# Important mathematical competences for students in the age of generative artificial intelligence: proof comprehension and evaluation, and ChatGPT

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**Abstract**

The rapid growth and popularity of generative artificial intelligence has influenced all aspects of life and education, including mathematics education. Specifically, generative Large Language Models (LLMs), notably ChatGPT, are becoming increasingly popular among students and are changing the landscape of what mathematical competences students need in order to function effectively in the age of generative artificial intelligence and interact productively with the new technologies. We contend that educational systems must adapt to the new realities by resetting priorities, reorganizing curricula, and retraining teachers. In this paper, we call attention to the increased importance gained by two competences in the area of proof, proof comprehension and proof evaluation, as students interact with ChatGPT. We view our discussion of these proof competences as a case in point of the pressing need for mathematics education researchers and practitioners to reflect on what constitute important mathematical competences for students in the age of generative artificial intelligence. This is a foundational step that can serve as the basis for discussions about reimagining the future shape and role of mathematics education in society, including the training of teachers.

**Keywords:** Proof; Proof comprehension; Proof evaluation; Proof competence; Artificial intelligence; ChatGPT

## PROOF AND PROOF COMPETENCES

The notion of *proof* has attracted mathematics education researchers' attention for decades, not least because of its central role in authentic mathematical activity (A. J. Stylianides et al., 2022; Weber & Melhuish, 2022) and the recognition that proof is hard-to-teach and hard-to-learn for many teachers and students, respectively (e.g., Harel & Sowder, 2007; G. J. Stylianides et al., 2017, 2024). While there are several perspectives on what proof means in mathematics education (e.g., Balacheff, 2002), several researchers generally agree that *proof* describes a special class of mathematical arguments that are mathematically robust (in terms of their underpinning statements and modes of reasoning and representation) but also conceptually accessible to the members of the respective (classroom) community (A. J. Stylianides, 2007). This perspective on proof helps pinpoint a major problem of students' learning in this area, namely, the common mismatch between arguments that students consider to be proofs and those that actually meet the standard of proof from a mathematical standpoint (Harel & Sowder, 2007; G. J. Stylianides et al., 2017).

This problem of students' learning of proof takes three main forms, each of which corresponds to a major proof-related competence: (1) students *constructing arguments* for or against mathematical claims that fall short of the standard of proof (*proof construction* competence); (2) students failing to comprehend key aspects of arguments that meet the standard of proof, such as proofs found in textbooks (*proof comprehension* competence); and (3) students failing to evaluate non-proof arguments as non-proofs or proof-standard arguments as proofs when presented with arguments of variable mathematical qualities (*proof evaluation* competence). Of these three competences, proof construction has attracted the most attention by mathematics education researchers thus far. This is reflected, for example, in the numerous hierarchical frameworks that researchers developed to categorize students' constructed arguments (for an overview of such frameworks, see Harel & Sowder, 2007; G. J. Stylianides et al., 2017).

Proof evaluation received considerable attention, with researchers often presenting to students arguments of variable mathematical qualities and judging students' proof evaluation competence based on which arguments students considered to be proofs. Attention to proof comprehension has been the least among the three proof competences in the related literature. Researchers developed proof comprehension frameworks (e.g., Mejía-Ramos et al., 2012; Yang & Lin, 2008, 2012), which they used to judge students' understanding of arguments that meet the standard of proof against several dimensions. However, to the best of our knowledge, proof comprehension has not been reported as a learning goal in classroom studies that aimed to enhance students' proof competences, which suggests that proof comprehension may receive limited attention by mathematics teachers and curricula alike.

## PROOF COMPREHENSION AND PROOF EVALUATION AS CRITICALLY IMPORTANT COMPETENCES IN STUDENTS' PROOF-RELATED INTERACTIONS WITH CHATGPT

The rapid growth and popularity of artificial intelligence has influenced all aspects of life and education, including how we think about mathematics education (for an edited volume in this area, see Richard et al., 2022). Specifically, generative Large Language Models (LLMs), notably ChatGPT, are becoming increasingly popular among students and, whether one likes it or not, ChatGPT is beginning to change the educational landscape (Lo, 2023) including our understanding of what it means to learn and teach mathematics (Wardat et al., 2023). Given the wide range of capabilities of ChatGPT across virtually all mathematical topics and its highly user-friendly and interactive nature, its potential impact on how we think about mathematics education is expected to be larger than the impact seen by prior technological advances like calculators, computer algebra systems, dynamic geometry software, and automated theorem provers. We contend that educational systems must seriously reflect on what constitute critical mathematical competences in the context of students' mathematical interactions with ChatGPT; this is a foundational step that can serve as the basis for discussions about reimagining the future shape and role of mathematics education in society in the age of generative artificial intelligence, including the training of teachers.

Mathematics education priorities related to proof competences are not immune to the new realities created by ChatGPT. Specifically, we argue that proof comprehension and proof evaluation, which as we saw in the previous section have received less attention in research and practice compared to proof construction, emerge as critically important in students' proof-related interactions with ChatGPT. This is not to say that proof construction is no longer important. Our suggestion, rather, is that the other two competences, especially proof comprehension, can no longer remain in the background.

ChatGPT can generate not only arguments that meet the standard of proof, like automated theorem provers, but also understandable explanations that can be read by a human, thus qualifying both as a *reasoning engine* and as an *explainer* as per Van Vaerenbergh and Pérez-Suay's (2022) classification of artificial intelligence systems for mathematics education. If it could be guaranteed that all arguments produced by ChatGPT were proofs, then proof evaluation could be considered obsolete and the emphasis could be placed on proof comprehension. However, this is not the case: ChatGPT also constructs arguments that are faulty in some way (Frieder et al., 2023; Wardat et al., 2023). According to Frieder et al. (2023), the mathematical performance of ChatGPT is currently well below the level of a graduate student in mathematics. Although ChatGPT's proof construction capabilities are likely to improve dramatically before too long, this is unlikely to change the fact that, for a student to interact productively and learn from/with ChatGPT when asking ChatGPT to prove a statement, the student must be able to critically engage with ChatGPT's purported proof: Is the presented argument valid? Are its underpinning assumptions relevant and appropriate? Is the proof method that was used (e.g., contradiction, mathematical induction) appropriately applied and fit for purpose? These and other questions one might ask are in the heart of proof evaluation and proof comprehension, with proof construction playing little if any role from the perspective of the student who interacts with ChatGPT as the onus for constructing a proof is on ChatGPT.[1]

## TWO "CONVERSATIONS" WITH CHATGPT TO ILLUSTRATE THE IMPORTANCE OF PROOF COMPREHENSION AND PROOF EVALUATION

To illustrate the importance of proof comprehension and proof evaluation in the age of generative artificial intelligence, we report on two "conversations" we had with ChatGPT from the perspective of a hypothetical student who has learned about the proof method of mathematical induction and was keen to apply this method to prove statements for which the method is atypical or inappropriate. We have a long-standing interest in students' understanding of proof by mathematical induction (G. J. Stylianides et al., 2007, 2016) whose framework (comprising of the base and inductive steps and the principle of

[1] The questions become even more pertinent when a prompt to ChatGPT to prove a statement deviates from typical ways of phrasing the statement or typical methods of proving the statement. In such cases, ChatGPT is more likely to generate a flawed or perplexed argument.

mathematical induction) is prone to student errors and misconceptions. It is beyond the scope of this short paper to analyze in detail our conversations with ChatGPT. We will simply outline the two conversations and present parts of them in Figures 1-3 to exemplify some key points. We will also provide the links to the full transcripts of the conversations so that interested readers can engage more fully with ChatGPT's responses to our prompts and the various issues that these responses raise from mathematical and educational standpoints.

## First conversation about the infinitude of prime numbers

In our first conversation with ChatGPT,[2] first we established that ChatGPT was familiar with the proof method of mathematical induction and then we challenged ChatGPT to use this proof method to prove the statement that there are infinitely many prime numbers. ChatGPT's response was as follows:

> The proof that there are infinitely many prime numbers actually relies on a contradiction argument rather than mathematical induction. However, I can certainly explain this classic proof by contradiction, originally attributed to Euclid.

ChatGPT then moved on to describe, and explain, the standard proof by contradiction. Following that, we posed again our challenge to ChatGPT: "Thank you, I know the proof by contradiction [for this statement] but I wonder whether you can help me come up with another proof using mathematical induction." ChatGPT's new response is provided in Figure 1.

As can be seen in the figure, ChatGPT acknowledged that "[m]athematical induction is typically not used to prove the infinitude of primes directly," but then took on our challenge: It noted that "there is an interesting approach that uses a related inductive argument to show something that implies the infinitude of primes," and it moved on to describe "an adaptation that uses an inductive flavor."

[2] The full transcript of the first conversation with ChatGPT is available here: https://chatgpt.com/share/63c4d59a-7a87-4475-baea-ef62b7c0659e

Mathematical induction is typically not used to prove the infinitude of primes directly, but there is an interesting approach that uses a related inductive argument to show something that implies the infinitude of primes. Here's an adaptation that uses an inductive flavor:

### Inductive Proof Concept: There Exists a Prime Greater Than Any Given Number

1. **Statement to Prove:** For any natural number $n$, there exists a prime number $p$ such that $p > n$.

2. **Base Case:** For $n = 1$, the next prime number is 2, which is greater than 1. So, the statement holds for $n = 1$.

3. **Inductive Step:**

    - **Inductive Hypothesis:** Assume that for some $k \geq 1$, there exists a prime number $p$ such that $p > k$. Let $p$ be this prime number.

    - **Inductive Conclusion:** We need to show that there exists a prime number $q$ such that $q > k + 1$.

      Consider the number $N = p_1 \cdot p_2 \cdot p_3 \cdots p_m + 1$, where $p_1, p_2, \ldots, p_m$ are all the primes less than or equal to $k + 1$. Note that $N$ is constructed to be greater than $k + 1$ and is not divisible by any of $p_1, p_2, \ldots, p_m$.

        - If $N$ is prime, then $N > k + 1$, and we have found a prime $q = N$ such that $q > k + 1$.
        - If $N$ is not prime, then $N$ must have a prime factor $q$. This prime factor $q$ must be greater than $k + 1$, because $N$ is not divisible by any of the primes $p_1, p_2, \ldots, p_m$ (all of which are less than or equal to $k + 1$).

      Therefore, in either case, we have shown that there exists a prime number $q$ such that $q > k + 1$.

4. **Conclusion:** By the principle of mathematical induction, for every natural number $n$, there exists a prime number greater than $n$. This implies that there are infinitely many prime numbers because for any given number, we can always find a prime number larger than it.

This proof combines elements of induction with the construction used in Euclid's proof to demonstrate that there cannot be a largest prime, thus showing that there are infinitely many primes.

**Figure 1.** ChatGPT's response to our challenge for it to prove the statement that there are infinitely many prime numbers using mathematical induction.

ChatGPT concluded by calling the argument a "proof" that "combines elements of induction with the construction used in Euclid's proof to demonstrate that there cannot be a largest prime, thus showing that there are infinitely many primes." Does this argument really do what ChatGPT claims? Is the argument a proof of the infinitude of primes? Does ChatGPT's argument establish the equivalence between the statement we asked it to prove and the one it aimed to prove? Is the use of the framework of mathematical induction suitable in this case, and if so, was it appropriately applied? We hope that a student (or indeed any user) who receives this response from ChatGPT will ask themselves these questions. The competences of proof comprehension and proof evaluation are in the heart of these questions.[3]

**Second conversation about the Pythagorean theorem**

In our second, separate conversation with ChatGPT,[4] we continued in the modus operandi of a student who sees mathematical induction as a panacea for proving all statements. Specifically, we asked whether ChatGPT could help us prove the Pythagorean theorem using mathematical induction. ChatGPT responded with an enthusiastic "Certainly!" and explained:

> To prove the Pythagorean theorem using mathematical induction, we need to establish a suitable framework for the induction process. However, it's important to note that the Pythagorean theorem itself is a geometric statement and is traditionally proved using geometric methods. Nonetheless, I'll present a structured proof using induction in a slightly different context to help illustrate the theorem.

ChatGPT's reference to "a structured proof using induction in a slightly different context to help illustrate the [Pythagorean] theorem" is ambiguous as to whether it actually claimed to have a proof for the Pythagorean theorem satisfying our requirements. ChatGPT proceeded with a formulation of the Pythagorean theorem and provided the argument that we present in Figure 2. As can be seen in the last paragraph of the response, under the section titled

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[3] In our first conversation with ChatGPT, we also challenged it to use mathematical induction to prove an open problem in mathematics, without revealing to it the name of the conjecture, being curious to see whether it would attempt to find a way to apply mathematical induction. ChatGPT correctly identified the open problem as Goldbach's Conjecture and explained that a proof using mathematical induction (or any other method) is not possible at this time.

[4] The full transcript of our second conversation with ChatGPT is available here: https://chatgpt.com/share/356c5236-86e8-4e01-a06d-274df09a28cc

"Conclusion," ChatGPT considered this argument to be a proof by mathematical induction:

> By proving the base case and the inductive step, we have shown through mathematical induction that the Pythagorean theorem holds for all right triangles constructed by incrementally increasing one leg while keeping the other leg constant. This completes the proof using induction.

However, it is unclear whether ChatGPT considered this to be a proof of the Pythagorean theorem for all right triangles or just those constructed with its described incremental procedure. This new proof by ChatGPT raises many questions that we hope a student (or indeed any user) who receives this response from ChatGPT would ask themselves: What does ChatGPT claim to prove precisely? Does ChatGPT actually prove what it purports to prove? Is the use of the framework of mathematical induction suitable in this case, and if so, was it appropriately applied? Again, the competences of proof comprehension and proof evaluation are in the heart of answering these questions or in deciding what new questions to pose next to ChatGPT. We invite readers to read the full conversation and engage with such questions.

We finish our illustrative discussion by describing our new question to ChatGPT during our second conversation, which focused on its work in the base case (top part of Figure 2). As can be seen in Figure 3, ChatGPT responded to our new question with an admission of having used circular reasoning and volunteered to offer a new approach to the problem, which can be seen in the full transcript. In relation to ChatGPT's new approach, which ChatGPT considered to be a mathematical induction proof of the Pythagorean theorem for all right angles, we explicitly asked ChatGPT about right triangles with decimal lengths of vertical sides. ChatGPT saw no issue with this set of right triangles and provided a reassuring response, as can be seen in the full conversation. Posing questions like these and engaging critically with ChatGPT's responses is a further illustration of the importance of the competences of proof comprehension and evaluation in the context of proof-related interactions with ChatGPT.

### Pythagorean Theorem Statement:

For any right triangle with legs of lengths $a$ and $b$, and hypotenuse of length $c$, the relationship $a^2 + b^2 = c^2$ holds.

### Induction Framework:

We'll use induction on the dimension of an increasing sequence of geometric constructions that maintain the properties of right triangles.

**Base Case:**

Consider a right triangle with legs of length 1 and 1. The hypotenuse, by the Pythagorean theorem, should be $\sqrt{1^2 + 1^2} = \sqrt{2}$. Let's prove this base case geometrically:

A 1x1 right triangle is half of a square with side length 1. The hypotenuse of this right triangle forms the diagonal of the square. The length of the diagonal of a square with side length 1 is $\sqrt{1^2 + 1^2} = \sqrt{2}$, which fits the Pythagorean theorem.

**Induction Step:**

Assume the Pythagorean theorem holds for a right triangle with legs $a$ and $b$. This means we assume $a^2 + b^2 = c^2$, where $c$ is the hypotenuse.

We need to prove that the theorem holds for the next increment in the geometric sequence, specifically for legs $a + 1$ and $b$.

**Inductive Step Construction:**

Consider a right triangle with legs of lengths $a + 1$ and $b$. According to our assumption:

1. For the triangle with legs $a$ and $b$, we have $a^2 + b^2 = c^2$.

Now consider a new right triangle where one leg is increased by 1, i.e., with legs $a + 1$ and $b$.

Using the Pythagorean theorem on this new triangle, the hypotenuse $c'$ should satisfy:
$(a + 1)^2 + b^2 = c'^2$

Expand and simplify the left side:
$(a + 1)^2 + b^2 = (a^2 + 2a + 1) + b^2$

Using our inductive hypothesis $a^2 + b^2 = c^2$:
$a^2 + b^2 = c^2$
$\Rightarrow (a^2 + b^2) + 2a + 1 = c^2 + 2a + 1$

Thus, we need to prove that:
$c^2 + 2a + 1 = c'^2$
Therefore:
$c'^2 = c^2 + 2a + 1$

Taking the square root on both sides, we get:
$c' = \sqrt{c^2 + 2a + 1}$

This shows that the hypotenuse length $c'$ for the new right triangle with legs $a + 1$ and $b$ maintains the relationship defined by the Pythagorean theorem, proving the inductive step.

### Conclusion:

By proving the base case and the inductive step, we have shown through mathematical induction that the Pythagorean theorem holds for all right triangles constructed by incrementally increasing one leg while keeping the other leg constant. This completes the proof using induction.

**Figure 2.** ChatGPT's response to our challenge for it to prove the Pythagorean theorem using mathematical induction.

In the base case, how did you derive that the length of the diagonal of a square with side length 1 is square root of 2? Have you used the Pythagorean theorem?

You're right, deriving the length of the diagonal of a square with side length 1 as $\sqrt{2}$ indeed uses the Pythagorean theorem. This makes it circular reasoning to use this in the base case of an induction proof for the Pythagorean theorem. Instead, let's look at a different approach.

To prove the Pythagorean theorem using mathematical induction, we need to frame it differently. One such approach is to prove a generalization involving areas, which can then be used to infer the Pythagorean theorem. Here's a different framework:

**Figure 3.** Our follow-up question to ChatGPT about its work in the base case, and ChatGPT's admission of circular reasoning.

## CONCLUDING REMARKS

In the concluding remarks of their edited volume on the links between artificial intelligence (AI) and mathematics education, Richard et al. (2022) noted their difficulty "to predict the [AI] developments that can be expected in 5 years, 2 years or even in the course of the current year" (p. 436). Indeed, few could have anticipated the launch of ChatGPT just a few months after the publication of Richard et al.'s volume and the impact that ChatGPT is already having on how we think about students' mathematical education. The emergence of ChatGPT makes the following reflections of Richard et al. (2022), including their concern about education's slow adaptation to technological advances, even more timely today:

> New tools require new curricula and new competencies for both teachers and students. However, in the field of education, this transformation is very slow, as if it often lags behind the development of civil society. Yet we are training tomorrow's learners, not yesterday's, at a time when the youth is entering technological complexity head-on with an ease that baffles many adults.
>
> (Richard et al., 2022, p. 435)

Our discussion in this article about the increased importance of proof comprehension and proof evaluation as students interact with ChatGPT is a case in point of the pressing need for mathematics education researchers and practitioners to seriously reflect on what constitute critical mathematical competences for students in the age of generative AI. We see this as a foundational step before discussions about reimagining the future shape and role of mathematics in society as well the reform of teacher education so that teachers (both prospective and practicing) are well-prepared to foster the critical competences among their students. We are, of course, aware that researchers and practitioners are often not the key decision makers regarding mathematics education reform; rather, policy decisions tend to be influenced more by political processes and ideologies (Wright, 2012). Yet the voices of researchers and practitioners can be strong, especially when they are well-reasoned and supported by evidence.

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