.04$), but that depicted by the solid line depletes the resource slowly so that it lasts for forty years, whereas that depicted by dashed line depletes it more rapidly so that it only lasts for 13 years.

The optimal savings rates for these depletion profiles are illustrated on the right panel, and follow equation (10) above. As in the exponential case, the savings rate is higher the faster is depletion. However, it is now the case that the savings rate increases over the course of depletion, since the present value, PVN_t , is gradually reducing while the flow N and the discount rate r are constant. Since PVN_t goes to zero as exhaustion approaches, the savings rate eventually rises to 100%. Different rates of depletion have a big impact, with the savings rate in the slow depletion case rising from 20% to 100%, while the fast depletion case gives the saving rate starting at 60% and rising to 100% in the final year. This

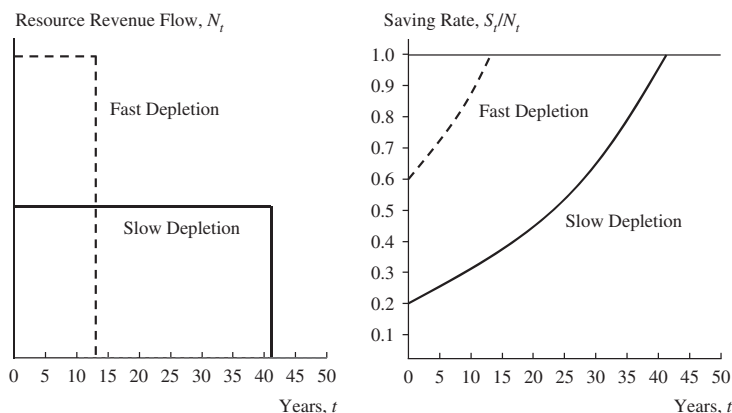


Figure 2: Savings rates from the PIH: constant depletion rates.

divergence between the implications of equations (9) and (10) points to the need to have as clear a view as possible about the time path of depletion, on which to base decisions about the fraction of flow revenue that is saved.

In summary, the optimal savings rate depends upon the extraction path. Three generalisations for policy appear warranted from the above discussion. First, for a given net present value of a resource discovery, the shorter its duration the higher should be the savings rate. Second, for an exponentially declining extraction path, the savings rate should be constant. Third, and more surprisingly, for a range of constant extraction paths the optimal savings rate should rise over time.

4. Long run savings: optimal saving in developing economies

In the PIH analysis of the preceding section all saving (asset accumulation) deriving from resource revenues was placed in a foreign assets, a SWF. This result is driven by the assumption that the economy has already expanded its capital stock to the point at which the marginal product of capital is equal to the constant and exogenous world interest rate. As we argued above, this is tantamount to assuming a high-income fully developed economy, with abundant capital.

Most poor countries have only asymmetric access to world capital markets: they can save more readily than they can borrow. Typically, as borrowers they are either entirely cut off or they can only borrow at a substantial premium over the world risk-free interest rate to compensate for perceived high risks of default.

To capture this we retain the model of Section 2, but now suppose that the interest rate faced by the economy is a function of the stock of foreign assets (or debt) that it holds, as in Bardhan (1967), Obstfeld (1982) and van der Ploeg and Venables (2011).¹⁰ We denote this $R(F)$. We will assume that if the country is not indebted then this is equal to the world interest rate, which we now denote r_* . If the country is indebted, then it exceeds the world rate, and is increasing in the level of debt, i.e.,

$$\text{If } F \geq 0, \quad R(F) = r_* \quad (11)$$

$$\text{If } F < 0, \quad R(F) > r_*, \quad R'(F) < 0.$$

Notice that while $R(F)$ is the rate of interest, the marginal cost of borrowing will exceed this if taking on more debt raises rates the country has to pay on existing debt. Thus, the marginal cost of borrowing is $r(F) = R(F) + FR'(F)$, which we assume is also decreasing in F (i.e., increasing in debt).

This characterisation of a developing economy changes answers to both our questions – the division of resource revenues between consumption and saving, and the assets which savings should be used to acquire – giving different answers from the PIH. We look first at the choice of assets, and then at the overall path of saving, investment and economic growth.

10 Van der Ploeg and Venables (2011) document empirical evidence for a relationship between foreign debt and the domestic interest rate. See also Akitobi and Stratmann (2008).

4.1 Domestic investment versus foreign assets

The economy has domestic capital K and foreign assets (or debt) F , and we call their sum assets, denoted $A \equiv K + F$. If the economy is asset scarce, such that the marginal return on domestic capital satisfies $Y'(K) > r^*$, then all assets should be held as domestic investment, and none in an SWF, i.e., $K \geq A$. However, such an asset scarce economy might borrow, i.e., choose $F < 0$. It should do so up to the point at which the return on domestic capital is equal to the marginal cost of borrowing,

$$Y'(K) = Y'(A - F) = r(F). \quad (12)$$

If some resource revenue is saved then the economy's assets increase. How should an increase, dA , be divided between foreign debt reduction, dF , and domestic investment, dK ? Differentiating equation (12),

$$dF = \left\{ \frac{Y''(A - F)}{Y''(A - F) + r'(F)} \right\} dA, \quad dK = \left\{ \frac{r'(F)}{Y''(A - F) + r'(F)} \right\} dA. \quad (13)$$

If the marginal cost of borrowing increases with indebtedness then $r'(F) < 0$, and some positive fraction of extra assets goes into domestic capital, $dK/dA \in (0, 1]$. In the limiting case where the country is shut out of capital markets $r'(F)$ is minus infinity and the whole of any increase in assets goes to domestic capital formation. At the other extreme, the PIH maintains the assumption that the marginal return/cost of lending/borrowing is constant so its derivative, $r'(F)$, is equal to zero and hence the domestic capital stock is unaffected by the change in assets, $dK = 0$, as we saw in Section 3.

This establishes that countries which are capital scarce should use resource revenues to increase the domestic capital stock (including human capital), and perhaps also pay down foreign debt. The case for increasing domestic investment is reinforced by several further arguments. One is that the resource boom may bring its own direct financing needs. For example, infrastructure projects may need to be brought forward, placing a demand on public funds. There may also be general equilibrium effects. The boom will likely bring a change in the structure of the economy, with some sectors booming and others contracting. This is likely to take the form of a reallocation of economic activity into non-tradable goods sectors and away from non-resource tradables (as suggested by the 'Dutch disease'). If the non-tradable sector is relatively capital-intensive this will mean that the capital-labour ratio in the economy as a whole will increase.

Rates of return should be adjusted for risk, including political risks such as the possibility that a liquid asset (such as foreign exchange holdings) might be more easily looted by a future government than illiquid assets (such as domestic infrastructure), and a diversified portfolio should take into account correlations between returns (van den Bremer et al. 2015). Rates of return should ideally be calculated to include the full social costs and benefits of investments and to take into account effects of investment on the domestic macro-economy, in particular if there is limited absorptive capacity, as discussed further in Section 5.

At the same time, a resource discovery is likely to reduce the economy's cost of borrowing, as resource revenues are perceived to be a form of collateral for loans. This too will tilt the use of resource funds more towards domestic capital formation rather than pay-down of foreign debt.

4.2 Saving versus consumption

The preceding sub-section makes the case that it is not appropriate for a developing economy to set up an SWF and lend to the rest of the world, although it may be appropriate for it to use resource wealth to pay down existing debt as well as investing in domestic assets. To complete the story we need to also look at total savings out of resource revenues and hence their implications for the growth of domestic income and consumption.

The framework of Section 2 together with our model of foreign borrowing, equation (11), gives a standard, albeit very simple, model of development. If inter-temporal efficiency holds, the rate of increase of consumption is given by the Euler equation (4), which becomes

$$g = \sigma(r(F) - \rho), \quad \text{with} \quad Y'(K) = r(F). \quad (14)$$

The right hand side now contains the marginal cost of borrowing, $r(F) > r^*$, and is positive (where the PIH assumed it was zero since $r^* = \rho$). This implies that consumption is rising through time, i.e., is relatively low initially. The rate of return is high, and as capital is accumulated and debt is reduced, so the rate of return falls, the rate of increase of consumption slows, and the economy converges to high income status. Thus, starting from a low base capital is accumulated, income rises and the rate of return falls, ultimately reaching a level similar to that in high income countries and supporting similar levels of income and consumption. This is illustrated in Figure 3, the upper panel giving consumption and the lower panel assets; the paths \tilde{C} and \tilde{A} are baseline paths, absent resource revenues.

What should resource revenue management look like in such a developing country? Resource revenues shift the budget constraint and create the possibility of increasing both current consumption and saving, this speeding up the rate of asset accumulation. More assets at any date mean a lower marginal cost of borrowing, $r(F)$, and hence a slower rate of increase in consumption. This is illustrated in Figure 3 by the solid lines C and A . Consumption jumps up at the date of discovery but then increases less rapidly than in the base case. Assets are accumulated more rapidly, more than offsetting the decline in the value of resource remaining.

A number of points come from this analysis. First, the consumption increment (the difference between the new path of C and the baseline \tilde{C}) is largest immediately after the windfall, and then declines. This is in sharp contrast to the PIH where the increment was constant through time. The intuition is that the current generation is poorer than future generations, so the consumption increment is skewed towards this poorer generation. If the windfall is very large there may be a 'permanent' element that continues in perpetuity, but for small windfalls the consumption paths illustrated in the figure converge.¹¹

At the same time as it brings forward consumption, the resource windfall also means that accumulated assets are greater at all dates, this bringing down the rate of return in the economy. Essentially, the optimal use of the resource revenue has the effect of bringing forwards development, as can be seen by comparing the consumption and asset accumulation

- 11 If $r(F) > r^*$ at the date the resource is depleted then the economy returns to a point on its original path, reaching this point sooner than without the resource. For a large windfall it is possible that capital accumulation is such that $r(F) = r^*$ at some date prior to exhaustion, beyond which the usual PIH applies. For a more technical treatment see [van der Ploeg and Venables \(2011\)](#).

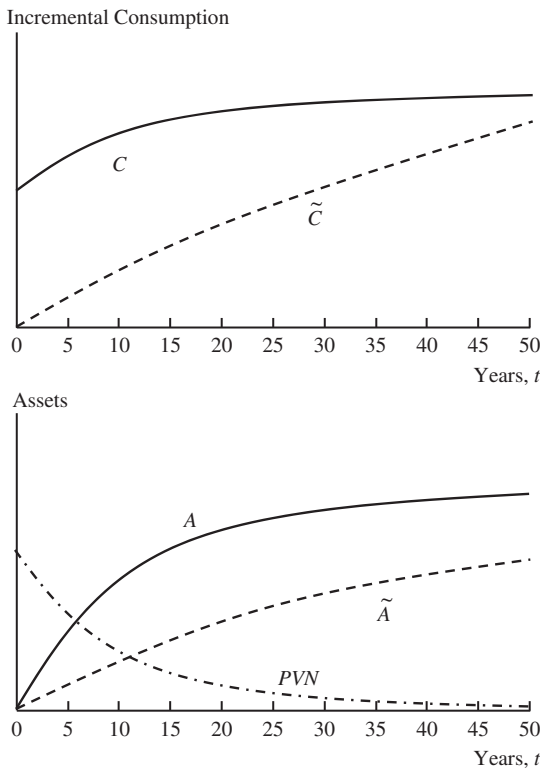


Figure 3: Optimal revenue management in a developing economy.

paths with and without the windfall in Figure 3. The share of resource revenues that should be saved depends, as before, on the precise path of resource revenues. However, the argument of this section tends to reinforce the message that saving should increase through time; the additional effect is the front-loading of the consumption increment.

In summary then, once the analytical framework is modified to include the key developing country features of capital scarcity and high borrowing costs, the policy messages are significantly different from those of the PIH. The consumption increment is front loaded, and resource revenues should be used for domestic investment and for paying off foreign debt, the domestic investment bringing forward the development path of the economy.

5. Efficient investment profiles: the case for a parking fund

The prioritisation of domestic investment rests on there being domestic projects that yield high returns (social returns, as well as private), and on projects being delivered in an efficient sequence and manner. Yet it is possible that – in the short run – there may be few high return projects ready to be implemented. And more generally, there will inevitably be a mismatch between the time profile of (efficient) investment and the time profile of resource revenues. Rather than spending revenues on inefficient projects, there is then a case for having a fund that plays the role of ‘parking’ resource revenues until they can be

efficiently used in the domestic economy. The case for parking funds until the economy is able to absorb extra spending efficiently involves both micro- and macro-economic arguments, and we go beyond the simple model of previous sections to review each in turn.

5.1 Micro-economics: project selection and supply dynamics

A country seeking to scale up investment is likely to encounter numerous bottlenecks. In the public sector there is unlikely to be a pipeline of good investment projects. There may be a lack of capacity to design and develop projects, project selection and cost-benefit processes may be weak, and so too the ability to procure, implement, and monitor projects. These problems have to be addressed before effective investments can be undertaken, a case for ‘investing-in-investing’ (Collier, 2010). Sequencing of projects also matters because the return to one project depends on other complementary projects that can be undertaken. For example, the return to private investment may be low, particularly if public investments in infrastructure and other aspects of productive capacity are lagging.¹²

At a more macro-level, additional domestic spending will increase demand for both traded and non-traded goods. As demand increases the economy will move up its supply curve of non-traded goods, creating both a quantity and a price response (‘Dutch disease’, see Corden and Neary, 1982). Since the supply curve will be less elastic in the short run than in the long run, upwards price pressure will be acute if spending increases rapidly. This is likely to be particularly true for ‘home-grown’ capital; while equipment can usually be imported, structures and human capital requires domestic capacity (e.g. in the construction and training professions), all of which take time to develop.¹³

There may be no market failures associated with these bottlenecks and price responses; it is simply a feature of the supply side of the economy that projects take time and have to be done in sequence. Nevertheless, this timing dictates the rate at which investment can be increased. If extra investment demand is met by a price increase rather than a quantity response then it is efficient to slow down the rate of investment until supply can catch up. This suggests that investment should be ramped up quite slowly. It follows that resource revenues should be ‘parked’ in foreign assets until they can be spent on the efficient investment path.

5.2 Macro-economics: exchange rate dynamics and policy responses

An economy undergoing a resource boom is likely to face numerous sources of extra demand. Spending of revenues – on consumption and investment – will be augmented by private investment in the resource sector and elsewhere. Changes in expectations held by households and by private firms may also change spending patterns. As we saw in the previous sub-section this is likely to lead to an increase in prices as well as in quantities supplied, and an increase in the price of non-tradables is likely to manifest itself as an appreciation of the real exchange rate. There are several potential macro-economic problems associated with this. One is the risk of inflation; the real appreciation may occur through nominal wage and price increases (rather than nominal exchange rate appreciation), posing a risk to price stability in the country. A second arises through movements in

12 In terms of equation (14), $Y'(K)$ is particularly large (negative) in the short run. Since the marginal product of capital falls more with domestic investment more should be devoted to foreign assets.

13 For example, it takes teachers to produce teachers; see van der Ploeg and Venables (2013).

the nominal exchange rate. If monetary policy is tight the nominal interest rate will rise. With an open capital account it is possible that the nominal exchange rate will ‘overshoot’, appreciating fast and then depreciating back to a long run value, along a path on which uncovered interest parity holds.

Outcomes depend on the exchange rate regime and the relative speeds of adjustment of different variables. Under a floating exchange rate, changes in the nominal exchange rate are likely to be fast, and not necessarily synchronised with levels of demand in the domestic economy. If exchange appreciation precedes real demands from resource spending, then anticipation effects can create a recession (Eastwood and Venables, 1982, Wills, 2015b).

Under an exchange rate peg these appreciations will happen through inflation. There is typically a lag between discovering oil and beginning production. This means the real exchange rate must appreciate twice: when households learn of the discovery and raise consumption, and when the oil revenues are eventually spent in the economy. The forward-looking component of this inflation will suppress output below its natural level, leading to a recession. If the central bank follows a simple Taylor rule that tightens aggressively against inflation it will exacerbate this recession (Wills, 2015b). Arezki et al. (2015) find empirical support for this in their study of large oil discoveries. They find that despite large increases in investment, the short run effect of a discovery on income is slightly negative (until production commences), while the private sector responds immediately to an oil discovery causing the current account to depreciate.

Well-designed fiscal and monetary policy can address these challenges. Fiscal policy can insulate the economy from large changes in oil revenues by placing them in a SWF – the ‘parking fund’ argument. Monetary policy that tracks changes in the natural rate of interest, caused by expected changes in production, can also stabilise inflation and the output gap (Wills, 2015b). However, the ability to do this is limited in the 75% of resource-dependent economies that have pegged exchange rates.

6. Volatility and stabilisation

The instability of commodity prices creates a third argument for using a SWF, to function as a stabilisation fund. The optimal size of such a fund will depend on how the government and the economy respond, *ex post*, to shocks that occur. Are shocks sufficiently costly that ‘insurance’ through a stabilisation fund is necessary, or can they be managed by other means? We note that one alternative way is through hedging strategies, as employed for example by Mexico. However, such strategies typically only offer insurance for a relatively short period, and we do not discuss them further.

6.1 How costly are shocks?

The benchmark case is (once again) an economy that has perfect access to world capital markets at a fixed rate of interest (independent of the shock). If resource price shocks are temporary around a stationary mean, then such an economy can smooth public consumption perfectly, saving when the price of the resource is high, and dissaving or borrowing when the price is low. The foreign assets of such an economy would fluctuate with the resource price but the economy would not need to purposefully set out to build a stabilisation fund. The expected value of the fund should be zero, as international capital markets provide all the smoothing that is needed.

This benchmark case breaks down for two reasons. One is because of the asymmetry of borrowing and lending costs. Borrowing during a sharp downturn in commodity prices is likely to be expensive, if not impossible. Borrowing rates will be high and, following the fall in prices, a developing resource exporter will likely be shut out of most borrowing options. The other is that oil price shocks are usually not temporary. In fact, most studies find the opposite – that the oil price follows a random walk, so that shocks are permanent (Bems and de Carvalho Filho, 2011; van den Bremer and van der Ploeg, 2013). Empirical evidence suggests that for horizons shorter than 6 months it is possible to beat a random walk forecast of the oil price out of sample (Alquist et al., 2013). Longer than six months it is better to assume that the oil price follows a random walk (Hamilton, 2009).

These arguments create the case for a stabilisation fund, but its size depends on the costs incurred if, in the event of a price fall, a country is unable or unwilling to borrow. The costs come through both micro- and macro- economic channels. The simplest argument for stabilisation is consumption smoothing; policymakers should save more in the face of oil price volatility because of precautionary savings. Precautionary savings stems not from risk aversion, but from ‘prudence’, which amounts to a positive third derivative of the utility function, as explored by Kimball (1990). Precautionary saving only takes into account the utility costs of failing to smooth consumption optimally, but problems are exacerbated by the fact that the fall in revenue falls principally on government.

As recent events illustrate, lower resource prices have severe implications for government spending, in some cases necessitating dramatic spending cuts. These may be disruptive of investment and other spending programmes and can be politically difficult to implement. However, low oil prices also make some interventions more politically feasible, such as cutting fuel subsidies or raising oil rent taxes. In both cases the pain of the policy is acute initially, whilst the benefits to public finances in the future can be considerable.

Perhaps most important, is the potential negative impact of a price fall on the macro-economy. While Section 5.2 considered the macroeconomic implications of an oil boom, let us now consider the effects of a bust. When the resource price falls, resource-exporters will experience a negative demand shock. Investors in the resource sector may be scaling back investment and production activity. Households may cut spending anticipating lower income, and firms in non-tradable sector may hold off investment as a result. At the same time there will be pressure on the government to cut spending.

Loose monetary policy should be able to perfectly offset a negative demand shock if such policy is unconstrained, though this will not be possible in many resource exporters. A negative demand shock will require the price of non-traded goods to fall relative to traded goods. This will only happen slowly with sticky prices, so for a period output will be below its natural level. Monetary policy can offset this by loosening, causing households to bring spending forward and the exchange rate to depreciate. However, many resource exporters face constraints on monetary policy so the fall in demand cannot perfectly be offset. On the household spending channel, low income resource exporters typically have underdeveloped financial markets, so changes in interest rates don’t readily affect household savings behaviour. Mortgage markets, a crucial channel of monetary policy transmission in high income countries, are typically thin. On the exchange rate channel, many resource dependent economies have poorly developed non-resource export sectors (a product of Dutch disease), which will take a long time to respond to a currency depreciation as investment is made.

Furthermore, 75% of resource-exporters peg their exchange rate which is a severe constraint on monetary policy's ability to respond to demand shocks.¹⁴

These constraints and the consequent potentially high costs of negative shocks make the case for creating a stabilisation fund which gives government the space to undertake counter-cyclical fiscal policy (e.g. moderate the pace of spending cuts) necessitated by a revenue downturn. The fund is even more important in countries with asymmetric borrowing and lending costs. Borrowing during a sharp downturn in commodity prices is likely to be expensive, if not impossible. Borrowing rates will be high and, following the fall in prices, a developing resource exporter will likely be shut out of most borrowing options. This asymmetry creates another case for a stabilisation fund, but its size depends on the costs incurred if, in the event of a price fall, a country is unable or unwilling to borrow.

6.2 How should you prepare for shocks?

How large should a stabilisation fund be, how rapidly should it be drawn down in the event of a negative shock, and what sort of assets should it hold?

The size of the fund will depend on the exposure to resource-price shocks and the costs when they happen (as discussed above). In general, the exposure to resource-price shocks is greatest at the beginning of the windfall when the majority of the resource is yet to be extracted. Longer and more back-weighted extraction paths will require larger stabilisation funds. Beliefs about the path of oil prices will also change the size of the fund – mean reversion makes resource prices less volatile in the future, reducing the size of the stabilisation fund.

One way of incorporating uncertainty into policy, while staying within the PIH framework, is to use a relatively high discount rate to value future resource revenues (see [van den Bremer et al., 2015](#)). If this is done then the present value of total above- and below-ground wealth will rise over time, as above-ground assets replace heavily discounted future resource revenues. The spending rule should be a constant share of total wealth, so should also rise accordingly (if oil price shocks are spanned by the market; if not then spending should rise as a share of the portfolio, to incorporate precautionary savings).

An extreme case of heavily discounting resource revenues is the 'bird-in-hand' rule. In this case future resource revenues are effectively discounted at an infinite rate, and all revenues are placed in foreign assets. The PIH then implies that the consumption increment should just be the interest on these above-ground assets (as done in Norway), which will start at zero and increase as assets are accumulated.¹⁵ Domestic spending of resource-revenues is thus heavily back-weighted, and unable to deliver domestic investment in the early years, which we have argued is efficient. The bird-in-the-hand rule is therefore not appropriate for poor countries because it exaggerates the risk that revenues might terminate earlier than expected, while completely ignoring the much larger risks to low-income societies from deferring growth.

If a stabilisation fund is in place, how rapidly should it be run down in the event of a negative shock? The answer depends on alternative instruments open to government

14 If the East African Monetary Union goes ahead this will also severely constrain the ability of the newly resource-rich members of the Union: Kenya, Tanzania and Uganda, to respond to commodity price shocks ([Wills and van der Ploeg, 2014](#)).

15 See [Barnett and Ossowski \(2003\)](#).

(including the effectiveness of monetary policy), the size of the fund, and on expectations about future resource prices. Paradoxically, if the oil price follows a random walk, and if there are no real or nominal rigidities, then the government should never draw down the capital in a stabilisation fund (Wills, 2015a). This follows from the permanent nature of shocks under a random walk – if the oil price is low then it is expected to remain low indefinitely, so government spending should adjust. In this case the role of the stabilisation fund, accumulated from precautionary savings, is to provide an additional stream of permanent income as compensation for bearing the risk of the oil price falling. In practice a combination of non-permanent oil price shocks and real costs of adjusting government spending mean that some temporary consumption of the capital in a stabilisation fund may be appropriate on occasions.

Finally, we note that the portfolio held by a stabilisation fund should meet two criteria. One is liquidity – it needs to be accessible, particularly since commodity price down turns are positively correlated with other asset price falls. Second, assets should be negatively correlated with commodity prices (van den Bremer et al., 2015). At present most SWFs invest in a diversified market portfolio, for example Norway's fund uses the FTSE Global All Cap Index as its equity benchmark (~7,500 stocks). This ignores subsoil oil. Aggregate risk can be reduced by altering the SWF portfolio in two ways. First, by leveraging up the holding of all assets to take account of the share of the portfolio below the ground. Second, by hedging oil price risk by holding relatively more (less) of assets that are negatively (positively) correlated with oil (after accounting for inter-asset correlations). Designing a portfolio in this way should also take into account transaction costs, short-sale constraints and time-varying correlations, so in practice it may be easier to simply vary the equity/bond mix in a SWF than the allocation to every stock.

7. Institutional arrangements

How should these SWFs be established in practice?

It is first important to draw a distinction between SWFs and foreign exchange reserves. If a resource-exporter pegs their exchange rate (as done by 75% of them) at an appropriate level, then they will accumulate foreign exchange reserves. These are similar to a de-facto SWF, though they are not equivalent (Wills and van der Ploeg, 2014). A SWF accumulates oil revenues in a foreign currency before they pass through the domestic economy. Foreign exchange reserves accumulate after the oil revenues have been spent domestically. The counterpart of these reserves is the net asset position of the private sector with the central bank. Therefore, while SWFs represent savings by the government, foreign exchange reserves are saved by the private sector. The share of GDP going to private, rather than public, consumption will correspondingly be lower if resource revenues are accumulated in foreign reserves.

It is important that SWFs have an investment mandate and spending rule that is appropriate for their purpose. An inter-generational fund should have a mandate to invest in assets with a long investment horizon, in line with the long-term strategy employed by many university endowment funds. The spending rule from an intergenerational fund should be relatively tight – avoiding dipping into the capital of the fund and instead spending a relatively fixed share of total wealth. In contrast, a parking fund should focus on assets with a range of maturities, as its capital will be drawn down slowly. Parking fund

spending rules should be limited in their use (domestic investment only) and their timing. The rule may even be linked to indicators such as inflation in the construction sector. Stabilisation funds, particularly in developing countries, should hold assets that can be liquidated at short notice if the oil price falls. Their spending rules must permit fast liquidation, contingent on the resource price, and should also include a mechanism for recapitalising the fund when the price rises again. All funds should design their investment portfolios to hedge the exposure to subsoil oil (van den Bremer et al., 2015).

In practice it may make sense for these funds to be managed within the same government department, but it is important that the quite distinct objectives – and corresponding portfolios of each – are made clear and kept distinct.

8. Concluding comments

In this paper we have addressed the two central questions of resource revenue management. What share of revenue should be used for current spending versus capital spending? And what sort of assets should be acquired, foreign or domestic? We argue that savings rates should be high and that domestic investment should be the priority for developing countries. Management of foreign assets can support this strategy, but it is important that there is clarity on the roles that foreign assets can play. We divide these into three elements.

8.1 Inter-generational transfer

The first role for foreign assets is to replace below-ground resources with above-ground financial assets. The financial assets are used to generate permanent income for future generations. The desirability of such funds depends principally on the long-run investment opportunities (or lack of them) in the domestic economy, and on political economy. When established these funds should focus on assets with long investment horizons, matching the permanent nature of the fund. For Norway, such a fund is sensible. The economy has a large domestic infrastructure stock and good access to international capital markets, implying that the rate of return on further domestic investment is unlikely to exceed the return that can be made on investments through a SWF. Furthermore, the Norwegian demographic of an aging population implies a need not for physical assets on which to employ workers, but for financial assets to generate an income flow in coming decades. Norwegian political economy is such that accumulated financial assets are unlikely to be raided by future governments – although even in Norway, the terms of withdrawal from the SWF are not beyond political debate.

These conditions are in stark contrast to those prevailing in most developing economies. As we argued above, capital scarcity means that investment in the domestic economy is a priority, although designing and implementing projects with high social return is difficult. This investment can help place the economy on an accelerated growth path, bringing forward economic growth and consumption benefits. If directed towards the non-traded sector this investment can also limit inflation from resource-financed demand. In developing countries demographics are such that capital is needed to support jobs for a growing work force, not flows of rent for pensioners. And concerns about future governments make it desirable to sink investments in physical structures and assets, rather than leave them in easily drawn down financial assets. The case for long run ‘inter-generational’ funds is therefore, we think, very weak for developing countries.

8.2 Parking

The second role of a SWF is to overcome the timing mismatch between receiving resource revenues and investing them efficiently in developing countries. The efficient path of investment will depend on the absorptive capacity of the economy, which must be slowly built up through a programme of ‘investing-in-investing’. This build-up may involve some borrowing in the early stages, to prepare for the flow of resource revenues, and it will also require that savings be ‘parked’ off-shore until they can be efficiently invested in the domestic economy. We use the term ‘parking’ in order to emphasise that this acquisition of foreign assets – while it may be substantial and take place over several years – is not an end in itself, but is just a way of uncoupling the precise time profile of revenues from that of spending in the domestic economy. Therefore these funds should invest in assets with a relatively shorter investment horizon than the intergenerational funds of developed countries.

8.3 Stabilisation

The third role of SWFs is to insulate the economy from short- to medium- run fluctuations in resource revenues; as we saw, under the PIH and other savings rules, spending in the domestic economy should be smoothed. It is possible, but difficult, for governments to transfer their exposure to volatile resource prices. Resource-extraction firms bear some risk, depending on the nature of their contracts, but are unwilling to bear it all. Financial derivatives can smooth short-run fluctuations but these are typically short-term and expensive. They can also be politically difficult – no finance minister wants to spend money on options that are not exercised or face criticism for selling the resource at a lower price than might (ex post) have been achieved.

Therefore, resource-exporters must self-insure by using stabilisation funds. These funds play two roles: providing an additional source of interest income when resource prices fall, and a stock of assets that can finance counter-cyclical fiscal policy (when monetary policy is constrained). To be successful they must have a robust and transparent policy rule, and a mandated specialist institution to implement it. Its success depends upon the political acceptability of periods in which resource revenues are substantially under-spent. Appropriate design depends upon context: for example, rules which are sufficiently simple to be understood by electorates may at times impose sub-optimal decisions, but complexity and discretion may not be robust to popular pressures. These funds should invest in liquid assets and have mechanisms to recapitalise when resource prices recover.

These three funds have different purposes, and so need separate legal structures and investment mandates. A developed country’s inter-generational fund should invest in long-horizon assets and spend a fixed share of above- and below-ground wealth. A developing country’s parking fund should invest in shorter-duration assets and have a rising limit for the amount spent each year, this constrained to be used for domestic investment. Stabilisation funds should focus on liquid assets and have spending and recapitalisation rules tied to the commodity price. All funds should design their portfolio with subsoil assets in mind – hedging resource-price volatility at the start of the windfall and rebalancing towards a diversified portfolio as the resource is exhausted. It may well make sense to manage these funds from within a single government body, but they should remain independent in line with their original aims.

There remain a number of issues that warrant further consideration. The first is a set of simple rules of thumb outlining the relative size of these funds and how quickly they should be accumulated, which can be used by policymakers. The second is to study in further detail the types of projects that warrant investment from developing country’s parking

funds most. The third is further work on how to strike the right balance between using a stabilisation fund for additional interest income and for counter-cyclical fiscal policy. The fourth is to provide more detail on asset allocation in these funds. This should include time-varying correlations between resource prices and financial markets; and taking into account the correlation of governments' future tax assets and spending liabilities with the funds, based on the demographic and industrial structure of the economy.

References

- Acemoglu D. (2009) *Introduction to Modern Economic Growth*. Princeton and Oxford: Princeton University Press.
- Akitobi B., Stratmann T. (2008) 'Fiscal policy and financial markets', *Economic Journal*, 118 (533): 1971–85.
- Alquist R., Kilian L. and Vigfusson R. J. (2013) 'Forecasting the price of oil', *Handbook of Economic Forecasting*, 2: 427–507.
- Arezki R., Ramey V., Sheng L. (2015) 'News Shocks in Open Economies: Evidence from Giant Oil Discoveries', OxCarre WP no. 153.
- Bardhan P. K. (1967) 'Optimum foreign borrowing', in K. Shell (ed.), *Essays on the Theory of Optimal Economic Growth*. Cambridge, Massachusetts: MIT Press, pp. 117–28.
- Barnett S., Ossowski R. (2003) 'Operational aspects of fiscal policy in oil-producing countries', IMF Working Paper WP/02/177, International Monetary Fund, Washington, D.C.
- Bems R. and de Carvalho Filho I. (2011) 'The current account and precautionary savings for exporters of exhaustible resources', *Journal of International Economics*, 84(1): 48–64.
- Collier P. (2010) *The Plundered Planet*. New York: Oxford University Press.
- Corden W. M., Neary J. P. (1982) 'Booming sector and de-industrialisation in a small open economy', *The Economic Journal*, 92(368): 825–48.
- Eastwood R. K., Venables A. J. (1982) 'The macro-economic implications of a resource discovery in an open economy', *Economic Journal*, 12: 285–99.
- Hamilton J. D. (2009) 'Understanding Crude Oil Prices', *The Energy Journal*, 30(2): 179–206.
- Kimball M. S. (1990) 'Precautionary saving in the small and in the large', *Econometrica*, 58(1): 53–73.
- Obstfeld M. (1982) 'Aggregate spending and the terms of trade: is there a Laursen-Metzler effect?', *Quarterly Journal of Economics*, 97: 251–70.
- van den Bremer T. S., van der Ploeg F. (2013) 'Managing and harnessing volatile oil windfalls', *IMF Economic Review*, 61(1): 130–167.
- van den Bremer T., van der Ploeg R., Wills S. (2015) 'The Elephant in the Ground: Managing oil and sovereign wealth', OxCarre Working paper 129, University of Oxford.
- van der Ploeg F. (2012) 'Bottlenecks in ramping up public investment', *International Tax and Public Finance*, 19(4): 509–38.
- van der Ploeg F., Venables A. J. (2011) 'Harnessing windfall revenues: optimal policies for resource rich developing economies', *Economic Journal*, 121: 1–31.
- van der Ploeg F., Venables A. J. (2013) 'Absorbing a windfall of foreign exchange: Dutch disease dynamics', *Journal of Development Economics*, 103: 229–43.
- Venables A. J. (2016) 'Using natural resources for development: why has it proven so difficult?', *Journal of Economic Perspectives*, 30(1): 161–84.
- Wills S. (2015a) 'Seven principles for managing resource wealth', OxCarre Working paper 154, University of Oxford.
- Wills S. (2015b) 'Optimal monetary responses to news of an oil discovery', OxCarre Working paper 121, University of Oxford.

Wills S., van der Ploeg R. (2014) 'Why Do So Many Oil Exporters Peg Their Currency? Foreign Reserves as a de-facto Sovereign Wealth Fund', *IMF conference proceedings: Macroeconomic Challenges facing Low Income Countries*.

Appendix

Using a simplified version of [Wills \(2015a\)](#), the social planner's problem is to choose consumption, C , and investment, I , in the face of exponentially declining oil revenues, N , to maximise utility,

$$\begin{aligned} J(F, K, t) &= \max_{C, I} \left[\int_t^\infty U(C_t) \exp(-\rho(\tau - t)) d\tau \right] \\ &\quad \text{s. t.} \\ \dot{F}_t &= r(F_t)F_t + N_t + Y(K_t) - C_t - I_t \\ \dot{K}_t &= I_t - \delta K_t \\ \dot{N}_t &= N_0 \exp(-\kappa t). \end{aligned} \quad (\text{A1})$$

We incorporate capital scarcity by assuming that interest rates may be elastic with respect to foreign assets F , $r(F_t) > r$, $r'(F_t) < 0$ for $F_t < 0$ and $r(F_t) = r$ otherwise. Output is produced using domestic capital and a fixed stock of labour, $Y_t(K_t)$ and $\bar{L} = 1$, and it is sold at the constant world price $P^* = 1$. The Hamiltonian for this problem is,

$$H \equiv U(C) + \lambda_1[r(F)F + N + Y(K) - C - I] + \lambda_2[I - \delta K] + \lambda_3[N_0 e^{-\kappa t}]. \quad (\text{A2})$$

This has the optimality conditions,

$$\begin{aligned} H_C &= U'(C) - \lambda_1 = 0 & H_I &= -\lambda_1 + \lambda_2 = 0 & \rho\lambda_1 - \dot{\lambda}_1 &= H_F = \lambda_1(r'(F)F + r(F)) \\ \rho\lambda_2 - \dot{\lambda}_2 &= H_K = \lambda_1 Y'(K) - \lambda_2 \delta & \rho\lambda_3 - \dot{\lambda}_3 &= H_O = \lambda_1. \end{aligned} \quad (\text{A3})$$

And the transversality condition $\lim_{t \rightarrow \infty} [\exp(-rt)\lambda_{1,t}F_t] = 0$. The Euler equation can be derived from the first-order conditions,

$$g = \sigma(r(F) + r'(F)F - \rho), \quad (\text{A4})$$

where g is the rate of change of consumption. Therefore, consumption will be delayed as long as the country faces a premium on its debt ($F < 0$), so the level of consumption will be rising. Whether income that is not consumed is used for domestic investment or foreign savings will depend on equating the marginal utility of an extra unit of foreign assets or of domestic capital,

$$\lambda_1 = \lambda_2 \quad \Rightarrow \quad (r(F) + r'(F)F) = Y'(K) - \delta. \quad (\text{A5})$$