

**Medieval Literature in Comparative Perspective:
Language and Number in India and the Western Mediterranean**

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Abstract

This paper provides for the first time a textual comparison of selected primary sources on medieval mathematics written in Sanskrit and Medieval Latin; by emphasising literary features instead of purely mathematical ones, it attempts to shed light on a neglected area in the study of scientific treatises which concerns lexicon and argument strategies. The methodological perspective takes into account the intellectual context of knowledge production of the sources presented; the medieval Indian and Latin traditions are historically connected, in fact, by one of the most fascinating episodes in the history of knowledge transfer across cultures: the transmission of the decimal place value system. This cross-linguistic analysis compares and contrasts the versatile textuality and richness of forms defining the interplay between language and number in medieval Sanskrit and Latin works; it employs interdisciplinary methods (Philology, History of Science, and Literary Studies) and challenges disciplinary boundaries by putting side by side languages and textual cultures which are commonly treated separately. The purpose in writing this research is to expand upon recent scholarship on the Global Middle Ages by embracing an Eastern literary culture and, in doing so, to promote comparative studies which include non-European traditions. This research is intended as a further contribution to

the field of Comparative Medieval Literature and Culture; it also aims to stimulate discussion on cross-linguistic and cross-cultural projects in Medieval Studies.

Keywords: Language, Number, Sanskrit, Arabic, Latin, Mathematical Writings, Comparative Medieval Literature.

1. Introduction

This paper investigates and compares treatises on arithmetic from late medieval India and Western Europe which have something in common: they were influential texts in their own literary traditions.¹ Although this research does not attempt to make claims about appropriation of knowledge, attention should be drawn to the fact that the two traditions are linked by one of the most important episodes in the history of the West: the transmission of the decimal place value system from India via Arabic sources. The process of adoption and adaptation of the new methods of calculation is well exemplified by the two Latin works presented here: the *Liber abaci* by Leonardo Pisano and the *Algorismus vulgaris* by John of Sacrobosco. A number of methodological assumptions underline my approach to these sources; first and foremost is the hypothesis that the introduction of the decimal positional numeration produced a paradigm shift which had equally clear implications for language. While the mathematical side of this story has received attention, there is much research still to be done on linguistic and textual features. Although a full treatment of this topic is beyond the scope of the present article, related research questions will be addressed.

The mathematical works which have been selected for a comparative analysis cover a period extending slightly less than 200 years (1050–1230 C.E.), and have been examined under three criteria: genre of mathematical writing, linguistic traits, and narrative strategies. I have opted to single out one specific aspect in each text that also allows a broader discussion on cultural

factors and the mathematical tradition to which the work belongs to. More precisely, the *bhūtsamkhyā* numeration system, brevity, and commentarial explanatory techniques are core characteristics of Sanskrit mathematical writings in their subdivision into root-texts and prose commentaries; in a like manner, the diffusion of Arabisms and the resemanticization of lexical items call attention to the friction between continuity and innovation, classical antiquity and new values that mathematical language exhibits in the medieval Latin tradition. My goal is not simply to identify (obvious) differences, but to use the comparison to uncover the way contemporary Eastern and Western traditions were shaped in their development by a variety of factors.

In investigating two dissimilar historical and geographical textual cultures, it is useful to draw attention to the most basic aspect of any such comparison, namely the intellectual tradition inside which these texts arose and, by way of illustration, to point to the characteristics of each. The analysis will unfold as follows: I will begin with a brief overview of mathematics in medieval India, and then present passages from Sanskrit treatises. Next, I will turn to two different works within the Latin tradition by first introducing the translation movement which contributed to the dissemination of the decimal place value system in the West. The conclusion will follow a brief comparative investigation.

Medieval Sanskrit and Latin mathematical works are here the object of a (concise) comparative analysis for the first time; this paper suggests, in fact, comparisons that have not been explored before. Despite the impossibility, for obvious reasons, of a detailed examination, it is hoped that the reader will hereby gain an insight into the relationship between language and number in the literary traditions discussed.

2. Mathematics as a Literary Subject in Medieval India

The medieval sources on arithmetic investigated in this paper were written in Sanskrit, the chief common language of literary culture in the Indian subcontinent approximately until the 16th–17th century C.E.² Sanskrit is an old Indo-Aryan language that has been widely used over three thousand years; for a long time it was the main lingua franca in use across South Asia.

The chief matrix of literate mathematical knowledge in Sanskrit was *jyotiṣa*, a term that includes the disciplines of mathematics, astronomy, and divination;³ the field of learning known as *jyotiḥśāstra* indicates literally ‘the science (*śāstra*) of heavenly bodies’.⁴ As far as we can tell from the extant Sanskrit sources, in India the treatment of topics on *gaṇita* ‘mathematics’ was incorporated into astronomical works; it was towards the second half of the first millennium CE then that mathematics developed as an independent literary subject.⁵ The first of the fully preserved astronomical texts that integrated both mathematical methods with astronomical explanations are those by the astronomer-mathematicians Āryabhaṭa and Brahmagupta. Āryabhaṭa (476 C.E.) occupies a special place in the history of Indian mathematics as the pattern set by him in mathematical investigations was emulated by the subsequent generations of mathematicians; mathematical rules, expressed in a highly condensed and cryptic form, are given in the *gaṇitapāda* section (the second chapter) of his astronomical work entitled the *Āryabhaṭīya*. Another leading figure, Brahmagupta dedicates the first ten chapters of the *Brāhmasphuṭasiddhānta* (628 C.E.) to the standard astronomical topics, and devotes two separate chapters to mathematics: in the 18th chapter, verses 30–35 give the first surviving explanation in Indian mathematics of positive and negative numbers, and of the arithmetic of zero.

The Sanskrit expression *gaṇitaśāstra* denotes the technical body of knowledge (*śāstra*) of mathematics (*gaṇita*) which appears to fully emerge after the 7th century C.E., when mathematics came to be subdivided into *pāṭīgaṇita* (‘mathematics of algorithms’) and

bījagaṇita (literally ‘mathematics by means of seeds’), which roughly correspond to arithmetic including geometry and algebra respectively. These are also known as *vyaktaṇita* ‘mathematics of visible (or known) numbers’ and *avyaktaṇita* ‘mathematics of invisible (or unknown) numbers’. Unfortunately, only a handful of medieval Sanskrit mathematical works have survived (although some in multiple copies); as sources have come down to us, the most influential mathematicians of medieval Sanskrit literature are Śrīdhara (ca. 8th–9th century C.E.), Mahāvīracārya (9th century C.E.), Śrīpati (ca. 11th century C.E.), and Bhāskarācārya (12th century C.E.); these authors wrote influential treatises which gave a broader shape to the didactic genre of mathematical literature.⁶ Works on arithmetic do not necessarily follow a similar organisation of the content and, notwithstanding that mathematicians show different literary styles, it is possible to identify inherent similarities in terms of vocabulary and topics treated. Texts on *pāṭīgaṇita* consist of rules and sample problems on ‘operations’ (*parikarman*) and ‘practices’ (*vyavahāra*), which are usually classified in the following way:⁷

- i) twenty ‘operations’ (*parikarman*): addition, subtraction, multiplication, division, square, square-root, cube, cube-root (all these operations are carried out with both integers and fractions), five classes for the simplification of compound fractions, rule of three, the inverse rule of three, rule of five, seven, nine, and eleven, and barter
- ii) eight ‘practices’ (*vyavahāra*): mixture, series, figures, excavations, piles, sawing, mounds of grain, and shadows of gnomons.

Authors normally provide numerous ‘sample problems’ (*uddeśaka*, *udāharaṇa*) on the ‘procedural rules’ (*karaṇasūtra*) enunciated which describe a variety of situations drawn from the animal world, the marketplace, and everyday life.

2.1 Literary Practices and Number Systems

In India, the ideal of oral learning connected to the veneration of Sanskrit texts to be uttered during rituals and worship includes different technical subjects. Since the beginning and at all levels of expression, Sanskrit didactic literature was deeply influenced by the tradition of oral learning rooted in the sacred texts called *Vedas* (literally ‘knowledge’); these were composed in *sūtras* ‘thread, string’, verses or short prose sentences which make them easier to memorise. It is in this context of knowledge production and consumption that the circulation of two different types of mathematical writings, namely verse-treatises and accompanying prose-commentaries, should be understood. Like other literary genres, mathematical works in verse were memorized by heart and orally transmitted.⁸ Treatises in verse were accompanied by long prose commentaries; without a commentary, the rules explained in the terse, aphoristic style of a *mūla*-text (i.e., the ‘root’- text) can be very hard to understand. Mathematical commentaries expand what is implied in a verse, provide synonyms, explain technical terms, and show the execution of the sample problems following each rule.

Sanskrit works employ different systems to express numbers which reflect the tension between orality and literacy, which are: 1) figures, 2) spelling out words to denote numerals, 3) the *bhūtasamkhyā* notation, and 4) the *kaṭapayādi* system. Here again the particular nature of Sanskrit literature and learning must be taken into account: the phonetic character of these systems, their conciseness, and coding power preserve the rhythm of the Sanskrit verse (*śloka*), avoid inelegant ways of naming numbers, and support a mnemonic function. Note that often authors combine these systems together in the same passage, it is common to find for instance: सप्तविंशतिः २७ ‘twenty-seven (*saptaviṃśati*), which [in figures] is 27’, thus a number denoted by both a verbal expression and figures, or—as it will be illustrated further on—a combination of the *bhūtasamkhyā* notation and cardinal numbers.

The *bhūtasamkhyā* notation is a word-numeral system where digits and numbers are given as words deriving from various areas of Indian culture, whereas in the *kaṭapayādi* digits from 0 to 9 are indicated by consonants of the Sanskrit alphabet; the latter represents a particular way of using code syllables—by which I mean syllables encoding mathematical values—at a symbolic and semantic level.⁹ Between the two, the *bhūtasamkhyā* seems to be older; it was employed throughout India from the early centuries of the Common Era onwards.¹⁰ The first traces of the *bhūtasamkhyā* system of numeration, in which words arranged as if they were digits of a whole number following a left-to-right direction, are found in the astrological text *Yavanajātaka* by Sphujidhvaja.¹¹

The *bhūtasamkhyā* was also popular in works on astronomy, music, and metrics; the particular combination of nominal numerals having decimal values was adopted for the sake of securing metrical convenience and avoiding unrefined ways of mentioning numerical expressions. The function of this verbal notation is to provide alternative terms for ordinary words, such as *eka* ‘one’, *dvi* ‘two’, and so forth, denoting cardinal numbers. This system enables the name of any object or being—derived from different sources of Indian culture including mythology, ritual, and cosmology—associated with a particular number to stand for it; in this sort of conceptual map, the lexicon is composed of ‘signs’, ‘signifying elements’ which translate cardinal numbers.¹² Let us look at some practical examples of the use of the *bhūtasamkhyā* in the *Gaṇitatilaka* by Śrīpati.

The *Gaṇitatilaka* is a Sanskrit mathematical text written by the astronomer-mathematician Śrīpati, who hailed from 11th century C.E. Maharashtra.¹³ His work has been handed down to us in a uniquely extant yet incomplete manuscript comprising the Sanskrit commentary by the Jaina monk Siṃhatilakasūri (13th century C.E.).¹⁴ The *Gaṇitatilaka* is a versified work on arithmetic; it consists of one hundred and thirty-three metrical stanzas containing procedural rules and sample problems which begin with basic arithmetic and end with investment

computations. In this text, the *bhūtasamkhyā* is employed in the formulation of some of the sample problems, for instance in the one below which is an exercise on ‘square’ (Sanskrit *kṛti*) of integers:

‘Tell [me] the square (*kṛtim*) of [the numbers] beginning with one [and] ending with nine (*ekādīnām navāntānām*), [also the square] of twelve, seventy-two (*dvāsaptates*), ninety-three, [and of the number composed of] one hundred and (literally) six and three (*trirasasya śatasya ca*).’¹⁵

In this verse, composed in the meter *anuṣṭubh*, Śrīpati uses different literary devices to express numbers in order to fit the requirements of the meter and thus the sequence of long and short syllables. The numbers 1 to 9 are denoted by the expression *ekādīnām navāntānām* (in the sentence in the genitive case because governed by the accusative noun *kṛtim*, ‘square’) which is literally ‘one (*eka*) and so forth (*ādīnām*) up to the end which is nine (*navāntānām*)’; the author employs also cardinal numerals, such as *dvāsaptati* ‘seventy-two’ (also in the genitive case for the same reason),¹⁶ and finally he expresses the number one hundred and sixty-three by mixing together the cardinal numerals *śata* (‘one hundred’) and *tri* (‘three’) with the number ‘six’ expressed by means of the *bhūtasamkhyā* notation, and indicated by the term *rasa*. The compound *trirasa* is literally ‘three’ (*tri*) and ‘taste’ (*rasa*); in the *bhūtasamkhyā* notation, *rasa* ‘taste’ denotes the number ‘six’ because in the Indian tradition there are six principal ‘tastes’.¹⁷

In general terms, if while composing a metrical stanza in a given meter the author has to mention, for instance, the number ‘five’ and cannot use the cardinal *pañca* because it does not fit the meter pattern, in order to preserve the scansion of the verse the author could use, just to mention some, the terms *iṣu* or *baṇa*, literally ‘arrow’, as in Sanskrit literature the arrows of the Indian Cupid (*Kāmadeva*) are said to be five. Similarly, the *Gaṇitatilaka* shows that the cardinals *saptasaptati* and *ekacatvarīṃśat* could be replaced by the compounds *śailaturāṅga*

and *rūpajaladhi* to denote the numbers 77 and 41 respectively: as illustrated by the commentator Siṃhatilakasūri, *śaila* is ‘seven’ since it indicates the seven mountains called Kulācalas,¹⁸ *turaṅga* stands for ‘seven’ because the horses of the sun’s chariot (the god Sūrya) are known to be seven;¹⁹ *rūpa* ‘form, shape’ is ‘one’ since everything has only its own one shape, *jaladhi* ‘ocean’ and indicates the number ‘four’ because it was believed that there were four oceans: the eastern, southern, northern, and western oceans.

In the mathematical work *Gaṇitasārasaṃgraha*—the first-known independent mathematical treatise in Sanskrit to survive in its entirety—the Jaina monk Mahāvīracārya (9th century C.E.) gives a list of words corresponding to numbers in *bhūtasamkhyā* system (1.53–63).²⁰ This list, found at the very beginning of the text, represents a practical device for students to interpret the coding-based association of the word-numerals system on which some of his sample problems are expressed. It is clear that the *bhūtasamkhyā* offers authors a set of imaginative, figurative options to create a verse that is both mathematically meaningful and poetically adorned. In brief, the number systems which have been described in this section are ingenious linguistic codes through which the versatility of the Sanskrit language has been used to enrich the communication of mathematical concepts.

2.2 Explanatory Strategies: Brevity and Glossing Techniques

Since the very beginning and in most genres, Sanskrit authors have attempted to achieve brevity as expression of elegance and literary merit. In order to compose texts in verse, Indian mathematicians had to be well versed in poetics and literary theory; their works, often rich in alliteration, attempt to convey the greatest amount of mathematical information in the shortest and most precise formulation possible. Content was elaborated in prose-commentaries and in the oral transmission from teacher to pupil.

Bhāskarācārya's *Līlāvatī* is the most famous Sanskrit mathematical work ever composed, it is a text on arithmetic consisting of 272 verses and which soon came to be considered the standard textbook; the *Līlāvatī* is also the most commented upon Sanskrit mathematical text.²¹ The poetic style of the work and the fact that the author himself provides a brief commentary on his own composition giving the solution of the sample problems may be partly responsible for the long-lasting fame of this treatise.²² Bhāskarācārya employs a refined, literary language that is rich in poetic ornamentations and figures of speech. I have mentioned earlier that brevity is a characteristic of Sanskrit works; there is a pressure to condense which is made possible by the fluidity of Sanskrit, a free word order language that allows infinite compounding possibilities. One should bear in mind that an English translation would hardly reflect the conciseness and precision of the Sanskrit formulation; in Sanskrit there are no articles and the verb 'to be' (*as-*) is almost always implied, scholars have often to supply terms (usually within square brackets) in order to make a translation comprehensible to a modern audience.

After the dedication to the Hindu god Gaṇeśa and a section on measuring units and numeration, in the *Līlāvatī* the first rule concerns addition and subtraction of integers.²³ In verse 12, Bhāskarācārya gives an extremely compact formulation of the methods concerning the two operations:

kāryaḥ kramād utkramato 'thavāṅkayogo yathāsthānakam antaram

'The sum (*yogo*) [as well as] the difference (*antaram*) of digits according to their notational places (*yathāsthānakam*) should be made (*kāryaḥ*) by the regular (*kramād*) or by the irregular order (*utkramato*).'

This rule explains the operations of addition and subtraction as the result of the 'sum' (*yoga*) and 'difference' (*antara*) of digits (*aṅka*) according to their 'position' (*sthāna*) in the place-value notation (i.e., units with units, tens with tens, and so forth); the addition and subtraction

should be thus executed by the ‘regular order’ (*krama*) or by the ‘reverse order’ (*krama*). The author does not elucidate the meaning of these expressions which later commentaries on the *Līlāvatī*, such as those by Gaṇeśa (ca. 1525 C.E.) and Śaṅkara and Nārāyaṇa (16th century C.E.), clarify to denote a right-to-left and left-to-right direction respectively, that is to say, starting from the unit at the very right or from the last digit at the very left.

From the perspective of argument structure, it is interesting to observe the way in which in half a verse the author expounds the two opposite operations of addition and subtraction; the neuter adverbial expression *yathāsthānakam* connects the final element *antara* (‘difference’) with the main clause where the subject *yoga* (‘sum’) is governed by the verbal adjective *kāryaḥ* (‘should be made’). In doing so, everything which concerns *yoga* the ‘sum’—such as the non-finite verb *kāryaḥ* the two ablative nouns *kramād utkramato*’ denoting the two methods the ‘regular order’ and the ‘reverse order’—holds true for *antara* (the ‘difference’) insofar as the condition of the notational position of digits (thus the compound *yathāsthānakam*) is met.²⁴ In brief, the author gives a straightforward formulation condensing the procedures of two operations in half a verse, in which only key items are stated: the two subjects (*yoga* ‘sum’ and *antara* ‘difference’), the verb (*kāryaḥ* ‘should be made’), the condition holding true for both procedures (*sthāna* ‘notational place’), and two nouns in the ablative case denoting the two methods shared by both procedures (*krama* ‘regular order’ and *utkrama* ‘reverse order’).

It has been mentioned that, given the compact style of texts in verse, one of the aims of a Sanskrit mathematical commentary is to provide synonyms, clarify technical terms, and expand upon the concise formulations of rules given in the root-text. These objectives are achieved by means of different types of gloss and explanatory techniques that are widespread in the Sanskrit commentarial tradition. To illustrate a glossing technique, I present below the translation of the rule of ‘square-root of fractions’ (*bhinnavargamūla*) given by Śrīpati followed by the explanation by the commentator Siṃhatīlakasūri:²⁵

chidvargamūlena hṛte 'mśavargamūle vibhinnam kṛtimūlam āhuḥ

‘They explain that when the square-root (*vargamūla*) of the numerator is divided by the square-root of the denominator, the square-root (*kṛtimūla*) of the fraction occurs.’

‘[This is] the explanation [of the verse above]: [in a given fraction] the denominator (*chit>chedana*) is the number [standing] underneath. Its “square” is the multiplication of two equal quantities; that “root” (*mūla>bīja*) refers to this [square]; [the square-root of the numerator] is divided (*hṛta>vibhājita*) by it. When the square-root of the numerator, that is, the “square-root” of the number above [is divided by it], there is the square-root of the fraction (*vibhinnam kṛtimūlam>bhinnavargamūlam*). [In the metrical stanza above] supply [the subject] ‘the learned’ [to the verb] explain.’

In this passage, the commentator elaborates on the concise and technical formulation enunciated by Śrīpati. Siṃhatilakasūri provides synonyms for the term ‘denominator’ (*chit* is glossed with *chedana*), explains that in a given fraction the denominator is the number which is above, expands upon ‘square’ by repeating again (the square has been already elucidated in the text) that the square is the multiplication of two equal quantities, provides a synonym of the verbal adjective *hṛta* ‘divided’, shows that *vibhinna*, which is used by Śrīpati as a predicative adjective (‘fractional, relating to the fraction’) can be explained by means of a *tatpuruṣa* compound where *bhinna* is a substantive: *vibhinnanam kṛtimūlam* literally ‘a fractional square-root’ is equal to saying *bhinnavargamūla* ‘the square-root of a fraction’.

3. Indo-Arabic Numerals in Latin Europe

This is not the place to discuss the complex question of the transmission of decimal numerals in the West; I sum up, instead the main facts. The nine Indian figures and zero which in *Nāgarī* are १·२·३·४·५·६·७·८·९·० are the base of the decimal place value system, in which a number is determined not just by its constituent signs but by the place of each sign. The positional system developed in India was transmitted by Arab scholars to Western Europe; the process of integration is characterised, *inter alia*, by the evolving appearance of the nine symbols in the Arabic, Latin, and Italian vernacular traditions before they were adapted for printing in Renaissance Italy.²⁶

Arabs established realms in Northern and Western India during the Islamic expansion of the early 8th century C.E., and after the rise of Islam, southern India's sea trade came to be dominated by Muslim Arab traders. The Indian base-ten place-value system reached Baghdad by the 9th century; Islamic authors wrote several treatises to describe the system (the Arabic expression *ḥisāb al-hindi* refers to Indian methods of calculation) and explain how to compute by it. The earliest known work on this system is a treatise on Indian reckoning (known through its Latin translations) by Muḥammad ibn Mūsā al-Kwārizmī (ca. 780–850 C.E.) who worked at the Bayt al-Ḥikmah (the 'House of Wisdom');²⁷ this text is no longer extant in Arabic but there are several Latin versions which were made in the 12th century C.E. It seems, however, that Indian numerals had become known in the West before the 12th century C.E.; by the end of the 10th century C.E., the Indo-Arabic numerals had made their appearance in Western Europe, but they had no representation of decimal values.²⁸ The oldest known examples of the nine symbols are in two Latin manuscripts written in monasteries in Northern Spain in the 10th century: the *Codex Vigilanus* (976 C.E.) and the *Codex Emilianus* (992 C.E.), where numerals from 1 to 9 are in the western Arabic form and are written from right-to-left, in accordance with the Arabic way of writing; these numbers were used only on counters of the abacus in the

tradition of Gerbert of Aurillac (10th century C.E.).²⁹ Decimal arithmetic and algebra began to circulate through the translation from Arabic into Latin of al-Khwārizmī's works. Robert of Chester, Gerard of Cremona, Guglielmo de Lunis, Adelard of Bath, and Plato of Tivoli contributed with their translations to disseminating knowledge and laying down new theoretical foundations; the 12th century was, in fact, a turning point for medieval mathematics in the West.³⁰

4. The Beginning of New Traditions

The mathematics of Latin Europe came from Roman sources and was taught within the framework of the seven liberal arts. During the Middle Ages, the liberal arts were central to university education. The *quadrivium* was composed of arithmetic, geometry, astronomy, and music and followed the preparatory work of the *trivium*, that is, grammar, dialectic, and rhetoric. Cassiodorus (5th century C.E.) and Isidore of Seville (6th century C.E.) are writers of vast importance in the history of the transmission of culture from late antiquity, their works—encyclopaedic in nature—enjoyed a long-lasting circulation in Western Europe. However, the most important book on theoretical mathematics all over Latin Christian Europe is Boethius' *De Institutione arithmetica*, which is a loose translation and elaboration of the Greek work *Arithmetic* of the Neopythagorean Nicomachus of Gerasa (1st century C.E.). Boethius' text contains essential elements of Pythagorean theory of numbers, such as the definitions of prime numbers, perfect numbers, and the division of natural numbers into even and odd. Boethius was one of the shaping figures of medieval Latin culture; the legacy of Boethius is a good starting point to understand the tension between tradition and innovation which distinguishes mathematical education in Renaissance Italy. As Marenbon has recently shown, Boethius had a great direct influence over an extraordinary range of intellectual life; his books on arithmetic

and music were widely read up to the Renaissance, remaining part of the syllabus despite the rise of new texts.³¹ Rather than a practical manual of calculation, *De Institutione arithmetica* comprises a philosophical discussion of numbers, their relationship and meanings. However, in the late medieval period the concomitant development of trade networks and rise of mercantile capitalism explains the surge of commercial arithmetic which witnessed a more practical approach to calculation.

In Spain and Italy, translation of Arabic mathematical texts was a complex, multidimensional process of knowledge transmission and transformation, and a major breakthrough in scientific practice.³² Following the introduction of the Indo-Arabic numerals and the translation of important mathematical works from Arabic into Latin, in Western Europe treatises started to appear containing the noun *algorismus*, a neologism which is the Latin adaptation of the Arabic name al-Khwārizmī.³³ The aim of these texts is to teach the new numerals and methods of computation which would progressively replace abacus calculations; *algorismus* came to denote the art of computing by means of the Indo-Arabic figures. Although the topics treated vary as does the amount of details provided by authors, in general terms *algorismi* are textbooks explaining the Indo-Arabic numerals and decimal arithmetic: the decimal place value system, operations with integers, decimal fractions, sexagesimal fractions, and arithmetical progression. The most popular *algorismus* is *Algorismus vulgaris* ('Common algorism') or *De arte numerandi* ('On the art of numeration') written by John of Sacrobosco; a significant number of extant manuscripts of this work attest to its popularity.³⁴

Alongside the algorism tradition, account books and treatises on commercial arithmetic became increasingly popular especially in Tuscany and Venice; the figure of the merchant-writer and entrepreneur arose together with the development of trading practices, for it was in large part Italian merchants who developed complex business techniques such as accounting and bookkeeping. By the end of the 13th century C.E., the development of the sedentary

merchant in Italy revolutionized the organization of commerce. The scale of trade routes, mercantile enterprises, and the flow of goods grew to extraordinary size throughout the Renaissance, marking the beginning of a commercial revolution. The *Liber abaci* of Leonardo Pisano and similar works on practical, business arithmetic grew out of this sophisticated environment of merchant networks, intellectual brokers, and cultural and linguistic intermediaries—all key factors in the production and circulation of new scientific knowledge.

4.1. Arabisms in the *Liber Abaci*

Although there is no general agreement on the role it played in the West in stimulating the transition from Roman to Indo-Arabic numerals, it is unanimously recognised that the Latin *Liber abaci* ('Book of Calculation') of Leonardo Pisano, also known as Fibonacci, represents a seminal work in the mathematics of the Middle Ages.³⁵ Fibonacci seems to have been the first to use the word *abacus/abbacus* (both spellings were found in Latin) to mean 'calculation', the *botteghe d'abbaco* ('abbaco schools') in Renaissance Italy and the *abbaco* mathematical literature teaching business arithmetic in the vernacular used this term in this very way.³⁶ This term should not thus be confused with the 'abacus' denoting the reckoning device.

The *Liber abaci* was completed before 1202 C.E., but the introduction makes it clear that the text which has come down to us is the revised copy (1228 C.E.) which Fibonacci dedicated to his contemporary Michael Scot (1175–1232 C.E.), an astrologer under the patronage of Frederick II and a significant influence on the organization of the intellectual life at the imperial court in Sicily.³⁷ Written in Medieval Latin, this work teaches methods of computation that make use of Indian positional numeration. It is a voluminous text enunciating in detail rules on

a wide range of topics from basic arithmetic to algebra, it also provides a variety of sample problems on commercial arithmetic and shows their execution step-by-step; it comprises 15 chapters dealing with whole numbers, fractions, business problems, algebra, and geometry. The *Liber abaci* is much more than an introduction to the new number system and the methods for working with it: this encyclopaedic work treats much of the known mathematics of the time on arithmetic, algebra, and problem solving.³⁸ Also, it is characterised by a large section on business practices involving financial transactions, money exchange, and barter of a variety of products.

In the dedicatory letter, Fibonacci explains that he became acquainted with the new science of numbers during his training in Northern Africa and travels in the Mediterranean basin, as his father was promoted to become the scribe responsible for the customs in Béjaïa (in modern-day Algeria). In the preface, Leonardo states how in his studies he has found the Indian number system and its methods (*modus indorum*) of calculation to be superior to all other methods, and that he wishes to bring these to the Italian people.³⁹ He also stresses that he will give proofs based upon Euclidean principles for the validity of the methods used. It is clear that the author is addressing himself to the broad audience of all the people who shared the Latin culture; Latin was the language of international trade as well the language of intellectuals, highly educated circles, architects, and other professionals.

Leonardo gives a table of contents for his entire book, which is then expanded into subsections at the head of each chapter with more detailed content lists. In the first chapter, the nine Indian figures (Latin *figura*) and zero are presented from a right-to-left direction following the Arabic convention. Notably, the author recognises the foreign origin of the science he is introducing also when in the incipit he explicitly mentions the ‘Indian figures’ (see *novem figure indorum*) and that the sign 0 is called *zephir* in Arabic (see *quod arabice zephirum appellatur*).

To do justice to the multifaceted literary and linguistic aspects of Leonardo's text, a book would need to be dedicated to expand upon what will be said here in a few lines; yet I will discuss the presence of some neologisms in the lexicon. Notwithstanding that Medieval Latin emerged from the formal Classical Latin and unlike vulgar Latin was preserved from major linguistic changes, as a literary language Medieval Latin's most significant feature is its rich mixture of old and new Latin words, and of old Latin words with new meanings—a phenomenon which occur in technical vocabularies too.⁴⁰ A parallel productive word-formation process took place deriving from both direct language contact and translation practices together with lexical and semantic expansion; I would like, in fact, to draw attention to the occurrence of some Arabisms used in the *Liber abaci* and then found in both the *algorismus* literary tradition and in the 14th–15th century *abbaco* vernacular literature:

- Latin *algorismus* < Arabic al-Khwārizmī
- Latin *elchatayam* < Arabic *al-khaṭa'ayn* 'two falsehoods', it denotes the method of double false position
- Latin *algebra* < Arabic *al-jabr* 'restoration'
- Latin *almuchabala* < Arabic *al-muqābala* 'confrontation'
- Latin *zephirum* < Arabic *ṣifr*, itself a transliteration of Sanskrit *śūnya* 'void'.

Strictly speaking, the Latin words show a close formal relation with the Arabic equivalents from which they were borrowed; an interesting feature is the presence of the initial /al/, some of the early loanwords frequently begin with this syllable because the Arabic definite article *al*—invariable for gender and number— was interpreted by speakers as an integral part of the noun, and therefore was borrowed with the noun it accompanied. These Latin terms are, in fact, the result of a distinct process of lexical borrowing based upon transliteration rather than literal translation, which is instead the case of Latin *res* 'thing' for Arabic *shay'* to denote an

unknown, *census* ‘wealth’ for Arabic *māl* to denote a square quantity, and *cubus* ‘cube’ for Arabic *kaʿb*— all occurring in the *Liber abaci* as well. The terms *algebra*, *census*, *almucabala*, *res*, and *cubus* are found in the Latin *Liber mahameleth* too, where the absence of the term *algorismus* may be an important indication of different original sources.⁴¹

Unquestionably, in the *Liber abaci* Arabisms are the result of a multilingual intellectual and social environment; from a linguistic perspective, due to the transfer from one language into another they embody strategies of morpho-phonological adaptation and semantic adjustment. These neologisms contributed to creating a specific mathematical vocabulary which turned into a shared and standardized lexicon.⁴²

4.2 A University Textbook: Sacrobosco’s *Algorismo Vulgaris*

When comparing Leonardo’s text with the probably slightly later (the exact dates remain uncertain) *Algorismus vulgaris* (also known as *Ars numerandi* or *Tractatus de arte numerandi*) of John of Sacrobosco (ca. 1195–ca. 1256 C.E.), it can be noticed that the latter exemplifies a different genre of mathematical writing and employs a distinct didactic style.⁴³ A quick overview of the syntactic patterns in the *Algorismus vulgaris* tells us that the author prefers a more formal, impersonal authorial voice. By choosing to write in a specific format, authors may have hoped to reach certain audiences; it is true, in fact, that given the range of options available to ancient writers, their narrative choices reflect authorial intention. Differences in presentation between the *Liber Abaci* and the *Algorismus vulgaris* suggest that they were written with different purposes in mind; for instance, the *Liber abaci* is rich in numerical tables, which are not characteristic of texts produced specifically for schools that were intended to be ‘heard’ rather than read, and while the *Liber abaci* is a voluminous text presenting numerous sample problems, Sacrobosco provides none. The particular style of presentation, in a

systematic format, is that of a matter-of-fact work;⁴⁴ although the Latin is more laconic than an English translation can indicate, yet modern readers can still appreciate its depth, elegance, and terseness.

Sacrobosco was a writer on quadrivial subjects, his best known work remain the astronomical treatise *Tractatus de Sphaera*. Nothing in the *Algorismus vulgaris*, a brief treatise describing decimal arithmetic, indicates where and when it was written.⁴⁵ *Algorismus vulgaris* was the first Latin work to be widely adopted in universities to teach Indo-Arabic numerals and new methods of calculation; it was one of the most popular Latin textbook on arithmetic in the Middle Ages.⁴⁶ In the incipit the author paraphrases Boethius—whom the author mentions further in the text—when he observes that everything from the very beginning has been formed in a pattern of numbers, and that *unde in universa rerum cognitione est ars numerandi compendiosa* ‘whenever the comprehension of things is concerned, the art of numbering is in force’.⁴⁷ Next, the author says:

‘A philosopher called Albus declared this concise science of numbering that is therefore named *algorismo* (*algorismus*), which translates as the “art of numbering” or the “introductory art into number” (*ars introductoria in numerum*)’.

Here the author gives the first etymological elucidation of ‘algorism’ as it was understood at that time, a neologism which I have mentioned earlier is found in Fibonacci’s *Liber abaci* and which is a Latinization of the Arabic name al-Khwārizmī. A brief survey of Van Egmond’s catalogue shows that this explanation was copied in most of the Italian vernacular *abbaco*;⁴⁸ in his mathematical text, Jacopo da Firenze explains, in fact, ‘algorism’ as an Arabic art, and that *algo* means ‘science’ and *rismus* is ‘number’.⁴⁹

Sacrobosco introduces then the topics of this ‘art of numbering’:

‘[...] Moreover, there are IX (i.e., nine) species of this art:⁵⁰ numeration (*numeratio*), addition (*additio*), subtraction (*subtractio*), having (*mediatio*), doubling (*duplatio*), multiplication (*multiplicatio*), division (*divisio*), progression (*progressio*), and extraction of roots (*radicum extractio*), which is twofold, since it [concerns] square numbers (*numeris quadratis*) and cube numbers (*cubicis*).’

Next, in the first section called *numeratio*, he introduces the nine figures and adds:

‘The tenth [figure], 0, is called teca (*theca*), circle (*circulus*), cipher (*cifra*),⁵¹ or the figure of nothing, since it signifies nothing. It holds a place and signifies for others, for without a cipher or ciphers a pure article cannot be formulated. And so it happens that any number can be represented by these nine significant figures.’

With respect to the terms used to denote ‘zero’, Cajori observes that *teca*—and he adds that Sacrobosco provides one of the earliest, if not the earliest, examples of such an usage—relates to the ways zero was represented in eleventh century astronomical manuscripts by means of letters of the Greek alphabet.⁵² Latin *circulus* ‘circle’ reminds us that in ancient India a zero was represented by a small dot called, in Sanskrit, *bindu* ‘dot’; Latin *cifra* is cognate with Greek *τζίφρα*, used for instance in *Maximus Planudes*’s 12th century work on arithmetic *The great calculation according to the Indians*. Latin *cifra* is a transliteration of Arabic *ṣifr*, which is in turn an elaboration of Sanskrit *śūnya*.

Another interesting item in Sacrobosco’s lexicon is *differentia* (literally ‘difference’) a word also used in the approximately contemporary *Dixit algorismi* and *Liber mahameleth* to denote a decimal place, a place-value where the *Liber abaci* has *gradus*. Sacrobosco himself explains

that *differentia* denotes the way in which a number—or better, the place value it occupies—differs from the preceding one. I take it as a technical term for ‘[decimal] order, rank’. In his analysis of the *Dixit algorismi*, at one point Folkerts observes: ‘it is not clear of which Arabic word *differentia* is a translation’.⁵³ To my mind, the use of this term is the result of a process of ‘resemanticization’, that is, a derivatory linguistic procedure in which a lexical item acquires a new signification. Let us consider the wider context: the *Dixit Algorismi* is a Latin reworking of the original Arabic text *Arithmetic* by al-Khwārizmī, we recall that in the translation new terms were created by means of transliteration. In the *Liber abaci*, Arabisms embody a process of appropriation of a foreign source. My basic thesis is that along with word creation—and, of course, plain translation—the foreign, technical Arabic vocabulary was transferred into Latin (the target language) by making already existing Latin terms semantically polysemous. More precisely, in evaluating a text or a topic with which one has not much familiarity, the answer is go to a familiar lexicon for comparison. This linguistic phenomenon allows the expansion of a technical vocabulary by integrating standard terms and assigning them a new meaning which is context-based.

To further elucidate my argument, I turn to medieval practices of lexical borrowing. It is interesting, in fact, that *differentia* is used in Medieval Latin texts on psalm tones and modes, where it indicates the endings which mark a transition between a psalm tone and an antiphon. *Differentia* denotes any of the final cadences of the antiphonal psalm tone; in works on the medieval Office, one finds *divisio* and *varietas* as its synonyms, another semantically-related key concept is *distinctio*, which in early Latin works denotes a tone in music. Note that all these terms would well replace *differentia* to denote a ‘place value’ without change in meaning: *differentia* is the ‘division’ of a number into digits where each occupies a place which is ‘different’ in value from the preceding, each digit holds a ‘distinguishing’ position, and represents thus a different ‘variety’ (digit, tens, hundreds, and so forth). Phillips argues that,

applied to choral recitation, *differentia* is a borrowing from Carolingian logical and dialectical studies, and speaks of ‘interdisciplinary borrowing,’ since medieval musical terminology in Latin shows a strong borrowing from other disciplines, especially arithmetic, grammar and rhetoric.⁵⁴ This process could also explain inconsistency and terminological confusion. After all, as subjects of study and fields of knowledge, mathematics and music have always been strictly interconnected, and in Greek and Sanskrit sources they (partly) share technical vocabularies.⁵⁵

More importantly to my point here is that *differentia* already occurs in Boethius’ *De arithmetica* as well as in *De institutione musica*; beside its lexical meaning (‘difference’), *differentia* designates the ratio of the ‘differences’ between terms in the harmonic mean;⁵⁶ Boethius defines an interval as the distance between a lower- and a higher- pitched note. The vast influence that Boethius had in the Middle Ages, as an authority who directly or indirectly tremendously affected mathematical and music conceptualizations and lexicons, can be hardly stressed enough.⁵⁷ Abdounur demonstrates that through the version of Adelard of Bath, the Boethian terminology for ratio predominated during the High Middle Ages, and that in the 14th century Oresme *Algorismus proportionum* uses *differentia* to express what we would call today ‘division of ratios’.⁵⁸ In Boethius’ *De divisione*, *differentia* is an essential concept, it indicates ‘that in respect of which we indicate that one thing differs from another (*Divisio*, 880b); this definition resembles Sacrobosco’s initial statement: *differentia vero quam per illam ostenditur qualiter figura sequens differe a precedente* or ‘[the meaning of the term] *differentia* is shown in this way: how [in the continually increasing series of decimal places] [the power of each] consecutive figure differs from the preceding’. In his text on ‘division’ (i.e., division into categories, classification), Boethius examines different kinds of division, distinguishes one from another, and points out the logical relations between what is being divided, classified, and its dividing elements; also, there is a certain use of mathematical ideas in this work. *Differentia*

is, together with *genus* and *species*, a key term to investigate the nature of related things; it can be seen thus again the connection with Sacrobosco's usage: *differentia* denotes a subdivision, a classification of numbers into hierarchies relating to their power of ten.

One final note from a comparative perspective: the corresponding Sanskrit term for Latin *differentia* as used mathematically by Sacrobosco is the aforementioned *sthāna*, which in mathematical works denotes, in fact, a 'place-value'; interestingly, in Sanskrit texts on musicology *sthāna* also occurs and denotes a 'tone, register'.⁵⁹ The reuse of terms, which is a process of accumulation of meanings is, in fact, a prevalent phenomenon in Medieval Latin and Sanskrit specialised literature. It would therefore not be surprising if, challenged by new concepts relating to decimal numbers, authors borrowed the term *differentia* from a well-established lexicon standardised by authoritative texts.⁶⁰

5. A Comparative Perspective

Sanskrit works on mathematics are characterised by the dichotomy between orality and textuality; this is embodied in the subdivision into verse-treatises and prose-commentaries. A Sanskrit mathematical text in verse is an oral text in that it was meant to be memorized by heart, but it is also a literary text as it imitates the form of a poetic text; metrical form renders a text poetic and easy to be learnt by heart. Sanskrit prose commentaries employ specific explanatory strategies and literary devices to expand upon the laconic formulation of verse-treatises. Although they meet different pedagogical needs, both types of writing display fascinating linguistic means by which the Sanskrit language has been adapted to express scientific ideas and discourse. Indian mathematical texts are highly original in more than one

way: composition in verse form, polysemous terms, and metaphorical expressions are unexpected processes for the exposition of technical matter.

The Latin mathematical works presented here were written in prose and the concise style shown by Sacrobosco differs from the Sanskrit in that, *inter alia*, it is not dictated by rigorous poetic and metric requirements. The *Liber abaci* treats topics in a comprehensive manner and expounds technical matter—such as rules, sample problems, and their execution—which, as mentioned, in the Sanskrit tradition is found in two types of writings, verse-treatises and prose-commentaries. Part of the algebra found in the *Liber abaci* was treated separately in the Sanskrit tradition, and demonstrations for mathematical statement were on rare occasions provided in commentaries; the reasoning behind a mathematical statement was probably orally transmitted. Also, unlike the Latin tradition built on Greek philosophical ideas of number, neither the Sanskrit nor the Arabic texts examined provide philosophical speculation on the nature of number.

The Sanskrit tradition did not happen to develop business arithmetic to the same extent which is shown in the *Liber Abaci*, and some of the topics were expounded more briefly. Fibonacci's treatise represents a type of writing in which commercial arithmetic is for the first time cultivated to such a degree in a Latin mathematical work; more than one-third of this work is devoted to business related-problems: barter computation, the alloying of money, conversion methods for weight and measures, partnership and distribution of profits, and so forth. This is not say that commercial arithmetic was a marginal topic in medieval India, but it may be the case that evidence of teaching contexts other than Sanskrit sources have not survived; we know next to nothing about mathematics teaching for merchants, but it must have existed.

Notwithstanding that the two textual traditions are historically related by the transmission of the Indian decimal value system, they exhibit numerous distinct elements. A variety of social, cultural, literary, and linguistic factors contribute to determining the final design of

works. To adequately interpret mathematical writings in all their dimensions it is fundamental to draw attention to the intellectual background of authors and to their pedagogical project. Forms of scientific knowledge have been generated in different cultural environments and are diversified because socially contextualized; mathematical works were strongly shaped by scholarly communities that developed conventions for reading them and working with them.

6. Conclusion

The analysis developed in this paper invites conclusions to be drawn on several levels. The focus on primary sources sharing a similar content (mathematical) and which can be grouped under the umbrella term of ‘didactic textbooks’ has demonstrated that they present internal diversity in style, which must be understood in light of the interface between text and context. For reasons of space, it has been not possible to examine more differences between individual cases, and thereby the uniqueness of each text, in more detail; my aim was to investigate some of the numerous distinguishing compositional traits which works from both traditions demonstrate. Brevity, poetic requirements, and commentarial strategies are some of the salient features of Sanskrit mathematical literature here represented by the *Gaṇitatilaka* and its commentary; by contrast, the Latin *Liber abaci* employs a semi-colloquial language and is partly structured as an interactive dialogue between the author and the reader, whereas the university textbook by Sacrobosco uses a formal didactic style.

I have also tried to bring to light the fact that the transmission of the decimal place value system from India via Arabic sources represents a paradigm shift which I claim was in equal measure mathematical, social, and linguistic. It was a fundamental, slow change in the basic concepts of number and calculation methods based on the prevailing framework of Boethian arithmetic and Roman numerals. In this regard, this investigation has underlined the role

played by the translation process in the origination of neologisms of which some are still part of modern mathematical vocabulary; I have argued that, in addition to the creation of new terms, a parallel phenomenon was the resemanticization of items from existing lexicons. Linguistically speaking, resemanticization serves economy and recognisability, two essential features of technical lexicons. Another intriguing aspect of this hypothesis is that the resemanticised term —*differentia* — draws a line of continuity with Boethius, the most influential author in the intellectual life of Western Europe during the Middle Ages, and notably that the term occurs in Sacrobosco's *algorismus*, which was a university textbook.

The transmission of the decimal place value system represents one of the most fascinating episodes of diffusion of Arabic and Indian learning; although limited in scope, this paper has tried to bring to light some of the ways in which new knowledge from the East was integrated with existing traditions in Medieval Europe. From a methodological point of view, one of my concerns has been to demonstrate that, when combined with other disciplines, such as literary history and the history of science, and applied to different languages, philology can foster global perspectives in the growing field of Comparative Medieval Literature. This is one of the most exciting fields in which still some uncertainty about best practices exists, but there can be no doubt that cross-linguistic and transcultural approaches are at the core of its foundations. Finally, it is hoped that the analysis of language in scientific works can be used to develop new research and bridge gaps largely obscured by disciplinary schisms.

Notes

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² According to the subdivision mostly adopted by modern scholars of South Asian History, the Indian medieval period lasted from the 5th to the 17th century approximately, and precedes the India modern era. As in mainstream Indological scholarship, here too ‘medieval’ represents a temporal framework rather than a categorization of cultural formations. I use the term ‘medieval’ only to refer to a historical period of Indian history as found in other numerous contributions, such as in Daud Ali, *Courtly culture and political life in early medieval India* (Cambridge: Cambridge University Press, 2004) and Sheldon Pollock, *The Language of the Gods in the World of Men: Sanskrit, Culture and Power in Pre-modern India* (Berkeley: University of California Press, 2006).

³ The most comprehensive work on the history of mathematics in India is by Kim Plofker, *Mathematics in India* (Princeton and Oxford: Princeton University Press, 2009).

⁴ The term *jyotiṣa* refers to a mix of astronomy, astrology, and calendrics; a thorough history of *jyotiḥśāstra* literature is found in David Pingree, *Jyotiḥśāstra. Astral and Mathematical Literature* (Wiesbaden: Otto Harrassowitz, 1981).

⁵ ‘[...] perhaps the first independent texts on *gaṇita* in its broader sense developed in parallel with early works on mathematical astronomy, or even before them, but simply failed to survive.’ Plofker, *Mathematics in India*, 122.

⁶ To this list of authors, we should add the anonymous text on arithmetic known as the Bakhshālī Manuscript (ca. 3rd–12th century C.E.?); edition, English translation, and commentary in Takao Hayashi, *The Bakhshālī Manuscript: an Ancient Indian Mathematical Treatise* (Groningen: Egbert Forsten, 1995). For an English translation of the *Gaṇitasārasaṃgraha* of Mahāvīrācārya, see Malur Raṅgācārya, *Ganita-Sara-Sangraha* of Mahāvīrācārya (Madras: Superintendent Government press, 1912). The chapters on mathematics of the *Brāhmasphuṭasiddhānta* of Brahmagupta and the *Līlāvātī* of Bhāskarācārya are translated in Henry T. Colebrooke, *Algebra with arithmetic and mensuration from the Sanscrit of Brahmagupta and Bhascara* (London: John Murray, 1973 [1817]).

⁷ Brahmagupta is the first known mathematician to have classified mathematical topics in this manner; see *Brāhmasphuṭasiddhānta* verses 12.8–9 and 12.14–54.

⁸ Some characteristics of Sanskrit mathematical commentaries are discussed in Agathe Keller, “On Sanskrit commentaries dealing with mathematics (fifth-twelfth century),” in *Looking at it from Asia: the processes that Shaped the Sources of History of Science*, ed. Florence Bretelle-Establet (Dordrecht; New York: Springer, 2010) and Plofker, *Mathematics in India*, 210–216.

⁹ It is called *kaṭapayādi* because the syllables *ka ṭa pa* and *ya* denote the number 1; see more in Annett Wilke and Oliver Moebus, *Sound and Communication: An Aesthetic Cultural History of Sanskrit Hinduism* (Berlin; New York: De Gruyter, 2011) 480–491.

¹⁰ An investigation of the *bhūtasamkhyā* and its relationship to culture and language is by Alessandra Petrocchi, “The Bhūtasamkhyā Notation: Numbers, Culture, and Language in Sanskrit Mathematical Literature,” in *On Meaning and Mantras: Essays in Honor of Frits Staal*, ed. George Thompson and Richard K. Payne (Berkeley: Institute of Buddhist Studies and BDK America, 2016).

¹¹ This text consists of an earlier translation into Sanskrit of a Greek astrological text; the emergence of Greco-Indian astrology is part of the cultural fusion that followed the collapse of Alexander's empire at the end of the 4th century B.C.E.

¹² Peirce's Sign Theory (1902) is an interesting account of signification, representation, reference and meaning; I will articulate a logical argument by using Pierce's definitions of 'sign', 'object' and 'interpretant' to describe the level of formalization used in the *bhūtasamkhyā* system. Let us take as an example Sanskrit *rasa* 'taste', which in mathematical literature stands for the number 'six' since there are 'six types of taste' in Sanskrit culture. We could say that *rasa* ('taste') is the 'sign', 'six tastes' are its 'object', the number 'six' is its 'interpretant'. The object determines the sign: since Sanskrit literature recognises six types of taste, 'taste' is the signifying element of the 'object' 'six tastes', and the number 'six' becomes the 'interpretant' of this relationship, that is to say, the translation, the development of the original sign 'taste'. The 'interpretant' is the understanding we reach of the sign/object relationship. In brief, because of the object 'six tastes', 'six' is the translation of the word and 'sign' 'taste'. The *bhūtasamkhyā*, as a conceptual map of 'signs' which are culturally shared, requires an interpretative process: when, in a mathematical verse, Indian students meet the word *rasa* or 'taste' understand that it is a 'sign' for 'six tastes', and its 'translation' is the number 'six'."

¹³ The Sanskrit compound *gaṇitatilaka* is literally 'ornament of mathematics'.

¹⁴ Sanskrit edition by Hariral R. Kāpadīā, *Gaṇitatilaka of Śrīpati with the commentary of Siṃhatilakasūri* (Baroda: Oriental Institutes, 1937). An English translation and analysis of the *Gaṇitatilaka* and its commentary is by Alessandra Petrocchi, *The Gaṇitatilaka and Its Commentary: Two Medieval Sanskrit Mathematical Texts* (London: Routledge, forthcoming 2019).

¹⁵ Sanskrit text in Kāpadīā, *Gaṇitatilaka of Śrīpati*, 9.

¹⁶ In terms of grammatical constructions, note that the added number (here *dvā*) is prefixed to the other; it must be mentioned however that equivalent substitutes are variously found in Sanskrit texts.

¹⁷ These are: sweet (*madhu*), sour (*amla*), salty (*lavaṇa*), pungent (*kaṭu*), bitter (*tikta*), and astringent (*kaṣāya*).

¹⁸ The geography of the *Pūranas* recognizes seven principal mountains.

¹⁹ This example is taken from the *Gaṇitatilaka*; Sanskrit text in Kāpadīā, *Gaṇitatilaka of Śrīpati*, 6.

²⁰ The list has been analysed in Petrocchi, *The Bhūtasamkhyā Notation*.

²¹ See Plofker, *Mathematics in India*, 182–196 for an overview of the contents of Bhāskarācārya’s mathematical texts; rules formulated in his works are also analysed in A.K. Bag, *Mathematics in Ancient and Medieval India* (Varanasi: Vidya Vilas Press, 1979); C.N Srinivasiengar, *The history of ancient Indian mathematics* (Calcutta: World Press, 1967), and Bibhutibhusan Datta and Avadresh N. Singh, *History of Hindu mathematics: a source book*, 2 vols (Bombay: Asia Publishing House, 1962 [1935]).

²² The translations of Bhāskarācārya’s *Līlāvātī* and *Bījagaṇita* by the colonial administrator and British Indologist Henry Thomas Colebrooke became classics of 19th century history of Indian mathematics. Here I rely on the Sanskrit edition by Venkateswara K. Sarma, *Līlāvātī of Bhāskarācārya with Kriyā-kramakarī* (Hoshiapur: VVBIS& IS, Panjab University, 1975).

²³ Gaṇeśa, one of the most worshipped deities in the Hindu pantheon, is revered as the remover of obstacles and the patron of arts and sciences.

²⁴ This is an *avyayībhāva* compound, where the first member is indeclinable, the final noun is a neuter accusative and takes an adverbial sense.

²⁵ I use the sign > to denote a gloss (given in the nominative case) provided by the commentator and which cannot be rendered in English by a different term; words within double quotation marks indicate terms used in the root-text and repeated in the commentary.

²⁶ A classic on Indo-Arabic numerals is by David. E. Smith and Louis. C. Karpinski, *The Hindu-Arabic numerals* (Boston: London: Ginn, 1911). A comprehensive investigation of numerical notation system is by Stephen Chrisomalis, *A comparative history of numerical notation* (Cambridge: Cambridge University Press, 2010).

²⁷ An important center of learning in medieval Iraq.

²⁸ On the Latin mathematics of Medieval Europe, see Victor J, Katz, ed., *Sourcebook in the Mathematics of Medieval Europe and North Africa* (Princeton and Oxford: Princeton University Press, 2016). This edited volume contains English translations of key mathematical texts from medieval Western Europe and North Africa written in one of the three scientific languages of that time: Latin, Hebrew, and Arabic. The mathematics of Medieval Islam is investigated in two volumes by Len Berggren, *Episodes in the Mathematics of Medieval Islam* (New York: Springer, 1986), and *Episodes in the mathematics of medieval Islam*, 2nd ed., (New York: Springer, 2016).

²⁹ From the late 10th century until the 12th century C.E., the main method of teaching practical arithmetic in Western Europe was that of the abacus with marked counters. The form of the numerals was not uniform and two main conventions have been recognised by scholars: one in the East and the other used in North Africa and Spain; they differ in the way in which some numerals were written (see figure 1-2-4-3 in Katz, *Sourcebook* , 24).

³⁰ There is no known Arabic manuscript preserving al-Khwārizmī's texts; all that we have are Latin texts based upon a lost translation. A critical edition of Gerard of Cremona and Robert of Chester's Latin translations of al-Khwārizmī's works on algebra is by Barnabas B. Hughes, "Gerard of Cremona's Translation of al-Khwārizmī's al-Jabr. A critical Edition," *Medieval*

Studies 48 (1986) and “Robert of Chester’s Latin translation of al-Khwārizmī’s al-Jabr. A new critical edition” (Stuttgart: Franz Steiner, 1989). Readers interested in the history of numerals and arithmetic in the Middle Ages can refer to the papers edited by Charles Burnett, ed., *Numerals and arithmetic in the Middle Ages* (Farnham: Ashgate Variorum, 2010).

³¹ John Marenbon, *Boethius* (Oxford University Press, 2003).

³² In the 11th and 12th centuries CE, Toledo was the principal center for the translation of Arabic scientific and philosophical texts into Latin; see Charles Burnett, “The Coherence of the Arabic-Latin Translation Program in Toledo in the Twelfth Century,” *Science in Context* 14, no. 1/2: (2001).

³³ Another Latin spelling in use was *Alchoarismi*; *algorismus* became *algorismo* in the Italian vernaculars, and *augrym* in Middle English. The *Dixit algorismi* (‘Algorismi said’) has been investigated by John N. Crossley and Alan S. Henry, “Thus Spoke al-Khwarizmi: A translation of the Text of Cambridge University Library Ms. Li. Vi. 5,” *Historia Mathematica* 17, (1990), and by Menso Folkerts, *Die älteste lateinische Schriftüber das indische Rechnen nach al-Ḥwārizmī. Edition Übersetzung und Kommentar von M. Folkerts unter Mitarbeit von Paul Kunitzsch* (München: Verlag der Bayerischen Akademie der Wissenschaften, 1997) and “Early Texts on Hindu-Arabic Calculation,” *Science in Context* 14, no. 1–2, (2001).

³⁴ *Algorismus vulgaris* was among the first printed books on mathematics. Another popular *algorismus* is by Alexander de Villa Dei; two other texts have been published and studied by Louis Charles Karpinski, “Two Twelfth century Algorisms,” *Isis* 3, (1921): 396–413. Early English translations of Alexander de Villa Dei and John of Sacrobosco’s texts were published by Robert Steele, *The Earliest Arithmetic in English* (London: Milford, 1922); Latin texts by James O. Halliwell-Phillipps, *Rara-mathematica: Or, A collection of treatises on the mathematics and subjects connected with them, from ancient inedited manuscripts* (London: Samuel Maynard, 1841) 2nd ed. A study of *algorimi* is by André Allard, “The Arabic origins

and development of Latin algorisms in the twelfth century,” *Arabic Sciences and Philosophy* 1 (1991).

³⁵ First published by Baldassarre Boncompagni, *Trattati d’aritmetica. I. Algoritmi de numero Indorum* (Roma: Tipografia delle scienze fisiche e matematiche, 1857).

³⁶ Robert Black, *Education and society in Florentine Tuscany* (Leiden, Boston: Brill, 2004) is a study on humanism and education in Medieval and Renaissance Italy.

³⁷ Sources of Fibonacci are discussed in Barnabas B. Hughes, “Fibonacci, Teacher of Algebra: An Analysis of Chapter 15.3 of Liber Abbaci,” *Mediaeval Studies* 66 (2004) and Jens Høyrup, “Leonardo Fibonacci and Abbaco culture: A proposal to invert the roles,” *Revue d’Histoire de Mathématiques* 11 (2005).

³⁸ Note that in the dedicatory letter Leonardo states the “the book is certainly more oriented towards practice than theory”, but one finds a practical analysis in his book which was neglected in earlier works on the abacus.

Leonardo Pisano wrote other books on mathematics, such as the *Liber quadratorum* (1225) and *Practica geometriae* (1223); see an English translation in Barnabas B. Hughes, *Finobacci’s De practica geometrie* New York: Springer, 2008) and Laurence Sigler, *The book of squares of Leonardo Pisano Fibonacci. An annotated translation into modern English* (Orlando; Boston: Academic Press, 1987).

³⁹ In Italy, Indo-Arabic and Roman numerals coexisted for a long time.

⁴⁰ A comparative study on Latin mathematical terminology is by André Allard, “La formation du vocabulaire latin de l’arithmétique médiévale,” in *Méthodes et instruments du travail intellectuel au moyen âge: études sur le vocabulaire*, ed. Olga Weuers (Turnhout: Brepols, 1990); a brief survey on Medieval mathematical lexicon is by Barnabas B. Hughes, “Mathematics and Geometry,” in *Medieval Latin: An Introduction and Bibliographical Guide*,

ed. Frank A.C. Mantello and Arthur G. Rigg (Washington, D.C.: Catholic University of America Press, 1996).

⁴¹ The *Liber mahameleth* is a work in Latin written in the 12th century based (mainly) on Arabic sources from Islamic Spain; see the critical edition by Anne-Marie Vlasschaert, *Le Liber mahameleth: édition critique et commentaires* (Stuttgart: Steiner, 2010), and an English translation by Jacques Sesiano, *Liber Mahameleth* (Cham, Switzerland: Springer-Verlag, 2014). Sesiano claims that the *Liber mahameleth* was written approximately 50 years before the *Liber Abaci*.

⁴² Spelling variations are a common phenomenon in Medieval Latin and the Italian vernaculars.

⁴³ A research paper on Sacrobosco is by Olaf Pedersen, “In quest of Sacrobosco,” *Journal for the History of Astronomy* 16, no. 3 (1985).

⁴⁴ It mentions Boethius towards the end of the extraction of square-roots. Notably, the English philosopher and Franciscan friar Roger Bacon in the *Opus Majus* uses the form *algoristicus* and repeats the same name of the rules given in Sacrobosco’s *Algorismus vulgaris*.

⁴⁵ Available online at: la.wikisource.org/wiki/De_Arte_Numerandi. An English translation is by Edward Grant, ed., *A Source Book in Medieval Science* (Cambridge, Mass.: Harvard University Press, 1974), 94–101.

⁴⁶ One should keep in mind that neither Sacrobosco’s nor other works which fall into the textual genre of the *algorismus* are a copy of al-Khwārizmī’s work.

⁴⁷ In brief, ‘pertinent’. This is a hidden quotation from Boethius’s *Arithmetica*, Book I Chapter III.

⁴⁸ Warren Van Egmond, *Practical mathematics in the Italian Renaissance: A catalogue of Italian abacus manuscripts and printed books to 1600* (Firenze: Istituto e museo di storia della scienza, 1980).

⁴⁹ An edition and English translation of Jacopo's text is by Jens Høyrup, *Jacopo da Firenze's Tractatus Algorismi and Early Italian Abbacus Culture* (Basel ; Boston : Birkhäuser, 2007).

⁵⁰ Note the use of Roman numerals in a treatise expounding Indo-Arabic numerals.

⁵¹ Cf. Arabic *ṣifr*. See the edition and French translation by André Allard, *Le grand calcul selon les indiens. Maxime Planude* (Louvain-la-Neuve: Université catholique de Louvain, 1981).

⁵² Florian Cajori, *A History of Mathematical Notations* (New York: Cosimo Classics, 2011 [1929]) 51.

⁵³ Folkerts, *Early Texts*, 22.

⁵⁴ Nancy Phillips, "Music," in *Medieval Latin: An Introduction and Bibliographical Guide*, eds. Frank A.C. Mantello and Arthur G. Rigg (Washington, D.C.: Catholic University of America Press: 1996), 299. Evans shows that, although his works do not explain practical calculation, medieval treatises on the abacus often quote Boethius verbatim, and borrow principles from dialectic and other medieval arts. See Gillen R. Evans, "Difficillima et ardua: theory and practice in treatises on the abacus, 950-1150," *Journal of Medieval History* 3 (1977).

⁵⁵ In the Pythagorean school, theory of music developed into elaborate treatments of ratios and proportions both in theoretical music and in mathematics. Music was part of the Latin *quadrivium*.

⁵⁶ Interestingly, *differentia* occurs in some medieval manuals on the abacus, where it denotes 'differences' between columns in relation to the decimal place occupied by the counters. Towards the end of the 12th century CE, abacus treatises were superseded by new methods of calculation using Indo-Arabic numerals and algorism treatises; the ideas of the fixed columns of the abacus have disappeared but most of its Latin terminology have survived. Several authors on the abacus borrow Boethius' vocabulary.

⁵⁷ The essays collected by Noel H. Kaylor and Philip E. Phillips, eds., *A companion to Boethius in the Middle Ages* (Leiden ; Boston : Brill, 2012) examine Boethius' influence in many aspects of the intellectual life in the Middle Ages. Black, *Education and society*, 222 observes that among school authors in 12th century Italy, Boethius appears to have occupied a particularly prominent role.

⁵⁸ Oscar J. Abdounur, "Ratios and Music in the Late Middle Ages: a Preliminary Survey," in *Music and mathematics: in late medieval and early modern Europe*, ed. Philippe Vendrix (Turnhout, Belgium Brepols Publishers, 2008).

⁵⁹ See, for instance, in the first chapter of the 13th century Sanskrit text on theory of music *Saṅgītaratnākara*; English translation by Ravindra K. Shringy, *Saṅgīta-ratnākara of Śārṅgadeva*. (Delhi: Motilal Banarsidass, 1978–1989).

⁶⁰ The mathematical part of Boethius's treatise does not mention, of course, the decimal place value system, and expounds a theorization of number based on the ideas of the Neopythagorean Nichomachus.

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