INSURANCE DESIGN FOR DEVELOPING COUNTRIES

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DANIEL J. CLARKE
MPhil MA FIA

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UNIVERSITY OF OXFORD.

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Professor Sujoy Mukerji
Professor Stefan Dercon

For my parents

Insurance Design for Developing Countries

Daniel J. Clarke, Balliol College
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Abstract

Over the last ten years there has been a renewed interest in providing agricultural insurance in developing countries. However, voluntary demand for unsubsidised insurance products has been low, particularly from the poorest farmers.

Chapter One presents a model of rational demand for hedging products, where there is a risk of contractual nonperformance. Demand is characterised and bounded for risk averse and decreasing absolute risk averse decision makers. For constant absolute and relative risk averse utility functions, demand is hump-shaped in the degree of risk aversion when the price is actuarially unfair, first increasing then decreasing, and either decreasing or decreasing-increasing-decreasing in risk aversion when the price is actuarially favourable. The apparently low level of demand for consumer hedging instruments, particularly from the most risk averse, is explained as a rational response to deadweight costs and the risk of contractual nonperformance. A numerical example is presented which suggests that some of the unsubsidised weather derivatives currently being designed for and marketed to poor farmers may in fact be poor products.

Chapter Two presents experimental evidence collected from a framed microinsurance lab experiment using poor subjects in rural Ethiopia. In line with the theoretical model of Chapter One, demand for actuarially unfair index insurance is hump-shaped in wealth, first increasing then decreasing. In contrast with recent field experiments where it is not possible to demonstrate that low demand for indexed insurance is ‘too low’, use of a laboratory experiment with an objectively known joint probability distribution allows normative statements to be made about the observed level of demand. The observed level of demand for index insurance in the experiment is higher than the decreasing absolute risk averse upper bound of Chapter One, suggesting that subjects bought ‘too much’ index insurance.

Chapter Three presents a vision of insurance design for the poor. Technically optimal arrangements involve insurance providers, such as microinsurers or governments, acting as reinsurer to groups of individuals who have access to cheap information about each other, such as extended families or members of close-knit communities, who in turn offer mutual insurance to each other.

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Statement of Authorship

Chapters I and III were sole authored. Chapter II was coauthored with Gautam Kalani. The experiments were designed and run by Daniel. Broadly, the theoretical analysis was led by Daniel and the empirical analysis was led by Gautam. Specifically, the Introduction and Conclusion were joint work, sections 2.3, 3 and 5 were led by Gautam and the remainder was led by Daniel.

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INTRODUCTION

Motivation. Over the last ten years there has been a renewed interest in providing agricultural insurance in developing countries. Early trials by the World Bank have been followed by a plethora of pilots, and attracted attention from governments, donors, private sector insurers and reinsurers, and academics.

The motivation for this program of work is sound: three quarters of the world's poor depend on agriculture for their livelihoods. Agriculture is a risky business and a bad harvest can devastate lives. By providing claim payments in the worst years, agricultural insurance could both reduce vulnerability and provide a foundation for production-boosting investments in agricultural that could help to lift hundreds of millions out of poverty.

However, despite the potential benefits from agricultural insurance, demand for unsubsidised products seems to be low, particularly from the poorest. This observation have been labeled a puzzle by academic economists who, by and large, believe that the products are fundamentally desirable but that either farmers must be making poor decisions (*behavioural preferences*) or that they don't have cash at the right time to pay the insurance premium (*credit constraints*).

Outline and contribution. This thesis comprises three chapters, each of which is presented as a distinct research paper; the first and third apply economic theory to inform insurance product design and the second reports on a set of laboratory experiments conducted in Ethiopia.

In Chapter One the shape and level of objective, rational demand for index insurance is characterised. The key to resolving the puzzle of low demand, as presented in Chapter One, is that most of the insurance products currently being sold to poor farmers are not indemnity insurance products; they are derivatives, providing only a hedge, not indemnification, against a bad harvest.

On purchasing a weather derivative a farmer's risk is reduced in those states of the world when the hedge performs, but the risk is increased in those states of the world when the hedge does not perform. In particular, on purchase of a weather derivative the worst that could happen gets worse; a farmer could lose her entire crop due to pestilence or localised weather conditions, but receive no claim payment from a weather derivative due to good weather having been observed at the contractual weather station. This risk of contractual nonperformance, or *basis risk* as it is referred to in derivative markets, suppresses objective rational demand, particularly from the most risk averse. As an extreme case, an infinitely risk averse, maximin, decision maker would optimally purchase zero cover since purchasing cover would worsen the worst that could happen. This result is similar to well known work by insurance theorists on objective rational demand for indemnity insurance when there is a chance that the insurer will not pay a claim due to insolvency or exclusion clauses.

In addition to basis risk, commercial insurance products are typically actuarially unfair; that is to say a policyholder would reasonably expect the premium to be higher than the expected claim income, to account for costs and profits of the insurer. The combination of basis risk and actuarially unfair premiums has a particularly interesting effect on rational demand. First, objective rational demand is hump-shaped in the degree of risk aversion with zero purchase from both the risk neutral, who don't like the product because it decreases mean wealth, and from the most risk averse, who don't like the product because it decreases the minimum possible wealth, leaving only materially positive purchase from those with intermediate levels of risk aversion.

It is also possible to derive rational upper bounds for the purchase of actuarially fair or unfair hedging products under either an assumption of risk aversion alone or risk aversion and decreasing absolute risk aversion (DARA). For the case of indemnity insurance, the bounds on demand are not interesting: a risk neutral consumer might quite reasonably purchase zero cover whilst an infinitely risk averse consumer might quite reasonably purchase full cover. Neither risk neutrality nor risk aversion are prima facie evidence of irrationality, and so the potential for generic financial advice is limited. However, the introduction of basis risk reduces the upper bounds for rational demand. Loosely speaking the bound for DARA arises because an individual who cares enough about the risk to want to purchase a sizeable hedge must, under the assumption of DARA, care enough about the downside risk of contractual nonperformance and the deadweight cost of hedging to limit the size of the hedge. I argue that these upper bounds for demand constitute a reasonable basis for objective, generic financial advice.

The paper concludes with a numerical example of weather derivatives designed for maize farmers and recently sold across a developing country. I calculate that, in the absence of subsidies, objective financial advice would recommend low or zero purchase for most farmers. Moreover, even if the products were heavily subsidised, objective financial advice would recommend low or zero purchase from the most risk averse.

Funded by a small grant (US$$10,000$) from the Microinsurance Innovation Facility I spent six weeks in Ethiopia in November and December 2009 with a team of three Ethiopian enumerators running a framed microinsurance laboratory experiment. The experiment involved 378 subjects from seven sites of the Ethiopian Rural Household Survey, for which there already existed seven rounds of detailed longitudinal data. Chapter Two presents the experiment and an analysis of the data collected.

The novelty of the experiment lies in the strong theoretical basis from Chapter One, which allows normative statements to be made about insurance purchase. For example, to be able to say anything meaningful about whether observed indexed insurance purchase is 'too low', rather than just saying that it is 'low', one must at the very least be able to argue that a well-informed financial advisor would advise a higher level of purchase. This turns out to be surprisingly difficult to do, particularly for real index insurance products where the contractual index is not perfectly correlated with the loss, and neither researcher nor consumer has a precise objective estimate of the joint probability distribution of losses and index claim payments.

Recent empirical work on agricultural insurance in developing countries has focused on analysing the purchase of real-world weather index insurance policies by poor farmers. However, without an objective joint distribution of losses and claim payments, analysis is limited to understanding who purchases rather than the more fundamental question of how observed purchase relates to 'rational' purchase.

In the lab experiment the shape of demand for insurance from rural Ethiopians is found to be consistent with DARA, under which demand for index insurance should be hump-shaped in wealth and higher background risk should be associated with greater (indirect) risk aversion. Both these results are consistent with recent evidence from field and lab experiments. However, the aggregate level of demand we observe is higher than can be rationalised by DARA; 66% of subjects purchased more insurance than would be optimal for *any* risk averse DARA expected utility maximiser. This result is consistent with recent lab experiments framed in the abstract and conducted with subjects from developing countries in which subjects subjectively underweight the probability of extreme events.

At first glance this appears to be the opposite of what has been found in index insurance field experiments; however, this is not an appropriate comparison. Unlike recent field experiments the present laboratory experiment has been designed with the specific aim of allowing normative statements to be made about the level of index insurance purchase. Whilst observed demand in field experiments has been low there has been no convincing attempt to demonstrate that it is 'too low'. If poor farmers are likely to underweight basis risk in their decision rules, we would expect observed demand for index insurance policies from poor farmers to be higher, not lower than that which would be advised by a well-informed financial advisor.

Chapter Three considers the optimal design of insurance arrangements for the poor. I argue that one key economic difference between microinsurance and conventional insurance is that the cost of formal loss adjustment, that is auditing and ex-post claim processing, is relatively much more expensive for an external insurer than an informal or semiformal mutual insurance organisation due to the presence of highly connected networks of individuals who have access to cheap information about each other, such as extended families or members of close-knit communities. I then show what this difference means for contract design.

For example, consider agricultural insurance to be sold in a particular rural village. Selling individual indemnity insurance products to each farmer, and paying claims to each farmer based on that farmer's own crop loss, is highly inefficient; the insurer has to visit the village and check individual claims even if overall the village has had a good harvest. Better would be to make the same claim payment to all policyholders, only paying if the village as a whole has a bad year, and rely on individuals in the village to pool idiosyncratic risk (the 'Stop Loss' insurance contract). Further, instead of visiting every farm in the bad years, it should be possible to accurately estimate the average yield in the village based on a sample of farms (the 'Sample-Based Index' insurance contract). Finally, if there is a cheaply available manipulation proof index, such as rainfall measurements from an automated rainfall station in or near the village, it may be possible to automatically pay claims in years in which the rainfall is particularly bad, without having to calculate the yield on a sample of farms (the 'Index Plus Gap' insurance contract).

This thesis has implications for the design and analysis of commercial hedging products sold to consumers, and particularly for agricultural insurance products sold to poor farmers. Chapter One demonstrates that the combination of unsubsidised premiums and basis risk can limit demand from objectively rational decision makers. Moreover, since basis risk particularly limits rational demand from the most risk averse, even in the presence of subsidies, academics and practitioners interested in providing insurance products that are of value to such individuals must design arrangements with low basis risk. Chapter Two argues that, whilst lab and field experiments are complementary, recent weather derivative field experiments do not advance our understanding of key questions such as how to design good products or whether clients make good financial decisions. Results from a framed lab experiment conducted in Ethiopia are presented in which such normative analysis is methodologically justified. Finally, Chapter Three presents a vision for the future of insurance for the poor in which formal insurance providers, such as microinsurers or governments, act as reinsurers to groups of individuals who have access to cheap information about each other, such as extended families or members of close-knit communities. In such arrangements the formal contract is designed to accurately capture aggregate shocks, and local risk pooling soaks up any residual basis risk.

A THEORY OF RATIONAL DEMAND FOR INDEX INSURANCE

DANIEL J. CLARKE*

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Abstract

Rational demand for hedging products, where there is a risk of contractual nonperformance, is fundamentally different to that for indemnity insurance. In particular, optimal demand is zero for infinitely risk averse individuals, and is nonmonotonic in risk aversion, wealth and price. For commonly used families of utility functions, demand is hump-shaped in the degree of risk aversion when the price is actuarially unfair, first increasing then decreasing, and either decreasing or decreasing-increasing-decreasing in risk aversion when the price is actuarially favourable. For a given belief, upper bounds are derived for the optimal demand from risk averse and decreasing absolute risk averse decision makers. The apparently low level of demand for consumer hedging instruments, particularly from the most risk averse, is explained as a rational response to deadweight costs and the risk of contractual nonperformance. A numerical example is presented for maize in a developing county which suggests that some unsubsidised weather derivatives, currently being designed for and marketed to poor farmers, may in fact be poor products, in that objective financial advice would recommend low or zero purchase from all risk averse expected utility maximisers.

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*DPhil student: Department of Economics, Centre for the Study of African Economies (‘CSAE’) and Balliol College, University of Oxford (**clarke@stats.ox.ac.uk**; http://www.stats.ox.ac.uk/∼clarke). This paper forms part of my DPhil thesis, which is supervised by Professors Sujoy Mukerji and Stefan Dercon; the work would not have been possible without their very generous assistance. A number of others have provided very useful comments on aspects of the paper; without implicating them in the shortcomings of the work, I thank Olivier Mahul, Ian Jewitt, Rocco Macchiavello, James Martin, Albert Park, Liam Wren-Lewis and Ruth Vargas Hill. I have presented the paper at the CSAE Research Workshop 2010, the CEAR Insurance for the Poor Workshop 2010 and the CSAE Conference 2011. ESRC funding is gratefully acknowledged.

1 INTRODUCTION

When should consumers use financial contracts to hedge against a potentially material loss, and when should they not? The question is not trivial to answer. Weighed against any benefit from hedging is both the deadweight cost of such a contract, typically passed on to the purchaser through an increased premium, and any risk that the net income will not accurately reflect an incurred loss. This risk of contractual nonperformance, or *basis risk* as it is referred to in derivative markets, renders the decision problem fundamentally different to an indemnity insurance purchase decision problem.

One particularly instructive hedging product is that of the weather derivative, which over the last ten years has begun to be sold by a variety of well-meaning institutions to poor farmers as *weather indexed insurance*. The rationale typically given is quite convincing: agriculture is an uncertain business, leaving households vulnerable to serious hardship (Dercon 2004, Collins et al. 2009), and traditional indemnity-based approaches to crop insurance were unsustainably expensive, plagued by moral hazard, adverse selection and high loss adjustment costs (Hazell 1992, Skees et al. 1999). By comparison, contracts conditional only on weather indices can be fairly cheap whilst still offering much needed protection against extreme weather events such as droughts (Hess et al. 2005).

The sale of weather derivatives to farmers in developing countries has led to careful empirical studies by academic economists who have in turn noted two puzzles. First, demand for such products has been lower than expected, prompting attention from empirical economists hoping to disentangle various candidate causes of low demand such as financial illiteracy, lack of trust, poor marketing, credit constraints, basis risk and price (Giné et al. 2008, Giné and Yang 2009, Cole et al. 2009, Cai et al. 2009). For example, Cole et al. (2009) report on a series of rainfall derivative trials in two Indian states in which only $5 - 10\%$ of households in study areas purchased cover, despite rainfall being overwhelmingly cited as the most important risk faced. Moreover, the vast majority of purchases were for one policy only, which the authors estimate would hedge only $2 - 5\%$ of household agricultural income.

The second empirical puzzle is that demand seems to be particularly low from the most risk averse (Giné et al. 2008, Cole et al. 2009). Guided by intuitions from indemnity insurance, which carry over to the mean variance model of Giné et al. (2008), these and other authors attribute this result to 'behavioural' decision making or external constraints:

> 'The most likely explanation [for demand falling with risk aversion] is that it is uncertainty about the product itself (Is it reliable? How fast are pay-outs? How great is basis risk?) that drives down demand.' (Karlan and Morduch 2009)

> 'Poor farmers on the other hand are not sufficiently well insured and would benefit from purchase of insurance, but they are severely cash and credit constrained, so that they cannot translate potential demand into purchases.'(Binswanger-Mkhize 2011)

This paper takes a different approach, arguing that the key to solving the puzzles is to note that the weather indexed insurance products currently being sold to farmers are derivatives, not indemnity insurance products.[1] Our model is one of rational demand, where the consumer is assumed to be a price taking risk averse expected utility maximiser with, for some results, decreasing absolute risk aversion (DARA).

One critical aspect of the model is the nature of the joint probability structure of the index insurance product and the consumer's loss. The net transfer from index

[1] Accountants have recently revisited the specific question of how to classify weather derivatives as part of the process of developing the International Financial Reporting Standards (International Accounting Standards Board 2007, pp.450-451). IFRS contains a principles-based distinction between an insurance contract 'in which an adverse effect on the policyholder is a contractual precondition for payment', and a derivative contract in which it is not. Under this definition, a weather derivative is a derivative, not any kind of insurance. A weather derivative is also classed as a derivative under US Generally Accepted Accounting Principles.

insurance is assumed be imperfectly correlated with the consumer's net loss, and so index insurance purchase both worsens the worst possible outcome and improves the best possible outcome; a consumer might incur a loss but receive no net income from the index insurance product or incur no loss but receive a positive net income. This model of basis risk is fundamentally different to the independent, additive, uninsurable background risk often considered by insurance theorists, under which purchase of indemnity insurance results in a contraction of net wealth in the sense of Rothschild and Stiglitz (1970).

The model is structurally similar to that of Doherty and Schlesinger's (1990) model of indemnity insurance with contractual exclusion clauses or a risk of insurer default; indeed theirs is a mathematical special case of ours in which the insurance premium is actuarially unfair and there is no risk of upside contractual nonperformance, where a claim payment is made even though no loss has been incurred. Whilst these authors interpreted the risk of contractual nonperformance as the perceived probability that an insured individual would not be indemnified against their loss due to insurer default or contractual exclusion clauses, we allow basis risk to act as another candidate cause.

Many of the results of Doherty and Schlesinger (1990) follow through to our model, namely that the risk of contractual nonperformance leads rational demand to be nonmonotonic in risk aversion, wealth and price. Demand for indexed insurance from infinitely risk averse, maximin decision makers, is shown to be zero and so demand cannot be everywhere increasing in risk aversion. For the classes of constant absolute and constant relative risk aversion we find demand for actuarially unfair indexed cover to be hump-shaped in the degree of risk aversion, first increasing then decreasing, and demand for actuarially favourable indexed cover to be either decreasing or decreasing-increasing-decreasing in risk aversion. Given these results it is perhaps not surprising that empirical economists have been finding a negative relationship between demand for weather derivatives and risk aversion for poor farmers in developing countries.

In the key theoretical contribution of this paper, we derive upper bounds for rational purchase of hedging instruments. For the case of indemnity insurance, that is insurance without basis risk, risk aversion and DARA alone cannot bound the purchase of indemnity insurance below full insurance; an infinitely risk averse individual would rationally purchase full insurance. However, tighter bounds may be derived for actuarially fair or unfair hedging products with basis risk, both under the restriction of risk aversion alone, and that of risk aversion and DARA. Loosely speaking the bound for DARA arises because an individual who cares enough about the risk to want to purchase a sizeable hedge must, under the assumption of DARA, care enough about the downside risk of contractual nonperformance and, for the case of an actuarially unfair price, the deadweight cost of hedging to limit the size of the hedge.

We close the paper by applying the framework to the numerical example of maize in a developing country. With a belief constructed from the empirical joint distribution function of yields and weather indexed claim payments we are able to characterise the level of optimal demand for the classes of risk averse, constant relative risk averse and decreasing absolute risk averse decision makers. In general we find low rational demand, and show that optimal demand from any risk averse expected utility maximiser is zero if the price for index insurance is more than 1.75 times the expected claim income. All pricing multiples calculated by Cole et al. (2009) for unsubsidised weather indexed insurance policies sold in India are greater or equal to 1.75, suggesting that the low observed demand for weather derivatives sold to poor farmers may be consistent with objective financial advice, rather than being a result of poor understanding, an unwillingness to experiment, or credit constraints.

The rest of the paper is organised as follows: Section 2 presents the model and characterises rational demand; Section 3 reports on a numerical analysis of rational demand for weather indexed insurance for a developing country; and Section 4 concludes.

Table 1. *Joint probability structure*

	$Index = 0$	$Index = I$	
$Loss = 0$	$1 - q - r$	$q + r - p$	$1 - p$
$Loss = L$	r	$p - r$	p
	$1 - q$	q	

2 THE MODEL

To capture the essence of rational purchase of index insurance consider the following model. A decision maker holds strictly risk averse preferences over wealth, with a von Neumann-Morgenstern utility function u satisfying $u' > 0$ and $u'' < 0$.[2] The decision maker is endowed with constant wealth w, is exposed to uninsurable zero mean background risk $\widetilde{z}$ with bounded support, and suffers a loss which can take the value L with probability p or zero with probability $1 - p$. There is also an index which can take the value I with probability q or zero with probability $1 - q$. The index is not necessarily perfectly correlated with the loss and so there are four possible joint realisations of the index and loss.

The index and loss are jointly statistically independent of background risk $\widetilde{z}$ and so, as usual, we may define the indirect utility function v by

$$v(x) = \mathbb{E}u(x + \widetilde{z}) \text{ for all } x \in \mathbb{R} \tag{1}$$

and reduce the problem to one with four states $s \in S = \{00, 0I, L0, LI\}$. Note that both risk aversion and decreasing absolute risk aversion of the direct utility function u are inherited by the indirect utility function v.[3]

For the purpose of talking about an increase or decrease in the risk of contractual nonperformance, or basis risk as it is more commonly known in the context of

[2] This restriction on the utility function is equivalent to assuming that, endowed with certain wealth, such an individual would never accept a non-degenerate lottery if the expected net gain from the lottery was nonpositive (Pratt 1964, Arrow 1965).

[3] The optimal decision of agent u with background risk of $\widetilde{z}$ is the same as for agent v who faces no background risk. $v^n(z) = \mathbb{E}u^n(x + \widetilde{z})$ where u^n is the nth derivative of u, and so v inherits the properties $v' > 0$ and $v'' < 0$. Moreover v inherits decreasing absolute risk aversion from u (Gollier 2001, p116).

derivatives, it is perhaps natural to consider changes in the joint distribution of loss and index whilst holding the marginal distributions fixed. In our four state model p and q fully specify the marginal distributions and with p and q fixed there is only degree of freedom in the joint probability distribution. We may therefore without loss of generality define basis risk parameter r as the joint probability that the index is 0 and the loss is L, and interpret an increase in r, without any change in p or q, as an increase in basis risk. Note that whilst this parameterisation effectively restricts a change in basis risk to be suitably symmetric, it does not restrict the level of basis risk; any four state joint probability distribution can be constructed by suitable choice of p, q and r.

The parametrisation of basis risk will have a natural interpretation in our model: r is the probability that an individual will incur a loss but the index will be good. For example, a farmer could lose her entire crop due to pestilence or localised weather conditions, but receive no claim payment from a weather derivative due to good weather having been observed at the contractual weather station. Regardless of how clever the design, weather derivatives are not able to accurately capture perils such as insects of disease, or localised weather events that can occur on the farmer's land without being observed at the contractual weather station.

Denoting the probability of each state s by π_s, we therefore have $\{\pi_{00}, \pi_{0I}, \pi_{L0}, \pi_{LI}\} = \{1 - q - r, q + r - p, r, p - r\}$ (see Table 1).[4] For an index realisation of I to be a signal that the loss has been L, we require that $\frac{\pi_{LI}}{\pi_{0I}} > \frac{\pi_{L0}}{\pi_{00}}$, that is $r < p(1 - q)$. We will also assume that basis risk r is strictly positive and that all states have nonnegative probability of occurrence, and so

$$0 < r < p(1 - q) \text{ and } p - q \leq r. \tag{2}$$

[4] Other specifications of basis risk are possible. One alternative specification would be that the probability of a loss was p and that the probability that the index was 'wrong', that is $\frac{\pi_{L0}}{\pi_{L0}+\pi_{LI}} = \frac{\pi_{0I}}{\pi_{0I}+\pi_{00}}$, was some ρ. However, under this specification $\mathbb{P}[Index = I] = p + (1 - 2p)\rho$ and so for $p \neq 1/2$ a change in basis risk parameter ρ would change the marginal distribution of the index in addition to increasing basis risk.

Table 2. *Four state framework*

State s	$L0$	LI	00	$0I$
Probability π_s	r	$p-r$	$1-q-r$	$q+r-p$
Wealth, no indexed cover	$w-L$	$w-L$	w	w
Wealth, indexed cover of αL	$w-P-L$	$w-P-L+\alpha L$	$w-P$	$w-P+\alpha L$

The loss is observable to the individual but not to the insurer, and so there is no market for indemnity insurance, with claim payment conditional only on the loss. However, the individual can purchase an indexed security that pays a proportion $\alpha \geq 0$ of the potential loss L when the index realisation is I. Loss and index are imperfectly correlated and so the indexed security only offers a hedge against the potential loss; indexed insurance does not pay a claim in the state $L0$. The indexed product is priced with a *multiple* of $m > 0$; that is the individual pays a premium of $P = qm\alpha L$ for cover of αL, receiving a claim payment of αL if the index realisation is I or zero if the index is 0.[5] For $m > 1$, $m = 1$, $m \geq 1$ and $m < 1$ the premium will be said to be actuarially unfair, actuarially fair and actuarially favourable, respectively. We will ignore the case in which $qm \geq 1$ for which zero indexed coverage is trivially optimal.

The individual's objective is therefore to choose a level of indexed coverage $\alpha \geq 0$ to maximise expected indirect utility:

$$\begin{aligned}\mathbb{E}V = (p-r)v(w-\alpha qmL-(1-\alpha)L) + (q+r-p)v(w-\alpha qmL+\alpha L) \\ + (1-q-r)v(w-\alpha qmL) + rv(w-\alpha qmL-L) \qquad (3)\end{aligned}$$

The first-order condition for an interior solution to this program, after cancelling $Lq(1-qm) > 0$, is:

$$Av'_{LI} + (1-A)v'_{0I} - B \times \left[Cv'_{00} + (1-C)v'_{L0}\right] = 0, \qquad (4)$$

[5] Using the standard insurance terminology, the corresponding *loading* would be $m-1$.

where for the sake of notational convenience,

$$A = \frac{p-r}{q}, \qquad B = \frac{m-qm}{1-qm}, \qquad C = \frac{1-q-r}{1-q}, \tag{5}$$

and v'_s denotes marginal indirect utility in state s. $\mathbb{E}V$ is concave in α and so the second-order condition is trivially satisfied. The parameter restrictions of equation (2) ensure that $p < A < 1$ and $1 - p < C < 1$ and therefore that $A + C > 1$. $B > 0$, with $B \gtreqless 1$ corresponding to $m \gtreqless 1$.

We begin by stating a basic result, for which we claim no novelty.

Proposition 1. *If the premium is actuarially unfair then full indexed coverage is never optimal, $\alpha^* < 1$. If the premium is actuarially fair then positive, partial indexed coverage is always optimal, $0 < \alpha^* < 1$. If the premium is actuarially favourable then positive indexed coverage is always optimal, $\alpha^* > 0$.*

Proof. If $\alpha \geq 1$ and $m \geq 1$ we have $B \geq 1$, $v'_{00} \geq \max(v'_{LI}, v'_{0I})$ and $v'_{L0} > \max(v'_{LI}, v'_{0I})$ and so the LHS of (4) is strictly negative, violating first-order condition (4). If $\alpha \leq 0$ and $m \leq 1$ the LHS of (4) is strictly positive since $B \leq 1$ and $A + C > 1$, violating first-order condition (4). □

In the model with $r > 0$ it is impossible to eliminate all uncertainty, due to the existence of an uninsurable basis risk. This may be compared to the well known results that for actuarially fair indemnity insurance, that is when $m = 1$, $p = q$ and $r = 0$, full insurance is optimal (Mossin 1968, Smith 1968), and remains optimal on addition of an independent background risk.

For $0 \leq \alpha < 1$, objective function (3) has strictly decreasing differences in $(\alpha; r)$, that is $\frac{\partial^2 \mathbb{E}V}{\partial \alpha \partial r} < 0$, yielding the following proposition:

Proposition 2. *The optimal level of coverage α^* is decreasing in basis risk r, and strictly decreasing if $\alpha^* > 0$.*

Whilst it is somewhat intuitive that risk aversion is sufficient to ensure that an increase in basis risk reduces demand, it bears mentioning that such a result doesn't hold for Doherty and Schlesinger's (1990) model of one-sided basis risk. In their model demand is nonmonotonic in the probability of insurer default, expect for the special cases of constant absolute risk aversion and quadratic utility. Whilst in both models an increase in basis risk reduces the correlation between the available cover and the loss, in our model it does so in a symmetric fashion, resulting in a mean preserving spread of wealth for any fixed α (Rothschild and Stiglitz 1970); the movements of probability mass from inner state 00 to outer state $L0$ and from inner state LI to outer state $0I$ each reduce the incentive to purchase indexed cover without changing the insurance premium. By comparison, in Doherty and Schlesinger's (1990) model, an increase in basis risk moves probability mass from inner state LI to outer state $L0$ and decreases the insurance premium, the combined effect of which is ambiguous.

2.1 Risk aversion

To appreciate the effect of basis risk on optimal demand for different levels of risk aversion, first consider extreme risk aversion. Ignoring background risk $\tilde{z}$, an infinitely risk averse, maximin, individual's objective is to maximise the lowest possible wealth realisation, and therefore full purchase of indemnity insurance ($r = 0, p = q$) is optimal so long as the net transfer to the individual is positive in the loss states ($qm < 1$); full insurance purchase increases the lowest possible wealth from $w - L$ to $w - qmL$. In stark contrast, an infinitely risk averse individual would optimally purchase zero indexed cover ($r > 0$) at any positive premium since purchase of cover of α would decrease the minimum wealth from $w - L$ to $w - L - \alpha qmL$.

Recall that if V' is a strongly more risk averse preference ordering than V in the sense of Ross (1981) then there exists $G : \mathbb{R} \rightarrow \mathbb{R}, G' < 0, G'' < 0$ and $\lambda > 0$ such that $V'(W) = \lambda V(W) + G(W)$. The following theorem characterises demand for indexed cover for extreme risk aversion.

Theorem 1. *Let $\{u_n\}_{n=1}^{\infty}$ be any sequence of utility functions satisfying:*

(i) *each u_n is everywhere twice differentiable with $u_n' > 0$;*

(ii) *u_{n+1} is more risk averse than u_n in the sense of Ross; and*

(iii) $\lim_n \left(\inf_{x \in \mathbb{R}} -u_n''(x)/u_n'(x)\right) = +\infty$.

Then optimal demand α_n^* from a decision maker with utility function u_n satisfies $\lim_n \alpha_n^* = 0$.

The purchase of indexed insurance reduces the difference in wealth between states LI and 00, where the hedge performs, but increases the difference in wealth between states $L0$ and LI, where the hedge does not perform. Even though the value to an infinitely risk averse individual of hedging states LI and 00 is high, the disutility arising from allowing a decrease in realised wealth in state $L0$ is so high as to justify zero cover.

One immediate implication of Theorem 1 is that an increase in risk aversion, either in the sense of Arrow-Pratt or even in the stronger sense of Ross (1981), does not necessarily lead to an increase in demand for indexed cover, echoing the earlier results of Schlesinger and Schulenburg (1987) and Doherty and Schlesinger (1990) for the case of a one-sided risk of contractual nonperformance. This differs from both classical models of demand for indemnity insurance with or without background risk and predictions from the mean variance model of demand for index insurance (Giné et al. 2008) in which optimal demand is increasing in risk aversion.

If the indirect utility function satisifies constant absolute risk aversion, that is $v'(x) \propto e^{-\gamma x}$ for some coefficient of absolute risk aversion γ, first order condition (4) may be rearranged to give the optimal cover as an explicit function of p, q, r, m, L and γ.

Lemma 1. *For any individual with indirect constant absolute risk aversion of $\gamma > 0$ the optimal level of indexed cover is:*

$$\alpha^*_{CARA}(\gamma) = \max\left[0, \frac{1}{\gamma L}\ln\left(\frac{A + (1-A)e^{-\gamma L}}{BCe^{-\gamma L} + B(1-C)}\right)\right] \tag{6}$$

For both the classes of constant absolute risk averse (CARA) and constant relative risk averse (CRRA) expected utility maximisers, the shape of optimal demand is characterised in the following theorem.

Theorem 2. *The following conditions on the optimal level of indexed cover hold both for the class of risk averse indirect utility functions that satisfy constant relative risk aversion and for the class of risk averse indirect utility functions that satisfy constant absolute risk aversion, where γ denotes the coefficient of relative or absolute risk aversion, respectively:*

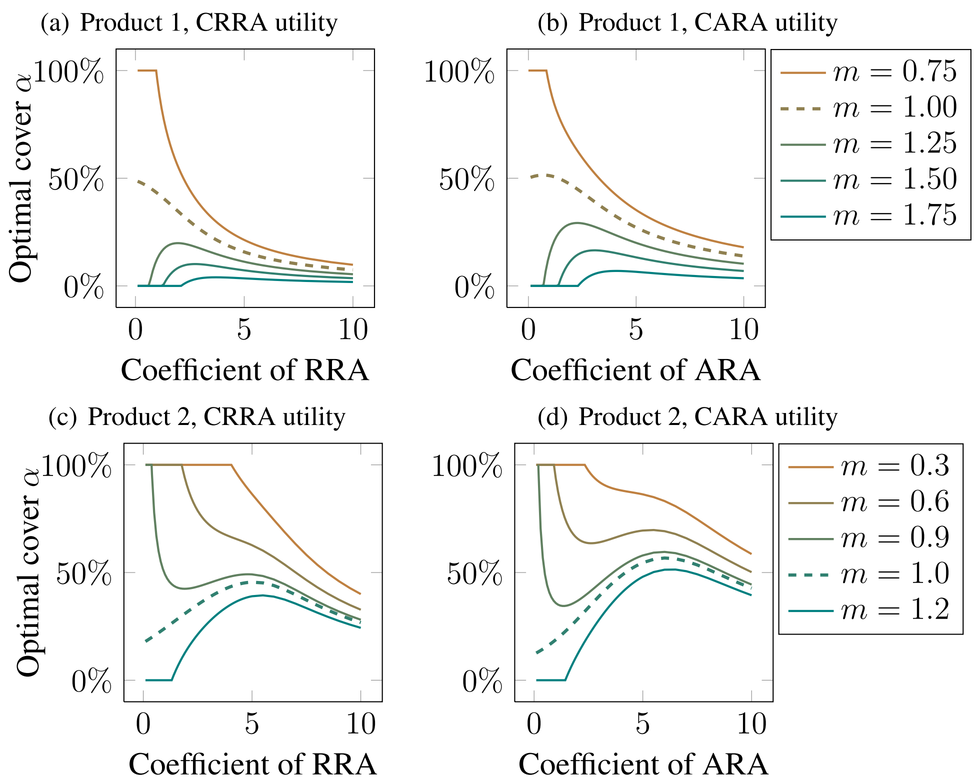
(i) Actuarially unfair products ($m > 1$): *$\alpha^*(\gamma) = 0$ for all $\gamma \in (0, \infty)$ if $r \geq p(1 - qm)$. Otherwise $\alpha^*(\gamma)$ is zero for $\gamma \leq \gamma_1$, strictly increasing for $\gamma_1 < \gamma < \gamma_2$ and strictly decreasing for $\gamma_2 < \gamma < \infty$ for some $0 < \gamma_1 < \gamma_2 < \infty$;*

(ii) Actuarially fair products ($m = 1$): *$\alpha^*(\gamma) > 0$ is either strictly decreasing for all $\gamma \in (0, \infty)$ or strictly increasing for $0 < \gamma < \gamma_1$ and strictly decreasing for $\gamma_1 < \gamma < \infty$ for some $0 < \gamma_1 < \infty$; and*

(iii) Actuarially favourable products ($m < 1$): *$\alpha^*(\gamma) > 0$ is either strictly decreasing for all $\gamma \in (0, \infty)$ or strictly decreasing for $0 < \gamma < \gamma_1$ and $\gamma_2 < \gamma < \infty$, and strictly increasing for $\gamma_1 < \gamma < \gamma_2$, for some $0 < \gamma_1 \leq \gamma_2 < \infty$.*

These relationships can be seen in Figure 1, which plots the optimal purchase of indexed cover for constant absolute and relative risk averse expected indirect utility maximisers against the respective coefficient of absolute and relative risk aversion for two different products.

Figure 1. *Rational hedging and risk aversion for CRRA and CARA*



Note: Assumptions underlying figures (a) to (d) are as follows. $p = q = 1/3$, $r = 1/9$ for product 1 (figures (a) and (b)) and $p = 1/20$, $q = 4/20$ and $r = 1/1000$ for product 2 (figures (c) and (d)). $w = 3L/2$ for CRRA (figures (a) and (c)) and $L = 1$ for CARA (figures (b) and (d)).

The result that the optimal level of actuarially unfair indexed cover is hump - shaped in the coefficient of relative or absolute risk aversion, first increasing then decreasing, may be understood as follows. For actuarially unfair premiums, $m > 1$, indexed cover decreases mean wealth and so risk neutral individuals would optimally purchase zero cover. Indexed cover also decreases the minimum possible wealth and so infinitely risk averse individuals would optimally purchase zero cover (Theorem 1). It is therefore clear that only individuals with intermediate levels of risk aversion might wish to purchase material amounts of actuarially unfair cover. For the cases of CARA and CRRA, this hump-shaped relationship is smooth.

For actuarially fair cover, $m = 1$, there are two factors affecting demand; an increase in cover both reduces the risk for states in which the hedge performs (00 and LI) and increases the risk for states in which the hedge does not perform ($L0$ and $0I$). For high risk aversion, the utility cost from basis risk will dominate decisions and demand will be decreasing in risk aversion. However, the benefit from the hedge when it performs may dominate for low risk aversion and so the optimal level of actuarially fair cover will either be decreasing or increasing then decreasing in risk aversion for CARA or CRRA utility.

For actuarially favourable premiums, $m < 1$, indexed cover increases mean wealth and so risk neutral individuals would optimally purchase as much cover as possible, but it decreases minimum wealth and so infinitely risk averse individuals would optimally purchase zero cover (Theorem 1 and Corollary **??**). Between these two extremes, the decision maker must trade off the benefits from the increase in expected wealth and from hedging when the contract performs (reduction in risk between states 00 and LI) against the cost of basis risk when the contract does not perform (particularly state $L0$). For CARA and CRRA, demand is either monotonically decreasing in risk aversion or first decreasing then increasing then decreasing in risk aversion. Panes (c) and (d) of Figure 1 provide a numerical example of demand 'decreasing with a hump', where the premium is only slightly lower than the actuarially fair premium and basis risk is low.

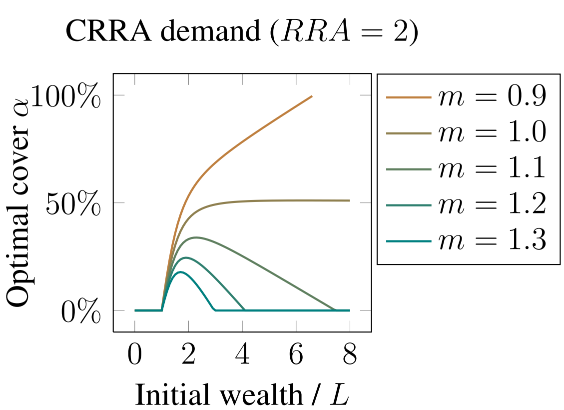
2.2 Wealth and price

Other comparative statics results do follow through from Doherty and Schlesinger (1990), as would be expected since their model is a reparameterised, special case of ours. To begin with, there is no monotonic relationship between demand and wealth w, pricing multiple m or loss L, even if one restricts preferences to satisfy increasing absolute risk aversion (IARA) or decreasing absolute risk aversion (DARA).

The first result stands in direct contrast to Mossin's (1968) observation that indemnity insurance is an inferior good under DARA. In models without basis risk, an increase in insurance purchase transfers wealth from high to low wealth states, subject to some deadweight cost. However, in the presence of basis risk an increase in indexed cover transfers wealth from the lowest and intermediate wealth states to the highest and other intermediate wealth states, and the restriction of DARA is no longer relevant for determining whether indexed cover is an inferior good.

The second result, that indexed cover may be a Giffen good, stands in direct contrast to the result of Mossin (1968) and Smith (1968) who show that that, in the absence of basis risk, demand for indemnity insurance is lower for $m > 1$ than for $m = 1$. This result also arises from indexed cover not being an inferior good, even under DARA. Although an increase in premium increases B, acting to decrease demand, it also uniformly decreases wealth, which has an ambiguous effect on optimal demand for reasons described in the previous paragraph. If one restricts preference to satisfy constant absolute risk aversion (CARA) this wealth effect disappears, ruling out the possibility that indexed cover is a Giffen good. However, without the restriction to CARA the effect on demand of an increase in m is ambiguous. Figure 2 plots the optimal purchase of indexed cover for constant relative risk averse expected utility maximisers with respect to initial wealth.

Figure 2. *Rational hedging and wealth ($p = q = 1/3$, $r = 1/9$)*



The third result follows trivially from the observation that demand is nonmonotone in loss L for CARA (see equation (6)), and therefore by extension for DARA or IARA. Although this result was not mentioned in Doherty and Schlesinger (1990), it also holds in the case of one-sided basis risk. By way of comparison, demand for indemnity insurance under DARA is monotonically increasing in loss L.

2.3 Bounds for rational demand

It is also possible to derive some strong results about the welfare implications of hedging. A financial advisor advising on the purchase of indemnity insurance cannot reasonably rule out any level of purchase without understanding a client's preferences; zero purchase may be advisable for a risk neutral client, and nearly full cover advisable for a risk averse client. As we will show in this section, this intuition does not follow through to the case of indexed cover with basis risk.

First, there are indexed products for which zero coverage is optimal for any risk averse expected utility maximiser.[6]

Theorem 3. *For any risk averse individual the optimal level of indexed cover is zero if* $\mathbb{E}[i|l = L] \leq m\mathbb{E}[i]$.

Proof. First note that the condition is equivalent to $r \geq p(1-qm)$ and, combined with the assumption of affiliation (2), can only hold if $m > 1$. Indirect utility function v inherits strict risk aversion from u and so $v'_{L0} \geq v'_{LI}$ and $v'_{00} \geq v'_{0I}$ for $\alpha \geq 0$.[3] Since $C > 1 - A$ and $B \geq 1$ for $m > 1$, $(1 - A)v'_{0I} - BCv'_{00} < 0$ and so first order condition (4) cannot hold unless $Av'_{LI} > B(1-C)v'_{L0}$. This in turn implies that $A > B(1 - C)$ which can be rearranged to give the restriction $\frac{p-r}{p} > qm$ or $\mathbb{E}[i|l = L] > m\mathbb{E}[i]$. □

[6] Theorem 3 holds in a more general setting with index $i \in [0, I]$ affiliated with loss $l \in [0, L]$ in the sense of Milgrom and Weber (1982).

Theorem 3 may be understood as follows. When $m \leq 1$, the condition of Theorem 3 can never hold due to our assumption that the index and loss are strictly positively correlated (equation (2)); with positive correlation and an actuarially fair or favourable price there will be a risk averse individual for whom positive purchase is optimal.

However, for the case of actuarially fair or unfair cover, purchase does not increase average wealth and so for positive purchase to be optimal for a risk averse individual it must at least result in an increase in average wealth in low wealth states, that is the states in which a loss has occurred. Purchase of cover of α increases the premium by αqmL and results in average claim income, conditional on a loss L having occurred, of $\alpha L\frac{p-r}{p}$. The condition ensuring that the average net gain in these low wealth states is positive on purchase of indexed cover is therefore $\alpha L\left(\frac{p-r}{p} - qm\right) > 0$, or equivalently $\mathbb{E}[i|l = L] > m\mathbb{E}[i]$.

When basis risk is low, for example when the only source of basis risk is a 1% chance of insurer default, the restriction of Theorem 3 is not much tighter than the restriction that the maximum possible claim payment αL is larger than the premium αqmL. However, when there is both a sizeable basis risk r and premium multiple m, the restriction of Theorem 3 is tighter.

For $\mathbb{E}[i|l = L] > m\mathbb{E}[i]$, risk aversion alone is not sufficient to justify an upper bound for demand tighter than full cover, even for actuarially unfair cover, since we cannot rule out the possibility that the individual is approximately risk neutral except for some interval between $w - \alpha qmL - (1 - \alpha)L$ and $w - \alpha qmL$ in which she is very risk averse. However, we may rule out such contrived cases by assuming that absolute risk aversion decreases with wealth. This assumption of decreasing absolute risk aversion (DARA) is equivalent to assuming that if the decision maker would accept some lottery given certain endowment of w she would accept the same lottery given certain endowment of $w' > w$. Regardless of whether this restriction is appropriate from a positive point of view, that is whether individual's actual decisions violate this assumption, we agree with

Arrow (1965) and Pratt (1964) who argued that it is normatively sound. Moreover we consider DARA, and by extension the upper bound of the following theorem, to be an appropriate basis for generic financial advice about products designed to reduce exposure to risk.

Under DARA we may derive the following upper bound on rational demand for indexed cover, identical to the former upper bound for products with $\mathbb{E}[i|l = L] \leq m\mathbb{E}[i]$ and tighter for products with $\mathbb{E}[i|l = L] > m\mathbb{E}[i]$.

Theorem 4. *For any strictly risk averse individual with decreasing absolute risk aversion the optimal level of actuarially fair or unfair indexed coverage is zero if* $\mathbb{E}[i|l = L] \leq m\mathbb{E}[i]$*, or otherwise bounded above by the unique* $\bar{\alpha}$ *that solves*

$$A\bar{\alpha}^{\bar{\alpha}}(1-\bar{\alpha})^{1-\bar{\alpha}} = (A + BC - 1)^{\bar{\alpha}} \times (B(1-C))^{1-\bar{\alpha}}$$
$$\bar{\alpha} \in \left(0, \frac{A + BC - 1}{A + B - 1}\right] \tag{7}$$

Although we relegate the full proof to the appendix, a sketch is as follows. Conditional on the marginal indirect utility of wealth in states LI and 00, v'_{LI} and v'_{00}, we first show that demand is bounded above by an indirect utility function which is risk neutral above v'_{00} and has constant absolute risk aversion of γ below v'_{00}. The maximisation problem is then to choose $\gamma \geq 0$ to maximise demand α, conditional on demand satsifying first order condition (4). Equation (7) defines this largest optimal α for any $\gamma \geq 0$.

Loosely speaking, the logic of the DARA upper bound of Theorem 4 is as follows: if an individual cares enough about the risk between states LI and 00 to want to purchase a sizeable hedge, the individual must care enough about the downside basis risk between states $L0$ and LI and the deadweight cost of hedging to limit the size of the hedge.

As presented in Clarke (2011, Chapter 2), Theorem 4 provides a one-decision problem test for DARA in experimental settings. So long as the randomisation device used to determine net transfers in an experiment is statistically independent of participant's background wealth, demand from participants in excess of the respective upper bound may be taken as a violation of risk aversion for products with $r \geq p(1-qm)$ or risk aversion and DARA for other products.

Table 3 presents values of $\bar{\alpha}$ when the probability of no claim being paid conditional on a loss having occurred is taken to be $\frac{1}{3}$.

Table 3. *Examples of upper bounds for rational purchase of indexed cover*

$\mathbb{P}[Loss] = \mathbb{P}[Claim]$ $(p = q)$	$\mathbb{P}[No\ Claim \vert Loss]$ (r/p)	$Premium\ Multiple$ (m)	$\bar{\alpha}$
$1/10$	$1/3$	6.0	3.9%
$1/8$	$1/3$	4.5	6.7%
$1/5$	$1/3$	3.0	4.5%
$1/4$	$1/3$	2.4	4.8%
$1/3$	$1/3$	1.8	5.4%

When premiums are actuarially fair, that is $m = 1$, the formula for the upper bound may be simplified.

Corollary 1. *For any strictly risk averse individual with decreasing absolute risk aversion the optimal level of actuarially fair indexed coverage is bounded above by* $\bar{\alpha} = \frac{p(1-q)-r}{(p-r)(1-q)}$.

Proof. For $m = 1$, $\bar{\alpha} = \frac{A+C-1}{A} = \frac{p(1-q)-r}{(p-r)(1-q)}$ is a solution to equation (7), and by Theorem 4 it is the unique solution. $\square$

2.4 Discussion

Bundling indexed insurance with credit

To many proponents, the main welfare gains to be derived from indexed insurance are through the relaxation of credit constraints. For example, it is often argued that weather or area yield indexed insurance can act as collateral against loans, increasing the creditworthiness of farmers and allowing them the opportunity to invest in appropriate inputs to increase agricultural productivity (Hazell 1992, Carter et al. 2007, Mahul and Stutley 2010).

This proposition can be analysed in the current framework. For example, suppose that a relaxation of credit constraints brought about by purchase of indexed insurance leads to a risk-free increase in wealth. The problem of how much of this bundle to purchase is then mathematically identical to the problem of how much indexed insurance to purchase, where indexed insurance is priced with a lower effective pricing multiple m than the standalone indexed insurance product. If the effective multiple is less than unity, one would expect risk neutral decision makers to purchase as much of the bundle as possible. However more risk averse decision makers would purchase less of the bundle, with demand suppressed by concerns about the increase in downside potential; for the case of bundled agricultural credit and weather indexed insurance, a farmer could take out a loan and purchase indexed insurance but have a bad year with no index insurance claim payment. The bundle would only be purchased by the infinitely risk averse if, as suggested by Carter et al. (2010), the effective pricing multiple m is nonpositive.

Beliefs and real insurance products

There are many interpretations to the model presented in this section. p, q and r could be known probabilities with $m = \frac{P}{q\alpha L}$ as the premium multiple consistent with the individual's objective belief q and the observed price P for cover of L. Alternatively (p, q, r) could be interpreted as a subjective decision-theoretic belief, after any subjective probability weighting.

For the case of indexed insurance r has been presented as the downside basis risk but other interpretations are possible. Schlesinger and Schulenburg (1987) and Doherty and Schlesinger (1990) consider a mathematical special case of ours in which $\pi_{0I} = 0$, that is where there is no upside basis risk. They interpret their model as one where an insurer sells indemnity insurance but with probability r a policyholder incurs a loss but does not receive indemnification from the insurer due to insurer insolvency or contractual exclusions. Mathematically, the cause of this risk of contractual nonperformance, whether it be downside basis risk, insurer insolvency or contractual exclusions, does not change the shape of optimal demand.

However, whilst insurer insolvency and contractual exclusions for consumer indemnity insurance products might be considered to have low probability in countries with robust capital adequacy requirements and consumer protection regulation, motivating Schlesinger and Schulenburg (1987) to end their paper with the caveat 'we might not expect to observe [demand falling with risk aversion] very often as a matter of practice', there may be a high risk of contractual nonperformance for indexed products, or products sold in environments without credible regulation. A negative relationship between demand and risk aversion has been well documented for the case of weather derivatives sold to poor farmers (e.g. Cole et al. 2009) and for a health microinsurance product in Kenya where consumers did not fully trust the insurer to pay valid claims (Dercon et al. 2011).

Gap insurance

One immediate implication of Theorem 1 is that gap insurance, which would make a payment in state $L0$ at least equal to the premium, would be valuable for the most risk averse if bundled with indexed cover (Doherty and Richter 2002). So long as the combined index/gap insurance premium was less than than maximum loss, an infinitely risk averse individual would optimally purchase full cover, as opposed to the zero cover of Theorem 1.

For the case of agricultural insurance it seems difficult to imagine a formal insurance company being able to offer an indexed product bundled with gap insurance at low cost. However, as suggested in Clarke (2011, Chapter 3), local institutions could perhaps supplement formal sector indexed insurance with local informal or semiformal risk pooling that provided gap insurance using cheap local information. Any such gap insurance would be of significant benefit to the most risk averse even if only a refund of premium was paid in state $L0$; the most risk averse would then optimally purchase full cover, rather than the zero cover of Theorem 1.

Hedging and the poverty headcount measure

There are, of course, sets of preferences which violate risk aversion or DARA, for which optimal purchase of indexed cover may exceed the upper bound of Theorem 4. Whilst we have argued above that such sets of preferences can reasonably be ignored for the purpose of providing generic financial advice to individuals, there is one particular class of preferences worthy of particular attention.

Suppose that policymakers choose policies to minimise the expected number of individuals below some poverty line $\mathcal{P}$. For individuals with wealth w and subject to loss L such that $\mathcal{P} \in [w - L, w - qmL)$, then purchase of cover $\alpha L \in \left(\frac{P-(w-L)}{1-qm}, L\right]$ would reduce their expected contribution to headcount poverty from p, the probability of a loss occurring, to r, the probability of a loss occurring and no indexed claim payment being made. This result arises from a well documented feature of the headcount poverty measure, whereby it is possible to reduce expected headcount poverty by transferring wealth from states below the poverty line to states also below the poverty line but with higher wealth (Sen 1979). Providing targeted subsidies for indexed financial products may therefore meet the objectives of policymakers, even if all rational individuals would strictly prefer the subsidy to be in the form of an actuarially equivalent uncontingent cash transfer.

3 NUMERICAL EXAMPLE

The methodology of Section 2 can be applied to real world data to construct a numerical analysis of objectively rational demand. By objective, we refer to a decision rule under which beliefs are based on an objective probability distribution and, as above, we use the term rational to mean DARA EUT preferences over roulette lotteries.[7]

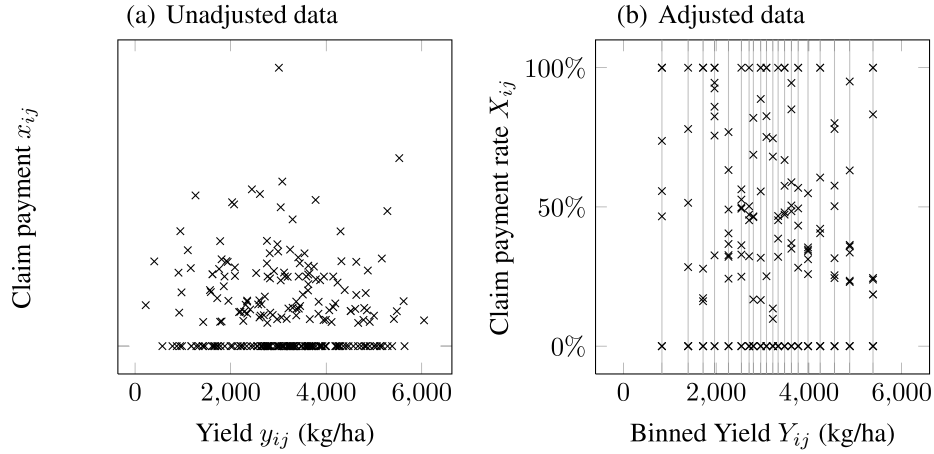
For example, suppose that a financial advisor is to provide advice to the following maize farmers in a developing country. In 2009, weather index insurance products designed for maize were sold in 31 nearby subdistricts in a developing country.[8] From 1999 to 2007, average maize yields within these subdistricts were recorded and weather readings were taken at the contractual weather station for the weather indexed insurance product. Let y_{ij} denote the yield in kilograms per hectare in year i for subdistrict j and x_{ij} denote the claim payment that would have been made, denominated in local currency, from one unit of the product for subdistrict j given the recorded weather readings for year i. 18 data items are missing, leaving a total of $n = 261$ complete (x_{ij}, y_{ij}) pairs, indexed by $ij \in D$ (see Figure 3(a)).

With up to nine years of matched historical data per product, the empirical joint distribution function is not a particularly useful belief for a objective EUT decision maker; optimal demand for indexed insurance is sensitive to the distribution of claim payments conditional on the yield, particularly for low yields, and the empirical conditional distribution is degenerate. With up to nine years of data, demand from the most risk averse will be driven by the year with the lowest yield for which either there was a claim payment greater than the premium in that year or there wasn't.

[7] Anscombe and Aumann (1963) differentiated *roulette lotteries*, where probabilities are known, and *horse lotteries*, where they are not. An objective financial advisor might combine an objective belief, constructed as a frequentist, with an individual's preferences over roulette lotteries to arrive at a recommended decision.

[8] The numerical example uses data for real weather derivatives sold across a developing country in 2009. However, for reasons of confidentiality the name of the country is not disclosed.

Figure 3. *Unadjusted and adjusted joint empirical distribution of yields and claim payments*



Another approach would be to assume homogeneity of the yield and claim payment distribution between subdistricts and use the empirical distribution of the 261 pairs as the decision maker's belief. However, there are two problems with this approach. First, although products were designed using a coherent methodology, the product details and the maximum possible claim payment in local currency (the *sum insured*) differed between products. Second, out of 261 observations, only 25 yield values have been recorded more than once, and so the empirical distribution of claim payments conditional on yields is still largely degenerate.

We address these problems in the following way (see Figure 3(b)). First, historical claim payment amounts x_{ij} are divided by the maximum historical claim payment for that product $\max_i(x_{ij})$ to give the historical claim payment rate $X_{ij} = \frac{x_{ij}}{\max_i(x_{ij})}$. This converts all claim payments histories to the same scale, from 0% to 100%, allowing histories from different products to be pooled.[9]

Second, all ij pairs are sorted in order of increasing yield y_{ij} and partitioned into

[9] The mean Pearson product-moment correlation coefficient for yields and claim payments over all 31 products is -16%. Pooling all data without adjusting gives a correlation coefficient of -6.8%. Pooling after adjusting the claim payments as described decreases this to -12.8% and averaging yield data within 5th percentile bins decreases it further to -13.6%. Adjusting claim payments using different rules, for example by dividing by the contractual sum insured for each product or the mean historical claim payment for each product gives similar results.

20 five-percentile bins, $k \in \{1 \ldots 20\}$.[10] For each ij pair in the kth bin, the binned yield Y_{ij} is the mean yield y_{ij} over all ij pairs in the bin. For example, for the first bin, containing the thirteen ij pairs with the lowest yields, Y_{ij} is calculated to be 831 kg/ha (see Table 4).

Table 4. *Illustration of binning of yield data for first bin*

y_{ij} rank	261	260	259	258	257	256	255	254	253	252	251	250	249
y_{ij} (kg/ha)	223	402	572	783	872	909	924	943	948	972	983	1104	1166
Y_{ij} (kg/ha)	831	831	831	831	831	831	831	831	831	831	831	831	831
X_{ij}	56%	74%	0%	0%	0%	100%	47%	0%	100%	100%	0%	100%	100%

We now interpret the adjusted joint probability distribution, displayed in Figure 3(b), as an objective belief about the joint distribution of yield and indexed claim payments for a representative maize farmer in our study area, and calculate how much index insurance such a farmer would purchase. Following the notation in the previous section, the maximum possible loss L a farmer could incur under our belief is just the difference between the maximum and minimum binned yield of $5,381 - 831 = 4,550$ kg/ha.

A maize farmer's objective is therefore to choose a level of coverage $\alpha \geq 0$, providing a maximum claim payment of αL, to maximise expected utility:

$$\mathbb{E}U = \frac{1}{n} \sum_{ij \in D} u(w + \tilde{z} + Y_{ij} + \alpha L(X_{ij} - m\bar{X})) \tag{8}$$

where $\bar{X}$ denotes $\frac{1}{n} \sum_{ij \in D} X_{ij}$, w is initial certain wealth, $\tilde{z}$ is a zero mean random background risk statistically independent of the joint distribution of (X, Y), m is the pricing multiple and u is the farmer's utility function, assumed to satisfy $u' > 0$ and $u'' < 0$.

The first-order condition for an interior solution may be written in terms of the

[10] All bins contain 13 (x_{ij}, y_{ij}) pairs except the 45th to 50th percentile bin which contains 14 products. For the three cases in which there are two pairs with equal yields, one of which must go in a higher bin than the other, and the claim payments are not equal, the allocation into bins is random.

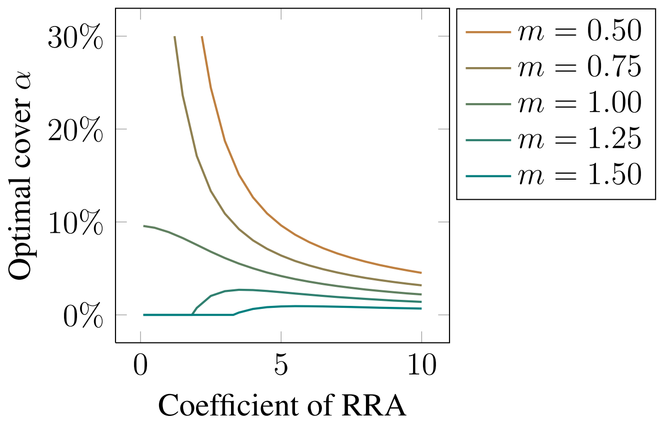
indirect utility function v (equation (1)) as

$$\sum_{ij \in D} L(w + X_{ij} - m\bar{X})v'(Y_{ij} + \alpha L(X_{ij} - m\bar{X})) = 0 \tag{9}$$

and the second-order condition is trivially satisfied.

Figure 4 presents the optimal level of demand for indexed insurance from a maize farmer with beliefs as displayed in Figure 3(b) and CRRA indirect utility function over net income from agriculture, that is with $w = 0$. Five different insurance multiples are considered: two actuarially favourable ($m = 0.5, 0.75$); two actuarially unfair ($m = 1.25, 1.50$); and actuarially fair ($m = 1$). As can be seen from Figure 4, and in line with Theorem 2, α is monotonically decreasing in the coefficient of relative risk aversion when $m < 1$ and is hump-shaped for $m \geq 1$. Moreover, the level of demand is low when the product is actuarially fair or unfair, with a maximum level of demand of 2.7% when $m = 1.25$ and 0.9% when $m = 1.5$. Ignoring the case of risk neutrality, for which the optimal level of cover is ambiguous, the maximum optimal level of demand is only 9.6% for an actuarially fair product with $m = 1$.

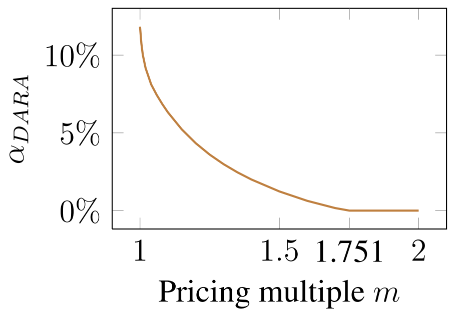
Figure 4. *Optimal purchase of index insurance for maize from CRRA decision makers*



Moreover, given the belief displayed in Figure 3(b), it is possible to numerically calculate the DARA upper bound, above which no DARA EUT decision maker would ever optimally purchase.[11] These rational bounds are displayed in Figure 5. As would be expected, the DARA upper bound is infinite for actuarially favourable products since a decision maker with very low levels of risk aversion could optimally purchase an infinite amount of indexed insurance. However, the DARA upper bound is decreasing in loading for a premium multiple $m \geq 1$ and is zero for $m \geq 1.751$.

In fact, DARA is not necessary for this result to hold; any strictly risk averse EUT decision maker would optimally purchase zero cover for $m \geq 1.751$ for the following reason, along the lines of Theorem 3.[12] The average unconditional claim payment is 29.7% and the average claim payment conditional on achieving the lowest possible binned yield is 52.0%. Moreover, the average claim payment conditional on achieving a binned yield less than or equal to any yield between 831 and $5,381$ kg/ha is no more than 52.0%. If the premium multiple were above 1.751, the insurance premium would be greater than the average claim payment conditional on achieving a binned yield less than or equal to any yield between 831 and $5,381$ kg/ha. The reasoning then follows that of Theorem 3.

Figure 5. *DARA upper bound for purchase of index insurance for maize*



In practice both fixed costs and variable costs seem to be high for small

[11] The procedure for calculating the DARA upper bound was as follows. First, the decision maker was assumed to be risk neutral for wealth above some level of wealth W and have constant absolute risk aversion of $\gamma \geq 0$ for wealth below W. W and γ were chosen to maximise the optimal level of demand α. Second, using this utility function as a starting point, the decision maker was free to choose levels of absolute risk aversion between each of the 261 ordered wealth realisations, consistent with DARA. The utility function was chosen to maximise the optimal level of demand α. In all cases, this α was virtually identical to the α calculated in the first step.

[12] For our empirical distribution the yield and indexed claim payments are negatively correlated but not affiliated, and so the conditions of Theorem 3 are not met. However, the joint distribution is sufficiently negatively correlated for a bound to be derived.

products marketed to poor farmers. Giné et al. (2007) and Cole et al. (2009) use historical weather data to estimate objective premium multiples m for unsubsidised products sold to poor farmers in India, with the former quoting an average premium multiple of 3.4 and the latter calculating multiples individually for seven products, ranging from 1.75 to 5.26. If the joint distribution between indexed claim payments and farmer yields for these products were the same as the joint distribution between indexed claim payments and area average yields for the present example of maize, the combination of basis risk and actuarially unfair prices with multiple above 1.75 would lead to zero optimal demand from any objective, risk averse EUT decision maker. The puzzle of low demand could therefore be resolved without requiring 'behavioural' decision makers or credit constraints.[13]

4 CONCLUDING REMARKS

Recent work on weather derivatives for the poor by academic economists seems to have been lacking a sound theoretical basis. This paper attempts to address this by presenting a model of rational demand for indexed products. The presence of basis risk is shown to alter or reverse many of the key results arising from the theory of rational demand for indemnity insurance (see Table 5, which extends Table 1 of Doherty and Schlesinger (1990)).

A numerical example is presented which suggests that the aggregate level and shape of observed demand for weather derivatives from poor farmers may be consistent with a model of rational demand, thereby offering explanations for two outstanding empirical puzzles without the need to resort to 'behavioural' preferences or credit constraints.

[13] This is not to say that models with behavioural preferences or credit constraints have no descriptive power for the analysis of index insurance purchase; Bryan (2010) and Cole et al. (2009) find strong relationships between purchase of indexed insurance and ambiguity aversion and credit constraints, respectively. Rather, one can explain the low demand, particularly for the risk averse, using nothing more than expected utility theory with an objective belief.

Table 5. *Indemnity and indexed insurance: a summary of key results*

Result without basis risk (Indemnity insurance, $r = 0$)	Result with basis risk (Indexed cover, $r > 0$)
Shape of rational hedging:	
	• α decreasing in basis risk
• More risk averse $\Rightarrow$ buy more cover	• More risk averse $\not\Rightarrow$ buy more cover
	• Unfair price ($m > 1$) $\Rightarrow$ CRRA/CARA cover hump-shaped (increasing then decreasing) in coefficient of RRA/ARA
	• Fair price ($m = 1$) $\Rightarrow$ CRRA/CARA cover decreasing or hump-shaped in coefficient of RRA/ARA
	• Favourable price ($m < 1$) $\Rightarrow$ CRRA/CARA cover decreasing or decreasing-increasing-decreasing in coefficient of RRA/ARA
• Infinitely risk averse $\Rightarrow \alpha = 1$	• Infinitely risk averse $\Rightarrow \alpha = 0$
• Fair price ($m = 1$) $\Rightarrow \alpha = 1$	• Fair price ($m = 1$) $\Rightarrow \alpha < 1$
• Positive loading ($m > 1$) $\Rightarrow$ buy less insurance	• Positive loading ($m > 1$) $\not\Rightarrow$ buy less cover
• Insurance is inferior for DARA utility	• Cover may not be inferior for DARA utility
• Larger potential loss $L \Rightarrow$ buy more insurance for DARA utility	• Larger potential loss L $\not\Rightarrow$ buy more cover for DARA utility
Level of rational hedging:	
• Positive loading ($m > 1$) $\Rightarrow$ DARA upper bound of $\bar{\alpha} = 1$	• Positive loading ($m > 1$) $\Rightarrow$ Risk averse upper bound of $\bar{\alpha}_{RA}$ and DARA upper bound of $\bar{\alpha}_{DARA} \leq \bar{\alpha}_{RA}$

To proponents, consumer hedging products based on cheaply observable, manipulation-free indices can offer partial cover at a lower deadweight cost than would be possible under traditional indemnity insurance contracts (Shiller 1993, Shiller 2003). However, the experience of selling weather derivatives to poor farmers suggests that the combination of deadweight costs and basis risk can render such products unattractive, particularly to the most risk averse, even if they offer a hedge against economically important risks.

Weather derivatives have changed the face of agricultural insurance for the poor by focusing the minds of academics and practitioners on a problem which, if suitably addressed, could lead to a substantial increase in welfare for many of the world's rural poor (Banerjee 2002, Collins et al. 2009, Karlan and Morduch 2009). Indices are not a silver bullet; designing a good agricultural insurance product for poor farmers will require more than just choosing the best functional form for a weather index, and implementing products with low basis risk will require more institutional capacity building than the installation of tamper-proof weather stations. However, indices have their uses, particularly in agricultural insurance.

One approach to reducing basis risk, presented in Clarke (2011, Chapter 3), is to combine indices that capture local aggregate shocks with local semiformal risk pooling. Groups of poor individuals, such as extended families or members of close-knit communities, are typically able to support relatively inexpensive pooling for locally idiosyncratic shocks. The role of the formal insurer should then be to 'reinsure' the group, paying claims when the group has suffered a large aggregate loss. The formal insurer could pay when the total group loss is large, as in Mexico's system of agricultural insurance through Fondos (Ibarra 2004), or when a statistical sample of losses indicates a large average loss has been incurred, as in India's system of area yield index insurance (Mahul et al. 2011). The combination of local sample-based indices and local risk pooling could lead to lower basis risk for farmers than other indexed approaches whilst offering lower cost than indemnity based approaches to formal insurance.

Regardless of how insurance for the poor develops it seems important for economists to be engaged with normative issues of welfare, generic financial advice and product design, in addition to positive issues such as the shape and level of observed demand. Paraphrasing Sen (1995), such engagement may help insurance companies and governments avoid the danger of designing products for the poor that end up being poor products.

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A Appendix

Proof of Theorem 1. We begin with the following lemma, proved in Klibanoff et al. (2005, Lemma 8).

Lemma 2. *Let Σ be a σ-algebra over a set Δ, η be a finitely additive probability measure on Σ, and ψ be a bounded σ-measurable function $\psi : \Delta \to I$ with I an interval in $\mathbb{R}$. Suppose $\{\phi_n\}_n$ is a sequence of twice differentiable real-valued functions $\phi_n : I \to \mathbb{R}$ satisfying $\phi_n'(x) > 0$ for all $x \in I$ and with Arrow-Pratt coefficients $A_n : I \to \mathbb{R}$ such that $\lim_n \left(\inf_{x\in I} A_n\right) = +\infty$ and $A_n(x) \leq A_{n+1}(x)$ for each $x \in I$ and each n. Then*

$$\lim_{n\to\infty} \phi_n^{-1}\left(\int_\Delta \phi_n(\psi)d\eta\right) = \operatorname{ess\,inf}\psi, \tag{A-1}$$

where $\operatorname{ess\,inf}\psi = \sup\{t \in \mathbb{R} | \eta(\{x|\psi(x) < t\}) = 0\}$

Define indirect utility functions as

$$v_n(x) = \mathbb{E}u_n(x + \widetilde{z}) \text{ for all } x \in \mathbb{R}, n \in \{1, \ldots, \infty\}.$$

From condition (ii), direct utility function u_{n+1} is more risk averse than u_n in the sense of Ross and so by Theorem 3 of Ross (1981) indirect utility function v_{n+1} is more risk averse than v_n in the sense of Pratt (1964).

From condition (iii) for any $k \in \mathbb{R}$ there is some n_k such that $-u_n''(x)/u_n'(x) \geq k$ for all $n \geq n_k$, $x \in \mathbb{R}$. So, for any $n \geq n_k$ and $x \in \mathbb{R}$,

$$\begin{aligned}
-\frac{v_n''(x)}{v_n'(x)} &= \frac{\int f(z) \times [-u_n''(x+z)]dz}{\int f(z)u_n'(x+z)dz} \\
&\geq \frac{k\int f(z)u_n'(x+z)dz}{\int f(z)u_n'(x+z)dz} \\
&= k
\end{aligned}$$

and so we have $\lim_n \left(\inf_{x\in\mathbb{R}} -v_n''(x)/v_n'(x)\right) = +\infty$.

Now, defining $\Delta = S$, $\phi_n = v_n$ and $\psi_s(\alpha) = w - \theta_s + \alpha(\iota_s - qmL)$, where ι_s and θ_s denote the realised index and loss respectively, Lemma 2 implies that for $\alpha \geq 0$

$$\lim_{n\to\infty} v_n^{-1}\left(\sum_S \pi_s v_n(\psi_s(\alpha))\right) = w - L - \alpha qmL \tag{A-2}$$

since $\psi(\alpha)$ is a discrete random variable with minimum of $w - L - \alpha qmL$.

Now consider random variables $\psi(0)$ and $\psi(\alpha)$, the random wealth on purchasing 0 or α units of indexed insurance. For any $\alpha \geq 0$ by (A-2) there exists n_α large enough so that, for all $n \geq n_\alpha$,

$$\left| v_n^{-1}\left(\sum_S \pi_s v_n(\psi_s(\alpha))\right) - (w - L - \alpha qmL) \right| < w - L - (w - L - \alpha qmL).$$

Hence, since $\psi_s(0) \geq w - L$ for all $s \in S$ and so $v_n^{-1}\left(\sum_S \pi_s v_n(\psi_s(0))\right) \geq w - L$,

$$v_n^{-1}\left(\sum_S \pi_s v_n(\psi_s(0))\right) > v_n^{-1}\left(\sum_S \pi_s v_n(\psi_s(\alpha))\right), \quad \text{for all } n \geq n_\alpha,$$

which in turn implies that a decision maker with utility function u_n prefers index insurance purchase of $\alpha^* = 0$ to $\alpha^* = \alpha > 0$ for all $n \geq n_\alpha$. $\square$

Proof of Theorem 2. For both constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) the proof will take the following form. First order condition (4) will give an equation defining the interior solution α as a function of γ, denoted $\alpha^*(\gamma)$. Totally differentiating first order condition (4) with respect to γ, setting $\frac{d\alpha}{d\gamma}$ to zero, and substituting in first order condition (4) will give a necessary condition for (α, γ) pairs such that $\frac{d\alpha^*(\gamma)}{d\gamma} = 0$. Such pairs will define α as an implicit function of γ, which we will denote by $\widetilde{\alpha}(\gamma)$. $\widetilde{\alpha}(\gamma)$ will be shown to either be strictly decreasing in γ or be strictly increasing for $\gamma < \Gamma$ and strictly decreasing for $\Gamma < \gamma$ for some $-\infty < \Gamma < \infty$. $\alpha^*(\gamma)$ is continuously differentiable and $\widetilde{\alpha}(\gamma)$ is continuous, and the limits as $\gamma \to 0^+$ and $\gamma \to \infty$ may be characterised using L'Hôpital's rule. Moreover, since $\alpha^*(\gamma)$ is continuously differentiable it must have zero gradient when crossing $\widetilde{\alpha}(\gamma)$, and therefore cannot cross $\widetilde{\alpha}(\gamma)$ from below when $\widetilde{\alpha}(\gamma)$ is increasing nor from above when $\widetilde{\alpha}(\gamma)$ is decreasing. These observations are combined to derive the stated conditions for each of $m > 1$, $m = 1$ and $m < 1$.

CARA: First consider the interior solution to first order condition (4) for the case of CARA

$$\alpha^*(\gamma) = \frac{1}{\gamma L} \ln \left(\frac{A + (1-A)e^{-\gamma L}}{BCe^{-\gamma L} + B(1-C)} \right)$$

(see Lemma 1). $\alpha^*(\gamma) \leq 0$ for all γ if $A \leq B(1-C)$ which, combined with the assumption that loss and index are postively correlated ($A + C > 1$), also implies that $B > 1$. This condition may be rearranged to give $r \geq p(1 - qm)$. For the remainder of the proof we consider the case $A > B(1-C)$ for which there exist $\gamma \in (0, \infty)$ for which $\alpha^*(\gamma) > 0$.

Differentiating $\alpha^*(\gamma)$ with respect to γ, evaluating it at some $\gamma \in (0, \infty)$ such that $\frac{d\alpha^*(\gamma)}{d\gamma} = 0$, and substituting in the first order condition results in the following equation for α:

$$\widetilde{\alpha}(\gamma) = \frac{(A + C - 1)e^{-\gamma L}}{(A + (1-A)e^{-\gamma L})(Ce^{-\gamma L} + (1-C))} \quad \text{(A-3)}$$

$\widetilde{\alpha}$ is continuous with strictly positive gradient for $\gamma \in (0, \Gamma)$ and strictly negative gradient for $\gamma \in (\Gamma, \infty)$ where $\Gamma := \frac{1}{2L} \ln \left(\frac{(1-A)C}{A(1-C)} \right) \in (0, \infty)$. We also have $\lim_{\gamma \to 0^+} \frac{\partial \widetilde{\alpha}(\gamma)}{\partial \gamma} = 1$, $\lim_{\gamma \to 0^+} \widetilde{\alpha}(\gamma) = A + C - 1$ and $\lim_{\gamma \to \infty} \alpha^*(\gamma) = 0$.

For $m > 1$, the optimal level of cover is zero when $\alpha^*(\gamma) \leq 0$, that is when $\gamma \leq \gamma_1 := \frac{1}{L} \ln \left(\frac{A+BC-1}{A+BC-B} \right)$, and strictly positive for $\gamma > \gamma_1$. $B > 1$ and so $0 < \gamma_1 < \infty$. Since $\alpha^*(\gamma)$ must have zero gradient when crossing $\widetilde{\alpha}(\gamma)$, it cannot cross $\widetilde{\alpha}(\gamma)$ from below when $\widetilde{\alpha}(\gamma)$ is increasing, that is when $\gamma < \Gamma$, nor from above when $\widetilde{\alpha}(\gamma)$ is decreasing, that is when $\gamma > \Gamma$. L'Hôpital's rule gives $\lim_{\gamma \to 0^+} \widetilde{\alpha}(\gamma) - \alpha^*(\gamma) > 0$ and so $\alpha^*(\gamma)$ can only cross $\widetilde{\alpha}(\gamma)$, that is have zero gradient, for at most one γ denoted γ_2 and satisfying $\gamma_2 \geq \Gamma$. Moreover, since α^* is continuously differentiable with $\alpha^*(\gamma) \leq 0$ for $\gamma \leq \gamma_1$, $\alpha^*(\gamma) > 0$ for $\gamma > \gamma_1$, and $\lim_{\gamma \to \infty} \alpha^*(\gamma) = 0$ there must be at least one $\gamma > 0$ such that $\frac{d\alpha^*(\gamma)}{d\gamma} = 0$. Therefore γ_2 exists and is unique.

For $m = 1$, both α^* and $\widetilde{\alpha}$ tend to $A + C - 1$ in the limit as $\gamma \to 0^+$ and the gradient of the former may be positive or negative, calculated to be $[A(1-A) - C(1-C)]L$ by repeated application of L'Hôpital's Rule:

$$\begin{aligned}\lim_{\gamma\to 0^+}\frac{d\alpha^*(\gamma)}{d\gamma} &= \lim_{\gamma\to 0^+}\frac{1}{\gamma}\left[\alpha^*(\gamma)-\frac{(1-A)}{Ae^{\gamma L}+(1-A)}+\frac{C}{C+(1-C)e^{\gamma L}}\right]\\ &= \lim_{\gamma\to 0^+}\frac{1}{\gamma}\left[\alpha^*(\gamma)-\frac{(1-A)}{Ae^{\gamma L}+(1-A)}+\frac{C}{C+(1-C)e^{\gamma L}}\right]\\ &\quad+\frac{A(1-A)Le^{\gamma L}}{(Ae^{\gamma L}+(1-A))^2}-\frac{C(1-C)Le^{\gamma L}}{(C+(1-C)e^{\gamma L})^2}\\ &= [A(1-A)L-C(1-C)]L\end{aligned}$$

For $\lim_{\gamma\to\infty}\alpha^*(\gamma)=0$ to hold, if $A(1-A)\le C(1-C)$ then $\alpha^*(\gamma)$ can never cross $\widetilde{\alpha}(\gamma)$ for $\gamma>0$ and if $A(1-A)>C(1-C)$ then $\alpha^*(\gamma)$ must cross $\widetilde{\alpha}(\gamma)$ exactly once for $\gamma>0$. $\alpha^*(\gamma)$ will therefore be strictly decreasing in γ for $\gamma\in(0,\infty)$ if $A(1-A)\le C(1-C)$ or strictly increasing for $\gamma\in(0,\gamma_1)$ and strictly decreasing for $\gamma\in(\gamma_1,\infty)$ if $A(1-A)>C(1-C)$.

For $0<m<1$, L'Hôpital's rule gives $\lim_{\gamma\to 0^+}\alpha^*(\gamma)=+\infty$ and so $\alpha^*(\gamma)$ can only cross $\widetilde{\alpha}(\gamma)$, with $\frac{\partial\alpha^*(\gamma)}{\partial\gamma}=0$ zero times, once at Γ, or twice, once at $\gamma_1\le\Gamma$ and once at $\gamma_2\ge\Gamma$. If they cross zero times, α^* is strictly decreasing over $(0,\infty)$, if once α^* is strictly decreasing over $(0,\infty)$ except at $\gamma_1=\gamma_2:=\Gamma$ where it has zero gradient, and if twice α^* is strictly decreasing over $(0,\gamma_1)$ and (γ_2,∞) and strictly increasing over (γ_1,γ_2).

CRRA: Now consider the case of CRRA. For risk averse CRRA with coefficient of relative risk aversion $\gamma\in(0,\infty)$, $v'(x)\propto x^{-\gamma}$ and we may rewrite first order condition (4) as $g(\alpha,\gamma)=0$, where

$$g(\alpha,\gamma)=Aw_{LI}^{-\gamma}+(1-A)w_{0I}^{-\gamma}-B\times\left[Cw_{00}^{-\gamma}+(1-C)w_{L0}^{-\gamma}\right]. \tag{A-4}$$

and $w_s>0$ for all states $s\in S$. Since CRRA utility is undefined if wealth in any state is negative, α^* is bounded by above by $\overline{\alpha}=\frac{w-L}{qmL}$ and below by $\underline{\alpha}=-\frac{w-L}{(1-qm)L}$. $\frac{\partial g}{\partial\alpha}<0$ due to the strict concavity of the objective function and $\lim_{\alpha\to\overline{\alpha}}g(\alpha,\gamma)=-\infty$ and $\lim_{\alpha\to\underline{\alpha}}g(\alpha,\gamma)=+\infty$ for all $\gamma\in(0,\infty)$ and so (A-4) defines α as an implicit function of γ, which we denote by $\alpha^*:\mathbb{R}^+\to(\underline{\alpha},\overline{\alpha})$. α^* is bounded and g is continuous in both its arguments and so α^* is continuous. Moreover, $\frac{\partial g}{\partial\gamma}$ is finite for $\alpha\in(\underline{\alpha},\overline{\alpha})$ and $\frac{\partial g}{\partial\alpha}<0$ and so $\alpha^*(\gamma)$ is continuously differentiable.

As for the case of CARA, $\alpha^*(\gamma)\le 0$ for all $\gamma>0$ if $A\le B(1-C)$, that is if $r\ge p(1-qm)$. This can be seen by noting that $r\ge p(1-qm)$ can only hold along with correlation equation (2) if $m>1$, in which case the sum of the first two terms of g has smaller magnitude than the sum of the second two terms for $\alpha\ge 0$. For the remainder of the proof we consider the case $A>B(1-C)$ for which there exist $\gamma\in(0,\infty)$ for which $\alpha^*(\gamma)>0$.

For small positive γ we may apply the Taylor expansion for e^x to FOC (A-4) to give:

$$\frac{1-B}{\gamma}=A\ln(w_{LI})+(1-A)\ln(w_{0I})-BC\ln(w_{00})-B(1-C)\ln(w_{L0})+O(\gamma) \tag{A-5}$$

The RHS of (A-5) is strictly increasing in α and so for the first order condition to hold we must have:

$$\lim_{\gamma\to 0^+}\alpha^*(\gamma)\begin{cases}=\underline{\alpha}<0 & \text{for } m>1, B>1\\ \in(\underline{\alpha},\overline{\alpha}) & \text{for } m=1, B=1\\ =\overline{\alpha}>0 & \text{for } m<1, B<1\end{cases} \tag{A-6}$$

Further,

$$\lim_{\gamma\to\infty}\alpha^*(\gamma) = 0^+ \tag{A-7}$$

for the following reason. (A-4) contains four terms, each with wealth in the respective state raised to the power of $-\gamma$. As $\gamma \to \infty$ the two terms with lowest wealth must dominate and for (A-4) to hold they must have opposite sign with sum approximately equal to zero. For any α, only the terms corresponding to states $L0$ and LI can have this property. We must therefore have $\lim_{\gamma\to\infty}\left(\frac{w_{L0}+\alpha L}{w_{L0}}\right) \times \left(\frac{B(1-C)}{A}\right)^{\frac{1}{\gamma}} = 1$. Since $A > B(1-C)$, the second term in brackets tends to unity from below and so the first term in brackets must tend to unity from above, with α also tending to zero from above.

Totally differentiating the first order condition with respect to γ at the optimal (unconstrained) α gives

$$\left.\frac{\partial g}{\partial \gamma}\right|_{\alpha=\alpha^*} + \left.\frac{\partial g}{\partial \alpha}\right|_{\alpha=\alpha^*} \times \frac{d\alpha^*(\gamma)}{d\gamma} = 0, \tag{A-8}$$

where

$$\begin{aligned}\frac{\partial g}{\partial \gamma} &= -A\ln(w_{LI})w_{LI}^{-\gamma} - (1-A)\ln(w_{0I})w_{0I}^{-\gamma} \\ &\quad + B \times \left[C\ln(w_{00})w_{00}^{-\gamma} + (1-C)\ln(w_{L0})w_{L0}^{-\gamma}\right],\end{aligned} \tag{A-9}$$

and since $\frac{\partial g}{\partial \alpha} < 0$, it must be that $\left.\frac{\partial g}{\partial \alpha}\right|_{\alpha=\alpha^*}$ and $\frac{d\alpha^*(\gamma)}{d\gamma}$ have the same sign.

Substituting in the FOC and multiplying by w_{00}^{γ} gives

$$\begin{aligned}h(\alpha,\gamma) &:= w_{00}^{\gamma}\left[\frac{\partial g}{\partial \gamma} + g \times \ln(w_{0I})\right] \\ &= A\ln\left(\frac{w_{0I}}{w_{LI}}\right)\left(\frac{w_{00}}{w_{LI}}\right)^{\gamma} - BC\ln\left(\frac{w_{0I}}{w_{00}}\right) \\ &\quad - B(1-C)\ln\left(\frac{w_{0I}}{w_{L0}}\right)\left(\frac{w_{00}}{w_{L0}}\right)^{\gamma}\end{aligned} \tag{A-10}$$

$h(\alpha,\gamma) = 0$ is a necessary condition for α,γ pairs such that $\frac{d\alpha^*}{d\gamma} = 0$ for $\gamma \in (0,\infty)$.

h is strictly decreasing in α for $\alpha \in [0,\overline{\alpha})$ and

$$\begin{aligned} h(0,\gamma) > 0\ \forall\gamma \in (0,\infty) \\ \lim_{\alpha\to\overline{\alpha}} h(\alpha,\gamma) = -\infty\ \forall\gamma \in (0,\infty),\end{aligned} \tag{A-11}$$

with the first inequality from our restriction to cases with $A > B(1-C)$, and so equation (A-10) defines α such that $\frac{d\alpha^*}{d\gamma} = 0$ as an implicit function of γ, which we denote by $\widetilde{\alpha} : \mathbb{R}^+ \to (0,\overline{\alpha})$. $\widetilde{\alpha}$ is bounded and h is continuous in both its arguments and so $\widetilde{\alpha}$ is continuous.

We may show that h is first strictly increasing then strictly decreasing in γ for $\alpha \in [0,1)$, and strictly decreasing in γ for $\alpha \geq 1$. Denoting $y_\alpha = \ln\left(\frac{w_{00}}{w_{LI}}\right)$, $z_\alpha = \ln\left(\frac{w_{00}}{w_{L0}}\right)$, $X_\alpha = BC\ln\left(\frac{w_{0I}}{w_{00}}\right)$, $Y_\alpha = A\ln\left(\frac{w_{0I}}{w_{LI}}\right)$ and $Z_\alpha = B(1-C)\ln\left(\frac{w_{0I}}{w_{L0}}\right)$, we may rewrite h as

$$h(\alpha,\gamma) = Y_\alpha e^{y_\alpha\gamma} - X_\alpha - Z_\alpha e^{z_\alpha\gamma} \tag{A-12}$$

where $z_\alpha \geq y_\alpha > 0$ for $\alpha \in [0,1)$ with strict inequality when $\alpha = 0$, and $z_\alpha > 0$, $y_\alpha \leq 0$ for $\alpha \geq 1$. The gradient of (A-12) with respect to γ is given by

$$\frac{\partial h}{\partial \gamma} = \left(y_\alpha Y_\alpha - z_\alpha Z_\alpha e^{(z_\alpha - y_\alpha)\gamma} \right) e^{y_\alpha \gamma}. \tag{A-13}$$

For the case of $\alpha \geq 1$ the first term is nonpositive and the second is strictly negative, and so $\frac{\partial h}{\partial \gamma}$ is strictly negative. For $\alpha \in [0,1)$, $\frac{\partial h}{\partial \gamma}$ is strictly positive for $\gamma < \Gamma$ and strictly negative for $\gamma > \Gamma$ where

$$e^\Gamma = \left(\frac{z_\alpha Z_\alpha}{y_\alpha Y_\alpha} \right)^{\frac{1}{z_\alpha - y_\alpha}} \tag{A-14}$$

For $m > 1$, the optimal level of cover is zero when $\alpha^*(\gamma) \leq 0$, that is when $\gamma \leq \gamma_1$, and strictly positive for $\gamma > \gamma_1$ where

$$\frac{A + BC - B}{(w-L)^{\gamma_1}} - \frac{A + BC - 1}{w^{\gamma_1}} = 0$$
$$\therefore \gamma_1 := \ln\left(\frac{A + BC - 1}{A + BC - B} \right) \Big/ \ln\left(\frac{w}{w-L} \right). \tag{A-15}$$

$B > 1$ and so $0 < \gamma_1 < \infty$. Equations (A-6) and (A-11) imply that $\lim_{\gamma \to 0^+} \widetilde{\alpha}(\gamma) - \alpha^*(\gamma) > 0$. The remainder of the proof for this case follows that for the case of $0 < m < 1$ for CARA.

For $m = 1$, $\lim_{\gamma \to 0^+} \alpha^*(\gamma) - \widetilde{\alpha}(\gamma) = 0$ since (A-5) and (A-10) become arbitrarily close for any $\alpha \in [0, \overline{\alpha})$ as $\gamma \to 0^+$. Following the case of $m = 1$ for CARA, if $\lim_{\gamma \to 0^+} \frac{d\alpha^*(\gamma)}{d\gamma} \geq 0$ then $\alpha^*(\gamma)$ can never cross $\widetilde{\alpha}(\gamma)$ for $\gamma > 0$ and otherwise $\alpha^*(\gamma)$ must cross $\widetilde{\alpha}(\gamma)$ exactly once for $\gamma > 0$. The remainder of the proof for this case follows that for the case of $m = 1$ for CARA.

If negative then $\alpha^*(\gamma)$ can never cross $\widetilde{\alpha}(\gamma)$ for $\gamma > 0$ and if positive it must cross $\widetilde{\alpha}(\gamma)$ if it is to satisfy $\lim_{\gamma \to \infty} \alpha^*(\gamma) = 0$. $\alpha^*(\gamma)$ will therefore be strictly decreasing in γ for $\gamma \in (0, \infty)$ if $A(1-A) \leq C(1-C)$ or strictly increasing for $\gamma \in (0, \gamma_1)$ and strictly decreasing for $\gamma \in (\gamma_1, \infty)$ if $A(1-A) > C(1-C)$.

For $0 < m < 1$, equations (A-6) and (A-11) imply that $\lim_{\gamma \to 0^+} \widetilde{\alpha}(\gamma) - \alpha^*(\gamma) < 0$. The remainder of the proof for this case follows that for the case of $0 < m < 1$ for CARA.

□

Proof of Theorem 4. Indirect utility function v inherits strict risk aversion and decreasing absolute risk aversion from u.[3] For $0 \leq \alpha \leq 1$ and any risk averse indirect utility function v that satisfies DARA there exist constants $\gamma_1 \geq \gamma_2 \geq \gamma_3 > 0$ such that $v'_{LI}(\gamma_1, \gamma_2, \gamma_3, \alpha) = v'_{L0}(\alpha) e^{-\gamma_1 \alpha L}$, $v'_{00}(\gamma_1, \gamma_2, \gamma_3, \alpha) = v'_{L0}(\alpha) e^{-\gamma_1 \alpha L - \gamma_2 (1-\alpha) L}$ and $v'_{0I}(\gamma_1, \gamma_2, \gamma_3, \alpha) = v'_{L0}(\alpha) e^{-\gamma_1 \alpha L - \gamma_2 (1-\alpha) L - \gamma_3 \alpha L}$. Substituting these into first-order condition (4) and rearranging gives:

$$e^{-\gamma_1 \alpha L} v'_{L0}(\alpha) \left(A + (1-A) e^{-\gamma_2 (1-\alpha) L - \gamma_3 \alpha L} - BC v'_{L0} e^{-\gamma_2 (1-\alpha) L} \right)$$
$$- B(1-C) v'_{L0}(\alpha) = 0 \tag{A-16}$$

The LHS of (A-16) is strictly decreasing in α due to the strict concavity of objective function (3). Morover, it is strictly decreasing in both γ_3 and γ_1 at the optimal level of cover $\alpha^*(\gamma_1, \gamma_2, \gamma_3)$. Setting γ_1 and γ_3 to be as low as possible, that is equal to γ_2 and 0 respectively, will therefore maximise the optimal cover α^*.

Denoting $0 \leq \Gamma = e^{-\gamma_2 L} \leq 1$, equation (A-16) becomes

$$A\Gamma^{\bar{\alpha}} - (A + BC - 1)\Gamma - B(1 - C) = 0, \tag{A-17}$$

and the requirement that Γ is chosen optimally becomes

$$-\bar{\alpha}A\Gamma^{\bar{\alpha}-1} + (A + BC - 1) = 0. \tag{A-18}$$

Rearranging equations (A-17) and (A-18) and recalling that $0 \leq \Gamma \leq 1$ gives:

$$0 \leq \left(\frac{B(1-C)}{1-\bar{\alpha}}\right)^{\frac{1}{\bar{\alpha}}} = \Gamma = \frac{\bar{\alpha}B(1-C)}{(A+BC-1)(1-\bar{\alpha})} \leq 1$$

which may be rearranged to give equation (7). Now, all that remains to be proven is that there is a unique solution to equation (7). Taking the natural logarithm of both sides of equation (7) and rearranging gives:

$$H(\bar{\alpha}) = [\bar{\alpha}\ln(\bar{\alpha}) + (1-\bar{\alpha})\ln(1-\bar{\alpha})] - \left[\ln\left(\frac{B(1-C)}{A}\right) + \bar{\alpha}\ln\left(\frac{A+BC-1}{B(1-C)}\right)\right] = 0 \tag{A-19}$$

$H(\alpha)$ is strictly convex for $0 < \alpha \leq \frac{A+BC-1}{A+B-1} < 1$ and therefore to prove uniqueness it is sufficient to show that $\lim_{\alpha\to 0^+} H(\alpha) > 0$ and $H(\frac{A+BC-1}{A+B-1}) \leq 0$. Now, using L'Hôpital's rule, $\lim_{\alpha\to 0^+} H(\alpha) = \ln\left(\frac{A}{B(1-C)}\right) > 0$ since $r < p(1-qm)$. Finally, $H(\frac{A+BC-1}{A+B-1}) = \ln\left(\frac{A}{A+B-1}\right) \leq 0$ since $B \geq 1$.

$\square$

MICROINSURANCE DECISIONS: EVIDENCE FROM ETHIOPIA

DANIEL J. CLARKE* GAUTAM KALANI†

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Abstract

We review experimental evidence collected from a framed microinsurance lab experiment using poor subjects in rural Ethiopia. In line with results from recent field experiments the demand for index insurance is increasing in wealth and decreasing with risk aversion at low levels of wealth. However, this relationship is nonlinear, consistent with the hump-shape prediction of decreasing absolute risk averse expected utility theory (DARA EUT). In contrast with recent field experiments where it is not possible to demonstrate that low demand for indexed insurance is 'too low', use of a laboratory experiment with an objectively known joint probability distribution allows normative statements to be made about the observed level of demand. The observed level of demand for index insurance in the experiment is higher than the DARA EUT upper bound, strengthening the case for research methods that allow comparison between the observed demand for indexed insurance products and demand which would be consistent with objective financial advice, without which it is not possible to make normative statements about the level of observed demand.

JEL codes: C91, D14, G22, O16.

*DPhil student: Department of Economics, Centre for the Study of African Economies ('CSAE') and Balliol College, University of Oxford (**clarke@stats.ox.ac.uk**; http://www.stats.ox.ac.uk/~clarke). This paper forms part of my DPhil thesis, which is supervised by Professors Sujoy Mukerji and Stefan Dercon; the work would not have been possible without their very generous assistance, nor that of Rocco Macchiavello and Abigail Barr, who offered useful suggestions throughout the project. A number of others have provided very useful comments on aspects of the paper; without implicating them in the shortcomings of the work, we thank Erlend Berg, Marcel Fafchamps, Glenn Harrison, James Martin, Catherine Porter, Simon Quinn, Amber Tomas, Ruth Vargas Hill and Liam Wren-Lewis. Solomon Ambessie, Zewdu Taddesse Demeke and Solomon Kidane provided invaluable assistance in conducting the experiment. I have presented the paper at the CEPR Development Economics Symposium 2009 Pre-Conference, a CESS Seminar and the CSAE Conference 2011. Financial support from the Microinsurance Innovation Facility and the Economic and Social Research Council (ESRC) and logistical support from the Centre for the Study of African Economies (CSAE) and the Ethiopian Development Research Institute (EDRI) are gratefully acknowledged.

†DPhil student: Department of Economics, Centre for the Study of African Economies ('CSAE') and Merton College, University of Oxford (**gautam.kalani@economics.ox.ac.uk**)

1 INTRODUCTION

Over the last ten years a variety of institutions have piloted the sale of weather indexed insurance policies to poor farmers, under which the net transfer between insurer and policyholders depends only on readings from a contractual weather station. However, despite the substantial welfare benefits that could arise from improved agricultural risk management, voluntary purchase of these products has been much lower than anticipated by proponents. This low demand has been referred to as a 'puzzle' in need of an explanation (Cole et al. 2009, Karlan and Morduch 2009).

Of course, to be able to say anything meaningful about whether observed index insurance purchase is 'too low', rather than just saying that it is 'low', we must at the very least be able to argue that a well-informed financial advisor would advise a higher level of purchase. This turns out to be surprisingly difficult to do, particularly for real index insurance products where the contractual index is not perfectly correlated with the loss, and neither researcher nor consumer has a precise objective estimate of the joint probability distribution of losses and index claim payments (Clarke 2011). Optimal demand from an expected utility maximiser is highly sensitive to both the expected claim payment and the claim payment distribution conditional on a large loss having occurred. Even if a researcher had, say, 20 years of matched data for both losses and index claim payments, the latter conditional distribution could not be objectively estimated with any degree of accuracy. In practice researchers are likely to have relevant matched data for significantly fewer than 20 years, rendering normative statements impossible.

If it is not possible to learn about the level of index insurance demand through natural field experiments with real index insurance products, we must instead look to environments in which the researcher has a greater degree of control. This paper reports on such an insurance experiment conducted in rural Ethiopia in which both index and losses were generated with known joint probability distribution.

While the use of a laboratory experiment entails a tradeoff between control and realism, we attempted to maximize external validity with decision problems framed as agricultural insurance purchase problems, payoffs of up to one week's income and an experimental design that yielded clear theoretical predictions. Moreover, subjects were chosen from households in the Ethiopian Rural Household Survey (ERHS), some of whom would be offered real weather indexed insurance policies in the subsequent two years as part of a pilot project conducted by the International Food Policy Research Institute (IFPRI) and the University of Oxford.

Drawing on best practice and based on extensive piloting, decision problems were designed to be easily understood by subjects, project choices and payoffs were described orally with the help of visual aids, randomisation devices were physical and generated salient probabilities using familiar mechanisms, session money was physical, and understanding was confirmed and tested throughout the session (Barr and Genicot 2008, Fischer 2010).

We considered both indemnity insurance, where the net transfer is a function of incurred losses, and index insurance, where the net transfer is correlated with, but not a function of, incurred losses. By considering both product types we are able to determine whether low demand for index cover can be explained by the overweighting of basis risk, that is, the risk that the net indexed transfer from insurer to policyholder does not match the incurred loss.

This experiment generated interesting results. Broadly speaking the *shape* of demand for insurance was consistent with decreasing absolute risk averse expected utility theory (DARA EUT). We find evidence for a hump-shaped relationship between index insurance purchase and wealth, with lower demand from the poorest and the richest and highest demand from subjects with intermediate levels of wealth. This hump shape is consistent with DARA EUT (Clarke 2011) and the finding that demand for index insurance increases with wealth at low levels of wealth is in line with the findings of recent field experiments on index insurance conducted by Giné et al. (2008) and Cole et al. (2009) in rural India. Results from the indemnity insurance and benchmark decision problems are also consistent with DARA EUT, with (indirect) risk aversion over game payoffs decreasing with wealth (Pratt 1964) and increasing with background risk (Gollier and Pratt 1996). Unlike Cole et al. (2009), we do not find strong evidence that schooling, understanding of the decision problems, or financial literacy increase index or indemnity insurance take-up.

However, the *level* of demand for index insurance is not consistent with DARA EUT. Specifically, in the index insurance decision problem, 66% of subjects purchased more insurance than would be optimal for *any* DARA EUT decision maker. At first glance this appears to be the opposite of what has been found in index insurance field experiments; however, this is not an appropriate comparison for two reasons. First, unlike recent field experiments the present laboratory experiment has been designed with the specific aim of allowing normative statements to be made about the level of index insurance purchase. Whilst observed demand in field experiments has been low there has been no convincing attempt to demonstrate that it is 'too low'. It may well be rational for demand for the products reported in Cole et al. (2009) to be low, and by extension the 'low' observed demand may be 'about right' or even 'too high' rather than 'too low' (Clarke 2011).[1] Second, our result is consistent with the results of traditional laboratory experiments framed in the abstract and with samples drawn from developing countries, which suggest an 'S-shaped' probability weighting function (Humphrey and Verschoor 2004a, Humphrey and Verschoor 2004b). For our decision problems, an 'S-shaped' probability weighting corresponds to underestimating the effect of basis risk and overpurchasing index insurance. If poor farmers are likely to underweight basis risk in their decision rules, we would

[1] Cole et al. (2009) use historical weather data for the products they report on to estimate that for every Rs. 1 of farmer premium, the expected claim payment to farmers is between Rs. 0.19 and 0.59. Combined with plausible estimates for the degree of basis risk, optimal demand from a constant relative risk averse (CRRA) EUT decision maker is close to zero.

expect observed demand for index insurance policies from poor farmers to be higher, not lower than that which would be advised by a well-informed financial advisor.

The rest of this paper is organised as follows. Section 2 presents the experimental design, including discussion of the decision problems and subjects. Section 3 analyzes the correlates of index insurance take-up, indemnity insurance take-up and risk aversion. Section 4 presents the comparison of the take-up in the experiment to the optimal level of take-up under EUT, and also compares with predictions of Quiggin's (1982) theory of Rank Dependent Utility (RDU). Section 5 outlines robustness checks and Section 6 concludes.

2 Experimental design

The following experiment was designed to examine the demand for different types of insurance. In each session, subjects made three decisions and at the end of the session played, and were paid for, only one of the three decision problems; in addition, all participants were payed a show-up fee of 5 birr. Each subject randomly selected the decision problem they would play (and be paid for) by choosing one out of three numbered tokens placed face down on a table. The daily wage for casual farm labour in the areas we ran the experiment was between 15 and 20 birr (1.2 to 1.6 USD). Minimum and maximum earnings in the experiment were 5 and 80 birr and mean realised earnings, including the show-up fee, was 40 birr.

The experiment included a benchmark decision problem, framed in the abstract, and four framed insurance decision problems. Only the benchmark and two of the four insurance decision problems are analysed in this paper. All 378 subjects played the benchmark decision problem, 258 played the index insurance decision problem, and 136 of these 258 also played the indemnity insurance decision

Table 1. *Individual Indemnity insurance purchase decision (T_M) and Benchmark decision (B)*

	Premium Choice (T_M)	Equivalent Choice in Benchmark (B)	Net Payoff (Ethiopian Birr)		Expected Payoff	Risk Aversion Range (CRRA)*
Loss:			50	0		
Probability:			$1/2$	$1/2$		
	0	F	15	65	40	$(-\infty, 1.036)$
	8	E	17	57	37	$(1.036, 1.285)$
	16	D	19	49	34	$(1.285, 1.715)$
	24	C	21	41	31	$(1.715, 2.698)$
	32	B	23	33	28	$(2.698, 8.553)$
	40	A	25	25	25	$(8.553, +\infty)$

* Risk aversion range denotes range of coefficients for which choice would be optimal for a subject with CRRA preferences over earnings from experiment, excluding show-up fee.

problem.[2] In total there were three types of sessions and each set of three problems was presented in two different orders, so as to enable control for order effects.

2.1 Benchmark

The benchmark decision problem (B) was as follows. Each subject was presented with a choice of six lotteries, shown in each row of Table 1. Alternatives were ordered to be increasing in both the average payoff and the variance around that payoff. Alternative A is the safe option, offering a certain amount, and alternative F has the highest payoff mean and variance. Following Barr and Genicot (2008) the gamble was framed in the gain domain and, whichever gamble was chosen, the payoff was determined by playing a game that involved guessing which of the author's hands contained a blue rather than a yellow counter. The decision problem was explained privately to each subject, who made a private decision. After making a decision, subject were seated separately and were not allowed to talk to each other.

[2] The two insurance decision problems not considered in this paper are group versions of the individual index and indemnity insurance decision problems described here, where subjects are randomly paired and aggregate losses are split between the pair. Clarke and Kalani (2011) contains a full description of the experiment.

This decision problem uses the Ordered Lottery Selection design of Binswanger (1980, 1981) to elicit risk preferences. Whilst alternative methodologies have become popular in recent years for experiments with standard samples (Harrison and Rutström 2008), the simplicity of the Ordered Lottery Selection design makes it well suited to nonstandard samples with low levels of formal education (Barr and Genicot 2008, Van Campenhout et al. 2008).

2.2 Insurance decision problems

All insurance purchase decisions were framed to be as similar as possible to a real insurance purchase decision, albeit in the controlled environment of the lab, with an objective probability structure, and with more time spent explaining and individually confirming understanding than would occur in the marketing process for a real product.

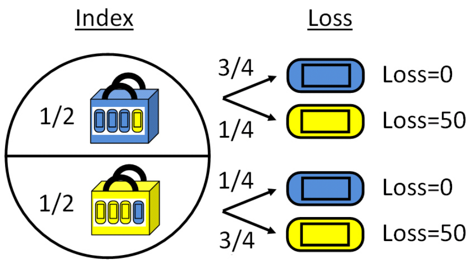
At the start of each insurance purchase decision problem, subjects were physically given 65 birr of game money and told that they might lose 50 birr. As referred to earlier, 50 birr was equivalent to between two and three days casual farm labour in the experimental sites.[3] Game money was smaller and more brightly coloured than Ethiopian currency, but was otherwise recognisably similar.

Enumerators spent 20 minutes explaining each insurance decision problem to the group of subjects, with an additional 10-20 minutes spent privately confirming understanding and recording decisions. Following common practice, we referred to both indexed and indemnity insurance as insurance, rather than referring to the former as a derivative.

Subjects were randomly partitioned into pairs and insurance purchase decisions

[3] For households in our experimental sites, the average monthly per household consumption was $1,520$ birr, a little higher than the $1,113$ birr average for the full ERHS sample. Accounting for household size, the average monthly per capita consumption in our experimental sites was 300 birr, varying from 181 birr in Indibir to 429 birr in Sirbana Godeti. 50 birr therefore represented between 28% and 12% of per capita monthly consumption.

Figure 1. *Two-Stage Probabilistic Structure for Insurance Decision Problems*

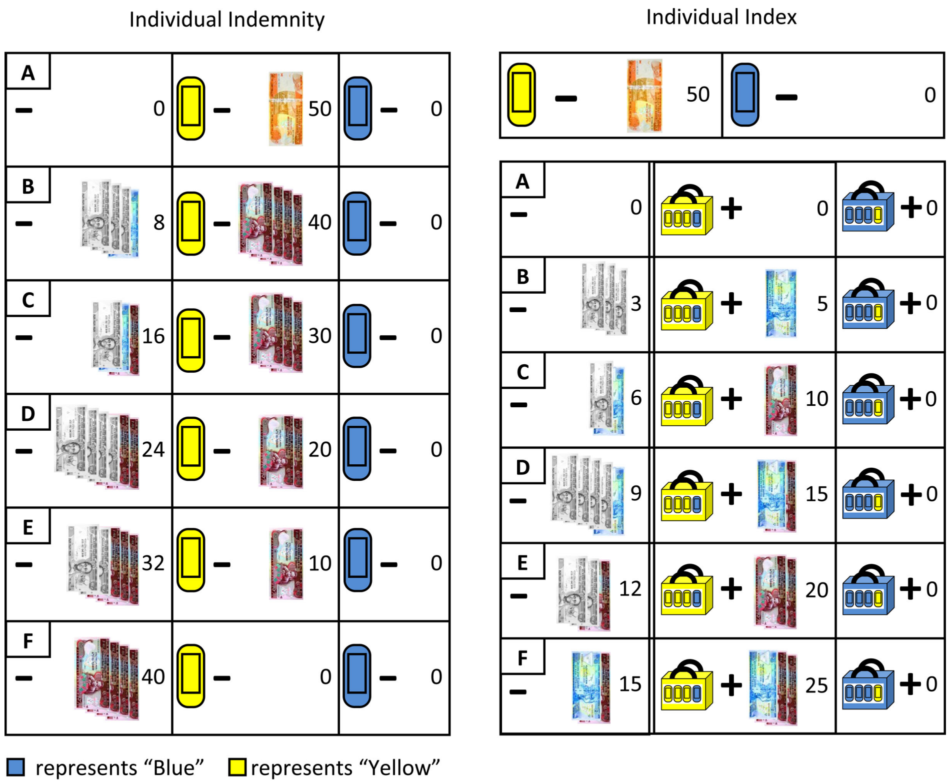


shared the following two-stage probability structure (see Figure 1). First, a fair wheel was spun to determine whether the blue or yellow bag would be used for the pair of subjects. The blue bag contained three blue tokens and one yellow, and the yellow bag contained three yellow tokens and one blue. Second, each member of the pair chose one token from the selected bag, with replacement. An outcome for a pair therefore comprised a bag and two tokens. For the decision problems analysed in this paper the pairs served no function, except that individuals were given the opportunity to ask questions of confirmation in pairs, and so subjects could hear their partner's questions of confirmation.

In addition to being given an explanation in terms of the wheel, bags and tokens, subjects were given an explanation for the probability structure in a real-world agricultural context. Subjects were told that the bags could be thought of as the weather, with the blue bag representing good weather and the yellow bag representing poor weather. The tokens were likened to the actual yield on a plot, with a blue token representing a good year (in terms of yield) for the owner and a yellow token representing a bad year. Bad (good) weather was likely to lead to a bad (good) year for the owner, but this was not always the case: there was one yellow token in the blue bag and one blue token in the yellow bag.

Given this probability structure, the treatments may be briefly summarised as follows. If a subject drew a yellow token they lost 50 birr. Indemnity insurance then corresponded to purchasing insurance against the drawing of a yellow token, and index insurance corresponded to purchasing insurance against the drawing of

Figure 2. *Presentation of the insurance decision problems to the experimental subjects*



a yellow bag. Indemnity insurance was priced with a loading of 60% and index insurance with a loading of 20%.[4] A complete description of the two insurance decision problems is as follows, with respective visual aids displayed in Figure 2.

Individual Indemnity (T_M): In the individual indemnity decision problem T_M a subject incurred a 50 birr loss if a yellow token was drawn, but could purchase between zero and five units of individual indemnity insurance against the loss occuring. One unit of indemnity insurance cost a premium of 8 birr and reduced the retained loss on drawing a yellow token by 10 birr. Each subject could therefore pay 0, 8, 16, 24, 32 or 40 birr to reduce the maximum loss to 50, 40, 30, 20, 10 or 0 birr, respectively (see Table 1). The gamble choices available to

[4] An insurance loading is defined as (Premium Charged)/(Expected Claim Income) $-$ 1. Loadings of 20% and 60% are low compared to reported commercial loadings for crop insurance, ranging from 70% to 430% for weather indexed insurance (Cole et al. 2009, Table 1) and 140% to 470% for indemnity insurance (Hazell 1992, Table 1). However, the 50% probability of claim payment in our experiment is much higher than that for commercial insurance products, and so these loadings cannot be directly compared. A higher loading reduces demand for either index or indemnity insurance by CRRA or RDU decision makers.

Table 2. *Individual Index insurance purchase decision (T_X)*

	Premium Choice	Net Payoff (Ethiopian Birr)				Expected Payoff	Risk Aversion Range (CRRA)
Index:		Good	Bad	Good	Bad		
Loss:		50	50	0	0		
Probability:		$1/8$	$3/8$	$3/8$	$1/8$		
	0	15	15	65	65	40	$(-\infty, 0.723)$ and $(3.888, +\infty)$
	3	12	17	62	67	39.5	$(0.723, 3.888)$
	6	9	19	59	69	39	N/A
	9	6	21	56	71	38.5	N/A
	12	3	23	53	73	38	N/A
	15	0	25	50	75	37.5	N/A

individuals in T_M were therefore numerically identical to those in B. However the framing of the choices was significantly different.

Individual Index (T_X): In the individual index insurance decision problem T_X, instead of being able to insure against drawing a yellow token (crop loss), subjects could only purchase between zero and five units of index insurance against a yellow bag being selected. One unit of index insurance cost a premium of 3 birr and led to a claim payment of 5 birr in the event of the yellow bag being selected, and zero otherwise (see Table 2). When describing T_X, substantial emphasis was placed on the 1 in 8 chance of incurring a 50 birr crop loss despite the weather being good and therefore no claim payment being due (that is, the downside basis risk).

The compound probability structure for insurance treatments was much more complex than the simple fair draw for the benchmark. Whilst experimental economists might argue that this could be too complex for subjects to understand, we consider it to be much less difficult to understand than the joint probability structure for a real weather indexed insurance policy; although a farmer might have a good understanding of the marginal loss distribution for their farm, they are unlikely to have a good understanding of the conditional distribution of weather indexed claim payments (Giné et al. 2007, Hill and Nobles 2010). Moreover, each stage of the randomisation device was chosen to have salient probabilities of $1/4$, $1/2$ and $3/4$.

2.3 Experimental sample and matched ERHS data

Our experiment involved 378 subjects from seven sites of the Ethiopian Rural Household Survey (ERHS), spanning three regions of the country.[5] In addition to subjects' choices in the various microinsurance decision problems, the experimental dataset contains information on literacy, schooling, financial literacy, understanding of the decision problems, occupation and various other demographic characteristics of the subjects. The ERHS dataset consists of seven rounds of detailed longitudinal data on various socioeconomic and demographic characteristics of households in the sample, such as asset ownership, income, consumption, membership in risk-sharing groups and household size.[6] It has been used extensively in studies regarding various aspects of the rural Ethiopian economy (e.g. Dercon and Krishnan 2000, Dercon 2004, Fafchamps and Quisumbing 2005). The combined dataset containing experimental and ERHS data provides a fairly extensive set of factors which could help explain the choices of subjects in the decision problems, and which could also be used to test which factors affect risk aversion and microinsurance take-up in rural Ethiopia.

[5] The seven ERHS sites chosen for the experiment were Sirbana Godeti, Korodegaga, Indibir, Milki, Komargefia, Karafino and Bokafia.

[6] The ERHS dataset is a longitudinal household dataset covering 19 villages and nearly 1600 households in rural Ethiopia, and the survey was conducted over seven rounds from 1994 to 2009. There were seven original sites chosen for the ERHS in 1989 because of the drought that affected Ethiopia in 1984-85; the rest were added in 1994 to achieve broad diversity of rural Ethiopia. The data collection was coordinated by the Economics Department at Addis Ababa University in collaboration with the Centre for the Study of African Economies at Oxford University and IFPRI. For the purpose of this paper, we use data mostly from the latest (seventh) round of the survey, conducted between April and August 2009, since the timing closely matches that of this experiment, which was conducted during November and December 2009. However, we also utilize certain panel aspects of the data, as will be described in the next section. For details on the survey, see Dercon and Krishnan (1998).

3 Correlates of insurance take-up and risk aversion

3.1 Theory

As is well known, optimal demand for the indemnity insurance in T_M is increasing in risk aversion (Pratt 1964). The benchmark decision problem B was constructed to be numerically identical to T_M, with a lower choice of letter corresponding to greater risk aversion (Table 1).

However optimal demand for index insurance is fundamentally different to that of indemnity insurance; the optimal demand in problem T_X is not monotonic, but rather hump-shaped in risk aversion, that is, first increasing and then decreasing in risk aversion (Clarke 2011). This hump shape is caused by the combination of actuarially unfair premiums, whereby premiums are greater than expected claim income, and basis risk, the risk that the income from index insurance will not accurately reflect the incurred loss. These factors cause demand to be low from both the risk neutral, for whom insurance purchase decreases mean income, and the very risk averse, for whom index insurance purchase decreases the minimum possible income. Only those with intermediate levels of risk aversion optimally purchase index insurance.

Further, EUT also provides predictions about the effect of wealth on the purchase of index insurance. Wealth affects index insurance purchase in two important ways. First, a household's wealth is indicative of the credit and liquidity constraints it faces, and financial constraints may play a key role in its decision to purchase insurance in the field (Giné et al. 2008). Second, household wealth is an important determinant of risk aversion, which in turn affects take-up of insurance. In an experimental setting where each subject is given 65 birr and the option to purchase insurance to offset adverse outcomes, the credit constraint effect of wealth is not expected to impact insurance take up, and wealth would only affect the purchase of index insurance through risk aversion. If subjects' preferences over aggregate wealth satisfy decreasing absolute risk aversion then the hump shape of demand relative to risk aversion is expected to carry over to wealth; both poor subjects (those with high risk aversion) and rich subjects (those with low risk aversion) would have low demand for index insurance due to basis risk and actuarially unfair premiums respectively, leaving higher demand only for those with intermediate wealth (Clarke 2011). Predictions from EUT differ from those arising from the mean variance model of Giné et al. (2008), under which purchase of index insurance takeup is expected to be monotonically increasing with risk aversion.

To test these theoretical predictions we include a measure of wealth and its square as determinants of index insurance take-up in T_X, that is, as explanatory variables in specifications with the respondent choice in T_X as the dependent variable.[7] If the coefficient estimate on the level term is significantly positive and that on the squared term is significantly negative, we can reject the hypothesis that index insurance take-up in T_X is monotonically decreasing in wealth, and this is indicative of the hump-shaped relationship between take-up and wealth.

For a particular decision problem, all subjects face the same risky outcomes and are given the same choices. However, subjects differ significantly in the risk they face in the natural course of their lives (outside of the experiment). Subjects' earnings in the experiments are statistically independent of this background risk and so if subjects have risk vulnerable preferences then, following Gollier and Pratt (1996), we may expect to associate a higher level of background risk with a higher level of indirect risk aversion over earnings from the experiment,

[7] Clarke (2011) also predicts that index insurance take-up is hump-shaped in risk aversion, and the impact of wealth on take-up in this model is driven purely by risk aversion. Though we can estimate the coefficient of relative risk aversion r from participants' choices in B, we cannot directly test for the existence of a hump-shaped relationship between index insurance take-up and r. This is because each choice in B corresponds to a range of implied r, and the only way to reduce the ranges to point values is to use a rather arbitrary averaging technique, as suggested by Binswanger (1981). In addition, including the level and squared terms of this discrete, quite arbitrarily chosen point value of r to test the nonlinear relationship seems inappropriate. Therefore, we prefer to test the prediction of Clarke (2011) that the relationship between index insurance take-up and the continuous variable wealth is hump-shaped. Further, given that we only expect the impact of wealth on take-up to be through risk aversion, testing the relationship between index insurance take-up and wealth can provide strong implications for the relationship between take-up and risk aversion.

and therefore a lower level of risk taken in T_M and B. In this paper, we are particularly interested in decision makers with utility functions that satisfy decreasing absolute risk aversion, a normatively robust framework for rational decision making (Pratt 1964, Arrow 1965) and sufficient condition for risk vulnerability (Gollier and Pratt 1996). To test the hypothesis that greater background risk is associated with more risk aversion over earnings from the experiment (and thus safer choices in the decision problems), we include a measure of the background risk faced by the participant as a determinant of respondent choice in the decision problems.

Following Giné et al. (2008), Cole et al. (2009) and Hill et al. (2010), we also include controls for membership in social groups and access to risk sharing networks, cognitive ability and understanding of the decision problems, and demographic characteristics of the participants, such as age and sex.

For each of the three decision problems, we apply both an Ordinary Least Squares (OLS) model and an Ordered Probit model to the data.[8] The Ordered Probit is included since the dependent variable is an ordinal, ordered response that is not continuous and so the linear structure of the OLS model may lead to an unbounded range for the coefficient estimates, even though the dependent variable can only take on values 1, 2, 3, 4, 5 or 6 (Wooldridge 2002).

3.2 Description of variables

The measure of household wealth used in this study is tropical livestock units (TLUs). TLUs are standardized tropical units of different types of livestock, and they are used as a measure of total livestock ownership in numerous studies set in the context of developing economies (e.g. Dercon 2004, Barrett and McPeak 2006).[9] Livestock typically accounts for over 90% of the value of

[8] The Ordered Probit model estimates coefficients using a maximum likelihood estimator – see Wooldridge (2002) for more details on the model.

[9] TLUs provide a single figure that expresses the total amount of livestock owned in a common unit. For the purposes of the ERHS, it is calculated using the following conversions: oxen=1, cows=0.70, bulls=0.75. horse=0.50, goat=0.10, sheep=0.10 and other similar values (Dercon 2004).

household assets in rural Ethiopia and is the most marketable asset in this region, and so is the most natural measure of household wealth (Dercon 2004). Table 3 provides the summary statistics for livestock ownership, as well as other ERHS and experiment variables used as explanatory variables to test which factors affect risk aversion and microinsurance take-up in rural Ethiopia.

The households of participants in the experiment (all of whom responded to decision problem B) own 10.5 TLUs, on average, slightly more than the mean for the complete sample, and all of them own at least some livestock; TLUs for households in our sample range from 8 to 31.5, as compared to 7 to 31.5 for the entire ERHS sample. (Table 3).[10] In Section 5 we show that our results are robust to an alternative measure of wealth, calculated as the first principal component from a principal component analysis of wealth measures in the ERHS survey.

We use the standard deviation of monthly household consumption over all seven rounds of the ERHS as a measure of the level of background risk to consumption faced by the household in recent past. Numerous studies use measures based on the standard deviation of consumption and income as measures of risk and shocks affecting rural households (e.g. Jalan and Ravallion 1998, Kamanou and Morduch 2004). In this case, the standard deviation of consumption is used rather than that of income because we are interested in the residual level of risk faced by the household after it has utilized all available consumption smoothing measures, and the ERHS income data are far less reliable than the consumption data (Porter 2008).[11] Consumption is measured as the total monthly household consumption in 1994 Ethiopian birr. It includes consumption of food, purchased food and non-investment non-food items (that is, excluding expenditure on

[10] Despite the use of land ownership as a measure of wealth in other studies of rural Ethiopia (e.g. Porter 2008), its suitability as a measure of wealth in rural Ethiopia is subject to debate. Dercon (2004) notes that in this region, all land is state-owned and subject to repeated redistribution – therefore, there is not much scope for long-term investment in landholdings, and livestock is the most suitable asset to proxy for wealth. This is why we use livestock, and not land, as the measure of wealth in our specifications.

[11] Hill et al. (2010) also include the standard deviation of household consumption as an explanatory variable in their specifications used to analyze the determinants of hypothetical take-up of index insurance by ERHS households.

Table 3. *Summary Statistics*

No. of Participants	T_X (258)		T_M (136)		B (378)		Full ERHS Round 7 Sample (1577)	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
ERHS Variables								
Total livestock units	10.81	2.788	10.88	3.061	10.54	2.900	9.216	1.974
Std. dev. of consumption[a]	546.2	329.1	516.0	363.5	528.8	335.4	375.6	298.5
No. of iddir	2.128	1.555	2.078	1.390	2.246	1.643	1.781	1.443
Can obtain 100 birr in emergency	0.829	0.377	0.831	0.376	0.827	0.379	0.754	0.431
If equb member	0.272	0.446	0.311	0.465	0.247	0.432	0.135	0.342
Household size	5.630	2.245	5.674	2.283	5.551	2.341	5.772	2.582
Experiment Variables								
Understanding	0.861	0.173	0.853	0.180	0.870	0.172		
How many decision problems paid for?	0.624	0.485	0.603	0.491	0.653	0.477		
Which decision problem paid for?	0.926	0.262	0.890	0.314	0.934	0.249		
Which colour token is bad?	0.977	0.151	0.963	0.189	0.976	0.153		
Wheel spin or bag draw first?	0.919	0.274	0.890	0.305	0.894	0.308		
No. of yellow tokens in yellow bag?	0.868	0.339	0.897	0.305	0.880	0.324		
Yellow or blue token draw more likely?	0.868	0.339	0.875	0.332	0.873	0.333		
Financial literacy	0.495	0.205	0.479	0.204	0.514	0.210		
$5 + 3 =?$	0.822	0.384	0.816	0.389	0.862	0.345		
$3 \times 7 =?$	0.523	0.500	0.507	0.502	0.545	0.498		
$\frac{1}{10}$th of $300 =?$	0.252	0.435	0.213	0.411	0.300	0.458		
5% of $200 =?$	0.0116	0.107	0.0147	0.121	0.0132	0.114		
Riskier to plant one crop or multiple crops?	0.868	0.339	0.846	0.363	0.852	0.356		
If literate	0.790	0.408	0.787	0.411	0.770	0.421		
Schooling obtained[c]	4.702	3.892	4.515	3.952	4.149	3.795		
Age[c]	46.60	15.86	48.90	15.79	45.14	15.94		
If female	0.322	0.468	0.331	0.472	0.325	0.469		
If household head	0.702	0.458	0.735	0.443	0.696	0.461		
If farmer	0.682	0.467	0.632	0.484	0.664	0.473		
Fraction of earnings kept	0.468	0.462	0.420	0.469	0.432	0.463		

[a] Measured in 1994 Ethiopian birr
[b] Measured in years

durables, health and education) – this consumption measure has been utilized by various other studies of consumption and poverty that have been conducted using ERHS data (Porter 2008). Table 3 shows that the inter-temporal standard deviation of monthly consumption is higher for households participating in the experiment – for these households, the mean standard deviation of around 530 birr represents a significant fraction (just under half) of the average monthly consumption measured in round seven of the ERHS (about 1100 birr). This indicates that subjects in the experiment face considerable risk to consumption, just like most households included in the survey.

Giné et al. (2008) and Cole et al. (2009) note that cognitive ability, financial literacy and understanding of financial products such as index insurance are instrumental in their proper valuation and rapid take-up. In conducting the experiment we went to great lengths to make all decision problems understandable to both literate and illiterate subjects, spending a great deal of time providing detailed explanations of each decision problem to the subjects, including the use of visual aids and tangible randomization devices. However, in spite of this, it is quite likely that people with different education levels and literacy status had different perceptions of the decision problems. Therefore, we include as explanatory variables both the number of years of schooling obtained by the experimental subjects and a literacy dummy, to test whether better cognitive ability leads to more index insurance take-up.[12] While 77% of the experimental subjects are literate, the average subject had only obtained around four years of formal schooling. However, years of formal schooling may be a poor proxy for education and the ability to solve the mathematical problems one encounters in everyday life (Cole et al. 2009). Therefore, in line with the work of Cole et al. (2009), we also include direct measures of understanding and financial literacy – these are the fraction of six questions relating to the understanding of the decision problems answered correctly and the fraction of

[12] At the end of each experiment we asked each subject to sign a receipt for their earnings and they were offered both a pen and a fingerprint card. The literacy dummy variable is one if the subject signed with a pen and zero if with a thumb print. Enumerators did not prompt.

five questions assessing probability and mathematical skills answered correctly (refer to Table 3 for more details on these questions). While the subjects exhibit a fairly good understanding of the decision problems (correctly answering 87% of the understanding questions, on average), the subjects answered only 51% of the financial literacy questions correctly, on average. Though this measure indicates a relatively low level of financial literacy, it is still higher than that measured by Cole et al. (2009), who find that respondents in their sample of rural inhabitants of Gujarat (India) correctly answer only 34% of financial literacy questions of a similar difficulty.

Giné et al. (2008) hypothesize when a new and unfamiliar technology or financial product, such as index insurance, is introduced to rural farmers, households draw inferences based on experience and familiarity with other similar products. In addition, the literature on technology adoption and diffusion indicates that households rely heavily on the large information flows between members of social groups in deciding whether to take-up new products (e.g Feder et al. 1985, Bandiera and Rasul 2006). Giné et al. (2008) claim that this argument also applies for new financial products, such as index insurance (which ERHS households have not previously been offered). Therefore, we hypothesize that farmers with larger social networks and more prior experience with financial products are more likely to purchase index insurance in the experiment. Three additional ERHS variables are included to capture the subjects' membership in social groups, prior experience with financial products such as savings and insurance, as well as access to informal insurance networks. These are dummy variables indicating whether the household is part of an equb group (a mutual savings association, similar to a ROSCA) and whether the household can obtain 100 birr (8 USD) within a week in case of an emergency (which is a proxy for the household's access to informal insurance, and therefore, the size of its informal risk-sharing network). Further, the number of iddir groups that the subject's household is a member of is used as an indicator of both the size of the household's social network as well as the household's, and thus the subject's, prior experience with informal insurance. Iddirs are informal insurance groups indigenous to Ethiopia that were originally formed to cope with the high cost of funerals – however, currently many iddirs also provide informal insurance and credit to its members when they experience other adverse shocks, such as fires, illnesses and loss of livestock (Hoddinott et al. 2005). Hoddinott et al. (2005) observe that iddir members have larger social networks, and also better access to informal insurance, than non-iddir members. Iddirs are quite widespread in rural Ethiopia, and over 85% of ERHS households are members of at least one iddir. Table 3 shows that households of experimental subjects are members of two iddir on average; on the other hand only 25% of the households are members of an equb group.[13] A large fraction (nearly 83%) of the households of experimental subjects report that they can obtain 100 birr within a week in case of an emergency.

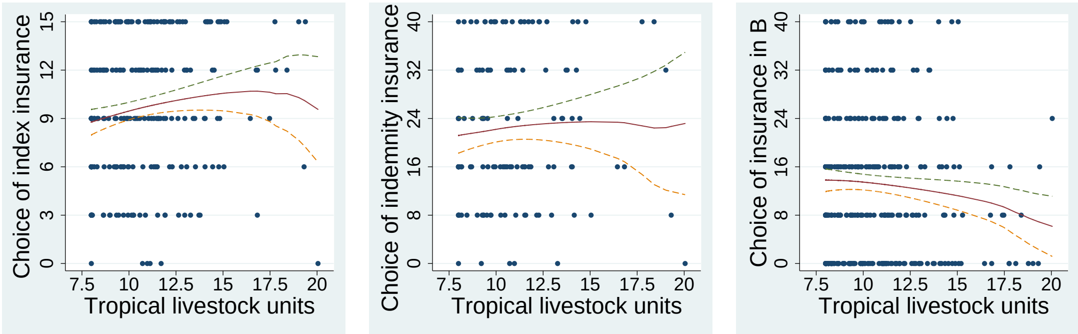
Other characteristics of the subjects that are expected to affect insurance take-up are also considered – these include sex (represented by the dummy variable 'If female' taking the value one if the subject is female), whether the subject's primary occupation is farming or not (indicated by the dummy variable 'If farming', which takes the value one if the subject is a farmer), whether the subject is a household head or not (indicated by the dummy variable 'If household head', which takes the value one if the subject is a household head), and household size.[14] These are similar to the demographic characteristics included by Giné et al. (2008) and Cole et al. (2009) in their studies of index insurance take-up in rural India. In addition to these variables, the self-reported fraction of the total earnings from the experiment that the subject intends to keep for himself is also included as a determinant of insurance take-up and risk aversion in the decision problems.

[13] Equbs are not as widespread as iddirs in rural Ethiopia, as the summary statistics indicate. Most households that are equb members are members of only one equb. Therefore, data on the number of equbs that a household is a member of is not collected in the ERHS, and we only use a dummy variable indicating whether the household is an equb member or not. On the other hand, iddir membership is extensive and many households are members of multiple iddirs (Hoddinott et al. 2005). Thus, to distinguish between households on the basis of iddir membership, we use the number of iddir (which is noted in the ERHS) rather than simply a dummy indicating whether the household is a member of at least one iddir.

[14] The average household size is greater than five for both the entire ERHS sample and the sub-sample participating in the experiment.

Figure 3. *Scatter plot and kernel regression: choice in decision problem against livestock owned*

(a) Decision problem T_X (b) Decision problem T_M (c) Decision problem B



Note: The figures show the point estimate and 95% confidence intervals for an Epanechnikov kernel with a bandwidth of 0.8 and trimming of 0.05. For figures (a) and (b), the y-axis indicates the amount of insurance chosen in T_X and T_M respectively. For figure (c), the numbers on the y-axis are the equivalent amounts of indemnity insurance purchase corresponding to the choices in B.

We hypothesize that those subjects keeping a larger fraction for themselves are expected to choose riskier options in the experiment (thus choosing lower levels of insurance cover) than subjects who intend to share their experimental earnings with others, and hence are wary about returning with very little (or no) money from the experiment other than the participation fee of 5 birr.

3.3 Results

The three graphs in Figure 3 display the scatter plots of the choices in the different decision problems versus tropical livestock units, in addition to a nonparametric kernel regression of choice in the decision problem against livestock units owned.[15]

The kernel regression line and the corresponding 95% confidence intervals in Figure 3(a) show that at low levels of wealth index insurance take-up is increasing in wealth. This is broadly in line with the results of Giné et al. (2008) and Cole et

[15] The kernel regression uses a Nadaraya-Watson estimator, and is a nonparametric technique to estimate the conditional expectation of the choice in the decision problem relative to the livestock units owned. See Li and Racine (2007) for more details on the kernel regression and Andersen et al. (2008) for use in the analysis of experimental data.

al. (2009), who find that take-up is decreasing with risk aversion in their samples of poor households in rural India.

Figure 3(a) also provides evidence for nonmonotonicity, with index insurance purchase first increasing then decreasing in wealth, as predicted by Clarke (2011) for DARA EUT. However, the kernel regression line does not provide conclusive proof of nonmonotonicity; it is possible to draw a straight line within the 95% confidence intervals. Index insurance take-up may not be decreasing at high levels of wealth because, in the sample of experimental subjects, there are no individuals who are so rich as to have very low levels (close to zero) of risk aversion. This is a reasonable assumption, given that the ERHS households are, in general, very poor (Dercon and Krishnan 1998, Porter 2008).

The kernel regression line and corresponding 95% confidence intervals in Figure 3(c) are monotonically decreasing with wealth, suggesting that decreasing absolute risk aversion is a characteristic of our sample in decision problem B. However, the kernel line in Figure 3(b) is relatively flat; the 95% confidence intervals are very wide, but do not rule out the possibility that risk aversion is decreasing with wealth. B and T_M are numerically identical and so we interpret differences between Figures 3(c) and 3(b) as framing effects, and the difference in sample size; only 136 subjects played T_M, as opposed to 378 who played B. When presented as an insurance problem individuals chose safer options than when presented in the abstract as a gain-frame lottery choice.

For a more formal analysis of the data, we consider the following reduced form specification:

$$
\begin{aligned}
\text{Decision Problem Choice}_i = \beta_0 &+ \beta_1\text{Livestock}_i + \beta_2\text{Livestock}_i^2 \\
&+ \beta_3\text{Std. dev. of consumption}_i + \beta_4\text{No. of iddir}_i \\
&+ \beta_5\text{Can Obtain 100 birr in emergency}_i + \beta_6\text{If equb}_i \\
&+ \beta_7\text{Household size}_i + \beta_8\text{Understanding}_i + \beta_9\text{Financial literacy}_i \\
&+ \beta_{10}\text{If literate}_i + \beta_{11}\text{Schooling}_i + \beta_{12}\text{Age}_i + \beta_{13}\text{If female}_i \\
&+ \beta_{14}\text{If household head}_i + \beta_{15}\text{If farmer}_i \\
&+ \beta_{16}\text{Fraction of earnings kept}_i + \epsilon_i \qquad (1)
\end{aligned}
$$

where i is the individual subject subscript. For each of OLS and Ordered Probit we consider four specifications: one with only the linear wealth term; one with the linear and squared wealth terms; one with the linear wealth term and all the other explanatory variables; and one with all the terms in specification (1).

For Table 8, the dependent variable is the choice in B, ordered to allow direct comparability between the numerically identical lottery choices B and T_M; a higher value of the dependent variable for either B or T_M corresponds to a safer lottery choice.

Tables 6-8 present the results for the estimation of these specifications. In Tables 6 and 7 the subject's choice is the dependent variable, with values ranging from 1 to 6 and higher values indicating more insurance purchased in T_X and T_M (refer to Tables 1 and 2). For Table 8, the dependent variable is the choice in B, ordered to allow direct comparability between the numerically identical lottery choices B and T_M; a higher value of the dependent variable for either B or T_M corresponds to a safer lottery choice. The OLS results are listed in the odd-numbered columns, while the Ordered Probit results are listed in the even-numbered columns.[16]

Standard errors clustered at the session level and robust to heteroskedasticity are used to construct the t-statistics reported in the tables for all specifications in this

[16] Note that the reported coefficient estimates for the Ordered Probit model are not marginal effects.

study. This allows for the possibility of correlation between the responses of subjects in the same session, or equivalently that the error term ϵ_i in specification 1 is correlated between individuals in a particular session. We might expect this to be the case as sessions differed in location and enumerator. The procedures for allowing for clustering also allow heteroskedasticity between and within clusters, as well as autocorrelation within clusters (Andersen et al. 2008).[17] However, these procedures still assume that the error is uncorrelated between subjects across different sessions.

As would be expected from the kernel regression line (Figure 3(a)) there is no statistically significant linear relationship between demand for index insurance and livestock (columns (1), (2), (5) and (6) of Table 6). However, when the quadratic term is added to the specification, the coefficient estimate of livestock is positive and significant (at the 1% and 5% levels in the OLS specifications and at the 1% level in the Ordered Probit specifications), and the coefficient estimate of livestock-squared is negative and significant at the 1% level (in all specifications). This, along with the kernel in Figure 3(a), is suggestive of a hump-shaped nonlinear relationship between the take-up of index insurance and wealth, consistent with DARA EUT (Clarke 2011).

The lower take-up by subjects from poor households also indicates that the low take-up observed among the most risk averse farmers by Giné et al. (2008) and Cole et al. (2009) may be a result of rational choice rather than poor understanding or irrationality on the part of rural consumers, as argued in Clarke (2011). It is also important to note that while we find the nonlinear relationship between index insurance take-up and wealth to be consistent with the hump-shape prediction of Clarke (2011), the levels of wealth are significantly greater than those predicted by the model. In this section we analyse only the shape, not the

[17] If the correlation of errors within sessions is not accounted for, the estimated standard errors may be quite wrong, leading to incorrect inference of coefficient estimates – this problem is magnified in small samples (Wooldridge 2002). Using clustered-robust standard errors to correct for this correlation of errors within groups is the norm when analyzing experimental data (see Harrison and Rutström 2008), and many experimental studies cluster at the session level (e.g. Barr 2003, Barr and Genicot 2008).

level, of insurance purchase and choices in B. This is in line with the work of Giné et al. (2008), who do not make theoretical predictions about the level of index insurance take-up in but rather focus on predicting, and empirically testing for, the shape of the relationship with respect to factors such as wealth and risk aversion.

In specifications (3), (4), (7) and (8) the coefficient estimates on the livestock (β_1) and livestock-squared (β_2) terms imply a maximum at 14.4, 13.8, 13.5 and 13.2, respectively.[18] These a little above the mean TLU for our sample, but well below the maximum of 31.5. These results therefore provide evidence for a hump shape, with demand for index insurance first increasing then decreasing with TLU, as opposed to just a concave relationship.

There is also some evidence that more literate subjects purchase more index insurance – the coefficient estimate on the 'If literate' variable is positive and statistically significant at the 5% or 10% level in all specifications. Unlike Cole et al. (2009), however, we do not find that other proxies for cognitive ability, such as financial literacy, significantly increase the take-up of index insurance. This may be an indication that what really matters for the take-up of index insurance is basic literacy, rather than the years of formal schooling obtained or financial literacy.

We also find some evidence, in line with the work of Cole et al. (2009), that a household's prior experience with other forms of insurance (such as informal insurance) and the size of its informal risk-sharing network increase the take-up of index insurance. The coefficient estimate of the dummy indicating whether the subject's household can obtain 100 birr in the case of an emergency is positive and significant at the 1% level in all specifications. However, there is no such relationship with iddir or equb membership.

For B we find the reverse story in terms of the relationship between risk taking and TLU; if the squared term is included in specifications then neither linear nor

[18] For $\beta_1 > 0$ and $\beta_2 < 0$ the quadratic reaches a maximum when livestock equals $-\frac{\beta_1}{2\beta_2}$.

squared term are significant, but if only the linear term is included the coefficient is statistically significant at the 1% level in all specifications (Table 8). Along with the kernel regression line in Figure 3(c), this provides evidence for DARA in that wealthier participants choose riskier lotteries in the benchmark decision problem B. For T_M, a numerically identical decision problem framed as an insurance purchase decision, we would expect to find similar coefficients to B. However, we are unable to reject the null hypothesis that there is no linear or quadratic relationship between insurance purchase and TLUs (Table 7). As already mentioned, this may be partly due to the small sample size.

For B, the coefficient estimate on the standard deviation of consumption is positive and significant at the 5% level in all specifications. These results are in line with Gollier and Pratt's (1996) theory of risk vulnerability, supporting the argument that subjects from those households that face more background risk choose safer lotteries in B, thus displaying a higher level of risk aversion over earnings in the decision problem. This is consistent with Harrison et al. (2007), which finds risk aversion to be increasing with the addition of independent background risk in an experiment involving numismatists at a coin show in Central Florida (USA).

The results in Tables 6 and 7 show that subjects who are not household heads tend to purchase more index and indemnity insurance. This may be because those subjects who are answerable to their household heads – and probably have to give most (or all) of their experimental earnings to their household heads – are averse to returning with little (or no) money from the experiment (other than the participation fee) and thus are more likely to choose safer options that imply more index and indemnity insurance take-up. Also in line with this explanation, Table 6 shows that the fraction of experimental earnings kept by the subject has a negative and statistically significant impact on index insurance take up. Further, it should also be noted that none of the statistically significant coefficient estimates in Tables 6-8 differ in sign between the OLS and Ordered Probit models. This gives us confidence in the robustness of the OLS results obtained in this section.

4 COMPARING THEORIES OF CHOICE

4.1 Theory

Having considered the shape of observed demand, we now consider the level. We allow two competing theories of decision under uncertainty to explain these data: expected utility theory (EUT) and rank dependent utility (RDU). We consider EUT to be an appropriate normative framework for making decisions about insurance purchase and therefore consider this to be our baseline model of decision under uncertainty. However, EUT is known to be somewhat limited as a positive theory of decision under uncertainty and so we will also compare EUT with RDU, where decision makers may weight objective cumulative probabilities.[19] In particular, we will see that these two theories differ in the predicted level of optimal index insurance purchase; relative to EUT, RDU can account for either a higher or lower level of purchase.

For each of the decision problems considered there are six possible choices, denoted $i \in \{1, \ldots, 6\}$, and N possible outcomes, denoted $k \in \{1, \ldots, N\}$, where N is 2 for B and T_M and 4 for T_X (see Tables 1 and 2).

4.1.1 Expected utility theory specification

For our EUT specification we assume that the indirect utility of income from the experimental session is given by:

$$U(x) = \begin{cases} \frac{x^{1-r}}{1-r} \text{ if } r \neq 1 \\ \ln(x) \text{ if } r = 1 \end{cases} \tag{2}$$

where x is the lottery prize and r is the coefficient of relative risk aversion.[20] With this constant relative risk aversion (CRRA) specification, $r > 0$, $r = 0$ and $r < 0$

[19] RDU was formally presented by Quiggin (1982, 1993), inspired by experiments conducted with Australian farmers, and is a leading theory of choice under uncertainty.
[20] The show-up fee of 5 birr is excluded from lottery prize x.

correspond to risk aversion, risk neutrality and risk loving, respectively. Under expected utility theory (EUT) the decision maker weights each possible lottery prize $x \in \{x_{i1}^d, \ldots, x_{iN}^d\}$ using objective probabilities $p_i^d(x)$ and so expected utility from choice $i \in \{1, \ldots, 6\}$ in decision problem $d \in \{T_M, T_X\}$ is:

$$EU_i^d = \sum_{k=1}^{N} p_i^d(x_{ik}^d) U(x_{ik}^d) \tag{3}$$

where probabilities and lottery prizes for each decision problem are given in Table 1.

Under CRRA it is never optimal to pay an insurance premium of more than 3 birr in T_X (see Table 2). It is important to note that such a limit on the amount of index insurance to purchase is not an artefact of our utility function specification in equation (2) but rather is a robust feature of preferences that satisfy the mild restriction of decreasing absolute risk aversion (DARA). We consider the DARA upper bound to be a normatively sound basis for advice from an objective financial advisor who, as in Clarke (2011), combines an objective probability distribution with their client's preferences over roulette lotteries. In this experiment the randomisation device had known probabilities and was statistically independent of participant's background wealth and so we may appeal to Theorem 4 of Clarke (2011) to calculate the DARA upper bound. In the notation of Clarke (2011), $p = q = \frac{1}{2}$, $r = \frac{1}{8}$ and $m = 1.2$, and so no risk averse expected utility maximising decision maker satisfying DARA over aggregate wealth would ever pay an insurance premium of more than 6 birr in T_X: if they cared enough about risk to want to purchase the cover, they would care enough about the downside basis risk to limit the size of the hedge. The DARA upper bound is therefore only slightly less restrictive than the CRRA bound.

The structural estimation model, along the lines of Harrison and Rutström (2008), is as follows. The expected utility EU_i^d from each potential choice i for each decision problem d is calculated according to equation (3). The index ∇EU_i^d is then calculated as follows:

$$
\begin{aligned}
eu_i^d &= \exp(EU_i^d) \\
\nabla EU_i^d &= \frac{eu_i^d}{eu_1^d + eu_2^d + eu_3^d + eu_4^d + eu_5^d + eu_6^d}
\end{aligned}
$$

The latent index ∇EU_i^d, based on latent preferences, is in the form of a probability, and thus can be directly linked to the observed choices. Following Harrison and Rutström (2008), ∇EU_i^d may therefore be interpreted as the probability of a subject choosing lottery choice i given decision problem d. To enable us to jointly consider both insurance decision problems, we make the strong assumption that the observed choices for individuals in T_M are independent of those in T_X. Given the observed choice y_a^d of individual a in decision problem d, the log-likelihood of the observed responses, conditional on EUT and CRRA utility, is:

$$
\ln L^{EUT}(r; y) = \sum_{a=1}^{136} \ln(\nabla EU_{y_a^{T_M}}^{T_M}) + \ln(\nabla EU_{y_a^{T_X}}^{T_X})
$$

The log-likelihood function $\ln L^{EUT}$ is maximized with respect to the constant r to yield the maximum likelihood estimate of the coefficient of relative risk aversion r.

4.1.2 Rank dependent utility specification

There are two components in the RDU specification: the utility function and the probability weighting function (Quiggin 1982). As for EUT, we assume the CRRA utility form defined in equation (2). However, instead of weighting outcomes with the objective probabilities $p_i^d(x)$, we assume that subjects instead weight cumulative probabilities according to a weighting function ω, as follows:

$$
RDU_i^d = \sum_{k=1}^{N} w_i^d(x_{ik}^d)U(x_{ik}^d)
$$

where, for each d, i, states are ranked from worst to best lottery prize, and $\omega(p)$ is some probability weighting function:

$$w_i^d(x_{ik}^d) = \begin{cases} \omega(p_i^d(x_{i1}^d)) \text{ for } k = 1 \\ \omega(p_i^d(x_{i1}^d) + \ldots + p_i^d(x_{ik}^d)) - \omega(p_i^d(x_{i1}^d) + \ldots + p_i^d(x_{i,k-1}^d)) \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{for } k > 1. \end{cases}$$

We consider two single-parameter functional forms for probability weighting function ω, the first due to Prelec (1998) and the second due to Tversky and Kahneman (1992). Both have well defined endpoints with $\omega(p) = 0$ for $p = 0$ and $\omega(p) = 1$ for $p = 1$, and

$$\omega^P(p) = \exp\{-(-\ln(p))^\phi\} \tag{4a}$$

$$\omega^{TK}(p) = p^\gamma/[p^\gamma + (1-p)^\gamma]^{1/\gamma} \tag{4b}$$

for $0 < p < 1$.

The usual case, for standard samples of university students, is that probability weighting follows an inverse S-shape, concave for low probabilities and convex for high probabilities (Gonzalez and Wu 1999). For equations (4a) and (4b) this corresponds to $0 < \gamma, \phi < 1$, and for index insurance decision problem T_X would lead to lower demand for index insurance than under subjective expected utility (SEU). Wakker et al. (1997) report on a set of 'probabilistic insurance' decision problems, which are similar to our index insurance purchase problems but with a lower probability of basis risk, and find an extreme aversion to basis risk from a standard sample of university students. They interpret this as providing evidence for an inverse S-shape probability weighting.

However, recent laboratory experiments, framed in the abstract and with nonstandard samples drawn from developing countries, have found S-shaped probability weighting, convex for low probabilities and concave for high probabilities (Humphrey and Verschoor 2004a, Humphrey and Verschoor 2004b).

This correponds to $\gamma, \phi > 1$, and for index insurance decision problem T_X would lead to a higher demand for index insurance than under SEU.

As for EUT, we may construct the log likelihood function $\ln L^{RDU}$ as follows:

$$
\begin{aligned}
rdu_i^d &= \exp(RDU_i^d) \\
\nabla RDU_i^d &= \frac{rdu_i^d}{rdu_1^d + rdu_2^d + rdu_3^d + rdu_4^d + rdu_5^d + rdu_6^d} \\
\ln L^{RDU}(r, \phi; y) &= \sum_{a=1}^{136} \ln(\nabla RDU_{y_a^{T_M}}^{T_M}) + \ln(\nabla RDU_{y_a^{T_X}}^{T_X})
\end{aligned}
$$

The log-likelihood function $\ln L^{EUT}$ is jointly maximized with respect r and ϕ in the case of Prelec probability weighting, and with respect to r and γ in the case of Tversky-Kahneman probability weighting, to yield the maximum likelihood estimates of these parameters.

4.2 Results

In this section we restrict attention to the 136 individuals who played both T_M and T_X. We consider three specifications (Table 4). Specification A is the EUT specification of Section 4.1.1 with no individual covariates.[21] The coefficient of relative risk aversion is estimated to be 0.777, broadly similar to that estimated by other studies and comparable to the coefficient of 0.891 recently estimated for Ethiopian subjects in a traditional laboratory experiment framed in the abstract (Harrison et al. 2010b) .

When probability weighting is introduced the joint maximum likelihood estimates for r and ϕ are 0.641 and 4.131 for Prelec's (1998) probability weighting function and estimates for r and γ are 0.926 and 2.019 for Tversky and

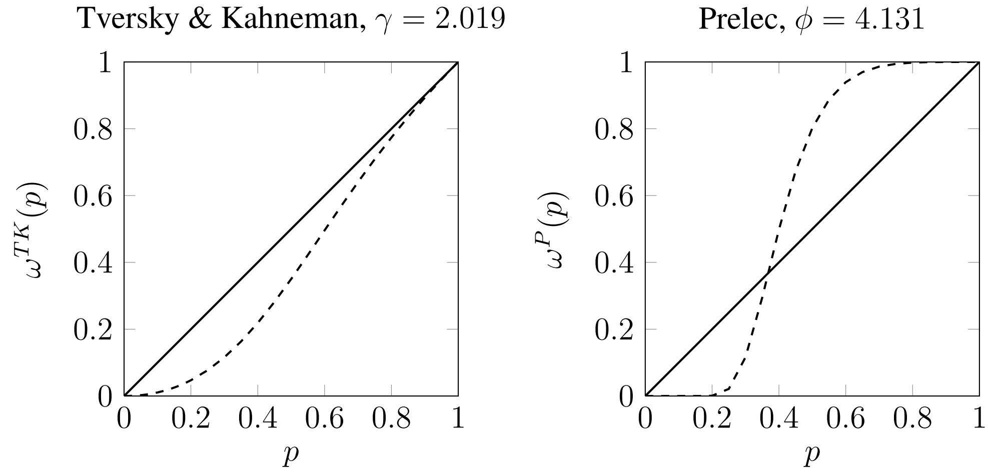
[21] The optimization routine did not converge when data from problems T_M and T_X were jointly analyzed using all or a substantial subset of the observable characteristics considered in Section 3. These convergence problems are probably due to the small sample size, with only two decisions for each of 136 subjects. The maximum likelihood approach with covariates seems to require a larger sample size to converge (Harrison and Rutström 2008), and even then the number of covariates is limited: Harbaugh et al. (2002) use only sex, age and order controls and Harrison et al. (2010b) using only sex, age, education, household size and location and order controls.

Table 4. *Maximum likelihood estimates of model parameters for insurance decision problems, with no individual covariates.*

Coefficient	Estimate	Standard Error	p-value	Lower 95% Confidence Interval	Upper 95% Confidence Interval
A. EUT: $U(x) = \frac{x^{1-r}}{1-r}$					
r	0.777	0.016	0.000	0.746	0.808
B. RDU with Prelec Probability Weighting Function ω^P					
r	0.641	0.083	0.000	0.479	0.804
ϕ	4.131	0.942	0.000	2.285	5.976
$H_0 : \phi = 1$			0.001		
C. RDU with Tversky & Kahneman Probability Weighting Function ω^{PT}					
r	0.926	0.006	0.000	0.913	0.928
γ	2.019	0.157	0.000	1.713	2.326
$H_0 : \gamma = 1$			0.000		

Reported standard errors are robust to heteroskedasticity and clustered at the session level.

Figure 4. *Alternative probability weighting functions*



Kahneman's (1992) probability weighting function. These parameter estimates imply S-shaped probability weighting functions, with convex weighting for low probabilities and concave weighting for high probabilities (see Figure 4). Moreover the S-shapes are statistically significant: Wald tests of the null hypotheses that there is no probability weighting ($\phi = 1$ and $\gamma = 1$) have p-values of 0.001 and 0.000 respectively, and we may convincingly reject the nulls in the direction of $\phi > 1$ and $\gamma > 1$.

This finding is not surprising given the data; a large number of participants purchased more index insurance than is consistent with EUT or RDU with an inverse S-shape. Moreover, it is consistent with the work of Humphrey and Verschoor (2004a) and Humphrey and Verschoor (2004b) who find an S-shaped probability weighting function in traditional laboratory experiments, framed in the abstract and with samples drawn from developing countries. Evidence for S-shaped probability weighting functions has also been found by Harbaugh et al. (2002) for another class of subjects with low levels of formal education, namely children. As one would expect, given the theoretical predictions of EUT and observed behaviour in index insurance treatments, we are able to reject EUT in favour of RDU with an S-shaped probability weighting function.

5 ROBUSTNESS CHECKS

The results of sections 3 and 4 withstand common robustness checks. One concern might be that outliers in the ERHS data on livestock ownership are driving the results involving this variable; this is often a worry when using survey data (Chambers and Skinner 2003). However, all the specifications in Section 3 produce similar results when the logarithm of tropical livestock units is used instead of the level. This indicates that the results obtained in Section 3 are probably not driven by outliers in the wealth measure.

If analysis is restricted to include only the first decision problem played by each subject, or only those 208 subjects who answered all the confirmation of understanding questions correctly, the results in sections 3 and 4 remain substantively the same. This suggests that neither order effects nor subjects with a low level of understanding are driving the results (Harbaugh et al. 2002).

Tropical livestock units is used as the main measure of wealth in our analysis, as is common for analyses using ERHS data. However, this measure may not discriminate well between subjects from wealthier households who may hold wealth in other forms. We therefore construct a composite wealth measure, using the ten main asset questions in the ERHS survey, and repeat the analysis for the index insurance decision problem T_X using this wealth measure. This wealth measure is constructed to be the first principal component arising from a principal components analysis (PCA) of these ten asset questions for the entire ERHS sample.[22] Table 5 presents the weights from the principal eigenvector, used to construct the index. As might be expected, all weights are the same sign, indicating that they are jointly positively correlated. Figure 5(a) shows that the resulting PCA wealth index is highly correlated with tropical livestock units, suggesting that tropical livestock units are a good proxy for wealth in our sample sites.

[22] Howe et al. (2008) provides an introduction to PCA as a tool for constructing wealth indices. PCA is not scale invariant and so analysis has been conducted using the correlation matrix. This is equivalent to using standardised variables.

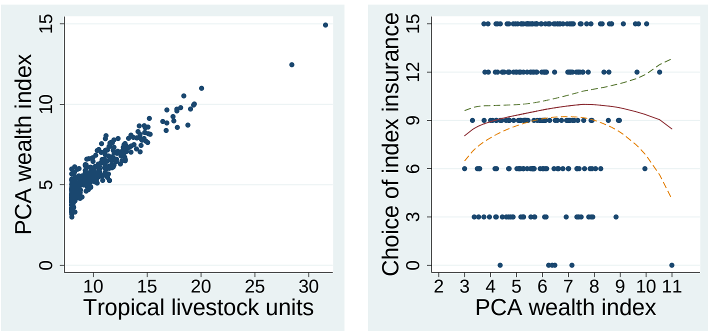
Table 5. *Weights assigned to each indicator in PCA wealth index*

Item	Item weight in PCA wealth index
Tropical livestock units	0.362
Land	0.295
If own bank account	0.268
If own hoe	0.272
If own plough	0.325
If own radio	0.397
If own gold	0.339
If own bicycle	0.214
If own cell phone	0.338
If currently storing any crop	0.313

Figure 5(b) and Table 9 present the scatter plots, nonparametric kernel regressions and correlates of index insurance takeup when this PCA wealth measure is used in place of tropical livestock units, and may be directly compared with Figure 3(a) and Table 6 respectively. As can be seen from the figure and table, all results remain substantively the same when TLUs is replaced with the PCA wealth index. The quadratic specifications (3), (4), (7) and (8) of Table 9 suggest that index insurance purchase is highest when the PCA wealth index is 7.1, 7.0, 7.0 and 7.0 respectively, again within one standard deviation of the mean PCA wealth index of 5.9 for our sample. This provides strong evidence that demand for index insurance is hump-shaped, not just increasing concave, in wealth.

Figure 5. *Results from principal components analysis wealth index*

(a) PCA wealth index versus tropical livestock units
(b) Scatter plot and kernel regression for T_X



Note: (b) presents point estimate and 95% confidence intervals for an Epanechnikov kernel with a bandwidth of 0.8 and trimming of 0.05.

Finally, although not reported in this paper, there were two other decision problems played by some subjects, similar to T_M and T_X but where individuals were randomly placed in risk-pooling pairs. The results of sections 3 and 4 hold if we add the decisions made in these problems (Clarke and Kalani 2011).

6 Conclusion

Lab and field experiments are complementary (Falk and Heckman 2009, Harrison et al. 2010a). However, for understanding whether the level of demand for indexed insurance is too low or too high, field experiments suffer from a critical problem: rational demand, by which we mean the demand that would be recommended by a well-informed financial advisor, is highly sensitive to beliefs about the joint distribution of the loss and the index, and an objective, unambiguous belief about this joint distribution is extremely difficult to obtain in the field (Clarke 2011). By contrast, in a lab experiment losses and the index can be generated by a known randomisation device with objective joint probability distribution, and it is therefore possible to make clear normative statements about the level, not just the shape, of observed demand.

In our lab experiment, we find the shape of demand for insurance from rural Ethiopians to be consistent with DARA EUT under which demand for index insurance should be hump-shaped in wealth (Clarke 2011), and higher background risk should be associated with greater (indirect) risk aversion (Gollier and Pratt 1996). Both these results are consistent with recent evidence from field and lab experiments (Cole et al. 2009, Harrison et al. 2007). However, the aggregate level of demand we observe is higher than can be rationalised by DARA EUT. This result is consistent with recent lab experiments framed in the abstract and conducted with subjects from developing countries (Humphrey and Verschoor 2004a, Humphrey and Verschoor 2004b).

Development economists and insurance practitioners are rightly excited about the potential for indexed insurance to substantially increase welfare for many of the world's rural poor (Skees et al. 1999, Karlan and Morduch 2009). However, voluntary purchase of unsubsidised products remains low. All too often this low demand is labeled a puzzle and attributed to poor 'behavioural' decision making on the part of farmers, without proper consideration of whether the product provides good value to rational farmers.

Regardless of how precisely insurance for the poor develops, it seems important for social scientists to be engaged with normative questions of interest to insurance providers, consumers and regulators. On their own, field experiments seem unlikely to advance our understanding of key questions such as how to design good products or whether clients make good financial decisions. To contribute to these questions we will also need normative theory and carefully designed lab experiments.

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Appendix

Table 6. *Correlates of Index Insurance Take-up in T_X*

Variables	(1) OLS	(2) O. Probit	(3) OLS	(4) O. Probit	(5) OLS	(6) O. Probit	(7) OLS	(8) O. Probit
Tropical livestock units	0.0346	0.0222	0.517***	0.452***	-0.0307	-0.0294	0.401**	0.412***
	(-0.600)	(-0.513)	(-4.830)	(-3.972)	(-0.510)	(-0.594)	(-2.548)	(-2.653)
Livestock squared			-0.0180***	-0.0164***			-0.0148***	-0.0156***
			(-5.570)	(-3.551)			(-3.235)	(-2.806)
Std. dev. of consumption					0.000224	0.000173	0.000137	0.000112
					(-0.780)	(-0.788)	(-0.479)	(-0.500)
No. of iddir					0.0724	0.0431	0.0338	0.00716
					(-0.845)	(-0.707)	(-0.395)	(-0.117)
Can obtain 100 Birr					0.749***	0.607***	0.700***	0.574***
					(-3.073)	(-3.337)	(-2.949)	(-3.228)
If equb					-0.14	-0.086	-0.166	-0.108
					(-0.528)	(-0.423)	(-0.629)	(-0.527)
Understanding					-0.0256	-0.103	0.0644	-0.0432
					(-0.031)	(-0.155)	(-0.078)	(-0.064)
Financial literacy					0.579	0.539	0.594	0.55
					(-0.912)	(-1.150)	(-0.970)	(-1.203)
If literate					0.511*	0.425**	0.487*	0.419*
					(-1.821)	(-1.969)	(-1.731)	(-1.927)
Schooling obtained					-0.000859	-0.000994	0.00775	0.00591
					(-0.032)	(-0.048)	(-0.299)	(-0.294)
Age					0.00322	0.00216	0.00217	0.000885
					(-0.434)	(-0.383)	(-0.301)	(-0.161)
If female					0.525*	0.377*	0.493	0.351
					(-1.851)	(-1.651)	(-1.700)	(-1.499)
If household head					-0.699***	-0.591***	-0.631***	-0.530***
					(-3.153)	(-3.400)	(-2.822)	(-2.990)
If farmer					0.413	0.311	0.397	0.301
					(-1.562)	(-1.467)	(-1.531)	(-1.438)
Household size					0.00937	0.00545	-0.0109	-0.0137
					(-0.222)	(-0.164)	(-0.267)	(-0.421)
Fraction of earnings kept					-0.429*	-0.391**	-0.409*	-0.380**
					(-1.886)	(-2.145)	(-1.829)	(-2.104)
Constant	3.819***		0.844		2.738**		-0.0292	
	(-6.255)		(-1.031)		(-2.547)		(-0.025)	
Threshold 1		-1.673***		0.862		-1.233		1.474
		(-3.463)		(-1.253)		(-1.398)		(-1.553)
Threshold 2		-0.786*		1.810**		-0.284		2.471**
		(-1.681)		(-2.554)		(-0.327)		(-2.572)
Threshold 3		-0.202		2.408***		0.38		3.141***
		(-0.452)		(-3.260)		(-0.458)		(-3.205)
Threshold 4		0.357		2.978***		0.992		3.759***
		(-0.758)		(-4.022)		(-1.154)		(-3.684)
Threshold 5		0.910**		3.542***		1.618*		4.393***
		(-2.013)		(-4.673)		(-1.935)		(-4.317)
Order Controls	No	No	No	No	Yes	Yes	Yes	Yes
Enumerator Controls	No	No	No	No	Yes	Yes	Yes	Yes
Location Controls	No	No	No	No	Yes	Yes	Yes	Yes
Observations	254	254	254	254	246	246	246	246
R-squared	0.004		0.049		0.188		0.210	
Log-likelihood		-423.1		-416.3		-383.7		-379.4

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$
Robust t-statistics based on standard errors clustered at the session level in parentheses.
Dependent variable is the respondent's choice in the specified decision problem.

Table 7. *Correlates of Indemnity Insurance Take-up in T_M*

Variables	(1) OLS	(2) O. Probit	(3) OLS	(4) O. Probit	(5) OLS	(6) O. Probit	(7) OLS	(8) O. Probit
Tropical livestock units	0.0519	0.036	-0.00574	-0.0233	0.0177	0.014	-0.209	-0.161
	(-1.170)	(-1.038)	(-0.032)	(-0.159)	(-0.420)	(-0.419)	(-0.677)	(-0.619)
Livestock squared			0.00203	0.00215			0.00729	0.00573
			(-0.364)	(-0.425)			(-0.785)	(-0.696)
Std. dev. of consumption					-0.0000202	-0.000108	0.0000207	-0.0000774
					(-0.044)	(-0.338)	(-0.044)	(-0.239)
No. of iddir					-0.123	-0.0867	-0.0837	-0.0576
					(-0.675)	(-0.634)	(-0.482)	(-0.435)
Can obtain 100 Birr					0.351	0.328	0.389	0.358
					(-1.214)	(-1.407)	(-1.273)	(-1.415)
If equb					-0.016	-0.0207	0.00267	-0.00485
					(-0.044)	(-0.083)	(-0.007)	(-0.020)
Understanding					-1.11	-1.081	-1.178	-1.116
					(-1.043)	(-1.257)	(-1.063)	(-1.263)
Financial literacy					1.618	1.157*	1.582	1.126
					(-1.740)	(-1.708)	(-1.636)	(-1.620)
If literate					0.147	0.174	0.175	0.193
					(-0.326)	(-0.495)	(-0.385)	(-0.544)
Schooling obtained					-0.048	-0.0434*	-0.0562	-0.0492
					(-1.417)	(-1.664)	(-1.424)	(-1.595)
Age					0.0046	0.00281	0.00438	0.00282
					(-0.486)	(-0.432)	(-0.481)	(-0.450)
If female					0.647*	0.377	0.672*	0.399
					(-1.987)	(-1.551)	(-1.986)	(-1.561)
If household head					-0.563*	-0.462**	-0.581*	-0.479**
					(-1.889)	(-2.256)	(-2.019)	(-2.489)
If farmer					0.165	0.0874	0.2	0.112
					(-0.445)	(-0.332)	(-0.513)	(-0.412)
Household size					-0.0104	-0.00953	-0.000947	-0.00291
					(-0.168)	(-0.209)	(-0.015)	(-0.060)
Fraction of earnings kept					-0.500	-0.369	-0.509	-0.375
					(-1.528)	(-1.587)	(-1.513)	(-1.586)
Constant	3.246***		3.614**		3.249**		4.745*	
	(-7.426)		(-2.789)		(-2.191)		(-1.922)	
Threshold 1		-1.299***		-1.669		-1.885*		-3
		(-2.828)		(-1.566)		(-1.793)		(-1.543)
Threshold 2		-0.363		-0.734		-0.884		-1.998
		(-1.031)		(-0.749)		(-0.853)		(-1.071)
Threshold 3		0.347		-0.0233		-0.0894		-1.201
		(-1.108)		(-0.024)		(-0.088)		(-0.649)
Threshold 4		0.716**		0.346		0.311		-0.798
		(-2.319)		(-0.352)		(-0.298)		(-0.421)
Threshold 5		1.248***		0.879		0.905		-0.201
		(-3.893)		(-0.892)		(-0.884)		(-0.107)
Order Controls	No	No	No	No	Yes	Yes	Yes	Yes
Enumerator Controls	No	No	No	No	Yes	Yes	Yes	Yes
Location Controls	No	No	No	No	Yes	Yes	Yes	Yes
Observations	132	132	132	132	126	126	126	126
R-squared	0.011		0.012		0.221		0.227	
Log-likelihood		-224.2		-224.1		-199.3		-198.9

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$
Robust t-statistics based on standard errors clustered at the session level in parentheses.
Dependent variable is the respondent's choice in the specified decision problem.

Table 8. *Correlates of Insurance Take-up in Benchmark Decision Problem* B

Variables	(1) OLS	(2) O. Probit	(3) OLS	(4) O. Probit	(5) OLS	(6) O. Probit	(7) OLS	(8) O. Probit
Tropical livestock units	-0.0597*** (-2.804)	-0.0480*** (-2.617)	0.034 (-0.440)	0.15 (-1.106)	-0.0838*** (-3.020)	-0.0782*** (-2.944)	-0.0894 (-0.831)	0.0534 (-0.385)
Livestock squared			-0.00332 (-1.486)	-0.00777 (-1.406)			0.00018 (-0.062)	-0.0049 (-0.918)
Std. dev. of consumption					0.000601** (-2.263)	0.000451** (-2.405)	0.000603** (-2.234)	0.000436** (-2.311)
No. of iddir					0.0344 (-0.545)	0.0304 (-0.677)	0.0347 (-0.547)	0.0259 (-0.562)
Can obtain 100 Birr					-0.245 (-1.434)	-0.181 (-1.498)	-0.245 (-1.411)	-0.194 (-1.618)
If equb					-0.463*** (-2.853)	-0.323*** (-2.759)	-0.463*** (-2.853)	-0.326*** (-2.788)
Understanding					-0.0125 (-0.020)	-0.0263 (-0.061)	-0.015 (-0.024)	-0.0293 (-0.068)
Financial literacy					-0.388 (-0.738)	-0.346 (-0.888)	-0.389 (-0.739)	-0.351 (-0.896)
If literate					-0.449* (-1.727)	-0.299* (-1.670)	-0.449* (-1.725)	-0.296* (-1.664)
Schooling obtained					0.0149 (-0.712)	0.00742 (-0.509)	0.0148 (-0.686)	0.00789 (-0.529)
Age					0.00846 (-1.355)	0.00658 (-1.458)	0.00847 (-1.333)	0.00633 (-1.388)
If female					-0.584** (-2.249)	-0.424** (-2.267)	-0.584** (-2.263)	-0.428** (-2.278)
If household head					-0.232 (-1.232)	-0.171 (-1.236)	-0.234 (-1.177)	-0.15 (-1.021)
If farmer					-0.265 (-1.031)	-0.225 (-1.221)	-0.263 (-1.009)	-0.231 (-1.251)
Household size					-0.0781* (-1.790)	-0.0516 (-1.535)	-0.0779* (-1.778)	-0.0555* (-1.663)
Fraction of earnings kept					-0.305 (-1.562)	-0.212 (-1.488)	-0.306 (-1.560)	-0.211 (-1.484)
Constant	3.275*** (-12.900)		2.683*** (-4.656)		5.954*** (-7.108)		5.991*** (-4.954)	
Threshold 1		-0.976*** (-4.491)		0.186 (-0.231)		-3.091*** (-4.684)		-2.310** (-2.101)
Threshold 2		-0.449** (-2.282)		0.716 (-0.897)		-2.527*** (-3.910)		-1.745 (-1.603)
Threshold 3		0.126 (-0.637)		1.292 (-1.614)		-1.895*** (-2.988)		-1.113 (-1.038)
Threshold 4		0.550** (-2.556)		1.717** (-2.141)		-1.426** (-2.212)		-0.643 (-0.597)
Threshold 5		0.962*** (-4.655)		2.131*** (-2.616)		-1.006 (-1.599)		-0.222 (-0.207)
Order Controls	No	No	No	No	Yes	Yes	Yes	Yes
Enumerator Controls	No	No	No	No	Yes	Yes	Yes	Yes
Location Controls	No	No	No	No	Yes	Yes	Yes	Yes
Observations	372	372	372	372	360	360	360	360
R-squared	0.012		0.014		0.143		0.143	
Log-likelihood		-612.5		-610.9		-565.7		-565.3

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$
Robust t-statistics based on standard errors clustered at the session level in parentheses.
Dependent variable is the respondent's choice in the specified decision problem.

Table 9. *Correlates of Index Insurance Take-up in T_X with PCA Wealth Measure*

Variables	(1) OLS	(2) O. Probit	(3) OLS	(4) O. Probit	(5) OLS	(6) O. Probit	(7) OLS	(8) O. Probit
PCA Wealth	0.0466	0.0299	0.918***	0.710**	-0.0738	-0.0681	0.793*	0.721*
	(-0.484)	(-0.420)	(-2.911)	(-2.499)	(-0.587)	(-0.662)	(-1.793)	(-1.776)
PCA Wealth squared			-0.0652**	-0.0512**			-0.0593*	-0.0544*
			(-2.546)	(-2.166)			(-1.927)	(-1.844)
Std. dev. of consumption					0.000169	0.000123	0.000109	0.0000808
					(-0.605)	(-0.581)	(-0.400)	(-0.390)
No. of iddir					0.118	0.0774	0.0679	0.0332
					(-1.352)	(-1.200)	(-0.791)	(-0.522)
Can obtain 100 Birr					0.745***	0.604***	0.664***	0.540***
					(-3.297)	(-3.448)	(-2.797)	(-2.935)
If equb					-0.0877	-0.0392	-0.126	-0.0738
					(-0.323)	(-0.190)	(-0.447)	(-0.341)
Understanding					-0.365	-0.36	-0.368	-0.372
					(-0.424)	(-0.521)	(-0.420)	(-0.526)
Financial literacy					0.553	0.535	0.476	0.468
					(-0.844)	(-1.098)	(-0.731)	(-0.954)
If literate					0.605**	0.493**	0.645**	0.535**
					(-2.179)	(-2.249)	(-2.275)	(-2.359)
Schooling obtained					0.00143	0.00116	0.000277	-0.0000969
					(-0.054)	(-0.057)	(-0.011)	(-0.005)
Age					0.00272	0.00182	0.0012	0.000288
					(-0.396)	(-0.350)	(-0.174)	(-0.055)
If female					0.498	0.359	0.477	0.344
					(-1.699)	(-1.526)	(-1.618)	(-1.441)
If household head					-0.723***	-0.614***	-0.660**	-0.557***
					(-3.036)	(-3.274)	(-2.757)	(-2.909)
If farmer					0.369	0.282	0.367	0.284
					(-1.309)	(-1.245)	(-1.298)	(-1.234)
Household size					0.00732	0.00499	-0.0106	-0.0114
					(-0.168)	(-0.142)	(-0.250)	(-0.338)
Fraction of earnings kept					-0.431*	-0.389**	-0.421*	-0.379**
					(-1.934)	(-2.197)	(-1.883)	(-2.130)
Constant	3.906***		1.168		3.095**		0.392	
	(-6.790)		(-1.140)		(-2.786)		(-0.276)	
Threshold 1		-1.717***		0.357		-1.498*		0.912
		(-3.832)		(-0.444)		(-1.656)		(-0.759)
Threshold 2		-0.834*		1.282		-0.55		1.896
		(-1.878)		(-1.488)		(-0.616)		(-1.520)
Threshold 3		-0.267		1.859**		0.111		2.565**
		(-0.637)		(-2.096)		(-0.129)		(-2.018)
Threshold 4		0.304		2.435***		0.737		3.194**
		(-0.702)		(-2.788)		(-0.841)		(-2.486)
Threshold 5		0.856**		2.991***		1.367		3.826***
		(-2.046)		(-3.292)		(-1.596)		(-2.942)
Order Controls	No	No	No	No	Yes	Yes	Yes	Yes
Enumerator Controls	No	No	No	No	Yes	Yes	Yes	Yes
Location Controls	No	No	No	No	Yes	Yes	Yes	Yes
Observations	244	244	244	244	237	237	237	237
R-squared	0.002		0.028		0.198		0.213	
Log-likelihood		-407.2		-403.6		-368.8		-366.2

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$
Robust t-statistics based on standard errors clustered at the session level in parentheses.
Dependent variable is the respondent's choice in the specified decision problem.

REINSURING THE POOR: GROUP MICROINSURANCE DESIGN AND COSTLY STATE VERIFICATION

DANIEL J. CLARKE*

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Abstract

This paper analyses collusion-proof multilateral insurance contracts between a risk neutral insurer and multiple risk averse agents in an environment of asymmetric costly state verification. Optimal contracts involve the group of agents pooling uncertainty and the insurer acting as reinsurer to the group, auditing and paying a claim only when the group or a sub-group has incurred a large enough aggregate loss. We interpret our models as providing support for insurance contracts between insurance providers, such as microinsurers or governments, and groups of individuals who have access to cheap information about each other, such as extended families or members of close-knit communities. Such formal contracts complement, and could even crowd in, cheap nonmarket insurance arrangements.

JEL codes: D14, D82, G22, O16.

*DPhil student: Department of Economics, Centre for the Study of African Economies ('CSAE') and Balliol College, University of Oxford (**clarke@stats.ox.ac.uk**; http://www.stats.ox.ac.uk/∼clarke). This paper forms part of my DPhil thesis, which is supervised by Professors Sujoy Mukerji and Stefan Dercon; the work would not have been possible without their very generous assistance. A number of others have provided very useful comments on aspects of the paper; without implicating them in the shortcomings of the work, I thank Keith Crocker, Ian Jewitt, Markus Loewe, Rocco Macchiavello, Olivier Mahul, Meg Meyer, Joe Perkins, Francis Teal, John Thanassoulis and Liam Wren-Lewis. I have presented the paper at the Gorman Student Research Workshop at the University of Oxford, the CSAE Research Workshop, an ESF SCSS Exploratory Workshop 2009, the CSAE Conference 2010, ERD Regional Conference 2010 and the CEAR Insurance for the Poor Workshop 2010. ESRC funding is gratefully acknowledged.

1 INTRODUCTION

Being poor in a poor country is risky. Not only is income from agriculture or the informal sector unpredictable, but the constant threat of health or mortality shocks leaves households vulnerable to serious hardship (Dercon 2004, Collins, Morduch, Rutherford and Ruthven 2009). In the absence of risk management tools the poor remain vulnerable to shocks; a typical response to a large income shock is to take children out of school (Jacoby and Skoufias 1997) and reduce nutritional intake (Behrman and Deolalikar 1988), particularly for girls (Behrman and Deolalikar 1990) and women (Dercon and Krishnan 2000).

One widely observed risk coping mechanism is that of risk pooling within families or communities. Nonmarket risk pooling can be informal (Morduch 2002) or through semiformal institutions that mutualise specific perils such as funerals (Dercon, De Weerdt, Bold and Pankhurst 2006), health (Jütting 2004) or fire (Cabrales, Calvó-Armengol and Jackson 2003). However, any risk pooling within a small community will be subject to a budget constraint, leaving households vulnerable to shocks that affect the whole community, and may also be constrained by the limited ability of households to commit to state contingent transfers.

Contracting with a formal institution, such as an insurer or government, could break budget and limited commitment constraints but is typically subject to large deadweight transaction costs and information asymmetries. In practice, formal insurance in poor countries is underdeveloped or nonexistent, leaving households exposed to shocks that are not diversifiable within their risk pooling network (Karlan and Morduch 2009, Section 7).

One major cause for the departure from first best risk sharing between formal insurers and poor individuals is the cost of ex-post claims processing, known as *loss adjustment*. Loss adjustment includes both the cost of verifying that claims are not fraudulent (*auditing*, to use the terminology of Townsend (1979)) and the cost of subsequently paying valid claims. In practice, loss adjustment costs are substantial for providers of formal indemnity-based insurance such as reinsurers, insurers and governments.[1] However, loss adjustment may be inexpensive within groups of individuals with intertwined economic or social lives, or within groups of firms in similar or complementary lines of work. Formal loss adjustment can generate a substantial variable deadweight cost and can therefore be a key determinant of the shape, not just the existence, of optimal insurance arrangements.

Much of the existing insurance design literature characterises efficient bilateral contracting between an insurer and a single policyholder. The baseline bilateral insurance contract in this literature is one in which the insurer receives a fixed premium from the policyholder and, in return, offers full marginal insurance below a deductible. Originally motivated by Arrow (1963) as the optimal contract when the insurer set premiums as a defined function of the actuarial value of a contract, it has since appeared as the optimal contract in models of ex ante moral hazard (Hölmstrom 1979) and costly state verification both without (Townsend 1979) and with (Picard 2000) sabotage. There is also a rich literature motivating and explaining more general forms of bilateral insurance contracts.[2]

Some authors have investigated multilateral insurance contracting. Arnott and Stiglitz (1991) consider the case of optimal contracting between an insurer and multiple agents under ex ante moral hazard where agents can side contract. However they only consider the case where the insurer sells a bilateral contract to each agent and do not consider the form of more general multilateral contracts. Ghatak and Guinnane (1999) and Rai and Sjostrom (2004) both consider models of multilateral credit contract design with the former allowing costly state verification and the latter instead allowing agents to send cross reports. However, both assume that agents are risk neutral and therefore ignore any demand for formal insurance.

[1] See for example Derrig (2002), from a special issue of the Journal of Risk and Insurance, focusing on insurance fraud.

[2] See Dionne, ed (2000) for an introduction to the literature.

This paper considers optimal multilateral contracting between a risk neutral insurer and two risk averse agents whose losses are affiliated and where the cost of loss adjustment, that is auditing and making transfers, is higher for the insurer than for agents. Auditing is modelled in Townsend's (1979) deterministic costly state verification framework; agents are required to send messages to the insurer, who only learns the true loss incurred by an agent by conducting an audit. In addition to not auditing or fully auditing the state of the world, the insurer may partially audit the state by auditing only one loss, rendering the problem a nontrivial multidimensional extension to Townsend (1979). The deadweight cost of loss adjustment by the insurer is assumed to increase in both the claim payment and the number of audits, and extends Arrow's (1963) single argument actuarial value cost function to two dimensions. By contrast, agents learn each other's losses and may make side transfers to each other at zero cost.

Agents are allowed to collude against the insurer and so, as is common in bilateral models of deterministic costly state verification, the aggregate net transfer from insurer to agents depends only on losses audited by the insurer; were the contract to depend on unaudited information, agents would always collude to send the jointly most favourable message, sharing the gains between themselves.

In the benchmark model we also allow each agents to increase their ex post loss by *sabotage*, without detection by the insurer. Allowing agents to conduct sabotage constrains the optimal contracts to feature no marginal overinsurance, since there will always be a no-sabotage contract that dominates any contract with sabotage.

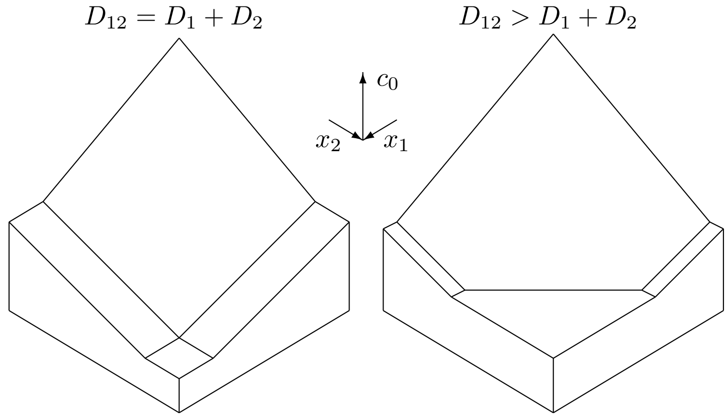
The economic problem is to design an arrangement that utilises the loss adjustment technology in an efficient fashion. Nonmarket insurance between agents is inexpensive but subject to a budget constraint. Formal insurance is not subject to a budget constraint but is expensive. As one might expect, efficient arrangements involve the agents offering mutual nonmarket insurance and the insurer offering protection for losses that are large for the group, relative to the corresponding increase in loss adjustment cost. The formal insurer therefore acts as reinsurer to the group who, in turn, can pool uncertainty between themselves at low informational and transactional cost.

The optimal solution in our benchmark model takes a particularly simple form, which we term a *Generalised Stop Loss* contract, named after the *Pure Stop Loss* contracts common in markets for reinsurance. For a group of two agents, an aggregate premium of p_0 is paid to the insurer, who in turn makes aggregate claim payment of $\max(0, x_1 - D_1, x_2 - D_2, x_1 + x_2 - D_{12})$ where x_1, x_2 are each agent's loss and D_1, D_2, D_{12} are single and double loss deductibles. This is the natural multidimensional extension to the bilateral insurance contract which offers full marginal insurance below a deductible.

Figure 1 shows an isometric projection of the total consumption of two agents who have jointly purchased a symmetric Generalised Stop Loss contract. The aggregate consumption schedule in the left pane could be achieved by both agents purchasing bilateral contracts from an insurer, each offering full marginal insurance against loss i above a deductible of D_i. However the consumption schedules in the middle and right pane could only be achieved by a contract where the claim payment to at least one of the agents is conditional on both losses. The key requirements for multilateral contracting to dominate bilateral contracting is that the group has access to cheap within-group loss adjustment technology, enabling cheap within-group monitoring and transfers. As is shown in section 6, formal insurers can use their contracting power to crowd in risk pooling within groups, and so within-group limited commitment or enforcement constraints do not invalidate the group-based approach to insurance.

When the insurer is able to detect sabotage for one loss, thereby removing the restriction of no marginal overinsurance for that loss, we find that optimal contracts are similar to Generalised Stop Loss contracts but with marginal overinsurance on the region for which only that loss is audited. The marginal overinsurance is optimal since a high loss incurred by one agent is associated with a high loss incurred by the second agent. This optimal contract is similar

Figure 1. *Aggregate net consumption of agents under a symmetric Generalised Stop Loss contract with single loss deductibles of $D_1 = D_2$ and double loss deductible of D_{12}*



to that of *model plot* area yield index agricultural insurance, in which the claim payout to all insured farmers within a locality depends on the average audited yield from a sample of model plots, chosen to be manipulation-free and statistically representative of the locality (Miranda 1991, Mahul 1999). In our framework such statistical sample-based indices are only ever optimal if the insurer can verify whether sabotage has occurred in the model plots. Moreover, the technology that allows the insurer to verify whether no sabotage has occurred is valuable, in that it allows a multilateral insurance contract to be put in place that strictly dominates all Generalised Stop Loss contracts.

The final model investigate the optimal contract when the insurer can condition claims on a costlessly observable index as well as auditable losses. The optimal contract is similar to the Index Plus Gap insurance policies of Doherty and Richter (2002), in which the indemnity-based element offers marginal insurance against losses net of indexed claim payments. In our multidimensional framework we find that the insurer pays an indexed payment and that the indemnity-based gap insurance is of the Generalised Stop Loss form, again based on losses net of the index claim payment. Pure index insurance is not optimal in general due to the possibility of the index realisation being good, but the aggregate group losses being high. In such joint states of the world, it may be optimal for loss adjustment to be conducted and a claim payment made (Chapter I).

These models offer a template for insurers wishing to offer efficient insurance arrangements to poor individuals. First, they should consider contracting with groups of individuals, economically and socially close enough to have access to a cheap loss adjustment technology and small enough to able to sustain risk pooling. Second they should use their contracting power to support nonmarket insurance between group members wherever possible. Third, they should think of their role as that of a reinsurer, offering protection when the group as a whole is unlucky. They should not offer cover for idiosyncratic shocks, as this would crowd out cheap within-group pooling. Fourth, both cheaply available indices that are affiliated with losses, and audit technologies that allow an insurer to verify when sabotage has taken place are valuable for the insurer, since they allow Pareto dominant insurance contracts to be signed between the insurer and agents.

Stop Loss Reinsurance in the Small and in the Large

These principles are already observable in insurance contracting around the world.

One example of pooling plus Stop Loss reinsurance is that of the Self-Insurance Funds (Fondos de Aseguramiento Agrícola) that have been operating in Mexico since 1988. Under the standardised legal framework farmers may join or create a Self-Insurance Fund, which allows farmers to pool agricultural uncertainty, but any such Fund must purchase Stop Loss reinsurance from a licensed insurance company. In 2004 Agroasemex, a reinsurer owned by the Mexican Government, sold pure Stop Loss products to 242 groups with total membership of more than 70,000 farmers (Ibarra 2004). The reinsurance product indemnifies each Self-Insurance Fund against total Fund losses above a Fund-level deductible. Unusually, it is a voluntary indemnity-based crop insurance scheme that has been self-funding in the medium term: as reported by Ibarra (2004), the total insurance payout by Agroasemex since inception was 57.7% of the total premium income by Agroasemex, above the break even net loss ratio of 75%.

The financial structure suggested in this paper for insurers and governments contracting with poor individuals is surprisingly similar to that observed between reinsurers and insurers or reinsurers and captive insurance companies in developed financial markets (Swiss Re 2002). One particularly famous arrangement that has existed since at least the 18th Century is the global structure of protection and indemnity (P&I) insurance for shipowners (Bennett 2000). The thirteen major P&I Clubs and the International Group of P&I clubs (IGP&I) coordinates not-for-profit mutual liability insurance for approximately 90% of the worldŠs ocean-going tonnage. The IGP&I then purchases one reinsurance policy from international reinsurers to protect shipowners worldwide against the IGP&I incurring a very large loss, which each shipowner is jointly and severally liable for. The theory of this paper provides a positive interpretation of such a structure if P&I Clubs, whose members have close economic ties, can conduct loss adjustment at a lower cost than the external reinsurer.

The rest of the paper is organised as follows. Sections 2 and 3 set up and characterise optimal contracts in our benchmark model. In Section 4 we allow the insurer to discriminate between sabotage and raw losses for one agent, in Section 5 we allow the insurance arrangement to be conditioned on a costlessly observable index, and in Section 6 we consider the case where agents cannot commit to a complete state contingent contract. Section 7 concludes. All proofs, unless otherwise noted in the text, are in the Appendix.

2 THE BENCHMARK MODEL

2.1 Preferences, Losses, and Information

Two agents contract with a single insurer. Preferences for agent i are represented by a twice differentiable utility function u_i, defined over own consumption c_i. Agents are strictly non-satiated and risk-averse, that is $u_i' > 0$ and $u_i'' < 0$ everywhere for all agents i. The insurer is risk-neutral, maximising expected profits.

Each agent i has initial wealth w_i and is subject to uncertain loss x_i which can take values in the interval $[0, \bar{x}]$. A state of the world x is a pair of losses $x = (x_1, x_2) \in X$ where $X = [0, \bar{x}] \times [0, \bar{x}]$. The exogenous cumulative density function of x is common knowledge and denoted by $F(x)$. F is atomless with full support on X, and losses are *affiliated* in the sense of Milgrom and Weber (1982); that is we can denote the probability density function by $f : X \to (0, \infty)$ where

$$f(x)f(x') \leq f(x \vee x')f(x \wedge x') \text{ for all } x, x' \in X \tag{1}$$

with $x \wedge x' = (min\{x_i, x_i'\})_{i=1}^2$ and $x \vee x' = (max\{x_i, x_i'\})_{i=1}^2$. Loss affiliation captures the notion that losses x_i and x_j are everywhere positively correlated, and will act to ensure that the optimal claim payment when only one loss is audited is nondecreasing in the loss.

Each agent i observes the realised joint state of the world x costlessly and may then increase own loss x_i by sabotage of $s_i \in [0, \bar{x} - x_i]$. Sabotage choices are observed by the other agent and then each agent sends a message m_i to the insurer. The insurer can observe zero, one or both losses by conducting costly audits. Following Picard (2000), the audit technology does not allow the insurer to distinguish between raw loss x_i and loss due to sabotage s_i, but is otherwise perfect; if the insurer audits agent i ($a_i = 1$), it discovers the total loss $x_i + s_i$; if the insurer doesn't audit agent i ($a_i = 0$), it discovers nothing.[3] We will find that allowing agents to conduct sabotage ensures that optimal contracts will feature no marginal overinsurance. (We relax the assumption that the insurer cannot discriminate between raw loss x and sabotage s in Section 4.)

Each agent i makes net transfer θ_i to the insurer and net side transfer τ_i to the other agent. Side transfers are not observable by the insurer. The resulting ex-post consumption of agent i is denoted by:

$$c_i = w_i - x_i - s_i - \theta_i - \tau_i \tag{2}$$

Loss adjustment, that is auditing and making transfers θ_i, incurs a deadweight cost for the insurer. We defer full specification of the deadweight loss adjustment cost function $\kappa(\cdot)$ until we have introduced and rewritten the incentive compatibility constraints in section 2.3.

2.2 Mechanisms and Side Contracts

The ex-post asymmetry in the cost of loss adjustment leads to an implementation problem similar to that considered by Townsend (1979). However, while Townsend (1979) considered the form of optimal contracts between an insurer and a single agent, we will consider the form of an optimal multilateral mechanism between an insurer and multiple agents.

A mechanism $G = (\{M_i, a_i, \theta_i\}_{i=1,2})$ will be offered to agents and the insurer by an independent mechanism designer, where each element has the following interpretation: M_i is the message space of agent i; $a_i : M_1 \times M_2 \rightarrow \{0, 1\}$ is the auditing function (audit (1) or no audit (0)); and $\theta_i : M_1 \times M_2 \times X \rightarrow \mathbb{R}$ is the net

[3] Our sabotage assumption may be considered as an extreme form of costly state falsification, where an agent can falsify a loss x_i as $x'_i \geq x_i$ at cost $x'_i - x_i$. Optimal insurance contracts under costly state falsification, where the marginal cost of falsification is less than unity can feature overpayment of small and underpayment of large claims (Bond and Crocker 1997, Crocker and Morgan 1998).

transfer from agent i to the insurer, and is restricted to vary only with messages and audited losses, not unaudited losses.[4] As is common in insurance models with costly state verification, the insurer is restricted to choose an audit rule that is a deterministic function of joint message m.[5] If either agent or the insurer reject G the insurer receives zero profit and each agent i receives reservation utility $\bar{u}_i$.

The form of the optimal mechanism will depend on the ability of agents to write binding side contracts with each other. If agents cannot contract with each other at all over messages m, side-transfers τ or the sabotage of losses s then the insurer can implement full insurance without any auditing in equilibrium:

Proposition 1. *If agents cannot side-contract then any consumption schedule $(c_1(x), c_2(x))$ such that each c_i is nonincreasing in x_i can be implemented by dominant strategies, with no auditing or sabotage in the equilibrium. If each c_i is strictly decreasing in x_i for all x_i then the implementation is unique.*

The insurer is able to fully extract information by setting up a Prisoner's Dilemma game to be played by the agents, where the insurer audits both agents whenever reports of the total loss disagree, rewards any truthful agent, and punishes untruthful agents. So long as each $c_i(x)$ is nonincreasing in own net loss then each agent will have no incentive to conduct sabotage, and if $c_i(x)$ is strictly decreasing in own loss then sabotage is never optimal.

If there is no deadweight loss adjustment cost when no audits are conducted, Proposition 1 implies that the first best, where full insurance is offered to both risk averse agents with no deadweight loss adjustment cost, can be implemented. Moreover, unique implementation is possible for a schedule that approximates full insurance.

[4] Formally, the restriction on θ_i is $\theta_i(m_1, m_2, x) = \theta_i(m_1, m_2, x')$ for all $(m_1, m_2) \in M,\ x, x' \in X$ such that $a_k(m_1, m_2) \times (x_k - x'_k) = 0$ for $k = 1, 2$.

[5] An exception is Fagart and Picard (1999), who characterise the optimal bilateral insurance contract under costly state verification when the audit rule is stochastic. However, in their model the global incentive compatibility constraints do not reduce to a local first order constraint except in the special case where the risk averse agent has constant absolute risk aversion. In another work Krasa and Villamil (2000) are able to show that optimal stochastic audit rules reduce to deterministic audit rules in a model of costly state verification with a form of renegotiation-proofness.

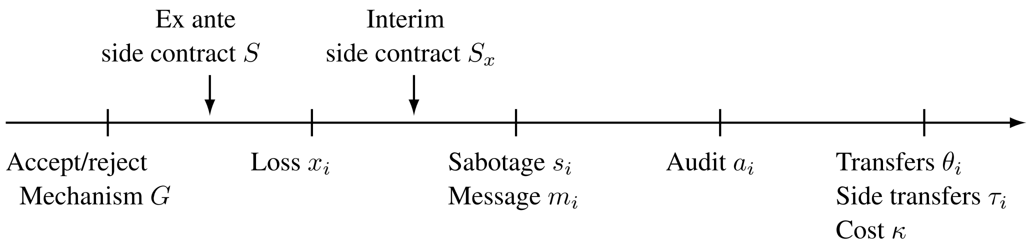
However, the assumption that agents cannot side contract at all seems unrealistic, particularly in situations where agents collectively have much to gain from conspiring against the insurer. We will henceforth assume that some degree of coordination between agents is possible. Such coordination could be modelled by an extensive form bargaining game between the agents over messages to be sent and corresponding side-transfers between agents. Following Laffont and Martimort (2000) and Rai and Sjostrom (2004) we abstract from this approach, and allow agents to sign binding side-contracts. We assume that the side-contracts are enforceable, although we do not specify a court of justice able to enforce such contracts.[6] Any enforcement constraints will be modelled in reduced form as restrictions on the side-contracts that may be signed. This is a modelling short cut, which allows us to capture in a static context the reputations of the agents which would guarantee self-enforceability in a repeated relationship. In the context of microinsurance, agents are likely to be able to threaten social sanctions, exclusion from nonmarket insurance or physical violence.

A side contract is a specification of the sabotage s_i to be performed by each agent, the message m_i to be sent by each agent to the insurer, and the net side-transfer τ_i from agent i to j, satisfying $\tau_1 + \tau_2 \geq 0$. By allowing agents to side contract, the mechanism designer cannot hope to elicit information by rewarding an individual agent, as that agent can commit to transfer any such reward to the other agent. The mechanism of Proposition 1 would unravel, with agents jointly choosing sabotage amounts and messages with the lowest aggregate transfer to the insurer, $\theta_0 = \theta_1 + \theta_2$, and the insurer never detecting the fraud.

Following Rai and Sjostrom (2004) we will consider two extreme classes of side contracts which differ in the ability of agents to pool uncertainty by committing to state contingent side transfers. A side contract where agents can fully commit to a state contingent transfer rule before the state of the world x is realised, is an *ex ante side contract* $S = (\{s_i, m_i, \tau_i\}_{i=1,2})$ where $s_i : X \to X$, $m_i : X \to$

[6] However, note that in many semiformal risk pooling arrangements there is an explicit process for arbitration and enforcement (Dercon et al. 2006, Jütting 2004, Cabrales et al. 2003).

Figure 2. *Timeline*



M_i and $\tau_i : X \to \mathbb{R}$. Under ex ante side contracting agents are able to sign Pareto optimal side contracts, performing sabotage and sending messages that maximise joint consumption $c_0 = c_1 + c_2$ and committing to a Pareto optimal consumption sharing rule. We consider the case of *interim side contracting* in Section 6, where agents are only able to sign binding contracts after the state of the world is realised. Under interim side contracting agents are able to collude against the insurer but are not able to commit to a Pareto optimal consumption sharing rule.

The timing is shown in figure 2.

2.3 Feasibility

Some restrictions on mechanisms are now in order. The mechanism designer will restrict attention to mechanisms which satisfy certain incentive compatibility and individual rationality (participation) constraints. Mechanism G and ex ante side contract S are together individually rational if expected utility for each agent exceeds the respective reservation utility, and the insurer receives nonnegative expected profit. Moreover, for a given mechanism G, an ex ante side contract S will be called incentive compatible if the agents could not sign a side contract that would be better for both agents:

(IR1) (For individual rationality under ex ante side contracting)

$\mathbb{E}[\theta_0] - \kappa \geq 0$ and $\mathbb{E}[u_i(c_i)] \geq \bar{u}_i$ for $i = 1, 2$.

(IC1) (For incentive compatibility of an ex ante side contract)

There is no other individually rational side contract which gives strictly higher expected utilities to both agents.

Under ex ante side contracting the revelation principle holds and allows us to narrow focus to direct mechanisms, where both agents report the joint total loss $x + s \in X$. We represent a direct mechanism as $G = (a, \theta)$, suppressing the message space $M = M_1 \times M_2$ which is taken to be $X \times X$. We will denote the outcomes when messages are truthful as indirect functions of the total loss $x + s$. That is, $a(x + s) := a(x + s, x + s)$, $\theta(x + s) := \theta(x + s, x + s, x + s)$ and $c(x + s) := w - x - s - \theta(x + s) - \tau(x + s)$ for all $x \in X$. For a given side-contract, the restrictions on mechanisms that guarantee that truth-telling messages are optimal for the agents will be called *incentive compatibility*:

(IC2) (For truthful report of the net loss)

There is an incentive compatible side contract with truthful messages, $m_1(x + s) = m_2(x + s) = x + s$ for all $x + s \in X$.

A mechanism G will be called *feasible* if there exists some ex ante side contract S which, together with G, satisfies (IR1), (IC1) and (IC2). For a feasible mechanism we will assume that all parties accept the mechanism and agents sign a side contract which, together with the mechanism, is individually rational and incentive compatible with truthful message rule $m(x) = (x, x)$ for all $x \in X$.

Optimality of a mechanism will be taken to mean constrained Pareto dominance in the following sense. A mechanism will be defined to *weakly dominate* another mechanism if for every feasible side contract for the latter mechanism, there is a feasible side contract for the former mechanism such that the expected utilities for all agents and the insurer are at least as large as in the other mechanism and side contract. The former *dominates* the latter if the weak dominance relation holds with strict inequality for at least one agent. A mechanism is defined to be optimal if it is not dominated by any other mechanisms. As usual, the use of

Pareto dominance in the definition of optimality allows us to abstract from the question of how the gains from trade should be split. As will become clear, the optimal form of the contract will not depend on the allocation of gains from trade.

We may now begin to characterise optimal feasible insurance mechanisms. First we will show that any feasible mechanism is weakly dominated by a mechanism with no marginal overinsurance, that is where

$$x_0 + \theta_0(x) \text{ is weakly increasing in each } x_i,\ i = 1, 2, \tag{3}$$

for $x_0 = x_1 + x_2$.

Lemma 1. *Any direct feasible mechanism under ex ante side contracting* (a, θ) *is weakly dominated by a direct feasible mechanism* (a', θ') *that satisfies condition* (3)

Under a feasible mechanism that satisfies condition (3) and for any incentive compatible side contract $S = (s, m, \tau)$ then $s(x) = 0$ almost everywhere.[7] Therefore without loss of generality we restrict attention to feasible mechanisms that satisfy condition (3) and incentive compatible side contracts in which $s(x) = (0, 0)$ for all $x \in X$.

Second we note that θ_0 can only vary with audited information, that is losses that the insurer has learned through auditing. If this were not the case, and θ_0 varied with messages sent by the agents even as the audited information remained the same, then any side contract that satisfied $(IC1)$ would specify that in each state of the world agents would send the joint message that resulted in the lowest $\theta_0 = \theta_1 + \theta_2$, violating $(IC2)$. We may therefore state the following result which extend's Townsend's (1979) rewrite of the bilateral incentive compatibility constraints to the multilateral case.

Lemma 2. *If* $a(x)$ *and* $\theta_0(x)$ *are feasible then there is a constant* p_0 *and functions*

[7] That is to say the set $\{x \in X | s_1(x) + s_2(x) > 0\}$ has zero measure.

$y_1(x_1)$, $y_2(x_2)$ and $z(x)$ such that

$$\theta_0(x) = p_0 - \max(y_1(x_1), y_2(x_2)) - z(x) \tag{4a}$$

$$a_i(x) = 1 \textit{ if } \begin{cases} y_i(x_i) > y_j(x_j) \textit{ or} \\ y_i(x_i) = y_j(x_j) > 0 \textit{ and } a_j(x) = 0 \textit{ or} \\ z(x) > 0 \end{cases} \tag{4b}$$

$$\textit{where } p_0, y_1(x_1), y_2(x_2), z(x) \geq 0 \textit{ for all } x \in X. \tag{4c}$$

The notation may be interpreted as follows. p_0 may be interpreted as the total premium paid by group members to the insurer, with $p_0 - \theta_0(x)$ as the total claim payment to group members in state x. $y_1(x_1)$ and $y_2(x_2)$ are minimum claim payments payable to the group based on either agent 1's or agent 2's losses respectively, such that if the state is x the total claim payment to the group is at least $\max(y_1(x_1), y_2(x_2))$. $z(x)$ is the additional claim payment payable in addition to $\max(y_1(x_1), y_2(x_2))$ if a second loss is audited.

A sketch of the proof is as follows. First, as is typical in models with deterministic auditing rules, θ_0 must be constant over the region where no audits occur. If not, the group would have an incentive to side contract so that equilibrium reports were of the zero audit state of the world with the lowest θ_0. Truthtelling would not be incentive compatible, violating (IC2). We may therefore write $\theta_0(x)$ for those x such that $a_1(x) = a_2(x) = 0$ as some constant p_0.

Second, unaudited states cannot result in a higher claim payment from insurer to agents than audited states, or the group would have an incentive to report a zero audit state, violating (IC2). Therefore $\theta_0(x) - p_0$ must be nonnegative for all x. Since expected insurer profits must be nonnegative, $p_0 \geq 0$.

Third, consider some x_1, x_1', x_2, x_2' such that in state (x_1, x_2') only loss 1 is audited and in state (x_1', x_2) only loss 2 is audited. For the group not to have the incentive to misreport either (x_1, x_2') or (x_1', x_2) when the true state was (x_1, x_2) it must be that $\theta_0(x_1, x_2) \leq \theta_0(x_1, x_2')$ and $\theta_0(x_1, x_2) \leq \theta_0(x_1', x_2)$. When only one agent is audited in state (x_1, x_2) it must be that $\theta_0(x_1, x_2) = \min(\theta_0(x_1, x_2'), \theta_0(x_1', x_2))$. For each x_i, if there is some x_j such that only agent i is audited, we define $y_i(x_i) := p_0 - \theta_0(x_1, x_2)$ and if there is no such x_j we define $y_i(x_i) := 0$. When only one agent is audited we may therefore write $\theta_0(x) = p_0 - \max(y_1(x_1), y_2(x_2))$.

Fourth, in any state x where both agents are audited the above inequality need not bind, and so we have $\theta_0(x) \leq p_0 - \max(y_1(x_1), y_2(x_2))$. Therefore there exists a $z(x) \geq 0$ such that (4a) holds.

2.4 Loss Adjustment Cost

We are now able to state our assumption about the cost of loss adjustment. Define $y(x) := \max(y_1(x_1), y_2(x_2))$. Then:

Assumption 1. *The deadweight loss adjustment cost to the insurer of a feasible mechanism $\{a, \theta\}$ is $\kappa(\mathbb{E}y, \mathbb{E}z)$ where $\kappa(0,0) \geq 0$ and $D_2\kappa(Y,Z) \geq D_1\kappa(Y,Z) > 0$ for all $Y, Z \geq 0$.*[8]

Assumption 1 is in effect an extension of Arrow's (1963) assumption that the insurer is willing to offer an insurance policy at a premium which depends only on the policy's actuarial value, that is the expected claim payment. In our environment the expected deadweight cost depends on the auditing structure as well as the actuarial value. The assumption $D_2\kappa \geq D_1\kappa > 0$ captures the notion that loss adjustment is costly and that it is at least as costly to increase the claim payment when two audits are necessary as when only one audit is necessary.

Suppose that the mechanism designer wanted to increase the claim payment in some state x where $y_i(x_i) > y_j(x_j)$. $y_i(x_i)$ could be increased, but this would necessitate increasing the minimum claim payment for all states x' in which agent

[8] The notation $D_i\kappa$ denotes the partial derivative of κ with respect to the ith argument.

i's loss was x_i. The designer could instead increase $z(x)$, without the need to increase claim payments in other states, but this increase would be subject to a larger deadweight cost than that from increasing $y(x)$ in state x.

Arrow's (1963) assumption is a special case of assumption 1, where $\kappa(\mathbb{E}y, \mathbb{E}z) = \kappa(\mathbb{E}y + \mathbb{E}z)$. A simple linear example of a loss adjustment function that satisfies assumption 1 is where the realised cost of loss adjustment in some state x is $\kappa_0 + \kappa_1 \times y(x) + \kappa_2 \times z(x)$ for constants $\kappa_2 \geq \kappa_1 > 0$ and $\kappa_0 \geq 0$.

3 THE BENCHMARK MODEL AND STOP LOSS CONTRACTS

Under ex ante side contracting we first reduce the multilateral contracting problem between two agents and an insurer to a bilateral problem between an insurer and a group representative agent. We parameterise optimal contracts by λ, the expected profit of the insurer, and show that any optimal mechanism maximises the expected utility of a group representative agent, subject to our rewritten incentive compatibility constraints. Denoting the expected profit of the insurer by $\pi(p_0, \mathbb{E}y, \mathbb{E}z) := p_0 - \mathbb{E}y - \mathbb{E}z - \kappa(\mathbb{E}y, \mathbb{E}z)$ and the group consumption by $c_0 := w_0 - p_0 - x_0 + \max(y_1, y_2) + z$ we have the following:

Lemma 3. *If a mechanism (a, θ) is optimal under ex ante side contracting then there exists constant λ and strictly increasing concave function u_0 such that $\theta_0 = p_0 - \max(y_1, y_2) - z$ is a solution to:*

$$\max_{p_0, y_1, y_2, z} \mathbb{E}u_0(c_0) \textit{ subject to } \pi = \lambda, \text{ (3) } \textit{and} \text{ (4c)} \tag{5}$$

Following Wilson (1968) we may interpret u_0 as a surrogate group utility function, defined over group consumption $c_0 = c_1 + c_2$, with corresponding belief $f(x)$. The objective function is therefore the expected utility of the group representative agent, and is an increasing linear function of the both realised utility of agent 1 and that of agent 2 under a Pareto optimal sharing rule.

Program (5) implicitly specifies a_i for each loss i of the state x, but only aggregate transfers θ_0 between insurer and group are specified: the program does not specify the split of insurance claim payments between agents. The specific allocation of claim payments does not matter because agents can contract with each other to undo any allocation of the consumption good determined by the insurer. Indeed, agents will contract so that net consumption profiles on the truthful equilibrium path satisfy the Borch (1962) rule.[9] Neither does the program specify the punishments levied on agents out of equilibrium, but without loss of generality we may assume that when any message is found to be incorrect, the aggregate net transfer to insurer from agents is p_0.

Our reduced problem is mathematically similar to those considered in Townsend (1979), but with the addition that the state can be partially audited; the insurer can choose to audit just one loss, and therefore only partially learn the state of the world, whereas in Townsend's models the state was one-dimensional and so any audit yielded the full state.

A mechanism (a, θ) is said to be a Generalised Stop Loss contract if

$$\theta_0(x) = p_0 - \max(0, x_1 - D_1, x_2 - D_2, x_0 - D_{12}) \text{ for almost all } x \in X \quad (6)$$

for some $D_1, D_2 \in [0, \bar{x}]$ and $D_{12} \in [0, 2\bar{x}]$. A Generalised Stop Loss contract offers full marginal insurance for any one loss above a single loss deductible of D_i, and full marginal insurance for the combined loss above a group loss deductible of D_{12}. Net consumption of the pair of agents for almost all $x \in X$ is:

$$c_0(x) = w_0 - p_0 - \min(x_1 + x_2, D_1 + x_2, x_1 + D_2, D_{12}) \quad (7)$$

[9] The Borch Rule states that $u_1'(c_1(x))/u_2'(c_2(x))$ is constant for almost all $x \in X$. If not, there would be a side-contract with truthful equilibrium consumption that strictly dominated the original side-contract.

Figure 1 illustrates the net consumption under a Generalised Stop Loss contract for different individual and group deductibles D_1, D_2 and D_{12}.

We are now in a position to state our main result.

Theorem 1. *Any optimal feasible mechanism is a Generalised Stop Loss contract.*

The solution methods we employ to prove Theorem 1 follow Winton (1995) and Gollier and Schlesinger (1996) in relying only on notions of second order stochastic dominance. The two key assumptions underlying our result are our loss adjustment cost assumption and the assumption that losses are affiliated. Without the affiliation assumption we could not guarantee that indemnity schedules would be increasing in the incurred loss.

An alternative loss adjustment cost assumption would be that the realised loss adjustment cost depended only on the number of audits conducted. However, this extension of Townsend's (1979) constant fixed audit cost assumption is not easy to work with in a model with incentive compatibility restrictions (4a) and (3). For example, Picard (2000) considered optimal contracting under these assumptions in the bilateral case but could only characterise the optimal indemnity schedule after having restricted attention to schedules in which the claims payment was weakly increasing in incurred loss. In our model Theorem 1 holds under the fixed claim cost assumption if the pdf of losses f is weakly decreasing in both its arguments, but the optimal contract seems more difficult to characterise under less restrictive assumptions.

Theorem 1 follows through if agents are unable to sign complete state contingent contracts on (s, m, θ) but may still commit to a sharing rule, $(c_1(c_0), c_2(c_0))$ as the sharing rule would align the agent's incentives for sabotage and message decisions. We will further relax the assumption about the ability of agents to commit to ex ante side contracts in section 6 and will consider optimal contracts when sabotage is not possible, or is separately observable by the insurer, in section 4.

Theorem 1 also extends to a setting with N agents: the insurer would offer full marginal insurance for losses above a suite of individual, subgroup and group deductibles.

We may derive a corollary to our main Theorem by appealing to the loss adjustment function considered by Arrow (1963). A mechanism (a, θ) is said to be a Pure Stop Loss contract if

$$\theta_0(x) = p_0 - \max(0, x_0 - D_{12}) \text{ for almost all } x \in X \tag{8}$$

for some $D_{12} \in [0, 2\bar{x}]$. A Pure Stop Loss contract offers full marginal insurance for combined losses above the group deductible D_{12}. It is a special case of a Generalised Stop Loss contract with $D_1 = D_2 = \bar{x}$ or $D_{12} < \min(D_1, D_2)$. Net consumption of the pair of agents for almost all $x \in X$ is:

$$c_0(x) = w_0 - p_0 - \min(x_1 + x_2, D_{12}) \tag{9}$$

Corollary 1. *For $\kappa(\mathbb{E}y, \mathbb{E}z) = \kappa(\mathbb{E}y + \mathbb{E}z)$ any optimal feasible mechanism under ex ante side contracting is a Pure Stop Loss contract.*

That is, when the loss adjustment cost only depends on the actuarial value and not the audit schedule, the insurer offers the group full marginal insurance for group losses above some aggregate group deductible. The intuition is as follows. When κ is additive in its arguments there is no need to ever audit just one agent as the additional cost of auditing the other agent is zero. Program 5 therefore reduces mathematically to that considered by Arrow (1963) where our representative group agent with preferences represented by u_0 takes the place of Arrow's policyholder.

3.1 A comparison with bilateral insurance

Instead of offering a multilateral contract to both agents, an insurance company could arrange for a bilateral insurance contract to be signed with each agent. (In a bilateral mechanism the net transfer from each agent θ_i to the insurer must depend only on that agent's loss x_i.) How does our optimal contract differ from a set of bilateral contracts? Any set of bilateral contracts offering full marginal insurance to each agent below an agent-specific deductible D_i may be written as a Generalised Stop Loss contract with $D_1 + D_2 = D_{12}$. That is, the aggregate transfer from agents to insurer satisfies $\theta_0(x) = p_0 - \max(0, x_1 - D_1) - \max(0, x_2 - D_2)$ for some deductibles $D_1, D_2 \in [0, \bar{x}]$. Such a bilateral contract is weakly dominated by the optimal multilateral contract, with strict dominance when in an optimal multilateral mechanism, $D_{12} \neq D_1 + D_2$.

A suite of bilateral contracts (left panel of Figure 1) is strictly dominated by a multilateral contract if either the deadweight loss adjustment cost of increasing transfers in the double audit region is high (right panel of Figure 1) or low (central panel of Figure 1). In practice, we might expect the latter to be more likely, as the cost of auditing a second loss from the small community may not be much higher than the cost of auditing just the first loss.

4 VERIFIABLE SABOTAGE AND SAMPLE-BASED INDEX INSURANCE

Suppose that during an audit the insurer could separately identify sabotage decisions s_1 and losses x_1 for agent 1, and condition transfer function θ on this information, but such separate identification was not possible for agent 2's losses. The indemnity schedule would no longer need to be restricted to feature no marginal overinsurance for x_1; the insurer could ensure that s_1 was always zero by setting θ_0 to be p_0 whenever s_1 was observed to be nonzero. However by the following lemma, the indemnity would still need to be restricted to feature no marginal overinsurance for loss x_2.

$$x_0 + \theta_0(x) \text{ is weakly increasing in } x_2 \tag{10}$$

Lemma 4. *Any direct feasible mechanism (a, θ) under ex ante side contracting where the insurer can separately identify sabotage decisions and losses for agent* 1*, but not for agent* 2*, is weakly dominated by a direct feasible mechanism (a', θ') that satisfies condition* (10)

Maximisation Program 5 would become:

$$\max_{p_0, y_1, y_2, z} \mathbb{E}u_0(c_0) \text{ subject to } \pi = \lambda, \text{(4c) and (10)} \tag{11}$$

and the following Theorem characterises the form of optimal mechanisms:

Theorem 2. *In any optimal feasible mechanism under ex ante side contracting with verifiable sabotage there exist constants $D_1, D_2 \in [0, \bar{x}]$ and $D_{12} \in [0, 2\bar{x}]$ such that for almost all x:*

1. $\theta_0(x) = p_0$ *for all* $x_1 + x_2 \leq D_{12}$ *and* $x_i \leq D_i$, $i = 1, 2$*;*
2. $x_1 - y_1(x_1)$ *is nonincreasing in* x_1 *for all* $x_1 > D_1$*;*
3. $y_2(x_2) = \max(x_2 - D_2, 0)$*;*
4. $z(x) = \max[0, x_1 + x_2 - \max(y_1(x_1), y_2(x_2)) - D_{12}]$*;*

The optimal contract therefore retains some features from the Generalised Stop Loss contract: the equality of consumption on the double audit region; the full marginal indemnification for agent 2's losses above a single loss deductible of D_2; and the shape of the zero audit region. However, marginal overinsurance is offered for the first loss, above a first loss deductible of D_1. This marginal overinsurance is for two reasons. First, losses are affiliated and so an increase in loss x_1 also implies that loss x_2 is likely to be greater. As an extreme example, suppose that losses are close to being perfectly correlated and $D_2\kappa(Y,Z) > D_1\kappa(Y,Z)$ for all $Y,Z > 0$. Then the double audit region will be the null set and on the single audit region for agent 1, $\partial y_1(x_1)/\partial x_1 \approx 2 > 1$. The second reason for marginal overinsurance is that, since y_2 is increasing, as y_1 increases the set $\{x_2 | y_1 \geq y_2(x_2)\}$ expands in the direction of states with larger losses. This provides further impetus for an additional increase in y_1.

In the case in which loss affiliation is strong this optimal contract may be likened to model plot area yield index agricultural insurance. The insurer would only ever conduct audits on model plots, and the aggregate transfer to agents would be a multiple of the average loss incurred on local model plots above the single loss deductible. The insurance claim payment is therefore a function of a statistical sample of local plots.

The optimal contract of Theorem 2 would only be suitable for an insurance company to offer if it could guarantee that plots could not be sabotaged without the knowledge of the insurer. If sabotage were possible, in some states of the world agents would want to create further damage, insofar as the insurer could not differentiate between the initial loss and the extra damage.

5 Indices and Stop Loss Gap Insurance

Suppose now that there is a index $v \in V = [0, \bar{v}]$ which is jointly affiliated with the losses and costless for the insurer and agents to observe. A state of the world is now a triplet $\omega = (x_1, x_2, v) \in \Omega = X \times V$ with probability density function $f(\omega)$ and the joint affiliation assumption may be written as:

$$f(\omega)f(\omega') \leq f(\omega \vee \omega')f(\omega \wedge \omega') \text{ for all } \omega, \omega' \in \Omega \tag{12}$$

A direct mechanism will be a pair (a, θ) as before, but where the audit rule and claims transfer function can also depend on the index.

We could again reduce attention to mechanisms which satisfy the no marginal overinsurance of incurred losses condition (3) and the following revised equations (4a) and (4c):

$$\theta_0(\omega) = p_0 - I(v) - \max(y_1(x_1, v), y_2(x_2, v)) - z(x, v) \tag{13a}$$

$$p_0, I(v), y_1(x_1, v), y_2(x_2, v), z(x, v) \geq 0 \text{ for all } \omega \in \Omega \tag{13b}$$

A natural extension to assumption 1 would be that the deadweight loss adjustment cost to the insurer would be denoted by $\kappa(\mathbb{E}I, \mathbb{E}y, \mathbb{E}z)$ where $\kappa(0, 0, 0) \geq 0$ and $D_i\kappa(I, Y, Z)$ is weakly increasing in $i = 1, 2, 3$ for all $I, Y, Z \geq 0$.

Doherty and Richter (2002) considered a similar model when there was only one loss and the friction was ex ante moral hazard, rather than costly loss adjustment. They considered insurance products which offered a claim payout that was a linear function of the index plus a linear function of the *gap*, which they defined to be the incurred loss minus the indexed payout. The optimal contracts in our setting will be the multidimensional extension of Doherty and Richter's (2002) index plus gap insurance.

Following the terminology of Doherty and Richter (2002) a mechanism (a, θ) is said to offer Index Plus Generalised Stop Loss Gap insurance if:

$$\theta_0(\omega) = p_0 - \max(I(v), x_1 - D_1, x_2 - D_2, x_0 - D_{12}) \text{ for almost all } \omega \in \Omega \tag{14}$$

for some $I(v) : V \to [0, \infty)$, $D_1, D_2 \in [0, \bar{x}]$ and $D_{12} \in [0, 2\bar{x}]$. Such a composite contract offers an indexed payout of $I(v)$ but if individual or joint losses are large enough there will be an indemnity based top-up of $\max(0, x_1 - I(v) - D_1, x_2 - I(v) - D_2, x_0 - I(v) - D_{12})$. Net consumption of the pair of agents for almost all $\omega \in \Omega$ is:

$$c_0(\omega) = w_0 - p_0 - \min(x_1 + x_2 - I(v), D_1 + x_2, x_1 + D_2, D_{12}) \tag{15}$$

Theorem 3. *Any optimal feasible mechanism under ex ante side contracting with a costlessly observable index offers Index Plus Generalised Stop Loss Gap insurance. Any optimal index claim function is weakly increasing in the index realisation.*

The general form of the optimal contract optimal contract in this case is an extension to the Generalised Stop Loss contract of section 3, but where indemnity payments are based on total audited losses net of the index payment $I(v)$.

A mechanism (a, θ) is said to offer Index Plus Pure Stop Loss Gap insurance if:

$$\theta_0(\omega) = p_0 - \max(I(v), x_0 - D_{12}) \text{ for almost all } \omega \in \Omega \tag{16}$$

Corollary 2. *For* $\kappa(\mathbb{E}I, \mathbb{E}y, \mathbb{E}z) = \kappa(\mathbb{E}I, \mathbb{E}y + \mathbb{E}z)$ *any optimal feasible mechanism under ex ante side contracting with a costlessly observable index offers Index Plus Pure Stop Loss Gap insurance*

That is to say, in the optimal contract the group receives an indexed payment $I(v)$ which depends only on the realised index and indemnity based payouts provide a floor of D_{12} on aggregate net loss $x_0 - I(v)$.

6 Crowding in

Although there is ample evidence that the poor find affordable ways to share risk within close knit groups, such as households, extended families or villages, such pooling is rarely observed to be Pareto optimal.[10] A side contract where agents can only commit to the transfer rule after the state of the world x is realised will

[10] There is ample evidence that mutual insurance within close knit groups, such as households, extended families or villages departs from first best (e.g. Townsend 1994, Udry 1994, Dercon 2002, Fafchamps and Lund 2003). Informal punishments such as exclusion from nonmarket insurance, social sanctions or physical violence may be able to induce small, but not large, insurance transfers (Coate and Ravallion 1993). Empirical investigations such as Ligon, Thomas and Worrall (2002) provide evidence for the existence of binding enforcement constraints in poor communities, causing mutual insurance arrangements to depart from first best in the direction of Coate and Ravallion (1993).

be called an *interim side contract* $S_x = (\{s, m, \tau\}_{i=1,2})$ where $s \in X$, $m_i \in M_i$ and $\tau_i \in \mathbb{R}$. Abusing notation, a collection of interim side contracts $\{S_x\}_{x \in X}$ will be denoted S. In the absence of incentives from the insurer, interim side contracting does not allow losses to be pooled between group members; by the interim stage the loss uncertainty has already been resolved and a lucky agent has no incentive to agree to make transfers to an unlucky agent. However, agents will still be able to collude against the insurer, by sending messages that maximise the total consumption and undoing any contractual split of consumption dictated by the split of θ_i.

Following Rai and Sjostrom (2004) we will show that any outcome that can be implemented under ex-ante side contracting can also be implemented under interim side contracting, so long as the insurer has the ability to inflict a large enough non-pecuniary punishment $\bar{q}$ on each agent. Let the non-pecuniary punishment for agent i be $q_i : M_1 \times M_2 \times X \rightarrow [0, \bar{q}]$. This could be the denial of future financial services or the cost of being 'hassled' by the insurer. Each agent i is assumed to have preferences defined over net consumption c_i and q_i with $u_i(c_i, q_i) = u_i(c_i - q_i)$. A mechanism under interim side contracting is taken to be a specification of (M, a, θ, q).

For a given mechanism G, an interim side contract S will be called incentive compatible if the agents could not sign a side contract that would be better for both agents in any state of the world. Individual rationality requires each side contract S_x to be accepted by both agents for each state $x \in X$. If the agents do not sign any side contract at the interim stage then they go on to play a Nash equilibrium of (G, x). Let $u_i(G, x)$ denote agent i's Nash equilibrium payoff.[11] $u_i(G, x)$ acts as the reservation utility for interim side contracting in state x, and so we may write the individual rationality and incentive compatibility constraints as:

[11] If there were multiple Nash equilibria of (G, x) we would assume that agents make some selection from the set of Pareto optimal Nash equilibria. However, the mechanism we construct has a unique Nash equilibrium.

(IR2) (For individual rationality under interim side contracting)

$\mathbb{E}[\theta_0] - \kappa \geq 0$, $\mathbb{E}[u_i(c_i)] \geq \bar{u}_i$ and $u_i(c_i(x)) \geq u_i(G, x)$ for $i = 1, 2$ and $x \in X$.

(IC3) (For incentive compatibility of an interim side contract)

There is no other individually rational side contract which gives strictly higher utilities to both agents in any state of the world.

A mechanism G will be called *feasible under interim side contracting* if there exists some ex ante side contract S which, together with G, satisfies (IR2), (IC2) and (IC3).

Assuming that the maximum punishment $\bar{q}$ is at least as large as the largest side transfer of the outcome we intent to implement, we may show the following:

Theorem 4. *Any outcome* (π^*, c_i^*, c_2^*) *from an optimal feasible mechanism under ex ante side contracting, and associated feasible ex ante side contract, is implementable by a feasible mechanism and interim side contract. Moreover, it is uniquely* ϵ*-implementable in the following sense: for any* $\epsilon > 0$ *there exists a mechanism* $G(\epsilon)$ *such that for any individually rational and incentive compatible side contract S,* $|c_1^*(x) - c_1(x)|, |c_2^*(x) - c_1(x)| \leq \epsilon$.

This result is equivalent to Proposition 3 of Rai and Sjostrom (2004) and relies on both the ability of the insurer to punish and that of agents to cross report. We show that Generalised Stop Loss contracts are still attractive even if agents cannot commit ex ante to pool uncertainty, so long as agents have a loss adjustment cost advantage and the insurer has sufficient ability to punish agents.

7 CONCLUSION

How is microinsurance, that is insurance for low-income people, economically different to conventional personal lines insurance? The key assumption modeled in this paper is that for microinsurance the cost of ex-post claims processing, known as *loss adjustment*, is lower for local nonmarket institutions relative to external formal sector insurers. Although multilateral credit contracts are now common, multilateral insurance contracts, where the claim payment to one policyholder explicitly depends on the losses incurred by other policyholders, are rarely observed in practice, with the exception of area yield agricultural insurance (Mahul and Stutley 2010). A normative interpretation of the above models provides support for particular multilateral insurance contract forms between premium-charging insurance companies or publicly funded social insurance schemes and poor individuals, namely the Stop Loss, Sample-Based Index and Index Plus Gap contract forms.

Development economists have a healthy suspicion of normative microeconomic theory brought about by the observation that the poor are usually more enterprising than the researcher. However, this suspicion is less well-founded in the context of formal and semiformal finance for the poor. The potential welfare gains from successful financial innovation are widely considered to be large (Banerjee 2002, Collins et al. 2009, Karlan and Morduch 2009). However, experimenting with financial innovations is costly and outcomes are difficult to evaluate, particularly for insurance contracts where payouts are typically expected to be made in only one year out of five. Financial innovations in developed markets have often been theory-led and the recent advances in positive microfinance theory leave economists well placed to make suggestions for improvements.

In their chapter on microfinance in the Handbook of Development Economics, Karlan and Morduch (2009) write: "[Micro-insurance] holds promise, but the field is young and no approaches have emerged so far that offer break-throughs akin to the original group-lending innovations that ignited the global explosion of microcredit". We wonder whether the contracts outlined in this paper might be useful here and there for formal contracting with the poor, particularly in the life, longevity and crop insurance classes of business.

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APPENDIX

Proof of Proposition 1. Consider the following direct mechanism. Let each agent report the total loss $s + x$, so $M_1 = M_2 = X$. Define $\theta_i^*(x) = w_i - x_i - c_i(x)$ where $(c_1(x), c_2(x))$ is the desired consumption schedule. If both agents report the same total loss x then neither agent is audited ($a_1(x, x) = a_2(x, x) = 0$) and transfers are such each agent's consumption follows the desired consumption schedule ($\theta_i(x, x, x') = \theta_i^*(x)$).

If agents report different states, then both agents are audited ($a_1(x', x'') = a_2(x', x'') = 1$ where $x' \neq x''$) and the insurer learns which agents have lied. If both agents have lied ($x' \neq x + s$ and $x'' \neq x + s$) then set $\theta_i(x', x'', x + s) = \theta_i^*(x + s) + \epsilon$ for some $\epsilon > 0$. If agent 1 tells the truth and agent 2 doesn't then set $\theta_1(x, x'', x + s) = \min(\theta_1^*(x''), \theta_1^*(x + s)) - \epsilon$ and $\theta_2(x + s, x'', x + s) = \theta_2^*(x + s) + \epsilon$. If agent 2 tells the truth and agent 1 doesn't then set $\theta_2(x', x + s, x + s) = \min(\theta_2^*(x'), \theta_2^*(x + s)) - \epsilon$ and $\theta_1(x', x + s, x + s) = \theta_1^*(x + s) + \epsilon$.

Under this mechanism misreporting the total loss ($m_i \neq x + s$) is strictly dominated by truthtelling ($m_i = x + s$) for both agents. If $c_i(x)$ is nonincreasing in x_i for $i = 1, 2$ then there is a Nash equilibrium where both agents choose $s_i = 0$ and then report truthfully, with no auditing on the equilibrium path. However, there are equilibria with sabotage by agent i in regions where $c_i(x)$ is constant over x_i. If $c_i(x)$ is strictly decreasing in x_i for $i = 1, 2$ then the no sabotage, truthtelling equilibrium is the unique equilibrium. □

Proof of Lemma 1. Define $\sigma : X \to X$ by $\sigma(x) = (x_1 + s_1(x), x_2 + s_2(x))$ $\forall x \in X$, $a' = a \circ (\sigma, \sigma)$, $\theta' = \theta \circ (\sigma, \sigma, (\sigma, Id))$, where θ is now a function of $m_1, m_2, (x, s_1)$.

Now for any feasible ex ante side contract $S = (s, \tau)$ we define $S' = (s', \tau)$ where $s'(x) = (0, 0)$ for all $x \in X$. Direct mechanism (a', θ') inherits feasibility from (a, θ) and under side contract S' both agents receive the same expected utility as under the original mechanism and S, but the insurer receives weakly higher expected profits as net transfers to agents are lower in states where $s(x) \neq (0, 0)$ under the original side contract. □

Proof of Lemma 2. For feasible (a, θ_0) define

$$p_0 := \max_{x \in X} \theta_0(x) \tag{A-1a}$$

$$y_i(x_i) := \min_{x_j \in [0, \bar{x}]} (p_0 - \theta_0(x)) \, , (i, j) \in \{(1, 2), (2, 1)\} \tag{A-1b}$$

$$z(x) := p_0 - \max(y_1(x_1), y_2(x_2)) - \theta_0(x) \tag{A-1c}$$

(A-1c) ensures that definitions (A-1a)-(A-1c) satisfy (4a).

To demonstrate that they satisfy (4c), first suppose $p_0 < 0$. Then $\theta_0(x) < 0$ for all $x \in X$, violating (IR1). So $p_0 \geq 0$. Since $\max_{x \in X} \theta_0(x) - \theta_0(x) \geq 0$, $y_i(x_i) \geq 0$ for $i = 1, 2$ and $x \in X$. Finally, substituting (A-1b) in to (A-1c) gives

$$\begin{aligned} z(x) &= -\max\left(\min_{x_1' \in [0, \bar{x}]} (-\theta_0(x_1', x_2)), \min_{x_2' \in [0, \bar{x}]} (-\theta_0(x_1, x_2')) \right) - \theta_0(x) \\ &= \min\left(\max_{x_1' \in [0, \bar{x}]} (\theta_0(x_1', x_2)), \max_{x_2' \in [0, \bar{x}]} (\theta_0(x_1, x_2')) \right) - \theta_0(x) \\ &\geq 0 \end{aligned}$$

where the final inequality arises from $\max_{x_1' \in [0, \bar{x}]} (\theta_0(x_1', x_2)) \geq \theta_0(x)$ and $\max_{x_2' \in [0, \bar{x}]} (\theta_0(x_1, x_2')) \geq \theta_0(x)$. Definitions (A-1a)-(A-1c) therefore satisfy (4c).

Now we show by contradiction that feasibility implies that 4b must hold for (A-1a)-(A-1c). First suppose that there is some $y_1(x_1) > y_2(x_2)$ such that $a_1(x_1, x_2) = 0$. $y_1(x_1) \leq p_0 - \theta_0(x_1, x_2)$ from (A-1b), with strict inequality if there was some $x_2' \in [0, \bar{x}]$ such that $\theta_0(x_1, x_2') > \theta_0(x_1, x_2)$. For (IC2) to hold $\theta_0(x_1', x_2) \leq \theta_0(x_1, x_2)$ for all $x_1' \in [0, \bar{x}]$ otherwise it would be optimal for agents to misreport state (x_1', x_2) as (x_1, x_2). Now, from (A-1b), $y_2(x_2) := \min_{x_1' \in [0,\bar{x}]}(p_0 - \theta_0(x_1', x_2)) = p_0 - \theta_0(x_1, x_2)$. However, combined with $y_1(x_1) \leq p_0 - \theta_0(x_1, x_2)$ it must be that $y_1(x_1) \leq y_2(x_2)$. Contradiction. The case with some $y_1(x_1) > y_2(x_2)$ such that $a_1(x_1, x_2) = 0$ follows trivially.

Second, suppose that there is some $x \in X$ such that $y_1(x_1) = y_2(x_2) > 0$ but $a_1(x) = a_2(x) = 0$. For (IC2) to hold $\theta_0(x') \leq \theta_0(x)$ for all $x' \in X$ otherwise it would be optimal for agents to misreport state x' as x. Therefore $p_0 = \theta_0(x)$ and $y_1(x_1) = y_2(x_2) = 0$ from (A-1a) and (A-1b). Contradiction.

Third, suppose that there is some $x \in X$ such that $z(x) > 0$ but $a_1(x) = 0$. For (IC2) to hold $\theta_0(x_1', x_2) \leq \theta_0(x_1, x_2)$ for all $x_1' \in [0, \bar{x}]$ otherwise it would be optimal for agents to misreport state (x_1', x_2) as (x_1, x_2). Now, from (A-1b), $y_2(x_2) := \min_{x_1' \in [0,\bar{x}]}(p_0 - \theta_0(x_1', x_2)) = p_0 - \theta_0(x_1, x_2)$ and so $z(x) = 0$ from (A-1c). Contradiction. The case with some $x \in X$ such that $z(x) > 0$ and $a_2(x) = 0$ follows trivially. □

Proof of Lemma 3. We may parameterise any constrained Pareto optimal mechanism and side contract by the expected utility of agent 2, μ, and the expected profit of the insurer, λ:

$$\begin{aligned}
\max_{p_0, y_1, y_2, z, c_1, c_2} \quad & \mathbb{E}u_1(c_1(x)) \\
& \text{subject to} \\
& \mathbb{E}u_2(c_2(x)) \geq \mu \\
& \pi(p_0, \mathbb{E}y(x), \mathbb{E}z(x)) \geq \lambda \\
& \text{(IC1), (IC2)}, \ y = \max(y_1, y_2), \ \theta_0 = p_0 - y - z \\
& \quad \text{and } c_1 + c_2 = w_0 - p_0 - x_0 + y + z
\end{aligned}$$

We may replace (IC1) and (IC2) with (3) and (4c). Both of these depend on aggregate consumption, but not the split of consumption between agents.

Denoting the Lagrangian multiplier on the budget constraint as $f(x) \times u_0'(c_0(x))$ for some function u_0', and that on the expected utility constraint for agent 2 as ν, the first order constraints for c_1 and c_2 yield:

$$u_1'(c_1^*(x)) = \nu u_2'(c_0^*(x) - c_1^*(x)) = u_0'(c_0^*(x)) \quad \forall x \in X$$

We may integrate u_0' to construct a function u_0 such that $u_0(c_0^*(x)) = u_1(c_1^*(x))$ for all x. u_0 is increasing and strictly concave from the strict increasing concavity of u_1 and u_2. The required result follows by substituting the expression for $u_0(c_0(x))$ into the objective function. □

Proof of Theorem 1. Throughout the proof we fix p_0, $\mathbb{E}y$ and $\mathbb{E}z$, and therefore the insurer's expected profit, and appeal to notions of second order stochastic dominance of aggregate consumption schedule c_0.[12] Suppose $\{p_0, y_1, y_2, z\}$ solved Program 5 but was not a Generalised Stop Loss contract.

First we show that:

$$z(x) = \max\left[0, x_1 + x_2 - \max(y_1(x_1), y_2(x_2)) - D_{12}\right] \tag{A-2}$$

for some D_{12}, offering a floor on consumption. Fix y_1 and y_2 and suppose that z does not satisfy equation (A-2). Define z' as in equation (A-2) for some D'_{12} chosen so that $\mathbb{E}z = \mathbb{E}z'$. Abusing notation, define random variables $c_0 = w_0 - p_0 - x_0 + y + z$ and $c'_0 = w_0 - p_0 - x_0 + y + z'$ with cdfs G_{c_0} and $G_{c'_0}$ respectively. By Gollier and Schlesinger (1996), c_0 is a mean preserving spread of c'_0 and so $G_{c'_0}$ strictly SSD G_{c_0}. c'_0 will therefore be strictly preferred by the representative group agent, whose preferences are strictly risk averse, and therefore by both agents.

To characterise optimal y_1, y_2 we appeal to the following Lemma to reduce attention to net consumption before the addition of z, $c_0^{-z} = w_0 - p_0 - x_0 + \max(y_1, y_2)$:

Lemma A1. *If c'_0 SSD c_0 and D'_{12}, D_{12} are set so that $\mathbb{E}\max(0, D'_{12} - c'_0) = \mathbb{E}\max(0, D_{12} - c_0)$ then $\max(c'_0, D'_{12})$ SSD $\max(c_0, D_{12})$. The dominance is strict iff the cdfs of $\max(c'_0, D'_{12})$ and $\max(c_0, D_{12})$ are not identical.*

Proof. $G_{c'_0}$ SSD G_{c_0} iff $\int_{-\infty}^{t}[G_{c_0}(c) - G_{c'_0}(c)]dc \geq 0 \ \forall t$. We are also given that D'_{12}, D_{12} are set so that

$$\int_{-\infty}^{D'_{12}} G_{c'_0}(c)dc = \int_{-\infty}^{D_{12}} G_{c_0}(c)dc \tag{A-3}$$

Now:

$$\begin{aligned}\int_{-\infty}^{t}[G_{\max(c_0,D_{12})}(c) - G_{\max(c'_0,D'_{12})}(c)]dc &= \int_{D_{12}}^{t} G_{c_0}(c)dc - \int_{D'_{12}}^{t} G_{c'_0}(c)dc \\ &= \int_{-\infty}^{t}[G_{c_0}(c) - G_{c'_0}(c)]dc \geq 0 \ \forall t\end{aligned}$$

Where the second equality is from equation (A-3) □

Consider the following definitions:

$$\begin{aligned}y'(x) &:= \max(y_j(x_j), x_i - D_i(x_j)) \\ y''(x) &:= \max(y_j(x_j), y''_i(x_i)) \text{ where } y''_i(x_i) = \max(x_i - D_i, 0)\end{aligned}$$

where D_i is chosen so that $\int_X f(x)[y''(x) - y(x)]dx = 0$ and $D_i(x_j)$ is chosen so that $\int_0^{\bar{x}} f(x)[y'(x) - y(x)]dx_i = 0$ for each x_j such that $\mathbb{E}[y(x)|x_j] > 0$. Due to our assumption (1) that losses are affiliated, $D_i(x_j)$ must be weakly increasing in x_j for those x_j such that $\mathbb{E}[y(x)|x_j] > 0$. We may therefore define $D_i(x_j)$ for those x_j for which $\mathbb{E}[y(x)|x_j] = 0$ to be no greater than $D_i(x'_j)$ for all $x'_j > x_j$ and no less than $D_i(x''_j)$ for all $x'_j < x_j$.

[12] Following Rothschild and Stiglitz (1970), second order stochastic dominance is defined as follows:

$$H \text{ SSD } G \text{ iff } \int_{-\infty}^{t}[G(c) - H(c)]dc \geq 0 \ \forall t$$

with strict SSD if there is a t for which the inequality is strict.

Let $c_0'(x) := w_0 - p_0 - x_0 + y'(x)$ and $c_0''(x) := w_0 - p_0 - x_0 + y''(x)$. We will show that $G_{c_0''}\ SSD\ G_{c_0'}\ strict\ SSD\ G_{c_0^{-z}}$ and that the new contract $\{p_0, y_i'', y_j, z\}$ is feasible and yields the same expected profit for the insurer as the original contract. This will provide us with our contradiction.

$\{p_0, y_i'', y_j, z\}$ is feasible and yields the same expected profit for the insurer by construction so all we need prove are the SSD relationships. That $G_{c_0'}\ strict\ SSD\ G_{c_0^{-z}}$ is straightforward: we perform a series of mean preserving contractions of consumption, one for each x_j, by moving claims mass to equalise net consumption in the highest net loss states. The strictness arises from the observation that if $G_{c_0'}$ equals $G_{c_0^{-z}}$ then the original contract must have been a Generalised Stop Loss contract.

Finally we show that $G_{c_0''}\ SSD\ G_{c_0'}$. Since $D_i(x_j)$ is weakly decreasing in x_j there is some x_j^* such that $D_i(x_j) \leq D_i$ for all $x_j \leq x_j^*$ and $D_i(x_j) \geq D_i$ for all $x_j \geq x_j^*$. To transform $c_0'(x)$ to $c_0''(x)$ we reduce claim payments for $x_j \leq x_j^*$ and increase claim payments for $x_j \geq x_j^*$. The minimum consumption in the region $x_j \leq x_j^*$ after reducing claim payments is $w_0 - p_0 - \min(D_i + x_j^*, \bar{x} + x_j^* - y_j(x_j))$. Moreover, the maximum consumption in the region $x_j \geq x_j^*$ after reducing claim payments is also equal to $w_0 - p_0 - \min(D_i + x_j^*, \bar{x} + x_j^* - y_j(x_j))$. We have moved claims mass from high net consumption states to low net consumption states and therefore $G_{c_0''}\ SSD\ G_{c_0'}$. □

Proof of Corollary 1. When κ is additive in its arguments, auditing two agents and making a claim payment costs the same as auditing only one and making the same claim payment and so without loss of generality we may assume that $y_1(x_1) = y_2(x_2) = 0$ for all $x \in X$ and instead write the optimal net transfer in terms of p_0 and $z(x) = p_0 - \theta_0(x)$ only. Theorem 1 holds, implying the required result. □

Proof of Lemma 4. Define $\sigma : X \to X$ by $\sigma(x) = (x_1, x_2 + s_2(x))\ \forall x \in X$, $a' = a \circ (\sigma, \sigma)$, $\theta' = \theta \circ (\sigma, \sigma, \sigma)$.

Now for any feasible ex ante side contract $S = (s, \tau)$ we define $S' = (s', \tau)$ where $s'(x) = (s_1(x), 0)$ for all $x \in X$. Direct mechanism (a', θ') inherits feasibility from (a, θ) and under side contract S' both agents receive the same expected utility as under the original mechanism and S, but the insurer receives weakly higher expected profits as net transfers to agents are lower in states where $s_2(x) \neq 0$ under the original side contract. □

Proof of Theorem 2. Parts 3. and 4. can be shown using the same logic as in the proof of Theorem 1. Appealing to Lemma A1 we may restrict attention to consumption before the addition of z, c_0^{-z}. Define $y(x) = \max(y_1(x_1), y_2(x_2))$ for some optimal y_1, y_2.

First we show that the region $y_1(x_1) = 0$ is a lower interval of x_1, and therefore that Part 1. of the Theorem holds. For each x_2 define $F_y(y|x_2)$ as the cdf of the random variable $y(x)$ conditional on the second loss. Consider the following definitions:

$$y'(x) := \sup_{y'}\{F_y(y'|x_2) \leq F_{x_1}(x_1|x_2)\} \text{ for all } x \in X$$
$$y''(x) := \max(y_1''(x_1), y_2(x_2))$$

where $y_1''(x_1)$ is the y_1'' that solves $\int_0^{\bar{x}} f(x) y'(x) dx_2 = \int_0^{\bar{x}} f(x) \max(y_1'', y_2(x_2)) dx_2$ or zero if there is no solution, for each x_1. Operation $y \to y'$ rearranges the claims mass to states with high x_1, keeping the conditional cdf constant for each x_2. Operation $y' \to y''$ shifts claims mass towards states with higher x_2 whilst making y incentive compatible, keeping $\mathbb{E}[y|x_1]$ constant.

By construction $y''(x)$ satisfies incentive compatibility and $G_{w_0-p_0-x_0+y''}$ SSD $G_{w_0-p_0-x_0+y'}$ SSD $G_{w_0-p_0-x_0+y}$ with strict dominance if the original y was not weakly increasing in both arguments for almost all x.

We will only provide a sketch proof to part 2. Suppose that there are some intervals $\beta = [\beta_1, \beta_2]$ and $\beta' = [\beta_1', \beta_2']$ such that $\beta_2' > \beta_1' > \beta_2 > \beta_1$ and for all $x_1 \in \beta, x_1' \in \beta'$ then $y_1(x_1') \geq y_1(x_1) > 0$ but $x_1' - y_1(x_1') > x_1 - y_1(x_1)$. By affiliation of losses and part 3. of this Theorem, the conditional cdf of consumption in region $\{x | x_1 \in \beta, y_1(x_1) > y_2(x_2)\}$ strictly SSD that of consumption in region $\{x | x_1 \in \beta', y_1(x_1) > y_2(x_2)\}$ and so we can strictly increase expected utility of the agents by decreasing $y_1(x_1)$ for all $x_1 \in \beta$ by some $\epsilon > 0$ and increasing $y_1(x_1)$ for all $x_1 \in \beta'$ by some $\epsilon' > 0$ where ϵ, ϵ' are chosen so that $\mathbb{E}y$ remains constant. □

Proof of Theorem 3. Proof of Theorem 3 closely follows that of Theorem 1 and so we only provide a sketch.

Conditional on function $I(v)$ and keeping $\mathbb{E}y_1, \mathbb{E}y_2, \mathbb{E}z$ constant the insurer must choose optimal y_1, y_2, z. If the insurer audits only agent i it learns the net loss $x_i - I(v)$ but does not learn x_j, and if the insurer audits both agents it learns the complete net loss $x_1 + x_2 - I(v)$. Following the same logic in the proof of Theorem 1 the double audit transfer will act to provide a floor on the net loss $x_1 + x_2 - I(v)$ and the single audit transfers will provide a floor on $x_i - I(v)$.

All that remains to be shown is that $I(v)$ is increasing in v. We may extend Lemma A1 to show that, conditional on y_1, y_2, z being of the form above, if $G_{w_0-p_0-x_0+I'(v)}$ SSD $G_{w_0-p_0-x_0+I(v)}$ then no mechanism with indexed payout function I can be optimal. Since v is jointly affiliated with x, any optimal indemnity schedule must be increasing in v for a.e. v or we can rearrange I to give a second order stochastically dominant consumption schedule for agents. □

Proof of Corollary 2. Proof follows that of Corollary 1. □

Proof of Theorem 4. Suppose the optimal direct mechanism is (a^*, θ^*) and associated ex ante side contract is (s^*, τ^*). There is no overinsurance (Lemma 1 holds) and so $s^*(x) = 0$ for all x.

Consider the following mechanism (M, a, θ, q) and a collection of feasible interim side contracts (s, m, τ). Under the mechanism each agent i must send a message (x^i, r^i) where x^i is i's report of the state of the world, $r^i = 1$ is a report that the other agent j has agreed to make the transfer $\tau_j^*(x)$ to agent i, and $r^i = 0$ is a report that agent j hasn't. The message space, audit rule, net transfer to insurer and punishment function are defined as follows for some small positive $\epsilon > 0$:

$$M_i := X \times \{0,1\}$$

$$a_i((x^1,r^1),(x^2,r^2)) := \begin{cases} a^*(x^i) & \text{if } x^1 = x^2 \text{ and } r^1 = r^2 = 1 \\ (1,1) & \text{otherwise} \end{cases}$$

$$\tilde{x}^i := \begin{cases} (x_1^i, x_2^i) & \text{if } a_1 \times (x_1^i - x_1) = 0, a_2 \times (x_2^i - x_2) = 0 \\ (x_1^i, x_2) & \text{if } a_1 \times (x_1^i - x_1) = 0, a_2 \times (x_2^i - x_2) \neq 0 \\ (x_1, x_2^i) & \text{if } a_1 \times (x_1^i - x_1) \neq 0, a_2 \times (x_2^i - x_2) = 0 \\ (x_1, x_2) & \text{if } a_1 \times (x_1^i - x_1) \neq 0, a_2 \times (x_2^i - x_2) \neq 0 \end{cases}$$

$$\theta_i((x^1,r^1),(x^2,r^2),x) := \begin{cases} \theta_i^*(\tilde{x}^i) & \text{if } r^i = 1 \\ \theta_i^*(\tilde{x}^i) + \min(0, \tau_i^*(\tilde{x}^i)) & \text{if } r^i = 0 \end{cases}$$

$$q_i((x^1,r^1),(x^2,r^2),x) := \begin{cases} \bar{q} & \text{if } \tilde{x}^i \neq x^i \\ \max(0, \tau_i^*(\tilde{x}^i)) + \epsilon & \text{if } \tilde{x}^i = x^i \text{ and } r^j \times r^i = 0 \\ 0 & \text{if } \tilde{x}^i = x^i \text{ and } r^j = r^i = 1 \end{cases}$$

So we define $\tilde{x}^i$ as the state of the world reported by agent i, x^i, corrected for any information discovered through auditing.

For a particular state x, consider the reservation utility of agent i. No matter what the message and transfer of agent j, agent i can receive utility of $u_i(c_i^*(x) - \epsilon)$, where $c_i^*(x) = w_i - x_i - \theta_i^*(x) - \tau_i^*(x)$. If $\tau_i^*(x) > 0$ then agent i can send message $m_i = (x, 1)$ to the insurer and make zero transfer to agent j. Agent i would transfer $\theta_i^*(x)$ to the insurer and receive a maximum punishment of $\tau_i^*(x) + \epsilon$. If $\tau_i^*(x) \leq 0$ then agent i can send message $m_i = (x, 0)$ to the insurer and make zero transfer to agent j. Agent i would transfer $\theta_i^*(x) + \tau_i^*(x)$ to the insurer and receive a maximum punishment of ϵ. In either case the utility of the agent is at least $u_i(c_i^*(x) - \epsilon)$.

Any interim side contract satisfying (IR2) must yield utility for each agent i of at least $u_i(c_i^*(x) - \epsilon)$. No matter what the joint message and transfer of agents in state x, the total consumption, net of punishments, cannot exceed $c_1^*(x) + c_2^*(x)$. To achieve this net consumption both agents must report $r^1 = r^2 = 1$ and $\tilde{x}^1 = \tilde{x}^2 = x$. Any interim side contract satisfying (IC3) must induce zero punishment and, given that utility must be $u_i(c_i^*(x) - \epsilon)$, consumption must be at least $c_i^*(x) - \epsilon$. □

Conclusion

Summary. This thesis was motivated by the desire to design good insurance products for poor individuals. Chapter One suggested that many of the weather derivatives currently being designed for and marketed to poor farmers may in fact be poor products, particularly for the most risk averse who would rationally purchase very little cover even in the presence of significant subsidies. Chapter Two presented results of a framed lab experiment conducted in Ethiopia in which normative analysis of both the shape and level of observed demand was methodologically justified. Finally, Chapter Three presented a vision of insurance design for the poor in which the formal insurance provider acted as reinsurer to groups of individuals with access to cheap information about each other, who in turn offered mutual insurance to each other.

Future directions. Weather derivatives have changed the face of agricultural insurance for the poor by focusing the minds of academics and practitioners on a problem which, if suitably addressed, could lead to a substantial increase in welfare for many of the world's rural poor.

What began with small scale pilots of innovative agricultural insurance products for farmers has already broadened to include improved risk management for organisations and governments that serve the poor. This is commendable: much risk management in developed countries happens behind the scenes without individuals having to purchase an insurance contracts from an insurer, and there is no reason to expect that optimal arrangements in developing countries would be different in this regard.

However, insuring the poor is going to be complex, no matter what the delivery channels. Insurance providers will need to be able to design, market, finance and deliver affordable products to large numbers of policyholders. Regulators will have to ensure that providers do not default on their obligations and take an active role in consumer protection. Governments may want to subsidise or promote products with a track record of reducing poverty. Consumers, many of whom will be illiterate and have low levels of formal education, will want to know how to make informed decisions. Academics will want to understand whether and precisely how insurance provision is having a positive impact on the lives of the poorest and provide thought leadership to providers, regulators, governments and consumers.

Regardless of how precisely insurance for the poor develops, it seems important for social scientists to be engaged with normative questions of interest to insurance providers, consumers and regulators. To contribute to these questions field experiments will need to be complemented by normative theory and carefully designed lab experiments.