

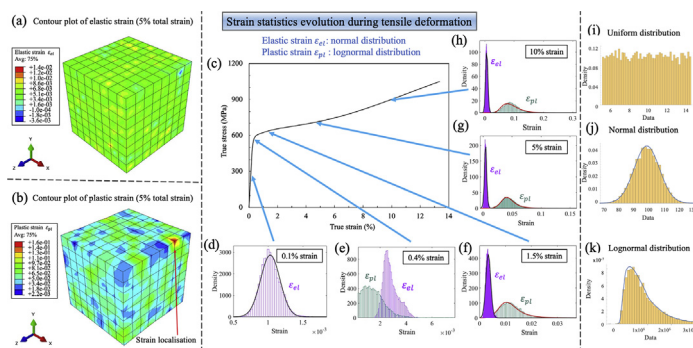
EXPRESS ARTICLE

Why is local stress statistics normal, and strain lognormal?

Jingwei Chen, Alexander M. Korsunsky*

MBLEM, Department of Engineering Science, University of Oxford, Parks Road, Oxford OX1 3PJ, United Kingdom

GRAPHICAL ABSTRACT



ARTICLE INFO

Article history:

Received 7 October 2020

Received in revised form 8 November 2020

Accepted 9 November 2020

Available online 11 November 2020

Keywords:

Crystal plasticity
Strain statistics evolution
Strain localisation
Normal distribution
Lognormal distribution

ABSTRACT

In the present study we elucidate the nature of local strain statistics evolution during tensile deformation in polycrystalline materials. A rate-independent formulation was implemented within a crystal plasticity framework by the means of representative volume element (RVE) analysis. Local elastic strain, as well as stress, were found to obey a normal distribution, whereas the statistics of local plastic strain conforms to a lognormal distribution. In line with experimental observations, the plastic strain becomes progressively localised and the local regions of large strains make significant contribution to the overall average strain increase. The results reveal the nature of strain inhomogeneity at the microscale and emphasize the fact that in metallic materials the elastic strain accumulation represents an additive process, whereas plastic deformation is a multiplicative process.

© 2020 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

Most polycrystals possess significant elastic and plastic anisotropy, leading to spatial inhomogeneity and pronounced orientation dependence of both elastic and plastic components of the total strain, $\varepsilon^{tot} = \varepsilon^{el} + \varepsilon^{pl}$. In the early stages of deformation far from strain localisation and the attainment of the critical condition (maximum bearing load), the macroscopic strain (Type I) obtained by averaging across multiple grains appears uniform and homogeneous. However, significant variation of local strain arises from early stages of deformation in

polycrystals. Local strain deviation from macroscopic average is further classified into Type II (grain level average) and Type III (intragranular deviation from grain average) that is highly inhomogeneous due to the difference in crystal orientation and intra-grain strain localisation due to plastic slip [1–3]. Thanks to the development of advanced imaging techniques and crystal plasticity theory it has become possible to investigate strain inhomogeneity both experimentally and numerically [4,5]. Several studies [4,5] reported observations that the statistics of total strain during deformation follow a lognormal distribution. To the best of the authors' knowledge, there is no statistical study performed separately on the distribution of local elastic and plastic strains at different loading stages and scales of consideration.

* Corresponding author.

E-mail addresses: jingwei.chen@eng.ox.ac.uk (J. Chen),
alexander.korsunsky@eng.ox.ac.uk (A.M. Korsunsky).

In the current study, we decomposed the total strain into elastic and plastic components. We discovered that the **elastic** strain obeys a **normal** distribution while **plastic** strain conforms to a **lognormal** distribution. We have been able to identify the universal underlying reason for this observation, namely that due to the respective magnitudes, the elastic strain accumulation represents an additive statistical process, whereas plastic deformation is a multiplicative process. We proposed this as the explanation for the above discovery. We explored this aspect of elastic-plastic polycrystal deformation with the intent of gaining better understanding of strain evolution and progress toward localisation that ultimately leads to failure, thus seeking to improve predictive modelling capability.

The local elastic and plastic strain distribution during tensile deformation were investigated in a polycrystalline nickel superalloy Haynes 282 using an implementation of crystal plasticity finite element method (CPFEM) [6]. A representative volume element (RVE) containing 600 grains and 15,000 integration points was employed. Random crystallographic orientation was assigned to the grain ensemble obtained by Delaunay tessellation, and the grain size was found to follow a lognormal distribution. The model was validated macroscopically by matching the experimental stress-strain curve and mesoscopically by matching neutron diffraction data for a polycrystalline Haynes 282 alloy. Figure (c) illustrates the simulated stress-strain curve of the alloy subjected to uniaxial tensile loading. Two contour plots in Figure (a) and (b) demonstrate the inhomogeneous elastic and plastic strain fields respectively at 5% total strain. Strong strain localisation was predicted by the model as indicated by the red arrow in Figure (b), with the maximum value of plastic strain reaching 16%, which is 3 times in excess of the macroscopic average strain (5%). The statistical distributions of the elastic and plastic strain components in the loading direction at the tensile strain of 5% are depicted in Figure (g). It is interesting to note that the local elastic strain conforms to a relatively narrow normal distribution, whereas the statistics of local plastic strain was described by a relatively broad lognormal distribution with a pronounced 'tail' at high strains. Failure in polycrystalline alloys frequently originates in highly localised fields where intergranular and intragranular strains and stresses reach large values. Our previous publication [7] highlighted the importance of the emergence of a "tail" of large size defects in defect size statistics during deformation evolution. The appearance of the large size "tail" was proposed as a criterion for the identification of the transition from micromechanical to macromechanical damage regime, i.e. from microcracking to large crack propagation phenomena that ultimately lead to failure. In the current work, we focused our attention on the evolution of *inelastic strain* at the early stage of material deformation process. Strain localization is an important phenomenon for initiation-dominant failure in polycrystals and precedes microcrack formation. Strong strain localisation corresponds to the long 'tail' in the lognormal strain distribution. We proposed that the fitting parameters (mean and standard deviation) of lognormal plastic strain statistics for a particular polycrystal determine the shape of long "tail" that ultimately will allow predicting the microcrack formation and the limit of structural integrity.

The evolution of elastic and plastic strain histograms at different deformation stages during tensile loading are presented in Figure (d-h). At a total strain of 0.1% (Figure (d)), the deformation is purely elastic, and the elastic strain follows a normal distribution. In the elastic-plastic transition region (Figure (e)), plastic slip gradually accumulates with the progressive activation of crystal slip systems, and the magnitude of plastic strain becomes comparable to that of the elastic strain. It is important to note that in this regime the statistics of elastic strain cannot be fitted by a normal function, indicating the strong interaction between elastic strain and the occurrence of plastic flow. In the plastic strain dominant region, Figures (f-h), the elastic strain is observed to conform to a normal distribution again, whilst plastic strain follows a lognormal distribution. Additionally, the lognormal distribution curve for plastic strain widens as the deformation occurs. This suggests that strain

becomes progressively localised and strain heterogeneity increases as deformation proceeds.

From the statistical point of view, a normal distribution reflects an additive influence of sources of variation [8]. On the contrary, a lognormal distribution emerges due to the multiplicative accumulation of multiple random variables. According to the central limit theorem, the summation of independent random variables tends toward a normal distribution. In the context of incremental deformation processes, a simplifying assumption is that the magnitude of small strain steps can be considered to follow a random distribution (Figure (i)), then a normal distribution of elastic strain arises after several small increments and is preserved during subsequent deformation (Figure (j)). When larger plastic strain is considered, small strain assumption no longer holds and multiplicative accumulation needs to be applied, leading to a lognormal distribution illustrated in Figure (k). The normal statistics of elastic strain and the lognormal distribution of plastic strain indicate that the elastic strain accumulation process is additive, whereas plastic deformation accumulation is a multiplicative process. When the total deformation is large (8–10%), the plastic strain becomes dominant, and hence the total strain appears to follow a lognormal distribution that has been reported in the literature [4,5]. This explains the reason why some researchers reported the total strain distribution closely obeying a lognormal distribution. We therefore argue that our findings agree with the literature reports, but go much further in explaining the fundamental mechanisms of strain accumulation. It is important to note that stress is directly related to *elastic* strain through generalised Hooke's law, explaining the apparent striking difference between the statistics of strain showing lognormal distribution, while stress follows normal distribution.

The distribution of elastic strain within crystals is known to be closely associated with diffraction line broadening. The histogram of grain-resolved elastic strains was found to follow a Gaussian shape for polycrystalline copper using X-ray diffraction [9], providing experimental evidence for the finding in this study. Due to the stress-strain relationship obeying generalised Hooke's law, this provides experimental evidence for the total stress also following a normal distribution. In polycrystals, the nature of plastic deformation at the microscale is governed by the generation of dislocations and the movement of existing dislocations. Experimental observation indicates that dislocation density distributions during tensile deformation can be described with a lognormal function [10]. Furthermore, significant dislocation accumulating in the vicinity of grain boundaries and triple junctions leads to strong strain localisation and pre-determines failure in these regions. It is important to note that plastic strain does not increase uniformly as deformation proceeds. Active slip systems accumulate more plastic strain than other slip systems with the total strain increase – it has been noted that slip activity on a given system leads to softening of other slip systems as reflected in negative lateral hardening [11]. Softening is the mechanism causing strain localisation and multiplicative accumulation resulting in the emergence of lognormal distribution of local plastic strain. The inhomogeneity of plastic deformation becomes progressively exaggerated as deformation proceeds with local areas of large strain continuing to deform significantly [4].

In summary, the considerations presented here use statistical analysis of the evolution of strain distributions during tensile deformation using crystal plasticity finite element method, and correlate them with experimental observations of the elastic and plastic strains. The small elastic strain (and stress) accumulation is found to be additive leading to a normal distribution, whereas larger magnitude plastic strain can be well described by a lognormal distribution and obeys a multiplicative accumulation process. The origins of local strain and stress statistics have been revealed fundamentally. We hypothesize that this finding is universal for polycrystalline materials with various deformation mechanisms and phase content when the volume of interest is above the minimum size threshold for a representative volume element that reflects the macroscopic homogeneity of deformation. The outcome

obtained from this observation is expected to provide useful insight into the relationship between polycrystalline microstructure and mechanical properties.

Data availability statement

The raw/processed data required to reproduce these findings cannot be shared at this time as the data also forms part of an ongoing study.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

References

- [1] A. Musienko, A. Tatschl, K. Schmidegg, O. Kolednik, R. Pippan, G. Cailletaud, Three-dimensional finite element simulation of a polycrystalline copper specimen, *Acta Mater.* 55 (2007) 4121–4136, <https://doi.org/10.1016/j.actamat.2007.01.053>.
- [2] M. Kamaya, Y. Kawamura, T. Kitamura, Three-dimensional local stress analysis on grain boundaries in polycrystalline material, *Int. J. Solids Struct.* 44 (2007) 3267–3277, <https://doi.org/10.1016/j.jisstr.2006.09.020>.
- [3] J. Chen, E. Salvati, F. Uzun, C. Papadaki, Z. Wang, J. Everaerts, A.M. Korsunsky, An experimental and numerical analysis of residual stresses in a TIG weldment of a single crystal nickel-base superalloy, *J. Manuf. Process.* 53 (2020) 190–200, <https://doi.org/10.1016/j.jmapro.2020.02.007>.
- [4] M. Tanaka, Y. Yoshimi, K. Higashida, T. Shimokawa, T. Ohashi, A multiscale approach for the deformation mechanism in pearlite microstructure: experimental measurements of strain distribution using a novel technique of precision markers, *Mater. Sci. Eng. A* 590 (2014) 37–43, <https://doi.org/10.1016/j.msea.2013.09.072>.
- [5] A. Tang, H. Liu, G. Liu, Y. Zhong, L. Wang, Q. Lu, J. Wang, Y. Shen, Lognormal distribution of local strain: a universal law of plastic deformation in material, *Phys. Rev. Lett.* 124 (2020) 1–6, <https://doi.org/10.1103/PhysRevLett.124.155501>.
- [6] A.M. Korsunsky, D. Dini, F.P.E. Dunne, M.J. Walsh, Comparative assessment of dissipated energy and other fatigue criteria, *Int. J. Fatigue* 29 (2007) 1990–1995, <https://doi.org/10.1016/j.ijfatigue.2007.01.007>.
- [7] A.M. Korsunsky, K. Kim, L.R. Botvina, An analysis of defect size evolution, *Int. J. Fract.* 128 (2004) 139–145, <https://doi.org/10.1023/B:FRAC.0000040976.04622.bb>.
- [8] GraphPad, Origin of the Gaussian Distribution, https://www.graphpad.com/guides/prism/7/statistics/origin_of_the_gaussian_distribution.htm 2020 (accessed 6 October 2020).
- [9] J. Oddershede, S. Schmidt, H.F. Poulsen, L. Margulies, J. Wright, M. Mosciacki, W. Reimers, G. Winther, Grain-resolved elastic strains in deformed copper measured by three-dimensional X-ray diffraction, *Mater. Charact.* 62 (2011) 651–660, <https://doi.org/10.1016/j.matchar.2011.04.020>.
- [10] J. Jiang, T.B. Britton, A.J. Wilkinson, Evolution of dislocation density distributions in copper during tensile deformation, *Acta Mater.* 61 (2013) 7227–7239, <https://doi.org/10.1016/j.actamat.2013.08.027>.
- [11] A. Manonukul, F.P.E. Dunne, High- and low-cycle fatigue crack initiation using polycrystal plasticity, *Proc. R. Soc. A Math. Phys. Eng. Sci.* 460 (2004) 1881–1903, <https://doi.org/10.1098/rspa.2003.1258>.