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POSITIVE HARMONY TRANSFORMATIONS AND EQUILIBRIUM SELECTION IN TWO-PLAYER GAMES

Daniel John Zizzo

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Manor Road Building, Oxford OX1 3UQ

Positive Harmony Transformations and Equilibrium Selection in Two-Player Games

By Daniel John Zizzo

University of Oxford

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Abstract

Game harmony is a generic game property that describes how harmonious (non-conflictual) or disharmonious (conflictual) the interests of players are, as embodied in the payoffs. It can be used to predict cooperation in two-player games. We show how, for large enough positive harmony transformations of the game, a utilitarian solution is always a Nash equilibrium, coincides with the Nash bargaining solution and acquires further desirable properties of payoff and risk dominance. Case-based reasoning and team reasoning are alternative mechanisms by which game harmony measures can successfully predict cooperative behavior.

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1. Introduction

Game harmony is a generic game property that describes how harmonious (non-conflictual) or disharmonious (conflictual) the interests of players are, as embodied in the payoffs. Think of car drivers driving from opposite sides of a road, and choosing whether to drive to the right or to the left of the road: if one of them drives to the right and the other to the left, they crash, which is bad for both of them, while they both gain equally from coordinating into either driving to the right or to the left. This is an example of a pure coordination game: a game where the interests of the players are perfectly aligned (harmonious). Conversely, a zero-sum game or other constant-sum game is one where the interests of the players are perfectly disharmonious, i.e. they go exactly in the opposite direction. The intuition of measures of game harmony is that one can try to classify games in a *simple* way according to the degree to which the interests of the players are harmonious or not. For example, in games such as Prisoner's Dilemmas or Chicken Games, there is partial conflict of interest but also partial alignment: the extent to which there is conflict or harmony of interests, as they are embedded in the payoffs, will depend on the particular game structure. The simplest measures of game harmony are an average of Pearson or Spearman correlation coefficients among payoff pairs, with higher numerical values indicating greater game harmony. In this paper our focus will be on the simplest possible measure for two-player games: the Pearson correlation coefficient between the payoffs of the two players.

We consider positive payoff transformations of the matrix, done simply by weighting in the utility function of an agent the payoff of the other agent according to some coefficient β, which changes according to the size of the payoff transformation. Payoff transformations of this kind have been used in a variety of settings, such as public good contribution (Collard, 1975), intergenerational altruism (Barro, 1974), workers' effort levels (Rotenberg, 1994) and social norms (Clark and Oswald, 1998). We show how positive payoff transformations may make a cooperative outcome more likely to be played out, and may help in equlibrium selection if multiple Nash equilibria are present. More specifically, we show that, for a large enough positive payoff transformations of the game, a utilitarian solution is always a Nash equilibrium, coincides with the Nash bargaining solution and acquires further desirable properties of payoff dominance and risk dominance. We also identify alternative mechanisms, based on case-based decision theory reasoning (Gilboa and Schmeidler, 1995, 2001) and team reasoning (Sugden, 1993; Bacharach, 1999, 2005), that explain how game harmony measures can successfully predict cooperative behavior. The idea in the first case is that more harmonious games are more likely to be assimilated to pure coordination game settings where cooperation is in order, and less likely to be assimilated to constant-sum games. In the second case, the idea is that higher game harmony is more likely to elicit team reasoning, i.e. a frame of mind where each player does his or her part to achieve the cooperative outcome, interpreted as the team's most preferred outcome. We illustrate our general analysis with examples based on a Prisoner's Dilemma, an Asymmetric Chicken Game and a coordination game.

We label payoff transformations as "harmony transformations", since we show that they can normally be mapped in an increase in game harmony, and so they may provide a rationale for why we may expect game harmony values to be associated with higher cooperation rates. Depending on the experimental dataset and definition of cooperative outcome used, Zizzo and Tan (2003) found that Pearson and Spearman correlation coefficients between simple game harmony measures and mean cooperation rates were between 0.557 and 0.962 ($P < 0.05$). They used both a dataset with some specific and well-known games games, including the PD, the Stag-Hunt, the Chicken, a coordination game and three variants of trust games, and a dataset with randomly generated games. Correlation coefficients between measured harmony and cooperation rates were above 0.8 in an experiment with games such as the Prisoner's Dilemma, the Stag-Hunt, a coordination game and three trust games, and above 0.55 in an experiment with random games. In a similar experimental condition with 3×3 games, eight out of ten of which were randomly determined (the remaining two were 3×3 variants of the PD), Pearson and Spearman correlation coefficients between simple game harmony measures and mean cooperation rate by game were in the 0.579-0.627 range ($P < 0.05$). Some 80% of the variance in behavior in Rapaport and Chamah's (1965) experimental dataset on the Prisoner's Dilemma play can also be explained with our simple basic measure of game harmony (Zizzo, 2003b).

Game harmony measures are related to, but simpler and more general, than Kelley and Thibaut's (1978) "index of correspondence", whereas Rapaport and Chammah's own measures are only applicable to 2×2 Prisoner's Dilemmas. Obviously ours is just one of many possible contributions to the problem of equilibrium selection when the payoff and risk dominant solutions do not coincide. As an example based on the theory of global games, Carlsson and van Damme (1993) show how in 2×2 games noisy observations of the game leads to the selection of the risk dominant equilibrium; this result may or may not generalize in the broader class of game considered by Frankel et al. (2003). As a different example based on the theory of team reasoning, Bacharach (1999) can be interpreted as providing a rationale for the choice of a payoff dominant solution even when there are "trembles" in team reasoning on the part of players.

This paper provides some theoretical ground for expecting why agents may tend to converge to cooperative outcomes, or to select one equilibrium rather than another, in more harmonious games. But, quite independently of any connection with harmony, it also clarifies in fairly general terms the relationship between standard payoff transformations and equilibrium selection concepts such as payoff dominance and risk dominance. In this sense, it is hoped that it may also be of some interest to readers not especially keen on the concept of game harmony.

The rest of this paper is organized as follows. Section 2 presents the theoretical setup used. Section 3 contains the main results of the paper on the relationship between harmony, Nash equilibrium, utilitarian solution, Nash bargaining solution, and risk and payoff dominance. Section 4 and section 5 are on the relationship between game harmony on the one side and case based reasoning and team reasoning, respectively, on the other. Sections 3 through 5 also present examples. Section 6 concludes.

2. Theoretical setup

2.1 The set of random games

Let F be a finite two-player game in normal form. Denote W_i the actions available to player i, so $W = W_1 \times W_2$ is the set of possible outcomes or states of the world, each of which we label by $w \in W$. There is no restriction, in principle, on the number of available actions. Payoffs are defined by $x_i : W \to \mathbb{R}$, a standard Von Neumann-Morgernstern utility function, and so x_{iw} is the payoff for player i in state of the world w.

Let $\Diamond_r k$ be an operator that rounds some real number k to a number k' with a given number of decimals r. In this paper a random game, Γ, is defined as a game $F : x_{i\tilde{w}} \Diamond_r U(0, 1)$: that is, payoffs are drawn from a uniform distribution standardized to the 0, 1 interval and then rounded to some arbitrary finite order r. We can then define a finite set of random games, A, as comprising all possible games that may be obtained by following this procedure. For example, if agents play simple 2×2 random games and $r = 3$, A is made of 1001^8 games. Let $Q \subset A$ be some subset of A for all elements of which some property q is true. In other words, each game $\Gamma \in Q$ has q. For example, q could be the property that the Nash equilibrium of each game in Q coincides with the cooperative outcome in each game. $P(Q \mid A)$ is defined as the fraction of games in A belonging to Q: it gives the probability that, if a game is randomly drawn from A, it has q.

2.2 Definition and properties of game harmony

We require that payoffs do change at least once across states of the world in W for each player. This is, intuitively, a requirement for us to be able to talk meaningfully of a game as a situation where strategic interaction is involved: if a player obtains the same utility regardless of her actions and of those of the other players, then there is no issue of strategic play for that player (Zizzo, 2003b, weakens this condition slightly). Let N be the number of players.

Condition 1 (Variability) $\forall\ i \in N \quad \neg\,[k_i : x_{iw} = k_i\ \forall w \in W]$

In deriving a measure of game harmony, we need to consider the N players who receive a payoff, since we are interested in the covariation between combinations - more specifically, pairs - of payoffs. For N players, the number of payoff pairs is $[N(N-1)]/2$. Let us label the payoffs x_{iw} for each payoff pair c as a_w, b_w for $w \in W$. The means and variances of the marginal distributions are μ_a, μ_b and σ_a^2, σ_b^2. σ_{a+b}^2 is the variance of the sum of the payoffs in each pair. We can now state a further condition and then provide a simple definition of harmony.

Condition 2 (Second Moments) $\exists\ Q \geq 0 : \sigma_a^2, \sigma_b^2, \sigma_{a+b}^2 < Q\ \forall\ i \in N$

Definition 1 *The cardinal harmony G of game Γ is the arithmetic mean of the Pearson correlations between the C pairs of Γ:*

$$G(\Gamma) = G(x_i, W) = \frac{1}{C}\sum_{c=1}^{C} r(a_w, b_w) = \frac{1}{C}\sum_{c=1}^{C} \frac{Cov(a_w, b_w)}{\sigma_a \sigma_b} \tag{1}$$

Since W is countable and $N = 2$ in this paper, for any game in A (1) reduces to the simple Pearson correlation between players payoffs and can be computed as follows:

$$G(\Gamma) = \frac{|W|\displaystyle\sum_{w=1}^{|W|} a_w b_w - \sum_{w=1}^{|W|} a_w \sum_{w=1}^{|W|} b_w}{\sqrt[2]{\left[|W|\displaystyle\sum_{w=1}^{|W|} a_w - \left(\sum_{w=1}^{|W|} a_w\right)^2\right]\left[|W|\displaystyle\sum_{w=1}^{|W|} b_w - \left(\sum_{w=1}^{|W|} b_w\right)^2\right]}} \tag{2}$$

In order for $G(\Gamma)$ to be defined, all we need is for Condition 1 to hold.

Proposition 1 *For any game $\Gamma \in A$, $G(\Gamma)$ is defined if Conditions 1 holds.*

Proof. See Appendix A. ■

One limitation of the simple measure we are considering is that it makes game harmony values sensitive to irrelevant duplication of strategies. Whenever this is considered a problem, it can be eliminated by transforming the normal form game to its *semi-reduced form* where strategically equivalent strategies get replaced by a single representative strategy (Ritzberger, 2002).

The higher $G(\Gamma)$, the higher the harmony of the game. As with two players game harmony values are just Pearson correlation coefficients, they range between -1 for constant-sum games and 1 for pure coordination game.

Definition 2 *Let a generic pure coordination game be defined by $CG : a_w = kb_w + h$ for all w and a generic constant-sum game be defined by $CS : \sum_{i=1}^{n} x_{iw} = q$ for all w, with k, h and q finite constants and $k > 0$.*

Proposition 2 *In pure coordination games $G(CG) = 1$.*

Proof. See Appendix A. ■

Proposition 3 *In constant-sum games and with $N = 2$, $G(CS) = -1$.*

Proof. See Appendix A. ■

We next characterize two basic properties of $G(\Gamma)$. The first property mirrors Axiom 2 of Luce and Raiffa's (1957, p. 287) desirable properties for decision-making rules.

Proposition 4 (Scale Independence) $G(\Gamma) = G(x_i, W) = G(k_i x_i + h_i, W)$ *for any finite $k_i > 0$ and h_i.*

Proof. See Appendix A. ■

Now consider what happens if a game Ψ is formed in terms of $k \times W$ states of the world W of the parent game Γ $\forall$ $k \in \mathbb{N}^{++}$, for example if the Battle of the Sexes game matrix Γ is duplicated to give Ψ (see Table 1).

This is a form of relabeling and, as such, it should not matter: see Luce and Raiffa's (1957) Axiom 11, p. 295. Since the semi-reduced normal form of Ψ is the same as that of Γ, finding the game harmony of the semi-reduced game would be sufficient to make the relabeling irrelevant. It turns out, however, that working on the semi-reduced form is not needed to get at least this invariance result and that, further, the invariance result can be generalized by allowing rescaling within each replica.

Proposition 5 (Replication) *Denote the operator* $\psi : \Gamma(x_i, W) \rightarrow \Psi = \gamma\Phi = \Gamma[x_i, k_i(\Phi), h_i(\Phi), \gamma \times W] \forall$ $\gamma \in \mathbb{N}^{++}$ *as the transformation of a parent game* Γ *into a game* Ψ *that is the aggregation of* γ*-replicas of* Γ*, where each replica* Φ *has payoffs* $k_i(\Phi)x_i + h_i(\Phi)$ *for any finite* $k_i > 0$ *and* h_i*. Then* $G(\Psi) = G(\Gamma)\ \forall\ \Psi$.

Proof. See Appendix A. ■

The scale independence and replication properties are both neutrality properties, and generalize to games with any N number of players (Zizzo, 2003b). Zizzo (2003b) has a discussion of further properties and also of alternative and more complex game harmony measures; in this paper, however, we shall stick to the simplest and most parsimonious of all measures, $G(\Gamma)$.

2.3 Harmony transformations

Define a *one-sided harmony transformation* mapping $x_{iw} \rightarrow x'_{iw}$ for any agent i and state of the world w and with $\beta_i \in [-1, 1]$ as

$$x'_{iw} = x_{iw} + \beta_i x_{jw} \tag{3}$$

If $\beta_i \in (0, 1]$, then the harmony transformation is positive; if $\beta_i \in [-1, 0)$, then it is negative. We label the case where $\beta_i > 0$ as a positive *harmony* transformation because $G(\Gamma') \geq G(\Gamma)$: there is an equivalence between a positive harmony transformation and an increase in game harmony, or, more precisely, a a positive harmony transformation corresponds to an increase in game harmony which operates purely through a change in β_i.

Proposition 6 (One-Sided Equivalence Result) *Assume that the untransformed payoffs* a_w *and* b_w *are given and that* $G(\Gamma')$ *and* $G(\Gamma)$ *are defined. Then* $G(\Gamma') \gtreqqless G(\Gamma) \leftrightarrow \beta_i \gtreqqless 0\ \forall\ i$*, and, if* $\beta_i < 1$ *and* $r(a_w, b_w) > -1$, $\partial\beta_i/\partial G(\Gamma') > 0$.

Proof. Since $G(\Gamma')$ is equal to

$$G(\Gamma') = r[a_w + \beta_i b_w, b_w + \beta_j a_w] \tag{4}$$

and $dv/da_w > 0$, $dv/db_w > 0$, and since (for $\beta_i \in (-1, 1)$) $\partial r/\partial \beta_i > 0$ (see Appendix A), it follows that $\partial G(\Gamma)/\partial \beta_i > 0$. Also, it is legitimate to apply the implicit function theorem by rewriting (4) as $G(\Gamma) - r[a_w + \beta_i b_w, b_w + \beta_i a] = 0$ and noting that $\partial r/\partial \beta_i > 0$ is well-defined, monotonic and continuous, so $\partial \beta_i/\partial G(\Gamma') > 0$. $G(\Gamma') \gtreqqless G(\Gamma) \leftrightarrow \beta_i \gtreqqless 0 \; \forall \; i$ follows. ■

Define a *two-sided harmony transformation* as one where $\beta_1 = \beta_2$ and so where

$$x'_{iw} = x_{iw} + \beta x_{jw} \forall i \; \& \; j \neq i \tag{5}$$

If $\beta \in (0, 1]$, then the harmony transformation is positive; if $\beta \in [-1, 0)$, then it is negative. A two-sided harmony transformation is equivalent to two one-sided harmony transformations being applied to a game together and under the restriction that $\beta_1 = \beta_2$.

Let us now conduct the following experiment. An operation $\Diamond : A \to A'$ maps each game Γ in A into a corresponding game Γ' in A', with each game being transformed according to a positive double-sided harmony transformation as defined by equation (5) and with $\beta \in (0, 1]$. The fraction of games in A' belonging to Q is $P(Q \mid A')$. In other words, if a game is randomly drawn from A', the probability that Γ' has q is $P(Q \mid A')$. Let $\triangle P(Q) = P(Q \mid A') - P(Q \mid A)$, i.e. the change in the fraction of games having q given $\Diamond$. We label $\triangle G(\Gamma) = G(\Gamma') - G(\Gamma)$.

Proposition 7 (Equivalence Result) *Assume that the untransformed payoffs a_w and b_w are given and that $G(\Gamma')$ and $G(\Gamma)$ are defined. Then $G(\Gamma') \gtreqqless G(\Gamma) \leftrightarrow \beta \gtreqqless 0 \; \forall \; i$, and, if $\beta < 1$, $\partial \beta/\partial G(\Gamma') > 0$.*

Proof. Consider again equation (4) and note that

$$\frac{dG(\Gamma')}{d\beta} = \frac{\partial G(\Gamma')}{\partial \beta_1}\frac{\partial \beta_1}{\partial \beta} + \frac{\partial G(\Gamma')}{\partial \beta_2}\frac{\partial \beta_2}{\partial \beta} \tag{6}$$

where $\beta_1 = \beta_2 = \beta$, and so $\partial \beta_1/\partial \beta = \partial \beta_2/\partial \beta = 1$. Therefore $dG(\Gamma')/d\beta = \partial G(\Gamma')/\partial \beta_1 + \partial G(\Gamma')/\partial \beta_2$, and, assuming $\beta_i \in (-1, 1)$, since $\partial G(\Gamma')/\partial\beta_1, \partial G(\Gamma')/\partial\beta_2 > 0$, then $dG(\Gamma')/d\beta > 0$, and using the same reasoning for the inverse function $d\beta/dG(\Gamma') > 0$ for $\beta_i \in (-1, 1)$. $G(\Gamma') \gtreqless G(\Gamma) \leftrightarrow \beta_i \gtreqless 0 \; \forall \; i$ follows. ■

The equivalence result stresses the correspondence between simple payoff transformation and perception of the game as more or less harmonious. It implies that, if for whatever reason (e.g., framing or simply different material payoffs) the agent perceives the game as more harmonious, she will behave as if she had more altruistic, or less envious, preferences. It also implies that, if one compares behavior in two games in A and one is the positive payoff transformation of another, we can assume harmony to be greater in the former than in the latter. The "size" of $\Diamond$ is determined by the size of β: the larger β is, the larger the size of the positive harmony transformation $\Diamond$. In the rest of this paper when we talk about harmony transformations we always refer to two-sided harmony transformations.

2.4 Equilibrium selection: some definitions

To investigate the relationship between game harmony and equilibrium selection, it is useful to define some solution concepts drawn both from cooperative and non-cooperative game theory. The utilitarian solution and the Nash bargaining solution have appeal in the definition of what outcome we may define as "cooperative", or at least in narrowing down the set of possible outcomes that we may define as such. We follow Harsanyi and Selten (1988) for the definition of payoff dominance and risk dominance as equilibrium selection criteria. A "general" payoff dominant outcome is one in relation to which the payoffs are better than any possible outcome, not just those in V, i.e. not just Nash equilibria outcomes; similarly, a Nash bargaining solution is one which has the global highest sum of payoffs, not just the highest sum of payoffs among Nash equilibria. We narrow down our attention to Nash equilibria in pure strategies.

Definition 3 *Define a utilitarian solution U as the i-th row and j-th column outcome (i, j) corresponding to the highest sum of payoffs:* $U : a^U_{ij} + b^U_{ij} \geq a_{kl} + b_{kl} \; \forall \; k, l.$

Definition 4 *The outcome (i, j) is the Nash bargaining solution iff it has the highest product of payoffs:* $a_{ij}b_{ij} \geq a_{kl}b_{kl} \forall \; k, l.$

Definition 5 *The outcome $(i, j) \in V$ is general payoff dominant iff $a_{ij} \geq a_{kl}$ and $b_{ij} \geq b_{kl}$ for any outcome* (k, l)

Definition 6 *The outcome* (i,j) *is a (pure) Nash equilibrium iff* $a_{ij} \geq a_{kj}$ *and* $b_{ij} \geq b_{il}$ $\forall k,l$*. Let* V *be the set of (pure) Nash equilibria. Then* $(a_{ij}, b_{ij}) \in V$*.*

Definition 7 *The outcome* $(i,j) \in V$ *is payoff dominant iff* $a_{ij} \geq a_{kl}$ *and* $b_{ij} \geq b_{kl}$ *for any outcome* $(k,l) \in V$*.*

Definition 8 *The outcome* $(i,j) \in V$ *is risk dominant iff* $a_{ij}b_{ij} \geq a_{kl}b_{kl}$ *for any outcome* $(k,l) \in V$*.*

3. Positive harmony transformations and equilibrium selection

3.1 Theory

If an action is a utilitarian solution U in Γ, the same outcome will be U in Γ': we need not worry that a positive harmony transformation will change the outcome (or set of outcomes) that can be characterized as utilitarian solution(s) to the game.

Proposition 8 *If* U *is a utilitarian solution to* Γ*, and* $\Diamond : \Gamma \rightarrow \Gamma'$ *is applied,* U *remains a utilitarian solution to* Γ'*.*

Proof. Since U is a utilitarian solution, $a_{ij}^U + b_{ij}^U \geq a_{kj} + b_{kj}$ $\forall$ $k \neq i$ and $a_{ij}^U + b_{ij}^U \geq a_{il} + b_{il}$ $\forall$ $l \neq j$. If $\Diamond$ is applied, the inequality relationships become $(1+\beta)(a_{ij}^U + b_{ij}^U) \geq (1+\beta)(a_{kj} + b_{kj})$ $\forall$ $k \neq i$ and $(1+\beta)(a_{ij}^U + b_{ij}^U) \geq (1+\beta)(a_{il} + b_{il})$ $\forall$ $l \neq j$, which still hold. ■

We can now prove some propositions on how, for large enough positive payoff transformations, the utilitarian solution becomes a Nash equilibrium and one that, in the presence of multiple equilibria, has some desirable properties.

Proposition 9 *For* $\Diamond$ *large enough, a utilitarian solution* U *is always a Nash equilibrium.*

Proof. Since U is a utilitarian solution, its corresponding payoffs $a_{ij}^U + b_{ij}^U$ are greater than the payoffs in any other state of the world, and so $a_{ij}^U + b_{ij}^U \geq a_{kj} + b_{kj}$ $\forall$ $k \neq i$ and $a_{ij}^U + b_{ij}^U \geq a_{il} + b_{il}$ $\forall$ $l \neq j$. Let $a_{Kj} = \max(a_{kj})$ and b_{Kj} be the corresponding payoff for player 2; let $b_{iL} = \max(b_{il})$ and a_{iL} be the corresponding payoff for player 1. Then $b_{ij}^U - b_{Kj} \geq a_{Kj} - a_{ij}^U$ and $a_{ij}^U - a_{iL} \geq b_{iL} - b_{ij}^U$. Assume that

U is not a Nash equilibrium. Then either $a^U_{ij} < a_{Kj}$ or $b_{il} < b_{iL}$ or both. Now apply $\Diamond$. We claim that, even if $a^U_{ij} < a_{Kj}$, $\exists\beta \in (0,1] : a^U_{ij} + \beta b^U_{ij} \geq a_{Kj} + \beta b_{kj}$. To see this, just solve this inequality for β: $\beta \geq (a_{Kj} - a^U_{ij})/(b^U_{ij} - b_{Kj})$. Since $b^U_{ij} - b_{Kj} > a_{Kj} - a^U_{ij}$, this will always hold for $\Diamond$ large enough. Similarly, even if $b_{il} < b_{iL}$, $\beta a^U_{ij} + b^U_{ij} \geq \beta a_{iL} + b_{iL}$ for $\Diamond$ large enough since $\exists\beta \in (0,1] : \beta \geq (b_{iL} - b^U_{ij})/(a^U_{ij} - a_{iL})$. ■

Proposition 10 *For $\Diamond$ large enough, a utilitarian outcome U is always a payoff dominant Nash equilibrium.*

Proof. We know from Proposition 9 that U is a Nash equilibrium for $\Diamond$ large enough; we now wish to prove that, for $\Diamond$ large enough, U payoff-dominates any other existing Nash equilibria even if $a^U_{ij} < a_{kl}$ or $b^U_{ij} < b_{kl}$ for some $k \neq i, l \neq j$. Assume that this is not the case for some $(k,l) \in V$ equilibrium. Let $a^U_{ij} < a_{kl}$. Then $b^U_{ij} > b_{kl}$ since U is a utilitarian solution and so $a^U_{ij} + b^U_{ij} \geq a_{kl} + b_{kl}$. The proof that U payoff dominates the (k,l) outcome then reduces to a proof that $a^U_{ij} + \beta b^U_{ij} \geq a_{kl} + \beta b_{kl}$ for $\Diamond$ large enough. Since $b^U_{ij} - b_{kl} \geq a_{kl} - a^U_{ij}$, this is always true as $\exists\beta \in (0,1] : \beta \geq (a_{kl} - a^U_{ij})/(b^U_{ij} - b_{kl})$. Similarly, if $b^U_{ij} < b_{kl}$ for some $k \neq i, l \neq j$, it is the case that $\beta a^U_{ij} + b^U_{ij} \geq \beta a_{kl} + b_{kl}$ for $\Diamond$ large enough since $\exists\beta \in (0,1] : \beta \geq (b_{kl} - b^U_{ij})/(a^U_{ij} - a_{kl})$. ■

Proposition 11 *For $\Diamond$ large enough, a utilitarian outcome U is always the general payoff dominant outcome.*

Proof. The proof is identical to that of Proposition 10, except that now (k,l) is not restricted to belong to V (see Appendix A for proof details). ■

Proposition 12 *For $\Diamond$ large enough, a utilitarian outcome U is always a risk dominant Nash equilibrium.*

Proof. We know from Proposition 9 that U is a Nash equilibrium for $\Diamond$ large enough; we now wish to prove that, for $\Diamond$ large enough, U risk-dominates any other existing Nash equilibria even if for some $k \neq i, l \neq j$, where outcome $(k,l) \in V$, it is the case that $a^U_{ij} b^U_{ij} < a_{kl} b_{kl}$. This entails proving that, for $\Diamond$ large enough, $(a^U_{ij} + \beta b^U_{ij})(\beta a^U_{ij} + b^U_{ij}) \geq (a_{kl} + \beta b_{kl})(\beta a_{kl} + b_{kl})$, i.e. $a^U_{ij} b^U_{ij} + \beta(a^U_{ij})^2 + \beta(b^U_{ij})^2 + \beta^2 a^U_{ij} b^U_{ij} \geq a_{kl} b_{kl} + \beta a^2_{kl} + \beta b^2_{kl} + \beta^2 a_{kl} b_{kl}$. Completing the squares, this can be re-written as $\beta(a^U_{ij} + b^U_{ij})^2 + (1-\beta)^2 a^U_{ij} b^U_{ij} \geq \beta(a_{kl} + b_{kl})^2 + (1-\beta)^2 a_{kl} b_{kl}$. Letting $K_u = (a^U_{ij} + b^U_{ij})^2 - (a_{kl} + b_{kl})^2$ and $K_r = a_{kl} b_{kl} - a^U_{ij} b^U_{ij}$, this implies $\beta K_u > (1-\beta)^2 K_r$ as the key condition. $K_u > 0$ since U is the utilitarian solution, and $K_r > 0$ since its alternative has a higher payoff product. The condition then does not hold for $\beta = 0$, but always holds for $\beta = 1$ since $K_u > 0$. Furthermore, if we write the value function $F = \beta K_u - (1-\beta)^2 K_r$, we find that $\partial F/\partial\beta = K_u + 2(1-\beta) > 0 \; \forall \; \beta \in [0,1]$. This means that $\triangle P(Q)$, the change in the fraction of games for which U coincides with the Nash bargaining solution given $\Diamond$, tends to increase as β increases. $\blacksquare$

Proposition 13 *For $\Diamond$ large enough, a utilitarian outcome U is always a Nash bargaining solution.*

Proof. The proof is identical to that of Proposition 12, except that now (k,l) is not restricted to belong to V (see Appendix A for proof details). $\blacksquare$

We can summarize the main results in this section in a single sentence.

Corollary 1 *For $\Diamond$ large enough, a utilitarian outcome U is always a Nash bargaining solution and a risk and payoff dominant Nash equilibrium.*

3.2 Example 1: The Prisoner's Dilemma

We now consider two simple examples of how harmony transformations affect the strategic structure of the game and hence the likely play by decision makers. As a first example, consider a standard Prisoner's Dilemma (PD) to which $\Diamond$ is applied: Table 2 displays both the untrasformed matrix PD and the harmony-transformed matrix PD', where $a, b, d, \beta \in (0,1]$ and $2a > d > a > b$. It follows that (C,C) is the utilitarian and Nash bargaining solution, but (D,D) is the unique Nash equilibrium of the game (and the strictly dominant strategy for both players). Propositions 9 and 11 imply that, when $\Diamond$ is large enough, (C,C) is also a Nash equilibrium and the payoff dominant outcome. If (D,D) remains a Nash equilibrium there is then an issue of an equilibrium selection. Whether (D,D) remains an equilibrium depends on whether $b + \beta b \geq \beta d$, i.e. on whether $b \geq [\beta/(1+\beta)]d$; this will occur for $\Diamond$ large enough only iff $b \geq (1/2)d$.

For (C,C) to be a (payoff dominant) Nash equilibrium and a general payoff dominant outcome, since $a + \beta a > b + \beta b$, the single condition that needs to hold is that $a + \beta a \geq d$, that is $\beta \geq (d-a)/a$, the critical treshold. This always holds for $\Diamond$ large enough since $(d-a)/a \in (0,1)$, and is independent of the b parameter. Let $G(PD'_{\min}), \beta_{\min} : G(PD'_{\min}) = G(PD' \mid \beta_{\min} = (d-a)/a)$, so $G(PD'_{\min})$ is the game harmony of the *minimally transformed* PD matrix, i.e. the one for which $\beta_{\min} = (d-a)/a$, the minimal value to make (C,C) a Nash equilibrium and payoff dominant outcome. Table 3 considers eight possible scenarios for different values of the cheater payoff d and the cooperation payoff a: it computes $\beta_{\min}$ treshold values and game harmony values for the untrasformed matrix $G(PD)$ and the minimally transformed matrix $G(PD'_{\min})$, assuming $b = 0.2$. Treshold values range from negative values for cases where the cooperation payoff is high relative to the cheater payoff (scenarios 2, 3, 6 and 8) and is as high as 0.833 in the worst case with the best relative payoff from cheating (scenario 6). Figure 1 plots how game harmony and tresholds vary for different values of β, allowing for negative as well as positive harmony transformations, i.e. expanding the range to $\beta \in [-1, 1]$ for generality of the analysis.

The Pearson correlation between $G(PD)$ and $G(PD'_{\min})$ is -0.697, indicating that the higher the initial harmony of the PD the smaller the positive harmony transformation that is required for the utilitarian and Nash bargaining solution (C, C) to become also a Nash equilibrium and payoff dominant outcome. Consistently with the Equivalence Result and Figure 1, there is a high Pearson correlation of 0.937 between $\beta_{\min}$ and $G(PD'_{\min})$.

3.3 Example 2: An Asymmetric Chicken Game

Consider the Asymmetric Chicken Game in Table 2, both in its untransformed form CK and its harmony-transformed form $CK\prime$. The game has parameters $a, b, c, d, \beta \in (0, 1]$ and with $a > b \geq c > d > 0, a+d > b+c$ (utilitarian condition) but $bc > ad$ (Nash bargaining condition). CK can be considered a form of two-sided trust game with two Nash equilibria, $(2, 1)$ and $(1, 2)$, where players can either coordinate on $(2, 1)$ as the more efficient utilitarian solution or on $(1, 2)$ as the more egalitarian Nash bargaining solution. The equilibrium selection problem is made more serious by the fact that $(2, 1)$ and $(1, 2)$ are not payoff dominant relative to one another: $(2, 1)$ favors player 1 while $(1, 2)$ favors player 2. CK' also has both $(2, 1)$ and $(1, 2)$ as Nash equilibria no matter the size of $\Diamond$, since $a + d, c + b > 2d > 0$.

Proposition 10, 12 and 13 ensure that, for $\Diamond$ large enough, the utilitarian solution $(2, 1)$ coincides with the Nash bargaining solution and the payoff dominant and risk dominant Nash equilibrium, as for Corollary 1, therefore addressing these difficulties. For $(2, 1)$ to be payoff dominant, we require $d + \beta a \geq b + \beta c$, and so $\beta \geq (b - d)/(a - c)$. Because of the utilitarian condition, this always holds for $\Diamond$ large enough. Let $G(CK'_P)$ be the game harmony of the CK matrix if subject to the minimum positive harmony transformation that is required to make the utilitarian solution payoff dominant. Let $\beta_P = (b - d)/(a - c)$ be the corresponding β

value. For $(2,1)$ to be risk dominant and the Nash bargaining solution we instead require $(a+\beta d)(d+\beta a) \geq (c+\beta b)(b+\beta c)$, which can be solved analytically and used to find the threshold value $\beta_R \in (0,1]$ at which this condition holds (see Appendix B). Let $G(CK'_R)$ be the game harmony of the CK matrix if subject to the minimum positive harmony transformation that is required to make the utilitarian solution also the risk dominant and Nash bargaining solution of the game.

Table 3 considers eight possible scenarios for different parameter values. $\beta_P < \beta_R$ in all cases, so there are three possible ranges: one where $\lozenge : CK \to CK'$ does not alter the properties of either $(2,1)$ or $(1,2)$, as $\beta < \beta_P$ ($\lozenge$ is small); a second one where the utilitarian solution is payoff dominant but not risk dominant, as $\beta_P \leq \beta < \beta_R$ ($\lozenge$ is of intermediate size); a third one where the utilitarian solution coincides with the risk dominant and Nash bargaining solution, as $\beta \geq \beta_R$ ($\lozenge$ is large). Figure 2 plots how game harmony and tresholds vary for different values of β, expanding again the range to $\beta \in [-1,1]$ for generality of the analysis.

The Pearson correlation coefficient between β_P and $G(CK'_P)$ is 0.934, whereas that between β_R and $G(CK'_R)$ is 0.730; there is a high correlation between β_P and β_R values (Pearson $r(\beta_P, \beta_R) = 0.886$). Unlike the PD, the higher $G(CK)$ the higher the threshold values (β_R and β_P) appear in our numerical calibrations (see Table 3). This may be because a high $G(CK)$ may be associated with a low efficiency parameter a relative to the b and c coefficients, whereas the thresholds are lowest are when a is highest relative to the other parameters, as in scenarios 5, 6 and 8,

4. A case-based approach

4.1 Theory

Consider a subset of games $M \subseteq A$. Thus, if $A \to A'$, $M \to M'$. Let q be the property that a cooperative strategy C exists and is chosen. X is a set of realized payoffs. x_0 is included within X as the default payoff if the act was not chosen. The set of *cases* is $C \equiv M \times W \times X$. When player i considers a current game, that player does so in terms of the game itself, possible strategies, and realized payoffs which spring from particular choices of strategy. We assume that the player considers only salient past experiences of realized outcomes. To make this concrete, given a subset of cases $C_s \subseteq C$, denote its projection P by H. So $H =$

$H(C_s) = \{\Gamma \in M \mid \exists W_i \in W, x_{iw} \in X, \text{ such that } (\Gamma, W_i, x_{iw}) \in C_s\}$ where H denotes the history of games, and $C_s \subseteq C$ denotes the subset of cases recorded as memory by the player in a given setting, such that (i) for every $\Gamma \in H(C_s)$ and $W_i \in W$, $\exists$ a unique $x_{iw} : (\Gamma, W_i, x_{iw}) \in C_s$, and (ii) for every $\Gamma \in H(C_s)$, $\exists$ a unique $W_i \in W : x_{iw} \neq x_0$. Memory is therefore a function that associates results to pairs of the form (*game*, *strategy*). For every memory C_s, and every $W_i \in H$, there is one strategy that was actually chosen at W_i with a payoff $x_{iw} \neq x_0$, and all other potential strategies are assigned the default payoff x_0. So, the agent has a memory of various games, and potential strategies, where one particular strategy was associated to each game. This produces a result $x_{iw} \neq x_0$, with other strategies having not been tried, so are given the generic result x_0. When faced with a random game, the player will examine her memory, C_s, for some similar problems encountered in history, H, and assign these past problems a value according to a similarity function $s(\mid \triangle G(\Gamma) \mid) \in [0, 1]$. We assume that $\partial s / \partial \triangle G(\Gamma) < 0$: similarity is a negative function of the difference in game harmony between a random game and the past problem. The lower the distance in game harmony between the games, the greater the similarity. We also assume that $s(0) = 1$. Past problems each have a remembered action with a given payoff, which can be aggregated according to the summation $\sum_{C_s} s(\mid \triangle G(\Gamma) \mid) x_{iw}$, where x_{iw} evaluates the payoff x_{iw} associated to each case $(\Gamma, W_i, x_{iw}) \in C_s$. Decision making is simply a process of examining past cases, assigning similarity values, summing, and then computing the act W_i to $\max_{W_i} U(C_s) = \sum_{C_s} s(\mid \triangle G(\Gamma) \mid) x_{iw}$.

Now consider the following setting. Let there be at most three games in memory in C_s: (1) a pure coordination game CG for which $G(CG) = 1$ and a pure strategy was chosen leading to the utilitarian outcome with payoff $c(CG)$; (2) the game Γ that is being played, where clearly $s(\mid \triangle G(\Gamma) \mid) = s(0) = 1$, and where a corresponding cooperative strategy leading to a payoff $c(\Gamma)$ may or may not have been chosen; (3) if characterized by a pure strategy, a constant-sum game CS for which $G(CS) = -1$ and where a pure non-cooperative strategy was chosen leading to payoff $nc(CS)$. We can interpret this limit case where only three games are present in memory as a decision setting where these are the only games that are salient and therefore potentially judged similar to the game currently being played, i.e. where $s(G(\Gamma)) = 0$ in relation to any other game. For any given game Γ we can compute a pure coordination game CG_Γ by applying a positive harmony transformation to it with $\beta = 1$ (defined as for equation 5), so that $\Gamma \rightarrow CG_\Gamma : (a_{ij}, b_{ij}) \rightarrow$

$(a_{ij} + b_{ij}, a_{ij} + b_{ij})$ $\forall i, j$; similarly, we can compute a constant-sum game CS_Γ by applying a negative payoff transformation to it with $\beta = -1$ (again defined as for equation 5), so that $\Gamma \to CS_\Gamma : (a_{ij}, b_{ij}) \to (a_{ij} - b_{ij}, b_{ij} - a_{ij})$ $\forall i, j$. We can interpret the memory of the agent as containing Γ, CG_Γ and CS_Γ. The limitation to a pure strategy in the context of the constant-sum game may be restrictive but is not essential, and may be relaxed in settings where it is plausible to think that the agent may simply believe the game as one of unpredictable choice. The restriction may be justified by the rationale of case-based decision-making as a "simple" algorithm for bounded-rational decision-makers, and therefore one not requiring the computation of mixed strategy equilibria for its implementation.

Proposition 14 *If $\Diamond : A \to A'$ is applied, q is the property that the agent chooses C if playing Γ or Γ', and the player's memory C_s contains $CG \to CG'$, the game being played and possibly $CS \to CS'$, with the same actions being associated to each game in memory before and after $\Diamond$ is applied, then $\triangle P(Q) \geq 0$.*

Proof. Assume that C was chosen not just in relation to CG but also in relation to game Γ. If $CS \notin C_s$, the player always chooses C. If $CS \in C_s$, the case-based decision-maker chooses C if she does not assimilate the game to CS, and so iff $\max[c(\Gamma), s(| 1 - G(\Gamma) |)c(CG)] > s(| G(\Gamma) + 1 |)nc(CS)$, or

$$\max[a_{ij}^U, s(| 1 - G(\Gamma) |)(a_{ij}^U + b_{ij}^U)] > s(| G(\Gamma) + 1 |)(a_{kl} - b_{kl}) \tag{7}$$

Let $f_1 = \max[a_{ij}^U, s(| 1 - G(\Gamma) |)(a_{ij}^U + b_{ij}^U)] - s(| G(\Gamma) + 1 |)(a_{kl} - b_{kl})$ (where clearly $a_{ij}^U + b_{ij}^U > a_{kl} - b_{kl}$).

Assume instead that C was not chosen in relation to game Γ: NC was chosen leading to payoff $nc(\Gamma)$. Then the player chooses C only if she assimilates Γ to CG. If $CS \notin C_s$, the player chooses C iff

$$s(| 1 - G(\Gamma) |)(a_{ij}^U + b_{ij}^U) > a_{kl} \tag{8}$$

If $CS \in C_s$, the condition is that $s(| 1 - G(\Gamma) |)(a_{ij}^U + b_{ij}^U) > \max[a_{kl}, s(| G(\Gamma) + 1 |)(a_{kl} - b_{kl})]$, i.e. $s(| 1 - G(\Gamma) |)(a_{ij}^U + b_{ij}^U) > a_{kl}$ once again. Let $f_2 = s(| 1 - G(\Gamma) |)(a_{ij}^U + b_{ij}^U) - a_{kl}$ (where $a_{ij}^U + b_{ij}^U > a_{kl}$).

Now consider the operation $\Diamond : A \to A'$. Note that $G(CG') = G(CG) = 1$, and, for any $\beta < 1$ (if $\beta = 1$ the variability condition fails, $G(CS')$ does not exist and we can assume $CS' \notin C_s$), $G(CS') = G(CS) = -1$.

Assume the player associates C to Γ. If $CS' \notin C_s$, the player always chooses C. If $CS' \in C_s$, the case-based decision-maker chooses C if she does not assimilate the game to CS, and so iff

$$\max[a_{ij}^U, 2s\,(\mid 1 - G(\Gamma') \mid)(a_{ij}^U + b_{ij}^U)] > (1+\beta)s\,(\mid G(\Gamma') + 1 \mid)(a_{kl} - b_{kl}) \tag{9}$$

since $c(CG') = (1+\beta)(a_{ij}^U + b_{ij}^U)$ and $c(CS') = (1+\beta)(a_{kl} - b_{kl})$. Let $f_1' = \max[a_{ij}^U, (1+\beta)s\,(\mid 1 - G(\Gamma') \mid)(a_{ij}^U + b_{ij}^U)] - (1+\beta)s\,(\mid G(\Gamma') + 1 \mid)(a_{kl} - b_{kl})$. Since $s\,(\mid 1 - G(\Gamma') \mid \geq s\,(\mid 1 - G(\Gamma) \mid$ and $s\,(\mid G(\Gamma') + 1 \mid) \leq s\,(\mid G(\Gamma) + 1 \mid)$, $f_1' \geq f_1$ and if we define $\triangle f_1 = f_1' - f_1$ it follows that $\triangle f_1 / \triangle G(\Gamma) \geq 0$.

Assume instead again that the player associates NC to Γ'. For C to be chosen, now we require

$$(1+\beta)s\,(\mid 1 - G(\Gamma') \mid)(a_{ij}^U + b_{ij}^U) > (a_{kl} + \beta b_{kl}) \tag{10}$$

and we can define $f_2' = (1+\beta)s\,(\mid 1 - G(\Gamma') \mid)(a_{ij}^U + b_{ij}^U) - (a_{kl} + \beta b_{kl})$. Since $s\,(\mid 1 - G(\Gamma') \mid) \geq s\,(\mid 1 - G(\Gamma) \mid)$ and $\beta(a_{ij}^U + b_{ij}^U) \geq \beta b_{kl}$, $f_2' \geq f_1$ and if we define $\triangle f_2 = f_2' - f_2$ it follows that $\triangle f_2 / \triangle G(\Gamma) \geq 0$.

We conclude that either the player always chooses C or, if not, whether she does so or not depends on whether either $f_1' > 0$ or $f_2' > 0$, depending on the setting. Since both $\triangle f_1 / \triangle G(\Gamma) \geq 0$ and $\triangle f_2 / \triangle G(\Gamma) \geq 0$, the application of the positive harmony transformation $\Diamond$ to $A \rightarrow A'$ leads to $\triangle P(Q) \geq 0$, as the number of games in which either $f_1' > 0$ or $f_2' > 0$ and so C is chosen is non-decreasing in $\Diamond$. ■

The proposition means that, if we apply a positive harmony transformation to a set of random games, there is a higher likelihood of cooperative play. The intuition is that games in A' are more likely to be assimilated to pure coordination game settings associated to cooperative actions, and less likely to be assimilated to constant-sum games.

4.2 Examples

Consider the Prisoner's Dilemma and Asymmetric Chicken Game analyzed in sections 3.2 and 3.3. Let $\Gamma \rightarrow CG_\Gamma, CS_\Gamma$ take the form of two harmony transformations over Γ (see equation 5), the first one with $\beta = 1$ to obtain CG_Γ and.the second with $\beta = -1$ to obtain CS_Γ. The first harmony transformation allows us to obtain two coordination games, CG_{PD} and CG_{CK}, from the PD and CK respectively. The second harmony transformation allows us to obtain two zero-sum games, CS_{PD} and CS_{CK}, from the PD and CK respectively. Table 2 shows these harmony-transformed games for the PD and the CK.

We assume that the cooperative strategy is that corresponding to the utilitarian solution in each game, and so to (C, C) for the PD and to $(2, 1)$ for the CK. By Propositions 10, 12 and 13, such utilitarian solution also coincides with the Nash bargaining solution, and payoff and risk dominant Nash equilibrium, in relation to CG_{PD} and CG_{CK}, and so we believe it plausible that a cooperative strategy would be undertaken in these games. In both CS_{PD} and CS_{CK} players should choose a mixed strategy and we shall assume that they are not in the memory of the agent. If the player associates action C to the PD and action 1 (if player 1) to the CK (action 2 if player 2), then the cooperative strategy is always chosen. Assume instead that the player associates action D (defection) to the PD and action 1 to CK. Note that in the PD $nc(PD) = b$ and $c(CG_{PD}) = 2a$. Therefore, in this case, the player chooses C iff $2s(\mid 1 - G(PD) \mid)a > b$, that is iff $s(\mid 1 - G(PD) \mid) > b/(2a)$. In the CK for player 1 $nc(CK) = c$ and $c(CG_{CK}) = a + d$; for player 2 $nc(CK) = b$ and again $c(CG_{CK}) = a + d$. So player 1 chooses C iff $s(\mid 1 - G(CK) \mid)(a + d) > c$, that is iff $s(\mid 1 - G(CK) \mid) > c/(a + d)$; player 2 chooses C iff $s(\mid 1 - G(CK) \mid)(a + d) > b$, that is iff $s(\mid 1 - G(CK) \mid) > b/(a + d)$. Table 3 presents critical values of similarity for which these conditions hold for the PD and for the CK, respectively, in relation to various parameter values combinations.

We now consider a positive harmony transformation $\Diamond : A \rightarrow A'$, as a result of which $PD \rightarrow PD'$, $CG_\Gamma \rightarrow CG'_\Gamma$, $CK \rightarrow CK'$ and $CK_\Gamma \rightarrow CK'_\Gamma$ (see Table 2). Clearly $G(PD') > G(PD)$ and $G(CK') > G(CK)$. Assume again that the player associates action D (defection) to the PD' and action 1 to CK'. In relation to the Prisoner's Dilemma now we have $s(\mid 1 - G(PD') \mid) > [b(1 + \beta)]/[2a(1 + \beta)]$ for PD', or $s(\mid 1 - G(PD') \mid) > b/(2a)$. Since $s(\mid 1 - G(PD') \mid) > s(\mid 1 - G(PD) \mid)$, there will be a range of parameter values $b/(2a) \in [s(\mid 1 - G(PD) \mid), s(\mid 1 - G(PD')] : s(\mid 1 - G(PD) \mid) < b/(2a) < s(\mid 1 - G(PD') \mid)$. That is, take the subset of all random games in A with the property that they satisfy the Prisoner's Dilemma structure, and call this subset $A_{PD} \subset A$. Now apply $\Diamond : A \rightarrow A'$ implying $A_{PD} \rightarrow A'_{PD}$. Then the percentage of games in which cooperation is observed is likely to be greater in A'_{PD} than in A_{PD}, since more games are likely to be assimilated to the pure coordination game benchmark.

In relation to the Asymmetric Chicken Game, following an analogous reasoning we have $s(\mid 1 - G(CK') \mid$

$) > c/(a+d)$ for player 1 and $s\left(\mid 1-G(CK')\mid\right) > b/(a+d)$ for player 2. Since $s\left(\mid 1-G(CK')\mid\right) >$ $s\left(\mid 1-G(CK)\mid\right)$, there will be a range of parameter values $b/(a+d), c(a+d) \in [s\left(\mid 1-G(CK)\mid\right), s\left(\mid 1-G(CK')\right]$: $s\left(\mid 1-G(CK)\mid\right) < b/(a+d), c(a+d) < s\left(\mid 1-G(CK')\mid\right)$. Take the subset of all random games in A with the property that they satisfy the Asymmetric Chicken Game structure, and call this subset $A_{CK} \subset A$. Now apply $\Diamond : A \to A'$ implying $A_{CK} \to A'_{CK}$ Then the percentage of games in which cooperation is observed will be greater in A'_{CK} than in A_{CK}.

This effect of positive harmony transformations on cooperation is purely based on the similarity metric among games. It works independently of whether the cooperative outcome is a Nash equilibrium or satisfies payoff or risk dominance, or a combination of these desirable properties.

5. Team reasoning and game harmony

5.1 Theory

Let $T_i(\Gamma)$ be a Boolean variable defining the *participation state* of player i in game Γ to one of two frames: if i is in an *individual reasoning* frame, then $T_i(\Gamma) = 0$; if i is in a *team reasoning* frame, then $T_i(\Gamma) = 1$. There are two differences between the frames. The first is a difference in utility payoffs; the second is a difference in reasoning:

1. If player i is in an individual reasoning frame ($T_i(\Gamma) = 0$), the set of payoffs for each state of the world $w \in W$ corresponds to her individual payoffs, i.e. $x_i : W \to \mathbb{R}$ as before. If i is in a team reasoning frame ($T_i(\Gamma) = 1$), the set of payoffs for each state of the world w is $y : W \to \mathbb{R}$, where y is the same to all members of the team and is therefore not i-specific. A *payoff transformation function* $Y : x_{iw} \to y(w)$ determines how individual payoff function are transformed in the team welfare function for each state of the world w. We assume, as in the previous sections, that the cooperative outcome is the utilitarian outcome $a^U_{ij} + b^U_{ij} \geq a_{kj} + b_{kj}\ \forall\ k \neq i$, but in addition, we assume that, if there are multiple equilibria and there is some commonly available heuristic that identifies one of them as salient, this will be the one picked up by team reasoners. There is experimental evidence showing that people do use salience as a device to identify focal points on which to coordinate their actions (e.g., Bacharach and Bernasconi, 1997), and team reasoning provides one possible cognitive mechanism to explain why this should work.

2. If player i is in an individual reasoning frame ($T_i(\Gamma) = 0$), the set of actions cognitively available to player i is W_i as before. If i is in a team reasoning frame ($T_i(\Gamma) = 1$), what matters to her is the set of actions available *to the team* $W(T) = W_1 \times W_2$. If $T_i(\Gamma) = 1$ for both players, $W(T) = W$, since the team can choose the vector of individual actions by the individual team players which is such as to achieve any state of the world. The team-reasoning player i chooses W_i so as to implement her part in $W(T)$.

In the standard formal set-up of team reasoning theory, it is assumed that what frame the agent is in is exogenously determined. We modify this assumption by assuming that the frame the agent is in is a function of the perceived harmony of the game. We postulate that $P(T_i(\Gamma) = 1) = f[G(\Gamma)]$, with $\partial P(T_i(\Gamma) = 1)/\partial G(\Gamma) > 0$.

Proposition 15 *If $\Diamond : A \rightarrow A'$ is applied and Q is the property that the agent chooses the cooperative outcome, then the probability of agents being team reasoners is higher and as a result $\triangle P(Q) \geq 0$.*

Proof. Since $\partial P(T_i(\Gamma) = 1)/\partial G(\Gamma) > 0$, and since $G(\Gamma') \geq G(\Gamma)$ by the definition of $\Diamond$, for each game pair of games (Γ, Γ') mapped from A into A', $P(T_i(\Gamma') = 1) > P(T_i(\Gamma) = 1)$, and so $\triangle P(Q) \geq 0$. ■

On the basis of this set of assumptions, game harmony increases cooperation by increasing the probability of team reasoning on the part of the different players. Put it differently, if players have reason - say, based on the material payoffs of the game - to assume their interests to be roughly aligned with those of other players, it is more likely that they will reason as a team and act accordingly to achieve the cooperative team profile of actions.

More formally, assume that $T_i(\Gamma) = 1 \; \forall i$. Then the team chooses the profile of action $W(T) : y(\pi) = \max y(w \in W)$, and so each team-reasoning player i chooses $W_i^\pi : \pi = (W_i^\pi, W_j^\pi) \; \forall j \neq i$: under the belief that $W_j = W_j^\pi \forall j \neq i$ (the other player does its part in the team effort), $y(W_i^\pi, W_j^\pi) \geq y(W_i, W_j^\pi) \; \forall W_i$ due to the definition of y, i.e. it is best for player i to act its own part in the cooperative effort.

5.2 Example: a coordination game

Consider the coordination game CO in Table 4, where $a > b, c$. Both $(1, 1)$ and $(2, 2)$ are pure Nash equilibria, and they cannot be differentiated using either payoff dominance, risk dominance or a utilitarian criterion, as they yield the same payoff to both players. We assume, however, that $(1, 1)$ is made more salient by some feature that is known by both players, and that this is known by both of them: in Table 4, we illustrate this by highlighting this in bold. Intuitively we may think that subjects should this into account, and team reasoning provides a way of formalizing this intuition by picking up $(1, 1)$ as the team's strategy profile. The game is *not* a pure coordination game unless $b = c$, since, as it is true in many real-world settings, the players may lose to a different extent from the failure to coordinate. For example two companies may wish to coordinate on one of two equally desirable deals, but if they do not one is better off than the other; or two research groups working a lot together may wish to coordinate on the use of either a MS Windows or an Apple operating system, but because of their differences one group stands to lose more than the other depending on what software platform is actually chosen.

The probability of agents choosing $(1, 1)$ will be a positive function of $P(T_i(CO) = 1)$, the probability that they are in the team reasoning frame. In turn, this depends on $G(CO)$, which depends on the values of the game parameters a, b, c and, if a positive (or negative) harmony transformation is applied, β (see Table 5). Table 5 exemplifies the analysis with eight sets of (b, c) values given $a = 1$. In scenarios 1 through 5 we set $b + c = 1$ and, starting from a pure coordination game scenario where $b = c = 0.5$, we move on to more and more disharmonious scenarios with uneven payoffs depending on the outcome that is achieved if coordination fails: Figure 3 plots how game harmony evolves for different values of β, showing large gaps in harmony for $\beta = 0$ that get progressively narrowed down the greater the positive harmony transformation that is applied. In scenarios 6 through 8, we vary the absolute value of b and c relative to a: if there is not much gain from coordination, as illustrated by Figure 3 game harmony climbs up less quickly as β goes up and so the probability of being in a team reasoning frame, and of coordinating successfully, increases.

6. Conclusions

Cooperative and non-cooperative game theory provide different notions of where agents may end up in playing many games. The Nash bargaining solution, but also the utilitarian solution in traditional welfare analysis, are obvious notions of what we may expect cooperation to achieve. The Nash equilibrium, and equilibrium refinements such as risk and payoff dominance, have instead been conceived as equilibrium outcomes of non-cooperative play.

In this paper we showed that, for large enough positive payoff or harmony transformations, the utilitarian solution may become a Nash equilibrium, and may be selected as the payoff and risk dominant outcome, and become the Nash bargaining outcome. In other words, positive harmony transformations may make cooperative play more attractive, though the degree to which this is going to be the case will depend on the size of the harmony transformation and on the game structure: we exemplified this with the Prisoner's Dilemma and an Asymmetric Chicken Game. Team reasoning may also make the coordination on one equilibrium rather than others easier, even if the equilibria cannot be differentiated on the basis of payoff or risk dominance considerations. Case-based reasoning and team reasoning provide further mechanisms by which we may expect higher cooperation rates to be observed in games characterized by higher harmony of interests between the players.

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Appendix A. Proposition proofs

Proof of Proposition 1. Condition 2 always holds for any game in Γ since W is a finite set. Condition 2 directly implies that the denominator of (1) is bounded, i.e. $\sigma_a\sigma_b \leq Q'$ for some finite constant Q'. Also, by Condition 1, $\sigma_a\sigma_b > 0$. Finally, we need to show that the numerator is also bounded: this follows by noting that $\sigma^2_{a+b} = \sigma^2_a + \sigma^2_b + 2Cov(a_w, b_w)$, so $Cov(a_w, b_w)$ is a linear function of bounded variables (by Condition 2), implying that it is itself bounded. ■

Proof of Proposition 2. In the light of the focus of the paper, we state the proof for $N = 2$ but the extension to $N > 2$ is straightforward (see Zizzo, 2003b). First, note that, since $a = kb_w + h$, $\sigma^2_a = Var(kb_w + h) = k^2\sigma^2_b$, so $\sigma_a = k\sigma_b$. Second, note that $Cov(a_w, b_w) = Cov(kb_w + h, b_w) = Cov(kb_w, b_w) = k\sigma^2_b$. It follows that

$$G(CG) = \frac{Cov(a_w, b_w)}{\sigma_a \sigma_b} = \frac{k\sigma^2_b}{k\sigma_b\sigma_b} = 1 \tag{11}$$

■

Proof of Proposition 3. If $N = 2$, then $x_{1w} + x_{2w} = q$, so $x_{1w} = q - x_{2w}$: we can re-write this simply as $a_w = q - b_w$. Now note that $\sigma^2_a = Var(q - b_w) = \sigma^2_b \to \sigma_a\sigma_b = \sigma^2_b$ and that $Cov(a_w, b_w) = Cov(q - b_w, b_w) = -\sigma^2_b$. Thus $G(CS) = -1$ (Zizzo, 2003b, also considers the case where $N > 2$). ■

Proof of Proposition 4 (Scale Independence). Consider $G(k_i x_i + h_i, W)$. Let k_a, h_a and k_b, h_b be the unit and additive constants, respectively, for the two players of each payoff pair $c \in C$.

$$\begin{aligned} G(k_i x_i + h_i, W) &= r(k_a a_w + h_a, k_b b_w + h_b) = \frac{Cov(k_a a_w + h_a, k_b b_w + h_b)}{k_a k_b \sigma_a \sigma_b} = \\ &= \frac{k_a k_b Cov(a_w, b_w)}{k_a k_b \sigma_a \sigma_b} = G(x_i, W) \end{aligned} \tag{12}$$

■

Proof of Proposition 5 (Replication). Consider each original payoff pair $a_w, b_w \to r(a_w, b_w)$ in the parent game Γ: with γ replicas, there are now $\gamma \times (a_w, b_w) \to \gamma r(a_w, b_w)$ for every $(a_w, b_w) \subset \Gamma$, since $r(k_a a_w + h_a, k_b b_w + h_b) = r(a_w, b_w)$ (by the proof of the scale independence property). Since, in Γ and each replica Φ, $r(a_w, b_w)$ is the same, $r(a_{\gamma w}, b_{\gamma w}) = r(a_w, b_w)$. Thus:

$$G(\Psi) = r(a_{\gamma w}, b_{\gamma w}) = r(a_w, b_w) = G(\Gamma) \tag{13}$$

■

Proof of Proposition 6 (One-Sided Equivalence Result). The proof is outlined in the text, but we need to prove here that $\partial r/\partial\beta_i > 0$ (for $\beta_i < 1$ and $r(a_w, b_w) > -1$). We prove this in relation to $\partial r/\partial\beta_a > 0$; the proof of $\partial r/\partial\beta_b > 0$ is symmetrical. Rescale payoffs (exploiting the scale invariance property) s.t. (a) $a, b_w \geq 1$, (b) $Var(a_w + \beta_a b_w) > 1$, (c) $a_w \leq b_w$ and (d) $b_w + \beta_b a \geq a + \beta_a b_w$ $\forall w \in W$. The rescaling is trivially feasible in relation to condition (a). Since $G(\Gamma')$ exists, the variability condition holds an so $Var(a + \beta_a b_w) > 0$, and by suitable multiplication of the payoffs by a constant condition (b) is also feasible. The rescaling is always possible in relation to conditions (c) and (d) since $\beta_i < 1$. We have:

$$r = \frac{|W|\sum_{w=1}^{|W|}(a_w+\beta_a b_w)(b_w+\beta_b a_w) - \sum_{w=1}^{|W|}(a_w+\beta_a b_w)\sum_{w=1}^{|W|}(b_w+\beta_b a_w)}{\sqrt[2]{\left[|W|\sum_{w=1}^{|W|}(a_w+\beta_a b_w)^2 - \left(\sum_{w=1}^{|W|}(a_w+\beta_a b_w)\right)^2\right]\left[|W|\sum_{w=1}^{|W|}(b_w+\beta_b a_w)^2 - \left(\sum_{w=1}^{|W|}(b_w+\beta_b a_w)\right)^2\right]}} \tag{14}$$

and if we now define $\tau_\alpha, \tau_\beta > 0$ as:

$$\tau_\alpha = |W|\sum_{w=1}^{|W|}(a_w+\beta_a b_w)^2 - \left(\sum_{w=1}^{|W|}(a_w+\beta_a b_w)\right)^2 \tag{15}$$

$$\tau_\beta = |W|\sum_{w=1}^{|W|}(b_w+\beta_b a_w)^2 - \left(\sum_{w=1}^{|W|}(b_w+\beta_b a_w)\right)^2 \tag{16}$$

and we label τ_γ the numerator of r, so $r = \tau_\gamma/\sqrt[2]{\tau_\alpha\tau_\beta} \leqslant 1$ by definition, then:

$$sgn(\partial r/\partial\beta_a) = |W|\sqrt[2]{\tau_\alpha\tau_\beta}\sum_{w=1}^{|W|}b_w(b_w+\beta_b a_w) - \sqrt[2]{\tau_\alpha\tau_\beta}\sum_{w=1}^{|W|}b_w\sum_{w=1}^{|W|}(b_w+\beta_b a_w)+ \tag{17}$$

$$-\frac{\tau_\gamma\sqrt[2]{\tau_\beta}}{\sqrt[2]{\tau_\alpha\tau_\beta}}(|W|-1)\sum_{w=1}^{|W|}b_w\sum_{w=1}^{|W|}(a_w+\beta_a b_w)$$

We claim that $\partial r/\partial\beta_a > 0$. In order for this to hold, it must be the case that:

$$|W|\sqrt[2]{\tau_\alpha\tau_\beta}\sum_{w=1}^{|W|}b_w(b_w+\beta_b a_w) > \sqrt[2]{\tau_\alpha\tau_\beta}\sum_{w=1}^{|W|}b_w\sum_{w=1}^{|W|}(b_w+\beta_b a_w)+r\sqrt[2]{\tau_\beta}(|W|-1)\sum_{w=1}^{|W|}b_w\sum_{w=1}^{|W|}(a_w+\beta_a b_w) \tag{18}$$

For any two random variates X and Y it is a fact that $E(XY) > E(X)E(Y)$ if X and Y are positively covarying; so since $\beta_b < 1$, $r(a_w, b_w) > -1$ and $a_w \leq b_w$, then b_w and $b_w + \beta_b a_w$ are positively covarying and so $\sum_{w=1}^{|W|} b_w(b_w + \beta_b a_w) > \sum_{w=1}^{|W|} b_w \sum_{w=1}^{|W|} (b_w + \beta_b a_w)$ and, noting that $r \leqslant 1$, (18) will hold if

$$(|W|-1)\sqrt[2]{\tau_\alpha}\sum_{w=1}^{|W|} b_w(b_w + \beta_b a_w) > (|W|-1)\sum_{w=1}^{|W|} b_w \sum_{w=1}^{|W|}(a_w + \beta_a b_w) \tag{19}$$

and, since $Var(a_w + \beta_a b_w) > 1$ implies $\sqrt[2]{\tau_\alpha} > 1$, this will always hold if

$$\sum_{w=1}^{|W|} b_w(b_w + \beta_b a_w) > \sum_{w=1}^{|W|} b_w \sum_{w=1}^{|W|}(b_w + \beta_b a_w) \tag{20}$$

which we have already shown to be true. ■

Proof of Proposition 10. We wish to prove that, for $\Diamond$ large enough, U payoff-dominates *any* other outcome (whether or not a Nash equilibrium), even if $a^U_{ij} < a_{kl}$ or $b^U_{ij} < b_{kl}$ for some $k \neq i, l \neq j$. Assume that this is not the case for any (k,l) outcome. Let $a^U_{ij} < a_{kl}$. Then $b^U_{ij} > b_{kl}$ since U is a utilitarian solution and so $a^U_{ij} + b^U_{ij} \geq a_{kl} + b_{kl}$. The proof that U payoff dominates the (k,l) outcome then reduces to a proof that $a^U_{ij} + \beta b^U_{ij} \geq a_{kl} + \beta b_{kl}$ for $\Diamond$ large enough. Since $b^U_{ij} - b_{kl} \geq a_{kl} - a^U_{ij}$, this always true as $\exists\beta \in (0,1] : \beta \geq (a_{kl} - a^U_{ij})/(b^U_{ij} - b_{kl})$. Similarly, if $b^U_{ij} < b_{kl}$ for some $k \neq i, l \neq j$, it is the case that $\beta a^U_{ij} + b^U_{ij} \geq \beta a_{kl} + b_{kl}$ for $\Diamond$ large enough since $\exists\beta \in (0,1] : \beta \geq (b_{kl} - b^U_{ij})/(a^U_{ij} - a_{kl})$. ■

Proof of Proposition 11. Assume that U is not a Nash bargaining solution: for some $k \neq i, l \neq j$, $a^U_{ij} b^U_{ij} < a_{kl} b_{kl}$. We need to prove that, for $\Diamond$ large enough, $(a^U_{ij} + \beta b^U_{ij})(\beta a^U_{ij} + b^U_{ij}) \geq (a_{kl} + \beta b_{kl})(\beta a_{kl} + b_{kl})$, i.e. $a^U_{ij} b^U_{ij} + \beta(a^U_{ij})^2 + \beta(b^U_{ij})^2 + \beta^2 a^U_{ij} b^U_{ij} \geq a_{kl} b_{kl} + \beta a^2_{kl} + \beta b^2_{kl} + \beta^2 a_{kl} b_{kl}$. Completing the squares, this can be re-written as $\beta(a^U_{ij} + b^U_{ij})^2 + (1-\beta)^2 a^U_{ij} b^U_{ij} \geq \beta(a_{kl} + b_{kl})^2 + (1-\beta)^2 a_{kl} b_{kl}$. Letting $K_u = (a^U_{ij} + b^U_{ij})^2 - (a_{kl} + b_{kl})^2$ and $K_r = a_{kl} b_{kl} - a^U_{ij} b^U_{ij}$, this implies $\beta K_u > (1-\beta)^2 K_r$ as the key condition. $K_u > 0$ since U is the utilitarian solution, and $K_r > 0$ since its alternative has a higher payoff product. The condition may then not hold for $\beta = 0$, but always holds for $\beta = 1$ since $K_u > 0$. Furthermore, if we write the value function $F = \beta K_u - (1-\beta)^2 K_r$, we find that $\partial F/\partial\beta = K_u + 2(1-\beta) > 0 \;\forall\; \beta \in [0,1]$. This means that $\triangle P(Q)$, the change in the fraction of games for which U coincides with the Nash bargaining solution given $\Diamond$, tends to increase as β increases. $\blacksquare$

Appendix B. Treshold for risk dominance in the Asymmetric Chicken Game

To find the treshold value for $(2,1)$ to be risk dominant we set $(a+\beta d)(d+\beta a)=(c+\beta b)(b+\beta c)$ and re-write this as $ad+\beta a^2+\beta d^2+\beta^2 ad-cb-\beta c^2-\beta b^2-\beta^2 bc=0$ or $\beta^2(ad-bc)+\beta(a^2+d^2-b^2-c^2)+ad-cb=0$, which can be solved for β_P as

$$\beta_P = \frac{b^2+c^2-a^2-d^2+\sqrt[2]{a^4-2a^2b^2-2a^2c^2-2a^2d+b^4-2b^2c^2-2b^2d^2+c^4-2c^2d^2}}{2(ad-bc)} \tag{21}$$

where we take the positive root since numerical calibrations show that it is the (only) root for which β is between 0 and 1.

Table 1. Duplication of Battle of the Sexes Game

Original Game		Duplicated Game			
2, 1	0, 0	2, 1	0, 0	2, 1	0, 0
0, 0	1, 2	0, 0	1, 2	0, 0	1, 2

Table 2. Prisoner's Dilemma, Asymmetric Chicken Game and derived matrices

	G (PD)		G (PD')	
	C	D	C	D
C	a, a	0, d	a + βa, a + βa	βd, d
D	d, 0	b, b	d, βd	b, b

	G (CG$_{PD}$)		G (CG'$_{PD}$)	
	C	D	C	D
C	2 a, 2 a	0, d	2 a (1 + β), 2 a (1 + β)	βd, d
D	d, 0	2 b, 2 b	d, βd	2b (1 + β), 2b (1 + β)

	G (CS$_{PD}$)		G (CS'$_{PD}$)	
	C	D	C	D
C	0, 0	- d, d	0, 0	- d (1 + β), d (1 + β)
D	d, - d	0, 0	d (1 + β), - d (1 + β)	0, 0

	G (CK)		G (CK')	
	1	2	1	2
1	d, d	c, b	d + βd, d + βd	c + βb, b + βc
2	a, d	0, 0	a + βd, d + βa	0, 0

	G (CG$_{CK}$)		G (CG'$_{CK}$)	
	1	2	1	2
1	2 d, 2 d	b + c, b + c	2 d (1 + β), 2 d (1 + β)	(b + c) (1 + β), b + c (1 + β)
2	a + d, a + d	0, 0	(a + d) (1 + β), (a + d) (1 + β)	0, 0

	G (CS$_{CK}$)		G (CS'$_{CK}$)	
	1	2	1	2
1	0, 0	b - c, c - b	0, 0	(b - c) (1 + β), (c - b) (1 + β)
2	a - d, a - d	0, 0	(a - d) (1 + β), (d - a) (1 + β)	0, 0

PD : $a, b, d \in (0, 1]$, $2a > d > a > b$. CK : $a > b > c > d$, $bc > ad$, $a + d > b + c$. The row player is player 1 and the column player player 2.

Table 3. Prisoner's Dilemma and Asymmetric Chicken Game scenarios

Prisoner's Dilemmas (PD)							
Scenario	a	b	d	G(PD)	G(PD'$_{min}$)	β_{min}	s$_{min}$
1	0.3	0.2	0.55	-0.928	0.636	0.833	0.333
2	0.4	0.2	0.55	-0.76	-0.205	0.375	0.25
3	0.5	0.2	0.55	-0.498	-0.333	0.1	0.2
4	0.5	0.2	0.7	-0.69	0	0.4	0.2
5	0.65	0.2	0.7	-0.393	-0.255	0.077	0.154
6	0.5	0.2	0.9	-0.761	0.833	0.8	0.2
7	0.65	0.2	0.9	-0.598	0.12	0.385	0.154
8	0.85	0.2	0.9	-0.303	-0.192	0.059	0.118

Asymmetric Chicken Games (CK)							
Scenario	a	b	c	d	G(CK)	G(CK'$_P$)	G(CK'$_R$)
1	1	0.6	0.4	0.2	0.245	0.403	0.962
2	1	0.5	0.5	0.2	0.4	0.578	0.963
3	0.8	0.6	0.4	0.25	0.418	0.732	0.997
4	0.8	0.5	0.5	0.25	0.596	0.696	0.965
5	1	0.5	0.3	0.1	0.05	0.239	0.905
6	1	0.4	0.4	0.1	0.192	0.336	0.912
7	0.85	0.5	0.3	0.15	0.17	0.398	0.942
8	0.85	0.4	0.4	0.15	0.351	0.516	0.965

Asymmetric Chicken Games (CK)				
Scenario	β_P	β_R	s$_1$	s$_2$
1	0.077	0.667	0.333	0.5
2	0.093	0.6	0.417	0.417
3	0.231	0.875	0.381	0.571
4	0.264	0.833	0.476	0.476
5	0.075	0.571	0.273	0.455
6	0.056	0.5	0.364	0.364
7	0.088	0.636	0.3	0.5
8	0.077	0.556	0.4	0.4

Game matrices are as in Table 2. $s_{\min}$ is the minimum similarity value required for PD' to be assimilated to CG_{PD}. s_1 and s_2 are the minimum similarity values for CK' to be assimilated to CG_{CK} by players 1 and 2, respectively.

Table 4. A coordination game (CO) matrix

G(CO)				G(CO')	
	1	2		1	2
1	a , a	c , b	1	a $(1+\beta)$, a $(1+\beta)$	c $+ \beta$b , b $+ \beta$c
2	b , c	a , a	2	b $+ \beta$c, c $+ \beta$b	a $(1+\beta)$, a $(1+\beta)$

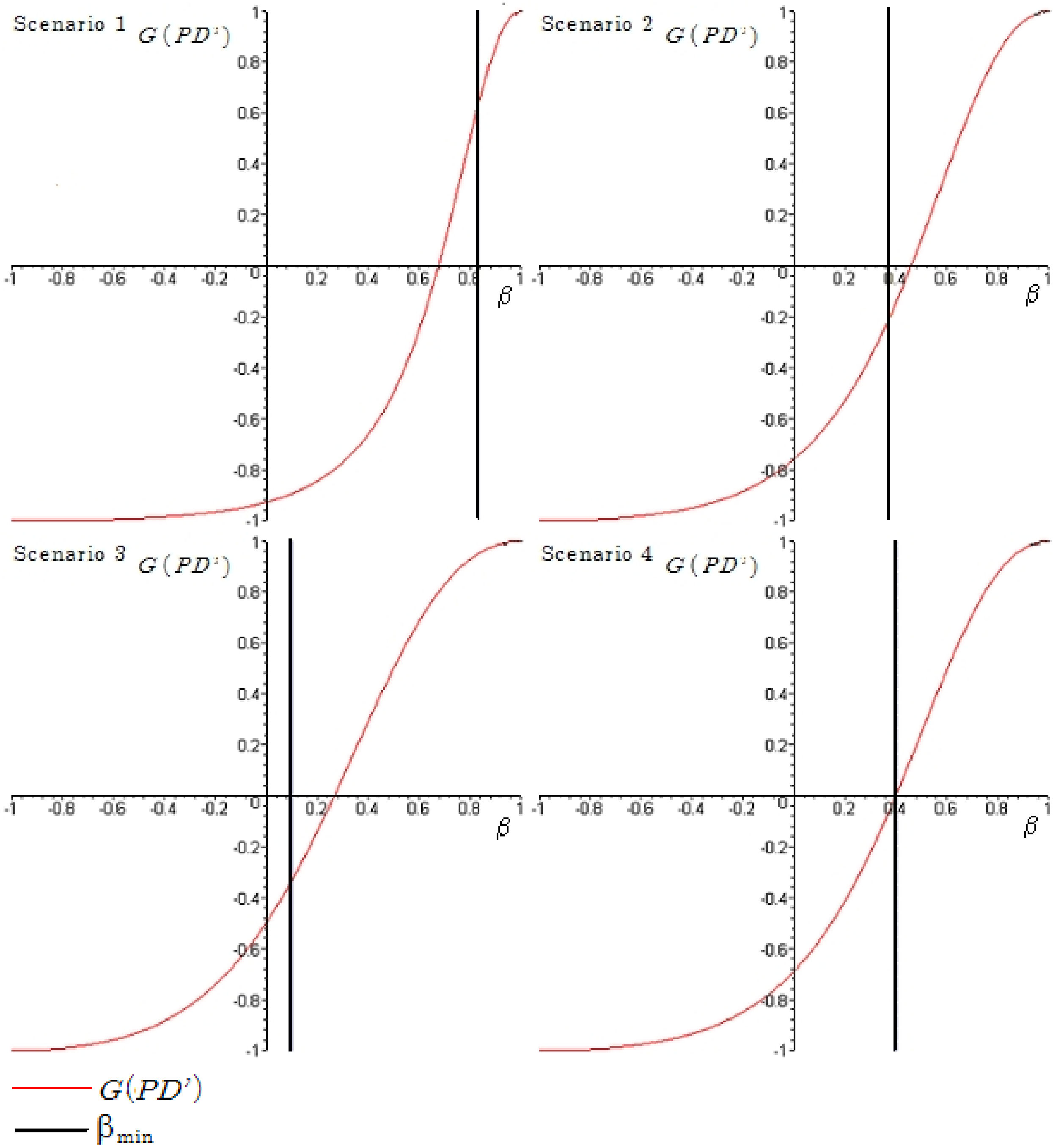
$a > b, c$. Bold characters refer to the salient equilibrium with team reasoning.

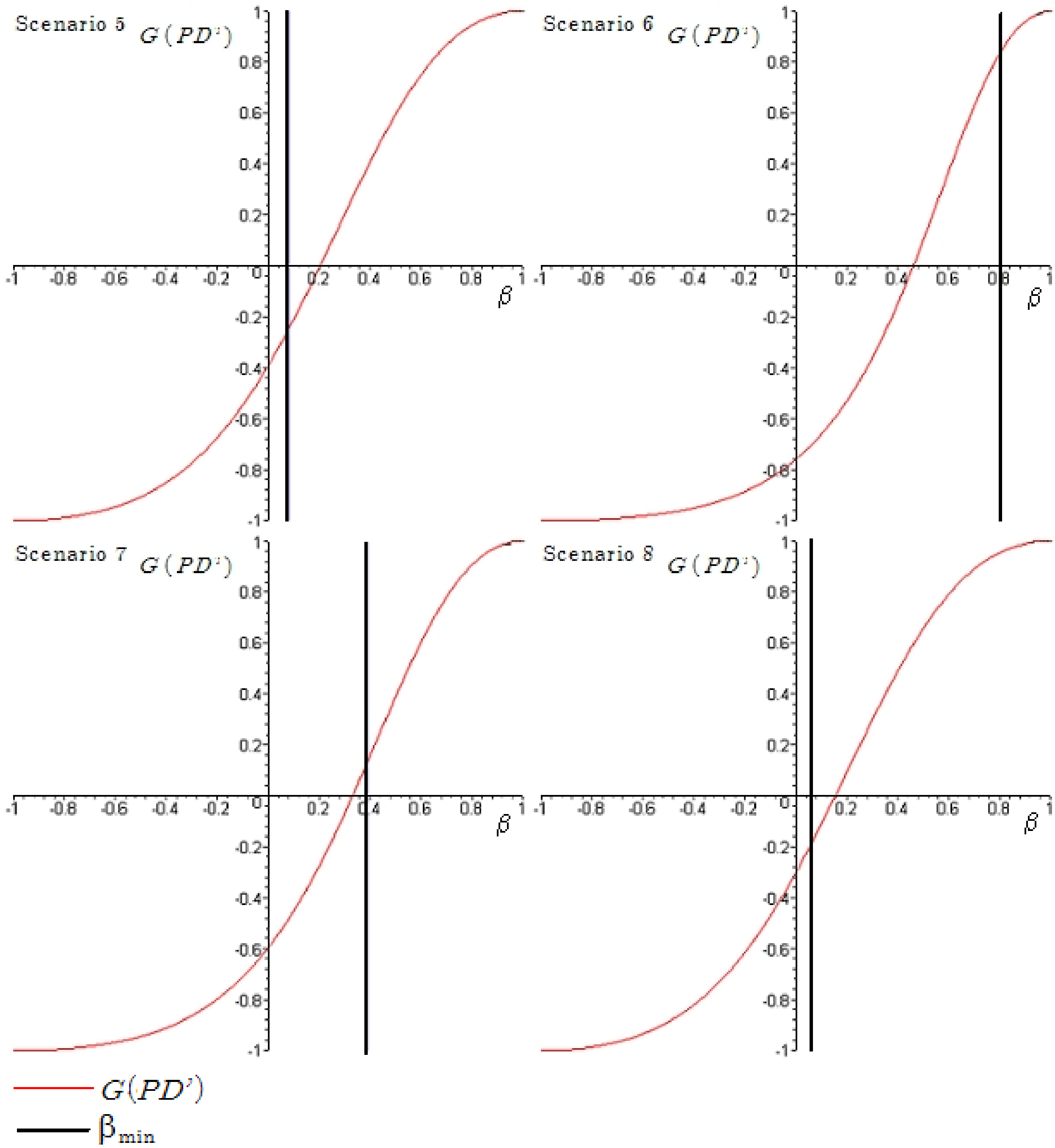
Table 5. Coordination game scenarios

Scenario	a	b	d	G(CO)
1	1	0.5	0.5	1
2	1	0.6	0.4	0.852
3	1	0.7	0.3	0.515
4	1	0.8	0.2	0.163
5	1	0.9	0.1	-0.123
6	1	0.9	0.8	0.636
7	1	0.5	0.4	0.967
8	1	0.2	0.1	0.986

Game matrices are as in Table 4.

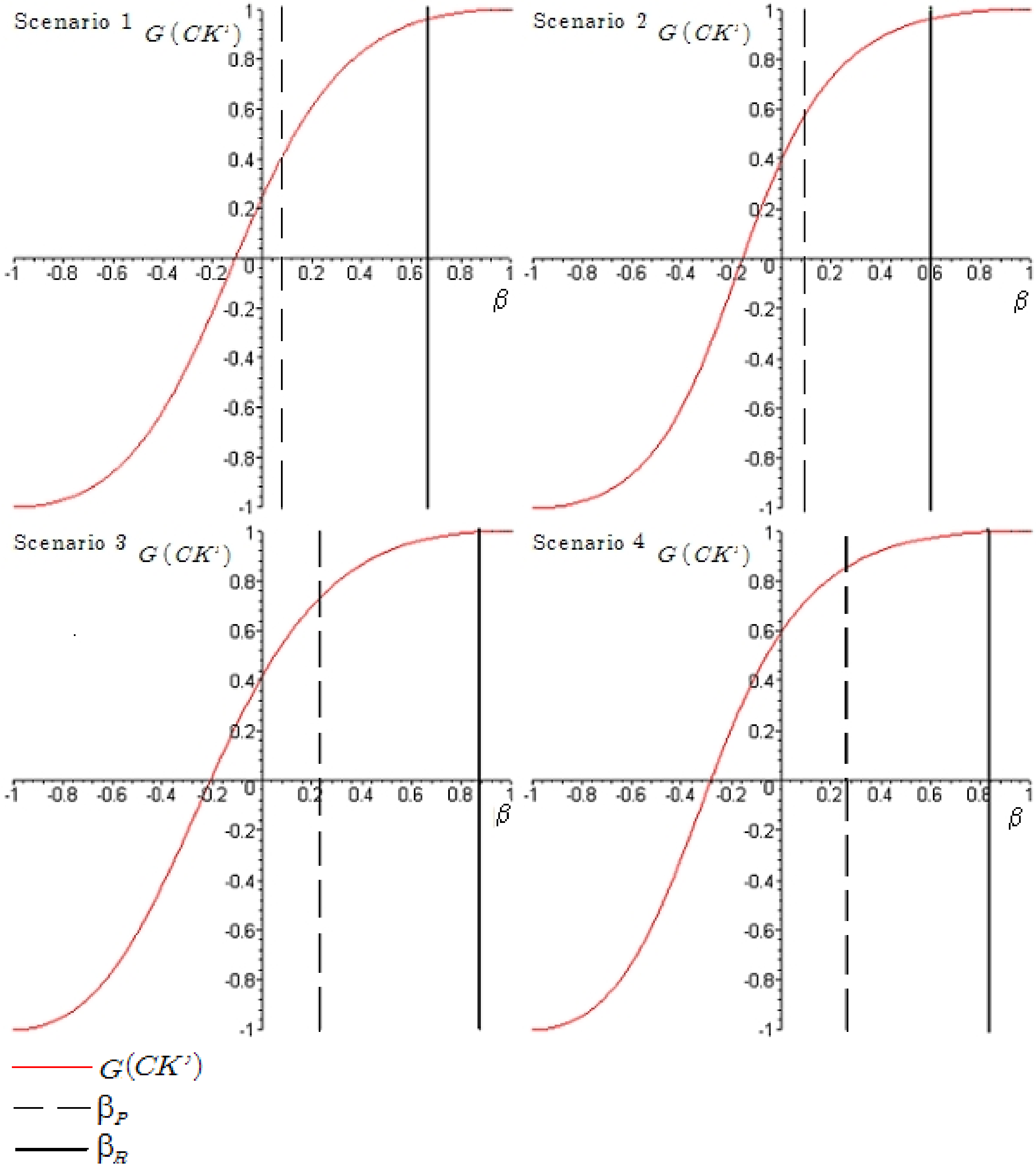
Fig. 1. Prisoner's Dilemma and harmony transformations: examples

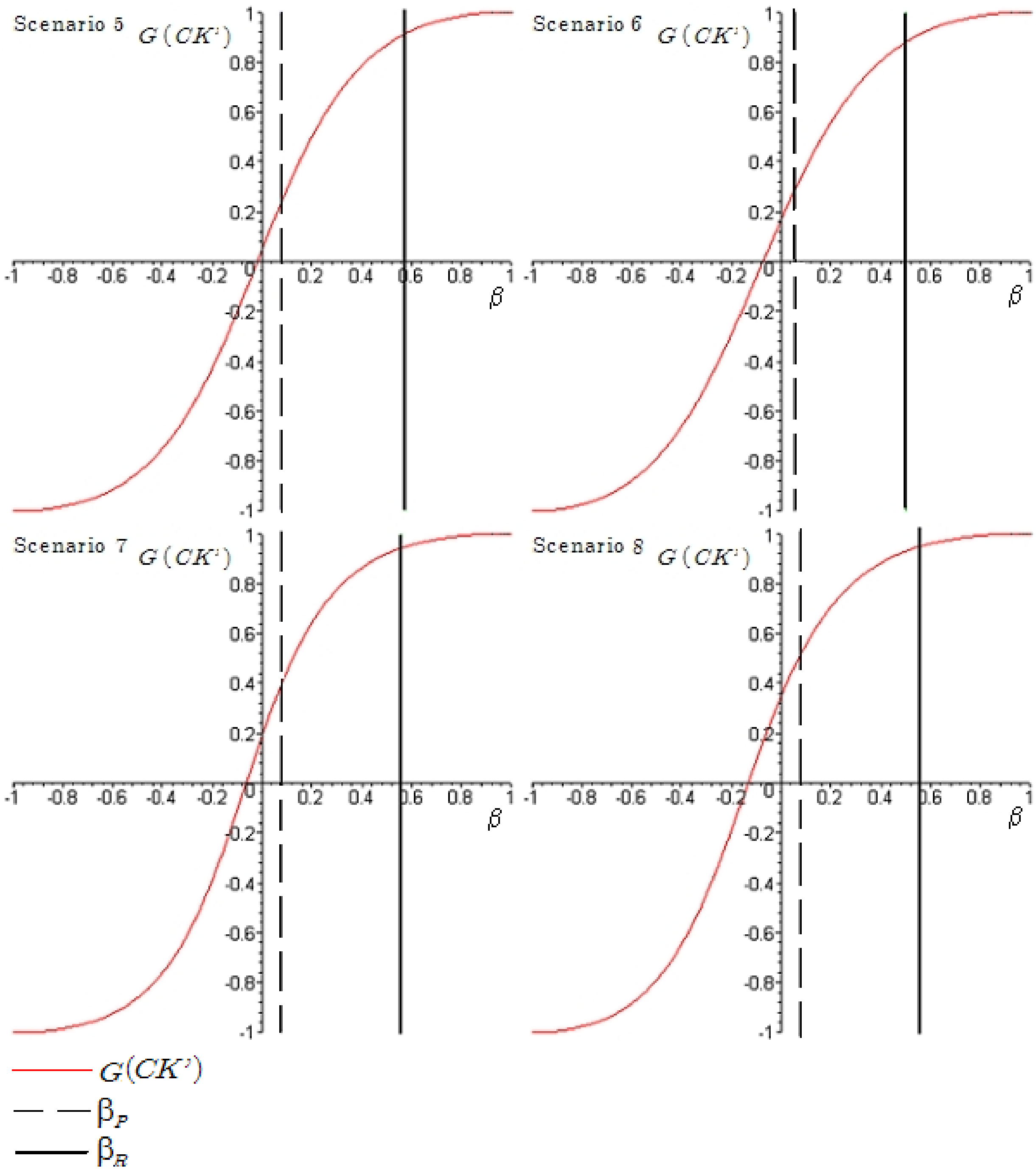




Scenarios are as specified in Table 5.

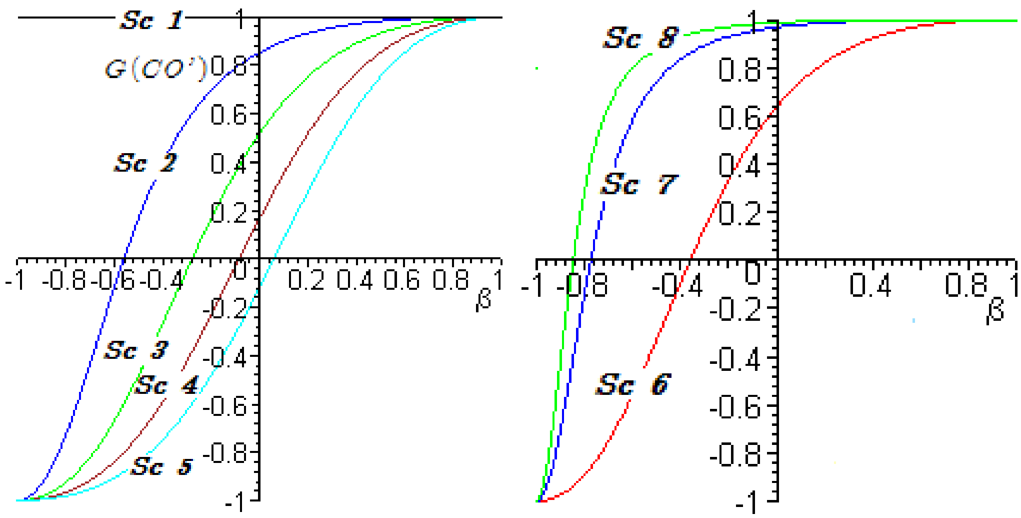
Fig. 2. Asymmetric Chicken Game and harmony transformations: examples





Scenarios are as specified in Table 5.

Fig. 3. Coordination game: examples



Sc x stands for scenario number x, as detailed in Table 5.