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DISCUSSION

Passive and active earth pressures in the presence of groundwater flow

N. BENMEBAREK, S. BENMEBAREK, R. KASTNER and A.-H. SOUBRA (2006). *Géotechnique* **56**, No. 3, 149–158

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I was very interested to see the two papers in the April issue of *Géotechnique* on the issue of earth pressure coefficients in the presence of steady-state seepage. I studied this topic, using the method of characteristics, for my undergraduate project at Cambridge University, but apart from the project report (Houlsby, 1975) the results have not been published before. The results lend support to the useful study by the authors.

The method used a standard application of the method of characteristics for effective stresses, as described by Sokolovski (1965). However, when the effective stresses and pore pressures are substituted for the total stresses in the equilibrium equations, these are written as

$$\frac{\partial \sigma'_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = -\frac{\partial u}{\partial x} \tag{2}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma'_{yy}}{\partial y} = -\gamma' - \frac{\partial u}{\partial y} \tag{3}$$

The body force in the $-y$ direction is therefore augmented by a term $\partial u/\partial y$, and there is also a body force in the $-x$ direction $\partial u/\partial x$. As long as the spatial variation of pore pressure u is specified, the resulting problem can be treated simply as one involving a variable body force. The method was programmed in Fortran, and the pore pressure could be specified either through an analytical expression or by interpolation from a grid of points.

Of particular relevance to the paper by Benmebarek *et al.* are some calculations of the stability of a sheet pile wall with essentially the same geometry that they study (see their Fig. 1). For the pore pressure distribution I used the elegant analytical solution presented by Schofield & Wroth (1968: pp. 59–62). I carried out active and passive pressure calculations for $\phi = 30°$ and $40°$; for angles of interface friction on the wall of zero, $\phi/2$ and ϕ; and for a variety of depths of water H (in the authors' notation). For simplicity I examined just the case $\gamma' = \gamma_w$ (the same as the authors).

My study was conducted in terms of the maximum hydraulic gradient at the soil surface i_0, which is related (using the analytical expression for the pore pressures) to the depth of water used by the authors by the expression $H/f = \pi i_0$, where f is the wall depth. The cases analysed in the two studies do not exactly correspond, but by interpolating linearly between the authors' figures for $\delta = \phi/3$ and $\delta = 2\phi/3$ it is possible to obtain the comparisons of active and passive earth pressure coefficients for the cases shown in Table 8 (which shows only a fraction of the possible comparisons). There is a very satisfactory agreement between the stress-characteristic solutions obtained by Houlsby (1975) and the figures quoted by the authors, which include FLAC analysis and limit equilibrium calculations by Soubra *et al.* (1999). The remarkably good comparison between the different methods lends confidence to both.

I took the study one stage further, and examined in two different ways the critical height of water on the upstream side of the sheet pile wall that would cause failure. The first method involved plotting against i_0 the horizontal components of the active and passive forces, including also terms from the water pressure both above and below the ground surface. The maximum allowable i_0 value was simply the point at which the two curves crossed (for lower i_0 values the active forces were calculated as less than the maximum passive resistance). This calculation would be relevant to a purely translational failure mechanism. The second calculation was the same in principle, but involved equating the moments about the toe of the sheet pile wall, thus representing a more realistic mechanism (although still not necessarily the most critical). The results of the two calculations, presented in terms of H/f, are given in Table 9, which shows that the second mechanism is more critical.

Table 8. Comparison of results by Houlsby (1975) and the authors

Case	H/f	ϕ	δ/ϕ	Houlsby (1975)	Range given by Benmebarek *et al.*
Active: K_a	0	30°	0	0·33	0·32–0·34
			0·5	0·30	0·30–0·32
		40°	0	0·22	0·21–0·26
			0·5	0·20	0·19–0·24
	2·5	30°	0	0·66	0·66–0·74
			0·5	0·61	0·60–0·63
		40°	0	0·44	0·42–0·54
			0·5	0·41	0·37–0·45
Passive: K_p	0	30°	0	3·00	2·98–3·06
			1·0	6·62	6·5–6·93
		40°	0	4·60	4·04–4·69
			1·0	18·90	18–20·01
	2·5	30°	0	0·53	0·51–0·54
			0·5	1·06	1·03–1·12
		40°	0	1·04	0·91–1·06
			0·5	3·08	2·94–3·57

Table 9. Critical values of *H*/*f* for failure of a sheet pile wall

ϕ	δ/ϕ	$(H/f)_{crit}$ (translation mechanism)	$(H/f)_{crit}$ (rotation mechanism)
30°	0	0·83	0·52
	0·5	1·09	0·67
	1·0	1·28	0·78
40°	0	1·17	0·69
	0·5	1·65	1·00
	1·0	2·05	1·25

Authors' reply

The authors would like to thank Professor Houlsby for his interest in our paper, and for his interesting remarks. His discussion raised two issues. The first concerns the comparison of the discusser's results for active and passive earth pressures using the slip-line method with the results presented in the paper using numerical FLAC simulations. The authors are pleased to see the good agreement between the two results. It should be noted here that a good agreement was also observed in the paper between FLAC simulation results and the upper-bound solutions presented by Soubra *et al.* (1999). The second issue raised by the discusser concerns the determination of the critical hydraulic gradient *H*/*f* leading to failure of the sheet pile using the global equilibrium equations of the wall. Both horizontal and moment equilibrium equations are considered by the discusser. This issue has not been examined in the paper. However, the authors' reply is as follows.

First, it should be noted that the seepage flow may induce failure of the sheet pile not only because of the non-verification of global equilibrium equations of the sheet pile, but also because of piping or heaving of the soil downstream of the sheet pile. A recent publication by Benmebarek *et al.* (2005) has shown that piping or heaving of the soil may occur for critical hydraulic gradients in the range 2·63–3·16 depending on the soil and soil/structure interface characteristics (ϕ, ψ, δ). For further details, the reader may refer to Benmebarek *et al.* (2005).

Second, examination of the fourth column of the discusser's Table 9 clearly shows that the moment equilibrium equation leads to smaller (i.e. more critical) hydraulic gradients than those obtained by Benmebarek *et al.* (2005). This means that the discusser's suggestion of considering the moment equilibrium equation is an essential task in the verification of the wall stability against failure due to seepage flow.

Note, however, that the discusser has considered a simplified mechanism for computation of the critical hydraulic head loss, as he assumed the rotation point to lie at the toe of the sheet pile. This may lead to approximate solutions and induce some error concerning this issue. However, this error may not be significant, because the moment arm of the net pressures acting below the rotation point is small. The error in the computation of the critical hydraulic head may possibly be more significant when considering the translation mechanism proposed by the discusser, because the wall stability cannot be obtained in this case without considering the passive pressures developed on the upstream side between the toe and the rotation point just above the toe of the sheet pile wall (cantilever wall failure mechanism) or without a strut maintaining the upper part of the wall.

The authors believe that the rotation mechanism (moment equilibrium) is more appropriate than the translation mechanism (horizontal force equilibrium) proposed by the discusser for a cantilever sheet pile. However, to rigorously identify the critical failure mode (horizontal translation, rotation, piping or heaving) and to quantify the corresponding critical hydraulic head, a more rigorous approach consists in gradually increasing the hydraulic head loss *H* until failure occurs using numerical simulations with FLAC or other finite element or finite difference codes.

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