CITATION:

Zhang, L., Stylianides, A. J., & Stylianides, G. J. (in press). Approaches to supporting and measuring mathematical problem posing: A systematic review of interventions in mathematics education. *International Journal of Science and Mathematics Education.*

Approaches to Supporting and Measuring Mathematical Problem Posing: A Systematic Review of Interventions in Mathematics Education

Abstract

Mathematical problem posing, a form of authentic mathematical inquiry and creation, has been acknowledged as important by educators and curriculum frameworks internationally and has been the focus of several intervention studies with students and teachers. However, the intervention components and measured outcomes of these prior studies varied, highlighting the diverse approaches researchers have taken to support and measure mathematical problem-posing competence. In this systematic review, we examined 39 intervention studies published over the past three decades, all aimed at developing mathematical problem-posing competence, and we identified nine common intervention components suggested or utilized within these studies. Also, we derived clusters of measured outcomes related to mathematical problem-posing competence used in the interventions. Our findings deepen understanding of approaches to supporting and measuring mathematical problem-posing competence, thus casting light on what works, including methods for measuring progress, in problem-posing interventions. We discuss implications of our findings for future research and practice.

Keywords: mathematical problem posing, competence, interventions, systematic review

Introduction

Mathematical problem posing, as a form of authentic mathematical inquiry and mathematical creation (Bonotto & Dal Santo, 2015), has been widely accepted as a vital intellectual activity of students' learning by educators, researchers, and curriculum frameworks internationally (Bonotto & Dal Santo, 2015; Cai et al., 2015; Chinese Ministry of Education, 2022; NCTM, 2000). The importance of problem posing in school mathematics is underpinned, for example, by recognition of the fact that mathematical advancements require creative imagination to ask new questions and view old questions from new angles (Ellerton & Clarkson, 1996) and by growing empirical evidence that problem posing has the potential to improve students' mathematical understanding, problem solving, and creativity (Bonotto & Dal Santo., 2015; Cai & Hwang, 2002). Given the importance of problem posing, there have been several interventions aiming at improving participants' mathematical problem-posing competence. While these interventions generally report a positive impact (Cai et al., 2020), the complex and multifaceted nature of mathematical problem posing (Cai & Hwang, 2020; Cai & Leikin, 2020; Kontorovich, 2020), as well as the frequently non-transparent mechanisms underlying the more or less effective interventions on developing this competence, make it difficult for researchers and practitioners to understand or replicate best practice.

In this systematic review, we took a step toward addressing these issues. Specifically, we aimed to qualitatively synthesize the various components and outcomes of prior interventions so as to deepen understanding about the approaches that researchers (together with practitioners) followed to support and measure mathematical problem-posing competence. In doing so, we aimed to cast light on what works, including the methods for measuring progress, in problem-posing interventions.

Research context

Mathematical problem-posing competence

A number of manifestations exist on what mathematical problem posing is. For example, it was described as *a logical process*, like "the process of formulating and expressing a problem within the domain of mathematics" (Cai & Hwang, 2020, p. 2) or "the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems" (Stoyanova & Ellerton, 1996, p. 519); as a *role-centered accomplishment* shaped by the norms of particular communities (Klinshtern et al., 2015; Kontorovich, 2020), like "a teacher-centered accomplishment that consists of constructing a problem that satisfies [certain] conditions" (Klinshtern et al., 2015, p. 463); as a *cognitive activity,* as a *research or instructional tool,* or as a *learning goal* (Cai & Leikin, 2020; Liljedahl & Cai, 2021); or as *a phenomenon* that "refers to all such manifestations" (Silver, 2013, p. 159).

The manifestation we adopt in this paper views mathematical problem-posing competence (Zhang et al., 2023) as a *learning goal* for mathematics instruction or training. Indeed, in several countries, recommendations for the reform of school mathematics have suggested the cultivation of mathematical problem-posing competence (Chinese Ministry of Education, 2022; NCTM, 2000). The emphasis on mathematical problem-posing competence is not solely due to its intrinsic importance and its role as a genuine form of mathematical inquiry and mathematical creation (Bonotto & Dal Santo, 2015), but rather it is also because of its presumed association with other learning accomplishments, including mathematical problem solving and mathematical creativity (Bonotto & Dal Santo, 2015; Cai et al., 2015).

Following the notion of competence proposed by Blömeke et al. (2015), mathematical problem-posing competence refers to the "criterion behavior" as well as the knowledge, cognitive skills, and affective-motivational dispositions that underline that behavior during engagement in the activity of mathematical problem posing (Zhang et al., 2023). Identifying or measuring a person's performance and characteristics underlying their problem posing behavior, and how these can be developed, are foregrounded in the realm of educational studies on competences (Blömeke et al., 2015). An implicit assumption is that these performances or characteristics are amenable to external interventions (Sternberg & Grigorenko, 2003), such as opportunities for students to learn in the classroom or for teachers to learn in training or professional development. This assumption has been supported by several studies that found students and teachers are capable of posing worthwhile mathematical problems and improve their mathematical problem-posing competence (Brown & Walter, 1983; Cai et al., 2015; Crespo, 2003). However, while these studies have provided valuable insights into the feasibility of external interventions for improving this competence, the field still lacks a systematic understanding of the approaches suggested or incorporated by these interventions to promote this competence.

Additionally, the problem-posing competence itself is conceptually assumed to involve a multitude of cognitive abilities and affect-motivation states that are changing throughout the duration of the performance and thus cannot be directly observed; rather, they must be inferred from the observed behavior (Blömeke et al., 2015). Although previous studies have offered perspectives on how competent problem posers should look like (Kontorovich & Koichu, 2016; Kontorovich, 2015; Kontorovich, 2020; Zhang et al., 2023) and how to measure mathematical problem posing (Silver & Cai, 1996; Zhang et al., 2022), there is still a significant lack of understanding regarding the anticipated observed changes in this competence resulting from the interventions.

As a way to deepen understanding of supporting mathematical problem-posing competence, we sought to examine what specific *intervention components* researchers suggested or employed to enhance this competence and to identify what criterion-behavior researchers considered in *measuring mathematical problem-posing competence* in intervention studies.

Interventions to enhance mathematical problem-posing competence

Following Stylianides & Stylianides (2013), we use the term “intervention” to broadly refer to a purposeful set of *actions* taken to improve a situation, in this case the mathematical problem-posing competence of individuals at any level, including school students and prospective or in-service teachers; these actions could be delivered in various settings (e.g., classrooms or laboratories). From a constructivism-oriented perspective, the question of "what works" stimulated our reflection on the various interventions (Danusso et al., 2010).

According to Harden and Thomas (2005), analyzing the "ideas" for *actions* regarding the core *components* of an intervention to affect outcome "X" helps answer the "what works" question. These "ideas" for actions could emerge from analyzing the reasons why participants have difficulties posing mathematical problems, such as participants lacking prior learning opportunities to engage in problem posing or explore problem posing situations (English, 1997). The interventions may provide, then, instructional practice or relevant resources in response to what participants are lacking. Also, the "ideas" for actions can be inspired by evidence demonstrating the value of strategies used for generating mathematical problems such as the "what-if-not" strategy (Brown & Walter, 1983), which involves participants listing the elements of the problem and then generating a new problem by asking "what if not the element *k*". These studies have provided necessary theoretical foundation for the development of interventions, including potential instructional practices, resources, or strategies that enable well-informed decisions about how to shape and organize the particular *set of actions* to improve problem-posing competence (Pressley et al., 2006).

In addressing the "what works" question, we aimed to identify specific *actions regarding intervention components* so as not only to show that these components were (potentially) effective (they "worked" or "could work"), but also to shed light on the mechanisms underlying the interventions (Cobb et al., 2003; Stylianides & Stylianides, 2013). Understanding these mechanisms is crucial in considering when to use the components and how to integrate them into the curriculum or the classroom. Specifically, this can potentially support educators (teachers) in making decisions about how to optimize or tailor these components to the needs of their students, determining what should remain unchanged, and what may be modified and how (Stylianides & Stylianides, 2013).

There are a few reviews related to mathematical problem posing; in their review of 13 reputable mathematics education journals, Lee (2021) found that only 6.5% of the articles with a focus on problem posing were review articles. There is also variation in the specific focuses of different reviews. For example, Rosli et al. (2014), Kul and Çelik (2020), and Wang et al. (2022) conducted a meta-analysis on the effects of engaging in problem posing activities on students' learning of mathematics. Other reviews examined the effects of engaging in problem posing activities on particular learning goals such as problem solving (Kopparla et al., 2019; Priest, 2009), mathematical attitudes and achievement (Bevan et al., 2019). The findings of these reviews are informative but insufficient in revealing what actions regarding the core components of an intervention might be effective, including methods of engaging participants in problem posing activities to enhance their mathematical problem-posing competence.

In our recent meta-analysis (Zhang et al., 2024), we summarized various intervention components aimed at fostering mathematical problem-posing competence and we examined overall intervention efficacy as well as the effects of moderators. While this meta-analysis provided valuable insights in relation to effective intervention components, it primarily focused on quantitative outcomes and did not dive into the nuanced context of how each intervention component was implemented across different educational settings. Additionally, this meta-analysis employed strict inclusion criteria, requiring that studies provided sufficient statistical evidence for effect size calculation, which resulted in a relatively small number of included studies (n=26).

In light also of recent reviews that emphasize the need to reveal possible actions and qualitative insights regarding intervention components beyond merely engaging participants in problem-posing activities (Bevan et al., 2019; Kopparla et al., 2019; Kul & Çelik, 2020; Priest, 2009; Rosli et al., 2014; Wang et al., 2022; Zhang et al., 2024), our systematic review aims to address this gap by incorporating a broader focus on exploring intervention components and expanding the evidence base to include qualitative studies. This approach allows us to uncover the complexities and mechanisms of how each intervention component is implemented in diverse educational contexts, thereby contributing to the evolving discussion of effective practices in fostering mathematical problem-posing competence. By doing so, we aim to provide a more comprehensive response to the "what works" question.

Measuring mathematical problem-posing competence

With regard to the *observed criterion-behavior* underlying mathematical problem posing, researchers agree that the development of participants' mathematical problem-posing competence can lead to positive observed cognitive outcomes, such as a higher quality and quantity of posed problems, and noncognitive outcomes, such as more positive affective-motivational dispositions that underlie the cognitive outcomes (Bicer et al., 2020; Cai & Leikin, 2020). In this review we specifically focused on exploring these outcomes.

From the point of view of posed problems, there are many conceptions of what constitutes a mathematical problem. In the early 1960s, Polya (1961) asserted that solving a problem is finding a way out of a difficulty, a way around an obstacle, or attaining an aim which was not immediately attainable. Information processing theories described a problem more generally as a path through the problem space, which consists of a representation of the initial state, goal state, and all intervening states (Newell & Simon, 1972). In addition, according to the level of cognitive demand for solving the problems, problems have been categorized as factual, reasoning, or open (Vacc, 1993). Given the diverse definitions of mathematical problems, a variety of criteria have been used to assess the quantity and quality of posed problems. The most common criteria are from the perspective of the solver of the posed problem, such as whether the problem is solvable (Silver & Cai, 1996), difficult (Cai et al., 2020), complex (Zhang et al., 2022), or open (Guberman & Leikin, 2013; Leavy & Hourigan, 2022). Others judged posed problems from the perspective of mathematicians (Crespo & Sinclair, 2008; Leavy & Hourigan, 2022) and discussed criteria using the "nutritious and tasty" metaphors, such as surprise, novelty, simplicity, and fruitfulness, that mathematicians often use in choosing "interesting" problems (Burton, 1999; Sinclair, 2004).

From the point of view of affective-motivational dispositions underlying problem posing, Cai and Leikin (2020) viewed affect in the context of problem posing as a system in which many constructs intertwine to make up the affective experience of a learner's participation with problem posing, including emotions, attitudes, beliefs, values, and motivations. In line with the research that viewed *problem posing as an instructional goal*, they presumed that associated affect, such as mathematical curiosity, interest, and enjoyment can lead to the discovery of new mathematics and the posing of new problems. Studies in this area are scarce but have seen a recent increase; they focused on particular kinds of affect in problem posing and used a variety of methods. For example, some studies employed self-designed questionnaires to evaluate the affective domain of participants engaging in problem-posing activities, such as their experienced difficulty or enjoyment of posing problems (Klein & Leikin, 2020; Li et al., 2020). Other studies used reflection meetings or semi-structured interviews to investigate the affective experiences of participants involved in problem posing (Kontorovich, 2020; Schindler & Bakker, 2020). Yet other studies examined the affective resources of expert problem posers, such as the sources of their motivation for posing problems for mathematical competitions (Kontorovich, 2015). Despite the aforementioned research, however, there remains a lack of knowledge about what affective-motivational dispositions researchers focused on to underpin and measure mathematical problem-posing competence. This lack of knowledge is not surprising given that there is no agreement on what attributes competent problem posers should embody (Kontorovich, 2020; Singer & Voica, 2017; Zhang et al., 2023).

To conclude, there is no standard measure of mathematical problem-posing competence. This follows from the lack of consensus on what constitutes a good mathematical problem (the product of problem posing) and on the affective-motivational dispositions that competent posers might possess, as well as their extent. This motivated further the need for our systematic review of researchers' expectations or main considerations in regard to measuring participants' problem-posing competence in the context of interventions that aimed to foster this competence.

The focus of this review

To address the "what works" question and guide the future design of effective interventions, we seek to shed light on the components that researchers suggested for inclusion, or actually incorporated, in interventions for improving mathematical problem-posing competence (RQ1). Additionally, to promote the field's understanding of researchers' expectations for their interventions, we review the measured outcomes of these interventions in relation to mathematical problem-posing competence (RQ2). Specifically, we undertook a systematic review to address the following research questions.

RQ1: What components were suggested or incorporated in interventions for enhancing participants' mathematical problem-posing competence?

RQ2: What measured outcomes related to mathematical problem-posing competence did interventions in this area focus on?

Methods

Literature search

Standardized guidelines for systematic reviews by Preferred Reporting Items for Systematic Reviews and Meta-analysis (PRISMA) (Page et al., 2021) were followed. We searched electronically the following databases in July 2021 as they were commonly used in previous literature reviews in mathematics education (Depaepe et al., 2013): Web of Science, Educational Resources Information Center (ERIC), PsycINFO, and Springer. To make the searching as comprehensive as possible, we set the query string with Boolean operators as follows, partially adapted from the searching word used in the reviews of Lo et al. (2017) and Wang et al. (2022): (math* OR algebra OR trigonometry OR geometry OR calculus OR statistics) AND ("problem posing" OR problem-posing).[1] In an effort to ensure all eligible publications were identified, the first 15 pages of Google Scholar search results (10 publications per page, ordered by relevance) were cross-referenced with the compiled inclusion bibliography, using the same search terms.

Inclusion and exclusion criteria. According to the framework provided by Cai and Leikin (2020) and Lijedahl and Cai (2021) for structuring a mathematics education research literature on

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[1] Given that each database relies on slightly different term entry formatting, we presented the precise search terms for each database as follows. Web of Science: TS=(math* OR algebra OR trigonometry OR geometry OR calculus OR statistics) AND TS=("problem posing" OR problem-posing); ERIC: AB=(math* OR algebra OR trigonometry OR geometry OR calculus OR statistics) AND AB= ("problem posing" OR problem-posing); PsycINFO: AB=(math* OR algebra OR trigonometry OR geometry OR calculus OR statistics) AND AB=("problem posing" OR problem-posing); Springer (google scholar): the exact phrase= mathematical problem posing, the exact phrase= mathematics problem posing, the exact phrase= math problem posing.

problem posing, we categorized the literature in the following three strands: research on *problem posing as a cognitive activity,* which focuses on understanding the nature of problem posing itself and its relationship with other constructs; research on *problem posing as a tool,* which investigates how problem posing can serve to improve one's learning of mathematics more generally; and research on *problem posing as a goal,* which concentrates on how one develops the competence for posing good problems, aligning with the focus of this study. We identified the last research strand – namely, *problem posing as a goal* – as the most relevant to our review, and we formulated the exclusion criteria so as to filter out intervention research belonging to the other two strands.

In particular, the inclusion criteria were as follows: the literature must (a) be peer-reviewed and published in journal articles or book chapters; (b) be printed in English; (c) be published between January 1990 and June 2021[2]; (d) include the term "mathematical problem posing" or "problem posing" (in the subject of mathematics) in the title and/or abstract/keywords; and (e) report on at least one type of intervention component related to mathematical problem posing along with case analysis or statistical evidence. The publications that satisfied these inclusion criteria were reviewed and their number was reduced using the following exclusion criteria: (a) book chapters if there were journal articles that reported the same data/analysis or findings (journal articles were typically more elaborate); (b) publications that reported research on *problem posing as a cognitive activity* or *problem posing as a tool*; and (c) duplicate publications or publications for which a copy could not be obtained.

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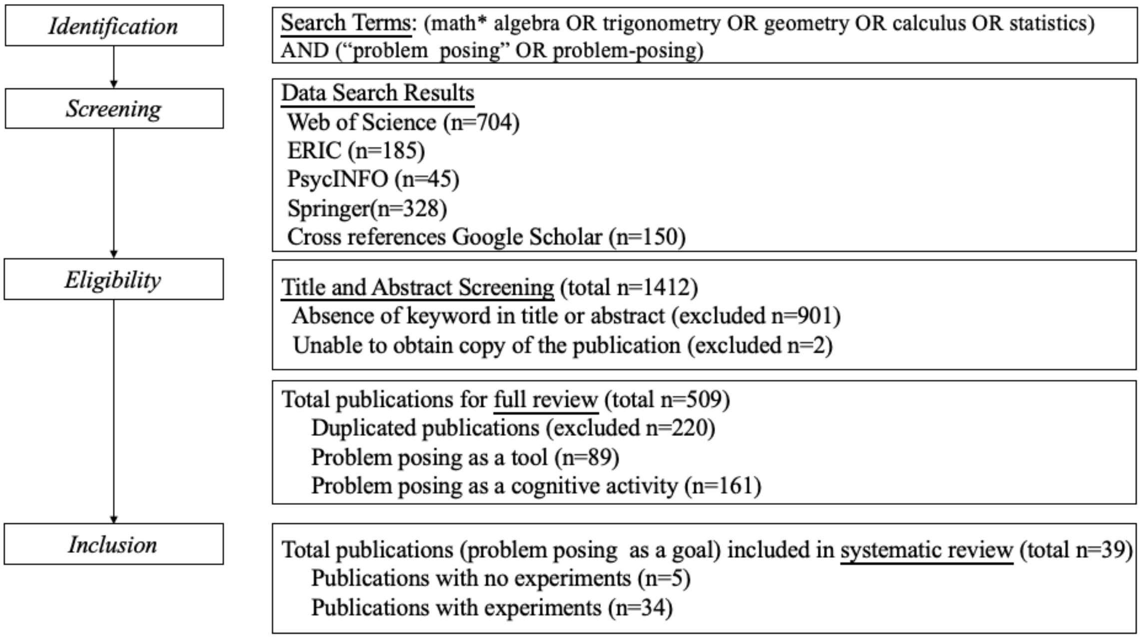
[2] We acknowledge that the period covered by this review not extending to the present day, an implication of logistical and financial constraints during the COVID pandemic as well as delays in preparing the findings of the review for publication, is a limitation of our study. Nevertheless, we believe the findings from the covered period provide valuable insights into the design of problem-posing interventions and lay the foundation for future research in this area.

Publications identified and selected. Using the inclusion and exclusion criteria, two research assistants (masters students majoring in mathematics education) classified 30 publications randomly selected from the total 1412 publications. The interrater agreement for judging whether these 30 articles should be included for full review was 100%. They then independently reviewed the remaining 1382 publications. The sample was reduced to 509 publications following title and abstract screening and removal of 2 publications for which a copy could not be obtained. These 509 publications were reviewed in full for qualitative and quantitative data extraction. This eligibility scan resulted in the exclusion of 220 duplicate publications, 89 publications that viewed problem posing as a tool (e.g., Bicer et al., 2020; Darhim et al., 2021), and 161 publications that viewed problem posing as a cognitive activity or as an independent variable (e.g., Van Harpen & Presmeg, 2013). Finally, a total of 39 publications met the inclusion criteria and were retained for systematic review. Among them, five publications presented only the suggested intervention with case analysis instead of an intervention within an experiment (Abramovich & Cho, 2015; Aydin & Monaghan, 2018; Contreras, 2007; Lavy & Bershadsky, 2003; Milinkovic, 2015), and 34 other studies reported experimental design information and particular statistical data regarding the treatment efficacy. Figure 1 presents the search flow summary.

FIGURE 1
Search flow summary



Data extraction and coding

Two trained research assistants extracted the following data (where available) from the included publications: (a) publication information including the study details (i.e., DOI, author names, publication year, country of origin) and type of publication (journal article, book chapter); (b) measured outcome resulting from the intervention including the type of outcome (i.e., the quantity of posed problems, the quality of posed problems, and affective aspects about problem posing); and (c) intervention components, duration, and participants.

Particularly, in terms of the data coding of the *intervention components* for RQ1, we considered the question, "How did participants experience the improvement of mathematical problem-posing competence?", according to the guidance for systematic reviews proposed by Harden and Thomas (2005). The information from the procedure/design of the methodology part in each study that conducted an experiment was parsed into discrete categories of intervention components. If the study suggested intervention components without reporting on an experiment, we extracted the data from the description of the suggested intervention components and any evidence or examples that were provided as rationale for the suggestions. Specifically, we adopted Bicer's (2021) definition of "instructional practices in mathematics education" and Boller et al.'s (2014) typology of "educational quality improvement interventions" to identify the *intervention components* that were suggested or incorporated in the interventions with the aim of improving participants' mathematical problem-posing competence. These components included *activity-based practice* that participants were required to experience (e.g., problem-posing activity), *method-based assistance* that helped participants pose problems (e.g., problem posing strategies, technology), and *environment-based support* that guided interaction (e.g., peer discussion). The constant comparative method (Strauss & Corbin, 2008) was adopted and used for identifying specific components that belonged to particular categories of intervention components. The studies were then described and mapped based on their (suggested) intervention components that influenced (or could influence) mathematical problem-posing competence. This process yielded a descriptive map showing the possible modes of the (suggested) intervention components in the studies.

According to the criteria of data extraction, all research members initially examined a random selection of two included articles. Results were discussed to ensure agreement and consistency in data extraction across research assistants. Two research assistants conducted the audit of the extracted data. The inter-rater reliability was 0.9 calculated by Cohen's kappa statistic[3] among

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[3] To calculate Cohen's Kappa, we first established a coding system with each paper evaluated on three distinct points, including (a) publication information, (b) measured outcomes, and (c) intervention components as mentioned in the first paragraph. After both research assistants independently coded the 39 papers, a contingency table was constructed for each coding point, summarizing the instances of agreement and disagreement between the raters. The observed agreement (Po) was then calculated as the proportion of total coding decisions where both raters agreed. The expected agreement (Pe), which represents the hypothetical probability of chance agreement, was estimated based on the marginal probabilities of each rater choosing each category independently. Finally, Cohen's Kappa was computed using the formula: $\kappa=(Po-Pe)/(1-Pe)$.

each coding point from 39 reviewed studies (Cohen, 1992). All disagreements were discussed until consensus was reached.

Results

Description of selected studies

In the Supplementary Material, we summarized all 39 studies that were included in the systematic review. Among them, five studies theoretically proposed some intervention components for enhancing problem-posing competence and provided examples to illustrate the rationale of those suggestions but reported no empirical results (Abramovich & Cho, 2015; Aydin & Monaghan, 2018; Contreras, 2007; Lavy & Bershadsky, 2003; Milinkovic, 2015). Also, one study of the 34 that reported experimental findings did not provide enough details for the experiment (Xia et al., 2007).

Figure 2 presents the participant age and intervention duration of the remaining 33 studies that reported experimental findings. These studies were published from 1996 (Silver et al., 1996) to 2021 (Ayvaz & Durmus, 2021; Cai & Hwang, 2021; Leavy & Hourigan, 2022[4]), but most of them (24/33) were published after 2010. The studies were conducted in fourteen different countries/districts: Australia, China, Cyprus, Indonesia, Ireland, Israel, Italy, Japan, Nigeria, Serbia, Sultanate of Oman, Taiwan, Turkey, and the USA. Also, they included a range of participants: kindergarten students, elementary school students, and secondary school students (mostly ages 5 to 18, 15 studies); university students preparing to become elementary or secondary school teachers (mostly ages 18-22, 14 studies); and in-service teachers (mostly ages over 23, 4 studies). The duration of the interventions ranged from less than one day (13 studies), between one

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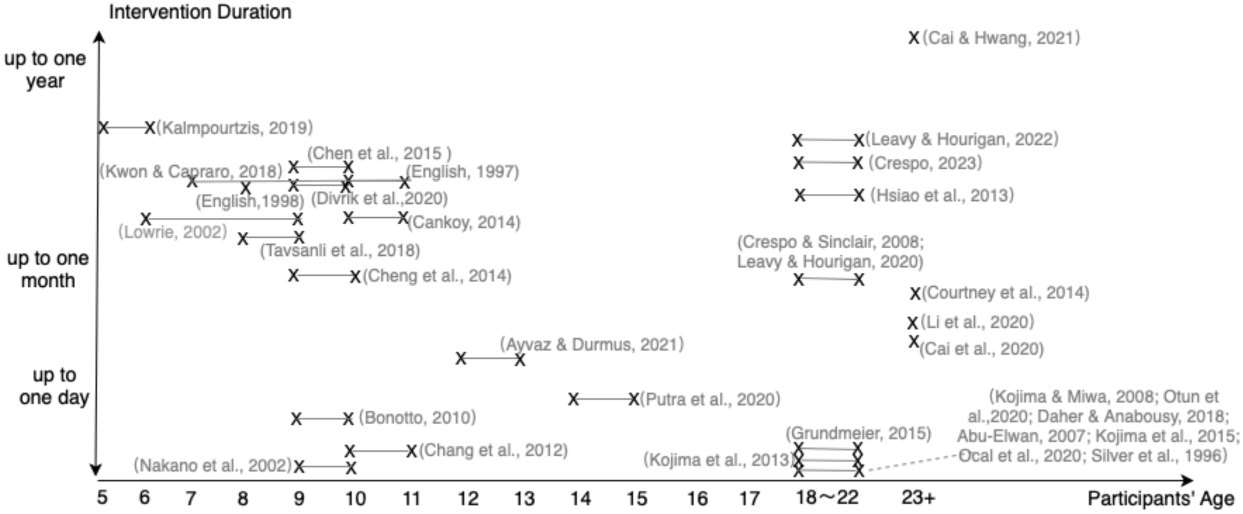
[4] Leavy & Hourigan (2022) was available online in 2021, so it was included in our review, even though it was officially published in 2022.

day and less than one week (2 studies), more than one week but less than a month (6 studies), and more than one month (12 studies).

FIGURE 2

Outline of the studies that reported experimental findings (N=33)



Intervention components for enhancing mathematical problem-posing competence

To address RQ1, we used evidence from the 39 studies and identified the following intervention components that the studies used or suggested for use when aiming to enhance participants' mathematical problem-posing competence (see last column of the table in Supplementary Material). The intervention components were categorized as follows (see Table 1). First, within *activity-based practice*, the intervention components included "overview of what problem posing is" (WPP) in 17.9% of the reviewed studies, "discussion of what 'good' problems are" (WGP) in 10.3% of the reviewed studies, "engagement with problem posing activities" (PPA) in 48.7% of the studies, and "evaluation of posed problems" (EPP) in 12.8% of the studies. Along with PPA, reviewed studies provided *method-based assistance* in the form of intervention component "comprehension of the problem posing situation" (CPPS) in 10.3% of the reviewed studies, "use of strategies involved in problem posing" (SPP) in 28.2% of the studies, "use of problem-posing examples" (PPE) in 10.3% of the studies, and "use of technology in problem posing" (TPP) in 23.1% of the studies. Finally, 38.5% of the reviewed studies attempted to create an *environment-based support* in the form of intervention component "creation of an interactive learning environment" (ILE). Next we elaborate on each intervention component separately.

TABLE 1

Frequency and proportion of publications from those included in the systematic review (N=39) that incorporated a particular category of intervention component

Categories of Intervention Components		No.
Activity-based Practice	Overview of what problem posing is (WPP)	7 (17.9%)
	Discussion of what 'good' problems are (WGP)	4 (10.3%)
	Engagement with problem posing activities (PPA)	19 (48.7%)
	Evaluation of posed problems (EPP)	5 (12.8%)
Method-based Assistance	Comprehension of the problem posing situation (CPPS)	4 (10.3%)
	Use of strategies involved in problem posing (SPP)	11 (28.2%)
	Use of problem-posing examples (PPE)	4 (10.3%)
	Use of technology in problem posing (TPP)	9 (23.1%)
Environment-based Support	Creation of an interactive learning environment (ILE)	15 (38.5%)

Overview of what problem posing is (WPP). Several researchers started their intervention giving an overview of problem posing (Cai et al., 2020; Cai & Hwang, 2021; Chen et al., 2015; English, 1998; Leavy & Hourigan, 2020, 2022; Li et al., 2020). Establishing knowledge of what problem posing is can be essential for participants' subsequent engagement in problem posing; indeed, Cai et al. (2020) found that only two of the 50 teachers in their study felt familiar with mathematical problem posing before their intervention (in the form of a workshop). In offering an overview of what problem posing is, several studies provided lectures to participants to discuss the nature of problem posing, problem structure for categorizing problems, types of problem posing activities, its meaning, and its significance for fostering student understanding (Cai et al., 2020; Cai & Hwang, 2021; Leavy & Hourigan, 2020; Li et al., 2020). Also, some empirical evidence about the significance of problem posing activities and their effect on students' understanding and learning of mathematics was presented in the lectures (Cai et al., 2020; Cai & Hwang, 2021).

Discussion of what "good" problems are (WGP). Scholars argued that value judgments are pervasive and important in a broad range of human activity (Johnson, 1993); this applies also to problem posing (Crespo & Sinclair, 2008). In terms of how to organize the discussion of what good problems are, two patterns emerged from our analysis of the various intervention designs: a *top-down approach* (one study) whereby posing problems followed discussion/development of consensus about criteria for good problems, and a *bottom-up approach* (three studies) whereby posing problems preceded discussion of criteria for good problems.

In terms of the *top-down approach* designed in the intervention of Chen et al. (2015), the orientation of what a good problem is that participants received was well defined by the researchers (i.e., whether it is solvable, clear, interesting, or complex). In the following unit when participants posed their own problems, certain criteria for self-evaluating the posed problems were used that we will discuss later in the section related to measured outcomes.

In terms of the *bottom-up approach* in intervention design (Cai et al., 2020; Crespo & Sinclair, 2008; Li et al., 2020), participants were asked to negotiate the meaning of "good" problems according to their posed problems to reach consensus among themselves about what should be considered as a good problem to pose. Yet, in the bottom-up approach the criteria for good problems did not always rely exclusively on participants' own discussion. Crespo and Sinclair (2008) incorporated specific criteria provided by the researchers, such as *surprise*, *novelty*, *simplicity*, and *fruitfulness* that mathematicians use in choosing "interesting" problems (Burton, 1999; Sinclair, 2004), to complement participants' discussion of good problems after they had posed their own problems. This latter practice is particularly relevant when participants might not think independently specific criteria.

Engagement with problem posing activities (PPA). A common intervention component, included in almost half of the reviewed studies (19 out of 39), was to set problem posing activities for participants to engage in. Participants commonly had experienced little or no problem posing activities in their prior education, which suggested they might not have an adequate foundation for problem generation (Cai & Hwang, 2021; English, 1997). A large and varied assortment of problem posing activities was presented to participants for them to experience and practice problem posing, which was viewed as prerequisite for improving their problem-posing competence (Leavy & Hourigan, 2020).

The design of problem posing activities in interventions varied depending on the feature of task format. One feature concerns the *types* of problem-posing tasks, including teachers engaging in their own mathematical problem posing and predicting the problems their students might pose (Cai & Hwang, 2021). The other feature concerns the *information format* of the problem-posing tasks, which can be in symbolic or non-symbolic contexts (English, 1998), with annotated arithmetic expressions or pure expressions (Cheng et al., 2014). Another feature concerns the *structure format* of the tasks and could include a structured, semi-structured, or free problem situation (Ayvaz & Durmus, 2021; Cankoy, 2014). Yet another feature concerns the *various prompts* given to participants in the problem-posing tasks: to pose “one easier, one moderately difficult, and one difficult problem” (Cai et al., 2020; Li et al., 2020), to pose “as many mathematical problems as possible” (Cai et al., 2020), or to pose “a mathematical problem you would like to solve” (Lowrie, 2002). An additional feature is the *sequence of problem posing*. Crespo and Sinclair (2008) asked half of their participants to explicitly explore the given mathematical situation prior to posing a problem and the other half to pose a problem spontaneously. Several researchers set a problem-posing task before, during, or after a problem-solving task (Grundmeier, 2015; Silver et al., 1996). The last feature we identified concerns the *source base*. Problem-posing tasks could be designed based on non-goal information, a given problem, a stimulus picture, a numerical answer, a symbolic expression, a mathematical story, a field trip site, a mathematical game, or realistic cultural artifacts (Bonotto, 2010; Cankoy, 2014; Chen et al., 2015; Courtney et al., 2014; Grundmeier, 2015; Leavy & Hourigan, 2022; Li et al., 2020).

Evaluation of posed problems (EPP). Peer-assessment or self-assessment of the problems posed by participants or presented by researchers are approaches that enable researchers to gather evidence of how participants judge their problem posing performance and their rationale behind problem selection and modification. These activities can also help participants improve their problem-posing competence. Accordingly, evaluation sessions have been used in several interventions, which could be put into two groups: *evaluation as learning* and *evaluation for learning*.

Evaluation as learning occurs when participants are their own assessors; they monitor their own perception of their posed problems or the problems presented by the researchers according to particular requirements or criteria, such as whether they like or dislike them (Lowrie, 2002), solvability and mathematical interest (Chen et al., 2015; Crespo & Sinclair, 2008), and so forth. Leavy and Hourigan (2022) combined *evaluation as learning* with *evaluation for learning* in their intervention with prospective teachers. In terms of evaluation *as* learning, the prospective teachers were each paired with two fourth-grade students and were asked to complete a pre-problem appraisal before sending their posed mathematical problems to their primary school children ‘penpals’ as part of a letter writing initiative. To do the appraisal, they used a range of strategies to reflect on the source of the problems and modifications made, as well as the rationale behind problem selection and modification, to analyze problem characteristics, to predict student strategies of solving the problem, and to rate the problem. Also, they were asked to reflect on their penpals’ response, which supported prospective teachers in critically evaluating their problems by comparing the children’s actual and predicted responses, thereby supporting the improvement of prospective teachers’ problem-posing competence *for* students’ learning.

Comprehension of the problem posing situation (CPPS). Hawkins (2000) argued that a student cannot formulate a problem spontaneously. Instead, there needs to be some time for the student to gain familiarity with a situation to push and pull at its constraints, to become aware of its various characteristics, possible tensions, etc. In other words, problem posing requires students to first understand what the problem posing situation is (Brown & Walter, 1993). Accordingly, four studies set a session of comprehension of the problem posing situation along with problem-posing activity in their intervention (Chen et al., 2015; Crespo & Sinclair, 2008; English, 1997, 1998). For instance, to help themselves identify the problem structure, students were encouraged to sort the problems by matching problems that would be solved in a similar way, or to specify the nature of the known information and the type of information that was unknown from the discussion of how the problems were similar and different (English, 1997; 1998).

Use of strategies involved in problem posing (SPP). Most of the strategies used in the problem posing intervention studies we reviewed focused on changing the elements or structure of the problem posing situation. These strategies were provided to the participants as scaffolding embedded in different ways. For example, the strategies were discussed and used by reversing knowns and unknowns, removing constraints, adding more knowns or more constrains or both, and using the what-if-not strategy (Chen et al., 2015; Daher & Anabousy, 2018; English, 1997; Putra et al., 2020). Beyond the assumption that modifying the components of a given problem can yield new problems, metacognitive strategies were also used to guide students' problem posing (Divrik et al., 2020), including checklists, error evaluation forms, and reflective journal writing forms. In addition, Otun et al., (2020) and Xia et al., (2007) incorporated strategies in problem-based instruction as a pedagogical approach.

Use of problem-posing examples (PPE). Students who have not typically engaged in problem posing may have difficulty using their experiences as solvers of ready-made teacher or textbook problems to discern what is expected of them when they are asked to pose their own problems (Cai & Hwang, 2021). To reduce this entry barrier, new problem posers can be supported by accompanying the problem-posing tasks presented to them with example problems that might be posed from the given situations. A series of studies conducted by Kojima and colleagues (Kojima & Miwa, 2008; Kojima et al. 2013, 2015) attempted to utilize the presentation of example problems as cases in supporting problem posing. They assumed that two attributes of problems were crucial: *surface features* such as contextual settings in problem situations, and *structural features* such as mathematical structures of solutions. They presented the example problems as a production task for participants that was generated by altering the situations or the solutions of the base problem.

Use of technology in problem posing (TPP). The use of technology in an intervention was often intended to help participants better engage in different components of the intervention, such as problem-posing examples (PPE) and strategy of problem posing (SPP). For example, Kojima et al. (2013) developed and implemented an e-learning system that presented examples of problem posing and supported learners in understanding the examples by having the learners reproduce them. Daher and Anabousy (2018) incorporated the what-if-not strategy with technology in their intervention, called “applet.” The applet not only provided the dragging functionality that gave students opportunities to use visual reasoning and generalize problems and relationships (Sinclair, 2004), but it also rapidly and efficiently took care of the technical work thus helping enrich and support the problem posing and conjecturing activities. Technology also contributed to better applying realistic or game scenarios to problem posing (Aydin & Monaghan, 2018; Chang et al., 2012; Daher & Anabousy, 2018). For instance, students were engaged in importing photographs of nature and of human constructions into GeoGebra and posing problems about mathematical properties of the object in the photograph (Aydin & Monaghan, 2018). The use of spreadsheet as a graphing software was found to be conducive for developing sophisticated problems about algebraic equations with parameters.

Creation of an interactive learning environment (ILE). Many of the reviewed studies (15 out of 39) aimed to develop an interactive learning environment that served as a context for other components of the intervention. We identified four typical interaction patterns in the reviewed interventions.

The first one was *collaboration with peers* and included activities in which participants could share and express their perceptions of different posed problems, compare and categorize the posed problems, and discuss and modify the posed problems with peers (Cai et al., 2020; Cai & Hwang, 2021; Crespo & Sinclair, 2008; English, 1997; Leavy & Hourigan, 2020; Li et al., 2020; Silver et al., 1996). Such an interaction pattern could not only increase participants’ awareness of different approaches to problem generation and foster their confidence in dealing with a wider range of problem types, but also help participants accumulate a richer basis for constructing new problems and questions (Brown & Walter, 1993).

The second interaction pattern was *support from prospective teachers to students* (Lowrie, 2002). It was based on the notion that it is quite difficult for young children to design appropriate problems without substantial practice (Ellerton, 1986) or guided questions (Lowrie, 2002), and that a prospective teacher would serve both as a role model for designing the problems and as an expert who would provide appropriate support.

The third interaction pattern was *feedback from students to prospective teachers*. Studies using letter writing exchanges between prospective teachers and primary students reported not only pedagogical benefits for prospective teachers in terms, for example, of insights into students' cognitive processes and affective dispositions towards mathematics, but also positive changes in prospective teachers' beliefs about mathematics as a discipline and views about worthwhile mathematical problems (Crespo, 2003; Leavy & Hourigan, 2022).

The last interaction pattern was *diagnosis from intelligent system to posers*. Nakano et al. (2002) created an intelligent learning system to communicate with problem posers, in which the problem posed by leaners could be diagnosed and corrected. The system could also lead learners to the next step of problem posing.

The measured outcomes related to mathematical problem-posing competence

To address RQ2, we categorized the interventions' measured outcomes into three clusters: those that measured the *quantity* or *quality* of the posed problems, and those that measured *affective-motivational disposition* underlying problem posing such as beliefs. Table 2 presents a summary of the criteria for assessing problem-posing competence within each cluster of measured outcomes. As expected, there is overlap between the criteria used in the various studies; the criteria marked in bold are meant as overarching statements.

TABLE 2

Clusters of measured outcomes and respective criteria for assessing problem-posing competence

Clusters	Criteria
Quantity	**the number of posed problems (10 studies)**
	- the number of posed problems (Silver et al., 1996*; Ayvaz & Durmus, 2021*; Courtney et al., 2014; Hsiao et al., 2013*)
	- the number of correct problems (Putra et al., 2020*)
	- the number of plausible problems (Grundmeier, 2015*)
	- the number of mathematical problems (Ayvaz & Durmus, 2021*)
	- the number of valid problems (Li et al., 2020*)
	- the appropriateness of posed problems (Chen et al., 2015*)
	- the number of reasonable problems (Cankoy, 2014*)
	the number of lesson episodes involving problem posing (2 studies) (Cai & Hwang, 2021; Cai et al., 2020)
Quality	**the solvability of posed problems (3 studies)**
	- solvable or unsolvable problems (Cankoy, 2014*; Ayvaz & Durmus, 2021*)
	- problems with sufficient information (Grundmeier, 2015*)
	the cognitive demand of posed problems (3 studies)
	- factual, reasoning, or open problems (Crespo & Sinclair, 2008*)
	- problem's level of cognitive demand (Leavy & Hourigan, 2022*; Courtney et al., 2014)
	the difficulty/complexity of posed problems (11 studies)
	- result-unknown, or start-unknown problems (Cankoy, 2014*)
	- multi-step problems (English, 1998*; Leavy & Hourigan, 2020*; Grundmeier, 2015*; Leavy & Hourigan, 2022; Li et al., 2020*; Cai et al., 2020)
	- the level/complexity of posed problems (Hsiao et al., 2013*)
	- altering problems with increased operations (Kojima et al., 2015*)
	- problems with different mathematical structure (English, 1997*)
	- linguistic and semantic complexity of a problem (Chen et al., 2015*)
	the openness of posed problems (3 studies)
	- multi-solution problems (Leavy & Hourigan, 2020*; Leavy & Hourigan, 2022; Courtney et al., 2014)
	the strategy variability of posed problems (9 studies)
	- different strategies used in posing problems (Kojima & Miwa, 2008*; Kojima et al., 2013*, 2015*; Grundmeier, 2015; English, 1997; Daher & Anabousy, 2018; Silver et al., 1996; Kwon & Capraro, 2018)
	- different solution-altering strategies (Kojima et al., 2013*; 2015*)
	the type of posed problems (12 studies)
	- curriculum strand (Leavy & Hourigan, 2020; Leavy & Hourigan, 2022)
	- problem type (Leavy & Hourigan, 2020; Li et al., 2020*; Daher & Anabousy, 2018; Ayvaz & Durmus, 2021*; Leavy & Hourigan, 2022; English, 1998; Crespo, 2020; Ocal et al., 2020; Lavy & Bershadsky, 2003; Contreras, 2007)
	the creativity of posed problems (9 studies)
	- flexibility related to strategies used to pose problems (Daher & Anabousy, 2018*)
	- originality of posed problems (Chen et al., 2015*; Chang et al., 2012*)
	- fluency of posed problems (Ayvaz & Durmus, 2021*)
	- flexibility of posed problems (Ayvaz & Durmus, 2021*; Chen et al., 2015*; Chang et al., 2012*; Daher & Anabousy, 2018*)
	- elaboration of posed problems (Chang et al., 2012*)
	the clarity of expression of posed problems (2 studies)
	- language and expression (Ayvaz & Durmus, 2021*)
	- accuracy (Chang et al., 2012*)
	Other (11 studies)
	- mean score of problem posing (Cai & Hwang, 2021*; Abu-Elwan, 2007; Tavsanli et al.,

	2018*; Divrik et al., 2020*; Otun et al., 2020; Nakano et al., 2002*; Kalmpourtzis, 2019; Xia et al., 2007) - problem comprehension (English, 1997*; Cheng et al., 2014*) - formulation ability in one-step and two-step problems (Cheng et al., 2014*)
Affective-motivational disposition	**Beliefs** about problem posing **(11 studies)** - beliefs about "what is a (good) problem" (Crespo & Sinclair, 2008; Leavy & Hourigan, 2020; Leavy & Hourigan, 2022) - beliefs about "what constitutes a problem" (Leavy & Hourigan, 2020) - confidence about teaching through problem posing (Cai & Hwang, 2021*; Cai et al., 2020*; Chen et al., 2015) - beliefs about the value of problem posing (Grundmeier, 2015; Li et al., 2020; Chen et al., 2015; Abu-Elwan, 2007) **Attitudes** towards problem posing **(2 studies)** (Chen et al., 2015*; English, 1997) **Mathematical disposition** on problem posing **(1 study)** (Cai & Hwang, 2021*) **Concern/ Challenges** with problem posing **(2 studies)** (Leavy & Hourigan, 2022; Li et al., 2020) **Familiarity** with problem posing **(1 study)** (Cai et al., 2020*)

*Note: * The data relevant to particular criteria in this study could be used in a meta-analysis.*

When calculating the *quantity of the posed problems* most of the researchers considered if the problems were correct, plausible, valid, appropriate, or reasonable. Such considerations are prerequisite for researchers' subsequent evaluation of the *quality* of the posed problems. Some researchers who investigated the impact of an intervention on teachers' problem-posing competence as well as their design of mathematics lessons that use a problem-posing approach (Cai & Hwang, 2021; Cai et al., 2020) considered the number of lesson episodes involving problem posing as a measure of teachers' incorporation of problem-posing activities in teaching.

Regarding the *quality of the posed problems*, the most popular criterion was "the difficulty or complexity of the posed problems", which was closely associated with the number and the kind of relations in the structure of the posed problems. Some researchers counted how many steps or operations could be used to solve a posed problem to reflect its difficulty (e.g., English, 1998; Kojima et al., 2015; Leavy & Hourigan, 2020) while others focused on the semantic relations representing the complexity of the problem (Chen et al., 2015; English, 1997). In addition, some researchers paid attention to the solvability of the posed problems (Ayvaz & Durmus, 2021; Cankoy, 2014; Grundmeier, 2015), the level of cognitive demand of the posed problems (Crespo

& Sinclair, 2008; Leavy & Hourigan, 2022), the openness of the posed problems to multiple solutions (Leavy & Hourigan, 2020; 2022), the variability of strategies used in posing the problems (e.g., English, 1997; Kojima et al., 2013; Kojima & Miwa, 2008), the creativity of the posed problems in terms of fluency, flexibility, originality, and elaboration (Chang et al., 2012; Chen et al., 2015; Daher & Anabousy, 2018), and the clarity of expression of the posed problems (Ayvaz & Durmus, 2021; Chang et al., 2012). Yet some studies focused on other aspects of the quality of problem posing. For instance, some researchers reported a final score to represent participants' performance in problem posing without reference to a detailed rubric (Cai & Hwang, 2021; Cheng et al., 2014). English (1997) and Cheng et al. (2014) measured students' problem comprehension, which was assumed to be a key component of problem-posing competence as well. In other cases, researchers drew attention to the type of posed problems, which could not be easily used to statistically discern changes in participants' problem-posing competence but rather to show the distribution of posed problems across various types.

Regarding *affective-motivational disposition* underlying problem posing, researchers were concerned about participants' beliefs about problem posing (Cai et al., 2020; Cai & Hwang, 2021; Chen et al., 2015; Crespo & Sinclair, 2008; Grundmeier, 2015; Leavy & Hourigan, 2020, 2022; Li et al., 2020), their challenges with problem posing (Leavy & Hourigan, 2022; Li et al., 2020), their attitudes towards problem posing (Chen et al., 2015; English, 1997), their familiarity with problem posing (Cai et al., 2020), and their mathematical disposition on problem posing (Cai & Hwang, 2021). Particularly, in terms of participants' beliefs of what a good problem is, results from Crespo and Sinclair (2008) and Leavy and Hourigan (2022) showed that prospective teachers shifted from exclusively attending to the pedagogical qualities of posed problems to attending also to some mathematical qualities. In terms of the challenges with problem posing faced by research participants, studies (e.g., Leavy & Hourigan, 2022; Li et al., 2020) reported that participants were concerned about whether the posed problems would make sense to their intended audience and whether other features of the problems, such as their level of difficulty, cognitive demand, and creativity, were suitable for the intended audience's prior knowledge, interest, or experience.

Discussion

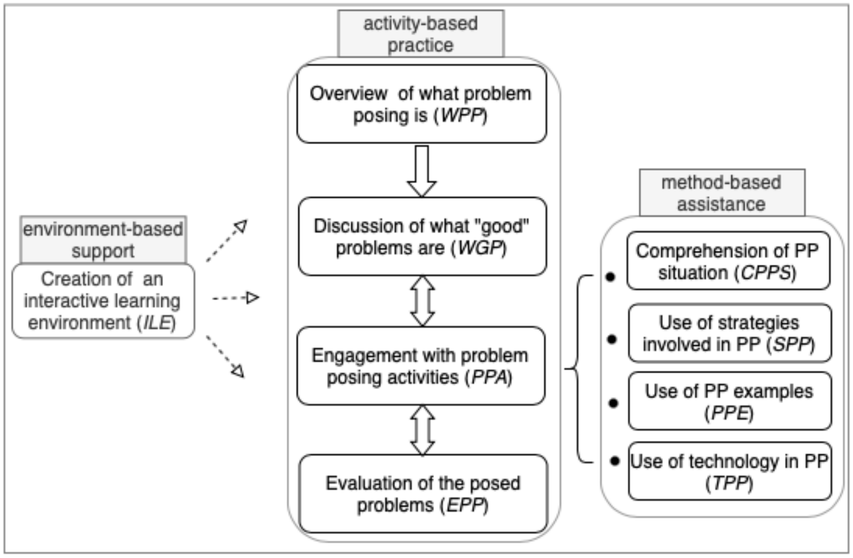
In this review we identified several components suggested or incorporated in interventions aiming to impact positively on the development of the mathematical problem-posing competence of students and (prospective or in-service) teachers. Also, we synthesized the measured outcomes in regard to the quantity of posed problems, the quality of posed problems, and the affective-motivational disposition underlying problem posing. Next, we discuss our results regarding each research question separately.

Components suggested or incorporated in interventions

Regarding RQ1, each of the nine intervention components we identified in our systematic review has its own potential for promoting participants' mathematical problem-posing competence. Based on how the different components tended to be used and structured in the reviewed studies, we present in Figure 3 a possible way to organize them that is consistent with many of the reviewed studies. While not a single study used the precise organizational structure that we present in the figure (partly because not all components were used in every intervention), the sequencing and relationships between components that were present tended to be as portrayed in the figure. This organizational structure helps address the "what works" question by visually summarizing (potentially) effective components and their possible relationships, providing a framework for how these components may be integrated in real educational settings to enhance mathematical problem-posing competence.

FIGURE 3

A possible overview organizational structure of the identified intervention components



In a nutshell, many of the reviewed studies (approximately 40%) provided an *environment-based support* for improving participants' mathematical problem-posing competence by creating an interactive learning environment (ILE), which could be embedded in any type of intervention component (e.g., Cai & Hwang, 2021; English, 1997). Participants who collaborated with a partner tended to learn more than those who worked alone (Archer-Kath et al., 1994). Also, group collaboration tended to support among participants a feeling of safety and appreciation, together with an increased interest in within-solution problem posing and an openness for trying out new things (Schindler & Bakker, 2020).

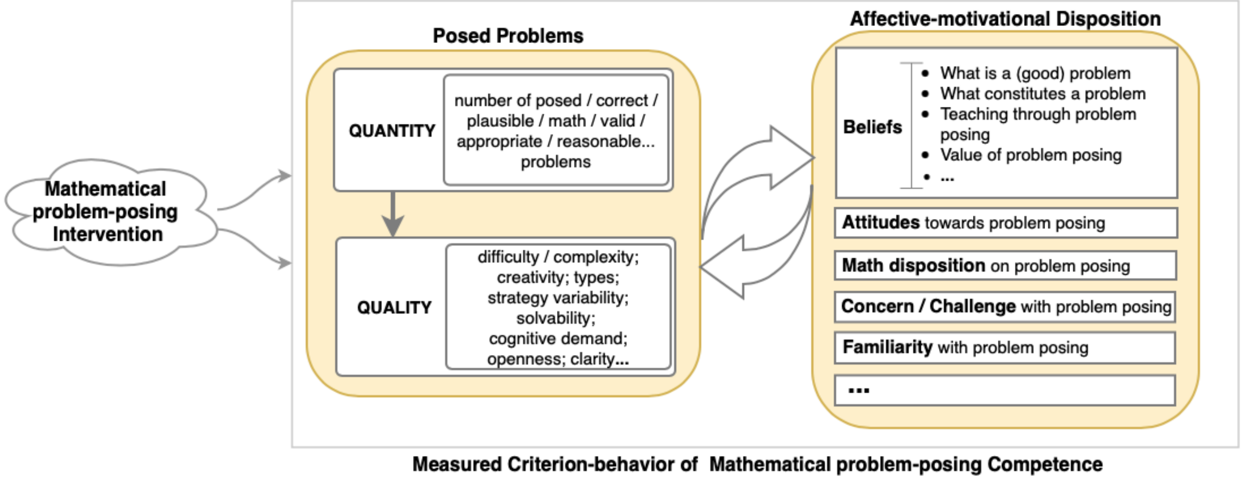
During all the interventions, participants were required to engage in particular *activity-based practice* including WPP, WGP, PPA, and EPP. Supporting participants to understand "what is problem posing" (WPP) and "what good problems are" (WGP) *prior* to engaging them in problem posing could be viewed as helping them acquire necessary prior knowledge of mathematical problem posing (e.g., Cai et al., 2020; English, 1998; Leavy & Hourigan, 2022). Problem solving may start better by first activating and differentiating relevant prior knowledge, even though there are several interdependent mechanisms underpinning the preparatory effects of problem solving (Kapur, 2015). Contrary to problem solving, problem posing is a comparatively newer endeavor for many participants. The prior knowledge activation of what mathematical problem posing is seems to be necessary to help participants understand what they need to do. However, there is likely to be a trade-off between knowledge activation and the extent to which the activated knowledge can restrict or direct participants' thinking. Evaluation of the posed problems *after* involving participants in posing problems, which could be considered as "looking back," might enable researchers to better understand the rationale behind participants' problem selection and modification. Also, engaging participants in self- or peer-assessment has the potential to nurture their confidence and independence in learning mathematics (NCTM, 2000) as well as foster reflection and metacognition (Siegesmund, 2016). Research indicates that students who are trained in self-assessment outperform their peers who do not receive such training (Enz & Serafini, 1995).

Along with engaging participants in problem-posing activities (PPA), some studies provided several kinds of *method-based assistance* for participants' problem posing such as comprehension of the problem-posing situation (CPPS), use of strategies involved in problem posing (SPP), use of problem-posing examples (PPE), and use of technology in problem posing (TPP). These afforded participants with opportunities to experience and practice posing problems with scaffolding. Regarding SPP, most of the strategies used in interventions were varying the elements or relations of the problems to generate new problems. Although such strategies help participants generate an increased number of problems, their contribution to participants' creative problem posing is unclear. Crespo and Sinclair (2008) argued that such strategies often seem compelling, because they permit an almost effortless generation of a new problem. However, these problem posing strategies do not necessarily determine the quality of newly posed problems.

Measured outcomes related to mathematical problem-posing competence

Regarding RQ2, we found several criteria that researchers considered as indicators of assessing changes in participants' problem-posing competence as a result of the interventions (see Figure 4). Participants were generally considered to have better mathematical problem-posing competence if they posed more problems or problems of a higher quality with respect to their difficulty, solvability, cognitive demand, creativity, type and so forth, or if they exhibited more positive affective-motivational disposition underlying problem posing.

FIGURE 4
An overview of measured criterion-behavior changes from interventions on enhancing mathematical problem-posing competence



The observed product of problem posing, i.e., the posed problems, relates to how we frame what constitutes a "good" problem. Although we identified several criteria that interventions view as describing "good" problems (e.g., the dominant criterion of difficulty/complexity, strategy variability, type, creativity), there is often a tension between mathematical and pedagogical perspectives that influences how one views "good" problems.

Take, for example, studies that explored prospective teachers' views of what good problems are (Crespo & Sinclair, 2008; Leavy & Hourigan, 2022). Their results showed that participants attended to various characteristics regarding the pedagogical qualities of the posed problems, which are likely to shape the problem-posing opportunities that these future teachers might be able to offer to their students (Costa et al., 2000). At the same time, however, participants might propose problems that incorporate many desirable problem features as examples of good problems after an intervention, such as multiple solution paths or multiple points of entry, but some of these problems might be too difficult for their intended audience (Crespo, 2003; Crespo & Sinclair, 2008; Leavy & Hourigan, 2020). From a pedagogical perspective, such problems cannot be identified as good problems if they are not solvable by their intended audience, even though, from a mathematical perspective, such problems can be viewed as good due to them being solvable by other audiences.

Also, some features used when judging the quality of a mathematical problem may be associated with different problem qualities depending on other factors. For instance, participants considered multi-step problems to be good given that such problems were more complex than one-step problems. However, many two-step problems merely involved application of more than one operation or arithmetical procedure to reach a solution, which did not influence the reasoning or higher order thinking required for their solution (Leavy & Hourigan, 2020).

Regarding affective-motivational dispositions underlying problem posing, a relatively small number of the intervention studies we reviewed (10 out of 39) ventured into evaluating or considering the affective changes of mathematical problem-posing competence, with a substantial focus (9 of the 10 studies) placed singularly on beliefs about mathematical problem posing. Notably, 7 of these 10 studies were published after 2015, highlighting a recent shift in research focus. The problems arising from this limited exploration of the affective aspects of mathematical problem-posing competence in the reviewed studies are exacerbated by their application of narrow research methods, which were dominated by self-designed questionnaires containing only one or two questions to measure a noncognitive outcome. Such approaches raise validity concerns, especially given the intricate nature of problem-posing competence.

Additionally, affective-motivational dispositions underlying the problem-posing activity and its product (the posed problems), are like a system, intertwined and cobbled together in response to problem-posing task demands (Blömeke et al., 2015). Despite the critical nature of this interconnection, only a few studies – predominantly recent ones – have focused on the affective aspects of mathematical problem-posing competence (e.g., Cai et al., 2020; Chen et al., 2015; Leavy & Hourigan, 2020). These studies have primarily concentrated on beliefs. There is a need to broaden the focus of future intervention designs to more comprehensively address affective aspects.

To conclude, assessment of mathematical problem-posing competence development, as required in intervention studies, presents a substantive theoretical and methodological challenge. Theoretically, the challenge is to conceptualize and define mathematical problem-posing competence – as the latent cognitive and affective-motivational factor underpinning specific performance in varying situations – in a reliable and valid way (Blömeke et al., 2015). The variety of criterion-behavior approaches used in the intervention studies we reviewed reflects not only the multifaceted nature of problem-posing competence but also its state of theorization in the field: the field still has not settled on a robust conceptualization of problem-posing competence and its various sub-components, particularly in regard with what constitute good posed mathematical problems and competent problem posers (Zhang et al., 2023). Methodologically, measuring mathematical problem-posing competence is at least as challenging as defining it. Our review revealed that current measurement instruments are primarily self-designed or task-based, focusing on evaluating individuals' changes in specific criteria related to mathematical problem-posing competence. However, assessing this competence typically demands criterion-referenced decisions, such as evaluating if certain competence levels have been achieved (Berry et al., 2011). Furthermore, unlike the abundance of standardized problem-solving tests (e.g., PISA, TIMSS), the field currently lacks a standardized test for mathematical problem-posing competence.

Concluding Remarks

Although mathematical problem posing as a construct is less established than its twin construct of mathematical problem solving, it has attracted increased research attention over the past years and gradually an important theoretical foundation has been developed in relation to it (Cai et al., 2015). By developing a deeper understanding of how mathematical problem-posing competence might be enhanced and measured, we as a field have better chances of providing researchers and practitioners with guidance to not only enhance participants' mathematical problem-posing competence but also other important competences that are believed to be associated with it, notably, mathematical problem solving and mathematical creativity.

One of the review's main findings are the nine typical intervention components we identified and organized in Figure 3. This organizational structure addresses the "what works" question, which is crucial for improving educational practice and further advancing research aiming at promoting mathematical problem-posing competence. Given the potential of each of these components to enhance participants' mathematical problem-posing competence, as well as our account of this potential as emerging from the reviewed studies that used and examined these components, practitioners can leverage selected intervention components to try and maximize the treatment efficacy while taking into account their aims, context, and participants. Take, for example, the intervention component of discussing what good problems are (WGP), which can be organized in either a top-down or a bottom-up form. Several studies found bottom-up approaches to be associated with better student learning outcomes as compared to more top-down oriented models (Arnold-Berkovits et al., 2019). At the same time, however, several other studies concluded that a top-down approach is still preferable for reducing the possible confusion resulting from a complicated learning process (Tsai & Beverton, 2007). So the impact of these two kinds of approaches on participants' problem-posing competence can vary (Skedsmo & Huber, 2019) and needs to be balanced against considerations of research context and participant needs. Similar things can be said about selection and use of the appropriate intervention dosage or types of intervention for optimizing participants' learning of mathematical problem posing according to their needs.

In addition to informing the design and implementation of interventions to foster mathematical problem-posing competence, our identification and discussion of particular intervention components can have implications for integrating mathematical problem posing into real classroom settings. While few studies have explored teaching models for integrating mathematical problem posing in actual classrooms – such as analyzing instructional flows through script writing (Kar et al., 2024) and teaching cases (Zhang & Cai, 2021) – these studies highlight common patterns. Typically, the process involves teachers presenting problem-posing situations and prompts, followed by students posing problems individually or in a group, with teachers then handling these posed problems. The quality of the pool of student-generated problems plays a crucial role in shaping the potential direction, breadth, and depth of students' learning opportunities. However, when handling students' generated problems, teachers often choose to focus on problems that align with the instructional goal(s) of the lesson, skipping over other problems (Kar et al., 2024; Zhang & Cai, 2021). While this focus on relevance is reasonable, it can become problematic, especially when students' problem-posing competence is limited.

Moreover, students often produce problems that are nonmathematical, irrelevant, unsolvable, unclear, or contain errors (Cai & Hwang, 2002; Zhang et al., 2022). This situation complicates teachers' ability to select problems that are both high-quality and relevant to the lesson goals, ultimately undermining the effectiveness of problem-posing instruction. By incorporating the illustrated intervention components within the instructional flows of regular classes – such as method-based assistance that helps students conceptualize the mathematical problem-posing situation (CPPS), using problem-posing strategies (SPP), or providing problem-posing examples (PPE) – teachers could enhance the likelihood of students generating high-quality problems. Our findings in this review can serve as a valuable resource in ongoing efforts to develop theoretically rich teaching models and practically effective and actionable strategies for integrating mathematical problem posing in real classrooms.

The other main finding of the review is the multiplicity of measured outcomes used in the various interventions (see Figure 4), which is an indication not only of the complex nature of problem-posing competence but also of an evolving field. Given that theoretical advances are limited by the measures we use, and in turn, our measures are limited by our theoretical understanding (Kersting et al., 2016), diverse views on assessing problem-posing competence, especially regarding its affective aspects, were to be expected across interventions. Yet one can also expect to see, over time, more convergence towards established and standardized measures of problem-posing competence, including measures that can be used not only by researchers in intervention studies but also by practitioners to assess learning and inform teaching.

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[5] Asterisks indicate articles included in the systematic review.

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Approaches to Supporting and Measuring Mathematical Problem Posing: A Systematic Review of Interventions in Mathematics Education

In these supplementary materials we present a summary of the studies included in the systematic review (Table 1).

TABLE 1

Summary of the studies included in the systematic review (N=39)

Research ID	Journal /chapter	Country /district	Details of Intervention	Duration	Participants[a]	Compo-nents[b]
English (1997)	Educ. Stud. Math.	Australia	- Problem exploration and problem sorting - Analysis problem structure and model new problems - Create new problems from given problem components - Transform a given problem into a new problem	10 weeks, 35mins per week	27 fifth graders	EPP PPA CPPS ILE SPP
Cai & Hwang (2021)	ZDM- Math. Educ.	USA	- Discuss the nature of PP - Engage in PP activities - Predict the problems students might pose - Discuss and modify lesson plans to integrate PP activities - Consider PP lessons designed by other teachers - Create and implement the PP teaching	4-8 workshops, up to whole year	103 primary /secondary teachers	WPP PPA ILE
English (1998)	J. Res. Math. Educ.	Australia	PP activities incorporated in each session - Explain the nature of PP - Interpret the problem situation - Engage in PP activities - Share ideas on other problems that could be created	Sixteen 45min sessions, two per week	54 8.1 years old students	WPP PPA CPPS ILE
Leavy & Hourigan (2020)	J. Math. Teach. Educ.	Ireland	- Give lectures underpinning PP and problem solving - Engage in series of PP and problem solving activities	3 weeks	415 prospective primary teachers	WPP PPA ILE
Li et al. (2020)	Educ. Stud. Math.	China	Four cycle workshops - Overview of PP - Engage in PP activities - Predict the problem students might pose - Write and modify the lessons plan integrating PP - Discuss lesson plans taught by other teachers	Four 3-day workshop	74 primary teachers	WPP WGP PPA ILE
Chen et al. (2015)	Chapter	China	- Explore the concept of PP - Explore the assessment criteria of PP	11 units, 90 mins	69 fourth graders	WPP WGP

			- Generate new problem from PP tasks - Transform a given problem into new problems - Practice on PP	per unit, one unit per week		PPA EPP SPP CPPS
Cai et al. (2020)	Int. J. Educ. Res.	USA	- Discuss of the nature of PP - Engage in PP activities - Predict the problems students might pose - Discuss and modify their lesson plans to integrate PP activities - Discuss PP lessons designed by other teachers	3 days workshop	50 primary teachers	WPP WGP PPA ILE
Ayvaz & Durmus (2021)	Think. Skills Creat.	Turkey	A 30-h action plan consisting of 18 problem posing activities - First action: 9 structured PP activities - Second action: 5 semi- structured PP activities - Third action: 4 free PP activities	30h	Six 7th grade gifted students	PPA
Silver et al. (1996)	J. Res. Math. Educ.	USA	Work by individual or pairs - Initially pose problems (before problem solving) - Pose problems (during problem solving) - Additionally posing problems (after problem solving)	45mins	53 secondary teachers; 28 prospective teachers[a]	PPA ILE
Crespo & Sinclair (2008)	J. Math. Teach. Educ.	USA	- Explore the situation first *VS* pose spontaneously - Evaluate their own posed problems - Discussion what makes a “good” or “interesting” problems	3 weeks	22 prospective teachers[a]	PPA CPPS EPP WGP ILE
Cankoy (2014)	Int. J. Sci. Math. Educ.	Cyprus	- Introduce a four-step problem solving procedure - Solve a set of textbook problems - Write problems for friends in free structured situations - Discuss about the quality of selected posed problems	5 weeks	30 fifth graders	PPA ILE
Grundmeier (2015)	Chapter	USA	- Engage in problem reformulation tasks - Engage in problem generation tasks	1 hour and 50 mins	19 prospective teachers[a]	PPA
Putra et al. (2020)	Int. J. Instr.	Indone-sia	- Experimental Class obtained scientific approach with What -if-not strategy - Control class obtained conventional Teaching	Less than one day	68 ninth graders	SPP
Leavy & Hourigan (2022)	J. Math. Teach. Educ.	Ireland	- Overview of PP and problem solving - Co-design problems - Analysis of penpal’ responses - Circle week with problem design - Solve a selection of mathematical problems - Discuss the feedback on problem design - Feedback and reflection	12 weeks	28 prospective primary teachers	WPP PPA EPP ILE

Lowrie (2002)	Math. Educ. Res. J.	Australia	Student teachers' support PP sessions - Pose a problem - Discuss approaches that would be required to solve the problem - Solve the problem - Reflect upon the manner in which they solved the problem - Gain insights into the children's perception of PP - Assist and encourage students to formulate problems	one hour per week for five weeks	25 first graders; 28 third graders; 53 undergraduates	PPA EPP ILE
Bonotto (2010)	chapter	Italy	- Introduce kinds of cultural artifacts - Analyze and interpret all the data present - Give assignment regarding the artifacts - Discuss answers and compare strategies	12h	2 fourth grade classes	PPA ILE
Kojima & Miwa (2008)	Int. J. Artif. Intell. Educ.	Japan	- Present example problem A and ask to solve it - Pose a problem from example problem A - Present with three cases problems and ask to solve it - Pose another problem from example problem A	-	112 undergraduates	PPE
Kojima et al. (2013)	Int. J. Artif. Intell. Educ.	Japan	- Training and learning in a system operation - Present the base and example problems to each condition groups - Pose problem by using system	45 mins	40 undergraduates	PPE TPP
Kojima et al. (2015)	Res. Pract. Technol. Enhanc. Learn.	Japan	- Instruct how to pose a novel problem from a base by a learning task - Reproduce problems according to examples - Evaluate the case problem	45mins	167 undergraduates	PPE TPP
Abramovich & Cho (2015)	Chapter	USA	-Digital technology of using spreadsheet -Reciprocal problem posing with technology	-	-	TPP
Aydin & Monaghan (2018)	Teach. Math. Its Appl.	Turkey	- Pose a paper-pencil problem ('before' problem) - Importing visual images into GeoGebra - Pose a 'after' problem inside of the GeoGebra software - Solve the 'after' problem	-	-	TPP
Milinković (2015)	Chapter	Serbia	- Pose problem by changing elements in the problem space - Pose problem by transformation of representation	-	-	SPP
Courtney et al. (2014)	J. Exp. Educ	USA	PP involved in 10 field trip sites - Design mathematics problems met students - Share problems with peers	2 weeks	68 K-12 teachers	PPA
Daher & Anabousy (2018)	Chapter	Israel	Groups differed in using of technology (applet) and use of what-if-not strategy - Introduce the technology or what-if-not strategy - Pose problem by using technology or strategy	-	79 prospective secondary teachers	TPP SPP

Cheng et al. (2014)	J. Comput. Educ	Taiwan	- Experimental group's PP by annotated expression - Control group's PP by pure expressions	3 weeks	29 fourth graders	PPA
Chang et al. (2012)	Comput. Educ.	Taiwan	- Try out the PP system - Pose problem (system-based or paper-based) - Verify and revise posed problem (system-based or paper-based) - Solve posed problem (system-based or paper-based) - Create new problems according to feedbacks (system-based or paper based)	160 mins	45 fifth graders	TPP
Abu-Elwan (2007)	Int. J. Technol. Math. Educ.	Sultanate of Oman	- take part in mathematical PP tasks - Introduce the WebQuests - Create new problem based on WebQuests	-	50 prospective teachers	TPP
Tavsanli et al. (2018)	Pegem. J. Educ. Instr.	Turkey	- PP activities with/without graphic organizers	4 weeks	38 third graders	TPP
Divrik et al. (2020)	Int. Electron. J. Elem. Educ.	Turkey	-Inquiry-based learning method with PP activities practice -PP/PS homework guide form supported by metacognitive strategies	9 weeks	63 fourth graders	PPA SPP
Otun et al. (2020)	J. Educ. Res. Dev. Area.	Nigeria	Solve-reflect-pose strategy(SRPS) [pedagogical approach] -engage in PS activities guided through Polya's PS steps -reflect on methods of solutions and students' misconceptions/ errors -engage in the activities on regeneration of similar problem	-	92 prospective teachers	SPP
Nakano et al. (2002)	Intell. Tutor. Syst.	Japan	An Intelligent learning environment (ILE) for PP - diagnose the problems posed by the learner - helps the learner to correct wrong problems - leads the learner in the next step of PP	40 mins	55 fourth graders	ILE
Kalmpourtzis (2019)	Br. J. Educ. Technol.	Greece	PP incorporated in the game design activities	3 months	34 kindergarten students	PPA
Hsiao et al. (2013)	Turkish. Online. J. Educ. Technol.	Taiwan	PP homework along with/without worked examples	6 weeks	107 undergraduates	PPE
Kwon & Capraro (2018)	J. Math. Educ.	USA	- Use concrete manipulatives in PP activities - Share and discuss PP process with peers	10 weeks	119 2nd to 5th grade students	SPP ILE
Crespo (2003)	Educ. Stud. Educ.	USA	Interactive work with pupils through letter writing and exchange regarding the posed problem for penpals to solve	11 weeks	34 prospective teachers	ILE

Ocal et al. (2020)	Redimat–J. Res. Math. Educ.	Turkey	Use of GeoGebra vs. paper-pencil test in PP	-	15 prospective teachers	TPP
Lavy & Bershadsky (2003)	J. Math. Behav.	Israel	PP in solid geometry using “what if not’ strategy	-	28 Prospective teachers	SPP
Contreras (2007)	Math. Educ.	USA	PP framework/strategy including proving, reversing, specializing, generalizing, and extending	-	Prospective teachers	SPP
Xia et al. (2007)	Front. Educ. China	China	“Situated creation and problem-based instruction” (SCPBI)[pedagogical approach] -create situations, pose problems, solve problems, and apply mathematics	-	Junior high school students	SPP

Notes: a Information was not available regarding whether the participants were prospective primary or secondary teachers. b WPP: “overview of what problem posing is”; WGP: “discussion of what ‘good’ problems are”; PPA: “engagement with problem posing activities”; CPPS: “comprehension of the problem posing situation”; SPP: “use of strategies involved in problem posing”; PPE: “use of problem posing examples”; TPP: “use of technology in problem posing”; EPP: “evaluation of posed problems”; ILE: “creation of an interactive learning environment.