Three Papers on Macroeconomics with Financial Frictions

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A thesis submitted for the degree of

Doctor of Philosophy in Economics

Trinity 2020

Acknowledgements

Foremost I thank Andrea Ferrero for all his guidance and support. I thank all the macroeconomics faculty at Oxford, especially Michael McMahon. I also thank my fellow macroeconomics DPhils, for many fun research chats and good friendships, with special thanks to Myrto Oikonomou and Alexander Haas. The Oxford macroeconomics group has been a great environment and made producing this thesis an intellectually stimulating and enjoyable experience.

I also must thank those who advised and mentored me outside of Oxford, in particular Andrea Tambalotti, Jeffrey Campbell, and José Dorich, among many others. I thank the CSWEP fellowship program for facilitating opportunities from which I have benefited greatly. For financial support I thank the Maple Leaf Trust's Canadian Centennial Scholarship Fund in conjunction with the Royal Bank of Canada.

On a personal note I would like to thank: Zoe Fannon, who has been a great friend over the last 6 years and tolerated many an interruption because I wanted to chat; Alexander Roberts, who after putting up with me during the job market somehow still is willing to follow me to Copenhagen; the members of the Oxford seconds swim team, because swimming with you made me a happier grad student.

To my family: my parents who have loved, supported, and understood me always. To my twin sister Jocienne, whose love and competition have made me better. To Gail and in loving memory of Bob.

All opinions and errors in this thesis are solely my own.

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Chapter 1: Financial frictions amplify the portfolio balance effect of QE. A costly state verification friction increases output growth by between 0.13 - 0.41 percentage points and increases inflation between 6 - 18 basis points more than the model without the friction. I find that overall that the Federal Reserve's second round of Large-Scale Asset Purchases (LSAPII) boosts output between 0.51% - 1.62%, which is the equivalent of a 83 - 278 basis point cut in the Federal Funds rate. Investors who arbitrage between long term government debt and corporate debt create a Portfolio Balance Channel in that the effects of QE spill over to the overall cost of corporate borrowing. This long term maturity preference of investors increases output growth by between 0.4 and 1.27% points, and inflation between 20 and 64 annualized basis points more than the model without this channel.

Chapter 2: From 2000-2006 US house prices and mortgage credit grew substantially. Simultaneously, the relative cost of mortgage credit fell – particularly for privately securitized mortgages – suggesting the importance of credit supply factors. This chapter explores two candidate (credit supply) shocks: an increase in the inflow of global savings into the US, and innovations in the securitization of mortgage credit. I model a two-layered mortgage market. This generates a novel *balance sheet effect*: changes in aggregate mortgage credit quantity are linked to changes in mortgage spreads via the interaction of financially constrained commercial banks and mortgage securitizers. Innovation in securitization matches mortgage credit market dynamics by directly relaxing the securitizers' financial constraint. Conversely, the inflow of global savings leads to a counter-factual increase of mortgage spreads through the balance sheet effect.

Chapter 3: In this chapter I show that the existence of unregulated financial institutions ("shadow banks") generates overborrowing (relative to the socially optimal level) in competitive equilibrium. This is because borrowers fail to internalize that an additional unit of borrowing, when the financial sector is well functioning, has adverse effects on the credit conditions they will face if a crisis arises. The social planner therefore has a motive to intervene to reduce borrowing in good times so as to limit the severity of a crisis. The discretionary planner's optimal allocation can be decentralized either using borrower based instruments or via regulation of financial intermediaries. The optimal regulation of commercial banks is pro-cyclical.

Chapter 1

The Portfolio Balance Channel of Quantitative Easing in a DSGE Model with Financial Frictions

1.1 Introduction

There is a tension between the generally small effects of the Federal Reserve's programs of Quantitative Easing[1] (QE) found in DSGE models and the relatively larger effects of QE shown in VAR/empirical work. This paper asks if the presence of financial frictions can amplify the portfolio balance effect of QE. I find that the second round of the Federal Reserve's Large Scale Asset Purchase Program (LSAPII) boosted output between 0.51% and 1.62%, the equivalent of a 83 - 278 basis point cut in the Federal Funds Rate.

At its most general the Portfolio Balance Channel of QE captures the idea that purchases of longer maturity US Treasuries increase the price of other, substitute assets. Krishnamurthy and Vissing-Jorgensen (2011) emphasize that the portfolio balance effect can be driven in multiple ways. This paper focuses on one specific mechanism: investors who arbitrage between the return on investing in corporate debt and the return on long term government bonds.

Quantitative Easing works in the model because households and investors have a preference for longer term assets - the "preferred habitat" characterized by Vayanos and Vila (2009). So the relative supply of long-to-short term bonds will impact the relative price of bonds. Krishnamurthy and Vissing-Jorgensen (2011) emphasize that the evidence

[1] I.e the Large Scale Asset Purchase Programs (LSAPs)

for preferred habitat is strongest for near-zero default risk assets. Longstaff, Mithal and Neis (2005) find that this includes corporate bonds of investment grade quality (rated above Baa). A narrow application of preferred habitat implies that a reduction in the relative supply of long vs short term government debt will reduce the term premium on government bonds. The model captures this with household who have a preferred ratio of long-to-short term government bonds. This preference is crucial to breaking Wallace's Irrelevance result (Wallace, 1981) meaning that QE has a role in this model. A wider interpretation of preferred habitat (extending it to corporate borrowing) implies that QE should also put downward pressure on corporate borrowing costs. In the model this is captured by investors who substitute between longer term government debt and longer term corporate debt. This means that QE will drive yields on corporate bonds. This is then amplified via a default risk channel - lower corporate borrowing costs and better economic conditions decrease default rates.

Vector autoregression (VAR) based estimates of the macroeconomic impacts of Quantitative Easing generally find larger quantitative impacts of QE than DSGE literature does. For example Baumeister and Benati (2013) find that the median impact of the Federal Reserve's second round of Large Scale Asset Purchases (LSAPII) was to boost GDP by 3% and increase inflation by 1%. For the UK Kapetanios, Mumtaz, Stevens and Theodoridis (2012) find that the peak effects of the Bank of England's first round of QE were a 1.5% increase in GDP and a 1.25% increase in inflation. In contrast the quantitative results in the DSGE literature on QE are muted. For example Chen, Curdia and Ferrero (2012) find that the Federal Reserve's second round of asset purchases had a slightly smaller effect than a surprise 25 basis point cut in the Federal Funds Rate. Their median results are that GDP increased by 0.13% and inflation increased by 3 basis points (both annualized). Harrison (2012) similarly finds small impacts of the Bank of England's QE program. One exception is Graeve and Theodoridis (2016): in a DSGE model with a complex fiscal block the authors find the Federal Reserve's 2011 Maturity Extension Program increased GDP by 0.6%. Key to their results is the response of the maturity structure of government debt to the QE policy. In contrast here I consider only the marginal change in the government's debt maturity structure (the reduction in long term bonds and the corresponding increase in reserves coming directly from purchases).

The model in this paper builds quantitative easing into a model based on Christiano, Motto and Rostagno (2014). The Christiano et al. (2014) model builds a costly state verification financial friction into a standard DSGE model a la Smets and Wouters (2007) and Christiano et al. (2005). The costly state verification friction comes from Bernanke, Gertler and Gilchrist (1999): lenders can only view the balance sheet of a defaulted non-financial firm by paying a cost. This generates an external finance premium that lenders must pay. Firms receive idiosyncratic productivity shocks. A shock to the standard deviation of the productivity distribution is called a "risk shock". Christiano et al. (2014) find that the variation of the risk shock over time is the most important driver of US business cycles. Del Negro and Schorfheide (2013) find the risk shock to have been an important factor during the Great Recession.

The model allows for QE to be non-neutral by incorporating preferred habitat preferences in two ways. One, households have a preferred ratio of long to short term government bonds - following Harrison (2012). Two, there are investors who only invest in long term government bonds or corporate debt. Broadening the role of preferred habitat in driving QE is motivated by the evidence in Krishnamurthy and Vissing-Jorgensen (2011) and Longstaff et al. (2005) which suggests that preferred habitat preferences for long term maturity bonds apply to high grade corporate debt in addition to long term Treasuries. Widening the preferred habitat preferences adds a Portfolio Balance Channel of QE and is the key contribution of this paper.

The paper proceeds as follows. Section 2 presents the model. Section 3 presents the data and calibration method. Section 4 presents the simulation of the Federal Reserve's second round of Large Scale Asset Purchases. Section 5 concludes.

1.2 The Model

1.2.1 Households

All households are identical and large in number. Each household holds a large number of entrepreneurs and every type of differentiated labor. Households consume, invest to produce raw capital (which is then sold to entrepreneurs), buy and hold long and short term government bonds, supply labor, and receive labor income.

The representative household has the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t - bC_{t-1}) - \psi_L \int_0^1 \frac{h_{it}^{1+\sigma_L}}{1+\sigma_L} di - \frac{\tilde{\nu}}{2} \left(\frac{B_t^L}{B_t} - \delta^b \right)^2 \right\}, \tag{1.2.1}$$

where β is the household's discount rate, C_t is per capita consumption, b is the habit parameter, ψ_L is the dis-utility weight on labor, σ_L is the Frisch elasticity of labor supply, and h_{it} is labor supply by labor type i. δ^b is the preferred ratio of long-to-short term government debt, $\tilde{\nu}$ governs the dis-utility from deviating from the preferred portfolio, and finally B_t and B_t^L are the market value of privately held short and long term government debt.

The final term in the household's utility function is the household's portfolio preference. Households have quadratic disutility over deviations from the preferred ratio of long-to-short term government debt (δ^b). I follow Harrison (2012), which builds on the work of Andrés et al. (2004). I calibrate δ^b to match the steady state ratio of long-to-short term government debt in the hands of the public. $\tilde{\nu} > 0$ implies that the term premium responds to changes in the household's relative holdings of short and long term government debt (this breaks Wallace's Irrelevance Result). I calibrate $\tilde{\nu} > 0$ to match the elasticity of the term premium to changes in the maturity structure of government debt.

The household's budget constraint is:

$$\begin{aligned} & P_t C_t + B_t + B_t^L + \frac{P_t}{\Upsilon^t \mu_{\Upsilon t}} I_t + Q_{\bar{K},t}(1-\delta)\bar{K}_t \\ & \leq \int_0^1 W_{it} h_{i,t} di + R_{t-1} B_{t-1} + R_t^L B_{t-1}^L + Q_{\bar{K},t} \bar{K}_{t+1}, \end{aligned} \tag{1.2.2}$$

where P_t is the price of the consumption good, I_t is the quantity of investment goods purchased by the household for a price $\frac{P_t}{\Upsilon^t \mu_{\Upsilon t}}$ ($\mu_{\Upsilon t}$ is a shock to investment technology, and Υ^t is trend growth in investment technology), $\bar{Q}_{\bar{K},t}$ is the price of raw capital, $\bar{K}_{t+1}$ is end of period t raw capital, δ is the depreciation rate of capital, W_{it} is the wage rate for labor type i, R_{t-1} is the rate paid on short term bonds issued in $t-1$ maturing in t, and R_t^L is the rate paid on long term bonds issued in $t-1$ maturing in t.

The model's treatment of the bond market is based on Harrison (2012). There are two types of government bonds: short, and long. Short bonds sell for a unit price at time t, and return R_t units of currency at time t+1. Long bonds are modeled as perpetuities that exist for an infinite number of periods (unless the government removes them from the market). They provide a coupon payment of 1 unit of currency each period, and have a value V_t at time t. In each period t, after making the coupon payment, the government rolls over its debt by purchasing the entire stock of long-term debt (B_{t-1}^c) at the market price V_t and issuing new consol bonds B_t^c which are purchased by households for the market price V_t. $R_t^L \equiv \frac{1+V_t^L}{V_{t-1}^L}$ is the gross return on a long bond sold at time $t-1$. $B_t^L \equiv V_t B_t^c$ is the total nominal value of long bonds at time t. Note the timing: V_t^L is unknown at time $t-1$; therefore R_t^L (the gross return on a long bond purchased at time $t-1$) is not known for certain until time t.

When investing in raw capital the household faces the following law of motion:

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t. \tag{1.2.3}$$

The investment adjustment cost function S has the functional form:

$$S(x_t) \equiv \frac{1}{2}\left\{ \exp\left[\sqrt{S''}(x_t - x)\right] + \exp\left[-\sqrt{S''}(x_t - x)\right] - 2 \right\}, \tag{1.2.4}$$

where $x_t \equiv \frac{I_t}{I_{t-1}}$ and $S'' \equiv S''(x)$ is a parameter calibrated to match the dynamics of investment.

1.2.2 Production Markets

1.2.2.1 Goods Market

Each intermediate good, $Y_{jt},\ j \in [0,1]$ is produced by a different monopolist according to the following production function:

$$Y_{jt} = \begin{cases} \epsilon_t K_{jt}^{\alpha}\big(z_t l_{jt}\big)^{(1-\alpha)} - \Phi z_t^*, & \text{if } \epsilon_t K_{jt}^{\alpha}\big(z_t l_{jt}\big)^{(1-\alpha)} > \Phi z_t^* \\ 0, & \text{otherwise} \end{cases} \qquad (1.2.5)$$

where the capital share $\alpha \in (0,1)$ and ϵ_t is a technology shock (that is covariance stationary). K_{jt} is the quantity of effective capital used by monopolist producer j, and l_{jt} the quantity of homogeneous labor employed by monopolist producer j. z_t is an effective labor shock which has a stationary growth rate. The proportional fixed cost Φz_t^* is such that the intermediate monopolistic producer earns zero profits in steady state. The detrending term z_t^* is described in more detail below.

There is a Calvo friction in the pricing of intermediate goods. Each period a random fraction of intermediate firms, $1-\xi_p$, can reoptimize their price P_{jt}. The remaining fraction ξ_p set their price as follows:

$$P_{jt} = \tilde{\pi}_t P_{j,t-1}, \qquad (1.2.6)$$

where inflation indexation is as follows:

$$\tilde{\pi}_t = \big(\pi_t^{target}\big)^{\iota}\big(\pi_{t-1}\big)^{1-\iota}. \qquad (1.2.7)$$

$\pi_{t-1} \equiv \frac{P_{t-1}}{P_{t-2}}$ is gross inflation. And π_t^{target} is the central bank's target inflation rate. ι is the price indexing weight on the inflation target.

The homogeneous final good Y_t, is produced by a competitive representative firm (with Dixit-Stiglitz technology):

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{1}{\lambda_f}}\, dj\right]^{\lambda_f},\ 1 \leq \lambda_f < \infty,\ j \in \big[0,1\big]. \qquad (1.2.8)$$

The homogeneous final good has two uses: consumption and investment. One unit of Y_t can be converted into one unit of the consumption good C_t, and thus (given perfect competition in the use of this technology), consumption has the price P_t. One unit of Y_t can also be converted into $\Upsilon^t \mu_{\Upsilon,t}$ units of the investment good, and thus (again given perfect competition in the use of the technology) has the price $\frac{P_t}{\Upsilon^t \mu_{\Upsilon,t}}$, where $\Upsilon > 1$.

There are two sources of growth in the model. First, the trend rise in the aforementioned technology for producing investment goods, Υ^t, and second, the effective labor shock z_t which has a stationary growth rate. The de-trending term z_t^* is a combination of both sources of growth:

$$z_t^* = z_t \Upsilon^{\left(\frac{\alpha}{1-\alpha}\right)t}. \tag{1.2.9}$$

z_t^* is used to normalize variables to find a non stochastic steady state. z_t^* is such that $\frac{Y_t}{z_t^*}$ will converge to a constant in the nonstochastic steady state of the model. $\mu_{z,t}^* \equiv \frac{z_t^*}{z_{t-1}^*}$ is the growth rate of z_t^*, which has the stationary growth rate μ_z^*.

1.2.2.2 Labor Market

Each differentiated labor type type $i \in [0,1]$ provides labor services h_{it} and is represented by a monopoly union that sets its wage rate W_{it} while facing a Calvo friction. Each period a fraction $1 - \xi_w$ of the monopoly unions can update the wage. The remaining fraction ξ_w set their wage as follows:

$$W_{it} = \left(\mu_{z,t}^*\right)^{\iota_\mu} \left(\mu_z^*\right)^{1-\iota_\mu} \tilde{\pi}_{wt} W_{i,t-1}, \tag{1.2.10}$$

where:

$$\tilde{\pi}_{wt} \equiv \left(\pi_t^{target}\right)^{\iota_w} \left(\pi_{t-1}\right)^{1-\iota_w}, \qquad 0 < \iota_w < 1. \tag{1.2.11}$$

Labor is aggregated via a Dixit-Stiglitz style aggregator by a competitive and representative labor contractor:

$$l_t = \left[\int_0^1 \left(h_{it}\right)^{\frac{1}{\lambda_w}} di\right]^{\lambda_w}, \qquad 1 \le \lambda_w. \tag{1.2.12}$$

The homogeneous labor aggregate l_t is sold to intermediate goods producers at the nominal wage W_t.

1.2.3 Entrepreneurs

Entrepreneurs have the role of turning raw capital (purchased from households) into effective capital (to be then sold to intermediate goods producers). They experience idiosyncratic productivity shocks in their ability to turn raw capital into productive capital. They finance the purchase of raw capital via debt, and entrepreneurs who experience low idiosyncratic productivity shocks default on their debt. Entrepreneurs' creditors can only observe the state of a defaulted entrepreneur's balance sheet by paying a proportional recovery cost. The expectation of paying this cost introduces a spread on entrepreneurial debt. This is the Costly State Verification (CSV) financial friction characterized in Bernanke, Gertler and Gilchrist, 1999 (henceforth BGG) and Christiano et al. (2014) (henceforth CMR).

Entrepreneurs' aggregate net worth is considered a proxy for the value of the stock market. Entrepreneurs can either be interpreted as being firms in the non-financial sector, or financial institutions with non-diversified holdings.

Entrepreneurs are classified by their net worth. An entrepreneur with net worth $N \geq 0$ is called an 'N-type' entrepreneur. The timing of one cycle in the life of an entrepreneur is as follows. Following production in period t, each entrepreneur gets a loan from a mutual fund. Each mutual fund is specialized. They make loans only to entrepreneurs of a specific level of net worth, but perfectly diversify by holding a large number of those loans. The entrepreneur combines the loan $B_{t+1}^{N,credit}$ (issued in period t and due in period $t+1$) with their own net worth to purchase raw capital ($\bar{K}_{t+1}^{N}$) at price $Q_{\bar{K},t}$:

$$B_{t+1}^{N,credit} + N = Q_{\bar{K},t}\bar{K}_{t+1}^{N}. \tag{1.2.13}$$

After raw capital is purchased each entrepreneur receives an idiosyncratic shock ω that determines the amount of effective capital they have, $\omega\bar{K}_{t+1}^{N}$. As in BGG and CMR, ω is distributed (independently across entrepreneurs and time) log-normally with a unit mean and a standard deviation $\sigma_t \equiv \sqrt{var(\log\omega)}$. The *risk shock* σ_t is simply the extent of cross-sectional dispersion of idiosyncratic productivity shocks experienced by entrepreneurs.

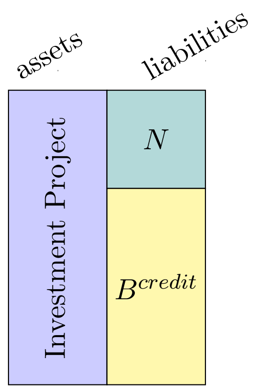


Figure 1.2.1: Entrepreneur's Balance Sheet

After realizing the risk shock entrepreneurs choose the utilization rate of effective capital u^N_{t+1} to maximize their return on capital: ωR^k_{t+1} at the end of period $t+1$. Entrepreneurs supply $u^N_{t+1}\omega\bar{K}^N_{t+1}$ units of effective capital to intermediate goods producers at the market rental rate r^k_{t+1}. The return on capital is defined as follows:

$$R^k_{t+1} \equiv \frac{[u_{t+1}r^k_{t+1} - a(u_{t+1})]\Upsilon^{-(t+1)}P_{t+1} + (1-\delta)Q_{\bar{K},t+1}}{Q_{\bar{K},t}}. \tag{1.2.14}$$

The choice of utilization is independent of net worth so the N superscript is dropped. The utilization cost of capital, $a(u_t)$, is increasing and convex:

$$a(u_t) \equiv \frac{r^k_t}{\sigma_a}\Big[\exp\Big(\sigma_a(u_t-1)\Big) - 1\Big]. \tag{1.2.15}$$

In addition to the utilization choice entrepreneurs must also choose the type of debt contract to accept. It is each entrepreneur's objective to maximize their expected net worth in the next period (t+1), which is as follows:

$$E_t\Bigg\{\int_{\bar{\omega}_{t+1}}^{\infty}\Big[R^k_{t+1}\omega Q_{\bar{K},t}\bar{K}^N_{t+1} - B^{N,credit}_{t+1}Z_{t+1}\Big]dF(\omega,\sigma_t)\Bigg\}$$
$$= E_t\Big[1-\Gamma(\bar{\omega}_{t+1})\Big]R^k_{t+1}L_tN. \tag{1.2.16}$$

where $\Gamma_t(\bar{\omega}_{t+1}) \equiv \Big[1-F_t(\bar{\omega}_{t+1})\Big]\bar{\omega}_{t+1} + G_t(\bar{\omega}_{t+1})$ is the fraction of expected earnings paid to the investor, $1-F_t(\bar{\omega}_{t+1})$ is the probability the entrepreneur experiences a idiosyncratic shock over the default threshold $\bar{\omega}_{t+1}$, and $F(.)$ is the cumulative distribution function of ω. Z_{t+1} is the gross nominal interest rate on debt. $G_t(\bar{\omega}_{t+1})$ is the expected value of the idiosyncratic shock in the population of defaulting entrepreneurs:

$$G_t(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega), \tag{1.2.17}$$

where $L_t \equiv \frac{Q_{\bar{K},t}\bar{K}^N_{t+1}}{N}$ is the entrepreneur's leverage. Entrepreneurs maximize (1.2.16) by choosing the conditions that characterize the debt contract from the available set of debt contracts that mutual funds are willing to provide. That is equation (1.2.19) described below. The conditions are: one, the level of the idiosyncratic shock ω below which they will default (this is $\bar{\omega}_{t+1}$), or equivalently the gross nominal interest rate on debt to be paid next period: Z_{t+1}, and two, the amount of leverage L_t they will take on.

The default threshold is defined as follows:

$$\bar{\omega}_{t+1} \equiv \frac{B^{N,credit}_{t+1} Z_{t+1}}{R^k_{t+1} Q_{\bar{K},t} \bar{K}^N_{t+1}}. \tag{1.2.18}$$

1.2.3.1 Mutual Funds & Investors

Specialized mutual funds make loans to N-type entrepreneurs, diversifying over type. They sell packaged entrepreneurial debt to investors. Investors are only willing to hold long term debt - they have preferred habitat preferences. Because of their preferences investors will only hold packaged entrepreneurial debt if its return is equal to the return on long term government debt[2]. Investors' preferences mean the long term government bond rate enters the mutual fund's zero-profit condition:

$$\Big[1 - F_t(\bar{\omega}_{t+1})\Big] Z_{t+1} B^{N,credit}_{t+1} + (1-\mu) \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega) R^k_{t+1} Q_{\bar{K},t} \bar{K}^N_{t+1} \equiv E_t R^L_{t+1} B^{N,credit}_{t+1}. \tag{1.2.19}$$

Putting the long term government bond rate into the zero profit condition (1.2.19) captures the portfolio balance channel in this model. When the rate on long term government debt falls, investors are willing to hold packaged entrepreneurial debt at a lower

[2]Clearly this is a strong assumption for two reasons. One, in the data there is a spread between corporate debt yields and US Treasury yields - meaning that there are likely frictions between government debt and corporate debt, and two, the empirical evidence that supports a preferred habitat preference over bond maturity does not extend to all corporate debt. The evidence in Longstaff, Mithal and Neis (2005) supports the preferred habitat only extending to corporate debt rated above Baa. This suggests that in future work adding a friction to investor arbitrage is important.

rate. This in turn relaxes the credit conditions that mutual funds are able to provide to entrepreneurs.

If an entrepreneur experiences an idiosyncratic shock ω below the threshold $\bar{\omega}_t$, then they will not be able to repay their debt to the investor and will declare bankruptcy. In this instance the mutual fund only knows that the entrepreneur is bankrupt, but does not observe the value of ω. Without further action by the mutual fund the entrepreneur could decide to transfer only a fraction of their remaining assets, $\omega R^k_{t+1} Q_{\bar{K},t} \bar{K}^N_{t+1}$, back to the mutual fund. In order to become fully informed about the assets a bankrupt entrepreneur has, the mutual fund must pay a cost that is a proportion μ of the final assets recovered. Thus the mutual fund only receives a fraction $(1-\mu)$ of the total assets of bankrupt entrepreneurs.

The following condition (1.2.20) characterizes the available menu of contracts entrepreneurs can choose from. This comes from using the definition of the default threshold (1.2.18) to substitute out $Z_{t+1} B^{N,credit}_{t+1}$ in the mutual funds' .

$$\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) = \frac{L_t - 1}{L_t} \frac{R^L_{t+1}}{R^k_{t+1}}. \tag{1.2.20}$$

1.2.3.2 Accelerator Effect

The characterization of the financial friction is from Christiano et al. (2014) (CMR) and is based on the costly state verification (CSV) financial accelerator mechanism in Bernanke, Gertler and Gilchrist, 1999 (BGG). BGG emphasize the following intuition behind the accelerating affects of the financial friction in the CSV class of models. The basic idea is that a fall in entrepreneurial net worth means that the entrepreneurs will have less inside funds to invest in the project. Therefore the investors that make loans to entrepreneurs face a greater agency cost when they finance the entrepreneurs. Essentially the entrepreneur has less “skin in the game”. The higher agency cost means investors charge a higher interest rate, so that the premium on external finance faced by entrepreneurs increases. Faced with a larger interest rate on loans, other things being equal, entrepreneurs will choose to purchase less capital. Because entrepreneurs play a key role in turning raw capital into effective capital used by producers the increase in the external finance premium will decrease output. The net worth of entrepreneurs is pro-cyclical. Therefore the external finance premium is counter-cyclical. Thus the interactions between investors and entrepreneurs via the hike in the external finance premium will serve to amplify the business cycle.

This also means that to the extent to which QE boosts entrepreneurial net worth it can have the reverse effect - lowering the external finance premium, boosting capital, and increasing output.

1.2.3.3 Aggregates

The aggregates are as follows:

Aggregate raw capital:

$$\bar{K}_{t+1} = \int_0^\infty \bar{K}_{t+1}^N f_t(N) dN. \tag{1.2.21}$$

Aggregate effective capital:

$$K_t = \int_0^\infty \int_0^\infty u_t^N \omega \bar{K}_t^N f_{t-1}(N) dF(\omega) dN = u_t \bar{K}_t. \tag{1.2.22}$$

Aggregate net worth:

$$N_{t+1} = \int_0^\infty N f_t(N) dN. \tag{1.2.23}$$

Aggregate credit:

$$B_{t+1}^{credit} = \int_0^\infty B_{t+1}^N f_t(N) dN = \int_0^\infty \Big[Q_{\bar{K},t} \bar{K}_{t+1}^N - N\Big] f_t(N) dN = Q_{\bar{K},t} \bar{K}_{t+1} - N_{t+1}. \tag{1.2.24}$$

Finally the evolution of aggregate net worth is:

$$N_{t+1} = \gamma \Big[1 - \Gamma_{t-1}(\bar{\omega}_t)\Big] R_t^k Q_{\bar{K},t-1} \bar{K}_t + W^e, \tag{1.2.25}$$

where γ is the fraction of entrepreneurs that continue each period (a fraction $1-\gamma$ exit and pay dividends to the household), and W^e is the transfer from the household to new entering entrepreneurs.

1.2.4 Government Policies

As in Chen et al. (2012) the government has an auto-regressive supply rule for the market value of de-trended long-term bonds ($b_t^L \equiv \frac{B_t^L}{P_t z_t^*}$):

$$\log\left(\frac{b_t^L}{b^L}\right) = \rho_{bL}\left(\frac{b_{t-1}^L}{b^L}\right) + u_t^{bL}. \tag{1.2.26}$$

u_t^{bL} is the sum of unanticipated and anticipated news shocks to the supply of long term government bonds:

$$u_t^{bL} \equiv \epsilon_t^{bL} + \xi_{1,t-1}^{bL} + ... + \xi_{8,t-8}^{bL}. \tag{1.2.27}$$

The government budget constraint is:

$$B_t + B_t^L = R_{t-1}B_t + R_t^L B_{t-1}^L + G_t - T_t, \tag{1.2.28}$$

where G_t is nominal government spending, and T_t is nominal government taxation.

Government spending is:

$$\frac{G_t}{P_t z_t^*} = g, \tag{1.2.29}$$

where the steady state level of real de-trended government spending, g, is calibrated to be 20% of steady state output.

The fiscal rule is adapted from Davig and Leeper (2006) and Eusepi and Preston (2011):

$$\frac{T_t}{P_t z_t^*} - \frac{G_t}{P_t z_t^*} = \kappa\left(\frac{b_{t-1} + b_{t-1}^L}{b^L + b}\right)^{\phi_T} \epsilon_t^T, \tag{1.2.30}$$

where κ is the steady state primary fiscal surplus, and ϕ_T is set high enough so that the primary surplus adjusts to satisfy the government inter-temporal budget constraint and where b_{t-1} and b_{t-1}^L are the real de-trended market value of short and long term bonds respectively ($b_t \equiv \frac{B_t}{P_t z_t^*}$).

1.2.5 Monetary Policy & Resource Constraint

1.2.5.1 Monetary Policy Rule

The central bank sets the short rate according to a backward-looking Taylor Rule:

$$\log\left(\frac{R_t}{R}\right) = \rho_m \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_m)\left[\phi_\pi \log\left(\frac{\pi_t}{\pi_t^{target}}\right) + \frac{\phi_y}{4}\left(\log\frac{Y_t}{Y_{t-1}} - \log\mu_z^*\right)\right] + \frac{1}{400}u_t^m, \tag{1.2.31}$$

where μ_z^* is the steady state growth of output. u_t^m is the sum of unanticipated and anticipated (news) monetary policy shocks:

$$u_t^m \equiv \epsilon_t^m + \xi_{1,t-1}^m + ... + \xi_{8,t-8}^m, \tag{1.2.32}$$

where ϵ_t^m is the unanticipated monetary policy shock. And $\sum_{p=1}^{p=8} \xi_{p,t-p}^m$ is the sum of anticipated monetary policy shocks.

1.2.5.2 The Resource Constraint

$$Y_t = G_t + C_t + \frac{I_t}{\Upsilon^t \mu_{\Upsilon,t}} + a(u_t)\Upsilon^{-t}\bar{K}_t + \Theta\frac{1-\gamma}{\gamma}(N_{t+1} - W^e) + D_t, \tag{1.2.33}$$

where $a(u_t)\Upsilon^{-t}\bar{K}_t$ is the aggregate capital utilization cost of entrepreneurs. $\Theta\frac{1-\gamma}{\gamma}(N_{t+1} - W^e)$ are the resources consumed by exiting entrepreneurs. $D_t \equiv \frac{\mu G_{t-1}(\bar{\omega}_t)R_t^k + Q_{\bar{K},t-1}\bar{K}_t}{P_t}$ are the resources expended on monitoring entrepreneurs.

1.3 Data and Calibration

The target period for calibration is 1985Q1 to 2007Q3. The data series used and mapping between the data and steady state targets are described in appendix 1.D.

The household's discount rate β is calibrated to match the period average of the Effective Federal Funds Rate. δ^b is set to match the period average of the ratio of long-to-short term government bonds held by the public, where long-term debt is defined as any government bond with over 1 year until maturity, and short-term debt includes reserves. Government spending in steady state is set to 20% of quarterly output, in line with the target in Christiano et al. (2014). ψ_L is set to target steady state hours worked (h) equal to 1. Steady state inflation is 2%. The calibrations corresponding with these

Table 1.1: Steady State Targets

Target	**Value**
Nominal Federal Funds Rate, annualized (R)	5.11
Ratio of long-term government bonds to annual output $\left(\frac{B^L}{4\times Y}\right)$	19.3%
Ratio of long to short bond holdings $\left(\frac{B^L}{B}\right)$	1.86
Ratio of government spending to quarterly output $\left(\frac{G}{Y}\right)$	20%
Hours worked (h)	1
Inflation, APR (π)	2%

targets are appendix 1.D table 1.D.2. The κ parameter in the fiscal rule (equation 1.2.30) is set to equal the steady state primary fiscal surplus.

$\nu \equiv \frac{1}{\lambda_z}\frac{\tilde{\nu}\delta^b}{b}(1+\delta^b)$ is the elasticity of the long rate to changes in the relative supply of long bonds (see appendix 1.E for further detail). The target range is a 3 to 15 basis point drop in the term premium in response to a $100 billion reduction in the supply of long-term bonds available to the public (holding short bonds constant). This range is discussed further in section 1.4.2.

The parameters in table 1.2 are fixed according to the calibration in CMR, or CMR posterior modes. All shock processes are specified as log AR(1)'s. Their persistence parameters are specified in table 1.3:

Table 1.2: Calibrated Parameters

Parameter	Description	Calibration
α	capital's share of output	0.4
b	habit parameter	0.74
Θ	fraction of assets consumed by exiting entrepreneurs	0.005
δ	depreciation rate of capital	0.025
ϵ	steady state value of the technology shock	1
$F(\bar{\omega})$	steady state probability of default	0.0056
γ	fraction of entrepreneurs who survive	0.985
ι	price indexing weight on inflation target	0.9
ι_μ	wage indexing weight on persistent technology growth	0.94
ι_w	wage indexing weight on inflation target	0.49
λ_f	markup in the product market	1.2
λ_w	markup in the labor market	1.05
μ	monitoring cost	0.21
μ_Υ	steady state value of $\mu_{\Upsilon,t}$	1
μ_z^*	mean growth rate of the unit root technology shock	1.0041
ϕ_π	parameter on inflation in the Taylor Rule	2.40
ϕ_y	parameter on output in the Taylor Rule	0.36
ϕ_T	feedback parameter in the fiscal rule, set according to Chen et al. (2012) posterior mean	1.3147
ρ_m	weighting of lagged short-rate in Taylor Rule	0.85
S''	parameter in the investment adjustment cost function	10.78
σ_a	curvature of utilization cost	2.54
σ_L	Frisch elasticity of labor supply	1
Υ	quarterly rate of investment-specific technological change	1.0042
w^e	lump sum transfer from household to the entrepreneur	0.005
ξ_p	Calvo price stickiness	0.74
ξ_w	Calvo wage stickiness	0.81

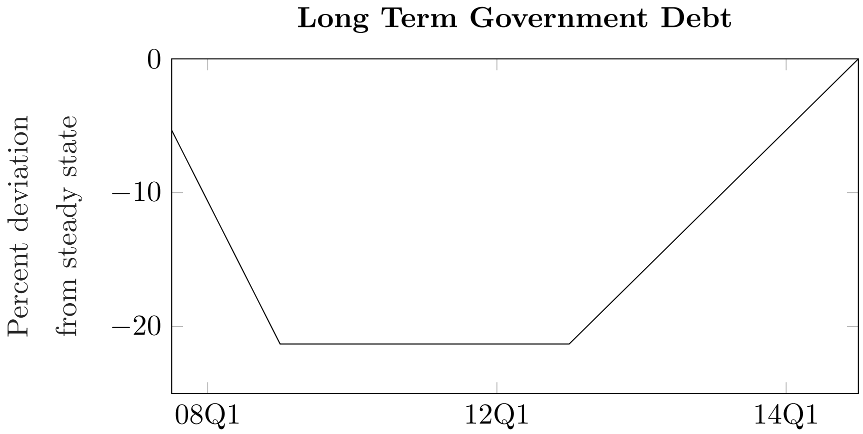
Table 1.3: Shock Autocorrelations

Parameter	Description	Calibration
ρ_{bL}	long-term bond supply	0
ρ_ϵ	transitory technology	0.81
$\rho_{\mu_z^*}$	persistent technology growth	0.15
ρ_σ	risk shock	0.97

1.4 Simulations

1.4.1 Simulating LSAPII

Figure 1.4.1: Simulated Path of Long-Term Government Debt During LSAPII



Following Chen et al. (2012) I simulate the second round of the Federal Reserve's quantitative easing program (LSAPII) with purchases taking place over two quarters, held for four quarters, and QE2 unwound over 8 quarters. The full path of purchases (figure 1.4.1) is known upon announcement of the program. I constrain the response of the federal funds rate for four quarters. I implement both the QE announced path and the forward guidance using news shocks. The mapping between the size of the $600 Billion in purchases and long bond supply shocks is described in appendix 1.D.

1.4.2 The Impact of LSAP II

Empirical estimates of QE's "stock effects" - i.e. the impact on bond prices from the semi-permanent reduction of bond supply within a given maturity - suggest that the Federal Reserve's various rounds of LSAP purchases reduced term premiums between 3 and 10 basis points per $100 billion of long term bond purchases. On the upper range of the estimates D'Amico and King (2013) find that LSAPI on average decreased yields within a given maturity by 1 basis point per $10 billion in long term bond purchases. On the lower range Hamilton and Wu (2012) find that a $400 billion purchase of long-term maturity government bonds could reduce the 10-year rate by 13 basis points when the policy rate is at the zero lower bound.

I calibrate ν to match these estimates of the elasticity of the term premium to purchases. This gives a range of 0.00074- 0.0025. Figures 1.4.2 and 1.4.3 show the range of results. The output growth peak is between 0.51% - 1.62%. Output 6 years after the start of the LSAPII program is between 0.26% - 0.80% above its steady state level. Inflation increases between 28 and 88 basis points. In terms of federal funds rate cut equivalence (the rate cut needed to achieve the same peak growth in output) the model suggests the LSAPII program was equal to between a 83 - 278 annualized basis point cut in the federal funds rate.

Figure 1.4.2: The Impact of LSAPII (a)

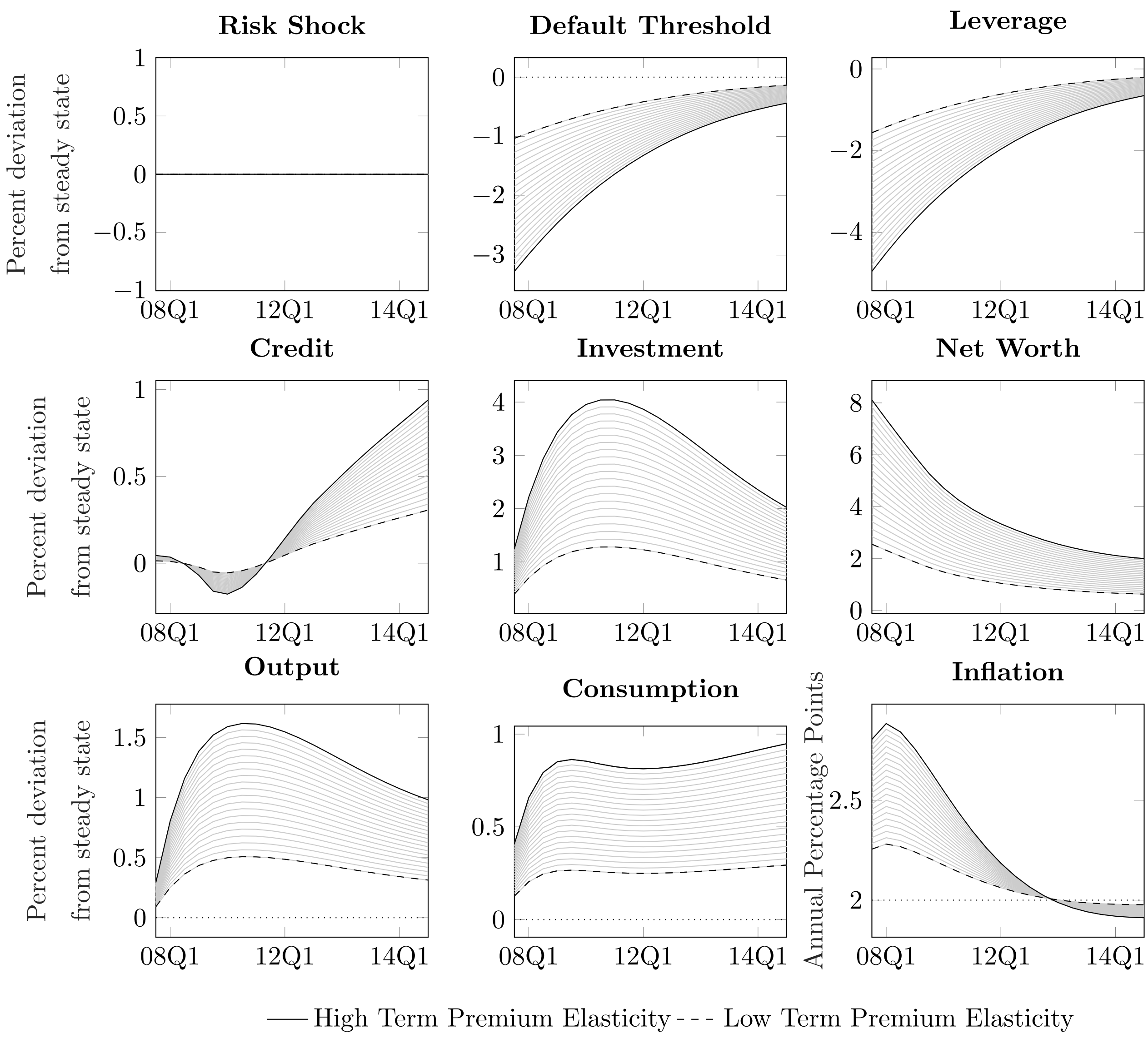
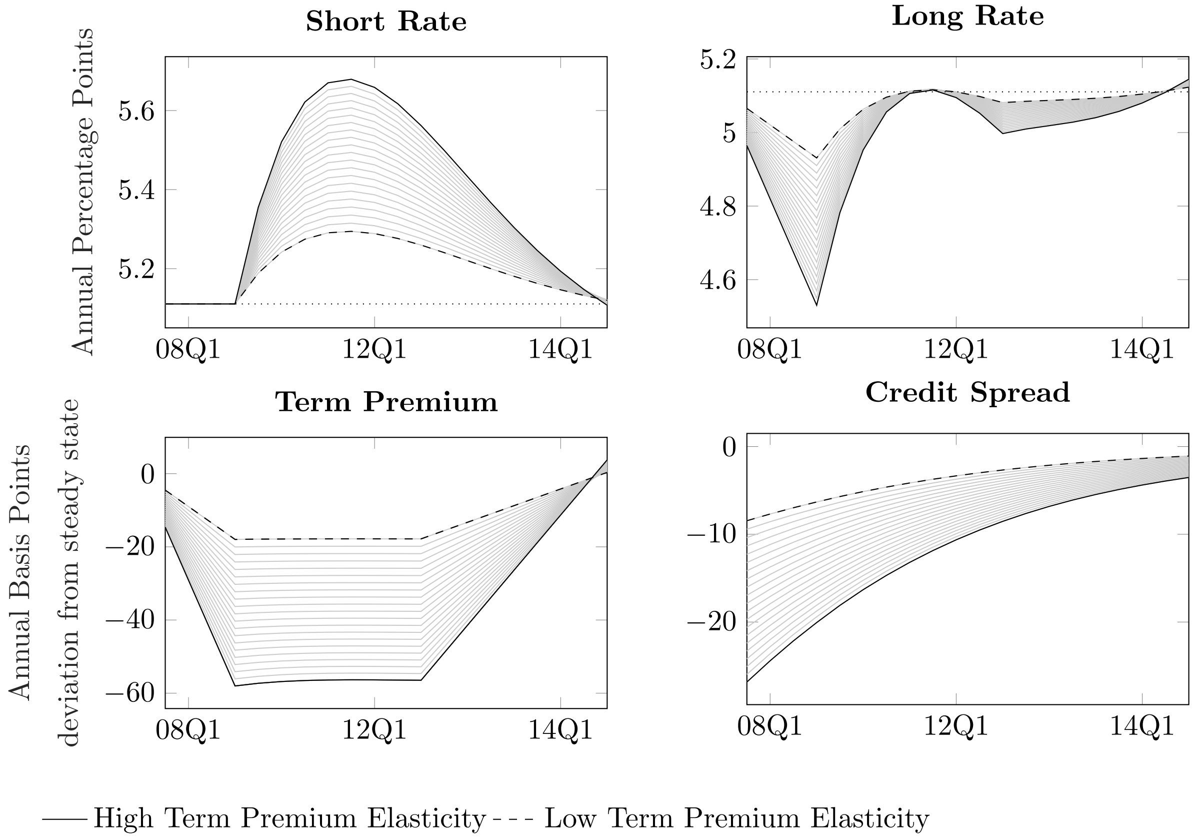


Figure 1.4.3: The Impact of LSAPII (b)



1.4.2.1 QE Mechanisms

In this paper the household's preference for an ideal ratio of long to short term government bonds in the household's utility function captures the idea that certain agents prefer to hold longer maturity assets ("preferred habitat") as emphasized by Vayanos and Vila (2009). This means per period returns on short vs long term assets are not fully arbitraged away, leading to a term premium for long term government bonds. The households have a narrow preferred habitat, restricted to only US Treasury bonds. This is the "Safety Channel" described by Krishnamurthy and Vissing-Jorgensen (2011): the preferred habitat is restricted to the near zero default risk assets. This channel breaks Wallace's Irrelevance result, meaning that the term premium falls in response to changes in the relative supply of long vs short term government debt. The term premium elasticity ν captures the strength of this channel and, as clearly can be seen in the lower left panel of figure 1.4.3, impacts how much the term premium drops in response to the LSAPII purchases. In a model with only this effect, for example Chen et al. (2012), QE acts solely via changing household's consumption and investment decisions via the long term bond Euler. As Chen et al. (2012) find this effect alone is quantitatively limited.

Adding the Entrepreneurial sector, and investors who arbitrage between long term government debt and packaged entrepreneurial debt, expands the role of preferred habitat. A decline in the long term government bond yield spills over into a general decline in the cost of corporate borrowing. Longstaff, Mithal and Neis (2005) show that this effect probably only exists for highly rated corporate debt (i.e. above Baa). In contrast to the Safety Channel (the spillover from Treasury yields to other near zero default risk assets, including high grade corporate debt) Krishnamurthy and Vissing-Jorgensen (2011) classify a general spillover effect as the "Duration Risk Channel". Corporate debt in this model does not distinguish between high and low rated debt because investors hold a perfectly diversified aggregate of debt. So by adding investors with strong long term maturity preferences I capture both the Safety Channel and Duration Risk Channel. Given the relatively weak empirical support for the Duration Risk Channel, including a friction in the arbitrage between corporate debt and US Treasuries could be an important dimension for future work.

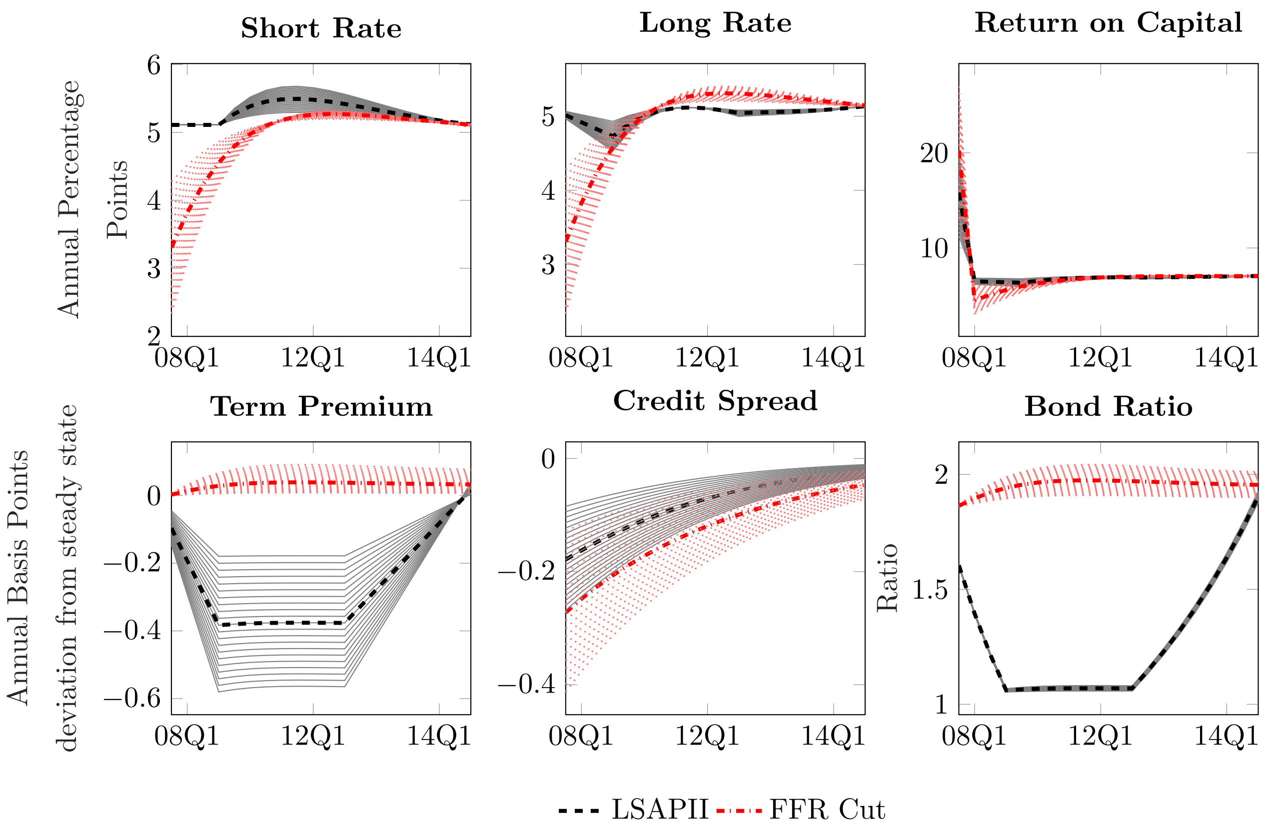
The Portfolio Balance channel also interacts with a "Default Risk Channel". Lower borrowing costs mean that fewer entrepreneurs default. This in turn, via the costly-state verification friction, lowers the spread charged on individual entrepreneurial debt contracts further, boosting net worth and accelerating the effects of QE via a reduction in the external finance premium (see section 1.4.5).

Borrowing cost declines stimulate entrepreneurial net worth and further amplify the impact of QE via accelerator effects. And QE has an inflationary impact, which means that *real* rates fall further.

I have a few additional comments about the dynamics of the entrepreneurs' credit conditions during QE. Both the leverage level and the default threshold fall. This on the surface suggests two opposite effects on the credit conditions, both easing (lower default threshold) and tightening (lower leverage). However, as pointed out in Christiano et al. (2014) the expectation that the price of capital will return to steady state has a muting effect on the response of credit under any shock. That is why credit responds by less than net worth, and so leverage falls in response to shocks that improve conditions for entrepreneurs.

1.4.3 LSAPII Impact vs FFR Cut Equivalence

Figure 1.4.4: Quantitative Easing vs a FFR Cut (a)

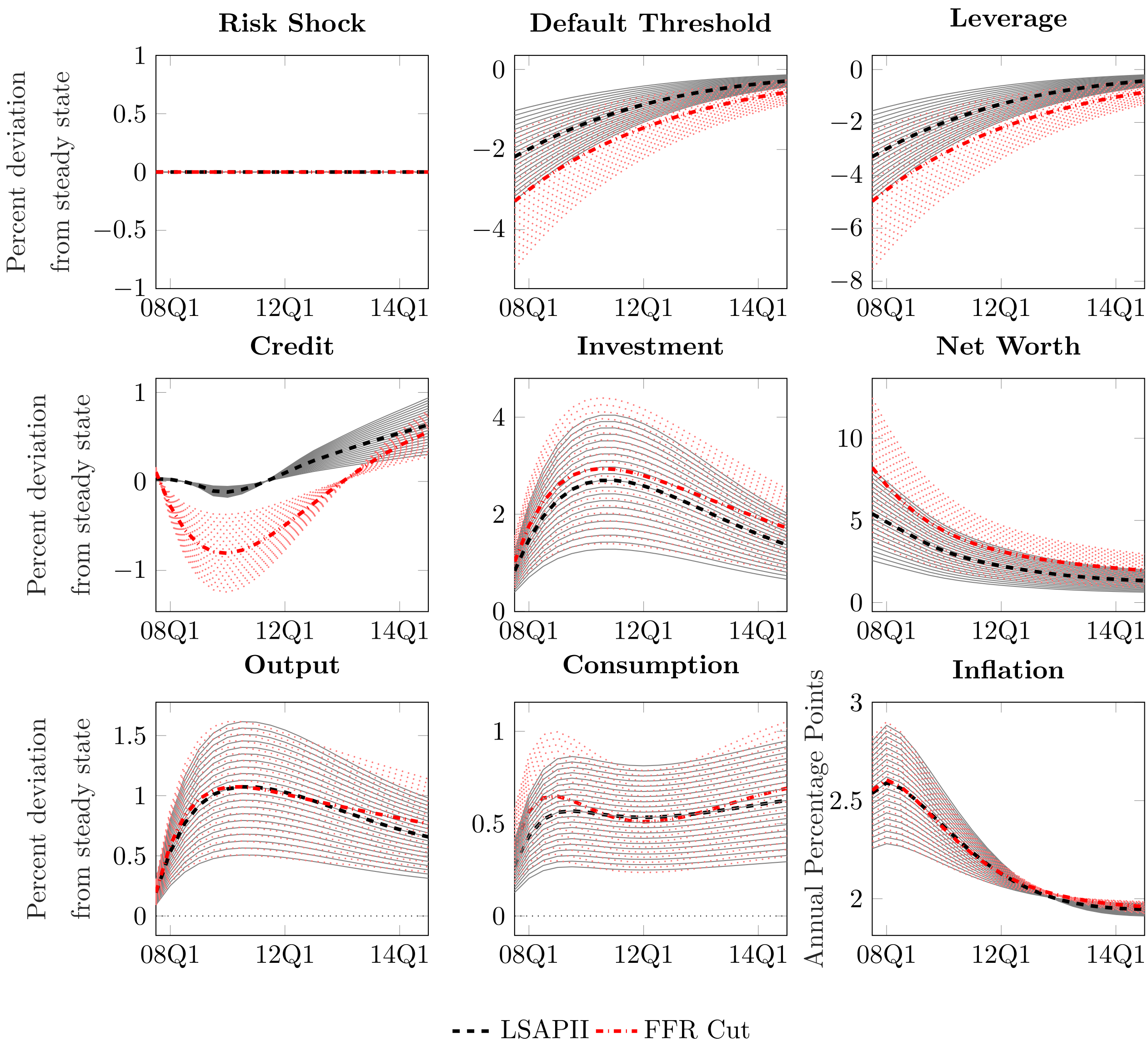


Note: The thick dashed lines correspond to the simulation at the median value for the portfolio preference term ν.

The simulated LSAPII program produces the same boost to output as a 83 to 278 basis point cut to the federal funds rate, corresponding to the lower and upper ranges for the strength of the portfolio preference (i.e. ν parameter calibration). Figures 1.4.4 and 1.4.5 show the response of the economy to the LSAPII program versus a federal funds rate cut (over the range of the portfolio preference parameter).

By target, output growth is the same across the LSAPII and the federal funds rate cut simulations. Note the thick dashed lines are the simulations at the median value of the term premium elasticity ν. Unsurprisingly inflation growth is roughly compatible across the two simulations. Investment responds slightly more to the federal funds rate cut (between 0.12 and 0.36 percentage points).

Figure 1.4.5: Quantitative Easing vs a FFR Cut (b)

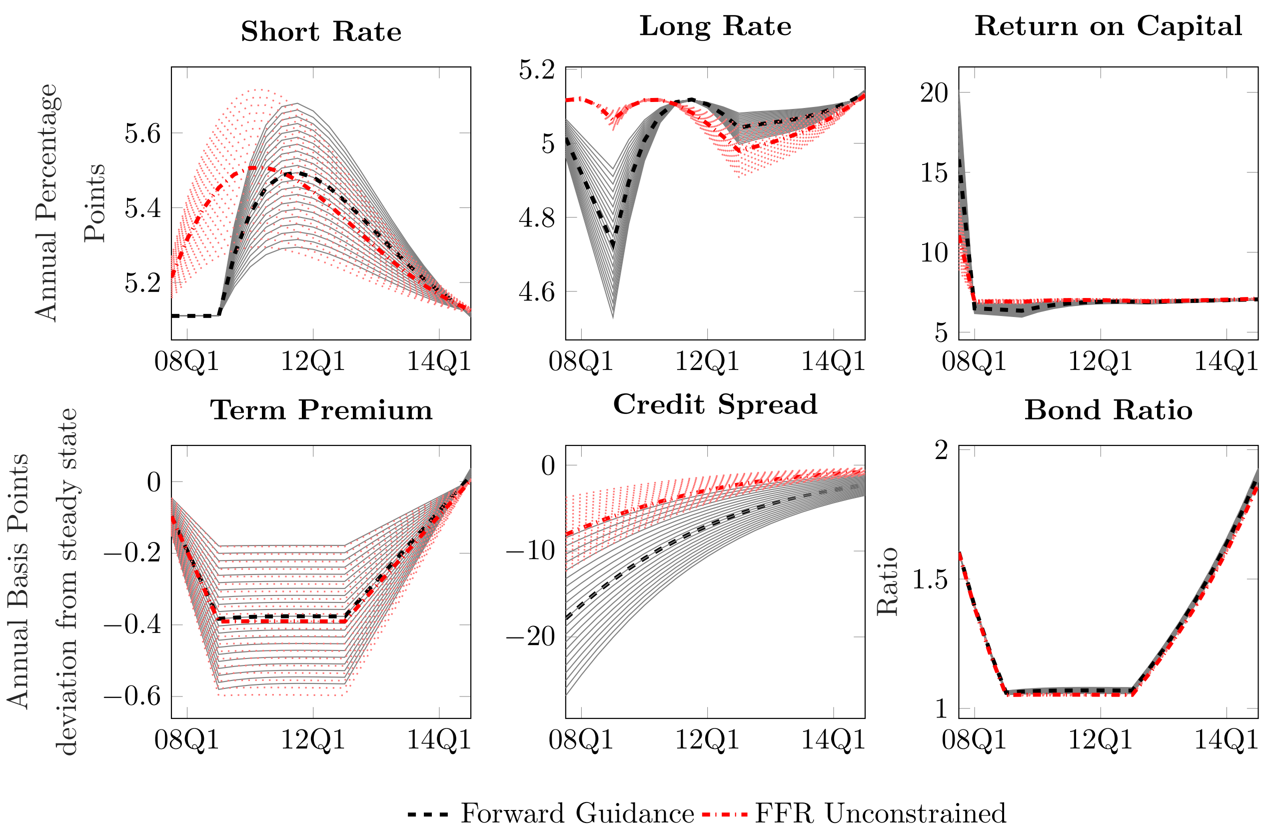


Note: The thick dashed lines correspond to the simulation at the median value for the portfolio preference term ν.

The impact on entrepreneurial credit conditions is substantially different between QE and the equivalent federal funds rate cut, highlighting the different channel through which these two policies act. Under quantitative easing credit drops slightly (between 0.06 - 0.18 % below its steady state level) once the forward guidance constraint on the federal funds rate is lifted. In contrast under the federal funds rate cut simulation credit drops between 0.37 - 1.24% below its steady state value (as the policy rate is normalized). The difference is that quantitative easing keeps long rates depressed for longer. This stimulates entrepreneurs' access to credit via the Portfolio Balance Channel, boosting credit quantity more than the federal funds rate cut scenario.

1.4.4 Importance of the ZLB Constraint

Figure 1.4.6: QE With and Without Forward Guidance (a)



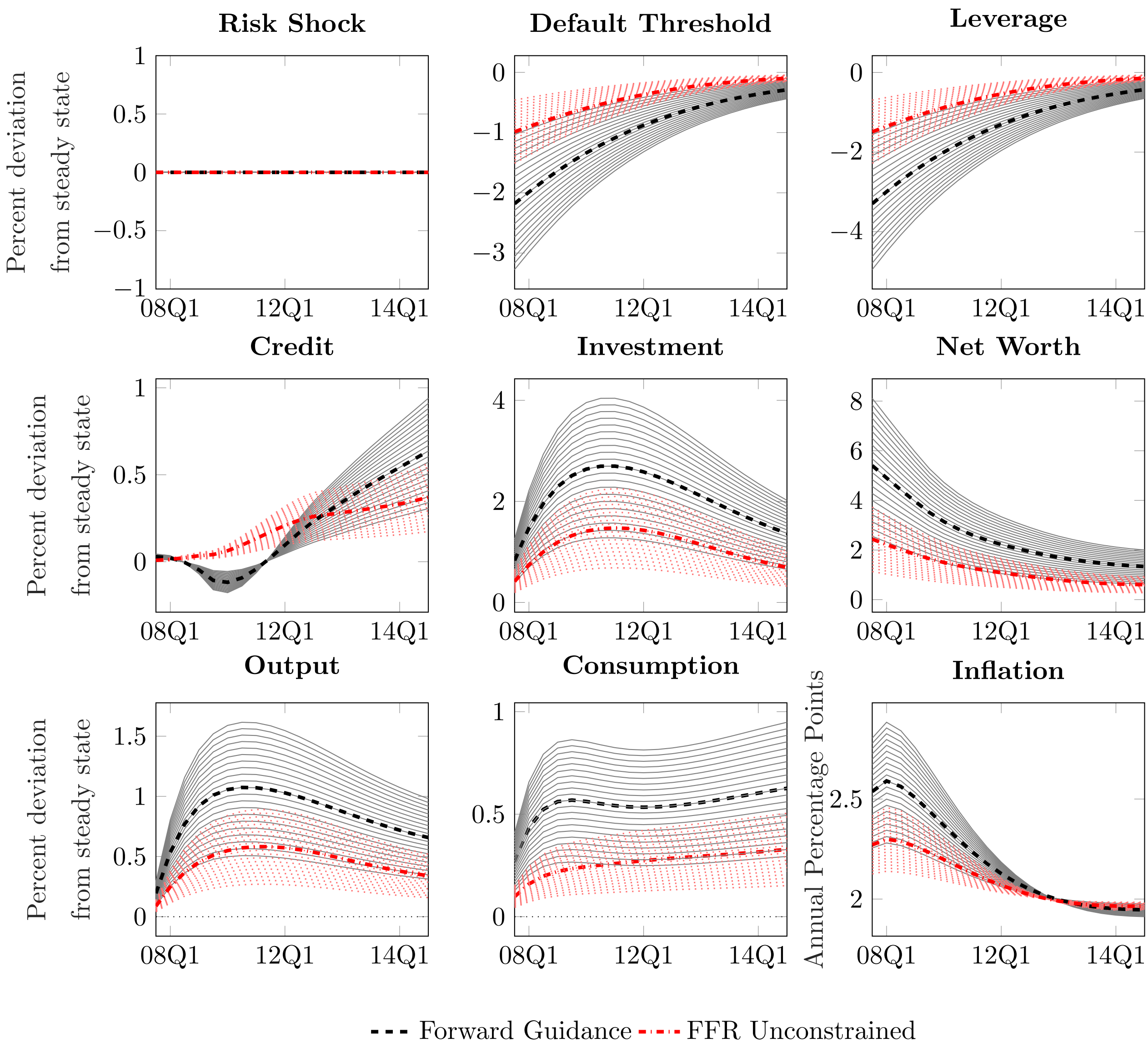
Note: The thick dashed lines correspond to the simulation at the median value for the portfolio preference term ν.

In the baseline LSAPII simulation ("Forward Guidance") news shocks are used to keep the federal funds rate at its steady state level. In the "FFR Unconstrained" simulation the federal funds rate is unconstrained. In figures 1.4.6 and 1.4.7 it is clear that the forward guidance magnifies the stimulus effect of QE. Output growth is between 0.24 and 0.73 percentage points higher under forward guidance than no forward guidance.

Inflation is between 14 and 43 annualized basis points higher under QE with forward guidance than QE without forward guidance.

The intuition is simply that QE is inflationary and expansionary. This means that if the short rate is not constrained the Taylor Rule drives the central bank to raise rates in response to the effects of the QE program. So removing the Forward Guidance policy which constrains monetary policy moderates QE's stimulus effect but does not qualitatively change the impact of QE on most series.

Figure 1.4.7: QE With and Without Forward Guidance (b)



Note: The thick dashed lines correspond to the simulation at the median value for the portfolio preference term ν.

The exception here is the response of credit. Credit has a different response under the QE with forward guidance ("Forward Guidance") than it does just under QE ("FFR Unconstrained"). Under the FFR unconstrained simulation the increase in the federal funds rate is front-loaded. And because the federal funds rate is never constrained the long term government bond rate does not experience as dramatic a decline. This substantially mutes the on impact response of entrepreneurial net worth (a 2.4% jump over steady state as compared to 5.4%, at the median ν value). A lower net worth means that, in the FFR unconstrained simulation, the entrepreneurs rely initially more on credit (and the drop in leverage is less pronounced).

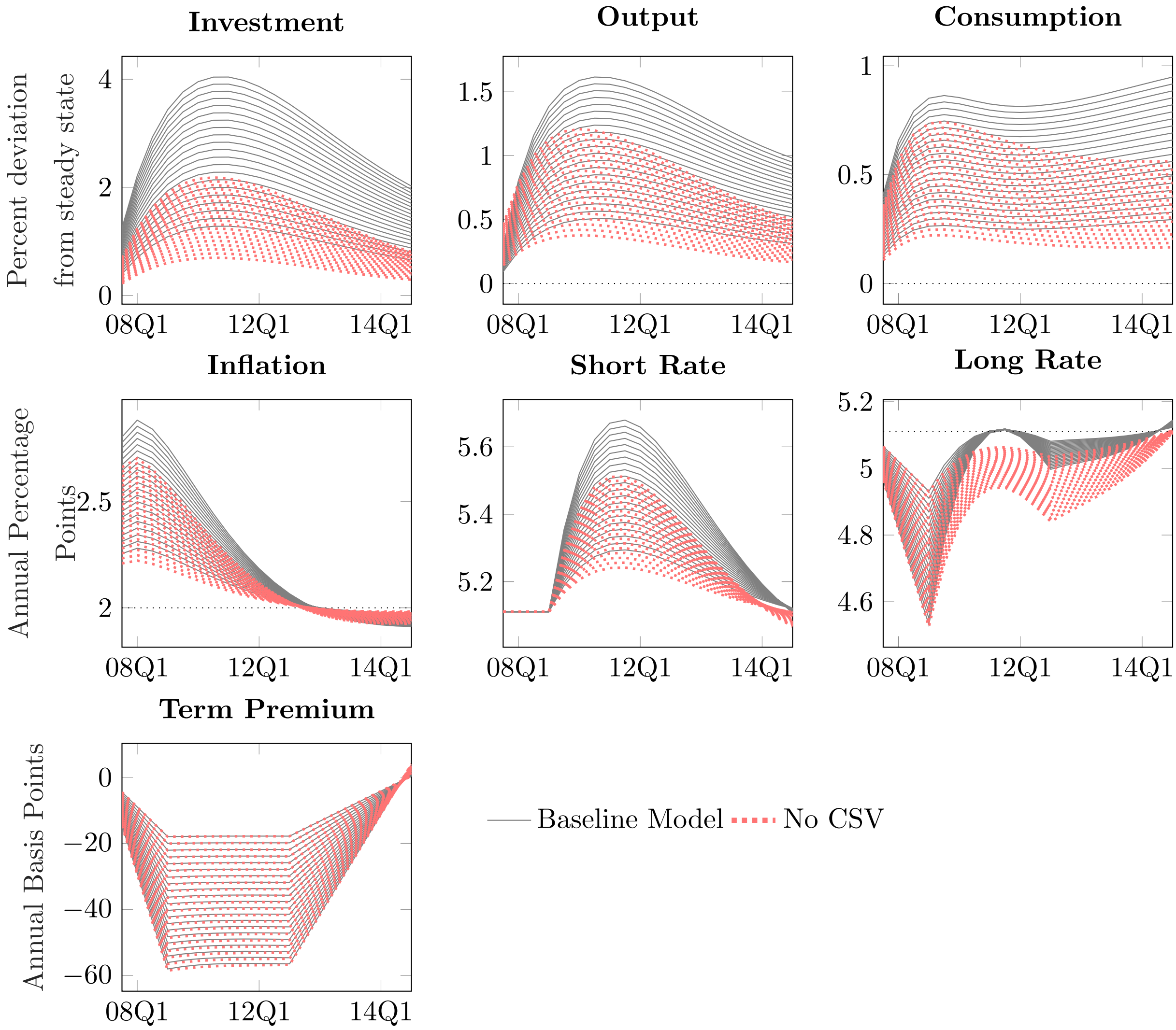
1.4.5 Amplifying of Quantitative Easing: Costly-State Verification

Output growth is between 0.13 and 0.41 percentage points greater with the Costly-State Verification (CSV) friction than without ($\mu = 0$, "No CSV"). Inflation is boosted between 6 and 18 annualized basis points by the CSV friction. Investment is between 0.58 and 1.86 percentage points higher with the CSV friction than without.

Note that in the No CSV model the spread between the expected return on capital and the long term government bond rate is zero[3]. So even in the No CSV results presented in figures 1.4.8 QE has a strong quantitative impact because the long term interest rate is directly related to the return on capital, and QE directly depresses the long term interest rate. Exploring the extent to which preferred habitat preferences link the long term interest rate to the return on capital, in future work, could be an important part of understanding QE transmission.

[3]The entrepreneur's optimal choice of the default rate is reduced to $E\left[R^k_{t+1} - R^L_t\right] = 0$. In the Christiano et al. (2014) model this condition will be $E\left[R^k_{t+1} - R_t\right] = 0$. Without the portfolio preference, this would be equivalent to a standard DSGE model (eg Christiano et al. (2005)) where households choose capital investment.

Figure 1.4.8: The Costly State Verification Friction Amplifies the Portfolio Balance Channel



1.4.6 Impact of Investors' Preferred Habitat Preferences

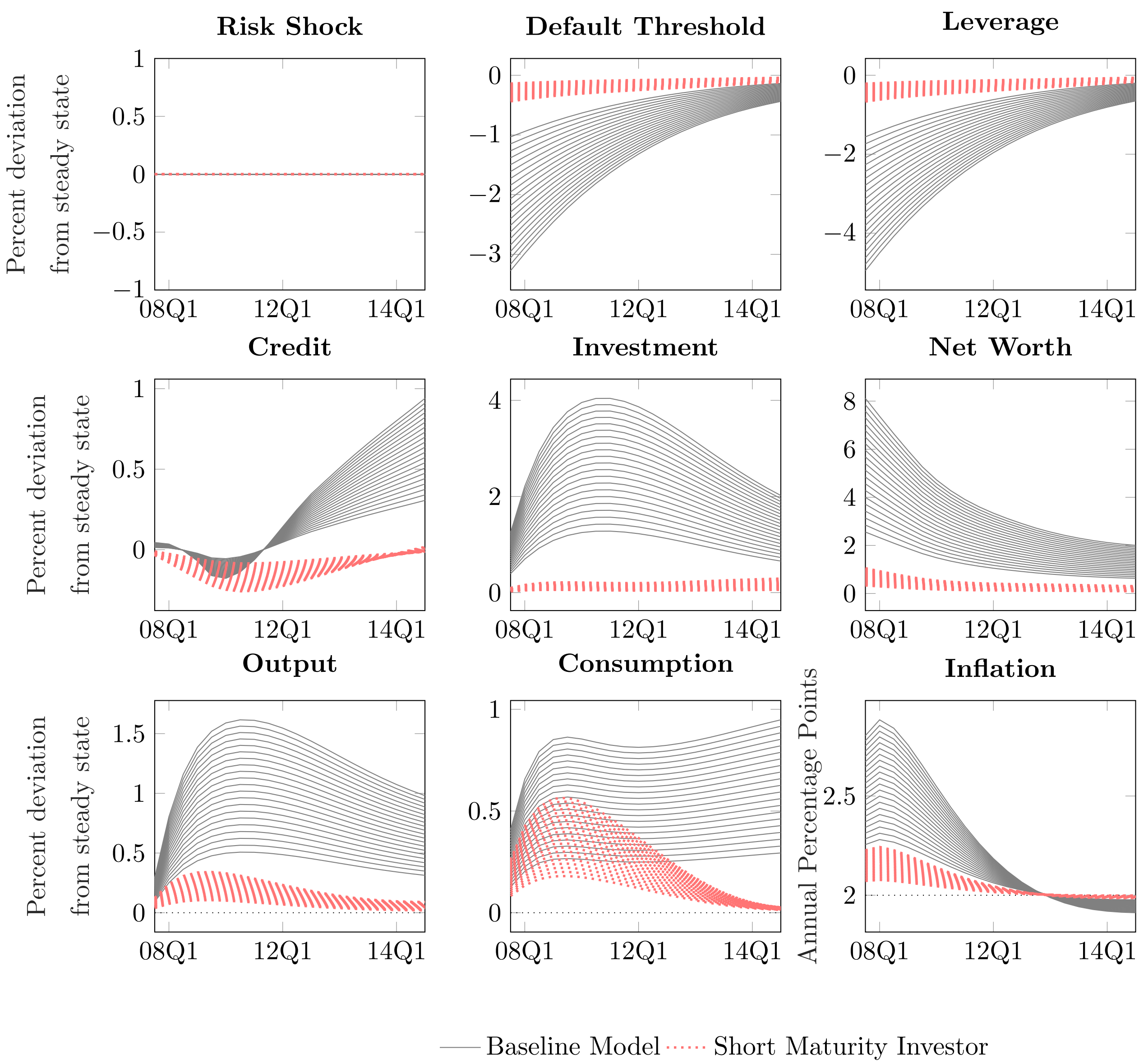


Figure 1.4.9: Impact of Investor Preferences (a)

The following repeat the baseline QE simulation (LSAPII + ZLB constraint) in the long vs short model. These results further emphasize the importance of the Duration Risk Channel to the quantitative impact of QE. Output growth is between 0.4 - 1.27 % points more when investors have a preferred habitat preference for long term assets ('the baseline model), versus when investors arbitrage between packaged corporate debt and the short term rate ("Short Maturity Investor"). Inflation is boosted between 20 and 64 annualized basis points more under the assumption of duration risk. Dropping investors' preferred habitat completely eliminates the lagged positive response of credit.

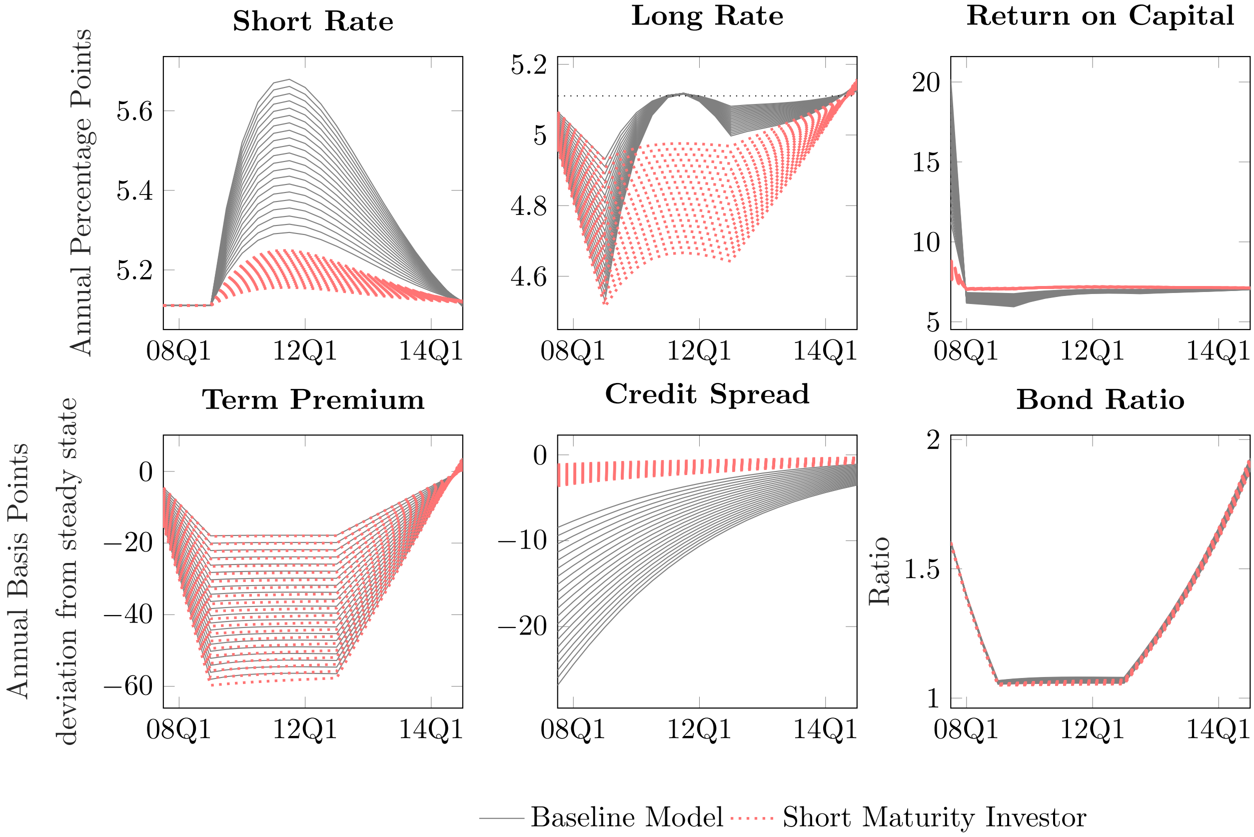


Figure 1.4.10: Impact of Investor Preferences (b)

Note: the credit spread in the short model is $Z - R$ and the credit spread in the long model is $Z - R^L$.

The only series that is not substantially dampened is consumption. QE does have a role without duration risk - in impacting the inter-temporal consumption decisions of households via the Euler equation for long bonds. But as shown by Chen et al. (2012) this effect is quantitatively limited. And these results indicate that the addition of the CSV financial friction alone does not substantially boost this effect.

1.5 Conclusion

In this paper I find that the second round of Federal Reserve Large Scale Asset Purchases (LSAPII) boosted output between 0.51% and 1.62% and inflation by 28 to 88 annualized basis points. Quantitatively this is closer to the larger macroeconomic effects found in vector autoregression assessments of QE than in the DSGE literature on QE. The key mechanism that contributes to the larger quantitative impact is that investors arbitrage between long term government debt and corporate bonds. This arbitrage exists because investors have preferred habitat preferences for long term bonds. This means that when Treasury yields fall as a result of QE the cost of corporate borrowing also falls, generating a portfolio balance effect.

Appendix

1.A Characterization of the Equilibrium

The equilibrium in this model requires equilibrium in the bond market, goods market (intermediate and final), and labour market. The resource constraint, household's budget constraint, and the investor arbitrage condition must hold. Lastly, interest rates must be such that households are willing to hold the available portfolio of bonds.

Lowercase variables are real detrended variables. So if X_t is a nominal variable then $x_t \equiv \frac{X_t}{P_t z_t^*}$.

Table 1.A.1: Notation Key

$q_t \equiv \Upsilon^t \frac{Q_{\bar{K},t}}{P_t}$	$y_{z,t} \equiv \frac{Y_t}{z_t^*}$	$i_t \equiv \frac{I_t}{z_t^* \Upsilon^t}$
$\tilde{w}_t \equiv \frac{W_t}{z_t^* P_t}$	$\bar{k}_t \equiv \frac{\bar{K}_t}{z_{t-1}^* \Upsilon^{t-1}}$	$\mu_{z,t}^* \equiv \frac{z_t^*}{z_{t-1}^*}$
$c_t \equiv \frac{C_t}{z_t^*}$	$b_t \equiv \frac{B_t}{z_t^* P_t}$	$b_t^L \equiv \frac{B_t^L}{z_t^* P_t}$
$g_t \equiv \frac{G_t}{z_t^*}$	$t_t \equiv \frac{T_t}{z_t^* P_t}$	$n_{t+1} \equiv \frac{N_{t+1}}{z_t^* P_t}$
$b_{t+1}^{credit} \equiv \frac{B_{t+1}^{credit}}{z_t^* P_t}$	$\lambda_{z,t} \equiv \lambda_t P_t z_t^*$	$v_t^L \equiv \frac{V_t^L}{z_t^* P_t}$

1.A.1 Auxiliary Expressions:

Aux 1: Index term in price updating for firms who cannot re-optimize

$$\tilde{\pi}_t \equiv \left(\pi_t^{\text{target}}\right)^{\iota} \pi_{t-1}^{\iota-1}.$$

Aux 2: $\tilde{\pi}_t$ ahead 1 period

$$\tilde{\pi}_{t+1} \equiv \left(\pi_{t+1}^{\text{target}}\right)^{\iota} \pi_t^{\iota-1}.$$

Aux 3: Definition of $K_{p,t}$:

$$K_{p,t} \equiv F_{p,t} \left[\frac{1 - \xi_p \left(\frac{\tilde{\pi}_t}{\pi_t} \right)^{\frac{1}{1-\lambda_{f,t}}}}{1 - \xi_p} \right]^{1-\lambda_{f,t}}.$$

Aux 4: $K_{p,t}$ ahead 1 period

$$K_{p,t+1} \equiv F_{p,t+1} \left[\frac{1 - \xi_p \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_{f,t+1}}}}{1 - \xi_p} \right]^{1-\lambda_{f,t+1}}.$$

Aux 5: Index term in wage updating for non-reoptimizing unions

$$\tilde{\pi}_{w,t} \equiv \left(\pi_t^{\text{target}} \right)^{\iota_w} \left(\pi_{t-1} \right)^{\iota_w - 1}.$$

Aux 6: $\tilde{\pi}_{w,t}$ ahead 1 period

$$\tilde{\pi}_{w,t+1} \equiv \left(\pi_{t+1}^{\text{target}} \right)^{\iota_w} \left(\pi_t \right)^{\iota_w - 1}.$$

Aux 7: Wage Inflation

$$\pi_{w,t} \equiv \pi_t \mu_{z,t}^* \frac{\tilde{w}_t}{\tilde{w}_{t-1}}.$$

Aux 8: $\pi_{w,t}$ ahead 1 period

$$\pi_{w,t+1} \equiv \pi_{t+1} \mu_{z,t+1}^* \frac{\tilde{w}_{t+1}}{\tilde{w}_t}.$$

Aux 9: **Definition of $K_{w,t}$:**

$$K_{w,t} \equiv \frac{\tilde{w}_t F_{w,t}}{\psi_L} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t} (\mu_z^*)^{\iota_\mu} (\mu_{z,t}^*)^{1-\iota_\mu}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w(\sigma_L+1)}.$$

Aux 10: $K_{w,t}$ ahead 1 period

$$K_{w,t+1} \equiv \frac{\tilde{w}_{t+1} F_{w,t+1}}{\psi_L} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t+1} (\mu_z^*)^{\iota_\mu} (\mu_{z,t+1}^*)^{1-\iota_\mu}}{\pi_{w,t+1}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w(\sigma_L+1)}.$$

1.A.2 Distributions

$$F_t(\bar{\omega}_{t+1}) \equiv CDF\left(\frac{\log(\bar{\omega}_{t+1}) + \frac{1}{2}\sigma_t^2}{\sigma_t} \right),$$

$$G_t(\bar{\omega}_{t+1}) \equiv CDF\left(\frac{\log(\bar{\omega}_{t+1}) + \frac{1}{2}\sigma_t^2}{\sigma_t} - \sigma_t \right),$$

$$G'_t(\bar{\omega}_{t+1}) \equiv PDF\left(\frac{\log(\bar{\omega}_{t+1}) + \frac{1}{2}\sigma_t^2}{\sigma_t} - \sigma_t\right)\frac{1}{\sigma_t},$$

$$\Gamma_t(\bar{\omega}_{t+1}) \equiv \bar{\omega}_{t+1}\Big[1 - F_t(\bar{\omega}_{t+1})\Big] + G_t(\bar{\omega}_{t+1}),$$

$$\Gamma'_t(\bar{\omega}_{t+1}) = 1 - F_t(\bar{\omega}_{t+1}).$$

1.A.3 Model Equations

Equation 1 (First order condition with respect to consumption):

$$E_t\left\{\frac{\mu^*_{z,t}}{c_t\mu^*_{z,t} - bc_{t-1}} - \frac{b\beta}{c_{t+1}\mu^*_{z,t+1} - bc_t} - \lambda_{z,t}\right\} = 0. \tag{1.A.1}$$

Equation 2 (First order condition with respect to the short bond):

$$E_t\left\{\nu\left(\frac{b_t^L}{b_t} - \delta^b\right)\frac{b_t^L}{b_t^2} - \lambda_{z,t} + \beta\frac{\lambda_{z,t+1}}{\pi_{t+1}\mu^*_{z,t+1}}R_t\right\} = 0. \tag{1.A.2}$$

Equation 3 (First order condition with respect to the long bond):

$$E_t\left\{-\nu\left(\frac{b_t^L}{b_t} - \delta^b\right)\frac{1}{b_t} - \lambda_{z,t} + \beta\frac{\lambda_{z,t+1}}{\pi_{t+1}\mu^*_{z,t+1}}R^L_{t+1}\right\} = 0. \tag{1.A.3}$$

Equation 4 (First order condition with respect to investment):

$$E_t\left\{\lambda_{z,t}\left(q_t - \frac{1}{\mu_{\Upsilon,t}}\right) - \lambda_{z,t}q_t\left[S\left(\frac{\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}\right) + S'\left(\frac{\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}\right)\frac{\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}\right]\right. \tag{1.A.4}$$
$$\left. + \beta\frac{\lambda_{z,t+1}q_{t+1}}{\mu^*_{z,t}\Upsilon}S'\left(\frac{\mu^*_{z,t+1}\Upsilon i_{t+1}}{i_t}\right)\left(\frac{\mu^*_{z,t+1}\Upsilon i_{t+1}}{i_t}\right)^2\right\} = 0.$$

Equation 5 (Firm's Production Function):

$$y_{z,t} = (p_t^*)^{\frac{\lambda_{f,t}}{\lambda_{f,t}-1}}\left[\epsilon_t\left(\frac{u_t\bar{k}_t}{\mu^*_{z,t}\Upsilon}\right)^{\alpha}\left(h_t(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}}\right)^{1-\alpha} - \phi\right]. \tag{1.A.5}$$

Equation 6 (Resource Constraint):

$$y_{z,t} = g_t + c_t + \frac{i_t}{\mu_{\Upsilon,t}} + \Theta\frac{1-\gamma}{\gamma}\big(n_{t+1} - w^e\big) + d_t + \frac{a(u_t)\bar{k}_t}{\Upsilon\mu^*_{z,t}}, \tag{1.A.6}$$

where $d_t \equiv \frac{\mu G_{t-1}(\bar{\omega}_t)R^k_t q_{t-1}\bar{k}_t}{\pi_t\mu^*_{z,t}}$.

Equation 7 (Rental Rate of Capital):

$$r^k_t = \alpha\epsilon_t\left(\frac{\mu^*_{z,t}\Upsilon h_t(w^*_t)^{\frac{\lambda_w}{\lambda_w-1}}}{u_t\bar{k}_t}\right)^{1-\alpha} s_t. \tag{1.A.7}$$

Equation 8 (Marginal Cost):

$$s_t = \frac{1}{\epsilon_t}\left(\frac{r^k_t}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha}. \tag{1.A.8}$$

Equation 9 (Optimal utilization of capital):

$$r^k_t = a'(u_t) = r^k\exp(\sigma_a(u_t-1)), \tag{1.A.9}$$

where $a(u_t) \equiv \frac{r^k}{\sigma_a}\big[\exp(\sigma_a(u_t-1)) - 1\big]$.

Equation 10 (Law of motion for capital):

$$\bar{k}_{t+1} = \frac{1-\delta}{\mu^*_{z,t}\Upsilon}\bar{k}_t + \Big[1 - S\Big(\frac{i_t\mu^*_{z,t}\Upsilon}{i_{t-1}}\Big)\Big]i_t, \tag{1.A.10}$$

Equation 11 (Rate of return on capital):

$$R^k_t = \frac{u_t r^k_t - a(u_t) + (1-\delta)q_t}{\Upsilon q_{t-1}}\pi_t, \tag{1.A.11}$$

Equation 12 (Entrepreneurs' FoC wrt $\bar{\omega}_{t+1}$):

$$E_t\Bigg\{\big[1-\Gamma_t(\bar{\omega}_{t+1})\big]\frac{R^k_{t+1}}{R^L_{t+1}} + \frac{\Gamma'_t(\bar{\omega}_{t+1})}{\Gamma'_t(\bar{\omega}_{t+1}) - \mu G'_t(\bar{\omega}_{t+1})}\Big[\frac{R^k_{t+1}}{R^L_{t+1}}\Big(\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}\Big) - 1\Big]\Bigg\} = 0. \tag{1.A.12}$$

Equation 13 (Evolution of Entrepreneurs' Net Worth):

$$n_{t+1} = \frac{\gamma}{\pi_t \mu^*_{z,t}} \left\{ R^k_t \Big(1 - \mu G_{t-1}(\bar{\omega}_t)\Big) - R^L_t \right\} \bar{k}_t q_{t-1} + w^e + \gamma \frac{R^L_t}{\pi_t \mu^*_{z,t}} n_t. \tag{1.A.13}$$

Equation 14 (Mutual Funds Zero-Profit Condition):

$$\frac{q_t \bar{k}_{t+1}}{n_{t+1}} \frac{R^k_{t+1}}{R^L_{t+1}} \Big[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) \Big] - \frac{q_t \bar{k}_{t+1}}{n_{t+1}} + 1 = 0. \tag{1.A.14}$$

Equation 15 (AR(1) for the supply of long term government bonds):

$$\log\left(\frac{b^L_t}{b^L}\right) = \rho_{bL} \log\left(\frac{b^L_{t-1}}{b^L}\right) + u^{bL}_t. \tag{1.A.15}$$

Equation 16 (Taylor Rule):

$$\log\left(\frac{R_t}{R}\right) = \rho_m \log\left(\frac{R_{t-1}}{R}\right) + \\ \Big(1 - \rho_m\Big) \left[\phi_\pi \log\left(\frac{\pi_t}{\pi^{target}_t}\right) + \frac{\phi_y}{4} \left(\log \frac{y_{z,t}}{y_z} - \log \frac{\mu^*_z}{\mu^*_{z,t}} \right) \right] + \frac{1}{400} \epsilon^m_t. \tag{1.A.16}$$

Equation 17 (Shock equation for σ_t):

$$\log\left(\frac{\sigma_t}{\sigma}\right) = \rho_\sigma \left(\frac{\sigma_{t-1}}{\sigma}\right) + \epsilon_{\sigma,t}. \tag{1.A.17}$$

Equation 18 (Φ_t = its value in steady state):

$$\phi = steady\_state(\Phi). \tag{1.A.18}$$

<u>Equations Related to Price Setting:</u>

Equation 19 (Law of motion for p^*_t):

$$p^*_t = \left[(1 - \xi_p) \Big(\frac{K_{p,t}}{F_{p,t}} \Big)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}} + \xi_p \Big(\frac{\tilde{\pi}_t}{\pi_t} p^*_{t-1} \Big)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}} \right]^{\frac{1-\lambda_{f,t}}{\lambda_{f,t}}}. \tag{1.A.19}$$

Equation 20 (Law of motion for $F_{p,t}$, relates to Calvo Frictions):

$$F_{p,t} = E_t\left\{\lambda_{z,t}y_{z,t} + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_{f,t+1}}}\beta\xi_p F_{p,t+1}\right\}. \tag{1.A.20}$$

Equation 21 (Law of motion for $K_{p,t}$):

$$K_{p,t} = E_t\left\{\lambda_{z,t}\lambda_{f,t}y_{z,t}s_t + \beta\xi_p\left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}}K_{p,t+1}\right\}. \tag{1.A.21}$$

Equation 22 (Law of motion for $F_{w,t}$, characterizes optimal wage setting):

$$F_{w,t} = E_t\left\{\lambda_{z,t}(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}}\frac{h_t}{\lambda_w} + \beta\xi_w(\mu_z^*)^{\frac{1-\iota_\mu}{1-\lambda_w}}(\mu_{z,t+1}^*)^{\frac{\iota_\mu}{1-\lambda_w}-1}\left(\frac{1}{\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}}\frac{\tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}}F_{w,t+1}\right\}. \tag{1.A.22}$$

Equation 23 (Law of motion for $K_{w,t}$):

$$K_{w,t} = E_t\left\{\left[(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}}h_t\right]^{1+\sigma_L} + \beta\xi_w K_{w,t+1}\left(\frac{\tilde{\pi}_{w,t+1}(\mu_{z,t+1}^*)^{\iota_\mu}(\mu_z^*)^{1-\iota_\mu}}{\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)}\right\}. \tag{1.A.23}$$

Equation 24 (Law of motion for w_t^*):

$$w_t^* = \left[(1-\xi_w)\left(\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}(\mu_z^*)^{1-\iota_\mu}(\mu_{z,t}^*)^{\iota_\mu}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right)^{\lambda_w} + \xi_w\left(\frac{\tilde{\pi}_{w,t}(\mu_{z,t}^*)^{\iota_\mu}(\mu_z^*)^{1-\iota_\mu}}{\pi_{w,t}}w_{t-1}^*\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}}. \tag{1.A.24}$$

Equation 25 (Entrepreneurs' balance sheet):

$$q_t\bar{k}_{t+1} = B_{t+1}^{credit} + n_{t+1}. \tag{1.A.25}$$

Equation 26 (Definition of leverage, L_t):

$$L_t = \frac{n_{t+1} + B_{t+1}^{credit}}{n_{t+1}}. \tag{1.A.26}$$

Equation 27 (Real government bonds):

$$b_t^L = v_t^L B_t^c. \tag{1.A.27}$$

Equation 28 (Long Rate):

$$R_{t+1}^L = \frac{1 + V_{t+1}^L}{V_t^L}. \tag{1.A.28}$$

Equation 29 (Entrepreneurial debt rate):

$$Z_{t+1} = R_{t+1}^k \bar{\omega}_{t+1} \frac{L_t}{L_t - 1}. \tag{1.A.29}$$

Fiscal Policy Block:

Equation 30 (Government Spending Rule):

$$g_t = g_{yz} steady\_state(y_z). \tag{1.A.30}$$

Equation 31 (Government Budget Constraint):

$$b_t + b_t^L = \frac{R_{t-1} b_{t-1}}{\pi_t \mu_{z,t}^*} + \frac{R_t^L b_{t-1}^L}{\pi_t \mu_{z,t}^*} + g_t - t_t. \tag{1.A.31}$$

Equation 32 (Fiscal Rule):

$$t_t - g_t = \kappa \left(\frac{b_{t-1} + b_{t-1}^L}{b + b^L} \right)^{\phi_T} \epsilon_t^T. \tag{1.A.32}$$

1.B News Shocks

The calibration method of the anticipated news shocks is based on Del Negro, Giannoni and Patterson (2013) and Lasen and Svensson (2011). This appendix section specifies where news shocks appear in the model, the news shock structure, and the process of calibrating news shocks to match a given path for the short rate.

1.B.1 Where is the News?

News in this model appears as shocks to the Taylor rule and shocks to the AR(1) rule for exogenous long-term bond supply. The backward-looking Taylor rule:

$$\log\left(\frac{R_t}{R}\right) = \rho_m \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_m)\left[\phi_\pi \log\left(\frac{\pi_t}{\pi_t^{target}}\right) + \frac{\phi_y}{4}\left(\log\frac{y_{z,t}}{y_s} - \log\frac{\mu_z^*}{\mu_{z,t}^*}\right)\right] + \frac{1}{400}u_t^m. \tag{1.B.1}$$

The long term government bond supply rule:

$$\log\left(\frac{b_t^L}{b^L}\right) = \rho_{BL}\log\left(\frac{b_{t-1}^L}{b^L}\right) + u_t^{BL}. \tag{1.B.2}$$

In the model simulations $\rho_{BL} = 0$, so that the entire path of bond purchases, holding, and unwinding in the quantitative easing program is set via surprise and news shocks.

1.B.2 News Shock Structure

A generic news shock has the following representation:

$$u_t = \epsilon_t + \xi_{1,t-1} + \xi_{2,t-2} + ... + \xi_{p,t-p}, \tag{1.B.3}$$

where ϵ_t is the unanticipated shock (eg the surprise monetary policy shock or long bond supply shock). And $\xi_{p,t-p}$ for $p \geq 1$ are the anticipated news shocks. The shock $\xi_{p,t-p}$ is observed by agents in period $t-p$, but does not affect the relevant sum of shocks until period t. In the AR(1) for long bonds these news shocks represent pre-announced QE purchases. In the Taylor rule they are a pre-announced path for the short rate targeted by the central bank (i.e. Forward Guidance).

1.B.3 Forward Guidance Implementation

Dynare has the following state-space representation of the model:

- s_t = m x 1 vector of states. (m = M_.npred + M_.nboth).
- x_t = n x 1 vector of controls. (n = M_.nstatic + M_.nfwrd).
- ϵ_t = w x 1 vector of shocks. (w = M_.exo_nbr).
- Φ = (n x m), the policy function.

State-space representation:

$$s_t = \underset{mxm}{A}\, s_{t-1} + \underset{mxw}{B}\, \epsilon_t, \tag{1.B.4}$$

$$x_t = \Phi s_t. \tag{1.B.5}$$

Substitute (1.B.4) into (1.B.5):

$$s_t = As_{t-1} + B\epsilon_t,$$

$$x_t = \Phi As_{t-1} + \Phi B\epsilon_t.$$

Define $\underset{nxm}{C} \equiv \Phi A$ and $\underset{nxw}{D} \equiv \Phi B$. And rewrite the state-space system:

$$s_t = As_{t-1} + B\epsilon_t,$$

$$x_t = Cs_{t-1} + D\epsilon_t.$$

Stack the system and collapse: $Y_t = \begin{bmatrix} s_t \\ x_t \end{bmatrix}, \qquad \Psi = \begin{bmatrix} A \\ C \end{bmatrix}, \qquad \Omega = \begin{bmatrix} B \\ D \end{bmatrix}.$

$$Y_t = \Psi s_{t-1} + \Omega\epsilon_t. \tag{1.B.6}$$

- $\Psi = oo{.}dr.ghx$ = (m+n) × m, matrix of coefficients that appears in the Dynare generated transition rule. (# of endogenous variables = m+n, by # of state variables = m).

- $\Omega = oo{.}dr.ghu$ = (m+n) × w, matrix of coefficients that appears in the Dynare generated transition rule. It has dimension (# of endogenous variables by # of shocks).

Let Z be a matrix ($m \times (m + n)$) that selects the state variables from the Y_t matrix, so that $s_t = ZY_t$. And define $M \equiv \Psi Z$. So we can rewrite 1.B.6 as:

$$Y_t = MY_{t-1} + \Omega\epsilon_t.$$

Split the shock vector:

$$\epsilon_t = \epsilon_t^1 + \epsilon_t^2. \tag{1.B.7}$$

Where ϵ_t^1 is a $w \times 1$ vector where all shocks, except the monetary policy forward guidance shocks are replaced with zeros. And ϵ_t^2 is a $w \times 1$ vector where all but the monetary policy forward guidance shocks are replaced with zeros. So can further rewrite 1.B.6 as:

$$Y_t = MY_{t-1} + \Omega\Big[\epsilon_t^1 + \epsilon_t^2\Big]. \tag{1.B.8}$$

Both the forward guidance shocks and the QE shock, that hit at $t = 1$, so $\epsilon_t = 0\ \forall t > 1$. Note that ϵ_1^1 is known (this vector contain the shocks to the long bond supply rule that introduce QE). Further note that ϵ_1^2 is the vector of shocks to be calibrated to produce the target path for the policy rate.

Using equation 1.B.8 note that:

$$Y_1 = MY_0 + \Omega\Big[\epsilon_1^1 + \epsilon_1^2\Big],$$

where Y_0 is the steady state (also known). Iterating forward:

$$\begin{aligned} Y_2 &= MY_1 \\ &= M^2Y_0 + M\Omega\Big[\epsilon_1^1 + \epsilon_1^2\Big]. \end{aligned}$$

Generally:

$$Y_t = M^tY_0 + M^{t-1}\Omega\Big[\epsilon_1^1 + \epsilon_1^2\Big]. \tag{1.B.9}$$

Let $\mathbf{R}^{FG}$ be the $t_{FG} \times 1$ target vector for the path of the policy rate, where t_{FG} is the number of periods that the policy rate is constrained (in the LSAPII simulation $t_{FG} = 4$). Let $\tilde{Z}$ be a $1 \times (n+m)$ row vector that selects the row of Y_t corresponding to the policy rate (in Dynare this is the DR ordering of R_t).

$$\begin{aligned} R_1 &= \tilde{Z}MY_0 + \tilde{Z}\Omega\epsilon_1^1 + \tilde{Z}\Omega\epsilon_1^2, \\ R_2 &= \tilde{Z}M^2Y_0 + \tilde{Z}M\Omega\epsilon_1^1 + \tilde{Z}M\Omega\epsilon_1^2, \\ R_3 &= \tilde{Z}M^3Y_0 + \tilde{Z}M^2\Omega\epsilon_1^1 + \tilde{Z}M^2\epsilon_1^2, \\ R_4 &= \tilde{Z}M^4Y_0 + \tilde{Z}M^3\Omega\epsilon_1^1 + \tilde{Z}M^3\Omega\epsilon_1^2. \end{aligned}$$

Stack:

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = \underbrace{\begin{bmatrix} \tilde{Z}MY_0 + \tilde{Z}\Omega\epsilon_1^1 \\ \tilde{Z}M^2Y_0 + \tilde{Z}M\Omega\epsilon_1^1 \\ \tilde{Z}M^3Y_0 + \tilde{Z}M^2\Omega\epsilon_1^1 \\ \tilde{Z}M^4Y_0 + \tilde{Z}M^3\Omega\epsilon_1^1 \end{bmatrix}}_{\mathbf{K}} + \underbrace{\begin{bmatrix} \tilde{Z}\Omega \\ \tilde{Z}M \\ \tilde{Z}M^2 \\ \tilde{Z}M^3\Omega \end{bmatrix}}_{\mathbf{J}} \epsilon_1^2. \tag{1.B.10}$$

Set the vector of policy rates equal to the target path $\mathbf{R}_{FG}$:

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = \begin{bmatrix} R_1^{FG} \\ R_2^{FG} \\ R_3^{FG} \\ R_4^{FG} \end{bmatrix} \equiv \mathbf{R}^{FG}. \tag{1.B.11}$$

$$\mathbf{R}^{FG} = \mathbf{K} + \mathbf{J}\epsilon_1^2. \tag{1.B.12}$$

and solve for ϵ_1^2:

$$\epsilon_1^2 = \mathbf{J}^{-1}\left(\mathbf{R}^{FG} - \mathbf{K}\right). \tag{1.B.13}$$

1.C The Role of Government Spending Rules

In this appendix I examine alternative government spending rules and show that changing the government spending rule has minimal quantitative impact on the results and qualitatively does not change the results. I present two alternative rules: one the Graeve and Theodoridis (2016) government spending rule, or two, a simple auto-regressive supply rule for the market value of de-trended short-term bonds ($b_t \equiv \frac{B_t}{P_t z_t^*}$):

$$\log\left(\frac{b_t}{b}\right) = \rho_b\left(\frac{b_{t-1}}{b}\right) + u_t^b, \tag{1.C.1}$$

where b is the steady state level of de-trended short-term government debt. Under the AR(1) for short term bonds a quantitative easing shock is captured as a combination of negative shocks that reduce the quantity of long-term government debt in the hands of the public, and positive shocks (u_t^b) to the supply of short term government debt in the hands of the public. The increase in short-term debt is set to be equal to the magnitude of the decrease in the market value of long-term government debt, reflecting the increased creation of reserves used to purchase long-term bonds.

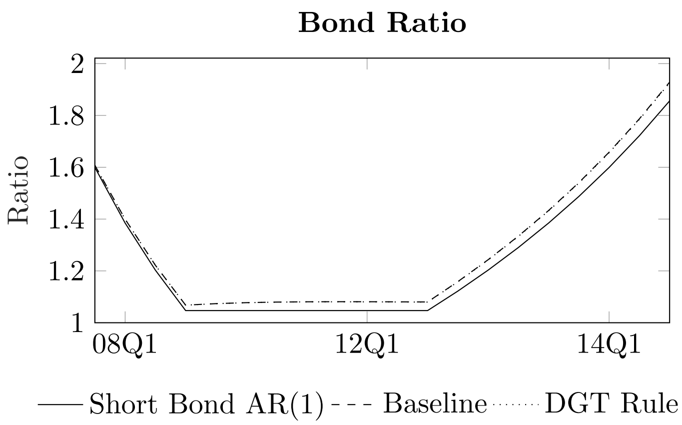


Figure 1.C.1: Effect of Government Spending Rules (a)

The Graeve and Theodoridis (2016) rule ($\phi_T = 0.04.$):

$$\frac{T_t}{P_t z_t^*} = \kappa \left(\frac{\frac{B_{t-1}}{P_{t-1} z_{t-1}^*} + \frac{B_{t-1}^L}{P_{t-1} z_{t-1}^*}}{b + b^L} \right)^{\phi_T} \epsilon_t^T. \tag{1.C.2}$$

Figure 1.C.1 shows that the fiscal policy rule has a minimal impact on the response of short bonds to QE purchases, and therefore a minimal impact on the impact of QE, as figures 1.C.2 and 1.C.3 indicate.

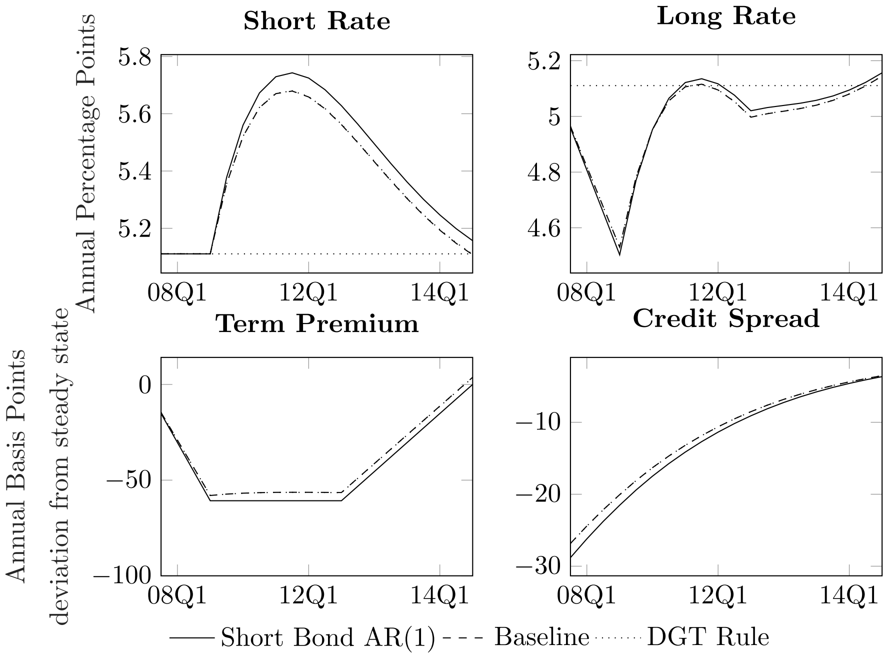


Figure 1.C.2: Effect of Government Spending Rules (b)

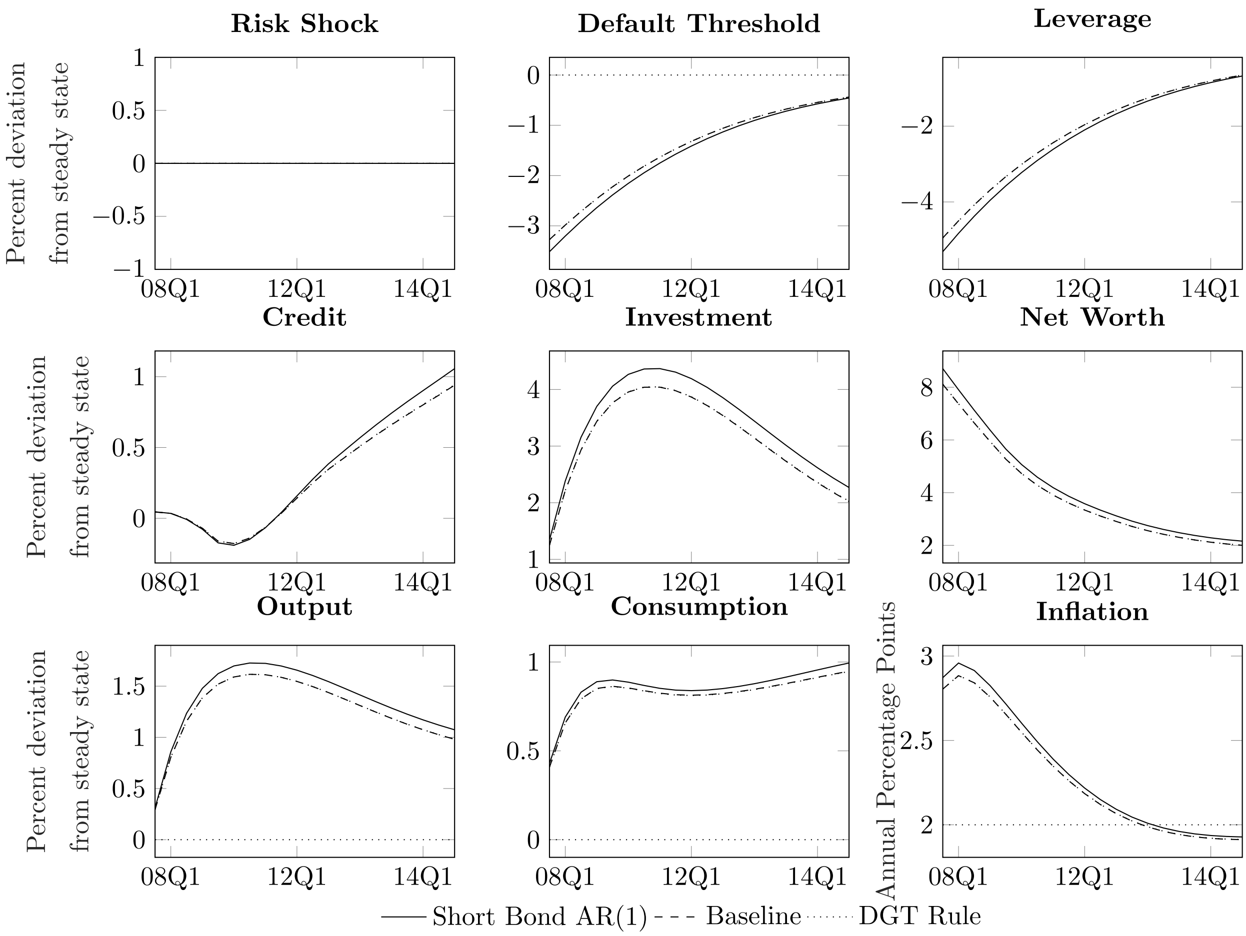


Figure 1.C.3: Effect of Government Spending Rules (c)

1.D Data Appendix

I use US quarterly data from 1985q1 to 2007q3 (the period before the NBER recession start date). The following series come from the Federal Reserve Bank of St. Louis' Federal Reserve Economic Data (FRED): Gross Domestic Product (GDP), and Effective Federal Funds Rate (FEDFUNDS). Data on the nominal value of privately-held marketable interest-bearing US public debt come from the Haver/DLX USECON database. The data are broken down by time until maturity. The long term government bonds are defined as bonds with over 1 year until maturity, and the short term government bonds are defined as bonds with less than 1 year left until maturity.

Table 1.D.1: Haver/DLX USECON Data Codes

Haver/DLX Code	**Data Series**
PDIMP	Total
PDIMPL	Less than 1 year left until maturity
PDIMP1	1 to 5 years left until maturity
PDIMP5	5 to 10 years left until maturity
PDIMP10	10 to 20 years left until maturity
PDIMP20	over 20 years left until maturity

The size of a \$100 billion USD purchase of long term bonds is $x\%$ of the steady state quantity of long term bonds, where x is calibrated as follows:

$$x = \frac{100}{bL\_yz \times \frac{\text{2007q3 GDP}}{4}}, \tag{1.D.1}$$

where bL_yz is the steady state ratio of long term bonds to (quarterly) output, and 2007q3 GDP is in annual terms. \$100 billion USD purchase of long term bonds is equivalent to 3.55% of steady state de-trended long bonds (b^L). So the \$600 billion USD purchase of long term bonds in LSAPII is equivalent to 21.3% of steady state de-trended long bonds.

Table 1.D.2: Parameters Corresponding to Targets

Parameter	Description	Calibration
β	discount rate	0.9964
b^L_{yz}	steady state ratio of long-term government bonds to output	0.77
δ^b	steady state ratio of long to short bond holdings	1.86
g_{yz}	steady state ratio of government spending to output	0.2
κ	steady state primary fiscal surplus	0.0143
π^{target}	steady state target inflation	1.005
ψ_L	disutility weight on labor	1.2126
ν	elasticity of the term premium to the bond ratio	0.00074-0.0025

1.E Derivation of ν

The $\nu \equiv \frac{1}{\lambda_z}\frac{\tilde{\nu}\delta^b}{b}(1+\delta^b)$ is the elasticity of the term premium with respect to the relative supply of long to short term government debt. The ν parameter governs the responsiveness of the term premium to changes in the relative supply of long versus short term bonds. The following shows the log-linearization of the key equations (Model equations 2 & 3) around the steady-state used to calibrate the partial equilibrium response.

First order condition with respect to the short bond:

$$f^a \equiv E_t\left\{\tilde{\nu}\left(\frac{b^L_t}{b_t}-\delta^b\right)\frac{b^L_t}{b^2_t}-\lambda_{z,t}+\beta\frac{\lambda_{z,t+1}}{\pi_{t+1}\mu^*_{z,t+1}}R_t\right\}=0.$$

First order condition with respect to the long bond:

$$f^b \equiv E_t\left\{-\tilde{\nu}\left(\frac{b^L_t}{b_t}-\delta^b\right)\frac{1}{b_t}-\lambda_{z,t}+\beta\frac{\lambda_{z,t+1}}{\pi_{t+1}\mu^*_{z,t+1}}R^L_{t+1}\right\}=0.$$

Log-linearizing around the steady state:

$$f^a \approx \lambda_z\Big[\hat{\lambda}_{z,t+1}-\hat{\lambda}_{z,t}+\hat{R}_t-\hat{\pi}_{t+1}-\hat{\mu}^*_{z,t+1}\Big]+\frac{\tilde{\nu}(\delta^b)^2}{b}(\hat{b}^L_t-\hat{b}_t), \tag{1.E.1}$$

$$f^b \approx \lambda_z\Big[\hat{\lambda}_{z,t+1}-\hat{\lambda}_{z,t}+\hat{R}^L_{t+1}-\hat{\pi}_{t+1}-\hat{\mu}^*_{z,t+1}\Big]-\frac{\tilde{\nu}\delta^b}{b}(\hat{b}^L_t-\hat{b}_t). \tag{1.E.2}$$

This implies:

$$\begin{aligned}\hat{R}^L_{t+1}-\hat{R}_t &= \frac{1}{\lambda_z}\frac{\tilde{\nu}\delta^b}{b}(1+\delta^b)\big[\hat{b}^L_t-\hat{b}_t\big]\\ &= \nu\big[\hat{b}^L_t-\hat{b}_t\big]. \end{aligned} \tag{1.E.3}$$

1.F Additional Results

1.F.1 Baseline Model vs Nested Model without the Default Risk or Wider Duration Preference

The "No CSV & Short Maturity Investor" model is a nested version of the baseline model. The state verification cost is set to zero ($\mu = 0$), meaning that there is no external finance premium for entrepreneurs. This removes the accelerator effect from variations in entrepreneurial net worth. Because credit conditions are neutral to the condition of the entrepreneurs' balance sheet, this shuts off the Default Risk Channel. Also in this nested model version the assumption that investors have a preferred habitat preference for long term assets is dropped. This means the long rate is replaced with the short rate in the mutual fund's zero profit condition[4].

Output is boosted in the baseline model of this paper relative to the No CSV & Short Maturity Investor model by 0.44% to 1.38%. Inflation is boosted between 23 to 71 annualized basis points. Investment growth is boosted between 1.29 - 3.97 percentage points. In the No CSV & Short Maturity Investor model the peak growth of output is between 0.07% - 0.23%, and inflation grows between 5 and 17 annualized basis points. These results are in the range of the small quantitative impact of LSAPII found in Chen et al. (2012), and further underline the importance not only of the financial friction in amplifying the impact of QE but also the Duration Risk Channel component of the Portfolio Balance Channel.

[4] The two changes imply that the entrepreneurs' first order condition with respect to the default threshold becomes $E\Big[R^k_{t+1} - R_t\Big] = 0$, making this model comparable to a model in which households directly invest in capital - eg Chen et al. (2012)

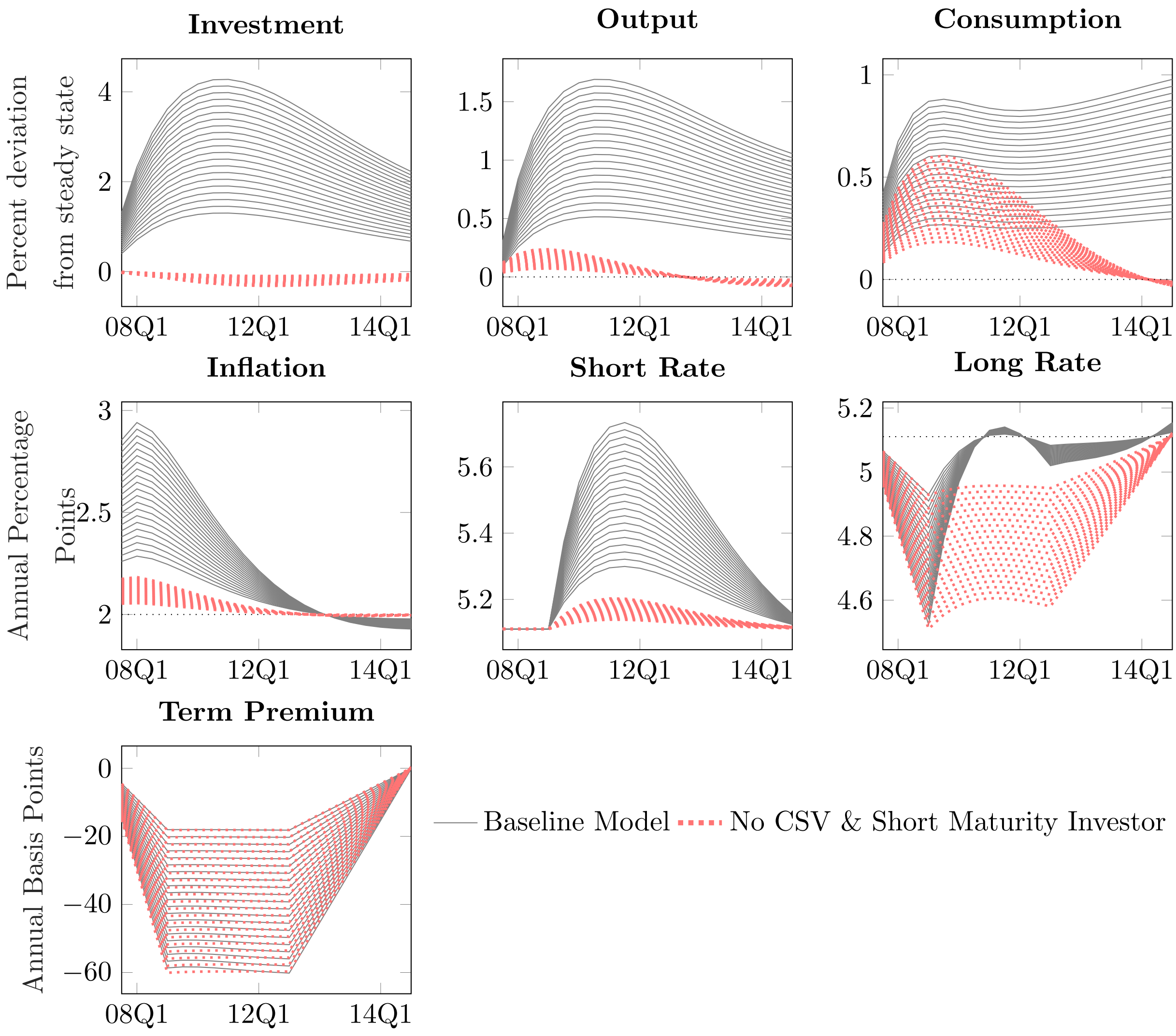


Figure 1.F.1: Baseline vs No Default Risk or Duration Preference

Chapter 2

Securitization and House Price Growth

2.1 Introduction

The Great Recession was preceded by an unprecedented 48% boom in US real house prices from 2000 to 2006. In contrast during the "Baby Boom" period from 1946 - 1964 real house price growth peaked at 33%. US mortgages were increasingly being held not by regulated commercial banks or the implicitly government backed Fannie Mae and Freddie Mac but by the shadow banking sector. The vehicle for this shift was private mortgage backed securitization[1]. The issuance of private mortgage backed securities grew from 126 Billion USD in 2000 to 1,145 Billion USD in 2006. This period was a culmination of a series of "innovations" in private securitization, including increased use of tranching[2] and other credit enhancements, which drove investor willingness to treat private sector issued mortgage backed securities as nearly substitutable to US Treasuries. My paper explores the role of a securitization driven credit supply expansion during this period. My key finding is that innovation in securitization drove at least 71% of the house price appreciation and 34 - 45% of the increase in non-conforming mortgage credit during this period.

The contribution of this paper is to explicitly model the private securitization of mortgage credit. By doing so I get a new *balance sheet effect* missed by standard models.

[1]Securitization is the process of a financial entity buying a group of mortgages and issuing an asset, the mortgage backed security (MBS), that pays out based on the underlying income stream from those mortgages as borrowers repay.

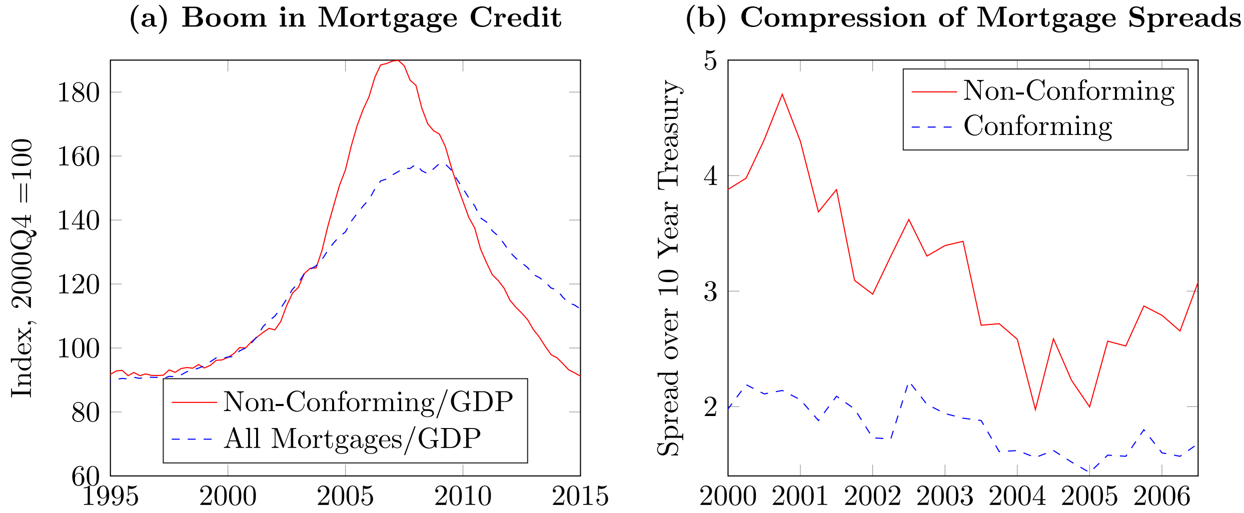
[2]When you buy a mortgage backed security you can buy the right to be paid off first (senior tranche) or last (equity tranche). Tranches are essentially your position in line to be paid back as borrowers repay their mortgages.

In my model financial intermediaries face constraints on the size and composition of their balance sheets. Shadow banks (the issuers of mortgage backed securities) face a constraint which limits the total quantity of mortgage credit they can hold, relative to the profits (i.e. spread) they make. They cannot exceed this limit because their liabilities (mortgage backed securities) would go from being perceived as risk-less to being perceived as too risky to hold. Commercial banks face a solvency constraint. This constraint forces them to diversify their assets by selling the (idiosyncratically risky) mortgages they issue and buying mortgage backed securities (which only have aggregate risk). The aggregate constraint on shadow banks is the ultimate constraint on aggregate mortgage credit that the financial sector can absorb at a given mortgage spread. Because of the balance sheet effect a relaxation of the constraint faced by shadow banks is needed to explain the decline in mortgage spreads and increase in total mortgage credit in the 2000 - 2006 U.S. data. This is "innovation in securitization".

The U.S. experience of the Financial Crisis opened up a debate as to whether positive shifts in credit demand or credit supply drove the boom in US house prices and mortgage debt between 2000 and 2006. On credit demand, researchers suggest that some non-financial factors, for example: optimism about future house prices (Kaplan, Mitman and Violante, 2017), or a speculative bubble (Shiller, 2007) drove an increase in house prices which in turn drove an increase in the demand for credit – to finance the purchase of more expensive housing. The credit supply view, advanced by Mian and Sufi (2017) and Justiniano, Primiceri and Tambalotti (2019), points to the large increase in the quantity of mortgage credit along with a decrease in the relative cost of mortgage credit, suggesting that the boom was driven by a positive shift in credit supply (Figure 2.1.1).

Capturing the balance sheet effect is key to distinguishing between different potential drivers of the expansion in credit supply. A generic credit supply shift, driven by an increase in savers' demand for deposits, drives commercial banks to increase the size of their balance sheet. Commercial banks issue more mortgage credit but also demand more mortgage backed securities. This is because the financial constraint faced by commercial banks penalizes them for holding their own originated mortgages. They choose to sell a

Figure 2.1.1: Credit Supply View - Key Stylized Facts



(a) All Mortgages/GDP: The total outstanding stock of mortgage credit relative to GDP, indexed to 2000Q4 levels. Non-Conforming/GDP: Estimated as "All Mortgages" minus those mortgages held by Government Sponsored Enterprises (GSEs) and GSE & Agency backed pools, relative to GDP, indexed to 2000Q4 levels. Source: Board of Governors of the Federal Reserve System (US), Z1 Financial Accounts of the United States, retrieved from DDP; www.federalreserve.gov/datadownload/, August 15, 2019.
(b) Conforming: Board of Governors of the Federal Reserve System (US), 30-Year Conventional Mortgage Rate (DISCONTINUED) [MORTG], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/MORTG, September 1, 2019. Non-Conforming: average mortgage rate at origination for the near-universe of privately securitized mortgages, source: Justiniano, Primiceri, & Tambalotti (2017).

portion of these mortgages and hold mortgage backed securities instead. In this way the generic credit *supply* shock translates into an increase in *demand* for mortgage backed securities - driving mortgage spreads up because of the balance sheet effect. Furthermore the balance sheet effect magnifies the positive impact on the mortgage spread that a credit demand shift ("Housing Demand" shock) would have.

Not only is the the innovation in securitization channel necessary to explain mortgage spread dynamics in the 2000 - 2006 data, it could have set the stage to amplify other potential factors driving credit dynamics during this period. I show that, in a world where mortgage securitizers face looser financial constraints, shifts in the demand for mortgage backed securities increase the quantity of credit more with a smaller increase in the mortgage spread. That is, the innovation in securitization driven boom could have amplified the mortgage credit response to other credit supply or credit demand shifts during this period. This suggests that the innovation in securitization driven boom story is complimentary to other candidate credit supply and demand explanations explored by the literature - as innovation in securitization amplifies the impact of these other shocks. The conclusion here is not that innovation in securitization was *the only* driver of mortgage credit market dynamics during this period, rather that innovation in securitization was necessary to set the stage for these other shocks.

I build a model in which idiosyncratic mortgage default risk generates the existence of the mortgage backed securitization market. I embed this model into the housing in DSGE framework originated in Iacoviello and Neri (2010). The key innovation is the addition of a two-layered mortgage securitizing financial sector comprised of mortgage issuing commercial banks and mortgage securitizing shadow banks. Shadow banks in this context are the Special Purpose Vehicles - the off-balance sheet entities owned by commercial or investment banks who bought and packaged non-conforming[3] mortgages into private mortgage backed securities. This captures the institutional reality of this period, missed by standard models, that by providing an outlet for commercial banks to move their own lending off their balance sheet shadow banks enabled commercial banks to loosen the regulatory constraints that would limit a credit supply boom. Models that do not account for securitization as an outlet for commercial banks will falsely reject the credit supply boom story.

The model captures the geographic dispersion of the US mortgage and housing markets and idiosyncratic mortgage default risk by incorporating an island structure. There is a continuum of islands, and each island has a borrower, saver, and commercial bank. Households can only interact with their island's commercial bank. In each period a proportion of islands receive a default shock which means borrowers on "hit" islands do not pay back a proportion of debt. Commercial banks can choose to either hold the mortgages they issue or mortgage backed securities. Shadow banks sit off-islands so can buy mortgages across islands and sell to commercial banks an asset (the mortgage backed security) that pays the average mortgage return across islands. Because of the idiosyncratic risk involved in retaining individual mortgages commercial banks hold mortgage backed

[3]Those mortgages that fell outside the standards required for securitization by the Government Sponsored Enterprises (Fannie Mae & Freddie Mac).

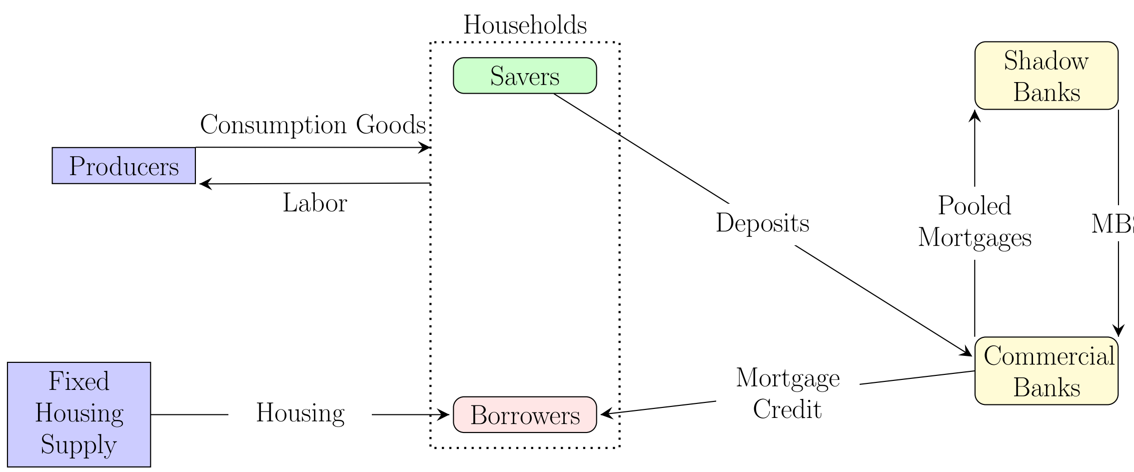
securities as insurance. Without this idiosyncratic risk commercial banks would prefer to hold the mortgages they originate (earning a greater return in expectation). Holding mortgage backed securities reassures savers that the deposits issued by commercial banks will be paid back even on the default "hit" islands. This allows commercial banks to intermediate more funds and expand total mortgage credit provision on island - and thus overall mortgage credit supply.

The paper proceeds as follows. Section 2 presents the model. Section 3 presents the calibration and simulation method. Section 4 explains the innovation in securitization mechanism. Section 5 presents and discusses the simulation results. And section 6 concludes.

2.2 The Model

2.2.1 Overview

Figure 2.2.1: The Model



I build a model in which idiosyncratic mortgage default risk micro-founds the existence of mortgage backed securitization, and embed this model into a simplified version of the housing in DSGE framework originated in Iacoviello and Neri (2010). The key innovation is the addition of a two-layered mortgage securitizing financial sector comprised of mortgage issuing commercial banks and mortgage securitizing shadow banks. Borrowers, savers, and commercial banks exist in geographically disperse locations (islands). A commercial bank can only take deposits from savers on their island and can only lend to borrowers on their island. Each period a proportion of islands receive a default shock, on these "bad" islands borrowers do not pay back a proportion (δ) of what they owe on their mortgage debt ($R_{M,t}b_t$).

The model overlays an island structure onto a RBC model. Each island contains a borrower household, a saver household, a commercial bank, and a producer who uses on island labor to produce output that is 1-for-1 convertible into the consumption good. Households can only interact with their local (on island) commercial bank.

Figure 2.2.2: Risky Mortgage Lending

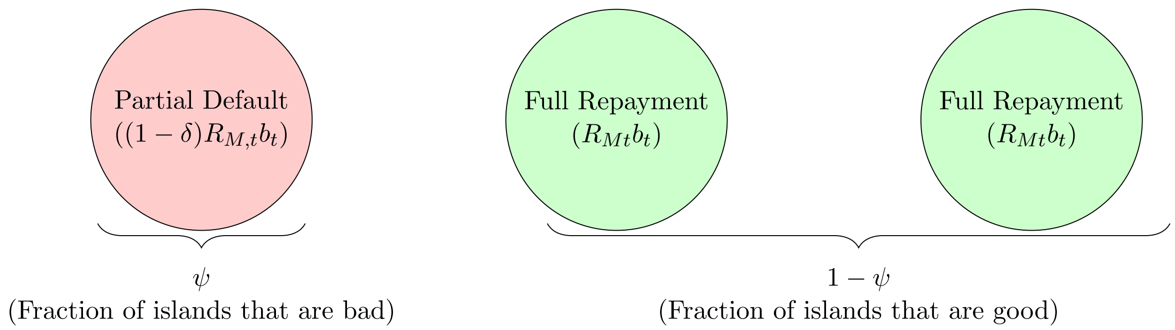


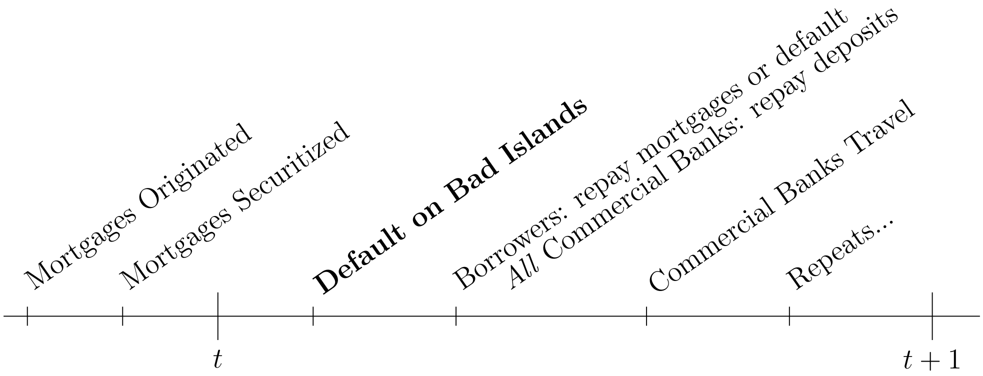
Figure 2.2.2 illustrates the island structure of default. The commercial banking sector on each island may only lend to households on their island. Every period a random fraction ψ of islands are hit by a default shock, similar to Gertler and Kiyotaki (2010)'s island-specific investment opportunity shock. On "bad" islands (those receiving a default shock) the borrower only repays a fraction $1-\delta$ of what they owe on their mortgage debt[4]. Where $R_{M,t}$ is the mortgage rate and b_t is the quantity of mortgage debt taken out by an individual borrower.

The timing is as follows (see Figure 2.2.3): prior to the start of the period mortgages are originated, commercial banks choose how to construct their balance sheet (between holding their own mortgages and holding mortgage backed securities). And shadow banks choose the quantity of pooled mortgages to buy and the quantity of mortgage backed securities to issue. These decisions jointly determine the mortgage spread. At the start

[4]This paper focuses on idiosyncratic risk, this framework could be extended to address aggregate mortgage market uncertainty by making ψ time-varying.

of a new period the islands realize their default status. On a good island the borrower repays in full, on a bad island the borrower defaults proportionally. Commercial banks across all islands repay deposits, then commercial banks travel across islands to equalize credit conditions on islands going into the next period[5].

Figure 2.2.3: Timing of Default



Note on "Travel": After deposits are repaid commercial banks move across islands to equalize credit conditions on all islands going into the next period. Essentially commercial banks are acting as a representative commercial banking sector but they cannot insure each other against adverse island shocks until *after* deposits are repaid.

Commercial banks can choose to retain the mortgages they issue on balance sheet (as "portfolio loans"), or to sell them to the off-island securitizing shadow bank. The shadow banking sector purchases mortgages from across all islands and packages them into "pass-thru" mortgage backed securities (which payoff based on the aggregate mortgage market return, averaged across islands). Shadow banks are able to divert funds, a la Gertler and Kiyotaki (2010) and Meeks et al. (2017), and therefore are subject to an incentive compatibility constraint.

2.2.2 Households

There are two types of households. Savers are the ultimate source of funding for mortgage debt. Borrowers are relatively impatient individuals who value housing and face a collateral constraint when obtaining mortgage credit. Each household type risk shares in a large family across islands an abstraction that focuses the idiosyncratic risk from island

[5]Essentially there is a representative commercial banking sector but commercial banks across islands cannot insure each other until *after* deposits on island are repaid.

specific default shocks entirely onto the financial sector in this model. Both household types have log preferences in consumption, and borrowers have additively separable log preferences in housing.

2.2.2.1 Savers' Problem

Savers are the previously wealthy individuals who have already bought a house and so do not price housing. They exist in the model to be the ultimate source of funding. Savers' are relatively patient (their discount factor $\tilde{\beta}$ is larger than the borrower's discount factor), they hold deposits, consume, and work. Their problem is:

$$\max_{\{\tilde{c}_t,\tilde{n}_t,d_t\}} \quad E_0 \sum_{t=0}^{\infty} (\tilde{\beta})^t \left[\ln \tilde{c}_t - \frac{(\tilde{n}_t)^{1+\eta}}{1+\eta} \right], \tag{2.2.1}$$

subject to their budget constraint:

$$\tilde{c}_t + d_t \leq R_{t-1} d_{t-1} + \tilde{w}_t \tilde{n}_t + Div_t. \tag{2.2.2}$$

Saver specific notation is denoted with tildes: $\tilde{c}_t$ denotes consumption of non-durable goods, $\tilde{n}_t$ labor hours, $\tilde{w}_t$ the wage rate, and d_t deposits (which pay the risk-free rate R_t). Finally Div_t denotes dividends received from commercial and shadow banks, as savers are the ultimate owners of financial institutions.

2.2.2.2 Borrowers' Problem

Borrowers are relatively impatient (discount factor: $\hat{\beta} < \tilde{\beta}$), they receive loans from commercial banks, consume, work, and purchase housing using a combination of current income and mortgage loans. Their problem is:

$$\max_{\{\hat{c}_t,\hat{h}_t,\hat{n}_t,b_t\}} \quad E_0 \sum_{t=0}^{\infty} (\hat{\beta})^t \left[\ln \hat{c} + j_t \ln \hat{h}_t - \frac{(\hat{n}_t)^{1+\eta}}{1+\eta} \right], \tag{2.2.3}$$

subject to their budget constraint:

$$\hat{c}_t + p_{h,t}\hat{h}_t + (1-\psi\delta) R_{M,t-1} b_{t-1} = b_t + (1-\psi\delta) p_{h,t} \hat{h}_{t-1} + \hat{w}_t \hat{n}_t, \tag{2.2.4}$$

and a collateral constraint:

$$R_{Mt} b_t \leq \bar{m}_t E_t p_{h,t+1} \hat{h}_t, \tag{2.2.5}$$

where $\bar{m}_t$ is the exogenous collateral value of housing, and $p_{h,t}$ is the price of housing. Borrower specific notation is denoted with hats: $\hat{c}_t$ denotes consumption of non-durable goods, $\hat{n}_t$ labor hours, $\hat{w}_t$ the wage rate, and b_t mortgage debt ($R_{M,t}$ is the mortgage rate). j_t is the borrower's housing preference - shocks to j_t are any factor unrelated to financing conditions that move house prices.

Borrowers in this model risk share: the aggregate (across island) value of non-defaulted housing and non-defaulted debt enters the borrower budget constraint (2.2.4). This means the model abstracts from potentially interesting heterogeneity between borrowers with different histories of default. This assumption is required for tractability outside of a heterogeneous agent model of borrowers. However, this treatment still allows commercial banks to face idiosyncratic risk from retaining their own lending, the focus of this paper.

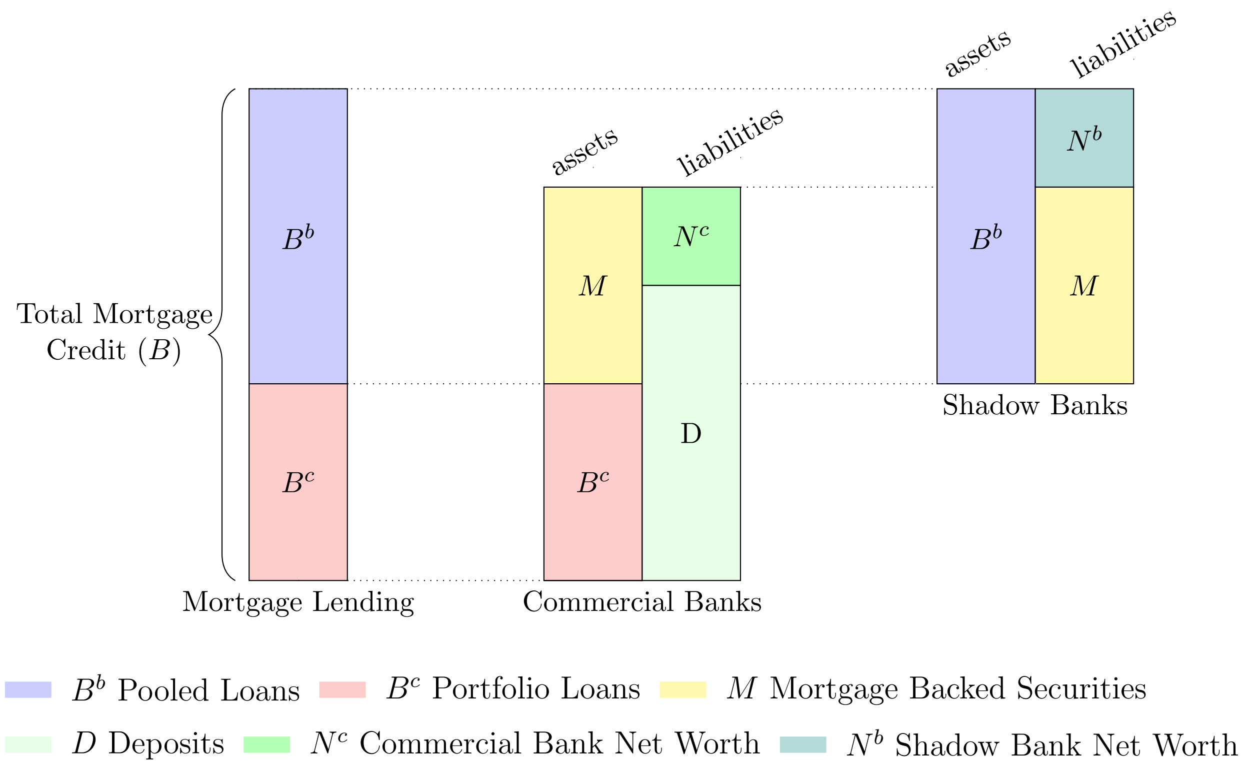
Credit demand shocks are captured in two ways. One, via housing demand shocks (positive shocks to the housing preference parameter j_t) which drive house prices up and therefore push borrowers to demand larger mortgage balances to finance the purchase of more expensive housing (via a collateral cycle effect this is possible). And two, housing collateral shocks (positive shocks to the collateral value of housing $\bar{m}_t$) which directly expand the borrowers ability to borrow via increasing the collateral value of their house.

2.2.3 Financial Sector

The models island structure motivates the existence of mortgage backed securities. Shadow banks sit off-islands so can buy mortgages from across all islands and sell (to commercial banks only) an asset (the mortgage backed security, MBS) that pays the average mortgage return across islands. Commercial banks demand MBS because holding MBS reassures savers deposits will be paid back allowing them to intermediate more funds and expand total mortgage credit provision on island.

Figure (2.2.4) provides an overview of the balance sheets of financial intermediaries. Capital letters indicate aggregate quantities of the following: mortgage lending (B), mortgages retained by commercial banks (portfolio loans, B^c), shadow bank held loans (B^b), commercial bank net worth (N^c), shadow bank net worth (N^b), deposits (D), and total

Figure 2.2.4: Financial Sector Balance Sheet



Note: Portfolio Loans (B^c) are the loans originated and then retained by an individual commercial bank, these loans are subject to island specific default risk. In contrast the Pooled Loans (B^b) are the loans purchased by shadow banks from across all islands, these loans are diversified so only have aggregate risk not island specific risk.

issuance of MBS (M). Note: MBS issued by shadow banks (M^b) is held entirely within the financial sector by commercial banks (M^c), so that $M = M^c = M^b$.

2.2.3.1 Commercial Banking Sector

Commercial banks are constrained by the savers willingness to make deposits. Savers will only make an additional deposit in their local commercial bank if they expect to be repaid in full even in the event of being on a "bad" (default hit) island. This "solvency constraint" requirement limits the ratio of portfolio mortgages to MBS the commercial bank can hold. Commercial banks can can relax the solvency constraint via the securitization process selling mortgages off their balance sheet and buying MBS which is diversified of their island specific risk.

There exists a continuum of commercial banks indexed by $c \in [0, 1]$. Each period commercial banks choose a specific island on which to locate for the purposes of mortgage lending and deposit taking, meaning that ex-ante islands have identical mortgage credit markets. In the following period the island's default status is realized. Commercial banks on all islands receive the same rate of return on MBS held, and must pay back deposits. Commercial banks on bad (default hit) islands are not fully repaid what is owed on mortgage debt. Commercial banks on good (non-defaulter) islands receive the full amount owed on mortgage debt and repay depositors. After repaying, commercial banks come together to redistribute net worth and travel across islands to equalize credit conditions. The solvency constraint is important because commercial banks can only risk share *after* deposits on island are repaid. Commercial banks continue with probability σ_c and die with probability $(1-\sigma_c)$. Upon death their net worth goes to saver households (the ultimate owners of all financial institutions). New commercial banks enter with transfers made by saver households. The entry and exit assumption is the standard assumption to ensure net worth is not accumulated to the point that the solvency constraint is slack.

The commercial bank's problem is to choose deposit volumes (d_t), on balance sheet loans (b_t^c), and MBS holdings (m_t^c) to maximize their continuation value (V_t^c) subject to their balance sheet identity & to the solvency constraint:

$$\max_{b_t^c, d_t, m_t^c} \quad V_t^c = E_t \tilde{\Lambda}_{t,t+1} \left\{ (1-\sigma_c) \left[(1-\psi) n_{t+1}^{c,good} + \psi n_{t+1}^{c,bad} \right] + \sigma_c V_{t+1}^c \right\}, \tag{2.2.6}$$

subject to their balance sheet identity:

$$b_t^c + m_t^c = n_t^c + d_t, \tag{2.2.7}$$

and the solvency constraint:

$$(1-\delta) R_{M,t} b_t^c + \bar{R}_{m,t} m_t^c \geq R_t d_t, \tag{2.2.8}$$

where $\tilde{\Lambda}_{t,t+1}$ is the patient households' stochastic discount factor. Individual commercial bank net worth is denoted by n_t^c. $R_{M,t}$, $\bar{R}_{m,t}$ and R_t, are the mortgage rate, the mortgage backed security rate, and the deposit rate respectively. Net worth is realized as follows on good and bad islands:

$$n_{t+1}^c = \begin{cases} R_{M,t} b_t^c + \bar{R}_{m,t} m_t^c - R_t d_t, & \text{if on a good island,} \\ (1-\delta) R_{M,t} b_t^c + \bar{R}_{m,t} m_t^c - R_t d_t, & \text{if on a bad island.} \end{cases} \tag{2.2.9}$$

The solvency constraint is the requirement that, when the banks island is hit with the default shock, its revenue on mortgage lending and MBS holdings must exceed or be equal to its obligation to depositors. Essentially the solvency constraint plays the role of a value-at-risk (VaR) constraint[6], where the probability of defaulting on deposits is 0.

Aggregate commercial banking sector net worth evolves according to:

$$N_t^c = (\sigma_c + \xi_c)\Big((1-\delta\psi)R_{M,t-1}B_{t-1}^c + \bar{R}_{m,t-1}M_{t-1}\Big) - \sigma_c R_{t-1}D_{t-1}, \tag{2.2.10}$$

where ξ_c is the proportional transfer saver households make to new entering commercial banks.

2.2.3.2 Shadow Banking Sector

Shadow banks are constrained by the market's willingness to hold their assets, the mortgage backed security (MBS). This constraint is the Gertler and Kiyotaki (2010) running away constraint. If this constraint exogenously loosens they are able to securitize more mortgage credit, which allows commercial banks to provide more mortgage credit this is the innovation in securitization credit supply shock.

Shadow Banks exist off-island. Each period they buy a perfectly diversified set of mortgages from every island and issue MBS which pay the average return on mortgage credit across islands. They die with probability $(1-\sigma_b)$ and survive with probability σ_b. They face an agency problem that follows that in Meeks et al. (2017) and Gertler and Kiyotaki (2010).

The shadow bank's problem is to purchased diversified (pooled) mortgage debt (b_t^b) and issue MBS (m_t^b) to maximize their continuation value (V_t^b) subject to their balance sheet identity and incentive compatibility constraint:

$$\max_{\{b_t^b, m_t^b\}} \; V_t^b = E_t\tilde{\Lambda}_{t,t+1}\Bigg[(1-\sigma_b)n_{t+1}^b + \sigma_b V_{t+1}^b\Bigg], \tag{2.2.11}$$

subject to their balance sheet identity:

$$b_t^b = m_t^b + n_t^b, \tag{2.2.12}$$

[6] Eg that in Adrian and Shin (2014).

the incentive compatibility constraint:

$$V_t^b \geq \theta_{b,t} b_t^b. \tag{2.2.13}$$

An individual shadow bank's net worth evolves according to:

$$n_{t+1}^b = \underbrace{(1-\psi\delta)R_{M,t}b_t^b}_{\text{return on the diversified mortgage pool}} - \bar{R}_{m,t}m_t^b. \tag{2.2.14}$$

The shadow bank's incentive compatibility constraint (2.2.13) captures the agency problem between a shadow bank and the commercial banks that holds the MBS the shadow bank issues. The literal interpretation of $\theta_{b,t}$ is as follows: each period the shadow bank is able to choose to close down and take away a fraction $\theta_{b,t}$ of the amount repaid on the mortgage debt the shadow bank owns. If the shadow bank chooses this they will never be trusted again, they close down and forfeit their continuation value V_t^b. This constraint (2.2.13) limits the quantity of MBS shadow banks can issue. Essentially $\theta_{b,t}$ indexes the trust that MBS holders place in shadow banks. A fall in $\theta_{b,t}$ captures financial innovation of the sort experience prior to the financial crisis.

Financial innovation in this context relates directly to "credit enhancements"[7]- the implicit or explicit agreements such as tranching that were used to reassure investors that MBS were nearly risk-free assets. An "innovation in securitization" shock (exogenous drop in $\theta_{b,t}$) captures in reduced form either: a) actual technological improvements in credit enhancements, or b) an increase in investor's perception about the ability of credit enhancements to minimize MBS credit risk. With innovation in securitization shadow banks can hold mortgage credit in greater quantities with a lower spread, meaning that the general equilibrium effect is lower mortgage spreads.

Aggregate shadow banking sector net worth evolves according to:

$$N_t^b = (\sigma_b + \xi_b)(1-\delta\psi)R_{M,t-1}B_{t-1}^b - \sigma_b\bar{R}_{m,t-1}M_{t-1}, \tag{2.2.15}$$

where ξ_b is the proportional transfer saver households make to new entering shadow banks.

[7] See Gorton and Souleles (2007) for a discussion of credit enhancements.

2.2.4 Production

Production is based on labor and labor is differentiated across borrower and saver types. This is a simplification of the Iacoviello and Neri (2010) framework.

Each island contains a goods producer who chooses saver ($\tilde{n}_t$) and borrower ($\hat{n}_t$) labor to maximize their profit (2.2.16) subject to their production function (2.2.17). Household members can travel costlessly across islands to work and consume, so that wages and prices equalize across islands (alternatively this can be considered as one aggregate producer), the producer problem is:

$$\max_{\tilde{n}_t, \hat{n}_t} \; Y_t - \left[\tilde{w}_t \tilde{n}_t + \hat{w}_t \hat{n}_t\right], \tag{2.2.16}$$

subject to:

$$Y_t = A_t \tilde{n}_t^{\alpha} \hat{n}_t^{1-\alpha}. \tag{2.2.17}$$

2.2.5 Market Clearing

Goods Market equilibrium:

$$Y_t - p_{h,t}\Big[\hat{h}_t - (1-\delta\psi)\hat{h}_{t-1}\Big] = \tilde{c}_t + \hat{c}_t. \tag{2.2.18}$$

Total Housing:

$$\bar{H} = \hat{h}_t. \tag{2.2.19}$$

Total lending:

$$B_t = B_t^c + B_t^b. \tag{2.2.20}$$

MBS market:

$$M_t^c = M_t^b. \tag{2.2.21}$$

2.3 Calibration & Simulation

2.3.1 Calibration

2.3.1.1 Macroeconomic Parameters

This set of parameters either match well established calibrations in the literature, or target an average of the 1990s data. $\tilde{\beta}$ is set to target the average real Fed Funds rate in

Table 2.1: Calibrated Parameters

Parameter	Value	Description
Macroeconomic Parameters		
$\tilde{\beta}$	0.9943	Saver's discount factor
$\hat{\beta}$	0.97	Borrower's discount factor
A	1	Steady state level of TFP
α	0.64	Labor income share of savers
η	1	Inverse of the Frisch elasticity of labor supply
Housing and Financial Parameters		
$\bar{H}$	1	Total inelastic supply of housing
σ_c, σ_b	0.95	Financial institution survival probability
$\bar{m}$	0.92	Housing collateral value
j	0.067	Borrower housing preference parameter
ψ	1.3%	Quarterly mortgage delinquency rate
δ	23.8%	On island default
θ_b	0.60	Fraction of pooled loans that are divertible
ξ_c	0.0028	Fraction of assets transfered to new commercial banks
ξ_b	0.0042	Fraction of assets transfered to new shadow banks

the 1990s data (of 2.28% annualized[8]). $\hat{\beta}$ is set to match the calibration in Iacoviello and Neri (2010). The relative impatience has a minimal effect on the co-movement of house prices and mortgage credit but does not effect the overall results appreciably. The level of TFP in steady state is normalized to 1. The calibration of the labor income share going to savers (α) comes from Justiniano et al. (2015) who identify borrowers as households whose liquid assets are less than two months of their total income - using the 1992, 1995 and 1998 Survey of Consumer Finances (SCF). Following Justiniano et al. (2015) the Frisch elasticity of labor supply $\left(\frac{1}{\eta}\right)$ is set to 1.

2.3.1.2 Housing and Financial Parameters

The supply of housing $\bar{H}$ is normalized to 1. σ_c and σ_b target an expected survival horizon for commercial and shadow banks of 5 years, consistent with the literature (eg Gertler and Kiyotaki, 2015). The collateral value of housing ($\bar{m}$) and the borrower's housing

[8]The average nominal Fed Funds rate (FEDFUNDS) in this period is 5.14% and the average growth in the Consumer Price Index (CPIAUSCSL) is 2.86%

preference parameter (j) jointly target a mortgage debt to borrower income ratio of 0.8[9], and a calculated loan-to-value (LTV) ratio of 90%[10].

2.3.1.3 Simulation Initialization Targets

The remaining parameters: $\delta, \theta_b, \xi_c, \xi_b$, and ψ which pertain most directly to the financial sector, jointly target the moments in the 2000 Q1 - 2000 Q4 data in table 2.2. This narrower target period better reflects the condition of the private securitization market, because the early 1990s were characterized by only a handful of private securitization deals. The first four parameters target the spread and balance sheet moments in table 2.2 and then ψ is set so that the product of ψ (the fraction of islands that are bad islands) and δ (the proportional default on bad islands) jointly target the fraction of mortgage dollars entering serious (90+ day) delinquency in the Quarterly Report on Household Debt and Credit (0.3180%).

Table 2.2: Simulation Initialization Targets

Target	**Value**
Mortgage Spread ($R_M - R$)	4.36%
Commercial bank asset composition ($\frac{M^c}{Bc}$)	0.05
MBS to Mortgages Ratio	0.04
Adjusted commercial bank leverage	4.6
% of Mortgage dollars entering serious delinquency	0.32%

Note: These targets are intended to match an average of 2000 Q1 - Q4 in the data, this is the starting point for the subsequent simulation.

The data series on the mortgage spread is the spread of the average mortgage rate in the PLSD over the 10 year US Treasury yield, using the adjusted for borrower quality series in Justiniano et al. (2017). The commercial bank asset composition is that of depository institutions (commercial banks and thrifts) in the US[11]. In the MBS to Mortgages Ratio ($\frac{M}{B}$) total mortgages (B) is measured as the sum of portfolio loans and loans held in the shadow banking sector (by ABS issuers & mortgage companies - in

[9]This follows the target in Justiniano et al. (2019): they identify borrowers as households whose liquid assets are less than two months of their total income - using the 1992, 1995 and 1998 SCF.

[10]This is a compromise between the higher documented LTV ratios in the non-conforming mortgage pool, and maintaining consistency with similar targets eg Justiniano et al. (2015) in the literature.

[11]These data come from the Flow of Funds Z1 release on U.S. Chartered Depository Institutions (Table L.111)

the Flow of Funds data). The quantity of loans held in the shadow banking sector is adjusted by percentage of MBS held in the commercial banking sector[12]. The "adjusted commercial bank leverage" is average commercial bank leverage during the calibration period (measure by the aggregate data in the Federal Deposit Insurance Corporation's Quarterly Bank Report) normalized by the percentage of assets on commercial banks' balance sheets that are either portfolio loans or private mortgage backed securities (as measured by the Flow of Funds data[13]).

2.3.2 Simulation Method

The following simulations involve large shocks (moving the model far away from the initial steady state) and multiple occasionally binding constraints. Therefore, I use a deterministic simulation method with the fully non-linear model. This preserves the integrity of the simulation even as it moves far away from the initial steady state. The non-linearity also allows for all relevant constraints to be occasionally binding. The approach to the deterministic simulation is the extended path approach of Fair and Taylor (1983), which is applied (and explained in more detail) in Christiano et al. (2015). Let z_t denote the $Nx1$ vector of endogenous variables determined at time t, and $\epsilon_t = \{\theta_{b,t}, j_t, \tilde{\beta}_t, \bar{m}_t\}$ the vector of exogenous deterministic variables realized at time t. Each period the agents realize an unexpected shock (to either $\theta_{b,t}$, j_t, $\tilde{\beta}_t$ or $\bar{m}_t$) and expect the economy to transition to a new steady state consistent with the realization of that shock. In t=1 the starting point of the deterministic simulation is the initial steady state, in $t \geq 2$ the starting point is the vector of endogenous variables in $t-1$.

2.4 The Innovation in Securitization Channel

Considering a simplified two-period version of the model with linear saver utility (see appendix 2.C.2) the shadow banks incentive compatibility constraint (2.2.13) can be expressed as follows (for simplicity I drop the timing subscript here, all variables refer to

[12]This makes the ratio of pooled loans to MBS consistent with the aggregate ratio
[13]Flow of Funds Z1 release on U.S. Chartered Depository Institutions (Table L.111)

the same period):

$$\tilde{\beta}\left\{\underbrace{\left[(1-\delta\psi)R_M-\bar{R}_m\right]}_{\text{Mortgage Spread}}B^b+\bar{R}_m N_1^b\right\}\geq\theta_b B^b. \tag{2.4.1}$$

The left hand side of (2.4.1) is the value shadow banks derive from choosing to honor the their liabilities from the mortgage backed securities they issued. And the right hand side of (2.4.1) is the value shadow banks get from choosing to divert (i.e. run away) with the pooled mortgages (B^b) on their balance sheet. θ_b is the fraction of pooled mortgages that are divertible. θ_b indexes the state of financial liberalization in the private mortgage backed securitization market (the lower θ_b is the more liberalized the securitization market is). An exogenous decrease in the divertible fraction (θ_b) is an "innovation in securitization" shock.

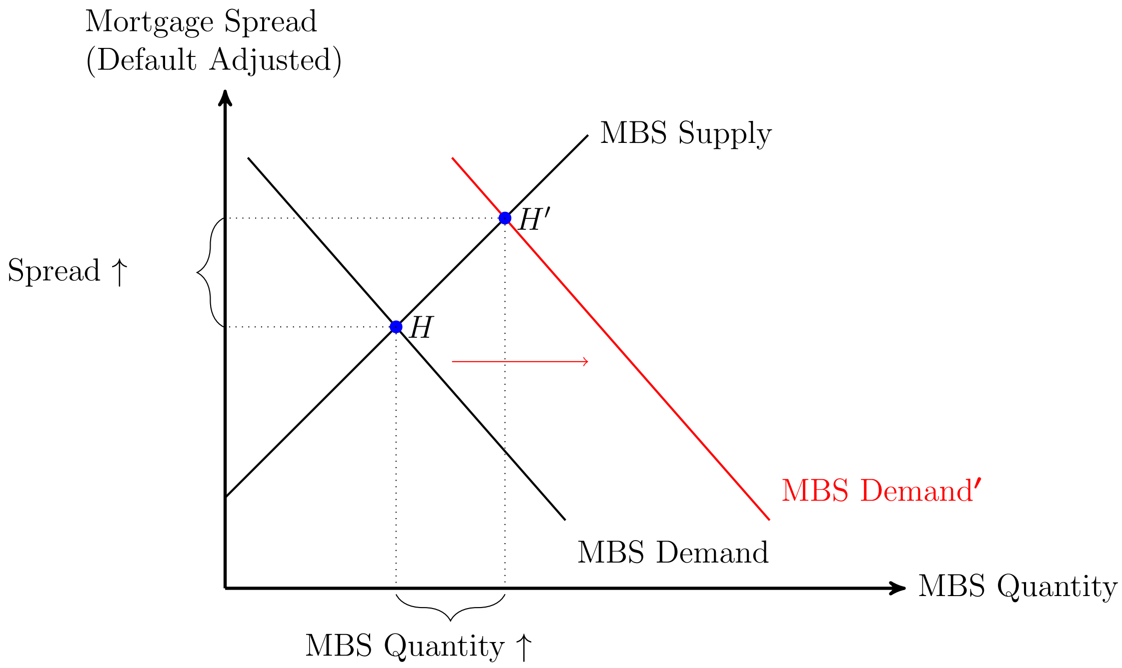
From the above it is clear to see that a 1 unit increase in pooled mortgages held by the shadow bank (driven by a 1 unit increase in mortgage backed securities issued) will increase the quantity of divertible loans by θ_b and continuation value for shadow banks (LHS) by:

$$\tilde{\beta}\times\text{(old) Spread}+\tilde{\beta}\frac{\partial\text{ Spread}}{\partial B^b}. \tag{2.4.2}$$

As long as the divertible fraction (θ_b) is sufficiently greater than zero, the mortgage spread must *increase* in response to any increase in the quantity of mortgage backed securities demanded.

Mortgage backed security demand shocks can be driven by shifts in housing demand or exogenous savings shocks. The exogenous savings shock (a shock to the saver's discount factor, $\tilde{\beta}_t$) captures Bernanke (2005)'s Global Savings Glut view. Both shocks put upward pressure on the size of the commercial banks' balance sheets (in the housing demand case because borrowers demand more mortgages, and in the exogenous savings case because savers demand more deposits). Because of the solvency constraint the commercial banks are limited in the ratio of mortgage loans they can retain to quantity of mortgage backed securities they must hold. Therefore both credit demand *and* the credit supply shocks that originate outside of the shadow banking sector operate as *MBS demand* shocks from

Figure 2.4.1: Mortgage Backed Securities Market - Partial Equilibrium Effect of an MBS Demand Shock



the shadow banks perspective and generate a counter-factual *increase* in the mortgage spread (see figure 2.4.1).

In contrast the innovation in securitization shock (an exogenous decrease in θ_b) directly decreases the quantity of divertible loans, because it makes the pooled mortgages (B^b) less divertible. This directly lowers the continuation value required for shadow banks to meet their incentive compatibility constraint (2.4.1), and means that shadow banks can respond by increasing the quantity of MBS they issue even while the mortgage spread in equilibrium falls (see figure 2.4.2). Finally, the pooled mortgage divertibility parameter (θ_b) indexes the liberalization of the shadow banking sector. For higher values of θ_b the shadow banking sector amplifies other shocks less, and for lower values of θ_b the shadow banking sector amplifies shocks more (see figure 2.4.3). This underlines the importance of correctly identifying the role of innovation in securitization as a potential driver of the boom in US house prices and mortgage debt. It also indicates that the innovation in securitization channel if present could have amplified the housing demand and alternative credit supply explanations that others have pointed to, to explain the US experience.

Figure 2.4.2: Mortgage Backed Securities Market - Partial Equilibrium Effect of an Innovation in Securitization Shock

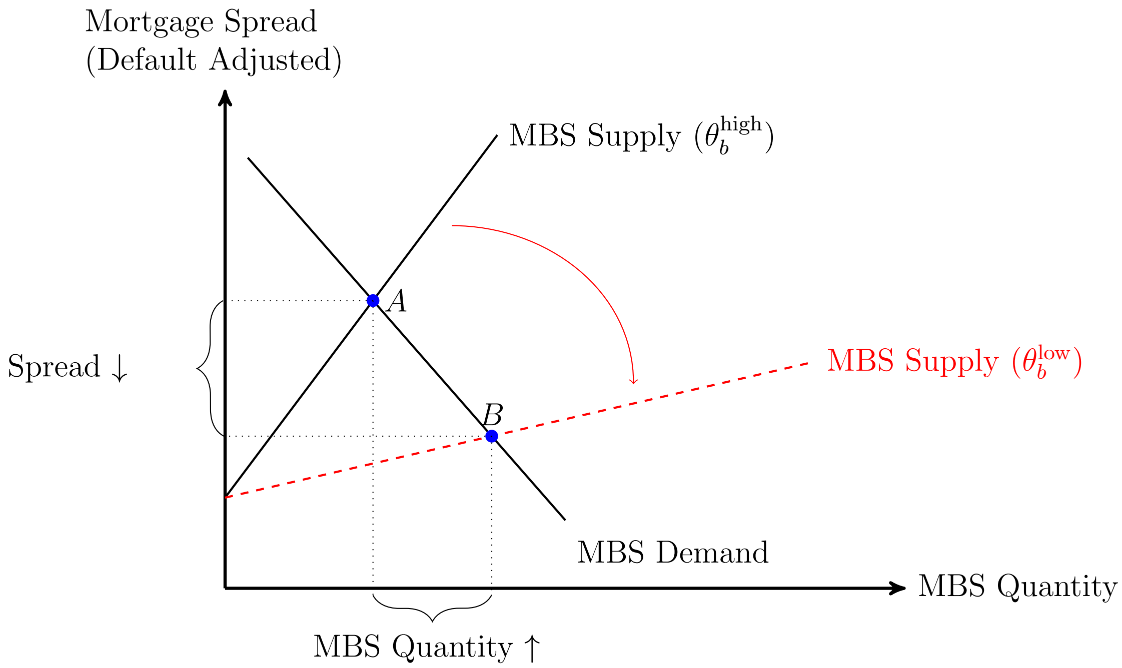
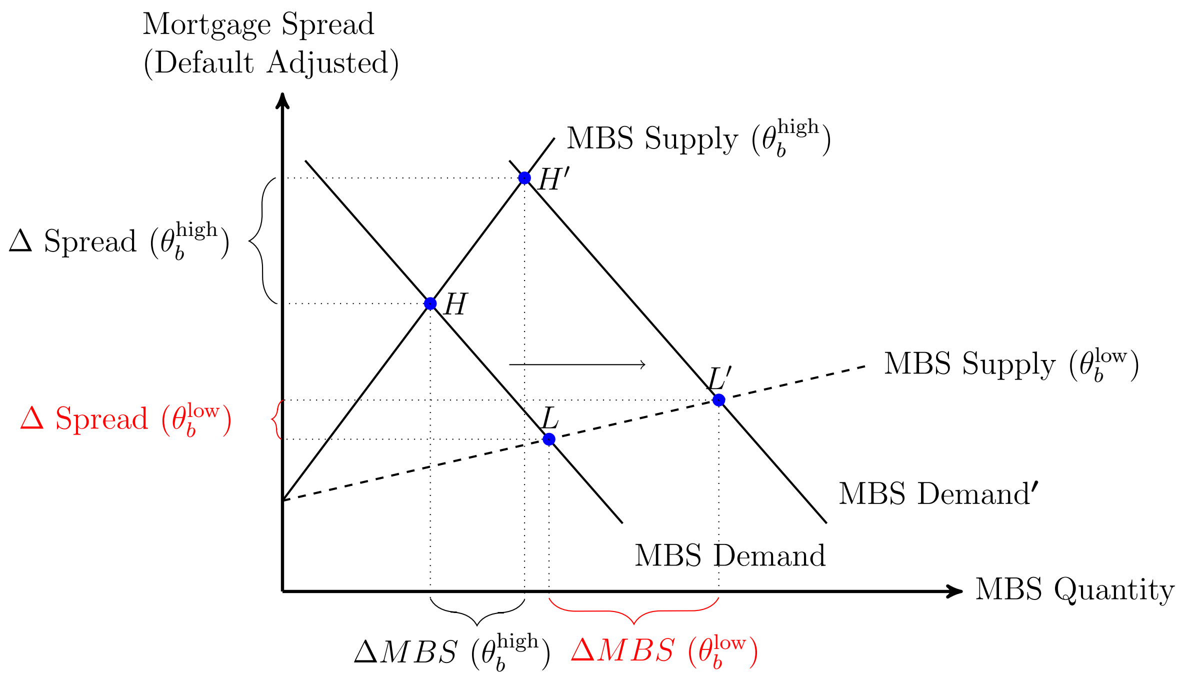


Figure 2.4.3: Mortgage Backed Securities Market - A More Liberalized Shadow Banking Sector Magnifies Other Shocks



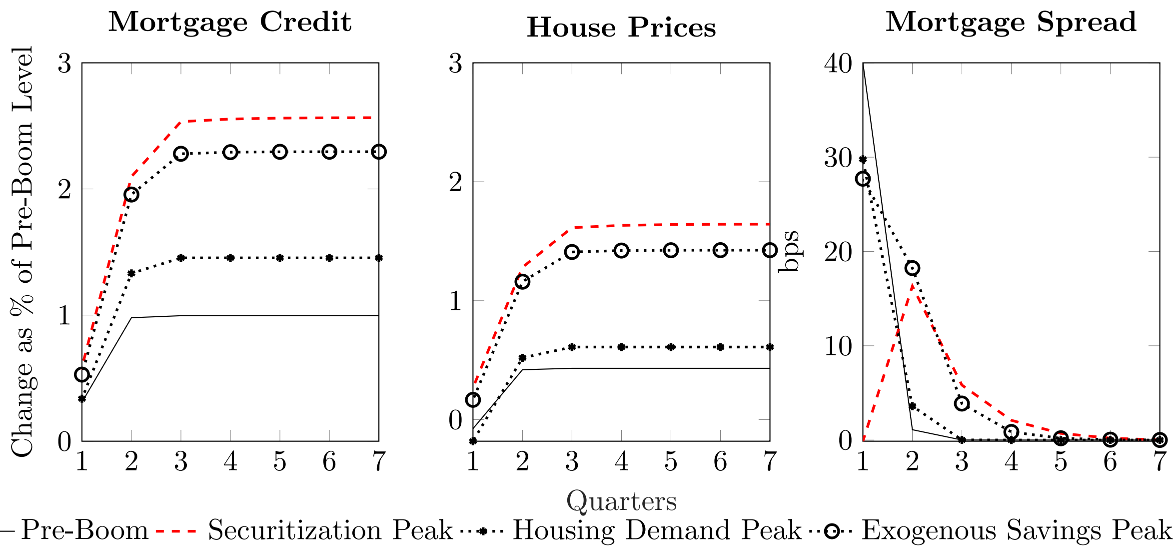
Figures 2.4.4, 2.4.5, and 2.4.6 make the qualitative amplification point in figure 2.4.3 quantitatively. In each figure respectively I calibrate a 1-off permanent shock to housing preference, housing collateral, and saver patience to generate a 1% increase in total

outstanding mortgage credit in the pre-boom (year 2000) version of the model. I then run the same shock at the Securitization Peak, Housing Demand Peak, and Exogenous Savings Peak (that is the model version consistent with the various 2006 Q4 versions of the model in the following horse-race). The qualitative point in figure 2.4.3 - for all three 1-off shocks the Securitization Peak amplifies the impact of the shock on house prices and mortgage credit the most relative to the other peaks.

Figure 2.4.4: Transmission of a Housing Demand Shock

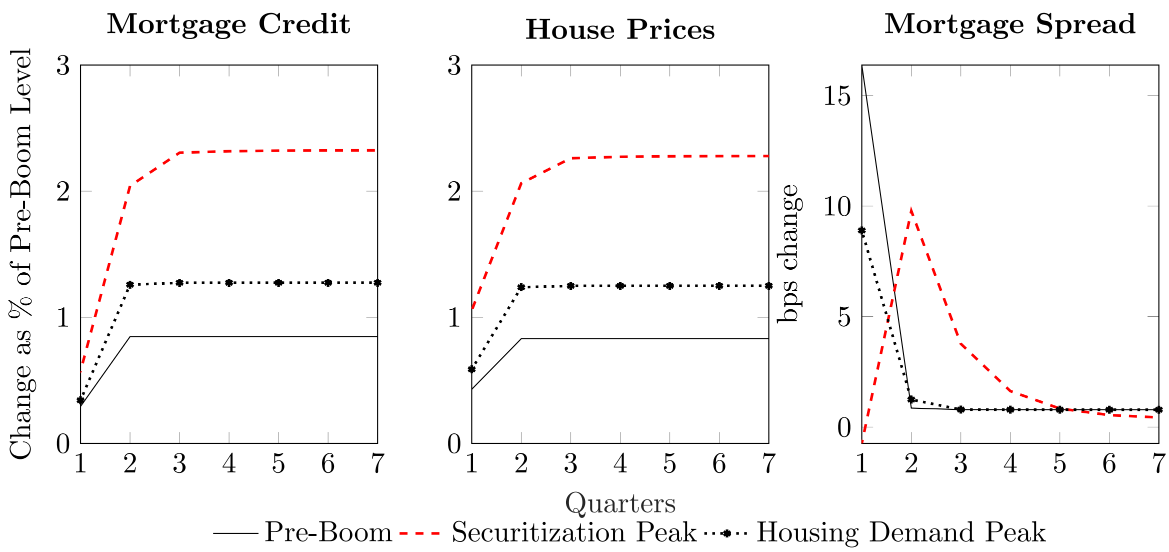
Note the housing demand shock is an exogenous increase in the borrowers' preference for housing parameter (j_t), in their utility function - equation (2.2.3). The mortgage spread is plotted in basis points deviation from steady state.

Figure 2.4.5: Transmission of a Housing Collateral Shock



Note: The housing collateral shock is an exogenous increase in $\bar{m}_t$, the housing collateral value in the borrowers' collateral constraint - equation (2.2.5).

Figure 2.4.6: Transmission of a Savings Shock



Note: The "savings shock" is an exogenous increase in saver time preference $\tilde{\beta}_t$.

2.5 Boom-Bust Simulation Results and Discussion

Two exercises are presented below. The first exercise is a horse-race between the following three candidate explanations of the boom. One, the "Securitization Boom": driven by negative shocks to $\theta_{b,t}$ (the "innovation in securitization" shocks). Two, the "Housing Demand Boom": driven by positive shocks to borrower housing preference, j_t. And three, the "Exogenous Savings Boom": driven by positive shocks to saver time preference, $\tilde{\beta}_t$. This is an alternative credit supply shock unrelated to shifts in the securitization sector, it is a way of capturing the Global Savings Glut argument put forward by Bernanke (2005). The second exercise is a quantitative assessment of the extent to which innovation in securitization drove house prices and mortgage debt.

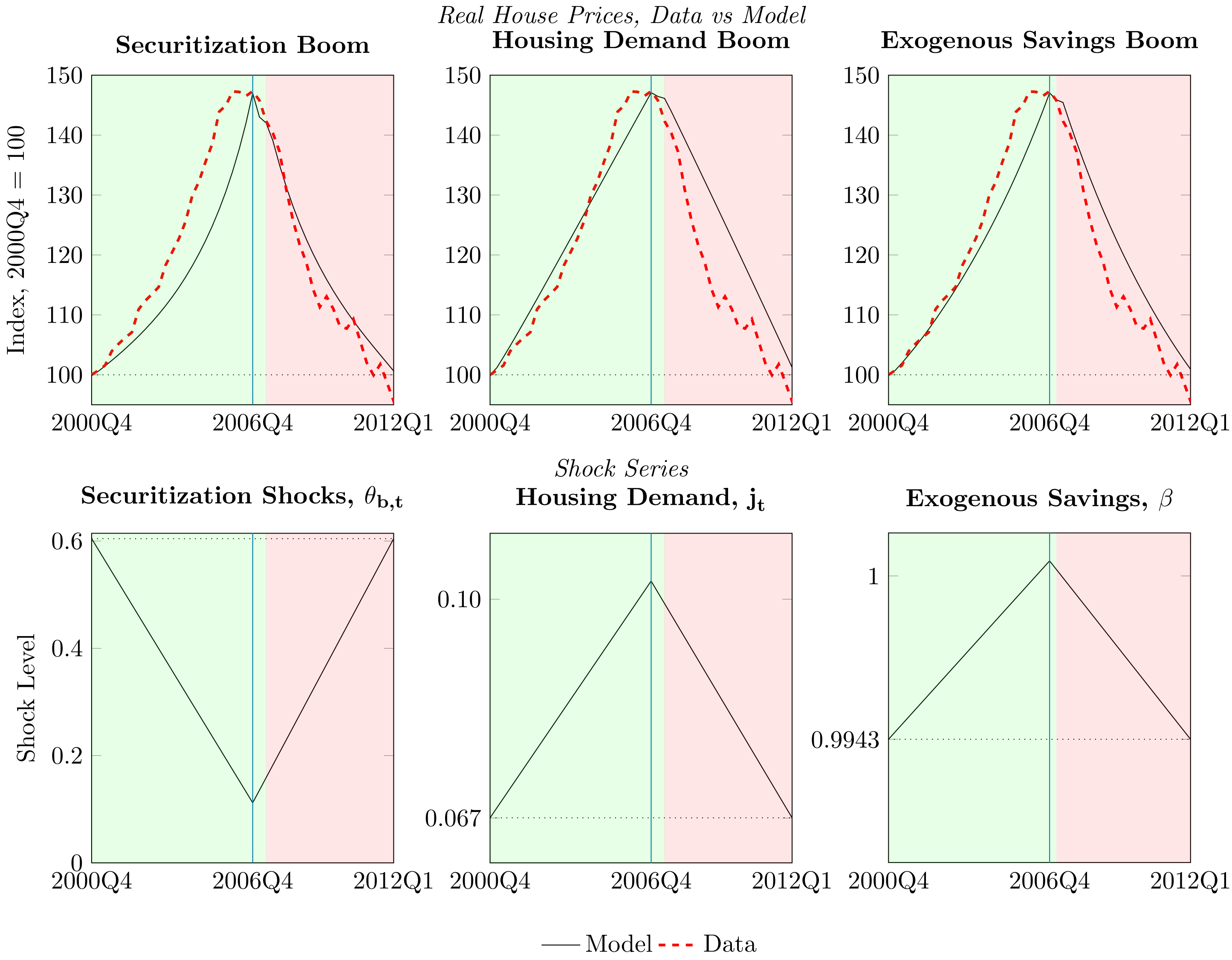
The key results here are that only innovation in securitization can explain the simultaneous increase in mortgage debt and decrease in the mortgage spread. Quantitatively I find that innovation in securitization drove between 71 - 100% of the appreciation in house prices and 34 - 45% of the increase in non-conforming mortgage debt during the boom period.

2.5.1 Horse-Race to Match House Price Growth

In each of the three competing simulations the individual shock series is calibrated to target the peak in house prices during the boom (47% in 2006 Q4 according to the Case-Shiller Real House Price Index). Figure 2.5.1 shows the target series and corresponding shock processes for each simulation in the horse-race. The goal here is to match the boom in house prices, and then ask how much of the bust can be matched by reversing the shock that drove the boom.

The key result, captured in figure 2.5.2 in this simulation is that only the innovation in securitization shocks can match (the direction & magnitude) of the spread between the mortgage rate and the risk free rate. Unsurprisingly the housing demand driven boom (a demand for credit shock) puts upward pressure on the mortgage spread. More interestingly the exogenous savings expansion, which operates like the inelastic credit supply shock in Justiniano et al. (2019) and in this closed economy model stands in for

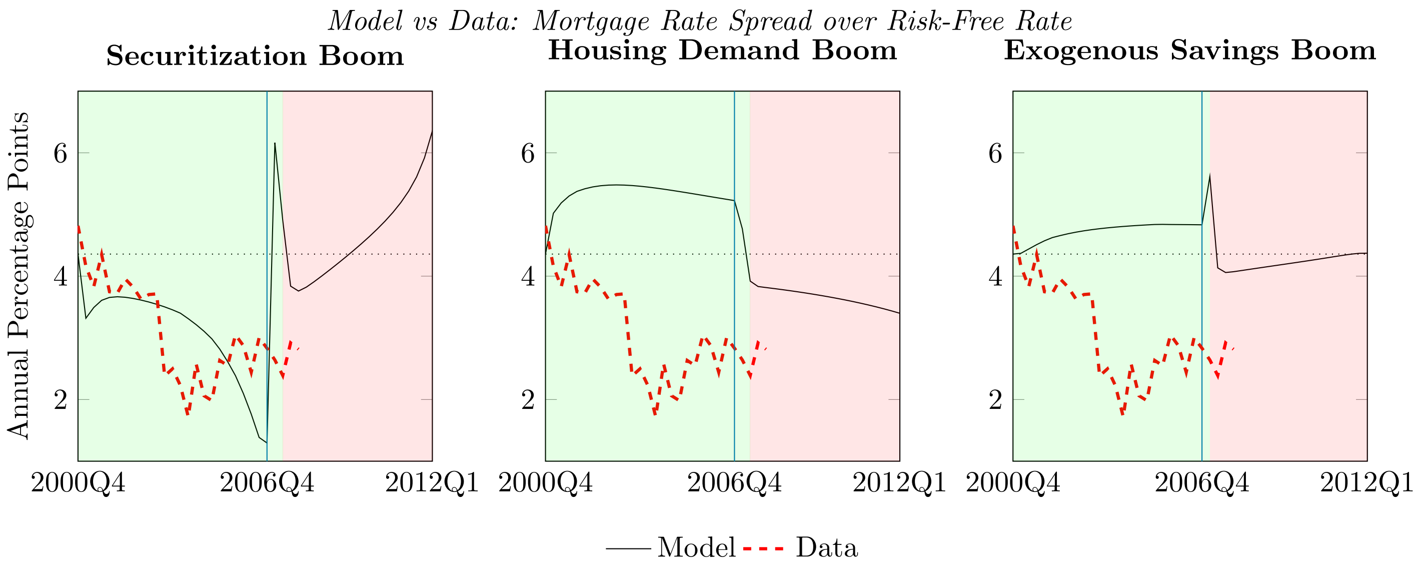
Figure 2.5.1: Matching Real House Price Growth



In this figure each column is a different simulation in the horse-race. The green shaded area indicates the housing collateral constraint is slack, the red shaded area indicates the housing collateral constraint binds. The vertical blue line is 2006Q4 (the peak period for real house prices according to the Case-Shiller Index).

an influx of foreign credit, also generates upward pressure on the mortgage spread. This is because the exogenous increase in savings drives deposits up, expanding the commercial banking sector's aggregate balance sheet. To expand their balance sheets commercial banks must hold more MBS to continue to meet the solvency constraint - this increases the *demand* shadow banks face for MBS, and because of the incentive compatibility constraint faced by shadow banks to increase quantity of MBS they issue they require an increased spread. This result underlines the necessity of modeling the securitization process, while the generalized credit supply expansion – "exogenous savings boom" – is *a credit supply shock*, the counter-factual implications suggest that it is likely not *the credit supply shock* that drove the US housing and mortgage market during the 2000s.

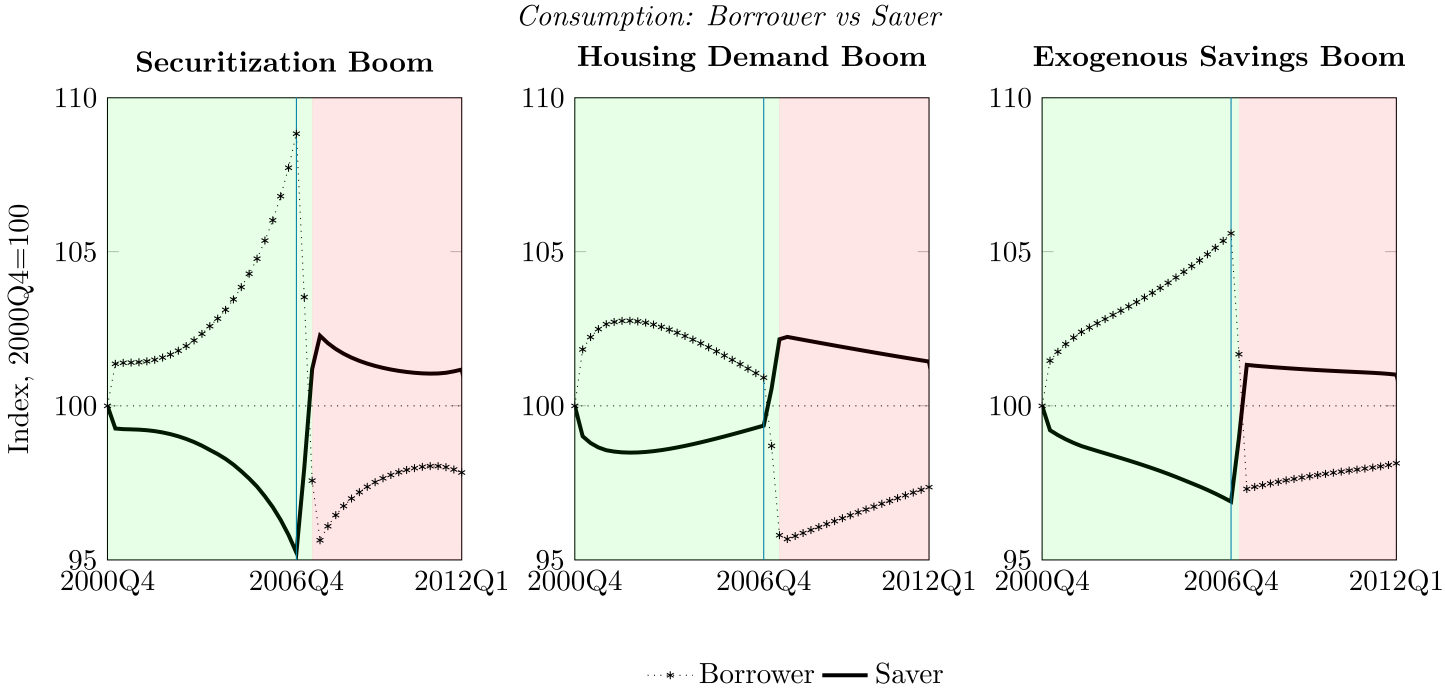
Figure 2.5.2: Only the Securitization Boom Explains Mortgage Spreads



In this figure each column is a different simulation in the horse-race. The green shaded area indicates the housing collateral constraint is slack, the red shaded area indicates the housing collateral constraint binds. The vertical blue line is 2006Q4 (the peak period for real house prices according to the Case-Shiller Index).

Additionally the Securitization boom generates the most volatility in borrower and saver consumption (figure 2.5.3) and is the only channel which generates a quantitatively reasonable response of several moments in the private securitization market (see Figure 2.B.2 in Appendix 2.B). Lastly the three candidate booms are indistinguishable in the response of mortgage debt and the borrower debt-to-annual income ratio (see Figure 2.B.1 in Appendix 2.B).

Figure 2.5.3: Consumption is Most Volatile in the Securitization Boom

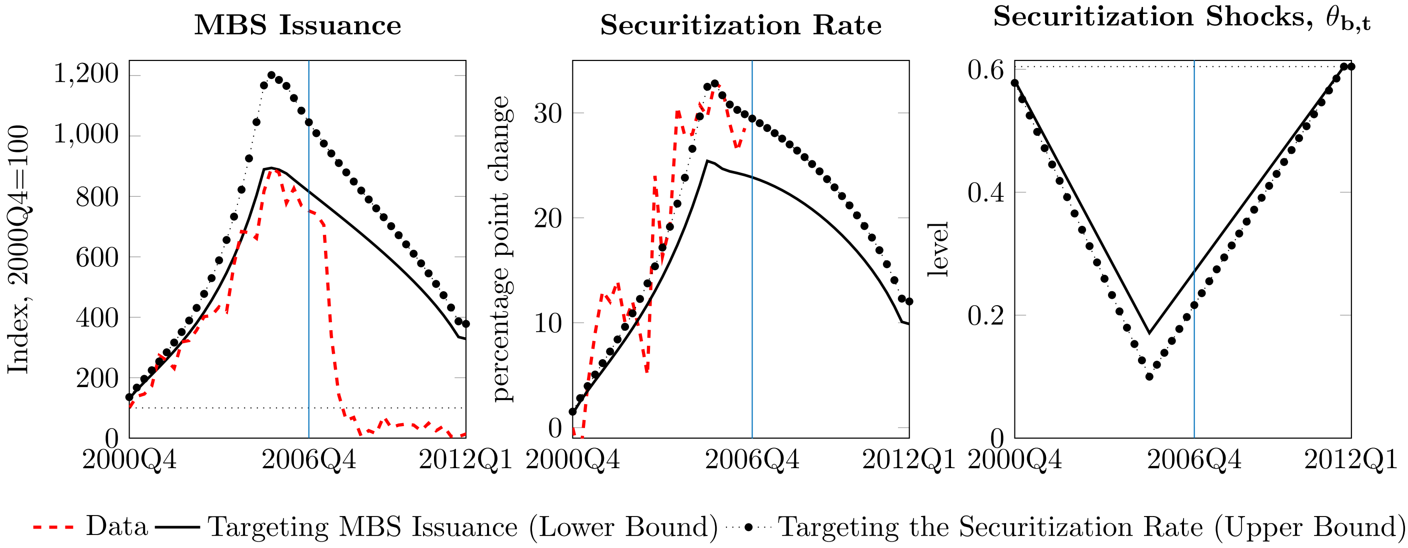


The green shaded area indicates the housing collateral constraint is slack, the red shaded area indicates the housing collateral constraint binds. The vertical blue line is 2006Q4 (the peak period for real house prices according to the Case-Shiller Index).

2.5.2 Measuring the Contribution of Innovation in Securitization

In this section I put upper and lower bounds on the model's prediction as to the extent to which innovation in securitization could have driven house price growth. The data series most closely related to the innovation in securitization shocks are: private mortgage backed security issuance and the securitization rate. Issuance is the flow of private mortgage backed securities produced by the shadow banking sector each quarter. This grew by over 790% between 2000 Q4 and its peak in 2005 Q3 (See the leftmost subplot in Figure 2.5.4), matching this data series gives a lower bound as to the magnitude of the innovation in securitization channel (see sold black line in Figure 2.5.4). In contrast matching the securitization rate gives an upper bound as to the magnitude of the innovation in securitization channel (see dotted black line in Figure 2.5.4). The securitization rate is the ratio of private mortgage backed security issuance to non-conforming mortgages originated in any given quarter. It is another flow series that is closely related to the constraints faced by the shadow banking sector when issuing private mortgage backed securities.

Figure 2.5.4: Matching Moments - Upper and Lower Bounds



Here the upper estimate of the securitization shock series (targeted to match the securitization rate - the ratio of MBS issued to mortgage debt originated) is the dot-dashed black line. The lower estimate of the securitization shock series is the solid black line (targeted to match the growth in private mortgage backed security issuance). The dashed red line is the data.

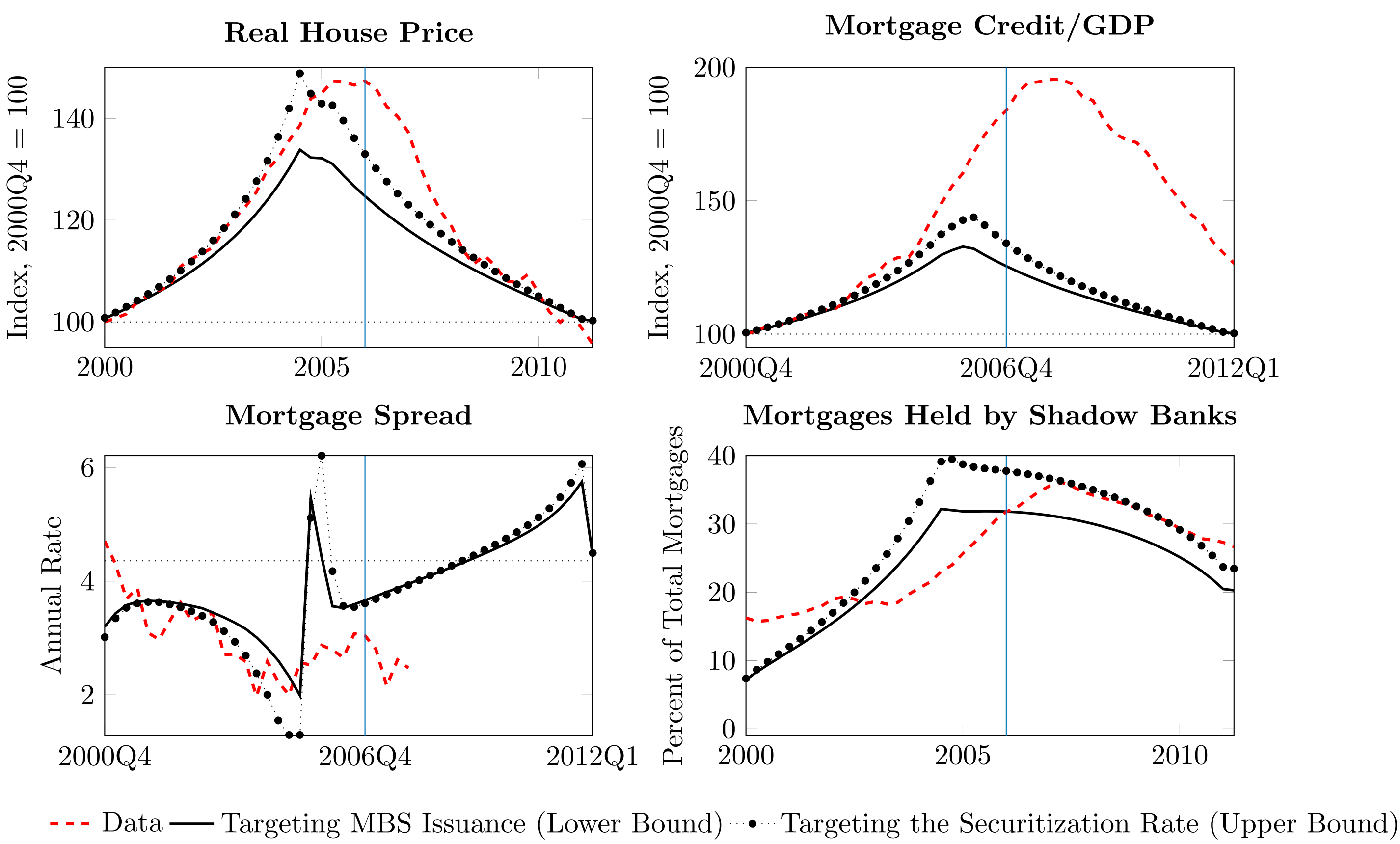
Figure 2.5.5 shows the upper and lower bounds the securitization rate target simulation and the MBS issuance target simulation put on the extent to which innovation in securitization drove the boom in house prices, mortgage credit, the mortgage spread, and the percentage of the stock of (non-conforming) mortgages held by the shadow banking sector. The results of the two simulations suggest that the innovation in securitization channel can explain between 72% and over 100% of the increase in real house prices seen in the data[14]. innovation in securitization drives between 34.3% to 45.8% of the increase in mortgage credit relative to GDP[15]. innovation in securitization explains at least 235 basis points of the 273 basis point drop in the mortgage spread found in the data (86% of the data)[16].

[14]The securitization rate target simulation overshoots the real house price peak by 3%.

[15]The multiplier between house price growth and mortgage credit growth does not match the data. This could be in part because the model's supply of housing is fixed, adding housing investment may bring the multiplier more in line with the data. This would not change the qualitative point made in the proceeding section.

[16]The securitization rate target simulation overshoots the data by 33 bps.

Figure 2.5.5: Quantifying the Innovation in Securitization Channel - Upper and Lower Bounds



Here the upper estimate of the securitization shock series (targeted to match the securitization rate - the ratio of MBS issued to mortgage debt originated) is the dot-dashed black line. The lower estimate of the securitization shock series is the solid black line (targeted to match the growth in private mortgage backed security issuance). The dashed red line is the data. On the mortgage spread subplot, the dotted red and blue lines are the adjusted for borrower quality mortgage spread series in Justiniano et al. (2019). Total mortgages refers to estimated total non-conforming mortgages outstanding.

2.6 Conclusion

In this paper I build a model in which the interaction of regulated commercial banks and the unregulated shadow banking sector is crucial. In the model the existence of mortgage backed securitization is based on idiosyncratic mortgage default risk. Shadow banks face a financial constraint on their balance sheet which relaxes over the boom period (2000 - 2006). This is "Innovation in Securitization". This innovation captures a number of factors including: the increased sophistication and use of tranching during this period, and increased market familiarity with private mortgage backed securitization (relative to the much older government associated securitization[17]). I find that this innovation was a primary driver of the increase in house prices and mortgage debt in the US between 2000 and 2006. The Innovation in Securitization shocks account for 71 - 100% of the increase in house prices and 34 - 45% of the increase in non-conforming mortgage debt observed in the data.

Additionally I show that other candidate explanations (a housing demand driven boom or a savings driven boom – the Global Savings Glut view) cannot on their own match the mortgage spread dynamics. For the housing demand driven boom the balance sheet effect amplifies the upward pressure on the mortgage spread. For the exogenous savings boom the balance sheet effect quantitatively reverses the initial negative impact on the mortgage spread. Because of the feedback driven by the balance sheet effect, capturing it shows that these two alternative explanations of the 2000 - 2006 US boom generate counter-factual implications for the mortgage spread. This is not to say that the model rules out housing demand and exogenous savings shocks as *amplifiers* of the securitization driven boom. I find that in a more liberalized[18] shadow banking sector the impact of housing demand and savings shocks on mortgage credit growth and house price growth are amplified relative to the pre-boom version of the model, and the response of the mortgage spread is moderated. The amplification effect is particularly strong for the transmission of savings shocks - suggesting securitization played an important role in amplifying the impact of inflows of foreign savings into the U.S. during this period.

[17]Mortgage backed securitization done by Fannie Mae, Freddie Mac, and Ginnie Mae.

[18]The model after a series of positive innovation in securitization shocks.

Appendix

2.A Model Equations

Auxiliary Expressions:

Aux 1:

$$\tilde{\Lambda}_{t,t+1} = \tilde{\beta}\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}.$$

Aux 2:

$$\Omega^c_{t+1} := \sigma_c\Big(\gamma^c_{t+1}R_{t+1} + v^c_{t+1}\Big) + (1-\sigma_c).$$

Aux 3:

$$\Omega^b_{t+1} := (1-\sigma_b) + \sigma_b(\mu^b_{M,t+1}\phi^b_{t+1} + \bar{v}^b_{m,t+1}).$$

Saver FOCs:

$$\tilde{\lambda}_t = \frac{1}{\tilde{c}_t}, \tag{2.A.1}$$

$$\tilde{\lambda}_t = \tilde{\beta}E_t\tilde{\lambda}_{t+1}R_t, \tag{2.A.2}$$

$$\tilde{n}^{\eta}_t = \tilde{\lambda}_t\tilde{w}_t. \tag{2.A.3}$$

Borrower FOCs:

$$\hat{\lambda}_t = \frac{1}{\hat{c}_t}, \tag{2.A.4}$$

$$\frac{j_t}{\hat{h}_t} - \hat{\lambda}_t p_{h,t} + \hat{\beta}E_t\Big[\hat{\lambda}_{t+1}(1-\psi\delta)p_{h,t+1}\Big] + \hat{\mu}_t\frac{\bar{m}_t E_t p_{h,t+1}}{R_{M,t}} = 0, \tag{2.A.5}$$

$$\hat{\lambda}_t - \hat{\beta}E_t\Big[\hat{\lambda}_{t+1}(1-\psi\delta)R_{M,t}\Big] - \hat{\mu}_t = 0, \tag{2.A.6}$$

$$\hat{n}_t^{\eta} = \hat{\lambda}_t \hat{w}_t, \tag{2.A.7}$$

$$\hat{c}_t + p_{h,t}\hat{h}_t + (1-\psi\delta)R_{M,t-1}B_{t-1} = B_t + (1-\psi\delta)p_{h,t}, \hat{h}_{t-1} + \hat{w}_t\hat{n}_t \tag{2.A.8}$$

$$R_{Mt}B_t \leq \bar{m}_t E_t p_{h,t+1}\hat{h}_t. \tag{2.A.9}$$

Production:

$$Y_t = A_t \tilde{n}_t^{\alpha}(\hat{n}_t)^{1-\alpha}, \tag{2.A.10}$$

$$\tilde{w}_t = \frac{\alpha Y_t}{\tilde{n}_t}, \tag{2.A.11}$$

$$\hat{w}_t = \frac{(1-\alpha)Y_t}{\hat{n}_t}. \tag{2.A.12}$$

Commercial Bank:

Solvency Constraint (binding if $\gamma_t^c \geq 0$):

$$(1-\delta)R_{M,t}B_t^c + \bar{R}_{m,t}M_t^c - R_t D_t \geq 0. \tag{2.A.13}$$

FOC wrt on balance sheet loans:

$$(v_{M,t}^c - v_t^c) + \gamma_t^c\Big((1-\delta)R_{M,t} - R_t\Big) = 0. \tag{2.A.14}$$

FOC wrt MBS:

$$(\bar{v}_{m,t}^c - v_t^c) + \gamma_t^c\Big(\bar{R}_{m,t} - R_t\Big) = 0. \tag{2.A.15}$$

Marginal Value on on-balance sheet loans:

$$v_{Mt}^c = E_t\tilde{\Lambda}_{t,t+1}\Omega_{t+1}^c(1-\psi\delta)R_{M,t}. \tag{2.A.16}$$

Marginal value on MBS:

$$\bar{v}_{m,t}^c = E_t\tilde{\Lambda}_{t,t+1}\Omega_{t+1}^c\bar{R}_{m,t}. \tag{2.A.17}$$

Marginal Value of Deposits:

$$v_t^c = E_t \tilde{\Lambda}_{t,t+1} \Omega_{t+1}^c R_t. \tag{2.A.18}$$

Aggregate net worth:

$$N_t^c = (\sigma_c + \xi_c)\Big((1-\psi\delta)R_{M,t-1}B_{t-1}^c + \bar{R}_{m,t-1}M_{t-1}\Big) - \sigma_c R_{t-1}D_{t-1}. \tag{2.A.19}$$

Balance sheet:

$$D_t + N_t^c = B_t^c + M_t^c. \tag{2.A.20}$$

Shadow Bank:

FOC wrt loans:

$$\mu_{M,t}^b = \frac{\lambda_t^b \theta_{b,t}}{1+\lambda_t^b}. \tag{2.A.21}$$

Incentive compatibility constraint (binding if $\lambda_t^b \geq 0$):

$$\phi_t^b \leq \frac{\bar{v}_{mt}^b}{\theta_{b,t} - \mu_{M,t}^b}. \tag{2.A.22}$$

Marginal Value of Loans (Note: $\mu_{M,t}^b := v_{Mt}^b - \bar{v}_{mt}^b$):

$$\mu_{M,t}^b = E_t \tilde{\Lambda}_{t,t+1} \Omega_{t+1}^b \Big[(1-\psi\delta)R_{Mt} - \bar{R}_{m,t}\Big]. \tag{2.A.23}$$

Marginal Value of MBS:

$$\bar{v}_{mt}^b = E_t \tilde{\Lambda}_{t,t+1} \Omega_{t+1}^b \bar{R}_{mt}. \tag{2.A.24}$$

Balance Sheet Identity:

$$B_t^b = N_t^b + M_t^b. \tag{2.A.25}$$

Aggregate Shadow Bank Net Worth:

$$N_t^b = (\sigma_b + \xi_b)(1-\psi\delta)R_{M,t-1}B_{t-1}^b - \sigma_b \bar{R}_{m,t-1}M_{t-1}^b. \tag{2.A.26}$$

Shadow Bank leverage:

$$\phi_t^b = \frac{B_t^b}{N_t^b}. \tag{2.A.27}$$

Market Clearing:

$$\bar{H} = \hat{h}_t, \tag{2.A.28}$$

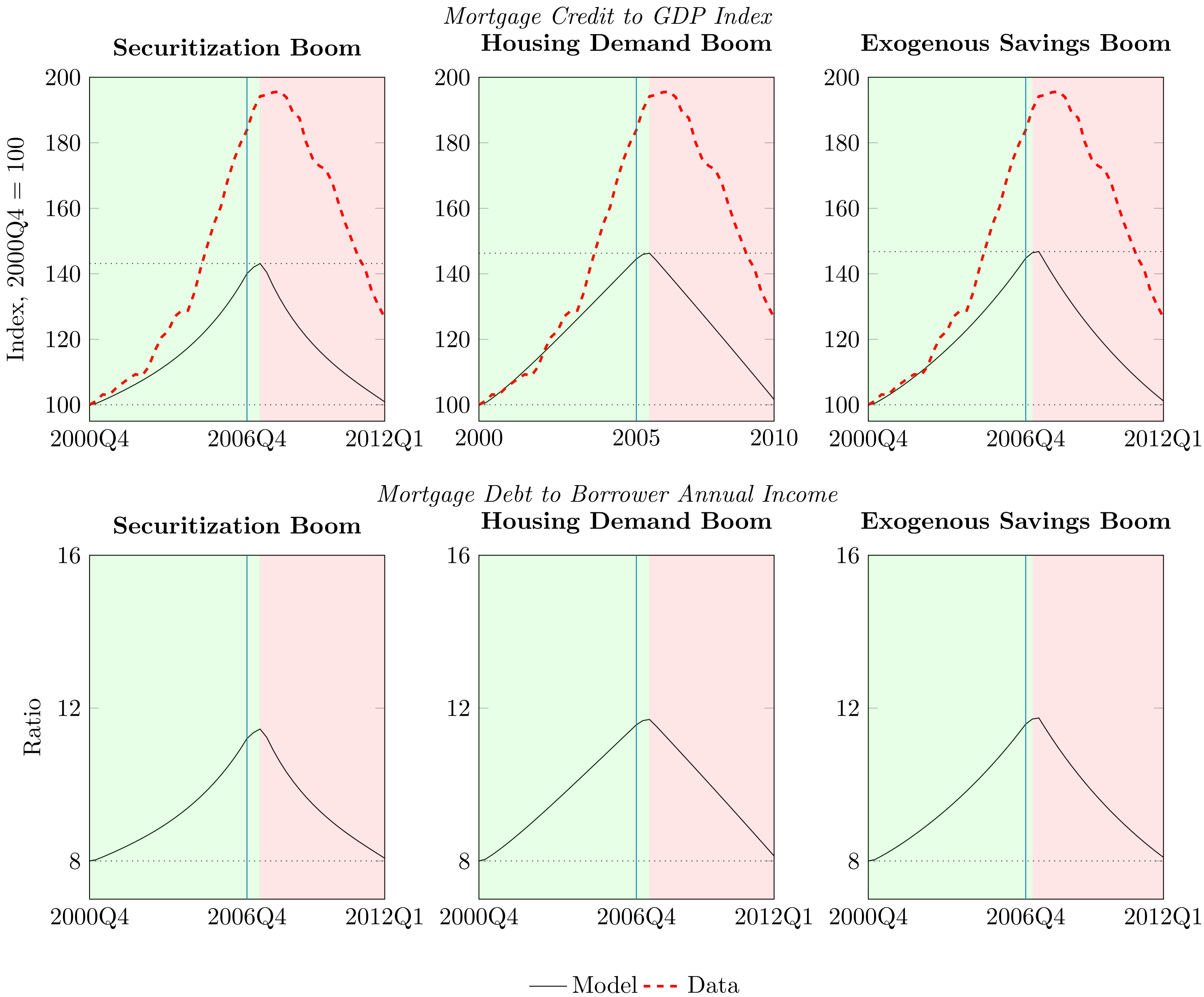
$$\tilde{c}_t + \hat{c}_t = Y_t - p_{h,t}\Big[\hat{h}_t - (1-\delta\psi_t)\hat{h}_{t-1}\Big], \tag{2.A.29}$$

$$M_t^c = M_t^b, \tag{2.A.30}$$

$$B_t = B_t^c + B_t^b. \tag{2.A.31}$$

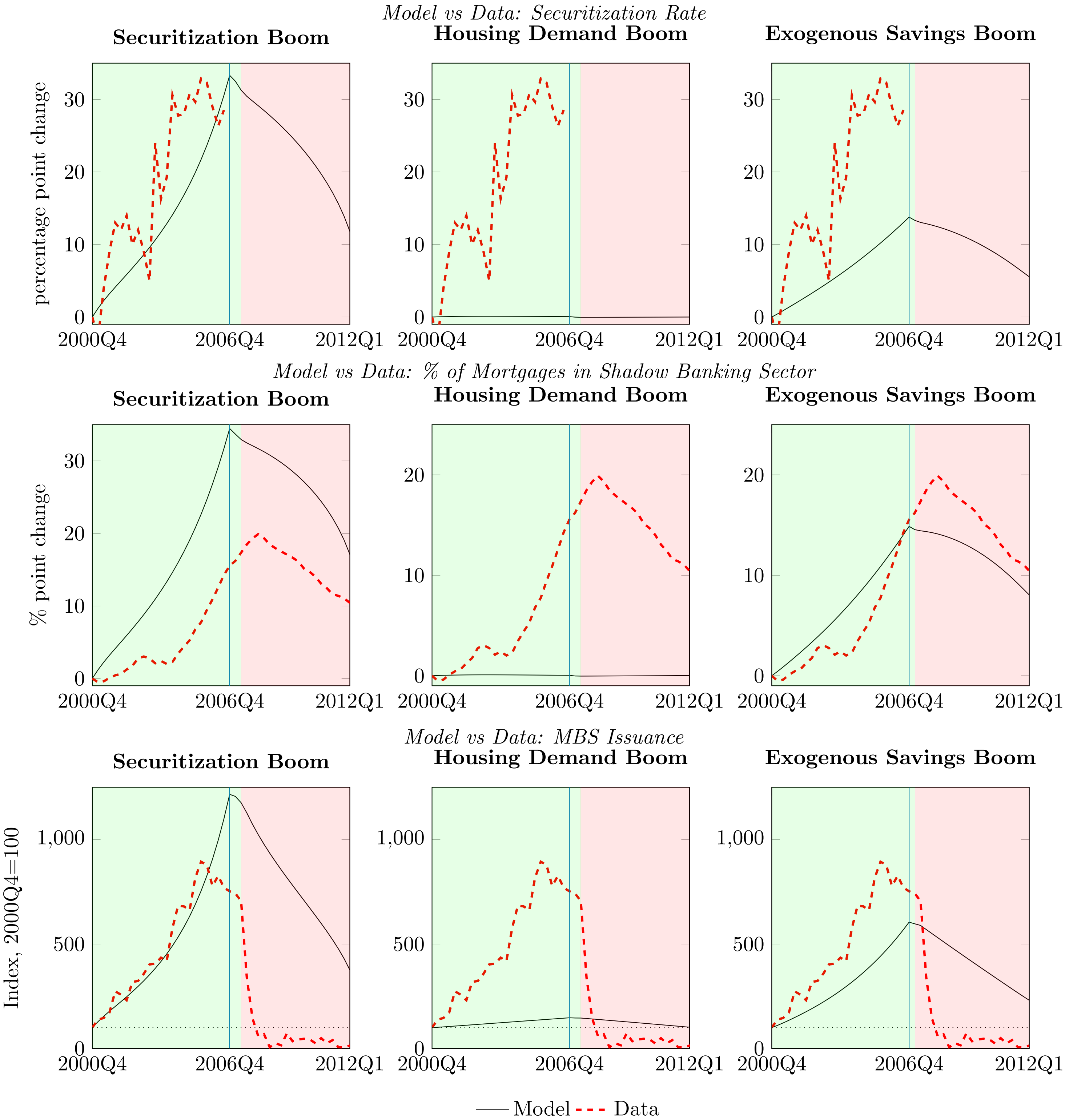
2.B Boom-Bust Simulation Additional Results

Figure 2.B.1: Candidate Booms are Indistinguishable on Credit Growth and Change in Borrower Indebtedness



The green shaded area indicates the housing collateral constraint is slack, the red shaded area indicates the housing collateral constraint binds. The vertical blue line is 2006Q4 (the peak period for real house prices according to the Case-Shiller Index).

Figure 2.B.2: Horse-Race: Only the Securitization Boom can Match Secondary Market Moments



The green shaded area indicates the housing collateral constraint is slack, the red shaded area indicates the housing collateral constraint binds. The vertical blue line is 2006Q4 (the peak period for real house prices according to the Case-Shiller Index). The securitization rate is the ratio of mortgage backed securities issued to mortgage debt originated in a given quarter, it is a flow measure of the extent to which mortgages were being originated and then quickly sold.

2.C Two Period Model

Date 1: Mortgages are originated (B_1), MBS is issued (M_1), $\theta_{b,1}$ is known.

- States: The individual and aggregate shadow bank net worth (n_1^b, N_1^b), individual and aggregate commercial bank net worth (n_1^c,N_1^c), the borrowers endowment ($\hat{\omega}_1$), and the saver's endowment ($\tilde{\omega}_1$).

- Controls: Saver consumption ($\tilde{c}_1$), borrower consumption ($\hat{c}_1$), borrower housing consumption ($\hat{h}_1$), borrower mortgage debt (b_1), individual and aggregate commercial banks mortgage holdings (b_1^c,B_1^c), individual and aggregate shadow banks pooled mortgage holdings (b_1^b,B_1^b), individual and aggregate commercial banks MBS holdings (m_1^c, M_1^c), individual and aggregate shadow banks MBS holdings (m_1^b, M_1^b).

- Prices: Deposit rate (R_1), mortgage rate (R_M), return on MBS $\bar{R}_m$, house price ($p_{h,1}$).

  Multipliers: on the housing collateral constraint ($\hat{mu}_1$), on the solvency constraint (γ_1^c), on the divertibility constraint (λ_1^b), on the savers budge constraint ($\tilde{\lambda}_1$), and on the borrowers budget constraint ($\hat{\lambda}_1$).

Date 2: Mortgages paid back or defaulted on, mortgage backed securities are paid back.

- Variables: Saver consumption ($\tilde{c}_2$), borrower consumption ($\hat{c}_2$), borrower housing consumption ($\hat{h}_2$), house price ($p_{h,2}$), the multiplier on the savers budge constraint ($\tilde{\lambda}_2$), and on the borrowers budget constraint ($\hat{\lambda}_2$).

<u>Symmetric equilibrium:</u>

- $\forall$ borrowers: $b_t = B_t, \hat{h}_t = \hat{H}_t = \bar{H}, \hat{c}_t = \hat{C}_t$.

- $\forall$ savers: $d_t = D_t, \tilde{c}_t = \tilde{C}_t$.

- $\forall$ CBanks: $b_t^c = B_t^c, m_t^c = M_t^c = M_t, n_t^c = N_t^c$.

- $\forall$ SBanks: $b_t^b = B_t^b, m_t^b = M_t^b = M_t, n_t^b = N_t^b$.

2.C.1 Household

2.C.1.1 Borrowers

$$\hat{U} \equiv \Big[u(\hat{c}_1) + j \log \hat{h}_1\Big] + \hat{\beta}\Big[u(\hat{c}_2) + j \log \hat{h}_2\Big].$$

Borrower Budget Constraints:

$$\hat{c}_1 + p_{h,1}\hat{h}_1 \leq \hat{y}_1 + \hat{\omega}_1 + B_1,$$
$$\hat{c}_2 + p_{h,2}\hat{h}_2 + (1-\psi\delta)R_{M,1}B_1 \leq \hat{y}_2 + (1-\psi\delta)p_{h,2}\hat{h}_1.$$

Housing Collateral Constraint:

$$R_{M,1}B_1 \leq \bar{m}_1 E_1 p_{h,2}\hat{h}_1, \qquad \text{(multiplier: } \hat{\mu}_1 \geq 0). \tag{2.C.1}$$

FOCs:

1. $$\frac{\partial \mathcal{L}_1}{\partial \hat{c}_1} = \hat{u}_{c,1} - \hat{\lambda}_1 = 0, \tag{2.C.2}$$

2. $$\frac{\partial \mathcal{L}_1}{\partial \hat{h}_1} = \frac{j_1}{\hat{h}_1} - \hat{\lambda}_1 p_{h,1} + \hat{\beta}E_1\Big[\hat{\lambda}_2(1-\psi\delta)p_{h,2}\Big] + \hat{\mu}_1\bar{m}_1E_1p_{h,2} = 0, \tag{2.C.3}$$

3. $$\frac{\partial \mathcal{L}_1}{\partial B_1} = \hat{\lambda}_1 - \hat{\beta}E_1\Big[\hat{\lambda}_2(1-\psi\delta)R_{M,1}\Big] - \hat{\mu}_1 R_{M,1} = 0, \tag{2.C.4}$$

4. $$\frac{\partial \mathcal{L}_2}{\partial \hat{c}_2} = \hat{u}_{c,2} - \hat{\lambda}_2 = 0, \tag{2.C.5}$$

5. $$\frac{\partial \mathcal{L}_2}{\partial \hat{h}_2} = \frac{j_2}{\hat{h}_2} - \hat{\lambda}_2 p_{h,2} = 0. \tag{2.C.6}$$

2.C.1.2 Savers ($\tilde{\beta} \geq \hat{\beta}$)

$$\tilde{U} \equiv u(\tilde{c}_1) + \tilde{\beta}^2 u(\tilde{c}_2).$$

Saver Budget Constraints:

$$\tilde{c}_1 + D_1 \leq \tilde{y}_1 + \tilde{\omega}_1,$$

$$\tilde{c}_2 \leq \tilde{y}_2 + R_1 D_1 + \Pi_2,$$

where $\Pi \equiv N_2^c + N_2^b$.

FOCs:

1.
$$\frac{\partial \mathcal{L}_1}{\partial \tilde{c}_1} = \tilde{u}_{c,1} - \tilde{\lambda}_1 = 0, \tag{2.C.7}$$

2.
$$\frac{\partial \mathcal{L}_1}{\partial D_1} = -\tilde{\lambda}_1 + \tilde{\beta} E_1 \left[\tilde{\lambda}_2 R_1 \right] = 0, \tag{2.C.8}$$

3.
$$\frac{\partial \mathcal{L}_2}{\partial \tilde{c}_2} = \tilde{u}_{c,2} - \tilde{\lambda}_2 = 0. \tag{2.C.9}$$

Linear saver utility implies:

$$R_1 = \frac{1}{\tilde{\beta}}, \tag{2.C.10}$$

and that their stochastic discount factor is:

$$\tilde{\Lambda}_{1,2} \equiv \tilde{\beta} \frac{\tilde{\lambda}_2}{\tilde{\lambda}_1} = \tilde{\beta}. \tag{2.C.11}$$

2.C.2 Shadow Banks

Net worth:

$$n_1^b \equiv \text{given},$$

$$n_2^b \equiv (1 - \delta\psi) R_{M,1} b_1^b - \bar{R}_{m,1} m_1^b. \tag{2.C.12}$$

Period 1 Problem:

$$
\begin{aligned}
V_1^b &= E_1 \tilde{\Lambda}_{1,2} n_2^b \\
&= E_1 \tilde{\beta} \Big[(1-\delta\psi) R_{M,1} b_1^b - \bar{R}_{m,1} m_1^b \Big] \\
&= E_1 \tilde{\beta} \Big[\Big((1-\delta\psi) R_{M,1} - \bar{R}_{m,1} \Big) b_1^b + \bar{R}_{m,1} n_1^b \Big]. \qquad (2.C.13)
\end{aligned}
$$

The problem is to max V_1^b subject to the ICC:

$$
V_1^b \geq \theta_{b,1} b_1^b, \qquad (2.C.14)
$$

$$
\mathcal{L}_1 = E_1 \Bigg\{ V_1^b + \lambda_1^b \Big[V_1^b - \theta_{b,1} b_1^b \Big] \Bigg\}.
$$

FOCs:

$$
\frac{\partial \mathcal{L}_1}{\partial b_1^b} = E_1 \Bigg\{ \tilde{\beta} \Big[(1-\delta\psi) R_{M,1} - \bar{R}_{m,1} \Big] (1 + \lambda_1^b) - \theta_{b,1} \lambda_1^b \Bigg\} = 0, \qquad (2.C.15)
$$

$$
\frac{\partial \mathcal{L}_1}{\partial \lambda_1^b} = E_1 \Bigg\{ V_1^b - \theta_{b,1} b_1^b \Bigg\}. \qquad (2.C.16)
$$

Aggregate Net Worth:

$$
N_2^b = (1-\delta\psi) R_{M,1} B_1^b - \bar{R}_{m,1} M_1^b. \qquad (2.C.17)
$$

Note: plugging in the value function guess (2.C.13) into the divertibility constraint (2.C.14) implies at the symmetric equilibrium:

$$
\tilde{\beta} \Big[\Big((1-\delta\psi) R_{M,1} - \bar{R}_{m,1} \Big) b_1^b + \bar{R}_{m,1} n_1^b \Big] \geq \theta_{b,1} B_1^b. \qquad (2.C.18)
$$

2.C.3 Commercial Banks

$$
n_2^c = \begin{cases} R_{M,1} b_1^c + \bar{R}_{m,1} m_1^c - R_1 d_1, & \text{if "good" (non-defaulter) island,} \\ (1-\delta) R_{M,1} b_1^c + \bar{R}_{m,1} m_1^c - R_1 d_1, & \text{if "bad" (defaulter) island.} \end{cases} \qquad (2.C.19)
$$

Aggregate Net Worth:

$$
N_2^c = (1-\delta\psi) R_{M,1} B_1^c + \bar{R}_{m,1} M_1^c - R_1 D_1. \qquad (2.C.20)
$$

Period 1 Problem:

$$\max_{b_1^c, d_1, m_1^c} V_1^c = E_1 \tilde{\Lambda}_{1,2} \Bigg\{ (1-\psi) n_2^{c,good} + \psi n_2^{c,bad} \Bigg\} \tag{2.C.21}$$

$$= E_1 \tilde{\beta} \Bigg\{ (1-\psi) n_2^{c,good} + \psi n_2^{c,bad} \Bigg\}, \tag{2.C.22}$$

subject to their balance sheet identity:

$$b_1^c + m_1^c = n_1^c + d_1, \tag{2.C.23}$$

and the solvency constraint:

$$(1-\delta) R_{M,1} b_1^c + \bar{R}_{m,1} m_1^c \geq R_1 d_1, \qquad (\gamma_1^c \geq 0). \tag{2.C.24}$$

$$\mathcal{L}_1 = E_1 \Bigg\{ V_1^c + \gamma_1^c \Big[((1-\delta) R_{M,1} - R_1) b_1^c + (\bar{R}_{m,1} - R_1) m_1^c + R_1 n_1^c \Big] \Bigg\}.$$

FOCs:

$$\frac{\partial \mathcal{L}_1}{\partial b_1^c} = E_1 \Bigg\{ \tilde{\beta} \big[(1-\psi\delta) R_{M,1} - R_1 \big] + \gamma_1^c ((1-\delta) R_{M,1} - R_1) \Bigg\} = 0, \tag{2.C.25}$$

$$\frac{\partial \mathcal{L}_1}{\partial m_1^c} = E_1 \Bigg\{ \tilde{\beta} \big[\bar{R}_{m,1} - R_1 \big] + \gamma_1^c \big[\bar{R}_{m,1} - R_1 \big] \Bigg\} = 0, \tag{2.C.26}$$

$$\frac{\partial \mathcal{L}_0}{\partial \gamma_0^c} = E_1 \Bigg\{ \Big[((1-\delta) R_{M,1} - R_1) b_1^c + (\bar{R}_{m,1} - R_1) m_1^c + R_1 n_1^c \Big] \Bigg\} = 0. \tag{2.C.27}$$

2.C.4 Resource Constraints

$$\tilde{c}_1 + \hat{c}_1 = \tilde{y}_1 + \hat{y}_1 + \tilde{\omega}_1 + \hat{\omega}_1 + N_1^c + N_1^b - p_{h,1} \hat{h}_1,$$

$$\tilde{c}_2 + \hat{c}_2 = \tilde{y}_2 + \hat{y}_2 - p_{h,2} \big[\hat{h}_2 - (1-\psi\delta) \hat{h}_1 \big].$$

Housing Supply:

$$\hat{h}_0 = \bar{H}, \tag{2.C.28}$$

$$\hat{h}_1 = \bar{H}, \tag{2.C.29}$$

$$\hat{h}_2 = \bar{H}. \tag{2.C.30}$$

(2.C.31)

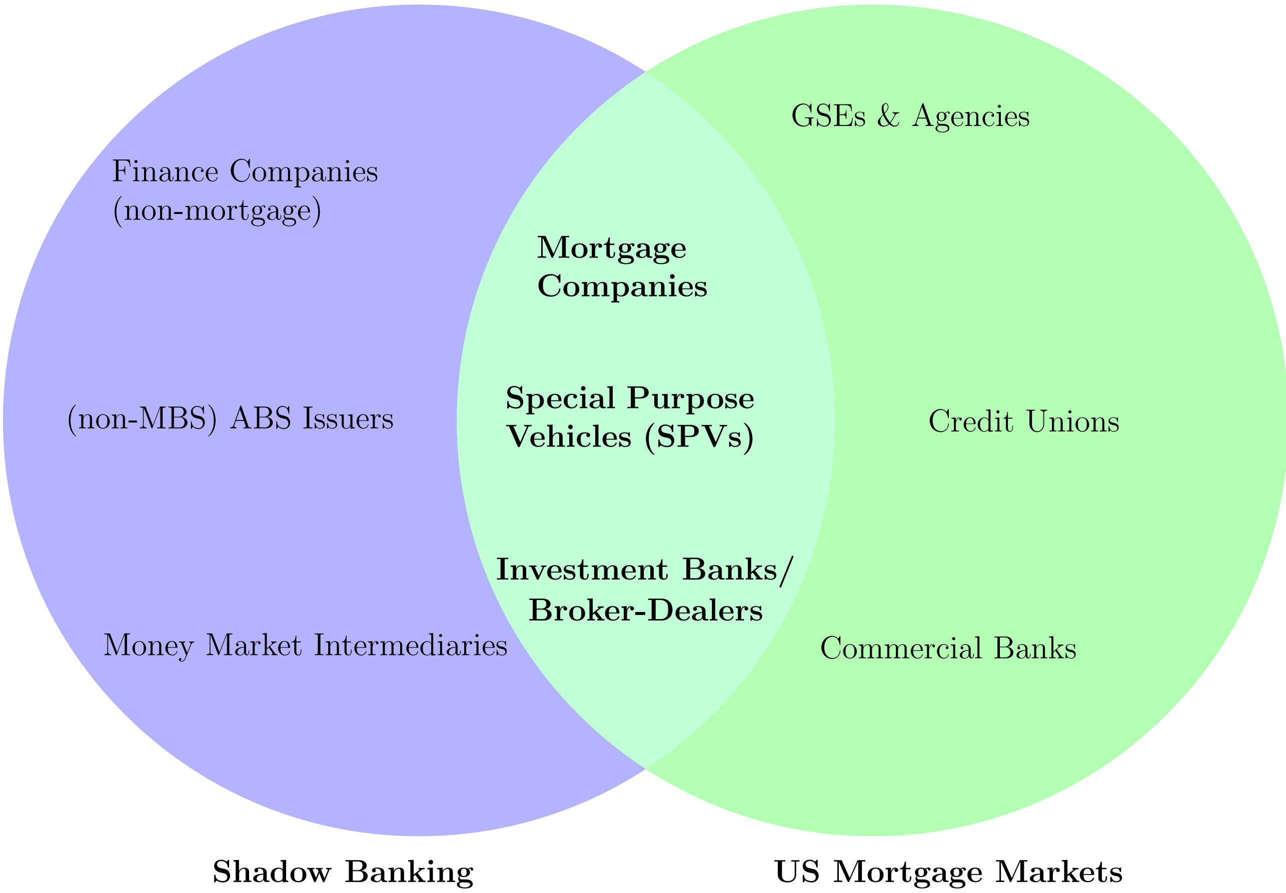
Debt Market:

$$B_1 = B_1^c + B_1^b. \tag{2.C.32}$$

(2.C.33)

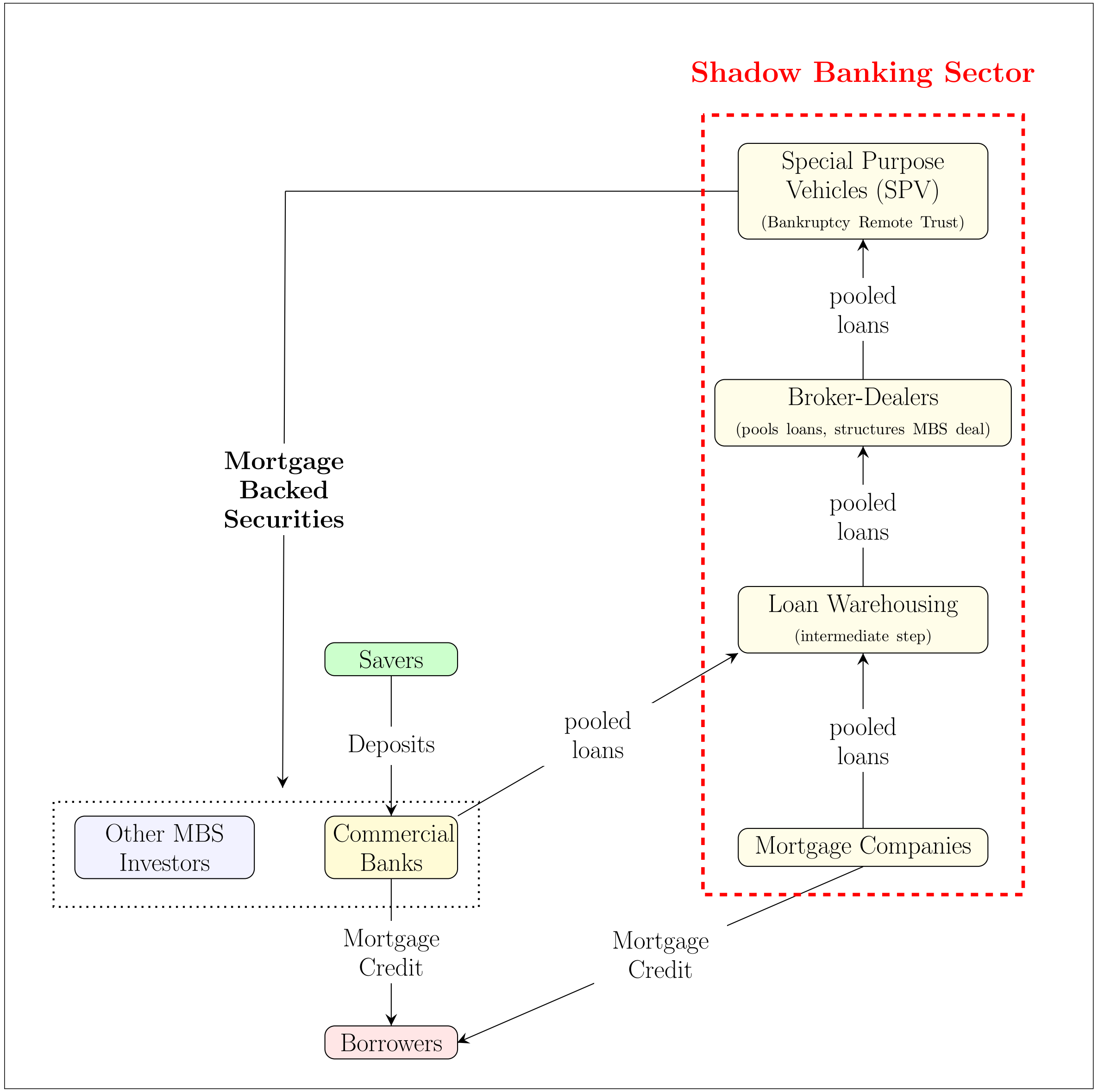
2.D Institutional Details Appendix

Figure 2.D.1: Overlap Between Shadow Banking and US Mortgage Markets



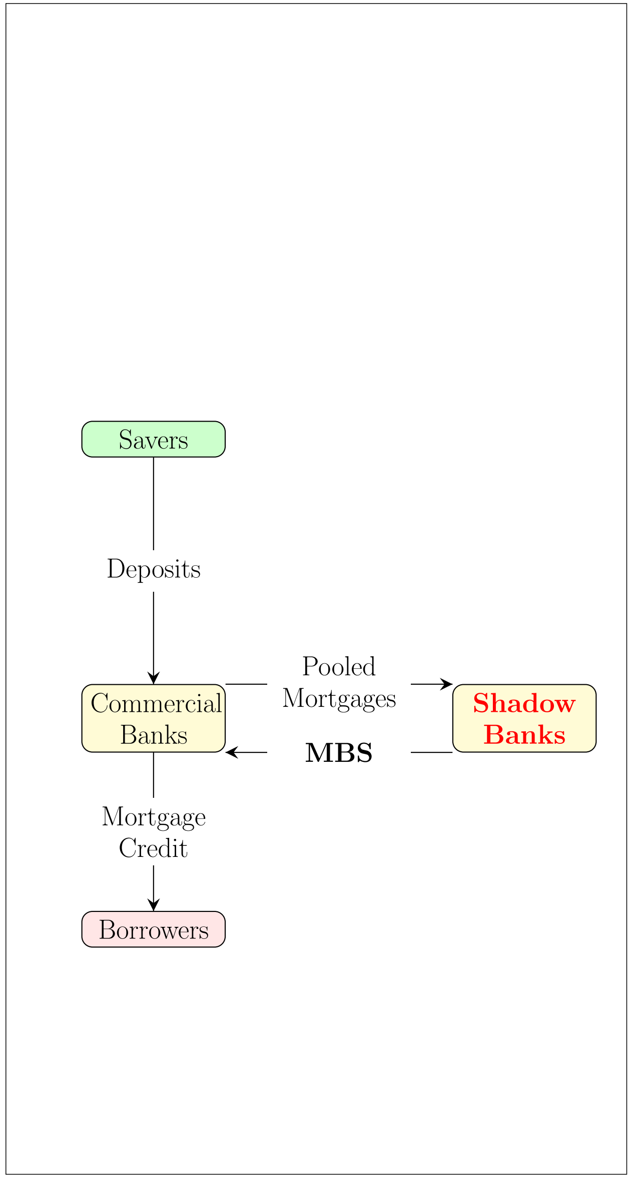
Note: this is a simplified characterization of the summary of the shadow banking sector presented by Pozsar et al. (2012).

Figure 2.D.2: Mortgage Securitization Process in Reality vs Model Simplification



(a) Shadow Banking Sector in Reality

This based on the detailed description of the securitization process in Ashcraft and Schuermann (2008)



(b) Model Simplification

Chapter 3

Overborrowing & Shadow Banking

3.1 Introduction

High default rate events generate distress in the financial sector. This point is obviously illustrated by the U.S. housing market experience during 2007-08. Though the full financial fallout of the Coronavirus crisis remains to be seen, the large rise in unemployment raises the possibility of widespread defaults. Losses that accumulate on the balance sheets of unregulated financial institutions ("shadow banks") can ultimately hit the balance sheets of commercial banks, either via direct ownership of off-balance sheet entities or via systemic factors.

This paper explores how the existence of unregulated financial institutions ("shadow banks") impacts credit conditions during "good times" and following high default rate events ("crises"). Specifically I ask: is the competitive equilibrium level of borrowing socially optimal when credit can leak from the regulated commercial banking sector into the unregulated shadow banking sector? And if no, what is the optimal policy intervention? I find that inefficient overborrowing arises in the competitive equilibrium. Individual borrowers do not internalize the impact that their borrowing has in good times on the aggregate credit conditions during a crisis. A constrained-efficient social planner wishes to lower borrowing in good times to minimize the deterioration in credit conditions during a crisis.

The constrained-efficient social planner's allocation can be decentralized without additional policy intervention using a direct credit limit placed on borrowers. This is reminiscent of the conforming loan limit in the US. If lump sum transfers are available the policymaker can alternatively use a macro-prudential debt tax to achieve optimality. Macro-prudential regulation of commercial banks is only optimal if the policymaker can use lump-sum taxes on commercial bank profits to make lump-sum transfers to borrowers. Furthermore the policy is pro-cyclical - it is optimal to loosen the constraint on commercial banks during good times and tighten it following a high default event.

The model in this paper is a three-period, modified, version of the model in **?**. There is a two-layered financial sector made up of commercial banks and shadow banks. In the initial period the state of the world is "good", i.e. commercial banks are well capitalized and shadow banks can easily raise funds. A shadow bank is any entity that provides credit to borrowers and issues liabilities but does not benefit from deposit insurance and is not subject to bank capital regulations. Tightening regulatory requirements on commercial banks has no effect on the cost or available quantity of credit during "good times". This is because shadow banks ultimately set the price of credit and (up to an aggregate limit) can supply the borrower with any amount of credit. In fact tightening regulatory requirements on commercial banks during "good times" is sub-optimal because it depresses commercial bank equity in a crisis.

A "crisis" arises when the aggregate default rate is sufficiently high such that in aggregate the shadow banking sector cannot meet its liabilities. In this event the commercial banking sector takes shadow banking liabilities back onto their balance sheet. This results in a loss to commercial bank equity and a collapse of the shadow banking sector (they are no longer able to issue liabilities that are perceived as risk free). The commercial bank micro-prudential regulation (which the planner cannot evade) is binding and impacts aggregate credit conditions, because credit can no longer leak into the shadow banking sector. This means that the decrease in commercial bank equity is socially costly. The supply of credit shrinks and credit becomes more expensive. The loss to commercial bank equity is worse the larger the aggregate borrowing in "good times" is. This effect is not internalized by individual borrowers, meaning that the constrained-efficient social planner has a motive to limit the accumulation of debt during good times.

In the model commercial banks ultimately own shadow banks – commercial banks honor the liabilities of shadow banks when shadow banks are unable to do so. This assumption is crucial to generating the overborrowing result, and motivated by the lack of true risk transfer that characterized the experience in the US financial system during the 2007-09 financial crisis. For example, Acharya, Schnabl and Suarez (2013) show that the losses asset-backed commercial paper (ABCP) conduits experienced were primarily born by the sponsoring commercial banks, not the investors in ABCP. During the financial crisis Merrill Lynch experienced losses when non-bank mortgage originators to which they had extended lines of credit failed (Financial Crisis Inquiry Commission, 2011, chap. 14, p. 257). Commercial banks including Merrill Lynch and Citigroup experienced losses on the super-senior tranches of the collateralized debt obligations (CDOs) they retained (Financial Crisis Inquiry Commission, 2011, chap. 14). Looking forward there is emerging concern about the collateralized loan obligation (CLO) market, the class of assets that securitizes corporate loans. Liu, Demarco and Schmidt-Eisenlohr (2020) estimate that in 2018 depository institutions held 18% of the domestic holdings of Cayman-issued U.S. CLO securities. The overall picture is clear: there are many ways in which losses in the shadow banking sector have the potential to accumulate on commercial banks balance sheets.

Much of the existing literature finds the existence of optimal policy interventions in pecuniary externalities arising from collateral constraints. Bianchi (2011) finds that in a partial equilibrium open economy context overborrowing arises in competitive equilibrium when borrowing must be collateralized by tradable and non-tradable output. Bianchi and Mendoza (2018) find that in a similar setup the optimal policy under commitment is time inconsistent, but time consistent optimal policies exist. Jeanne and Korinek (2019) find that credit booms and busts are amplified by a borrowing constraint linking asset prices and debt accumulation. Optimal Pigouvian taxes exist and are countercyclical.

Recent experience in the US housing market suggests that financial conditions available to borrowers, including the collateral constraints borrowers face, can be shifted themselves by changes in the unregulated portions of the financial sector. For example in **?** I show that the dynamics of residential mortgage credit expansion in the U.S. during

2000-2006 are matched by innovations in the securitization process for mortgage credit that took place in the shadow banking sector. This motivates my focus here on the role of the financial sector in creating credit conditions that may be sub-optimal.

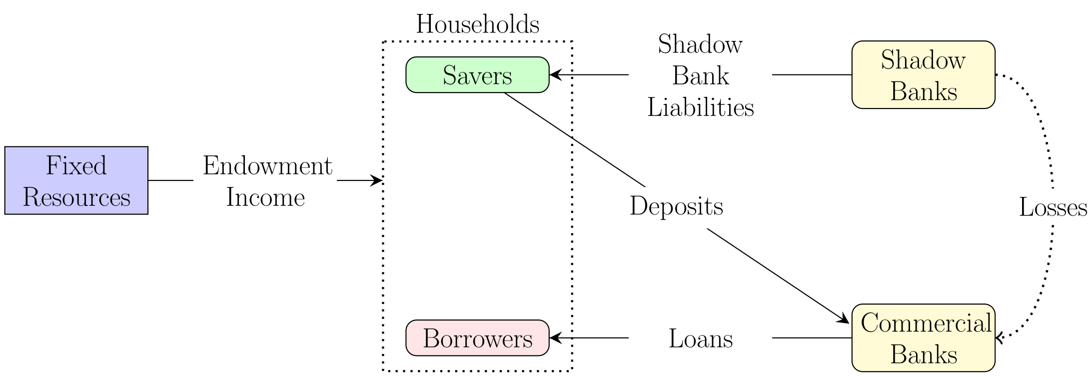
The paper proceeds as follows. Section 3.2 presents the model. Section 3.3 presents the constrained efficient social planner's problem. Section 3.4 presents optimal policies that decentralize the planner's problem. Section 3.5 Concludes.

3.2 The Model

3.2.1 Overview

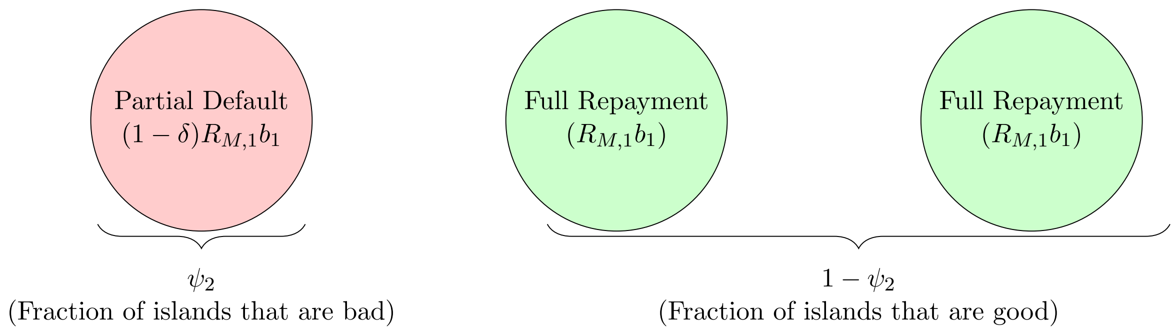
This is a simplified and modified three-period version of the model in chapter 2. There is a two-layered financial sector comprised of loan issuing commercial banks and loan securitizing shadow banks. Borrowing is sub-optimally high in period 1 because individual borrowers do not internalize the negative impacts of an additional unit of borrowing on the aggregate credit market in period 2. There is a continuum of ex-ante identical islands of measure 1. Each island contains borrower households, saver households, and a commercial bank. Households can only interact with their local (island specific) commercial bank, as commercial banks have a specific expertise in lending to borrowers on their island. The island structure reflects the geographic segmentation of certain US credit markets, for example the residential mortgage market. Shadow banks exist off island and can purchase a diversified pool of loans from across all islands.

Figure 3.2.1: The Model



Note: Commercial banks ultimately own shadow banks. This means that when shadow banks cannot honor their liabilities commercial banks take on those losses.

Figure 3.2.2: Risky Lending



There are three discrete time periods: $t = 1, 2, 3$. At the start of period 1 commercial banks originate loans (b_1), take deposits (d_1) and choose how many of their own loans to retain (b_1^c). Shadow banks choose the quantity of pooled loans to buy (b_1^b) and the quantity of "risk-free" liabilities to issue (m_1) and sell to saver households. At the start of period 2 a fraction of islands (ψ_2) get hit with a default shock, similar to Gertler and Kiyotaki (2010)'s island-specific investment opportunity shock. On a good island borrowers repay in full. On bad islands a fraction (δ) of borrowers fully default. Commercial banks across all islands repay deposits and exit if they are on bad islands. If the aggregate default ($\delta\psi_2$) is sufficiently high shadow banks will not be able to repay the holders of their liabilities. Commercial banks then take these liabilities onto their balance sheet and make the savers whole. In this case shadow banks permanently close down.

At the end of period 1 commercial banks travel across islands to equalize credit conditions across islands going into period 2[1]. Borrowers who default are permanently unable to take out loans. Figure 3.2.2 illustrates the island structure of default, $R_{M,1}$ is the interest rate on period 1 lending.

After households and commercial banks have traveled across islands in period 2 the process of loan origination repeats. Commercial banks originate loans (b_2), take deposits (d_2) and choose how many of their own loans to retain (b_2^c). If shadow banks still operate they choose the quantity of loans to buy (b_2^b) and the quantity of liabilities to issue (m_2). In period 3 borrowers repay in full, and financial liabilities are repaid. Financial sector

[1]Otherwise, because of commercial bank exit, the default hit islands would have no credit provision.

capital is then paid to savers as dividends.

The overborrowing result is driven by borrowers taking out loans in period 2. If a high default event occurs the financial sector is constrained in aggregate by commercial banking regulation. More borrowing in period 1 means that the commercial bank's balance sheet is in worse condition in period 2, and therefore borrowers face tighter credit conditions. Individual borrowers do not internalize the impact an additional unit of their period 1 borrowing has on aggregate credit conditions in period 2 if a crisis arises, driving borrowing above the socially optimal level in period 1.

3.2.2 Households

There are two types of households. Savers are the ultimate source of funding for loans. Borrowers are individuals whose endowment income increases over time, so are motivated to borrow to smooth consumption. Once borrowers default they are excluded from accessing financial markets.

3.2.2.1 Savers' Problem

Across all islands there is a continuum of measure 1 of identical savers. Savers have linear utility, and receive an endowment income in periods 1 and 2. Savers can hold deposits (d) or shadow bank liabilities (m). The saver considers both deposits and shadow bank liabilities to be risk-free[2]. As a result savers consider these assets perfect substitutes. The savers discount factor is one, so the risk-free rate R is one.

An individual saver's utility function is:

$$E_1\tilde{U} \equiv E_1\left\{\tilde{c}_1 + \tilde{c}_2 + \tilde{c}_3\right\}. \tag{3.2.1}$$

[2]In the model deposits are risk-free because commercial banks are regulated so that they are able to repay deposits in the event that they receive the island specific default shock. Shadow bank liabilities are risk-free from the point of view of savers because they receive implicit guarantees that the owner commercial bank will honor the shadow bank's liability in the event of shadow bank collapse. Section 3.2.3 explains the institutional background for these assumptions in more detail.

The individual saver's budget constraints are:

$$\tilde{c}_1 \leq \tilde{y}_1 - (m_1 + d_1), \tag{3.2.2}$$

$$\tilde{c}_2 \leq R(m_1 + d_1) - (m_2 + d_2) + \tilde{y}_2, \tag{3.2.3}$$

$$\tilde{c}_3 \leq R(m_2 + d_2) + \pi_3. \tag{3.2.4}$$

Saver specific notation is denoted with tildes: $\tilde{c}_t$ denotes the saver's consumption of non-durable goods and $\tilde{y}_t$ is saver endowment income. π_3 is the final period dividend received from commercial and shadow banks, as savers are the ultimate owners of financial institutions.

In the first two periods, $t = 1, 2$ the saver chooses deposits (d_t) and shadow bank liabilities (m_t) to maximize their utility. In the final period, $t = 3$, the saver simply consumes the income they receive from their asset holdings.

3.2.2.2 Borrowers' Problem

There is a continuum of ex-ante identical borrowers, of measure 1. The individual borrower's utility function is concave. Borrowers take out loans and consume. Their endowment income is increasing over time, so they are motivated to borrow to smooth consumption. A fraction $(\delta\psi_2)$ of borrowers in period 2 default fully. The borrowers that default are locked out of the loan market, so only consume their endowment income in periods $t = 2, 3$.

An individual borrower's utility function is:

$$E_1\hat{U} \equiv E_1\big\{\hat{u}(\hat{c}_1) + (1 - \delta\psi_2)\big[\hat{u}(\hat{c}_2) + \hat{u}(\hat{c}_3)\big] + \delta\psi_2\big[\hat{u}(\hat{y}_2) + \hat{u}(\hat{y}_3)\big]\big\}. \tag{3.2.5}$$

The individual borrower's budget constraints are:

$$\hat{c}_1 \leq b_1 \qquad\qquad , \tag{3.2.6}$$

$$\hat{c}_2 \leq \hat{y}_2 + b_2 - R_{M,1}b_1, \tag{3.2.7}$$

$$\hat{c}_3 \leq \hat{y}_3 - R_{M,2}b_2. \tag{3.2.8}$$

Borrower specific notation is denoted with hats: $\hat{c}_t$ denotes consumption of non-durable goods, $\hat{y}_t$ is borrower endowment income. b_t is the loans borrowers take out, and $R_{M,t}$ is the lending rate.

In periods 1 and 2, $t = 1, 2$, borrowers choose consumption ($\hat{c}_t$) and borrowing (b_t) to maximize expected utility subject to their budget constraints. In period 2 only borrowers who have not defaulted continue to make inter-temporal optimizing decisions.

3.2.3 Financial Sector

The financial sector consists of commercial banks and shadow banks. Both commercial and shadow banks issue liabilities that savers view as risk-free. Savers believe commercial bank deposits to be risk-free because commercial banks face a solvency constraint - they must construct their balance sheet so they are always able to repay deposits in full (especially when their island is hit with a default shock). This captures in reduced form that commercial banks face both capital regulation and deposit insurance. Savers believe shadow bank liabilities to be risk-free because they recognize that the commercial banking sector is attached to the shadow banking sector and will step in to honor the liabilities issued by shadow banks if shadow banks cannot.

The assumption that all shadow banking liabilities were viewed as fully risk-free prior to 2008 is a simplification, but it reflects the reality pre-2008 that the sponsoring institutions (often commercial banks) of Special Purpose Vehicles (SPVs, i.e. shadow banks) often supplied credit enhancements to the holders of shadow banking liabilities (eg Mortgage Backed Securities, Asset Backed Commercial paper, etc). Credit enhancements are essentially promises by the entity or sponsor of the entity issuing a liability to take on a specified portion of the default risk associated with the return on the underlying assets the shadow bank liability is based on. Gorton and Souleles (2007) discuss credit enhancements in the context of credit card securitization, and mention cases where the sponsor implicitly (not contractually) supported the securitized asset. Ashcraft and Schuermann (2008) discuss credit enhancements used in subprime mortgage backed securitization. Acharya et al. (2013) emphasize that a subset of asset-backed commercial paper (ABCP) credit enhancements were structured so that the sponsoring commercial bank did not face additional regulatory capital requirements but still bore default risk from the underlying assets. In the model savers anticipate commercial bank intervention to support shadow bank liabilities, but commercial banks are not penalized for this support in the regulatory constraint (the solvency constraint). This captures the implicit nature of the credit enhancement.

There is a continuum of ex-ante identical islands, a fraction of which experience default in period 2. Commercial banks locate themselves on an island for the duration of each period. They have an expertise in lending to borrowers on their island. Shadow banks have no attachment to specific islands. Shadow banks can purchase loans from across all islands. Commercial banks hold their island's idiosyncratic default risk, whereas shadow banks hold the aggregate default risk. Commercial banks face a regulatory constraint, shadow banks by definition exist beyond the reach of regulation. Commercial banks ultimately own shadow banks, but this is not reflected in the regulatory constraint. A "crisis" occurs when losses on the shadow banks' balance sheets are too big for shadow banks to repay their liabilities. In this event commercial banks will take shadow bank liabilities back onto the commercial banks' balance sheet. By taking the losses of shadow banks onto their balance sheets commercial banks suffer losses to their own net worth. This has a negative impact on overall credit conditions. Figure (3.2.3) provides an overview of the balance sheets of financial intermediaries.

3.2.3.1 Shadow Banking Sector

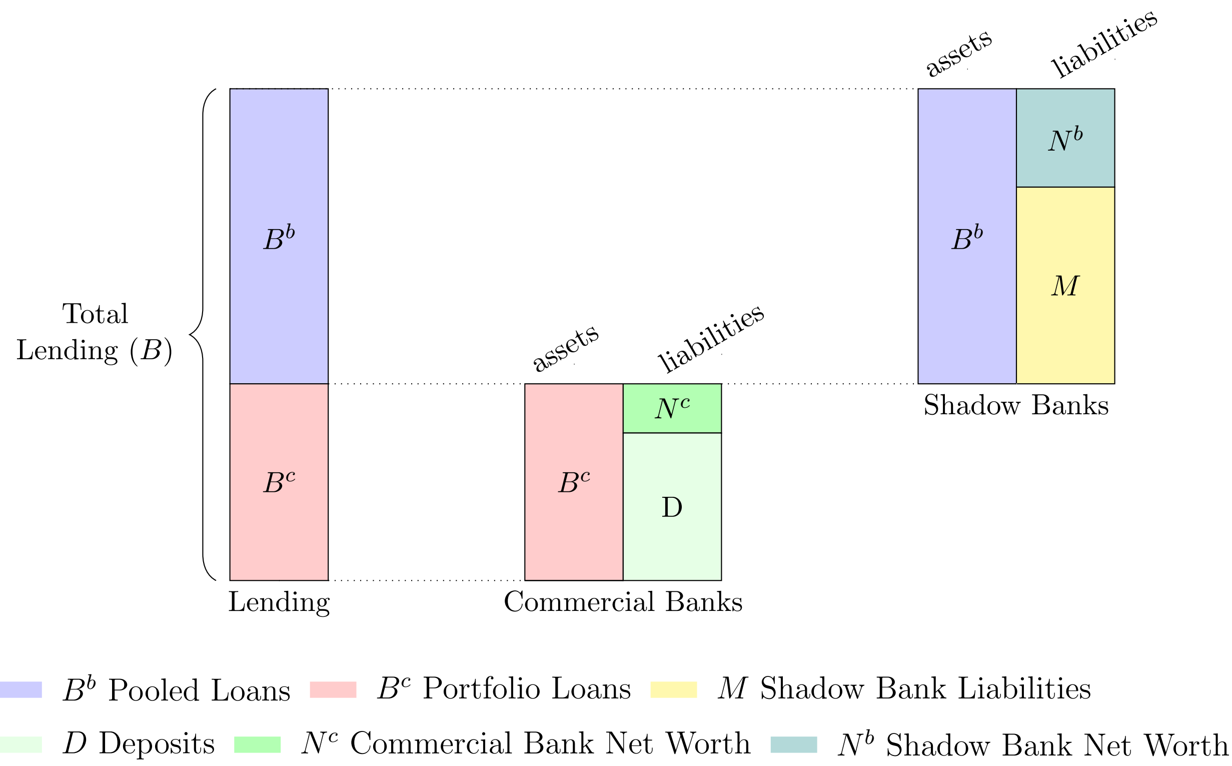
Shadow banks in this model are the special purpose vehicles (SPVs), owned by investment and commercial banks, that bought loans and packaged them into asset backed securities - reflecting the reality during the lead up to the Financial Crisis. They operate independently to commercial banks, but ultimately commercial banks own shadow bank losses if the shadow bank is unable to repay liabilities and collapses.

Shadow banks exist off-island. They buy a perfectly diversified set of loans from every island and issue liabilities which pay a promised risk-free rate to savers.

Period 2 Problem

Shadow banks only continue to operate in period 2 if they can repay the holders of the

Figure 3.2.3: Financial Sector Balance Sheet



Note: Portfolio Loans (B^c) are the loans originated and then retained by an individual commercial bank. These loans are subject to island specific default risk. In contrast the Pooled Loans (B^b) are the loans purchased by shadow banks from across all islands. These loans are diversified, so have only aggregate risk not island specific risk.

liabilities they issue in full. That is if the following condition holds:

$$(1 - \delta\psi_2)R_{M,1}b_1^b \geq Rm_1. \tag{3.2.9}$$

So in period 2 if this condition is met (i.e. if $n_2^b \geq 0$) shadow banks continue to operate. Again they purchase diversified loans (b_2^b) and issue liabilities (m_2) that pay the risk-free rate (R) to maximize the expected value of the final dividend payment made to savers ($E_2 n_3^b$) subject to their balance sheet identity (3.2.11). Maximizing final period dividends is equivalent to the standard assumption of maximizing continuation value in an infinite horizon model. See for example Gertler and Kiyotaki (2010). Their problem is:

$$\max_{b_2^b, m_2} \; E_2 n_3^b, \tag{3.2.10}$$

subject to:

$$b_2^b = m_2 + n_2^b. \tag{3.2.11}$$

An individual shadow bank's net worth evolves according to:

$$n_3^b = \underbrace{R_{M,2} b_2^b}_{\text{return on diversified loans}} - Rm_2. \tag{3.2.12}$$

Period 1 Problem

In period 1 shadow banks purchase loans from commercial banks across all islands (b_1^b) and liabilities (m_1) that promise a risk-free return (R) to maximize the expected value of the final dividend payment made to savers ($E_1 n_3^b$) subject to their balance sheet identity (3.2.14). Their problem is:

$$\max_{b_1^b, m_1} E_1 n_3^b, \tag{3.2.13}$$

subject to:

$$b_1^b = m_1 + n_1^b. \tag{3.2.14}$$

An individual shadow bank's net worth evolves according to:

$$n_2^b = \underbrace{(1 - \delta\psi_2) R_{M,1} b_1^b}_{\text{return on diversified loans}} - Rm_1. \tag{3.2.15}$$

3.2.3.2 Commercial Banking Sector

Commercial banks issue deposits and make loans. Commercial banks retain a portion of those loans on their balance sheet as portfolio loans and sell the rest to shadow banks. Commercial banks are constrained by the savers' willingness to make deposits. Savers will only make an additional deposit in their local commercial bank if they expect to be repaid in full in the event of being on a "bad" (default hit) island. This "solvency constraint" requirement limits the quantity of portfolio loans the commercial bank can hold. This is the regulatory constraint[3].

There exists a continuum of commercial banks indexed by $c \in [0, 1]$. In periods 1 and 2 commercial banks choose a specific island on which to locate for the purposes of lending and deposit taking. Meaning that ex-ante islands have identical credit markets. In period 2 the island's default status is realized. Commercial banks on bad (defaulter)

[3] A similar constraint would result from the combination of deposit insurance and leverage constraint, as the solvency constraint can be re-written as a leverage constraint.

islands are not fully repaid what is owed on the loans they issued, but they still repay deposits. Commercial banks on good (non-defaulter) islands receive the full amount owed on loans made and repay depositors. After repaying deposits commercial banks on bad islands exit. Any remaining equity of these commercial banks is transferred (lump-sum) to continuing commercial banks. After default and exit in period 2 continuing commercial banks travel across islands to equalize credit conditions before loans are issued.

Period 2 Problem

In period 2 the commercial bank's problem is to choose deposits (d_2) and on balance sheet loans (b_2^c) to maximize the expected value of the final dividend payment made to savers ($E_2 n_3^c$) subject to their balance sheet identity (3.2.17) and the solvency constraint (3.2.18). Their problem is:

$$\max_{b_2^c, d_2} \; E_2 n_3^c, \tag{3.2.16}$$

subject to:

$$b_2^c = n_2^c + d_2, \tag{3.2.17}$$

$$(1-\delta) R_{M,2} b_2^c \geq R d_2. \tag{3.2.18}$$

Net worth is realized as follows on good and bad islands[4].

$$n_3^c = \begin{cases} R_{M,2} b_2^c - R d_2, & \text{if on a good island,} \\ (1-\delta) R_{M,2} b_2^c - R d_2, & \text{if on a bad island.} \end{cases} \tag{3.2.19}$$

Period 1 Problem

In period 1 the commercial bank's problem is to choose deposits (d_1), and on balance sheet loans (b_1^c), to maximize the expected value of the final dividend payment made to savers, ($E_1 n_3^c$) subject to their balance sheet identity (3.2.21) and the solvency constraint (3.2.22). Their problem is:

$$\max_{b_1^c, d_1} \; E_1 n_3^c, \tag{3.2.20}$$

[4]Note: For simplicity I assume there is no risk of default on debt issued in period 2 (i.e. that $\psi_3 = 0$). The solvency constraint remains, reflecting the reality that commercial banks are always regulated.

subject to:

$$b_1^c = n_1^c + d_1, \tag{3.2.21}$$

$$(1-\delta)R_{M,1}b_1^c \geq Rd_1. \tag{3.2.22}$$

Net worth (before shadow bank liabilities are taken onto balance sheet) is as follows:

$$n_2^c = \begin{cases} R_{M,1}b_1^c - Rd_1, & \text{if on a good island,} \\ (1-\delta)R_{M,1}b_1^c - Rd_1, & \text{if on a bad island.} \end{cases} \tag{3.2.23}$$

3.2.4 Aggregation

3.2.4.1 Consumption & Endowments

Aggregate borrower consumption[5]:

$$\hat{C}_1 = \hat{c}_1, \tag{3.2.24}$$

$$\hat{C}_2 = (1-\delta\psi_2)\hat{c}_2, \tag{3.2.25}$$

$$\hat{C}_3 = (1-\delta\psi_2)\hat{c}_3. \tag{3.2.26}$$

Aggregate saver consumption levels:

$$\tilde{C}_t = \tilde{c}_t, \qquad t = 1,2,3. \tag{3.2.27}$$

Aggregate endowment of borrowers:

$$\hat{Y}_t = \hat{y}_t, \qquad t = 1,2,3. \tag{3.2.28}$$

Aggregate endowment of savers:

$$\tilde{Y}_t = \tilde{y}_t, \qquad t = 1,2,3. \tag{3.2.29}$$

3.2.4.2 Loans & Assets

Aggregate quantity of loans:

$$B_1 = b_1, \tag{3.2.30}$$

$$B_2 = (1-\delta\psi_2)b_2. \tag{3.2.31}$$

[5] $\hat{C}_2$, $\hat{C}_3$ is the aggregate consumption of the borrowers who did not default in period 2.

Aggregate quantity of shadow bank liabilities:

$$m_1 = M_1, \tag{3.2.32}$$

$$m_2 = M_2. \tag{3.2.33}$$

Aggregate quantity of deposits:

$$d_1 = D_1, \tag{3.2.34}$$

$$d_2 = D_2. \tag{3.2.35}$$

3.2.4.3 Aggregate Budget Constraints

Aggregate budget constraints of borrowers:

$$\hat{C}_1 = B_1, \tag{3.2.36}$$

$$\hat{C}_2 = (1 - \delta\psi_2)\hat{Y}_2 + B_2 - (1 - \delta\psi_2)R_{M,1}B_1, \tag{3.2.37}$$

$$\hat{C}_3 = (1 - \delta\psi_2)\hat{Y}_3 - R_{M,2}B_2. \tag{3.2.38}$$

Aggregate budget constraints of savers:

$$\tilde{C}_1 = \tilde{Y}_1 - (M_1 + D_1), \tag{3.2.39}$$

$$\tilde{C}_2 = \tilde{Y}_2 + R(M_1 + D_1) - (M_2 + D_2), \tag{3.2.40}$$

$$\tilde{C}_3 = R(M_2 + D_2) + \Pi, \tag{3.2.41}$$

where $\Pi \equiv N_3^c + N_3^b = R_{M,2}B_2 - R(M_2 + D_2)$, is the aggregate dividends paid from financial intermediaries to savers at the end of period 3.

3.2.4.4 Debit Limits

Non-negativity constraints on savers consumption imply the aggregate debt limits:

$$B_1 \leq \tilde{Y}_1 + N_1^c + N_1^b, \tag{3.2.42}$$

$$B_2 \leq \tilde{Y}_2 + (1 - \delta\psi_2)R_{M,1}B_1. \tag{3.2.43}$$

3.2.4.5 Net Worth

Both commercial and shadow banks are ex-ante identical. So:

$$N_1^c = n_1^c, \tag{3.2.44}$$

$$N_1^b = n_1^b. \tag{3.2.45}$$

Evolution of aggregate shadow banking sector net worth:

$$N_2^b = (1 - \delta\psi_2) R_{M,1} B_1^b - RM_1, \tag{3.2.46}$$

$$N_3^b = \Big[R_{M,2} B_2^b - RM_2 \Big]. \tag{3.2.47}$$

Evolution of aggregate commercial banking sector net worth:

$$N_2^c = \begin{cases} (1-\psi_2)\Big[R_{M,1}B_1^c - RD_1\Big] + T_2^c, & \text{if } N_2^b \geq 0, \\ (1-\psi_2)\Big[R_{M,1}B_1^c - RD_1\Big] + T_2^c + (1-\psi_2)N_2^b, & \text{if } N_2^b < 0, \end{cases} \tag{3.2.48}$$

$$N_3^c = \Big[R_{M,2} B_2^c - RD_2 \Big]. \tag{3.2.49}$$

Where T_2^c is the transfer of equity from exiting commercial banks to continuing commercial banks (all aggregate losses or gains remain within the financial sector).

$$T_2^c = \begin{cases} \psi_2\big[(1-\delta)R_{M,1}B_1^c - RD_1\big], & \text{if } N_2^b \geq 0, \\ \psi_2\big[(1-\delta)R_{M,1}B_1^c - RD_1\big] + \psi_2 N_2^b, & \text{if } N_2^b < 0. \end{cases} \tag{3.2.50}$$

3.2.5 Resource Constraint & Debt Market Clearing

Debt market clearing:

$$B_1 = B_1^c + B_1^b, \tag{3.2.51}$$

$$B_2 = B_2^c + B_2^b. \tag{3.2.52}$$

Aggregate commercial bank balance sheet:

$$B_1^c = N_1^c + D_1, \tag{3.2.53}$$

$$B_2^c = N_2^c + D_2. \tag{3.2.54}$$

Aggregate shadow bank balance sheet:

$$B_1^b = N_1^b + M_1, \tag{3.2.55}$$

$$B_2^b = N_2^b + M_2. \tag{3.2.56}$$

The resource constraints (see appendix 3.C.5 for derivation):

$$\hat{C}_1 + \tilde{C}_1 = \tilde{Y}_1 + N_1^b + N_1^c, \tag{3.2.57}$$

$$\hat{C}_2 + \tilde{C}_2 = \hat{Y}_2 + \tilde{Y}_2, \tag{3.2.58}$$

$$\hat{C}_3 + \tilde{C}_3 = \hat{Y}_3. \tag{3.2.59}$$

3.2.6 Conditions for a Crisis

A crisis occurs when shadow banks are unable to fully repay the liabilities they have issued on their own. In this case commercial banks take shadow bank liabilities back onto their balance sheet and honor them. Following this event investors (i.e. savers) are unwilling to purchase shadow bank liabilities, so shadow banks shut down.

In period 1 loans are originated (B_1) and asset backed securities (M_1) and deposits (D_1) are issued. The island specific default (δ) is known but the aggregate default rate in period 2 ($\delta\psi_2$) is not known. There is a probability distribution over outcome of the island specific shock ψ_2:

$$\psi_2 = \begin{cases} \psi_L, & \text{wp } (1-\mathbb{P}) \\ \psi_H, & \text{wp } \mathbb{P} \end{cases}, \tag{3.2.60}$$

where ψ_L is the low realization of defaults and ψ_H is the high realization of defaults ($\psi_L < \psi_H$). The distribution of defaults is exogenous: a borrower does not make a decision to default and the planner cannot change the distribution of defaults. This simplification maintains tractability. The social planner in this framework does not have control over micro-prudential regulation that influences the distribution of borrower quality.

This assumption maintains tractability. It also reflects that there are certain high default events that a planner cannot ex-ante moderate. For example pre-2008 the planner in this paper could not change micro-prudential standards for mortgage lending but could set bank capital regulation. In the Coronavirus case, this assumption reflects that the planner cannot control the disease process but can recognize the possibility of a high default event and regulate to moderate the fallout.

A crisis occurs if the realization of defaults is such that shadow banks cannot repay holders of ABS in full ($N_2^b < 0$). This is equivalent to:

$$\left[(1-\delta\psi_2)R_{M,1} - R\right]B_1^b + RN_1^b \leq 0. \tag{3.2.61}$$

I will assume (see appendix 3.C.3 for details) that aggregate shadow banking sector net worth is sufficiently small such that:

$$N_2^b \text{ is } \begin{cases} < 0 & \text{when } \psi_2 = \psi_H, \\ \geq 0 & \text{when } \psi_2 = \psi_L. \end{cases} \tag{3.2.62}$$

This assumption means that a crisis will always occur when high defaults are realized. Whereas the social planner can only moderate the severity of a crisis, they cannot change the probability of a crisis.[6] In the following I will use "crisis" and "high default realization" interchangeably.

3.3 The Social Planner

The social planner is constrained efficient and operates under discretion, similar to the approach of Bianchi and Mendoza (2018). The planner must obey the resource constraints and the financial sector structure. The planner each period takes the actions (policy functions) of the subsequent period's planner as given. The planner does not set the market price (interest rate) on loans, but does internalize how the aggregate quantity of borrowing affects interest rates (the planner takes the optimality conditions of financial intermediaries as additional constraints[7]).

The planner internalizes the effect an additional unit of borrowing in good times has on credit conditions during a crisis. In period 1, because shadow banks operate freely, the financial sector is unconstrained - commercial banking regulation does not limit the quantity of credit available. The spread set by the shadow bank (see appendix 3.C.3) is sufficiently low such that during a high default event shadow banks cannot repay their liabilities. During a crisis commercial banks take on shadow bank aggregate losses,

[6] Relaxing this assumption in future work would add an additional dimension along which to motivate optimal policy.

[7] In Bianchi and Mendoza (2018) and other models with collateral constraints the borrower's Euler would replace the financial sector optimality conditions in the planner's set of constraints.

reducing commercial bank aggregate net worth. This is socially costly because it shifts the period 2 credit supply curve left. As a result the credit conditions borrowers face in the aftermath of a crisis worsen the larger the net worth loss to the commercial banking sector is. Overborrowing occurs because borrowers fail to internalize the negative externality of an additional unit of borrowing in period 1 on the credit conditions they will face if a crisis occurs in period 2. This gives the social planner an opportunity to intervene. This occurs even though the planner and the borrower know the true probability distribution of defaults.

Appendix 3.D.1 shows the full derivation of the social planner's problem. The following summarizes the planner's problem recursively.

3.3.1 Period 3 Social Planner

The period 3 social planner (sp3) takes the aggregate net worth of the financial sector as given (N_3) as well as the final repayment of debt ($R_{M,2}B_2$). The planner maximizes the weighted sum of borrower utility[8]:

$$\max_{\hat{C}_3,\tilde{C}_3} \quad (1-\delta\psi_2)\hat{u}(\hat{c}_3) + \delta\psi_2\hat{u}(\hat{y}_3), \tag{3.3.1}$$

subject to:

$$\hat{C}_3 = (1-\delta\psi_2)\hat{c}_3, \tag{3.3.2}$$

$$\tilde{C}_3 = \tilde{c}_3, \tag{3.3.3}$$

$$\tilde{C}_3 \geq R(M_2 + D_2) + \Pi, \tag{3.3.4}$$

$$\tilde{C}_3 \geq 0, \tag{3.3.5}$$

$$\tilde{C}_3 + \hat{C}_3 = (1-\delta\psi_2)\hat{Y}_3, \tag{3.3.6}$$

where $\Pi = N_3$ is the aggregate dividend payout from financial institutions to savers. Section 3.2.4 summarizes the aggregation.

[8] $\tilde{C}_3$ is aggregate consumption of savers; $\hat{C}_3 = (1-\delta\psi_2)\hat{c}_3$ is the aggregate consumption of borrowers who have not defaulted.

The planner chooses the aggregate consumption of (not defaulted) borrowers and savers, to maximize the weighted sum of borrower utility (3.3.1), subject to aggregation (3.3.2), (3.3.3), the representative[9] saver's participation constraint (3.3.4), the non-negativity constraint on saver consumption (3.3.5), and the resource constraint (3.3.6) . Period 1 borrowers who have defaulted are excluded from the financial markets thereafter, they consume their endowments in periods 2 and 3. The planner cannot make transfers to these agents.

This planner's sole objective is the borrowers' welfare. The planner is constrained, by the saver participation constraint 3.3.4, to ensure that the saver remains on the same indifference curve they would be in the competitive equilibrium at the market prices that prevail at the planner's allocation. Essentially the planner cannot steal from the saver to increase the borrowers' welfare.

Appendix 3.D.1.1 solves the period 3 planner's problem. The planner's optimal allocation gives the minimum resources to the savers necessary to satisfy the participation constraint and everything else to the not defaulted borrowers. The planner's policy functions are[10]:

$$\tilde{C}_3^{sp3} = R_{M,2} B_2, \tag{3.3.7}$$

$$\hat{C}_3^{sp3} = (1 - \delta\psi_2)\hat{Y}_3 - R_{M,2} B_2. \tag{3.3.8}$$

3.3.2 Period 2 Social Planner

The period 2 social planner (sp2) knows the realization of the default state (ψ_2). They take the net worth of financial intermediaries (N_2^c, N_2^b), the asset positions of savers (D_1, M_1), and the borrower debt level from the proceeding period (B_1) as given, as well as the policy functions of the period 3 planner. The period 2 planner maximizes the weighted sum of borrower utility[11]:

$$\max_{\hat{C}_2, \tilde{C}_2, B_2} \quad (1 - \delta\psi_2)\hat{u}(\hat{c}_2) + \delta\psi_2 \hat{u}(\hat{y}_2) + \Big[(1 - \delta\psi_2)\hat{u}(\hat{c}_3^{sp3}) + \delta\psi_2 \hat{u}(\hat{y}_3)\Big], \tag{3.3.9}$$

[9] Individual savers are ex-ante and ex-post identical so can be treated as a representative agent.

[10] Where the evolution of aggregate commercial (3.2.49) and shadow banking sector (3.2.47) net worth have been use to substitute out M_2, D_2, & Π.

[11] As before weighted across not defaulted and defaulted borrowers.

subject to:

$$\hat{C}_2 = (1 - \delta\psi_2)\hat{c}_2, \tag{3.3.10}$$

$$\tilde{C}_2 = \tilde{c}_2, \tag{3.3.11}$$

$$\tilde{C}_2 \geq R(M_1 + D_1) - (M_2 + D_2) + \tilde{Y}_2, \tag{3.3.12}$$

$$B_2 \leq \tilde{Y}_2 + (1 - \delta\psi_2)R_{M,1}B_1, \tag{3.3.13}$$

$$\hat{C}_2 + \tilde{C}_2 = (1 - \delta\psi_2)\hat{Y}_2 + \tilde{Y}_2, \tag{3.3.14}$$

$$R_{M,2} = \begin{cases} R, & \text{if } \psi_2 = \psi_L, \\ \max\left(R, \frac{R}{1-\delta}\left[1 - \frac{N_2^{c,H}(B_1)}{B_2}\right]\right), & \text{if } \psi_2 = \psi_H, \end{cases} \tag{3.3.15}$$

$$B_2 = \begin{cases} N_2^c + D_2, & \text{if } \psi_2 = \psi_H, \\ N_2^c + N_2^b + D_2 + M_2, & \text{if } \psi_2 = \psi_L, \end{cases} \tag{3.3.16}$$

where (3.3.10) is aggregate (not defaulted) borrower consumption, (3.3.11) is aggregate saver consumption, (3.3.12) is the representative saver's participation constraint, (3.3.13) is the feasibility limit on debt (derived from the non-negativity constraint on saver consumption), and (3.3.14) the resource constraint. The period 2 planner is also constrained by the market pricing of the interest rate (3.3.15), as well as the consolidated financial sector balance sheet[12] (3.3.16).

The period 2 planner's resulting optimality condition[13] is:

$$\hat{u}_{c,2} = \left[R_{M,2} + \frac{\partial R_{M,2}}{\partial B_2}\right]\hat{u}_{c,3}. \tag{3.3.17}$$

When $\frac{\partial R_{M,2}}{\partial B_2} = 0$ the planner's optimality condition collapses to the borrower's private optimality condition:

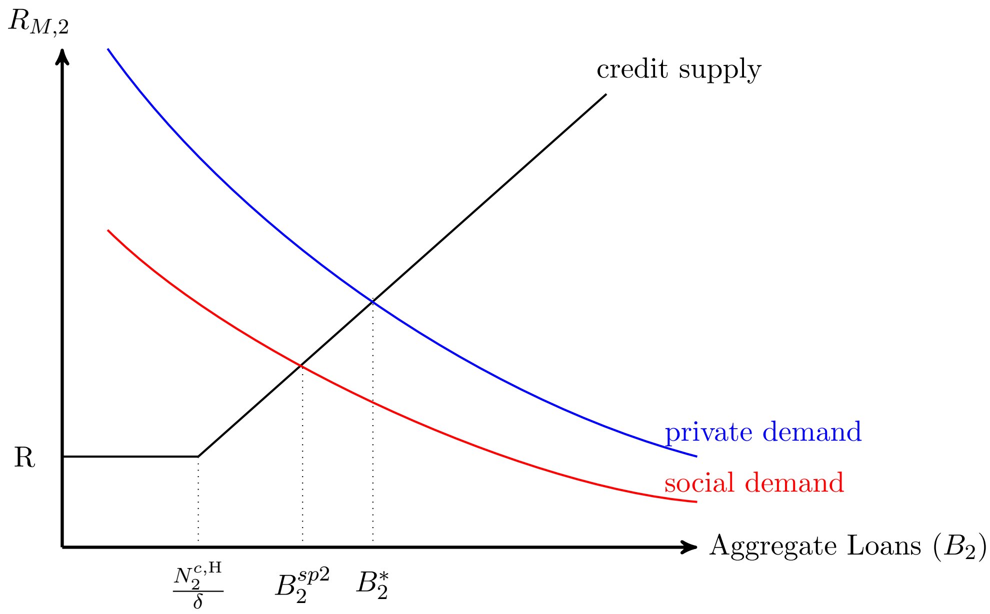
$$\hat{u}_{c,2} = R_{M,2}\hat{u}_{c,3}. \tag{3.3.18}$$

$\frac{\partial R_{M,2}}{\partial B_2} = 0$ holds either when the solvency constraint is slack or when the realization of defaults is low. When the financial sector is unconstrained there is no motive for the planner to deviate from the competitive equilibrium.

[12]This comes from combining (3.2.52), (3.2.54), and (3.2.56).

[13]See appendix 3.D.1.2 for the derivation.

Figure 3.3.1: Period 2 Social Planner - Different Credit Demand Curve



Note: B_2^{sp2} is the period 2 social planner's optional choice of loans. B_2^* is the competitive equilibrium level of loans.

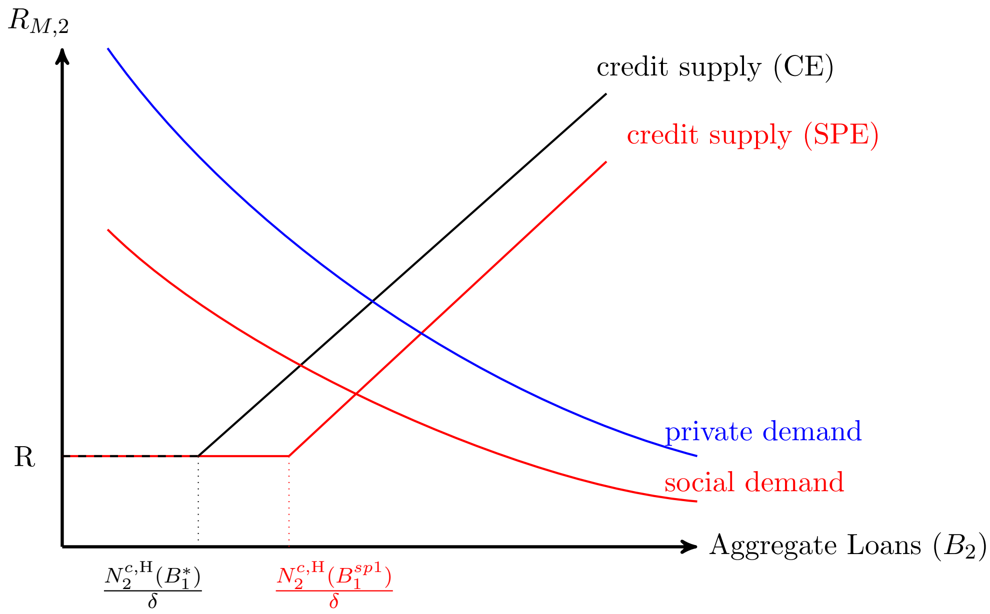
$\frac{\partial R_{M,2}}{\partial B_2} > 0$ in a crisis (a high realization of defaults). Shadow banks have collapsed. So the financial sector is only made up of regulated commercial banks. The solvency constraint on commercial banks generates an upward sloping credit supply curve. Individual borrowers do not internalize the impact their period 2 borrowing has on the equilibrium price of credit. Therefore the period 2 planner has a motive to intervene to hold back borrower credit demand to moderate the cost of credit. Figure 3.3.1 captures this difference between private versus socially optimal credit demand.

In summary, conditional on a crisis, the competitive equilibrium level of borrowing in period 2 is also sub-optimally high. This is because an individual borrower fail to internalize that an additional unit of their borrowing results in a higher equilibrium interest rate.

3.3.3 Period 1 Social Planner

The period 1 social planner (sp1) takes the policy functions of the future planners as given. In particular the planner recognizes that in period 2, if a crisis occurs, the interest rate schedule faced by the period 2 planner is a function of the commercial banking

Figure 3.3.2: Period 1 Social Planner - Holds Back Credit to Shift out t=2 Credit Supply Curve



Note: CE = competitive equilibrium, SPE= social planner's equilibrium. B_1^{sp1} is the period 1 social planner's optional choice of loans. B_1^* is the competitive equilibrium level of loans.

sector's net worth. Boosting the commercial banking sector's net worth during a crisis is socially valuable because it shifts the credit supply curve rightward, meaning that the trade-off between credit quantity and cost is more favorable to the borrower (see figure 3.3.2). The period 1 planner can shift out the (crisis) period 2 credit supply curve by limiting the quantity of loans taken out by borrowers in period 1.

The period 1 planner's problem is:

$$\max_{\hat{C}_1,\tilde{C}_1,B_1} \; \hat{u}(\hat{c}_1) + E_1\left\{(1-\delta\psi_2)\hat{u}(\hat{c}_2^{sp2}) + \delta\psi_2\hat{u}(\hat{y}_2) + \left[(1-\delta\psi_2)\hat{u}(\hat{c}_3^{sp3}) + \delta\psi_2\hat{u}(\hat{y}_3)\right]\right\}, \tag{3.3.19}$$

subject to:

$$\hat{C}_1 = \hat{c}_1, \tag{3.3.20}$$

$$\tilde{C}_1 = \tilde{c}_1, \tag{3.3.21}$$

$$\tilde{C}_1 \geq \tilde{Y}_1 - (M_1 + D_1), \tag{3.3.22}$$

$$B_1 \leq \tilde{Y}_1 + N_1^c + N_1^b, \tag{3.3.23}$$

$$\hat{C}_1 + \tilde{C}_1 = \tilde{Y}_1 + N_1^c + N_1^b, \tag{3.3.24}$$

$$R_{M,1} = \frac{R}{1 - \delta\psi_L}, \tag{3.3.25}$$

$$B_1 = N_1^c + N_1^b + D_1 + M_1. \tag{3.3.26}$$

The period 1 planner maximizes expected borrower utility subject to aggregate borrower consumption (3.3.20), aggregate saver consumption (3.3.21), the saver's participation constraint (3.3.22), the feasibility limit on debt (3.3.23, derived from a non-negativity constraint on saver consumption), and the resource constraint (3.3.24). They are also constrained by the market pricing of the interest rate, i.e. the shadow bank's optimality condition (3.3.25) and the aggregate financial sector balance sheet[14] (3.3.26). The solvency constraint of the commercial bank does not impact aggregate credit conditions when shadow banks operate freely, so does not present as a constraint on the period 1 planner. The resulting optimality condition of the period 1 planner is:

$$\hat{u}_{c,1} = \mathbb{P}\left(\hat{u}_{c,2}^{\mathrm{H}}\left[(1 - \delta\psi_H)R_{M,1} - \underbrace{\frac{\partial N_2^{c,H}}{\partial B_1}}_{>0}\right]\right) + (1 - \mathbb{P})\hat{u}_{c,2}^{\mathrm{L}}(1 - \delta\psi_L)R_{M,1}, \tag{3.3.27}$$

where the superscripts H & L refer to the high and low default states and commercial banking sector net worth in a crisis is[15]:

$$N_2^{c,\mathrm{H}} = \big[(1 - \delta\psi_H)R_{M,1} - R\big]B_1 + R(N_1^c + N_1^b). \tag{3.3.28}$$

So:

$$\frac{\partial N_2^{c,\mathrm{H}}}{\partial B_1} = (1 - \delta\psi_H)R_{M,1} - R = R\frac{\delta(\psi_L - \psi_H)}{1 - \delta\psi_L} < 0, \tag{3.3.29}$$

[14]This comes from combining (3.2.51), (3.2.53), and (3.2.55).
[15]From combining (3.2.48) and (3.3.26).

where the final equality comes from the shadow bank's optimality condition for pricing loans (3.C.24). Because the crisis realization of commercial bank net worth is decreasing in the aggregate quantity of period 1 loans, the planner has a motive to lower the level of debt in period 1. To see this, compare the planner's optimality condition (3.3.27) to the borrower's Euler condition for loans in competitive equilibrium:

$$\hat{u}_{c,1} = \mathbb{P}\hat{u}^{H}_{c,2}(1 - \delta\psi_H)R_{M,1} + (1 - \mathbb{P})(1 - \delta\psi_L)R_{M,1}\hat{u}^{\mathrm{L}}_{c,2}. \tag{3.3.30}$$

By assuming quasi-linear borrower utility it is possible to show analytically that the *level* of period 1 debt chosen by the planner is lower than the competitive equilibrium level of debt (see appendix 3.E).

To summarize: overborrowing in period 1 arises because individual borrowers do not internalize the impact an additional unit of their borrowing has on tightening credit conditions in period 2 (if a crisis occurs). The externality is always negative when shadow banks are present. In contrast if shadow banks do not exist both overborrowing and under-borrowing could occur (see appendix 3.D.1.4). The sign of the externality would depend on the tightness of commercial banking regulation.

3.4 Decentralized Policy

The planner has a number of instruments available to decentralize their optimal allocation. The set of instruments include: a tax on loans, a credit limit on borrowers, balance sheet taxes on financial intermediaries, and "stressing" the solvency constraint faced by commercial banks. Because the planner operates under discretion they do not have the ability to commit to policies in the future. Credit limits effectively decentralize the borrower's optimal allocation without the need to rely on any other policy instrument. The optimality of a tax on loans or financial intermediary balance sheets relies on the availability of lump-sum transfers, so that the revenue raised by the tax can be rebated to the borrower. Policy implemented via non-tax regulation of financial intermediaries relies on the availability of both lump-sum taxes and lump-sum transfers. The policy authority must be able to lump-sum tax the relevant financial intermediary and transfer the revenue to borrowers. Appendix 3.F contains the proofs of the optimality of the decentralizing policies.

3.4.1 Decentralizing the Period 1 Social Planner's Choice

The financial sector is unconstrained in this period. Any regulation or tax on commercial banks will simply push credit further onto shadow banks' balance sheets. It will have no impact on aggregate credit quantity or the equilibrium cost of credit (this is set by the optimality condition of unconstrained shadow banks). Regulating commercial banks is ineffective. The social planner is left with two options: regulating borrowers or regulating shadow banks[16].

3.4.1.1 Regulating Borrowers in Period 1

Option 1: Debt Tax on Borrowers

The social planner's period 1 allocation can be decentralized with a tax, τ_1 on the income generated by borrowing. In this case the borrower's Euler equation for loans becomes:

$$(1-\tau_1)\hat{u}_{c,1} = \mathbb{P}\hat{u}^{H}_{c,2}(1-\delta\psi_H)R_{M,1} + (1-\mathbb{P})\hat{u}^{\mathrm{L}}_{c,2}(1-\delta\psi_L)R_{M,1}. \tag{3.4.1}$$

The period 1 planner's optimal allocation is achievable when the tax is set as follows (and all revenue raised by the tax is rebated to the borrower in period 1):

$$\tau_1 \equiv \frac{-\mathbb{P}\hat{u}^{H}_{c,2}\frac{\partial N^{c,H}_{2}}{\partial B_1}}{\mathbb{P}\hat{u}^{H}_{c,2}\left[(1-\delta\psi_H)R_{M,1} - \frac{\partial N^{c,H}_{2}}{\partial B_1}\right] + (1-\mathbb{P})\hat{u}^{L}_{c,2}(1-\delta\psi_L)R_{M,1}} > 0. \tag{3.4.2}$$

Option 2: Credit Limit

Because the social planner's optimal level of period 1 debt is lower than the competitive equilibrium level it can also be decentralized using a credit limit, i.e. that the policy authority imposes that an individual borrower cannot take out a loan larger than a specified amount $\bar{b}_1$. When this limit binds, the borrower's Euler equation for loans

[16] This is a hypothetical regulation because depending on what is considered a shadow bank it may not be realistically possible to regulate.

becomes:

$$\hat{u}_{c,1} = \mathbb{P}\hat{u}^{H}_{c,2}(1-\delta\psi_H)R_{M,1} + (1-\mathbb{P})\hat{u}^{\mathrm{L}}_{c,2}(1-\delta\psi_L)R_{M,1} + \hat{\mu}_1, \tag{3.4.3}$$

where $\hat{\mu}_1$ is the multiplier on the credit limit:

$$b_1 \leq \bar{b}_1 \qquad\qquad (\hat{\mu}_1 \geq 0). \tag{3.4.4}$$

The optimal allocation is decentralized when the credit limit is set to the social planner's optimal level of debt (borrower's are ex-ante identical and of mass 1):

$$\bar{b}_1 \equiv B_1^{sp1}. \tag{3.4.5}$$

Under the credit limit policy no additional (lump-sum) tax or transfer are necessary to achieve the optimal allocation.

3.4.1.2 Regulating Shadow Banks in Period 1

If, hypothetically, the policymaker was able either to tax shadow banks per loans they buy (τ_1^b), or on revenue raised from issuing asset backed securities (τ_1^m), the policymaker could increase the interest rate on loans implied by the shadow banks optimality condition:

$$R_{M,1}^{tax} = \frac{R}{1-\delta\psi_L}\frac{1+\tau_1^b}{1-\tau_1^m}. \tag{3.4.6}$$

The tax enters the borrower's Euler equation for loans (where $R_{M,1}$ is the pre-tax[17] level of the interest rate):

$$\hat{u}_{c,1} = \left[\mathbb{P}\hat{u}^{H}_{c,2}(1-\delta\psi_H)R_{M,1} + (1-\mathbb{P})\hat{u}^{\mathrm{L}}_{c,2}(1-\delta\psi_L)R_{M,1}\right]\frac{1+\tau_1^b}{1-\tau_1^m}. \tag{3.4.7}$$

Assuming the planner is able to store the tax revenues and the period 2 planner rebates the tax revenues lump-sum in the following period (and this is anticipated by borrowers) then the optimal allocation is achieved by setting:

$$\frac{1+\tau_1^b}{1-\tau_1^m} = \frac{1}{1-\tau_1}. \tag{3.4.8}$$

So either $\tau_1^m = \tau_1 > 0$, or $\tau_1^b = \frac{\tau_1}{1-\tau_1} > 0$.

[17] $R_{M,1} = \frac{R}{1-\delta\psi_L}$.

3.4.2 Decentralizing the Period 2 Social Planner's Choice

The period 2 planner's allocation only differs from the competitive equilibrium if the crisis state ($\psi_2 = \psi_H$) is realized. In the crisis state, shadow banks have collapsed, so the planner can either regulate borrowers or regulate commercial banks.

3.4.2.1 Regulating Borrowers in Period 2

Option 1: Debt Tax on Borrowers

The social planner's period 2 allocation can be decentralized with a tax, τ_2 on the income generated by borrowing. In this case the borrower's Euler equation for loans becomes:

$$(1 - \tau_2)\hat{u}_{c,2} = R_{M,2}\hat{u}_{c,3}. \tag{3.4.9}$$

The period 2 planner's optimal allocation is achievable when the tax is set as follows, and all revenue raised by the tax is rebated to the borrower in period 2:

$$\tau_2 \equiv \frac{N_2^{c,H}}{B_2^{sp2,H}} > 0. \tag{3.4.10}$$

So the optimal per unit tax on borrowing is the inverse of the optimal commercial bank leverage at the period 2 social planner's optimal allocation.

Option 2: Credit Limit

Because the social planner's optimal level of period 2 debt is lower than the competitive equilibrium level it can also be decentralized using a credit limit, i.e. that the policy authority imposes that an individual borrower cannot take out a loan larger than a specified amount $\bar{b}_2$. When this limit binds, the borrower's Euler equation for loans becomes:

$$\hat{u}_{c,2} = R_{M,2}\hat{u}_{c,3} + \hat{\mu}_2 \tag{3.4.11}$$

where $\hat{\mu}_2$ is the multiplier on the credit limit:

$$b_2 \leq \bar{b}_2 \qquad\qquad (\hat{\mu}_2 \geq 0). \tag{3.4.12}$$

The optimal allocation is decentralized when the credit limit is set to the social planner's optimal level of per-borrower debt :

$$\bar{b}_2 \equiv \frac{B_2^{sp2,H}}{1-\delta\psi_H}. \tag{3.4.13}$$

Under the credit limit policy no additional (lump-sum) tax or transfer is necessary to achieve the optimal allocation.

3.4.2.2 Regulating Commercial Banks in Period 2

Option 1: Balance Sheet Taxes

The period 2 policymaker can set a tax on the balance sheet items of commercial banks, either using a tax per unit of loan issued (τ_2^b), or a tax per unit of deposits raised (τ_2^d). Under the tax, if the solvency constraint binds, the interest rate on loans is:

$$R_{M,2}^{tax} = \frac{R/1(1-\tau_2^d)}{1-\delta}\left[1+\tau_2^b - \frac{N_2^{c,H}}{B_2^{sp2,H}}\right]. \tag{3.4.14}$$

The optimal allocation can be achieved either using the loan tax only or the deposit tax only. In both cases the optimal level of the tax is also the inverse of commercial bank leverage at the period 2 social planner's optimal allocation:

$$\tau_2^b, \tau_2^d = \frac{N_2^{c,H}}{B_2^{sp2,H}}. \tag{3.4.15}$$

Option 2: Stressed Solvency Constraint

I will assume that the commercial bank can introduce a "stress" term into the solvency constraint (it can tighten the constraint but it cannot loosen it). So an individual commercial bank's problem in period 2 becomes:

$$\max \quad E_2 n_3^c = R_{M,2}^s b_2^c - Rd_2, \tag{3.4.16}$$

subject to:

$$b_2^c = n_2^c + d_2, \tag{3.4.17}$$

$$(1-\delta^s)R_{M,2}^s b_2^c \geq Rd_2. \tag{3.4.18}$$

The resulting interest rate schedule is:

$$R^s_{M,2} = \frac{R}{1-\delta^s}\left[1 - \frac{N_2^{c,H}}{B_2}\right]. \tag{3.4.19}$$

The period 2 planner's optimal allocation is achieved when the interest rate is set to $\frac{R}{1-\delta}$. This requires that the "stressed" solvency constraint term (δ^s) is set to:

$$\delta^s = \delta + (1-\delta)\frac{N^{c,H}}{B_2^{sp2,H}}. \tag{3.4.20}$$

To achieve the optimal allocation of the period 2 planner excess profits of commercial banks must be lump-sum taxed and lump-sum transferred to borrowers:

$$T_3 \equiv \left[R^s_{M,2} - R_{M,2}\right] B_2^{sp2} \tag{3.4.21}$$

3.5 Conclusion

In this paper I show that when an unregulated shadow banking sector exists along side the traditional commercial banking sector the competitive equilibrium level of borrowing is sub-optimally high. This is because private borrowing exhibits a negative externality. During a crisis (a high default event) the average return on loans held by shadow banks is not sufficient to repay their liabilities. Because the commercial and shadow banking sectors are implicitly connected this results in losses to commercial banks' aggregate net worth. During a crisis losses to commercial bank aggregate net worth are worse the more borrowing that is accumulated on shadow bank balance sheets during good times. A discretionary social planner is therefore motivated to intervene to hold back borrowing during good times. The social planner's optimal allocation can be decentralized using a number of different policy instruments. All instruments, except for credit limits (eg the conforming loan limit) placed on borrowers, require the existence of lump sum taxes or transfers.

Appendix

3.A Model Equations - Competitive Equilibrium

Households:

Borrower aggregate budget constraints:

$$\hat{C}_1 = B_1, \tag{3.A.1}$$

$$\hat{C}_2 = (1-\delta\psi_2)\hat{Y}_2 + B_2 - (1-\delta\psi_2)R_{M,1}B_1, \tag{3.A.2}$$

$$\hat{C}_3 = (1-\delta\psi_2)\hat{Y}_3 - R_{M,2}B_2. \tag{3.A.3}$$

The resource constraints:

$$\hat{C}_1 + \tilde{C}_1 = \tilde{Y}_1 + N_1^b + N_1^c, \tag{3.A.4}$$

$$\hat{C}_2 + \tilde{C}_2 = (1-\delta\psi_2)\hat{Y}_2 + \tilde{Y}_2, \tag{3.A.5}$$

$$\hat{C}_3 + \tilde{C}_3 = (1-\delta\psi_2)\hat{Y}_3. \tag{3.A.6}$$

Borrower's first order conditions[18] are:

$$\hat{u}_{c,1} = \mathbb{P}(1-\delta\psi_H)R_{M,1}\hat{u}_{c,2}^H + (1-\mathbb{P})(1-\delta\psi_L)R_{M,1}\hat{u}_{c,2}^L, \tag{3.A.7}$$

$$\hat{u}_{c,2} = R_{M,2}\hat{u}_{c,3}, \tag{3.A.8}$$

where the superscripts H & L refer to the high and low default states.

Financial Sector:

Shadow banks' FoCs wrt diversified loans:

$$(1-\delta\psi_L)R_{M,1} = R, \tag{3.A.9}$$

$$R_{M,2} = R \qquad \text{(only if shadow bank survives).} \tag{3.A.10}$$

[18]Note the FoCs hold at the individual borrower consumption levels ($\hat{c}$).

Commercial banks' FoCs:

$$\gamma_1^c = \frac{R(R_{M,1} - R)(1 - E_1\psi_2)}{R - (1-\delta)R_{M,1}}, \tag{3.A.11}$$

$$\gamma_2^c = \frac{R_{M,2} - R}{R - (1-\delta)R_{M,2}}. \tag{3.A.12}$$

Solvency constraints:

$$(1-\delta)R_{M,1}B_1^c \geq RD_1 \qquad (\gamma_1^c \geq 0), \tag{3.A.13}$$

$$(1-\delta)R_{M,2}B_2^c \geq RD_2 \qquad (\gamma_2^c \geq 0). \tag{3.A.14}$$

Debt limits:

$$B_1 \leq \tilde{Y}_1 + N_1^c + N_1^b \qquad (\Xi_1 \geq 0), \tag{3.A.15}$$

$$B_2 \leq \tilde{Y}_2 + (1 - \delta\psi_2)R_{M,1}B_1 \qquad (\Xi_2 \geq 0). \tag{3.A.16}$$

Evolution of aggregate shadow banking sector net worth:

$$N_2^b = (1 - \delta\psi_2)R_{M,1}B_1^b - RM_1, \tag{3.A.17}$$

$$N_3^b = \Big[R_{M,2}B_2^b - RM_2\Big]. \tag{3.A.18}$$

Evolution of aggregate commercial banking sector net worth:

$$N_2^c = \begin{cases} (1-\psi_2)\Big[R_{M,1}B_1^c - RD_1\Big] + T_2^c, & \text{if } N_2^b \geq 0, \\ (1-\psi_2)\Big[R_{M,1}B_1^c - RD_1\Big] + T_2^c + (1-\psi_2)N_2^b, & \text{if } N_2^b < 0, \end{cases} \tag{3.A.19}$$

$$N_3^c = \Big[R_{M,2}B_2^c - RD_2\Big]. \tag{3.A.20}$$

Aggregate transfer from bad island commercial banks:

$$T_2^c = \begin{cases} \psi_2\big[(1-\delta)R_{M,1}B_1^c - RD_1\big], & \text{if } N_2^b \geq 0, \\ \psi_2\big[(1-\delta)R_{M,1}B_1^c - RD_1\big] + \psi_2 N_2^b, & \text{if } N_2^b < 0. \end{cases}$$

Loan market clearing:

$$B_1 = B_1^c + B_1^b, \tag{3.A.21}$$

$$B_2 = B_2^c + B_2^b. \tag{3.A.22}$$

Aggregate commercial banking sector balance sheet:

$$B_1^c = N_1^c + D_1, \tag{3.A.23}$$

$$B_2^c = N_2^c + D_1. \tag{3.A.24}$$

Aggregate shadow banking sector balance sheet:

$$B_1^b = N_1^b + M_1, \tag{3.A.25}$$

$$B_2^b = N_2^b + M_1. \tag{3.A.26}$$

Rates:

Note that: financial sector optimization implies the following about rates:

$$(1 - \delta\psi_L)R_{M,1} = R,$$
$$R_{M,2} = \begin{cases} R, \text{ if } N_2^b \geq 0, \\ \frac{R}{1-\delta}\left[1 - \frac{N_2^c}{B_2}\right], \text{ if } N_2^b < 0 \text{ and solvency constraint binds.} \end{cases}$$

3.B Model Equations - Social Planner's Equilibrium

Households:

Borrower aggregate budget constraints:

$$\hat{C}_1 = B_1, \tag{3.B.1}$$

$$\hat{C}_2 = (1-\delta\psi_2)\hat{Y}_2 + B_2 - (1-\delta\psi_2)R_{M,1}B_1, \tag{3.B.2}$$

$$\hat{C}_3 = (1-\delta\psi_2)\hat{Y}_3 - R_{M,2}B_2. \tag{3.B.3}$$

The resource constraints:

$$\hat{C}_1 + \tilde{C}_1 = \tilde{Y}_1 + N_1^b + N_1^c, \tag{3.B.4}$$

$$\hat{C}_2 + \tilde{C}_2 = (1-\delta\psi_2)\hat{Y}_2 + \tilde{Y}_2, \tag{3.B.5}$$

$$\hat{C}_3 + \tilde{C}_3 = (1-\delta\psi_2)\hat{Y}_3. \tag{3.B.6}$$

The t=1 social planner's optimality condition[19] are:

$$\hat{u}_{c,1} = \mathbb{P}\left(\hat{u}_{c,2}^H\left[(1-\delta\psi_H)R_{M,1} - \frac{\partial N_2^{c,H}}{\partial B_1}\right]\right) + (1-\mathbb{P})\hat{u}_{c,2}^{\mathrm{L}}(1-\delta\psi_L)R_{M,1},$$

where $N_2^{c,H}$ is commercial bank net worth under the high default regime.

The t=2 social planner's optimality condition:

$$\hat{u}_{c,2} = \begin{cases} \hat{u}_{c,3}\frac{R}{1-\delta}, & \text{if defaults are high,} \\ \hat{u}_{c,3}R_{M,2}, & \text{if defaults are low.} \end{cases} \tag{3.B.7}$$

Financial Sector:

Shadow banks' FoCs wrt diversified loans:

$$(1-\delta\psi_L)R_{M,1} = R, \tag{3.B.8}$$

$$R_{M,2} = R \qquad \text{(only if shadow bank survives).} \tag{3.B.9}$$

Commercial banks' FoCs:

$$\gamma_1^c = \frac{R(R_{M,1}-R)(1-E_1\psi_2)}{R-(1-\delta)R_{M,1}}, \tag{3.B.10}$$

$$\gamma_2^c = \frac{R_{M,2}-R}{R-(1-\delta)R_{M,2}}. \tag{3.B.11}$$

[19]Note the social planner's optimality conditions hold at the representative individual borrower consumption levels ($\hat{c}$).

Solvency constraints:

$$(1-\delta)R_{M,1}B_1^c \geq RD_1 \qquad (\gamma_1^c \geq 0), \tag{3.B.12}$$

$$(1-\delta)R_{M,2}B_2^c \geq RD_2 \qquad (\gamma_2^c \geq 0). \tag{3.B.13}$$

Debt limits:

$$B_1 \leq \tilde{Y}_1 + N_1^c + N_1^b \qquad (\Xi_1 \geq 0), \tag{3.B.14}$$

$$B_2 \leq \tilde{Y}_2 + (1-\delta\psi_2)R_{M,1}B_1 \qquad (\Xi_2 \geq 0). \tag{3.B.15}$$

Evolution of aggregate non-banking sector net worth:

$$N_2^b = (1-\delta\psi_2)R_{M,1}B_1^b - RM_1, \tag{3.B.16}$$

$$N_3^b = \Big[R_{M,2}B_2^b - RM_2\Big]. \tag{3.B.17}$$

Evolution of aggregate commercial banking sector net worth:

$$N_2^c = \begin{cases} (1-\psi_2)\Big[R_{M,1}B_1^c - RD_1\Big] + T_2^c, & \text{if } N_2^b \geq 0, \\ (1-\psi_2)\Big[R_{M,1}B_1^c - RD_1\Big] + T_2^c + (1-\psi_2)N_2^b, & \text{if } N_2^b < 0, \end{cases} \tag{3.B.18}$$

$$N_3^c = \Big[R_{M,2}B_2^c - RD_2\Big]. \tag{3.B.19}$$

Transfers:

$$T_2^c = \begin{cases} \psi_2\big[(1-\delta)R_{M,1}B_1^c - RD_1\big], & \text{if } N_2^b \geq 0, \\ \psi_2\big[(1-\delta)R_{M,1}B_1^c - RD_1\big] + \psi_2 N_2^b, & \text{if } N_2^b < 0. \end{cases}$$

Loan market clearing:

$$B_1 = B_1^c + B_1^b, \tag{3.B.20}$$

$$B_2 = B_2^c + B_2^b. \tag{3.B.21}$$

Aggregate commercial banking sector balance sheet:

$$B_1^c = N_1^c + D_1, \tag{3.B.22}$$

$$B_2^c = N_2^c + D_1. \tag{3.B.23}$$

Aggregate shadow banking sector balance sheet:

$$B_1^b = N_1^b + M_1, \tag{3.B.24}$$

$$B_2^b = N_2^b + M_1. \tag{3.B.25}$$

Rates:

Note that: financial sector optimization implies the following about rates:

$$
\begin{aligned}
&(1-\delta\psi_L)R_{M,1} = R \\
&R_{M,2} = \begin{cases} R, \text{ if } N_2^b \geq 0, \\ \frac{R}{1-\delta}\left[1 - \frac{N_2^c}{B_2}\right], \text{ if } N_2^b < 0 \text{ and solvency constraint binds.} \end{cases}
\end{aligned}
$$

3.C Competitive Equilibrium Appendix

3.C.1 Saver's Problem

Period 1 problem:

$$\max_{d_1,m_1} \; E_1\tilde{U}_1 \equiv \tilde{c}_1 + \tilde{c}_2 + \tilde{c}_3,$$

subject to:

$$\tilde{c}_1 \leq \tilde{y}_1 - (m_1 + d_1),$$

$$\tilde{c}_2 \leq R(m_1 + d_1) - (m_2 + d_2) + \tilde{y}_2,$$

$$\tilde{c}_3 \leq R(m_2 + d_2) + \pi,$$

$$\tilde{c}_1 \geq 0 \qquad (\kappa_1 \geq 0),$$

$$\tilde{c}_2 \geq 0 \qquad (\kappa_2 \geq 0),$$

$$\tilde{c}_2 \geq 0 \qquad (\kappa_3 \geq 0).$$

The saver's FoCs wrt d_1 and m_1 are:

$$1 + \kappa_1 = R. \tag{3.C.1}$$

Period 1 problem:

$$\max_{d_2,m_2} \; E_2\tilde{U}_2 \equiv \tilde{c}_2 + \tilde{c}_3,$$

subject to:

$$\tilde{c}_2 \leq R(m_1 + d_1) - (m_2 + d_2) + \tilde{y}_2,$$

$$\tilde{c}_3 \leq R(m_2 + d_2) + \pi,$$

$$\tilde{c}_2 \geq 0 \qquad (\kappa_2 \geq 0),$$

$$\tilde{c}_2 \geq 0 \qquad (\kappa_3 \geq 0).$$

The saver's FoCs wrt d_2 and m_2 are:

$$1 + \kappa_2 = R. \tag{3.C.2}$$

Where κ_1 and κ_2 are non-negativity constraints on saver's period 1 and 2 consumption. The saver FoCs imply that saver consumption must be non-negative[20]. In aggregate this is:

$$\tilde{C}_1 = \tilde{Y}_1 - (M_1 + D_1) \geq 0, \tag{3.C.3}$$

$$\tilde{C}_2 = \tilde{Y}_2 + R(M_1 + D_1) - (M_2 + D_2) \geq 0. \tag{3.C.4}$$

Plugging in the financial sector balance sheets and the laws of motion for aggregate net worth[21], the saver consumption non-negativity constraints imply the debt feasibility limits:

$$B_1 \leq \tilde{Y}_1 + N_1^c + N_1^b, \tag{3.C.5}$$

$$B_2 \leq \tilde{Y}_2 + (1 - \delta\psi_2) R_{M,1} B_1. \tag{3.C.6}$$

3.C.2 Borrower's Problem

The borrower's lagrangian in period 1 is:

$$\begin{aligned}\mathcal{L}_1 = E_1 \Big\{ & \hat{u}(\hat{c}_1) + (1 - \delta\psi_2)\big[\hat{u}(\hat{c}_2) + \hat{u}(\hat{c}_3)\big] + \delta\psi_2\big[\hat{u}(\hat{y}_2) + \hat{u}(\hat{y}_3)\big]\Big\} \\ & + \hat{c}_1 \Xi_1 + \hat{c}_2 \Xi_2 + \hat{c}_3 \Xi_3 - \hat{\lambda}_1\big[\hat{c}_1 - b_1\big] - \hat{\lambda}_2\big[\hat{c}_2 - \hat{y}_2 - b_2 + R_{M,1} b_1\big] - \hat{\lambda}_3\big[\hat{c}_3 - \hat{y}_3 + R_{M,2} b_2\big].\end{aligned} \tag{3.C.7}$$

Where Ξ_t is the non-negative constraint on consumption at time t.

The borrower's first order conditions in period 1 are:

$$\frac{\partial \mathcal{L}_1}{\partial \hat{c}_1} = E_1\Big\{\hat{u}_{c,1} - \hat{\lambda}_1 + \Xi_1\Big\} = 0, \tag{3.C.8}$$

$$\frac{\partial \mathcal{L}_1}{\partial \hat{c}_2} = E_1\Big\{(1 - \delta\psi_2)\hat{u}_{c,2} - \hat{\lambda}_2 + \Xi_2\Big\} = 0, \tag{3.C.9}$$

$$\frac{\partial \mathcal{L}_1}{\partial b_1} = E_1\Big\{\hat{\lambda}_1 - R_{M,1}\hat{\lambda}_2\Big\} = 0. \tag{3.C.10}$$

[20] Saver aggregate consumption in period 3 is $\tilde{C}_3 = R_{M,2} B_2$, using the definition of Π the law of motion for aggregate financial sector net worth (3.C.48). It is obvious to see that the non-negative constraint on $\tilde{C}_3$ never binds.

[21] Using the evolution of net worth rewrite:

$$\begin{aligned}\tilde{C}_2 &= R(M_1 + D_1) - (M_2 + D_2) + \tilde{Y}_2 = R(M_1 + D_1) - B_2 + (N_2^b + N_2^c) + \tilde{Y}_2 \\ &= R(M_1 + D_1) - B_2 + (1 - \delta\psi_2) R_{M,1} B_1 - R(M_1 + D_1) + \tilde{Y}_2 \\ &= -B_2 + (1 - \delta\psi_2) R_{M,1} B_1 + \tilde{Y}_2.\end{aligned}$$

The borrower's lagrangian in period 2 is:

$$\mathcal{L}_2 = \hat{u}(\hat{c}_2) + \hat{u}(\hat{c}_3) + \hat{c}_2\Xi_2 + \hat{c}_3\Xi_3 - \hat{\lambda}_2\big[\hat{c}_2 - \hat{y}_2 - b_2 + R_{M,1}b_1\big] - \hat{\lambda}_3\big[\hat{c}_3 - \hat{y}_3 + R_{M,2}b_2\big]. \tag{3.C.11}$$

The borrower's first order conditions in period 2 are:

$$\frac{\partial \mathcal{L}_2}{\partial \hat{c}_2} = E_2\Big\{\hat{u}_{c,2} - \hat{\lambda}_2 + \Xi_2\Big\} = 0, \tag{3.C.12}$$

$$\frac{\partial \mathcal{L}_2}{\partial \hat{c}_3} = E_2\Big\{\hat{u}_{c,3} - \hat{\lambda}_3 + \Xi_3\Big\} = 0, \tag{3.C.13}$$

$$\frac{\partial \mathcal{L}_1}{\partial b_2} = E_1\Big\{\hat{\lambda}_2 - R_{M,2}\hat{\lambda}_3\Big\} = 0. \tag{3.C.14}$$

The borrower's lagrangian in period 3 is:

$$\mathcal{L}_3 = \hat{u}(\hat{c}_3) + \hat{c}_3\Xi_3 - \hat{\lambda}_3\big[\hat{c}_3 - \hat{y}_3 + R_{M,2}b_2\big].$$

The borrower's first order condition in period 3 is:

$$\frac{\partial \mathcal{L}_3}{\partial \hat{c}_3} = E_2\Big\{\hat{u}_{c,3} - \hat{\lambda}_3 + \Xi_3\Big\} = 0. \tag{3.C.15}$$

Focusing on the equilibrium where the non-negativity constraints don't bind:

$$\hat{u}_{c,1} = \mathbb{P}(1 - \delta\psi_H)R_{M,1}E_1\hat{u}^{\mathrm{H}}_{c,2} + (1 - \mathbb{P})(1 - \delta\psi_L)R_{M,1}E_1\hat{u}^{\mathrm{L}}_{c,2}, \tag{3.C.16}$$

$$\hat{u}_{c,2} = R_{M,2}\hat{u}_{c,3}. \tag{3.C.17}$$

Notation: $\mathbb{P}$ is the probability that defaults are high ($\psi_2 = \psi_H$), and $\hat{u}^{\mathrm{H}}_{c,2}$, $\hat{u}^{\mathrm{L}}_{c,2}$ are an individual borrower's marginal utility consumption in the high and low default case respectively.

3.C.3 Shadow Bank's Problem

3.C.3.1 Period 2 Problem

If a shadow bank survives to period 2, their problem is[22]:

$$\max_{b_2^b, m_2} \; n_3^b, \tag{3.C.18}$$

[22]The expectation is dropped because there is no uncertainty between t=2 and t=3.

subject to the balance sheet identity and evolution of net worth:

$$b_2^b = m_2 + n_2^b, \tag{3.C.19}$$

$$n_3^b = R_{M,2} b_2^b - R m_2. \tag{3.C.20}$$

Plugging the balance sheet into the evolution of net worth, get:

$$n_3^b = \big[R_{M,2} - R\big] b_2^b + R n_2^b.$$

So the first order condition of their unconstrained maximization problem is:

$$R_{M,2} - R = 0. \tag{3.C.21}$$

3.C.3.2 Period 1 Problem

$$\max_{b_1^b, m_1} \; E_1 n_3^b,$$

subject to the balance sheet identity and the evolution of net worth:

$$\begin{aligned}
b_1^b &= m_1 + n_1^b, \\
E_1 n_3^b &= E_1 \Big\{ \big[R_{M,2} - R\big] b_2^b + R n_2^b \Big\}, \\
E_1 n_2^b &= E_1 \Big\{ (1 - \delta\psi_2) R_{M,1} b_1^b - R m_1 \Big\}.
\end{aligned}$$

Plugging in the balance sheet identity:

$$E_1 n_3^b = E_1 \Big\{ \big[R_{M,2} - R\big] b_2^b + R \Big(\underbrace{\big[(1 - \delta\psi_2) R_{M,1} - R\big] b_1^b + R n_1^b}_{= n_2^b} \Big) \Big\}.$$

The shadow bank pays a dividend to the savers only if they survive (i.e. if $n_2^b \geq 0$). So the expectation of the final dividend payment is the probability of survival times the expected payment conditional on survival. The shadow bank only survives if the following condition holds:

$$\underbrace{(1 - \psi_2\delta) R_{M,1} b_1^b}_{\text{return on diversified lending}} \geq \underbrace{R m_1}_{\text{amount owed on liabilities}}. \tag{3.C.22}$$

If the endowed net worth (n_1^b) is sufficiently small (I assume this holds) then shadow banks always default when defaults are high ($\psi_2 = \psi_H$). Therefore the probability of shadow bank survival is $1 - \mathbb{P}$. And the expectation of final period dividends is:

$$
\begin{aligned}
E_1 n_3^b &= (1-\mathbb{P})E_1\Big\{\big[R_{M,2}-R\big]b_2^b + Rn_2^b|\psi_2=\psi_L\Big\}\\
&= (1-\mathbb{P})E_1\Big\{\big[R_{M,2}-R\big]b_2^b + R\Big(\big[(1-\delta\psi_2)R_{M,1}-R\big]b_1^b + Rn_1^b\Big)|\psi_2=\psi_L\Big\}.
\end{aligned}
$$

So the shadow bank's problem at t=1 is the unconstrained maximization problem:

$$
\max_{b_1^b} \quad (1-\mathbb{P})E_1\Big\{\big[R_{M,2}-R\big]b_2^b + R\Big(\big[(1-\delta\psi_2)R_{M,1}-R\big]b_1^b + Rn_1^b\Big)|\psi_2=\psi_L\Big\}.
$$

$t=1$ FoC:

$$
(1-\mathbb{P})E_1\Big\{R\big[(1-\delta\psi_2)R_{M,1}-R\big]|\psi_2=\psi_L\Big\}=0. \tag{3.C.23}
$$

This implies that:

$$
R_{M,1}=\frac{1}{1-\delta\psi_L}R. \tag{3.C.24}
$$

3.C.4 Commercial Bank's Problem

3.C.4.1 Period 2 Problem

$$
\max_{b_2^c,d_2} \; E_2 n_3^c.
$$

subject to the solvency constraint, balance sheet identity and evolution of net worth:

$$
\begin{aligned}
&(1-\delta)R_{M,2}b_2^c - Rd_2 \geq 0,\\
&b_2^c = n_2^c + d_2,\\
&E_2 n_3^c = E_1\Big\{R_{M,2}b_2^c - Rd_2\}.
\end{aligned}
$$

An individual commercial bank's Lagrangian is (using the balance sheet identity to substitute out deposits, d_2):

$$
\mathcal{L}=E_2\Big\{\big[R_{M,2}-R\big]b_2^c + Rn_2^c\Big\} + \gamma_2^c\Big(\big[(1-\delta)R_{M,2}-R\big]b_2^c + Rn_2^c\Big).
$$

First order conditions:

$$
\frac{\partial\mathcal{L}}{\partial b_2^c} = R_{M,2}-R+\gamma_2^c\big[(1-\delta)R_{M,2}-R\big]=0, \tag{3.C.25}
$$

$$
\frac{\partial\mathcal{L}}{\partial \gamma_2^c} = \big[(1-\delta)R_{M,2}-R\big]b_2^c + Rn_2^c \geq 0 \qquad \text{(Holds with equality when } \gamma_2^c>0\text{)}. \tag{3.C.26}
$$

3.C.4.2 Period 1 Problem

$$\max_{b_1^c, d_1} \quad E_1 n_3^c,$$

subject to the solvency constraint, balance sheet identity and evolution of net worth:

$$\begin{aligned}
&(1-\delta)R_{M,1}b_1^c - Rd_1 \geq 0, \\
&b_1^c = n_1^c + d_1, \\
&E_1 n_3^c = E_1\Big\{(1-\psi_2)\Big(\big[R_{M,2} - R\big]b_2^c + Rn_2^c\Big)|\text{survival}\Big\}, \\
&E_1 n_2^c = E_1\Big\{(1-\psi_2)\Big[R_{M,1}b_1^c - Rd_1\Big]|\text{survival}\Big\}.
\end{aligned}$$

Note that the commercial bank only survives to t=2 if they are on a good island, with probability $1 - \psi_2$.

Substitute the balance sheet identity into the solvency constraint and the evolution of net worth:

$$\begin{aligned}
&\big[(1-\delta)R_{M,1} - R\big]b_1^c + Rn_1^c \geq 0, \\
&E_1 n_3^c = (1 - E_1\psi_2)E_1\Big\{\Big(\big[R_{M,2} - R\big]b_2^c + Rn_2^c\Big)|\text{survival}\Big\}, \\
&E_1 n_2^c = (1 - E_1\psi_2)E_1\Big\{\Big[\big[R_{M,1} - R\big]b_1^c + Rn_1^c\Big]|\text{survival}\Big\}.
\end{aligned}$$

Plug the expression for expected net worth in period 2 into the final dividend expression:

$$E_1 n_3^c = (1 - E_1\psi_2)E_1\Big\{\Big(\big[R_{M,2} - R\big]b_2^c + R\Big(\big[R_{M,1} - R\big]b_1^c + Rn_1^c\Big)\Big)|\text{survival}\Big\}. \quad (3.C.27)$$

An individual commercial bank's Lagrangian is then:

$$\begin{aligned}
\mathcal{L} = &(1 - E_1\psi_2)E_1\Big\{\Big(\big[R_{M,2} - R\big]b_2^c + R\Big(\big[R_{M,1} - R\big]b_1^c + Rn_1^c\Big)\Big)|\text{survival}\Big\} \\
&+ \gamma_1^c\Big(\big[(1-\delta)R_{M,1} - R\big]b_1^c + Rn_1^c\Big).
\end{aligned}$$

First order conditions:

$$\frac{\partial \mathcal{L}}{\partial b_1^c} = (1 - E_1\psi_2)E_1\Big\{R\big[R_{M,1} - R\big]|\text{survival}\Big\} + \gamma_1^c\big[(1-\delta)R_{M,1} - R\big] = 0, \qquad (3.C.28)$$

$$\frac{\partial \mathcal{L}}{\partial \gamma_2^c} = \big[(1-\delta)R_{M,1} - R\big]b_1^c + Rn_1^c \geq 0 \qquad \text{(Holds with equality when } \gamma_1^c > 0\text{)}. \qquad (3.C.29)$$

Note that $E_1\Big\{R\big[R_{M,1}-R\big]|\text{survival}\Big\} = R\big[R_{M,1}-R\big]$, rearranging the FoC wrt loans get:

$$\gamma_1^c = \frac{R\Big[R_{M,1}-R\Big](1-E_1\psi_2)}{R-(1-\delta)R_{M,1}}. \tag{3.C.30}$$

So the solvency constraint is always binding in period 1 and is slack in period 2 under no crisis. In a crisis a binding solvency constraint implies that the loan rate in period 2 is a function of lending in period 2 ($B_2 = B_2^c$ because shadow banks collapse and therefore do not lend in a crisis):

$$R_{M,2}(B_2) = \frac{R}{1-\delta}\left[1-\frac{N_2^c}{B_2}\right], \tag{3.C.31}$$

(as long as $\gamma_2^c > 0$).

This implies that:

$$\frac{\partial R_{M,2}(B_2)}{\partial B_2} = \frac{R}{1-\delta}\frac{N_2^c}{(B_2)^2} > 0. \tag{3.C.32}$$

So the social planner in period 2 will have a motive to moderate the quantity of period 2 debt so that in aggregate borrowers face a more favorable interest rate. Similarly because $\frac{\partial N_2^{c,\text{crisis}}}{\partial B_1} < 0$ (see below) the social planner in period 1 will have a motive to moderate the quantity of period 1 debt so that the the interest rate schedule in period 2 is more favorable to borrowers.

3.C.5 Resource Constraints

Add the borrower and saver budget constraints together:

$$\hat{C}_1 + \tilde{C}_1 = \tilde{Y}_1 + B_1 - (M_1 + D_1), \tag{3.C.33}$$

$$\hat{C}_2 + \tilde{C}_2 = \tilde{Y}_2 + (1-\delta\psi_2)\hat{Y}_2 + B_2 - (M_2 + D_2) - \Big[(1-\delta\psi_2)R_{M,1}B_1 - R(M_1 + D_1)\Big], \tag{3.C.34}$$

$$\hat{C}_3 + \tilde{C}_3 = (1-\delta\psi_2)\hat{Y}_3 + N_3 - \Big[R_{M,2}B_2 - R(M_2 + D_2)\Big]. \tag{3.C.35}$$

And the aggregate balance sheet of financial sector:

$$N_1^c + N_1^b + M_1 + D_1 = B_1^c + B_1^b, \tag{3.C.36}$$

$$N_2^c + N_2^b + M_2 + D_2 = B_2^c + B_2^b. \tag{3.C.37}$$

Aggregate evolution of shadow bank net worth:

$$N_2^b = (1 - \psi_2 \delta) R_{M,1} B_1^b - RM_1, \tag{3.C.38}$$

$$N_3^b = \Big[R_{M,2} B_2^b - RM_2\Big]. \tag{3.C.39}$$

Aggregate evolution of commercial bank net worth, when defaults are low:

$$N_2^{c,\mathrm{L}} = (1 - \psi_2)\Big[R_{M,1} B_1^c - RD_1\Big] + T_2^c = (1 - \psi_2 \delta) R_{M,1} B_1^c - RD_1, \tag{3.C.40}$$

$$N_3^c = \Big[R_{M,2} B_2^c - RD_2\Big]. \tag{3.C.41}$$

Aggregate evolution of commercial bank net worth, when defaults are high (i.e. during a crisis):

$$\begin{aligned} N_2^{c,\mathrm{H}} &= (1 - \psi_2)\Big[R_{M,1} B_1^c - RD_1\Big] + T_2^c + (1 - \psi_2) N_2^b \\ &= (1 - \delta\psi_2) R_{M,1} B_1 - R(M_1 + D_1) = \big[(1 - \delta\psi_2) R_{M,1} - R\big] B_1 + RN_1, \end{aligned} \tag{3.C.42}$$

$$N_3^c = \Big[R_{M,2} B_2^c - RD_2\Big]. \tag{3.C.43}$$

Transfers:

$$T_2^c = \begin{cases} \psi_2\big[(1 - \delta) R_{M,1} B_1^c - RD_1\big], & \text{if } N_2^b \geq 0 \\ \psi_2\big[(1 - \delta) R_{M,1} B_1^c - RD_1\big] + \psi_2 N_2^b, & \text{if } N_2^b < 0. \end{cases}$$

Note, because the optimality condition of the shadow bank at t=1, $(1-\delta\psi_L) R_{M,1} = R$. So in a crisis commercial bank equity ($N_2^{c,H}$) is declining in period 1 debt ($\psi_H > \psi_L$):

$$\frac{\partial N_2^{c,H}}{\partial B_1} = (1 - \delta\psi_H) R_{M,1} - R < 0. \tag{3.C.44}$$

Given the above can write:

$$N_3 = N_3^c + N_3^b = \Big[R_{M,2} B_2 - R(M_2 + D_2)\Big],$$

$$N_2 = N_2^c + N_2^b = (1 - \delta\psi_2) R_{M,1} B_1 - R(M_1 + D_1).$$

$$B_2 - (M_2 + D_2) - \Big[(1 - \psi_2\delta) R_{M,1} B_1 - R(M_1 + D_1)\Big],$$

$$= N_2 - \Big[(1 - \psi_2\delta) R_{M,1} B_1 - R(M_1 + D_1)\Big] = 0.$$

So the resource constraints become:

$$\hat{C}_1 + \tilde{C}_1 = \tilde{Y}_1 + N_1^b + N_1^c, \tag{3.C.45}$$

$$\hat{C}_2 + \tilde{C}_2 = \tilde{Y}_2 + \hat{Y}_2, \tag{3.C.46}$$

$$\hat{C}_3 + \tilde{C}_3 = \hat{Y}_3. \tag{3.C.47}$$

Law of motion for aggregate financial sector net worth:

$$\begin{aligned} N_3 = N_3^c + N_3^b &= \Big[R_{M,2}B_2 - R(M_2 + D_2)\Big], \\ &= \big[R_{M,2} - R\big]B_2 + R(N_2^c + N_2^b) = \big[R_{M,2} - R\big]B_2 + RN_2^b, \\ &= \big[R_{M,2} - R\big]B_2 + R\big[(1 - \delta\psi_2)R_{M,1} - R\big]B_1 + R^2(N_1^b + N_1^c). \end{aligned} \tag{3.C.48}$$

3.D Social Planner Appendix

Note: in the following, because initial shadow banking sector aggregate net worth is assumed to be sufficiently small, a crisis always occurs when the high default state of the world is realized in period 2: $\psi_2 = \psi_H$ (i.e. condition 3.2.61 holds). And a crisis never occurs in the low default state: $\psi_2 = \psi_L$. The subscript H is used to denote the high default state of the world in period 2. This is interchangeable with a crisis event. The subscript L is used to denote the low default state of the world in period 2.

3.D.1 Social Planner Under Discretion

3.D.1.1 $t = 3$ Social Planner

$N_3, R_{M,2}, B_2$ are taken as given by the planner. The planner's problem[23] is:

$$\max \qquad (1-\delta\psi_2)\hat{u}\left(\frac{\hat{C}_3}{1-\delta\psi_2}\right) + \delta\psi_2\hat{u}(\hat{y}_3), \tag{3.D.1}$$

subject to the saver participation constraint[24]:

$$\tilde{C}_3 \geq R_{M,2}B_2, \tag{3.D.2}$$

the non-negativity constraint on saver consumption:

$$\tilde{C}_3 \geq 0, \tag{3.D.3}$$

And the t=3 resource constraint:

$$\tilde{C}_3 + \hat{C}_3 = (1-\delta\psi_2)\hat{Y}_3. \tag{3.D.4}$$

The social planner's allocation is obvious and trivial (the non-negativity constraint is always satisfied because debt is non-negative).

The social planner's optimal (aggregate) allocations are:

$$\tilde{C}_3^{sp3} = R_{M,2}B_2,$$

$$\hat{C}_3^{sp3} = (1-\delta\psi_2)\hat{Y}_3 - R_{M,2}B_2.$$

[23] Where $\hat{c}_3 = \frac{\hat{C}_3}{1-\delta\psi_2}$ has been substituted in.

[24] The participation constraint (3.3.4) has been simplified using the laws of motion for commercial and shadow banking net worth: (3.2.49) and (3.2.47)

3.D.1.2 $t = 2$ Social Planner

The t=2 social planner knows the realization of ψ_2. They take the states N_2^c, N_2^b, and the policy functions of the t=3 social planner $(\hat{C}_3^{sp3}, \tilde{C}_3^{sp3})$ as given. Their problem[25] is:

$$\max_{\hat{C}_2, B_2} \quad (1-\delta\psi_2)\Big[\hat{u}\Big(\frac{\hat{C}_2}{1-\delta\psi_2}\Big) + \hat{u}\Big(\frac{\hat{C}_3^{sp3}}{1-\delta\psi_2}\Big)\Big] + \delta\psi_2\Big[\hat{u}(\hat{y}_2) + \hat{u}(\hat{y}_3)\Big], \qquad (3.D.5)$$

subject to:

$$\hat{C}_3^{sp3} = (1-\delta\psi_2)\hat{Y}_3 - R_{M,2}B_2,$$

The consolidated resource and saver participation constraint[26]:

$$\hat{C}_2 \leq (1-\delta\psi_2)\Big[\hat{Y}_3 - R_{M,1}B_1\Big] + B_2, \qquad (3.D.6)$$

The non-negativity constraint on saver consumption (can be rewritten as):

$$B_2 \leq \tilde{Y}_2 + (1-\delta\psi_2)R_{M,1}B_1, \qquad (3.D.7)$$

and the interest rate schedule:

$$R_{M,2} = \begin{cases} R, & \text{if } \psi_2 = \psi_L \\ \max\Big(R, \frac{R}{1-\delta}\Big[1 - \frac{N_2^{c,H}(B_1)}{B_2}\Big]\Big), & \text{if } \psi_2 = \psi_H \end{cases}. \qquad (3.D.8)$$

Where commercial banking sector net worth in a crisis is:

$$N_2^{c,\mathrm{H}} = (1-\delta\psi_H)R_{M,1}B_1 - R(M_1 + D_1) = \big[(1-\delta\psi_H)R_{M,1} - R\big]B_1 + R(N_1^c + N_1^b).$$

Substituting the period 3 planner's policy function and the participation constraint into the utility function, we get the period 2 planner's lagrangian:

$$\mathcal{L} = \hat{u}\Big(\frac{(1-\delta\psi_2)\hat{Y}_2 + B_2 - (1-\delta\psi_2)R_{M,1}B_1}{1-\psi_2\delta}\Big) + \hat{u}\Big(\frac{(1-\delta\psi_2)\hat{Y}_3 - R_{M,2}B_2}{1-\psi_2\delta}\Big)$$
$$+ \gamma^{b2}\Big[\tilde{Y}_2 + (1-\delta\psi_2)R_{M,1}B_1 - B_2\Big]. \qquad (3.D.9)$$

[25] Where $\hat{c}_2 = \frac{\hat{C}_2}{1-\delta\psi_2}$ and $\hat{c}_3^{sp3} = \frac{\hat{C}_3^{sp3}}{1-\delta\psi_2}$ have been substituted in.

[26] Note that the savers' participation constraint $\tilde{C}_2 \geq R(M_1+D_1)-(M_2+D_2)+\tilde{Y}_2$ has been simplified as follows. First note that the aggregate balance sheet implies $M_2 + D_2 = B_2 - (N_2^b + N_2^c)$. Second note that the aggregate evolution of net worth implies: $N_2^b + N_2^c = (1-\delta\psi_2)R_{M,1}B_1 - R(M_1+D_1)$, which holds in both the crisis and non-crisis states. So $R(M_1+D_1)-(M_2+D_2) = R(M_1+D_1) - B_2 + (N_2^b + N_2^c) = (1-\delta\psi_2)R_{M,1}B_1 - B_2$. Then use RC2: $\tilde{C}_2 = \tilde{Y}_2 + \hat{Y}_2 - \hat{C}_2$ to substitute out saver consumption and get the consolidated constraint

$$\frac{\partial \mathcal{L}}{\partial B_2} = \hat{u}_c\left(\frac{(1-\delta\psi_2)\hat{Y}_2 + B_2 - (1-\delta\psi_2)R_{M,1}B_1}{1-\delta\psi_2}\right)\frac{1}{1-\delta\psi_2}$$
$$-\left[R_{M,2} + \frac{\partial R_{M,2}}{\partial B_2}B_2\right]\hat{u}_c\left(\frac{(1-\delta\psi_2)\hat{Y}_3 - R_{M,2}B_2}{1-\delta\psi_2}\right)\frac{1}{1-\delta\psi_2} - \gamma^{b2} = 0.$$

Focusing on equilibrium where the feasibility limit on debt does not bind, the planner's optimality condition is:

$$\hat{u}_{c,2} = \left[R_{M,2} + \frac{\partial R_{M,2}}{\partial B_2}B_2\right]\hat{u}_{c,3}. \tag{3.D.10}$$

Recall:

$$\frac{\partial R_{M,2}}{\partial B_2} = \begin{cases} 0, & \text{if } \psi_2 = \psi_L \\ \frac{R}{1-\delta}\frac{N^{c,H}}{(B_2)^2}, & \text{if } \psi_2 = \psi_H \text{ and the solvency constraint binds.} \end{cases} \tag{3.D.11}$$

When the default realization is low, the planner's optimality condition is identical to the competitive equilibrium optimality condition of the borrowers: $\hat{u}_{c,2} = R_{M,2}\hat{u}_{c,3}$. The planner cannot improve on the private market outcome. Note in this case $R_{M,2} = R$.

When the default realization is high, the planner's optimality condition can differ from the competitive equilibrium optimality condition:

$$\hat{u}_{c,2} = \begin{cases} \frac{R}{1-\delta}\hat{u}_{c,3}, & \text{if } B_2^H > \frac{N_2^{c,H}}{\delta}, \\ R\hat{u}_{c,3}, & \text{if } B_2^H \leq \frac{N_2^{c,H}}{\delta}. \end{cases} \tag{3.D.12}$$

Specifically, when the solvency constraint binds, the planner has a motive to hold back demand for loans in order to lower the total repayment cost of loans.

The period 2 planner's policy function can be expressed in terms of the period 1 planner's choice variables and the period 2 planner's choice of debt (the consolidated participation and resource constraint always binds because the planner wants to give the borrower as much consumption as possible):

$$\hat{C}_2^{sp2} = (1-\delta\psi_2)\hat{Y}_2 - (1-\delta\psi_2)R_{M,1}B_1 + B_2^{sp2}. \tag{3.D.13}$$

Where B_2^{sp2} is the period 2 planner's optimal choice of loans.

3.D.1.3 $t = 1$ Social Planner – With Shadow Banks

The period 1 planner does not know the realization of ψ_2. They take the policy functions of future planners as given. Their objective is[27]:

$$\max_{\hat{C}_1,\tilde{C}_1,B_1} \quad \hat{u}(\hat{C}_1) + E_1\Bigg\{(1-\delta\psi_2)\Bigg[\hat{u}\Bigg(\frac{\hat{C}_2^{sp2}}{1-\delta\psi_2}\Bigg) + \hat{u}\Bigg(\frac{\hat{C}_3^{sp3}}{1-\delta\psi_2}\Bigg)\Bigg] + \delta\psi_2\Big[\hat{u}(\hat{y}_2) + \hat{u}(\hat{y}_3)\Big]\Bigg\}, \tag{3.D.14}$$

subject to:

$$\tilde{C}_1 \geq \tilde{Y}_1 - (M_1 + D_1), \tag{3.D.15}$$

$$B_1 \leq \tilde{Y}_1 + N_1^c + N_1^b, \tag{3.D.16}$$

$$\hat{C}_1 + \tilde{C}_1 = \tilde{Y}_1 + N_1^c + N_1^b, \tag{3.D.17}$$

$$R_{M,1} = \frac{R}{1-\delta\psi_L}, \tag{3.D.18}$$

$$B_1 = N_1^c + N_1^b + D_1 + M_1. \tag{3.D.19}$$

The t=1 social planner's lagrangian, where the consolidated resource constraint (3.D.17), saver participation constraint (3.D.15), and aggregate balance sheet (3.D.19) and the future planners' policy functions are substituted in, is:

$$\begin{aligned}
\mathcal{L} = \hat{u}\big(B_1\big) + \mathbb{P}\Bigg((1-\delta\psi_H)\Bigg[\hat{u}\Bigg(\frac{(1-\delta\psi_H)\hat{Y}_2 + B_2^{sp2} - (1-\delta\psi_H)R_{M,1}B_1}{1-\delta\psi_H}\Bigg)& \\
+ \hat{u}\Bigg(\frac{(1-\delta\psi_H)\hat{Y}_3 - R_{M,2}B_2^{sp2}}{1-\delta\psi_H}\Bigg)\Bigg] + \delta\psi_H\Big[\hat{u}(\hat{y}_2) + \hat{u}(\hat{y}_3)\Big]\Bigg)& \\
+ (1-\mathbb{P})\Bigg((1-\delta\psi_L)\Bigg[\hat{u}\Bigg(\frac{(1-\delta\psi_L)\hat{Y}_2 + B_2^{sp2} - (1-\delta\psi_L)R_{M,1}B_1}{1-\delta\psi_L}\Bigg)& \\
+ \hat{u}\Bigg(\frac{(1-\delta\psi_L)\hat{Y}_3 - R_{M,2}B_2^{sp2}}{1-\delta\psi_L}\Bigg) + \delta\psi_L\Big[\hat{u}(\hat{y}_2) + \hat{u}(\hat{y}_3)\Big]\Bigg)& \\
+ \gamma^{b1}\Big[\tilde{Y}_1 + N_1^c + N_1^b - B_1\Big].&
\end{aligned}$$

[27]Where $\hat{c}_1 = \hat{C}_1$, $\hat{c}_2^{sp2} = \frac{\hat{C}_2^{sp2}}{1-\delta\psi_2}$, and $\hat{c}_3^{sp3} = \frac{\hat{C}_3^{sp3}}{1-\delta\psi_2}$ have been substituted in.

Note that under high default:

$$R_{M,2}B_2 = \begin{cases} RB_2, & \text{if the solvency constraint is slack: } B_2^H \leq \frac{N_2^{c,H}}{\delta}, \\ \frac{R}{1-\delta}\Big[B_2 - N_2^{c,H}\Big], & \text{if the solvency constraint binds: } B_2^H > \frac{N_2^{c,H}}{\delta}. \end{cases} \tag{3.D.20}$$

If the solvency constraint is slack under the t=2 social planner's choice of debt, then there is no difference between the t=1 social planner's choice of B_1 and the competitive equilibrium, focusing on the case where the solvency constraint is binding:

$$\begin{aligned} \mathcal{L} = \hat{u}\big(B_1\big) + \mathbb{P}\Bigg((1-\delta\psi_H)\Bigg[\hat{u}\Bigg(\frac{(1-\delta\psi_H)\hat{Y}_2 + B_2^{sp2,H} - (1-\delta\psi_H)R_{M,1}B_1}{1-\delta\psi_H}\Bigg) \\ + \hat{u}\Bigg(\frac{(1-\delta\psi_H)\hat{Y}_3 - \frac{R}{1-\delta}\Big[B_2^{sp2,H} - N_2^{c,H}\Big]}{1-\delta\psi_H}\Bigg)\Bigg] + \delta\psi_H\Big[\hat{u}(\hat{y}_2) + \hat{u}(\hat{y}_3)\Big]\Bigg) \\ + (1-\mathbb{P})\Bigg((1-\delta\psi_L)\Bigg[\hat{u}\Bigg(\frac{(1-\delta\psi_L)\hat{Y}_2 + B_2^{sp2,L} - (1-\delta\psi_L)R_{M,1}B_1}{1-\delta\psi_L}\Bigg) \\ + \hat{u}\Bigg(\frac{(1-\delta\psi_L)\hat{Y}_3 - RB_2^{sp2,L}}{1-\delta\psi_L}\Bigg) + \delta\psi_L\Big[\hat{u}(\hat{y}_2) + \hat{u}(\hat{y}_3)\Big]\Bigg) \\ + \gamma^{b1}\Big[\tilde{Y}_1 + N_1^c + N_1^b - B_1\Big]. \end{aligned}$$

Where

$$N_2^{c,\mathrm{H}} = \big[(1-\delta\psi_H)R_{M,1} - R\big]B_1 + RN_1 \implies \quad \frac{\partial N_2^{c,\mathrm{H}}}{\partial B_1} = (1-\delta\psi_H)R_{M,1} - R < 0.$$

FoC wrt B_1

$$\begin{aligned} \hat{u}_{c,1} = \mathbb{P}(1-\psi_H\delta)\Bigg(\hat{u}_{c,2}^H\Big[(1-\delta\psi_H)R_{M,1} - \frac{\partial B_2^{sp2,\mathrm{H}}}{\partial B_1}\Big]\frac{1}{1-\psi_H\delta} \\ + \hat{u}_{c,3}^H\frac{R}{1-\delta}\Big[\frac{\partial B_2^{sp2,\mathrm{H}}}{\partial B_1} - \frac{\partial N_2^{c,H}}{\partial B_1}\Big]\frac{1}{1-\psi_H\delta}\Bigg) \\ + (1-\mathbb{P})(1-\psi_L\delta)\Bigg(\hat{u}_{c,2}^{\mathrm{L}}\Big[(1-\psi_L\delta)R_{M,1} - \frac{\partial B_2^{sp2,\mathrm{L}}}{\partial B_1}\Big]\frac{1}{1-\psi_L\delta} \\ + \hat{u}_{c,3}^{\mathrm{L}}R\frac{\partial B_2^{sp2,\mathrm{L}}}{\partial B_1}\frac{1}{1-\psi_L\delta}\Bigg) + \gamma^{b1}. \end{aligned}$$

Rearranging (and focusing on equilibrium where the feasibility constraint does not bind ($\gamma^{b1} = 0$):

$$\hat{u}_{c,1} = \mathbb{P}\Bigg(\hat{u}^{H}_{c,2}\Big[(1-\delta\psi_H)R_{M,1} - \frac{\partial B_2^{sp2,\mathrm{H}}}{\partial B_1}\Big] + \hat{u}^{H}_{c,3}\frac{R}{1-\delta}\Big[\frac{\partial B_2^{sp2,\mathrm{H}}}{\partial B_1} - \frac{\partial N_2^{c,H}}{\partial B_1}\Big]\Bigg)$$
$$+ (1-\mathbb{P})\Bigg(\hat{u}^{\mathrm{L}}_{c,2}\Big[(1-\psi_L\delta)R_{M,1} - \frac{\partial B_2^{sp2,\mathrm{L}}}{\partial B_1}\Big] + \hat{u}^{\mathrm{L}}_{c,3}R\frac{\partial B_2^{sp2,\mathrm{L}}}{\partial B_1}\Bigg).$$

Recall the t=2 social planner's FoCs:

$$\hat{u}^{\mathrm{H}}_{c,2} = \hat{u}^{\mathrm{H}}_{c,3}\frac{R}{1-\delta},$$
$$\hat{u}^{\mathrm{L}}_{c,2} = \hat{u}^{\mathrm{L}}_{c,3}R.$$

Plug the above into the FoCs:

$$\hat{u}_{c,1} = \mathbb{P}\Bigg(\hat{u}^{H}_{c,2}\Big[(1-\delta\psi_H)R_{M,1} - \frac{\partial B_2^{sp2,\mathrm{H}}}{\partial B_1}\Big] + \hat{u}^{\mathrm{H}}_{c,2}\Big[\frac{\partial B_2^{sp2,\mathrm{H}}}{\partial B_1} - \frac{\partial N_2^{c,H}}{\partial B_1}\Big]\Bigg)$$
$$+ (1-\mathbb{P})\Bigg(\hat{u}^{\mathrm{L}}_{c,2}\Big[(1-\psi_L\delta)R_{M,1} - \frac{\partial B_2^{sp2,\mathrm{L}}}{\partial B_1}\Big] + \hat{u}^{\mathrm{L}}_{c,2}\frac{\partial B_2^{sp2,\mathrm{L}}}{\partial B_1}\Bigg).$$

Canceling like terms, get the t=1 social planner's FoC:

$$\hat{u}_{c,1} = \mathbb{P}\hat{u}^{H}_{c,2}\left[(1-\delta\psi_H)R_{M,1} - \frac{\partial N_2^{c,H}}{\partial B_1}\right] + (1-\mathbb{P})\hat{u}^{\mathrm{L}}_{c,2}(1-\psi_L\delta)R_{M,1}.$$

3.D.1.4 $t = 1$ Social Planner – Without Shadow Banks

If shadow banks do not exist then the t=1 social planner internalizes a positive relationship between aggregate loan quantity (B_1) and the interest rate on loans ($R_{M,1}$) when the solvency constraint binds:

$$R_{M,1} = \max\left(R,\ \frac{R}{1-\delta}\left[1 - \frac{N_1^c}{B_1}\right]\right). \tag{3.D.21}$$

As before the social planner's Lagrangian is:

$$\mathcal{L} = \hat{u}\big(B_1\big) + \mathbb{P}\Bigg((1-\delta\psi_H)\Bigg[\hat{u}\Bigg(\frac{(1-\delta\psi_H)\hat{Y}_2 + B_2^{sp2,H} - (1-\delta\psi_H)R_{M,1}B_1}{1-\delta\psi_H}\Bigg)$$
$$+ \hat{u}\Bigg(\frac{(1-\delta\psi_H)\hat{Y}_3 - \frac{R}{1-\delta}\Big[B_2^{sp2,H} - N_2^{c,H}\Big]}{1-\delta\psi_H}\Bigg)\Bigg] + \delta\psi_H\Big[\hat{u}(\hat{y}_2) + \hat{u}(\hat{y}_3)\Big]\Bigg)$$
$$+ (1-\mathbb{P})\Bigg((1-\delta\psi_L)\Bigg[\hat{u}\Bigg(\frac{(1-\delta\psi_L)\hat{Y}_2 + B_2^{sp2,L} - (1-\delta\psi_L)R_{M,1}B_1}{1-\delta\psi_L}\Bigg)$$
$$+ \hat{u}\Bigg(\frac{(1-\delta\psi_L)\hat{Y}_3 - RB_2^{sp2,L}}{1-\delta\psi_L}\Bigg) + \delta\psi_L\Big[\hat{u}(\hat{y}_2) + \hat{u}(\hat{y}_3)\Big]\Bigg)$$
$$+ \gamma^{b1}\Big[\tilde{Y}_1 + N_1^c + N_1^b - B_1\Big].$$

And their FoC wrt B_1 is:

$$\hat{u}_{c,1} = \mathbb{P}(1-\delta\psi_H)\Bigg(\hat{u}^H_{c,2}\Bigg[(1-\delta\psi_H)\Big[R_{M,1} + \frac{\partial R_{M,1}}{\partial B_1}B_1\Big] - \frac{\partial B_2^{sp2,\mathrm{H}}}{\partial B_1}\Bigg]\frac{1}{1-\delta\psi_H}$$
$$+ \hat{u}^H_{c,3}\frac{R}{1-\delta}\Big[\frac{\partial B_2^{sp2,\mathrm{H}}}{\partial B_1} - \frac{\partial N_2^{c,H}}{\partial B_1}\Big]\frac{1}{1-\delta\psi_H}\Bigg)$$
$$+ (1-\mathbb{P})(1-\delta\psi_L)\Bigg(\hat{u}^{\mathrm{L}}_{c,2}\Bigg[(1-\delta\psi_L)\Big[R_{M,1} + \frac{\partial R_{M,1}}{\partial B_1}B_1\Big] - \frac{\partial B_2^{sp2,\mathrm{L}}}{\partial B_1}\Bigg]\frac{1}{1-\delta\psi_L}$$
$$+ \hat{u}^{\mathrm{L}}_{c,3}\frac{R}{1-\delta}\Big[\frac{\partial B_2^{sp2,\mathrm{L}}}{\partial B_1} - \frac{\partial N_2^{c,L}}{\partial B_1}\Big]\frac{1}{1-\delta\psi_L}\Bigg) + \gamma^{b1}.$$

Note that:

$$N_2^c = (1-\psi_2)\Big[R_{M,1}B_1 - RD_1\Big] + T_2^c = \big[(1-\delta\psi_2)R_{M,1} - R\big]B_1 + RN_1^c, \tag{3.D.22}$$

(where $B_1 = B_1^c$ because only commercial banks exist).

So:

$$\frac{\partial N_2^c}{\partial B_1} = (1-\delta\psi_2)R_{M,1} - R \lesseqgtr 0. \tag{3.D.23}$$

Further note:

$$R_{M,1} + \frac{\partial R_{M,1}}{\partial B_1}B_1 = \frac{R}{1-\delta}. \tag{3.D.24}$$

Using the above and the t=2 social planner's FoC to make substitutions the FoC becomes:

$$\hat{u}_{c,1} = \mathbb{P}\hat{u}^H_{c,2}\Bigg[(1-\delta\psi_H)R_{M,1} + \Bigg((1-\delta\psi_H)\frac{\partial R_{M,1}}{\partial B_1}B_1 - \big[(1-\delta\psi_H)R_{M,1} - R\big]\Bigg)\Bigg]$$
$$+ (1-\mathbb{P})\hat{u}^{\mathrm{L}}_{c,2}\Bigg[(1-\delta\psi_L)R_{M,1} + \Bigg((1-\delta\psi_L)\frac{\partial R_{M,1}}{\partial B_1}B_1 - \big[(1-\delta\psi_L)R_{M,1} - R\big]\Bigg)\Bigg] + \gamma^{b1}.$$

The case for planner intervention depends on the level of the interest rate. There are 4 possible cases:

1. The solvency constraint is slack: $R_{M,1} = R$ and $\frac{\partial R_{M,1}}{\partial B_1} = 0$, $\frac{\partial N_2^{c,j}}{\partial B_1} < 0$ for both $j = L, H$. The planner has a motive to reduce debt in order to limit period 2 losses to commercial bank equity.

2. The solvency constraint binds: $R_{M,1} > R$ and $\frac{\partial R_{M,1}}{\partial B_1} > 0$. The planner has two motives for intervention:

   (a) Because $\frac{\partial R_{M,1}}{\partial B_1} > 0$, the planner wants to limit debt to lower the cost of credit.

   (b) $\frac{\partial N_2^{c,j}}{\partial B_1}$

      i. If the equilibrium interest rate is sufficiently low such that $\frac{\partial N_2^{c,j}}{\partial B_1} < 0$, for both $j = L, H$. The planner has a motive to reduce debt in order to limit period 2 losses to commercial bank equity.

      ii. If $(1 - \delta\psi_L)R_{M,1} - R = 0$, then $(1 - \delta\psi_H)R_{M,1} - R < 0$. So the planner still has a motive to reduce debt in order to limit period 2 losses in the high default state.

      iii. If $(1 - \delta\psi_L)R_{M,1} - R > 0$, and $(1 - \delta\psi_H)R_{M,1} - R < 0$,then the planner faces a trade-off between period 2 commercial bank net worth in the low default state (motive to increase B_1) and in the high default state (motive to decrease B_1).

      iv. If both $(1 - \delta\psi_L)R_{M,1} - R > 0$, and $(1 - \delta\psi_H)R_{M,1} - R > 0$ then the planner faces a trade-off between moderating the cost of credit (motive to decrease B_1) and increasing period 2 commercial bank net worth (motive to increase B_1).

The overall point here is that, unlike the world with shadow banks, in the world without shadow banks there is ambiguity to the sign of the period 1 planner's intervention.

3.E Analytical Solutions

3.E.1 Quasi-Linear Utility

$$\hat{U} \equiv \log \hat{c}_1 + (1-\delta\psi_2)\big[\log \hat{c}_2 + \hat{c}_3\big] + \delta\psi_2\big[\log \hat{y}_2 + \hat{y}_3\big]. \tag{3.E.1}$$

Using this functional form the borrower's first order conditions are:

$$\frac{1}{\hat{c}_1} = \mathbb{P}(1-\delta\psi_H)R_{M,1}\frac{1}{\hat{c}_2^H} + (1-\mathbb{P})(1-\delta\psi_L)R_{M,1}\frac{1}{\hat{c}_2^L}, \tag{3.E.2}$$

$$\frac{1}{\hat{c}_2} = R_{M,2}\big[1+\Xi_3\big]. \tag{3.E.3}$$

Where $\Xi_3 \geq 0$ is the multiplier on the non-negativity constraint for the borrower's period 3 consumption.

Note that this means the t=2 social planner's FoC (under a crisis) is:

$$\frac{1}{\hat{c}_2} = \frac{R}{1-\delta}. \tag{3.E.4}$$

3.E.1.1 If the non-negativity constraint is slack ($\Xi_3 = 0$)

Plugging the borrower's t=2 FoC into the borrower's t=1 FoC, and plugging in $B_1 = \hat{C}_1 = \hat{c}_1$

$$\frac{1}{B_1^*} = \mathbb{P}(1-\delta\psi_H)R_{M,1}R_{M,2}^* + (1-\mathbb{P})(1-\delta\psi_L)R_{M,1}R. \tag{3.E.5}$$

Where '*' indicates the competitive equilibrium level. Compare to the social planner's FoC:

$$\begin{aligned}\frac{1}{B_1^{sp1}} &= \mathbb{P}\left(\frac{R}{1-\delta}\Big[(1-\delta\psi_H)R_{M,1} - \underbrace{\frac{\partial N_2^{c,crisis}}{\partial B_1}}_{>0}\Big]\right) + (1-\mathbb{P})(1-\psi_L\delta)R_{M,1}R \\ &= \mathbb{P}\frac{R}{1-\delta}R + (1-\mathbb{P})(1-\psi_L\delta)R_{M,1}R.\end{aligned}$$

Subtracting the t=1 social planner's FoC from the borrower's FoC we get:

$$\begin{aligned}\frac{1}{B_1^*}-\frac{1}{B_1^{sp1}}&=\mathbb{P}\left[(1-\delta\psi_H)R_{M,1}R_{M,2}^*-\frac{R^2}{1-\delta}\right]\\&=\mathbb{P}\left[\frac{1-\delta\psi_H}{1-\delta\psi_L}R\frac{R}{1-\delta}\left[1-\frac{N_2^{c,H}}{B_2}\right]-\frac{R^2}{1-\delta}\right]\\&=\mathbb{P}\frac{R^2}{1-\delta}\left[\underbrace{\frac{1-\delta\psi_H}{1-\delta\psi_L}}_{<1}\underbrace{\left[1-\frac{N_2^{c,H}}{B_2}\right]}_{<1}-1\right]<0,\end{aligned}\tag{3.E.6}$$

which implies $\frac{1}{B_1^*}<\frac{1}{B_1^{sp1}}$ and therefore $B_1^{sp1}<B_1^*$.

3.E.1.2 If the non-negativity constraint binds ($\Xi_3>0$)

If the non-negativity constraint on period 3 consumption binds in competitive equilibrium, then $R_{M,2}b_2=\hat{y}_3$ and there are clear analytical solutions for B_2^L and B_2^H. Recall that: $b_2=\frac{B_2}{1-\delta\psi_2}$. So:

$$R_{M,2}B_2=\begin{cases}\hat{Y}_3(1-\delta\psi_L), & \text{if } \psi_2=\psi_L,\\ \hat{Y}_3(1-\delta\psi_H), & \text{if } \psi_2=\psi_H.\end{cases}\tag{3.E.7}$$

In the high case:

$$R_{M,2}B_2=\frac{R}{1-\delta}\left[B_2-N_2^{c,H}\right].\tag{3.E.8}$$

So we get:

$$B_2^L=\frac{(1-\delta\psi_L)\hat{Y}_3}{R},\tag{3.E.9}$$

$$B_2^H=\frac{1-\delta}{R}(1-\delta\psi_H)\hat{Y}_3+N_2^{C,H}.\tag{3.E.10}$$

So the borrower's choice of consumption in period 2 is (in aggregate):

$$\hat{C}_2^L=(1-\delta\psi_L)\left[\hat{Y}_2-R_{M,1}B_1\right]+\frac{(1-\delta\psi_L)\hat{Y}_3}{R},\tag{3.E.11}$$

$$\begin{aligned}\hat{C}_2^H&=(1-\delta\psi_H)\left[\hat{Y}_2-R_{M,1}B_1\right]+\frac{1-\delta}{R}(1-\delta\psi_H)\hat{Y}_3+N_2^{C,H},\\&=(1-\delta\psi_H)\left[\frac{(1-\delta)\hat{Y}_3}{R}+\hat{Y}_2\right]-RB_1+R(N_1^c+N_1^b).\end{aligned}\tag{3.E.12}$$

So the borrower's t=1 FoC is:

$$\frac{1}{B_1^*} = \frac{\mathbb{P}(1-\delta\psi_H)R_{M,1}}{\hat{C}^{*,H}/(1-\delta\psi_H)} + \frac{(1-\mathbb{P})(1-\delta\psi_L)R_{M,1}R}{\hat{C}^{*,L}/(1-\delta\psi_L)}. \tag{3.E.13}$$

And the social planner's

$$\frac{1}{B_1^{sp1}} = \frac{\mathbb{P}R}{\hat{C}^{sp2,H}/(1-\delta\psi_H)} + \frac{(1-\mathbb{P})(1-\psi_L\delta)R_{M,1}}{\hat{C}^{sp2,L}/(1-\delta\psi_L)}.$$

Note:

$$\frac{\partial \hat{C}_2^L}{\partial B_1} = -(1-\delta\psi_L)R_{M,1}, \tag{3.E.14}$$

$$\frac{\partial \hat{C}_2^H}{\partial B_1} = -R. \tag{3.E.15}$$

Note $(1-\delta\psi_H)R_{M,1} = \underbrace{\frac{1-\delta\psi_H}{1-\delta\psi_L}}_{<1} R < R$. So at the social planner's optimal choice of period 1 debt B_1^{sp1} the LHS of the borrower's FoC (3.E.13) is greater than the RHS. This implies again that $B_1^* > B_1^{sp1}$.

3.F Decentralized Policy

3.F.1 Decentralizing the Period 1 Social Planner's Choice

3.F.1.1 Regulating Borrowers in Period 1

Option 1: Debt Tax on Borrowers A tax on the funds generated by borrowing enters the borrower's problem as follows:

$$\max_{\hat{c}_1, b_1} \; E_1\hat{U}_1 \equiv E_1\Big\{\hat{u}(\hat{c}_1) + (1-\delta\psi_2)\big[\hat{u}(\hat{c}_2) + \hat{u}(\hat{c}_3)\big] + \delta\psi_2\big[\hat{u}(\hat{y}_2) + \hat{u}(\hat{y}_3)\big]\Big\}.$$

Individual Budget Constraints:

$$\hat{c}_1 \le b_1(1-\tau_1) + \underbrace{\hat{T}_1}_{\equiv \tau_1 b_1}, \qquad \hat{c}_2 \le \hat{y}_2 + b_2 - R_{M,1}b_1, \qquad \hat{c}_3 \le \hat{y}_3 - R_{M,2}b_2.$$

The tax revenues are rebated lump-sum (T_1) to the borrower. In general lump-sum transfers are essential to ensure that the tax does not reduce borrower utility at the social planner's optimal level of debt. The borrower's Euler under the tax is:

$$\begin{aligned}(1-\tau_1)\hat{u}_{c,1} &= \mathbb{P}\hat{u}^H_{c,2}(1-\psi_H\delta)R_{M,1} + (1-\mathbb{P})\hat{u}^{\mathrm{L}}_{c,2}(1-\psi_L\delta)R_{M,1} \\ \Longrightarrow \hat{u}_{c,1} &= \frac{1}{(1-\tau_1)}\Big[\mathbb{P}\hat{u}^H_{c,2}(1-\psi_H\delta)R_{M,1} + (1-\mathbb{P})\hat{u}^{\mathrm{L}}_{c,2}(1-\psi_L\delta)R_{M,1}\Big].\end{aligned}$$

The social planner's first order condition is:

$$\hat{u}_{c,1} = \mathbb{P}\hat{u}^H_{c,2}R + (1-\mathbb{P})\hat{u}^{\mathrm{L}}_{c,2}(1-\psi_L\delta)R_{M,1}.$$

The optimal tax is set so that the decentralized Euler is equivalent to the social planner's optimality condition. This requires:

$$\begin{aligned}(1-\tau_1) &= \frac{\mathbb{P}\hat{u}^H_{c,2}(1-\delta\psi_H)R_{M,1} + (1-\mathbb{P})\hat{u}^{\mathrm{L}}_{c,2}(1-\delta\psi_L)R_{M,1}}{\mathbb{P}\hat{u}^H_{c,2}R + (1-\mathbb{P})\hat{u}^{\mathrm{L}}_{c,2}(1-\delta\psi_L)R_{M,1}} \\ \Longrightarrow \tau_1 &= \frac{\mathbb{P}\hat{u}^H_{c,2}\big[R - (1-\delta\psi_H)R_{M,1}\big]}{\mathbb{P}\hat{u}^H_{c,2}R + (1-\mathbb{P})\hat{u}^{\mathrm{L}}_{c,2}(1-\delta\psi_L)R_{M,1}}.\end{aligned} \tag{3.F.1}$$

Finally:

$$\tau_1 = \frac{-\mathbb{P}\hat{u}^H_{c,2}\frac{\partial N_2^{cH}}{\partial B_1}}{\mathbb{P}\hat{u}^H_{c,2}R + (1-\mathbb{P})\hat{u}^{\mathrm{L}}_{c,2}(1-\delta\psi_L)R_{M,1}}. \tag{3.F.2}$$

Option 2: Credit Limit The credit limit modifies the borrower's problem as follows:

$$\max_{\hat{c}_1, b_1} \quad E_1\hat{U}_1 \equiv E_1\Big\{\log(\hat{c}_1) + (1-\psi_2\delta)\big[\log(\hat{c}_2) + \log(\hat{c}_3)\big] + \psi_2\delta\big[\log(\hat{y}_2) + \log(\hat{y}_3)\big]\Big\}.$$

Individual Budget Constraints:

$$\hat{c}_1 \leq b_1, \qquad \hat{c}_2 \leq \hat{y}_2 + b_2 - R_{M,1}b_1, \qquad \hat{c}_3 \leq \hat{y}_3 - R_{M,2}b_2.$$

Borrowing limit:

$$b_1 \leq \bar{b}_1, \qquad\qquad (\hat{\mu}_1 \geq 0). \tag{3.F.3}$$

The borrower's Euler for loans when the credit limit binds is:

$$\hat{u}_{c,1} = \mathbb{P}\hat{u}^H_{c,2}(1-\psi_H\delta)R_{M,1} + (1-\mathbb{P})\hat{u}^{\mathrm{L}}_{c,2}(1-\psi_L\delta)R_{M,1} + \hat{\mu}_1. \tag{3.F.4}$$

The credit limit is simply set equal to the optimal level of period 1 debt:

$$\bar{b_1} \equiv B_1^{sp1}. \tag{3.F.5}$$

3.F.1.2 Regulating Shadow Banks in Period 1

A shadow bank's period 1 problem is equivalent to maximizing the discounted expected net worth the next period:

$$\max_{b_1^b, m_1} \quad RE_1 n_2^b,$$

subject to their balance sheet identity (which is modified by the tax):

$$(1+\tau_1^b)b_1^b = (1-\tau_1^m)m_1 + n_1^b. \tag{3.F.6}$$

Given the evolution of individual shadow bank net worth:

$$RE_1 n_2^b = (1-\mathbb{P})RE_1\left\{R\left(\left[(1-\delta\psi_2)R^{tax}_{M,1} - R\frac{1+\tau_1^b}{1-\tau_1^m}\right]b_1^b + \frac{R}{1-\tau_1^m}n_1^b\right)|\psi_2 = \psi_L\right\},$$

FoC implies:

$$R^{tax}_{M,1} = \underbrace{\frac{R}{1-\delta\psi_L}}_{=R_{M,1}}\frac{1+\tau_1^b}{1-\tau_1^m}.$$

Let $R_{M,1}^{tax}$ be the equilibrium interest rate under a tax. This means the borrower's Euler is:

$$\begin{aligned}\hat{u}_{c,1} &= \mathbb{P}\hat{u}_{c,2}^{H}(1-\delta\psi_H)R_{M,1}^{tax} + (1-\mathbb{P})\hat{u}_{c,2}^{\mathrm{L}}(1-\delta\psi_L)R_{M,1}^{tax} \\ &= \left[\mathbb{P}\hat{u}_{c,2}^{H}(1-\delta\psi_H)R_{M,1} + (1-\mathbb{P})\hat{u}_{c,2}^{\mathrm{L}}(1-\delta\psi_L)R_{M,1}\right]\frac{1+\tau_1^b}{1-\tau_1^m}.\end{aligned} \tag{3.F.7}$$

So to achieve optimality set $\frac{1+\tau_1^b}{1-\tau_1^m} = \frac{1}{1-\tau_1}$. This is reliant on lump-sum transfers from shadow banks to borrowers in period 2 so that the period 2 level of consumption is not reduced by the increase in the interest rate. The required lump-sum transfer is:

$$T_2^b \equiv \left[R_{M,1}^{tax} - R_{M,1}\right]B_1^{sp1} = \frac{R}{1-\delta\psi_L}\left[\frac{1+\tau_1^b}{1-\tau_1^m} - 1\right]B_1^{sp1}. \tag{3.F.8}$$

The policymaker lump-sum transfers the in-period revenue of taxes to savers:

$$\tilde{T}_1 \equiv \tau_1^b B_1^b + \tau_1^m M_1. \tag{3.F.9}$$

3.F.2 Decentralizing the Period 2 Social Planner's Choice

Focus on the crisis state of the world (a planner has no reason to intervene in the non-crisis state).

3.F.2.1 Regulating Borrowers in Period 2

Option 1: Debt Tax on Borrowers Surviving borrowers' problem:

$$\max \quad \hat{u}(\hat{c}_2) + \hat{u}(\hat{c}_3),$$

subject to:

$$\hat{c}_2 = (1-\tau_2)b_2 + \hat{y}_2 - R_{M,1}b_1 + \underbrace{\hat{T}_2}_{=\tau_2 b_2}, \tag{3.F.10}$$

$$\hat{c}_3 = \hat{y}_3 - R_{M,2}b_2. \tag{3.F.11}$$

First order condition:

$$(1-\tau_2)\hat{u}_{c,2} = R_{M,2}\hat{u}_{c,3}. \tag{3.F.12}$$

Compare to the t=2 social planner's FoC:

$$\hat{u}_{c,2} = \Big[R_{M,2} + \frac{\partial R_{M,2}}{\partial B_2} B_2\Big] \hat{u}_{c,3}. \tag{3.F.13}$$

$\frac{\partial R_{M,2}}{\partial B_2} > 0$ as long as the solvency constraint on commercial banks binds. Set the tax such that:

$$\frac{R_{M,2}}{(1-\tau_2)} = \Big[R_{M,2} + \frac{\partial R_{M,2}}{\partial B_2} B_2\Big]. \tag{3.F.14}$$

Note that $R_{M,2} = \frac{R}{1-\delta}\Big[1 - \frac{N_2^{cH}}{B_2}\Big]$, and $R_{M,2} + \frac{\partial R_{M,2}}{\partial B_2} B_2 = \frac{R}{1-\delta}$. So the above expression simplifies to:

$$1 - \tau_2 = \frac{R_{M,2}}{R/(1-\delta)} = 1 - \frac{N_2^{cH}}{B_2}. \tag{3.F.15}$$

The optimal tax in equal to the inverse of commercial bank leverage at the social planner's optimal allocation:

$$\boxed{\tau_2 = \frac{N_2^{cH}}{B_2^{H,sp2}}}. \tag{3.F.16}$$

Option 2: Credit Limit The credit limit modifies the surviving borrower's problem as follows:

$$\max \quad \hat{u}(\hat{c}_2) + \hat{u}(\hat{c}_3).$$

Individual Budget Constraints:

$$\hat{c}_2 \leq \hat{y}_2 + b_2 - R_{M,1} b_1, \qquad \hat{c}_3 \leq \hat{y}_3 - R_{M,2} b_2.$$

Borrowing limit:

$$b_2 \leq \bar{b}_2 \qquad\qquad (\hat{\mu}_2 \geq 0). \tag{3.F.17}$$

The borrower's Euler for loans when the credit limit binds is:

$$\hat{u}_{c,2} = R_{M,2} \hat{u}_{c,3} + \hat{\mu}_2. \tag{3.F.18}$$

The credit limit per borrower is simply set to achieve the optimal level of debt in aggregate:

$$\bar{b}_2 \equiv \frac{B_2^{sp2,H}}{1 - \delta \psi_H}. \tag{3.F.19}$$

3.F.2.2 Regulating Commercial Banks in Period 2

Option 1: Balance Sheet Taxes An individual commercial bank's problem in period 2:

$$\max \quad E_2 n_3^c = R_{M,2}^{tax} b_2^c - R d_2,$$

subject to:

$$(1+\tau_2^b) b_2^c = n_2^c + (1-\tau_2^d) d_2, \tag{3.F.20}$$

$$(1-\delta) R_{M,2}^{tax} b_2^c \geq R d_2. \tag{3.F.21}$$

Using the budget constraint to substitute out for d_2:

$$\max \quad \left[R_{M,2}^{tax} - \frac{1+\tau_2^b}{1-\tau_2^d} R \right] b_2^c + \frac{R n_2^c}{1-\tau_2^d}, \tag{3.F.22}$$

subject to the solvency constraint:

$$\left[(1-\delta) R_{M,2}^{tax} - \frac{1+\tau_2^b}{1-\tau_2^d} R \right] b_2^c + \frac{R}{1-\tau_2^d} n_2^c \geq 0. \tag{3.F.23}$$

FoC:

$$\left[R_{M,2}^{tax} - \frac{1+\tau_2^b}{1-\tau_2^d} R \right] + \gamma_2^c \left[(1-\delta) R_{M,2}^{tax} - \frac{1+\tau_2^b}{1-\tau_2^d} R \right] = 0. \tag{3.F.24}$$

Implies:

$$\gamma_2^c = \frac{R_{M,2}^{tax} - \frac{1+\tau_2^b}{1-\tau_2^d} R}{\frac{1+\tau_2^b}{1-\tau_2^d} R - (1-\delta) R_{M,2}^{tax}}. \tag{3.F.25}$$

Aggregate solvency constraint:

$$\left[(1-\delta) R_{M,2}^{tax} - \frac{1+\tau_2^b}{1-\tau_2^d} R \right] B_2 + \frac{R}{1-\tau_2^d} N_2^{c,H} \geq 0. \tag{3.F.26}$$

So under the balance sheet taxes the interest rate on loans is:

$$R_{M,2}^{tax} = \max \left(R, \frac{R/(1-\tau_2^d)}{1-\delta} \left[1 + \tau_2^b - \frac{N_2^{c,H}}{B_2} \right] \right). \tag{3.F.27}$$

$R_{M,2}^{tax}$ enters the borrowers budget constraint and FoC, so to achieve optimality solely through this tax the social planner will need to set $R_{M,2}^{tax} = \frac{R}{1-\delta}$. This means the following must hold:

$$\frac{1}{1-\tau_2^d}\left[1+\tau_2^b-\frac{N_2^{c,H}}{B_2}\right]=1. \tag{3.F.28}$$

$$\tau_2^d=0 \implies \tau_2^b=\frac{N_2^{c,H}}{B_2^{sp2,H}}, \tag{3.F.29}$$

$$\tau_2^b=0 \implies \tau_2^d=\frac{N_2^{c,H}}{B_2^{sp2,H}}. \tag{3.F.30}$$

This taxation strategy is feasible as long at the solvency constraint binds: $\gamma_2^c>0$ at the optimal tax. Given that $R_{M,2}^{tax}=\frac{R}{1-\delta}$, the multiplier is:

$$\gamma_2^c=\frac{\frac{1}{1-\delta}-\frac{1+\tau_2^b}{1-\tau_2^d}}{\frac{1+\tau_2^b}{1-\tau_2^d}-1}. \tag{3.F.31}$$

The denominator is always positive. So the feasibility of the taxation strategy relies on:

$$\frac{1}{1-\delta}>\frac{(1+\tau_2^b)}{(1-\tau_2^d)}. \tag{3.F.32}$$

(3.F.32) holds in the deposit tax case if:

$$\delta>\frac{N_2^{c,H}}{B_2^{sp2,H}}. \tag{3.F.33}$$

And (3.F.32) holds in the loan tax case if:

$$\frac{\delta}{1-\delta}>\frac{N_2^{c,H}}{B_2^{sp2,H}}. \tag{3.F.34}$$

The optimality of the commercial bank balance sheet tax is reliant on lump-sum transfers from commercial banks to borrowers in period 3 so that the period 3 level of consumption is not reduced by the increase in the interest rate. The required lump-sum transfer is:

$$T_3^c\equiv\left[R_{M,2}^{tax}-R_{M,2}\right]B_2^{sp2}. \tag{3.F.35}$$

The policymaker lump-sum transfers the in-period revenue of taxes to savers:

$$\tilde{T}_2\equiv\tau_1^bB_2^c+\tau_2^dD_2. \tag{3.F.36}$$

Option 2: Regulate Commercial Banks - Stressed Solvency Constraint An individual commercial bank's problem in period 2:

$$\max \quad E_2 n_3^c = R_{M,2}^s b_2^c - Rd_2,$$

subject to:

$$b_2^c = n_2^c + d_2, \tag{3.F.37}$$

$$(1-\delta^s) R_{M,2}^s b_2^c . \geq Rd_2 \tag{3.F.38}$$

Where $\delta^s > \delta$ captures that the solvency constraint is being stressed. Assume there is some floor on δ^s is δ.

Using the budget constraint to substitute out for d_2:

$$\max \quad \Big[R_{M,2}^s - R \Big] b_2^c + Rn_2^c. \tag{3.F.39}$$

subject to:

$$\Big[(1-\delta^s) R_{M,2}^s - R \Big] b_2^c + Rn_2^c \geq 0. \tag{3.F.40}$$

FoC:

$$\Big[R_{M,2}^s - R \Big] + \gamma_2^c \Big[(1-\delta^s) R_{M,2} - R \Big] = 0. \tag{3.F.41}$$

Implies:

$$\gamma_2^c = \frac{R_{M,2}^s - R}{R - (1-\delta^s) R_{M,2}}. \tag{3.F.42}$$

Aggregate SC:

$$R_{M,2}^s = \frac{R}{1-\delta^s} \left[1 - \frac{N_2^{c,H}}{B_2} \right]. \tag{3.F.43}$$

Social planner is targeting the interest rate that enters the borrower's FoC: $R_{M,2}^s = \frac{R}{1-\delta}$. So set δ^s such that:

$$\frac{R}{1-\delta} = \frac{R}{1-\delta^s} \Big[1 - \frac{N_2^{c,H}}{B_2} \Big]. \tag{3.F.44}$$

Which implies:

$$\delta^s = \delta + (1-\delta)\frac{N^{c,H}}{B_2^{sp2,H}}. \tag{3.F.45}$$

Note: $\delta^s > \delta$. The excess profits of commercial banks must in period 3 be lump-sum transferred to borrowers:

$$T_3 \equiv \left[R^s_{M,2} - R_{M,2}\right] B_2^{sp2}. \tag{3.F.46}$$

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