Judah ha-Cohen and the Emperor's Philosopher

Dynamics of Transmission at Cultural Crossroads

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by

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Abstract

In his Hebrew encyclopaedic compendium *Midrash ha-Ḥokhmah*, the thirteenth-century Toledan scholar Judah ben Solomon ha-Cohen reports of a correspondence, held in Arabic, that he had with an unnamed philosopher who belonged to the court of the Holy Roman Emperor Frederick II in Italy. The present work investigates the different ways in which this correspondence helped transmit knowledge between scholars from different cultural and geographical settings. First, a critical edition, translation, and analysis are rendered of the two problems discussed in the text, which concern the construction of the five regular polyhedra and the calculation of oblique ascensions. The correspondence is then placed within the framework of other accounts of scholars who reportedly received imperial inquiries. It is shown that its subject matter was of interest to both the court and the scholarly community, and can be linked to the work of Frederick's correspondents Leonardo Fibonacci in Italy and the school of Ibn Yūnus in Mosul, and to the work of later scholars - Campanus of Novara and Muḥyī al-Dīn al-Maghribī. The unnamed philosopher, who is proved wrong in the correspondence, is in all likelihood Theodore of Antioch. An analysis of the terminology used in the Hebrew translation of the lost Arabic original shows that Judah created a unique mathematical and astronomical vocabulary, which changed during his working life. It is influenced by that of Jacob Anatoli, but Judah's terminology is generally much closer to that of his predecessor Ibn Ezra. It is then shown that the interreligious collaboration recorded in the correspondence is typical for the appropriation of Greek learning in the Middle Ages, but its placement within the framework of the *Midrash ha-Ḥokhmah* is influenced by interreligious polemics. Here, it serves to prove the superiority of the Jewish religion.

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Chapter One

Introduction

The *Midrash ha-Ḥokhmah* ('Exposition of Wisdom')[1] by Judah ben Solomon ha-Cohen of Toledo is the earliest extant Hebrew encyclopaedia of logic, philosophy, mathematics, astronomy and astrology. Originally composed in Arabic in the 1230s, the work was translated into Hebrew by the author himself around the year 1247. The *Midrash ha-Ḥokhmah* contains summaries and abbreviations of the most important philosophical and scientific works circulating at the time, covering Aristotelian logic, natural philosophy and metaphysics, geometry, astronomy, cosmology and astrology. In addition, it contains three treatises dedicated to traditional Jewish learning.

It is in the mathematical section of the compendium that Judah ha-Cohen reports of a correspondence with the court of Frederick II (1194-1250), Holy Roman Emperor of the Staufen dynasty, who was known for sending questions about a wide variety of subjects to scholars residing in many different cultural contexts. At the age of eighteen, so Judah alleges, he received a letter written in Arabic by the 'philosopher to the emperor' (his identity remains unknown), who asked him several scientific questions. He answered the philosopher, adding questions of his own, so initiating a correspondence that went on for several years. Two of the questions that were posed to Judah are discussed in detail in the *Midrash ha-Ḥokhmah*: one on Euclidean geometry, and one on astronomy. The discussions

[1] The tile of the work has been translated in various ways in different publications, as both the Hebrew root *d-r-sh* ('to examine, expound') and the word *ḥokhmah* ('wisdom, discipline, learning') can have several different meanings. Resianne Fontaine suggested that it might reflect the Arabic *Ṭalab al-ḥikmah*, which is the title of Saadya Gaon's *Commentary on Proverbs*. She proposes the following translations: 'The exposition of science'; 'The search for wisdom'; 'Inquiry into science/wisdom'. See Fontaine, "The Early Reception of Aristotle through Averroes in Medieval Jewish philosophy," 212. Like the Arabic *ḥikmah*, the Hebrew term *ḥokhmah* has to be understood in a much broader sense than the English notion of 'science'.

were passed on to Frederick, who was reportedly very pleased with Judah's answers. Judah finally travelled to Italy ten years later and resided at the emperor's court, where he translated the *Midrash ha-Ḥokhmah* from the Arabic original into the Hebrew language. The discussions give us insight into one of the ways in which the transmission of knowledge took place in the thirteenth century at this most crucial of geographical and intellectual boundaries.

The Arabic original of the *Midrash ha-Ḥokhmah* is lost. The complete Hebrew version can be found in two manuscripts,[2] but there are over forty manuscripts which contain excerpts from the work.[3] Judah's account of the correspondence with the emperor's philosopher can be found only in four of these: Cambridge, Cambridge University Library, MS. Add. 1737; Oxford, Bodleian Library, MS. Mich. 400; Parma, Biblioteca Palatina, MS. 2769; Vatican City, Biblioteca Apostolica Vaticana, MS. Ebr. 338. The portions of the *Midrash ha-Ḥokhmah* that were copied most often are a treatise on the hidden meaning of the letters of the Hebrew alphabet and Judah's Hebrew adaptation of Ptolemy's astrological work *Tetrabiblos*. The latter was published in 1886 by Jacob Shapiro.[4] An analysis and Italian translation of this part of the *Midrash* were recently made by Marienza Benedetto.[5] The first treatise of the *Midrash ha-Ḥokhmah*, which is the commentary on Genesis, Psalms and Proverbs, was edited by David Goldstein in 1981.[6] Colette Sirat published several passages from the *Midrash ha-Ḥokhmah* that illustrate

[2] Vatican City, Biblioteca Apostolica Vaticana, MS. Ebr. 338 and Oxford, Bodleian Library, MS. Mich. 551. However, the latter manuscript misses a few quires in the middle of the work.
[3] A list of 43 manuscripts can be found in Manekin, "Steinschneider on the Medieval Hebrew Encyclopedias," 475–479.
[4] *Otot ha-shamayim*. There are many mistakes in this printed version, which were already pointed out by Steinschneider: *Verzeichniss der hebräischen Handschriften der Königlichen Bibliothek zu Berlin*, 2:125.
[5] Benedetto, *Un enciclopedista ebreo*.
[6] Goldstein, "The Commentary of Judah ben Solomon Hakohen."

Judah's critique of Aristotle and his understanding of *qabalah* as superior oral tradition,[7] as well as extracts from the second treatise on the letters of the Hebrew alphabet.[8] In 2001, she rendered the Hebrew text (with a French translation) of a passage that can be found at the end of this treatise, in which Judah describes polemical questions on the nature of the Jewish people that were posed to him by a presumably Christian philosopher.[9] Resianne Fontaine is currently preparing a critical edition and English translation of the introduction and the natural philosophical portion of the *Midrash ha-Ḥokhmah.*

The fact that Judah ha-Cohen corresponded with the emperor's court has attracted considerable attention in modern research. However, the contents and structure of the correspondence have never been explored. The first scholar to do extensive research on the *Midrash ha-Ḥokhmah* was Moritz Steinschneider. In 1893 he published his monumental work *Die hebraeischen Übersetzungen des Mittelalters und die Juden als Dolmetscher: Ein Beitrag zur Literaturgeschichte des Mittelalters meist nach handschriftlichen Quellen.* The first paragraph of the work deals with Hebrew encyclopaedias, first of which is the *Midrash ha-Ḥokhmah.*[10] The most exhaustive description of the author and the work can be found in Steinschneider's Berlin catalogue. There he gives a full account of the contents and structure of the *Midrash*, cites beginnings and endings of chapters, identifies some of its sources and compares the readings of different manuscripts.[11] Steinschneider's notes and observations on the subject, published in various books and catalogues, became the basis

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[7] Sirat, "Juda b. Salomon ha-Cohen"; ead., "La Qabbale d'après Juda B. Salomon ha-Cohen"; ead., "Les traducteurs juifs à la cour des Rois de Sicile et de Naples."
[8] Sirat, "L'explication des lettres selon Juda b. Salomon ha-Cohen."
[9] Sirat, "A la cour de Frédéric II Hohenstaufen: une controverse philosophique entre Juda ha-Cohen et un sage chrétien." See also Fontaine, "Religious Polemics in a Philosophical Encyclopedia: Judah ha-Cohen on 'The Chosen People.'"
[10] Steinschneider, *Die hebraeischen Übersetzungen des Mittelalters*, 1–4.
[11] Steinschneider, *Verzeichniss der hebräischen Handschriften der Königlichen Bibliothek zu Berlin*, 2:121–126.

of all research that was done after him.[12] Despite having such a prominent place in one of Steinschneider's major studies, the *Midrash ha-Ḥokhmah* received little attention in the following decades.

It was only in the 1970s that further research on the *Midrash ha-Ḥokhmah* was carried out. Shalom Rosenberg noted that the philosophical writings of Hillel of Verona and Moses of Salerno were influenced by Judah ha-Cohen's terminology.[13] In 1975 David Goldstein demonstrated that both Immanuel ben Solomon of Rome and Rabbenu Bahya ben Asher knew the *Midrash ha-Ḥokhmah* and utilised the work in their biblical commentaries.[14] Georges Vajda and G. Sermoneta also mentioned the *Midrash ha-Ḥokhmah* in their articles.[15] The work of Colette Sirat was particularly important in making the work known to a wider audience. Apart from collecting and arranging all the information on the subject that can be gathered from Steinschneider's notes, she also did some valuable research on Judah's philosophy and use of sources, the importance of the notion of *qabalah* in his work, and his relationship with Christian scholars at the court of Frederick II.[16]

In the following years the work received increased scholarly attention. Mauro Zonta identified additional sources for the structure and doctrines presented in the

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[12] See, for example, Steinschneider, *Catalogus ... Lugduno-Batavae*, 53–60; id., *Die Mathematik bei den Juden*, d 110–111; id., *Die arabische Literatur der Juden*, 162; id., *Die hebræischen Handschriften der K. Hof- und Staatsbibliothek in Muenchen*, 118; Blumenfeld, *Otsar neḥmad*, II:233–236.
[13] Rosenberg, "Joseph Baruch Sermoneta (ed.), Moses ben Solomon, 'Un glossario filosofico ebraico-italiano del XIII secolo' (1969)."
[14] Goldstein, "The Citations of Judah ben Solomon ha-Cohen in the Commentary to Genesis of Rabbenu Bahya ben Asher"; id., "The commentary of Immanuel ben Solomon of Rome on chapters I-X of Genesis."
[15] Vajda, "La question disputée de l'essence et de l'existence vue par Juda Cohen, philosophe juif de Provence"; Sermoneta, "Federico II e il pensiero ebraico nell'Italia del suo tempo."
[16] Sirat, "Juda b. Salomon ha-Cohen"; ead., "La Qabbale d'après Juda B. Salomon ha-Cohen"; ead., "L'explication des lettres selon Juda b. Salomon ha-Cohen."; ead., *A History of Jewish Philosophy in the Middle Ages*, 250–255.

philosophical part of the *Midrash ha-Ḥokhmah.*[17] But the largest contribution towards understanding the philosophical part of the encyclopaedia was made by Resianne Fontaine. She investigated Judah's critical attitude towards Aristotle and the inclusion of non-Aristotelian thinkers into his work, analysed Judah's views on chosen topics in natural philosophy in comparison with other Jewish thinkers, and examined Judah's Hebrew scientific terminology, which is heavily influenced by Arabic.[18] In 2000, a conference on medieval Hebrew encyclopaedias took place in Jerusalem. The papers of this conference, which contained some discussion of the *Midrash ha-Ḥokhmah,* were published in the volume *The Medieval Hebrew Encyclopedias of Science and Philosophy (Dordrecht, 2000).* There, Resianne Fontaine discusses in detail Judah's sources and use of sources for the first part of the *Midrash ha-Ḥokhmah*, and investigates the question if the anachronistic term 'encyclopaedia' can be applied to the work.[19] Two articles in the volume deal with the mathematical/astronomical section of the *Midrash ha-Ḥokhmah*, of which the correspondence with the philosopher forms a part. Tony Lévy investigated the structure, language and sources of Judah's Hebrew rendering of Euclid's *Elements*.[20] Y. Tzvi Langermann examined the contents and structure of the three sections of the *Midrash ha-Ḥokhmah* that pertain to the science of the heavens: the parts on astronomy, on astrology, and the treatise on the letters of the alphabet.[21] Following the conference, Judah's polemic discussion with a Christian philosopher on the status of the Jews was analysed

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[17] Zonta, *La filosofia antica nel Medioevo ebraico*, 200–204.
[18] Fontaine, "Aristotle vs. Galen in the Zoological Part of R. Judah ben Salomon's 'Midrash ha-Hochmah'"; ead., "The Inhabited Parts of the Earth according to Medieval Hebrew Texts"; ead., "Red and Yellow, Blue and Green: The Colours of the Rainbow according to Medieval Hebrew and Arabic Scientific Texts"; ead., "The Facts of Life: The Nature of the Female Contribution to Generation according to Judah ha-Cohen's Midrash ha-Hokhma and Contemporary Texts"; ead., "Arabic Terms in Judah ben Solomon ha-Cohen's *Midrash ha-Hokhmah*."
[19] Fontaine, "Judah ben Solomon ha-Cohen's 'Midrash ha-Hokhmah': Its Sources and Use of Sources."
[20] Lévy, "Mathematics in the *Midrash ha-Hokhmah* of Judah ben Solomon ha-Cohen."
[21] Langermann, "Some remarks on Judah ben Solomon ha-Cohen."

by both Colette Sirat and Resianne Fontaine.[22] The role of Jewish translators at the court of Frederick II and his successors was investigated by Mauro Zonta.[23] Marienza Benedetto dedicated her doctoral thesis to Judah's adaptation of Ptolemy's *Tetrabiblos*.[24] But it is mainly the work of Resianne Fontaine that has furthered our understanding of the encyclopaedia. In preparation of an edition of the natural-philosophical part of the *Midrash ha-Ḥokhmah* she investigated the presentation of Aristotle's writings on different topics in the work,[25] furthered our insight into Judah's use and presentation of sources,[26] identified works that cite Judah's work,[27] explored the theme of the 'three worlds' that plays a major role throughout the encyclopaedia,[28] and investigated the role that the *Midrash ha-Ḥokhmah* fulfilled in being an encyclopaedic work.[29] While the natural philosophical part of the *Midrash ha-Ḥokhmah* has received a lot of scholarly attention, no further research on its mathematical/astronomical part, which includes the correspondence, has been conducted in recent years.

[22] Sirat, "A la cour de Frédéric II Hohenstaufen: une controverse philosophique entre Juda ha-Cohen et un sage chrétien"; Fontaine, "Religious Polemics in a Philosophical Encyclopedia: Judah ha-Cohen on 'The Chosen People.'"
[23] Zonta, "Traduzioni filosofico-scientifiche ed enciclopedie ebraiche alla corte di Federico II e dei suoi successori (secolo XIII)."
[24] Benedetto, *Un enciclopedista ebreo*.
[25] Fontaine, "The Early Reception of Aristotle through Averroes in Medieval Jewish philosophy: The Case of the Midrash ha-Hokhmah"; ead., "Aristotle's 'De Anima' in a Hebrew encyclopedia"; ead., "The First Survey of the 'Metaphysics' in Hebrew."
[26] Ead., "Abraham ibn Daud and the Midrash ha-Hokhmah: a Mini-Discovery."
[27] Ead., "An Unexpected Source of Meir Aldabi's *Shevile Emunah*."
[28] Ead., "The Theme of the Three Worlds in the 'Midrash ha-Hokhma.'"
[29] Fontaine and Berger, "On Pre-modern Hebrew and Yiddish Encyclopedias."

Judah ben Solomon ha-Cohen

What little information we have about Judah ben Solomon ha-Cohen,[30] stems mainly from the *Midrash ha-Ḥokhmah* itself.[31] Judah was born and raised in Toledo, which had been under Arab rule from the eighth to eleventh century. After the town had been conquered by Alfonso VI in 1085, its population still consisted of a mix of Mozarabs, or 'Arabised' Christians, Muslims and Jews. Arabic remained the language of culture and religion for a large part of the population. At the time Judah was born, around the year 1215, Toledo's Jewish community had become the most prominent in the Kingdom of Castile and one of the most important in Spain. Around 12,000 Jews lived in the city; they would have constituted about a third of the population.[32] Documents from the period show that Toledo was not only a centre of Jewish learning, but that Jews also held high positions at the royal court and played an important role in a Christian translation movement that had started in the twelfth century. Christian scholars from Latin Europe came to visit the town in the search of Arabic philosophic writings. They translated these works into Latin, often with the help of Jews.[33] Judah ha-Cohen was the descendant of one of the most

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[30] Judah ben Solomon ha-Cohen is in some publications also referred to as 'Ibn Matqah'. Moritz Steinschneider, for example, lists the author's name as 'Jehuda b. Salomo Kohen (1247) ex familia מתקה, Toletano', stating that the appellation *ibn Matkah* was already used by Joseph ha-Yevani, a 14th century scholar (Steinschneider, *Catalogus ... Lugduno-Batavae*, 54). However, the name 'Ibn Matqah' can be found in only one of the over 40 surviving manuscripts that contain parts of the *Midrash ha-Ḥokhmah*; a 16th century manuscript, which has the name in an annotation. The authenticity of that name is thus very doubtful. See Fontaine, "Matkah, Judah ben Solomon Ha-Kohen"; Sirat, "Juda b. Salomon ha-Cohen," 40.

[31] Collette Sirat has gathered the available information in "Juda b. Salomon ha-Cohen," 40–45. See also "La Qabbale d'après Juda B. Salomon ha-Cohen"; ead., "Les traducteurs juifs à la cour des Rois de Sicile et de Naples"; ead., *A History of Jewish Philosophy in the Middle Ages*, 250–255.

[32] It is estimated that about 37,000 people lived in Toledo at the end of the eleventh century. See Roth, "New Light on the Jews of Mozarabic Toledo," 196; Beinart, "Toledo," 22.

[33] On Toledo as the centre of the translation movement, see Burnett, "The Coherence of the Arabic-Latin Translation Programme in Toledo in the Twelfth Century."

prominent Jewish families at the time. In the astrological part of the *Midrash ha-Ḥokhmah* he writes:[34]

> *If Saturn and Mercury are [in conjunction] with the Sun in the ascendant [...], the new-born will be slow of speech, and of a slow tongue.*[35] *The author said: This came true for me. And on the day that I was born, my teacher,*[36] *my mother's father Rabbi Ziza ibn Shoshan,*[37] *had made this [astrological] judgement about me.*

By confirming the veracity of this astrological rule, Judah informs us that he apparently had some kind of speech impediment, but also, and more importantly, that his maternal grandfather was a certain Ziza ibn Shoshan. To a medieval reader the name Ibn Shoshan would certainly be familiar, as the family, which can be traced back to the eleventh century, was one of the most important and influential families in Toledo.[38] In the twelfth century, members of the family had held positions in the local government and were highly esteemed by the Christians. We do not have much information on Judah's maternal grandfather, but a Ziza ben Abraham ibn Shoshan is mentioned in a Toledan document of 1205, as intervening in the will of a certain Tusi, daughter of Solomon ben R. Falcon.[39] More information is available on Judah's relative Abu ʿUmar Joseph ibn

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[34] Vatican, MS. Ebr. 338, fol. 287v-288r: ואם שבתי וכוכב עם השמש בצומח או בשביעי והיה כוכב מערביי והביטו שניהם לירח יהא כבד פה וכבד לשון. אמר המחבר. וזה נתקיים בי וביום שנולדתי דן עלי כן זקני אבי אמי ר׳ זיזא אבן שושן נ׳ב׳ת׳ו׳י׳י׳א׳. The text can also be found in the printed edition of the astronomical part of the *Midrash ha-Ḥokhmah*: *Otot ha-shamayim*, 16. An Italian translation of the edition was rendered by Benedetto, *Un enciclopedista ebreo*, 210. See also Gutwirth, "History, Language, and the Sciences in Medieval Spain," 515.

[35] Cf. Ex. 4,10: *And Moses said unto the LORD, O my Lord, I am not eloquent, neither heretofore, nor since thou hast spoken unto thy servant: but I am slow of speech, and of a slow tongue*.

[36] Throughout his encyclopaedia, Judah uses the Hebrew term זקן in the same sense as the Arabic *shaykh*; it can mean 'old man', but also 'teacher'. See Langermann, "Some remarks on Judah ben Solomon ha-Cohen," 374.

[37] In the printed edition the grandfather's name is given as 'Zira': זקני אבי מורי זירא בן שושן (*Otot ha-shamayim*, 16). This appears to be an error, since all manuscripts we consulted give the grandfather's name as זיזא ('Ziza'). Furthermore, all of them read אבי אמי ('my mother's father') instead of אבי מורי ('my father and teacher'). Cf. Oxford, MS. Mich. 551, fol. 184r; Milan, Biblioteca Ambrosiana, MS. J 17 Inf., fol. 93v; Prague, Jewish Museum, MS. 264; Parma, Biblioteca Palatina, MS. 2769.

[38] On the importance of the family, see Roth, "New Light on the Jews of Mozarabic Toledo"; id., "Ibn Sūsan Family"; Avneri, "Ibn Shoshan." A scholarly treatise by a later member of the family, David ben Solomon ibn Shoshan (*fl.* late 15th cent.), is discussed by Y. Tzvi Langermann, "David Ibn Shoshan on Spirit and Soul."

[39] Gutwirth, "'Entendudos': Translation and Representation in the Castile of Alfonso the Learned," 393.

Shoshan (1135–1205), who was *almoxarife* (treasurer) in the court of Alfonso VIII of Castile. In 1203, he founded one of Toledo's many synagogues. He carried the title *ha-nasi* ('the prince') and was highly respected in the Jewish community. Abu ʿUmar Joseph ibn Shoshan was also praised by the Hebrew poet and translator Judah al-Harizi (1165–1225), the Provençal Talmudic scholar Abraham ben Nathan of Lunel (c. 1155–1215), and the Talmudic commentator, thinker, and poet Meir ben Todros ha-Levi Abulafia (c. 1170–1244). The latter also married one of Abu ʿUmar's daughters. It is therefore no surprise that Meir Abulafia became Judah's teacher. In the zoological section of the *Midrash ha-Ḥokhmah* Judah writes:

> *Some of the animals with blood have a gall-bladder, and some do not. Aristotle forgot to mention another species that does not have a gall-bladder, namely the dove, as is mentioned in the Jerusalem Talmud. Thus I heard from my teacher and master Meir ha-Levi of blessed memory.*[40]

Rabbi Meir Abulafia was in fact the most renowned Spanish rabbi in the first half of the thirteenth century. His most extensive literary work was a commentary on the Talmud called *Sefer Prate Pratin* ('Book of Minute Details'), but he is best known for his role in the Maimonidean controversy on the topic of resurrection of the dead. Although sincerely admiring Moses Maimonides (1135-1204), Abulafia was the first European scholar to publicly criticise him, in particular for the view of 'the world to come' that was displayed in Maimonides' *Mishneh Torah*, as he believed that Maimonides did not affirm the resurrection of the body.[41] Judah ha-Cohen received a profound religious education from his teacher. Maimonides appears to have been an inspiration for the study of philosophy:

> *I acknowledge that the* Guide to Righteousness *[Maimonides] planted in my heart a burning desire to study the books of the philosophers, until I had*

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[40] ויש למקצת החי הדמיי מרה ואין למקצתן. ושכח אריסטו להזכיר עוד מין א׳ שאין לו מרה והוא היונה כמ׳ שהזכיר בירושלמי. כן שמעתי מפי מורינו ורבינו מאיר הלוי ז׳צ׳ל'. Oxford, MS. Mich. 551, fol 73r. The passage was first discussed by Septimus, *Hispano-Jewish Culture in Transition*, 20.

[41] On Meir Abulafia see ibid.

> *achieved what little I have. I truly know that the intention of the Guide was merely to cause those who erred after the words of Aristotle to return and to hold fast to our holy Torah. One ought not suspect the lamp of the exile of Ariel of the things of which some people of this generation suspected him.*[42]

Judah's statement refers to Maimonides' philosophical work, the *Guide for the Perplexed*. Originally composed in Arabic, the work was first translated into Hebrew by Samuel ibn Tibbon (c. 1165–1232) in 1204, shortly before Maimonides' death. As soon as the Hebrew version became available, it triggered discussions on the usefulness and dangers of the study of Greek philosophy. Opponents claimed that the occupation with secular disciplines would ultimately lead to heresy, while its supporters argued the opposite, namely that it would corroborate faith. As we will see, Judah probably wrote the Arabic version of his encyclopaedia in the late 1230s, a few years after the discussion about the permissibility of the study of philosophy in general, and Maimonides' *Guide for the Perplexed* in particular, had reached a high point among Jewish scholars in Spain and Southern France. In the *Midrash ha-Ḥokhmah* Judah delineates the usefulness but also the limits of the study of philosophy – it can therefore be interpreted as Judah ha-Cohen's own contribution to the discussion.[43]

But what 'little knowledge' Judah had achieved of the secular sciences must also have brought him a considerable reputation amongst non-Jewish scholars, since already at the young age of eighteen he received a letter with several questions on scientific topics from the court of Emperor Frederick II. Judah informs his readers that he answered these

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[42] Oxford, MS. Mich. 551, fol. 145r. The passage was first translated by Septimus, *Hispano-Jewish Culture in Transition*, 98. See also Sirat, "Juda b. Salomon ha-Cohen," 44; Fontaine, "Judah ben Solomon ha-Cohen's 'Midrash ha-Hokhmah': Its Sources and Use of Sources," 194.

[43] On the Maimonidean controversy see Septimus, *Hispano-Jewish Culture in Transition*, 75–103. The influence of the controversy on the *Midrash ha-Ḥokhmah* has been discussed by R. Fontaine. See, for example, "Judah ben Solomon ha-Cohen's 'Midrash ha-Hokhmah': Its Sources and Use of Sources," 206; ead., "The Early Reception of Aristotle through Averroes in Medieval Jewish philosophy: The Case of the Midrash ha-Hokhmah," 219.

questions, adding some questions of his own, which initiated a correspondence between them. The contents and circumstances of the exchange will be discussed in the following section. Regarding Judah's personal history, the following point is important: He notes that ten years after the correspondence took place he travelled to 'the Emperor's lands' and saw the emperor's court, and that it was in Tuscany that he translated the *Midrash ha-Ḥokhmah* into the Hebrew language, which he had composed in Arabic while in Spain. Within the zoological section of his encyclopaedia Judah relates that in 1245, he visited the emperor's court in Lombardy:[44]

> *With its trunk the elephant brings food and water to its mouth. It can wrap it around a tree and uproot it with it. Its design is suitable for this kind of work, since it has great power and strength. I saw it in Lombardy at the Emperor's court in the year [5]005 [i.e. 1245].*

Two years later, Judah ha-Cohen is translating the *Midrash ha-Ḥokhmah* into Hebrew:[45]

> *This is the perfect beauty of the Book of Astronomy of al-Biṭrūjī,[46] who was very discerning, and whose time was close, about 30 years before our time now, which is the year 5007 [i.e. 1247].[47]*

The biographical information that is provided by Judah thus makes it possible to calculate an approximate date of birth; he travelled to Italy ten years after his first correspondence with the philosopher, which he had at the age of eighteen. Therefore, he must have been about 28 years of age when he left Toledo. As he informs us that he was at the emperor's

[44] Oxford, MS. Mich. 551, fol. 69v: וחוטם הפיל יקרב המאכל והמשתה עד פיו וידרוך אותו על האילן ויתלשנו בו ובריתו נכונה לזה הפועל לפי שיש לו כח וחזק גדול. וכבר ראיתיו בלומברדיאה בחצר המלך שנת חמש. The passage is translated by Sirat, "Les traducteurs juifs à la cour des Rois de Sicile et de Naples," 176; ead., "Juda b. Salomon ha-Cohen," 42.

[45] Oxford, MS. Mich. 551, fol 161v: זהו מכלל יופי מספר התכונה לאלבטרוגי׳ שהיה נבון הרבה וזמנו קרוב לזמננו זה שהיא שנת שבע וה׳ אלפים כמ׳ ל׳ שנה. See Sirat, "Juda b. Salomon ha-Cohen," 42.

[46] The 'Book of Astronomy' is the *Kitāb fī al-hayʾa* by the Andalusian scholar Nūr al-Dīn Abū Isḥāq al-Biṭrūjī (fl. 12th century). In English speaking sources, the work is usually referred to as *On the Priciples of Astronomy*. For an introduction to the author and his work, see Mancha, "al-Biṭrūjī."

[47] It turns out that 1217 is the year in which Michael Scot rendered a Latin translation of the work, while staying in Toledo. It is therefore possible that the date Judah gives in this passage is that of the translation, not that of al-Biṭrūjī's death. On the topic see Sirat, "Juda b. Salomon ha-Cohen," 42; Samsó, "al-Bitrūjī al-Ishbīlī, Abū Ishāq," 33.

court in Lombardy in 1245, he must have been born in 1217 at the latest. If, of course, he had arrived in Italy some time before 1245, his date of birth must be changed accordingly. No mention of Judah can be found after the year 1247.

Judah ha-Cohen was thus an accomplished scholar stemming from a well-known, influential family who left his native country and settled in Italy. There he encountered different cultural settings, not only when visiting the imperial court - but also amongst the Jewish community. While the Jews of Toledo had for centuries been confronted with and also contributed to the philosophical and scientific accomplishments of Islamic culture, the Jews in Northern Italy and Southern France had had no access to philosophical writings rendered in Arabic. Only when Hebrew translations of philosophical works reached the Jews in the Latin-speaking world, were they able to study the so-called 'foreign sciences'. It was mainly Maimonides' *Guide for the Perplexed* that sparked a keen interest in the study of Greek philosophy within those Jewish communities. Confronted with philosophical concepts that were entirely new to them, the Jewish communities of Southern France and Northern Italy were eager to study the works Maimonides referred to in his writings, and to understand the philosophical and scientific ideas that were underlying Maimonides' thought. Judah ha-Cohen on his part was very familiar with that philosophic tradition; he had even composed an encyclopaedic work in Arabic. To the Jews he encountered in the Latin West, who were eager to get access to Greek philosophy in the Arabic tradition, his work had the potential to become a very useful source of information. Judah notes:

> *Initially I had composed this book in the Arabic language, and when it so happened that I went down to Tuscany, my friends urged me to translate it into the holy tongue, and I did so according to my ability.*[48]

His claim appears to be more than just a trope; the Jewish communities in Italy were eager to get information on philosophic thought and asked Judah to make his own source of information accessible to them.

The correspondence with the emperor's philosopher

Judah's account of the correspondence with the Emperor's philosopher is found in the second Part of the *Midrash ha-Ḥokhmah*. After his abridgement of Euclid's *Elements* Judah writes:[49]

> *The author said: In the time of my youth the sage, the Philosopher to the Emperor, the Great King, Emperor Frederick (may his glory be exalted), sent me questions — I was still in my own land in the midst of the exile of Jerusalem which is in Spain. The first one was: How do we construct each of the five [regular] solids around a given sphere, and how do we construct the given sphere inside each of them and also around each one of them, [and provide each case] with a proof? I responded — I was eighteen year years old at the time — and this was what I said. Who could have told me when I answered [the questions] in the country of my birth, in the community of the town of Toledo, before my teacher and master, my father (may his rest be glorious - his soul shall dwell at ease and his seed shall inherit the earth), that it would turn out that I would go down to Tuscany. There I translated [the questions] from Arabic into the Hebrew language, with this book that I composed when I was still there [i.e. in Toledo]. Everything that the Merciful does is for the best. Blessed be God every day.*

Judah had thus received a set of questions from the court of Frederick II (1194-1250), one of the most famous and controversial emperors of the Middle Ages. Frederick was highly interested in the arts, philosophy and the natural sciences. He engaged in numerous

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[48] Oxford, MS. Mich. 551, fol. 123v: ובתחלה חברתי זה הספר בלש׳ ערבי וכשנתגלגל הדבר וירדתי לטוסקנא פיסו ממני חברי להעתיקו בלש׳ הקודש ועשיתי כפי היכולת.

[49] See my edition, paragraphs 1-2.

building projects, founded the University of Naples, commissioned translations of scientific texts from Arabic into Latin, as well as inviting scientists to his court and engaging in discussions with them. His unquenchable thirst for knowledge is legendary - already in his own time he was known as *stupor mundi*, amazement of the world. Particularly striking is the way he had lists of philosophic and scientific questions sent to renowned scientists all over the world.[50]

Leonardo of Pisa, also known as Fibonacci, (ca. 1170-after 1240), who had studied Arabic mathematics in North Africa and Syria, is just one example. He had been invited to the court where he engaged in mathematical discussions with the emperor and his philosophers.[51] Leonardo relates that the court official John of Palermo had prepared a series of difficult mathematical questions for him. Having discussed them at the meeting, Leonardo later presented some problems and their solutions in writing, in order to send them to the emperor. A few of these questions he published in his works *Flos* and *Liber quadratorum.* He even dedicated the latter work to the emperor. Furthermore, Leonardo sent a letter with mathematical problems and their solutions to another court official who was highly trained in Arabic mathematics, Theodore of Antioch (c.1195-1250). The revised version of his major work, the *Liber abaci*, was dedicated to Frederick's court philosopher Michael Scot (c.1175-c.1235). This physician, alchemist and astrologer of Scottish origin was certainly the first person Frederick would turn to with his questions.[52] Before joining the court of the emperor, he had spent several years in Judah's hometown Toledo, where he had translated different scientific books from Arabic to Latin. In Italy

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[50] On the scientific culture at the court of Frederick II, see Abulafia, *Frederick II*, 251–289; Stürner, *Friedrich II*, 2. Der Kaiser 1220–50:342–457; Haskins, *Studies in the History of Mediaeval Science*, 242–271; Sirat, "Les traducteurs juifs à la cour des Rois de Sicile et de Naples."
[51] See Rashed, "Fibonacci et les mathématiques arabes."
[52] On Michael Scot, see Haskins, op. cit., 272–298; Burnett, "Michael Scot and the Transmission of Scientific Culture from Toledo to Bologna via the Court of Frederick II Hohenstaufen."

Michael worked as Frederick's court astrologer, continued translating and composed scientific works of his own. In his *Liber introductorius*, an encyclopaedic work in three major parts, Michael mentions numerous questions that were posed to him by the emperor; he gives a list of some fifty queries. The topics range from hydrology and volcanology to the whereabouts of the hereafter and God's residence. Michael answers most of them, not without praising the emperor for his great learning.[53] But both Frederick and his court astronomer would also turn to one of Judah's co-religionists with their questions. In his collection of homilies *Malmad ha-Talmidim* the Jewish scholar Jacob Anatoli (c.1194-c.1256) reports of twenty-one exegetical questions that he discussed with Michael Scot. At times, the emperor himself also joined these discussions.[54]

But Frederick did not restrict his search for knowledge to Europe. A list of questions on philosophy, geometry and mathematics was sent to the Ayyubid sultan al-Kāmil (1180-1238) in Egypt, who forwarded them to his sages.[55] Another set of questions was sent to one of Theodore of Antioch's teachers, the mathematician Kamāl al-Dīn ibn Yūnus (d. 1242) in Mosul.[56] But the most famous questions answered by Muslim scholars are probably the "Sicilian questions" on Aristotelian philosophy that were sent to the North-African philosopher Ibn Sabʿīn (1217- c. 1270).[57] Although the authenticity of some of these reports has been questioned recently, and some of the questions actually stem from

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[53] The list has been edited by Grebner, "Der 'Liber Nemroth', die Fragen Friedrichs II. an Michael Scotus und die Redaktionen des 'Liber particularis.'"
[54] The questions were published by Sirat, "Les traducteurs juifs à la cour des Rois de Sicile et de Naples."
[55] See Gabrieli, *Arab Historians of the Crusades*, 270.
[56] Hasse, "Mosul and Frederick II Hohenstaufen: Notes on Atiraddin al-Abhari and Siragaddin al-Urmawi."
[57] An Arabic edition with German translation was made by Akasoy, *Philosophie und Mystik in der späten Almohadenzeit*.

older works,[58] the mass of different sources makes it clear that the emperor had a habit of sending questions to scientists of Christian, Jewish and Muslim descent.

The 'emperor's philosopher' who contacted Judah thus appears to have acted on Frederick's orders. Unfortunately, Judah does not mention his name, but the fact that he refers to him as *philosopher*, or *one who is called 'philosopher'* throughout his account of the correspondence, suggests that *philosopher* was actually an official title that this person held. Furthermore, the correspondence was carried out in Arabic. Only a few scholars that held official positions at Frederick's court had the required knowledge of the topic and of the Arabic language to conduct such a correspondence. These were Michael Scot, and the two scholars that reportedly had discussed mathematical problems with Leonardo of Pisa: John of Palermo, and Theodore of Antioch.

The question that was sent to Judah ha-Cohen relates to the construction of the five regular polyhedra, which are the tetrahedron, cube, octahedron, dodecahedron and icosahedron. The construction of these solids inscribed in spheres is described in detail in book XIII of Euclid's *Elements*. The apocryphal books XIV and XV of this work, which were probably authored by Hypsicles and Isidore of Miletus, respectively, further discuss the properties of these solids. Judah, however, is asked to solve problems that are not discussed by Euclid: Instead of having the sphere circumscribe the solids, he is asked to have the solids circumscribe a given sphere, and in addition to construct a sphere inside and outside of given solids.

The importance of the regular polyhedra to ancient and medieval scholars alike lies in the fact that they play a special role in the cosmology of the ancient Greek philosopher

[58] See ibid., 107–124; Grebner, "Der 'Liber Nemroth', die Fragen Friedrichs II. an Michael Scotus und die Redaktionen des 'Liber particularis.'"

Plato (c. 427-347 BC).[59] This is why the regular polyhedra are also referred to as platonic solids. In his *Timaeus* Plato argues that the four elements (earth, water, air, fire) and the world as a whole had geometrical structures that were defined by the five regular polyhedra. Physical bodies were composed from polyhedral particles, which were bounded by mathematical surfaces. He pictured transformations between elements as the disassembly and rearrangement of these surfaces. Plato's polyhedral theory was rejected by Aristotle (384-322 BC), who discusses it in length in book III of his *De caelo*. As the early Latin translations and commentaries on Plato's *Timaeus* did not contain his theory on regular polyhedra, scholars in Christian Europe had little knowledge of Plato's theory until Aristotle's works on natural philosophy were translated into Latin. The first Latin translation of Aristotle's *De caelo* was made (from a 9th-century Arabic version) by Gerard of Cremona (c. 1114-1187), who worked as an Arabic-Latin translator in Toledo. Some fifty years later, a second source of information on Plato's theory was made available in Latin. The Andalusian scholar Ibn Rushd (1126-1198)[60] had written more than thirty Arabic commentaries on Aristotle's work. It was Frederick's court astronomer Michael Scot who translated the long commentary on *De caelo* into Latin, and with it Aristotle's and Ibn Rushd's discussions of the polyhedral theory. It may well have been this translation that triggered the philosopher's interest in platonic solids. In any case, the construction of regular polyhedra was not just a mathematical problem, as it played an important role in cosmology.

Having answered the philosopher's question on regular polyhedra, Judah writes:[61]

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[59] An introduction to Plato's theory and its reception in antiquity and the middle ages is given by Sanders, "The Regular Polyhedra in Renaissance Science and Philosophy," 10–77.
[60] For introductions on Ibn Rushd's life and works, see Arnaldez, "Ibn Rushd"; Iskandar, "Ibn Rushd, Abū'l-Walīd Muḥammad ibn Ahmad ibn Muḥammad."
[61] Edition, paragraph 31.

This is how I managed to answer that 'emperor's philosopher' when I was eighteen years old. In addition, I myself asked other questions, and what we will write now is one of them. It was to give a clear geometric proof for the tables of ascensions for places in sphaera obliqua*. The so-called 'philosopher' answered me at that time in Arabic in the following words. I have translated it into the Hebrew language.*

The question that Judah asks the philosopher relates to astronomy. He refers to tables of *oblique ascension*: they are tables by which one can calculate arcs of the celestial equator that rise with arcs of the ecliptic depending on the position of the observer on the earth.[62] Knowing which part of the celestial equator was rising at the same time as a certain part of the ecliptic was important for timekeeping as well as astrology. Tables containing these data had already been made in antiquity by the famous astronomer Ptolemy (2^{nd} century C.E.). They had continuously been revised and improved by scientists in Islamic countries, who introduced Indian and Iranian methods of calculation into Greek astronomy. Many of these tables were distributed in astronomical handbooks, called *zīj* in Arabic. Z*ījes* contained different kinds of data tables needed for the calculation of the positions of the stars and the planets, but they also covered other topics such as timekeeping and astrological computations. These compilations were quite popular in the Arab world. Over 200 different *zījes* were produced between the eighth and the fifteenth centuries. At the time of the correspondence, the 1230s, only a few *zījes* were available in Latin, the most famous being the Toledan Tables.[63]

In his answer the philosopher uses spherical geometry in order to find the so-called equation of daylight, or ascensional difference, which is the difference between the rising time of a certain part of the ecliptic as seen from the terrestrial equator and its rising

[62] A short introduction to medieval mathematical astronomy can be found in Van Brummelen, *The Mathematics of the Heavens and the Earth*, 1–8. On ascensions see King, "al-Maṭāliʿ." An introductory discussion of tables of ascensions can be found in Evans, *The History & Practice of Ancient Astronomy*, 109–127.
[63] See Kennedy, "A Survey of Islamic Astronomical Tables." The work has been updated by King and Samsó, "Astronomical Handbooks and Tables from the Islamic World (750-1900): An Interim Report."

time as seen from any other locality in the northern hemisphere. Judah presents the philosopher's explanation and then writes:[64]

> *I then supplied him with several answers in relation to this text and to the diagram he had made, constructed on the other page.*[65] *I answered him regarding this matter: On error and on errors that are not appropriate for a student who is called 'philosopher' like you: [...]*

What follows is a mocking, even sarcastic, refutation of the philosopher. Judah's critique is based on one major point. Firstly, the philosopher had claimed that two particular spherical triangles are *similar*, i.e. that they contain the same angles respectively. Then he had claimed that those triangles have different sizes. In spherical geometry, however, that is impossible. There is no such thing as *similar* triangles on one given sphere: if two triangles on the same sphere contain the same angles, then the triangles are *equal*. In other words, the philosopher had confused plane and spherical geometry. This explains Judah's sarcasm: his colleague had made such a fundamental mistake that he did not deserve to be called 'philosopher'. If he truly believed that the same rules applied to spherical and plane triangles, Judah concludes, "he has to help us with this *novelty* that has come about *under the sun*." Of course, this request for help is ironic, for it is well-known that *there is nothing new under the sun (Eccl. 1:9).* He ends his account of the correspondence with the following words:[66]

> *The author said: When the things were reported before the Emperor, the King, Emperor Frederick (may his glory be exalted), he was very pleased with the answers that I had given to the one who is called 'philosopher' before him. In addition, there was much debate between us about many things, and about many questions and answers, but we cannot elaborate any further on these matters in this book. Furthermore, about ten years later it so happened that I went down to the emperor's lands; I saw the quality of his actions and his affairs, his philosophers, his scribes, his wise men, his judges, his officers, the food of his*

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[64] Edition, paragraph 43.
[65] Unfortunately, the philosopher's diagram was not transmitted in the *Midrash ha-Ḥokhmah*.
[66] Edition, paragraphs 48-49.

table and the seating of his servants.[67] *Everything depends on the stars.*[68] *I pray to God, may He be praised and exalted every day, that He may return me to my father's house and the country of my birth in peace and happiness. May thus be His will, Amen. May God say so.*

The *Midrash ha-Ḥokhmah*

Judah ha-Cohen states that his compendium is divided into two parts and three treatises. The first part of the book is dedicated to the 'world of generation and corruption', or 'physical sciences'. The second part of the work is dedicated to 'the world of the spheres' or 'mathematical sciences'. Within that structure there is a lengthy introduction, summaries or abridgements of fifteen Aristotelian treatises, one work of Euclid, two of Ptolemy and the Arabic astronomical text of al-Biṭrūjī, in addition to three original treatises on Jewish traditions, an account of a polemical discussion with a gentile scholar, and the account of the correspondence with the imperial court on mathematical and astronomical problems. The structure is as follows:[69]

INTRODUCTION

I. PHYSICAL DISCIPLINES ('the world of generation and corruption')

LOGIC

1. Aristotle's *Categoriae*
2. Aristotle's *De interpretatione*
3. Aristotle's A*nalytica priora*

[67] Cf. 1 Kings 10:15.
[68] The Hebrew term Judah uses here is מזל, which originally denotes 'constellation'. In this context, it applies to divine influence that is exerted upon humankind via the spheres of the stars.
[69] A table comparing the structure of the work with the classification of the sciences given in the introduction can be found in Fontaine, "Judah ben Solomon ha-Cohen's 'Midrash ha-Hokhmah': Its Sources and Use of Sources," 209–210.

4. Aristotle's *Analytica posteriora*

NATURAL PHILOSOPHY

5. Aristotle's *Physica*
6. Aristotle's *De caelo*
7. Aristotle's *De generatione et corruptione*
8. Aristotle's *Meteorologica*
9. Aristotle's *De partibus animalium*
10. Aristotle's *De generatione animalium*
11. Aristotle's *De anima*
12. Aristotle's *De sensu et sensibilibus*
13. Aristotle's *De memoria et reminiscentia*
14. Aristotle's *De longitudine et brevitate vitae*
15. Aristotle's *Metaphysica*
16. A treatise in which verses from Genesis, Psalms and Proverbs are explained

II. MATHEMATICAL DISCIPLINES ('the world of the spheres')

17. Euclid's *Elements* (I-VI, X-XIII)
18. The account of the correspondence with the emperor's philosopher
19. Ptolemy's *Almagest* (I-IX)
20. al-Biṭrūjī's *On the Principles of Astronomy*
21. Ptolemy's *Tetrabiblos*
22. A treatise on the secret knowledge contained in the shapes and numerical values of the letters of the Hebrew alphabet
23. An account of a polemic discussion with a gentile scholar about the status of the Jews as 'chosen people'

24. A treatise explaining Talmudic aggadot according to secret knowledge

Judah's encyclopaedia thus contains both summaries and adaptations of philosophical and scientific works, and original treatises that relate to Jewish tradition. In addition, there are two short accounts of discussions with non-Jewish scholars. For his abridgements of Greek works, Judah does not seem to have used (Arabic translations of) the original treatises, but rather adaptations and commentaries by Arabic-speaking authors. In doing so, he availed himself of the latest scholarly products of the surrounding non-Jewish society. Sirat, Fontaine and Zonta have shown that the sections on logic and the natural sciences of the *Midrash ha-Ḥokhmah* mainly follow the structure and doctrines of the middle commentaries on Aristotle by the Andalusian scholar Ibn Rushd (1126-1198).[70] The Arabic version of the *Midrash ha-Ḥokhmah* was compiled only three decades after the Ibn Rushd's death, and its Hebrew translation was the earliest work that made Ibn Rushd's works available in a comprehensive way to a readership that did not know Arabic.[71] In addition to the middle commentaries, Ibn Rushd's epitomes on Aristotle and the encyclopaedic work *Shifāʾ* by Ibn Sīnā (c. 980-1037)[72] were also used by Judah.

Apart from these main sources, which Judah more often than not does not mention explicitly, there are short quotations of other authorities scattered throughout the work.[73] These include Aristotle's predecessors Plato and Socrates, which are only briefly referred to, Aristotle's Greek commentators Alexander of Aphrodisias (fl. 2nd century CE)[74]

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[70] Sirat, "Juda b. Salomon ha-Cohen," 46; Fontaine, "Judah ben Solomon ha-Cohen's 'Midrash ha-Hokhmah': Its Sources and Use of Sources," 194–196; Zonta, *La filosofia antica nel Medioevo ebraico*, 200–204.
[71] The topic is further discussed by Fontaine, "The Early Reception of Aristotle through Averroes in Medieval Jewish philosophy: The Case of the Midrash ha-Hokhmah."
[72] On Ibn Sīnā's life and works, see, for example, Goichon, "Ibn Sīnā."
[73] These sources are listed by Fontaine, "Judah ben Solomon ha-Cohen's 'Midrash ha-Hokhmah'," 194–196.
[74] On Alexander, see Merlan, "Alexander of Aphrodisias."

and Themistius (317-c. 388 CE),[75] and medical authorities such as Galen and Hippocrates. As regards Muslim scholars, Judah cites the philosopher Abū Naṣr al-Fārābī (c. 870-950)[76] and the Andalusian polymath Ibn Bājja (c. 1085–1138)[77] in addition to Ibn Sīnā and Ibn Rushd. The only Jewish thinkers Judah quotes are Abraham ibn Ezra (1089–1164)[78] and Abraham ibn Daud (c. 1110–1180).[79] Furthermore, he mentions his teacher Meir Abulafia, and Moses Maimonides. But although he acknowledges that it was the latter's works that led him to study philosophy, he does not cite any specific passages from his writings.

The fact that Judah gives his readers a state-of the-art outline of Aristotelian philosophy does not mean that he was a fervent adherer of Aristotle. On the contrary, Judah is very critical, sometimes even hostile, towards both Aristotle and his commentator Ibn Rushd. Some of his doctrines, such as the eternity of the world, Judah rejects vehemently, and throughout his summaries of Aristotle's works he makes critical remarks, points out contradictions, and cites sources that have counterarguments to Aristotle's teachings. Judah also disapproves of ibn Rushd's uncritical attitude towards Aristotle, as the commentator defends Aristotle's opinions even if they have been proven wrong.[80]

The second part of the *Midrash ha-Ḥokhmah* also presents authoritative sources combined with current learning of Judah's time. Firstly, Judah tries to give his readers the basic

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[75] On Themistius, see Verbeke, "Themistius."
[76] An introduction to his life and work is given by Mahdi, "al-Fārābī, Abū Naṣr Muḥammad ibn Muḥammad ibn Ṭarkhān ibn Awzalagh." On his philosophy, see Janos, "al-Fārābī, philosophy."
[77] On Ibn Bājja's life and work, see Dunlop, "Ibn Bādjdja"; Pines, "Ibn Bājja, Abū Bakr Muḥammad ibn Yaḥyā ibn al-ṣa'igh."
[78] An introduction to Ibn Ezra's life and works with an extensive bibliography can be found in Simon and Jospe, "Ibn Ezra, Abraham ben Meir." On his life, see also Roth, "Abraham ibn Ezra - Highlights of his Life." On his scientific writings, see Sela, *Abraham ibn Ezra and the Rise of Medieval Hebrew Science*.
[79] On Ibn Daud, see Fontaine, "Ibn Daud, Abraham ben David Halevi"; Sirat, *A History of Jewish Philosophy in the Middle Ages*, 141–155. Ibn Daud is also quoted in the margins of one copy of the *Midrash ha-Ḥokhmah*. See Fontaine, "Abraham ibn Daud and the Midrash ha-Hokhmah: a Mini-Discovery."
[80] Fontaine, "Judah ben Solomon ha-Cohen's 'Midrash ha-Hokhmah': Its Sources and Use of Sources," 204; Sirat, "Juda b. Salomon ha-Cohen," 52.

understanding of geometry that is needed in order to comprehend spherical astronomy. Thus the astronomical part of the encyclopaedia starts with an adaptation of Euclid's *Elements,* the standard work on geometry throughout the middle Ages. Judah includes in his epitome only the books that are immediately relevant for the mastery of the *Almagest*: books I-VI, which deal with surface geometry, and XI-XIII, which deal with spatial geometry. Tony Lévy demonstrated that his adaptation is possibly based on the mathematical part of Ibn Sīnā's *Shifāʾ*, and a commentary on the *Elements* by the Persian mathematician al-Nayrīzī (9th-10th cent.),[81] but Judah may also have consulted other sources that were dependant on the Ḥajjāj-tradition, which is the earliest (only partly) preserved Arabic translation of the *Elements*.[82]

Having laid the mathematical foundations, Judah summarises the standard work on spherical astronomy, Ptolemy's *Almagest*. His rendering of the work contains numerous critical comments, and citations of authoritative astronomers. Tzvi Langermann noted that apart from the Greek authors Menelaus, Theodosius and Hipparchus, Judah mentions the Egyptian polymath Ibn al-Haytham (c. 965–c. 1040),[83] and the astronomers Abū Isḥāq al-Zarqālī (*d.* 1100)[84] and Jābir ibn Aflaḥ (*fl.* 12th cent.),[85] both of whom were active in al-Andalus. In his treatise on the letters of the Hebrew alphabet he also cites al-Zarqālī's patron and collaborator Ṣāʿid al-Andalusī (1029-1070).[86] The only Jewish

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[81] On al-Nayrīzī, see Sabra, "al-Nayrīzī, Abu'l-'Abbās al-Faḍl ibn Ḥātim."
[82] Lévy, "Mathematics in the *Midrash ha-Hokhmah* of Judah ben Solomon ha-Cohen," 309–312. An overview of the different known Arabic traditions is given by de Young, "The Arabic Textual Traditions of Euclid's Elements."
[83] On Ibn al-Haytham's life and works, see Sabra, "Ibn al-Haytham, Abū ʿ Alī al-Ḥasan ibn al-Ḥasan"; Vernet, "Ibn Al-Haytham." On works attributed to him, see also Sabra, "One Ibn al- Haytham or Two? An Exercise in Reading the Bio- bibliographical Sources."
[84] On al-Zarqālī, see Puig, "Zarqālī: Abū Isḥāq Ibrāhīm ibn Yaḥyā al-Naqqāsh al-Tujībī al-Zarqālī."
[85] On Jābir ibn Aflaḥ, see Calvo, "Jābir ibn Aflaḥ: Abū Muḥammad Jābir ibn Aflaḥ." A short overview of the most important theorems found in his work is given in Lorch, "Jābir ibn Aflah al-Ishbīlī, Abū Muḥammad."
[86] See Richter-Bernburg, "Ṣāʿ id al-Andalusī: Abū l-Qāsim Ṣāʿ id ibn abī l-Walīd Aḥmad ibn ʿ Abd al-Raḥmān ibn Muḥammad ibn Ṣāʿ id al-Taghlibī al-Qurṭubī"; Martinez-Gros, "Ṣāʿ id al-Andalusī."

authority Judah mentions in the astronomical part of the encyclopaedia is the otherwise unknown Toledan astronomer David ibn Naḥmias.[87] With his remarks that were often very critical of Ptolemy, Judah followed the trend of Andalusian scholars to rebel against Ptolemaic astronomy. Thus, his adaptation contains only the first nine books of the *Almagest*, while books X-XIII, which deal with planetary astronomy, are left out. Instead of presenting Ptolemy's models for the motions of the naked eye planets, Judah summarises a work that presents an alternative planetary theory: *On the Principles of Astronomy* by Nūr al-Dīn al-Biṭrūjī, who was also an Andalusian scholar.[88] The astronomer had encountered contradictions between the metaphysics of Aristotle and the astronomical system of Ptolemy, and tried to combine the two into a new model. Two areas were of particular concern: Ptolemy's system of epicycles, which accounts for the retrograde motion of the planets, and the theory that the centre of the planetary orbits is not the centre of the earth, both contradict the principle that perfect motion is simple and circular. Al-Biṭrūjī introduced a planetary model in which the motions of the entire cosmos are the result of one single force that is transmitted downward from the outermost orb. Instead of using epicycles, he argued that planetary motion is caused by the circular movement of the poles of the spheres to which the planets were attached. Judah's Hebrew adaptation of *On the Principles of Astronomy* is the earliest source from which Jews who did not know Arabic could study the work. Having explained the mathematical aspects of the motions of the celestial spheres, Judah goes on to explain the practical implications of these motions for humankind. For this, he once again turns

[87] Langermann, "Some remarks on Judah ben Solomon ha-Cohen," 373–379.
[88] An edition and English translation of the original Arabic text and Moses ibn Tibbon's Hebrew translation were made by B.R. Goldstein, *On the Principles of Astronomy*.

to Ptolemy; the last adaptation of an authoritative work that appears in his encyclopaedia is his version of Ptolemy's astrological work *Tetrabiblos*.

Judah's summaries and adaptations of philosophical and scientific works thus combine the works of the most authoritative Greek authors - Aristotle, Euclid and Ptolemy - with the latest scholarship. State-of-the-art works that Judah cites were mainly authored by Muslim authorities, such as Ibn Rushd, Ibn Bājja, al-Biṭrūjī, al-Zarqālī and Jābir ibn Aflaḥ, but occasionally he also mentions Jewish thinkers, such as Abraham ibn Ezra, Abraham ibn Daud and David ibn Naḥmias. All of the authors in this list were active, for at least parts of their lives, in Judah's place of origin, the Iberian Peninsula. Coming to the lands of Emperor Frederick II, Judah was confronted with a Jewish readership that did not read Arabic, and that thus was not aware of the scientific developments that had taken place in the Arabic-speaking world. By translating his encyclopaedia from the Arabic, Judah made these sources, many for the first time, available in Hebrew. But we have already seen that the *Midrash ha-Ḥokhmah* is by no means a simple, uncritical account of Aristotelian philosophy. Moreover, Judah ha-Cohen never meant for the work to be just that.

Judah's attitude towards Greco-Arabic science and philosophy and their relationship with traditional Jewish learning comes to the fore in those parts of the *Midrash ha-Ḥokhmah* that are original treatises by Judah ha-Cohen himself and not abridgements and paraphrases of earlier authorities. He does not present his worldview in one coherent system. Bits and pieces are scattered throughout his introduction, the exegetical treatise at the end of the first part of the work, the introduction to the adaptation of Ptolemy's *Tetrabiblos*, the treatise on the hidden meaning of letters of the Hebrew alphabet, the account of the encounter with the presumably Christian philosopher, and the treatise on Talmudic aggadot. Two themes play an important role in these parts of the encyclopaedia: the system of the three worlds, that is, the sublunar world, the world of the spheres, and the spiritual world, and the notion of *qabalah*, that is, secret divine knowledge that God revealed to Moses and the Israelites on Mount Sinai, and which was handed down from generation to generation.[89]

The worldview that is presented in the *Midrash ha-Ḥokhmah* is neo-platonic: The world of the sphere and the world of generation and corruption emanated from the spiritual world and are dependent on it. The spiritual world consists of pure light (which is different in essence from the light of the sun), which emanates via the world of the spheres to the sublunar world.[90] In his introduction Judah explains that the three different branches of the learning refer to the three different worlds: physics concerns the sublunar world, astronomy and astrology study the world of the spheres, and metaphysics the divine world. Like the subjects they concern, these disciplines exist in hierarchical order, the knowledge of the divine world being the highest and most important. However, while the philosophers of the nations have acquired knowledge of the two lower worlds, true knowledge of the divine spiritual world cannot be gained through the study of secular learning, that is, Aristotle's metaphysics; it can only be received and transmitted from person to person, as *qabalah*. Without this knowledge, which was revealed to the Jews alone, no true wisdom can be achieved.[91] While the study of the two lower worlds is the task of the nations - that is, the gentiles - Israel was chosen for knowledge of the upper world. This divine science comprises knowledge of the two lower worlds, but understanding of the lower worlds alone does not lead to any knowledge of the spiritual

[89] The notion of *qabalah* is explained in Sirat, "La Qabbale d'après Juda B. Salomon ha-Cohen." The role of the three worlds is discussed by Fontaine, "The Theme of the Three Worlds in the 'Midrash ha-Hokhma.'"
[90] Goldstein, "The Commentary of Judah ben Solomon Hakohen," lines 7–20. The passages are discussed by Fontaine, "The Theme of the Three Worlds in the 'Midrash ha-Hokhma,'" 433–434.
[91] Sirat, "La Qabbale d'après Juda B. Salomon ha-Cohen," 191–193.

world. Thus, the knowledge of the upper world that was revealed by God originally also comprised physical and mathematical knowledge, but this information was lost in the course of time.[92] To regain this knowledge, Jews have to study the philosophical and scientific books of the nations.

The theme of universal knowledge that the Jews once held and that eventually was lost can be found in the writings of many Jewish thinkers, especially from the Iberian Peninsula.[93] For Judah, this idea is certainly more than just a trope, an apology for his occupation with secular learning in the first place. This becomes clear in the first treatise. In his commentary on verses from Genesis, Psalms and Proverbs Judah tries to reveal the knowledge of secular learning that is hidden in the bible and in traditional Jewish literature. He explains that by studying the 'divine wisdom', man can make his rational soul ascend to the highest world. But in order to be able to do so, he needs to master physics, so that he can let form prevail over matter, and thus subdue his evil inclination.[94] After that it is necessary to study the mathematical disciplines.[95] To Judah, the study of secular learning is thus not only permissible, but even necessary. But there is a limit to the study of these disciplines, since too much occupation with them will lead to the denial of god, while too little occupation with them will lead to an incorrect understanding of the divine world.[96] There is also a limit to the knowledge of the divine wisdom that God grants to those that are perfect regarding ethical and intellectual virtues. Gentiles are

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[92] Ead., "Juda b. Salomon ha-Cohen," 49.
[93] It can be found, for example, in the thought of Judah ha-Levi, Abraham Ibn Ezra, and Maimonides. See Langermann, "Some remarks on Judah ben Solomon ha-Cohen," 368; "Science and the Kuzari," 507–508; Sela, *Abraham ibn Ezra and the Rise of Medieval Hebrew Science*, 104–106.
[94] Fontaine, "The Theme of the Three Worlds in the 'Midrash ha-Hokhma,'" 434–435.
[95] Goldstein, "The Commentary of Judah ben Solomon Hakohen," line 545. On the connection between mathematics and 'divine science' see also Sirat, "La Qabbale d'après Juda B. Salomon ha-Cohen," 195.
[96] Fontaine, "The Theme of the Three Worlds in the 'Midrash ha-Hokhma,'" 435.

unworthy of this knowledge altogether.[97] He argues that the nations dominate the lower world, while the spiritual world is Israel's true realm. In fact, the lower world was only created to give man the opportunity to act the correct way, and thus to become a citizen of the divine world, which Judah identifies with the world to come.[98]

In short, Judah considers the study of secular learning as helpful, even necessary, for the study of the divine wisdom, which was revealed by God and handed down from generation to generation. But secular knowledge, if understood wrongly, can also be harmful. It is only the divine wisdom which enables man to actualise his potentiality, to elevate his rational soul to the divine world, the world to come. Judah does not give us an indication of how the world to come is related to the coming of the Messiah. However, at the very end of his treatise on the secret meaning of the letters of the Hebrew alphabet, he presents a calculation, based mainly on astrological considerations, which predicts the coming of the Messiah for the year 1260. This date is only thirteen years away from the time when he translated the encyclopaedia from Arabic into Hebrew, the year 1247. Tzvi Langermann therefore suggests that Judah may have compiled his entire encyclopaedia in order to prepare his coreligionists for the impending redemption.[99] In addition, R. Fontaine argues that not only the date of 1260, but also the importance of the theme of three worlds in Judah's writings, were influenced by a similar calculation that was very popular amongst Christians in Judah's time. The theologian and mystic Joachim of Fiore (c. 1135–1202), for example, had divided history into three stages, the last of which was the time of the Holy Spirit. This would be an era of spiritual enlightenment, in which

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[97] Thus, although Judah holds al-Biṭrūjī in high esteem, he writes at the beginning of his Hebrew rendering of *On the Principles of Astronomy*: "Know that a great secret was revealed to him, and if he had been Jewish, he would have been worthy of the divine science." Oxford, MS. Mich. 551, fol. 161v: ודע כי נגלה לו סוד גדול ואלו היה יהודי היה ראוי לחכמה אלהית.

[98] Fontaine, "The Theme of the Three Worlds in the 'Midrash ha-Hokhma,'" 436, 439.

[99] Langermann, "Some remarks on Judah ben Solomon ha-Cohen," 389.

mankind was to come in direct contact with God. He predicted the beginning of this time for the year 1260.[100] Thus, if Judah, too, had been influenced by Christian messianic belief and expected the era of redemption, the beginning of the world to come, to be immediately imminent, it is perfectly plausible that the main goal of his encyclopaedia was to prepare his coreligionists for that time. In fact, the whole internal structure of the *Midrash ha-Ḥokhmah* suggests that Judah is trying to lead his readers towards the treatise about the hidden meaning of the Hebrew alphabet, which ends with one important conclusion: the beginning of a new era in the year 1260.

Thus, although the *Midrash ha-Ḥokhmah* is one of the first comprehensive works that made Aristotelian philosophy available in Hebrew, it was never meant to be a simple reference work. That explains why Judah admonishes his readers not to copy only parts of the work:

> *Every man who wishes to copy this book is not allowed to copy only one matter and to leave the rest aside, but he must copy the book as it is, from the beginning to the end, letter for letter, word for word, and he must quote the source as he finds it written in the book.*[101]

For Judah, the different parts of the book can only be understood correctly in the context of the entire work – that is, it was intended as a coherent work that guides the reader toward an understanding of divine knowledge, and which leads to a conclusion. But if this is the case, it is not clear what purpose the account of the correspondence with the emperor's philosopher is supposed to fulfil within the compendium. As Langermann already observed, there are two items that that appear to be out of place, as they do not seem to fit into the overall structure of the work: the discussion with the Christian philosopher on the status of the Jews and the account of the correspondence. He

[100] Fontaine, "The Theme of the Three Worlds in the 'Midrash ha-Hokhma,'" 441–443.
[101] Goldstein, "The Commentary of Judah ben Solomon Hakohen," lines 1–4.

therefore speculates that they may never have been meant to be integral parts of the encyclopaedia, but rather served as 'appendices' to the work. This leads him to pose the question: *'is everything Judah ever wrote to be considered part of the Midrash ha-Ḥokhmah?'*[102]

In other words, the fact that these two accounts of interactions with non-Jewish philosophers appear to break the coherence of the work suggests that they originally were not a part of the work. Furthermore, Judah does not mention them in his introduction. On the other hand, the two items in question fit perfectly the aim that Judah states he had in writing the encyclopaedia:

> *We already mentioned in the introduction of the book that the aim of this work is to make known the extent of the usefulness of the works of the nations, so that the Israelite not be ignorant of them, lest they grow haughty toward him with their sciences. This work is there for him to know how to answer a heretic within his own science and to return one who erred regarding their books to our holy Torah.*[103]

As we have seen, the philosophical, mathematical and astronomical portions of the *Midrash ha-Ḥokhmah* certainly equipped its readers with the latest scientific knowledge *so that the Israelite not be ignorant of them,* while the sections devoted to Jewish learning guided these readers towards a higher knowledge that was to be found in the bible, and thus *returned them to the holy Torah.* But the third aim that Judah mentions - preventing the nations from priding themselves on their learning and looking down on the Jews - is achieved through the two parts of the works that seem to be out of place: the correspondence on geometry and astronomy, and the dispute on the place of the Jews in this world. In both cases, Judah refutes unnamed non-Jewish philosophers. In the case of the correspondence, he does so using solely secular learning; he thus refutes the

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[102] Langermann, "Some remarks on Judah ben Solomon ha-Cohen," 387–388.
[103] Oxford, Bodleian Library, MS. Mich. 551, fol. 45v. The translation is based on that rendered by Septimus, *Hispano-Jewish Culture in Transition*, 98. See also Fontaine, "Judah ben Solomon ha-Cohen's 'Midrash ha-Hokhmah': Its Sources and Use of Sources," 202.

philosopher *within his own science*. The question how the account of the correspondence relates to the rest of the *Midrash ha-Ḥokhmah* remains therefore open: on the one hand, it does not fit the structure of the compendium as a whole, on the other hand, it does fit one of its proclaimed aims.

Reflections on the Correspondence

The correspondence between Judah ben Solomon ha-Cohen and the emperor's philosopher exemplifies two dimensions of knowledge transfer that occurred in thirteenth-century Europe. On the one hand, an exchange of mathematical/astronomical knowledge took place between an Andalusian Jew, the heir of an illustrious Jewish-Muslim intellectual culture, and a Christian royal court in Latin Europe - a court that hosted the most accomplished Christian mathematicians of the time. On the other hand, about a decade after the original exchange had taken place, this correspondence was published in Hebrew for a Jewish audience in Latin Europe – an audience that was eager to study Arabic scientific and philosophical writings to which they had previously not had access. The account of the correspondence that is found in the *Midrash ha-Ḥokhmah* thus documents both an intercultural and an intracultural transmission of knowledge.

As regards the *inter*cultural exchange with the emperor's court, an analysis of the mathematical and astronomical contents of the questions that were posed and the methods used in their answers can give us insight into the quantity and quality of mathematical knowledge that were present in two very different parts of Europe. In addition, the topic of the philosopher's question and its relationship with scientific inquiries that Fredrick sent to other scholars may give us further insight into the comparative availability of mathematical knowledge in different parts of Europe. Both the philosopher's and Judah's motives for posing their questions merit attention. The correspondence appears to document an open exchange of ideas, but it also may have had a polemical dimension. Was the philosopher's question original, or had it already been discussed previously? In other words, did the philosopher already know the answer to the question when he posed it, and was he trying to test the reportedly learned youth from Toledo? Similarly, Judah's motive for asking the philosopher about *oblique ascensions* needs to be analysed. The problem he requested the philosopher to solve was by no means a difficult one: it had already been solved in Ptolemy's *Almagest* and, using different methods, by generations of Arab astronomers after him. Had Judah as a youth not yet studied these works and was genuinely interested in a solution, or did he already know how to solve the problem? Thus, was he simply testing the skills of his learned correspondent? Furthermore, as the problem would have been easy to solve by a trained mathematician, why did the philosopher get the answer so wrong? Was his mistake made out of ignorance, or was the answer in turn another test? The answers to these questions will hopefully also get us closer to uncovering the identity of Judah's unnamed correspondent.

With regard to the *intra*cultural exchange documented in the correspondence Judah's motives again merit further attention. He reports that an entire list of queries was sent to him, and that he sent several questions in return. Yet he published only two of the questions that were discussed. The fact that he translated these questions into Hebrew indicates his desire to educate his fellow Jews on these topics, but his choice of literary genre suggests that his intentions lay beyond the mere transmission of scientific contents. This hints at the possibility of a polemical nature to his account, in that it may have been intended to show to other Jews the intellectual defeat which Judah had managed to inflict on his gentile adversary.

Furthermore, the relationship of Judah's account of the correspondence with his encyclopaedic compendium *Midrash ha-Ḥokhmah* will be given further consideration. Was the correspondence, as Tzvi Langermann suggested, meant only as an appendix, or was it an integral part of the work? In addition to the structure and content of the encyclopaedia, Judah's language may also provide clues in this respect. If the Hebrew mathematical terminology of the correspondence differs greatly from that of the parts that precede and follow it within the encyclopaedia, that may signify that they were translated at different stages of Judah's working life. But even if no differences can be detected, Judah's mathematical terminology can reveal the intracultural transfer of knowledge that took place in thirteenth-century Italy. It was only in the 1230s that Arabic works on mathematics and astronomy began to be translated into Hebrew. About a hundred years later, all the standard works that Jews would study for the following several centuries had been translated. Judah was thus part of a line of translators who established a Hebrew mathematical language that would be in use until the beginning of the 20th century. First in this line was another Jew who reports of contacts with Emperor Frederick, Jacob Anatoli (c.1194-c.1256). A comparison of the mathematical vocabularies of Judah and Jacob will be explored in order to reveal possible contacts between the two scholars and to contrast Judah's and Jacob's different approaches in making mathematical knowledge available to the Jews in the Latin world.

The correspondence of Judah ben Solomon ha-Cohen with the unidentified philosopher of Emperor Frederick II is a prime example of a multifaceted interchange that took place in Europe in the thirteenth century. Through it, letters travelled from one cultural setting to another, disseminating ideas that both the Christian audience at the court and the Jewish audience in Christian Europe were eager to explore. This doctoral thesis will examine some of the mathematical as well as philosophical ideas that were of interest to both Christians and Jews and what Judah ha-Cohen's role in spreading these ideas amongst them was. In doing so, it will explore the unique dynamics of one instance of Jewish-Christian intellectual cooperation in advancing Greek works that had been preserved in Arabic.

Chapter Two

Judah ben Solomon ha-Cohen's Correspondence with the Emperor's Philosopher: text and translation

The mathematical contents of the correspondence

In the correspondence two separate geometrical problems are being discussed, one concerning Euclidean geometry, the other spherical astronomy.

The first question

The first question, which is posed by the emperor's philosopher, consists of three different problems. Judah describes it thus:

> *How do we construct each of the five [regular] solids around a given sphere, and how do we construct the given sphere inside each of them and also around each one of them, [and provide each case] with a proof?*

The first problem is therefore to circumscribe the five platonic solids around a given sphere. Judah solves it by using the constructions of the five regular polyhedra that is found in Euclid's *Elements*. He thus first constructs a regular polyhedron inscribed in a given sphere, using *Elements* XIII.13-17. With the help of lines drawn from the centre of the sphere to the angles of the solid, he then constructs a second polyhedron that encloses both the sphere and the original polyhedron. In modern terms, his method could be described as uniform scaling of the original solid, with the centre of the sphere serving as scaling centre. He then proves that the faces of the newly constructed solid all touch the sphere.

The second problem is to inscribe the given sphere inside each of the platonic solids. The question is strangely worded; one would expect the problem to be to inscribe a sphere in a *given* platonic solid. A *given* sphere can only be inscribed in a platonic solid, if the solid has exactly the right dimensions for the given sphere to fit inside. Instead of exploring the exact ratio between solid and sphere that is needed to achieve this, Judah simply makes an identical copy of the platonic solid that he has just constructed in answer to the first question, knowing that Euclid's given sphere will fit inside. He then finds the centre of the solid, proves that it is also the centre of the sphere to be inscribed in it, and constructs the sphere. He does not need to prove that this sphere does in fact touch the polyhedron, as he already did this in answer to the first question.

The third problem is to circumscribe the given sphere around each of the platonic solids. Again, this is only possible if the solids have the correct dimensions in regard to the given sphere. However, this problem is already solved in *Elements* XIII.13-17.[1] Instead of simply referring to Euclid, Judah makes an exact copy of the polyhedron constructed in the *Elements,* but without the circumscribing sphere. He then finds the centre of the polyhedron, and constructs the circumscribing sphere around it.

The second question

The second question discussed in the correspondence concerns spherical astronomy. This time it is Judah ha-Cohen who asks the question and the emperor's philosopher who answers it. Judah describes the question as:

> *[T]o give a clear geometric proof for the tables of ascensions for places in sphaera obliqua.*

[1] See T.L. Heath, *The Thirteen Books of Euclid's Elements*, 3:467–503.

In his answer, the philosopher tries to show how to calculate the ascensional difference, that is, the difference between the right and the oblique ascension of a given arc of the ecliptic, while also demonstrating why this difference has to be subtracted from the right ascension when the given arc of the ecliptic lies north of the celestial equator, and to be added to the right ascension when the arc of the ecliptic lies south of the equator.

In order to do so, he constructs a diagram in which the oblique ascensions for both a northern arc and a southern arc of the ecliptic are shown. Unfortunately, none of the four manuscripts that contain the text contain said diagram. However, based on the philosopher's text and Judah's critique of it, it is possible to make the following modern reconstruction (here also with modern mathematical notation):

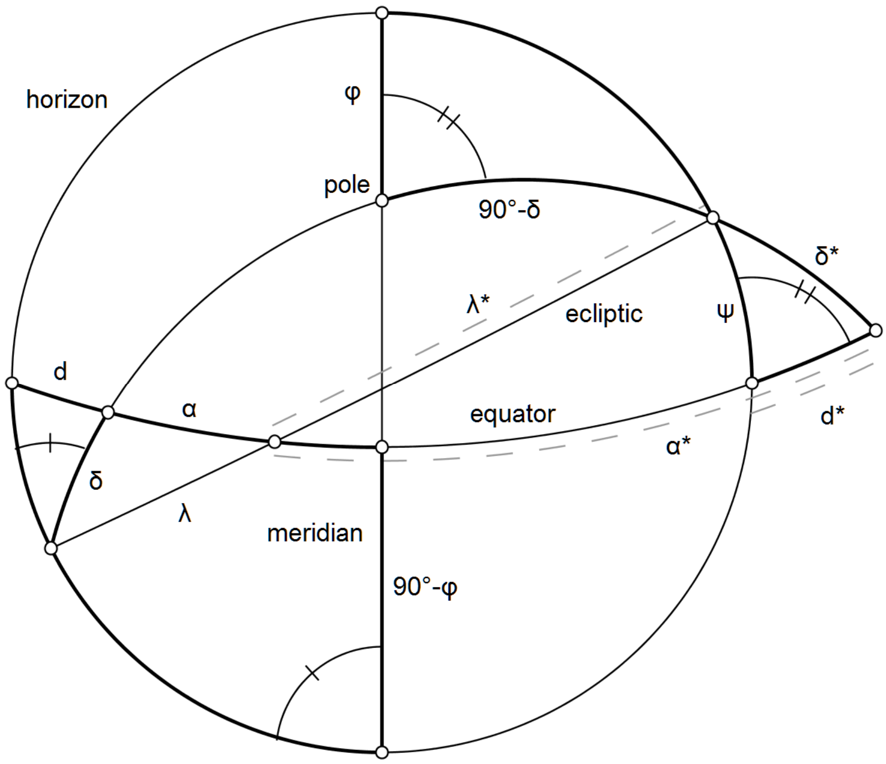

Figure 1

In figure 1, the large circle is the horizon, and *λ* and *λ** are two arcs of the ecliptic. If *λ* is rising on the horizon, then arc *α* of the celestial equator is its right ascension, that is, its rising time as it would be seen by an observer who is standing on the terrestrial equator. But if the observer is north of the terrestrial equator (at a place with latitude φ), then arc d, the ascensional difference, has to be added to the right ascension (α) in order to find the oblique ascension (α+d) of arc λ. If λ* is rising on the horizon, then its right ascension (α*) is larger than its oblique ascension (α*-d*). Thus, the ascensional difference (d*) has to be subtracted from the right ascension. Arc δ is the declination (i.e. the vertical distance to the celestial equator) of the point of the ecliptic that is on the horizon, and ψ is its rising amplitude (i.e. its horizontal distance to the celestial equator).

The philosopher tries to find a formula to calculate the ascensional difference (d and d*), both for the case that the rising arc of the ecliptic (λ) is situated to the south of the equator (d), and for the case that this arc (λ*) is situated to the north of it (d*). But by wrongly applying the concept of 'similar triangles' to spherical geometry, the he arrives at two incorrect formulas.

He (erroneously) claims that the two spherical triangles marked by thick lines on the left hand side are *similar* (a concept that does not apply to spherical triangles). Thus, he (wrongly) argues that the two angles marked in these triangles are equal. He needs to establish that these angles are equal in order to be able to use theorem III.2 of Menelaus' *Sphaerica:*[2]

In the notion that arc a lies opposite angle A in triangle ABC: If ABC and $A'B'C'$ are two spherical triangles and if A=A' and C=C', or C is supplementary to C', then

[2] This modern notation is taken from Kline, *Mathematical Thought from Ancient to Modern Times*, 1:121. In the original Greek, Menelaus used chords instead of sines. Due to the influence of Indian astronomy, medieval Arab astronomers shifted from using the chord function to the use of the sine function (see the entry *Sine, cosine, sinus totus* in Chapter Four). The philosopher apparently used a medieval Arabic adaptation of the theorem.

$$\frac{\sin c}{\sin a} = \frac{\sin c'}{\sin a'}$$

Applying the theorem, he arrives at the (incorrect) formula:

$$\frac{\sin d}{\sin \delta} = \frac{\sin 90°}{\sin(90° - \varphi)}$$

He then rearranges it thus:

$$\sin d = \frac{\sin \delta \cdot \sin 90°}{\cos \varphi}$$

Similarly, he (erroneously) claims that the two spherical triangles marked on the right hand side are *similar*, and that therefore the two marked angles in them are equal. Applying *Sphaerica* III.2, he arrives at the formula:

$$\frac{\sin \psi}{\sin d^*} = \frac{\sin(90° - \delta)}{\sin \varphi}$$

Which he rearranges thus:

$$\sin d^* = \frac{\sin \psi \cdot \sin \varphi}{\cos \delta}$$

It turns out that even if the actual diagram the philosopher was using should differ from our reconstruction, his results, which are found in the text, are nonetheless wrong. This becomes clear when we compare them with the correct formula for calculating the ascensional difference:[3]

$$\sin d = \tan\delta \cdot \tan\varphi$$

Or, if we substitute the tangent:

$$\sin d = \frac{\sin\delta}{\cos\delta} \cdot \frac{\sin\varphi}{\cos\varphi}$$

In his answer to the text, Judah spots the philosopher's mistake. By proving that the spherical triangles are not equal, he establishes that the marked angles cannot be equal, therefore *Sphaerica* III.2 cannot be applied and the philosopher's formulae are wrong.

[3] King, "al-Maṭāliʿ."

Manuscripts

The following manuscripts were used for the edition:

P Parma, Biblioteca Palatina, MS. 2769 (IMHM F 13618), fols. 136v-140r. This is a paper manuscript that was written in the fourteenth century in Oriental semi-cursive script. The whole manuscript consists of 214 folios. It contains an incomplete copy of the *Midrash ha-Ḥokhmah* (from the section on *De partibus animalium* to almost the end of the *Tetrabiblos*). It begins with quire number 9, which suggests that it used to be a complete copy.

V Vatican City, Biblioteca Apostolica Vaticana, MS. Ebr. 338 (IMHM F 377), fols. 206v-210v. This is a paper manuscript that was written in the fifteenth century in Sephardic semi-cursive script. The whole manuscript consists of 323 folios. It contains a complete copy of the *Midrash ha-Ḥokhmah*. In the second half of the codex (including ff. 206v-210v) corrosive ink has leaked through the pages.

O Oxford, Bodleian Library, MS. Mich. 400 (IMHM F 19291), fols. 35r-38v. This is a paper manuscript that was written in the mid-fifteenth century in Byzantine-Sephardic semi-cursive script. The manuscript consists of Judah ha-Cohen's rendering of Euclid's *Elements* and his account of the correspondence. It was bound together with other manuscript parts (different codicological units, written on different kinds of paper and in different hands), which contain eleven mathematical and astronomical treatises.

C Cambridge, Cambridge University Library, MS. Add. 1737 (IMHM F 17492), fols. 220v-224r. This is a vellum and paper manuscript (outer and inner folios of quires are vellum) that was written in the fifteenth century in Byzantine semi-cursive script. The whole manuscript consists of 252 folios. It contains an incomplete copy of the *Midrash ha-*

Ḥokhmah (from the middle of the introduction to the middle of the sixth chapter of the *Almagest*).

Editorial principles

Because of the difficult mathematical content of the text, each of the four manuscripts contains many scribal errors, especially regarding the names of points in the diagrams that are being discussed. The text presented here is therefore a composite text made from the four different versions found in the four manuscripts. It is based on MS. P, which is the earliest copy of the text which also contains the fewest scribal errors. Whenever the four manuscripts do not all agree on one reading of the text, the preferred reading was the one deemed to be more mathematically accurate. In some cases, when the readings given by all manuscripts appear to be copyists' errors and are in logical contradiction to other parts of the text, a new reading that is consistent with the text was offered between square brackets [].

The Hebrew apparatus is intended to record, in an abbreviated manner, all the readings in the Hebrew manuscripts consulted that differ from the text printed here. Differences in orthography (*scriptio plene* versus *defectiva*, word endings in -*īn* versus -*īm*) were only recorded when a variant may result in a different reading. Letters in geometrical figures were transliterated in analogy to the system proposed for Arabic diagrams by Edward Kennedy.[4] I have added punctuation to the Hebrew text, and, to facilitate references, have divided it into paragraphs.

[4] Kennedy, "Transcription of Arabic Letters in Geometric Figures."

The translation is meant to help readers understand the text with as much fidelity to the Hebrew original as possible. Explanatory comments as regards Hebrew expressions, variants and the mathematical theorems applied in the text can be found in the footnotes to the translation.

Although all four manuscripts contain diagrams in Judah ha-Cohen's version of Euclid's *Elements*, none of them contain the diagrams that are being discussed in the correspondence. As none of the manuscripts contain empty spaces that would suggest that these diagrams were meant to be copied at a later stage, it appears that they were all copied from texts that did not contain the diagrams either. However, it was possible to make (modern) reconstructions of these diagrams, which are presented in the English translation for ease of reading.

Sigla

-	word or expression is missing in the manuscript
+	word or expression is added after incipit
iter.	expression is repeated
O(*ac.*)	before correction (in the same hand)
O(*pc.*)	after correction (in the same hand)

Judah ben Solomon ha-Cohen's Correspondence with the Emperor's Philosopher

Hebrew text

[1] **אמר המחבר** : ובעת[1] נערותי שלח לי[2] החכם[3] הפילוסוף של קיסר המלך הגדול האנפיראור פדריק, ירום הודו[4], שאלות, בעודי בארצי תוך גלות ירושלם אשר בספרד[5]. האחת מהן היא[6] : כיצד נחוק על כדור ידוע כל אחד מהחמשה[7] גופנים, וכיצד נחוק בכל אחד[8] מהם הכדור הידוע, וגם על[9] כל אחד מהם, במופת?

[2] והשיבותי לו[10] אז באותו זמן, ואני[11] מבן שמנה עשרה[12] שנה, בזה הנוסח שנאמר עתה. ומי ימלל לי, כאשר השבתי[13] עליהן בארץ מולדתי, בקהל מדינת טליטלה[14], לפני מורי אדוני אבי מ״כ נבתוי״א[15], כי היה מתגלגל הדבר וירדתי לארץ טושקאנה[16], והעתקתים שם מערבי ללשון עברי, עם זה הספר שחברתי בעודי שם. וכל[17] דעביד רחמנא לטב. וברוך ייי יום יום[18].

[3] **וזה נוסח התשובה:** כיצד נעשה כל אחת[19] מחמש[20] צורות על הכדור המוצע? נעשה תחלה כל אחת מהצורות החמש[21] בתוך הכדור, כמו שהזכיר אקלידס[22]. ואחר כן נגיע מרכז הכדור בזויות[23] הצורה בקוים ישרים, ונוציא אותן הקוין על יושר[24] באין חקר.

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1 ובעת] בעת C
2 שלח לי] שלחני O
3 החכם] – COV
4 המלך הגדול האנפיראור פדריק, ירום הודו] – COV
5 בארצי תוך גלות ירושלם אשר בספרד] בספרד CO בארץ ספרד V
6 מהן היא] – COV
7 מהחמשה] מחמשה CO
8 אחד] אחת O
9 על] – PV
10 והשיבותי לו] והשיבותיו V והשבותיו CO
11 ואני] – CO
12 שמנה עשרה] כ׳ C יח׳ O
13 כאשר השבתי] כשהשבתי CO
14 בארץ מולדתי, בקהל מדינת טליטלה] בארצי במדינת טוליטלה V בארצי בֵטוּלֶדוּ O בארצי C
15 לפני מורי אדוני אבי מ״כ נבתוי״א] – C לפני אדוני אבי מ״כ נבתוי״א O
16 וירדתי לארץ טושקאנה] – C וירדתי לטושקאנה V וירדתי בטוסקאנה נ׳ט׳ O
17 וכל] +מה C
18 וברוך ייי יום יום] – COV
19 אחת] אחד O
20 מחמש] מהחמש CO
21 החמש] – CO
22 אקלידס] אקלידש O
23 בזויות] בזוית O
24 יושר] – O

[4] ואחר כן נגיע מרכז הכדור במרכז שטח אחד משטחי הצורות. כיצד? בצורת בעלת ארבע, ובעלת שמונה, ובעלת עשרים נגיע הקו עד המרכז משולש אחד מן המשולשות[25] הסובבות בכדור, וכן במרכז המרובע מצורת המכעב, וכן במרכז מחומש אחד[26] מצורת בעלת שתים עשרה[27].

[5] ודע כי מרכז השטח - רצוני לומר מרכז העגולה הסובבת עליו. והדבר נראה כי מרכז השטח הוא[28] הנקודה שיפגשו עליה הקוין השוין היוצאין מזויות השטח בשטח.

[6] ונגיע מרכז השטח במרכז הכדור. והדבר נראה כי אותו הקו המוגע הוא נצב על השטח על זויות נצבות[29], כמו שנבאר אחר זה בעזרת שדי ית'. ונוציא זה[30] הקו עד שטח[31] הכדור, ונקים על ראשו עמוד מחוץ לכדור, ממשש בה.

[7] ונבאר תחלה כי הזוית[32] שיסובבוה[33] השני קוין המוצאין ממרכז הכדור - האחד שהוא מגיע עד זוית הצורה, והאחר[34] שיגיע עד המרכז השטח משטחי הצורה - כי היא קטנה מנצבת. וכשהדבר כן, הלכך בהכרח יפגש זה הקו, שהוציאנוהו נוגע, עם הקו המוגע בין מרכז הכדור וזוית הצורה. הלכך יפגשו מחוץ לכדור.

[8] ונחלק[35] מהקוין האחרים המוגעים בין מרכז הכדור [וזויות][36] הצורה קוין שוין לזה הקו, ונגיע בין ראשי הקוין קוין ישרים. נאמר כי בזה המעשה השלמנו עשיית הצורות שרצינו על הכדור המוצע.

[9] **ונמשיל באותיות:** תהיה תחלה צורה בעלת ארבע עשויה[37] בתוך הכדור, כמו[38] שבאר אקלידס. ויהא[39] מרכז הכדור נ̇, וזויות הצורה ח̇ ט̇ כ̇ ל̇, ומרכזי המשולשות ד̇ ה̇ ז̇ ס̇. ונגיע קוי ח̇נ̇ נ̇ט̇ נ̇כ̇ נ̇ל̇, ונוציאם על יושר באין חקר. ונגיע נ̇ד̇, ונוציאנו על יושר עד שיפגש שטח הכדור על מ̇.

[25] מן המשולשות] מהמשולשות CO
[26] אחד] אחת PV
[27] שתים עשרה] יב̇ CO
[28] הוא] היא V
[29] זויות נצבות] זוית נצבת O
[30] זה] – PV
[31] שטח] השטח COV
[32] הזוית] הזויות CPV
[33] שיסובבוה] שיסובבנה CO
[34] והאחר] והאחד C
[35] ונחלק] וכחלק C
[36] וזויות] וזוית COPV
[37] עשויה] עשרה V
[38] כמו] – O
[39] ויהא] ויהיה C

[10] נבאר תחלה כי זוית נ̇ד̇ח̇ נצבת. כיצד? כשנחוק בעגולה הגדולה קו[40], והוא אלכסון העגולה הסובבת על משולש ח̇ט̇כ̇[41], ונוציא ממרכזה ועד זה הקו קו נ̇ד̇, וחילקו[42] בשני חצאין על נקודת ד̇, שהיא מרכז המשולש. הלכך יחצבנו על זויות נצבות. הלכך זוית נ̇ד̇ח̇[43] נצבת, והוא[44] מה שרצינו.

[11] והואיל ונתבאר[45] זה, נוציא מנקודת מ̇ קו, יהא עמוד על מ̇ד̇נ̇. ולפי שנתבאר כי זוית נ̇ד̇ח̇ נצבת, לפיכך תהא זוית [ד̇נ̇ח̇][46] קטנה מנצבת[47]. הלכך שני קוי נ̇מ̇ ח̇נ̇[48] בהכרח יפגשו. ונוציאם עד שיפגשו על נקודת ע̇.

[12] ואחר כן נחצוב מקוי נ̇ט̇ [נ̇כ̇][49] נ̇ל̇, המוצאין לאין חקר, קוין שוין לקו נ̇ח̇ע̇[50], והן נ̇ט̇צ̇ נ̇כ̇פ̇[51] נ̇ל̇ס̇. ואחר כן[52] נגיע קוי ע̇צ̇ ע̇פ̇ ע̇ס̇ צ̇פ̇ פ̇ס̇ צ̇ס̇[53].

[13] ונבאר כי אלו [הששה][54] כלן שוין. כיצד? לפי שזויות ח̇נ̇ט̇ ח̇נ̇כ̇ ח̇נ̇ל̇[55] ט̇נ̇כ̇[56] [כ̇נ̇ל̇][57] ל̇נ̇ט̇ שוין[58], וקוי נ̇ע̇[59] נ̇צ̇ נ̇פ̇ נ̇ס̇ שוין, הלכך קוי ע̇צ̇ ע̇פ̇ ע̇ס̇ צ̇פ̇[60] פ̇ס̇ ס̇צ̇[61] שוין. לפיכך הארבעה[62] משולשות שוין.

[14] ואחר כן[63] נבאר כי כל אחד[64] משטחי זו הצורה[65], הנעשת מחוץ לכדור, הוא נכחי לכל אחד משטחי העשוי תוך הכדור.

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40 קו] הקו PV
41 ח̇ט̇כ̇] ח̇ט̇ב̇ CO
42 וחילקו] וחלקו V
43 נ̇ד̇ח̇] – CO
44 והוא] והיא C
45 ונתבאר] ונתברר C
46 ד̇נ̇ח̇] נ̇ד̇ח̇ COPV
47 מנצבת] מנצבם O
48 ח̇נ̇] נ̇ח̇ CV
49 נ̇כ̇] נ̇ד̇ COPV
50 נ̇ח̇ע̇] ח̇נ̇ט̇ C
51 נ̇כ̇פ̇] נ̇ב̇פ̇ O
52 כן] כך C
53 צ̇ס̇] ס̇צ̇ CO
54 הששה] השלשה COPV
55 ח̇נ̇ל̇] ח̇כ̇ל̇ C
56 ט̇נ̇כ̇] ט̇נ̇ב̇ CO
57 כ̇נ̇ל̇] ק̇ה̇ל̇ C ח̇ה̇ל̇ OPV
58 שוין] שוה V
59 נ̇ע̇] נ̇ט̇ C
60 צ̇פ̇] צ̇ב̇ O
61 ס̇צ̇] ס̇פ̇ PV פ̇כ̇ C
62 הארבעה] הארבע CO
63 כן] כך V
64 אחד] +ט̇ C
65 זו הצורה] הצורה הזו C

[15] ונמשיל תחלה ונאמר כי שטח משולש ע׳צ׳פ׳ נכחי לשטח משולש ח׳ט׳כ׳[66]. כיצד? לפי שמשולש נ׳ע׳צ׳ הוציאו מאחת[67] מצלעותיו אל השנית[68] קו שחצבן ביחס אחד, והוא ח׳ט׳, כי יחס ע׳ח׳[69] אל ח׳נ׳ כיחס צ׳ט׳ אל ט׳נ׳, הלכך ח׳ט׳ כנגד ע׳צ׳. וכן נבאר כי קו ח׳כ׳[70] כנגד ע׳פ׳. ובצורת ט׳ו׳ ממאמר[71] י׳א׳ מספר היסודות לאקלידס[72] יהא משולש [ח׳כ׳ט׳][73] כנגד משולש[74] ע׳צ׳פ׳, והוא מה שרצינו.

[16] ועוד נבאר כי השלשה משולשות[75] הנשארים הם ממששין[76] לכדור זה[77]. וזה יתבאר כשנבאר כי הקוין המוגעין בין מרכזי[78] כל[79] המשולשות[80] ובין מרכז הכדור כולן שוין לקו מ׳ד׳נ׳[81]. כיצד[82]? כשנחצוב קוי [השלשה][83] משולשות בשני חצאין, ונגיע מקום חתך במרכז הכדור[84], ויהיו הקוין המוצאין ממרכז הכדור עד חצאי הקוין שוין. כיצד? כשנחלק שני קוי צ׳פ׳ פ׳ס׳[85] בשני חצאין על שתי נקודות [ת׳ ו׳][86], נאמר כי הן שוין.

[17] והמופת : לפי שקו צ׳פ׳ שוה לקו ס׳פ׳, וקו פ׳ת׳ לקו פ׳ו׳, וקו פ׳נ׳ היוצא[87] מזוית הצורה עד מרכז הכדור שותף, וזוית[88] נ׳פ׳ת׳[89] שוה לזוית נ׳פ׳ו׳ - לפי שמשולש צ׳נ׳פ׳ שוה למשולש[90] ס׳נ׳פ׳ - הלכך תושבת נ׳ת׳ כמו נ׳ו׳[91], והוא מה שרצינו.

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[66] ח׳ט׳כ׳] ח׳ט׳ב׳ C
[67] מאחת] מאחד CO
[68] השנית] השני CO
[69] ע׳ח׳] ט׳ח׳ C
[70] ח׳כ׳] ח׳ב׳ V
[71] ממאמר] מן מאמר COV
[72] י׳א׳ מספר היסודות לאקלידס] י׳א׳ אקלידדס C י׳א׳ אקלידס O
[73] ח׳כ׳ט׳] ח׳נ׳ט׳ COPV
[74] משולש] משו P
[75] משולשות] משולשת O
[76] ממששין] ממשמשים V ממשין O
[77] זה] – CO
[78] מרכזי] מרכז CV
[79] כל] – COV
[80] המשולשות] המשולשת CPO
[81] מ׳ד׳נ׳] מ׳ד׳כ׳ P
[82] כיצד? כשנחצוב קוי [השלשה] משולשות] – C
[83] השלשה] הששה OPV
[84] במרכז הכדור] המרכז O
[85] פ׳ס׳] פ׳צ׳ C
[86] ת׳ ו׳] ת׳ו׳ ז׳נ׳ OPV ז׳נ׳ C
[87] היוצא] הוצא C
[88] וזוית] ווית O
[89] נ׳פ׳ת׳] נפת C
[90] למשולש] – P
[91] נ׳ו׳] ט׳ C

[18] ובכיוצא[92] בזה נבאר כי משולש [עׄנׄוׄ][93] שוה למשולש עׄנׄתׄ. ונהוג בו כן תמצא כי קׄנׄ שוה למׄנׄ[94]. ולכן מׄנׄ הוא יוצא [ממרכז][95] הכדור עד סיבובה. וקו קׄנׄ יוצא ממרכז הכדור - הלכך יגיע עד סיבובה[96]. לפיכך משולש [עׄפׄסׄ][97] ימשש בכדור.

[19] וכן יתבאר בשאר המשולשות - ובכיוצא[98] בזה גם בארבע הצורות[99], כשנתמשל בהן כמו שהמשלנו בזה - כי כל אחד משטחיו[100] הן נוגעין בכדור.

[20] וכן יתבאר כי כל אחד[101] משטחי הצורה החיצונית[102] הוא נכחי לדומה לו מן הצורה[103] הפנימית - היא הצורה[104] העשויה בתוך הכדור. כיצד[105]? לפי שהיה השטח[106] נוגע[107] בכדור, יהא הקו המוגע בין מרכז הכדור ומקום הנגיעה, שהוא מרכז השטח, עמוד על השטח, כמו שנתבאר במאמר ראשון מן תאודוסיוס[108]. הלכך יסוב עם הקוין המוצאין[109] בו[110] בזויות נצבות. [וכשזוית][111] עׄקׄנׄ נצבת, יתבאר כי משולש עׄפׄסׄ נכחי למשולש[112] [חׄכׄלׄ][113]. וכן יתבאר בשאר המשולשות ובשאר שטחי החמש צורות[114].

[21] **וכיצד נעשה** הכדור המוצע בתוך הצורה מחמש צורות? כמו שנספר: נעשה גופני שוה לצורה[115] הגופנית הנזכרת, כמו שנספר בעזרת שדי ית׳. ואחר כן[116] נקים על מרכז שטחיו עמודים בתוך הצורה,

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92 ובכיוצא] וכיוצא C
93 עׄנׄוׄ] עׄטׄ C עׄכׄוׄ OPV
94 למׄנׄ] לׄמׄנׄ O
95 ממרכז] המרכז COPV
96 וקו קׄנׄ יוצא ממרכז הכדור - הלכך יגיע עד סיבובה] – O
97 עׄפׄסׄ] עׄפׄהׄ COPV
98 ובכיוצא] וכיוצא C
99 הצורות] צורות O
100 משטחיו] משטחין C מן שטחיו V
101 אחד] אחת O
102 החיצונית] החיצוני O
103 מן הצורה] מהצורה COV
104 היא הצורה] – COV
105 כיצד] כי O
106 השטח] השטח השטח V
107 נוגע] נגע).O(ac
108 תאודוסיוס] תאורסיות C תיאודוסיוס V
109 המוצאין] המוציאין O
110 בו] כי PV
111 וכשזוית] ובשויות CO ובהשוית PV
112 למשולש] למשמש CO
113 חׄכׄלׄ] חׄטׄלׄ COPV
114 צורות] הצורות O
115 לצורה] לצורת O
116 ואחר כן] ואחרכן C

ונוציאם על יושר עד שיפגשו על נקודה אחת. ואחר כן נוציא אחד מאותם הקוין על יושר עד שיהא הקו כלו כפל הראשון[117]. ונאמר כי אותו הקו הוא אלכסון הכדור.

[22] אבל עשית[118] הגופני שוה לגופני מה שנתבאר בספר אקלידס[119], הוא[120] השוה השטחים בלבד. אך עשיית שאר הגופניות הוא כמו שנספר: נוציא קו שוה לאחד מקוי הצורה, ואחר כן נעשה על ראשו זוית גופנית שוה לזוית הצורה, ונוציא הקוין שוין לקוין הסובבין באותה זוית[121]. ואחר כן נעשה גם[122] על ראשי הקוין המוצאין זויות גופניות שוות לאותן[123]. וכן נעשה עד אשר תגמר[124] הצורה.

[23] וכשנעשה הצורה, נוציא ממרכזי[125] שטחיהן קוין נצבים עליהם[126], יהיו מוצאין בתוך הצורה, ונפלש אותן על יושר עד שיפגשו. והדבר נראה כי כלן יפגשו על נקודה אחת בתוך הצורה, ואותה נקודה היא מרכז הכדור.

[24] ונבאר תחלה כי אלו העמודים הם נכחיים לאותן העמודים שבצורה הראשונה[127], המוצאת ממרכז הכדור עד סבובה. המשל בצורת[128] בעלת ארבע: יהא משולש אׄבׄגׄ מזו הצורה שעשינו עתה שוה למשולש אׄבׄגׄ מהצורה הראשונה[129], ומשולש בׄגׄדׄ למשולש בׄגׄדׄ מן האחרת[130]. ונחלק בׄגׄ בשני חצאין על שׄ, ונגיע מרכז משולש בׄגׄדׄ, והוא חׄ, בנקודת שׄ.

[25] והדבר נראה כי, לפי שהיתה זו הצורה השנית שוה לצורה הראשונה, ושטחיה לשטחיה, כי שני קוי זׄשׄ חׄשׄ מזו הצורה שוין לשני קוי זׄשׄ חׄשׄ מהצורה האחרת, וגם כל אחד שוה[131] לחברו, וזוית זׄשׄחׄ מזו הצורה כזוית זׄשׄחׄ מהאחרת, הלכך תושבת זׄחׄ מהאחת כתושבת זׄחׄ מהאחרת. ויהיו שתי זויות שׄזׄחׄ שׄחׄזׄ

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117 הראשון] האחת PV
118 עשית] עשיית CO
119 בספר אקלידס] באקלידס COV
120 הוא] הוא הוא O
121 ואחר כן נעשה על ראשו זוית גופנית שוה לזוית הצורה, ונוציא הקוין שוין לקוין הסובבין באותה זוית] – PV
122 גם] – O
123 לאותן] לאותה COP
124 אשר תגמר] שתגמר CV שתגמור O
125 ממרכזי] מרכז CPV
126 עליהם] על קו PV עליהן C
127 הראשונה] האחת PV
128 בצורת] בצורה CO
129 הראשונה] האחת PV
130 מן האחרת] מהאחרת COV
131 שוה] שנה C

שלמעלה[132] מתושבת זו הצורה שוות לשתי זויות[133] שׂזֹח שֹחֹז שלמעלה[134] מתושבת משולש הצורה האחרת, כל אחת לנכחיתה.

[26] ותהא הנקודה שנפגשו עליה העמודין בתוך[135] הצורה נקודת נֹ[136]. ולפי שהיתה זוית נֹזֹשׂ מזו הצורה כזוית נֹזֹשׂ מהצורה האחרת, לפי שכל אחת מהן נצבת, נפיל ממנה שתי זויות שׂזֹח שֹחֹז השוות. תשאר שתי זויות נֹזֹח נֹחֹז שוות, וכל אחת מהן שוה לחברתה[137] מן המשולש[138] שדומה לו[139] מהצורה האחרת. ותנהוג בו[140] כראוי ותמצא המשולש שוה למשולש.

[27] הלכך יהיו כל אחד משני צלעי נֹזֹ נֹחֹ[141] מהצורה האחת שוה לדומה[142] לו מהצורה האחרת.והן הקוין המוצאין מנקודת פגישת העמודים בתוך הצורה עד שטח הכדור.וכמו כן הן באותה הצורה האחרת יוצאין[143] ממרכז הכדור עד שטחה. הלכך מרכז הכדור בזו הצורה השנית הוא[144] נקודת נֹ.

[28] ונוציא זֹנֹ על יושר עד שיהא מכופל[145], והוא זֹנֹקֹ, ונעגיל[146] עליו חצי עגולה עד שתחזור למקומה. ואותה היא הכדור המוצע, והוא מה[147] שרצינו.

[29] **אבל איכות עשיית הכדור** המוצע על כל אחת מאלו הצורות החמש הוא כשנעשה כל אחת [מהצורות][148] שוה לאותה שעשה אקלידס, כלומר[149] שיהא הכדור סובב[150] על הצורה. ואחר כן נוציא העמודים ממרכזי[151] שטחיו עד שיפגשו כלן תוך הכדור על נקודה אחת. ואחר כן נגיע בין זויות הצורה ובין אותה הנקודה קוין ישרים. והדבר נראה כי מרכז הכדור הוא אותה נקודה.

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132 שלמעלה] של מעלה P
133 לשתי זויות] לשתים נצבות PV
134 שלמעלה] של מעלה P
135 בתוך] שבתוך C
136 נֹ] גֹ C
137 לחברתה] לנכחיתה COV
138 מן המשולש] מהמשולש COV
139 לו] לזו V
140 בו] – O
141 נֹזֹ נֹחֹ] נֹזֹחֹ (.ac) O
142 לדומה] ודומה O
143 יוצאין] חצאין V
144 הוא] היא O
145 מכופל] מוכפל C
146 ונעגיל] ונעגל C
147 מה] – O
148 מהצורות] מהצורה COPV
149 כלומר] כלו׳ O
150 סובב] הסובב C
151 ממרכזי] ממרכז PV

[30] וגם נוציא אחד[152] מאותן הקוין אשר מזוית[153] הצורה עד נקודת הפגישה על יושר, עד שיהא הקו כלו כפל הראשון[154], כי אותו הקו הוא אלכסון הכדור, ונעגיל עליו חצי עגולה. ובזה נשלם עשיית הכדור על כל[155] אחת מהחמש צורות, והוא מה שרצינו. שבח לאל יתברך.[156]

[31] **זה הוא**[157] מה שעלה בידי להשיב לאותו הפילוסוף של קיסר, ואני מבן שמנה עשרה[158] שנה. ועוד שאלתי[159] אני בשאלות אחרות, וזה[160] שנכתוב[161] עתה אחת מהן. והיא לתת מופת ברור מגימטריא[162] על לוחות המעלות במקומות[163] שהכדור נוטה עליהן. והשיבני אז אותו[164] הנקרא פילוסוף[165] בזה הנוסח בערבי[166], והעתקתיהו[167] בלשון עברי[168].

[32] לפי[169] שכבר באר תלמי על יתרוני המעלות בכדור הנצב והציע להן לוחות ראויין כפי מופתיו, והדבר ידוע כי מה שיעלה מגלגל המישור עם כל קשת מהקשתות הצפוניות בשכון מן הארץ הוא יותר קטן ממה שיעלה בכדור הנצב. ואיני[170] אומר[171] אי זו קשת שתזדמן[172], אך אי זה חלק מוצע מראש טלה ועד כ֗ט֗ מבתולה צומח באופקים הצפוניים ונגד מה שיעלה עמו מגלגל המישור קטן ממה[173] שיעלה עמו באופק הסתו.

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152 אחד] אחת O
153 אשר מזוית] שמזוית COV
154 הראשון] האחד PV
155 כל] – O
156 שבח לאל יתברך] שיל׳ יתי׳ V +אמן O
157 זה הוא] זהו C זהוא O
158 שמנה עשרה] כ֗ C י֗ח֗ O
159 ועוד שאלתי] ושאלתי CO
160 וזה] וזו C
161 שנכתוב] שאכתוב O
162 מגימטריא] מגמטרייא C מגימיטריא O
163 במקומות] בנקודות PV
164 אותו] אותו אותו V
165 פילוסוף] פלוסוף C
166 בזה הנוסח בערבי] בלשון ערבי PV
167 והעתקתיהו] והעתקתיו V
168 בלשון עברי] בעברי CO
169 לפי] ולפי V
170 ואיני] ואי O
171 אומר] אומ׳ V
172 שתזדמן] שיזדמן O אשר תזדמן V
173 ממה] מן מה V

[33] והמשל כי מעלות סוף מזל תאומים במקום שרחבו ארבעים[174] הן[175] סׄחׄ[176] לׄדׄ, ובכדור הנצב [צׄ][177], ומעלות כׄטׄ ממזל בתולה[178] קׄעׄחׄ[179], ובכדור הנצב קׄעׄטׄ. ושעור החסרון בכאן כשעור התוספת בחלקים שהן מתחלת מזל[180] מאזנים עד סוף מזל[181] דגים.

[34] נאמר זה ואיני שוכח כי קיבוץ מעלות כל שתי[182] קשתות שריחוקן[183] שוה מנקודת היפוך אחד ידוע באופקים הנוטים שוה לקיבוץ מעלות אותן הקשתות בכדור הנצב. וגם לא אשכח כי חצי עגולת המזלות יעלה עם חצי עגולת גלגל המישור. אך מפני מה היו [המעלות][184] באופקים הנוטין לכל קשת מחצי[185] גלגל המזלות הצפוני פחות ממעלות הכדור הנצב?

[35] הוא מפני כי נטיית[186] עגולת גלגל המישור באופקים הצפוניים מקדקד הראש הוא יותר מנטיית החלקים הצפוניים[187] כמו שהם יותר קרובים לקדקד הראש מהחלקים[188] הדרומיים[189]. ויתחייב בעבור כן כשנדע שעור התוספת והגרעון, נדע יתרוני המעלות בכדור הנוטה.

[36] ונבאר בזו[190] הצורה כי זה היתרון הוא[191] ידוע: **להיות** אׄבׄגׄדׄ[192] עגולת האופק, ואׄחׄגׄ[193] חצי היום, ובׄחׄמׄ[194] גלגל המישור על סדן[195] טׄ, והׄכׄדׄ[196] גלגל המזלות. ותהא נקודת [הׄ][197] החלק שמגלגל המזלות על

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[174] ארבעים] מׄ P
[175] הן] – O
[176] סׄחׄ] טׄחׄ C
[177] צׄ] פׄ C סׄ OPV
[178] ממזל בתולה] מברזלה C מבתולה O
[179] קׄעׄחׄ] קׄפׄחׄ O
[180] מזל] – CO
[181] מזל] – CO
[182] שתי] שני COV
[183] שריחוקן] שריחוקו O
[184] המעלות] המעגלות COPV
[185] מחצי] מחציי CO
[186] כי נטיית] שנטיית CO
[187] מקדקד הראש הוא יותר מנטיית החלקים הצפוניים] – PV
[188] מהחלקים] מהחלקיים O
[189] הדרומיים] הנגדיים C הנגביים O הגביים (.ac) O
[190] בזו] בזאת V
[191] הוא] – CO
[192] אׄבׄגׄדׄ] אׄבׄגׄ O
[193] ואׄחׄגׄ] ואׄהׄגׄ V
[194] ובׄחׄמׄ] ובׄחׄהׄ C
[195] סדן] סכן C סדק O
[196] והׄכׄדׄ] וכׄדׄרׄ (.ac) O והׄכׄדׄרׄ O והׄקׄדׄ PV
[197] הׄ] קׄ COPV

האופק, ויהא החלק הדרומיי[198]. ונרצה לידע כמה[199] אפשר להוסיף על מעלות אותו החלק בכדור הנצב, עד שיהיו המעלות באותו מקום.

[37] הלכך נוציא מן הסדן עד אותו החלק קשת טוֹהֹ. לפיכך יהיו שני משולשי[200] מוֹהֹ [מֹחֹאֹ][201] [דומין][202], לפי שזוית מֹ שותפת[203] ושתי זויות וֹ חֹ[204] נצבות. תשאר זוית הֹ שוה[205] לזוית אֹ.

[38] הלכך יחס ג׳יב מוֹ לג׳יב [וֹהֹ][206] כיחס ג׳יב מֹחֹ[207] לג׳יב חֹאֹ[208]. לכן מֹחֹ ידוע, לפי שהוא רבע עגולה. ו[וֹהֹ][209] ידוע, לפי שהוא נטיית נקודת הֹ מגלגל המזלות על[210] גלגל המישור. ו[חֹאֹ][211] ידוע, לפי שהוא תשלום רחב העיר. ומֹוֹ[212] מוסכל[213].

[39] הלכך אלו הארבעה[214] שיעורים מתיחסין יחס הראשון[215] מוסכל אל השני הידוע כיחס השלישי הידוע אל הרביעי הידוע. ונכפול ג׳יב הנטייה באלג׳יב הגדול, ונחלקנו על ג׳יב תשלום רחב העיר. יעלה לנו [וֹמֹ][216] היתרון ידוע. ונצרפנו אל קשת [וֹכֹ][217] הידועה[218], והיא מעלת[219] נקודת הֹ בקו המישור, יצא מעלות נקודת הֹ בעיר.

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198 הדרומיי] הנגביי COV
199 כמה] במה P
200 משולשי] המשולשי C
201 מֹחֹאֹ] מֹחֹטֹ COPV
202 דומין] – COPV
203 שותפת] לזוית פֹּתֹ O משותפת V
204 וֹ חֹ] וחֹ O
205 שוה] – V
206 וֹהֹ] מֹהֹ COPV
207 מֹחֹ] מֹסֹ O
208 חֹאֹ] הֹאֹ O
209 וֹהֹ] וזה CP וזֹהֹ OV
210 על] עד C
211 וחֹאֹ] וטֹאֹ O וטֹחֹ CPV
212 ומֹוֹ] ומֹ O
213 מוסכל] מוסבל C
214 הארבעה] הדֹ C
215 הראשון] האחד PV
216 וֹמֹ] ומֹ COPV
217 וֹכֹ] וטֹ COPV
218 הידועה] הידוע O
219 מעלת] מעלות CO

[40] **אך כשהחלק צפוני**, לנקודת דׄ על האופק. נוציא מהסדן[220] קשת [טׄדׄ][221] עד גלגל המישור, ויפגשנו[222] על זׄ. ויהיו שני משולשי דׄבׄזׄ דׄטׄגׄ[223] דומין, לפי ששתי[224] זויות דׄ[225] מהן המתחצבות הן שוות, ושתי זויות [גׄ זׄ][226] נצבות. ישארו שתי זויות[227] [בׄ טׄ][228] שוות.

[41] הלכך יחס ג׳יב דׄבׄ[229] לג׳יב זׄבׄ כיחס ג׳יב דׄטׄ לג׳יב טׄגׄ. לכן דׄבׄ ידוע, לפי שהוא רחב המזרח. וזׄבׄ מוסכל, לפי שהוא היתרון בין מעלות השיווי ומעלות העיר. וטׄדׄ ידוע, לפי שהוא תשלום הנטייה. וטׄגׄ רחב העיר, והוא ידוע[230].

[42] ונכפול ג׳יב רחב המזרח בג׳יב רחב העיר[231] ונחלקנו על ג׳יב תשלום הנטייה. יעלה בידנו[232] ג׳יב היתרון ידוע. הלכך קשתו, והיא בׄזׄ[233], ידועה[234]. ונגרענה מן קשת[235] זׄכׄ[236] הידועה, והיא[237] מעלת[238] נקודת [דׄ][239] בקו המישור, ישאר [בׄכׄ][240] ידוע, והיא מעלת[241] נקודת דׄ בעיר, וזה מה שרצינו.

עד כאן[242] דברי אותו הפילוסוף[243].

[43] **והשבתיו**[244] אז בזה הנוסח כמה תשובות, והצורה שעשה[245] חקוקה בדף האחר. והשבתיו[246] בדבר: על חטא[247] ועל חטאים שאינן ראויין לתלמיד המכונה פילוסוף כמוך.

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[220] מהסדן] מהסכן C
[221] טׄדׄ] טׄדׄ טׄזׄהׄ CPO טׄדׄ טׄהׄ V
[222] ויפגשנו] ויפגשו O
[223] דׄטׄגׄ] דׄטׄעׄ C
[224] ששתי] ששני PV
[225] דׄ] – O
[226] גׄ זׄ] דׄ זׄ COPV
[227] זויות] זוית (ac.) O
[228] בׄ טׄ] מׄ טׄ CPV – O
[229] דׄבׄ] בׄבׄ (ac.) VO דׄבׄ (pc.) O
[230] והוא ידוע] – (ac.) C
[231] והוא ידוע. ונכפול ג׳יב רחב המזרח בג׳יב רחב העיר] – O
[232] בידנו] בידינו (pc.) CO
[233] בׄזׄ] בׄדׄ O
[234] ידועה] ידוע O
[235] מן קשת] מקשת CO
[236] זׄכׄ] זׄבׄ CO
[237] והיא] והוא CO
[238] מעלת] מעלות CO
[239] דׄ] הׄ COPV
[240] בׄכׄ] בׄדׄ COPV
[241] מעלת] מעלות CO
[242] עד כאן] עׄכׄ V
[243] אותו הפילוסוף] זה הפלוסוף O הפילוסוף CV
[244] והשבתיו] והשבותיו O והשיבותי V
[245] שעשה] עשה V
[246] והשבתיו] והשיבותיו V
[247] חטא] חט C – O

[44] חדא מה שאמרת: "יהיו שני משולשי מׄוׄהׄ מׄחׄאׄ דומין, לפי שזוית [מׄ][248] שותפת ושתי זויות [וׄ חׄ][249] נצבות. תשאר זוית הׄ שוה לזוית אׄ." - יאמר נא הפילוסוף מתי נתבאר כי, כשישוו שתי זויות מן משולש[250] בעל הקשתות לשתי זויות מהמשולש האחר[251], כי הזוית הנשארת מהאחד שוה לזוית הנשארת מהאחר - כי זה לא נתבאר כי אם במשולשות בעלות הקוין הישרים בלבד.

[45] ועוד כי הצעת[252] המשולש האחד[253] יותר גדול מהאחר[254], כמו שהוא בצורה. וזה שוא ואי אפשר כפי מה שהצעת מהשוית[255] זויות המשולש האחר[256]: כי יתחייב כשיהיו זויות המשולש[257] האחד שוות לזויות המשולש האחר[258], כל אחת לדומה[259] לה, שיהיו השני משולשין שוין, כמו שנתבאר[260] בצורת יׄזׄ ממאמר ספר[261] מילאוש[262].

[46] הלכך, כשנציע הסברא התנאיי[263] כן[264]: אם יהיו זויות ממשולש מׄוׄהׄ שוות לזויות משולש מׄחׄאׄ, כל אחת לדומה לה, יתחייב להיות משולש מׄוׄהׄ[265] שוה למשולש מׄחׄאׄ[266]. ונשנה[267] כנגד המאוחר, והוא כי אין משולש מׄוׄהׄ שוה למשולש מׄחׄאׄ. ויוליד לנו כנגד המוקדם, והוא כי אין זויות משולש מׄוׄהׄ שוות אל זויות[268] משולש מׄחׄאׄ.

[47] הלכך לא תהא זוית הׄ שוה לזוית אׄ[269]. הלכך לא יהא יחס גׄיׄבׄ מׄוׄ לגׄיׄבׄ וׄהׄ[270] כיחס גׄיׄבׄ מׄחׄ[271] לגׄיׄבׄ חׄאׄ[272] - כי היה כן אלו היתה זוית הׄ שוה לזוית אׄ, ובתנאי שיהיה בקיבוץ שני קשתי מׄאׄ מׄהׄ יחד פחות

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[248] מׄ] אׄ COPV
[249] וׄ חׄ] נׄ חׄ OPV נׄ צׄ C
[250] מן משולש] ממשולש COV
[251] האחר] ואחר V
[252] הצעת] המוצע O
[253] האחד] האחר O
[254] מהאחר] מהאחד C
[255] מהשוית] מהשויית C
[256] האחר] האחד P
[257] האחר: כי יתחייב כשיהיו זויות המשולש] – V
[258] האחר] האחד C
[259] לדומה] דומה PV
[260] שנתבאר] נשנתבאר C
[261] ספר] מספר C – O
[262] מילאוש] מיליאוש C מילאוס O מיליאוס V
[263] התנאיי] התנאי C והתנאי PV
[264] כן] – (ac.) O
[265] שוות לזויות משולש מׄחׄאׄ, כל אחת לדומה לה, יתחייב להיות משולש מׄוׄהׄ] – CPV
[266] מׄחׄאׄ] מחה C
[267] ונשנה] ונשנא O
[268] אל זויות] לזויות COV
[269] הלכך לא תהא זוית הׄ שוה לזוית אׄ] iter. O
[270] וׄהׄ] והׄ O
[271] מׄחׄ] – C
[272] חׄאׄ] הׄאׄ O

מחצי עגולה, כמו שנתבאר במאמר[273] שלישי מספר מיליאוש[274]. הלכך הדבר נראה כי זה המופת לא יתכן כי אם במשולשות של[275] קוין[276] ישרים, לא של קשתות. ואם הוא בעיני הפילוסוף[277] דין אלו ואלו שוה, יש לו[278] להועיל לנו[279] בזה החדש שנעשה תחת השמש.

עד כאן.[280]

[48] **אמר המחבר** : וכשהרצו[281] הדברים לפני הקיסר, המלך האנפראור פדריק, ירום הודו[282], שמח מאד בתשובות אשר השיבתי לאותו הנקרא פילוסוף לפניו[283]. ועוד היה[284] בינינו[285] משא ומתן הרבה על דברים רבים ועל[286] שאלות רבות ותשובות, ואין להאריך בזה הספר מאלו הדברים[287] יותר.

[49] ועוד אחר[288] כן[289] כמו עשר שנים נתגלגלו עלי דברים וירדתי לארצות הקיסר[290]. וראיתי תכונת מעשיו ועניניו וחכמיו וסופריו וזקניו[291] ושופטיו ושוטריו ומאכל שולחנו[292] ומושב עבדיו. והכל תלוי במזל. ואני מתפלל לאל, יתעלה ויתברך[293] בכל יום, להחזירני[294] אל בית אבי[295] ואל ארץ מולדתי[296] שליו[297] ושמח. כן יהי רצון מלפניו. אמן. כן יאמר האל.[298]

273 במאמר] ממאמר O
274 מספר מיליאוש] ממילאוש C ממלאוש (ac.) O ממילאוש (pc.) O
275 של] על PV
276 קוין] קנין C – O
277 הפילוסוף] הפלוסוף (ac.) CO
278 לו] – CO
279 להועיל לנו] להועילנו CO
280 עד כאן] – CO
281 וכשהרצו] כשהרצו C וכהרצו V
282 המלך האנפראור פדריק, ירום הודו] – COP
283 אשר השיבתי לאותו הנקרא פילוסוף לפניו] – COP
284 היה] היו C
285 בינינו] בנינו O
286 משא ומתן הרבה על דברים רבים ועל] – COP
287 מאלו הדברים] – COP
288 ועוד אחר] ואחר COP
289 כן] iter. O
290 הקיסר] קיסר C של קיסר O
291 וסופריו וזקניו] וזקניו וסופריו O
292 שולחנו] שלחנו P
293 יתעלה ויתברך] יתברך CO יתי ויתברך V
294 להחזירני] להחזרני O
295 אל בית אבי] – COP
296 ואל ארץ מולדתי] לארצי CO לארצי ולמולדתי P
297 שליו] שלו COP
298 כן יהי רצון מלפניו. אמן. כן יאמר האל] – O עמ״י עש״ו C כן יהי רצון מלפניו. אמן. כן יאמר ייי P

Judah ben Solomon ha-Cohen's Correspondence with the Emperor's Philosopher

English translation

[1] **The author said**: In the time of my youth the sage, the Philosopher to the Emperor, the Great King, Emperor Frederick (may his glory be exalted), sent me questions — I was still in my own land in the midst of the exile of Jerusalem which is in Spain.[1] The first one was: How do we construct each of the five [regular] solids around a given sphere, and how do we construct the given sphere inside each of them and also around each one of them, [and provide each case] with a proof?

[2] I responded — I was eighteen year years old at the time — and this was what I said. Who could have told me when I answered them [i.e. the questions] in the country of my birth, in the community of the town of Toledo, before my teacher and master, my father (may his rest be glorious - *his soul shall dwell at ease and his seed shall inherit the earth*),[2] that it would turn out that I would go down to Tuscany. There I translated them from Arabic into the Hebrew language, with this book that I composed when I was still there. *Everything that the Merciful does is for the best*.[3] Blessed be God every day.

[3] **This is the text of the answer:**

[*1. Construction of the circumscribed solid*]

How do we construct each of the five figures around the given sphere? First, we construct each of the five figures inside the sphere, in the way that Euclid mentioned.[4] Then we join the centre of the sphere and the angles of the figure with straight lines, and we produce these lines indefinitely.

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[1]Cf. Obad. 1:20.
[2]The Hebrew נבתוי״א is an acronym of Psalm 25:13.
[3]Berakhot 60b. In the introduction to his Hebrew rendering of Ptolemy's *Tetrabiblos*, Judah sees this phrase as referring to the astrological 'judgements of the stars' which carry out God's will on earth. See *Otot ha-shamayim*, 2a.
[4]See *Elements* XIII. 13-17.

[4] Then we join the centre of the sphere with the centre of one of the faces of the figures. How? In [the case of] the tetrahedron, octahedron and icosahedron we draw the line to the centre of one of the triangles that are contained in the sphere, and likewise [we do] with the centre of the quadrangle of the figure of the cube, and likewise with the centre of one pentagon of the dodecahedron.

[5] Know that the centre of the face is the centre of the circle that is circumscribed about it. It is obvious that the centre of the face is the point in which the equal lines meet that are drawn from the angles of the face in the [same] plane.

[6] We join the centre of the face with the centre of the sphere. It is obvious that this joining line stands on the face at right angles, as we will show afterwards, with the help of the Almighty, blessed be He. We produce the line up to the surface of the sphere, and erect on its end a perpendicular outside of the sphere, which touches it.

[7] First, we show that the angle that is enclosed by the two lines that were drawn from the centre of the sphere (the one that goes to the angle of the figure, and the other that goes to the centre of one of the faces of the figure), is smaller than a right angle. If this is the case, then this line that we have drawn tangentially will necessarily meet the line joining the centre of the sphere and the angle of the figure.[5] Thus they meet outside of the sphere.

[8] From the other lines joining the centre of the sphere and the angles of the figure we cut off lines equal to this line, and we join the ends of the lines with straight lines. We say

[5] Judah does not mention that this is only the case if the two lines lie in the same plane. See also paragraph [11] below.

that herewith we have completed the construction of the figures that we wanted around the given sphere.

[9] **We give an example with letters:**

First, let a tetrahedron be constructed within the sphere, in the way that Euclid showed.[6]

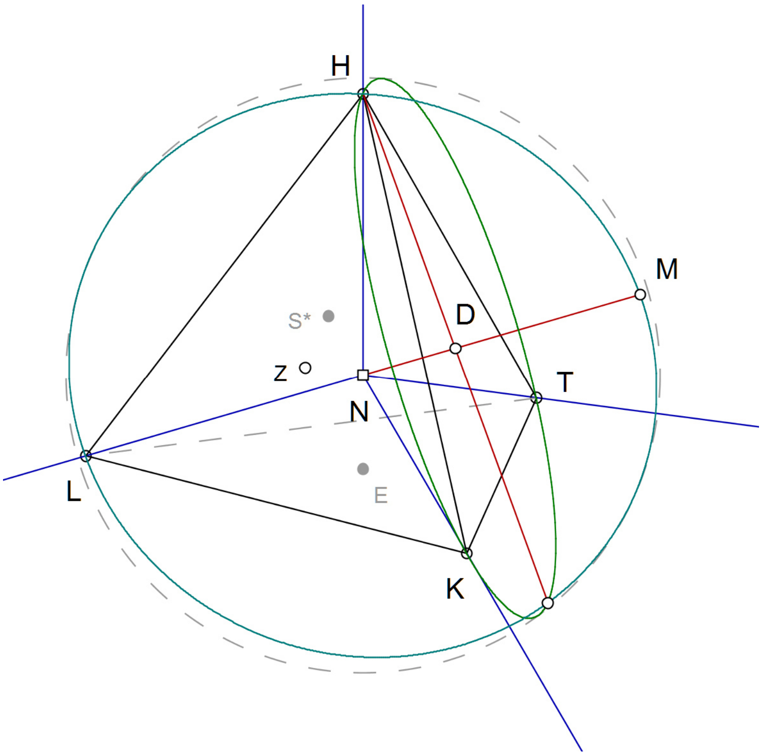

Figure 2: Modern reconstruction.

Let the centre of the sphere be *N*, and the angles of the figure *H*, *T*, *K*, *L*, and the centres of the triangles *D*, *E*, *Z*, *S*.[7] We join the lines *HN*, *NT*, *NK*, *NL*, and produce them indefinitely. We join *ND* and produce it until it meets the surface of the sphere in *M*.

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[6] *Elements* XIII.13: *To construct a pyramid, to comprehend it in a given sphere; and to prove that the square on the diameter of the sphere is one and a half times the square on the side of the pyramid.* All my citations of Euclid's *Elements* follow the translation by David E. Joyce, "Euclid's *Elements*."

[7] Judah uses the letter *S* to for two different points: here it is the centre of a triangle. However, the point is not used in the construction that follows. In paragraph [12] *S* is defined as an angle of the tetrahedron constructed around the sphere.

[10] First, we show that angle *NDH* is a right angle. How? If in the great circle[8] we draw a line which is the diameter of the circle circumscribed about triangle *HTK*, and from its centre to this line we draw line *ND*, [then] it cuts it into two halves on point *D*, which is the centre of the triangle. Therefore it intersects it at right angles, thus angle *NDH* is a right angle, and that is what we wanted.[9]

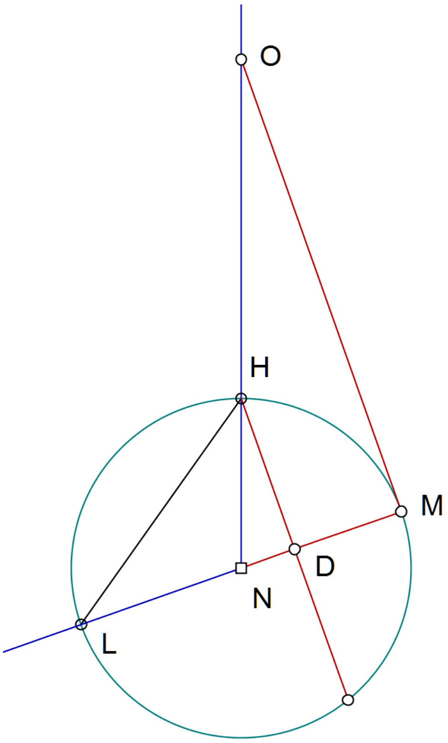

Figure 3: Modern reconstruction.

[11] Since this has been shown, we draw a line from point *M* - let it be perpendicular to *MDN*.[10] As it has been shown that angle *NDH* is a right angle, therefore angle [*DNH*] will be

[8]Judah is referring to the great circle in plane *NDH* (see figure 2).
[9]*Elements* III.3: *If a straight line passing through the center of a circle bisects a straight line not passing through the center, then it also cuts it at right angles; and if it cuts it at right angles, then it also bisects it.*
[10]From the following, it is clear that this perpendicular line must lie within plane *NDH*. As Judah does not mention this fact, it is possible that he used a two-dimensional diagram, similar to figure 3 above, in which the perpendicular to *NDM* is necessarily also parallel to line *DH*.

less than a right angle.[11] Therefore the two lines [the perpendicular to] *NM*,[12] [and line] *NH* will necessarily meet.[13] We produce them until they meet in point *O*.

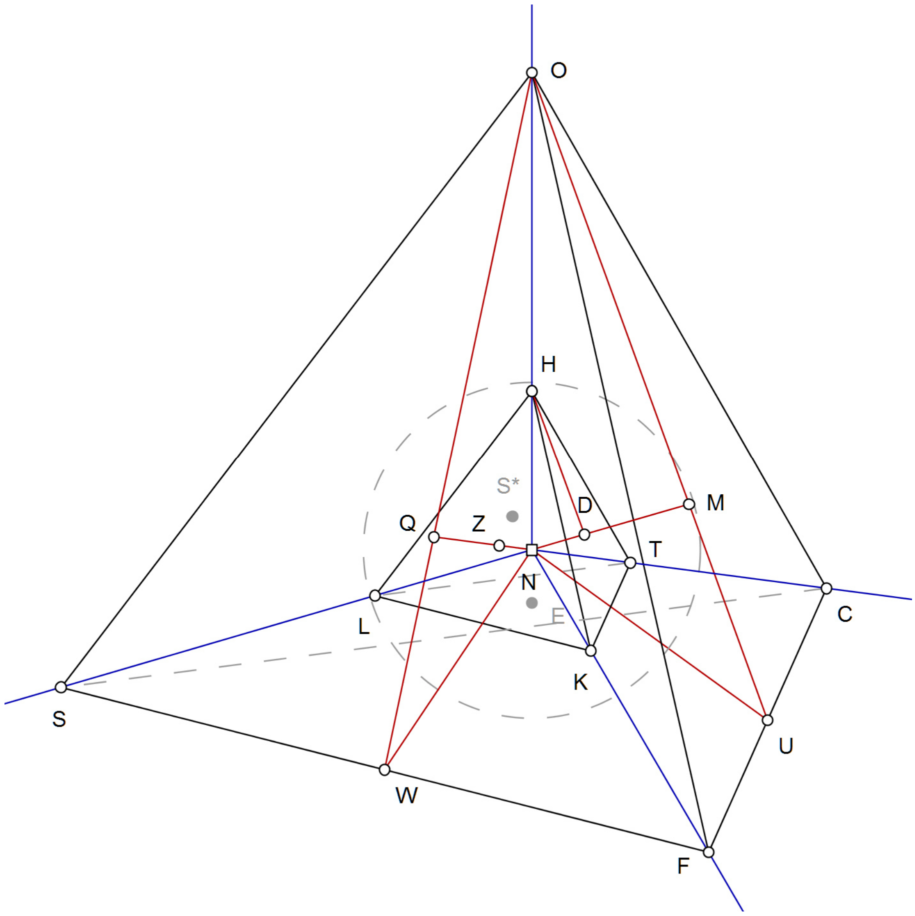

Figure 4: Modern reconstruction.

[12] Then we cut off from the lines *NT*, *N[K]*, *NL*, which were produced indefinitely, lines equal to line *NHO*; these are *NTC*, *NKF*, *NLS*. Then we join the lines *OC*, *OF*, *OS*, *CF*, *FS*, *CS*.

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[11]*Elements* I.17: *In any triangle the sum of any two angles is less than two right angles.*
[12]Although all manuscripts read 'the two lines *NM*, *NH*', it is clear that Judah refers to the perpendicular to *NM*. See also paragraph [7]: "If this is the case, then this line that we have drawn *tangentially* will necessarily meet the line joining the centre of the sphere and the angle of the figure."
[13]For his proof, Judah makes use of *Elements* I postulate 5: *That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.* The postulate is only applicable if all lines lie in the same plane, a fact that Judah fails to mention. (Cf. paragraph [7].)

[13] We show that these [six] are all equal. How? Since angles *HNT*, *HNK*, *HNL*, *TNK*, [*KNL*], *LNT* are equal and lines *NO*, *NC*, *NF*, *NS* are equal, therefore lines *OC*, *OF*, *OS*, *CF*, *FS*, *SC* are equal. Therefore the four triangles are equal.[14]

[14] Then we show that each of the faces of this figure that was constructed outside of the sphere is parallel to each of the faces of the one constructed inside the sphere.

[15] First, we give an example; we say that the plane of triangle *OCF* is parallel to the plane of triangle *HTK*.[15] How? Since in triangle *NOC* a line was drawn from one of its sides to the other, namely *HT*, which cuts them in the same ratio (since the ratio *OH* to *HN* is like the ratio *CT* to *TN*), therefore *HT* is parallel to *OC*.[16] Likewise we show that line *HK* is parallel to *OF*. And [according to] figure 15 of book XI of the *Elements* by Euclid,[17] triangle [*HKT*] is parallel to triangle *OCF*, and that is what we wanted.

[16] Furthermore, we show that the three remaining triangles touch this sphere.[18] This is shown if we show that the lines that were drawn between the centres of the triangles and the centre of the sphere are all equal to line *MDN*. How? If we cut the lines of the [three] triangles into two halves, and we join the point of section with the centre of the sphere,

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[14]*Elements* I.4: *If two triangles have two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, then they also have the base equal to the base, the triangle equals the triangle, and the remaining angles equal the remaining angles respectively, namely those opposite the equal sides.*

[15] In the following, Judah first proves that face *OCF*, which was constructed with the help of point *M* on the sphere's surface, is parallel to face *HTK* of the inner tetrahedron. As the remaining three faces of the outer tetrahedron were constructed in a different way, he then proves that they, too, are parallel to the remaining faces of the inner tetrahedron.

[16]*Elements* VI.2: *If a straight line is drawn parallel to one of the sides of a triangle, then it cuts the sides of the triangle proportionally; and, if the sides of the triangle are cut proportionally, then the line joining the points of section is parallel to the remaining side of the triangle.*

[17]*Elements* XI.15: *If two straight lines meeting one another are parallel to two straight lines meeting one another not in the same plane, then the planes through them are parallel.*

[18] It is obvious that face *OCF* touches the sphere, since it was constructed with the help of point *M* on the sphere's surface. Judah now proves that the remaining outer faces also touch the sphere.

the lines that were drawn from the centre of the sphere to the halves of the lines will be equal. How? If we cut the two lines *CF*, *FS* into two halves on the two points [*U*, *W*], we say that they are equal.

[17] And the proof is: Since line *CF* is equal to line *SF*, and line *FU* to line *FW*, and line *FN*, which goes from the angle of the figure to the centre of the sphere, is common, and angle *NFU* is equal to angle *NFW* (since triangle *CNF* is equal to triangle *SNF*), therefore base *NU* is like *NW*,[19] and that is what we wanted.

[18] Similarly we show that triangle [*ONW*] equals triangle *ONU*. Practise it, and you will find that *QN* equals *MN*; but *MN* goes from the centre of the sphere to its circumference. And line *QN* [also] starts at the centre of the sphere - therefore it reaches its circumference. Therefore triangle [*OFS*] touches the sphere.

[19] Likewise it is shown for the rest of the triangles - and similarly also for the four [other] figures, if we make examples for them like we made examples for this one - that each of its faces touches the sphere.

[20] Likewise it is shown that each face of the outer figure is parallel to its equivalent from the inner figure, which was constructed inside the sphere.[20] How? Since the face touches the sphere, the line that was drawn between the centre of the sphere and the point of contact (which is the centre of the face) is perpendicular to the face, as is shown

[19] *Elements* I.4 (see notes to paragraph [13] above).

[20] In paragraph [15] Judah proved only that faces *OCF* and *HTK* are parallel. As *OCF* was constructed in a way that differs from the construction of the other three outer faces, he now proves that they, too, are parallel to the inner faces.

in the first book of Theodosius.[21] Therefore it makes right angles with the lines that meet it.[22] [And if angle] *OQN* is a right angle, it is shown that triangle *OFS* is parallel to triangle [*HKL*].[23] Likewise it is shown for the rest of the triangles and for the rest of the faces of the five figures.

[*2. Construction of the given sphere inscribed inside the solids*]

[21] How do we construct the given sphere inside one of the five figures? As we will explain. We construct a solid [that is] equal to the afore-mentioned solid figure,[24] as we will explain, with the help of the Almighty, blessed be He. Then we erect on the centres of its faces perpendiculars inside the figure. We produce them until they meet in one point. Then we produce one of these lines, until the whole line is doubled. We say that this line is the diameter of the sphere.

[22] But the construction of a solid equal to a[nother] solid that is shown in Euclid's book, [applies] only [to] the parallelepiped.[25] But the construction of the rest of the solid figures is [done] as we will explain. We draw a line equal to one of the lines of the figure. Then we construct on its end a solid angle equal to the angle of the figure, and we draw lines

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[21]*Sphaerica* I.4. See *Autolykos: Rotierende Kugel und Aufgang und Untergang der Gestirne. Theodosios von Tripolis: Sphaerik*, 82: *If a sphere touches a plane which does not cut the sphere, then the line joining the centre of the sphere with the point of contact is perpendicular to the plane.*

[22]*Elements* XI definition 3: *A straight line is at right angles to a plane when it makes right angles with all the straight lines which meet it and are in the plane*. Judah forgets to mention that by saying 'the lines that meet it' he refers to the lines in the plane of one face.

[23] According to the proof in paragraph [10], *HZN* is a right angle. That faces *OFS* and [*HKL*] are parallel, follows from *Elements* XI.14*: Planes to which the same straight line is at right angles are parallel. And HZN is also a right angle*.

[24]I.e. we copy the circumscribed solid that was constructed in answer to the first question – this way it is guaranteed that the given sphere will 'fit' into the solid.

[25]It appears that Judah is referring to Elements XI.27: *To describe a parallelepipedal solid similar and similarly situated to a given parallelepipedal solid on a given straight line*. However, these solids are *similar*, not equal.

equal to the lines that confine that angle. Then we construct also on the ends of the drawn lines solid angles equal to them. Thus we continue until the figure is completed.

[23] When the figure is constructed, we draw from the centres of [its] faces lines that are perpendicular to them. Let them be drawn inside the figure. We extend them rectilinearly until they meet. It is obvious that they all meet in one point inside the figure, and this point is the centre of the sphere.[26]

[24] First, we show that these perpendiculars are parallel to those perpendiculars that are in the first figure, which was drawn from the centre of the sphere to its circumference.[27]

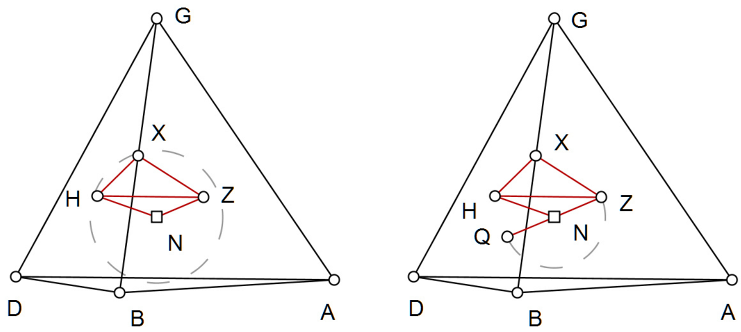

Figure 5: Modern reconstruction.

And the example is the tetrahedron: Let triangle *ABG* of this figure that we have constructed now be equal to triangle *ABG* of the first one, and triangle *BGD* to triangle *BGD* of the other. We cut *BG* into two halves on *X*, and we join the centre of triangle *BGD*, which is *H*, with point *X*.

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[26] In the following, Judah proves that the perpendiculars inside the polyhedron that he has just constructed do indeed meet, and that their meeting point is the centre of the sphere to be inscribed in it. He does that by proving that these perpendiculars are equal to the lines that lead from the centre of the sphere to the faces of the polyhedra in his first construction.
[27] I.e. the solid constructed in answer to the first question: the circumscribed solid that was constructed with the help of a line coming from the centre of the sphere.

[25] It is obvious that - as this second figure is equal to the first one, and its surfaces to its surfaces - that the two lines *ZX*,[28] *HX* of this figure are equal to the two lines *ZX*, *HX* of the other figure, and each one is also equal to the other, and angle *ZXH* of this figure is like angle *ZXH* of the other. Therefore base *ZH* of the one is like base *ZH* of the other. And the two angles *XZH*, *XHZ* that are above the base in this figure are equal to the two angles *XZH*, *XHZ* that are above the base of the triangle in the other figure, each one to its equivalent.

[26] Let the point in which the perpendiculars meet inside the figure be point *N*. As angle *NZX* of this figure is like angle *NZX* of the other figure (as each of them is a right angle),[29] we subtract from it the two equal angles *XZH*, *XHZ*. There remains that the two angles *NZH*, *NHZ* are equal, and each one of them is equal to its equivalent in the triangle similar to it in the other figure. Use it correctly and you will find that the triangle is equal to the triangle.

[27] Therefore each of the two sides *NZ*, *NH* of the one figure is equal to its equivalent of the other figure. These are the lines that were drawn from the meeting point of the perpendiculars inside the figure to the surface of the sphere. And in that other figure they also go from the centre of the sphere to its surface. Therefore the centre of the sphere in this second figure is point *N*.

[28] We produce *ZN* rectilinearly until it is doubled, that is *ZNQ*, and we carry a semicircle around it until it returns to its [original] position. This is the given sphere, and that is what we wanted.

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[28]Judah has not defined point *Z*. From the context it is clear that this point must be the centre of another face of the tetrahedron.
[29]This was proven in paragraph [10].

[*3. Construction of a sphere circumscribed about the solids*]

[29] But in order to construct the given sphere around each of those five figures, we construct each figure equal to that which Euclid constructed, that is to say that the sphere would be enclosing the figure.[30] Then we draw the perpendiculars from the centres of its surfaces, until they all meet inside the sphere in one point. Then we join the angles of the figure and this point with straight lines. It is obvious that the centre of the sphere is this point.

[30] We also produce one of those lines that [go] from the angle of the figure to the meeting point, until the whole line is doubled, as this line is the diameter of the sphere, and we carry a semicircle around it. Herewith the construction of the sphere around each of the five figures is complete, and that is what we wanted. Praise to the blessed God.

[31] **This is how I managed** to answer that 'emperor's philosopher' when I was eighteen years old. In addition, I myself asked other questions, and what we will write now is one of them. It was to give a clear geometric proof for the tables of ascensions for places in *sphaera obliqua*. The so-called 'philosopher' answered me at that time in Arabic in the following words. I have translated it into the Hebrew language.

[32] Ptolemy already showed the differences of the ascensions in *sphaera recta* and prepared appropriate tables for them according to his proofs. And it is known that [the arc] of the equator that rises with each northern arc [of the ecliptic] in the inhabited part of the world is smaller than [the arc of the equator] that rises in *sphaera recta*. I am not

[30] In answer to the second question, the polyhedron circumscribed about the sphere (the first construction) was copied. In answer to the third question, Euclid's original solid (which is meant to be inscribed in the given sphere) is copied.

saying any arbitrary arc, but any given part from the beginning of Aries to 29° of Virgo ascending in the northern horizons,[31] and in reverse, [the arc] of the equator that rises with it is smaller than [the arc of the equator] that rises with it in the autumnal horizon.[32]

[33] The example is that at a place whose latitude is 40°[33] the ascensions of the end of the zodiacal sign of Gemini are 68°34', and in *sphaera recta* [90°],[34] and the ascensions of 29° of the zodiacal sign of Virgo are 178°, and in *sphaera recta* 179°. The magnitude of the deficit here equals the magnitude of the excess for the parts that [extend] from the beginning of the zodiacal sign of Libra to the end of the zodiacal sign of Pisces.

[34] We say this, and I am not forgetting that the sum of the ascensions for every two arcs whose distance is equal from one known solstice point in *sphaera obliqua* equals the sum of the ascensions for these arcs in *sphaera recta*.[35] I am also not forgetting that half of the ecliptic circle rises with half of the circle of the equator. But why are the ascensions in *sphaera obliqua* for each arc of the northern half of the ecliptic circle less than the ascensions in *sphaera recta*?

[35] This is because the distance of the circle of the equator to the zenith[36] in the northern horizons is [as much] larger than the distance of the northern parts [of the ecliptic to the zenith], as [the parts of the equator] are closer to the zenith than the southern parts [of

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[31] I.e., the ecliptic is north of the celestial equator.
[32] I.e., the ecliptic is south of the celestial equator.
[33] The data given here for the oblique ascensions of 30° of Gemini and 29° of Virgo could help identify the astronomical tables the author was using, and thus help reconstruct the transmission history of astronomical knowledge in Europe. However, within the framework of this dissertation it was not possible to determine where these data originate.
[34]The right ascension of 30° of Gemini is per definition 90°, that is why the values found in the different manuscripts (60° or 80°) were replaced with the correct value.
[35]Ptolemy, *Almagest*, II.7, (H 119): *[...] if two arcs of the ecliptic are equal and are equidistant from the same solstice, the sum of the two arcs of the equator which rise with them is equal to the sum of the rising-times [of the same two arcs of the ecliptic] at sphaera recta.* (Translation by Toomer, *Ptolemy's Almagest*, 91–92.)
[36]Cf. Almagest ii.1 (*Ptolemy's Almagest*, 75.)

the ecliptic]. From this it necessarily follows that if we know the magnitude of the excess and the deficit, we know the ascensional differences in *sphaera obliqua*.

[36] We will show with this diagram that this difference is known.

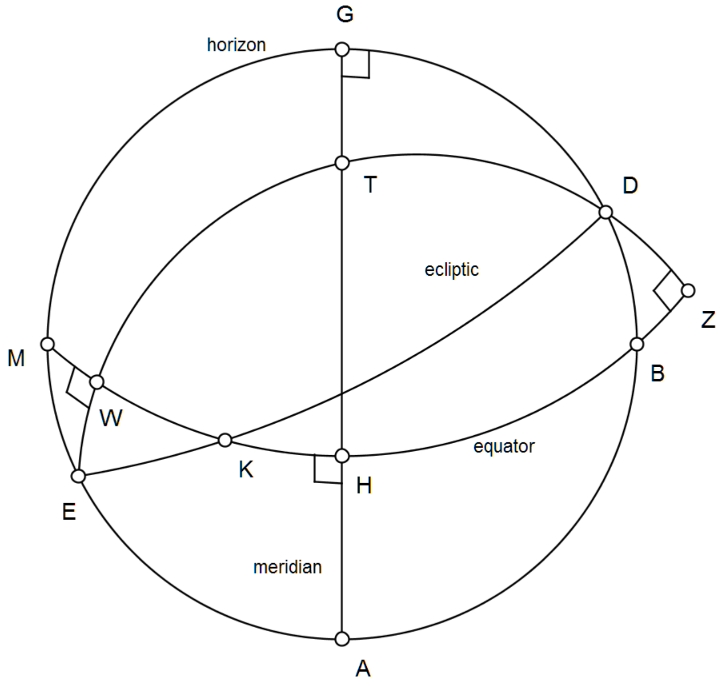

Figure 5: Modern reconstruction of the philosopher's diagram.

Let *ABGD* be the circle of the horizon, and *AHG* the meridian, and *BHM* the equator on pole *T*, and *EKD* the ecliptic. Let point [*E*] be the part of the ecliptic that is on the horizon, and let it be the southern part. We want to know how much can be added to the ascensions of this part *in sphaera recta,* until the ascensions are at this place.

[37] Therefore we draw from the pole to this part [of the ecliptic] arc *TWE*. Thus the two triangles *MWE*, [*MHA*] will be [similar],[37] since angle *M* is common and the two angles *W*, *H* are right angles. There remains that angle *E* is equal to angle *A*.[38]

[38] Therefore the ratio of Sine[39] *MW* to Sine [*WE*] is like the ratio of Sine *MH* to Sine *HA*.[40] But *MH* is known, since it is a quarter circle. And [*WE*] is known, since it is the declination of point *E* of the ecliptic to the equator. [*HA*] is known, since it is the complement of the latitude of the town. And *MW* is unknown.[41]

[39] Therefore these four quantities relate to each other [the following way]: the ratio of the first (unknown) to the second (known) is like the ratio of the third (known) to the fourth (known). We multiply the sine of the declination by the *sinus totus*[42] and we divide it by the cosine of the latitude. We get as a result that the difference *WM* is known.[43] We add it to the known arc [*WK*], which is the ascension of point *E* on the equator. The result is the ascensions of point *E* in the town.

[40] But if the part [of the ecliptic] is northern, [let] point *D* [be] on the horizon. We draw arc [*TD*] from the pole to the equator. It meets it at *Z*. And the two triangles *DBZ*, *DTG* will be similar, since their two angles *D*, which are vertically opposed, are equal, and the two angles [*G*], *Z* are right angles. There remains that the two angles [*B*], *T* are equal.

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[37]As Judah cites this part of the text verbatim in his response below (paragraph [44]), it is clear that the word 'similar' was left out due to a scribal error. The comparison of both texts also made it possible to correct the scribal errors regarding the names of points which are erroneous in all manuscripts.

[38]The philosopher tries to find the ascensional difference (arc *MW*) by use of prop. III.2 of Menelaus' *Sphaerics*. This theorem would only apply if angles E and A were equal. He tries to prove this by claiming that the two spherical triangles *MWE* and *MHA* are similar.

[39] The sine function in medieval Arabic and Hebrew writings differs from the modern function in that it is usually not calculated from a unit circle (with the radius R=1), but with a base circle with the radius R=60. On the relationship between the two, see Kennedy, "A Survey of Islamic Astronomical Tables," 139.

[40] In a modern notation: Sin *MW* / Sin *WE* = Sin *MH* / Sin *HA*.

[41] $MH = 90°$ $WE = \delta$ $HA = 90° - \varphi$ $MW = d$.

[42] That is, the Sine of 90°.

[43] $\text{Sin } d = (\text{Sin } \delta \times \text{Sin } 90°) / \text{Cos } \varphi$.

[41] Therefore the ratio of Sine *DB* to Sine *ZB* is like the ratio of Sine *DT* to Sine *TG*. But *DB* is known, since it is the rising amplitude. *ZB* is unknown, since it is the difference between the right ascensions and the ascensions in the town. *TD* is known, since it is the complement of the declination. *TG* is the latitude of the town, which is known.[44]

[42] We multiply the sine of the rising amplitude by the sine of the latitude of the town and divide it by the cosine of the declination. We get as a result that the sine of the difference is known.[45] Thus its arc, which is *BZ*, is known. We subtract it from the known arc *ZK*, which is the ascension of point [*D*] on the equator. There remains that [*BK*] is known, which is the ascension of point *D* in the town, and that is what we wanted.

Thus far the words of that philosopher.

[43] I then supplied him with several answers in relation to this text and to the diagram he had made, constructed on the other page.[46] I answered him regarding this matter:

On error and on errors[47] that are not appropriate for a student who is called ‘philosopher’ like you:

[44] Firstly, you said: “The two triangles *MWE*, *MHA* will be similar, as angle [*M*] is common and the two angles [*W*], *H* are right angles. There remains that angle *E* is equal to angle *A*.” - May the philosopher please say when it was shown that, if two angles of a triangle which consists of arcs are equal to two angles of another triangle, that the remaining

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[44] Sin *DB* / Sin *ZB* = Sin *DT* / Sin *TG*. $DB = \psi$ $ZB = d$ $TD = 90° - \delta$ $TG = \varphi$.

[45] $\text{Sin } d = (\text{Sin } \psi \times \text{Sin } \varphi) / \text{Cos } \delta$.

[46] Unfortunately, the philosopher’s diagram was not transmitted in the *Midrash ha-Ḥokhmah.*

[47] The phrase על חטא ועל חטאים is a play on the prayer *ʿAl Ḥeṭ* (‘For the Sin’) that is said on the Day of Atonement. Each verse of the confession opens with the words על חטא – ‘For the sin’. Then follows the plea ‘For all these, God of pardon, pardon us, forgive us, atone for us’. The verses that follow each start with the words ועל חטאים – ‘And for the sins’. Apparently Judah considers his opponent’s mistake to be a great sin for which he should ask for forgiveness.

angle of the one is equal to the remaining angle of the other - as this has been shown only for triangles that consist of straight lines.

[45] Furthermore, you claimed that the one triangle is larger than the other, as is the case in the diagram. That is useless and impossible according to what you claimed about the equality of the angles of the other triangle: Since if the angles of the one triangle are equal to the angles of the other, each one to its counterpart, then the two triangles will necessarily be equal, as is shown in proposition 17 of the [first] book of Menelaus.[48]

[46] Therefore, if we put the conditional syllogism this way:

'If the angles of triangle *MWE* are equal to the angles of triangle *MHA*, each one to its counterpart, then triangle *MWE* will necessarily be equal to triangle *MHA*.'

And, we deny the consequent, namely:

'Triangle *MWE* is not equal to triangle *MHA*.'

We get as result the denial of the antecedent, namely:

'The angles of triangle *MWE* are not equal to the angles of triangle *MHA*.'

[47] Therefore angle *E* is not equal to angle *A.*[49] Therefore the ratio of Sine *MW* to Sine *WE* is not like the ratio of Sine *MH* to Sine *HA* - since it would [only] be so if angle *E* were equal to angle *A*, on the condition that the sum of the two arcs *MA*, *ME* together were less than half a circle, as is shown in the third book of Menelaus.[50] Therefore it is obvious that this

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[48]That is, Menelaus, *Sphaerics*, prop. I.17. A German translation of the text was rendered by Max Krause, *Die Sphärik von Menelaos aus Alexandrien in der Verbesserung von Abū Naṣr Manṣūr ibn ʿAlī ibn ʿIrāq*, prop. I.17.

[49]The remaining two angles of the triangles are equal respectively. But as the triangles are not equal, angle E cannot be equal to angle A.

[50]*Sphaerics*, prop. III.2. An English translation of this proposition is rendered, for example, by Kline, *Mathematical Thought from Ancient to Modern Times*, 1:121: "The second theorem of Book III, in the notion that arc a lies opposite angle A in triangle ABC, states that if ABC and A'B'C' are two spherical triangles and if A=A' and C=C', or C is supplementary to C', then sin c / sin a = sin c' / sin a'."

proof is only possible for triangles of straight lines, not of arcs. But if in the philosopher's eyes logic dictates that both are equal, he has to help us with this novelty that has come about under the sun.[51]

Thus far.

[48] **The author said**: When the things were reported before the Emperor, the King, Emperor Frederick (may his glory be exalted), he was very pleased with the answers that I had given to the one who is called 'philosopher' before him. In addition, there was much debate between us about many things, and about many questions and answers, but we cannot elaborate any further on these matters in this book.

[49] Furthermore, about ten years later it so happened that I went down to the emperor's lands; I saw the quality of his actions and his affairs, his philosophers, his scribes, his wise men, his judges, his officers, the food of his table and the seating of his servants.[52] Everything depends on the stars.[53] I pray to God, may He be praised and exalted every day, that He may return me to my father's house and the country of my birth in peace and happiness. May thus be His will, Amen. May God say so.

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[51]This plea for help is, of course, ironical. Judah plays with the verse Eccl. 1:9: *What has been will be again, what has been done will be done again; there is nothing new under the sun.*
[52] Cf. 1 Kings 10:15.
[53] The Hebrew term Judah uses here is מזל, which originally denotes 'constellation'. In this context, it applies to divine influence that is exerted upon humankind via the spheres of the stars. Judah repeats the phrase 'הכל תלוי במזל' in the introduction to his Hebrew version of the Tetrabiblos (אם הכל תלוי במזל מה תועיל תפלתו של אדם. *Otot ha-shamayim*, 2a.) The idiomatic expression was used by many medieval authors, such as Abraham ibn Ezra (*Commentary on Job* 28:1) and the author of the Zohar (Numbers 134a).

Chapter Three

Mathematics in an intercultural exchange

Frederick II (1194-1250) was one of the most famous and controversial emperors of the Middle Ages. He was highly interested in the arts, philosophy and the natural sciences, engaged in numerous building projects, founded the University of Naples, commissioned translations of scientific texts from Arabic into Latin, invited scientists to his court and engaged in discussions with them. His unquenchable thirst for knowledge is legendary - already in his own time he was known as *stupor mundi*, amazement of the world. Particularly striking is the way he had lists of philosophic and scientific questions sent to renowned scientists all over the world.[1] Thus, when Judah ha-Cohen relates that it was this famous emperor who sent him mathematical questions when he was only eighteen years old, he forms part of a series of famous scholars who claim to have engaged in discussions with Frederick. While three of these scholars, Michael Scot, Jacob Anatoli, and Leonardo Pisano, resided in Italy and claim to have met the emperor in person, five scholars are reported to have been given letters containing the emperor's questions: the Sufi Ibn Sabʿīn, the jurist Shihāb al-Dīn al-Qarāfī, the polymath Kamāl al-Dīn ibn Yūnus, and the mathematicians and astronomers ʿAlam al-Dīn Qayṣar and Athīr al-Dīn al-Abharī. The authors of these accounts come from many different cultural settings, and not all of them had historical accuracy as a goal when rendering their reports. Thus, when dealing with this material, one has to keep in mind what the authors' intentions were, how well informed their reports are, and if these reports are historically verified. In this chapter,

[1] On the scientific culture at the court of Frederick II, see Abulafia, *Frederick II*, 251–289; Stürner, *Friedrich II*, 2. Der Kaiser 1220–50:342–457; Haskins, *Studies in the History of Mediaeval Science*, 242–271; Sirat, "Les traducteurs juifs à la cour des Rois de Sicile et de Naples."

we will examine each single report of questions being asked by the emperor and try to analyse its historicity – separately, and in the context of other accounts and historical circumstances. As a second step, we will do the same with Judah ha-Cohen's correspondence and try to place it within the different extant accounts of questions.

Questions in Frederick's immediate environment

Michael Scot

The first account of questions being asked by Frederick personally comes from the emperor's famous court astrologer Michael Scot (d. 1235). Little is known about Michael's origins. He was probably born in Scotland and received a thorough education before he travelled south to Spain and Italy.[2] His name is first mentioned in 1215, when he accompanied Rodrigo, the archbishop of Toledo, to the Fourth Lateran Council in Rome.[3] While staying in Toledo he rendered numerous Latin translations of Arabic works, such as al-Biṭrūjī's *On the Principles of Astronomy*,[4] Aristotle's *De animalibus*, and Ibn Rushd's Long Commentary on Aristotle's *De caelo et mundo*.[5] In 1220 at the latest, Michael left Toledo for Italy, where he joined the court of Frederick II.[6] It seems to have been the practice of astrology that gained him the emperor's trust. Michael reports that he used his astrological skills to predict the right times for attacks on rebelling cities, but he also

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[2] On Scot, see Haskins, *Studies in the History of Mediaeval Science*, 272–298; Stürner, *Friedrich II*, 2. Der Kaiser 1220–50:400–422; Burnett, "Michael Scot and the Transmission of Scientific Culture from Toledo to Bologna via the Court of Frederick II Hohenstaufen"; Ackermann, "*Habent sua fata libelli* - Michael Scot and the Transmission of Knowledge Between the Courts of Europe"; Grebner, "Der 'Liber Nemroth', die Fragen Friedrichs II. an Michael Scotus und die Redaktionen des 'Liber particularis.'"

[3] Burnett, op. cit., 102. The council lasted from 1215 to 1217.

[4] In the *Midrash ha-Ḥokhmah*, Judah ha-Cohen relates that al-Biṭrūjī lived 30 years before the year 1247. This is, in fact, the year in which the Latin translation, not the original work, was rendered. He may therefore have known Michael's Latin translation.

[5] On his role as a translator, see Burnett, op. cit.

[6] It is not clear exactly when he joined Frederick's service. The fact that he received honourable mentions in several papal letters, and abruptly ceases to be mentioned in papal documents after 1227, the year in which Frederick was excommunicated, implies that at that point at the latest Michael Scot was associated with the emperor.

advised the emperor on more personal matters, such as the right times to have a blood-letting. In Frederick's employment Michael also continued his translation work from the Arabic, and composed original writings, such as tractates on alchemy, and a commentary on John de Sacrobosco's astronomical work *De spera*. Apparently at the emperor's bidding, he also completed his major work *Liber introductorius*, an encyclopaedic work in three parts, which was designed to introduce novices to the art of astrology. In the second part of this work, the *liber particularis*, he gives a list of 55 queries[7] that were posed to him by the emperor:

> *When Frederick, emperor of Rome and always enlarger of the empire, had long meditated according to the order which he had established concerning the various things which are and appear to be on the earth, above, within and beneath it, on a certain occasion he privately summoned me, Michael Scot, faithful to him among all astrologers, and secretly put to me at his pleasure a series of questions concerning the foundations of the earth and the marvels within it, as follows: "My dearest master, we have often and in diverse ways listened to questions and solutions from one and another concerning the heavenly bodies, that is the sun, moon, and fixed stars, the elements, the soul of the world, peoples pagan and Christian, and other creatures above and on the earth, such as plants and metals; yet we have heard nothing respecting those secrets which pertain to the delight of the spirit and the wisdom thereof, such as paradise, purgatory, hell, and the foundations and marvels of the earth. Wherefore we pray you, by your love of knowledge and the reverence you bear our crown, explain to us the foundations of the earth [...]"*[8]

The questions that follow concern seven broader topics: the positioning of the earth, the heavens, god and angels, hell and hereafter, relations in size, sweet and salt water, and phenomena concerning wind, fire and smoke. Michael answers many of the questions, not without praising the emperor for his great learning.

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[7] There are, in fact, two versions of the work: first, a short version was composed, which was later redacted into a long one. The short version contains 49, the long one 55 questions. See Grebner, "Der 'Liber Nemroth', die Fragen Friedrichs II. an Michael Scotus und die Redaktionen des 'Liber particularis.'"

[8] Translation by Haskins, "Science at the Court of the Emperor Frederick II," 689. Haskins published the Latin original in *Studies in the History of Mediaeval Science*, 292.

At first sight Michael's account appears to be very reliable; the *Liber introductorius* was composed for Frederick, during his lifetime, and may therefore have been read by the emperor himself. Michael Scot would not have fabricated a story of questions being asked by his sovereign in the knowledge that this same sovereign would read his work and thus expose the account as a lie. However, there is no evidence to suggest that Frederick actually read the work, or intended to read it. Michael may have included the set of questions into his book in order to demonstrate his own importance, being the person the emperor would turn to with all questions on difficult scientific and philosophical matters. The nature of the questions on the other hand may also demonstrate the learnedness of the emperor himself, who personally engaged in the study of philosophy and metaphysics.

Furthermore, it turns out that not all of the questions listed in the work were original to the emperor. Nineteen of them were taken over, most of them verbatim, from an illustrated astrological-astronomical compendium by the name *Liber Nemroth.*[9] Author and origin of the work are unknown, but it can be traced back to the 8th century. The questions reproduced in the *liber particularis* can all be found in a chapter entitled *de fundamento terre* ('on the foundation of the earth') of the *Liber Nemroth.* Whereas in the original it is the student Ioanton who is asking his teacher Nemroth these questions, in Michael's version the enquirer is the emperor - which would make Michael Scot the learned teacher.

But this does not necessarily mean that Michael's account is wrong; as Frederick himself was interested in arts and sciences, the emperor may have come upon these questions

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[9] Grebner, "Der 'Liber Nemroth', die Fragen Friedrichs II. an Michael Scotus und die Redaktionen des 'Liber particularis.'"

during his studies and copied them from the source. Perhaps he posed them to his court astrologer in order to compare his answers to those given in the *Liber Nemroth*. The questions could thus have functioned as a test of Michael's skills as a philosopher. Frederick may also have instructed another scholar at his court to find suitable questions to test his court philosopher, instead of reading the *Liber Nemroth* himself. The presence of other learned men besides Michael at the imperial court does in fact support the theory that Michael's account is genuine. While the emperor himself may not have had the time to read the work dedicated to him, there was a chance that someone in his environment would, and would thus spot a possible lie about questions being asked by the emperor. The risk of detection thus appears to be too high for Michael to include a blatant lie in his work.

Thus, despite the fact that some questions Michael reports of are not original, his account may still be truthful. This opens the possibility that the same might be the case with other questions allegedly posed by the emperor.

Jacob Anatoli

Not a list of questions being posed, but personal discussions with the emperor are reported by the Jewish scholar Jacob ben Abba Mari Anatoli (c.1194-c.1256).[10] The son-in-law of the famous translator Samuel ibn Tibbon was originally from Provence, but he left his home and established himself at the court of Emperor Frederick II in Naples in 1230/1. There, he translated several scientific works from Arabic into Hebrew. In his original composition, the collection of homilies *Malmad ha-talmidim* ('the goad for students'),[11] he reports of numerous discussions on exegetical questions that he had with *the great*

[10] On Anatoli, see Lévy, "The Establishment of the Mathematical Bookshelf," 440–441.
[11] Published by the Mekize Nirdamim in 1866: Anatoli, *Malmad ha-talmidim*.

Christian scholar named Michael, with whom I was associated for a period,[12] who is none other than Michael Scot. Throughout his work, he recounts twenty-one interpretations of biblical verses that he attributes to the astrologer. On two occasions also the emperor would join their discussions. Thus, Anatoli relates one instance in which the three men discussed Maimonides' *Guide for the Perplexed*, disagreeing on a certain interpretation of a passage in the second book: *Our master Emperor Frederick explained why the sage took the word snow to signify prime matter. […] The sage with whom I was associated held a different view […].* [13]

Anatoli's account appears to be genuine. His translations of scientific works show that he spent several years in Naples, that he had access to Latin sources and that he was in the emperor's employ, although it is not certain what kind of work he performed for Frederick. Furthermore, that Maimonides' work was indeed discussed at the imperial court is also documented by Michael Scot: between the first and the second redaction of his compendium *Liber introductorius*, Michael added a passage to the work in which he explicitly cites Maimonides.[14]

Leonardo Fibonacci

An account of mathematical questions being discussed in Frederick's presence is given by Leonardo Pisano, known as Fibonacci.[15] Born (c. 1170) in the city of Pisa, he had moved

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[12] Sirat, "Les traducteurs juifs à la cour des Rois de Sicile et de Naples," 171.
[13] Ibid., 172.
[14] Grebner, "Der Transfer mathematischen Wissens aus dem Orient und der Hof Friedrichs II. Der Asymptotentraktat und sein personelles wie epistemisches Umfeld," 222; Hasselhoff, *Dicit Rabbi Moyses*, 37–40.
[15] Leonardo's writings were published by Boncompagni, *Scritti di Leonardo Pisano*. The contents are being discussed by Cantor, *Vorlesungen über Geschichte der Mathematik*, 2:3–52. On Leonardo's relation to Arabic mathematics and his contribution to Western mathematics see Rashed, "Fibonacci et les mathématiques arabes"; id., "Fibonacci et le prolongement latin des mathématiques arabes." An English translation of both articles can be found in Rashed, *Classical Mathematics from al-Khwarizmi to Descartes*, 411–444.

to North Africa to be instructed in Arabic-Indian calculation on the behest of his father, who was an officer at the customhouse in Bugia (Béjïa) for the Pisan merchants. He continued his studies on business trips to Egypt, Syria, Greece, Sicily and Provence. As a result, he composed his major work *Liber abaci*, which deals with Arabic and Indian numerals and calculation, in 1202. The revised version of the work, which was completed in 1228, Leonardo dedicated to Michael Scot. A second major work, the *Practica geometrie*, was written in 1220 and dedicated to a certain master Dominick.[16] A few years later, possibly in 1226,[17] he composed the *Liber quadratorum* ('book of Squares'), a book on Diophantine equations which he dedicated to the emperor himself. In the introduction to the work he recounts how he was introduced to the court by master Dominick and got involved in a discussion with the emperor's philosopher John of Palermo:

> *After being brought to Pisa by Master Dominick to the feet of your celestial majesty, most glorious prince, Lord F[rederick], I met Master John of Palermo; he proposed to me a question that had occurred to him, pertaining no less to geometry than to arithmetic: find a square number from which, when five is added or subtracted, always arises a square number. Beyond this question, the solution of which I have already found, I saw, upon reflection, that this solution itself and many others have origin in the squares and the numbers which fall between the squares. When I heard recently from a report from Pisa and another from the Imperial Court that your sublime majesty deigned to read the book I composed on numbers, and that it pleased you to listen to several subtleties touching on geometry and numbers, I recalled the question proposed to me at your court by your philosopher. I took upon myself the subject matter and began to compose in your honor this work which I wish to call The Book of Squares.*[18]

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[16] An English translation with an introduction that focusses on Leonardo's learning was made by Hughes, *Fibonacci's De Practica Geometrie*.

[17] The book is dated 1225 in the introduction, but Frederick had visited Pisa for the first time in 1226. The discrepancy may be due to fact that Leonardo used the Pisan calendar. See Muccillo, "FIBONACCI, Leonardo."

[18] Translation by L.E. Sigler: Fibonacci, *The Book of Squares*, 3. In a modern mathematical notation, the problem posed is: $x^2+5=y_1^2$, $x^2-5=y_2^2$.

As stated in the introduction, Leonardo discusses the question posed by the philosopher, along with similar problems. Appended to the work is also a mathematical question that was posed to him by another court official, Theodore of Antioch:

> *The question proposed to me by Master Theodore, Philosopher to the Emperor I wish to find three numbers which added together with the square of the first number make a square number. Moreover, this square, if added to the square of the second number, yields thence a square number. To this square, if the square of the third number is added, a square number similarly results.*[19]

Another account of the audience in Pisa can be found in the treatise *Flos* ('flower'), which deals with different questions on algebra, arithmetic and geometry. Here, Leonardo reports that in the emperor's presence, John of Palermo had engaged him in a long discussion on various mathematical problems. The answers to some of these, along with other questions that had reached him from the emperor's court, he put down in writing and sent them directly to Frederick. At the end of the work, there is a letter addressed to Theodore of Antioch, discussing three additional mathematical problems.

Leonardo's account, too, appears to be very reliable, as he recounts problems that were raised while the emperor was present, and whose answers he worked out for Frederick to read. In both the *Liber quadratorum* and the *Flos* he addresses the emperor personally, and he even gives us the name of a messenger that was employed to bring some of the answers to the emperor: *In a similar way I shall also solve one of the two questions that I sent to Your Majesty through your footboy Robertinus.*[20] Furthermore, while we have little information on master Dominick who introduced Leonardo to the emperor (and none at

[19] Ibid., 107. In modern notation the problem reads: $x + x^2 + y + z = r^2$; $r^2 + y^2 = s^2$; $s^2 + z^2 = t^2$.

[20] My translation. The Latin text can be found in *Scritti di Leonardo Pisano*, II:236.

all on the footboy Robertinus), it is well-documented that both John of Palermo, who discussed these problems with Leonardo, and Theodore of Antioch, to whom Leonardo addressed two letters, had official positions at the imperial court.

John of Palermo's name is first mentioned in 1221, when he wrote an official document for the emperor.[21] Nineteen years later, in February 1240, John is listed as one of two envoys whom Frederick sent on a mission to Tunis. He may have been of ill health at the time, as his companion was instructed to choose another envoy in case illness prevented John from going. The last official mention of John is made two months later, in April 1240, when Frederick urgently requested John's presence before him at Orta for unspecified services, and put barque at his disposal. John was not only well-versed in mathematics, but also in the Arabic language. This becomes evident from his Latin translation of an anonymous Arabic work on the hyperbola and its asymptote, entitled *De duabus lineis semper approximantibus sibi invicem et numquam concurrentibus* ('On two lines always approaching each other but never meeting').[22]

We have more information regarding Theodore of Antioch,[23] who fulfilled several functions at the imperial court; he was active as an astrologer, a translator of scientific

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[21] On John of Palermo, see Clagett, *Archimedes in the Middle Ages*, 1980, 4:33–34.
[22] The Arabic original of the treatise is lost. The Latin translation was edited by Clagett, *Archimedes in the Middle Ages*, 1980, 4:33–61. See also Freudenthal, "Maimonides' 'Guide of the Perplexed' and the Transmission of the Mathematical Tract 'On Two Asymptotic Lines' in the Arabic, Latin and Hebrew Medieval Traditions.
[23] On Theodore of Antioch, see Burnett, "Master Theodore, Frederick II's Philosopher"; Kedar and Kohlberg, "The Intercultural Career of Theodore of Antioch." See also Burnett, "Antioch as a Link between Arabic and Latin Culture in the Twelfth and Thirteenth Centuries."

works and imperial documents, but also acted as the emperor's physician.[24] He is first mentioned in Latin sources in the autumn of 1238, when he engages in a discussion with Dominican friars during the siege of Brescia. A year later he is said to have cast a horoscope for the Emperor while staying in Parma. In the register of Frederick II's acts from 1239-40, which is the only one extant, Theodore is mentioned several times:[25] In December 1239 Frederick puts a boat in Pisa at his disposal, as he is returning from a journey.[26] Two months later he is sent a blank sheet with the emperor's seal and ordered to draft a letter in Arabic to the king of Tunis, giving the credentials of two messengers being dispatched to the ruler. As stated above, one of these envoys was John of Palermo.[27] Theodore must have died before November 1250, as at that time Frederick gives a vineyard in Messina, which 'Master Theodore our philosopher held so long as he lived', to the son of his stable master.[28]

As a writer, Theodore composed both translations from the Arabic and original works. He translated the Prologue to Ibn Rushd's Great Commentary on Aristotle's *Physics* at the bidding of some scholars in Padua (perhaps in 1239).[29] At the emperor's bequest, he translated an Arabic work on falconry, the *Moamin*, into Latin, which Frederick himself revised in 1240-1.[30] He may also have translated a second book on falconry, the *Ghatrif*.[31]

[24] This is confirmed by Petrus Hispanus (1210/20-1277), who was later to become Pope John XXI, who mentions that one of his masters had been Theodore, 'the Emperor's physician'. Kedar and Kohlberg, op. cit., 168.
[25] Ibid., 169.
[26] Ibid.
[27] The instructions, with an English translation, can be found in Burnett, "Master Theodore, Frederick II's Philosopher," 257.
[28] Burnett, op. cit., 226.
[29] Kedar and Kohlberg, op. cit., 168.
[30] On this translation, see, for example Akasoy, "Zu den arabischen Vorlagen des Moamin."
[31] Abeele, *La fauconnerie au Moyen Âge*, 29.

Being also a trained physician, Theodore also dedicated a letter on regimen (*Epistola Theodori philosophi ad imperatorem Fridericum*) to Frederick.[32]

Leonardo Fibonacci thus discussed mathematical problems with Frederick and two of his scholars who were both trusted, proficient in the mathematical sciences and also in the Arabic language. This last point becomes important if we take a look at the questions that Leonardo was asked to answer; the question mentioned at the beginning of the *Liber quadratorum* reads in a modern mathematical notation: $x^2+5=y^2$, $x^2-5=z^2$. It turns out that this problem appears verbatim in several much earlier Arabic treatises on Diophantine equations. In his *Kitāb al-badīʿ fī al-ḥisāb* (*Magnificent book on calculation*), for example, the mathematician and engineer Abū Bakr al-Karajī (c. 953–c. 1029) formulates the problem this way: *If one says a square and if one adds to it five units, one has a square, and if one subtracts five units, one has a square.*[33] A second question asked by John of Palermo is transmitted in the booklet *Flos.* Here, Leonardo is asked to solve an equation, which in a modern notation reads: $x^3 + 2x^2 + 10x = 20$.[34] Again, the very same equation appears in a much earlier Arabic work, the *Treatise of Algebra* by al-Khayyām (1048-1131).[35] None of the Arabic sources that the questions originate from have ever been translated into Latin, and as Leonardo's methods for solving the problems differ considerably from those of his predecessors, he himself does not seem to have had a direct access to any of these sources. It appears that it was the emperor, or rather his philosophers, who had found the questions in Arabic works and posed them to the Pisan mathematician. How they could have come in contact with these writings we will discuss at a later stage. What is important at this point is the

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[32] The letter, with an English translation, was published by Burnett, "Master Theodore, Frederick II's Philosopher," 267–274.
[33] Rashed, *Classical Mathematics from al-Khwarizmi to Descartes*, 420.
[34] The Latin text can be found in *Scritti di Leonardo Pisano*, II: 228.
[35] Rashed, op. cit., 427.

fact that like in the case of Michael Scot, at least some of the questions posed were taken verbatim from earlier works. As to the reasons why the court philosophers may ask questions that had already been treated by previous mathematicians, different scenarios are conceivable. They may have encountered the questions in their studies without having access to the solutions. The questions may therefore have been asked out of genuine curiosity. On the other hand, Theodore of Antioch and John of Palermo may already have known the answers to the given problems. The questions would have served to test Fibonacci's mathematical skills. In any case, Fibonacci himself appears to have been unaware of the fact that the problems had already been solved by previous mathematicians.

In conclusion, the three accounts about Frederick engaging in scientific debate by Michael Scot, Jacob Anatoli and Leonardo Fibonacci give us valuable information on Frederick and his method of formulating questions: Philosophical and scientific works by Christian, Jewish and Muslim authors were studied at the imperial court. Frederick was personally interested in theological, philosophical and mathematical problems, and proficient enough in the subjects he studied to discuss these problems with experts, or at least to follow their discussions. Furthermore, at least some of the questions being posed to scholars by the emperor, or his philosophers, were not original but stemmed from earlier works on the topic; in the case of Michael Scot from a Latin source, in the case of Leonardo Fibonacci from Arabic sources. The emperor's interest may thus have been to advance scholarship by finding new solutions to known problems. But the questions may also have served as a test of the recipients' skills and learnedness.

Questions sent to Muslim scholars

The reports discussed above were rendered by people in Frederick's immediate environment. They appear to be reliable, as these scholars' involvement with the imperial court is testified by many different sources, and the risk of detection in case of a lie, which may have led to them losing the emperor's favour, would have been high. But scholars who did not belong to Frederick's entourage and who lived in distant places did not have to fear immediate consequences in case they were proven to have fabricated stories about answering the emperor's questions. In addition, most of the accounts of Frederick's queries reaching places outside of Italy were not written by the recipients themselves, the only exception being Judah ha-Cohen's account of the correspondence and the *Sicilian Questions* by Ibn Sabʿīn. Yet, the historicity of Ibn Sabʿīn's account has recently been called into question. In the following, we will therefore list all extant reports of the emperor's questions reaching scholars outside of Italy, which (except for Judah ha-Cohen) all happen to be Muslim scholars, and try to judge in how far they can be considered to be historical.

Ibn Sabʿīn

As mentioned above, the most famous set of questions that were reportedly sent out by Frederick II are the *Sicilian Questions* (*al-Masāʾil al-Ṣiqilliyya*) that were answered by the philosopher and Sufi ʿAbd al-Ḥaqq ibn Sabʿīn of Murcia (c.1217-c.1270).[36] Born near Murcia in the Ricote valley, Ibn Sabʿīn left his hometown during a period of political turmoil in 1238, settling first in Granada, then in the African port town Ceuta. There he wrote his two philosophical works: the *Sicilian Questions* (written between 1238 and 1242)

[36] On Ibn Sabʿīn see Akasoy, *Philosophie und Mystik in der späten Almohadenzeit*, 3–35. The book also contains an edition and German translation of the *Sicilian Questions*. See also Akasoy, "Ibn Sab'īn's *Sicilian Questions*."

and the *Budd al-ʿārif* ('Knowledge of the Gnostic'). The *Sicilian Questions* survive in only one manuscript, kept in the Bodleian Library.[37] In the prologue to the text, the author (or perhaps one of his students) gives an account of what prompted Ibn Sabʿīn to compose the work:

> *The Shaykh [...] Quṭb al-Dīn [...] ibn Sabʿīn [...] replied to the questions of the King of the Romans, the Emperor, the ruler of Sicily. He had (initially) sent a copy of the questions to the East, to Egypt, to Syria, to Iraq, to Asia Minor and to Yemen. But when the responses of the Muslim sages had come back, they had not satisfied him. Then he asked if there were appropriate scholars in Tunisia. He was told that this was not the case. He inquired about Morocco and al-Andalus and was told that there was a man known as Ibn Sabʿīn. So he turned to the Caliph al-Rashīd from the dynasty of ʿAbd al-Muʾmin with his questions. The Ruler of the Faithful wrote to his governor in Ceuta, namely Ibn Khalāṣ, to see to the mentioned man so that he would reply to the questions. The King of the Romans had sent a ship with his envoy and a lot of riches. Ibn Khalāṣ had the Imam Quṭb al-Dīn come and gave him the questions at the behest of the caliph. Quṭb al-Dīn laughed [...] and promised to answer the questions. Ibn Khalāṣ gave him the precious gifts that the envoy of the king of the Romans had brought. [But] Quṭb al-Dīn rejected them and did not accept them, saying: '[I answer the questions] in the hope of God's reward in the hereafter and for the triumph of the Muslim community' [...] Quṭb al-Dīn answered [the questions], and when the answer reached the king, it satisfied him. He sent a great gift, which, however, was sent back to him like the first one. Thereupon the Christian recognized his own inadequacy. God lets Islam triumph, granting it victory over the Christian community due to ultimate demonstrative proofs. Thanks be to God, the Lord of all the world.*[38]

The four questions that Frederick allegedly asked are only transmitted in Ibn Sabʿīn's answers. The first one relates to Aristotle's belief in the eternity of the world, the second one regards the nature of the *Divine Science,* the third one relates to the nature and number of categories, and in the fourth and final question Ibn Sabʿīn is asked to give proofs for the immortality of the soul. Ibn Sabʿīn answers these queries, beginning with a harsh critique of the wording and methodology of the emperor's questions.

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[37] Oxford, Bodleian Library, MS. Hunt. 534.

[38] Akasoy, *Philosophie und Mystik in der späten Almohadenzeit*, 340 (Arabic), 411–412 (German translation).

Anna Akasoy was the first scholar to doubt the historicity of this account:[39] At the time that Ibn Sabʿīn allegedly answered the questions, around 1240, he was only in his early twenties and certainly not such a renowned scholar that his fame had reached the Emperor's court. It is hardly conceivable that Frederick could not find a single Muslim philosopher besides Ibn Sabʿīn to answer the questions to his satisfaction. In addition, it is unlikely that the Almohad caliph would have asked Ibn Sabʿīn of all people to answer the questions, as his family had links with Ibn Hūd, the man who had started a revolt against the Almohad court. Furthermore, it was the governor of Ceuta, Ibn Khalāṣ, who in 1242 banished the Sufi from the city. But also the text of the answers itself raises doubts about the historicity of Ibn Sabʿīn's account. Firstly, it is difficult to imagine that Ibn Sabʿīn would have sent a ruler, albeit a foreign one, such a harsh critique of his wording and methodology. More importantly, more than anything else the *Sicilian Questions* appear to be a handbook on philosophy written for students: rather than dealing with specific queries that might have interested a foreign ruler, the treatise discusses very general introductory questions about philosophy in the form of questions and answers – a popular genre in Arabic literature.

As to the reasons Ibn Sabʿīn may have forged a connection with the imperial court, Akasoy argues that living in a port town that attracted traders from all over the Mediterranean, Ibn Sabʿīn may have been well aware of the fact that Frederick was interested in Aristotelian philosophy. By claiming that the emperor consulted Ibn Sabʿīn on these matters, the Sufi may have tried to increase his own authority. The connection with Frederick may also have made the work more attractive to potential readers. In

[39] Ibid., 107–124; ead., "Ibn Sabʿīn's *Sicilian Questions*," 121–123.

addition, the claim may also have served as a justification to deal with Aristotelian ideas in a climate hostile to philosophy. It is therefore possible, if not likely, that Ibn Sabʿīn's claim of having composed the *Sicilian Questions* on behalf of the emperor is fictitious. But if this is the case, we have to analyse also the other accounts of Frederick's questions being sent to Muslim scholars regarding the points in question: Do the author's texts and external circumstances indicate that their accounts are reliable?

Shihāb al-Dīn al-Qarāfī

Another North-African scholar who reports of natural-philosophical questions being posed by the emperor is the Egyptian jurist and legal theoretician Shihāb al-Dīn al-Qarāfī (1228-1285).[40] Al-Qarāfī is best known for his legal writings, which spread throughout the Muslim world, but he also composed a work on natural philosophy, or more precisely, on optical phenomena, with the title *Kitāb al-istibṣār fī-mā tudrikuhu al-abṣār* ('Reflection on what is perceived by sight'). It is in this work that he introduces several optical questions that were supposedly asked by Frederick. In the introduction to the work he writes:

> *Thus [God] keeps the Muslim people from the error of inferiority, when they are asked strange questions, so that the enemies of pure faith do not imagine, due to a lack of response, that they take the first rank regarding perfection and ability [...] Once, at the time of al-Malik al-Kāmil, the emperor, king of the Franks in Sicily wrote seven very difficult, extremely tricky questions to test the Muslims; he was a clever man of great knowledge, acumen and understanding. I heard they answered some of them; if all, I do not know. That they could answer the questions and confirm the accuracy of the answers is because back then there were many people who had good understanding and scholars of the [Muslim] nation who were in agreement.*[41]

Al-Qarāfī discusses three of the emperor's questions in his treatise. The first one concerns straight objects that are partly immersed in water: Why do they appear to be bent?[42] The

[40] On al-Qarāfī, see Jackson, "S͟hihāb Al-Dīn Al-Ḳarāfī."

[41] Wiedemann and Meyerhof, "Über ein optisches Werk des Aḥmad al-Qarāfī," 123 (Arabic), 8 (German).

[42] Ibid., 28–29. The Arabic text is on p. 107.

second one concerns a star, Canopus, which at its rising appears to be bigger than at its culmination; as it rises in the south, which is dry, the reason for its bigger appearance cannot be moisture, with which the same phenomenon is explained regarding the sun.[43] The third question regards the eyes themselves: why does someone suffering from cataracts see black filaments and flashes of light and gnats outside his eye, even when there is nothing there?[44]

The question whether al-Qarāfī renders a true account of three questions that were asked by the emperor is difficult to answer. The circumstances reported in the introduction seem to be much more plausible than those reported by Ibn Sabʿīn; the emperor did indeed have a good relationship with the Ayyūbid Sultan al-Malik al-Kāmil, who reigned between 1218 and 1238. This is attested, for example, by the historian Ibn Wāṣil (1207-1298), who in 1261 was sent to the court of Frederick's son Manfred as an ambassador:[45]

> *The Emperor was a sincere and affectionate friend of al-Malik al-Kāmil, and they kept up a correspondence until al-Kāmil died—God have mercy on him! — and his son al-Malik al- ʿAdil Saif ad-Din Abu Bakr succeeded him. With him too the Emperor was on sincerely affectionate terms and maintained a correspondence. When al- ʿAdil died in his turn and his brother al-Malik aṣ-Ṣāliḥ Najm ad-Din Ayyūb succeeded him, relations were unchanged: al-Malik aṣ-Ṣāliḥ sent to the Emperor the learned shaikh Sirāj ad-Din Urmawī, now qadi of Asia Minor, and he spent some time as the Emperor's honoured guest and wrote a book on logic for him. The Emperor loaded him with honours. After this, still in high favour, he returned to al-Malik aṣ-Ṣāliḥ.*[46]

But al-Qarāfī's introduction gives us little verifiable information apart from the fact that Frederick corresponded with al-Malik al-Kāmil. The author states himself that his information on the questions is not first hand – he was not the recipient of the questions,

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[43] Ibid., 37. The Arabic text is on p. 99.
[44] Ibid., 43. The Arabic text is on p. 93.
[45] English translation by Gabrieli, *Arab Historians of the Crusades*, 277. On his time at the imperial court he reports: *It is the author, Jamal ad-Din ibn Wasil, who speaks: I saw these parts when I was sent as ambassador of the Sultan al-Malik az-Zahir Rukn ad-Din Baibars, of blessed memory, to the Emperor's son, Manfred by name.* (Ibid., 268.)
[46] Ibid., 276. Al-Malik aṣ-Ṣāliḥ ruled between 1240 and 1249.

and he is unsure whether all of them were answered. In addition, the aim of the introduction is clearly not to give a historical account. Rather, al-Qarāfī's motives for mentioning the foreign emperor appear to be both apologetic and polemical: One is not only allowed to, but one has the duty to engage in the study of secular sciences in order to be able to defend the honour of the Muslim nation against foreigners. What foreigner could be more suitable to demonstrate the importance of the questions discussed in al-Qarāfī's work than the famous Roman emperor?

The questions reported by al-Qarāfī could certainly have been asked by the emperor; Frederick was interested in natural philosophy, and he laid special emphasis on precise observation in his own scientific work, the book on falconry *De arte venandi cum avibus*. He may thus have observed the phenomena described in the questions and wondered how they came about. But in this regard, the second of the three questions turns out to be problematic; the star Canopus, second-brightest star in the night sky, cannot be seen at all from any place north of Catania, and even in Southern Sicily it would not have risen on the horizon by more than 1°.[47] Thus, a Sicilian observer would certainly not have noticed a difference in the star's magnitude between its rising and its culmination, for even when passing the meridian the star would hardly have risen at all. Furthermore, if it were to be sighted from Southern Sicily, it would appear to rise in the Mediterranean Sea, not the dry desert land that is referred to in the question. Both facts suggest that the question did not originate at the imperial court, but that it was authored by someone living further to the south than the emperor. But as we have seen in the case of Michael

[47] This was first pointed out by Schramm, "Frederick II of Hohenstaufen and Arabic Science," 305. However, Schramm's translation of the question itself appears to be inaccurate.

Scot and Leonardo Fibonacci, by the fact alone that the query is not original, we cannot exclude the possibility that stemmed from the imperial court.

All in all, al-Qarāfī's claim that three questions listed in his work were asked by the emperor, can neither be verified nor disproved. His work, written in the style of question and answer with an apologetic introduction, reminds us a lot of Ibn Sabʿīn's *Sicilian Questions*. The phenomena discussed could certainly have been studied by the emperor, but al-Qarāfī simply does not give us enough verifiable information on the historical circumstances to decide either way.

Kamāl al-Dīn ibn Yūnus

While the two reports discussed above concern natural-philosophical questions, all other accounts of queries being despatched into the Arab world by Frederick concern questions that were answered by skilled mathematicians, who were all connected to the school of a famous scholar in Mosul, Kamāl al-Dīn ibn Yūnus (1156-1242). The polymath Ibn Yūnus was one of the most respected and sought-after scholars of thirteenth-century Islam. Based in Mosul, he was a famous theologian, but was also an expert in the mathematical sciences and astronomy, Greek philosophy, and medicine.[48] Of his works, only the mathematical ones have come down to us. The famous author of biographies Ibn Khallikān (1211–1282), whose father had been a personal friend of Ibn Yūnus, reports that he was knowledgeable in all branches of science and philosophy, and that his theological knowledge included not only Muslim, but also Christian and Jewish scriptures, so that Jews and Christians regularly came to listen to his biblical exegesis.[49] The polymath

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[48] On Ibn Yūnus, see, for example, Hasse, "Mosul and Frederick II Hohenstaufen: Notes on Atiraddin al-Abhari and Siragaddin al-Urmawi."
[49] *Ibn Khallikan's Biographical Dictionary*, 3:468.

reportedly received letters on mathematical and astronomical problems from Baghdad and Damascus. He answered them promptly, surpassing the senders' skills and expectations by far.[50] It is the physician and historian Ibn Abī Uṣaybiʿah (1203-1270) who recounts an occasion on which Ibn Yūnus was visited by the emperor's emissary:[51]

> *It is said that Kamāl al-Dīn ibn Yūnus knew magic. An example of this was related to me by the Judge Najm al-Dīn ibn al-Kuraydī who said, 'The Judge Jalāl al-Dīn al-Baghdādī, the student of Kamāl al-Dīn ibn Yūnus, used to live near ibn Yūnus in the school. Once, an emissary came to the merciful king Badr al-Dīn Luʾluʾ the governor of Mosul, from the Emperor the king of the Franks, who was a master in the sciences. The emissary had with him certain questions about astrology and the like and sought the answers from Kamāl al-Dīn ibn Yūnus. The governor of Mosul sent a message to Kamāl al-Dīn informing him of this and saying that he should wear beautiful clothes and prepare a splendid salon for the emissary knowing that Ibn Yūnus used to wear rough clothes without affectation and had no knowledge of the things of the world. Ibn Yūnus said, 'Yes.'*
>
> *Jalāl al-Dīn relates saying, 'I was with him when he had been told that the emissary of the Franks had arrived and was close to the school. Ibn Yūnus sent some students to meet the emissary and when he arrived we looked and saw that the room was adorned with the most beautiful and finest Byzantine carpets with a group of slaves and servants in fine clothes. The emissary entered and the Shaykh Kamāl al-Dīn greeted him and wrote the answers to all the questions. When the emissary had gone all that we had seen before vanished so I said to the Shaykh, 'Master, how wonderful were the splendours and the servants we saw a short time ago!' He smiled and said, 'Baghdādī, that is science!'*

The focus of this account is not the meeting with the emissary and the questions being asked but the great authority and skill of Ibn Yūnus, which appear to be legendary; the

[50] Ibid., 3:470–471; Suter, "Beiträge zu den Beziehungen Kaiser Friedrichs II. zu zeitgenössischen Gelehrten," 5. D. Raynaud surmises that the question sent from Damascus was sent by the next scholar in our list, ʿAlam al-Dīn Qayṣar, and pertained to the problem posed to him by the emperor. While this is certainly possible, the source mentions neither Qayṣar's name, nor the fact that the problem was posed by Frederick. See Raynaud, "Le tracé continu des sections coniques à la Renaissance," 341–343.

[51] Translated by Alasdair Watson, in *ALHOM* (*A Literary History of Medicine, the ʿUyūn al-anbāʾ fī ṭabaqāt al-aṭibbāʾ of Ibn Abī Uṣaybiʿah*), forthcoming. See also Suter, op. cit., 6. Raynaud, op. cit., 341.

polymath was so sought-after that even the king of the Franks sent an envoy to him. As the visitor's high rank made it necessary to quickly produce a suitable meeting place, the polymath created the illusion of a place filled with all kinds of riches, which disappeared as soon as the guest had left. While the report in itself might have been embellished by either Ibn Yūnus' student or the historian, it is quite conceivable that Frederick's emissary met the polymath and asked him scientific questions on behalf of the emperor, as many different sources report of contacts between the imperial court and Ibn Yūnus' school in Mosul.

Thus, one of Ibn Yūnus' students was Sirāj al-Din Urmawī (1198-1283), author of the book on logic *Maṭāliʿ al-anwār* ('The Rising of Lights').[52] His name was already mentioned above, in the historian Ibn Wāṣil's description of the good relations between the court of Hohenstaufen and the Ayyubid sultans. In the last part of his account he mentions that Sirāj al-Din Urmawī was sent to the emperor's court and wrote a book on logic for him. As said above, Ibn Wāṣil is a reliable witness, as he had spent some time at the imperial court himself. Furthermore, he even dedicated a treatise on logic to Frederick's son Manfred, the *Risāla al-anbrūriyya* ('the imperial treatise'), thus he would certainly have been aware of previous works on logic written at the imperial court.

Also a second student of Ibn Yūnus' has already been mentioned in this chapter: Frederick's court philosopher Theodore of Antioch. While the Latin sources do not give us any information about his upbringing, it is an Arabic source that sheds light on

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[52] The *Maṭāliʿ al-anwār* consisted in fact of two parts of which only the first one, dealing with logic, has come down to us. The lemmata of the second part, which dealt with metaphysics and natural philosophy, were only transmitted in a commentary. See Hasse, "Mosul and Frederick II Hohenstaufen: Notes on Atiraddin al-Abhari and Siragaddin al-Urmawi," 152. That al-Urmawī was a student of Ibn Yūnus we know from the historian al-Subkī: *He studied in Mosul under Kamaladdīn ibn Yūnus. He was born in the year 594 <i.e. 1198 A.D.> and died in the year 682 <i.e. 1283 A.D.> in Konya.* Translation by Hasse, op. cit., 149.

Theodore's life before his sojourn in Italy and on the circumstances of his death. The Syriac Bishop and writer Gregory Barhebraeus (Abu al-Faraj ibn al-ʿIbrī; 1225/6-1286), renders a biography of Theodore in his *Mukhtaṣar taʾrikh al-duwal (*'Short history of the dynasties').[53] According to Barhebraeus, the *ḥaqīm* (i.e. 'philosopher') *Thādhūrī of Antioch* was a Jacobite Christian, who, having studied Latin, Syriac and some philosophy in Antioch, moved to Mosul in order to study under the famous polymath Kamāl al-Dīn ibn Yūnus. There, he studied the works of al-Fārābī and Ibn Sīnā, along with Euclid's *Elements* and the *Almagest.* He moved back to Antioch for a short while, but returned to Kamāl al-Dīn ibn Yūnus to deepen his knowledge. Afterwards he studied medicine in Baghdad. He served the Sultan ʿAlāʾ al-Dīn of Konya in Asia Minor, who ruled from 1219-37,[54] then he travelled to the sultan's enemies in Armenia and served Constantine of Lampron (d. 1261), the father of king Hayton (ruled 1226-1270). From there, he joined an envoy of Emperor Frederick and travelled to his court. The Emperor held him in high esteem and even assigned him a parcel of land. But at some point, Theodore wanted to return home, which the emperor did not allow. In the emperor's absence, Theodore boarded a ship travelling to Acre. When an unfavourable wind brought the ship to the port that the emperor was stationed in, Theodore committed suicide by poison out of shame. Whether or not the report of Theodore's suicide is true, Barhebraeus's account appears to be reliable for the most part; being a Jacobite Christian himself who lived in Antioch in 1244, he may even have had first-hand information on Theodore.

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[53] Burnett, "Master Theodore, Frederick II's Philosopher," 264–265 (Arabic), 228–229 (English). An English translation can also be found in Kedar and Kohlberg, "The Intercultural Career of Theodore of Antioch," 175–176. A German translation was rendered by Suter, "Beiträge zu den Beziehungen Kaiser Friedrichs II. zu zeitgenössischen Gelehrten," 8–9.
[54] Burnett, op. cit., 232.

Frederick's connections with Kamāl al-Dīn ibn Yūnus thus consisted not only of letters and ideas being exchanged, but of actual people travelling from Mosul to Italy. In this light, the story about Frederick's emissary visiting the sage in Mosul becomes much more credible. Further evidence of strong connections between the Mosul school and the imperial court is given by accounts about two of Ibn Yūnus' students receiving enquiries from the emperor, ʿAlam al-Dīn Qayṣar and Athīr al-Dīn al-Abharī.

ʿAlam al-Dīn Qayṣar

It is once again the historian Ibn Wāṣil who reports that one of Frederick's questions was answered by Ibn Yūnus' student ʿAlam al-Dīn Qayṣar (1178-1251).[55] The mathematician and astronomer ʿAlam al-Dīn Qayṣar, known as Taʿāsīf, was born in Upper Egypt. He studied mathematics in Egypt and Syria before he travelled to Ibn Yūnus. After one year in Mosul he returned to Syria, teaching in Hama, and he also built observational towers and astronomical instruments for its Ayyubid ruler. The question that Ibn Wāṣil reports of was posed during Frederick's crusade in 1228-1229, while the emperor negotiated a truce and free access to Jerusalem with the Ayyūbid Sultan al-Malik al-Kāmil:[56]

> *The amir Fakhr al-Dīn ibn al-Shaykh conducted the negotiations for [al-Malik al-Kāmil], and many conversations and discussions took place between them, during which the Emperor sent to al-Malik al-Kāmil queries on difficult philosophic, geometric and mathematical points, to test the men of learning at his court. The Sultan passed the mathematical questions on to Shaykh ʿAlam al-Dīn Qayṣar, a master of that art, and the rest to a group of scholars, who answered them all. Then al-Malik al-Kāmil and the Emperor swore to observe the terms of the agreement and made a truce for a fixed term.*

Ibn Wāṣil's account once again appears to be reliable, for not only had the historian spent time at the imperial court, but he also knew the mathematician ʿAlam al-Dīn Qayṣar

[55] On Qayṣar, see Sabra, "Simplicius's Proof of Euclid's Parallels Postulate," 8.
[56] Gabrieli, *Arab Historians of the Crusades*, 270. A very similar account is given by the Egyptian historian al-Maqrīzī (1364-1442). See Raynaud, "Le tracé continu des sections coniques à la Renaissance," 341.

personally; he had assisted him in the construction of the observatory and astronomical instruments in Syria. But if his report on ʿAlam al-Dīn is trustworthy, there is no reason to doubt the rest of his account: not only mathematical questions, but also philosophical ones were asked, which were answered by different scholars. This query may, in fact, be the same one that Shihāb al-Dīn al-Qarāfī speaks of in his optical work: we recall that seven extremely difficult questions were supposedly sent to al-Malik al-Kāmil, some of which were answered by excellent scholars. Still, this does not make al-Qarāfī's account in itself more reliable; he may have known Ibn Wāṣil's report and used it as a source for his own introduction. Thus, his statement that some of the questions were answered, but he does not know if all of them were, would reflect the fact that he only knew what Ibn Wāṣil wrote in his account: the mathematical questions were answered by ʿAlam al-Dīn Qayṣar.

Athīr al-Dīn al-Abharī

The second pupil of Ibn Yūnus who is reported to have answered a question posed by Frederick was the famous mathematician, astronomer and philosopher Athīr al-Dīn al-Abharī (d. between 1263 and 1265).[57] He wrote, amongst others, the mathematical and astronomical works *al-Usṭuqussāt fī al-handasa* ('Elements of geometry')[58], *Talkhīṣ al-Majisṭī* ('Epitome of the *Almagest*'), *Ghāyat al-idrāk fī dirāyat al-aflāk* ('The utmost understanding in knowledge of the spheres'), and at least one *zīj* (astronomical handbook). His philosophical compendium *Hidāyat al-ḥikma* ('Guidance in philosophy'), which covered logic, natural philosophy and metaphysics, became very well known, and was

[57] On his life and works see Eichner, "al-Abharī, Athīr al-Dīn."
[58] Sabra, "Thabit B. Qurra on Euclid's Parallels Postulate," 14.

commented upon by numerous Muslim scholars.[59] One of al-Abharī's pupils, the cosmographer and geographer al-Qazwīnī (c. 1203- 1283), relates another instance of Frederick's mathematical questions reaching Mosul:[60]

Among the wonderful things I heard about him [Ibn Yūnus] was that at the time of al-Malik al-Kāmil the Franks sent problems whose solution they required to Syria, among them were medical, philosophical and mathematical ones; the medical and philosophical ones the scholars of Syria solved themselves, for the mathematical ones they were no match. But al-Malik al-Kāmil demanded that they all be solved, thus he sent them to Mosul to Mufaḍḍal b. ʿOmar al-Abharī, our teacher. He had no equal in the geometrical sciences, but the solution was too difficult for him; he showed the problem to Shaykh Ibn Yūnus, who thought about it and solved it.

The problem is this: Let there be a given arc. Draw its cord and extend it beyond the arc, and construct on the extended cord a square whose area is equal to that of the part of the arc. The figure is the following:

Al-Mufaḍḍal found the proof for it, made a treatise of it and sent it to Syria to al-Malik al-Kāmil. When I came to Syria, I found the most excellent scholars in amazement at the treatise; they also praised the discovery of the proof, because it was a rare product of the time.

Similarly to the report about ʿAlam al-Dīn Qayṣar, the account above was given by a witness who knew the places and scholars in question personally, in this case by a pupil of al-Abharī's. Al-Qazwīnī had resided in Damascus around 1233, and moved to Mosul before the year 1240.[61] If we believe him that he saw al-Abharī's treatise in Syria, this would have taken place within this period. This assumption is supported by the fact that al-Malik al-Kāmil ruled in Egypt from 1218 until his death in 1238, but it was only during the last year of his life that he extended his rule over Damascus. As al-Qazwīnī explicitly

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[59] Hasse, "Mosul and Frederick II Hohenstaufen: Notes on Atiraddin al-Abhari and Siragaddin al-Urmawi," 150.
[60] The Original Arabic can be found in *Zakarija ben Muhammed ben Mahmud el-Cazwini's Kosmographie*, 2:310. A German translation was rendered by Suter, "Beiträge zu den Beziehungen Kaiser Friedrichs II. zu zeitgenössischen Gelehrten," 3. See also Hasse, op. cit., 147–148.
[61] Raynaud, "Le tracé continu des sections coniques à la Renaissance," 342.

mentions that all of Frederick's questions were sent to Syria and returned to al-Malik al-Kāmil, the query must have been sent around that year. Furthermore, the sultan did not send the questions directly to Ibn Yūnus but to his pupil al-Abharī; this implies that the latter must already have been a well-established scholar at the time, while Ibn Yūnus may have been considered too old or too respected to be asked directly.[62] This, too, would suggest that the enquiry was made in the year 1238, and that al-Qazwīnī's account is reliable.

In addition, al-Qazwīnī's report is the only one in which the content of a mathematical question supposedly asked by the emperor is given. It appears that the sage was asked to achieve no less than the squaring of a circle segment, which, as we know today, is not possible for all segments by means of only a compass and a ruler. No such treatise by al-Abharī has come down to us. However, D. Raynaud has pointed out that there is a treatise by al-Abharī whose very first diagram looks almost identical to that recorded by al-Qazwīnī.[63] According to the first few lines, it was *composed by the Imām Athīr al-Dīn [...] al-Abharī in the course of his study with Shaykh Kamāl al-Dīn ibn Yūnus on the Treatise on the Perfect Compass.*[64] As al-Qazwīnī was not a mathematician and he reportedly saw the treatise he is describing before he met al-Abharī in Mosul, he may thus have been slightly mistaken when recounting the problem. But before looking into this treatise in some more detail, we can already conclude that al-Qazwīnī's report appears to be the most reliable one among those found in Arabic sources, as both the historical circumstances and the mathematical content being related can be verified.

[62] Hasse, "Mosul and Frederick II Hohenstaufen: Notes on Atiraddin al-Abhari and Siragaddin al-Urmawi," 147–148.
[63] Raynaud, "Le tracé continu des sections coniques à la Renaissance," 344.
[64] Rashed, *Geometry and Dioptrics in Classical Islam*, 828.

In conclusion, it has become obvious that among the Arabic reports about Frederick sending out questions, the ones regarding Ibn Yūnus and his pupils are the most reliable ones. While some of these accounts show signs of literary embellishment, there are several dependable sources that report of mathematical questions being answered by Ibn Yūnus and his pupils ʿAlam al-Dīn Qayṣar and Athīr al-Dīn al-Abharī, and of Ibn Yūnus' pupils Sirāj al-Din Urmawī and Theodore of Antioch travelling to the emperor's court in Italy. In contrast, the report about optical questions by Shihāb al-Dīn al-Qarāfī contains only third-hand information which cannot be verified, while there is some doubt about the veracity of the report found in Ibn Sabʿīn's *Sicilian Questions* on natural philosophy and metaphysics. Considering that the reports talk of mathematical and astronomical questions being answered by Ibn Yūnus and his pupils, and that also Fibonacci's account of mathematical problems being posed by the emperor and his philosophers (among which is even one of Ibn Yūnus' students) is trustworthy, it is thus clear that it is in particular the reports on mathematical questions that can be considered to be historical. Furthermore, Theodore of Antioch's connection with both Mosul and the emperor's court gives a possible explanation for the fact that Leonardo Fibonacci was asked questions that could be found in Arabic sources only; having studied mathematics and astronomy under Ibn Yūnus, he could have brought these sources with him. In fact, Theodore's master in Mosul did compose a treatise on Diophantine equations, one of the topics covered by the questions posed to Fibonacci: the *Risāla fī bayān annahu lā yumkin an yūjad ʿadadān* *murabbaʿān fardān majmūʿhuma murabbaʿ* ('treatise on the proof that there cannot be two squares of odd numbers whose sum is a square').[65]

In addition, it is also the content of the one mathematical question being discussed in the Arabic sources that shows a connection between the school in Mosul and the imperial court. As stated above, al-Qazwīnī's account contains a diagram that strongly resembles that of an actual treatise that al-Abharī composed under the supervision of his master. It is entitled *On the Compass of Conic Sections* (*Fī birkār al-quṭūʿ*)[66] and describes how to construct compasses with which not only circles can be drawn, but also ellipses, parabolas and hyperbolas – curves that result from cutting a cone in different ways. Although we know of no Arabic or Latin version of the work present at the imperial court, it is the described instrument itself that links the Frederick's court with the school of Mosul; the exact compass described by al-Abharī suddenly appears about 200 years later in the works of Renaissance scholars and artists such as Lorenzo della Volpaia, Leonardo Da Vinci and Michelangelo Buonnarroti. D. Raynaud has shown that there is a possible line of transmission that goes straight back to the court of Frederick II and the school of Ibn Yūnus in Mosul.[67] But if al-Abharī's *On the Compass of Conic Sections* is the treatise that was sent to Frederick, it does not appear to be the answer to a specific mathematical problem that was posed by the emperor but rather to his wish to learn more about the construction of a specific instrument unknown in the Latin West, as well as the mathematics behind it.

[65] Rashed, *Classical Mathematics from al-Khwarizmi to Descartes*, 419.
[66] The treatise was edited and translated into English by Rashed, *Geometry and Dioptrics in Classical Islam*, 828–847.
[67] Raynaud, "Le tracé continu des sections coniques à la Renaissance."

That Frederick may have been interested in the construction and use of a *Compass of Conic Sections* is supported by his interest in the properties of hyperbolae; as mentioned above, his philosopher John of Palermo translated an anonymous Arabic treatise on the relationship between a hyperbola and its asymptote, which are constructed through a conic section, into Latin: *De duabus lineis semper approximantibus sibi invicem et numquam concurrentibus.*[68] John's translation (and a commentary on it that was probably made at the university of Paris shortly after the translation was finished) is the only medieval Latin treatise referring to conic sections extant in the Latin West that was not written in an optical context.[69] As to the reasons why the emperor may have been interested hyperbolae and an instrument for their representation, there is also a plausible explanation: the first sentence of the Arabic treatise on the hyperbola and its asymptote that was translated into Latin by John of Palermo is quoted by Maimonides in his *Guide for the Perplexed*.[70] He uses the example of asymptotic lines to illustrate that there are things that are known to be true, but which the mind cannot imagine.[71] As we have seen, that the *Guide* was studied at the imperial court, even by Frederick himself, is attested by the Jewish scholar Jacob Anatoli.

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[68] The treatise was edited and translated into English by Clagett, *Archimedes in the Middle Ages*, 1980, 4:33–61.

[69] Ibid., 4:34.

[70] The mathematical theorems used in the Arabic treatise go back to the treatise on conic sections by Apollonius of Perga (c. 262–c. 190 BCE). However, Maimonides does not appear to quote an Arabic version of Apollonius' *Conics*, but the Arabic original of *De duabus lineis*. See Freudenthal, "Maimonides' 'Guide of the Perplexed' and the Transmission of the Mathematical Tract 'On Two Asymptotic Lines' in the Arabic, Latin and Hebrew Medieval Traditions," 114-5. See also Grebner, "Der Transfer mathematischen Wissens aus dem Orient und der Hof Friedrichs II. Der Asymptotentraktat und sein personelles wie epistemisches Umfeld," 222.

[71] The impossibility of imagining two lines getting closer to each other but never meeting, the hyperbola and its asymptote, was already used as an epistemological argument by Gemenius (1st century BCE). The same argument was also used by mathematicians writing in Arabic in tenth and eleventh centuries. See Freudenthal, op. cit., 116-120.

Judah ha-Cohen's account

Having discussed the sources that mention Frederick's questions to scholars in and outside of Italy, it is now possible to determine the place of Judah ha-Cohen's account of the correspondence with the emperor's philosopher within this framework. On the one hand, Judah's question deals with mathematics, a subject that was demonstrably of interest to the imperial court. We know that the emperor engaged in mathematical discussions between 1226 (Fibonacci) and 1238 (al-Abharī). While it is not clear when exactly the correspondence supposedly took place, Judah's account makes it clear that it must have been during this period. On the other hand, Judah's account has a lot in common with that of the Sufi Ibn Sabʿīn, which may have been purely fictitious. Both Judah and Ibn Sabʿīn grew up in the Iberian Peninsula; the former under Christian rule in Toledo, the latter under Muslim rule in Murcia. In fact, the two scholars are the only Andalusian ones among those who claim to have received the emperor's queries. Both relate that they received the emperor's questions at a very young age; Judah at eighteen, Ibn Sabʿīn at twenty-three. Both addressed the alleged authors of the questions rather condescendingly; Ibn Sabʿīn admonishes his addressee to word his questions more carefully, while Judah even ridicules the philosopher's mathematical skills.

Furthermore, Akasoy already pointed out that Ibn Sabʿīn's writings show parallels to those of Jewish Andalusian scholars.[72] These parallels can also be found regarding Judah's *Midrash ha-Ḥokhmah*. Both Judah and Ibn Sabʿīn were on the one hand well versed in Aristotelian philosophy, but on the other hand also very critical of it. They used similar sources in their philosophical writings, such as Alexander of Aphrodisias, Galen,

[72] Akasoy, *Philosophie und Mystik in der späten Almohadenzeit*, 73–84.

Themistius, al-Fārābī, Ibn Bājja, Ibn Sīnā and Aristotle's commentator Ibn Rushd. Of the latter, both were exceptionally critical, although heavily relying on his commentaries on Aristotle's writings as a source.[73] Judah describes Ibn Rushd as defending Aristotle, even when he is obviously wrong, *as if his soul were attached to Aristotle's.*[74] Thus, he writes in the natural philosophical part of his *Midrash ha-Ḥokhmah:*[75]

> *Aristotle brings theoretical arguments without reflection, and Ibn Rushd declares Aristotle's opinion to be true, as is his custom regarding all those who doubt him, even regarding those who bring arguments that are true to the sight of the eye [...] as if Aristotle were an angel of God that must not be criticised.*

We find a very similar criticism of Ibn Rushd in Ibn Sabʿīn's *Budd al-ʿārif:*

> *This man was absolutely crazy about Aristotle. He worshipped him and followed him almost blindly in his views of sense perception and first intelligibilia. Had he heard that according to the Wise [i.e. Aristotle] one can stand and sit at the same time, he would have repeated this with full conviction [...] Ibn Rushd is absolutely incapable, his knowledge is small, he has stupid ideas, and he is unintelligent. Yet he is a good man, who does not interfere (in things which do not concern him), he is just and aware of his limited capacities. Then again, he did not rely on his own endeavour since he followed Aristotle blindly.*[76]

While it is improbable that Judah knew Ibn Sabʿīn's writings, the similarity of their views makes it clear that both Judah ha-Cohen and Ibn Sabʿīn grew up in a similar intellectual milieu – as Andalusian scholars they studied similar sources and came to similar conclusions regarding these sources. Is it therefore possible that it was this milieu, the literary tradition and intellectual history of the Iberian Peninsula, which prompted Judah to describe a fictitious correspondence with a famous foreign ruler, similar to Ibn Sabʿīn's account? Association with the emperor would certainly also have increased Judah ha-

[73] Regarding Judah's sources, see Sirat, "Juda b. Salomon ha-Cohen," 46; Fontaine, "Judah ben Solomon ha-Cohen's 'Midrash ha-Hokhmah'", 194–196; Zonta, *La filosofia antica nel Medioevo ebraico*, 200–204. Ibn Sabʿīn's sources are analysed by A. Akasoy, "Ibn Sab'īn's *Sicilian Questions*," 128–135; ead., *Philosophie und Mystik in der späten Almohadenzeit*, 177–331.
[74] Sirat, "Juda b. Salomon ha-Cohen," 52.
[75] Ibid.
[76] Translation by Akasoy, "Ibn Sab'īn's *Sicilian Questions*," 134–135.

Cohen's authority, his being young and being able to refute his opponent even more so. But the parallels between Judah ha-Cohen and Ibn Sabʿīn's writings might also serve to substantiate the historicity of the *Sicilian Questions*; as both scholars apparently belonged to comparable intellectual circles, the imperial queries may have reached both young men through established scholars. In Ibn Sabʿīn's case, the introductory nature of the questions may have prompted more established scholars to give the young man a chance to prove his skills. In in Judah's case, the transmitted question required mathematical skills at expert level, which Judah clearly possessed.

Be that as it may, there are also obvious differences between Judah's case and that of Ibn Sabʿīn. It is quite conceivable that even as an eighteen-year-old youth Judah himself might have received a letter from the imperial court; while he would not have been a famous scholar at the time, his mother's family, the Ibn Shoshan family, was one of the most influential Jewish families of Toledo. His teacher Meir Abulafia being a famous Jewish scholar and his grandfather being an astrologer, Judah appears to be a worthy recipient of the emperor's queries. Furthermore, Frederick had a direct connection to Toledo via his court astrologer, Michael Scot. Michael had worked in the town as a translator and even reverted to the services of Jews for this task. But more importantly, both the subject-matter and the nature of the question that Judah discusses differ immensely from the questions treated by Ibn Sabʿīn: Judah's query is not a general introductory question but a very advanced one, and his answer to it does certainly not read like a student handbook.

Moreover, the subject of the question is mathematics. All accounts of mathematical questions being sent out by the emperor that we saw so far turned out to be quite reliable. Scholars belonging to the imperial court were interested in mathematics, studied Arabic mathematical writings and posed questions that they found in these writings to scholars in Europe. Within this framework, one of the emperor's questions may have reached Judah ben Solomon ha-Cohen in Toledo. But if this is the case, there should be sources for questions posed to Judah, or at least some explanation as to why the question may have been important to the court.

It turns out that different mathematical works were available at the court which could have triggered the question. In order to examine these possibilities, we should first take a look at what Judah's question entailed.

The question reported by Judah consists of three parts: 1. How to construct the five regular polyhedra around a given sphere? 2. How to construct the given sphere inside each of the regular polyhedra? 3. How to construct the given sphere around the five regular polyhedra?

The answer to the third part of the question is already given in book XIII of Euclid's *Elements*; in propositions 13-18 it is shown how to construct the regular polyhedra and to comprehend them in a given sphere. To the thirteen original books later the so-called books XIV (a treatise by Hypsicles, 2nd century BCE) and XV (partly stemming from Isidore of Miletus, 6th century CE) were added, which discuss the relationships between the regular solids. In the twelfth and thirteenth centuries several Latin translations and reworkings of the *Elements* were rendered, mostly made from different Arabic translations of the work, but also a translation of a Greek version of the text was made.[77]

[77] An overview of the different translations can be found in Busard, *Campanus of Novara and Euclid's Elements*, 1–40.

Both Arabo-Latin and Greek-Latin versions were known and used in Frederick's cultural environment.[78] It is therefore conceivable that the emperor's philosopher studied books XIII-XV of the *Elements* and came upon a question that so far had not been treated in any of these books: How to construct the solids *around* a sphere, or how to construct a sphere *inside* the solids. But having studied the *Elements*, he would certainly have been aware of the fact that the third of his questions had already been solved by Euclid. Once again, it appears that at least one of the problems posed to Judah was not simply asked out of general curiosity, but in order to be able to compare Judah's answer to that which was already known. This question thus would have served to test Judah's skills.

In this context it has to be stressed that, similar to the questions reported by Ibn Sabʿīn, to a modern scholar the question that Judah received would appear to be strangely worded: Instead of asking how to construct *a sphere*, Judah's correspondent asks how to construct *a given sphere* inside and around each solid. This can only be achieved if the platonic solids have the correct dimensions in relation to the given sphere to start with; it is impossible to construct a *given sphere* inside and around *any* regular body. But although Judah does not shy away from criticising his opponent harshly in the second part of the correspondence, he does not mention the strange wording of the philosopher's question, which would have been a great opportunity for criticism. On the contrary, in his answer he goes to great lengths to construct the solids in a way that the given sphere will 'fit' inside or around each of them.

The wording of the question hints at another source that the enquirer may have studied. In the twelfth century Gerard of Cremona had rendered the treatise *Verba filiorum Moysi*

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[78] The translation from the Greek had actually been made in Sicily in the late 12th century. Leonardo Fibonacci used it along with translations from the Arabic in his work. (Ibid., 30.)

filii Sekir, which is a Latin translation of an Arabic treatise on the measurement of areas and volumes of certain plain and solid figures by three brothers, called the Banū Mūsā (10th century). The corollary to the very first proposition of the work states *that in the case of every [regular] body containing a sphere, the multiplication of the radius of the sphere by one third the surface area of the body containing the sphere is the volume of the body.*[79] The text thus mentions the platonic solids containing spheres, but neither the proposition nor its corollary state how to construct this sphere inside the regular body. The *Verba filiorum* was used by Leonardo Fibonacci in his *De practica geometrie*. In the sixth chapter of the work, Fibonacci deals with the dimensions of different bodies, such as parallelepipeds, pyramids, spheres, columns and cones, and their relations to each other. In the third part of the chapter, he examines the ratios of nested solids:[80]

> *Given that a column is inscribed in a cube of the same height and a sphere within the column, let a double cone be inscribed so that the common base is in the center of the column and the two vertices touch its basal centers. [...] Moreover, in the double cone inscribe a solid octahedron [...] I will therefore explain the ratios of all these bodies to one another.*

In the following, he calculates the ratios of these solids, and also the volumes of a dodecahedron and an icosahedron inscribed in a given sphere. In part, these calculations pertain to the problem that is posed to Judah: In the passage above Fibonacci describes a sphere inscribed in a given cube, but he does not explain how to construct this sphere inside the cube in the first place. Furthermore, as the ratio of the cube to the sphere is known, it is easy to calculate the ratio of a circumscribed cube to a given sphere, but that would not solve the problem of how to construct this cube. The question that Judah was asked arises from applying these considerations not only to the cube, but to all regular

[79] Clagett, *Archimedes in the Middle Ages*, 1964, 1:249.
[80] Translation by Hughes, *Fibonacci's De Practica Geometrie*, 330.

polyhedra. The fact that the enquirer demanded that a 'given sphere' be constructed, might thus be explained by the fact that he had the ratios between a given sphere and the platonic solids in mind when posing the question: What is the ratio of a given sphere to a platonic solid circumscribed about it, what is its ratio to an inscribed solid, and how does one construct these nested bodies? As Fibonacci was well acquainted with the emperor and his mathematicians, it may have been the *Practica geometrie* that gave rise to the question sent to Judah. But if this is the case, it seems very odd that this question was sent to an eighteen-year-old youth in Spain instead of the author of the work in question. Even if it was not Fibonacci's work that prompted the question it seems peculiar that he, being a very capable mathematician, would not have been asked before the enquirer turned to foreign lands for help.

A problem extremely similar to the second question described by Judah, namely how to construct a sphere (not a given sphere) inside each of the regular polyhedra, is treated within 20 years of Judah's report by two different mathematicians in their reworkings of Euclid's *Elements*: Campanus of Novara and Muḥyī al-Dīn al-Maghribī.

Campanus of Novara (d. Viterbo 1296) is the author of the Latin version of Euclid's *Elements* that would become the most popular one in the late Middle Ages. His rendition of the work became the *editio princeps* in 1482, and it was reprinted at least thirteen times in the fifteenth and sixteenth centuries. Little is known about Campanus' life, but he appears to have composed his version of the *Elements* between 1255 and 1261.[81] His

[81] On Campanus' life, see Toomer, "Campanus of Novara." A critical edition of his version of the *Elements* was rendered by Busard, *Campanus of Novara and Euclid's Elements*.

adaptation of book XV of the work contains some propositions that cannot be found in the Latin translations of his predecessors. The very last proposition (XV.13) reads:[82]

> *Having constructed each of the regular solids, to inscribe a sphere in it.*
>
> *From the thirteenth book it is clear that each of these five solids can be inscribed in a sphere. Now it will therefore become apparent that in reverse, a sphere can be inscribed in each of them. For from the centre of the circumscribing sphere to all the bases of each of them go perpendiculars, which necessarily fall into the centres of the circles that circumscribe those bases. And since all of the circles that circumscribe them are equal, these perpendiculars will be equal.*[83] *Thus if, according to the size of one of them [i.e. the perpendiculars], you describe a circle on the centre of the circumscribing sphere, and you carry its semicircle around until it returns to the place from where it started to move, you prove clearly from corollary 15 of the third [book]*[84] *that the sphere which is described by the motion of this semicircle touches all the bases of the given solid in the meeting points with the perpendiculars. For the sphere cannot touch more of the bases of the solid than the semicircle touches which is carried around it, if it is moved. Hence it is evident that we inscribed a sphere in the given solid, just as was proposed.*

Campanus thus solves the same problem that Judah is asked to solve, the only difference being that Judah first has to determine the right dimensions for the polyhedra in order to fit a *given* sphere into them. After Judah has constructed the polyhedra in the right size, he constructs the sphere in the same way as Campanus: both drop perpendiculars to the centres of the solid's faces, and get the diameter of the circumscribed sphere by doubling one of the perpendiculars. Because Judah assumes that the centre of the circumscribing sphere is unknown, he drops the perpendiculars into the centre of the sphere, while Campanus drops them from the centre of the sphere. Their proofs, however, differ considerably. Unlike Judah, Campanus makes use of theorem I.6 of

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[82] The Latin original can be found in Busard, op. cit., 529–530.
[83] Theorem I.6 of Theodosius' *Sphaerica.*
[84] Elements III.16, corollary: From this it is manifest that the straight line drawn at right angles to the diameter of a circle from its end touches the circle.

Theodosius' *Sphaerica*.[85] While Judah mentions the work in his refutation of the philosopher, he does not use it in his proof. However, the same theorem is used, albeit in a different way, by the second author we mentioned before, Muḥyī al-Dīn al-Maghribī.

About Muḥyī al-Dīn al-Maghribī's life even less is known than about Campanus. His was active around 1260-1265, first in Syria and later in Marāgha.[86] Among his mathematical works is a reworking of Euclid's *Elements*, of which only extracts have ever been published.[87] It is in book XIV of the work that Muḥyī al-Dīn relates propositions XIII.13-17 of the original Euclid, i.e. the construction of the regular polyhedra inside a given sphere. After each construction, he also demonstrates how to construct a sphere inside each individual platonic solid. Thus, propositions XIV.2, XIV.4, XIV.6, XIV.8, XIV.10, and XIV.12 describe the construction of a sphere within a pyramid, a cube, an octahedron, an icosahedron and a dodecahedron, respectively.[88] As a premise to book XIV, Muḥyī al-Dīn explains Theodosius I.6, which he uses in each of the constructions. Like Judah, he first finds the centre of the inscribed sphere in each proposition, but unlike both Judah and Campanus, he does that by connecting opposite angles of the solid in all cases but the tetrahedron.

Both Campanus' and Muḥyī al-Dīn al-Maghribī's reworkings of the *Elements* were rendered about fifteen years after Judah made his Hebrew translation of the correspondence, and the original exchange must have happened at least ten years before

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[85] See Czwalina, *Autolykos: Rotierende Kugel und Aufgang und Untergang der Gestirne. Theodosios von Tripolis: Sphaerik*; Theodosius, *Sphaerica*.
[86] Tekeli, "Muḥyi 'l-Dīn al-Maghribī."
[87] Book XV was edited and translated by Hogendijk, "An Arabic text on the comparison of the five regular polyhedra." Interestingly, Muḥyī al-Dīn's source text was also used in a Hebrew text on the regular polyhedra. See Langermann, "Hebrew Texts on the Regular Polyhedra." Al-Maghribī's proof of the parallel postulate was edited and translated by Sabra, "Simplicius's Proof of Euclid's Parallels Postulate," 15–17, 21–24.
[88] Oxford, Bodleian Library, MS. Bodl. Or. 448, fols. 148r-153v.

that. They can therefore not be considered to be sources for the question. Neither can Judah's answer be seen as a direct source for the two other scholars, as all three use different methods to solve the problem. But the fact that within twenty-five years three different scholars discuss a problem which, as far as we can tell, has never before been treated in mathematical writings makes it clear that there must have been some event in the mid-thirteenth century that triggered this sudden interest in circumscribed spheres. It is very likely that the court of Frederick II played some role in spreading this interest: Campanus of Novara was active in Italy, Frederick's land, shortly after the emperor's death. Muḥyī al-Dīn al-Maghribī does not seem to have had direct links with the emperor, but he studied under yet another of Ibn Yūnus' pupils, the Persian polymath Nasīr al-Dīn al-Tūsī. Judah ha-Cohen, finally, reports that the problem was posed by the emperor's philosopher. Thus, the question may have originated at the imperial court through the study of writings on geometry, and it may have been transmitted to the Arab world through Frederick's connections with Muslim mathematicians.

On the other hand, the question may have stemmed from an Arabic source that was transmitted to the court via the school of Ibn Yūnus. It turns out that a potential source can be found among the writings of Ibn Yūnus himself. The theoretical foundations for constructing the circumscribed solids without making use of the inscribed solids had already been laid in the tenth century by the Persian mathematician and astronomer Abū al-Wafāʾ al-Būzjānī (940-998) in the treatise *Kitāb fī mā yaḥtāju al-ṣāniʿ min al-aʿmāl al-handasiyya* ('Book on those Geometric Constructions which are Necessary for Craftsmen').[89] In the eleventh chapter of the work he describes different methods to

[89] The work was described by Woepcke, "Analyse et extrait d'un recueil de constructions géométriques par Aboûl Wafâ." A list of subsequent studies and editions is given in Raynaud, "Abu al-Wafa' Latinus? A Study of Method," 35–36. For the current work, we relied on the 1979 edition by Ṣ.A. ʿAlī.

divide the surface of a sphere into four, six, twenty, and twelve equal parts.[90] From there it would have been easy to construct the five regular polyhedra inside and outside of the sphere. Ibn Yūnus in turn wrote a commentary on the treatise, called *Sharḥ al-aʿmāl al-handasiyya li-Abū al-Wafāʾ* ('Commentary on the Geometric Constructions by Abū al-Wafāʾ').[91] In this work, he also discusses Abū al-Wafāʾ's constructions of polyhedra.[92]

Regarding the portions of Abū al-Wafāʾ's treatise that deal with two-dimensional geometry, Dominique Raynaud has demonstrated that many of the constructions were echoed by Renaissance scholars in the Latin West. However, none of the Western scholars that dealt with the exact same problems as Abū al-Wafāʾ appear to have had a direct access to (Latin translations of) his work, nor to Ibn Yūnus' commentary on it.

Regarding the construction of polyhedra that Judah describes in the correspondence the situation is similar: the problem could be solved using Abū al-Wafāʾ's treatise, but Judah does not appear to have known it. But via the school of Ibn Yūnus and the court of Frederick II, Abū al-Wafāʾ's and Judah ha-Cohen's works are in fact connected. As we have seen, Frederick and his philosophers appear to have had a habit of asking scholars questions that they had encountered in earlier sources. The philosopher may therefore have known either Abū al-Wafāʾ's treatise or Ibn Yūnus' commentary on it, and his question to Judah may have been a direct reaction to it. Whatever the case, the fact that the problem of constructing a sphere inside the regular polyhedra is later treated both in

[90] Abū al-Wafā' al-Būzjānī, *Mā yaḥtāju ilayhi al-ṣāniʿ min ʿilm al-handasah*, 160–169.
[91] The treatise is preserved in one manuscript: Mashhad, Astān Quds Central Library, MS. 5357. The manuscript and its contents (only regarding two-dimensional constructions) are described in Raynaud, "Abu al-Wafa' Latinus? A Study of Method."
[92] Ibid., 25.

Italy and in the Arab world strongly suggests that Judah is correct in stating that it is connected with the court of Frederick.

But if the contents of the correspondence lead us to believe that Judah's account is truthful, who was his correspondent, 'the emperor's philosopher'? He must have fulfilled three criteria: As letters were exchanged in the Arabic language, the philosopher must have known Arabic. Secondly, he had sufficient knowledge of mathematics and astronomy to compose the question on regular polyhedra, and to answer, albeit incorrectly, Judah's question on oblique ascensions. Thirdly, Judah's correspondent must have held some kind of official position at the imperial court. We have already discussed the three men in the emperor's employ that fulfilled these criteria: Michael Scot, John of Palermo, and Theodore of Antioch.

It is conceivable that Michael Scot, who had spent an important part of his working life as a translator in Judah ha-Cohen's hometown, established the contact with the learned descendant of a prolific Toledan Jewish family. During his time as a translator he had certainly met Jewish scholars; his Latin translation of al-Biṭrūjī's work, for example, he had rendered with the help of a Jew called 'Abuteus levita'. But regarding his translations from the Arabic, recent scholarship suggests that he may have received more than just a little help from his Toledan collaborators, since in his original work *Liber de signis* (part of the *Liber introductorius*) he displays very little knowledge of the Arabic language.[93] Considering his apparently quite limited language skills, it is very doubtful that he would have been able to draft an official letter in Arabic, let alone one containing a sophisticated mathematical question.

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[93] Ackermann, *Sternstunden am Kaiserhof*.

Furthermore, the dating of the correspondence presents a problem; Michael died before the year 1236. We find Judah at the emperor's court in Lombardy in the year 1245, and he states that the correspondence took place about ten years before he visited the emperor's lands. We do not know when Judah left Spain and travelled to Italy. If Judah's visit to the court in 1245 was his very first one, Michael Scot died in the year of the correspondence.

In addition, Judah speaks of his opponent as 'that (emperor's) philosopher', but he also refers to him as 'the *so-called* 'philosopher'', or even 'the one *who is called 'philosopher' before him* [i.e. the emperor]''. When speaking to his opponent directly, he addresses him as 'a student who *is called* 'philosopher' like you', bids him '[m]ay the philosopher please say…', and argues 'if in the philosopher's eyes logic dictates …' In doing so, Judah makes it clear that his addressee carried the official title 'philosopher', but that his treatise on the oblique ascensions causes Judah to doubt whether he actually deserves this title. While Michael Scot held the position of 'astrologer' at Frederick's court, he was never *called 'philosopher' before him*; in imperial documents he appears as *magister*, *astrologus*, or *astrologus domini Frederici Rome imperatoris*, never as *philosophus*.[94]

The second mathematician at the court, John of Palermo, is not called 'emperor's philosopher' in official documents, but he is addressed as such in the writings of Leonardo Fibonacci. Unfortunately, our information about him is very limited; his questions to Leonardo show that he was versed in algebra and arithmetic, but this does not mean that he was also an expert in Euclidian geometry and astronomy. John may thus have been the scholar who made such a big mistake that Judah doubts that he should be called 'philosopher', but the argument is *ex silentio*.

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[94] Burnett, "Master Theodore, Frederick II's Philosopher," 248.

There was, however, a learned scholar at the emperor's court, who did officially carry the title 'philosopher': Theodore of Antioch, who is called *Theodorus philosophus (et) fidelis noster*, or *philosophus noster* in all imperial documents.[95] As we have seen, he had studied both mathematics and astronomy under Ibn Yūnus. If it was in fact the study of Ibn Yūnus' commentary on Abū al-Wafāʾ that triggered the question on platonic solids, it is very conceivable that it was Theodore who posed the question to Judah. In any case, he might have been the one who brought knowledge about the treatise, or about the problems being discussed in it, to the imperial court. But having studied in Mosul, he must have been an expert in mathematics and astronomy. It seems unlikely that he would not have been able to answer Judah's question on oblique ascensions correctly. There are, however, several Latin sources that describe Theodore's intellectual shortcomings in other areas of expertise.

Thus, the jurist and writer Rolandino of Padua (1200–1276) reports that when Frederick left Padua in May 1239 to punish some enemies, Theodore cast a horoscope for him, predicting a favourable time to start the journey. He stood on the city tower holding an astrolabe, but as the sky was obscured by clouds, he erred regarding what time of night it was, and thus regarding the ascendant, giving the emperor directions to leave the city at an inauspicious moment, which led to the emperor's defeat.[96]

The Dominican Etienne de Salagnac (d. 1291) reports that in the autumn of 1238, while Frederick was besieging Brescia, Theodore baffled the Dominican friars of the city with philosophical arguments, to which they did not know how to reply. When Roland of

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[95] Ibid.

[96] Burnett, "Master Theodore, Frederick II's Philosopher," 256–257. See also Kedar and Kohlberg, "The Intercultural Career of Theodore of Antioch," 171.

Cremona, Dominican professor of theology in Paris, heard of their defeat, he rushed to the emperor's camp, found Theodore, and challenged him to a philosophical debate. In this debate he utterly defeated Theodore and restored his order's honour. From that day on, Etienne concludes, *that philosopher held him in great honour*.[97]

When Judah described the philosopher's defeat in his correspondence, he may thus have been inspired by similar stories about Theodore of Antioch that were circulating in Italy. It should also be noted that in the second Latin account, Theodor is called 'that philosopher' (*philosophus ille*), an expression which is very similar to Judah's expression אותו הפילוסוף, which he uses twice in the correspondence. It is therefore also conceivable that Judah was indeed referring to Theodore of Antioch when describing his exchange with 'the Philosopher to the Emperor'. This opens the possibility that at least the answer to the second question discussed in Judah's account may be fictitious. In order to demonstrate his own importance and intellectual superiority, Judah may have simply invented the answer and ascribed it to the famous Emperor's Philosopher, drawing on similar stories about Theodore's great skills and great failures that circulated at the time.

However, Judah does not simply state that he defeated the philosopher, he also provides his readers with the contents of the exchange. An examination of these contents makes it clear that Judah discusses an actual attempt at a geometric proof. It seems unlikely that he would take upon himself to make a geometric construction, which requires expert astronomical knowledge, and then to apply wrong methods of calculation, just in order to demonstrate his superior knowledge.

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[97] Burnett, op. cit., 255–256. See also Kedar and Kohlberg, op. cit., 171.

Whatever is the case, we have seen that the interest in mathematics at the imperial court is well-documented, both in Latin and in Arabic sources. Mathematical knowledge was exchanged between Frederick's court and the school of Ibn Yūnus in Mosul, which are linked through the scholars Theodore of Antioch and Sirāj al-Din Urmawī. There are in fact strong indications that Judah's correspondent was Theodore of Antioch. The fact that one of the mathematical problems Judah describes was treated a few decades later both in a Latin and an Arabic treatise, makes it clear that the topic of the correspondence was certainly of interest to scholars of the time.

Chapter Four

Transfer of knowledge through transfer of language: the Hebrew terminology of the correspondence

While the contents of the correspondence can give us insight into the transmission of knowledge between different communities, the medium used to express these contents, Hebrew technical language, can shed light on the way this knowledge was spread within the Jewish community itself. As regards his technical language in the *Midrash ha-Ḥokhmah*, Judah ha-Cohen remarks:[1]

> *Initially I had composed this book in the Arabic language. When it so happened that I went down to Tuscany, my friends persuaded me to translate it into the holy tongue, and I did so according to my ability. And each man whose heart prompted him to study it 'shall bless himself in the God of truth',[2] to judge me favourably; and may he not accuse me because of the mistakes, be it in the subject matter or in the words. Because of the shortcomings of our language nowadays I was confined in the translation of terms, and I said something that is close and can be understood from the context. And the Lord, blessed be he, knows my intention, as I referred to his name in what I did.*

At the time when Judah translated the correspondence with the Emperor's philosopher into Hebrew, around the year 1247, a Hebrew mathematical terminology had only just begun to develop. Traditional Hebrew literature simply did not deal with purely abstract mathematical questions, and had thus not developed a mathematical *scientific language*.[3] "Science requires that the boundary of the semantic area of a specific word be clear, distinct,

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[1] Oxford, MS. Mich. 551, fol. 123v-124r; MS. Pococke 343, fol. 89r: ובתחלה חברתי זה הספר בלשׁ׳ ערבי וכשנתגלגל הדבר וירדתי לטוסקנא פיסו ממני חברי להעתיקו בלשׁ׳ הקודש ועשיתי כפי היכולת ויתברך באלהי אמן כל איש אשׁ׳ ידבנו לבו לעיין בו לדונני לכף זכות ואל יאשימני על השגיאות בין בענין בין במילות ולפי קוצר לשוננו עתה נדחקתי בהעתקת השמות ואמרתי מה שקרוב להבין כפי הענין והשם יתב׳ יודע דעתי כי לשמו נתכוונתי במה שעשיתי.

[2] Isaiah 65:16.

[3] On the development of Hebrew mathematical terminology, see Sarfatti, *Mathematical Terminology in Hebrew Scientific Literature of the Middle Ages*, and recent research by Ilana Wartenberg: "Mathematical Terminology: Medieval and Modern"; ead., "The Birth of the Medieval Hebrew Mathematical Language as Manifest in Ibn Al-Aḥdab's 'Epistle of the Number'"; ead., "Mathematics in Judaism."

and well known to all those who use it," writes Gad Sarfatti in his *Mathematical terminology in Hebrew scientific literature of the Middle Ages*.[4] In this chapter we will investigate Judah ben Solomon ha-Cohen's role in the development of such a clear and distinct Hebrew mathematical terminology - a terminology that was developed when Greek and Arabic science started to be discussed in Hebrew. Understanding the development of Hebrew mathematical literature in the Greek tradition would be impossible without the pioneering work of Gad Sarfatti and the subsequent studies of Tony Lévy. Relying on their ground-breaking work, we are able to analyse the Judah's terminology as regards Euclidean and spherical geometry, the topics of the correspondence, and investigate how it was influenced by the writings of other medieval scholars writing in Hebrew.

Before the eighth century, there were no Hebrew writings dedicated exclusively to pure mathematics, as we find them in Ancient Greece.[5] The Hebrew bible referred to mathematical topics, such as arithmetical problems (e.g. census, tributes and chronological computations) and geometric descriptions (e.g. the Temple and the division of the land), only indirectly. As regards terminology, the discussions show hardly any development of a mathematical vocabulary that is distinct from everyday language. In the Mishnah and Talmud (first to seventh centuries) we find many discussions of problems that required mathematic knowledge (such as the computation of the new moon, the measuring of the Sabbath boundary, the division and growth of inheritance etc.), but mathematics were used only as a tool for Halakhic decisions; the transmission of mathematical knowledge in itself was not of primary importance. Thus mathematical discussions were scattered throughout these writings. They contained hardly any unambiguous mathematical language, and do not

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[4] Sarfatti, *Mathematical Terminology*, vii.

[5] For a detailed study of the history of Hebrew mathematic terminology see Sarfatti, *Mathematical Terminology*. A short overview can be found in Wartenberg, "Mathematical Terminology: Medieval and Modern."

appear to employ abstract terms for even basic geometric figures, such as ‘point’, ‘circle’ or ‘straight line’. In the *gaonic* period (sixth to eleventh centuries) a specific Hebrew terminology for calculations of the calendar was fully developed, but still there were no fixed terms for many other mathematical notions. It was probably during this period that the first Hebrew work of purely mathematical content was composed, the *Mishnat ha-midot*. The work remained unknown to later authors and was only rediscovered in the nineteenth century.[6] As a rule, the *geonim* would turn to Arabic as their language of choice when dealing with questions related to ‘scientific’ rather than religious learning. The same was true for generations of Jewish authors after them; ever since a large corpus of Greek literature had been translated, often via Syriac, into the Arabic language in the eighth and ninth centuries, Arabic had become the language of science and philosophy in countries under Muslim rule. In medieval Spain the predominant role of Arabic did not change even after the Reconquista; Judah ha-Cohen was born and raised in Toledo, which had been under Christian rule since the eleventh century, but his scientific learning he had acquired from Arabic sources. Thus it was only when he travelled to the Latin West that he considered translating his own scientific writings from the Arabic into Hebrew. When he did so, there were only a few mathematical works written in Hebrew that he could possibly rely on as a source for mathematical terminology.

The first author known by name who composed mathematical works in Hebrew was Abraham Bar Hiyya (c.1070-c.1136).[7] The ‘father of Hebrew mathematics’ was born and raised

[6] The author and date of composition have yet to be identified. Tzvi Langermann suggested that it was written in the eighth or ninth century as part of a set of Hebrew writings of scientific nature and thus marks the beginning of Hebrew scientific literature. See Langermann, “On the Beginnings of Hebrew Scientific Literature and on Studying History through ‘Maqbilot’ (Parallels).”
[7] On Bar Hiyya’s life and work see Lévy, “Les débuts de la littérature mathématique hébraïque: la géométrie d’Abraham bar Hiyya (XIe-XIIe siècle).”

in Northern Spain, possibly in the Arab kingdom (*ṭā'ifa*) of Zaragoza.[8] The fact that he was also known by the title *Savasorda*, the Latinised form of the Arabic *ṣāḥib al-shurṭa* ('captain of the bodyguard', 'chief of police'), suggests that he may have held a court position, while his Hebrew title *nasi* ('prince') indicates also an official position in the Jewish community. It was in Barcelona, which was under Christian rule at the time, that he composed the first Hebrew works on mathematics and astronomy, as well as works on calendar calculation and philosophy, intended for a readership who did not know Arabic. Of his encyclopaedic work *Yesode ha-tevunah u-migdal ha-emunah* ('Foundations of Understanding and Tower of Faith') only sections on geometry, arithmetic, and optics have been preserved.[9] His *Ḥibbur ha-meshiḥah weha-tishboret* ('Treatise on Mensuration and Calculation')[10] was translated into Latin in 1145 as *Liber embadorum* by Plato of Tivoli, with whom he also collaborated on different Arabic-Latin translations. Bar Hiyya's astronomical oeuvre consists of the treatises *Tsurat ha-arets* ('Form of the Earth')[11], *Ḥeshbon mahalekhot ha-kokhavim* ('Calculation of the Courses of the Stars'),[12] astronomical tables, and a treatise on calendar reckoning, the *Sefer ha-ʿibur* ('Book of Intercalation').[13] In order to convey the mathematical ideas and concepts found in Arabic mathematical works, Bar Hiyya had to coin a completely new Hebrew mathematical vocabulary. His mathematical terms were, like the topics of his works, based on Arabic writings. That is why at times he would simply take over Arabic loan words,

[8] Argued by Millás Vallicrosa, "La Obra Enciclopédica de R. Abraham bar Hiyya."
[9] An edition and Spanish translation was rendered by J.M. Millás Vallicrosa, *La obra enciclopédica Yesode Ha-Tebuna U-Migdal Ha-Emuna de R. Abraham Bar Hiyya Ha-Bargeloní.* See also Rubio, "The First Hebrew Encyclopedia of Science: Abraham Bar Hiyya's *Yesodei ha-Tevunah u-Migdal ha-Emunah.*"
[10] An edition was rendered by M. Guttmann, *Ḥibur ha-meshiḥah weha-tishboret.* A Catalan translation of this edition was made by J.M. Millás Vallicrosa, *Llibre de geometria: hibbur hameixihà uehatixbòret.*
[11] Published by Jonathan b. Joseph in Offenbach in 1720: Hiyya, *Sefer tsurat ha-arets.* Translated into Spanish by J.M. Millás Vallicrosa, *La obra Forma de la tierra de R. Abraham Bar Hiyya ha-Bargeloní.*
[12] Edited and translated by J.M. Millás Vallicrosa, Hiyya, *Séfer Ḥešbón mahlekot ha-kokabim.*
[13] Published by H. Filipowski, *Sefer ha-ʿibur.*, London, 1851.

but more often he used Biblical and Mishnaic Hebrew vocabulary on which he bestowed a new, scientific meaning.[14]

The Hebrew mathematical works of Bar Hiyya were followed about a generation later by the works of Abraham ibn Ezra (1092—1167).[15] Born in Toledo or perhaps in Tudela in Navarre, Ibn Ezra spent a large part of his life travelling through Europe. The grammarian, poet, biblical commentator, and astronomer-astrologer wrote numerous treatises on different topics, many of which also included mathematical discussions. Entirely dedicated to mathematics was his *Sefer ha-mispar* ('Book of the Number'),[16] an arithmetic textbook, written in Italy in 1146 or earlier. Translated into Latin perhaps already during Ibn Ezra's lifetime, it was one of the first works to introduce the arithmetic of al-Khwārazmī[17] into the Latin West. In addition, Ibn Ezra authored the short mathematical treatise *Sefer ha-eḥad* ('Book of the Unit'),[18] which dealt with the attributes of numbers, sometime before 1148. He may also be the author of the recently discovered geometrical treatise *Sefer ha-midot* ('Book of Geometry'), which is extant in one Hebrew and one Latin manuscript.[19] His astronomical works include the *Sefer ṭa'ame ha-luḥot* ('Book of the Reasons behind Astronomical tables'), of which only a Latin version has come down to us,[20] and the treatise *Keli ha-neḥoshet* ('The Instrument of Brass')[21] that described the use of the astrolabe. With the help of a disciple Ibn Ezra also translated it into Latin. Like Bar Hiyya, Ibn Ezra wrote a treatise on calendar

[14] On Bar Hiyya's terminology, see Sarfatti, *Mathematical Terminology*, 61–130; Efros, "Studies in Pre-Tibbonian Philosophical Terminology"; Efros, "More about Abraham B. Hiyyas Philosophical Terminology."
[15] Numerous works have been published on Ibn Ezra's different writings. On Ibn Ezra's scientific works see Sela, "Abraham Ibn Ezra's scientific corpus" and his monograph *Abraham ibn Ezra and the Rise of Medieval Hebrew Science*.
[16] Edited and translated into German by Moritz Silberberg, *Sefer Ha-Mispar*.
[17] On al-Khwārazmī, see, for example, Vernet, "Al-Khwārazmī."
[18] Published in 1867 by S. Pinsker: *Sefer ha-Echad*.
[19] Lévy and Burnett, "*Sefer ha-Middot*: A Mid-Twelfth-Century Text on Arithmetic and Geometry Attributed to Abraham ibn Ezra."
[20] Edited by Millás Vallicrosa, *El libro de los fundamentos de las tablas astronómicas de R. Abraham Ibn 'Ezra*.
[21] Edited by in 1845 by H. Edelmann, *Sefer Keli neḥoshet*.

reckoning, the *Sefer ha-ʿibur* ('Book of Intercalation').[22] In addition, he also rendered Hebrew translations of Arabic scientific works by authors like Māshāʾallāh and Ibn al-Muthannā.[23] His Hebrew mathematical terminology was influenced by that of Bar Hiyya, but Ibn Ezra showed a much stronger tendency for 'purism': There were no Arabic loan words in his mathematical terminology, and the Hebrew words used to express mathematical concepts were predominantly Biblical.[24]

Although Abraham Bar Hiyya and Abraham ibn Ezra had set the basis for Hebrew mathematical writing, for a long period of time no original mathematical works were produced in Hebrew. Maimonides (1135-1204), for example, did write original treatises dedicated to purely mathematical subjects, but these treatises were written in Arabic.[25] His major philosophical work, the *Guide for the Perplexed*, in which abstract mathematical and astronomical problems are raised and discussed, was written in Judaeo-Arabic. His major Hebrew composition, the *Mishneh Torah*, was written with the intention to reach the widest Jewish audience possible. Imitating Mishnaic and Rabbinic Hebrew, Maimonides did for the most part not apply an unambiguous scientific language when touching upon scientific questions in his legal code. Although he slightly altered the meanings of existing words and introduced new terms, the specific meaning of the words he used became clear mainly through the context, not so much through a clear-cut Hebrew scientific terminology.[26]

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[22] Edited in 1874 by S.Z. Halberstam, *Sefer ha-'ibbur*.
[23] See Sela, *Abraham ibn Ezra and the Rise of Medieval Hebrew Science*, 75–78. An edition of one of his Hebrew translations was rendered by Goldstein, *Ibn al-Muthannā's commentary*.
[24] On Ibn Ezra's terminology, see Sarfatti, *Mathematical Terminology*, 130–155; Sela, "Abraham Ibn Ezra's Special Strategy." See also his glossary and remarks on terminology in *The Book of Reasons*. A comparison of Bar Hiyya's and Ibn Ezra's astrological terminology has been made by Rodríguez Arribas, "The Terminology of Historical Astrology according to Abraham bar Hiyya and Abraham ibn Ezra."
[25] On Maimonides' mathematical works in Arabic see Langermann, "The Mathematical Writings of Maimonides."
[26] See Sarfatti, *Mathematical Terminology*, 156–165.

Whereas Bar Hiyya and Ibn Ezra had composed original mathematical works in Hebrew, the following generations of scholars concentrated their efforts on translating. The classical Greek works on mathematical sciences and their Arabic commentaries and enhancements that Jewish scholars would study, or, as Tony Lévy puts it, the 'Hebrew mathematical bookshelf', were all translated from the Arabic into Hebrew within a century, between the 1230s and the 1330s.[27] When Judah ben Solomon ha-Cohen translated the *Midrash ha-Ḥokhmah* and his exchange with the Emperor's philosopher into Hebrew in the 1240s, he stood therefore right at the beginning of a line of translators who established a Hebrew mathematical language that would be in use until the beginning of the 20th century.

The translation of mathematical books and treatises was part of a larger translation movement of scientific and philosophical works from Arabic into Hebrew, which had been initiated in the middle of the twelfth century by Judah ibn Tibbon (c. 1120–1190).[28] Judah had left his hometown Granada following the arrival of the Almohads in Muslim Spain and had settled in the town of Lunel in Southern France. There at the bidding of friends he started to translate philosophical and ethical works from Arabic into Hebrew. His son Samuel and his grandson Moses continued his line of work. Samuel (c. 1165–1232) rendered the most popular translation of Maimonides' *Guide for the Perplexed*, but it was only the third generation of translators who made purely mathematical works available in Hebrew.

Two of these translators were Judah's contemporaries: Jacob Anatoli and Moses ibn Tibbon. Jacob Anatoli (c.1194-c.1256) was the son-in-law of Samuel ibn Tibbon. Having received his

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[27] A detailed listing of the translators and the works being rendered in Hebrew in this period was first published by Lévy, "The Establishment of the Mathematical Bookshelf." See also his articles "The Hebrew Mathematics Culture (Twelfth-Sixteenth Centuries)" and "Hebrew Mathematics in the Middle Ages: An Assessment."
[28] A thorough study of the emergence of Hebrew science and the translations of scientific works into Hebrew has been made by Freudenthal, "Les sciences dans les communautés juives médiévales de Provence: Leur appropriation, leur role."

scientific training from Samuel in Provence, he established himself at the court of Emperor Frederick II in Naples in 1230/1. With the support of the emperor he worked on translations of logical and astronomical writings. By 1236 he had translated Ptolemy's *Almagest*, Ibn Rushd's *Epitome of the Almagest*, and al-Farghānī's *Astronomy*. Lévy's research suggests that he might also be the author of a Hebrew version of Euclid's *Elements*, which would make this version the earliest.[29]

Jacob's brother-in-law and nephew Moses ibn Tibbon was active between 1240 and 1283. Based in Provence, he also visited his uncle Jacob in Naples, as some of his writings were rendered there in 1246.[30] Moses translated al-Biṭrūjī's *Astronomy* in 1259. In 1270 he finished his Hebrew translation of Euclid's *Elements*. His translations of Ibn al-Haytham's commentary on the definitions of Euclid's *Elements* and of al-Fārābī's commentary were probably done in the same year. One year later Moses added to these works translations of Theodosius's *Spherics* and al-Hassar's *Arithmetic.* A Hebrew translation of Ibn Aflaḥ's *Revision of the Almagest* followed in 1274.

Another Hebrew version of Euclid's *Elements* was finished in 1289 by Jacob ben Makhir (c.1236-1306), who was probably a nephew of Moses, and was also active in Provence. Jacob was a professional astronomer and author of original astronomical writings that were translated into Latin. As a translator he provided Hebrew versions of the *Data*, and perhaps the *Optics* by Euclid, Autolycos' *On the Rotating Sphere*, the *Spherica* by Theodosius and the *Spherica* by Menelaus, all rendered between 1271 and 1273.

[29] See Lévy, "Une version hébraïque inédite des 'Eléments' d'Euclide."
[30] He translated the *Introduction to Phenomena* by Gemenius and copied Jacob Anatoli's autograph manuscript of the Hebrew translation of Ibn Rushd's *Epitome of the Almagest*. See Lévy, "The Establishment of the Mathematical Bookshelf," 441.

The last great translator of mathematical writings of the period was Qalonymos ben Qalonymos of Arles (1289-after 1329). Between 1309 and 1317 he translated numerous mathematical works into Hebrew, by authors including Ibn al-Haytham, Archimedes, Eutocius, Thābit ibn Qurra, Jābir ibn Aflaḥ and Nicomachus of Gerasa.

Thus when Judah sojourned at the emperor's court in Italy between 1245 and 1247, there were two other Hebrew translators active in the region, Moses ibn Tibbon and Jacob Anatoli, the latter being financed by Frederick II.[31] These two translators had both family and scientific relations. Together, the three scholars were the first to translate mathematical literature from Arabic into Hebrew. It is certainly conceivable that Judah ha-Cohen and Jacob Anatoli, the two Jewish scholars in the emperor's service, had scientific and personal contacts. It is also conceivable that Moses ibn Tibbon, who visited his uncle Jacob exactly when Judah ha-Cohen was working on the translation of his *Midrash ha-Ḥokhmah*, was introduced to Judah. Yet none of these scholar's writings explicitly mention any exchanges between Judah and the other two. If there were any personal or scientific contacts between them, they can be traced only indirectly. It is by means of an analysis of the scientific language the three scholars used that one might posit a connection between them. As no thorough study has been done yet exploring Judah's mathematical terminology,[32] the examination of the mathematical vocabulary used in the correspondence with the philosopher might be a first step towards pinpointing possible connections between the scholars. If they knew each other's translations, they may have been influenced by the

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[31] On the relationship between Judah ha-Cohen, Jacob Anatoli and Moses ibn Tibbon see Sirat, "Les traducteurs juifs à la cour des Rois de Sicile et de Naples."
[32] A preliminary analysis of Judah's terminology was rendered by Tony Lévy, "Mathematics in the *Midrash ha-Hokhmah* of Judah ben Solomon ha-Cohen."

mathematical language of their respective treatises. Furthermore, the terminology of the correspondence can help to shed light on the formation history of the *Midrash ha-Ḥokhmah* itself. A comparison between the vocabularies used in the correspondence and in the rest of the work might help to determine whether they were translated within the same period of Judah's working life, and if the correspondence was thus supposed to be an integral part of the work.

Taking on trust Judah's statement in the introduction to the correspondence, the Hebrew text of the correspondence is a translation of several Arabic texts written by two different authors: the original of the first text, namely the discussion of platonic solids circumscribed about spheres, was written by Judah himself. The second text, a geometrical model for the calculation of oblique ascensions, was composed by the philosopher. The last part of the correspondence, i.e. the refutation of the philosopher's model, was again written by Judah ha-Cohen. These parts differ from each other not only in their alleged authorship, but also in the mathematical divisions that are being treated. Whereas the first part deals with Euclidean geometry, the second part of the text is dedicated to spherical geometry/astronomy. Since the subjects of these two parts of the correspondence differ, so, naturally, does the terminology that is used to express the mathematical contents. When analysing the scientific terminology of the Hebrew translation, it is therefore necessary to treat each of the two parts separately.

As a first step the part of the correspondence dealing with Euclidean geometry will be analysed. The mathematical terms will be compared to the terminology of Abraham Bar Hiyya and Abraham ibn Ezra and then to the vocabulary used in other parts of the *Midrash ha-Ḥokhmah* itself. Then the astronomical vocabulary will be analysed in a similar way.

Euclidean Geometry

Judah ha-Cohen derived much of his mathematical vocabulary from Abraham Bar Hiyya. A comparison with G. Sarfatti's analysis of Bar Hiyya's terminology[33] shows that the basic terms of Euclidean geometry that both authors use are almost identical. To this basic vocabulary belong the terms נקודה (*nequdah*) for 'point', קו (*qav*) for line, קו ישר (*qav yashar*) for 'straight line', זוית (*zavit*) for 'angle', צלע (*tselaʿ*) for 'side' (of a triangle), כדור (*kadur*) for 'sphere', שטח (*sheṭaḥ*) for 'surface' and 'plane', אלכסון (*ʾalakhson*) for 'diameter', מופת (*mofet*) for 'proof' , מרכז (*merkaz*) for 'centre', משולש (*meshulash*) for 'triangle', תושבת (*toshevet*) for 'base', עמוד (*ʿamud*) for 'perpendicular', and נצב (*nitsav*) for 'right (angle)'. Like Bar Hiyya, Judah uses the Hebrew verbs יצא (*yatsa'*) and הוציא (*hotsi'*) in the sense of the Arabic خرج and أخرج: a straight line 'departs' from a point, and can be 'caused to depart', that is, 'being drawn'.

Judah ha-Cohen shows no hesitation to use words of biblical and Mishnaic origin (like *qav, mofet* and *nequdah)* together with loan translations from the Arabic (like *ʿamud* and *sheṭaḥ*) and Arabic loan words themselves (like *merkaz*). But the great influence of Arabic on Judah's translation is not restricted to Arabic loan words and loan translations. It can also be felt in his sentence structure, and in expressions such as החמש צורות (*ha-ḥamesh tsurot*; *the five figures*), with an initial ה, that reflect an influence of vernacular Arabic. At times Judah confuses Hebrew and Arabic words that are phonetically similar. So when he writes the Hebrew word לכן (*lakhen*), meaning 'therefore' in Hebrew,[34] he uses it in the sense of the Arabic word لكن (*lākin*), which has the opposite meaning of the Hebrew term, namely 'but' or 'however'. This interference from the Arabic on the Hebrew translation can also be found

[33] Sarfatti, *Mathematical Terminology*, 61–130.
[34] Edition, paragraphs 18, 38, 41.

in other parts of the *Midrash ha-Ḥokhmah*. The confusion of *lakhen* and *lākin*, for example, is also present in the translation of Ptolemy's *Almagest.*[35]

In order to convey the meaning of 'therefore', Judah often employs an Aramaic term, namely הלכך (*hilkakh*). The word had entered the Hebrew language as a technical term used in Jewish legalistic reasoning. Neither Abraham Bar Hiyya nor Abraham Ibn Ezra appear to make use of this term in their mathematical writings. Judah's usage of the word may point to the fact that his writing style was influenced by a thorough rabbinical training that he received under his teacher Meir Abulafia.

In this context it is noteworthy that in the astronomical part of the correspondence Judah uses another word that is used as a technical term in rabbinical writings. The word denoting 'geometry' in the correspondence is גימטריא (*gemaṭriya*). The term, which originates from the Greek word *geometria,* was used in rabbinical literature to refer to one of the hermeneutic rules for interpreting the Torah.[36] But already in rabbinical times the plural *gemaṭriyot* was also used to simply denote 'calculations'.[37] Thus, the expression תקופות וגמטריאות (*tequfot we-gemaṭriyot*; 'heavenly revolutions and *gemaṭriyot*') which can be found in the Talmud is replaced with תקופות וחשבונים (*tequfot we-ḥeshbonim*; 'heavenly revolutions and calculations') in *Midrash Bereshit Rabbah.*[38]

The use of this technical term in the sense of 'geometry' distinguishes Judah from both Abraham Bar Hiyya and Abraham ibn Ezra. When referring to Greek scientific disciplines in his encyclopaedic work *Yesode ha-Tevunah u-Migdal ha-Emunah*, Abraham bar Hiyya does not

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[35] MS. Vatican 338, fol. 215v "ולכן מרובע א״ד הוא רביע מרובע", fol. 218v "ולכן ז׳א׳ וא׳ב׳ ידועין", fol. 223v " לכן ב׳א׳ ידוע לפי שהוא נטיית קשת ב׳ז׳", fol. 226v "ולכן זוית ד׳ה׳ג׳ שוה לזוית א׳ז׳ב׳", etc.
[36] In the system of *gematria* a word or group of words is interpreted according to the numerical value of its letters.
[37] Such is the case in Avot 3:18.
[38] The expressions can be found in Suk. 28a and BB 134a, and in Bereshit Rabbah 26:5 and 32:6, respectively. See Derovan, Sholem, and Idel, "Gematria," 24.

draw a connection between the Hebrew technical term *gemaṭria* and the science of geometry. He transliterates the Greek loanword as גיאומטריא (*giʾomeṭriya*) in the introduction*,* while he uses Hebrew expressions to refer to 'geometry' elsewhere in his work.[39] Abraham Ibn Ezra, too, prefers Hebrew expressions for 'geometry' in most of his writings.[40] But in the introduction to his translation of Ibn al-Muthannā's commentary on al-Khwārazmī's tables he states the following: 'All the proofs which Ptolemy—who is also called Talmay—brought forth in his great book called the *Almagest* are complete. No scholar can contradict his proofs, for they are taken from the science of measurement called *geometria* in Greek, and *gemaṭriyot* by the sages of Israel.'[41] Like Judah after him, Ibn Ezra thus saw a connection between the science of geometry and the Hebrew term *gemaṭriya*. But while Judah uses the term in the singular form, Ibn Ezra explicitly states that the sages used the word *gemaṭriyot*, in the plural, for 'geometry'.

However, while Judah's usage of the word *gemaṭriya* may point to the fact that he was very familiar with rabbinical literature, which influenced his mathematical vocabulary, he was certainly not the first medieval author to use the term in this sense. Thus, already Maimonides employed the word גימטריה (*gemaṭriyah*) to denote 'geometry' in his *Mishneh Torah.*[42] In the astronomical writings of Judah's contemporary Jacob Anatoli the word גימטריא (*gemaṭriya*) also appears frequently in the same sense.[43]

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[39] Sarfatti, *Mathematical Terminology*, 92.
[40] He uses חכמת המדות in *Sefer Ha-Mispar*, 45; and *The Book of Reasons*, 182.
[41] "וכל הראיות שהביא בטלמיוס הוא הנקרא תלמי בספרו הגדול הוא נקרא אלמגסטי הם ראיות גמורות אין יכול שום חכם לחלוק עליהם כי ראיות הם מחכמת השיעור והטעם חכמת המדות שקורין בלשון יון יומטריאה וחכמי ישראל הקדושים קוראים אותו גימטריאות", *Ibn Al-Muthannā's Commentary*, 104. The translation is based on Goldstein's English translation, p. 149, with a few changes.
[42] Sarfatti, *Mathematical Terminology*, 159.
[43] See, for example, Lay, "L'abrégé de l'Almageste, Hebrew text, pp. 4-6, sentences 49, 51, 55, 56.

Not only the use of certain rabbinical terms, also the influence of Arabic vocabulary makes Judah's mathematical terminology very different from that of Abraham ibn Ezra, who in his quest for a 'pure' Hebrew language rejected Arabic loan words altogether, and preferred biblical terminology to Mishnaic Hebrew. Thus in Ibn Ezra's *Sefer ha-Mispar* ('Book of the Number') the term used for 'base' is not *toshevet*, which is a loan translation from the Arabic word قاعدة (*qāʿidah*), but the biblical word מכונה (*mekhonah*). An angle is not the Mishnaic word *zavit* but the biblical מקצוע (*miqtsoaʿ*), and a right angle is מקצוע ישר (*miqtsoaʿ yashar*).[44]

But Judah ha-Cohen's vocabulary is also distinct from that of Bar Hiyya. There are small grammatical nuances in the word choice, such as the term for 'right angle', being זוית נצבה (*zavit nitsavah*) in Bar Hiyya's work, and זוית נצבת (*zavit nitsevet*) in the correspondence. There are also more significant differences. Although employed by both Bar Hiyya and Ibn Ezra, Judah does not use the Hebrew term מעוקב (*meʿuqav*) for 'cube' but instead introduces the word מכעב (*mukaʿab*). Instead of a Hebrew loan translation he thus prefers a simple transliteration of مكعب, the Arabic word for 'cube'. The remaining regular polyhedra have Hebrew names. Tetrahedron, octahedron, dodecahedron and icosahedron are called צורת בעלת ארבע (*tsurat baʿalat arbaʿ; the figure of four*), צורת בעלת שמנה (*tsurat baʿalat shmoneh; the figure of eight*), צורת בעלת שתים עשרה (*tsurat baʿalat shtem ʿesreh; the figure of twelve*) and צורת בעלת עשרים (*tsurat baʿalat ʿesrim; the figure of twenty*), respectively. The parallelepiped, a prism made up of six parallelograms, is called השוה השטחים (*ha-shaweh ha-shṭaḥim*; *[the solid] of regular surfaces*). *Shaweh* does in this case not mean 'equal', but rather 'regular', 'equilateral'.

[44] Ibn Ezra, *Sefer Ha-Mispar*. For an analysis of Ibn Ezra's terminology, see Sarfatti, *Mathematical Terminology*, 130–156; Sela, "Abraham Ibn Ezra's Special Strategy."

The word was used in the same sense by Abraham ibn Ezra, who for example called a 'cube' גוף שווה (*guf shaweh; regular solid*).[45]

While in the case of 'cube', Judah replaces an existing Hebrew term with an Arabic loan word, in the case of 'diameter' he does the opposite. Instead of using the loan word קוטר (*qoṭer*) from the Arabic قطر, which was introduced by Bar Hiyya, he uses the Greek loanword אלכסון (*alakhson*), which in Mishnaic Hebrew denotes 'diagonal'. Both words appear in Bar Hiyya's works side by side. For the terms 'circle', 'figure', and 'solid' Judah chooses the words עגולה (*ʿagulah*), צורה (*tsurah*) and גופני (*gufani*) respectively. Abraham Bar Hiyya uses for 'circle' the word עגול (*ʿigul*), whereas עגולה (*ʿagulah)* is mostly used as an adjective (meaning 'circular'), for 'figure' he uses the words שטח (*sheṭaḥ*) and תמונה (*temunah*), and for 'solid' גולם (*golem*) or גוף (*guf*). Judah's term *gufani* is of course the adjective derived from *guf*, but he uses it both as a noun and as an adjective. The 'five solids' are called *ha-ḥamishah gufanim*, and a solid angle is *zavit gufanit* (Bar Hiyya: *zavit gelumah*). Like Bar Hiyya, Judah uses the Biblical Hebrew root משש in the sense of the Arabic root مسّ for 'to touch', but he uses it in the piʿel (מישש, *mishesh*) instead of the qal.

Judah's mathematical language has several peculiarities. An example is the verb נגע (*nagaʿ*) which, like *mishesh*, also denotes 'to touch'. It is not clear what the difference between *mishesh* and *nagaʿ* precisely is, as he is using the words to express exactly the same mathematical concepts. So when he begins a proof writing: *Furthermore, we demonstrate that the three remaining triangles* touch *this sphere*,[46] the word expressing 'to touch' is *mishesh.* But when at the end of the same proof he states: *And likewise it is demonstrated [...] that each of its*

[45] Sarfatti, *Mathematical Terminology*, 142.
[46] ועוד נבאר כי השלשה משולשות הנשארים הם ממשמשים לכדור זה (paragraph 16).

surfaces touches *the sphere*,[47] his word of choice is *nagaʿ*. It seems that he is using the two words interchangeably, as synonyms.

Similarly, the adjective נכחי (*nokheḥi*) and the preposition כנגד (*ke-neged*) both express the concept of 'parallel'. Again, Judah begins a proof using the word *nokhekhi* ('*We demonstrate that each of the surfaces of this figure [...] is* parallel *to each of the surfaces of the one constructed inside the sphere*')[48] and ends it using *ke-neged* ('*Triangle [HKT] is* parallel *to triangle OCF, and that is what we wanted*').[49] As the words belong to different word classes, and the preposition *ke-neged* cannot be used as an adjective (as in 'parallel lines'), these words do not act as real synonyms, but the example shows that Judah's terminology is not fixed. The same holds true for the word Judah uses for 'to cut, intersect'. While the 'point of section' is called מקום חתך (*meqom ḥatakh*),[50] the verb he uses for 'to cut' is not *ḥatakh*, but חצב (*ḥatsav*).[51]

Seemingly interchangeable are also the Hebrew expressions Judah uses for 'given' and 'to construct'. When describing the contents of the letter he received from the emperor's philosopher, he reports that the question posed to him was: *How do we* construct *each of the five [platonic] solids on a* given *sphere*.[52] He begins his discussion of the platonic solids by repeating the question: *How do we* construct *each of the five figures on the* given *sphere*.[53] But whereas in the philosopher's question the verb חקק (*ḥaqaq; to draw a circle, engrave, legislate*) is used to express the term 'to construct', in Judah's answer this role is fulfilled by the verb עשה (*ʿasah; to do, make*). The word 'given' is in the first sentence expressed by ידוע (*yaduaʿ; known*), in the second one by מוצע (*mutsaʿ; offered, suggested*). This might suggest that Judah is

[47] וכן יתבאר [...] שכל אחד מן שטחיו הן נוגעין בכדור (paragraph 19).
[48] נבאר כי כל אחד משטחי זו הצורה [...] הוא נכחי לכל אחד משטחי העשוי תוך הכדור (paragraph 14).
[49] יהא משולש [ח̇כ̇ט̇] כנגד משולש ע̇צ̇פ̇, והוא מה שרצינו (paragraph 15).
[50] Paragraph 16.
[51] Paragraphs 10, 15.
[52] כיצד נחוק על כדור ידוע כל אחד מהחמשה גופנים (paragraph 1).
[53] כיצד נעשה כל אחת מחמש צורות על הכדור המוצע (paragraph 3).

using the words as synonyms, but it may also be caused by the use of different expressions in the Arabic original. As the first sentence is a Hebrew translation of the philosopher's letter and the second a translation of Judah's answer, it is conceivable that they used different terms in the Arabic, which Judah faithfully translated.

There were in fact different Arabic words used at the time to express the concept of 'to construct'. In the present case, *ʿasah* might be a literal translation of the Arabic عمل (*ʿamila*; to do, make), and *ḥaqaq* of the verb رسم (*rasama*; to draw, sketch, lay down as a rule). Judah uses the word *ḥaqaq* in his own discussion of the philosopher's question: *If we* construct *a line in the great circle [...]*.[54] But here a line is being drawn, not a solid figure. When mentioning the construction of the platonic solids Judah always uses *ʿasah*. The word *yaduaʿ* on the other hand does not appear at all in Judah's discussion of the philosopher's question – the 'given sphere' is always called *kadur mutsaʿ*. This, too, might hint at the possibility that Judah and the philosopher used a different vocabulary in their Arabic writings.

This leads to the question to what extent Judah ben Solomon stayed faithful to the terminology of the Arabic original he was translating. While Judah himself does not further discuss his methodology in his encyclopaedia, it is clear that medieval Hebrew translators were well aware of the problems posed by different translation techniques. In principle, there were two concurring methods of translation: on the one hand, the global meaning of a sentence of even passage could be translated into another language, which would lead to text that is easy to read in the target language, although the original words of the author might be somewhat obscured. On the other hand, a translator could stay faithful to the author by translating every sentence word for word, which might however lead to a somewhat clumsy

[54] כשנחוק בעגולה הגדולה קו (paragraph 10).

and obscure text in the target language. Drawing on the considerations of Muslim thinkers regarding the translations into Arabic that had been made centuries earlier, medieval Hebrew scholars in general preferred the 'transfer of meaning' to a literal translation, as can be seen from many writings discussing methodology.[55] In practice, however, their translations were mostly just the opposite, namely a slavish word for word translation of the Arabic source. "This is true to such an extent that it is often possible – and not particularly difficult – to reconstruct with some accuracy the Arabic source text from its medieval Hebrew translation," concludes Steven Harvey in his discussion of the Arabic into Hebrew translation movement.[56] But while a Hebrew translation may closely follow the syntax of its Arabic original, one and the same Arabic technical term might sometimes be translated with different Hebrew expressions, as has been demonstrated by Joost van der Lijn in a recent study.[57] Be that is it may, in general the terminology of Hebrew translators appears to be more coherent and consistent, the less 'literary' and the more 'scientific' the translated text is.[58] As we have seen, mathematical writings could be labelled as scientific texts *par excellence*, as they convey abstract ideas in a very special language. It is therefore no surprise that Hebrew translations of mathematical writings follow their Arabic sources very closely, the translators taking care to assign one specific Hebrew term to each Arabic technical term. For the most part Judah's Hebrew terminology forms no exception; it is generally very precise and consistent. How can interchangeable terms in his vocabulary then be explained? As stated above, the inconsistencies may be the result of different wordings in his Arabic source

[55] Zwiep, *Mother of Reason and Revelation*, 63–77. See also Harvey, "The Introductions of Thirteenth-Century Arabic-to-Hebrew Translators of Philosophic and Scientific Texts"; and Rothschild, "Motivations et méthodes des traductions en hébreu du milieu du XIIe à la fin du XVe siècle."
[56] Harvey, "Arabic Into Hebrew: The Hebrew Translation Movement and the Influence of Averroes Upon Medieval Jewish Thought," 265.
[57] van der Lijn, "Can Source Texts Be Reconstructed from Translations? Some Lexical Analysis of *Hovot ha-Levavot*."
[58] Toury, "Translation and Reflection on Translation: A Skeletal History for the Uninitiated," xviii–xix.

text. On the other hand, they may also be owed to Judah's approach to translation, which in this case aimed at translating the meaning, not the language. Nonetheless, the inconsistencies may not be the result of a conscious choice at all. In her study of the Arabic terminology in the parts of the *Midrash ha-Ḥokhmah* dealing with natural philosophy and metaphysics, Resianne Fontaine detected many signs of what she called Judah's "relative clumsiness or incompetence as a translator".[59] As we have seen, this 'incompetence' is also detectable in the correspondence, where Judah's Hebrew is frequently influenced by Arabic grammar and syntax. He may therefore not have been aware of the fact that he gave the same Arabic technical terms different Hebrew translations. As the Arabic original of the correspondence is lost, the question whether Judah faithfully translated different Arabic expressions or opted for a less literal translation, and whether this was the result of a conscious choice or of his shortcomings as a translator, cannot be answered sufficiently at this point.

As regards Judah's terminology in the philosophical part of his encyclopaedia, Fontaine observes that – irrespective of the question whether he was skilled as a translator – Judah appears to have made the conscious choice not to employ the technical Hebrew vocabulary that was established by Samuel ibn Tibbon, for example in his translation of Maimonides' *Guide for the Perplexed*. Fontaine argues that this avoidance of Tibbonid terminology might be due to Judah's critical attitude towards Aristotelianism; while Samuel ibn Tibbon was a strong supporter of Aristotelian philosophy, Judah maintained that it ultimately failed to provide true knowledge. In the correspondence, however, Judah does employ a mathematical term that was first introduced by Samuel. To express 'ratio' or 'proportion', Abraham Bar Hiyya had used different Hebrew words, namely ערך (*ʿerekh*), הקשה (*haqashah*)

[59] Fontaine, "Arabic Terms in Judah ben Solomon ha-Cohen's *Midrash ha-Hokhmah*," 127.

and קצב (*qetsev*); Abraham ibn Ezra had also relied on the term *ʿerekh.* Judah uses a completely different word for this concept, namely the Mishnaic term יחס (*yaḥas,* from the biblical יחש), which originally expressed genealogical relationship. While the word had already been employed by Saadiah Gaon to denote 'relation', it was first used in a mathematical sense by Samuel ibn Tibbon in his Hebrew translation of the *Guide for the Perplexed*.[60] Other mathematical terms used in Samuel's translation that are also being used by Judah, include *nequdah, qav yashar, zavit, tselaʿ, kadur, sheṭaḥ, ʾalakhson, mofet* and *merkaz.* But as we have seen, all of these latter terms had already been used previously by Abraham Bar Hiyya. For other mathematical terms found in the Hebrew translation of the *Guide*, like שטח (*sheṭaḥ*; 'figure'), תכלית (*takhlit*; 'extremity'), גשם (*geshem*; 'solid'), קוטר (*qoṭer*; 'diameter') and תחתית (*taḥtit*; 'base'), Judah used completely different Hebrew terms. Thus Judah apparently knew Samuel ibn Tibbon's Hebrew translation and borrowed the newly introduced technical term *yaḥas* from it. Of the other mathematical terms found in the Hebrew *Guide*, Judah used many that had already been introduced earlier, while he introduced new Hebrew terms for others. As some of these terms had also been used by Bar Hiyya before, it is difficult to determine whether Judah's choice was influenced by the wish to avoid using Samuel's terminology. What can be said is that Judah obviously felt the need to find different Hebrew expressions for some mathematical terms that had already been introduced into the Hebrew language.

Judah ha-Cohen thus makes use of earlier mathematical works in his terminology, but he does not take over their vocabulary uncritically. While a large part of his terminology corresponds with that of Abraham Bar Hiyya, similarities with that of Samuel ibn Tibbon and, to a much lesser degree, Abraham ibn Ezra are also discernible. Characteristic is the use of

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[60] Sarfatti, *Mathematical Terminology*, 180.

different terms carrying the same meaning side by side. This may on the one hand be caused by differences in terminology of the Arabic texts Judah is translating, but there are also clear examples for competing Hebrew terms for one and the same mathematical concept being used simultaneously.

To give a full overview of the terminology, the mathematical terms that appear in Judah ha-Cohen's Hebrew translation of his discussion of the platonic solids are listed alphabetically:

Hebrew	English
אלכסון	diameter
באין חקר	indefinitely
גופני	solid
דומה	similar
זוית	angle
זוית גופנית	solid angle
זוית נצבת	right angle
חצי עגולה	semicircle
ידוע	known, given
יושר	straight
יחס	ratio
כדור	sphere
כנגד	parallel

כפל double

מופת proof

מוצע given

מחומש pentagon

מכעב cube

מקום חתך point of section

מקום נגיעה point of contact

מרובע square

מרכז centre

משולש triangle

נוגע tangent

נכחי parallel

נצב perpendicular

נקודה point

סיבוב circumference

עגולה circle

עגולה גדולה great circle

עמוד perpendicular (*n.*)

צורה figure

צורה גופנית	solid figure
צורת בעלת ארבע	tetrahedron
צורת בעלת עשרים	icosahedron
צורת בעלת שמנה	octahedron
צורת בעלת שתים עשרה	dodecahedron
צלע	side
קו	line
קו ישר	straight line
ראש	end, extremity
שוה השטחים	parallelepiped
שווה	equal
שותף	common
שטח	surface, plane, face
תושבת	base

The discussion contains the following verbs:

Root	Verb	English translation
באר	ביאר	to demonstrate, to prove
	נתבאר	to be demonstrated, proved

חלק	חילק	to cut off
חצב	חצב	to cut, to intersect
חקק	חקק	to construct
יצא	יצא	to depart (line)
	הוציא	to draw (a line)
משש	מישש	to touch
נגע	הגיע	to join, to draw (a line)
	נגע	to touch
נפל	הפיל	to subtract (angles)
סבב	סבב ב	to be inscribed in
	סבב על	to be circumscribed about
עגל	העגיל	to erect, describe a circle
עשה	עשה	to construct
פגש	פגש	to meet *(trans.)*
	נפגש	to meet (*intrans.*)
קום	הקים	to erect

The Euclidean Terminology of the *Midrash ha-Ḥokhmah*

Having analysed the mathematic vocabulary of the correspondence, it is now possible to compare it to the vocabulary Judah uses elsewhere in the *Midrash ha-Ḥokhmah*. Most appropriate for the task is Judah's Hebrew translation of Euclid's *Elements*. The Greek work (4th century BCE) was the mathematical book most widely studied in the Islamic world. The *Midrash ha-Ḥokhmah* contains a Hebrew translation of an earlier Arabic version, both rendered by Judah ha-Cohen himself. Without a doubt he depended on Arabic sources for his compilation, but what exactly these sources were and to which extent he relied on them is difficult to make out at this point, as a lot of research still needs to be carried out regarding the transmission history of the *Elements* in Arabic. Our understanding of the Arabic Euclid tradition is complicated by the fact that the original Greek text was translated into Arabic several times. The first translation was made by al-Ḥajjāj ibn Yusuf ibn Maṭar (fl. between 786–833). He reworked his own translation several years after rendering it. It is not clear to what extend this second version differed from the first one, or whether al-Ḥajjāj was translating from Greek or Syriac manuscripts. These two versions constitute the so-called Ḥajjāj tradition, which is partly lost. Another translation was rendered by Isḥāq ibn Ḥunayn (d. 910). Isḥāq's translation was in turn revised by Thābit ibn Qurra (d. 901). These two texts constitute the so-called Isḥāq/Thābit tradition. Based on these traditions numerous Arabic scholars made abbreviations, commentaries or expositions of (parts of) the *Elements*.[61] Tony Lévy's research suggests that Judah ha-Cohen's abbreviation of the *Elements* draws on sources

[61] An overview of the different known Arabic traditions is given by de Young, "The Arabic Textual Traditions of Euclid's Elements." See also Brentjes, "The Relevance of Non-Primary Sources for the Recovery of the Primary Transmission of Euclid's Elements into Arabic." A critical edition of the entire Arabic text has yet to be made. An edition of the arithmetic books of the *Elements* (thus the books that were not included in the *Midrash ha-Ḥokhmah*) was rendered by G. de Young in his unpublished doctoral dissertation "The Arithmetic Books of Euclid's Elements in the Arabic Tradition." Book V was edited by J.W. Engroff in his unpublished thesis "The Arabic Tradition of Euclid's Elements: Book V." A transcription of the beginning of the first book of the *Elements* in the Isḥāq/Thābit tradition can be found on the website of the Oslo Arabic Seminar "Arabic Text of Euclid's Elements. Translated by Isḥāq ibn Ḥunayn and Revised by Thābit ibn Qurra. Text from the Uppsala Manuscript."

in the Ḥajjāj tradition, among which are possibly a mathematical part of Ibn Sīnā's *Shifāʾ*, entitled *ʾUṣūl al-handasah* ('The Foundations of Geometry'), and the *Commentary on the Elements* by the astronomer and mathematician Abu al-'Abbās al-Nayrīzī (9th-10th cent.).[62] Judah does not mention his sources by name, nor does his text always agree with the Euclidean text presented in either or both of these two works. He may in fact have used an Arabic source common to both Ibn Sīnā and al-Nayrīzī. Furthermore, Judah reworked the Euclidean text himself. In the *Midrash ha-Ḥokhmah* he renders an abridged version of books I –VI and XI – XIII. As he states himself,[63] his aim is to prepare his readers for the study of Ptolemy's *Almagest,* for which the omitted books (on theoretical arithmetic and rational and irrational magnitudes) that are not relevant. In doing so, he not only renders abbreviations of the books on plain and solid geometry, but he also adjusts the text according to his goal; he reformulates and rearranges propositions and proofs, re-introducing concepts stemming from the omitted parts, or even exchanging Euclid's proofs with those found in the A*lmagest*.[64] While his efforts prove that he was a very skilled mathematician, his reworking of the text makes it difficult to compare his Hebrew version to possible Arabic sources. As the Arabic original of the *Midrash ha-Ḥokhmah* is lost, his method of translating can only be determined indirectly. What can be analysed is the consistency of his mathematical terminology throughout the work, including the correspondence.

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[62] Lévy, "Mathematics in the *Midrash ha-Hokhmah* of Judah ben Solomon ha-Cohen." See also his overview "Les éléments d'Euclide en Hébreu (XIIIe-XVIe siècles)," 81. An edition based on an incomplete and contaminated manuscript of al-Nayrīzī's commentary was published by Besthorn and Heiberg, *Codex leidensis, 399, 1*. A critical edition of book I of al-Nayrīzī's commentary was published by R. Arnzen: *Abū l-ʿAbbās an-Nayrīzīs Exzerpte aus (Ps.-?) Simplicius' Kommentar zu den Definitionen, Postulaten und Axiomen in Euclids Elementa I*. An English translation was rendered by Anthony Lo Bello: *The Commentary of Al-Nayrizi on Book I of Euclid's Elements of Geometry* and *The Commentary of Al-Nayrizi on Books II-IV of Euclid's Elements of Geometry*.
[63] MS. Vatican Ebr. 338, fol. 189v.
[64] Lévy, "Mathematics in the *Midrash ha-Hokhmah* of Judah ben Solomon ha-Cohen."

For the largest part the Hebrew mathematical language used in Judah's translation of the *Elements* and that of the correspondence are in agreement. It is clear that both texts were rendered by the same translator. But there are also differences in terminology. These differences concern several terms that had already been labelled as problematic in the correspondence. Our starting point for a comparison of vocabulary will be Judah's Hebrew translation of the beginning of book I. The Greek original of the *Elements* begins with a set of definitions, postulates and common notions in which fundamental terms of Euclidean geometry are being presented and defined in a logical and structured order. In Judah's Hebrew version these rules are combined in *46 premises (haqdamot)*.[65] The first five premises read thus:[66]

1. הנקודה היא שאין לה חלק.
2. הקו הוא אורך בלא רחב ושתי קצוותיו שתי נקודות.
3. הקו הישר הוא המתוח בהקבלת כל אחת משתי נקודות שבשתי קצוותיו האחת כנגד האחרת.
4. הפשוט הוא שיש לו אורך ורוחב בלבד. ושתי קצוות הפשוט קוים.
5. הפשוט המתוח הוא המתוח בהקבלות הקוים שבשתי קצוותיו מקצתן למקצת.

The English translation of these definitions reads:

1. A point is that which has no part.
2. A line is breadthless length, and the two extremities of a line are points.
3. A straight line is a line which stretches out in such a way that any two points on its two extremities lie parallel to one another.

[65] This presentation differs from both Ibn Sīnā's and al-Nayrīzī's versions. See: Lévy, "Mathematics in the *Midrash ha-Hokhmah* of Judah ben Solomon ha-Cohen," 310.
[66] Parma, Biblioteca Palatina, MS. 2769 (De Rossi 421), fol. [99r].

4. A surface is that which has length and breadth only, and the two extremities of a surface are lines.
5. A plane surface is a surface which lies evenly with the straight lines on its two extremities.

Already the second definition deviates from the terminology of the correspondence. While the terms for ‘line’ and ‘point’ are consistent with the correspondence, the word meaning ‘end’, or ‘extremity’ is קצה (*qatseh*). But when describing the ends of lines in the correspondence, Judah calls them ראש (*rosh*).[67] Also the following definition shows differences. Again, the word for ‘extremity’ is *qatseh*, but here Judah also uses a different word for ‘parallel’ than the two terms described above. In the correspondence, a parallel line was called *nokheḥi* or *ke-neged*. Here the noun for ‘being parallel’ is הקבלה (*haqbalah*). The forth definition treats the term ‘surface’, which in the correspondence was always called *sheṭaḥ*. Here he uses the word פשוט (*pashuṭ*). Again, the ‘extremities’ of the surface are called *qitswot*.

In contrast, the following five definitions show a great resemblance to the terminology of the correspondence:

6. הזוית השטוחה היא נטיית שני קוין האחד אל השני וריווחן על שטח וחיבורן לא על יושר.
7. כשהשני קוין הסובבין בזוית ישרים יאמר לאותה הזוית ישרת הקוין.
8. כשיעמד קו ישר על קו ישר והיו השתי זויות שמשני צדדי הקו העומד שוות כל אחת מהן היא זוית נצבת.
9. הקו העומד על הקו יאמר לו עמוד על הקו שהוא עומד עליו.
10. הזוית שהיא יותר גדולה מנצבת תקרא מרווחת והקטנה מנצבת תקרא חדה.

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[67] ‘על ראשו’, ‘ראשי הקוין’ (paragraph 22).

The English translation reads:

6. A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.
7. When the two lines containing the angle are straight, the angle is called rectilineal.
8. When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is right.
9. The line standing on the other is called a perpendicular to that on which it stands.
10. The angle greater than a right angle is called an obtuse angle, and an angle less than a right angle is called an acute angle.

In definition 6 the word used for 'plane' changes to *sheṭaḥ*, which is used in the correspondence, and accordingly the 'plane angle' is called *zavit sheṭuḥah*. The expression לא על יושר (*lo ʿal yosher*; *not in a straight line*) is the negative equivalent of the expression על יושר (*ʿal yosher*; in a straight line) that Judah frequently uses in the correspondence.[68] 'To contain' an angle is just like in the correspondence expressed by the verb סבב (*savav*), a 'right angle' is called *zavit nitsevet*, a 'perpendicular' is called *ʿamud*, and an angle 'less than a right angle' is קטנה מנצבת (*qeṭanah mi-nitsevet*).[69]

The following definitions, too, resemble the language that Judah used to translate his discussion with the philosopher. The terms *tsurah*, *ʿagulah*, *merkaz*, '*alakhson*, *ḥatsi ʿagulah*, *meshulash*, *tselaʿ* and *merubaʿ* are defined in the exact way that Judah uses them in the correspondence. It seems that after the first few sentences, the author may have changed his mind about his wording and chosen a different terminology. But premise 32 shows yet another deviation, both from the terminology of the correspondence and from that of the earlier definitions. It reads:

[68] Cf. paragraph 3: ונוציא אותן הקוין על יושר באין חקר; paragraph 9: ונוציאם על יושר באין חקר. ונגיע נֹדּ, ונוציאנו על יושר; also paragraphs 21, 23, 28 and 30.
[69] Cf. paragraphs 7 and 11.

32. והקוין הנגדים הם הקוין שיהיו על שטח אחד ואם יוצאו בשני הצדדין לא יפגשו וא׳ע׳פ׳ שיוצאו לאין סוף.

32. Parallel lines are lines which, being in the same plane and being produced in both directions, do not meet one another, even though they are being produced indefinitely.

'Parallel lines' are here קוין נגדיים (*qavin negdiyim*). As observed above, 'parallel' is expressed by two different terms in the discussion of the platonic solids, but not by the adjective *negdi*, which is derived from *neged*. Since in *premise* 5 Judah chose the word *haqbalah* for 'being parallel', here one would expect a word derived from the same root, namely *maqbil* (מקביל). Instead, he chooses a term that has appeared in neither of the texts before.

Of special interest is also the last part of the sentence, *being produced indefinitely* (יוצאו לאין סוף; *yuts'u le-'en sof*). The term *'en sof* literally means 'without end'. In the correspondence Judah uses a different term to express the notion of 'infinite' or 'indefinite'. There, lines are 'being produced' באין חקר (*be-'en ḥeqer*), which literally means 'without examination'.[70] While the expression *'en sof* is used with the same meaning by Abraham Bar Hiyya in his mathematical writings,[71] the biblical expression *'en ḥeqer* appears to be specific to Judah's own mathematical terminology. It looks as though the two terms were used by Judah interchangeably, yet in different places; one in the correspondence, the other in the translation of the *Elements*. But the term *ḥeqer* does also appear in his translation of the *Elements*, in *premise* 35:

35. ושיוציא קו ישר שיש לו חקר מקו ישר סמוך לו.

35. To produce a finite straight line continuously in a straight line.

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[70] Cf. paragraphs 3, 9, and 12.
[71] Sarfatti, *Mathematical Terminology*, 76.

‘Finite’ is here translated as שיש לו חקר (*she-yesh lo ḥeqer*; *which is examinable*). The same expression, *she-yesh lo ḥeqer,* is also used in the part of the encyclopaedia dealing with natural philosophy. Resianne Fontaine argues that this expression, which seems to be used exclusively by him, is one of the terms that Judah deliberately chose in order to distinguish his writings from those of Samuel ibn Tibbon. In his translation of Maimonides’ *Guide for the Perplexed* Samuel uses the expression בעל תכלית (*baʿal takhlit*) to express the concept of ‘finite’. Again, it is difficult to determine with certainty what caused the differences in terminology between the correspondence and the translation of the *Elements* on the one hand, and within different sentences of the *Elements* on the other.

One possible explanation is that the differences reflect the fact that different expressions were used in Judah’s Arabic *vorlage*. In fact, the Greek text of the *Elements* has two different terms to express the concepts of ‘finite, determinate’ and ‘infinite, indeterminate’. In definition 23 of the first book, which is the equivalent to Judah’s *premise* 32, the term *apeiron* denotes ‘indeterminate’, while in postulate 2 (equivalent to *premise* 35) the term *peperasmenon* denotes ‘finite’.[72] The inconsistencies we encounter in Judah’s 46 *premises* may therefore be owed to the text he translated. However, there is no evidence to suggest that Judah read the original Greek text or even knew Ancient Greek. Furthermore, all Arabic translations and adaptations of the *Elements* that we consulted are consistent in their terminology: the word *apeiron* in definition 23 is translated as ‘without end’, and the word *peperasmenon* in the second postulate is translated as ‘with end’. The term that denotes ‘end, extremity’ in both cases is one and the same in the Isḥāq/Thābit version of the text, in al-Nayrīzī’s commentary on the *Elements* and in Ibn Sīnā’s adaptation of the work: *nihāya* (نهاية),

[72] See *Euclid’s Elements of Geometry*, 7.

which originally means ‘utmost possible point’.[73] Thus, in all likelihood Judah’s use of two different terms, *sof* and *ḥeqer,* to denote ‘end’ or ‘examination’, was not caused by his *vorlage.* This may point to the fact that when translating the Arabic text, Judah simply used whichever term first came to his mind, being unaware of any inconsistencies. But it is also possible that the change of expressions bears witness to the chronological development of Judah’s own, exclusive, mathematical terminology.

Another difference in expressions can be found in the wording of the first proposition of book I of the Elements, which follows immediately after the 46 *premises.* There we read: *And from the point of section, which is G, we draw the two lines GB and GA.*[74] Here ‘point of section’ is called נקודת החצבות (*nequdat ha-ḥatsivut*). In the correspondence the same root, *ḥatsav,* was also used to express ‘to cut’, but there the term for ‘point of section’ was *meqom ḥatakh*, not *nequdat ḥatsivut.*

As book I of the *Elements* treats only plane geometry, we have to turn to other books in order to compare the terms used for other concepts. The term for ‘ratio’ can be found in book V, which treats proportions. Just like in the correspondence, the word used to express ‘ratio’ is *yaḥas*. So we read in proposition V.4: *We say that the ratio of HB to ZD is like the ratio of AB to GD.*[75] The wording is exactly like that found in the correspondence: *Since the ratio OH to HN is like the ratio CT to TN.*[76]

Solid figures are discussed in books XI to XIII. Propositions 25 to 39 of book XI concern parallelepipeds. Throughout, the Hebrew term being used is הגופני הנגדיי השטחים (*ha-gufani*

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[73] See “Arabic Text of Euclid’s Elements. Translated by Isḥāq ibn Ḥunayn and Revised by Thābit ibn Qurra. Text from the Uppsala Manuscript”; *Abū l-ʿAbbās an-Nayrīzīs Exzerpte aus (Ps.-?) Simplicius’ Kommentar zu den Definitionen, Postulaten und Axiomen in Euclids Elementa I*, 35, 44; Ibn Sīnā, *al-Riyāḍīyāt*, 1: Uṣūl al-handasah:18–19.
[74] ונוציא מנקודת החצבות והיא ג׳ שני קוי ג׳ב׳ ג׳א׳.
[75] Fol. [116v]: נאמר כי יחס ה׳ב׳ אל ז׳ד׳ כיחס א׳ב׳ אל ג׳ד׳.
[76] כי יחס עׄחׄ אל חׄנׄ כיחס צׄטׄ אל טׄנׄ (paragraph 15).

ha-negdiye ha-shṭaḥim; the solid with parallel surfaces).[77] The term of the correspondence, *ha-shaweh ha-shṭaḥim*, does not appear in these propositions.

Regular polyhedra are treated in book XIII. In propositions 13-17, it is explained how to construct the five regular polyhedra and to circumscribe spheres about them. In Judah's version, the beginnings of these propositions read:

13. כיצד נעשה מחודד בעל ארבע תושבות משולשות שוות הצלעים יסוב עליו כדור מוצע [...]
How do we construct a pyramid having four equilateral triangles as bases, which a given sphere comprehends.

14. כיצד נעשה מכעב יסוב עליו כדור ידוע
How do we construct a cube, which a known sphere comprehends.

15. כיצד נעשה גופני בעל שמונה תושבות משולשות שוות הצלעים יסובבנו כדור ידוע
How do we construct a solid having eight equilateral triangles as bases, which a known sphere comprehends.

16. כיצד נעשה צורה בעלת עשרים תושבות משולשות יסובבנה כדור ידוע
How do we construct a figure having twenty equilateral triangles as bases, which a known sphere comprehends.

17. כיצד נעשה כופני בעל שתים עשרה תושבות מחומשות שוות הצלעים יסובבנו כדור ידוע
How do we make a solid having twelve equilateral pentagons as bases, which a known sphere comprehends.

The terms used here for the polyhedra are similar to those used in the correspondence, but the terminology is more precise; instead of just being called 'of four [eight, twelve, etc.]', the solids are exactly defined by the number and form of their sides. The difference may be owed

[77] Fols. [123v]-[126v].

to the different Arabic originals Judah is translating. Since in the discussion in the correspondence he treats the definition of the polyhedra as known, Judah does not have to make precise descriptions and shortens the terms. An exception is the cube; like in the correspondence it carries the Arabic name *mukaʿab*. Here, too, Judah does not use the Hebrew term *meʿuqav* that was employed by his predecessors. Another point of interest is the fact that the solids are called both גופני (*gufani*; *solid*), as in prop. 15 and 17, and צורה (*tsurah*; *figure*), as in prop. 16. Furthermore, the 'given' sphere is first called מוצע (*mutsaʿ*; *given*), but afterwards ידוע (*yaduaʿ*; *known*). Exactly the same phenomenon we saw in the correspondence; in the philosopher's question the five solids were called חמשה גופנים (*ḥamishah gufanim; five solids*) and the sphere כדור ידוע (*kadur yaduaʿ; known sphere*), but in Judah's answer they were called חמש צורות (*ḥamesh tsurot; five figures*) and כדור מוצע (*kadur mutsaʿ; given sphere*).

The phenomenon of (seemingly) interchangeable words is not restricted to these terms. In the correspondence we saw that there was no clear distinction between *nagaʿ* and *mishesh*. Both were used to express 'to touch'. The same is true of the translation of Euclid's *Elements*. Preposition XII.17 begins: *[There are] two spheres on one centre, and we want to inscribe in the greater sphere a polyhedral solid which does not touch the lesser sphere at its surface.*[78] The word used for 'to touch' is *nagaʿ*. But in proposition III.16 we find the sentence*: From this it is manifest that the straight line drawn at right angles to the diameter of a circle from its extremity touches the circle.*[79] Here the word for 'to touch' is once more *mishesh*. Furthermore, the word expressing 'extremity' in this sentence is *rosh*, the same term that was used in the correspondence. But *qatseh*, the term denoting 'extremity' in the definitions of book I, can also be found within

[78] Fol. [130v]: שני כדורים על מרכז אחד. ונרצה לעשות בכדור הגדול צורת רבת תושבות שלא תגע בשטח הכדור הקטן.
[79] Fol. [129r]: ושם נתבאר הקו היוצא מראש האלכסון על זוית נצבת כי הוא ממשש לעגולה.

the chapters on solid geometry. The first definition of book XI reads in Judah's version: *A solid is that which has length, breadth, and depth, and the extremities of a solid are surfaces.*[80] Here, an 'extremity' is once more called *qatseh*. The 'solid' is not *gufani* but *guf*, a term that appears neither in the correspondence nor in the rest of the translation of the *Elements*. Both use the term *gufani*. Remarkable is also the fact that although premise I.4 introduced the term *pashuṭ* to denote 'surface', the term used here is *sheṭaḥ*. In fact, throughout the rest of the book the term for 'surface' is *sheṭaḥ*, like in the correspondence. It seems that after using the word *pashuṭ* in the initial definition, Judah changed his mind and opted for *sheṭaḥ* instead.

Lastly, in the correspondence the words *ke-neged* and *nokheḥi* denoted 'parallel', while in the translation of book I (*premise* 3) the words *negdi* and *haqbalah* appeared. In the rest of the Hebrew translation of Euclid, 'parallel' is in fact expressed through several Hebrew terms side by side. A good example can be found in book I. Propositions I.27-31 all treat parallel straight lines. In proposition I.27 and the beginning of I.28 the Hebrew term for 'parallel' is *ke-neged*,[81] and *le-neged* in I.31.[82] It is *negdi* in the second part of I.28[83] and in I.29.[84] In I.30 we find the verb *hiqbil* alongside *negdi* in one and the same sentence: *If the two lines AB [and] GD are parallel to line HZ, then lines AB [and GD] are also parallel [to one another].*[85] While in the first part of the sentence '[the lines] are parallel' is expressed through *yaqbilu [ha-qavin]*, in the second part of the sentence the parallel lines are called *negdiyim*. Within the work, Judah's preferred term for the concept appears to be *neged*, be it as a preposition or as the derived adjective *negdi*. It seems that only when he wishes to use 'being parallel' as a verb form, he is forced to derive a verb from a different root: *hiqbil*. If this is the case, the word *haqbalah* in

[80] Fol. [120rv]: הגוף שיש לו אורך ורחב וקומה וקצוי הגוף שטחים.
[81] Fols. [102v, 103r]: נאמר כי קו א׳ב׳ כנגד ג׳ד׳.
[82] Fol. [103r]: כשנרצה להעביר על נקודת א׳ קו ה׳ז׳ לנגד קו ב׳ג׳.
[83] Fol. [103r]: יהיו השני קוין נגדיים.
[84] Fol. [103r]: אם שני קוין נגדיים.
[85] Fol. [103r]: כשיקבילו שני קוי א׳ב׳ ג׳ד׳ לקו ה׳ז׳ גם שני קוי א׳ב׳ [ג׳ד׳] נגדיים.

the *46 premises* of book I would have to be read as *in their being parallel*. Be that is it may, the one term that cannot be found in propositions I.27-31 is *nokheḥi,* which appears solely in the correspondence. The absence of this term constitutes a genuine difference between the terminology of the Hebrew translation of the *Elements* and that of the correspondence.

In summary it can be said that the terminology of the correspondence shows a great resemblance to that of the Hebrew translation of the *Elements*, but significant differences can be made out. Striking is the different vocabulary used for ‘extremity’, ‘plane, surface’, ‘parallel’, ‘infinite’, ‘parallelepiped’ and ‘point of section’. But it turns out that regarding these terms, the translation of the Elements itself cannot be seen as a homogenous text. ‘Extremity’ is called *qatseh* in the definitions of books I and XI, while in the propositions it is called *rosh*, the term used in the correspondence. ‘Plane’ is defined as *pashuṭ* in book I, while in the rest of the work it is expressed by *sheṭaḥ*, the same Hebrew word used in the correspondence. ‘Parallel’ is expressed by the terms *negdi* and *hiqbil* in the 46 *premises*, terms which appear also in the rest of the work, but there alongside *ke-neged*. In the correspondence, too, there are competing terms expressing ‘parallel’, but they are *ke-neged*, which does not appear in the 46 premises, and *nokheḥi*, which takes the place of the term *negdi* used the Hebrew version of the *Elements*. ‘End’ is expressed in book I both by *sof* and *ḥeqer* side by side, while in the correspondence the term *ḥeqer* is used. ‘Parallelepiped’ naturally does not appear in the first books of the Elements. The term used in book XI, *gufani ha-negdiye ha-shṭaḥim,* differs considerably from the *gufani ha-shaweh ha-shṭaḥim* of the correspondence. While both texts agree on the verb *ḥatsav* meaning ‘to cut’, the correspondence deviates from this terminology in calling a ‘point of section’ *meqom ḥatakh*, while in the *Elements* the term corresponds with the verb form: *nequdat ḥatsivut*. Finally, while in the definitions of book XI a ‘solid’ is called *guf*, the term used elsewhere in the book is *gufani*.

It is very unlikely that all of these discrepancies are owed to differences in terminology of Judah's Arabic sources. While his abilities as a translator may be debatable, it also seems unlikely that he simply did not feel the need for creating a clear and consistent mathematical language, especially when we consider his substantial mathematical skills. Rather, the findings suggest that the differences in terminology reflect a gradual development of Judah's specific mathematical terminology. In this case, the correspondence and the abbreviation of the *Elements* may have been translated at different points in time. But the findings also indicate that the *Elements*, too, were probably translated in different stages; the premises of book I (and perhaps the definitions of book XI) being apart from the rest of the work. The differences in terminology would therefore reflect three distinct stages in the development of Judah's mathematical language.

The Euclidean Terminology of MS. Hunt. 46

When comparing the terminology of the correspondence with that of the *Midrash ha-Ḥokhmah*, we were referring to the seven manuscripts that contain (large parts of) the mathematical part of the work. However, in the Biblioteca Palatina in Rome and in the Bodleian Library in Oxford there are two manuscripts that also contain the beginning of the *Elements*.[86] Both manuscripts hold a short text consisting only of the *46 premises* and proposition I.1, in other words, they contain book I of the Elements to proposition 1. Since in structure and formulation their text is identical with that of the known copies of the *Midrash ha-Ḥokhmah*, Tony Lévy has suggested that they contain a different Hebrew translation of the

[86] Oxford, Bodleian Library, Ms. Hunt. 46, fols. 13r-14r, and Rome, Biblioteca Casanatense, MS. Ebr. 2916, fols. 182v-183r.

Arabic original, be it of the Arabic *Midrash ha-Ḥokhmah* or of an Arabic composition by another author that served as Judah's *vorlage*.[87] When comparing this version of the text with that of the correspondence, we find that their terminology is in fact much closer to that of the correspondence than that found in the known copies of the *Midrash ha-Ḥokhmah*.

In the Oxford copy of this text the first five definitions of book I read thus:[88]

1. הנקודה היא שאין לה חלק.
2. הקו הוא ארך כל רחב ושני ראשי הקו נקודות.
3. הקו הישר הוא המתוח בנוכח כל אחת מהשתי נקודות שבשני ראשיו האחת כנגד האחרת.
4. השטח הוא שיש לו ארך ורחב בלבד. ראשי השטח קוים.
5. הפשוט המושטח הוא המתוח בהקבלת הקוים שבראשיו ממקצתם למקצת.

The great similarity between this text and the premises in the *Midrash ha-Ḥokhmah* is striking. But here, in definitions 2, 3 and 5, the 'extremities' of lines and surfaces are not called *qatseh* but ראש (*rosh*), as is the case in the correspondence. In definition 3 the term for 'being parallel' is not *haqbala*, but נוכח (*nokhaḥ*), derived from the same root that Judah uses in his discussion of the platonic solids. The term for 'surface' in definition 4 is not *pashuṭ* but, just like in the correspondence, *sheṭaḥ*. Only in definition 5 the terminology changes; instead of *sheṭaḥ* we find *pashuṭ* for 'surface', and instead of *nokhaḥ*, 'being parallel' is called *haqbalah*. These are the exact terms that were found in the translation of the *Elements* in the *Midrash ha-Ḥokhmah*.

The following *premises* (6 to 10) read:

6. הזוית השטוחה היא הטיית שני קוים האחד אל האחד והרוחתם על שטח וחבורת על זולת יושר.
7. כשהשני קוים הסובבים בזוית ישרים יאמר לאותה הזוית ישות [?] הקוין.

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[87] Lévy, "Mathematics in the *Midrash ha-Hokhmah* of Judah ben Solomon ha-Cohen," 303–304.
[88] Oxford, Ms. Hunt. 46, fol. 13r.

8. כשיקום קו ישר [על קו ישר] והיו השתי זויות שמצדי הקו העומד שות בין כל אחת מהן היא זוית נצבת.

9. הקו הקם יאמרו לו העמוד על הקו שהוא קם עליו.

10. הזוית שהיא גדולה מנצבת יאמר לה המרווחת. ושהיא קטנה מנצבת יאמר לה החדה.

Again there are deviations from the terminology of the *Midrash ha-Ḥokhmah*: הטייה (*haṭayah*) instead of נטייה (*neṭiyah*) for 'inclination', הרוחה (*harwaḥah*) instead of ריווח (*riwuaḥ*) for 'distance', and על זולת יושר (*ʿal zulat yosher*) instead of לא על יושר (*lo ʿal yosher*) for 'not straight'. But these differences concern only nuances. In the first two examples the words differ only in the stem, while the roots are the same, the third example uses a different word for the negation, while the noun is not changed. As these words do not appear in the correspondence, the differences cannot give us any clues about the authenticity of the text. But in definitions 8 and 9 the text shows another similarity to the terminology of the correspondence: the verb expressing that a perpendicular is 'standing' on a line is קם (*qam*), instead of the verb עמד (*ʿamad*) that was used in the version of the *Midrash ha-Ḥokhmah*. When describing the construction of a perpendicular in the correspondence, Judah writes: *We produce the line up to the surface of the sphere, and erect on its end a perpendicular*.[89] Here, too, he uses the root קום (*qum)* to express the action of 'making a line stand', not עמד (*ʿamad*).

The premises that follow show little difference to the terminology of the *Midrash ha-Ḥokhmah*. Again, most of these differences concern stems, while the roots remain unchanged. In this text, too, the term used for 'parallel' in *premise* 32 is *negdi*. The word '*heqer'*, as we have said typical of Judah's very own terminology, also appears in *premise* 35; but here the clause שיש לו חקר (*she-yesh lo ḥeqer*) is replaced with בעל חקר (*ba ʿal ḥeqer*).

[89] ונוציא הקו עד שטח הכדור ונקים על ראשו עמוד (paragraph 6).

Finally, in proposition 1 the text of the additional two manuscripts deviates from both the terminology of the correspondence and from that of the *Midrash ha-Ḥokhmah.* The term 'point of section' is here expressed neither by *meqom ḥatakh* as in the correspondence, nor by *nequdat ḥatsivut* as in the *Midrash ha-Ḥokhmah.* It is called נקודת כריתות (*nequdat kritut*). To express 'to cut, intersect' in this case the root כרת (*karat*) was used – a term not appearing in the texts we have seen before.

All in all the similarities between the terminology of the Oxford manuscript and that of the correspondence are so great that it is safe to assume that both translations were rendered by the same scholar, Judah ben Solomon ha-Cohen. When comparing the correspondence to the translation of the *Elements* in the *Midrash ha-Ḥokhmah*, we found that major differences between the two texts lay in the terms they used for 'extremity', 'plane, surface', parallel', 'parallelepiped', and 'point of section'. While the term 'parallelepiped' does not appear in this second version of the 46 *premises*, the others do. In this manuscript the word for 'extremity' is *rosh,* as is the case in the correspondence. Like in the correspondence, the term for 'plane' is *sheṭaḥ.* The word for 'parallel' is *nokheaḥ*, a word that does not appear in the *Midrash ha-Ḥokhmah* at all, but that is used in the correspondence. Only *nequdat kritut*, the term for 'point of section', differs from both the terminology of the correspondence and that of the *Midrash ha-Ḥokhmah.* To this list can be added the terms for 'to stand' and 'to erect', *qam* and *heqim;* the correspondence and the second version of the 46 *premises* use the root קום to express these terms, while in the *Midrash ha-Ḥokhmah* the root used is עמד.

The main differences in the terminology are shown in the following table:

Term	MH, I.1	MH, main text	MS. Oxford	Correspondence
extremity	*qatseh*	*rosh*	*rosh*	*rosh*

Term	MH, I.1	MH, main text	MS. Oxford	Correspondence
surface	*pashuṭ*	*sheṭaḥ*	*sheṭaḥ, pashuṭ*	*sheṭaḥ*
parallel	*negdi, hiqbil*	*negdi, hiqbil, ke-neged*	*nokheaḥ, hiqbil*	*ke-neged, nokheaḥ*
point of section	*nequdat ḥatsivut*		*nequdat kritut*	*meqom ḥatakh*
to stand	*ʿamad*	*ʿamad*	*qum*	*qum*
parallelepiped		*negdiye ha-shṭaḥim*		*shaweh ha-shṭaḥim*

If we assume that the differences in the terminology were not added by later copyists, then it is clear that the table reflects different stages in the development of Judah's mathematical terminology. It is difficult to make out in what way this terminology developed. If Judah translated the parts of the text in the order that they appear in the *Midrash ha-Ḥokhmah*, the first stage of the vocabulary would be that of MH I.1, followed by MH, main text and then by the correspondence. The transition would have been from *qatseh* to *rosh,* from *pashuṭ to sheṭaḥ, from negdi, hiqbil to ke-neged, nokheaḥ* and from *ʿamad* to *qum.* Due to its similarity to the correspondence, MS. Oxford could then form the last stage. The appearance of the terms *pashuṭ* and *hiqbil* in MS. Oxford would contradict these results only seemingly; they could be interpreted as interference of MH I.1, on which this second translation was based. But also the opposite is possible; the correspondence and MS. Oxford could be the earlier stages in the development, of which MH I.1 formed the last stage. The differences in MH I.1 and MH, main text could have been caused by the different types of text they present; when translating definitions literally, Judah used a new vocabulary, but when paraphrasing he fell back to the vocabulary he had used earlier.

In summary, it can be stated that the analysis of the mathematical terminology used in the correspondence yields the following results: The terminology that Judah uses is for the greatest part based on that of Abraham Bar Hiyya. He may have acquired this terminology by studying Bar Hiyya's works directly, or by reading Samuel Ibn Tibbon's Hebrew translations of different Arabic texts containing mathematical terminology, of which he was certainly aware. But Judah also made changes to the terminology of his predecessors and introduced his own specific vocabulary. This vocabulary was not static, but changed gradually in the course of time. We can make out at least three different stages in the development of Judah's terminology, but it is not possible to decide in which order these stages took place. However, it is possible to conclude that the Hebrew translation of the mathematical part of the correspondence and the Hebrew translation of the mathematical part of the *Midrash ha-Ḥokhmah* in all probability were not carried out within the same period of Judah's working life.

Spherical astronomy

In the second part of the correspondence, Judah professedly renders a Hebrew translation of the philosopher's discussion of an astronomical problem, with which Judah himself clearly disagrees. But once again, it is not the astronomical content of the discussion that will be analysed in this chapter but the language that is used to transfer this content from one cultural setting to another. Judah's Hebrew astronomical terminology can give us clues as to the strategy he followed when finding Hebrew equivalents for Arabic technical terms, as it reflects on possible Hebrew sources that he consulted when forming his own vocabulary. Comparing the terminology used in the correspondence with that used elsewhere in the *Midrash ha-Ḥokhmah* will help in reconstructing the formation history of the encyclopaedia.

Once again this research depends heavily on the study of Arabic astronomy, since the first astronomical texts written in Hebrew relied entirely on Arabic sources. At the time the correspondence between Judah and the emperor's philosopher took place, the vocabulary of astronomical terms in Arabic had been standardized for a large part. Although different authors might have used slightly different Arabic expressions to convey the same concept, most of the technical vocabulary remained the same in different treatises. It is therefore possible to reconstruct the Arabic astronomical terms that Judah is translating with some accuracy, even though the Arabic original of the correspondence is lost.

Judah's question to the emperor's philosopher pertains to a standard problem of spherical astronomy: the geometrical proof for the differences between right and oblique ascensions, that is, the differences between the rising times of certain arcs of the ecliptic depending on the position of the observer. In his answer the philosopher makes use not only of arithmetical terms, such as *to add, to subtract, to multiply, to divide, unknown*, but he also refers to numerous concepts of spherical astronomy: *arc, beginning of Aries, ascendant, right* *ascensions, oblique ascensions, celestial sphere, circle, declination, degree, distance, ecliptic, celestial equator, terrestrial equator, horizon, latitude of a locality, meridian, pole, to rise, rising amplitude, sine, cosine, sinus totus, solstice, sphaera obliqua, sphaera recta, zenith,* and *zodiacal sign*.

When finding Hebrew equivalents for these astronomical terms, there were only a few texts available in Hebrew that Judah could rely on as a source. The first Hebrew works dedicated exclusively to astronomy were authored by Abraham Bar Hiyya (c.1070-c. 1136) and Abraham ibn Ezra (1092—1167). While philosophical and scientific works rendered in Hebrew after the pioneering work of these two authors did touch on astronomical subjects, the next works that have come down to us which deal solely with astronomy, be it original compositions or translations of Arabic works, were rendered more than 60 years after Ibn Ezra's death. It was between 1231 and 1235 that Jacob Anatoli, son-in-law of Samuel ibn Tibbon, translated Ptolemy's *Almagest* from the Arabic into Hebrew as *Ḥibbur ha-gadol ha-niqra al-Magesti.* As Mauro Zonta has shown, Anatoli relied not only on an Arabic source in his translation but also consulted the Latin translation made by Gerard of Cremona when the Arabic text was difficult to understand.[90] At the time, Anatoli was residing in Naples, where he had been employed as a physician by Emperor Frederick II. In his *Malmad ha-Talmidim* ('A Goad to Scholars'), a collection of homilies containing allegorical and philosophical exegesis, Anatoli even reports of several discussions on philosophical subjects that he had with both the emperor and his court philosopher Michael Scot. In addition to the *Almagest*, Anatoli rendered Hebrew versions of Ibn Rushd's *Epitome of the Almagest*, which is lost in the Arabic original, and al-Farghānī's *Astronomy*. It was at the same court, about a decade later, that the

[90] Zonta, "La tradizione Ebraica dell'*Almagesto* di Tolomeo."

next astronomical work was rendered in Hebrew: the astronomical part of Judah ha-Cohen's *Midrash ha-Ḥokhmah*.

Thus, if we leave aside any treatises that only marginally touched upon astronomical questions while mainly dealing with other topics, there were only three authors we know of who rendered complete astronomical works in Hebrew before Judah ha-Cohen translated the *Midrash ha-Ḥokhmah* into Hebrew. Two of them had been active more than a century before him, while the third was Judah's contemporary. Both in the correspondence and in the other parts of his encyclopaedia dealing with the subject, Judah shows a thorough knowledge of spherical astronomy. As a skilled theoretical astronomer, he would have been interested in any Hebrew astronomical works that were accessible to him. Yet, the only Jewish authority that is explicitly mentioned in the astronomical part of the *Midrash ha-Ḥokhmah* is the otherwise unknown Hispano-Jewish astronomer David ibn Naḥmias, for whose teachings Judah's encyclopaedia is our only source.[91] If Judah relied on works entirely dedicated to astronomy as an inspiration for his own astronomical writings, we might find parallels in the vocabularies used by the three authors mentioned above. While Judah ha-Cohen and Jacob Anatoli could draw on earlier works when finding Hebrew expressions for astronomical terms, their predecessors had to create a completely new Hebrew astronomical vocabulary. In order to better understand Judah's strategy when creating his Hebrew terminology, we will compare the technical terms that he uses in the correspondence with the astronomical vocabularies of Abraham Bar Hiyya, Abraham ibn Ezra and Jacob Anatoli. Furthermore, in order to establish whether Judah's terminology is consistent throughout his encyclopaedia,

[91] Langermann, "Some remarks on Judah ben Solomon ha-Cohen," 375.

we will compare the technical terms of the correspondence with that of the rest of the astronomical part of the *Midrash ha-Ḥokhmah*.

Comparison

As regards mathematics, Abraham Bar Hiyya and Abraham ibn Ezra were interested in different branches of these sciences. While Bar Hiyya dedicated his mathematical treatises to geometry and some arithmetic, Ibn Ezra's mathematical works deal with arithmetic and combinatorial analysis. Therefore, a direct comparison between the technical vocabularies of Abraham Bar Hiyya, Abraham ibn Ezra, Jacob Anatoli and Judah ha-Cohen was impossible. But as the astronomical question that Judah poses to the Christian philosopher pertains to fundamental principles of spherical astronomy which are treated by Bar Hiyya, Ibn Ezra and Anatoli alike, it is possible to make a comparison of the astronomical terminology used by the four authors. For the Arabic terminology, we consulted articles on Arabic astronomical terminology by E.S. Kennedy, D.A. King, W. Hartner, and F. De Blois. In addition, we consulted E.W. Lane's *Arabic-English lexicon*,[92] and the glossary of astronomy that can be found in the astronomical handbook *al-Zīj al-jāmiʿ* by the Iranian astronomer and mathematician Kūshyār ibn Labbān (written in 1020-5).[93] For Bar Hiyya's terminology we consulted his astronomical treatises *Tsurat ha-ʾArets* ('The Shape of the Earth') and *Ḥeshbon Mahalakhot ha-Kokhavim* ('Computation of the Motions of the Stars).[94] While the first treatise describes the form and structure of the universe, the second treatise is dedicated to astronomical calculations. This will be our main source for Bar Hiyya's astronomical terminology. For Ibn Ezra's terminology

[92] Lane, *Arabic-English Lexicon*, 1863, repr., Cambridge 1984.
[93] Edited and translated by Mohammad Bagheri, "Kūshyār ibn Labbān's Glossary of Astronomy."
[94] The first treatise was printed in Offenbach in 1720: Hiyya, *Sefer tsurat ha-arets*. A Spanish translation was rendered in 1956 by J.M. Millás Vallicrosa: *La obra Forma de la tierra de R. Abraham Bar Hiyya ha-Bargeloní*. The second treatise was edited and translated into Spanish by Millás Vallicrosa in 1959: *Séfer Ḥešbón mahlekot ha-kokabim*.

the following works were used: Firstly, his treatise on the use of the astrolabe, *Sefer Keli Neḥoshet*. The author rendered three different versions of the treatise, of which two exist only in manuscript form. For our study we used the first version, which was written in Lucca in 1146 and printed in Königsberg in 1845.[95] Our second source is his Hebrew translation of a commentary on the astronomical tables of Abū Jaʾfar Muḥammad b. Mūsā al-Khwārazmī (first half of the 9th century). The commentary was authored, probably in the tenth century, by the Andalusian scholar Ibn al-Muthannā. Ibn Ezra translated it into Hebrew in 1160, while he was staying in England.[96] In doubtful cases, also the two versions of his astrological work *Sefer ha-Ṭeʿamim* ('The Book of Reasons') were consulted. The earliest version was written in Beziers in 1148.[97] For Jacob Anatoli's astronomical vocabulary we consulted three of his works, which were all rendered in Naples between 1231 and 1235: firstly, his Hebrew translation of Ptolemy's *Almagest.* The text exists only in manuscript form.[98] Mauro Zonta has shown that Anatoli used both Arabic and Latin sources in his translation: the structure of his text follows the Latin version of Gerard of Cremona, but when translating, Anatoli followed the Arabic sources of Gerard's version. When the Arabic text was difficult to understand, he amended the text according to the Latin version. Like Euclid's *Elements*, the *Almagest*, too, was translated into Arabic several times. The oldest translation was made by the otherwise unknown al-Ḥasan ibn al Quraysh at the beginning of the ninth century. This translation appears to be lost. A second translation was made directly from the Greek by Ḥajjāj ibn Yūsuf ibn Maṭar in 827. A third translation was made by Isḥāq ibn Ḥunayn, which again was revised

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[95] Ibn Ezra, *Sefer Keli neḥoshet*. For a study of the different versions of the text see Sela, *Abraham ibn Ezra and the Rise of Medieval Hebrew Science*, 28–36.
[96] The text was edited and translated by B.R. Goldstein: *Ibn al-Muthannā's commentary*.
[97] Both versions were edited and translated by Shlomo Sela: *The Book of Reasons*.
[98] We consulted the manuscript Paris, Bibliothèque nationale, MS. hébr. 1019 (henceforth: MS. Paris 1019).

by Thābit ibn Qurra.[99] Following Gerard of Cremona's Latin version, Anatoli's text appears to be a mixture of the latter two Arabic translations.[100] The second source for Anatoli's vocabulary is his Hebrew translation of the *Compendium of the Science of the Stars* by the Arab philosopher Abū al-ʿAbbās Aḥmad ibn Muḥammad ibn Kathīr al-Farghānī (active in the second half of the 9th century). The Hebrew text exists only in manuscript form.[101] The Arabic text was printed in Amsterdam in 1669.[102] In this translation, too, Anatoli consulted both the Arabic original and Gerard of Cremona's Latin version. Our third source is his Hebrew translation of the *Compendium of the Almagest* by the Andalusian scholar Ibn Rushd (1126-1198). The Arabic original of the work is lost. The Hebrew translation consists of two major parts. The first part of the work has been edited by J. Lay in her doctoral thesis *L'abrégé de l'Almageste, attribué à Averroès, dans sa version hébraïque: étude de la première partie* (Paris, 1991). The second part exists only in manuscript form.[103] Lastly, the terminology of the correspondence will be compared with that of the astronomical part of the *Midrash ha-Ḥokhmah*. It consists of two treatises: Judah's paraphrases of Ptolemy's *Almagest* and of al-Biṭrūjī's *Principles of Astronomy*.[104] As the question that is raised in the correspondence, that is, the geometrical proof for oblique ascensions, is discussed in the second book of the *Almagest*, Judah's Hebrew rendering of this work will be our main point of focus. It has yet to be determined to which tradition of the Arabic *Almagest* Judah's Hebrew version is closest. In any case, Judah's Hebrew rendering appears to be less of a translation, but rather a paraphrase that sometimes rearranges the original text. Following the general structure of

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[99] An overview of these traditions and the existing manuscripts can be found in Zonta, "La tradizione Ebraica dell'*Almagesto* di Tolomeo," 327.
[100] Ibid., 342.
[101] We consulted the manuscripts Paris, Bibliothèque nationale, MS. hébr. 1022 (henceforth: MS. Paris 1022); and Mantua, Comunità Israelitica, MS. ebr. 4 (henceforth: MS. Mantua 4).
[102] This version was reprinted in 1986: *Jawāmiʿ ʿilm al-nujūm*.
[103] We consulted MS. Mantua 4.
[104] We consulted the manuscripts Vatican City, Biblioteca Apostolica Vaticana, MS. Ebr. 338 (henceforth: Ms. Vat. 338), and Oxford, Bodleian Library, MS. Mich. 551 (henceforth: MS. Mich. 551).

the *Almagest*, Judah tries to convey the astronomical theories and mathematical principles that can be found in the work, rather than the concrete data. Thus, he leaves out Ptolemy's tables and star catalogue and concentrates on transmitting the geometrical proofs that form the basis of these tables.

In the following, the Arabic terms that were commonly used to express the notions that appear in the correspondence will be presented. They will be compared the terms frequently used by Abraham Bar Hiyya, Abraham ibn Ezra, Jacob Anatoli, and lastly, Judah ha-Cohen himself in the books under consideration.

General arithmetical terms

The general arithmetical terminology of the four authors does not differ greatly in their astronomical writings. All four authors use the verb כפל as a translation of the Arabic verb ضرب (*ḍaraba*; 'to multiply'), and חילק as a translation of قسم (*qasama*; 'to divide'). The Arabic verb denoting 'to add' is زيد (*zāda*). Abraham Bar Hiyya uses the terms הוסיף, חיבר, צירף and קיבץ. Abraham ibn Ezra appears to use only the first two verbs, הוסיף and חיבר, in the writings under consideration, while Jacob Anatoli and Judah ha-Cohen make use of the first three terms, הוסיף, חיבר and צירף. Slight differences between the authors can be found regarding the term for 'to subtract', نقص (*naqaṣa*) in Arabic. Abraham Bar Hiyya uses the verbs גרע and הוציא for this notion. Abraham ibn Ezra, too, uses the verb גרע, but הוציא does not appear in his astronomical writings. Instead, he uses the verb חיסר as an alternative. Jacob Anatoli applies all three of these verbs, but in addition, he uses the rather unusual verb הפיל to denote 'to subtract'.[105] Judah ha-Cohen frequently applies the verb גרע, but on at least two occasions,

[105] The term הפיל does not appear in any text analysed in G. Sarfatti's *Mathematical Terminology*.

once in the correspondence and once in his summary of the *Almagest*, he also uses the verb הפיל.

There is another uncommon arithmetical term used by both Jacob Anatoli and Juda ha-Cohen. The Arabic term for ‘unknown’ is مجهول (*majhūl*). The participle is derived from the verb جهل (*jahila*), which originally denotes ‘being ignorant, silly, foolish’. We could not find an equivalent Hebrew term in Abraham Bar Hiyya’s writings. Abraham ibn Ezra refers to unknown quantities as מבוקש (‘sought, desired, demanded’),[106] which appears to be a translation of the Arabic term with the same meaning, مطلوب (*maṭlūb*). Jacob Anatoli, however, uses the term מוסכל to refer to an unknown quantity.[107] Like the term *majhūl*, the Hebrew participle is derived from a verb that originally denotes ‘to be foolish’, which is the biblical Hebrew verb סכל. While the verb הוסכל was already used by Judah and Samuel ibn Tibbon,[108] Jacob Anatoli appears to be the first person to use the participle in a mathematical sense. Judah ha-Cohen takes over Jacob Anatoli’s term. מוסכל appears both in the correspondence and in his summary of al-Biṭrūjī’s *Principles of Astronomy*.[109]

Zodiacal sign

The Arabic term برج (*burj*, plural بروج *burūj*) originally means ‘tower’, but in astronomy it refers to the twelve signs of the zodiac. The Hebrew translation of this term used by all four authors is מזל. This does not come as a surprise since the term had been in use long before

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[106] “והנה הרביעי מאלה המספרים איננו ידוע והוא המבוקש”, *Ibn al-Muthannā’s commentary*, 121. “והנשאר הוא המבוקש”, *Sefer Ha-Mispar*, 7.

[107] MS. Mantua 4, fol. 76r, ll. 22, 23.

[108] Judah ibn Tibbon uses the word in his Hebrew translation of Judah ha-Levi’s *Kuzari*. He translates the sentence “לאן אלממכן אלמחץ׳ מגהול בטבעה” as “כי האפשר הגמור מוסכל בטבעו”. Levi, *Sefer ha-Kuzari*, 120; “Kitāb al-radd wa-al-dalīl fī al-dīn al-dhalīl,” iv.1. In his translation of Maimonides’ *Guide for the Perplexed* Samuel renders “ואללה תעאלי לא חכת׳יר פי חקיקה וג׳ודה פיפהם מנה שי ויג׳הל שי אכ׳ר” as “והשם יתעלה אין רבוי באמתת מציאותו שיובן ממנו דבר ויוסכל דבר אחר”. *Dalālat al-ḥā’irīn*, 99 (I.60).

[109] “לפיכ׳ יהא ערך גיב קשת א׳ב׳כ׳ הידועה לגיב קשת ב׳כ׳ הידועה כערך ג׳יב קשת א׳ל׳ הידועה גם כן לג׳יב קשת ט׳ל׳ המוסכלת”, MS. Mich. 551, fol. 164v.

the first spherical astronomical treatises were written in Hebrew. Already in the bible the term מזלות in the plural denotes 'lodging places of the sun, zodiacal sign'.[110] In rabbinic writings the singular referred to 'zodiacal sign', or 'planet', but it also gained the additional meaning of 'destiny' – a connotation that the Arabic term *burj* does not have.

Celestial sphere, circle

One of the most important terms in spherical astronomy is the term denoting 'sphere' itself. It seems somewhat surprising that there are two different terms denoting the concept in medieval Arabic. When referring to the mathematical concept of 'sphere', scientific writers usually used the term كرة (*kura*). However, when discussing the 'celestial sphere', they would apply the word فلك (*falak*). This second term did in fact have several meanings; apart from 'celestial sphere', the word could also refer to 'the revolving of the heavens', or to any kind of 'circuit'. Accordingly, it also was applied to denote 'celestial circle', alongside the term دائرة (*dāʾira*), which was used for 'circle' in a mathematical sense. Medieval scientific writers did not always use these different terms consistently. The eleventh century Iranian polymath al-Bīrūnī writes regarding the nomenclature: "*dāʾira* ['circle'] and *falak* are two terms that denote the same thing and are interchangeable; but sometimes *falak* refers to the globe (*kura*), in particular when it is moveable [...]"[111] The Arabic term فلك البروج (*falak al-burūj*; 'the *falak* of the zodiacal signs'),[112] for example, could thus refer to both the sphere and the circle of the ecliptic, that is, the apparent path of the sun on the celestial sphere. When explicitly referring to the ecliptic circle, some authors would add the word *dāʾira* to form the expression and دائرة فلك البروج (*dāʾirat falak al-burūj*; 'the circle of the *falak* of the zodiacal

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[110] 2 Kings 23,5.
[111] Hartner, "Falak."
[112] This expression is regularly used, for example, by al- Farghānī in the *Jawāmiʿ ʿilm al-nujūm*.

signs'),[113] or they would simply write دائرة البروج (*dāʾirat al-burūj*; 'the circle of the zodiacal signs').[114]

Abraham Bar Hiyya uses three different Hebrew terms to discuss celestial spheres and circles. When explicitly referring to celestial spheres, he employs the term רקיע, which in biblical Hebrew denotes the 'firmament of heaven'. When writing about celestial circles, he employs the words אופן and גלגל. In Biblical Hebrew both terms denoted 'wheel', but the word גלגל came to be used to refer to both a round celestial body, like the sun, and a celestial sphere in Talmudic times.[115] The 'sphere of the ecliptic' is thus called רקיע המזלות in his writings.[116] In order to refer to the 'ecliptic circle', he employs the expression אופן המזלות, 'the wheel of the zodiacal signs'.[117] However, the term גלגל המזלות does appear in other treatises, such as the *Sefer ha-Ibbur*.[118] Only the term עגול, which he uses to denote 'circle' in his treatises on geometry, does not appear in his astronomical writings. While there is no doubt about the fact that רקיע denotes 'celestial sphere', there is some confusion amongst scholars regarding the correct meaning of the terms אופן and גלגל in Bar Hiyya's writings. A reasonable definition of the two terms is given by J.M. Millás Vallicrosa in his Spanish translation of *Sefer Heshbon Mahalakhot ha-Kokhavim.* He maintains that אופן denotes 'circle', and גלגל 'revolution' or 'orbit'.[119] Apparently Bar Hiyya was aware of the three different meanings of the Arabic

[113] "Kūshyār ibn Labbān's Glossary of Astronomy" No. 15.
[114] This expression is used in "Kūshyār ibn Labbān's Glossary of Astronomy", No. 21.
[115] Sarfatti, *Mathematical Terminology*, 97.
[116] "והרקיע השמיני החופה את כלם הוא רקיע המזלות הסובב על הארץ ממזרח למערב", *Séfer Ḥešbón mahlekot ha-kokabim*, 7.
[117] *Séfer Ḥešbón mahlekot ha-kokabim*, index: אופן המזלות (p. 141).
[118] *Sefer ha- ʻibur*, 10.
[119] *Séfer Ḥešbón mahlekot ha-kokabim*, index (p. 141). This matches the definition given by Bar Hiyya himself: "וכל עיגול אשר ברקיע סובב ומתגלגל אנו קורין אותו גלגל, וכל עגול שאנו זוכרין אותו מדרך הקפתו וסבובו אנו קוראין אותו אופן." *Sefer ha- ʻibur*, 10. I. Efros interprets the two terms differently. He defines אופן as "*orbit*, hence different from *galgal* (q. v.), which denotes a sphere." ("Studies in Pre-Tibbonian Philosophical Terminology," 135). However, this definition is definitely wrong regarding Bar Hiyya's *Sefer ha-Ibbur*, where he describes that the celestial equator, גלגל המישר, equals the horizon for inhabitants of the North Pole. In this case, גלגל thus has to be some kind of circle, and not a sphere. "ובמקום אשר הוא רחיק מקו השוה צ׳ חלקים יהיה קוטב הצפון עומד נגד ראשו ויהיה גלגל המישור לו אופן מפריש בין הנראה מן הרקיע ובין הנסתר," *Sefer ha- ʻibur*, 12.

word *falak* and chose three different Hebrew words to render the meanings ‘sphere’, ‘circuit’, and ‘revolution’.

The authors that came after Bar Hiyya did not consistently distinguish between these three meanings of the Arabic term *falak*. In fact, the inconsistency of the Arabic nomenclature is constantly reflected in Hebrew astronomical writings. Abraham ibn Ezra uses the Hebrew word גלגל as a translation of *falak*, and the word עגולה as a translation of *dāʾira.* In his translation of Ibn al-Muthannā’s *Commentary,* Ibn Ezra makes a distinction between גלגל המזלות, ‘the sphere of the ecliptic’,[120] and עגולת גלגל המזלות, ‘the circle of the sphere of the ecliptic’, or in short, ‘the ecliptic circle’.[121] However, like the Arabic *falak*, the term גלגל can also refer to a ‘circle’ in Ibn Ezra’s writings. Thus, in his *Sefer ha-Ṭeʿamim*, the term הגלגל הגדול does not mean ‘great sphere’ but ‘great circle’.[122]

Jacob Anatoli, too, uses the word גלגל as a translation of *falak*, while his Hebrew translation of *dāʾira* differs slightly from Ibn Ezra’s term. Instead of עגולה he uses the word עגול, which is the word for ‘circle’ (in a mathematical sense) that was introduced by Abraham Bar Hiyya. Thus his Hebrew translation of *falak al-burūj* is גלגל המזלות,[123] but his Hebrew expression for ‘ecliptic circle’ is עגול המזלות.[124] He does not use the combination עגול גלגל (‘the circle of the sphere’) when referring to the ecliptic, but the expression does appear regarding the meridian circle.[125] Furthermore, Anatoli also uses the Hebrew word עגול in itself as a translation of the Arabic *falak*. In his translation of al-Farghānī’s *Astronomy* he consistently

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120 “והטעם כי גלגל המזלות הגדול שהוא גלגל הכוכבים העליונים הוא הגדול בכל הגלגלים הוא עליון עליהם”, *Ibn al-Muthannā’s commentary*, 109.
121 “והנה נעשה עגולת גלגל המזלות”, *Ibn al-Muthannā’s commentary*, 108.
122 See *The Book of Reasons* glossary, “great circle” (p. 382).
123 MS. Mantua 4, fol. 169r, l. 8; cf. *Jawāmiʿ ʿilm al-nujūm*, 40, l. 2.
124 MS. Paris 1019, fol. 23v, l. 14.
125 “ונצייר עגול גלגל חצי היום”, MS. Paris 1019, fol. 23r, l. 27.

renders the expression الفلك المائِل (*al-falak al-māʾil*; 'the inclined sphere') as העגול הנוטה (*ha-ʿagul ha-noṭeh*; literally 'the inclined circle').[126]

Like Abraham ibn Ezra and Jacob Anatoli before him, Judah ha-Cohen uses the גלגל to translate the Arabic word *falak*. As a translation of the Arabic word for 'circle', *dāʾira,* Judah employs עגולה, which is the word that was used by Ibn Ezra. The expressions he uses for 'ecliptic' in both the correspondence and his abbreviation of the *Almagest* are thus גלגל המזלות and עגולת המזלות. Regarding the meaning of these terms no consistent distinction is made: both refer to the ecliptic circle.[127]

Ecliptic

As stated above, the Arabic term for 'ecliptic', that is, the sun's annual path in the sky, is فلك البروج (*falak al-burūj*; 'the *falak* of the zodiacal signs'). As the term *falak* is ambiguous, the word for 'circle', دائرة (*dāʾira*), is sometimes used in front of, or instead of, the term فلك (*falak*). However, there is also another expression for this notion: منطقة البروج (*minṭaqat al-burūj*; 'the belt of the zodiacal signs'). It is also called منطقة فلك البروج (*minṭaqat falak al-burūj;* 'the belt of the *falak* of the zodiacal signs'). This term, too, is somewhat ambiguous, as it can refer to both the 'ecliptic circle' and the 'zodiac'. As the 'zodiac' is a zone or 'belt' that extends to the north and south of the ecliptic circle, the 'ecliptic' itself is sometimes also called 'the circle running through the middle of the zodiac', or simply 'the middle of the zodiac', in Arabic: وسط البروج (*wasaṭ al-burūj*).[128]

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[126] MS. Paris 1022, fol. 18r, l. 12; MS. Mantua 4, fol. 169r, l. 22. Cf. *Jawāmiʿ ʿilm al-nujūm*, 40, l. 19.

[127] Thus, when writing about 'each arc of the northern half of the ecliptic' in the correspondence, Judah is obviously referring to the ecliptic circle, but he uses the word גלגל instead of עגולה: "לכל קשת מחצי גלגל המזלות הצפוני", edition, paragraph 34.

[128] Hartner, "Minṭaḳat al-Burūdj̲."

In Abraham Bar Hiyya's writings we find the terms רקיע המזלות ('the sphere of the ecliptic'), אופן המזלות ('the circle of the ecliptic'), and גלגל המזלות ('the orbit of the ecliptic'). Abraham ibn Ezra uses (עגולת) גלגל המזלות, '(the circle of) the sphere of the ecliptic'. But he also employs the peculiar expression חשב אפודת גלגל המזלות ('the belt of the dress of the zodiac'), which he sometimes shortens to חשב האפודה ('the belt of the dress') or to אפודת הגלגל (the dress/belt of the sphere).[129] This term appears to be a translation of the Arabic منطقة فلك البروج. Jacob Anatoli's terminology is less obscure. He refers to the 'ecliptic' as גלגל המזלות ('the sphere/circle of the zodiacal signs'), עגול אמצע המזלות ('the circle of the middle of the zodiacal signs') or אמצע גלגל המזלות ('the middle of the sphere/circle of the zodiacal signs'). Judah ha-Cohen uses the terms גלגל המזלות ('the sphere/circle of the zodiacal signs') and עגולת המזלות ('the circle of the zodiacal signs') throughout the *Midrash ha-Ḥokhmah*. However, in the summary of the *Almagest* we find in addition Ibn Ezra's term אפודת גלגל המזלות ('the dress of the sphere of the zodiacal signs') and the term אמצע אפודת המזלות ('the middle of dress of the zodiacal signs').[130]

Pole

The Arabic term for 'pole [of a sphere]' is قطب (*quṭb*),[131] a word that originally denotes 'axis, pivot of a mill'. Abraham Bar Hiyya introduces the concept into the Hebrew language by forming the loanword קוטב.[132] In contrast, Abraham ibn Ezra employs the word סדן to convey the same meaning.[133] The word originally denotes 'block', as in 'the block of a millstone'. He

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[129] The term חשב אפודתו ('the belt of his dress') is of biblical origin (Ex. 28,8 and 39,5). It refers to the ceremonial dress of the high priest Aaron. On Ibn Ezra's use of the term see Sela, *Abraham ibn Ezra and the Rise of Medieval Hebrew Science*, 137–139.

[130] "ומצא בהבטה בכלי המשרטטת שהיה ריחוק מרכז הירח מאפודת גלגל המזלות בלקות האחד מ׳ח׳ רגעים וחצי לפי שמצא ריחוקו מנקודת הקשר בגלגל הנוטה תשעה חלקים ושליש. ומצא בלקות השני החציי ריחוק מרכז הירח מהקשר שמונה חלקים פחות חומש. לפיכך היה ריחוקו מאמצע אפודת המזלות מ׳א׳ רגעים פחות שליש", MS. Vat. 338, fol. 239rv.

[131] See, for example, "Kūshyār ibn Labbān's Glossary of Astronomy", No. 14.

[132] Sarfatti, *Mathematical Terminology*, 92.

[133] "הכוכבים שהם קרובים על הסדן שאינם נשקעים תחת הארץ", *Sefer Keli neḥoshet*, 23. "שיש אומרים כי סדני גלגל המזלות יעלו גם ירדו שמונה מעלות", *The Book of Reasons*, 50.

thus uses a Hebrew word with a similar original meaning to that of the Arabic term instead of introducing a loanword. However, the generations of translators that followed Ibn Ezra, among them Jacob Anatoli, used the loanword that had been introduced by Abraham bar Hiyya. Judah ha-Cohen on the other hand consistently uses Ibn Ezra’s term סדן throughout the *Midrash ha-Ḥokhmah*.

Meridian

The Arabic expression denoting ‘meridian’ is دائرة (فلك) نصف النهار (*dāʾirat (falak) niṣf al-nahār;* literally ‘the circle (of the sphere of) half the day’).[134] As we have seen, the four authors differ in their translations of ‘circle’ and ‘sphere’, but they all use the same Hebrew expression, חצי היום, as a literal translation of ‘half the day’.[135]

Celestial equator

The Arabic term for the ‘celestial equator’ is دائرة معدل النهار or فلك معدل النهار (*dāʾirat /falak muʿaddil al-nahār*; ‘the circle of the equalizer of the day’).[136] The Arabic noun معدل in this expression is derived from the root عدل, which originally denotes ‘to act justly, rightly’. The participle in the second stem signifies ‘making straight’ or ‘making even’. Abraham Bar Hiyya translates the expression معدل النهار (‘making even of the day’) with the biblical Hebrew noun מישור, from the root ישר (‘straight’), which originally signified ‘uprightness’, ‘justice’,

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[134] “Niṣf al-Nahār”; “Kūshyār ibn Labbān’s Glossary of Astronomy,” No. 17.

[135] “דע איזה מעלה שתרצה ושים אוחה על קו חצי היום.” *Sefer Keli neḥoshet*, 18. “וערך מצעד המזלות על הארץ באופן המפריש על קו השוה הוא ערך מעברם על קצת חצי השמים המסמן חצי היום לכל הארץ.” *Séfer Ḥešbón mahlekot ha-kokabim*, 21. For Jacob Anatoli, see MS. Paris 1022, fol. 17b, l. 8. Cf. *Jawāmiʿ ʿilm al-nujūm*, 39, l. 5. For Judah ha-Cohen, see edition, paragraph 36; MS. Vatican 338, fol. 214v, l. 18-19.

[136] The term is sometimes transliterated as *muʿaddal al-nahār* (for example, in the IE article “Niṣf al-Nahār.”). This suggests that the word معدل is a passive participle with the meaning ‘made straight, even’. But since all authors under consideration translate the term as an active participle (‘making straight, even’) into Hebrew, we have transliterated it as *muʿaddil*. This transliteration can, for example, be found in W. Hartner’s article “Falak.” The term also appears in “Kūshyār ibn Labbān’s Glossary of Astronomy”, No. 14.

alongside ‘plain’, ‘level’. His Hebrew versions of ‘circle of the celestial equator’ are thus אופן המישור and גלגל המישור.[137]

Abraham ibn Ezra translates the term differently. In his *Sefer Keli Neḥoshet* he uses the participle משוה, from the biblical root שוה (‘to be equal’), to translate the Arabic word معدل. The ‘circle of the equalizer of the day’ is thus עגולת משוה היום. In addition, he refers to the celestial equator as עגולת ראש טלה ומאזנים, ‘the circle of the beginnings of Aries and Libra’.[138] However, in both the *Sefer ha-ʿIbur* and *Sefer ha-Teʿamim*, he calls the celestial equator קו הצדק, which literally means ‘line of justice’.[139] This translation of might seem unusual, but in fact Hebrew root צדק has a very similar meaning to the Arabic root عدل, from which the word معدل is derived. While عدل originally denotes ‘to act justly, rightly’, צדק denotes ‘to be right, true, just’. The two translations משוה היום and צדק are thus both to some extend literal, but they render different aspects of the Arabic original into Hebrew. Jacob Anatoli follows the first translation made by Ibn Ezra. The ‘equalizer of the day’ is consistently referred to as משוה היום in all of his translations.[140] As regards Judah ha-Cohen’s Hebrew term for ‘celestial equator’, we find discrepancies between his translation of the correspondence with the philosopher, his summary of the *Almagest*, and his Hebrew rendering of al-Biṭrūjī’s *Principles of Astronomy.* In the correspondence and in the *Principles of Astronomy*, he refers to the celestial equator as גלגל המישור and עגולת גלגל המישור.[141] He thus uses the expression that was introduced by Abraham Bar Hiyya. However, this is not the case in his Hebrew summary

[137] He uses אופן המישור in the *Séfer Ḥešbón mahlekot ha-kokabim* (see index). גלגל הממישור is used in the *Sefer ha-ʻibur*.
[138] “עגולת ראש טלה ומאזנים שהוא עגולת משוה היום”, *Sefer Keli neḥoshet*, 19.
[139] “נקראו הששה מזלות שמאליים כי השמש הולך עליהם והוא בצד שמאל כנגד קו הצדק, והפך זה הדרומיים”, *The Book of Reasons*, 36. See also note 4 on p. 112.
[140] See, for example, MS. Paris 1022, fol. 17v, l. 19. Cf. *Jawāmiʿ ʿilm al-nujūm*, 39, l. 15–16.
[141] In his summary of al-Biṭrūjī’s work, we find, for example, the sentence “והוא כמ׳ כפל נטיית עגולת השמש על גלגל המישור” (MS. Mich. 551, fol. 162r).

of the *Almagest*. There, he refers to the celestial equator as עגולת (מזל) טלה ('the circle of (the sign of) Aries'), or as צידוק היום ('the justification of the day'). Both expressions are very similar to the ones that were used by Abraham ibn Ezra. Like Ibn Ezra, Judah uses the root צדק to translate the Arabic root عدل, but he stays closer to the original; the noun צידוק is derived from the *piʿel*, which is the equivalent of the second stem in Arabic, and Judah faithfully translates النهار as היום. However, he seems to have been aware of the fact that his choice of צדק as a translation for عدل was rather unusual, as he explicitly mentions the Arabic term when introducing his Hebrew terminology: *'The poles of the daily motion are the poles of the 'celestial equator [tsidduq ha-yom]', which we have called 'the circle of the sign of Aries [ʿagulat mazal ṭaleh]', [in Arabic:] muʿaddil al-nahār.'*[142]

Terrestrial equator

Distinct from the celestial equator, which is a circle on the celestial sphere, is the terrestrial equator, which is a circle on the surface of the earth. In Arabic this circle was usually referred to as خط الاستواء (*khaṭṭ al-istiwāʾ*; 'the line of evenness').[143] Both Abraham bar Hiyya and Abraham ibn Ezra render the term as קו השוה ('the line of evenness') in Hebrew. They translate the Arabic root س و ى ('being equal'), of which the noun استواء is derived, with its Hebrew cognate שוה.[144] Jacob Anatoli uses the same root in his Hebrew translation, but he derives the noun signifying 'evenness' from the *piʿel*; his Hebrew translation thus reads קו השווי in his translation of the *Almagest* and Ibn Rushd's *Compendium*. As al-Farghānī refers to

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142 "וסדני התנועה היומית הן סדני גלגל צידוק היום שקראנוהו עגולת מזל טלה מעדל״ אל נהאר״", MS. Vat. 338, fol. 214v.
143 Miquel, "Istiwāʾ." See also "Kūshyār ibn Labbān's Glossary of Astronomy," 61.
144 "השער הרביעי בפירוש מעלות המזלות על אופן המפריש לשוכני קו השוה", *Séfer Ḥešbón mahlekot ha-kokabim*, 21. "השער השבעה עשר בידיעת כמה מעלות יעלה כל מזל בקו השוה", *Sefer Keli neḥoshet*, 20.

the terrestrial equator as 'the circle of evenness', دائرة الاستواء, Anatoli faithfully translates it as עגול שווי.[145]

Judah ha-Cohen, however, uses a completely different Hebrew root to denote 'evenness'. Both in the correspondence and in his summary of the *Almagest* he renders the Arabic expression as קו המישור,[146] thus applying the same word that he used to refer to the celestial equator. While his predecessors employed two distinct Hebrew words to translate the Arabic nouns معدل and استواء, Judah translates both terms with one and the same Hebrew word, מישור.

Horizon

The Arabic word denoting 'horizon' is أفق (*ʾufq*; originally: 'side, part, region'). The 'circle of the horizon', دائرة الافق (*dāʾirat al- ʾufq*), is defined as the 'dividing [circle] between the visible [part] of the [celestial] sphere and [its] hidden [part] and its pole is the zenith'.[147] Following this definition, Abraham Bar Hiyya calls it אופן המפריש ('the separating wheel') in Hebrew.[148]

Abraham ibn Ezra uses the same expression in his writings.[149] But in his *Sefer Keli Neḥoshet* he also employs the loanword אופק.[150] However, the Arabic word is used in a different sense in this book: Once it appears in combination with another Arabic word, אקלים ('climate'), in order to clarify Ibn Ezra's Hebrew translation of 'the seven climates'.[151] In all other instances

145 MS. Paris 1022, fol. 17v, l. 16. Cf. *Jawāmiʿ ʿilm al-nujūm*, 39, l. 17.

146 In his version of the *Almagest* (MS. Vat. 338, fol. 220r-v) he writes: "וגם יקרה בזה המקום והוא קו המישור כי עגולת האופק והיא עגולת חצי היום תחלק לעולם לעגולת הנגדיות לעגולת מזל טלה בשני חצאין והיום והלילה אצלם שוין לעולם." ("It also happens at this place, that is, the terrestrial equator, that the horizon, which is the meridian, always cuts the circles that are parallel to the celestial equator into two halves, and day and night are always equal for them.")

147 "Kūshyār ibn Labbān's Glossary of Astronomy", No. 16.

148 "האופן הנטוי מפריש להם בין הנסתר ובין הנגלה מן הרקיע וזה גדר אופן המפריש" *Sefer tsurat ha-arets*, 5v. See also *Séfer Ḥešbón mahlekot ha-kokabim*, index: אופן המפריש (p. 141).

149 "סבב האופן המפריש כל שעה חמש עשרה מעלות", *The Book of Reasons*, 238.

150 *Sefer Keli neḥoshet*, 9–11, 19.

151 "ויכתבו שם שבעה גבולי הארצות כי הישוב הוא בפאת צפון והוא נחלק לז׳ חלקים נקראים אופקים או אקלימים", *Sefer Keli neḥoshet*, 10. On this passage see also Sela, *Abraham ibn Ezra and the Rise of Medieval Hebrew Science*, 108–9.

it is used as a technical term given to a part of the astrolabe.[152] In contrast, Jacob Anatoli fully accepts the loanword אופק in his writings. Thus he faithfully translates *dāʾirat al-ʾufq* as עגול האופק whenever he refers to the 'circle of the horizon'. Judah ha-Cohen, too, consistently uses the Arabic loanword in both the correspondence and his summary of the *Almagest*. Thus he renders the 'circle of the horizon' as עגולת האופק. However, he is aware of the fact that the word is originally Arabic. This can be seen from the fact that in one instance he simply transliterates the word אפאק (*ʾāfāq*), which is the Arabic plural form of 'horizon', although he usually uses the Hebrew plural of the word, אופקים.[153] Furthermore, while throughout the first few books of his summary of the *Almagest* Judah consistently uses the term אופק, at the end of the seventh book he deviates from his own terminology; when briefly listing the contents of the eighth book (and explaining why he will not translate it) he refers to the 'horizon' as אופן המפריש, which is the term used by Bar Hiyya and Ibn Ezra.[154]

Arc

Originally denoting 'bow', the Arabic term قوس (*qaws*) gained the meaning 'arc of a circle' in scientific writings. Its Hebrew equivalent קשת ('bow') underwent the same transformation in meaning. Its use as 'arc' in a mathematical sense is already documented in the *Mishnat ha-Middot*.[155] All four authors use this term consistently throughout.

Declination

The 'declination', that is the distance between a celestial body and the celestial equator, is called ميل (*mayl*; originally 'leaning, inclination') in Arabic.[156] Abraham Bar Hiyya translates

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[152] "והגשר הראשון מהם הוא נקרא האופק", *Sefer Keli neḥoshet*, 11.
[153] "וראיה על מה שהזכרנו כי השטחין המוצאין ממראיתנו בכל מקום והן הנקראין אפאק יחצבו לעולם כדור השמים כלו בחצאין", MS. Vat. 338, fol. 214r.
[154] "ואחר כן דבר בראיית הכוכבים הקיימין וסתרתן כפי האופן המפריש", MS. Mich. 551, fol. 159v, l. 28-29.
[155] Sarfatti, *Mathematical Terminology*, 58.
[156] King, "al-Mayl." See also "Kūshyār ibn Labbān's Glossary of Astronomy" Nos. 58, 59.

the term as נמיכות (‘lowness’).[157] Abraham ibn Ezra after him renders the term as נטייה (‘spreading; inclination’).[158] Both Jacob Anatoli and Judah ha-Cohen take over Ibn Ezra’s translation and thus consistently render ‘declination’ as נטייה.[159]

Ascendant

The ‘ascendant’ is the point of the ecliptic that is rising above the horizon at a given moment. In Arabic it is called الطالع (*al-ṭāliʿ*; ‘that which rises’).[160] The participle is derived from the verb طلع (*ṭalaʿa*), which is used to indicate that arcs of celestial circles ‘rise’ above the horizon. Abraham Bar Hiyya uses the Hebrew verb עלה (‘to rise’) as a translation of the Arabic verb. When referring to the part of the ecliptic that ‘is rising at a given moment’, however, he makes use of a participle that is derived from a different root; the ‘ascendant’ is called הצומח in his writings. The verb צמח denotes ‘to grow, sprout’, as well as ‘to break forth, shine’. Bar Hiyya’s term for ‘ascendant’ thus indicates that the part of the ecliptic that is ‘breaking forth’ from the eastern horizon is at the same time starting to ‘shine’. Abraham ibn Ezra uses both the verb עלה and the participle הצומח in exactly the same way as Bar Hiyya. Jacob Anatoli, too, uses the verb עלה as a translation of طلع. But the term הצומח cannot be found in his writings. Instead, he uses the term העולה when referring to the ‘ascendant’. Like its Arabic counterpart, it is simply the present participle of the verb ‘to rise’. Judah ha-Cohen, however, uses both the verb עלה and the participle הצומח in the way that they were used by Bar Hiyya and Ibn Ezra, both in the correspondence and in his summary of the *Almagest*.

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157 “השער השלישי בפירוש הקשת הנאחזת בין שני אופני המישור והמזלות הנקרא קשת הנמיכות וערך הנמיכות הזה לכל מעלה והקשת הזה נקראת קשת הנמיכות וארכה לדעת בטלמיוס כ״ג מעלות” *Séfer Ḥešbón mahlekot ha-kokabim*, 19. “ומעלה [...] וני״א שברי מעלה” *Sefer tsurat ha-arets*, 5v.

158 “מה אלה השנים לוחות” *Sefer Keli neḥoshet*, 19. “השער הארבע עשר. בידיעת נטיית איזה מעלה או איזה כוכב שתרצה [...]” “שכתב מחמד מן מוסי בספרו לחתכות היתר והנטייה ומה טעם היתר והנטייה [...]” *Ibn al-Muthannā’s commentary*, 124.

159 For Anatoli’s use of the word see, for example, MS. Paris 1022, fol. 18r, l. 11. Cf. *Jawāmiʿ ʿilm al-nujūm*, 40, l. 17. For Judah’s use see edition, paragraph 38; MS. Vat. 338, fol. 220v, l. 7.

160 See King, “al-Ṭāliʿ”; Kennedy, “A Survey of Islamic Astronomical Tables,” 140; “Kūshyār ibn Labbān’s Glossary of Astronomy,” No. 82.

Sphaera recta and sphaera obliqua

A fundamental concept in spherical astronomy is the notion of *sphaera recta* and *sphaera obliqua,* the ‘right sphere’ and the ‘oblique sphere’. Depending on the location of the observer, equal arcs of the ecliptic take different amounts of time to rise on the horizon. S*phaera recta* refers to the celestial sphere as it is perceived by an observer standing on the terrestrial equator. S*phaera obliqua* is the celestial sphere as it is perceived by an observer to the north of the equator. The Arabic terms commonly used for these concepts are الفلك المستقيم (*al-falak al-mustaqīm;* ‘the right, straight sphere’) and الفلك المائِل (*al-falak al-māʾil*; ‘the inclined sphere’), respectively.[161] Less common are the alternative expressions الكرة المنتصبة (*al-kura al-muntaṣiba; ‘*the upright sphere’) for ‘sphaera recta’ and الكرة المائلة (*al-kura al-māʾila;* ‘the inclined sphere’) for ‘sphaera obliqua’.[162]

Abraham Bar Hiyya renders the two expressions as האופן הישר (‘the straight circle’) and האופן הנוטה (‘the inclined circle’), respectively.[163] Abraham ibn Ezra refers to *sphaera recta* as גלגל היושר (‘the sphere of straightness’).[164] Like Bar Hiyya, he uses the Hebrew root ישר to translate the Arabic term مستقيم, but instead of an adjective, he uses the noun derived from this root. We could not find an explicit mention of the concept of *sphaera obliqua* in the writings we studied, but in a slightly different context Ibn Ezra uses the term הגלגל הנוטה for ‘a sphere that is inclined’.[165] Jacob Anatoli’s terminology resembles that of Abraham Bar Hiyya. To denote the ‘right sphere’ he uses the expressions הכדור הישר and הגלגל הישר in his translation

[161] See Hartner, “Falak.” These terms are used, for example, by al-Farghānī, *Jawāmiʿ ʿilm al-nujūm*, 40, l. 11, 19.

[162] The term الكرة المنتصبة (*al-kura al-muntaṣiba*) is used, for example, in the Iṣḥāq-version of the Almagest, while in the Ḥajjāj-version the ‘right sphere’ is referred to as الكرة المستقيمة (*al-kura al-mustaqīma*). See Ptolemy, *Der Almagest*, 135–136. In the astronomical part of the Shifāʾ Ibn Sina refers to “المطالع حيث الكرة منتصبة”, *al-Riyāḍīyāt*, 4: ʿIlm al-hayʾah:105.

[163] *Séfer Ḥešbón mahlekot ha-kokabim*, index (p. 141).

[164] *The Book of Reasons*, 265, note 12.

[165] “כי יש למשרתים גלגלים מהם גלגל הנוטה”, *Ibn al-Muthannā’s commentary*, 109.

of the *Almagest*, and העגול הישר in his translation of al-Farghānī's *Astronomy*.[166] Accordingly, he refers to *sphaera obliqua* as הכדור הנוטה and הגלגל הנוטה in his translation of the *Almagest*, while in his translation of al-Farghānī he translates الفلك المائِل as העגול הנוטה.[167] The use of both כדור and גלגל to denote 'sphere' in his rendering of the *Almagest* could be due to the fact that he found two different Arabic words (such as فلك and كرة) in the original. However, one has to keep in mind that he does not consistently translate the term فلك as גלגל, since in his Hebrew version of al-Farghānī's *Astronomy* he renders it as עגול ('circle').

Once again, Judah ha-Cohen's terminology regarding these expressions differs from that of his predecessors. In the correspondence and some parts of his summary of the *Almagest*, he calls the 'right sphere' הכדור הנצב and the 'oblique sphere' הכדור הנוטה. While in all other cases he refers to spheres as גלגל, he applies the Hebrew word כדור in this expression. Furthermore, he does not make use of the root ישר when referring to *sphaera recta*, but he uses the participle נצב ('standing [upright]') instead. It is possible that he simply chose to translate the Arabic participle مستقيم ('straight') in a different way than his predecessors. But in the mathematical part of the *Midrash ha-Ḥokhmah* Judah consistently translates the Arabic term مستقيم with the Hebrew term ישר. It seems to be more than mere coincidence that the word נצב is derived from the same root as the Arabic word منتصب, which denotes 'upright' in the expression الكرة المنتصبة (*al-kura al-muntaṣiba;* 'the upright sphere'). Both the Hebrew נצב and the Arabic نصب originally denote 'to set up, to put up'. In all likelihood the expression הכדור הנצב is thus a translation of الكرة المنتصبة, while Judah's predecessors translated the term الفلك المستقيم.

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166 הכדור הישר: MS. Paris 1019, fol. 16r, ll. 7, 8. הגלגל הישר: ibid., fol. 3v, l. 25. העגול הישר: MS. Paris 1022, fol. 18, l. 5.
167 הכדור הנוטה: MS. Paris 1019, fol. 17r, l. 10. הגלגל הנוטה: ibid., l. 10. העגול הנוטה: MS. Paris 1022, fol. 18, l. 12.

In his summary of the *Almagest*, Judah uses yet another Hebrew expression for the concept, which also differs from the terms used by his predecessors. In the second chapter he consistently uses the expressions גלגל המישור ('the sphere of uprightness') and הגלגל הנוטה ('the inclined sphere') to denote *sphaera recta* and *sphaera obliqua*, respectively.[168] Regarding *sphaera recta*, he thus deviates from the terminology of the correspondence and employs the root ישר to denote 'uprightness'. As the same root was used by his predecessors to translate the expression الفلك المستقيم, it seems reasonable to assume that this deviation reflects differences in the terminology of the Arabic originals that he is translating. This assumption is supported by the fact that in later chapters of the translation of the *Almagest* the expression הכדור הנצב is used.[169] The latter term is thus a translation of الكرة المنتصبة, while גלגל המישור is a translation of الفلك المستقيم. However, the expression גלגל המישור appears also in the correspondence and in Judah's Hebrew rendering of al-Biṭrūjī's *Principles of Astronomy*, but as we have seen, there it is used as a translation of فلك معدل النهار, the 'celestial equator'. One might thus argue that in this respect, Judah's terminology is ambiguous. But this is only the case if we regard the correspondence and the translation of the Almagest as parts of the same work. If we understand them to be two distinct works, the terminology of each one is consistent, since in the *Almagest* Judah uses two completely different terms to translate 'celestial equator'.

[168] For example, in the sentence " והעולה מזו הצורה השנית הוא כי צמחי תאומים וסרטן שניהם יחד בגלגל המישור הוא כקיבוץ צממחיהן בגלגל הנוטה כלומר בכל אופק עיר ועיר. וקבוץ צמחי שור ואריה במישור כקבוץ צמחים בנוטה.", MS. Vat. 338, fol. 221v. ("And the result of this second diagram is that the ascensions of Gemini and Cancer [added] together in sphaera recta are like the sum of their ascensions in sphaera obliqua, that is to say, in each horizon of each locality. And the sum of the ascensions of Taurus and Leo in [sphaera] recta is like the sum of the ascensions in [sphaera] obliqua.")

[169] For example, in MS. Vat. 338, fol. 230r: "לפי שצמחי שני מזלות אלו בכדור הנצב".

Ascensions

The distinction between *sphaera recta* and *sphaera obliqua* is important for the measurement of 'ascensions', or 'rising times', which are called المطالع (*al-maṭāliʿ*; plural of المطلع *al-maṭlaʿ*) in Arabic.[170] This technical term is derived from the verb طلع (*ṭalaʿa*; 'to rise'). 'Ascensions' are arcs of the celestial equator that rise simultaneously with given arcs of the ecliptic circle, usually on the eastern horizon. The lengths of these arcs depend on the position of the observer on the earth. 'Right ascensions' refer to rising times in places where the celestial equator is perpendicular to the local horizon, which is the case for all places that lie on the terrestrial equator. They were commonly called المطالع في الفلك المستقيم (*al-maṭāliʿ fī al-falak al-mustqīm;* 'the ascensions at *sphaera recta*'), or مطالع خط الاستواء (*maṭāliʿ khaṭṭ al-istiwāʾ*; 'the ascensions belonging to the terrestrial equator')[171] in scientific Arabic. 'Oblique ascensions' refer to these arcs of the equator at all other places. They were commonly called مطالع البلد (*maṭāliʿ al-balad*; 'the ascensions of the country/town'), or المطالع في الفلك المائِل (*al-maṭāliʿ fī al-falak al-māʾil*; 'the ascensions at *sphaera obliqua*'). The values of the oblique ascensions depend on the terrestrial latitude of the observer.

Abraham Bar Hiyya refers to 'ascensions' as מצעדים, a biblical Hebrew word that originally means '[foot]steps'.[172] The word is derived from the root צעד ('to step, to go on slowly'), whose cognate root in Arabic, صعد, means 'to ascend'.[173] He translates 'right ascensions' as המצעדים באופן הישר, 'ascensions at *sphaera recta*',[174] or as מצעדים בקו השווה, 'ascensions at the

[170] See King, "al-Maṭāliʿ."
[171] "Kūshyār ibn Labbān's Glossary of Astronomy", No. 62.
[172] He defines מצעד thus: "אין אנו חושבים מעלת המזלות מהקיף אופן עצמו כי אם מן הערך העולה בהעלותם מאופן המישור ואנו קוראים לעליתם הזאת מצעד המזלות", *Séfer Ḥešbón mahlekot ha-kokabim*, 21.
[173] The Hebrew root may initially have had the same meaning. See *Gesenius' Hebrew and Chaldee Lexicon to the Old Testament Scriptures*, 714.
[174] "אם תהיה חקירתך בלוחות המצעד באופן הישר יהיה המספר ההוא סכום המצעדים באופן הישר מראש מזל גדי עד המעלה ההיא", *Séfer Ḥešbón mahlekot ha-kokabim*, 27.

equator'.[175] 'Oblique ascensions' are called מצעדים באופנים הנוטים, 'ascensions of the inclined spheres',[176] or מצעדי האקלים, 'ascensions of the geographical region'.[177] Abraham ibn Ezra uses the same term as Bar Hiyya to denote 'ascensions'. He calls 'right ascensions' מצעדי גלגל היושר, 'ascensions at *sphaera recta*',[178] and he also refers to מצדעים על הקו השוה 'ascensions at the terrestrial equator'.[179] Oblique ascensions are מצעדי הארץ, 'ascensions of the country'.[180] Jacob Anatoli translates the word مطالع differently. He calls 'ascensions' עליות.[181] This noun is derived from the Hebrew verb denoting 'to rise', which is עלה. Anatoli translates 'ascensions at *sphaera recta*', المطالع في الفلك المستقيم, as העליות בעגול הישר, and 'ascensions at *sphaera obliqua*', المطالع في الفلك المائِل, as העליות בעגול הנוטה.[182]

Judah ha-Cohen's terminology is once again different. In the correspondence, he refers to 'ascensions' as מעלות. Like his predecessors' terms, the noun is formed in analogy to the Arabic original, which literally means 'something that rises'. Like Jacob Anatoli, Judah derives the term from the verb עלה ('to rise'), but he gives it a slightly different form. Judah refers to 'right ascensions' as המעלות בכדור הנצב ('the ascensions at *sphaera recta*'), or as מעלות השווי ('the ascensions of [the line of] evenness', i.e. 'the ascensions belonging to the terrestrial equator'). One has to note, however, that in the latter case Judah deviates from his own terminology, as the term for 'equator' he uses elsewhere is קו המישור, not קו השיווי.

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175 "ואומר כי מצעדי המזלות האלה נבדל הוא ממצעדן בקו השוה, מהם מוסיף על מצעד קו השוה ומהם גורע", *Séfer Ḥešbón mahlekot ha-kokabim*, 25.

176 "ואם תהיה חקירתך בלוחות באופנים הנוטים יהיה המספר אשר מצאת לפני המעלה סכום המצעדים באופן ההוא [...]", *Séfer Ḥešbón mahlekot ha-kokabim*, 27.

177 "הבא במצעדים ההם אל לוחות מצעדי האקלים", *Séfer Ḥešbón mahlekot ha-kokabim*, 104. The term אקלים itself is a loanword from the Arabic إقليم, which means 'region'.

178 "רחוקה מקו התהום שתי שעות גם ארבעים ושלשה חלקים במצעדי גלגל היושר", *The Book of Reasons*, 104.

179 "ומצדעיו חסרים כי הוא יעלה בכל מקום פחות ממה שיעלה על הקו השוה בארץ", *The Book of Reasons*, 342.

180 "ואם היה הכוכב במעלה הצומחת, נתן למבטו השלישית מאה ועשרים מעלות במצעדי הארץ", *The Book of Reasons*, 94.

181 "ויקראו אלו הזמנים מסבוב משוה היום שהם כשיעברו המזלות בעלו העגולים עליות המזלות", MS. Paris 1022, fol. 18r. Cf. *Jawāmiʿ ʿilm al-nujūm*, 40, l. 7–9.

182 MS. Paris 1022, fol. 18r, ll. 17-20. Cf. Jawāmiʿ ʿilm al-nujūm, 41, ll. 6-8.

‘Oblique ascensions’ are referred to as המעלות בנקודות שהכדור נוטה עליהן (‘ascensions at points on which the sphere is inclined’), or as מעלות העיר (‘ascensions of the town’).

In his Hebrew summary of the *Almagest*, however, he uses a completely different term to denote ‘ascensions’. There he employs the word צמחים when referring to rising-times. ‘Right ascensions’ are thus called הצמחים בגלגל המישור (‘the ascensions in *sphaera recta*’), ‘oblique ascensions’ are called הצמחים בגלגל הנוטה (‘the ascensions in *sphaera obliqua*’).[183] He also refers to צמחים בזו העיר המוצעת (‘ascensions in this given town’).[184] While the word he uses in the correspondence is derived from the verb ‘to rise’, the term that is found in the Almagest is derived from the root צמח (‘to grow, sprout; to break forth, shine’), which Judah employs to denote ‘ascendant’. In both cases, Judah thus forms terms in analogy to the Arabic terminology, as the words ‘to rise’, ‘ascendant’ and ‘ascensions’ are all derived from the same root in Arabic. In this respect, Judah ha-Cohen’s vocabulary (and also that of Jacob Anatoli) reflects the Arabic terminology much more closely than that of Bar Hiyya and Ibn Ezra, whose terms for ‘ascensions’ and for ‘to rise’ are derived from different roots. However, Judah seems to be aware of the fact that his terminology deviates from that of his predecessors, since after introducing the term צמחים in the second chapter of the *Almagest*, he explains: *The meaning of ‘tsomḥim’ is ‘maṭāliʿ’ in Arabic. They are the arcs of the celestial equator, which is the ‘justification of the day’ (tsiduq ha-yom), that rise with each arc of the ecliptic.*[185]

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[183] “והעולה מזו הצורה השנית הוא כי צמחי תאומים וסרטן שניהם יחד בגלגל המישור הוא כקיבוץ צממחיהן בגלגל הנוטה. כלומר בכל אופק עיר ועיר. וקבוץ צמחי שור ואריה במישור כקבוץ צמחים בנוטה.”, MS. Vat. 338, fol. 221v.

[184] “כשנגרע מצמחיהן בגלגל המישור שיעור צמחי טלה בזו העיר המוצעת ישארו צמחי בתולה בזו העיר”, MS. Vat. 338, fol. 221v.

[185] “פירוש צמחים מטאלע בערבי. והן הקשתות העולות מעגולת טלה שהיא צידוק היום עם כל קשת מן גלגל המזלות.”, MS. Vat. 338, fol. 221v.

Beginning of Aries

The Arabic expression اول الحمل (*awwal al-ḥamal*; 'the first of Aries') refers to the first point of the zodiacal sign of Aries, which is vernal equinox.[186] Abraham Bar Hiyya and Abraham ibn Ezra both translate it as ראש טלה ('the head/beginning of Aries'). Jacob Anatoli usually uses the only slightly different expression ראשית טלה ('the beginning of Aries'). Perhaps he did not want to confuse the term with the name of the star Alpha Arietis, which is called رأس الحمل (*rāʾs al-ḥamal*; 'the head of [the constellation] Aries') in Arabic. Judah ha-Cohen, however, uses the term introduced by Bar Hiyya and Ibn Ezra throughout the *Midrash ha-Ḥokhmah.*

Degree

The common Arabic term for 'degree [of a circle]' is درجة (*daraja;* originally 'stair, step'). However, in the different Arabic translations of Ptolemy's *Almagest*, the word جزء (*juzʾ*, plural *ajzāʾ*; 'part') is frequently used for this notion.[187] Also in later scientific writings both the terms درجة and جزء could be used to denote 'degree'.[188] Abraham Bar Hiyya translates the former term as מעלה ('ascent, step')[189] and the latter as חלק ('part').[190] Abraham ibn Ezra appears to prefer the use of only מעלה in his writings.[191] Jacob Anatoli consistently uses the term מעלות ('steps') in his Hebrew version of al-Farghānī's *Astronomy,* although the Arabic original uses جزء ('part').[192] In his translation of Ibn Rushd's *Compendium*, however, he uses

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[186] See King, "al-Maṭāliʿ." See also "Kūshyār ibn Labbān's Glossary of Astronomy" No. 25, 29, 34, etc.
[187] Ptolemy, *Der Almagest*, 165–160.
[188] Hartner, "Minṭaḳat al-Burūdj." Kūshyār Ibn Labbān, for example, uses both terms in is glossary: جزء in No. 67, درجة in Nos. 75-77.
[189] "ונמצא הרקיע כלו נחלק לש״ס מעלה וכל מעלה ומעלה מהם לששים חלקים", *Séfer Ḥešbón mahlekot ha-kokabim*, 6.
[190] "נמצא הגלגל כלו נחלק לש״ס חלקים [...] ונקודת אמצעית המחצית הזאת נוטה לצפונה מגלגל המישור כגון נ״ד חלקים", *Sefer ha-ʿibur*, 10.
[191] "והנה נחלק זה העגול לארבעה חלקים שוים [...] וזה הרביעי נחלק לתשעים חלקים יקראו מעלות", *Sefer Keli neḥoshet*, 7. "חלקו הגלגל לשלש מאות וששים מעלות", *The Book of Reasons*, 28.
[192] MS. Paris 1022, fol. 10v, l. 12. Cf. *Jawāmiʿ ʿilm al-nujūm*, 20, ll. 12–13.

either the word חלק to denote ‘degree’, or the word מדרגה (‘step’).[193] The latter term appears to be an alternative translation of درجة, as both words are derived from the same root. Judah ha-Cohen follows the terminology of Abraham Bar Hiyya. He uses the term חלק for ‘degree’ in the correspondence (while the word מעלה denotes ‘ascension’, as mentioned above) and in his rendering of the *Almagest*, and מעלה in his summary of al-Biṭrūjī’s *Astronomy*.

Solstice

The Arabic term for ‘solstice point’ is نقطة الانقلاب (*nuqṭat al-inqilāb*; ‘the point of inversion’).[194] Abraham Bar Hiyya does not use a direct translation of this expression in his writings. Instead, re refers to the solstice points as ראש (מזל) גדי (‘the beginning of [the sign of] Capricorn’) and ראש (מזל) סרטן (‘the beginning of [the sign of] Cancer’).[195] Abraham Ibn Ezra uses the Hebrew word הפוך (‘inversion’) to denote ‘solstice’.[196] Jacob Anatoli uses the same root; he refers to the ‘solstice point’ as מתהפך (‘inverting’) and as נקודת היפוך (‘point of inversion’).[197] The latter term is also used by Judah ha-Cohen, both in the correspondence and in his summary of the *Almagest*.[198]

Zenith

The Arabic expression for ‘zenith’, that is, the point of the celestial sphere directly above the observer, is سمت الرأس (*samt al-rāʾs*; ‘the direction of the head’).[199] Abraham Bar Hiyya

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[193] Lay, “L’abrégé de l’Almageste,” index: degré.
[194] The term ‘solstice’ does not appear in Kūshyār ibn Labbān’s glossary, but it is used, for example, by Ibn Sīnā in the astronomical part of his *Shifāʾ*.
[195] “ואם תחפוץ לדעת המעלה שהכוכב עובר עמה על הצי השמים תדע תחלה מרחק הכוכב מראש מזל גדי או מראש מזל סרטן”, *Séfer Ḥešbón mahlekot ha-kokabim*, 35.
[196] “עגולת הפוך הקייצי”, *Sefer Keli neḥoshet*, 10.
[197] MS. Paris 1019, fol. 17v, l. 18, fol. 23r, l. 3.
[198] Thus in MS. Vat. 388, fol. 215r, he gives the following definition of ‘solstice point’: “והיו שתי נקודות החיצוב על עגולת המזלות עם עגולת מזל טלה הם נקודות הצידוקים ושתי נקדות חיצוב עגולת חצי היום עם גלגל המזלות נקדות ההפוכין”.
[199] “Kūshyār ibn Labbān’s Glossary of Astronomy”, No. 16 and 18.

translates the term as נוכח הראש ('the opposite of the head').[200] We could not find a translation of the term in Abraham ibn Ezra's writings. However, the first word in the Arabic expression, سمت (*samt*) is a technical term in its own right. It denotes the direction of a celestial object on the horizon, called 'azimuth'. Ibn Ezra translates the term as קדקד ('top of the head, vertex').[201] Jacob Anatoli apparently struggles to find an appropriate Hebrew expression for 'zenith'; in his translation of al-Farghānī's Astronomy he first calls the 'zenith' הנקודה שהיא כנגד הראש ('the point that is opposite the head'), then simply הראש ('the head'), then נקודת הראש ('the point of the head'). In the end he goes over to using Bar Hiyya's term נכח הראש ('the opposite of the head'). In all of these cases we find one and the same expression in the Arabic original: سمت الرأس.[202] Finally, in his translation of the *Almagest* he uses solely the expression נכח הראש. Judah ha-Cohen uses a very different expression to translate the Arabic term. Both in the correspondence and in his summary of the *Almagest* he refers to the 'zenith' as קדקד הראש ('the vertex of the head').[203] Apparently this translation was influenced by Ibn Ezra's terminology.

Rising amplitude

The 'rising amplitude' (or 'ortive amplitude') is the arc of the horizon between the east point and the rising point of the sun on a given day. In Arabic scientific writings it was referred to as سعة المشرق (*saʿat al-mashriq*; 'width of the east').[204] Abraham Bar Hiyya translates the

[200] "ודע כי אין אופן המישור עובר על נוכח ראשי שוכני הארץ כ״א על ראשי השוכנים תחתיו לבדם", *Sefer tsurat ha-arets*, 5v. "הרוחב הנזכר כאן הוא ערך הקשת הנאחזת מהאופן הנטוי על קטבי המהלך בין אופן המישור ובין נקודת נוכח הראש במקום ההוא", *Séfer Ḥešbón mahlekot ha-kokabim*, 31.
[201] "גם יש כלים שיחוקו על הגשרים קוים חותכים הגשרים ונקראים קוי הקדקד ונקראים ג״כ אלסמת", *Sefer Keli neḥoshet*, 11. On the concept of 'azimuth', see King, "al-Samt"; Lane, *Arabic-English Lexicon*, 1422.
[202] MS. Paris 1022, fol. 10v, l. 12 - fol. 11r, l. 14. Cf. *Jawāmiʿ ʿilm al-nujūm*, 21, l. 3 – 22, l. 6.
[203] In his version of the *Almagest*, we find it for example in the following sentence: "ואחר כן יחפש לדעת כמות הקשת מעגולת מזל טלה העולה עם חלק ידוע מגלגל המזלות במקום שעגולת טלה עוברת על קדקד הראש" (MS. Vat. 338, fol. 218v).
[204] "Kūshyār ibn Labbān's Glossary of Astronomy" No. 66.

term as מרחב המזרח ('width of the east').[205] We could not find a Hebrew rendering of 'rising amplitude' amongst Abraham ibn Ezra's writings. Jacob Anatoli takes over Bar Hiyya's term in his translation of the *Almagest*.[206] Judah ha-Cohen renders the expression slightly differently. Throughout the *Midrash ha-Ḥokhmah* he refers to the 'rising amplitude' as רחב המזרח ('width of the east').[207] As Bar Hiyya's and Judah's terms for 'width' are both derived from the same root, the difference in terminology is only minimal.

Latitude

The (geographical) latitude of a given locality was referred to as عرض البلد (*ʿarḍ al-balad*; 'the width of the country/town') in Arabic.[208] Abraham Bar Hiyya translates the term as רוחב המקום or מרחב המקום ('the width of the place').[209] As the two terms denoting 'width' are derived from the same root, the two expressions show only a minimal stylistic difference. Abraham ibn Ezra, too, uses both רחב and מרחב for 'width', but he chooses a slightly different translation for the Arabic word بلد (*balad*; 'region, country, town'). In both versions of his *Sefer ha-Teʿamim* he refers to the 'latitude of a locality' as מרחב הארץ ('the width of the land/country'),[210] while in his treatise on the astrolabe he opts for רוחב העיר ('the width of the town').[211] The latter term is also used by Jacob Anatoli. He refers to the 'latitude' as רחב העיר ('the width of the town'), or simply as רחב ('width').[212] The same is the case for Judah ha-

[205] "ואם תהיה לך החילוף בין הימים ידוע ונכון, ורצונך לדעת מרחב מזרח המעלות במקום ההוא"; *Séfer Ḥešbón mahlekot ha-kokabim*, 33.
[206] MS. Mantua 4, fol. 76r, l. 1.
[207] He defines the term thus: "שיעורי קשתי עגולת האופק שבין עגולת מזל טלה וגלגל הנטיה והוא הנקרא רחב המזרח" (MS. Vat. 338, fol. 218v).
[208] "Kūshyār ibn Labbān's Glossary of Astronomy" No. 65.
[209] E.g. "הרוחב הנזכר כאן הוא ערך הקשת הנאחזת מהאופן הנטוי על קטבי המהלך בין אופן המישור ובין נקודת נוכח הראש במקום ההוא, והוא הנקרא רוחב המקום מקו השוה..."; *Séfer Ḥešbón mahlekot ha-kokabim*, 31. "והנשאר בידך הוא מרחב המקום ההוא מקו השוה"; *ibid.*, 32.
[210] "והדרך השני להיות הבתים מתוקנים כפי מרחב הארץ ומצעדי המזלות עליה"; *The Book of Reasons*, 96. "בעבור מרחב הארץ, ככה תהיינה צורות בני האדם"; ibid., 218.
[211] "בידיעת לקיחת רוחב כל עיר. דע כי הרוחב הוא כמה מקומה רחוק מקו השוה לצד צפון." *Sefer Keli neḥoshet*, 22.
[212] MS. Mantua 4, fol. 167b, l. 26. Cf. Farghānī, *Jawāmiʿ ʿilm al-nujūm*, 35, l. 6–10.

Cohen, who consistently uses the expression רחב העיר ('the width of the town') in both the correspondence and the summary of the *Almagest*.[213]

Distance

The Arabic word for 'distance' is بعد (*buʿd*). Abraham Bar Hiyya, Abraham ibn Ezra and Jacob Anatoli all render the term as מרחק ('distance') in their writings.[214] In addition, Anatoli uses the term רחק.[215] Judah ha-Cohen refers to 'distance' as ריחוק in the correspondence, while in the summary of the Almagest he uses both ריחוק and מרחק, sometimes even in one and the same sentence.[216] As all three nouns are derived from the same root, differences between the authors thus pertain to the style, not the translation technique. It has to be noted, however, that Judah's terminology appears to be less consistent.

Sine, cosine, sinus totus

Medieval Arab scientists adopted the concept of 'sine' from Indian astronomy.[217] In ancient Greece, trigonometric calculations had been carried out with the help of tables of arcs and 'chords', which are segments of a circle.[218] Since astronomical calculations required doubling arcs of great circles and then halving the resulting chords, Indian astronomers replaced 'chords' with 'half-chords', or 'sines', which made astronomical calculations much easier to perform.[219] Arab astronomers took over the concept; in their writings they transliterated the

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[213] For example, in MS. Vat. 338, fol. 218v: "וכשתרצה להביט בזה הכלי גובה הסדן על מקום ידוע והוא הנקרא רחב העיר […]".
[214] See, for example, *Séfer Ḥešbón mahlekot ha-kokabim*, 19; *Ibn al-Muthannā's commentary*, 108.
[215] Lay, "L'abrégé de l'Almageste," index: distance.
[216] "יבאר כי אינה יוצאת מהמחוור ריחוקה שוה מכל אחד מן השני סדנין בין שמרחקה שוה בין מזרח ומערב או שהיא נוטה לאחד מהן", 212v-213r.
[217] The sine function in medieval Arabic writings differs from the modern function in that it is usually not calculated from a unit circle (with the radius R=1), but with a base circle with the radius R=60. On the relationship between the two, see Kennedy, "A Survey of Islamic Astronomical Tables," 139.
[218] For a detailed explanation of the chord function and its use in ancient Greece, see Van Brummelen, *The Mathematics of the Heavens and the Earth*, 40–6.
[219] Van Brummelen, *The Mathematics of the Heavens and the Earth*, 95–6.

Sanskrit word for ‘sine’, *jyā,* or *jīva* (‘bowstring, half-chord’)[220] as جيب (*jība*).[221] As the Arabic form جيب, written without vowel signs, looked the same as the existing word denoting ‘breast’, or ‘breast-pocket’, it was later misread as *jayb*, which became the standard pronunciation of جيب.[222] The ‘cosine’ was usually referred to as جيب التمام (*jayb al-tamām*; ‘the sine of the complement’).[223] In this mathematical context, التمام (*al-tamām*; ‘the completion’) refers to the complementary angle to a given angle, so that both angles together add to 90°. The ‘total sine’ or ‘sinus totus’, that is, the sine of 90°, was called الجيب الأعظم, (*al-jayb al-aʿẓam*; ‘the greatest sine’), جميع الجيب (*jamīʿ al-jayb*; ‘all of the sine’), or الجيب كله (*al-jayb kulluhu*; ‘the whole sine’).[224] An alternative expression for ‘sine’ was the term ‘half-chord’, نصف الوتر (*niṣf al-watar;* ‘half of the chord’).[225] This expression was sometimes shortened to الوتر (*al-watar*; ‘the chord’, originally ‘the bow-string’) alone, so that the word could denote both ‘chord’ and ‘sine’, depending on the context.[226]

Abraham Bar Hiyya translates the Arabic word for ‘chord’, وتر (*watar*; ‘bowstring’) with its Hebrew equivalent מיתר (‘bowstring’). His expression for ‘sine’ is ‘half-chord’, המיתר המחוצה (literally ‘the halved chord’).[227] But like in the Arabic, Bar Hiyya also uses the word מיתר alone to denote ‘sine’. Thus the term he uses for ‘cosine’ is מיתר שארית (the ‘sine of the complement’, literally: ‘the chord the remnant’).[228]

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[220] The Sanskrit word *jyā*, or *jīva,* is in turn a shortened version of *jyā-ardha*, meaning ‘half-chord’. Ibid., 96.
[221] See “Kūshyār ibn Labbān’s Glossary of Astronomy”, No. 5.
[222] The English word ‘sine’ stems from the Latin ‘sinus’, which is a literal translation of *jayb,* ‘breast’. See von Braunmühl, *Vorlesungen über Geschichte der Trigonometrie*, 1:49–50; De Blois, “Tard̲j̲ama.”
[223] The term can be found in “Kūshyār ibn Labbān’s Glossary of Astronomy”, No. 9.
[224] Kennedy, Kunitzsch, and Lorch, *The Melon-Shaped Astrolabe in Arab Astronomy*, 226.
[225] “Kūshyār ibn Labbān’s Glossary of Astronomy”, No. 4.
[226] Sarfatti, *Mathematical Terminology*, 108.
[227] “ ורוב מעשה חשבונם או קרוב מכלו הם משתמשים במיתריהם המחוצה והמיתר המחוצה הוא המחצית מיתר כפל הקשת אשר הוא מיתר מחוצה לה”, *Séfer Ḥešbón mahlekot ha-kokabim*, [15]. See also Sarfatti, *Mathematical Terminology*, 107–9.
[228] “מיתר שארית הרוחב והוא מיתר העודף אשר בין צ׳ מעלות ובין מרחב המקום”, *Séfer Ḥešbón mahlekot ha-kokabim*, 25.

Abraham ibn Ezra's term for 'chord' differs slightly from Bar Hiyya's. In his *Sefer ha-Mispar* he translates the term as יתר ('cord, rope'), which is of course derived from the same root as وتر and מיתר. The term 'half-chord' is rendered as חצי היתר ('half of the chord') in his calculations. Unlike Bar Hiyya, Ibn Ezra does not assign the additional meaning of 'sine' to the word that denotes 'chord'. When explicitly referring to the concept of 'sine', he uses a loanword from the Arabic instead. In his Hebrew translation of Ibn al-Muthannā's commentary on the astronomical tables of al-Khwārazmī, Ibn Ezra transliterates the Arabic term جيب as ייבא.[229] The text does not contain vowel signs, but the fact that Ibn Ezra added an *alef* at the end of the word suggests that he intended it to be pronounced as *yība,* or *jība.* The Hebrew expression he uses for *sinus totus* is אלייבא כלו ('the whole sine'), which is a literal translation of the Arabic expression الجيب كله (*al-jayb kulluhu*).[230] Since Ibn Ezra usually avoided introducing Arabic loanwords into the Hebrew language, his use of the Arabic term *jība* is remarkable. While he considered the concept of 'half-chord' to be translatable into the Hebrew language, he felt that for the single term conveying the same idea there was no equivalent; perhaps he was aware of the fact that the word *jība* and the concept it represented were of foreign origin.[231]

Jacob Anatoli on the other hand uses the same terminology as Abraham Bar Hiyya. Thus, in his translation of Ibn Rushd's *Compendium of the Almagest* he uses the word מיתר for 'chord',[232]

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229 "והוא חצי יתר צ׳ הוא הנקרא בלשון ערבי ייבא", *Ibn al-Muthannā's commentary*, 125.
230 "הנה אלייבא כלו יהיה ידוע כמו חצי אלכסון", *Ibn al-Muthannā's commentary*, 126.
231 It could be argued that in the translation Ibn al-Muthannā's commentary he was forced to use the Arabic term, since it appeared in the Arabic original as a technical term. However, in the case of the technical term 'the head and tail of the dragon' he renders first a literal Hebrew translation *rosh ha-tanin u-zenavo*, but then adds: "Abraham said: and this is called in the Holy tongue *naḥash bariaḥ*." (*Ibn al-Muthannā's commentary*, 155. See also Sela, *Abraham ibn Ezra and the Rise of Medieval Hebrew Science*, 124–6.) But in the case of 'sine' he did not add a biblical Hebrew equivalent of the word *jība.*
232 Lay, "L'abrégé de l'Almageste," index: ytr.

while in his translation of the *Almagest* itself, the term מיתר ישר ('the straight chord/sine'; a translation of the Arabic term *al-jayb al-mustawi*) denotes 'sine'.[233]

Once again, Judah ha-Cohen's terminology resembles that of Abraham ibn Ezra. In his summaries of the *Almagest* and al-Biṭrūjī's *Principles of Astronomy* he consistently uses Ibn Ezra's term יתר to denote 'chord'.[234] Like Ibn Ezra, he simply transliterates the Arabic word جيب when referring to the 'sine'. However, Judah's transliteration differs from Ibn Ezra's, as he renders the term as ג׳יב (*jayb*), both in the correspondence and in the summary of the al-Biṭrūjī's *Astronomy*. When referring to the '*sinus totus*', he speaks of אלג׳יב הגדול ('the great *al-jayb*'). The expression appears to be a translation of the Arabic term الجيب الأعظم ('the greatest sine'). Judah's term for 'cosine' is ג׳יב התשלום ('the sine of the complement'). Like the term تمام that denotes 'complement' in Arabic, the word תשלום is derived from a root that originally denotes 'to be whole, complete'. Judah's term is thus a more literal translation of the Arabic original than the expression מיתר שארית that was used by Abraham Bar Hiyya.

Results

The following table gives an overview of the development of the Hebrew astronomical vocabulary from Abraham Bar Hiyya to Judah ha-Cohen.

[233] MS. Mantua 4, fol. 67v, ll. 12-13.

[234] An example is his rendering of *Almagest* I.10: "שער שביעי. בידיעת שיעורי יתרי חלקי העגולה. הוציא תחלה כמות יתר המעושר והמחומש החקוקין בעגולה", MS. Vat. 338, fol. 215r. However, in the calculation that follows the term מיתר is used twice. Then the text immediately switches over to the term יתר. If this deviation was not added by a later scribe, Judah thus changed his terminology from that of Abraham Bar Hiyya to that of Abraham ibn Ezra.

English	Arabic	Abraham Bar Hiyya	Abraham ibn Ezra	Jacob Anatoli	Judah ha-Cohen	
					Midrash ha-Ḥokhmah	correspondence
to add	زيد	הוסיף, חיבר, צירף, קיבץ	הוסיף, חיבר	הוסיף, חיבר, צירף	הוסיף, חיבר, צירף	
arc	قوس	קשת	קשת	קשת	קשת	
ascendant	الطالع	הצומח	הצומח	העולה	הצומח	
ascensions	مطالع	מצעדים	מצעדים	עליות	צמחים	מעלות
beginning of Aries	اول الحمل	ראש טלה	ראש טלה	ראשית טלה	ראש טלה	

English	Arabic	Abraham Bar Hiyya	Abraham ibn Ezra	Jacob Anatoli	Judah ha-Cohen	
					Midrash ha-Ḥokhmah	correspondence
celestial equator	معدل النهار	מישור	משוה היום, צדק, [עגולת ראש טלה ומאזנים]	משוה היום	צידוק היום, [עגולת מזל טלה]	מישור
celestial sphere/ circle	فلك	רקיע, גלגל, אופן	גלגל	גלגל, עגול	גלגל	
circle	دائرة	[עגול], in astronomy: אופן, גלגל	עגולה	עגול	עגולה	
chord	وتر	מיתר	יתר	מיתר	יתר	
cosine	جيب التمام	מיתר שארית			ג׳יב התשלום	

English	Arabic	Abraham Bar Hiyya	Abraham ibn Ezra	Jacob Anatoli	Judah ha-Cohen	
					Midrash ha-Ḥokhmah	correspondence
declination	ميل	נמיכות	נטייה	נטייה	נטייה	
degree	درجة, جزء	מעלה, חלק	מעלה	מעלה, מדרגה, חלק	מעלה, חלק	
distance	بعد	מרחק	מרחק	מרחק, רחק	מרחק, ריחוק	
to divide	قسم	חילק	חילק	חילק	חילק	
ecliptic	(دائرة) فلك البروج, منطقة البروج, وسط البروج	רקיע המזלות, גלגל המזלות, אופן המזלות	(עגולת) גלגל המזלות, חשב אפודת גלגל המזלות, חשב האפודה, אפודת הגלגל	גלגל המזלות, עגול אמצע המזלות, אמצע גלגל המזלות	גלגל המזלות, עגולת המזלות אפודת גלגל המזלות, אמצע אפודת המזלות	
horizon	دائرة الافق	אופן המפריש	אופן המפריש	עגול האופק	עגולת האופק [אופן המפריש]	

English	Arabic	Abraham Bar Hiyya	Abraham ibn Ezra	Jacob Anatoli	Judah ha-Cohen	
					Midrash ha-Ḥokhmah	correspondence
latitude	عرض البلد	רוחב המקום, מרחב המקום	מרחב הארץ, רוחב העיר	רחב העיר, רחב	רחב העיר	
meridian	نصف النهار	חצי היום	חצי היום	חצי היום	חצי היום	
to multiply	ضرب	כפל	כפל	כפל	כפל	
oblique ascensions	المطالع في الفلك المائِل, مطالع البلد	מצעדים באופנים הנוטים, מצעדי האקלים	מצעדי הארץ	העליות בעגול הנוטה	הצמחים בגלגל הנוטה, הצמחים בעיר	המעלות בנקודות שהכדור נוטה עליהן, מעלות העיר
pole	قطب	קוטב	סדן	קוטב	סדן	
right ascensions	المطالع في الفلك المستقيم, مطالع خط الاستواء	המצעדים באופן הישר, מצעדים בקו השווה	מצעדי גלגל היושר, מצדעים על הקו השוה	העליות בעגול הישר	הצמחים בגלגל המישור	המעלות בכדור הנצב, מעלות השווי
to rise	طلع	עלה	עלה	עלה	עלה	

<table>
<tr><th rowspan="2">English</th><th rowspan="2">Arabic</th><th rowspan="2">Abraham Bar Hiyya</th><th rowspan="2">Abraham ibn Ezra</th><th rowspan="2">Jacob Anatoli</th><th colspan="2">Judah ha-Cohen</th></tr>
<tr><th>Midrash ha-Ḥokhmah</th><th>correspondence</th></tr>
<tr><td>rising amplitude</td><td>سعة المشرق</td><td>מרחב המזרח</td><td></td><td>מרחב המזרח</td><td colspan="2">רחב המזרח</td></tr>
<tr><td>sine</td><td>جيب,
نصف الوتر,
الوتر</td><td>המיתר המחוצה,
מיתר</td><td>ייבא,
חצי היתר</td><td>מיתר ישר</td><td colspan="2">ג׳יב</td></tr>
<tr><td>sinus totus</td><td>الجيب الأعظم,
جميع الجيب,
الجيب كله</td><td></td><td>אלייבא כלו</td><td></td><td colspan="2">אלג׳יב הגדול</td></tr>
<tr><td>solstice</td><td>(نقطة) الانقلاب</td><td>[ראש גדי, ראש
סרטן]</td><td>הפוך</td><td>מתהפך,
נקודת היפוך</td><td colspan="2">נקודת היפוך</td></tr>
<tr><td rowspan="2">sphaera obliqua</td><td rowspan="2">الفلك المائِل,
الكرة المائلة</td><td rowspan="2">האופן הנוטה</td><td rowspan="2">[הגלגל הנוטה]</td><td rowspan="2">הכדור הנוטה,
הגלגל הנוטה,
העגול הנוטה</td><td colspan="2">הכדור הנוטה</td></tr>
<tr><td>הגלגל הנוטה</td><td></td></tr>
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<table>
<tr><th rowspan="2">English</th><th rowspan="2">Arabic</th><th rowspan="2">Abraham Bar Hiyya</th><th rowspan="2">Abraham ibn Ezra</th><th rowspan="2">Jacob Anatoli</th><th colspan="2">Judah ha-Cohen</th></tr>
<tr><th>Midrash ha-Ḥokhmah</th><th>correspondence</th></tr>
<tr><td rowspan="2">sphaera recta</td><td rowspan="2">الفلك المستقيم,
الكرة المنتصبة</td><td rowspan="2">האופן הישר</td><td rowspan="2">גלגל היושר</td><td rowspan="2">הכדור הישר,
הגלגל הישר,
העגול הישר</td><td colspan="2">הכדור הנצב</td></tr>
<tr><td>גלגל המישור</td><td></td></tr>
<tr><td>to subtract</td><td>نقص</td><td>גרע,
הוציא</td><td>גרע,
חיסר</td><td>גרע,
הוציא,
חיסר,
הפיל</td><td colspan="2">גרע,
הפיל</td></tr>
<tr><td>terrestrial equator</td><td>خط الاستواء</td><td>קו השוה</td><td>קו השוה</td><td>קו השווי</td><td colspan="2">קו המישור</td></tr>
<tr><td>unknown</td><td>مجهول</td><td></td><td>מבוקש</td><td>מוסכל</td><td colspan="2">מוסכל</td></tr>
<tr><td>zenith</td><td>سمت الرأس</td><td>נוכח הראש</td><td>[سمت = קדקד]</td><td>הנקודה שהיא כנגד הראש,
הראש,
נקודת הראש,
נכח הראש</td><td colspan="2">קדקד הראש</td></tr>
</table>

English	Arabic	Abraham Bar Hiyya	Abraham ibn Ezra	Jacob Anatoli	Judah ha-Cohen	
					Midrash ha-Ḥokhmah	correspondence
zodiacal sign	برج	מזל	מזל	מזל	מזל	

Discussion

The table shows that each of the four authors under consideration uses his own idiosyncratic terminology, but the vocabularies in use did not develop independently. Being a pioneer in Hebrew astronomical writing, Abraham Bar Hiyya's approach in finding Hebrew words for foreign concepts was pragmatic; he used biblical and post-biblical Hebrew roots and adjusted their meanings as required. Thus, his Hebrew expressions for 'latitude', 'meridian', 'rising amplitude', 'sine', 'terrestrial equator', and 'zenith' are literal translations of the Arabic expressions that were used in scientific writings. As Gad Sarfatti puts it, "words were obtained from Hebrew lexical material shaped in Arabic moulds, or, putting it in another way, words of 'Hebrew body and Arabic soul'."[1] If necessary, Bar Hiyya does not refrain from even taking over the Arabic 'body' of scientific terms, thus introducing the loanword קוטב for 'pole' into the Hebrew language. However, he does not always follow the Arabic uncritically. Regarding the ambiguous term فلك, Bar Hiyya makes a distinction between רקיע ('sphere'), אופן ('circle') and גלגל ('orbit') that does not seem to exist in Arabic scientific writings. Furthermore, he translates the composite expression denoting 'celestial equator', معدل النهار, with one biblical Hebrew term, מישור. Similarly, he does not translate the term for 'horizon', دائرة الافق, literally, but uses the expression אופן המפריש instead.

Abraham ibn Ezra knew Bar Hiyya's astronomical writings.[2] Yet, his terminology differs considerably from that of his predecessor. While he takes over some terms, such as מצעדים for 'ascensions', אופן המפריש for 'horizon', or קו השוה for 'terrestrial equator', he finds slightly different expressions for 'chord', 'declination', or 'sphaera recta'. Although

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[1] Sarfatti, *Mathematical Terminology*, x.
[2] See Sela, *Abraham ibn Ezra and the Rise of Medieval Hebrew Science*, 99.

some of his terms, like עגולת גלגל המזלות for دائرة فلك البروج, and משוה היום for معدل النهار follow the Arabic more closely, he rejects the loanword קוטב for 'pole' and uses the Hebrew word סדן instead. In addition, he introduces biblical terminology, like the first part of the expression חשב אפודת גלגל המזלות for منطقة فلك البروج ('ecliptic') and the term צדק for معدل النهار ('celestial equator'). In general, Ibn Ezra's Hebrew expressions follow the Arabic more closely, but the lexical material that he 'shapes in Arabic moulds' is preferably biblical. This deviation from Bar Hiyya's terminology may partly be explained by the fact that Ibn Ezra was very critical of Bar Hiyya's astronomical treatises. But, more importantly, Ibn Ezra followed a completely different strategy when creating his own technical terminology. Shlomo Sela has demonstrated that it was mainly ideological considerations that prompted him to create a puristic Hebrew vocabulary; his strategy as a translator was influenced by his view of the Hebrew language. He believed that the bible contained authentic scientific terms that pointed to scientific truths, which had since been forgotten amongst Jewish scholars.[3] In the introduction to the third version of his *Sefer Keli Neḥoshet*, for example, he states that Hebrew, being the first of all languages, was also the most comprehensive one. The bible covered only a tiny part of the original Hebrew vocabulary, and when the Jewish people were exiled and started to speak the languages of the surrounding nations, the Hebrew terminology used for subjects not covered by the bible was simply forgotten.[4] The topos of scientific knowledge that the Israelites once held and that has been forgotten can also be found in the writings of other Jewish scholars of the Iberian Peninsula, like Judah ha-Levi and Maimonides.[5] Ibn Ezra

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[3] Sela, *Abraham ibn Ezra and the Rise of Medieval Hebrew Science*, 140–3; "Abraham Ibn Ezra's Special Strategy," 85–87.

[4] The passage and its English translation can be found in id., *Abraham ibn Ezra and the Rise of Medieval Hebrew Science*, 105, 339.

[5] See Langermann, "Some remarks on Judah ben Solomon ha-Cohen," 368; "Science and the Kuzari," 507–508.

differs from these authors in that he finds traces of this forgotten knowledge in the actual words that are used in the bible. Thus he applies biblical words in his scientific writings in order to 'revive the semi-extinguished holy tongue, to restore an original scientific meaning that had been forgotten, a task which involved mining the biblical text to rediscover a set of original Hebrew scientific terms.'[6] In his view the biblical terms which he applied in his scientific writings were, in fact, relics of the original Hebrew language that he tried to revive. It might be somewhat surprising, then, that Ibn Ezra did not try to find a biblical Hebrew term for the notion of 'sine'. But this indicates that he was aware of the fact that the concept was created long after the bible was written.

Jacob Anatoli does not appear to follow an ideological incentive when creating his astronomical terminology. His lexicon is first and foremost based on that of Abraham Bar Hiyya, but he stays much closer to the original Arabic expressions in his translations. Thus, he takes over Bar Hiyya's terms נכח הראש for 'zenith', מיתר for 'chord, sine' and קוטב for 'pole', but not his distinction between אופן ('circle'), גלגל ('orbit') and רקיע ('sphere'). Instead of the term אופן המפריש, which was used by both his predecessors to denote 'horizon', he uses the loan translation עגול האופק. However, the word that denotes 'circle' in this expression, עגול, stems from Bar Hiyya. In addition to the term מעלות for 'degrees', he also uses the loan translation מדרגות, which is even closer to the Arabic درجات. His terms for 'ascensions' and 'ascendant', too, deviate from the terminology that was established by Bar Hiyya and Ibn Ezra. Instead of using the terms of מצעדים and הצומח, respectively, which are derived from different Hebrew roots, he imitates the Arabic word formation in his terminology. Thus, as the Arabic terms for 'ascensions' and

[6] Sela, *Abraham ibn Ezra and the Rise of Medieval Hebrew Science*, 143.

'ascendant', مطالع and الطالع, are derived from the verb طلع ('to rise'), he uses the terms עליות and העולה, respectively. Like the Arabic terms, both words are derived from the same verb denoting 'to rise', which is עלה in Hebrew. In fact, by closely following the Arabic text and forming new expressions in analogy to the Arabic original, Anatoli adopts the terminology and translation technique of his father-in-law and mentor Samuel ibn Tibbon. He thus continues the tradition that was started by Samuel's father Judah.

Judah ben Solomon ha-Cohen appears to have been acquainted with that tradition. Like Jacob Anatoli, uses the verb הפיל for 'to subtract', employs the loanword אופק for 'horizon', and generally follows the Arabic syntax and terminology very closely. More importantly, he translates the Arabic term مجهول as מוסכל – a term that was introduced into the Hebrew language by Judah ibn Tibbon and that was first used in a mathematical sense by Jacob Anatoli. To this list should be added four terms that we already encountered in the mathematical part of the correspondence: יחס ('ratio'), נכחי ('parallel'), מקום חתך ('point of section'), and גימטריא ('geometry'). All four terms are used consistently, with exactly the same meaning as in the correspondence, in the astronomical translations of Jacob Anatoli.[7] Judah was thus certainly aware of his writings and influenced by them.

Regarding the notion of 'ratio', however, it turns out that Judah also uses an alternative term, especially in the non-mathematical sections of the *Midrash ha-Ḥokhmah*: in his introduction and the treatises on biblical verses and the letters of the Hebrew alphabet he uses the term ערך.[8] As we have seen, this is the term employed by both Abraham Bar

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[7] See, for example, Lay, "L'abrégé de l'Almageste," index: intersection (lieu de l'); parallèle; rapport.

[8] In his introduction, for example, he writes: ערך חכמה אלהית לשאר החכמות כערך האור האמיתי שאין בו חושך מאור האש הנקרא חושך (Oxford, Bodleian Library, MS. Pococke 343, fol. 8v; MS. Michael 551, fols. 5v). See also Goldstein, "The Commentary of Judah ben Solomon Hakohen," lines 914–915: ותהיה כאור חזק ערכו מזו [...] ההשגה שעלה לה בכאן בזה העולם כערך אור השמש מאור נגה.

Hiyya and Abraham ibn Ezra to denote 'ratio', while the term יחס was introduced by Samuel ibn Tibbon. It appears that Judah was acquainted with both expressions and decided to use יחס only in a strictly mathematical sense.

But despite his familiarity with the Tibbonide tradition, he employs a vocabulary that generally resembles much more that of Abraham ibn Ezra than that of the translator family. Thus, he uses עגולה (instead of עגול) for 'circle', סדן (instead of קוטב) for 'pole', יתר (instead of מיתר) for 'chord', and קדקד הראש (instead of נכח הראש) for 'zenith'. Like Ibn Ezra, he uses a transliteration of the Arabic word جيب (instead of the Hebrew term for 'chord') to denote 'sine'. In fact, between the correspondence and the summary of the *Almagest*, a shift towards Ibn Ezra's vocabulary can be made out. Judah calls the 'ecliptic' גלגל המזלות and עגולת המזלות in the correspondence. Both terms are also used in his translation of the *Almagest*, but in addition Judah employs the expressions אפודת גלגל המזלות and אמצע אפודת המזלות. The use of אפודה, which originally referred to the garment of the high priest, as a translation of منطقة, the Arabic term for 'belt', goes back to Abraham ibn Ezra. Like Jacob Anatoli, Judah consistently uses the Arabic loanword אופק for 'horizon' in both the correspondence and the *Almagest*. But in addition, Ibn Ezra's term אופן המפריש appears the latter work.

The similarity to Ibn Ezra's vocabulary suggests that Judah ha-Cohen had studied Ibn Ezra's astronomical works closely. Given his background as a rabbinical scholar, he may have been familiar with Ibn Ezra's terminology through the study of his non-scientific writings, such as his biblical commentaries. Judah shared Ibn Ezra's belief that the Jews had once been in possession of scientific knowledge that was lost in the course of time.[9]

[9] Sirat, "Juda b. Salomon ha-Cohen," 49.

Yet, unlike Ibn Ezra, Judah does not actively try to 'revive' the scientific vocabulary that was allegedly used by earlier generations. Thus, in the three treatises dedicated to traditional Jewish learning he tries to show that hidden astronomical knowledge can be found in the Bible and Talmud, but he does not use the words whose hidden meanings he unravels in his astronomical writings.[10] Judah's use of both Ibn Ezra's and Jacob Anatoli's vocabularies appears to be purely pragmatic. However, he seems to be aware of the fact that some of his expressions might be difficult to understand for a reader who is not familiar with Ibn Ezra's writings. For example, when employing the expression צידוק היום for 'celestial equator', Judah adds the original Arabic term he is translating.

Regarding the relationship between the correspondence and the astronomical writings in the *Midrash ha-Ḥokhmah*, it has to be noted that Judah's terminology is not consistent. There are three fundamental notions for which the vocabulary differs completely between the correspondence and the rest of the encyclopaedia. These are 'celestial equator', 'sphaera recta', and 'ascensions'. Judah calls the 'celestial equator' גלגל המישור and עגולת גלגל המישור in both the correspondence and his summary of the *Principles of Astronomy*. This expression is in fact the only scientific term that he explicitly takes over from Abraham Bar Hiyya's terminology. However, in his summary of the *Almagest* he uses the expressions עגולת מזל טלה and צידוק היום for 'celestial equator'. These terms resemble the expressions עגולת ראש טלה ומאזנים and קו הצדק, which were used by Ibn Ezra. Thus, Judah does not simply employ an additional term on top of the terminology already in use, but he completely replaces the technical term used in his other writings. Still, the expression גלגל המישור also appears in the summary of the *Almagest*, but with a

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[10] Judah argues, for example, that the 'pole' is called כוותא דרקיעא ('window of the sky') in the Talmud. See Goldstein, "The Commentary of Judah ben Solomon Hakohen," lines 460–474. However, he does not apply this term in his abbreviations of astronomical books.

completely different meaning. There it denotes 'sphaera recta', as opposed to הגלגל הנוטה, 'sphaera obliqua'. Between the correspondence, the summary of the *Almagest*, and the summary of the *Principles of Astronomy*, the meaning of the term גלגל המישור thus switches from 'celestial equator' to 'sphaera recta' and back to 'celestial equator'. The third notion that is expressed in a different way is 'ascensions'. Like Jacob Anatoli, Judah deviates from Bar Hiyya's and Ibn Ezra's term מצעדים. In the correspondence he uses the word מעלות for 'ascensions'. The term differs only slightly from Anatoli's term עליות. But in the rest of the *Midrash ha-Ḥokhmah* the word מעלות refers to 'degrees', while 'ascensions' are called צמחים.

Differences in the use of vocabulary between the correspondence and the summaries of the *Almagest* and the *Principles* can also be made out regarding two of the additional terms that Judah ha-Cohen has in common with Jacob Anatoli: נכחי ('parallel') and מקום חתך ('point of section'). These two expressions were already found to be problematic in the discussion of the geometrical terminology, since in the summary of the *Elements* Judah employs different terms for these notions. As regards Judah's astronomical writings, neither מקום חתך nor the verb חתך in general appear in his versions of the *Almagest* and the *Principles of Astronomy*. Instead, Judah uses the term that we already encountered in his summary of the *Elements*, נקודת חצבות, or the similar expression נקודת חיצוב. Concerning the notion of 'parallel', Judah uses the adjective נגדי throughout the *Almagest* and the *Principles of Astronomy*, while the adjective נכחי, which did not appear in his translation of Euclid's *Elements* at all, denotes 'opposite'. However, there is one exception. At the very beginning of his paraphrase of the *Almagest*, Judah states that the fixed stars always move in parallel circles. The Hebrew expression he uses is על גלגלים נכחיים זה לזה.[11]

[11] MS. Vat. 338, fol. 211r.

Since after this sentence the use of נגדי for 'parallel' and נכחי for 'opposite' is consistent throughout the treatise, it seems as though Judah changed his mind after translating the first passage.

All in all, the vocabulary of the correspondence differs from that of the astronomical writings of the *Midrash ha-Ḥokhmah* to such a degree that the same conclusion must be reached as regards the mathematical writings of the encyclopaedia; the correspondence was rendered at a different point in Judah's working-life than the astronomical summaries. Furthermore, the correspondence shows a greater similarity with Jacob Anatoli's terminology than the summary of the *Almagest*, while the latter work shows a greater resemblance to the vocabulary of Abraham ibn Ezra. In this sense, the turn towards Ibn Ezra's terminology that is apparent between the correspondence and the *Almagest*, could in fact also be interpreted as a turn away from Jacob Anatoli's terminology.

At this point the question arises whether the correspondence, the summary of the *Almagest* and the summary of the *Principles of Astronomy* were, in fact, translated in chronological order. If the correspondence was translated after the other works, the development described above would be in the opposite direction; Judah would have replaced terms that he had taken over from Ibn Ezra's writings with those he found in the works of Jacob Anatoli.

The question cannot be answered with certainty. However, the consistent use of terms like עגולה, סדן, ג׳יב, and קדקד הראש within both the correspondence and the other astronomical writings indicates that Judah ha-Cohen was influenced by Ibn Ezra's terminology at every stage of his translation enterprise. But when translating the correspondence, he took over terms from Jacob Anatoli's vocabulary, such as מקום חתך

and נכחי, which otherwise do not (or only once) appear in his writings. This hints at the possibility that Judah encountered Jacob Anatoli's translations of astronomical texts only when he had already started translating the *Midrash ha-Ḥokhmah* into Hebrew, and that through the study of these texts his own terminology shifted towards that of Anatoli.

Regarding the order in which Judah translated different parts of the *Midrash ha-Ḥokhmah* into Hebrew, the word מעלות, which denotes 'ascensions' in the correspondence, is of special interest. Judah must have been aware of the fact that the term already denoted 'degrees' in all scientific writings available to him. In contrast, the term he uses in the *Almagest*, צמחים, is unambiguous, and could thus be considered a better and therefore also a later translation. However, a different order of events is indicated by Judah's use of the term גלגל המישור. In both the correspondence and the summary of the *Principles of Astronomy*, its use is consistent and unambiguous. Only in the translation of the *Almagest* the term receives a different meaning.

Although the vocabulary in each separate abbreviation and summary appears to be consistent, Judah changes his terminology back and forth in between different parts of the encyclopaedia. But if we assume that the correspondence was translated *after* the completion of Hebrew version of the *Midrash ha-Ḥokhmah*, there would be a continuous development: The meaning of גלגל המישור would have shifted from 'sphaera recta' to 'celestial equator' between the translations of the *Almagest* and the *Principles of Astronomy*. This new meaning would later also be applied in the correspondence. However, in that case Judah would have exchanged an unambiguous term denoting 'ascensions' for a term that already had a different meaning in his own (and his predecessors') writings.

This shift in terminology between different summaries does not seem to reflect a chronological order of translation; rather, the encyclopaedia appears to be a mixture of different versions of the text. Regarding the Hebrew summary of the *Elements*, we have already seen that Judah rendered two different translations. The same seems to be the case in the astronomical part of his encyclopaedia. While the terminology remains almost constant within each treatise, the astronomical part as a whole gives the impression of a work in progress, made up of translations whose vocabularies were in different stages of completion.

Conclusion

The comparison of the astronomical vocabularies of Abraham Bar Hiyya, Abraham ibn Ezra, Jacob Anatoli and Judah ha-Cohen shows that each author uses his own distinct and identifiable vocabulary, but that these vocabularies are interlinked. Abraham Bar Hiyya's terminology laid the basis for all following translations. On the one hand it is pragmatic; Hebrew words are formed and given new meanings according to the Arabic words they represent. On the other hand, Bar Hiyya tried to find unequivocal Hebrew expressions for terms that were ambiguous in Arabic. His successors followed the structure of the Arabic scientific terminology more closely, but they used different Hebrew words to resemble that structure. In accordance with his belief that the language of the bible contains remnants of lost scientific knowledge, Abraham ibn Ezra avoids the use of Arabic loanwords and uses preferably biblical Hebrew terms for astronomical concepts. Jacob Anatoli does not follow this approach. His terminology is based on that of Abraham Bar Hiyya, but his vocabulary is much closer to the Arabic than that of his predecessors. He does not refrain from using Arabic loanwords, and his loan translations closely imitate Arabic scientific terminology. Judah ha-Cohen is clearly influenced by both Jacob Anatoli's and Abraham ibn Ezra's astronomical writings. Of the different astronomical texts contained in the *Midrash ha-Ḥokhmah*, the correspondence with the philosopher contains the terminology that is closest to that of Jacob Anatoli. It may have been a first tentative translation, while the other astronomical treatises in the encyclopaedia reflect later stages in the formation of Judah's unique terminology. However, it is also possible that the correspondence was translated only after the Hebrew version of the *Midrash ha-Ḥokhmah* was finished. In that case, Judah may initially have been familiar with mainly Abraham ibn Ezra's astronomical terminology, but through the study of Jacob Anatoli's translations his vocabulary eventually changed. Be that as it may, its vocabulary differs from that of the abbreviations of the *Almagest* and the *Principles of Astronomy* to such an extent that it was certainly translated at a different stage of Judah's working life.

Chapter Five

Writing for a Jewish readership: the place of the correspondence within the *Midrash ha-Ḥokhmah*

While the previous chapters dealt with the correspondence itself and with the mode of its Hebrew translation, in this chapter we shall look at not *how*, but *why* Judah decided to translate his literary encounter with the emperor's philosopher into Hebrew. It is obvious that the translation was intended for a Jewish audience – in all probability for Judah's Italian coreligionists who were unable to read the Arabic original. But this does not answer the question as to why Judah deemed the correspondence to be important enough to be presented to a Jewish readership. Or rather, why Judah deemed two selected problems that were dealt with in his exchange with the emperor's philosopher to be suitable reading material for his Italian Jewish audience. Closely related to this question is the question whether initially Judah intended to publish the correspondence as part of the *Midrash ha-Ḥokhmah* and what function it fulfils within the framework of the encyclopaedia.

The place of religion in the original correspondence

The Arabic correspondence on mathematics and astronomy that Judah ha-Cohen had with the emperor's philosopher appears to be an example of a phenomenon which B. Goldstein described as *science as a 'neutral zone' for interreligious cooperation*.[1] Since the very beginning of the appropriation of Greek science in the Muslim world scholars of different

[1] Goldstein, "Science as a 'Neutral Zone'"; Grasshoff, "Contextualizing the History of Islamic Sciences," 302–305.

religions and denominations had cooperated in translating texts into Arabic and in developing their knowledge and understanding of ancient texts. This cooperation could take different forms. On the one hand, patrons might employ skilled translators and scholars of different faiths to their own; on the other hand, exchanges of knowledge and collaboration on scientific projects would take place between scholars of different religions. Thus, one of the most renowned translators of the Abbasid period (750-950) was the Christian physician Ḥunayn ibn Isḥāq (809 – 873). As we saw earlier, his son Isḥāq ibn Ḥunayn (d. 910/11) made Arabic translations of Ptolemy's *Almagest* and Euclid's *Elements*. These translations were revised by the Sabian scholar Thābit ibn Qurra (826-901). Thābit, one of the most famous astronomers of the time, participated in joint research projects with prominent Muslim astronomers. As D. Gutas put it: *The support for the translation movement cut across all lines of religious, sectarian, ethnic, tribal, and linguistic demarcation. Patrons were Arabs and non-Arabs, Muslims and non-Muslims, Sunnīs and Shī'ites, generals and civilians, merchants and land-owners, etc.*[2] This cooperation across religious boundaries as regards sciences continued in the following centuries, and it appears to have been especially strong in the disciplines of medicine and astronomy.[3]

But the phenomenon of interreligious collaboration in scientific matters was not exclusive to the Muslim world. It was again a translation movement, this time from Arabic to Latin, that in the twelfth century gave rise to a similar interreligious exchange of ideas in Christian Europe. The centre of this Arabic-to-Latin translation movement was Judah ha-Cohen's hometown of Toledo. Already under Muslim rule, from the eighth to the eleventh centuries, the city had been a centre of Arabic learning, and Arabic had

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[2] Gutas, *Greek Thought, Arabic Culture*, 5.
[3] Grasshoff, "Contextualizing the History of Islamic Sciences," 303; Savage-Smith, "The Universality and Neutrality of Science," 171–175.

remained the language of culture and education for a large part of the population after the Christian conquest in 1085. Because Arabic scientific works remained available even after many among the Muslim elite had left Toledo in the wake of the Christian conquest, scholars from the Christian West came to the city in search for works that were otherwise unattainable. A demand for Latin translations also arose among the newly established cathedral clergy that came from Latin Europe and could therefore not read Arabic. These Latin translations were often made under collaboration of Christian and Jewish scientists. The Jewish scholar Abraham ibn Daūd, for example, had fled Cordoba at the arrival of the Almohads (1148) and settled in Toledo in 1160. There, he not only wrote original philosophical and astronomical works in Arabic, but he also collaborated with the Christian archdeacon Dominicus Gundissalinus in the Latin translation of one book of Ibn Sīnā's encyclopaedia Shifāʾ. Ibn Daūd translated the Arabic original into the vernacular, which was then translated into Latin by Gundissalinus.[4] Likewise, a Jew called 'Abuteus levita' helped Michael Scot translate al-Biṭrūjī's *On the Principles of Astronomy* into Latin in 1217.[5] A year later, some astrological texts were translated into Latin by Salio, a canon of Padua, with the help of a learned Jew called David.[6] But collaboration between Jewish and Christian scholars, especially in the mathematical and astronomical sciences, was not restricted to Toledo. In Barcelona, the 'father of Hebrew mathematics', Abraham bar Ḥiyya, had cooperated with Plato of Tivoli in translating several astronomical works into Latin. About a century later, in Castile, the Jewish scholars Judah ben Moses ha-Cohen

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[4] For the latest research on Abraham ibn Daūd, see the dossier *New Perspectives on Abraham Ibn Daud* in *Aleph* 16, no. 1, 1-106. On his identity with the translator Avendauth, see Szilagyi, "A Fragment of a Book of Physics from the David Kaufmann Genizah Collection (Budapest) and the Identity of Ibn Daud with Avendauth"; Freudenthal, "Abraham Ibn Daud, Avendauth, Dominicus Gundissalinus and Practical Mathematics in Mid-Twelfth Century Toledo". See also Burnett, "Communities of Learning in Twelfth-Century Toledo"; id., "The Coherence of the Arabic-Latin Translation Programme in Toledo in the Twelfth Century."
[5] Id., "Michael Scot and the Transmission of Scientific Culture from Toledo to Bologna via the Court of Frederick II Hohenstaufen," 109.
[6] Chabás, "Interactions between Jewish and Christian Astronomers in the Iberian Peninsula," 149.

and Isaac ibn Sid would be among the astronomers who composed a new set of astronomical tables under the patronage of king Alfonso X. The Alfonsine tables, which contain data that start with 1 January 1252, would become the most popular astronomical tables throughout Europe.

As we saw in the previous chapters, the intellectual climate at the court of Frederick II bore similar characteristics. In his quest for scientific and philosophical knowledge, Frederick sent queries to Christian, Muslim and Jewish scholars alike, and whenever possible he engaged in personal discussions with them. In many respects the imperial court could therefore indeed be regarded as a 'neutral zone' in which scholars could discuss scientific problems as equals, regardless of their confession. Judah's correspondence with Frederick's philosopher is only one of many examples for this interchange across denominational boundaries.

Did Judah therefore publish his scientific exchange with a gentile philosopher in order to promote interreligious cooperation in the sciences? As a Toledan scholar, he must certainly have been aware of the collaboration that took place both in his hometown and elsewhere, and also of the fact that scientific ideas had been exchanged between sages of different faiths and denominations since the beginnings of Arabic mathematics and astronomy. He may therefore have chosen to translate a mathematical and an astronomical problem into Hebrew in order to demonstrate to his Italian coreligionists that as regards these sciences, religious boundaries were of no importance.

In his account of the correspondence, the fact that his opponent was a Christian does not seem to be important at all to Judah, for never does he mention the fact that his correspondent was not Jewish. However, we should bear in mind that he does not give his readers a full account of all the questions and answers that were allegedly being discussed during his exchange with the imperial court, but only a very limited selection. In his response to the first question, asked by the philosopher, he is able to display his expert knowledge in Euclidean geometry. But it is the second question, which Judah sets, that reveals the fact that the relationship between him and his correspondent was not solely characterised by mutual respect and collaboration; Judah asks his opponent a question on oblique ascensions – a problem that requires expert knowledge in astronomy, but, for an expert, should not have been too difficult to answer. Yet, although the philosopher had obviously studied the most important astronomical textbook of the day, Ptolemy's *Almagest*, he was unable to answer the question correctly. On the contrary, in his response he makes a mistake so grave that Judah takes the opportunity to ridicule him in his rebuttal and to go so far as to question whether his correspondent truly deserves to be called 'philosopher'. Judah thus chooses not to present selected problems that were discussed by two equally skilled experts, but instead presents his readers with his opponent's failure and his own triumph. His intellectual superiority over a figure of such a high standing as the emperor's philosopher certainly demonstrated his own expertise, and it increased his own status as an intellectual authority, regardless of religious boundaries.

However, Judah published the Hebrew translation of the two questions not as an independent treatise, but within the framework of his encyclopaedic work *Midrash ha-Ḥokhmah*. As he states himself, *[...] I translated them from Arabic into the Hebrew language, with this book that I composed when I was still there [i.e. in Toledo].*[7] Whether or not the exchange was part of the Arabic original remains at this point subject for speculation, but it is clear that its Hebrew translation was not intended to be read independently of the

[7] Edition, paragraph 2.

encyclopaedia. It is possible that within the framework of the *Midrash ha-Ḥokhmah*, the correspondence may have fulfilled a different function than simply promoting interreligious collaboration in scientific studies or establishing Judah's authority in the matter. In fact, within the encyclopaedia Judah's attitude towards both the 'foreign sciences' and gentile scholars is very ambiguous. In order to determine the function and meaning of the correspondence within the work, we shall first give an overview of the contents and structure of the encyclopaedia, the historical circumstances of its composition, and its aim.

Contents and structure of the *Midrash ha-Ḥokhmah*

A recurring theme in the *Midrash ha-Ḥokhmah* is the division of the universe into three worlds: that of generation and corruption, that of the celestial spheres and the spiritual world. Accordingly, the encyclopaedia is divided into two parts: the first one deals with the world of generation and corruption, the second one with the world of the celestial sphere. Knowledge of the divine world is only hinted at through three treatises, one following the first part, two following the second part of the work.

In the first part of the encyclopaedia Judah gives summaries of Aristotle's works on logic, natural science, and metaphysics according to Ibn Rushd's interpretation.[8] Throughout these works, Judah's critical attitude towards both Aristotle and his commentator Ibn Rushd comes to the fore. Thus, at times he inserts his own short comments pointing out perceived inconsistencies and contradictions in the Aristotelian system, but he also devotes two longer passages, one in the introduction and one right after his summary of the *Metaphysics*, to a more detailed criticism of Aristotelian doctrines and principles. At

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[8] See table in Chapter One for details.

the end of this first part of the work, Judah presents a treatise in which he explains the meaning of selected biblical verses from Genesis, Psalms and Proverbs. He tries to demonstrate that the knowledge that can be gained from studying the so-called 'external sciences' was presupposed in the Bible and that this secular scientific knowledge was also held by the sages of Mishnah and Talmud. It forms part of the 'divine wisdom' – an all-encompassing knowledge of the divine world which God revealed to Israel alone.

The second, mathematical-astronomical, part of the *Midrash ha-Ḥokhmah* is structured in a similar way. Again, Judah first gives summaries of scientific books and then adds his own original work in order to demonstrate that Israel's sages had always been privy to the knowledge contained in secular works. It contains a reworked version of books I-VI and XI-XIII of Euclid's *Elements*, the account of the correspondence with the emperor's philosopher, the first nine books of Ptolemy's *Almagest*, al-Biṭrūjī's *On the Principles of Astronomy*, and Ptolemy's *Tetrabiblos*. Then follow two treatises, one on the secret meaning of the shapes and numerical values of the letters of the Hebrew alphabet, which is followed by an account of a polemic disputation with a gentile sage, and a treatise on the secret knowledge that comes to the fore in Talmudic aggadot. It is in these treatises that Judah explains how the astronomical/astrological knowledge found in the second part of the work forms part of the divine wisdom given to Israel, since the way God's command emanates from the higher worlds to the world of generation and corruption is already represented in the names, shapes, numerical values and order of the letters of the Hebrew alphabet. The 'true order' of the planets (that is, their order in al-Biṭrūjī's planetary system) had also been part of this wisdom, and was reflected in the order of the Hebrew letters. The final treatise on Talmudic aggadot is devoted to exposing some aspects of this 'divine wisdom' which were known to the sages of Mishnah and Talmud. In the introduction to this treatise Judah explains that this wisdom goes far beyond the realms of Aristotelian *Physics* and *Metaphysics*, that it is secret and can only be transmitted orally from person to person.[9] Thus, Judah tries to hint at the secret meaning of Mishnaic sayings, such as '*Maʿaseh Be-reshit* (the work of creation) must not be explained before two',[10] or 'Ten things were created on the first day'.[11]

The *Midrash ha-Ḥokhmah* is therefore not simply a handbook for the study of science and philosophy, for it demonstrates the relationship between these 'foreign sciences' and religious learning. Judah's enterprise of delineating both the benefits and the limits of scientific and philosophical knowledge should be understood in its historical context.

The Maimonidean controversy

Born in the second decade of the thirteenth century, Judah received his education in a period in which philosophical teachings became available to Jewish communities in the Latin West that previously had not had access to this philosophical tradition. It was with the circulation of Maimonides' *Mishneh Torah,* especially its first part, the *Sefer ha-Madaʿ*, as well as the Hebrew translation of his philosophical work *Guide for the Perplexed*, that several discussions broke out among Jewish intellectuals about the philosophical doctrines found in these works, which would then turn into discussions about the permissibility of the study of philosophy itself.

A first dispute, in which Judah's teacher Meir ben Todros Abulafia (c. 1170-1244) played an active role, erupted in the early years of the thirteenth century. In the *Sefer ha-Madaʿ* Maimonides appeared to promote the view that the 'world to come' was not a period in history in which the dead would be bodily resurrected, but the immortal existence of the

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9 Oxford, MS. Mich. 551, fol. 198v.
10 Mishnah, Hagigah 2:1.
11 Babylonian Talmud, Hagigah 12a.

intellect in proportion to its attainment of knowledge. From this Meir Abulafia inferred that Maimonides denied resurrection of the body as a halakhic principle. He opposed this opinion vehemently and sought affirmation of his views from the scholars of Lunel (southern France), who, to his disappointment, chided him for disagreeing with Maimonides. The dispute only ended when Samuel ibn Tibbon circulated his Hebrew translation of Maimonides' *Treatise on the Resurrection of the Dead* (*Maqalah fī Teḥiyat ha-Metim*, composed in 1190–91) among scholars, and Abulafia became satisfied that Maimonides did affirm bodily resurrection.

Between the years 1230 and 1235 a second controversy regarding Maimonides' works, this time about the permissibility of rationalist philosophy altogether, erupted within the Jewish communities of Southern France and Spain. As a venerable scholar and communal leader in Toledo, Judah's teacher Meir Abulafia played an important role also in this debate.[12] Whereas previous discussions about the validity of science and philosophy had taken place within Judeo-Arabic culture, it was the entry of Spanish rationalist philosophy into Latin-Jewish culture that triggered this new heated debate, which turned into an inter-communal battle characterized by polemical propaganda, excommunications, and charges of informing.

Rationalist philosophy was eagerly taken on by intellectuals in Provence, with Samuel ibn Tibbon, the translator of Maimonides' *Guide*, as one of the most fervent adherers of philosophical ideas that valued universal reason over religious tradition. In the years after becoming available in Hebrew, Maimonides' *Sefer ha-Mada*ʿ and *Guide for the Perplexed* had turned into basic textbooks for the emerging Provencal rationalism. A leading critic

[12] For a detailed analysis of Abulafia's role in the conflict, see Septimus, *Hispano-Jewish Culture in Transition*, 61–74.

of this development was Solomon ben Abraham of Montpellier.[13] Together with his pupils he engaged in a forceful dispute with the advocates of philosophic learning. In his struggle he turned to the tosafist schools of northern France for support, which consequently condemned the study of the *Guide* and *Sefer ha-Mada*ʿ. The rationalists in turn sent letters north defending Maimonides and the study of philosophy, and engaged in a campaign to excommunicate Solomon and his pupils. In Toledo, the most powerful Jewish community in Castile, Judah ha-Cohen's teacher Meir Abulafia was approached by both proponents and opponents of philosophical rationalism. While he declared his firm opposition to the ban on Solomon ben Abraham and strongly supported his cause, he considered efforts to ban the study of philosophy altogether to be futile. At the height of the conflict, under unclear circumstances, the *Sefer ha-Mada*ʿ and *Guide for the Perplexed* attracted the attention of the inquisition, which led to the public burning of these books by the Dominicans in Montpellier in 1232.

It was precisely during these years of turmoil within the Jewish communities, in the early 1230s, that Judah ha-Cohen composed the Arabic original of the *Midrash ha-Ḥokhmah*. That he was well aware of the vigorous discussions concerning Maimonides' works taking place at the time becomes clear from a statement he makes at the end of the first part of his encyclopaedia:

> *I acknowledge that the* Guide to Righteousness *planted in my heart a burning desire to study the books of the philosophers, until I had achieved what little I have. I truly know that the intention of the Guide was merely to cause those who erred after the words of Aristotle to return and to hold fast to our holy Torah. One ought not suspect the lamp of the exile of Ariel of the things of which some people of this generation suspected him.*[14]

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[13] Not to be confused with Solomon ben Abraham Adret of Barcelona (*c.* 1235-*c.* 1310).

[14] Oxford, MS. Mich. 551, fol. 145r: לפיכ׳ אני מודה כי המורה לצדקה שם בלבי אש בוערת לעיין בספרי הפילוסופים עד שעלה בידי זה המעט. וידעתי באמת כי לא היתה כוונת המורה אלא להשיב ולהחזיר מי ששגג בדברי ארסטו להחזיק בתורתנו שלא לחשוד נר גלות אריאל במה שחשדוהו מקצת אנשי הדור. The passage was first translated by Septimus, *Hispano-*

The notion that a well-rounded education should include science and philosophy had become commonplace in the Andalusian tradition that Judah ha-Cohen grew up in. It was not the rationalist tradition as a whole, but certain internal developments that gave Spanish anti-rationalists cause for alarm. They criticised a new radical form of nonliteral exegesis that imposed philosophical naturalism upon the biblical text, relying solely on philosophical reasoning and denying the possibility of a literal interpretation when it contradicted Aristotelian philosophy. Regarding the study of philosophy, they denied not its permissibility, but its primacy over biblical and Talmudic studies.

In his encyclopaedia Judah delineates the benefits and limitations of philosophical studies. The *Midrash ha-Ḥokhmah* can therefore be seen as Judah ha-Cohen's contribution to the discussion about the value and permissibility of the study of secular science and philosophy that was taking place at the time. Having been brought up in the Andalusian tradition, he deemed philosophical learning an essential part of education, but like his teacher he opposed the new radical kind of Aristotelianism that was burgeoning in Spain and Provence. Thus, while on the one hand giving an overview of the most important philosophical and mathematical/astronomical works of his time, he at the same time tries to demonstrate the limits of philosophical study and to establish the superiority of religious learning.

Jewish Culture in Transition, 98. See also Sirat, "Juda b. Salomon ha-Cohen," 44; Fontaine, "Judah ben Solomon ha-Cohen's 'Midrash ha-Hokhmah': Its Sources and Use of Sources," 194.

The aim of the *Midrash ha-Ḥokhmah*

In the introduction to the *Midrash ha-Ḥokhmah*, Judah himself describes the relationship between religious and philosophical knowledge, as well as his aim in writing the encyclopaedia. Having explained the concept of the three worlds, he describes the scientific curriculum of his time, enumerating the different disciplines, their order, and the books from which they need to be studied. Immediately after this general introduction, however, he describes the limits of philosophical and scientific knowledge. According to Judah, the secular sciences are helpful in understanding the two lower worlds, which are the world of generation and corruption and the world of the celestial spheres, but perception and reason alone cannot lead to understanding of the divine world, which is not perceived by the senses. Thus, he argues, even if one were to know Aristotle's books on metaphysics by heart, all that can be learned from them is that there is one incorporeal prime mover, and that there are separate intellects governing the spheres.[15]

Profound knowledge of the divine world and of the 'essence' of the world of the spheres can only be achieved by revelation. God gave this deeper knowledge to Moses on Mount Sinai, and it was handed down from generation to generation in an unbroken tradition – a process which he (and others, such as Nahmanides) called *qabalah* ('tradition'). During the millennia that followed, Israel held on to this most important knowledge, but knowledge of the two lower worlds, which is included in the all-encompassing divine knowledge, was slowly lost:[16]

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[15] Cf. Oxford, Bodleian Library, MS. Mich. 551, fol. 4v. An English translation of the passage can be found in Sirat, *A History of Jewish Philosophy in the Middle Ages*, 253.
[16] Oxford, Bodleian Library, MS. Pococke 343, fol. 8v and MS. Michael 551, fols. 5v-6r. MS. Pococke reads: ואמ׳ יתרון לחכמה מן הסכלות כיתרון האור מן החושך כלומ׳ ערך חכמה אלהית לשאר החכמות כערך האור האמיתי שאין בו חושך מאור האש הנקרא חושך לפי שאותה חכמה היא אור בהיר וזאת החכמה לפי רום מעלתה וחשיבותה ושהיא כוללת

And [scripture] says: 'there is more gain in wisdom than in folly, as there is more gain in light than in darkness'.[17] That is to say, the relation of the divine wisdom to the rest of the [scientific and philosophical] disciplines is like the relation of the true light in which there is no darkness to the light of the fire, which is called 'darkness' - since this wisdom is bright light. And because of its sublimeness and its importance, and as it comprises all disciplines, and as it is the essence that everything is dependent upon, God revealed this wisdom only to Israel alone, as it is said: 'He has revealed His word to Jacob, His laws and decrees to Israel. [He has not dealt so with any nation: and as for His judgments, they have not known them.]'[18] – Since the sages of the nations of the world - the wisdom of their philosophers only extended to those disciplines. But those about whom it is said 'a wise and understanding people',[19] they had root and branch.

But since 'the wisdom of our wise men perished, and the understanding of our prudent men was hid',[20] of these disciplines 'only two or three berries in the top of the uppermost bough'[21] were left in our hands, so that the glory of the Israelite sage nowadays consists only of the knowledge of those books that we mentioned, or some of them; up until it became clear among the people of the world that there was no wisdom in Israel and that 'my people is foolish',[22] since it was the Greeks alone who wrote down the [scientific and philosophical] disciplines, and these were translated to other languages as their treatises became known. This is because of the detachment of [God's] providence from Israel, like Isaiah said: 'Therefore, behold, I will proceed to do a marvellous work among this people, even a marvellous work and a wonder: for the wisdom of their wise men shall perish, and the understanding of their prudent men shall be hid'.[23]

That is to say, it is a marvellous wonder that that the true wisdom left the people about whom it is said 'a wise and understanding people' and so forth,[24] and that they returned to rely on the treatises of the nations. 'The waters failed from the

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כל החכמות והיא עיקר שהכל תלוי בו מה שגלה הק׳ב׳ה׳ ממנה לא גלה כי אם לישר׳ בלבד שנ׳ מגיד דבריו ליעקב חק׳ ומש׳ לישראל וג׳ לפי שחכמי אומות העולם לא הגיעה חכמת הפילוסוף מהם כי אם לאותן החכמות בלבד. אך מי שנאמר בהם עם חכם ונבון עמהם הוא שורש וענף. אבל בעבור שאבדה חכמת חכמינו ובינת נבונינו נסתתרה לא נשאר בידינו מאותן החכמות שנים שלשה גרגרים בראש אמיר עד שהיתה תפארת החכם הישראלי בזה הזמן אינה כי אם בידיעת אותן הספרים שהזכרנו או מקצתן עד שנתבאר אצל בני עולם כי לא היתה חכמה בישראל וכי אויל עמי לפי שהחכמות לא כתבום כי אם היוונים והועתקו ללשונות אחרות כפי שנתפרסמו חבוריהם וזה מריחוק ההשגחה בישראל כמו שאמר ישעיה ע׳ה׳ הנני יוסיף להפלא את העם הזה הפלא ופלא ואבדה חכמ׳ חכ׳ ובי׳ נבו׳ תסתתר. כלומ׳ כי זה פלאי פלאות להיות העם שנאמ׳ בהן עם חכם ונבון וגו׳ שסרה ממנו החכמה האמיתית וישובו להתלות בחיבורי האומות. אזלו מים מני ים וחזר להתמלאות משקתות המים שני׳ כי נטשת עמך בית יעקב כי מלאו מקדם ועוננים כפלש׳ וכילדי נכר׳ ישפיקו. שאין ערך חיבורי האומות לחכמת ישראל הודאית אלא כערך אור הנר לאור החמה ביום יהיה שבעתים. ומפני זה נשענתי בייי׳ אלהי ישראל ונערתי חצני אני יהודה כהן הקטן מקטני ספרד וחברתי זה הספר וקראתי שמו מדרש החכמה וכללתי בו עניני כל אותם הספרים שהזכרתי בעבור שני דברים. האחד מי שירצה לעיין באותן החכמות ידע תחלה מכאן מה התועלת העולה בידו מאותן הספרים וגם יעיין בזה החיבור ברמזים שמחכמה אלהית שלנו והמבין יבין ואם יזכה יבחין. והתועלת השנית להיות מי שכבר הטריח את מחשבתו ואבד את זמנו באותן הספרים יהיה לו חיבור זה מחזירנו להתעסק בתורתינו הקדושה תמימה מחכימת פת׳ ישר׳ משמ׳ לב ברה מאי׳ עיני׳ טהור עומ׳ לעד אמת.

[17] Ecclesiastes 2:13.

[18] Psalm 147:19-20.

[19] Deuteronomy 4:6.

[20] Cf. Isaiah 29:14.

[21] Isaiah 17:6.

[22] Jeremiah 4:22.

[23] Isaiah 29:14.

[24] Deuteronomy 4:6.

sea'[25] and [Israel] returned to be filled from the water trough, as it is said: 'For you have rejected your people, the house of Jacob, they are full of things from the east and of fortune-tellers like the Philistines, and they strike hands with the children of foreigners'.[26] Since the relation of the treatises of the nations to the absolute wisdom of Israel is like the relation of the light of a candle to the light of the sun on the day that it is sevenfold.[27]

That is why I leaned on the Lord, the God of Israel, and I 'shook the fold of my garment',[28] I, Judah ha-Cohen, the small among the small ones of Spain, and I composed this book and called it Midrash ha-Ḥokhmah ['Exposition of Wisdom'], and I collected in it the contents of all the books that I mentioned, for two reasons: One is that whoever wants to study these disciplines will first know from here what the benefit is that he will get from these books, and he will also study in this treatise the hints that [come] from our divine wisdom. And he who understands will understand, and if he deserves it he will discern. The second benefit is that he who already bothered his thinking and wasted his time on these books will have this treatise to return him to absorption in the study of our holy Torah.

In this passage Judah establishes the relationship between philosophical teachings and religion: the knowledge attained by the philosophers was already included in the all-encompassing 'divine wisdom' that God revealed to Israel alone. However, a large part of this knowledge was lost to the Jews. The study of the 'external sciences' has therefore become a useful and even necessary tool for understanding and re-discovering the true and absolute wisdom that is found in the Jewish religion. While the study of philosophy alone cannot lead to that wisdom, a true understanding of the 'divine wisdom' is only possible by re-discovering its forgotten knowledge, which can be achieved through the study of philosophy. His goal in writing the encyclopaedia is therefore to demonstrate the usefulness of the 'external sciences', but also to show the superiority of religious knowledge.

[25] Job 14:11.
[26] Isaiah 2:6.
[27] Cf. Isaiah 30:26.
[28] Cf. Nehemiah 5:12.

We should also pay attention to the biblical verses Judah invokes in this passage. When describing Israel's 'divine wisdom', he quotes prophetic books traditionally connected with the coming of the Messiah. Israel's loss of 'divine wisdom', for example, was already foretold by Isaiah. It did therefore not happen by accident, but it is an essential part of Israel's salvation history.[29] The image of the 'light of the sun on the day that it is sevenfold' evokes Isaiah 30:26: *Moreover, the light of the moon will be as the light of the sun, and the light of the sun will be sevenfold, as the light of seven days, in the day when the Lord binds up the brokenness of his people, and heals the wounds inflicted by his blow.* Numerous Jewish sources quote this verse in connection with the coming of the Messiah.[30] Both Israel's apparent lack of wisdom and its rediscovery are in Judah's view therefore connected with Israel's redemption and the messianic age.

By claiming that all important knowledge could be found in an ancient Jewish tradition that had since been lost, Judah makes use of a *topos* that was well-established among Jewish intellectuals from the Iberian Peninsula. Similar views were expressed by scholars such as Abraham ibn Ezra, Judah ha-Levi, Maimonides and Nahmanides.[31] Furthermore, some of the biblical verses Judah cites are also used by Maimonides in the introduction to the *Mishneh Torah*.[32] However, in the *Midrash ha-Ḥokhmah* Judah goes to great lengths to prove this claim. He gives a systematic and detailed account of the philosophical and scientific knowledge of his day that is unparalleled by his predecessors, and then in an

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[29] A similar view is expressed in the Babylonian Talmud, Shabbat 138b, where Isaiah 29:14 is interpreted to indicate that "[t]here will be a time when the Law will be forgotten by Israel." The discussion is immediately followed by an argument that, amongst others, relates to the end of the exile.
[30] See, for example, Babylonian Talmud, Pesachim 68a and Sanhedrin 91b; Pirqe de-Rabbi Eliezer 51:1.
[31] See Langermann, "Some remarks on Judah ben Solomon ha-Cohen," 368–369; id., "Science and the Kuzari," 507–508; Sela, *Abraham ibn Ezra and the Rise of Medieval Hebrew Science*, 140–143; id., "Abraham Ibn Ezra's Special Strategy," 85–87.
[32] Isaiah 29:14 and Nehemiah 5:12.

equally thorough way tries to demonstrate how this knowledge can be found in Jewish sources.

But not only the relationship between philosophical and religious learning, also the way the Jews are perceived by the nations plays an important role in the passage quoted above. Thus, although it was the Jews who received divine, all-encompassing wisdom through revelation, to Judah's dismay Israel nowadays appears to be foolish and devoid of wisdom, while the other nations are conceived as wise. That this perceived inferiority as regards secular learning is one of the reasons why Judah composed his encyclopaedia, can be seen in yet another passage in the *Midrash ha-Ḥokhmah*. Judah repeats his goals in writing the encyclopaedia in a passage following his abridged version of the first four books of Aristotle's *Organon* (that is, *Categoriae, De interpretation,* A*nalytica priora* and *Analytica posteriora*):

> *We already mentioned in the introduction of the book that the aim of this work is to make known the extent of the usefulness of the works of the nations, so that the Israelite not be ignorant of them, lest they grow haughty toward him with their sciences. This work is there for him to know how to answer a heretic within his own science and to return one who erred regarding their books to our holy Torah.*[33]

In this second passage, Judah explicitly mentions the superiority of the nations regarding science and philosophy as a reason for composing his work. His statement, however, should be regarded with caution. The notion that Jews need to engage with the sciences in order be able to keep up with the gentiles and to disprove their claims of superiority

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[33] Oxford, Bodleian Library, MS. Mich. 551, fol. 45v: וכבר הזכרנו בפתיחת הספר כי עניין זה החיבור אינו כי אם להודיע תכלית תועלת חיבורי האומות ושלא יהא הישראלי רק מהן כדי שלא יתגאו עליו בחכמותיהן ויעלה לו החיבור לידע ולהשיב לאפיקורוס מתוך חכמתו ולהחזיר מי ששגג בספריהן לתורתינו הקדושה. The translation is based on that rendered by Septimus, *Hispano-Jewish Culture in Transition*, 98. See also Fontaine, "Judah ben Solomon ha-Cohen's 'Midrash ha-Hokhmah': Its Sources and Use of Sources," 202.

was a common trope among medieval Jewish scholars – an excuse for practicing ‘foreign wisdom’ in the first place. Similar arguments were also used by Christians and Muslims as an apology for engaging in non-religious matters that could possibly pose a threat to one’s own religion. Thus, several recipients of emperor Frederick’s letters appear to have seen the authority of their own religion challenged by his questions. Ibn Sabʿīn answered the queries that he allegedly received from the emperor *in the hope of God’s reward in the hereafter and for the triumph of the Muslim community.*[34] Al-Qarāfī introduces the questions that were supposedly asked by Frederick by saying: *Thus [God] keeps the Muslim people from the error of inferiority, when they are asked strange questions, so that the enemies of pure faith do not imagine, due to a lack of response, that they take the first rank regarding perfection and ability.*[35] Leonardo Fibonacci, on the other hand, contents himself with stating in the introduction to his *Liber abaci* that he wrote the book on the Indian numeral system *in order to have those eager for this knowledge instructed in this method that is perfect above all, so that henceforth the nation of the Latins [i.e. the Italians] shall not be found ignorant of it, as was the case until now.*[36]

Moreover, Judah’s statement regarding the aim of his encyclopaedia shows many similarities with parts of a text written by the other Jewish scholar who was in Emperor Frederick’s services, Jacob Anatoli. In 1232, at the height of the Maimonidean controversy, Anatoli had introduced his translation of Ibn Rushd’s Middle Commentaries

[34] Akasoy, *Philosophie und Mystik in der späten Almohadenzeit*, 340.
[35] Wiedemann and Meyerhof, “Über ein optisches Werk des Aḥmad al-Qarāfī,” 123 (Arabic), 8 (German).
[36] [...] ut extra, perfecto pre ceteris modo, hanc scientiam appetentes instruantur, et gens Latina de cetero, sicut hactenus, absque illa minime inueniatur. *Scritti di Leonardo Pisano*, 1.

on Porphyry's *Isagoge* and on the first four books of Aristotle's *Organon* in a very similar way:[37]

> *There is the additional consideration that logic is indispensable if one is to stand up to opponents who employ sophistic arguments. Our rabbis warned about that, saying: "Be diligent in studying the Torah [so that you will know] what to answer the heretic,"[38] and in their warning they meant not merely the heretic, but everyone who opposes the truth. It is evident that without studying the science of logic none of us will be able to hold his ground against clever representatives of the other nations who oppose us. Since I, Jacob, the son of Abba Mari b. Samson b. Anatolio of blessed memory, saw how numerous are the wicked fools who presume against us in an argumentative and dialectic way, I became zealous at them, and there was aroused in me the desire to translate this science as far as lay in my power.*

Both Jacob and Judah argue that the Jews should study secular sciences or, in Jacob's case, logic, in order to be able to rebut gentile attacks on the Jewish faith. Both use the Mishnaic dictum that one should 'know what to answer the heretic' to demonstrate that this study is in accordance with Jewish tradition. Furthermore, both statements are found in their translations of the same four books of Aristotle's *Organon*. Jacob Anatoli writes it as part of the introduction to his Hebrew translation, while Judah makes his remark at the end of his Hebrew summary of the works. As we have seen, Judah seems to have known Jacob's translations of astronomical works, and it is therefore likely that he also knew his translation of these works on logic.

However, for both Judah ha-Cohen and Jacob Anatoli the notion of gentile attacks on the Jewish faith that warranted the translation of scientific and philosophical works into Hebrew may have been more than just a mere commonplace. Jewish intellectuals were indeed confronted with Christian critiques of their religion in their interactions with

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[37] Translation by Herbert A. Davidson: Ibn Rushd, *Middle Commentary on Porphyry's Isagoge*, 3–4. On this introduction see also Harvey, "The Introductions of Thirteenth-Century Arabic-to-Hebrew Translators of Philosophic and Scientific Texts," 227.
[38] Mishnah, Pirqe Avot 2:14.

gentile scholars. In addition to the free scholarly exchange we find in the correspondence with the emperor's philosopher, the *Midrash ha-Ḥokhmah* contains a prime example of Christian-Jewish polemics: the disputation which is found in the second part of the work. Following the treatise on the letters of the Hebrew alphabet, Judah reports on a discussion on the status of the Jewish people he allegedly had with a gentile scholar. In this disputation Judah's opponent uses arguments taken from Aristotelian philosophy in order to demonstrate the inferiority of the Jewish faith. Judah refutes his opponent, also making use of philosophical arguments.

The disputation with the gentile scholar and the correspondence with Frederick's philosopher are the two only texts found in the encyclopaedia which transmit dialogues that Judah allegedly had with other scholars, be it Jewish or gentile. However, the forms and contents of these dialogues differ greatly. In order to determine what exactly the relationship between the two texts might be, we shall take a closer look at its form and contents.[39]

Judah's disputation with a gentile scholar

Judah's opponent begins the discussion by asking: *If your god has chosen you, and you are a holy nation, and his sons, and all the other things that you claim about yourselves, why does it look as though you are a burden to the world?*[40] Drawing on a Midrash that explains that God had to create a womb for Sarah in order to enable her to conceive Isaac, he then argues that the Jews' existence was accidental from the very beginning, as opposed to the other

[39] An edition of the Hebrew text with a French summary and commentary were published by C. Sirat in 2001: "A la cour de Frédéric II Hohenstaufen: une controverse philosophique entre Juda ha-Cohen et un sage chrétien." Independently, R. Fonaine published an English summary and analysis of the text in the same year: "Religious Polemics in a Philosophical Encyclopedia: Judah ha-Cohen on 'The Chosen People.'"

[40] אם אליכם בחר בכם [השם ית] ואתם גוי קדוש ובניו ושאר כיוצא בזה שאתם אומרים בעצמיכם למה נראה כאלו אתם מטרידים על העו'[לם]. Sirat, op. cit., 73.

nations, whose existence is essential. Thus, Jacob received his father's blessing only through trickery. The Jews became a nation only after much toil and hardship and even famous figures, like Abraham, Joseph, or King Solomon, were not particularly rich or powerful compared with other rulers. All of this points to the fact, thus Judah's opponent, that the Jews are a burden to the world and the world does not need them. If the Jews were indeed a holy nation, then members of other nations would be happy to serve Jewish individuals. In addition, God put so many restrictions upon the Jews regarding worldly pleasures, food, sexual intercourse, and other bodily needs that the Jews are like *prisoners in chains who do not do anything without permission.*[41]

Judah opens his rebuttal with the sentence: *If the entire creation consisted solely of this world of generation and corruption, how just and correct would your words be.*[42] He first addresses the problem of the nations having more worldly power than the Jews. In analogy to the human body, the universe is divided into three worlds: the highest, pure and holy one, the intermediary world of the spheres, and the lowest world which collects the filth. These worlds can be compared to different parts of a royal palace; the inner chamber is reserved for the king and his spouse, in the middle part the king dines and meets his advisers, and the outer part can be accessed by the multitude. Each part has its own guard. If the king orders his guards to prevent noblemen from entering the palace, even the guards of the outmost part of the palace can thus have a seemingly higher status than aristocrats. Similarly, as the lower two worlds belong to the nations, their status is seemingly higher than that of the Jews, whose domain is the highest world. In fact, the world of generation and corruption over which the nations rule, was only created as a

[41] כללו של דבר כי אתם כמו אסורים בזקים שאינכם עושים דבר כי אם ברשות. Sirat, op. cit., 74.
[42] אלו לא היה כל המצוי כי אם זו עולם ההויה וההפסד כמה היו דבריך טובים ונכוחים. Ibid.

preparation and gatehouse to the highest world, which is the Jews' domain. But as there has to be justice and balance in nature, the Jews, who were given the divine world, cannot also rule over the lowest world. Furthermore, Judah explains that the hardships that Jews have to endure in this world do not make them inferior: Although the Jews are under the yoke of the nations, their faith has prevailed through the ages, while the gentiles come up with new religions over and over again. In addition, he compares the Jews' situation to that of Adam, who was in a better position when he was naked than when he was clothed, as that was when he was expelled from paradise.

In a next step, Judah himself puts forward another argument, namely the fact that the nations are more versed in secular sciences and philosophy than the Jews. But, he asserts, deeds are more important than reasoning, and regarding their deeds the Jews are much more holy than the other nations, who use philosophy and science for vain and evil purposes, such as gain of authority, or lust, or the making of gold - an impossible feat. Then he addresses the problem of the Jews being much fewer in number than the nations. Just as there is more arid than arable land, and more animals than humans, there are more gentile nations than the holy nation. And just as one sun is enough for the entire earth, the existence of one holy nation suffices. Finally, he addresses his adversary's claim that God's commandments restrict the Jews so much that they are comparable to prisoners. He compares the Jews to the biblical nazirite, who, due to the restrictions he imposes upon himself, is consecrated to God and in that state becomes like the High Priest.[43]

[43] While R. Fontaine interpreted Judah's lengthy discussion of the nazirite as a critique on Christian ascetic orders, C. Sirat saw it as a defence of Jewish ascetism that was burgeoning in kabbalistic circles. In my opinion, the discussion is neither; Judah merely addresses his opponent's final claim that the Jews are like prisoners.

Judah's opponent then argues that according to rabbinical exegesis someone who does not own land is not human. Thus, the nations are humans while the Jews are not. Judah replies that the earth is there merely to produce food. But under the yoke of the nations, the Jews have enough food to survive without working the earth, which allows them more time to worship God.

Finally, Judah's adversary asks why it is then that the Jews pray for their own kingdom and land. Judah explains that there are two reasons for this. On the one hand, it is difficult for some people to devote their life to God wholeheartedly while their mind is preoccupied by their state of oppression. On the other hand, a large proportion of the commandments could only be observed in the land of Israel.

Christian-Jewish polemics and scientific collaboration

Although Judah does not mention his adversary's religion, it is obvious that he is a Christian. Resianne Fontaine and Colette Sirat showed that the main arguments are deeply rooted in Christian-Jewish polemics: The notion that the Jews' servitude contradicts their claim of being God's chosen people figures prominently in Christian polemical treatises against Judaism.[44] Closely related is the argument that the Jews do not possess their own land and can therefore not be regarded as a nation. The Midrash discussed above about the divine womb prepared for Sarah in order to enable her to give birth to Isaac was used by Christian theologians as an answer to Jewish critique of the concept of virgin birth. Judah's reply to the sage's claims also contains arguments that were commonly used in Christian-Jewish polemics. His statement that gentile sages used

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[44] Sirat, "A la cour de Frédéric II Hohenstaufen: une controverse philosophique entre Juda ha-Cohen et un sage chrétien," 62; Fontaine, "Religious Polemics in a Philosophical Encyclopedia: Judah ha-Cohen on 'The Chosen People,'" 100–101; Chazan, *Daggers of Faith: Thirteenth-Century Christian Missionizing and Jewish Response*, 53–56; Berger, "Introduction to the Jewish Christian Debate in the High Middle Ages: A Critical Edition of the Nizzahon Vetus with an Introduction, Translation, and Commentary," 87.

the sciences to pursue immoral goals reflects the fact that each side accused the other of immorality. The assertion that the adherents of the other religion are like animals is likewise commonly found in Christian-Jewish polemical texts. Finally, Judah ends his answer with a quotation from Isaiah 53 - a chapter that Christian theologians interpreted as a prediction of Christ's suffering. Judah stresses that the prophecy alludes to the reward granted to Moses.[45]

Judah rebuts his opponent's claims that the position of the Jews is inferior and indeed hopeless, not by invalidating his arguments one by one, but by establishing a completely different worldview to that of his opponent; based on the notion of the three worlds he argues that the Jews' real domain is the highest world, which he identifies with the world to come. Thus, their seemingly inferior status in the world of generation and corruption is in fact a sign of their superiority. Their servitude is only a temporary phenomenon, which assures them of their moral superiority, and of their high status in the highest world. He thus responds to his opponent by using philosophical arguments and reasoning combined with the notion of the 'divine wisdom' that was revealed to Israel. Judah concedes that the Christian has argued correctly according to his level of understanding: *how just and correct would your words be.* It is the understanding of the divine world, which is only available to the Jews, that enables Judah to invalidate his opponent's claims of superiority.

The Christian scholar's identity cannot be established – it is certainly possible that the disputation in its current form is purely fictitious. But his familiarity with Jewish exegesis reflects the new polemic approach that Christians took in the thirteenth century, which

[45] On Isaiah 53 see, for example, Berger, "Introduction to the Jewish Christian Debate in the High Middle Ages: A Critical Edition of the Nizzahon Vetus with an Introduction, Translation, and Commentary," 80–81.

was characterised by the study of Jewish post-biblical literature.[46] Sirat argued that the polemic discussion, or at least similar discussions of the same genre, may have taken place at the court of Frederick II. On the one hand, the gentile philosopher's ideas are those studied at the court. On the other hand, the haughty tone of the gentile sage, as well as the reference to the possession of horses as a sign of nobility, make it plausible that Judah met his opponent at Frederick's court. To this can perhaps be added the fact that Judah mentions hunting as a pastime of landowners: *Hunting and all the other things that you do are not necessary for the worship of the Lord*.[47] In addition, Sirat interprets Judah's attack on alchemy as a critique not only of Christian practice, but also of Jacob Anatoli, his fellow Jew at Frederick's court.[48] The scholar was reportedly a skilled alchemist who instructed both Michael Scot and Vincent de Beauvais in this discipline.[49] It should be noted here that Anatoli not only practised alchemy together with Michael Scot, but, as we saw in Chapter Three, also biblical exegesis. The fact that Judah's opponent was familiar with rabbinic exegesis may thus also indicate that he belonged to Frederick's entourage.

Fontaine, however, is doubtful whether the disputation should be linked solely to Judah's experiences at Frederick's court. Judah could have collected different polemical arguments that he had come across over a period of time, both in Toledo and in Italy, and

[46] Fontaine, "Religious Polemics in a Philosophical Encyclopedia," 103–104.
[47] כלום צריך לעבודת השם יתב׳ לצוד ציד ושאר הדברים שאתם עושים. Sirat, "A la cour de Frédéric II Hohenstaufen," 77.
[48] However, instead of having specific practices or alchemists in mind, Judah may simply have taken over the argument from alchemy that he encpuntered in Judah ha-Levi's *Kuzari*. There, in his rebuttal of Aristotelian philosophy, Christianity and Islam, the Jewish scholar explains that all science originated with the Jews, and that alchemists and necromancers are in error and misled by any accidental results they get from their experiments. See Patai, *The Jewish Alchemists,* 147-149.
[49] Sirat, "A la cour de Frédéric II Hohenstaufen," 64.

put them together in a way that enabled him to put forward his own view of the three worlds and the Jews' dominion in the divine world as a response.

But regardless of the question where Judah might have been confronted with Christian accusations, his familiarity with Christian polemical arguments makes it clear that he must have engaged in disputations of the sort at some point. Gad Freudenthal has argued that Christian-Jewish polemics was in fact one of the driving factors that prompted the Arabic-to-Hebrew translation movement of scientific works that started in the twelfth century. He sees Jewish interest in science and philosophy as an indirect consequence of the Arabic-to-Latin translation movement that had started a few decades earlier.[50] The study of philosophy caused Christian scholars in the twelfth century to use new, rationalist, arguments in interreligious debates. Thus, instead of simply relying on scripture in their attempts to disprove Judaism, Christians scholars used arguments taken from Aristotelian philosophy: fundamental notions of Judaism contradicted not scripture, but reason, and were incompatible with what was known about the world through science and philosophy. Jewish authorities felt the need to rebut these Christian claims within the same framework of rationalist thinking. In order to do that, they needed to avail themselves of the necessary intellectual tools, that is, logic and philosophy. In addition, Jewish scholars had a sense of intellectual inferiority when confronted with these newly acquired sciences. As a result, they felt the need to create a comparable culture, which would at the same time form an alternative to the dominant one. They found this alternative in the products of Judeo-Arabic writings transmitted in the Hebrew language.

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[50] Freudenthal, "Arabic into Hebrew: The Emergence of the Translation Movement in Twelfth-Century Provence and Jewish-Christian Polemic." See also his "Arabic and Latin Cultures as Resources for the Hebrew Translation Movement: Comparative Considerations, Both Quantitative and Qualitative."

Freudenthal argues that rationalist arguments were put forward only where a free informal exchange between Christians and Jews was possible. Jews were confronted with this new form of polemics not in formal disputations, but in informal settings where open discussions between Christian and Jewish scholars occurred. As we have seen, this was the case in those places where Jews and Christians collaborated in the translation of scientific texts: in Toledo, Provence, and Italy. In fact, Judah's correspondence with the philosopher, followed by his sojourn at the imperial court, is a prime example of such a free and open discussion between a Christian and a Jew that took place within the 'neutral zone' of science. Leaving the question aside whether polemics was one of the factors that caused the Jewish interest in Aristotelian philosophy, it was certainly a phenomenon that accompanied this free and open exchange between scholars in Provence and Italy.

The disputation and the correspondence thus represent two very different kinds of scholarly exchange that was prompted by the study of Greco-Arabic science and philosophy. Both dialogues take place between a Jewish and a Christian intellectual. But whereas in the correspondence the opening question is of purely scientific character and the recipient is addressed as an equal within the realm of science, the opening question of the disputation makes it clear that the recipient is not only regarded as inferior, but his very right of existence is challenged. While the correspondence is written in the form of question and answer, the disputation has the form of accusation and defence. However, both end with Judah successfully refuting his opponent. In the disputation, Judah not only renders arguments that disprove the Christian scholar's claims, he also establishes that the Jews' perceived inferiority is in fact a sign of their actual superiority over gentiles. In addition to the arguments brought forward by the Christian scholar, Judah himself addresses the fact that the Jews are less versed in science and philosophy than gentiles as a sign of perceived inferiority. Here, he argues that the Jews' moral behaviour is more important than their philosophical erudition. In the correspondence, however, he makes sure to demonstrate his own superiority, at least as regards mathematical sciences, over his opponent. Using Freudenthal's theory, we could suggest that it was open exchanges like that of the correspondence that prompted the harsh attacks that we find in the disputation. Where a free exchange between Jews and Christians was possible, it led both to a close interreligious collaboration in the areas of science and philosophy and to interreligious polemics.

The correspondence and the *Midrash ha-Ḥokhmah*

But the disputation and the correspondence are also linked in a different way. They are the only two texts found in the *Midrash ha-Ḥokhmah* whose proper place in the encyclopaedia has been called into question. It was Tzvi Langermann who first pointed out that the two texts seem to disrupt the overall structure and goal of the work they appear in.[51] He argues that in both contents and structure the *Midrash ha-Ḥokhmah* is a coherent work that reflects Judah's twofold aim of giving a comprehensive account of science and philosophy and demonstrating that this knowledge was part of the Jewish tradition from the very beginning. Given that the two accounts of interactions with Christians neither serve this goal nor fit within the structure of the work, he surmises that they were perhaps meant to be 'appendices' to the work.

In addition, Judah's treatise on the letters of the Hebrew alphabet, which immediately precedes his account of the disputation in the *Midrash ha-Ḥokhmah*, concludes with a calculation which, based on exegetical and astrological considerations, predicts the

[51] Langermann, "Some remarks on Judah ben Solomon ha-Cohen," 387–388.

coming of the Messiah for the year 1260. This prompted Langermann to suggest that that the entire encyclopaedia was set up to prepare its readers for the coming age of redemption, which was 'dangerously close'.[52] This calculation seems to have been influenced by the teachings of the Christian Joachim of Fiore (c. 1135–1202), which were very popular in Italy at the time of Judah's sojourn there.[53] The theologian and mystic had divided history into three stages that were ruled by the three persons of the Trinity. The era of the 'Old Testament' had been under the rule of the Father, the era of the 'New Testament' was ruled by the Son, and a third era, that of the Holy Spirit, was soon to begin, in the year 1260.[54] Spirituality increased during each era, and in the final epoch the Holy Spirit would engulf all men, leading to an age of spiritual enlightenment and angelic-like perfection in which Jews and Muslims would convert to Christianity. Resianne Fontaine pointed out that not only the importance of the year 1260, but also the theme of the three worlds that is found in Judah's encyclopaedic work shows similarities to the teachings of Joachim of Fiore. Since throughout the encyclopaedia Judah identifies the divine, spiritual world with the world to come, it seems that like Joachim's followers he expected an age of spiritual enlightenment to be imminent.

Regarding the placement of the disputation in the encyclopaedia, both Colette Sirat and Resianne Fontaine came to the conclusion that the dialogue forms an integral part of the *Midrash ha-Ḥokhmah*. Firstly, the same arguments Judah uses in the discussion also appear in other parts of the encyclopaedia. The theme of the three worlds is one of the leading ideas repeated throughout the work, and it is even represented in the structure of the

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[52] Ibid., 389.
[53] A detailed investigation of how Joachim's views may have influenced Juda is given by Fontaine, "The Theme of the Three Worlds in the 'Midrash ha-Hokhma,'" 441–443. The fact that Judah's expectation coincides with Joachim's was first noticed by Sirat, "Juda b. Salomon ha-Cohen," 48.
[54] For more information on Joachim's thought see, for example Reeves, *The Influence of Prophecy in the Later Middle Ages*; ead., *Joachim of Fiore and the Prophetic Future*.

composition. The themes of justice and balance in nature, the Jews' special status as belonging to the higher world, and the nations' claim that Jews are lacking in knowledge of the sciences are also discussed in various contexts in the encyclopaedia. In addition, the Hebrew expressions used in the discussion can also be found elsewhere in the work.[55]

However, both Sirat and Fontaine argue that the disputation did not form a part of the original Arabic encyclopaedia that was composed in Toledo. According to Sirat, it reflects discussions Judah had at the court of Frederick II, and therefore must have been added to the work after Judah had left his hometown. Fontaine observed that while the disputation forms an integral part of the work as regards its content, structure, language and purpose, it is not mentioned in the introduction to the encyclopaedia where Judah outlines the contents of the work. This leads her to conclude that Judah added it to the work when he translated the text into Hebrew around 1247, in order to provide his Italian coreligionists with ammunition against Christian polemical accusations.

In addition, Fontaine pointed out that the disputation immediately follows the prediction made for the year 1260, and that it is the theme of the three worlds with the Jews' dominion being the highest one which is most dominant in Judah's argumentation. This leads her to suggest that the disputation with the Christian scholar, too, might serve the goal of preparing Judah's coreligionists for the arrival of the Messiah.[56]

But regardless of the question if Judah had the imminent dawn of a new era in mind when he rendered the Hebrew version of the *Midrash ha-Ḥokhmah,* it is clear that the disputation

[55] Sirat, "A la cour de Frédéric II Hohenstaufen," 58–61; Fontaine, "Religious Polemics in a Philosophical Encyclopedia," 104–106.
[56] Fontaine, "The Theme of the Three Worlds in the 'Midrash ha-Hokhma,'" 441; ead., "Religious Polemics in a Philosophical Encyclopedia," 105–106.

with the Christian scholar forms an integral part of the work. Can the same be said about the correspondence with Frederick's philosopher?

Regarding the original Arabic encyclopaedia, this question cannot be answered with certainty. However, the correspondence certainly forms an integral part of the Hebrew version of the *Midrash ha-Ḥokhmah*. As we have seen, the theme of gentile scholars attacking the Jewish faith and accusing Israel of being uneducated plays a dominant role in the encyclopaedia. It is mentioned in the introduction to the work, repeated when Judah reformulates his aim in writing the encyclopaedia, and also plays a role in his polemic debate with a Christian scholar. But also within other parts of the encyclopaedia Judah deals with Christian attacks against Judaism and the role of gentile scholars.[57] In fact, Judah's explicit aim in writing the encyclopaedia was not only to demonstrate the usefulness of the external sciences and the superiority of religious learning, but also to prove gentile scholars wrong who pride themselves of their sciences and grow haughty towards the Jews. Together with the disputation, the correspondence serves as an example of a case where this goal is achieved: the gentile philosopher is ultimately refuted and ridiculed. It demonstrates to Judah's readers that the Jews could be equal, even superior, to the nations in their mastery of the sciences. Judah himself had achieved this feat even as an eighteen-year-old: an emperor's philosopher, a representative of the wisdom of the nations, had been beaten at his own game, the sciences, by a Jew. After Judah's refutation, this representative could certainly not pride himself on 'his' sciences any more.

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[57] For example, in his treatise on the letters of the Hebrew alphabet Judah refutes the Christian view that the final letter *mem* that appears in the middle of the word לםרבה in Isiah 9:4 signifies Mary's womb and thus predicts the coming of Jesus. See Langermann, "Some remarks on Judah ben Solomon ha-Cohen," 381–382.

But also regarding the contents and structure of the encyclopaedia, the correspondence forms an integral part of the work. While the problems being discussed do not appear elsewhere, they do fit the structure of the mathematical curriculum that Judah describes in his introduction:[58]

> *The discipline of mathematics:*
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> *Regarding this circular body that turns eternally [i.e. the highest sphere], we have grasped only its movement - and this movement is circular, as is demonstrated in natural philosophy, and it is also clear from observation - therefore it is only possible for us to calculate it and to demonstrate the movements of the bodies that move in it and their properties and details through the knowledge of the properties of the circles that are made by the circular motion of the sphere and their intersection with each other, and this knowledge is only possible through the discipline of geometry. That is why we first need knowledge of the planes, and they are described in the first six treatises of the book of Euclid; and after that knowledge of solid figures, and they are in treatises 11, 12 and 13 of it. And after that [we need] knowledge of the circles that are made on the surface of the sphere, and they are [described] in the book of Theodosius; and after that the knowledge of the triangles consisting of arcs that are made on the surface of the sphere, and the 'sector figure', and they are in the book of Menelaus.*
>
> *The discipline of mathematics is divided into seven parts: arithmetic, geometry, music, [mechanical] devices, optics, statics, and the discipline of the celestial sphere [i.e. astronomy].[59] In this our composition we have only mentioned the part of geometry and the discipline of the celestial sphere.*

[58] Oxford, MS. Mich. 551, fol. 3a: חכמת הלימודים: לפי שזה הגוף העגול [ה]סובב תמיד לא השגנו ממנו כי אם תנועתו בלבד וזאת התנועה היא בסיבוב כמו שנתבאר בחכמת הטבע וגם נתבאר כן בהבטה לפי כך לא היה אפשי לנו חשבונה ולעמוד על ביאור תנועת הגופים המתנענעין בהן ואיכותיהן ופרטיהן אלא בידיעת איכות העגולות הנעשות על ידי סיבוב הכדור וחיצובן אלו על אלו ואי אפשר ידיעתן אלא בחכמת גימטריא לפי כך הוצרכנו בתחלה לידיעת השטחים והם מבוארים בששה מאמרים הראשונים מספר אקלידס ואחר כן בידיעת הצורות הגופניות והן במאמר י״א י״ב י״ג ממנו. ואחר כן בידיעת העגולות הנעשות על שטח הכדור והן בספר תאודוסיוס ואחר כן בידיעת המשולשות בעלות הקשתות הנעשות על שטח הכדור וצורת החצב והן בספר מילאוש. וזו חכמת הלימודים נחלקת לשבעה חלקים. המספר וגימטריא ומוסיקא ותחבולות והבטות והרמת הכבדים וחכמת הגלגל. ולא הזכרנו מכל אלו בחבורינו זה כי אם חלק גימטריא וחכמת הגלגל בלבד. וחכמת הגלגל שני חלקים. ידיעה ומעשה. חלק הידיעה הוא ידיעת תכונות תנועות השמים והכוכבים ולקות מה שילקה מהם. ומדת הארץ ומקצת הכוכבים. וזולת זה מהדברים שנתבארו בזאת החכמה. ותלמי כלל הכל בי״ג מאמרות של ספר אלמג׳סטי. שני מאמרות הראשונים ממנו הזכיר בהם כללות השמים והארץ ושינוי ארך הימים והצל בכל מקום כפי רחבו מגלגל המישור. ג׳ בתנועת השמש. [...] י״ג׳ בתנועתם ברוחב. וחלק המעשה הוא פעולות אלו הגופנים העליונים בזה העולם התחתון שכמו שמי השמים נעבדים לשמים שהוא העליון כך הארץ נעבדת לשמי השמים והכל מכח הק׳ב׳ה׳ כיצד מאמר הק׳ב׳ה׳ נאצל תחלה על אותו עולם הנקרא שמים ומהשמים נאצל על שמי השמים ומשמי השמים על הארץ. ותלמי חיבר בזה העניין הנקרא דיני הכוכבים הארבעה מאמרות. א׳ בכללות זאת החכמה. ב׳ בחילוק הארץ ומה שנפרט בו כל מקום ממנה. ועל מה יורו לקיות המאורות והקיבוצים והנכחים כפי טבע הכוכבים והמזלות. ג׳ בחי הנולד ודעותיו. ד׳ בעשרו והצלחתו.

[59] This classification of the mathematical sciences goes back to al-Fārābī (c. 870-950). On the classification of these sciences by medieval Jewish authors, see Wolfson, "The Classification of Sciences in Mediaeval Jewish Philosophy," 299–304. See also Lévy, "Mathematics in the *Midrash ha-Hokhmah* of Judah ben Solomon ha-Cohen," 301; Zonta, "The Reception of al-Fârâbî's and ibn Sînâ's Classifications of the Mathematical and Natural Sciences in the Hebrew Medieval Philosophical Literature," 376.

The discipline of the sphere [has] two parts: theory and practice. The theoretical part is the knowledge of the properties of the movements of the heavens and the planets, and eclipses of those [planets] that have eclipses, and the measurement of the earth and some of the planets, and other things that are demonstrated in this discipline. Ptolemy collected everything in thirteen chapters of the book Almagest. In the first two chapters of it he mentioned the basic rules of the heavens and the earth and the changes in the lengths of the days and the shadows at every place according to its latitude from the equator. Three [is] on the movement of the sun. [...] Thirteen is on their motion in latitude.

The practical part is the agency of these upper bodies in this lowest world. Since just as the 'heaven of heavens'[60] *serves the 'heavens', which is the highest, thus the earth serves the 'heaven of heavens', and everything [happens] through God's power. How? God's command emanates first upon that world which is called 'heaven' and from 'heaven' it emanates to the 'heaven of heaven' and from 'heaven of heaven' to the earth. Ptolemy composed on that subject, which is called 'judgements of the stars', the 'four treatises' [Tetrabiblos]. The first [is] on the basic rules of that discipline. Two is on the division of the earth and the details of each place of it, and what the lunar and solar eclipses indicate, and conjunctions and oppositions, according to the nature of the stars and the zodiacal signs. Three on the life of a new-born and his opinions. Four on his wealth and success.*

Judah describes the role of the 'mathematical disciplines', the standard textbooks and the order in which they should be studied. But a comparison between the books Judah lists above and the actual contents of his encyclopaedia makes it clear that in the *Midrash ha-Ḥokhmah* Judah deviates from this curriculum. On the one hand, he replaces the last books of Ptolemy's *Almagest* with al-Biṭrūjī's *Principles of Astronomy*. But more importantly, he states that after Euclid's *Elements*, but before the *Almagest,* one should study spherical geometry from the books of Theodosius and Menelaus. The 'book of Theodosius' refers to the *Sphaerica* by Theodosius of Bithynia (second century BCE).[61] The work describes general properties of circles on the surface of a sphere. Menelaus of Alexandria (first

[60] Cf. Deuteronomy 10:14: *Behold, to the Lord your God belong heaven and the heaven of heavens, the earth with all that is in it.* Judah takes this verse to refer to the three worlds: 'heaven' to the divine world, 'heaven of heavens' to the world of the celestial spheres, and 'earth' to the sublunary world.

[61] An English translation of the work was rendered by Kunitzsch and Lorch, *Sphaerica*. For a short overview of the work, see Van Brummelen, *The Mathematics of the Heavens and the Earth*, 49–53.

century CE) also wrote a book called *Sphaerica*, in which he lays the theoretical basis for the calculations that Ptolemy made in his *Almagest*. The first theorem of the third book had in the Islamic world become known as the 'transversal figure' or 'sector figure' (*shakl al-qaṭṭāʿ*).[62] Together, Theodosius' and Menelaus' treatises laid the theoretical foundations that were necessary for understanding mathematical astronomy. In the Greco-Arabic tradition they belonged to the 'intermediary books'- that is, books to be studied after the *Elements* and before the *Almagest*. However, in the *Midrash ha-Ḥokhmah* Judah does not present any summaries of these works. In place of the 'intermediary books' we find his account of the correspondence with Frederick's philosopher set between Euclid's *Elements* and Ptolemy's *Almagest*. Although the correspondence cannot replace these works, its subject matter does fit the criterion of 'intermediary'. While the first question deals with a problem that can be solved after the study of Euclid's *Elements*, the second question relates to a basic problem of spherical astronomy that can be solved only with the help of the 'intermediary books'. In fact, the works of both Theodosius and Menelaus are introduced in the correspondence. When answering the philosopher's question, Judah explicitly makes use of theorem I.4 of Theodosius' *Sphaerica*.[63] In his answer to the second question, the philosopher in turn uses proposition III.2 of Menelaus' work on spherical astronomy. In his refutation, Judah mentions this fact and quotes the proposition, and in addition he introduces proposition I.17.[64]

[62] Menelaus' *Sphaerica* is available in an Arabic edition with a German translation in *Die Sphärik von Menelaos aus Alexandrien in der Verbesserung von Abū Naṣr Manṣūr ibn ʿAlī ibn ʿIrāq*. An overview of the work and its reception in the Arabic tradition can be found in Van Brummelen, *The Mathematics of the Heavens and the Earth*, 53–68, 173–178. See also Lévy, "Mathematics in the *Midrash ha-Hokhmah* of Judah ben Solomon ha-Cohen," 300–301.
[63] See edition, paragraph 20.
[64] Edition, paragraphs 45 and 47.

From a didactic point of view, however, the discussion of the astronomical question cannot be regarded as 'intermediary'; both its subject matter and its technical language would have been beyond any reader who had just finished studying the *Elements*. Judah and the philosopher start their discussion *in medias res* – there is no introduction explaining that Judah's question pertains to a problem in astronomy, no clarification of technical terms, such as *right* and *oblique ascensions*, and no explanation of the mathematical concepts used, such as *sine* and *cosine*. A novice to the discipline of astronomy would therefore have been at a complete loss when reading the second part of the correspondence. Furthermore, while Judah successfully proves the philosopher wrong and identifies his mistakes, he does not supply his readers with a correct solution to the problem. What the novice reader could therefore have learned by reading the text is merely that there is a difference between Euclidian and spherical geometry, and that he should never apply the rules of one discipline to the other, lest he make a fool of himself.

But the passage on the mathematical sciences found in the introduction does not only demonstrate how the correspondence fits into the structure of the encyclopaedia. It also shows that the subject matter of the debate plays an important role in the *Midrash ha-Ḥokhmah*. Through mathematics and astronomy, it is possible to predict the movements of the planets, which are the instruments carrying out divine directives. In fact, it is only through the mathematical sciences that one can get an understanding of how God's command emanates from the highest to the lowest world. They are thus a tool for a more important kind of knowledge: God's agency in the world of generation and corruption. This knowledge is of course closely related to the secret knowledge that was revealed to Israel: divine wisdom. In his treatise on biblical verses, he makes this even more clear: *Just like the discipline of mathematics is needed for the divine wisdom, thus only an expert in* *mathematical sciences can truly understand the divine wisdom.*[65] Regarding the Muslim scholar al-Biṭrūjī, whose astronomical system Judah prefers to that of Ptolemy, he writes:

> *Know that a great secret was revealed to him, and if he had been Jewish, he would have been worthy of the divine wisdom. As [scripture] says: 'In the heart of the understanding ['navon'] rests wisdom [i.e. divine wisdom]'.*[66] *As we explained in our exegesis of Proverbs, [scripture] calls the mathematical disciplines 'binah', and the one who knows them is called 'navon'.*[67]

By indicating his own mathematical abilities and even more, proving a gentile philosopher wrong, Judah thus demonstrates to his readers that he himself has the prerequisite skills for understanding the highest kind of knowledge, divine wisdom, while, perhaps unsurprisingly, the gentile philosopher does not. In this way his account of the philosopher's failure also serves as an example of the fact that it is the Jews alone who are capable and worthy of understanding the divine world. If Judah indeed wished to prepare his readership for the imminent coming of the Messiah, the fact that his opponent was not only a gentile, but also a representative of the Holy Roman Empire, may add an additional dimension to the philosopher's defeat, for it could be interpreted as a first indication of the fact that the gentiles' rule would soon come to an end.

Be that as it may, it is clear that while the original Arabic correspondence with the emperor's philosopher appears to have been a free scholarly exchange between two experts in mathematical sciences regardless of their religion, its Hebrew rendering within the framework of the *Midrash ha-Ḥokhmah* was influenced by polemical encounters between Jews and Christians. The refutation of the philosopher's claims thus not only

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[65] Goldstein, "The Commentary of Judah ben Solomon Hakohen," lines 544–546: וכמ׳ שחכמת הלימודים היא צריכה לחכמה אלהית כמ׳ כן לא יוכל לידע חכמת אלהית על בוריה אלא חכם הלימודים. On the connection between mathematics and 'divine science' see also Sirat, "La Qabbale d'après Juda B. Salomon ha-Cohen," 195.

[66] Proverbs 14:33.

[67] Oxford, MS. Mich. 551, fol. 161v: ודע כי נגלה לו סוד גדול ואלו היה יהודי היה ראוי לחכמה אלהית שנ׳ בלב נבון תנוח חכמה כמ׳ שפירשנו בפטרון משלי שהוא קורא לחכמת הלימודים בינה והיודעה נקרא נבון. See also Langermann, "Some remarks on Judah ben Solomon ha-Cohen," 386.

demonstrates Judah's own expertise in astronomy, but also the fact that Jews could and should be able to prove gentile scholars wrong, especially since a thorough understanding of the mathematical sciences was a prerequisite for a deeper understanding of the 'divine wisdom' revealed to Israel alone.

Conclusion

The correspondence between Judah ha-Cohen and the Emperor's philosopher exemplifies the different ways in which knowledge was transmitted between scholars from different cultural and geographical settings during the thirteenth century.

In Chapter Two I prepared a critical edition and English translation of the text, analysed its mathematical contents, and reconstructed the diagrams that were being discussed. I argue that in the astronomical section of the correspondence, the unnamed philosopher had provided one single diagram that showed the oblique ascensions for both northern and southern parts of the ecliptic, and that Judah ha-Cohen correctly identified a mistake in the philosopher's method to calculate these ascensions.

The original exchange between Judah and the philosopher was typical of the intercultural exchange of knowledge that was initiated at the court of Emperor Frederick II. The emperor's interest in mathematics is well-documented - in both Christian and Muslim sources. The *Midrash ha-Ḥokhmah* ('Exposition of Wisdom') by Judah ha-Cohen provides a Jewish source as well that reports of imperial inquiries on mathematical questions.

In Chapter Three I demonstrated that the subject of the first, mathematical, question that is discussed in the correspondence included in Judah's encyclopaedia can be linked to the work of both Leonardo Fibonacci in Italy and Ibn Yūnus in Mosul. The fact that one of the problems posed to Judah was treated by both the Christian scholar Campanus of Novara and the Muslim scholar Muḥyī al-Dīn al-Maghribī in their adaptations of Euclid's *Elements* within twenty-five years after the initial correspondence between Judah and the imperial court, makes it clear that the discussion involved issues that were highly topical at the time.

Of the recipients of Frederick's queries, Judah ha-Cohen is the only one who records that he responded to the questions with questions of his own, which were then answered in return. The discussion of the second, astronomical, question reveals the intellectual challenges that the study of spherical astronomy posed in thirteenth-century Europe. I maintain that the free and open nature of the exchange is an excellent example of the phenomenon of 'science as a neutral zone': both in the Muslim world and in the Latin West scholars of different denominations and religions cooperated in the appropriation of Greek learning.

In order to make the knowledge that resulted from this cooperation available to a Jewish readership that was unacquainted with the Greco-Arabic tradition, Judah ha-Cohen had to convey it in Hebrew - a language that had only just begun to be used for mathematical and astronomical writings. My analysis of Judah's mathematical vocabulary in Chapter Four demonstrates that he struggled with the task, since he made various attempts to translate the beginning of his Hebrew version of the *Elements*, using different mathematical vocabularies. However, over time he changed only nuances and subtle differences in his mathematical terminology, which may reflect the fact that a basic Hebrew mathematical vocabulary was already in use in Judah's time of the mid-thirteenth century. My comparison of Judah's astronomical terminology with the vocabularies of Abraham Bar Hiyya, Abraham ibn Ezra and Jacob Anatoli has established that Judah's astronomical vocabulary changed considerably during his working life. In the Hebrew translation of the correspondence, as well as a few other sections of the *Midrash ha-Ḥokhmah*, it shows some similarity to that of the astronomical writings of Jacob Anatoli. Since Jacob belonged to Frederick II's entourage it is not surprising that Judah studied his contemporary's writings. Judah does not, however, take over entirely Jacob's astronomical vocabulary, but rather he develops his own unique terminology. Moreover, other sections of the *Midrash ha-Ḥokhmah* show significant similarities with the astronomical writings and vocabulary of Abraham ibn Ezra.

I argue that Judah essentially tried to avoid the use of astronomical terms employed by members of the Ibn Tibbon family, as he was quite critical of the philosophical ideas that Jacob Anatoli and his father-in-law Samuel ibn Tibbon adhered to. Judah's belief in the superiority of religious learning over philosophical reasoning comes to the fore throughout the *Midrash ha-Ḥokhmah*. At the same time, it is clear that Judah ha-Cohen was very conscious of the fact that in Italy Christian scholars were far more advanced in the acquisition of scientific and philosophical knowledge in the Greco-Arabic tradition than his Jewish coreligionists. Judah felt the need to educate his fellow Jews in these disciplines, particularly, since he encountered a new form of harsh anti-Jewish polemics that was based on philosophical, rationalist arguments during his scholarly exchanges with Christian intellectuals.

I demonstrated that taken in its own right, the original correspondence with the gentile philosopher is a prime example of interreligious cooperation between highly skilled experts that took place in thirteenth-century Europe. In Chapter Five I argue that within the framework of the *Midrash ha-Ḥokhmah* the correspondence acquires a different meaning. Here, the focus lies on the fact that Judah's opponent was a gentile, and that this gentile was refuted. For Judah ha-Cohen, the philosopher's defeat proved that Jews were able and required to surpass the gentiles in their learning, especially since the mastery of mathematics and astronomy was a prerequisite for a deeper understanding of secret religious knowledge that is only really accessible to Jews.

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