I consider the workings, under free entry, of some plausible mechanisms enforcing product quality when it is not readily observed by buyers, but is chosen by sellers: there is a moral hazard problem to be overcome. In equilibrium, uninformed buyers can often take price as indicating quality. This is however not the same as Spence's signalling notion, since quality is chosen by sellers. It is fruitful to consider price as a commitment, which changes the quality-incentives in a common-knowledge way.

In the introduction, I note briefly much of the relevant literature, including my M. Phil. thesis.

In Chapter 1, on Repeat-Sales, I consider a continuous model of that incentive to quality. With strong simplifying assumptions, I am able to solve for 'competitive' equilibrium, which involves a bunding of buyers at the minimum quality level, price premia for higher qualities, and consequent inefficiencies.

The second chapter is the theory of cheating. It is possible, but costly, for buyers to be 'vigilant'; but equilibrium cannot involve their all being vigilant. There are various externalities between buyers. Different prices can exist in equilibrium, with different honesty levels. I identify a key feature of the cost function, which (together with buyers' tastes, etc.) determines equilibrium.

In Chapter 3, I consider introductory offers and their role in entry and quality-signalling, when buyers are very rational. When buyers are identical, a single entrant will never use introductory offers. If there are many entrants, a low first-period price will be observed, but it conveys no information. Only some quality levels can (credibly) be promised by an entrant, and the result is that an incumbent can make positive profits but prevent entry.
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18 November, 1980.
Introduction

This thesis is addressed to the question: What happens in a market for a good whose quality is chosen by sellers and is not easily observed by buyers?

Most markets have this feature, at least to some extent. The buyer of a car, a hotel room, a meal, a stockbroker's advice, has to trust the seller; what justifies this trust? The law goes some way, enforcing some minimum standard, 'fitness for purpose;' but generally buyers assume that at least the more expensive brands will have higher quality than that. We can ask, is it rational for buyers to make such an assumption? What are the motives that deter sellers from cutting costs by cutting quality? In this work, I consider several answers that many people would suggest: the expectation of repeat-sales to satisfied customers; the similar 'reputation' mechanism; the presence in the market of some 'experts,' i.e. buyers who can (and do) distinguish quality; liability and guarantees; introductory offers.

Scitovsky [1944], considered informally what might happen in a world where ignorant buyers take price as indicative of quality - a central theme of this thesis, and a frequent occurrence, as Leavitt [1954], and Gabor and Granger [1966] demonstrated empirically.

Scitovsky did not, however, consider what counterbalances the incentive to cheat, which I am most concerned with.

The careful study of markets with qualitative uncertainty really began with George Akerlof's famous [1970] 'Market for Lemons' article. Owners of different-quality used cars, indistinguishable to buyers,
consider selling. Assuming a single price in the market (because buyers cannot distinguish), Akerlof pointed out the 'adverse-selection' problem, which might even become so severe as to destroy the market. However, he did not consider the corresponding 'moral-hazard' problem, since in his model quality was exogenously distributed.

Still in the context of exogenously-distributed quality, an important advance, the idea of signalling, was made by Spence [1974], who took the labor market as his canonical example. Because high-quality sellers can gain privately by distinguishing themselves from their low-quality colleagues, they may try to find ways of showing off - indulging in acts or displays which show ability. It is necessary for equilibrium that the lower-ability workers (low-quality sellers) should not find it worthwhile to imitate them, and thus be taken for high-ability. An equilibrium can be established in which, for instance, the amount of education obtained (the 'signal') enables a prospective employer (the uninformed buyer of labor) to infer, correctly, the ability of a worker; the resulting wage-as-a-function-of-education schedule is just such that it pays each worker to make the educational choice which ends up confirming the expectations of buyers. What lies behind this is that education is more costly for the lower-ability worker.

Joseph Stiglitz [1976], and Charles Wilson [1980], have considered whether price can be a signal of quality in such a market. It turns out it can; the cost of raising price may be a reduction in the probability of selling. This is costlier to owners of low-quality items, since (presumably) they have a lower use-value. The 'law of supply and demand' must be repealed too - but the forces behind that law no longer operate when sellers may be unable to sell at the highest price, and buyers fear lower quality at a low price).
Riley [1975, 1980] pointed out a difficulty with the existence of signalling equilibria. Essentially, if a buyer offers a price above the 'going price' for some low signal level, it turns out he can do better than by adhering to the price schedule.

Despite that objection, the notion of signalling and of signalling equilibrium has made its way firmly into the minds of theorists in the area. Methodologically, we have to say that even if convincing precise models of the signalling process which are 'true economic equilibria' (no agent can do better) have yet to come, still we believe that Spence's models illuminate an important 'way the world works.' In my view that is the best an economic theorist can hope for, since any model is wide open to objections (for instance on the grounds of oversimplification) however perfectly it may achieve internal consistency, or consistency with "global maximizing" postulates - which themselves do not have an absolute claim on our belief.

The reader will note that the Riley objection, which involves choosing a price and level of signal off the schedule, does not seem to make sense in my case, where price is signal. Whether there is some analogous objection, I do not know.

Another strand, in a different skein, is the securities-pricing literature. There is an exogenous distribution of returns to some asset; some traders have or acquire some information about the return, and then the asset (in fixed supply) is traded. When will the price of the asset reflect (all) the information about the return? - more precisely when will it be a sufficient statistic for the information on the return?
Sufficient conditions for the existence of such a 'revealing equilibrium' are given in Grossman [1976]. Grossman and Stiglitz [1980] take a different approach, letting the set of 'informed traders' be endogenous; in their model if price revealed all information, nobody would appropriate returns on information-gathering. Equilibrium must (when information is costly) involve a price system which does not reveal all information, unless of course there are other motives for information-gathering (see Bray [1980]).

An interesting and provocative model of quality-signalling is Philip Nelson's [1974]. There, being a better buy (a matter of price and quality) makes repeat purchases more likely; thus the marginal reward to advertising is greatest for the best buys, and so the best buys advertise the most. This argument, however, ignores the fact that, countervailingly, the best buys will have the smallest 'margins' (price minus marginal cost), and this makes their incentive to advertise smaller. Schmalensee [1978] shows how the conclusion depends on elasticities, in a model in which firms choose quality and advertising, while price is fixed.

All the work discussed above has a common feature which makes it not directly relevant for my purpose. In every model, an exogenous distribution of quality is assumed. But in many markets for goods and services, quality is chosen by the seller. Instead of the adverse selection problem of Akerlof, we have a moral hazard problem: what will induce the quality-chooser to choose correctly, in the absence of a properly quality-contingent contract? Quality may be unobservable (as in the usual moral hazard literature) because other factors get mixed in before observation. Equally, the quality may (after use) be
observed only by seller and buyer; and neither may have the proper incentives to report correctly. For whatever reason, quality-contingent contracts are assumed away, and we are left with the problem of seeing how quality is provided in the absence of explicit and enforceable contractual provision. Klein and Leffler [1979] indeed take 'quality' as a metaphor for any aspect of a trade that is not governmentally enforced.

Quality-contingent contracts are not unknown. One might argue that the custom of tipping in restaurants is of this nature (although one could dispute whether there is a contract). Perhaps more clearly, we observe both legal (compulsory) liability rules, and voluntary guarantees or warranties. If a product may break down or cause an accident, with some probability which is chosen by the seller and not observed by the buyer, then we can influence the producer's incentives, or he can publicly influence his own, by placing responsibility on him should something go wrong.²

This has two effects: the incentive effect, and an insurance effect. Some insurance is compulsorily bundled with the product. (The insurance aspect, ignoring the incentive effect, is considered by Heal [1977].) If the buyer does not want the insurance, or if the seller is risk-averse and therefore reluctant to sell it (and unable to reinsure - buy liability insurance - because of the moral hazard problem), this inevitably leads to non-optimality. Two instruments are needed; Spence [1977] considers liability from seller to buyer and from seller to State.
Walter Oi [1973] emphasises the compulsory-insurance aspect, and belittles the informational problem; not surprisingly this leads him to the conclusion that seller liability can be a bad thing. He also points out that if buyers differ in the loss from an accident (generally, valuation of quality), then a liability rule which fails to allow for that will lead to improper choices.

More problems arise when acts of the buyer can influence the probability of an accident. Then something more is needed, such as a doctrine of 'contributory negligence'. Presumably the need to retain some incentive for buyer care is the reason why most consumer durables have such risibly short warranty lives, by comparison with buyers' reasonable expectations of the actual life of the item. There is also a moral-hazard problem involved in promising a warranty payment which exceeds the value of the item, and this puts another damper on the effectiveness of guarantees as quality-certificates. In short, it is only in special cases that guarantees or liability will fully solve the quality problem, and I feel justified in ignoring them in what follows - not that I claim they are of truly negligible importance, but rather that an important problem remains.

Reputation

Different sellers are often judged likely to provide different qualities, even if their price, advertising, etc. is identical. There is such a thing as reputation. Buyers tend to expect a seller to provide quality today similar to what he provided yesterday: the past is not forgotten.
Why, we may ask, do buyers have such expectations? And, given that they do, what effect does that have on a seller's quality incentives? Putting the two together, can reputation be an equilibrium phenomenon?

Why should we expect a seller's previous quality to help predict his current quality? First, quality may be a once-only choice. Then, of course, knowing what he did in the past is equivalent to knowing what he is doing now. This puts us into the world of 'Introductory Offers' (Chapter 3). Less stringently, there may be some 're-tooling cost' associated with changing quality.

Another possibility is that sellers actually differ in some relevant parameter which is not directly observable: discount rate, for instance, or marginal cost of quality. Then there can be a signalling equilibrium, in which each period's quality choice signals the unobservable parameter (perhaps imperfectly), and is therefore useful in predicting later quality choice. The papers of Paul Milgrom and John Roberts [1980] and David Kreps and Robert Wilson [1980] define and prove existence of similar equilibria.

Whatever their reason (if any), there can be little doubt that buyers do often expect quality to be predictable from past quality. Therefore a seller who provides high quality will build up a valuable asset, his reputation, which will enable him to charge a higher price and/or get more sales in future. Naturally, this constitutes a quality incentive. What will be the properties of markets in which reputations are important?
In principle, reputation should be modeled by describing fully who believes what about a seller's quality. This will be very complicated, and will lead to a rich and intractable model in general. In practice, one must simplify (it seems) drastically. Apart from the work of Avner Shaked and John Sutton [1980], who emphasize the importance of extreme expectations, I believe all work has been conducted on the assumption that reputation varies on the same single dimension as quality. Taking a control-theory viewpoint, this ensures that (under mild assumptions) any optimal path will converge. Generally, the point of convergence will be a lower quality than the seller would choose if buyers had perfect information. (See Shapiro [1980a]).

The other aspect of reputation that has received attention [Ephraim Toaff, 1978; Benjamin Klein and Keith Leffler, 1979; Carl Shapiro 1980b] is the 'valuable asset' approach. First, if a good reputation does not yield profits, then no seller will enter and build such a reputation, assuming it is a costly thing to do. Second, if equilibrium is to involve the maintenance of reputations by those who have them, some profits must be made, because there is always the option to cheat on the reputation. Toaff, in a two-quality model, and Shapiro, in a continuous-quality model (begun with the present author), showed how price must be related to quality in any such equilibrium. (If information flow is such that word of a new high-quality seller spreads very fast, and news that he is now lowering quality spreads slowly, there can be no such free-entry equilibrium.)
In Shapiro's simple model, price \( p \) is related to quality \( q \) and minimum quality \( q_0 \) by

\[
p = c(q) + r[c(q) - c(q_0)]
\]

where \( r \) is the discount rate and \( c(.) \) the unit cost function.

Evidently, with \( r > 0 \), there are misallocations compared with the first-best. More interestingly, minimum-quality standards should in general be imposed which stop some buyers from getting their preferred quality. The reason is that their loss is balanced by the gain of those who wish to buy higher quality and find the prices coming down towards costs.

Klein and Leffler [1979] consider a model in which sellers choose whether to remain honest (once they have their reputation) or to 'cheat', which involves taking one-period gains and then leaving the market. The authors concentrate on the ways in which, by manipulating his costs, a seller can commit himself to remaining honest.

Essentially, they argue, any firm-specific capital good which provides benefits only while the seller is in business, will serve this purpose. Notice that their argument depends on each seller having to leave the market after being dishonest, rather than merely losing his good reputation. Also, the claim that 'sunk costs' (as, for instance, absurd advertising) can serve this purpose, has an interesting feature they do not point out. There is, as usual, an equilibrium in which bygones are bygones; what they work towards showing, is that there may be another equilibrium, in which bygones remain relevant. To have thrown money away creates a flow of benefits which encourage the firm to remain honest, if and only if buyers so believe. Much interesting work remains to be done in this area.
Because my M.Phil. thesis (Farrell, [1979]) is obviously of special relevance to this work, being its precursor, and because it may be less accessible than the other references, I will now describe its contents somewhat more fully than I have summarized other works.

I began with a model ('the Advertising model') in which there are two exogenously-determined quality levels, 'good' and 'bad,' produced at unit costs of $c_g$ and $c_b$ respectively ($c_g > c_b$). New buyers flow into the market, and choose a price to pay (on the basis of the average quality prevailing at each price). They then choose a seller at that price, in a fashion influenced by "advertising" in a simply-specified way. If they pick a 'bad' brand, they try again; if it was a 'good' brand, they go on buying a unit each period until they 'die.' Different buyers have different expected lifetimes, and indeed that is all that distinguishes them and causes them to choose different prices.

The result of this is that 'good' firms make more sales, over time, from a unit of 'advertising' than do 'bad' firms at the same price. This is what stops 'bad quality' being a dominant strategy in this model.

Assuming that firms did not change either price or quality (an assumption I later discussed - p. 51), I used comparison of steady state profits (not, of course, the correct procedure, but (I hoped) qualitatively reasonable) for the two qualities at each price, with a zero-profit condition, to derive the level of advertising and the proportion ($y$) of 'bad' firms (weighted by advertising). The solution emerged in the form of a first-order differential equation for $y$ (p. 13), which unfortunately I was unable to solve explicitly. Other
analytical disadvantages were that the needed end-condition for the
differential equation was elusive (p.25), and that comparative-static
results were difficult to obtain. The essential purpose of the model,
however, was to indicate how it might be that high quality could be
made viable, by allowing for the savings in selling costs. Closely
related was the result, which came out clearly, that the lowest price
at which high quality can be sold at all is bounded below (p. 15) by

\[ c_g \exp\left(\frac{1 + r - \mu}{\mu}\right) \]

where \( r \) is the discount rate, and \( \mu \) is the largest probability that a
current satisfied customer will return next period. Although I did not
follow this up explicitly, it has obvious implications - most clearly
visible in the case where the social surplus is maximized at 'good'
quality, but the extra price premium for high quality is so large that
buyers prefer to go to low quality. Notice incidentally that, while \( r \)
looks like an ordinary discount rate, in fact it is the discount rate
from one purchase to the next. Thus, 'rare purchases' will make high
quality more expensive.

Finally, I undertook a limited stability analysis of the
equilibrium. For a single price, I analyzed the stability of
equilibrium as 'good' and 'bad' firms react to profits and losses.
The conclusion was that if 'bad' firms react faster to profit
opportunities (by increasing scale) than do 'good' firms, there is instability at the single-price level. This is because a reduction in average quality increases the number of unattached buyers per firm.

Leaving the Advertising Model, I considered next arguments of the firm, "Because information on quality can be got, and some buyers will go to the trouble of getting it, it will not be worthwhile for bad-quality sellers to enter." I pointed out that, in a plausible formalization of the argument, some increasing returns to scale (over some range) were necessary to sustain non-degenerate zero-profit equilibrium. I then briefly analyzed a model with two exogenous qualities, and a fixed cost for each seller. While I was able to solve the model at any one price, the problems arising when multiple prices were allowed became too severe. This remains true: see the latter part of Chapter 2 below.

Next, I asked where the 'two qualities' hitherto assumed had come from. One answer I gave, the 'threshold-quality' model, foreshadowed the 'Bargains & Ripoffs' version in Chapter 2 of the present work. The other answer was an early version (like the Advertising model, done in steady-state terms) of my Repeat-Sales model (Chapter 1 below).

The thesis concluded with some speculations. First, I asked what will happen if changes of quality are permitted. This really amounted
to pointing out the inappropriateness of the steady-state analysis in the Advertising and 'Differentiable' (repeat-sales) models. There is of course likely to be some cost of changing quality, to simplify or complicate the problem. Next, I discussed changes of price. In the Advertising model, the constant-price assumption was binding and unnatural. In a sense, my work on Introductory Offers (Chapter 3 below) is a response to that. This strengthens the incentive to quality and the extent to which price signals quality, but raises some new questions.

Then, I turned to the question of how growth and uncertainty in a market affect quality incentives. The essential point there is that firm-specific capital, such as goodwill or reputation, has a different value in a market in which the marginal firm is closing, from that it has if the marginal firm is opening. If neither exit nor entry is taking place, there is a range of possible values. These differences affect the 'cost of quality.'

To those observations I have nothing more precise to add, but I draw the reader's attention to the remarks in the Appendix to the Repeat-Sales model (Chapter 1) below.

Finally I pointed out a question which has not been properly treated in the economics-of-information literature. Papers in this tradition show the existence of a problem, and indicate one possible (usually partial) solution. It is not generally asked which information-transfer mechanism(s) is or are likely to come about in a given market, nor whether that 'choice' has any optimality properties. (And not only must one 'informational
structure' be compared with another, but mixtures and combinations must be considered.) For instance, in adverse-selection problems, the 'better' agents have an incentive not to let a 'pooling' situation persist, and they may try to introduce signalling - but what else might happen?

Having thus sketched a research program, I must confess that I have not yet followed it up. Instead, in this work, I have considered three individual mechanisms in somewhat greater detail. First, the repeat-sales model (Chapter 1 of the current work). Here I abandon the steady-state comparison, and instead formulate a control-theoretic version of the model. Unfortunately, only in a simple case can I find an explicit solution; but that case reveals features which we can reasonably expect to persist in general. First, and most important, the prospect of repeat sales is valuable enough to counteract the lure of cost-cutting only if sales have a sufficiently high (positive) shadow price, which means that market price must exceed marginal cost of production of the relevant quality. Second, the price premium (that excess) increases with the discount rate. Third, it increases with the propensity of buyers to leave exogenously. Fourth, the minimum quality will attract a 'bunch' of buyers of different tastes, in general, because no price premium is required at that quality. This has implications for the optimal choice of quality standard.

The second model in this work is that of the effect of having some well-informed buyers on market quality. There, (Chapter 2), the principal conclusions are as follows. First, with costly information, equilibrium will never involve enough buyers being informed to produce a perfect market. Second, the usual externality from informed to uninformed (as in price-search models) applies, as does an externality
between the uninformed. Third, partly because of an incentive effect, and perhaps partly because of selection, different prices can involve different levels of honesty in equilibrium. All this depends on some properties of technology, which I can clearly identify, as well as on buyers' information costs in a somewhat less straightforward way.

Fourth, the effects on equilibrium and on welfare of changes in a minimum quality standard are not straightforward, and neither is the effect of a change in technology. In particular, it seems that a reduction in costs can be a welfare-reducing change.

In Chapter 3, I consider a model which was designed to address two questions. First, what is the effect of the ability of a high-quality seller to seize some of the surplus of his buyers by raising price? And, second, can quality be effectively signalled by introductory offers? The literature, such as it seems to be, on introductory offers, suggested the consideration of another question: Can such offers be used to overcome any difficulty a new firm may have in entering a market with quality uncertainty? The results, briefly, are as follows: First, a single potential entrant will, if buyers are fully rational in a sense I make precise, never avail himself of the opportunity to make introductory offers. Second, if there are many (potential) entrants, 'introductory offers' will be seen, but it is not they that signal quality. Third, there are limits (determined by the level of surplus supplied by the incumbent) on the quality levels that can be promised by the 'offers' mechanism. And fourth, the incumbent can choose his price so that profitable entry is impossible, even though profits are being made.
Chapter 1: Repeat-sales.

1. Introduction

One mechanism which is often informally mentioned as ameliorating the problem of imperfect quality information, is that of prospective repeat sales. The idea is that a seller who provides high quality can expect to find his customers returning for more. This upward shift in his future demand curve is good for his profits; and the prospect of that provides an incentive for him to increase quality above what it would otherwise be.

In this paper, I examine the effects of a simply-specified mechanism of repeat sales on quality and price, in a formulation which I believe to be as close to perfect competition as one can come with the imperfect information assumed. Equilibrium generally involves a range of prices and qualities, with uninformed buyers correctly regarding price as indicative of quality. For all qualities above the minimum, however, the price is too high. Entry dissipates these quasi-profits, but does not reduce the price at any quality level.

The imposition of a minimum-quality standard has interesting effects on equilibrium and welfare. In particular, the standard should always be set so that some buyers regard it as excessively high. The reason, briefly, is that some buyers who under full information would prefer a higher quality, will choose to go to the minimum because it ends up costing them discretely less than even a slightly higher quality. These buyers benefit from an increase in the standard, and that effect is set off against the loss to those who feel the standard is now too high. Other comparative-static results are derived.
Some of the results carry over at least plausibly to the somewhat more general formulation I relegate to an Appendix, where scale affects average costs and repeat-buying behavior is more complicated. Some interesting questions of signalling by sales or by age of firm, arise in that context.
A good, which can be produced at any of a range of quality levels, is sold in a market with free entry and many sellers. Each seller sets his quality, which affects costs in the obvious way, and quality is not certified or guaranteed; any statements a seller makes about his quality are ignored by buyers. Buyers have very poor information on quality: Not only can they not observe a seller's quality before buying from him, but they cannot even accurately evaluate the quality of a brand they have consumed. What I have in mind here is that quality represents some kind of reliability, or probability of a 'satisfactory' outcome in a particular use or trial.

Thus a buyer wishing to behave optimally in this market will find it hard, in general: He would have to solve the 'n-armed bandit' problem, and indeed if we allow sellers to alter quality over time, it is worse yet. I regard it as unreasonable to model buyer behavior as if such a problem were solved; and instead assume a simple (and in my view, fairly plausible) behavioral rule. (Perhaps some of the wrath of optimalists at such a step may be turned away by the observation that the rule I propose is in fact optimal in equilibrium, though not in general.) This rule is simply that he buys from a seller until the product fails or breaks down, whereupon he leaves that seller. This is the rule considered by Smallwood and Conlisk [1979]. Their model considered quality competition at only one, exogenously given, price; so this paper can be viewed as a widening of theirs. There are some
other differences.\textsuperscript{2} An effect of such behavior is that the time-path of sales, for any seller, depends only on his own quality choices.\textsuperscript{3} This is an important simplification of the problem, with the usual virtues and vices of simplifications.

While quality is hidden from buyers, price is not. This assumption seems to be a fairly good representation of the world. Much has been made, in the last twenty years, of the costs of searching for the best price, and consequent price structures; but for most goods and services one can get any price quote at the cost of a telephone call. While even that search cost will support some (perhaps interesting) price dispersion, it seems clear that the problem of quality uncertainty is much more serious.

Price is freely observable to buyers. But not all buyers necessarily buy at the lowest price on offer. Their reasoning, which is both common in real life\textsuperscript{4} and confirmed in equilibrium in this model, is that sellers at higher prices are likely to provide higher quality. I would emphasize, however, that this will not be a signalling model in the sense introduced by Spence [1973], and used by Stiglitz [1976], and C. Wilson [1980] in a context apparently similar to this one. Their models rest on the assumption that there is some exogenous distribution of quality - some men are able, some are not. The essence of the current work, however, is that sellers choose their quality. And while in Spence's models, the informed agents choose the signal on the basis of their exogenous quality, I prefer to think of the sellers as choosing first the 'signal' (price) and then on that basis choosing quality. Thus we can think of it as a model of
commitment, in the sense of Schelling [1960], but in a market setting, not in bargaining. Price affects the incentives to quality, and that fact is common knowledge; therefore in announcing his price, a seller conveys to buyers what quality it will be in his interests to choose. Because of that, he can persuade them to pay a price not the lowest.

Before getting down to detail, I will describe schematically the nature of the equilibrium I consider. Buyers observe many sellers, at (perhaps) different prices, offering the good for sale. They have (perhaps rational) expectations concerning the distribution of quality conditional on each observed price; but they cannot observe individual sellers' quality levels. Each buyer, given his expectations and his tastes, chooses the best prospect (for him) among the prices. He selects a seller at random at that price, and then remains a customer of that seller until the product breaks down, or 'disappoints' him; then he abandons that seller (and perhaps the market).

Taking as given that behavior of buyers, and the flow of new customers he will get at any given price, each seller tries to maximize his profits by choice of quality. Higher quality costs more, but causes his sales to grow faster (or shrink less). It turns out not to be worth his while, in the simple model, to consider exit or a change in price after entry. It turns out that his optimal choice of quality is an increasing function of the price he charges. Intuitively, this is because higher price makes repeat sales more valuable.

If we wish to make the rational-expectations assumption, the optimal quality choice feeds back into buyers' expectations of quality given price. And finally, free entry at all price levels determines
the number of new customers coming to any firm at any active price-level, and drives entrants' profits to zero.
3. **Formal Statement of the Model:**

1. There is an exogenous flow of new buyers into the market. These new buyers differ in their tastes; specifically they differ in their willingness to trade higher quality for more 'other consumption', i.e. lower price.

2. Each new buyer chooses a price at which to buy, selects a seller at random amongst those selling at that price, and buys from him at a unit rate, until a breakdown occurs, or some exogenous reason causes him to leave the market. The probability of a breakdown in a short time interval \((t,t+dt)\) is \((1-q)dt\), where \(q\) is the seller's quality at time \(t\). The probability of leaving exogenously in the same interval is \(k dt\).

3. Entry is free at all prices. A seller at price \(p\) gets a flow of new customers equal to (say) \(n(p)\). Sellers believe themselves unable to influence \(n(p)\) for any \(p\). In fact \(n(p) = N(p)/S(p)\), where \(N(p)\) is the total flow of new customers at price \(p\) (determined by buyers' tastes and quality expectations), and \(S(p)\) is the number of sellers at price \(p\). There is a fixed set up cost of \(F\).

4. A seller who changes his price loses all his customers. (For a justification of this simplifying assumption, see note 5.)
5. Each seller chooses at each instant a value $q_t$ between $q_0$ and 1. ($q_0$ is a legal minimum quality standard.) The cost of producing at rate of sales $x$ and quality $q$ is $C(x,q)$, which I assume is just $xc(q)$, where $c' > 0$, $c'' > 0$.

6. A seller at price $p$ maximizes his profits by solving the following problem:

Maximize $\int_0^\infty e^{-rt}x_t(p-c(q_t))dt$  \hspace{1cm} (1)

where $x_t = f(x_t,q_t) = n(p) - (1 - q_t + k)x_t$
and $x_0 = 0$

(Notice that this assumes no exit; it is clear that, provided $p > c(q_0)$, exit is never better than the best staying-in strategy; and no firm will enter at a price less than $c(q_0)$.)
4. Solution of the Model

I consider the problem (1) for a seller at any price p. This leads to quality choices by the seller - in fact, in this simple model, quality does not vary over time. There is then what I call the skeleton quality-curve: i.e. a relationship that tells us what quality a seller will choose at any price. In general, a variety of prices p will induce qualities q(p) such that some buyers regard the pair (p, q(p)) as optimal (among the choices available), and we expect those prices to be active in (rational-expectations) equilibrium.

First, then, I describe the solution to (1).

Lemma: In the solution $q_t$ to (1), $q$ is independent of $t$.

Proof: Write $V(x,n)$ for the maximized present discounted profits starting from sales-level $x$, and with new-customer flow $n$. Then because both $x(p-c(q))$ and $n-(1-q+k)x$ are homogeneous of degree 1 in $(x,n)$, we have

$$aV(x,n) = V(ax,an)$$

for any $a > 0$.

Therefore for any $x_1, x_2$

$$V(x_1,n) - V(x_2,n) = a^{-1}[V(ax_1,an) - V(ax_2,an)]$$
\[ a^{-1}[\alpha(x_1-x_2)V_x(a_{x_1},x_2) + o(\alpha(x_1-x_2))] \]

\[ = (x_1-x_2)V_x(0,0) \]

letting \( \alpha \to 0 \).

So \( V \) is affine in \( x \). But that means that the shadow value \( V_x \) of extra sales is independent of \( x \).

In the Hamiltonian

\[ H = x(p-c(q)) + V_x f(x,q) \]

we find

\[ H_q = -xc'(q) + V_x x \]

so that, when \( H_q = 0 \), \( H_{qx} = 0 \) also (because \( V_{xx} = 0 \)), and so the optimal \( q \) is independent of \( x \). It is clear from (1) that \( q \) could depend on \( t \) only via \( x \).

**Remark:** This Lemma, which makes the model tractable, uses the assumption that the probability of a given customer leaving the firm is independent of how long he has been a customer. This is not really plausible, but it is convenient, as it enables us to find explicitly what quality will be provided at any price.

Now (3) will describe for us the optimal choice of \( q \), if we can evaluate \( V_x \). To do that, I write the dynamic-programming equation:
\[ rV = x(p-c(q)) + V_x^f(x,q) \]  

and differentiate with respect to \( x \), remembering that the optimal \( q \) does not change, and \( V_{xx} = 0 \):

\[ rV_x = p-c(q) + V_x f_x \]  

where \( q \) is the optimal choice. From (5),

\[ V_x = \frac{p - c(q)}{r - f_x} \]  

Substituting (6) in (3) and equating to zero for the first-order condition on quality choice, and dividing through by \( x \), we have

\[-c'(q) + \frac{p-c(q)}{r+k+1-q} = 0 \]  

Equation (7) describes for us the choice of \( q \) that will be made at price \( p \), and given parameters \( r \) and \( k \). Accordingly, it describes the skeleton price-quality curve. More conveniently, however, we can rewrite it as
\[ p = p(q) = c(q) + (r + k + 1 - q)c'(q) \]  \hspace{1cm} (8)

In that form, it defines the price \( p(q) \) at which any quality \( q \) will be forthcoming: in other words, the equilibrium price of quality \( q \).

**Proposition** All qualities are (potentially) available, at a high enough price. That price always exceeds marginal cost of production.

The rate at which price changes when quality increases is
\[ c''(q)(r+k+1-q) > 0; \] thus\(^7\) higher price and higher quality go together, but the increase in price accompanying an increase in quality bears no simple relation to \( c'(q) \), the increase in production cost. Finally, increases in \( k \) or \( r \) lead to lower quality at each price, i.e. higher price for a given quality.

**Proof:** From (8).

**Comment:** There is here a certain pedagogic problem. It should be clear from the analysis that the result "\( p > c(q) \) when \( q > q_0 \)" does not by any means rest on the existence of the fixed set-up cost \( F \) mentioned above: rather, it is because the shadow value of extra sales would vanish if \( p = c(q) \), and therefore this (and any other) mechanism purporting to give quality incentives via increasing sales, could not work.\(^8\)

However, precisely because \( p > c(q) \) in any equilibrium with above-minimum quality \( q \), we cannot have such an equilibrium with free entry
without some fixed cost, whether a set-up or a fixed operating cost. That is one reason for introducing the fixed cost. Another is to show how, in the presence of quality uncertainty, a tricky problem may actually be ameliorated. In traditional analysis, with a fixed cost and constant marginal cost, free entry, and price-conscious buyers, there is no Bertrand equilibrium. Here, the imperfect information restores equilibrium. A seller who cuts his price will (quite rationally) be seen as cutting quality, and he will not necessarily increase sales. With full information, the variability of quality would not help; it is the lack of information that does it.

Having found the skeleton quality-curve (8), I will now describe equilibrium. Buyers have (derived) indifference curves in (p,q) space - not all the same, if buyers' tastes, incomes, etc. differ. On the skeleton quality-curve each buyer has an optimal point, generally unique. For each type of buyer, we expect sellers to offer the optimal price. Thus, the prices observed will be those which are optimal for some buyers. There is in general no reason to expect the set of prices observed to be connected, even with a 'connected' continuous distribution
of types of buyer and the most favorable concavity assumptions on $c(\cdot)$ (as assumed) and on indifference curves, as shown. We cannot expect concavity of $p(\cdot)$. In fact $p''(q) = c'''(q)(r+k+1-q) - c''(q)$.

There is one important modification to be made in this analysis. Equation (8) describes the part of the curve where quality $q$ is an interior optimum choice, given $p$. But sellers can convincingly offer to sell the minimum quality $q_o$ (enforced by technology, law, limited quality observability, or otherwise) at any price above $c(q_o)$. Here the assumption of a fixed cost is a nuisance — without it, we could just say $q_o$ is offered at $c(q_o)$. With the fixed cost, we have to say that equilibrium will presumably involve sellers of $q_o$ playing mixed strategies on price in the range $(c(q_o), p(q_o))$, and making expected quasi-profits just sufficient to cover fixed cost. (For a description of a mixed-strategy equilibrium in a similar setting, see Varian [1980].) Notice that in general $p(q_o) > c(q_o)$, so there is a nontrivial distribution of prices for $q_o$. With a large number of
firms selling $q_0$, it seems the best price available should be near $c(q_0)$ most of the time (Varian [1980]). Thus it seems we should not go far wrong in pretending, for simplicity, that $q_0$ is available at a price $p_1(q_0)$, where $p_1(\cdot)$ is slightly above $c(\cdot)$, but below $p(\cdot)$.

We can then consider an extended version of Figure 1, in which the curve $p = p_1(q)$ is plotted as well as the curve $p = p(q)$. Equilibrium is then the following: The government, or nature, determines the minimum quality $q_0$. Then $(p_1(q_0), q_0)$ is available, as is the entire skeleton quality-curve. Equilibrium will generally involve a 'bunching' of different buyer-types at $q_0$, because even a very slightly higher quality costs discretely more to buy. (This is true unless $p(q_0) = p_1(q_0) = c(q_0)$; but $p(q_0) = c(q_0) + c'(q_0)(1 + k + r - q_0)$, and so we have a jump unless $c'(q_0) = 0$, which by assumption ($c'' > 0$) could never happen unless $q_0 = 0$, and not in general then.)
5. Welfare Remarks

The distortion of price away from marginal cost leads as usual to inefficiencies. We can be more specific.

Suppose buyers have utility functions given by
\[ u = -p + \alpha q + \beta \]
if they buy the good, and zero (say, without loss of generality) from not entering the market. Here \( \alpha \) and \( \beta \) are parameters differing among buyers.

A buyer of type \((\alpha, \beta)\) ought, socially, to buy the quality \(q(\alpha, \beta)\) given by

\[ c'(q(\alpha, \beta)) = \alpha \]

if that leads him to positive utility when the price is \(c(q(\alpha, \beta))\); and he ought to leave the market otherwise. Both these decisions are distorted. If he buys, it will be the quality \(\tilde{q}(\alpha, \beta)\) given by

\[ p'(\tilde{q}(\alpha, \beta)) = \alpha \]

i.e. \[ c''(\tilde{q}(\alpha, \beta))[1 + r + k - \tilde{q}(\alpha, \beta)] = \alpha \]

- or else he will buy \(q_0\) - and this choice may be either too high or too low. The fact that prices always exceed marginal cost tells us
unambiguously that too many buyers will leave the market.

Now let us consider the optimal choice of minimum-quality standard $q_o$. The lower is $q_o$, down to some point, the better it suits those with low values of $\alpha$. But because $q_o$ is available at a price lower than $p(q_o)$, there will generally be a bunching of consumer types buying $q_o$. Therefore the setting of $q_o$ must represent a compromise among all those types.

Because the curve $p = p(q)$ does not depend on $q_o$, there is not a further 'external' effect, as in Shapiro [1980b], where the prices of all qualities are affected by a change in $q_o$, via a change in the payoff to cheating. Here, the relevant cheating is marginal, and so the skeleton price-quality curve does not depend on $q_o$. 
6. Conclusion

In a very simple model, I have demonstrated how the prospect of repeat sales leads to quality incentives even if buyers' quality information is very poor. On the other hand, the model also indicates that quality above the minimum will be available only at a price premium above its marginal cost, and this leads to welfare losses both from substitution to the wrong quality, and from buyers not buying at all. One result of this is that it may be beneficial to set a minimum-quality standard high enough that some buyers will be forbidden to purchase their preferred quality. This is in contrast with the case of perfect information, where such a policy is bad.

I used simplifying assumptions to deduce the quality choice made by a seller at any price. In equilibrium, all sellers at any price $p$ have the same quality $q(p)$. Thus any rule for switching around among such sellers is optimal in equilibrium for an individual buyer; this includes both complete loyalty and utter caprice, as well as the rule I consider. But if sellers differ (say) in discount rate or cost function, or if my simplifying assumptions do not hold, then dispersion of quality will reappear at each price, and a rule (like mine) which makes a buyer likely to stay longer with a seller of relatively high quality, is sensible.

In the Appendix I consider the problem with a more general cost function and more general expression for $dx/dt$. The intractability of that model, on which I spent too much time, confirms the wisdom of making simplifying assumptions. Economic models - especially partial-equilibrium models! - cannot hope to be complete. The best we can do is to aim for clarity.
Appendix: A more general formulation of the repeat-sales model

I now consider what can be said about the role of repeat-sales in a more general framework – specifically, when the cost function and the equation determining rate of change of sales are more general. The results are disappointing; we can predict convergence to a steady state, but even to do comparative static analysis on the steady state turns out to be difficult.

Formulation of the seller's problem

Production at rate $x$ and quality $q$ costs $C(x,q)$, where I assume $C_q > 0$, $C_{qq} > 0$, $C_x > 0$.

Sales $x$ change at a rate determined by sales level and by quality:

$$\dot{x} = f(x,q)$$

(18)

Then the seller has the following control-theory problem:

Choose $(q_t)$ to maximize

$$\int_0^\infty e^{-rt} \left[ px_t - C(x_t,q_t) \right] dt$$

where $\dot{x} = f(x,q)$ and $x_0 = 0$

(19)
Convergence Assured

Since time occurs explicitly only in the discount factor, and the
discount rate is constant, the optimal choice of \( q \) depends only on
\( x \), not on \( t \) explicitly. Therefore \( x \) follows a first-order differential
equation

\[
\dot{x} = f(x, q(x)).
\]

I assume that, for small \( x \), this is positive - i.e. it is worth
building up sales at least a little, once you are in, and that for
large \( x \), \( f \) is negative - intuitively, the exogenous flow of new
customers cannot keep up with the leavers, however good quality is.
Then \( x \) converges to the first value \( x^* \) for which \( f(x^*, q(x^*)) = 0 \).

Notice that for this argument I have assumed that \( f(., q(\cdot)) \) is
continuous. It is certainly reasonable to assume \( f \) is continuous in
each argument; and \( q(\cdot) \) is continuous if the Hamiltonian is
continuous in \( x \) and strictly concave in \( q \), which is assured if
\( C_{qq} > 0 \) and \( f_{qq} < 0 \). Accordingly, I make these assumptions.
Discontinuity of \( q(\cdot) \) seems to lead only to technical problems, not to
interesting nonconvergence of \( x \).

Since \( x \) converges, and \( q(\cdot) \) is continuous, \( q \) also converges, on
the optimal path.

Comparative-statics of steady-state: Use of \( q(\cdot) \) curve

The optimal path, we know, follows the curve \( q = q(x) \), i.e. optimal
choice of \( q \) given \( x \), as far as the first intersection with \( f = 0 \),
converging to that steady-state point.

In order to make comparative-static statements about \((x^*, q^*)\), we must know how the curves \(q=q(x)\) and \(f=0\) shift when we change parameter values.

If we write

\[
H(q;x,v;p,r,k,\ldots) = px - C(x,q) + vf(x,q)
\]

(11)

then we know

\[
q(x) \text{ maximizes } H(\cdot;x,v(x);\ldots)
\]

(12)

where \(v(x)=V'(x)\) is the shadow value of sales.

If we assume concavity of \(H\) in \(q\), then the maximization condition (12) can be replaced by the first-order condition,

\[
H_q = 0
\]

(13)

Accordingly, \((x^*, q^*)\) is defined by the intersection of the two curves \(f=0\) and \(H_q=0\). Let \(s\) be any parameter affecting \(f\) or \(H\). We have

\[
\begin{bmatrix}
H_{qq} & H_{qx} \\
q & x
\end{bmatrix}
\begin{bmatrix}
dq^* \\
ds
\end{bmatrix}
+
\begin{bmatrix}
H_{qs} \\
df_s
\end{bmatrix} = 0
\]

(14)
where - most importantly - we must remember that \( v \) varies also when we change the parameter \( s \), so that by \( H_{qx} \) I mean

\[
\frac{\partial^2 H}{\partial q \partial x} + \frac{\partial^2 H}{\partial q \partial v} \frac{dv}{dx} = -C_{qx} + vf_{qx} + v'f_q
\]  

(15)

and similarly for \( H_{qs} \) of course.

Now we know that the intersection that interests us has the property that \( q=q(x) \) meets \( f=0 \) from above. That enables us to sign the determinant of the matrix in equation (14): the determinant is positive. Therefore the changes in \( q^* \) and \( x^* \) have the signs respectively of

\[
-f_{xs} + H_{xq} s
\]

(16)

and

\[
-f_{qs} + H_{qs} s
\]

(17)

Can we sign (16,17)?
Lemma: With free entry and zero profits at each of a range of prices, defining equilibrium we have

(a): $f_p < 0$

and (b): $f_r > 0$ if $p_x - C$ increases along the optimal path.

Proof: With a given $f$, entry must be more attractive at a higher price. Hence, to equilibrate, we must have (a).

As for (b), this follows from the result that

if $\int e^{-rt} R_t dt = 0$ and $\dot{R} > 0$ then

$$\frac{d}{dt} \int e^{-rt} R_t dt \leq 0$$

(To see that, integrate by parts.) Since profits fall from zero on an increase in $r$ for a given level of $f$, there must be a compensating rise in $f$.

Proposition: If the assumptions of the above Lemma hold, and if

$$H_{qx} \leq 0$$

$$H_{qp} \geq 0$$

$$H_{qr} \leq 0$$

(18)
then \( \frac{dq^*}{dp} > 0 \) and \( \frac{dq^*}{dr} < 0 \).

**Proof:** From the assumption and the Lemma, applied to (16, 17).

Consider the expression (15) for \( H_{qX} \). Substituting the equation
\[
v = \frac{C_q}{f_{qX}}
\]
- this holds at all points on the optimal path - into (15), we get

\[
H_{qX} = -C_{qX} + C_q f_{qX}/f_q + v' f_q
\]  \( \tag{18} \)

Let us recall how this looks in the simple example treated in the main text. There, \( f_q = x, f_{qX} = 1 \); thus we get, from the form of \( f \),

\[
H_{qX} = (-C_{qX} + \frac{C_q}{x}) + v' f_q
\]  \( \tag{19} \)

At the same time, the example treated above made assumptions ensuring that \( v' = 0 \) and \( C_{qX} = \frac{C_q}{x} \), so that in the end \( H_{qX} \) vanished. This of course corresponds to the fact that \( q \) was independent of \( x \), which (the reader will recall) made the analysis simple.

As an aside, I turn to an interesting interpretation of the sign of \( H_{qX} \), which is of course the sign of \( q'(x) \) also. If it is
negative, this says that new sellers provide relatively high quality, and reduce quality as they build up towards steady state. If on the other hand \( q(x) \) increases with \( x \), this corresponds to new sellers being less reliable than old-established ones.

Thus, in a limited way, some progress has been made towards understanding in what cases we are likely to see new sellers a better bet than old—casual observation suggests that cheap restaurants fall in this category, for instance—and when, conversely, sellers may wish to advertise their long-standing establishment.

Clearly for any such conclusion we would have to modify the model so that age (or sales) was observable—notice that in this model it was not—and also so that buyers do not all go to those sellers at a particular price with 'the best' value of \( x \). But it seems to me of interest as a suggestion.

As for signing \( H_{qs} \), we have

\[
H_{qs} = v_s f_q + v_f s q
\]  

(20)

if \( s \) does not affect \( C \). Indeed, we might reasonably assume \( f_{sq} = 0 \) also (the parameters affect only the flow of new customers, say), and then we have

\[
H_{qs} = v_s f_q
\]  

(21)
We have now reduced the problem of signing \( \frac{dq}{dp} \) and \( \frac{dq}{dr} \) to the problem of investigating \( v' \) and \( v_s \), i.e. \( V_{xx} \) and \( V_{xs} \). Unfortunately, the investment of much time in that problem has not paid off. It seems, intuitively, that \( v_p > 0 \), for instance; and yet it is not so clear, because we have to remember that \( f \) is also affected by the change in \( p \). A priori, the most promising approach seemed to be to work on the equation

\[
rV = px - C + V_x f
\]

(22)

differentiating variously. But success did not follow. The simple model of the main text really expresses the interesting ideas already.
Does the presence of the strong help protect the weak? If there are expert buyers as well as ignorant buyers in a market, what is the effect on quality? What determines how much expertise there will be, and how much cheating?

In this chapter I consider these questions in a fairly simple model of quality choice. 'Experts,' the 'vigilant' or 'informed' buyers, know all sellers' prices and qualities; the uninformed know only prices. First, in a general game-theoretic model in which sellers choose to be 'honest' (high quality) or 'dishonest' (low quality), I show how protection is imperfect, but in general occurs - the marginal amount of protection being an externality, one corresponding to a well-known externality in the theory of price-search.

Then, in the context of free-entry equilibrium, I show why fixed costs are important. (By 'fixed costs' I actually mean only increasing returns in the relevant range and below.) The reason is that small-scale low-quality (dishonest) operations have to be made unattractive. I describe a concept of equilibrium analogous to that used in the previous chapter, and describe the nature of equilibrium. Existence I prove in a very special case only; in general this may be a problem. Finally, some welfare statements, or at least cautionary remarks, can be made. In particular, some not-obvious effects of minimum-quality standards are pointed out.
1. **A Model of Vigilance and Cheating**

When is honesty the best policy? If one is 'trading' with a vigilant and careful 'opponent', who will detect dishonesty and punish one for it, honesty is likely to be optimal. Yet if honesty prevails, vigilance flags; and when vigilance sleeps, the wicked flourish. What determines the equilibrium of vigilance and of honesty? What are the welfare properties of such an equilibrium? In this section, I present a very simple model of trust and cheating: cheating is more profitable than honesty if it goes undetected, and vigilance has its cost. A Nash or market degree of trust and of cheating results. This model provides a mental framework for the remainder of the paper, which is concerned with the same question in a particular context - namely, price and quality choice in market equilibrium, where information on quality is available but costly.

**The Model:** It is easiest to present this first as a two-player game. The 'buyer' can be vigilant or trusting; the 'seller' can be honest or dishonest. Their payoffs are as follows:

<table>
<thead>
<tr>
<th>Buyer's Choice</th>
<th>Seller's Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Honest</td>
</tr>
<tr>
<td>Vigilant</td>
<td>(H, u-c)</td>
</tr>
<tr>
<td>Trusting</td>
<td>(H, u)</td>
</tr>
</tbody>
</table>

Here the seller's payoffs (written first) are H (the gain from honest trade) if he is honest, and D (the dishonest gain) if he is dishonest and the buyer is trusting. If he is detected in dishonesty, he gets 0:
this can be viewed as a normalization, though it is especially natural if we think of the seller not making a sale. The buyer's cost of vigilance is c, and his loss from being cheated is k; an honest trade is worth u to him.

**Proposition.** Assume $k > c > 0$ and $D > H > 0$. Then there is no Nash equilibrium in pure strategies.

**Proof:** Best response to honesty is trust ($c > 0$) to dishonesty is vigilance ($k > c$) to vigilance is honesty ($H > 0$) to trust is dishonesty ($D > H$)

A unique Nash equilibrium in mixed strategies exists: it is given by the equations

$$H = tD \quad \text{(seller equilibrium)} \quad (1)$$

$$(1-d)(u-c) - dc = (1-d)u - dk \quad \text{(Buyer equilibrium)} \quad (2)$$

(2) gives $c = dk \quad (3)$

Here $t$ and $d$ are the frequencies with which trust and dishonesty, respectively, are played.
The seller's expected payoff is $H$; the buyer's is

$$(1-d)(u-c) - dc = (1-d)u - c$$

$$= (1 - c/k)u - c$$

(4)

In particular, if $(1 - c/k)u < c$, the buyer will refuse (if he can) to play: the chance of mutually beneficial $(u > 0, H > 0)$ trade has been destroyed by (potential) dishonesty.

As usual, an alternative to thinking of this as a two-player game with mixed strategies is to think of it as played by a continuum of identical buyers against a continuum of identical sellers. (Notice that this interpretation makes the players' behavior, in taking as given $t$ or $d$, much more compelling a description of reality than if we consider it a two-player game.) This suggests the generalization to heterogeneous buyers and sellers, which enables us to see some aspects not visible in the homogeneous case. Equations (1) and (3) are still true of the marginal sellers and buyers. If $H/D$ has distribution function $S(\cdot)$ among sellers, while $c/k$ has distribution function $B(\cdot)$ among buyers, equilibrium satisfies

$$d = S(t) \quad \text{(seller equilibrium)}$$

(5)

$$t = 1 - B(d) \quad \text{(buyer equilibrium)}$$

(6)
We can infer uniqueness of equilibrium from the equation
\[ t = 1 - B(S(t)) \] (derived from (5), (6)), which has a left-hand side increasing in \( t \), and a right-hand side non-increasing. Existence of equilibrium follows from the Kakutani theorem applied to the correspondence \( (t,d) \to (1 - B(d), S(t)) \) which takes \([0,1] \times [0,1]\) to itself.\(^1\) Local stability (in a reasonable dynamic process) is easy to establish.\(^2\)

Welfare remarks. The values of \( t \) and \( d \) can be regarded as describing the behavior of all agents: thus, all sellers with \( H/D \leq t \) are dishonest and those with \( H/D > t \) honest; all buyers with \( c/k \leq d \) are vigilant, and the others trusting. Therefore, \( t \) and \( d \) determine the expected payoff to each agent. So any welfare function dependent only on individuals' payoffs can be regarded as a function of \( (t, d) \) alone.

The buyer equilibrium curve (6) is the result of each buyer maximizing his payoff given \( d \). With \( d \) given, therefore, the buyer-equilibrium value of \( t \) maximises any Paretian buyers'-welfare function: there are no externalities between buyers given \( d \). On the other hand, buyers are worse-off if \( d \) increases, so the contours of a buyers'-welfare function look like this:
A similar argument shows us the contours of a Paretian sellers' welfare function: [Figure 3].

Notice that the buyer and seller equilibrium curves are therefore the reaction curves of fictitious agents who choose respectively \( t \), \( d \) and whose payoffs are the buyers' (respectively, sellers') welfare levels. (These agents are of course not fictitious in the two-player interpretation.) Equilibrium is the Nash equilibrium of this 'fictitious' game. This enables us to make various welfare remarks:
(1) Equilibrium is not Pareto-optimal: a small increase in \( t \) and reduction in \( d \) will improve both the buyers' welfare and the sellers' welfare levels.

(2) Small movements from equilibrium down the seller equilibrium curve are desirable for buyers' welfare. All movements up it are undesirable for buyers. Thus, if 'the buyers' can be Stackelberg leader the outcome involves less trust and less dishonesty than the Nash equilibrium. It is of course also preferred by buyers.

In the many-buyer interpretation, this corresponds to the following externality: Each buyer decides on his strategy taking \( d \) as fixed. But in fact buyers' decisions do affect \( d \) (assuming sellers stay on their reaction-curve), and this affects other buyers' payoffs. Those inframarginal-trusting buyers would like to bribe/subsidize the marginal buyers to choose vigilance.

Of course, this is analogous with the usual 'externality from search' in the price-dispersion models where the distribution of price depends on (sellers' perceptions of) buyers' readiness to search.

Perfect correction of the externality (with respect to the welfare function used) corresponds to 'the buyers' seizing the Stackelberg leadership.

(3) Such movement down the seller equilibrium curve is displeasing to the sellers.

(4) A shift of the sellers' equilibrium curve downwards - as for instance if some change increases each seller's value of \( H/D \) - makes buyers better-off.
2. An Example

In the above game, we had exogenous random meetings between sellers and buyers; and the terms of trade (the specifications of $H$, $D$, $u$, and $k$) were exogenous to the model. Suppose we think of the trade as being the selling of an item whose quality is detectable only to the vigilant; and suppose there are just two qualities available, good (G) and bad (B), which cost $c(G)$, $c(B)$ respectively to make. Then if all trades take place at an exogenously given price $p$, we have an example of the situation described above. We can write

\[
\begin{align*}
H &= p - c(G) \\
D &= p - c(B)
\end{align*}
\] (7)

\[
\begin{align*}
u &= v(G) - p \\
k &= p - v(B)
\end{align*}
\] (8)

We can calculate, given perhaps heterogeneity of the functions $c(\cdot)$ and $v(\cdot)$, the equilibrium levels of $t$ (trust) and $d$ (dishonesty). We can undertake comparative-static exercises. Perhaps the most interesting of these is the question: what happens to buyers' welfare if $p$ increases?
With identical buyers, welfare is given by (4):

\[ W = (1 - \frac{c}{k})u - c \]

\[ = (1 - \frac{c}{p-v(B)})(v(G) - p) - c \]

and we obtain

\[ \frac{dW}{dp} = -k^2 + ck + cu \]

\[ = -(p - v(B))^2 + c(v(G) - v(B)) \]  

which means that for low prices \( p \), specifically when

\[ p < v(B) + \sqrt{c(v(G) - v(B))} \]  

we have welfare increasing with price. Of course, the meaningful range of prices only includes those greater than \( c(G) \), and it is not guaranteed that the calculated optimal price (the right hand side of (10)) satisfies that restriction. However, it is entirely possible that prices too little in excess of \( c(G) \) are too low.

When buyers are not identical, the answer obviously depends on the welfare function chosen. (In this model, the informed buyers cannot be helped by a price increase, although if \( G \) were to improve as a result it can happen that a Pareto improvement would follow.)

What is the intuition here? Quite simply, by increasing \( p \), we increase \( H/D \) (see (7)) for each seller. At the same time we reduce \( c/k \) (see (8)) for each buyer. Thus \( S(\cdot) \) shifts down, and the buyer reaction curve shifts left, leading to a reduction in equilibrium \( d \), which weighs against the obvious effect of the increased price on buyers' welfare.
3. **Endogeneity of Price**

Let us consider a market system, in which rather than random meetings between seller and buyer, we observe sellers setting prices and buyers looking over the various sellers' offers. In traditional analysis, i.e. with information on quality supposed to be perfect, this system is supposed to drive price to market-clearing marginal cost in the short run, and minimum cost in the long run. What can we expect to see under the current assumptions?

Buyers will be aware that the incentives to quality depend on price. This dependence occurs in two ways: not only does price affect $S(\cdot)$ and thus the outcome for given $B(\cdot)$, but also, if there are several prices around, some selection may be occurring, so that the functions $B(\cdot)$ determining equilibrium differ at different prices.

And sellers, aware that buyers regard price not merely as foregone consumption of other goods, but potentially as a quality indicator, will not (as in the traditional case) all try to offer the lowest price.

Thus equilibrium may involve more than one price; and even if there is a single price, it may not be simply related to costs.

In the example above, the optimal price from buyers' point of view, if they all pay the same price, was calculated (see(10)). Will that price be equilibrium?, we can ask.

What do I mean by equilibrium? In the spirit of the previous chapter, I propose the following definition:
Equilibrium is a set $P$ of 'active prices' at which trade takes place; and for each buyer a choice of price and of vigilance/trust; and for each seller a choice of price and of honesty/dishonesty, or of quality. The following conditions should hold:

(a) Given the honesty level (distribution of quality) at each price of $P$, each buyer is making an optimal choice.

(b) Given the sales he will make from each strategy, each seller is choosing optimally.

These conditions of course are entirely straightforward. They assume that agents know the values of $t$ and $d$ at each price, but that does not seem unreasonable. However, we have to ask what happens if a seller were to charge a price not in $P$. In order to claim an equilibrium, we should impose a further condition:

(c) Sellers believe that no strategy outside $P$ will be more profitable than their choices within $P$ - a zero profit level, if we are considering free entry.

And, although it rests on shakier arguments than the 'rational-expectations' element of (a) and (b), we will also require:

(d) Sellers are correct in that belief.

What determines the profit of a seller charging a new price? Clearly, it is the sales level he gets - the number of buyers prepared to try the new price. At least for uninformed buyers, this is determined by their beliefs about his quality. Thus, in (c)-(d), there is an assumption about what buyers will believe about the quality of a seller who starts charging an unaccustomed price.

Evidently, if buyers were always optimistic about such a seller's quality, equilibrium could not exist unless perhaps with all
conceivable prices being active. Equally, if buyers are always very pessimistic about such mavericks, no set $P$ would be ruled out by the superiority of prices not in $P$. We would like some intermediate, reasonable case. The natural step seems to be to assume:

(e) Buyers are correct in their quality expectations regarding any seller charging a new price."

In the presentation so far, I have not explained why any informed buyer should ever choose to pay a price greater than the lowest at which $G$ is obtainable. I now consider what equilibrium is like, with that observation; then I describe reasons why we may wish to consider other models, in which the informed will choose various prices. Then I describe a model in which moreover the 'good' quality varies according to the price and is endogenous.

If informed buyers know which brands sell $G$, they will all patronise the cheapest $G$-seller. Thus, perfect competition will prevail as far as the informed buyers are concerned. Let $p(G)$ be the minimum average cost of producing $G$. (As the reader will see shortly, we must abandon constant costs if we hope to have a nontrivial free-entry equilibrium.) Then the informed buyers buy $G$ at price $p(G)$. Likewise let $p(B)$ be the minimum average cost for $B$. Then uninformed buyers can buy $B$ at price $p(B)$. 
However, it cannot be an equilibrium for there to be $G$ sold at price $p(G)$ and for all uninformed buyers to buy $B$ at $p(B)$. For if the latter holds, then all sellers at $p(G)$ sell $G$; and then no buyer need become informed. It may be an equilibrium for everyone to buy $B$ at $p(B)$; or, more interestingly, we may have a mixed equilibrium in which a proportion $h$ of the sellers at $p(G)$ sell $G$ (i.e. are 'honest'). Then, the payoff to being informed is

$$v(G) - p(G) - c$$  \hspace{1cm} (11)

Payoff to being uninformed and paying $p(B)$ is

$$v(B) - p(B)$$  \hspace{1cm} (12)

and payoff to being uninformed and taking one's chance at price $p(G)$ is

$$hv(G) + (1-h)v(B) - p(G)$$  \hspace{1cm} (13).

Suppose buyers differ in their valuation function $v(.)$ and in information cost $c$. Let $i$ be the proportion becoming informed (choosing (11)), and $t$ the proportion being uninformed but paying $p(G)$ (choosing (13)). The determination of $i$ and $t$ as functions of $h$ is clear in principle, and the following diagram shows who goes where:
We have a further equilibrium condition, derived from free entry. It is this which requires decreasing costs.

If trade at \( p(G) \) is occurring at all, and as I have shown it cannot be that all buyers at \( p(G) \) are informed in equilibrium, then it must be that a seller at \( p(G) \) who sold B would make positive sales. In free-entry equilibrium, he cannot make positive profits, and so at the sales level he gets, \( p(G) \) must (at most) just cover average costs for B. But \( p(G) \) by definition covers average costs for G at optimum scale, and I assume G-costs are higher than B-costs. The only way out is that a B-seller at \( p(G) \) cannot sell as much as he would like, and in fact can sell only

\[
x(B,p(G)) = \inf\{x: AC(x,B) < p(G)\}
\]

(14)
Hence we know, from free entry, how much a B-seller at \( p(G) \) sells. We also know how much a G-seller at \( p(G) \) sells: the optimal scale for G.

On the other hand, we can also calculate the ratio between these sales levels just from \( h \) and \( i/t \); it is

\[
\frac{i/t + h}{h} \quad \text{(15)}
\]

[Proof: Suppose there are altogether \( B \) buyers and \( S \) sellers at \( p(G) \). Then the sales level of a G-seller is

\[
iB/hS + tB/S
\]

and that of a B-seller is

\[
tB/S.
\]

Hence the ratio is as given.]

Thus we see how \((i,t)\) is determined from \( h \), and how \( h \) is determined from \((i,t)\). These of course correspond to the buyer and seller reaction curves discussed earlier. Equilibrium is a fixed point of the composite function \( h \circ (i,t) \circ h \) which goes from [0,1] to [0,1]. Existence of equilibrium is guaranteed provided that function is continuous. If the set of buyers indifferent between two of (11,12,13) always has measure zero, then the map \( h \circ (i,t) \) is continuous. If \( t \) is never zero, the other half of the function is also continuous.
Uniqueness of equilibrium follows from the fact that the function in question is decreasing, as the reader can see by checking how the borders in the diagram above move when h increases, and using (15).

Stability, heuristically, also follows. If h becomes too large, i will tend to fall and t to rise, which will reduce h. A fuller stability analysis would have to consider just how buyers react to changing h, and how sellers react to disequilibrium profitabilities. Also, G is sold at p(G) competitively only in long-run equilibrium, and that would complicate matters.

Qualitative properties of this equilibrium include:

(1) There is a positive externality from informed to uninformed.

(2) There is a negative externality - congestion of the opportunity to free-ride on the effects of the informed - between the uninformed.

(3) In general, some of the uninformed will pay the wrong price, e.g. will pay p(B) and get B even though v(G)-v(B)>p(G)-p(B).

(4) There is production inefficiency - the B-sellers at p(G) are operating at inefficiently small scale. This is necessary for the viability of equilibrium: it is the method by which the cost savings from cheating are dissipated.

(5) If the cost structure happens to be such that at all positive output levels, average cost of B is less than p(G), then no equilibrium can exist in which G is provided. This is true irrespective of tastes, and holds true however small the positive information costs c are. In particular, it may happen that all buyers would be better off committing themselves to buying information; in this model they cannot do so.
Now I ask what might in fact cause informed buyers not all to go to the same price? I consider two answers. First, it might be that the assumption of perfect information at a single cost is inappropriate. If, instead, it costs a buyer (say) \( \gamma \) to inspect a given brand, then the expected cost of searching at price \( p \) until a \( G \) brand is found is \( (p+\gamma/(1-h(p))) \). If informed buyers differ in \( \gamma \), they will minimise that expected cost at different prices, in general. I discussed such a model in [Farrell, 1979].

The second answer is the one I follow here. It may be that instead of a single exogenous good quality \( G \), the quality obtained by an informed buyer depends on the price. That will obviously mean that informed buyers will no longer all go to the same price.

I describe a model, generalising the one above, in which there is perfect competition for the custom of the informed, while the uninformed can get some minimum quality at its competitive price, or else take their equilibrium chance at a higher price. As above, in equilibrium there is some minimum-quality sold at every price, and the cost savings made by the low-quality sellers by lowering quality are just balanced by the inefficiency cost of producing at too low an output.
Assumptions.

I assume a legally or otherwise enforced minimum quality level, $q_0$, but suppose that at least some buyers would (under full information) prefer to pay for a higher quality. Buyers behave as described above. Sellers are ex-ante identical, and face an average cost function $C(x,q)$, where $x=$output and $q=$quality. They can set quality at any level $q \geq q_0$, and they can set price at will; buyers' optimization and free entry of sellers determine the sales level $x(p,q)$ of a seller of quality $q$ at price $p$. I write $p(q)$ for $\min[C(x,q):x>0]$, the minimum average cost of $q$ (which I assume exists), and $q(p)$ for the inverse function. There is free entry.

I assume for now that equilibrium exists in which some buyers are informed, and consider what such an equilibrium will be like.
Analysis

To avoid trivialities, I assume that there are informed buyers in equilibrium, and I consider what equilibrium will be like. (The existence problem is not trivial.)

Lemma 1: Each informed buyer gets his most-preferred point on the full-information perfectly-competitive price-quality schedule.

This follows from the fact that the informed are indeed perfectly informed. I also assume away any problems of there being only a small number of buyers of any given type. In other words, I consider a 'large' market.

Lemma 2: At every price $p$ in $P$, quality $q_0$ is sold in equilibrium.

For if not, then no buyer intending to pay that price need be informed - which means that minimum quality would then be a dominant strategy for any seller at that price.

Lemma 3: At any price $p$, only qualities $q_0$ and $q(p)$ are sold.

For, if the highest quality at price $p$ were less than $q(p)$, it would pay a seller to enter at price $p$ and with quality slightly higher than the current maximum. And if a seller's quality is not the highest, he will want to reduce it to $q_0$, since that loses him no sales.

The reader familiar with Salop and Stiglitz [1977] will recognise these results as analogous to their two-price equilibrium. They also dealt with some other kinds of equilibrium, which are not treated here.
Lemma 4: Define

\[ x_0(p) = \inf \{ x : C(x, q_0) \leq p \} \]  

(16)

and

\[ x(q) = \text{optimal scale for producing } q. \]

(I assume these exist.)

Then, (i) Unless \( x_0(p) > 0 \), price \( p \) cannot be charged in equilibrium, so quality \( q(p) \) and higher qualities cannot be sold in equilibrium, regardless of tastes.

(ii) If \( x_0(p) > 0 \), then seller equilibrium requires

Sales of \( q(p) \)-seller = \( x(q(p)) \)

Sales of \( q_0 \)-seller = \( x_0(p) \)

which together imply

\[ 1 + \frac{1}{h} \frac{i/t}{1+i/t} = \frac{x(q(p))}{x_0(p)} \]  

(17)

where \( h \) is the honesty rate, i.e. the proportion of sellers who provide \( q(p) \), and \( i/t \) is the ratio between the number of buyers who are informed and the number who are uninformed, at price \( p \).
We can re-write (17) as

\[ h = \frac{j/t}{1 - i/t} \left[ \frac{x(q(p))}{x_0(p)} - 1 \right]^{-1} \]  

(18)

in which form it represents the seller reaction function.

Part of the content of the statement is that the right hand side of (18) is less than one. However, as a reaction function, strictly, \( h \) is the minimum of 1 and the right side of (18). (In equilibrium, \( h \) can never be 1, because it would then pay no buyer to be informed, but that cannot support any positive \( h \)).

Part (i) follows in the same way as presented earlier. It gives a necessary condition for the existence of an equilibrium in which high quality is produced; and that in terms of technology only. Notice that it is in a sense a limiting case of (ii).

The relatively difficult part of this equilibrium story is the buyers' reaction function. It is easy enough to see how buyers will react if, as above, there is only a small number of prices. It is much harder to visualise when there are perhaps many prices, and the attractiveness of each option is determined by how many people are taking it and some other: it is a kind of multiple-activity congestion model. Qualitatively, we may expect the features of the equilibrium outlined above to persist. Before proceeding further, I ask whether equilibrium will indeed exist.
Theorem: Suppose

(a) There is a finite set \( P = \{p_1, \ldots, p_n\} \) of prices such that, for every buyer \( i \), the price \( p \) maximizing \( v(q(p)) - p \) is one of the \( p_j \).

(b) For each \( j \), the set of buyers such that
\[
v(q(p_j)) - p_j - c \geq v(q(p_k)) - p_k \quad (k \neq j)
\]
has positive mass.

(c) For any \( n \) honesty levels \( h_1, \ldots, h_n \) in \([0,1]\), the set of buyers for whom 'the best' among the \( 2n+1 \) available buying strategies [pay any price in \( P \) or else \( p(q_o) \), being informed or uninformed] is not unique, has zero mass.

Then equilibrium exists.

Proof: We can restrict our attention, because of assumption (a), to the prices in \( P \) together with \( p(q_o) \). There are \( n \) relevant honesty levels \( h \), and \( n \) relevant values of \( i \) and \( t \), the proportions of buyers being informed and uninformed respectively at a given price. We must look for a fixed point of the composite reaction function, which takes the vector \( h \) to buyers' choices and thence determines \( h \).

The latter part of the function, the sellers' reaction function, is given by
\[ h = \min \left\{ 1, \frac{i/t}{1 - i/t} \left( \frac{x(q(p))}{x_o(p)} - 1 \right)^{-1} \right\} \]

and is therefore continuous provided that we do not hit \( i=0=t \); that is the purpose of assumption (b), which says that even if \( p_K \) had an \( h \) of 1, there would still be some who would prefer \( p_J \). That is a strong and not very plausible assumption; in particular, if \( p_K \) is just slightly above \( p_J \), it cannot reasonably hold if costs and tastes are smooth. Thus in some sense our buyers must fall into discrete groups.

The other half of the function is continuous provided something like assumption (c) holds. What we have to avoid is the discontinuous response to \( h \) of buyers' choices.

Once we have continuity, we need only apply Brouwer's theorem.

In view of the strength of the assumptions used, one may reasonably ask whether this is a worthwhile existence proof. In general, the informed can pick their own favorite prices, but if one is uninformed, one has to stick to the prices favored by the informed (or else go to \( p(q_0) \)). This means that when an uninformed buyer, differing in tastes from the informed, decides to become informed (as a result of a small adverse shift in \( h \)), the set of prices available to the uninformed changes, and so the numbers choosing various other prices may change drastically. This seems to pose serious problems for any existence proof.
Let us consider, in this model, the welfare and positive effects of increasing the minimum-quality standard. There are several effects, and it seems that no general conclusions can be drawn. The value of the exercise lies in displaying some effects which might not otherwise be noticed.

(1) First, the available point \( (p(q_0), q_0) \) in price-quality space moves. In general, there will be many different buyer types who are buying at that price. The envelope theorem tells us that we can evaluate the welfare change directly due to this effect, simply by considering the utility changes of those who are buying at \( p(q_0) \).

This shift may also cause some who were paying \( p(q_0) \) to pay another price now, or vice versa. This represents an externality, because it will influence the equilibrium \( h \) at the different prices.

(2) Because of the change in \( q_0 \)-surplus \( v(q_0) - p(q_0) \), even assuming the set of other active prices does not change, the buyer reaction function which determines from the vector \( h \) of honesty levels, the numbers of buyers choosing each strategy, will shift.

The buyer reaction function shifts again, because the payoff to being uninformed, paying a price above \( p(q_0) \), and being unlucky, has changed; so the expected value of being uninformed, for any \( h \), has changed. Intuitively, we would expect to find more buyers being uninformed at prices which have relatively low \( h \), than before. But it is not clear how to trace through the effects of such a shift.
(3) At the same time, the seller reaction function shifts. Assuming that, at each price \( p \), \( x_0(p) \) increases with an increase in \( q_0 \), the expression (15) has to fall: thus \( h \) is higher for any value of \( i/t \). The intuition is that, since the cost of 'low quality' has gone up, the 'cheating' strategy is less attractive than before (at any level of \( i/t \)).

Effects (2) and (3) tell us that there will be a hard-to-predict change in the equilibrium honesty-vector \( h \), and in the proportions of buyers at the various prices who are informed. Now buyer welfare can be written, using simple surplus for each buyer:

\[
\max_i \{ \max_j \{ \max \{ v(q(p_j)) - p_j - c_i, h_j v(q(p_j)) + (1-h_j)v(q_0) - p_j, 0 \} \} \]

where \( i \) indexes buyers, and \( j \) prices. When each \( h_j \) is a complicated function of \( q_0 \), it is not clear what we can say, except that of course these effects all depend on the imperfect information and its consequence, the congested freeloding.
And finally, to make confusion utterly confounding, I note:

(4) If the increase in \( q_0 \) makes it no longer worthwhile to be informed, for some entire taste-class of the previously informed, then the set of active prices itself may shrink. Similarly, new active prices may be introduced. These things can happen even when 'the informed' are perfectly-informed, as here; it will be all the more likely when, for instance, the informed are searching brand by brand and so care about \( h \) themselves.

Finally, I note that cost-reducing technological change can reduce welfare in this model. The reason is that, if for instance the main impact is on the cost of producing small quantities of low-quality items, then \( x_0(p) \) will fall for each \( p \). Hence, the seller equilibrium or reaction function shifts adversely, and the uninformed are worse-off. Of course, it is only so simple if there is no effect on the cost of providing higher quality, and if nobody will buy an item in the knowledge that it will be \( q_0 \). However, the effect is there even in the general case. Some people might argue, for instance, that the invention of cheap canned food has so reduced the cost to restaurateurs of providing lousy meals, that it is on net a deleterious innovation.
1. Introduction

In markets where quality uncertainty is important, it is widely believed that there is an advantage to being first. If a "me too" firm tries to enter, it must overcome an apparent disadvantage arising from the fact that, while buyers don't know its quality, they do know the incumbent's. Just whether, and if so in what sense, this is a barrier to entry, is (like much discussion involving that phrase) unclear, but it seems to be a problem worth studying.

One technique which seems to be used to overcome this problem, is the introductory offer. This consists of offering the new brand at a low "introductory" price initially, thus persuading buyers to give it a try; after that, the price is raised to its normal level.

The analysis by Schmalensee [1980], and previous work, takes the following model of buyers' expectations: There is some exogenous distribution of quality for this new brand. What price is low enough to make it worth while to try it?

As Schmalensee points out, in studying the possible existence of entry barriers created by imperfect information, it is desirable to assume rational buyers. If, for instance, all buyers were afflicted with a nameless dread at the very idea of buying a new brand, it would be unsurprising to find entry barriers. Schmalensee's own analysis makes buyers rational given their quality expectations; but
those expectations are not rationally derived. The present paper, by contrast, makes buyers fully, indeed quite implausibly, rational. They understand the seller's profit-maximization problem, and correctly predict his unobservable action (quality choice). This incidentally leads to the removal of uncertainty (as distinct from imperfect information) from the model. Thus the question of whether in some sense buyers are "risk-averse in quality" does not arise.

The focus is on the entrant's choice of quality, which is assumed to be initially unobservable by buyers. High quality costs more to make, so there is a built-in incentive to cheating. The question becomes, what incentives are there working against that; and what can an entrant do to persuade buyers that he will not cheat them?

Here, I study the possibility that buyers may reason as follows. If the initial price is very low, there will be scant profits to be made from cheating, since the seller will lose all his customers and will not have profited greatly even in the initial period. Especially if the 'normal price' is deemed likely to be very profitable, it will be better for the seller not to cheat, i.e. he will supply high enough quality to induce repeat-buying. On such an argument (which I make precise), buyers infer the seller's choice of quality from his announced prices.

I consider equilibrium in what seems to be the simplest model in which the questions can be asked. The principal results are as follows: If buyers are identical, and ignoring income effects, a single firm contemplating entry will never choose to make an "introductory offer." Such offers do appear when several firms enter at once, but the low
introductory price conveys no information: it is the normal price that enables buyers to infer quality. The entrant's choice of quality is either socially optimal (if the incumbent provides sufficiently little surplus) or too low (if not). In the former case, entry is in no sense barred, but in the latter case it may be; and the incumbent firm can ensure that the second case arises, and that entry is barred, while still making positive profits.
2. The Model

I consider what seems to be the simplest model in which the relevant questions can be asked. There are two periods; in the first period, entry can occur, and an 'introductory price' can be charged; the second period represents the rest of time, during which the benefits of entry may be reaped.

I consider partial equilibrium in a market for a single good whose quality (a scalar) can vary. Out of somewhere - law, custom, cost structure, or a limited degree of consumer vigilance - I assume a minimum quality level \( q_0 \).

All consumers are identical. The utility function of a consumer has the form

\[
(q_1 + c_1) + k(q_2 + c_2)
\]

where \( q_1, q_2 \) are the qualities of the good he consumes in the two periods - he always buys just one unit - and \( c_1, c_2 \) represent 'other consumption.' This 'other consumption' good can be traded across the two periods at rate \( k \). All this is intended to ensure that, in terms of qualities and prices, he maximizes

\[
(q_1 - p_1) + k(q_2 - p_2)
\]

where \( p_1, p_2 \) are the prices he pays in the two periods.
There is an 'established brand' which provides quality $\bar{q}$ at price $\bar{p}$, giving surplus $\bar{u}$, and this is known. I assume $\bar{u}$ does not change if an entry occurs.  

To provide quality $q$ for the buyers, of whom I assume for simplicity there are a unit mass, costs $c(q)$, where $c$ is taken to be increasing, differentiable, and convex (i.e. $c'' > 0$). The social surplus from the production of quality $q$, $q - c(q)$, is (I assume) maximized at a quality level $q^* > q_o$, giving surplus $q^* - c(q^*) = u^* > u_o = q_o - c(q_o)$.

Entrants are taken to maximize the profit function

$$\Pi = s^i(p^i - c(q)) + hs(p - c(q))$$

where $q$ is quality, $p^i$ is the 'introductory offer' price, $p$ is the 'normal' (second period) price, $h$ a factor reflecting discounting and the relative length of the second period (like $k$, of course), and $s^i$ and $s$ the corresponding sales levels. Because all buyers are identical, we can take it that $s^i$ and $s$ assume only the values 0 and 1; and the analysis avoids boring technicalities of openness if I give the entrant an infinitesimal advantage and suppose that, if indifferent, buyers patronize him.

On the entry of a new seller, his introductory price and normal price are announced. Each buyer then reflects on what quality the seller is likely to provide. He may or may not buy from the entrant in the first period; if he does, he observes the quality. The quality cannot be changed between the first period and the second.

All of the above facts and functions are common knowledge.
3. Entry of a single firm

Suppose then that the established firm provides surplus $\bar{u}$, and a single firm has a chance to enter. It announces a normal price of $p$ and an introductory price of $p^i$. What beliefs will rational buyers have about the entrant's quality?

In the second period, when quality $q$ is known, and price $p$ is charged, buyers will continue buying if $q \geq p + \bar{u}$. Therefore quality, being chosen by the profit-maximizing seller, will either be the minimum, $q_o$, or the value $p + \bar{u}$ which just induces repeat-sales. If minimal quality is supplied, profits are

$$p^i - c(q_o)$$

(1)

If 'threshold' or 'honest' quality $p + \bar{u}$ is supplied, profits are

$$p^i - c(p+\bar{u}) + h[p - c(p+\bar{u})]$$

(2)

Since I am assuming rational buyers who know they face a profit-maximizing entrant, it follows that buyers believe the entrant is 'honest', i.e. quality is $p + \bar{u}$, if (2) exceeds (1), i.e. if

$$hp - (1+h)c(p+\bar{u}) \geq -c(q_o)$$

(3)

and believe $q = q_o$ otherwise.
Proposition 1: If \( \bar{u} > q_\infty - c(q_\infty) \), i.e. the incumbent provides consumer surplus greater than the social surplus from minimum-quality production, then entry at prices \((p^i, p)\) with \(p\) not satisfying (3), is never profitable.

Proof: The most profitable quality choice, if \(p\) fails (3), is \(q_\infty\). Then profits are zero if buyers do not buy; if they do, we know \(p^i \leq q_\infty - \bar{u}\). But then first-period profits are

\[
p^i - c(q_\infty) \leq q_\infty - \bar{u} - c(q_\infty) < 0 \text{ by assumption.}
\]

Repeat-buying is induced only if \(p < q_\infty - \bar{u}\), and the same argument applies.

Remark: Obviously, if \(\bar{u} < q_\infty - c(q_\infty)\), then entry at quality \(q_\infty\) and a price between \(c(q_\infty)\) and \(q_\infty - \bar{u}\) is possible and profitable: the problem of imperfect quality information provides no obstruction if entry at \(q_\infty\) is good enough to improve on the incumbent's offer.

As a result of the above Proposition and Remark, it is sensible to restrict our attention to entry with \(p\) satisfying (3), or, as I call it, honesty-implying entry. I next investigate the condition on \((p^i, p)\) which - assuming (3) holds - means that buyers will actually sample the entrant's brand.
Proposition 2: When (3) holds, buyers will buy the entrant's brand if
and only if

\[ p^i \leq p \]  \hspace{1cm} (4)

Proof: Given that (3) holds, so that \( q = p + \bar{u} \), the buyer gets utility

\[ (p+\bar{u}) - p^i + k[p+\bar{u}-p] \]

if he buys from the entrant; this is to be compared with \((1+k)\bar{u}\) if he
sticks to the incumbent. The result follows immediately.

The entrant's problem is therefore:

Maximize (2) subject to (3) and (4).

Proposition 3 (Theorem): We will always observe \( p^i = p \): in other
words, no introductory offers occur.

Proof: \( p^i \) does not occur in (3), while (2) increases with \( p^i \);
thus, given the optimal \( p \), \( p^i \) will always be set equal to \( p \).
Remark: This result is intuitively clear, for the following reason. Whatever value of \( p \) is chosen (satisfying (3)), buyers know they will get just \( \bar{u} \) surplus in the second period, and hence they know quality is \( p+\bar{u} \). But given that, they will buy in the first period at any price up to \( p \), and the seller gains nothing by letting them have it cheaper. Thus, there are no introductory offers under these assumptions.$^5$

I now consider what price and quality level an entrant would in fact choose, as a function of \( \bar{u} \), and then ask whether, at that quality with the corresponding price, he would find it profitable to enter.

Equation (3) describes the normal prices an entrant can set, if buyers are to believe his quality is 'honest' (\( q=p+\bar{u} \)) instead of dishonest (\( q=q_0 \)). Which qualities can he thus promise?

Proposition 4: The quality \( q \) can be promised (I say \( q \) is plausible) if and only if

\[
c(q) - \alpha q \leq \alpha c(q_0) - \alpha \bar{u}
\]

where \( \alpha = h/(1+h) \).

Proof: Simply substitute \( q=p+\bar{u} \) into (3).
Now define \( q_1 = q_1(\alpha) \) by

\[
q_1 \text{ minimises } c(q) - \alpha q
\]

so that \( c'(q_1) = \alpha \). (6)

**Proposition 5:** The set \( Q \) of plausible qualities is nonempty if and only if \( q_1 \in Q \), i.e.

\[
c(q_1) - \alpha q_1 \leq (1-\alpha)c(q_0) - \alpha \bar{u}
\]

(7)

**Proof:** By (5) and the definition of \( q_1 \).

**Remark:** By convexity of \( c(\cdot) \), if \( Q \) is nonempty, it is a closed interval containing \( q_1 \).

**Proposition 6:** If either \( q \in Q \) or \( q = q_0 \), the profit from entering at the price \( p = q - \bar{u} \) which promises \( q \) is

\[
\Pi(q) = (1+h)(q - c(q) - \bar{u})
\]

(8)

**Proof:** Trivial.

If \( Q \) is nonempty, and assuming that \( c' \) is not bounded above by \( \alpha \), we can define a quality level, the upper boundary point of \( Q \), by

\[
q^e \geq q_1
\]

\[
c(q^e) - \alpha q^e = (1-\alpha)c(q_0) - \alpha \bar{u}
\]

(9)
Proposition 7 (Theorem): An entrant's best choice of quality is

\begin{align*}
q^* & \text{ if } q^* \in Q \quad (q^* \text{ satisfies (5)}) \\
\max(q_o, q^e) & \text{ if } q^* \notin Q \text{ but } Q \text{ is nonempty } ((7) \text{ holds}) \\
q_o & \text{ if } Q \text{ is empty.}
\end{align*}

Proof: By definition, any quality in Q is more profitable as an honest-entry strategy than is entry at \( q_o \) as a fly-by-night. When Q is nonempty, \( q \) is chosen in Q to maximise (8); if \( q^* \in Q \), that is certainly the best choice.

But if \( q^* \notin Q \), since we know \( q_1 < q^* \) (\( c' \) monotonic and \( a < 1 \)), all of Q lies below \( q^* \). By the concavity of (8), then, the best choice is the largest member of Q, i.e. \( q^e \) — provided this is legal.

If Q is empty, or if \( q^e < q_o \) (all of Q is illegal), then of course \( q_o \) is the only option.
\( q_1 > q_0 : \) if \( Q \neq \emptyset \) then entry possible (at \( \min [q_e, q^*] \)).

\[
c = \alpha q - \alpha \bar{u} + (1 - \alpha) c(q_0)
\]

\( Q(\bar{u}) \)

\( q_1 < q_0 : \) if \( q_e < q_0 \) then entry blocked even if \( Q \neq \emptyset \).

\[
c = \alpha q - \alpha \bar{u} + (1 - \alpha) c(q_0)
\]

\( Q(\bar{u}) \)
Notice that, as \( \bar{u} \) rises, (5) becomes more stringent. Thus, perhaps paradoxically, as the incumbent serves buyers better, the entrant's best choice of quality moves further from the optimum. (The reason is that the price needed to assure high quality from an entrant is sufficiently high, and with high \( \bar{u} \), the margin is sufficiently low, that cheating would become irresistibly attractive.) Yet if \( \bar{u} \) is high, it will not be profitable to provide equal or better surplus by producing an inefficient quality. This suggests that sufficiently high \( \bar{u} \) will prevent (profitable) entry. That turns out to be true. Moreover, \( \bar{u} \) can be set discretely below \( u^* \), the socially maximal surplus, so that an incumbent can be getting positive profits, and still entry is prevented.

**Proposition 8:** If \( \bar{u} \) is such that \( q^* \in Q \), then entry at \( q^* \) and corresponding price \( p = -\bar{u} + q^* \) is profitable. The condition for \( q^* \) to be in \( Q \) can be written

\[
\bar{u} \leq u^* - (1-a)/a \left[ c(q^*) - c(q_o) \right]
\]  

**Proof:** (10) comes from manipulating (5) with \( q = q^* \). Then compare (8).

**Proposition 9:** Suppose \( Q \) is nonempty. Then

(i) If \( q_1 \geq q_o \), then \( q^e \geq q_o \).

(ii) If \( q_1 \leq q_o \), then

\[ q^e \geq q_o \text{ if and only if } \bar{u} \leq u_o = q_o - c(q_o). \]

**Proof:** (i) \( q^e \geq q_1 \geq q_o \).

(ii) The function \( q + c(q) - aq \) is minimised at \( q_1 \), and
rises monotonically after that. Therefore, since \( q^e \geq q_1 \) by definition, and \( q_0 \geq q_1 \) by assumption, \( q^e - q_0 \) has the sign of
\[
[c(q^e) - aq^e] - [c(q_0) - aq_0] \\
= [c(q_0) - a[c(q_0) + \bar{u}]] - [c(q_0) - aq_0] \\
= a[q_0 - c(q_0) - \bar{u}].
\]

**Proposition 10:** If \( Q \) is nonempty, the profits from entry at \( q^e \) are
\[
\pi(q^e) = \alpha^{-1}[c(q^e) - c(q_0)] \\
\tag{11}
\]

**Proof:** From (8) and (9).

**Proposition 11 (Theorem):**

(i) If \( q_1 \geq q_0 \), then entry is prevented if and only if \( \bar{u} \) is such as to make \( Q \) empty, i.e., if and only if
\[
\bar{u} \geq [q_1 - c(q_1)] - (1/h)[c(q_1) - c(q_0)] \\
\tag{12}
\]

(ii) If \( q_1 \leq q_0 \), then entry is prevented if and only if
\[
\bar{u} \geq u_0 = q_0 - c(q_0). \\
\tag{13}
\]

**Proof:** (i) By Prop. 9(i) and Prop. 10, it is necessary for entry-prevention that \( Q \) be empty, which is equivalent to \( q_1 \in Q \), or (12). When \( Q \) is empty, however, entry is blocked provided (13) holds; but using the definition (6) of \( q_1 \) it turns out that (12) implies (13).
(ii) By Prop. 9(ii), (13) ensures that Q lies entirely within the illegal region. Hence we need only consider low-quality entry; but that is unprofitable, by (13). And (13) is always necessary for entry-prevention, by Prop. 6.

Proposition 12 (Comments): (i) Both (12) and (13) allow room for profits, i.e. both bounds are strictly below $u^*$.

(ii) An increase in $q_o$ reduces entry-preventing profits and increases buyer surplus.

(iii) An increase in $h$ is desirable for buyers if $q_1 > q_o$.

(Proof of (iii): The change $\frac{\partial U}{\partial h}$ in the right-hand side of (12) is

$$- h^{-2}[c(q_1) - c(q_o)] + (aq_1/\partial h)[1 - c'(q_1)(1+h)/h]$$

of which the second term vanishes, by $(8')$.)

We must ask two further questions about this 'entry-barrier' theorem. First, given the structure of the model, can the 'incumbent' (who was once an entrant, of course) in fact choose $\bar{U}$? And second, do his profits do more than compensate him for losses he may have had to take in becoming the established brand?

To answer these questions, we must add a previous period to the model, a period in which the first firm entered and had to persuade buyers to buy the new product. To analyze the introduction of a completely new product in terms which assume the cost-function is common knowledge, seems absurd. Therefore, although I sketch such an approach, I keep it brief.
On entering, the first firm faced an exogenous $\bar{u}$ of zero, i.e. 
$\epsilon = u^*$ (in effect). That means he faced a lenient version of (3). He 
could charge any of a relatively wide range of prices, and get buyers 
to buy his product. Thus, entry for the first seller was relatively 
easy.

If however we really thought of his entry as analogous to the 
second entrant's, we would know he would necessarily provide $\bar{u} = 0$, 
just enough to keep buyers buying. But then, of course, he would 
generally be open to entry. The model suggests a strategic choice of 
$(\bar{p}, \bar{q})$ for the first firm: set $\bar{p}$ such that buyers will buy, and set $\bar{q}$ so 
as to provide an entry-deterring but profitable value of $\bar{u} = \bar{q} - \bar{p}$.

Note: (1) If buyers are aware that he is doing that, they will be 
more ready to buy the first firm's product, which gives him more 
freedom.

(2) If the second firm were also contemplating further potential 
entry in the event of his wrestling the market from the first firm, then 
he would no longer necessarily set $q$ so that $q - p = \bar{u}$; rather, he 
might have an incentive to set higher $q$, as a more effective entry-
deterring strategy. Evidently, this calls for a new model with a 
continual stream of (potential) entrants, each aware of his followers. 
I have not worked out such a model, but the following two ideas are 
suggested:
(i) that even for a given product, with no innovation (new products or cost savings), we should think of competition, where there is imperfect quality information, as a Schumpeterian process: competition consists in trying to seize the entry-deterring monopoly.

It seems possible that this would actually lead us to the optimum - but it would certainly depend just how buyers formed their quality expectations. If $\bar{u} < u^*$, there is profit to be made, and perhaps more effective entry-deterrence to be had, from a higher value of $u$; the question is whether (as above) the possibility of cheating would make buyers too incredulous for such entry to be possible.

(ii) that it seems possible that buyers may be better off if all firms do try to deter entry, than if only some or none do.

(3) The choice of $\bar{q}$ by the first firm may not be optimal - given that he chooses $\bar{u}$ strategically, and $\bar{p}$ with an eye on the condition (3), there seems no reason to expect $\bar{q} = q^*$. This is unclear.
4. Entry of Many Firms

Suppose, in a two-period model like that above, many firms (no longer just one) can enter in the first period. What difference will this make?

It turns out that it makes a substantial qualitative difference: we observe introductory offers. The reason for this is that it seems reasonable to continue assuming that, whichever 'new brand,' if any, is successful, it will offer zero surplus (compared with the incumbent, who is then still buyers' alternative) in the second period. But with the competitive entry, some surplus is offered to buyers: it all has to be in the first period, via a low price then. (But notice, that this is not informative, and does not seem to conform to our intuitive idea sketched in the introduction above.)

What happens is as follows. The incumbent's brand gives surplus \( \bar{u} \). There are many entrants, each offering a \((p^1, p)\) price-combination, and unobservably choosing quality.

If buyers buy a brand, and discover its quality, they will re-buy in period 2 on just the same condition as above - namely, \( q - p \geq \bar{u} \). Notice that I am assuming that after making no sales in the first period the other 'entrants' either have dropped out, or are regarded by buyers as a poor risk. (This latter indeed would be rational, for a firm entering and knowing it will not be sampled in period 1 but may be in period 2, will always set \( q = q_0 \).)

Accordingly, provided \( \bar{u} > u_0 \), entrants are effectively constrained by (3). Subject to (3), an entrant can make any offer
(p^i, p), and buyers will expect (correctly) to get utility p - p^i (see derivation of equation (4)) from his brand. Thus the brand with the greatest value of p - p^i, subject to (3), will be the entrant who succeeds in capturing the market, if any one does.

With 'many' entrants acting noncollusively and aware of the buyers' reasoning, we expect to find all the entrants clustering at the point (p^i, p) which solves:

\[
\begin{align*}
\text{Maximize } & p - p^i \\
\text{subject to } & (3) \\
\text{and } & \Pi > 0.
\end{align*}
\]

Case where (3) binds.
Case where (3) does not bind.

Again, quality is optimally set if and only if (3) does not (strictly) bind; if non-optimal, quality is too low. Although the value of $p^i$ is different, everything else is the same as with one entrant: the possibility or impossibility of break-even entry, the 'normal' price $p$, and the quality chosen.

Thus, to summarize, we have

Proposition 13 (Theorem): With many potential entrants in the first period, entry is possible under exactly the same conditions as with one potential entrant, and entrants will choose the same quality level as in Prop. 7. However, if entry does occur, we will observe $p^i < p$ in general; this is the way in which the consumer surplus from the entry competition is given to buyers.
In fact, we can calculate $p^i$. Suppose first that (3) binds, so that entry is at quality $q^e$. Then $p^i$ is determined by the zero-profit condition, along with the fact that $p = q - \bar{u}$:

$$0 = p^i - c(q^e) + h[q^e - \bar{u} - c(q^e)]$$

$$= p^i - c(q_0) \text{ using the definition of } q^e.$$

Another way of deriving this is to note that, at $q^e$ and $p^e = q^e - \bar{u}$, the profits are just equal to the profits from one-period selling of $q_0$ at price $p^e$. To hand back these profits requires an initial-period price reduction of $p^e - c(q_0)$, which gives us

$$p^i = c(q_0)$$

Now suppose (3) does not bind, i.e. $q^* \in Q$ and entry occurs at $q^*$. Then $p^i$ is defined by

$$0 = p^i - c(q^*) + h[q^* - \bar{u} - c(q^*)]$$

so $p^i = c(q^*) - h[u^* - \bar{u}]$.

But the condition (10) for $q^* \in Q$ is equivalent to

$$h[u^* - \bar{u}] \geq c(q^*) - c(q_0)$$

and therefore $p^i \leq c(q_0)$, with strict inequality if $q^* \in \text{int}(Q)$. 


Intuitively, the zero-profit condition, along with plausibility, tells us that the profits from dishonest operation cannot be positive; thus $p^i \leq c(q_0)$. If entry takes place at $q^e$, then honest profits are equal to dishonest profits; hence $p^i = c(q_0)$. If $q^e \in \text{int}(Q)$, then dishonest profits are strictly less, so $p^i < c(q_0)$, by just the amount of the difference.
5. Choice of $q_0$ and of $h$

We can imagine $q_0$ as set by law. In that case, what are the considerations to be borne in mind in choosing $q_0$?

In this simple model, the optimal $q_0$ is of course $q^*$. When consumers' preferences are identical, we can always do away with the sort of informational problem considered in this work, 'simply' by legislating optimal quality as a well-enforced minimum; then we get a first-best outcome. For the problem to be interesting, we must regard the model as giving us hints about how a more complicated system (society itself) would respond. In particular, since tastes differ, the above first-best solution does not exist.

The insight provided by the model is the following. An increase in $q_0$ makes the dishonest strategy less attractive, and thus makes (3) and (5) more lenient. This means that a wider range of prices $p$ will be honesty-inducing, and a wider range of qualities plausible, and in particular we have a greater chance of allowing entry and allowing socially optimal quality to be produced.

As $q_0$ is raised, first towards $q_1$ and then beyond and towards $q^*$, we first observe the entry-deterring incumbent having to raise $\bar{T}$ according to (12), and then according to (13).

Now consider choice of $h$. If we consider a model which is really continuous time, and we let $T$ be the length of time before quality information is disseminated and the normal price introduced, the maximand is
where \( \mu^i, \mu \) are the payoff flows (to seller or buyer) in the introductory and normal periods respectively. The above expression is

\[
\int_0^T \mu^i e^{-rt} dt + \int_T^\infty e^{-rt} dt
\]

Thus increases in \( r \) and in \( T \) both decrease \( h \). Therefore policies affecting discount rate, or more plausibly affecting information flow and thus \( T \), will affect \( h \). So we can consider the proper value of \( h \).

For the case where there is just one potential entrant, increases in \( h \) make (3) more lenient

\[
(\text{proof: } \frac{\partial}{\partial h} [hp - (1+h)c(p+\bar{u})] = h[(p - c(p + \bar{u})]
\]

\[
= c(p + \bar{u}) - c(q_o) \text{ if (3) holds}
\]

with equality

\[ \geq 0 \text{ since } q_o \text{ is minimum.} \)
Therefore, increases in \( h \) make entry easier, make entry-deterrence harder, and make entrants more likely to choose 'correct' quality \( q^* \). So in this case \( h \) should be made as large as possible.

For the other case considered above, all the effects just mentioned still apply, but it is also true that the smaller is \( h \), the longer can the buyers enjoy the low price of the introductory offer. However, the zero-profit condition also shifts, so the net result is unclear. In any case, I would not wish to push this interpretation too hard, because really we ought to consider a many-period model.
6. Normal Price not Announced

So far, I have assumed that potential entrants can and do announce \((p^i, p)\); \(p\) can be taken by buyers as indicating something about quality (in fact, that \(q = \bar{u} + p\) if \(p\) satisfies (3), and that \(q = q_o\) if not). What happens if, in the first period, only \(p^i\) is announced?

The natural assumption for buyers to make is then that the seller will choose \(p\) and \(q\) to maximize profits, given his choice of \(p^i\). This means that if there are values of \(p\) satisfying (3), the seller will choose that one (with its associated \(q = \bar{u} + p\)) which maximizes profits (2). (For, if \(p\) satisfies (3), that means it is more profitable to offer \((p^i, p)\) with quality \(\bar{u} + p\) than to offer quality \(q_o\).) If no value of \(p\) satisfies (3), buyers will correctly believe that \(q = q_o\).

The conclusion, therefore, is that nothing is changed if \(p\) is not announced in the first period. This is essentially because buyers can correctly guess what the value of \(p\) would have been. Obviously, this is to take ultra-rationality very seriously!
7. Conclusions and Further Questions

This was a model inspired by two curiosities. First, there was the question of what happens to results such as Schmalensee's (that imperfect information coupled with economies of scale can result in entry barriers) when one pushed rationality to its limit. The answer is a little surprising: barriers are always present if the incumbent chooses $\bar{u}$ properly, and this is independent of scale economies. (However, it seems possible that this result would change if we went from a two-period to a many-period model.)

The second question was, in studying the repeat-sales mechanism (as a quality incentive), we should ask how the argument is affected by the firm's ability to confiscate (part of) its satisfied customers' surplus, by raising price. This problem was inadequately dealt with in Farrell [1979], and the present model is an attempt to improve that. From this point of view the conclusion seems to be that the mechanism goes on working: if $\bar{u}$ is sufficiently low, optimal quality $q^*$ is produced.

Driving the results is the strong rationality assumption made about buyers. While this is certainly unrealistic, it is unclear in what direction the results should be modified to take account of limited rationality.
Probably the most desirable extension will be to turn from a two-period to a many-period model. This will enable us to capture the possibility of strategic (entry-preventing) decisions by the entrant as well as by the incumbent. It will be interesting to see whether the entry barrier displayed above survives that change. If so, then I claim that this is an important complement to the less absurdly-rational models of Schmalensee and others. If not, it will be an interesting illumination of how the possibility of seizing a monopoly leads to a less monopolistic outcome than before.
Concluding Remarks.

In this thesis I have considered several of the many mechanisms by which an important and ubiquitous problem might be overcome by 'the market'. All, it seems, can work; none perfectly. This may leave room for judicious intervention, either by improving information (which we can analyse as shifting $B(.)$ in Chapter 2, or increasing $h$ in Chapter 3, for instance), or by imposing minimum quality standards, or by other means.

One question which comes to mind is, Why not simply have some reputable body test each brand on the market and announce its quality? And a related question: Can a firm come into existence which makes its money by selling such information to buyers? This is one view of the role of retailers, who are often undercut by mail-order opportunities or 'discount warehouses'. Retailers try to charge for their informational services by setting somewhat higher prices. Of course, there is an appropriability problem: it is open to a buyer to collect information at a full-service retailer's, and then make his purchase elsewhere. Another question is what motivates a retailer to perform his checking function conscientiously. A further problem is that many important aspects of quality - e.g. reliability (Ch.1) or most especially durability of a new product - cannot easily be checked by the 'agency'. Undoubtedly, there is an important role for such agencies, as the continued existence of Consumer Reports, Which?, and the Good Food Guide suggests. Equally, it seems clear that the problem is not completely solved by those bodies. Indeed, Chapter 2 above may be viewed as a preliminary analysis of what will happen when such a certifying agency exists. The question of when, and just how,
an agency can create and maintain credibility, and overcome the free-rider problem, remains open. But we can note that when a certifying firm sells its information to a buyer, it is by that very act creating a public good which is a substitute for its product. This special problem suggests that such firms may have a hard time.

When quality is as simple as it has been made to appear in this work, there are some fairly simple policies that will clearly be very beneficial. For example, if all buyers really had (as Ch.3 assumes) identical tastes, the enforcement of the optimal quality level as a minimum quality standard would give a first-best allocation. If buyers differ only in the trade-off between quality and other consumption, as assumed above throughout, then a set of price-dependent quality standards can be made to do the job — although obviously this is considerably more complicated. The objection is that in reality buyers are unlikely to be so well-agreed on what is good. I have modeled what might be called hierarchical quality, but generally buyers will differ in preferences on the vector of characteristics which a product involves. The resulting mixture of signalling and matching problems will be more complicated than either separately.

Thus, the considerable homogeneity of buyers assumed above is a matter of convenience. By contrast, the assumption of ex-ante seller homogeneity is the harder road. If in fact sellers differ in relevant characteristics, it is in some sense easier for the market to persuade them to do the different things which are (with heterogeneous buyers) desirable.
In this work I have taken some steps towards what seems - from the paucity of the literature - to be a little-studied problem: that of enforcing unwritten contracts. In a bilateral monopoly framework, there is the literature on supergames; but in a market framework little is known. What are the principles to emerge from this work? First, future-oriented incentives are likely to be imperfect where discounting prevails. Second, sales-oriented incentives require a disparity between price and marginal cost for their very efficacy; this introduces inevitable inefficiencies into a market where such mechanisms are required. Third, some technologies make fly-by-night operation almost irresistible (e.g. nearly constant returns, in Chapter 2). Fourth, there are nontrivial considerations in the welfare economics of minimum quality standards. Fifth, the notion of a public commitment (incentive change) to high quality is important, and in a range of models a higher price can achieve that. Finally, how different such a market can look from the classical model!
Notes on Introduction.

(1) See Leffler, Long and Russell [1974].

(2) Spence, in [1977], points out that, in his equilibrium, the guarantee level signals quality, and price is related to the guarantee level. Thus, price signals quality in a sense; but it cannot be an equilibrium for buyers to take it as a signal, or all sellers would just raise prices to exploit them.

If it is easier to check price than to examine a guarantee, then it also cannot be an equilibrium for buyers to refrain from using the price-quality relationship. So, in Spence's model thus amended, there is no equilibrium. I attribute that to his constant-returns assumption. Along the lines of my Chapter 2, we could define an equilibrium in which some ('informed') buyers check the guarantees, while others simply infer quality from price. Some ('honest') firms will provide the expected quality in terms of the guarantee; others, who will sell only to the uninformed, supply minimum quality. The dishonest sellers will sell less than the honest, and if we introduce decreasing costs, we may get an equilibrium. Indeed, checking on some reliable indicator of quality (such as guarantees) may be the best interpretation of the 'becoming informed' of Chapter 2.
(1) In the context of durability, a well-known popular argument runs the other way, saying that future demand is lessened by selling highly durable items, because replacement demand is less. In a 'perfect' world, Swan [1970] showed that even a monopolist will not be led to choose sub-optimal durability by this consideration. Schmalensee [1974] (among others) has reexamined the question.

(2) Smallwood-Conlisk did not have my flow of new buyers. Because of that, and because they considered oligopolistic market structure, it mattered whether dissatisfied buyers were 'strongly' or 'weakly' dissatisfied - they never left the market. By contrast, I assume dissatisfied buyers leave the market, but since I am considering markets with many sellers, it would not affect quality choice - the variable of interest - if I made some other assumption, provided buyers cease to be loyal to a disappointing brand.

(3) That statement holds only if we assume that disappointed buyers leave the market. However, even if they go to other buyers, it remains true that the way in which sales depend on a seller's own quality is unaffected by others' choices. It is this that achieves the simplification. For more on this choice of assumption, see note (2).

(4) See e.g. Gabor-Granger [1966], Scitovsky [1944].
(5) This is a consequence of a rationality assumption. Suppose a seller at price \( p \) changes to price \( p' \). Then his incentives for quality are just the same as other sellers' who charge \( p' \). But these buyers, his customers, chose \( p \) (with its associated quality gamble) over \( p' \) (with its). Therefore they will leave, and find someone charging \( p \).

Obviously this is a strong claim. For one thing, a seller's customers have (by definition, with the behavior postulated) had nothing but good from him. One might expect them to believe better of him, therefore, than the price would indicate; this may lead them to expect high quality from him at the new price. It might also be that nobody was selling at \( p' \) when his customers chose \( p \) . . . - all sorts of objections can be adduced. I simply wish to rule price changes out of consideration in this simple model.

(6) Since \( c'' > 0 \), the Hamiltonian is strictly concave in \( q \), and so use of the first-order condition is unassailable, provided an interior maximum exists. To ensure that, I assume further that \( c^\rightarrow \) as \( q^\rightarrow 1 \), and that if the optimal choice were \( q = q_o \), buyers would realize the fact and not buy, unless perhaps at a price very near \( c(q_o) \); we deal with that case separately.

(7) The explicit formula (8) in the text makes it easier to think of \( \frac{dp}{dq} \). However, the conceptual atmosphere of the model suggests looking at \( dq/dp \) in (7), which comes from \( q \max H(x,q;p) \) with (6) substituted
\[
H_{qp} = \frac{\partial V}{\partial p} f_q > 0
\]

and therefore, using the second-order condition \(H_{qq} < 0\), we have \(\frac{dq}{dp} > 0\).

Likewise, we have \(H_{qr} < 0\), \(H_{qk} < 0\), and so increases in \(k\) or \(r\) are bad for quality.

(8) This assumes that small quality changes are to give rise to small sales changes. In the "Salop-Stiglitz" framework of my informed/uninformed (Ch.2) model, that fails to be true, and we can have price equal to marginal cost - but that is in a sense very special, because the dependence of sales on quality is discontinuous, and indeed that model is rigged to provide that result.

(9) Strictly, we would want to define "other consumption:" \(c\), and write

\[
u = c + \alpha q
\]
subject to a budget constraint

\[ p + c \leq \beta \]

which reduces to the form in the text.
This is a continuous function, so the Brouwer theorem applies, unless $E(\cdot)$ or $S(\cdot)$ contains atoms. In that case, by $S(t)$ I mean $[S(t^-), S(t)]$, where $S(t^-) = \lim_{\epsilon \to 0} S(t-\epsilon)$; and by $1-B(d)$ I mean $[1-B(d), 1-B(d^-)]$.

Assume
\[
\begin{align*}
\text{sign } (d) &= \text{sign } (S(t)-d) \\
\text{sign } (t) &= \text{sign } ((1-B(d)) - t)
\end{align*}
\]

Then linearizing about equilibrium $(t^*, d^*)$,
\[
\begin{align*}
\dot{t} &= -At' - Bd' \\
\dot{d} &= Ct' - Dd'
\end{align*}
\]
for some positive numbers $A, B, C, D$.

Local stability is equivalent to both the eigenvalues of the matrix
\[
\begin{bmatrix} -A & -B \\ C & -D \end{bmatrix}
\]

having negative real parts. The characteristic equation of that matrix is
\[
(-A -x)(-D -x) - (-B)C = 0
\]
or,
\[
(AD + BC) + (A+D)x + x^2 = 0
\]
whose roots have sum $-(A+D)$ and product $AD+BC$; hence we have local stability.

Here I assume differentiability.
(4) With a large number of buyers, dishonest entry at a new price will always be profitable for a single firm, and 'honest' entry too, if 'any' uninformed (or, respectively, 'any' uninformed) buyers try the new price. Thus, if a new price is to be unattractive, it is necessary that no buyers would sample it. This makes it difficult to say what the true honesty level really would be.

One solution to this quandary is to suppose, as I do below, that all informed buyers go to their optimum price; and then it is just a matter of seeing where the uninformed go. Thus, if $p$ is not the best possible price for any informed buyers, all buyers correctly believe that all sellers at that price will be dishonest.
Introductory Offers: Notes.


(2) When buyers have identical tastes, there is no loss of generality in taking quality to be scalar.

(3) It seems plausible that if the incumbent is allowed to change his price after entry occurs, then entry-prevention is made easier and profitable entry harder.

If it is not an incumbent with the same product, technology, etc., but rather u is the utility level buyers can get without this new product, the assumption is more compelling.

(4) In the model as presented, there is no efficiency loss due to the monopolistic position of the incumbent, as buyers had inelastic demands. This clearly is special, and one would expect the monopolistic effect to persist in a more general model in which it would create an efficiency loss.

(5) How much does this depend on the special assumptions of identical buyers and of a simple utility function? Let us relax first the form of the utility function, and consider instead a general

\[ u(c^i,c;q^i,q) \]

where \( q, c \) are 'quality' and 'other consumption' in the second period, and \( q^i, c^i \) are the same in the introductory period. I continue to
assume identical buyers.

What is the 'threshold' quality level here, corresponding to \( q = p + \tilde{u} \) in the text? I propose the level \( q(p, p^i, w, p, q) \) defined as the smallest quality \( q \) such that

\[
\max\{u(c^i, w - p^i - kp - c^i; q, q) - u(c^i, w - p^i - kp - c^i; q, \tilde{q})\} \geq 0.
\]

The idea is that, assuming \( c^i \) is chosen on the assumption that \( q \) will appear, when \( q \) does appear it will just induce repeat-buying. \( W \) here is the buyer's wealth.

One feature of the model in the text is that \( p^* \) does not affect \( q(p) \equiv p + \tilde{u} \). When is that true here? The answer comes by observing that \( p^i \) appears only with \( W \). In other words, the mechanism by which a low \( p^i \) might suggest a high \( q \) is only that "if \( p^i \) is low, buyers will be richer in the second period, and that may make them more willing to pay more for higher quality." It seems clear that this is not the effect we wish to model.

The other way in which a low \( p^i \) might suggest high quality, is by making dishonest behavior less attractive, and thus implying \( q(p, \ldots) \) insted of \( q_0 \). When will this occur here, as it never does in the text? I claim that, provided buyers are identical, \( p^i \) will cancel out from the equivalent of (3) in general, provided it does not affect \( q(p, \ldots) \). To the extent that a low \( p^i \) (via the income effect) raises \( q(p, \ldots) \), it will actually cause fewer prices \( p \) to satisfy the equivalent of (3).
Now what happens if buyers are not identical? Then the following becomes possible: Different $p^i$ will cause different types of buyers to buy in the introductory period, and this will affect the function which relates second-period demand to actual quality. Hence different $p^i$ may go with different quality choices ($p$, etc., being held constant), and $p^i$ may convey information.

Thus, suppose there were two groups in the buyer population, with utility functions equivalent to

$$u_1 = (q^i - \theta_1^i p^i) + k(q - \theta_1 p)$$
$$u_2 = (q^i - \theta_2^i p^i) + k(q - \theta_2 p)$$

where say $\theta_1 < \theta_2$. Then if normal price is $p$, there are three quality levels worth considering: $q_0$, $\bar{u}_1 + \theta_1 p$, and $\bar{u}_2 + \theta_2 p$

(Where $u_j = q - \theta_j p$). Assume $p > \bar{p}$, so that the third quality level is the highest. If group 2 is large, this may be the best choice for the seller: but only if group 2 buys in the first period, which requires
\[
\bar{u}_2 + \theta_2 p - \theta_2^i p^i > \bar{q} - \theta_2^i p
\]

or, \[
\bar{q} - \theta_2 \bar{p} + \theta_2 p - \theta_2^i p^i > q - q_2^i p
\]

or, \[
\theta_2 (p - \bar{p}) + \theta_2^i p > \theta_2^i p^i
\]

So if, for instance, \( \theta_2 > \theta_2 \) and \( p > \bar{p} \), \( p^i \) must be less than \( p \) for group two to buy; and thus it may be rational for buyers to believe

\[
q = u_2 + \theta_2 p \quad \text{if} \quad p^i \text{ is low}
\]

\[
q = u_1 + \theta_1 p \quad \text{if} \quad p^i \text{ is high}
\]

So I conclude that differences between buyers may cause \( p^i \) to convey information about \( q \); but with identical ultra-rational buyers, \( p^i \) conveys no information, unless we wish to count the income-effect.
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List of Notation: Chapter 3: Introductory Offers

\[ p \]  price
\[ p^i \]  introductory price
\[ \pi \]  profits
\[ h \]  relative importance (to entrant) of the second period
\[ \alpha \]  \( \alpha = h/(1+h) \)
\[ q \]  quality
\[ q_o \]  minimum quality
\[ q^* \]  socially optimal quality
\[ Q \]  set of plausible qualities (defined in eq. (5))
\[ q_i \]  \( c'(q_i) = \alpha \)  (see eq. (6))
\[ q_e \]  largest element of \( Q \)
\[ c(q) \]  cost of quality \( q \)
\[ c \]  is used intermittently for "other consumption"
List of Notation: Chapter 2

Section 1:  
- $H,D$: seller's gains from honest and dishonest trade  
- $u$: buyer's gain from honest trade  
- $c$: buyer's cost of vigilance  
- $k$: buyer's loss from dishonesty.  
- $t$: proportion of buyers who are 'trusting'  
- $d$: proportion of sellers who are 'dishonest'  
- $S(\cdot)$: distribution function of $H/D$  
- $B(\cdot)$: Distribution function of $c/k$

Section 2:  
- $p$: price  
- $G,B$: qualities 'good', 'bad'  
- $c(\cdot)$: unit cost function

Section 3:  
- $v(\cdot)$: buyer's valuation function  
- $h$: proportion of 'honest' sellers.  
- $i,t$: proportion of buyers at price $p(G)$ who are 'informed' and 'trusting', respectively  
- $q_o$: minimum quality  
- $C(x,q)$: average cost function  
- $p(q)$: minimum average cost of $q$  
- $q(p)$: inverse function to $p(q)$  
- $x_o(p)$: smallest scale at which a $q_o$-seller can break even at price $p$
List of Notation: Chapter 3: Introductory Offers

\( p \) \hspace{1cm} \text{price}

\( p^i \) \hspace{1cm} \text{introductory price}

\( \pi \) \hspace{1cm} \text{profits}

\( h \) \hspace{1cm} \text{relative importance (to entrant) of the second period}

\( a \) \hspace{1cm} \( a = h/(1+h) \)

\( q \) \hspace{1cm} \text{quality}

\( q_0 \) \hspace{1cm} \text{minimum quality}

\( q^* \) \hspace{1cm} \text{socially optimal quality}

\( Q \) \hspace{1cm} \text{set of plausible qualities (defined in eq. (5))}

\( q^c \) \hspace{1cm} \text{largest element of Q}

\( c(q) \) \hspace{1cm} \text{cost of quality q}

\( c \) \hspace{1cm} \text{is used intermittently for "other consumption"}