

The Metaphysics of Properties and Relations

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To my parents
and my sister

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Abstract

This thesis defends heterodox views on central topics in the metaphysics of properties and relations. The first four chapters develop stances on traditional problems. One is the relation regress, which derives from the intuitive view that related things are related because they instantiate a relation. Chapter 1 argues that no extant solution to the regress is adequate. The next two chapters turn to questions that figure in both contemporary and ancient discussions. Chapter 2 considers the question whether properties are either immanent or transcendent and defends a third view: not all properties are immanent, although some may be and others not. Chapter 3 argues that three prominent accounts of the universal-particular distinction associated with Plato and Aristotle are inadequate. Chapter 4 approaches the metaphysics of properties from the perspective of their relation to objects. I reject the commonly held view that properties are constituents of their bearers and raise further arguments against two theories of objects that assume it: substratum theory and bundle theory. The remaining chapters revisit themes from the earlier metaphysical chapters in the context of higher-order logic. In response to the Russellian paradox for properties, Chapter 5 defends a view I call ‘higher-order nominalism’, which says that generalisations involving properties and relations ought to be formalised using higher-order quantification into predicate position. The view denies a tacit assumption of most contemporary work in the metaphysics of properties and relations – that properties and relations are things, in the Fregean sense that they lie in the range of the first-order quantifiers. Chapter 6 develops the implications of this paradigm shift for two earlier topics: the immanence-transcendence debate and the universal-particular distinction. In contrast to existing views, I argue that higher-order logic does not solve these issues. Finally, Chapter 7 argues that some higher-order quantification is expressible in English.

Introduction

Properties are to 1-place predicates as relations are to n -place predicates, for $n > 1$. For the most part, this slogan animates the approach to the metaphysics of properties and relations in this thesis. The slogan may be strained by the fact that the concept of direction or order is, at least on the face of it, sensibly applied to relations, though not to properties. But there are many points of analogy, and the problems of which this thesis treats have a general application to both properties and relations.¹

In philosophy, one finds both the conventions that a ‘property’ is a 1-place relation and that a ‘relation’ is an n -place property, for $n > 1$. Both conventions are upheld in various chapters of the thesis. Some problems, like the relation regress, have historically been presented in terms of relations; others, like the property paradox, in terms of properties. But they may be understood as instances or versions of more general problems, relevant to the metaphysics of both properties and relations. Thus they are understood here – though for the sake of continuity with the existing literature, the terminology of the original problems has been preserved. Accordingly, where relations are foregrounded, properties are understood to be a special case of them, of arity $n = 1$. Where properties are the focus, relations are identified as a special case of relations, of arity $n > 1$.

The relation regress derives from a traditional and intuitive explanation of relatedness, according to which related things are related because they instantiate a relation. The

¹ There is also a natural extension of this slogan: properties are to 0-place predicates (i.e. sentences) as properties are to 1-place predicates and relations are to n -place predicates, for $n > 1$. But the analogy is further strained by the fact that propositions are, inter alia, the objects of the propositional attitudes, like belief. Though some of the issues of which this thesis treats have a natural extension to propositions also, they will not be foregrounded here.

regress will be familiar to those familiar with Bradley's regress of relations, though the more anodyne label 'the relation regress' has been preferred, to sidestep issues of Bradleyan exegesis. The regress is held by many philosophers to be the central problem in the metaphysics of properties and relations. Its influence may be seen in the number and variety of solutions it has inspired. The regress has been used in arguments for the existence of God, alternative theories of meaning, alternative theories of properties and relations, and non-classical systems of logic, which allow, e.g., for true contradictions and failures in the transitivity of identity. Bradley himself took this or a similar regress to motivate monism.

Despite its prominence, a systematic comparison and evaluation of the various answers to the regress remains to be given. Chapter 1 aims to fill that lacuna and thus establish the dialectic on the regress. It presents a range of views and explains exactly how they attempt to solve the regress and why they are ultimately inadequate. The conclusion to which this exercise trends is that the relevant notion of explanation is indeterminate, and that there are therefore insufficient grounds to adjudicate between a few remaining solutions – for example, that relatedness has no explanation or that relatedness has an explanation, though it does not terminate. Progress must be bought here at the expense of general appeal. A theory of explanation may supply reasons to adjudicate between these solutions, but unless the theory is uncontroversial, it will spoil the intuitive appeal of the view that related things are related because they instantiate a relation. Current debates on forms of explanation, such as grounding, truthmaking, and conceptual analysis, suggest that the theory will not be uncontroversial. In that case, the relation regress will cease to be a problem of general philosophy. That may be a welcome consequence. The bottom line here is that there are hard questions about explanation that need answering.

The next three chapters develop stances on three traditional debates in the metaphysics of properties. Chapter 2 considers the question whether properties are immanent or transcendent. The question has an ancient pedigree. Plato held a theory of forms on which they were ‘separate from’ their instances, and in this transcendent; Aristotle criticised this theory, and held instead that all forms were ‘in’ their instances, and in this sense immanent. Today one finds several distinct accounts of these notions in terms, respectively, of metaphysical dependence, parthood, or location, though a common thread to each of them is that the existence of an immanent property is predicated on the existence of its bearers, though the existence of a transcendent property is not. Here the account in terms of location is taken. A property is immanent if and only if it is not located in space and time, and a property is transcendent if and only if it is located in space and time. It is further held that a property is located in space and time if and only if it is located wherever it is instantiated. It follows, as on other accounts of immanence, that properties are immanent only if there are no uninstantiated properties.

There are a couple reasons for taking the location account. First, it has come to be standard in contemporary debates on immanence and transcendence. Second, and relatedly, it is the received account in higher-order metaphysics, which forms the topic of Chapters 5 to 6. With that said, the argument of Chapter 2 generalises to other accounts of immanence and transcendence, insofar as the crux of this argument is that uninstantiated properties in addition to instantiated properties are needed for at least some of the intended applications of property theory in metaphysics and semantics. The rest of the chapter criticises prominent arguments against transcendence, though it also observes that there are no obvious reasons to prefer a theory of transcendent properties over a hybrid theory, say, on

which instantiated properties are immanent, and uninstantiated properties transcendent. Thus the chapter concludes against immanence, and neither for nor against transcendence.

There is an intuitive distinction between properties and objects like ourselves, tables, and chairs, which has traditionally been characterised as the distinction between universals and particulars. But absent an account of what universals and particulars are, the universal-particular distinction appears to do no work that is not done by the anterior property-object distinction. The need for an account is especially salient for trope theory, which is defined by the premise that properties are particular as opposed to universal. Chapter 3 introduces three standard accounts of the universal-particular distinction, along with their defects. As it stands, no account of the distinction has ever carried conviction. However, it is granted that a necessary, if not sufficient, condition for being a trope is that it cannot be had by more than one thing at a time. With this datum in hand, a range of standard arguments for trope theory are criticised.

The previous two chapters approached the metaphysics of properties by considering two questions about what properties are like. Chapter 4 approaches the topic from the perspective of their relation to objects. On a traditional conception of the relation between properties and objects, properties are ‘parts’ or ‘constituents’ of their bearers. Objects, in turn, are seen as ‘wholes’ or ‘complexes’ whose decomposition into more basic parts it is the task of a theory of objects to describe. There are two prominent theories in this tradition: substratum theory and bundle theory. They differ insofar as substratum theory holds that the parts of an object include something in addition to its properties, this being the underlying ‘substratum’ in which the properties inhere. This is denied by bundle theory, according to which objects are mere ‘bundles’ of properties.

The chapter first raises problems for the view that the relation between properties and objects can be understood in terms of mereology. The relation in which properties stand to their bearers cannot be identified with the familiar relation of parthood in which an arm stands to a body, for they differ in their logical properties. The former is neither reflexive nor transitive, though the latter is both reflexive and transitive. These preliminary observations cast into doubt the conceptual framework assumed by both substratum theory and bundle theory. Nevertheless, the rest of the chapter considers arguments for and against both theories, on their terms. It is seen that without this conceptual framework, there is little to motivate either theory. Substratum theory is further mired by complications concerning its central premise that a substratum is something that bears properties, though it lacks any properties ‘in itself’. Bundle theory, in turn, was historically developed in reaction to substratum theory: rejecting the notion of a substratum, it retains the idea that objects have a decomposition into more basic parts, these being the object’s properties. Some problems associated with bundle theory are examined, the most prominent of them being the argument from the possibility of distinct indiscernible objects – a possibility which bundle theory appears to be unable to recognise.

The analogy stated in the opening of this introduction suggests the following comprehension principle: for every predicate or open formula defined for the language, there is a property (or relation) with the same extension. The principle is said to be ‘naïve’, both because it reflects our intuitive, untutored conception of properties, but also because it leads to a paradox, by an argument analogous to Russell’s paradox for sets. Since its discovery, the obvious structural analogies between the property paradox and the set paradox have fostered the expectation that a solution to the former may be ‘adapted’ from a solution to the latter, on which work is, by comparison, more advanced. Chapter 5 puts

flesh on this idea with reference to the standard solution to the set paradox, as it is embodied in contemporary set theory. Two ways of adapting contemporary set theory to a solution to the property paradox are considered. The first, set nominalism, identifies properties with sets. The second formulates property-theoretic analogues of the set-theoretic axioms that give expression to the iterative conception of sets that, arguably, motivates contemporary set theory. Both are ultimately rejected. Properties and sets are wanted for different kinds of work, which place different formal and informal desiderata on property theory and set theory.

Instead, a higher-order solution to the property paradox is defended, on which the work of first-order quantification over properties is done instead by higher-order quantification into predicate position. The solution is intuitive, given the close connection between properties and predicates described by the natural language equivalence between sentences of the form ‘ x is F ’ and ‘ x has the property of being F ’. It is also theoretically well-motivated. It satisfies higher-order analogues of three key property-theoretic desiderata: (1) non-extensionality, (2) plenitude, in the sense that there is at least a property corresponding to every predicate, and (3) closure under Boolean operations. The use of higher-order logic in this context is also inferentially integrated with developments in other areas of inquiry. Higher-order logic has been applied to the formalisation of mathematical theories and concepts, to theories of logical consequence for languages with unrestricted quantifiers, and more. To palliate post-Quinean suspicion of any but first-order quantification, three arguments for higher-order logic are described in this chapter: the arguments from natural language, arithmetic, and set theory. A note on type theory, which describes the syntax of higher-order formal languages, concludes the chapter.

The higher-order resolution of the property paradox is consonant with higher-order metaphysics – an approach to metaphysics characterised by the application of higher-order logic to problems in metaphysics. It has seen increasing traction in recent years, though current views on higher-order metaphysics cluster around two poles: either higher-order logic has no significance for metaphysics, or it solves (or dissolves) the central problems in the metaphysics of properties and relations. Against these extremes, a moderate view is taken here. Higher-order logic solves some problems in the metaphysics of properties and relations, such as the property paradox, but not all.

In support of this view, Chapter 6 returns to the immanence-transcendence debate and the universal-particular distinction, which formed the topics of Chapters 2 and 3, respectively. Contrary to existing views, it argues that higher-order logic does not solve or dissolve these issues. True, if the work of quantification over properties is done instead by higher-order quantification into predicate position, the questions whether properties are immanent or transcendent and whether they are universal or particular are made redundant, for they employ quantification over properties. But any nominalist theory that rejected the existence of properties would have the same consequence. Higher-order resources are of no special significance in this regard. Moreover, the same resources allow the formulation of higher-order analogues of these questions, whose resolutions turn on considerations that are themselves analogous to those that arise in the original, first-order debates. Thus the turn to higher-order logic does not exonerate us from the first-order debates.

With that said, the applications of higher-order logic in this thesis share an important premise with other views in higher-order metaphysics. On the intended interpretation of the higher-order quantifiers, they are *sui generis*, neither objectual nor substitutional.

Formal semantics for such languages have been formulated that are, arguably, faithful to this interpretation. But insofar as such semantics themselves employ higher-order quantifiers, they leave open an interpretative question. Namely, how are we to understand the higher-order quantifiers?

An obvious method for learning any language, formal or informal, is by translation into a more familiar language. For example, on the usual translation scheme of the language of first-order logic into English, the first-order quantifiers $\exists x$ and $\forall x$ translate to ‘something’ and ‘everything’, respectively. Let F stand for ‘... is French’; $\exists x Fx$ thus translates to ‘Something is French’. By applying this translation scheme to a range of more or less simple sentences of first-order logic, we come to understand the first-order quantifiers. The trouble is that there are no generally accepted translation schemes for the languages of higher-order logic. Though there have been many attempts at capturing higher-order constructions in natural language, and English in particular, they are generally taken to be unsuccessful.

The intelligibility of a language cannot be contingent on the existence of translation schemes for the language into an already understood language; that would lead to an infinite regress. In some cases, we learn by direct method, and there is no reason, in principle, that the same method cannot be applied to the languages of higher-order logic. But despite this, the lack of translation schemes for higher-order languages has promulgated skepticism that such languages are intelligible. Chapter 7 thus renews the attempt to capture higher-order constructions in English, with a focus on second-order constructions in particular. A translation scheme for the language of second-order logic is defended that satisfies two desiderata: it is faithful to the intended interpretation of the

language, and it is general, in the (avowedly contextual) sense that it captures at least those sentences that are structurally analogous to the first-order sentences captured by the usual first-order translation scheme. If the first-order scheme is in good standing, then the second-order scheme ought to be, too.

This thesis contains 48,303 words, exclusive of bibliographic references.

Chapter 1

Related/Relation

1 The relation regress

‘Relation’ is the colloquial expression, but it is ambiguous. It can refer to the thing that relates, to the state of being related, or, in some cases, to the thing that is related – as in ‘friends and relations’. In this chapter, I reserve ‘relation’ for the thing that relates, and use ‘relatedness’ for the state of being related. The third sense of ‘relation’ will be irrelevant, but, if so desired, it may also be naturally disambiguated – as ‘members of family’.

Traditionally, relatedness has been explained in terms of the instantiation or holding of relations:

Explanation of Relatedness

If any things are related, then they are related
because they instantiate a relation.

$$\forall x_1 \dots \forall x_n (\rho x_1 \dots x_n \rightarrow \exists y (\rho x_1 \dots x_n \succ I x_1 \dots x_n y)),$$

where ρ is a schematic predicate letter whose substitution instances are any predicate, including complex predicates formed by λ -abstraction.² Thus formalised, Explanation of Relatedness (ER) is a schema, of which there is one instance for each polyadic predicate, simple or complex, defined for the language. I stands for ‘... instantiate(s) ...’, and n may be any natural number.

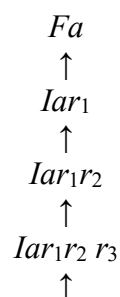
² For an introduction to λ -abstraction, see Bacon (2024: 50-5). Briefly, the symbol λ is used to form complex predicates from sentences. If $x_1, \dots, x_n$ are variables, and Φ is a sentence, then $\lambda x_1 \dots x_n. \Phi$ is an n -place predicate. $\lambda x. \Phi$ may be read ‘is an x such that Φ ’. Such predicates are called ‘ λ -abstracts’. Thus $\lambda xy. Lxyab$ may be read ‘Achilles and Briseis are such that he loves her’ (with the obvious dictionary).

It is standardly assumed that whatever it is in virtue of which related things are related is a ‘relation’, but this assumption is not essential to ER and is in fact rejected by some authors, for which reason it is suppressed in the formalisation.

As is well known, instances of ER lead to an infinite regress. Consider an arbitrary case of relatedness: a is related to b . By ER, there is a relation r_1 such that: a is related to b because a and b instantiate r_1 . But being instantiated by some things is a way of being related to them, and, since the aim is to explain relatedness, an explanation is now wanted of a and b ’s instantiating r_1 . By parity of reasoning, there must be a relation r_2 such that: a and b instantiate r_1 because a , b , and r_1 instantiate r_2 . Plainly, this is the start of an infinite regress.

In other words, the aim is to explain relatedness, which is expressed by statements of the form $\rho x_1 \dots x_n$, for any n . Since the explanations given by instances of ER are themselves of this form, these explanations also express relatedness that is to be explained. And so an infinite regress ensues.

An instance of the regress is pictorially represented here. The simplest case is taken, where ρ is a simple monadic predicate. Note that $\uparrow$ represents ‘because’, and the arrow points away from the explanans to the explanandum.



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The relation regress has exercised an outsized influence on metaphysics, logic, and the philosophy of language, as witnessed by the number and variety of views it has inspired. To take but a few examples, it has been used in arguments for the existence of God, dialetheism and the failure of the transitivity of identity, and alternative theories of properties and relations. Bradley (1893), with whom the regress is often associated, himself took this or a similar regress to motivate monism, or the view that there is just one thing.³

To my knowledge, however, there has yet to be a systematic comparison and evaluation of the various answers to the regress. This chapter aims to fill that lacuna and thus establish the dialectic on the regress. It presents a range of views and explains exactly how they attempt to solve the regress and why they are ultimately inadequate. The solutions are organised according to the stances they adopt on ER. Section 2 criticises views on which ER has no true instances. Section 3 criticises views on which ER has some true and some false instances. And section 4 criticises views on which ER has only true instances. Section 5 concludes the chapter with a question: what is the relevant notion of explanation?

³ All of these arguments are discussed below. Bradley (1893) has historically formed the point of entry into this topic, though he also takes up the regress in his (1883) and (1911). Some degree of rational reconstruction is needed in order to extract the relation regress as it is presented here from his writings. Bradleyan exegesis not being of especial concern here, I have abstained from using the title ‘Bradley’s regress’. Other authors have taken up the exegetical issues. Perovic (2014), for example, attempts to reconstruct Bradley’s ‘original’ regress and ultimately identifies three in his (1893). None of them is what I call the ‘relation regress’ here.

2 Rejection

On one salient solution to the regress, ER has no true instances: ER is, in this sense, invalid. In this category falls the popular view that higher-order logic solves or dissolves the regress; its proponents reject all instances of ER.⁴ However, given the intuitive appeal of ER, and the fact that there are solutions to the regress that allow that it has at least some true instances, some account is naturally sought of the invalidity of ER. There are two possible accounts. Either relatedness has no explanation, or it has an explanation, though it is not captured by ER. The trouble is that it is not clear to which of these two alternatives the higher-order solution is committed.

Let me flesh out the solution. A formal introduction to higher-order logic may be found in Shapiro (1991) or Bacon (2024). Briefly, where first-order quantifiers bind variables in singular term position, higher-order quantifiers bind variables in other syntactic positions. Second-order quantifiers, in particular, bind variables in predicate position. The distinction between first- and second-order quantification may be illustrated by the pair: $\exists x Fx$ and $\exists X Xa$. In $\exists x Fx$, the variable x occupies the position of a singular term, which is to say that only a singular term may occur in the position occupied by x in Fx to form a grammatical sentence. Fa is grammatical, where FG , say, is not. In $\exists X Xa$, the variable X occupies the position of a predicate.

Now recall the diagnosis of the regress in section 1. An explanation of relatedness in terms of the instantiation of relations introduces yet more relatedness to be explained. That is, any way of describing how a relation ‘stands to’ its relata will employ relational

⁴ See Frege (1892: 54-5), Russell (1927: 252-3), Dummett (1981: 174-6), and Trueman (2021: 129-37), though as Jones (2023) points out, Trueman’s solution does not essentially appeal to higher-order logic.

vocabulary, and thus re-introduce the same phenomenon of relatedness that was to be explained.

However, this assumes that relations lie in the range of the first-order quantifiers. (The formalisation of ER makes this explicit.) If relations lie in the range of the first-order quantifiers, then they are denoted by singular terms; for a first-order quantifier is one that quantifies into singular term position. And if relations are denoted by singular terms, then a predicate is needed to relate a term for a relation with terms for its relata in order to yield a grammatical sentence. Predicates that do the job include: ‘instantiates’, ‘holds of’, ‘obtains between’, and ‘relates’.

By contrast, if relations do not lie in the range of the first-order quantifiers, but rather in the range of the higher-order quantifiers, matters appear different. In this case, a predicate is not needed to describe how a relation stands to its terms. Compare: $\exists x Iabx$ and $\exists X Xab$. Concatenation does the work of a relational predicate of instantiation. Thus, it is claimed, the regress is solved: ‘The word for a relation does have to be related to the words for its terms. But this is a linguistic, not a metaphysical, fact, and the regress does not have to go any further’ (Russell 1927: 252-3).

Not so fast. Granted that a word for a relation does need not be related to the words for its relata, it does not follow that the relation is not related to its relata – or that there is no explanation of this phenomenon. True, if relations cannot be denoted by singular terms and thus cannot lie in the range of the first-order quantifiers, then ER is invalid. But this consequence does not dissolve or address the theoretical motivation underlying ER: to explain relatedness. Furthermore, the resources of higher-order logic can be used to

formulate alternative explanations of relatedness that do not employ first-order quantification over relations, and these explanations equally lead to an infinite regress.

To show this, second-, third-, and fourth-order analogues of ER are formulated here. Still higher-order analogues of ER may be inferred on a similar pattern. Note that superscripts on predicate letters and variables indicate their level. In particular, a *first-level* predicate is a predicate of singular terms; a *second-level* predicate is a predicate of first-level predicates; a *third-level* predicate is a predicate of second-level predicates; and so on.⁵

Explanation 2 $\forall x_1 \dots \forall x_n (\rho^1 x_1 \dots x_n \rightarrow \exists X^2 (\rho^1 x_1 \dots x_n > X^2 \rho^1 x_1 \dots x_n))$

Explanation 3 $\forall X^1 \forall x_1 \dots \forall x_n (\rho^2 X^1 x_1 \dots x_n \rightarrow \exists X^3 (\rho^2 X^1 x_1 \dots x_n > X^3 \rho^2 X^1 x_1 \dots x_n))$

Explanation 4 $\forall X^2 \forall X^1 \forall x_1 \dots \forall x_n (\rho^3 X^2 X^1 x_1 \dots x_n \rightarrow$
 $\exists X^4 (\rho^3 X^2 X^1 x_1 \dots x_n > X^4 \rho^3 X^2 X^1 x_1 \dots x_n))$

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Given such higher-order analogues of ER, a regress ensues, of which an instance is:

$$\begin{array}{c} F^1 a \\ \uparrow \\ I^2 F^1 a \\ \uparrow \\ I^3 I^2 F^1 a \\ \uparrow \\ I^4 I^3 I^2 F^1 a \\ \uparrow \end{array}$$

⁵ Chapter 5 section 5 gives a more rigorous exposition of the assignment of levels in relational type theory, which describes the syntax of higher-order formal languages.

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If proponents of the higher-order solution to the relation regress accept Explanation 2, Explanation 3, and so on, then they face a higher-order analogue of the relation regress. If they reject Explanation 2, Explanation 3, and so on, they owe us an account of the alleged invalidity of these higher-order analogues. There are two possible accounts. Either there is no explanation of relatedness, or there is an explanation of relatedness, though it is not given by higher-order analogues of ER. To the defence of neither alternative do higher-order resources appear relevant.

It may further be observed that even in a higher-order framework, Russell (*op. cit.*)'s claim that relations, unlike the words by which they are expressed, do not have to be related to their terms is not obviously true.

In a higher-order language with λ -abstraction, relatedness may be expressed as follows:

$\lambda Xx_1 \dots x_n. Xx_1 \dots x_n$, for any n . (Where $n = 1$, this yields the standard higher-order expression of instantiation: $\lambda Xx. Xx$.) Now a natural principle governing λ -terms is:

Extensional β -equivalence $\lambda y_1 \dots y_n. \phi a_1 \dots a_n \leftrightarrow \phi a_1/y_1 \dots a_n/y_n$,

where $y_1, \dots, y_n$ are any distinct variables, $a_1, \dots, a_n$ are any terms, and $\phi a_1/y_1 \dots a_n/y_n$ is a sentence that results from replacing each free occurrence of y_i in ϕ with an occurrence of the corresponding a_i in such a way that no free variables in any a_i become bound. Each a_i and y_i may be of any syntactic type, including (but not limited to) the types of predicates and singular terms. Thus an instance of Extensional β -equivalence is $\lambda Xxy. Xxy(Rab) \leftrightarrow$

Rab , which implies $\neg\lambda Xxy.Xxy(Rab) \rightarrow \neg Rab$.⁶ This is a sense in which R must be related to its terms. Since R was arbitrary, the argument generalises to other relations.

Let me therefore set aside the idea that higher-order logic solves the regress and return to the idea put forward at the start of this section: ER has no true instances. In the absence of any other salient, non-regressive explanations of relatedness, it appears that the path of least resistance is to hold that ER has no true instances because there is no explanation of relatedness to be had; relatedness is, in this sense, primitive. But how to defend this?

There is an obvious reason to hold that relatedness has no explanation: it stops the regress. The problem is that the same rationale justifies other, mutually incompatible, solutions to the regress. For example, a solution that restricts the range of substitution instances for ER to exclude predicates, like ‘instantiates’, that describe how relations ‘stand to’ their terms also stops the regress. In fact, such solutions embody the most prominent line of response to the regress (section 3). In this dialectical context, further argument is needed to support the view that relatedness is primitive, and that it is for this reason ER is invalid.

For example, it may be argued that the rejection of all instances of ER, as opposed to only some, yields a more unified theory of relatedness. It also avoids the introduction of relations and is, in this respect, ontologically economical. But to this, it may be replied that a theory that permits at least some instances of ER explains more than a theory that does not and is thus to be preferred for its explanatory power. Moreover, if relations do

⁶ Parentheses are interpolated in λ -terms for readability. For discussion (and defence) of Extensional β -equivalence and related principles for λ -terms, see Dorr (2016: 52-8) and Bacon (2024: 57-86). As Dorr points out, Extensional β -equivalence is uncontroversial in extensional contexts.

exist – if there are other reasons to postulate relations – then considerations of ontological economy mean little.

Without pretending to have reached a resolution of these issues, I move on to another category of solutions to the regress in order to fill out the picture.

3 Restrictions

As mentioned, the most prominent solutions to the regress allow that some, but not all, instances of ER are false. In particular, they allow that there is a property, e.g., such that: Socrates is wise because he instantiates it. But they deny that there is a relation (i.e. a relation of instantiation) such that: Socrates instantiates wisdom because Socrates and wisdom instantiate it. A pictorial representation of this view is as follows:

Socrates is wise.
↑
Socrates instantiates wisdom.

In terms of ER, this means that the range of substitution instances for the schematic predicate letter p is restricted to exclude all and only those predicates that describe how a relation stands to its terms and vice versa, such as ‘instantiates’, ‘holds of’, ‘obtains between’, or ‘relates’.

The challenge for such solutions is to justify the stated restrictions to ER. Crucially, the challenge cannot be met with the observation that the restrictions would stop the regress, for the same rationale justifies other, mutually incompatible solutions to the regress, such as those that reject every instance of ER (section 2).

There have been many attempts to meet this challenge. The most prominent of them appeal to: the tie of instantiation (section 4.1), metaphysical glue (4.2), the metaphor of saturation (4.3), the identification of properties with ways (4.4), non-transferable tropes (4.5), God (4.6), and dialetheism and the non-transitivity of identity (4.7). The rest of this section criticises each of these attempts in turn. It will be seen that most of these solutions suffer from the same general defects, including, but not limited to, a reliance on metaphors or figures of speech.

3.1 The tie of instantiation

Solution. There is no such relation as instantiation. Rather, instantiation is a ‘nexus’ or a ‘tie’ (Johnson 1921: 211-2, Strawson 1959: 167, Bergmann 1967: 9). Sometimes Armstrong also held this view; see his (1978a: 108-11, 1989b: 108-10).

Reply. In the absence of an account of nexuses or ties, the claim that instantiation is a ‘nexus’ or a ‘tie’ amounts to nothing more than a change in label. The claim is also liable to mislead, insofar as it suggests that there is something, i.e. instantiation, that ‘ties’ a relation to its terms, even if it does not ‘relate’ them. That way lies another regress, one formulated in terms of ties and tying rather than relations and relating (Lewis 2002: 7).

Armstrong (1978a: 110) defends the tie of instantiation by comparing it to the ‘formal distinction’ that supposedly distinguishes the members of the Holy Trinity: the Father, the Son, and the Holy Ghost are distinct, but they are not related. For another example, he points to the size and shape of an object. These examples entail further restrictions to ER.

For example, they entail that even if distinct a , b , and c satisfy the predicate $\lambda xyz. (x \text{ is the shape of } z, \text{ and } y \text{ is the size of } z)$, they are not related.

A problem for Armstrong is that the stated exceptions do not generalise into a significant notion of relatedness. Compare the general notion of relatedness assumed at the outset, on which things are related if they satisfy *any* relational predicate, simple or complex. It may well be that on Armstrong's restricted notion of relatedness, relatedness has a non-regressive explanation given by the instances of ER. But unless that notion is significant or bears a significant relationship to the initial, general notion, this consequence does not address the theoretical motivation underlying the original regress.

Armstrong (1978a: 107) also describes instantiation as a 'more intimate union' than relation. To this, van Cleve (1994: 583) replies: 'But there is no union so intimate it ceases to be a relation, except perhaps for identity, and Armstrong does not think the union of universal and particular is a case of identity'.

3.2 Metaphysical glue

Solution. Relations hold of their terms without being related to them by a further relation, just as glue holds things together without being glued to them by some more glue (Grossman 1992: 55-6). See also Broad (1933: 85), who describes instantiation as 'metaphysical glue'.

The glue metaphor may be better appreciated with some historical context. Bradley (1893: 33) had presented the regress as a chain of links:

... we are forced to go on finding new relations without end. The links are united by a link, and this bond of union is a link which also has two ends; and these require each a fresh link to connect them with the old. The problem is to find how the relation can stand to its qualities, and this problem is insoluble.

On the present solution, the regress suggests an incorrect metaphorical conception of relations. A relation is not another link in the chain; it is the glue that holds things together.⁷

Reply. The problem, of course, is that this just trades one metaphor for another. With or without its historical context, the glue metaphor does not justify the claim that there are no relations in virtue of which relations like loving or being taller than hold of their terms.

A better reply to Bradley may be found in Russell, who comments that the chain analogy should arouse suspicion, for it shows, if it is valid, that finite chains are impossible, and yet, they do exist: ‘There is not a word in his argument which would not apply to physical chains’ (1927: 252). A third link is not needed in order to connect two links. The analogy fails. It thus supplies no reason to suppose that another relation is needed to relate a given relation to its terms.

Metaphors aside, the basic idea here is that relations hold of their terms without being related to them by a further relation. But this seems false, even if the substitution instances of ER are restricted to exclude those predicates that describe how a relation stands to its

⁷ See also Blanshard (1984: 215): ‘He [i.e. Bradley] is thinking of a relation as if it were another term, as if *A-R-B* were three beads on a string... But *R* is not the same sort of being as its terms’. Bradley’s analogy brings to mind Leibniz’s claim that relations, if they existed, would be ‘in two subjects, with one leg in one, and the other in the other, which is contrary to the notion of accidents’ (*Fifth Letter to Clark*, 47).

terms (and vice versa). Say that justice is comparable to courage. By ER, there is a relation (i.e. the relation of being comparable to) such that: justice is comparable to courage because justice and courage instantiate it. So it is false that relations hold of their terms without being related to them by a further relation.

There are two moves that can be made in response here. First, it can be held that the substitution of 'is comparable to' in the schema ER yields a false instance of ER. Of course, examples like the above are readily multiplied; there are many other relations that satisfy many other predicates. By parity of reasoning, it must ultimately be held, on this view, that there are many predicates, besides those that describe how a relation stands to its terms, that fall outside the scope of ER. The problem is that the resulting restrictions to ER will not obviously generalise into a significant notion of relatedness (see the criticism of Armstrong in section 4.1).

Rather than restrict the range of substitution instances for ER, an alternative move is to restrict its leading quantifiers $\forall x_1 \dots \forall x_n$, so that no relations lie in their range. On this view, cases of relatedness in which the related things are relations have no explanation, or at any rate no explanation corresponding to instances of ER. But why not? One answer might be that this avoids the introduction of higher-order relations, or relations that hold of relations and properties. But that justification would be plausible only if higher-order relations were in bad standing.

In any case, the glue metaphor does not justify the posited restrictions to ER.

3.3 Unsaturated properties

Solution. Properties are not related to their bearers, for properties are, by nature, ‘unsaturated’, and they are saturated directly by objects (= non-properties) (Armstrong 1980: 447, Lowe 2000: 513-4).⁸

Reply. The solution is often criticised insofar as it supplies no reasons to hold that properties, as opposed to objects, are unsaturated. In at least one respect, properties and objects appear equally unsaturated: only in conjunction with other items do they form propositions (Ramsey 1925: 403, 408, Bergmann 1958: 209, MacBride 2005: 606).

Alternatively, for all that the metaphor of saturation tells us, objects might be unsaturated, and saturated in turn by properties, which are themselves saturated (cf. Dummett 1981: 54-80).

The criticisms are well-taken, if the saturated-unsaturated distinction is intended to yield an account of the object-property distinction. But in the present context, it is beside the point. The metaphor of saturation, whether it is applied to properties or to objects, is just that: a metaphor. It does not justify the claim that properties are not related to their bearers or that there is no relation in virtue of which properties are related to their bearers. Thus it fails to meet the challenge of justifying the posited restrictions to ER.

⁸ The metaphor of saturation originates in Frege (1892: 54), who used it as an attempt to express type distinctions in natural language, and in particular the view that properties (concepts) are not objects. However, contemporary authors who appeal to the metaphor of saturation in this context typically do so without intending to invoke those associations. For Armstrong (*op. cit.*) and Lowe (*op. cit.*), for example, properties are objects, in the Fregean sense that they lie in the range of the first-order quantifiers.

Metaphors aside, the claim that properties are not related to their bearers does not obviously correspond to a significant notion of relatedness (see the criticism of Armstrong in section 4.1).

3.4 Ways

Solution. Properties are ways, and a way something is is not related to the thing that is that way (Levinson 1978: 1-7, Seargent 1985: 110-32, Armstrong 1997: 25, 30-1, Heil 2003: 126, 2012: 12-23, 2021: 41-2, Lowe 2006: 90-1).⁹

Reply. A way something is and the thing that is that way satisfy the relational predicate $\lambda xy. (x \text{ is a way } y \text{ is})$. Given the general notion of relatedness assumed by ER, on which the satisfaction of a relational predicate is sufficient for relatedness, the way and the thing are related. By ER, it follows that there is a relation r such that: the way and the thing that is that way are related because the way and the thing instantiate r . The present solution must therefore be understood to have isolated an alternative notion of relatedness, on which the satisfaction of some relational predicates, like $\lambda xy. (x \text{ is a way } y \text{ is})$, is not sufficient for relatedness. But granted that there is such a notion, it is not obvious that it is significant. And unless this notion is significant, the present solution does not address the theoretical motivation underlying the regress – to explain the general and significant

⁹ Seargent also suggests that Bradley's regress is the product of a category error – the error of treating properties as if they were things and not ways (113). By comparison, Armstrong demurs at the suggestion that ways are not 'entities', though he does 'accept that it is wrong to *substantialize* universals. To think of them as ways that things are or stand to each other is to *de*-substantialize them' (30). All this raises the question what things, entities, and substances are. Neither author can adopt the Fregean convention that a 'thing' ('entity') is whatever is in the range of the first-order quantifiers, on pain of inconsistency.

notion of relatedness assumed by ER. (Again, this is a version of the criticism of Armstrong in section 4.1.)¹⁰

3.5 Non-transferable tropes

Solution. This solution appeals to trope theory, and in particular to the notion of non-transferable tropes (Wieland and Betti 2008, Maurin 2010, 2012, Simons 2010).¹¹ For context, trope theory is the theory that properties exist and are tropes, where a ‘trope’ is a property that is particular, as opposed to universal. The theory thus assumes the cogency of the universal-particular distinction, though it must be said that no account of the distinction has ever carried conviction (see Chapter 3). In any case, it is standardly held that a necessary, if not sufficient, condition for being an n -ary trope is that it cannot be had by more than n things. A one-place trope of redness, for example, cannot be had by more than one red thing; other red things each have their own tropes of redness. A trope is then said to be ‘non-transferable’ if and only if necessarily, if it exists, then it is had by the things by which it is actually had.¹²

¹⁰ The identification of properties with ways can also be challenged. ‘way’, like ‘manner’, is more commonly understood to refer to how something is done or occurs. See Simons (1994: 564-5) for further ‘linguistically motivated’ reservations about the identification of properties with ways.

¹¹ Minor modifications have been made to the versions defended by the cited authors in the interests of clarity, unity, and strength. See the following two notes. The trope-theoretic solution to the regress was earlier raised by Armstrong (1997: 117), though he rejected it since he rejected non-transferable tropes (and also trope theory in general). Perovic (2016) reworks the solution by replacing tropes with Armstrongian states of affairs, which are also particulars as opposed to universals (see Armstrong 1997: 126-7). Her solution is subject to criticisms analogous to those raised below.

¹² Simons (*op. cit.*: 202) gives a different characterisation of non-transferability, according to which non-transferable tropes ‘depend individually on the things they connect’. He uses a clap for an example: ‘that clap could not have existed or come about without those two hands existing’. In other words: necessarily, if the clap exists, then the two hands exist, though the converse does not hold, for the two hands could have existed without ever being joined in that particular clap. However, the view that tropes are non-transferable in Simon’s sense does not entail that necessarily, a given relation trope holds of its terms, though his solution seems to depend on this (see MacBride 2011b: 173). Simons wants something stronger: necessarily, if a clap exists, then it is a clap *of* those two hands. This is in line with the characterisation of non-transferability presented above. That said, neither characterisation helps to answer the regress, for the reasons stated in the following *reply*.

The crux of this solution is that relations are non-transferable tropes.¹³ By definition, it follows that relations necessarily relate the things they actually relate. In other words, a relation could not have failed to relate the things it actually relates. That being the case, a relation does not need to be related to the things it relates by a further relation, in order to ensure that it does relate them. The conclusion is that a relation is not related to its terms by a further relation.

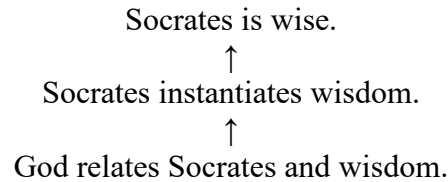
Reply. The conclusion is not implied, and does not appear to be otherwise supported, by the claim that relations do not ‘need’ to be related to their terms or that they are non-transferable tropes. If relations are indeed not related to their terms by a further relation, that would stop the regress; but this stipulation would be subject to the same criticisms of the previous three sections – namely, that it failed to correspond to a significant notion of relatedness. In any case, the apparatus of non-transferable tropes and more generally trope theory are idle wheels here.

3.6 God

Solution. If Socrates is wise, then there is a property (i.e. wisdom) such that: Socrates is wise because he instantiates it. However, it is not the case that there is a relation (i.e. instantiation) such that: Socrates instantiates wisdom because Socrates and wisdom

¹³ To be precise, Maurin does not hold that all relation tropes are non-transferable – only that instantiation tropes are (or compresence tropes, if trope theory is combined with a bundle theory of objects as opposed to a substance or substratum theory). Her reasoning is that if all tropes are non-transferable, then ‘nothing strictly speaking moves or changes’ and ‘nothing could have been other than it actually is’ (2010: 317-21, 2012: 803-4). But there are some missing premises here: the argument, at least as it is presented, is not valid. In any case, the issue is irrelevant in the present context, for which reason it has been elided in the main text. The regress is not solved by holding either that all tropes are non-transferable or that only some are.

instantiate it. Rather, if any things instantiate a property or relation, they do so because God relates the property or relation to its terms (Vallicella 2000, 2002).



According to Vallicella, only God, or a being with ‘the power of contingent self-determination’, can ensure that relations are instantiated without regress. The power must be ‘contingent’, since in many cases, the instantiation of a relation is contingent; and ‘self-determined’, since otherwise something else would be needed to explain the fact that God relates a given relation and its terms.

Reply. An equally rigorous solution would appeal to magic to do the requisite work. In the absence of argument for the claim that there is such a power, or that there is a being in possession of it, the view constitutes no conceptual or explanatory advance over one that simply stipulates that relatedness in general or the instantiation of properties and relations in particular cannot be explained.

3.7 Logic of paradox: Dialetheism and the non-transitivity of identity

Solution. The last solution to be considered in this section is addressed to the unity regress, a regress that derives from a principle analogous to ER:

Explanation of Unity

If some things are unified, then they are unified because something unifies them.

$$\forall x_1 \dots \forall x_n (U_1 x_1 \dots x_n \rightarrow \exists y (U_1 x_1 \dots x_n \succ U_2 x_1 \dots x_n y))$$

U_1 stands for ‘... are unified’, U_2 for ‘... are unified by ...’, and $n > 1$. Assuming that if x_1 , ..., and x_n are unified by y , then x_1 , ..., x_n and y are themselves unified, Explanation of Unity (EU) leads to an infinite regress structurally analogous to the relation regress.

Unity (or ‘unifiedness’) is not relatedness. Things can be related without being unified, in any salient sense. If Achilles dislikes Agamemnon, then they are related, but they are not thus unified. However, on many views, the complex fact that Achilles dislikes Agamemnon is a unity, a whole composed of parts. If this unity could be explained without regress, we might reasonably expect an analogous, non-regressive explanation of relatedness. For this reason, among others, Priest (2014)’s solution to the unity regress is considered here.¹⁴

A Priest (2014: 14) understands it, EU poses a dilemma. Let a ‘gluon’ be whatever it is that constitutes the unity of some thing. On the one hand, a gluon must be a thing, since we can refer to it using singular terms, like ‘it’, and anything we can refer to using a singular term is a thing. (Implicit here is the Fregean convention, noted above, that a ‘thing’ is whatever is in the range of the first-order quantifiers.) On the other hand, if a gluon is a thing, then its unity with the things it unifies must also be explained. Its postulation thus re-creates the problem of unity; it does not solve it. But its postulation must solve the problem, since it has been identified as that which constitutes the unity of a thing. So whatever it is that constitutes the unity of a thing must not be a thing.

¹⁴ Priest (2014: 9-14) himself assimilates the unity regress to Bradley’s regress of relations.

Priest's solution to this dilemma is to take both hands: a gluon both is and is not a thing.

The solution implies dialetheism, or the view that there are true contradictions.

However, this is not yet a solution to the regress. As long as the instances of EU are true, a regress follows – even if those instances are also false.

So the story continues. According to Priest (2014: 16-7), the regress is 'generated by the thought that a gluon is distinct from the things it unifies', for this suggests that 'there is room, as it were, for something to be inserted' between the gluon and these things – in other words, that 'there is a metaphysical space' between them. Priest thus rejects the thought. A gluon is identical with each of the things it unifies. Moreover, it is by being identical to these things that it unifies them.

$$\begin{array}{c} U_1ab \\ \uparrow \\ U_2abg \\ \uparrow \\ g = a \wedge g = b \end{array}$$

The solution implies that identity is not transitive, for the original things a and b are, by stipulation, not identical – thus an explanation is sought for their unity.¹⁵

Reply. This consequence may be, will by many be, taken as a reductio of the view in question. But even with this criticism suspended, it is still not clear that Priest has solved

¹⁵ Both dialetheism and the non-transitivity of identity are features of Priest's Logic of Paradox, which he had earlier employed in an answer to the logical paradoxes, i.e. the set-theoretic paradoxes, such as Russell's, and the semantic paradoxes, such as the liar (see his 1979). For present purposes, we need concern ourselves only with the question whether Priest's answer to the unity regress is correct and can thus be leveraged into a solution to the relation regress.

the regress. For it is not clear that a gluon's non-transitive 'identity' with each of the unified things suffices for unity.

Priest (2014: 17) tries to show that it does suffice by offering a version of the glue analogy. Suppose two bricks are glued together. Clearly, the two bricks do not become one. But according to Priest, the molecules of the glue and the molecules of the bricks 'become physically indissoluble'. And thus he claims that the gluon's terms do not become identical to each other, but 'the gluon bonds with each [term] in the most intimate way, by being identical with it'. (The expression 'gluon' is thus understood to allude, at least in part, to the capacity of its referents to glue.)

The problem is that this is merely metaphor: gluons are not glue. The metaphor itself is vitiated by the fact that the comparison case never obtains: the molecules of some glue and each brick it glues never become physically indissoluble. Just consider the molecules in the interior of the brick or on any sides on which glue has not yet been applied. Even if *some* molecules of the glue and *some* molecules of each brick it glues become physically indissoluble, physical indissolubility is not identity. The analogy yields no traction on the claim that the non-transitive 'identity' between a gluon and each of the things it unifies explains their unity. Priest's metaphorical talk of there being no 'room' or 'metaphysical space' between the gluon and its terms also relies upon our intuitive understanding of identity, which minimally satisfies transitivity. Intuitively, there is 'no room' between x and y , if x is identical to y . But the relation between a gluon and the things it unifies is not identity, as it is intuitively understood.

So Priest has not solved the unity regress. As far as the relation regress is concerned, a non-transitive identity relation is also of no avail. Even if it were supposed that a relation relates its terms by being (non-transitively) identical to them, being identical to some things is a way of being related to them, and by ER, it follows that there is a relation (i.e. identity) such that: the given things are identical because they instantiate it. And so we are off on the regress again. The regress can be stopped only by imposing some restrictions on ER, but such restrictions are not entailed, and do not seem to be otherwise supported, by failures in the transitivity of identity or by dialetheism.

4 Acceptance

A third and final category of solutions to the regress simply accepts it. Each instance of ER is true. Solutions under this heading divide into two categories. In one, the regress is taken as a reductio of the assumption that any things are related (section 5.1). In the other, it is denied that the regress is ‘vicious’, in some sense to be explained. Two accounts under this sub-heading will be examined. The first appeals to the doctrine of the ontological free lunch (5.2). The second says that the instances of ER constitutes adequate explanations of every instance of relatedness (5.3).

4.1 Monism

Solution. If any things are related, there must be an explanation of their relatedness. The salient explanation is given by instances of ER, but they lead to an infinite regress. A regressive explanation of relatedness is an inadequate explanation, but there are no other salient explanations of relatedness. Thus the assumption is false: no things are related.

Moreover, since any two things both exist, and existing together is a way of being related, it follows, by a further reductio, that there are not two things. At most, there exists one thing.

A view like this was held by Bradley (1893), and it has continued to find contemporary proponents, e.g. in Della Rocca (2013, 2020).

Reply. The argument to the conclusion that no things are related assumes that if relatedness cannot be explained without regress, it does not obtain.¹⁶ Yet the assumption is unjustified.

Could it be an instance of a more general principle – namely, that if something cannot be explained without regress, it does not obtain or exist? However, on any theory, some things cannot be explained without either regress or circularity. Suppose, for example, that monism is true. Given ER and any true monadic predication Fa implied by monism, it follows that that there is an r_1 such that: Fa because Iar_1 , where I stands for ‘instantiates’. If r_1 is not a , then monism is false. If r_1 is a , then by ER, there is an r_2 such that: Iar_1 because Iar_1r_2 . If r_2 is not r_1 or a , then monism is false. If r_2 is r_1 is a , then by ER, there is an r_3 such that: Iar_1r_2 because $Iar_1r_2r_3$. And so on, ad infinitum. The regress is avoided only by assuming that there are no true monadic predications implied by this theory or by holding that no such predications correspond to instances of ER. The former is to give up monism, and the latter is to give up the principle that all things admit of explanation without regress – assuming, as above, that there are no other explanations of relatedness.¹⁷

¹⁶ The assumption is made explicit in Della Rocca (2013: 199-207).

¹⁷ Incidentally, Bradley also restricted ER by holding relatedness to imply diversity. If x and y are related, then x is not y . By contraposition, if x is y , x is not related to y . So identity is not something to be explained by instances of ER, since ER concerns the explanation of relatedness. But in the absence of argument, this is

4.2 The ontological free lunch

Solution. Armstrong, who at various points defended the view that properties are tied as opposed to related to their bearers (1978a: 108-11, 1989b: 108-10), that properties are unsaturated (1980: 447), and that they are ways (1997: 30-1), also claimed that, even if a regress was conceded, it would be ‘harmless’, on the basis of his doctrine of the ontological free lunch (1997: 118-9). This doctrine says that what supervenes is no addition of being to what it supervenes upon (1997: 12-3). Since all of the relations in the regress further to the first supervene upon it, they constitute no addition of being. Note that in Armstrong’s sense of supervenience, x supervenes upon y if and only if the existence of y entails the existence of x (1997: 11).¹⁸

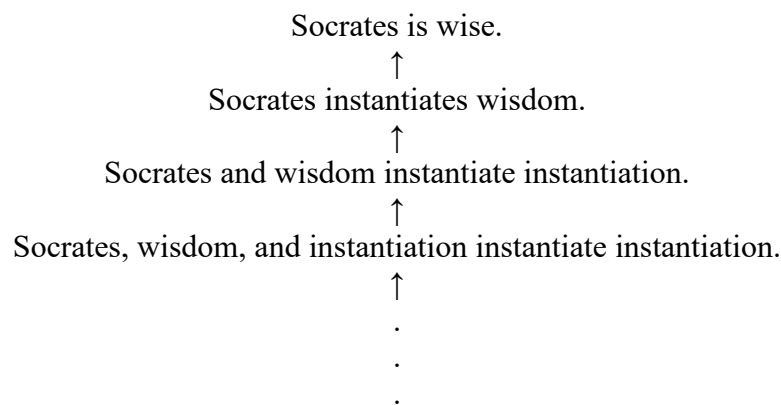
Reply. The crux of the doctrine lies in the idea that there is no ‘cost’ involved in what supervenes upon another. Since the existence of {Socrates} is entailed by the existence of Socrates and so supervenes upon him, there are no problems posed by the existence of {Socrates}: it is just a corollary of the existence of Socrates. However, the doctrine is both unjustified and unintuitive. If God exists, she does so necessarily, so God supervenes upon everything. But God’s supervenience does not imply or seem otherwise to suggest that there is no cost in God’s existence. It does not make theism ontologically harmless.¹⁹

nothing more than a terminological stipulation about the scope of ‘relatedness’. See also Russell (1910: 376)’s reply to Bradley: ‘I can only say that no reason is alleged for holding this view [i.e. that relatedness implies diversity], and that for my part I see no reason to hold it’.

¹⁸ This is not standard. It is more usual to hold, after Davidson (1970: 214), that x supervenes upon y if and only if there could be no difference in x without a difference in y . For theories of supervenience in this vein, see e.g. Kim (1984) or Bader (2012).

¹⁹ By comparison, Rodriguez-Pereyra (2001: 400) puts pressure on the claim that what supervenes is ‘no addition of being’ to what it supervenes upon. As he understands it, x is no addition of being to y only if x is y . But since supervenience does not entail identity, the claim must be false. For a more sympathetic, if still critical, reading of this claim in terms of truthmaker theory, see Lewis (1992: 203).

It is also not clear that the doctrine is relevant in this context, for it is not clear that the harm of the regress consists in its implied additions to ontology. Suppose that Socrates instantiates wisdom. By ER, it follows that Socrates and wisdom instantiate a relation, which we might call ‘instantiation’. By ER again, it follows that Socrates, wisdom, and instantiation instantiate a relation. This relation may be the same relation of instantiation introduced by the previous application of ER. The formulation of ER does not rule out this possibility, and neither, for that matter, do contemporary views on the metaphysics of relations.²⁰ It may thus be the very same relation in virtue of which *any* things instantiate a property or relation.



This possibility, however, does not seem to vindicate the regress.

Ironically, the irrelevance of the doctrine of the ontological free lunch in this context is crystallised by a comparison that Armstrong draws to support his case. Another regress that he claims is ‘harmless’ is the truth regress, which follows from instances of the following schema:

²⁰ Previous philosophers have assumed that relations cannot have a variable number of argument places, but that assumption is no longer standard. After Leonard and Goodman (1940), relations that admit of variable argument places are called ‘multigrade’. On the topic of multigrade relations, see Morton (1975), Mundy (1989), or MacBride (2005: 568-95).

Explanation of Truth

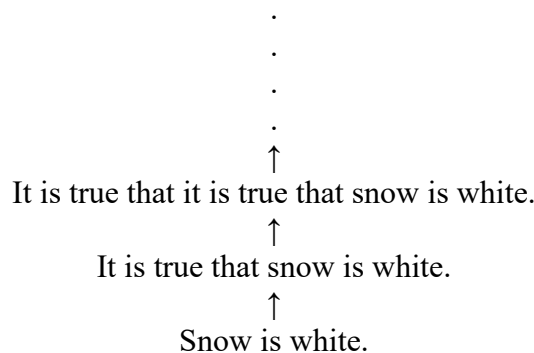
It is true that p because p ,

where p is a schematic sentence letter whose substitution instances are any sentence.

Instances of Explanation of Truth (ET) are: it is true that snow is white, it is true that it is true that snow is white, and so on.

As Armstrong (1997: 119) puts it, ‘we are not forced to postulate a richness in the world to account for this potential infinity of statements. Why cannot we deal with the regress of instantiation [i.e. the relation regress] in the same way?’.

The answer to that question is delivered by a pictorial representation of the truth regress:

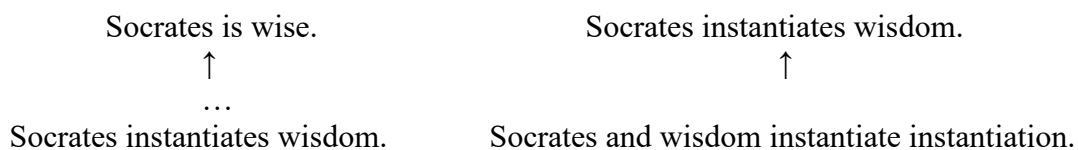


The potential infinity of statements in this regress is accounted for by a single statement, ‘Snow is white’. By contrast, the potential infinity of statements in the relation regress arises in the attempt to account for a single statement. Simply put, the truth regress terminates; the relation regress does not.

4.3 Two regresses

Solution. Every case of relatedness is adequately explained by an instance of ER (Wolterstorff 1970: 101, Loux 2006: 34-6, Orilia 2006, 2007, Gaskin 2008: 368-9). The present solution understands the theoretical ambition motivating ER to be the explanation of every case of relatedness, as opposed to relatedness tout court. It thus solves the regress by holding that each case of relatedness introduced by the regress is explained by a single instance of ER. True, each instance of ER introduces another case of relatedness that must be explained, in line with the stated ambition. But that consequence does not undermine the adequacy of the given explanation.

A pictorial representation of this view is as follows:



Reply. The following analogy may support this solution to the regress. Someone asks for an explanation of A. It is clear, from the context, that the explanation sought is causal. Let it be said that B caused A. An explanation of B is then sought. Let it be said that C caused B. An explanation of C is then sought. And so on, seemingly without end. The explanatory demand may always be reiterated – even if it cannot always be met – but this does not undermine the original explanation of A by B or the series of explanations that follow.

From this perspective, it seems plausible that the regress is benign. But even so, the question remains whether relatedness is not better taken as primitive.

It is also not obvious that ER does concern the explanation of particular cases of relatedness, as opposed to relatedness tout court – though in the present context, it does not much matter which is the ‘correct’ reading of ER. What the present solution brings to light is that there are at least two regresses concerning relatedness and ER. One ensues when instances of ER are taken to explain particular cases of relatedness. It is plausibly argued, as above, that this regress is benign, though it remains to be determined whether a benign regress is better than no regress.

The other ensues when instances of ER are used to explain the general phenomenon of relatedness, as expressed by *any* statement of the form $px_1 \dots x_n$, for any n . Since instances of ER appeal to cases of the very phenomenon to be explained, they appear to exacerbate, rather than elucidate, the problem. Assuming that explanation in the sense at stake is irreflexive and that the explanatory target is the general phenomenon of relatedness, no instances of ER constitute adequate explanations thereof, for instances of ER explain relatedness in terms of relatedness.²¹

There are thus two outstanding questions for the present solution: is a benign regress better than no regress? and is there an explanation of relatedness, tout court?

5 The explanatory project

To an extent, progress can be made on the relation regress on the basis of more or less dialectical considerations. An adequate solution to the regress cannot be embodied in a

²¹ See Broad (1933: 86): ‘we are now embarked on the self-evidently impossible task of explaining, by means of particular relational judgments, that general relational form which is presupposed by all relational judgments whatever.’ MacBride (2011b: 168) and Jones (2018a: 827) express similar views.

theory of relatedness whose sole motivation is that it avoids regress, for there are many, mutually incompatible, ways of avoiding the regress. Yet most existing solutions to the regress ultimately offer no other reasons to prefer them over their competitors. Instead, they fall back on metaphors, introduce resources – like higher-order logic or the ‘tie’ of instantiation – in terms of which analogous regresses may be formulated, or make stipulative restrictions to the scope of ‘relatedness’, without establishing the significance of the restricted notion of relatedness to which these stipulations correspond (sections 2 and 3).

But at a point, topic-specific considerations become necessary. The intuitive slogan that related things are related because they instantiate a relation appeals to a notion of explanation (‘because’) that is not fully determinate from the context. For this reason, several solutions to the regress appear equally viable. Two are salient: either (1) ER is rejected, and relatedness is held primitive, or (2) ER is accepted, and the ensuing regress is held to be benign (section 4).

If ER is taken to concern the explanation of relatedness tout court, rather than particular cases thereof, then it may be argued that the regress to which it leads is not benign.

Relatedness cannot be explained in terms of relatedness. This assumes that explanation, in whatever sense is at stake, is irreflexive. This assumption can be justified: explanations are expected to be informative, and reflexive explanations generally are not. But this, too, has been challenged (see Dorr 2016: 42-3).²²

²² Another standard assumption is that explanation is factive. If A because B, then: A is true, and B is true. If it is further held in this case that $\succ$ (‘because’) entails $\leftrightarrow$ (‘if and only if’), a contradiction may be derived by the argument of Russell’s paradox (see Chapter 5). That chapter defends a higher-order solution to the property paradox, but higher-order logic alone cannot defuse the relation regress, for the reasons described in section 3.

The bottom line is that there are hard questions about explanation here, which must be answered for the debate over the relation regress to progress.

In recent years, some authors have attempted to situate ER and the regress in a more precise explanatory framework. Thus the relevant notion of explanation has been identified as that of ‘truthmaking’ or ‘grounding’.²³ So far, however, lip service only has been paid to the relevant notions of explanation. As the discussion of these authors in the preceding pages illustrates, their views ultimately rely on no special features of truthmaker theory or metaphysical grounding.

Incidentally, these authors also maintain an ‘intuition’ that relatedness requires truthmakers or grounds, though it is not obvious that intuitions can sensibly be invoked to motivate this demand for explanation, especially once the relevant theories of truthmaking or grounding are in place.²⁴ Current controversies over these and other theories of explanation suggest that once the notion of explanation relevant to ER is specified, ER will lose its intuitive appeal.²⁵ Philosophers who do not buy into the posited frameworks will feel no reason to endorse ER, thus understood. The ensuing regress will then cease to become a problem of general philosophy.

²³ For truthmaking, see Armstrong (1997), Vallicella (2000, 2002), Orilia (2006, 2007), Maurin (2010, 2012), Simons (2010), and Perovic (2016). For grounding, see Della Rocca (2013, 2020).

²⁴ Cf. Armstrong (1997: 116): ‘We are asking what in the world will ensure, make true, underlie, serve as the ontological ground for, the truth that *a* is F.’ The various explanation words serve to crystallise an intuition that the truth in question is to be explained. This appeal to intuition is typical of contemporary and historical presentations of the regress.

²⁵ It is not trivial to formulate the relevant notion of truthmaking; see e.g. Restall (1996), Milne (2005), Rodriguez-Pereyra (2005), and MacBride (2014). There is also considerable disagreement about the correct formulation of grounding among grounding theorists; see e.g. Schaffer (2009), Rosen (2010), Fine (2012), and Bader (2020).

Chapter 2

The Immanence-Transcendence Debate

1 Introduction

According to the traditional view, a property is *transcendent* if and only if it is separate from its bearers, and *immanent* if and only if it is in its bearers. The notion of transcendence is associated with Plato, and in particular with his theory of forms. Its central premise was that *F* (e.g. beautiful, just) things are *F* to the extent that they partake in the form of the *F* itself. Such forms were said to be ‘separate’ from the things that partook in it. Aristotle in turn accepted the existence of the forms but criticised the notion that they were separate from, as opposed to ‘in’, their bearers – or, as we might more naturally say in English, ‘of’ or ‘had by’ their bearers.²⁶ For Aristotle, the existence of a form was predicated upon the existence of an object which had that form.²⁷

The relation between the notion of a form and our notion of a property is an interesting topic, but, in the present context, an irrelevant one. My concern lies with properties and, in particular, the utility of the notions of transcendence and immanence for a theory of properties. So how exactly are the notions of being ‘separate from’ and ‘in’ an object to be understood in this context? There are several answers to this question in the contemporary literature, which appeal, respectively, to the notions of dependence, parthood, location, or

²⁶ Though we do also use ‘in’ in the sense of ‘had by’, as in ‘The most important quality in a person is kindness’.

²⁷ Thus the notion of immanence is associated with Aristotle, but it bears mention that the notion is already found in Plato, who speaks of forms being ‘in’ their bearers or ‘immanent’ (ἐνόντων, *Parmenides* 130b). In the passage cited he says that it is from the immanence of the opposites themselves (i.e. the forms) that things get their names (i.e. ‘large’, ‘small’). For this aspect of Plato’s theory of forms, see e.g. Adam (1902: 3.402c16 n.), Cornford (1939: 74-81f.), Demos (1948), or Morrison (1977).

some combination of these.²⁸ Despite the diversity of such answers, they share a common thread. They imply that transcendence is consistent with the existence of uninstantiated properties, and immanence inconsistent with it.

This chapter focuses on the contemporary account of immanence and transcendence in terms of location.

Immanence Every property is located somewhere.

$$\forall x (Px \rightarrow \exists r Lxr)$$

Transcendence Every property is not located somewhere.

$$\forall x (Px \rightarrow \neg \exists r Lxr)$$

There are two reasons for this focus. First, it is the predominant account in the literature. Second, it is the received account in higher-order discussions of the immanence-transcendence debate (e.g. Jones 2018b, Trueman 2021). The chapter thus develops a stance on a debate central to traditional, first-order metaphysics of properties, and it also sets up the discussion of higher-order metaphysics of properties in Chapter 6.

With that said, the argument of this chapter generalises to other accounts of immanence and transcendence, insofar as the crux of this argument is that uninstantiated properties are needed for the reasons that properties in general are needed. Since immanence, whether it

²⁸ For accounts of transcendence and immanence in terms of dependence, see Lowe (2006), Correia (2005), or Costa (2021); parthood and location, see Armstrong (1978a: 64-76, 103-7) or Lewis (1986b: 64); location only, see Hawthorne (1995) or Sider (1995). Sider also criticises accounts of transcendence and immanence that appeal to both parthood and location: if the spatiotemporal component of these accounts is correct, then ‘dispute over the mereological component looks pointless’ (367).

is understood in terms of parthood, dependence, or location, implies that there are no uninstantiated properties, immanence is false. Meanwhile, transcendence is, on all accounts, consistent with the existence of uninstantiated properties.

There are, however, other arguments that have often been raised for and against immanence and transcendence, as these notions are understood in terms of location. These arguments will also be considered, and criticised, in this chapter. Section 2 focuses on immanence, and section 3 on transcendence.

2 Immanence

Immanence says that properties are located somewhere, which raises the question: where? It is universally held that if properties are located anywhere, then they are located where their bearers are.

Property Location	Every property is located wherever it is instantiated.
	$\forall x (Px \rightarrow \forall r (Lxr \leftrightarrow \exists y (Iyx \wedge Lyr)))$

It must be said, however, that Property Location (PL) is rarely justified, and such justification as can be found in the literature is weak. For example, it has been claimed that similarity between objects can be explained in terms of the identity of (some of) their properties, if properties are both universal and immanent:

By taking properties to be universals and universals to be wholly present in each of their instances, you can account for the fact that similar objects behave similarly and at the same time ground similarity in identity. Heil 2013: 132; similarly Armstrong 1978a: 68-9

But contrary to the claim made in this passage, no work is done by the assumption that properties are wholly present, or more generally located in, each of their instances. For all that has been said, similarity between objects may be ‘grounded in’ or explained in terms of the identity of one or more of transcendent properties.

Another author looks to ordinary language to establish that at least some properties, i.e. colours, are objects of ostension:

... if I say ‘This is green’ while pointing in the direction of a tree, and then, pointing in the direction of my carpet, say ‘And this is green, and this is identical with that,’ then I would be pointing at a quality. And so would a father who in teaching his child says, ‘Here’s green, and here’s green, and here’s green again.’ Wolterstorff 1960: 185

Since one cannot point at something that is not there, Wolterstorff concludes that at least some properties are located where their instances are. There are, however, several points at which this argument can be contested.

First, the assumption that colours are properties (qualities) is not uncontroversial.²⁹ This point is of some importance, for the claim that properties are the objects of ostension

²⁹ Dummett (1981: 484-94), e.g., argues that they are concrete objects, as opposed to either abstract objects or Fregean concepts.

appears less rather than more plausible when applied to properties other than the colours, such as justice or personhood.

Second, the scenario Wolterstorff describes is less natural than he suggests. I take it that an ordinary speaker would not say ‘This is identical to that’, while pointing in the direction of two green things, in order to communicate their identity of colour. Instead, she would say something like: ‘This is the same colour as that’. Similarly, a father teaching his child what green is would more naturally say, ‘This is green’, as he pointed to various green things. The ‘is’ in such sentences is to be understood as the ‘is’ of predication, as opposed to identity.

In general, the passage fails to supply convincing evidence that children learn what green is by being shown the relevant property, as opposed to various things that instantiate the property. More generally, it fails to show that we ever attribute location to properties. Indeed, the unnaturalness of claims explicitly and unambiguously attributing location to properties, such as ‘Green is located here’ or ‘Wisdom is located where Socrates is’, militates against the prospects of discerning a commitment to PL, whether restricted to colours or otherwise, in ordinary thought and discourse.

Of course, even if such a commitment could be discerned, it could be overturned on the basis of other, theoretical considerations.

The passages considered here are not exhaustive, but they are representative. Arguments for PL are generally weak – a feature which may be symptomatic of the weakness of arguments for immanence more generally. A commitment to neither PL nor immanence

appears to be justified with respect to the standard theoretical virtues, such as explanatory power or simplicity. A commitment to neither PL nor immanence renders property theory simpler, if simplicity is measured, for example, by the number of logically independent sentences a theory entails. It also does not render property theory any more able to explain the relevant phenomena, at least if existing arguments on this score, like those by Heil and Wolterstorff above, are any indication.

In fact, there is an important respect in which a commitment to immanence and PL renders a theory of properties able to explain less. For it is an immediate consequence of the conjunction of these premises that there are no uninstantiated properties, though uninstantiated properties are needed for at least some of the reasons that properties, in general, are needed. The case for uninstantiated properties is made at length in Chapter 5.

The conjunction of immanence and PL also implies that everything is located somewhere (assuming everything instantiates some property). But prominent mathematical and scientific theories imply that there are things that are not located anywhere, such as numbers, sets, and spatiotemporal points. Given a toss-up between immanence and PL, on the one hand, and our best mathematical and scientific theories, on the other, there appears little reason to settle the issue in favour of immanence and PL – unless there are good reasons to reject any of the alternatives to immanence, such as transcendence.

In the next section, I criticise standard arguments against transcendence. But before moving on, it is worth mentioning three other arguments that have been raised against immanence and PL. All three are arguments from intuition and, are, for this reason, inconclusive. Replies are advanced below on behalf of the proponent of immanence.

Argument 1. Given PL, multiply instantiated properties (e.g. being a person) are located in multiple places at a time. Now multiple location is not unique to properties. The United States is an example of an object (= non-property) that is multiply located – witness Alaska and Hawaii. But where objects are located in multiple, disjoint places at a time by having distinct proper parts in those places, properties cannot be multiply located in this way, on pain of falsifying PL. PL draws a material equivalence between property location and instantiation. If proper parts of properties are located wherever the property bearers are, then, as Plato observed in the *Parmenides* (131a-e), this equivalence does not hold. For this reason, immanent properties are said to be ‘wholly present’ wherever they are instantiated.

For many philosophers, the primary objection to PL, and thus to immanence, is the very notion that anything could be wholly present in multiple places at once (e.g. Heil 2003: 132-6; Lowe 2006: 98-9).

Reply. Lewis (1983: 345) has argued that the unintuitive character of the notion of multiple whole presence stems from intuitions pertaining to particular objects, like ourselves, tables, and chairs, as opposed to properties. Intuitively, such objects cannot be wholly present in multiple places at once, but properties lie outside the scope of such intuitive principles.

I am not convinced by this argument – partly because I am not sure that we do have any intuitions concerning the application of theoretical notions like multiple whole presence, and partly because I suspect that if we do have such intuitions, they are not thus limited in scope. The relevant principle seems to be that *nothing* can be wholly present in multiple

places at once. But be that as it may, a better reply may be made on behalf of the defender of immanence. It is that fit with intuition is not the only or even the primary measure of a philosophical thesis.

The problem is that there appear no good reasons to locate either properties or relations in space and time, for the reasons described above. A commitment to PL or immanence does not appear to be justified with respect to the standard theoretical virtues: it does not increase the explanatory power or simplicity, e.g., of property theory. Moreover, the conjunction of PL and immanence implies that there are no uninstantiated properties, though uninstantiated in addition to instantiated properties are needed for at least some of the intended applications of a theory of properties.

In any case, the problem here does not lie with the multiple whole presence of properties, but elsewhere. The next two arguments are seen to follow a similar pattern.

Argument 2. Assuming a plenitudinous conception of properties, on which for every predicate there is a distinct property with the same extension, PL implies that there are at least as many properties located where an object is as there are predicates true of the object.³⁰ It has been objected that this would render spacetime ‘altogether too populated’ (Sider 1995: 13). Relatedly, assuming that immanent properties are also parts, in some sense, of their bearers, it has been deemed ‘just absurd’ to suppose that an object has such parts for all of its countless abundant properties (Lewis 1986b: 67).

³⁰ Some restrictions must be made to this conception to avoid property-theoretic versions of Russell’s paradox for sets. See Chapter 5.

Reply. It is interesting that Lewis should draw the line of absurdity here, rather than at the multiple whole presence of properties. In any case, it has been observed that overpopulation is curbed on more sparse conceptions of properties, such as a conception according to which there are only enough properties to characterise reality ‘completely and without redundancy’.³¹ However, it remains an open question how exactly the notion of a ‘sparse’ property is to be cashed out (see Dorr and Hawthorne 2003).

More importantly, even if properties are sparse, PL still generates a conflict with ordinary counts of the numbers of things located at any given place at a time. Located where an electron is, for example, there will also be its sparse property of being negatively charged. How many more things must there be than ordinary counts suggest for there to be ‘too many’? One plausible answer is that if a theory predicts that there is even one more thing located at a place than ordinary counts suggest, that is too many. By these lights, a sparse conception of properties will not avail an immanent theory of properties any more than a plenitudinous conception.

The deference of immanence would be better served, not by a distinction between sparse and plenitudinous properties, but rather by the platitude that fit with intuition is not the only or primary measure of a philosophical thesis – though this will not save her theory in the end.

Argument 3. It has long been held that PL has no natural generalisation to relations. In particular, the obvious suggestion, that relations are located where their terms are, is often

³¹ Armstrong (1978a, 1978b) is a prominent example of a philosopher who combines immanence with a sparse conception of properties.

rejected out of hand, presumably on the basis that it is too unintuitive.³² Insofar as properties and relations are things of the same general kind, differing primarily in the numbers of their argument places, a unified theory of properties and relations is desirable. If relations cannot be located anywhere in space and time, properties ought not to be located, either. So immanence is false.

Reply. Another platitude to the rescue: one philosopher's modus ponens is another's modus tollens. Assuming that a unified theory of properties and relations is desirable, and properties are located where their bearers are, relations ought to be located where their relata are. Of course, the question I have been asking is: are properties located where their bearers are?

3 Transcendence

Transcendence does not imply that there are no uninstantiated properties, and given natural background assumptions, it does not imply that everything is located in space and time. But a theory's consistency with certain theoretical desiderata does not establish its truth. In the present case, transcendence and immanence are mutually exclusive, but not exhaustive, theories (assuming there are properties). A third theory of properties is also possible, according to which some properties are located, and others are not. Such a theory

³² See e.g. Simons (2010: 207) on the problem of locating relations as opposed to properties: 'Of course one could locate them with their terms, but that feels like a cheat.' Russell (1912: 56), observing that the relation of being north of did not exist any more in Edinburgh than in London, since it related the two and was 'neutral' between them, determined that there was no particular place or time that the relation could be found. He was thus led to deny location to relations, though he allowed it to properties like whiteness (1911: 3). But granted that the relation does not exist any more in Edinburgh than in London, this leaves open the possibility that the relation is located in both places equally. MacBride (1998: 211) also observes that it does not follow from the fact that uninstantiated universals are not 'found' anywhere in space and time that they are not located anywhere in space and time.

would also be consistent with the aforementioned desiderata, but inconsistent with transcendence. So the falsity of immanence does not entail the truth of transcendence.

That said, the rest of this section criticises three standard arguments against transcendence – and more generally against any theory of properties that implies that there are at least some transcendent properties. The arguments are shown to share a common flaw. They make essential use of vague or inchoate premises, and on the most salient precisification of these premises, the arguments are either (1) no longer sound or (2) consistent with transcendence. A challenge is thus raised for critics of transcendence to re-formulate these arguments. Until this challenge is met, transcendence, or any theory that entails the existence of transcendent properties, cannot reasonably be determined false on their basis.

3.1 Argument from naturalism

Perhaps the single most prominent case against transcendence rests on the claim that:

Naturalism Everything is located in space and time.³³

Whatever intuitive force this claim holds, however, it can be challenged by considering a number of things that have no obvious location: sets, numbers, space, time, gravity, and so on. Events, too, are not clearly located anywhere in space, even if they are commonly

³³ In truth, the label ‘Naturalism’ does duty for a number of different views. These are separable into two general categories: ontological and methodological. Naturalism as stated in the main text falls in the former category, though formulations of ontological Naturalism differ. Armstrong (1978a: 126, 1989b: 99, 1997: 5-6, 38-43), for example, describes it as the hypothesis that ‘the world is a single spatiotemporal system’. He does not defend the hypothesis. By contrast, methodological Naturalism is, roughly, the view that knowledge is only attainable by the methods of natural science. Quine (1981: 21) describes it as ‘the recognition that it is within science itself, and not some prior philosophy, that reality is to be identified and described’.

attributed locations in time. There are theoretical reasons to postulate all of these things. Our best mathematical, scientific, and philosophical theories quantify over and refer to them, in a way that may be ineliminable both in practice and in principle.³⁴ As long as that remains a live possibility, appeals to Naturalism in arguments against transcendence can hardly be deemed conclusive.

3.2 Argument from causal powers

Assuming that spatiotemporal location is a necessary, if not sufficient, condition on the possession of causal powers, transcendent properties must lack causal powers. However, according to an ancient philosophical doctrine sometimes known as the Eleatic principle, nothing lacks causal powers. A contradiction follows, which may be taken as a *reductio* of one of the following three assumptions: (1) spatiotemporal location is a necessary condition on the possession of causal powers, (2) there are transcendent properties, or (3) nothing lacks causal powers.

(iii), or the Eleatic principle, is sometimes defended by an argument like the following.

Something without causal powers would be empirically undetectable, and therefore unknowable; and what cannot be known cannot be held to exist. However, both claims are dubious at best. Empirical detection is not the only method by which knowledge is attained. Scientific and philosophical theories regularly employ methods, such as inference

³⁴ Armstrong writes that it must be possible for the truth conditions of mathematical statements to be given ‘in some way or other which does not appeal to entities outside Nature’ (1978a: 131), but he does not elaborate. Earlier Quine had given up the view that everything is in space and time precisely on the basis that science required mathematics required set theory, and sets are not located in space and time (see his 1978: 43; cf. Goodman and Quine 1947, where the authors attempt, unsuccessfully, to give a nominalist reduction of set theory). Field (2016) is another naturalist nominalist who attempts to do science without numbers, but his arguments are concerned only with a part of physics and have no clear generalisation to the rest of physics or science, which is not to say that the arguments he does give are sound.

to the best explanation, that cannot be reduced to empirical detection, and such theories are generally taken to constitute advances in human knowledge. Moreover, the possession of causal powers cannot be the only *raison d'être* for everything. There are other ways in which things manifest theoretical utility and thus supply us with reasons to postulate them. Sets and numbers are the usual paradigms here.

In light of such paradigms, the Eleatic principle may appear the most vulnerable premise in the argument from causal powers. But it is worth observing that (i) also is not invulnerable. What, exactly, does it mean to have causal powers?

A plausible answer is that: for any x , x has causal powers if and only if possibly, x enters into causal relations. Now on standard theories of causation, the terms of causal relations are events or states of affairs – things of the kind identifiable by formulas of the form ' x 's being F (at a time t)'.³⁵ In other words, only events or states of affairs enter into causal relations.

Now if (i) only events or states of affairs enter into causal relations, and (ii) something has causal powers if and only if possibly, it enters into causal relations, then (iii) transcendent properties lack causal powers, since transcendent properties are neither events nor states of affairs. This is what the critic of transcendence wanted; but the trouble is that her account of causal powers proves too much. For it also follows from (i) and (i) that (iv) immanent

³⁵ See e.g. Kim (1973), Bennett (1988), Mellor (1995), and Armstrong (1997). Terminology differs between these authors. Kim identifies causal relata as 'events', Armstrong uses 'states of affairs', and Bennett and Mellor 'facts'. But for Bennett and Mellor, facts are just states of affairs that obtain – and for Armstrong, such are the only states of affairs. In general, each of these authors identifies causal relata as things that can be described by formulas of the form ' x 's being F (at t)'. Note that other authors also employ the terminology of events, facts, and states of affairs differently. Lewis (1986a), for example, identifies causal relata as events, but identifies events as properties of spacetime regions. Given his view of properties and his modal realism (for which see his 1986b), events are thus classes of spacetime regions in different possible worlds.

properties, as well as objects like ourselves, tables, and chairs, lack causal powers, since such things are neither events nor states of affairs. Given the Eleatic principle, none of these things exist.

In other words, the present account of causal powers entails that there are no transcendent properties only if the Eleatic principle is true. The problem is that the conjunction of this account and the Eleatic principle entails too much: it entails not only that there are no transcendent properties, but more generally that there are no properties or objects like ourselves, tables, and chairs. And the Eleatic Stranger does not deny that he himself exists.

The question now is whether there is an alternative, plausible account of either (a) causal powers, (b) the metaphysics of causation, or (c) the metaphysics of states of affairs or events, which will entail (other things being held fixed) that transcendent properties do not have causal powers, while immanent properties and objects like ourselves, tables, and chairs do.³⁶ In the context, such accounts may be regarded ‘plausible’ only if they are at least as plausible as the Eleatic principle itself – otherwise, the more dialectically sound move would be to give up the principle. But to give up the Eleatic principle is once again to allow the possibility that there are transcendent properties.

An answer to this question will not be pursued here. That onus rests on proponents of the argument from causal powers against transcendence.

³⁶ A natural first thought may be to look to Davidson (1963, 1967), who also identifies the terms of the causal relation as events, though for him events are ‘unrepeatable, dated individuals such as the particular eruption of a volcano...’ (1970: 209, see also his 1969). Ultimately, however, this will not help proponents of the argument from causal powers against transcendence. Objects like volcanoes are not events on this theory, so, assuming (1) the account of causal powers given in the main text, according to which something has causal powers if and only if possibly, it enters into causal relations, and (2) the Eleatic principle, it follows that volcanoes do not exist, which is false. For a criticism of other accounts of causal powers in the context of the immanence-transcendence debate, see Sider (1995: 372-4).

3.3 The access problem

Earlier I said that uninstantiated properties are needed for at least some of the reasons that properties, in general, are needed. One such reason concerns the semantics of predicates. Properties have traditionally been expected to serve as the semantic values of such expressions. To fulfil this role, there must be at least a property corresponding to every predicate – including predicates, like ‘is red and not red’, that are true of nothing.³⁷ The values of such predicates are naturally held to be uninstantiated as opposed to instantiated properties. Since no immanent properties are uninstantiated, immanent properties are unsuited to serve as the semantic values of predicates.

However, transcendent properties have long been thought to fare no better in this role. For such properties would be ‘outside the causal swim’, and thus inaccessible to thought and discourse. And if they are inaccessible to thought and discourse, then they could not serve as the semantic values of our expressions.³⁸

It should be plain that these ideas are undermined by an argument very similar to that raised in the previous section. Given standard theories of the metaphysics of causation, neither properties nor objects like ourselves, tables, and chairs enter into causal relations. In that sense, they are ‘outside the causal swim’. But objects like ourselves, tables, and chairs are evidently accessible to thought and discourse. So being inside the causal swim cannot be a necessary condition on being accessible to thought and discourse. Proponents of this version of the access problem owe us a more careful formulation and defence of

³⁷ Again, some exceptions must be made for versions of Russell’s paradox, as discussed in Chapter 5.

³⁸ For a recent formulation of this argument, see Trueman (2021: 137).

their views in order to establish a case that transcendent properties are ineligible to serve as the semantic values of predicates.³⁹

Suppose it is said, for example, that:

Causal Reference An expression e refers to some x only if a causal relation obtains between an utterance of e and x .

Something like this constraint appears to be alluded to by various expressions of the access problem (see e.g. Benacerraf 1973: 671). It is worth mentioning, however, that the view is not commonly held in contemporary philosophy of language. In that context, ‘causal theory of reference’ refers primarily to a view associated with Kripke (1980), who argued that a particular use of a name refers to an object x if and only if this use is related, by a causal chain of reference-preserving links, to the event of x ’s first being given or acquiring the name. In this case, the causal relation obtains between the initial ‘baptism’ and later uses of the name. It is not a relation to the bearer of that name.⁴⁰

Of course, prominence is not necessary for truth, but closer inspection also casts in doubt the assumption that transcendent properties are ill-suited for the needs of a causal theory of reference, in the sense described by Causal Reference. For starters, given standard theories of the metaphysics of causation, Causal Reference implies that expressions only

³⁹ The access problem was originally put forward in the philosophy of mathematics against a Platonist conception of numbers, on which numbers exist and are not located in space and time; see Benacerraf (1973). There is an extensive literature on this problem, for which see e.g. Hale and Wright (2002) and the references therein.

⁴⁰ Devitt (1981: 25-64) and Fodor (1987: 97-127) are exceptions to the rule. By ‘causal theory of reference’ they do mean a theory on which reference involves a causal relation between the referent and the term.

refer to events, but this is false; the referent of ‘Socrates’, for example, is not an event, but a person. Causal Reference might thus be revised to:

Causal Reference* An expression e refers to some x only if a causal relation obtains between an utterance of e and an event involving x .

Is this account plausible? Whether or not it is, it does not preclude reference to transcendent properties, so it does not help in this context regardless.

The point is this. Absent a convincing version of the access problem, there is no immediate bar to allowing transcendent as opposed to immanent properties to serve as the semantic values of predicates.

Analogous remarks apply to an epistemological version of the access problem, according to which (1) knowledge is a causal relation, and (2) there can be no causal relations to, and hence no knowledge of, transcendent properties. For one, the causal theory of knowledge is no longer a dominant theory of knowledge in epistemology.⁴¹ For another, standard theories of the metaphysics of causation imply that there are no causal relations to objects like ourselves, tables, and chairs. Granted that we do have knowledge of such objects, standing in causal relations cannot be a necessary condition on being an object of knowledge.

⁴¹ The causal theory of knowledge was prominently defended by Goldman (1967), though he later (1976) gave up the view in the light of ‘fake barns’ problem cases. For further criticism of the view, see e.g. Gendler and Hawthorne (2005).

Chapter 3

The Universal-Particular Distinction

1 An intuitive distinction

There is an intuitive distinction between properties and objects like ourselves, tables, and chairs. No properties are objects, and no objects are properties. Philosophers have for a long time tried to capture this intuitive distinction in other terms, and they have traditionally done so by appeal to a distinction between universals and particulars.

Accordingly, it is said that all properties are universals, and all objects like ourselves, tables, and chairs are particulars. However, in order for the distinction between universals and particulars to do any work that is not done by the more familiar distinction between properties and objects, an account is needed of what universals are and what particulars are. This chapter presents three standard accounts of the universal-particular distinction, along with their defects. A final section criticises trope theory, which turns the traditional view on its head; on this theory, all properties are particular, as opposed to universal. As with Chapter 2, this chapter serves two purposes. One, it introduces a topic central to traditional, first-order metaphysics of properties. And two, it sets up the discussion of this topic from the perspective of higher-order metaphysics to be found in Chapter 6.

2 Three accounts

2.1 Predication

The first account of the universal-particular distinction considered here originates in Aristotle's *Categories* (1a20-2).

Universal[1] A universal is something that can be predicated of something.

Particular[1] A particular is something that cannot be predicated of anything.

Given that properties are supposed to be universals, the intuitive idea underlying this account of a universal is that properties are what we predicate of things. By contrast, objects like ourselves, tables, and chairs are the subjects of predication; they are not themselves predicated of anything.

The link between property instantiation and predicate satisfaction is suggested by the apparent equivalence between sentences like ‘Socrates is wise’ and ‘Socrates has the property of wisdom’. Famously, however, Ramsey argued that such equivalence actually undermined this account of the universal-particular distinction. Take the sentences: (1) ‘Socrates is wise’ and (2) ‘Wisdom is a property of Socrates’, which is apparently equivalent to ‘Socrates has the property of wisdom’.

Now of one of these sentences “Socrates” is the subject, of the other “wisdom”; and so which of the two is subject, which predicate, depends upon what particular sentence we use to express our proposition, and has nothing to do with the logical nature of Socrates or wisdom, but is a matter entirely for grammarians. In the same way, with a sufficiently elastic language any proposition can be so expressed that any of its terms is the subject. Hence there is no essential distinction between the subject of a proposition and its predicate, and no fundamental classification of objects can be based upon such a distinction. 1925: 404

Now there are a few points at which this argument may be challenged. For one, neither ‘Socrates’ nor ‘wisdom’ are predicates; they lack a main verb (Simons 1991: 152). So we have yet to be given an example in which Socrates himself is predicated of something, i.e. an example in which Socrates appears not as a particular, as we might expect, but as a universal. From the other angle, we have also yet to be given an example in which something that we might expect to be a universal is not predicable of anything.

Ramsey’s argument also relies on a comprehension principle that, in full generality, leads to a version of Russell’s paradox (see Chapter 5).

Nevertheless, the underlying spirit of his argument has seemed persuasive to many philosophers. The distinction between properties and objects is a metaphysical distinction, and a metaphysical distinction cannot be based on a linguistic distinction, between what can and cannot be predicated.

2.2 Instantiation

A second account of the universal-particular distinction seeks to answer the above criticism, by drawing the distinction in metaphysical terms. Incidentally, it, too, originates in Aristotle’s *Categories* (1a23-9).

Universal[2] A universal is something that can be instantiated by something.

Particular[2] A particular is something that cannot be instantiated by anything.

This account of the universal-particular distinction is often interpreted as the metaphysical analogue of the first account. Note, however, that instantiation is not strictly analogous to predication, in the sense of being isomorphic to. There are false, but meaningful, predicates, and no analogue of this at the metaphysical level.

It is for this reason that, on this account, properties fall on both sides of the universal-particular distinction. Some properties, like being red and being a horse, are classed as universals. Others, like being a round square and being self-distinct, are classed as particulars; these are the necessarily uninstantiated properties. The account is thus in direct conflict with the starting assumption that all properties are universals. Even if that assumption is abandoned, as it is on trope theory, it is universally held to be an adequacy constraint on an account of the universal-particular distinction that it does not imply that properties fall on both sides of the distinction. By these lights, the present account is inadequate.

2.3 Multiple instantiation

A third account of the universal-particular distinction originates, this time, in Plato.

Universal[3]	A universal is something that can be instantiated by more than one thing at a time.
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Particular[3]	A particular is something that cannot be instantiated by more than one thing at a time.
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The intuitive datum underlying this account of the universal-particular distinction is that every F thing instantiates one and the same property of being F . So whenever there is more than one F thing, the property of being F will be instantiated by more than one thing at a time. A property is, in this sense, a ‘one over many’ (*Republic* 5.476a5-8).

However, it was pointed out in the previous section that being instantiable is not a necessary condition on being a property: some properties are not instantiable. That being so, being instantiable + C , where C is some condition, will also fail to describe a necessary condition on being a property. Since on the present account, C is not trivial, there are properties in addition to the necessarily uninstantiated ones that fail to satisfy the condition for being a universal and are instead classed as particulars. These are the properties that can be instantiated but cannot be instantiated at once. Examples include the properties of being the tallest person in the room now and being the last dodo.⁴² So the third account of the universal-particular distinction is also inadequate, insofar as it fails to characterise a mutually exclusive distinction between properties and objects.

3 Trope theory

Other accounts of the universal-particular distinction have been given, but none has ever managed to carry general conviction. Either they are inadequate, in the sense that they fail to characterise a mutually exclusive distinction between properties and objects, or, as

⁴² There is a sense in which these properties, being the tallest person in the room and being the last dodo, can be instantiated by more than one thing at a time. For example, either you or I could have been the tallest person in the room. You could have grown a few inches, and so achieved that distinction; likewise, so could I, other heights remaining fixed. So being the tallest person in the room is a property that could have been instantiated by either of us now. However, it could not have been instantiated by both of us at the same time. These subtleties do not affect the necessarily uninstantiated properties, which are still classed as particulars on this account.

MacBride (2005: 603) puts it, they are ‘divorced from whatever theoretical insight (or oversight) may have supplied their original motivational force’. One interesting suggestion, in this context, is that we were wrong to assume, as we did, that all properties are universals. According to a prominent theory of properties, all properties are particular, as opposed to universal.

Let a ‘trope’ be a property that is particular, as opposed to universal (after Williams 1953). Trope theory is thus the theory that properties exist and are tropes. The theory raises the logical possibility that all there are are particulars: properties are particulars, as are the objects that instantiate them. Trope theorists may be absolved of the obligation to give an account of the universal-particular distinction, insofar as they do not recognise any universals; but they are still reasonably expected to give an account of the notion of a particular, or more specifically, a trope. Again, no such account has carried general conviction.⁴³

However, trope theorists are united in the view that tropes cannot be had by more than one thing at a time. Thus a redness trope cannot be had by more than one red thing at a time; other red things each have their own tropes of redness. Notice that this is a necessary, but not a sufficient, condition for being a trope. Socrates cannot be had by anything, in the relevant sense; therefore, he cannot be had by more than one thing at a time. But he is not a property; therefore, he is not a trope.

⁴³ See Ehring (2011) for a fairly recent, and involved, attempt to give such an account on behalf of trope theory.

Nevertheless, given this minimal constraint on the notion of a trope, trope theory can be shown to be unmotivated, for reasons independent of the absence of an adequate account of what tropes are. Four prominent arguments for trope theory will be criticised in the rest of this chapter. Notably, all of them assume both immanence and Property Location (Chapter 2). That is, they assume both that properties are located and that they are located wherever they are instantiated. Immanence and PL were criticised in Chapter 2. However, let these criticisms be suspended here for the sake of argument. For even when all of its assumptions are granted, the case for trope theory remains demonstrably weak.

3.1 Argument from multiple location

Argument. Assume PL. Assume also that properties are universal, in the sense that they can be instantiated by multiple things at a time. Specifically, assume that every *F* thing instantiates a single property of *F*-ness. Then properties instantiated by multiple objects at once will be located in multiple places at once. Specifically, given standard assumptions about the manner in which properties are located, they will be ‘wholly present’ in multiple places at once (Chapter 2). But nothing can be wholly present in multiple places at once. Contradiction. Trope theorists take this argument to be a reductio of the premise that properties are universal (e.g. Campbell 1981: 477, Heil 2003: 130-6).

Reply. The trouble is that the same argument may equally be taken to be a reductio of PL. Moreover, no argument is made in support of the premise that nothing can be wholly present in multiple places at a time, other than that its negation is implausible (*op. cit.*). But as far as fit with intuition is concerned, trope theory does not have much of a leg to

stand on, either. For the theory denies, what we ordinarily believe, that properties like being wise or being a person are had by more than one thing at a time.

3.2 Argument from change

Argument. Tropes are needed for an account of change, for an object changes if and only if one or more of its properties come or cease to be, and universals do not come or cease to be (Martin 1980: 9).

Reply. Grant that universals do not come or cease to be.⁴⁴ Even so, no reasons have been given, and no reasons appear to hand, to prefer the trope-theoretic account of change over the following: an object changes if and only if it comes or ceases to instantiate a property. This account of change is consistent with the view that properties are universals; it is as simple as the trope-theoretic account; and it is at least as intuitive.

3.3 Argument from causation

Argument. Tropes are needed for an account of causation. Consider a singular causal statement like ‘The heat of the stove caused the burn on my arm.’ Such statements describe a causal relation between two conditions: the heat of the stove and the burn on my arm. These conditions are properties, though they are not the universal properties shared by distinct objects. Rather, they are particular properties, which are specific to a place and time. In short, they are tropes (Campbell 1981: 480).

⁴⁴ The thought here seems to be that since the very same universal can be instantiated at different times, it does not come or cease to be when one of its bearers exist; therefore, it never comes or cease to be. Strictly speaking, the conclusion does not follow, but this point is inessential to the argument of the main text.

Reply. The trope-theoretic account of causation is primarily motivated by our intuitive judgments of what is ‘involved’ in singular causal statements like the above. Intuitively, the heat of the stove and the burn on my arm are both particulars, or specific conditions at a place and time. The problem is that these intuitive judgments are equally vindicated by more standard theories of the metaphysics of causation, on which the terms of the causal relation are identified, not as tropes, but rather as events or states of affairs (see Chapter 2). According to such theories, the event of the stove’s being hot caused the event of my arm’s being burned, which is at least as intuitive as the trope-theoretic view that the particular hotness of the stove caused the particular burn on my arm.⁴⁵

3.4 Argument from ontological economy

Trope theory leaves open the possibility that all there are are particulars. In fact, trope theorists usually hold something more – namely, that all there are are particular properties, i.e. tropes. For trope theorists usually hold a bundle theory of objects, according to which objects like ourselves, tables, and chairs are mere ‘bundles’ of properties (Chapter 4). The ontological economy afforded by the combination of trope theory and bundle theory has been touted as a significant virtue of both views – to be compared, for example, with a two-category theory on which there are universal properties and particular objects instantiating them.⁴⁶ In retrospect, considerations of ontological economy may also

⁴⁵ By Campbell’s own admission, there are other singular causal statements that are better interpreted in terms of events, such as ‘The short circuit caused the fire’. He is thus led to suggest a trope-theoretic metaphysics of events, on which they are ‘trope-sequences, in which one condition gives way to others. Events, on this view, are changes in which tropes replace one another’ (*op. cit.*). It is difficult to evaluate this claim without further guidance, but it seems that in practice, we will often be unable to identify which tropes, and thus which changes, are involved in simple causal statements like the above. A complex account of causation looms.

⁴⁶ After Williams (1953: 7) and Campbell (1981: 481-3), trope theory is standardly packaged with a bundle theory of objects, though there are exceptions. Martin (1980), Heil (2003), and Lowe (2006), for example, all view objects as complexes partly constituted by a substratum in which tropes inhere. Substratum theory is examined with bundle theory in Chapter 4.

suggest a reason to prefer the trope-theoretic accounts of causation and change: they appeal only to one kind of thing (i.e. tropes), where other accounts appeal to at least two (i.e. universal properties and particular objects).

However, the claim to comparative ontological economy and thus to virtue ought to be tempered.

First, bundle theory may be, and in fact often is, combined with a theory of universals: objects are thus held to mere bundles of universals, as opposed to tropes. Assuming that a universal-theoretic bundle theory is at least as plausible as a trope-theoretic bundle theory, trope theory is inessential to the ontological ‘reduction’ of objects to (bundles of) properties.

Second, a distinction may be drawn between qualitative and quantitative ontological economy. Qualitative ontological economy is measured by how many kinds of things there are, where quantitative ontological economy is measured by how many things there are. The ontological economy claimed for trope theory is qualitative. Trope theory does not introduce any universals, and a trope-theoretic bundle theory postulates just one kind of thing in its account of reality, i.e. tropes. However, this qualitative economy is offset by the theory’s comparative quantitative excess. Where a theory of universals postulates a single universal of *F*-ness instantiated by each *F* thing, trope theory postulates a multiplicity of *F*-ness tropes, with a distinct trope had by each *F* thing.

Third, trope theory’s qualitative ontological economy is also offset by its ideological complexity, which becomes obvious when trope theory is applied to the theoretical work

to which theories of properties have traditionally been applied. A case in point is found in standard trope-theoretic accounts of similarity. On the traditional, universal-theoretic account, if any things are similar, then they are similar because they instantiate some common property or properties. Thus similarity between distinct objects is explained in terms of the identity of one or more of their properties. But since trope theory denies that distinct objects ever instantiate a common property, it rules out this account. Instead, most trope theorists accept similarity as primitive. If any things are similar, then they are similar because they have similar tropes. Moreover, *F* things are similar because they each have exactly similar tropes (of *F*-ness).⁴⁷

One measure of the ideological complexity of a theory is the number of primitive notions the theory accepts. Trope theory cannot explain similarity in other terms, where a theory of universals can. So trope theory must accept at least one more primitive notion than a theory of universals. It may also be added that if similarity between tropes is in turn explained in terms of the similarity of *their* tropes, an infinite regress ensues.

The best arguments for trope theory have been examined and found wanting. Now to the criticisms of this section, let the criticisms of immanence and PL (Chapter 2) and bundle theory (Chapter 4) be added. The prospects are dim for trope theory. There seems little reason to deny, what is affirmed in ordinary thought and discourse, that many properties are universal, in the sense of being instantiable by distinct objects at a time. Nevertheless,

⁴⁷ There are other trope-theoretic accounts of similarity. One has it that objects are similar because they have similar tropes, and similar tropes are similar because they instantiate a common universal (Cook Wilson 1926: 670-6). Another has it that objects are similar because they have tropes that belong to the same class (Ehring 2011: 175-202). In general, one can concoct trope-theoretic versions of any nominalist or realist accounts of similarity. However, such trope-theoretic accounts typically forego the claim to a one-category ontology, from the perspective of at least one categorial scheme. For example, the two accounts just described imply that there are universals and that there are classes, respectively. The alternatives, for any plausible version of trope theory, seem to be either to accept similarity as primitive or forego the claim to a one-category ontology.

in the spirit of the previous chapter, the criticisms of this chapter may be interpreted as challenges for the trope theorist: to re-formulate the arguments from multiple location, change, causation, or ontological economy or to put forward other, better arguments in favour of trope theory altogether.

Chapter 4

Theories of Objects

1 Parthood*

Substratum theory and bundle theory share a common assumption. Material objects like ourselves, tables, and chairs have a decomposition into more basic parts. These parts include their properties. The two theories differ insofar as substratum theory alone posits among the parts of an object something in addition to the object's properties: an object is constituted by its properties and a further thing, an underlying 'substratum', in which the properties inhere. By comparison, bundle theory says that an object is constituted solely by its properties; objects are, in this sense, mere 'bundles' of properties.

Let 'parthood*' designate the relation in which properties stand to their bearers, according to substratum theory and bundle theory. It is standardly assumed that parthood* is the very relation described in such familiar statements as 'The wrist is a part of the arm'; that is, parthood* is parthood. Hence the name. However, that assumption is falsified by the simple observation that the two relations differ in their logical properties.

For one, parthood is a transitive relation. If the wrist is a part of the arm, and the arm is a part of the body, then the wrist is a part of the body. It is also reflexive. Everything is a part of itself.

However, parthood* is neither reflexive nor transitive. Both substratum theory and bundle theory imply the following constraint on parthood*:

Parthood* Constraint A property p is a part* of x if and only if x instantiates p .

Since instantiation is neither reflexive nor transitive, neither is parthood*. To see that instantiation is irreflexive: the property of being wise, e.g., does not instantiate itself. To see that it is not transitive: the property of being a property is instantiated by the property of being wise, and the property of being wise is instantiated by Socrates, but the property of being a property is not instantiated by Socrates.

So parthood* is not parthood, and the relation between properties and objects cannot be subsumed under mereology.⁴⁸ Parthood* is also not instantiation, for substratum theory holds that substrata are parts* of objects in the same sense in which properties are parts* of objects, but substrata are not instantiated by objects. Parthood*, if it exists, is a relation distinct from other relations recognised by our ordinary conceptual scheme. Like every other theoretical postulate, it ought to be justified. However, substratum theorists and bundle theorists alike have tended to assume, rather than defend, the view that properties are parts* of their bearers, and that objects have a decomposition into more basic parts*.

These assumptions are not innocuous. Standard arguments for substratum theory depend on these assumptions. And historically, bundle theory was defended in reaction to and rejection of substratum theory. These points are substantiated in what follows. Section 2 focuses on substratum theory. Section 3 focuses on bundle theory.

⁴⁸ Cf. Williams (1953: 9-10), who claims that a trope is a part of its bearer ‘in the very primitive sense of “part” dealt with by recent calculi of individuals’. In a footnote, he cites Leonard and Goodman (1940)’s calculus of individuals. But it may be proven in this calculus that parthood is both reflexive and transitive (see theorems I.3 and I.31 on p. 49).

2 Substratum theory

The prefix ‘sub-’ in ‘substratum’ alludes to the idea that a substratum is that which ‘underlies’ the properties of an object or is that in which the properties of an object ‘inhere’. Substrata have also been called ‘thin’ or ‘bare’ particulars, in allusion to the metaphor that they are merely ‘clothed in’ as opposed to constituted by properties. By contrast, the familiar objects of ordinary thought and discourse, which include ourselves, tables, and chairs, are ‘thick’ particulars. They are constituted by properties and the thin particulars in which those properties inhere.

Substratum theory has sometimes been motivated by a certain analysis of predication. Take a simple subject-predicate sentence like ‘Socrates is wise’. According to traditional grammar, the subject of the sentence is Socrates, and what is predicated is the property of being wise. But it has appeared to some philosophers that the real subject of predication must be something that does not have the property, or else nothing has been said: ‘if you predicate what is *not* different, you say nothing at all’ (Bradley 1893: 20; see also Loux 2006: 94). Since Socrates does have the property of being wise, the real subject of predication must be something else: an underlying substratum that lacks the property.

The argument from predication is, however, invalid. Grant, as Bradley claims, that the subject of predication must be something different from what is predicated of it, on pain of saying nothing. It does not follow that the subject of the sentence ‘Socrates is wise’ must be something other than Socrates. Socrates has the property of being wise, but he is not identical to the property. So it remains open that something has indeed been said in predicating wisdom of Socrates.

Substratum theory has also been defended on the basis of metaphysical considerations, and it is on this basis that both this theory and bundle theory will be considered in what follows. As stated in section 1, substratum theory assumes that objects have a decomposition into simpler parts*, among which are included their properties. Intuitively, however, an object is not just a collection of properties. Martin (1980: 7) describes the relevant intuition as follows:

An object is not just a group of properties, because properties are not themselves objects to be grouped. An object, therefore, stands in need, not only of a set of properties as an ingredient, but also the ingredient of a bearer of whatever properties are borne.

Granted that an object is not just a collection of properties, and that there must be something that bears the properties of an object, it does not follow that the thing that bears the properties of an object is distinct from the object in question. The intuitive motivation for the introduction of substrata relies on the further premise that properties are ‘ingredients’, or parts*, of their bearers – and more generally, on the assumption that an object has a decomposition into more basic parts*, among which number the object’s properties. That being the case, it is natural enough to hold that an object’s properties cannot be ‘all there is’ to an object. But what is missing is an argument for the assumption that an object has a decomposition into more basic parts*.

The intuitive appeal to the notion of a substratum is further undermined by the problems to which it gives rise. Substrata are supposed to be the bearers of properties. The trouble is that objects like ourselves, tables, and chairs also bear properties. A chair, for example, bears the property of being a chair. Substrata must thus be distinguished from the material objects of which they are substrata, and to this purpose, substratum theorists typically say

that a substratum lacks any properties ‘itself’ or ‘in itself’. It is not always clear how these qualifications are to be understood, though the salient interpretation, in light of the given ideology, is that substrata lack any properties as parts*.⁴⁹ Given Parthood* Constraint (PC), it follows that substrata do not instantiate any properties. (So ‘bearing’ a property must not be equated with ‘instantiating’ a property.)

Thus arises the traditional epistemological worry for substratum theory. If a substratum does not instantiate any properties, it is ‘something I know not what’ (Locke 1689, II, XXIII, ii); it is ‘something unknown and invisible’ (Hume 1739: I, IV, iii). In more words, we know things by their properties. If a substratum instantiates no properties, we can have no knowledge of substrata. And where there can be no knowledge of a thing, there seems little reason to postulate it. Occam’s razor then enjoins us not to postulate it.⁵⁰ Two contemporary responses to this worry are considered here.

2.1 A regress

In response, some contemporary substratum theorists have denied that substrata lack any properties. On one account, substrata are characterised by the ‘leading property’ of being a

⁴⁹ ‘in itself’ can also be used to mean ‘intrinsically’ or ‘essentially’. Thus Sider (2006: 389-94) criticises the suggestion that a substratum does not have any properties ‘in itself’, in the sense that it does not have any properties intrinsically. Loux (2006: 105-7) criticises the suggestion that a substratum does not have any properties essentially.

⁵⁰ Loux (2006: 90) describes another, broadly Humean argument against substratum theory. The argument has two premises. (1) Everything there is can, in principle, be an object of direct experience. (2) Direct experience consists in the apprehension of some property of a thing. Since substrata have no properties, by (2), they cannot be objects of direct experience. By (1), there are no substrata. The obvious problem with this argument is that (1) imposes too high a standard on existence. Many of the things postulated by physics are not objects of direct experience, and given human perceptual limitations, cannot be the objects of direct experience. Those are poor grounds for dismissing the postulates of physics. Perhaps the qualification ‘in principle’ can be refined to rule out this counterexample, but it is doubtful that there is a version of this premise that is invulnerable to refutation. Anyway, few philosophers today would be interested to advance one.

bearer of properties (Martin 1980: 7). In light of this concession, one might wonder whether substrata also have properties such as not being a person or not being red. Given the intuitive principle that x does not instantiate a property p if and only if x instantiates the property $\neg p$, it follows that substrata do have such properties. However, Martin is silent on this point.

Given PC, a substratum instantiates the property of being a bearer of properties if and only if the property is a part* of the substratum. Substratum theory further holds that every object is constituted by its properties and a further thing, an underlying ‘substratum’, in which the properties inhere. By parity of reasoning, we should expect that every substratum is constituted by the properties it instantiates and a further substratum in which these properties inhere. These further substrata must themselves have properties that inhere in still further substrata. And so on, ad infinitum. The present revision to substratum theory thus leads to an infinite regress.

This regress may be prevented by denying that PC applies to the properties instantiated by substrata or by denying, more generally, that substrata have any parts*. Some justification is then wanted for this denial, other than that it would prevent a regress. For the same justification would motivate the claim that material objects like ourselves, tables, and chairs lack any parts*, from which it follows that substratum theory is false.

2.2 The redundancy of substrata

A second response to the worry that there can be no knowledge of substrata says that substrata have all the properties that are ordinarily attributed to objects (Sider 2006). Thus

a substratum may be red, round, and juicy; it may be a tomato. But it does not have these properties as parts*, on which basis it is distinguished from the object, or thick particular, of which it is the substratum.

The view encounters the following dilemma. If thick particulars instantiate these properties also, then we have two tomatoes, e.g., where we thought there was one: the thick particular and the substrata, which are distinct. The problem generalises. There will be many more tomatoes, chairs, and so on than ordinary discourse recognises.

If thick particulars do not instantiate these properties, then the introduction of substrata is made redundant. Substrata were introduced in a particular analysis of material objects, in order to serve as the bearers of *their* properties. Now it has emerged that material objects do not instantiate any properties. Instead, substrata play the role of material objects: they are people, tables, chairs, and so forth. But on this view, substrata lack any properties as parts and simply instantiate them. This is to abandon the project of giving an ontological reduction of material objects into more basic parts*, though it is in the context of this project that substratum theory was originally motivated. In particular, since the thick particular is now inessential to an account of what there is, the notion of parthood*, in terms of which the thick particular is characterised, also becomes redundant: there are simply substrata, properties, and a relation of instantiation in which they stand. But then the notion of a substratum does no work that is not done by the familiar notion of an object. It could simply be said that there are objects, properties, and a relation of instantiation in which they stand.

3 Bundle theory

Historically, the bundle theory of objects was defended in reaction to and rejection of substratum theory (see Hume 1739: I, iv, 2). Philosophers denied the existence of substrata, without denying or otherwise challenging the assumption that an object had a decomposition into more basic parts*, among which numbered the object's properties. Given these choice points, either (1) an object must be composed of something other than a substratum in addition to its properties, or (2) it must be composed solely of its properties. The latter alternative forms the central premise of bundle theory. The rest of this section examines three problems for bundle theory.

3.1 Immanent parthood*

It is standardly assumed that the properties in a bundle must be located where their bearers are (so e.g. Hawthorne 1995: 191). That is,

Immanent Parthood*	If bundle theory is true, then: a property x is a part* of y
Constraint	only if x is located in space and time.

It must be said that Immanent Parthood* Constraint (IPC) is not obviously true. It is at least logically possible that an object has transcendent properties as parts* and nothing else, given what we are told of the relevant notion of parthood*. However, it is true that parthood* is a theoretical notion, and to this extent, its properties are somewhat open to stipulation. So let IPC be granted. Chapter 2 criticised an immanent theory of properties, on the basis that it ruled out the existence of both uninstantiated properties and non-

located things (e.g. sets, numbers, spatiotemporal points). These problems are now inherited by bundle theory.

3.2 The difficulty of imperfect community

On bundle theory, objects are mere bundles of properties. But not every bundle of properties is an object. For example, a bundle including both the properties of being red and being a cloud will not be an object, since no (actual) object instantiates both properties. Only certain bundles compose objects; and a problem for bundle theory is to identify which.

This problem cannot be settled by saying that all and only the properties instantiated by a given object are parts* of that object, on pain of circularity. Bundle theory aims to give an analysis of objects into their more basic parts*, but the analysis becomes circular if these parts* are identified only with reference to the objects of which they are parts*. ⁵¹

Russell (1948: 256-69) answered the problem by appeal to a distinct relation, which he called 'compresence'. Informally, it is the relation that holds between any properties of a single object; but this is to be taken as an informal gloss only, for it is in terms of this notion that non-trivial, necessary and sufficient conditions for being an object are to be described. A better grasp of the relation may be acquired by considering some of its formal characteristics. For example, compresence is reflexive and symmetric, but not transitive. For a counterexample to transitivity: there is an apple that has the properties of being red

⁵¹ By comparison, substratum theory implies that all and only the properties that inhere in a given substratum are parts* of a single object. Insofar as the relevant properties are identifiable with reference to the substratum, as opposed to the material object (= thick particular), the analysis of material objects is, arguably, non-circular.

and being round, and an orange that has the properties of being round and not being red; but nothing has the properties of being red and not being red. Thus there are properties *a*, *b*, and *c* such that: *a* is compresent with *b*, and *b* with *c*, though *a* is not compresent with *c*.

Now a ‘complete complex of compresence’ is a plurality such that (i) every member of the plurality is compresent with every other and (ii) no non-member is compresent with every member. The proposal is that every complete complex of compresence composes a single object.

It has been objected that this solution is vitiated by a version of ‘the difficulty of imperfect community’ (Armstrong 1978a: 98-100, 1989b: 71-2).⁵² Suppose that an object *x* has properties *a* and *b* but not *c*, *y* has *b* and *c* but not *a*, *z* has *a* and *c* but not *b*, and that nothing has all of *a*, *b*, and *c*. Then *a* is compresent with *b*, *b* with *c*, and *c* with *a*. By definition, *a*, *b*, and *c* form a complex of compresence. Either this complex is complete, or it is not complete. If it is complete, then it composes an object. But by assumption, no object has all of *a*, *b*, and *c*. Contradiction. If it is not complete, then let properties be added to the complex until it is complete. The resulting complete complex of compresence must compose an object; but by assumption, no object has all of *a*, *b*, and *c*. Contradiction. Thus, Armstrong concluded that Russell’s solution was incorrect.

However, his objection mistakes compresence for a binary relation that holds at most between two things.⁵³ Russell is explicit that it is not: compresence ‘holds between two or more qualities’ (1948: 260). In light of his views on the metaphysics of relations, Russell

⁵² Goodman (1966: 119-25) first introduced the difficulty as a challenge for Carnap (1928)’s analysis of properties in terms of basic concrete units, or ‘erlebs’, and a primitive binary relation on them.

⁵³ Hochberg (2001: 105-6) also makes this point.

is naturally read here to have held that there is a distinct compresence relation for every adicity, as opposed to a single multigrade relation with variable adicity. But either reading dissolves the difficulty of imperfect community, for either reading allows that *a* and *b* may be compresent, *b* and *c* compresent, and *a* and *c* compresent, while all of *a*, *b*, and *c* are not compresent. It follows that there is no complete complex of compresence including all of *a*, *b*, and *c*.⁵⁴

That said, there may be another problem for Russell's proposal. By stipulation, compresence holds between any properties of a single object. As mentioned, this informal gloss cannot be taken as a definition of compresence: it appeals to the notion of an object, which it is the aim of bundle theory to analyse. Moreover, the fact that compresence is a reflexive, symmetric, and non-transitive relation does not suffice to distinguish compresence from all other relations, for there are many relations with these formal properties – being of a similar height is one. Of course, being of a similar height does not hold between the properties of a single object, but to distinguish this relation from compresence on this basis is again to appeal to the notion of an object. It appears that compresence must remain a theoretical primitive, in terms of which the notion of object is analysed. The problem is that it is not clear what advantages there are to a theory in which compresence is primitive, as opposed to one in which the familiar notion of object is.⁵⁵

⁵⁴ A third view, on which compresence is a property of plurals, also answers the difficulty. Campbell (1981: 481-3) has argued that the difficulty of imperfect community may be resolved by taking properties to be tropes, for if properties are tropes, compresence is an equivalence relation. Objects may then be identified with bundles of tropes, all and only whose constituents are members of a given equivalence class. Campbell concludes that the ease with which bundles are constructed on a trope-theoretic version of the bundle yields a point for trope theory. Against Campbell, a resolution of the difficulty of imperfect community does not necessitate an alternative metaphysics of properties.

⁵⁵ An example of such a theory would be substance theory, according to which the notion of a 'substance' or individual object is held primitive. It denies the basic assumption of both substratum theory and bundle theory, that objects have a decomposition into parts that are, in some sense, more basic. For an introduction to substance theory, see Loux (2006: 107-17).

3.3 The identity of indiscernibles

The third and final problem for bundle theory discussed here concerns the principle of the identity of indiscernibles.

On bundle theory, objects have no parts* other than their properties. It follows that:

Bundle Theorem Necessarily, objects x and y have the same parts* if
and only if x and y instantiate the same properties.

The theorem holds of necessity, insofar as bundle theory is a metaphysical theory whose premises hold of necessity. Now it is standardly held that:

Identity of Parts* Necessarily, if objects x and y have exactly the same
parts*, then x is y .

Bundle Theorem (BT) and Identity of Parts* (IP) imply:

Identity of Indiscernibles Necessarily, if objects x and y instantiate exactly the
same properties, then x is y .⁵⁶

However, certain thought experiments seem to show that the principle of the Identity of Indiscernibles (PII) is false (Black 1952: 156). Imagine a world in which there are only two iron spheres of exactly the same mass, size, shape, and colour, located a metre apart.

⁵⁶ There are other versions of this principle. For an extended discussion of some salient versions, see Rodriguez-Pereyra (2022: 5-60). Rodriguez-Pereyra (2014) discusses the principle in the context of Leibniz's philosophy.

These spheres share all their properties as well as their relations to each other (and thus, to everything else). If such a world is possible, then:

Difference of Indiscernibles

Possibly, there are distinct objects that instantiate exactly the same properties.

Clearly, Difference of Indiscernibles (DI) and PII imply a contradiction. By reductio ad absurdum, one of BT, IP, PII, and DI must be false.⁵⁷

The rest of this section criticises six bundle-theoretic solutions to this problem. The first five deny DI. In particular, they deny that thought experiments like Black's demonstrate the possibility of distinct indiscernible objects. Either there are properties that are not had by both the iron spheres in question, or contrary to the initial stipulation, there are not two spheres, but one. The sixth and last solution denies IP.

3.3.1 Locations

Solution. The two spheres differ in their locations; thus they do not share all their properties.

Reply. By stipulation, the iron spheres in Black's world share exactly the same spatiotemporal relations to each other and to everything else. The solution works by

⁵⁷ By comparison, substratum theory does not imply the identity of indiscernibles. The theory leaves open the possibility of there being distinct objects that instantiate exactly the same properties, which are nevertheless distinguished by their distinct substrata. Loux (2006: 101-4) concludes on this basis that substrata are needed for the individuation of objects. However, the individuation of substrata remains primitive on his view.

pointing out that the stipulation may have failed to consider the regions or places at which these iron spheres are located. If so, there exist things other than the spheres, with respect to which they may be distinguished. That is, the spheres are distinguishable by their locations only if there are distinct places at which the spheres are located. Thus the solution implies that there are such things as places, and more generally that space(time) exists. This is not uncontroversial.⁵⁸

That controversy aside, the present solution may be refuted even if it is allowed that space exists. For consider a world in which there are just two indiscernible spacetime points. These points may be either unoccupied or occupied by objects that are themselves indiscernible. In either case, these points cannot be distinguished by their locations in spacetime, for they have none. So it looks as if the appeal to locations will not work after all.⁵⁹

3.3.2 Haecceities

Solution. The two spheres differ in their non-qualitative, as opposed to qualitative, properties. By stipulation, the spheres share their mass, size, shape, and colour. Such properties are ‘qualitative’. The ‘non-qualitative’ properties, by contrast, are properties like being identical to this sphere or being Socrates. Non-qualitative properties are also called ‘haecceities’, after the Latin ‘haec’, meaning ‘this’. The idea is that a haecceity is a

⁵⁸ Spacetime substantivalism is the view that spacetime exists. It is opposed to spacetime relationalism. On the latter view, spacetime does not exist, and claims ostensibly about spacetime are to be parsed as claims about material objects and their possible spatiotemporal relations. See Pooley (2013) for an introduction to this debate.

⁵⁹ Famously, Leibniz defended relationalism about space as well as the principle of the identity of indiscernibles and its converse, the indiscernibility of identicals. Thus these principles are also known as Leibniz’s Laws.

property that distinguishes an object as ‘this’ object, or the individual object that it is. Accordingly, it cannot be had by any object other than the object by which it is in fact had.⁶⁰

Reply. An appeal to haecceities is incongruous with the bundle-theoretic aim of analysing individual objects like the spheres in question into their more basic parts*. By all appearances, haecceities are individuated by the objects of which they are haecceities. To use haecceities to individuate their bearers in turn renders the bundle-theoretic analysis of objects circular.

3.3.3 Bundles of tropes

Solution. Properties are tropes, as opposed to universals. Tropes cannot be had by more than one object, so necessarily, if x and y have exactly the same properties, x is y . Thus trope theory implies PII. However, trope theory leaves open the possibility of there being distinct objects that have exactly similar, yet numerically distinct, tropes, which on standard trope-theoretic assumptions, is just what it takes for the objects in question to be, in all qualitative respects, exactly similar (so Campbell 1981: 481-3). Thus trope theory yields a plausible diagnosis of our conflicting intuitions concerning PII and the possibility of a world like Black’s.

Reply. Chapter 3 argued that trope theory does not measure up to the explanatory power, simplicity, or quantitative ontological economy of a theory of universals. Granted, there

⁶⁰ The notion of a haecceity is due to Duns Scotus (*Quaestiones in libros metaphysicorum*, VII. xii. schol. 3, cf. *Ordinatio*, II.3.1.2, 57).

has now emerged an argument for trope theory which was not then considered: it may redeem bundle theory from a conflict with certain intuitive thought experiments. But the other problems for trope theory, which were identified in that chapter, remain.

It is also worth mentioning that some accounts of the individuation of tropes undermine the trope-theoretic solution to this problem. For example, it is commonly held that exactly similar tropes are individuated by the objects of which they are tropes (e.g. Lowe 1998: 206). For the reasons described in section 3.3.2 on haecceities, this would render the bundle-theoretic analysis of objects circular: objects are individuated by their distinct tropes, which are themselves individuated by their distinct bearers. Alternatively, tropes have also been individuated by their locations in spacetime (e.g. Schaffer 2001: 248-56). But assuming that places or spatiotemporal points are themselves subject to a bundle-theoretic analysis, the tropes that compose them must themselves be individuated. However, they cannot be individuated by their locations, for they have none. This is a version of the problem raised in section 3.3.1 on locations.⁶¹

3.3.4 From conceivability to possibility

The previous three solutions each denied DI by denying that the iron spheres in Black's world had exactly the same properties. The next two solutions deny DI by denying, instead, that there is more than one sphere in his world.

⁶¹ By comparison, Campbell (1991: 54) relinquishes the individuation of tropes by location and adopts primitivism instead after considering the possibility of non-located tropes.

Solution. Black may have mistaken the content of his imagination. Perhaps what he took to be two indiscernible spheres located a foot apart was really a single sphere located in a curved or symmetric space, a foot apart from itself. There are distinct indiscernible objects in Black's world only if space is Euclidean, but it remains to be shown that space *is* Euclidean in Black's world. The moral: 'No matter how vivid your imagination, it remains a question how correctly to describe the content of your imagination' (Hacking 1975: 251).

Reply. The moral is misleading. In at least some cases, part of what is imagined is stipulated. If I imagine Trump wearing a wig, then it is Trump – not someone else, like his long-lost twin or a look-alike – that I imagine wearing a wig. If Max Black claims to have imagined two qualitatively indiscernible spheres, then the challenge his imagined scenario poses for bundle theory cannot be dismissed by saying that Black has really imagined something else. The content of his imagination is, to some extent, determined by his initial description of it, and not solely by the corresponding mental picture (cf. Rodriguez-Pereyra 2022: 76). To give an alternative description of the content of Black's imagination is thus to change the world in question.⁶²

The issue is not whether Black imagined a world with two qualitatively indiscernible spheres. The issue is whether the world he imagined is metaphysically possible. If it is possible, then it is a counterexample to PII; if it is not possible, then it is not a counterexample to PII.

⁶² On imagination, see Williamson (2016) and the contributions in Badura and Kind (2021).

There is a more general question here, concerning the relation between conceivability and possibility: does the former entail the latter?⁶³

Without pretending to answer the general question, it seems that in the particular case at hand, Black's world may be shown to be metaphysically possible, by an argument due to Adams (1979: 17-9). Consider a world in which there are two iron spheres, exactly similar to each other save for a small dent on one. In this world, there are certainly distinct spheres: they have different properties. This world is surely possible. But if it is, then surely a world with two indiscernible objects is also possible. For the dented sphere might not have been dented. Thus, it seems, we can infer the possibility of two indiscernible objects from the uncontested possibility of two almost indiscernible objects.

The modal principle in question is S4, which says that what is possibly possible is itself possible. S4 is a standard, if not universally, accepted principle of metaphysical modality.⁶⁴ In light of the above argument, a solution that simply denies that Black did imagine a world with two indiscernible spheres is seen to be beside the point; and a solution that denies that such a world is possible requires further defence.⁶⁵

⁶³ There is an extensive literature on this topic, for which see e.g. Yablo (1993), Chalmers (1996: 52-71), Jackson (1998: 67-86), and Gendler and Hawthorne (2002).

⁶⁴ The standard logic of metaphysical modality is the system S5, which includes the modal principle S4. For a dissenting voice, see Salmon (1986), who rejects S4 in light of the puzzles of material constitution. Williamson (2013: 30-80) discusses S4, S5, and related principles in the context of metaphysical modality.

⁶⁵ Rodriguez-Pereyra (2017) raises some doubts for Adam's argument. Rodriguez-Pereyra (2022: 75-100) defends alternative arguments for the metaphysical possibility of Black's world.

3.3.5 Multiply located bundles

Solution. Black's world is not metaphysically possible, given both a bundle theory of objects and an immanent theory of properties on which properties are located wherever they are instantiated. On this theory of properties, multiply instantiated properties are multiply located. And if properties can be multiply located, it must also be allowed that bundles of properties can be multiply located too (Hawthorne 1995: 195). That is, if the property of being iron can be located a foot apart from itself, then surely the bundle of being iron and being a sphere can also be located a foot apart from itself. The following thus becomes possible: there is a bundle of properties located a foot apart from itself, and nothing else. Black's world is such a world.

Reply. Assuming that properties can be multiply located, it does not follow that bundles of properties can also be multiply located. A bundle of properties is not just a plurality of properties. It has those properties as parts*, and the parts are also supposed to stand in a special relation, called 'compresence, with each other. A bundle of properties can cease to exist even while the properties continue to exist; the plurality cannot (nor, for that matter, can the mereological sum of those properties). In light of the distinction between bundles of properties and pluralities of properties, it is open to claim that bundles cannot be multiply located, though their parts can. The multiple location of bundles is not entailed by the multiple location of properties, in the way that Hawthorne claims.

That said, the relevant notions of parthood* and compresence are so threadbare that it is, arguably, open to stipulation that bundles can be multiply located, and that any world in which there appear to be indiscernible objects is really a world in which there is a single object, multiply located. Then Black's world is impossible, and DI is false.

It should be noted, however, that this solution will generate conflicts with significant parts of our ordinary thought and discourse.

The most obvious conflicts involve counting claims. Suppose there are what we would ordinarily describe as two drops of water on the window. They share all their intrinsic properties. They differ in their relational properties, such as their relative proximity to each corner of the window. But since the iron spheres in Black's world are not differentiated by appeal to any relational differences, it seems that these drops of water ought not be differentiated by such differences either. Thus it will be held, by parity of reasoning, that there are not two drops on the sill after all, but only one, located a certain distance apart from itself. Assuming such situations do obtain, Hawthorne's solution predicts that our ordinary counts in such situations will be false.

Other sources of conflict are claims concerning location. The present solution cannot recognise the possibility of two indiscernible spacetime points. For this reason, Hawthorne and Cover (1998) suggest that if there are any spacetime points at all, there is only one: a bundle comprising pointhood and instanthood, at certain distances from itself. It appears to be a consequence of this view that everything that is located at all must be located in this point, for there is no where else to be. So everything that is located at all is located in the same place, a consequence which conflicts with ordinary and scientific views of location (see Rodriguez-Pereyra 2004: 75).

3.3.6 Bundles and their instances

The sixth and last solution considered here differs from the others, insofar as it does not deny DI, but rather IP.

Solution. Material objects like ourselves, tables, and chairs are not bundles of properties, but *instances* of bundles of properties (Rodriguez-Pereyra 2004: 77-8). Wherever there is a bundle of properties, there is also an instance of the bundle. Each instance is entirely composed of the properties of the bundle, but it is distinct from the bundle. Unlike the bundle, the instance cannot be multiply located. So a bundle that is multiply located has multiple instances, one in each place that the bundle is. The upshot is that objects are entirely composed of properties, without being identical to bundles of properties. If a bundle of properties has more than one instance, then there are distinct objects with exactly the same parts*. Such is the situation in Black's world. IP is false.

Reply. The account above follows Rodriguez-Pereyra's presentation of the solution, but there is a minor inconsistency. The solution demands that each instance is entirely composed of the properties that compose the bundle, and that each instance is distinct from the bundle. But the conjunction of these claims implies that if a bundle has even *one* instance, there are distinct particulars with exactly the same constituents: the bundle and its instance.

The central premise of bundle theory is that objects are composed of the properties they instantiate. Since the bundle and its instances are composed of the very same properties, they instantiate the very same properties. But the fact that the bundle and its instances can be distinguished at all implies, by PII, that they differ in their properties. Rodriguez-

Pereyra rejects PII on behalf of bundle theory, on the basis that distinct instances of a single bundle may instantiate exactly the same properties.⁶⁶ But PII aside, it is clear that the instances do not instantiate the very same properties: they have different locations, for one.

It is equally clear that the bundle and its instances do not instantiate the very same properties. It is the instances that this solution identifies with material objects like ourselves, tables, and chairs; so the instance of a given bundle, say, will be a chair, while the bundle lacks that property. Moreover, the instance of a bundle has the property of being an instance; the bundle lacks that property. Conversely, the bundle has the property of being a bundle, which the instance lacks. The bundle can also be multiply located; the instance cannot. And so on.

The central premise of this version of bundle theory is that material objects are instances of bundles of properties. This implies that there exist both bundles and instances thereof. A point might be docked against this solution for lack of ontological economy. That aside, the previous criticisms show that if this version of bundle theory is to be maintained, some revisions must be made. One option is to deny that sortals, locations, modal profiles, and so on are properties, and thus to maintain that bundles and their instances do have exactly the same properties. It is expected that there will be some principled basis for distinguishing the properties from the non-properties, independent of the needs of this

⁶⁶ More precisely, he rejects a version of PII restricted to universals, according to which for any x, y : if x and y instantiate exactly the same universals, x is y . Correlatively, bundle theory is identified as the theory that objects are composed of exactly the universals they instantiate. It is understood that the universals form a subset of the properties. Thus haecceities are not universals, where the salient properties of the spheres in Black's world are: diameter, temperature, colour, shape, and size (*op. cit.*: 72). This distinction does not affect the argument of the main text. All the properties mentioned subsequently are universals, by Rodriguez-Pereyra's lights.

theory. Another option is to deny that instances have exactly the same properties as the bundles of which they are instances, though in this case, it is unclear in what sense they are ‘instances’ of the relevant bundles at all.

Finally, it bears mention that Rodriguez-Pereyra (2004: 74) criticises Hawthorne’s solution on the basis that it cannot accommodate the possibility of there being two indiscernible spheres. But his version of bundle theory also cannot accommodate certain intuitive possibilities, such as the possibility of there being just one iron sphere, and nothing else – for where there is the sphere, there is also the bundle of which it is an instance.

Chapter 5

Higher-order Nominalism

1 The property paradox

Properties are to 1-place predicates as relations are to n -place predicates, for $n > 1$. This intuitive slogan suggests a plenitudinous conception of properties and relations, according to which there is at least a property or relation for every n -place predicate, for $n \geq 1$. The intuitive plausibility of the slogan stems, at least in part, from the natural language equivalence between sentences of the form ‘ x is F ’ and ‘ x has the property of being F ’. Such sentences are held to be equivalent, in a sense that implies, at least, their material equivalence. This equivalence generalises to the following principle:

Naïve Comprehension for Properties	For any formula, there is something such that it is a property and anything instantiates it if and only if the thing satisfies the formula.
---	---

$$\exists y (Py \wedge \forall x_1 \dots \forall x_n (Ix_1 \dots x_n y \leftrightarrow \phi x_1 \dots x_n)),$$

where P stands for ‘... is a property’, I stands for ‘... instantiate(s) ...’, and ϕ is any formula, provided that y does not occur free in ϕ .

Naïve Comprehension for Properties (NC-P) is a schema, of which there is an instance for each formula ϕ defined for the language, provided that y does not occur free in ϕ .

Assuming that the language has the standard logical connectives, some formulas will not have the form of a predication, as in Fxy , but the form of conditionals, conjunctions,

disjunctions, and negations, as in $Fx \rightarrow Gx$, $Fx \wedge Gy$, $Fx \vee Gy$, and $\neg Fx$. The collective instances of NC-P thus embody a plenitudinous view of properties, on which there is at least a property corresponding to any arbitrary formula, simple or complex.

The pejorative moniker ‘naïve’ alludes not only to the principle’s intuitive appeal, but also to the inconsistency of some of its instances. To see this, consider the predicate ‘does not instantiate itself’. An instance of NC-P for this predicate is ‘There is a property such that anything instantiates it if and only if the thing does not instantiate itself’, which is an instance of an inconsistent propositional form.

Formally, a contradiction may be derived as follows. Let ϕ be $\neg Ixx$:

- | | |
|---|----------------------------|
| 1. $\exists y (Py \wedge \forall x (Ixy \leftrightarrow \neg Ixx))$ | NC-P |
| 2. $Pa \wedge \forall x (Ixa \leftrightarrow \neg Ixx)$ | Existential Elimination |
| 3. $\forall x (Ixa \leftrightarrow \neg Ixx)$ | Conjunction Elimination |
| 4. $Iaa \leftrightarrow \neg Iaa$ | Universal Elimination |
| 5. Iaa | Assumption |
| 6. $\neg Iaa$ | 4, 5, Modus Ponens |
| 7. $\neg Iaa$ | 5, 6, Reductio ad Absurdum |
| 8. Iaa | 4, 7, Modus Ponens |
| 9. $\exists y (Py \wedge \forall x (Ixy \leftrightarrow \neg Ixx))$ | 7, 8, Reductio ad Absurdum |

So NC-P, at least in full generality, is not valid: it has some false instances.⁶⁷

⁶⁷ Russell’s paradox is often presented using the law of excluded middle. By LEM, $Iaa \vee \neg Iaa$, and from either assumption, a contradiction follows, by 4 and modus ponens. The proof in the main text is stronger. It derives a contradiction from a theorem, without the use of LEM. It thus valid even in intuitionistic logic.

The obvious solution is to deny the paradox-inducing instances of NC-P. Predicates that entail contradictions when substituted for ϕ do not correspond to a property with the same extension; they are exceptions to NC-P. Assuming, moreover, that no predicate corresponds to a property with a different extension, such predicates do not correspond to a property at all.

But the solution has not won general approval. As Russell wrote, such exceptions are not ‘such as even the cleverest logician would have thought of if he had not known of the contradictions’ (1959: 80). There exists a presumption that an adequate solution to the paradox will have a motivation independent of the paradox, one which the present solution lacks (also Dummett 1991: 188-9).

That presumption deserves elaboration. In one sense, the constraint that an adequate solution will have a motivation independent of the paradox is too strong. Contemporary theories of vagueness are not motivated independently of the sorites paradox; theories of truth are not motivated independently of the liar paradox; contemporary set theory and other non-standard theories of sets are not motivated independently of Russell’s paradox for sets. If such theories had been put forward in a context in which the paradoxes remained unknown, none of them would have carried conviction. But those are not grounds on which to reject such theories.

The point must be that the final resolution of the paradox cannot be embodied in a theory of properties whose *sole* motivation is that it avoids paradox. The property paradox is a legitimate motivation for restricting comprehension. The question is how, exactly, comprehension is to be restricted. For example, a theory of properties that denied the

paradox-inducing instances of NC-P as well as some others would also avoid contradiction. In the presence of multiple possible restrictions on comprehension, some method of choosing between them is wanted. Of relevance here are the usual theoretical virtues: explanatory power, simplicity, integration with other theories, and so on. A theory of properties ought to avoid the property paradox, and it ought to exhibit, so far as possible, the theoretical virtues. And in particular, it ought to be adequate for its intended applications.

The problem for any solution to the property paradox that restricts NC-P is that any such restrictions disrupt at least some of the intended applications of a theory of properties.

NC-P gives at least partial expression to our intuitive conception of properties, which draws a close connection between property instantiation and predicate satisfaction. This connection influences the theoretical work to which properties are applied. Most obviously, properties have traditionally been expected to serve as the semantic values of predicates, like ‘is just’, and abstract singular terms, like ‘justice’, which are formed from predicates by a general device of nominalisation.⁶⁸ For this purpose, there ought to be at least a property corresponding to every meaningful predicate.

However, the paradoxical predicate ‘does not instantiate itself’ is meaningful. The meaningfulness of this predicate may be deduced from the principle of compositionality, which says that the meaning of a complex expression is a function of the meanings of its constituents and their manner of composition. The sentence ‘This chair does not instantiate itself’ is true; and since truth presupposes meaning, it is meaningful. By compositionality,

⁶⁸ See e.g. Lewis (1983) or Oliver (1996) for an introduction to the role of properties in semantics.

its constituents must also be meaningful; and this includes the constituent predicate ‘does not instantiate itself’. From another angle, compositionality also implies that meaningful predicates are closed under complementation. Since ‘instantiates itself’ is meaningful, so is ‘does not instantiate itself’.

The principle of compositionality is a standard assumption in contemporary philosophy of language. A solution to the property paradox that denied that ‘does not instantiate itself’, or any other meaningful predicate, determined a property would thus disrupt the intended applications of a theory of properties in semantics.⁶⁹

It is a common misconception that plenitudinous properties have application primarily to semantics, where sparse properties are more appropriate for metaphysics. Even if a sparse conception of properties is needed in metaphysics, a plenitudinous conception of properties is needed also.⁷⁰ Properties are used, *inter alia*, to individuate things. The principle of the Indiscernibility of Identicals is a prime example. It says that if x is y , then x and y have exactly the same properties. Socrates is Greek, and Cicero is not; by NC-P, Socrates has a property Cicero lacks; by the Indiscernibility of Identicals, Socrates is not Cicero. To restrict NC-P is to restrict applications of Leibniz’s Laws that are essential to metaphysical theorising (Williamson 2013: 226-9).

Other things being equal, it may be supposed that a solution to the paradox that restricts NC-P to rule out all and only its paradox-inducing instances is to be preferred over other

⁶⁹ On compositionality, see e.g. Montague (1974), Partee (2004), and Pagin and Westerståhl (2010).

⁷⁰ It remains an open question how exactly a sparse conception of properties is to be spelled out, but it is generally understood that such a conception will entail that there are far fewer properties than predicates. The distinction between plenitudinous (abundant) properties and sparse properties is drawn in Lewis (1986b: 59-61), who acknowledges properties of both conceptions.

solutions, insofar as it embodies the least set of restrictions on comprehension.

However, there are at least two reasons to believe that this view may be too simplistic.

First, it is possible that there is no ‘least set’ of restrictions on comprehension – that certain instances of NC-P are individually consistent, but jointly entail a contradiction. This would be a property-theoretic version of the ‘bad company’ problem in the philosophy of mathematics, according to which certain instances of the Naïve Comprehension Principle for Sets and other abstraction principles are individually consistent, but jointly entail a contradiction.⁷¹

Second, it remains to be seen whether another solution to the property paradox is embodied in a theory of properties that better satisfies its intended applications. For this reason, the rest of this chapter examines two other potential solutions to the property paradox. Section 2 examines, and ultimately rejects, the prominent view that a solution to the property paradox can be ‘adapted’ from a solution to Russell’s paradox for sets. The crux of this argument is that set theory and property theory have distinct intended applications and are thus constrained by distinct formal and informal desiderata. Section 3 motivates a higher-order resolution of the paradox that satisfies the property-theoretic desiderata identified in section 2. In support of this solution, section 4 makes a case for higher-order logic. A note on the relational type theory assumed in applications of higher-order logic in this and other chapters of the thesis occupies section 5.

⁷¹ On the bad company problem, see e.g. Boolos (1997), Weir (2004), Linnebo (2018), and Studd (2023). Assuming Naïve Comprehension for both Sets and Properties, sets can be simulated either as (1) one-place properties, each of which is had by just one set, or (2) relations whose instances are ordered pairs of members of the set. Either alternative will yield a one-one correspondence between sets and properties. Thus naïve set theory can be simulated within naïve property theory, with the qualification that property theory, unlike set theory, will not be extensional. But since extensionality is not needed for Russell’s paradox or its property-theoretic versions, it is *prima facie* possible that a version of the bad company problem can be simulated for property theory. I leave the development of this topic for another occasion.

2 A comparison with set theory

The property paradox is a version of Russell's paradox for sets, which derives from an instance of the following principle:

Naïve Comprehension for Sets	For any formula, there is something such that it is a set and any thing is a member of it if and only if the thing satisfies the formula.
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$$\exists y (Sy \wedge \forall x (x \in y \leftrightarrow \varphi x)),$$

where S stands for '... is a set', and y does not occur free in φ .

As with NC-P, the moniker 'naïve' alludes both to the intuitive appeal of Naïve Comprehension for Sets (NC-S) and to the inconsistency of some of its instances. One such instance is 'There is a set such that anything is a member of it if and only if the thing is not a member of itself'. Formally, a contradiction may be derived by steps analogous to those used to formulate the property paradox, with $x \notin x$ for φ .

Given the obvious structural analogies between the two paradoxes, there arose the general idea that a solution to the property paradox could be adapted from a solution to the set paradox, on which work is, by comparison, far more advanced (e.g. Bealer 1982: 86, van Inwagen 2004: 134). It is worth noting, however, that there are at least two dimensions along which this general idea may be varied in its application.

First, there is more than one existing solution to the set paradox. The standard solution is embodied in contemporary set theory, which is Zermelo-Frankel set theory (ZF). But there are also other, non-standard theories of sets that describe different solutions to the set paradox. There is thus a choice of theories on which to model a solution to the property paradox.

Second, there is more than one way in which a given solution to the set paradox may be ‘adapted’ to a solution to the property paradox. On one extreme, properties might be identified with sets. Let *set nominalism* be the view that properties are sets. Short of this extreme, some, but not all, features of a solution to the set paradox might be replicated within a solution to the property paradox.

A proper evaluation of the idea that a solution to one paradox can be adapted to a solution to the other would demand its development along both dimensions just mentioned.⁷²

However, the present chapter is content merely to undercut its intuitive appeal. For this purpose, it will suffice to assume ZF. Other, non-standard theories of sets are non-standard precisely because they are not generally acknowledged to constitute adequate solutions to the set paradox – a fact which, I take it, suffices to undermine the intuitive appeal of adapting any one of these solutions to the property case. The rest of this section considers, and ultimately rejects, two possible ways of adapting ZF to a solution to the property paradox. Section 2.1 considers set nominalism; incidentally, the criticisms raised here are not specific to ZF and pertain to other set theories. Section 2.2 considers a view on which

⁷² Cf. van Inwagen (2004: 134), who claims that one can ‘easily adapt’ any one of the existing solutions to the set paradox to a solution to the property paradox, with the results being ‘neither more nor less intellectually satisfying than the originals’. He does not elaborate.

an iterative conception of sets, to which ZF arguably gives expression, is adapted to a theory of properties.

2.1 Set nominalism

The obvious objection to the identification of properties with sets is that sets are extensional, and properties are not.

Extensionality is written into our intuitive conception of sets. Intuitively, a set is just a collection of things; they are built up out of their members, or as Cocchiarella (1989: 78) puts it, a set ‘has its being in the objects that belong to it’. Thus no two sets have exactly the same members: $\forall x \forall y (Sx \wedge Sy \wedge \forall z (z \in x \leftrightarrow z \in y) \rightarrow x = y)$ is an axiom of ZF. It has been argued that this axiom enjoys an epistemological status not shared by other set-theoretic principles: the principle is analytic, true by virtue of the meanings of the words contained in it, and a theory that denied it would not be recognised as a theory of sets (Boolos 1971: 229-30).

By contrast, properties are not thought to be ‘built up’ out of their instances; rather, they ‘characterise’ them. Intuitively, therefore, they are not extensional: there are distinct properties with exactly the same instances. The classic examples, due to Quine (1951), are the properties of being cordate and being renate, but there are many others, including having size and having shape, being red and being red and not a flying pig, and so on.

Relatedly, some of the intended applications of property theory demand that properties are not extensional. Standard semantic theories, for example, imply that at least some co-

extensive predicates, like ‘is cordate’ and ‘is renate’, do not have the same semantic values. So if properties are to serve as the semantic values of predicates, sameness of extension ought not to imply sameness of property.

The one rider to this is modal realism. Sameness of extension is not sameness of property, for which reason properties cannot be identified with sets – unless some of their members are possibilia. It should be noted that the requisite conception of possibilia is one on which every object is in exactly one possible world. The most prominent such theory is Lewis’s modal realism, on which all possible worlds are equally real, and *de re* possibility is interpreted in terms of an object’s counterparts in different possible worlds. Since in some possible worlds, there are things that are cordate but not renate, and in others, there are things that are renate but not cordate, the set of all and only the cordate things is not the set of all and only the renate things. On such a theory, the identification of properties with sets of their instances does not have the consequence, e.g., that the property of being renate is the property of being cordate (Lewis 1986b: 50-69).

There are two quick objections to the appeal to modal realism in this context. The first objection is to modal realism itself: despite its prominence, the theory is generally rejected. Second, it implies that sameness of *intension* is sameness of property, though this, too, is controversial.⁷³

Matters of grain aside, the identification of properties with sets may obviate the formulation of a distinctive, property-theoretic version of the set paradox, but it

⁷³ It is widely held that properties are *hyperintensional*, or individuated more finely than by their intensions; see e.g. Bader (2013), Hale (2015: 189), and Rodriguez-Pereyra (2022: 10). For pushback and a defence of intensionality, see Williamson (2021, 2022).

foregrounds the need for a solution to the set paradox itself. Suppose that solution is embodied in contemporary set theory, i.e. ZF. There are reasons, further to extensionality, that ZF cannot fulfil the intended applications of a theory of properties. They are described in the following section.

2.2 An iterative conception of properties

As mentioned, we have an intuitive conception of sets, according to which a set is just some collection of things. That conception is embodied in naïve set theory. A different, iterative conception of sets is arguably embodied in ZF – a conception according to which, roughly speaking, every set is formed at some stage and has as members only individuals or sets formed at still earlier stages. In ZF, the process of set formation may be understood in terms of an iterative, two-step process. The null set, pairing, unions, and powerset axioms, and the axiom of infinity generate sets. Instances of the separation axiom schema then identify subsets of those sets corresponding to formulas.

Separation $\forall z \exists y \forall x (x \in y \leftrightarrow (x \in z \wedge \varphi)),$

where y does not occur free in φ . That is, for any formula and any set z , there is a subset of z whose members are all and only those things that satisfy the formula.⁷⁴

The consistency of contemporary set theory can only be proven in a formal mathematics strictly stronger than set theory, for reasons connected to Gödel's proof of the

⁷⁴ The idea that ZF embodies an iterative conception of sets involves a degree of idealisation, for it includes the axioms of replacement, which 'do not seem to us to follow from the iterative conception' (Boolos 1971: 228). The separation axioms are also provable from the replacement axioms. For an introduction to contemporary set theory, see Enderton (1977).

incompleteness of second-order logic. If set theory were inconsistent, that stronger theory would be inconsistent too, so anything could be proven in it. It remains possible that instances of Separation may give rise to a contradiction, perhaps in combination with the other axioms of set theory. What is true is that a contradiction cannot be derived from Separation in the standard way. Consider, for example, the substitution of the paradoxical $x \notin x$ for ϕ . This instance of Separation says only that a given set z has a subset containing all of the members of z that are not members of themselves. Since another axiom of ZF – regularity – implies that no set is a member of itself, this subset is just z itself.

With that said, any consistent solution to the set paradox will have to restrict NC-S in some way. But as pointed out above, the final resolution of the set paradox cannot be embodied in a theory whose sole motivation is that it avoids the paradox – not least because the same motivation would motivate also other, mutually inconsistent, theories. I will not delve here into the motivations for ZF, in light of its intended applications in mathematics. The proposal that a solution to the set paradox be adapted to a solution to the property paradox is presumably made on the assumption that the former is adequate; and, for present purposes, that assumption need not be questioned here. But it is worth noting that there is, arguably, some intuitive motivation to the iterative conception of sets, to the extent that it seems strange that any set could contain, or be a member of, itself (so Boolos 1971: 218-20). But there is nothing strange in the idea that some properties should instantiate themselves; examples include the property of being a property and the property of being thought of. There thus appears to be no comparable, intuitive motivation for an iterative conception of properties.

This preliminary observation suggests not only that ZF itself may be inadequate for the intended applications of a theory of properties, but that a theory of properties developed around an analogous, iterative conception of properties may lack a well-motivated, or clearly intended, model (cf. Bealer 1982). And in fact, there are other features, distinctive of ZF, that are undesirable from the perspective of a theory of properties.

One is the lack of a universal set. Suppose, for reductio, that there is such a set u . By Separation, there exists a set s such that $\forall x: \{x \in u: x \notin x\}$. By the argument of Russell's paradox, however, $s \in s$ and $s \notin s$. Therefore, there is no universal set u . A theory of properties informed by ZF should have an analogous, property-theoretic axiom schema of separation, by which it could similarly be shown, by the argument of the property paradox, that there is no universal property. But the existence of a universal property – e.g. the property of being self-identical or the property of being a thing – is essential to the intended applications of a theory of properties, more so even than the Russell property of being a non-self-instantiator.

To see this, consider again the intended applications of a theory of properties in semantics, according to which properties are the semantic values of predicates and abstract singular terms. A case might reasonably be made that the Russell predicate 'does not instantiate itself' does not often appear in discourse outside the metaphysics of properties, and thus that a semantic theory of such discourse need not attribute to it a semantic value. However, the predicates 'is self-identical' and 'is a thing' are frequently used in a range of theoretical and ordinary contexts. A semantic theory that could not accommodate such universal predicates would surely be inadequate.

ZF also implies that sets are not closed under complementation. As earlier mentioned, one axiom of ZF is union, which says that for any two sets, there is the union of those sets. Now the union of a set y and its complement $\{x: x \notin y\}$ would be the set of all sets, but as mentioned above, there is no such set. So sets are not closed under complementation.⁷⁵

Properties, by contrast, ought to be closed under all Boolean operations, including complementation. As mentioned above, for at least some of the intended applications of a theory of properties, there ought to be at least a property corresponding to every meaningful predicate. Since the meaningful predicates are closed under the Boolean operations, properties also must be closed under the same operations. Granted, there cannot be a property corresponding to such paradox-inducing predicates as ‘does not instantiate itself’, so such predicates must be excluded from the class of meaningful predicates; comprehension cannot be completely general, on pain of contradiction. However, general closure conditions on the class of meaningful predicates are implied by compositionality; and, given e.g. the semantic applications of a theory of properties, that implies that properties are closed under analogous conditions.

Note that the existence of a universal property may also be derived from such closure conditions – specifically, closure under complementation and disjunction. For the disjunction of any property p and its complement $\neg p$ will be a universal property, $p \vee \neg p$ (assuming the law of excluded middle).

⁷⁵ There are non-standard theories of sets that allow a universal set and closure under complementation – Quine (1937)’s New Foundations, the system of Church (1974), and positive set-theory (Skala 1974) are examples. However, all such theories have other restrictions on set formation to prevent Russell’s paradox from occurring that rule out the existence of certain sets. For an overview of set theories with a universal set, see Forster (1995).

If there are restrictions on properties and relations analogous to those on sets in ZF, there also will be no relation of membership. For assuming that relations correspond to sets of ordered pairs, there is no set of ordered pairs that contain all pairs of the form $\langle x, y \rangle$ such that $x \in y$, for the same reason that there is no set of all sets. Thus the basic relation in mathematics will not exist, if relations are assimilated to sets.

The criticisms raised in this section against a property theory based on ZF hold, whether or not sets are allowed to contain possibilia. For the criticisms raised in this section pertain to ZF's resolution of the set paradox, and extensionality is irrelevant to the formulation of the set paradox. They thus suggest that a solution to the property paradox that is embodied in a property theory based on ZF will not be adequate, whether properties are identified with sets or more loosely conceived as iterative.

To summarise, an adequate answer to the property paradox will be embodied in a theory of properties that is both consistent and adequate for its intended applications. A theory of properties adequate for certain of its intended applications will satisfy at least the following desiderata:

Non-extensionality	Properties do not satisfy an extensional criterion of individuation.
Closure	Properties are closed under Boolean operations.
Plenitude	Properties are plenitudinous, in the sense that every formula determines a property.

Any answer to the property paradox that identifies properties with sets will not satisfy Non-extensionality, unless it is combined with a certain form of modal realism. But even given modal realism, a theory of properties modelled on contemporary set theory will not satisfy Closure or Plenitude.

These observations should considerably undercut the appeal of searching for a resolution to the property paradox in contemporary set theory. Granted, there are other set theories, and it is possible that some such theory might serve as a more suitable model for the development of property theory than ZF. However, it should now be clear that property theory and set theory have distinct applications and are, accordingly, constrained by distinct desiderata. That suggests that a resolution of the property paradox should be sought in light of the intended applications of property theory itself, rather than in a comparison with contemporary set theory.

3 A higher-order resolution

The chapter began with the following observation: sentences of the form ‘ x is F ’ and ‘ x has the property of being F ’ are ordinarily held to be equivalent, in some sense that implies their material equivalence. Such equivalences can be taken to motivate a general comprehension principle, according to which there is a property corresponding to every meaningful predicate. But they can equally be taken to motivate a nominalist paraphrase strategy, on which talk of properties is, in principle, eliminable, along with the implied existence of any properties.⁷⁶

⁷⁶ In this vein, it has been argued that the meanings of sentences of the form ‘ x has the property of being F ’ are more perspicuously expressed by simple predications of the form ‘ x is F ’, insofar as the latter do not imply the existence of a property in addition to the subject x . The purpose of the more prolix expression, and

An immediate challenge for such a strategy arrives in the form of property-involving statements that are not of the form ‘ x has the property of being F ’. Familiar examples include:

- (1) Justice is a virtue.
- (2) Red is more similar to orange than to yellow.
- (3) It has that property.
- (4) They share some properties.
- (5) There is something such that: it is a property, and anything instantiates it if and only if it is red.

(1) and (2) are statements in which properties appear in the subject position. (3) refers to a property using a demonstrative, and thus without explicitly identifying the property in question. (4) and (5) generalise over properties. All of these statements represent important kinds of property-involving statements; none of them have obvious nominalist paraphrase.⁷⁷

In this respect, the property predicate appears to fulfil an expressive need much like the truth predicate in English: to enable the expression of generalisations and other statements that could not otherwise be expressed. Consider the statement ‘Everything Socrates said is true’. If we knew what Socrates said, we could restate it without using the truth predicate: we could repeat what Socrates said, and say that Socrates said this and nothing else. But since we do not know all that he said, the truth predicate enables us to express what

talk of properties in general, is held to be purely pragmatic: it serves to shift emphasis from the subject of the sentence, x , to the predicate (so Hofweber 2007).

⁷⁷ See Pap (1959), Jackson (1977), Armstrong (1978a, 1980), or Lewis (1983) for further discussion of this point.

appears to be an otherwise inexpressible generalisation. Thus the truth predicate increases the expressive power of a language, and the property predicate appears to serve a similar function.⁷⁸

One salient point of difference, however, is that where the truth predicate is used to simulate generalisation into embedded sentence position, the property predicate is used to simulate generalisation into the position of the predicate. The close link between property instantiation and predicate satisfaction is already apparent in the natural language equivalence between statements of the form ‘ x is F ’ and ‘ x has the property of being F ’. Strictly speaking, however, quantification over properties is not a form of quantification into predicate position. In (4), for example, ‘some properties’ occupies the position of a term, as opposed to a predicate. As it happens, English and other natural languages appear to lack unambiguous devices for expressing quantification into predicate position (Chapter 7). But those expressive limitations do not extend to the formal languages of higher-order logic.

As mentioned in Chapter 1, first-order quantifiers bind variables in singular term position. By contrast, higher-order quantifiers bind variables in other syntactic positions. Second-order quantifiers, in particular, bind variables in (first-level) predicate position. The syntactic distinction between quantification into singular term position and predicate position is key to the higher-order resolution of the property paradox. For example, the second-order analogue of NC-P is:

⁷⁸ See Quine (1970, 1992) on the expressive utility of the truth predicate and Hofweber (2006) for a comparable account of the property predicate.

Second-order Comprehension $\exists X \forall x_1 \dots \forall x_n (Xx_1 \dots x_n \leftrightarrow \varphi x_1 \dots x_n),$

where φ is any formula, provided that X does not occur free in φ .

Unlike the instances of NC-P, the instances of Second-order Comprehension (SC) are consistent. Indeed, instances of SC are axioms of standard second-order deductive systems (Shapiro 1991: 80-1). And there are analogous, still higher-order comprehension principles, which govern quantification into the position of the predicates of predicates, the predicates of predicates of predicates, and so on. Their instances are presumed axioms of still higher-order deductive systems.

Standard type theory, which describes the syntax of higher-order formal languages, imposes certain restrictions on well-formedness. Such type distinctions form the crux of the higher-order resolution to the property paradox. For example, the problematic notion of non-self-instantiation cannot even be expressed under standard type-theoretic assumptions. In a second-order language with λ -abstraction, the predicate ‘instantiates itself’ is regimented as $\lambda XX. XX$, and its negation ‘does not instantiate itself’ as $\lambda \neg XX. XX$. Both these strings are ungrammatical. (Compare the expression of ‘instantiates’, which is grammatical: $\lambda Xx. Xx$.)⁷⁹

However, it has been argued that the property paradox cannot motivate such type distinctions: ‘when faced with an argument from plausible premises to a contradiction, we don’t normally think that one available response is to conclude that some of the arguments

⁷⁹ There are non-standard type theories, such as cumulative type theory, that relax this restriction; for further discussion, see Linnebo and Rayo (2012) and Florio and Jones (2021).

are *ungrammatical*' (Goodman 2024: 92-3).⁸⁰ But when faced with an argument from plausible premises to a false conclusion, we do normally think that one available response is to conclude that some of its premises have been incorrectly regimented – that the syntactic structure of such premises is other than it has been assumed. A classic example is: 'The king of France is not bald; therefore, the king of France exists'. On the assumed, naïve reading, the negation takes narrow scope relative to the definite description. On an alternative, Russellian reading, it takes wide scope, and on this reading, the argument is invalid (see Russell 1905). The same idea animates the higher-order resolution of the property paradox. A naïve regimentation of an intuitive comprehension principle for properties uses first-order quantifiers. An alternative regimentation uses higher-order quantifiers, and its instances do not lead to contradiction.

A related criticism of the higher-order resolution to the property paradox has been raised by Hofweber (2022). Granted that higher-order quantification is not quantification over properties (propositions, relations), Hofweber (2022: 42) argues that questions involving quantification over properties cannot be given answers using higher-order quantification instead, on pain of a 'mismatch' between question and answer: 'The answer doesn't address the question we asked, at least not directly'. He does not apply this precept to the property paradox specifically, but its implications are obvious. NC-P is one answer to the question: 'Which predicates or open formulas determine properties?'. Its instances lead to a contradiction. The higher-order solution to the paradox rejects NC-P and replaces it with analogous, higher-order comprehension principles. But if Hofweber is correct, the original

⁸⁰ A similar stance is taken by Button and Trueman (2024: 249). Trueman (2021) also defends a higher-order conception of properties, relations, and propositions, but he does not use the paradoxes to motivate his view (6-7).

question cannot be answered in this way. There can be no higher-order resolution of the property paradox, this being a first-order problem.

Critically, however, Hofweber's argument proceeds on the assumption that there is no 'significant relationship' between quantification over properties and higher-order quantification (*op. cit.*). If that were true, his precept would be plausible. But I have argued that it is not. There is a significant relationship between higher-order quantification and first-order quantification over properties. Namely, higher-order quantification can do the work of first-order quantification over properties, without being reducible to it. Call this view *higher-order nominalism*.⁸¹

Higher-order nominalism is intuitive. Given the close connection between properties and predicates, it is eminently plausible that quantification over properties be regimented by quantification into predicate position. There are also formal reasons to expect that higher-order logic can do the work of property theory. Recall the three property-theoretic desiderata identified in section 2: a theory of properties adequate for at least some of its intended applications in metaphysics and semantics ought to satisfy Non-extensionality, Closure under Boolean operations, and Plenitude. Higher-order nominalism satisfies higher-order correlates of each of these desiderata. For higher-order logic does not imply that the 'items' in the range of the higher-order quantifiers are extensional, and higher-

⁸¹ There is some terminological discrepancy here between authors. Jones (2018b) and Trueman (2021), e.g., describes the same view as a form of realism, on the basis that that higher-order existential quantification expresses (a kind of) existence. By contrast, Williamson (2013: 60) and Liggins (2021: 10021-4), e.g., argue that existence is an exclusively first-order notion. The latter convention is adopted here.

order comprehension principles imply both that such items are closed under Boolean operations and that they are plenitudinous, in the relevant sense.⁸²

Furthermore, the property paradox supplies a reason to reject the terms of the original question. The paradox shows that NC-P has some false instances. Some explanation is sought for the failure of this otherwise intuitive principle. On the present view, this failure stems, in part, from the general device of nominalisation implicit in NC-P, by which predicates of the language like ‘... is just’ are transformed into singular terms like ‘justice’ or ‘being just’. The terms of the question ‘Which predicates or open formulas determine properties?’ presuppose this device, and are for this reason suspect.

The first adequacy condition for a solution to the property paradox was that it could not be embodied in a theory of properties whose sole motivation was that it avoids paradox. The present, higher-order resolution of the paradox also settles this concern.

4 A case for higher-order logic

Some recent criticisms of the higher-order resolution of the property paradox were addressed in the previous section. But from a historical standpoint, the salient objection to the higher-order resolution of the property paradox is its use of higher-order logic itself. Certain characteristic features of first-order logic, such as completeness, do not hold for higher-order logic on the standard semantics (see Shapiro 1991: 80-8). For these reasons, first-order logic became the dominant logic for mathematics and philosophy, and the use

⁸² A sparse conception of properties can also be accommodated within a higher-order framework by means of suitable restrictions on the higher-order quantifiers, so that not every predicate defines a property. See e.g. Williamson (2013: 226-9) for discussion of this point.

of higher-order logic came to be regarded with suspicion. But matters were not always thus, and recent years have seen a resurgence of interest in higher-order logic, with fruitful applications to metaphysics, mathematics, and semantics. This section provides a brief case for higher-order logic.

In fact, the formal systems of almost all the early founders of modern mathematical logic, including Frege (1879), Peano (1889), and Russell and Whitehead (1910), were higher-order. First-order logic only appeared as a separate system when it was identified as an interesting sub-system of higher-order logic, notably by Hilbert and Ackerman (1928), who raised the question of its metalogical properties, like completeness. That question was answered by Gödel (1929, 1931), who in effect showed that first-order logic has a sound and complete axiomatisation, and later, that second-order logic (under the standard semantics) has not. Gödel's results were the catalyst for the historical emergence of first-order logic. From a practical standpoint, many logicians and philosophers of mathematics preferred to work with a formally axiomatisable consequence relation, but theoretical considerations related to Gödel's results were also adduced to privilege the use of first-order logic by Quine and others. Notably, the 'remarkable concurrence' of two distinct notions of logical consequence, one semantic, and the other proof-theoretic, suggested that first-order logic represented 'a solid and significant unity', which marked it off from other formal languages (see Quine 1970: 91).⁸³

⁸³ Both the semantic and proof-theoretic notions of logical consequence have straightforward extensions from first- to second-order logic. But once they are extended in this way, they cease to be equivalent; that is, we no longer have the completeness result. Boolos (1975), e.g., criticises Quine's view that completeness is of particular significance in determining the scope of logic. For an extended narrative of the historical emergence of first-order logic, see Moore (1988) or Ferreiros (2001).

4.1 Argument from natural language

More significantly, the completeness result was also taken to suggest that the first-order semantics provided better models of ordinary discourse than the standard second-order semantics (Quine 1970: 95-102). This view rested upon two premises. One was that ‘logic is inseparable from translation’. That is, the logical truths are just those sentences of a language that are true under any likely translation, or interpretation, of that language. The second was that ‘logic is obvious; potentially obvious’. Logical truths are obvious, like instances of the schema ‘If ϕ , then ϕ ’, or they are potentially obvious, to the extent that they may be deduced from obvious truths by a finite number of iterations of certain obvious inference rules, like modus ponens. By Gödel’s completeness and incompleteness theorems, both premises are verified by first-order semantics, but not by the standard second-order semantics.

Against Quine, however, there are examples that suggest that the standard second-order semantics better models at least some parts of ordinary discourse than the first-order semantics. Logical consequence in first-order semantics is *compact*: a conclusion follows from an infinite set of premises only if it follows from a finite subset of that set. Logical consequence in natural language, however, seems not to be. Consider the following argument:

There is at least one star.

There are at least two stars.

There are at least three stars.

.

.

.

There are infinitely many stars.

The conclusion ‘There are infinitely many stars’ seems to follow from the set of all sentences of the form ‘There are at least n stars’, where n is replaced by a numeral. But it does not follow from any finite subset of this set. Thus the argument cannot be captured in first-order logic; in particular, the conclusion has no first-order formalisation. (Suppose it did. Then the formalisation of the above argument would be an argument whose conclusion follows from an infinite set of premises but does not follow from any finite subset of this set. That is, first-order consequence would be non-compact, but, as Gödel showed, it is compact. By *reductio ad absurdum*, the conclusion has no first-order formalisation.)

The argument can, however, be captured in second-order logic, which has a non-compact consequence relation. The following is a second-order sentence which is true in just those models with infinite domains: $\exists X (\forall x \exists y Xxy \wedge \forall x \forall y (Xxy \rightarrow \neg Xyx) \wedge \forall x \forall y \forall z (Xxy \rightarrow (Xyz \rightarrow Xxz)))$. Thus the notion of infinity can be defined in second-order logic, but not in first-.

4.2 Argument from arithmetic

In fact, there are many significant notions, such as countability, well-ordering, progression, ancestral, identity, and most, that can be defined in second-order logic, but

not in first-.⁸⁴ Second-order logic is strictly expressively more powerful than first-order logic.⁸⁵

Take, for another example, the intuitive statement of the principle of mathematical induction: if a property is had by zero and by the successor of every number by which it is had, then it is had by all numbers. The principle is standardly axiomatised in first-order arithmetic as the following schema:

First-order Induction $\varphi 0 \wedge \forall x (\varphi x \rightarrow \varphi sx) \rightarrow \forall x \varphi x$

The instances of the schema say, collectively, that induction holds for every property expressible by a formula, φ , in the language of first-order arithmetic. The problem is that the language of first-order arithmetic, like other languages, has a finite vocabulary, and is thus subject to certain expressibility constraints, though the numerical properties for which we expect induction to hold are not thus subject. More precisely, there are only countably many formulas in the language of first-order arithmetic, and thus countably many instances of the induction schema, though there are uncountably many subsets of the natural numbers, each of which will correspond to one or more properties.⁸⁶ Thus the collective instances of the schema fall short of capturing the intended generality of the intuitive principle.

⁸⁴ For second-order formulations of some of these notions, see Shapiro (1991: 97-109). Note that identity can be *expressed* in first-order logic with identity, by stipulating that this logical constant holds between objects if and only if they are identical. The point is that it cannot be defined in other terms.

⁸⁵ This is not to say that the second-order characterisation of these notions is, ultimately, the best or correct characterisation. For example, it has been argued that an infinitary logic or a plural logic is better for these purposes; see Paseau and Griffiths (2022) and Oliver and Smiley (2016), respectively. But second-order logic is, at least, still in the running here.

⁸⁶ For proofs that there are only countably many formulas in a language with a finite vocabulary, but uncountably many subsets of the natural numbers, see, respectively, pp. 34 and 12 of Smith (2013).

This generality is, however, captured by a single sentence of second-order logic, under its intended interpretation:

Second-order Induction

$$\forall X ((X 0 \wedge \forall x (Xx \rightarrow Xsx)) \rightarrow \forall x Xx)$$

From an epistemic standpoint, it has thus been argued that we accept the instances of the first-order schema because we accept the second-order axiom (Kreisel 1967: 148).

The second-order axiom is also simpler and stronger than the collective instances of the first-order schema. The superior strength of second-order induction is related to the categoricity of second-order arithmetic. A theory is *categorical* if and only if all of its models are isomorphic. Let L2A be the second-order language whose signature contains the symbols for zero, successor, addition, and multiplication; that is, $A = \{0, s, +, \cdot\}$. Second-order arithmetic, or AR, is the conjunction of the axioms of first-order Peano arithmetic, which comprise some basic rules of arithmetic, together with the second-order induction axiom. Let N be the model of L2A whose domain is the set of natural numbers, and which assigns zero to 0, and the successor, addition, and multiplication functions to s, +, and · respectively. Then $N \models \text{AR}$. Indeed, N is the intended model of AR. It is also the only model, in an important sense: it is the only model up to isomorphism, which is to say that AR is categorical. This result may be derived using certain instances of the second-order induction axiom (see Shapiro 1991: 82-3 for the proof).

By comparison, first-order arithmetic, which is obtained from second-order arithmetic by replacing the second-order induction axiom with the first-order schema, is not categorical (Shapiro 1991: 110-3 for a proof sketch). In this sense, the standard second-order

semantics, but not the first-, suffice to pin down the structure of the natural numbers.

There are essential features of our understanding of arithmetic that are, arguably, missing from some models of first-order arithmetic.

4.3 Argument from set theory

The intuitive idea to the axiom schema of Separation is that any subset of a given set x forms a set. But the instances of the schema fall short of expressing this idea. The Separation axioms only identify those subsets of a given set x that correspond to formulas of the language of first-order set theory. Given standard cardinality assumptions, there are only countably many formulas of first-order set theory, and thus only countably many sets defined by the collective instances of the schema. But there are uncountably many subsets of any infinite set. So there are subsets of some sets that are not defined by any instances of Separation.

By contrast, the intuitive idea to Separation is expressible using the resources of second-order logic. On the standard second-order semantics, second-order quantifiers range over the entire powerset of the first-order domain. Thus the intended generality of the Separation axioms is captured, instead, by a single universally quantified sentence:

Second-order Separation $\forall z \forall X \exists y \forall x (x \in y \leftrightarrow (x \in z \wedge Xx))$

In this connection, it has been argued that we accept first-order ZF because we accept a simpler and stronger theory, consisting of a finite number of axioms, some of which

require for their adequate expression second-order quantifiers (Boolos 1984: 441). This is second-order ZF.

It has further been argued that there are essential features of our contemporary understanding of set theory that are missing from some models of first-order ZF, but present in models of second-order ZF. One such feature is well-foundedness. A binary relation R is *well-founded* if and only if there is no infinite sequence $a_0, a_1, \dots$ such that $Ra_1a_0, Ra_2a_1, \dots, Ra_{i+1}a_i, \dots$ all hold – i.e. there are no ‘infinite descending R -chains’. The well-foundedness of the iterative hierarchy is a standard assumption in the literature on the philosophical underpinnings of Zermelo set theory (see e.g. Parsons 1977 and Studd 2013). Accordingly, the intended models of set theory are those in which the membership relation is well-founded. It may be shown, using the axioms of infinity and second-order replacement, that every model of second-order set theory is well-founded. By contrast, a model of first-order set theory may or may not be well-founded. If it is, it will have other anomalous features, which prevent its being the intended model (see Shapiro 1991: 113-6 for the technical details).

The upshot is that the higher-order resolution to the property paradox is inferentially integrated with developments in other areas of inquiry. There are, in addition, other arguments for higher-order logic, further to those described in this chapter. For example, higher-order quantifiers have been used in theories of logical consequence for unrestricted first-order languages (Williamson 2003, Rayo and Williamson 2003), modality (Dunaway 2013, Bacon 2018), real definition (Dorr 2016), and truth (Künne 2003: 333-74, Rumfitt 2014). Of course, like other arguments in philosophy, such arguments may be challenged.

(One challenge to the arguments from mathematics and set theory is considered, and rejected, in Chapter 7 section 1.) But collectively they make the case that higher-order logic is useful. That case ought to address, if not answer, what might for many philosophers still form the chief objection to the higher-order resolution of the property paradox – namely, its use of higher-order logic.

Recently, it has also been argued that higher-order logic solves or dissolves some traditional problems related to the metaphysics of properties and relations. This is the topic of the following chapter. A brief note on type theory concludes this chapter.

5 A note on type theory

The following is a simple **relational type theory**.⁸⁷ There are two basic types, e and t , the types of singular terms and sentences respectively (so-called because they are the types that name *entities* and are *truth-evaluable*). Complex types are recursively defined in terms of simpler types. If $\tau_1, \dots, \tau_n$ are types, then $(\tau_1, \dots, \tau_n)$ is the type of all expressions formed from a sentence by the omission of one or more occurrences of expressions of types $\tau_1, \dots, \tau_n$. (e) is thus the type of all expressions formed from a sentence by the omission of one or more occurrences of the same singular term – i.e. a unary predicate. (e, e) is the type of all expressions formed from a sentence by one or more occurrences of two singular terms – i.e. a binary predicate. (t) is the type of a unary sentential connective, like

⁸⁷ The theory is adapted from Church (1940), who derived the simple theory of types from the ramified theory of types presented in Russell and Whitehead (1910). In a letter to Frege, Russell mentions Frege's hierarchy of levels as a source of inspiration for his own theory. Frege's hierarchy is essentially the same doctrine as that described by the simple theory of types, formulated in terms of his notion of 'incomplete' or 'unsaturated' expressions; see Dummett (1981: 34-54).

‘it is necessary that’, and (t, t) is the type of a binary sentential connective, like ‘and’ or ‘or’.

A formula (in prefix notation) is then a string of the form $t, t_1, \dots, t_k$, where each t_i is a term (i.e. a variable or constant) with type τ_i , for $i = 1, \dots, k$, and t is a term with type $(\tau_1, \dots, \tau_n)$.

Define the *level* of types e and t as 0. The level of $(\tau_1, \dots, \tau_n)$ is the least integer to exceed the level of each of $\tau_1, \dots, \tau_n$. That is, if $\tau_1, \dots, \tau_n$ are of level $k_1, \dots, k_n$ respectively, the level of $(\tau_1, \dots, \tau_n)$ is $k + 1$, where $k = \max(k_1, \dots, k_n)$. $((e))$ is thus the type of a second-level predicate whose argument-place demands a first-level unary predicate, and $((e, e))$ the type of a second-level predicate whose argument-place demands a first-level binary predicate.⁸⁸

A relationally typed higher-order language is assumed in applications of higher-order logic in this thesis.

In the literature on higher-order metaphysics, one also sees a **functional type theory**.

Again, there are two basic types, e and t , and complex types are defined by recursion. If σ and τ are types, then there is another functional type $\sigma \rightarrow \tau$, that takes something of type σ as argument and returns something of type τ . For instance, a unary predicate like ‘... is an object’ is of type $e \rightarrow t$, since it takes a singular term to return a sentence; a binary

⁸⁸ Functional expressions have been omitted for simplicity, but their inclusion is straightforward. If τ is a type of level n , let $[\tau]$ be the type of an expression formed by the omission of one or more occurrences of a single expression of type τ from a singular term (as opposed to a sentence). For example, one- and two-place first-level functional expressions are of type $[e]$ and $[e, e]$ respectively, and a second-level predicate whose argument-place demands a functional expression is of type $[[e]]$.

predicate like ‘... loves ...’ is of type $e \rightarrow (e \rightarrow t)$, since it takes two singular terms in succession to return a sentence; and so on. Predicates of higher type are also recognised. A predicate of a predicate is of type $(e \rightarrow t) \rightarrow t$, since it takes a predicate to return a sentence.

The Appendix to Dorr (2016) shows how to translate functionally typed expressions into relationally typed expressions, and vice versa. As he there points out, however, there is a sense in which functionally typed languages postulate arbitrary syntactic structure, with no correlate at the metaphysical level. Functionally typed expressions can only take one argument at a time. Thus expressions like ‘loves’ or ‘and’, which intuitively take two arguments at once, must be translated into a functionally typed language as expressions that satisfy this constraint. A decision must therefore be taken as to the order in which such expressions take their arguments, though there seems to be no corresponding asymmetry in reality. For the same reason, there is no obvious way of assigning levels to functional types, so that the same expressions may be assigned the same levels in functional as in relational type theory.

Chapter 6

Higher-order Metaphysics of Properties

1 Introduction

Recent years have seen a surge of interest in higher-order metaphysics, an approach to metaphysics characterised by the application of higher-order logic to issues in metaphysics.⁸⁹ A central plank of this approach is higher-order nominalism, or the view that the work of first-order quantification over properties can be done instead by higher-order quantification into predicate position. Chapter 5 applied this view to a resolution of the property paradox. This chapter further explores the impact of this view on the metaphysics of properties.

A pivotal theme of higher-order metaphysics is the idea that higher-order logic solves or dissolves most or all of the issues or problems traditionally associated with the metaphysics of properties. The perceived higher-order resolution or dissolution of these issues has in turn been claimed an advantage of the higher-order approach. By going higher-order, we cut out the ‘noise’ of unproductive metaphysical debates (see Jones 2018b, Trueman 2021: 123-47, and Bacon 2024).

However, this theme can be tempered. True, higher-order nominalism justifies the rejection of any problems that assume that there are properties, for it denies that assumption. But so does any nominalist theory that denies the existence of properties.

Moreover, some first-order, property-theoretic problems have higher-order analogues that

⁸⁹ See e.g. Williamson (2013), Dorr (2016), Goodman (2017), Bacon (2018), Jones (2018b), Liggins (2021), Trueman (2021), Fritz (2022), and the contributions in Fritz and Jones (2024).

are no more easily resolved than the originals. Chapter 1 provided a case in point. On one prominent view, higher-order logic dissolves the regress. If relations, roughly speaking, are denoted only by predicates as opposed to singular terms, then a further predicate (e.g. ‘instantiates’) is not needed to describe how a relation stands to its terms: concatenation does the work of a relational predicate of instantiation. But as it was then pointed out, higher-order logic can be used to formulate a higher-order analogue of the relation regress that is no more easily resolved than the original, first-order regress, insofar as they turn on analogous issues.

This chapter revisits the topics of Chapters 2 and 3 from a higher-order perspective.

Section 2 argues that there is a higher-order analogue of the first-order debate between immanence and transcendence. It then shows that this higher-order debate reaches an analogous resolution on the basis of analogous considerations. Section 3 argues that there is no meaningful higher-order analogue of the question whether properties are universal or particular, since the terms of the original question remain ill-defined. In this case also, the higher-order approach does not enjoy any unique advantages over the first-order, property-theoretic approach.

With that said, it remains true that higher-order nominalism yields an intuitively plausible and theoretically well-motivated resolution of the property paradox. In this significant respect, the higher-order approach does enjoy an advantage over the first-order, property-theoretic approach (*pace* Hofweber 2022). Thus a moderate view of the impact of higher-

order logic on the metaphysics of properties is taken here. Higher-order logic solves some problems traditionally associated with the metaphysics of properties, but not all.⁹⁰

2 The immanence-transcendence debate, redux

2.1 The first-order debate

The main results of the first-order debate are summarised here in order to underscore the analogy with the higher-order debate.

As in Chapter 2, let P stand for ‘... is a property’, L for ‘... is located in ...’, and I for ‘... instantiates ...’.

First-order Immanence

Every property is located somewhere.

$$\forall x (Px \rightarrow \exists r Lxr)$$

First-order Transcendence

Every property is not located somewhere.

$$\forall x (Px \rightarrow \neg \exists r Lxr)$$

It is also universally held that if properties are located anywhere, they must be located where their bearers are, though, as pointed out in Chapter 2, this constraint is rarely justified.

⁹⁰ The observation of an analogy between first- and higher-order debates cuts both ways. For example, Lederman (2024) argues that a natural account of the propositional attitudes conflicts with certain ‘core principles’ of higher-order logic, concluding that ‘propositional attitudes raise foundational questions for higher-order metaphysics’. However, analogous questions arise in the first-order case also (e.g. Frege-Kripke puzzles), where first-order quantification over propositions is employed instead of quantification into sentence position.

Property Location

Every property is located wherever it is instantiated.

$$\forall x (Px \rightarrow \forall r (Lxr \leftrightarrow \exists y (Iyx \wedge Lyr)))$$

The conjunction of First-order Immanence and PL implies:

$$(1) \quad \forall x (Px \rightarrow \exists y Iyx)$$

There are no uninstantiated properties.

However, uninstantiated in addition to instantiated properties are needed for at least some of the intended applications of property theory. So assuming those applications are correct, First-order Immanence is false. Meanwhile, First-order Transcendence is consistent with the existence of uninstantiated properties, though this does not suffice to establish its truth. As it stands, there appear no good reasons either to affirm or reject First-order Transcendence, when compared with other theories of property location that are also consistent with the existence of uninstantiated properties – such as a hybrid theory on which instantiated properties are immanent, and uninstantiated properties are transcendent. So the first-order debate is settled against First-order Immanence and neither for nor against First-order Transcendence.

2.1 A second-order analogue

The predicates ‘... is immanent’ and ‘... is transcendent’ are first-level predicates. A *first-level* predicate is a predicate whose argument-place can only be filled by a singular term in order to yield a grammatical sentence. The predicates ‘... is located’ and ‘... is not

located', in terms of which immanence and transcendence are standardly defined, are also of first level. The question whether properties are immanent or transcendent thus presupposes that properties lie in the range of the potential reference of singular terms – which is to say that it presupposes that properties lie in the range of the first-order quantifiers.

That presupposition is false, if higher-order nominalism is true. But higher-order nominalism uses higher-order quantifiers, and such quantifiers may allow the formulation of higher-order analogues of the original, first-order question.

Are there such analogues? First-order Immanence and First-order Transcendence were defined in terms of location, so their higher-order analogues ought to be defined in terms of higher-order location. It has, however, been argued that there is no such notion, and thus that there is no higher-order analogue of the immanence-transcendence debate. In particular, higher-order location has no useful theoretical applications, so there are no reasons to introduce such a notion (Jones 2018b: 819-22, Bacon 2024: 56).

But contrary to these claims, higher-order location does appear to be useful. Here are two preliminary arguments in support of this.

Given higher-order nominalism, statements that appear to refer to or quantify over properties must be interpreted otherwise. For example, property attributions of the form '*x* has the property of being *F*' are interpreted as simple predications of the form *Fx*. And property generalisations like '*x* and *y* share all their properties' are interpreted using higher-order quantifiers into predicate position, as in $\forall X (Xx \leftrightarrow Xy)$. Thus property-

involving statements are not rejected out of hand for presupposing, falsely, that there are properties; rather, they are given a more charitable, higher-order interpretation. It is consistent with this *modus operandi* that property-involving statements that appear to attribute location or non-location to properties are given a higher-order interpretation. The salient interpretation is one on which a locational predicate is applied, not to a singular term, but to a first-level predicate or first-order variable, as in L^2F^1 or $\forall X^1 L^2X^1$. (Superscripts on predicate letters and variables indicate their level.) Such a predicate would be a predicate of predicates; that is, a predicate of second level or higher. And it would express a higher-order analogue of location.

The introduction of higher-order location is further motivated by higher-order accounts of *agglomeration*, or the phenomenon in which a single predicate appears to have as arguments terms referring to both properties and objects (= non-properties). An example is ‘She talked about Socrates and justice’. On the standard, first-order regimentation of this sentence, a single three-place predicate, ‘... talked about ... and ...’, takes as arguments ‘she’, ‘Socrates’, and ‘justice’. But assuming that ‘justice’ is a property term, i.e. an expression that attempts to refer to a property, this regimentation is incompatible with higher-order nominalism. The salient higher-order regimentation of this sentence decomposes the sentence into two predications, one involving a first-level predicate that takes singular terms as arguments and another involving a second-level predicate that takes a singular term in its first argument-place and a first-level predicate in its second: ‘She⁰ talked about¹ Socrates⁰, and she⁰ talked about² is just¹. If this account of

agglomeration is to have the requisite generality, then it will imply that there are higher-order analogues of all first-level predicates, including those that concern location.⁹¹

In particular, a second-order analogue of the first-level predicate L^1 for ‘... is located in ...’ would be a second-level two-place predicate L^2 that takes first-level predicates in its first argument-place and singular terms referring to places in its second. (In the language of relational type theory, it would be a predicate of type $(e, (e))$.) Now the salient second-order analogue of PL, formulated in terms of L^2 , is:

Second-order Location

$$\forall X \forall r (L^2 X r \leftrightarrow \exists x (Xx \wedge L^1 x r))^{92}$$

In addition, the salient second-order analogues of First-order Immanence and First-order Transcendence are, respectively:

Second-order Immanence

$$\forall X \exists r L^2 X r$$

Second-order Transcendence

$$\forall X \neg \exists r L^2 X r$$

⁹¹ Like the argument previous, this argument for higher-order location assumes that higher-order nominalism is correct, and thus that agglomeration is to receive an account compatible with higher-order nominalism. It is worth mentioning that agglomeration has also been taken in the opposite direction, to undermine higher-order nominalism (Hofweber 2022: 38). He argues that in a sentence like ‘She talked about two things: Socrates and justice’, the quantifier ‘two things’ must be understood to range over a single domain including both objects and (first-order) properties, for only thus is it true that there are two things she talked about. By contrast, the higher-order nominalist will deny that this sentence is, strictly speaking, true. Suffice it to say that matters here are not settled, but the argument in the main text demands only a *prima facie* case for higher-order location.

⁹² This constraint on second-order location is also suggested by Trueman (2021: 137-9). However, he goes on to observe that this constraint is consistent with second-order multiple location, i.e. $\exists X \exists r_1 \exists r_2 (L^2 X r_1 \wedge L^2 X r_2 \wedge r_1 \neq r_2)$, though the ‘usual laws’ governing first-order location imply that nothing can be multiply located. Thus he concludes that the second-order relation described by this constraint is not an analogue of first-order location. However, Trueman’s assumption that nothing can be multiply located is inconsistent with standard views of property location, as expressed e.g. in PL, and the truth of such views are part of what is at stake here.

Notice that Second-order Immanence and Second-order Transcendence are mutually exclusive, but not exhaustive. The one is not the negation of the other. This is analogous to the first-order case.

In the second-order case, the conjunction of Second-order Immanence and Second-order Location jointly imply:

$$(2) \quad \forall X \exists x Xx,$$

which is the second-order analogue of (1). Roughly speaking, second-order existence implies instantiation.

Now the negation of (2) is a theorem of second-order logic:

$$(-2) \quad \neg \forall X \exists x Xx$$

It is straightforward to derive (-2) from an instance of Second-order Comprehension, e.g. $\exists X \forall x (Xx \leftrightarrow Fx \wedge \neg Fx)$. Chapter 5 showed that the most prominent arguments for second-order logic assume Second-order Comprehension: a fully general comprehension principle is needed for the intended applications of second-order logic in mathematics and semantics, and it is needed for the higher-order resolution of the property paradox.

Assuming those arguments are correct, Second-order Comprehension is valid: its instances are true. So by reductio, Second-order Immanence is false.

By contrast, Second-order Transcendence is consistent with the collective instances of Second-order Comprehension. But it does not follow that Second-order Transcendence is true. Other theories are also consistent with the collective instances of Second-order Comprehension, but inconsistent with Second-order Transcendence; thus consistency with Second-order Comprehension cannot be sufficient for truth. An example of such a theory is:

Some Second-order Immanence $\exists X \exists r L^2 Xr$

The conjunction of Some Second-order Immanence and

Some Second-order Transcendence $\exists X \neg \exists r L^2 Xr$

is the second-order analogue of a hybrid theory of property location on which some properties are (first-order) immanent and others (first-order) transcendent. It, too, is consistent with the collective instances of Second-order Comprehension, and inconsistent with full Second-order Transcendence.

As it stands, there appear no good reasons either to affirm or reject Second-order Transcendence, when compared with other theories of second-order location that are also consistent with Second-order Comprehension.

Thus the second-order debate reaches a conclusion exactly analogous to the first-order debate. The salient point of contention in both the first- and second-order debates is whether or not a fully general comprehension principle is, in fact, needed for the intended

applications of first-order property theory and second-order logic, respectively. I have argued that it is. Given the relevant comprehension principles and the relevant location constraints, Immanence cannot be maintained, in either the first- or second-order case, without revisions to logic.

3 The universal-particular distinction, redux

3.1 The first-order distinction

The universal-particular distinction is intended to draw a fundamental classification between things. Traditionally, it was held that properties were universals, and objects like ourselves, tables, and chairs were particulars, but this has since been denied. Let *universal theory* be the view that properties are universals. By contrast, *trope theory* is the view that properties are particular.

Arguments for and against universal theory and trope theory were discussed in Chapter 3. But as it was there pointed out, a significant debate between universal theory and trope theory presupposes a significant distinction between universals and particulars, though no account of the distinction has ever carried conviction. For example, the once-standard account that a universal is something that can be instantiated by more than one thing, where a particular is something that cannot be thus instantiated, is incompatible with universal theory, for it implies that some properties are universals (e.g. being red), and others particulars (e.g. being the last dodo, being self-distinct).

From the perspective of trope theory, the idea that a particular is something that cannot be had by more than one thing at a time may yield a necessary condition for being a trope, but it does not yield a sufficient condition. Socrates cannot be had by anything, in the relevant sense; therefore, he cannot be had by more than one thing. So he is also a particular, but since he is not a property, he is not a trope.

3.2 A second-order analogue

The formulation of a second-order ‘analogue’ of a first-order question implies that the original question is itself meaningful. But it is not clear that the first-order question whether properties are universal or particular is indeed meaningful, for it is not clear that there is a meaningful distinction between universals and particulars.

A different view is taken by Jones (2018b: 825-7), who argues that the question does have a second-order analogue, and that this analogue is ‘readily resolved’ in favour of the second-order analogue of universals theory. The argument proceeds from the account of the universal-particular distinction described above. A universal is something that can be instantiated by distinct things, where a particular is something that cannot be thus instantiated. More precisely, a universal of being F is instantiated by every F thing. First-order trope theory and universals theory are thus defined as follows:

Trope 1 $\Box \forall x_1 \dots x_n \forall y (Ix_1 \dots x_n y \rightarrow \Box \forall z_1 \dots z_n (x_1 \dots x_n \neq z_1 \dots z_n \rightarrow \neg Iz_1 \dots z_n y))$

Universal 1 $\exists y \Box \forall x_1 \dots x_n (Ix_1 \dots x_n y \leftrightarrow \phi x_1 \dots x_n),$

where y does not occur free in ϕ . As Jones points out, the range of substitution instances for ϕ must be further restricted in order to avoid a version of Russell's paradox. Note that $x_1, \dots, x_n \neq z_1, \dots, z_n$ abbreviates $(x_1 \neq z_1 \wedge \dots \wedge x_1 \neq z_n) \vee \dots \vee (x_n \neq z_1 \wedge \dots \wedge x_n \neq z_n)$.

The second-order analogues of Trope 1 and Universal 1 are, respectively:

Trope 2 $\Box \forall x_1 \dots x_n \forall X (Xx_1 \dots x_n \rightarrow \Box \forall z (x_1 \dots x_n \neq z_1 \dots z_n \rightarrow \neg Xz_1 \dots z_n)),$

Universal 2 $\exists X \Box \forall x_1 \dots x_n (Xx_1 \dots x_n \leftrightarrow \phi x_1 \dots x_n),$

where X does not occur free in ϕ .

Jones then points out that sentences of the form $\Box \forall x_1 \dots x_n (Fx_1 \dots x_n \leftrightarrow \phi x_1 \dots x_n)$ are true.

And since each instance of Universal 2 follows from such a sentence by second-order existential generalisation, each instance of Universal 2 is true.

By contrast, Trope 2 is 'inconsistent with one of the most basic aspects of our ordinary and scientific world-views: many distinct objects can be of the same type' (*op. cit.*: 826). For example, many distinct apples are all red. Let F stand for ' $\dots$ is red', and a and b name distinct red things. The sentence

(i) $Fa \wedge Fb \wedge a \neq b$

is therefore true. However, Trope 2 implies:

(ii) $Fa \rightarrow (a \neq b \rightarrow \neg Fb)$.

(i) and (ii) are inconsistent. By reductio, Trope 2 is false.

In general, Trope 2 cannot be true in a world in which there are two or more things that satisfy a given predicate. Trope 1, however, can; it is thus consistent with (i).

Thus Jones resolves the second-order debate against Trope 2 and in favour of Universals 2, while the first-order debate remains unresolved:

Whereas the first-order debate about tropes and universals is intractable and contentious, the parallel second-order debate is readily resolved because: (a) the second-order theory of universals follows from uncontroversial truths by relatively uncontroversial principles of second-order logic; (b) the second-order theory of tropes is inconsistent with some of the most central components of our world-view. 2018b: 827

However, a closer analogy may be discernible between the first- and second-order debates than Jones allows.

First, the non-paradoxical instances of Universal 1 also follow from what are, from the perspective of our ordinary world-view, uncontroversial truths: sentences of the form

$\Box \forall x_1 \dots x_n (Ix_1 \dots x_n a \leftrightarrow Fx_1 \dots x_n)$, where a names the property of being F .

Second, Trope 1 is also inconsistent with some arguably central components of our world-view: that some properties are instantiated by distinct things.

Third, the relevant principles of second-order logic, which license existential generalisation into predicate position, are more controversial than Jones claims. In some circles, second-order logic is more controversial than first-order universal theory (see e.g. Bealer 1982).

These points aside, there is also a sense in which both the first- and second-order debates are moot. Notice that Trope 1 is consistent with some instances of Universal 1, and Trope 2 is consistent with some instances of Universal 2. The relevant instances are those derived by the substitution of predicates satisfiable by just one thing, like ‘... is the last dodo’. If necessarily, just one object satisfies a given predicate, then Trope 1 is consistent with the corresponding instance of Universal 1; and the same goes for Trope 2 and Universal 2. It follows that the properties corresponding to the satisfaction of these predicates are both tropes and universals.⁹³

But the universal-particular distinction was intended to draw a fundamental classification between things. If it is to serve its intended purpose, an adequate account of the distinction ought at least to imply that no universals are tropes, and no tropes are universals; that is, the distinction ought to be mutually exclusive. By this minimal constraint, Universal 1 and Trope 1 fail to constitute an adequate account of the universal-particular distinction, and Universal 2 and Trope 2 fail to constitute an adequate, second-order analogue of the distinction.

Relatedly, Universal 1 cannot accommodate the existence of necessarily uninstantiated properties. The obvious fix is to use a predicate expressing the notion of a property: $\exists y (Py$

⁹³ Jones (2018b: 826) acknowledges this complication but ultimately dismisses it.

$\wedge \square \forall x_1 \dots x_n (Ix_1 \dots x_n y \leftrightarrow \phi x_1 \dots x_n)$). But insofar as the universal-particular distinction is also intended to characterise the property-object distinction in other terms, this formulation of Universal 1 is inadequate – it appeals to the notion of a property.

The upshot is that there is no higher-order analogue of the universal-particular distinction, if there is no meaningful universal-particular distinction in the first place.

This chapter argued that the immanence-transcendence debate has a meaningful second-order analogue, which is settled by considerations themselves analogous to the considerations that settle the original, first-order debate. Immanence is inconsistent with certain theoretical desiderata (i.e. a fully general comprehension principle) and is therefore false. Transcendence is consistent with the same desiderata, and there are no obvious reasons to deny it; but those are insufficient reasons to affirm Transcendence. So the debate is settled against Immanence, in both the first- and second-order case, but neither for nor against Transcendence.

By contrast, it was suggested that the universal-particular debate does not have a meaningful second-order analogue. The reason for this contrast is simple. The terms of the original immanence-transcendence debate are clearly defined: First-order Immanence and Transcendence are precisely formulable theses. The terms of the universal-particular debate are not. In the absence of a clear distinction between universals and particulars, there can be no meaningful second-order ‘analogue’ of the distinction.

Chapter 7

A Translation of Second-order Languages

1 The intended interpretation

In Chapter 5, a case was presented for higher-order logic. According to the argument from set theory, the intended generality of Separation was better captured by a single sentence of second-order logic,

Second-order Separation $\forall z \forall X \exists y \forall x (x \in y \leftrightarrow (x \in z \wedge Xx)),$

rather than by the collective instances of the first-order axiom schema,

First-order Separation $\forall z \exists y \forall x (x \in y \leftrightarrow (x \in z \wedge \varphi)),$

where y does not occur free in φ . Separation was intended to ensure the existence of arbitrary subsets of the sets generated by the other set-theoretic axioms, though the collective instances of First-order Separation ensured only the existence of those sets corresponding to formulas of the language of first-order set theory. Since there are only countably many formulas in this language, there are only countably many sets whose existence is ensured by the schema, though there are uncountably many subsets of any infinite set. By contrast, Second-order Separation under the standard semantics ensures the existence of a subset corresponding to every property.

A ‘revenge’ problem is now raised. Second-order Comprehension is itself a schema. Its collective instances tell us that there is at least a ‘higher-order property’ corresponding to every second-order formula, but this leaves out all the higher-order properties that have no corresponding formula. Although Second-order Separation is non-schematic, Second-order Comprehension is not. Thus second-order ZF is no more expressively powerful than first-order ZF (Sider ms.: 12-7). A similar problem may be posed for the argument from arithmetic for higher-order logic (see Sider ms.: 20-3).

The answer to this problem must be that Second-order Comprehension delivers on some property-theoretic desiderata – for instance, closure under Boolean operations – but not all. The main work in the mathematical and set-theoretic applications of second-order logic is done by the intended interpretation of the second-order quantifiers, according to which they range over all properties of objects in the first-order domain.

Critically, this interpretation is not given by the standard model-theoretic semantics, in which the second-order quantifiers are interpreted as quantifiers ranging over all subsets of the first-order domain. For one, properties are not sets. As explained in Chapter 5, properties are non-extensional, and they are closed under Boolean operations; sets are neither. Furthermore, the higher-order resolution to the property paradox demands that second-order quantifiers are not reducible to restricted or unrestricted first-order quantifiers.

There are other inadequacies of a set-theoretic model theory for second-order logic, which are inherited from the inadequacies of a set-theoretic model theory for *first*-order logic. On the intended interpretation of the language of first-order set theory, the first-order

quantifiers range over all sets. So the domain of a model corresponding to this interpretation should be a set containing all sets, but by the argument of Russell’s paradox, there is no such set. Thus the model theory is not faithful to the intended interpretation of the language of first-order set theory. For the same reason, no set-theoretic semantics will be faithful to the intended interpretation of any higher-order languages with unrestricted first-order quantifiers. To compound the problem, there are more subsets of the domain than members (by Cantor’s theorem). Thus the second-order quantifiers, which on the standard set-theoretic semantics range over all the subsets of the first-order domain, are interpreted as ranging over more things than the first-order quantifiers range over, since they range over the members. This underscores the failure of the model theory to interpret the first-order quantifiers as absolutely unrestricted (Williamson 2013: 235).⁹⁴

Thus arguments from versions of Russell’s paradox and Cantor’s theorem show that there is a semantic distinction corresponding to the syntactic distinction between first- and second-order quantification.⁹⁵ The standard model-theoretic semantics for second-order logic is best understood as an approximation to the intended interpretation – approximate, to the extent that there is a set for every property, i.e. its extension, and adequate for some

⁹⁴ The model-theoretic approach to semantics, for defining logical truth and consequence, was developed by Tarski (see Tarski and Vaught 1956). Tarski (1936) also developed an alternative, type-theoretic approach to semantics, on which a formula ϕ is logically true if and only if the result of replacing all non-logical terms in ϕ with appropriate variables is satisfied by all variable assignments. Earlier Tarski (1935) had defined satisfaction (relative to a variable assignment) for formulas of order n in a language of order $n + 1$. The type-theoretic approach does not specify domains of quantification, which means that quantifiers can be given an unrestricted reading – in particular, the quantifiers of the language of first-order set theory can range over all sets. However, the lack of domains also has some anomalous consequences; for a prime example, $\exists x \exists y (x \neq y)$ is counted a logical truth, since the quantifiers are interpreted as ranging over all objects, and there is more than one thing. For a comparison of the type-theoretic and model-theoretic strategies, see Williamson (forthcoming).

⁹⁵ For a more detailed exposition, see e.g. Williamson (2003: 452-60; 2013: 235-40), Rayo and Williamson (2003: 331-8), Linnebo (2006), and Linnebo and Rayo (2012). For a dissenting view, see Pickel (2024); Williamson (2024) replies.

of second-order logic's most significant applications in mathematics, but inadequate for others.

For a time, the most prominent criticism of second-order logic was that it is 'set theory in sheep's clothing' (Quine 1970: 66-8). The criticism may be better appreciated in light of a few other Quinean doctrines, which are here relayed. Quine concedes, first, that the salient interpretation of the second-order quantifiers is one on which they range over properties (1970: 66-7). This is consonant with the close connection between properties and predicates observed in Chapter 5. But Quine also held that there was 'no entity without identity'; that is, we should not admit into our ontology any entities that did not obey a satisfactory (i.e. extensional) criterion of individuation (1960: 67, also 8-10, 1969: 23, and elsewhere).

The next most salient reading interpreted the second-order quantifiers as first-order quantifiers over sets. On this reading, the second-order sentence $\exists X Xx$, e.g., says that there is a set of which some object is a member. That being the case, a more perspicuous formulation would be given in the language of first-order set theory, as in $\exists y (x \in y)$, where the symbol for set-theoretic membership indicates the range of the first-order quantifiers.⁹⁶ Thus the use of second-order formulas disguises set theory's 'staggering existential assumptions' in what appears to be a purely formal apparatus of quantification, continuous with first-order quantification. This ran afoul of a third Quinean doctrine, that logic should presuppose no ontology, or as modest an ontology as possible. In

⁹⁶ Though as Boolos (1975: 512) points out, translating second-order formulas into the language of first-order set theory à la Quine can result in the loss of validity or consequence. For example, $\exists X \forall x Xx$ is valid, but $\exists y \forall x (x \in y)$ is not, as Russell's paradox showed us.

presupposing sets, second-order logic was not a part of logic, but mathematics (1970: 61-79).

The scope of logic is not to our purpose here.⁹⁷ But with this fuller picture, the significant points of disagreement may be marked as follows. The non-extensional nature of properties is not a bar to their admission into ontology, but an argument for it; it is one reason that properties do work that sets cannot (Chapter 5). But strictly speaking, higher-order quantifiers do not range over either properties or sets, for the reason that it does not range over any objects – an ‘object’ being whatever is in the range of the first-order quantifiers. On the intended interpretation of second-order logic, it is neither set theory nor property theory, though the ‘items’, roughly speaking, over which second-order quantifiers range are naturally held to be non-extensional.

Where a first-order, objectual interpretation of the quantifiers has been found wanting, it has often been assumed that the only alternative is a substitutional interpretation (e.g. Parsons 1971). On such an interpretation, a second-order generalisation $\forall X \phi$ is true if and only if every uniform substitution of a predicate of the language for the variable X in ϕ yields a true sentence. However, it should be clear that a substitutional interpretation is neither the interpretation of second-order logic intended here. For one, it would undermine the arguments from arithmetic and set theory for second-order logic (Chapter 5). These arguments presuppose that there are more ‘items’ in the range of the second-order quantifiers than there are predicates. Thus $\forall X \phi$ may fail to be true, though every uniform substitution of a predicate of the language for X in ϕ yields a truth (see Tarski 1936). The

⁹⁷ For criticism of Quine’s views here, see Boolos (1975); for a recent defence, see MacBride (2024).

intended interpretation of second-order logic is not hostage to the expressive limitations of the language, contrary to a substitutional interpretation.

So the intended interpretation of the language of second-order logic is *sui generis*, irreducible to either an objectual or a substitutional interpretation. For the reasons described above, the set-theoretic model theory is not fully faithful to this interpretation. A more faithful semantics would be formulated in a higher-order meta-language, with higher-order quantifiers irreducible to first-order quantifiers over sets or properties.⁹⁸

Still, there remains an interpretative question. How are we to understand the second-order quantifiers? From the perspective of one who sought to understand higher-order quantification in general, a formal semantics for second-order logic in a still higher-order meta-language would be beside the point – for she would need to understand the higher-order quantifiers in the meta-language.

An obvious informal approach to understanding a novel language is by translation or paraphrase into a more familiar language. Formal languages are no exception. On the usual translation scheme for the language of first-order logic into English, the first-order quantifier $\exists x$ translates, roughly, to ‘something’ and $\forall x$ to ‘everything’; the propositional connective $\wedge$ translates to ‘and’ and $\neg$ to ‘not’ or ‘it is not the case that’; and so on. The first-order symbolism is learned by applying this scheme to some simple sentences of the language of first-order logic. For example, supposing that F stands for the predicate ‘... is French’, $\exists x Fx$ translates to ‘Something is French’, and $\forall x Fx$ to ‘Everything is French’.

⁹⁸ Such semantic theories may be found in Boolos (1985), Rayo and Uzquiano (1999), Rayo and Williamson (2003), McGee (2003), Rayo (2006), and Linnebo and Rayo (2012). Alternatively, Wright (2007), e.g., has argued that the meaning of the quantifiers is grounded in the patterns of inference licensed by their introduction and elimination rules; for criticism, see Williamson (2013: 259).

The problem is that the second-order quantifiers do not seem to be as amenable to translation. It has already been pointed out above that second-order quantification is not first-order quantification over properties. Note, in this connection, that the English expression ‘some property’ occupies the position of a first- as opposed to second-order quantifier: the position it quantifies into is that of a noun or noun phrase as opposed to a predicate. For the same reason, Frege’s well-known account of the distinction between first- and second-order quantification as the distinction between quantification over objects and over concepts is also inadequate. Concepts are objects, in the abovementioned, Fregean sense that they lie in the range of the first-order quantifiers. Hence Frege’s paradox of the concept *horse* that is not a concept, but an object.⁹⁹

The general consensus is that English and other natural languages lack genuinely predicative quantifiers. There are thus no established translation schemes for the language of second-order logic into natural language.

This lacuna has, in turn, promulgated skepticism that second-order quantifiers are intelligible without reducing them to first-order quantifiers (e.g. Resnik 1988 and Hazen 1993; see also Hofweber 2022). Against such skepticism, Williamson (2003: 459-60, 2013: 259-60) has pointed out that the lack of a translation scheme need not preclude our understanding. Not all language acquisition can be achieved by translation into another, more familiar language; that would lead to an infinite regress. We have to start

⁹⁹ The paradox is described in Frege (1892: 46). For discussion, see e.g. Dummett (1981: 211-7), Wiggins (1984), Wright (1998), MacBride (2011a), and Jones (2016).

somewhere; we learn, at least in some cases, by direct method. There is no reason, in principle, that the same method may not be applied to the second-order quantifiers.¹⁰⁰

Goodman (2024: 77-80) adds that sentences like:

(i) $\exists x \exists y (x \neq y)$

(ii) $\forall x \forall y (x = y)$

prompt a range of glosses, none of which are structurally identical to the original sentences. For example, (i) may be glossed, *inter alia*, as ‘There are at least two things’ or ‘Some things are distinct from each other’; (ii) as ‘There is at most one thing’ or ‘All things are identical to one another’. He argues that the diversity of the glosses as well as their structural discrepancy from the original sentences show that we do not understand the original sentences by exploiting any ‘mechanical translation’ of the first-order symbolism into English. By analogy, he concludes, second-order sentences ought to be intelligible without exploiting any mechanical translation of the second-order symbolism into English.

But though it is agreed that sentences like (i) and (ii) may be understood without exploiting any mechanical translation of the symbolism into English, I deny that this shows that either first- or second-order languages are intelligible without a translation manual.

¹⁰⁰ Williamson (*op. cit.*) also argues that the theoretical fruits of higher-order resources constitute abductive evidence of their intelligibility. Barwise and Etchemendy (1993) make a case for learning the language of first-order logic by direct method.

As I said, an obvious informal approach to understanding any novel language, including any formal language, is by translation or paraphrase into a more familiar language. ‘novel’ is the operative word. At some point, we cease to require translations and can understand new sentences of a given language directly. At this point, the language has ceased to be novel and become familiar, and the crutch of a translation manual is longer needed. We can understand other and more complex sentences without it. In the present context, there is a salient disanalogy between the languages of first- and second-order logic, which Goodman’s argument underplays. Namely, there is a translation scheme adequate for the purposes of faithfully rendering some simple sentences of the language of first-order logic into natural language, one with which most, if not all, practitioners of first-order logic are acquainted, where there is not a comparable translation scheme for the language of second-order logic.

This is not to deny that the language of second-order logic can be learned, as Williamson claims, by direct method. But it raises an important point. Translation schemes for the language of second-order logic have often been dismissed on the basis that they cannot handle a wide range of second-order constructions or even that they do not generalise to all cases. But those are unnecessarily demanding standards, for the purposes of understanding a language. In comparison, the usual translation scheme for first-order logic also cannot handle all sentences, and it cannot even handle all relatively simple sentences. For example, the sentence $\exists x \exists y Nxy$, where N stands for ‘is next to’, lacks an entirely felicitous English translation. Incidentally, the more structurally complex sentence $\exists x \exists y (Nxy \wedge x \neq y)$ does have a ready translation: ‘Something is next to something else’. Thus the usual translation scheme for the language of first-order logic into English does not always predict simple translations for comparatively simple sentences of first-order logic.

It is an interesting question just how much a translation scheme has to be able to capture in order to be deemed adequate for the purposes of understanding the language. In the present context, a workable, if defeasible, criterion is that a second-order translation scheme is adequate if it can capture at least the second-order sentences that are structurally analogous to the first-order sentences capturable by the usual first-order translation scheme.

Summarising, then, a translation scheme is adequate for the purposes of conveying the intended interpretation of the second-order quantifiers if and only if:

Faithful It is faithful to their intended interpretation, and

General It is general, to the extent that it captures at least the second-order sentences that are structurally analogous to the first-order sentences capturable by the usual first-order translation scheme.

The rest of this chapter reconsiders a few translation schemes for the language of second-order logic in this light. To establish the dialectic, Section 2 considers the well-known scheme due to Boolos, on which second-order quantifiers are interpreted as plural quantifiers. The chief defect of this scheme is that it is not faithful. Section 3 then develops a translation scheme that is faithful but not general. It exploits the intuitive distinction between *what* there is and *how* it is in order to deliver faithful translations of a limited range of second-order sentences (see Rayo and Yablo 2001). Finally, section 4 defends a translation scheme that is both faithful and general. It is based on the observation, found in Sellars (1960), Prior (1971), and others, that the English expression

‘something’ can be used to express both nominal and non-nominal quantification. The last part of section 4 applies this scheme to the related project, raised by Dummett (1981), of giving a direct characterisation of the ‘items’ over which the monadic second-order quantifiers range.

2 A plural reading

For decades, the most prominent strategy for an interpretation of higher-order quantification that did not reduce it to first-order quantification was Boolos’ (1984, 1985). The basic idea is that quantification into unary predicate position can be interpreted as quantification into plural term position. Thus an atomic formula Xx is translated, roughly, as ‘it is one of them’, and the second-order quantifiers $\forall X$ and $\exists X$ as ‘any things are such that’ and ‘some things are such that’, respectively. For example, the second-order formula $\forall X (\forall x Xx \rightarrow \exists x Xx)$ is read ‘Any things are such that if everything is one of them then something is one of them’. The propositional connectives and the first-order existential quantifiers $\forall x$ and $\exists x$ receive the usual treatment; to this extent, Boolos’ translation scheme is an extension of the usual scheme for translating first-order formulas into English.

A plural reading of higher-order quantification appears to satisfy the desideratum that such quantification be understood as non-first-order. There are sentences in English that have a second-order formalisation, on this reading, but no first-order equivalent. A well-known example is the Geach-Kaplan sentence ‘Some critics admire only one another’, which can be formalised in second-order logic as: $\exists X (\exists x Xx \wedge \forall x \forall y (Xx \wedge Axy \rightarrow x \neq y \wedge Xy))$. The same sentence cannot be formalised in first-order logic with identity (for Kaplan’s proof,

see Boolos 1984: 57). Examples like these suggest that plural quantification in English cannot be reduced to singular quantification.

There are other features that commend a plural interpretation of the second-order quantifiers. One is that second-order quantification, on this interpretation, incurs no further ontological commitments than first-order quantification over objects. In particular, it incurs no independent commitment to sets or classes.¹⁰¹ To use a now famous example, when you have some cheerios, you are not eating a set of them – you are just eating THE CHEERIOS (Boolos 1984: 448-9). The existence of some things does not logically entail the existence of some set of them.

A plural reading is also adequate for some of the most prominent applications of second-order logic to the formalisation of mathematical theories and concepts. For example, a model of first-order logic interprets the instances of the first-order axiom schema of Separation as generalisations only over the countably many subsets expressible by formulas in the language of first-order set theory, whereas a standard model of second-order logic interprets the second-order axiom of Separation as a generalisation over all the uncountably many subsets of the infinite first-order domain, as intended. That is one motive for going second-order; and for these purposes, a plural reinterpretation of second-order quantifiers is adequate also, since for any non-empty subset of the domain there are its members. For similar reasons, a formulation of the axiom of mathematical induction as a first-order schema with countably many instances does not capture its intended

¹⁰¹ This contention has been challenged. Resnik (1988: 77), for example, claims ‘dissenting logical/linguistic intuitions’: ‘I find that I cannot process many sentences containing plural quantifications without understanding them in terms of collections’. Hazen (1993) argues that a reading of second-order quantification in terms of classes is superior to a plural reading. See also Rouilhan (2002) and Linnebo (2003).

generality, whereas its formulation as a second-order axiom over standard models of second-order logic does, which in turn ensures the categoricity of second-order arithmetic. For these purposes, a plural reinterpretation is adequate also (Williamson 2013: 242-3).¹⁰²

That said, there are reasons to doubt that a plural reading is the intended reading of second-order quantification. The most obvious one is that plural terms are not predicates – they lack a main verb. The plural term ‘horses’ and the predicate ‘... is a horse’ belong to distinct grammatical categories (Simons 1997: 258). On this reading, monadic second-order quantification is a form of objectual quantification, for the position quantified into is that of a term.¹⁰³

Second, plurals, like sets, are extensional. If every cordate is a renate and every renate is a cordate, then the cordates are the renates. The following sentence is thus true on a plural interpretation:

$$\forall X \forall Y (\forall x (Xx \leftrightarrow Yx) \rightarrow X = Y).$$

However, it is typically denied that the sentence is true on the intended interpretation, for the same reasons that it is typically denied that the property of being cordate is the property of being renate.

¹⁰² Oliver and Smiley (2016: 243-302) present a system of plural logic powerful enough to serve as a background logic for arithmetic and set theory.

¹⁰³ Rayo and Yablo (2001: 86) designate plural quantification a form of first-order quantification, but there are reasons to withhold the honorific ‘first-order’ from plural quantifiers. Granted, plural logic is like first-order logic, to the extent that both plural and first-order quantifiers range over objects. However, plural logic is more like second-order logic in the way it behaves – features like compactness that are characteristic of first-order logic fail both for second-order logic and plural logic. As alluded above, it is just these features of plural logic on which its applications in mathematics depend. In Quinean terms, there is no ontological difference between first-order quantification and plural quantification, but there is an ideological difference.

Third, the translation rule for the existential quantifier requires modification. The plural ‘some things are such that’ implies ‘at least one thing is such that’, but the second-order existential quantifier has no such existential import.¹⁰⁴ On the intended reading, a sentence like $\exists X \neg \exists x Xx$ can be true though nothing has the property in question. To handle this, Boolos adds an extra clause to his translation scheme for the second-order quantifier: $\exists X \dots Xx \dots$ is read ‘Some things are such that $\dots x$ is one of them $\dots$ or $\dots \neg x = x \dots$ ’ (1984: 444). But the clause is ad hoc, and the need for such a clause suggests a mismatch between the plural and the intended reading of the quantifier.

Fourth, as mentioned at the start, Boolos’ translation scheme applies only to monadic second-order logic, and not to full second-order logic. However, full second-order logic is strictly expressively more powerful than monadic second-order logic, and the aforementioned mathematical and set-theoretic applications of second-order logic demand full second-order logic. The reason for this limitation to Boolos’ scheme is simple: a plural reading of the monadic second-order quantifiers has no natural generalisation to the polyadic quantifiers. Consider, for example, the sentence $\exists X Xab$. On the most straightforward extension of Boolos’ translation scheme to quantification into binary predicate position, this sentence reads: ‘There are some things such that a is one of them and b is one of them’. The problem is that this translation leaves out the ‘relation’ in which a stands to b . An alternative translation that seeks to re-introduce this relation is non-sensical: ‘There are some things such that a bears them to b ’.

¹⁰⁴ The usual assumption in logic is that ‘some things’ implies ‘at least one thing’ and not ‘at least two things’, but the account is easily adjusted to the alternative.

The usual remedy here is to regiment quantification into n -place predicate position as quantification over n -tuples, and so construe polyadic second-order quantifiers monadically (see Resnik 1988: 86 and Rayo and Yablo 2001: 75-7). Thus $\exists X Xab$ is read ‘Some pairs are such that $\langle a, b \rangle$ is one of them’. The problem with this is that second-order existential quantification is given first-order existential import on this reading, to some tuple or tuples, though, as mentioned above, it has no such import on the intended reading.¹⁰⁵ The problem is compounded for theories whose domains have not been assumed to contain sets or to be closed under ordered pairing, for thus a plural reading generates ontological and ideological costs, beyond the commitments of the theory itself. Indeed, if the domain of a theory is closed under ordered pairing, there will be infinitely many things if there are at least two things (see Williamson 2013: 244 for the proof).

It may here be objected, on behalf of the plural scheme, that there are no ready translations for many first-order sentences with more than one quantifier, and that the project of capturing second-order quantification beyond the monadic case is fraught with exactly the same difficulties as that of capturing first-order sentences with more than one quantifier. However, the cases are not quite analogous. There are natural translations of at least some first-order sentences with more than one quantifier. It has already been pointed out that $\exists x \exists y (Nxy \wedge x \neq y)$ is readily translated as ‘Something is next to something else’. $\exists x \forall x Nxy$ and $\forall x \forall y Nxy$ also have simple, faithful translations. The examples are easily multiplied. A plural reading of second-order quantification, however, appears unable to capture *any* second-order constructions involving polyadic second-order quantifiers.

¹⁰⁵ Lewis, Burgess, and Hazen identify concrete pair-surrogates in the appendix to Lewis (1991), but the proposal is fairly stipulative, as the authors concede. For example, certain metaphysical conditions must be met, among which is the condition that there are ‘infinitely many atoms [and] not too much atomless gunk’ (121).

These four points indicate that a plural reading is not the intended reading of the higher-order quantifiers. Thus a plural translation scheme for the language of second-order logic is not faithful to its intended reading. Assuming that plural quantification is a *sui generis* form of quantification, it would be most perspicuously regimented in a language with distinctive plural quantifiers: $\exists xx$ and $\forall xx$.¹⁰⁶

A fifth and final criticism that is sometimes made of a plural reading of second-order logic is worth dispelling, to clarify the scope of the present project. It is that a plural interpretation of second-order logic has no natural generalisation to higher orders, though $n+1$ -order quantification in the metalanguage is necessary to define logical consequence for an n -order object language (Williamson 2003: 457, 2013: 244).

Attempts to answer this criticism have appealed to ‘superplurals’, these being expressions that stand to plural terms as plural terms stand to singular terms (so Rayo 2006). A putative example in English is ‘these people and those people’, as in ‘These people and those people play against each other’ or ‘These people and those people and these other people play against each other’.¹⁰⁷ It is not uncontroversial that these are indeed examples of superplurals in English, but even so, they are not examples of superplural quantification. The challenge of extending a plural interpretation to third and still higher orders remains standing.

A better answer to Williamson’s criticism is that if it were a mark against a translation scheme for a language of a given order that it had no natural generalisation to languages of

¹⁰⁶ Oliver and Smiley (2016: 60) observe that Boolos’ unusual representation of plurals within the syntax of second-order logic was influenced by his prior interest in second-order logic.

¹⁰⁷ The examples are due to Linnebo and Nicolas (2008). For earlier suggestions see Landman (1989), Rosen and Dorr (2002), and Oliver and Smiley (2005).

still higher orders, it should be a mark against the usual translation scheme for first-order languages that it has, apparently, no natural generalisation to still higher-order languages. But it is not. So a translation scheme adequate for the purposes of communicating the intended interpretation of a language need not be extendible to other, higher-order languages.

That said, for the four reasons identified above, a plural reading is not faithful to the intended interpretation of the second-order quantifiers. Nevertheless, the discussion of the plural reading has not been fruitless. It has highlighted four constraints on a faithful translation scheme for second-order logic. It also serves another, more dialectical purpose. It allows that there may be quantifiers that are not first-order, in a philosophical climate once hardened by Quine against any but first-order quantifiers. Thus it opens up the possibility that higher-order quantifiers may also be understood in such a way that they, too, are distinct from first-order quantifiers.

3 Scheme 1: Somehow, however

The first criterion for a faithful translation scheme of second-order logic, which Boolos' plural scheme was seen to fail, is that it interprets the second-order quantifiers as predicative quantifiers. As it happens, there are predicative quantifiers in English. For example, in the inference from 'Socrates is in Athens' to 'Socrates is somewhere', we quantify into the predicative complement. This suggests the idea of translating $\exists X$ and $\forall X$ as 'somewhere' and 'everywhere', respectively, whenever the second-order quantifiers are restricted to locations. A range of second-order sentences thus becomes capturable in English, of which the simplest examples are:

$$(1^*) \quad \exists X Xa$$

a is somewhere.

$$(2^*) \quad \forall X Xa$$

a is everywhere.

Alternatively, the restriction in range can be made explicit using a second-order predicate, say L , that returns a true sentence only if completed with a first-level predicate ascribing location.

$$(3^*) \quad \forall x \exists X (LX \wedge Xx)$$

Everything is somewhere.

Admittedly, the syntactic structure of the original sentence in (3*) does not align with the structure of the translation: the former has, and the latter lacks, a conjunction. However, it was observed in section 1 that the usual translation scheme for first-order logic also predicts translations whose structures deviate from the original first-order sentences. To see this again, consider the usual translation of the following first-order sentence:

$$(4^*) \quad \exists x (x = a)$$

a exists.

The original has, and the latter lacks, an identity.

So deviations in structure between translation and original sentence cannot be held against this fragmentary second-order translation scheme, without raising an objection also to the usual first-order translation scheme.

Note that for ordinary purposes, the difference between ‘at some place’ or ‘somewhere’ is purely verbal. Sentences like ‘Socrates is at some place’ and ‘Socrates is somewhere’ are held to be equivalent, in a sense that entails, at least, their material equivalence. In the present context, however, there is a vital difference between the expressions. The first quantifies into the predicative complement, as opposed to name position – thus ‘somewhere’ cannot be instantiated using a name for a place alone. The name must be accompanied by a preposition that indicates the ‘relation’ to the place, as in ‘Socrates is somewhere – namely, in Athens’. By contrast, the expression ‘at some place’ is most naturally read as a first-order quantifier over places. It is instantiable using a name for a place alone.

However, we are after a general translation scheme for second-order logic. In light of the above, one natural idea is to capture other forms of restricted second-order quantification using the remaining ‘wh-’ question words: ‘who’, ‘what’, ‘when’, and ‘why’. A moment’s reflection, however, frustrates this idea. The remaining question words do not form idiomatic second-order quantifiers, whether prefixed or appended with the determiners ‘some’ and ‘every’: ‘somewhen’ and ‘somewhy’, e.g., are not words in English. In this case, we only have first-order quantifiers like ‘at some time’ or ‘for some reason’.

As it stands, there is also no single question word in English that prompts any description of the subject in question, as opposed to particular descriptions concerning, e.g., the

subject's location or purpose.¹⁰⁸ Prior (1971) similarly observed that there is no question word in English that prompts an answer that is a complete sentence. Instead, such questions are expressed by an inversion of word order; thus compare 'She will stay' and 'Will she stay?'. But we do describe the asking of such questions by using the word 'whether', as in 'I asked whether she would stay'. In this light, Prior suggests that we 'simply concoct' the propositional quantifiers 'anywhether' and 'somewhether', thus translating, for example, $\forall P (P \rightarrow P)$ as 'If anywhether, then thether' (37). The invention of the requisite quantifiers may serve to underscore the point that the second-order quantifiers are not first-order. However, it does not serve the present aim, of establishing a translation scheme for second-order logic into English.

The closest approximation in English to an expression that prompts any description of the subject in question is: 'how'. In phrases like 'how things are', 'how' can be taken to cover a range of conditions things satisfy. This observation appears to form the basis of an alternative translation scheme defended by Rayo and Yablo (2001), on which $\exists X$ and $\forall X$ and are translated, roughly, as 'somehow' and 'however'.

Some quirks of their scheme deserve mentioning. First, the key expression in their translations of polyadic second-order quantifiers is 'somehow things relate'. Thus they translate

¹⁰⁸ Incidentally, the same is true of ancient Greek. On a prominent reading of the *Categories*, Aristotle arrived at his list of categories through considering the different questions that might be asked about a substance and noticing that only a limited range of answers can be appropriately given to any particular question (so Ackrill 1963: 78-9). Standard English translations of the various categories, such as 'quantity' and 'quality', efface the fact that each of the non-substance categories are introduced by adjectives, adverbs, and infinites, as opposed to nouns. 'category' itself is a transliteration of *κατηγορία*, the Greek word for 'predicate'.

(5*) $\exists X Xab$

as ‘Somehow things relate such that a is related to b ’. However, the scheme returns false translations of true sentences, such as:

(6*) $\exists X \forall x \forall y \neg Xxy$.

According to Rayo and Yablo, this says ‘Somehow things relate such that nothing is so related to anything’, but this is a contradiction, where (6*) is a second-order logical truth. To avoid this consequence, they stipulate that ‘somehow things relate’ expresses ‘not that things *are* so related but their classifiability as so related or not’ (84). However, this stipulation is inconsistent with ordinary usage.

A second quirk of Rayo and Yablo’s scheme is due to the fact that they seem to regard the translation of the monadic second-order quantifiers as more, rather than less, difficult than the translation of the polyadic quantifiers. To capture the former, they introduce the verb ‘determined’. The sentence

(7*) $\exists X \forall x (Xx \rightarrow Fx)$

is thus translated as ‘Somehow a thing is determined such that only F s are so determined’.

But the verb ‘determined’ introduces irrelevant noise – like the implication that there is something by which things are determined.¹⁰⁹

¹⁰⁹ Williamson (2013: 258 n.) criticises Rayo and Yablo’s scheme on the basis that ‘somehow’ is ‘uncomfortably close to a first-order quantifier over ways’, though he grants that it may be read as a second-order quantifier. The discussion of the quantifiers ‘at some place’ and ‘somewhere’ is relevant here.

Simpler translations of both monadic and polyadic second-order constructions seem possible, however, if the aim for properly idiomatic translations is abandoned.

(1) $\exists X Xa$

Somehow a is.

(2) $\forall X Xa$

However a is.

(3) $\exists X (Xa \wedge \neg Xb)$

Somehow a is and b is not.

(4) $\exists X \neg \exists x Xx$

Somehow everything is not.

(5) $\exists X Xab$

Somehow a is to b .

(6) $\forall X (Xab \rightarrow Xbc)$

However a is to b , b is to c .

With the exception of (6), none of these translations are entirely natural.

The translation of (6) is, arguably, faithful to the intended interpretation of the second-order quantifiers. In general, the present scheme arguably satisfies at least the four criteria for faithfulness that the plural scheme was seen to fail. It is predicative. It is not necessarily extensional. There are no ad hoc clauses. And it captures some polyadic second-order constructions, using the ‘to’ which we use when we verbalise the scheme of analogy.

The problem is that it is not general. It does not predict natural translations for a range of second-order sentences comparable to the range of first-order sentences capturable by the usual first-order translation scheme. (1)-(5) bear witness to this, as do the more complex translations given by Rayo and Yablo.

4 Scheme 2: Something, everything

An observation, which is found in Sellars (1960), Prior (1971), and many others, forms the basis of the third and final scheme to be considered here. In some natural languages, including English, the same verbal expressions may be used to express both nominal and non-nominal quantification. Examples are ‘something’, ‘everything’, ‘what’, and ‘whatever’. In the inference from ‘That apple is red’ to ‘Something is red’, we quantify into name position. In the inference from ‘She is wise, and I am not’ to ‘She is something I am not’, we quantify into the predicative complement, this being the expression that follows the copula in the predicate. Both forms of inference are common in English.¹¹⁰

Nominal Something is red – namely, that apple.

Predicative She is something I am not – namely, wise.

Nominal She has everything I want.

Predicative She is everything I want to be.

Nominal She has what I want.

¹¹⁰ Sainsbury (2018) gives a convincing array of examples to show that ‘something’ is used in English to quantify into a wide range of positions, further to the positions of names and predicative complements.

Predicative She is what I want to be.

We are after quantification into predicate position, as opposed to the predicative complement. But it is arguable that the copula lacks either semantic content or logical significance, in which case the distinction between predicates and predicative complements may be elided (see Dummett 1981: 214). The argument standardly rests on two comparisons. First, the copula is not the only expression in English that seems to lack content or logical significance. The pronoun ‘it’, as used e.g. in ‘It is raining’, is another. There it supplies a subject where the sense of the sentence requires none; and indeed, in some languages, like Korean, the same thing can be said without a grammatical subject. Second, the copula is superfluous in some natural languages. In ancient Greek, for example, there exists a distinct word for the copula, but it may be, and customarily is, dropped. Instead, concatenation does the work of the copula.

Admittedly, both comparisons only indirectly support the logical insignificance of the copula – the first, by pointing to other expressions apparently of this kind, and the second, by pointing to languages in which the copula is dropped.¹¹¹

There are also more direct arguments in favour of the elision of predicates and predicative complements in this context. One assumes the analogue of the principle of Extensional β -conversion for English: that sentences of the form ‘Socrates is wise’ are materially equivalent to sentences of the form ‘Socrates is such that he is wise’. From the latter

¹¹¹ By comparison, Wiggins (1984) gives an account of second-order quantification in which the copula plays an essential role. In particular, the function of the copula is to combine with a ‘concept-word’ to produce an ‘unsaturated expression’ (i.e. predicate) that will combine with a ‘saturated expression’ (singular term) to produce a complete sentence. Wiggins’ discussion of the copula makes much of Frege’s metaphor of saturation, but, as Williamson (2003: 458) points out, the metaphor is misleading.

‘Socrates is something’ follows, so that the range of variation in the predicative complement is equivalent to the range of variation in the predicate: there is no loss of generality by quantifying into the predicative complement, as opposed to the predicate (Williamson 2013: 255).¹¹²

Another argument points to the fact that standard type theories, which describe the syntax of higher-order formal languages, do not distinguish predicative complements from predicates. For the purposes of capturing higher-order constructions, it therefore seems legitimate to elide the grammatical distinction between predicates and predicative complements in English.

The translation scheme given here thus assumes that quantification into the predicative complement is equivalent to quantification of the predicate.

The two important expressions for the purposes of capturing existential and universal second-order quantification are ‘something’ and ‘everything’. Thus $\exists X$ translates as ‘something’, and its dual $\forall X$ translates as ‘everything’. It is easiest to see how the scheme works with some examples. Translations will be improved upon as I go, in the usual fashion.

(1) $\exists X Xa$.

There is something a is. That is, a is something.

¹¹² Incidentally, there may be a reading on which the natural language analogue of Extensional β -conversion is invalid. For example, the sentence ‘Helen is such that she works hard’ may be interpreted as saying something more than ‘Helen works hard’ – something to the effect that Helen’s character or disposition leads her to work hard. But as long as there is a reading on which the two sentences say the same thing, the point in the main text holds.

$$(2) \quad \forall X Xa$$

a is everything.

$$(3) \quad \exists X (Xa \wedge \neg Xb)$$

a is something that *b* is not.

$$(4) \quad \exists X \neg \exists x Xx$$

There is something that nothing is.

$$(5) \quad \exists X Xab$$

a is something to *b*.

$$(6) \quad \forall X (Xab \rightarrow Xbc)$$

Everything *a* is to *b*, *b* is to *c*.

The scheme is faithful to the intended interpretation. In particular, it satisfies the four desiderata that a plural scheme failed to satisfy. It is predicative. It is not necessarily extensional. There are no ad hoc clauses. And it can be extended to capture some polyadic second-order constructions.

On this scheme, the same verbal expressions translate the first- and second-order existential quantifiers, and the same is true of the first- and second-order universal quantifiers. However, the risk of ambiguity is minimal, for the occurrence of the expression, whether in predicative or name position, will generally indicate whether it expresses first- or second-order quantification. (4) is a good example of this. Of course, where there are multiple occurrences of first- and second-order quantifiers, translations will become strained. But that is true also where there are multiple occurrences of first-order quantifiers only.

In some cases, like (2), it will not be obvious from the translation alone whether the quantifier is first- or second-order. The same sentence ‘a is everything’ could also be regimented with a first-order quantifier, as $\forall x (x = a)$. But this fact cannot be leveraged against the second-order translation, without also forming grounds to criticise the first-order translation.

The examples given show that the present scheme is able to handle simple sentences of second-order logic, in a fashion analogous to the usual translation scheme for the language of first-order logic. Accordingly, the scheme should suffice to yield a purchase on the intended interpretation of the second-order quantifiers. If we understand the use of expressions ‘something’ and ‘everything’ in English as they are used to express predicative quantification, then we understand the second-order quantifiers.

Necessary and sufficient conditions on an adequate translation scheme for the language of second-order logic were identified in section 1 of this chapter. It had to be faithful to the intended interpretation of the language. And it had to be general, in the sense that it captured a range of second-order sentences, comparable to the range of first-order sentences capturable by the usual first-order translation scheme. Three translation schemes for second-order logic were evaluated in this light: Boolos’ plural translation scheme; Scheme 1, which used the predicative quantifiers ‘somehow’ and ‘however’; and Scheme 2, which used the general quantifiers ‘something’ and ‘everything’. It was shown that the plural scheme was not faithful, and Scheme 1 was not general. Nevertheless, reflection on the inadequacies of both schemes helped to elucidate the intended reading of the second-

order quantifiers and further desiderata for a translation scheme adequate for the purposes of understanding the language of second-order logic.

It was then argued that Scheme 2 was both faithful and general. The scheme does not extend to every sentence of the language of second-order logic, but a translation scheme adequate for the purposes of learning a language does not need to capture every sentence of that language. In the first-order case also, the usual translation scheme does not extend to every sentence of the language, nor was it intended to. The essential expressions ‘something’ and ‘everything’ are also ambiguous, insofar as they can be used to express both first- and second-order quantification; but if this is a criticism of Scheme 2, then it is a criticism of the usual first-order translation scheme also.

4.1 The ‘range’ of monadic second-order quantification

A project related to that of capturing the second-order quantifiers in natural language is that of characterising the ‘items’ over which it ranges. As pointed out, it cannot be said that it ranges over properties, characteristics, or concepts, since such things, if they exist, lie in the range of the first-order quantifiers. For the same reason, it cannot be said to range over ‘things’. Does this contradict the translation scheme just defended? No. In that sentence ‘things’ occupies the position of a term; contrast it with the position of ‘something’ in ‘There is something she is not’. In general, the formula ‘The second-order quantifiers range over ...’ can only be grammatically completed with a term, which means that no grammatical completions of this formula will yield a true sentence. So the question ‘What do the second-order quantifiers range over?’ ought to be rejected, for it involves a presupposition failure: that there is anything over which they range.

Nevertheless, Dummett (1981: 211-19) argued that it was still possible to characterise the ‘range’ of monadic second-order quantification. For context, the unrestricted first-order quantifiers are said, by convention, to range over ‘things’ (‘objects’, ‘entities’). In this light, the predicate ‘... is a thing’ appears to be an expression such that: filling its argument-place with an expression of the kind which syntax demands, i.e. a singular term, never returns a false sentence.¹¹³ Dummett then pointed out that the analogous expression, for the second-order case, would be an expression such that: filling its argument-place with a first-level predicate never returns a false sentence. Such an expression would be a predicate of second level. In other words, the desired expression would be one that stood to first-level predicates as the first-level predicate ‘... is a thing’ stood to singular terms.

It is clear that the predicate ‘... is a property’, e.g., is unsuitable for this purpose, for its argument-place is open only to singular terms, as opposed to predicates.

The expression Dummett supplied was:

Dummett's Formula ... is something which everything is or is not.

The gap is intended to be filled by a ‘predicative expression’, which he defines as an expression that is formed from a grammatical predicate (i.e. an open sentence with a single argument place) by either (i) dropping the copula or (ii) if the main verb is anything other than the copula, converting that verb into the participial form of the same tense (1981: 216). The resulting sentence says that the law of excluded middle holds for the predicative

¹¹³ It is assumed that the predicate returns a sentence that is neither true nor false for any singular terms that lack reference, such as 'Pegasus'. This assumption could be challenged, but it is inessential to the argument of the main text.

‘item’, as in ‘Red is something which everything is or is not’. Dummett thus presents his formula symbolically as: ‘For every x , $\varphi(x)$ or not $\varphi(x)$ ’.

The proposal assumes the law of excluded middle, but the assumption is not essential to the proposal. Another formula that serves the same purpose is: ‘... is something which everything is, if it is’. Filled by a predicative expression, the resulting sentence says that the law ‘ φ if φ ’ holds for the corresponding item, as in ‘Red is something which everything is, if it is’.¹¹⁴

Now predicative expressions are not predicates, but a case has already been made that the copula is logically insignificant; the previous translation scheme depends on this. But Dummett’s proposal assumes more than the logical insignificance of the copula in order to handle predicates whose main verbs are other than the copula – see clause (ii) of his formation rule for predicative expressions, as stated above. One consequence is that some predicative expressions formed according to this rule cannot appear in predicates. Examples include ‘having eaten’, which is formed from the predicate ‘has eaten’; ‘was eating’; ‘will be eating’. To this extent, it can be doubted whether these expressions deserve to be called ‘predicative’ expressions.

More importantly, these expressions cannot be inserted into Dummett’s formula to return a grammatical sentence, never mind a sentence that is not false, contrary to his proposal.

The underlying problem is that participial conversion on verbs not in the present tense

¹¹⁴ This formula may be preferred on the basis that the validity of ‘ φ if φ ’ is less controversial than that of excluded middle, though it, too, is not universally accepted. In Kleene (1938)’s three-valued system, for example, instances of ‘ φ if φ ’ evaluate to # if φ itself evaluates to #. Incidentally, Dummett himself rejected the law of excluded middle (see his 2000 on intuitionistic logic).

yields expressions that cannot function grammatically as nouns, and therefore cannot occur grammatically in the argument-place of Dummett's formula.

Furthermore, changes of meaning occur when a verb is converted into the participial form of the same tense, which means that the meaning of the original verb is never quite captured on Dummett's proposal. The difference between saying 'She reads' and 'She is reading' is that the former says that reading is something she does (not necessarily at this moment), whereas the latter says that she is now reading. The predicates '... reads' and '... is reading' thus have different meanings, but the meaning expressed by the former is never quite captured on Dummett's proposal. Participial conversion can also produce an expression that is morphologically identical to an expression that is not a participle. An example is 'loving', which can be interpreted either as the present participle of the verb 'loves' or as an adjective, in which case it expresses a disposition as opposed to an action. (Notice that there is no comparable adjective for 'hating'; it can only be interpreted as the present participle of the verb 'hates'.)

A few simple revisions to Dummett's proposal circumvent these issues. First, modify his formula so that the argument-place occurs at the tail, rather than the head:

Dummett's Formula (Revised) Something which everything is or is not ...

Since the copula has been assumed to be logically insignificant, the revised formula still counts as a predicate. Second, abandon Dummett's notion of a predicative expression. Instead, let a predicative expression be any predicate, simple or complex, that is prefaced by the copula. Assuming the natural language analogue of Immediate β -equivalence,

predicates that are not prefaced by a copula, such as 'runs', are equivalent to complex predicates that are prefaced by the copula, such as 'is such that it runs'. It is then naturally held that the meaning of the original simple predicates is captured by inserting the corresponding complex predicates into the revised formula.

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