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# *Roman Law on the Just Price in Nicolaus Bernoulli's Mathematics*

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**Abstract—**It must be rare that discoveries which transform mathematics also undermine legal rules. Yet this is precisely what happened when probability was first developed in the second half of the 17th century and the first decades of the following one. The focus of this article is a doctoral thesis in law written in 1709 by Nicolaus Bernoulli, an important mathematician of the age. He highlighted the dramatic implications of the new mathematics of probability for a rule which was fundamental to contemporary contract law in continental Europe. This article reconstructs a remarkable story about the place of mathematics in the history of contractual justice and the place of contractual justice in the history of mathematics.

**Keywords:** Roman law, contract law, just price, *laesio enormis*, legal history, European private law

## *1. Introduction*

It must be rare that discoveries which transform mathematics also undermine legal rules. Yet this is precisely what happened when probability was first developed in the second half of the 17th century and the first decades of the following one. The focus of this article is a doctoral thesis in law which highlighted the dramatic implications of the new mathematics for a rule which was fundamental to contemporary contract law in continental Europe. It is a remarkable story about the place of mathematics in the history of contractual justice and the place of contractual justice in the history of mathematics. The title of the thesis, which was defended and published in Basel in 1709, is *De Usu Artis Conjectandi in Jure* (The Art of Conjecturing in Law) and its author is Nicolaus Bernoulli (1687–1759), an important mathematician of the age.[1]

Two years before the outbreak of the French Revolution of 1789, one of its most illustrious intellectual figures, Nicolas de Condorcet, explained that Bernoulli's thesis was a 'landmark in the history of knowledge … in particular because it

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[1] N Bernoulli, *De Usu Artis Conjectandi in Jure: Dissertatio Inauguralis Mathematico-Juridica* (Basel 1709).

shows … how many serious mistakes were made by lawyers who were trying to resolve the same issues without mathematics'.[2] Indeed, Condorcet undertook the laborious task of translating the thesis into French. Significantly, he broke off his translation—mid-sentence—once he had reached the part where Bernoulli explained the dramatic implications of his findings for the Roman law on the just price, the subject of this article.[3] Bernoulli showed how the mathematics of probability—which was in gestation at that time—could be used to value chances and consequently he argued that the just price, which requires equality between what one gives and receives under a contract, should no longer be 'excluded' from contracts of chance such as the sale of a life annuity or the right to whatever fish one can catch in a river in one year.[4] As this conclusion in relation to the scope of the just price was 'contrary to the wishes of almost all legal scholars', Bernoulli's thesis exposed as unfounded a central legal principle of the time.[5]

Bernoulli's insight had, indirectly, an important impact on English law, which, unlike continental legal systems, had never absorbed Roman law on the just price.[6] However, by Condorcet's time, Bernoulli's insight had still not been heeded by continental lawyers; they were still making 'many serious mistakes' by excluding or simply overlooking the impact of the mathematics of probability on the law on the just price. From the second half of the 18th century, the Roman law which had been examined, applied and adapted across continental Europe (the *ius commune*) began to fragment into a constellation of particular Codes for particular territories.[7] Under the Bavarian Civil Code (the *Codex Maximilianeus Bavaricus Civilis*) of 1756, the just price 'had no place' in contracts of chance.[8] The Prussian Civil Code (the *Allgemeines Landrecht für die Preussischen Staaten*), which was enacted in 1794, the year of Condorcet's death, excluded life annuity contracts—the quintessential contract of chance—from the remit of the just price.[9] The French Civil Code—the great *Code Napoléon*[10]—which was introduced 10 years later gave a prominent role to the just price following days of

[2] M-J-A-N de Caritat, marquis de Condorcet, *Discours sur l'Astronomie et le Calcul des Probabilités* (1787) in A Condorcet O'Connor and MF Arago *Œuvres de Condorcet*, vol I (Firmin Didot 1847) 482, 497–8. All translations in this article are my own. Where appropriate (ie when the use of particular words is of significance), I also provide the text in the original language.

[3] M-J-A-N de Caritat, marquis de Condorcet, 'Imprimé sur le calcul des probabilités' (MS 875, undated, Bibliothèque de l'Institut de France) 200, 213.

[4] N Bernoulli, *Dissertatio* (n 1) 23, 38.

[5] ibid 23.

[6] C Kennefick, 'The Contribution of Contemporary Mathematics to Contractual Fairness in Equity, 1751–1867' (2018) 39 Journal of Legal History 307, 308–10.

[7] F Wieacker, *Privatrechtsgeschichte der Neuzeit: Unter Besonderer Berücksichtigung der Deutschen Entwicklung* (2nd edn, Vandenhoeck & Ruprecht 1967) 322–47. All such Codes were composed in the vernacular (including the *Codex Maximilianeus Bavaricus Civilis*, notwithstanding its Latin title).

[8] pt IV, ch IV, s 4. See also pt IV, ch XII, s 6 (on wagers), which does not expressly use the language of the just price. The general provisions on the just price appear in pt IV, ch III, ss 8 and 19–20.

[9] pt I, title 11, s 6, art 610—without expressly using the language of the just price—provides that it is 'exclusively' the parties' agreement which determines the amount of the capital sum and the annuities. The provisions on the just price appear earlier: pt I, title 11, s 1, arts 58–69.

[10] The French Civil Code formally acquired this title in 1807, lost it during the Restoration, reacquired it during the Second Empire and lost it again *de facto* when that regime, led by Napoléon III, was replaced by the Third Republic: J-L Halpérin, *Le Code Civil* (2nd edn, Dalloz 2003) 117–23.

debates in which Napoléon Bonaparte himself played a major role;[11] however, it too provided that the just price did not apply to life annuity contracts.[12] In 1811, the Austrian Civil Code (the *Allgemeines Bürgerliches Gesetzbuch*) was even more emphatic: the just price was excluded from all contracts of chance.[13] Indeed, the French and Austrian Civil Codes are the only ones still in force today which contain express provisions on the just price; and they are, therefore, also the only ones which exclude it from contracts of chance.[14]

The fate of Bernoulli's insight in continental European law after 1709 is an important story for law and it has yet to be told. However, the logically prior story—of the genesis and content of the thesis itself—is the one which is reconstructed here. The thesis has attracted the attention of mathematicians; indeed, the principal modern study of the history of probability devotes several pages to it.[15] However, the thesis is almost entirely unknown in law.[16] The purpose of this article is to highlight the originality and significance of Bernoulli's contribution to the Roman law on the just price and, by doing so, to fill a gap in existing accounts of the just price in legal literature.[17]

## 2. *The Just Price of Roman Law*

The just price is a major idea in intellectual history. It has influenced economics, moral theology and moral philosophy,[18] as well as law and mathematics, the subject of this article. The origin of the just price is rather inauspicious. C.4.44.2, a constitution of the *Codex*—one part of the compilation which came to be known as Emperor Justinian's *Corpus Iuris Civilis*—provides a remedy for someone who

[11] P-A Fenet, *Recueil Complet des Travaux Préparatoires du Code Civil*, vol XIV (Marchand du Breuil 1827) 33–78; P-A Fenet, *Recueil Complet des Travaux Préparatoires du Code Civil*, vol XIII (Marchand du Breuil 1827) 48–9, 95–111. Indeed, the just price was discussed in five of the 102 debates in the *Conseil d'Etat* on the Code: 3 November 1803, 17 November 1803, 22 December 1803, 12 January 1804 and 28 January 1804. Contractual justice, economics and revolutionary politics combined to produce strikingly vehement and polarised discussions, thus confuting John Austin's assumption that 'rescission of a contract for inadequacy of consideration' is one of the legal questions 'which do not try the passions': J Austen, *The Province of Jurisprudence Determined and The Uses of the Study of Jurisprudence* (first published 1832 and 1863 respectively, Weidenfeld & Nicholson 1954) 374. Nothing in this passage or its wider context suggests that Austen was not talking about law regardless of place and time.

[12] art 1976—without expressly mentioning the language of the just price—provides that in life annuity contracts, the price can be whatever 'pleases the contracting parties'. The provisions on the just price are set out earlier in arts 1674–85.

[13] art 1268.

[14] The French Civil Code was in force in some parts of Germany until it was supplanted in 1900 by the BGB (the *Bürgerliches Gesetzbuch*): Wieacker (n 7) 331.

[15] A Hald, *History of Probability and Statistics and their Applications before 1750* (Wiley 1990) 375–8 (albeit using only an unpublished translation of the thesis into English).

[16] There appears to be only one study which focuses on the legal dimension: R Carvais, 'Anticipation et réception d'une thèse de droit. "De Usu Artis Conjectandi in jure" de Nicolas Bernoulli (Bâle, 1709)' (2006) 2 Journal Electronique d'Histoire des Probabilités et de la Statistique 15: <www.jehps.net/juin2006.html> accessed 20 August 2024.

[17] Existing studies of the just price do not include this mathematical angle. Two recent contributions to the history and theory of this subject respectively are W Decock, *Theologians and Contract Law: The Moral Transformation of the Ius Commune (ca. 1500–1650)* (Nijhoff 2013) 507–604; J Reyes, *Just Price Theory: A Reassessment* (Hart Publishing 2023).

[18] R de Roover, 'The Concept of the Just Price: Theory and Economic Policy' (1958) 18 Journal of Economic History 418; JW Baldwin, 'The Medieval Theories of the Just Price, Romanists, Canonists and Theologians in the Twelfth and Thirteenth Centuries' (1958) 49 Transactions of the American Philosophical Society 8; Decock (n 17) 507–604.

sells land at less than half of its 'just price': the purchaser must pay the difference or return the land to the seller.[19] From this very specific text—ostensibly instructions from Emperors Diocletian and Maximian to private individuals who had requested answers to particular questions—copious tomes of legal thinking have emerged over the millennium since the rediscovery of Roman law in Bologna at the end of the 11th century. The 'ambiguity' of the constitution undoubtedly explains, in part at least, the great destiny to which it has been called.[20]

> We might ask, as have thousands of others, whether the rule applied to the sale of movables? To a sale for too high a price? If so, how was 'too high' to be calculated? Only to sale, or also to other contracts?... One might even ask how the true value was to be calculated?[21]

Attempts to answer this last question played a decisive role in the development of the mathematics of probability, as I shall explain in the next section.

In addition to generating a seemingly infinite series of questions about its scope—and origins[22]—this constitution became the foundation of a more philosophical consideration: a theory of contractual justice which tells us why and which contracts should be recognised as binding and enforced. According to the idea of the just price, what one party gives under a contract must be equal in value to that received from the other because the purpose of contracts (other than gifts) is not to impoverish one party at the expense of the other.[23] Once again, the story begins in the Middle Ages: philosophical thinking about justice in contract law flourished from the 13th century with the fertile confluence in the work of Thomas Aquinas of Roman law and Aristotle's *Nicomachean Ethics*, the complete text of which had, at that point, recently been translated into Latin.[24] The just price *qua* equality in contracts was a central part of prominent theories of contractual justice until the end of the Enlightenment.[25] It is omnipresent in the works of Samuel von Pufendorf's *De Jure Naturae et Gentium* (1672) and Robert-Joseph Pothier's *Traité des Obligations* (1761), at the start and the end of that period respectively.[26] There are, therefore, at least two sides to the idea of the just

[19] The last line of C.4.44.8 is also of relevance. In the *Codex*, both Constitutions are attributed to Emperors Diocletian and Maximian (C.4.44.2 in 285 and C.4.44.8 in 293), but for centuries there has been debate about their true origins: see eg A Watson, 'The Hidden Origins of Enorm Lesion' (1981) 2 Journal of Legal History 186. A recent contribution is J Platschek, 'Bemerkung zur Datierung der laesio enormis' (2011) 128 Zeitschrift der Savigny-Stiftung für Rechtsgeschichte: Romanistische Abteilung 406. The Digest (533) and the Institutes (533) constitute the two other parts of the *Corpus Iuris Civilis*.

[20] Watson (n 19) 193.

[21] ibid 190.

[22] While it was Christian Thomasius who first raised doubts about the authenticity of C.4.44.2 and C.4.44.8 in a thesis in 1706, this debate really began when Otto Gradenwitz published an article in 1889 in which he argued that the two constitutions were interpolations: AJB Sirks, 'La Laesio Enormis en Droit Romain et Byzantin' (1985) 53 Tijdschrift voor Rechtsgeschiedenis 291, 291. See C Thomasius, *Dissertatio Juridica Inauguralis, De Aequitate Cerebrina L. II. C. De Rescind.Vendit. et eius Usu Practico* (Halle 1706).

[23] J Gordley, *The Philosophical Origins of Modern Contract Doctrine* (Clarendon Press 1991) 14.

[24] O Langholm, *The Legacy of Scholasticism in Economic Thought: Antecedents of Choice and Power* (CUP 1998) 15–42; Gordley (n 23) 10–68.

[25] The just price was eclipsed by the will theory in the 19th century: Gordley (n 23) 161–229.

[26] S Pufendorf, *De Jure Naturae et Gentium* (Junghans 1672) ; R-J Pothier, *Traité des Obligations: Selon les Règles tant du For de la Conscience, que du For Extérieur* (Debure l'aîné 1764). (The first edition of Pothier's treatise was published in 1761, but it is this 'new' edition of 1764 which is most commonly cited.) See also Decock (n 17) 597–8.

price: a wide understanding, also known as commutative justice, which pertains to the philosophy of contract law, and a narrow understanding, a practical rule, which determines the scope of its application in particular circumstances. Both are of significance for the purposes of this article. Since the era of the glossators, the school of legal scholars which emerged soon after the rediscovery of Roman law at the end of the 11th century, the narrow version has also been known as *laesio enormis*,[27] which essentially means 'very significant injury'. A third side of the just price is, clearly, its numerical expression, to which I now turn.[28]

## 3. *The Just Price and the Mathematics of Probability*

In theory and in practice, the application of the just price required the computation of a numerical value—or, rather, a range of numerical values.[29] That exercise depended on various assumptions relating to what counted as relevant factors for this purpose. By exploring and articulating these assumptions, scholars developed concepts which have had a significant influence on economics, eg monopolies and theories of value.[30] One finding was particularly important for the purposes of this article: by the 16th century, the dominant view was that the just price was the market price (ie the mean of all individual evaluations).[31] The just price could not, therefore, be equated with the cost of production or a figure which varied according to the social status of the parties.[32]

This search for particular numerical values is also what connects mathematics with the just price. Indeed, this major idea in intellectual history looms large in the emergence of the mathematics of probability in the second half of the 17th century.[33] Probability was first conceived in order to answer the obvious numerical question raised by the just price in contracts involving chance: what is the value of the opportunity of gain and the risk of loss? An exchange of letters between Blaise Pascal and Pierre de Fermat in 1654 is generally accepted to constitute the origin of probability as a branch of mathematics.[34] Significantly, the problem to which they directed their great minds was the value of one party's position in a partly executed contract.[35] The context was a thought experiment on a game of chance involving two or more players seeking a certain outcome by

[27] R Zimmermann, *The Law of Obligations: Roman Foundations of the Civilian Tradition* (OUP 1996) 259.

[28] The label of the just price is used to designate all three meanings throughout this article. When the context does not make it clear which one is intended, a more specific designation is used.

[29] Variations on the noun 'latitude' and the phrase 'absence of indivisibility' appear frequently in the literature: Decock (n 17) 526–7.

[30] Langholm (n 24).

[31] De Roover (n 18) 424 (unless it was 'fixed by law'); See also Decock (n 17) 520–1. In situations of market failure, the just price would be the price set 'by law' by the relevant public authority: De Roover (n 18) 425; Decock (n 17) 525.

[32] De Roover (n 18) 424.

[33] E Coumet, 'La Théorie du Hasard Est-Elle Née par Hasard?' (1970) 25 Annales: Histoire, Sciences Sociales 574; L Daston, *Classical Probability in the Enlightenment* (Princeton UP 1988) 6–33. In both works, the emphasis is on the influence of law on mathematics rather than *vice versa*, and the discussion of the just price is very general.

[34] I Hacking, *The Emergence of Probability: A Philosophical Study of Early Ideas about Probability, Induction and Statistical Inference* (2nd edn, CUP 2006) 11; Daston (n 33) 15. These letters were not published at the time. Three of the letters were published in 1679 and the final four, together with the first three, in 1779: Hald (n 15) 45.

[35] M Le Guern (ed), *Blaise Pascal: Œuvres Complètes*, vol I (Gallimard 1998) 145–66.

throwing a die or dice over several rounds. If the game is interrupted once it has begun but before all the rounds have been completed, what is the value of each party's chance of winning? The answer to that question determines how the stake should be divided. A fundamental assumption in their reasoning on this question is that what one receives should be equal to the price paid: Pascal, in one of his letters in this series to Fermat, expressly refers to the computation of the 'just value'.[36]

The just price became more prominent in the contributions to probability which immediately followed that of Pascal and Fermat. Having learned of (but not having read) this famous epistolary exchange when he was in Paris in 1655, Christiaan Huygens published his own thoughts in 1657 on the calculation of probabilities in certain games of chance.[37] It is 'the first printed textbook of probability',[38] and the just price is manifestly the underlying conceptual framework. Huygens's principal purpose—outlined in the introduction—is to determine how the stake should be divided or 'for what price I can fairly (*aequum*) sell my place' in 'a game which we have agreed to leave unfinished', viz a partly executed contract.[39] There are also several references to a 'just game'.[40]

The just price is even more central to the tract on the valuation of life annuities, composed by Johann de Witt in 1671.[41] Unlike the preceding and subsequent contributions to the emerging mathematics of probability, this one had a political purpose. De Witt, who 'was called a second Euclid' for his contribution to another area of mathematics,[42] was the leader of the principal province in the Dutch confederal republic and, as war was imminent, he tried to find an effective way to raise money for the state.[43] His study of life annuities is also noteworthy as constituting the first attempt to apply probability outside games of chance. It is replete with references to 'an equal Contract',[44] and the principal objective was the pursuit of 'the just value' or 'the perfect and just value' of the chances.[45]

However, the just price is conspicuously *absent* from the most important contribution to probability after the epistolary exchange between Pascal and Fermat: the *Ars Conjectandi* of Jacob Bernoulli, the uncle of Nicolaus.[46] Calculating

[36] ibid 146, 158 (29 July 1654).

[37] Hacking (n 34) 93–4. The original text was in Dutch. Van Schooten translated it into Latin 'by no means to Huygens's satisfaction': ibid 93. There is no difference of significance between the Latin text and the Dutch text in relation to the matters highlighted in this paragraph: F van Schooten, *Mathematische Oeffeningen, Begrepen in Vijf Boecken: Waer by Gevougt Is Een Tractaet Handelende van Reeckening in Speelen van Geluck door Christianus Hugenius; desen Druck Vermeerdert met Een Korte Verhandeling van de Fondamenten der Perspective* (van Goedesbergh, 1659) 489–500.

[38] Hacking (n 34) 92.

[39] C Huygens, 'De Ratiociniis in Ludo Aleae' in F van Schooten, *Exercitationum Mathematicarum* (Elzevir 1657) 521, 521.

[40] ibid 522, 523, 524. See Daston (n 33) 24–6.

[41] J de Witt, 'Waerdye van Lyf-renten naer Proportie van Los-renten' (Scheltus 1671) in *Die Werke von Jacob Bernoulli*, vol 3 (Birkhäuser 1975) 327–50. Similarly, Daston notes that 'De Witt's vocabulary is even more legalistic than Huygens's': 28.

[42] Hacking (n 34) 114.

[43] Daston (n 33) 25–9.

[44] De Witt (n 41) 329, 330, 332.

[45] ibid 334, 346.

[46] J Bernoulli, *Ars Conjectandi* (Thurneysen 1713).

probabilities in games of chance depends on the ability to know *a priori* all possible outcomes, which figure becomes the denominator.[47] The chance of a die landing on 3, when cast, is 1/6, as there are six sides, each of which is equally likely to land facing upwards.[48] However, determining with precision all possible outcomes 'is hardly ever feasible outside games of chance'.[49] Hence, the need to find 'another way'.[50] When all possible outcomes are unknown, can the probability of a particular outcome be derived from observed instances of what has actually happened? In other words, could the figure of 1/6 be obtained *a posteriori* by observing many instances of a person casting a die? This is precisely the question which Jacob Bernoulli (hereafter Jacob) addressed in his magnum opus, which, although completed by the time of his death in 1705, was not published until 1713.[51]

Jacob's answer to this question allowed probability to soar as an intellectual discipline: it 'places [him] in the pantheon of probabilists'.[52] While probability had, previously, been used to draw conclusions about the future from existing data,[53] such applications had, at that point, no sound theoretical foundation. However, in what is now known as the Law of Large of Numbers,[54] Jacob succeeded in devising a rigorous theoretical proof which could underpin the use of observed frequencies to estimate underlying probabilities.[55] It is vital to note that reasoning *a posteriori* in this way does not lead to an answer which mirrors exactly that yielded when all possible outcomes are known *a priori*.[56] Reverting to the example of the chance of obtaining a 3 when casting a die, Jacob's theorem cannot tell us that the underlying probability is precisely 1/6; rather, it can tell us the probability that the mean number of the observed outcomes will be within specified parameters. After all, I may cast the die 6, 60, 600, 6000 or more times without obtaining a 3. Nonetheless, the more times the die is cast, the higher the

[47] ibid 223. Equiprobability—that all possible outcomes are equally probable—is another assumption, although where this is absent one can adjust accordingly the number of possible outcomes when one knows the extent to which some outcomes are more likely than others: ibid 218–19.

[48] ibid 223–4. Of course, the laws of Newtonian mechanics apply to throwing a die or tossing a coin. In a seminal article, it has been shown that the side of a coin which is facing upwards when tossed is more likely (50.8%) than the other one to land facing upwards: P Diaconis, S Holmes and R Montgomery, 'Dynamical Bias in the Coin Toss' (2007) 49 Society for Industrial and Applied Mathematics Review 211.

[49] J Bernoulli, *Ars* (n 46) 223.

[50] ibid 224.

[51] Disharmony in the family is one of the reasons for which the treatise was not published until eight years after Jacob's death: ED Sylla, 'Introduction' in J Bernoulli, *The Art of Conjecturing: Together with Letter to a Friend on Sets in Court Tennis* (ED Sylla tr, Johns Hopkins UP 2006) 1, 12–13, 60–2.

[52] Hacking (n 34) 154.

[53] See the letters, in 1669, between Christiaan Huygens and his brother, Lodewijk: Hald (n 15) 106–10. The other instances, in chronological order, are the appendix to de Witt's tract in 1671 (although he does not reveal his calculations), the letters between de Witt and Johannes Hudde and an article by Edmond Halley in *Philosophical Transactions* in 1694 which has been described as 'pathbreaking': de Witt (n 41) 349–50; Hald (n 15) 125–31, 378. In addition to being Mayor of Amsterdam, Hudde was an eminent mathematician whose work 'prepared the way for the differential calculus of curves': Hacking (n 34) 114.

[54] The label was coined by Siméon-Denis Poisson 'in the 1830s', when he extended Jacob's theorem to situations in which the underlying probabilities are not constant: Sylla (n 51) 41–3.

[55] Jacob thereby connected what we now call 'mathematical probability and statistics': Daston (n 33) 125, 226–53.

[56] The terms '*a priori*' and '*a posteriori*' are used by Jacob when describing his research in a letter to Leibniz and in the treatise itself: A Weil, C Truesdell and F Nagel (eds), *Der Briefwechsel von Jacob Bernoulli* (Birkhäuser 1993) 117 (3 October 1703); J Bernoulli, *Ars* (n 46) 224–6.

probability that the mean of the observed outcomes will vary from 1/6, the underlying probability, by less than a specified margin (which, as Jacob stated, can be as 'narrow' as one desires).[57]

The absence of the just price in Jacob's work is not just conspicuous, as mentioned above; it is also very significant. Jacob realised that his theorem could best be deployed by applying it to a wide range of 'civil, moral and economic' questions. Indeed, this is the title of the fourth part of his treatise,[58] which he described in a letter to Gottfried Wilhelm Leibniz, one of the leading figures of the Enlightenment, as 'the most important part'.[59] However, Jacob's knowledge of law—and, in particular, contracts—appears to be minimal and certainly insufficient to reap the benefit of his mathematical breakthrough in this field. Notwithstanding the title of the fourth part, the only application of Jacob's theorem was to a hypothetical problem to which it is unlikely that a solution would ever be needed in practice: the determination of the likely ratio of white counters to black counters in an urn containing 5000 counters of one colour or the other.[60] There is, certainly, a legal dimension to some of the hypothetical scenarios which are used to explain in very general terms his approach to the art of conjecturing: is a particular person responsible for killing another?[61] Did a thief sell an item which was stolen to another person?[62] Did a notary fraudulently attest to an incorrect date in a written contract?[63] In addition, the treatise contains two other terms which are obviously legal in nature: judge[64] and magistrate.[65] Nevertheless, neither the questions which a lawyer might ask nor the concepts which a lawyer might use to answer them are present in these discussions. Furthermore, there are no references to legal sources and there is no analysis of the application of

[57] J Bernoulli, *Ars* (n 46) 226. To continue my example, I would expect to see a 3 on circa 1000 occasions if the die is rolled 6000 times. Using a Python code to roll a die 6000 times on four occasions, the author and David Kohan Marzagao obtained the following results for the number 3: 1023, 983, 1011 and 982.

[58] J Bernoulli, *Ars* (n 46) 210.

[59] *Briefwechsel* (n 56) 116 (3 October 1703).

[60] J Bernoulli, *Ars* (n 46) 238–9, although the problem is first outlined in ch IV: 225–6. He found that 25,550 experiments were required in order to make it 1000 times more likely that the mean was within rather than without 1/50 of the underlying ratio of 3:2: 238–9. After 25,550 experiments had been completed, another 5708 would, at each stage to infinity, increase the likelihood by a factor of 10: 31,258 experiments would make it 10,000 times more likely and 36,966 would make it 100,000 times more likely: ibid. A great statistician of the 20th century, Karl Pearson, subsequently argued, in a scathing tone, that these figures were seriously inflated; he also noted that, in the 18th century, Abraham De Moivre had succeeded in reducing Jacob's figures: K Pearson, 'James Bernoulli's Theorem' (1925) 17 Biometrika 201. De Moivre played a role in the development of a version of the just price in contracts of chance in English law: see Kennefick (n 6) 307–13, 325–8. Having fled to England from France as a Huguenot refugee, De Moivre was 'the leading mathematician in England after Newton'; he was naturalised in 1706: D Bellhouse, *Abraham De Moivre: Setting the Stage for Classical Probability and its Applications* (CRC Press 2011) 15–23, 175, 41.

[61] J Bernoulli, *Ars* (n 46) 214, 218, 222.

[62] ibid 214. Jacob notes here simply that when the other person is present, the answer can be known with certainty by asking him, and a judge would be 'inept' to resort to probabilities derived from the circumstances: 214.

[63] ibid 222–3. Much of this discussion is, effectively, a translation of two paragraphs of *La Logique ou l'Art de Penser: Contenant, outre les Règles Communes, Plusieurs Observations Nouvelles Propres à Former le Iugement* (Paris 1662) 459–60. Jacob does not, however, refer to his source. He does refer to it, though, two pages later, when discussing a different point: J Bernoulli, *Ars* (n 46) 225. *La Logique* is a major philosophical treatise which was first published in 1662 and is said to have been composed by two Jansenists, Antoine Arnauld, 'perhaps the most brilliant theologian of his time', and Pierre Nicole, although Pascal also 'participated in writing' it: Hacking (n 34) 25. The treatise influenced the development of the mathematics of probability: Daston (n 33) *passim*.

[64] J Bernoulli, *Ars* (n 46) 217.

[65] ibid 217.

any legal rule, whether Roman or other, to which probability could provide an answer. In short, there is no demonstration of exactly how probability could be usefully applied to law; there is simply—but significantly—the intuition that it can be done.

There is, however, one very short paragraph in the *Meditationes*, Jacob's notebook, which indicates an ability to identify a relevant legal question to which probability could make a contribution. The topic is the liability of a borrower to a lender-owner under a contract of *commodatum* (a gratuitous loan of a thing for use). In contrast to a *mutuum* (a gratuitous loan of a thing for consumption), ownership of the thing does not pass to the borrower under a *commodatum*.[66] In a *commodatum*, it is, therefore, necessary to determine whether the borrower or the lender-owner should bear the consequences of the destruction of the thing when the borrower has met the required standard of care.[67] In general, Roman law provides that it is the lender-owner who should bear the loss in such circumstances.[68] Jacob wonders whether such a borrower should be liable in situations where the thing would not have perished had it stayed with the lender-owner, ie had it not been lent at all.[69] This distinction is precisely the same as that drawn by Pufendorf in his famous treatise *De Jure Naturae et Gentium*,[70] and unsurprisingly, Pufendorf is indeed named in the very next sentence in which Jacob notes that while Pufendorf answers this question in the affirmative, 'other jurists' take a 'contrary' view.[71] In contrast to Pufendorf's reason for drawing this distinction,[72] Jacob's is firmly probabilistic. Jacob acknowledges that while it may seem 'unjust' that such 'a large loss' (the liability to the lender-owner) might be borne for what is essentially 'a fleeting benefit' (the use of the thing for a defined period), accidental destruction is so rare that borrowing the thing is analogous to having odds of winning 100 times greater than those of losing.[73] This very short passage is significant as it is the only one in the *oeuvre* of Jacob which demonstrates some knowledge (and understanding) of a legal rule, if only via the scholarship of one of the leading jurists of that or any age.[74]

It is clear, nonetheless, that this general paucity of legal knowledge weighed heavily on Jacob. By October 1703, the treatise had still not been published, even though Jacob had been refining his ideas on probability for 'many years'.[75] At this point, Jacob outlined the contents of his proposed treatise in a letter to Leibniz and implored him to send 'legal material … [to] which you think [these

[66] Instit.3.14.1–2.
[67] The contract of sale demonstrates that ownership is not always coterminous with the risk of destruction of the things as the liability lies with the purchaser even before he has become the owner of the thing by delivery (*traditio*): Instit.3.23.3.
[68] Instit.3.14.2. There are further refinements in the Digest. See Zimmermann (n 27) 192–7.
[69] 'Aus dem Meditationes von Jakob Bernoulli' in *Die Werke* (n 41) 21, 67 (Text 163,VI).
[70] Pufendorf, *De Jure Naturae et Gentium* (n 26) 638–9.
[71] 'Meditationes' (n 69) 21, 67 (Text 163,VI).
[72] Pufendorf, *De Jure Naturae et Gentium* (n 26) 638–9.
[73] 'Meditationes' (n 69) 21, 67 (Text 163,VI).
[74] I have found no evidence of epistolary or other exchanges between Jacob and Pufendorf.
[75] *Briefwechsel* (n 56) 117 (3 October 1703).

thoughts] could usefully be applied'.[76] An accomplished legal scholar,[77] Leibniz was certainly capable of fulfilling Jacob's wish, but, doubting the value of Jacob's proposed project, Leibniz did not oblige.[78] Jacob also repeatedly asked Leibniz to send him a copy of de Witt's tract on life annuities.[79] These entreaties too were to no avail.[80]

Learning or practical experience in law was a denominator common to many of the leading protagonists in the emergence of probability: in addition to being very distinguished mathematicians, de Fermat was a judge in Bordeaux and Huygens, Leibniz and de Witt had studied law.[81] Jacob was one of the striking exceptions. His degrees were in philosophy and theology, and for most of his career he was professor of mathematics in Basel.[82] Jacob's lack of legal knowledge was manifestly an impediment to the flowering of his mathematics in suitable applications. Indeed, this was precisely the gap into which Nicolaus Bernoulli stepped.

## *4. Nicolaus Bernoulli*

In 1687, Nicolaus Bernoulli (hereafter Nicolaus)[83] was born into a very distinguished family of mathematicians from Basel.[84] We have encountered his uncle Jacob already; Jacob was professor of mathematics in Basel from '1687 until his death in 1705'[85] and Nicolaus was one of his students.[86] Johann Bernoulli, another uncle of Nicolaus, was appointed professor of mathematics in Basel in 1705 and he assumed the role of Nicolaus's 'scientific adviser':[87] indeed, in the preface to his doctoral thesis in law, Nicolaus thanks his 'most illustrious uncles,

[76] ibid.

[77] See eg K Luig, 'Leibniz's *Elementa Iuris Civilis* and the Private Law of his Time' in JW Cairns and OF Robinson (eds), *Critical Studies in Ancient Law, Comparative Law and Legal History* (Hart Publishing 2001) 267; A Artosi and G Sartor, 'Leibniz as a Jurist' in MR Antognazza (ed), *The Oxford Handbook of Leibniz* (OUP 2016) 1; S Meder, *Der Unbekannte Leibniz: Die Entdeckung von Recht und Politik durch Philosophie* (Böhlau 2018).

[78] *Briefwechsel* (n 56) 123–4 (Letter from Leibniz to Jacob, 3 December 1703).

[79] ibid 129 (20 April 1704); 132 (2 August 1704); 134 (15 October 1704); 138 (28 February 1705). Some copies of de Witt's tract had been 'printed for private circulation': Hald (n 15) 123. Hacking (n 34) states that there was 'only one edition of thirty copies': 118.

[80] Leibniz stated that he could not find his copy of the tract in his papers, but he promised to send it as soon as it had been located: *Briefwechsel* (n 56) 135 (28 November 1704); 143 (undated, April 1705). It is likely that Jacob never received it. In one of his last letters to Jacob, who died on 16 August 1705, Leibniz stated that he had not yet found his copy, but he renewed his promise to send it to Jacob: ibid 143, 147 (undated, April 1705), 150. In July of that year, in letters which I have not seen, Leibniz urged Jacob to try to find a copy in Holland and Jacob confirmed that he had done so without success: ibid 147.

[81] Kennefick (n 6) 309–10.

[82] AP Yushkevich (B Secker (tr)), 'Nicholas Bernoulli and the Publication of James Bernoulli's *Ars Conjectandi*' (1987) 31 Theory of Probability and its Applications 286, 287; Sylla (n 51) 7.

[83] As Sylla (n 51) observes, confusion is caused 'not only by the many Bernoullis who made significant scientific and mathematical contributions but also by the fact that each individual Bernoulli may appear to multiply because his name may be given in Latin, German, French, or English': 4. She follows the usage of the *Bernoulli Edition* and I do so too here.

[84] Sylla (n 51) 2–4. Nicolaus died in 1759.

[85] Sylla (n 51) 7.

[86] Yushkevich (n 82) 286.

[87] ibid 287.

Jacob and Johann Bernoulli', for having led him to what he calls 'the divine science' of mathematics.[88] Like his brother Jacob, Johann Bernoulli made very significant contributions to mathematics, particularly to calculus, which had only recently been invented. Indeed, Jacob and Johann Bernoulli, 'after Newton and Leibniz, may be regarded as the creators of the calculus'.[89]

In Nicolaus's education and career, law and mathematics were combined in a way which distinguishes Nicolaus from even the illustrious Leibniz.[90] Nicolaus studied both law and mathematics at the University of Basel: for his Master of Arts degree, Nicolaus defended orally a part of Jacob's work on infinite series, while, as we have noted, Nicolaus's doctoral thesis was 'legal-mathematical' ('*mathematico-juridica*').[91] Nicolaus subsequently obtained academic positions in both law and mathematics: his first professorship was in mathematics in Padua in 1716;[92] in 1722, he was appointed professor of logic in Basel, before moving 'to the more prestigious chair in law' in 1731.[93] Although not as eminent as his uncles, Johann and Jacob, Nicolaus made a number of notable contributions to mathematics, one of which was an improved version of Jacob's theorem.[94]

Nicolaus's doctoral thesis in law, *De Usu Artis Conjectandi in Jure: Dissertatio Inauguralis Mathematico-Juridica*, which was composed and defended in Basel in 1709, is at the heart of his distinctive intellectual trajectory in law and mathematics. The originality of Nicolaus's thesis has, however, been questioned by historians of mathematics. It has been said that 'one might wonder why neither Nicolaus nor the professors who accepted his doctoral thesis asked whether he was being overly reliant on Jacob's unpublished book'.[95] Others are more forthright, dismissing parts of Nicolaus's thesis as 'plagiarism' and 'copied and pasted'.[96] There

[88] N Bernoulli, *Dissertatio* (n 1) 3.

[89] Hald (n 15) 221. Another prominent relative was Nicolaus's cousin Daniel Bernoulli (1700–82), the son of Johann Bernoulli. One reason for which Daniel Bernoulli is remembered, which is especially relevant to this article, is his innovative solution, published in 1738, to the St Petersburg Paradox: 'Specimen Theoriae Novae de Mensura Sortis' in *Commentarii Academiae Scientiarum Imperialis Petropolitanae*, vol V (the year is marked 1730–31, but the volume is published only in 1738) 175. This was a hypothetical problem, devised by Nicolaus in 1713, requiring the valuation of a place in a game of chance from which the expected gain is infinite: PR de Montmort, *Essay d'Analyse sur les Jeux d'Hazard* (2nd edn, Quillau 1713) 402 (Letter from Nicolaus to Montmort, 7 September 1713). Daniel Bernoulli's solution has been discussed by many eminent minds in the last three centuries and constitutes the basis of the concept of marginal utility in economics. See J Dutka, 'On the St Petersburg Paradox' (1988) 39 Archive for History of Exact Sciences 13; Daston (n 33) 70–108. Some legal dimensions have been considered: D Teira, 'On the Normative Dimension of the St. Petersburg Paradox' (2006) 37 Studies in History and Philosophy of Science 210. Daniel also made a very significant contribution to hydrodynamics principally in the form of what is now known as 'Bernoulli's law'; indeed, 'He coined the word *hydrodynamica*': O Darrigol, *Worlds of Flow: A History of Hydrodynamics from the Bernoullis to Prandtl* (OUP 2005) 1-9, 4.

[90] See n 77 above on the significance of law and mathematics in Leibniz's education and career.

[91] Yushkevich (n 82) 287; the full title of the doctoral thesis is *De Usu Artis Conjectandi in Jure: Dissertatio Inauguralis Mathematico-Juridica* (n 1).

[92] Yushkevich (n 82) 301 (with the help of Leibniz). Nicolaus resigned in 1719: A Robinet, *L'Empire Leibnizienne: La Conquête de la Chaire de Mathématiques de l'Université de Padoue* (Lint 1991) 259.

[93] Sylla (n 51) 10.

[94] Hald (n 15) 264–7, 280–5. A mark of Nicolaus's standing in mathematics is his election to a fellowship of the Royal Society on 11 March 1714, on the proposal of Isaac Newton (1643–1727): Bellhouse (n 60) 106.

[95] Sylla (n 51) 57.

[96] O Sheynin, 'Nicolas Bernoulli as a Statistician' (2016) 15 Śląski Przegląd Statystyczny/ Silesian Statistical Review 265, 267. Meusnier refers to Nicolaus's use of Jacob's 'manuscript (without mentioning it)': 'Nicolas, Neveu Exemplaire' (2006) 2 Journal Electronique d'Histoire des Probabilités et des Statistiques 5 <www.jehps.net/juin2006.html> accessed 20 August 2024.

is, however, also a favourable view. Nicolaus's thesis has been described as 'not a compilation but rather an original work by a talented scholar, erudite in both mathematics and jurisprudence', and there is praise for his 'originality in taking up and solving new problems'.[97] Even the more favourable view acknowledges, nonetheless, the very significant influence of Jacob's treatise and personal notebook (the *Meditationes*) on Nicolaus's thesis.

A reminder of the timeline of events is appropriate here: while Nicolaus's thesis was completed four years before the publication of Jacob's treatise, the text of the latter was exactly as it had been left by Jacob on his death in 1705.[98] When composing his thesis, Nicolaus was aware of the content of Jacob's treatise; indeed, he cites from the unpublished manuscript of Jacob at several points and indicates with precision the location of several points in the original source.[99] However, there is, plainly, also some unacknowledged use of Jacob's ideas. Two examples from the material discussed in the preceding section suffice. First, Nicolaus repeats almost verbatim Jacob's juxtaposition of the ability to determine the probability of outcomes *a priori* in games of chance with the need to find 'another way' in almost all other circumstances.[100] Another example is Nicolaus's use of the *commodatum* question discussed in the previous section.[101] Although Nicolaus provides more detail,[102] the substance of all the points—and even some of the language used to express them—can be traced to Jacob's very short paragraph on this question in his notebook.[103]

In this period, it is not unusual to find theses—in law and in other disciplines—written entirely or partly by the student's supervisor; the oral defence was considered to be the principal part of the process.[104] Indeed, as already mentioned, Nicolaus obtained his Master of Arts degree by orally defending a text on infinite series written by Jacob.[105] So prevalent was this practice that it is often impossible to ascribe theses to particular authors in this period.[106] There is no indication,

[97] Yushkevich (n 82) 290; Hald (n 15) 375. Kohli's overall assessment is similar; indeed, he describes one result as 'wholly novel': K Kohli, 'Kommentar zur Dissertation von Niklaus Bernoulli: De Usu Artis Conjectandi in Jure' in *Die Werke* (n 41) 541, 541, 545.

[98] See n 51.

[99] See in particular ch I of N Bernoulli, *Dissertatio* (n 1), which set outs the general ideas underpinning the thesis. See also letters from Nicolaus to de Montmort: *Essay*, 2nd edn (n 89) 308, 313–14 (26 February 1711); 388, 393 (23 January 1713).

[100] Compare N Bernoulli, *Dissertatio* (n 1) 8 with the text above from n 48 to n 50.

[101] Compare ibid 55–6 with the text above from n 65 to n 74.

[102] For example, Nicolaus states that in addition to Pufendorf, the French jurist Antoine Mornac is in favour of imposing liability on the borrower in this situation: ibid 55. Nicolaus also, helpfully, makes explicit much of what is implicit in the compressed language in Jacob's very short paragraph on this question. Jacob, for example, concludes by stating that 'doing someone a favour should not lead to loss': 'Text 163' (n 69). Nicolaus amplifies this point by stressing the gratuitous nature of *commodatum*: because the lender can receive no remuneration, it is unjust for him to bear the loss, and because the borrower is not compelled to pay for the use of the thing, it is not unjust for him to bear the loss: N Bernoulli, *Dissertatio* (n 1) 55–6.

[103] 'Meditationes' (n 69) 21, 67 (Text 163, VI).

[104] E Horn, *Die Disputationen und Promotionen an den Deutschen Universitäten vornehmlich Seit dem 16 Jahrhundert* (Harrassowitz 1893) 51–72.

[105] See the text above from n 90 to n 91.

[106] Horn (n 104).

however, that Nicolaus's doctoral thesis was not—or even not meant to be—entirely his own work. In contrast to many other theses of this period,[107] only Nicolaus's name—that of the student—appears on the title page. Furthermore, while Jacob (who had died four years earlier) and Johann Bernoulli are mentioned on the very next page, along with the Rector of the University of Basel, they are specifically said to be the persons to whom the thesis is dedicated. Indeed, it is not even clear who was the supervisor (*der Präses*) of Nicolaus's doctoral thesis.

Notwithstanding the similarities with parts of Jacob's work, the principal argument in this article is that the originality of Nicolaus's thesis can be affirmed emphatically when one pays adequate attention to its legal dimension.[108] Indeed, one historian of mathematics perceptively noted that the significance of the thesis even for mathematics was that it 'demonstrated convincingly that the problems and methods for studying games of chance could be used to solve actual real life questions'.[109] Lacking the legal knowledge to identify relevant questions, this was precisely what Jacob could not do, as we saw in the preceding section. Furthermore, having applied this new mathematics to specific legal issues, Nicolaus was able to make insightful judgements about law, particularly the just price. Therefore, the advances in both mathematics and the law are the consequence of Nicolaus's innovative use of his legal knowledge. We are now in a position to turn the spotlight on this aspect of Nicolaus's thesis.

## 5. *De Usu Artis Conjectandi in Jure: Dissertatio Inauguralis Mathematico-Juridica (1709)*

Nicolaus's thesis was submitted for the doctoral degree in law at the University of Basel in 1709. It is, in essence, an application of the mathematics of probability to questions in contract law and succession law.[110] While, as we have seen, it is mathematicians rather than lawyers who have been enchanted by this thesis for over 300 years, its style and content make it very much a piece of legal literature.

First, this is evident in the expert use of primary and secondary sources. Nicolaus demonstrates a wide knowledge of the primary sources—particularly the Digest, but also the *Codex*—across the 56 pages of the thesis. His wide knowledge of legal scholarship on the particular issues which are discussed is also plain. While the majority of the jurists who appear in the thesis are Germans of the 16th and 17th centuries and two of them—Gottlieb Gerhard Titius and Nicolaus Christoph von Lincker—were still alive when the thesis was written, one finds in this study the views of jurists from all over continental Europe and from every century from the 13th to the 18th. Consequently, the major schools

[107] See eg Johann Samuel Stryk and Georg Fürbringer, *Dissertatio Juridico Inauguralis De Emptione Spei* (Halle and Magdeburg 1714). The contribution, if any, of Stryk—the supervisor—to this thesis is not clear.

[108] Indeed, in what seems to be the only other study of this thesis which focuses on law, Carvais describes it as 'intellectual brilliance': Carvais (n 16).

[109] Yushkevich (n 82) 291.

[110] The one exception is a discussion in the short final chapter of the reliability of testimony and the determination of guilt from certain facts: N Bernoulli, *Dissertatio* (n 1) 54–5.

of continental legal scholarship are represented: the *mos italicus*, *mos gallicus* and natural law.[111] The provision of long lists of jurists to support particular legal points is a further feature of Nicolaus's thesis which is common to civilian scholarship of this period; a notable example is that of the six jurists who are mobilised to support the uncontroversial view that the sale of a future thing should be distinguished from the sale of a chance.[112] Indeed, in a clear sign that jurists and mathematicians read—and write—differently, Condorcet, in his translation of Nicolaus's thesis, pointedly fails to reproduce all the lists of jurists in the original text; instead, Condorcet adds 'etc' after some names, translates others compendiously as 'several jurists' or erases them entirely.[113] The legal sources are plainly not what Condorcet considers to be significant.

The second feature of Nicolaus's thesis which firmly marks it out as a piece of legal literature is that it is with legal rather than mathematical disputes that the author engages in depth. As mentioned earlier, Condorcet stated that Nicolaus's thesis showed 'how many serious mistakes were made by lawyers who were trying to resolve the same issues without mathematics'.[114] Nicolaus's findings lead him to disapprove of both secondary and primary sources. One notable example is Nicolaus's scathing assessment of Titius's interpretation of the *Lex Falcidia*, a rule requiring one-quarter of the estate of a deceased to be reserved for the heir.[115] It may not be easy to calculate the value of the quarter when the deceased has made legacies which depend on the length of a life or lives, such as life annuities or the right to live in a house for one's life. The relevant primary source, D.35.2.68, is, in Nicolaus's view, a 'clear' text, and Titius's view is denounced as 'absurd' and 'ridiculous';[116] Nicolaus's trenchant conclusion is, nonetheless, that the relevant passage in the Digest 'cannot be used' as its presumptions of life expectancy are out of line with the findings of the probabilistic analysis which he has undertaken in chapter II.[117] In contrast to the treatment of lawyers, Jacob and Huygens are cited throughout the thesis, but Nicolaus does not engage in any analogous evaluation of their work.[118]

[111] See Carvais (n 16), 5–6, 17–18.

[112] N Bernoulli, *Dissertatio* (n 1) 23. On this legal question, see the text below from n 128 to n 137.

[113] Condorcet (n 3) 209, 211, 212.

[114] See the text to n 2.

[115] N Bernoulli, *Dissertatio* (n 1) 39–41. On the other hand, Nicolaus speaks highly of Jean Domat (1625–96), who, he says, writes 'elegantly': ibid 41.

[116] N Bernoulli, *Dissertatio* (n 1) 40, 41.

[117] ibid 41–2. Further examples are his dismissal of the views of other jurists on the length of the period after which a missing person should be declared dead and of D.5.4.3 on the consequences for the estate of a deceased of the unknown number of children, if any, to whom a pregnant wife may give birth: ibid 19–21, 53.

[118] N Bernoulli, *Dissertatio* (n 1) 4, 5, 7, 8, 9, 15, 16, 48. Nicolaus refers cursorily and dismissively to Juan Caramuel y Lobkowitz, the only other mathematician in this thesis; when discussing the justice of the Genoese lottery in ch VII, Nicolaus states that it 'would take too long to show here' the errors of Caramuel, who also finds that the lottery is 'unjust': ibid 49–50. Caramuel was an influential polymath of the 17th century: on his 'original contributions' to mathematics, see J Vernet, 'Caramuel y Lobkowitz, Juan' in C Gillispie (ed), *Dictionary of Scientific Biography*, vol III (Scribner 1971) 61. Significantly, the discussion cited by Nicolaus appears in a part of Caramuel's mathematical treatise ('syntagma six' which Nicolaus, however, wrongly describes as 'syntagma seven') in which the terms 'contract' and 'just game' are omnipresent: I Caramuel, *Mathesis Biceps: Vetus et Nova*, vol 2 (Campania 1670) 921–1036. Huygens's influence may have been significant in this respect; the latter's treatise on probability, which Caramuel thought had been composed by 'Longomontanus … a Danish astronomer who lived from 1562 to 1647', is reproduced in the same 'syntagma six' of the *Mathesis Biceps*: I Todhunter, *History of the Mathematical Theory of Probability from the Time of Pascal to That of Laplace* (Macmillan 1865) 45.

It is worth adding here that despite the clear legal purpose of Nicolaus's thesis, it is not without mathematical merit. The significant step from a mathematical perspective is his detailed probabilistic treatment of data on lives—*a posteriori* reasoning from observations, as discussed above in section 3.[119] These data were, effectively, third hand: as Nicolaus's source in the thesis indicates, he draws on Jacob's very brief report and use of the data which were reproduced in a summary of John Graunt's non-probabilistic but very significant examination of mortality figures in England, principally in London.[120] This summary of Graunt's book, published in the *Journal des Sçavans* in 1666,[121] had been Jacob's starting point, 20 years later, in one paragraph of what is anyway a short tract on logic and miscellaneous topics.[122] Before Nicolaus's thesis appeared, probabilistic analyses of data on lives had been published by Halley and de Witt, but it appears that Nicolaus developed his methods independently of these studies.[123] Indeed, some of Nicolaus's mathematical work in this thesis was done in 'an original way'.[124]

We are now in a position properly to assess Nicolaus's treatment of the just price. It is, plainly, central to the thesis. At the end of chapter II, in which the mathematical foundations of probability are outlined, Nicolaus states that this mathematics is of 'the greatest utility especially in our law', and one of the three main applications which he mentions is the following: the determination of 'the just price of a contract based on the length of life, whether and when the remedy of the law 2. C. *de rescind. vend.*[125] is applicable to such contracts'.[126] This question is discussed principally in chapter IV,[127] which, significantly, is the centrepiece of the thesis: at 17 pages, it is by far the longest chapter of the thesis[128] and

[119] Not all of Nicolaus's probabilistic analysis in this thesis is based on data. The applications of probability to law in chs VIII and IX are not based on data; the length of lives is irrelevant to the questions which are examined therein and Nicolaus has no other data. Parts of chs VI and VII are also in this category.

[120] N Bernoulli, *Dissertatio* (n 1) 9, 15. Nicolaus thinks, erroneously, that the data are from London and Paris or just Paris: ibid 8–9, 16. On Graunt's book, which was first published in 1662 and 'had immense influence', see Hald (n 15) 81–105, 103. Nicolas also procured from 'a friend' some fresh data from an unspecified city in Switzerland: *Dissertatio* (n 1) 16. Nicolaus provides only a very brief summary of the results of his probabilistic analysis of this data and notes that they suggest longer lifetimes: ibid. He does not use these data in any other part of his thesis.

[121] *Journal des Sçavans* (1666) XXXI 613, 615–16 (anonymous author). If Nicolaus had used the data in this article rather than relying only on Jacob's summary, he would have had some data with which to answer, in part at least, the question outlined in n 117 on the number of children, if any, to whom a pregnant wife may give birth. As the article reports, Graunt had found that a pregnancy was 20 times as likely to result in a live birth than a miscarriage or stillbirth: ibid 615.

[122] J Bernoulli, *Theses Logicae de Conversione et Oppositione Enunciationum cum Adnexis Miscellaneis* (Basel 1686) 283 [31]. Jacob's principal objective in this paragraph is to provide an answer, for the purposes of a dowry contract, to the question of how much more likely it is that a girl of 16 will outlive a man of 56, but he does not show how he arrives at this result. Jacob's reference to 'Paris and London' may, in part, have led to the error made by Nicolas highlighted in n 121.

[123] Apart from Halley's article of 1694, of which Nicolaus was 'unaware', and de Witt's tract, which was, as we have seen, not readily available, in no published work could one find a probabilistic analysis of data on lives: the letters between Christiaan and Lodewijk Huygens were 'not published until 1895': see nn 53 and 79, and the text above from n 78 to n 79; Hald (n 15) 378, 106. None of the trio is mentioned in Nicolaus's thesis.

[124] Yushkevich (n 82) 290.

[125] This was the contemporary way of citing C.4.44.2.

[126] N Bernoulli, *Dissertatio* (n 1) 17.

[127] In ch VI, the method by which one can calculate the 'just' price (or rather the premium) in a contract insuring life for a fixed period is outlined: N Bernoulli, *Dissertatio* (n 1) 46, 49.

[128] The next longest, ch II, contains only 11 pages.

constitutes 31% of its content; it is also the only chapter in which one finds an equally significant amount of legal and mathematical analysis.

The title of chapter IV is 'The Sale of a Chance and in Particular the Sale of Life Annuities'. Nicolaus begins by defining the sale of a chance (*emptio spei*).[129] First, he refers to the three classic examples discussed together in D.18.1.8.1, the text which provides that such sales are valid:[130] the prospective catches of the fisherman and the hunter and the prospective catch of largesse distributed to crowds.[131] In all these cases, it is the chance and not the outcome which is sold; thus, neither the seller nor the purchaser can invoke the just price on the ground that more or less than what was expected was, in fact, caught.[132] So far, so orthodox. Nicolaus also distinguishes the sale of a chance (*emptio spei*) from what he calls '*emtio rerum futurarum*' (the sale of a future thing),[133] or what we now generally call *emptio rei speratae* (the sale of an expected thing).[134] In the latter case, the contract is conditional on the thing materialising in the future—for example, as D.18.1.8.1 provides, a calf being born—so if this does not happen, the contract is retroactively annulled.[135] In contrast, in the sale of a chance, the contract is binding regardless of whether any 'thing' ultimately materialises. This discussion too is entirely orthodox and Nicolaus mobilises impressively long lists of leading jurists to support these propositions of law.[136]

What is more innovative is his treatment of the sale of life annuities as a species of the sale of a chance: he describes the former as the 'principal and most commonly used kind today' of the latter.[137] The sale of life annuities was certainly discussed by contemporary jurists in their commentaries on the relevant provisions of the Digest (principally D.18.1.8),[138] but it is significant that Pufendorf, who, in 1672, in the different style of natural law, had carved out the conceptual category of 'contracts containing chance' ('*de contractibus qui aleam continent*'), did not include within it the sale of life annuities.[139] Indeed, Nicolaus extends what he says 'about the sale of a chance in general' to a particular contract which was common in the Italy of his time under which, in exchange for a sum paid by a father when his daughter is born, the other party repays five or four times the sum if the daughter reaches 16 years of age; the objective is that the latter sum would constitute the daughter's dowry.[140]

[129] N Bernoulli, *Dissertatio* (n 1) 22–3.

[130] D.18.1.8pr provides that they are valid sales even though '*sine re*', ie even though 'there is no thing'. As Daube noted, sale required a 'thing' and things could be incorporeal, but by the 6th century, Roman lawyers had not 'reached the stage where the chance of obtaining a haul—no matter whether it would come off or not—might be described as such a thing': D Daube, 'Purchase of a Prospective Haul' in *Studi in Onore di Ugo Enrico Paoli* (Le Monnier 1955) 203, 204.

[131] Nicolaus adds to this trio the sale of what one hopes to inherit. The practice of 'praetors or consuls throwing largesse to the crowd' appears in the Institutes: Instit.2.1.46. 'This throwing of presents to the populace became more frequent from the end of the Republic': Daube (n 130) 206.

[132] N Bernoulli, *Dissertatio* (n 1) 23.

[133] ibid.

[134] Zimmermann, *The Law of Obligations* (n 27) 245–9.

[135] N Bernoulli, *Dissertatio* (n 1) 23.

[136] ibid.

[137] ibid 38, 24.

[138] See eg J Voet, *Commentarius ad Pandectas*, vol I (De Hondt 1698) 808.

[139] Pufendorf, *De Jure Naturae et Gentium* (n 26) 680–5.

[140] N Bernoulli, *Dissertatio* (n 1) 38, 37–8. Nicolaus's conclusion is that the just price requires 5.457 times the original sum to be repaid: ibid 38.

What is entirely novel, though, is Nicolaus's emphatic answer to the question, posed at the end of chapter II, whether the just price should apply to the sale of chances: it 'should not be completely banished as nearly all the jurists wish'.[141] His innovative contribution to law in this chapter is to demonstrate that the just price of chances—particularly those relating to the length of lives—can be calculated, and consequently the reason for excluding such sales from the remit of the rule is now otiose.

Surprisingly, given the copious lists of sources which are provided for other propositions of law in the thesis, Nicolaus leaves the reader bereft of authority for the claim that the just price is excluded from sales of chances. Nonetheless, it is plain that the contemporary legal position was precisely that set out by Nicolaus. A brief glance at the commentaries on the Digest of Johannes Voet, one of the leading scholars of the age, suffices to demonstrate this point.[142] He excludes various contracts from the remit of the just price and notably 'the sale of a chance, such as the cast of a net … or when a life annuity is sold, given that the length of any individual human life is entirely uncertain'.[143] This exclusion is particularly notable, as in all other respects Voet interprets the rule quite extensively; in his view, it is not limited to the case of the sale of land at less than half of the just price which is set out in C.4.44.2. For example, Voet states that the rule can be invoked by the purchaser and not just by the vendor; it applies to valuable moveables and not just land; it can be invoked in some contracts other than sale, such as *permutatio* (barter)[144] and *emphyteusis*,[145] and the period within which the claim must be brought is 30 years rather than four.[146]

Of more significance, perhaps, are the two jurists to whom Nicolaus refers in support of his controversial contention that the just price should apply to sales of chances: Johannes Brunnemann and Pufendorf.[147] Brunnemann's work supports Nicolaus's claim (even though it does not say quite what Nicolaus reports), but no support at all is to be found in that of Pufendorf.

Nicolaus appears to have used Brunnemann's commentaries on C.4.44.2 in the latter's *Commentarius in Duodecim Libros Codicis Justinianei*.[148] In the first and second editions of this treatise, Brunnemann says many things about the just price, this 'most noble and vaunted' rule,[149] but there is no discussion of the specific

[141] ibid 23.

[142] Voet (n 138). On Voet, see R Zimmermann, 'Römisch-hollandisches Recht—ein Überblick' in R Feenstra and R Zimmermann (eds), *Das Römisch-hollandische Recht: Fortschritte des Zivilrechts im 17. Und 18. Jahrhundert* (Duncker & Humblot 1992) 9, 39–42. Given the eminence of Voet, it is surprising that he is entirely absent from Nicolaus's thesis. There were, though, two representatives of the Roman–Dutch legal tradition (which flowered in the 17th and 18th centuries) in Nicolaus's thesis: in addition to Hugo Grotius, one finds Johan van den Sande, whose work, which drew on the decisions of the courts of Friesland, was frequently cited by German jurists in the 17th and 18th centuries: N Bernoulli, *Dissertatio* (n 1) 45; see also Stryk and Fürbringer (n 107) 34 (who describe as a 'memorable case' the same one cited by N Bernoulli, *Dissertatio* (n 1) 44).

[143] Voet (n 138) 808.

[144] A *permutatio* is not a sale as a thing is exchanged for a thing instead of a price for a thing: Zimmermann, *The Law of Obligations* (n 27) 250–1, 532–7.

[145] On which, see ibid 358–9.

[146] Voet (n 138) 804, 807–8.

[147] N Bernoulli, *Dissertatio* (n 1) 24.

[148] Nicolaus's citation is 'ad l. 2. C. de rescind. vend. num. 19'. I have been unable to find any para 19 or anything corresponding to '19' in any edition of Brunnemann's *Commentarius in Duodecim Libros Codicis Justinianei*.

[149] J Brunnemann, *Commentarius in Duodecim Libros Codicis Justinianei* (Leipzig, 1663) 351.

case of the sale of a chance here. In the third edition of the treatise, the final one produced in Brunnemann's lifetime, he does refer to the sale of a chance: he states that the just price applies to such contracts and gives the example of a purchaser paying 'more than double the price which has been paid by another'.[150] While this sentence is not entirely consistent with Nicolaus's description of Brunnemann's view—there is no reference to fraud or mistake, and the sale of a chance is wider than the prospective catch of a fisherman to which Nicolaus refers[151]—and it describes the claim of a purchaser only, it certainly supports Nicolaus's position that the just price should apply to sales of chances. Indeed, given that the extension of the just price to the purchaser was itself a contested point,[152] the reference to the purchaser can be taken implicitly to include the seller too. Furthermore, although it is not a source cited by Nicolaus, Brunnemann's commentary on the Digest, provides further support for Nicolaus's contention and, importantly, in the context of a contract based on the length of a life.[153] In Brunnemann's observations on D.18.1.8.1, there is a brief but favourable reference to the just price: Brunnemann states that where a usufruct of feudal land is ceded for 1000 a year for the life of the transferee, the just price ('*laesio enormis*') applies if the land was never capable of yielding such a sum annually.[154] Thus, like Nicolaus, Brunnemann is in favour of applying the just price to sales of chances, and particularly to contracts based on the length of lives.[155] However, unlike Nicolaus, Brunnemann provided no practical way of achieving this objective.

Albeit with some hesitation, Nicolaus states that support for the view that the just price should apply to sales of chances can also be found in Pufendorf's *Elementorum Jurisprudentiae Universalis Libri II*.[156] Nicolaus tells us that the just

[150] J Brunnemann, *Commentarius in Duodecim Libros Codicis Justinianei* (Leipzig 1672) 487. In the 'Preface to the Reader', Brunnemann states that this edition 'expands and enriches' the text of the previous editions. Another motivation, he explains, was to correct the serious errors in the previous editions. He indicates that he had no role in the second edition, which was published 'in France, specifically in Lyon' (presumably the one produced in 1669), and he laments the fact that this edition did not correct the errors in the first. Indeed, the title page of the third edition warns the reader that the French edition is 'deficient and defective'.

[151] N Bernoulli, *Dissertatio* (n 1) 24. It is possible that Nicolaus was using an edition published after Brunnemann's death. In the edition published in 1679, there is a relevant sentence in a s 9 (rather than 19): J Brunnemann, *Commentarius in Duodecim Libros Codicis Justinianei* (Leipzig 1679) 487. However, this sentence covers only one of the points which Nicolaus attributes to Brunnemann and it is conspicuously absent from the three editions published in Brunnemann's lifetime.

[152] Domat, for example, limited the claim to vendors of immoveables: J Domat, *Les Loix Civiles dans leur Ordre Naturel*, vol I (Coignard 1689) 150–2.

[153] J Brunnemann, *Commentarius in Quinquaginta Libros Pandectarum* (Leipzig 1674). The first edition was published in 1670. This second edition was published two years after his death, but Samuel Stryk, in the preface, tells us that the text is entirely that of Brunnemann, who had been able only to amend the text relating to the first eleven Books of the Digest.

[154] ibid 696–7. When discussing C.4.44.2 (in his commentary on D.18.5), Brunnemann extends the rule on the just price to events after the conclusion of the contract in at least one instance: a contract for the annual provision of a specified amount of grain in exchange for a lump sum. He explains that fluctuations in the price of grain in, for example, the sixth or seventh year could provide a basis for judicial intervention on the basis of the just price: ibid 719. This example is puzzling precisely because he also expresses the view that contracts for the sale of chances cannot be rescinded simply because the outcome is more favourable to one party than another regardless of the justice of the price at the time of the conclusion of the contract, eg when the purchaser of a life annuity 'has a very long life': ibid 719.

[155] However, once again, contrary to Nicolaus's description of Brunnemann's view, there are no qualifications of this statement relating to error and fraud here.

[156] S Pufendorf, *Elementorum Jurisprudentiae Universalis Libri II* (Vlacq 1660).

price for Pufendorf is the contract price, but the contract price is just if 'we assume that the parties had regard to what ordinarily happens when they were determining the price'.[157] Crucially, Nicolaus thinks that Pufendorf too has made this assumption: 'and this seems to have been the view of Pufendorf too'.[158] However, Nicolaus's hesitation is justified as this is not quite what Pufendorf states in his treatise. In Pufendorf's view, the price agreed for the sale of 'the right to fish in a river or the expected yield from gardens, vines etc' is considered to be equal ('*pro aequali*') to the chance even though what is subsequently caught in the river or grown in the garden or on the vines is of greater or lesser value than the price paid.[159] Yet this issue is adjacent to rather than within the remit of the just price: Pufendorf is discussing what happens *after* the conclusion of the contract rather than the justice of the contract at the time when it is concluded. Indeed, as Nicolaus previously explains, what is sold is the chance, so that if nothing materialises, 'the purchaser cannot say that he is injured' ('*de laesione conqueri non potest*') because *ab initio*, from the conclusion of the contract, 'he could have been a winner or a loser and nobody can be considered to have been injured' ('*nemo videatur laedi*') when each side has the same chance of winning or losing.[160] Furthermore, while Pufendorf states that the price of such chances is set by having regard to 'what ordinarily is obtained', he does not prescribe that this *must* always be the case.[161] It does not quite match Nicolaus's probabilistic analysis (rudimentary in this case), according to which the just price or 'the value of the right to fish for a year in a river is calculated by dividing the sum of what was caught in previous years by the number of [those] years'.[162] Overall, Pufendorf's *Elementorum* does not bear the weight which Nicolaus seeks to place upon it.[163]

Nicolaus's contention in chapter IV that the just price can and should be applied to sales of chances is all the more innovative given that, as he says, it goes against the views of 'nearly all jurists'.[164] With regard to a life annuity contract, which Nicolaus rightly considers to be a species of sales of chances, these jurists had much to say. Nicolaus explains that the legal literature indicates two approaches. One view was that all such contracts, regardless of the price stipulated, were unlawful on the basis that they could circumvent the usury laws or that they led to one party seeking the death of another person.[165] The majority of jurists, however, eschewed this approach and determined in an arbitrary way the price at which they ought to be sold. The range of practices in this regard was remarkably varied: Nicolaus reports that in just one treatise—that of the eminent French jurist of the 16th century, Jean Dumoulin—six different opinions are

[157] N Bernoulli, *Dissertatio* (n 1) 24.
[158] ibid.
[159] Pufendorf, *Elementorum* (n 156) 177.
[160] N Bernoulli, *Dissertatio* (n 1) 23.
[161] Pufendorf, *Elementorum* (n 156) 177.
[162] ibid 23. Of course, Nicolaus did not have any data on this point; he simply outlines the method to apply once one has the relevant data.
[163] There is also nothing of note on this question in Pufendorf's other major work on law, *De Jure Naturae et Gentium* (n 26).
[164] N Bernoulli, *Dissertatio* (n 1) 23.
[165] ibid 25.

noted.[166] Yet Nicolaus contends that he has found 'the true and genuine method of valuing life annuities';[167] therefore, the just price can apply to the sale of such interests.[168] 'The justice of the contract requires that [the purchaser and the seller] are equally exposed to the risk of losing and that cannot happen unless the price is set according to the probable and likely length of life.'[169]

Clearly, much turns on Nicolaus's ability to show the reader precisely how the just price can be calculated in life annuity contracts so that this new mathematics can be used in practice. His starting point is to draw an analogy between the investment of a capital sum at 5% annual interest (the legal limit at the time) and the purchase price of the life annuity.[170] However, because, in the latter case, 'the capital sum cannot be recovered', each annuity must exceed the ordinary rate of interest which the capital sum would have earned in that year.[171] The objective is that the capital sum 'decrease from year to year until it is extinguished', and this is achieved by notionally deducting the annual excess from the capital sum.[172] Drawing on the probabilistic analysis of data on lives which he had undertaken in chapter II, Nicolaus then demonstrates over several pages his calculations of the just price of life annuities for a variety of ages. While it is easy to follow most steps in his approach—it is a 'geometric progression' incorporating life expectancy as the variable of time—the calculations are, plainly, laborious even with the use of logarithms.[173] Ultimately, having abruptly changed course,[174] Nicolaus presents, in a tabular form, his results on the just price of life annuities from birth to age 76 using intervals of decades from age 6 to 76.[175] The calculations underpinning one result in the table—the purchase of 1000 a year on the life of a person currently aged 16—are then explained and set out in detail: the just price of 10,593 is achieved via a brief arithmetic application of the probability of death in each subsequent decade to previous results which he obtained on the value of annuities in each year from 1 to 100.[176] Even if one could not follow the underlying mathematics, one could, nonetheless, by using this table, obtain quite easily the just price of any particular life annuity contract.[177]

In the concluding sentence of this very significant chapter, Nicolaus reminds the reader of its strikingly innovative finding for all sales of chances, including contracts based on the length of lives: the just price can be invoked by either the

[166] ibid 26.
[167] ibid 29.
[168] ibid 38.
[169] ibid 26.
[170] ibid 29.
[171] ibid.
[172] ibid.
[173] ibid 29–31.
[174] Three-quarters of the way through ch IV, Nicolaus realises that one of his assumptions is wrong: ibid 31. Hald concludes that Nicolaus was 'correct' to alter his approach: (n 15) 128.
[175] N Bernoulli, *Dissertatio* (n 1) 33.
[176] ibid 33.
[177] Nicolaus proceeds to show how the same approach can be used to obtain the just price of the dowry contracts, mentioned above, which were commonly concluded by Italian fathers of the time. See the text from n 139 to n 140.

seller who receives less than half of the just price or the purchaser who pays more than double the just price.[178]

## 6. *Quod Erat Demonstrandum*

A mathematical proof of a theorem could end here with these famous words. This article, however, requires a more detailed *envoi*. We have seen that Nicolaus's application of probability to the just price is both novel and significant. In particular, he rightly concluded that the rule should be extended to contracts of chance because the reason for its exclusion was otiose: it was now possible to determine the just price using the mathematics of probability. The case he made was particularly strong in contracts relating to the length of lives, such as life annuities; probabilistic analysis of available data (however imperfect) led to more precise determinations of the just price than was possible in other instances, such as maritime insurance contracts, for which his probabilistic analysis was based on *a priori* assumptions of a hypothetical kind.[179]

In 1706, Christian Thomasius, 'the founding father of the Enlightenment in its specifically German version',[180] vehemently and colourfully denounced the just price in a dissertation of almost 80 pages which was reprinted throughout the 18th century and widely disseminated.[181] Thomasius noted that the sale of chances was one situation in 'which according to the consensus of legal scholars our rule does not apply'.[182] Yet, three years later, in his doctoral thesis, Nicolaus demolished the foundation of this position. Following two pages of consecutive questions on the scope of the just price, Thomasius famously quipped: 'See how many heads has this hydra.'[183] The consequence of Nicolaus's thesis was to add one very large head to the mythical monster.

Nicolaus's first contribution to law in his doctoral thesis of 1709 was, effectively, his last. A summary comprising 12 pages was published in *Acta Eruditorum*, one of the principal journals of the Enlightenment, in 1711.[184] However, it contained

[178] N Bernoulli, *Dissertatio* (n 1) 38. Thus, while Nicolaus favoured extending the rule to the purchaser, he adopted the geometric rather than the arithmetic method of calculating the 'injury': ibid 24. The geometric method was less favourable to the purchaser as the threshold for invoking the just price was higher: one and a half times the just price sufficed for the arithmetic method. This was yet another contested aspect of the just price. See eg Voet (n 138) 804, who favours applying the arithmetic method to a purchaser's claim.

[179] The just price in insurance contracts is discussed in ch VI of Nicolaus's thesis (n 1).

[180] R Zimmermann, 'Christian Thomasius, the Reception of Roman Law and the History of the *Lex Aquilia*' in M Hewett (ed, tr), *Larva Legis Aquiliae: The Mask of the Lex Aquilia Torn off the Action for Damage Done: A Legal Treatise by Christian Thomasius (1655–1728)* (Hart Publishing 2000) 49.

[181] Thomasius (n 22). It was reprinted several times over the course of the 18th century and influenced the debates on the just price in the French Civil Code in 1803: Fenet, vol XIV (n 11) 35–44, 129. Indeed, the great German jurist, Karl Friedrich von Savigny, ridiculed this use of Thomasius's thesis. He said it 'came by chance into the hands of one Councillor and it is really touching to see the admiring way in which it was received, cited and discussed'; he added that Germany 'could, of course, furnish similar and better scholarship to be used on other points': *Vom Beruf unsrer Zeit für Gesetzgebung und Rechstwissenschaft* (Mohr und Zimmer 1814) 64–5. The 'Councillor' to whom Savigny refers is Théophile Berlier.

[182] Thomasius (n 22) 76.

[183] ibid 46–7. In a similar vein, he referred to the diversity of opinions on these questions as 'the vast Augean stable': 51.

[184] *Acta Eruditorum* (1711), Supplementa, vol IV (1711) 159. The author appears to be Nicolaus, given that Jacob is described as 'Dn Patrui p.m.' and 'Cl. Patruus p.m.', which are best translated here as 'my uncle': 159. The same terms are also used in the original thesis. The journal '*Acta Eruditorum* was founded by Leibniz in 1682': Hald (n 15) 172.

nothing new; indeed, it contained much less law than the original: the Digest is cited only once, and all the legal scholars have been excised.[185] Nicolaus produced four further legal texts in 1714, 1720, 1722 and 1731, when professorships in law became vacant at the University of Basel.[186] Each of these texts contained a short series of unsupported claims, called 'theses', which were subsequently defended orally in public.[187] No single thesis was longer than a paragraph and many comprised one line only. Significantly, the entirety of thesis XXXII in the text of 1714 is that 'the just price has no place in contracts of chance'.[188] These theses were, manifestly, not contributions to knowledge.

Nicolaus 'disappear[ed] from the scene of mathematical probability' in 1713.[189] Fittingly, however, in that year, one of his last publications in this field was triggered by a discussion in a contemporary legal treatise of the just price—the just stake—in a contract of chance. In an epistolary exchange with Nicolaus, Pierre Rémond de Montmort reports having received recently from Paris a book on 'ethics' in which the albeit 'judicious' author 'seems to make mistakes in the places where he speaks of the use of geometry in determining probabilities in games'.[190] Montmort then proceeds to provide an example concerning the determination of the just price in a game in which one player has two turns whereas the other has only one.[191] Nicolaus's reply to this specific point is lengthy: it is spread over three pages in Montmort's treatise.[192] First, he identifies the book as the *Traité du Jeu* and the author as 'Mr Barbeyrac, who is at present Professor of Law in Lausanne

[185] ibid 169. Therefore, in contrast to the thesis, the focus of the summary is, unquestionably, mathematical rather than legal. The decision to focus on mathematics seems to be deliberate. Nonetheless, law was not an unsuitable topic for the *Acta Eruditorum*, a journal which seems to have embraced all fields of knowledge. Indeed, in this very supplement, there is a review of a treatise on public law which had been published in 1710 by Justus Henning Böhmer, a distinguished jurist of the age: 508–18. In addition, this supplement contained a review of a book published in 1705 in London containing reprints of 'two pamphlets … and the *Elementa jurisprudentiae*' by Richard Zouch (1590–1661), former Regius Professor of Civil Law in Oxford: 118, 122–3. The anonymous reviewer is particularly interested in Zouch's discussion of wager of battle: 122–3.

[186] *Theses Juridicae Miscellaneae* (Basel 1714); *Theses Juridicae Controversae* (Basel 1720); *Theses Juridicae Miscellaneae* (Basel 1722); *Theses Iuridicae Miscellaneae* (Basel 1731). These four texts match the account of Nicolaus's life which is given in JH Brucker, *Die in den Schwachen Kräftige Gnade Gottes Betrachtet den 2 Christmonats 1759: Zu St. Peter, den Hohansehnlicher und Zahlreicher Gleich-Bestassung des Edlen, Ehrenvesten und Hochgelehrten Nicolaus Bernoulli B. Rechten Doctors und Öffentlichen Lehres bey unserer Hohen Schule* (Basel 1759) 23, 26–8. This text appears to reproduce the funeral service of Nicolaus Bernoulli. The author is described on the title page as 'pastor of the same place'.

[187] Nicolaus's purpose in 1720, 1722 and 1731 was to obtain the professorship in law: *Die in den Schwachen* (n 186) 27–8. He was not successful until 1731: see the text from n 92 to n 93. However, in 1714, Nicolaus was not competing for the professorship in law; this text is described on the title page as a 'practice' undertaken 'on the occasion of a vacancy': *Theses Juridicae Miscellaneae* (Basel 1714). See also the account in *Die in den Schwachen* (n 186) 26.

[188] *Theses Juridicae Miscellaneae* (Basel 1714) 7.

[189] Hald (n 15) 393. The reason for this retreat is unclear. It has been said that Nicolaus practised law and had 'very little spare time left for mathematical research' on his return to Basel, but no evidence is provided to support this claim: Yushkevich (n 82) 301. Moreover, Nicolaus did make further contributions in other fields of mathematics. He published, for example, four articles in *Acta Eruditorum* between 1719 and 1720, and 'In his letters to Euler (1742–1743) … he solved the problem of the sum of reciprocal squares … which had confounded Leibniz and Jacob': JO Fleckenstein, 'Bernoulli, Nikolaus I' in C Gillispie (ed), *Dictionary of Scientific Biography*, vol 2 (Scribner 1970) 56, 57. Nicolaus's retreat from probability in 1713 may be related to the diminishing interest which Montmort took in this subject: Hald (n 15) 393. It may be significant too that De Moivre also stopped writing to Nicolaus in 1714: Bellhouse (n 60) 107.

[190] Montmort, *Essay*, 2nd edn (n 89) 320 (10 April 1711).

[191] ibid 320–1 (10 April 1711).

[192] ibid 332–4 (10 November 1711).

in Switzerland'.[193] Barbeyrac was one of the most eminent jurists of the age;[194] indeed, in his own century, he was cited even in English cases.[195] Disagreeing with Montmort's assessment of the 'particular example' in Barbeyrac's treatise, Nicolaus affirms that he sides with Barbeyrac and provides detailed mathematical reasoning for this view.[196] Montmort's response is equivocal. He states that in order to justify Barbebyrac's position, it suffices that Nicolaus approves the relevant passage, and he adds that 'it is because there are many views on this point that he [Barbeyrac] is not wrong'.[197]

This epistolary exchange between Nicolaus and Montmort encapsulates the contemporary place of discussions of law and mathematics on the just price: they took place almost exclusively amongst mathematicians. Barbeyrac's *Traité du Jeu*, which was published in 1709, the same year as the publication of Nicolaus's thesis and one year after the first edition of Montmort's treatise, is the exception.[198] Alone amongst legal scholars of the time, Barbeyrac expressly recognises the contribution of mathematics to gaming contracts: in elegant prose, he acknowledges 'the grand and beautiful efforts of the mathematical mind' in this respect.[199] Strikingly, he even highlights, in a long footnote, some recent mathematical developments in this field by referring to Jacob and the treatises by Huygens and Montmort, without, though, mentioning Montmort's name.[200] However, Barbeyrac's understanding of the mathematics appears to be quite superficial. A scrupulous scholar, Barbeyrac cites his sources systematically, and they are, unsurprisingly, secondary sources: an obituary of Jacob from 1705 and a review of Montmort's treatise in a journal.[201] When the second edition of Barbeyrac's treatise appeared almost two decades later, he had still not acquired a deeper knowledge of probability, although the list of mathematicians in the footnote had

[193] ibid 332 (10 November 1711).

[194] Like the mathematician Abraham De Moivre, Barbeyrac was a Huguenot refugee from France: P Meylan, *Jean Barbeyrac, 1674–1744 : Et les Débuts de l'Enseignement du Droit dans l'Ancienne Académie de Lausanne* (Rouge 1947) 29–67.

[195] Barbeyrac is, for example, cited in his lifetime by Lord Talbot in *Barbuit's Case in Chancery* (1736) Tal 281, 283; 25 ER 777, 778 (on the status of consuls). After Barbeyrac's death, he is mentioned by Aston J in *Millar v Taylor* (1769) 4 Burr 2303, 2340–1; 98 ER 201, 221–2 (a seminal case on copyright).

[196] Montmort, *Essay*, 2nd edn (n 89) 332–4 (10 November 1711). Nicolaus does, however, find that Barbeyrac has erred in his reasoning in one instance in the treatise: ibid 333. This discussion leads Nicolaus, in the same letter, to propose four other problems: 334. Indeed, Montmort finds this discussion equally stimulating; he subsequently poses a problem 'which is very similar to the case of Mr Barbeyrac': 337, 346 (1 March 1712).

[197] ibid 346 (1 March 1712).

[198] J Barbeyrac, *Traité du Jeu: Où l'on Examine les Principales Questions de Droit Naturel et de Morale qui Ont du Rapport à Cette Matière*, 2 vols (Humbert 1709); PR de Montmort, *Essay d'Analyse sur les Jeux de Hazard* (Quillau 1708).

[199] Barbeyrac, *Jeu* (n 198) 126–7. By 1731 at the latest, Barbeyrac is joined by Johann Ulrich von Cramer (1709–72), a notable jurist and philosopher of the 18th century, who, in a thesis on contracts of chance, refers *inter alia* to Jacob, Huygens, Halley, Montmort and De Moivre: *Specimen Novum Juris Naturalis de Aequitate in Probabilibus: Exemplo Emptionis Spei Illustrata, Notatis Simil Differentiis illius Juris a Jure Civili Commune* (Marburg 1713) preface. Notably, Nicolaus is not mentioned.

[200] Barbeyrac, *Jeu* (n 198) 126. In fact, Montmort's name does not appear on the title page of the first edition of this treatise.

[201] ibid.

grown longer and by then included an 'English treatise' published in London in 1718 by Abraham De Moivre.[202]

It is puzzling that Barbeyrac did not embrace the new mathematics of probability and grasp its implications for gaming and other contracts. Clearly, he was aware of even the most recent developments in the field. Furthermore, he was not uninterested in other aspects of mathematics: on his death, there were several treatises on mathematics and physics in his personal library, including ones by luminaries such as Galileo Galilei, René Descartes and Isaac Newton.[203] Significantly, neither edition of Montmort's treatise nor Nicolaus's thesis feature in the inventory of the books in Barbeyrac's possession when he died.[204] It is wholly unsurprising in this context that Barbeyrac concludes that there is 'no use' for the mathematics of probability in gaming contracts.[205] After listing spurious reasons of a practical kind,[206] he advances a theoretical argument: 'morality does not require such precision: the most just stake lies here within a range and is not indivisible as in mathematics'.[207] One hears here the echoes of tomes of legal learning: the supposed indivisibility of the just price was a common refrain in the scholarly literature.[208]

This brief account of Barbeyrac's treatment of the mathematics of probability serves to reinforce the uniqueness and significance of Nicolaus's contribution to knowledge. Nicolaus imaginatively and skilfully tied together the latest developments in mathematics to Roman law on the just price law and drew the important and logical conclusion that the existing legal position was no longer tenable.

[202] Barbeyrac, *Traité du Jeu, où l'on Examine les Principales Questions de Droit Naturel et de Morale qui Ont du Rapport à Cette Matière*, vol I (2nd edn, Humbert 1737) 146–7. The footnote now also included a reference to the second edition of Montmort's treatise, but once again this knowledge seems to have been gleaned exclusively from a secondary source: a review by Leibniz. It is, therefore, entirely possible that Barbeyrac never knew about the discussion of his own treatise in that very book. As noted above (n 60), De Moivre's mathematics played a role in the development of a version of the just price in English law.

[203] *Bibliotheca Barbeyraciana* (Spandaw 1744) 61–5.

[204] Barbeyrac would, however, have had access to the summary of Nicolaus's thesis in 1711 in the *Acta Eruditorum* (n 184) as the former possessed all volumes, 'including the supplements', of this journal from 1682 to 1743: *Bibliotheca Barbeyraciana* (n 203) 120. Several theses in law also feature in his private library, but their authors are not identified in the inventory: 149.

[205] Barbeyrac, *Jeu* (n 198) 126.

[206] For example, he states that even players who are mathematicians are 'unlikely to apply their learning to such a thing as a game as greed for profit does not leave much time for abstract ideas and discussions': 127.

[207] Barbeyrac, *Jeu* (n 198) 127.

[208] Decock (n 17) 526–7.